A STATISTICAL MODEL FORMULATION FOR POWER SYSTEMS

by

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ABSTRACT

An investigation has been undertaken to ascertain how readily a power system lends itself to statistical modelling. A nonlinear state variable model has been derived in terms of measurable states. This model is linear in its coefficients which are evaluated by the least squares fitting technique of regression analysis. The statistical model's performance is evaluated by comparison of its predicted system responses with those predicted by Park's formulation, and with those produced by a laboratory power system model.

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NOMENCLATURE

Prime Mover

L_{af}	a d.c. motor coefficient; where $\omega_m L_{af}$ is the speed voltage coefficient
r_a	armature resistance
r_s	series resistance in armature circuit
R	total resistance in armature circuit
$\frac{P}{2}$	pole pairs
i_f	field current
i_a	armature current; controls mechanical torque output
ω_m	mechanical speed
T_{dc}	electrical torque in d.c. motor
D_{dc}	d.c. motor damping coefficient

Mechanical System

J	moment of inertia of prime mover - generator set
F	friction coefficient
T_f	torque loss due to friction; $T_f = F\omega_m$

Regulator-Exciter

T_A	regulator time constant
K_{A1}	regulator gain on reference voltage input
K_{A2}	regulator gain on terminal voltage feedback
u_1	regulator-exciter reference voltage

Synchronous Machine

D	synchronous generator damping coefficient
R_f	field resistance
T'_d	d-axis transient short circuit time constant
T'_{do}	d-axis transient open circuit time constant
T''_{do}, T''_{qo}	d- and q-axis subtransient open circuit time constants
x_{ad}	mutual reactance between stator and rotor in d-axis
x_d, x_q	d- and q-axis synchronous reactances
x'_d	d-axis transient reactance
x''_d, x''_q	d- and q-axis subtransient reactance
x_e	equivalent reactance of local load and transmission system ($x'_d + x$)
i_d, i_q	d- and q-axis current
i_{fd}	field current
P	real power output of the machine
Q	reactive power output of the machine
T_e	energy conversion torque of synchronous generator
T_i	mechanical torque on the rotor
v_d, v_q	d- and q-axis voltages
v_t	machine terminal voltage
v_{fd}	field voltage
v_F	a voltage proportional to field voltage
v_{FR}	a voltage proportional to field current
ψ_d, ψ_q	d- and q-axis flux linkages
ψ_{fd}	field flux linkage

ψ_F	flux proportional to field flux linkage
ω	electrical angular speed
ω_o	synchronous speed, 377 rad/sec
$\Delta\omega$	per unit speed variation
δ	torque angle (between q-axis and infinite bus voltage)

Transmission System

r	series resistance
x	series reactance
G	shunt conductance
B	shunt susceptance

Statistical Model

X	independent variable
Y_i	value of dependent variable observed
$\hat{Y}_i$	value of dependent variable predicted by model
$\bar{Y}$	mean value of observations of dependent variable
ε_i	residuals $(Y_i - \hat{Y}_i)$
β	population coefficients
b	sample coefficients
σ^2	population variance
R	multiple regression coefficient

1. INTRODUCTION

Requirements for exact and easily-updated power system models are ever increasing with modern complex systems. One modelling problem arises when attempting to find a detailed representation of multimachine systems. For theoretical models, the analysis is very involved [1], [2] and is limited to only a few machines [2]. Another important problem in modelling a system is updating the model as the system changes. For theoretical models, system parameters are measured off-line and if they change during system operation, for example when a line is lost, the model is no longer exact.

A statistically derived model may find application in dynamic state estimation. Present static state estimation schemes and tracking algorithms for system security assessment [3] - [9] do not require a system model. However, if state estimation techniques are expanded to state prediction for use with local controllers then a system model will no doubt be required. As data is obtained for the estimation scheme, it is convenient to also use this data for statistical derivation of the system model.

A statistical approach to modelling is investigated in this thesis. It is introduced in an attempt to overcome the problems of theoretical models in maintaining an up-to-date system model and possibly to facilitate modelling more complex multimachine systems. A statistical model has the following inherent advantages. It may be set to retain only the most statistically significant system variables, with insignificant variables readily eliminated. Secondly, the statistically-derived equations are determined using the particular system configuration operating as it will be when the model is employed. Also statistical modelling lends itself to real-time updating, thus allowing the mathematical representation of the system to be updated as parameters change, for example, in response to system load changes.

Two ultimate aims of this project are: firstly, to model systems for which accurate theoretical representations are not obtainable; and secondly, to obtain a mathematical modelling scheme which is feasible for on-line modelling and parameter updating. The research reported in this thesis is concerned with the intermediate aim of investigating the proposed statistical scheme using a well defined system in an off-line environment.

This project involves setting up a laboratory model power system, and deriving two mathematical representations of this system. One is a statistical representation which is nonlinear in the state variables, but linear in their coefficients which are statistically estimated. This model constitutes the major portion of original research. The other is a theoretical representation based upon Park's formulation of the synchronous machine equations. Its purpose is to allow a performance comparison of the newly-developed statistical model with the classical theoretical model. Both of these are checked against the laboratory system performance. Another facet of this research is the data acquisition required to collect observations from the laboratory system for use in deriving the statistical model. The basic interface and computer was available but the interface required modification before being used. As the monitored power system signals contain considerable undesired noise, this project also entails signal conditioning and filtering. Data acquisition required development of PDP-8 software as well as extensive data handling and checking programs written for the computing center IBM 360/Model 67.

The organization of the work is as follows. Chapter 2 contains a presentation of the ideas of regression analysis as it is applied to the statistical modelling in this project. In Chapter 3, the two mathematical nonlinear state variable representations of a power system (theoretical and statistical) are developed. The laboratory model power system and the data acquisition system are discussed briefly in Chapter 4. A comparison is made in Chapter 5 of the responses from the statistical model, the theoretical model, and the laboratory system. Testing the data for validity of assumptions is also discussed. Chapter 6 includes the conclusions derived from this work as well as a few guide lines for further investigations.

2. STATISTICAL MODELLING USING REGRESSION ANALYSIS

There are advantages to obtaining experimental models of systems using statistical techniques. Insignificant variables can be detected by various statistical tests, thus yielding a model containing only significant variables. Also, since the model is formed by data acquired from the actual system, the statistical approach lends itself to on-line modelling or on-line updating of the system model. Another advantage of using actual system data is that the models are more readily expressed in terms of measurable states.

Describing a system's behaviour statistically is accomplished by monitoring system performance, and deriving an equation to "best" describe this observed performance. A common mathematically convenient method of determining the "best" equation is to perform a least squares fit to data comprised of measurements of system variables. This technique, which is one method of fitting a line to a set of observations or data points, simply minimizes the sum of squares of the errors. If Y is the dependent variable, which the model will eventually be used to predict, then the error is the distance measured parallel to the Y-axis between the given data point and the fitted line.

Regression analysis is one technique of performing a least squares fit. This method of statistical analysis has been chosen for the thesis partly because computer programs are readily available, but mainly because regression analysis provides many tests for checking system data and for testing the model produced. When choosing a modelling scheme, one must consider that systems are defined to greater and lesser degrees. At one extreme are completely deterministic systems for which all theory, and therefore the model, is completely defined. At the other extreme are the "blackbox" systems for which there exists no theory defining system performance from which a model may be derived. A power system is somewhere mid-way because even though the theory is known from which a model may be derived, the parameters of this model will change as the system operating point and the system configuration change.

Regression analysis can treat any system as a "blackbox" and use data to construct a model by trial and error, but this method requires extensive analysis and may yield a model which allows little or no insight into the physical structure and operation of the system. However, if the form of the system model is constrained such that it has physical meaning for that system, then regression analysis may be used to identify the parameters of this model with much less analysis required than for the trial-and-error blackbox approach. In this thesis the mathematical model is constrained to be of a form derived from theory using Park's representation of a synchronous machine (Chapter 3), and the coefficients in these equations are estimated using regression analysis on measurements of state variables made during system operation.

2.1 Features of Regression Analysis

The basic concepts inherent in regression analysis are now introduced. The theoretical details of this statistical modelling technique are not included because they are not required for the application of regression analysis computer programs. What is required, though, is an understanding of underlying assumptions and of the basic mechanisms of the analysis in order to accurately interpret the results obtained. By way of definition, when concerned with the dependence of a random variable Y on a quantity X which is a variable but not a random variable, an equation that relates Y to X is usually called a regression equation.

Regression analysis is applied to determine the relationship between a dependent variable Y and one or more independent variables X_1, X_2, ... X_n where X_i may be a simple system variable or a function of one or more variables. The analysis uses many measurements of the independent variables and corresponding dependent variable to determine the coefficients in the relationship. For example let a system be described by

$$Y = \beta_o + \beta_1 X_1 + \beta_2 X_2 + \varepsilon , \tag{2.1}$$

then many observations of Y, X_1, and X_2 are subjected to regression analysis to obtain estimates of the linear coefficients β_o, β_1 and β_2. In equation (2.1) ε represents the error the model will make when used to predict Y and, as it is different for each Y observed, it is not measurable. The population coefficients β_o, β_1, and β_2 can not be found exactly without examining all possible Y, X_1 , and X_2 values, however they are estimated in regression analysis by the sample coefficients b_o , b_1 and b_2. The mathematical model obtained then may be written

$$\hat{Y} = b_o + b_1 X_1 + b_2 X_2 \tag{2.2}$$

where $\hat{Y}$ denotes the Y values obtained when using the model for prediction.

The procedure used for regression analysis may be explained for the simple two-variable case by plotting points relating observed values of Y and X on a set of axes. A straight line is drawn through these points such that the sum of the squares of the distances (parallel to the Y-axis) between the points and the line is minimized. The equation of this line then defines the coefficients b_o and b_1. Multiple regression including many independent variables consists of a similar process except the straight line is replaced by hyperplanes in multi-dimensional space.

The choice of independent variables and therefore the form of the model chosen depends upon prior knowledge of the physical system unless a blackbox approach is being used in which case the independent variables are guessed.

2.2 Assumptions in Regression Analysis

In the model $Y_i = \beta_o + \beta_1 X_i + \varepsilon_i$, $i = 1, 2, \ldots n$ describing each measured value of the dependent variable, it is assumed that:

(1) ε_i is a random variable with mean zero and variance σ^2 (unknown), that is, $E(\varepsilon_i) = 0$, $V(\varepsilon_i) = \sigma^2$.

(2) ε_i and ε_j are uncorrelated for $i \neq j$ so that $\mathrm{cov}(\varepsilon_i, \varepsilon_j) = 0$.

Therefore $E(Y_i) = \beta_o + \beta_1 X_i$, $V(Y_i) = \sigma^2$.

and Y_i and Y_j, $i \neq j$ are uncorrelated.

(3) In addition to (1), ε_i is a normally distributed random variable. That is, $\varepsilon_i \simeq N(o, \sigma^2)$.

Therefore ε_i, ε_j are not only uncorrelated, but necessarily independent.

Knowing the assumptions governing the errors (and therefore the data) in regression, one is able to test the model after it is derived to be sure that it adequately explains the behaviour evident in the observed data. Also the data itself may be checked to verify whether or not it does meet the assumptions inherent in this analysis. The lack of fit of the model may be expressed analytically for the simple regression case [10]; however, in the multiple regression which will be used, lack of fit is investigated by plotting the residuals (or ε_i values). Verification that the data meets required assumptions is also accomplished by means of residual plots. The examination of residuals is explained thoroughly in Chapter 3 of Draper and Smith [10].

2.3 Significance of Regression Equation

After the form of the equation is established and regression analysis is used to evaluate the coefficients, the usefulness of the regression equation as a predictor of system performance is checked. This is accomplished by comparing the multiple regression coefficient, R, with tabulated values which give significant R values for the number of variables and the number of observations used. The multiple regression coefficient is defined by

$$R^2 = \frac{\text{sum of squares due to regression}}{\text{sum of squares total}} = \frac{SS_{Reg}}{SS_{Total}} \qquad (2.3)$$

where $$SS_{Reg} = \sum_{i=1}^{n} (Y_i - \bar{Y})^2 \qquad (2.4)$$

$$SS_{Total} = \sum_{i=1}^{n} (\hat{Y}_i - \bar{Y})^2 \qquad (2.5)$$

for which the various Y values are illustrated in Figure 2.1. If R is not greater than the tabulated value for a desired level of significance, then the regression model is not useful because Y could just as well be described by its mean value $\bar{Y}$.

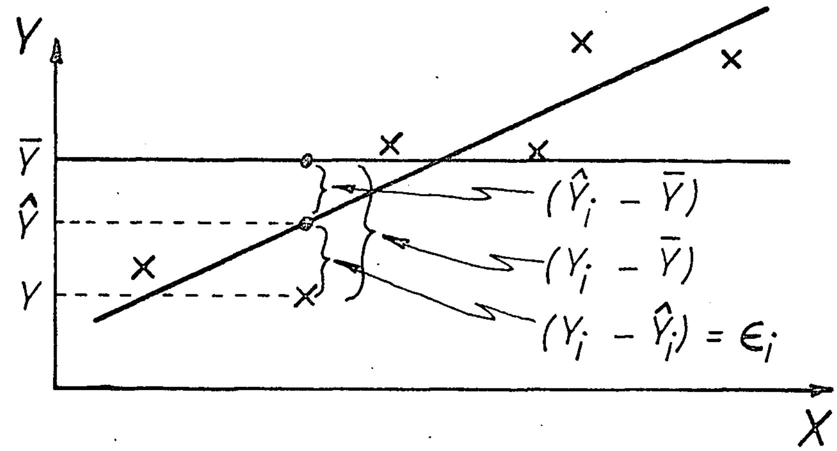


Figure 2.1 Example of Regression Model

2.4 Significance of Coefficients

When multiple regression is employed to identify coefficients in an assumed model, it is possible to have this analysis omit any independent variable or combination of independent variables which are found to be insignificant in the data sample from which the model is being derived. This is done by using an F-test to check the statistical significance of the coefficients.

For example in the model

$$\hat{Y} = b_o + b_1X_1 + b_2X_2 \tag{2.6}$$

the question of whether $b_1 = 0$ or not may be answered by investigating two models, one including b_1X_1 and one omitting that term. That is, consider equations (2.6) and (2.7)

$$\hat{Y} = b_o + b_2X_2 \tag{2.7}$$

and measure the contribution of b_1 as though it were added to the model last. This entails the use of a partial F-test for b_1. The partial F-test which is outlined in detail in Chapter 2 of Draper and Smith [10] involves finding the difference of the sums of squares due to regression in models (2.6) and (2.7). This type of F-test is used during the building up procedure for a regression model to omit statistically insignificant variables from the resulting model.

2.5 Regression Analysis in Power Systems

To obtain a physically meaningful system model, regression analysis will be used only to identify coefficients in an assumed form of a power system model. The form of this statistical model is outlined in Chapter 3 after development of Park's formulation on which it is based.

In using regression analysis to evaluate the coefficients of a discrete state variable model, the dependent variable is chosen to be the particular state considered at time t_{k+1} and the independent variables are the states and functions of states as defined by the form of the model at time t_k, where the measurements of state variables are acquired at the uniform interval of $t_{k+1} - t_k = \Delta t$. When developing the model, the state variables are measured and subjected to the regression analysis which, by least squares fitting to the acquired data, estimates the linear coefficients in the nonlinear state variable equations. To obtain a system model consisting of four state variable equations, four separate regression analyses are required.

3. MATHEMATICAL MODELS - THEORETICAL AND STATISTICAL

A theoretical state variable model for a one-machine infinite bus power system is derived. The model consists of a third order representation of the synchronous machine approximated from Park's equations, and a first order voltage regulator. Based on this formulation, the form of the statistical state variable model including only measurable states is derived.

Initial development of Park's representation [11] closely follows the development as outlined by Vongsuriya [12] and Dawson [13], except that the torque angle, δ , is defined as the angle between the q-axis and the infinite bus or reference voltage at the beginning of the derivation. Other deviations from the references quoted include an approximation of the synchronous machine electrical damping to compensate for neglecting amortisseur winding effects in Park's representation and the derivation of a damping expression for the d.c. machine.

