A STUDY OF THE RELATION BETWEEN TEACHER AND STUDENT UNDERSTANDING OF LIMIT CONCEPTS TAUGHT IN GRADE EIGHT

by

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ABSTRACT

The purpose of this study was to investigate if a relationship exists between the understandings of students and those of their teachers for a specific concept in mathematics. A review of literature revealed that no study had attempted to examine the relation between student and teacher understanding of a specific concept in mathematics although several had investigated the relation between teacher and student understanding of general mathematical concepts, usually in arithmetic. The single concept chosen for the present study was intuitive limit concepts as prescribed for Mathematics 8 students in British Columbia schools. The following null hypothesis was established and tested: For Mathematics 8 classes of better students there is no significant correlation between teacher understanding of intuitive limit concepts and student understanding of intuitive limit concepts.

Measures of understanding were obtained by the use of two testing instruments constructed by the investigator, one for students and one for teachers. The preliminary student test constructed was checked for content validity and given a trial use. The reliability of the test was calculated and an item analysis made to determine which items to use in the final form of the test. The teacher

test constructed used hypothetical answers to student test items. Teacher test items were taxonomized according to Bloom. Fourteen classes of Mathematics 8 students of better ability and their teachers were tested using the final form of each test. Class means for student tests were adjusted by analysis of covariance to allow for initial differences in intelligence and mathematics achievement. Calculation of the coefficient of correlation between these adjusted means and teacher scores gave a result of 0.09. This correlation was not significant. Thus the null hypothesis tested was accepted.

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THE PROBLEM

I. GENERAL STATEMENT OF THE PROBLEM

The proposition that one cannot teach what one does not understand would appear to be a most reasonable assumption. Does a teacher need to understand a concept in mathematics in order that his students can develop an understanding of this same concept? The purpose of this investigation is to make a preliminary enquiry into the validity of this assumption.

II. DEFINITIONS OF TERMS

Mathematics 8 is the mathematics course prescribed for all grade eight students in British Columbia Schools.[1]

Mathematics 8 classes of better students refers to classes identified as such by the administration of each participating school.

Student understanding of limits is a measure determined by a test of intuitive limit concepts prepared as part of the study.

Teacher understanding of limits is a measure determined by the accuracy of marking hypothetical answers prepared for the student test.

Intuitive limit concepts are concepts related to the following ideas developed in Mathematics 8:

1. The set of rational numbers is identified with the set of repeating decimals.

2. The set of irrational numbers is identified with the set of infinite, non-repeating decimals.

3. Rational and irrational numbers are dense and can be compared to each other using the real number line.[2]

III. THE QUESTION

An answer to the following question will be sought: When students' results are compared with those of their teacher, is there a relationship between student understanding of intuitive limit concepts and teacher understanding of these concepts?

FOOTNOTES

[1]British Columbia Department of Education, Secondary School Mathematics, 1966, (Victoria: British Columbia Department of Education, 1966), p. 7.

[2]Charles F. Brumfiel, Robert E. Eicholz, and Merrill E. Shanks, Introduction to Mathematics, (Reading, Massachusetts: Addison-Wesley Publishing Company, 1963), pp. 147-172.

CHAPTER II

REVIEW OF THE LITERATURE

I. INTRODUCTION

A search of literature relevant to the problem reveals that both experts and classroom teachers have indicated a need for improved understanding by teachers of mathematical concepts. Investigators show concern over lack of such understanding, primarily at the elementary school level. No investigations were found concerning mathematical understandings of secondary school mathematics teachers. A variety of studies have attempted to find a relationship between teacher and pupil understandings of mathematical concepts. Some of these have related the academic background in mathematics of teachers to the achievement in mathematics of their classes. Others have related teacher gain in understanding from in-service education programs to the gain in understanding made by their pupils over several months. A few have described the relationship between teacher scores on tests of general mathematical understanding and the achievement of their pupils. No studies have attempted to relate the understanding of a specific mathematical concept held by a teacher to that of students who were to learn this concept from that teacher.

II. EXPRESSED NEED FOR TEACHER UNDERSTANDING

The assumption that teachers must understand mathematical concepts as a necessary condition for their pupils to learn mathematics is widely held by teachers and experts. Instructors of an aircraft mechanics hydraulics course are reported to rate the effectiveness of fellow instructors on the basis of their knowledge of subject matter.[1] Yet, for the 3,000 students and 121 instructors involved, the investigators found no significant relationship between student gains in achievement adjusted for initial differences and instructor knowledge of hydraulics. Outlining his views on the minimum mathematical background needed by teachers, Newsom states, "All too frequently teachers in the elementary grades are hardly a jump ahead of their alert students . . ."[2] A 1958 survey of secondary mathematics teachers in three states found that 85 per cent of those participating wanted more workshops and 33 per cent indicated that advanced mathematics courses would help them in their teaching.[3] Arguing the need for greater emphasis upon the measurement of pupil understanding of mathematical concepts, Dutton concludes, "Then, and only then, will many classroom teachers begin to examine their own understandings of mathematical concepts . . ."[4] A review of recent research by Brown and Abell led them to conclude, "Many prospective teachers of elementary school mathematics do not have the understanding of basic mathematical concepts that experts agree they should have."[5] There is no doubt about the emphasis on the need for the teacher to understand mathematical concepts. Does research support this need?

