MEASUREMENT OF THE PROPAGATION CHARACTERISTICS OF
SHIELDED AND UNSHIELDED DIELECTRIC-TUBE WAVEGUIDES

by

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ABSTRACT

Accurate measurements of the propagation coefficient of the \( \text{HE}_{11} \) mode on polythene-tube waveguides in air and surrounded by a polyfoam shield are reported. These were carried out at X-band frequencies using a cavity-resonance method. The results obtained confirm previous theoretical predictions although there is an element of uncertainty concerning the exact dielectric properties of the commercial grade polythene tubes used. The measurements also yielded the phase coefficient of the \( \text{HE}_{11} \) mode which was confirmed by measurement of the radial decay of the electric field outside the tube.

Enclosing the dielectric-tube in a low-density, low-loss polyfoam shield resulted in only a slight degradation of the attenuation characteristics of the waveguides.

Measurements of the phase characteristics of the higher order \( \text{TE}_{01} \) and \( \text{TM}_{01} \) modes on the tube at frequencies close to cutoff are also reported.
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LIST OF SYMBOLS

\( a_i, b_i \) = constants

\( A_n(p_{ij}), B_n(p_{ij}) \) = functions of Bessel functions

\( c_i \) = \( jb_{i}/a_{i} \)

\( E_{zi}, E_{ri}, E_{\theta i} \) = longitudinal, radial, azimuthal components of electric field, respectively, in medium \( i \)

\( f \) = frequency

\( f_r \) = resonant frequency

\( h_i \) = wave number of medium \( i \)

\( H_{zi}, H_{ri}, H_{\theta i} \) = longitudinal, radial, azimuthal components of magnetic field, respectively, in medium \( i \)

\( I_n(p_{ij}) \) = modified Bessel function of the first kind

\( J_n(p_{ij}) \) = Bessel function of the first kind

\( k_0 \) = phase coefficient of free space

\( K_n(p_{ij}) \) = modified Bessel function of the second kind

\( \lambda \) = number of half wavelengths in resonator

\( L \) = length of resonator

\( m, n \) = mode subscripts

\( N, N_{1} \) = total power loss per unit length and power loss per unit length in medium \( i \), respectively

\( N_{g} \) = power flow

\( N_{p}, N_{p1} \) = total power loss in each end plate and power loss in each plate in medium \( i \), respectively

\( p_{ij} \) = \( h_{i}r_{j} \)

\( Q \) = quality factor

\( Q_{\lambda} \) = loaded Q factor

\( Q_{u} \) = unloaded Q factor

\( r \) = radial co-ordinate

\( r_1, r_2 \) = inner and outer radius of tube respectively
\( R \) = normalized resistive component
\( R_m \) = resistive component of wave impedance of a metal
\( S_A, S_B, S_{AB}, T_A \) = integrals of functions of Bessel functions
\( S_I, S_K, T_I, T_K \) = integrals of functions of modified Bessel functions
\( \tan \delta_i \) = loss tangent of medium \( i \)
\( v_0, v_g, v_p \) = speed of light in free space, group velocity and phase velocity, respectively
\( W, W_i \) = total energy storage per unit length and total energy storage per unit length in medium \( i \), respectively
\( Y_n(p_{ij}) \) = Bessel function of the second kind
\( z \) = longitudinal co-ordinate
\( Z_0 \) = impedance of free space
\( \alpha \) = attenuation coefficient of tube
\( \beta \) = phase coefficient of tube
\( \beta_i \) = coupling coefficient
\( \Delta f \) = bandwidth
\( \varepsilon_{ri} \) = relative permittivity of medium \( i \)
\( \theta \) = azimuthal co-ordinate
\( \lambda \) = free space wavelength
\( \lambda_c, \lambda_g, \lambda_r \) = cutoff, guide and resonant wavelength, respectively
\( \mu_{ri} \) = relative permeability of medium \( i \)
\( \rho \) = \( r_1/r_2 \)
\( \omega \) = angular frequency
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1. INTRODUCTION

During the last forty years or so, many investigators have considered the problem of surface-wave propagation along dielectric tubes. In 1932, Zachoval obtained the characteristic equation for TM modes and solved this graphically for a range of tube parameters. Two years later, the existence of these modes was verified by Liska, whose measurements of guide wavelength showed good agreement with Zachoval's theory. In 1949, Astrahan obtained the characteristic equations for TE and hybrid modes and measured values of guide wavelength for the HE, TM, and TE modes which agreed very well with theory. At about the same time, Jakes gave expressions for the attenuation coefficients of TM and TE modes and measured the attenuation of the TM and TE modes on polystyrene tubes. A technique for obtaining the attenuation coefficient of any mode was outlined by Unger in 1954 using a method similar to Jakes, but the analysis was completed only for the HE mode on tubes with small diameter to wavelength ratios. Mallach made a rough estimate of the attenuation of the HE mode by measuring the radius at which the magnitude of the electric field fell to 1/e of its value at the tube surface. In 1968 Kharadly and Lewis completed a comprehensive study of the possible usefulness of the dielectric tube as a low-loss waveguide. They concluded that a moderately thin-walled tube propagating the dominant HE mode could have propagation characteristics greatly superior to those of conventional metallic waveguides at millimeter-wave frequencies. Also, they proposed a method for overcoming the problems of supporting the tube and the degradation of performance due to adverse weather conditions or nearby obstacles. This consisted of embedding the tube in a layer of low-density, low-loss dielectric of sufficient radial
extent that a negligible portion of the wave was carried outside this dielectric.