A third order machine representation was chosen for two reasons. Firstly, Dawson [13] concluded that for many system studies (except subtransient and switching phenonmenon) a third order representation is sufficient. Secondly, to find higher order models statistically requires much faster system sampling and thus much more data acquisition apparatus than that readily available for this project.

3.1 Synchronous Machine State Variable Equations

Detailed derivations of Park's equations are numerous. As a third order representation is used, the derivation starts with the third order approximation of Park's equations [12], [13]. To obtain this simplified form of Park's model, the following assumptions are made.

(1) Subtransient time constants are neglected.

(2) The induced voltages and the voltages due to speed variations are neglected because they are small compared to the speed voltages due to cross excitations.

(3) The relatively small d-axis damper leakage time constant and armature resistance are neglected.

These assumptions reduce Park's equations for a synchronous machine in d-q coordinates to:

$$v_d = -\psi_q \omega_o \tag{3.1}$$

$$v_q = \psi_d \omega_o \tag{3.2}$$

$$\psi_d = \frac{x_{ad}}{\omega_o R_F} \frac{v_{fd}}{(1 + T'_{do}p)} - \frac{x_d(1 + T'_d p)}{\omega_o(1 + T'_{do}p)} i_d \tag{3.3}$$

$$\psi_q = -\frac{x_q}{\omega_o} i_q \tag{3.4}$$

The mechanical behaviour of the machine is expressed by the torque equation:

$$T_i = Jp^2\delta + Dp\delta + T_e \tag{3.5}$$

and the expression for the rotor angle:

$$\theta = \omega_o t + \delta \tag{3.6}$$

Equation (3.6) is represented in Figure 3.1 where the rotating reference is chosen to coincide with the infinite bus voltage phasor.

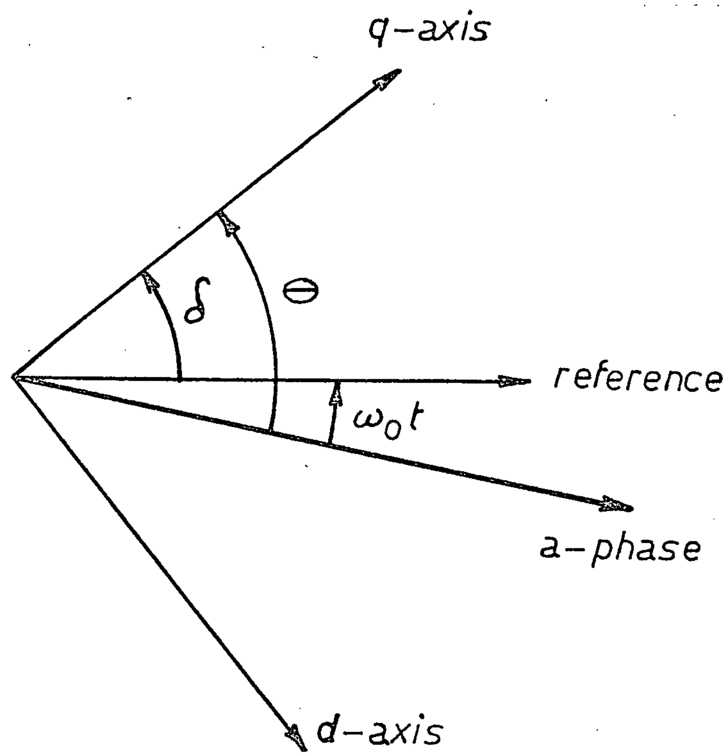


Figure 3.1 Rotor Angular Position

From equations (3.1) to (3.4) and (3.5) and (3.6) the synchronous machine dynamics can be expressed in terms of one electrical and two mechanical state variable equations. According to the development in Appendix 3A, the state variable formulation of the machine dynamics may be written as:

$$p\psi_F = v_F - v_{FR} \tag{3.7}$$

$$p\delta = \omega_o \Delta\omega \tag{3.8}$$

$$p\Delta\omega = \frac{1}{J\omega_o}(T_i - Dp\delta - T_e). \tag{3.9}$$

Auxiliary equations required include the energy conversion torque

$$T_e = i_q\psi_d - i_d\psi_q \quad , \tag{3.10}$$

the terminal voltage

$$v_t^2 = v_d^2 + v_q^2 \quad , \tag{3.11}$$

the power and reactive power output

$$P = v_d i_d + v_q i_q \tag{3.12}$$

and

$$Q = v_q i_d - v_d i_q \ . \tag{3.13}$$

Also required are equations to evaluate v_{FR}, i_d and i_q . These can be solved from equation (3.14) which is formed by combining equations (3A.7), (3A.2), and (3.4).

$$\begin{bmatrix} \psi_F \\ \psi_d \\ \psi_q \end{bmatrix} = \begin{bmatrix} T'_{do} & -T'_{do}(x_d - x'_d) & 0 \\ 1/\omega_o & -x_d / \omega_o & 0 \\ 0 & 0 & -x_q/\omega_o \end{bmatrix} \cdot \begin{bmatrix} v_{FR} \\ i_d \\ i_q \end{bmatrix} \tag{3.14}$$

Measurable States

In defining the form of the statistical model based on Park's formulation, it is required that all states be measurable. It may be shown that the immeasurable state, ψ_F, may be replaced by the measurable field current, i_{fd}, as follows. From equations (3.4) and (3A.11)

$$\psi_F = T'_{do} x_{ad} i_{fd} - T'_{do}(x_d - x'_d) i_d \tag{3.15}$$

and

$$i_d = \frac{1}{T'_{do} x'_d} \psi_F - \omega_o \frac{1}{x'_d} \psi_d \tag{3.16}$$

which gives ψ_F in terms of i_{fd}, i_d and i_q. Equations (3.15) and (3.16) are expanded further using auxiliary equations when outlining the form of the statistical model.

Torque

From d.c. machine theory [14] the torque developed by a shunt d.c. motor is described as

$$T_{dc} = k_v i_a \tag{3.17}$$

where [15]

$$k_v = \frac{P}{2} L_{af} i_f . \tag{3.18}$$

The torque input to the synchronous machine, T_i, in equations (3.5) and (3.9) is equal to the electrical conversion torque of the d.c. machine in equation (3.17) minus the mechanical torque loss in the system. Mechanical torque loss $T_f = F(\omega_m) \cdot \omega_m$ is determined experimentally by evaluating the friction term, F, where

$$F = \frac{T_{dc}}{\omega_m} = \frac{P}{2} \frac{L_{af} i_a i_f}{\omega_m} \tag{3.19}$$

when using the d.c. motor prime mover to rotate the synchronous generator (with no load) at various speeds near synchronous speed [15]. Then,

$$T_i = T_{dc} - T_f . \tag{3.20}$$

Damping

The description of the damper winding circuits for the synchronous generator is not included in the machine equations for the third order representation. However, the damping effect may be approximated [16], [17], [18] by

$$D(\delta) = D_1 \sin^2\delta + D_2 \cos^2\delta \tag{3.21}$$

where $$D_1 = v_o^2 \frac{(x_d' - x_d'')}{(x_e + x_d')^2} T_{do}'' \tag{3.22}$$

$$D_2 = v_o^2 \frac{(x'_q - x''_q)}{(x_e + x'_q)^2} T''_{qo} \quad . \tag{3.23}$$

The d.c. motor simulating the prime mover has an electrical damping which is dependent on the change of torque with motor speed, and may be determined as follows. For a d.c. shunt motor [14]

$$T_{dc} = k_v i_a \tag{3.17}$$

$$e = k_v \omega_m \tag{3.24}$$

and in the armature circuit shown in Figure 3.2,

$$v = R i_a + e. \tag{3.25}$$

Substituting gives

$$v = \frac{R}{k_v} T_{dc} + k_v \omega_m. \tag{3.26}$$

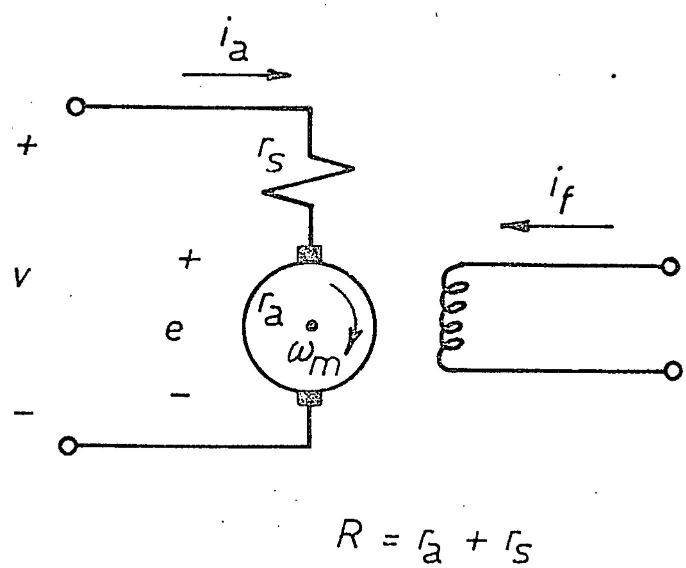


Figure 3.2 D.C. Shunt Motor Circuit

Assuming constant applied voltage, differentiating equation (3.26) yields

$$\frac{R}{k_v} dT_{dc} = - k_v d\omega_m \ . \tag{3.27}$$

Hence the d.c. motor damping defined as

$$D_{dc} = \frac{dT_{dc}}{d\omega_m} \tag{3.28}$$

may be expressed as

$$D_{dc} = \frac{- k_v^2}{R} \ . \tag{3.29}$$

3.2 Voltage Regulator-Exciter Equation

A voltage regulator-exciter was modelled by an amplidyne as shown in Figure 3.3. Assuming a single time constant representation, the amplidyne equation appears as

$$\frac{v_{fd}}{v_i} = \frac{K}{1 + T_A p} \tag{3.30}$$

where

$$K v_i = K_{A1} u_1 - K_{A2} v_t \ . \tag{3.31}$$

Therefore,

$$p v_{fd} = \frac{1}{T_A} (-v_{fd} + K_{A1} u_1 - K_{A2} v_t) \tag{3.32}$$

describes the exciter in state variable form where the control, u_1, is the exciter reference voltage. This then produces a fourth order model for the synchronous generator and its voltage regulator.

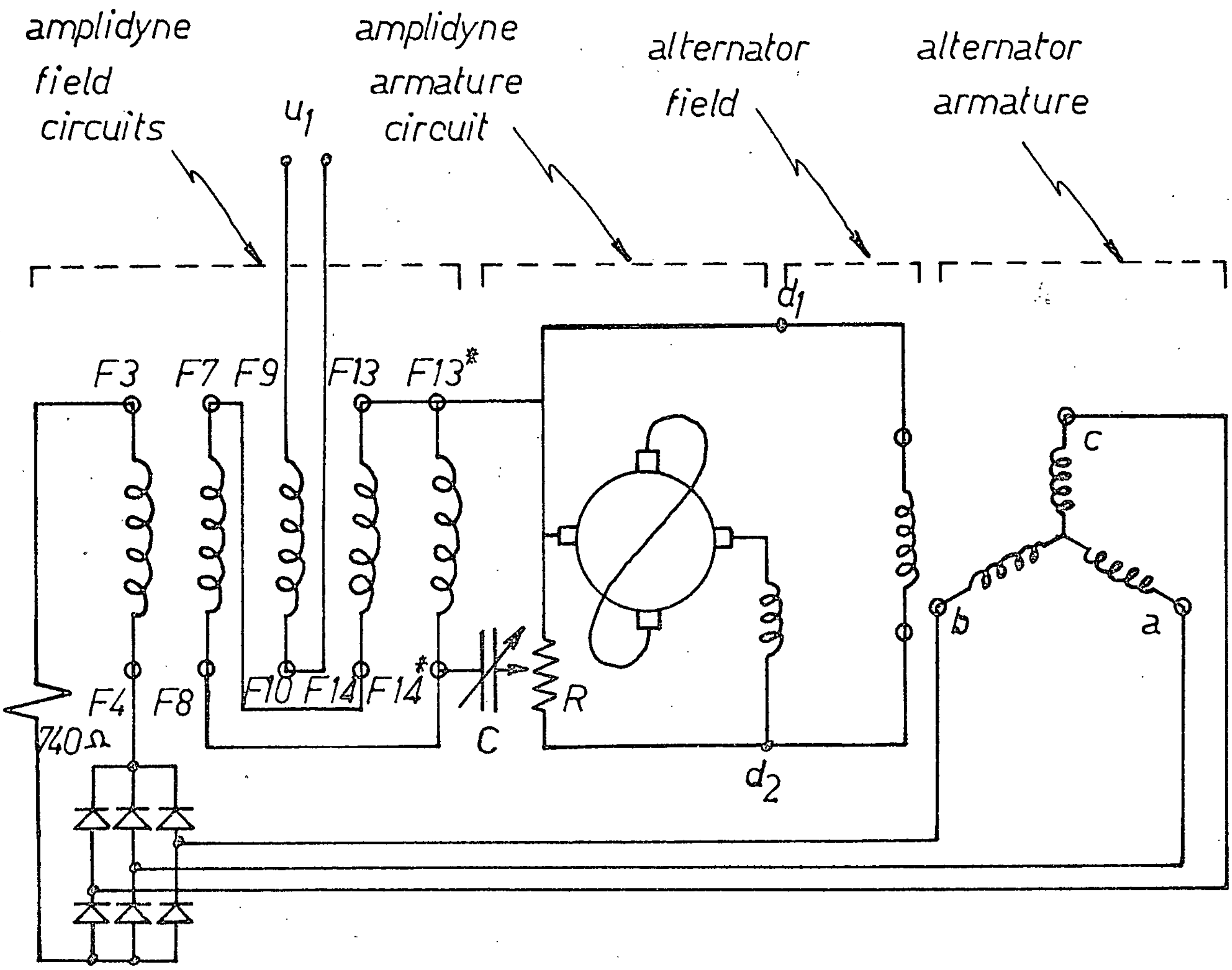


Figure 3.3 Voltage Regulator - Exciter

Rating of Field Circuits

coil label	# of turns	resistance at 25° C	maximum current
F3 - F4	1780	980	0.12
F7 - F8	390	43	0.6
F9 - F10	85	2.6	2.2
F13 - F14	400	56	0.5

3.3 One-machine Infinite Bus System

The power system modelled is shown schematically in Figure 3.4.

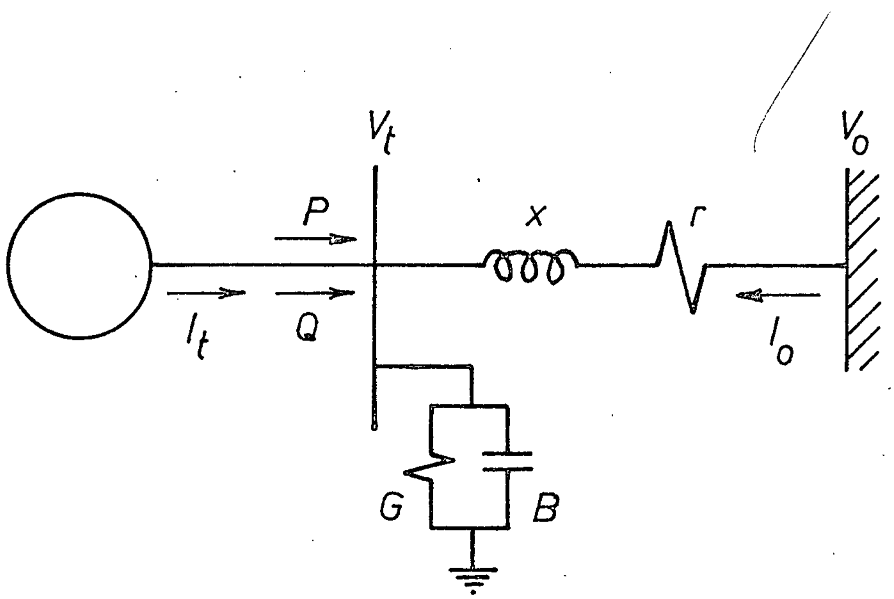


Figure 3.4 Power System Schematic

A state variable description of the generator and its exciter is expressed by equations (3.7) to (3.9) and (3.32) along with their associated auxiliary equations. The transmission system may be described by the external voltage and current relationships

$$[I] = [Y]\,[V]\,. \tag{3.33}$$

The q-axis position has been described by equation (3.6) as

$$\theta = \omega_o t + \delta \tag{3.6}$$

which is demonstrated in Figure 3.1.

