THE AGGREGATE PRODUCTION FUNCTION AND TECHNOLOGICAL CHANGE IN CANADIAN AGRICULTURE, 1935-65

by

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B.Sc., University of Alberta, 1966

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
in the Department
of
Agricultural Economics

We accept this thesis as conforming to the
required standard

THE UNIVERSITY OF BRITISH COLUMBIA
June, 1970

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ABSTRACT

A study was undertaken to investigate the macro production relationships in Canadian primary agriculture during the 1935-65 period. Specifically, the problem was to measure simultaneously the rate of disembodied technological change and technological change embodied in machinery and implements, and material inputs.

To estimate technological change, regression estimates were obtained for a linear homogeneous Cobb-Douglas production function, where real gross agricultural output per person employed was the dependent variable, and a time index, weather index, and the annual flow of real capital services (including material inputs) per person employed were the independent variables. The data, which consisted of time series of thirty-one annual observations, was derived mainly from publications of the Dominion Bureau of Statistics. The rate of disembodied technological change was estimated directly by specifying a term which allowed for shifts in the production function over time. To measure the rate of embodied technological change, which was assumed to be capital-augmenting in the vintage sense, several alternative values for the improvement in the productive quality of machinery and implements, and material inputs were imposed on the original data series. Based on

these alternatives, a matrix of regression results was obtained, and the true value of the rate of embodied technological change was inferred by choosing the "best" regression. In addition, several alternative models were investigated.

When disembodied and embodied technological change were specified simultaneously, the "best" estimate of the annual rate of disembodied technological change was 1.76 per cent, while embodied technological change in material inputs was estimated at 3.5 to 4.0 per cent annually. There was no evidence of a positive rate of embodied technological change in machinery and implements in any of the regressions. However, it was concluded that this a priori unexpected result should be considered substantially biased.

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ACKNOWLEDGEMENTS

The author wishes to gratefully acknowledge the generous assistance and guidance provided by his thesis supervisor, Dr. G. R. Winter. In addition, the stimulating intellectual environment provided by the staff and fellow graduate students at the University of British Columbia was extremely motivating and enjoyable.

Special appreciation is extended to the author's wife, Maria, whose assistance, patience and encouragement contributed immensely to the successful completion of this study.

CHAPTER I

PRODUCTIVITY AND TECHNOLOGICAL CHANGE

I. PURPOSE OF THE STUDY

The purpose of this study was to apply macroeconomic concepts to the measurement of technological change in Canadian agriculture during the period 1935-65. An attempt was made to measure three separate kinds of technological change: technological change reflected in the improved productive quality of machinery and implements, technological change reflected in the improved productive quality of material inputs, and all other technological change derived as a residual.

Technological change may be regarded as an advance in technology which is: (1) knowledge used by productive units (firms or farms in this case) regarding the principles of physical, biological and social phenomena; (2) knowledge regarding the application of these principles to production such as the application of genetics to the development of better livestock or new varieties of crops; and (3) knowledge regarding the day-to-day operations of production such as management techniques.

Technological change is an important and perhaps the most important factor responsible for economic growth.

Economists have made significant attempts since the mid 1950's to measure the effect of the rate of technological change on a nation's rate of economic growth. Solow, for example, found that almost ninety per cent of the long-term increase in output per unit of labor input in the United States was attributable to technological change with the remaining ten per cent attributable to increases in the quantity of capital employed.[1] Although these results were extremely rough, more recent studies have confirmed that the effect of technological change on productivity over time has been substantial.

II. PRODUCTIVITY

Since the eighteenth century, economists and policy makers have been interested in productivity. Essentially, productivity is a measure which expresses the relationship between output and the resources utilized in its production. More precisely, it is the ratio of output to a single input or to a composite of inputs. For example, the volume of output per man-year, and the number of bushels per acre are expressions of productivity. These ratios are measures of performance relating the volume of output produced to

[1]R.M. Solow, "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, 39:312-20, August, 1957.

the volume of inputs used. Productivity, however, is not synonymous with efficiency, since productivity simply expresses a physical relationship between output and input while efficiency implies an optimum level of performance in a productive situation in terms of the combination of inputs to produce a given level of output.

As a description of a technical relationship between output and inputs, productivity is a characteristic of the individual economic unit, and its changes, therefore, indicate that the productive resources within the unit have been reorganized so as to affect output. Alternatively, productivity changes may arise from all sourcesincluding shifts in production and employment of resources between units having different levels of productivity as well as productivity advances within individual units. This second concept is more suitable for most economic and policy analysis at the macro level.

For conceptual as well as practical reasons, labor productivity, that is, output per unit of labor input, has been the most commonly studied measure of productivity, since labor usually represents a major proportion of value added in production, labor input is relatively easy to measure, and changes in labor productivity are directly related to changes in real income per capita. In recent years, there has been an increasing volume of empirical work on productivity at all levels of aggregation which has contributed to an extensive knowledge of the trends and magnitudes of productivity change. However, considerable scope remains for further investigation of the causes and sources of productivity increases.

In a broad sense, changes in productivity may result from three sources: (1) the nature and rate of technological change; (2) factor substitution in response to changes in relative input prices; and (3) economies of scale or increases in the utilization of existing productive capacity.

The effects of changing factor proportions on productivity are easily shown in Figure 1 where

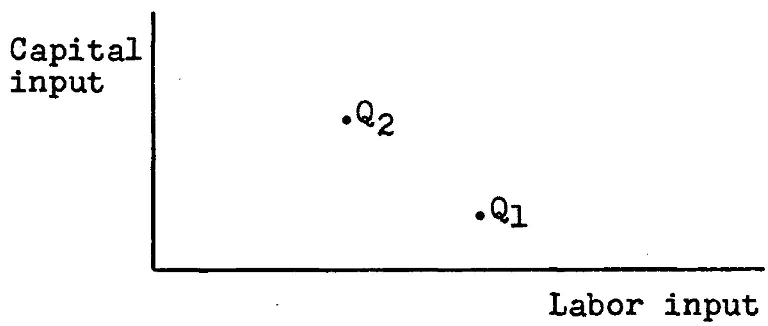

Figure 1. Factor proportions and productivity change.

Q_1 and Q_2 represent the same level of output, but each is produced by a different, although technically efficient, combination of capital and labor. In response to a change in relative input prices, a shift from Q_1 to Q_2 would result in an increase in labor productivity as shown by an increase in the output-labor ratio and a decrease in capital productivity.

Economies of scale exist when the percentage change in required inputs is less than the percentage change in the resultant output, when all inputs are increased in the same proportion. In this situation, it is obvious that productivity increases as output increases, since the output-input ratio increases.

III. TECHNOLOGICAL CHANGE

A production function shows, for a given level of technology, the maximum output level which can be obtained from given amounts of inputs. Technological change results in a shift in the production function over time. In Figure 2,

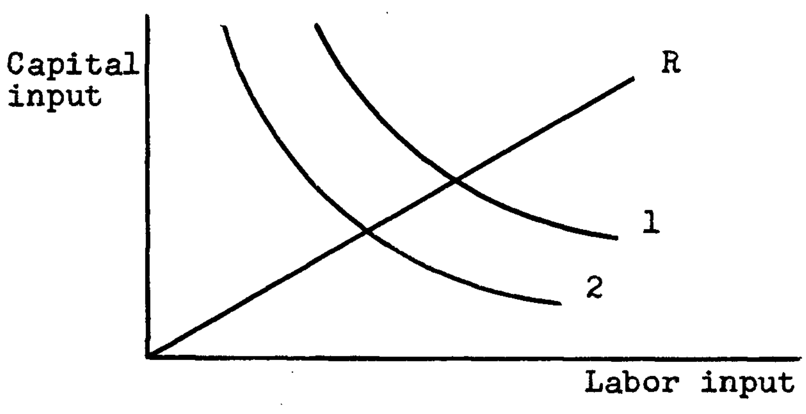

Figure 2. Technically efficient combinations of labor and capital inputs at two levels of technology.

a shift in the production indifference curve from position 1 to position 2 indicates that an increase in productivity has occurred, since smaller amounts of capital and labor are now required to produce the same level of output.[2] In the usual case, and for purposes of this study, this increased productivity is defined as the result of disembodied technological change.

The increase in productivity shown in Figure 2 is not the result of economies of scale, since the output level is unchanged. Factor (input) substitution is also eliminated as a possible source of increased productivity, since the level of output can always be produced at technology level 2 by a smaller combination of inputs employed in the same proportion, as shown by a ray (R) through the origin, than at technology level 1.

An implicit assumption in Figure 2 is that the productive quality of the inputs, labor and capital, does not improve over time. This homogeneity of inputs is implied because the production indifference curves for two instances in time are drawn on the same indifference curve map. Consequently, a second type of technological change, namely embodied technological change, has been eliminated from Figure 2. Embodied technological change is defined

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[2]A production indifference curve is defined as a locus of technically efficient input combinations all of which are capable of producing the same level of output.

as a change in the productive quality of one or all of the inputs used in the production process. For example, technological change may be embodied in labor as a result of improved health, higher educational attainments and training programs. Similarly, technological change may be embodied in capital in the form of improved designs.[3] Embodied technological change, therefore, gives rise to productivity increases as a result of increased output levels corresponding to inputs measured in "efficiency" units.

Technological change may also be classified as neutral, labor-saving, or capital-saving. This topic is discussed in Chapter II. Chapter II also outlines techniques whereby it is possible to estimate the rate of movement of the production function over time by a single number. This is often used as a measure of disembodied technological change.

IV. PRODUCTIVITY CHANGE IN CANADIAN AGRICULTURE

Several empirical studies have attempted to measure productivity change in Canadian agriculture over the past three or four decades. In order to indicate the extent of productivity change in Canadian agriculture, and the attempts which have been made to identify the sources of

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[3] H.A.J. Green, "Embodied Progress, Investment, and Growth," American Economic Review, 56:138-51, March, 1966.

productivity changes, a few of these studies are briefly discussed below.

The first of the recent studies on productivity in Canadian agriculture was completed by Lok in the late 1950's.[4] Lok examined, for Canadian agriculture as a whole, the relationship between annual percentage changes in total productivity and real net return per farm over the years 1926-57. Lok concentrated on the estimation of a total productivity index. He aggregated constant dollar series for individual inputs into a single constant dollar index measuring total input, which was then divided into a constant dollar index of total output.

Lok devoted considerable attention to the discrepancies between productivity indexes when prices of different periods were used to weight the classes of outputs and inputs in the construction of constant dollar series.[5] As a result of this enquiry, he presented six indexes showing total productivity change in Canadian agriculture during 1926-57.[6] The estimates ranged from

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[4]Siepko H. Lok, An Enquiry into the Relationships Between Changes in Overall Productivity and Real Net Return per Farm, and Between Changes in Total Output and Real Gross Return, Canadian Agriculture, 1926-1957, Economics Division, Canada Department of Agriculture, Technical Publication 61/13 (Ottawa: 1961).

[5]*Ibid*., pp. 10-11.

[6]Lok, *op. cit*., table 6, p. 76.

a low increase of 19.8 per cent to a high of 59.1 per cent over the period 1926-57. Although Lok made no attempt to quantify the sources of these estimated productivity increases, he did offer some general reasons such as research and education, economies of scale for individual firms, and greater adherence to the principle of comparative advantage.[7]

Furniss has used similar methods and basic data sources as did Lok to estimate productivity change during the period 1935-60.[8] He estimated that total agricultural productivity increased by 60 per cent over this period, which is equivalent to an annual growth rate of 1.9 per cent.[9] This compared with an annual growth rate of 2.2 per cent during the 1946-60 period.[10]

Furniss also investigated individual factor productivities using the constant dollar method and output-individual input ratios. He found that labor productivity increased by 183 per cent during the 1935-60 period. Similarly, the productivity of land and buildings increased

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[7]Lok, op. cit., pp. 20-21.

[8]I. F. Furniss, "Productivity of Canadian Agriculture, 1935-1960: a Quarter Century of Change," Canadian Journal of Agricultural Economics, 12, No. 2: 41-53, 1964.

[9]Ibid., p. 42.

[10]Ibid., p. 51.

by 42 per cent over the 1935-60 period, but only 14 per cent over the 1946-60 period. In 1960, the ratio of total output to capital inputs (all other inputs) was 36 per cent less than in 1935, and similar to the 1946 level.[11] Like Lok, Furniss made no attempt to quantitatively explain these estimated productivity changes in terms of technological change, economies of scale, and factor substitution. He did, however, indicate the nature of the changes in input proportions over the time period studied, and suggested that substantially increased inputs of purchased feed, seed, fertilizers and pesticides had made an important contribution to increased total agricultural productivity.

Mackenzie also has investigated productivity in Canadian agriculture.[12] Unlike Lok and Furniss, who investigated productivity change related to a gross measure of agricultural output, Mackenzie examined net labor

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[11] Ibid., pp. 43-44.

[12] W. Mackenzie, "The Terms of Trade, Productivity and Income of Canadian Agriculture," Canadian Journal of Agricultural Economics, 9, No. 2: 1-13, 1961; W. Mackenzie, "The Impact of Technological Change on the Efficiency of Production in Canadian Agriculture," Canadian Journal of Agricultural Economics, 10, No. 1: 41-53, 1962; and W. Mackenzie, "Regional Changes in Income, Terms of Trade and Productivity within Canadian Agriculture," Canadian Journal of Agricultural Economics, 11, No. 2: 41-51, 1963.

productivity change in agriculture by deriving a measure of value added output in real terms (gross outputs less material inputs) per unit of labor input.[13] A comparison of Mackenzie's estimates with those of Furniss, indicates that net labor productivity has increased much less than gross labor productivity. This suggests that purchased inputs have contributed substantially to the phenomenal labor productivity increases estimated by Furniss. Mackenzie[14] extended his estimates of net labor productivity changes to a total net productivity index for Canadian agriculture by aggregating inputs into a measure of total input in a manner suggested by Kendrick.[15] On this basis, Mackenzie estimated that total net productivity for Canadian agriculture increased by 37.0 to 43.8 per cent from the 1944-48 period to the 1954-58 period.

In a recent Doctoral dissertation, Li attempted to explain the increases in labor productivity in Canadian agriculture.[16] This is the only study which has specifically attempted to explain productivity changes in terms

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[13] W. Mackenzie, 1961, p. 7.

[14] W. Mackenzie, 1962, p. 43.

[15] J.W. Kendrick, "Productivity Trends in Agriculture and Industry," <u>Journal of Farm Economics</u>, 40:1554-64, December, 1958.

[16] Lew-king Li, "Technological Change in Canadian Agriculture" (unpublished Doctoral dissertation, University of Manitoba, Winnipeg, 1968).

of technological change, economies of scale, and factor substitution. Using the Solow or geometric method, Li estimated the rate of disembodied technological change on the basis of both net value added and gross measures of output.[17] He estimated that disembodied technological change has proceeded at an annual rate of 3.1 per cent in the agricultural sector as a whole for the period 1946-65.[18] Over the same period, net labor productivity increased by 176 per cent with 75.2 per cent of this increase attributable to technological change and the remainder, 24.8 per cent, attributable to increases in the capital-labor ratio.

Productivity increases have been well demonstrated for Canadian agriculture. The main results of the above studies are summarized in Table I. However, interpretive analysis in the terms suggested in this chapter have only recently begun. Important aspects of the problem of understanding productivity changes have not yet been investigated.

V. THE PROBLEM

In the real world, it is difficult to isolate the effects of technological change, factor substitution and economies of scale on changes in productivity. However,

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[17]R. M. Solow, "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, 39:312-20, August, 1957.

[18]Li, *op. cit.*, p. 112.

TABLE I

SUMMARY OF ESTIMATED PRODUCTIVITY CHANGE IN CANADIAN AGRICULTURE

Author	Productivity measure	Average annual percentage growth rate
Lok	total productivity, 1926-57	0.5-1.5
Furniss	total productivity, 1935-60	1.9
	total productivity, 1946-60	2.2
	labor productivity, 1935-60	4.1
	land and buildings productivity, 1935-60	1.4
	land and buildings productivity, 1946-60	0.9
	capital productivity, 1935-60	-2.6
	Capital productivity, 1946-60	0.0
Mackenzie	total net productivity, 1944-58	2.3
Li	disembodied technological change, 1946-65	3.1
	net labor productivity, 1946-65	5.2

one way to gain a better understanding of productivity changes over time is to separate the productivity changes into the broad source components of technological change, factor substitution, and economies of scale. This concept is shown in Figure 3. In time period t, 50 units of output

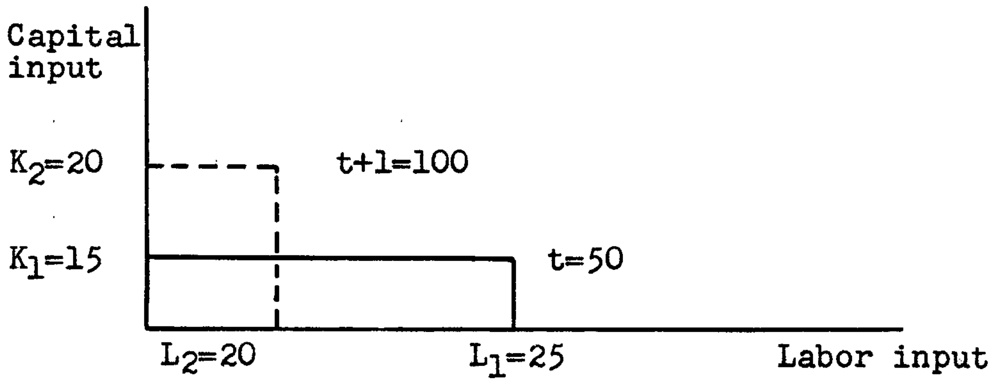

Figure 3. Productivity changes over time.

are produced with 25 units of labor and 15 units of capital. In the subsequent time period, t+1, 100 units of output are produced with 20 units of capital and 20 units of labor.

Productivity has increased from time period t to time period t+1. Both capital and labor productivity ratios have increased and it is conceivable that total productivity has also increased, although this cannot be ascertained from the limited information. Furthermore, it is impossible to discuss why the increases in productivity have occurred. Input substitution has occurred and may have contributed to the increase in productivity, especially labor productivity. However, disembodied technological change and/or economies of scale may also have contributed to productivity change.

The broad changes which have occurred in Canadian agriculture are similar to those portrayed in the simple example above. Productivity has increased in the agricultural sector. There has been a substitution of capital for labor, and agricultural labor productivity has increased more rapidly than in any other major sector of the Canadian economy during the post war period.[19] When the real world

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[19]The Dominion Bureau of Statistics has estimated that output per person employed in Canadian agriculture has increased by 5.5 per cent annually during 1946-67. This compares with 2.8 per cent for the commercial nonagricultural industries, and 4.3 per cent for the nonagricultural goods-producing industries. See: Canada, Dominion Bureau of Statistics, Aggregate Productivity Trends, 1946-67. (Ottawa: Queen's Printer, 1968).

agricultural sector is considered, the changing quality of inputs over time further complicates efforts to attribute the sources of productivity gains. Embodied technological change must also be considered, and it would seem reasonable on a priori grounds to postulate that the productive quality of many agricultural inputs has increased over time.

