PRACTICE VERSUS GRAPHICAL REPRESENTATION FOR MAINTAINANCE OF BASIC ARITHMETIC COMPETENCIES: FIRST YEAR PRIMARY

by

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ABSTRACT

Educators such as Edith Biggs in Britain and Vincent Glennon and the Cambridge Conference on School Mathematics in the United States have suggested that the amount of time children spend on direct practice of newly learned skills and understandings can be greatly reduced. The Americans propose an integration of this practice with the presentation and learning of new topics. The British favour an activity approach, where new learnings are put to immediate use, and the need for acquisition and perfection of mathematical competencies becomes obvious to the children. A few American research studies have substantiated the merits of reduced practice, at the intermediate level.

This study explores the place of practice for maintenance of the basic competencies of First Year Primary children in British Columbia at the end of the school year. The competencies chosen for study were 1) Numeration: reading, writing and understanding of base ten numerals ≤ 99, and 2) Computation: addition and subtraction operations with sums and minuends ≤ 10.

The new material, chosen to be presented as an alternative to direct practice, was Graphical Representation, a unit developed from the Nuffield Project booklet, Pictorial Representation [1]. Two schools in the Vancouver area were used,

the first with a class of 54 children and the second with 34. Parallel pre-tests and post-tests in the basic competencies were administered. During a three week intervening interval, the investigator taught the children, who were divided into groups, by random selection, as follows:

In the first school, three groups of 18 children were instructed respectively in Graphical Representation, in review and practice, using familiar materials, and in geometry, involving no use of numbers (control group). In the second school, two groups of 17 children were instructed in Graphical Representation, and in review and practice, respectively.

At the end of the experiment, there was no significant difference in the tested numeration competencies of the two experimental groups in their respective schools. The control group showed a slightly lower achievement. Time did not permit a retention test.

In the first school, where computational efficiency was low, the results slightly favoured the review and practice group, over the other groups. In the second school, there was no significant difference between the two groups, regarding progress in computational skills.

Within its limitations, this study demonstrates the possibility of maintaining basic competencies, while introducing new topics, at the first year level.

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CHAPTER I

THE PROBLEM, THE RESEARCH DESIGN, AND DEFINITION OF TERMS

STATEMENT OF THE PROBLEM

This is a study of the effects of practice versus the introduction of a graphing unit on the maintainance of basic arithmetic competencies of six and seven year old British Columbia children at the end of First Year Primary.

GENERAL VIEW

Today's world is one of change, at an ever accelerating pace. Educators must prepare children for a world as yet unknown. To this end they strive continuously to improve curricula and methods, in all areas of instruction. Mathematics was one of the first curricula to undergo drastic revision. The vanguard of reformers had long been levelling the criticism that there was too much rote learning and drill in the "traditional" programs. More recently the "new" mathematics programs have been criticized for not providing sufficient drill for mastery of basic skills. There is justification for both criticisms and a problem exists to establish a balance between these two positions.

In the United States, Vincent Glennon[1] and Marshall Stone[2] have sought examination of the problem of synthesis of curriculum and method, with due regard to child development and psychology. The latter said:

> We need fundamental studies in psychology and mathematics, and ultimately on the combination of these findings into a coherent and efficient program of instruction.[3]

In Britain and Canada, Edith Biggs and James MacLean have said:

> Our experience with young children has shown that we do not yet know how much mathematics children can learn.[4]

The general problem that this study concerns itself with can be expressed thus:

> How and to what extent can we increase both the quality and the quantity of mathematics education while at the same time "Maintaining reasonable computational skills".[5]

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[1]Vincent J. Glennon, "...and now Synthesis: a theoretical model for mathematics education", The Arithmetic Teacher, vol. 12, no. 2, (February, 1965) pp. 134-141.

[2]Marshall H. Stone, "What is Expected of Research", in Needed Research in Mathematical Education, Howard Fehr, ed., A Summary Report of a Conference held at Greystone, New York, October 30-31, 1965, (New York: Teachers' College Press, Teachers' College, Columbia University).

[3]Ibid. p.3.

[4]Edith E. Biggs and James R. MacLean, Freedom to Learn, (Toronto, Canada: Addison-Wesley, 1969), p.4.

[5]Ibid., p.4.

SPECIFIC AREA OF INVESTIGATION

In particular an attempt was made to incorporate the practice needed for maintenance of the basic arithmetic skills and understandings of First Year Primary with the learning of new material from the Teacher's Guide, Pictorial Representation, [1].[6] The resultant maintenance of competency was compared with that of a group of children who spent an equal allotment of time on traditional year-end review and practice. A control group was exposed to no arithmetic during the same period, and competency maintenance assessed and compared with the other groups.

HYPOTHESES

Main Hypotheses

It was hypothesized that the group involved in Graphical Representation would maintain a level of competence comparable to that of the review and practice group, during a three week period. It was further hypothesized that both groups would do better than the control group, with respect to maintenance and/or improvement of the basic competencies under examination. The specific hypotheses will be found in Chapter IV, pages 46 to 49.

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[6] Nuffield Mathematics Project, Pictorial Representation [1], (London: W. & R. Chambers and John Murray, 1967, available from Longmans Canada Ltd., Don Mills, Ontario).

Supplementary Hypotheses

It was hypothesized that children of lesser mathematical ability might profit more from the novel program than the more able children. Sufficient statistical data for this analysis was only available for some of the children in the original experiment. The specific supplementary hypotheses regarding the effect of ability will be found in Chapter IV, pages 61 to 63.

RESEARCH DESIGN

Two schools were available to the investigator. School #1 had 56 children, in a large team teaching situation. School #2 had a single class of 34 children. Both schools used a modern meaningful approach to mathematics, but teaching methods were very different. To enable comparisons between groups which had previously been taught the same way, the classes were divided so that both experimental methods were used in both schools. To make the numbers viable, a control group was set up in School #1 only.

Two areas of basic competence were chosen for study: 1) Numeration, involving reading, writing and understanding of base ten numerals <100.

2) Addition and subtraction operations. Pretests and parallel post-tests in these areas were ad-

ministered to the children in the two schools. An attempt was also made to measure ability to obtain information from a block chart, as developed in the unit on Graphical Representation.

To minimize Hawthorne effects, all teaching and testing was carried out by the investigator. The three teaching groups were set up as follows:

Group A (experimental) was introduced to the unit Pictorial Representation, [1],[7] with the investigator incorporating use of the skills under study, wherever possible.

Group B (experimental) was instructed, as a review unit, in practice of the above skills, using, as far as possible, the methods and materials to which the children were accustomed.

Group C (control) was instructed in an open ended geometry unit, care being taken to avoid counting or addition and subtraction.

Children were assigned to their respective groups with the aid of a computer-generated list of random numbers. The resultant research design is shown in Figure 1.

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[7] Op. cit.

	School #1	School #2
New Material (Graphical Representation)	A_1	A_2
Review and Practice	B_1	B_2
Control (Geometry)	C_1	

FIGURE 1 RESEARCH DESIGN

DEFINITION OF TERMS

Algorithm. A computational recipe.

Concept. A concept is an abstraction formed by generalization from particulars.

Curriculum. Subject matter or course content.

Drill or practice. Repeated use of a skill or repeated recall of facts with the express purpose of learning or improving the skill or of re-enforcing retention of facts.

Developmental Activities. Those activities of teacher and class intended to increase understanding of the number system, processes or operations, and the general usefulness of number and quantity in everyday experiences.[8] Projects, constructions and use of manipulative materials would be included.

Meaning Theory. Teaching understanding of computational process, before formal presentation of an algorithm.

Method. Includes how curriculum is presented, taught and evaluated.

Number 'facts'. Formerly items of 'tables'.

Number Sentence. A symbolic expression involving numbers, the relationships $<$, $>$, or = and frequently an operator such as + or -.

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[8] Karl G. Zahn, "Use of class time in eighth-grade arithmetic," The Arithmetic Teacher, vol.13, no.2, (Feb.1966).

Equation. A number sentence involving the equivalence relationship.

Structure. "How things are related."[9]

Primary (School) in North America, includes Kindergarten to Grade 3, children of ages 5-9. In Europe, generally includes children of ages 5-12. (France: ages 2-12).

Elementary (School). North American term, includes Kindergarten to Grade 7 (±1), i.e. pre-High School.

Intermediate Grades. North American term, includes Grades 4-6 or 4-7, ages 9-12 or 9-13.

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[9]Jerome S. Bruner, The Process of Education, (New York: Vintage Books, 1960), p.7.

CHAPTER II

SURVEY OF THE LITERATURE

INTRODUCTION

Very little experimental research on the place of practice has been reported in recent literature, in spite of several specific suggestions that this be done. In fact there has been a "dearth of any hard research" at the primary level.[1]

RESEARCH AT FIRST YEAR PRIMARY LEVEL

The lack of research at this level was a strong motivating force for this study. The importance of early childhood years is being increasingly recognized by psychologists and educators around the world. Modern scientific investigations appear to back up the old Jesuit saying "Give me a child until he is seven . . . " At present there is considerable activity at the kindergarten and earlier levels, and many Piagetian type studies are being carried out with primary school children. For example, Reidesel's list of "Research Contributions", a "representative selection"

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[1]Vincent J. Glennon and Leroy G. Callahan, Elementary School Mathematics, A Guide to Current Research, (Association for Supervision and Curriculum Development, National Education Association, 1968), p.75.

in The Arithmetic Teacher of March, 1970[2] included at least thirty Piaget oriented studies. On the other hand, there were only five studies dealing specifically with the first grade. None had any application to the problem of the place of practice. Any other list or summary yielded the same general picture.[3] Also, at the Northwest Mathematics Conference in Tacoma, in 1968, several speakers were heard to deplore this lack.

THE PLACE OF PRACTICE--CHANGING EMPHASES

"Practice makes perfect" runs an old saying and for centuries, in numerous fields of learning, practice has been an important and necessary follow-up of the presentation of new topics and skills.

During the early years of the twentieth century, drill was considered part of the cognitive process. The research of E. L. Thorndike and others confirmed that drill did improve competence in computation, a much needed skill in those days. Buswell and Judd, in their Summary of Educational Investigations Relating to Arithmetic listed thirty-three studies

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[2]C. Alan Riedesel, "Recent Research Contributions to Elementary School Mathematics", The Arithmetic Teacher, vol. 17, no. 3, March, 1970.

[3]See Bibliography for other lists examined. e.g. J. Fred Weaver and Glenadine Gibb, "Mathematics in the Elementary School", Review of Educational Research, vol. 34, no. 3, June, 1964.

which were unanimous in reporting the benefits of drill.[4]

Later, the "meaning theory" of learning, advocated by Brownell[5] and others replaced that of drill. Suydam and Riedesel, in their Interpretive Study of Research and Development in Elementary School Mathematics,[6] reported consistency of findings that the"meaning method" was superior to "tell and drill", for retention and transfer.

When Gestalt theories and methods reached North America, Wheeler suggested, in the Tenth Yearbook of the National Council of Teachers of Mathematics, that practice (the newer name for drill) could be eliminated altogether from the classroom.[7] This suggestion does not seem to have

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[4]G. T. Buswell and C. H. Judd, Summary of Educational Investigations Relating to Arithmetic, (Chicago: University of Chicago Press, 1925).

[5]cf. study by William A. Brownell and Harold E. Mosher Meaningful vs. Mechanical Learning, A Study in Grade Three Subtraction, Duke University, Studies in Education, vol. 8, 1949, pp. 1-207.

[6]Marilyn N. Suydam and C. Alan Riedesel, Interpretive Study of Research and Development in Elementary School Mathematics, Final Report, Project No. 8-0586, U.S. Office of Education, Department of Health, Education and Welfare, (University Park, Pa.: The Pennsylvania State University, June 30, 1969).

[7]NCTM, The Teaching of Arithmetic, Tenth Yearbook, (Washington, D.C.: National Council of Teachers of Mathematics, 1935) as quoted by T. R. McConnell in "Recent Trends in Learning Theory: Their Application to the Psychology of Arithmetic", Arithmetic in General Education, Sixteenth Yearbook, (Washington, D.C.: National Council of Teachers of Mathematics, 1941), pp. 268-289.

found many advocates. Is it that practical experience in classrooms has not provided any supporting evidence which would indicate the worth of experiment in this direction?

The Swiss epistemologist, Jean Piaget, whose work is being increasingly recognized in the United States, speaks specifically of practice, saying that "repetitive behavior" is an essential part of the learning process.[8]

By mid-century, educators were emphasizing the importance of teaching concepts and structures.[9]

As children were taught new curricula, with new approaches, it began to appear that, when tested with old-type computational achievement tests, they did better than children on more "conventional" programs. David Page said: "It now appears that higher proficiency in computation is an almost

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[8] J.H. Flavell, The Developmental Psychology of Jean Piaget, (New Jersey: D. Van Nostrand Co., Inc., 1963), p. 57.

[9] e.g. Jerome S. Bruner, "The Importance of Structure", The Process of Education, (New York: Vintage Books, Random House, 1960), Chapter II, pp. 17-32, and Esther J. Swenson, "Arithmetic for Pre-school and Primary Grade Children", The Teaching of Arithmetic, Fiftieth Yearbook, Part II, National Society for the Study of Education, (Chicago: University of Chicago Press, 1951), p. 74.

automatic adjunct."[10] Marguerite Brydegaard[11] and Hugh Peck[12] reported similar results, the latter especially among children of lower ability.

However, the general application of these new programs by teachers not specifically trained in their use, was frequently much less successful. Demands for new learning theories were made; researchers continued to study how children learn, and educators strove to improve teachers' training.[13]

But the questions remain: How should children get practice? When? How much is necessary?

THREE PERTINENT SUGGESTIONS

Three closely related proposals as to the place of practice, when trying to answer the basic problem of in-

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[10]David Page, "General Information Release." University of Illinois Arithmetic Project", May, 1962, as quoted by C. D. Hopkins, "An Experiment on Use of Arithmetic Time in Fifth Grade", (Doctoral Dissertation, Indiana University, 1965), Dissertation Abstracts, vol. 26: 5291, 1966. Microfilm: R2616/4.

[11]A study by Marguerite Brydegaard referred to by Herbert F. Spitzer and Paul C. Burns in"Mathematics in the Elementary School", Chapter II of Review of Educational Research, "The Natural Sciences and Mathematics", vol. 31, no. 3, June, 1961, p. 249.

[12]Peck, Hugh A., "An Evaluation of Topics in Modern Mathematics", The Arithmetic Teacher, vol.10, no.5, May,1963.

[13]See for example, "Cambridge Conference on Teacher Training", Goals for Mathematical Education of Elementary School Teachers, (Boston: Houghton Mifflin Co., 1967).

creasing quantity and quality of instruction, have recently been proposed by Vincent J. Glennon, The Cambridge Conference on School Mathematics, and Edith Biggs.

Glennon Proposals and Related Research

In 1951, Glennon and Buswell proposed that studies be made to determine the optimum division of arithmetic time between practice and concept development (developmental activities).[14] Brownell, in The Arithmetic Teacher of October, 1956, also called for a balance between teaching for meaning or understanding, and training in computational skills.[15] Very few researchers seem to have taken up this particular challenge. In their Interpretive Study of Research and Development in Elementary School Mathematics.[16] Suydam and Riedesel report four such studies, all at the intermediate level, by Shipp and Deer, Shuster and Pigge, Zahn and Hopkins.

Shipp and Deer, in a large, carefully planned study, in grades four, five and six, tested understandings, accuracy

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[14]G. T. Buswell, "Needed Research in Arithmetic", The Teaching of Arithmetic, Fiftieth Yearbook, Part II, National Society for the Study of Education (Chicago: The University of Chicago Press, 1951), p. 291.

[15]William A. Brownell, "Meaning and Skill-Maintaining the Balance", The Arithmetic Teacher, vol. 3, no. 6, October, 1956.

[16]Marilyn N. Suydam and C. Alan Riedesel, op.cit.

with skills, and problem solving.[17] They found a definite trend towards higher achievement when a greater percentage of class time was spent on developmental activities. The largest proportion of time thus allotted was seventy-five percent.

Shuster and Pigge obtained similar results with a study of addition and subtraction of fractions at the grade five level.[18] Zahn, in "Use of Class Time in Eighth Grade Arithmetic" concluded that fifty percent or more of class time spent on developmental activities, resulted in higher achievement.[19] Hopkins, in his study of "Experimental Use of Time in Fifth Grade", concluded that the amount of drill time should be considerably reduced.[20] Most interestingly he found that the greatest gains in computational proficiency,

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[17]Donald Eugene Shipp, "Experimental Study of Achievement and Time Allotment", Dissertation Abstract, vol. 19 (1), p. 492, 1958, and D. E. Shipp and G. M. Deer, "Use of Class Time in Arithmetic", The Arithmetic Teacher, vol. 7, no. 3., March, 1960, pp. 117-121.

[18]Albert H. Shuster and Fred L. Pigge, "Retention Efficiency of Meaningful Teaching", The Arithmetic Teacher, vol. 12, no. 1, January, 1965, pp. 24-31.

[19]Karl George Zahn, "Use of Class Time in Eighth Grade Arithmetic", The Arithmetic Teacher, vol. 13, no. 2, February, 1966. See also Dissertation Abstracts, vol. 26, p. 6459, May, 1966.

[20]C. D. Hopkins, "An Experiment on the Use of Arithmetic Time in Fifth Grade," Doctoral Dissertation, R2616/4, Dissertation Abstracts, vol. 26, no. 5291, 1966.

when practice time was reduced, were made by the students of lower ability.

In the brief summary of their report, published in The Arithmetic Teacher, March 1970,[21] Suydam and Riedesel say that "Drill and practice are necessary for computational accuracy". This seems an oversimplification of the above reported studies, which were examining proportion of time spent on drill rather than the question 'to drill or not to drill'. They are possibly referring to a study by Ivor G. Meddleton[22] at the fourth grade level, which, in their Interpretive Report,[23] they say confirmed that "systematic, short review work in basic mathematics produces significantly higher levels of achievement".

More recently, Glennon and Leroy G. Callahan made a stronger suggestion than the Glennon and Buswell one when they wrote:

> Because of the sequential development of a sound mathematics education program, much of the practice on previously learned skills can be 'built-in' to subsequently learned materials.[24]

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[21]Suydam and Riedesel, op.cit.

[22]Meddleton, Ivor G. "An Experimental Investigation into the Systematic Teaching of Number Combinations in Arithmetic", British Journal of Educational Psychology, vol. 26: 117-127, June 1956.

[23]Suydam and Riedesel, op.cit.

[24]Vincent J. Glennon and Leroy G. Callahan, Elementary School Mathematics, A Guide to Current Research, (Association for Supervision and Curriculum Development, National Education Association, 1968), p. 81.

Brownell also seems to have had this in mind when he did his studies on fifth grade readiness for long division.[25] Roberta Chivers, et al, have incorporated 'built-in' practice in their text book series, Number Patterns, for example, with the use of parentheses to review addition and multiplication 'facts' during the development of the 'facts' for larger numbers.[26]

Glennon and Callahan[27] reported one study by Lelon Capps, "Division of Fractions", which they say showed the efficiency of one method of presentation over another, in that it provided extra practice in multiplication and hence maintained this skill. Capps himself was more cautious, concluding that there were many facets to his problem of which was the more desirable method to use, facets which called for examination before either method was advocated or condemned.[28]

In spite of these studies, a very recent survey by

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[25]Brownell, William A. "Arithmetic Readiness as a Practical Classroom Concept", Elementary School Journal, September, 1951 and"Effects of Practising a Complex Arithmetic Skill upon Proficiency in its Constituent Skills", Journal of Educational Psychology, February, 1953.

[26]Roberta Chivers, J. E. Smith, Eric D. MacPherson, Alfred P. Hanwell, Number Patterns, Books I and II, (Toronto; Holt, Rinehart and Winston, 1966).

[27]Glennon and Callahan, op.cit.

[28]Lelon R. Capps, "Division of Fractions", The Arithmetic Teacher, vol. 9, no.1, January, 1962, pp. 10-16.

Milgram showed that in forty-six intermediate classrooms in Pennsylvania, three quarters of class time was being spent on drill or practice.[29]

Proposals of the Cambridge Conference on School Mathematics

In 1963, the Cambridge Conference on School Mathematics recommended a step-up of content to gain three years over the period from kindergarten to the end of secondary school; it was proposed that this be accomplished by "a new organization of subject matter . . . replacing the unmotivated drill of classical arithmetic by problems which illustrate new mathematical concepts."[30]

This aspect of replacing repetitive practice with use of skills in a new situation does not seem to have been specifically examined to any great extent in North America. At Nova School, in Dade Park, Florida, an attempt is being made to put the overall curriculum proposals of the Cambridge conference into practice for bright pupils. Professor Fitzgerald reported on this in 1965;[31] at that time the

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[29]Joel Milgram, "Time Utilization in Arithmetic Teaching", The Arithmetic Teacher, vol.16, no.3, March, 1969.

[30]Cambridge Conference on School Mathematics, Goals for School Mathematics, (Boston: Houghton Mifflin Company, 1963) p.7.

[31]See report of discussion following Professor Willoughby's report, "Curriculum Experimentation", in Howard A. Fehr, Needed Research in Mathematics Education, A summary report of a conference held at Greystone, New York, Oct. 30 and 31, 1965, (New York: Teachers' College Press, Teachers' College, Columbia University, 1965).

experience was exposing problems needing further research, among them problems concerned with concept formation, non-verbal instruction, individualized instruction and diagnosis.

At the next meeting, the Cambridge Conference on Teacher Training, in 1966, it was reported that:

> Experimental work of an even more drastic nature than the '<u>Goals</u>' proposals already has been started in the United States, and the mathematicians of at least one European country, Denmark, have embarked on formal development of a school mathematics program that seems to parallel closely much of the '<u>Goals</u>' outline.[32]

At the time of this writing, there are not yet any published results of these endeavours, in journals directed towards mathematics educators.

<u>The Role of Practice in Primary Schools</u>.

For an account of results of this type of approach with primary aged children, we turn to Miss Edith Biggs. With James R. Maclean, in <u>Freedom to Learn</u>, she said:

> If we can use the natural experiences of children to develop the basic ideas of mathematics in the field of number, measurement and shape, while maintaining reasonable computational skills, we will avoid much of the criticism being directed at many of the 'modern mathematics' programs.[33]

And again, under the topic Meaningful Practice:

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[32]Cambridge Conference, op.cit., p. 7.

[33]Edith E. Biggs and James R. MacLean, <u>Freedom to Learn</u>, (Toronto, Canada: Addison-Wesley, 1969), p. 4.

> The practice of computational skills is based upon the practical activities in which the children are engaged. In this way the pupils understand and appreciate the need for polishing and expanding these skills.[34]

The basis for these statements is found in the British publication Mathematics in Primary Schools, prepared by Miss E. E. Biggs,[35] and in the UNESCO report of L. G. W. Sealey.[36]

The latter describes the "upsurge of reform" which took into account "newer knowledge of child development and the on-going work in cognitive studies".[37]

Miss E. E. Biggs, H.M.I., and her colleagues led "the first of the organized national efforts to improve early mathematics learning".[38] Mathematics in Primary Schools, although definitive in its 'Position', could be called an interim report. In the chapter "Research in children's method of learning"[39] Miss Biggs reviews the research of

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[34] Ibid., p. 13.

[35] The Schools Council, Mathematics in Primary Schools, Curriculum Bulletin No. 1, Second edition, (London: Her Majesty's Stationery Office), 1967.

[36] L. G. W. Sealey, "An Outline of Curricular Changes in Great Britain", in J. D. Williams, Mathematics Reform in the Primary School (Hamburg: UNESCO Institute for Education, 1967), pp. 106-114.

[37] Ibid., p. 106.

