HYDRAULIC GEOMETRY OF GREEN AND BIRKENHEAD RIVERS: SOUTHWESTERN COAST MOUNTAINS, BRITISH COLUMBIA

by

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ABSTRACT

Green and Birkenhead Rivers are located in the southwestern Coast Mountains of British Columbia, and the drainage in both basins is still strongly controlled by glacial features left after the retreat of the Vashon ice sheet. River slopes are imposed on the upland streams while the slope of the main valley streams is at least partly imposed by the glacial topography. Discharge in the streams is dominated by snowmelt during the summer though peak daily discharges frequently occur in autumn during autumn storms.

At-a-station hydraulic geometry curves were determined by least square regression analysis for five sections from Water Survey of Canada gauging records. Velocity shows a more rapid than usual rate of adjustment, and resistance decreases more rapidly than the average as discharge increases. Residual values appear to be distributed about the regression lines in a systematic manner suggesting that the channel form fluctuates systematically over time. Similar results were found for ten other sections in the southwestern Coast Mountains.

Downstream hydraulic geometries were determined for Green River and Birkenhead River. Bankfull discharge was assumed to have a constant recurrence interval of 2.33 years for both basins. Channel width shows a greater than usual increase in the downstream direction while velocity appears to remain constant or decrease.

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SYMBOLS USED IN TEXT

A	Channel cross sectional area
a	Coefficient in relation $w=aQ^b$
b	Exponent in relation $w=aQ^b$
c	Coefficient in relation $d=cQ^f$
D_{84}	Grain diameter equal to or larger than 84% of the bed particles
d	Mean depth, A/w
e	Error term of regression equation
f	Exponent in relation $d=cQ^f$
f	Darcy-Weisbach resistance factor
g	Coefficient in relation $A=gQ^h$
h	Exponent in relation $A=gQ^h$
k	Coefficient in relation $v=kQ^m$
M	Rank of each yearly peak flood
m	Exponent in relation $v=kQ^m$
N	Number of years of record in flood frequency analysis
n	Number of observations
Q	Stream discharge
$Q_{2.33}$	Stream discharge with a 2.33 year recurrence interval
R	Hydraulic mean depth
R^2	Coefficient of determination from regression analysis
s	Water surface slope
t	Coefficient in relation $s=tQ^z$
v	Mean velocity
w	Water surface width

y	Exponent in relation $ff \propto Q^y$
z	Exponent in relation $s=tQ^z$
α	Intercept of regression equation
B	Slope of regression equation
6^2	Variance

Chapter 1

RIVER BASIN CHARACTERISTICS

1-1 INTRODUCTION

All natural streams are subject to a wide range of discharges. Since each channel cross section must transmit the water passed into it from upstream, the hydraulic characteristics of the channel must adjust to accommodate each new discharge. The discharge is essentially independent of the stream section while the remaining hydraulic parameters, sediment size and concentration, width, depth, velocity, slope, and roughness are dependent, to varying degrees, on the discharge.

The mutual adjustment of these parameters has been termed the hydraulic geometry of stream channels (Leopold and Maddock, 1953). Though this term was first used by Leopold and Maddock in 1953, similar studies had been carried out for many years before by engineers investigating irrigation canals (cf. for example Kennedy, 1895; Lacey, 1929; Inglis, 1949). The objective of these investigations was to design a canal in alluvial material which was stable; one in which there was no significant scour or deposition for the limited range of flows the canal carried. Such a canal was said to be "in regime".

Leopold and Maddock extended the canal investigations of channel form to natural streams in the midwestern United States. Since their original paper, other hydraulic geometry studies have been carried out in a wide variety of river environments. These studies have found that, though streams carry a wider range of flows than canals, many streams are in regime or in a state of "quasi-equilibrium".

Hydraulic geometry studies concentrate on the mean response of the hydraulic parameters of a stream channel as reflected in the channel form. The processes which lead to the varying channel forms embody complex effects which are not adequately reflected in the mean channel response. The analysis of the stream's hydraulic geometry makes no attempt to enumerate these various processes. However, by looking at the mean response of the channel, some general statements about the channel forming processes can be made. The mean characteristics remain of some scientific and a great deal of practical interest.

This thesis will examine the hydraulic geometry of two streams in the Coast Mountains of southwestern British Columbia. Except for the work by Day (1970), the hydraulic geometry of streams in similar high energy environments has been neglected.

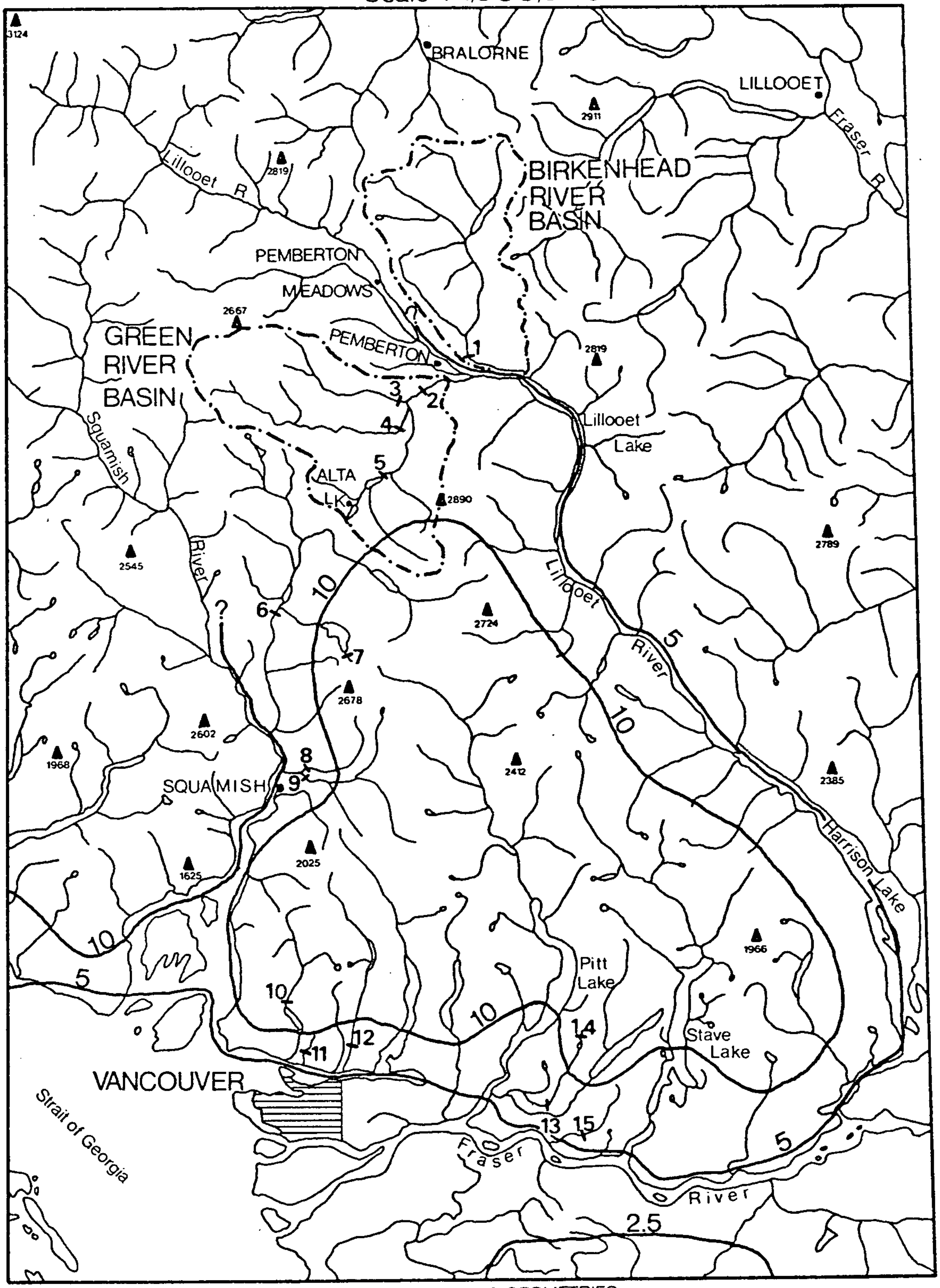
1-2 STUDY AREA

The two river basins chosen for this investigation were Green River basin and Birkenhead River basin. Both basins are located near Pemberton, British Columbia, approximately 145 km north of Vancouver, British Columbia (fig. 1). Birkenhead River drains the region to the north of Pemberton and flows south into Lillooet Lake, 15 km east of Pemberton. Green River drains the area to the southwest of Pemberton and flows northward receiving two major tributaries from the west, Soo River and Rutherford Creek, before it joins Lillooet River 5 km east of Pemberton.

These two basins were chosen because of their accessibility by comparison with the majority of basins in the Coast Mountains. They also constitute part of the larger Lillooet River basin where work on

Figure 1, Location of Study Area

Scale 1:1,000,000



GAUGING STATIONS USED FOR AT-A-STATION HYDRAULIC GEOMETRIES

1 BIRKENHEAD RIVER	6 CHEAKAMUS RIVER	11 MOSQUITO CREEK
2 GREEN RIVER	7 SENTINEL CREEK	12 SEYMOUR RIVER
3 RUTHERFORD CREEK	8 MASHITER CREEK	13 NORTH ALOUETTE RIVER
4 SOO RIVER	9 MAMQUAM RIVER	14 JACOBS CREEK
5 GREEN RIVER AT GREEN LAKE	10 CAPILANO RIVER	15 KANAKA CREEK

ISO-DISCHARGE CONTOURS: MEAN ANNUAL DISCHARGE $\times 10^{-2} (m^3/s)/km^2$

Source: Topographic, N.T.S. 1:1,000,000 map sheet NM-9/10

Iso-discharge Contours after Slaymaker

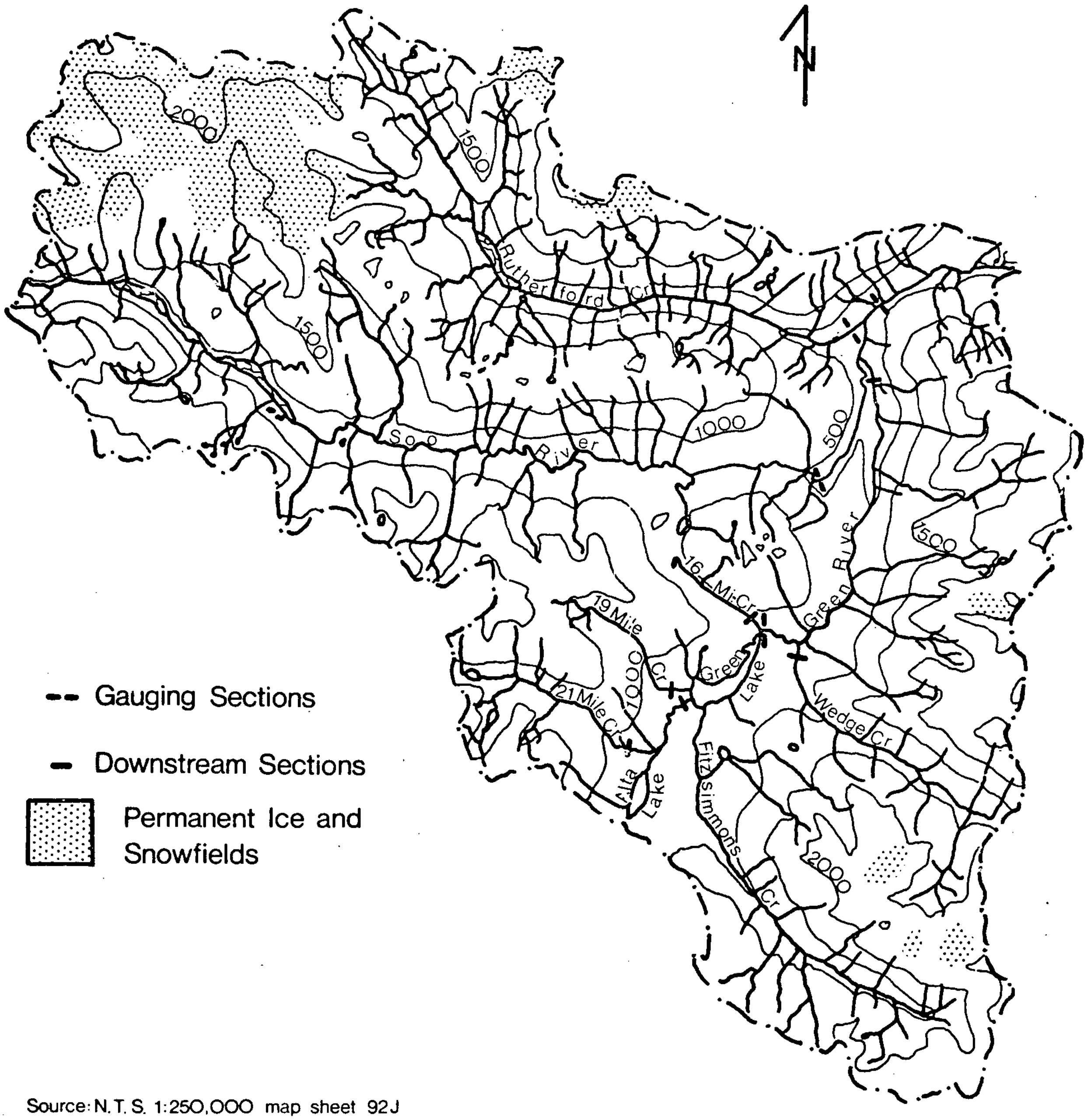
sediment sources and fluvial sedimentation is being carried out (Gilbert, 1972; Slaymaker and Gilbert, 1972). Discharge data were also available for both streams. Four gauging stations have been located in the Green basin at various times (fig. 2). These were established in the mid 1910's and abandoned in the late 1940's. The station on Birkenhead River was established in 1945 and was in operation until 1971 (fig. 3). In addition to the data collected in the main study area, data from ten other gauging stations in the southwestern Coast Mountains were collected in order to examine regional similarities in sectional geometry (fig. 1).

1-3 GEOLOGY

Little detailed geologic work has been done in either basin. From an aerial reconnaissance of the area with occasional sampling of surface exposures, Mathews (personal communication) mapped the bedrock of the region: fig. 4 presents a simplification of his map. A small area of Tertiary and Quaternary volcanics is found in the Birkenhead basin with the rest of the bedrock in the basin being divided about equally between the two major groups. Only about one third of the Green basin is underlain by the metavolcanic and metasedimentary rocks, and none of the more recent volcanics are found. The major glacial valleys in both basins are covered with Quaternary unconsolidated materials.

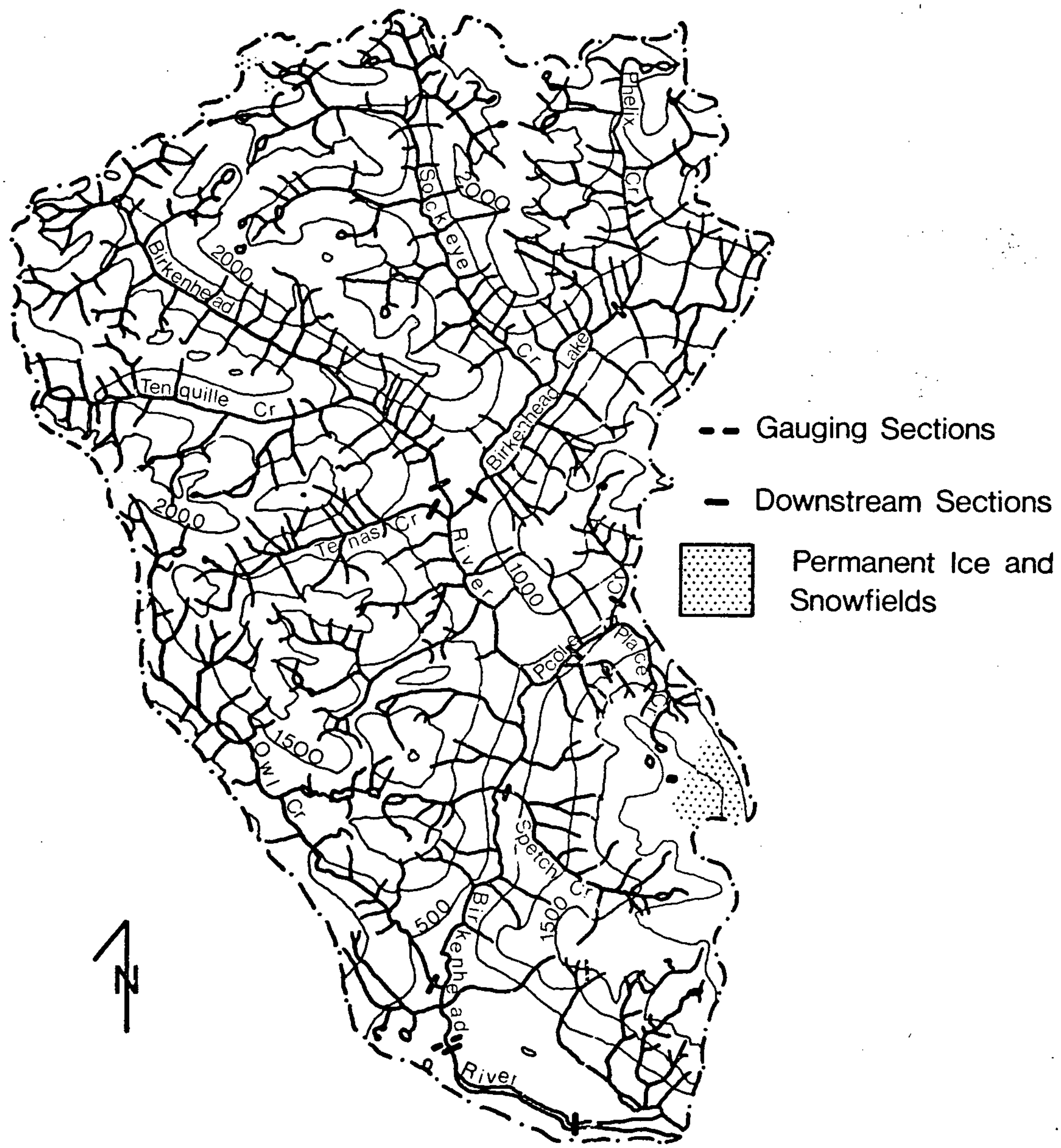
The bedrock geology would appear to have only a minor influence on the streams within the region. Some reaches of the streams appear to flow along contacts between the two major rock types, and a short section of the Birkenhead basin divide follows a contact; however, most streams appear unaffected by changes in bedrock. There are no marked changes in channel pattern as streams cross contacts; the drainage density does not appear to change as the rock type changes; and stream