The external system and the machine must be referred to a common reference in order to derive physical system quantities from the state variable model outlined. Expression of external system quantities in terms of d-q coordinates is presented in Appendix 3B using Park's transformations [12], to yield the following result.

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \begin{bmatrix} k_1 & k_2 \\ -k_2 & k_1 \end{bmatrix} \cdot \begin{bmatrix} v_o \sin\delta \\ v_o \cos\delta \end{bmatrix} + \begin{bmatrix} K_1 & K_2 \\ -K_2 & K_1 \end{bmatrix} \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} \tag{3.34}$$

where

$$k_1 = \frac{1 + rG - xB}{(1 + rG - xB)^2 + (xG + rB)^2} \tag{3.35}$$

$$k_2 = \frac{xG + r\,B}{(1 + rG - xB)^2 + (xG + rB)^2} \tag{3.36}$$

$$K_1 = k_1 r + k_2 x \tag{3.37}$$

$$K_2 = -k_1 x + k_2 r \;. \tag{3.38}$$

An expression for v_d and v_q is available in terms of i_d and i_q and state variables (equation 3.34). Also i_d and i_q may be expressed in terms of ψ_d and ψ_q and state variables (equation 3.14). Therefore an additional expression is required giving ψ_d and ψ_q in terms of state variables only, thus allowing all auxiliary variables to be evaluated at any time knowing the state of the system at that time. The development in Appendix 3C yields the following desired result.

$$\begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \cdot \begin{bmatrix} v_o \sin\delta \\ v_o \cos\delta \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \cdot \begin{bmatrix} \psi_F \end{bmatrix} \tag{3.39}$$

where

$$M_1 = \frac{\omega_o}{\Delta} \left[\frac{K_1 k_1}{x_q} + (K_2 \frac{1}{x_q} - 1) k_2 \right] \tag{3.40}$$

$$M_2 = \frac{\omega_o}{\Delta} \left[\frac{K_1 k_2}{x_q} - (K_2 \frac{1}{x_q} - 1) k_1 \right] \tag{3.41}$$

$$M_3 = \frac{\omega_o}{\Delta} [- \frac{K_1 k_2}{x_d'} + (K_2 \frac{1}{x_d'} - 1)k_1] \tag{3.42}$$

$$M_4 = \frac{\omega_o}{\Delta} [\frac{K_1 k_1}{x_d'} + (K_2 \frac{1}{x_d'} - 1)k_2] \tag{3.43}$$

$$N_1 = \frac{\omega_o}{\Delta T_{do}' x_d'} [\frac{K_1^2}{x_q} + (K_2 \frac{1}{x_q} - 1)K_2] \tag{3.44}$$

$$N_2 = \frac{\omega_o}{\Delta T_{do}' x_d'} [- \frac{K_1 K_2}{x_d'} + (K_2 \frac{1}{x_d'} - 1)K_1] \tag{3.45}$$

$$\Delta = \frac{K_1^2 \omega_o^2}{x_q x_d'} + \omega_o^2 \frac{(x_d' - K_2)(x_q - K_2)}{x_q x_d'} . \tag{3.46}$$

Therefore from equations (3.14), (3.34) and (3.39), the auxiliary system variables may be found.

The initial states of a power system defined by v_d, v_q, i_d, i_q, v_o, and δ are determined from the steady-state operating conditions as outlined by Vongsuriya [12]. This operating point is described by the machine terminal voltage, v_t, the real power, P, and reactive power, Q, output of the machine.

3.4 Formulation of Statistical Model

To obtain a physically meaningful system model, regression analysis is used to evaluate coefficients in an assumed form of a power system model. This form is derived using Park's formulation just outlined. It must, however, be in terms of measurable state variables so that as the system is operating the data acquisition system can record observations of these state variables directly. If the model were in terms of immeasurable state variables, then theortical auxiliary equations would be required to obtain the state variable value from the measurements; thus resulting in a model which no longer has experimentally determined coefficients. Because the infinite bus voltage, v_o, is measurable and is controllable on the laboratory system, it is expressed explicitly in the equations defining the form of the statistical model. This allows the model to be used to predict system response to a voltage dip from a neighbouring system.

Choice of State Variables

For the theoretical model developed, the choice of ψ_F, δ, $\Delta\omega$, and v_{fd} as state variables is convenient both for the state space model derivation and for evaluation of initial conditions. Before using this model to define the form of the statistical model, ψ_F must be replaced by a measurable state.

In section 3.1 it is stated that i_{fd} may be chosen to replace ψ_F as a state variable thus giving a model in terms of measurable states only. The derivation consists of substituting equation (3.16) into (3.15) and eliminating ψ_d using

$$\psi_d = M_1 v_o \sin\delta + M_2 v_o \cos\delta + N_1 \psi_F \tag{3.47}$$

from equation (3.39). The resulting expression is

$$\psi_F = F_1 i_{fd} + F_2 (M_1 v_o \sin\delta + M_2 v_o \cos\delta) \tag{3.48}$$

where

$$F_1 = \frac{T'_{do} x'_d x_{ad}}{x_d - \omega_o T'_{do}(x_d - x'_d) N_1} \tag{3.49}$$

$$F_2 = \frac{\omega_o T'_{do}(x_d - x'_d)}{x_d - \omega_o T'_{do}(x_d - x'_d)N_1} . \tag{3.50}$$

Differentiating equation (3.48) yields

$$p\psi_F = F_1 pi_{fd} + \omega_o F_2(M_1 v_o \cos\delta\Delta\omega - M_2 v_o \sin\delta\Delta\omega). \tag{3.51}$$

Substituting equations (3.48) and (3.51) into (3.7) gives a state equation for the measurable state variable i_{fd} as

$$pi_{fd} = \frac{1}{F_1}[-x_{ad}i_{fd} + \frac{x_{ad}}{R_F}v_{fd}$$

$$- \omega_o F_2(M_1 v_o \cos\delta\Delta\omega - M_2 v_o \sin\delta\Delta\omega)]. \tag{3.52}$$

The torque expressions in equation (3.9) also require expansion in terms of measurable quantities. From equation (3.17), $T_i \propto i_a$. From equation (3.10)

$$T_e = i_q\psi_d - i_d\psi_q \tag{3.10}$$

where i_d and i_q are expressed in terms of ψ_d and ψ_q by equation (3.14) so that

$$T_e = -\omega_o \frac{1}{x_q}\psi_q\psi_d - \frac{1}{T'_{do}x'_d}\psi_F\psi_q + \omega_o \frac{1}{x'_d}\psi_d\psi_q . \tag{3.53}$$

Substituting equations (3.39) and (3.48) into (3.53) gives

$$T_e = A_1 i_{fd}^2 + A_2 i_{fd} v_o \sin\delta + A_3 i_{fd} v_o \cos\delta$$

$$+ A_4 v_o^2 \sin\delta\cos\delta + A'_5 v_o^2 \sin^2\delta + A'_6 v_o^2 \cos^2\delta \tag{3.54}$$

where the coefficients are not expanded because their values are not required when defining the form of the statistical model. From trigonometry

$$\cos^2\delta = 1 - \sin^2\delta \tag{3.55}$$

which reduces (3.54) to

$$T_e = A_1 i_{fd}^2 + A_2 i_{fd} v_o \sin\delta + A_3 i_{fd} v_o \cos\delta + A_4 v_o^2 \sin\delta\cos\delta + A_5 v_o^2 \sin^2\delta. \qquad (3.56)$$

This formulation leads to additional nonlinearities due to products of state variables appearing in the expression. It is nevertheless an acceptable form of solution for the statistical modelling scheme used here and is readily handled by forming the desired product at each sampling time and then obtaining the desired linear coefficients by regression analysis.

The form of the model to be exploited, then, is defined by equations (3.8), (3.9), (3.32), (3.52) and auxiliary equations (3.17) and (3.56). The difference equation counterparts of these differential state equations are used because the discrete sampling environment of the data acquisition system more easily facilitates a difference equation representation. In the pertinent state equations there are no intercept terms present. However for the regression analysis, the intercept b_o is included. To assume that statistically $b_o = 0$ requires considerable investigation. This intercept value is thus retained in all the regression equations for this project to account for effects of measurement offsets, even though it is desired that b_o in fact equal zero. Maintaining the offset term, b_o, and expressing v_o explicitly, the discrete state variable equations for regression analysis are as follows:

$$i_{fd}(k+1) = b_{10} + b_{11} i_{fd}(k) + b_{12} v_{fd}(k) + b_{13} v_o(k)\cos\delta(k)\Delta\omega(k) + b_{14} v_o(k)\sin\delta(k)\Delta\omega(k) \qquad (3.57)$$

$$\delta(k+1) = b_{20} + b_{21}\delta(k) + b_{22}\Delta\omega(k) \tag{3.58}$$

$$\begin{aligned}\Delta\omega(k+1) = \; & b_{30} + b_{31}\Delta\omega(k) + b_{32}v_o^2(k)\sin^2\delta(k) \\ & + b_{33}i_{fd}^2(k) + b_{34}v_o^2(k)\sin\delta(k)\cos\delta(k) \\ & + b_{35}i_{fd}(k)v_o(k)\sin\delta(k) + b_{36}i_{fd}(k)v_o(k)\cos\delta(k) \\ & + b_{37}i_a(k)\end{aligned} \tag{3.59}$$

$$v_{fd}(k+1) = b_{40} + b_{41}v_{fd}(k) + b_{42}u_1(k) + b_{43}v_t(k). \tag{3.60}$$

Auxiliary Equation

A major consideration in this modelling involves the non-linearities in equation (3.60) which arise when substituting for v_t. The terminal voltage, v_t, is measurable and thus can be used to evaluate the coefficients in equation (3.60). However, when the model is used as a predictor, an expression is required to express v_t as a function of state variables, that is, $v_t = f(i_{fd}, \delta, \Delta\omega, v_{fd})$.

From the terminal condition expressed by equation (3.11)

$$v_t^2 = v_d^2 + v_q^2. \tag{3.11}$$

Substitution for v_d and v_q from equations (3.1) and (3.2) gives

$$v_t^2 = \psi_q^2\omega_o^2 + \psi_d^2\omega_o^2 \tag{3.61}$$

and a further substitution of equation (3.39) gives

$$\begin{aligned}v_t^2 = \; & \omega_o^2(M_1v_o\sin\delta + M_2v_o\cos\delta + N_1\psi_F)^2 \\ & + \omega_o^2(M_3v_o\sin\delta + M_4v_o\cos\delta + N_2\psi_F)^2.\end{aligned} \tag{3.62}$$

Expressing ψ_F in equation (3.62) by equation (3.48) and substituting equation (3.55) yields an expression of the form

$$\begin{aligned} v_t^2 &= B_1 v_o^2 \sin\delta + B_2 i_{fd}^2 + B_3 v_o^2 \sin\delta\cos\delta \\ &\quad + B_4 i_{fd} v_o \sin\delta + B_5 i_{fd} v_o \cos\delta \end{aligned} \tag{3.63}$$

where the values of the coefficients are not required for defining the form of the statistical model. Including an offset term, the desired auxiliary regression equation is found to be

$$\begin{aligned} v_t^2(k) &= b_{50} + b_{51} v_o^2(k)\sin^2\delta(k) + b_{52} i_{fd}^2(k) \\ &\quad + b_{53} v_o^2(k)\sin\delta(k)\cos\delta(k) \\ &\quad + b_{54} i_{fd}(k) v_o(k)\sin\delta(k) \\ &\quad + b_{55} i_{fd}(k) v_o(k)\cos\delta(k). \end{aligned} \tag{3.64}$$

If this auxiliary equation is substituted directly into equation (3.60) then equation (3.60) is no longer linear in its coefficients. However, because v_t is a measurable quantity, equation (3.64) may be treated as an auxiliary equation and regression analysis will estimate the linear coefficients. When the model is used as a predictor, v_t^2 may be found at each time desired and its square root used in equation (3.60).

Equations (3.57) to (3.60) and (3.64) then describe the form of the state variable model of the system with all equations linear in their coefficients. Regression analysis is used to estimate the unknown linear coefficients in each of the five equations, thus yielding a state variable nonlinear system model. This model is tested in Chapter 5 by investigating it statistically as well by comparing its performance with that of the theoretical model and the laboratory system.

4. LABORATORY POWER SYSTEM AND DATA ACQUISITION SYSTEM

A laboratory model of a one-machine infinite bus power system has been assembled. This model is used not only to check the responses predicted by both Park's model and the statistical model but also to supply the data required in producing the statistical model. This latter function requires considerable measurement and data acquisition apparatus.

The laboratory model consists of a four pole d.c. motor simulating a prime mover and driving a small six pole three-phase synchronous generator. An inductive three-phase balanced transmission system connects the machine terminals to the infinite bus. Voltage regulation is accomplished using an amplidyne in the exciter circuit, with a reference voltage on one input of the amplidyne and feedback from the generator terminal voltage on another input coil (Figure 3.3). To collect observations for the statistical modelling, a computerized data acquisition system is employed. The computer interface allows measurement of a number of analog signals through a multiplexer and an analog-to-digital (A/D) converter. An optical shaft-angle encoder monitors the shaft position, from which shaft speed may be derived by differentiation. The central processor used in the data acquisition is a Digital Equipment Corporation (DEC) PDP-8/L.

The following three sections supply further details for the power system laboratory model, the data acquisition hardware and the data acquisition software.

4.1 Power System Laboratory Model

Figure 4.1 illustrates the laboratory system configuration. Specifications for the d.c. motor, the synchronous generator and the amplidyne are displayed in Table 4.1.

Table 4.1

Laboratory Model Machine Specifications

Machine	Specification
Synchronous Generator output:	5 KVA 220 volts 13 amps 90% pf 60 Hz 1200 rpm
exciter:	3.2 amps 125 volts
D.C. Motor	5.6 Kw 115 volts 56 amps 850/1200 rpm
Amplidyne input:	220/440 volts 7.2/3.6 amps 3-phase - 60 Hz 1725 rpm
output:	1.5 Kw 125 volts 12 amps

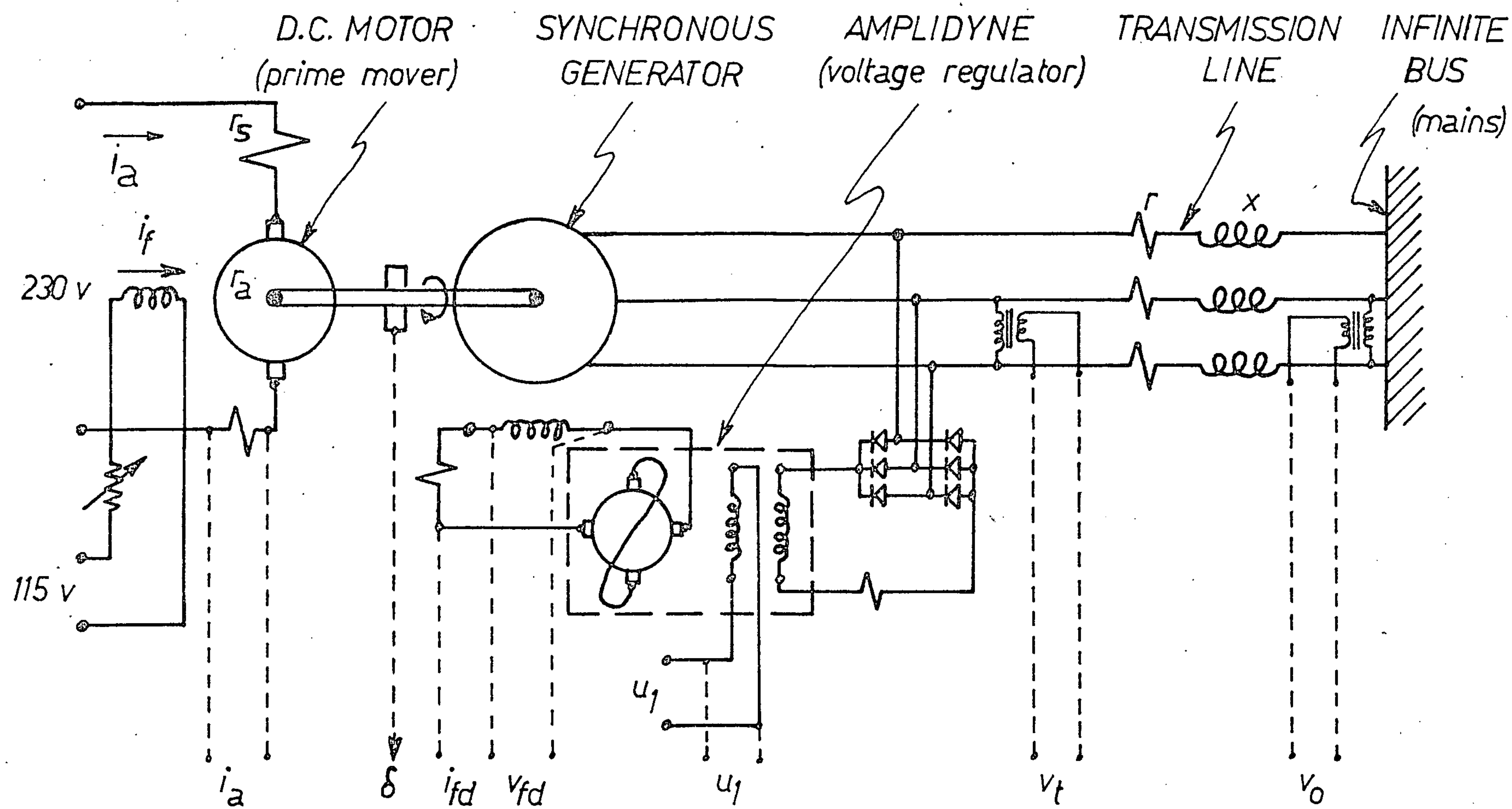


Figure 4.1 Laboratory System Configuration

Notes - broken lines indicate measurement points.
- for amplidyne detail see Figure 3.3.