This study was designed to investigate technological change in the aggregate primary agricultural sector in Canada during the time period 1935-65. Specifically, the problem was: (1) to measure the rate of disembodied technological change; (2) to measure the rate of technological change which has been embodied in agricultural machinery and implements; and (3) to measure the rate of technological change which has been embodied in material inputs. This study did not investigate all of the possible sources of productivity change. An assumption was made regarding economies of scale, and the influences of the substitution of capital for labor were not estimated. Conceptually, the measurement of embodied technological change should have been extended to include all inputs. A priori, it would be reasonable to expect that the quality of the labor force has improved over time. However, because of data limitations and the lack of suitable methods of analysis, this study was limited to the consideration of embodied technological change in two inputs only.

Chapter II is a review of the theoretical framework and related empirical studies regarding the measurement of technological change. Various models are interpreted and evaluated in terms of their contribution to this study. The model and data, including methods of derivation, manipulation and assumptions, used in this study are outlined in Chapter III. The results of the analysis and related discussion are presented in Chapter IV. Finally, Chapter V presents a summary and the main conclusions and implications of the study.

CHAPTER II

CONCEPTUAL FRAMEWORK FOR MEASURING TECHNOLOGICAL CHANGE

This chapter reviews the theory which is pertinent to the measurement of technological change, and the methods and approaches which have been developed. This review is not exhaustive, but rather it concentrates on those methods and studies which have provided the background for, and contributed most to, the methods used in this study, which are outlined in the following chapter.

I. NEUTRALITY OF TECHNOLOGICAL CHANGE

Economists distinguish among three types of technological change as it affects the shift in the production function: neutral, labor-saving, and capital-saving technological change.

In many methods, estimation of the rate of technological change involves accurate specification both of the aggregate production function and of the form of technological change. In addition, it has been customary, for reasons of theoretical and empirical convenience, to assume that technological change is neutral. When considering the question of neutrality, the usual procedure has been to make assumptions about the way in which technological change affects relationships between certain variables which are derived from the production function.[1] Technological change is then neutral if its effects do not alter the relationship between the chosen variables. Because there are several possible pairs of variables which may be chosen, alternative definitions or various forms of neutral technological change are possible. The most widely used and best known of these are the "Hicks" and "Harrod" definitions of neutral technological change.[2]

Technological change is neutral in the Hicks sense if the ratio of the marginal product of labor to the marginal product of capital is unchanged when the capital-labor ratio is unchanged. When disembodied technological change is assumed to be an exogenous function of time (t), the production function

$$Y=f(K,L,t)$$

is implied. If technological change is Hicks-neutral, the function becomes

$$Y=A(t)f(K,L)$$

where A(t) is any function of time. This is the general

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[1]M.J. Beckmann and R. Sato, "Aggregate Production Functions and Types of Technical Progress: a Statistical Analysis," American Economic Review, 59:88-101, March,1969.

[2]A more complete treatment, from which the following discussion is drawn, of the implications of the two definitions is found in: F. Hahn and R. Matthews, "The Theory of Economic Growth: a Survey," Economic Journal, 74: 779-901, December, 1964.

form of the production function used to estimate disembodied technological change in this study, and, therefore, Hicks-neutral technological change was implicitly assumed.

The Harrod definition is based on the comparison of points on the two production functions where the marginal product of capital is constant. Technological change is neutral in the Harrod sense if the capital-labor ratio which results in a constant marginal product of capital after technological change, also causes the capital-output ratio to remain constant. With two inputs, Harrod-neutral technological change is shown algebraically as

$$Y=f(K,A(t)L)$$

which indicates that Harrod-neutral technological change may be described as "labor-augmenting", since the labor force is measured in efficiency units, $A(t)L$. Technological change may also be "capital-augmenting". In this case the general form of the production function is

$$Y=f(A(t)K,L)$$

which is the mirror image of Harrod-neutral technological change with K and L reversed. Capital-augmenting technological change is a useful concept in the study of vintage-capital models, and it is equivalent to the concept of embodied technological change in capital which was used in this study. The concept is more fully discussed in Section III below.

Technological change is neutral in both the Hicks and Harrod sense when the elasticity of substitution between labor and capital is unity.[3] The Cobb-Douglas production function, which was used in this study, possesses this property and, therefore, unequivocal neutrality was implicitly assumed.

Recently, Beckmann and Sato have generalized the concept of technological neutrality by extending the principle that technological change is neutral when the relationship between a specific pair of variables is invariant through time, to relationships between variables other than those considered in the Hicks, Harrod, and Solow definitions.[4] Under the Beckmann and Sato scheme technological change is Hicks-neutral when the relationship between the marginal rate of substitution and the capital-labor ratio is constant, Harrod-neutral when the relationship between the capital-output ratio and the interest rate does not change, and Solow-neutral when the relationship between output per worker and the wage rate is invariant. After examining relationships between output-capital ratios, output-labor ratios, capital-labor ratios, interest rates, wage rates, marginal rates of

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[3] Ibid., p. 829.

[4] Beckmann and Sato, op. cit., p. 90.

substitution, and labor's share, Beckmann and Sato suggested a number of interesting new types of technological change. They applied regression analysis to time series data for the U.S., Japanese, and German private non-farm economies in order to empirically investigate the implications of their formulations of technological neutrality. They concluded that:[5] (1) the traditional types of Hicks, Harrod, and Solow neutrality were for all countries at least as good as the unconvential types of neutrality; (2) Solow-neutral technological change performed particularly well; (3) general factor-augmenting technological change did not give a substantially improved explanation of observed data when compared with single-factor-augmenting technological change; and (4) irrespective of how technological change was specified, the estimated production function turned out to be close to a Cobb-Douglas or CES function.[6]

II. DISEMBODIED TECHNOLOGICAL CHANGE

The rate of growth of total factor productivity is conventionally defined as the difference between the rates

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[5] Ibid., p. 95.

[6] The CES production function is a more general function which allows the elasticity of substitution to be estimated. It also contains efficiency and distribution parameters so that both neutral and non-neutral technological change can be estimated. Although the CES function has been the most commonly applied in recent work, it was not employed in this study because, being a more general form, it is more difficult to estimate than the Cobb-Douglas form.

of growth of real output and real input, where the rates of growth of real output and input are the weighted averages of the rates of growth of individual products and inputs.[7] Under various assumptions (including neutrality, perfect competition, and constant returns to scale) a change in total factor productivity may be identified with a shift in the production function, and changes in real output and input not accompanied by a change in total factor productivity may be associated with movements along the production function. Technological change is also defined as a shift in the production function, and the terms technological change and total factor productivity have tended, in practice, to be used interchangeably.[8] Much of the empirical work during the 1950's and early 1960's concentrated on this simple concept of deducting the contributions of increased capital and labor inputs to increased output, and attributing the "residual" growth in output to disembodied technological change or total factor productivity. The arithmetic index and the Solow model were the most common methods employed.

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[7]D.W. Jorgenson and Z. Griliches, "The Explanation of Productivity Change," The Review of Economic Studies, 34:249-83, July, 1967.

[8]This interchangeability depends upon the assumption of constant returns to scale, and is valid only if economies or diseconomies of scale do not exist.

The arithmetic index which has been used by Abramovitz[9] and Kendrick[10] may be defined as

$$C=Y/(wL+iK)$$

where Y is output, w is the real wage rate in the base period, L is labor input (in physical units) in a given year, i is the real return to capital in the base period, and K is capital input (in physical units) in a given year.[11] Perfectly competitive equilibrium is implied since the weights, w and i, represent the marginal products of labor and capital, respectively. The critical assumption, however, is that the marginal products of the inputs are changed only by technological change and always in the same proportion. Therefore, the marginal products are assumed to be independent of the ratio of the quantities of the inputs, which is a very restrictive assumption and not reasonable over a longer period where substantial changes may be expected in the capital-labor ratio. The so called constant dollar method which, as noted in Chapter I, has been used by Lok, Furniss, and Mackenzie to estimate productivity change in Canadian agriculture

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[9]M. Abramovitz, "Resource and Output Trends in the United States Since 1870," American Economic Review, 46:5-23, May, 1956.

[10]J.W. Kendrick, Productivity Trends in the United States (Princeton: Princeton University Press, 1961).

[11]E.D. Domar, "On Total Productivity and All That," Journal of Political Economy, 70:599, December, 1962.

is a special formulation of the arithmetic index. The constant dollar index may be defined as

$$C=y/(l+k)$$

where y, l and k are the values of output, labor and capital, respectively in base year prices. Aside from the normal assumption of competitive equilibrium, the basic difficulty with this method (as well as with the arithmetic index) is that it is not suited to either a linear or an exponential world.[12] For example, if the values of output and of inputs are linear with respect to time, C will gradually approach a constant while if they are exponential, the relative rate of growth of C will approach the difference between the rates of growth of output and of the fastest growing input.

Solow derived, from very general assumptions, a somewhat better measure of the rate of movement of the production function than the arithmetic index.[13] Solow's derivation was based on a linear homogeneous production function of the general form

$$Y=A(t)f(K,L).$$

Solow defined technological change as "any kind of shift"

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[12] *Ibid.*, p. 607.

[13] R. M. Solow, "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics*, 39:312-20, August, 1957.

in the production function, and the term A(t) measures the cumulated effect of shifts over time.[14] With the further assumptions of perfect competition and neutral technological change, technological change between two periods is given by

$$\bar{A}=\bar{Y}-W_k\bar{K}-W_l\bar{L}$$

where $\bar{A}$, $\bar{Y}$, $\bar{L}$, and $\bar{K}$ are the percentage rates of change per unit of time of disembodied technological change, output, labor input and capital input, respectively. W_k and W_l are the shares of capital and labor in output which, under the assumptions of this model, will be equal to the elasticity of output with respect to capital and labor, respectively. Given time series data on $\bar{Y}$, $\bar{L}$, $\bar{K}$, W_k and W_l, $\bar{A}$ can be estimated.

The interpretation of the Solow model is straightforward: disembodied technological change is equal to the change in the output which is not accounted for by the changes in capital and labor. Thus, as Domar has clearly pointed out, disembodied technological change is estimated in this method as a residual, and for this reason he prefers to call it the "Residual."[15]

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[14] *Ibid.*, p. 312.

[15] E. D. Domar, "On the Measurement of Technological Change," *Economic Journal*, 71:712, December, 1961.

Upon applying the above model to the U.S. non-farm economy for the period 1909-49, Solow concluded that: (1) technological change was neutral on average; (2) technological change proceeded at an annual rate of about one per cent for the first half of the period and two per cent for the last half; and (3) 87.5 per cent of the increase in gross output per man-hour could be attributed to technological change, and the remaining 12.5 per cent to increased use of capital.[16]

Lave has applied the Solow model to U.S. agriculture and concluded that: (1) technological change in agriculture was twice as rapid as in the private non-farm sector; and (2) technological change accounted for 60 to 73 per cent of the increase in output per man-year during the 1850-1950 period with the remainder, 27 to 40 per cent, attributable to increases in capital.[17] As noted in Chapter I, Li applied the Solow model to Canadian agriculture for the period 1946-65.

Domar has provided a more general interpretation of the Solow model - the geometric index.[18] Since any linear homogeneous production function with constant factor shares

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[16]Solow, op. cit., p. 320.

[17]L. B. Lave, Technological Change: Its Conception and Measurement. (New Jersey: Prentice-Hall, Inc., 1966), pp. 47-57.

[18]Domar, op. cit.

is of the Cobb-Douglas type, Domar identified $L^{W_1}K^{W_k}$ as a weighted geometric mean. It follows that, if both L and K are index numbers with a common base, then $L^{W_1}K^{W_k}$ is a geometric index of inputs, each weighted by its share in output in the base period. Disembodied technological change is then the ratio between geometric indexes of outputs and inputs.[19]

The geometric index is simply a geometric index number with constant factor shares as weights, and therefore differs from the Solow model which uses current factor shares as weights. However, with this assumption, Domar has derived an index which circumvents the underlying assumption of an aggregate production function with the accompanying implications. Furthermore, since relative factor shares appear to have been quite stable over time and relative prices have not, the geometric index seems to better approximate reality than does the arithmetic index.

The above models provide only indirect measures of disembodied technological change. They do not isolate the effects of "pure" technological change alone, but include in the measures all increases in output not accounted for by the growth of explicitly recognized inputs and, therefore, must be treated conceptually as residuals. Moreover, these

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[19] Ibid., p. 713.

methods are based on the notion that technological change is disembodied, that is, that all technological change consists of better methods and organization which improve the output performance of the inputs. The inputs are assumed to be homogeneous over time. However, many changes in technology must be embodied in new capital if they are to be utilized. In the above methods, capital does not change in quality, form, or composition, nor does it act as a vehicle for the introduction of technological change into the productive process. The methods reviewed in the next section were developed in an attempt to explicitly recognize this concept, and to provide estimates of the rate of embodied technological change.

III. EMBODIED TECHNOLOGICAL CHANGE

The concept of embodied technological change developed from the notion that capital investment and technological advance influence each other in such a way that their separation is meaningless, if not impossible. If technological change cannot be implemented without introducing new kinds of capital, then capital investment may be regarded as the vehicle of technological advance, and capital, therefore, cannot be considered homogeneous. In this approach, technological change is embodied in new capital, and may be regarded as a progressive reduction in the cost of producing capital, or alternatively, as a progressive improvement of the quality of capital.[20] Therefore, capital goods embody the technology of their date of construction, and those built at different dates ("vintages") are qualitatively dissimilar. A separate production function is required for each vintage and total output is the sum of output from all vintages in use.[21]

In 1959, Solow reconstructed his earlier disembodied model to make allowance for embodied technological change.[22] Solow began by interpreting his disembodied model as a linear homogeneous Cobb-Douglas function

$$Q=Be^{\lambda}L^{a}K^{1-a}$$

where e^{λ} is an exogenous shift function which measures the rate of neutral disembodied technological change.[23] Solow's embodied method is based on a vintage model of production. Output at time t, $Q_v(t)$, from the surviving capital equipment of vintage v is given by a linear homogeneous Cobb-

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[20]F. Hahn and R. Matthews, "The Theory of Economic Growth: a Survey," Economic Journal, 74:843, December, 1964.

[21]Ibid., p. 837.

[22]R. M. Solow, "Investment and Technical Progress," Mathematical Methods in the Social Sciences, K. J. Arrow, editor (Stanford: Stanford University Press, 1960), pp. 89-104.

[23]This is a more restrictive formulation than the 1957 model, since disembodied technological change is now assumed to advance at a constant rate, λ, over time, and factor shares are also assumed constant over time.

Douglas production function

$$Q_v(t)=Be^{\lambda v}L_v(t)^a K_v(t)^{1-a}.$$

Embodied technological change, represented by λ, is assumed to be uniform, approximately exponential over time, and capital-augmenting (Solow-neutral). Therefore, all technological progress appears as a steady improvement in the quality of capital goods at the rate $\lambda/(1-a)$.[24] To emphasize obsolescence rather than depreciation, capital is assumed to be subject to a constant force of mortality, m, and the average length of life of capital is 1/m.[25] Labor is homogeneous and the allocation of labor to capital of various vintages is assumed to equate the marginal productivity of labor in all uses. As Solow demonstrated, it is then possible to derive a measure of "equivalent capital" at time t, J(t), by summing the surviving capital goods of past vintages inclusive of time t, weighted according to their vintage. Output at time t, Q(t), is then given by

$$Q(t)=Be^{-m(1-a)}L(t)^a J(t)^{1-a}.$$

Using exogenous estimates of a (elasticity of output with respect to labor) and m, Solow estimated the value of

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[24]Solow, op. cit., p. 91.

[25]The theory does not require an explicit assumption about depreciation. However, the vintage composition of the stock of capital is required, and since such information is not usually available, it must be derived by employing an assumption about depreciation. See: Ibid., p. 93.

λ from time series of output, labor, and gross investment. For the U.S. private sector, 1919-53, he found that λ equalled about 0.025 which was substantially larger than the estimated value of 0.015 from his disembodied model. However, the difference is in the expected direction, since in the embodied model only new capital benefits from technological advance rather than all capital goods as in the disembodied model.

In 1962, Solow presented a slightly different method for estimating capital embodied technological change while drawing a distinction between actual and potential output.[26] In this model all technological advance is embodied in new capital goods, and the rate of embodied technological change is, therefore, synonymous with the rate of improvement in the productivity of capital goods, λ. Assuming that labor and capital of various vintages are allocated so that output is maximized, that is, the marginal productivity of labor is equal in all uses, the equivalent stock of capital in year t, J(t), is

$$J(t)=\sum_{v=-\infty}^{t} (1+\lambda)^{v} B(t-v) I(v)$$

where I(v) is gross investment in year v, and B(t-v) is the amount surviving in a later year t. Potential output, P(t), is then a function of the equivalent stock of capital

[26] R. M. Solow, "Technical Progress, Capital Formation, and Economic Growth," American Economic Review, 52:76-86, May, 1962.

and the available labor supply, L(t), and is given by

$$P(t)=F(J(t),\ L(t)).$$

No explicit term representing technological change is required because it is contained in J.[27]

However, actual output, A(t), is less than potential output because of unemployment and idle capital. If u(t) is the unemployment rate, then

$$A(t)=f(u)F(J(t),\ L(t)).$$

To derive empirical estimates of λ and u, Solow used a linear homogeneous Cobb-Douglas production function

$$A=B10^{b+cu+du^2}J^aL^{1-a}.$$

Solow fitted the function using various estimates of the equivalent stock of capital which were derived by using various values for the improvement factor λ. In an effort to determine whether the rate of productivity improvement differed between plant and equipment, different values of λ were used for each component. The criteria for determining the best estimate of λ were the goodness of fit and low standard errors of the regression coefficients.

Solow's above model provided two concepts used in this study: (1) the distinction between potential and

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[27] *Ibid.*, p. 77.

actual output; and (2) the possibility that different kinds of capital may experience different rates of embodied technological change.

IV. SIMULTANEOUS EMBODIED AND DISEMBODIED TECHNOLOGICAL CHANGE

The methods outlined in Section II depend on the assumption that technological advance increases the productivity of old and new capital goods in the same way and in the same proportion. On the other hand, methods described in Section III are based on the opposite assumption that technological advance can be introduced into the production process only through new capital investment. In the real world, the truth most probably lies somewhere between these two extremes.

The disembodied and embodied approaches to technological change were synthesized by Phelps in a linear homogeneous Cobb-Douglas production function

$$Q(t)=Ae^{ut}J(t)^{a}L(t)^{1-a}$$

where u is an estimate of neutral disembodied technological change, and J is Solow's equivalent capital stock (which embodies technological change at rate λ).[28] In this model,

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[28]E. S. Phelps, "The New View of Investment: a Neoclassical Analysis," Quarterly Journal of Economics, 76:549-67, November, 1962.

disembodied technological advance occurs if $u > 0$ and $\lambda = 0$, while embodied technological advance occurs if $u=0$ and $\lambda > 0$. When both u and λ are positive, both kinds of technological change occur jointly.

Intriligator extended the Solow and Phelps models empirically in two ways: (1) embodied and disembodied technological change were estimated jointly rather than separately as in the Phelps method; and (2) technological change embodied in improved quality of labor as well as improved quality of capital was estimated.[29] Intriligator derived his model by adding Solow's unemployment function relating actual and potential output to Phelps' embodied and disembodied model. His model, therefore, is a linear homogeneous Cobb-Douglas production function relating actual output, Q(t), to equivalent capital, J(t), and equivalent labor, M(t),

$$Q(t)=Ae^{ut}e^{b+cu+du^2}J(t)^aM(t)^{1-a}$$

where u is a measure of neutral disembodied technological change, and J and M are capital and labor inputs, respectively, weighted for quality change (embodied technological change).

