POWER SYSTEM STABILITY STUDIES
USING LIAPUNOV METHODS

by

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ABSTRACT

The transient stability of power systems is investigated using Liapunov's direct method. Willems' method is applied to three- and four-machine power systems with the effect of damping included. The distribution of damping among the machines of a multi-machine system is studied, and optimum ratios are derived. An extension of Willems' method is used to include governor action in the system representation. Finally, the effect of flux decay on stability regions is studied using Chen's method.

TABLE OF CONTENTS

Page

LIST OF ILLUSTRATIONS

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NOMENCLATURE

$\underline{x}$	Vector of state variable
$\dot{\underline{x}}$	Time derivative of $\underline{x}$
V	Liapunov function
$\dot{V}$	Time derivative of V
V_m	Value of V defining stability region
t	Time
δ	Angle between quadrature axis of synchronous machine and infinite bus or a reference frame rotating at synchronous speed in the case of multimachine systems
δ_o	Steady state value of δ
δ^u	Value of δ at the unstable equilibrum position
H	Inertia constant in KW - Sec/KVA
M	$H/(\pi f)$
f	System frequency = 60c/s
α	Damping coefficient
R	α/M, Relative damping constant of synchronous machine
P_m	Mechanical power input to synchronous machine
P_e	Electrical power output of synchronous machine
E'_q	Instantaneous voltage proportional to field flux of synchronous machine
E_B	Voltage of infinite bus
E	Steady state internal voltage of synchronous machine
X_{12}	Total reactance between synchronous machine and infinite bus
X'_d	Transient reactance of synchronous machine
X_e	Reactance of transmission line

X_d	Synchronous reactance of synchronous machine
T'_o	Open circuit transient time constant of synchronous machine
n_1	$(X_e + X_d)/T'_o(X_e + X'_d)$
n_2	$(X_d - X'_d)E_B/T'_o(X_e + X'_d)$
0	The null matrix
I_n	The unit matrix
s	Laplace operator
X1Y	Product of three matrices, X and Y are n x n matrices and 1 is an n x n matrix with all elements equal to 1.

INTRODUCTION

Since the early days of a.c. electric power generation and utilization, oscillations of power flow between synchronous machines have been known to be present. The possiblity of such oscillations and the tendency of a system to lose synchronism appears to be more prevalent in large systems. The stability characteristics of a power system during transient disturbances may be assessed from its mathematical model: a set of nonlinear differential equations, known as the swing equations. These equations describe the power system dynamics, their order depending on the detail of representation used for the synchronous machines and associated control apparatus. Several methods are available for the solution of the transient stability problem. For simple configurations under the usual assumptions of constant input, no damping and constant voltage behind transient reactance, the equal area criterion or the phase plane method may be used. When the study involves a large number of machines or when it is necessary to take into account such refinements as transient saliency, field decrement, exciter action and damping, stability studies are usually investigated through step-by-step numerical integration of the system differential equations until the critical switching time is found. Such a method is cumbersome and very costly since an almost prohibitive amount of computation is required in its execution. Thus the need increases for the development of more direct methods for studying stability. During the past few years the application of the second method of Liapunov to the problem of power system transient stability using models of varying degree of complexity for the power systems has been found useful and straight forward. The approach involves choice of a suitable Liapunov function to estimate the region of asymptotic stability around the equilibrum state of the post fault system and the critical switching time can be obtained by carrying out only one forward integration of the swing equations.

The difficulty in the application of Liapunov's direct method is that in general there is no obvious way to choose a suitable Liapunov function. In many cases involving a physical (mechanical or electrical) system the energy stored in the system appears to be a natural candidate. Gless [13] studied 1-, 2-, and 3- machine systems representing the machines in the simple form of a constant voltage behind synchronous reactance, neglecting all losses, damping, flux decaying and considering a constant input. El-Abiad and Nagappan [14] considered a multi-machine system including in their model losses and constant damping.

Siddique [16] considers a single machine system taking into account field decrement and simplified governor and regulator action.

Other applications were made using formalized construction procedures, Yu and Vongsuriya [15] employed Zubov's method to develop a Liapunov function for one machine infinite bus system using a second order model for the machine and including a damping coefficient which is a function of the angular displacement of the machine. Rao [17] used Cartwright's [20] procedure to construct a V-function for a single machine taking into account the transient saliency effect, a constant damping factor and a governor action represented by a single time constant. Rao also applied this method to a simplified 3-machine system. The variable gradient method [21] was applied by Rao and Desarkar [19] to a one-machine system including the effect of the field-flux linkage changes.

Pai, Mohan and Rao [18] applied Popov's theorem on the absolute stability of nonlinear systems using Kalman's procedure [4] to construct a Lure-type Liapunov function for a one machine system with and without governor action. The generalized Popov criterion [8] for multivariable feedback systems was used by J.L. Willems [9, 10] to develop a Liapunov function for n-machine power system.

In this thesis the stability of single-machine as well as multi-machine power systems is investigated using two different procedures to construct suitable Liapunov functions. In Chapter I Willems' method is applied to a three machine power system taking into account the damping effect. A four machine system is considered in Chapter II and the best distribution of damping ratios is obtained by maximizing the hypervolume enclosed by the Liapunov function. Willems' method is extended in Chapter III to study a three machine system including governor action. In Chapter IV Chen's method is applied to a single machine infinite-bus system taking into account the decay in field flux linkage.

CHAPTER I

GENERALIZED POPOV'S CRITERION AND WILLEMS' METHOD

The stability study of automatic feedback control systems containing single memoryless nonlinearities, figure 1.1, was initiated by Lure. Normally the nonlinearity is confined to a sector of the first and third quadrants as shown in figure 1.2. Popov [1] made a most important contribution to the problem by giving sufficient conditions for absolute stability which are completely dependent on the frequency response of the linear part of the system. A procedure for constructing Liapunov functions for such systems was introduced by Kalman [4].

Recently Anderson [6], [8] developed a theorem generalizing Popov's criterion and Kalman's procedure to investigate the stability of feedback control systems containing more than one nonlinearity. The theorem relates the concept of a positive real matrix to the concept of minimal realization of a matrix of transfer functions [7]. Liapunov functions based on Anderson's theorem were constructed by Willems [10] for multimachine power system stability studies. Willems' method is applied in this chapter to a three machine power system.

1.1 Generalized Popov's Criterion [8]

Automatic feedback control systems with multi-nonlinearities, figure 1.3 and figure 1.4, can be descirbed mathematically in state variable form by

$$\begin{aligned} \dot{x} &= Ax - Bf(\varepsilon) \\ \varepsilon &= Cx \end{aligned} \tag{1.1}$$

where

x n vector

ε m vector

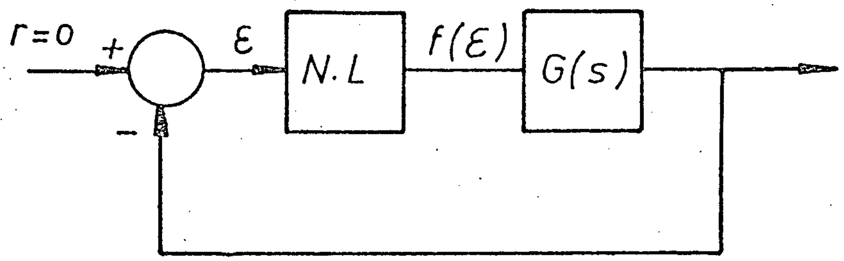

Fig. 1.1 Automatic Feedback Control System Containing Single Memoryless Nonlinearity

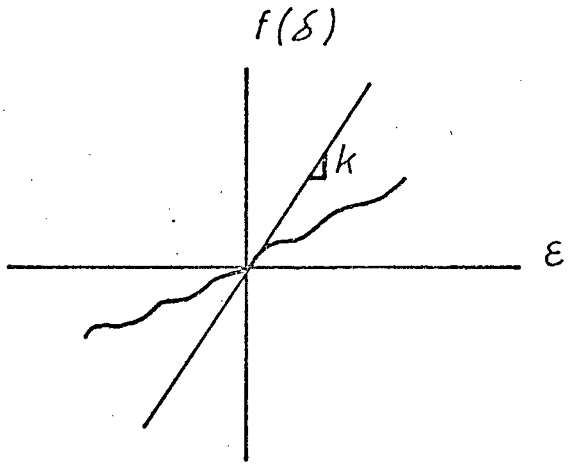

Fig. 1.2 Nonlinearity Confined to a Sector of the First and Third Quadrant

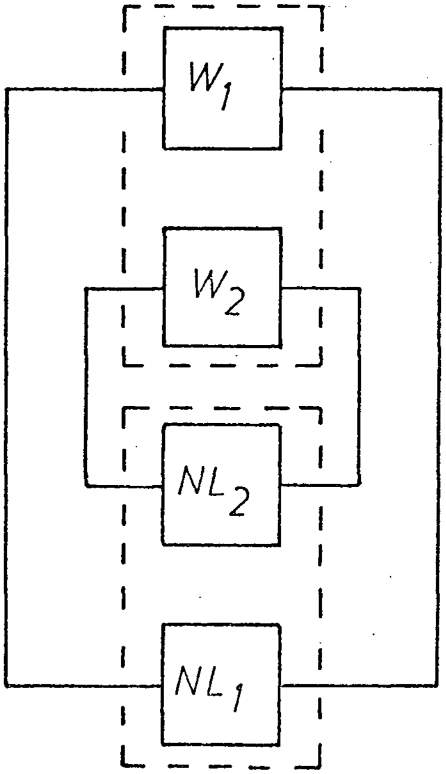

Fig. 1.3 Automatic Feedback Control System With Multi-nonlinearity

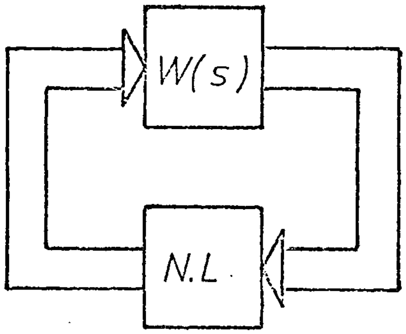

Fig. 1.4 Automatic Feedback Control System With Multi-nonlinearity

A $n \times n$ asymptotically stable matrix

B $n \times m$ matrix

C $m \times n$ matrix

$f(\varepsilon)$ m vector satisfying the sector condition

$$0 \leq f_i(\varepsilon_i) < k_i \varepsilon_i^2$$

$$f_i(0) = 0 \qquad i = 1,2, \ldots, m$$

Theorem [6]

If there exist real diagonal matrices

$$N = \mathrm{diag}\,(n_1, n_2, \ldots, n_m)$$

$$Q = \mathrm{diag}\,(q_1, q_2, \ldots, q_m)$$

$$K = \mathrm{diag}\,(k_1, k_2, \ldots, k_m)$$

with $n_m \geq 0$, $q_m \geq 0$, $n_m + q_m > 0$ such that

$$Z(s) = NK^{-1} + (N + Qs)\, W(s)$$

is a positive real matrix

where

$W(s) = C(sI - A)^{-1}B$ $(n \times n)$ matrix of stable rational transfer function and $W(\infty) = 0$ then the system is stable. The stability of system (1.1) can be determined by the Lure type Liapunov function

$$V(x,\varepsilon) = x^T P x + 2Q \int_0^{Cx} f(\varepsilon)^T \, d\varepsilon \tag{1.2}$$

where P is a positive definite symmetric matrix determined

$$\begin{aligned} PA + A^T P &= -LL^T \\ PB &= C^T N - LW_0 + A^T C^T Q \\ W_0^T W_0 &= 2NK + QCB + B^T C^T Q \end{aligned} \tag{1.3}$$

where L, W_0 are auxiliary matrices of order $(n \times n)$, $(n \times m)$.