III. MATHEMATICAL UNDERSTANDINGS OF TEACHERS

In one of the first investigations of mathematical understandings possessed by student teachers and teachers of elementary school, Glennon prepared his own eighty-item test of basic mathematical understandings and found that the average student teacher knew about 43 per cent and the average teacher about 55 per cent of the understandings tested.[6] Yet these understandings were basic to the computational processes taught in grades one to six. Glennon described his findings as presenting a not very optimistic picture. Using their own eighteen-item test of arithmetic understandings with 322 teachers and student teachers attending summer sessions at three universities, Orleans and

Wandt found that few of the concepts tested were understood by a large percentage of the group.[7] Glennon's test was later used by Weaver who reports similar conclusions to those of Glennon on the basis of his findings.[8] Fulkerson tested students in an arithmetic methods class with a forty-item test of the knowledge he thought prospective teachers of arithmetic should possess.[9] He reports, ". . . far too many of the 158 prospective elementary teachers studied . . . have an insufficient knowledge of arithmetic to teach the subject effectively."[10] Using a fifty-item test based partly on Glennon's test, Kenney found the median for 356 teachers who took the test to be 29.7.[11] He indicates that a higher degree of mastery of understanding is needed by these teachers. Kipps used her own carefully designed test of basic mathematical understandings and obtained a mean of 68 per cent for the 310 elementary teachers who wrote the test.[12] On the basis of her findings, she feels it is necessary for teachers to improve their knowledge of the concepts tested.

The studies cited share the claim of Orleans and Wandt that for children to acquire real understanding of arithmetic, ". . . it would seem obvious that the teachers of arithmetic must possess the understandings that they are transmitting to their students."[13] However the caution given by Sparks must not be ignored. In 1961, he observed that no research was available to ". . . indicate that a better comprehension of mathematical concepts on the part of the elementary school teacher results in better achievement on the part of students."[14] A similar statement could be applied to high school teachers. A recommendation for further research given by Sparks asks, "What is the relationship between pupil achievement and teacher knowledge?"[15]

IV. TEACHER ACADEMIC PREPARATION AND PUPIL ACHIEVEMENT

Several studies have investigated the relationship between the academic preparation in mathematics of teachers and pupil achievement in mathematics. An extensive study of school organizations with and without specialist teachers in science and mathematics is reported by Gibb and Matala.[16] They found no evidence that children learned mathematics more effectively with than without a specialist teacher. Leonhardt compared grade ten geometry classes ranking high in mathematical achievement with those ranking low and found that mathematics teachers in the high-ranking schools had studied more undergraduate mathematics courses than those in the low-ranking schools.[17] He also found that the former usually held a major in mathematics whereas the latter did not. In addition, students in high-ranking classes believed that their teachers knew the subject matter better. Neill used three criterion measures to assess the performance of classes of academically talented grade seven pupils being given one of five selected mathematics programs.[18] While he found that pupil characteristics contributed more to the variance in pupil performance than teacher characteristics, the latter did make a contribution with the length of academic preparation of the teacher contributing most. The effects of teacher variables were also less marked than the effects of the different programs used. In another study comparing student achievement in arithmetic reasoning and computation over a nine-year period (kindergarten to grade eight) with the high school and college mathematics preparation of their teachers, Rouse computed multiple regression statistics for the various combinations but found no high correlations between student achievement and the academic background of their teachers.[19] Thus, the evidence available is inconclusive.

V. IN-SERVICE EDUCATION AND PUPIL ACHIEVEMENT

One method of obtaining evidence which supports the need for the teacher to understand mathematical concepts his students are to learn is to compare the achievement of students whose teachers have taken in-service education courses in mathematics with those students whose teachers have not taken such courses. Houston and DeVault used a voluntary thirteen-hour in-service education program on mathematical concepts related to the elementary school program which was provided for 102 intermediate grade teachers.[20] Both teachers and their pupils were tested before and after the in-service program. Although no significant relationship had been found between the initial teacher scores for understanding and the change in pupil scores for understanding, a significant relationship (.01) was found between final teacher scores for understanding and change in pupil scores for understanding. A significant relationship (.01) was also found between change in teacher scores for understanding and change in pupil scores for understanding. Dickens compared mean changes in mathematical understanding for grade four, five, and six pupils whose teachers participated in a sixteen-hour in-service education program with those whose teachers had no organized in-service education program.[21] Although teachers from the in-service education program made significant gains in mathematical understanding, the comparison between the two groups of pupils showed a significant difference at grade six, but not for grade four or five. Studies in this direction again give inconclusive evidence concerning the relationship between the mathematical understanding of teachers and their pupils.