So far as is known, no accurate measurements of the attenuation characteristics of the dominant $\text{HE}_{11}$ mode on dielectric-tubes have been made. This seems surprising in view of the fact that this mode is the one most likely to be used in practice.

The objectives of the investigation reported here were therefore:

(i) to obtain experimental data on the attenuation and phase coefficients of the $\text{HE}_{11}$ mode on commercially available polythene tubes from direct measurements, using the cavity-resonance method.

(ii) to ascertain experimentally the effect of shielding the tube with low-density, low-loss polyfoam.

Chapter 2 reviews briefly some of the features of surface-wave propagation on dielectric-tube waveguides. The theory in this chapter is drawn from reference 7. In Chapter 3, the theory underlying the cavity-resonance method for measuring the attenuation coefficient of low-loss waveguides is discussed. This is followed in Chapter 4 by a description of the experimental apparatus used. Experimental results for the propagation characteristics of the $\text{HE}_{11}$ mode on polythene tubes in air and surrounded by a polyfoam shield are given in Chapter 5, together with results for the phase coefficient of the $\text{TE}_{01}$ and $\text{TM}_{01}$ modes at frequencies close to cutoff. Conclusions drawn from this investigation and suggestions for further work are contained in Chapter 6.
2.1 Field Components

The tube configuration of interest is shown in figure 2.1. It consists of two coaxial dielectric regions of infinite length and relative permittivities \( \varepsilon_{r1} \) and \( \varepsilon_{r2} \) embedded in a third infinite dielectric of relative permittivity \( \varepsilon_{r3} \), where

\[
\begin{align*}
\varepsilon_{r2} &> \varepsilon_{r1} \\
\varepsilon_{r2} &> \varepsilon_{r3}
\end{align*}
\]

In all cases, it will be assumed that the relative permeability of the \( i \)th region, \( \mu_{ri} \), is unity. Propagation is assumed in the \( z \)-direction, with \( t-\theta-z \) dependence of the form \( \exp j(\omega t-n\theta-\beta z) \) in the lossless case.

Figure 2.1 The Dielectric-Tube Waveguide
Under these conditions, omitting the factor exp \( j(\omega t-n\phi-\beta z) \), the field components are given by

\[
E_{z1} = a_1 I_n(h_1r)
\]

\[
E_{r1} = \frac{\beta}{h_1} a_1 I_n'(h_1r) + \frac{n\mu_{r1} k_0 Z_0}{h_1^2 r} b_1 I_n(h_1r)
\]

\[
E_{\theta 1} = \frac{n\beta}{h_1^2 r} a_1 I_n(h_1r) - j\frac{\mu_{r1} k_0 Z_0}{h_1} h_1 I_n'(h_1r)
\]

\[
H_{z1} = b_1 I_n(h_1r)
\]

\[
H_{r1} = -\frac{n \varepsilon_{r1} k_0}{h_1^2 r} a_1 I_n(h_1r) + \frac{\beta}{h_1} b_1 I_n'(h_1r)
\]

\[
H_{\theta 1} = j\frac{\varepsilon_{r1} k_0}{h_1^2 r} a_1 I_n'(h_1r) + \frac{n\beta}{h_1^2 r} b_1 I_n(h_1r)
\]

\[
E_{z2} = a_2 \left[ J_n(h_2r) + \frac{a_4}{a_2} Y_n(h_2r) \right] \Delta = a_2 A_n(h_2r)
\]

\[
E_{r2} = -j\frac{\beta}{h_2} a_2 A_n'(h_2r) - \frac{n\mu_{r2} k_0 Z_0}{h_2^2 r} b_2 B_n(h_2r)
\]

\[
E_{\theta 2} = -\frac{n\beta}{h_2^2 r} a_2 A_n(h_2r) + j\frac{\mu_{r2} k_0 Z_0}{h_2} b_2 B_n'(h_2r)
\]

\[
H_{z2} = b_2 \left[ J_n(h_2r) + \frac{b_4}{b_2} Y_n(h_2r) \right] \Delta = b_2 B_n(h_2r)
\]

\[
H_{r2} = \frac{n \varepsilon_{r2} k_0}{h_2^2 r} a_2 A_n(h_2r) - j\frac{\beta}{h_2} b_2 B_n'(h_2r)
\]

\[
H_{\theta 2} = -j\frac{\varepsilon_{r2} k_0}{Z_0 h_2} a_2 A_n'(h_2r) - \frac{n\beta}{h_2^2 r} b_2 B_n(h_2r)
\]
\[ E_{z3} = a_3 K_n(h_3 r) \]
\[ E_{r3} = j \frac{\mu_3}{h_3} a_3 K_n(h_3 r) + \frac{n \mu_3 k_0 z_0}{h_3^2 r} b_3 K_n(h_3 r) \]
\[ E_{\theta 3} = \frac{\mu_3}{h_3^2 r} a_3 K_n(h_3 r) - j \frac{\mu_3 k_0 z_0}{h_3} b_3 K_n(h_3 r) \]
\[ H_{z3} = b_3 K_n(h_3 r) \]
\[ H_{r3} = -\frac{n \epsilon_{r3} k_0}{h_3^2 r z_0} a_3 K_n(h_3 r) + j \frac{\beta}{h_3} b_3 K_n'(h_3 r) \]
\[ H_{\theta 3} = j \frac{\epsilon_{r2} k_0}{h_3^2 z_0} a_3 K_n'(h_3 r) + \frac{n \delta}{h_3^2 r} b_3 K_n(h_3 r) \]

where, from the wave equation

\[ h_1^2 = \beta^2 - \mu_1 \epsilon_1 k_0^2 \]
\[ h_2^2 = \mu_2 \epsilon_2 k_0^2 - \beta^2 \]
\[ h_3^2 = \beta^2 - \mu_3 \epsilon_3 k_0^2 \]

The symbols appearing in equations 2.2 and 2.3 are defined in the list of symbols.