Figure 2
Green River Basin
Scale 1:250,000
Contour Interval 500 m



Source: N.T.S. 1:250,000 map sheet 92J

Figure 3
Birkenhead River Basin
Scale 1:250,000
Contour Interval 500m

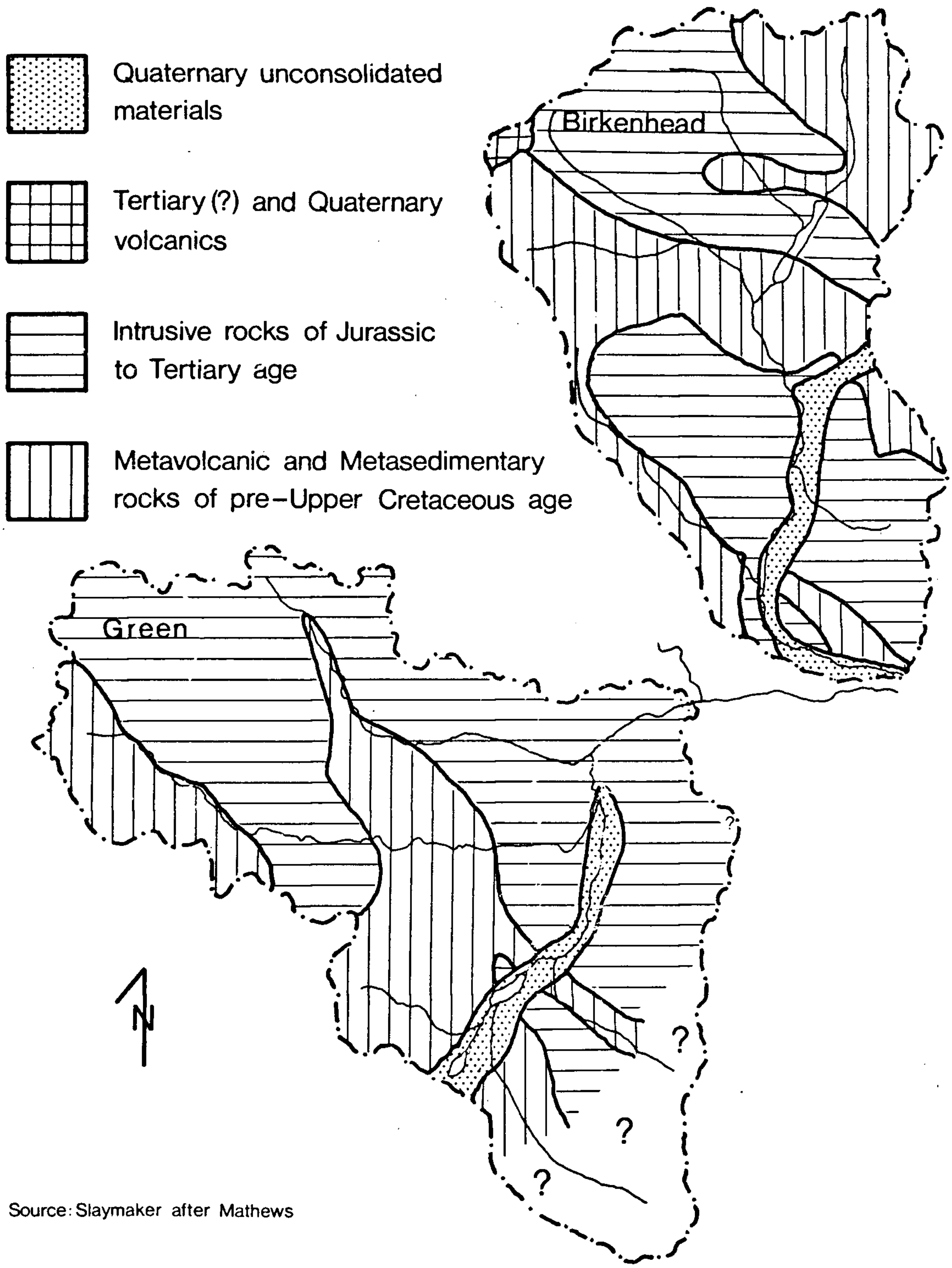


Source: N.T.S. 1:250,000 map sheet 92J

Figure 4

Geology of the Green and Birkenhead Basins

Scale 1:400,000



Source: Slaymaker after Mathews

patterns are not obviously related to the structure of the area.

Glaciation has left the most marked imprint on the character of the drainage within each basin. Mathews (1958), working in the Mount Garibaldi area 15 km to the southwest of the Green basin, found that the maximum height of the Vashon glaciation was approximately 2100 m. The Vashon Stade began 25,000 B.P. and the Coast Mountains appear to have been continuously glaciated until the final retreat 10,000 B.P. (Ryder, 1972). Only traces of this last regional glaciation are found, though well developed cirques found below the maximum height of glaciation may pre- or post-date the Vashon maximum.

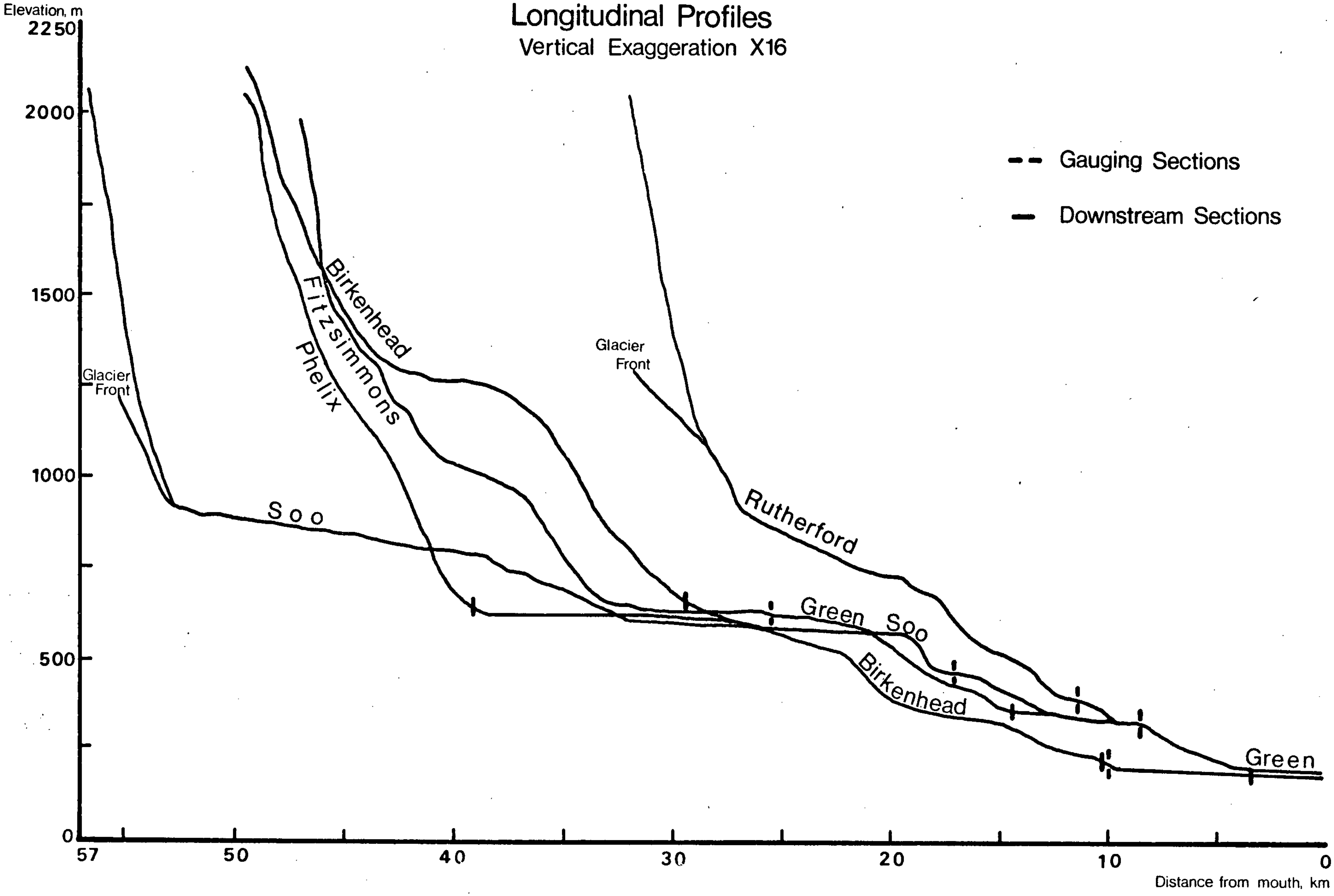
In the lower reaches of the four major streams in the Green and Birkenhead basins, Green, Soo, Rutherford, and Birkenhead, water falls are found as well as canyon like sections. These features are an indirect result of glaciation and are also found on many of the smaller tributaries. Hanging valleys are another feature common to the region.

The effect of glaciation on the longitudinal profiles of the major streams is seen in fig. 5. Soo, Green, and Birkenhead all have low slopes at approximately 650 and 850 m with a sharp increase in slope below this level. Lower slopes along Green and Birkenhead Rivers also appear at approximately 500 m. Mathews (1958) noted a similar stepped profile for Cheakamus and Monmouth valleys. The sharp breaks between the relatively smooth sections of the profiles do not appear to be related to changes in bedrock. The sharp decrease in slope at the mouth of both Green and Birkenhead Rivers is a result of the post-glacial valley fill of Lillooet River. The other breaks in slope may be a result of base level control on the depth of erosion of the tributary glaciers by the main valley glaciers, but this is not clear. Whatever

Figure 5

Longitudinal Profiles

Vertical Exaggeration X16



the precise origin, it is clear that the rivers do not control the gradient of their long profiles. A similar stepped pattern can also be seen in the hypsometric integrals (fig. 8).

Drainage divides at the heads of the major glacial valleys in both basins are poorly defined. A swamp at the southwestern end of Alta Lake acts as the divide at the head of the main glacial valley in the Green basin. Place Creek, which enters the main glacial valley of the Birkenhead basin very near the divide, appears to have flowed into the adjoining basin at one time: abandoned channels flowing towards Gates Lake in the adjoining basin are still apparent. Though fluvial modifications have taken place, the glacial topography still dominates the streams within both basins.

1-4 MORPHOMETRY

Following the techniques outlined by Strahler (1952, 1956, and 1957), stream order and lengths in each basin were recorded (Table I). Basin area, area covered by permanent ice and snowfields, and basin perimeter were measured. A random sample of 110 points within the Birkenhead basin and 150 points within the Green was taken, and at each point elevation and aspect were recorded. The 150 points within the Green basin were divided proportionately among the three subbasins for which there were also discharge records. In the Soo basin, 50 random points were sampled; 38 points were sampled in the Rutherford and Green at Green Lake basins and 24 in the remaining area.

All of the data were collected from the N.T.S. 1:250,000 topographic map with a contour interval of 500 ft. This is the only map which covers the entire area; however, drainage on this map is shown in considerably more detail than on the 1:50,000 provisional N.T.S.

TABLE I

SUMMARY OF MORPHOMETRIC DATA

		Birkenhead	Green	Soo	Rutherford	Green at Green Lake
Basin area, km^2		656	841	268	180	171
Glacial area, km^2		4.8	70.8	31.7	28.0	5.4
Percent glacierized		0.7	8.4	11.8	15.5	3.2
Perimeter, km		115	155	91	73	68
Number of streams	order 1	380	245	69	83	35
	2	85	58	16	20	10
	3	24	8	3	2	2
	4	4	3	1	1	1
	5	1	1			
Mean stream length, km	order 1	0.92	1.21	1.45	0.96	1.43
	2	1.65	2.13	1.72	1.81	2.11
	3	3.02	6.56	5.83	5.00	11.25
	4	8.28	19.38	28.13	16.25	
	5	31.87	8.34			
Geometric mean bifurcation ratio		4.42	3.96	4.10	4.36	3.27
Drainage density, km/km^2*		0.96	0.70	0.73	0.94	0.57
Circularity ratio		0.623	0.440	0.407	0.424	0.465
Relief, m		2305	2680	2255	2286	1990
Mean elevation, m		1390	1385	1387	1442	1420
Hypsometric integral		0.522	0.466	0.477	0.538	0.391

* Total length of streams/unglacierized area.

maps. Gilbert (personal communication) was able to determine from sampling air photos and some ground reconnaissance that first order streams on the map were probably no greater than second order. If this is the case, the unusually low drainage density (Table I) suggests that much of the runoff occurs as interflow or in unconcentrated flow over bedrock. The high cover of permanent ice and snowfields also influences the drainage density, but the data quoted in Table I applies only to areas of the basin not covered by permanent ice and snowfields. The area of permanent ice and snowfields in the Birkenhead basin is approximately 3.8 sq. km greater on the provisional maps than on the N.T.S. topographic map. Though this is a large percentage increase, the actual increase is small and it likely indicates the order of magnitude by which the area of permanent ice and snowfields in the other basins would increase with larger scale maps.

The Birkenhead system seems to follow the laws of stream order and stream length reasonably closely (figs. 6, 7); however, Green River and its sub-basins do not appear to follow these relations as closely. This is likely a result of the larger glacial area. A stream coming from a glacier can only be counted as a first order stream though it may be draining an area which could support a second or third order stream. Though the geology of the Birkenhead basin is more varied, there does not appear to be a change in drainage density between the two major rock types (sec. 1-2). The geometric mean bifurcation ratios are all close to the random value of four predicted by Shreve (1966) and also tend to indicate a lack of strong structural control.

Though the maximum relief in the Birkenhead basin is lower than in the Green basin, its mean elevation is slightly greater and the hypso-

Figure 6
Number of Streams to Stream Order

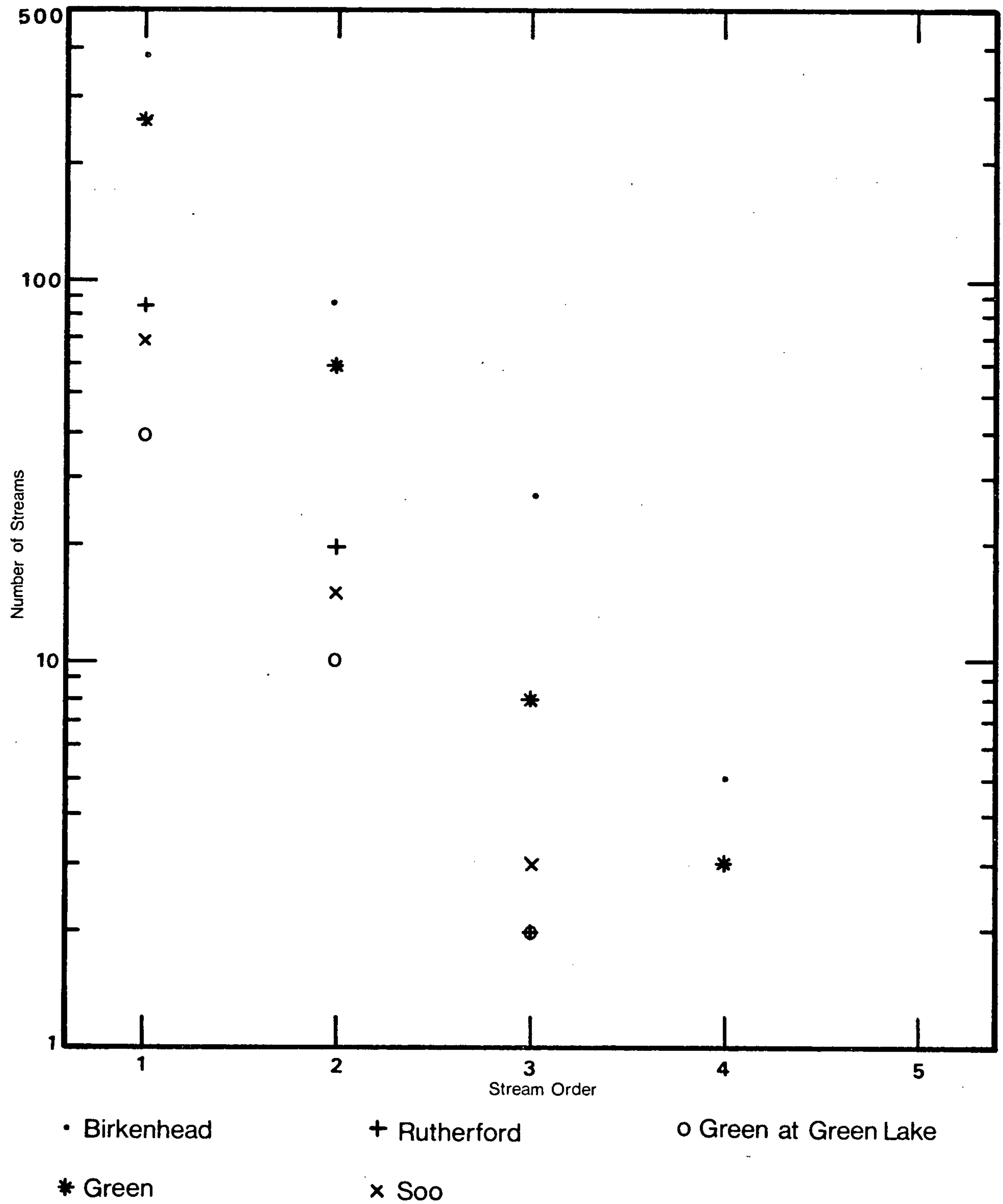
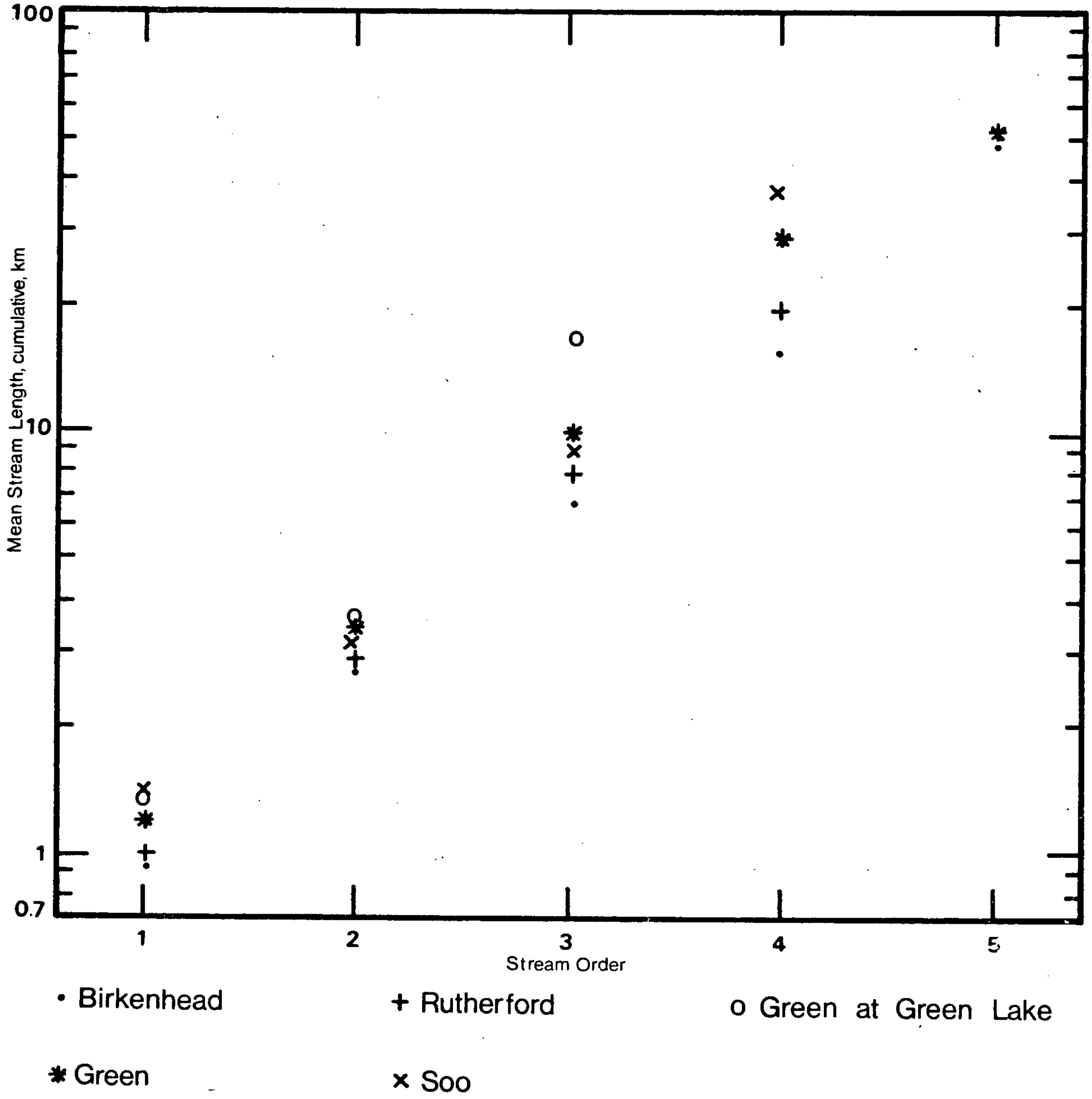