1.2 Willems' Method [10]

Willems applied the above technique to estimate the transient stability regions for multimachine power systems.

Assuming that

1. The flux linkages are constant during the transient period
2. The damping power is proportional to the slip velocity
3. The mechanical power inputs to the machines are constant
4. Armature and transmission line resistances are neglected.

The differential equations describing the motion of the machines can be put in the form

$$M_i \frac{d^2\delta_i}{dt^2} + \alpha_i \frac{d\,\delta_i}{dt} + P_{ei} - P_{mi} = 0 \quad \text{for} \quad i = 1,2, \ldots n \tag{1.4}$$

$$\text{with } P_{ei} = G_i E_i^2 + \sum_{\substack{j=1 \\ i \neq j}}^{n} E_i E_j Y_{ij} \sin(\delta_i - \delta_j) \qquad i = 1,2, \ldots n$$

where E_i = internal voltage of the ith machine

G_i = local load conductance

Y_{ij} = transfer admittance between the ith and the jth machine

At equilibrum

$$\frac{d\,\delta_i}{dt} = \omega_i = 0, \qquad \frac{d^2\delta_i}{dt^2} = \dot{\omega}_i = 0, \qquad P_{mi} = P_{ei}$$

Let $x = \begin{bmatrix} \omega \\ \sigma \end{bmatrix}$ 2n vector

where ω, σ are column vectors with components

$$\omega = [\omega_1, \omega_2, \ldots \omega_n]$$

$$\sigma = [\sigma_1, \sigma_2, \ldots \sigma_n]$$

$$\sigma_1 = \delta_1 - \delta_1^o, \qquad \sigma_2 = \delta_2 - \delta_2^o, \ldots \quad \sigma_n = \delta_n - \delta_n^o$$

Although the state variable vector x has 2n components the actual order of the system is (2n - 1) since only the differences between the rotor angles appear in the system equations.

Let

$$M = \text{diag}\,(M_i) \qquad (n \times n) \text{ matrix}$$
$$R = \text{diag}\,(-\alpha_i) \qquad (n \times n) \text{ matrix} \tag{1.5}$$

D = an $(m \times n)$ matrix such that the vector $\varepsilon = D\sigma$ has its components

$$\varepsilon_1 = \sigma_1 - \sigma_2, \quad \varepsilon_2 = \sigma_1 - \sigma_3, \ldots \quad \varepsilon_{n-1} = \sigma_1 - \sigma_n,$$

$$\varepsilon_n = \sigma_2 - \sigma_3, \quad \varepsilon_{n+1} = \sigma_2 - \sigma_4, \ldots \quad \varepsilon_m = \sigma_{n-1} - \sigma_n$$

where $\quad m = \dfrac{n(n-1)}{2}$

Define the function $f(\varepsilon)$ as

$$f_i(\varepsilon_i) = E_p E_q Y_{pq} \,(\sin\,(\varepsilon_i + \varepsilon_i^o) - \sin\,\varepsilon_i^o) \quad i = 1,2,\ldots m \qquad \text{Where} \tag{1.6}$$

p, q are the indices of the component of σ on which ε_i is dependent. Let ε_i^o be the value of ε for $\delta_i = 2\delta_i^o$ and define the matrices A, B and C as

$$A = \begin{bmatrix} M^{-1}R & 0_{nn} \\ I_n & 0_{nn} \end{bmatrix} \qquad (2n \times 2n) \text{ matrix}$$

$$B = \begin{bmatrix} M^{-1}D^T \\ 0_{nm} \end{bmatrix} \qquad (2n \times m) \text{ matrix} \tag{1.7}$$

$$C = \begin{bmatrix} 0_{mn} & D \end{bmatrix} \qquad (m \times 2n) \text{ matrix}$$

The differential equations (1.4) become equivalent to

$$\dot{x} = Ax - Bf(\varepsilon)$$
$$\varepsilon = Cx \tag{1.1}$$

The stability of system (1.1) is determined by a Liapunov function of the form (1.2).

The time derivative $\dot{V}$ is given by

$$\dot{V} = -(x^T L - f(Cx)^T W_0^T)\,(L^T x - W_0 f(Cx)) - 2x^T C^T N f(Cx) \tag{1.8}$$

The next step is to find the matrix P of equation (1.2). Since by definition

$$CB = B^TC^T = 0_{mm}$$

and choosing

$$N = 0_{mm}$$

$$Q = I_m$$

then substituting in equation (1.3) results in

i) $$W_0 = 0_{mm} \tag{1.9a}$$

$$PA + A^TP = -LL^T \tag{1.9b}$$

$$PB = A^TC^T \tag{1.9c}$$

ii) $Z(s) = sC(sI - A)^{-1}B$ is positive real if all the damping constants are nonnegative

iii) equation (1.8) reduces to

$\dot{V} = -x^TLL^Tx$ which is negative semidefinite

Let $$P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix} \tag{1.10}$$

where P_1, P_2, P_3 are (n x n) square matrices.

Thus equation (1.9c) is equivalent to

$$P_1M^{-1}D^T = D^T \tag{1.11a}$$

$$P_2M^{-1}D^T = 0_{nm} \tag{1.11b}$$

and from the negative semidefinitness of

$$PA + A^TP = \begin{bmatrix} P_1M^{-1}R + RM^{-1}P_1 + P_2 + P_2^T & RM^{-1}P_2^T + P_3 \\ P_2M^{-1}R + P_3 & 0_{nn} \end{bmatrix}$$

we get

$$P_3 = -P_2M^{-1}R = -RM^{-1}P_2^T \tag{1.12}$$

Since matrix D^T contains $m = \frac{n(n - 1)}{2}$ columns with each column containing only two nonzero elements, +1 on the ith row and -1 on the jth row, the solution of the equation $YD^T = 0_{nm}$, where Y is an unknown symmetric (n x n) matrix, is $Y = \mu 1$ where μ is a scalar constant and 1 is an (n x n) matrix with all elements equal to 1. Applying the above reasoning to (1.11a) results in

$$(M^{-1}P_1M^{-1} - M^{-1})D^T = 0_{nm}$$

$$P_1 = M + \mu M1M \tag{1.13}$$

which is positive definite if $\mu_1 > \mu_o$ where μ_o is the solution of the det. $/M + \mu_o M1M/ = 0$

$$\therefore \quad \mu_o = \frac{-1}{\sum_{i=1}^{n} M_i} \tag{1.14}$$

from equations (1.11b) and (1.12)

$$R^{-1}P_3R^{-1}D^T = 0_{nm}$$

and hence

$$P_3 = \gamma R1R$$

$$P_2 = -\gamma R1M \tag{1.15}$$

where γ is a scalar constant and is taken equal to zero

hence $$P = \begin{bmatrix} P_1 & 0_{nn} \\ 0_{nn} & 0_{nn} \end{bmatrix} \tag{1.16}$$

The matrix $PA + A^TP$ is negative semidefinite if, and only if, the matrix $Z(\mu) = 2R + \mu(M1R + R1M)$ is negative semidefinite .

$Z(\mu)$ is negative semidefinite for certain values of μ

$$\mu_0 \le \mu_1 \le 0 \tag{1.17}$$

where μ_1 is the solution of the det $|Z(\mu)| = 0$ which is equivalent to

$$\mu^2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{1}{4} \left(M_j\sqrt{\frac{\alpha_i}{\alpha_j}} - M_i\sqrt{\frac{\alpha_j}{\alpha_i}}\right)^2 - \mu\left(\sum_{i=1}^{n} M_i\right) - 1 = 0 \tag{1.18}$$

Equation (1.18) has a positive and a negative solution for μ, the negative one being μ_1.

Substituting the value of P in equation (1.2) we obtain

$$V(x) = \omega^T M\omega + \mu\, \omega^T M1M\omega + 2\int_0^{Cx} f(\varepsilon)^T d\varepsilon \tag{1.19}$$

with its derivative

$$\dot{V}(x) = 2\omega^T R\omega + 2\mu\omega^T M1R \tag{1.20}$$

1.3 Stability Regions

Since the derivative of the Liapunov function is negative semidefinite [9] the boundary of the transient stability region can be obtained by solving the equations

$$\frac{\partial V(x)}{\partial \omega_i} = 0 \qquad \frac{\partial V(x)}{\partial \delta_i} = 0 \qquad \text{for } i = 1,2, \ldots n \tag{1.21}$$

The first equation gives $\omega_1 = \omega_2 = \ldots = \omega_n = 0$. The second equation gives the closest equilibrum state (necessarily unstable) to the origin x^u. The region bounded by the closed surface

$$V(x) = V(x^u)$$

and containing the origin is a stable region.

1.4 Numerical Example

Consider the three machine system shown in figure 1.5. The differential equations describing the motion of the system are

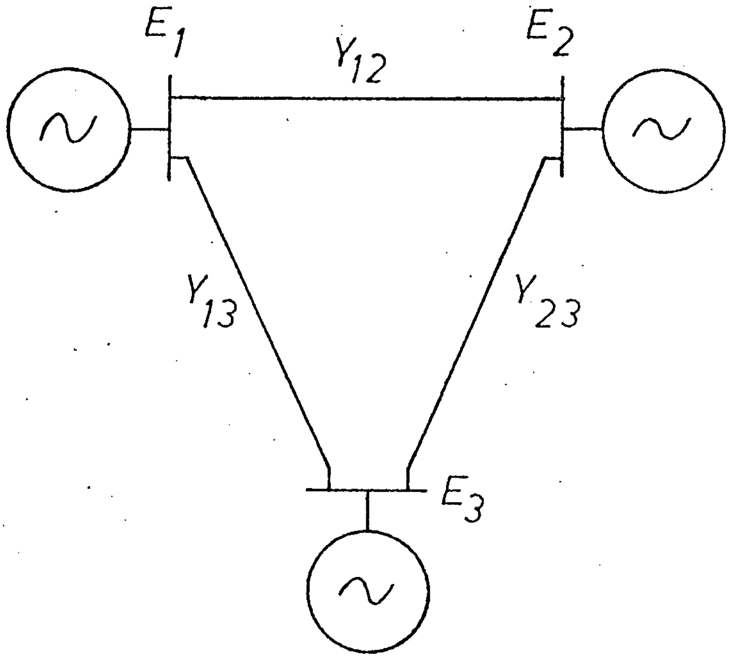

Fig. 1.5 A Three-Machine Power System

$$
\begin{aligned}
M_1 \frac{d^2\delta_1}{dt^2} + \alpha_1 \frac{d\delta_1}{dt} + P_{e1} &= P_{m1} \\
M_2 \frac{d^2\delta_2}{dt^2} + \alpha_2 \frac{d\delta_2}{dt} + P_{e2} &= P_{m2} \\
M_3 \frac{d^2\delta_3}{dt^2} + \alpha_3 \frac{d\delta_3}{dt} + P_{e3} &= P_{m3}
\end{aligned}
\tag{1.22}
$$

Let the state variable vector be

$$x = \left(\frac{d\delta_1}{dt}, \frac{d\delta_2}{dt}, \frac{d\delta_3}{dt}, \quad \delta_1 - \delta_1^o, \delta_2 - \delta_2^o, \quad \delta_3 - \delta_3^o \right)^T$$

