A PIAGETIAN ANALYSIS OF INTELLECTUAL PERFORMANCE ON FIRST-YEAR UNIVERSITY PHYSICS EXAMINATIONS

by

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ABSTRACT

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The Problem

This thesis is addressed to a problem in classroom practice identified as formative evaluation, that is, the development of a method of utilizing Piaget's theory of intellectual development to evaluate the intellectual performance of students contending with the formal concepts and methods of inquiry presented in a first year physics course with a view to improving instruction in the course.

The method of studying the problem required a good understanding and analysis of the relevant aspects of Piaget's theory so that it could be reformulated in such a way as to be unable in identifying formal behaviour of individuals' answers to specially selected examination items. This resulted in the formulation of a methodology in the form of an inventory of descriptors, and a method of analysis for identifying behaviour at the final stage of Piaget's developmental sequence, namely the formal operations stage.

Method of Study

The inventory of descriptors was used to identify formal operational behaviour of students performance on selected Piaget tasks, providing information concerning their maximum potential level of intellectual development. It was then used to identify physics examination items which required formal operations for their solution and the formal operational behaviour displayed by students in responding to the selected items thereby providing information concerning the actual level of intellectual performance displayed by students in classroom situations. A comparison of identified intellectual behaviours

provided information concerning the actual level of intellectual performance displayed by students in classroom situations. A comparison of identified intellectual behaviours provided information concerning the usefulness of such an instrument in education.

Results of the Study

It was concluded that the inventory of descriptors adequately described and identified intellectual behaviour at the formal operations level, both in student performance on the Piaget tasks, and in student performance on selected items from the physics examination paper. The inventory of descriptors proved to be of potential value to formative evaluation in the classroom situation.

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My great respect and grateful thanks go to Dr. Walter Boldt who supervised, advised and encouraged me, and above all, taught me so much. Thanks are due also to Dr. Clifford Anastasiou for his valuable advice, to my committee for agreeing to be there, and to my fellow students who always proved most stimulating.

To

my baby
Fraser John
who nearly made
this thesis impossible

and

Peter
my husband
who saved it
so often.

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CHAPTER I

THE PROBLEM AND ITS CONTEXT

The problem being investigated in this study is described in the context of educational evaluation. A discussion of the relevant aspects of educational evaluation is given first followed by a description of the general and specific problem being considered. The method of study is presented in outline, and the scope and limitations of the study are discussed. Finally, for purposes of clarification, a description is given of the terms used in this study.

A. CONTEXT OF THE STUDY

The philosopher Urmson (1969) defines evaluation as the intention to make a judgment about a phenomenon. According to Smith (1962, p. 1), a precondition for judging a phenomenon is understanding it. To understand and judge a phenomenon such as intellectual development, for example, requires that the phenomenon be observed, analyzed into it's various elements, and the elements classified into a set of descriptive categories to which normative criteria can be applied (Westbury, 1970, p. 251). Piaget's theory of intellectual development is often seen as constituting such a set of descriptive categories.

The role of educational evaluation is twofold, and is described by Scriven (1967) as formative evaluation which provides feedback information for an ongoing educational program such as curriculum development or methods of instruction, and summative evaluation which assesses the worth of a new educational program after its completion.

Teachers often need to perform the functions of both kinds of evaluation in the classroom situation. The function of formative evaluation is to provide the teacher with information for the improvement of a course of instruction while the teaching is being done. Summative evaluation serves the function of establishing the merits of a course of instruction upon its completion. Formative evaluation includes, among other things, an evaluation of the level of intellectual performance required of students to perform classroom tasks successfully and an evaluation of the intellectual skills the learner is capable of performing. By careful matching of the intended level of intellectual functioning required to perform a task with observed intellectual capability and performance, the teacher can optimize conditions for learning, predict possible sources of difficulty in performing the task, and identify possible sources of difficulty with the task itself.

The evaluation of intellectual behaviour, according to Piaget, would include a description of the operational mechanisms governing intellectual behaviour, (piaget, 1953, p. xviii) and not simply obtaining a measure of intellectual skills by means of various tests. This approach to evaluating intellectual behaviour appears to have an advantage in that it enables one to judge the educational value in the learner's interaction with the environment by understanding the intellectual processes by which the learner copes with his environment.

Piaget's creative theorizing, however, poses a methodological problem when it comes to dealing with problems of classroom practice, such as the problem in formative evaluation just described. It is the methodological problem, then, that is the focal point of the present study.

In summary, the study has been oriented in a general area of educational evaluation - Scriven's theoretical view of evaluation, and a specific area, formative evaluation involved in classroom practice. It should be added that the study presented here was carried out within the context of the Physics Education Evaluation Project of the University of British Columbia (1970/1971).

B. STATEMENT OF THE PROBLEM

General Problem

The general problem to be investigated in this study is the development of a method for utilizing Piaget's theory of intellectual development for evaluating the intellectual performance of students contending with formal concepts and methods of enquiry as presented in a first-year university physics course.

In the classroom situation an educational theory is much needed for guiding teachers in handling the problems involved in the evaluation of intellectual performance. Skager and Broadbent (1968) discuss these problems and advocate that evaluative techniques should be based on a theoretical model in order to give generality and validity to the evaluative criteria. Unfortunately, current educational theory provides little direct help for teachers in dealing with problems related to the learning of complex subject matter in areas such as the sciences (Easley, 1969).

Cognitive psychology has touched on aspects of this problem (Ausubel, 1968), but there is not yet a theory of cognitive development formulated in such a way that it can be used to meet the problems that arise in classroom practice (Easley, 1969). Of the available theories pertinent to this area of concern, Piaget's theory appears to be the most useful.

The theory appears at least potentially useful to the classroom teacher for providing a means of identifying the way in which the intellect of the infant evolves into the intellect of the adult. Easley (1969) points out that Piaget's theory could be used to judge the educational value of a child's interaction with the environment by providing an understanding of the way in which the child copes with his environment. It could be used to understand the way in which informal conceptual structures and intuitive methods of inquiry used by the child evolve into the more formal structures and modes of inquiry used by adults and that teachers seek to help students attain.

Piaget describes developmental changes in the internal intellectual structures which are manifest in the psychological functioning of an individual performing tasks especially designed by Piaget for this purpose. Analysis of performance on the Piaget tasks provides information concerning the person's maximum intellectual potential for investigating and understanding his world.

Unfortunately Piaget's theory lacks sufficiently clear formulation to guide classroom teachers in analyzing and interpreting performance on the Piaget tasks and, more seriously, lacks a methodology for evaluating intellectual performance on classroom tasks.

The Specific Problem

Students in a course in first year Physics at the University of British Columbia have an average chronological age of about 18 years, and have all passed either one or two High School physics courses, (a Physics 110 course requirement). Presumably students in this course could be expected, in Piagetian terms, to have reached the final stage of intellectual development, namely the stage of formal operations. This stage involves the ability to use formal, logical, hypothetical-deductive thought in coping with intellectual problems. Other factors being equal, students in the Physics 110 course should be able to answer correctly physics examination items which can be shown to require formal operational thought for their solution. Conversely, it would be expected that students who do not prove capable of using formal operational thought would fail to respond correctly to these examination items.

The specific problem investigated in this study is the development of a method for utilizing Piaget's theory of intellectual development for evaluating intellectual behaviour of students in the Physics 110 course coping with items on a final examination for the course, and to explore the usefulness of the method for meeting problems in science teaching.

C. METHOD OF STUDY

The problem of the study was investigated in five stages.

Stage one involved an analysis of Piaget's theory of intellectual development with special emphasis on the concrete and formal operations stages. Descriptions, or elements termed "descriptors" of psychological functioning related to theoretical constructs of processes and structure at these stages of development, were identified. The descriptors together with the related constructs constituted the theoretical basis on which the remainder of the study was developed. The results of Stage One of the study are presented in Chapter II.

Stage two of the study was the development of an inventory for making a Piagetian analysis and interpretation of intellectual performance. An important reason for the development of the inventory of descriptors was to meet the problem of facilitating the analyses of student performance on the Piaget tasks and selected examination items. A measure of consistency would be ensured if the analyses were made in terms of descriptions of adequate behaviour acceptable for classification into a particular stage of development.

Concerning the problems involved in analysing the performance of individuals Piaget writes:

> The important thing is to find a number of rules of interpretation which will unite the maximum of flexibility with the maximum of strictness, in so far as these two requisites can be reconciled ... we must find out what rules must be followed to avoid problems of premature judgment. (Piaget, 1929, p.23).

The rules which Piaget subsequently discusses (Piaget, 1929, introduction) were inadequate for identifying formal operational thought in the performance of an individual. The major concern of this study was therefore to develop an inventory of descriptors for identifying formal operational thought which in effect serve as "rules of interpretation" advocated by Piaget. The inventory of descriptive categories representing different facets of intellectual performance at the formal operations stage was developed, and the descriptors elicited in Stage one were classified into the categories of the inventory. Exemplars of psychological functioning to which the descriptors were thought to apply were obtained from the Piagetian literature (Inhelder and Piaget, 1958), and were included in the inventory for reasons of clarity and ease of application in analysis and interpretion of intellectual performance. The development of the inventory is described in the first part of Chapter III.

In Stage three, data were collected on the performance of Physics 110 students on Piaget tasks at the formal operations stage and on selected items on the final course examination. Since most students at the first year university level were expected to be at the formal operations stage of intellectual development, application of the inventory to performance data on the Piaget tasks was seen as a way of checking the validity of the inventory. In addition performance data on the Piaget tasks was seen as an indication of maximum intellectual potential of the students performing the tasks. The performance data on the examination items were intended for use in demonstrating the applicability of the inventory to evaluating intellectual performance on a typical classroom task. Selection and descriptions of students and tasks, as well as procedures for collecting and recording task performance is reported in the second part of Chapter III.

A clinical technique was used in the administering of the Piaget tasks as any standardization of administration is a process to which Piaget strongly opposed. If subjects are to be stimulated into displaying their intuitive competence and weakness in intellectual thought, they must be given the freedom to follow their thoughts to conclusions. This freedon may be curtailed in the administering of standardized tasks, and there is always the danger that such tests tap only the surface of a subject's cognitive skills, and would "not provide a reliable index of the real quality of his understanding" (Ginsburg and Opper, 1969, p.227). In view of these objections to standardization, a clinical approach was used in the administration of the Piaget tasks which was basically unscheduled (Piaget, 1929, p. 7-23) (See Chapter II. p. 10). The analyses of student performance were, however, standardized in that the descriptors and their exemplars were used as standards against which performance was compared.

In Stage four of the study, an attempt was made to illustrate the applicability of the inventory to analyzing and interpreting performance data. Transcripts of videorecordings of task performance were analysed and interpreted by means of the inventory. The inventory was applied to the examination items by analysing first the responses expected by the instructor and a tutor for the course, and secondly to the actual responses to the items made by the students involved in the study. Details of the analyses and summaries of the results are presented in Chapter IV of the study.

Stage five included a comparison of the results of analysis of performance on the Piaget tasks with the actual performance on the examination items. Implications of the comparisons for the purposes of formative evaluation and improvement of classroom practice are suggested, and a critique of the usefulness of the inventory for these purposes is presented. The results of Stage five of the study are presented in Chapter V.

D. SCOPE AND LIMITATIONS OF THE STUDY

The study was confined to a consideration of the formal operations stage of Piaget's theory. The inventory developed, therefore, is applicable only to Piaget tasks, classroom tasks, and students at this level of intellectual behaviour.

The inventory developed was used to analyse and interpret performance on tasks that involved both verbal and non-verbal responses. The inventory therefore, can be applied to classroom tasks of both kinds provided that a record of performance is available for analysis.

The inventory has been applied by persons both trained and untrained in Piaget theory, although considerable familiarity with the inventory was required before application. The inventory, therefore could be useful to teachers who are willing to thoroughly familiarize themselves with the inventory.

Piaget's theory itself, being an original contribution to epistemoloty, presents difficulties for establishing the construct validity of the inventory in that it is constantly being modified and articulated. Piaget's older volumes use a somewhat different vocabulary from his more recent writings. Furthermore, it has been drawn to the writer's attention that the English translations of Piaget's theory are sometimes inaccurate. The theory itself, is primarily philosophical in nature and often very difficult for untrained individuals to understand. These difficulties with the theory pose a serious problem in developing a valid inventory which can be easily used in the classroom situation. Even for a trained person, use of the inventory is difficult and time-consuming. Further, while the information attainable through the use of the inventory is detailed and in-depth, it is not clear at present how a teacher might go about using this information for improving instruction. Without the assistance of a person trained in Piagetian theory, a training program using both written materials and demonstrations would seem desirable before attempting to use the inventory.

The study is exploratory in nature, and attempts to find a way of using Piaget's theory for meeting a problem of classroom practice in evaluation without deviating too far from Piaget's view of retaining as much flexibility in procedural rules as possible. Retaining this flexibility raises difficulties in developing a reliable instrument. Standardization of procedures for administering tasks and applying the inventory, therefore, has not been undertaken. The reliability of the instrument, in terms of consistency of application and results across interviewers, consequently, is much more dependent on a thorough understanding of Piagetian theory than it is on following procedural rules.

E. DESCRIPTION OF TERMS

In an attempt to clarify Piaget's theory of intellectual development, definitions of some of the terms used by Piaget introduced in this thesis are provided. (See also Furth, 1969, pp 259-265).

1. Accomodation - The modification of structures of a biological organism by assimilated elements. Likewise the modification of intellectual structures to deal with assimilated "objects of knowing".

2. Adaptation - In a biological sense involves a balanced state between an organism and its environment - in a Piagetian sense it involves the establishing of a state of equilibrium between assimilation and accomodation.

3. Assimilation - The integration of external elements into evolving or completed structures of an organism. Likewise, an "object of knowing" is incorporated by the existing intellectual structures as "knowable".

4. 16 Binary Operations - The total number of propositional statements which are the product of two given propositions - form a commutative group, subject to the laws of the INRC group of operations - constitute a "structures whole".

5. Concrete Operations - Characteristics of the first stage of operational intelligence - concerned with a limited extension of empirical reality.

6. Elementary Groupments - Incomplete logical systems governed by the five operations of composition, inversion, identity, associativity, and tautology, of which the operation of tautology is a restrictive condition.

7. Epistemology - The theoretical study of the nature of knowledge.

8. Equilibration - In a biological sense involves an internal regulatory process characteristic of most biological systems - in a Piagetian sense it applies to the process of the development of the intelligence in which a balance is established between assimilation and accomodation. Can also be described as the process by which the individuals environmental interactions are balanced with his autoregulations.

9. Equilibrium - The state of balance between assimilation and accomodation.

10. Formal Operations - Mature, logical, hypothetical - deductive thought characteristic of the final stage of operational intelligence - Develops from concrete operations as a result of the co-ordinating of the elementary groupments into a single system.

11. Formal Operational Schemes - See Operational Schemata.

12. INRC Group of Operations - The means for mentally transforming data about the real world so that they can be organized and used selectively. A group of four transformations also called operations, i.e. identity operation, negation operation, reciprocity operation and correlation operation, which when applied to particular propositional statements gives use to the formal operatory schemes - Serve as heuristic principles.

13. Intellectual Behaviour - The behaviour manifest in an individual as a result of his stage of intellectual development.

14. Intellectual Structures - The internal organization of interrelated and co-ordinated forms of "knowing actions" or schemes which organize reality in terms of concepts such as object, cause, space and time.

15. Intelligence - The total number of possible intellectual co-ordinations or transactions that characterize the adaptive behaviour of the individual towards his environment.

16. Logic - In Piagetian sense - formal system which can be used to describe the intellectual structures of intelligent behaviour - A system of operations carried out on propositional statements.

17. Logic - According to others - "the laws of thought" (Bode); "the theory of inquiry" (Dewey); a system of syntactical calculi" (Russell and Whitehead); a theory of proof as "natural deduction" (Vaughan).

18. Logical Operation - An internal transformation of one propositional statement into another - subject to the laws of a perfect mathematical group, i.e. INRC group of operations.

19. Operation - A reversible, internalized mental action which is co-ordinated with others in an integrated structure - analogous to a logical operation or an internal transformation. If used as a structure, should be termed operational scheme.

20. Operational Schemata - (Inhelder and Piaget, 1958)
Formal Operational Schemes - (Piaget 1969, p. 140 - 144)
The term formal operational schemes has recently replaced the older term operational schemata. However, as constant reference is made to the Inhelder and Piaget (1958) text in this thesis, it is important to note that their meanings are the same. (Recently Piaget has used the term Schema to mean simply the figurative aspect or image of an object). A co-ordinated set of higher order schemes which imply the diverse possibilities implicit in the propositional logic - based on the combinational system - obey the conditions of the INRC group of operations - considered to be examples of equilibrium between the intellectual processes of assimilation and accomodation.

21. Propositional Logic - Operations performed on propositional statements which arise from the commutative group of the 16 Binary Operations, which are subject to the laws of the INRC group of operations.

22. Psychological Functioning - Overt intellectual behaviour which results from and reflects the internal intellectual structures of intelligence.

23. Scheme - The internal general form of a specific knowing activity. Schemes become co-ordinated into higher order schemes, e.g. formal operational schemes, and are collectively included in the term intellectual structures.

24. Stages - Consecutive identifiable periods of the intellectual development of a child.

25. Structured Whole - See 16 Binary Operations.

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25. Structured Whole - See 16 Binary Operations.

CHAPTER II

THEORETICAL BACKGROUND OF THE STUDY
PIAGET'S THEORY OF INTELLECTUAL DEVELOPMENT

The focal point of this Chapter is the presentation of aspects of Piaget's theory of intellectual development relevant to the study. The relevant aspects are the intellectual structures and corresponding psychological functioning of individuals at the concrete and formal operations stages of development. The first part of the Chapter presents the general theoretical framework developed by Piaget, and the aspects of major concern in the theory. The remainder of the Chapter presents a description of Piaget's clinical method used in the study, followed by a more detailed description of the salient aspects of structures and functioning of the intellect at the concrete and formal operations stages.

A. THE PIAGETIAN FRAMEWORK

Piaget address himself to the problem of 'What is Intelligence?' He claims that "every psychological explanation comes sooner or later to lean either on biology or on logic." (Piaget, 1950, p. 3). It is to both these two sources of knowledge that Piaget turns to answer the problem.

Theoretical Constructs

Piaget uses biological concepts at the most general level of his theory to describe processes by which one form of intelligence develops into another. At a more specific level he makes use of symbolic logic as an instrument or technique to analyse and describe psychological behaviour and the structure of thought at different levels of intellectual development.

Piaget defines intelligence as follows:

> Intelligence constitutes the state of equilibrium towards which all the successive adaptations of a sensorimotor and cognitive nature, as well as all assimilatory and accomadatory interactions between the organism and the environment (are directed). (Piaget, 1950, p. 11).

Piaget is describing the development of the epigenetic system in which human intelligence evolves through a series of adaptations or unstable equilibria, towards a final stable equilibrium. Each equilibrium is considered to be analogous to the adaptation of an organism to its environment, involving both environmental interactions and auto-regulations of that organism. In other words, the adaptation of intelligence depends "as much on progressive internal co-ordinations as on information acquired through experience." (Piaget, 1970, p. 703). The final stable equilibrium results when a real balance exists between the individual's environmental interactions and his auto-regulations. Each intermediate, unstable equilibrium develops from the one prior to it, and enables a further equilibrium to be reached. The whole process is called equilibration.

Every knowing activity has a general internal intellectual structure which Piaget refers to as a <u>scheme</u>. It is through the elaboration of schemes that the child develops the ability to deal with his environment. To explain the elaboration of schemes Piaget uses the analogy of accommodation and assimilation of food in an organism or of energy in photosynthesis and explains further his use of the term equilibrium. Assimilation, from the biological point of view, is:

> ...the integration of external elements into evolving or completed structures of an organism...e.g. the assimilation of food consists of a chemical transformation that incorporates it into the substance of the organism; (photosynthesis is the) integration of radiation energy in the metabolic cycle of the plant (p. 706-707).

The incorporating of environmental data is through the process of assimilation. Just as an organism can only assimilate those materials which it can deal with or utilize, so the intellect assimilates only those objects, activities, experiences or ideas with which the schemes can deal. Analogous to the way in which the organism confers a quality on the materials assimilated, <u>the assimilating scheme confers a quality on that which it assimilates</u>. For example a baby has a very simple scheme, the grasping scheme. It assimilates objects as "graspable" and confers the quality of "graspability" on the "object of knowing." If, as a result of the interaction <u>the scheme becomes modified</u> to deal with that particular "object of knowing," then that process is called accomodation. The balance between these two processes describes the adaptation of the scheme to the environment with which it interacts.

As a result of the process of adaptation, a process of differentiation becomes evident. Piaget says that intellectual development can be seen as a process in which the mental organization of a child changes from being undifferentiated through progressive stages of differentiation to clearly and highly differentiated mental processes. (Piaget, 1969, p. 152). As the mental processes become more differentiated, they also become correspondingly more co-ordinated.

The processes of equilibration (including assimilation and accomodation) and differentiation and co-ordination, constitute the general adaptive patterns of a biological nature, applied by Piaget to the problem of epistemology. They will not be considered further in this study. Instead, the processes by which the functioning of the intellectual structures develop will be discussed since this is of greater immediate importance to the present study.

Piaget has attempted to describe the intellectual structures in terms of the logical operations characteristic of logical systems referred to as groups and lattices. Piaget explains his position on the use of the logical operations to describe the structures of the intellect by taking the view that symbolic logic "is the mirror of thought and not vice versa." (Piaget, 1950, p. 27) i.e. logic is the result of man's intellectual thought and can therefore be used to describe, in some measures at least, those intellectual structures which it reflects.

As the child develops intellectually he progresses through a series of identifiable intellectual stages. Whether or not a child has reached a particular stage (or substage) of intellectual development depends on his behaviour when he is faced with particular tasks.

The nature of these behaviours constitute a necessary link between Piaget's theoretical constructs and reality. Piaget describes his theory with the term "axiomatics" by which he means, "axiomatics (can) replace the inductive science which forms the essential link to reality." (p. 28). The psychological functioning or behaviour of an individual is considered to be the empirical, observable manifestation of his internal intellectual structures.

The Piaget Tasks

The tasks are designed to produce inductive, experimental evidence which substantiates the deductive aspects of Piaget's theory i.e. the theoretical constructs. They are based on actions involving the manipulation of simple apparatus. The level of psychological functioning of the subjects is determined from both verbal and overt responses made when attempting a task.

Piaget describes the psychological functioning displayed in performing the tasks as operational. In the psychological sense, "operations are actions internalized, reversible, and co-ordinated into systems characterized by laws...which apply to the system as a whole." (Piaget, 1953, p. 8). In the logical sense, operations refer to the symbolic manipulations involved in logic. Piaget explains psychological functioning which is the manifestation of the intellectual structures in terms of logical operations. This will be explained in greater detail later in this Chapter.

The tasks have been specially designed so as to illuminate different and particular psychological functionings of individuals at different stages of development.

The Stages

The psychological functioning (or behaviour) of individuals has been observed and described by Piaget and his co-workers as consisting of "four main stages...which extend over the period from birth to maturity." (Piaget, 1953, p.9). These stages are the sensory-motor stage (0-2 years), the pre-operational stage (2-5 years), the concrete operations stage (5-11 years), and the formal operations stage (11-15 years and beyond). The ages given are approximate, the important fact being that the order of appearance of the stages is constant.

At about 15 years of age a child should optimally be capable of mature, logical, hypothetical-deductive thought. The manifestation of this sort of psychological functioning is found at the formal operations stage. Rudimentary, incomplete forms of adult psychological functioning typifies the concrete operations stage. It is possible for the developmental sequence of some individuals to stop at the concrete operations stage, or to show evidence of having made the transition to the final stage without having fully attained the level of psychological functioning characteristic of that stage. Factors such as experience, social environment and neuro-physiological conditions play a role in the rate and extent of development.

B. PIAGET'S CLINICAL METHOD

Piaget and Inhelder's investigative technique has been primarily descriptive and analytical. They advocate that a clinical interviewing technique be used in the administration of the tasks, and that the interviewer should attempt to elicit the maximum potential of the subject without unwittingly "teaching" him the answer. The procedure consists of the interviewer showing the apparatus to the subject and then posing the problem to be solved. The interviewer uses his discretion as to the extent of his questioning and its nature. The questioning technique should, however, be as unscheduled as possible in order to provide freedom for the subject to answer to the best of his ability in his own way.

There are a number of drawbacks to the use of the clinical method which have earned Piaget a measure of criticism. Firstly, the method is time-consuming, resulting in a limitation to the number of tests performed and number of subjects used. The necessity of allowing the subjects the opportunity for spontaneous questions and answers makes it difficult to standardize the administration of the Piaget Tasks. It is possible to ensure that the selection of the Piaget Tasks and the conditions are the same, but the questions asked can only be approximately the same in each case. In the introduction to The Child's Conception of the World (1929), Piaget indicates that the subjects' answers should be related to a scale or schedule that serves as a standard of comparison, both qualitatively and quantitatively. While this would serve the real need of standardizing the interpretation or analysis of subject performance on the tasks, no such scale or schedule seems to be available, and the major concern of this study is addressed to this need. The development of a means for standardizing the analysis and interpretation of subject performance on the Piaget Tasks may also be applicable to other situations involving subject performance, for example student performance in educational evaluation.

