ACCOUNTING: FROM AN INFORMATION SYSTEMS PERSPECTIVE

by

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ABSTRACT

The author extended the synthesis of the so-called accounting spread sheet into a more compact and mathematically rigorous formulation. This formulation was applied to an example in the form of a computerized accounting information system.

The systematic approach used bridges the communication gap between the accounting profession and the quantitatively oriented computer specialists who design computer based accounting systems.

The use of tensor analysis and coordinate transformations in accounting theory was also explored. The author believes this to be an important area for further research.

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I. INTRODUCTION (JUSTIFICATION OF STUDY)

This thesis develops specialized insights into computerized accounting information systems.

The author is guided by a conceptual framework (Will, 1968) outlining information systems characteristics of accounting. Research in accounting (Mattessich, 1957; 1964; Ijiri, 1965, 1967; ...) is viewed in the confines of organizational theory (March and Simon, 1958; Cyert and March, 1963; ...) and some of the accumulated knowledge is applied.

An attempt is also made to utilize some of the interdisciplinary research within the fields of systems theory, organizational development and information technology (Ackoff, 1960; Blumenthal, 1969; Emery, 1969; Forrester, 1961; Von Neumann, 1955; ...).

Accounting has historically used information technology of the times to provide the organization with an important component of what is currently referred to as Management Information Systems. Accounting thought and practice is essentially steeped in management and systems sciences.

"It is hoped that a better understanding of the information systems characteristics of accounting will be helpful in developing better information systems ". (Will, 1968)

A. Study in perspective:

Accounting information systems are considered to be essential entities in the overall framework of managerially useful information systems.

From the practical viewpoint, the author extended the synthesis of the so-called accounting spread sheet (Mattessich, 1957; Kohler, 1963; Ijiri, 1965) into a more compact and mathematically rigorous formulation, and applied it to an example in the form of a computerized accounting system.

From the theoretical viewpoint, the author believes that accounting systems may be interrelated through the use of tensors and transformation of coordinates. However these thoughts have to be further researched and applied to an example.

B. Purpose of thesis:

The contemporary problems of an operational accounting information system can be attacked through the practical application of interdisciplinary research.

The thesis will

i. provide a perspective on the relevance of accounting information to organizations;

ii. develop a systematic analysis of financial accounting information from the computer specialist's viewpoint;

iii. use the extended matrix representation of the accounting spread sheet in generating the financial statements.

A systematic approach is used to bridge the communication gap between the accounting profession and the quantitatively oriented computer specialists who develop computer based information systems. The extended accounting spread sheet provides both the accountant and systems analyst with common ground from which to work.

A mathematical expression of accounting gives the systems analyst perspective on the problem, and allows him to logically apply information technology and computer science to the computerization of accounting systems. The mathematical expressions and the use of algorithmic techniques may reduce the misunderstandings that arise in the attempts to computerize accounting systems. The accountant , on the other hand, may gain insight into modern information generating technology.

The use of analystical tools and information technology as well as the realization that the systematic procedures of accounting have dimensions not necessarily tied to traditional methods of recording activities, will result in the development of new approaches to accounting information systems.

C. Nature of the problem:

Both the theoretical and practical facets of the problem have to be considered.

Theoretical considerations deal with the

i. implications of accounting information systems on organizational development;

ii. viewing accounting information systems in the broader field of management information systems;

The practical considerations depend on the ability to develop pragmatic and workable information systems for use by organizations.

D. Chapter organization:

Chapters two through four point out that information has important characteristics, and that it is a resource utilized by all organizations.

Chapters five and six highlight the specialized insights attained by applying mathematical techniques to the matrix representation of the accounting spread sheet. Chapter six, in particular, applies the findings to a simple accounting system using periodic inventory valuation.

Chapter seven explores the use of tensor analysis and coordinate transformations in accounting theory; the author believes that this is an important area for further study.

Appendix one lists the explicitly programmed version of the accounting system example; it is programmed in the PL/1 computer language, and is easily extendable to any number of accounts within a given chart of accounts.

E. Definition of terms:

Accounting spread sheet: a worksheet providing a two-way analysis or classification and storage of costs or other accounting data.

Algorithm: a step-by-step procedure that always yields a solution to a problem in a finite number of steps.

Coordinate: a set of numbers used to specify the location of a point in space.

Endogenous: system dependent (variable).

Exogenous: system independent (variable).

Pragmatic: concerned with the practical consequences of actions or beliefs.

Principal: object or phenomenon measured.

Semantic: concerned with the study of meaning in language.

<u>Surrogate</u>: a substitute.

<u>Syntactic</u>: concerned with the way in which elements are combined to form classes, sets and numbers.

<u>Tensor</u>: a generalization of a vector; may be the result of vector multiplication.

<u>Tensor analysis</u>: deals with the study of vector multiplication, and with the manner in which vector products transform from one system of coordinates to another.

II. AN OVERVIEW OF THE FIRM

The information systems architect must be aware of the complexities of information systems design. The information system designed will have to take into account the limitations of an organization. These limitations are usually associated with the behavioral aspects of the firm, with the structural characteristics of the firm, and with the problems of coordination.

A. Behavioral aspects:

The behavioral theory of the firm has been formulated by Cyert and March (1963). Their concepts are very useful in dealing with the limitations of information systems within the confines of the organization and subject to its relational peculiarities.

Their focal points of uncertainty avoidance, problemistic search, quasi-resolution of conflict, and organizational goals and learning are briefly outlined,

i. Uncertainty avoidance: the dominant coalition negotiates its environment by attempting to transform uncertainty and risk into certainty equivalents.

ii. Problemistic search: the solution is biased by the urgency with which it is needed; it is motivated by the problem, and may tend to be simple-minded.

iii. Quasi-resolution of conflict: the group or coalition of individuals controlling the organization are in dynamic balance with each other, and the decision rules incorporated into the firm do not provide optimal decisions, but result in what is acceptable or feasible.

iv. Organizational goals and learning: the reconciliation of the organization to its performance will be based on the expectations of the coalitions and their choice of decisions.

The response to the dysfunctions and opportunities facing the dominant coalition to some extent leaves the organization dependent on the environment, the coalitions, and their aspiration levels. Bounded rationality, resulting from the quasi-resolution of conflict and the environmental uncertainties, tends to emphasize sequential attention to goals. In short, in a situation of bounded rationality, the expectations of a firm and its choice of alternative actions will be highly influenced by the availability of information, and will depend on its format.

B. Structural characteristics:

Considerable amount of literature exists on the subject of organizational structure; and only one of the many constructs is presented.

The behavioral aspects of the firm can be complemented by a rational, albeit idealized, view of its structural components depicting the organization as a pyramidal structure with sides representing the various functions (financial, production, marketing, ...) and layered into a policy-oriented apex, an administrative midsection, and a transactional/operational base (intimately related to the logistics of the organization). The figure summarizes the main features of the construct,

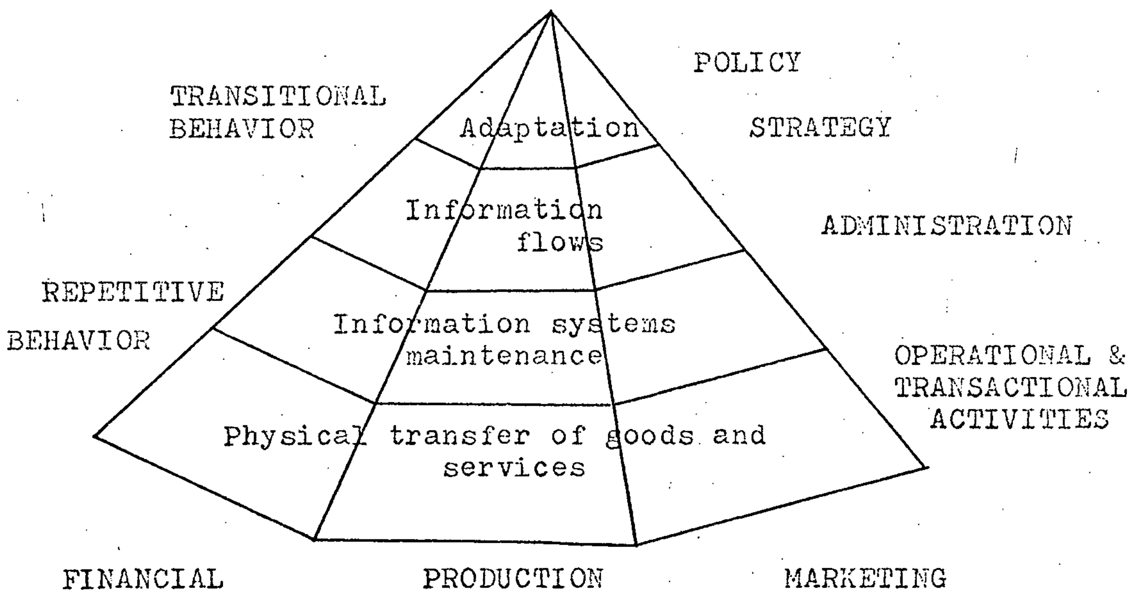

FIGURE. 1. The pyramidal structure of an organization

In very broad terms, one may say that the adaptive and behavioral aspects dominate the upper half of the pyramidal structure, due to required adaptation to unforeseen circumstances which result in transitional behavior. The lower half or 'technological core' is more predictable, due to the higher incidence of repetitive behavior and the shorter time spans of control involved. At this level the potential importance of computational information and algorithmic procedures can be emphasized.

Emery (1969), in considering the hierarchical nature of organizations, the need for interaction among organizational subunits, and the coordination decisions facing the managers and administrators, suggests that the degree of coordination will depend on the costs of improved information technology. Here information technology is seen affecting organizational structure, and is considered to be the interconnecting tissue between organizational subunits.

C. Problems of coordination:

Emery (1969), using the Simon-Ando model of nearly decomposable systems, applied it to the organizational hierarchy.

Briefly, interactions between organizational subunits are coordinated through the transmission of information, and is the result of planning and/or goal setting. The organization is seen factoring its global objectives into a hierarchy of subobjectives. He also brings in the notion of the implicit tradeoff between local and global objectives, and introduces the time dimension as the measure of the sub-unit's independence.

" ... Because of the limited information-handling ability

> of both humans and information processing equipment, the organization <u>must</u> be constituted as a nearly decomposable system. This is achieved by combining closely related activities and decoupling them from hierarchically more distant activities. The macro-characteristics of the organization are governed by relatively aggregate plans issued by higher level managers. Within the constraints imposed by this information, a lower level manager then pursues his (changing) goals more or less independently.
>
> This scheme has the essential advantage of economizing on coordination. Higher level managers adjust lower level constraints without having to know their detailed implications. Lower level managers are relatively isolated from the rest of the organization, and can carry on their activities without constant attention to most of the detailed activities of other parts of the organization."(Emery, 1969; pp.32-33)

The activities of the organization provide the managerial levels with the events to be captured and retained in the form of information for possible future use. The need for consistent accumulation of managerially useful information can be satisfied by the use of operationally oriented information systems. The need for coordination of disparate local activities is an important factor favoring management information systems designed to fulfill specific objectives.