Amplidyne

In an attempt to minimize undesired system noise an amplidyne, rather than an available SCR exciter, was chosen to model the voltage regulator exciter circuit. However, it appears that amplidyne-induced noise is plentiful and is in fact more difficult to filter than short rise time peaks induced by the SCR exciter.

The regulator-exciter circuit was designed with a relatively long open-loop time constant, T_A, as seen in Table 4.2. This was done because shorter time constants, of the magnitude of the generator open circuit time constant, cause the exciter-generator combination to be unstable.

D.C. Motor

An attempt is made to decrease the inherent damping of the d.c. motor, thus making it more realistically model a prime mover, typically with low p.u. damping. From the analysis of Chapter 3 it is seen that d.c. motor damping is inversely proportional to the resistance in the armature circuit as described by:

$$D_{dc} = \frac{-k_v^{\,2}}{R} . \qquad (3.29)$$

Therefore a series resistance is placed in the armature circuit to increase its total resistance, R. Then 230 volts d.c. is applied to the circuit to maintain approximately 115 volts across the armature and thus maintain correct armature current. Input torque to the generator is controlled by the armature current in the d.c. motor. Torque is then evaluated according to equation (3.17), that is,

$T_{dc} = k_v i_a$. The advantage of this control over using field current control is that fluctuations of field current in turn cause changes in armature current requiring that both be monitored to evaluate torque.

System Parameters

Synchronous machine electrical parameters required for the theoretical model based on Park's representation are determined using standard techniques as outlined in Chapter 7 of IEEE Test Code [19]. Table 4.2 shows the measured values for the system parameters. They are displayed in p.u. as well as in MKS units although MKS units have been used throughout this thesis. The base values used are: 125 volts r.m.s., 8 amps r.m.s., and 377 radians/sec.

The d.c. machine coefficient L_{af} is found from a plot of the speed voltage coefficient, $\omega_m L_{af}$, which is defined as open circuit voltage/field current [15].

The system inertia, J, is evaluated using the retardation test. The friction damping term, F, is evaluated using equation (3.19) [15]

$$F = \frac{P}{2} \frac{L_{af} i_a i_f}{\omega_m} \tag{3.19}$$

by operating the synchronous generator at no load and driving it with the d.c. motor at various speeds near synchronous speed.

A simplified single time constant model is used for the amplidyne and the time constant is found by monitoring amplidyne output for a step input. The steady state gain of each input is also found from measured input and output voltages near the estimated operating point.

Table 4.2

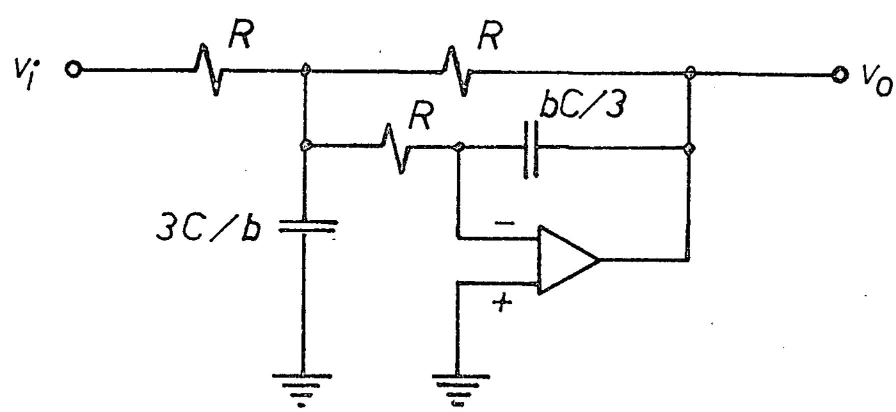
Laboratory System Parameters

Parameter	MKS units	per unit
DC machine parameter		
L_{af}	$.565 \ \frac{\text{volts-sec}^2}{\text{amps rad}}$	
Mechanical Parameters		
J	$.62 \text{ kg} - \text{m}^2$	
F	$-.00081\ \omega_m + .1637\ \frac{\text{joule-sec}}{\text{rad}^2}$	
Synchronous Machine Parameters		
x_d	9.03 ohms	1.00 pu
x_q	5.47 ohms	0.60 pu
x_d'	2.00 ohms	0.22 pu
T_{do}'	0.282 secs	
x_{ad}	125 ohms	13.85 pu
R_F	20.4 ohms	2.26 pu
Exciter-Regulator Parameters		
K_{A1}	113	
K_{A2}	5.59	
T_A	1.4 secs	
Transmission Line Parameters		
r	.0313 ohms	0.0035 pu
x	1.77 ohms	0.196 pu
G	0	0
B	0	0

4.2 Data Acquisition Hardware

Measurements

A number of the analog inputs required filtering before entering the A/D converter at the interface. Active filters have been selected to perform the low pass filtering which entails attenuating noise at frequencies as low as 360 Hz without interfering with system responses or with the 60 Hz terminal waveforms. The schematic of the Philbrick filters [20] used for these voltage inputs is shown in Figure 4.2. Filter performance curves are displayed in Figure 4.3, labelled with the input signals to which each is applied. Table 4.3 displays a measure of ripple on the filtered signals which are submitted to the A/D converter.



$$V_o = -\frac{1}{1 + b(RCp) + (RCp)^2} V_i$$

Figure 4.2 Active Filter Schematic

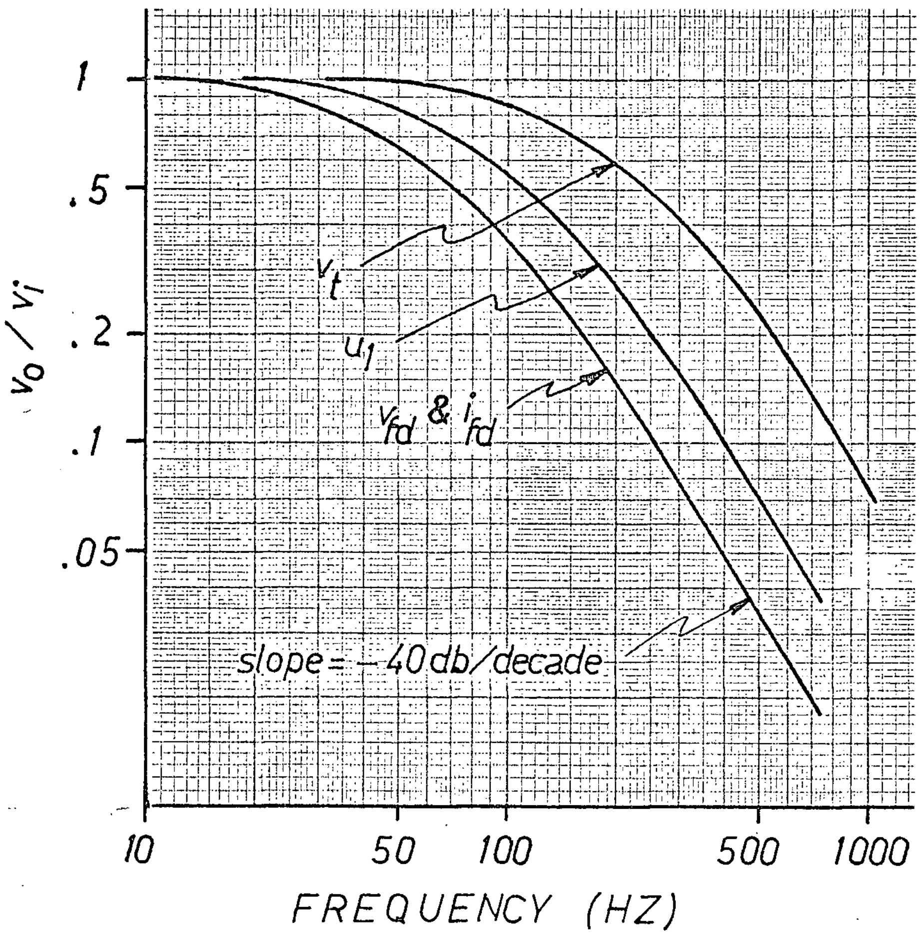


Figure 4.3 Filter Performance

Table 4.3

Noise at A/D Input

Input Signal	Percentage Noise (ripple)
synch. machine field volts (v_{fd})	1.3%
synch. machine field current (i_{fd})	1.0%
terminal voltage (v_t)	negligible
exciter reference voltage (u_1)	negligible
infinite bus voltage (v_o)	negligible
D.C. motor armature current (i_a)	1.1%

Direct current is measured by detecting and amplifying the voltage drop across a series resistance. Alternating current is measured by means of a current transformer. Both d.c. and a.c. voltages are fed directly to the interface with appropriate attenuation or amplification and filtering.

A digital shaft encoder is fixed to the accessible end of the synchronous machine shaft. This optical encoder (DRC Model 29 manufactured by Dynamics Research Corporation) outputs square pulses as the shaft rotates. Each revolution of the shaft produces 1500 pulses which are fed to the interface where they are used to measure the rotor position with respect to some reference. Further details are supplied when describing the interface.

Interface

The primarily TTL interface, designed to be compatible with a DEC PDP-8/L computer, was constructed prior to the start of this research project. However, this untried interface had a number of bugs including design deficiencies which were to be rectified before this project could proceed. Firstly, a short description of the interface is presented, and then a look is taken into the problems encountered.

Much of this interface is standard. It consists of an A/D converter (DEC #A811 with 0.1% F.S. accuracy) following a multi-channel (18 connected) FET multiplexer to measure analog signals. For computer control functions using analog signals, four digital-to-analog (D/A) converters are provided. The RTL system designed to interpret the optical shaft-angle encoder output is not standard and requires some attention. This logic allows measurement of the rotor angle in electrical units while the encoder itself is detecting angle in mechanical units. The result for the six pole synchronous machine being monitored is that the 1/1500 resolution for 2π mechanical radians provides only 1/500 resolution for 2π electrical radians.

To achieve the electrical angle measurement, the shaft-angle encoder output pulses are counted by a modulo-500 counter which is read by transferring its contents to a read buffer at a rate specified by a reference pulse train. At the start of a mechancial revolution, the counter is reset to zero and it then counts 500 pulses (1/3 revolution) when it resets itself. In the meantime, the reference pulse has gated the counter to the read buffer to determine shaft position at the time of the reference pulse. This read rate may be specified by one of the two available shaft encoders, by the mains frequency, or by a crystal oscillator if an absolute reference is desired. In this project one state variable is the angle δ between the q-axis and the infinite bus (mains) voltage. This can be measured directly by using a reference pulse train of mains frequency to gate the modulo-500 counter contents into the read buffer.

In an attempt to minimize construction cost, the original design of the interface incorporated a number of schemes to reduce hardware expenditure. One example of this which led to a dangerous design was in the multiplexer channel selection decoding logic. Table 4.4(a) outlines a portion of the decoding scheme used for multiplexer channel selection. It is noted that if inadvertently, through

Table 4.4

Multiplexer Channel Selection Decoding

(a) Original Channel Selection Scheme

Channel #	Word Addressing Channel												Octal Address
	0	1	2	3	4	5	6	7	8	9	10	11	
1	1						1		1				4050
2	1						1			1			4044
3	1							1			1		4022
4	1							1				1	4021
5		1					1		1				2050
6		1					1			1			2044
7		1						1			1		2022
8		1						1				1	2021

(b) Modified Channel Selection Scheme

Channel #	Word Addressing Channel												Octal Address
	0	1	2	3	4	5	6	7	8	9	10	11	
0													0
1												1	1
2											1		2
3											1	1	3
4										1			4
5										1		1	5
6										1	1		6
7										1	1	1	7

program error or hardware fault, bits 0 and 1, for example, are both high, two channels may be selected at once. The result is a short circuit through two of the FET switches and thus the destruction of one or more multiplexer channels. As it is believed that the cost of detecting and replacing shorted FET's outweighed the gain of reduced hardware for decoding, a fail-safe decoding scheme is implemented. The resulting code which provides activation of only one possible channel for every 12 bit binary number is outlined in Table 4.4(b).

Another minor interface change which is made for programming convenience is the installation of switches on all the interrupt lines. Thus when programming with interrupt on, undesired interrupts are easily disabled. This prevents loss of computing time in unnecessary servicing of interrupts.

4.3 Data Acquisition Software

Software performance is dictated by both the laboratory system and the statistical modelling scheme. A certain number of observations of each variable is required for statistical analysis. For this analysis to produce an accurate mathematical model, it is further required that the observations be spaced close enough in time to follow the fastest laboratory system response desired to be represented. The form of the mathematical model further defines the software by specifying which variables must be monitored.

Data Storage

When working with a basic 4-K memory computer without high speed storage devices, data storage is a problem when a large number of observations of many variables is required. For the laboratory system, time constants of interest were expected to be greater than 1/10 second and therefore it was decided to sample all system variables once each period of the mains voltage waveform. Furthermore, it was decided to store eight input signals per sampling time giving 480 samples per second. With only 3500 storage locations available it is seen that at one observation per word storage, the system may only be monitored for approximately 7.5 seconds. Thus a more intense packing procedure is desired for data storage.

The packing scheme adopted uses a full word to store the first value of each input, and thereafter only stores the deviation from this operating point. A problem arises because for a 10 bit A/D output the 6 bit packing allows only 6.25% deviation when operating at full scale of $\pm$ 5 volts input. By working at less than full scale, 10% deviations may comfortably be stored. This small allowable deviation requires software checking for overflows when packing deviations. A packing scheme which allows larger deviations but which is more susceptible to errors is to store the differences between successive readings. Due to the possibilities of error (if one value is wrong, all those following are wrong) in the latter scheme, the deviations from an operating point were stored. The small allowable deviations are acceptable except in deriving speed from angle measurements where limited resolution of the shaft-angle encoder presents a problem. By storing two observations per computer word, the system's eight measurement points may be interrogated each cycle of the mains for approximately 15 seconds. This provides acquisition of an adequate number of observations for the statistical analysis.