Intriligator's method of estimation was similar to that used by Solow in his 1962 model, that is, the

[29] M. D. Intriligator, "Embodied Technical Change and Productivity in the United States, 1929-1958," Review of Economics and Statistics, 47:65-70, February, 1965.

production function was estimated using the alternative capital and labor input series based on various assumed levels of embodied technological change. The results of the various regressions were then compared in order to choose among the assumed values for embodied technological change. Using data for the U.S. private sector, 1929-58, Intriligator concluded that embodied and disembodied technological change must be treated simultaneously.[30]

Thus, Intriligator provided a method, which was used in this study, for determining the rates of disembodied and embodied technological change simultaneously. However, many of the stringent assumptions of the previous Solow models were necessarily retained: (1) disembodied technological change is Hicks-neutral and proceeds at a constant rate; (2) embodied technological change is both capital and labor-augmenting; (3) the production function is linear homogeneous and Cobb-Douglas; (4) the economy is in a state of perfectly competitive equilibrium; and (5) the marginal productivity of labor is equated over all vintages of capital. The validity and implications of two of these assumptions, neutrality and constant returns to scale, have been subject to considerable scepticism and investigation. If they are not valid, biased estimates of

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[30] *Ibid.*, p. 69.

disembodied technological change will result. This is discussed in the following section.

V. ECONOMIES OF SCALE AND NON-NEUTRALITY

Walters has clearly pointed out that if economies of scale are present in the aggregate economy, its effects will be included in the measure of disembodied technological change.[31] In view of its importance, Walters investigated the assumption of constant returns to scale by estimating the unrestricted Cobb-Douglas function

$$Q=Ae^{ut}K^{a}L^{b}$$

with similar data to that used by Solow in his 1957 paper. Walters found that the sum of a+b was significantly greater than one, thus indicating economies of scale. According to his estimates, 27 to 35 per cent of the increase in output in the U.S. private non-farm sector could be attributed to economies of scale with a consequent reduction in the proportion attributable to disembodied technological change. However, as Walters indicated, the implications of economies of scale in the aggregate production function are not clear, and his results cannot be regarded as overwhelming evidence against the hypothesis of constant returns to scale.

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[31]A. A. Walters, "A Note on Economies of Scale," Review of Economics and Statistics, 45:425-27, November, 1963.

Ferguson has expressed similar views.[32] On the basis of a study of the U.S. manufacturing sector, 1929-63 he concluded that in aggregate studies covering long periods of time, a production function which is homogeneous of degree one is likely to provide economically more meaningful results even though these results may be statistically less significant than in the case of homogeneity of degree greater than one (economies of scale).[33] Ferguson also argued that homogeneity of degree greater than one should not be interpreted to mean that the aggregate economy is subject to economies of scale. In addition, Kislev has suggested that many of the estimated aggregate production functions for U.S. agriculture, which have shown significant economies of scale, are biased in the direction of overestimating economies of scale.[34]

The neutrality assumption most commonly questioned has been that of Hicks-neutral disembodied technological change. For example, Resek questioned this assumption in Solow's 1957 paper, as well as the method which Solow used

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[32]C.E. Ferguson, "Substitution, Technical Progress, and Returns to Scale," American Economic Review, 55:296-305, May, 1965.

[33]*Ibid.*, pp. 303-05.

[34]Y. Kislev, "Overestimates of Returns to Scale in Agriculture - A Case of Synchronized Aggregation," Journal of Farm Economics, 48:967-83, November, 1966.

to test the neutrality assumption.[35] Resek suggested that if technological change is non-neutral in the Hicks sense, then some of the observed increases in output per man could be attributed to the interaction of capital or labor and technological change.

One method of relaxing the assumption of Hicks-neutrality is to allow for other types of neutral technological change such as Harrod-neutral and Solow-neutral which are non-neutral or biased in the Hicks sense. To the extent that other types of neutrality are also taken into account in a method or model, the assumption of neutrality would likely lead to less biased results than in the case where only Hicks-neutral disembodied technological progress is a possibility. An example of this approach is a study by David and van de Klundert of the private domestic sector of the U.S. economy, 1899-1960.[36] They employed a homogeneous of degree one CES production function incorporating both labor and capital-augmenting technological change. The relative rates of labor and capital augmentation can then be related to the usual concepts of neutral, labor-saving, and capital-saving technological advance in the

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[35] R. W. Resek, "Neutrality of Technical Progress," Review of Economics and Statistics, 45:55-63, February, 1963.

[36] P. A. David and T. van de Klundert, "Biased Efficiency Growth and Capital-Labor Substitution in the U.S., 1899-1960," American Economic Review, 55:357-94, June, 1965.

Hicks sense.[37] From the empirical application of their model, they concluded that technological progress in the private domestic sector of the U.S. economy was labor-saving in the Hicks sense.

A quite different approach to non-neutrality and economies of scale has been suggested by Brown and Popkin.[38] They attempted to attribute changes in output over any discrete time period to the weighted change in inputs, economies of scale, and neutral and non-neutral technological change. The method consisted of fitting a Cobb-Douglas production function to various time periods in order to identify time periods called "technological epochs," in which there was only neutral technological change. Within each epoch, the influences on output of neutral technological change, economies of scale and increased inputs were estimated, and the changes in the parameters of the estimated production function between epochs were then used to measure output change attributable to non-neutral technological change.

However, the abrupt shift from one epoch to another is an approximation, since it is likely that a shift in

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[37] Ibid., pp. 362-63.

[38] M. Brown and J. Popkin, "A Measure of Technological Change and Returns to Scale," Review of Economics and Statistics, 44:402-11, November, 1962.

technology occurs gradually over time. Thus, the production function is misspecified at the ends and beginnings of all periods. However, if a study covers a long period of time, the misspecification as a result of using epochs compared with only one time period would be of a lesser degree. Using data for the U.S. private non-farm sector, 1890-1958, Brown and Popkin found evidence of economies of scale, and showed that the effects on output of non-neutral disembodied technological change were extremely small compared with the effects of neutral technological change.

In summary, the evidence against the assumption of constant returns to scale is not conclusive. The question of assuming neutrality is really a question of specifying the correct type of technological change. However, it must be noted that if the assumptions are not valid, biased estimates of technological change result. The use of these assumptions in this study is discussed in the following chapter.

VI. SOME PROBLEMS AND ALTERNATIVES

The models and methods outlined in the previous sections provided the basic concepts which were used in this study. As an aid to interpreting the results of this study, it is useful to briefly outline the basic problems and objections to these methods, and some alternative approaches.

From an analytical point of view, disembodied technological change has been treated as an exogenous variable which is not explained by any economic phenomenon. It has been called the "Residual" and "a measure of our ignorance."[39] The embodiment hypothesis (technological change embodied in factor inputs) was an attempt to relate part of this residual to qualitative change in factor inputs. These attempts have also been criticised. Although he conceded that the embodiment hypothesis is a potentially fruitful method of analysis, Griliches has argued that in practice it turns out to be a mere "relabelling of an already empty box."[40] Moreover, if the assumption that technological change proceeds at constant exponential rates is dropped, Jorgenson has shown that it is often impossible to distinguish capital-embodied from disembodied technological change on the basis of available data.[41] However, David and van de Klundert have defended the embodiment approach.[42]

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[39]Domar, op. cit., p. 709.

[40]Z. Griliches, "Technological Change and Economic Theory: Discussion," American Economic Review, 55:344, May, 1965.

[41]D. W. Jorgenson, "The Embodiment Hypothesis," Journal of Political Economy, 74:1-17, February, 1966.

[42]David and van de Klundert, op. cit., pp. 357-59.

They argued that it is possible to infer the rate of factor augmentation from conventional measures of inputs and outputs, and that this may be used to place prior restrictions on further attempts to empirically identify the sources of capital and labor augmentation.

The conception and estimation of an aggregate production function raises numerous theoretical and practical problems, although these are less troublesome when the methods are applied to one sector such as agriculture rather than to the whole economy.[43] There has been considerable discussion in the literature on the relevant concept of capital as it relates to the production function. Harcourt has recently provided a useful review of the controversies.[44]

In addition to the above conceptual problems, there are numerous difficulties in obtaining accurate measurements of inputs and outputs which are required for any empirical analysis. The measurement of aggregate capital is particularly difficult because: (1) it is usually purchased not hired; (2) it is durable; and (3) its cost is ambiguous.[45]

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[43]For a discussion see: L. B. Lave, _Technological Change: Its Conception and Measurement_ (New Jersey: Prentice-Hall, Inc., 1966), pp. 13-15; 37-38; 140-41.

[44]G. C. Harcourt, "Some Cambridge Controversies in the Theory of Capital," _Journal of Economic Literature_, 7:369-405, June, 1969.

[45]E. D. Domar, "On Total Productivity and All That," _Journal of Political Economy_, 70:602, December, 1962.

Errors of measurement will bias any estimate of technological change. Errors may arise from: (1) errors in the time series; (2) non-homogeneity of the series over time; and (3) errors stemming from the economy's not always being in long-run equilibrium.[46]

The alternative approaches, largely inspired by Denison, Griliches and Jorgenson, are attempts to directly explain a large portion of the residual. Denison attempted to identify the important elements of quality change in labor inputs.[47] Increases in output not accounted for by increased amounts of inputs or quality changes of inputs were attributed to changes in total factor productivity. Growth in total factor productivity was then ascribed to particular sources that could be identified and quantified such as resource shifts, economies of scale and the effect of demand pressures. In this way Denison was able to explain a large part of the residual. The Economic Council has applied Denison's methods to explain the growth of output in the Canadian economy.[48] Similar methods have

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[46] Lave, op. cit., p. 63.

[47] Edward F. Denison, The Sources of Economic Growth in the United States and the Alternatives Before Us, CED Supplementary Paper No. 13 (New York: Committee for Economic Development, 1962).

[48] Economic Council of Canada, The Challenge of Growth and Change, Fifth Annual Review (Ottawa: Queen's Printer, 1968), pp. 7-61.

been used by the Council to explain the growth in labor productivity in Canadian agriculture.[49]

Griliches attempted to explain productivity change in the U.S. agricultural sector by estimating a cross-sectional production function.[50] The Cobb-Douglas function estimated was homogeneous of degree greater than one with six independent variables: livestock expense, other current expense, machinery, land, buildings and man-years of labor. Griliches then adjusted the time series data on inputs for changes in quality, and combined these by using weights derived from the estimated production function. On this basis he was able to account for all of the observed increases in total agricultural productivity, 1940-60.

More recently, Jorgenson and Griliches have examined the hypothesis that if quantities of output and input are measured accurately, growth in total output is largely explained by growth in total input.[51] Within the framework of social accounting, the hypothesis becomes that if real output and real input are accounted for accurately,

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[49]Ibid., pp. 63-75.

[50]Z. Griliches, "The Sources of Measured Productivity Growth: United States Agriculture, 1940-60," Journal of Political Economy, 71:331-46, August, 1963.

[51]D. W. Jorgenson, and Z. Griliches, "The Explanation of Productivity Change," Review of Economic Studies, 34:249-83, July, 1967.

the observed growth in total factor productivity is negligible.

In summary, these alternative approaches attempt to make the residual disappear by constructing new measures of the growth of the various inputs which will, when taken together, fully account for the observed growth of output. This approach is questionable to the extent that it is tantamount to tampering with the data. Moreover, David and van de Klundert have questioned whether this represents an alternative approach. They suggest that it would be more sensible to begin by trying to identify the form which factor augmentation has taken, and then proceed to tackle the intriguing, but quite distinct question of the sources of such augmentation.[52]

The methods used in this study, which are presented in the following chapter, draw heavily on the models for measuring embodied and disembodied technological change which are reviewed in earlier sections of this chapter.

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[52]David and van de Klundert, op. cit., p. 358.

CHAPTER III

METHOD OF ANALYSIS AND MEASUREMENT OF VARIABLES

This chapter presents the basic model and data series which were used to estimate the rates of disembodied and embodied technological change in Canadian agriculture, 1935-65. The chapter consists of two parts: (1) Section I contains a description of the basic model, a definition of variables, and a discussion of the implied assumptions; and (2) Section II outlines the sources, methods, and assumptions which were employed in the derivation of the time series data used to estimate the model's coefficients. The actual data series are presented in Table V which is found in the Appendix. Also found in the Appendix is Table VI which presents a list of symbols representing variables, parameters, and coefficients used in this study.

I. MODEL FOR ESTIMATING TECHNOLOGICAL CHANGE

The basic model for this study was a linear homogeneous Cobb-Douglas production function of the form

$$Q = Ae^{ut}e^{f+gw+hw^2}L^{1-a}K^{a}$$

where: (1) Q represents annual gross output in the primary agricultural sector at base period prices;

(2) L represents the number of persons employed in the agricultural sector;

(3) K represents the equivalent annual flow of material inputs (intermediate goods purchased from other sectors of the economy), and capital services including livestock, land, buildings, and machinery and implements all measured at base period prices;

(4) t is a time index, 1, 2, ..., 31, representing the years during the time period 1935-65;

(5) w is an annual index which was designed as a proxy for environmental influences on output, and measures the observed deviations from the expected long-term trend of a weighted composite of crop yields;[1]

(6) e^{ut} is a shift function designed to measure the annual percentage rate (100u) of neutral disembodied technological change; and

(7) $e^{f+gw+hw^2}$ is a function designed to relate actual output, Q, to potential output through the weather index, w.

The model was estimated in natural log-linear form using the least-squares regression technique, which provided estimates of: the constant, lnA'(=lnA+f); the disembodied technological change coefficient, u; the w and w^2 coefficients, g and h; and the elasticity of output with respect to capital, a. The time series data consisted of 31 observations on Q, t, w, L, and K for each of the years 1935-65.

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[1]For purposes of this study, the index is called a "weather index".

The rates of technological change embodied in material inputs, and in machinery and implements, β and λ respectively, were measured indirectly in the manner suggested by Solow and Intriligator, and reviewed above in Chapter II. A priori values of β and λ were used to construct alternative series for the K variable.[2] Thus, there was an alternative time series for K for each possible combination of β and λ. The model was estimated using each of these alternatives which resulted in a matrix of regression equations where each regression was computed on the basis of a different time series for K. The real world values of β and λ were then inferred by choosing the "best" regression equation using the criteria of goodness of fit, significance levels of the estimated coefficients and low standard errors.

The above model is based on several important assumptions: (1) disembodied technological change was Hicks-neutral and proceeded at a constant rate;

(2) embodied technological change proceeded at a constant rate and was capital-augmenting in the vintage sense, which implies that the productive quality of material inputs improved at an annual rate of 100β per cent, and that the productive quality of new gross investment in

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[2]Section II below outlines the detailed method used to derive the alternative K series.

machinery and implements improved at an annual rate of 100λ per cent, that is, machinery and implements purchased in any year were 100λ per cent more productive than those purchased in the preceding year;

(3) the Cobb-Douglas production function was linear homogeneous in labor and capital, which implies constant returns to scale and unitary elasticity of substitution between capital and labor;

(4) the agricultural sector was in a state of perfectly competitive equilibrium;

(5) inputs of labor, livestock, land, and buildings were homogeneous over time;

(6) capital inputs were utilized at a constant rate; and

(7) labor was allocated so that its marginal product was equated over all vintages of machinery and implements.

The construction of the data series, particularly for the flow of capital services and the derivation of the equivalent stock of machinery and implements, required several additional assumptions. However, these are more conveniently discussed in the following section, which deals specifically with measurement of the variables.

The above assumptions are highly restrictive. Assumptions (4), (6) and (7) were necessary because of the lack of feasible operational alternatives. Assumption (5) is a serious deficiency in this study, since it is unreasonable to expect that the productive quality of these inputs has remained unchanged over the entire period, 1935-65. However, within the context of the general approach of this study, alternative methods for measuring embodied technological change in more than two factors are not available. In a recent study of technological change in Canadian agriculture, 1946-65, Li empirically investigated the hypotheses of constant returns to scale, Hicks-neutral disembodied technological change and unitary elasticity of substitution between capital and labor. He did not find any statistical evidence which would reject any of these hypotheses.[3] However, since this study covered a longer time period, a dummy variable was devised to investigate whether the parameters and coefficients of the production function changed significantly during the time period under study. The assumption of constant returns to scale was also relaxed in an alternative model. Therefore, it was possible to compare the regression results under assumptions of economies or diseconomies of scale and constant returns to scale. The specific models used to investigate the stability of the production function, and to relax the assumption of constant

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[3]Lew-king Li, "Technological Change in Canadian Agriculture," (unpublished Doctoral dissertation, University of Manitoba, Winnipeg, 1968), pp. 76-87.

returns to scale are outlined in detail in the following chapter.

It is well known that variations in climatic conditions account for substantial year-to-year variation in agricultural production, especially in the output of field crops. One method of allowing for this involves directly adjusting the field crops component of output with a weather index. This method has been used by Li, and was used as an alternative in this study.[4] Such a procedure, however, results in the use of an independently calculated measure of potential output to estimate the production function. However, since the production function itself is intended to provide an estimate of potential output, it is somewhat circular to impose an independently calculated measure of potential output at the outset.[5] Therefore, it is more logical to introduce an expression into the production function which would relate potential and actual (observed) output. The basic functional form, $e^{f+gw+hw^2}$, used in this study is similar to the one used by Solow to relate potential and actual output in the U.S. economy through the unemployment rate.[6] This particular expression may

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[4]Ibid., pp. 43-44.

[5]R. M. Solow, "Technical Progress, Capital Formation, and Economic Growth," American Economic Review, 52:77, May, 1962.

[6]Ibid., p. 78.

duplicate the left half of the normal curve, and, a priori, it seemed to possess the right general shape. However, as outlined in the following chapter an alternative functional form was also investigated.

II. MEASUREMENT OF THE VARIABLES

Annual time series data for gross output (Q), weather index (w), labor input (L), and the flow of capital services (K), were required to estimate technological change. The main data sources were publications of the Dominion Bureau of Statistics. The series on output and capital services were measured at 1935-39 constant prices. This base period was chosen of necessity, since the official price indexes, which were used as deflators, are constructed on the base period, 1935-39=100. A description of the derivation of the required time series follows.

Gross Agricultural Output (Q)

Since the specification of the production function included material inputs, the relevant concept of output was gross output rather than a measure of value-added production. Gross agricultural output consists of three components: (1) cash receipts from the sale of farm products (excluding inter-farm transfers); (2) income in kind; and (3) changes in farm-held inventories of field crops and livestock. Each of these components was further subdivided into field crops, livestock and livestock products, and forest and maple products which, in turn, were deflated by the appropriate price index.[7] Total cash receipts, income in kind, and inventory changes for livestock and products, and for field crops were deflated by the animal products and field products components, respectively, of the Canadian farm products price index.[8] Total cash receipts and income in kind from forest and maple products were deflated by the lumber and timber component of the general wholesale price index.[9] Gross output at base period prices was obtained as the sum of the deflated livestock, field crops, and forest and maple products output.

One variant of the gross output series was constructed by dividing the total field crop component by the weather index described in the following section. This had the effect of increasing gross output in years of unfavorable

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[7]For a detailed description of the various income components see data source: Canada, Dominion Bureau of Statistics, <u>Handbook of Agricultural Statistics, Part II: Farm Income - 1926-65</u> (Ottawa: Queen's Printer, 1967).