Figure 7
Mean Stream Length to Stream Order



metric integral is larger. The large difference in maximum relief in the two basins is due to Wedge Mountain which, at 2900 m, is considerably higher than other peaks in the area. This high point also tends to make the hypsometric integral slightly atypical, but the lower integral and lower mean elevation of the Green do reflect the larger area covered by lower glacial valleys. Slopes with a southern aspect are more frequent in the Birkenhead basin while northern slopes are dominant in the Green (fig. 8). This leads to a more rapid snow melt in the Birkenhead basin (Table III, fig. 9a). The more nearly circular shape of the Birkenhead may allow a more rapid concentration of storm and meltwater runoff.

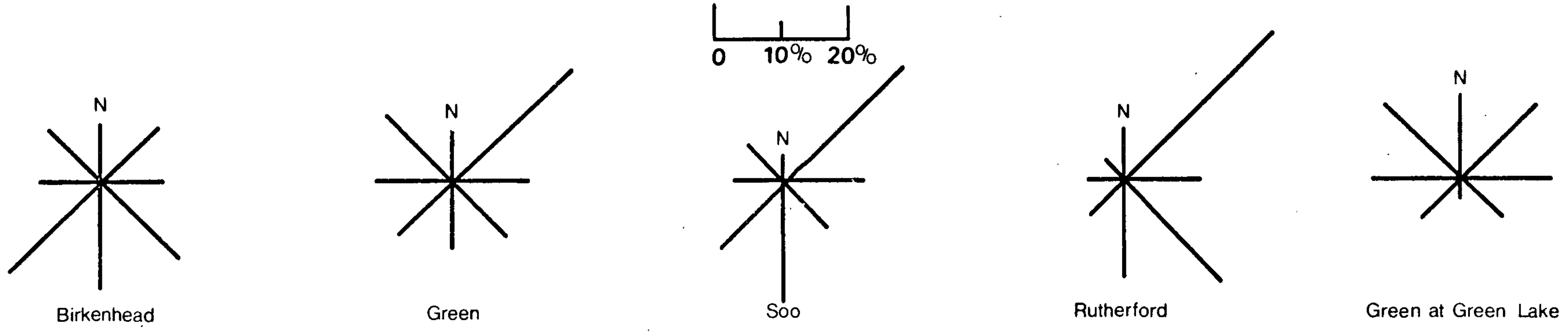
1.5 PRECIPITATION

Because of the mountainous terrain of the two basins, precipitation can be expected to vary with elevation and exposure throughout each basin. A general trend of decreasing precipitation from western to eastern borders of the basins should also be expected due to the dominant west to east storm path and the orographic effect of the mountains. Only one weather station, Alta Lake, is located in the study area, but two others have been in operation nearby (fig. 1). There is a trend for a decrease in precipitation from west to east (Table II). However, the data are so limited that the whole area must be regarded as hydrologically homogeneous for discharge purposes (c.f. ch. 4).

Estimated mean annual precipitation in the Green basin ranges from 750 mm to a high in the mountains which may exceed 3750 mm while the maximum precipitation in the Birkenhead basin probably does not exceed 2500 mm. The majority of the precipitation falls between October and February as snow (Table II).

Figure 8, Morphometry

Slope Orientations



Hypsometric Integrals

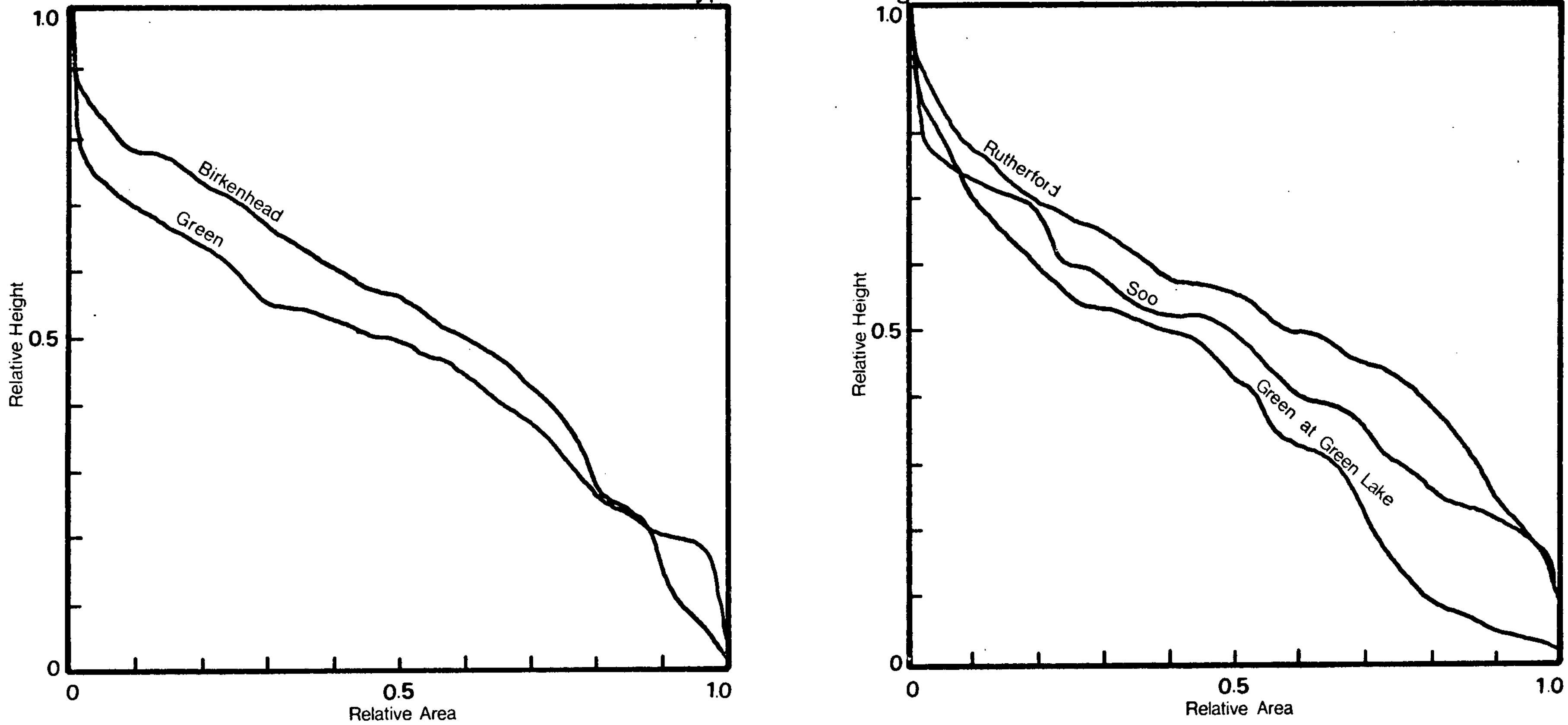


TABLE II

MEAN MONTHLY PRECIPITATION (mm) AND TEMPERATURE (°C) AT THREE SELECTED STATIONS NEAR THE STUDY AREA[1]

	Alta Lake			Pemberton Meadows			Bralorne		
	Precip.	%Precip.	Temp.	Precip.	%Precip.	Temp.	Precip.	%Precip.	Temp.
January	223.0	15.4	-4.3	122.9	13.3	-4.8	65.0	9.9	-7.5
February	162.5	11.3	-2.3	102.1	11.0	-1.7	54.6	8.3	-4.3
March	114.8	7.9	0.4	67.6	7.3	3.3	44.5	6.8	0.6
April	92.2	6.4	4.9	37.9	4.1	8.8	32.3	4.9	4.4
May	47.8	3.3	8.9	32.8	3.5	13.1	32.3	4.9	8.7
June	58.9	4.1	12.7	27.7	3.0	16.2	38.6	5.9	12.0
July	30.5	2.1	14.9	22.9	2.5	18.1	29.7	4.5	14.4
August	52.6	3.6	14.5	27.9	3.0	17.3	30.0	4.6	13.8
September	89.9	6.2	11.9	59.7	6.4	13.4	40.4	6.2	10.7
October	169.9	11.8	6.3	125.7	13.6	7.9	79.5	12.1	4.7
November	185.2	12.8	1.2	126.8	13.7	2.1	95.5	14.6	-1.3
December	217.7	15.1	-1.7	173.0	18.7	-2.6	112.3	17.2	-6.1
Total	1445.0	100.0		927.0	100.0		654.0	100.0	
Mean	120.4		5.6	77.3		7.6	54.6		4.2

1 Normal precipitation and temperature for the period 1931 - 1960 as computed by the Department of Transport Meteorological Branch in the Monthly Record.

TABLE III

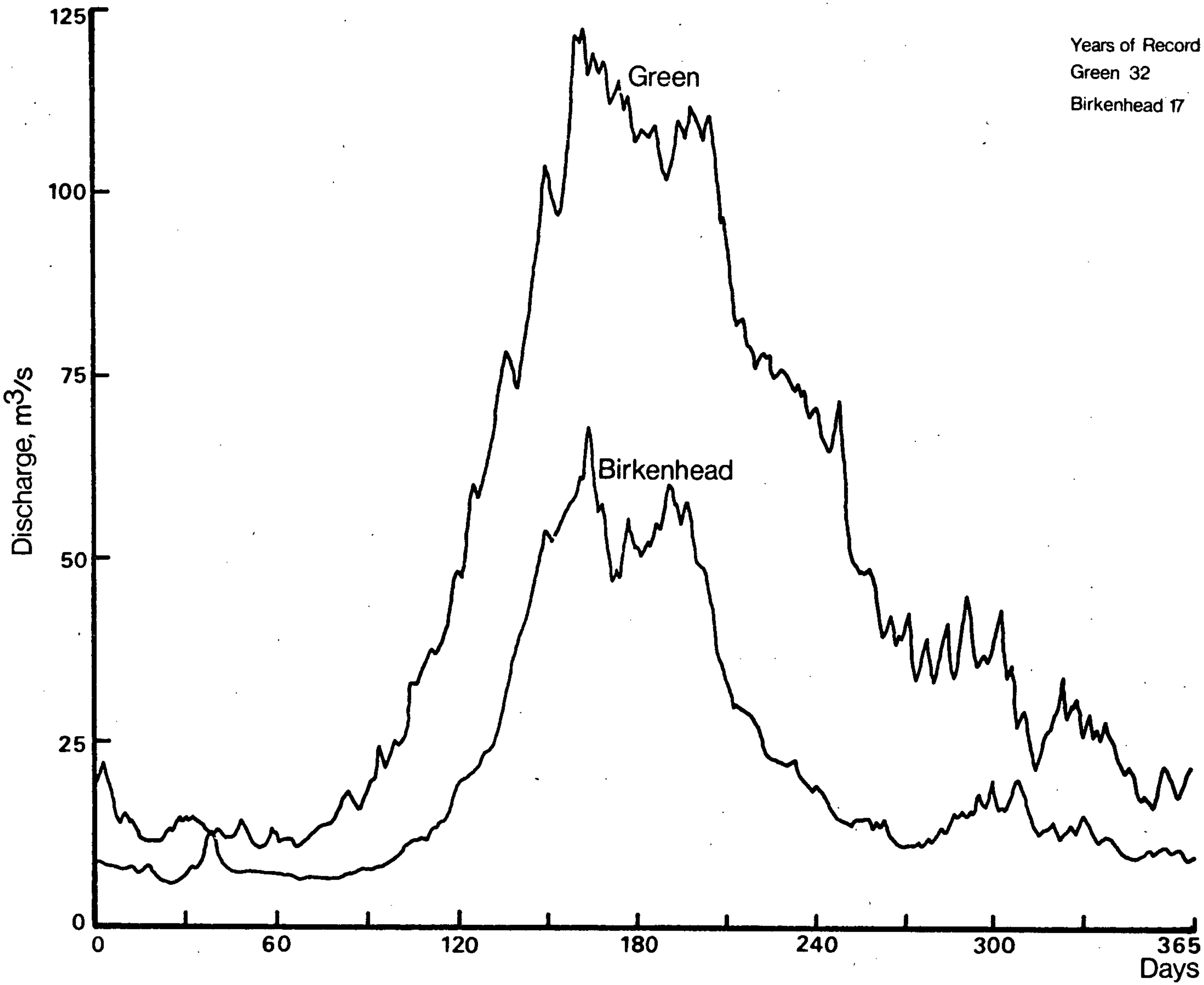
MEAN MONTHLY DISCHARGE PER SQUARE KILOMETER[1]

	Birkenhead	Green	Soo	Rutherford	Green at Green Lake
January	11.9	18.5	19.7	18.0	18.0
February	13.5	16.2	19.0	18.9	17.1
March	11.3	17.9	20.5	17.2	17.9
April	21.5	39.6	51.6	37.4	36.5
May	67.6	93.9	125.2	95.4	83.1
June	110.3	141.2	167.1	154.1	114.7
July	86.9	135.3	157.8	154.5	102.9
August	40.9	94.7	116.4	94.4	54.0
September	25.3	61.1	81.8	64.9	32.9
October	24.3	50.1	71.2	59.5	37.7
November	23.1	33.6	43.5	32.2	28.3
December	17.7	25.8	31.4	30.8	26.2

1 Discharge is in $m^3/sec \times 10^{-3}$.
Underlined discharges are not significantly different from the similarly underlined values of that month at the 5% level when Duncan multiple range tests are applied to the data.

Figure 9a

Mean Daily Discharge



1-6 DISCHARGE

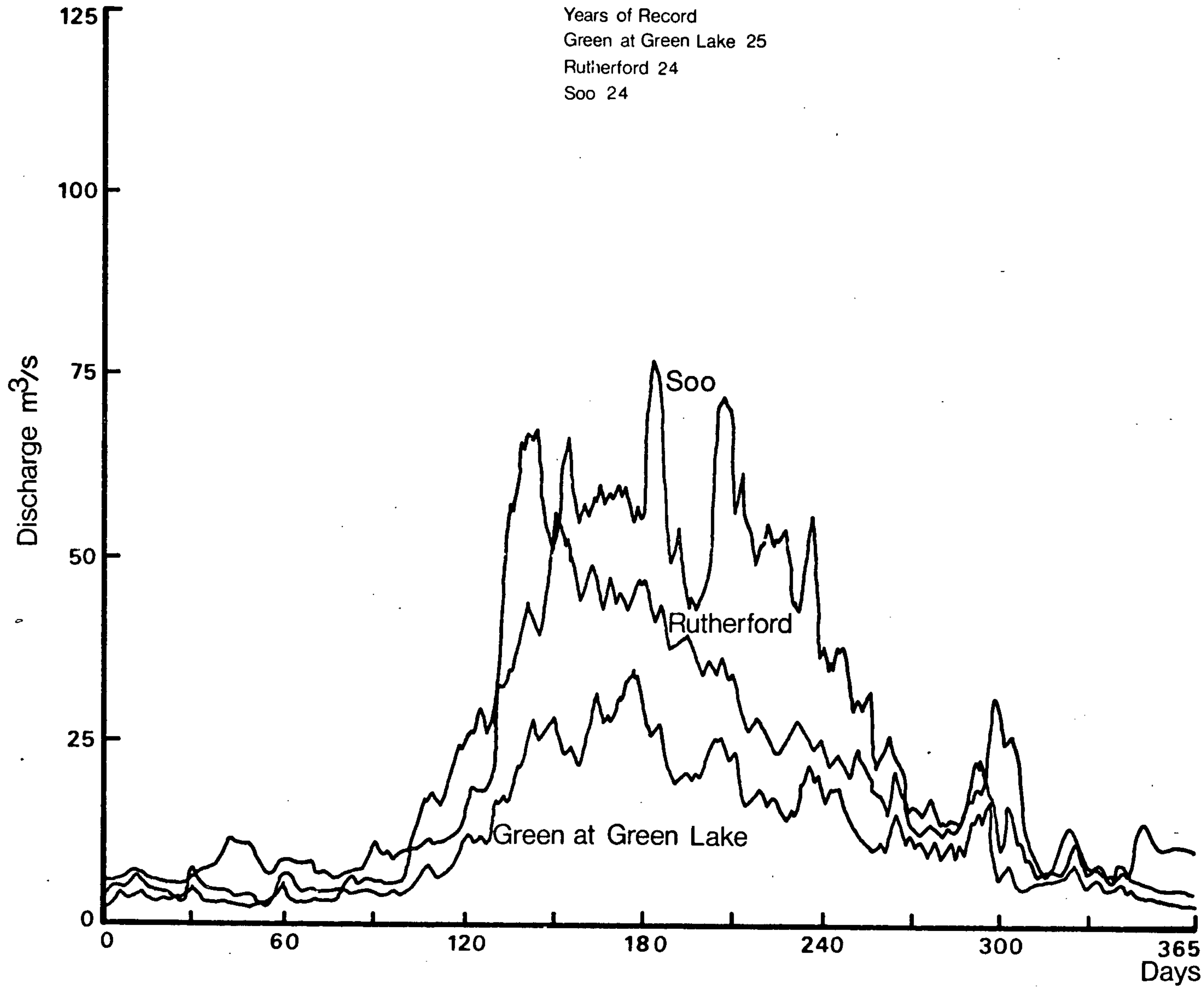
The discharge of the streams in the two basins follows a yearly regimen which would be expected from the nature of the precipitation input and the temperature regimen (figs. 9a, b). Minimum discharge is recorded during the late winter and maximum flows are generally associated with periods of maximum snow melt in the late spring and early summer. Periods of high flow also occur during the autumn though the general trend is for decreasing discharge. These autumn peaks are associated with warm rain storms from the coast which bring heavy precipitation to the area and melt much of the snow that has already accumulated. Storms of this type account for the peak individual flows recorded in each basin. It would appear that these storms are more intense in the western basins for over half of the yearly peak flows in the Soo occur during the autumn while less than one fifth of the Birkenhead yearly peak flows have occurred at this time of year.