Following the steps described in section 1.2, the system equations (1.22) become

$$\dot{x} = Ax - Bf(\varepsilon)$$

$$\varepsilon = Cx$$

where

$$A = \begin{bmatrix} -\frac{\alpha_1}{M_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\alpha_2}{M_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\alpha_3}{M_3} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{M_1} & \frac{1}{M_1} & 0 \\ -\frac{1}{M_2} & 0 & \frac{1}{M_2} \\ 0 & -\frac{1}{M_3} & -\frac{1}{M_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$f_1(\varepsilon_1) = E_1E_2Y_{12}\ (\sin\ (\varepsilon_1 + \varepsilon_1^o) - \sin\ \varepsilon_1^o)$$

$$f_2(\varepsilon_2) = E_1E_3Y_{13}\ (\sin\ (\varepsilon_2 + \varepsilon_2^o) - \sin\ \varepsilon_2^o)$$

$$f_3(\varepsilon_3) = E_2E_3Y_{23}\ (\sin\ (\varepsilon_3 + \varepsilon_3^o) - \sin\ \varepsilon_3^o)$$

with
$$\varepsilon_1 = X_4 - X_5 \qquad \varepsilon_1^o = \delta_1^o - \delta_2^o$$
$$\varepsilon_2 = X_4 - X_6 \qquad \varepsilon_2^o = \delta_1^o - \delta_3^o$$
$$\varepsilon_3 = X_5 - X_6 \qquad \varepsilon_3^o = \delta_2^o - \delta_3^o$$

The Liapunov function is then given by

$$\begin{aligned} V(x) = {} & M_1X_1^2 + M_2X_2^2 + M_3X_3^2 + \mu\,[M_1X_1 + M_2X_2 + M_3X_3]^2 \\ & + 2[E_1E_2Y_{12}(\cos\ \varepsilon_1^o - \cos\ (X_4 - X_5 + \varepsilon_1^o) - (X_4 - X_5)\ \sin\ \varepsilon_1^o) \\ & + E_1E_3Y_{13}(\cos\ \varepsilon_2^o - \cos\ (X_4 - X_6 + \varepsilon_2^o) - (X_4 - X_6)\ \sin\ \varepsilon_2^o) \\ & + E_2E_3Y_{23}(\cos\ \varepsilon_3^o - \cos\ (X_5 - X_6 + \varepsilon_3^o) - (X_5 - X_6)\ \sin\ \varepsilon_3^o)] \end{aligned} \qquad (1.23)$$

The system studied has the following data

$E_1 = 1.174\ \underline{|22.64^o}$ p.u.	$Pm_1 = 0.8$ p.u.	
$E_2 = 0.996\ \underline{|2.61}^o$ p.u.	$Pm_2 = 0.3$ p.u.	
$E_3 = 1.06\ \underline{|-11.36}^o$ p.u.	$Pm_3 = -1.1$ p.u.	
$H_1 = 3$ KW. sec/KVA	$\frac{\alpha_1}{M_1} = 10$	$Y_{12} = 1.13375$ p.u.
$H_2 = 7$ KW. sec/KVA	$\frac{\alpha_2}{M_2} = 7$	$Y_{13} = 0.52532$ p.u.
$H_3 = 8$ KW. sec/KVA	$\frac{\alpha_3}{M_3} = 3$	$Y_{23} = 3.11850$ p.u.

A sudden 3-phase symmetrical short circuit to ground occurs on the transmission line connecting machines 2 and 3 of Figure (1.5) close to bus 3. The unstable equilibrum state nearest to the origin is calculated and was found to be at

$$X_4 - X_5 = 2.61168 \text{ rad}$$
$$X_4 - X_6 = 2.95275 \text{ rad}$$
$$X_1 = X_2 = X_3 = 0.0$$

with $V_m = 3.36$.

The critical clearing time obtained from the above V-function was found to be between 14-15 cycles.

Figure (1.6) shows the function $V = V_m$ plotted in the two dimensions $(X_4 - X_5)$, $(X_4 - X_6)$ for $X_1 = X_2 = X_3 = 0$.

The actual critical clearing time obtained from forward integration of the swing equations using Runge Kutta method is 20 cycles.

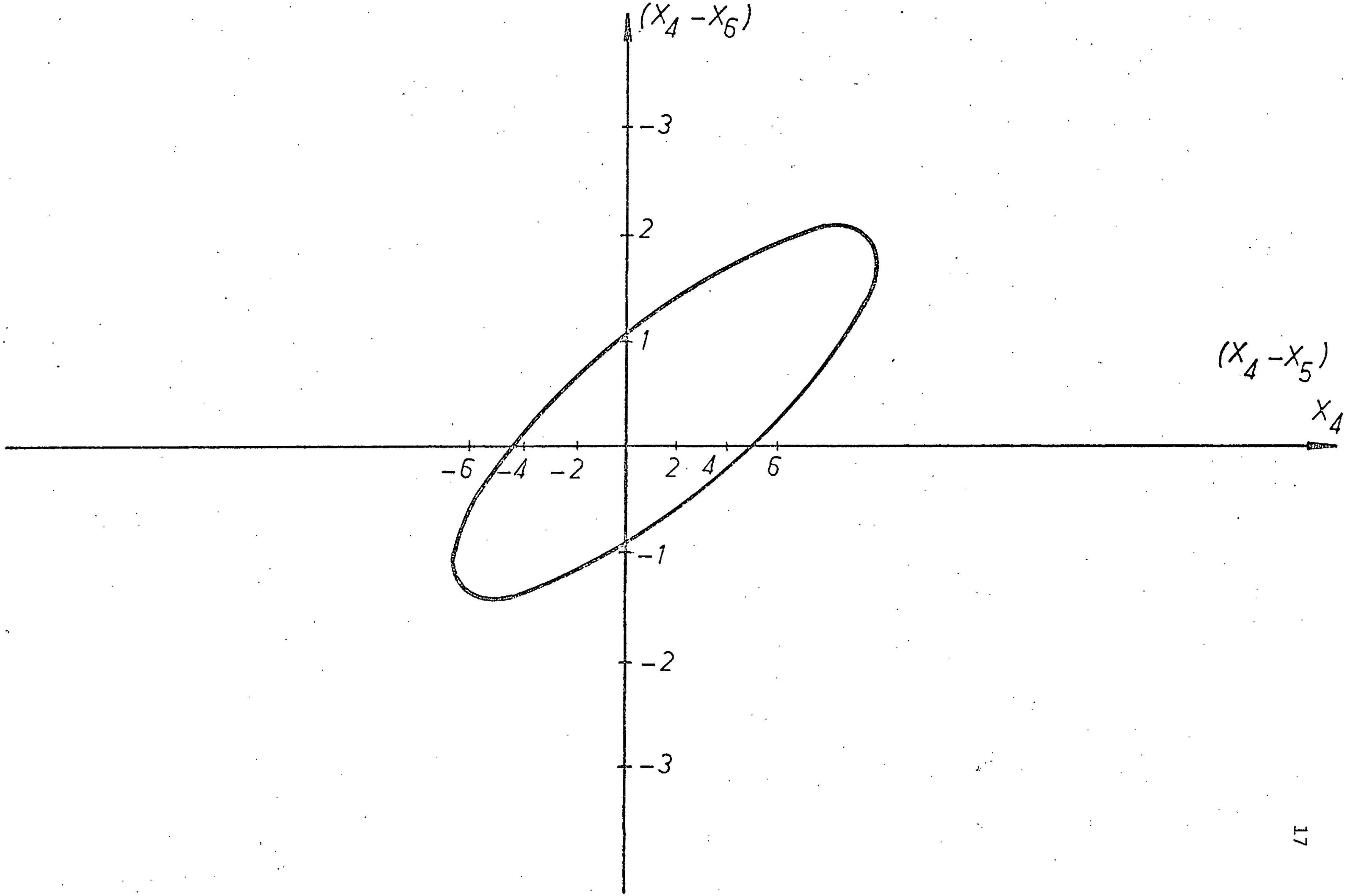

Fig. 1.6 Stability Region $V = V_m$ for $X_1 = X_2 = X_3 = 0$

CHAPTER II

OPTIMUM DISTRIBUTION OF DAMPING FOR MAXIMUM TRANSIENT STABILITY REGION

In this Chapter Willems' method described in Chapter I is applied to a four machine power system and a study is made to find the optimum distribution of damping that maximizes the region of stability.

2.1 System Equations

Under the same assumptions made in Chapter I the system equations are

$$M_i \frac{d^2 \delta_i}{dt^2} + \alpha_i \frac{d \delta_i}{dt} + P_{ei} = P_{mi} \qquad i = 1,2,3,4 \tag{2.1}$$

where

$$\begin{aligned}
P_{e1} &= A_1 \sin(\delta_1 - \delta_2) + A_2 \sin(\delta_1 - \delta_3) + A_3 \sin(\delta_1 - \delta_4) \\
P_{e2} &= A_1 \sin(\delta_2 - \delta_1) + A_4 \sin(\delta_2 - \delta_3) + A_5 \sin(\delta_2 - \delta_4) \\
P_{e3} &= A_2 \sin(\delta_3 - \delta_1) + A_4 \sin(\delta_3 - \delta_2) + A_6 \sin(\delta_3 - \delta_4) \\
P_{e4} &= A_3 \sin(\delta_4 - \delta_1) + A_5 \sin(\delta_4 - \delta_2) + A_6 \sin(\delta_4 - \delta_3)
\end{aligned} \tag{2.2}$$

and

$$A_1 = E_1E_2Y_{12} \qquad A_2 = E_1E_3Y_{13} \qquad A_3 = E_1E_4Y_{14}$$

$$A_4 = E_2E_3Y_{23} \qquad A_5 = E_2E_4Y_{24} \qquad A_6 = E_3E_4Y_{34}$$

Following the same steps described in section 1.2 to represent (2.1) in the form

$$\begin{aligned}
\dot{x} &= Ax - Bf(\varepsilon) \\
\varepsilon &= Cx
\end{aligned} \tag{2.3}$$

the results are

$$x = (\omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_4 \quad \delta_1 - \delta_1^o \quad \delta_2 - \delta_2^o \quad \delta_3 - \delta_3^o \quad \delta_4 - \delta_4^o)^T \tag{2.4}$$

$$\varepsilon^o = (\delta_1^o - \delta_2^o \quad \delta_1^o - \delta_3^o \quad \delta_1^o - \delta_4^o \quad \delta_2^o - \delta_3^o \quad \delta_2^o - \delta_4^o \quad \delta_3^o - \delta_4^o)^T \tag{2.5}$$

$$\varepsilon = (X_5 - X_6 \quad X_5 - X_7 \quad X_5 - X_8 \quad X_6 - X_7 \quad X_6 - X_8 \quad X_7 - X_8)^T \tag{2.6}$$

$$f_i(\varepsilon_i) = A_i(\sin(\varepsilon_i + \varepsilon_i^o) - \sin \varepsilon_i^o) \tag{2.7}$$

$$A = \begin{bmatrix} -\frac{\alpha_1}{M_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\alpha_2}{M_2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\alpha_3}{M_3} & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \tag{2.8}$$

$$B = \begin{bmatrix} \frac{1}{M_1} & \frac{1}{M_1} & \frac{1}{M_1} & 0 & 0 & 0 \\ -\frac{1}{M_2} & 0 & 0 & \frac{1}{M_2} & \frac{1}{M_2} & 0 \\ 0 & -\frac{1}{M_3} & 0 & -\frac{1}{M_3} & 0 & \frac{1}{M_3} \\ 0 & 0 & -\frac{1}{M_4} & 0 & -\frac{1}{M_4} & -\frac{1}{M_4} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2.2 Construction of Liapunov Function