Further drawbacks in the clinical method arise in situations where the subject is reticent, and does not communicate all his thoughts due to shyness or feelings of inferiority, or where he feels that certain events are too obvious to be commented upon or even cases of disinterest. It is occasionally difficult, in the case of young children, to distinguish play (or romanticizing) from belief. Confusion in a subject can be the result of previous teaching rather than an indicant of his intellectual structures.

While it is clear that there are problems involved in the clinical method used by Piaget, it is probably true that this is the most authentic method for observing the maximum potential of an individual's intellectual functioning, especially when compared with responses to standard tests.

C. STAGE OF CONCRETE OPERATIONS

An individual who reaches the concrete operations stage of intellectual development is able to use operational thought which was not possible at the earlier stages. Operations are "the means for mentally transforming data about the real world so that they can be organized and used selectively in the solution of problems." (Inhelder and Piaget, 1958, p. XIII). Piaget uses the word "operation" in this context to emphasize that the individual performs mental actions which are derived from the "interiorisation" of physical actions. For example, the operation of addition can be performed both physically and mentally, and the mental operation is the result of the interiorisation of the physical operation. Piaget also emphasizes that operations are reversible, and "constitute set-theoretical structures." (Piaget, 1970, p. 705). For example, the addition operation is reversed by the subtraction operation. Particular logical operations, vis. operations of combination, associativity, inversion, identity and closure apply to the logical system of addition and subtraction as a whole, constituting what Piaget calls the "set-theoretical structures."

Intellectual Structures

When describing the intellectual structures of the concrete operations stage in terms of the logical systems of classes and relations, Piaget noticed certain restrictions. The logical system of classes and relations normally form a perfect mathematical set described as a lattice and a group, having logical operations of combination, associativity, inversion, identity and closure (see Table 4). At the concrete operations stage, however, the operation of tautology is also present, and it is this operation which restricts the operation of closure, resulting in a failure to constitute a formal logic or perfect group and lattice structures. (Piaget, 1953, p. 17). The psychological functioning of individuals at this level is correspondingly limited, e.g. in the lack of ability to generalize and the restriction to the handling of concrete subject matter. Piaget has introduced the term elementary groupement to describe the limited kind of logic used by individuals at this stage.

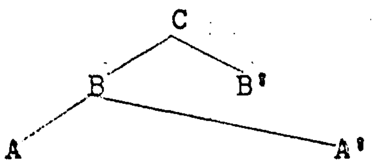
Elementary Groupements of Classes

There are two kinds of systems of classes:

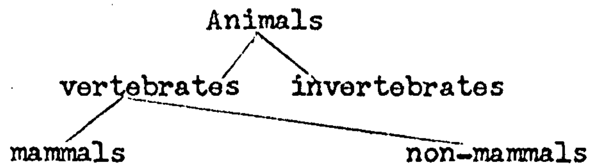
1. Additive Classes are typified by dichotomous biological classifications (see Table 1). A class of objects having certain properties, e.g. a class of all animals, is designated the letter C. The sub-classes of C are designated the letters B and B', and represent vertebrates and invertebrates respectively. Similarly, the subclasses of B are designated the letters A and A', which represent mammals and non-mammals respectively. The sub-classes of B' are not designated, forming what is known as a semi-lattice.

TABLE 1

The Dichotomous Division of Classes into Sub-Classes forming the Additive Groupement of Classes.



Schematic

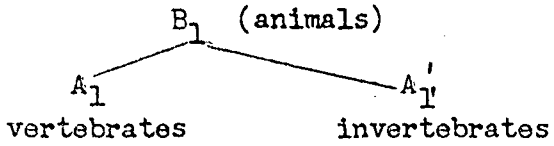


Example

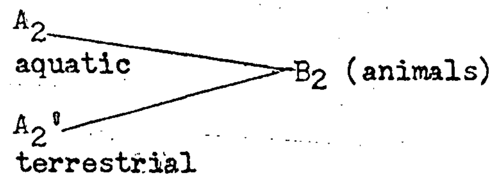
2. Multiplicative Classes result from the multiplication of two classes designated B_1 and B_2, in which all the objects in B_1 are contained in B_2 and vice versa. For example, B_1 represents animals divided into sub-classes of A_1, vertebrates, and A_1', invertebrates. Class B_2 represents animals divided into sub-classes of A_2, aquatic animals, and A_2', terrestrial animals. The product of B_1 and B_2 consists of four different combinations of animals according to their habitat and their structural type (see Table 2). This system of multiplicative classes conforms to the operations characteristic of a perfect group.

TABLE 2

A Two-Way Classification of Classes forming the Multiplicative Groupment of Classes.



A_1 A_2	A_1' A_2
A_1 A_2'	A_1' A_2'



A_1 A_2 = vertebrates, aquatic.

$A_1'A_2$ = invertebrates, aquatic.

A_1 A_2' = vertebrate, terrestrial.

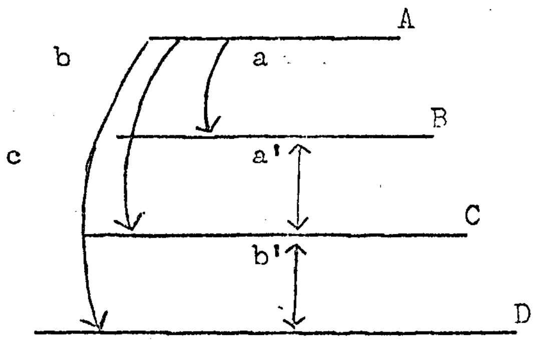
$A_1'A_2'$ = invertebrate, terrestrial.

Elementary Groupment of Relations

The operations involved in the elementary groupements of relations also involve the operations of composition, inversion, identity, tautology, and association (see Table 4) used in the system of serial ordering. For example, a number of objects such as wooden rods may be arranged in order of increasing or decreasing length. The relationships between each two consecutive rods and between each rod and the first rod constitute the system of relations (see Table 3).

TABLE 3

The System of Serial Ordering Forming the Groupement of Relations.



a, b, and c = the relation expressing the difference between A and B, A and C, A and D.

a' and b' = the relation expressing the difference between B and C, and C and D.

The details of the operations of composition, inversion identity, tautology and association at the concrete operations stage will not be discussed in further detail as the present study is more concerned with the stage of formal operation. The reader is referred to Piaget's Traité de Logique (1949) for further information. A summary is provided in Table 4.

Characteristics of the Intellectual Structures

For the purposes of this study the three most important characteristics of the logical structures of the concrete operations stage are:

1. The different operations cannot be integrated with each other. This means that the individual lacks the ability to link operations together to give a truly logical interpretation of his experience.

TABLE 4

Summary of the Operations Involved in the Groupments of Classes and Relations. (The designation of the letters is given in Tables 1, 2, and 3)

Operation	Groupment of Additive Classes	Groupment of Multiplicative Classes	Groupment of Relations
Composition any element in a set may be combined with any other element of the same set, and the logical result is itself an element included in that set	$A + A' = B$ $B + B' = C$	$B_1 \times B_2 =$ $A_1 A_2 + A_1 A_2'$ $+ A_1'A_2 + A_1'A_2'$	$a + a' = b$
Inversion for each element in a set, there is only one element which when added to it effectively nullifies the original operation	$-A-A' = -B$ $A = B - A'$ and $A' = B - A$	$B_1 \times B_2 : B_2 = B_1$ (where: B_2 means eliminating B_2)	$a + (-a) = 0$
Identity for each element in a set, there is only one element which when added to another in the set leaves it unchanged	$A + (0) = A$	$Z : B_1 = Z$ (where Z is the most general class)	$a + (0) = a$
Tautology when an element in a set is added to itself the result is the same element (i.e. the effect is not cumulative)	$A + A = A$	$(B_1B_2) \times$ $(A_1A_2') = A_1A_2'$	$a + a = a$
Associativity elements of a set are put together in different ways to give the same result	$(A + A') + B =$ $A + (A' + B)$	$(B_1B_2) \times (A_1A_2')$ $= (B_1B_2) \times (A_1A_2')$	$(a+a') + b'$ $= a + (a' + b')$
Closure one element in a set when added to another produces a third element which is included in the set	-	-	-
	See Table 1	See Table 2	See Table 3

2. Inversion (or negation) is the most important and the most clearly defined operation. It has the characteristic "of being able to return to (the) original state or starting point...and results from the co-ordination of the actions of combining, dissociating, ordering and the setting up of correspondences." (Piaget, 1953, p. 13). It allows the individual to construct a proposition and then to reverse the direction. This reversibility of thought enables the individual to conserve (or hold invariant under transformation), weight, volume, distance, etc. Inversion involves an annulment of the composition operation to give the original class, in the case of the groupements of classes. For in the inversion operation in the groupement of relations, however, the result of eliminating the difference is a null difference relationship. This gives rise to a statement of equivalence or symmetry, which is better described as a reciprocal operation.

3. The operation of tautology is restrictive, and results in an incomplete operation of associativity, forming an imperfect group and a semi-lattice. A perfect mathematical group has the property of closure, which is not found here due to the restrictions of the tautological operation.

D. PSYCHOLOGICAL FUNCTIONING AT THE CONCRETE OPERATIONS STAGE.

Empirically Oriented Thought

The individual at this stage of development is capable of solving some problems, but in a rather limited way. Piaget says: "Concrete operations consist of nothing more than a direct organization of immediately given data." (Inhelder and Piaget, 1958, p. 249). The empirical or immediately given data may be concrete objects or events, e.g. discs of different colours or shapes, or may be graphic representations or even imaginative representations of actual objects. The important point here is that the objects need to be such that the individual can easily dissociate the objects' properties from his own actions, e.g. different colours or different lengths. These properties "can be objectified more readily" (p. 249) than properties such as weight and density, which cannot be easily represented in drawings or in the imagination. In this way it can be understood why concrete operations can be described as "nothing more than a limited extension of empirical reality." (p.250). At this stage, form such as change in shape or weight cannot be divorced from the subject matter, because the logic available to the individual is insufficiently flexible. The individual is constantly using the empirical facts as the starting point in his reasoning.

Non-Integrated Thought

The individual has no means by which he can integrate his thoughts. The propositions he makes are not integrated within a system having the effect of limiting their usefulness. He can handle the logic of classes and relations, but the mental operations he performs "function only with reference to observations or representations regarded as true, and not on the basis of a mere hypothesis." (Piaget and Inhelder, 1969, p. 132). Furthermore, he is only able to integrate his thoughts by "bringing classes or relations together by a class inclusion or contiguous linkage which moves from one element to the next." (Inhelder and Piaget, 1958, p. 274). In other words the reasoning process is constituted by a step by step progression of thoughts or propositions. These may be obtained simply by "decomposing and recomposing the content of propositions", (p.292) or from combinations obtained by simple "trial and error", (p. 311) or even "haphazard" variations of given empirical data (piaget, 1953, p.19). The individual thus collects all the information concerning the problem with which he establishes an approximation of "the whole picture" and can at least establish the invariance in the empirical facts under transformation.

The lack of integration in concrete operational thought is further seen in the way in which the individual "is only able to introduce or eliminate (a) variable in order to see if (that variable) itself plays an active role, and not as a means of studying the other (variables)." (Inhelder and Piaget, 1958, p.285). Moreover, the limited intellectual ability is observed in the way in which he fails truly to separate variables. The limitation is particularly clear "in cases where a factor cannot be physically separated." (p.284). Once again the empirical nature of the individual's thought is apparent.

Examples of the limitation of unintegrated operations can be shown firstly in the way in which the individual at this stage attempts to understand the concept of proportionality between the weights and lengths on a balance. He may search for a common denominator of the two relations of weights and lengths and instead of suggesting the proportion $W/W' = L'/L$ he may think the relation is additive, resulting in an inequality of differences where $W - W' = L' - L$. He interprets the problem in concrete terms, applying the elementary groupments of additive classes to the relationship between weight and length. (W and L represent weight and length on the one side of the balance and W' and L' represent weight and length on the other side).

Secondly, the individual is able to increase the weight of a pendulum so as to establish the effect of the weight factor. As he lacks the ability to integrate the logical operations he performs, he does not consider the possible effect of different lengths of the pendula while experimenting on the effect of weight.

Non-Generalizable Thought

A further limitation of concrete operational thought is "that it cannot be immediately generalized to all physical properties." (p. 249). The generalizations which are made, are simply "hypotheses which do no more than outline plans for possible action." (p. 251). They are better described as solutions for particular problems or cases which are empirically based, and are an approximation of ' the whole picture'.

E. STAGE OF FORMAL OPERATIONS

An individual at the formal operations stage is capable of using logical hypothetical-deductive thought.

Intellectual Structures

The two main intellectual structures of formal operational thought can be described in terms of operations characteristic of the Combinational System and the INRC Group. The integration of the Combinational System and INRC Group result in what Piaget calls a structured whole which constitutes a truly formal logic. The individual is now able to use propositional logic, and specific combinations of propositional logic which form the formal operational schemes.

The Combinatorial System

The structure of formal operational thought is based on the dual structure of a complete lattice and a perfect group. The operations of the structure of the groupements of classes and relations (previously separated in the semi-lattice and group systems) become integrated, forming the system of the 16 Binary Operations. This is a combinatorial system which is derived from a generalization of multiplicative classification in which both the multiplication of the elements and the resultant products are considered. In addition to the single "n x n" combinations of elements, the products thereby obtained are combined in pairs, triplets, one set of four and a set of zero combinations, giving a "set of all subsets." There are a total of 16 Binary Operations obtained in this way. (See Table 5).

TABLE 5.

The 16 Binary Operations Derived from the Generalized Multiplicative Classifications.

B_1 (animals)

A_1 vertebrates	A_1' invertebrates		
A_1A_2 (1)	$A_1'A_2$ (2)	A_2 aquatic	B_2 (animals)
A_1A_2' (3)	$A_1'A_2'$ (4)	A_2' terrestrial	

$A_1+A_1' = B_1 = A_2+A_2' = B_2$

i.e. vertebrates (A_1) + invertebrates (A_1') constitute the class of animlas (B_1) and aquatic animals (A_2) + terrestrial animals (A_2') constitute the class of animals (B_2).

The multiplication of $B_1 \times B_2$ gives 16 possible binary combinations: (using the numerical representations given in Table 5)

1, 2, 3 and 4	=	The 'n x n' combinations (giving a total of 4)
12, 13, 14, 23, 24, 34, 123, 134, 124, 234, 1234, and 0.	=	The 'set of all subsets' combinations (giving a total of 12).

The 16 Binary Operations are related to each other in the sense that each one can be derived from the others by operations conforming to those of the complete lattice and perfect group. (The restrictive operation of tautology does not apply to the 16 Binary Operations). Piaget's books, Traité de Logique (1949) and Logic and Psychology (1953) give further details on this subject.

The INRC Group of Operations

The combinatorial system is structures by four operations or transformations. Any one combination can be transformed into any other by means of an operation. The operations are those of identity (I), negation (N), reciprocity(R), and correlation (C). They form a group called the INRC group which has a particular structure and properties i.e. the group is commutative. The interrelationships of the INRC group are shown in Table 6.

Properties of the INRC Group

Identity Operation (I)leaves the original proposition unchanged. i.e. $I(pvq) = (pvq)$. The identity operator is also the resultant of every operation and its inverse, i.e. $N(pvq) = (\bar{p}.\bar{q})$.

Negation Operation (N) changes both the signs and the operator of the original proposition i.e. $N(pvq) = (\bar{p}.\bar{q})$.

Reciprocal Operation (R) changes the sign of each statement in the proposition, but keeps the same operator, i.e., $R(pvq) = (\bar{p}v\bar{q})$.

Correlate Operation (C) changes the operator of the proposition, i.e., $C(pvq) = (p.q)$.

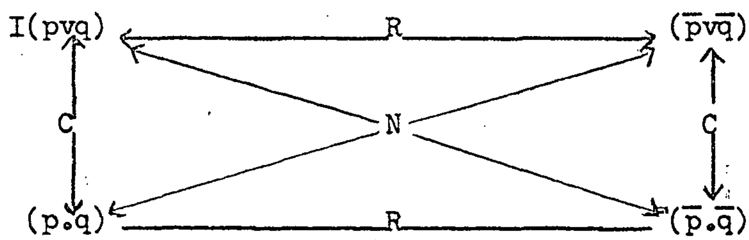
Key: v = either, or both (disjunction),

. = and (conjunction)

$\bar{}$ = not (negation)

TABLE 6

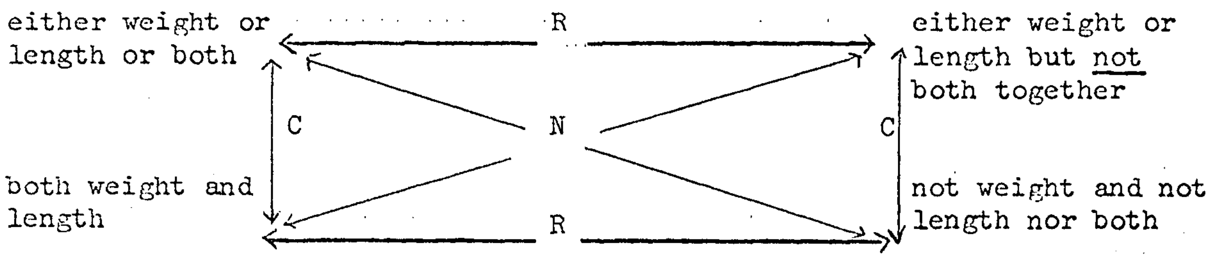
The Interrelationships of the INRC Group, using the Binary Combination (pvq) as the Initial Proposition.



In the task requiring the determination of factors affecting oscillations of a simple pendulum, (pvq) represents the statement that either weight (p) of bob or length (q) of suspending string or both, influence the rate of oscillation. The interrelationships between the INRC transformations is shown in Table 7 using this example:

TABLE 7

An Example of the Interrelationships of INRC Group of Operations in the Simple Pendulum Task.



The INRC group of operations allow the individual to perform operations on propositions themselves without regard to the reality of the content. The mind is no longer limited to operating with the logic of classes and relations characteristic of the concrete operations stage. "The propositional operations ... form a single system such that it is possible to move with accuracy from anyone of its sixteen elements to each of the others." (Inhelder and Piaget, 1958, p.303). The INRC group of operations provide the means for doing this. Piaget refers to these operations as 'Second Order Operations' or 'Interpropositional Operations'.

Propositional Logic

"Propositional logic is a logic of all possible combinations, whether these combinations arise in relation to experimental problems or purely verbal questions" (Inhelder and Piaget, 1958, p. 253). Verbal statements are in fact substituted for objects, and are symbolically represented by letters such as p, q, etc. When the symbolic representations of the verbal statements are connected by what is called a combinatorial operation such as conjunction (.), disjunction (v), implication (⊃) etc., they become propositional variables, e.g. in the propositional statements (p.q), (pvq) and (p⊃q). The logic of "all possible combinations" for the propositional variables p, $\overline{p}$, q and $\overline{q}$ gives rise to sixteen possible combinations forming the combinatorial system, or 16 Binary Operations. The derivation of these has been shown in Table 5. Each propositional statement has particular implications, e.g. $(p \supset q) = (p.q)v(\overline{p}.q)v(\overline{p}.\overline{q})$, which can be represented by a Venn diagram. The propositional statements based on the 16 Binary Operations are given in Table 8, including the implications of each.

Formal Operational Schemes

Formal operational schemes are sets of propositional statements which relate to a particular concept such as proportionality and are strongly linked to each other within the combinatorial system and the INRC group of operations. They are to be considered as representations of the cognitive process of equilibration between assimilation and accomodation. The links between the operations involved in dealing with the concept of proportions, for instance, are established by the individual as a result of his need to interpret the concept in the course of his experiences. "When the need is felt, he

TABLE 8

Propositional Statements Based on the
16 Binary Operations.

Operation	Propositional Statement
1. Disjunction	$(pvq) = (p.q)v(p.\overline{q})v(\overline{p}.q)$
2. Conjoint Negation	$(\overline{p.q}) = (\overline{p}.\overline{q})$
3. Conjunction	$(p.q) = (p.q)$
4. Incompatibility	$(\overline{p.q}) = (p.\overline{q})v(\overline{p}.q)v(\overline{p}.\overline{q})$
5. Implication	$(p \supset q) = (p.q)v(\overline{p}.q)v(\overline{p}.\overline{q})$
6. Non-Implication	$(p \not\supset q) = (p.\overline{q})$
7. Converse Implication	$(q \supset p) = (p.q)v(p.\overline{q})v(\overline{p}.\overline{q})$
8. Negation of Implication	$(q \not\supset p) = (\overline{p}.q)$
9. Equivalence	$(p=q) = (p.q)v(\overline{p}.\overline{q})$
10. Reciprocal Exclusion	$(\overline{p=q}) = (p.\overline{q})v(\overline{p}.q)$
11. Independence	$p\,[q] = (p.\overline{q})v(p.q)$
12. Inverse of Independence	$p\,[\overline{q}] = (\overline{p}.q)v(\overline{p}.\overline{q})$
13. Independence	$q\,[p] = (\overline{p}.q)v(p.q)$
14. Inverse of Independence	$q\,[\overline{p}] = (p.\overline{q})v(\overline{p}.\overline{q})$
15. Tautology	$p*q = (p.q)v(p.\overline{q})v(\overline{p}.q)v(\overline{p}.\overline{q})$
16. Contradiction	$[0]$ = nothing true.

manages to work them out spontaneously." (p.308), providing he is capable of using propositional logic.

There are eight formal operational schemes described by Piaget and Inhelder (1958), namely, combinatorial operations, proportions, co-ordination of two systems of reference, multiplicative compensation, concept of mechanical equilibrium, notion of probability, correlation, and forms of conservation which cannot be empirically verified, e.g. inertia.

In the example of the formal operational schemes of proportionality necessary for understanding the simple balance, the INRC operations are related to each other as shown below:

$$\frac{\text{Increase of wt \& dist. on left arm of balance}}{\text{Increase of wt. \& dist. on right arm of balance}} = \frac{\text{Increase of wt. or distance on left arm of balance}}{\text{Increase of wt. or distance on right arm of balance}} \qquad (1)$$

If an increase of weight and distance on the left arm of the balance is designated by p and q respectively, and an increase of weight and distance on the right arm of the balance is designated p' and q' respectively, then the proportionality schema can be written with the propositional statements representing each of the statements in (1) as follows:

$$\frac{(p.q)}{(p'.q')} = \frac{(p \vee q)}{(p' \vee q')} \qquad \text{...(2)}$$

Applying the INRC group of operations described on page   to the propositional statement (p.q); it is shown that each propositional statement in (2) can be obtained as follows:

$$I(p.q) = (p.q)$$

$$N(p.q) = (p'vq')$$

$$R(p.q) = (p'.q')$$

$$C(p.q) = (pvq)$$

Thus, by substituting the INRC operations into (2) and omitting the propositional statements the following relationship is obtained:

$$\frac{I}{R} = \frac{C}{N} \qquad (3)$$

The proportion involves 'second order operations' (or interpropositional operations) where relations are established between relations, an important feature of operational schemata.

F. PSYCHOLOGICAL FUNCTIONING AT THE FORMAL OPERATIONS STAGE

Five distinctive features of psychological functioning at the formal operations stage have been selected for the purpose of the present study. Firstly, there is the ability to reason by hypothesis, i.e. to reason from a proposition that is assumed, or seems to be a likely explanation or theory. Secondly, there is the ability to think within a framework of related possible ideas. Thirdly, there is the ability to integrate ideas. The fourth characteristic is that of establishing the relationships between the ideas, and thereby consolidating the hypothesis. Lastly, deductive reasoning involves the ability to apply a general principle or hypothesis to particular cases in order to verify the hypothesis. Thinking at the formal operations stage is often referred to as hypothetical-deductive thought. Each of the aspects will be described in further detail below.

Hypothetical Thought

There are different aspects involved in hypothetical thinking which are discussed in order of the most general to the most specific.

An individual is capable of accepting unproven facts as true, or assuming what seems to be a likely explanation, and which he is able to investigate or think about in a systematic logical way. In order to hypothesize, the individual must be able to consider all the logical possibilities including those which may not be physically possible (e.g. in the concept of density it may be hypothesized that weight and volume are independent of each other. In the physical sense, however, volume and weight cannot be separated). Hypothetical thought also involves the ability to deal with objects by using "verbal elements" rather than the objects themselves (Inhelder and Piaget, 1958, p. 252). In other words, an individual uses exclusively hypothetical terms when verbally formulating information which cannot be imagined. "Verbal statements which are simply substituted for objects are used in the construction of propositional logic." (p. 252).