[8]Price index data source: Canada, Dominion Bureau of Statistics, <u>Prices and Price Indexes</u> (Ottawa: Queen's Printer, various issues).

[9]Price index data source: <u>Ibid</u>. This was not the most ideal price deflator, but a more suitable alternative was not available.

weather conditions, and reducing the measure of output in years of better than average weather conditions. The data series for gross output, Q, and gross output adjusted for weather influences, Q', are shown in Table V, columns 2 and 3, respectively.

Weather Index (w)

This variable was designed as a proxy for environmental effects on agricultural output. The concept used to construct the index was suggested by Stallings.[10] He employed time series of crop yields from experimental plots where as many variables as possible were held constant. A trend was estimated to account for changing soil fertility and seed quality over time. The crop yield variation about the estimated trend provided an indication of the year-to-year influence of weather on yields. Such a method involves two basic assumptions: (1) all variations in yield due to non-weather influences not correlated with weather are randomly and normally distributed with an expected value of zero; and (2) the trend of yields is linear, and can be removed by the simple regression of yield on time.

In view of the nonavailability of suitable experimental plot data, and the very aggregated nature of this

[10] James L. Stallings, "A Measure of the Influence of Weather on Crop Production," Journal of Farm Economics, 43:1153-59, December, 1961.

study, average annual yields were used to construct the index. In this respect the method used in this study differed from that suggested by Stallings. The significance of this departure in method is that additional factors influencing yields, which could be held constant in an experimental plot, were included in the weather index developed for this study. For example, the weather index may include such non-weather influences as annual variations in seed and fertilizer application, cultural practices, and crop damage by pests. The weather index, therefore, is an "ex post" measure of all influences on crop yields after removal of the long-term linear trend. However, this did not seem to be a serious limitation for purposes of this study. Firstly, it is reasonable to assume that the effects of technological change on crop yields were accounted for by the estimated linear trend. Therefore, the weather index would not remove the effect of technological change which this study attempted to measure. Secondly, since the correlation of the weather index with the other explanatory variables, labor and capital, was negligible, the weather index would not explain any of the annual variation in output properly attributable to changes in labor and capital inputs.

The actual weather index was constructed by estimating a simple regression of the form

$$y=a+bx$$

for each crop considered, where y is the average yield in bushels per acre, and x is a time index representing each of the years 1935-65. The crops considered were all wheat, oats, and barley in each of the Prairie Provinces. A weather index was computed for each crop in each province from the regression results by dividing the observed yield by the predicted yield value. The nine individual indexes were then combined into a single aggregate weather index by weighting each according to their value of production as a proportion of the total value of production of all three crops in the three provinces.[11] The aggregate weather index is shown in Table V, column 4.

Labor (L)

Labor input was measured in man-years on the basis of the number of persons employed annually in agriculture as reported in the Labour Force Survey. The number of persons employed includes those paid and unpaid, fourteen years of age and over. Regular quarterly and monthly surveys were not initiated until 1945. Prior to this, only annual estimates based on the number of persons employed at the beginning of June are available. There-

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[11]Source of yield and value of production data for 1935-62: Canada, Dominion Bureau of Statistics, Handbook of Agricultural Statistics, Part I: Field Crops (Ottawa: Queen's Printer, 1964); and for 1963-65: Canada, Dominion Bureau of Statistics, Quarterly Bulletin of Agricultural Statistics (Ottawa: Queen's Printer, various issues).

fore, to ensure a continuous and comparable series, the employment estimates used for the years 1946-65 were the June estimates rather than the annual averages. This is a potential weakness in that agriculture is characterized by considerable seasonal fluctuation in employment. However, a comparison of the June estimates with the annual averages for the 1946-65 period shows that, while the June estimates were slightly larger in magnitude, the trend and year-to-year changes diverged very little.

Although the Labour Force Survey is the only source of employment data for agriculture, the estimates have severe limitations which must be recognized. In addition to the sampling error of the survey itself, a simple measure such as the number of persons employed fails to take into account the changing structure and quality of the agricultural labor force. The average hours of work per week have been declining. Therefore, other things being equal, a simple measure of labor input such as the number of persons employed would be biased upwards in the later years. The age and sex composition of the labor force is ignored. The proportion of people employed who are in the younger age groups has been declining. Therefore, to the extent that older workers are less productive because of their age, the labor input estimates could be biased upwards in the later years. On the other hand, however, it is very likely that the quality of the labor force has improved over time, as a result of improved health standards and skills. This would result in a downward bias in the measurement of labor input. Ideally, the labor input series should have been adjusted for these influences. However, for purposes of this study such adjustments were not attempted because of the lack of appropriate information on the relationships involved. Any adjustments made would tend to be highly arbitrary and, therefore, questionable. Moreover, the effects of the various influences are to some extent offsetting.

Li attempted to account for the changing age and sex composition of the agricultural labor force by developing the concept of a man-equivalent.[12] However, a comparison of Li's data with the labor input series used in this study for the 1946-65 period reveals that the average annual percentage decline in the two series differed by less than 0.2 per cent, and the year-to-year movements were very similar. This was interpreted as an additional indication that, given data limitations, detailed adjustments to the labor input series were not justified.

Thus, the measure of annual labor input used in this study was the June estimate of the total number of

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[12] Li, op. cit., p. 45.

persons employed in agriculture.[13] The series is shown in Table V, column 5.

Flow of Capital Services (K)

Measurement of the flow of capital services required six data series at base period prices: (1) quantity of livestock and poultry on farms; (2) stock of buildings; (3) amount of building depreciation; (4) amount of land input; (5) stock of machinery and implements; and (6) quantity of material inputs. The method of aggregating these components into a single measure of the flow of capital services is outlined following a brief discussion of the derivation of each series.

Livestock and poultry. The value of livestock and poultry on farms at base period prices was derived by dividing the current value of livestock and poultry by the animal products component of the Canadian farm products price index.[14] The resulting series is shown in Table V,

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[13]Source for 1935-45: Canada, Dominion Bureau of Statistics, Canadian Labour Force Estimates, 1931-45, Reference Paper No. 23 (Ottawa: Queen's Printer, 1958); and for 1946-65: Canada, Dominion Bureau of Statistics, The Labour Force (Ottawa: Queen's Printer, various issues).

[14]Current values of livestock and poultry data source: Canada, Dominion Bureau of Statistics, Quarterly Bulletin of Agricultural Statistics (Ottawa: Queen's Printer, various issues). Price index data source: Canada, Dominion Bureau of Statistics, Prices and Price Indexes (Ottawa: Queen's Printer, various issues).

column 7.

Buildings. Since the value of the stock of buildings was not available separately, it was necessary to derive the series from published estimates of gross investment and the depreciation figures which are discussed below.[15] The net stock of agricultural buildings at base period prices in year t+1 was defined as the net stock in year t plus gross investment in year t+1 minus depreciation in year t, all at base period prices. Gross investment at base period prices was obtained by deflating the current dollar estimates of gross investment by the building materials component of the price index numbers of commodities and services used by farmers.[16] The real net stock of buildings series is shown in Table V, column 8.

Building depreciation. The published estimates for building depreciation do not include depreciation on

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[15] New construction (gross investment) data source for 1935-48: O. J. Firestone, Private and Public Investment in Canada 1926-1951, Department of Trade and Commerce (Ottawa: King's Printer, 1951), p. 154; and for 1949-65: Canada, Dominion Bureau of Statistics, Private and Public Investment in Canada, Outlook and Regional Estimates (Ottawa; Queen's Printer, various issues).

[16] Price index data source: Canada, Dominion Bureau of Statistics, Price Index Numbers of Commodities and Services Used by Farmers (Ottawa: Queen's Printer, various issues).

buildings located on rented farms. Therefore, it was necessary to adjust the published estimates.[17] It was assumed that the average value of buildings on rented farms, part owner - part tenant farms, and owner operated farms was equal. It was further assumed that one-half of the farms classified as part owner - part tenant had buildings on the rented portion. Using these assumptions, an annual adjustment factor was defined as the percentage of total farms operated by tenants plus one-half of the percentage of part owner - part tenant farms. The adjustment factor was calculated from Census of Agriculture data for census years, and values for intercensal years were interpolated. The building depreciation series adjusted to include rented farms was then obtained by dividing the published series by one minus the adjustment factor, which increased the published estimates by approximately fifteen per cent. To arrive at depreciation at base period prices, the adjusted series was deflated by the building materials price index.[18] The final series is shown in Table V, column 9.

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[17]Source of published estimates: Canada, Dominion Bureau of Statistics, Handbook of Agricultural Statistics, Part II: Farm Income - 1926-65 (Ottawa: Queen's Printer, 1967).

[18]Source of price index data: Canada, Dominion Bureau of Statistics, Price Index Numbers of Commodities and Services Used by Farmers (Ottawa: Queen's Printer, various issues).

Land. The total agricultural land area in Canada increased by less than seven per cent from 1931 to 1966. In fact, between 1941 and 1961 there was a slight decline in total agricultural land area. However, the ratio of improved to unimproved land increased substantially from 1.1 in 1935 to 1.6 in 1965. The land input series developed for this study was an attempt to account for the shift towards improved land. For census years the acreage of improved and unimproved land was taken from the Census of Agriculture, and estimates for the intercensal years were interpolated. The value of total agricultural land at base period prices was then calculated by multiplying the acreage estimates by the average value per acre during 1935-39 for improved and unimproved land, respectively.[19] The resulting series is shown in Table V, column 10.

Material inputs. Material inputs refers to the goods and services other than durable capital which are purchased from the non-agricultural sector of the economy and consumed in the process of production. The material input

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[19] The average values per acre for 1935-39 were derived by dividing the total value of land and buildings (less the value of buildings as derived for this study) by the number of acres. This resulted in a 1935-39 average value per acre of $19.60 and $4.20 for improved and unimproved land, respectively. Source of total value of land and buildings data: Canada, Dominion Bureau of Statistics, Quarterly Bulletin of Agricultural Statistics (Ottawa: Queen's Printer, various issues).

series at base period prices was derived as the sum of six items: (1) total machinery expenses (excluding machinery repairs) deflated by the price index for gasoline, oil, and grease; (2) fertilizer and lime expense deflated by the price index for fertilizer; (3) feed expense deflated by the price index for feed; (4) machinery repair expense deflated by the price index for farm machinery; (5) building repairs (adjusted to include buildings on rented farms by employing the adjustment factor outlined above under building depreciation) deflated by the building materials price index; and (6) the sum of other crop and livestock expense, electricity, telephone, and miscellaneous expense deflated by the price index for hardware items.[20] This procedure resulted in a time series of material inputs at base period prices which embodies no technological change, that is, $\beta = 0$. The basic series is shown in Table V, column 12.

Material input series embodying technological change at various rates were derived from the basic series by assuming that technological change occurred at a constant

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[20]Source of all material inputs expense data: Canada, Dominion Bureau of Statistics, Handbook of Agricultural Statistics, Part II, Farm Income - 1926-65 (Ottawa: Queen's Printer, 1967). Source of all price index data: Canada, Dominion Bureau of Statistics, Price Index Numbers of Commodities and Services Used by Farmers (Ottawa: Queen's Printer, various issues).

annual rate of 100β per cent. Augmented material input series for various values of β were easily derived by multiplying each observation, n, of the $\beta=0$ series by $(1+\beta)^n$ where n=0, 1, 2, ..., 30.

To test the sensitivity of the method of estimating embodied technological change to the price indexes used to deflate the current dollar value of material inputs, an alternative series for material inputs was derived by deflating the total current dollar value of material input expenses by the general wholesale price index.[21] The general wholesale price index was chosen for this purpose because it is often used as a reference level against which to compare the movements of other price indexes. The alternative series for material inputs when $\beta=0$ is shown in Table V, column 13. Based on this alternative, material input series embodying varying rates of technological change were constructed in the manner outlined in the preceding paragraph.

Stock of machinery and implements. Since a purpose of this study was to measure embodied technological change in machinery and implements, knowledge about the vintage composition of the stock of machinery and implements was

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[21]Source of wholesale price index data: Canada, Dominion Bureau of Statistics, Prices and Price Indexes (Ottawa: Queen's Printer, various issues).

required. Assuming that machinery and implements purchased in any year were 100λ per cent more productive than those purchased in the preceding year, the equivalent stock of machinery and implements was defined as the sum of the surviving machinery and implements of different vintages, after weighting each vintage by the appropriate rate of embodied technological change. Thus, derivation of the equivalent stock of machinery and implements embodying technological change at an annual rate of 100λ per cent required a time series of past gross investment measured in volume terms, that is, at base period prices, and knowledge about the service life of machines and implements, that is, knowledge about the rate of replacement of old investment goods.

Gross investment at base period prices was obtained by deflating the current dollar estimates of gross investment (Table II, column 2) by the farm machinery component of the price index numbers of commodities and services used by farmers.[22] The resulting series of annual gross investment at base period prices for 1921-65 is shown in Table II, column 3. To test the sensitivity of the method of measuring embodied technological change to the price deflator, gross investment was also deflated by the USDA index of

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[22]Source of price index data: Canada, Dominion Bureau of Statistics, Price Index Numbers of Commodities and Services Used by Farmers (Ottawa: Queen's Printer, various issues).

TABLE II

GROSS INVESTMENT IN MACHINERY AND IMPLEMENTS, CANADIAN AGRICULTURE, 1921-65

Year	Gross investment at current prices*	Gross investment at 1935-39 prices: Deflated by DBS price index	Gross investment at 1935-39 prices: Deflated by adj. USDA price index
(1)	(2)	(3)	(4)
	--- thousands	of	dollars ---
1921	59140	53088	50590
1922	28725	31952	30237
1923	42240	45468	42753
1924	32105	31353	31261
1925	33790	34515	33489
1926	69200	70902	68719
1927	88600	90872	87377
1928	116200	119057	115164
1929	100300	102872	99405
1930	72000	74227	72217
1931	26700	28135	26202
1932	23400	24867	22180
1933	15500	16830	15784
1934	30600	32347	32797
1935	34400	36021	35318
1936	44200	45194	44964
1937	62900	64712	62837
1938	67400	64745	64808
1939	63000	60811	59886
1940	82600	78072	74684
1941	88400	81027	78929
1942	71500	62500	60287
1943	39400	33646	32059
1944	72900	61675	57949
1945	90400	78540	71069
1946	131600	110774	104527
1947	214900	170150	159067
1948	280500	198093	178095
1949	352395	222612	193517
1950	389640	236002	197286
1951	423065	226480	206072
1952	449805	230197	227980
1953	440255	223821	220017
1954	296050	149596	148993
1955	323745	162850	160747
1956	371495	177409	176902

TABLE II (continued)

Year (1)	Gross investment at current prices* (2)	Gross investment at 1935-39 prices: Deflated by DBS price index (3)	Deflated by adj. USDA price index (4)
	--- thousands	of	dollars ---
1957	325655	145512	151750
1958	347620	146861	153339
1959	410650	165318	175867
1960	423065	166430	174532
1961	389640	149059	150266
1962	456490	170269	163969
1963	547215	200519	191401
1964	612155	218940	209499
1965	683780	240007	227547

*Source of gross investment data for 1921-25: Kenneth Buckley, Capital Formation in Canada 1896-1930 (Toronto: University of Toronto Press, 1955), pp. 131-32; for 1926-48: O. J. Firestone, Private and Public Investment in Canada 1925-1951, Department of Trade and Commerce, (Ottawa: King's Printer, 1951); and for 1949-65: Canada, Dominion Bureau of Statistics, Private and Public Investment in Canada, Outlook and Regional Estimates (Ottawa: Queen's Printer, various issues).

farm machinery prices paid by farmers.[23] Before the USDA index was used as a deflator, however, it was converted to a 1935-39 base and adjusted for the changing value of the U.S. dollar in Canadian funds. Gross investment as deflated by the adjusted USDA index is shown in Table II, column 4.

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[23]Source of price index data: United States Department of Agriculture, Agricultural Statistics (Washington: Government Printing Office, various issues).

Four basic hypotheses about the rate of replacement of old investment goods have been used in total factor productivity studies: (1) accounting depreciation is set equal to replacement; (2) gross investment in some earlier period is set equal to replacement; (3) a weighted average of past investment with weights derived from studies of the survival curves of individual pieces of equipment is set equal to replacement; and (4) each investment generates a series of replacement investments over time.[24] For purposes of this study, hypothesis (2) was adopted, and the stock of machinery and implements was measured by assuming a service life of thirteen years after which the machine or implement is discarded.[25] Thus, the stock of machinery and implements is a thirteen-year moving sum of past gross investment at base period prices. This method is analogous to that used by Hood and Scott,[26] and to that suggested by Griliches.[27]

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[24] D. W. Jorgenson and Z. Griliches, "The Explanation of Productivity Change," Review of Economic Studies, 34:255, July, 1967.

[25] A service life of thirteen years was also used by Hood and Scott. See: W.C. Hood and A. Scott, Output, Labour, and Capital in the Canadian Economy, Royal Commission on Canada's Economic Prospects (Ottawa: Queen's Printer, 1957), p. 473.

[26] Ibid., pp. 234-37.

[27] Z. Griliches, "Measuring Inputs in Agriculture: a Critical Survey," Journal of Farm Economics, 42:1417, December, 1960.

The equivalent stock of machinery and implements in year t, J(t), was defined as

$$J(t) = \sum_{v=t-12}^{t} (1+\lambda)^{v} I(v)$$

where I(v) is the amount of gross investment in year v at base period prices. For $\lambda=0$, the stocks of machinery and implements derived from gross investment deflated by the DBS price index and the USDA price index are shown in Table V, columns 14 and 15, respectively.

Two assumptions are inherent in this method: (1) all machinery and implements have the same service life which is constant over time; and (2) gross investment at base period prices is an unbiased measure of the quantity of machinery and implements actually brought into production during any given year. Implicit in the assumption of a common service life for all machinery and implements is the condition that the service lives of various kinds of machinery and implements can be averaged into one representative figure, and that annual gross investments consist of constant proportions of the various kinds of machinery and implements.[28] In order to provide an indication of the sensitivity of the method to assumptions about the service life of machinery and implements, stocks of machinery and implements based on alternative assump-

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[28]Hood and Scott, _op_. _cit_., p. 239.

tions about the service life and rate of replacement of old investment goods were also constructed.[29] However, the results of the regression models using the various alternatives followed the same pattern, and differed only slightly in degree from the results using the basic assumption of a thirteen-year service life.[30] The issue involved in the assumption that gross investment at base period prices is a measure of the quantity of machinery and implements actually brought into production, is the separation of the values of transactions in new investment goods into a price and a quantity component. This is commonly achieved by deflating current dollar estimates by an appropriate price index. However, an error in this separation will affect the magnitude of the flow of capital services, and result in a biased measure of technological

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[29]The alternatives were: (1) a fifteen-year sum of past gross investment; (2) a service life function, $1-(t/15)$, which allows for a constant proportion of gross investment in any year to be discarded in each successive year; (3) a service life function, $1-(t^2/15^2)$, which allows for an increasing proportion of gross investment in any year to be discarded; and (4) a service life function, $1-(\sqrt{t}/\sqrt{15})$, which allows for a decreasing proportion of gross investment in any year to be discarded in each successive year.