VI. TEACHER UNDERSTANDING AND PUPIL ACHIEVEMENT

Three studies have avoided interference from in-service education in making a direct examination of the relationship between measures of teachers' mathematical understandings and measures of pupils' mathematical understandings. Bassham sought objective evidence to support or refute the assumption that good teacher understanding of basic mathematical concepts is a necessary condition for the promotion of satisfactory pupil growth in arithmetic.[22] Using twenty-eight sixth-grade teachers and their classes he compared a measure of teacher understanding of basic mathematical concepts with the arithmetic scores of their pupils controlled for initial differences in arithmetic achievement, reading achievement, mental ability, and interest in arithmetic. The correlation computed between these two sets of data was found to be significant (.05). The correlation between the same two sets of data restricted to those in each class above the mean intelligence score was highly significant (.01). However, the correlation for those in each class below the mean intelligence score was not significant. Thus, Bassham found the relationship to be dependent upon the level of pupil intelligence. A similar study by Lampela involving seventy teachers of grades four, five, and six found no significant relationship between either teacher understanding or change in teacher understanding of mathematical concepts and the change in pupil understanding of mathematical concepts over a five-month period.[23] Peskin investigated a number of relationships between teacher understanding and attitude and student understanding and attitude in regular seventh-grade mathematics classes.[24] She used three criterion measures in arithmetic for both teachers and students and three in geometry. Fifty-five teachers and 565 of their students chosen at random were involved from nine junior high schools. Using partial correlation techniques to remove the effects of initial differences in mathematics achievements of the groups, significant correlations (.05) were found between teacher understanding scores and student achievement scores in arithmetic and geometry. Peskin found that teachers with high attitude and understanding scores had students who achieved highest but that teachers with low understanding scores had students with the next best results. Furthermore, those teachers with high understanding but low attitude scores had students whose achievement was poorest. Obviously there is much more to learn about the relationship between teacher understanding and student achievement in the junior high school.

VII. LIMITS AS A TOPIC IN JUNIOR HIGH SCHOOL

Bing points out, "The notion of limit is a very important one in mathematics. . . . A student may do well in arithmetic, algebra, and even geometry . . . without understanding limits, but he must learn this concept in order to go far in mathematics."[25] Since limit is such an important topic in mathematics, the study of intuitive limit concepts is recommended for the secondary school college preparatory program by the Commission on Mathematics of the College Entrance Examination Board.[26] High school and college teachers surveyed by Leissa and Fisher are highly favorable to this recommendation.[27] The present course prescribed for Mathematics 8 includes an intuitive introduction to limit concepts similar to that recommended by the Commission on Mathematics.[28]

Two studies give evidence that intuitive limit concepts can be learned successfully by junior high school students. Smith provided three hours of instruction in limit concepts for students in grades seven, nine, and eleven to determine whether or not the pupils involved could benefit from this experience.[29] He found that they could on the basis of scores from a specially prepared limits test. Equating groups on the basis of mental age he found the mean scores for groups who had the special instruction to be significantly higher than those for groups which had no instruction. Dessart investigated the feasibility of teaching some aspects of convergence and divergence of infinite series to superior grade eight students.[30] Different presentations were made to different groups but all were kept intuitive and precise definitions avoided. The majority of students showed a satisfactory gain in understanding regardless of the presentation used. Thus grade eight students of superior ability can learn concepts of convergent and divergent infinite series.

VIII. THE HYPOTHESIS

Since intuitive limit concepts are a new topic in the grade eight mathematics curriculum in British Columbia, many teachers at this level have had no previous experience with teaching these ideas. Since the topic does not occur in any earlier mathematics course it is assumed that students will have had no previous experience with the topic. If the teacher has little or no understanding of the concepts to be presented, it is possible that such a teacher might provide no opportunity for better students to develop an understanding of intuitive limit concepts. This topic, therefore, provides a unique opportunity to measure the relationship between teacher understanding and student understanding of a single mathematical concept.

The null hypothesis tested will be: For Mathematics 8 classes of better students there is no significant correlation between teacher understanding of intuitive limit concepts and student understanding of intuitive limit concepts.

FOOTNOTES

[1]Joseph E. Morsh, George C. Burgess, and Paul N. Smith, "Student Achievement as a Measure of Instructor Effectiveness," Journal of Educational Psychology, 47:86, February, 1956.

[2]C. V. Newsom, "Mathematical Background Needed by Teachers," The Teaching of Arithmetic, Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: University of Chicago Press, 1951), p.232.

[3]Kenneth E. Brown, "Teaching Load and Qualifications of Mathematics Teachers," The Mathematics Teacher, 53:9, January, 1960.

[4]Wilbur H. Dutton, Evaluating Pupils' Understanding of Arithmetic (Englewood Cliffs, New Jersey: Prentice Hall Incorporated, 1964), p. 104.

[5]Kenneth E. Brown and Theodore L. Abell, "Research in the Teaching of High School Mathematics." The Mathematics Teacher, 59:56, January, 1966.

[6]Vincent J. Glennon, "A Study in Needed Redirection in the Preparation of Teachers of Arithmetic," The Mathematics Teacher, 42:393, December, 1949.

[7]Jacob S. Orleans and Edwin Wandt, "The Understandings of Arithmetic Possessed by Teachers," Elementary School Journal, 53:507, May, 1953.

[8]J. Fred Weaver, "A Crucial Problem in the Preparation of Elementary School Teachers," Elementary School Journal, 56:260, February, 1956.

[9]E. Fulkerson, "How Well Do 158 Prospective Elementary Teachers Know Arithmetic?," The Arithmetic Teacher, 7:141, March, 1960.

[10]*Ibid.*, p. 146.

[11]Russell A. Kenney, "Mathematical Understandings of Elementary School Teachers," The Arithmetic Teacher, 12:433, October, 1965.

[12]Carol Kipps, "Elementary Teachers' Ability to Understand Concepts Used in New Mathematics," The Arithmetic Teacher, 15:368, April, 1968.

[13]Orleans and Wandt, op. cit., p. 501.

[14]Jack N. Sparks, "Arithmetic Understandings Needed by Elementary School Teachers," The Arithmetic Teacher, 8:402, December, 1961.