Thinking Within a Framework of Related Possible Ideas

An hypothesis which "seems to be a likely explanation" of a problem must be based on a limited number of possibilities which are logically related to each other. The individual intuitively appreciates the framework which limits the scope of the possibilities.

The individual establishes a particular framework of related possible ideas by first recognizing the most significant variables (operative factors) involved in the hypothesis. For example, in a problem concerning floating objects, the individual may hypothesize that volume and weight are important. The operative factors are firstly that the greater the weight of the object the less the ability to float, and secondly, the greater the volume of the object, the greater the ability to float. The individual can use these operative factors as foundations for the whole framework of possible ideas.

The limitations to the number of possibilities involved are determined by: (1) The logic of the system. Within a particular system, only a certain number of possible propositions can be made. In a binary combinational system (e.g. increase and decrease in volume) there are 16 possible combinations or propositions which form the "related possible ideas". (2) The relevance of the possibilities. Some of the possibilities may be neglected because they have been found to be irrelevant and unimportant, or are obviously so.

The individual must appreciate the implications of particular ideas in order to check their validity against the observable or given facts. He is then able to select those ideas that are valid and summarize them in a statement.

Integrating the Ideas - Control of the Variables

The "related possible ideas" must be integrated by the individual in order to appreciate the implications of each. This integration or co-ordination of operations is essentially the process by which the individual controls the variables involved in the hypothesis. In instances where the variables involved are not physically separable, it is possible to establish the effect of individual variables by using the concept of "all other things being equal". By keeping all the variables equal except one, the individual nullifies their effect, and is free to investigate the effect of one variable at a time.

Establishing the Relationship Between Ideas

The integration of the "related possible ideas" enables the individual to identify the relationships that exist between them. This gives coherence and structure to the hypothesis, making it logically sound.

Deductive Thought

The hypothesis is applied to particular proven cases or facts which are test cases for its verification. The individual must be able to recognize appropriate facts which are relevant to the hypothesis, if it is correct.

G. SUBSTAGES A AND B OF THE FORMAL OPERATIONS STAGE

Substage A is considered a transitional stage between concrete and formal operations. The intellectual structures are present, but in a latent form, and are therefore not functioning adequately. The intellectual structures at Substage B are, however, well established and functional. Piaget maintains that the ability to use the 16 Binary Operations and the INRC transformations develops as a whole. Some aspects of propositional logic and some of the operational schemata may remain latent simply because the individual has not had the opportunity or experience required to make them functional.

The differences in psychological functioning between Substages A and B of the formal operations stage are a matter of degree (Inhelder and Piaget, 1958, p. 120). The individual at Substage A is described as hesitant, inconsistent, relatively unsystematic and uncoordinated, while the individual at Substage B is more certain, consistent, systematic, co-ordinated and capable of sporadic elaborations on what he is doing.

H. SUMMARY

This Chapter covers those aspects of Piaget's theory of intellectual development which are important for this study. The basic theoretical constructs and their implications in the concrete and formal operations stages are discussed in some detail. A good understanding of this part of the theory is required for understanding the inventory and its formulation which is presented in Chapter III. While the inventory is, at this stage, concerned only with identifying formal operational behaviour, it is less meaningful to discuss formal operational behaviour without first discussing concrete operational behaviour which theoretically must always precede it in the developmental sequence.

CHAPTER III

PROCEDURE AND DATA USED IN THE STUDY

A. DEVELOPMENT OF AN INVENTORY FOR MAKING PIAGETIAN INTERPRETATIONS OF INTELLECTUAL PERFORMANCE: FORMAL OPERATIONS STAGE.

The inventory is intended to act as a model and guide for interpreting intellectual behaviour of individuals at the formal operations stage. In the discussion of Piaget's clinical method of administering the Piaget tasks (Chapter II, p. 18), it was emphasized that such a schedule was needed, especially as the clinical method of administering the Piaget tasks obviates any standardized procedures. The inventory is intended as an aid to the classroom teacher for classifying intellectual performance of individuals into Piagetian-like categories of intellectual behaviour in order to indentify their level of intellectual development.

Basis for Development

The salient aspects of Piaget's theory concerned particularly with the stage of formal operations were summarized and analyzed in Chapter 2, pages . It was found that the psychological functioning of individuals could be divided into five main categories representing aspects of Piaget's concept of formal operational thought. (Chapter II, pp 29). The categories are taken to constitute a model of intellectual behaviour characteristically used by an individual at this stage of intellectual development.

Each category is delineated by descriptors of the psychological functioning of individuals at this stage. The descriptors are based on material found in Piaget's writings.

Each descriptor is illustrated, for the purposes of clarification, by two exemplars, obtained from Piaget's descriptions of subjects performing Piaget's tasks. The exemplars depict details of psychological functioning characteristic of the descriptors and, at the same time, illustrating the intellectual structures which underlie and are made manifest by the functions performed. The structures exemplified are the 16 Binary Operations (constituting the combinatorial system) and the INRC group of operations. (See Chapter II, pp. 30-33).

Description of the Inventory

The inventory presented on page 44, is divided into three parts. Part one (1.0) gives a brief description of two main characteristics of the intellectual structure of formal operational thought termed in Piagetian writings as the 16 Binary Operations (1.1) and the INRC Group of Operations (1.2). Part two (2.0) contains five categories of psychological functioning of individuals at the formal operations stage (2.1, 2.2, 2.3, 2.4, and 2.5). Each category is delineated by descriptors to be used for interpreting intellectual performance on selected Piaget tasks and classroom examination items (2.11, 2.12, 2.21, 2.22, 2.23 etc.). Part three (3.0) contains six descriptors (3.11, 3.12, 3.13, 3.21, 3.22, and 3.23) for use in distinguishing between substage A (3.1), the transition stage, and substage B (3.2), the well established stage of formal operations. These descriptors differ from those in the second part (2.00) in the extent to which the functions can be performed and are considered to be a further refinement of the descriptors pertaining to the formal operations stage. (See Chapter II, p. 39).

The inventory is intended for use in interpreting the psychological functioning involved in intellectual performance. To illustrate the use of the inventory, actual performances were recorded on videotape. Transcrips of the recordings were made and then condensed into synopses which were subsequently analyzed for behaviours that could be adequately described by the descriptors. A summary was made of the actual descriptors used and an attempt made to establish whether or not the individual was using formal operational thought, and if so, the substage A or B, at which he was functioning. An individual was considered to be using formal operational thought when the descriptors could be used to describe actually observed behaviour. Since all the descriptors of the inventory, taken as a whole, describe a model of intellectual performance at the formal operations stage, the extent to which the descriptors match actual performance provides information about what aspects of intellectual functioning were used in a particular task. Utilization of the inventory was extended to a comparison of the intellectual performance of which an individual was potentially capable as indicated by performance on the Piaget tasks with performance on examination items.

Piaget tasks are specifically aimed at eliciting the maximum potential of an individual whereas this is not necessarily so for intellectual tasks often given in the classroom, for example. For this reason, the study first illustrates application of the inventory to actual performance on selected Piaget tasks at the formal operations stage. The user of the inventory can then compare the intellectual behaviours elicited by a non-Piagetian task with intellectual behaviours that a person is maximally able to perform at the formal operations stage of intellectual development.

The Inventory

The following categories, descriptors, and examplars constitute the inventory developed and described above.

INVENTORY FOR MAKING PIAGETIAN INTERPRETATIONS
OF INTELLECTUAL PERFORMANCE:
FORMAL OPERATIONS STAGE

Part 1.0: Brief Description of the Logical Structures of the Formal Operations Stage.

1.1 The 16 Binary Operations

All the possible products of two propositions are compiled and the relationship between the elements form a commutative group. (Piaget, 1953, p. 37).

1.2 The INRC Group of Operations

The laws pertaining to the combined structure of the lattice and the group, enabling an individual to transform one proposition into another within the combinatorial system. (See Inhelder and Piaget, 1958, p.134).

Part 2.0: Categories of Psychological Functioning at the Formal Operations Stage.

CATEGORY 2.1: HYPOTHETICAL THOUGHT

2.11 ABILITY TO ACCEPT UNPROVEN FACTS AS HYPOTHETICALLY TRUE, IN ORDER TO DEDUCE THE REAL FROM THE POSSIBLE.
(Piaget 1953, p.18, 19. Inhelder & Piaget, 1958, p. 251).

2.111 I: "How do you know you have to bring the weight toward the centre?"
S: "The idea just came to me, I wanted to try."
(Inhelder and Piaget, 1958, p. 173).
The S has a possible idea or theory in mind which is within the limited framework of the possible interactions of the given variables of increase and decrease of weight on both sides of the balance (designated by p, p, p', p'), and increase and decrease of distance from the fulcrum on both sides (q, q, q' and q'). The interaction of these variables constitutes a combinatorial system.

2.112 S: "If I bring it (weight) in halfway, the value of the weight is cut in half. I know but I can't explain it." (p.173).
The S intuitively appreciates that his hypothesis involves a transformation of a reciprocal nature.

2.12 ABILITY TO CONSIDER THE LOGICAL POSSIBILITIES INDEPENDENT OF THE CONTENT (Inhelder & Piaget, 1958, 252, 293)

2.121 S explains the balance task by saying: "The distance and the weights; it's a system of compensations." (p. 174). S is considering the combinations involved in a general sense which also holds true for the empirical facts. He is not restricted to actually manipulating the apparatus to obtain combinations.

2.122 The statement: "it's a system of compensations" (p. 174) in the balance task involves the S in carrying out the INRC transformations on the possible combinations from which he devises a compensatory relationship, $\frac{I}{R} = \frac{C}{N}$ (p.177).

or $\frac{I\ (p.q)}{R\ (\overline{p}.\overline{q})} = \frac{C(pvq)}{N(\overline{p}v\overline{q})}$ (p. 178)

(where p and $\overline{p}$ designate increase and decrease of weight on one side, and q and $\overline{q}$ designate an increase and decrease of distance on the same side).

CATEGORY 2.2: THINKING WITHIN A FRAMEWORK OF RELATED POSSIBLE IDEAS

2.21 ABILITY TO INTUITIVELY INTEGRATE THOUGHTS WITHIN A SYSTEM OF RELATED POSSIBLE STATEMENTS (Piaget, 1953, p. 39).

2.211 "At the same time that the subject combines the (four different colourless liquids) given in the experimental context, he also combines the propositional statements which express the results of these combinations of facts, and in this way mentally organizes the system of binary operations consisting in conjunctions, disjunctions, exclusions etc." (Inhelder & Piaget, 1958, p. 122). For example, if p and $\overline{p}$ designate the presence and absence of the colour reaction, and q and $\overline{q}$ designate the presence and absence of liquid 4, then a statement may be made that liquid 4 is incompatible with the presence of the colour, i.e. (p/q). Also, if q and $\overline{q}$ are changed to designate the presence or absence of liquid 2, a statement can be made that liquid 2 has no effect on the colour reaction, and is neutral, i.e. (p*q), which is a tautological statement (p. 118-119).

2.212 The statement that p is incompatible with q, i.e. (p/q) (where liquid 4 is designated by q), has particular implications, i.e. $(p/q) = (p.\overline{q})\ v\ (\overline{p}.q)\ v\ (\overline{p}.\overline{q})$. In order to reach the statement p/q, the S must be able to integrate his thoughts which involves using the INRC transformations, e.g. $I\ (p.\overline{q}) = R(\overline{p}.q)$.

2.22 ABILITY TO FORMULATE THE OPERATIVE FACTORS INVOLVED AND ARRANGE EXPERIMENT OR THOUGHT SEQUENCE ACCORDINGLY
(Piaget, 1958, p. 19, Inhelder & Piaget, 1958, p. 250).

2.221 S: "The greater the distance, the smaller the weight should be." (p. 174). In order to say this, S must first have considered all the possible interactions involved in the combinatorial system of variables. He then can arrange his thought sequence in order to experiment, e.g. S: "If I replaced this weight (1 unit) with that one (2 units), it would only go halfway up." (p. 175).

2.222 The statement S: "The greater the distance, the smaller the weight should be," (p. 175) is a proposition which involves the transformation of reciprocity e.g. $I(p.\overline{q}) = R(\overline{p}.q)$, i.e. an increase in weight (p) together with a decrease in distance ($\overline{q}$) has the same effect as a decrease in weight ($\overline{p}$) with an increase in distance (q) on the same side of the balance.

2.23 ABILITY TO INFER THE IMPLICATIONS OF THE STATEMENTS (WITHIN THE FRAMEWORK OF IDEAS), AND SELECT THE TRUE STATEMENTS AND DISCARD THE FALSE, AND SYNTHESIZE A STATEMENT OF NECESSARY AND POSSIBLE CONDITIONS.
(Piaget, 1953, pp. 19, 39).

2.231 In the liquids task, the implications of the statement (p/q) are $(p.\overline{q}) \vee (\overline{p}.q) \vee (\overline{p}.\overline{q})$. (See 2.211 and 2.212 for the meaning of the symbols). S realizes that either colour appears in the absence of liquid 4, or colour disappears in the presence of liquid 4, or colour is absent in the absence of liquid 4, or all possibilities. He then selects these for their validity with respect to the observable facts and then clearly states a synthesis or summary of his reasoning and observations, e.g. S: "Liquid 4 cancels it all (i.e. the colour)." (Inhelder & Piaget, 1958, p. 117).

2.232 In selecting the true statements and discarding the false, the S uses the INRC transformations, e.g. the proposition; $(p/q) = (p.\overline{q}) \vee (\overline{p}.q) \vee (\overline{p}.\overline{q})$ (See 2.211 and 2.212) is checked by carrying out the identity transformation on each statement and comparing it with the observable facts. Each statement would be selected as true in this case, thus confirming the proposition; (p/q).

CATEGORY 2.3: INTEGRATING THE IDEAS - CONTROL OF THE VARIABLES

2.31 ABILITY TO SEPARATE THE VARIABLES BY NEUTRALIZING OR ELIMINATING FACTORS WHICH CANNOT BE PHYSICALLY SEPARATED
(Inhelder & Piaget, 1958, p. 284).

2.311 S: "It's the length of the string that makes it go faster or slower, the weight doesn't play any role." (p. 75). In the pendulum task, the variables are; modification or lack of modification of length (p and p); a modification or lack of modification of weight (q and q), and a modification or lack of modification of the frequency of oscillation in the pendulum (x and x). S: "varies the length of the string with equal weights" (p.76), thereby neutralizing the effect of weight while observing the effect of length. Student also uses different weights on equal string lengths, thereby neutralizing the effect of length. In both cases, weight cannot be physically separated from length. The truth statement made by student is composed of four combinations of the variables.

$$(p.q.x)v(p.\bar{q}.x)v(\bar{p}.q.\bar{x})v(\bar{p}.\bar{q}.\bar{x})$$

This means that with, or without, a modification in weight, a modification in length results in a modification in frequency of oscillation and vice versa. This is better written; p[q]x. In addition, student also holds the other two variables constant, i.e. modification in amplitude (r) and modification in initial force applied (s). Thus the whole truth statement is; p[q.r.s.]x. There are sixteen true combinations (a combinatorial system) implied in this statement (see p. 77).

2.312 In order to neutralize or eliminate factors the student must manipulate the binary operations, i.e. perform the INRC transformations. The decision to hold all the variables constant and then to vary one at a time so as to establish its role involves the transformation of negation, i.e. student may hypothesize weight influences the frequency of the pendulum.

i.e. (using the designation given in 2.71),

$$(p \supset x) = (p.x)v(\bar{p}.x)v(\bar{p}.\bar{x})$$

However in the experimentation student discovers that weight has no effect on frequency of the pendulum, i.e. $(p*x) = (p.x)v(\bar{p}.x)v(p.\bar{x})v(\bar{p}.\bar{x})$. Student realizes that negation is involved, i.e. $(p \supset x) = N\ (\bar{p}.x)$ and therefore discounts his hypothesis $(p \supset x)$ in favour of the experimental evidence that $(p*x)$, saying; "Nothing has changed". (p. 76).

2.32 ABILITY TO CONTROL THE VARIABLES BY STUDYING THE ROLE OF ONE FACTOR BY VARYING ANOTHER (Inhelder & Piaget, 1958, p.285).

2.321 Pendulum task: S: "When it's smaller (i.e. length is shorter), the weight goes faster. It's because I didn't put on the same weight (that I didn't prove anything). Now I'll put on the same weight". (p. 74). Student must therefore actively control the weight variable by ensuring there are equal weights on both pendula. As in 2.311, the 16 Binary Operations must be considered.

2.322 Student experiments with two variables (weight and length) at once, and may hypothesize $(p.q) \supset x$, i.e. weight and length affect the frequency of the pendulum. The possible interaction between weight and length produces a cumbersome number of possibilities to be tested. S may therefore change the hypothesis $(p.q) \supset x$ into $(p \vee q) \supset x$, i.e. weight or length or both affect the frequency of pendulum. This involves a correlation transformation

$$\text{i.e.} \quad (p.q) = C\,(p \vee q)$$

which will alter the subsequent reasoning pattern.

CATEGORY 2.4: ESTABLISHING THE RELATIONSHIP BETWEEN THE IDEAS

2.41 ABILITY TO INTERPOLATE MEANING BETWEEN THE SUCCESSIVE STATEMENTS, ESTABLISHING RELATIONS BETWEEN RELATIONS
(Inhelder & Piaget, 1958, p. 254, 279).

2.411 S initially establishes the relationship between the weights and distances on one side at a time. In order to do this, he works within a combinatorial system, e.g. using p, $\overline{p}$, q and $\overline{q}$, q' and $\overline{q}'$, (see 2.111), he establishes $(p.\overline{q})$ and $(\overline{p}.q)$ will give a balance with $(p'.\overline{q}')$ and $(\overline{p}'.q')$.

2.412 In order to explain the balance system, S has to work out the relationship between these propositions for which he uses INRC group of operations to establish the existence of a compensatory relationship, which in this case is; $\frac{I}{R} = \frac{C}{N}$. (See Inhelder & Piaget, 1958, p. 176-181 for further details).

CATEGORY 2.5: DEDUCTIVE THOUGHT

2.51 ABILITY TO PREDICT THE REAL SITUATION IF THE HYPOTHETICAL CONDITION WAS FULFILLED, AND BY OBSERVING THE CONSEQUENCES, VERIFY THE HYPOTHESIS
(Inhelder & Piaget, 1958, p. 251, 279).

2.511 Angles of incidence and reflection task; S hypothesizes; "The more the target approaches the plunger, the more the plunger must (of necessity) also approach the target." (p. 11). S then predicts a real situation by saying: "For example, if there were a (perpendicular) line here, the ball would come back exactly the same way." (p. 11-12). S then verifies the law by putting the plunger at 45°, followed by "several angles chosen at random" and demonstrates the law of equality of angles of incidence and reflection. In order to establish the statement that the angle of incidence (p) implies the angle of reflection (q), i.e. $(p \supset q)$, S has to consider the four possibilities, i.e. $(p.q) \vee (p.\overline{q}) \vee (\overline{p}.q) \vee (\overline{p}.\overline{q})$. This involves the combinatorial system, and S must establish that $(p.\overline{q})$ never occurs in order to say; $(p \supset q) = (p.q) \vee (\overline{p}.q) \vee (\overline{p}.\overline{q})$
Thus S verifies his hypothesis that $(p \supset q)$ by demonstrating $(p.\overline{q})$ never happens.

2.512 The negation transformation of the hypothesized statement ($p \supset q$) (as in 2.511), is ($p.\bar{q}$, i.e. ($p \supset q$) = $N(p.\bar{q})$. In order to make sense of the possible combinations, S thus uses the INRC group of operations, and, in addition, compiles the logical consequences of a statement such as ($p \supset q$). The verification of such a statement can then be demonstrated empirically by checking the validity of each consequence, which involves using the identity transformation.

3.0 PSYCHOLOGICAL FUNCTIONING AT SUBSTAGES A AND B OF THE FORMAL OPERATIONS STAGE

3.1 SUBSTAGE A

3.11 Approach is hesitant, uncoordinated, unsystematic and uncertain, resulting in non-rigorous proofs and a tendency to jump to conclusions. (Inhelder and Piaget, 1958, pp. 62, 116, 120, 292, 294 310, 311)

3.12 Ability to use concept of "all other things being equal" in a rudimentary manner. (p. 43)

3.13 Tendency to make proofs and generalizations which are restricted to empirical facts. (p. 11, 58).

3.2 SUBSTAGE B

3.21 Approach is systematic, integrated, sure and organized, resulting in exhaustive and rigorous proofs, without jumping to conclusions. (pp. 63, 121, 292, 294, 311).

3.22 Ability to use concept of "all other things being equal" in a general sense. (p. 43-44, 277)

3.23 Tendency to make logical proofs and generalizations based on concept of "logical necessity." (p. 11)

B. STUDENT PERFORMANCE ON THE PIAGET TASKS

Selection of Piaget Tasks

The tasks used by Piaget and his co-workers are described in Inhelder and Piaget's book, The Growth of Logical Thinking from Childhood to Adolescence (1958). The tasks fall into two categories; those which require propositional logic for their solution; and those which involve formal operational schemes. Both propositional logic and formal operational schemes are involved in formal operational thought, and both evolve from the 'structured whole' or unified system of 16 Binary Operations.

Four tasks were selected for purposes of illustrating a possible use of the inventory; two from each category identified above. The Pendulum Oscillation task and the Angles of Incidence and Reflection task require propositional logic for their solution, while the Combination of Liquids task and Balance task involve particular formal operational schemes. The restriction to four tasks was due to the limited time available for the interviews. Only three of the eight formal operational schemes described by Piaget are involved in the Combination of Liquids and Balance tasks. Piaget, however, maintains that once an individual has reached the stage of formal operations he is potentially capable of using all the formal operational schemes. (Piaget, 1953, Inhelder & Piaget, 1958, p. 308). An individual's lack of experience in specific fields may result in particular formal operational schemes remaining latent. The interviewer, using the clinical method, may be able to elicit the functioning of latent formal operational schemes.

The four tasks selected for the purposes of this study are described below in terms of the apparatus involved. The nature of the task, the concept involved, and its relevance to Piaget's theory.

Angles of Incidence and Reflection Task

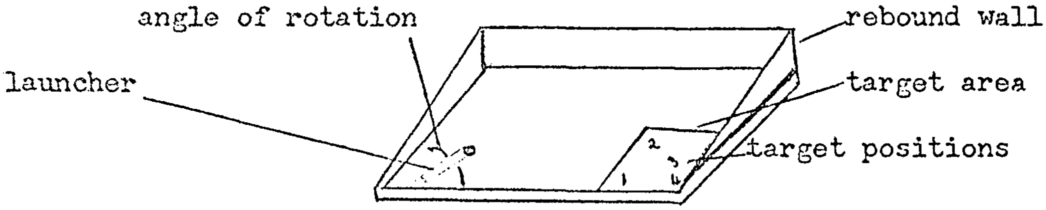


Diagram 1. Pin-ball Apparatus

Apparatus. A square board with a hard plastic-lined rebound wall is used as a type of pin-ball apparatus. Balls are launched with a device consisting of a tube and a spring plunger. The launcher can be pivoted in a restricted area around a fixed point, so that the target cannot be hit directly. The ball is fired against the rebound wall which causes it to reflect toward the target area of the board. A target (a second ball) is placed at position 1, and subsequently moved through positions 2, 3 and 4 by the interviewer (I).

Nature of the Task. The subject (S) is instructed to aim ball in the launcher at the target in position 1. After hitting the target in position 1, the investigator moves it to positions 2, 3 and 4 each time asking S to predict the direction which which the launcher should be aimed, and to provide reasons for his predictions. S is asked to hit the target in each position.

Concept and Relevance. The apparatus involves the reflection of a ball from a hard surface. Under ideal conditions, the angle of incidence will equal the angle of reflection. If the target is moved to position 2, the launcher must be aimed at a new point on the rebound wall, a little to the right of the original point. In this case the angles of incidence and reflection remain equal, but are both increased in size. The same thing happens when the target is moved to position 3. Position 4 lies on the line of reflection of the ball aimed at the target at position 3, therefore no change in direction of the launcher is made (See diagram 2).

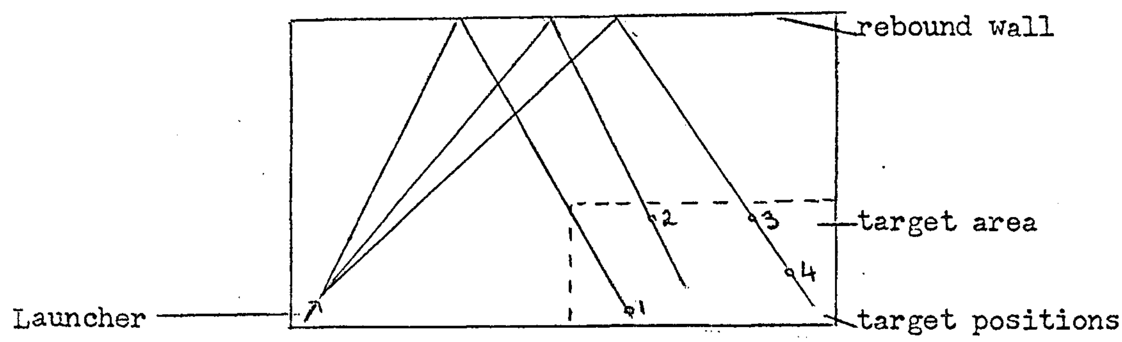


Diagram 2. Positions of target on pin-ball apparatus

A person at the formal operations stage is capable of the propositional logic required to perform the task successfully. i.e. he can make use of the propositional logic involved in reciprocal implications (Chapter II,p. 21). The S is able to reason from the generalization of the law of incidence and reflection to a particular case and is able to verbalize the law in so far as he recognized this necessity. The S at the concrete operations stage can also perform the task, but resorts to a trial and error technique without indicating the formal solution to the problem.