According to the expression for P given in section 1.2

$$P = \begin{bmatrix} P_1 & 0_{nn} \\ 0_{nn} & 0_{nn} \end{bmatrix}$$

where

$$P_1 = M + \mu M1M$$

$$= \begin{bmatrix} M_1^2 + \mu M_1^2 & \mu M_1M_2 & \mu M_1M_3 & \mu M_1M_3 \\ \mu M_1M_2 & M_2^2 + \mu M_2^2 & \mu M_2M_3 & \mu M_2M_4 \\ \mu M_1M_3 & \mu M_2M_3 & M_3^2 + \mu M_3^2 & \mu M_3M_4 \\ \mu M_1M_4 & \mu M_2M_4 & \mu M_3M_4 & M_4^2 + \mu M_4^2 \end{bmatrix}$$

The Liapunov function is then given by

$$\begin{aligned} V(x) = {} & M_1X_1^2 + M_2X_2^2 + M_3X_3^2 + M_4X_4^2 + \mu\,[M_1X_1 + M_2X_2 + M_3X_3 + M_4X_4]^2 \\ & +2\,[A_1(\cos\varepsilon_1 - \cos(X_5 - X_6 + \varepsilon_1^o) - (X_5 - X_6)\sin\varepsilon_1^o) \\ & +A_2(\cos\varepsilon_2 - \cos(X_5 - X_7 + \varepsilon_2^o) - (X_5 - X_7)\sin\varepsilon_2^o) \\ & +A_3(\cos\varepsilon_3 - \cos(X_5 - X_8 + \varepsilon_3^o) - (X_5 - X_8)\sin\varepsilon_3^o) \\ & +A_4(\cos\varepsilon_4 - \cos(X_6 - X_7 + \varepsilon_4^o) - (X_6 - X_7)\sin\varepsilon_4^o) \\ & +A_5(\cos\varepsilon_5 - \cos(X_6 - X_8 + \varepsilon_5^o) - (X_6 - X_8)\sin\varepsilon_5^o) \\ & +A_6(\cos\varepsilon_6 - \cos(X_7 - X_8 + \varepsilon_6^o) - (X_7 - X_8)\sin\varepsilon_6^o)\,] \end{aligned} \tag{2.9}$$

2.3 Numerical Example

The system chosen as an example

is shown in Figure 2.1.

$E_1 = 1.004\ \underline{|0.0013 \text{ rad}}$ p.u.

$E_2 = 1.0410\ \underline{|0.103 \text{ rad}}$ p.u.

$E_3 = 1.1900\ \underline{|0.197 \text{ rad}}$ p.u.

$E_4 = 1.070\ \underline{|0.0772 \text{ rad}}$ p.u.

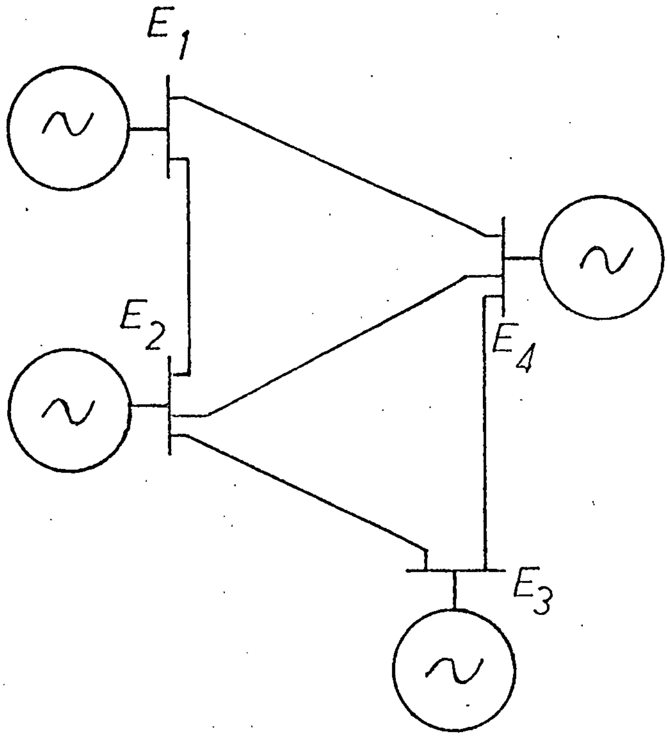

Fig. 2.1 A Four-Machine Power System

$P_{m1} = 0.332$ P.u.	$M_1 = 75350$ P.u.	$D_1 = 1.0$ P.u.
$P_{m2} = 0.1$ P.u.	$M_2 = 1130$ P.u.	$D_2 = 12.0$ P.u.
$P_{m3} = 0.3$ P.u.	$M_3 = 2260$ P.u.	$D_3 = 2.5$ P.u.
$P_{m4} = 0.2$ P.u.	$M_4 = 1508$ P.u.	$D_4 = 6.0$ P.u.

A sudden 3-phase symmetrical short circuit to ground occurs close to bus 3 on the transmission line connecting machines 3 and 4 of Figure 2.1. The following table gives the stable equilibrum state of the post-fault system and the unstable equilibrum state closest to the stable one.

Internal bus	δ, radians (stable)	δ, radians (unstable)
1	0.05630	0.06610
2	0.15013	0.20136
3	0.21430	3.0820
4	0.02497	-0.02425

The exact critical clearing time was calculated using Runge Kutta and was found to be at 30 cycles.

When calculating the value of the Liapunov function at the unstable equilibrum state and $X_1 = X_2 = X_3 = X_4 = 0.0$ we obtain a value for $V_m = 3.155$ which gives a clearing time of 25 cycles.

Figure 2.2 shows the function $V(x) = V_m$ plotted in the three dimensional space $(X_5 - X_6)$, $(X_5 - X_7)$, $(X_5 - X_8)$ with the components $X_1 = X_2 = X_3 = X_4 = 0.0$.

2.4 Optimum Damping Distribution

The optimum distribution of damping ratios ($\frac{\alpha_i}{M_i}$) is obtained by finding the relative values of $\frac{\alpha_i}{M_i}$ to maximize the hypervolume enclosed by the Liapunov function that defines the stability region of the system. Considering the V-function (2.9) for a four machine system

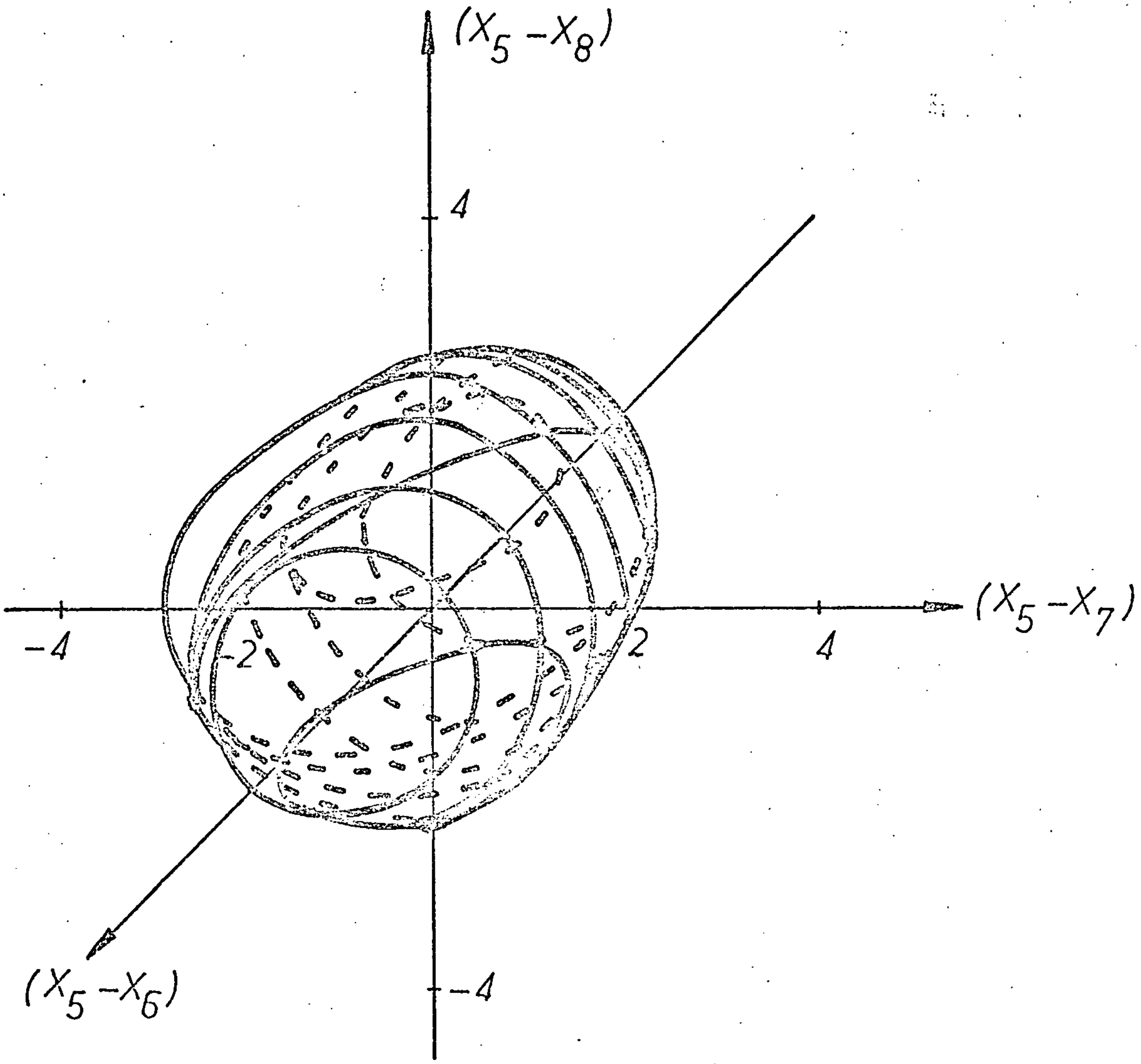

Fig. 2.2 Stability Region $V = V_m$ for $X_1 = X_2 = X_3 = X_4 = 0$

$$V = \omega^T(M + \mu M1M)\omega + 2\int_o^{Cx} f(\varepsilon)^T\, d\varepsilon$$

where μ is given by

$$f_R\,\mu^2 - 4k_1\mu - 4 = 0$$

which gives

$$\mu = 2\left(\frac{k_1 - \sqrt{k_1^2 + f_R}}{f_R}\right)$$

where

$$f_R = \frac{M_1M_2}{R_1R_2}(R_1 - R_2)^2 + \frac{M_1M_3}{R_1R_3}(R_1 - R_3)^2 + \frac{M_1M_4}{R_1R_4}(R_1 - R_4)^2 + \frac{M_2M_3}{R_2R_3}(R_2 - R_3)^2 + \frac{M_2M_3}{R_2R_4}(R_2 - R_4)^2 + \frac{M_3M_4}{R_3R_4}(R_3 - R_4)^2$$

$$k_1 = M_1 + M_2 + M_3 + M_4$$

It is noticed that the integral part of V does not depend on the damping coefficients and therefore the hypervolume enclosed by the quadratic part alone is to be considered. This volume is given by [22]

$$H_p = \pi^2\, V_q^{\,2}/2\sqrt{|A|}$$

where

$$V_q = \omega^T A\omega$$

$$A = M + \mu M1M$$

$$|A| = \text{determinant of matrix } A = k_2(1 + \mu k_1)$$

$$k_2 = M_1M_2M_3M_4$$

$$V_q = N_1 + \mu N_2$$

$$N_1 = M_1\omega_1^2 + M_2\omega_2^2 + M_3\omega_3^2 + M_4\omega_4^2$$

$$N_2 = (M_1\omega_1 + M_2\omega_2 + M_3\omega_3 + M_4\omega_4)^2$$

Thus $$H_p = \frac{\pi^2}{2}(N_1 + \mu N_2)^2/\sqrt{k_2 + \mu k_1k_2}$$

For maximum volume

$$\frac{\partial H_p}{\partial R_i} = 0 \qquad i = 1,2,3,4 \tag{2.10}$$

but

$$\frac{\partial H_p}{\partial R_i} = \frac{\partial H_p}{\partial \mu}\cdot\frac{\partial \mu}{\partial f_R}\cdot\frac{\partial f_R}{\partial R_i}$$