Sampling a.c. Signals

Without reconstructing the waveform sampled there are two methods of obtaining the magnitude of a.c. waveforms. They are to filter the signal and record one sample, or to continuously sample the signal and store the peak value. As filtering a 60 Hz waveform to an acceptable ripple causes measurement response time constants longer than the system time constants, the a.c. signals are continuously sampled and their peak values stored. This too has potential measurement error as the peak value may occur between samples. The data acquisition program allows for three desired a.c. waveforms to be sampled with 245 μsec between samples. This results in approximately 0.11% error in detecting the peak values, which is in the range of the 0.1% error in the A/D converter. Many more a.c. signals could be cross-sectioned before obtaining sampling errors of the magnitude of the noise ripple on the signals.

PDP-8 Program Outline

The PDP-8/L program, which is outlined by a flowchart in Appendix 4A, begins by reading a grounded multiplexer channel and storing the resulting A/D offset read. Another channel connected to an accurately known d.c. supply is read and the A/D output stored. These offset and calibration values later allow for correction of A/D readings and for conversion of stored binary numbers back to voltage levels on the system. The program then stores an A/D reading for each variable thus describing the system operating point. It then begins to record observations and pack in half words their deviation from the appropriate operating point value. A computer interrupt at each period of the mains-voltage waveform then initiates the following procedure. Pack the deviations from operating point for the previous sampling interval in the computer. Sample rotor position and then all d.c. variables in succession. Sample all a.c. variables successively and continuously until the next interrupt, storing the peak magnitude of each a.c. waveform.

As the deviations from operating point are stored, they are checked to see whether or not they may in fact be packed into six binary bits. When an overflow in packing the deviation from operating value in a half word occurs an overflow counter is incremented and the program will halt at some preset allowable number of overflows. If the overflow is positive, the largest possible positive deviation is stored, when negative, the most negative deviation is stored. This minimizes the error resulting from using the overflowed value as a valid observation in the statistical analysis. After the computer memory is filled the data stored is punched on paper tape using a single odd parity bit for each frame.

Off-line Data Handling

The information on the paper tape from the PDP-8 is converted onto magnetic tape for use on the computer center IBM 360/ Model 67. This data is then supplemented with measured attenuations of amplifications external to the interface for each variable and with the accurate value of the d.c. calibration voltage. A Fortran program uses this supplementary information and the paper tape information to reconstruct all system voltage and current values monitored. The result is 861 observations of eight system variables for use in the statistical modelling scheme. These observations are stored on magnetic tape, readily accessible for analysis or output.

5. PERFORMANCE OF THE STATISTICAL MODEL

The proposed statistical model is investigated by checking it statistically using the data acquired. Also its performance as a predictor is investigated by comparison with the performance of the theoretical model as well as with the laboratory system. Whether the statistical model is a good predictor or not is easily determined by comparison with the laboratory system only. However, further investigation of some aspects of the model may be more easily performed in a computer using the proven theoretical model. For example, a computer comparison of mathematical model responses may be used to decide whether or not the statistical model has identified a particular system time constant.

Section 5.1 describes the system operating points investigated and how the data is acquired for the statistical analysis. These important points require consideration before beginning the investigation of the statistical model.

5.1 System Data for Statistical Model Derivation

The laboratory power system is run at three different operating points. Its operation is monitored using the PDP-8/L computer and associated interface discussed in Chapter 4. At each operating point, responses to steps in each of u_1, i_a and v_o are monitored to be used as comparisons for the responses of the mathematical models. Also system response is recorded for small random variations in the three inputs u_1, i_a and v_o. This is necessary to obtain the data required to estimate the coefficients in the equations defining the statistical model. Applied perturbations are required at the three inputs because these signals are generally very well regulated and thus do not provide the amount of variation necessary in the statistical analysis. The perturbations are introduced on the control inputs independently and randomly. Simultaneously, the system response is monitored by recording values of the system state variables at regular time intervals. Separate sets of state variable observations are collected as perturbations are applied to u_1 and i_a, then to u_1 and v_o, and finally to i_a and v_o in three test runs. Hard copy of the data taken for each test is produced on paper tape by the PDP-8/L computer and is then processed using an IBM 360/Model 67 computer. Gains external to the PDP-8 interface, such as ratio of potential transformers, amplifiers and attenuators, are measured using meters. This data is supplied to the IBM 360 on cards to supplement the paper tape information. Thus the system voltage and current values may be reconstructed within the IBM 360 for any sampling time. These reconstructed voltage and current values are stored on magnetic tape until they are required for analysis or plotting.

The three operating points for which data is acquired are summarized in Table 5.1.

Table 5.1

System Operating Points Investigated

Variable	Units	Operating Point		
		A	B	C
P	watts	1230	1635	405
Q	vars	953	- 381	1660
v_t	volts	125	125	124
i_{fd}	amps	1.75	1.25	2.0
δ	degrees	29.5	45.4	3.6
v_{fd}	volts	36.8	26.6	42.8
u_1	volts	6.5	6.49	6.55
i_a	amps	15.5	18.9	9.4
v_o	volts	109	133	94

In Table 5.1 P, Q, and v_t completely describe the initial conditions of the system for the theoretical representation. The values of i_{fd}, δ, v_{fd} and v_o are calculated from the theoretical model initial conditions as a preliminary check on the theoretical model. Calculated and measured initial conditions are shown in Table 5.2. It must be noted that meter accuracy is generally not better than 2 - 3%.

Table 5.2

Steady State Values

Variable	Units	Steady State Values		
		Experimental	Theoretical	% Difference
i_{fd}	amps	1.235	1.172	5.1%
δ	degrees	46.0	43.4	5.9%
v_{fd}	volts	26.85	23.91	11%
v_t	volts	126	125	.80%
v_o	volts	133	132	.76%

5.2 Statistical Investigations of the Regression Model

Two important statistical investigations of the model are performed using residual plots. One is a check of whether or not the assumptions inherent in regression analysis (Chapter 2) have been violated. The other is a test of lack of fit, indicating how well the particular form of equation describes the data to which it is fitted. The philosophy used in interpreting residual plots is similar to that evident in testing hypotheses. That is, if a plot indicates an assumption is violated, one concludes the assumption is violated, while if a plot indicates the assumption holds, one concludes only that the assumption has not been violated. Residual plots then give evidence of lack of fit and violation of assumptions, but do not confirm that the model is perfectly adequate or that the assumptions have been completely satisfied.

The normality assumption for the residuals (ε_i, i = 1, 2, ...n) is investigated using an overall plot of residuals as shown for a typical case in Figure 5.1. As well as an overall normality distribution for ε_i, it is also required that the ε_i be normally distributed at any instant of time or over any interval of time. Rather than using a number of normality plots, the distribution of ε_i with time is plotted as shown in Figure 5.2. Since this plot indicates uniform distribution with time, then the results of the overall plot may be assumed to hold during any interval of time.

It is noted that the distribution of ε's in this thesis depends on how the perturbations of u_1, i_a and v_o are administered. Here the inputs are varied manually and an attempt is made to induce random fluctuations. It is more desirable to have controlled signals on the system inputs so that regulated perturbations could be administered, thus producing a better gaussian error distribution. Also, poor resolution in speed detection requires larger than desirable perturbations from nominal values.

The assumption of constant variance is readily checked by plotting ε_i against the predicted value as shown in Figure 5.3. Using a similar plot, lack of fit is investigated by plotting residuals for each regression equation against each of the independent variables as well as against the predicted value. An example is shown in Figure 5.4 where lack of fit of the v_t term in the equation for v_{fd} (Equation 3.60) is checked. It is noted that although the normality assumption appears to be violated, variance and lack of fit may be investigated qualitatively using these residual plots.