Except for Green at Green Lake, the frequency distribution curves of the peak discharges for each basin are similar for the more frequent events (fig. 10). The lower slope of the curve for Green at Green Lake results from the damping effect of Green Lake on the magnitude of peak flows. The Birkenhead has a steeper slope for extreme events which may be related to its more circular shape: it does appear as if the more circular basins have a steeper slope for extreme events.

The mean monthly discharge per km^2 was computed for each basin (Table III). Soo River has the largest discharge intensity in each month while the Birkenhead has the lowest. This may reflect the different period of record for each basin; but if it is assumed that no trend exists, this decrease in discharge from west to east reflects

Figure 9b

Mean Daily Discharge



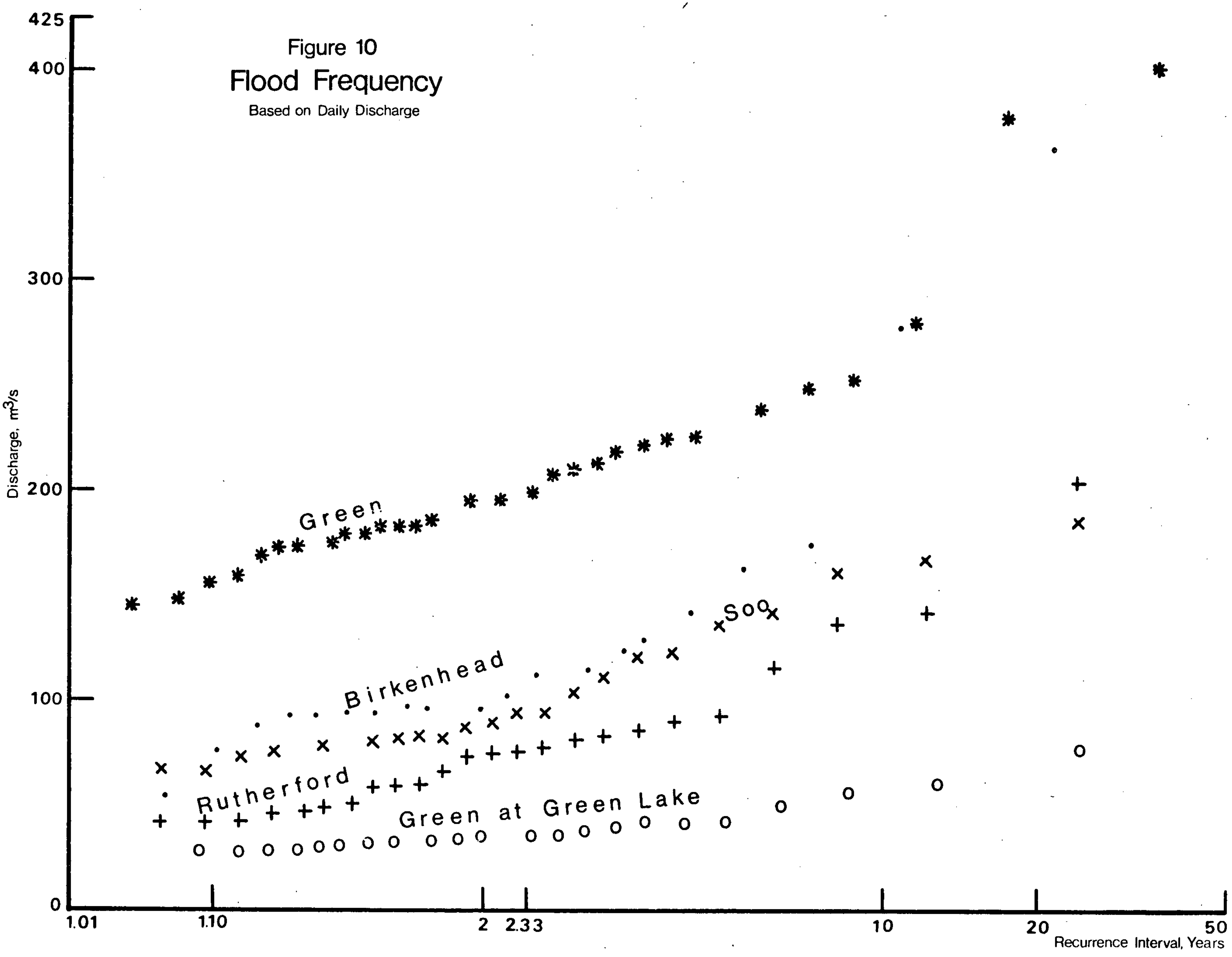


Figure 10
Flood Frequency
Based on Daily Discharge
Discharge, m3/s
425
400
300
200
100
0
Green
Birkenhead
Soo
Rutherford
Green at Green Lake
1.01
1.10
2
2.33
10
20
50
Recurrence Interval, Years

the decrease in precipitation. Birkenhead River has a sharper peak in maximum discharge in June which may reflect the more rapid snow melt, smaller accumulation of snow, and smaller glacial area. Only Rutherford Creek has an increase in discharge intensity between June and July reflecting its greater glacial cover.

Chapter 2

DATA COLLECTION AND FIELD TECHNIQUES

2-1 INTRODUCTION

The mutual adjustment of the dependent hydraulic parameters to the varying discharges of a stream section may be considered in two different ways. The discharge of a channel section will vary through time; it will also vary with the location of the channel section within the channel system. The relationships between discharge and the dependent parameters as discharge changes through time at a given channel section are termed at-a-station hydraulic geometry (Leopold and Maddock, 1953). The relationships which consider the adjustment of the dependent parameters with increasing discharge downstream are termed downstream hydraulic geometry.

Since there are two distinct types of hydraulic geometries for stream channels, two different groups of data need to be collected for a complete analysis (cf. Leopold and Maddock, 1953). Ideally the sections used for the downstream analysis would also be used for the sectional analysis so that the sectional and downstream hydraulic geometries would be integrated into a "three dimensional" picture of channel adjustment. However, this ideal approach is often impractical.

The measurements made at each section can be approached in two ways. The reach in which the section is located can be considered homogeneous, and a number of measurements along the reach are made so that the average sectional form of that reach is determined. Such an approach minimizes the effect of chance occurrences such as a tree fall or a bank slump on the sectional geometry; however, only the average character of the reach will be recorded. The movement of a bar through the reach would not be noticed. The second approach considers the section as a two dimensional transect across the river. The movement of a bar through this section would be detected, but the section may not be representative of that portion of the river system due to some chance occurrence which had modified the two dimensional transect.

The second approach was the one chosen for this thesis. Data collected at gauging stations with cableways is of the two dimensional type. Since the data for the at-a-station analysis came from gauging records, the second approach seemed the most suited. However, distinct problems do arise when cableways are not exclusively used (sec. 2-2). By adopting the two dimensional approach, special care had to be taken in the choice of the location of the downstream sections; but less time was needed to measure each section. The pool and riffle sequence is a distinct feature of all streams in the study area, and it would be meaningless to compare the form of pools in one section of the system to the form of riffles in another section. An attempt was made to measure all downstream sections at a point which was transitional between a pool and riffle. An attempt was also made to choose sections which, from visual inspection, appeared to be representative of the channel system in that area.

2-2 SECTIONAL DATA COLLECTION

No data for the determination of at-a-station hydraulic geometries were collected by the author. However, data were available for five sections within the basins; and on the basis of this data five sets of at-a-station hydraulic geometries were determined. These five sections were gauging stations of the Water Survey of Canada (figs. 2, 3); here a large number of measurements of some of the hydraulic parameters at varying discharges exist. Several times each year during the operation of each station width, mean velocity, and cross sectional area were measured, and the discharge was computed. The number of observations range from 97 at Rutherford Creek to 140 at Green River (Table V). Measurements of channel slope and bed material size were made by the author.

Data gathered at a gauging station may not be the best suited for the construction of sectional hydraulic geometry; however, the length of record will document systematic changes in the channel form if they exist. Since the data were collected by the Water Survey in order to establish the gauge height-discharge relationship, the exact cross section at which the measurements were made was not critical. This may mean that some measurements were made at slightly different points along the channel from others, and this may account for some of the variance of the data (Wolman, 1955). This error will be greatest for streams which may be waded for slightly different sections will be more suited for measurement at different discharges.

From the Water Survey station descriptions, this error is most likely to be found in the data for Green River at Green Lake. Measurements at this site were taken by wading and from a cableway. The Birkenhead section may also be affected by this since some wading measurements were made; however, it appears that most measurements at the site were made from a cableway. Measurements at Soo and Green were taken from a cableway while those at Rutherford appear to have been taken from a bridge.

In addition to these five sections, data for ten other gauging stations in the southwestern Coast Mountains were obtained from the

Water Survey of Canada (fig. 1, sec. 3-4). An attempt was made to select stations which represented varying basin sizes and at which a reasonably large number of observations had been made. Unfortunately the selection was very limited. The number of observations range from 17 for Mamquam River to 123 for Seymour River (Table VI). The smaller streams (Sentinel and Jacobs Creeks) are waded while cableways are used to make measurements of the larger streams (Seymour, Capilano, Mamquam, and Cheakamus Rivers, and Mashiter Creek). The other sections are measured by wading at low flows and from cableways or bridges at high flows. No additional data on slope or material size was collected at these sites.

2-3 DOWNSTREAM DATA COLLECTION

The data used for the computation of the downstream hydraulic geometries was collected by the author during the summer of 1970. Measurements were made at sixteen sections, six on Green River and its tributaries (figs. 2, 3). The sections measured were chosen in an attempt to represent a variety of basin sizes. However, accessibility proved to be a major limiting factor in the choice of the actual field sites. Except for the portion of the river between the gauging station and the confluence with Green River, Soo River was completely inaccessible. The discharge along this accessible portion generally makes wading impractical. Some of the upstream portions of Rutherford Creek are accessible during the summer, but only when the discharge precludes wading. Sockeye Creek, which flows into Birkenhead Lake, was accessible during the summer, but the velocity of the stream made wading impossible. Some other accessible sites could not be used because of man made modifications of the channel.

At each measurement site bankfull width, bankfull sectional area, and water surface slope were measured and a sample of the bed and bank material was taken. Bankfull flow was chosen as the discharge to be considered at each section because of its reasonably constant recurrence interval (sec. 4-2) and because of the clear evidence of this flow that is generally left throughout the channel system (cf. Leopold and Skibitzke, 1967). The value of bankfull discharges at each site was derived from the relationship between basin area and discharge. The bankfull mean velocity was computed from the continuity equation, v=Q/A.

The cross section measured at each of the sixteen sites was at a point which was transitional between a pool and a riffle. This was done so that the variation in the dependent hydraulic parameters due to the pool and riffle sequence would be removed from the data. The height of the bankfull flow along each bank was determined independently from evidence such as debris and changes in vegetation. In all cases the two points were within 15 cm of the same height. It was assumed that the straight line which connected these two points and which was perpendicular to the direction of flow represented the bankfull water surface and the bankfull width. This distance was measured using a metric tape. The distance from this line to the bed of the stream was then measured with a Philadelphia rod at a number of points. The number of depth measurements and their spacing varied with the width of the section and the intricacy of the bottom topography: the average number of measurements was approximately thirty. Because of the generally large size of the bed material, it was assumed that extensive scour did not take place at higher flows. These measurements and the bankfull width were used to compute the bankfull cross sectional area. Mean bankfull depth was then defined as d=A/w, where A is the channel cross sectional area and w is the water surface width.

Water surface slope was measured over a distance of eight to ten times the bankfull width. Though slope does change as discharge increases, this change is small (Wolman, 1955), and the mean water surface slope over this distance at lower flows may be assumed to be equal to that of bankfull discharge. Slope changes in this area are likely to be even less significant for the channel slope is at least partially imposed by the glacial topography. At low flows, however, there is a greater difference between the water surface slope of pools and riffles than at higher flow when the slope of the pools increases while that of the riffles decreases giving a more uniform slope to the whole reach. By measuring the average slope over the suggested distance of eight to ten times the bankfull width, at least one pool and riffle sequence will be included so that the average slope at bankfull will be obtained (Leopold and Skibitzke, 1967). The slope was surveyed using a theodolite and the Philadelphia rod.

The sample of sediment size at each site was made by stretching the metric tape diagonally from one bank to the other. Generally this line was over a distance of 100 meters and an arbitrary sample interval along the tape was chosen (usually one meter). The intermediate axis of the particle directly under each sample point on the tape was measured. One hundred measurements were made at each site.

No attempt was made to determine the error introduced into the measurements by the field methods used. Instrument errors should be very small, and the major sources of error are due to the assumptions and estimations which must be made. Miller (1958) made successive measurements of bankfull width and mean bankfull depth along straight sections of a stream with no tributaries at low flow. He found that the standard deviation of the measurements was as great as 15% of the mean width and 22% of the mean depth. These measurements, however, were apparently made without regard to their position within the pool and riffle sequence. The pool and riffle sequence will account for a portion of the variance found by Miller. By measuring the section at approximately the same point within the sequence, the error should be decreased. In absence of better information, Miller's results can be viewed as a probable upper limit of the error.

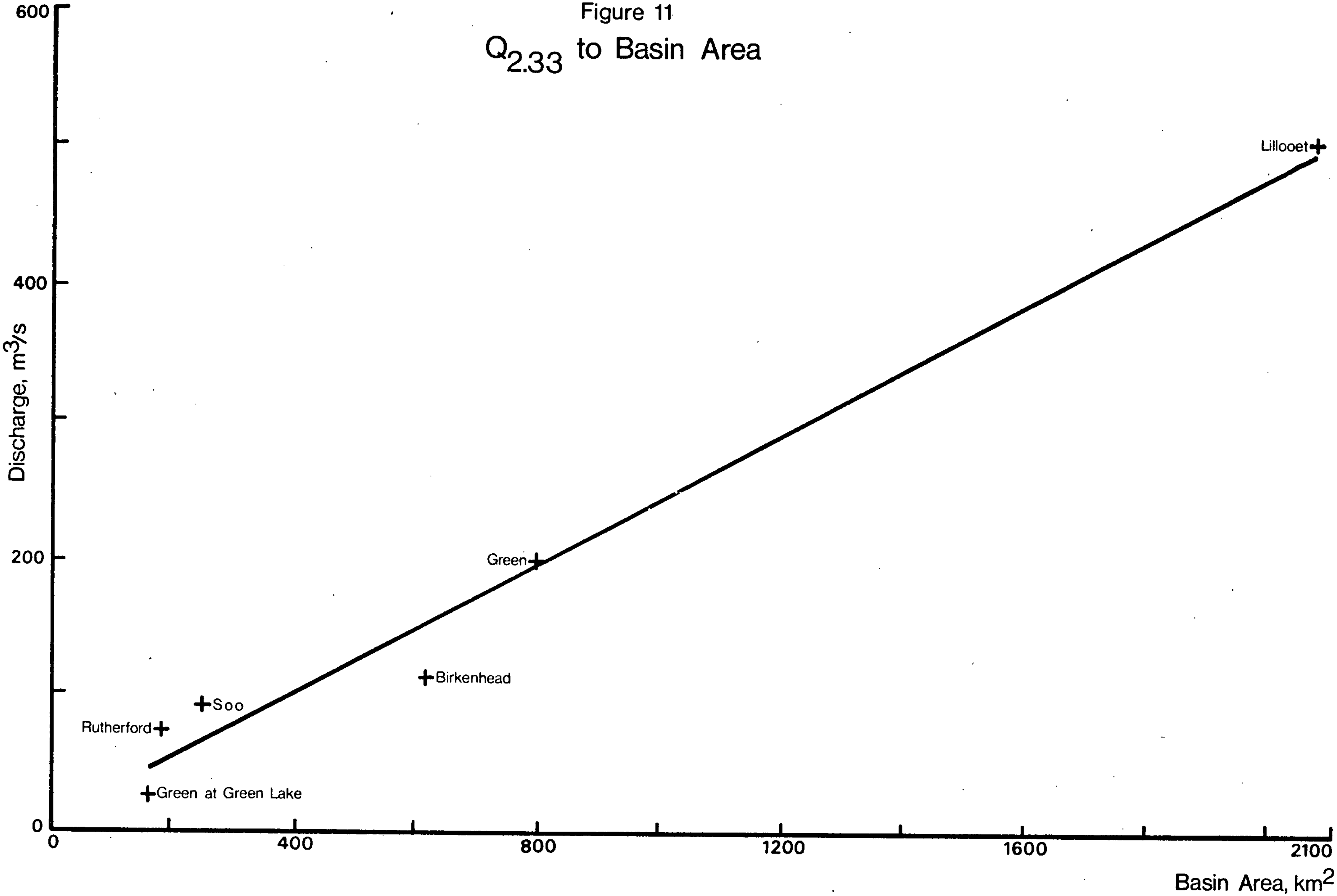
The bankfull discharge was not actually measured at any of the sites. The five gauging stations within the basins and a sixth on Lillooet River were used to establish a relationship between basin area and bankfull discharge. For this computation, the assumption that each basin was hydrologically similar had to be made. Flood recurrence intervals were calculated from the series of annual peak discharges using the formula

$$\text{recurrence interval} = N+1/M$$

where N equals the number of years of record and M is the rank of each annual peak within the series (fig. 10). Least square regression was used to fit an equation to the data which related bankfull discharge to basin area (fig. 11). Though other workers (cf. Miller, 1958; Brush, 1961) have found that a power law relationship gives the best fit to the data, a linear relationship was found to give a better fit for this area. Though R^2 for the regression was very high (0.98), the standard error of estimate was 28.55 and the predicted values from the equation deviated by a maximum of 30% (Birkenhead River) from the observed values. This is due in part to the very small number of sample points and the assumption of hydrologic similarity. The deviations of the observed

Figure 11

$Q_{2.33}$ to Basin Area



values from the regression equation reflect the hydrologic conditions of each basin (Tables II, III; figs. 9a, b) and the deviation of the estimated discharges at each site from the actual bankfull discharges may be as great as the deviations of the observed values.