[30]Since the regression results based on the alternative assumptions did not affect the conclusions of the study, they are not reported.

change.[31]

Aggregation of the flow of capital services. The annual flow of capital services from the stocks of land, buildings and livestock was derived by assuming an annual rate of return on the investment at base period prices. The flow of capital services from the stock of buildings also included depreciation. A method suggested by Griliches was employed to convert the stock of machinery and implements into an annual flow of services.[32] Assuming that there is no deterioration with age, and that the flow of services is constant over the life span of all machines and implements, then the annual flow of services equals an annuity for the service life at the rate of return. As Griliches points out, under these assumptions the annuity equals the sum of annual interest and depreciation charges, with the interest charges falling and the depreciation charges rising as the machine ages.[33]

Thus, the annual flow of capital services derived for this study was the arithmetic sum of four components: (1) the value of livestock, plus the value of land, plus

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[31]This also applies to all of the other variables in this study where the quantity has been measured by deflating current dollar estimates by a price index.

[32]Griliches, loc. cit.

[33]Ibid.

the stock of buildings, all at base period prices multiplied by the rate of return;[34] (2) building depreciation at base period prices; (3) material inputs at base period prices; and (4) the stock of machinery and implements at base period prices multiplied by the factor for a thirteen-year annuity at the rate of return.[35]

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[34]An annual rate of return of six per cent was assumed. An alternative rate of eight per cent was also employed. However, the regression results using an eight per cent return followed the same pattern and differed only slightly in degree from the results using the basic assumption of a six per cent return. The alternative results are not reported.

[35]The factor for a thirteen-year, six per cent annuity is 0.11296.

CHAPTER IV

EMPIRICAL RESULTS

I. INTRODUCTION

The multiple regression routine of the "UBC TRIP" computer program was used to provide least-squares regression estimates.[1] The basic model was estimated in natural logarithmic form

$$\ln(Q/L)=\ln A+ut+f+gw+hw^2+\ln(K/L)^a+\ln \mu$$

where μ is a disturbance term about which the usual assumptions were made.[2] The output of the program included: (1) the estimated regression coefficients; (2) the standard errors of each regression coefficient; (3) the F-ratio and associated probability for each regression coefficient;[3] (4) the standard error of the estimate, S; (5) the coeffi-

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[1] J. H. Bjerring and R. H. Hall, UBC TRIP (Triangular Regression Package) (Vancouver: University of British Columbia, Computing Centre, 1968).

[2] The usual assumptions are that μ_i are random variables with zero expectation and constant variance, and are pairwise uncorrelated. See: J. Johnston, Econometric Methods (New York: McGraw-Hill Book Company, Inc., 1963), p. 107.

[3] The F-ratio is equivalent to the more common "t" test. See: Ibid., pp. 123-25. The associated probability is the level at which the estimated regression coefficient is significantly different from zero. See: Bjerring, op. cit., pp. 48-49.

cient of multiple determination, R^2; and (6) the Durbin-Watson d statistic, "d". There are two constant terms, lnA and f, in the above model. However, only one constant term, lnA'=lnA+f, was estimated. If desired, lnA' can be separated by noting that actual output must equal potential output when the weather index equals one, which implies

$$f+g(1.0)+h(1.0)^2=0.$$

Therefore, f can be determined, and thus lnA, from estimates of g and h. All other coefficients are presented in the same units as they appeared in the models.

The method of determining the rate of embodied technological change in machinery and implements and in material inputs, λ and β respectively, involved estimating a regression equation for each possible combination of the a priori imposed values for λ and β. Thus, if m alternative values of λ and n alternative values of β were imposed, there would be m×n possible combinations resulting in a m×n matrix of estimated equations. The regression estimates for the various models are presented in the Appendix.[4] For each equation the estimated regression coefficients, R^2, standard error of the estimate (S), and the Durbin-Watson

[4]Each page of a particular table shows the regression estimates for several values of β given a specific value of λ, that is, the regression estimates for an individual equation are shown as a column with the column heading and table heading identifying the β and λ value, respectively.

d statistic ("d") are shown. All values of "d" are insignificant (that is, reject the hypothesis of serial correlation) at the one per cent level of significance unless otherwise noted. The standard errors of the regression coefficients are shown in parentheses. The associated probability of the F-ratio for each coefficient is shown immediately below each standard error.

The results of five models are reported. Model I was the basic model outlined in Chapter III. In Model II all technological change was assumed to be embodied in machinery and implements and material inputs. The assumption of constant returns to scale was relaxed in Model III, while two alternatives for relating actual and potential output were investigated in Model IV. Finally, Model V was an attempt to assess the stability of the production function over time by introducing a dummy variable into the model.

II. RESULTS OF THE BASIC MODEL

Three alternative sets of data were employed to estimate the production function

$$Q=Ae^{ut}e^{f+gw+hw^2}L^{1-a}K^{a}.$$

The regression results for each alternative set of data are discussed individually as Models I(a), (b) and (c).

Model I(a) (Table VII)

In this model, material inputs were deflated by the individual price indexes (Table V, column 12), and gross investment in machinery and implements by the DBS price index (Table V, column 14). β values ranging from 0.0 to 0.08 were used to construct alternative series for material inputs, thus embodying technological change at the corresponding annual rates of 0.0 to 8.0 per cent. Likewise, λ values ranging from -0.01 to 0.03 were used to construct alternative stocks of machinery and implements embodying technological change at the corresponding annual rates of -1.0 to 3.0 per cent. The matrix of regression results for the various combinations of λ and β are shown in Table VII. (Table VII consists of nine pages; each page reports the results for a specific λ value).

Table III shows the R^2 values and standard errors of the estimate for all combinations of λ and β. It is evident from Table III that: (1) for a given λ value, R^2 increased and then decreased as the value of β increased; and (2) for a given β value, R^2 decreased as the value of λ increased. Some individual numerical results are summarized as follows:

	λ value	β value
Highest R^2 (.9877)	-0.01	0.035
Lowest R^2 (.9847)	0.03	0.0

TABLE III

COEFFICIENTS OF MULTIPLE DETERMINATION AND STANDARD ERRORS OF THE ESTIMATE FOR MODEL I(a)*

λ \ β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
-0.01	.9871 (.0563)	.9875 (.0554)	.9876 (.0552)	.9876 (.0552)	.9877 (.0551)	.9876 (.0552)	.9875 (.0554)	.9873 (.0558)	.9871 (.0563)	.9869 (.0567)
.0	.9865 (.0577)	.9870 (.0566)	.9871 (.0564)	.9872 (.0562)	.9873 (.0560)	.9873 (.0560)	.9873 (.0560)	.9872 (.0562)	.9870 (.0565)	.9868 (.0569)
.005	.9861 (.0584)	.9867 (.0573)	.9868 (.0570)	.9869 (.0568)	.9870 (.0566)	.9871 (.0564)	.9871 (.0563)	.9871 (.0564)	.9870 (.0567)	.9868 (.0570)
.01	.9858 (.0591)	.9864 (.0580)	.9865 (.0577)	.9866 (.0574)	.9867 (.0572)	.9868 (.0570)	.9869 (.0567)	.9869 (.0567)	.9869 (.0569)	.9867 (.0572)
.015	.9855 (.0597)	.9860 (.0586)	.9862 (.0584)	.9863 (.0581)	.9864 (.0578)	.9866 (.0576)	.9867 (.0572)	.9868 (.0571)	.9867 (.0571)	.9867 (.0573)
.02	.9852 (.0603)	.9857 (.0593)	.9859 (.0590)	.9860 (.0588)	.9861 (.0585)	.9863 (.0582)	.9865 (.0578)	.9866 (.0575)	.9866 (.0574)	.9866 (.0575)
.025	.9850 (.0608)	.9854 (.0600)	.9855 (.0597)	.9857 (.0594)	.9858 (.0591)	.9859 (.0589)	.9862 (.0583)	.9863 (.0580)	.9864 (.0578)	.9864 (.0578)
.03	.9847 (.0613)	.9851 (.0606)	.9852 (.0603)	.9853 (.0601)	.9855 (.0598)	.9856 (.0595)	.9859 (.0590)	.9861 (.0585)	.9862 (.0582)	.9863 (.0581)

*Standard errors of the estimate are shown in parentheses.

	λ value	β value
Lowest S (.0551)	-0.01	0.035
Most significant u (0.35 per cent level)	0.03	0.0
Most significant a (0.39 per cent level)	-0.01	0.035

Thus, the "best" regression based on the criteria of goodness of fit, low standard error of the estimate and significance level of the a coefficient occurred when λ=-0.01 and β=0.035. This implies embodied technological change in material inputs at the annual rate of 3.5 per cent, and in machinery and implements at the negative annual rate of 1.0 per cent. A negative rate of embodied technological change in machinery and implements was disturbing and contradicted a priori expectations. This should most likely be considered as a spurious result for various reasons which are discussed below. The specific negative value of λ which would have given the "best fit" was not determined, since in view of the questionable nature of the results, the model was not extended to include higher negative values for λ.

When λ=0.0, that is, no embodied technological change in machinery and implements, the "best" regression (R^2=.9873) occurred when β=0.035 to 0.05. The a coefficient was most significant when β=0.04. This indicates embodied technological change in material inputs at an annual rate of approximately 4.0 per cent. These results, therefore, support the hypothesis of substantial embodied technological change in material inputs.

The u coefficient, which is an estimate of the annual rate of disembodied technological change (100 u), had the largest value and was most highly significant in the regressions when β=0.0. This was the expected result, because if embodied technological change is not specified, then all increases in total factor productivity would show up as disembodied technological change. However, with respect to λ, the a priori expected results did not occur. The largest and most significant value of u occurred when λ=0.03, rather than when λ=0.0. This was further evidence which suggested that the results with respect to embodied technological change in machinery and implements must be regarded as suspect.

When no embodied technological change was specified (λ=β=0.0), u=0.0246 indicating an annual rate of disembodied technological change of 2.46 per cent. In this regression the u coefficient was highly significant (1.44 per cent level) with a relatively low standard error (.0095). When λ=0.0 and β=0.04 (the "best" regression), u=0.0113. Thus, when embodied technological change was specified the estimate of disembodied technological change declined, which supports the conclusion that a large portion of increases in total factor productivity can be attributed to technological change embodied in material inputs. It should be noted, however, that the estimates of u became increasingly imprecise and less significant as β increased.

The a coefficient was highly significant with low standard errors in all regressions. The g coefficient was not significant, while h was significant at about the 15 per cent level. Although the test for serial correlation was inconclusive for some regressions, serial correlation was not considered to be a problem.

Model I(b) (Table VIII)

This model was an attempt to assess the sensitivity of the method to an alternative deflator for gross investment in machinery and implements. In this model gross investment was deflated by the adjusted USDA price index (Table V, column 15). A matrix of regression results was obtained for the various combinations of λ and β as outlined for Model I(a) above. The regression results when λ=0.0 are reported in Table VIII.

The regression results of Model I(b) exhibited the same trends as Model I(a) with only slight differences in the magnitudes of the estimated coefficients and statistical measures. The main results are summarized as follows:

	λ value	β value
Highest R^2 (.9879)	-0.01	0.025-0.035
Lowest R^2 (.9853)	-0.03	0.0
Lowest S (.0547)	-0.01	0.02 -0.035

	λ value	β value
Most significant u (0.73 per cent level)	0.03	0.0
Most significant a (0.31 per cent level)	-0.01	0.025-0.035

Thus, the general conclusions drawn from the results of Model I(a) are applicable to this model as well. In view of the questionable nature of the results with respect to λ, only the regressions in which λ=0.0 are reported.

When λ=0.0, the "best" regression occurred when β=0.035, although R^2=.9875 in all regressions when β=0.025 to 0.05. Thus, this model gave a slightly lower estimate of embodied technological change in material inputs (approximately 3.5 per cent compared with 4.0 per cent in Model I(a)). The u coefficient was almost identical in Models I(a) and (b), (0.0113 compared with 0.0115). The a coefficient, however, was slightly more significant in Model I(b) than in Model I(a).

Thus, the results of Model I(b) were very similar to those of Model I(a), which suggests that the method was not particularly sensitive to the alternative deflator for gross investment in machinery and implements. It was concluded that the results of Model I(b) did not provide significant evidence for preferring the adjusted USDA price index over the DBS index for purposes of this study.

Model I(c) (Table IX)

This model was an attempt to assess the sensitivity of the method to an alternative deflator for material inputs. In this model, all material inputs were deflated by the general wholesale price index (Table V, column 13). A matrix of regression results was obtained for the various combinations of λ and β as outlined for Model I(a) above. The results when λ=0.0 are reported in Table IX.

Like Model I(b), the results of Model I(c) showed the same trends as Model I(a). The main results are summarized as follows:

	λ value	β value
Highest R^2 (.9880)	-0.01	0.025-0.035
Lowest R^2 (.9856)	0.02	0.0
Lowest S (.0543)	-0.01	0.025-0.03
Most significant u (0.13 per cent level)	0.02	0.0
Most significant a (0.26 per cent level)	-0.01	0.025-0.03

In view of the questionable nature of the results with respect to λ, only the regressions in which λ=0.0 are reported.

When λ=0.0, the "best" regression occurred when β=0.035 to 0.04, although R^2=.9877 in all regressions when β=0.03 to 0.04. Thus, this model gave an estimate of embodied technological change in material inputs which was identical to the estimates of Models I(a) and (b). However, the u coefficient was considerably greater in magnitude (0.0168 compared with 0.0113), and was more highly significant (10.72 per cent level compared with 38.53), than in Model I(a).

Thus, the results of Model I(c) were similar to those of Models I(a) and (b). The most notable difference was the larger and more highly significant estimate for disembodied technological change in Model I(c). The results with respect to β, however, suggest that the method was not sensitive to the alternative deflator for material inputs, and it was concluded that the results of Model I(c) did not provide significant evidence for preferring the general wholesale price index over the individual price indexes as a deflator for material inputs for purposes of this study.

Discussion of Model I

On a priori grounds, the results of Models I(a), (b) and (c) with respect to λ were unexpected. There are two possible interpretations: (1) the rate of embodied technological change in machinery and implements was, in fact, negative during the 1935-65 period; or (2) the method failed to provide an unbiased estimate of embodied technological change. Although this study did not provide sufficient evidence upon which to base a choice between these explanations, there are strong reasons to suspect that the latter interpretation is the more plausible and realistic.

Several factors could account for a biased or spurious estimate of embodied technological change. First, there was the problem of obtaining an accurate measurement of the real stock of machinery and implements. Since individual machines and implements are extremely heterogeneous, they must be aggregated in value terms, and then deflated by a price index to arrive at a measure of the stock in volume terms.[5] There are two alternatives for measuring capital goods in value terms. Capital goods may be valued in terms of input costs, or in terms of their ability to produce (either on the basis of output or capacity). The first alternative attributes all increases in output to changes in the productivity of capital itself, while the second alternative attributes all increases in output to changes in productivity in the production of capital goods. Clearly, neither of these extreme alternatives was satisfactory for purposes of this study. The crucial problem was the separation of such elements as design improvements and serviceability (embodied technological change) from changes in the cost of production; that is, the separation of increases in the productivity of machines and implements from increases in productivity in the production of machines and implements. On the assumptions that the suppliers of machinery and implements

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[5]The following discussion is largely drawn from: Richard Ruggles and Nancy Ruggles, "Concepts of Real Capital Stocks and Services," Output, Input and Productivity Measurement (Studies in Income and Wealth, Vol. 25. Princeton: Princeton University Press, 1961), pp. 387-411.

do not have monopolistic power in the product markets, and that the price charged continues to reflect chiefly the price of inputs available to the agricultural machinery and implement industry, the appropriate measure of machinery and implements for purposes of this study was gross investment deflated by a price index in which quality changes of the machines and implements have been accounted for. However, for purposes of constructing a price index, quality changes are difficult to define and measure. Therefore, it is important to note that the price deflator used in this study may have been biased. If so, there would be errors of measurement in the data series for real gross investment in machinery and implements.

However, even if the estimate of real gross investment was unbiased, the difficult problem of measuring the stock of machinery and implements and the flow of services would remain. In this regard, the restrictive nature of the assumptions used in this study is outlined in Chapter III.

Secondly, there was the potential problem of changing rates of utilization of machinery and implements over time. This may have been particularly important in this study in view of the cycles in gross investment. As shown in Table II, column 3, real gross investment reached peaks in 1928, 1952 and 1965, and troughs in 1933 and 1958.

This resulted in a stock of machinery and implements which actually declined during 1938-43 and 1959-65, but experienced very rapid growth during 1944-58. Since agricultural output maintained a steady upward trend throughout the entire period, either other inputs were substituted for machinery and implements in the short run, and/or the utilization rate of machinery and implements was a variable. The model could accommodate substitutability between capital and labor, but it did not take into account varying utilization rates for capital.

Thirdly, the assumption of a constant rate of embodied technological change in machinery and implements may have been inappropriate. It is possible that a cyclical pattern during the 1935-65 period may have obscured a long-term trend in the rate of embodied technological change.

Fourthly, it was possible that the time series data did not provide enough independent variation to allocate simultaneously with a high degree of confidence the increases in total factor productivity to three sources - disembodied technological change, and embodied technological change in both material inputs and machinery and implements. In fact, there was reason to suspect that the model could not choose between alternative combinations of embodied technological change in machinery and implements and material inputs. For example, the correlation coefficient between the flow of capital services, K, when $\lambda=0.0$ and $\beta=0.02$ on the one hand, and when $\lambda=0.02$ and $\beta=0.0$ on the other, was 0.9988 indicating an almost exact linear relationship between the two measures of K. Therefore, the regression results for the two alternatives would not provide sufficient information to use as a basis for choice between the alternative combinations of the rates of embodied technological change. This suggests that the model may not have been capable of estimating the rate of embodied technological change in more than one factor.

Thus, there were many reasons to suggest that the estimate of a negative rate of embodied technological change in machinery and implements may have been substantially biased.

III. MODEL WITH ALL TECHNOLOGICAL CHANGE EMBODIED

In Model I technological change, both embodied and disembodied, was specified to occur simultaneously. Of course, when $\lambda=\beta=0.0$ all technological change was assumed to be disembodied. For Model II a function of the form

$$Q=Ae^{f+gw+hw^2}L^{1-a}K^a$$

was estimated. Therefore, all technological change was assumed to be embodied in material inputs and machinery and implements. A matrix of regression results for the various combinations of λ and β was estimated using the same data series as in Model I(a). The results when λ=0.0 are reported in Table X.

<u>Model II</u> (Table X)

The regression results of Model II indicate a positive rate of embodied technological change in material inputs. However, like Model I, the "best" regression occurred when λ=-0.01, indicating a negative rate of embodied technological change in machinery and implements. Therefore, for reasons outlined above, only the results when λ=0.0 are reported.

A comparison of the results of the "best" regression when λ=0.0 in Model II, with those of Model I(a), reveals the following points: (1) Model II provided a higher estimate for the rate of embodied technological change in material inputs (5.0 per cent compared with 4.0 per cent); (2) R^2 was higher in Model I(a) (.9873 compared with .9870); and (3) the elasticity of output with respect to capital, a, was larger (0.5416 compared with 0.4572), more highly significant, and had a substantially lower standard error in Model II than in Model I(a).