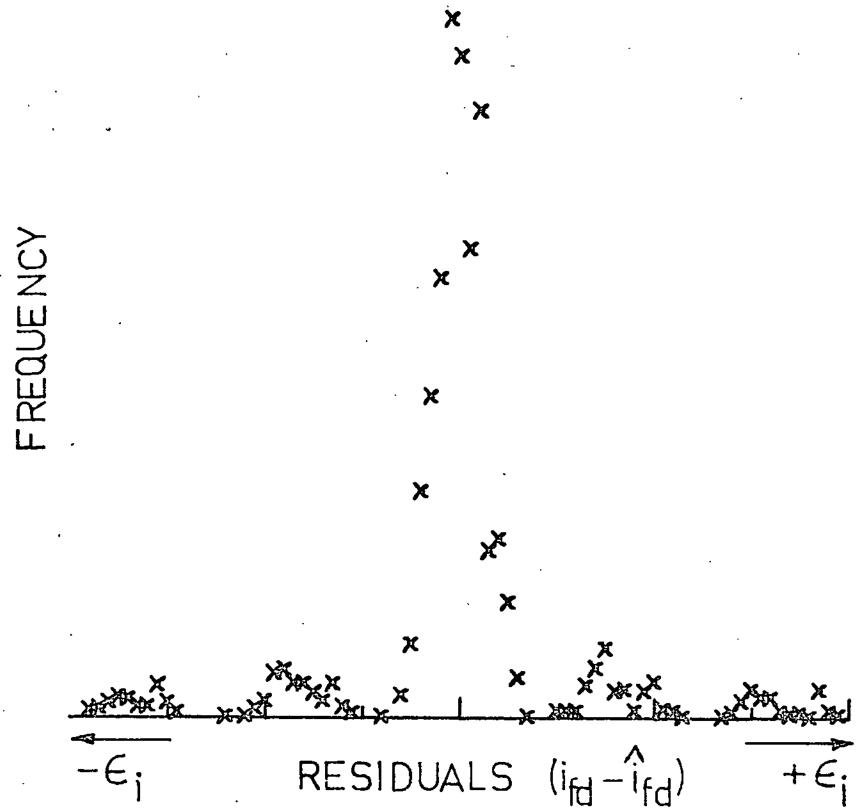


Figure 5.1 Overall Plot of Residuals

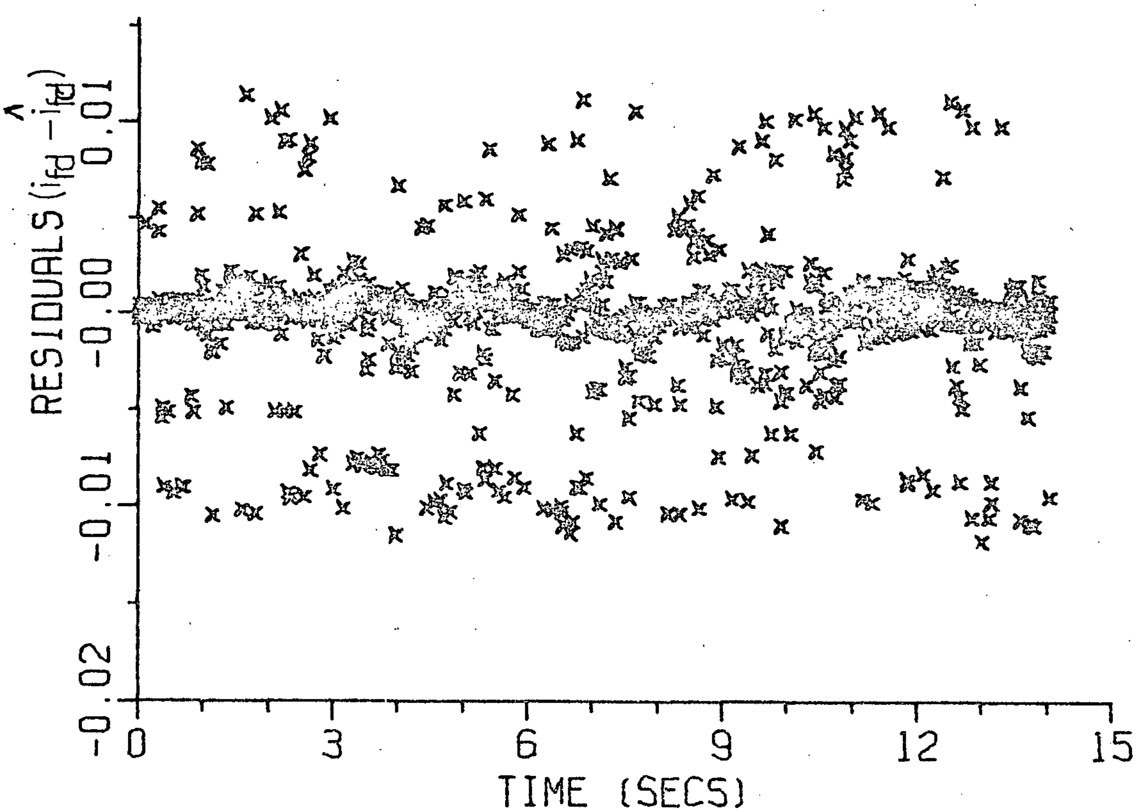


Figure 5.2 Distribution of Residuals with Time

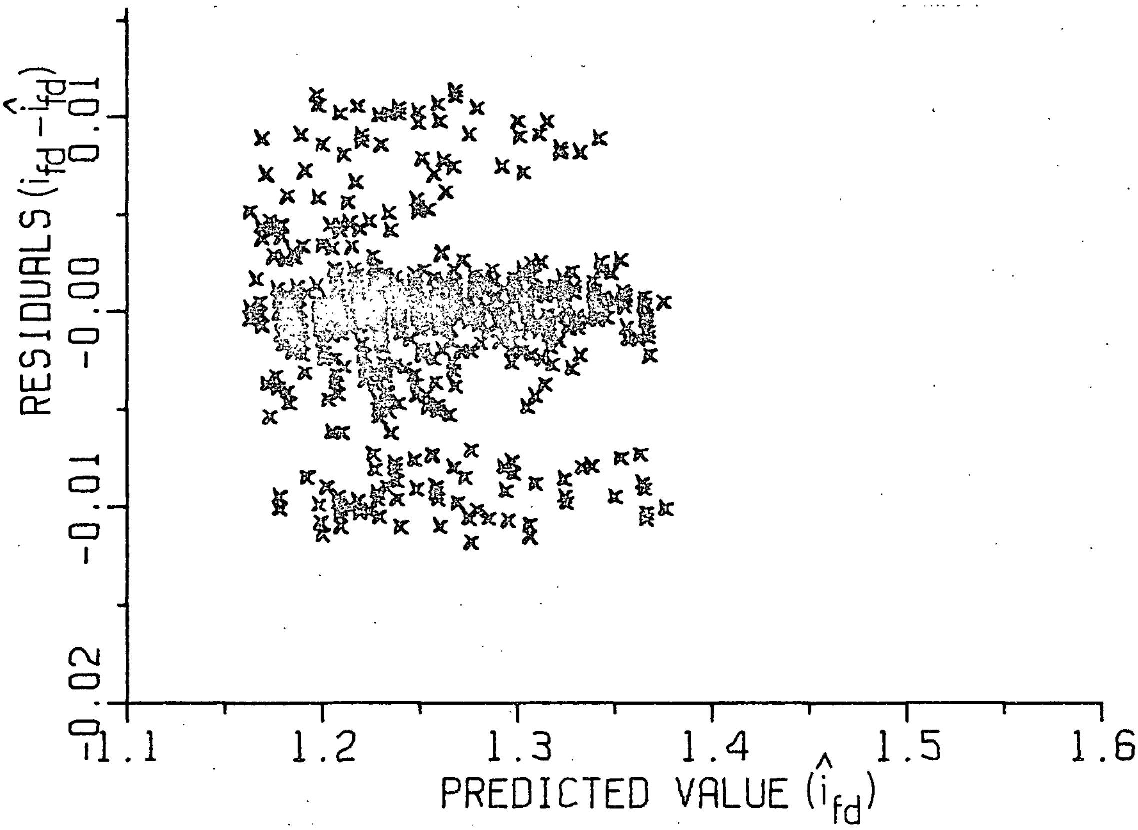


Figure 5.3 Example of Variance of Residuals

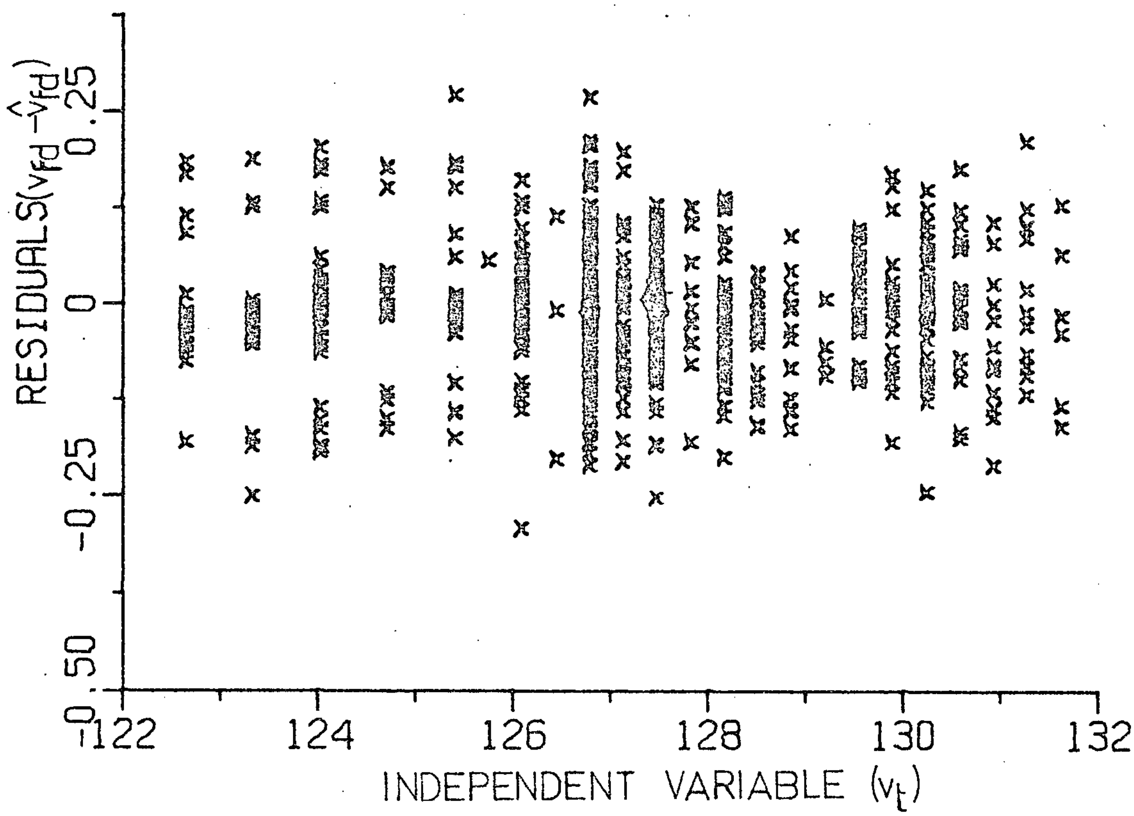


Figure 5.4 Example of Test for Lack of Fit

5.3 Model Responses to Step Inputs

The performance of the theoretical and statistical models were compared with each other and with the laboratory system response. For the comparisons, step inputs were applied to the regulator reference voltage, u_1, the d.c. motor armature current, i_a, and the infinite bus voltage, v_o, as outlined in section 5.1.

Effect of Operating Point

From Figures 5.5, 5.6 and 5.7 it is evident that the statistical model yields a good steady state response, but is not capable of predicting the most rapid fluctuations in the system's dynamic behaviour. It does, however, provide a close approximation to the system's dynamic response. By comparing Figure 5.6 and 5.7(a) it is observed that the statistical models give similar prediction of response whether or not they are used at the same operation point at which they are derived. This indicates that the model sufficiently explains system nonlinearities. This is further emphasized by comparing the coefficients for a typical case (equation 3.57) given in Table 5.3. It is seen that the most significant coefficients (b_{11} and b_{12} in this case) vary by only small amounts of the operating point changes. Figure 5.5 displays a response of a model derived from data at all three operating points. This less accurate predictor yields an acceptable response when compared with the steady state response obtained using Park's formulation.

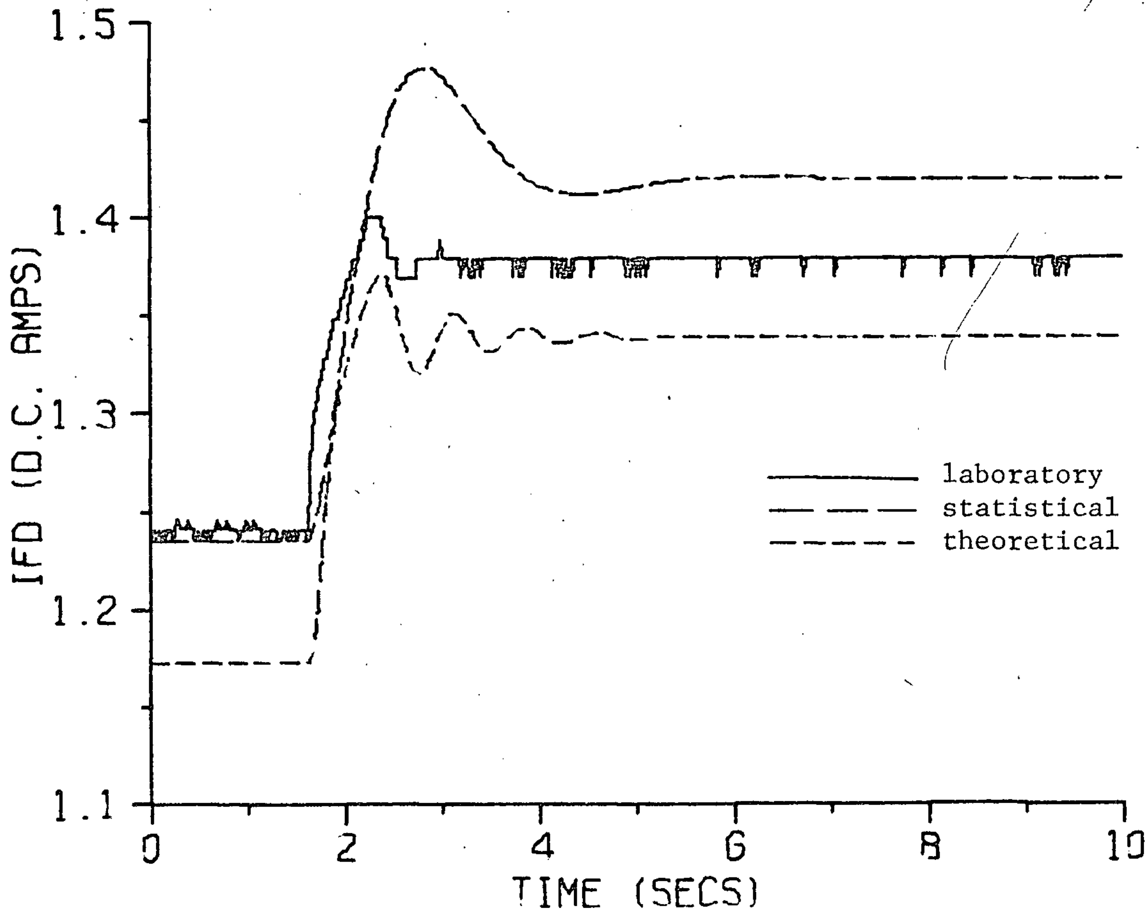


Figure 5.5 Response to step in u_1 at operating point B
- Statistical model from operating points A, B and C combined

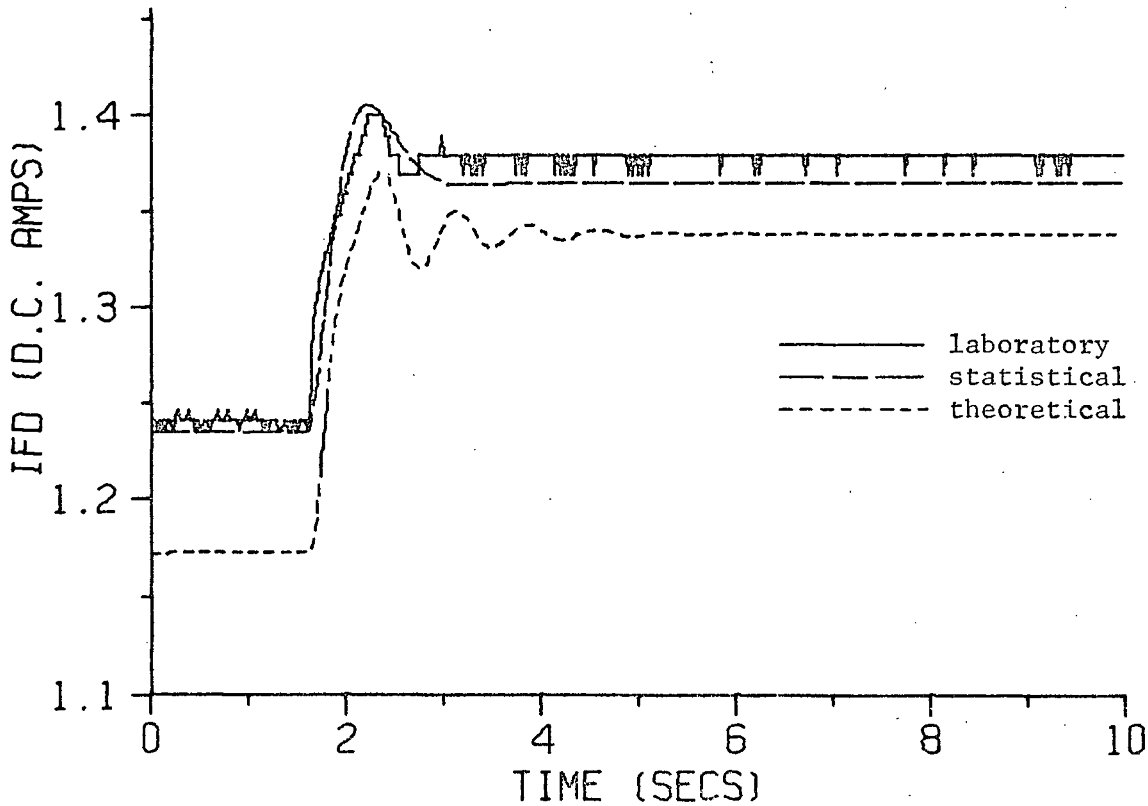


Figure 5.6 Response to step in u_1 at operating point B
- Statistical model from operating point A

Table 5.3

Example of Variation in Coefficients

Operating Point	R^2	Coefficients			
		b_{11}	b_{12}	b_{13}	b_{14}
B	.9930	.932	.0031	.0014	
C	.9917	.904	.0043		.0028
A+B+C	.9995	.929	.0033		.0034

The set of plots shown in Figure 5.7 indicate the predicted responses of all the measurable state variables for a step in u_1 using a statistical model with coefficients estimated at the same operating point as the step is applied. The model used is found at operating point B. The coefficients describing this model (equations (3.57) to (3.60) and (3.64)), excluding those coefficients which are non-significant to a 5% level, are as follows.

$b_{10} = .00084$, $b_{11} = .932$, $b_{12} = .00311$, $b_{13} = .00136$, $b_{14} = 0.0$

$b_{20} = 0.0$, $b_{21} = 1.0$, $b_{22} = 1.0$

$b_{30} = -.039$, $b_{31} = -.156$, $b_{32} = 0.0$, $b_{33} = 0.0$, $b_{34} = 0.0$,
$b_{35} = -.000478$, $b_{36} = .0000325$, $b_{37} = -.00481$

$b_{40} = -.357$, $b_{41} = .959$, $b_{42} = .971$, $b_{43} = -.0380$

$b_{50} = 2548.$, $b_{51} = .307$, $b_{52} = -1334.$, $b_{53} = .359$, $b_{54} = -14.7$,
$b_{55} = 96.7$.

This model was found using a 60 Hz sampling rate by combining three sets of data runs at the same operating point (B) as outlined in section 5.1. Similar statistical model performance was observed using step inputs of i_a and v_o.

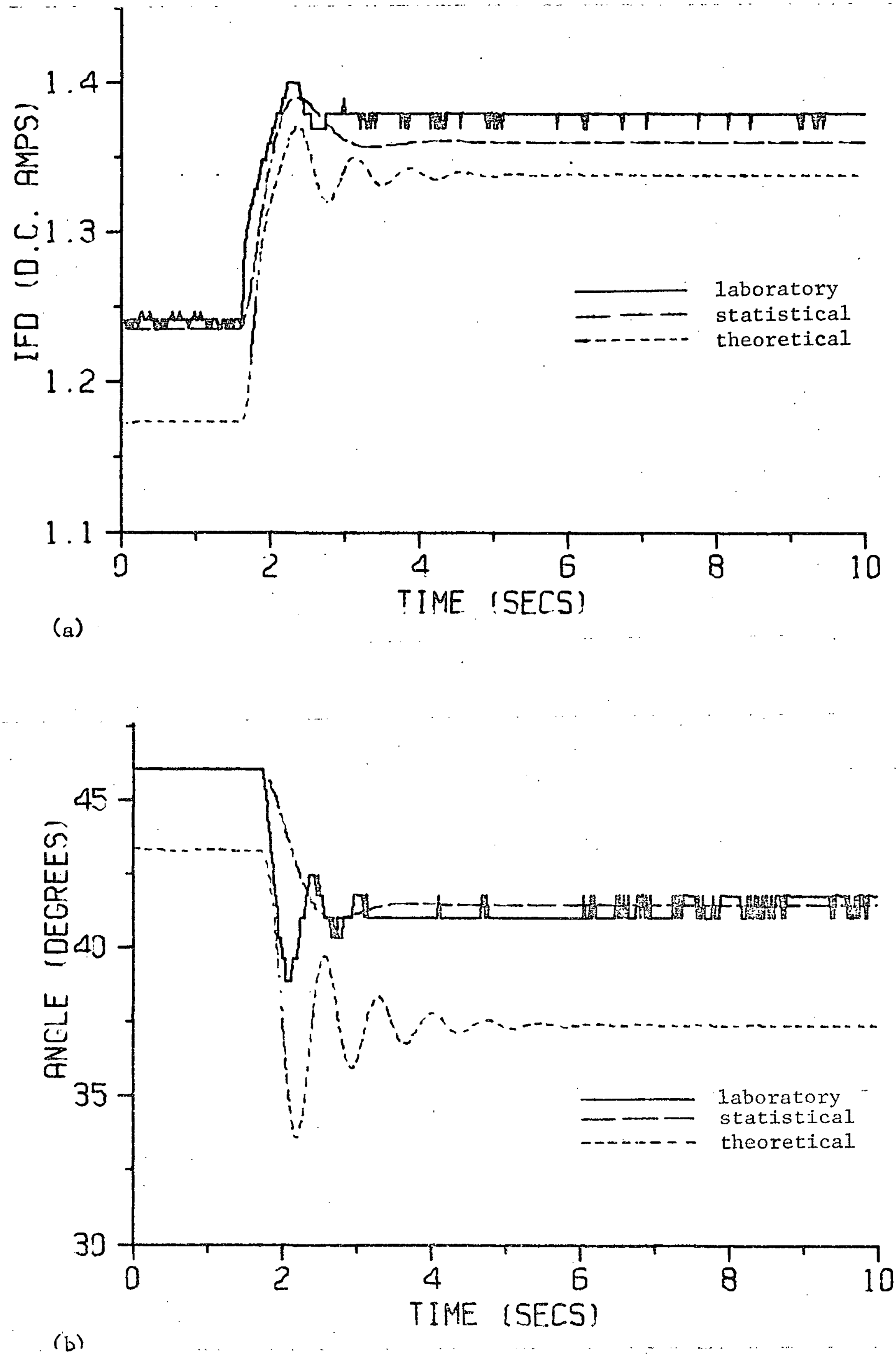


Figure 5.7 Responses to step in u_1 at operating point B
- Statistical model from operating point B