[15]Ibid., p. 403.

[16]E. G. Gibb and D. M. Matala, "Study of the Use of Special Teachers in Science and Mathematics in Grades Five and Six," School Science and Mathematics, 61:569-572, November, 1961; 62:565-585, November, 1962.

[17]Earl Albert Leonhardt, "An Analysis of Selected Factors in Relation to High and Low Achievement in Mathematics" (unpublished Doctoral thesis, The University of Nebraska, 1962), p. 228.

[18]Robert Dudley Neill, "The Effects of Selected Teacher Variables on the Mathematics Achievement of Academically Talented Junior High School Pupils" (unpublished Doctoral thesis, Columbia University, 1966), p. 2.

[19]W. M. Rouse, "A Study of the Correlation between the Academic Preparation of Teachers of Mathematics and the Mathematics Achievement of their Students in Kindergarten through Grade Eight" (unpublished Doctoral thesis, Michigan State University, 1967).

[20]W. R. Houston and M. V. DeVault, "Mathematics In-service Education: Teacher Growth Increases Pupil Growth," The Arithmetic Teacher, 10:243, May, 1963.

[21]Charles H. Dickens, "Effects of In-service Training in Elementary School Mathematics on Teachers' Understanding and Teaching of Mathematics" (unpublished Doctoral thesis, Duke University, 1966).

[22]Harrell Bassham, "Teacher Understanding and Pupil Efficiency in Mathematics: A Study of Relationship," The Arithmetic Teacher, 9:383, November, 1962.

[23]Roland Mitchell Lampela, "An Investigation of the Relationship between Teacher Understanding and Change in Pupil Understanding of Selected Concepts in Elementary School Mathematics" (unpublished Doctoral thesis, University of California, Los Angeles, 1966).

[24]Anne Stern Peskin, "Teacher Understanding and Attitude and Student Achievement and Attitude in Seventh Grade Mathematics" (unpublished Doctoral thesis, New York University, 1964), p. 2.

[25]R. H. Bing, "Point Set Topology," Insights into Modern Mathematics, Twenty-third Yearbook of the National Council of Teachers of Mathematics (Washington: The National Council of Teachers of Mathematics, 1957), p. 314.

[26]College Entrance Examination Board, Report of the Commission on Mathematics, Appendices (New York: College Entrance Examination Board, 1959), pp.64-73.

[27]Arthur W. Leissa and Robert C. Fisher, "A Survey of Teachers' Opinions of a Revised Mathematics Curriculum," The Mathematics Teacher, 53:116, February, 1960.

[28]British Columbia Department of Education, Secondary School Mathematics, 1966 (Victoria: British Columbia Department of Education, 1966), p. 7.

[29]Lehi T. Smith, "Could We Teach Limits?," The Mathematics Teacher, 54:344, May, 1961.

[30]Donald Joseph Dessart, "A Study of Programmed Learning with Superior Eighth Grade Students" (unpublished Doctoral thesis, University of Maryland, 1961).

CHAPTER III

DESIGN OF THE STUDY

I. PREPARATION OF THE TESTS

A preliminary unspeeded test of intuitive limit concepts was prepared and the forty items classified by a mathematician into the following categories determined by the researcher: limit of a sequence, limit of a series, limit of a function, least upper bound, and greatest lower bound. This test was given to two Mathematics 8 classes of better students in a large urban school district of British Columbia. Both classes had been taught the limit concepts prescribed for Mathematics 8. Administration of the test was by the regular mathematics teacher who was provided with directions to follow. A maximum time of forty minutes was allowed as ample time for most students to complete the test. Teachers were asked to note if this time was inadequate. Each item was marked as right or wrong. The University of British Columbia IBM 7044 computer was used to complete an item analysis of the test and to calculate the reliability of the test using Kuder-Richardson formula 20.[1] Since the test is unspeeded, this method of rational equivalence is appropriate to use in determining reliability.[2]

Several hypothetical student answers were prepared for each of the items on the preliminary test for students. Some were correct, others incorrect. Incorrect answers were prepared to appear correct to a person unfamiliar with limit concepts. Four or more hypothetical answers were usually given for each item. These sets of answers were then used in conjunction with the preliminary student test as a test of teacher understanding of limits. The forty items for teachers obtained in this manner were taxonomized according to Bloom by an expert in mathematics education.[3] To establish a suitable marking scheme, ten teachers of Mathematics 8 in two large urban secondary schools took the test. They were told that some of the choices given were incorrect and asked to mark all answers with which they would agree. Two marking schemes were tried. One gave zero for each item on the student test if any incorrect answer was chosen and one mark if all correct answers were chosen. The other gave zero for each item if any incorrect answer was chosen, one if only some correct answers were chosen, and two if all correct answers were chosen. With either scheme, questions one to six were given one if correct, otherwise zero. The marking scheme chosen for the final test was the one giving the greater distribution of scores.

A final form of the student test of intuitive limit concepts was constructed using all items of the preliminary test having a point biserial greater than 0.20.[4] A copy of this test is included in Appendix A. Corresponding items of the preliminary teacher test were used to construct the final teacher test. A copy of this test is included in Appendix B.