The higher estimate of embodied technological change in Model II was expected, since any technological change which was estimated in Model I(a) as disembodied technological change was measured as embodied technological change in Model II. The higher R^2 in Model I(a) provided some evidence that embodied and disembodied technological change should be treated simultaneously, although this evidence must be considered weak in view of the lower standard error of the estimate, more highly significant a, and lower standard error of a in Model II. As expected, the elasticity of output with respect to capital was higher in Model II, because all increases in total factor productivity were assumed to be the result of embodied technological change. The relatively greater indeterminancy of the estimates of a in Model I(a) compared with Model II was most likely due to the very high correlation between the flow of capital services, K, and time, t, in Model I(a). Of course, this problem of multicollinearity did not arise in Model II, since t was not a specified variable.

IV. MODEL TO RELAX THE ASSUMPTION OF CONSTANT RETURNS TO SCALE

All other models reported in this study were homogeneous of degree one, that is, constant returns to scale was assumed. Model III was an attempt to assess the sensitivity of the method to the assumption of constant returns to scale. In Model III labor was introduced into the production function as an independent variable. The function

$$Q=Ae^{ut}e^{f+gw+hw^2}L^bK^a$$

was estimated using the same data series as in Model I(a). Estimates were obtained for λ=0.0 only, and are reported in Table XI.

Model III (Table XI)

A comparison of the results of Model III with the results of Model I(a) (when λ=0.0) shows the following: (1) Model III provided a slightly higher estimate of the rate of embodied technological change in material inputs (4.0 to 5.0 per cent compared with 4.0); (2) Model III gave a substantially higher estimate of the rate of disembodied technological change (1.99 to 1.88 per cent compared with 1.13); (3) the R^2 values were lower in all regressions in Model III (.9513 compared with .9873 in the "best" regression); (4) the a coefficient was substantially lower in Model III, had large standard errors and was not significant; and (5) serial correlation in the disturbance term may have been a problem in Model III, since the Durbin-Watson test was inconclusive in all regressions.

The relatively large standard errors for u and a were evidence of the problem of multicollinearity in Model III. The three explanatory variables, t, K and L, were highly correlated. For example, the correlation coefficient between labor and the flow of capital services when λ=β=0.0 was -0.9506. This increased to -0.9676 when λ=0.0 and β=0.08.

In Model III the sum of a+b was less than one, which indicated the possibility of diseconomies of scale in the aggregate production function. However, this result was not statistically significant.[6]

In summary, the assumption of constant returns to scale resulted in a slightly lower estimate of the rate of embodied technological change in material inputs, and a more substantially lower estimate of the rate of disembodied technological change. However, there was insufficient statistical evidence to reject the hypothesis of constant returns to scale.

V. ALTERNATIVE MODELS TO RELATE POTENTIAL AND ACTUAL OUTPUT

The principal means for relating potential and actual output used in this study is outlined in Chapter III. Two alternative forms were investigated. These are discussed individually as Models IV(a) and (b).

Model IV(a) (Table XII)

As described in Chapter III, a measure of potential output, Q' (Table V, column 3), was derived from actual

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[6]The F-test employed is outlined in: Gerhard Tintner, Econometrics (New York: John Wiley and Sons, Inc., 1952), pp. 89-91. Substitution of the appropriate values gave F=1.27. For one and twenty-five degrees of freedom, the critical values of the F distribution are 7.77 and 4.24 for the one and five per cent levels of significance, respectively.

gross output by dividing the field crops component by the weather index. The production function

$$Q' = Ae^{ut}L^{1-a}K^{a}$$

was then estimated using the same data series as in Model I(a) for t, L and K. The results when λ=0.0 are reported in Table XII.

A comparison of the results of Model IV(a) with the results of Model I(a) (when λ=0.0) reveals the following: (1) the "best" regression in Model IV(a) occurred when β=0.05 compared with β=0.04 in Model I(a); (2) the R^2 values were lower and the standard errors of the estimate were larger in Model IV(a) (.9784 and .0675 compared with .9873 and .0560 in the "best" regressions); and (3) the u and a coefficients were very similar in magnitude and level of significance.

Thus, directly adjusting gross output for weather influences resulted in a slightly larger estimate of the rate of embodied technological change in material inputs. However, on the basis of goodness of fit and low standard errors of the estimate, this method was inferior to the method of relating actual and potential output which was used in Models I, II, III and V.

<u>Model IV(b)</u> (Table XIII)

Another function which seemed a priori to have the right general shape for relating potential and actual output is the logarithmic, reciprocal function

$$y=e^{f-g/w}.$$

Therefore, the production function

$$Q=Ae^{ut}e^{f-g/w}L^{1-a}K^{a}$$

was estimated using the same data series as in Model I(a). The results when λ=0.0 are reported in Table XIII.

Compared with Model I(a), Model IV(b): (1) gave a lower estimate of the rate of embodied technological change in material inputs (3.0 per cent compared with 4.0); and (2) provided estimates of the rate of disembodied technological change which were lower and also less significant. The R^2 values were lower and the standard errors of the estimate were higher in Model IV(b) compared with Model I(a).

Thus, the use of the logarithmic, reciprocal function to relate potential and actual output resulted in a lower estimate of both the rate of disembodied and embodied technological change. However, on the basis of goodness of fit and low standard errors of the estimate, this method was also inferior to the principal means for relating actual and potential output used in Models I, II, III and V.

VI. TESTING THE PRODUCTION FUNCTION FOR STABILITY OVER TIME

Since agriculture experienced significant structural changes during the time period under study, it was desirable to test whether the estimated regression relationships were stable over time. The technique involved the use of a dummy variable, D, which took the value zero for all years prior to a given date, and the value one for all subsequent years. On the basis of the most highly significant dummy variable, the 1935-65 period was divided into two sub-periods, and the hypothesis that the estimated regression coefficients were equal in all three time periods was tested by computing the appropriate F-ratio.

<u>Model V</u> (Table XIV)

Using the same data as in Model I(a) when $\lambda=\beta=0.0$, the production function

$$Q=Ae^{ut}e^{f+gw+hw^2}L^{1-a}K^{a}e^{jD}$$

was estimated for each of the dummy variables, D_1 to D_{13}, listed in Table IV. A significant regression coefficient for D implies that a significant change occurred in at least part of the relationship from one period to the other. The results of ten regressions with the most highly significant D coefficients, j, are reported in Table XIV. In three regressions, D_2, D_5 and D_6, j was significant at the five per cent level, and in one regression, D_6, j was significant at the one per cent level. In addition, the regression with the dummy variable D_6 also had the highest R^2, lowest standard error of the estimate, and the most highly significant value for a. Therefore, it was concluded that a significant shift occurred between the 1935-49 and 1950-65

TABLE IV

DUMMY VARIABLES SPECIFIED FOR MODEL V:

$Q=Ae^{ut}e^{f+gw+hw^2}L^{1-a}K^{a}e^{jD}$, $\lambda=\beta=0.0$, 1935-65

Dummy variable	Zeros	Ones
D_1	1935-40	1941-65
D_2	1935-45	1946-65
D_3	1935-46	1947-65
D_4	1935-47	1948-65
D_5	1935-48	1949-65
D_6	1935-49	1950-65
D_7	1935-50	1951-65
D_8	1935-51	1952-65
D_9	1935-52	1953-65
D_{10}	1935-53	1954-65
D_{11}	1935-54	1955-65
D_{12}	1935-55	1956-65
D_{13}	1935-60	1961-65

periods in at least part of the true production function relationship.

In view of this result, the period under study was divided into two subperiods, 1935-49 and 1950-65. Model I(a) was estimated for each subperiod when $\lambda=\beta=0.0$. The regression results are shown in Table XV.

The regression results indicate that disembodied technological change occurred at a more rapid annual rate during the 1950-65 period than during the 1935-49 period (3.08 per cent compared with 1.09). In addition, the u coefficient was highly significant in the regression for the 1950-65 period, but not significant in the regression for the 1935-49 period. However, the elasticity of output with respect to capital was more highly significant in the regression for the 1935-49 period than in the regression for the 1950-65 period.

To test the hypothesis that the true set of regression coefficients was equal in all three time periods, 1935-49, 1950-65 and 1935-65, that is, that the observations for the 1950-65 period belonged to the same relationship as those for the 1935-49 period, an F-test was performed.[7] On the basis of the test, there was insufficient statistical evidence to reject the hypothesis of equal sets of regression coefficients for all three time periods.

This result should not be interpreted as a contradiction of the results obtained when the dummy variable was

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[7]The test is outlined in: Johnston, op. cit., pp. 136-37. Substitution of the appropriate values gave F=2.47. For five and twenty-one degrees of freedom, the critical values of the F distribution are 4.04 and 2.68 for the one and five per cent levels of significance, respectively.

introduced into the model. The F-ratio tested the hypothesis that the entire set of regression coefficients was equal in all three time periods, that is, that the regression relationship taken as a whole did not shift significantly over time, while the significant regression coefficient for D indicated that all or part of the regression relationship shifted between the two subperiods. Therefore, interpreting the results of the F-test and dummy variable together, it was concluded that only part of the relationship shifted significantly over time. Although it was not possible to determine precisely which part(s) of the relationship underwent a significant shift, the regression results for the two subperiods (Table XV) suggest that the constant term and u were significantly different in the two periods. Both the constant term and u were almost three times as large with relatively much lower standard errors in the 1950-65 period than in the 1935-49 period.

CHAPTER V

SUMMARY AND CONCLUSIONS

I. SUMMARY

Productivity increases in Canadian agriculture over the past three or four decades have been well demonstrated. However, only recently have research efforts been directed specifically towards identifying the sources of the observed productivity gains. The purpose of this study was to identify the kinds and magnitudes of technological change which have contributed to total productivity gains in Canadian agriculture.

More specifically, the problem was to measure: (1) the rate of disembodied technological change; (2) the rate of technological change embodied in agricultural machinery and implements; and (3) the rate of technological change embodied in material inputs. Disembodied technological change was defined as a shift in the production function. Therefore, the rate of disembodied technological change was a measure of the effect on output of new technology which can be implemented with reliance on the existing resources or inputs. General improvements in farm management and decision-making, adoption of soil-testing practices, and more efficient feed rations for livestock are obvious examples of disembodied technological change. Embodied technological change, on the other hand, is a measure of the effect on output of improved technology which must be implemented in conjunction with improved or new kinds of inputs. In particular, an attempt was made to measure the rate of embodied technological change corresponding to changes in the productive quality of machines and implements. Intuitive examples of technology which might be expected to be embodied in machines and implements include: power options, electric starters, improved tillage implements, and entirely new machines such as hay conditioners, side delivery rakes and machinery for handling specialty crops. Similarly, the rate of embodied technological change in material inputs was an attempt to measure the corresponding changes in the productive quality of material inputs. Intuitive examples are improved seed varieties, more effective herbicides and pesticides, improved fertilizers, and feed additives.

The analysis was carried out for the aggregate primary agricultural sector in Canada for the 1935-65 period. To measure technological change, regression estimates were obtained for a linear homogeneous Cobb-Douglas production function where gross agricultural output per person employed was the dependent variable, and a time index, weather index, and the annual flow of capital services (including material inputs) per person employed were the independent variables. The data required was derived from publications of the Dominion Bureau of Statistics, and consisted of time series of thirty-one annual observations. Gross output and the flow of capital services were measured at 1935-39 base period prices, while labor input was measured as the number of persons employed in agriculture.

Hicks-neutral disembodied technological change was measured by specifying an exponential shift function, e^{ut}, in the production function. Embodied technological change was assumed to be capital-augmenting in the vintage sense so that machines and implements, and material inputs purchased in any year were 100λ and 100β per cent, respectively, more productive than those purchased in the preceding year. Alternative data series for the flow of capital services were constructed by imposing several values for λ and β. A matrix of regression results was obtained, and the true values of λ and β were inferred by choosing the "best" regression on the criteria of goodness of fit, significance levels of the estimated coefficients and low standard errors.

Several alternative models were investigated: (1) all technological change was assumed to be embodied; (2) the assumption of constant returns to scale was relaxed; (3) alternative means of relating actual and potential output were investigated; and (4) the stability of the production function relationship over time was tested. Two hypotheses were statistically tested: (1) constant returns to scale; and (2) equal sets of regression coefficients in the three time periods, 1935-65, 1935-49 and 1950-65.

II. CONCLUSIONS

Many rigorous assumptions are implied in the models as well as in the derivation of the data series, especially the stock of machinery and implements and the flow of capital services. Therefore, the results obtained must be interpreted carefully.

It is particularly important to note that the rate of technological change was assumed to be constant over time. Therefore, the estimated values must be interpreted as long-term trends. Also, there were undoubtedly errors of measurement in the variables. These errors could arise from several sources: (1) the flow of capital services and gross output were measured in constant dollars, and therefore, it was implicitly (and heroically) assumed that the agricultural sector was in long-run equilibrium; (2) problems of aggregation of economic units (farms) may have been present, since the analysis and data employed were for the aggregate agricultural sector taken as a whole; (3) the utilization rate of inputs was assumed constant over time; and (4) aggregation over products and inputs may have introduced errors, since non-homogeneous products and inputs were aggregated into single measures of gross output, labor input, and capital input. In addition, there was the problem of aggregating capital inputs which have imputed rates of return and those which have market returns. If errors of measurement were present, and it seems likely that they were, a dependence between the disturbance term and the observed values of the explanatory variables would exist, which invalidates one of the basic assumptions of the linear regression model. Thus, the regression estimates would be biased.[1]

In addition, the results of the study are based on the assumption that the model was correctly specified, that is, that the model was a true expression of the production relationships which actually existed in the aggregate agricultural sector. Of course, this is an untestable assumption which can only be qualitatively judged on the basis of conformity with existing knowledge and theory, and whether the results are reasonable on such a priori grounds.

With the preceding qualifications kept in mind, the following main conclusions appear to be justified.

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[1]J. Johnston, Econometric Methods (New York: McGraw-Hill Book Company, Inc., 1963), pp. 148-50.

1. This study confirmed that there has been substantial technological change in Canadian agriculture. When all technological change was assumed to be disembodied, it was estimated to be about 2.46 to 2.70 per cent per year. On the criteria of goodness of fit and most significant u coefficient, the "best" estimate of disembodied technological advance (when embodied technological change was not specified) was 2.70 per cent annually during the period 1935-65.

2. When all technological change was assumed to be embodied, the "best" estimate was an annual rate of embodied technological change in material inputs of about 5.0 to 6.0 per cent.

3. When both disembodied and embodied technological change were specified simultaneously, the estimates of the annual rate of disembodied technological change ranged from 1.13 to 1.76 per cent, while embodied technological change in material inputs was estimated at 3.5 to 4.0 per cent annually. On the criteria of goodness of fit and most significant u coefficient, the "best" estimate of the annual rate of disembodied technological change was 1.76 per cent. It should be noted, however, that in these "best" regressions the disembodied technological change coefficient, u, had a relatively large standard error and was not significant.

4. There was no evidence of a positive rate of embodied technological change in machinery and implements in any regressions. It was concluded that the disturbing and a priori unexpected result of a negative rate of embodied technological change in machinery and implements should probably be considered substantially biased, for several reasons discussed.

5. The results of this study suggest that both disembodied and embodied technological change should be treated simultaneously. This is particularly evident when a comparison is made between regressions in which all technological change was specified as disembodied (that is, $\lambda=\beta=0.0$), and those where both disembodied and embodied technological change were specified. In all cases the R^2 values were higher and standard errors of the estimate lower when disembodied and embodied technological change were specified simultaneously. The evidence is not as conclusive, however, when the simultaneous case is compared with the one in which all technological change was specified as embodied. Although the R^2 values were slightly higher in the simultaneous specification, the standard error of the estimate was slightly lower when all technological change was assumed to be embodied.

6. There was insufficient statistical evidence to reject the hypothesis of constant returns to scale. How-

ever, the assumption of constant returns to scale did result in slightly smaller estimates of the rate of both disembodied and embodied technological change.

7. A comparison of the regression results for the 1935-49 and 1950-65 subperiods strongly suggests that disembodied technological change occurred at a more rapid rate during the 1950-65 period. Moreover, this result is generally compatible with the results of previous studies. However, there was no statistical evidence which would result in the rejection of the hypothesis that the regression relationship, when taken as a whole, was equal in the two subperiods. Therefore, the production function as a whole appeared to be stable during the period under study.

III. IMPLICATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

The macro theory of production is closely related to the theory of economic growth. This study was an investigation of the macro production relationships in Canadian agriculture, and as such it was an attempt to contribute to the existing knowledge about the sources of increased agricultural output per unit of input. In this regard, the single most interesting result was the large role attributed to technological change embodied in material inputs by all models investigated. However, there are no direct policy implications which can be drawn from this study.

The following suggestions are offered for further research. Firstly, the present models could be refined in several ways. Alternative functional forms such as the CES production function could be investigated. The measurement of capital was particularly troublesome, and much additional research is needed to improve these estimates. Similarly, the measurement of labor input was extremely rough and the possibility of technological change embodied in labor should be investigated. In addition, it would be desirable to recognize the regional differences in Canadian agriculture by disaggregating the analysis on a geographical basis. Secondly, it would be extremely interesting to bring such variables as the rate of research and development into the analysis. Little is known about the determinants of technological change, and any meaningful policy variables must focus on the factors which influence the rate of technological change. Nelson has made some useful observations on this topic.[2] Thirdly, there is the cost of technological change relative to its benefits. In other words, there is the difficult and important question of the allocation of resources to technology generating activities from a

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[2]See: M. Brown, The Theory and Empirical Analysis of Production (Studies in Income and Wealth, Vol. 31. New York: Columbia University Press, 1967), pp. 479-99.

social point of view. Fellner has advanced some interesting ideas in this area.[3]

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[3]W. Fellner, "Trends in the Activities Generating Technological Progress," American Economic Review, 60:1-29, March, 1970.