Pendulum Oscillation Task

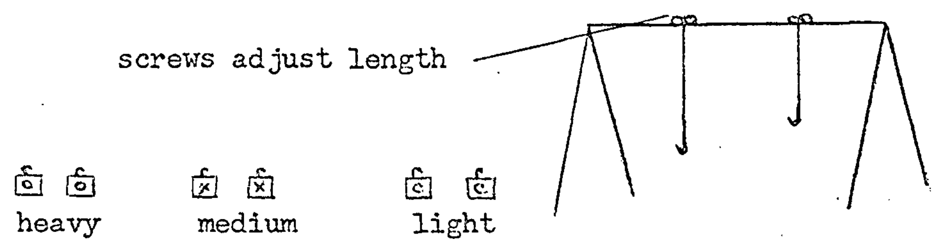


Diagram 3. Pendulum apparatus.

Apparatus. Two pendula consist of two adjustable strings, suspended from a wooden frame with terminal hooks on which different bobs may be hung. The bobs consist of three pairs of wooden blocks; a heavy pair (O), a medium weight pair (X) and a light pair(C). The blocks differ only in weight, not in size or shape.

Nature of the Task. The student is asked to establish which of the four variables (weight of the bob, length of the string, amplitude of swing, effect of initial force on bob) affect the frequency of oscillation of the pendulum.*

Concept and Relevance. The frequency of oscillation of a pendulum at least to a first approximation, is dependent only on the length of the string. In solving the problem the student has to separate and control the variables in order to establish which of them affects the frequency of oscillation. This involves using the concept of " all other things being equal." The variables must then be selected or excluded according to their established effects. This process particularly demonstrates the operation of exclusion, an operation in propositional logic (Chapter II, p. 34).

A student who is not at the formal operations stage is unable to separate and control the variables by using the concept of "all other things being equal." He may vary two factors at once, e.g. weight of bob and length of string, and falsely conclude from his results that only one of the factors affect the frequency of oscillation of the pendulum.

Combination of Liquids Task

Apparatus. Four similar bottles, each containing colourless and odourless

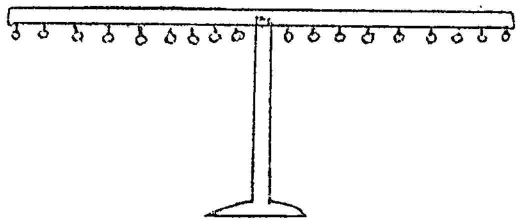
* On occasions the student was asked for the period of oscillation of the pendulum, but this does not alter the task.

liquids, are labelled 1, 2, 3 and 4. Bottle 1 contains dilute sulphuric acid; bottle 2 contains water; bottle 3 contains dilute hydrogen peroxide; bottle 4 contains sodium thiosulphate solution. A fifth bottle labelled 'g' is described as an indicator, and contains a solution of potassium iodide. A supply of small beakers is provided in which the S can combine the liquids. The hydrogen peroxide (3) reacts with the potassium iodide (g) in an acid medium (1) to give a yellow solution (i.e. 1 + 3 + g). The water (2) is neutral, and has no effect on the reaction, but the sodium thiosulphate solution (4) acts as a bleach, and removes any yellow colour that may be formed.

Nature of the Task. The S is shown a beaker containing a 'mystery' colourless liquid which is a combination of (1 + 3) previously prepared by I. S is told that the mystery liquid is obtained somehow, from the given liquids. A few drops of g are added to this liquid, and a deep yellow colour results. S is asked to reproduce the mystery liquid using bottles 1, 2, 3 and 4 as he wishes, and test for it using the indicator. When S accomplishes this, he is asked to distinguish between the effects of liquids 2 and 4, and if possible to guess what they might be.

Concept and Relevance. The solution to this task involves combining the four given liquids in different ways. A combinatorial system is involved, giving rise to 16 possible combinations (Chapter II, p. 34). The S intuitively uses relationships of implication, disjunction and exclusion in solving the problem. It is important that I establish that S is capable of making the systematic combinations rather than using a haphazard, trial and error technique, not characteristic of formal operational thought. I must also ascertain that S is capable of using the formal operation schemes of combinations to establish the role of liquids 2 and 4, which involves using the INRC group of operations (Chapter 2, p. 19).

Balance Task



box of weights

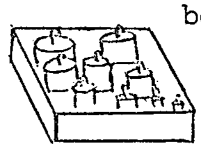


Diagram 4. Balance apparatus

Apparatus. A uniform rod supported by a central fulcrum has nine hooks, uniformly spaced, on each side on which varying weights (standard type) can be hung.

Nature of the Task. S is asked to set up a variety of balanced systems, using the weights supplied. If S fails to do more than set up equal weights at equal distances on both sides of the balance, I then semi-structures the situation by setting up one side and asking S to balance the other side in a variety of ways. S is asked to explain the relationship between weight and distance, for a balance in equilibrium.

Concept and Relevance. I observes S and establishes whether S is aware of and capable of using and understanding the relationship $W/W' = L'/L$, where W and W' are the weights and L and L' are the distances of the weights on the fulcrum. The formal operational schemes of proportionality and mechanical equilibrium are involved in this task (Chapter II, p. 35-36). A person who is not at the formal operations stage would not only be unable to verbalize the law, but would tend to resort to trial and error experimentation. He may successfully use the formula $W/W' = L'/L$ without understanding it. It is essential that I establish the level of understanding of S.

Selection of Students

Fifteen student volunteers from the first year physics course (Physics 110) at the University of British Columbia were invited to take part in the study. They were each asked to do the four Piaget tasks and their performance was recorded on videotape. The performances of three students were then selected from the original fifteen for detailed analysis and interpretation using the inventory. The selection of the three students was based on the success of the interview technique. The elimination of students was due either to incomplete interviews or to unsatisfactory use of the clinical method of interviewing on the part of the interviewer.

Administration of Piaget Tasks

Each interview lasted approximately two hours, the first fifteen minutes of which were spent in informal talk, intended primarily for establishing a friendly, relaxed atmosphere. The interviewer asked questions concerning the student's background in physics, his appreciation of physics, and his plans for his future career. The information was recorded in an informal questionnaire but was not used subsequently, and is therefore not included in the thesis.

The clinical method of interviewing was used. The interviewer introduced the apparatus, posed the problem to be solved and then assumed a sort of non-directive role in which the student was allowed to follow his own line of reasoning. The interviewer asked for clarifications, explanations, justifications and indications of procedure on the part of the student.

The Angles of Incidence and Reflection task involved more direction from the interviewer as more specific questions were asked concerning the different positions of the target.

The Piaget tasks were designed to show the maximum intellectual potential of the individuals. In some cases the interviewer found it necessary to prompt the student to think further, while at the same time taking care to resist the temptation to "teach" rather than "probe."

Method of Recording Piaget Task Performance

All the interviews were recorded on videotape, using the apparatus and facilities of a TV studio. Transcripts were then made from videotapes in which both the comments and actions were noted. The transcripts were tabulated in order to facilitate the analysis and interpretations using the inventory.

Method of Analysing Piaget Task Performance

Transcripts of student performance on the Piaget tasks were first condensed into synopses which summarized the intellectual performance displayed. One full transcript is given in Appendix A, which was cross-keyed to the appropriate synopses for checking purposes. The synopses were analyzed for statements and actions which could be adequately interpreted with one or more of the descriptors in the inventory. The appropriate descriptor(s) used to interpret the statements and actions were recorded, and the inventory code number provided. A summary was made of the descriptors used to interpret the student's performance in each task, separately, and all the tasks taken together. A final summary showed the overall Piaget task performance of all three students.

C. STUDENT PERFORMANCE ON SELECTED EXAMINATION ITEMS

Selection of Examination Items

The final examination in the first year Physics 110 course at the University of British Columbia was used as an example to show the applicability of the inventory to non-Piagetian task situations. In order to assess whether the examination could potentially elicit intellectual performance at the formal operations stage the inventory was used to select examination items requiring formal operational thought for their solution. Before the inventory could be used, it was necessary to eliminate all items which could not be classified into any of the categories of the inventory.

The class instructor and a class tutor were asked to give their own versions of the correct solutions to each item. They were also asked to indicate items involving only recall of information presented in the lectures, tutorials, or textbook.

The solutions to the items involving more than simple recall were then analyzed for reasoning sequences which could be interpreted using one or more of the descriptors. The descriptors used to interpret the expected intellectual performance were recorded and the code number given. A summary was made of the descriptors used to interpret the expected intellectual performance for each selected item.

The complete physics examination and a table showing reasons for selection of individual items is given in Appendix B.

Method of Analysing Student Performance on Selected Examination Items

Student written responses to the selected examination items were recorded, as well as the mark credited them by the examiners. The responses were then analysed for statements and reasoning sequences which could be interpreted using one or more of the descriptors. The descriptors used to interpret the intellectual performance displayed were noted and the code number given.

A summary was made of the descriptors used for each student on each item and on all the items taken together.

D. COMPARISON OF PIAGET TASK PERFORMANCE WITH EXAMINATION ITEM PERFORMANCE AND ASSESSMENT OF RESULTS

Method of Comparison

The analyses and interpretations of the students' overall performances on the Piaget tasks and on the selected examination items were compared for congruency. The appropriate descriptors for each student were summarized in terms of their overall performance on Piaget Tasks and overall performance on the selected examination items. The comparison was considered congruent when adjacent rows either both contained descriptors, or were both empty. Conversely, a description was considered non-congruent when two adjacent rows differed markedly in the number of descriptors listed.

Method of Assessment

The inventory for making Piagetian interpretations of intellectual performance at the formal operations stage was developed for analysing individuals' intellectual performance on the Piaget tasks as well as on selected classroom examination items. Firstly the extent of agreement (congruency) between the model of formal operational thought as represented by the descriptors of the inventory and actual performance of individuals on the Piaget tasks was examined for implications regarding the intellectual functioning potentially available to the individual. Secondly the extent of agreement (congruency) between the model (i.e. the descriptors of the inventory) and actual performance of individuals on the selected examination items was examined for implications regarding the intellectual functioning demonstrated by individuals in a classroom examination situation. Thirdly, the extent of agreement between the Piaget task performance and selected examination item response for each individual was examined in order to compare the intellectual functioning displayed in responding to the examination items with the intellectual functioning of which the individual was potentially capable at that time.

These three comparisons provide information which allows for speculation concerning the intellectual demands of the examination items, and the calibre of the students' intellectual responses to the examination items.

The data thus obtained were used to speculate on the applicability of Piaget's theory of intellectual development to science education evaluation, by means of the inventory formulated for this purpose.

CHAPTER IV

APPLICATION OF THE INVENTORY FOR ANALYSING STUDENT PERFORMANCE ON PIAGET TASKS AND SELECTED EXAMINATION ITEMS

Introduction

The chapter is divided into two parts. Part A contains synopses of the performances of three students on each of the four Piaget tasks and an analysis of each student performance in terms of formal operation thought using the inventory given in Chapter III. Part B contains firstly, the expected responses by instructors of the course for two selected course examination items and an analysis of the expected responses in terms of formal operational thought. Secondly, this part contains an analysis of the responses of the three students to the two selected examination items in terms of formal operational thought apparently used in responding to the examination items. Summaries of the analyses are given for each student on the Piaget tasks and selected examination items. Overall summaries of the student performance on all the Piaget tasks and on both selected examination items are given at the end of Part A and Part B respectively.

A. THE PIAGET TASKS

Method of Analysis and Reporting of Results

The information synopsized in this part of the chapter was obtained from written transcripts made from videorecordings of actual task performance. A full transcript of the performance of Student L. W. on all the Piaget tasks is given in Appendix A, and is coded for cross-reference to the corresponding synopses given in this section. The reader may use these transcripts for the purpose of checking the accuracy of the synopses. Transcripts for the remaining two students have not been included in the Appendix for reasons of brevity.

The synopses record all the statements and actions of the students considered significant for the task at hand. Repetitions and asides were omitted, and lengthy explanations were summarized. The synopses include descriptions of student performance that reflect as accurately as possible the mode of thought used by the students when doing the tasks.

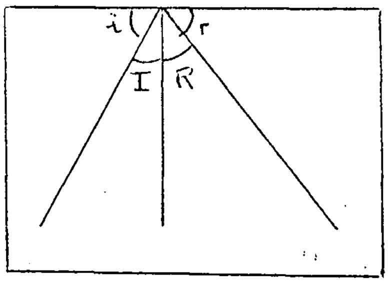
The analyses were made by carefully reading the synopses and extracting and collating from them those sections which could be described by one or more of the descriptors of the inventory. The analysed sections were then placed in a table with the appropriate descriptor listed alongside. For purposes of clarity, some of the descriptors were elaborated to indicate in which way they were found to be appropriate to the analysed statements or actions. The elaborations are indicated by means of parentheses. The analyses given in the table are cross-keyed for reference to the appropriate line of the synopses. Tables 9-12, 14-17, and 19-22 contain the results of the analyses of student performance on the Piaget tasks.

Summaries of the analyses of student performance on the four Piaget tasks are presented in table form. The tables contain the descriptors used to describe performance on each task. An overall summary of the descriptors used to describe student performance on all the tasks is also presented. Tables 13, 18 and 23 give the summaries for individual tasks and Table 24 presents the overall summary of student performance across tasks.

The summaries are compiled in this way in order to facilitate a comparison between the students intellectual performance on the Piaget tasks and performance on the selected examination items (Table 38).

In some instances of student performance a descriptor could only partially be applied. In order to indicate a doubt about the applicability of the entire descriptor to such a case, the descriptor code number used to identify the descriptor applied is qualified by means of parentheses. In Table 15 code number (2.11) is a case in point. When the overall summaries were made the qualified descriptors were not noted when the same descriptors was found adequate in describing other aspects of student performance in the same task.

For convenience certain abbreviations and symbols have been adopted in recording the data. The symbols $\angle I$ and $\angle R$ refer to the angles of incidence and reflection with respect to the normal to the rebound wall, whereas, $\angle i$ and $\angle r$ refer to the angles of incidence and reflection with respect to the reflecting surface or rebound wall.



Different liquids in the combination of liquids tasks were referred to by 1, 2, 3, and 4, and g (the indicator) while combinations are indicated by expressions such as (1 + 3 + g).

In the balance task the arrangements of the weights on the hooks on both the left and right side of the fulcrum are written as equations. For example, 100gm x 8L = 50gm x 4R means that a 100gram weight was hung on the eighth hook from the fulcrum on the left side of the fulcrum to balance a 50gram weight hung on the fourth hook from the fulcrum on the right side.

The dashes (--) indicate hesitations in speech by the student, whereas dots (...) indicate that words have been omitted.

TABLE 9

SYNOPSIS AND ANALYSIS OF PERFORMANCE ON THE ANGLES OF INCIDENCE AND REFLECTION TASK: STUDENT J.V.

Synopsis

The student (S) explains firstly that, "you have to keep the angles the same" and then demonstrates with a pencil held perpendicular to the rebound wall that a particular point on the rebound wall must be found so that the ball rebounds. Student has some difficulty in verbalising exactly what she means and recalls in Grade 11 she was told to think in terms of the angles to the perpendicular. Student attempts to explain again the necessity of the two angles ($\angle I$ and $\angle R$) being the same.

Student aims at target 1 and correctly adjusts the launcher to give a near hit. Student correctly adjusts launcher to the right for hitting target 2, and explains easily that the total angle ($\angle I$ + $\angle R$) is now greater, the point of rebound must be shifted to the right so that the angles (of incidence and reflection) are equal.

After further questioning by the investigator (I), student maintians that the angles of incidence and reflection are always equal when a hit is made. Student is a little confused about the changing size of the total angle with different positions, but is capable of working it out. Otherwise student clearly understands the proportionality, reciprocity and equality operations involved in the task.

Analysis

Aspects of Student Performance	Descriptors Used	Descriptors Code Numbers
1. Student immediately resorts to a generalization of the equality of angles of incidence and reflection, and the role of the perpendicular, due partly to recall (5). Student confidently states later on that the angles are always equal for any position of the launcher (9, 10).	1.a. Tendency to make logical proofs and generalizations based on concept of "logical necessity".	3.23
	b. Ability to consider the logical possibilities (of the equality of the angles) independent of the content (before handling the apparatus).	2.12

Table 9 (continued)

Aspects of Student Performance	Descriptors Used	Descriptors Code Numbers
2. Student has some difficulty in verbally explaining the concept, but is competent in adjusting the launcher direction for different target positions. Student is capable after some thought (6) of explaining the change of size of the angles and change in point of rebound, in order to keep the angles of incidence and reflection equal (11).	2.a. Ability to intuitively and explicitly integrate thought within a system of related possible statements (concerning angle equality, angle size, and point of rebound.)	2.21
	b. Ability to formulate operative factors involved (in concept of the equality of the angles in order to think through the implications for the angle size and point of rebound).	2.22
	c. Ability to infer the implications of the statements (of the equality of the angles), and select the true and discard the false statements in order to synthesise a statement of the necessary and possible conditions (i.e. to establish the change in angle size and change in rebound point corresponding to the change in launches direction).	2.23
3. Student clearly understands the proportionality, reciprocity and equality operations involved in the task (12).	3. Ability to interpolate meaning between the statements (concerning equality of angles, change in size, change in rebound point), establishing relations between relations. (i.e. proportionality, reciprocity and equality.)	2.41

TABLE 10

SYNOPSIS AND ANALYSIS OF PERFORMANCE ON THE
OSCILLATION OF A PENDULUM TASK:
STUDENT J.V.

Synopsis

S tests for the effect of length on the frequency of oscillation, and says, "Put the same mass on both and displace them the same amount, I hope." She speculates that "the short one would ... be faster." After experimenting, S concludes that the frequency of oscillation "varies one over the length somehow." S then tests the effect of different masses, and makes the lengths of the strings equal. She concludes that mass has no effect on the frequency of oscillation. When testing for the effect of impetus, S ensures that amplitude is constant as well as length. At first S cannot decide on the effect of impetus, but after establishing that different amplitudes did not affect the frequency, she repeated the impetus test. S refers to the fact that she has two different masses, but adds that "that doesn't affect it". S concluded that "frequency varies inversely to the length." "It doesn't vary with mass or amplitude" ... and impetus "doesn't seem to make any difference." S thus successfully completed the task and excluded each of the noneffective variables.

Finally I gives S a brass and a wood cylinder and asks if the frequency of oscillation would be affected. S replies, "No, if you keep all the other variables constant," and is sufficiently convinced that no further testing is required.

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S immediately recognizes the problem involved and first tests the effect of length on the frequency of oscillation, and explicitly ensures that the mass and amplitude are held constant. (1-2).	1. a. Ability to use concept of "all other things being equal" in a general sense.	3.21
	b. Ability to separate the variables (not being tested) by neutralizing their effect (through making them equal on both pendula).	2.31
	c. Ability to control the variables by studying the role of one factor, (e.g. mass) by varying the others, (such as length, amplitude and impetus).	2.32

TABLE 10 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
	d. Ability to intuitively integrate thoughts within a system of related possible statements (concerning the interaction of the four variables).	2.21
	e. Ability to formulate operative factors involved (in that the effect of each factor has to be tested independently to make sense out of experimentation), and arrange experiment accordingly.	2.22
	f. Approach is systematic, integrated, sure, resulting in exhaustive and rigorous proofs, without jumping to conclusions.	3.22
2. S concludes that the frequency of oscillation varies inversely to the length of the pendulum (3). S establishes that neither mass, impetus, nor amplitude have any effect on the frequency of oscillation.	2. Tendency to make logical generalizations based on concept of "logical necessity."	3.23
3. S is sufficiently convinced that mass does not affect the period of oscillation that she indicates that using different masses while testing for the effect of impetus will not make any difference (9). Also S does not feel it necessary to retest the effects of the brass and wood cylinders (14)	3. Ability to consider the logical possibilities (implications) of the non-effect of mass on frequency of oscillation) independent of the content (ie. the fact that the masses were actually different).	2.12

TABLE 11

SYNOPSIS AND ANALYSIS OF PERFORMANCE ON THE COMBINATION OF LIQUIDS TASK: STUDENT J.V.

Synopsis

S begins by adding the indicator to liquids 1, 2, 3 and 4 separately. After hesitating because she thought the mystery liquid was said to come from one bottle, she says, "So I will have to try all possible combinations." She says she will combine (3 + 4), (1 + 2), (1 + 3), (2 + 4), (1 + 4), (2 + 3), then (1 + 2 + 3), (4 + 2 + 3), and (1 + 2 + 3 + 4). I advises S to write down the possible combinations, which she does in an orderly, systematic fashion. S then adds liquid combinations, each time referring to her list of possibilities. She takes (1 + g), then (3 + g) and (4 + g), then (2 + 3 + g). S thinks, then adds (1 + 3 + g) and gets positive reaction. S asks, "Shall I keep trying for more?" and is told to find out "to her satisfaction" the answer to the problem.

S tries a combination of (2 + 4 + g), then (1 + 4 + g), getting no reaction. S then thinks, and decides to add (3 + g) and (1 + 2). S comments that the colour should change, because the mixture already contains (1 + 3), "unless 2 stops it from changing."

S then combines (4 + 3 + 2 + g) and proposes to combine (1 + 2 + 4 + g). S is asked how many possible combinations there are. She records 14 possibilities, omitting zero and (1 + 2 + 3 + 4), and comments that she can remember doing something similar in Grade 9.

S is asked to identify 2, and to differentiate it from 4. S replies that 2 "doesn't seem to react with anything else" and proposes to add (1 + 3 + 4 + g) to see if 4 is the same as 2. She combines them, gets no reaction, and concludes that 4 is different from 2, because 2 with (1 + 3 + g) changed colour, and 4 with (1 + 3 + g) stopped the colour change.

S refuses to guess what liquid 2 might be, and adds that "you could get to know more about chemistry working around here."

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S recognizes that a number of liquid combinations are possible. She writes them out systematically and proceeds to test each possibility methodically	1. a. Ability to consider logical possibilities, (i.e. combinations), independent of the content, (i.e. independent of the actual combinations).	2.12

Table 11 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
referring to the list (1-5).	b. Ability to intuitively integrate thoughts within a system of related statements (i.e. possible combinations of liquids).	2.21
	c. Approach is systematic, integrated and sure, testing is rigorous, without jumping to conclusions.	3.21
2. S hypothesises the role of 2 when added to (1 + 3 + g), commenting that the colour should change, unless 2 stops it (from changing)." (11).	2.a. Ability to formulate operative factors involved (in considering the role of 2) and arrange experiment accordingly.	2.22
	b. Ability to infer the implications of the statements (concerning the possible role of 2).	2.23
	c. Tendency to make logical generalizations based on concept of "logical necessity."	3.23
3. S establishes the difference between 2 and 4 with controlled experimentation. i.e. S first establishes that (1 + 3 + 2 + g) produces the colour reaction, and then compares the effect with that of 2 by adding (1 + 3 + 4 + g). (15-19).	3.a. Ability to control the variables by studying the role of one factor (i.e. 2 or 4) by varying another (i.e. in this case using (1 + 3 + g) in both cases.)	2.32
	b. Ability to interpolate meaning between successive statements, (e.g. concerning the difference between 2 and 4) establishing relations between relations.	2.41

TABLE 12

SYNOPSIS AND ANALYSIS OF PERFORMANCE ON THE BALANCE TASK: STUDENT J. V.

Synopsis

S begins by balancing 100 gm x 8L = 50 gm x 4R, but immediately recognizes her mistake and corrects herself with 50 gm x 8L = 100 gm x 4R. S writes the formula $F_1 \times D_1 = F_2 \times D_2$, and explains her balance saying "the distance in this case (left side) should be twice as large as that one (right side). The mass is half as large so they should be equal." I asks for further variations, S tries 80 gm x 1L = 40 gm x 2R and explains saying that the formula always works. I puts 30 gm x 2L, S thinks and then balances it with 20 gm x 3R, and when I suggests changing round the masses, S replies that it couldn't be done as it would be heavier on the right side. On being asked for reasons, S says "I don't know," but on further questioning adds that the formula means "that the further away from the fulcrum that you put a certain weight, the more it would bring it down...the weight of the ruler would also be acting...don't know exactly why."