[38] Ibid., p. 107.

[39] The Schools Council, op.cit., pp. 5-9.

Piaget, Dienes, and others, and summarizes in part:

> 4) Practice is necessary to fix a concept once it has been understood, therefore practice should follow, and not precede discovery.[40]

Specific research work is not quoted for this principle, probably because of its place in Piaget's scheme, but further on she says:

> Those teachers who have already organized the work so that the children learn by their own efforts all bear testimony to the decrease in the amount of computational practice children require in order to attain and maintain computational efficiency. Many teachers have found that they can safely reduce this practice to two periods (and some to one) each week and that the children's efficiency in computation has improved and not declined in consequence.[41]

One direct outcome of this reform movement in Great Britain is the Nuffield Mathematics Project which is developing a new primary school mathematics curriculum on a national scale. The Project is being developed in close collaboration with L'Institut des Sciences de l'Education in Geneva.[42] The latter has provided a team which is developing "Individual Check-ups" to replace individual tests.

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[40] Ibid., p. 9.

[41] Ibid., p. 41.

[42] Nuffield Mathematics Project, Pictorial Representation [1] (London: W. & R. Chambers and John Murray, 1967, available from Longmans Canada Ltd., Toronto), p. 1.

THE PLACE OF GRAPHICAL REPRESENTATION

The importance of early introduction to concepts of graphing is being increasingly recognized in many countries and in any curriculum movement that is working for integration of science and mathematics programs. The film "Maths Alive" from the United Kingdom showed this vividly.[43] Mme. Picard, in "Curricular Change in the French Primary Schools", referred to the use of "representations", that is: sketches, diagrams, tables and graphs.[44] This use is based on Piaget's studies and on experiments in classes in France; it provides for the child a concrete representation of an abstract mental operation, a representation which "precedes symbolization and facilitates it". Furthermore, Mme. Picard says:

> Another purely educational but not unimportant advantage is that young children have great difficulty in expressing their thought in words but experience real satisfaction in being able to express themselves by means of graphs, diagrams, or schemata; often, after inspecting a representation which they have made, they find it possible to express their thoughts verbally.[45]

In the United States, John R. Mayor has recently reiterated proposals for the integration of science and

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[43]"Maths Alive", Film, coloured, 30 min. by British Petroleum Company, obtainable Canada: Learning Materials Service Unit, Dept. of Education, 559 Jervis St., Toronto 5, Ontario.

[44]Nicole Picard, "Curricular Change in the French Primary Schools", in J. D. Williams ed., Mathematics Reform in the Primary Schools, (Hamburg: UNESCO Institute for Education, 1967), p. 75-76.

[45]Op.cit., p. 76.

mathematics.[46] One of the two first steps in this direction would be the early introduction of graphing. He mentions the MINNEMAST program in Minnesota as one example where this is already being done.

From a local and practical point of view it was thought that the introduction of such a relevant and 'open-ended' topic would give interested practising teachers and their students a new tool for their mathematics and science studies in the coming year.

CONCLUSION

Since the proposal by Glennon and Buswell in 1951,[47] a number of studies have been reported at the intermediate level, designed to determine the optimum division of arithmetic time between practice and concept development. The conclusion is unanimous that classroom time spent on practice can be greatly reduced. No controlled experimental research studies appear to have been reported at the primary level. Miss Biggs states that teachers in primary schools in England have been able to considerably reduce practice.[48]

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[46] John R. Mayor, "Science and Mathematics: 1970's -- a Decade of Change", The Arithmetic Teacher, vol. 17, no. 4, April, 1970.

[47] Glennon and Buswell, op. cit., p. 291.

[48] The Schools Council, op. cit., p. 41.

The present study attempts to incorporate sufficient practice for the maintenance of certain competencies into the learning of new material (Graphical Representation), as suggested by the Cambridge Conference on School Mathematics in 1963,[49] and by Glennon and Callahan in 1967,[50] while keeping in mind Miss Biggs' remarks and the above mentioned results with intermediate children.

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[49]Cambridge Conference on School Mathematics, op. cit., p. 7.

[50]Glennon and Callahan, op. cit., p. 81.

CHAPTER III

PROCEDURES

INTRODUCTION

This study attempts to compare the maintenance of arithmetic competencies of groups of children, previously taught by a common method, who spent a period of time in one of the following three ways:

a) an introductory unit on graphical representation

b) traditional year-end review and practice

c) a geometry unit involving no arithmetic skills

Two important areas of the British Columbia curriculum were chosen as 'basic skills' to be evaluated.

I. Numeration: ability to recognize, read and write numerals for numbers $\leqslant 99$ with understanding of base ten numeration.

II. Computation: addition and subtraction operations in equation form with sums and minuends $\leqslant 10$.

The investigator taught all groups, to reduce Hawthorne effects.

MATERIALS USED

a) An introduction to graphs was chosen as the new material to be introduced using as teacher's guide Pictorial Representation [1],[1] from the series of teacher's guides for the Nuffield Mathematics Project. Examination of this booklet indicated that the material could be used, with careful planning, in place of the regular year-end review of addition and subtraction operations and facts, and of base ten numeration of numbers <100, including concepts of greater than and less than, all of which are stressed in the present curriculum in British Columbia. See Appendix A for detailed plan of the unit.

b) As far as possible, methods and materials were those to which the children were accustomed. See Appendix A for detailed plan of the unit.

c) An open-ended geometry unit was planned, using Elementary School Mathematics, Books I and II,[2] and

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[1]Nuffield Mathematics Project, Pictorial Representation [1], (London: W. & R. Chambers and John Murray, 1967, available from Longmans Canada Ltd., Don Mills, Ontario).

[2]Robert E. Eicholz, et al, Elementary School Mathematics Second Edition, Books I and II, (Don Mills, Ontario: Addison-Wesley Canada Ltd., 1969).

tesselating as described in the Nuffield Project booklet, Shape and Size, ▽2▽.[3] See Appendix A for detailed plan of unit.

ORGANIZATION

Two schools were available, the first with 54 children, and the second with 34 children. In the first school, designated School #1, the children had been in a single large class under a team teaching situation, thus providing the common method, up to the time of the investigation. All three methods were used in this school, the children being chosen at random to form the Groups A_1, B_1, and C_1, corresponding to the three treatments, with 18 members to a group.

In the second school, designated School #2, two groups, A_2 and B_2, with 17 members each, were set up, using random selection; this provided for comparison between the two experimental treatments.

Three weeks of teaching were planned, but, in School #1, three days were lost to other school activities and pre-testing took an extra day. This left only eleven days, and caused considerable loss of continuity. Planned teaching periods of twenty-five minutes were seldom that long,

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[3]Nuffield Mathematics Project, Shape and Size, ▽2▽ Nuffield Mathematics Project, (London: W. & R. Chambers and John Murray, 1967, available from Longmans (Canada) Ltd., Toronto).

owing to administrative procedures, and physical arrangements involved in class change-overs. Group A most frequently received its full time allotment, the other two groups completing their time with assigned seat work under the supervision of the two regular teachers.

TABLE I

GROUPING OF STUDENTS AT START

OF EXPERIMENTAL PERIOD

	School #1			School #2	
Group	A_1	B_1	C_1	A_2	B_2
No. students	18	18	18	17	17
No. boys	9	9	9	10	10
No. girls	9	9	9	7	7

In school #2, the full three weeks teaching session was utilized, each group receiving approximately the twenty-five minutes planned for them each day. The lesson for Group A_2 was alternated daily between first and second session.

EVALUATION INSTRUMENTS

Basic Competencies, Test I and II

New instruments were developed and subsequently

referred to as Test I and Test II, to test competency in Numeration and Computation respectively (marked Part I and Part II respectively in the children's booklets).

Parallel power tests, Forms A and B, were constructed to include extension of topics to items not normally encountered by the children until their second or third year. It was thought that this would give a better picture of the achievement level of individual pupils and of the class as a whole.

Reading matter was restricted to:

a) words and symbols for equality and inequality

b) words and symbols for tens and ones

c) symbols for addition and subtraction.

The first of these required three versions, symbolic: $<$, $>$, $=$, the printed word, and the printed word in i.t.a. script.

For details of test construction, see Test Manual, Appendix C. The tests will be found in Appendices D and E.

Graphical Representation, Test III

Achievement in "Graph Reading" or obtaining information from a block chart was the special skill chosen for testing. The unit being taught to Groups A_1 and A_2 was based on the Teachers' Guide: Pictorial Representation [1], from the series developed by the Nuffield Mathematics

Project in Great Britain.[4] 'Check-up Guides' were not yet commercially available for this topic; no answer was received from the head of the project to the letter requesting help in this area. Hence tests were improvised, using a chart-sized copy of "Block Chart of Pets in Class I" adapted from Pictorial Representation [1].[5] (Appendix F). A pre-test, consisting of eight oral questions (see Appendix G), was administered to the children in School #2, to a third of the class at a time. The children in this group printed their answers on lined newsprint. This pre-testing was not carried out in School #1, owing to limitations of time and of class organization. Informal oral testing carried out in groups revealed that ability to read a block chart was almost non-existent. For example, although the children knew that there were more dogs than cats, no one in any group challenged the general concensus that there were 10 dogs and 5 cats, rather than 14 and 7, respectively.

The same chart was used for the post-test with additional questions added, making fourteen questions in all. The format of Test III as presented in School #1 was unfortunate, psychologically. The questions, which

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[4] Op. cit.

[5] Ibid., p. 18.

were to be read out by the examiner, were printed on the page. This proved rather overpowering for poorer readers and for those who had not yet made the transition from i.t.a. script to standard orthography. Consequently the children had rather more trouble than expected finding the correct place on the paper. "This is the hardest test I have ever written!" was one comment. The child was thanked for this information and assured that no other child would ever have to write it (see Appendix H). A new format was devised for School #2, with a small copy of the chart appearing on the paper, as well as little pictures, and appropriate spaces to complete the answers to the oral questions (see Appendix I). The test attempted to assess the following skills:

1. Ability to obtain information directly from the chart.
2. Ability to construct number sentences and equations expressing the above information and showing how it was obtained.
3. Ability to construct other number sentences and equations suggested by the data set out on the chart.

PRE-TESTING

Form B of Tests I and II was used as pre-test of

basic competencies. (See Appendix D)

School #1

The investigator tested the children in small groups. A group of slower children was taken first, followed by the rest of the boys and then the rest of the girls. These children had never been tested in any way before, so difficulty was experienced in obtaining independent responses. More time was needed than anticipated. Three full sessions and part of the first teaching day were necessary.

The individual oral testing was accomplished in two afternoons; the examiner was in a corner of the classroom where the remainder could not hear, and while the class proceeded with its regular lessons, each child tested would quietly send the next one for his or her turn.

The Graph Reading test (Test III) was presented as a short oral discussion session at the beginning of the first teaching period for each group.

School #2

Tests I and II, Form B, were administered to the whole class at once, with the assistance of the teacher to assure that all the children understood instructions, were placing responses in correct places and were answering all questions that they were able to. The tests were completed in two and one half sessions. Difficulty in obtaining independent responses was experienced in a few cases, due to seating

arrangements.

Test III was administered to a third of the class at a time.

The individual oral testing was completed in one afternoon, with the exception of a few absentees who were tested later in the week. Use of the foyer assured privacy and independent responses.

TEACHING ACTIVITIES

Graphing unit as carried out in School #2

The activities of Group A_2 will be described first as this group accomplished more than Group A_1.

1. Comparison of Number of Boys and Girls in Group

a) One block per child, blocks were lined up and comparisons made. The symbols $>$ and $<$ were introduced and used to print pertinent number sentences on the chalkboard. Sizes of families were compared. An attempt to develop facts of nine was not too successful, the children not being ready to relate arithmetic skills to actual situations.

b) Paper squares, one per child, pink for girls and blue for boys, were pasted up in two rows on blank chart paper. A chart story was elicited with word and number sentences. Working with partners, the children then wrote their own number stories and word sentences

about the chart.

2. Birthday Chart

Names of months were entered on chart paper, providing twelve horizontal rows. A starting line was drawn and each child pasted up his self portrait, all portraits being drawn on the same sized paper. With partners, the children wrote their own word and number sentences.

The next day, portraits from Group B_2, drawn during an art period, were added to the chart. This gave larger numbers and a different pattern of distribution for Group A_2 to work with. The stories now written by the children showed a developing ability to obtain information from their chart.

3. Pets

a) Picture Charts. A chart was prepared listing the species of pets the children actually had at home. This chart had no standard size boxes or starting line. Outline drawings were provided, which the children coloured, cut out and pasted up. They observed that the line of dogs was longer than that of the cats but there were more cats. The ensuing discussion resulted in an unpasting and re-arranging to obtain a one-to-one correspondence. Then it was observed that 'tadpoles' was such a long word that the first bowl was under the fourth dog. At the next session various suggestions were tried. By this time, the other

group had added their pets, without discussion, and this further confounded things. However, the children themselves suggested, at this point, that they start again with a new chart.

The number of dogs and cats each being > 10, incidental review of tens and ones, and of counting by fives and tens was carried out, chiefly by an elicitation method. With coloured lines for the fives and tens, the children had a quick way of determining the exact number of any particular kind of animal.

b) Vertical Block Graph. A double length chart was prepared as graph paper with one inch squares; the zero line, the fives and tens lines were distinctively marked. To provide an activity involving larger numbers, pets were grouped according to the number of legs each had. Coloured squares were provided for the children to paste up. This gave numbers in the forties and fifties and offered much opportunity for variety of equations and inequalities, and for application of knowledge of tens and ones.

c) New Pet Chart. The children pinned rather than pasted up their pictures, so that they could more easily make desired alterations in position.

d) Contamination. Without prior consultation with the investigator, the teacher assigned at least two work-

sheets of computational practice to this group.

Graphing Unit as Carried out in School #1

1. Comparison of Number of Boys and Girls in Group

a) One counter per child, counters were lined up and comparisons made. The symbols $>$ and $<$ were introduced and appropriate number sentences elicited and printed on the chalkboard. Working in pairs, and using large round counters or one inch linoleum tiles, the children compared the sizes of their families.

b) Self-portraits were drawn on common-sized pieces of rectangular paper, and pasted up on chart paper. Individual writtern work was started using a worksheet for guidance and the chart for information. The children seemed quite at home combining word and number sentences.

c) Paper squares, pink and blue, were used for a new chart, following the permanent departure of one member of the group. Working in pairs, the children also produced their own smaller charts. Questions guided their discovery of the importance of lining up the squares to show a one-to-one correspondence. One child proposed and printed a question for the others to answer.

2. Birthday Chart

a) Self-portraits were drawn and pasted up as described for School #2. Stories about the results were written and discussed. The chart clearly showed a less

mature group, chronologically, than in the other school. This fact, coupled with the lack of continuity of teaching days, could account for the impression that the majority of children gained little from the discussions. After the children from Group B_1 had drawn self-portraits for seat work, and added them to the chart, further work was done by Group A_1 with the larger numbers thus provided.

b) Graph paper. Individual birthday charts were prepared, and the children coloured a square opposite the appropriate month for each member of the class. Stories were written in the space provided. Time did not permit adequate assessment with each child of the originality or comprehension of this written work. During the colouring, it became apparent that the children did not know the relationship between a written numeral and tens and ones. The birthday chart was used as a starting point, but it was necessary to use concrete objects, and reteach this topic.

c) Vertical Block Graph. A long chart was prepared, similar to that used for Pets in School #2. Coloured squares were pasted up, vertically, in two columns, comparing the number of birthdays from January to July with those from August to December. The tens lines were marked for easier counting, but, if time had permitted, another week could have been spent exploring and consolidating.

Review and Practice Unit as Carried Out in School #1

Considerable re-teaching was involved here, as work with the children soon substantiated the evidence of the pre-tests that very few of them were competent in the basics assessed in Tests I and II. See Table II.

TABLE II

SCHOOL #1 ACHIEVEMENT AND 'MASTERY' SCORES

PRE-TEST

Test Topic	Maximum Score	Grade One 'Mastery' Score	Class Average Score	Number Attaining 'Mastery'
I Numeration	42	35	24.2	3
II Addition-subtraction	40	32	24.6	3

Number of pupils tested = 56.

a) The addition and subtraction facts of 7, 8, 9, 10 were reviewed, using both equations and vertical form, but no equations with more than two addends. The investigator's method involved somewhat more use of concrete objects than the demonstration method to which the children were accustomed.

b) Considerable time was devoted to numeration, especially to discovery and teaching of the relationship between the printed numeral and the number it represents.

Concrete objects were used. Then some work was done circling pictured objects into groups of tens and units.

Organizing for and assigning the seat work, as requested by the class teachers, cut down the time available for developmental work. However, this meant that the children carried out their usual practice activities, with counters available and their own teachers to supervise.

Review and Practice Unit as Carried Out in School #2

a) Inequality signs were introduced.

b) Addition, subtraction and multiplication facts of 9 and 10 were developed with the use of blocks to which the children were accustomed. Review of earlier facts was incorporated into the activity lessons and daily worksheets. These included the use of parentheses.

c) Balancing equations, e.g. 5+3=2+__ provided for the more able, but was not pushed with the others.

d) Numeration review was carried through with a familiar type of worksheet, with the exception of one recent transferee who required work with objects to count and group.

e) Rote counting by ones, fives and tens, and counting of pictured money were reviewed.

f) Adding and subtracting of one from any number less than one hundred was practiced briefly.

Geometry Unit as Carried Out in School #1

From the plan shown in Appendix A, the following topics were developed:

a) Familiarity with two-dimensional shapes.

b) Creating and extending patterns.

c) Background for later work with congruence.

d) Review of open and closed curves and location of points in relation thereto.

e) Geometric shapes from a collection of points.

CONFOUNDING FACTORS

Computational Practice

Group A_2 (Graphical Representation) were assigned at least two worksheets for computational practice. This was done by the teacher without consulting the investigator.

Exposure to Graphing Unit

In School #1, all groups were exposed to the results of the work of the Graphical Representation Group. However, this was on display in a partitioned Arithmetic Corner, and the other groups were kept occupied with their lessons when there. Group B_1 (Computation) also added their portraits to the birthday chart, but this was done without comment or discussion.

In School #2, only the group being taught was in the classroom during a lesson. However, in the self-contained

classroom, the other group was exposed to the results of the activities for the rest of the day. This could have had an effect on alert yougsters, regarding their knowledge of Graphical Representation. Group B_2 also contributed, without comment or discussion, to the birthday chart, and also to the pet chart.

POST-TESTING

Form A of Tests I and II were used as post-test of basic competencies. (See Appendix E)

School #1

The individual oral testing was smoothly accomplished with the aid of one of the teachers and a duplicate set of flash cards, following the last teaching session.

Several unfortunate aspects marred the atmosphere for the pencil and paper testing which commenced on the final Monday of the school year. Academic activity had virtually ceased through the school and families began premature withdrawal of their children. Group A_1, which had lost one member at the end of the first week, lost three more. Arrangements had been made to administer the tests to half the class at a time, with the assistance of one teacher, as described on page 32. The other teacher was to keep the other children occupied. Unfortunately, she took them outside to play, creating considerable diversion

of attention among those remaining. On the second day, one child in Group A_1 printed S T O P across his set of addition and subtraction equations, and stopped. This was not discovered until it was too late to rectify, and his results for Test II had to be discarded. Test III was the last to be administered, after recess on Tuesday morning. Perhaps because of the distress due to the poor format discussed earlier, the teachers told the children that it would not count on their report cards. As other activities were planned for the afternoon and the remaining days, the children were aware that this was the last work that would be expected of them for the year.

School #2

The pencil and paper post-testing was commenced during the regular arithmetic time on the Monday, and completed during the long pre-recess period on the Wednesday. The whole class was tested at one time, with the teacher assisting as previously indicated. Desks were arranged to encourage independent work.

The individual oral testing ran smoothly in the foyer, during Monday afternoon. The brighter youngsters had to be reassured that there was no catch to it, when they were presented with the task of reading numerals and completing equations.

CHAPTER IV

STATISTICAL ANALYSIS OF RESULTS

CHOICE OF ANALYSIS OF COVARIANCE

The initial score for each test was a measure of skills achieved through a combination of ability and of the effect of eight to nine months learning, whereas the final score measured the additional effect of three and a half weeks of instruction, including the learning which took place during the writing of the pre-test. Hence it was reasonable to expect that the initial scores which marked the experimental starting point would show a high positive correlation with the final scores. Analysis of covariance tests on each pair of pre-test and post-test means would determine whether such a relationship did indeed exist, and, if so, would provide the necessary data for removing the effects of the pre-test score by calculation of adjusted means. The unequal cells at the end of the experiment precluded the use of a nested design, analysis of variance, on the adjusted means. (See Table III for n's in each group). Hence, orthogonal contrasts between adjusted means using t-tests were planned.

TABLE III

GROUPING OF STUDENTS

AT END OF EXPERIMENTAL PERIOD

	Group	A_1	B_1	C_1	A_2	B_2
TEST I	No. of students	14	17	15	16	17
	No. boys	8	8	8	10	10
	No. girls	6	9	7	6	7
TEST II	No. of students	13	17	18	16	17
	No. boys	7	8	9	10	10
	No. girls	6	9	9	6	7
TEST III	No. of students	13	17	18	14	16
	No. boys	7	8	9	9	10
	No. girls	6	9	9	5	6

Analysis of covariance can be used when there exists a linear 'regression' relationship between initial and final scores and when there is a common regression line for all groups (populations). The BMDX82 computer program for one-way analysis of covariance from the Department of Bio-Mathematics, School of Medicine, University of California at Los Angeles, was made available through the kindness of Dr. S. S. Lee and was used on the IBM 360, Model 67 computer at the University of British Columbia. This statistical program provides tests for slope existence, commonality of regression lines and equality of means after adjustment to remove effects of initial differences. It also provides a t-test matrix for adjusted group means and t-tests for any desired contrasts. A short Fortran program was used to determine the exact probabilities of the F values corresponding to the t's. For the one-way analysis of variance, and contrasts, required to analyse the data for Graphical Representation in School #1, a companion program, BMDX64, was used. This gave the F values for the desired orthogonal contrasts between means.

Assumptions

Underlying the above analyses are the assumptions of normal distribution of the populations sampled and of homogeneity of variance. These are reasonable assumptions since the whole population of the first year was involved for each

school; the smaller group was over thirty and the subgroups within schools were randomly selected. Furthermore, as pointed out by Hays, while an F-test for equality of variance is quite sensitive for non-normality, this apparently makes little difference in tests concerning means.[1]

MAIN HYPOTHESES

Test I Numeration

The hypothesis matrix shown in Table IV provides tests for the following null hypotheses, all concerning the adjusted means of the post-test scores.

TABLE IV

HYPOTHESIS MATRIX

A_1	B_1	C_1	A_2	B_2
1	-1	0	0	0
0	0	0	1	-1
1	1	0	-1	-1
-1	-1	4	-1	-1

H(1,1): There will be no significant difference between the mean score for Test I of Groups A_1 and B_1.

[1]Hays, William L., Statistics for Psychologists (New York: Holt, Rinehart and Winston, 1966), p. 352.

H(1,2): There will be no significant difference between the mean scores for Test I of Groups A_2 and B_2.

H(1,3): There will be no significant difference between the mean scores for Test I of the treatment groups of School #1 and of School #2, i.e. A_1+B_1 will not differ significantly from A_2+B_2.

H(1,4): There will be no significant difference between the mean scores for Test I of all treatment groups and the mean score of the control group.

Test II Computation

Using the same hypothesis matrix as shown in Table IV, the null hypotheses were as follows:

H(2,1): There will be no significant difference between the mean scores for Test II of Groups A_1 and B_1.

H(2,2): There will be no significant difference between the mean scores for Test II of Groups A_2 and B_2.