Upon setting \( n=0 \) (no \( \theta \)-variation), equations 2.2 separate into two sets corresponding to the circularly symmetric modes designated \( \text{TM}_{0m} \) and \( \text{TE}_{0m} \). For \( n \neq 0 \), equations 2.2 describe inseparable combinations of TE and TM modes which are designated hybrid modes. In general, one or other of the component parts of a hybrid mode is dominant. If the TE portion is dominant, the mode is designated \( \text{HE}_{nm} \); if the TM component is dominant, it is termed \( \text{EH}_{nm} \). The nature of TE or TM dominance and the significance of the subscript \( m \) in the mode designation is discussed fully in reference 7.
2.2 Mode Spectrum

2.2.1 Characteristic Equations

By matching the axial and tangential field components in media 1 and 2 at \( r=r_1 \) and those in media 2 and 3 at \( r=r_2 \), eight homogeneous equations in eight unknowns, \( a_i, b_i, i=1-4 \), are obtained. These may be solved to give the following characteristic equations for the hybrid modes:

\[
\left( \frac{\varepsilon r_2 A_n'(p_{22})}{p_{22}^2 A_n(p_{22})} + \frac{\varepsilon r_3 K_n'(p_{32})}{p_{32}^2 K_n(p_{32})} \right) \left( \frac{\mu r_2 B_n'(p_{22})}{p_{22}^2 B_n(p_{22})} + \frac{\mu r_3 K_n'(p_{32})}{p_{32}^2 K_n(p_{32})} \right) = \left[ \frac{n^2}{k_0} \left( \frac{1}{p_{22}} + \frac{1}{p_{32}} \right) \right]^2 \]  \ ....... 2.4.a

and

\[
\left( \frac{\varepsilon r_2 A_n'(p_{21})}{p_{21}^2 A_n(p_{21})} + \frac{\varepsilon r_1 I_n'(p_{11})}{p_{11}^2 I_n(p_{11})} \right) \left( \frac{\mu r_2 B_n'(p_{21})}{p_{21}^2 B_n(p_{21})} + \frac{\mu r_1 I_n'(p_{11})}{p_{11}^2 I_n(p_{11})} \right) = \left[ \frac{n^2}{k_0} \left( \frac{1}{p_{21}} + \frac{1}{p_{11}} \right) \right]^2 \]  \ ....... 2.4.b

Equation 2.4.a is applicable for EH\(_{nm}\) modes and equation 2.4.b is applicable for HE\(_{nm}\) modes. The ratio \( a_4/a_2 \) is given by equation 2.5. The ratio \( b_4/b_2 \) is obtained from equation 2.5 by interchanging \( \varepsilon_{ri} \) and \( \mu_{ri} \). The characteristic equations for the TE\(_{0m}\) and TM\(_{0m}\) modes are obtained by setting \( n=0 \) in equation 2.4.b.

A typical spectrum of modes on a polythene tube in free space \((\varepsilon_{r1}=\varepsilon_{r3}=\mu_{r1}=\mu_{r3}=1, \varepsilon_{r2}=2.26, \text{ and } \rho=r_1/r_2=0.5)\) is shown in figure 2.2.

The main features of the mode spectrum are:

(i) The HE\(_{11}\) mode has no lower cutoff frequency.

(ii) Unlike the case for the dielectric rod (\( \rho=0 \)), the TE\(_{0m}\) and TM\(_{0m}\) modes do not have the same value of \( r_2/\lambda \) at cutoff. This is also true for HE\(_{1,m+1}\) and EH\(_{1m}\) modes.