Thus to satisfy equation (2.10)

$$\frac{H_p}{\partial \mu} = 0 \quad \text{or} \quad \frac{\partial \mu}{\partial f_R} = 0 \quad \text{or} \quad \frac{\partial f_R}{\partial R_i} = 0 \qquad i = 1,2,3,4$$

$$\frac{\partial H_p}{\partial \mu} = \frac{\pi^2}{4} \; \frac{k_2 (N_1 + \mu N_2)}{k_2 + \mu k_1 k_2} \; (3k_1 N_2 \mu + 4N_2 - k_1 N_1)$$

which when equating to zero gives

$$\mu = - \frac{N_1}{N_2} \quad \text{or} \quad \mu = \frac{k_1 N_1 - 4 N_2}{3k_1 N_2}$$

Both answers are rejected since the value of μ depends on the values of state variable

$$\frac{\partial \mu}{f_R} = \frac{f_R - 2k_1 \sqrt{k_1^2 + f_R} + 2k_1^2}{f_R^2 \sqrt{k_1^2 + f_R}} \neq 0$$

The third possibility is that

$$\frac{\partial f_R}{\partial R_1} = \frac{M_1}{R_1^2} \left\{ \frac{M_2}{R_2} (R_1^2 - R_2^2) + \frac{M_3}{R_3} (R_1^2 - R_3^2) + \frac{M_4}{R_4} (R_1^2 - R_4^2) \right\} = 0$$

$$\frac{\partial f_R}{\partial R_2} = \frac{M_2}{R_2^2} \left\{ - \frac{M_1}{R_1} (R_1^2 - R_2^2) + \frac{M_3}{R_3} (R_2^2 - R_3^2) + \frac{M_4}{R_4} (R_3^2 - R_4^2) \right\} = 0$$

$$\frac{\partial f_R}{\partial R_3} = \frac{M_3}{R_3^2} \left\{ - \frac{M_1}{R_1} (R_1^2 - R_3^2) - \frac{M_2}{R2} (R_2^2 - R_3^2) + \frac{M_4}{R_4} (R_3^2 - R_4^2) \right\} = 0$$

$$\frac{\partial f_R}{\partial R_4} = \frac{M_4}{R_4^2} \left\{ - \frac{M_1}{R_1} (R_1^2 - R_4^2) - \frac{M_2}{R_2} (R_2^2 - R_4^2) - \frac{M_3}{R_3} (R_3^2 - R_4^2) \right\} = 0$$

which gives

$$R_1 = R_2 = R_3 = R_4$$

Thus for a maximum region of stability <u>the damping ratios of all machines should be equal.</u>

CHAPTER III

EXTENSION OF WILLEMS' METHOD TO INCLUDE GOVERNOR ACTION

In this chapter Willems' construction procedure is extended to develop a Liapunov function for multimachine power systems including governor action.

3.1 System Equations

Assuming that flux linkages are constant, resistances are neglected, damping power is proportional to slip velocity and governor response may be represented by a single time lag transfer function,

$$\frac{\Delta P_m}{\omega} = \frac{-K}{1 + T_S} \tag{3.1}$$

The equations of the ith machine are

$$\begin{aligned} \frac{d\delta_i}{dt} &= \omega_i \\ M_i \frac{d\omega_i}{dt} &= -\alpha_i \omega_i - P_{ei} + P_{moi} + \Delta P_{mi} \\ \frac{d\Delta P_{mi}}{dt} &= -\frac{1}{T_i} \Delta P_{mi} - \frac{K_i}{T_i} \omega_i \end{aligned} \tag{3.2}$$

where

Pmoi is the value of the mechanical input at steady state.

Defining the vector

$$X = [\omega_1, \omega_2 \ldots \omega_n, \Delta P_{m1}, \Delta P_{m2}, \ldots \Delta P_{mn}, \delta_1 - \delta_1^o, \delta_2 - \delta_2^o, \ldots \delta_n - \delta_n^o]^T$$

and the matrices (3.3)

$$A = \begin{bmatrix} M^{-1}R & M^{-1} & 0_{nn} \\ Y & Z & 0_{nn} \\ I_n & 0_{nn} & 0_{nn} \end{bmatrix}$$

$$B = \begin{bmatrix} M^{-1}D^T \\ 0_{nm} \\ 0_{nm} \end{bmatrix} \qquad C = \begin{bmatrix} 0_{mn} & 0_{mn} & D^T \end{bmatrix} \tag{3.4}$$

where $Y = \mathrm{diag}\ [-\frac{k_i}{T_i}]$

$Z = \mathrm{diag}\ [-\frac{1}{T_i}]$

M, R and D are as defined in chapter I.

Equation (3.2) takes the form

$$\dot{x} = Ax - Bf(\varepsilon)$$
$$\varepsilon = Cx \qquad (3.5)$$

3.2 Construction of Liapunov Function

Applying Popov's generalized criterion, system (3.5) is stable if $(N + Qs)C\ (Is - A)^{-1}B$ is positive real matrix.

Taking $N = 0_{mm}$ and $Q = I_m$ then $sC(Is - A)^{-1}B$ is positive real if the damping constants are nonnegative. A suitable Liapunov function for such a system is

$$V = x^T Px + 2\int_0^{Cx} f(\varepsilon)^T\, d\varepsilon$$

where P is determined from the requirement that

$PA + A^T P$ be negative semidefinite and that (3.6a)

$PB = A^T C^T$ (3.6b)

$$\text{Let } P = \begin{bmatrix} P_1 & P_{12}^T & P_{13}^T \\ P_{12} & P_2 & P_{23}^T \\ P_{13} & P_{23} & P_3 \end{bmatrix} \qquad (3.7)$$

Substituting (3.7) into (3.6b) gives

$$P_1 M^{-1} D^T = D^T$$
$$P_{12} M^{-1} D^T = 0_{nm} \qquad (3.8)$$
$$P_{13} M^{-1} D^T = 0_{nm}$$

Substituting for matrix A in equation (3.6a) one has

$$\begin{bmatrix} P_1 M^{-1}R + P_{12}^T Y + P_{13}^T + M^{-1}RP_1 + YP_{12} + P13 & P_1 M^{-1} + P_{12}^T Z + M^{-1}RP_{12}^T + YP_2 + P_{23} & M^{-1}RP_{13}^T + YP_{23}^T + P_3 \\ P_{12}M^{-1}R + P_2 Y + P_{23}^T + M^{-1}P_1 + ZP_{12} & P_{12}M^{-1} + P_2 Z + M^{-1}P_{12}^T + ZP_2 & M^{-1}P_{13}^T + ZP_{23}^T \\ P_{13}M^{-1}R + P_{23}Y + P_3 & P_{13}M^{-1} + P_{23}Z & 0_{nn} \end{bmatrix} \qquad (3.9)$$

Setting all off diagonal elements of (3.9) equal to zero give

$$P_1M^{-1} + P_{12}^TZ + M^{-1}RP_{12}^T + YP_2 + P_{23} = 0_{nn} \tag{3.10}$$

$$M^{-1}RP_{13}^T + YP_{23}^T + P_3 = 0_{nn} \tag{3.11}$$

$$M^{-1}P_{13}^T + ZP_{23}^T = 0_{nn} \tag{3.12}$$

Equation (3.8), (3.10), (3.11) and (3.12) are solved for P_1, P_2, P_3, P_{12}, P_{13} and P_{23} to give

$$\begin{aligned}
P_1 &= M + \mu M1M \\
P_{12} &= \gamma_1\, M1M \\
P_{13} &= \gamma_2\, M1M \\
P_{23}^T &= -\gamma_2\, Z^{-1}1M \\
P_3 &= \gamma_2(YZ^{-1}M^{-1} + M^{-1}R)M1M \\
P_2 &= Y^{-1}(\gamma_2 Z^{-1}1M - \gamma_1(M1MZ + M^{-1}RM1M) - \mu M1 - I)
\end{aligned} \tag{3.13}$$

where γ_1 and γ_2 are constant scalars and μ is given by

$$\mu^2 \left[\sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{1}{4}\left(M_j\sqrt{\frac{\alpha_i}{\alpha_j}} - M_i\sqrt{\frac{\alpha_j}{\alpha_i}}\right)^2\right] - \mu \sum_{i=1}^{n} M_i - 1 = 0 \tag{3.14}$$

Choosing γ_1 and γ_2 to be equal to zero, matrix P reduces to

$$P = \begin{bmatrix} P_1 & 0_{nn} & 0_{nn} \\ 0_{nn} & P_2 & 0_{nn} \\ 0_{nn} & 0_{nn} & 0_{nn} \end{bmatrix} \tag{3.15}$$

where $\quad P_1 = M + \mu M1M$

$\quad\quad\quad P_2 = -Y^{-1}(\mu M1 + I)$

Thus for a three machine system, the Liapunov function is

$$V(x) = M_1X_1^2 + M_2X_2^2 + M_3X_3^2 + \mu(M_1X_1 + M_2X_2 + M_3X_3)^2$$

$$+ \frac{T_1}{K_1}X_4^2 + \frac{T_2}{K_2}X_5^2 + \frac{T_3}{K_3}X_6^2 + \mu(X_4 + X_5 + X_6)\left(\frac{M_1T_1}{k_1}X_4 + \frac{M_2T_2}{k_2}X_5 + \frac{M_3T_3}{k_3}X_6\right)$$

$$+ 2E_1E_2Y_{12}(\cos\varepsilon_1 - \cos(X_7 - X_8 + \varepsilon_1^o) - (X_7 - X_8)\sin\varepsilon_1^o)$$

$$+ 2E_1E_2Y_{13}(\cos\varepsilon_2 - \cos(X_7 - X_9 + \varepsilon_2^o) - (X_7 - X_9)\sin\varepsilon_2^o)$$

$$+ 2E_2E_3Y_{23}(\cos\varepsilon_3 - \cos(X_8 - X_9 + \varepsilon_3^o) - (X_8 - X_9)\sin\varepsilon_3^o)$$

3.3 Numerical Example

The same numerical example of Chapter I is considered. With governor action taken into account the equations describing the machine dynamics are

$$\frac{d\delta_i}{dt} = \omega_i$$

$$\frac{d\omega_i}{dt} = -\alpha_i\omega_i - P_{ei} + P_{moi} + \Delta P_{mi}$$

$$\frac{d\Delta P_{mi}}{dt} = -\frac{K_i}{T_i}\omega_i - \frac{1}{T_i}\Delta P_{mi} \qquad i = 1,2,3$$

System Data

$E_1 = 1.174 \;\underline{|22.64^o}$ P.u. $\qquad \frac{\alpha_1}{M_1} = 10.0$

$E_2 = 0.996 \;\underline{|2.61}$ P.u. $\qquad \frac{\alpha_2}{M_2} = 7.0$

$E_3 = 1.006 \;\underline{|-11.36}$ P.u. $\qquad \frac{\alpha_3}{M_3} = 3.0$

$P_{mo1} = 0.8$ p.u. $\qquad T_1 = 0.2$ sec $\qquad Y_{12} = 1.13375$ p.u.