H(2,3): There will be no significant difference between the mean scores for Test II of the treatment groups of School #1 and of School #2.

H(2,4): There will be no significant difference between the mean scores for Test II of all treatment groups and the mean score of the control group.

Test III Graphical Representation

Statistical comparison between the schools was not

made for Test III. They would have been meaningless owing to the previously noted differences in teaching time, content covered and test format.

School #1

There were no pre-test scores for School #1. Hence orthogonal contrasts between post-test means were planned, using the Hypothesis Matrix shown in Table V.

TABLE V

HYPOTHESIS MATRIX SCHOOL #1 TEST III

A_1	B_1	C_1
1	1	-2
1	-1	0

The null hypotheses were as follows:

H(3,1): There will be no significant difference between the mean scores for Test III of the two treatment groups and that of the control group.

H(3,2): There will be no significant difference between the mean scores for Test III of Groups A_1 and B_1.

School #2

Both pre-test and post-test scores were available for School #2. There was also reason to believe there was a sufficient difference in mean ability between the two groups to make advisable the use of analysis of covariance for this

data.

The null hypothesis was as follows:

H(3,3): There will be no significant difference between the mean scores for Test III of Groups A_2 and B_2.

TABLE VI

SUMMARY OF TEST RESULTS: NUMERATION

Group	A_1	B_1	C_1	A_2	B_2
Maximum possible score	42	42	42	42	42
No. cases per group	14	17	15	16	17
Pre-test:					
Max. observed	38	35	39	35	37
Min. observed	15	19	8	21	20
Mean	24.57	25.76	24.0	30.938	29.471
Standard deviation	6.653	4.790	7.910	3.714	4.361
Post-test:					
Max. observed	38	40	40	40	40
Min. observed	14	22	16	30	20
Mean	28.643	30.588	27.60	35.937	33.176
S. D.	5.719	4.515	6.685	3.193	4.733
Adjusted mean	30.286	31.445	29.621	33.382	31.588
Std. error	0.942	0.845	0.920	0.909	0.857

ANALYSIS OF DATA

Main Hypotheses Regarding Numeration Skills

For convenience of reference Table VI summarizes the results of the tests administered. All groups showed improvement from pre-test to post-test, hence it is a matter of examining relative improvement between groups. Table VII shows the results of testing for use of analysis of covariance.

TABLE VII

TESTS FOR USE OF ANALYSIS OF COVARIANCE

TEST I : NUMERATION

Source of Variance	D.F.	Sum of Sq.	Mean Sq.	F-Value	Prob.
Equality of adj. cell means	4	106.361	26.590	2.219	0.074
Zero slope	1	1013.687	1013.687	84.605	0.000
Error	73	874.646	11.981		
Equality of slopes	4	35.052	8.763	0.720	0.584
Error	69	839.594	12.168		

An examination of this table shows that the probability of no regression slope is virtually zero and the probability of equal slopes for the different groups is

greater than 50%. Hence the use of analysis of covariance is justified. The probability of 7.4% that the adjusted cell means are equal indicates a 'borderline' probability of finding, among the contrasts, any important differences due to the different treatments.

Table VIII shows the probabilities that the null hypotheses, H(1,1) to H(1,4), are true.

TABLE VIII

TESTS OF MAIN HYPOTHESES

TEST I: NUMERATION

Hypothesis	t-value	D.F.	$t^2=F$	Probability
H(1,1)	-0.925	73	0.8556	0.361
H(1,2)	1.4818	73	2.1957	0.139
H(1,3)	-1.7225	73	2.9671	0.085
H(1,4)	-1.9986	73	3.9944	0.047

Conclusions : Numeration

School #1. H(1,1): The negative t-value indicates that the 'practice' group, B_1, did better than the experimental group, A_1, but there is a 36% probability that this could occur by chance, hence H(1,1) is accepted.

School #2. H(1,2): Experimental group A_2 did better than 'practice' group B_2 but with a probability as

large as 14% this can merely be considered as indicative of a possible trend towards higher achievement in numeration of the graphical representation students.

Across schools. H(1,3): The treatment groups of School #2, A_2+B_2, did better than those of School #1, A_1+B_1, with only 8% probability, suggesting a trend which corroborates the subjective judgment of the investigator, that the mean achievement level of School #2 was higher than that of School #1.

H(1,4): The control group did not gain as much as the experimental groups, the probability of this difference having occurred by chance being less than 5%. A re-test, after an opportunity for review and reteaching, would have helped to determine whether this apparently lower gain was significant.

Main Hypotheses Regarding Computational Skills

The results of the tests for computational skills are summarized in Table IX. All groups except A_1 showed improvement on the post-test. Table X gives the results of the tests for the used of Analysis of Covariance. The probability of no regression slope is virtually zero. The 3.4% probability that the slopes are the same is a borderline case but was considered sufficient to justify

TABLE IX

SUMMARY OF TEST RESULTS

TEST II : COMPUTATION

Group	A_1	B_1	C_1	A_2	B_2
Max. possible score	40	40	40	40	40
No. cases per group	13	17	18	16	17
Pre-test:					
Max. observed	34	31	32	40	32
Min. observed	17	15	12	20	19
Mean	26.154	25.471	23.944	29.500	27.29
S. D.	4.562	4.200	4.976	4.803	3.584
Post-test:					
Max. observed	36	33	34	40	40
Min. observed	17	20	12	28	20
Mean	24.462	26.412	25.167	33.750	31.765
S.,D.	5.027	3.607	5.555	3.531	5.333
Adjusted mean	24.646	27.069	26.881	31.616	31.159
Std. error	0.991	0.870	0.872	0.936	0.870

TABLE X

TESTS FOR USE OF ANALYSIS OF COVARIANCE

TEST II : COMPUTATION

Source of Variance	D.F.	Sum of Sq.	Mean Sq.	F-Value	Prob.
Equality of adj. cell means	4	499.076	124.769	9.788	0.000
Zero slope	1	721.893	721.893	56.634	0.000
Error	75	956.004	12.747		
Equality of slopes	4	128.495	32.124	2.756	0.034

TABLE XI

TESTS OF MAIN HYPOTHESES

TEST II : COMPUTATION

Hypothesis	t-value	D.F.	t^2=F	Probability
H(2,1)	-1.8403	75	3.3867	0.066
H(2,2)	0.3629	75	0.1317	0.717
H(2,3)	-5.9092	75	34.9186	0.000
H(2,4)	-1.7433	75	3.0392	0.082

the use of the 'common slope' factor in the use of analysis of covariance and calculation of the adjusted means. The probability of 0.000 that the adjusted cell means are the same indicates there is some difference of note, either between schools or between treatments.

Table XI shows the probabilities that the null hypotheses, H(2,1) to H(2,4) are true.

Conclusions : Computation

H(2,1): The negative t-value confirms that the 'practice' group B_1 did better than the experimental group A_1 with a probability of 6.6% of this happening by chance. This trend would correspond to expectations in the wake of the actual teaching that was carried out with group B_1.

H(2,2): With the effect of initial differences being removed by analysis of covariance, there is nothing significant about the difference betweeen the results for groups A_2 and B_2, hence H(2,2) is accepted.

H(2,3): The students in School #2 showed a significantly higher achievement than the combined experimental groups of School #1, $A_1 + B_1$; hence H(2,3) is rejected.

H(2,4): All the experimental groups combined showed a trend toward greater improvement than the control group, the probability being 8.2%. The greater part of this difference is attributable to the superior performance of the pupils of School #2 as can be seen by an examination of the

t-test matrix for adjusted group means, shown in Table XII.

TABLE XII

t-TEST MATRIX FOR ADJUSTED GROUP MEANS

TEST II : COMPUTATION

Group	A_1	B_1	C_1	A_2	B_2
A_1	0.0				
B_1	-1.840	0.0			
C_1	-1.70	0.155	0.0		
A_2	-5.095	-3.504	-3.563	0.0	
B_2	-4.936	-3.309	-3.433	0.363	0.0

TABLE XIII

SUMMARY OF TEST RESULTS

TEST III : GRAPHICAL REPRESENTATION

Group	A_1	B_1	C_1	A_2	B_2
No. cases per group	13	17	18	14	16
Pre-test:					
Max. poss. score				9	9
Max. observed				7	7
Min. observed				0	0
Mean				3.643	2.625
S.D.				2.437	2.419
Post-test:					
Max. poss. score	16	16	16	16	16
Max. observed	9	8	5	13	13
Min. observed	1	1	0	5	1
Mean	4.539	4.059	2.333	9.425	7.250
S.D.				2.564	3.550
Adjusted mean				8.930	7.687
Std. error				0.605	0.565

Main Hypotheses Regarding Graphical Representations

For reasons indicated in the section setting out the main hypotheses (pages 47-48), hypothesis testing for graphical representation was carried out separately for the two schools. The test results for both schools are summarized in Table XIII.

TABLE XIV

ANALYSIS OF VARIANCE SCHOOL #1

TEST III : GRAPHICAL REPRESENTATION

Source of Variance	D.F.	Sum of Sq.	Mean Sq.	F-Value	Prob.
Mean	1	624.553	624.553	173.303	0.000
Overall treatment	2	43.745	21.872	6.069	0.005
Contrasts:					
H(3,1)	1	43.160	43.160	11.976	0.001
H(3,2)	1	1.695	1.695	0.470	0.503
Error	45	162.172	3.604		

Conclusions - School #1

Table XIV shows the F-Values and the probabilities for the hypotheses proposed.

H(3,1): The combined experimental groups of School #1 showed a significantly higher achievement than the control group, the probability of this happening by chance being

0.5%; H(3,1) is rejected.

H(3,2): There was no significant difference between mean scores for Test III of Groups A_1 and B_1; H(3,2) is accepted.

Conclusions - School #2

The results of the tests for the use of analysis of covariance are given in Table XV.

TABLE XV

TESTS FOR USE OF ANALYSIS OF COVARIANCE

TEST III : GRAPHICAL REPRESENTATION SCHOOL #2

Source of Variance	D.F.	Sum of Sq.	Mean Sq.	F-Value	Prob.
Equality of adj. cell means	1	11.018	11.018	2.2031	0.146
Zero slope	1	139.397	139.397	27.873	0.000
Error	27	135.031	5.001		
Equality of slopes	1	9.580	9.580	1.895	0.167
Error	26	125.452	4.825		

H(3,3): The probabilities being virtually zero for the zero slope test and 17% for equal slopes, the use of analysis of covariance would be justified but the probabi-

TABLE XVI

TABULATION OF SCORE GAINS

TEST I : NUMERATION SCHOOL #2

	A_2 High	A_2 Other	B_2 High	B_2 Other
	7	10	6	8
	5	8	6	6
	5	7	6	6
	5	7	5	5
	4	6	4	5
	4	5	1	5
	3	3		5
	3	-2		1
				0
				-1
				-3
Total	36	44	28	37
n	8	8	6	11
Mean	4.5	5.5	4.7	3.4

lity of 15% that the adjusted cell means are equal indicates that no statistical significance should be attached to the higher achievement of group A_2. The experiment had of course been contaminated by the participation of group B_2 in building the charts and in their being exposed to the results of the work of the others. H(3,3) is rejected. However, the observed trend indicates that a longer range study with better control and more adequate evaluation could be worthwhile.

SUPPLEMENTARY HYPOTHESES: ABILITY GROUPINGS SCHOOL #2

Reliable data on the general mathematical abilities of the students in School #2 were available from a teacher with long experience who had kept regular records of daily work and periodic testing. Hence plans were made for statistical testing of the effects of ability on progress in the three areas of Numeration, Computation and Graphical Representation. Originally, the children were classified roughly as 'high', 'above average', 'average', and 'below average'. For statistical purposes each treatment group was subdivided into 'high' and 'other'. Score gains on the numeration tests for these groupings are shown in Table XVI. It was hypothesized that the slower children of Group A_2 had made greater progress than those of Group B_2 or those of higher ability in Group A_2.

Numeration

Analysis of covariance was carried out on the test results for the four 'ability' groupings as shown in Table XVII. Orthogonal contrasts were planned according to Table XVIII, leading to Hypotheses H(4,1), H(4,2) and H(4,3). The hypothesis testing is shown in Table XIX.

TABLE XVII

TESTS FOR USE OF ANALYSIS OF COVARIANCE

ABILITY GROUPINGS SCHOOL #2

TEST I : NUMERATION

Source of Variance	D.F.	Sum of Sq.	Mean Sq.	F-Value	Prob.
Equality of adj. cell means	3	39.822	13.274	1.792	0.170
Zero slope	1	177.421	177.421	23.957	0.000
Error	28	207.364	7.406		
Equality of slopes	3	17.330	5.777	0.760	0.530
Error	25	190.034	7.601		

An examination of Table XVII shows that the probability of no regression slope is virtually zero and the probability of equal slopes for the different groups is greater than 50%. Hence the use of analysis of covariance is justified.

TABLE XVIII

HYPOTHESIS MATRIX : ABILITY GROUPINGS SCHOOL #2

A_2 High	A_2 Other	B_2 High	B_2 Other
1	1	1	-3
1	-1	0	0
1	1	-2	0

Supplementary Hypotheses

Using the hypothesis matrix shown in Table XVIII, the following null hypotheses were formulated:

H(4,1): There will be no significant difference between the combined mean scores of all children in Group A_2 and of the more able children in Group B_2 and the other children in Group B_2.

H(4,2): There will be no significant difference between the mean score of the more able children in Group A_2 and the other children in Group A_2.

H(4,3): There will be no significant difference between the mean score of all children in Group A_2 and that of the more able children in Group B_2.

TABLE XIX

TESTS OF SUPPLEMENTARY HYPOTHESES

TEST I : NUMERATION ABILITY GROUPINGS SCHOOL #2

Hypothesis	t-value	D.F.	t^2=F	Probability
H(4,1)	2.273	28	5.164	P=0.029
H(4,2)	0.447	28	0.1998	0.662
H(4,3)	0.281	28	0.079	0.772

Conclusions: Numeration, Ability Groupings

H(4,1): There was a significant difference between the combined mean scores of the more able children in Group B_2 and all children in Group A_2 as opposed to the other children in Group B_2. The probability of chance occurrence being 2.9%; H(4,1) is rejected.

H(4,2): There was no significant difference between the mean score of the more able children in Group A_2 and the other children in Group A_2; H(4,2) is accepted.

H(4,3): There was no significant difference between the mean score of all children in Group A_2 and that of the more able children in Group B_2; H(4,3) is accepted.

The above acceptances and rejection point to the conclusion that all children in Group A_2 made approximately equal progress and progress equal to the more able children in Group B_2, whereas the other children in Group B_2 made

TABLE XX

TABULATION OF SCORE GAINS

TEST II : COMPUTATION SCHOOL #2

	A_2 High	A_2 Other	B_2 High	B_2 Other
	12	8	9	9
	8	7	7	9
	4	5	6	8
	4	4	4	8
	4	4	3	4
	3	3	1	3
	2	1		3
	-3	0		2
				1
				0
				-3
Total	34	32	30	44
n	8	8	6	11
Mean	4.25	4.0	5.0	4.0

TABLE XXI

TESTS FOR USE OF ANALYSIS OF COVARIANCE

TEST II : COMPUTATION : ABILITY GROUPINGS SCHOOL #2

Source of	D.F.	Sum of Sq.	Mean Sq.	F-Value	Prob.
Equality of adj. cell means	3	88.532	29.511	2.651	0.062
Zero slope	1	68.264	68.264	6.132	0.019
Error	28	311.704	11.132		
Equality of slopes	3	26.737	8.912	0.781	0.518
Error	25	284.968	11.399		

TABLE XXII

TESTS OF SUPPLEMENTARY HYPOTHESES

TEST II : COMPUTATION : ABILITY GROUPINGS SCHOOL #2

Hypothesis	t-value	D.F.	$t^2=F$	Probability
H(5,1)	2.238	28	5.008	0.032
H(5,2)	2.039	28	4.156	0.049
H(5,3)	-0.784	28	0.614	0.446

inferior progress. This indicates strengthening of skills, hopefully due to new insights gained from application of skills to the task of obtaining information from a graph. Also it should be noted that individual children in Group A_2 made unexpectedly spectacular gains. These results would appear to merit further investigation with larger and less homogeneous groups.

Computation

A similar analysis using the same hypotheses, renumbered as H(5,1), H(5,2) and H(5,3) was carried out regarding computational progress in School #2. The gain scores are tabulated in Table XX and Table XXI summarizes the tests for analysis of covariance.

An examination of Table XXI shows that the probability of no regression slope is only 6.2% and the probability of equal slopes for the different groups is greater than 50%. Hence the use of analysis of covariance is justified.

Conclusions: Computation, Ability Groupings

H(5,1): There was a significant difference between the combined mean scores of all the children of high ability in Group B_2 and all children in Group A_2 as opposed to the other children in Group B_2, the probability of chance occurrence being 3.2%; H(5,1) is rejected.

H(5,2): There was a significant difference between

TABLE XXIII

TABULATION OF SCORE GAINS

TEST III : GRAPHICAL REPRESENTATION SCHOOL #2

	A_2 High	A_2 Other	B_2 High	B_2 Other
	10	9	6	9
	7	7	6	7
	7	7	5	7
	7	6	4	6
	5	4	3	5
	5	2	2	5
	5			4
	2			3
				3
				1
				1
Total	48	35	31	51
n	8	6	6	11
Mean	6.0	5.82	5.18	4.64

the mean score of the more able children in Group A_2 and the other children in Group A_2, the probability of chance occurrence being 4.9%; H(5,2) is rejected.

H(5,3): There was no significant difference between the mean score of all children in Group A_2 and that of the more able children in Group B_2; H(5,3) is accepted.

There is little significance in the acceptance of H(5,3) (as compared with H(4,3)) since the combined results of H(5,1) and H(5,2) are indicative of the higher achievement of the more able children in both groups. Also this experiment was contaminated by practice work given to A_2. Hence these results can in no way be considered conclusive.

Graphical Representation - Ability Groupings

It was suspected that the more able youngsters in Group B_2 would have made gains in skills of graphical representation by virtue of exposure to a greater extent than the other members of their group. The tabulation of score gains shown in Table XXIII shows them in the following rank order: A_2 High, A_2 Other, B_2 High, B_2 Other. However, the standard error of adjusted mean scores is approximately 1.0 and the difference between A_2 High and B_2 Other adjusted means is less than 2.0. (See Table XXIV). Hence we can expect no statistically significant results from this data. The statistical results of Analysis of Covariance and Hypothesis Testing similar to H(4,1), H(4,2), H(4,3) are

shown in Tables XXV and XXVI.

TABLE XXIV

ADJUSTED MEAN SCORES

TEST III : GRAPHICAL REPRESENTATION

ABILITY GROUPINGS SCHOOL #2

Group	No. of Pupils	Group Mean	Adj. Group Mean	Standard Error
A_2 High	8	11.125	9.497	1.1005
A_2 Other	6	7.167	8.319	1.027
B_2 High	6	9.500	7.838	1.116
B_2 Other	10	5.900	7.508	0.932

TABLE XXV

TESTS FOR USE OF ANALYSIS OF COVARIANCE

TEST III: GRAPHICAL REPRESENTATION

ABILITY GROUPINGS SCHOOL #2

Source of Variance	D.F.	Sum of Sq.	Mean Sq.	F-Value	Prob.
Equality of adj. cell means	3	13.993	4.664	0.883	0.465
Zero slope	1	40.052	40.052	7.582	0.010
Error	25	132.057	5.282		
Equality of slope	3	41.518	13.839	3.363	0.037
Error	22	90.539	4.115		

TABLE XXVI

TESTS OF SUPPLEMENTARY HYPOTHESES

TEST III : GRAPHICAL REPRESENTATION

ABILITY GROUPINGS SCHOOL #2

Hypothesis	t-value	D.F.	t^2=F	Probability
all A_2+B_2 High vs. B_2 Other	0.849	25	0.723	0.408
A_2 High vs. A_2 Other	0.864	25	0.747	0.400
all A_2 vs. B_2 High	0.736	25	0.543	0.474

SUMMARIZED RESULTS, CONCLUSIONS AND IMPLICATIONS

Numeration

Summary of statistical results. No difference of statistical significance was found between the two experimental groups in School #1.

The graphical representation group in School #2 showed a somewhat higher achievement than the drill and practice group. Statistically this can be considered a trend only, as probability of chance occurrence was 14%.

School #2 showed a trend toward higher achievement than the combined experimental groups of School #1, the probability of chance occurrence being 8%.

The control group in School #1 showed less gain in achievement than the combined results of all the experimental groups. This was statistically significant at the 0.047 level.

Conclusions: Working with numbers on the new material for the time of the experiment did not lower the basic skills tested as compared to skill maintenance on a re-teaching and review program.

The difference between schools is not unusual and could in part be responsible for the lower achievement of the control group, members of which were all drawn from

School #1. This experiment gives no guidance as to whether the poorer achievement of the control group was educationally significant.

Implications. Graphical representation as used in this study served as an adequate vehicle for review and maintenance of basic numeration skills. At the same time the children gained some familiarity with a new topic which in addition to its intrinsic value offered new insights and applications of their skills.

To determine whether the lower achievement of the control group was educationally detrimental, further experimental work would be necessary. For example, a re-test after a review and practice period would have supplied some information regarding this particular experiment. More desirable would be fresh experimentation to determine the optimum time to leave numerical skills unpracticed, for the purpose of pursuing other mathematic topics such as geometry.

Computation

Summary of statistical results. The review and practice group in School #1 showed somewhat better achievement than the graphical representation group, the probability of chance occurrence being 6.6%.

No difference of statistical significance was found

between the two experimental groups in School #2.

The experimental groups in School #2 did very much better than their counterparts in School #1, the probability of chance occurrence being virtually zero.

The control group in School #1 showed somewhat less gain in achievement than the combined results of all the experimental groups, the probability of chance occurrence being 8.2%. This appears to be mainly accountable for by the difference in the schools. Examination of the t-test matrix indicates this group was on a par with the review and practice group, School #1, when compared with School #2.

Conclusions. From the results of School #1, it would appear that reteaching of concepts of basic operations, when these have not been adequately mastered, is a more effective method of improving computational skills than incidental use, as in the graphical representation unit.

In School #2, where the children had good concepts of numeration and basic operations, and well developed skills, the group which spent most of its time on graphical representation progressed just as well as that which devoted considerable time to development of higher'facts', those of 9 and 10, in this case. The small amounts of practice which were given to the former group could have had a bearing on these results.

The difference between the schools justifies the different treatments resorted to during the course of the experiment.

The achievement of the control group was comparable to that of the experimental groups in the same school.

Implications. The experience in School #1 is an indication of the importance of spending adequate time on 'developmental activities'. From the results in School #2 it would appear that small amounts of direct practice were just as beneficial for maintainence of competencies as the spending of a major proportion of the time on such activities.

The achievement of the control group is a further indication that the amount of practice can be reduced.

Graphical Representation

Summary of statistical results.

School #1. Combined experimental groups showed a higher achievement than the control group. This was statistically significant at the 0.05 level.

There was no significant difference between the achievement of the graphical representation group and the review and practice group.

School #2. There was no significant difference between the achievement of the two experimental groups.

Conclusions. No conclusions can be drawn regarding the achievement of the children with respect to graphical representation. The only statistically significant result showed a higher achievement for the combined experimental groups of School #1 over the control groups, but the experiment was contaminated by the exposure of the review and practice group to the construction of graphs.

Supplementary Hypotheses Ability Groupings, School #2

Numeration. All children of the graphical representation group combined with the more able children of the review and practice group showed a higher achievement than the other children in the second group, statistically significant at the 0.029 level.

The more able and the less able children of the graphical representation group made similar progress.

The whole of the graphical representation group showed similar progress to that of the more able children in the second group.