I moves one mass further out and asks "What am I doing?" She says, "Well you're increasing their weight almost, except you're not really...You're making the ability to bring it down on that side greater." On being asked to balance 30 gm x 4L, S says "that's 120 gm so that would be 20 gm x 6R." I adds an extra 10 gm to the 30 gm that is balanced (30 gm x 4L = 20 gm x 6R), S says "You're just increasing the weight, so I could add to it."

On further questioning S shows she understands very well the proportions, multiplicative compensation and mechanical equalibrium involved in the balance task. Although S could not give exact reasons, in terms of force for example, for explaining the balance, it was evident that her reasoning was clear and well organized. She worked rapidly and easily.

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S shows ability to accurately balance the apparatus in each example provided by I, and S explains say-	1. a. Ability to intuitively integrate thoughts within a system of related possible statements (concerning increase and decrease of	2.21

Table 12 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Number
ing: "The distance in this case (left) should be twice as large as that one (right). The mass is half as large, so they should be equal." (3) "The further away from the fulcrum that you put a certain weight, the more it would bring it down, you're making the ability to bring it down on that side greater" (11).	weight and distance on both sides.)	
	b. Ability to formulate operative factors involved (i.e. interaction of distance and weight), and arrange experiment accordingly. (i.e. set up the balances.)	2.22
	c. Ability to consider the logical possibilities (of the interaction of distance and weight) independent of the actual apparatus.	2.12
	d. Tendency to make logical generalizations based on the concept of "logical necessity."	3.23
	e. Ability to predict the real situation (using the general formula), and by observing the consequences (check on its validity).	2.51
2. S understands the concepts of proportionality, mechanical equilibrium and multiplicative compensation, e.g. in saying that if the weight is increased on one side, the weight on the other side can either be increased or moved outwards (8-11).	2. Ability to interpolate meaning between statements concerning the increase and decrease of weights and distances, establishing relations between relations, (i.e. in formula W/D' = W'/D.)	2.41
3. S refers to the formula $F_1 \times D_1 = F_2 \times D_2$. While she cannot verbalize exactly what it means in terms of force, her reasoning is nevertheless clear and organized when using weight and distance (2,17).	3. Approach is systematic, integrated and sure, showing no tendency to jump to conclusions.	3.2;

TABLE 13

SUMMARY OF PERFORMANCE ON PIAGET TASKS: STUDENT J.V.

Task	DESCRIPTORS USED														
	Formal Operations									Substage A			Substage B		
	2.11	2.12	2.21	2.22	2.23	2.31	2.32	2.41	2.51	3.11	3.12	3.13	3.21	3.22	3.23
Angles		+	+	+	+			+							+
Pendulum		+	+	+		+	+						+	+	+
Liquids		+	+	+	+		+	+					+		+
Balance		+	+	+					+				+		+
Combined		+	+	+	+	+	+	+	+				+	+	+

The table shows Student J.V. is capable of using formal operational thought at the substage B level.

TABLE 14

SYNOPSIS AND ANALYSIS OF PERFORMANCE ON THE ANGLES OF INCIDENCE AND REFLECTION TASK: STUDENT B.H.

Synopsis

S begins by explaining that the midpoint between the firing ball and the target ball must be found, and extrapolated to the rebound wall to form the point at which the launcher must be aimed. As explanation, S says, "The angle of incidence equals the angle -- well, it rebounds symmetrically off -- well, the rebounds are equal -- er -- the angles are equal." The angle of incidence equals the angle of reflection."

On being asked to demonstrate these angles, S hazily indicates ∠I and ∠R. As he seems muddled, he is asked again, and the second time he describes the angles made with the rebound wall (i.e. ∠i and ∠r). S admits he is recalling his experience with the light experiment and with playing pool.

S is asked to hit the ball at target 1. He sets up the launcher using imaginary angles, fires, misses to the right, S explains he is trying to hit the halfway mark. S correctly resets the launcher to the left, fires and misses.

S explains his actions saying, "Well if the angles are supposed to be equal, and if you make this angle (∠i) smaller, then you have to have a smaller angle here (∠r). Then it will go off in that direction right. If this angle (∠i) is larger then this angle is larger (∠r), and it (ball) should come further in (to left)."

I moves the target to position 2, S correctly readjusts the launcher to the right and explains as above. I moves the target to position 3, and S says, "I have to move it (launcher) to the left...(thinks) -- er -- No, not to the left, to the right."

On being asked what happens when the launcher is aimed at 90°, S says, "it should bounce right back."

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S quotes the general law "The angle of incidence equals the angle of reflection." (4) and is able to use the law in subsequent reasoning (11-13).	1. Ability to consider logical possibilities independent of the content (i.e. general use of law incidence and reflection.)	2.12

Table 14 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
2. S aims the launcher in the correct directions for hitting the target ball. (15-17).	2. Ability to intuitively integrate thoughts within a system of related possible statements (concerning sizes of the angles of incidence and reflection, and the direction of the fired ball).	2.12
3. S explains his actions saying, "Well if the angles are supposed to be equal, and if you make this angle (∠i) smaller, then you have to have a smaller angle here (∠r). Then it will go off in that direction (right). If this angle (∠i) is larger then this angle is larger (∠r), and it (ball) should come further in (to left" (11-14).	3.a. Ability to formulate, operative factors involved (i.e. size of angles, direction of ball), and arrange experiment and thought sequence accordingly.	2.22
	b. Ability to infer the implications of the statements (concerning size of angles, direction of ball), select the true statements and discard the false, and synthesize a statement of necessary and possible conditions.	2.23
	c. Ability to interpolate meaning between the successive statements, establishing relations between relations (reciprocal implication and equality of the angles).	2.41
	d. Ability to predict the real situations if the hypothetical condition was fulfilled (i.e. the equality of the angles), and by observing the consequences verify the hypothesis.	2.51
	e. Tendency to make logical proof and generalization based on concept of "logical necessity" (i.e. equality of the angles, ∠i and ∠r).	3.23

TABLE 15

SYNOPSIS AND ANALYSIS OF PERFORMANCE ON THE
OSCILLATION OF PENDULUM TASK:
STUDENT B.H.

Synopsis

Student begins by announcing that he intends to establish the effect of weight on the frequency, and proposes using different weights and equal lenths. He predicts that the heavier one should decrease the frequency. As explanation, student says, "In order to compare -- you have to have something -- that is constant."

While experimenting, although there is very little difference in frequencies of the two bobs, student tends to rely on his own convictions (hypothesis) rather than the experimental evidence. On being given the wood and brass cylinders student still maintains the light one has a higher frequency of oscillation, but admits that "...it doesn't seem to be as much as I thought it would be." Student then shortens both strings, tries again and concludes "They're oscillating at a higher frequency." and later adds, "Shortening the strings seems to decrease the accuracy of it ...if you decrease the length, the frequency seems to be closer together -- they swing more in time." Student admits that he has not justified his hypothesis that weight affects the frequency of oscillation.

Student then proposes to establish the effect of length of string on the frequency and experiments with one pendulum only, using the brass bob on a long string and a short string. Student claims he is simply getting an impression of the effect. He concludes: "As length decreases, frequency increases," and reasons that the different effect of different weights would be "harder to see" when using short strings.

Student is asked about the other variables. He proceeds to test the effect of impetus, taking care to ensure the lengths are the same, by using one pendulum, but inadvertently uses very different amplitudes, which is pointed out. Student is asked how he is measuring the difference, and he replies "just by the speed and the distance it moves." Instructor suggests that student use two pendula, and student them makes the bobs the same. After many tries student claims that he can tell nothing from his experimentation. He reasons that amplitude doesn't seem to have any effect, and if one pendulum is given "some sort of acceleration -- (thinks) -- that shouldn't affect it either -- if the amplitude didn't matter -- giving it a push is the same as changing the amplitude."

Student is asked to demonstrate conclusively his hypothesis that both length and weight affect the frequency of oscillation. Student suggests, "If I put a lighter mass on a small string, and a heavier mass on a long string, then it should amplify the effect." Student tests his hypothesis, and concludes: "The short string and light mass seem to have a much higher frequency than the large string." Instructor suggests interchanging the bob. Student says the effect sould be about the same, "depending on how they vary." Student then refutes his statement, saying that the short

TABLE 15 (continued)

string with heavy bob "will still have a higher frequency," explaining that "the difference in weight isn't that much, and the difference in the string is." Student is asked to prove conclusively his hypothesis concerning the effects of length and mass, He attempts to do this, but insists on varying both the length and weight together, and becomes more and more confused. Student eventually asks "What did I say?" then says "I thought mass didn't have anything to do with it." "I was working on an assumption -- I was trying to go back in my memory and I guess my memory was wrong".

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S tends to rely on his own convictions (hypothesis) rather than the experimental evidence (4). S finally claims "I was working on an assumption -- I was trying to back in my memory, and I guess my memory was wrong." (31).	1.a. Ability to accept unproven facts as hypothetically true, but without deducing the real from the possible.	(2.11)
	b. Ability to consider the logical possibilities (of weight and length affecting the frequency of oscillation), independent of the content, (i.e. the experimental evidence).	2.12
2. S reasons that if the amplitude has no effect on the frequency, then impetus will also have no effect, but does not succeed in demonstrating his point (19-30).	2.a. Ability to consider logical possibilities (concerning effect of amplitude and impetus), independent of the content.	2.12
	b. Ability to intuitively integrate thoughts (concerning effect of amplitude and impetus) within a system of related possible statements.	2.21
	c. Ability to formulate operative factors involved, (i.e. in amplitude and impetus) and arrange thought sequence accordingly (but not his experiment).	(2.22)

TABLE 15 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
	d. Ability to infer the implications of the statement, (i.e. that amplitude has no effect on the frequency) and select the true statements and discard the false, and synthesize a statement of necessary and possible conditions (i.e. con-concerning effect of impetus on frequency).	2.23
	e. Ability to interpolate meaning between the successive statements, establishing relations between relations, (i.e. interconnections between amplitude and impetus and their effect on frequency of oscillation).	2.41
3. S proposes establishing the effect of weight on frequency, by using different weights and equal lengths (1), but does not actually do it. S tests effect of impetus taking care to ensure the lengths are equal, forgetting to check amplitude (15). S attempts to check effects of weight and length on frequency by varying both together (29)	3. a. Indicated ability to separate the variables (of length and weight), but neutralising the effect of length (by making lengths equal).	(2.31)*
	b. Ability to control the variables by studying the role of one factor (weight) by varying another (length).	(2.32)*
	c. Inability to actually separate or control variables.*	

TABLE 15 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
4. S suggests, "If I put a lighter mass on a small string and a heavier mass on a long string, then it should amplify the effect" (24).	4.a. Ability to interpolate meaning between successive statements concerning effects of weight and length, establishing relations between relations (even though based on an inaccuracy.)	2.41
	b. Ability to predict the real situation if the hypothetical condition was fulfilled, but unable to verify hypothesis from observing consequences, due to basic inaccuracy.	(2.51)
5. S shortens both strings while testing effect of different weights on frequency (7). S claims he measures the effect "by simply getting an impression of the effect" (12) and by measuring "just the speed and the distance it moves (17). S concludes, "As the length decreases, the frequency increases" and reasons that the different effect of different weights would be "harder to see" when using short strings (14). S tests effect of impetus by specifically keeping a constant length, but inadvertently varying amplitude (15). S concludes; "The short string and light mass seem to have a much higher frequency than the large mass and long string" (24). S insists on varying both length and weight together, becoming more and more confused (29).	5.a. Approach is hesitant, uncoordinated, unsystematic and uncertain, resulting in non rigorous proofs and a tendency to jump to conclusions.	3.11
	b. Ability to use concept of "all other things being equal" in a rudimentary manner.	3.12

TABLE 16

SYNOPSIS AND ANALYSIS OF PERFORMANCE ON THE COMBINATION OF LIQUIDS TASK: STUDENT B.H.

Synopsis

S begins by suggesting that he "will try each one and see if they turn yellow with the indicator. I guess it's by trial and error. It can be a mixture can it?" S tries (4 + g) and (3 + g) then (2 + g) and (1 + g) and (3 + 4 + g). He decides "It's going to be a mixture. Well I guess I could try a mixture of all four plus indicator" and mixes (1 + 2 + 3 + 4 + g). Then tries (4 + g) + (2 + g) then (2 + g) + (3 + g) + (1 + g) .

On being asked if S can say anything more about the liquids, he replies: "Well, something must happen between those three, so the indicator can show that." S is asked if 1, 2, and 3 are all important and he decides to test (1 + 2 + 4 + g), commenting: "I'm just trying to think of a way-of all the possibilities." I suggests that he writes them down. He writes 1, 2, 3 4, 12, 13, 14, 23, 24, 34, 123, 345, 134 and 124, commenting; "I haven't tried them all."

S then adds 4 to (1 + 2 + 3 + g) and comments "It's not that." He then tries (1 + 2 + 4 + g) and (2 + 3 + 4 + g), and in passing notes that (1 + 3 + g) give the positive reaction.

S is asked about the role of 2. S replies "1, 2 and 3 first showed colour, 1 and 3 showed colour, so 2 can't have any effect."

On being asked to distinquish between 2 and 4, S tries (1 + 3 + g), gets the colour, adds 4, and finds colour is removed. He concludes, "2 is different from 4 in that 2 in combination with 1 and 3 gives the yellow colour but 4 in combination with 1 and 3 does not. Therefore they're different." S suggests 2 could be water.

S is asked how many possibilities there are. He makes guesses such as 10 and suggests 4^2, but notes that he has 14 on his list. S fails to remember the formula, and also forgets that he actually mixed (1 + 2 + 3 + 4 + g) twice but does not have the combination of all four on his list.

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. After making several combinations, S comments, "I'm just trying to think of a way--of all the possibilities" (8),	1.a. Ability to consider the logical possibilities, (i.e. the total number of possible combinations),	2.12

TABLE 16 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
and systematically writes down 14 of the 16 possibilities (9). S guesses that the total number of possibilities may be 4^2 (19).	independent of the content, (i.e. of actually doing the experiments).	
	b. Ability to intuitively integrate thoughts within a system of related possible statements (i.e. combinations of liquids).	2.21
2. After making several combinations and identifying (1 + 3 + g) as the solution, S is asked about the role of 2, and also to distinquish between 2 and 4. He comments "1, 2 and 3 first showed colour, 1 and 3 showed colour, so 2 can't have any effect." Also "2 is different from 4 in that 2 in combination with 1 and 3 gives the yellow colour, but 4 in combination with 1 and 3 does not. Therefore they're different." (15).	2.a. Ability to formulate the operative factors involved and arrange experiment and thought sequence accordingly. (i.e. by comparing (1 + 3 + 2 + g) with (1 + 3 + 4 + g)).	2.22
	b. Ability to infer the implications of the statements, and select the true statements and discard the false, and synthesize a statement of necessary and possible conditions (concerning the role of 2 and 4).	2.23
	c. Ability to control the variables by studying the role of one factor (liquids 2 and 4 respectively) by varying another, (in this case, keeping (1 + 3 + g) as a constant).	2.32
	d. Ability to interpolate meaning between successive statements (combinations), establishing relations between relations (i.e. roles of 2 and 4).	2.41
	e. Ability to use concept of "all other things being equal" in a general sense.	3.22

TABLE 16 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
3. S twice combines all 4 liquids but fails to list them in the total number of possibilities (20). The sequence of performed combinations is not systematic. S guesses the total number of possible combinations as 10, and 42, but sticks to 14 which is the total number he listed (19).	3.a. Approach is hesitant, uncoordinated, uncertain, (i.e. when actually combining liquids) ... with a tendency to jump to conclusions (hazards guesses as to total number of combinations).	3.11
	b. Tendency to make generalizations which are restricted to empirical facts, (i.e. number of combinations actually listed).	3.13

TABLE 17

SYNOPSIS AND ANALYSIS OF PERFORMANCE
ON THE BALANCE TASK:
STUDENT B.H.

Synopsis

S begins by balancing the apparatus with 100 x 4L = 100 x 4R, and explains saying that "the same weights are equidistant from the fulcrum." He then adds 50 gm x 3L and 50 gm x 5R to the previous balance system, and on being asked if it will balance, S replies "No," and rearranges the weight so that they are equidistant from the fulcrum.

On being asked for more interesting variations, S puts 50 gm x 4L = 100 gm x 2R. He explains saying, "You take twice the distance of that side, and half the weight. This side is half the distance of that and twice the weight. Then it should be balanced."

S balances 30 gm x 2L with 30 gm x 2R, then 60 gm x 1R, suggests 15 gm x 4R, and is finally advised to use the 20 x 3R, and gives a muddled explanation saying, "As long as you've got an equal number, say with 20 gm at 3, would be equal to the same thing as 30 gm at 2." S is pushed for further explanation and says "As you change the distance, you're getting, ah--farther away from the fulcrum, you're getting more force."

S is asked to explain in terms of force. He says "...the farther you are away from the fulcrum, the less weight you need -- to provide the same force -- on the other side." On being asked to prove this, S suggests a balance system with 100 gm x 1L = 10 gm x 10R. S then sets up 30 gm x 2L = 10 gm x 6R, and prepares to substitute 10 gm x 6R for 60 gm x 1R. He explains saying "If you need less weight as you go out, then if you go in you will need more weight." On being asked if the weight in the balance 30 gm x 2L = 20 gm x 3L can be interchanged, S replies "No," and attempts to explain saying, "Well it was balanced initially, and now you have taken one weight off one side and put it on the other, so it won't be balanced." "You haven't got a constant on both sides. You call it a constant 'K' I guess, and you multiply the weight times the distance, and now you can't. You have got more weight at the same distance, so it won't be balanced."

S is asked to balance 30 gm x 3L. He suggests 15 gm x 6R, hesitates and reconsiders, mumbles about 50 gm and 25 gm, and eventually suggests 45 gm x 2R, or 10 gm x 9R.

S is asked what would happen if the 10 gm weight on the left side were moved further inwards. S replies first that the left side will go down, reconsiders, says that the right side will go down, and explains saying "because you're--going in--uh--decreasing the distance" so on my side right it remains constant. Your equivalent weight is going to become smaller." On being asked what he should do to maintain the balance if more weight is added to the 30 gm x 3L, S says that he could add an equivalent weight to his side, or he could move his existing weight outwards."

TABLE 17 (continued)

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S explains the balance in terms of the constant 'K' (implicit reference to the formula W'/D = W/D'), as well as in terms of force (20-21,14).	1. Ability to consider the logical possibilities independent of the content, (i.e. use of general concepts).	2.12
2. S is capable of setting up balances involving different weights and distances, e.g. 50 gm x 4L = 100 gm x 2 R (1), and 100 gm x 1L = 10 gm x 10R (15).	2.a. Ability to intuitively integrate thought within a system of related possible statements (concerning variables of weight and distance on both sides of the fulcrum).	2.21
	b. Ability to formulate operative factors involved, (i.e. the relationship between the variables), and arrange experiment accordingly.	2.22
3. S explains the balances with such statements as "You take twice the distance of that side, and half the weight. This side is half the distance and twice the weight. Then it should be balanced," and "As you change the distance, you're getting, ah--farther away from the fulcrum, you're getting more force." Also "... the farther you are away from the fulcrum, the less weight you need to provide the same force--as on the other side," and, "If you need less weight as you go out, then, if you go in, you will need more weight." (16).	3.a. Ability to infer the implications of the statements (concerning the increase and decrease of weights and distances) and select the true statements and discard the false, and synthesize a statement of the necessary conditions (for balance to occur).	2.23
	b. Ability to interpolate meaning between the successive statements establishing relations between relations, (i.e. W'/D = W/D').	2.41
	c. Tendency to make logical generalizations based on concept of "logical necessity" (implied in formula W'/D = W/D').	3.23

TABLE 17 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
4. S attempts a balance with 50 gm x 3L = 50 gm x 5R (2), but reconsiders and corrects himself making a symmetrical arrangement. S explains that interchange of weights in 30 gm x 2L = 20 gm x 3R system won't balance saying "Well, it was balanced initially, and now you have taken one weight off one side and put it on the other, so it won't be balanced." (19). S hesitates and has to reconsider when thinking of a system for balancing 30 gm x 3L (23). S is confused about the effect on the fulcrum of moving a weight inwards, but clarifies later. (25-30).	4.a. Approach is hesitant, uncoordinated unsystematic and uncertain, resulting in non-rigorous proofs and a tendency to jump to conclusions.	3.11

TABLE 18

SUMMARY OF PERFORMANCE ON PIAGET TASKS: STUDENT B.H.

Task	DESCRIPTORS USED														
	Formal Operations									Substage A			Substage B		
	2.11	2.12	2.21	2.22	2.23	2.31	2.32	2.41	2.51	3.11	3.12	3.13	3.21	3.22	3.23
Angles		+	+	+	+			+	+						+
Pendulum	(+)	+	+	(+)	+	(+)	(+)	+	(+)	+	+				
Liquids		+	+	+	+		+	+		+		+		+	
Balance		+	+	+	+			+		+					+
Combined	(+)	+	+	+	+	(+)	+	+	+	+	+	+		+	+

The table shows Student B.H. was capable of using formal operational thought predominantly at substage A level.

TABLE 19

SYNOPSIS AND ANALYSIS OF PERFORMANCE ON THE ANGLES OF INCIDENCE AND REFLECTION TASK: STUDENT L.W.

Synopsis

S begins by explaining that it is necessary to aim for a point on the rebound wall in such a way that if the total angle of rebound is divided into two equal angles (2), the line dividing the angles will be perpendicular to the rebound wall (4). S is trying to state the law of the equality of the angles of incidence and reflection (2,4,6). S explains further that if the rebound wall were rotated, the position of the perpendicular would change, the angle of incidence becomes greater as does the angle of reflection (8). S also stipulates that the surface must be flat for the law to hold.

S devises a technique for proving that the angles are equal using a glass sheet, or carbon paper and paper to trace the path of the fired ball (12).

S is asked what happens to the angles when the launcher position is changed, for example to the right. She replies that "the angle it hits at will be wider, ...but it will hit somewhere over here (to right) and you will have to bring your target over here (to right)" (16). S adds: "If you moved it the other way, the angle is getting smaller until you get it perpendicular with (the rebound wall) in which case it will go and come right back again, and your target would have to be even with (the launcher)" (18).

Tape ended before interview was concluded.

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S attempts to verbalize the law of equality of angles of incidence and reflection, and seems to understand the basic concept (1-4).	1. Tendency to make ...generalizations based on concept of "logical necess-ity," (concerning the equality of angles of incidence and reflection).	3.23

Note: Synopses of student L.W. are cross-referenced to transcripts in Appendix A.

TABLE 19 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
S discusses the apparatus in terms of the position of the rebound wall and the smoothness of the surface rather than the position of the launcher (4-6).		
2. S indicates that if the launcher is moved to the right, the angles of incidence and reflection will be "wider," and the rebound point will move to the right (10-11). Conversely, if the launcher is moved to the left, the angles get smaller until the lines of incidence and reflection coincide with the perpendicular, when the ball "will go and come right back again." (14).	2.a. Ability to intuitively integrate thoughts within a system of related possible statements (concerning sizes of angles and points of rebound).	2.21
	b. Ability to formulate operative factors involved and arrange thought sequence accordingly, (e.g. in explaining effect of different positions of launcher).	2.22
	c. Ability to infer the implications of the statements, (concerning size of angles and positions of rebound points) select the true statements and discard the false, and synthesize a statement of necessary and possible conditions.	2.23
	d. Ability to interpolate meaning between the successive statements (concerning size of angles, and points of rebound), establishing relations between relations (i.e. reciprocal implications).	2.41

TABLE 20

SYNOPSIS AND ANALYSIS OF PERFORMANCE ON THE
OSCILLATION OF A PENDULUM TASK:
STUDENT L.W.

Synopsis

S begins by making the lengths the same in order to establish the effect of different weights. She uses the heavy and light bobs, saying "This shows it most obviously." She also takes care that the bobs are dropped from the same point (2). S concludes weight has no effect on the period of oscillation (3).

S then determines the effect of force (impetus) on the period of oscillation. She first pushes both pendula equally establishing that the period is the same, i.e. all variables are being held constant (8). S then pushes the right pendulum and drops the left pendulum. She says, in surprise, "They both turn at the same time, but I thought force had something to do with it "(8). S is asked if she can justify the results, she thinks, then says "I guess so! If you use F = ma. You had more force, so acceleration was greater, masses are the same, so that should mean they are equal in period (10).

S then checks the effect of amplitude, ensuring that the lengths and weights are equal (14). S explains that in doing experiments a control is necessary, and the "control should always be constant ...so that you can compare them "(16). S concludes that amplitude does not affect the period of oscillation (2), and explains the results saying, "Well this one (right pendulum) has a greater amplitude, but it has a greater speed too, so that it's...they both have the same period "(24). She also explains in terms of the formula F = ma (30).

S predicts "the shorter the string the greater the period "(31), and "if this one (right pendulum) was half that one (left pendulum) it should take half as long for this (right pendulum) to get back to the point as this one does (left pendulum)"(33). S swings the pendula to demonstrate what she is saying.