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A P P E N D I X

TABLE V

TIME SERIES DATA FOR CANADIAN AGRICULTURE, 1935-65

Year	Gross output at 1935-39 prices: Unadjusted	Adj. for weather influence	Weather index	Number of persons employed
(1)	(2)	(3)	(4)	(5)
	- thousands of	dollars -		thousands
1935	714692	742933	0.918	1298
1936	678246	817834	.631	1319
1937	649721	857883	.522	1339
1938	750782	766809	.954	1359
1939	951000	853852	1.251	1379
1940	953247	880568	1.191	1344
1941	858138	897597	.893	1224
1942	1254375	982733	1.705	1139
1943	960235	930630	1.100	1118
1944	1181643	1109792	1.153	1136
1945	954172	1023127	.838	1144
1946	1047257	1052000	.990	1271
1947	1065622	1182537	.804	1172
1948	1118516	1162991	.930	1186
1949	1087086	1277670	.740	1114
1950	1005254	1016709	.975	1066
1951	1164233	1079363	1.210	991
1952	1351133	1092384	1.418	927
1953	1419083	1240862	1.235	911
1954	1117393	1342909	.719	906
1955	1208839	1152614	1.107	880
1956	1355879	1202803	1.258	808
1957	1150249	1224544	.868	777
1958	1182370	1238907	.872	746
1959	1219542	1282430	.892	739
1960	1289950	1294529	.993	690
1961	1137814	1509708	.535	712
1962	1413262	1404225	1.013	694
1963	1537396	1432982	1.149	695
1964	1509317	1563203	.934	679
1965	1608468	1523514	1.107	649

TABLE V (continued)

Year (6)	Livestock and poultry (7)	Net stock of buildings (8)	Deprecia- tion on buildings (9)	Land (10)
	--- thousands of dollars at 1935-39 prices ---			
1935	574396	1351600	54064	2061640
1936	612201	1306375	47224	2077880
1937	569502	1268903	42502	2093700
1938	560188	1236533	45045	2107140
1939	634961	1202311	42166	2119180
1940	639664	1171179	37722	2131920
1941	499441	1145869	34060	2140159
1942	519636	1122314	31813	2151800
1943	678595	1101082	33240	2159920
1944	651395	1080617	31482	2168040
1945	612398	1064009	31559	2176160
1946	593449	1053283	33795	2184280
1947	573854	1041234	34325	2191980
1948	472120	1029240	31180	2199680
1949	516500	1029270	30034	2207380
1950	521528	1028256	28844	2214240
1951	595575	1027105	25888	2222534
1952	645360	1031220	26184	2232720
1953	590031	1036225	28551	2244900
1954	555845	1036990	28993	2255400
1955	596788	1036171	31040	2266180
1956	576233	1036401	32197	2275497
1957	586229	1033005	32858	2284940
1958	677764	1031552	35982	2290960
1959	720340	1029046	37854	2296980
1960	711098	1023844	39172	2306920
1961	737124	1036476	41700	2317120
1962	718105	1051472	42958	2331980
1963	769765	1065067	44270	2346680
1964	810433	1075297	45253	2361380
1965	726664	1086587	48361	2376500

TABLE V (concluded)

Year	Material inputs, $\beta=0$		Gross stock of machinery and implements, $\lambda=0$	
	Deflated by individual price indexes	Deflated by wholesale price index	Deflated by DBS price index	Deflated by adj. USDA price index
(11)	(12)	(13)	(14)	(15)
	--- thousands of dollars at 1935-39 prices ---			
1935	169411	174961	707466	682666
1936	174075	177669	707192	684877
1937	182278	182353	740551	716453
1938	199377	196460	770781	747772
1939	225497	213413	760690	738939
1940	225657	208516	747890	726246
1941	246423	223922	709860	690011
1942	286355	272181	669488	650893
1943	336784	314609	628907	610735
1944	346837	324221	662447	642482
1945	367195	340750	716120	691371
1946	413327	371739	810064	780114
1947	454735	372595	947867	906384
1948	411694	354737	1109939	1049161
1949	413859	358113	1287357	1197714
1950	411939	349656	1458647	1332163
1951	434667	342790	1620382	1473427
1952	434064	376873	1789768	1641521
1953	445886	387063	1935517	1786854
1954	457683	400376	2004086	1856918
1955	471260	411014	2104436	1957378
1956	513563	439299	2248199	2102221
1957	497298	429753	2332036	2196022
1958	536079	462155	2400357	2278292
1959	556777	488396	2454901	2349632
1960	559201	495272	2451181	2365097
1961	560004	499659	2402147	2344468
1962	577179	519177	2349804	2314920
1963	612057	546143	2314321	2309035
1964	644093	578178	2306781	2312462
1965	664227	595164	2316591	2312029

TABLE VI

SYMBOLS USED IN THE MODELS

Symbol	Derivation	Meaning
Q	Variable	Gross agricultural output (thousands of 1935-39 dollars)
Q'	Variable	Gross agricultural output adjusted for weather influences (thousands of 1935-39 dollars)
L	Variable	Number of persons employed (thousands)
w	Variable	Weather index (potential=actual output when w=1.0)
K	Variable	Annual flow of capital services (thousands of 1935-39 dollars)
t	Variable	Time index (1, 2, ..., 31)
D	Variable	Dummy variable (ones and zeros for specified time periods)
λ	Parameter	Rate of embodied technological change in machinery and implements
β	Parameter	Rate of embodied technological change in material inputs
e	Number	$e=\lim_{n\to\infty}[1+(1/n)]^n$ and lne=1
A	Estimated	Constant coefficient (lnA'=lnA+f)
f	Estimated	Constant coefficient
u	Estimated	Regression coefficient for rate of disembodied technological change
g	Estimated	Regression coefficient for w
h	Estimated	Regression coefficient for w^2
j	Estimated	Regression coefficient for D
a	Estimated	Elasticity of output with respect to capital
b	Estimated	Elasticity of output with respect to labor

TABLE VII

REGRESSION ESTIMATES FOR MODEL I(a): $Q=Ae^{ut}e^{f+gw+hw^2}L^{1-a}K^a$, $\lambda=-0.01$, 1935-65

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	3.0982	3.0320	3.0549	3.1006	3.1616	3.2387	3.4251	3.6303	3.8337	4.0210
	(1.0002)	(.9602)	(.9441)	(.9256)	(.9408)	(.8837)	(.8383)	(.7929)	(.7495)	(.7096)
u	.0225	.0153	.0137	.0124	.0113	.0104	.0094	.0090	.0091	.0094
	(.0092)	(.0109)	(.0113)	(.0117)	(.0120)	(.0123)	(.0129)	(.0133)	(.0137)	(.0141)
	.0208	.1695	.2336	.2975	.3577	.4092	.4794	.5125	.5206	.5171
g	.1055	.1052	.1054	.1053	.1052	.1049	.1038	.1024	.1006	.0988
	(.2299)	(.2260)	(.2254)	(.2251)	(.2250)	(.2252)	(.2262)	(.2277)	(.2296)	(.2315)
	.6540	.6494	.6481	.6479	.6480	.6494	.6539	.6602	.6677	.6754
h	.1522	.1541	.1547	.1554	.1561	.1569	.1587	.1604	.1621	.1637
	(.1078)	(.1059)	(.1056)	(.1054)	(.1053)	(.1054)	(.1058)	(.1065)	(.1073)	(.1082)
	.1665	.1539	.1513	.1489	.1468	.1449	.1422	.1404	.1394	.1387
a	.4925	.5062	.5030	.4959	.4861	.4736	.4430	.4089	.3750	.3437
	(.1682)	(.1622)	(.1597)	(.1567)	(.1533)	(.1499)	(.1424)	(.1350)	(.1277)	(.1211)
	.0069	.0044	.0041	.0040	.0039	.0040	.0045	.0054	.0068	.0085
R^2	.9871	.9875	.9876	.9876	.9877	.9876	.9875	.9873	.9871	.9869
S	.0563	.0554	.0552	.0552	.0551	.0552	.0554	.0558	.0563	.0567
"d"	1.540	1.589	1.601	1.611	1.619	1.624	1.630	1.629	1.626	1.621

TABLE VII (continued), λ=-0.005

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	3.2746	3.1447	3.1479	3.1736	3.2166	3.2778	3.4382	3.6290	3.8249	4.0105
	(.9933)	(.9638)	(.9493)	(.9322)	(.9134)	(.8927)	(.8486)	(.8033)	(.7591)	(.7179)
u	.0236	.0162	.0146	.0131	.0118	.0108	.0095	.0089	.0089	.0092
	(.0093)	(.0111)	(.0115)	(.0118)	(.0122)	(.0125)	(.0131)	(.0135)	(.0139)	(.0142)
	.0173	.1511	.2135	.2789	.3430	.3992	.4809	.5213	.5331	.5295
g	.1044	.1048	.1051	.1052	.1053	.1051	.1043	.1029	.1012	.0993
	(.2331)	(.2286)	(.2277)	(.2272)	(.2268)	(.2268)	(.2273)	(.2285)	(.2301)	(.2319)
	.6614	.6543	.6522	.6511	.6503	.6510	.6539	.6598	.6668	.6745
h	.1528	.1543	.1547	.1553	.1559	.1566	.1582	.1600	.1617	.1633
	(.1039)	(.1071)	(.1067)	(.1064)	(.1062)	(.1061)	(.1063)	(.1069)	(.1076)	(.1084)
	.1704	.1582	.1554	.1528	.1506	.1484	.1452	.1428	.1412	.1402
a	.4621	.4864	.4865	.4828	.4761	.4663	.4402	.4086	.3760	.3450
	(.1668)	(.1626)	(.1603)	(.1576)	(.1546)	(.1512)	(.1440)	(.1366)	(.1292)	(.1224)
	.0099	.0059	.0054	.0050	.0048	.0048	.0051	.0059	.0072	.0088
R^2	.9868	.9873	.9874	.9874	.9875	.9875	.9874	.9873	.9871	.9869
S	.0571	.0560	.0558	.0557	.0556	.0556	.0557	.0560	.0564	.0568
"d"	1.515	1.563	1.575	1.587	1.597	1.605	1.615	1.618	1.618	1.615

TABLE VII (continued), λ=0.0

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	3.4487	3.2685	3.2562	3.2602	3.2838	3.3282	3.4587	3.6305	3.8163	3.9983
	(.9824)	(.9634)	(.9514)	(.9368)	(.9198)	(.9006)	(.8583)	(.8135)	(.7689)	(.7270)
u	.0246	.0173	.0155	.0139	.0125	.0113	.0096	.0089	.0087	.0090
	(.0095)	(.0112)	(.0116)	(.0120)	(.0124)	(.0127)	(.0133)	(.0137)	(.0141)	(.0144)
	.0144	.1323	.1910	.2574	.3246	.3853	.4798	.5302	.5476	.5456
g	.1030	.1039	.1043	.1048	.1050	.1050	.1045	.1033	.1017	.0998
	(.2360)	(.2312)	(.2302)	(.2294)	(.2288)	(.2285)	(.2285)	(.2294)	(.2307)	(.2323)
	.6688	.6603	.6577	.6554	.6538	.6536	.6548	.6596	.6661	.6736
h	.1536	.1546	.1549	.1553	.1558	.1565	.1579	.1596	.1613	.1630
	(.1106)	(.1083)	(.1078)	(.1074)	(.1071)	(.1069)	(.1069)	(.1073)	(.1079)	(.1086)
	.1735	.1619	.1592	.1566	.1542	.1519	.1481	.1452	.1431	.1417
a	.4322	.4649	.4676	.4675	.4642	.4572	.4362	.4079	.3771	.3468
	(.1647)	(.1623)	(.1605)	(.1582)	(.1555)	(.1524)	(.1456)	(.1382)	(.1308)	(.1238)
	.0138	.0080	.0071	.0065	.0060	.0058	.0059	.0065	.0076	.0092
R^2	.9865	.9870	.9871	.9872	.9873	.9873	.9873	.9872	.9870	.9868
S	.0577	.0566	.0564	.0562	.0560	.0560	.0560	.0562	.0565	.0569
"d"	1.495*	1.538	1.551	1.564	1.575	1.585	1.600	1.607	1.609	1.608

*Test for serial correlation is inconclusive at one per cent level of significance.

TABLE VII (continued), λ=0.005

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	3.6323	3.4140	3.3827	3.3691	3.3722	3.3968	3.4924	3.6415	3.8120	3.9862
	(.9675)	(.9598)	(.9509)	(.9389)	(.9239)	(.9070)	(.8671)	(.8237)	(.7791)	(.7366)
u	.0258	.0185	.0166	.0149	.0133	.0120	.0100	.0089	.0086	.0087
	(.0096)	(.0113)	(.0118)	(.0122)	(.0125)	(.0129)	(.0135)	(.0139)	(.0143)	(.0146)
	.0117	.1114	.1658	.2295	.2979	.3632	.4713	.5333	.5598	.5624
g	.1013	.1025	.1032	.1038	.1044	.1045	.1045	.1035	.1020	.1003
	(.2389)	(.2340)	(.2328)	(.2318)	(.2310)	(.2304)	(.2299)	(.2304)	(.2314)	(.2327)
	.6772	.6677	.6643	.6613	.6587	.6574	.6568	.6604	.6659	.6726
h	.1546	.1551	.1553	.1556	.1559	.1564	.1577	.1593	.1609	.1626
	(.1120)	(.1096)	(.1090)	(.1086)	(.1081)	(.1079)	(.1076)	(.1078)	(.1082)	(.1088)
	.1761	.1655	.1629	.1603	.1579	.1554	.1512	.1478	.1453	.1435
a	.4009	.4398	.4456	.4485	.4486	.4450	.4299	.4056	.3774	.3485
	(.1620)	(.1615)	(.1602)	(.1584)	(.1560)	(.1533)	(.1469)	(.1398)	(.1324)	(.1254)
	.0193	.0110	.0096	.0086	.0078	.0073	.0069	.0073	.0082	.0097
R^2	.9861	.9867	.9868	.9869	.9870	.9871	.9871	.9871	.9870	.9868
S	.0584	.0573	.0570	.0568	.0566	.0564	.0563	.0564	.0567	.0570
"d"	1.478*	1.515	1.527	1.540	1.552	1.564	1.582	1.593	1.599	1.601

*Test for serial correlation is inconclusive at one per cent level of significance.

TABLE VII (continued), λ=0.01

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	3.8218	3.5755	3.5298	3.4979	3.4821	3.4868	3.5456	3.6624	3.8161	3.9781
	(.9485)	(.9523)	(.9467)	(.9371)	(.9255)	(.9109)	(.8750)	(.8337)	(.7901)	(.7470)
u	.0270	.0199	.0180	.0162	.0145	.0130	.0106	.0092	.0086	.0085
	(.0096)	(.0115)	(.0119)	(.0123)	(.0127)	(.0130)	(.0137)	(.0141)	(.0145)	(.0148)
	.0093	.0905	.1384	.1974	.2644	.3318	.4517	.5304	.5665	.5769
g	.0992	.1008	.1016	.1024	.1032	.1036	.1040	.1036	.1023	.1007
	(.2417)	(.2369)	(.2356)	(.2344)	(.2334)	(.2327)	(.2317)	(.2316)	(.2323)	(.2333)
	.6863	.6763	.6724	.6688	.6652	.6630	.6605	.6618	.6664	.6723
h	.1557	.1559	.1559	.1561	.1562	.1566	.1576	.1590	.1606	.1622
	(.1133)	(.1110)	(.1104)	(.1098)	(.1093)	(.1089)	(.1084)	(.1083)	(.1086)	(.1091)
	.1780	.1686	.1662	.1637	.1614	.1589	.1545	.1507	.1477	.1454
a	.3686	.4120	.4202	.4262	.4294	.4292	.4204	.4015	.3763	.3495
	(.1586)	(.1600)	(.1593)	(.1579)	(.1561)	(.1538)	(.1481)	(.1413)	(.1342)	(.1270)
	.0269	.0154	.0134	.0116	.0103	.0094	.0085	.0084	.0091	.0103
R^2	.9858	.9864	.9865	.9866	.9867	.9868	.9869	.9869	.9869	.9867
S	.0591	.0580	.0577	.0574	.0572	.0570	.0567	.0567	.0569	.0572
"d"	1.465*	1.494*	1.505*	1.517	1.530	1.542	1.563	1.578	1.587	1.592

*Test for serial correlation is inconclusive at one per cent level of significance.

TABLE VII (continued), λ=0.015

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	4.0086	3.7466	3.6915	3.6453	3.6147	3.5980	3.6172	3.7002	3.8273	3.9767
	(.9257)	(.9403)	(.9377)	(.9322)	(.9235)	(.9120)	(.8809)	(.8427)	(.8007)	(.7581)
u	.0282	.0214	.0195	.0176	.0159	.0142	.0114	.0096	.0087	.0084
	(.0097)	(.0115)	(.0120)	(.0124)	(.0128)	(.0132)	(.0139)	(.0144)	(.0148)	(.0151)
	.0074	.0715	.1114	.1637	.2253	.2930	.4222	.5167	.5686	.5868
g	.0969	.0987	.0996	.1005	.1014	.1022	.1032	.1032	.1023	.1009
	(.2444)	(.2397)	(.2385)	(.2372)	(.2361)	(.2351)	(.2337)	(.2332)	(.2334)	(.2341)
	.6957	.6856	.6816	.6774	.6736	.6701	.6656	.6649	.6677	.6727
h	.1569	.1568	.1568	.1568	.1568	.1570	.1577	.1588	.1603	.1619
	(.1145)	(.1123)	(.1117)	(.1111)	(.1106)	(.1101)	(.1093)	(.1091)	(.1091)	(.1094)
	.1793	.1711	.1690	.1668	.1645	.1623	.1578	.1537	.1502	.1475
a	.3368	.3826	.3924	.4007	.4065	.4098	.4077	.3946	.3739	.3493
	(.1546)	(.1578)	(.1576)	(.1568)	(.1556)	(.1538)	(.1489)	(.1427)	(.1358)	(.1288)
	.0367	.0215	.0186	.0161	.0142	.0126	.0106	.0100	.0103	.0113
R^2	.9855	.9860	.9862	.9863	.9864	.9866	.9867	.9868	.9867	.9867
S	.0597	.0586	.0584	.0581	.0578	.0576	.0572	.0571	.0571	.0573
"d"	1.455*	1.476*	1.485*	1.496*	1.508*	1.520	1.542	1.561	1.574	1.581

*Test for serial correlation is inconclusive at one per cent level of significance.

TABLE VII (continued), λ=0.02

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	4.1926	3.9287	3.8655	3.8109	3.7637	3.7311	3.7108	3.7544	3.8506	3.9805
	(.8998)	(.9247)	(.9252)	(.9227)	(.9176)	(.9094)	(.8844)	(.8505)	(.8106)	(.7692)
u	.0294	.0231	.0212	.0194	.0175	.0157	.0126	.0103	.0090	.0084
	(.0098)	(.0116)	(.0121)	(.0125)	(.0129)	(.0133)	(.0140)	(.0146)	(.0150)	(.0153)
	.0058	.0546	.0866	.1298	.1851	.2483	.3820	.4930	.5625	.5933
g	.0943	.0960	.0970	.0980	.0992	.1001	.1017	.1024	.1020	.1009
	(.2470)	(.2426)	(.2414)	(.2402)	(.2389)	(.2378)	(.2360)	(.2349)	(.2346)	(.2350)
	.7055	.6962	.6920	.6879	.6833	.6793	.6725	.6694	.6700	.6736
h	.1583	.1580	.1579	.1578	.1577	.1577	.1580	.1589	.1601	.1616
	(.1157)	(.1137)	(.1131)	(.1125)	(.1119)	(.1113)	(.1104)	(.1099)	(.1097)	(.1098)
	.1799	.1729	.1712	.1692	.1673	.1651	.1609	.1567	.1529	.1498
a	.3056	.3514	.3626	.3723	.3807	.3868	.3913	.3849	.3695	.3482
	(.1500)	(.1550)	(.1552)	(.1550)	(.1544)	(.1532)	(.1493)	(.1439)	(.1374)	(.1306)
	.0495	.0303	.0262	.0227	.0196	.0173	.0139	.0123	.0119	.0125
R^2	.9852	.9857	.9859	.9860	.9861	.9863	.9865	.9866	.9866	.9866
S	.0603	.0593	.0590	.0588	.0585	.0582	.0578	.0575	.0574	.0575
"d"	1.448*	1.462*	1.469*	1.478*	1.488*	1.499*	1.522	1.542	1.558	1.569

*Test for serial correlation is inconclusive at one per cent level of significance.