Individual children of lesser ability made spectacular gains in numeration skills after working on the graphical representation unit.

Conclusions. The fresh approach of graphical representation appears to have helped the numeration skills and understandings of less able children to a greater extent than a routine review.

Implications. With one class divided into four groupings the numbers are small for adequate statistical analysis. The above results point to the need for further investigation with larger numbers and less homogeneous groups than this school provided.

Computation. Higher achievement appeared to go hand in hand with higher ability. As the experiment had been contaminated, in addition to the small numbers involved, judgment was withheld in this case.

Graphical representation. The groupings showed the following rank order of achievement: A_2 more able, A_2 less able, B_2 more able, B_2 less able, but there were no results of statistical significance. The experiment was contaminated, the numbers small and the evaluation instrument inadequate. Hence judgment was withheld in this case.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

INTRODUCTION

This study has confirmed in a limited way for two particular groups of primary children the results reported by Miss Biggs in Mathematics in Primary Schools;[1] as stated earlier, she claims that with a planned activity approach where children explore mathematical aspects and possibilities of their world, computational skills can be maintained with a considerable reduction in time spent on routine practice.

It has indicated the possibility that, with an adequately controlled approach, primary children would show the same increased gains in computational efficiency, when more time is spent on developmental activities, as did their intermediate counterparts, discussed in Chapter II.

In line with the suggestions of the Cambridge Con-

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[1]Schools Council for the Curriculum and Examinations, Mathematics in Primary Schools, Curriculum Bulletin #1, second edition, (London: Her Majesty's Stationery Office, 1967) p. xv, p. 49.

ference on School Mathematics[2] and Glennon and Callahan,[3] new material, namely Graphical Representation, was introduced to two groups of First Year children, without any significant loss in basic arithmetic competencies.

SUMMARY OF FINDINGS AND IMPLICATIONS

At the start of this study, it was hypothesized that arithmetic competencies of primary children at the end of first year would be equally well-maintained during a three week period spent on a unit of Graphical Representation, and during an equivalent time spent on routine review and practice. Further, it was hypothesized that maintenance would be superior to that of children who were exposed to no arithmetic activities during the same period. The arithmetic competencies tested were Numeration (reading, writing and understanding of base ten numerals $\leqslant 99$) and Computation (addition and subtraction with sums and minuends $\leqslant 10$).

Numeration

Statistical analysis of the data on pre-testing and post-testing (see Chapter IV) showed that working with

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[2]Cambridge Conference on School Mathematics, Goals for School Mathematics, (Boston: Houghton Mifflin Company, 1963), p. 7.

[3]Vincent J. Glennon and Leroy G. Callahan, Elementary School Mathematics, A Guide to Current Research, (Association for Supervision and Curriculum Development, National Education Association, 1968), p. 81.

numbers on the new material for the time of the experiment did not lower the tested numeration competencies for the time of the experiment. The control group showed somewhat poorer achievement, but the data from this experiment was insufficient to assess educational significance of this.

The implication drawn from these results is that graphical representation, as used in this study, served as an adequate vehicle for review and maintainance of basic numeration competencies. In addition, the children gained some familiarity with a new topic which offered new insights and application of their skills, besides its own intrinsic merits.

Computation

Statistical analysis of the data on pre-testing and post-testing (see Chapter IV) showed that the review and practice group in School #1 showed somewhat better achievement than the graphical representation group. The children in this school had not, for the most part, mastered the concepts of the basic operations being tested. Hence, it was concluded that the reteaching of these concepts, which was carried out with the review and practice group, was more effective than the incidental use, as in the graphical representation group, for initial mastery.

In School #2, where the children had good concepts and well-developed skills, the two groups progressed equally

well. The graphical representation group had done a small amount of practice, hence the implication here is that small amounts of direct practice were just as beneficial as a major proportion of the time so occupied. In addition, the graphical representation group had the advantage, mentioned above, of the new material.

Graphical Representation

The attempted assessment of new learning by the graphical representation groups produced no results of statistical significance.

Limitations

The chief limitations of this study can be classified under Time, Place, School Differences, Contamination and Evaluation Instruments.

Time: It is impossible from this short experiment to draw any conclusions regarding long range effects. The teaching period was only three and one half weeks and there was no follow-up retention test at a suitable later interval.

Place: While there was considerable range of ability in the classrooms used, the children were drawn from a fairly homogenous socio-economic background.

School differences: As indicated, the teaching methods in the two schools had been so different that a divergence of methods during the experiment was inevitable. In addition, the different atmosphere and attitude at the

end of the term undoubtedly affected the final test results.

When it came to the graphical representation test, the children in School #1, lacking an adequate understanding of basic operations, were unwilling to attempt unknown addition or subtraction equations. This resulted in extremely low mean scores.

Contamination: In both schools, all children were exposed to the results of the work done by the children in the Graphical Representation group. The Drill and Practice groups in particular contributed to the graphs that were built. This effect would have been more severe in School #2, where the results were on display in the main classroom. In School #1, the charts were in the 'Arithmetic Corner' and not continuously within sight of the children.

In addition, in School #2, the Graphical Representation group was given two or three practice sheets in computation by their own teacher. It was originally intended that there be no practice for them.

The Drill and Practice group in School #2 were introduced to the 'facts' for nine and ten, which were new to them. However, this can be considered more as an extension of work already mastered since no new concepts or methods were used.

Evaluation Instruments: The evaluation instruments for graphical representation were untried and probably inadequate.

SUGGESTIONS FOR THE FUTURE

In retrospect, this study has many of the earmarks of a pilot or feasibility study. Useful conclusions can even be drawn from unforseen occurrences. For example, it would appear that small amounts of practice are better than none at all. Any conclusions from this study have to be confined to the population involved. However, the fact that neither Groups A or C showed any serious losses in numeration or computational skills over the period of time involved would indicate that the topic is worth pursuing further. Both the graphing material and the geometry would lend themselves to a long-term project; work in either can be interspaced with periods of instruction and development in the basic skill.

One possible follow-up study would eliminate the contamination experienced in the present investigation by having pairs of classes, each pair being taught by one teacher or investigator, over a prolonged period. One class would follow a basic program (as had been done in School #2 before the arrival of the investigator) while the other class of each pair would have practice time cut drastically in order to include other topics. Graphical representation is one such topic with the added attraction that much review and re-enforcement (practice) can be woven

into its development.

Before any follow-up research is carried out, though, it is imperative that contact be made with Britain and investigators be brought up to date with research there, relative particularly to Miss Biggs' statement, previously referred to, that teachers have been able to considerably reduce practice time.[4]

NON-STATISTICAL OUTCOMES REGARDING GRAPHICAL REPRESENTATION

Graphical representation is an important topic in its own right in our society today. Children are obviously able to learn a good deal regarding this at a considerably younger age than they do in our current British Columbia curriculum. (Grades 4 and 5). If introduced in the primary grades, it is essential that every child be actively involved in the learning process according to Piagetian principles. With such involvement, and careful planning, an alert teacher can bring in a considerable amount of review of basic skills. Often new insights into meanings of concepts will be gained and new uses for basic skills will be discovered by the children. An example from this project was the practical application of knowledge of 'tens and ones' to interpretation of data presented on a vertical bar graph.

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[4] Op. cit., p. 49.

During the development of the unit in School #2, there was a noticeable redirection of the children's thinking from 'blocks and numbers games' to real life situations, a hoped-for outcome of most mathematics programs today.

Graphical representation also lends itself well to correlation of mathematics with subjects such as science and language. It is a good vehicle for team work and socializing, the importance of which is being increasingly recognized in the learning processes of young children.

The spurts of achievement noted among individual children of average or lower ability would merit further investigation. The numbers were too small for adequate statistical analysis, but this phenomenon has been noted in studies of older children. (See Chapter II, p. 15-16).

The unit had advantages for the teacher too. She got to know the children more quickly, found she had less work to do, and that the variety and challenge were much more interesting than 'review'.

<u>A note of caution</u>. In the course of the work in School #1, it appeared that Graphical Representation was not suitable for teaching the basic concepts of our number system. However, no claim to the contrary was made at any time, nor is implied in the Nuffield Guide used. The investigator would have been well advised, had time permitted, to have brought these children up to a certain standard of

proficiency before embarking on the new material.

CONCLUDING REMARKS

L. G. W. Sealey, in his discussion of "Some Problems Involved in the Induction (sic.) of New Approaches into Primary Schools," points to the difficulties of evaluating a new approach, but says, "it is often possible to point to cases in which children taught by such-and-such a new approach have not suffered in respect of attainment on tests of a traditional nature."[5] It is claimed that this has indeed been done in this study.

When discussing the tremendous difficulties involved in any attempt at scientifically controlled experimental research, Mr. Sealey says:

> . . . while we may not be able to obtain evidence of the relative value of a new approach, we should appreciate, where the approach continues to survive, the importance of its survival as evidence of its viability.[6]

This seems to be happening, at the time of writing, with the approach in primary education, of more activity and

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[5]Sealey, L. G. W., "Some Problems Involved in the Induction of New Approaches into Primary Schools," in J. D. Williams, ed., Mathematics Reform in the Primary School, A report of a meeting of experts held in Hamburg during January, 1966, (Hamburg: UNESCO Institute for Education, 1967) p. 54.

[6]Ibid., p. 56.

exploration, and less practice, not only surviving but spreading.[7]

The results of this study are admittedly limited, but they do show that it is possible to teach more topics than at present, and at the same time maintain basic competencies.

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[7]Two Canadian examples of this trend are W. W. Bates and D. Inglis, Mathaction (Toronto: Copp Clark and Co. Ltd., 1970), and Roberta Chivers et al., Project Mathematics, (Toronto: Holt and Rinehart and Winston of Canada Ltd., 1970).

APPENDICES

APPENDIX A

PLANS FOR TEACHING

A general plan was drawn up for each group that was to be taught. Observation and pre-test results revealed that groups B_1 and B_2 would require quite different treatment over and above the difference between traditional and i.t.a. orthography. The same plan was used to start off with, for both A groups, but it evolved quite differently in the two classrooms.

A suggested outline was kept prepared for about a week at a time, but the detailed daily planning was done every day, taking into account what had developed during the class sessions. This was particularly true of the graphing sessions which lent themselves well to a group version of Piaget's 'clinical method'.[1]

General Plan--A Groups--Graphical Representation

Aim: To develop abilities and understandings in the areas of (a) construction and (b) interpretation of pictorial and block charts.

Evaluation of Unit:

(a) Construction to be carried out as group activity

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[1]See Flavell, J.H. The Developmental Psychology of Jean Piaget, (New Jersey; D.Van Nostrand Co., Inc., 1963), p.29.

accompanied and followed by discussion. It is hoped children will suggest further activity. No formal evaluation will be attempted.

(b) Interpretation: Evaluation will be attempted with a short test (Test III) in which a chart is displayed and the children are asked to print answers to orally presented questions which ask for:

(i) information directly obtainable from and pertaining to the chart.

(ii) construction of number sentences and equations expressing the above information and how it is obtained.

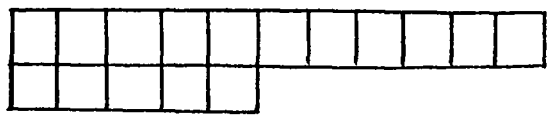
(iii) construction of other number sentences and equations suggested by the data set out on the chart.

General Outline--as adapted from Pictorial Representation [1] [2].

Stage One: Comparison of 2 rows (columns), one object per child.

Topic: Comparison of number of boys to number of girls.

(a) Concrete objects (e.g. blocks, milk bottle tops). Compare the number of boys to the number of girls in the group. e.g.: diagram.



[2] Nuffield Mathematics Project, Pictorial Representation [1], (London: W. & R. Chambers and John Murray, 1967, available from Longmans Canada Ltd., Don Mills, Ontario).

Variations: comparison of boys and girls in the whole class or in the families of the group.

Introduce the symbols $>$ and $<$.

Discuss results and develop appropriate number sentences on large chart of chalkboard.

(b) Transition to more permanent form of recording: one inch squares, blue for boys, pink for girls, to be pasted onto a chart.

Children, working with partners to be encouraged to develop own number and word sentences.

Stage Two: Increase in number of data, from comparison of two rows to comparison of several rows.

Topic: Birthdays. Prepare charts with a row for each month. Provide uniform pieces of paper on which each child will draw a self portrait to be pasted in the appropriate place.

Children to be encouraged to develop number and word sentences that tell about the chart, particularly as it pertains to their own birthday, or that of a friend. Elicit problems for the children to pose to each other.

Stage Three: Transition to block chart.

Topic: Pets.

(a) Pet Chart. Horizontal rows prepared for each type of pet owned by a pupil. Pupils to colour, cut out and paste their pets in appropriate rows. Hopefully a hap-

hazard pasting will reveal that the longest row does not have the most pets and a fresh start will be suggested with each pet in one-to-one correspondence with one above it. Chart can then be ruled vertically and use of special lines for 5's and 10's ('to save counting') can be developed. This will provide incidental review of counting by 5's and 10's and of how many 10's and ones are represented by a given number.

(b) Block Type Chart--Vertical (Column)

Arrangement. New chart, with unit squares, different colours for each pet, to be 'piled up' in columns.

Stage Four: Use of squared paper, squares coloured in.

Topics: Birthdays, pets.

Stage Five: (if time, which is doubtful). Abstract representation by strips and by bar line.

General Plan--Group B_1--Review and Practice

Review and Practice of

(a) Addition and subtraction facts of 7, 8, 9, 10, using both equation form and vertical form. Addition equations with more than two addends were quite unfamiliar to these children and therefore omitted.

(b) Counting by ones, fives and tens.

(c) Simple word problems, oral and printed, which involved facts and operations being reviewed.

(d) Numeration: units and tens and relationship

between printed numeral and number it represents: also relationship to counting money, arranged in dimes and pennies.

Materials and Methods

Flannel board and chalk board demonstrations, involving a few children, while others watched. Various types of counters available for individual manipulation and solution, and checking of printed exercises (dittoed worksheets).

Coloured sticks and pegs, especially for numeration.

Evaluation: Test I, II and III as described in Chapter III.

General Plan--Group B_2--Review and Practice

Teach: symbols $>$, $<$.

Having been introduced, unexpectedly as 'the lady from the University' who was going to do 'experiments' and teach new things, it was deemed essential for the establishment of good rapport to introduce something new! Also this was requested by the teacher.

Review and Practice of

(a) Addition, subtraction and multiplication facts up to 8.

(b) Equations with more than 2 addends, sums $\leqslant$ 8.

(c) 'Balancing the equation' e.g. 5+3=2+_.

(d) Use of parenthesis as in Number Patterns I[3].

(e) Numeration: units and tens and relationship between printed numeral and the number in represents.

(f) Counting by ones, 5's and 10's.

(g) Counting money: dimes, nickels and pennies.

(h) 'Extensions' such as 57+1=_, 43-1=_, i.e. adding and subtracting one to any number $<$ 100.

Materials and Methods

Children to work singly, using one inch coloured cubes. In oral lessons equations will be elicited and printed on the chalkboard. Children to be encouraged to make their own discoveries. Practice work sheets to be done with teacher assistance available. Expectation of amount to be completed to be adjusted to individual abilities.

Evaluation: Test I, II and II as described in Chapter III.

General Plan--Group C_1--Geometry

General Topic: Two dimensional geometric shapes.

Proposed sub-topics, materials and methods:

(a) Develop familiarity with circles, ovals, triangles and rectangles. Manipulation and sorting of cardboard shapes of various colours and sizes. Tracing and colouring, cutting, sorting and pasting, extending

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[3]Chivers, et al, Number Patterns Book I (Toronto: Holt, Rinehart and Winston, 1966).

given patterns, and creating new ones.

(b) Develop background for later work with congruence. Comparison of sizes and colours of similar shapes.

(c) Review open and closed curves and related topics such as point, line, inside, outside, on, between. Group work at chalkboard and flannel board. Individual work with dittoed work sheets.

(d) Geometric shapes from a collection of points, or (polygonal) with pegboards and elastics.

(e) Folding regular shapes into equal parts (preparation for fractions).

(f) Tesselation. Cutting and pasting of regular geometric shapes, seeking to discover which will cover an area without leaving gaps.

Source Books: (a) to (e) adapted from Elementary School Mathematics, Second Edition, Book 1, Teacher's Edition.[4]

(f) adapted from Shape and Size, Nuffield Mathematics Project.[5]

Evaluation: No evaluation of geometry was planned as these children were to be subjected to Tests I, II and III.

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[4] Robert E. Eicholtz, et al, Elementary School Mathematics, Second Edition, Book 1, Teacher's Edition, (Addison-Wessley (Canada) Ltd., 1969, Don Mills, Ontario).

[5] Nuffield Mathematics Project, Shape and Size Nuffield Mathematics Project (London: W & R Chambers and John Murray, 1967, available from Longmans (Canada) Ltd., Toronto).

APPENDIX B

SAMPLE LESSON PLANS

School #1, Thursday, June 4.

Group B_1: (15 minutes) Review of addition and subtraction facts of 8: Have counters and worksheets ready, in a circle on the floor, for each child. Brief review and assign seat work; allow a start to be sure pupils understand what is expected. Children take counters and worksheets to their desks, to complete. (See Figure 2, page 97)

Group C_1: (25 minutes) Patterns from geometric shapes. On flannel board, start a pattern, using cardboard circles, triangles and rectangles. Have children complete. Repeat with a new pattern. Have children develop a pattern from scratch. Follow-up of yesterday's lesson: Did anyone find an oval shape at home? Seat work assignment: cut and paste shapes to complete a pattern begun on a worksheet. (See Figure 3, p. 98)

Group A_1: (25 minutes) Paste pictures of selves, drawn yesterday, on a common size of paper, onto chart paper, the girls in one row, the boys in another. Develop equations, word sentences and inequalities. Start the group off on a worksheet to follow up this development. (See Figure 4, p.99)

Group B_1: (10 minutes) Children return to arithmetic corner for checking of work, and of corrections on yesterday's work.

name_____

3	4	4	2	7	6	4	5	2
+5	+4	+3	+5	+1	+2	+2	+3	+5
___	___	___	___	___	___	___	___	___

8	8	8	8	8	8	8	8
-2	-4	-6	-8	-1	-3	-5	-7
___	___	___	___	___	___	___	___

6	2	8	4	3	8	8	4
+1	+6	-3	+3	+5	-7	-6	+4
___	___	___	___	___	___	___	___

Draw 3 cats. Draw 5 dogs.
Hou meny animals hav yœ nou?
Print the eekwæshun.

FIGURE 2

SAMPLE WORKSHEET FOR GROUP B_1

Redrawn from half of an 8½"x14" worksheet.

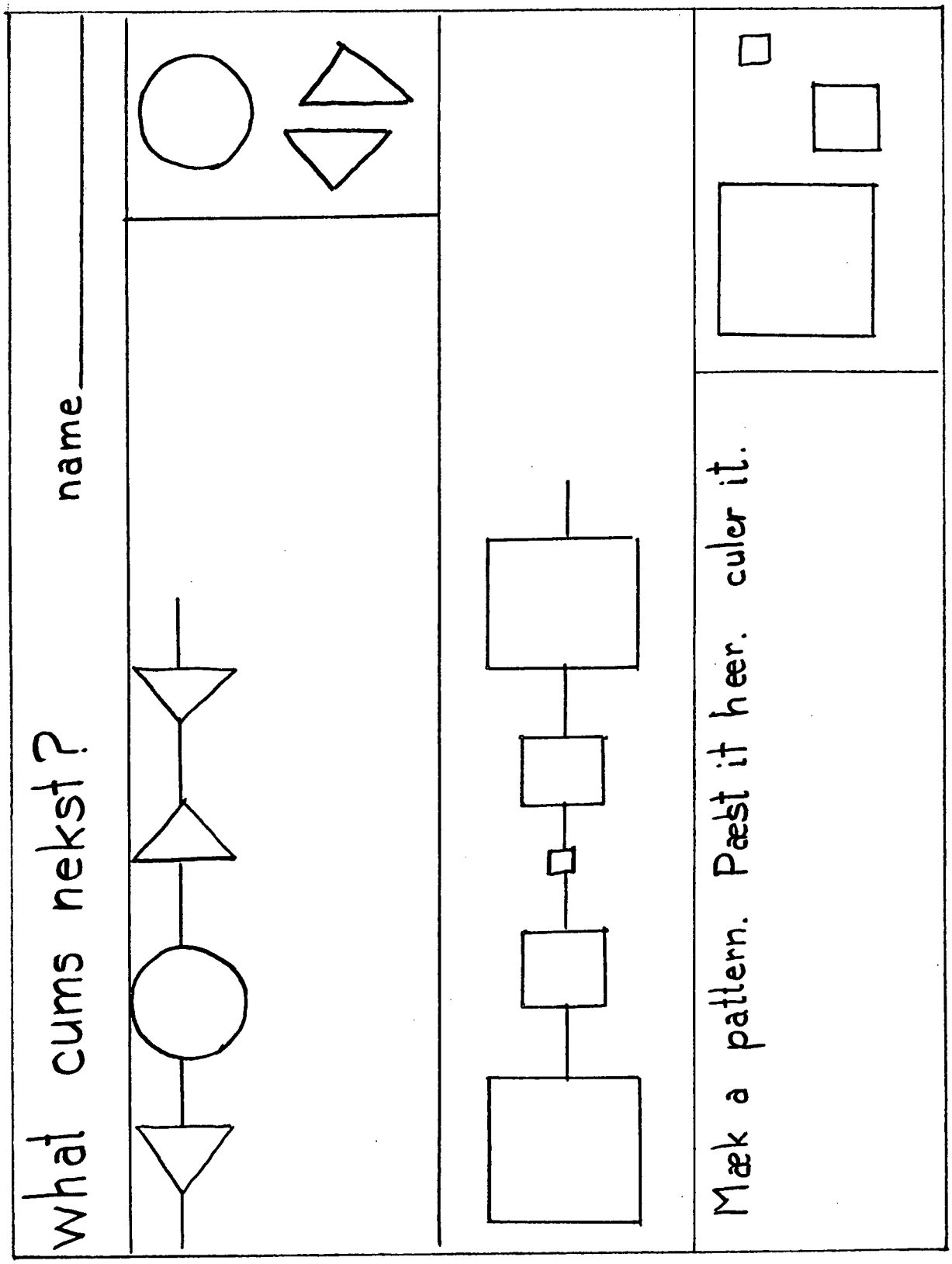

FIGURE 3

SAMPLE WORKSHEET FOR GROUP C_1

Redrawn from half of an 8½"x14" worksheet.