Historically, information systems created and used by organizations revolve around well established accounting practices and procedures, which do not require the investment of large monetary funds for their maintenance. However, the rapid introduction of computers into all sectors of North American activity, has increased the use of information technology and organized intelligence to a level where one cannot view them as free commodities. The use of information and organized intelligence has to be considered as a legitimate factor of production involving costs of collection and dissemination.

In summary, information is found to be an important ingredient used by all organizations and it influences their behavior. Information can be generated and disseminated through the use of information systems, and these to some extent reflect the structure of the organization.

III. ORGANIZATIONAL DEVELOPMENT

The approach to organizational development usually involves environmental research, positional audits, identification of challenges, forecasts on premises previously established, and proposal of goals or objectives. The descriptive diagnosis based on the current states of the organization is then used to develop a prescriptive analysis of required organizational climates, ways of work, interpersonal relationships, communication and information systems.

Organizational development may require an overall-system, planned-change effort in order to cope with the alterations needed by the organization.

A. Short and long run considerations:

The long term aspirations of the organization are assumed to be really those of the dominant coalition members. The members are expected to give the organization its global objectives and its stated rationality. The verbalized objectives, useful in guiding the organization's specific activities and growth may be called intelligence functions.

The objectives and rationality of a firm change with time, and a distinction between short and long run behavior should be made. The short run considerations constrains the intelligence functions, and direct them to the fulfillment of local objectives through purposeful activity. The long run considerations alter the firm's behavior, by requiring it to conform to the modifying forces of the environment and to the outcomes of organizational assessment.

In practice, the global objectives are revised through periodically issued directives and guidelines. These alter the pseudo-bureacratic conditions within the organization, its activities and the control and allocation of its resources.

B. <u>The worth of information:</u>

In the previous chapter, the worth of information was emphasized from several viewpoints. The generation of information and its availability were found to be influenced by the behavioral aspects of the firm, to the firm's structural characteristics, and to the problems of coordination between organizational subunits. It was pointed out that

i. information is the interconnecting tissue between organizational subunits;

ii. information flows are major determinants of organizational structure, and vice versa;

iii. information generation involves costs of collection and dissemination;

iv. information can be generated and disseminated through the use of information systems;

v. the degree of coordination within a firm is dependent on the costs of improved information technology;

vi. availability of information influences the expectations of the firm, and its choice of alternatives.

Under the circumstances, it is felt that the development of information systems should conform to the overall objectives of organizational development.

C. <u>Financial information in perspective:</u>

Organizational development, among other considerations, concentrates primarily on improving the current state of the organization. In most organizations, wealth determination is an essential activity. It provides the organization with information that is useful in assessing the financial health of the organization.

Financial information, obtained through wealth determination, supplies the firm with indicators measuring both the current state of the organization, and its ability to survive. The knowledge of financial profitibility and liquidity is the operational goal of wealth and income determination.

This knowledge has for centuries been provided by the double entry, bookkeeping accounting information system. In this perspective, financial information and organizational development are intimately related.

D. Conceptual framework for information system design:

Will (1969) developed premises, pragmatic in nature, which consider information systems as means of extending managerial capabilities.

The following figure graphically conveys the concept of managerial authorities defining and applying intelligence functions (or processes) within the confines of organizational rationality.

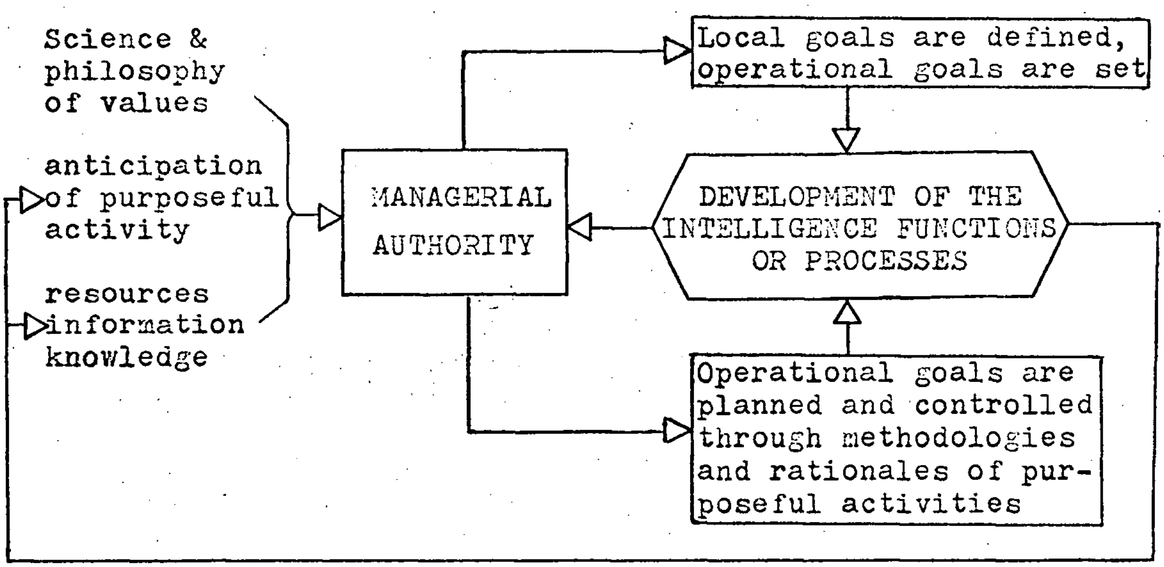

FIGURE 2. Managerial authorities developing intelligence functions.

In such an approach, one assumes that rationality is imposed upon the function or process. This is in line with the concept of rationality as being imposed upon the 'world' by the enquirer himself. Such an approach need not seek 'optimality' according to some given law, and can instead allow that the rationality be imposed by the managerial authorities in accordance with their inclinations.

The managerial authorities are able to obtain goal related knowledge and information by using information handling and system design technology. Figure 3 shows a conceptual approach useful in information systems design. Both figures 2 and 3 reflect the conceptual framework developed by Will (1969); however, they are this author's interpretations of his work.

E. Information systems design:

The multifaceted pyramidal structure of an organization suggests that information systems design requires an integrated approach to interfacing the various facets of subunits and the levels of the pyramid. However, the complexity of the organization, and the near decomposability of its subunits, vitiates the use of communal information through well-defined, interre-

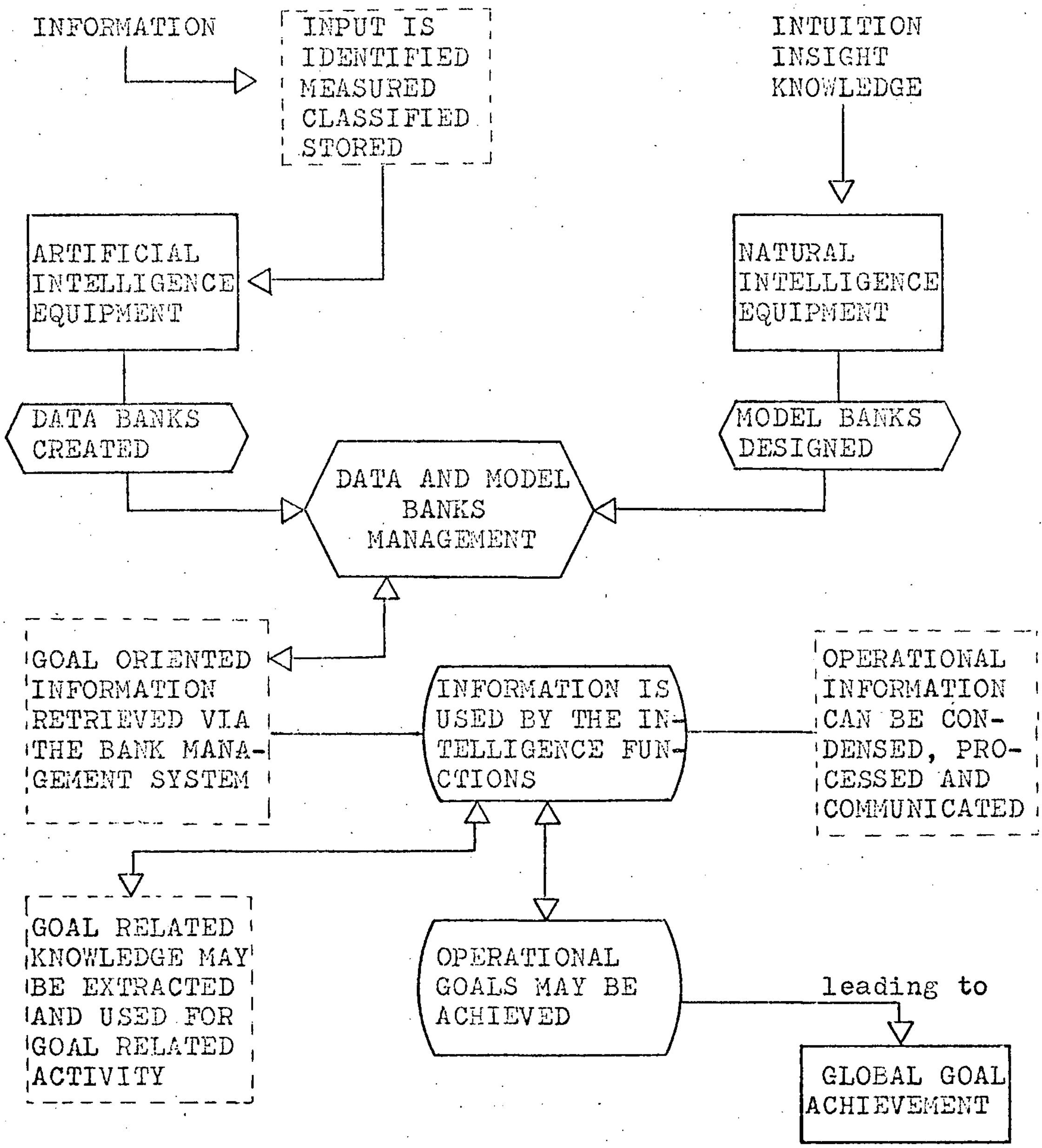

FIGURE 3. Conceptual approach to systems design

lating data bank files. It is felt that data banks must be created with the intelligence function in mind and must be oriented toward some objective.

Information systems have situational and relative characteristics, and the idea of measurement is imbedded in them. They (information systems) provide precise causal relations, and serve purposeful needs. The output of information systems is reproducible, repetitive and structured; but its use is not.

Information systems design attempts to facilitate the flow of information to, from and through the organizational hierarchy. In practice, they adapt to the operational activities of the organization and involve all phases of logistic control. One may emphasize that the control is not normally exercised by recurring caricatures of managers (as profit maximizers), but by individuals whose goal formulation incorporates besides risk and uncertainty, the typical human inclinations and ambitions that cannot be assumed away in the design and development of information systems.

Current information technology permits one to create data and model banks for use throughout the organization. The data banks can provide the users with information on financial demographic, simulative, and economic factors, among others.

The model bank can transform raw, unaggregated data (as well as information), into operationally useful information, while filtering out the insignificant information.

F. <u>The characteristics of information:</u>

What is information?