S summarizes, "Force, no difference, mass the same, as long as length was the same. Amplitude didn't make any difference. Length of string did make a difference " (37).

TABLE 20 (continued)

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S attempts to justify the results in terms of the formula F = ma (9, 14).	1. Ability to consider the logical possibilities (concerning the effect of the variables), independent of the content.	2.12
2. S predicts and demonstrates "The shorter the string, the greater the period" and "If this one (right pendulum), was half that one (left pendulum), it should take half as long for this (right pendulum), to get back to the point as this one does (left pendulum) " (17-20).	2a& 3a. Ability to intuitively integrate thoughts within a system of related possible statements (concerning effect of length on period of oscillation).	2.21
	2a& 3b. Ability to formulate operative factors (involved with length variable) and arrange experiment and thought sequence accordingly.	2.22
	2 & 3c. Ability to infer the implications of the statements (concerning length variable), and select the true statements and discard the false, and synthesize a statement of the necessary and possible conditions (concerning the effect of the variable on the frequency of oscillation).	
	2d. Ability to predict the real situation if the hypothetical condition (that length affects the period of oscillation) was fulfilled, and by observing the consequences verify the hypothesis.	2.51

TABLE 20 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
3. S takes care to vary one variable while keeping all the others constant, e.g. S experiments with weight (1) force (impetus) (5), amplitude (11), then length (17). S explains that in doing experiments a control is necessary, and the "control should be always constant...so that you can compare them " (12).	3d. Ability to separate the variables by neutralizing the effect of factors such as weight, length, impetus, and amplitude, which cannot be physically separated in the pendulum.	2.31
	e. Ability to control the variables by studying the role of one factor by varying others, (i.e. keeping them constant in this case).	2.32
	f. Approach to task is systematic, integrated, and organized, resulting in exhaustive rigorous proofs, without jumping to conclusions.	3.21
	g. Ability to use the concept of "all other things being equal" in a general sense.	3.22
4. S explains her results for impetus variable saying, "If you use F = ma, you had more force, so acceleration was greater, masses are the same, so that should mean they equal in period " (8-10). S explains her results for the amplitude variable saying; "Well, this one (right pendulum) has a greater amplitude, but it has a greater speed too, so...they both have the same period " (14).	4. Ability to interpolate meaning between the successive statements, (concerning force, mass and acceleration, and amplitude and speed), establishing relations between relations.	2.41

TABLE 21

SYNOPSIS AND ANALYSIS OF PERFORMANCE ON
THE COMBINATION OF LIQUIDS TASK:
STUDENT L.W.

Synopsis

S begins by adding g to 1, 2, 3 and 4 (4). S then adds (1 + g) + (2 + g) then (1 + g) + (3 + g), and obtains the positive colour reaction (8). On being asked if she is sure about it, S says, "Well, I could try the last one," adds (1 + g) + (4 + g) and concludes the mystery liquid was (1 + 3 + g) (12). S checks her results by remixing (1 + 3 + g) (14), and also (2 + g) + (4 + g) (16).

On being pushed for more information about the liquids S says "Well, keep going on different combinations to see which one works " (19). S says she has forgotten how to figure out the total number of combinations, but writes out "4 singles, 6 doubles, 4 triples, 1 all four " (25).

S is asked about liquids 2 and 4 (26). S replies that neither (4 + g) nor (2 + g) nor (2 + 4 + g) give a reaction, "so there's nothing of what you're trying to determine in any of those two " (27). S is asked if what she has said proves conclusively that 2 and 4 have nothing to do with the reaction (28). S replies "you might have to do other---. They (1 + 3) might contain something and you'd need something to set off the reaction. 2 and 4 may need another liquid or something to mix in with it -- just to set off the reaction...You'd have to do other experiments "* (29).

S is asked if she can distinguish between 2 and 4, and although I pushes, S does not attempt any further experimentation, and simply summarizes that the right combination is 1 and 3, "Anything with just (1 + g) would turn a tiny bit yellow. With just 3 in it, or 2 and 4, it would be clear. I think that's all " (34).

*The I missed a valuable clue, and should have asked S what further experimentation she would consider relevant (29). S lacked motivation to experiment extensively.

TABLE 21 (continued)

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S is able to account for 15 of the 16 possible combinations, and writes them as "4 singles, 6 doubles, 4 triples, and 1 all four " (5). S experiments by systematically adding (1 + g), (2 + g), (3 + g) (4 + g), (1 + 2 + g), (1 + 3 + g) (1 + 4 + g) and (2 + 4 + g) ( ). S mentions further experimentation, but does not specify (1-4,13).	1.a. Ability to intuitively integrate thoughts within a system of related possible statements, (concerning the possible liquid combinations).	2.21
	b. Ability to formulate operative factors involved and arrange experiment and thought sequence accordingly.	2.22
	c. Approach is systematic, integrated, sure and organized.---	(3.21)

TABLE 22

SYNOPSIS AND ANALYSIS OF PERFORMANCE
ON THE BALANCE TASK:
STUDENT L.W.

Synopsis

S begins by balancing the apparatus with 200 gm x 8L = 200 gm x 8R (6). She explains saying, "You've added more weight to one side so you have to add more equal weight to the other side to keep a balance on the fulcrum. It doesn't matter how much weight you add, but you have to add it the same to both sides to (get a) balance." (10). "It should be the same weight and distance from the centre." (12).

On being asked to give more variations, S maintains her argument about having equal weights and equal distances on both sides of the fulcrum (16). S is asked again for variations "using different weights on each side" (17), and she responds by setting up 500 gm x 9L =(200 gm + 200 gm + 100 gm) x 9R, and says "Well, you can add it -- you can have 200 gm + 200 gm and another 100 gm to make up the 500 gm, but you can't put them in different places," (20), and, "You can have any combination you want to make up the weight on the opposite side, but they have to be the same distance " (22) and gives a further example, 400 gm x 9L =(200 gm + 200 gm) x 9R (24).

S is asked if 100 gm can be made to balance with 50 gm (27). S replies "Well, if you put 200 gm x 9L and 100 gm x 5R -- No! -- the other way round (i.e. 200 x 5L = 100 gm x 9R) it should work. The 200 gm on this side is at the half distance, and this one (100 gm x 9R) has half the weight and twice the distance from the fulcrum. It should still balance " (28). I asks if this is a general rule. S replies: "Yes -- it should be." (30). I releases the apparatus, the left side drops, S comments, "No, it's not!" (32). S changes the balance to 200 gm x 4L = 100 gm x 8R, saying "Well, before this (R) was 9 and this (L) was 5, so this (I) was more." (32).

S is asked to balance 40 gm x 5L using 50 gm (33) which she places at 4, i.e. 40 gm x 5L = 50 gm x 4R. She explains saying, "You've got them in the same ratio, this is 40 at the 5th hook which equals 200 and 50 at the 4th hook which equals 200 as well." (38), "Distance and mass should be in a balance." (40).

S correctly balances 30 gm x 2L with 20 gm x 3R, saying "It's the same ratio, the greater the distance, the smaller the weight you need to balance " (44). On being asked if the two weights could be interchanged (49), S responds by changing the weights and the distances, i.e. 30 gm x 3L = 30 gm x 2R (50).

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S eventually balances 200 gm x 4L = 100 gm x 8R (17) and 40 gm x 5L = 50 gm x 4R (19) and 30 gm x 2L = 20 gm x 3R (23). In explanation S explains the first balance by saying that the 200 gm is at half the distance, of the 100 gm and is twice its weight (14). "You've got them in the same ratio " (20). "Distance and mass should be in balance " (21), and finally "It's the same ratio, the greater the distance the smaller the weight you need to balance (the other side) "(23).	1.a. Ability to intuitively integrate thought within a system of related possible statements, (concerning weights and distances on both sides of the fulcrum).	2.21
	b. Ability to formulate operative factors involved (i.e. W'/D = W/D') and arrange experiment and thought sequence accordingly, (i.e. setting up balances and reasoning).	2.22
	c. Ability to interpolate meaning between the successive statements, establishing relations between relations, (concerning proportionality and mechanical equilibrium).	2.41
2. S initially insists that a balance must be established with equal weights and equal distances, but shows that she is capable of setting up other balances (1-4 12). S suggests firstly that 200 gm x 9L = 100 gm x 5R (12), then changes the weights around to 200 gm x 5L = 100 gm x 9R (14) but on seeing that no balance was obtained, changed the distances to 200 gm x 4L = 100 gm x 8R (17).	2.a. Approach is hesitant, uncoordinated and uncertain.	3.11

TABLE 23

SUMMARY OF PERFORMANCE ON PIAGET TASKS: STUDENT L.W.

Task	DESCRIPTORS USED														
	Formal Operations									Substage A			Substage B		
	2.11	2.12	2.21	2.22	2.23	2.31	2.32	2.41	2.51	3.11	3.12	3.13	3.21	3.22	3.23
Angles			+	+	+		+								+
Pendulum		+	+	+		+	+	+	+				+	+	
Liquids			+	+									(+)		
Balance			+	+				+		+					
Combined		+	+	+	+	+	+	+	+	+			+	+	+

The table shows Student L.W. is capable of using formal operational thought predominantly at substage B level, but with some indication of using substage A level.

Comment: Student L.W. seemed to be capable of adequately performing all the tasks. She needed, however, to be pushed and seemed to be self restricted to the easiest answers.

TABLE 24

OVERALL SUMMARY OF STUDENT PERFORMANCE ON ALL PIAGET TASKS

Student	DESCRIPTORS USED														
	Formal Operations									Substage A			Substage B		
	2.11	2.12	2.21	2.22	2.23	2.31	2.32	2.41	2.51	3.11	3.12	3.13	3.21	3.22	3.23
J.V.		+	+	+	+	+	+	+	+				+	+	+
B.H.	(+)	+	+	+	+	(+)	+	+	+	+	+	+		+	+
L.W.		+	+	+	+	+	+	+	+	+			+	+	+

The table shows that all students were capable of using formal operations. Student L.W. and J.V. seemed to function mainly at the substage B level of formal operations while B.H. tended to function more at the substage A level.

B. THE EXAMINATION ITEMS

Method of Analysis and Reporting of Results

Two course examination items were selected from the Physics 110 examination on the basis that the responses expected require formal operational thought. The selected examination items and responses expected by the course instructor and class tutor to these items are given in Tables 25 and 26 . The Tables include the instructor's best guesses about the information to be recalled in responding according to expectation, and the maximum mark obtainable for each item. The Tables give a detailed analysis of the expected performance, using the inventory, and display the formal operational thought expected for each item. The entire examination is given in Appendix B, and brief reasons for the selection of the individual items given in Appendix C.

The actual responses given by the students to the selected examination items were transcribed from their examination papers. The marks credited to each student on each item by the examiners were also transcribed.

The analyses of the expected responses and actual student responses to the selected test items were made in the same way as they were for the students performance on the Piaget tasks. The responses were read carefully and those sections which could be described by one or more of the descriptors of the inventory were placed in a table with the appropriate descriptor(s) listed alongside. (See Tables 28, 29, 31, 32, 34 and 35) Where necessary, the descriptors were elaborated to indicate in which way the descriptors were found to be appropriate to the responses given. The elaborations are indicated by parentheses. As in the analyses of student performance on the Piaget tasks, qualified descriptors could be used if cases arose where a descriptor was found to be nearly adequate.

As in the case of the Piaget task performances, summaries were compiled for the expected responses and for the responses of each student on both the selected examination items. (See Tables 27, 30, 33, and 36) The summary of the overall performance across items was made so as to display the descriptors which were found to be appropriate to the students' performance, at least once in their responses to both the selected examination items (See Table 37).

TABLE 25

EXPECTED RESPONSE AND ANALYSIS OF PERFORMANCE ON SELECTED EXAMINATION ITEM I

Item 1 (maximum marks, 8)

Nuclear Energy can be converted into heat by nuclear fusion as well as by nuclear fission. Could one not make the best use of these processes by first splitting atoms (nuclear fission, heat will be produced), and then re-uniting the parts again (nuclear fusion, heat will be produced)? Repeating this cycle over and over again, one would have an inexhaustible energy source. Explain, in terms of the binding energies of nuclei, why this process is impossible.

Expected Response

Nuclear fission gains energy by splitting heavy nuclei. The result of this fission is medium sized nuclei. Nuclear fusion gains energy by uniting light nuclei. The result of this fusion is medium sized nuclei. As medium sized nuclei have the maximum binding energy per nucleon, energy has to be provided to either split or unite medium sized nuclei. If one splits heavy nuclei one obtains energy. The same energy has to be invested to reunite the nuclei. Thus the overall gain is zero, as predicted by the law of conservation of energy.

Analysis

Aspects of Expected Performance	Descriptors Used	Descriptor Code Numbers
1. S is asked to explain why the given hypothesis that a continuous cycle of nuclear fission and nuclear fusion would produce an inexhaustible energy source would be impossible.	1. Ability to accept unproven facts (concerning nuclear fusion and nuclear fission) as hypothetically true, in order to deduce the real from the impossible. (i.e. the actual effects of nuclear fusion and nuclear fission).	2.11

TABLE 25 (continued)

Aspects of Expected Performance	Descriptors Used	Descriptor Code Number
2. S must consider the logical possibilities involved in nuclear fusion and nuclear fission as separate processes and as integrated processes, in terms of the binding energy of the nuclei.	2. Ability to consider the logical possibilities involved, independent of the content (i.e. as a theoretical consideration).	2.12
3. S has to consider the interaction of factors such as nuclear fusion and fission, heavy, light and medium atoms, the energy produced and the energy gained, and high and low binding energies.	3. Ability to intuitively integrate thoughts within a system of related possible statements (concerning interaction of the factors).	2.21
4. S must reason that nuclear fission occurs when splitting heavy nuclei, and that nuclear fusion occurs when uniting light nuclei, and that both processes result in energy gained and medium sized nuclei.	4. Ability to formulate operative factors involved (in nuclear fusion and fission) and arrange thought sequence accordingly.	2.22
5. S must infer the implications of the statements concerning nuclear fusion and fission, and reason that as medium sized nuclei have the maximum binding energy per nucleon, energy would have to be provided to either split or unit them, and that the overall gain in energy would be zero.	5a. Ability to infer the implications of the statements, select the true statements and discard the false, and synthesize a statement of necessary and possible conditions (concerning nuclear fusion and fission).	2.23
	b. Ability to interpolate meaning between the successive statements (concerning	2.41

TABLE 25 (continued)

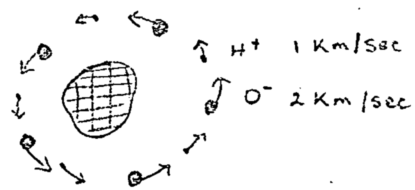
Aspects of Expected Performance	Descriptors Used	Descriptor Code Numbers
	the gain or loss of energy), establishing relations between relations, (i.e. relations between the nuclear fusion and fission processes).	
	c. Ability to predict the real situation if the hypothetical condition (that nuclear fusion and fission could give an inexhaustible supply of energy) was fulfilled, and by observing the consequences (reject) the hypothesis.	2.51
6. S must approach this problem systematically in order to reason and prove the falsehood of the given hypothesis.	6. Approach is systematic, integrated, sure and organized, resulting in exhaustive and rigorous proofs without jumping to conclusions.	3.21
7. S may eventually rely on or state the law of conservation of energy which effectively falsifies the hypothesis.	7. Tendency to make logical proofs and generalizations based on concept of "logical necessity," (i.e. law of conservation of energy).	3.23

TABLE 26

EXPECTED RESPONSE AND ANALYSIS OF EXPECTED PERFORMANCE
ON SELECTED EXAMINATION ITEM 2

Item 2 (maximum marks, 12)

Space explorers discover a ring of charged particles orbiting around a mysterious cloud. The ring consists of positive hydrogen ions and negative oxygen ions, circulating in the same direction. The speed of the hydrogen ions is 1 km/sec, the speed of the oxygen atoms is 2 km/sec, the radius of orbit is the same for both kinds of particles. The number of particles per cubicmeter is too small to allow the ions to combine. For the same reason, no electric or magnetic forces between the ions could account for the motion. The explorers discuss the following explanations to account for the circular orbits of the ions. Try to rule out as many of these explanations as possible. Give your reasons.



a. The circular orbits are due to gravitational attraction by a massive star within the cloud: Could be / /; Cannot be / /

b. The circular orbits are due to a charged object hidden in the cloud: Could be / /, Cannot be / /.

c. The circular orbits are due to a magnetic field at right angles to the plane of the orbits: Could be / /, Cannot be / /

Note A certain amount of recall is involved in answering this item:

1. The S must know the dynamics of satellite motion, i.e. attractive force = $M_{Sat.}\frac{v^2}{r}$ (mass x centripetal acceleration).
2. The properties of the proposed forces (gravitational, coulomb, magnetic) e.g. the magnetic force is given by $F = \frac{qvB}{c}$ and the relative directions of (qv), B and F by the Right Hand Rule.
3. Like charges repel each other while unlike charges attract each other.

Expected Response

None of the three hypotheses account for the circular orbits of the ions.

TABLE 26 (continued)

a. If the hypothesis were true, the O^- and H^+ ions would move with the same speed, i.e.

$$\frac{GmM}{r^2} = \frac{mv^2}{r} \quad \text{therefore} \quad \frac{GM}{v^2} = \frac{\text{constant}}{v^2}$$

Thus implications of hypothesis are inconsistent with given information.

b. If the hypothesis were true, only one of the ions (negative or positive) would be attracted to the centre object, depending on its charge. The other one would not move in orbit, as it would be repelled by the centre object.

$$\frac{qQ}{r^2} = \frac{mr^2}{r}$$

Thus implications of hypothesis are inconsistent with given information.

c. If the hypothesis were true, the ions could not move in the same direction due to difference in charge. One kind would move clockwise while the other kind would move counter-clockwise.

$$\frac{qvB}{c} = \frac{mr^2}{r}$$

Thus implications of hypothesis are inconsistent with given information.

Analysis

Aspects of Expected Performance	Descriptors Used	Descriptor Code Numbers
1. S has to accept the three hypothetical explanations of the given problem, and deduce from each its truth or falsehood in terms of the recalled physics information.	1. Ability to accept unproven facts (involved in suggested explanations) as hypothetically true in order to deduce the real from the possible.	2.11

TABLE 26 (continued)

Aspects of Expected Performance	Descriptors Used	Descriptor Code Numbers
2. S has to consider variables of the equality or inequality of radius of ions from object, of mass, charge, direction of current flow, and velocity of ions.	2. Ability to consider the logical possibilities independent of the content (i.e. theoretically).	2.12
3. These variables must be considered by S in terms of their interactions as noted in the relevant information (formulae and laws) that must be recalled for this item.	3. Ability to intuitively integrate thoughts within a system of related possible statements (concerning the variables).	2.21
4. For each hypothesis S has to formulate the appropriate and relevant variables so as to apply the recalled formulae or laws.	4. Ability to formulate operative factors involved and arrange thought sequence accordingly.	2.22
5. S must establish the implications of possible statements made in the terms of the recalled formulae or laws so as to be able to state the conditions arising out of each suggested hypothesis.	5a. Ability to infer the implications of the statements, select the true statements and discard the false, and synthesize a statement of necessary and possible conditions.	2.23
S must decide whether or not the implications of the suggested hypotheses agree with the given facts.	b. Ability to interpolate meaning between the successive statements (made in each hypothesis) establishing relations between relations (i.e. in judging the agreement).	2.41
S thus can verify or reject the suggested hypothesis by checking his deduced implications against the given conditions.	c. Ability to predict the real situation (using knowledge of physics formulae and laws) if the hypothetical condition was fulfilled, and by observing the consequences verify (or reject) the hypothesis.	2.51

TABLE 26 (continued)

Aspects of Expected Performance	Descriptors Used	Descriptor Code Numbers
6. S must think through the implications of each hypothesis carefully and systematically, and use recalled physics knowledge as the logical reason for accepting or rejecting each hypothesis.	6a. Approach is systematic, integrated, sure and organized, resulting in exhaustive and rigorous proofs, without jumping to conclusions.	3.21
	b. Tendency to make logical proofs and generalizations based on concept of "logical necessity."	3.23

TABLE 27

SUMMARY OF EXPECTED PERFORMANCE ON SELECTED EXAMINATION ITEMS

Selected Items	DESCRIPTORS USED														
	Formal Operations									Substage A			Substage B		
	2.11	2.12	2.21	2.22	2.23	2.31	2.32	2.41	2.51	3.11	3.12	3.13	3.21	3.22	3.23
1	+	+	+	+	+			+	+				+		+
2	+	+	+	+	+			+	+				+		+
Combined	+	+	+	+	+			+	+				+		+

The table shows that both selected examination items require formal operational thought at the substage B level for the expected responses. Descriptors 2.31, 2.32 and 3.22 are generally not applicable to written responses to examination items. They would, however, be more applicable to laboratory situations.

TABLE 28

STUDENT RESPONSE AND ANALYSIS OF PERFORMANCE ON
SELECTED EXAMINATION ITEM 1:
STUDENT J.V.

Student Response (marks obtained 8/8)

Once a heavy atom has undergone nuclear fission to become a medium-sized atom, and once a light atom has undergone nuclear fusion to become a medium-sized atom, they have become medium-sized atoms with the binding energy of the nuclei of these medium-sized atoms so great that the same amount of energy would be needed to extract or produce any energy by either splitting them or reuniting them with another nucleus so there could be no inexhaustible supply of energy. 5

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S did not accept and reason from the hypothesis that there could be "an inexhaustible energy source." But S effectively refuted the hypothesis by first dealing with the implications of nuclear fusion and nuclear fission, then linked the two as suggested in the hypothesis with the resulting contradiction.	1. Ability to accept unproven facts as hypothetically true in order to deduce the real from the possible.	2.11
2. S considered the logical possibilities of nuclear fusion and nuclear fission separately and concluded that "there could be no inexhaustible supply of energy." (5).	2. Ability to consider the logical possibilities independent of the content(of nuclear fusion and nuclear fission.)	2.12

TABLE 28 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Number
3. S integrated factors, i.e. nuclear fusion and nuclear fission, heavy, medium and light atoms, energy produced or "extracted," and binding energy of the nuclei in order to reach her conclusion.	3. Ability to intuitively integrate thoughts within a system of related possible statements (concerning implications of nuclear fusion and fission).	2.21
4. S arranged her argument as a function of the energy needed and produced. She stated that both splitting a heavy atom and uniting a light atom results in "medium sized atoms."(2).	4. Ability to formulate operative factors involved and arrange thought sequences accordingly.	2.22
5. S states "...the binding energy of these medium-sized nuclei (is) so great that the same amount of energy would be needed to extract or produce any energy by either splitting them or reuniting them with another nucleus." (3).	5a. Ability to infer the implications of the statements, select the true statements and discard the false, and synthesize a statement of necessary and possible conditions.	2.23
	b. Ability to interpolate meaning between the successive statements (concerning the production (gain) or extraction (loss) of energy) establishing relations between relations.	2.41
S simply refutes the hypothesis by saying "...so there could be no inexhaustible supply of energy" (5) indicating that she has found the hypothesis to be incompatible with the logical implications of the premises on which the hypothesis is based.	c. Ability to predict the real situation if the hypothetical condition was fulfilled and by verifying the consequences (reject) the hypothesis.	2.51

TABLE 28 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
6. S systematically considers the logical implications of the given premises in order to refute the suggested hypothesis.	6. Approach is systematic, integrated, sure and organized resulting in exhaustive and rigorous proofs without jumping to conclusions.	3.21
7. S proves logically that there "could be no inexhaustible supply of energy" by showing in effect, the law of conservation of energy.	7. Tendency to make logical proofs and generalizations based on concept of "logical necessity."	3.23

TABLE 29

STUDENT RESPONSE AND ANALYSIS OF PERFORMANCE ON SELECTED EXAMINATION ITEM 2: STUDENT J.V.

Student Response (marks obtained 8/12)

(a) "could be." Both oxygen and hydrogen atoms have mass, and so could be attracted by a massive star if there is one.

(b) "cannot be." If the charged objects hidden in the cloud was of a net positive charge, it would attract only the O^- ions and would repel the H^+ ions, or, if the object was of a net negative charge it would attract only the H^+ ions and would repel the O^- ions. However, the cloud seems to attract positive and negative ions.

(c) "cannot be." The H^+ ions and O^- ions are circulating in the same direction. Therefore there are two directions of current flow. Using the Right Hand Rule 1 of these ions but not both would exert a force to the center of the clouds on only one set (O^- or H^+).

Note No credit was given for (a).