II. ADMINISTRATION OF THE TESTS

Approval to give the final test to all Mathematics 8 classes of better students in a second large urban school district of British Columbia was obtained from its District Superintendent of Schools. By contacting the principals of each school having classes of Mathematics 8, fourteen classes from four schools were identified as Mathematics 8 classes of better students. A timetable was then established to permit all classes to be tested in a single week. Arrangements were made with each principal for the regular mathematics teacher to remain in the classroom while the tests were administered by two examiners experienced in classroom work. To ensure uniformity in the administration of the final tests, the examiners were instructed together in its administration. Written instructions were also provided both examiners. These are included in Appendix C.

Classroom sets of tests, including a teacher test, were provided the examiners in unmarked envelopes at the start of the week chosen for testing in May, 1969. The testing was done in the regular mathematics classroom with the mathematics teacher assisting in the distribution of materials. On the basis of experience with the preliminary test, forty minutes was allowed for students to write the final test. Shortly after the student test had begun, the examiner asked the mathematics teacher to answer the sheet on notational agreements (the final teacher test) without reference to any textbook. If the teacher asked whether he was writing a test, he was told, "Yes." Each teacher was assured that there was no way by which he, his class, or his school could be identified in the investigation. While the tests were being written, the examiner prepared a class list of those writing the student test. After collection, the tests were returned to the unmarked envelope together with the class list.

III. COLLECTION OF DATA

After leaving the classroom, the examiner either used a list of the required data prepared by the school staff or used the permanent record card of each student who wrote the test to record on the class list the student's school district stanine scores for both the Lorge-Thorndike Verbal Intelligence Test (Form E) and the School District Mathematics Test (Grade Seven). These tests were written in November, 1967, and June, 1968, respectively. The class list was returned to the envelope which was then sealed. At the end of the one-week testing period, the fourteen unmarked envelopes were returned to the investigator for marking.

A score of student understanding of limit was obtained by marking each item as right or wrong and recording the total number right for each student. A score of teacher understanding of limit was obtained by totalling the marks obtained for each item using the two-one-zero marking scheme tried with the preliminary test. This data was recorded on the class list, including the teacher's score.

IV. TREATMENT OF DATA

Analysis of covariance was used to calculate adjusted means of student understanding of limit for each class using data from the Lorge-Thorndike and School District Mathematics tests as covariates. By using analysis of covariance, the class mean scores for the student test were adjusted to allow for initial class differences in intelligence and mathematics achievement. The calculations were made by the University of British Columbia IBM /360 computer using the MFACO program prepared by Dempster and Starkey.[5] The product-moment correlation was then calculated between adjusted class means and teacher understanding of limit scores using the formula:[6]

$$r = \frac{\Sigma XY - NM_x M_y}{\sqrt{(\Sigma X^2 - NM^2_x)(\Sigma Y^2 - NM^2_y)}}$$

The coefficient of correlation obtained was then checked for significance.

FOOTNOTES

[1]Henry E. Garrett, _Statistics in Psychology and Education_ (New York: David McKay Company, 1958), p. 341.

[2]_Ibid._, p. 353.

[3]Benjamin S. Bloom (ed.), _Taxonomy of Educational Objectives, the Classification of Educational Goals, Handbook I: Cognitive Domain_ (New York: David McKay Company, 1956), pp. 201-207.

[4]Garrett, _op. cit._, p. 368.

[5]J. R. H. Dempster and G. E. Starkey, _MFACO: Analysis of Covariance_ (Vancouver: University of British Columbia Computing Centre, 1968).

[6]Garrett, _op. cit._, p. 142.

CHAPTER IV

ANALYSIS OF THE DATA

I. PRELIMINARY STUDENT TEST DATA

Table I indicates the categorization of items according to limit topics for the preliminary and final student tests. A satisfactory distribution of items among the five limit topics is indicated for both tests. In the preliminary test, the concept of a limit of a sequence is included in nine items, limit of a series in twenty-one items, limit of a function in four items, least upper bound in seventeen items, and greatest lower bound in sixteen items. Nineteen of these items include two limit concepts and four items include three concepts. The item analysis of the preliminary test indicated thirty-five items with a point biserial correlation greater than 0.20 which were used to make the final test. The correlation for each preliminary test item is given in Table I. In the final test, limit of a sequence is included in seven items, limit of a series in nineteen items, limit of a function in four items, least upper bound in sixteen items, and greatest lower bound in fourteen items. Seventeen of the final test items are categorized under two topics and four under three topics. Kuder-Richardson formula 20 gave a reliability coefficient

TABLE I

POINT BISERIAL CORRELATION AND LIMIT TOPIC CATEGORIZATION OF TEST ITEMS

Item number preliminary test (final test)	Point biserial correlation	Limit of a sequence	Limit of a series	Limit of a function	Least upper bound	Greatest lower bound
1(1)	0.21		x			
2(2)	0.21		x			
3(3)	0.54		x			
4	0.15		x			
5(4)	0.29		x			
6(5)	0.37		x			
7(6)	0.68		x			
8(7)	0.66		x			
9(8)	0.58		x			
10(9)	0.52		x			
11(10)	0.66		x			
12	0.17		x			
13	0.12	x			x	
14(11)	0.47	x			x	
15(12)	0.39	x			x	
16(13)	0.32	x				x
17(14)	0.33	x				x
18	0.00	x				x
19(15)	0.39	x				
20(16)	0.31	x				
21(17)	0.46	x				
22(18)	0.62		x		x	
23(19)	0.55		x		x	
24(20)	0.44		x		x	
25(21)	0.38		x		x	
26(22)	0.25		x		x	
27(23)	0.36		x		x	x
28(24)	0.36		x		x	x
29(25)	0.37		x		x	x
30(26)	0.44				x	x
31(27)	0.55				x	x
32(28)	0.37		x		x	x
33(29)	0.28				x	
34(30)	0.56				x	x
35(31)	0.30				x	x
36	-0.08					x
37(32)	0.71			x		x
38(33)	0.66			x		x
39(34)	0.61			x		x
40(35)	0.50			x		x