> " ... if the notion of management is related to such intelligence tasks as goal formulation (consisting of goal planning and goal setting), and goal (achievement) control, such insights ought to be related to the concept of information. ... If the information system reflects the real system perfectly, such that goal pursuit and goal attainment can be determined with a high degree of accuracy, reliability, and predictability, then the information is considered relevant. Relevance is thus an indicator of the degree of identity between the information system and the underlying real system. ...
>
> It is now possible to call classified phenomena or their surrogates (measurements) data and to relate this data definition to the pragmatic definition of information by postulating that the data descriptions (data names) selected for the measurements reflect the goal systems such that goal variables and parameters are identifiable within goal-sub-goal relations. ...
>
> To realize that information is fundamentally a three-dimensional (pragmatic, semantic, syntactic) concept is simple but to incorporate this insight into an

information systems definition means to apply the <u>structural</u> and <u>procedural</u> systems notions to the concepts of information. Information is then considered as the output of data transmutation processes and is identical to desired knowledge which provides insights into a problem or a particular problem solution ..." (Will, Dec.,1969; Management Science, pp.B169-71)

The pragmatic, semantic and syntactic dimensions of information are useful in analysing accounting information systems.

IV. ACCOUNTING INFORMATION CHARACTERISTICS

The operational goals of wealth and income determination are essentially the knowledge of financial profitability and liquidity. The objectives have for centuries been partially satisfied through the use of double entry, bookkeeping accounting systems.

A. Overview:

Accounting systems are pragmatic because the generated information is used in attaining knowledge of financial profitability and liquidity. The goal orientation of accounting information is also relevant as an indication of the identity between the surrogate and the mapped principal.

The goal orientation of an overall accounting system determines to a large measure the processing flow of double entry accounting entries. The accounting systems will provide users with consistent methods for recording valuations. Essentially, the characteristics of accounting systems require that

i. surrogate measures of the principals be established;

ii. surrogate structure be defined;

iii. the states of the surrogate measures be maintained over time;

iv. the states of the surrogate measures be alterable by consistent rules.

The problem of identifying the principal, when the accounting valuation is the surrogate, is a problem faced by the managerial authorities. They have the option of adopting structure of surrogates, dispensing with them altogether, or altering them. In all cases, the managerial authorities make an attempt to conform to their own objectives, and measure the principal accordingly.

The structure of the surrogates is largely dictated by the classifications used in reflecting the principal. Also, goal orientation affects the measuring, recording and processing methodology used in an accounting system. In short, the accounting system is expected to supply relevant information in accordance with the required organizational rationality.

B. Accounting activities:

From an information systems perspective, double entry accounting information has semantic, syntactic and pragmatic dimensions. In fact, from such a perspective, 'double entry' is no longer relevant, as it implies that an amount is recorded twice. Double entry and double classification are implicit in the structure of computerized accounting systems; peculiar classificational schemes are accepted as distinguishing features of accounting systems, which set them apart from other computerized, non-accounting systems.

In light of operational systems theory, accounting activities have to be viewed from an internal and external activity viewpoint. The characteristic distinctions between the two types of activity are significant in information system design. External accounting entries are based on events occuring outside of the boundaries of the system, and must be considered to be the exogenous variables that alter the states of the surrogates. The internal accounting entries, on the other hand, may be generated on an a priori goal oriented assumption. These assumptions may reflect the expressed one-to-one correspondences between states of the surrogates (accounts) whose levels are altered to reflect accruals and adjustments. The use of exogenous and endogenous variables are extremely useful in the attempts to reduce accounting information systems to algorithms.

The problems of human communication faced in describing the attributes of given measurements limit the semantic dimensions of accounting information. Also, the economic goal orientation of accounting information will effectively subordinate the syntactic dimension of accounting information to the classificational considerations long embedded and institutionalized into various chart of accounts.

C. <u>Use of accounting information:</u>

Global objectives are necessarily vague, and they have to be reduced to surrogates capable of measure. Measured surrogates are then used to assess the fulfillment of local objectives relating to the overall goals.

The adaptibility of organizational behavior to changes in response to local goals, is an indicator of the affective pressures applied. It is the ability to generate behavioral changes originally predicted, which result in the success of stated objectives, that has to be measured. In both cases, accounting information is used to generate the relevant measures. Here, the indicators of affective pressures and of the predicted behavior are usually financial in nature.

Accounting information provides the base for much of the systematic planning attempted. It ensures that members of an organization are supplied with periodic, factual and searching analyses on the behavior of the organization. It offers administrators with alternative allocations of resources, and focuses on key behavioral and/or performance problems affecting the well-being of the organization and of its dominant coalition.

Accounting measurement has dominated the rationality of most economically oriented organizations, by providing the managerial authorities with reliable financial information with which to assess their performance.

The use of accounting information is a major influencing factor on organizational behavior, and any attempt to clarify accounting system methodology (in information systems perspective) will add to the study of organizational development.

V. THE ACCOUNTING SPREAD SHEET

According to Mattessich (1964), the idea of presenting accounting in matrix form can be traced to Gomberg's "geometrical" presentation of bookkeeping methods back in 1927. The matrix form is also known to American business accounting under the name of "spread sheet" (Kohler, 1963).

Kohler's spread sheet is " a worksheet providing a two-way analysis or recapitulation of costs or other accounting data" and it achieves dual classification with a single entry (Mattessich, 1964; p.90).

A. Matrix representation:

Mattessich (1957; 1964, pp. 75-77) interpreted every transaction as a separate matrix, in order to " reveal the structural relations of accountancy in terms of a general and universally accepted language (of mathematics)".

As a result, accounting matrices have become more frequently referenced; and Ijiri (1960; 1963, pp. 82-137) has extended the usefulness of this representation by developing mathematical expressions for it, and by tying it in with linear and goal programming (Charnes, Cooper and Ijiri, 1963).

The matrix formulation of the accounting spread sheet is used to relate the " ... fundamental relation (of asset and equity partitioning) ..." (Ijiri, 1963, p.90) to a square matrix W identified with the so-called spread sheet of double-entry accounting.

The square matrix W, s x s, with elements w_{ij} representing the total amount of simple transactions whose debit entries are all made to the ith account and whose credit entries are all made to the jth account, is related to the changes in the asset and equity accounts Δu, by the following mathematical expression (Ijiri, 1963),

$$(W - W^*) \times e = \Delta u \qquad (1)$$

where, W^* is the transpose matrix of W;

e is a column vector, s x 1, whose elements are all equal to 1;

Δu is the resultant vector, s x 1, containing all numerical changes to the beginning balance vector u.

Some of the shortcomings of such an exposition lies in the convenient but unnecessary restriction of the chart of accounts to the Asset and Equity portion of an accounting system's chart of accounts.

The beginning balances are stored separately from the matrix W, and the current balances are calculated by adding two different vectors $\check{u}$ and Δu to obtain $\hat{u}$, the final balance vector,

$$\check{u} - \Delta u = \hat{u} \qquad (2)$$

Other methods employed by Kemeny, et al.(1962) use additional rows and columns of the matrix for the beginning and ending balances.

In either case, the computerization of the accounting matrix presents a minor irritant, due to the separation of the beginning balances from the transaction entries.

The author has formulated a set of mathematical expressions that allow the beginning balances to be incorporated into the accounting matrix through the use of its diagonal elements. This approach does not detract from the mathematical exposition of Ijiri (1963), and in fact enhances it.

B. Matrix accounting:

The author shows that there is considerable potential in the matrix representation for the development of computerized accounting information systems,

i. the matrix representation contains the chart of accounts used;

ii. the methodology of matrix representation lets one create or define transformation mechanisms for the formulation of one matrix accounting system in terms of another.

In non-technical words, more than one spread sheet matrix can be used, their interdepencies can be mathematically formulated, and subsequently computerized.

VI. EXTENDING THE ACCOUNTING SPREAD SHEET

The role of the extended accounting matrix in the development of accounting information systems will be briefly discussed in the first section. In the second section the specialized insights of the author, obtained by applying mathematical techniques to the matrix representation of the accounting spread sheet, will be descriptively explained. The theoretical formulation is then given in the third section.

A. Role of the extended accounting matrix in the development of accounting information systems:

The extended accounting matrix lends itself to computer programming and to the definition of computer based files. In this respect, it has widespread applicability in the development of accounting information systems.

Also, the matrix representation of an accounting system has data and model bank characteristics,

i. as a data bank, the elements of a matrix can be viewed as storage devices, retaining information on the accounts of the system;

ii. as a model bank, the application of mathematical techniques can be viewed as dependent on the explicit structural characteristics of the matrix.

B. Descriptive analysis of matrix accounting applied to the spread sheet:

Accounting accounts may be diagrammatically represented by a number of arrays and extended into matrices. The matrices contain the aggregate information about the status of the accounts of the system.

i. Beginning balances: the chart of accounts can be shown to be an array with as many rows or columns as there are accounts. The rows define the debit sides and the columns define the credit sides of the respective accounts (in accordance with the Ijiri notation). Since debiting and crediting of an account by the same transaction is not practised, the diagonal of the resulting matrix can be reserved for the beginning balances of the accounts (this is a breakthrough in the matrix representation of the spread sheet).

The beginning matrix T_o is as follows,

$$\begin{array}{cc} & \text{C r e d i t s} \\ \begin{array}{c} \text{D} \\ \text{e} \\ \text{b} \\ \text{i} \\ \text{t} \\ \text{s} \end{array} & \begin{bmatrix} \text{BEGINNING} & 0 \\ 0 & \text{BALANCES} \end{bmatrix} \end{array} = T_o \quad (3)$$

ii. Postings: the exogenous transactions, or postings, are the components w_{ij} developed by Ijiri (1963), and consist of linear aggregation of one or more similar simple transactions to given non-diagonal elements,

$$\begin{bmatrix} 0 & & & & & w_{ij} \\ & 0 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & 0 & \\ w_{ij} & & & & & 0 \end{bmatrix} = W \quad (4)$$

iii. Accruals and portfolio changes: the exogenous transactions w_{ij} mirror events ocurring in the "real" world. They have been supplemented by endogenous variables to reflect the events imposed upon the system by the rationality and methodology of the specific scheme. These are represented by the a_{ij}'s first developed in this thesis; these are similar to the w_{ij}'s, except that they are generated through internal computation, and according to preset rules,

$$\begin{bmatrix} 0 & & & & a_{ij} \\ & 0 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & 0 & \\ a_{ij} & & & & 0 \end{bmatrix} = A \qquad (5)$$

iv. The time dimension: time is implicit in all three matrices,

a. at time p_o, the opening balance is T_o

b. at some subsequent time p_1, all w_{ij} have been posted for the time period $p_1 - p_o$;

c. at time p_1, the accruals and portfolio changes are made.

v. The composite T-matrix: by superimposing (through matrix addition) the contents of the three matrices $T_{o,}$ W and A the author arrives at a single matrix called the T-matrix. This matrix consists of the accounts of the system and reflects changes to them (both endogenous and exogenous). In other words, the set of all mapped principals for a given accounting system can be contained in the T-matrix, where,

$$T = T_o + W + A \qquad (6)$$

vi. Matrix extraction: subsets of the mapped principals can be extracted from the T-matrix. They are then used to arrive at the traditional financial statements. It is important to view the T-matrix as consisting of all relevant accounts of an account structure, and not limited to the balance sheet or the income statement accounts. The matrix extracted is defined in accordance to the financial statement requirements, and consists only of those accounts that are of interest.