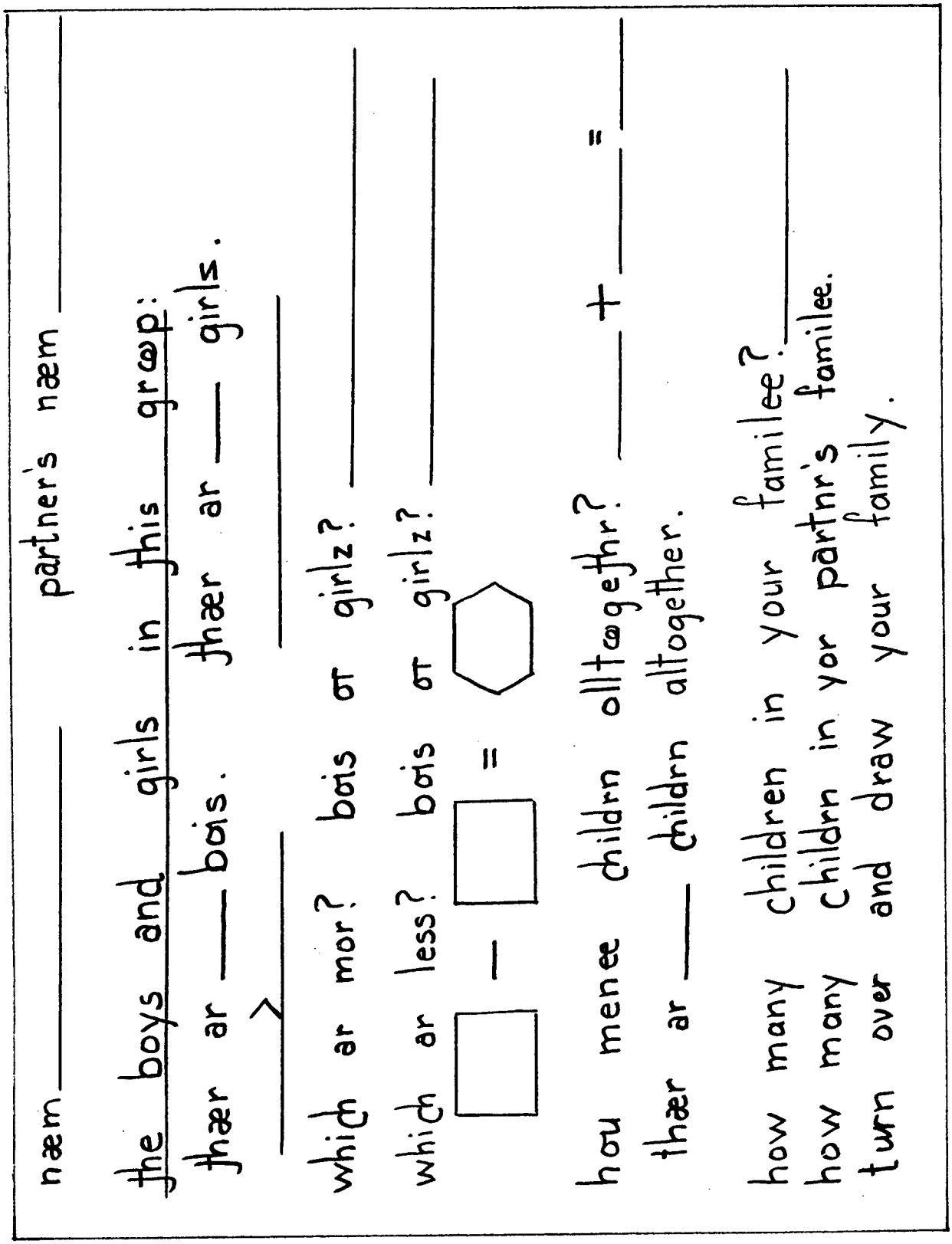

FIGURE 4

SAMPLE WORKSHEET FOR GROUP A_1

Redrawn from half of an 8½"x14" worksheet.

School #2 Wednesday, June 3.

Group A_2: (25 minutes) a) Materials: each child to get enough one inch cubes for every member of his family. Reteach $>$ and $<$ by comparing number of boys to number of girls in family. Have three or four children write these inequalities on the chalkboard.

b) Materials: blue paper squares for boys, pink for girls. Paste in two rows on lined chart paper. Print one or two sentences and equations--words and symbols.

c) Seat work: (see Figure 5, p. 101) Arrange partners, must be from a different reading group. Assign seat work, to be carried out in foyer, while other group has lesson in room. This proved to be too ambitious for one day. Hence:

Thursday, June 4: Re-do part of worksheet as class project. "We are learning to write explanations that need sentences and equations." This lesson to follow Group B_2.

Group B_2: (25 minutes) Materials: nine one inch cubes for each child. Introduce symbols $>$ and $<$. Review addition and subtraction facts of 9. Children use blocks to demonstrate their discoveries, and dictate equations for printing on the chalkboard. Start worksheet with $>$ and $<$. (See Figure 6, p. 102)

Thursday, June 4: This group comes first. Work on 9 and symbols $>$ and $<$ using worksheet that was just started yesterday.

Red Group Name _____

Partner's name _____

A. The Boys and Girls in our Group

There are ___ boys. There are ___ girls

___ > 6 ___ < ___

There are more _____ than _____ .

There are fewer (less) _____ than _____ .

___ + ___ = ___ . There are ___ children altogether.

B. Our families

___ + ___ + ___ = 9

_____ _____ _____ have 9 children altogether.

C. Other ways to make 9: ___ + ___ = 9

___ + ___ + ___ = 9 ___ + ___ = 9 ___ + ___ = 9

D. My family and my partner's family.

_____ and I have ___ people in our families.

___ + ___ = ___ ___ < ___ ___ > ___

I have more / less than my partner

Turn over to print more stories and equations.

FIGURE 5

SAMPLE WORKSHEET FOR GROUP A_2

Redrawn from an 8½x11" worksheet.

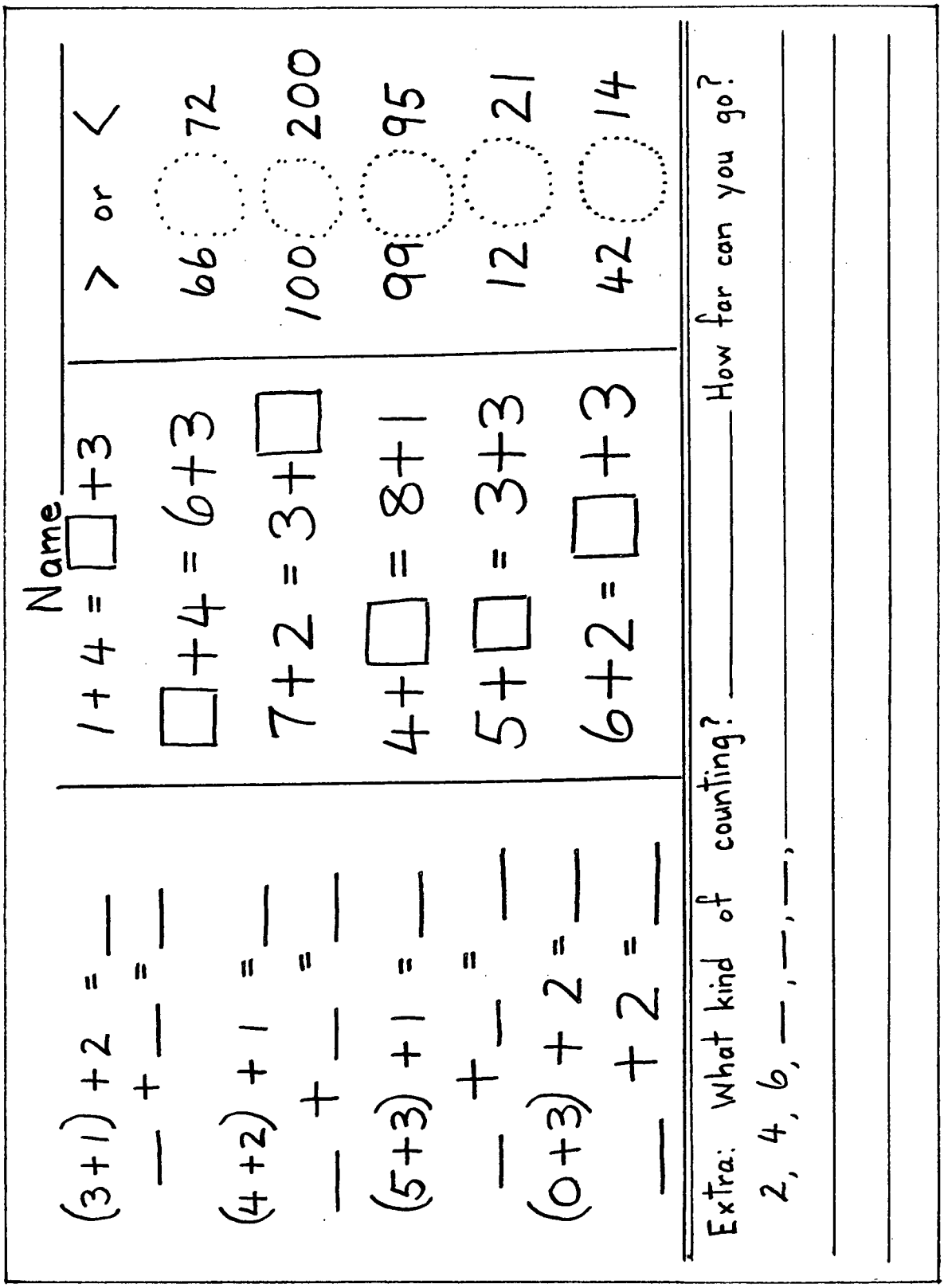

FIGURE 6

SAMPLE WORKSHEET FOR GROUP B_2

Redrawn from half of an $8\frac{1}{2}$"x14" worksheet.

APPENDIX C

TEST

MANUAL

POWER TEST OF BASIC ARITHMETIC SKILLS

-FIRST YEAR PRIMARY-

Forrest Johnson

University of British Columbia

November, 1970

C

CONTENTS

C

POWER TEST OF BASIC ARITHMETIC SKILLS

-FIRST YEAR PRIMARY-

DIRECTIONS FOR ADMINISTERING TEST

GENERAL DESCRIPTION OF TEST

This test is composed of two parts, intended to evaluate skills and understandings in the areas of

I. Numeration

II. Addition and Subtraction Operations and Facts

The skills are separated so that a weakness in one will not be masked by strength in the other as in traditional tests. An attempt was made during preliminary tryouts to subdivide Numeration into: a) printing of numerals, b) recognition and reading of numerals, c) understanding of base ten numeration system. The resultant rather lengthy test showed such a high degree of internal reliability (KR-20 = 0.9503) that this subdivision was abandoned. The three sub-skills appeared, on the basis of test items selected, to be closely interrelated.

The test was specifically designed for use in British Columbia schools, towards the end of the first year of formal schooling. Part I is based chiefly on numbers <40 with a smaller percentage of items using numbers between 40 and 100. A few selected items using numbers greater than 100 are included to provide some degree of assessment for advanced pupils. Estimated time requirement for Part I is 30-40 minutes, which should be divided into two sessions.

Part II employs chiefly examples with sums or minuends ≤ 8; a small percentage of items have sums or minuends of 9 or 10. Some assessment of the advanced pupils is again attempted with selected items using larger numbers (<100), and for the final items, vertical in place of equation form. The estimated time requirement for Part II is 15-20 minutes.

A short individual oral testing session requires about one or two minutes with each child.

The information on the cover is optional. It may be completed and/or the picture coloured, if the home-room teacher so desires,during the individual oral testing by the examiner.

If the home-room teacher is carrying out the testing it is advisable to do so in small groups. The assistance of a second persons is important if all members of an average sized class are to be tested at one time; this is to ensure that all children are working in the right place, understand the instructions and are doing as much as they are comfortably able to do,.

Throughout the testing the examiner should use terms familiar to the class being tested: e.g. 'numeral' or 'number'. Children will need to be encouraged and reassured that they are not expected to be able to do all the examples. Some of them are 'about interesting things you will be learning next year'. Teachers, too, need to be reminded that this test covers considerably more content than that to which they are expected to have exposed their classes in the first year at school.

Two parallel forms have been developed, Form A and Form B. Page four, which tests the concepts of 'greater than', 'less than', 'equal to', has two alternate versions: 4* is designed for those children who have been instructed with the symbols rather than the words. 'i.t.a.4' is provided for children who have been receiving instruction in the Pitman Initial Teaching Alphabet. Poor readers may, of course, be given help with the reading. An 'i.t.a.' form has also been provided for page five.

'Official' STOP signs are provided periodically, for formal breaks between sittings. However, the test is designed so that a break may be taken at the end of any page. Double lines, part way down a page, are a signal to wait for new

instructions. In general the material on any one page is related, so that a break in mid-page is inadvisable.

Repetition of essential instructions is of course necessary at this age level.

DETAILED INSTRUCTIONS

INDIVIDUAL ORAL TESTING[1] FORMS A AND B

The children will come, individually, to the examiner to read aloud the following numerals (Part I) and equations (Part II) prepared on flash cards. Scoring space is provided on the cover. A tick in the first column indicates a correct response, a tick in the second column indicates an error; scoring is done later in the third column.

FORM A

sample)	7	38)	2+4=6
1)	21	39)	8-3=5
2)	68	40)	9+0=9[2]
3)	12		
4)	159		

FORM B

sample)	9	38)	3+2=5
1)	21	39)	8-2=6
2)	76	40)	7+0=7[2]
3)	12		
4)	138		

Most children are readily satisfied that it is the examiner's turn to make a pattern, and are happy to see all the ticks appear. The examiner's remarks are of course adjusted to suit the child. More able students have at times had to be reassured that there was 'no trick, just some easy numbers to read aloud.'

---

[1] Need not precede pencil and paper testing.

[2] Equations of form 8=3+5 were dropped as many children read from right to left. (Mathematically correct.)

Others need encouragement. On the other hand, a hesitant child can be moved past the large number, for example, if he really doesn't want to try it, by saying 'All right, let's try this equation!'(number sentence). Responses of 'thank you','that's fine', etc. can keep the test moving whatever the child answers.

Scoring

When scoring, an item is considered correct(one mark) if a child has corrected himself spontaneously. 'Is' may be substituted for 'equals' if that is the familiar form.

Item 4: Reading of digits only is not acceptable.

Item 38: 'Plus', 'add', or 'and' are acceptable.

Item 39: 'O' may be read as 'nothing' or 'zero' but 'oh' is not acceptable.

Item 40: 'Take away' or 'minus' are acceptable.

PENCIL AND PAPER SESSIONS FORM A

The area(s) being tested by each group of items is indicated within square brackets before the oral instructions are given.

PART I NUMERATION

N. B. REPEAT ALL ESSENTIAL DIRECTIONS! THIS IS NOT A TEST OF LISTENING SKILLS!

Page 2

Items 5-6 [Recognizing printed numerals (aural to reading) with understanding.]

5. See the baseball bat in the first box and the row of numerals that come after it. Put your finger on the baseball bat. Put a big X on the greatest (highest) number in that box.

6. See Snoopy's house and the row of numerals that come after it. Put your finger on Snoopy's house[3]. Now put a big X on the numeral one hundred thirty-six.

---

3. Use the technique 'put your finger on---' whenever advisable to ascertain that the pupils are working in the correct place.

Items 7-11 [Aural to printing of numerals with understanding of place value.]

7. See the box with the little Christmas tree in the corner. Print the numeral 28 in the box with the Christmas tree.

8. See the box with the little stickman in the corner. Print the numeral 17 in the box with the stickman.

9. See the box with the little fish in the corner. Print the numeral that is one less than 30 (that comes before 30) in the box with the little fish.

10. See the box with the little button in the corner. Print the numeral that comes between 33 and 35 in the box with the little button.

11. See the box with the little apple in the corner. Print the numeral that means four tens and six ones. Print the numeral that means 4 tens and 6 ones in the box with the apple.

Items 12-15 [Matching numeral to picture, involving counting of pictured objects and recognition of numerals <100.] (Use item 13 as a 'sample')[4].

13. Put your finger on the box with the leaves in it. How many leaves are there? Yes, that's right. Now put the point of your pencil inside the box with the leaves. Do you see this long column of numerals? (Demonstrate). Draw a line from the box of leaves to the numeral that tells exactly how many leaves there are. (Check that this item is correctly done.)

12-15. Now there are some more boxes of objects (pictures) for you to match with the correct numeral, BUT BEWARE! SOMEONE HAS PUT IN TOO MANY NUMERALS! Here is a little hint: If there are lots of things for you to count, you will find that they are in groups of fives or tens, to help you. (Do not labour this point.)

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[4] See note, page 14 re: revision of Form A.

12. See all the seagulls flying. Can you find out how many there are altogether? Then draw a line to the right numeral. BE SURE YOUR LINE STARTS INSIDE THE BOX.

14. Now you can match the cherries to their numeral. Be sure your pencil starts INSIDE the box.

15. Now you can match the stars with the right numeral. (If a child claims he cannot find the correct numeral it can be suggested that he print the numeral he is looking for beside the picture).

Item 16 [Reading numeral and printing its name.]

sample: (Put the numeral 6 on the chalkboard and print the word 'six'. Have the children do the same in the space provided).

16. Now do you see this big numeral at the bottom of the page? If you know what it is called don't say a word! Just print its name the way it sounds to you. (Spelling will not be marked but the ability to put down sound symbols which indicate that the child knows the correct name. Scoring hint given later. See page 24).

Page 3

Items 17-18 [Choosing correct picture for printed numeral. Multiple choice.]

17. See the tiny baseball in the margin. Put your finger on the baseball. Now look at the numeral next to the baseball. After it there is a row of boxes with baseballs in them. One box has just as many baseballs as the numeral says. Put a big X on the box that has as many baseballs as the numeral says. Don't forget to look and see if there is a quick way of counting by groups.

18. See the tiny seed in the margin. Put your finger on the seed. Now look at the numeral next to the seed. After it there is a row of boxes with seeds in them. One box had so many seeds that they had to be put into envelopes.

There are ten seeds in each envelope. Put a big X on the box that has as many seeds as the numeral says. Don't forget to look and see if there is a quick way of counting by groups.

Items 19-22 [Number sequences. Counting without pictures.]

sample. Now it is your turn to count without any pictures and find out what numerals are missing. BEWARE! EACH ROW IS DIFFERENT and some rows are quite tricky! I think everyone can do the row that comes after the tea cup. (sample.) Put your finger on the little tea cup. What kind of counting is it? That's right, counting by ones. Let's read together: 3, 4, something, 6, 7, something, something, 10, period. Now, please print the correct numerals on the lines where we read 'something'. (Demonstrate on the chalkboard if necessary and make sure the children complete this line correctly).

19. Now put your finger on the little feather. Very quietly, to yourself, read the numbers that come after it. Figure out what kind of counting it is. Then fill in the missing numbers like we did before.

20. Now look at the row with the star. It might be a different kind of counting. See if you can figure it out and put in the missing numbers.

21. The row with the balloon is started but not finished. Please finish it carefully.

22. The row with the wizard's hat needs to be finished too. Please finish it carefully.

Items 23-24 [Making pictures to match the numeral.]

Now it is your turn to make pictures!

23. See the chair with the numeral beside it, in a little box. Shh, don't tell what it says! In the long empty box beside the chair I want you to make just as many chairs as the numeral says. Make the chairs in the long empty box.

24. See the little star (baseball)[5] with the numeral beside it. Please draw as many stars (baseballs) as the numeral says. Try to fit your stars into the long box. (Overflow may of course be placed below). (A few children can profit from a suggestion of grouping in fives or tens as they draw).

TIME FOR A BREAK

Page 4

Items 25-28 [Concepts of greater than, less than, equals, one greater than, one less than.]

(Three versions are provided: 1) reading the words, as in Seeing Through Arithmetic,[6] 2) reading the words in i.t.a.printing, 3) symbols. (Page 4*). (Samples are provided to be done on the chalkboard by the examiner, and by the children in their booklets).

25-26 [Printing correct phrase or symbol.]

sample 1. Let us read what it says in the boxes (what the symbols say) at the top of this page. After the moon it says 8, and then there is a long line (dotted circle) and then 6. Now what can we print on the line (in circle) to make a sentence that tells the truth? That's right! Eight is greater than six. Please print 'is greater than' on the line (in the circle).

sample 2. Now look at the sun. Which box (symbol) are we going to use to make this a true sentence? That's right! 48 is less than 84.

25 and 26. Now there are two sentences for you to do, as quietly as you can. There is the mushroom sentence and the sentence that starts with a little flower. You will have to use one of the boxes (symbols) again. One of the boxes (symbols) has to be used twice. Now see if you can make both rows tell the truth.

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[5] See note, page 14 re: revision of Form A.

[6] Maurice L.Hartung,et al., Seeing Through Arithmetic, Book I, Teacher's Edition (Toronto: W.J.Gage Ltd., 1965)

DON'T FORGET TO REPEAT IMPORTANT INSTRUCTIONS!

27-28. [Choosing correct phrase or symbol. Multiple choice.]

sample 3. See the little pear? Beside it we can read 21+1. How much does that make? All right, remember in your head how much 21+1 is. Now we have to find out which phrase (symbol) in the middle fits with 21+1 and with the numeral at the end of the row to make a true sentence. Let's try '21+1 is greater than 22'. No? (Continue in this vein until pupils are satisfied with 'is equal to'). Good, now put a ring around 'is equal to'and read the sentence to yourself to make sure it is true.

sample 4. Let's try the row that starts with a lemon and do it just the same way. (Elicit as before until item is completed).

27 and 28. Now here are two more jobs for you to do just the same way, all by yourself. Look at the row that starts with the racing car. Read quietly and carefully and think what numeral the first expression stands for. Now read each phrase (symbol) in the middle and the numeral at the end. Put a ring around the phrase (symbol) in the middle which makes the line tell the truth.

Now try the row that starts with the sail boat. Read quietly and carefully. Think what numeral the first expression stands for. Now read each phrase (symbol) in the middle and the numeral at the end. Put a ring around the phrase (symbol) in the middle which makes the line tell the truth.

Page 5. [Place value .] (i.t.a.5 has 'wuns' for 'ones').

Items 29-33. [Pictures, hundreds, tens and ones. Numerals to print.]

29. Here are some pens, tied up in bundles. There are ten pens in each bundle. Some pens were left over. In the box at the end of the row print the numeral that tells how many pens there are altogether.

30, 31, 32. Here are some logs. They were too heavy to put into bundles so someone piled them up. There are ten logs in each pile. Some logs were left over. Please print how many piles of ten there are and how many ones left over. Then printhow many logs there are altogether.

33. HERE IS A REALLY TRICKY ONE! (Very few first year children are ready for this one). I'D LIKE YOU TO TRY IT. There are lots and lots of boxes of erasers. Every box has ten erasers in it. There are so many boxes that they had to be piled up. DO YOU SEE THE LITTLE ERASER THAT WAS LEFT OVER? Now see if you can figure out how many erasers there are altogether. If you can, don't say a word! Just print the right numeral in the box at the end of the row.

Items 34-37. [___tens ___ones = ____. Completion, from words to numeral and vice versa .]

sample. See the box with the little bell in the corner. Beside the bell it says, (pause), "Three tens and nine ones equals ?' Yes, that's right, thirty-nine. Print 39 on the line.

34-35. Now see the little pussy cat. He has two more jobs for you to do all by yourself. Read them carefully and print the numeral that the tens and ones stand for, each time.

sample. See the box with the butterfly in the corner. This time there is a numeral, and you are to print how many tens and how many ones that numeral stands for. Seventy-three is ? Yes, that's right, seven tens and ? yes, three ones.

36-37. Now the butterfly has two more jobs like that for you to do all by yourself.

Items 38-39. [Understanding of place value, including hundreds, with numbers $<$ 10,000 .]

38. Put your finger on Charlie Brown's baseball cap. Now look at the row of numerals that comes after it. One of those numerals has a four in the hundreds place and a four in the ones place. Put a big X on the numeral that has a four in the hundreds place and a four in the ones place. Be sure that you make only one X please.

39. Put your finger on the baseball. See the big numeral in the box beside it. Now look carefully at the row of numerals that come after the box. One of the numerals in the row tells how many ones the numeral (digit) eight in the box stands for. Put a big X on the numeral in the row which tells how many ones the numeral (digit) 8 in the box stands for. Just make one X please.

Items 40-42. [Equations requiring understanding of base ten numeration .]

40-42. Put your finger on the flag. Between the flag and the stop sign are three equations. The numbers look pretty big but if you READ them carefully, one at a time, you should be able to figure out the right numeral to put in the box, to make each equation tell the truth. You shouldn't have to do any counting. Remember to do some thinking before you fill in a box. If you are not quite sure, try again. (Do not allow excessive time for the children who make rows of dots or otherwise try to count).