(iii) As \( \rho \to 1 \), the phase characteristics of the TE\(_{0m}\) and HE\(_{1m}\) mode become indistinguishable, as do those of the TM\(_{0m}\) and EH\(_{1m}\) modes, thus
\[
\frac{a_4}{a_2} = \left( \frac{J_n(p_{21})}{Y_n(p_{21})} \right) \left\{ \left( \frac{Y_{n}(p_{22})}{Y_{n}(p_{21})} - \frac{J_{n}(p_{22})}{J_{n}(p_{21})} \right) \left[ \left( \frac{I_{n}(p_{11})}{p_{11}I_{n}(p_{11})} - \frac{J_{n}(p_{11})}{p_{11}J_{n}(p_{11})} \right) + \frac{\varepsilon_{r1}I_{n}(p_{11})}{p_{11}I_{n}(p_{11})} + \frac{\varepsilon_{r2}Y_{n}(p_{21})}{p_{21}Y_{n}(p_{21})} \right] - \left( \frac{n^2g_2}{\mu_{r1}k_0} \right) \left( \frac{1}{2} + \frac{1}{2} \left( \frac{p_{11}}{p_{21}} \right) \right)^2 \right\} \\
+ \frac{\mu_{r2}}{\mu_{r1}} \left[ \left( \frac{J_{n}(p_{21})Y_{n}(p_{22})}{p_{21}J_{n}(p_{21})Y_{n}(p_{21})} - \frac{Y_{n}(p_{21})J_{n}(p_{22})}{p_{21}Y_{n}(p_{21})J_{n}(p_{21})} \right) \left( \frac{\varepsilon_{r2}Y_{n}(p_{22})}{p_{22}Y_{n}(p_{22})} + \frac{\varepsilon_{r3}K_{n}(p_{32})}{p_{32}K_{n}(p_{32})} \right) \right] \\
+ \left( \frac{1}{2} + \frac{1}{2} \left( \frac{p_{11}}{p_{21}} \right) \right) \left( \frac{Y_{n}(p_{21})}{p_{21}Y_{n}(p_{21})} - \frac{J_{n}(p_{21})}{p_{21}J_{n}(p_{21})} \right) \left( \frac{\varepsilon_{r2}Y_{n}(p_{22})}{p_{22}Y_{n}(p_{22})} + \frac{\varepsilon_{r3}K_{n}(p_{32})}{p_{32}K_{n}(p_{32})} \right) \right\} \\
\]
providing the physical distinction between HE and EH modes.

(iv) As $\rho\rightarrow 1$, the $n=0$ and $n=1$ modes appear in widely separated clusters, each cluster consisting of four modes ($HE_{1m}$, $TE_{0m}$, $TM_{0m}$ and $EH_{1m}$).

(v) The $HE_{1m}$ and $TE_{0m}$ phase characteristics intersect at some value of $r_2/\lambda$. In most cases, for values of $r_2/\lambda$ greater than that at the intersection, the differences in the two curves are too small to be seen graphically. However, the degeneracy of the $HE_{12}$ and $TE_{02}$ modes for $\rho=0.5$ can be seen in figure 2.2.

\[ \text{Figure 2.2 Mode Spectrum of Polythene Tube, } \rho=0.5 \]
2.2.2 Cutoff Conditions

Lossless surface-wave propagation on the dielectric-tube requires that all quantities appearing in equations 2.3 be real and positive. If \( \mu_{r3} \epsilon_{r3} = \mu_{r1} \epsilon_{r1} \), then cutoff occurs when \( h_3 = 0 \) and \( h_1 = 0 \), or generally, when \( p_{32} = 0 \) and \( p_{11} = 0 \). Hence by applying small argument approximations to certain of the Bessel functions in equations 2.4.a-b, the following cutoff conditions are obtained.

\[
\frac{J_0(p_{22})}{Y_0(p_{22})} = \begin{cases} \frac{\epsilon_{r1} p_{21} J_0(p_{21}) - 2 \epsilon_{r2} J_1(p_{21})}{\epsilon_{r1} p_{21} Y_0(p_{21}) - 2 \epsilon_{r2} Y_1(p_{21})} & \text{TM modes} \\ \frac{p_{21} J_0(p_{21}) - 2 J_1(p_{21})}{p_{21} Y_0(p_{21}) - 2 Y_1(p_{21})} & \text{TE modes} \end{cases}
\]

\[ J_{1}(p_{22}) = J_{1}(p_{21}) \quad \text{HE}_{11} \text{ mode} \]

\[ \frac{J_{1}(p_{22})}{Y_{1}(p_{22})} = \frac{J_{1}(p_{21})}{Y_{1}(p_{21})} \quad \text{HE}_{1m} \text{ modes} \quad m > 1 \]

\[ \left[ \frac{J_{1}(p_{21})}{J_{1}(p_{22})} - \frac{Y_{1}(p_{21})}{Y_{1}(p_{22})} \right] = \left[ \frac{J_{1}(p_{21})}{J_{1}(p_{22})} - \frac{Y_{1}(p_{21})}{Y_{1}(p_{22})} \right] \left[ \frac{1}{p_{21}} - \frac{p_{21} \epsilon_{r1}}{(\epsilon_{r1} + \epsilon_{r2})} \right] \quad \text{EH}_{1m} \text{ modes} \quad m > 1 \]

At cutoff \( p_{22} \) is given by

\[ p_{22} = 2\pi \frac{r_2}{\lambda_c} \sqrt{\mu_{r2} \epsilon_{r2} - \mu_{r3} \epsilon_{r3}} \]

from which the value of \( r_2/\lambda_c \) can be determined.

The variation of \( r_2/\lambda_c \) with \( \rho \) for the TE\(_{01} \), TM\(_{01} \), EH\(_{11} \) and HE\(_{12} \) modes on a polythene tube (\( \epsilon_{r2} = 2.26 \)) in free space is shown in figure 2.3.
Figure 2.3 Cutoff Conditions; TE\textsubscript{01}, TM\textsubscript{01}, EH\textsubscript{11} and HE\textsubscript{12} Modes
3. CAVITY-RESONANCE METHOD FOR MEASURING ATTENUATION

3.1 Introduction

The cavity-resonance method appeared to be the one most suitable for directly measuring the small attenuation coefficient of the $HE_{11}$ mode on dielectric-tube waveguides. The main advantages of the method are that only a fairly short length of waveguide is needed and the problems of accurate measurement of power levels or substituted attenuation are avoided. The relationship between attenuation coefficient and the $Q$ factor of a cavity formed from a section of the waveguide and two metallic end plates is discussed in the next section.