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S accepts each hypothesis in turn and deduces the implications of each. The implications of hypothesis (a) are inaccurate, but are accurate for hypotheses (b) and (c).	1. Ability to accept unproven facts as hypothetically true in order to deduce the real from the possible.	2.11

TABLE 29 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
2. S considers the variables of mass, charge, attractive and repulsive forces, and directions of current flow.	2. Ability to consider the logical possibilities independent of the content.	2.12
3. S considers the variables of charge in terms of the attractive and repulsive forces, and the directions of current flow in terms of the Right Hand Rule.	3. Ability to intuitively integrate thoughts within a system of related possible statements.	2.21
4. For hypothesis (b), S correctly formulates the variables of charge, attraction and repulsion, and for hypothesis (c), the variables of directions of current flow, magnetic force and Right Hand Rule. S incorrectly formulates variables of mass and attraction and repulsion for hypothesis (a), and reasons from these points.	4. Ability to formulate operative factors involved and arrange thought sequence accordingly.	2.22
5. S considers the implications of the hypotheses in terms of the recalled formulae or laws, and is able to state the conditions for each hypothesis, i.e. Hypothesis (b), S states that the cloud would attract either the positive or the negative ions, not both (5); Hypothesis (c), S states that the magnetic field would exert a force on only one set of ions (O^- or H^+) but not both. (8,9).	5a. Ability to infer the implications of the statements, select the true statements and discard the false, and synthesize a statement of necessary and possible conditions.	2.23
	b. Ability to interpolate meaning between the successive statements establishing relations between relations (for each hypothesis).	2.41

TABLE 29 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
S rejects hypotheses (b) and (c) by checking her deduced implications against the given conditions which are that both positive and negative ions are attracted by the cloud, (therefore hypothesis (b) is rejected), and that both the types of ions are travelling in the same direction, (therefore hypothesis (c) is rejected).	5c. Ability to predict the real situation if hypothetical condition was fulfilled, and by observing the consequences (reject) the hypothesis.	2.51
6. S reasons systematically and methodically through the implications of hypotheses (b) and (c), and uses recalled physics knowledge as the logical basis for rejecting the hypothesis. Reasoning for hypothesis (a) was incomplete, and therefore inaccurate.	6a. Approach is systematic, integrated, sure and organized resulting in exhaustive and rigorous proofs, without jumping to conclusions (applicable to hypotheses (b) and (c).	3.21
	b. Tendency to make logical proofs and generalizations based on concept of "logical necessity."	3.23

TABLE 30

SUMMARY OF PERFORMANCE ON SELECTED EXAMINATION ITEMS: STUDENT J.V.

Selected Items	DESCRIPTORS USED														
	Formal Operations									Substage A			Substage B		
	2.11	2.12	2.21	2.22	2.23	2.31	2.32	2.41	2.51	3.11	3.12	3.13	3.21	3.22	3.23
1	+	+	+	+	+			+	+				+		+
2	+	+	+	+	+			+	+				+		+
Combined	+	+	+	+	+			+	+				+		+

The table shows that Student J.V. used formal operational thought at the substage B level in her responses to the selected examination items.

TABLE 31

STUDENT RESPONSE AND ANALYSIS OF PERFORMANCE ON
SELECTED EXAMINATION ITEM 1:
STUDENT B.H.

Student Response (Mark obtained 5/8)

The binding energy of an atom increases with atomic weight, to a certain point, around 80. Then the binding energy decreases, presumably due to electrostatic repulsive forces. At 80, binding energies are greatest, splitting apart or uniting nuclei would require great amounts of energy. However, the light elements and very heavy elements have smaller binding energies, since less force is keeping them together they are easier to split and to unite.

But when say U^{238} is split it produces lighter nuclei of higher binding energies which require great amounts of energy to reunite. This is the same as H when it fuses, only it produces heavier nuclei with higher binding energies which are very hard to split. Thus it would be impossible to have a continuous cycle since both reactions go to nuclei with high binding energies which cannot be fused or split (practically).

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S sets about disproving the given hypothesis concerning nuclear fusion and fission, and concludes his argument saying, "Thus it would be impossible to have a continuous cycle." (8).	1. Ability to accept unproven facts as hypothetically true in order to deduce the real from the possible.	2.11
2. S considers the effect of high and low binding energies with respect to heavy and light elements, and fission and fusion.	2. Ability to consider the logical possibilities involved independent of the content. (i.e. theoretically)	

TABLE 31 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
3. S considers the interaction of factors such as the ease of nuclear fusion, and nuclear fission, heavy, light and medium elements, and high and low binding energies.	3. Ability to intuitively integrate thoughts within a system of related possible statements (concerning the effects of nuclear fusion and nuclear fission).	2.21
4. S reasons that when the binding energy of elements is high, it is very difficult to split or fuse the nuclei as great amounts of energy would be required. The opposite is true of light and heavy elements. S attempts to explain that the fusion of light elements, e.g. H and the fission of heavy elements, e.g. U^{238}, both result in medium sized nuclei with high binding energies. The argument is implicit rather than explicit.	4. Ability to formulate operative factors involved and arrange thought sequence accordingly.	2.22
5. S reasons that both the fusion of light elements and the fission of heavy elements produce nuclei with high binding energies which require great amounts of energy to unite or split apart, "Thus it would be impossible to have a continuous cycle." (8).	5a. Ability to infer the implications of the statements, select the true statements and discard the false and synthesize a statement of necessary and possible conditions.	2.23
	b. Ability to interpolate meaning between the successive statements concerning the gain or loss of energy, establishing relations between relations (i.e. between nuclear fusion and nuclear fission).	2.41

TABLE 31 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
	c. Ability to predict the real situation if the hypothetical condition was fulfilled, and by observing the consequences (reject) the hypothesis.	2.51
6. Approach to problem is not very coherent although accurate.	6. Approach is uncoordinated and unsystematic, resulting in (seemingly) non-rigorous proof --- .	3.11
7. S does not comment on the conservation of energy aspect, but is more concerned with the impracticability of splitting nuclei which have a high binding energy.	7. Tendency to make proofs and generalizations which are restricted to empirical facts.	3.13

TABLE 32

STUDENT RESPONSE AND ANALYSIS OF PERFORMANCE ON
SELECTED EXAMINATION ITEM 2:
STUDENT B.H.

Student Response (marks obtained 4/12)

(a) "Cannot be." If it were a gravitational attraction, then the force exerted on the particles would be $F = \frac{Gm_1m_2}{r^2}$, where m_1 is mass of particle and m_2 is mass of cloud. Since the oxygen atoms are heavier than the hydrogen ions, there would be a greater F_G and hence they would be closer to the cloud than the H^+ ions. Since they are in a ring then it couldn't be a gravitational attraction.

(b) "Cannot be." If this were the case then those particles that had the same charge as that of the object in the cloud would be repelled and would not stay in orbit. Both charges would not be present in the orbit if it were a charged object within the cloud.

(c) "Could be." If there were a continuous magnetic field at right angles to the plane orbits, then there would be an electric field at right angles to the magnetic field. The particles would then be kept in a circular orbit and their velocity would depend on their mass.

Note Only reasoning in (b) was given credit.

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S accepts each of the three hypothetical explanations and is prepared to make deductions from each. However accurate reasoning is displayed for hypothesis (b) only.	1. Ability to accept unproven facts as hypothetically true in order to deduce the real from the possible.	2.11
2. S considers the variables of mass, distance from cloud, charge, attraction and repulsion, and (direction of current flow), and velocity of the particles.	2. Ability to consider the logical possibilities independent of the content.	2.12

TABLE 32 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
3. For hypothesis (b) S is able to integrate the variables of charge with the variables of attraction and repulsion of ions. For hypotheses (a) and (c) S is incorrect in his reasoning but he nevertheless attempts to integrate the variables.	3. Ability to intuitively integrate thoughts within a system of related possible statements.	2.21
4. For hypothesis (b) S reasons in terms of the attraction and repulsion of differently charged ions. For hypothesis (a) S incorrectly reasons in terms of the distance from the centre, which is not the operative factor in this case.	4. Ability to formulate operative factors involved and arrange thought accordingly.	2.22
5. For hypothesis (b), S infers that "Both charges would not be present in the orbit if it were a charged object within the cloud." (6). For hypotheses (a) and (c) S makes certain inferences but they are incorrect. S rejects hypothesis (b) by checking his deduced implications against the given conditions, i.e. that both positive and negative charged ions are in orbit around the cloud.	5a. Ability to infer the implications of the statements, select the true statements and discard the false, and synthesize a statement of necessary and possible conditions.	2.23
	b. Ability to interpolate meaning between the successive statements (made in hypothesis (b) establishing relations between relations (i.e. in judging the agreement of the hypothesis with the given conditions).	2.41
	c. Ability to predict the real situation if the hypothetical condition was fulfilled,	2.51

TABLE 32 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
	and by observing the consequences (reject) the hypothesis.	
6. For hypothesis (b) S carefully and methodically deduces the implications and logically rejects the hypothesis. For hypotheses (a) and (c), S selects the wrong operative factors or incorrectly infers the implications of the hypothesis, giving some evidence of uncertain, non-rigorous thinking.	6a. Approach is systematic, integrated, sure and organized, resulting in exhaustive and rigorous proofs, without jumping to conclusions, (applicable to hypothesis (b) only).	3.21
	b. Tendency to make logical proofs ... based on concept of "logical necessity" (applicable to hypothesis (b) only).	3.23
	c. Approach is ... uncertain, resulting in non-rigorous proofs and a tendency to jump to conclusions (applicable to hypotheses (a) and (c)).	3.11

TABLE 33

SUMMARY OF PERFORMANCE ON SELECTED EXAMINATION ITEMS: STUDENT B.H.

Selected Items	DESCRIPTORS USED														
	Formal Operations									Substage A			Substage B		
	2.11	2.12	2.21	2.22	2.23	2.31	2.32	2.41	2.51	3.11	3.12	3.13	3.21	3.22	3.23
1	+	+	+	+	+			+	+	+		+			
2	+	+	+	+	+			+	+	+			+		+
Combined	+	+	+	+	+			+	+	+		+	+		+

The table shows Student B.H. used formal operational thought at both the substage A and substage B levels. This indicates that B.H. was not fully capable of using formal operational thought at all times.

TABLE 34

STUDENT RESPONSE AND ANALYSIS OF PERFORMANCE ON SELECTED EXAMINATION ITEM 1: STUDENT L.W.

Student Response (marks obtained 5/8)

This is impossible because only heavy nuclei can go through the process of nuclear fission and extremely light nuclei only can go through the process of nuclear fusion. If it were possible to get a material whose nucleus had a binding energy at the meeting point of nuclear fusion and fission, it might be possible to split this nucleus, and then reunite it to produce an inexhaustible energy source, but there is no element as of yet with a nuclear binding energy at exactly this point.

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S hypothesizes that "a material whose nucleus had a binding energy at the meeting point of nuclear fusion and fission," could be split and reunited "to produce an inexhaustible energy source." (4). This hypothesis is inaccurate as such a "material" would have the highest nuclear binding energy, and could not be split or reunited to form an inexhaustible supply of energy.	1a. (Ability to hypothesize...but in this case the hypothesis is inaccurate and therefore does not contribute to the problem).	(2.11)
	b. Ability to consider logical possibilities independent of the content.	2.12
2. S rejects the suggested hypothesis because "only heavy nuclei can go through the process of nuclear fusion and extremely light nuclei only can go through	2. Approach is hesitant, uncoordinated, unsystematic and unsure, resulting in non-rigorous proofs and a tendency to jump to conclusions.	3.11

TABLE 34 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Number
the process of nuclear fusion." (2). It is not clear on what grounds S rejects the hypothesis except that she has accurately given the conditions for nuclear fusion and fission. This could be a case of simple recall, or of jumping to conclusions.		
Note S may not have recalled information concerning medium sized nuclei.		

TABLE 35

STUDENT RESPONSE AND ANALYSIS OF PERFORMANCE ON SELECTED EXAMINATION ITEM 2: STUDENT L.W.

Student Response (marks obtained 0/12)

(a) "Could be." A massive star of no charge one way or the other would not attract the ions. If there were no forces acting on the ions they would go off into space, but the gravitational force of the large central mass would keep changing the direction of n. of the ions, thus the movement would be orbital.

$$F_c = \frac{mv^2}{R}, \quad F_g = \frac{GMm}{r^2}$$

(b) "Cannot be." If the object was positively charged the negative oxygen ions would be attracted, and if it were negatively charged the positive hydrogen ions would be attracted. Since however the ions stay in constant orbit the object in the cloud cannot be charged.

(c) "Could be." Using the Right Hand Rule, as the direction of the current constantly changes the magnetic field will turn at the same rate always remaining at right angles to the plane of the orbits, and thus the force will continually change.

Note Response (b) closely approximates the examiner's expected response. However no credit was given.

Analysis

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
1. S accepts each of the three hypothetical explanations of the given problem, and is prepared to make deductions from each. However accurate reasoning is displayed only for (b).	1. Ability to accept unproven facts as hypothetically true in order to deduce the real from the possible.	2.11

TABLE 35 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
2. S considers the variables of charge, (mass) attraction (and repulsion), direction of the current.	2. Ability to consider the logical possibilities independent of the content (i.e. theoretically)	2.12
3. For hypothesis (b) S is able to integrate the variables of positive and negative charge with the variables of attraction and repulsion of ions. For hypotheses (a) and (c) S is incorrect in her reasoning, but nevertheless attempts to integrate the variables.	3. Ability to intuitively integrate thoughts within a system of related possible statements.	2.21
4. For hypothesis (b) S reasons in terms of the attraction of negative and positive ions to the object, depending on the charge of the object. For hypothesis (c) S reasons in terms of the direction of the current and the Right Hand Rule, but is insufficiently rigorous in her reasoning.	4. Ability to formulate operative factors involved and arrange thought sequence accordingly.	2.22
5. For hypothesis (b), S infers that "if the object was positively charged, the negative oxygen ions would be attracted, and if it were negatively charged the positive hydrogen ions would be attracted" (6). For hypotheses (a) and (c), S attempts certain inferences which are incorrect and not relevant.	5a. Ability to infer the implications of the statements, select the true statements and discard the false, and synthesize a statement of the necessary and possible conditions.	2.23
	b. Ability to interpolate meaning between the successive statements (made in hypothesis (b)) establishing relations between relations (i.e. in judging the agreement of	2.41

TABLE 35 (continued)

Aspects of Student Performance	Descriptors Used	Descriptor Code Numbers
S rejects hypothesis (b) by checking her deduced implications against the given conditions, and concludes, "Since however the ions stay in constant orbit, the object in the cloud cannot be charged." (7).	the hypothesis with the given conditions).	
	c. Ability to predict the real situation if the hypothetical condition was fulfilled and by observing the consequences verify (or reject) the hypothesis.	2.51
6. For hypothesis (b), S carefully and methodically deduces the implications and logically rejects the hypothesis. For hypothesis (a), S selects the wrong operative factors, and for hypothesis (c) S is unable accurately to deduce the implications, giving evidence of uncertain non-rigorous thinking.	6a. Approach, for hypothesis (b) is systematic, integrated, sure and organized, resulting in exhaustive and rigorous proofs, without jumping to conclusions.	3.21
	b. Tendency to make logical proofs...based on concept of "logical necessity." (hypothesis (b) only)	3.23
	c. Approach is unsystematic...unsure, resulting in non-rigorous proofs and a tendency to jump to conclusions. (hypotheses (a) and (c))	3.11

TABLE 36

SUMMARY OF PERFORMANCE ON SELECTED EXAMINATION ITEMS: STUDENT L.W.

Selected Items	DESCRIPTORS USED														
	Formal Operations									Substage A			Substage B		
	2.11	2.12	2.21	2.22	2.23	2.31	2.32	2.41	2.51	3.11	3.12	3.13	3.21	3.22	3.23
1	(+)	+								+					
2	+	+	+	+	+			+	+	+			+		+
Combined	+	+	+	+	+			+	+	+			+		+

The table shows student L.W. used formal operational thought at substage A and substage B, indicating that she was not fully capable of using formal operational thought at all times.

TABLE 37

OVERALL SUMMARY OF STUDENT PERFORMANCE ON BOTH SELECTED EXAMINATION ITEMS

Student	DESCRIPTORS USED														
	Formal Operations									Substage A			Substage B		
	2.11	2.12	2.21	2.22	2.23	2.31	2.32	2.41	2.51	3.11	3.12	3.13	3.21	3.22	3.23
J.V.	+	+	+	+	+			+	+				+		+
B.H.	+	+	+	+	+			+	+	+		+	+		+
L.W.	+	+	+	+	+			+	+	+			+		+

The table shows student J.V. used formal operations at the substage B level, while both students B.H. and L.W. used formal operation thoughts at substages A and B levels indicating that they were not fully capable of using formal operational thought at all times.

CHAPTER V

DISCUSSION OF RESULTS, SUMMARY AND CONCLUSIONS

The final Chapter contains the comparison of the results of the analyses of student performance on the Piaget tasks with the analyses of the actual student performance on the examination items. The inferences made from the comparisons are presented. The implications of the comparisons are discussed in the context of both formative evaluation and the improvement of classroom practice. A critique of the usefulness of the inventory is followed by suggestions for further research. The Chapter ends with a summary of the problem and the conclusions of the study.

A. COMPARISON OF STUDENT PERFORMANCE ON PIAGET TASKS AND SELECTED EXAMINATION ITEMS

The overall results of student performance on the Piaget tasks and selected examination items are displayed in a table so as to facilitate the comparison between them (See Table 38). The comparison was made by looking for congruency between the descriptors used to describe performance on the Piaget tasks and selected examination items, for each student in turn. The results were considered congruent when the same, or nearly the same descriptors were identified for a student in both the Piaget tasks and selected examination items. Conversely, the results were considered non-congruent when the descriptors identified for the same student in the Piaget tasks and selected examination items did not agree.

Three comparisons are considered. Firstly, the comparison of the inventory of descriptors with student performance on the Piaget tasks provides information which identifies the potential level of intellectual development of the students, and is a summary of Table 24. Secondly, the comparison of the inventory of descriptors with student performance on selected examination items provides information about the actual level of intellectual development displayed in non-Piagetian conditions, and is a summary of Table 37. Thirdly, the comparison of the students potential and actual levels of intellectual development, the latter displayed in non-Piagetian conditions, is a summary of information obtained from Table 38. Finally, Table 38 is used to provide information concerning the adequacy of the inventory of descriptors in describing student performance on Piaget tasks and selected examination items.

Summary of Comparisons

1. Inventory of descriptors compared with student performance on Piaget tasks, i.e. potential level of intellectual development.

Student J.V. displayed intellectual functioning at the formal operations, substage B level. Both students B.H. and L.W. displayed potential intellectual functioning at the formal operations level, with indications of operating at the substage B level as well as at the substage A level, from which it could be inferred that their formal operational thought was not fully developed (Table 24).

2. Inventory of descriptors compared with student performance on selected examination items, i.e. actual intellectual performance displayed in non-Piagetian conditions.

Student J.V. displayed actual intellectual functioning at the formal operations, substage B level. Both students B.H. and L.W. displayed actual intellectual functioning that was both at substage A and B levels, from which it could be inferred that they did not actually use fully developed formal operational thought (Table 37).

3. Student potential level of intellectual development compared with student actual intellectual performance displayed in non-Piagetian conditions.

All three students showed congruency between actual intellectual performance and potential level of intellectual development (Table 38).

TABLE 38

COMPARISON OF STUDENT PERFORMANCE ON THE PIAGET TASKS AND SELECTED EXAMINATION ITEMS

Student	DESCRIPTORS USED															OVERALL PERFORMANCE
	Formal Operations									Substage A			Substage B			
	2.11	2.12	2.21	2.22	2.23	2.31	2.32	2.41	2.51	3.11	3.12	3.13	3.21	3.22	3.23	
J.V.		+	+	+	+	+	+	+	+				+	+	+	Piaget Tasks
	+	+	+	+	+			+	+				+		+	Exam. Items
B.H.	(+)	+	+	+	+	(+)	+	+	+	+	+	+		+	+	Piaget Tasks
	+	+	+	+	+			+	+	+		+	+		+	Exam. Items
L.W.		+	+	+	+	+	+	+	+	+			+	+	+	Piaget Tasks
	+	+	+	+	+			+	+	+			+		+	Exam. Items

This table shows the congruency between intellectual performance observed on the Piaget tasks and intellectual performance observed in the responses to the selected examination items for students J.V., B.H., and L.W.

4. Adequacy of inventory of descriptors in describing student performance on Piaget tasks and on selected examination items.

Descriptor 2.11 was not found to be useful in theoretical situations, i.e. on the examination items. Descriptors 2.31, 2.32, 3.12 and 3.22 were not found useful in practical experimental situations, i.e. in the Piaget tasks. The fifteen descriptors however, seemed to cover adequately both theoretical and practical situations.

In conclusion, it is apparent: -

a) that the inventory of descriptors can be used to identify and select examination items that require formal operational thought for their solution;

b) that these selected examination items can be used as a means of evaluating student performance in examinations in terms of the level of intellectual development displayed by the students in the classroom situation;

c) that Piaget's theory of intellectual development can be further developed in order to become of practical use in the classroom.

B. IMPLICATIONS OF THE COMPARISONS

Formative Evaluation

The inventory of descriptors is an instrument designed to identify intellectual behaviour of individuals at the formal operations stage, and can be applied to both verbal and non-verbal behaviour. Such an instrument could prove valuable in the formative evaluation involved in curriculum design.

The designs of new science curricula are very often based on one or more theories of education. An independent method of evaluating the intellectual progress of students using the curricula is advantageous, especially if the evaluative criteria are based on an acceptable theory such as Piaget's theory of intellectual development. The inventory of descriptors for indentifying the formal operations level of development could be used in the formative evaluation of curriculum development in three ways;

1) evaluation of the suitability of subject matter being presented to different age groups in the terms of the intellectual standards required to master the concepts.
2) evaluation of the suitability of different methods of instruction for different age groups, and
3) evaluation of the intellectual demands made by different forms of examinations, e.g. multiple choice, essay questions, practical examinations. etc.

Classroom Practice

The inventory of descriptors could be used by teachers in the classroom situation to evaluate a course of instruction in progress in order to improve it. This could be effected by;

1) evaluating the level of intellectual performance required of students in order to carry out classroom tasks and unserstand new concepts,
2) evaluating the potential intellectual skills of the students in order to ascertain the level at which they are able to function,
3) matching the selected classroom task against the identified intellectual capacity of the students.

Teachers could thus optimize conditions for learning by tailoring the courses to meet the intellectual needs of the students. They could anticipate the source of possible difficulties for the students in performing the tasjs and understanding concepts, and could take steps to present difficult subject matter in such a way as to minimize the difficulties.

C. CRITIQUE OF THE USEFULNESS OF THE INVENTORY

The inventory of descriptors has been shown by this study to identify successfully intellectual behaviour of individuals at the formal operational level from both performance on the Piaget tasks as well as performance in non-Piagetian conditions. The inventory can be applied to behaviour that is both verbal and non-verbal in nature, provided that it is recorded.

The advantage of this method of identifying intellectual behaviour is that it analyses behaviour in depth, producing more thorough, and meaningful information than might otherwise be the case. The instrument identifies the maximum intellectual potential of individuals as well as the actual intellectual level displayed by the individual in different circumstances.

The flexibility provided in the administration of the Piaget tasks allows for freedom in handling different personalities in different conditions. The inventory of descriptors provides a means of standardizing the interpretation of the data in such a way as to add a measure of reliability to the results. The validity of the instrument is ensured by the fact that it adheres as closely as possible to the Piaget texts.

The format of the inventory serves both as a means of helping potential users of the inventory to understand the descriptors, as well as to demonstrate their coherence and relationship with Piaget theory both generally and specifically.

There are two main disadvantages with this method of identifying intellectual levels. Firstly, the technique involves in-depth analyses of behaviour. The clinical technique of administering the Piaget tasks requires time and patience. In addition, the application of the inventory to the data to be analysed is time consuming, as fine details and nuances of behaviour must be considered. The complex nature of the inventory compounds the difficulty in applying it. The method thus limits the number of individuals whose behaviour can be analysed, and makes it impossible for mass testing. Moreover, the method does not lend itself to quantification.

The second disadvantage is found in the need for users of the inventory to be familiar with Piaget theory and to be well trained in the application of the inventory to the data. This requirement further limits the use of the inventory in the classroom.

In summary, it is apparent that the methodology developed in this study is at present directed towards in-depth analysis of the intellectual performance of a limited number of individuals. It is applicable to subject matter such as that found in a classroom situation as well as to student performance on classroom tasks.

D. FURTHER RESEARCH

The inventory of descriptors is designed at present to identify intellectual behaviour at the formal operations level. Further research is required to refine the instrument itself. The interrelationships between the theoretical constructs, descriptors and exemplars, should be established with more rigour. The methodology for analysing relatively instructional and structural data on intellectual performance, represented by the inventory, require considerable development in order to make it possible for classroom teachers to determine existing and changing modes of thought and the possible reason for these shifts. This would serve to improve the logical validity of the inventory, as well as to render it more usable and reliable and therefore more useful.

An inventory of descriptors of behaviour at the concrete operations stage would make the analyses of behaviour more convincing. If an individual is shown not to be performing at the formal operations stage, it is more meaningful to reinforce the results by identifying his behaviour as being at the concrete operations stage. The importance of the methodology developed in this study lies in it's potential usefulness for the classroom. If the inventory could provide a way of systematically analyzing intellectual performance with a view to discriminating between individuals at the formal operations stage and the concrete operations stage, it could facilitate the improvement of classroom instruction. Similarly if the pre-operational stage and sensory motor stage could be represented by an inventory of descriptors, the theory of Jean Piaget, instead of being potentially significant for classroom practice, could become of practical importance.