- BREAK -

- END OF SECOND SITTING -

PART II

ADDITION AND SUBTRACTION

Page 6.

Items 1-4. [Action pictures. Corresponding equations to be completed .]

1. See the box with the little chickens in it. Something is happening in the picture. Underneath is a number story that tells what is happening. But it isn't finished. Look carefully at the picture and then print a numeral in the box, to finish the equation so that it tells what happens in the picture.

2. Now look at the bunny rabbits. Can you see what is happening? Shh! Don't tell! Just read the equation to yourself and then finish it to match the picture.

3. See the ladies with the feathers in their hats? Those extra lines mean that some of them are walking away. There are two numerals missing in the equation for this picture. Please print the right numerals in both boxes so that your equation tells what happens.

4. There are some balls sitting on this table. There is a ledge around it so that they won't fall off. Some more balls are falling down and going to land on the table. Please print a numeral in the box and a numeral in the triangle so that the equation tells what will happen.

Items 5-9. [Choice of + or - sign in simple equations .]

sample. Put your finger on the floppy hat in the next box. This sign says ? (Print + on board) Yes, plus, or add. This sign says ? (Print - on board) Yes, minus or take away. Now here is an equation with a hole in the middle. (Put sample on board). The dotty ring means that a sign is missing.

Maybe it is a plus sign; maybe it is a minus sign. Let's try reading the equation with a plus sign and see if it works. No? Well, how about five minus four equals one? Good! Now you finish the equation right under the hat on your paper.

5-9. Now there are five more equations to finish just the same way. Read each one carefully and see if you need to put a plus or a minus sign in the hole to make the equation tell the truth. Please stop when you come to the double line.

Items 10-21. [Addition and subtraction facts $\leqslant 10$.]

The rest of this page is work you know pretty well. See how many of these equations you can finish correctly. Some are addition and some are subtraction (take away) so BE CAREFUL! Please don't use your counters (blocks, rods,etc.) just for today. I want to see how many of these equations you are ready to do in your head. There are three columns of equations to do. Don't stop until you have finished the whole page.

Page 7

Items 22-25. [Equations with three addends. Items 24 and 25 have = in beginning position: $9=5+\triangle+\triangle$.]

See the box with the tree in the corner. It has some equations that are a little longer. There is one with two triangles to be filled in. The triangles are the same shape so they want the same numeral in them. (If this is totally unfamiliar, demonstrate with a different example, e.g. $\triangle+1+\triangle=7$). There are four equations in this box. Please see if you can make each one of them tell the truth.

Items 26-37. [Second and third year work .]

(Children should be encouraged to do what they know how to, but counting or the

drawing of pictures for more than one or two examples should be discouraged. Alternatively, picture making can be used to interpret a child's understanding of concepts and operations but in that case he should not be credited with mastery.)

Put your finger on the giant slide. It marks the start of the kind of work that you might be going to do next year. Maybe you already know how to do some of them. If you do, please do them quietly. Do as many as you can. But don't worry, you will be learning how to do all of them some day soon. (Papers may be collected as children reach their limit, or they may colour the cover while their classmates work a little longer).

Items 38-40. Reading equations orally from flash cards.

See section on Individual Oral Testing, page 3 .

THAT'S ALL! THANK YOU VERY MUCH!

AMENDMENTS TO FORM A

In the light of experience during administration and item analysis of Form A, the following amendments were subsequently made to Part I.

On page two, item thirteen, which was actually used as a sample during testing, was placed in the position of item twelve and re-labeled 'sample'. This necessitated re-numbering all subsequent items of Part I, there being now only forty-one items in all. Former item twenty-four was also changed to read 34 ⊘'s (baseballs).

PENCIL AND PAPER SESSIONS, FORM B

The area(s) being tested by each group of items is indicated, within square brackets, before the oral instructions are given.

PART I NUMERATION

N. B. REPEAT ALL ESSENTIAL DIRECTIONS! THIS IS NOT A TEST OF LISTENING SKILLS!

C

Items 5-9. [Aural to printing of numerals with understanding of place value .]

5. See the box with the pussy cat in the corner. Put your finger on the pussy cat. Print the numeral 27 in the box with the pussy cat.

6. See the box with the ice-cream cone in the corner. Put your finger on the ice-cream cone[7]. Print the numeral 16 in the box with the ice-cream cone.

7. See the box with the little flower in the corner. Print the numeral that is one less than 40 (comes before 40) in the box with the little flower.

8. See the box with the tennis ball in the corner. Print the numeral that comes between 63 and 65 in that box.

9. See the box with the party hat in the corner. Print the numeral that means 3 tens and 5 ones in that box.

Items 10-11. [Recognizing printed numerals (aural to reading) with understanding .]

10. Put your finger on Charlie Brown's baseball cap. See the row of nmmerals that come after it in the same box. Put a big X on the greatest(highest) numeral in that box.

11. Put your finger on the football. See the row of numerals that come after the football. Put a big X on the numeral one hundred twenty-seven.

Items 12-14. [Matching numeral to picture, involving counting of pictured objects and recognition of numerals <100 .]

sample. Put your finger on the box with the tea cups in it. How many tea cups are there? Yes, that's right. Now put the point of your pencil inside the tea cup box. Do you see this long column of numerals? (Demonstrate). Now draw a line from the tea cup box to the numeral that tells exactly how many tea cups there are. (Check that this item is correctly completed).

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[7] Use the technique 'Put your finger on---' whenever advisable, to ascertain that the pupils are working in the correct place.

C

12-14. Now there are some more boxes of objects (pictures) for you to match with the correct numeral, BUT BEWARE! SOMEONE HAS PUT IN TOO MANY NUMERALS! Here is a little hint: If there are lots of things for you to count you will find that they are in groups of fives or tens, to help you. (Do not labour this point).

12. See all the little stars. Can you find out how many stars there are altogether? Fine, now draw a line to the right numeral. BE SURE YOUR LINE STARTS INSIDE THE BOX.

13. Now you can match the little seeds to their numeral. Be sure your pencil starts INSIDE the box.

14. Now you can match the marbles with the right numeral. (If a child claims he cannot find the correct numeral it can be suggested that he print the numeral he is looking for beside the picture).

Item 15. [Reading a numeral and printing its name .]

sample. Put the numeral 4 on the chalkboard and print the word 'four'. Have the children do the same in the space provided.

15. Now do you see this big numeral at the bottom of the page? If you know what it is called, don't say a word! Just print its name, the way it sounds to you. (Spelling will not be marked but the ability to put down sound symbols which indicate that the child knows the correct name. Scoring hint given later. See page 24).

Page 3.

Items 16-17. [Choosing correct pictures for printed numeral. Multiple choice .]

16. See the seagull flying in the margin. Put your finger on the seagull. Now look at the numeral in the box beside the seagull. After it there is a row of boxes with seagulls in them. One box has just as many seagulls as the numeral

says. Put a big X on the box that has as many seagulls as the numeral says. Don't forget to look and see if there is a quick way of counting by groups.

17. See the little cherry in the margin. Put your finger on the cherry. Now look at the numeral next to the cherry. After it there is a row of boxes with cherries in them. There were so many cherries in one box that they had to be put into baskets. There are ten cherries in each little basket. Put a big X on the box that has as many cherries as the numeral says. Don't forget to look and see if there is a quick way of counting by groups.

Items 18-21. [Number sequences. Counting without pictures. Completion .]

sample. Now it is your turn to count without any pictures and find out what numerals are missing. BEWARE! EACH ROW IS DIFFERENT! and some rows are quite tricky! I think everyone can do the row that comes after the balloon. (sample) Put your finger on the balloon. What kind of counting is it? That's right! Counting by ones. Let's read together: 3, 4, something, 6, 7, something, 10, period. Now please print the correct numerals on the lines where we read 'something'. (Demonstrate on the chalkboard if necessary, and make sure the children complete this line correctly).

18. Now put your finger on the little Christmas tree. Very quietly, to yourself, read the numbers that come after it. Figure out what kind of counting it is. Then fill in the missing numbers like we did before.

19. Now look at the row with the button. It might be a different kind of counting. See if you can figure it out and put in the missing numbers.

20. The apple row is started but not finished. Please finish it carefully.

21. The row with the little stick man needs to be finished too. Please finish it carefully.

Items 22-23. [Making pictures to match the numeral .]

22. Now it is your turn to make pictures! See the baseball bat with the

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numeral beside it. Shh! Don't tell what it says! In the long empty box beside the bat I want you to make just as many bats as the numeral says. Make the bats in the long empty box. Please don't put any tape or other trimming on the handles, until you have made all the bats.

23. See the little X with a numeral beside it? Please draw as many X's as the numeral says. Try to get your X's to fit into the long box. (Overflow may of course be placed below). (A few children can profit from a suggestion of grouping in fives and/or tens as they draw).

- TIME FOR A BREAK -

Page 4.

Items 24-27. [Concepts of greater than, less than, equals, one greater than, one less than .]

(Three versions are provided: 1) reading the words, as in Seeing Through Arithmetic,[8] 2) reading the words in i.t.a.printing, 3) symbols. (page 4 *) Samples are provided to be done on the chalkboard by the examiner, and by the children in their booklets).

24-25. [Printing correct phrase or symbol .]

sample 1. Let us read what it says in the boxes (what the symbols say) at the top of this page. After the feather it says 9, and then there is a long line (dotted circle) and then 7. Now what can we print on the line (in the circle) to make a sentence that tells the truth? That's right! Nine is greater than seven. Please print 'is greater than' on the line (in the circle).

sample 2. Now look at the tea cup. Which box (symbol) are we going to use to make this a true sentence? That's right! 57 is less than 75.

25 and 26. Now there are two sentences for you to do, as quietly as you can. There is the star sentence, and the sentence that starts with the wizard's

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8 Maurice L.Hartung, et al., Seeing Through Arithmetic, Book I, Teacher's Edition, (Toronto: W.J.Gage Ltd., 1965)

hat. You will have to use one of the boxes (symbols) again. One of the boxes (symbols) had to be used twice. Now see if you can make both rows tell the truth.

DON'T FORGET TO REPEAT IMPORTANT INSTRUCTIONS!

26-27. [Choosing correct phrase or symbol. Multiple choice .]

sample 3. See the little moon. Beside it we can read 31+1. How much does that make? All right, remember in your head how much 31+1 is. Now we have to find out which phrase (symbol) in the middle fits with 31+1 and with the numeral at the end of the row to make a true sentence. Let's try '31+1 is greater than 32'. No? (Continue in this vein until pupils are satisfied with 'is equal to'). Good, now put a ring around 'is equal to' and read the sentence to yourself to make sure it is true.

sample 4. Let's try the row that starts with a sun and do it just the same way. (Elicit as before until item is completed).

26 and 27. Now here are two more jobs for you to do just the same way, all by yourself. Look at the row that starts with a sailboat. Read quietly and carefully, and think what numeral the first expression stands for. Now read each phrase (symbol) in the middle and the numeral at the end. Put a ring around the phrase (symbol) in the middle which makes the line tell the truth.

Now try the row that starts with the mushroom. Read quietly and carefully. Think what numeral the first expression stands for. Now read each phrase(symbol) in the middle and the numeral at the end. Put a ring around the phrase (symbol) in the middle which makes the line tell the truth.

Page 5. [Place value .] (i.t.a.5 has 'wuns' for 'ones').

Items 28-32. [Pictures: hundreds, tens and ones. Numerals to print .]

28. Here are some pencils tied up in bundles. There are ten pencils in each bundle. Some pencils were left over. In the box at the end of the row,

print the numeral that tells how many pencils there are altogether.

29, 30, 31.. Here are some logs. They were too heavy to put into bundles so someone piled them up. There are ten logs in each pile. Some logs were left over. Please print how many piles of ten there are, and how many ones left over. Then print how many logs there are altogether.

32. HERE IS A REALLY TRICKY ONE! (Very few first year children are ready for this one). I'D LIKE YOU TO TRY IT. There are lots and lots of chalkboard brushes. Every box has ten brushes in it. There are so many boxes that they had to be piled up. DO YOU SEE THE BRUSHES THAT WERE LEFT OVER? Now see if you can figure out how many chalkboard brushes there are altogether. If you can, don't say a word! Just print the correct numeral in the box at the end of the row.

Items 33-36. [____tens ____ones = ____. Completion, from words to numeral and vice versa .]

sample. See the box with the pear in the corner. Beside the pear it says (pause) 'Three tens and nine ones equals ?' Yes, that's right, thirty-nine. Print 39 on the line.

33-34. Now underneath are two more jobs for you to do all by yourself. Read them carefully and print the numeral that the tens and ones stand for, each time.

sample. See the box with the grapefruit in it. This time there is a numeral and you are to print how many tens and how many ones that numeral stands for. Thirty-five is ? Yes, that's right, three tens and five ones.

35-36. Now underneath the grapefruit are two more jobs for you to do, just like that one, all by yourself.

Items 38-39. [Understanding of place value, including hundreds, with numbers $< 10,000$.]

37. Put your finger on Snoopy's night cap. Now look at the row of numerals that comes after it. One of those numerals has a three in the hundreds place and a

three in the ones place. Put a big X on the numeral that has a three in the hundreds place and a three in the ones place. Be sure that you make ONLY ONE X please.

38. Put your finger on the jet plane. See the big numeral in the box beside it. Now look carefully at the row of numerals that comes after the box. One of the numerals in that row tells how many ones the numeral (digit) seven in the box stands for. Put a big X on the numeral in the row which tells how many ones the numeral (digit) seven in the box stands for. JUST MAKE ONE X please.

Items 39-41. [Equations requiring understanding of base ten numeration .]

39-41. Put your finger on Snoopy's dog house. Between Snoopy's dog house and the stop sign are three equations. The numbers look pretty big but if you READ them carefully, one at a time, you should be able to figure out the right numeral to put in the box to make each equation tell the truth. You shouldn't have to do any counting. Remember to do some thinking before you fill in a box. If you are not quite sure, try again. (Do not allow excessive time for the children who make rows of dots or otherwise try to count).

- BREAK -

- END OF SECOND SITTING -

PART II

ADDITION AND SUBTRACTION

Page 6.

Items 1-4. [Action pictures. Corresponding equations to be completed .]

1. See the box with the bunny rabbits in it. Something is happening in the picture. Underneath is a number story that tells what is happening. But it isn't finished. Look carefully at the picture and then print a numeral in the box to finish the equation so that it tells what happens in the picture.

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2. Now look at the picture of the birds. Can you see what is happening? Shh! Don't tell! Just read the equation to yourself. Then finish it to match the picture.

3. Now look at the funny little boys. Can you see which way some of them are running? Good! Now be careful! There are two numerals missing in the equation for this picture. Please print the right numerals in both boxes so that your equation tells what happens.

4. There are some balls sitting on this table. But some of them have fallen off. That is what those lines behind them mean. Please print a numeral in the box and a numeral in the triangle so that the equation tells what has happened.

Items 5-9. [Choice of + or - sign in simple equations .]

sample. Put your finger on the spinning top in the next box. This sign says ? (Print + on board). Yes, plus, or add. This sign says ? (Print - on board). Yes, minus, or take away. Now here is an equation with a hole in the middle. (Put sample on board). The dotty ring means that a sign is missing. Maybe it is a plus sign; maybe it is a minus sign. Let's try reading the equation with a plus and see if it works. No? Well, how about five minus three equals two? Good! Now you finish the equation right under the top on your paper.

5-9. Now there are five more equations to finish just the same way. Read each one carefully and see if you need to put a plus or a minus sign in the hole to make the equation tell the truth. Please stop when you come to the double line.

Items 10-21. [Addition and subtraction facts $\leqslant 10$.]

The rest of this page is work you know pretty well. See how many of these equations you can finish correctly. Some are addition and some are subtraction (take away), so BE CAREFUL! Please don't use your counters (blocks, rods, etc.)

just for today. I want to see how many of these equations you are ready to do in your head. There are three columns of equations to do. Don't stop until you have finished the whole page.

Page 7.

Items 22-25. [Equations with three addends. Items 24 and 25 have = in beginning position: 7=3+△+△ .]

See the box with the butterfly in the corner. It has some equations that are a little longer. There is one with two triangles to be filled in. The triangles are the same shape so they want the same numeral in them. (If this is totally unfamiliar, demonstrate with a different example, e.g. △+2 +△ =8). There are four equations in this box. Please see if you can make each one of them tell the truth.

Items 26-37. [Second and third year work .]

(Children should be encouraged to do what they know how to, but counting, or drawing of pictures for more than one or two examples should be discouraged. Alternatively, picture making can be used to interpret a child's understanding of concepts and operations, but in that case, he should not be credited with mastery).

Put your finger on the beach ball. It marks the start of some kind of work that you might be going to do next year. Maybe you already know how to do some of them. If you do, please do them quietly. Do as many as you can, but don't worry, you will be learning how to do all of them some day soon. (Papers may be collected as children reach their limit, or they may colour the cover while their classmates work a little longer).

Items 38-40. [Reading equations orally from flash cards .]

(See section on Individual Oral Testing, pages 3-4).

THAT'S ALL! THANK YOU VERY MUCH!

SCORING, PENCIL AND PAPER SESSIONS

General Remarks

One point is given for each item correct throughout the test. No part marks are awarded. If a digit is reversed, e.g. '6' for '2' that is not considered an error. However, if two digits are in reverse order, e.g. '21' for '12', that is incorrect. An utterly ambiguous answer should be marked wrong. (It is a disservice to a child to give him credit for such a response). If more than one response is marked, in the multiple choice items, that is incorrect, unless an obvious attempt has been made to eliminate all but one.

Specific Items, Part I

Printing names of numbers, item 15 or 16

Correct spelling is not necessary but the child must be able to print sound symbols which indicate he knows the correct number name and can commit it to paper in word form.

Examples:

Acceptable: hundrtanin, hunderdsivn, wun hudrdsen.

Not Acceptable: one oh nin, wun hunderdsevnty.

Number sequences, items 19-22 or 18-21

Reversal of a single digit is acceptable, but two digits in reverse order is an error. Complete row must be correct.

Making pictures to match the numeral, items 23-24 or 22-23

The child's intent is important. The boxes are provided as a convenient guide not a restriction, and no penalty should be attached to overflow. One quick way of marking the large number of stars (balls or X's) is to run a marking pencil under or around five at a time, until the answer is obviously correct or not.

Greater than, etc. page 4, items 25-26 or 24-25

Intent is what should be marked, not correct spelling.

Place value

Corresponding item numbers and responses are as follows:

Form A		Form B	
item 30	2 tens	item 29	2 tens
item 31	5 ones	item 30	4 ones
item 32	25 logs	item 31	24 logs

SUGGESTED AMENDMENTS TO FORM B

In the light of experience during administration of Form B, the following amendments to Part I are considered advisable, to get the children off to a good start.

On page two, place items ten and eleven first, re-numbering them as five and six. Items five to nine would then follow and be re-numbered as items seven to eleven.

TEST CONSTRUCTION

The Preliminary Form of the test, containing 75 items in Part I (numeration skills) and 56 items in Part II (Computation Skills) was administered to four classes containing children with a wide range of ability, achievement and socio-economic background from two school districts in the Lower Mainland of British Columbia.

Part I(Pr.) was completed by 86 children.

Part II(Pr.)was completed by 113 children.

Time table problems prevented the completion of Part I by all of the children in one of the schools.

Computer assisted item analysis was carried out on the results of this testing on the IBM 360, Model 67, at the University of British Columbia (Program TIA). Two parallel forms, Form A and Form B, were developed with the aid of this analysis. Form B follows very closely actual items used in the Preliminary Form. Form A which contains many alternate (parallel) items was cross-validated in another school district in the Lower Mainland on two classes in different schools, containing children with a wide range of ability, achievement and socio-economic background.

Test I(A) was completed by 47 children.

Test II(A) was completed by 49 children.

Computer assisted item analysis was carried out as before.

Reliability

Computer assisted item analysis gave the following KR-20 values for the Preliminary Form:

Test I(Pr.) : KR-20=0.9530

Test II(Pr.) : KR-20=0.9390

Computer assisted item analysis of Form A results gave the following KR-20 values:

Test I(A) : KR-20=0.9435

Test II(A): KR-20=0.9291

The slight reduction in KR-20 values was to be expected since the tests had been considerably shortened in order to obtain a reasonably practical length for first year students.

Content Validity

During test construction the following texts and teacher's guides were consulted: Seeing Through Arithmetic[1], Elementary School Mathematics[2], Number Patterns[3], and the Greater Cleveland Mathematics Program[4]. The arithmetic section of the Metropolitan Achievement Tests, Primary I Battery[5] suggested a format pleasing to young children. A few items similar to those in this test were included in Part I, thus: two multiple choice items on recognition of printed numerals, and knowledge of "greatest" number (aural to printed) and four items involving writing of numerals from oral instructions. The final forms also contain two items on place value, similar to items in Ashlock's Test of Understanding of Selected Properties of a Number System: Primary Form[6] which was not available

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1 Maurice L.Hartung, et al., Seeing Through Arithmetic, Book I, Teacher's Edition, (Toronto: W.J.Gage Ltd., 1965).

2 Robert E.Eicholz, et al., Elementary School Mathematics, Second Edition, Teacher's Edition, Books I and II, (Don Mills, Ontario: Addison-Wesley (Canada) Ltd., 1969).

3 Roberta Chivers, et al., Number Patterns, Books I and II, Teacher's Edition, (Toronto: Holt Rinehart & Winston, 1966).

4 Educational Research Council of Greater Cleveland, Greater Cleveland Mathematics Program, Teacher's Guide for First Grade (Chicago: Science Research Associates, Inc., 1961, 1962).

5 Metropolitan Achievement Tests, Primary I Battery, Form B. (New York: Harcourt, Brace and World, Inc., 1959).

6 Robert B.Ashlock and Ronald C.Welch, "A Test of Understandings of Selected Properties of a Number System: Primary Form", Bulletin of the School of Education, Indiana University, Vol.42, No.2, March 1966.

at the time of construction of the Preliminary Form. Teachers were very helpful during the trials in pointing out items which might be ambiguous to the children. They commented favourably on the comprehensiveness of the Preliminary Form but agreed with the administrator that it was too long to be practical.