In view of the variation in the frequency of bankfull flow, the exact recurrence interval of bankfull flow in this area is not known (sec. 4-2). However, the similarity of the flood frequency curves (fig. 10) indicates that the recurrence interval is approximately the same for all stations. Birkenhead River at the gauging section was observed to be very near bankfull on one occasion. The discharge at that time equalled a flow with a recurrence interval of approximately 2.33 years. On the basis of this observation and the frequent use of $Q_{2.33}$ as bankfull by other authors, a flow with this recurrence interval was chosen for the downstream hydraulic geometry analysis. Because of the lack of data, no rating curves could be constructed for the sites in order to check this value of bankfull recurrence nor could any other check be used.

2-4 DATA ANALYSIS

In the past it has been generally assumed that in self-formed alluvial channels the hydraulic geometry relations are power laws (e.g. $\rho=\mu\theta^{\beta}$) (Leopold and Maddock, 1953). For this reason, logarithmic transformations of the data were made and least square regression was used to relate the dependent parameter to discharge. The significance of the correlation coefficient and the B coefficient (the exponent in the hydraulic geometry equation) was tested using the table of significant values of the correlation coefficient from Yamane (1967) and the and the F ratio respectively. That the relationships do not follow power laws is a possibility, and the assumption that they do is a bias that is reflected in this thesis. A number of linear relationships were also computed using the regression model, and in well over 80% of the cases the power law yielded a higher degree of explanation. That yet another relationship exists is a possibility which was not investigated.

It should also be noted that many of the channels in this area are not self-formed alluvial channels but have fixed boundaries. The material which forms the channel boundaries would be placed in motion only during exceptionally high flows. This is likely the case for ten of the downstream sections and at least two of the gauging sections, Rutherford Creek and Green River. It may also hold for Birkenhead River and Green River at Green Lake. At the other sections it is quite likely that the slope is at least partly imposed. Channels with fixed boundaries may not adjust according to a power law though it would appear from the analysis of the data that they do.

2-5 REGRESSION ANALYSIS

Least square regression is used throughout this thesis to determine the power equation which best fits the data. This technique can be viewed as simply a means of fitting an equation to data or as a statistical technique from which certain inferences about the data may be made. Until recently few workers have attempted to do more than determine the equation which best fits the data by using "visual regression": the line which appears to best fit the data is simply drawn through the scatter of data points. As the data scatter increases, the chance of drawing by eye the line to give the equation which minimizes the variance decreases (c.f. Table IV, Miller's data). If the worker has a preconceived bias of

TABLE IV

A COMPARISON OF HYDRAULIC GEOMETRY EQUATIONS DERIVED FROM LINES FITTED BY HAND AND BY LEAST SQUARE REGRESSION[1]

Stream	a	b	R^2	c	f	R^2	k	m	R^2
Miller[2], Pecos River, hand fitted	1.20	0.59		0.38	0.30		2.10	0.13	
regression equation	0.27	0.58	0.85	-0.78	0.23	0.50	6.61	0.25	0.01
Brush[3], Little Juniata, hand fitted	1.15	0.55		0.28	0.38		2.60	0.07	
regression equation	1.29	0.53	0.98	0.25	0.39	0.99	3.17	0.07	0.30
Brush, Standing Stone, hand fitted	0.41	0.65		0.42	0.25		4.30	0.10	
regression equation	0.50	0.63	0.98	0.39	0.27	0.79	4.52	0.12	0.22
Brush, Shaver, hand fitted	1.50	0.47		0.29	0.34		1.90	0.21	
regression equation	1.49	0.47	0.95	0.31	0.33	0.81	2.24	0.20	0.55

[1] $w=aQ^b$

$d=cQ^f$

$v=kQ^m$

[2] Miller, 1958

[3] Brush, 1961

what the equation should be, it may be reflected in the line he draws to describe the equation. By using least square regression to determine the equation, this possibility of worker bias is removed, and the equation will minimize the squared deviations of the data from the regression line.

The least square regression model is also a statistical technique from which inferences may be drawn, but as such it is only as powerful as the degree to which the data used fit its underlying assumptions. The general form of the model is $Y_i=\alpha+BX_i+e_i$. The X_is are observable fixed independent variables which can be measured without error, α is an unknown location parameter, and B is an unknown parameter reflecting the effect of X. Y_i is a random dependent variable associated with X_i, and e_i is an unobservable random variable with a mean of zero and a variance of 6^2. This variance remains constant for all X_i, and e_i and e_j are uncorrelated if $i\neq j$ (cf. Krumbein and Graybill, 1965). The value of α and of B may be found so that Y is related to X. This relationship depends not only on the values of Y and X but also on the conditions set upon the relationship. In the least square model the calcualted functional relationship must pass through the means of X and Y, and the squared deviations from the regression line are minimized.

The hydraulic geometry data seems to fit the underlying assumptions of regression only poorly. Discharge (X) can not be measured without some degree of error. This error may be small at a section, but in the downstream direction it may vary by as much as 30% from the "real" value. Discharges at a section may be considered to be reasonably independent of each other if sufficient time is allowed between each measurement. For most of the collected discharges there were at least several weeks between readings; however, these discharges can not be considered completely independent for discharge is a function of basin size, basin characteristics, and climatic factors. In the downstream direction the dependence of the discharges is even stronger. The discharge at P is clearly dependent on the discharge at P-ΔP, some point upstream of P.

The deviations from the calculated regression lines are due to both measurement errors and variance within the parameters. These sources of variance may be assumed to be essentially random (Scheidegger and Langbein, 1966). Therefore it can be assumed that the mean value of the deviations about each X is zero. However, the variance of the deviations about each X will not be the same for all Xs. The possible values of the dependent parameters will clearly cover a larger range for a discharge of 1,000 m^3/s than for one of 10 m^3/s though the logarithmic transforms will tend to reduce this.

It is also likely that the deviations about X_i are partially correlated with those about X_j. If an unusual amount of scour occurs during one high flow, the values of the dependent parameters will reflect this event at the following low flows. Mean depth may show a consistent positive deviation from the regression equation until another high flow changes the channel section. The correlation among deviations will be greater at a section than in the downstream direction if the downstream measurements are made a reasonable distance apart. The sectional deviations may reflect the last formative discharge. The downstream sections may also reflect the last formative discharge, but each section will have responded slightly differently to this flow due to differences in the character of the section.

Since the data only poorly fits the assumptions of regression, inferences and probabilistic statements which can be made from the analysis can not be applied to the data in a rigorous manner or with a great deal of confidence. R^2 can still be used as a measure of the degree to which the equation fits the data (Yamane, 1967), but confidence intervals constructed about the coefficients or tests that B is different from zero, for example, can not be rigorously applied. It would appear that the technique does aid in discriminating between two similar, but different sets of data. The change in the gauging section for Green River at Green Lake is clearly seen in the residuals (fig. 13) even though equations for the two sets of data can not be considered statistically different. However, least square regression would appear to be most useful as a means of fitting an equation to the data in as objective a fashion as possible.

Chapter 3

AT-A-STATION HYDRAULIC GEOMETRY

3-1 HYDRAULIC GEOMETRY EQUATIONS

Width, cross sectional area, mean depth, and mean velocity were related to discharge at the five gauging stations, Green River, Soo River, Rutherford Creek, Green River at Green Lake, and Birkenhead River. The exponents of the hydraulic geometry relations compare favorably with values found by workers in other areas (Tables V, VII; fig. 12). The average value of the exponent in the $w=aQ^b$ relationship, b (0.11), is lower than that found by some workers but is similar to the average value for 158 stations in the United States computed by Leopold, Wolman, and Miller (1964). Mean depth changes in a manner similar to depth changes elsewhere, but velocity appears to vary more rapidly than is usual.

Width changes appear to reflect the local geology of the section to a greater extent than do the other parameters. The lower increases in width are associated with channels that are downcutting and have no upper level to their banks which can be associated with fluvial action. Green River has downcut into bedrock along one bank and morainal material along the other. The river is essentially flowing between the walls of a canyon which greatly restrict any rapid changes in width. The Rutherford Creek section is also downcut into glacial material which restricts its width. Below this "canyon" section, the creek spreads out and some braiding takes place. Though it is not clear where sectional

TABLE V

AT-A-STATION HYDRAULIC GEOMETRY[1]

Station	n	a	b	R^2	c	f	R^2	k	m	R^2	g	h	R^2	y
Rutherford Creek	97	9.57	0.08	0.21	0.42	0.33	0.59	0.25	0.59	0.92	3.99	0.41	0.85	-0.86
Rutherford Creek, 1948 to 1941	33	11.57	0.06	0.30	0.28	0.40	0.92	0.31	0.54	0.93	3.21	0.46	0.91	-0.68
Rutherford Creek, 1941 to 1934	26	9.07	0.11	0.74	0.49	0.29	0.76	0.22	0.60	0.94	4.48	0.40	0.87	-0.91
Rutherford Creek, 1934 to 1924	38	7.89	0.12	0.63	0.56	0.26	0.82	0.23	0.62	0.98	4.40	0.38	0.95	-0.98
Green River at Green Lake	113	11.25	0.18	0.21	0.31	0.34	0.43	0.30	0.48	0.77	3.37	0.52	0.81	-0.62
Green River at Green Lake, 1948 to 1936	62	10.38	0.14	0.52	0.37	0.37	0.71	0.28	0.49	0.78	3.58	0.52	-0.79	-0.60
Green River at Green Lake, 1936 to 1920	51	13.20	0.22	0.38	0.23	0.36	0.60	0.34	0.42	0.82	3.00	0.57	0.91	-0.48
Soo River	109	17.87	0.11	0.53	0.23	0.47	0.88	0.24	0.42	0.91	4.11	0.48	0.93	-0.37
Green River	140	19.70	0.06	0.39	0.26	0.47	0.87	0.20	0.46	0.86	5.11	0.53	0.89	-0.46
Birkenhead River	136	22.44	0.12	0.54	0.20	0.36	0.86	0.25	0.48	0.81	4.50	0.48	0.93	-0.59

1 $w=aQ^b$ $v=kQ^m$ $ff \propto Q^y$

$d=cQ^f$ $A=gQ^h$ n number of observations

2 All B and R^2 values are statistically different from 0 at the 99% level.

TABLE VI

AT-A-STATION HYDRAULIC GEOMETRY OF ADDITIONAL COAST MOUNTAIN SECTIONS[1]

Station	n	a	b	R^2	c	f	R^2	k	m	R^2	g	h	R^2	y
8GA-56	21	5.88	0.34	0.52	0.21	0.26	0.54	0.81	0.39	0.87	1.25	0.61	0.95	-0.51
8GA-49	37	5.91	0.25	0.53	0.22	0.19	0.38	0.75	0.57	0.91	1.33	0.43	0.85	-0.95
8MH-108	39	7.34	0.27	0.83	0.30	0.30	0.51	0.41	0.37	0.77	2.21	0.57	0.75	-0.65
8GA-57	21	9.39	0.30	0.33	0.23	0.22**	0.28	0.47	0.48	0.79	2.11	0.52	0.81	-0.75
8MH-76	67	10.29	0.27	0.73	0.28	0.33	0.70	0.34	0.37	0.70	2.91	0.60	0.83	-0.42
8MH-66	74	11.08	0.23	0.61	0.26	0.29	0.72	0.35	0.47	0.75	2.87	0.53	0.79	-0.66
8GA-10	67	13.65	0.19	0.52	0.29	0.33	0.89	0.25	0.48	0.85	3.96	0.53	0.88	-0.62
8GA-30	123	18.96	0.08	0.49	0.59	0.27	0.63	0.10	0.62	0.82	11.08	0.35	0.76	-0.97
8GA-17	39	22.65	0.08	0.91	0.51	0.26	0.71	0.09	0.66	0.95	11.70	0.34	0.83	-1.07
8GA-54	17	24.50	0.04*	0.04*	0.27	0.39	0.68	0.15	0.57	0.92	6.63	0.43	0.88	-0.74

8GA-56 Sentinel Creek above Garibaldi Lake, 1966 to 1968
8GA-49 Mosquito Creek near North Vancouver, 1964 to 1970
8MH-108 Jacobs Creek above Jacobs Lake, 1965 to 1968
8GA-57 Mashiter Creek near Squamish, 1966 to 1969
8MH-76 Kanaka Creek near Webster Corner, 1960 to 1971
8MH-66 North Alouette River near Haney, 1960 to 1971
8GA-10 Capilano River near North Vancouver, 1954 to 1970
8GA-30 Seymour River near North Vancouver, 1945 to 1971
8GA-17 Cheakamus River at Garibaldi, 1961 to 1967
8GA-54 Mamquam River above Mashiter Creek, 1966 to 1968

1 $w=aQ^b$

$d=cQ^f$

$v=kQ^m$

$A=gQ^h$

$ff \propto Q^y$

n number of observations

* Not statistically different from 0 at 95% level

** Not statistically different from 0 at 99% level

TABLE VII

COMPARISON OF THE EXPONENTS OF AT-A-STATION HYDRAULIC GEOMETRIES[1]

Streams	b	f	m	h	y
Green and Birkenhead stations, mean	0.11	0.39	0.49	0.48	-0.58
Additional Coast Mountain stations, mean	0.21	0.32	0.50	0.49	-0.73
Mountain streams, mean[2]			0.55	0.45	
Baffin Island sandurs, mean[3]	0.22	0.31	0.48	0.52	-0.65
Midwestern U.S., mean[4]	0.26	0.40	0.34		
Average of 158 U.S. streams[5]	0.12	0.45	0.43		
Brandywine Creek, Pennsylvania[6]	0.04	0.41	0.55		
Ephemeral streams in semi-arid U.S.[7]	0.29	0.36	0.34		
Non-cohesive river, theory[8]	0.50	0.23	0.27		

1 $w=aQ^b$
$d=cQ^f$
$v=kQ^m$
$A=gQ^h$
$ff \propto Q^y$

2 Day, 1970

3 Church, 1970

4 Leopold and Maddock, 1953

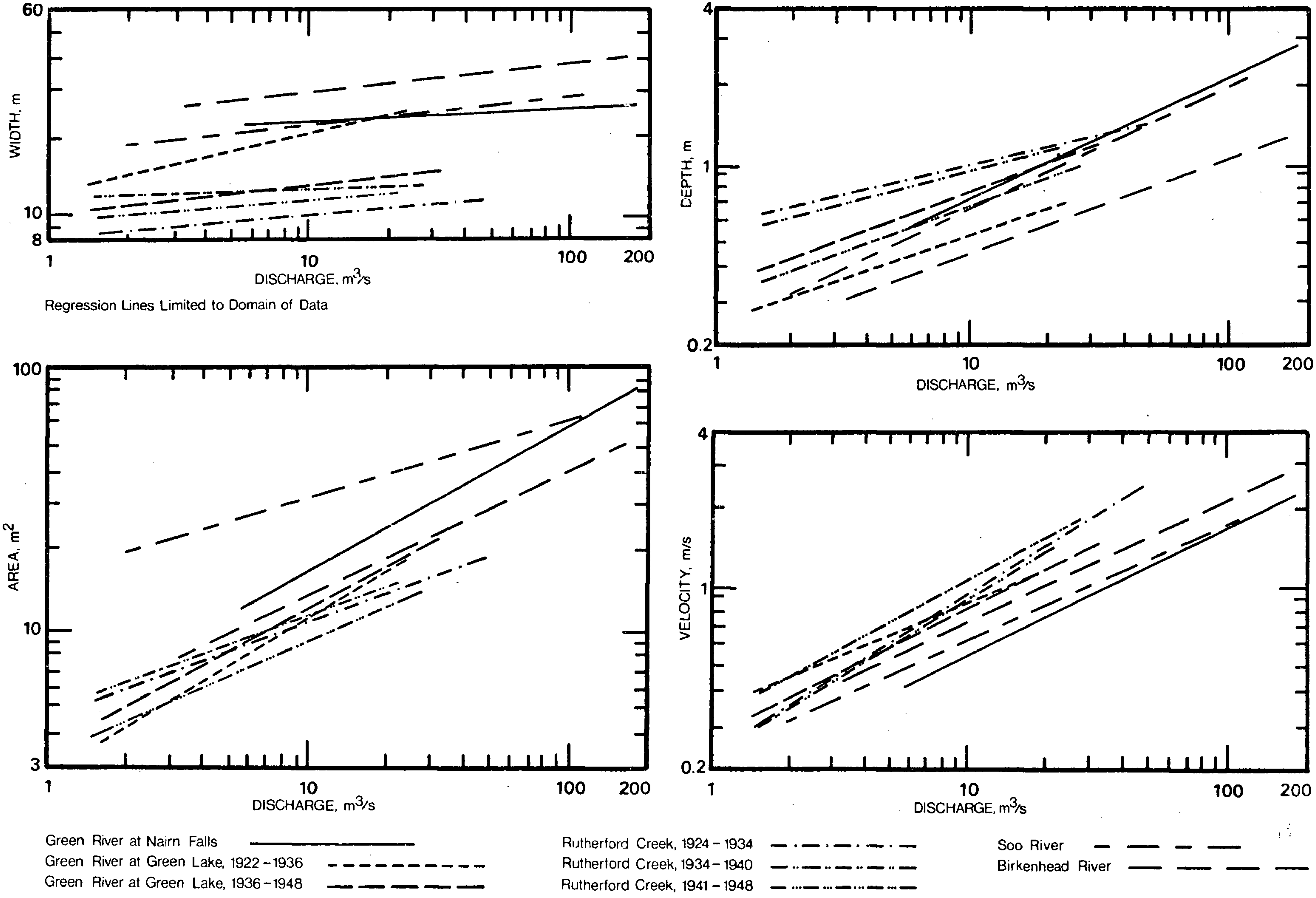
5 Leopold, Wolman, and Miller, 1964

6 Wolman, 1955

7 Leopold and Miller, 1956

8 Langbein, 1964

Figure 12, At-A-Station Hydraulic Geometry



measurements were made, it may be that the earlier measurements were made in the lower, wider reaches while the last group of measurements were made in the "canyon". The more rapid increases in width are associated with sections where the banks appear to be less dependent on the local geology and more on fluvial deposits.