for the preliminary test of 0.87. Since the purpose of the test results is to distinguish between the means of similar classes, the reliability coefficient exceeds the criteria of Garrett who states that reliability coefficients of 0.50 or 0.60 are adequate for such purposes.[1] Teachers noted that the forty minute time allotment was sufficient for students to complete the test. Thus, the preliminary student test was shown to include the limit topics intended, provide a sufficient number of items for a final test, be more than adequate in reliability, and permit virtually all students to answer every item.

II. PRELIMINARY TEACHER TEST DATA

The taxonomization of the preliminary teacher test indicated nineteen items at the level of knowledge, eighteen at the level of comprehension and three at the level of application. No items were assigned to the three highest educational goals described by Bloom, analysis, synthesis, and evaluation. The complete taxonomization appears in Table II. It indicates that the preliminary test requires a teacher to recognize or recall the limit concepts previously categorized and to respond to test items by translating, interpreting, or extrapolating from

TABLE II

TAXONOMIZATION OF TEACHER TEST ITEMS*

Item number preliminary test (final test)	Knowledge	Comprehension	Application
1(1)		x	
2(2)		x	
3(3)	x		
4		x	
5(4)		x	
6(5)		x	
7(6)		x	
8(7)		x	
9(8)		x	
10(9)	x		
11(10)		x	
12		x	
13	x		
14(11)	x		
15(12)		x	
16(13)		x	
17(14)		x	
18		x	
19(15)		x	
20(16)		x	
21(17)		x	
22(18)	x		
23(19)	x		
24(20)	x		
25(21)			x
26(22)			x
27(23)	x		
28(24)			x
29(25)	x		
30(26)	x		
31(27)	x		
32(28)	x		
33(29)	x		
34(30)	x		
35(31)	x		
36	x		
37(32)	x		
38(33)	x		
39(34)	x		
40(35)		x	

*No items were classified at the level of analysis, synthesis, or evaluation.

the information given. On this basis, the teacher test was judged to be a suitable instrument for measuring teacher understanding of limit concepts.

The two marking systems tried for the teacher test showed little difference. Because the two-one-zero system gave a slightly greater distribution of marks and avoided ties, it was used for the final teacher test.

III. FINAL TEST DATA

In the final testing program, complete data was gathered for all mathematics teachers of the fourteen classes tested and for 332 of the 462 students who wrote the limits test. Table III gives teacher scores and the corresponding class means adjusted by the MFACO program for initial class differences in intelligence and mathematics achievement. Teacher scores range from 22 to 59 with a mean of 42.7. Adjusted class mean scores range from 12.4 to 19.0 with a mean for the fourteen classes of 15.5. The product-moment correlation calculated between teacher scores and adjusted class means is 0.09.[2] Using Table XXV of Garrett with twelve degrees of freedom this correlation is not significant.

TABLE III

TEACHER SCORES AND ADJUSTED CLASS MEANS

Class	Teacher score	Adjusted class mean
A	22	15.9
B	37	18.9
C	55	12.4
D	41	13.3
E	57	19.0
F	46	16.1
G	32	13.3
H	43	16.1
I	53	17.9
J	59	15.7
K	39	13.9
L	33	15.3
M	55	14.7
N	27	15.8
Range	22-59	12.4-19.0
Mean	42.7	15.5

FOOTNOTES

[1]Henry E. Garrett, *Statistics in Psychology and Education* (New York: David McKay Company, 1958), p. 351.

[2]See Appendix D for computation.

CHAPTER V

SUMMARY AND CONCLUSIONS

I. SUMMARY

A preliminary student test of intuitive limit concepts was constructed, checked for content validity, and given a trial use. The reliability of the test was calculated and an item analysis made to determine which items to use in a final form of the test. A teacher test of understanding was constructed using student test items together with hypothetical answers and its items were taxonomized according to Bloom. Fourteen classes of Mathematics 8 students of better ability and their teachers were tested for understanding of limit concepts using the final forms of the two tests constructed. After tests were marked, class means were adjusted by analysis of covariance to allow for initial differences in intelligence and mathematics achievement. A coefficient of correlation was calculated between these adjusted means and the teacher scores. The resulting correlation of 0.09 was not significant. Thus the null hypothesis was accepted: For Mathematics 8 classes of better students there is no significant correlation between teacher understanding of intuitive limit concepts and student understanding of intuitive limit concepts.

II. CONCLUSIONS

Acceptance of the null hypothesis in this investigation suggests that students are not prevented from understanding intuitive limit concepts if their teacher does not understand the topic well. Similarly, a teacher's understanding of intuitive limit concepts gives no assurance that his students will attain a greater understanding of the topic than students of a teacher with less understanding of the same topic. Although not able to teach the intuitive limit concept because of his own lack of understanding, a teacher may be very able to create an atmosphere which fosters the learning of this concept by his students from textbooks, supplementary publications, and other material and human resources. Should these conclusions be substantiated by further studies, there are definite implications regarding the subject-matter mastery expected of teachers-in-training.