Concurrent Validity

Concurrent validity of the Preliminary Form was established by comparison of results of one class with those provided by an able and experienced teacher. Her class list showed the children grouped and approximately ranked according to her testing and observation throughout the year. Correlation was very high between the two sets of results, the only discrepancies being in the 'middle zone'; those were the children who vary the most from day to day and cannot be too firmly assessed at this age. (See Scatter Diagram, Figure I)

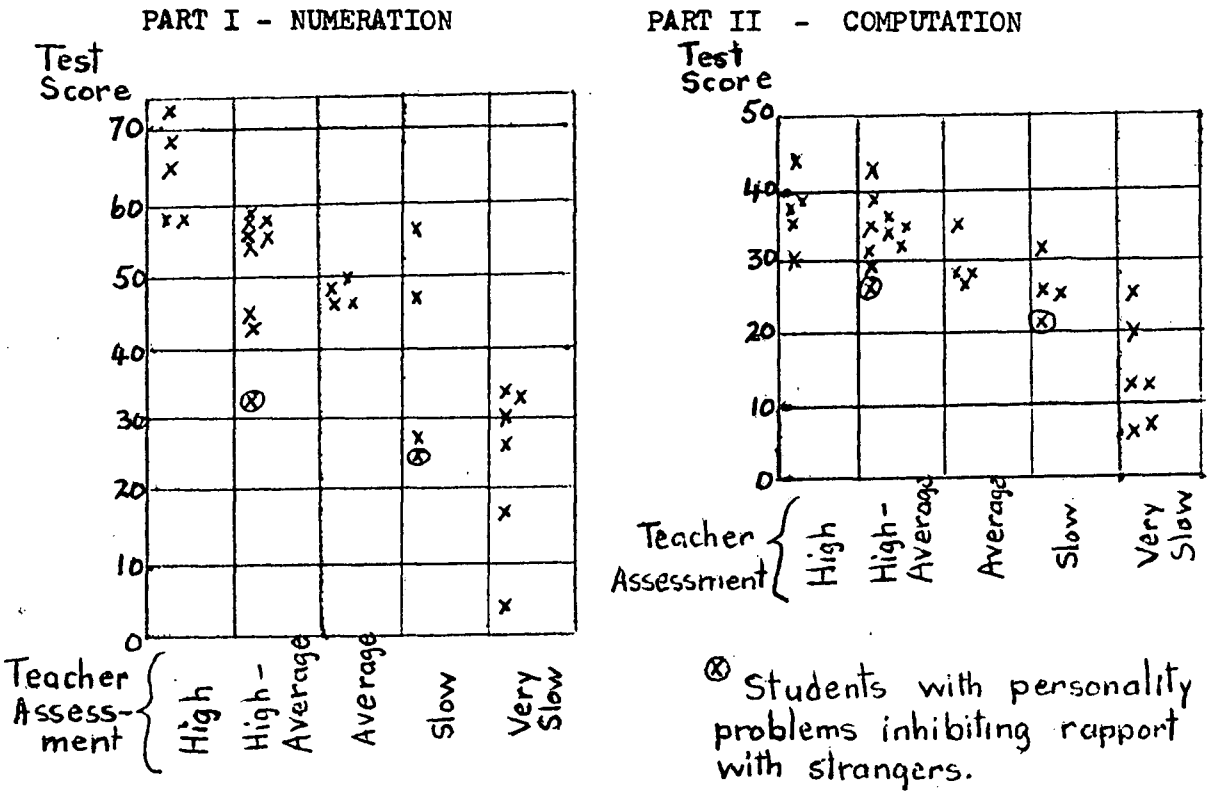

Figure I Concurrent Validity - Preliminary Form

Concurrent validity of Form A was affirmed by very high correlation with the year-end results already established in both schools where corss-validation was carried out: "To a child" said one teacher.

Concurrent validity was similarly established for Form B with a class of 34 children in another school district.

Summary of Areas Tested Revised Forms A and B

PART I: NUMERATION

Items 1-4. Oral reading of printed numerals.

Item 5. Recognition of printed numerals (aural to reading).

Item 6. As item 5 but includes concept of greatest number.

Items 7-11. Aural to printing of numerals.

Item 9. 'one less than'.

Item 10. 'between'.

Item 11. Place value, 'tens' and 'ones'.

Items 12-14. Matching numeral to pictures, involving counting of pictured objects and recognition of numerals 100.

Item 15. Reading a numeral between 100 and 110 and printing its name.

Items 16-17. Choosing correct pictures for printed numeral, multiple choice. Ability to count by fives and tens can be used.

Items 18-21. Number sequences. Counting without pictures, completion.

Items 22-23. Making pictures to match the numeral.

Items 24-27. Concepts of greater than, less than, equals, one greater than, one less than, using either the printed words or the symbols.

Items 28-32. Pictures: hundreds, tens and ones. Numerals to print.

Items 33-36. Completion from words to numerals and vice cersa of ___tens___ones=___.

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Items 37-38. Understanding of place value, including hundreds with numbers $< 10{,}000$. Multiple choice from oral instructions.

Items 39-41. Equations requiring understanding of base ten numeration.

PART II COMPUTATION

Items 1-4. Action pictures. Corresponding equations to be completed.

Items 5-9. Choice of + or - sign in simple equations.

Items 10-21. Addition and subtraction facts $\leqslant 10$, in equation form.

Items 22-25. Equations with three addends. Items 24 and 25 have = in beginning position, e.g. $7=3+\triangle+\triangle$.

Items 26-31. Addition and subtraction facts 11-18, in equation form.

Items 32-35. Extensions of addition and subtraction facts <10, in equation form.

Items 36-37. Addition and subtraction of two digit numerals in vertical computational form, no carrying or borrowing.

Items 38-40. Oral reading of complete printed equations.

ADDENDA

When Form A was administered, the items now numbered 10 and 11 preceded the present items 5 through 9. This gave the children a much better start, psychologically. It is recommended that, if these tests are used again, items 10 and 11 be placed first, on page 2 of either form, followed by the present items 5 through 9.

F. J.

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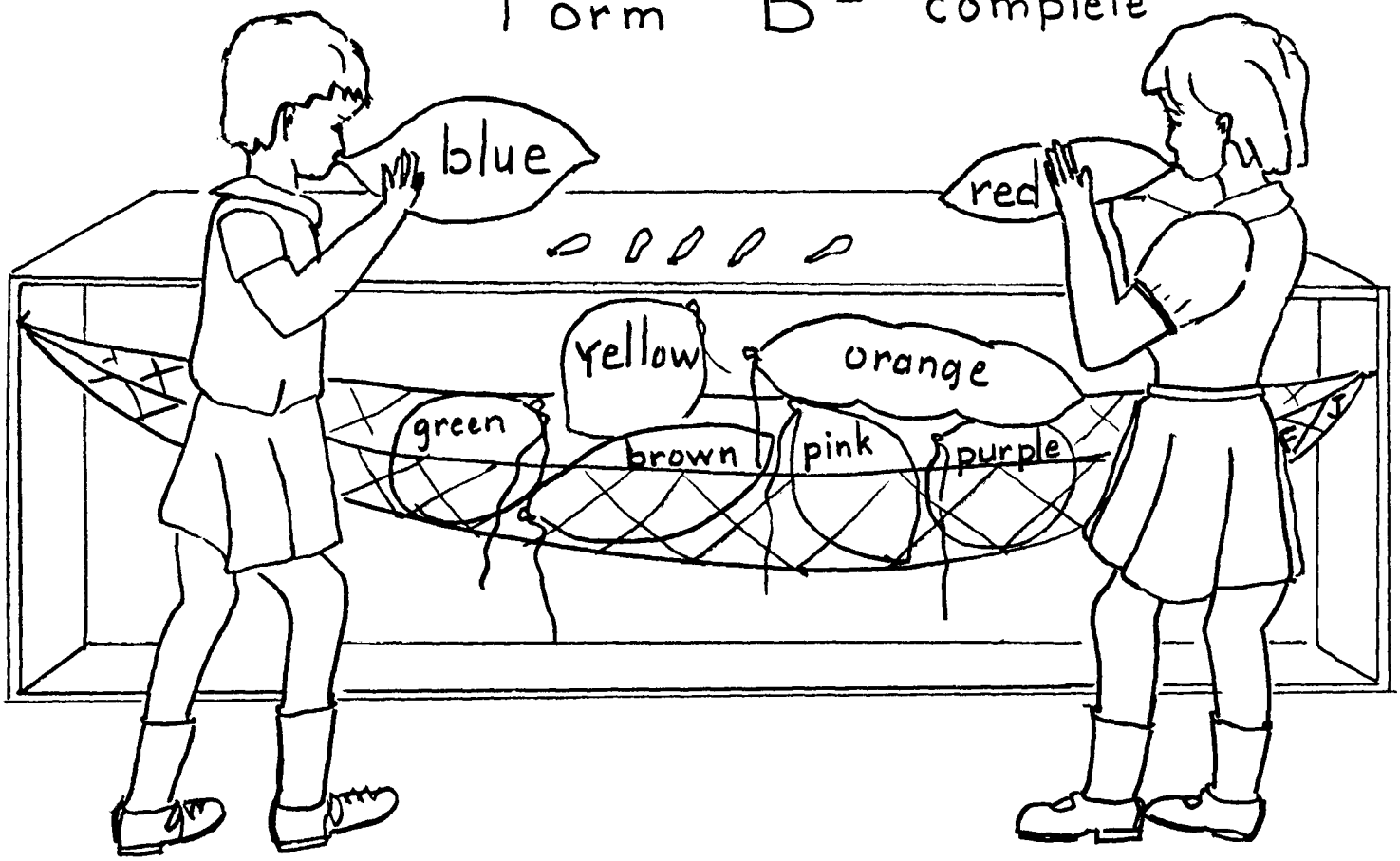
First Year - Primary

Power Test of Certain Arithmetic Skills

-Form B- complete



My name is _____

Teacher _____

School _____ I

Age _____

Division _____ II

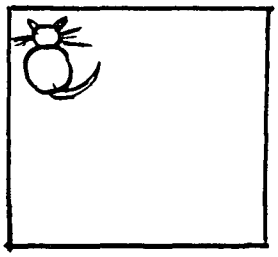
Individual Oral Testing				Final Score
Sample				
1				
2				
3				
4				
38				
39				
40				

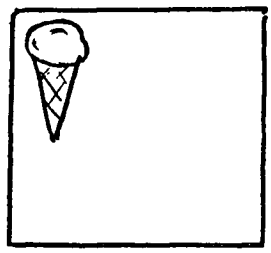
-Forrest Johnson-
June 1970

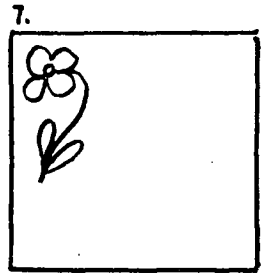
Form B

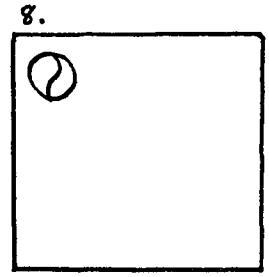
Part I

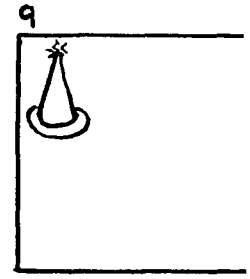
D 2.

5. 

6. 

7. 

8. 

9. 

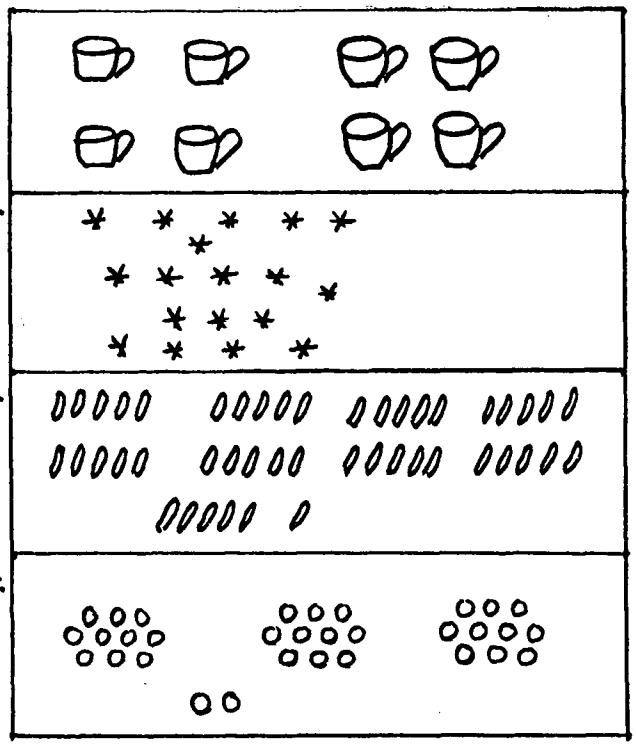
10. 

13 6 26 15

11. 

27 127 113 137

sample



12.

13.

14.

18

81

8

46

32

64

23

F.J.

sample

4

15

107

Form B

D 3.

16.	25					
17.	17					

sample.

2, 3, __, 5, 6, __, __, 9.

18. 35, __, 37, 38, __, __, 41.

19. 24, 23, 22, __, 20, __, __, 17, __.

20. 96, 97, 98, 99, __, __, __.

21. 30, 40, 50, __, __, __.

22	7 /'s	
23	24 x's	

Form B

D4

is greater than	is less than	is equal to

sample 1. 9 _____ 7

sample 2. 57 _____ 75

24. 38 _____ 38

25. 41 _____ 14

---

sample 3. 31 + 1 — is greater than / is less than / is equal to — 32

sample 4. 82 − 1 — is greater than / is less than / is equal to — 28

26. 15 − 1 — is greater than / is less than / is equal to — 15

27. 46 + 1 — is greater than / is less than / is equal to — 47

Form B

D 4*

> or < or =

sample 1. 9 ◯ 7	sample 2 57 ◯ 75
24. 38 ◯ 38	25. 41 14

sample 3. 31 + 1 > = < 32

sample 4. 82 − 1 > = < 28

26. 15 − 1 > = < 15

27. 46 + 1 > = < 47

Form B

D)ita 4

is græter than	is less than	is eekwaul tœ

sampl 1. 9 _____ 7

sampl 2. 57 _____ 75

24. 38 _____ 38

25. 41 _____ 14

---

sampl 3. 31+1 — is græter than / is less than / is eekwaul tœ — 32

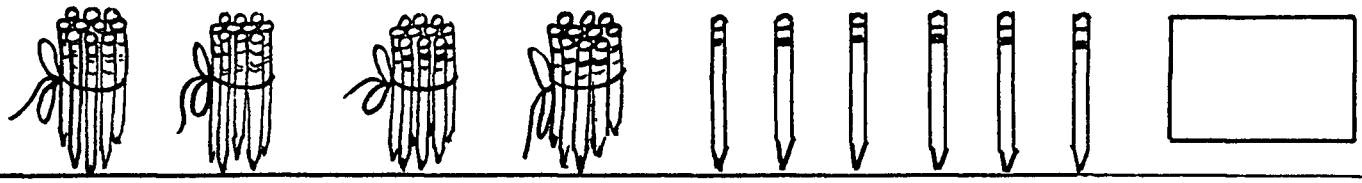
sampl 4. 82−1 — is græter than / is less than / is eekwaul tœ — 28

26 15−1 — is græter than / is less than / is eekwaul tœ — 15

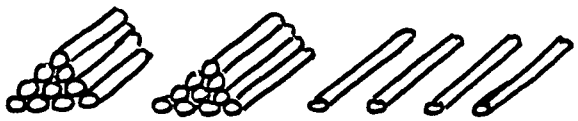
27. 46+1 — is græter than / is less than / is eekwaul tœ — 47

Form B [For ita : 'ones' → 'wuns'] D5

28.

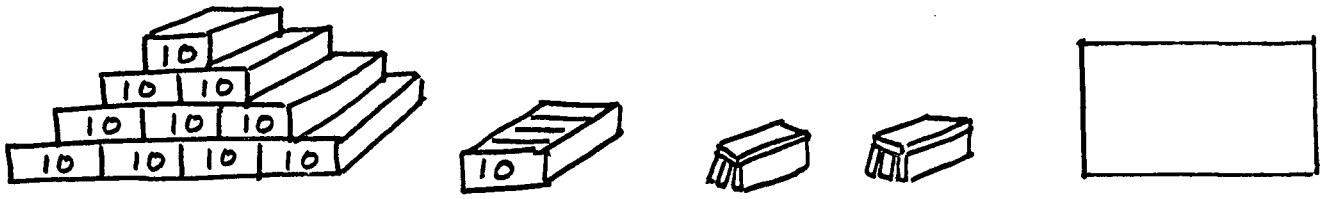


29.
30.
31.



___ tens ___ ones = ___ logs

32.



sample

5 tens 8 ones = ____

33. 1 ten 2 ones = ____

34. 2 tens 10 ones = ____

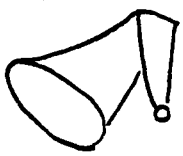
sample

35 = ___ tens ___ ones

35. 83 = ___ tens ___ ones

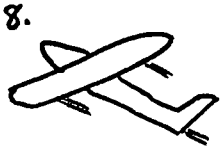
36. 41 = ___ tens ___ ones

37.



334 33 343 433

38.

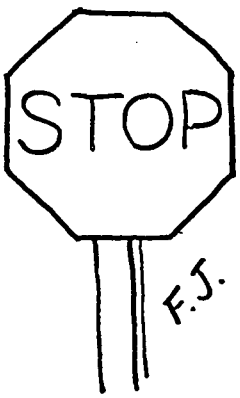


6578 7 70 700 7000

39. 46 = 40 + ☐

40. 32 = ☐ + 2

41. 20 + 10 + 10 + 3 = ☐

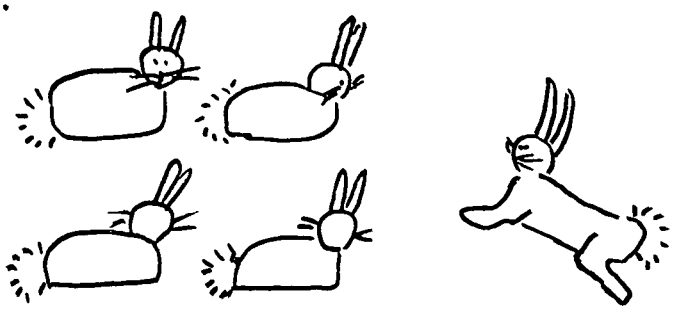


Form B

Part II

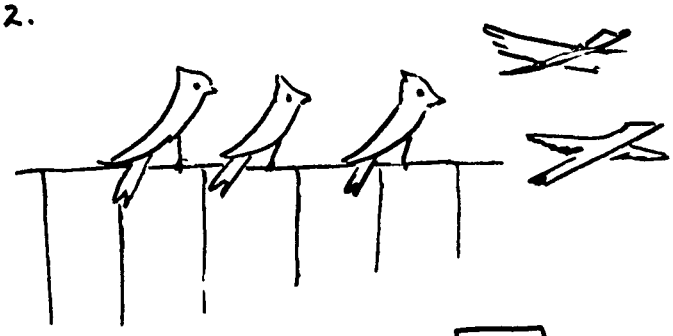
D6

1.



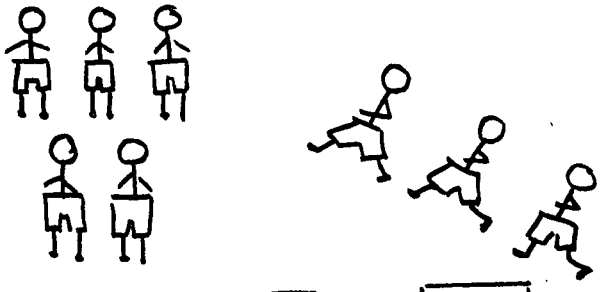
4 + 1 = □

2.



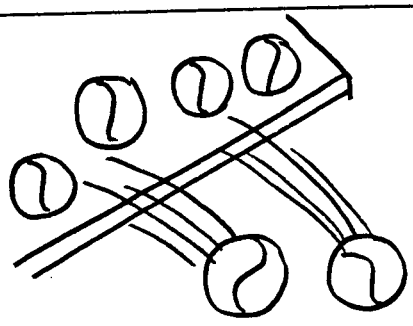
5 − 2 = □

3.



5 + ⬡ = □

4.



6 − □ = △

+ or − ?

sample 5 ◯ 3 = 2

5. 6 ◯ 1 = 5

6. 3 ◯ 4 = 7

7. 5 ◯ 3 = 2

8. 7 ◯ 1 = 8

9. 9 ◯ 5 = 4

10. 1 + 3 = ___

11. 6 + 0 = ___

12. 5 + 5 = ___

13. 3 + 2 = ___

14. 5 − 1 = ___

15. 7 − 4 = ___

16. 4 − 0 = ___

17. 9 − 7 = ___

18. 6 − 5 = ___

19. 2 + 7 = ___

20. 8 − 6 = ___

21. 4 + 6 = ___

F.J.

Form B

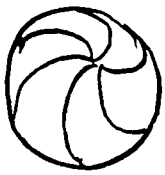
D 7

22. $3 + 4 + 1 = \square$

23. $2 + 2 + 2 = \square$

24. $6 = 2 + \square + 1$

25. $7 = 3 + \triangle + \triangle$



26. $7 + 5 =$ ___

27. $8 + 3 =$ ___

28. $15 - 6 =$ ___

29. $12 - 8 =$ ___

30. $9 + 7 =$ ___

31. $17 - 9 =$ ___

32. $23 + 3 =$ ___

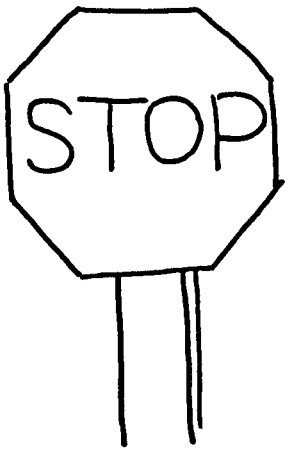
33. $34 + 2 =$ ___

34. $47 - 5 =$ ___

35. $64 - 2 =$ ___

36.
$$\begin{array}{r} 22 \\ +\ 31 \\ \hline \_\_\_ \end{array}$$

37.
$$\begin{array}{r} 36 \\ -\ 14 \\ \hline \_\_\_ \end{array}$$

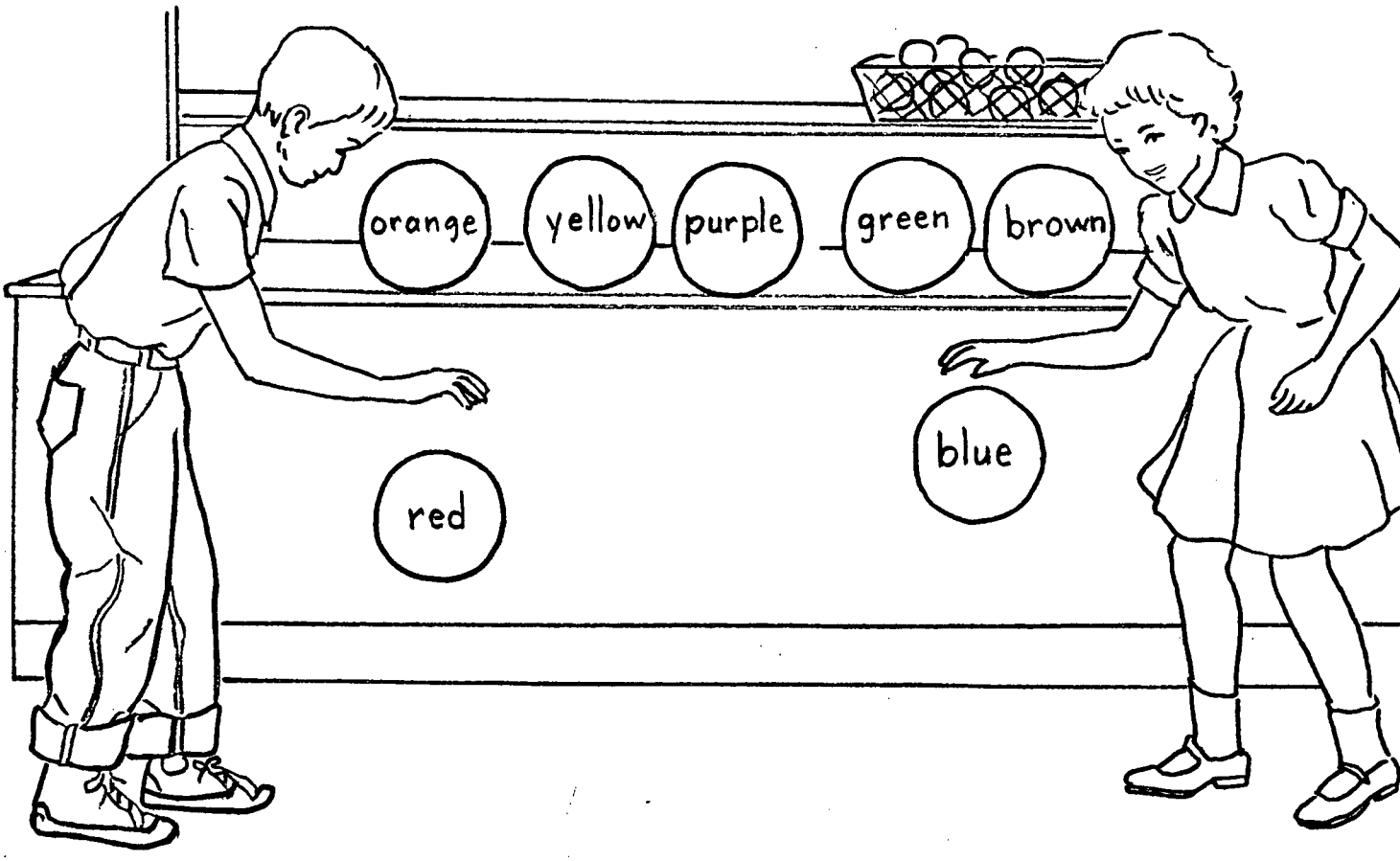


F.J.