3.2 Relation Between Attenuation Coefficient and $Q$ Factor

Adopting the nomenclature of reference 7, the $Q$ factor of the resonator is given by

\[ Q = \frac{\omega WL}{2N + NL} = \frac{\omega LN / v}{2N + 2L \alpha N g} \]  \hspace{1cm} (3.1)

where

\[ W = N g / v_g, \quad N = 2 \alpha N g \]

Then

\[ \frac{1}{Q} = \frac{2 \alpha v_g}{\beta v_p} + \frac{2N}{\omega WL} \]  \hspace{1cm} (3.2)

where

\[ \beta = \omega / v = 2 \pi / \lambda_g \]

For very long resonators, the second term in equation 3.2 can be neglected and the expression for $Q$ becomes

\[ Q \approx \frac{\beta}{2 \alpha} \left( \frac{v_p}{v_g} \right) \]  \hspace{1cm} (3.3)
Then the attenuation coefficient $\alpha$ is given by

$$\alpha = \frac{\beta}{2Q} \left( \frac{v_p}{v_g} \right)$$ \hspace{1cm} 3.4

In previous experimental investigations of surface waveguides, values of $\alpha$ have been obtained by measuring $Q$ and $\beta$ and using the transmission-line formula,

$$\alpha = \frac{\beta}{2Q}$$ \hspace{1cm} 3.5

which assumes $v_p/v_g = 1$ in equation 3.4. This assumption may lead to significant errors. As an example, the factor $v_p/v_g$ for the dominant $HE_{11}$ mode on a polythene tube ($\varepsilon_r = 2.26$) waveguide has been calculated and is shown in figure 3.1. Inspection of this figure shows that equation 3.5 is valid for such waveguides when the phase-velocity reduction is very small or very large, but can lead to appreciable errors for intermediate values.

In the present investigation, no provision was made for measuring $v_g$ and hence the ratio $v_p/v_g$ together with the term in equation 3.2 involving end plate losses were evaluated using the theory given below.

Figure 3.1 Characteristics of Dielectric-Tube Waveguide, $HE_{11}$ Mode
3.2.1 Relation Between Attenuation Coefficient and Q Factor for Surface-Wave Resonator

From equation 3.2,

\[
\alpha = \frac{\beta}{2} \left( \frac{V_p}{V_g} \right) \left( \frac{1}{Q} - \frac{2N_p}{\omegaWL} \right) = \frac{\beta}{2} \left( \frac{V_p}{V_g} \right) \left( \frac{1}{Q} - \frac{2(N_{p1} + N_{p2} + N_{p3})}{\omegaL(W_1 + W_2 + W_3)} \right)
\]

where, for the dominant HE_{11} mode,

\[
W_1 = \frac{a_2 a_2^* \pi \epsilon \rho_1}{4 \nu_0 \mu_0 h_1^4} \left( \frac{\lambda_2^2(p_{21})}{\lambda_1^2(p_{11})} \right) \left[ h_2^2 T_A + \left[ \beta^2 + (k_0 \rho Z_0 c_1)^2 \right] S_I - 4 \beta k_0 \rho Z_0 c_1^2 (p_{11}) \right]
\]

\[
W_2 = \frac{a_2 a_2^* \pi \rho_2}{4 \nu_0 \mu_0 h_2^4} \left[ h_2^2 T_A + \beta^2 S_A + (k_0 \rho Z_0 c_2)^2 S_B + 4 \beta k_0 \rho Z_0 c_2 S_{AB} \right]
\]

\[
W_3 = \frac{a_2 a_2^* \pi \rho_3}{4 \nu_0 \mu_0 h_3^4} \left( \frac{\lambda_2^2(p_{22})}{\lambda_1^2(p_{32})} \right) \left[ h_3^2 T_A + \left[ \beta^2 + (k_0 \rho Z_0 c_3)^2 \right] S_K + 4 \beta k_0 \rho Z_0 c_3^2 S_{AB} \right]
\]

\[
N_{p1} = \frac{2 a_2 a_2^* \pi \rho_1}{h_1^4} \left( \frac{\lambda_2^2(p_{21})}{\lambda_1^2(p_{11})} \right) \left[ \frac{(k_0 \rho Z_0 r_1)^2}{2} S_I + c_2^2 \beta^2 S_I - 4 c_1 \beta S_K + \frac{(k_0 \rho Z_0 r_1)^2}{2} \right] \times \frac{1^2(p_{11})}{1^2(p_{11})}
\]

\[
N_{p2} = \frac{2 a_2 a_2^* \pi \rho_2}{h_2^4} \left[ \frac{(k_0 \rho Z_0 r_2)^2}{2} S_A + c_2^2 \beta^2 \frac{\lambda_2^2(p_{21})}{B_1^2(p_{21})} S_B + 4 c_2^2 \beta \frac{(k_0 \rho Z_0 r_2)^2}{Z_0} \left( \frac{A_1(p_{21})}{B_1(p_{21})} \right) S_{AB} \right]
\]

\[
N_{p3} = \frac{2 a_2 a_2^* \pi \rho_3}{h_3^4} \left( \frac{\lambda_2^2(p_{22})}{\lambda_1^2(p_{32})} \right) \left[ \frac{(k_0 \rho Z_0 r_3)^2}{2} S_K + c_3^2 \beta^2 S_K + 4 c_3^2 \beta \frac{(k_0 \rho Z_0 r_3)^2}{Z_0} \right] \times \frac{1^2(p_{32})}{1^2(p_{32})}
\]

The functions \( S_I, S_A, S_B, S_{AB}, S_K, T_A, T_I \) and \( T_K \) are integrals of functions of Bessel functions which are defined and evaluated in reference 7.