At this point it is opportune to draw the reader's attention to the very recent work of Professor Klaus G. Witz (1971). This work may well represent a major step forward in the direction described above and is recommended to the reader interested in a more rigorous methodology for applying Piagetian theory to problems of classroom practice.

E. SUMMARY AND CONCLUSIONS

This thesis is addressed to a problem in classroom practice indentified as formative evaluation, that is, the development of a method of utilizing Piaget's theory of intellectual development for evaluating the intellectual performance of students contending with the formal concepts and methods of inquiry presented in a first year physics course with a view to improving instruction in this course.

The method of studying the problem required a good understanding and analysis of the relevant aspects of Piaget's theory so that it could be re-formulated in such a way to be usable in identifying formal behaviour of individuals in answering specially selected examination items. This resulted in the formulation of a methodology in the form of an inventory of descriptors, and a method of analysis designed for identifying behaviour at the final stage of Piaget's developmental sequence, namely, the formal operations stage.

The inventory of descriptors was used to identify formal operations; behaviour of students performance on selected Piaget tasks, providing information concerning their maximum potential level of intellectual development. It was then used to identify physcis examination items which required formal operations for their solution and the formal operational behaviour displayed by students in responding to the selected items, thereby providing information concerning the actual level of intellectual performance displayed by students in classroom conditions. A comparison of identified intellectual behaviours provided information concerning the usefulness of the instrument as well as information concerning the potential usefulness of such an instrument in science education.

It was concluded that the inventory of descriptors adequately described and identified intellectual behaviour at the formal operations level, both in student performance on the Piaget tasks, and in student performance on selected items from the physics examination paper. The inventory of descriptors may well be of potential value to formative evaluation in the classroom situation.

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APPENDIX A

TRANSCRIPTS OF STUDENT PERFORMANCE ON THE PIAGET TASKS

Complete transcripts of the performance of student L.W. on the four Piaget tasks are given. The transcripts are taken literally from the video tapes made of the interview, and descriptions of student actions are included.

The ennumeration of the comments enables the reader to make cross-reference to the appropriate synopses given in the text for the purpose of checking their accuracy.

TRANSCRIPT OF STUDENT PERFORMANCE ON THE ANGLES OF INCIDENCE AND REFLECTION TASK: STUDENT L.W.

Interviewer	Student
1. (Shows apparatus and poses the problem.)	2. Well you aim it so it hits there (rebound wall). If you split the angle up between there and there -- so both those angles are equal, (i.e. $\angle I$ and $\angle R$).
3. How do you split the angle?	4. You bisect the angle -- like the angle that this comes in on, it has -- uhm -- you take the perpendicular to this (rebound wall), and if it (ball) goes in on this angle here ($\angle I$), it must come out (rebound) at the same angle ($\angle R$).
5. Why did you take the perpendicular?	6. Uhm, well you want to find out what angle it comes in on, and if you take --. Well if this (wall) was turned around a bit -- well, it always has to be perpendicular to the surface.
7. If this (rebound wall) was turned around a little, what would happen?	8. Then the perpendicular would come out at an angle like this (demonstrates). So then the angle ($\angle I$) would be greater and it ($\angle R$) would come out -- greater.
9. OK. Is this a rule which holds for all cases?	10. Uh -- yes, if something comes in on a certain angle, providing the surface is flat, it is. If it hits here in waves, or something -- it will come off at a different angle.
11. Can you prove it?	12. Well, I'm not a very good shot (laughs) uhm -- Well, if you were to -- if this (board) was glass or something, (takes a piece of paper) and you put this (paper) underneath and drew a line out perpendicular to it (rebound wall), and you drew a straight line which would meet at this point here (line of incidence), then get a piece of carbon paper and put it on top of the paper, and see the path that the ball makes on the carbon paper when coming out, and you measure those two angles, and they should be equal.
13. OK, I get the idea now that they should be equal, but what happens when you start changing the position of the launcher?	

14. You have to change the position of your target too.

15. And what about the angles, are they still the same?

16. Well the angle that it hits, it will be wider, -- but it will hit somewhere over here (to right) so you will have to bring your target over here (to right).

17. And the other way?

18. If you moved it the other way, the angle is getting smaller until you get it perpendicular with the (rebound wall) in which case it will go and come right back again and your target would have to be even with it (i.e. the launcher).

19. OK, what about the other angles?

End of Tape.

TRANSCRIPT OF STUDENT PERFORMANCE ON THE OSCILLATION OF THE PENDULUM TASK: STUDENT L.W.

Interviewer	Student
1. (Shows apparatus, and poses the problem.)	
	2. You put them (the pendula) both the same length (adjusts string) and try to determine if weight is a factor. Take a light and heavy one. This shows it most obviously. Uh -- you pull it back -- How shall I determine this? (Uses ruler as starting point).
	3. They're both turning at the same time. This one "C" (light bob) has a shorter distance to go, but they're still turning at the same time, so the period is still the same.
4. What did you mean by "C" has a smaller distance to go?	
	5. Well, this one (heavy bob) is still going further so it's going faster. This one (light bob) is going a smaller distance.
	6. If you want to determine -- to see if the length does anything -- now let's see -- (begins shortening) -- Well, you could try the force, but this wouldn't be very accurate because you don't know if your forces are the same. (Puts a heavy bob on both pendula).
7. You don't need to do it too high.	
	8. (Pushes pendula equally). That was the same force, and they are travelling at very much the same amplitude -- I mean (Drops left pendulum, pushes right pendulum) Let's try it. (Unintelligible remark of surprise) They both turn at the same time, but I thought that -- force had something to do with it! They're both turning at the same time, so force must be -- I didn't put any force on this one (left pendulum), and this one I did (right pendulum).
9. Can you justify that? Are you happy with it?	
	10. Uhm (thinks) No, not really when -- I guess so! If you use F = ma -- you had more force -- so acceleration is greater.

Masses are the same -- so that should mean they are equal in period, like they both travel at the same time. I can't remember any other --

11\. Well, have we finished?

12\. No, we have length and (removes weights) amplitude.

13\. (Suggests S use a ruler if she wishes).

14\. (Shortens right pendulum then lengthens it to make it equal the length of the left pendulum). Measuring amplitude, you need the same weight.

15\. How do you know that?

16\. Well, when you're doing experiments, we already said that mass had nothing to do with it, so you should -- well, you see -- this is your control. Your control should be always constant. And this should be the one to vary (i.e. right pendulum), and if you're comparing them and not the actual weights, -- something else -- in a system, these (lengths) should be the same. So that you can compare them.

17\. Uh hum.

18\. So, pull this (right pendulum) back to a further amplitude. (Swings them and watches).

19\. Uh hum.

20\. Well, they're turning at the same time so the period must still be equal.

21\. Something else?

22\. (Swings weights again giving the right pendulum larger amplitude). This one (right pendulum) is taking longer now, but you can't tell if you've given it a tiny bit of force or not. I said force didn't have anything to do with it either. Let's see, uhm. (Swings them again).

23\. (Indicates that only initial swings should be considered.)

24\. Well this one (right pendulum) has a greater amplitude but it has a greater speed too so that it's -- You can justify it that way that they both have the same period.

25\. OK - that makes sense does it?

26\. Uhm.

27\. And the force?

28\. What did I say about the force? It didn't -- it didn't matter about force.

29\. Can you use the same explanation to justify force?

30\. Uhm, Well, we used the same mass. If you use same mass, then mass is constant -- You're applying a greater force to this, so the acceleration will be greater within a certain period of time. If you use F = ma, and the force is greater and the a is greater, m is constant, then it should be the same ratio. (Shortens right pendulum and puts on both heavy weights. Allows both weights to drop).

31. I'd say, the shorter the string the greater the period.

32. Shorter the string, the greater the period?

33. Uh hum. (Watches) -- Well, the shorter the string -- Oh, if this (right pendulum) was half that (left pendulum) it should take half as long for this (right pendulum) to get back to the point as this one does (left pendulum).

34. Uh hum. (Noncommital).

35. If they (the pendula) are down here, with the same length and same weight you get the same period. If this one is shorter, the faster it goes.

36. OK. So now can you summarize?

37. Force, no difference. Mass, the same as long as length was the same. Amplitude, didn't make any difference. Length of string did make a difference. It frustrates me because I can't remember. I know we take this stuff in Physics to determine what's dependent on one another and when you can't remember it -- If you're doing that for the first time I think you'd accept what you saw, but knowing in doubt before, I was just wondering -

38. Was that confusing you?

39. Yes -- uh hum -- I think this is -- I think if you've had it before but never learned it thoroughly enough to remember it, well then, it confuses you the second time.

TRANSCRIPT OF STUDENT PERFORMANCE ON THE COMBINATION OF LIQUIDS TASK: STUDENT L. W.

Interviewer	Student
1. (Shows apparatus and poses question).	2. Shall I do it? - You took it (the liquid) from one of the beakers?
3. I'm not telling you exactly where it comes from!	4. (Adds each liquid singly in beakers. Adds indicator to each. Adds more g to (1 + g). I'd say it was this one, (1 + g).
5. It will go pencil yellow, darker than that.	6. (Adds more indicator and shakes each). I'm assuming that it's changing -- when -- Can I use as many of these ...?
7. Yes.	8. (Tries ((1 + g) + 2 + g)) and ((1 + g) + (3 + g)). Is that the colour?
9. Yes, that's positive.	10. Then it's a combination of this and this and this and plus indicator (i.e. (1 + 3 + g)).
11. Are you quite sure about it?	12. Uh -- Well, I could try the last one. ((1 + g) + (4 + g)) I'd say those, (1 + 3) and the g (i.e. (1 + 3 + g)).
13. Uh hum (noncommital).	14. I'll see if it doesn't turn when I mix them up. (Tries (1 + 3 + g) again and gets color).
15. Is that the only way you could get the color? Is there anything else you could tell me? Could you tell me if any of the others are important as well?	16. Well, I mixed the other two in these combinations here (1 + 4 + g), and (1 + 2 + g) and they didn't work. And here I have the indicators in by themselves ((2 + g) and (4 + g)) and they didn't work as well. So it had to be a combination and it was this combination that turned it (1 + 3 + g).
17. Uh hum --	
18. Just say for the argument that it hadn't turned with (1 + 3 + g), what would you have done then?	19. You mean on mixing? Well, keep going on different combinations to see which one works.
20. How many combinations?	21. (Thinks) Uhm, forgotten how you figure that ---
22. Could you write them out?	23. Writes: (4 singles, 6 doubles, 4 triples, 1 all four) So that's 15.

24. 15?

25. Yea - 4 singles, 6 doubles, 4 triples, and 1 all four.

26. All right, good. Well, what about liquids 2 and 4? What can you tell me about the possibilities from what you've done already?

27. Well, when this one (4) doesn't react with g, and whatever you were trying for is not in this one (2) and you also know that when you mix the 2 together that there is no reaction shown off by the g. So there's nothing of what you're trying to determine in any of those two. Also, depending on the g you've got, acid, base, or water.

28. Well, we haven't any technique for identifying acids or bases. So, let's see. On what would you base your hypothesis that they both (2 and 4) have nothing to do with the reaction? Do you feel that what you've said proves it conclusively?

29. You might have to do other -- They (1 + 3) might contain something and you'd need something to set off the reaction. 2 and 4 may need another liquid or something to mix in with it -- just to set off the reaction. This doesn't prove anything. You'd have to do other experiments.

30. Uh hum. Can you distinguish between 2 and 4?

31. As they are, no, I don't think so. With another indicator, you might be able to. (4 + g) has turned off clear (cloudy). (2 + g) is completely clear. This (4 + g) is not much changed, it's off-color in (2 + 3 + g) but not enough. I suppose you could distinguish them in that way, but it's not enough for what we want. It may have been that I put too much indicator in, or more of 2 or 3 than I should have. But there's not enough change to say anything different about them. I would have to do further tests.

32. (Corrects S for she is naming wrong beakers)

33. Can you make any predictions? You've got 16 possible combinations there - could you predict for each one what the reaction would be?

34. Well, anything with 1 + 3 + g would turn yellow, and there'd have to be the combination of 1 and 3. Anything with just 1 + g would turn a tiny bit yellow. With just 3 in it, or 2 or 4, it would be clear. I think that's all.

TRANSCRIPT OF STUDENT PERFORMANCE ON THE BALANCE TASK: STUDENT L.W.

Interviewer	Student
1. (Shows apparatus and explains problem.) Set up a balanced system.	
	2. This - uhm - so you can determine - ?
3. Just so you can balance both sides.	
	4. Uhm. (Tries).
5. What have you got there?	
	6. (Puts 200gm x 8L = 200gm x 8R and adds a 10gm x 7L).
7. Uh hum - Why did you do that?	
	8. Before, the balance was slightly this way (slanted to right) and I was wondering - now it's gone - this side is heavier but that's because this is closer to the end. (S concerned about small unbalance in apparatus).
9. Yes, it's a crude apparatus - don't worry about that. Can you explain what's going on here?	
	10. You've added more weight to one side so you have to add more equal weight to the other side to keep a balance on the fulcrum. It doesn't matter how much weight you add but you have to add it the same to both sides to balance.
11. Does it matter where you put it?	
	12. No, if you put it here on this side (R) you have to put the same distance on this side (L). It should be same weight and distance from centre.
13. Give me some more variations now.	
	14. If you want to prove distance you put this further out. That's more weight - no balance - That's why it has to be the same distance.
15. Do the weights change?	
	16. No, the weight hasn't changed. There's more weight at end of balance -- what's pulling it down is -- see, you've no longer got the fulcrum in the centre of the system, so it will be unbalanced. It's as if you've got more weight on this side. So you've got to put it to the same distance. Doesn't matter how close you bring it in to the fulcrum. It has still always got to be the same (distance); if you had -- it doesn't matter how small weights. Size of the weight shouldn't matter if it's balanced. (Moves weights to various equal positions on ruler, and keeps a balance).

17. Can you set up something using different weights on each side?

18. You can have -- (puts 500gm x 9L = ((200gm + 200gm + 100gm x 9(R)).

19. Just explain what you're going to do.

20. Well, you can add it -- You can have 200gm + 200 gm and another 100gm to make up the 500gm, but you can't have the -- you can't put them in different places. It has to be the same place to do it (balance).

21. Uh hum.

22. You can have any combination you want to make up the weight on the opposite side but they have to be the same distance.

23. Uh hum.

24. And if you wanted a 400gm on this side (L) and make it equal, you could put the other 200gm there to make it equal, and it wouldn't matter. (i.e. 400gm x 9L = (200gm + 200gm) x 9R)).

25. OK.

26. If you're going to have all the weight on one (L) hook - you have to have it all on one hook here (R) too.

27. Is it possible to use different weights? Can I put 100gm on my side (L) and 50gm on your side (R)? Could you work out a balance.

28. Well, if you put 200gm here at 9L and 100gm x 5R -- no! -- the other way around (i.e. 200gm x 5L = 100gm x 9R) it should work. The 200 lbs. on this side (R) is at half the distance and this one (100gm x 9R) here has half the weight and twice the distance from the fulcrum. It should still balance.

29. Is this a general rule?

30. Yes (thinks) It should be...

31. (Releases balance and the left side drops). Can you see what's the matter? Can you adjust left side now? Any reason?

32. No, it not (a general rule)! -- uhm (thinks, then changes balance to 100gm x 8R = 200gm x 4L). Well, before this (R) was 9 and this (L) one was 5 - so this (L) was more.

33. Fine (Takes off weights) I'm going to use 40 gm, how about that? (Puts on 40gm x 5L and gives S 50gm to balance on other side).

34. (Hangs 50gm at 4, i.e. 40gm x 5L = 50gm x 4R).

35. Are you sure?

36. It should be -- uhm (Thinks).

37. Any reason?

38. I think so. Well, you've got them in the same ratio, this is 40gm at the 5th hook which equals 200, and 50gm at the 4th hook which equals 200 as well.

39. Yes?

40. Distance and mass should be in balance.

41. There is a formula for this, but let's not worry about formulae. Can you explain to me in terms of ratios... I think you've probably got it. Let's give you another possibility. Put this one there (30gm x 2L), can you balance that without being exactly symmetrical?

42. That's 30gm ... (Puts 20gm x 3R).

43. How do you know?

44. You have the same ratio of weight. This one is 30gm at 2(L). Here 20gm at 3(R). It's the same ratio, like the greater the distance, the smaller the weight that you need to balance.

45. Just for interest, if I were to start counting from the edges (1, 2, 3, 4, 5, 6,) would that be important.

46. No - it's the distance from the fulcrum.

47. Yes, is that important?

48. Distance from fulcrum is. It doesn't matter what you've got out here.

49. Another thing - could we swap these, my 30gm and your 20gm? Would that be alright?

50. You'd have to treat them like ... (changes distance so that 20gm x 3L = 30gm x 2R).

51. Oh - you have to change the distance?

52. Yes.

APPENDIX B

PHYSICS 110 EXAMINATION PAPER
UNIVERSITY OF BRITISH COLUMBIA
APRIL 1970

1.a) Give a definition of FORCE (Classical Mechanics), as an equation:

b) Explain the symbols used:

2.a) Give a definition of MOMENTUM (Classical Mechanics), as an equation:

b) Explain the symbols used:

3.a) Give a definition of WORK (Classical Mechanics), as an equation:

b) Explain the symbols used:

4.a) Give a definition of ELECTRIC POTENTIAL DIFFERENCE, as an equation:

b) Explain the symbols used:

5. An object moves on a circle with constant speed.

a) Give an equation for the force required to keep it in orbit:
b) Explain the symbols used:

6. Give a relationship between the wavelength λ, the frequency f, and the speed of propagation v of a wave:

7. When the potential energy of a system increases there will also be an increase in mass. Give an equation governing this mass increase:

8. Somebody claims that telepathic signals are a special kind of waves. What general kind of experiment would he have to perform to demonstrate that they actually are <u>waves</u> (in the meaning of the word "waves" as used in physics)?

9. Nuclear Energy can be converted into heat by nuclear fusion as well as by nuclear fission. Could one not make the best use of these processes by first splitting atoms (nuclear fission, heat will be produced), and then re-uniting the parts again (nuclear fusion, heat will be produced)? Repeating this cycle over and over again, one would have an inexhaustible energy source.

Explain in terms of the binding energies of nuclei, why this process is impossible.

<u>YOU HAVE THE CHOICE TO OMIT QUESTION 10 OR QUESTION 11</u>

10. A radioactive sample explodes in a laboratory. Immediately after the explosion a Geiger counter in the room records 960 counts/sec. One day

later, the same counter records 240 counts/sec. Assuming that a safe level of radiation as indicated by the counter would be 1 count/sec., what would be a reasonable estimate of the number of days since the explosion for people to safely re-enter the laboratory?

(Give reasons with your answer).

YOU HAVE THE CHOICE TO OMIT QUESTION 10 OR QUESTION 11

11. A microphone stands at some distance from a wall. At some greater distance, a loudspeaker emits sound. The frequency of the sound is steadily increased beginning from zero, while the intensity of the sound is kept constant. When the frequency is below 1,500 cycles/sec., sound will be picked up by the microphone. At 1,500 cycles/sec., no sound is received. When the frequency is increased further, the microphone will pick up the sound again.

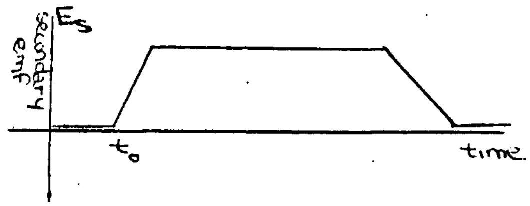
a) Explain this phenomenon and b) predict at which higher frequency there will be the next minimum so that no sound will be received by the microphone.

b) ANS: NEXT MINIMUM OCCURS AT: __________cycles/sec.

YOU HAVE THE CHOICE TO OMIT ONE OF THE QUESTIONS 12, 13 OR 14

12. An indirect way for measuring currents (frequently used to measure strong current pulses) is to pass the current, I_p through the primary coil of a transformer and to display the secondary emf, $_s$, with an oscillscope.

The current begins to flow at time $_+$. The oscilloscope trace of the secondary emf,$_s$ looks like this:



After careful consideration, give a qualitative graph of the primary current I_p.

YOU HAVE THE CHOICE TO OMIT ONE OF THE QUESTIONS 12, 13, or 14

13. A proton moves, with steadily increasing speed, along a straight line (x axis of a coordinate system). The only forces involved in this motion are caused by electric and/or magnatic fields. These electric and/or magnetic fields are constant in time and homogeneous in space.

Which one or more of the fields listed below has to be, or could be, present? (disregard signs. "along x axis" means: in direction of + or -x)

There has to be an electric field along the x axis ☐

There could be an electric field along the x axis ☐

There has to be an electric field along the y axis ☐

There could be an electric field along the y axis /‾‾/

There has to be an electric field along the z axis /‾‾/

There could be an electric field along the z axis /‾‾/

There has to be a magnetic field along the x axis /‾‾/

There could be a magnetic field along the x axis /‾‾/

There has to be a magnetic field along the y axis /‾‾/

There could be a magnetic field along the y axis /‾‾/

There has to be a magnetic field along the z axis /‾‾/

There could be a magnetic field along the z axis /‾‾/

If you would rather answer in a different way, please do so on the back of the preceding page.

YOU HAVE THE CHOICE TO OMIT ONE OF THE QUESTIONS 12, 13 OR 14

14. Assume you bought a 100 Watt A.C. power supply specified to supply currents at a frequency of 50 cycles/sec. How could you test for the frequency using nothing else but some wire, and a calibrated stroboscope with adjustable frequency?

(Please explain your answer with the aid of a drawing.)

YOU HAVE THE CHOICE TO OMIT ONE OF THE QUESTIONS 15, 16 OR 17

15. (A Spy vs. Spy episode) Passing the Black Spy's spaceship with a relative speed of 80% of the speed of light, the White Spy triggers a time bomb hidden in the Black Spy's ship. If the explosion is to occur 1,000 m distance from the White Spy's ship, at what time interval should the time bomb be set?

YOU HAVE THE CHOICE TO OMIT ONE OF THE QUESTIONS 15, 16 OR 17

16. Space explorers discover a ring of charged particles orbiting around a mysterious cloud. The ring consists of positive hydrogen ions and negative oxygen ions, circulating in the same direction. The speed of the hydrogen ions is 1 km/sec, the speed of the oxygen atoms is 2 km/sec, the radius of orbit is the same for both kinds of particles. The number of particles per cubic meter is too small to allow the ions to combine. For the same reason, no electric or magnetic forces between the ions could account for the motion.

The explorers discuss the following explanations to account for the circular orbits of the ions. Try to rule out as many of these explanations as possible.

a) The circular orbits are due to gravitational attraction by a massive star within the cloud, ☐ could be, ☐ cannot be.
Give reasons for your choice.

b) The circular orbits are due to a charged object hidden in the cloud. ☐ could be, ☐ cannot be.
Give reasons for your choice.

c) The circular orbits are due to a magnetic field at right angles to the plane of the orbits. ☐ could be, ☐ cannot be.
Give reasons for your choice.

YOU HAVE THE CHOICE TO OMIT ONE OF THE QUESTIONS 15, 16 OR 17

17. Design a device to (indirectly) measure the wavelength of a given ultraviolet spectral line. No use may be made of interference (as e.g. by using gratings, slits, standing wave patterns) or of refraction (e.g. as by using a prism).

Give a drawing of your design in sufficient detail. Explain what you observe with this device, and how you obtain the wavelength of the spectral line from your observations.

APPENDIX C

BASES OF SELECTION AND REJECTION OF EXAMINATION ITEMS

Examination Item	Selected	Rejected	Reasons for Selection or Rejection
1 - 7		+	Simple RECALL of formulae.
8		+	Student required to recall information on the properties of waves in general, The student is required to select the appropriate recalled information to fit the facts - RECALL.
9	+		Involves FORMAL OPERATIONS. Item concerns hypothetical question involving nuclear energy
10		+	FORMAL OPERATIONS NOT INVOLVED
11		+	FORMAL OPERATIONS NOT INVOLVED
12		+	Involves FORMAL OPERATIONS. Involves conversion of secondary emf into primary current. Insufficient analysable material in student response.
13		+	Involves FORMAL OPERATIONS. Involves selection of correct response to given conditions. Insufficient analysable material in student response.
14		+	Involves FORMAL OPERATIONS. Involves design of method for testing frequency of current under particular conditions. Student L.W. did not respond.
15		+	Involves FORMAL OPERATIONS. Item based on understanding of relativity and a calculation insufficient analysable material in student response.
16	+		Involves FORMAL OPERATIONS. Involves selection of appropriate hypothesis to fit given situation.
17		+	FORMAL OPERATIONS NOT INVOLVED.