First Year - Primary

Power Test of Certain Arithmetic Skills

-Form A- (revised)



My name is _____

Teacher _____

School _____

Age _____

Division _____

-Forrest Johnson-
June 1970

	Individual Oral Testing			Final Score
	Sample			
I	1			
	2			
	3			
	4			
II	38			
	39			
	40			

Form A
-revised

Part I

E 2.

5. 

6. 

7. 

8. 

9. 

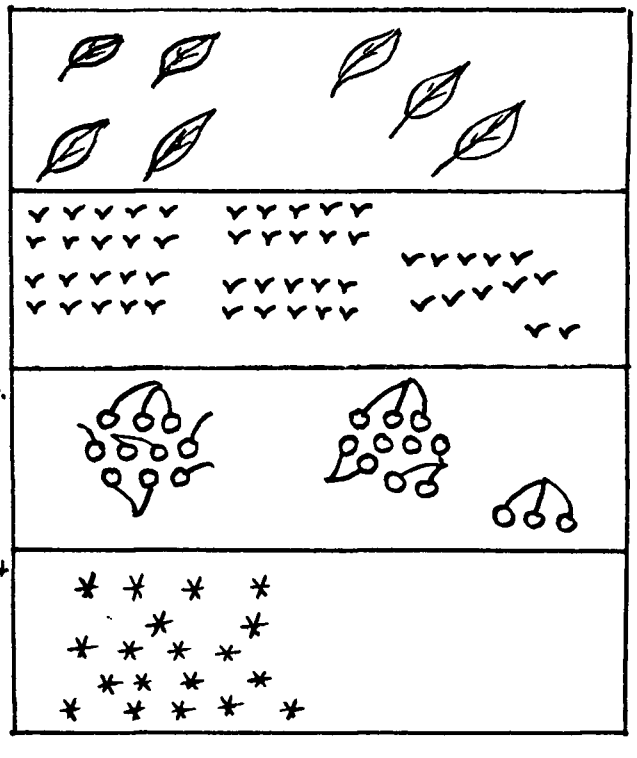
10. 

13 7 27 16

11. 

136 113 26 126

sample



12.

13.

14.

25

52

91

7

19

32

23

F.J.

sample

6

15.

109

Form A
revised

E 3.

16.	24					
17.	16					

sample.

3, 4, ___, 6, 7, ___, ___, 10.

18. 73, ___, 75, ___, 77, 78, ___, ___, 81.

19. 34, 33, 32, ___, 30, ___, ___, 27, ___.

20. 40, 50, 60, ___, ___, ___.

21. 97, 98, 99, ___, ___, ___.

22.	8 ⊢'s	
23.	34 O's	

STOP

Form A
-revised

E 4

is greater than	is less than	is equal to

sample 1. 8 _____ 6

sample 2. 48 _____ 84

24. 57 _____ 57

25. 31 _____ 13

---

sample 3. 21 + 1 — is greater than / is less than / is equal to — 22

sample 4 62 − 1 — is greater than / is less than / is equal to — 26

26. 18 − 1 — is greater than / is less than / is equal to — 18

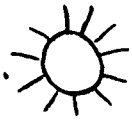
27 34 + 1 — is greater than / is less than / is equal to — 32

Form A
revised

E)ita 4

is græter than	is less than	is eekwaul tœ

sampl 1.  8 _____ 6

sampl 2.  48 _____ 84

24.  57 _____ 57

25.  31 _____ 13

---

sampl 3.  21 + 1 — is græter than / is less than / is eekwaul tœ — 22

sampl 4.  62 − 1 — is græter than / is less than / is eekwaul tœ — 26

26.  18 − 1 — is græter than / is less than / is eekwaul tœ — 18

27.  34 + 1 — is græter than / is less than / is eekwaul tœ — 32

Form A
- revised

E)4*

> or < or =

sample 1. 8 ◯ 6	sample 2. 48 ◯ 84
24. 57 ◯ 57	25. 31 ◯ 13

sample 3. 21 + 1 > = < 22

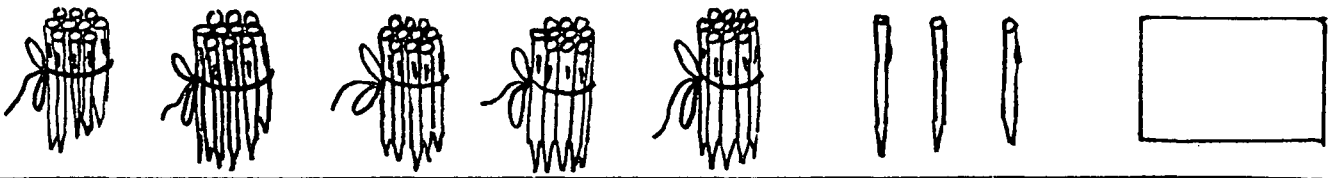
sample 4. 62 − 1 > = < 26

26 18 − 1 > = < 18

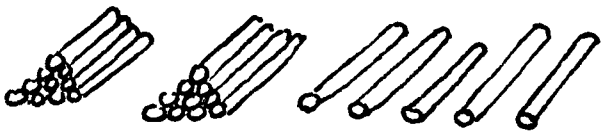
27. 34 + 1 > = < 35

ita E 5

28.

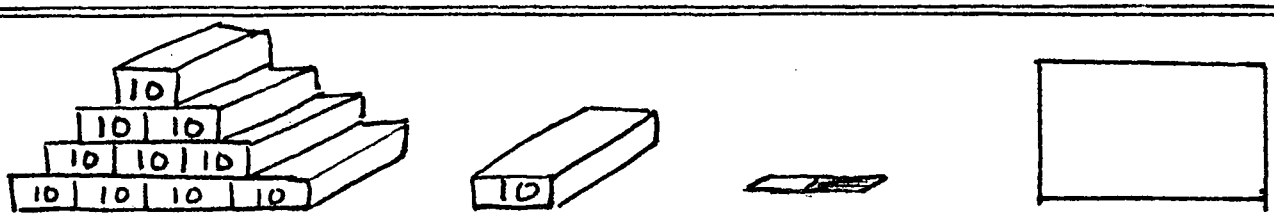


29.
30.
31.



___ tens ___ wuns = ___ logs

32.



sampl

3 tens 9 wuns = ___

33.

1 ten 3 wuns = ___

34. 4 tens 10 wuns = ___

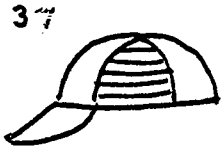
sampl

73 = ___ tens ___ wuns

35 31 = ___ tens ___ wuns

36 48 = ___ tens ___ wuns

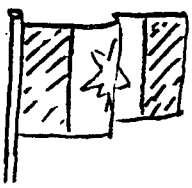
37



44 443 344 434

38.

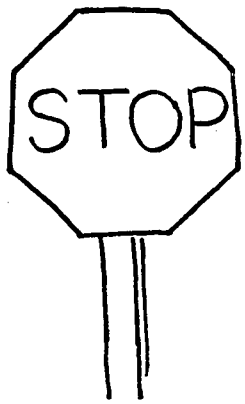
5487 8 80 800 8000



39. 49 = 40 + ☐

40. 26 = ☐ + 6

41. 30 + 10 + 10 + 4 = ☐

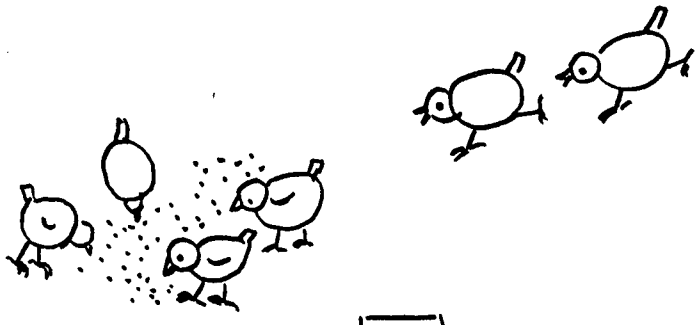


Form A

Part II

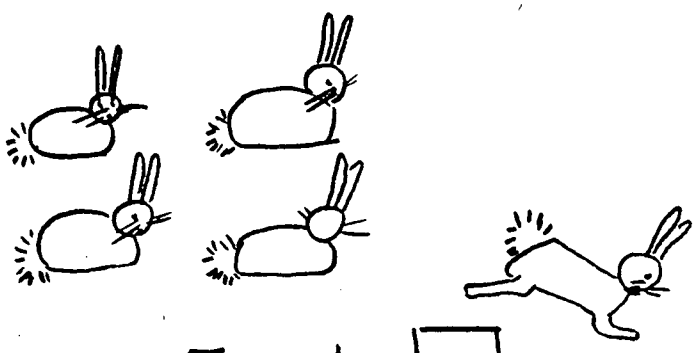
E 6.

1.



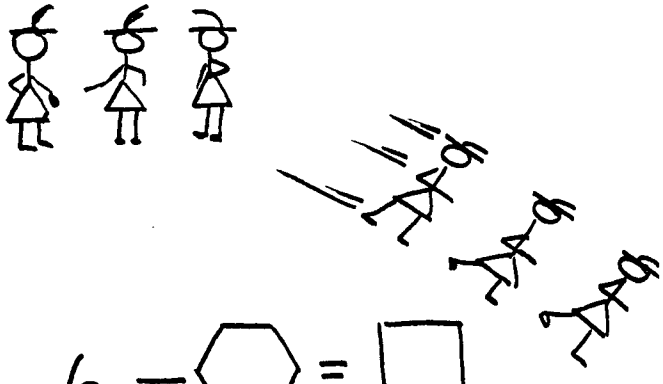
4 + 2 = □

2.



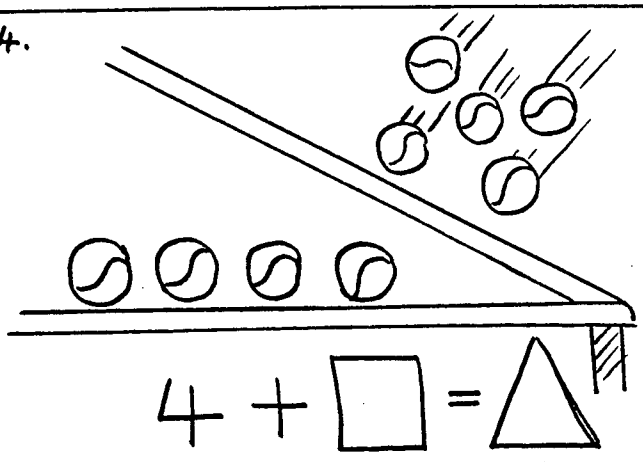
5 − 1 = □

3.

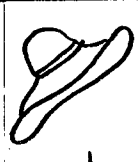


6 − ⬡ = □

4.



4 + □ = △



+ or − ?

sample 5 ◯ 4 = 1

5. 7 ◯ 5 = 2

6. 6 ◯ 1 = 7

7. 6 ◯ 1 = 5

8. 7 ◯ 3 = 4

9. 3 ◯ 5 = 8

10. 2 + 1 = ___

11. 3 + 3 = ___

12. 0 + 6 = ___

13. 2 + 7 = ___

14. 4 − 1 = ___

15. 8 − 6 = ___

16. 3 − 0 = ___

17. 10 − 5 = ___

18. 3 + 2 = ___

19. 7 − 4 = ___

20. 10 − 4 = ___

21. 5 + 4 = ___

Form A.

E 7.

22. $2 + 2 + 2 = \square$

23. $4 + 3 + 1 = \square$

24. $9 = 5 + \triangle + \triangle$

25. $7 = 2 + \square + 1$

26. $8 + 3 =$ ___

27. $9 + 7 =$ ___

28. $12 - 8 =$ ___

29. $7 + 5 =$ ___

30. $15 - 6 =$ ___

31. $17 - 9 =$ ___

32. $43 + 3 =$ ___

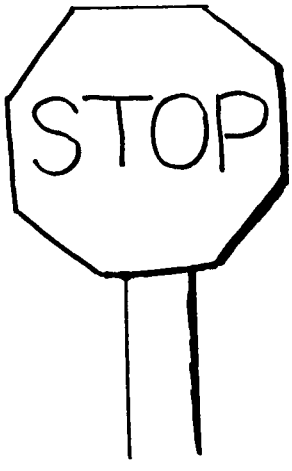
33. $24 + 2 =$ ___

34. $37 - 5 =$ ___

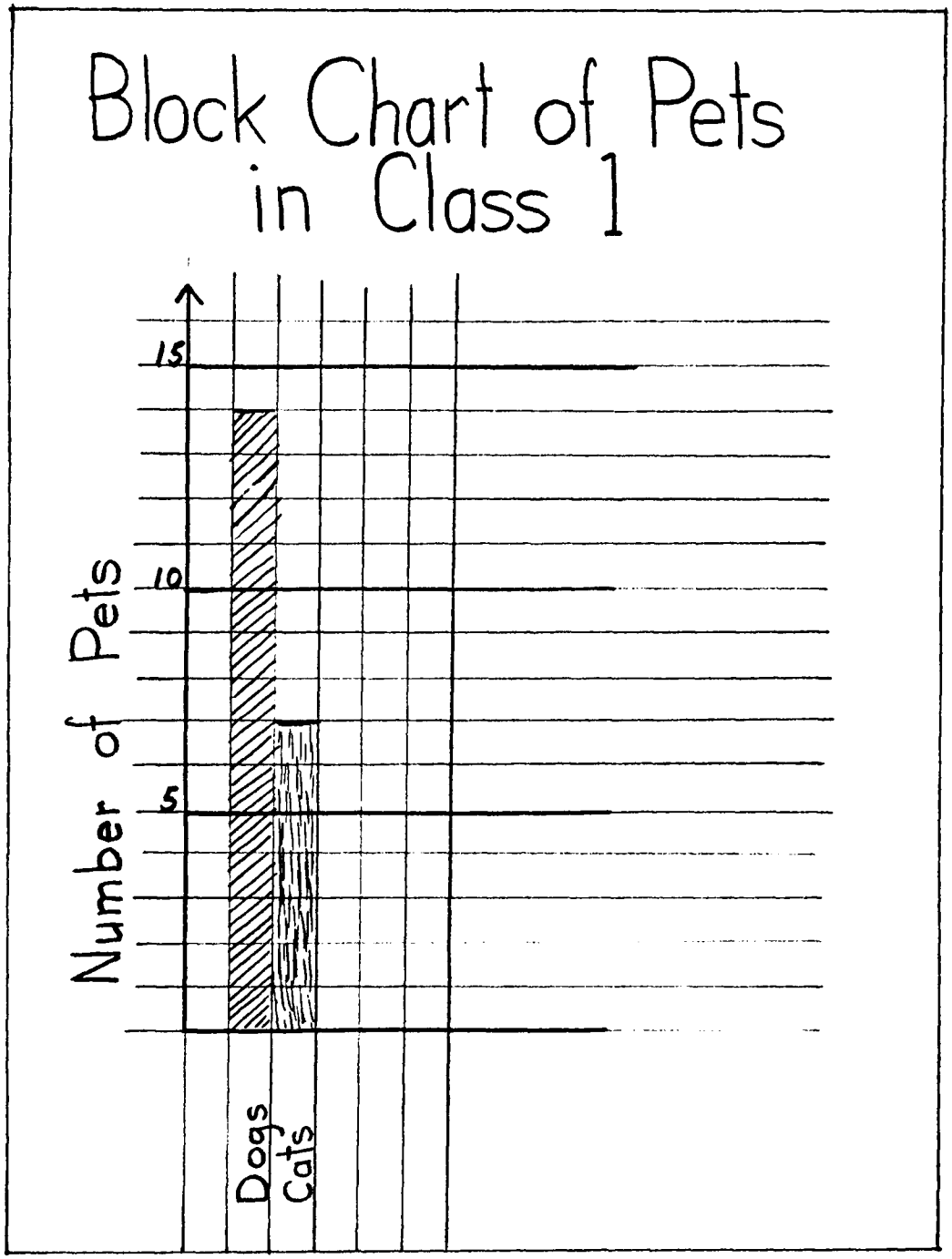
35. $64 - 3 =$ ___

36.
$$\begin{array}{r} 22 \\ +\ 31 \\ \hline \_\_\_ \end{array}$$

37.
$$\begin{array}{r} 36 \\ -\ 14 \\ \hline \_\_\_ \end{array}$$



APPENDIX F



BLOCK CHART FOR GRAPHICAL REPRESENTATION TESTS

Redrawn from a 22"x28" chart.

APPENDIX G

GRAPHICAL REPRESENTATION TEST III

PRE-TEST QUESTIONS

With the block chart shown in Appendix F on display, the following questions were put orally to the children. They printed their answers on lined newsprint paper.

1. How many dogs did the children in Class 1 have altogether?
2. How many cats did they have?
3. Which were there more of, dogs or cats?
4. How many more were there?
5. How many pets were there altogether?
6. Write a number sentence to prove what you said in question 5.
7. Do you think they had any chickens? (This particular school had chickens at that time) ("Don't know" was considered a possible correct response.)
8. Why do you say that?

APPENDIX H

GRAPHICAL REPRESENTATION TEST III

POST-TEST SCHOOL #I

With the block chart shown in Appendix F on display, an 8½ x 14" worksheet with the following questions on it was distributed to the children. Each question was read orally, and repeated as necessary by the examiner. Adequate lined spaces were provided for answers.

1(a) How many dogs did the children in this class have?
1(b) Print an equation to show how you could find this out from the chart, without counting them all.

2(a) How many cats did the children in this class have?
2(b) Print and equation to show how you could find this out from the chart without counting them all.

3(a) Which were there more of? Dogs or cats?
3(b) Print a number sentence that tells this.

4(a) How many more were there?
4(b) Print an equation to show how to get this result.

5(a) How many pets were there altogether?
5(b) Print an equation to show how to get this result.

6(a) Do you think this class had any white rats for pets?
6(b) Why do you say that?

7. Print any other number sentences or equations that this chart tells you about, (that you can tell from this chart).

APPENDIX I

GRAPHICAL REPRESENTATION TEST III

POST-TEST SCHOOL #2

An 8½ x 14" worksheet similar to Figure 7, page 158, was distributed to the children. The block chart, as shown in Appendix F, was also on display. The following questions were asked orally by the examiner, and repeated as necessary.

1. How many dogs did the children in this class have?
2. Print an equation to show how you could find this out from the chart, without counting them all. (use of tens and ones)
3. How many cats were there?
4. Print an equation to show how you could find this out without counting them all.
5. Which were there more of? Dogs or cats? Put a ring around the right picture.
6. Print a number sentence that tells this.
7. Print another number sentence that tells this too.
8. How many more were there?
9. Print a number sentence to show how you could find this out.
10. How many pets were there altogether?
11. Print an equation to show how to get this.
12. Do you think this class had any white rats for pets?
13. Why do you say that?

14.
15. Print any other equations that you can about the pets on this chart.
16.

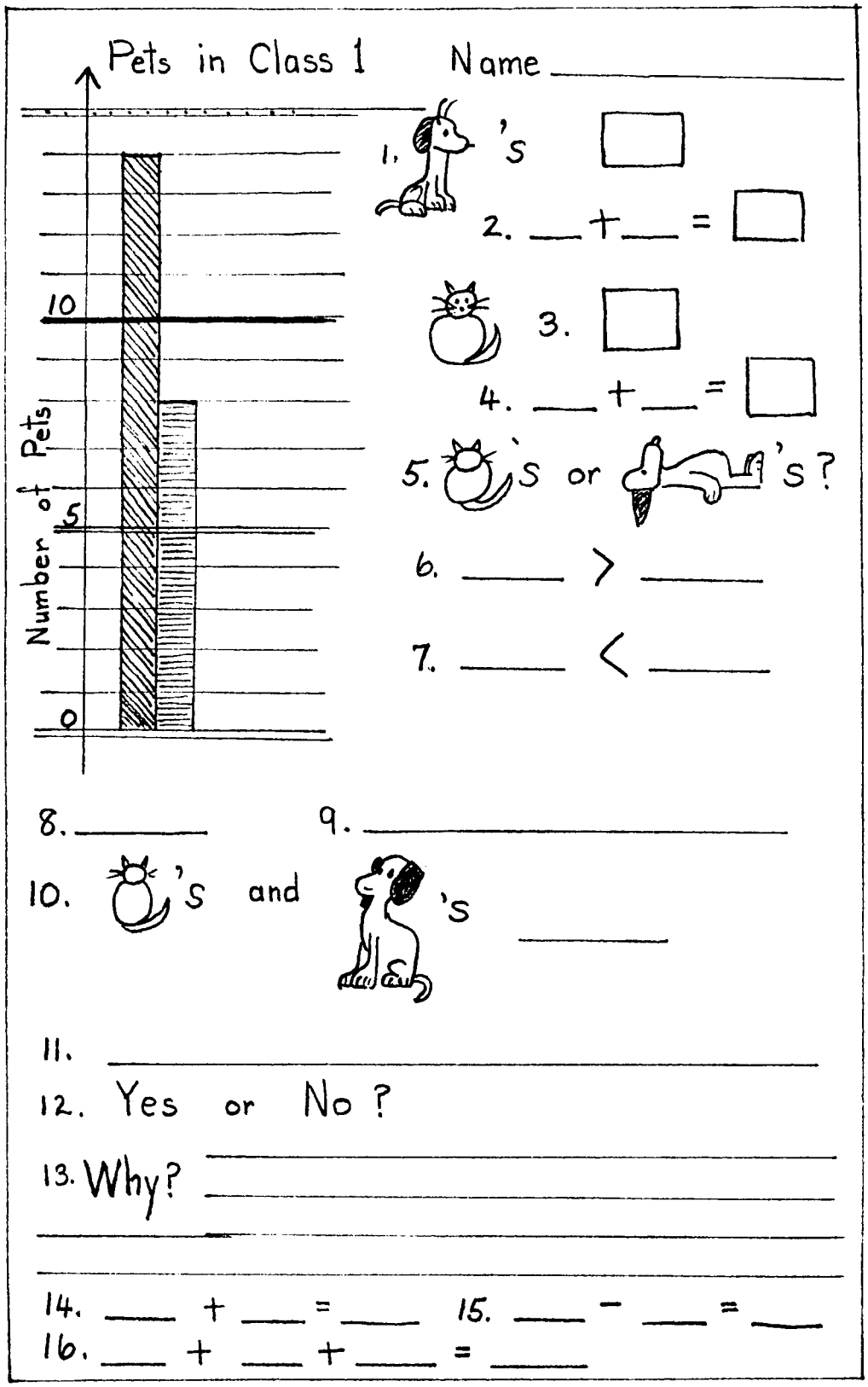


FIGURE 7

WORKSHEET FOR GRAPHICAL REPRESENTATION

TEST III SCHOOL #2

Redrawn from an 8½"x14" worksheet.

BIBLIOGRAPHY

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