The rate of change of velocity appears to be associated with sectional characteristics. The most rapid increase in velocity is associated with the section where the water surface slope is the steepest, Rutherford Creek. Soo River has the least rapid increase in velocity and the lowest slope. There appears to be a trend for a decrease in the rate of change of velocity with a decrease in slope.

The rate of change of depth does not appear to follow any general trend in sectional characteristics; it is more dependent upon the way in which width and velocity vary. From the continuity equation it can be seen that the sum of the exponents of width, depth, and velocity must equal 1. Since velocity and width appear to be more dependent on sectional characteristics, depth must adjust to these values in order to maintain the continuity relation; however, depth is not totally dependent on width and velocity, but does affect these parameters. For all sections the sum of the exponents is approximately 1.

3-2 RESISTANCE PROPERTIES

The Darcy-Weisbach nondimensional expression for total flow resistance is

$$ff \propto gRs/v^2,$$

where g is gravitational acceleration, s is slope, v is mean velocity and R is hydraulic mean depth which is approximately equal to d, mean depth, in wide channels. From the hydraulic geometry relationships,

$$ff \propto (Q^{f}) \cdot (Q^{z}) / (Q^{2m})$$
$$\propto Q^{f+z-2m} = Q^{y}$$

The exponent, y, of the above relation was computed for each gauging section. Assuming z equal to zero (sec. 2-2), the average value of y for the five sections is -0.63. This rate of change is approximately twice as great as that found for streams in noncohesive materials (Leopold, Wolman, and Miller, 1964). Church (1970) found a similar value in sandur channels on Baffin Island; and from the average values of the exponents for sections along Brandywine Creek (Wolman, 1955), the resistance there decreased slightly more rapidly (-0.69).

The bed and bank materials of the Green and Birkenhead sections are typical of those associated with channels with noncohesive boundaries, but it may be that the large size of the material tends to cause the channels' resistance to respond in a manner more typical of channels with cohesive boundaries. Sand size material is the most easily eroded while smaller material, such as that found along the banks of Brandywine Creek, and larger material are much less easily put into motion by flowing water. It appears that as the sediment size increases beyond sand size, the channel responds in a manner more typical of a cohesive channel. Soo River, where the bed and banks are nearest to sand size, has a channel resistance response most like other channels in small noncohesive material while Rutherford Creek, where the largest material is found, has a rate of change in resistance similar to Brandywine Creek.

Since the value of y is derived from observed relations among velocity, depth, and slope rather than directly measured, it reflects the influence of a number of resistance features such as the size of the individual roughness elements, bed and bank features such as bars or riffles, and channel bends (Leopold, Wolman, and Miller, 1964). The significance of any one resistance feature can not be determined from this type of analysis; however, their general influence can be discussed. In fully turbulent flow, as is found in most streams, the resistance due to the roughness elements is a function of the size of the roughness elements and the depth of flow. Wolman (1955) showed that the total resistance for a natural stream bears the same relation to relative roughness as occurs in pipes or experimental channels. He found that as the ratio of depth of flow to bed particle size increases the resistance decreases and that for Brandywine Creek the equation is $1/ff = 2\log(d/D_{84})+1.0$, where D_{84} is the grain diameter equal to or larger than 84% of the bed particles. The semilogarithmic form of the relationship indicates that a stream with large roughness elements will have a greater decrease in resistance than will a stream with smaller roughness elements for a similar change in flow depth. The depth of flow for Soo and Rutherford is of the same order of magnitude (0.1-1 m), but the size of the dominant bed particles of Rutherford (400 mm) is an order of magnitude larger than those of Soo (30 mm). Therefore a more rapid decrease in resistance with increasing discharge would be expected for Rutherford because of the size of the roughness elements.

The rate of change of depth relative to width will also be important in determining the rate of change of resistance. In channels with cohesive banks or coarse bank material, width will tend to increase more slowly than in channels with sand size noncohesive banks. With a similar velocity change, depth would increase more rapidly in channels with lower width increases, leading to a more rapid decrease in resistance. Other factors are also operating to change the resistance due to the roughness elements and the other resistance features. Church (1970) suggested that the rapid decrease in resistance observed in sandur channels may be due to a "live" bed at higher flows, an increase in "straight through" flow which decreased the influence of the pool and riffle sequence, and a damping of turbulence by rapidly increasing sediment discharge. These factors may also be operating in the sections of Green and Birkenhead Rivers.

3-3 DATA VARIANCE

The degree to which the data is scattered about the mean condition reflected by the hydraulic geometry equations is seen via the coefficients of determination which are derived from the regression analyses and are a measure of the degree to which the equation fits the data. The scatter is consistently greatest about the width relationships (Table V). Langbein (1964), in his theoretical derivation of the hydraulic geometry equations, has considered width to be essentially independent at a section in cohesive channels and not adjustable in any short periods of time. The higher variance in the width relations and the more distinctive residual groupings tend to support this consideration.

Depth has an intermediate amount of scatter for it reflects the adjustment of both width and cross sectional area. The partial independence of width will tend to increase the variance of the mean depth. Cross sectional area and velocity have the least variance about their mean condition for they are largely dependent on discharge and may adjust rapidly over a short period of time. Cross sectional area can adjust to fluctuations in discharge and width by changes in the water depth. If the channel is slightly wider after a high flow than is usual, a decrease in mean depth can maintain essentially the same cross sectional area. Velocity can also adjust rapidly to changes in discharge; and if the cross sectional area is able to adjust in the same way for varying discharges, velocity must also vary only slightly about its mean rate of adjustment.

When the residual values of the regressions were listed as an ordered sequence in time, two types of distinctive groupings of positive and negative residual values appeared. The first type of grouping appears to reflect distinct changes in the cross section at which the measurements were made. For Green River at Green Lake two sub-groupings of residuals were present with the break in groupings appearing in 1936 (fig. 13). In that year the gauging section was relocated a short distance downstream. The width residuals in each group are larger and more distinctively grouped than are the cross sectional area residuals. This pattern would tend to indicate the stronger dependence of sectional area on the discharge of the stream. Rutherford Creek shows a similar, though less distinctive, residual grouping. Three sub-groupings appear, one of which may be a result of the second highest recorded discharge which occurred in 1940. It is not clear whether or not the actual gauging section was ever moved.

The data for these two sections were divided on the basis of the residual groupings and separate hydraulic geometries were computed. Groupings once again appeared in the residuals when they were listed as an ordered sequence in time, but the pattern observed was similar to the pattern seen in the data for the three other sections. This second type of grouping showed a number of positive residuals followed by a number of negative ones (fig. 14). This pattern existed in all relations though it was not distinctive throughout. It suggests that the data do not fluctuate about the regression lines in a totally random way. Rather

Figure 13

Residuals of Sectional Area and Width
Green River at Green Lake

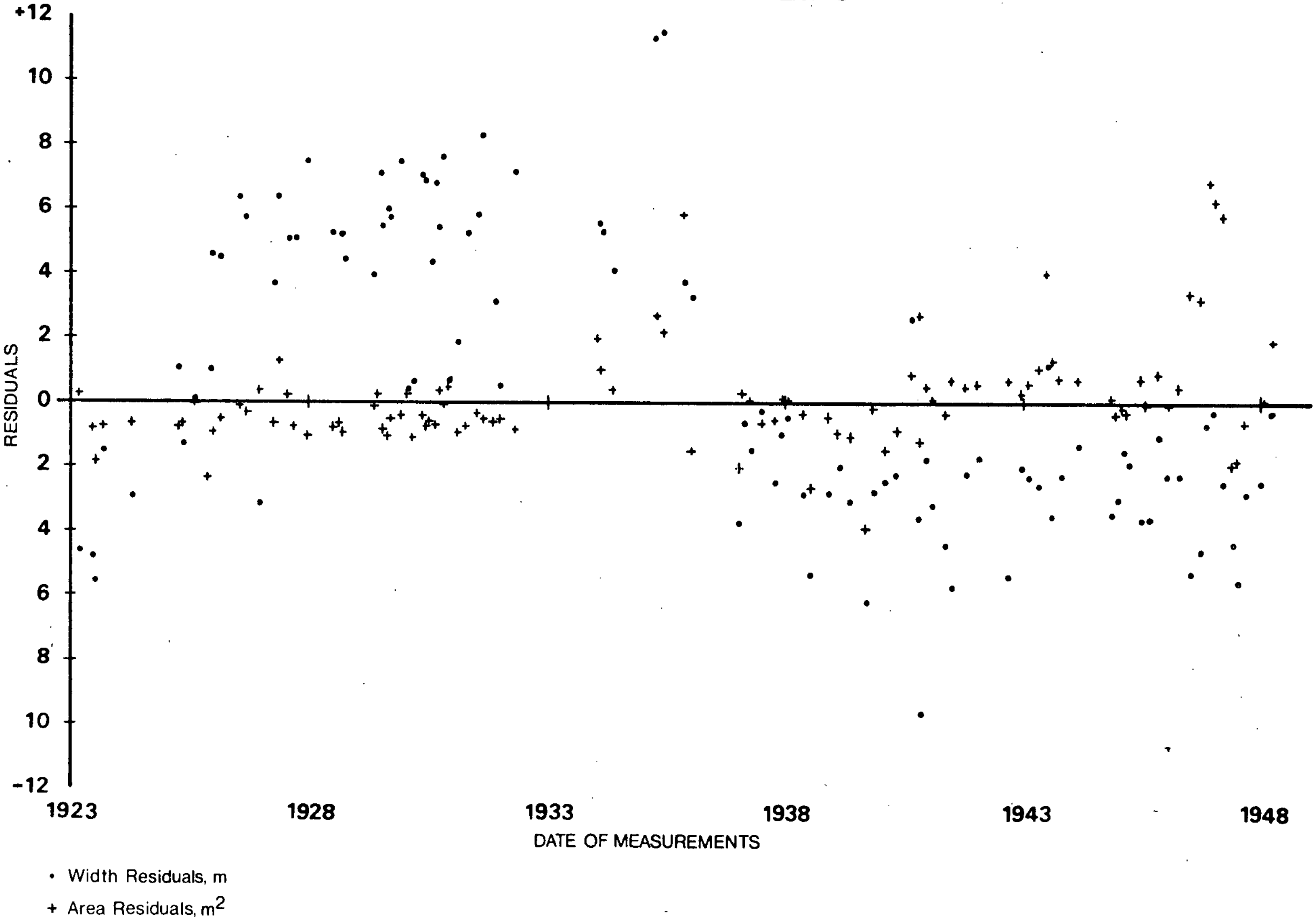
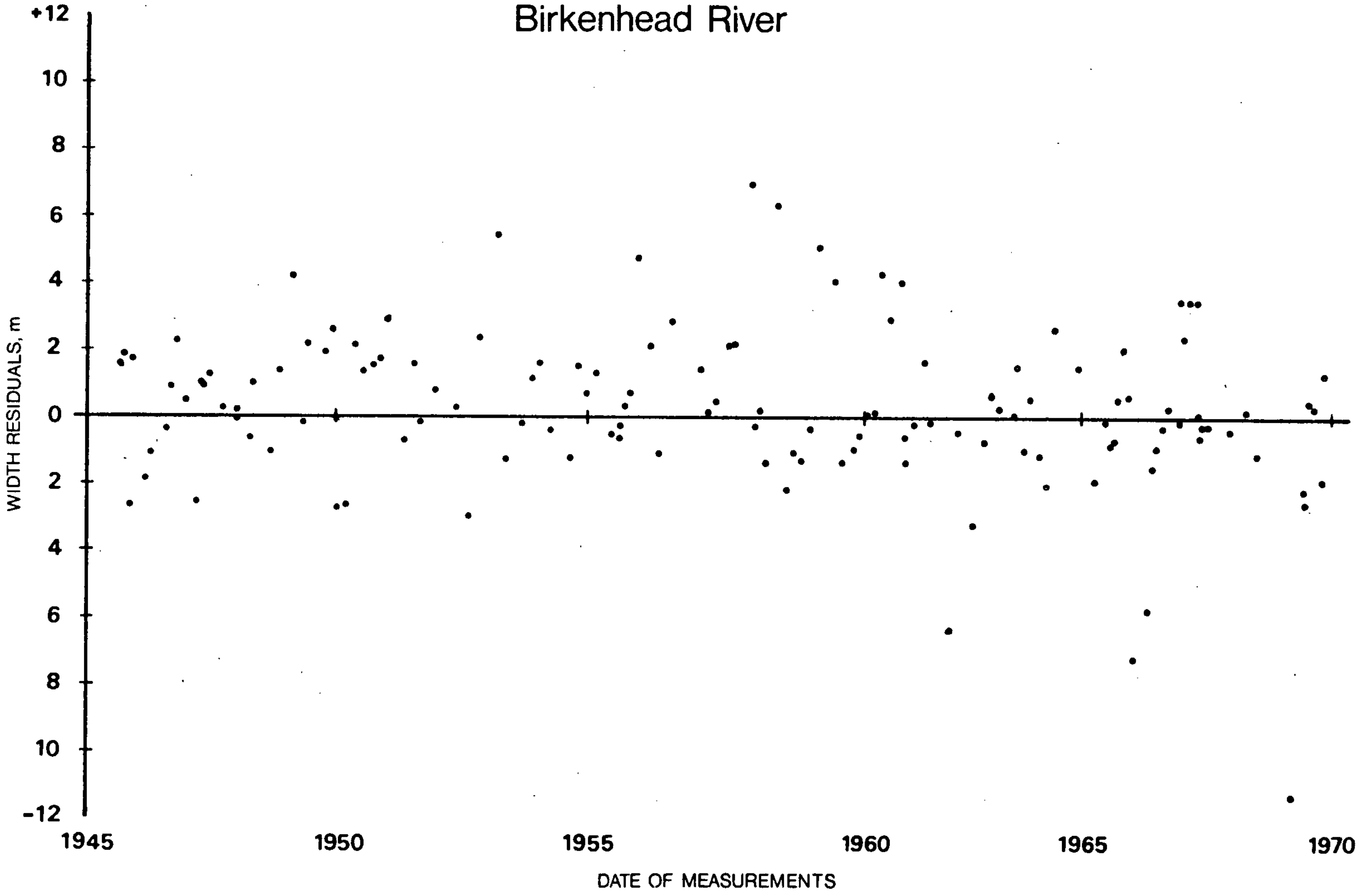


Figure 14

Residuals of Sectional Width

Birkenhead River



the sections appear to fluctuate slowly about the mean condition expressed by the regression line, a type of behavior which Langbein and Leopold (1964) have suggested is typical of a channel which has reached equilibrium. Due to disturbances introduced into the channel, the hydraulic parameters are seldom at the exact state of equilibrium. Instead the disturbances cause adjustments among the parameters so that they are continually fluctuating about some mean condition. The hydraulic geometry relations can be taken as the mean condition, and the observed values of the parameters can be seen to fluctuate about the relations through time in an apparently nonrandom manner.

3-4 ADDITIONAL COAST MOUNTAIN SECTIONS

In addition to the five sections within the Green and Birkenhead basins, ten other sections on streams in the coastal mountains of southwestern British Columbia were considered in order to see if the Green and Birkenhead sections were characteristic of the area (Table VI, figs. 15a-d). For these stations, velocity proved to be the most rapidly varying parameter in all cases, varying slightly more rapidly on the average than within the Green and Birkenhead where velocity was also the most rapidly varying parameter for three of the five sections. Except for the results of Church (1970) and Day (1970) this dominance of velocity variation in noncohesive materials had not been reported before in the literature. Width changes were more rapid for the additional stations while depth changed less rapidly, leading to a rate of change in cross sectional area only slightly less than for the Green and Birkenhead sections. Changes in the resistance factor were also high for these streams with an average value of -0.75 and a range of values similar to the Green and Birkenhead range. The scatter of the