$P_{mo2} = 0.3$ p.u. $\qquad T_2 = 0.22$ sec $\qquad Y_{13} = 0.5232$ p.u.

$P_{mo3} = -1.7$p.u. $T_3 = 0.25$ sec $Y_{23} = 3.11856$ p.u.

$$K_1 = K_2 = K_3 = 0.0$$

The unstable equilibrum state close to the stable one is calculated by solving the equations

$$\frac{\partial V(x)}{\partial \omega_i} = 0.0 \tag{3.17}$$

$$\frac{\partial V(x)}{\partial \Delta P_{mi}} = 0.0 \tag{3.18}$$

$$\frac{\partial V(x)}{\partial \delta_i} = 0.0 \qquad i = 1,2,3 \tag{3.19}$$

Equations (3.17) and (3.18) gives

$$X_1 = X_2 = X_3 = X_4 = X_5 = X_6 = 0.0$$

Equation (3.19) gives

$$X_7 - X_8 = 2.61168 \text{ rad}$$

$$X_7 - X_9 = 2.95275 \text{ rad}$$

with $V_m = 3.36$

The critical clearing time obtained from the above V-function was found to be between 15-16 cycles while the exact clearing time obtained from the forward integration of the swing equations using Runge Kutta was at 20 cycles.

3.4 Concluding Remarks

It is obvious from the material presented in this chapter that the generalized Popov's criterion can be successfully applied to power systems including governor action. On the other hand the same method failed when applied to a power system taking into account the flux decay in the field circuits of the synchronous machines. The reason behind this failure is that the nonlinearities introduced when considering flux decay are different in form from those considered by Popov. Thus it is concluded that Popov's theorem is applicable to higher order power systems as long as the nonlinearities retain the same form.

CHAPTER IV

A LIAPUNOV FUNCTION FOR A POWER SYSTEM INCLUDING FLUX DECAY (CHEN'S METHOD)

When including the effect of flux decay in the field circuit of synchronous machines the power system can not be represented in a suitable form for application of the generalized Popov's criterion. A new method, developed by Chen, based on the use of an auxiliary exact differential equation derived from the given nonlinear differential equation representing the system, is applied in this chapter. The method is employed to construct a Liapunov function for a third order model of a synchronous machine connected to an infinite bus with the effects of flux decay in the field included.

4.1 Chen's Method [11], [12]

Consider a set of n first order autonomous differential equations

$$\dot{x} = f(x) \tag{4.1}$$

where both $\dot{x}$ and $f(x)$ are n dimension vectors, all $f_i(x)$, $i = 1,2, \ldots .n$ together with their first partial derivatives are defined and continuous in some region Ω of the state space E_n and the point $x = 0$ is an equilibrum point also in Ω.

Define $\quad g_i = f_1 + f_2 + \ldots . + f_{i-1} - f_{i+1} \ldots . - f_n \qquad (4.2)$

then $\quad \sum_{i=1}^{n} g_i \dot{x}_i = 0$

which gives

$$g^T \dot{x} = 0 \text{ or } g^T dx = 0 \tag{4.3}$$

Equation (4.3) is said to be an exact differential equation in Ω if there is some single-valued differentiable function $U(x)$ defined and continuous together with its first partial derivatives in some neighborhood of every point in Ω such that

$$dU(x) = g^T \cdot dx$$

$$\frac{dU(x)}{dt} = \nabla U(x)^T \cdot \dot{x} = g^T \cdot \dot{x} \qquad (4.4)$$

which results in

$$\frac{\partial U(x)}{\partial x_i} = g_i \qquad (4.5)$$

$$\frac{\partial g_i}{\partial X_j} = \frac{\partial g_j}{\partial X_i} \qquad i,j = 1,2, \ldots .n$$

and $\qquad U(x) = \int_c g^T \cdot dx$

which is independent of any integration path C contained in the domain of U(x). Thus equation (4.5) is a necessary and sufficient condition for the exactness of (4.3). If equation (4.3) is not exact the U(x) does not exist. A function h(x) can be added to g(x) such that $(g + h)^T \cdot dx$ is an exact differential which gives

$$\frac{dU(x)}{dt} = (g + h)^T \cdot \dot{x} \qquad (4.6)$$

$$\frac{dU(x)}{dt} = (g + h)^T \cdot f$$

$$\frac{dU(x)}{dt} = h^T f$$

For equation (4.6) to be an exact differential equation, then

$$\frac{\partial U(x)}{\partial X_i} = g_i + h_i$$

and $$\frac{\partial(g_i + h_i)}{\partial X_j} = \frac{\partial(g_j + h_j)}{\partial X_i} \qquad i, j = 1, 2, \ldots .n \qquad (4.7)$$

The function U(x) can be evaluated by the line integral

$$U(x) = \int_C (g + h)^T dx \qquad (4.8)$$

For an integration between limits o and x, (4.8) gives

$$U(x) - U(o) = \int_o^x (g(y) + h(y))^T \cdot dy \qquad (4.9)$$

It remains then to select h(x) such that U(x) has the characteristics of a Liapunov function. These are given by

a) $(g + h)^T \cdot \dot{x}$ is an exact differential or

$$\frac{\partial(g_i + h_i)}{\partial X_j} = \frac{\partial(g_j + h_j)}{\partial X_i} \qquad i,j = 1,2, \ldots .n$$

b) $h^T f = \frac{dU(x)}{dt}$ is negative definite or semidefinite

c) $U(x)$ is positive definite

Let $h = \theta(g) + \nabla\psi(x)$ (4.10)

where $\theta(g)$ is a known function of $g(x)$. For n=3 $\theta(g)$ is given by

$$\theta(g) = \begin{bmatrix} -\int(\frac{\partial g_1}{\partial X_3} - \frac{\partial g_3}{\partial X_1})\, dX_3 - \int[(\frac{\partial g_1}{\partial X_2} - \frac{\partial g_2}{\partial X_1}) - \int \frac{d}{dX_3}(\frac{\partial g_1}{\partial X_3} - \frac{\partial g_3}{\partial X_1})dX_3]\, dX_2 \\ -\int(\frac{\partial g_2}{\partial X_3} - \frac{\partial g_3}{\partial X_2})\, dX_3 \\ 0 \end{bmatrix} \tag{4.11}$$

and $\psi(x)$ is a scalar function that has to be selected such that conditions b and c are satisfied. Substituting for $h(x)$ from equation (4.10) gives

$$U(x) = \int_o^x (g(y) + h(y))^T dy = \int_o^x (g + \theta(g))^T dy + \int_o^x \nabla\psi(y)^T dy$$

$$U(x) = W(x) + \psi(x) - \psi(0) \tag{4.12}$$

If $\psi(x)$ is chosen such that $\psi(0) = 0$, then

$$U(x) = W(x) + \psi(x) \tag{4.13}$$

conditions b and c can be restated as

b) $\dot{U}(x) = \dot{W}(x) + \dot{\psi}(x)$ is negative semidefinite

c) $U(x) = W(x) + \psi(x)$ is positive definite

where $W(x)$ is directly evaluated from the system equation (4.1) with $\psi(x)$ serving as correction function to give $U(x)$ its desired characteristics.

4.2 Estimation of Stability Regions

For locally stable systems a closed form solution for the undetermined coefficients of the function $\psi(x)$ does not exist. In this case the stability region is estimated by generating a Liapunov function in a series form after expanding the system nonlinearities into polynomial form. Thus $W(x)$ can be written as the sum of homogeneous polynomials

$$W(x) = F_{o2}(x) + F_{o3}(x) + \ldots\ldots F_{om}(x) \tag{4.14}$$

where $F_{oj}(x)$ is a jth degree homogenous polynomial. Thus

$$U(x) = F_{o2}(x) + F_{o3}(x) + \ldots\ldots F_{om}(x) + \psi(x)$$

and

$$\dot{U}(x) = - G_{o2}(x) - G_{o3}(x) \ldots\ldots G_{m+s}(x) + \dot{\psi}(x) \tag{4.15}$$

where G_{oj} is also a jth degree homogeneous polynomial and s is the highest degree of $f(x)$. Then the final Liapunov function is obtained in the following steps

1. Starting with $\psi(x) = 0$, check the positive definitness of $F_{o2}(x)$ and $G_{o2}(x)$. If both are positive definite then $U(x)$ is a Liapunov function.
2. If $F_{o2}(x)$ and $G_{o2}(x)$ are not positive definite, consider a quadratic function $\psi_1(x) = F_{12}(x)$ with undetermined coefficients, thus

   $$U_1(x) = [F_{o2}(x) + F_{12}(x)] + F_{o3}(x) + \ldots\ldots F_{om}(x)$$
   $$\dot{U}_1(x) = -G_{12}(x) - G_{13}(x) \ldots\ldots G_{1(m+s)}(x)$$

   The unknown coefficients of $\psi_1(x)$ are determined from

i) $F_{o2}(x) + F_{12}(x)$ is positive definite

ii) $G_{12}(x)$ is positive definite

3. For a second approximation a homogeneous third order polynomial $F_{23}(x)$ is added to $U_1(x)$ to give

   $$U_2(x) = [F_{o2}(x) + F_{12}(x)] + [F_{o3}(x) + F_{23}(x)] + F_{o4}(x) + \ldots\ldots F_{om}(x)$$
   $$\dot{U}(x) = -G_{12}(x) - G_{23}(x) \ldots\ldots G_{2(m+s)}(x)$$

   The coefficients of $F_{23}(x)$ are determined by setting $G_{23}(x) = 0$

4. Further approximation can be made as required.

4.3 System Equations

For a single machine connected to an infinite bus the system equations including the effect of flux decay in the field [16] are

$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = \frac{1}{M}\left(P_{mi} - \frac{E'_q E_B}{X_{12} + X'_d}\ \sin(X_1 + \delta^o)\right)$$

$$\dot{X}_3 = -\ \eta_1 X_3 - \eta_2(\cos(X_1 + \delta^o) - \cos\delta^o) \tag{4.16}$$

where $X_1 = \delta - \delta^o$ $\quad X_2 = \dot{X}_1 = \dot{\delta}$ $\quad X_3 = E'_q - E$

In order to apply Chen's method, equations (4.16) are expanded as follows :