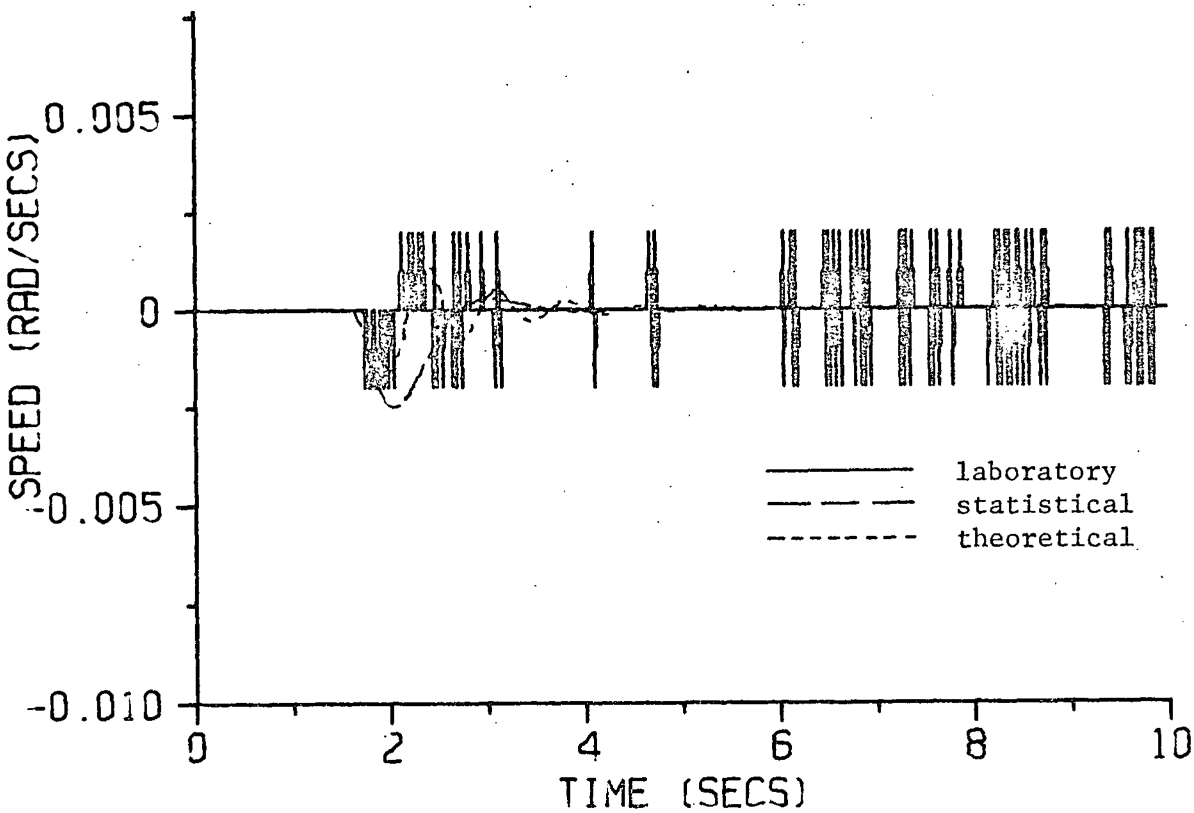


Figure 5.7(c)

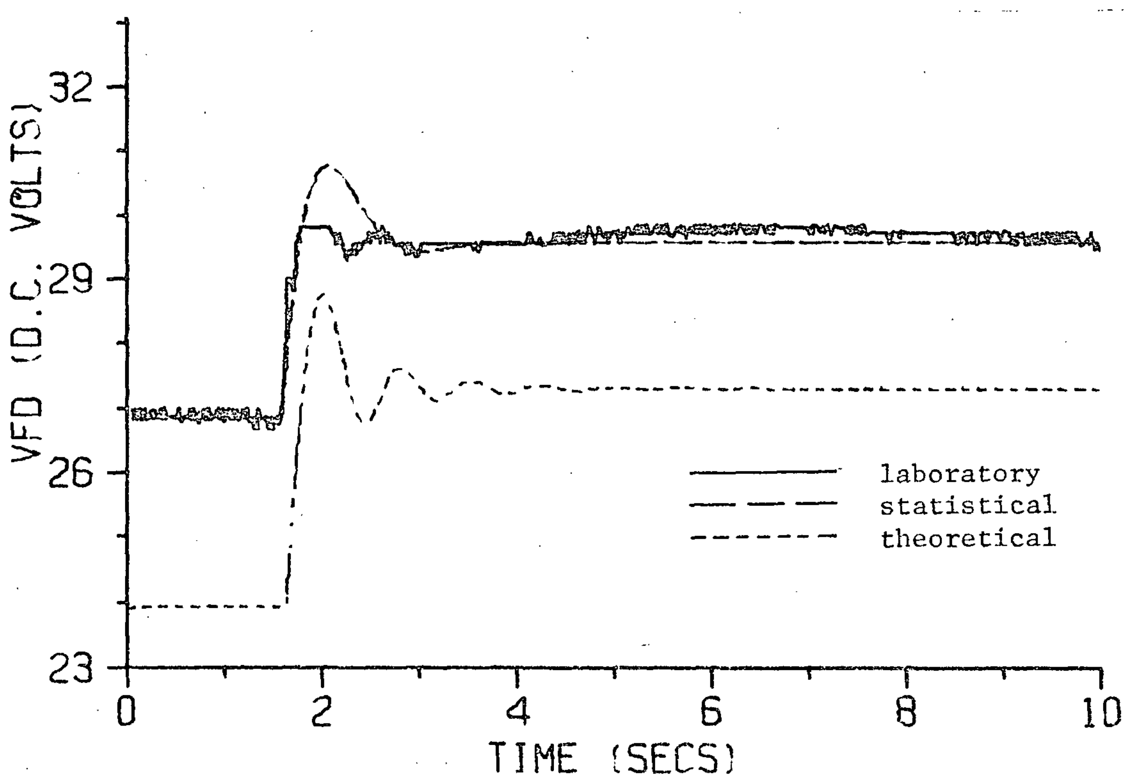


Figure 5.7(d)

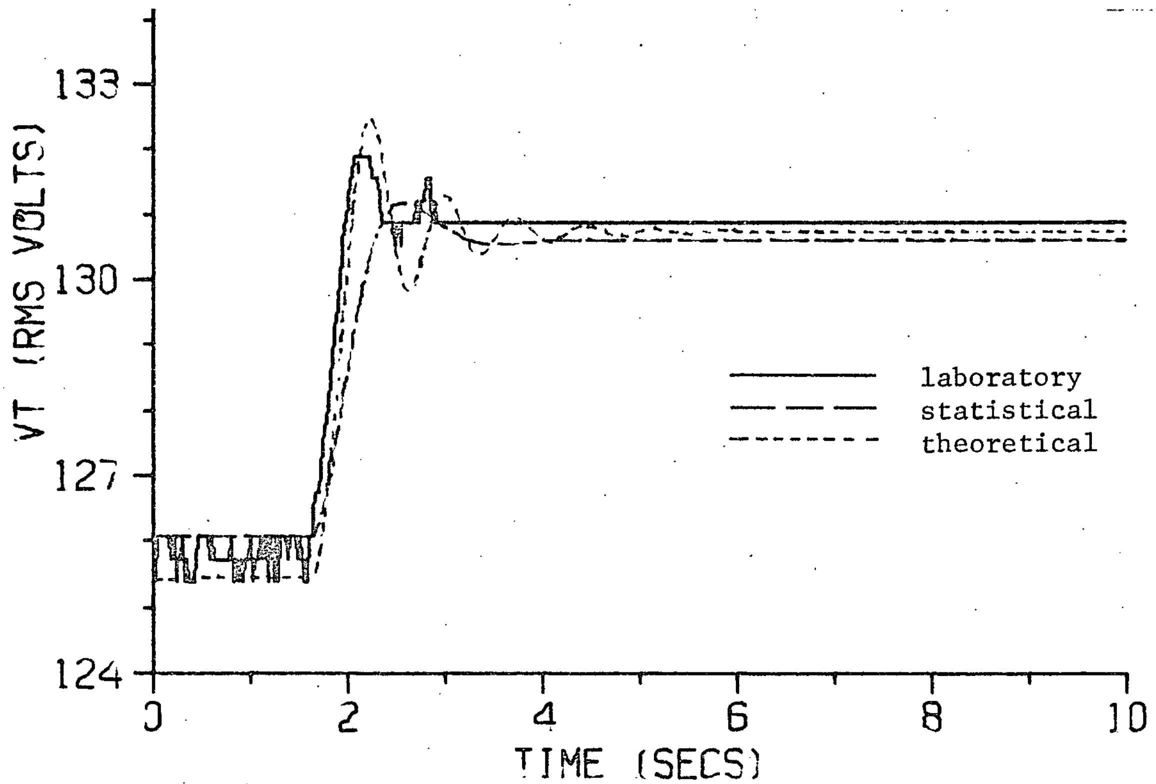


Figure 5.7(e)

Effect of Sampling Rate

Figures 5.8 and 5.9 indicate the effect of changes of sampling interval when obtaining data to form a model. The statistical model used to produce the response in Figure 5.7(a) used samples collected at each period of the mains. For the model producing Figure 5.8 the samples were taken every second period and in Figure 5.9 every fifth period. For this range of sampling frequency, change in sampling rate had negligible effect on the model produced.

In forming the various statistical models from different sets of data, note was taken of changes in the value of the multiple regression coefficient, R (see section 2.3). It is noted that as more samples were used to estimate the coefficients in the model, the R^2

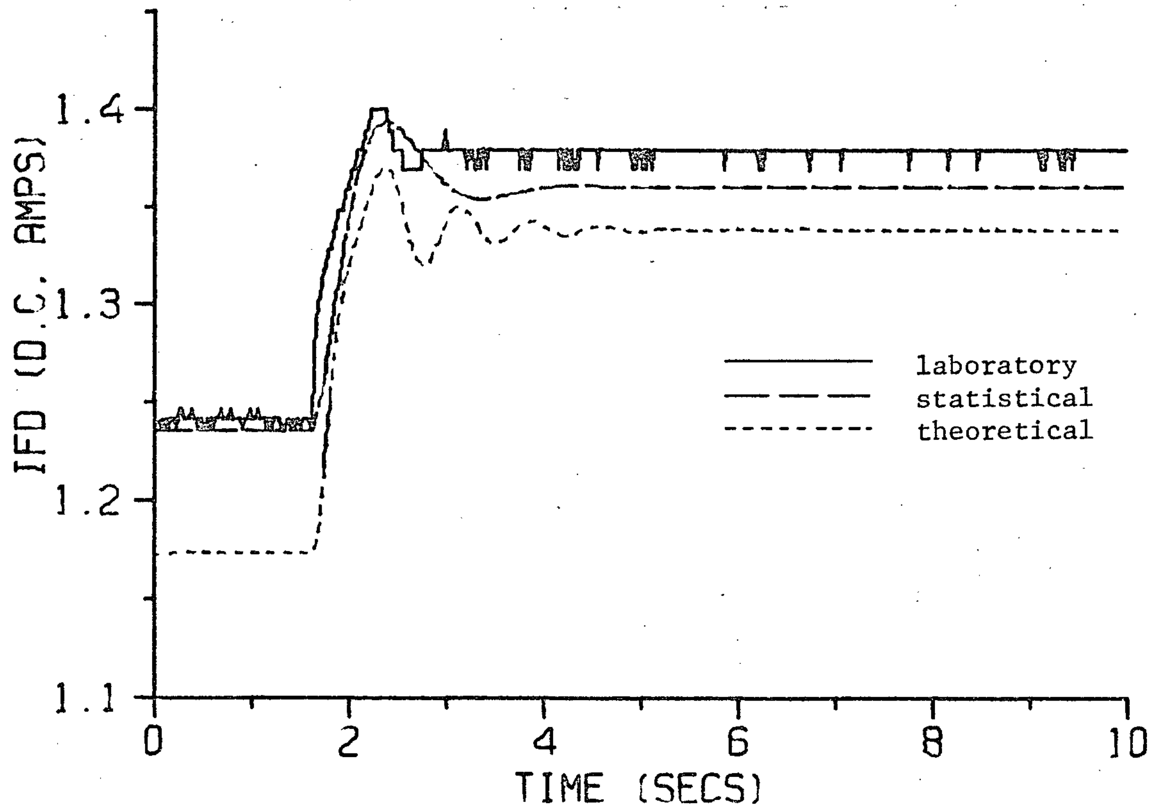


Figure 5.8 Response with statistical model found using 0.033 second sampling interval

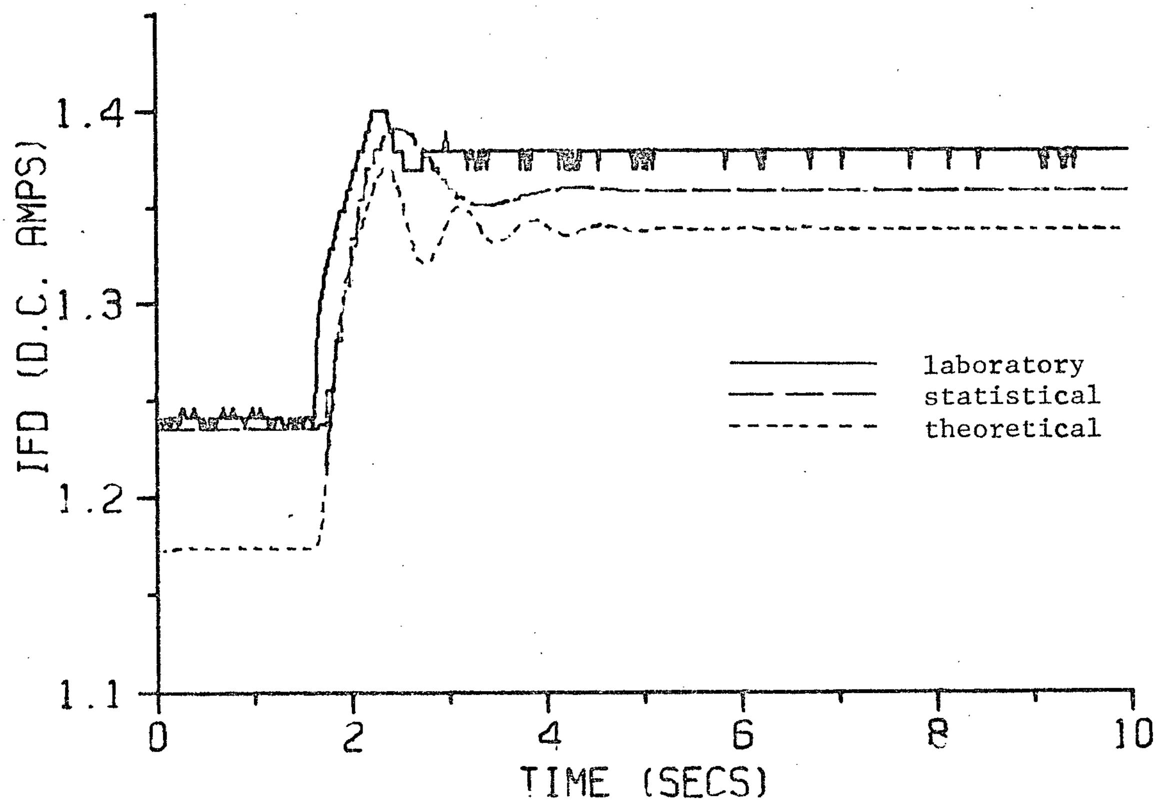


Figure 5.9 Response with statistical model found using 0.083 second sampling interval

value tended to increase (e.g. Table 5.3). However, for equation (3.58) describing speed, R^2 was only slightly significant when sampling at each period of the mains (.016 sec), but for most sets of data this R^2 increased ten fold when sampling every second period (.033 sec). In the modelling scheme, speed is found by differentiating angle (i.e. $\omega(k) = \delta(k+1)-\delta(k)$) which is monitored by the shaft encoder and yields poor resolution after conversion to electrical units. The effect is that small deviations with poor resolution produce a set of data which the particular form of equation chosen does not adequately describe. In practice, though, as long as the multiple regression coefficient is significant, say by four times the tabulated value, an order of magnitude increase in R^2 does not appreciably affect the model derived.

Generally, then, it is found that the derived statistical model yields good steady state prediction, but it responds slower than the system when predicting dynamics. Discrepancies observed in the theoretical model performance at steady state are within meter error tolerances as the theoretical and statistical models define their operating point using a different set of variables and therefore different meters. Lack of resolution in speed measurement (see Figure 5.7(c) created the major problem in the statistical modelling.