Before concluding that no correlation exists between teacher and student understanding of the same topic in mathematics, certain limitations of the investigation should be noted. First, this study concerns only a single topic in a single grade--intuitive limit concepts in grade eight. Further investigations seem warranted for other topics at different grade levels. Secondly, because intuitive limit concepts is a topic not prescribed for any mathematics course prior to grade eight, the present study did not attempt to measure student growth in understanding of the topic in grade eight. However, the use of pre-tests and post-tests for students might now be warranted as a check on the assumption made in this study. Finally, the use of only intelligence and mathematics achievement scores as covariates may be insufficient. In particular, a measure of student attitude to mathematics might be included as a covariate in future investigations. Although beyond the scope of the present study, the role of teacher understanding of a specific topic, teacher attitude to the specific topic, teacher attitude to students, and teacher strategies used to develop the topic are four interacting variables which require further investigation as to their relation to student understanding of the specific topic.

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APPENDICES

APPENDIX A

FINAL STUDENT TEST

MATHEMATICS 8 NAME:________________ DIVISION:_____

DIRECTIONS: This test is to determine your understanding of an idea in mathematics. You will probably find that some of the questions are different from those you have seen before. Try them anyway. By thinking and experimenting you will probably be able to answer most questions.

NOTE: If a question has <u>no correct answer</u> write NONE in the answer space.

PART A: Use the symbol for greater than (>), less than (<), or equals (=) to make each of the following true.

1. 0.666... 0.666

2. 0.666... 0.666666

3. 0.666... $\frac{2}{3}$

4. 0.667 $\frac{2}{3}$

5. 0.666 $\frac{2}{3}$

PART B:

6. ADD
$$\begin{array}{r} 0.9999\ldots \\ +\underline{0.9999\ldots} \\ \underline{\qquad\qquad} \end{array}$$

7. ADD
$$\begin{array}{r} 0.39999\ldots \\ +\underline{0.49999\ldots} \\ \underline{\qquad\qquad} \end{array}$$

8. ADD
$$\begin{array}{r} 0.472222\ldots \\ +\underline{0.318888\ldots} \\ \underline{\qquad\qquad} \end{array}$$

9. SUBTRACT
$$\begin{array}{r} 5.0000\ldots \\ -\underline{4.9999\ldots} \\ \underline{\qquad\qquad} \end{array}$$

10. SUBTRACT
$$\begin{array}{r} 3.000000\ldots \\ -\underline{0.262626\ldots} \\ \underline{\qquad\qquad} \end{array}$$

PART C: A number line may help you with these questions. Consider each set of numbers as continuing in the pattern shown.

In the first two lists the numbers get closer and closer to 1. For each list, what is the <u>smallest number</u> which the numbers keep getting closer to?

11. 0.9, 0.99, 0.999, 0.9999, ... 11.__________

12. $\frac{1}{4}$, $\frac{3}{8}$, $\frac{7}{16}$, $\frac{15}{32}$, $\frac{31}{64}$, ... 12.__________

In the next two lists the numbers get smaller and smaller. What is the <u>largest number</u> that is still smaller than any number in each list?

 NAME:____________________

PART D: Think of each of the following as an endless list of additions (or subtractions) in which the pattern continues as shown. Give the sum (or difference) of each endless list.

18. $0.6 + 0.06 + 0.006 + 0.0006 + \ldots$ 18.__________

19. $9 + 0.9 + 0.09 + 0.009 + 0.0009 + \ldots$ 19.__________

20. $1 + 2 + 4 + 6 + 8 + 16 + \ldots$ 20.__________

21. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ 21.__________

22. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \ldots$ 22.__________

SUBTRACT:

23. $1 - 0.9 - 0.09 - 0.009 - 0.0009 - \ldots$ 23.__________

24. $1 - \frac{1}{2} - \frac{1}{4} - \frac{1}{8} - \frac{1}{16} - \ldots$ 24.__________

25. $1 - \frac{9}{10} - \frac{9}{100} - \frac{9}{1000} - \frac{9}{10000} - \ldots$ 25.__________

PART E: Think of the set of all real numbers less than 5.

26. Give a number in the set which is greater than 4.9 but less than 5. 26.__________

27. Give a number in the set which is greater than 4.9999 but less than 5. 27.__________

28. Give a number in the set which is greater than 4.999... but less than 5. 28.__________

29. What is the largest number in the set of all real numbers less than 5? 29.__________

PART F: Think of the set of all real numbers greater than 15.

30. Give a number in the set which is greater than 15 but less than 15.1 30.__________

31. Give a number in the set which is greater than 15 but less than 15.111... 31.__________

PART G: If you replace $\triangle$ with larger and larger numbers then:

32. the value of $\frac{1}{\triangle}$ gets closer and closer to what number? 32.__________

33. the value of $\frac{5}{\triangle}$ gets closer and closer to what number? 33.__________

34. the value of $\frac{1}{5 \times \triangle}$ gets closer and closer to what number? 34.__________

35. the value of $\frac{(2 \times \triangle) + 1}{\triangle}$ gets closer and closer to what number? 35.__________

END

APPENDIX B

FINAL TEACHER TEST

STUDENTS FROM DIFFERENT CLASSES MAY HAVE USED A VARIETY OF NOTATIONAL AGREEMENTS TO DETERMINE THEIR ANSWERS. SOME ARE MATHEMATICALLY INCORRECT. PLEASE CIRCLE ALL OF THE FOLLOWING ANSWERS WITH WHICH YOU AGREE.