Table 3.1 shows a sample output from a computer program called QFACTOR which was used to obtain the unloaded Q factor of the surface wave resonator, the factor \( \frac{V_p}{V_g} \) and the attenuation coefficient of the
dominant $HE_{11}$ mode by solving equation 2.4.b for $k_0(\beta)$, where the phase coefficient $\beta$ was decided by the number of half wavelengths contained in the length of the cavity.
<table>
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<th>R2/λ</th>
<th>Ko/Beta</th>
<th>VP/VG</th>
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<th>Alpha (dB/ft)</th>
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Table 3.1 Sample Output from QFACTOR
3.3 Relation Between Unloaded Q and Loaded Q

In practice the loaded Q factor, $Q_L$, of a cavity resonator is given by

$$ Q_L = \frac{f_r}{\Delta f} \quad \text{..................} 3.9 $$

where $f_r$ is the resonant frequency and $\Delta f$ is the bandwidth at the half-power points of the transmission characteristic.

To determine the bandwidth $\Delta f$, either the amplitude or the phase of the transmission characteristic of the resonator can be used (figure 3.210).

The unloaded Q can be obtained by measuring the loaded Q and the coupling coefficients of the cavity input and output apertures. In the case where there are two coupling apertures, the unloaded Q is given by

$$ Q_u = Q_L (1+\beta_1+\beta_2) \quad \text{............} 3.10 $$

In the case where the output coupling $\beta_2$ is negligibly small, equation 3.10 becomes

$$ Q_u \approx Q_L (1+\beta_1) \quad \text{............} 3.11 $$

To obtain the coupling coefficient $\beta_1$, it is necessary to measure the
input impedance of the resonator at resonance. If the normalized resistive component $R$, which is equal to the coupling coefficient $\beta_1$, is found to be greater than unity, the cavity is overcoupled. If $R$ is found to be less than unity, the cavity is undercoupled and if $R$ is found to be unity the cavity is critically coupled.
4. EXPERIMENTAL APPARATUS

4.1 Introduction

Although dielectric-tube waveguides would be most advantageously used at millimeter-wave frequencies, it was more convenient to conduct the present investigation at X-band frequencies. This placed less stringent tolerance requirements on the dimensions of the tube, making it possible to use commercially available tubes.

The general layout of the microwave apparatus is shown in figure 4.1.

To improve the frequency stability of the X-13 klystron, the latter was water cooled and a klystron synchronizer (FEL Model 136-AF) was used. For measurement of the Q factor of the surface-wave resonator, it was necessary to measure the bandwidth Δf of the resonator Q curve accurately. This was facilitated by use of a beat-frequency technique which made it possible to measure frequencies in the X-band range with an error of not more than ± 50 KHz. By comparison, the ordinary reaction type of cavity frequency meter has a typical accuracy of ± 1MHz in the same frequency range.

Details of the components of the surface-wave resonator are given in the following sections.
Figure 4.1 Layout of Apparatus
4.2 Surface-Wave Resonator

The surface-wave resonator, shown in figures 4.2-4.5, consisted of a length of dielectric tube [1]* approximately 1.78m long bounded at both ends by flat, circular, aluminum plates, 0.61m in diameter and 1.2 cm in thickness, mounted at right angles to the waveguide. Since it was desirable to use as long and as straight a tube as possible in order to obtain accurate measurements of the attenuation coefficient, it was necessary to devise some method of adequately supporting and tensioning the tube. This was achieved by passing the ends through holes in the end plates of the resonator and radially gripping the tube walls between these plates and close fitting, circular, short-circuiting plugs, [2 and 3], inside the tube. Leakage of energy outside the resonator through the dielectric-filled, annular apertures thus formed in the end plates was prevented by the use of annular short-circuiting plungers [4] at each end of the dielectric tube. The end plates of the resonator were kept parallel and in alignment by four tie rods [5]. Alignment of the end plates was carried out using a laser in a manner similar to that used for aligning optical cavities. Table 4.1 shows details of the polythene tubes used in the investigation.

The other end plate of the resonator had a number of holes [7], 0.13 cm in diameter, lying along a radius of the plate, through which was inserted a small wire probe sensitive to the longitudinal component of the electric field within the resonator. By moving the probe from one sampling hole to another the radial field decay could be investigated. Normally, all the holes in the end plate, except the one containing the probe, were closed by tightly fitting aluminum plugs.

* The numbers given in the text correspond to those appearing in figures 4.3-5.
For the measurement of the radial decay of the radial component of the electric field inside the resonator, another probe, mounted on a modified slotted-line carriage, was moved radially across some cross-sectional plane inside the resonator.