6. CONCLUSION

An investigation has been undertaken to ascertain how readily a power system lends itself to statistical modelling. A nonlinear state variable model has been derived. This model is linear in its coefficients which are evaluated by the least squares fitting technique of regression analysis. The form of the statistical model is based on Park's formulation of synchronous machine dynamics, with the unmeasurable state describing field flux, ψ_F, replaced by the field current, i_{fd}. As the expression for v_t is nonlinear in the coefficients as well as in the states, an auxiliary equation was introduced to allow prediction of v_t^2 and thus v_t may be calculated at any time.

An existing interface to a PDP-8 computer was modified and used for data acquisition. Signal conditioning networks were designed to eliminate undesired ripple and to obtain required signal levels for the interface. The software was designed to pack observations in half words by storing deviations from an operating point, thus allowing an adequate number of observations to be stored in the minimal memory available. Data handling software was also developed for the IBM 360. A program was written to interpret the logged data, taking it from paper tape, reconstructing the values of system signal levels, and storing them on magnetic tape with appropriate headings. Other programs were written to transform the data by combining values at each observation for regression analysis input; to catalogue and store intermediate statistical results on magnetic tape; to calculate and plot residuals; and to solve and plot the statistical model responses and the theoretical model responses as well as the system responses to various inputs. All the data acquired has been retained and, in conjunction with the data handling routines developed, this data may provide a starting point for further work on similar projects.

The statistical models identified produce very accurate prediction of the system steady state response. The models were not sensitive to operating points, as those produced from data at one operating point provided accurate prediction at another. When used to predict dynamic performance of the system to step inputs, the statistical model failed to predict the fastest system fluctuations. It did, however, predict the time for the new steady state operating point to be reached and is therefore a good dynamic model for many practical applications.

For further research, the following improvements are suggested. More rapid sampling of system variables is required to provide better dynamic prediction. More resolution is required in the speed measurement. This may be achieved by sensing speed directly or by having greater accuracy in the angle measurements. To achieve a better normal distribution of residuals it is desirable to perturb the system using controlled random inputs. However, the apparent violation of the normality assumption in this work did not appear to have a significant effect on the results. An extension of this work to more closely track system dynamics, preferrably in an on-line environment, would be of practical interest. The extension of the modelling scheme to multimachine systems would provide a valuable contribution as the scheme would then be of greater practical interest.

APPENDIX 3A

A third order state variable model may be derived from the simplified Park's equations (3.1) to (3.4) along with the mechanical equations (3.5) and (3.6). Equation (3.3) may be rearranged as [12]

$$\psi_d = \frac{x_{ad}}{\omega_o R_F} \frac{v_{fd}}{(1 + T'_{do}p)} - \frac{x_{ad}}{\omega_o} \frac{(1 + T'_d p)}{(1 + T'_{do}p)} i_d \tag{3.3}$$

$$\psi_d = \frac{x_{ad}}{\omega_o R_F} \frac{v_{fd}}{(1 + T'_{do}p)} - \left(\frac{A}{\omega_o} + \frac{pB}{\omega_o(1 + T'_{do}p)} \right). \tag{3A.1}$$

Solving for A and B and collecting terms gives

$$\psi_d = \frac{v_{FR}}{\omega_o} - \frac{x_d i_d}{\omega_o} \tag{3A.2}$$

where $$v_{FR} \triangleq \frac{v_F + p \{ x_d(T'_{do} - T'_d) \} i_d}{(1 + T'_{do}p)} \tag{3A.3}$$

and $$v_F \triangleq \frac{x_{ad}}{R_F} v_{fd}. \tag{3A.4}$$

It can be shown that [12] $x'_d \triangleq \frac{T'_d}{T'_{do}} x_d$ allowing v_F to be written as:

$$v_F = p \{ T'_{do} [v_{FR} - (x_d - x'_d)i_d] \} + v_{FR} \tag{3A.5}$$

or $$v_F = p\psi_F + v_{FR}. \tag{3A.6}$$

Thus equation (3.7) is found from (3A.6)

$$p\psi_F = v_F - v_{FR} \tag{3.7}$$

where

$$\psi_F \triangleq T'_{do} [v_{FR} - (x_d - x'_d) i_d] \tag{3A.7}$$

Substituting (3A.4) into(3A.6) gives

$$v_{fd} = p(\psi_F \frac{R_F}{x_{ad}}) + R_F (\frac{v_{FR}}{x_{ad}}) \tag{3A.8}$$

which when compared to the field voltage equation

$$v_{fd} = p\psi_{fd} + R_F i_{fd} \tag{3A.9}$$

gives $$\psi_F \frac{R_F}{x_{ad}} = \psi_{fd} \tag{3A.10}$$

and $$\frac{v_{FR}}{x_{ad}} = i_{fd} \tag{3A.11}$$

which are useful relationships when referring state equation variables to actual system quantities.

The electro-mechanical relationship in equation (3.8) is derived from the expression for the rotor angle in (3.6)

$$\theta = \omega_o t + \delta. \tag{3.6}$$

Differentiating gives

$$p\theta = \omega_o + p\delta \tag{3A.12}$$

or $$p\delta = \omega_o \Delta\omega \tag{3A.13}$$

where

$$\Delta\omega = \frac{p\theta - \omega_o}{\omega_o} = \frac{\omega - \omega_o}{\omega_o} \tag{3A.14}$$

is a per unit change in speed.

APPENDIX 3B

For a one-machine infinite bus system, the transformation of transmission system quantities to the d-q coordinate system is straightforward. A simplification from Vongsuriya's derivation [12] exists because the infinite bus voltage corresponds to the rotating reference and therefore at steady state is at the angle δ from the q-axis. Projections onto the d and q axes are then all that is required to express v_o in terms of Park's system. That is,

$$v_{od} + jv_{oq} = v_o \sin\delta + jv_o \cos\delta . \tag{3B.1}$$

From the system diagram in Figure 3.4

$$[\ 1 + (r + jx)\ (G + jB)\ ]\ v_t = v_o + (r + jx)\ i \tag{3B.2}$$

where in Park's system

$$v_t = v_d + jv_q \tag{3B.3}$$

$$i = i_d + j\ i_q \tag{3B.4}$$

$$v_o = v_{od} + jv_{oq} . \tag{3B.5}$$

Substituting (3B.1) for v_o in (3B.2) gives v_o in Park's system, and equation (3B.3) and (3B.4) into (3B.2) puts v_t and i in Park's system thus giving the system equation

$$[\ 1 + (r + jx)(G + jB)\ ]\ [v_d + jv_q] = v_o \sin\delta + jv_o \cos\delta + (r + jx)(i_d + j\ i_q) \tag{3B.6}$$

which by expanding and separating real and imaginary parts can be written as equation (3.34).

APPENDIX 3C

Equation (3.39), which expresses ψ_d and ψ_q in terms of state variables only, may be developed as follows. In (3.34) substitute expressions for v_d, v_q, i_d and i_q in terms of fluxes, that is, use equations (3.1), (3.2) for v_d and v_q and use equation (3.14) for i_d and i_q

$$i_d = \frac{1}{T'_{do}x'_d}\psi_F - \omega_o \frac{1}{x'_d}\psi_d \tag{3C.1}$$

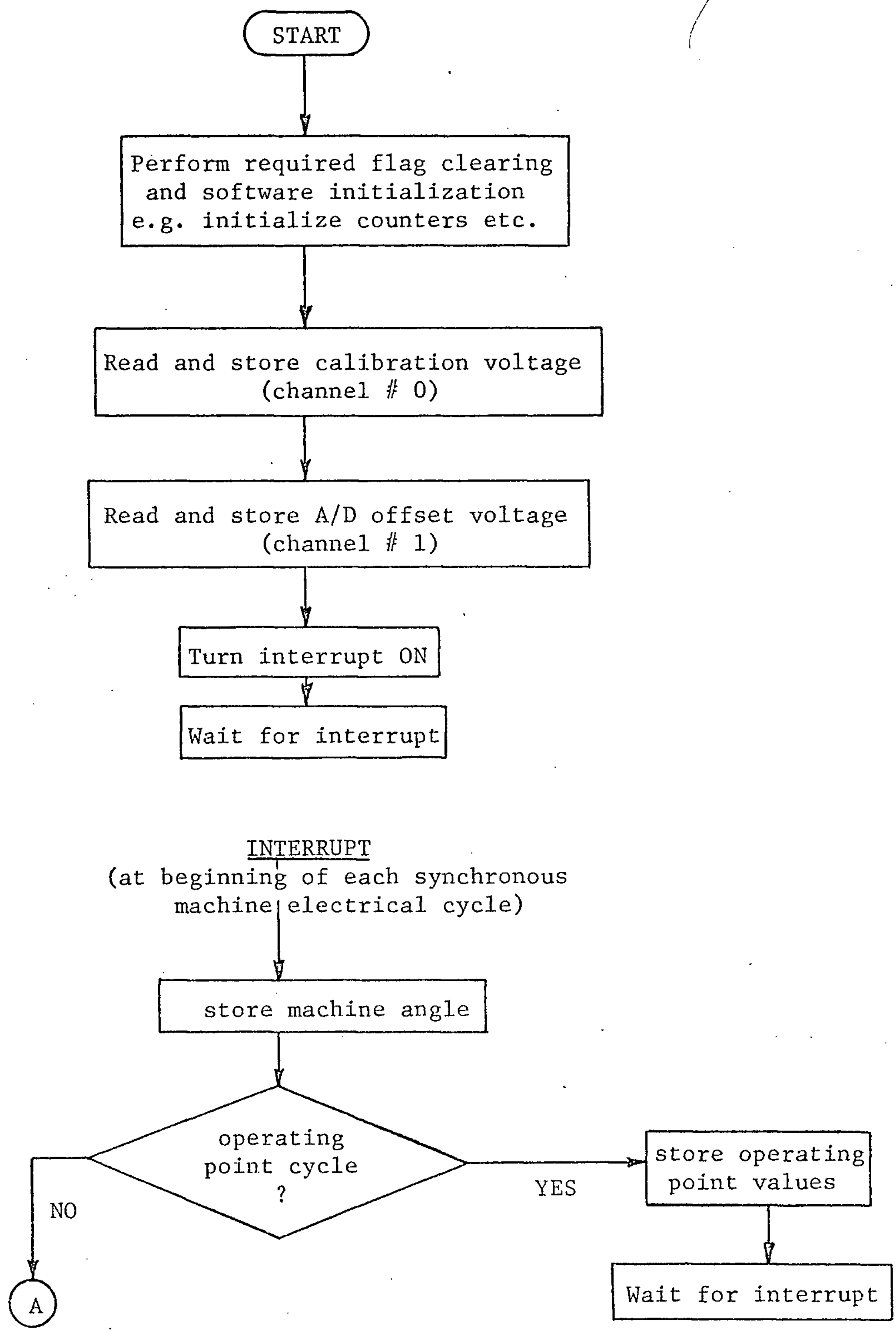
$$i_q = -\omega_o \frac{1}{x_q}\psi_q . \tag{3C.2}$$

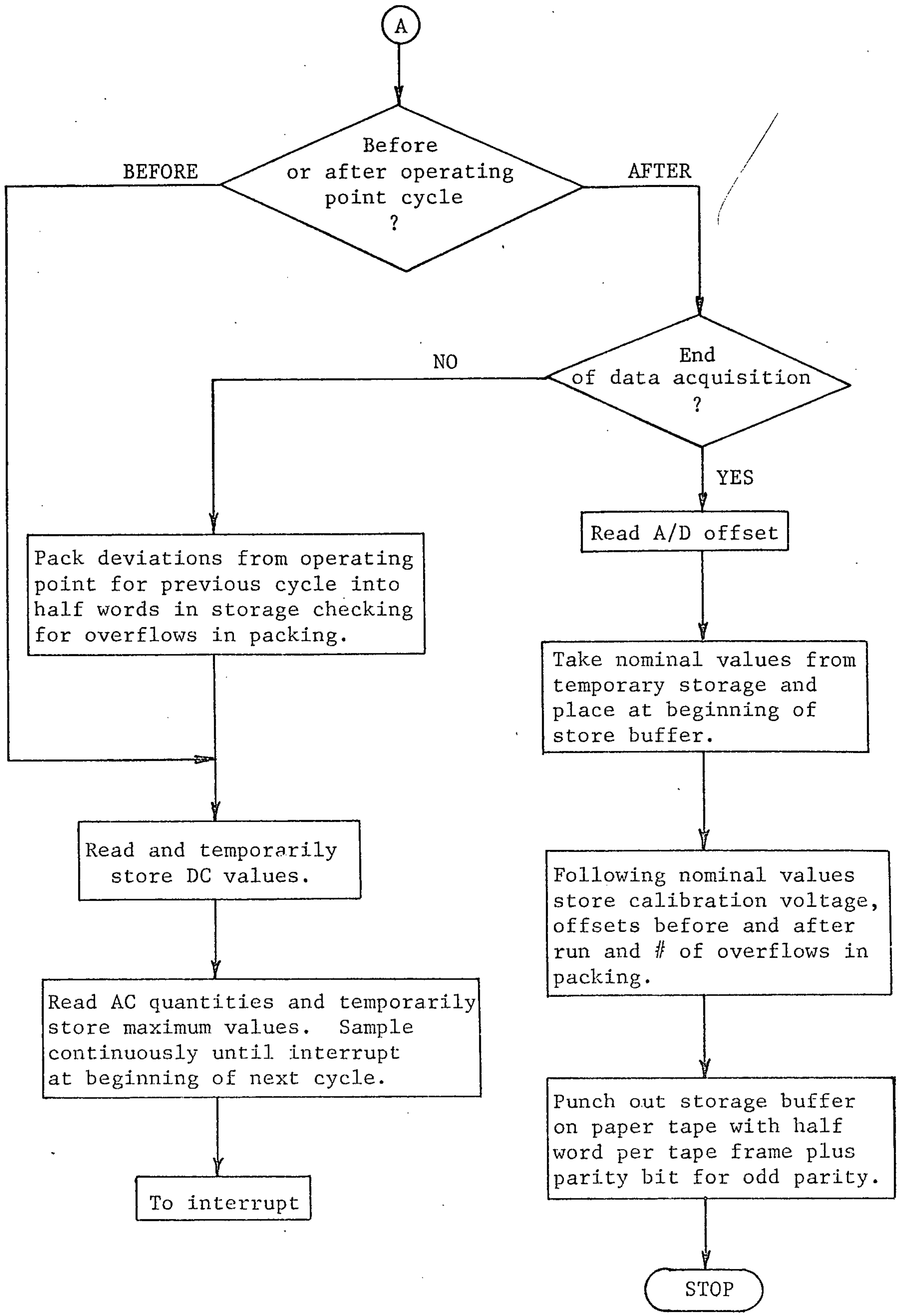
The equation resulting from these substitutions is

$$\begin{bmatrix} -\psi_q\omega_o \\ \psi_d\omega_o \end{bmatrix} = \begin{bmatrix} k_1 & k_2 \\ -k_2 & k_1 \end{bmatrix} \cdot \begin{bmatrix} v_o\sin\delta \\ v_o\cos\delta \end{bmatrix} + \begin{bmatrix} K_1 & K_2 \\ -K_2 & K_1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{x'_dT'_{do}}\psi_F - \frac{1}{x'_d}\psi_d \\ \omega_o \frac{1}{x_q}\psi_q . \end{bmatrix} . \tag{3C.3}$$

APPENDIX 4A

Flowchart for Data Acquisition Program




A
Before or after operating point cycle ?
BEFORE
AFTER
End of data acquisition ?
NO
YES
Read A/D offset
Pack deviations from operating point for previous cycle into half words in storage checking for overflows in packing.
Take nominal values from temporary storage and place at beginning of store buffer.
Read and temporarily store DC values.
Following nominal values store calibration voltage, offsets before and after run and # of overflows in packing.
Read AC quantities and temporarily store maximum values. Sample continuously until interrupt at beginning of next cycle.
Punch out storage buffer on paper tape with half word per tape frame plus parity bit for odd parity.
To interrupt
STOP

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