The derived T-matrix is a composite, and is the end result of a number of previous operations. It is quite possible to set up non-financial T-matrices preceding the financial T-matrix. As a suggestion, the notion of composite T-matrix (resulting from the amalgamation of several disparate matrices) can be extended to inventories (opening inventories, issues/receipts, commitments/on-orders). It can also be extended to price and quantity purchased, and to price and quantity sold matrices; as well as to the logistics preceding the accounting measurements.

In this respect, accounting and management science interface, and the multidimensionality of a given simple financial transaction can be utilized to better advantage by both disciplines.

C. Theoretical analysis of the extended matrix accounting approach:

The need for an extended matrix accounting theory is dictated by the lack of a consistent approach to accounting from an operational computer information system perspective. The equations formulated by the author help remedy the situation.

i. Theoretical development of the T-matrix: the T-matrix is an ordered array of amounts consisting of aggregated accounting entries on the off-diagonal elements, and of opening balances on the main diagonal elements. The T-matrix is a structure encompassing all of the accounts of the chart of accounts, and is an n x n element matrix. The n stands for the total number of accounts within a given chart of accounts,

$$T = T_o + \left[t_{ij}\right] \qquad (7)$$

where

$$T_o = \left[t_{ii}\right] \times \left[e'_{ii}\right] \qquad (8)$$

$$\left[t_{ij}\right] = W + A \qquad (9)$$

$$= \left[w_{ij}\right] + \left[a_{ij}\right] \qquad (9a)$$

<u>Key to symbols used</u>:

- T - the T-matrix
- T_o - the opening balance matrix
- $\left[t_{ij}\right]$ - the transactions entry matrix, consisting of off diagonal ($i \neq j$) elements
- $\left[t_{ii}\right]$ - the unsigned opening balance matrix, consisting only of the diagonal ($i = j$) elements
- $\left[e'_{ii}\right]$ - unit matrix, with positive diagonal elements for assets and expenses, negative for equity, revenue and summary accounts

$\left[w_{ij}\right]$ - the exogenous transactions matrix $(i \neq j)$

$\left[a_{ij}\right]$ - the endogenous transactions matrix $(i \neq j)$

(where the transactions refer to the usual accounting entries used in financial accounting).

ii. The algorithm: the outstanding balances are calculated by means of the following expression

$$T \times e = (T_o + (T_{ij} - T_{ji})) \times e = t_f \qquad (10)$$

where

t_f is the ending balance vector, n x 1, whose elements become the components of $\left[t_{ii}\right]$ of the following accounting period.

iii. Verification of posting: the accuracy of the computational process is checked by making use of the following relationship,

$$e_n^T \times T_o = e_n^T \times T = 0 \qquad (11)$$

where

e_n^T is the transpose of a column vector consisting of all elements equal to 1.

iv. Changes to the T-matrix: a summary aggregation of all changes to the accounting system by entries affecting it are obtained through vector subtraction,

$$t_f - t_o = \Delta t \tag{12}$$

The equations (7), (10), (11) and (12) form the core of the author's theoretical development of the extended matrix approach to accounting systems. It is interesting to note that the above approach allows one to think in terms of imposed boundary conditions (the beginning balance T_o), and in terms of so-called natural boundary conditions (the chart of account structure). The concepts of "boundary" are quite important from a systems viewpoint, and help to isolate in one's mind the essential characteristics of an accounting information system.

D. <u>Example of an accounting information system for computers</u>:

The findings and analysis presented in this thesis will be illustrated by an example. The example represents a simple accounting system that uses periodic inventory valuation. It was extracted from a popular textbook by Gordon and Shillinglaw (1969), and then used to bring out the practicality of the extended theory of matrix accounting.

In the example, the statement of Financial Position as of December 31, 19x7 was given as

BALANCE SHEET

ASSETS			EQUITIES	
Cash on hand		4,000	Accounts payable	20,000
Accounts receivable		24,000		
Inventory		16,000	Capital stock	35,000
Furniture	15,000			
less				
Depreciation	4,000	11,000		
		55,000		55,000

and the chart of accounts was defined as follows,

Account number	Account name	Initial balances Debit	Initial balances Credit
1	cash on hand	4,000	
2	accounts receivable	24,000	
3	inventory	16,000	
4	furniture	15,000	
5	depreciation-furniture		4,000
6	accounts payable		20,000
7	capital stock		35,000
8	retained earnings		
9	sales revenue		
10	miscellaneous revenue		
11	cost of goods sold		
12	administrative expenses		
13	miscellaneous expenses		
14	summary accounts		

The accounting entries to be posted were the following,

1. Merchandise purchased

inventory	98,000	
accounts payable		98,000

2. Expenses payable

miscel. expenses	19,000	
accounts payable		19,000

3. Cash sales

cash on hand	45,000	
sales revenue		45,000

4. Credit sales

accounts receivable	127,000	
sales revenue		127,000

5. Salaries

administrative expenses	50,900	
cash on hand		50,900

6. Accumulated depreciation

administrative expenses	1,000	
depreciation-furniture		1,000

7. Furniture sold

cash on hand	50	
furniture		50

8. Gain on sale

cash on hand	30	
miscel. revenue		30

9. Write-off

depreciation-furniture	350	
furniture		350

10. Collection

cash on hand	118,000	
accounts receivable		118,000

11. Payments

accounts payable	114,100	
cash on hand		114,100

12. Ending inventory was determined at $18,000.

The above information is enough to generate the financial statements from. In order the calculations are reasonably clear to follow, all the relevant vectors and matrices will be explicitly listed.

Referring to equation (8), t_{ii} and e'_{ii} are defined in the following manner,(in 000's)

$$\left[t_{ii}\right] = \begin{bmatrix} 4 & & & & & & & & & & & & & \\ & 24 & & & & & & & & & & & & \\ & & 16 & & & & & 0.0 & & & & & & \\ & & & 15 & & & & & & & & & & \\ & & & & 4 & & & & & & & & & \\ & & & & & 20 & & & & & & & & \\ & & & & & & 35 & & & & & & & \\ & & & & & & & 0 & & & & & & \\ & & & & & & & & 0 & & & & & \\ & & & & & & & & & 0 & & & & \\ & & 0.0 & & & & & & & & 0 & & & \\ & & & & & & & & & & & 0 & & \\ & & & & & & & & & & & & 0 & \\ & & & & & & & & & & & & & 0 \end{bmatrix}$$

and,

(in units),

$$
\left[e'_{ii}\right] = \begin{bmatrix}
1 &&&&&&&&&&&&& \\
& 1 &&&&&&&&&&&& \\
&& 1 &&&&&&& 0.0 &&&& \\
&&& 1 &&&&&&&&&& \\
&&&& -1 &&&&&&&&& \\
&&&&& -1 &&&&&&&& \\
&&&&&& -1 &&&&&&& \\
&&&&&&& -1 &&&&&& \\
&&&&&&&& -1 &&&&& \\
&&&&&&&&& -1 &&&& \\
&& 0.0 &&&&&&&& 1 &&& \\
&&&&&&&&&&& 1 && \\
&&&&&&&&&&&& 1 & \\
&&&&&&&&&&&&& 1
\end{bmatrix}
$$

The matrix multiplication for t_{ii} x e'_{ii} gives the T_o matrix with the following values in 000's,

$$
T_o = \begin{bmatrix}
4 &&&&&&&&&&&&& \\
& 24 &&&&&&&&&&&& \\
&& 16 &&&&&&& 0.0 &&&& \\
&&& 15 &&&&&&&&&& \\
&&&& -4 &&&&&&&&& \\
&&&&& -20 &&&&&&&& \\
&&&&&& -35 &&&&&&& \\
&&&&&&& 0 &&&&&& \\
&&&&&&&& 0 &&&&& \\
&&&&&&&&& 0 &&&& \\
&& 0.0 &&&&&&&& 0 &&& \\
&&&&&&&&&&& 0 && \\
&&&&&&&&&&&& 0 & \\
&&&&&&&&&&&&& 0
\end{bmatrix}
$$

The matrix W is formed by direct entries of the w_{ij}'s into the respective positions of the matrix,

$$w_{ij} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \end{array} \begin{array}{c} \begin{array}{cccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \end{array} \\ \begin{bmatrix} 0 & 118 & & .05 & & & & & 45 & .03 & & & & \\ & 0 & & & & & & & 127 & & & & & \\ & & 0 & & & 98 & & & & & & & & \\ & & & 0 & & & & & & & & & & \\ & & & .35 & 0 & & & & & & & & & \\ 114.1 & & & & & 0 & & & & & & & & \\ & & & & & & 0 & & & & & & & \\ & & & & & & & 0 & & & & & & \\ & & & & & & & & 0 & & & & & \\ & & & & & & & & & 0 & & & & \\ & & & & & & & & & & 0 & & & \\ 50.9 & & & & 1 & & & & & & & 0 & & \\ & & & & & 19 & & & & & & & 0 & \\ & & & & & & & & & & & & & 0 \end{bmatrix} \end{array}$$

At the beginning of the period p_o, the matrix T_o was generated in accordance with the structural peculiarities of the chart of accounts. During the interval between p_o and p_1, transaction entries were entered into the W matrix. This will go on until such time p_1 when the financial statements are requested.

At p_1 the closing inventory valuation will be required, in order that the closing entries can be made; the endogenous matrix A is then generated.

The ending inventory valuation of $18,000 allows one to form the endogenous transaction entry number 13 as follows,

$$\text{COST OF GOODS SOLD} = \text{BEGINNING INVENTORY} - \text{ENDING INVENTORY} - \text{PURCHASES} \quad (13)$$

or,

$$CGS = 16 - 18 - 98 = 96$$

and

13. Cost of goods sold valuation

cost of goods sold	96,000	
inventory		96,000

Up to here, the matrix T was discussed in terms of the component matrices T_o, W and A. It is now assumed that the principles have been sufficiently well outlined, and the attention will be focused on the T-matrix as defined in equation (6).

The T-matrix, at time p_1, will have the folowing elements,

$$
\begin{array}{cc}
 & \begin{array}{cccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \end{array} &
\left[\begin{array}{cccccccccccccc}
4 & 118 & & .05 & & & & & 45 & .03 & & & & \\
 & 24 & & & & & & & 127 & & & & & \\
 & & 16 & & & 98 & & & & & & & & \\
 & & & 15 & & & & & & & & & & \\
 & & & .35 & -4 & & & & & & & & & \\
114.1 & & & & & -20 & & & & & & & & \\
 & & & & & & -35 & & & & & & & \\
 & & & & & & & 0 & & & & & & \\
 & & & & & & & & 0 & & & & & \\
 & & & & & & & & & 0 & & & & \\
 & & 96 & & & & & & & & 0 & & & \\
50.9 & & & & 1 & & & & & & & 0 & & \\
 & & & & & 19 & & & & & & & 0 & \\
 & & & & & & & & & & & & & 0
\end{array}\right]
\end{array}
$$

At this point, the mathematical expression, equation (1), can be expressed in terms of Δt_{int}, an intermediate vector consisting of net transaction balances of all active accounts,

$$(T - T^T) \times e = \Delta t_{int} \tag{14}$$

and can be applied to the revenue and expense account numbers 9, 10, 11, 12 and 13. The outstanding balances in these accounts will then be transfered by means of endogenous transaction entries into the summary account number 14.