<table>
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<tr>
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<th>$r_2$(cm)</th>
<th>$\rho=r_1/r_2$</th>
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<td>1.270</td>
<td>0.750</td>
</tr>
<tr>
<td>II</td>
<td>1.270</td>
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<td>0.800</td>
</tr>
<tr>
<td>III</td>
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<td>1.905</td>
<td>0.833</td>
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Table 4.1 Details of Polythene Tubes
Figure 4.2 General View of Surface-Wave Resonator
Figure 4.3 Surface-Wave Resonator

scale 1/10
Figure 4.4 Details of Cavity End Plate Showing HE_{11} Mode Exciter

scale 1/1
Figure 4.5 Details of Cavity End Plate

scale 1/1
4.3 Mode Exciters

Excitation of the dominant $HE_{11}$ mode on the dielectric tube was achieved by means of a small circular aperture [6] fed by a circular waveguide, which also formed one of the tube tensioning plugs [3] mentioned in section 4.2.

Excitation of the $TE_{01}$ mode was achieved by replacing the annular short-circuiting plunger [4] at the input end of the resonator by two polystyrene-filled rectangular waveguides of transverse dimensions 0.8 cm by 1.3 cm (figure 4.7), which butted up against the exposed end of polythene tube III. The waveguides were excited $180^\circ$ (figure 4.6.b) out of phase by using the set-up shown in figure 4.6.a. The dimensions of the tapered waveguides were such as to equalize the phase velocities of the $TE_{10}^{\circ}$ mode of the exciter and the $TE_{01}^{\circ}$ mode of the surface waveguide. This arrangement could also be used to excite the dominant $HE_{11}$ mode by feeding both dielectric waveguides in phase. An alternative method of exciting the $TE_{01}$ mode, that due to
Astrahan\textsuperscript{3}, is shown in Figure 2.6.c. This was tried in the present investigation, but proved to be unsatisfactory, since it excited both the $\text{HE}_{11}$ and $\text{TE}_{01}$ modes simultaneously.

For excitation of the $\text{TM}_{01}$ mode, the circular waveguide and aperture were replaced by a section of coaxial line, having a tapered inner conductor. This is shown in figure 4.8.
Figure 4.7 $\text{TE}_{01}$ Mode Exciter

Figure 4.8 $\text{TM}_{01}$ Mode Exciter
5. RESULTS

5.1 Dependence of Cavity Q Factor on Size of Coupling Aperture

Figure 5.1 shows the dependence of the cavity Q factor of the \( \text{HE}_{11} \) mode on coupling aperture size for tube II at a frequency of 8.328 GHz. For aperture sizes of less than about 5 mm, both the loaded Q factor and the unloaded Q factor became virtually constant and the coupling coefficient \( \beta_1 \) was smaller than 0.01. Hence the amount of cavity loading for this range of aperture sizes was negligible. Table 5.1 shows the actual size of aperture used with each particular size of tube. In all cases, the apertures were small enough to ensure that the errors in the measurements were small.

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<td>6.4</td>
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<tr>
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Table 5.1 Details of Coupling Apertures

![Figure 5.1 Measured Dependence of Cavity Q Factor of \( \text{HE}_{11} \) Mode on Coupling Aperture Diameter, tube II, \( f=8.328 \text{GHz} \)]

- \( Q \) experimental points for unloaded Q factor
- \( \Delta \) experimental points for loaded Q factor
5.2 Measurement of Guide Wavelength

Measurement of the wavelength along the surface of the dielectric-tube mounted inside the resonator was carried out using a perturbation method similar to that described by Barlow and Karbowiak\textsuperscript{11}. Essentially the method involved the determination of the number of half wavelengths contained in the length of the resonator when the latter was resonant at a known frequency. This was achieved with the aid of a small aluminum bead supported in close proximity to the dielectric waveguide by a cotton thread stretched transversely between two parallel nylon running cords mounted longitudinally outside the resonator and diametrically opposite one another. By simultaneous axial movement of the running cords the small bead was made to traverse the length of the resonator, remaining throughout at approximately the same distance from the dielectric-waveguide. While no appreciable disturbance of the field was produced by the cotton thread, some energy was scattered by the bead except when it was situated at a node of the electric field. Thus the output of the probe connected to the resonator exhibited successive variations as the bead was moved along the dielectric waveguide, and it was only necessary to count the number of oscillations in the probe output in traversing the length of the resonator. The number of maxima corresponded to the number of nodes in the longitudinal field distribution of the resonator and the wavelength was therefore determined. Thus, the accuracy of the method was dependent on the precision with which the length of the resonator could be measured. In the present investigation, this was achieved to an accuracy better than $\pm 1 \text{ mm}$, leading to an error in the measurement of guide wavelength of not more than 1 part in 1780.

Figure 5.2 shows a typical probe output, obtained when the field perturbing
Figure 5.2 Measurement of Guide Wavelength of HE_{11} Mode by Field Perturbing Bead Method, Tube III, $f=8.323$ GHz, $\lambda=101$. 
bead was moved along the length of the resonator. From this, it was deduced that there were 101 half wavelengths in the length of the resonator at a frequency of 8.323 GHz.

The measured and theoretically predicted variation of the guide wavelength of the HE_{11} mode with r_2/λ is shown in figures 5.3.a-c for tubes I, II and III. The experimental results agree well with particular theoretical curves computed for values of relative permittivity in the range 2.26 to 2.31. (The exact dielectric properties of the commercially available polythene tubes used were not known.)

As a check on the cutoff frequencies of the higher order TE_{01} and TM_{01} modes, the latter were individually excited on tube III, using the appropriate mode exciter and the variation of guide wavelength with r_2/λ at frequencies close to cutoff was measured. Good agreement between experiment and theory was obtained using the particular value of relative permittivity for tube III found before, ε_r = 2.26. These results are shown in figure 5.3.c.