Put $\quad K_1 = E_B/M(X_{12} + X'_d)$

$\quad K_2 = K_1 E$

Thus

$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = -K_1 X_3 \sin(X_1 + \delta^o) - K_2(\sin(X_1 + \delta^o) - \sin\delta^o)$$

$$\dot{X}_3 = -\ \eta_1 X_3 - \eta_2(\cos(X_1 + \delta^o) - \cos\delta^o)$$

which when expanded gives

$$\dot{X}_1 = X_2$$

$$\dot{X}_2 = -\sum_{i=1}^{\infty} p_i X_1^i - X_3 \sum_{i=1}^{\infty} q_i X_1^{i-1} \tag{4.17}$$

$$\dot{X}_3 = -\ \eta_1 X_3 - \sum_{i=1}^{\infty} r_i\ X_1^i$$

where

$$p_i = \frac{K_2}{i!}\ \sin\left(\delta^o + \frac{i\pi}{2}\right)$$

$$q_i = \frac{K_1}{i!} \sin\left(\delta^o + \frac{(i-1)}{2}\pi\right)$$

$$r_i = \frac{\eta_2}{i!}\ \sin\left(\delta^o + \frac{(i-1)}{2}\pi\right) \tag{4.18}$$

4.4 Construction of Liapunov Function

Applying Chen's method to the system equations (4.17) gives

$$g(x) = \begin{bmatrix} \sum_{i=1}^{n} (p_i + r_i)X_1^i + X_3 \sum_{i=1}^{n} q_i X_1^{i-1} + \eta_1 X_3 \\ \sum_{i=1}^{n} r_1 X_1^i + X_2 + \eta_1 X_3 \\ -\sum_{i=1}^{n} p_i X_1^i - X_3 \sum_{i=1}^{n} q_i X_1^{i-1} + X_2 \end{bmatrix} \tag{4.19}$$

$$\theta(g) = \begin{bmatrix} -\eta_1 X_3 - \sum_{i=1}^{n} [(q_i + ip_i)X_3 + \frac{i}{2} q_{i+1} X_3^2 - ir_i X_2]\, X_1^{i-1} \\ (1 - \eta_1)X_3 \\ 0 \end{bmatrix} \tag{4.20}$$

$$W(x) = \frac{1}{2} X_2^2 + 2X_2X_3 - \frac{1}{2} q_1 X_3^2 + \sum_{i=2}^{n+1} \left(\frac{p_{i-1} + r_{i-1}}{i}\right) X_1^i + \sum_{i=2}^{n+1} (2X_2 r_{i-1} - 2X_3 p_{i-1})\, X_1^{i-1} - X_3^2 \sum_{i=3}^{n+1} q_{i-1}\, X_1^{i-2} \tag{4.21}$$

4.5 Numerical Example [16]

The system taken as an example is shown in Figure 4.1 with the following data

$E = 1.02$ P.u.

$\delta^o = 0.42$ rad

$M = 147 \times 10^{-4}$ P.u.

$X'_d = 0.3$ P.u.

$E'_q = 1.03$ P.u.

$E_B = 1.0$ P.u.

$X_e = 0.2$ P.u. on base 25 MVA

$X_d = 1.15$ P.u.

$T_o'' = 6.6$ sec

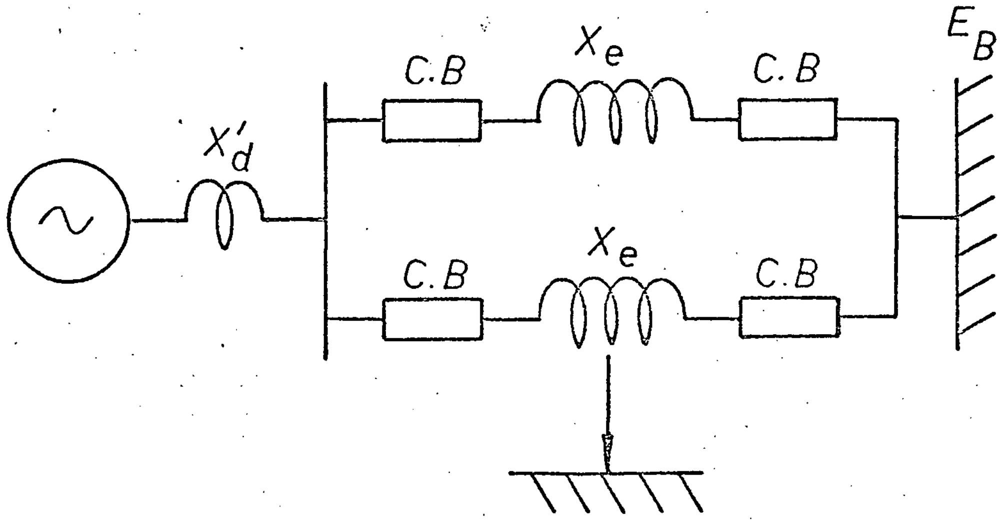

Fig. 4.1 Single Machine - Infinite Bus Power System

A three phase short circuit at the middle of one transmission circuit as shown in Figure 4.1 occurs, the corresponding V and $\dot{V}$-functions are

$$W(x) = \frac{1}{2} X_2^2 + 2X_2X_3 - \frac{1}{2} q_1X_3^2 + \sum_{i=2}^{n+1} \left(\frac{p_{i-1} + r_{i-1}}{i}\right) X_1^i$$

$$+ \sum_{i=2}^{n+1} (2X_2r_{i-1} - 2X_3p_{i-1}) X_1^{i-1} - X_3^2 \sum_{i=3}^{n+1} q_{i-1} X_1^{i-2}$$

$$\dot{W}(x) = - 2\eta_1X_2X_3 - q_1(\eta_1 - 2)X_3^2 - X_2 \sum_{i=2}^{n+1} r_{i-1} X_1^{i-1}$$

$$+ 2X_2^2 \sum_{i=2}^{n+1} (i - 1) r_{i-1} X_1^{i-2} - X_2X_3 \sum_{i=2}^{n+1} (2(i - 1)p_{i-1} + q_{i-1})X_1^{i-1}$$

$$+ X_3 \sum_{i=2}^{n+1} (q_1r_{i-1} + 2(\eta_1 - 1)p_{i-1}) X_1^{i-1}$$

$$+ 2X_3^2 \sum_{i=3}^{n+1} (\eta_1 - 1)q_{i-1} X_1^{i-2} - X_2^2X_3 \sum_{i=3}^{n+1} (i - 2) q_{i-1} X_1^{i-3}$$

It is seen that F_{o2} and G_{o2} are not positive definite and successive approximations are needed.

4.5.1 The First Approximation

A quadratic function $F_{12}(x) = X^TAx$ is added to W(x). When solving for $\{F_{12}(x) + F_{o2}(x)\}$ positive definite and G_{12} positive definite we get

$$A = \begin{bmatrix} -1.45 & -0.109 & 153 \\ -0.109 & -0.01 & -1.0 \\ 153 & -1.0 & 50.5 \end{bmatrix}.$$

The first approximate stability region boundary is obtained by calculating $\alpha_1 = \min \{U_1(x)/\dot{U}_1(x) = 0$, except the origin$\}$ where $U_1(x) = W(x) + F_{12}(x)$. The critical clearing time is calculated from the above V-function and was at 0.05 sec.

4.5.2 The Second Approximation

A third order homogeneous polynomial $F_{23}(x)$ is added to $U_1(x)$

$$F_{23}(x) = a_1X_1^3 + a_2X_2X_1^2 + a_3X_2^2X_1 + a_4X_2^3 + a_5X_3X_1^2 + a_6X_3^2X_1 + a_7X_3^3 + a_8X_2^2X_3 + a_9X_2X_3^2 + a_{10}X_1X_2X_3$$

Solving for the unknown coefficients introduced by $F_{23}(x)$ from the condition

$$G_{23}(x) = 0$$

gives

$a_1 = 0.1472$ $\qquad a_6 = 71.03$

$a_2 = -0.31947$ $\qquad a_7 = 1.9328$

$a_3 = 0.224456 \times 10^{-6}$ $\qquad a_8 = 0.08046$

$a_4 = -0.46473 \times 10^{-3}$ $\qquad a_9 = -0.042757$

$a_5 = -15.752$ $\qquad a_{10} = -0.04443$

The second approximate stability region boundary is obtained by calculating $\alpha_2 = \min \{U_2(x)/\dot{U}_2(x) = 0, \text{ except the origin}\}$ where $U_2(x) = U_1(x) + F_{23}(x)$ which gives a critical clearing time of 0.1 sec.

4.5.3 The Third Approximation

A complete fourth order homogeneous polynomial $F_{34}(x)$ is added to $U_2(x)$

$$F_{34}(x) = a_1X_1^4 + a_2X_1^3X_2 + a_3X_1^3X_3 + a_4X_1^2X_2^2 + a_5X_1^2X_3^2 + a_6X_1^2X_2X_3 + a_7X_1X_2^3 + a_8X_1X_3^3 + a_9X_1X_2X_3^2 + a_{10}X_1X_2^2X_3 + a_{11}X_2^4 + a_{12}X_2^3X_3 + a_{13}X_2^2X_3^2 + a_{14}X_2X_3^3 + a_{15}X_3^4$$

The unknown constants introduced by $F_{34}(x)$ are calculated from the condition

$$G_{34}(x) = 0.0$$

which gives

$a_1 = -40.694$	$a_6 = 0.41651$	$a_{11} = -0.0025408$
$a_2 = 0.047175$	$a_7 = -0.40783$	$a_{12} = 0.003362$
$a_3 = -118.09$	$a_8 = -67.118$	$a_{13} = -0.61292$
$a_4 = 0.64395$	$a_9 = 0.058045$	$a_{14} = 0.72708$
$a_5 = -112.98$	$a_{10} = -0.56244$	$a_{15} = -26.650$

Also the third approximate stability region boundary is obtained by computing $\alpha_3 = \min \{U_3(x)/\dot{U}_3(x) = 0$, except the origin$\}$
where $U_3(x) = U_2(x) + F_{34}(x)$
which gives a critical clearing time of 0.083 sec. The actual critical clearing time is calculated by integrating the system equations using the Runge-Kutta method and is found to be at 0.5 sec. Figures 4.2.1, 4.2.2 and 4.2.3 show the function $V(x) = V_{max}$ for the first, second and third approximations respectively plotted in the three dimensional space X_1, X_2 and X_3.