Figure 15a

At-A-Station Hydraulic Geometry of Ten Streams in Southwestern British Columbia

Sectional Width

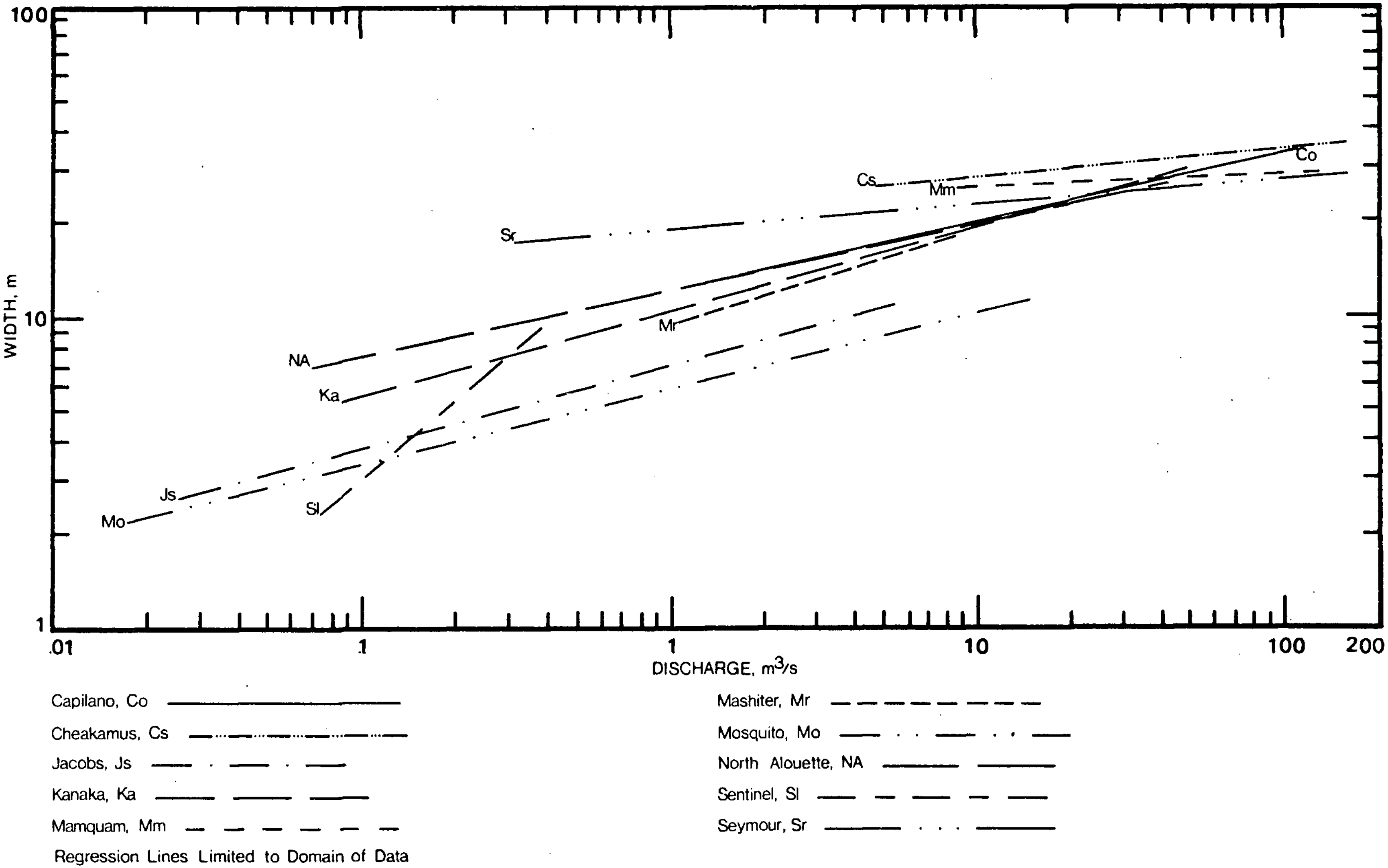


Figure 15b, Mean Sectional Depth

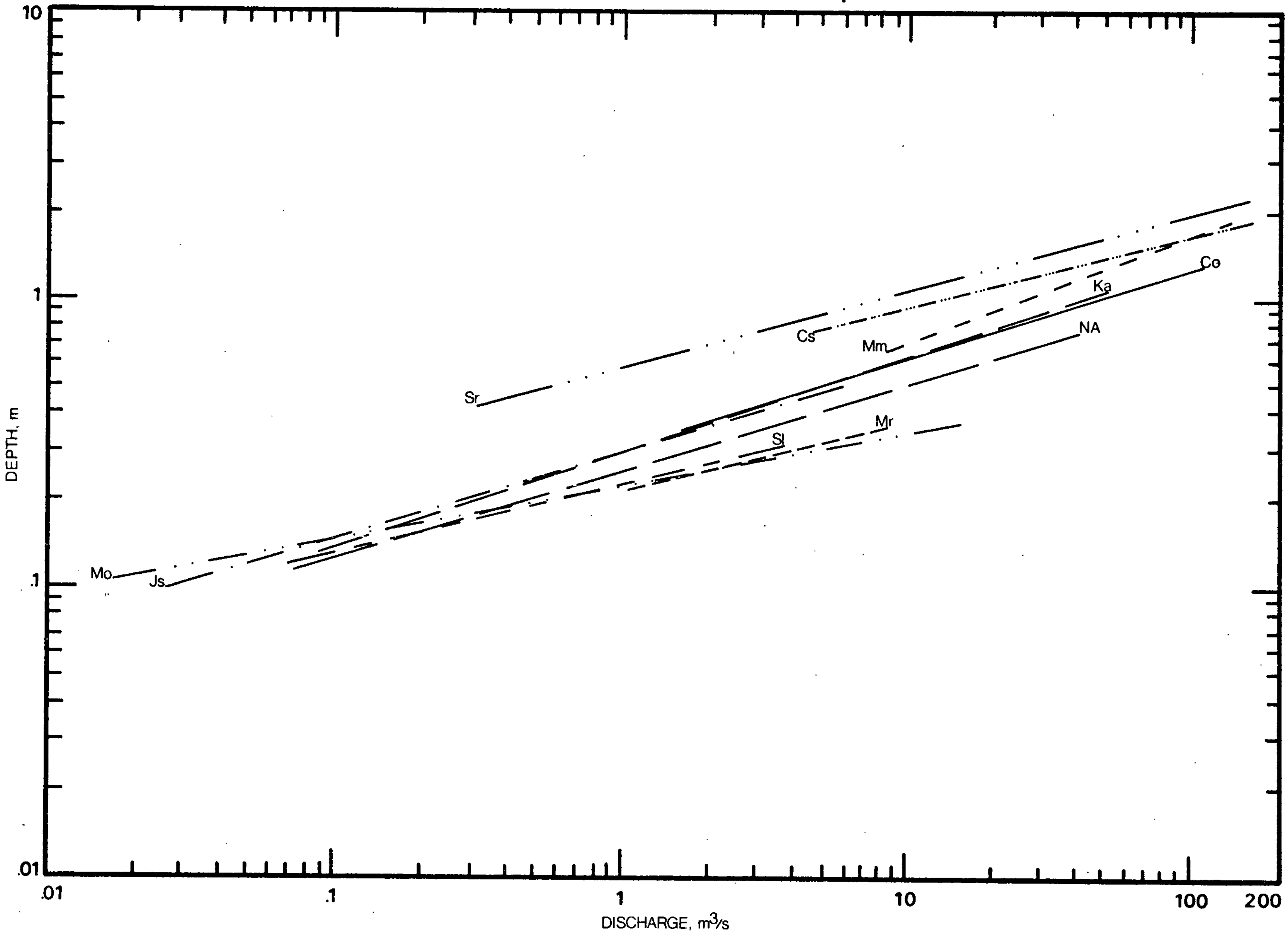


Figure 15c, Mean Sectional Velocity

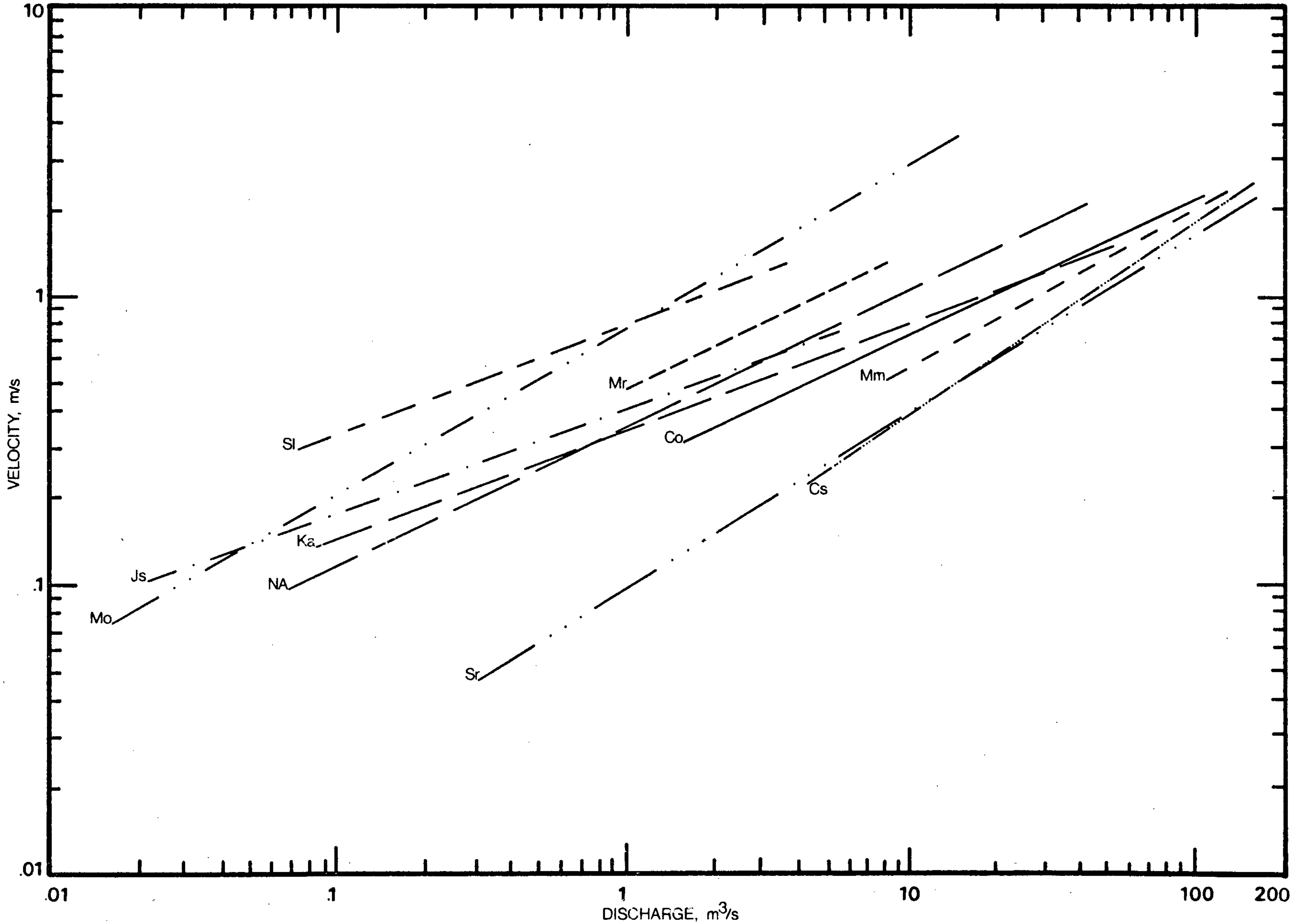
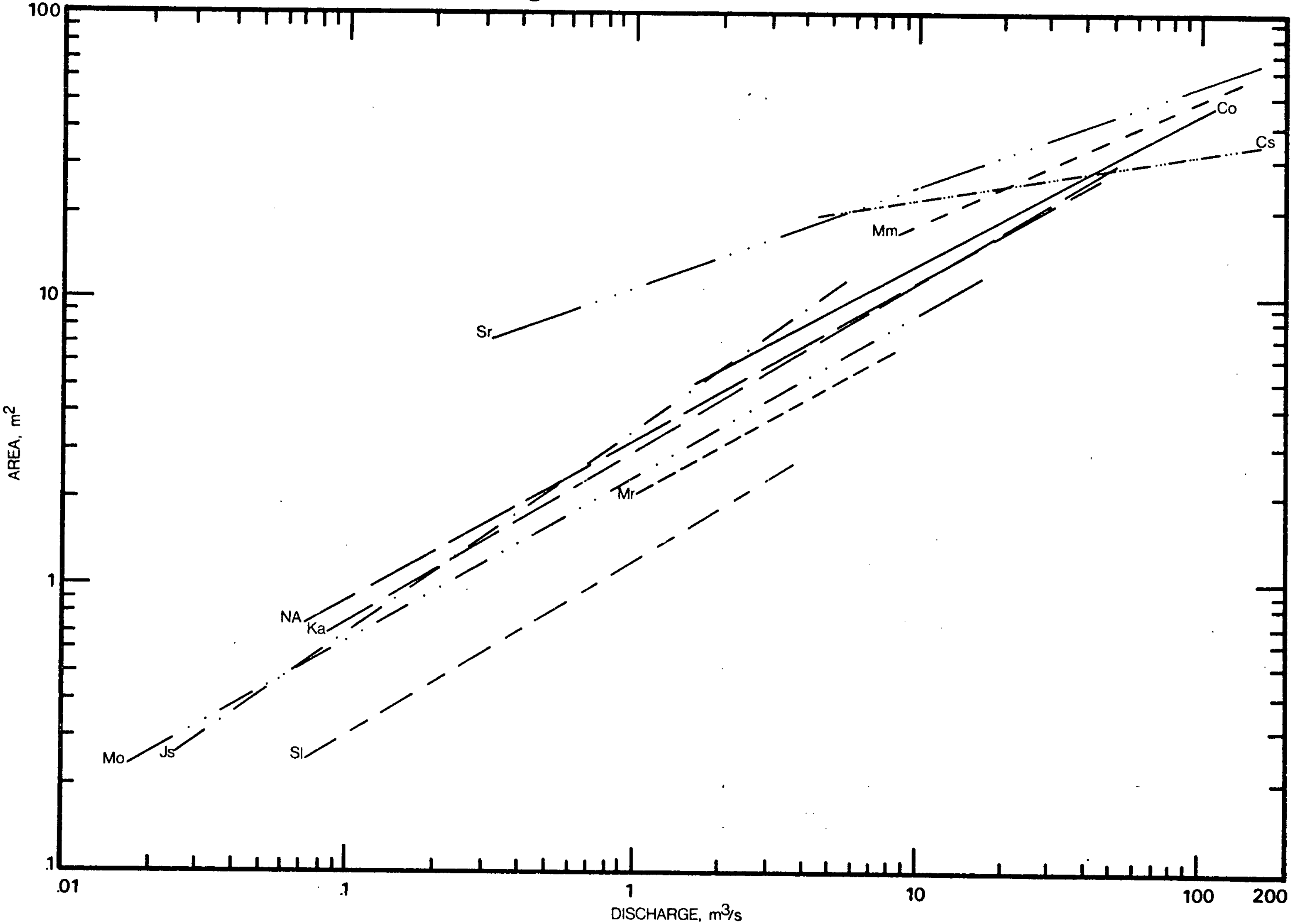


Figure 15d, Sectional Area



data was also similar to that found at the Green and Birkenhead sections. Width and mean depth continue to have the greatest scatter though width did not always have the lowest R^2. At all sections residual groupings of the second type were found.

These hydraulic geometry relations are similar to those found for the Green and Birkenhead sections and would tend to indicate that these are typical of the regional relations within the Coast Mountains. There is a considerable range in the values of the exponents, and adjustment of the hydraulic parameters in the study area would appear to be slightly different from other regions with lower rates of width adjustment, a dominant rate of velocity adjustment and a high rate of decrease in resistance (Table VII).

Chapter 4

DOWNSTREAM HYDRAULIC GEOMETRY

4-1 FLOW FREQUENCY AND BANKFULL DISCHARGE

Downstream hydraulic geometry considers the changes in the hydraulic parameters as the discharge increases downstream. In most streams, discharge will increase downstream because the size of the contributing basin is increasing. To accommodate this increasing flow the channel must also adjust. Since the downstream relations are only concerned with changes in the channel form caused by changes in section location within the system, the time variation of flow must be eliminated. This can be done by considering flows of equal frequency at each section (Leopold and Maddock, 1953).

It would be desirable to analyze a system within which a flow with a two year recurrence interval affects each channel section at essentially the same time, but this is rarely the case. Therefore, flows occurring at different times but with the same recurrence interval are considered similar in so far as channel forming characteristics are concerned. Any frequency of flow could be considered for the downstream analysis, but each will give slightly different results (Wolman, 1955). Since the parameters being considered relate the shape of the channel to discharge, a frequency of flow which fills the entire channel would appear to be a meaningful flow to consider. Bankfull discharge meets the above consideration and appears to be one of the important channel forming flows (Leopold, 1962) though other flows are also as important

(Dury, 1969).

The frequency of bankfull flow has been computed for many sections on a wide variety of streams and has been found to have a recurrence interval between one and three years (cf. Kilpatrick and Barnes, 1964). The climate of the basins considered accounts for some of the variability in bankfull recurrence from region to region though Kilpatrick and Barnes found a range in bankfull recurrence between basins in the southern piedmont of the United States. The flood series considered is also important. Nixon (1959), working in England and Wales, found a recurrence interval of only six months for bankfull discharge. The value is low because he considered all floods above an arbitrary level rather than only the annual peak flow. Henderson (1966) shows that the recurrence interval of 1.4 years computed by Leopold and Wolman (1957) in the southwestern United States would be nine months if a flood series such as that used by Nixon had been used instead of the annual flow. Others such as Brush (1961), working in central Pennsylvania, and Miller (1958), working in the mountains of New Mexico, have found that the recurrence interval is 2.33 years.

Another factor which influences the frequency of bankfull flow is the location of the section within the channel system. There is evidence to suggest that bankfull may occur more frequently in the headwaters of a stream (Dury, 1961). When the main channel reaches bankfull, it is likely that a heavy input has been experienced throughout the entire basin and that many of the tributaries have also reached bankfull. However, a localized input, such as a thunderstorm or locally rapid snow melting, may bring a tributary to bankfull without a large effect on the main stream. Within the study area two reaches, Green River above

Green Lake and lower Poole Creek, were observed to be at or very near bankfull on three occasions during the spring and summer of 1970. The Green section was also observed to be near bankfull in the spring of 1971. On only one of these occasions were the other streams in the area near bankfull. At the other times the flow in the other tributaries was only moderately high. Both reaches have a low slope relative to their upstream reaches and both are near their local base level. The apparently high frequency with which these sections approach or reach bankfull may be a result of these factors.

In spite of the deviations from a constant recurrence interval, bankfull flow still has the great advantage that clear evidence of this flow is generally left and can be identified during times of lower flows (Leopold and Skibitzke, 1967). It would be impractical to attempt to measure the hydraulic parameters at a high flow at a large number of sections for the duration of these flows is relatively short. Instead the bankfull width and bankfull cross sectional area can be measured from evidence of bankfull flow. Bankfull mean depth is computed from the relation, $d=A/w$, and bankfull mean velocity can be found from the continuity equation, $v=Q_{bankfull}/A$ (sec. 2-3).