The closing entry transactions are as follows,

14. Closing entry - sales revenue

sales revenue	172,000	
summary account		172,000

15. Closing entry - misc. revenue

misc. revenue	30	
summary account		30

16. Closing entry - cost of goods sold

summary account	96,000	
c.g.s.		96,000

17. Closing entry - administrative expenses

summary account	51,900	
admin. expenses		51,900

18. Closing entry - miscel. expenses

summary account	19,000	
misc. expenses		19,000

After the closing entries are made, the equation (14) is applied to the summary account number 14, in order to determine the retained earnings balance. The balance is then used to create the last entry,

19. Transfer to retained earnings

summary account	5,130	
retained earnings		5,130

Though the above outline does no more than skim through the mechanics of an accounting system using periodic inventory valuation, it is clear that the T-matrix has at this moment all the entries required to generate the new balance sheet, and that the income statement was available as soon as the cost of goods sold was known. In fact, the computerized version of the example, stores the balances of the revenue and expense accounts as they become known. This is an important point, since endogenous transactions are created through an iterative process.

Assuming, for the moment, that the endogenous transactions 14 through 19 have not yet been recorded in the T-matrix, the calculation of

$$(T - T^T) \times e = \Delta t_{int} \qquad (14)$$

will provide one with an intermediate vector whose components are the net transaction balances of all exogenously activated accounts. The components of the matrix $(T - T^T)$ are not explicitly shown in this discussion. They are similar to the components of a matrix developed further in the section, and shown on page 50.

The net transaction balances of all activated accounts are given by Δt_{int}, and are as follows,

$$\begin{bmatrix} -1.92 \\ 9 \\ 2 \\ -.40 \\ -.65 \\ -2.90 \\ 0 \\ 0 \\ -172 \\ -.03 \\ 96 \\ 51.9 \\ 19 \\ 0 \end{bmatrix}$$

the vector components -172, -.03 are net balances of the revenue accounts; similarly, 96, 51.9 and 19 are the net balances of the expense accounts (in 000's), and the difference between the linear aggregation of the accounts will result in an amount equal to the retained earnings.

The income statement, can therefore be generated,

INCOME STATEMENT

SALES REVENUE			172,000
Less	COST OF GOODS SOLD	96,000	
	ADMINISTRATIVE EXP	51,900	
	MISCELLANEOUS EXP	19,000	
		166,900	
INCOME prior to extraordinary item			5,100
Gain on MISCELLANEOUS REVENUE			30
			5,130

It will be noted that the accounts involved in the Income Statement have zero beginning balances. This is because they are aggregation of entries to accounts that are set to zero at the end of an accounting period. The traditional trial balance and adjusted trial balance can also be made available from the T-matrix. But because they are essentially worksheets, and convenient only to manual processing of accounting information, they have been bypassed in the computerized information systems context.

Now we return to the T-matrix where all the endogenous transactions have been posted. The entries at the time of the determination of the balance sheet are arranged in the T-matrix in the following manner,

$$\begin{array}{cc}
 & \begin{array}{cccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \end{array} &
\begin{bmatrix}
4 & 118 & & .05 & & & & & 45 & .03 & & & & \\
 & 24 & & & & & & & 127 & & & & & \\
 & & 16 & & & 98 & & & & & & & & \\
 & & & 15 & & & & & & & & & & \\
 & & & .35 & -4 & & & & & & & & & \\
114.1 & & & & & -20 & & & & & & & & \\
 & & & & & & -35 & & & & & & & \\
 & & & & & & & & & & & & & \\
 & & & & & & & & & & & & & 172 \\
 & & & & & & & & & & & & & .03 \\
 & 96 & & & & & & & & & & & & \\
50.9 & & & & 1 & & & & & & & & & \\
 & & & & & 19 & & & & & & & & \\
 & & & & & & & 5.13 & & & 96 & 51.9 & 19 &
\end{bmatrix}
\end{array}$$

where the entries into account number 14 are all endogenously derived entries a_{ij}; the application of equation (10) to the T-matrix generates the ending balance vector t_f, whose elements are then used in printing out the balance sheet (and in supplying the T-matrix with a new set of beginning balances for the following period).

The elements of $(T_o + (T_{ij} - T_{ji}))$ are shown below,

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	4	118		.05		-114.1			45	.03		-50.9		
2	-118	24							127					
3			16			98					-96			
4	-.05			15	-.35									
5				-.35	-4							-1		
6	114.1		-98			-20							-19	
7							-35							
8														-5.13
9	-45	-127												172
10	-.03													.03
11			96											-96
12	50.9				1									-51.9
13						19								-19
14								5.13	-172	-.03	96	51.9	19	

Multiplying the above T-matrix by a row vector e, gives the following vector t_f,

$$\begin{bmatrix} 2.08 \\ 33 \\ 18 \\ 14.6 \\ -4.65 \\ -22.9 \\ -35 \\ -5.13 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The statement of Financial Position as of December 31, 19x8 will be given as

BALANCE SHEET

ASSETS			EQUITIES	
Cash on hand		2,080	Accounts payable	22,900
Accounts receivable		33,000		
Inventory		18,000	Capital stock	35,000
Furniture	14,600			
less			Retained earnings	5,130
Depreciation	4,650	9,950		
		63,030		63,030

VII. STATE OF THE ART

Accounting theory has evolved primarily out of the practical application of accounting systems to help managements to cope with real world problems. Thus for an accounting valuation scheme to be successful, the particular method of aggregation must be chosen in accordance with the way the aggregate will be used. Though the determination of rationality is not a problem of accounting *per se*, the reflected rationality requires explicit structural relationships to emerge; and hence the value of mathematical analysis in developing computerized accounting information systems.

Accounting transactions can be viewed as reflecting activities involving measurement in general. They are seen as being independent of the particular coordinate system used in describing, classifying and aggregating them. Mathematical analysis allows one to develop alternative coordinate systems, or if they already exist, provide an explicit formulation of a coordinate system in terms of another, subject to coordinate transformation equations, and consistent with information systems technology.

The rest of the chapter explores some of the qualities of tensors, and of transformation coordinates. Their use in accounting is commented upon.

A. simple transactions:

Events are recorded in accounting systems by assigning debit and credit dimensions to the amount reflecting the event. The debit and credit dimensions assigned are restricted by the account names used within an account structure; it is quite clear that the account structure encompasses the sum total of the one-to-one reciprocal correspondences between all dimensions of all accounts, for that given accounting system.

The interrelatedness existing between the amount A, the debit and credit dimensions x^j and x^k, and the account structure, is not unique. It is quite possible that the amount A have different dimensions x^j and x^k in different accounting systems. For example, debit cash and credit inventory as versus debit cash and credit sales revenue. In other words, the simple accounting transactions involving amount A are independent of the accounting system imposed upon the amount A. The accounting system used in classifying and aggregating the transactions can then be looked upon as a specific coordinate system reflecting certain structural characteristics.

It also follows that in amount A reflects a valuation of a principal, it cannot be changed by the accounting system used in recording the transaction.

If the valuation of the assets is recorded through amounts A exogenous to the accounting system, then the linear aggregate of accounting quantities is governed by the inputs to the given accounting system, and the valuation is reflected through the specific account structures used, however, the valuation will remain unchanged or invariant. The linear aggregate of $100 will always equal $100 in spite of any chart of accounts used.

Let the set of arrayed numbers, representing the linear aggregate of accounting quantities, within a given account structure, be such that its sum total will equal the linear aggregate of an another accounting system. When this condition holds, then the sets of arrayed numbers from the different accounting systems, using the same input transaction amounts A, can be subjected to coordinate transformation, and interrelating their account structures. The explicit formulation of one coordinate system in terms of another can be made available through the specification of coordinate transformation equations.

Sections B and C of this chapter will introduce the concepts of arithmetic n-space, transformation of coordinates, covariant and contravariant tensors, all of which may be relevant to accounting theory.

B. Arithmetic n-space and transformation of coordinates:

The intention here is not to emphasize a semi-Euclidean method of ideally representing measureable magnitudes; but to point out that the idea of magnitude and of perceived dimensions of the geometric tradition can be superseded by the abstract, spatial development of the variable relation-values between points in space. This geometry is partly based on the position of points in space that is not necessarily three-dimensional (a manifold of points), and partly on the analysis of numbers defined through point positions in space. By replacing lengths and magnitudes by positions carries with it a purely spatial and no longer material conception of extension.

In three dimensional space, a point is a set of three values determined by specifying a particular frame of reference or coordinate system. The Cartesian, the cylindrical and the spherical coordinate systems are the most commonly used frame of references. With them, the same point can be expressed in terms of (x, y, z), or (x, r, Θ), or $(r, \Theta, \emptyset)$. Also the three frames of reference are mathematically related to each other by derived sets of equations.

Coordinate systems of more than three dimensions follow by analogy, and may be used in locating an n-dimensional point within a space of n-dimensions.

Any set of objects which can be placed in a one-to-one reciprocal correspondence with an arithmetic n-space, will result in a coordinate system. The one-to-one correspondence between the elements or points of the n-space and the arithmetic n-space used, can be chosen in many ways, and reflects the nature of the problem.

As an example, consider a point P corresponding to the n-tuple $(x^1, x^2, \dots x^n)$,

$$\text{if } y^i = y^i\,(x^1, x^2, \dots x^n) \qquad i = 1, 2, \dots n \quad (15)$$

and assuming that x^i can be solved for, so that

$$x^i = x^i\,(y^1, y^2, \dots y^n) \qquad i = 1, 2, \dots n \quad (16)$$

where y^i and x^i are single valued.

Then the point P can be put into correspondence with the n-tuple $(y^1, y^2, \dots y^n)$. The point P has not changed, but a new method for attaching numbers to the point has been made available. The equation (15) is called a transformation of coordinates system of equations, and results in a new coordinate system being defined.

C. Contravariant and covariant tensors:

The abstract, spatial development of the variable relation-values between points relies in part on the theory of tensor analysis. In this section, the definitions of contravariant and covariant tensors are given, and related to the previous section on coordinate transformations.

In general, any set of n quantities $A^1, A^2, \ldots A^n$ in a coordinate system $(x^1, x^2, \ldots x^n)$ can be related to n other quantities $\bar{A}^1, \bar{A}^2, \ldots \bar{A}^n$ in another coordinate system $(\bar{x}^1, \bar{x}^2, \ldots \bar{x}^n)$ by the transformation equations

$$\bar{A}^p = \frac{\partial \bar{x}^p}{\partial x^q} A^q = \sum_{q=1}^{n} \frac{\partial \bar{x}^p}{\partial x^q} A^q \qquad p = 1, 2, \ldots n \qquad (17)$$

which are defined as the components of a contravariant vector or contravariant tensor of the first rank, or of the first order.