TABLE VII (continued), λ=0.025

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	4.3683	4.1121	4.0460	3.9829	3.9277	3.8808	3.8241	3.8303	3.8912	3.9962
	(.8715)	(.9046)	(.9081)	(.9090)	(.9073)	(.9027)	(.8846)	(.8562)	(.8199)	(.7803)
u	.0306	.0248	.0230	.0211	.0193	.0174	.0140	.0113	.0095	.0086
	(.0099)	(.0116)	(.0121)	(.0125)	(.0130)	(.0134)	(.0142)	(.0148)	(.0153)	(.0156)
	.0045	.0407	.0653	.1002	.1460	.2025	.3341	.4563	.5446	.5911
g	.0915	.0931	.0941	.0952	.0964	.0976	.0998	.1010	.1013	.1006
	(.2493)	(.2454)	(.2442)	(.2430)	(.2418)	(.2406)	(.2385)	(.2370)	(.2362)	(.2362)
	.7153	.7070	.7031	.6987	.6941	.6896	.6810	.6758	.6739	.6759
h	.1597	.1594	.1592	.1589	.1588	.1586	.1586	.1591	.1601	.1614
	(.1168)	(.1149)	(.1144)	(.1138)	(.1132)	(.1126)	(.1116)	(.1109)	(.1104)	(.1104)
	.1800	.1741	.1726	.1710	.1694	.1675	.1637	.1597	.1558	.1523
a	.2758	.3201	.3317	.3428	.3525	.3609	.3715	.3715	.3621	.3451
	(.1451)	(.1514)	(.1521)	(.1525)	(.1524)	(.1518)	(.1491)	(.1447)	(.1388)	(.1323)
	.0655	.0420	.0366	.0317	.0275	.0239	.0186	.0156	.0143	.0143
R^2	.9850	.9854	.9855	.9857	.9858	.9859	.9862	.9863	.9864	.9864
S	.0608	.0600	.0597	.0594	.0591	.0589	.0583	.0580	.0578	.0578
"d"	1.444*	1.451*	1.456*	1.463*	1.471*	1.480*	1.501*	1.523	1.541	1.556

*Test for serial correlation is inconclusive at one per cent level of significance.

TABLE VII (continued), λ=0.03

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	4.5352	4.2911	4.2248	4.1612	4.0988	4.0434	3.9587	3.9253	3.9496	4.0238
	(.8410)	(.8812)	(.8873)	(.8914)	(.8928)	(.8918)	(.8807)	(.8587)	(.8273)	(.7904)
u	.0318	.0264	.0248	.0231	.0212	.0194	.0158	.0127	.0104	.0090
	(.0099)	(.0116)	(.0121)	(.0126)	(.0130)	(.0135)	(.0143)	(.0150)	(.0155)	(.0159)
	.0035	.0300	.0483	.0748	.1117	.1589	.2796	.4095	.5149	.5803
g	.0884	.0900	.0909	.0920	.0932	.0945	.0971	.0991	.1001	.1000
	(.2514)	(.2479)	(.2469)	(.2458)	(.2446)	(.2434)	(.2411)	(.2393)	(.2380)	(.2375)
	.7254	.7180	.7144	.7105	.7058	.7012	.6915	.6840	.6796	.6792
h	.1613	.1608	.1606	.1603	.1600	.1598	.1595	.1596	.1602	.1613
	(.1178)	(.1161)	(.1156)	(.1151)	(.1145)	(.1139)	(.1128)	(.1119)	(.1113)	(.1110)
	.1795	.1747	.1734	.1722	.1708	.1693	.1660	.1623	.1585	.1549
a	.2475	.2897	.3012	.3123	.3232	.3330	.3482	.3548	.3517	.3399
	(.1398)	(.1472)	(.1484)	(.1493)	(.1497)	(.1497)	(.1483)	(.1449)	(.1399)	(.1339)
	.0849	.0571	.0503	.0442	.0383	.0334	.0254	.0204	.0177	.0167
R^2	.9847	.9851	.9852	.9853	.9855	.9856	.9859	.9861	.9862	.9863
S	.0613	.0606	.0603	.0601	.0598	.0595	.0590	.0585	.0582	.0581
"d"	1.442*	1.443*	1.446*	1.451*	1.457*	1.464*	1.483*	1.503*	1.524	1.540

*Test for serial correlation is inconclusive at one per cent level of significance.

TABLE VIII

REGRESSION ESTIMATES FOR MODEL I(b): $Q=Ae^{ut}e^{f+gw+hw^2}L^{1-a}K^{a}$, $\lambda=0.0$, 1935-65

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	3.2307 (.9738)	3.1352 (.9485)	3.1461 (.9355)	3.1745 (.9209)	3.2165 (.9034)	3.2761 (.8846)	3.4326 (.8431)	3.6204 (.8005)	3.8125 (.7580)	3.9952 (.7181)
u	.0226 (.0094) .0221	.0157 (.0110) .1630	.0142 (.0114) .2242	.0128 (.0118) .2894	.0115 (.0122) .3536	.0105 (.0125) .4103	.0092 (.0130) .4927	.0087 (.0135) .5338	.0086 (.0139) .5477	.0089 (.0143) .5461
g	.1123 (.2314) .6363	.1100 (.2276) .6375	.1097 (.2270) .6376	.1093 (.2266) .6368	.1089 (.2263) .6389	.1083 (.2263) .6406	.1069 (.2268) .6457	.1049 (.2282) .6534	.1028 (.2298) .6617	.1007 (.2315) .6700
h	.1498 (.1084) .1758	.1522 (.1066) .1619	.1529 (.1063) .1588	.1537 (.1061) .1559	.1544 (.1059) .1534	.1553 (.1059) .1510	.1571 (.1061) .1471	.1591 (.1067) .1444	.1610 (.1074) .1424	.1627 (.1082) .1412
a	.4693 (.1634) .0078	.4878 (.1599) .0052	.4866 (.1579) .0048	.4824 (.1556) .0046	.4759 (.1528) .0044	.4664 (.1498) .0045	.4409 (.1430) .0048	.4099 (.1360) .0056	.3780 (.1290) .0068	.3475 (.1224) .0084
R^2	.9870	.9874	.9875	.9875	.9875	.9875	.9875	.9873	.9871	.9869
S	.0566	.0557	.0556	.0555	.0554	.0554	.0556	.0559	.0563	.0567
"d"	1.532	1.572	1.583	1.593	1.602	1.609	1.618	1.620	1.619	1.617

TABLE IX

REGRESSION ESTIMATES FOR MODEL I(c): $Q=Ae^{ut}e^{f+gw+hw^2}L^{1-a}K^{a}$, $\lambda=0.0$, 1935-65

β	0.0	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	3.6030	3.4889	3.5180	3.5627	3.6228	3.8032	3.9636	4.1233	4.2673
	(.8502)	(.8064)	(.7905)	(.7732)	(.7548)	(.7071)	(.6695)	(.6347)	(.6028)
u	.0270	.0197	.0185	.0176	.0168	.0165	.0159	.0158	.0157
	(.0079)	(.0094)	(.0097)	(.0100)	(.0102)	(.0104)	(.0108)	(.0111)	(.0115)
	.0021	.0450	.0645	.0860	.1072	.1232	.1482	.1657	.1805
g	.1399	.1440	.1436	.1429	.1416	.1441	.1401	.1357	.1318
	(.2333)	(.2271)	(.2265)	(.2261)	(.2261)	(.2270)	(.2281)	(.2295)	(.2310)
	.5607	.5384	.5385	.5400	.5434	.5381	.5512	.5662	.5798
h	.1379	.1377	.1383	.1392	.1402	.1408	.1433	.1460	.1484
	(.1092)	(.1063)	(.1060)	(.1058)	(.1057)	(.1061)	(.1065)	(.1072)	(.1078)
	.2158	.2040	.2006	.1971	.1936	.1932	.1872	.1817	.1774
a	.4037	.4244	.4199	.4128	.4030	.3722	.3458	.3194	.2956
	(.1414)	(.1345)	(.1320)	(.1291)	(.1261)	(.1178)	(.1116)	(.1058)	(.1005)
	.0081	.0040	.0038	.0037	.0037	.0040	.0046	.0056	.0067
R^2	.9870	.9876	.9877	.9877	.9877	.9876	.9875	.9873	.9872
S	.0567	.0552	.0551	.0550	.0550	.0552	.0555	.0559	.0562
"d"	1.612	1.690	1.703	1.712	1.720	1.721	1.722	1.719	1.715

TABLE X

REGRESSION ESTIMATES FOR MODEL II: $Q=Ae^{f+gw+hw^2}L^{1-a}K^{a}$, $\lambda=0.0$, 1935-65

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	.9269	1.8038	2.0018	2.1899	2.3679	2.5358	2.8419	3.1109	3.3464	3.5523
	(.1821)	(.1519)	(.1468)	(.1425)	(.1389)	(.1360)	(.1316)	(.1289)	(.1271)	(.1260)
g	.2024	.1571	.1498	.1436	.1385	.1343	.1283	.1244	.1218	.1201
	(.2566)	(.2343)	(.2309)	(.2283)	(.2264)	(.2252)	(.2242)	(.2246)	(.2258)	(.2274)
	.4426	.5150	.5291	.5416	.5527	.5628	.5785	.5907	.6000	.6075
h	.1073	.1305	.1346	.1382	.1412	.1439	.1481	.1511	.1534	.1552
	(.1203)	(.1098)	(.1083)	(.1070)	(.1061)	(.1056)	(.1051)	(.1053)	(.1059)	(.1066)
	.3842	.2437	.2224	.2051	.1916	.1811	.1671	.1593	.1553	.1534
a	.8578	.7133	.6806	.6495	.6201	.5923	.5416	.4970	.4579	.4237
	(.0229)	(.0173)	(.0163)	(.0154)	(.0145)	(.0138)	(.0126)	(.0116)	(.0107)	(.0100)
	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
R^2	.9829	.9858	.9862	.9865	.9868	.9869	.9870	.9870	.9868	.9867
S	.0636	.0580	.0572	.0565	.0561	.0558	.0555	.0556	.0559	.0563
"d"	1.175*	1.383*	1.425	1.461	1.491	1.514	1.545	1.559	1.563	1.562

*Test for serial correlation is inconclusive at one per cent level of significance.

TABLE XI

REGRESSION ESTIMATES FOR MODEL III: $Q=Ae^{ut}e^{f+gw+hw^2}L^bK^a$, $\lambda=0.0$, 1935-65

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	7.9914	6.6457	6.3933	6.1298	6.0039	5.9191	6.0063	6.3616	6.7965	7.2655
	(3.9918)	(4.0902)	(4.1326)	(4.1479)	(4.1294)	(4.1144)	(4.0219)	(3.8433)	(3.6421)	(3.4580)
u	.0320	.0257	.0240	.0223	.0210	.0199	.0188	.0194	.0207	.0226
	(.0113)	(.0150)	(.0160)	(.0170)	(.0177)	(.0185)	(.0196)	(.0200)	(.0202)	(.0202)
	.0088	.0961	.1428	.1984	.2460	.2929	.3476	.3452	.3157	.2730
g	.1759	.1600	.1566	.1530	.1509	.1492	.1492	.1527	.1575	.1630
	(.2427)	(.2418)	(.2416)	(.2414)	(.2412)	(.2411)	(.2411)	(.2413)	(.2414)	(.2417)
	.4816	.5209	.5297	.5388	.5442	.5484	.5484	.5394	.5270	.5132
h	.1171	.1258	.1279	.1302	.1316	.1329	.1337	.1324	.1301	.1272
	(.1143)	(.1141)	(.1141)	(.1141)	(.1141)	(.1142)	(.1144)	(.1146)	(.1147)	(.1149)
	.3165	.2804	.2726	.2641	.2587	.2543	.2524	.2579	.2670	.2787
b	.5014	.4892	.4895	.4910	.4947	.4996	.5119	.5241	.5333	.5388
	(.1734)	(.1721)	(.1712)	(.1701)	(.1690)	(.1681)	(.1672)	(.1677)	(.1692)	(.1715)
	.0077	.0086	.0082	.0077	.0070	.0064	.0052	.0045	.0042	.0043
a	.1212	.2313	.2506	.2701	.2779	.2820	.2688	.2351	.1968	.1578
	(.3113)	(.3197)	(.3217)	(.3208)	(.3166)	(.3123)	(.2977)	(.2759)	(.2527)	(.2316)
	.7005	.4826	.4490	.4124	.3924	.3787	.3787	.4067	.4492	.5088
R^2	.9500	.9507	.9509	.9511	.9512	.9513	.9513	.9511	.9509	.9506
S	.0574	.0570	.0569	.0568	.0567	.0567	.0567	.0568	.0569	.0570
"d"*	1.565	1.564	1.567	1.572	1.577	1.582	1.590	1.593	1.594	1.593

*Test for serial correlation is inconclusive at one per cent level of significance in all regressions.

TABLE XII

REGRESSION ESTIMATES FOR MODEL IV(a): $Q'=Ae^{ut}L^{1-a}K^{a}, \lambda=0.0$, 1935-65

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	3.6856	3.5147	3.4985	3.5009	3.5218	3.5626	3.6865	3.8550	4.0384	4.2188
	(1.1384)	(1.1258)	(1.1125)	(1.0955)	(1.0749)	(1.0510)	(.9966)	(.9380)	(.8794)	(.8240)
u	.0232	.0158	.0140	.0123	.0108	.0095	.0076	.0067	.0064	.0065
	(.0112)	(.0134)	(.0139)	(.0143)	(.0148)	(.0151)	(.0158)	(.0163)	(.0166)	(.0169)
	.0449	.2460	.3245	.4040	.4787	.5427	.6373	.6856	.7039	.7048
a	.4416	.4730	.4766	.4769	.4742	.4680	.4483	.4208	.3905	.3605
	(.1953)	(.1940)	(.1920)	(.1894)	(.1862)	(.1824)	(.1737)	(.1642)	(.1546)	(.1456)
	.0302	.0204	.0185	.0170	.0160	.0153	.0147	.0154	.0167	.0187
R^2	.9774	.9780	.9781	.9782	.9783	.9784	.9784	.9784	.9783	.9781
S	.0691	.0682	.0680	.0678	.0677	.0676	.0675	.0676	.0678	.0681
"d"	1.850	1.890	1.901	1.912	1.921	1.928	1.936	1.937	1.934	1.928

TABLE XIII

REGRESSION ESTIMATES FOR MODEL IV(b): $Q=Ae^{ut}e^{f-g/w}L^{1-a}K^{a}$, $\lambda=0.0$, 1935-65

β	0.0	.02	.025	.03	.035	.04	.05	.06	.07	.08
lnA'	3.5670	3.4818	3.4973	3.5303	3.5829	3.6566	3.8430	4.0645	4.2930	4.5091
	(1.2718)	(1.2668)	(1.2551)	(1.2392)	(1.2193)	(1.1956)	(1.1403)	(1.0786)	(1.0155)	(.9550)
u	.0186	.0112	.0096	.0081	.0068	.0058	.0045	.0041	.0044	.0051
	(.0125)	(.0151)	(.0157)	(.0163)	(.0168)	(.0173)	(.0182)	(.0188)	(.0194)	(.0198)
	.1449	.4685	.5542	.6301	.6921	.7379	.7935	.8106	.8046	.7869
g	-.3176	-.3197	-.3206	-.3216	-.3226	-.3237	-.3257	-.3274	-.3288	-.3299
	(.0469)	(.0467)	(.0467)	(.0467)	(.0468)	(.0468)	(.0471)	(.0475)	(.0479)	(.0483)
	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
a	.5188	.5363	.5346	.5300	.5220	.5104	.4804	.4441	.4063	.3702
	(.2186)	(.2188)	(.2172)	(.2149)	(.2119)	(.2083)	(.1997)	(.1899)	(.1799)	(.1702)
	.0238	.0201	.0196	.0194	.0195	.0201	.0222	.0257	.0306	.0366
R^2	.9749	.9752	.9752	.9752	.9752	.9752	.9750	.9748	.9745	.9742
S	.0772	.0767	.0767	.0767	.0767	.0767	.0770	.0774	.0778	.0783
"d"	1.874	1.898	1.905	1.910	1.914	1.917	1.918	1.913	1.906	1.899

TABLE XIV

REGRESSION ESTIMATES FOR MODEL V: $Q=Ae^{ut}e^{f+gw+hw^2}L^{1-a}K^{a}e^{jD}$, $\lambda=\beta=0.0$, 1935-65

D	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_{10}	D_{11}	D_{13}
lnA'	3.0299	3.8387	3.4769	3.4531	3.0012	2.1505	2.5205	3.0274	2.8945	3.0478
	(1.0150)	(.9437)	(.9667)	(.9444)	(.9461)	(.9038)	(1.1566)	(1.1064)	(1.1095)	(1.0178)
u	.0192	.0320	.0274	.0280	.0244	.0190	.0193	.0224	.0215	.0196
	(.0101)	(.0096)	(.0095)	(.0093)	(.0089)	(.0081)	(.0100)	(.0099)	(.0099)	(.0101)
	.0673	.0027	.0079	.0057	.0105	.0257	.0610	.0309	.0377	.0615
g	.0813	.1258	.0983	.0990	.0814	.1349	.0973	.0851	.1100	.1752
	(.2328)	(.2225)	(.2322)	(.2269)	(.2218)	(.1979)	(.2312)	(.2382)	(.2355)	(.2395)
	.7272	.5833	.6779	.6690	.7152	.5084	.6796	.7218	.6485	.4775
h	.1549	.1376	.1525	.1545	.1642	.1478	.1619	.1564	.1469	.1229
	(.1089)	(.1044)	(.1088)	(.1063)	(.1040)	(.0927)	(.1085)	(.1113)	(.1105)	(.1117)
	.1638	.1969	.1702	.1552	.1230	.1195	.1446	.1689	.1930	.2816
a	.5037	.3635	.4273	.4299	.5076	.6448	.5874	.5047	.5246	.4967
	(.1705)	(.1586)	(.1621)	(.1584)	(.1587)	(.1509)	(.1938)	(.1865)	(.1858)	(.1700)
	.0066	.0291	.0137	.0114	.0038	.0003	.0055	.0117	.0089	.0071
j	.0541	-.0787	-.0538	-.0690	-.0844	-.1335	-.0719	-.0399	-.0471	.0501
	(.0399)	(.0378)	(.0394)	(.0390)	(.0398)	(.0384)	(.0497)	(.0472)	(.0442)	(.0386)
	.1840	.0455	.1811	.0854	.0418	.0020	.1569	.4104	.2976	.2038
R^2	.9874	.9885	.9874	.9880	.9885	.9909	.9875	.9868	.9871	.9873
S	.0568	.0544	.0568	.0555	.0542	.0484	.0566	.0581	.0576	.0570
"d"	1.551*	1.683	1.736	1.717	1.895	1.659	1.692	1.444*	1.434*	1.620

*Test for serial correlation is inconclusive at one per cent level of significance.

TABLE XV

REGRESSION ESTIMATES FOR MODEL I(a):

$Q=Ae^{ut}e^{f+gw+hw^2}L^{1-a}K^{a}, \lambda=\beta=0.0,$
FOR THE 1935-49 AND 1950-65 SUBPERIODS

	1935-49 subperiod	1950-65 subperiod
lnA'	1.1424 (1.4696)	3.2510 (1.3939)
u	.0109 (.0120) .3872	.0308 (.0134) .0406
g	.1925 (.2775) .5096	-.0701 (.3980) .8395
h	.1179 (.1243) .3680	.2608 (.2023) .2222
a	.8119 (.2449) .0077	.4461 (.2340) .0804
R^2	.9706	.9738
S	.0539	.0481
"d"	2.122	1.513*

*Test for serial correlation is inconclusive at one per cent level of significance.