4-2 HYDRAULIC EQUATIONS

Width shows the most rapid increase within the channel systems of Green and Birkenhead Rivers while mean depth increases less rapidly (Table VIII, fig. 16). Mean velocity tends to decrease slowly in the downstream direction, and slope and the Darcy-Weisbach resistance factor decrease rapidly. The increase in width is more rapid than in other areas (Table IX). Leopold and Maddock (1953) and Brush (1961) did find values as high as these, but their average values (0.50 and 0.55) are

TABLE VIII

DOWNSTREAM HYDRAULIC GEOMETRY[1]

Stream	n	a	b	R^2	c	f	R^2	k	m	R^2
Green River	6	1.94	0.60*	0.70**	0.23	0.40***	0.42***	2.16	-0.01***	0.00***
Birkenhead River	10	1.00	0.79	0.94	0.17	0.44	0.94	6.04	-0.23*	0.48*

Stream	g	h	R^2	t	z	R^2	y
Green River	0.45	1.00**	0.63**	1.19	-1.29**	0.69***	-0.87
Birkenhead River	0.22	1.17	0.89	2.93	-1.51	0.60	-0.61

1 $w=aQ^b$ $v=kQ^m$ $s=tQ^z$

$d=cQ^f$ $A=gQ^h$ $ff \propto Q^y$

n number of observations

* Not statistically different from 0 at the 99% level

** Not statistically different from 0 at the 95% level

*** Not statistically different from 0 at the 90% level

TABLE IX

COMPARISON OF THE EXPONENTS OF DOWNSTREAM HYDRAULIC GEOMETRIES[1]

Streams	b	f	m	z
Green River	0.60	0.40	-0.01	-1.29
Birkenhead River	0.79	0.44	-0.23	-1.51
Brandywine Creek, Pennsylvania[2]	0.42	0.45	0.05	-1.07
Appalachian streams[3]	0.55	0.36	0.09	
Average values, midwestern U.S.[4]	0.50	0.40	0.10	-0.75
Ephemeral streams in semiarid U.S.[5]	0.50	0.30	0.20	-0.95
Appalachian plateau streams[6]	0.36	0.20	0.44	
Theory[7]	0.53	0.10	0.37	-0.75

1 $w=aQ^b$
$d=cQ^f$
$v=kQ^m$
$s=tQ^z$

2 Wolman, 1955

3 Brush, 1961

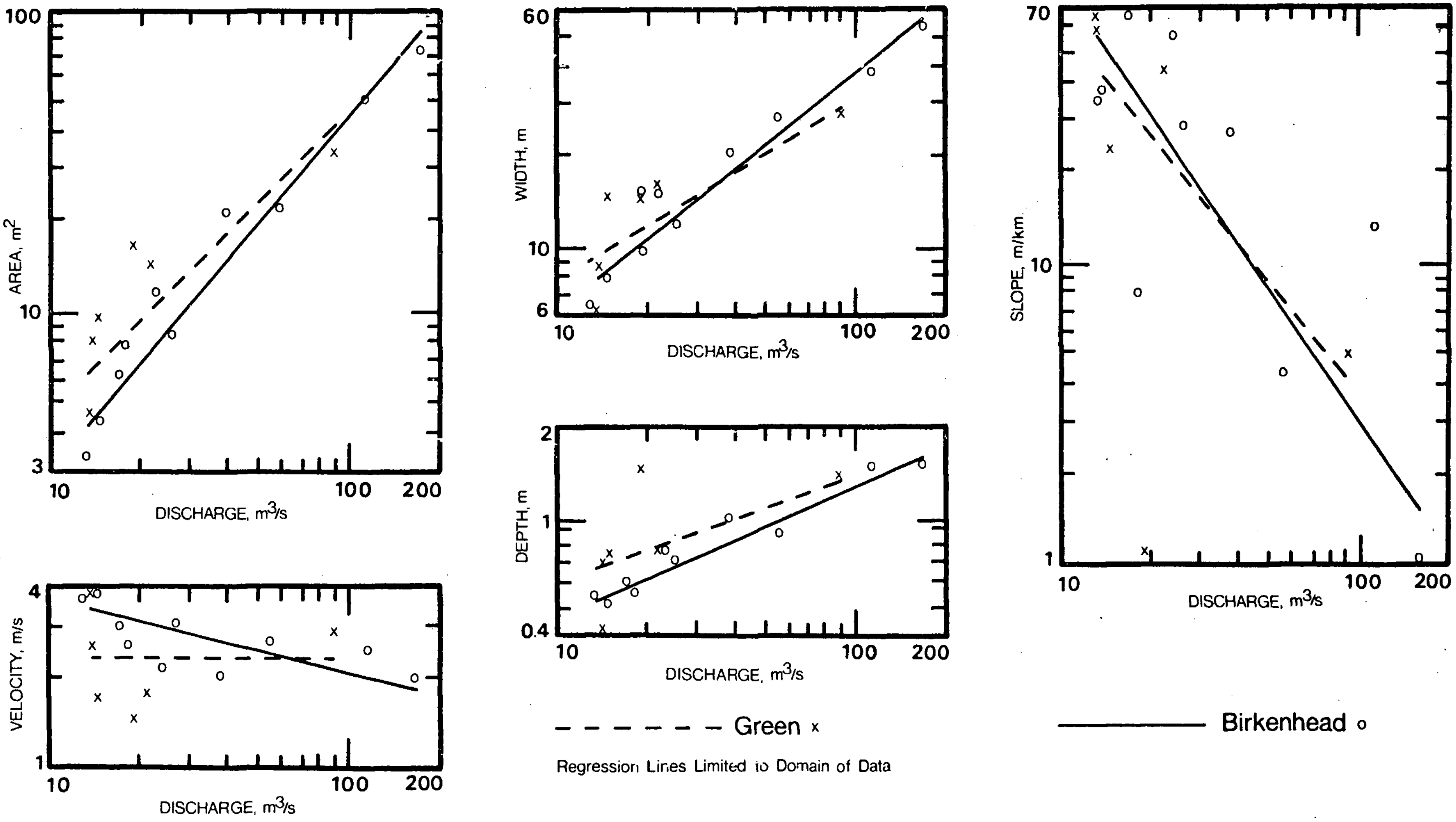
4 Leopold and Maddock, 1953

5 Leopold and Miller, 1956

6 Coates, 1969

7 Langbein, in Leopold, Wolman, and Miller, 1964

Figure 16
Downstream Hydraulic Geometry



well below the average value of 0.71 for Green and Birkenhead Rivers. This rapid increase in width is probably a result of the changing nature of the banks downstream. In their upstream reaches, channels are controlled by large bed and bank material while downstream reaches are less confined because of smaller material which can be moved more often by the streams.

Depth changes slightly more rapidly than in most other areas considered. Nevertheless, channels in this area appear to have a more rapidly increasing width to depth ratio than in other areas indicating that the headwater channels are relatively narrow and deep while downstream reaches are relatively wide and shallow. This too is a result of the changing nature of the banks. The rapid decrease in resistance is partly a result of the change in the size of the bed and bank material. It is also a result of the increase in depth. This leads to an increase in the ratio of depth of flow to particle size which leads to a decrease in resistance (sec. 3-2).

Velocity shows the greatest departure from previous studies. For each set of data, velocity shows a decrease in the downstream direction. Brush (1961) did find that several of the mountain streams he studied also showed a decrease in velocity, but most other workers have found that velocity increases. The average value of the velocity exponent for the Green and Birkenhead is -0.12; all other workers have found that the average change of velocity in an area is positive.

There are two possible reasons why the change of velocity is negative. It may be a result of the physical characteristics of the basins, or it may simply be a result of measurement errors. A third alternative is that the velocity at bankfull remains essentially constant.

Since bankfull velocity is derived from the relation, v=Q/A, it will reflect errors in the estimation and measurement of A and the estimation of Q (sec. 4-3). These errors may be sufficient to obscure the true relationship. The coefficient of determination, R^2, is 0.00 for Green River and 0.48 for Birkenhead River, and these low values may reflect the errors of measurement.

The low R^2 values may also result from the fact that no systematic change in velocity takes place. It has been suggested (Dury, 1969) that the change in velocity is in fact zero in the downstream direction. Neither the correlations between velocity and discharge nor the slopes of the relationship are statistically different from zero (Table VIII). This is the case for the Birkenhead only at the 99% significance level; but since the data so poorly fit the assumptions of the statistical model (sec. 2-5), the significance level should be considered lower (Yamane, 1967). Similarly low R^2 values were found when regressions were run on one set of data collected by Miller (1958) and three sets collected by Brush (1961) (Table IV). Neither the correlations nor the slopes of these relations were statistically different from zero. These low R^2 values tend to reinforce the impression in the literature of consistently greatest scatter about the downstream velocity-discharge relationships and would suggest that there is in fact no systematic change in velocity.

Finally, the physical characteristics of the basins may lead to an actual decrease in velocity. The average decrease in water surface slope in the downstream direction is -1.40. This decrease in slope is much more rapid than in other areas (Table IX) and is a result of glaciation and the partial independence of slope from discharge. Though the slope in the lower reaches is probably partly dependent on discharge, the headwater reaches can be assumed to be independent (Day, 1970). Leopold (1953) suggested that the observed increase in velocity downstream was a result of the increase in depth and the decrease in resistance. These changes were sufficiently great to compensate for the decrease in slope which would tend to reduce velocity. In this area the rapid decrease in slope would appear to be sufficiently greater than the decreasing resistance and increasing depth to cause a reduction in velocity downstream.

4-3 DATA VARIANCE

The variance of the data about the regression lines indicates that the hydraulic geometry equations do not completely describe the change of the dependent parameters with discharge. There are several sources of variance which lead to deviations of the observed values from the regression line. Measurement errors are one source, but this should be relatively small. A larger source is the estimations which must be made. Deviations which exist in the actual values of the hydraulic parameters also contribute to overall variation. Interdependence among these parameters is high, and a change in one will cause changes in each of the others. Adjustments in the system are not instantaneous, and the system is seldom at its mean condition. Since this condition can not be precisely determined, a certain degree of inherent variance must be expected (Langbein and Leopold, 1964). Natural inhomogeneities, such as tree falls or bank slumps, provide another source which may be indistinguishable from the inherent variance.

It is not possible to divide the variance among the different sources, but an intuitive impression of the degree to which each factor may contribute to the overall variation can be established. The largest scatter (lowest R^2) occurs about the velocity-discharge relations. A large part of this may be a result of the estimations which must be made in order to determine the value of velocity. The error in discharge (sec. 2-3) is reflected in both the dependent and independent variables of the regression and would tend to increase the scatter. In the other regressions, the discharge error is only reflected in the independent variable, discharge. Though this large possible error in discharge, and hence velocity, may account for a large portion of the scatter about the velocity-discharge relations for Green and Birkenhead Rivers, other studies where the error in discharge estimates was smaller also have large scatter.

Another cause of the large variance in the velocity relationships is that velocity is the most sensitive of the hydraulic parameters. It may adjust almost instantly to disturbances in the channel while adjustment of the other parameters will take longer. For this reason, velocity will fluctuate rapidly about its mean with a large relative range of fluctuation. The partial independence of slope has a greater effect on velocity than on the other parameters.

Width shows the best fit for the two sets of data. This also appears to be true in most other studies of downstream hydraulic geometry. Width variance is least because it is the least sensitive of the parameters and can be measured with the least error. Evidence of bankfull flow is relatively clear so that only a small error will result from its estimation.

Depth gives the next best fit for the study area and in general for the data from other areas. Measurement error will be larger for a small error in both sectional area and width can lead to a larger error in depth. The assumption that the water surface is level between the banks may not be correct, and some scour or deposition may take place at bankfull flow. Adjustments in the channel at lower flows will result in a larger error in the determination of depth. The position of the cross section within the pool and riffle sequence will also lead to a larger variance in depth.

The variance in slope is only slightly greater than the variance in depth. This would seem to be largely a result of natural inhomogeneities (the partial independence of slope) between the different sections.

Variance in the Green River relations is consistently greater than in the Birkenhead River relations. At the 99% significance level none of the slopes or the correlations of the Green relations are different from zero. Though this might suggest that the Green is very different from the Birkenhead and that the hydraulic geometry relations do not hold, it is more likely a result of the small number of sections measured.

Chapter 5

QUASI-EQUILIBRIUM IN COAST MOUNTAIN STREAMS

5-1 INTRODUCTION

A stream which has reached a state of quasi-equilibrium is neither rapidly aggrading nor degrading, and the hydraulic parameters are adjusted so that a change in one will be negated by adjustments in the other parameters (Mackin, 1948). Such a stream can be viewed as an open system which has reached a steady state with material and energy passing through it but with no significant changes taking place within the system (Chorley, 1962). This condition may apply to an entire channel system, or it may apply to several sections within the system. If a quasi-equilibrium has been established within the system, it will be reflected not only in the downstream channel adjustments but also in the sectional adjustments.

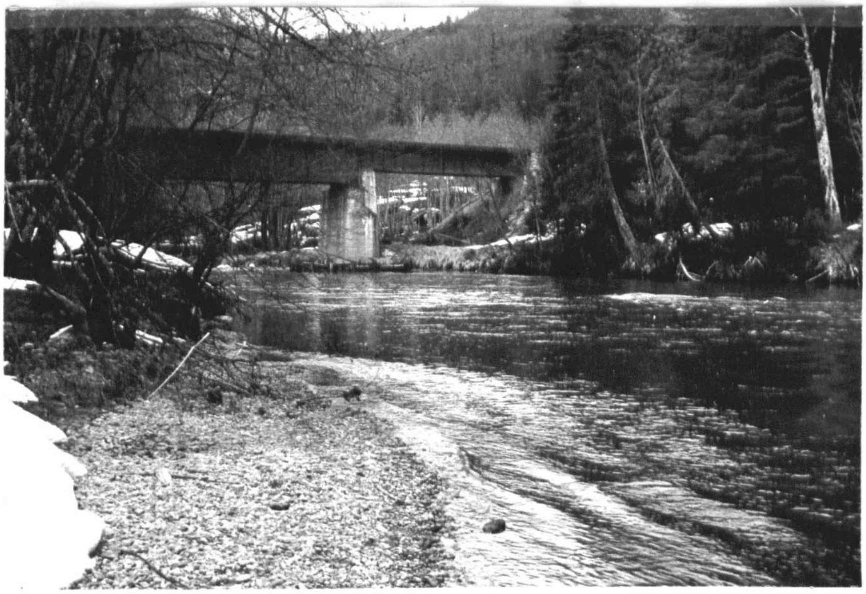
5-2 QUASI-EQUILIBRIUM AND COAST MOUNTAIN STREAMS

Though theoretical values for the hydraulic geometry of a stream at equilibrium have been derived (Leopold and Langbein, 1962; Langbein, 1964), it is unlikely that these results will apply to Coast Mountain streams. The assumption was made in the theoretical derivation that the streams had self-formed alluvial channels. This is hardly the case in the Coast Mountains where slope is clearly imposed on the upland channels and width and resistance are probably partly imposed and where slope is at least partly imposed on the channels in the main valleys. For this reason it is unlikely that any self-imposed equilibrium exists throughout either the Green or Birkenhead system.

The lack of a self-imposed equilibrium, however, does not imply that rapid changes are taking place within each system or that a quasi-equilibrium does not exist. The stability of the at-a-station hydraulic geometry relations over a reasonably long period of record suggest that these sections are stable. In addition, the fluctuations of the parameters about the mean condition expressed by the hydraulic geometry relations occur in a seemingly systematic manner over time. If these channel sections are at equilibrium, it would imply that the reaches within which they are located are also at equilibrium. An equilibrium has been established which accommodates the imposed conditions.

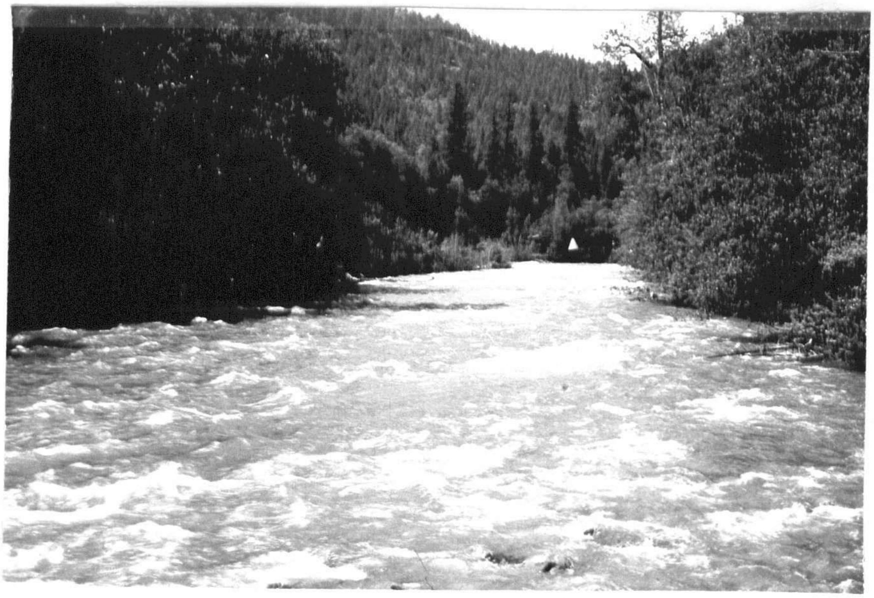
Though a quasi-equilibrium may exist for reaches within each basin, it is unlikely that one has been established for each complete system. Water falls, which are common on these streams, are clearly a non-equilibrium feature. Reports of rapid degradation along some reaches have been made, and evidence of aggradation and degradation exists along other reaches. The change in the conditions imposed on the channels would also suggest that different channel adjustments take place in different areas. Instead of one type of channel adjustment which brings an equilibrium to the entire channel system, it would appear that a number of different equilibrium reaches have been formed which have not yet been integrated into one balanced system. This situation exists because of the strong influence that glacially derived features still exert over the drainage system (sec. 1-3) and because of the post-Pleistocene change in the imposed conditions. It is not clear how many different types of equilibrium reaches are in the area. There may be only two, one of which is found in the upland channels and one which is located in the main valley channels.

At all events, the evidence of the hydraulic geometry suggests streams in this region have established reaches which adjust in an orderly fashion to the geologic conditions imposed upon them.



PHOTOGRAPH 1
Soo River, looking upstream near gauging section

PHOTOGRAPH 2
Rutherford Creek, looking upstream near gauging section



PHOTOGRAPH 3
Birkenhead River, looking upstream at gauging section and downstream section during near bankfull discharge

PHOTOGRAPH 4
Birkenhead Lake River between Birkenhead Lake and confluence with Birkenhead River, looking upstream at downstream section

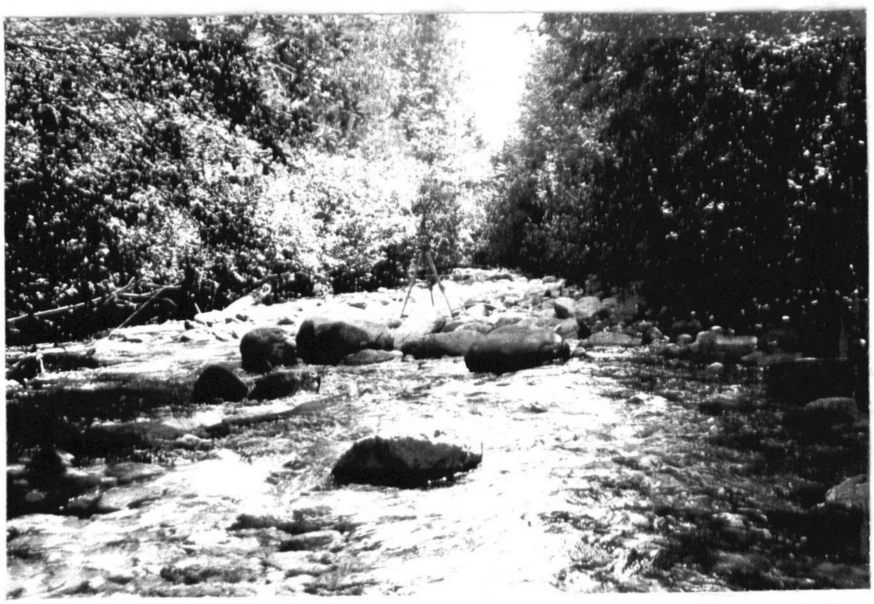
PHOTOGRAPH 5
Birkenhead River above confluence with Tenas Creek, looking downstream at downstream section

PHOTOGRAPH 6
Owl Creek, looking upstream at downstream section

PHOTOGRAPH 7
Tenas Creek, looking upstream at downstream section

PHOTOGRAPH 8
Upper Poole Creek section, looking upstream at downstream section

PHOTOGRAPH 9
Green River above Green Lake, looking downstream at downstream section



PHOTOGRAPH 10
21 Mile Creek, looking downstream at downstream section

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