1. > 2. > 3. = 4. > 5. <

6.	2.000...	1.9999...8	1.999...	1.9998	1.9999	
7.	0.8998	0.79999...	0.8999...	0.89999...8	0.9	
8.	0.79111...0	0.79111...	0.791110...	0.8000...		
9.	0	0.0001...	0.000...1	0.1111...		
10.	2.737373...74	2.737373...	2.747474...	2.737374...		
11.	0.99999	0.999...	$\frac{9}{10}$	0.999...9	1	NONE
12.	$\frac{63}{128}$	0.4999...	$\frac{1}{2}$	1	0.5	NONE
13.	0.000...1	$\frac{1}{12}$	0	$\frac{1}{1000...}$	NONE	
14.	2.111...	$2\frac{1}{7}$	2	2.000...1	1.999...	NONE
15.	0.66665	$0.6666\frac{1}{2}$	0.666...	0.666...6	$\frac{2}{3}$	NONE
16.	$3\frac{7}{8}$	4	4.111...	3.999...	NONE	
17.	$0.1111\frac{1}{2}$	0.111...1	0.111...	$\frac{1}{9}$	0.11115	NONE
18.	0.6666	0.666...	0.7	0.666...6	NONE	
19.	9.999...	9.9999	10	9.999...9	NONE	
20.	1000...0	69	999...9	37	NONE	
21.	$2\frac{15}{16}$	2	1.999...	$2\frac{31}{32}$	NONE	
22.	$1\frac{5}{6}$	1.999...	2	$1\frac{6}{7}$	NONE	
23.	0.111...	0.0001	0.000...1	0	NONE	
24.	$\frac{1}{16}$	0	$\frac{1}{1000...}$	$\frac{1}{32}$	NONE	
25.	$\frac{111...}{1000...}$	0	$\frac{1}{1000...}$	$\frac{1}{1000}$	NONE	
26.	4.9111...	4.95	4.999...	4.99	NONE	
27.	4.999989999...	4.9999...	4.99999	4.99995	NONE	
28.	4.9999...	4.999...9	4.99991111...	4.999998	NONE	
29.	4.9	4.999...9	4.999...	4.99999...	NONE	
30.	15.01	15.111...	15.0111...	15.0999...	NONE	
31.	15.01	15.0999...	15.1	15.0111...	NONE	
32.	0.999...	0.111...	0.000...1	1.0	0	NONE
33.	5.0	0	0.111...	4.999...	0.000...1	NONE
34.	0	0.1999...	0.222...	0.2	0.000...1	NONE
35.	0	0.000...1	1.999...	2.0	3.0	NONE

APPENDIX C

DIRECTIONS FOR ADMINISTRATION OF FINAL TESTS

1. Introduction: "I'm here today to give you a test which is part of a University of B. C. study of how well grade eight students understand a certain idea in mathematics."

2. Distribution: Distribute foolscap and test papers face-down.

   "When you receive your copy of the test, leave it face-down on your desk. The foolscap is for rough work but will be collected."

   Check that everyone has a test paper.

3. Name and division: "When you turn the test paper over please PRINT your <u>full</u> <u>name</u> and <u>division</u> <u>number</u> at the top of page one. Do this now." (pause)

   "Now turn to page two and again <u>print</u> your name at the top."

4. Directions: "Turn to the directions on page one and read them to yourself while I read them aloud."

   (Read from your copy of test, including NOTE.)

   "Time will be provided for most of you to finish but it will be no longer than <u>40</u> <u>minutes</u>. If you do finish, check your solutions before placing your paper face-down until time is up. Once you begin, no questions will be answered. Is there anyone not sure of what he is to do?"

(Answer from directions if necessary.)

"BEGIN."

(Note time plus 40 minutes to "STOP".)

5. Notation: While the students write the test, have their mathematics teacher complete the spirit-stencilled sheet on notational agreements without using a text-book. Assure him that there will be no way of identifying school, class, or teacher, but his completion of the answers is a key part of the study. ASK HIS COOPERATION IN NOT DISCUSSING THE TEST WITH HIS COLLEAGUES.

6. Collection: When 40 minutes has elapsed, announce, "STOP." Collect test papers first, then foolscap. Place all in envelope with teacher answers. DO NOT IDENTIFY ENVELOPE. Any unused papers must be brought away in the "EXTRA" envelope.

APPENDIX D

COMPUTATION OF PRODUCT-MOMENT CORRELATION BETWEEN TEACHER SCORES (X) AND ADJUSTED CLASS MEANS (Y)

$$r = \frac{\Sigma XY - NM_xM_y}{\sqrt{(\Sigma X^2 - NM^2_x)(\Sigma Y^2 - NM^2_y)}}$$

$$= \frac{9375.0 - 14(42.8)(15.6)}{\sqrt{(27451 - 14(42.8)^2)(3456.9 - 14(15.6)^2)}}$$

$$= \frac{9375.0 - 9347.5}{\sqrt{(27451 - 25645.8)(3456.9 - 3407)}}$$

$$= \frac{27.5}{\sqrt{(1805.2)(49.9)}}$$

$$= \frac{27.5}{\sqrt{90079.48}}$$

$$= \frac{27.5}{300.1}$$

$$= 0.09$$