Figure 5.3.b shows the effect of surrounding tube II by a low-density polyfoam shield of cross-sectional dimensions 50 cm by 50 cm. As can be seen, the dispersion characteristics of the shielded tube are little different from those of the unscreened tube. The experimental results agreed most closely with the theoretical ones when the relative permittivity of the polyfoam shield was assumed to be 1.041.
Figure 5.3.a  Experimental and Theoretical Phase Characteristics of HE_{11} Mode on Polythene Tube I

- Experimental points
- Theoretical curve for $\varepsilon_{r_2} = 2.31$
Figure 5.3.b Experimental and Theoretical Phase Characteristics of $HE_{11}$ Mode on Shielded and Unshielded Polythene Tube II

(i) unshielded tube
- experimental points
- theoretical curve for $\varepsilon_r = 2.28$

(ii) shielded tube
- experimental points
- theoretical curve for $\varepsilon_r = 2.28$, $\varepsilon_r = 1.041$
Figure 5.3.c Experimental and Theoretical Phase Characteristics of HE_{11}, TE_{01}, and TM_{01} Modes on Polythene Tube III

(i) HE_{11} mode
- experimental points
- theoretical curve for $\varepsilon = 2.26$

(ii) TE_{01} mode
- experimental points
- theoretical curve for $\varepsilon = 2.26$

(iii) TM_{01} mode
- experimental points
- theoretical curve for $\varepsilon = 2.26$
5.3 Measurement of Radial Decay of Electric Field

As a check on the dispersion characteristics of the HE_{11} mode on the unscreened tube, the radial decay of the longitudinal and radial components of the electric field was measured when the resonator was resonant in the HE_{1,1,101} mode at a frequency of 8.328 GHz. The results obtained for tube II are plotted in figures 5.4.a-b together with the theoretical curves computed from equations 2.2.c for $\varepsilon_{r2}=2.28$.

![Graph showing the radial decay of $E_{z3}$ for HE_{11} Mode, Tube II, f=8.328GHz](image)

**Figure 5.4.a** Radial Decay of $E_{z3}$ for HE_{11} Mode, Tube II, f=8.328GHz
- **O** experimental points
- **---** theoretical curve for $\varepsilon_{r2}=2.28$
Figure 5.4.b Radial Decay of $E_{r3}$ for HE$_{11}$ Mode, Tube II, $f=8.328$ GHz
experimental points
theoretical curve for $\varepsilon_r=2.28$
5.4 Measurement of Attenuation Coefficient

The cavity-resonance method was used to measure the relatively small attenuation coefficient of the polythene-tube waveguide. The evaluation of the attenuation coefficient from the measured Q factor of the resonator was carried out using equation 3.6.

The measured and theoretically predicted variation of the attenuation coefficient with $r_2/\lambda$ of the HE$_{11}$ mode is shown in figures 5.5.a-c for the same polythene tubes used previously. Assuming that the tubes had the values of relative permittivity found in section 5.3, it was found that the experimental points agreed with the theoretical curves for loss tangents in the range 0.00058 to 0.00085. Figure 5.5.b shows the effect of surrounding tube II by the polyfoam shield. It can be seen that the attenuation coefficient of the shielded tube is only slightly higher than that of the unshielded tube. The experimental results agree best with the theoretical ones when the loss tangent is taken to be 0.00007.
Figure 5.5.a Experimental and Theoretical Attenuation Characteristics of HE_{11} Mode on Polythene Tube I

- Experimental points
- Theoretical curve for $\epsilon_{r2}=2.31$, $\tan\delta_2=0.00085$
Figure 5.5.b Experimental and Theoretical Attenuation Characteristics of HE_{11} Mode on Shielded and Unshielded Polythene Tube  

(i) Unshielded tube.  
- experimental points  
- theoretical curve for \( \epsilon_r^2 = 2.28, \tan\delta_2 = 0.00068 \)

(ii) Shielded tube  
- experimental points  
- theoretical curve for \( \epsilon_r^2 = 2.28, \epsilon_{r3} = 1.041, \tan\delta_3 = 0.00007 \)
Figure 5.5.c Experimental and Theoretical Attenuation Characteristics of $HE_{11}$ Mode on Polythene Tube III

- $\alpha$ (dB/m)
- $r_2/\lambda$
- Experimental points
- Theoretical curve for $\varepsilon_r=2.26$, $\tan\delta=0.00058$
6. CONCLUSIONS

Accurate measurements of the attenuation coefficient of the \( HE_{11} \) mode on certain dielectric-tube waveguides have been made using a cavity-resonance method. The results obtained confirm previous theoretical predictions although there is an element of uncertainty concerning the exact dielectric properties of the commercial grade polythene tubes used. The measurements also yielded the phase coefficient of the \( HE_{11} \) mode which was confirmed by measurement of radial decay of the electric field outside the tube. Enclosing the dielectric tube in a low-density, low-loss polyfoam shield resulted in only a slight degradation of the attenuation characteristics of the waveguide.

Areas in which future theoretical and experimental work on dielectric tube waveguides might be carried out include the following:

(i) Effects of discontinuities and bends on the propagation characteristics of the \( HE_{11} \) mode

(ii) Measurement of \( \nu_g \) of the \( HE_{11} \) mode

(iii) Coupling between waveguides

(iv) Development of efficient mode exciters and filters

(v) Extension of all these topics to millimeter-wave frequencies
REFERENCES