Similarly, n quantities $A_1, A_2, \ldots A_n$ in a coordinate system $(x^1, x^2, \ldots x^n)$ relate to n other quantities $\bar{A}_1, \bar{A}_2, \ldots \bar{A}_n$ in another coordinate system $(\bar{x}^1, \bar{x}^2, \ldots \bar{x}^n)$ by the transformation equations

$$\bar{A}_p = \frac{dx^q}{d\bar{x}^p} A_q = \sum_{q=1}^{n} \frac{dx^q}{d\bar{x}^p} A_q$$

$$p = 1, 2, \ldots n \qquad (18)$$

which are defined as the components of a covariant vector or covariant tensor of the first rank or first order.

The tensor is not just the set of components in one coordinate system, but an abstract quantity which is represented in each coordinate system $(x^1, x^2, \ldots x^n)$ by the set of components A^q or A_q. If, for example, the components of a contravariant tensor are known in one coordinate system, then the components are known in <u>all</u> other allowable systems by the equation (17). The coordinate system does not give a new vector, it changes the components of the same vector; in other words, the contravariant tensor is an invariant under a coordinate transformation (an object of any kind which is not changed by transformations of coordinates is called an invariant).

D. <u>Use of coordinate transformations in accounting</u>:

If accounting measurements are not perceived only in terms of magnitude and dimensions of the geometric tradition, and if it is supplemented by the definition of numbers through point-positions in space, a purely spatial conception of accounting systems may be attempted. Such a formulation would be of major importance to accounting theory.

Accounting entries of the simple type are independent of the particular coordinate system used in describing and aggregating them mathematically. Their invariance permits tensor analysis to be applied to the theory of accounting.

As an example, let $(x^1, x^2, \ldots x^n)$ and $(\bar{x}^1, \bar{x}^2, \ldots \bar{x}^n)$ be coordinates of a point in two different frames of reference, and accepting the existence of n-dimensional space, n independent relationships between the coordinates of the two systems can be set up,

$$x^i = x^i (\bar{x}^1, \bar{x}^2, \ldots \bar{x}^n) \qquad i = 1,2, \ldots n \qquad (19)$$

or,

$$\bar{x}^i = \bar{x}^i (x^1, x^2, \ldots x^n) \qquad i = 1,2, \ldots n \qquad (20)$$

Once the relations are defined, an explicit and mathematically valid methodology becomes available to accounting encompassing the accounting spread sheet, the incidence matrix and the net-work formulation.

The accounting equation (Assets = Equity) mirrors a given accounting entry in more than one coordinate system, and thus it may be expressed in the components of the contravariant or covariant tensors of the first rank.

The amount of the entry is independent of the particular chart of accounts used, or imposed upon it. The structural and functional considerations of the chart of accounts can be distinguished from the purely algorithmic processing of the transaction entries. The application of structural and procedural systems notions to the concepts of information (accounting or other) will be needed in order to create accounting information systems.

VIII. CONCLUSIONS

The thesis drew upon existing knowledge, and added to it by extending the application of computerized accounting systems.

The tangible contributions made to accounting from an information systems perspective are summarized in the first section. A discussion of the systematic development of accounting systems for computers and directions for further research and development conclude the thesis.

A. Summary:

The thesis provided perspective on

i. the relevance of accounting information to organizations, by directing the reader's attention to the behavioral aspects and structural characteristics of the firm, and to the problems of coordination facing an organization;
ii. information systems generated in conformance with the objectives of organizational development;
iii. the characteristics of information and information systems.

The thesis then

i. traced the development of the accounting spread sheet in matrix form;
ii. enhanced the mathematical exposition of matrix

accounting by formulating a new series of mathematical expressions that synthesized the previous work in the area, and extended its (matrix) applicability to computerized accounting information systems;

iii. applied the findings and analysis to a simple computerized accounting system using periodic inventory valuation;

iv. explored the use of tensors and coordinate transformation equations in the field of accounting.

B. <u>Direction for further research:</u>

The systematic development of accounting systems for computers will help

i. bridge the communication gap between the accounting profession and the quantitatively oriented computer specialists;

ii. improve the quality of the documentation available on existing computerized accounting systems;

iii. focus attention of the practitioners to the advantages of using mathematically developed algorithms.

The matrix representation can be extended to a number of areas closely related to the accounting process. Non financial T-matrices can be defined and incorporated with the financial T-matrix. This interface between accounting and management science should be exploited, since it will help clarify their common objectives, and improve the understanding of accounting as an important ingredient of management science.

The use of the matrix representation for accounting systems operating on common input data, can allow the interdependencies between the accounting systems to be mathematically formulated, and computerized. This practical area of research can be applied to existing accounting systems. Finally, explicit and implicit formulation of structural relationships between accounting systems should be attempted. Examples of auch relationships will do much to advance the use of tensors and coordinate transformations.

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<u>Appendix one</u>:

A computer program for the accounting system example

(Please refer to chapter VI, section D)