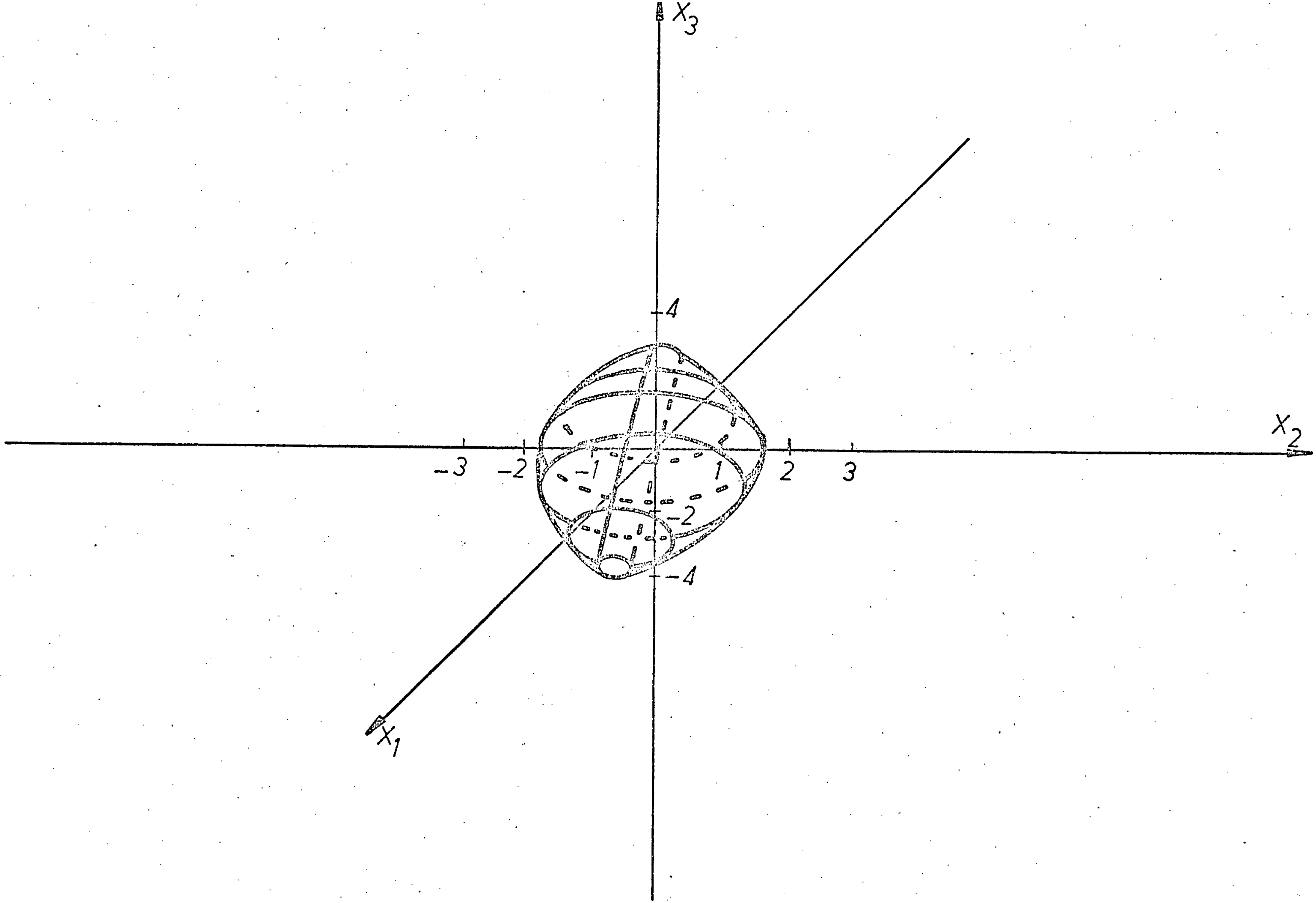

Fig. 4.2.1 First Approximation of Stability Region

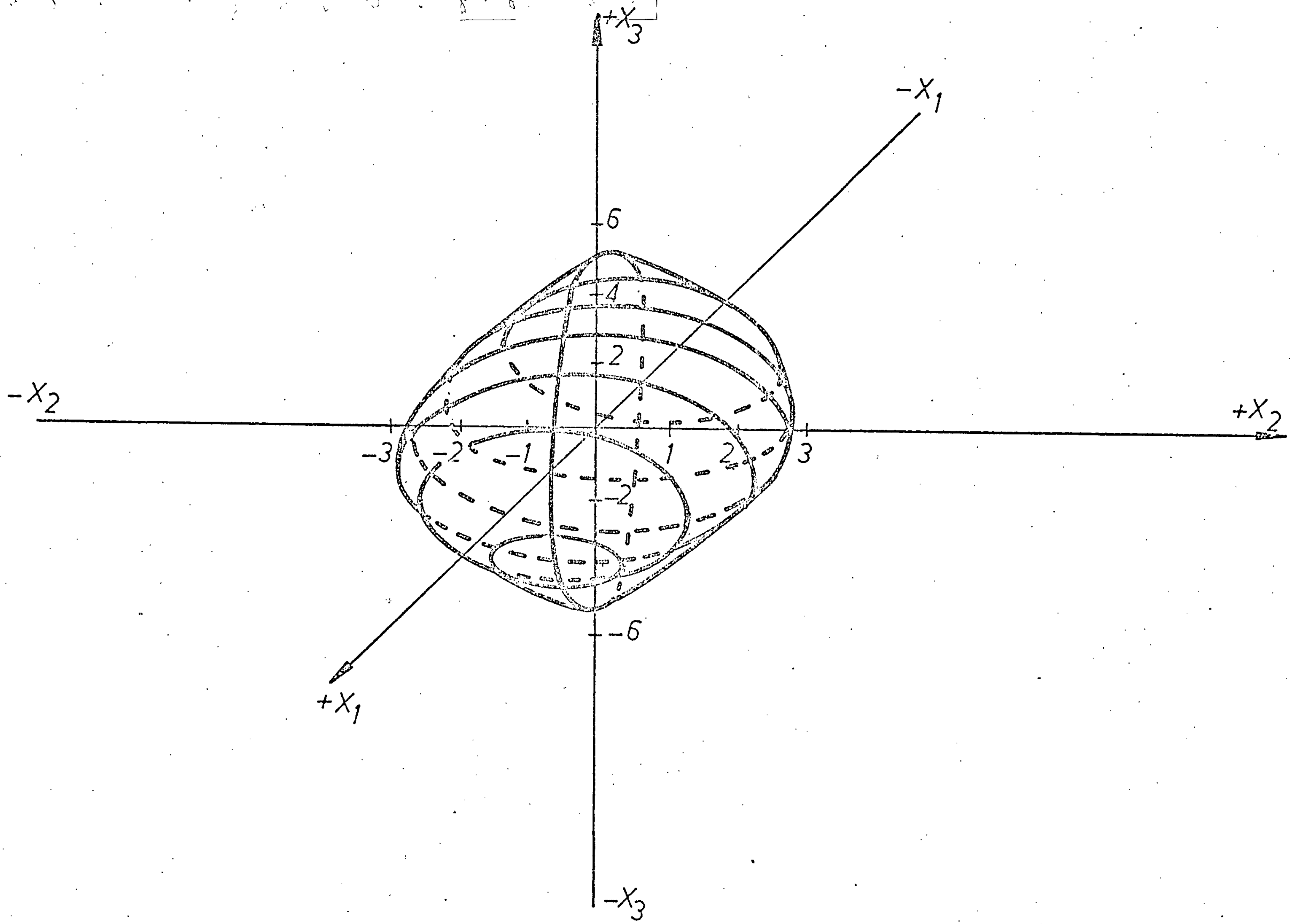

Fig. 4.2.2 Second Apprxoimation of Stability Region

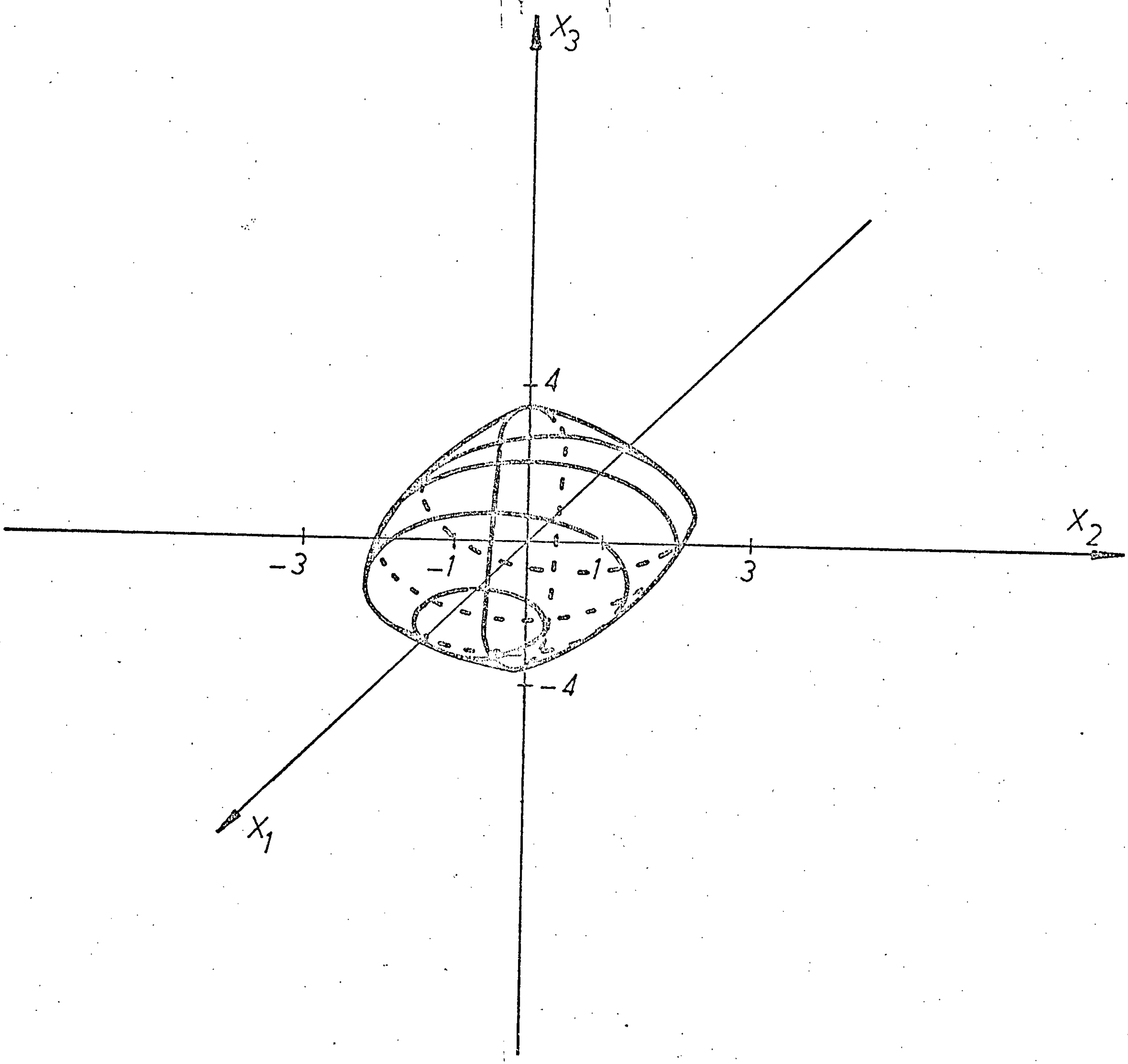

Fig. 4.2.3 Third Approximation of Stability Region

CONCLUSIONS

Two methods for constructing Liapunov functions have been applied to study the transient stability of power systems. In Chapter I Willems' method was applied to a three machine system in which each machine was represented by a second order nonlinear differential equation. The optimum distribution of damping ratios among different machines in a multimachine power system was investigated in Chapter II by maximizing the hypervolume enclosing by quadratic part of the Liapunov function. Governor action was included in the representation of the power systems in Chapter III and Willems' method was extended to enable construction of Liapunov functions for such systems. The effect of field flux decay is considered in Chapter IV and Chen's method was employed to construct a Liapunov function for a third order model of a single machine connected to an infinite bus. From these studies it is concluded that:

1. The efficiency of Willems' method, based on the generalized Popov criterion, is not affected by the number of machines included in the power system studied nor by the introduction of a governor
2. For a maximum region of stability, the damping ratios of all the machines should be equal.
3. Willems' method cannot be applied when the effects of flux decay are included.
4. Chen's method is applicable when power systems are represented in detail but it yields very restrictive results unless a large number of successive approximationsis performed. It is also shown that the stability region estimated using this method does not increase monotonically with the number of approximations.

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