A A A A A A A A A A A A A

SIMON FRASER UNIVERSITY

COMPUTING CENTRE

MVT REL 17

JUNE 1969

```
STMT LEVEL NEST
                        /* PROGRAM ACCT    - V.MATVEIEF  MARCH 1970  */
   1             A00ACCT:  PROC  OPTIONS (MAIN) ;

                        /* DECLARE INPUT AND OUTPUT FILES */
   2    1               DCL  TRANMST   FILE  OUTPUT ;  /* TRANSACTION TAPE,STREAM I/O */
   3    1               DCL  SYSIN     FILE  INPUT  ;  /* CARD INPUT,STREAM I/O  */
   4    1               DCL  SYSPRINT FILE  OUTPUT ;  /* PRINT OUTPUT,STREAM I/O */

                        /* DECLARE STORAGE AREAS   */
   5    1               DECLARE   FMLB  LABEL ;
   6    1               DECLARE   FMLB2 LABEL ;
   7    1               DCL  ROW_NO (18)           PIC'99' ;
   8    1               DCL  SIGN_DIAG (18)        CHAR(1) ;
   9    1               DCL  NAMES (18)            CHAR(25) ;

  10    1               DCL  DIAG_ELEMENTS(18)     PIC'SZZZZZZZZZZV.99' ;
  11    1               DCL  DV1 (18)              PIC'SZZZZZZZZZZV.99' ;
  12    1               DCL  DV2 (18)              PIC'SZZZZZZZZZZV.99' ;
  13    1               DCL  DV3 (18)              PIC'SZZZZZZZZZZV.99' ;
  14    1               DCL  DATE1                    PIC'999999' ;
  15    1               DCL  DEBIT_ACCOUNT            PIC'99' ;
  16    1               DCL  CREDIT_ACCOUNT           PIC'99' ;
  17    1               DCL  AMOUNT                   PIC'SZZZZZZZZZZV.99' ;
  18    1               DCL  REF                      CHAR(3) ;
  19    1               DCL  EXPLANATION              CHAR(20) ;
  20    1               DCL  DATEAREA                 PIC'999999' ;
  21    1               DCL  INCOME_STMT (18)         PIC'SZZZZZZZZZZV.99' ;
  22    1               DCL  (MATRIX_SIZE, M)         PIC'99' ;
  23    1               DCL (NO_CARDS_DIAG, L )       PIC'99' ;
  24    1               DCL  INITIAL_RUN              CHAR(1) ;
  25    1               DCL  DATE_OF_STMT             PIC'99/99/99' ;

  26    1               DCL  MATRIX(18,18)            PIC'SZZZZZZZZZZV.99' ;
  27    1               DCL  SUM                      PIC'SZZZZZZZZZZV.99' ;
  28    1               DCL  F1              CHAR(77) ;
  29    1               DCL  F2              CHAR(52) ;
  30    1               DCL  F3              CHAR(8) ;
  31    1               DCL  F4              CHAR(35) ;
  32    1               DECLARE  ASSET_TOTAL          PIC'SZZZZZZZZZZV.99'  INIT(0) ;
  33    1               DECLARE  EQUIT_TOTAL          PIC'SZZZZZZZZZZV.99'  INIT(0) ;
  34    1               DECLARE  NET_INCOME           PIC'SZZZZZZZZZZV.99'  INIT(0) ;
  35    1               DCL  ASSET_EQUIT (18)         PIC'SZZZZZZZZZZV.99' ;
  36    1               DCL  CONST                    CHAR(30)  INIT(' ') ;

  37    1               DO I = 1 TO 18 ;
```

```
STMT LEVEL NEST

 38    1    1         ROW_NO (I) = 0 ;
 39    1    1         SIGN_DIAG(I) = ' ' ;
 40    1    1         NAMES(I) = ' ' ;
 41    1    1         DIAG_ELEMENTS(I) = 0 ;
 42    1    1         DV1(I) = 0 ;
 43    1    1         DV2(I) = 0 ;
 44    1    1         DV3(I) = 0 ;
 45    1    1         INCOME_STMT(I) = 0 ;
 46    1    1         ASSET_EQUIT(I) = 0 ;
 47    1    1         END ;
 48    1              DO I = 1 TO 18 ;
 49    1    1         DO J = 1 TO 18 ;
 50    1    2         MATRIX(I,J) = 0 ;
 51    1    2         END ;
 52    1    1         END ;

 53    1              ON ENDFILE (SYSIN)       GOTO A20 ;

                      /* START MAIN PROCEDURE   */
                      /* READ CONTROL CARD      */
 55    1              PUT EDIT    ( '*****************************************' ,
                                    '*       INPUT DATA TO ACCOUNT PROGRAM     *' ,
                                    '* ASSETS AND EQUITIES FOR BALANCE SHEET   *' ,
                                    '*****************************************')
                                   (X(30),A, SKIP(2),X(30),A,SKIP(2),X(30),A,SKIP(2),
                                   X(30),A          ) ;
 56    1              PUT  SKIP(4) ;
 57    1              GET EDIT (MATRIX_SIZE, NO_CARDS_DIAG,INITIAL_RUN,
                                DATE_OF_STMT,F1)  (F(2),F(2),A(1),F(6),A(69));
 58    1              PUT DATA (MATRIX_SIZE, NO_CARDS_DIAG,INITIAL_RUN,
                                DATE_OF_STMT    ) ;
                      /* INITIAL_RUN = 1 FOR CREATING MASTER
                                        BLANK OTHERWISE                        */
 59    1              IF  INITIAL_RUN = ' '  THEN GOTO A05   ;

                      /* READ IN NAMES */
 61    1              PUT  SKIP(4) ;
 62    1              M = MATRIX_SIZE ;
 63    1              L = NO_CARDS_DIAG ;
 64    1              DO  I = 1 TO M  ;
 65    1    1         GET EDIT  ( ROW_NO (I), SIGN_DIAG(I), NAMES(I) , F2)
                                ( F(2) , A(1), A(25) , A(52) ) ;
 66    1    1         PUT DATA  ( ROW_NO (I), SIGN_DIAG(I), NAMES(I))      ;
 67    1    1         PUT  SKIP(1) ;
 68    1    1         END ;
```

```
STMT LEVEL NEST

                                  /* READ IN DIAGONAL ELEMENTS - 6 ELEMENTS ON EACH CARD */
  69    1                         K1 = 1 ;
  70    1                         K2 = 6 ;
  71    1                         DO I= 1 TO L  ;
  72    1     1                   GET EDIT((DIAG_ELEMENTS (K) DO K= K1 TO K2 ) ,F3 )
                                                    ((6) F(12,2)          , A(8) );
  73    1     1                   K1 = K2 + 1 ;
  74    1     1                   K2 = K2 + 6 ;
  75    1     1                   END ;
  76    1                         PUT  SKIP(4) ;
  77    1                         PUT DATA ((DIAG_ELEMENTS(K) DO K = 1 TO M ));

                                  /* PUT DIAGONAL_ELEMENTS INTO MATRIX WITH CORRECT SIGNS */
  78    1                         DO I=1 TO M ;
  79    1     1                   IF SIGN_DIAG (I) = 'N'
  80    1     1                      THEN MATRIX(I,I)= DIAG_ELEMENTS(I) * (-1) ;
  81    1     1                      ELSE MATRIX(I,I)= DIAG_ELEMENTS(I) ;
  82    1     1                   DIAG_ELEMENTS(I)  = MATRIX(I,I) ;
  83    1     1                   END ;
  84    1                         PUT PAGE ;

  85    1           A05        :    /* READ IN TRANSACTION  CARDS   */
                                  GET EDIT (DATE1,DEBIT_ACCOUNT,CREDIT_ACCOUNT,AMOUNT,REF,
                                            EXPLANATION  ,F4 )
                                           ( F(6), F(2), F(2), F(12,2) , A(3), A(20)
                                            ,  A(35) ) ;
  86    1                         PUT DATA (DATE1,DEBIT_ACCOUNT,CREDIT_ACCOUNT,AMOUNT,
                                            EXPLANATION )          ;
  87    1                         PUT  SKIP(2) ;
  88    1                         IF DEBIT_ACCOUNT = CREDIT_ACCOUNT THEN GOTO  A10 ;
  90    1                         CALL BOOWTR_TRAN  ;
  91    1                         MATRIX(DEBIT_ACCOUNT, CREDIT_ACCOUNT) = AMOUNT ;
  92    1                         GOTO A05 ;
  93    1           A10        :
                                  DV1 (CREDIT_ACCCUNT) = AMOUNT ;
  94    1                         CATEAREA = CATE1 ;
  95    1                         GOTO A05 ;
  96    1           A20        :
                                  PUT  EDIT  ( '*****************************************' ,
                                               '**    OUTPUT DATA FROM ACCOUNT PROGRAM   *' ,
                                               '** ASSETS AND EQUITIES FCR BALANCE SHEET  *',
                                               '** INCOME STATEMENT                       *',
                                               '*****************************************')
                                           (PAGE,SKIP(2),X(30),A,SKIP(2),X(30),A,SKIP(2),
                                           X(30),A,SKIP(2),X(30),A,SKIP(2),X(30),A ) ;
  97    1                         PUT  SKIP(4) ;
```

```
STMT LEVEL NEST

  98     1                      PUT DATA ((DV1(I) DO I = 1 TO M ));
  99     1                       CALL COOSUBVAR1 ;
 100     1                       CALL DOOACCT_ALGOR ;
 101     1                      PUT EDIT ( '***** MATRIX AFTER IST ACCOUNTING ALGORITHM *****')
                                          (PAGE, X(30),A          ) ;
 102     1                      PUT  SKIP(4) ;
 103     1                      PUT DATA ((( MATRIX(I,J) DO J=1 TO M) DO I= 1 TO M )) ;
 104     1                      DO I = 1 TO M ;
 105     1     1                INCOME_STMT(I) = DV2 (I) ;
 106     1     1                END  ;
 107     1                      PUT  SKIP(4) ;
 108     1                      PUT DATA (( DV2        (I)  DO I = 1 TO M ));
 109     1                       CALL EOOSUBVAR2 ;
 110     1                       CALL DOOACCT_ALGOR ;
 111     1                      PUT EDIT ( '***** MATRIX  AFTER 2ND ACCOUNTING ALGORITHM *****')
                                          (PAGE, X(30),A          ) ;
 112     1                      PUT  SKIP(4) ;
 113     1                      PUT DATA ((( MATRIX(I,J) DO J=1 TO M) DO I= 1 TO M )) ;
 114     1                      PUT  SKIP(4) ;
 115     1                      PUT DATA ((DV2(I) DO I = 1 TO M ));
 116     1                       CALL FOOSUBVAR3 ;
 117     1                       CALL DOOACCT_ALGOR ;
 118     1                      PUT EDIT ( '***** MATRIX AFTER 3RD ACCOUNTING ALGORITHM *****')
                                          (PAGE, X(30),A          ) ;
 119     1                      PUT DATA ((( MATRIX(I,J) DO J=1 TO M) DO I= 1 TO M )) ;
 120     1                      PUT  SKIP(4) ;
 121     1                      PUT DATA ((DV2(I) DO I = 1 TO M ));
 122     1                       CALL GOOFIN_DIAG ;
 123     1                      PUT EDIT ( '*****  END PRODUCT OF ACCOUNT PROGRAM *****' ,
                                            '*****  ASSETS AND EQUITIES FOR BALANCE SHEET',
                                            '*****' )
                                          ( PAGE, X(30),A, SKIP(2),X(30),A,A ) ;
 124     1                      PUT  SKIP(4) ;
 125     1                      PUT DATA ((DV3(I) DO I = 1 TO M ));
 126     1                       CALL HOOFIN_MATRIX ;
 127     1                      II = 0 ;
 128     1                      DO I = 1 TO M ;
 129     1     1                ASSET_EQUIT(I) = DIAG_ELEMENTS(I) ;
 130     1     1                END ;
 131     1           A30      :
                                ASSET_TOTAL = 0 ;
 132     1                      EQUIT_TOTAL = 0 ;
 133     1                      FMLB = FM1 ;
 134     1                      II = II + 1 ;
 135     1                      IF II = 2 THEN CONST= 'BASED ON T(F)' ;
 137     1                      PUT EDIT ('STATEMENT  OF  FINANCIAL  POSITION  ( AS OF',
                                          'DEC.31.1970)', CONST, 'BALANCE       SHEET' ,
                                          'A S S E T S ($)' , 'E Q U I T I E S ($)'  )
```

```
STMT LEVEL NEST

                                (PAGE, X(30),A,A, X(4), A, SKIP(4),
                                           X(50), A,  SKIP(4) ,
                                           X(27), A, X(48), A ) ;
 138    1                       PUT SKIP(2) ;
 139    1                       PUT EDIT (NAMES(1),ASSET_EQUIT(1), NAMES(6),
                                      ASSET_EQUIT (6) ) (R( FMLB) ) ;
 140    1                       PUT EDIT ( NAMES(2) , ASSET_EQUIT(2) ) ( R (FMLB) ) ;
 141    1                       PUT EDIT ( NAMES(3), ASSET_EQUIT(3), NAMES(7) ,
                                           ASSET_EQUIT(7) ) (R (FMLB) ) ;
 142    1                       PUT EDIT ( NAMES(4), ASSET_EQUIT(4)     ) ( R (FMLB) ) ;
 143    1                       PUT EDIT (NAMES(5), ASSET_EQUIT(5), NAMES(8),
                                           ASSET_EQUIT(8) ) ( R(FMLB)) ;
 144    1                       FMLB= FM2 ;
 145    1                       PUT EDIT ( '_______________' , '_______________' ) (R(FMLB)) ;
 146    1                       DO I = 1 TO 5 ;
 147    1     1                 ASSET_TOTAL = ASSET_TOTAL + ASSET_EQUIT(I) ;
 148    1     1                 END ;
 149    1                       DO I = 6 TO 8 ;
 150    1     1                 EQUIT_TOTAL = EQUIT_TOTAL + ASSET_EQUIT (I) ;
 151    1     1                 END ;
 152    1                       PUT EDIT ( ASSET_TOTAL, EQUIT_TOTAL)
                                      ( SKIP(2), X(40), P'SZZZZZZZZZZV.99',
                                      X(46), P'SZZZZZZZZZZV.99' ) ;
 153    1                       DO I = 1 TO M ;
 154    1     1                 ASSET_EQUIT(I) = DV3(I) ;
 155    1     1                 END ;
 156    1                       IF II= 2 THEN GOTO A35 ;
 158    1                       GOTO A30 ;
 159    1             A35 :
                                 PUT EDIT ( 'STATEMENT  OF  FINANCIAL  POSITION  ( AS OF',
                                      'DEC.31.1970)', CONST, 'INCOME   STATEMENT')
                                      (PAGE, X(30),A,A, X(4),A, SKIP(4) ,
                                           X(50),A, SKIP(4)) ;
 160    1                       FMLB = FM3 ;
 161    1                       FMLB2 = FM4 ;
 162    1                       PUT EDIT ( NAMES(9), INCOME_STMT(9)) (R(FMLB2)) ;
 163    1                       PUT EDIT ( NAMES(11),INCOME_STMT(11)) (R(FMLB)) ;
 164    1                       PUT EDIT ( NAMES(12),INCOME_STMT(12)) (R(FMLB)) ;
 165    1                       PUT EDIT ( NAMES(13),INCOME_STMT(13)) (R(FMLB)) ;
 166    1                       ASSET_TOTAL = INCOME_STMT(11) + INCOME_STMT(12) +
                                INCOME_STMT(13) ;
 167    1                       EQUIT_TOTAL = INCOME_STMT(9) + ASSET_TOTAL ;
 168    1                       NET_INCOME = EQUIT_TOTAL + INCOME_STMT(10) ;
 169    1                       PUT EDIT ('_______________' ) (SKIP(2),X(40),A) ;
 170    1                       PUT EDIT (ASSET_TOTAL) (SKIP(2),X(40),P'SZZZZZZZZZZV.99' ) ;
 171    1                       PUT EDIT ('INCOME PRIOR TO EXTRAORDINARY ITEM', EQUIT_TOTAL)
                                      (SKIP(2), X(10),A, X(56), P'SZZZZZZZZZZV.99' ) ;
 172    1                       PUT EDIT( NAMES (10), INCOME_STMT(10) ) (R(FMLB2) ) ;
```

```
STMT LEVEL NEST

 173     1                     PUT EDIT( '_____________' ) ( SKIP(2), X(100),A) ;
 174     1                     PUT EDIT('NET INCOME', NET_INCOME)
                               (SKIP(3),X(10)    ,A,X(80), P'SZZZZZZZZZZV.99' ) ;
 175     1          FM1        : FORMAT ( SKIP(2), X(10), A, X(5), P'SZZZZZZZZZZV.99',
                                          X(16), A, X(5), P'SZZZZZZZZZZV.99' ) ;
 176     1          FM2        : FORMAT ( SKIP(2), X(40), A, X(46), A) ;
 177     1          FM3       : FORMAT ( SKIP(2), X(10) ,A, X(5), P'SZZZZZZZZZZV.99' ) ;
 178     1          FM4       : FORMAT ( SKIP(2), X(10), A, X(65), P'SZZZZZZZZZZV.99' ) ;


 179     1          BOOWTR_TRAN : PROC ;    /* WRITE A TRANSACTION RECORD */
 180     2                     PUT FILE (TRANMST) EDIT (DATE1,DEBIT_ACCOUNT,CREDIT_ACCOUNT,
                                                  AMOUNT, REF, EXPLANATION)
                                    ( F(6), F(2), F(2), F(12,2) , A(3), A(20)  ) ;

 181     2                     END  BOOWTR_TRAN ;


 182     1          COOSUBVAR1 : PROC ;
                               /* VARIABLE SUBROUTINE 1 -  MATRIX (11,3)  */
 183     2                     MATRIX(11,3) = MATRIX(11,3) + MATRIX(3,3) + MATRIX(3,6)
                                            - DV1(3) ;
 184     2                     AMOUNT        = MATRIX(11,3) ;
 185     2                     DATE1         = DATEAREA      ;
 186     2                     REF           =   '  ' ;
 187     2                     EXPLANATION   =  'COST OF GOODS SOLD' ;
 188     2                     CALL  BOOWTR_TRAN ;

 189     2                     END COOSUBVAR1 ;


 190     1          DOOACCT_ALGOR : PROC  ;
                               /* ACCOUNTING ALGORITHM   */
 191     2                     DO I = 1 TO M ;
 192     2      1              DV2(I) = 0 ;
 193     2      1              END ;
 194     2                     DO I=1 TO M  ;
 195     2      1              SUM= 0 ;
 196     2      1              DO J=1 TO M  ;
 197     2      2              IF I= J  THEN GOTO  D05  ;
 199     2      2              SUM= SUM +  MATRIX(I,J) - MATRIX(J,I) ;
 200     2      2   D05       :END ;
 201     2      1              DV2 (I) = SUM ;
 202     2      1              END ;
 203     2                     END  DOOACCT_ALGOR  ;


 204     1          EOOSUBVAR2 :  PROC ;   /* VARIABLE SUBROUTINE 2  */
```

```
STMT LEVEL NEST

 205     2                MATRIX(9,14) = - DV2(9) ;
 206     2                DATE1 = DATEAREA ;
 207     2                REF    = '    ' ;
 208     2                EXPLANATION = 'CLOSING ENTRY - REV' ;
 209     2                DEBIT_ACCOUNT = 9 ;
 210     2                CREDIT_ACCOUNT= 14 ;
 211     2                AMOUNT = -DV2(9) ;
 212     2                CALL  BOOWTR_TRAN ;


 213     2                MATRIX(10,14)= - DV2(10) ;
 214     2                DEBIT_ACCOUNT = 10 ;
 215     2                AMOUNT = - DV2(10) ;
 216     2                CALL  BOOWTR_TRAN ;


 217     2                MATRIX (14,11) =   DV2(11) ;
 218     2                EXPLANATION = 'CLOSING ENTRY - EXP' ;
 219     2                DEBIT_ACCOUNT = 14 ;
 220     2                CREDIT_ACCOUNT = 11 ;
 221     2                AMOUNT =  DV2(11) ;
 222     2                CALL  BOOWTR_TRAN  ;


 223     2                MATRIX(14,12) =   DV2(12) ;
 224     2                CREDIT_ACCOUNT = 12 ;
 225     2                AMOUNT =  DV2 (12) ;
 226     2                CALL  BOOWTR_TRAN ;


 227     2                MATRIX(14,13) =  DV2(13) ;
 228     2                CREDIT_ACCOUNT= 13 ;
 229     2                AMOUNT =  DV2(13) ;
 230     2                CALL  BOOWTR_TRAN ;


 231     2                END   EOOSUBVAR2 ;


 232     1            FOOSUBVAR3 : PROC ;   /* SUBROUTINE  VARIABLE 3  */
 233     2                MATRIX(14,8) =  - DV2 (14)  ;
 234     2                EXPLANATION  = 'EARNINGS' ;
 235     2                AMOUNT = -DV2 (14) ;
 236     2                CALL  BOOWTR_TRAN ;

 237     2                END FOOSUBVAR3 ;
```

```
STMT LEVEL NEST

 238     1           GOOFIN_DIAG :  /* FINAL VALUES OF DIAGONALS   */ PROC ;
 239     2                  DO I=1 TO M ;
 240     2     1            DV3 (I) = MATRIX (I,I) +  DV2(I) ;
 241     2     1            END ;

 242     2                  END  GOOFIN_DIAG ;

 243     1           HOOFIN_MATRIX :  PROC  ;   /* FINAL MATRIX  */
 244     2                  MATRIX   = 0 ;
 245     2                  DO I=1 TO M ;
 246     2     1            MATRIX (I,I)  = DV3 (I) ;
 247     2     1            END ;

 248     2                  END  HOOFIN_MATRIX ;
 249     1                  END   A00ACCT ;
```

```
*****************************************
*        INPUT DATA TO ACCOUNT PROGRAM     *
* ASSETS AND EQUITIES FOR BALANCE SHEET    *
*****************************************

MATRIX_SIZE=14          NO_CARDS_DIAG=03        INITIAL_RUN='1'         DATE_OF_STMT=31/12/70;

ROW_NO(1)=01            SIGN_DIAG(1)=' '        NAMES(1)='CASH ON HAND                ';
ROW_NO(2)=02            SIGN_DIAG(2)=' '        NAMES(2)='ACCOUNTS RECEIVABLE         ';
ROW_NO(3)=03            SIGN_DIAG(3)=' '        NAMES(3)='INVENTORY                   ';
ROW_NO(4)=04            SIGN_DIAG(4)=' '        NAMES(4)='FURNITURE                   ';
ROW_NO(5)=05            SIGN_DIAG(5)='N'        NAMES(5)='DEPRECIATION - FURN         ';
ROW_NO(6)=06            SIGN_DIAG(6)='N'        NAMES(6)='ACCOUNTS PAYABLE            ';
ROW_NO(7)=07            SIGN_DIAG(7)='N'        NAMES(7)='CAPITAL STOCK               ';
ROW_NO(8)=08            SIGN_DIAG(8)='N'        NAMES(8)='RETAINED EARNINGS           ';
ROW_NO(9)=09            SIGN_DIAG(9)='N'        NAMES(9)='SALES REVENUE               ';
ROW_NO(10)=10           SIGN_DIAG(10)='N'       NAMES(10)='MISC REVENUE               ';
ROW_NO(11)=11           SIGN_DIAG(11)=' '       NAMES(11)='COST OF GOODS SOLD         ';
ROW_NO(12)=12           SIGN_DIAG(12)=' '       NAMES(12)='ADMINISTRATIVE EXP         ';
ROW_NO(13)=13           SIGN_DIAG(13)=' '       NAMES(13)='MISCELLANEOUS EXP          ';
ROW_NO(14)=14           SIGN_DIAG(14)=' '       NAMES(14)='SUMMARY - REV & EXP        ';

DIAG_ELEMENTS(1)=+      4000.00         DIAG_ELEMENTS(2)=+      24000.00        DIAG_ELEMENTS(3)=+      16000.00
DIAG_ELEMENTS(4)=+     15000.00         DIAG_ELEMENTS(5)=+       4000.00        DIAG_ELEMENTS(6)=+      20000.00
DIAG_ELEMENTS(7)=+     35000.00         DIAG_ELEMENTS(8)=+           .00        DIAG_ELEMENTS(9)=+           .00
DIAG_ELEMENTS(10)=+         .00         DIAG_ELEMENTS(11)=+          .00        DIAG_ELEMENTS(12)=+          .00
DIAG_ELEMENTS(13)=+         .00         DIAG_ELEMENTS(14)=+          .00;
```

```
DATE1=010169   DEBIT_ACCOUNT=03   CREDIT_ACCOUNT=06   AMOUNT=+    98000.00   EXPLANATION='MERCHANDISE PJRCHASD';
DATE1=020169   DEBIT_ACCOUNT=13   CREDIT_ACCOUNT=06   AMOUNT=+    19000.00   EXPLANATION='MISC EXP PAYABLE    ';
DATE1=030169   DEBIT_ACCOUNT=01   CREDIT_ACCOUNT=09   AMOUNT=+    45000.00   EXPLANATION='CASH SALES          ';
DATE1=040169   DEBIT_ACCOUNT=02   CREDIT_ACCOUNT=09   AMOUNT=+   127000.00   EXPLANATION='CREDIT SALES        ';
DATE1=050169   DEBIT_ACCOUNT=12   CREDIT_ACCOUNT=01   AMOUNT=+    50900.00   EXPLANATION='SALARIES EXPENSE    ';
DATE1=060169   DEBIT_ACCOUNT=12   CREDIT_ACCOUNT=05   AMOUNT=+     1000.00   EXPLANATION='ACCUM DEPRECIATION  ';
DATE1=070169   DEBIT_ACCOUNT=01   CREDIT_ACCOUNT=04   AMOUNT=+       50.00   EXPLANATION='FURNITURE SOLD      ';
DATE1=070169   DEBIT_ACCOUNT=01   CREDIT_ACCOUNT=10   AMOUNT=+       30.00   EXPLANATION='GAIN ON FURV SOLD   ';
DATE1=070169   DEBIT_ACCOUNT=05   CREDIT_ACCOUNT=04   AMOUNT=+      350.00   EXPLANATION='ACCUM DEP ONFURN DEL';
DATE1=080169   DEBIT_ACCOUNT=01   CREDIT_ACCOUNT=02   AMOUNT=+   118000.00   EXPLANATION='COLLECTION OF CASH  ';
DATE1=090169   DEBIT_ACCOUNT=06   CREDIT_ACCOUNT=01   AMOUNT=+   114100.00   EXPLANATION='PAYMENTS TO SJPPLIER';
                                                                  373,430
DATE1=310169   DEBIT_ACCOUNT=03   CREDIT_ACCOUNT=03   AMOUNT=+    18000.00   EXPLANATION='ENDING INVENTORY    ';
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STATEMENT OF FINANCIAL POSITION (AS OFDEC.31.1969)

BALANCE SHEET

ASSETS ($)			EQUITIES ($)		
CASH ON HAND	+	4000.00	ACCOUNTS PAYABLE	-	20000.00
ACCOUNTS RECEIVABLE	+	24000.00			
INVENTORY	+	16000.00	CAPITAL STOCK	-	35000.00
FURNITURE	+	15000.00			
DEPRECIATION - FURN	-	4000.00	RETAINED EARNINGS	+	.00
	+	55000.00		-	55000.00

STATEMENT OF FINANCIAL POSITION (AS OFDEC.31.1970) BASED ON T(F)

INCOME STATEMENT

SALES REVENUE			-	172000.00
COST OF GOODS SOLD	+	96000.00		
ADMINISTRATIVE EXP	+	51900.00		
MISCELLANECUS EXP	+	19000.00		

	+	166900.00		
INCCME PRICR TO EXTRAORDINARY ITEM			-	5100.00
MISC REVENUE			-	30.00

NET INCCME			-	5130.00

STATEMENT OF FINANCIAL POSITION (AS OFDEC.31.1970) BASED ON T(F)

BALANCE SHEET

A S S E T S ($)			E Q U I T I E S ($)		
CASH ON HAND	+	2080.00	ACCOUNTS PAYABLE	-	22900.00
ACCOUNTS RECEIVABLE	+	33000.00			
INVENTORY	+	18000.00	CAPITAL STOCK	-	35000.00
FURNITURE	+	14600.00			
DEPRECIATION - FURN	-	4650.00	RETAINED EARNINGS	-	5130.00
		------------			------------
	+	63030.00		-	63030.00