in the Department
of
Electrical Engineering

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Electrical Engineering
The University of British Columbia Vancouver 8, Canada

Date $\qquad$ Sept 26 1973


#### Abstract

A wealth of experiments have been performed studying image encoding techniques as applied to non-time varying or single-frame images. However, to date little work has been done to apply these techniques to time varying images, with most of such works emphasizing various ad hoc redundancy reduction techniques.

In this work, a computer based experimental system is implemented which makes more methodological studies of time varying images possible. Particular attention is devoted to obtaining very accurate interframe registration and uniform quantization of the images. Using this system, a selection of 35 mm movie film images are digitized and stored on computer magnetic tape in a format compatible with many other computing installations, providing a standard data base for future experiments.

An often used model for ..describing picture data is the stationary Gauss-Markov model. In this work, the appropriateness of this model for describing time varying images is studied by comparing the autocorrelation functions as described by the model and as obtained by computation from the picture data. These results indicate that the autocorrelation function is best described by a function which is separable in the time dimension and nonseparable in the spacial dimensions.

A number of DPCM communication systems are then studied as a vehicle for evaluating the effect of using the Gauss-Markov model. These results indicate that, for the sample images studied here, the estimated performance using the Gauss-Markov model is good when the model is a good fit to the first data point of the computed autocorrelation function.


Page
I. INTRODUCTION
1.1 Motivation of Study ..... 1
1.2 Review of Previous Work ..... 2
1.3 Scope of Thesis ..... 4
II. THE IMAGE DIGITIZER
2.1 Introduction ..... 5
2.2 Film Transport and Hardware ..... 6
2.3 Interframe Registration Technique ..... 13
2.4 Automatic Intensity Control for the Flying Spot Scanner ..... 18
2.5 The Digitized Images ..... 20
III. MODELS FOR IMAGES
3.1 Introduction ..... 24
3.2 Stationary Models for Monochromatic Time Varying Images ..... 24
3.3 Closeness of Fit Criterion ..... 27
3.4 The Source Images ..... 27
3.4.1 Subject One ..... 33
3.4.2 Subject Two ..... 35
3.5 Measured Model Accuracy ..... 35
IV. THE DPCM COMMUNICATION SYSTEM
4.1 Introduction ..... 53
4.2 System Mode1 ..... 53
4.3 Optimum Linear One-Dimensional Prediction ..... 56
4.4 The Optimum Linear Two-Dimensional Predictor ..... 61
4.5 Signal-to-Noise Ratio Measurement ..... 63
v. CONCLUSION ..... 70
APPENDIX ..... 71
REFERENCES ..... 108

## LIST OF ILLUSTRATIONS

Figure Page
2.1A Photograph of 35 mm film transport - front view ..... 8
2.1B Photograph of 35 mm film transport - top view ..... 9
2.1C Photograph of image digitizing system - layout ..... 10
2.1D Photograph of image digitizing system - close-up ..... 11
2.2 Flow diagram for the film advance algorithm ..... 14
2,3 Standard dimensions for 35 man motion picture film ..... 16
2.4 Sketch of video waveform from single scan along perforations of 35 mm film and the associated decision waveform generated ..... 17
2.5 Flow diagram for a general film processing algorithm ..... 19
2.6 Flow diagram for beam intensity control algorithm ..... 21
2.7 Flow diagram for the image digitizing algorithm ..... 22
3.1A Photographs of selected franes from experimental image film strips ..... 28
3.1B Photographs of selected frames from experimental image film strips ..... 29
3.1C
Photographs of selected frames from experimental image film strips ..... 30
3.1 D Photographs of selected frames from experimental image film strips ..... 31
3.1E Photographs fo selected frames from experimental image film strips ..... 32
3.2 Average temporal correlation at a time lag of one frame versus the frame number of the first frame for Subject One ..... 34
3.3 Average temporal correlation at a time lag of one frame versus the frame number of the first frame for Subject Two ..... 36
3.4A Correlation in x-direction for (a) subject one (b) subject two ..... 40
3.4B Correlation in y-direction for (a) subject one (b) subject two ..... 41

## Page

3.4 C Correlation in time direction for (a) subject one (b) subject two ..... 42
3.4D Correlation in $x$-y-direction for (a) subject one (b) subject two ..... 43
3.4 E Correlation in $x-t$ direction for (a) subject one (b) subject two ..... 44
3.5A Predicted and computed diagonal autocorrelation functions for (a) subject one (b) subject two ..... 47
3.5B Predicted and computed diagonal autocorrelation functions for (a) subject one (b) subject two ..... 48
3.6A Predicted and computed diagonal autocorrelation functions for (a) subject one (b) subject two ..... 51
3.6B Predicted and computed diagonal autocorrelation functions for (a) subject one (b) subject two ..... 52
4.1 A DPCM communication system ..... 54
4.2 $\sigma e^{2} / \sigma x^{2}$ versus prediction coefficient for one-dimen- sional linear predictor ..... 574.3 Signal-to-noise Ratio (SNR) versus predictioncoefficient for one-dimensional linear predictor59
4.4 SNR degradation from optimum versus prediction coef- ficient deviation from optimum for one-dimensional predictor ..... 60
4.5 Nearest previous samples for $\mathrm{x}_{\mathrm{io}}$ used for prediction ..... 64
I Exponential constants for least square fitted curves and the resulting error measure . . . . . . . . 45
Model accuracy ( $\mathrm{D}=-10 \log _{10}$ (MSE)) in predicting dia- gonal autocorrelation based on (a) exponentially fitted curves (b) computed data points ..... 49III Signal-to-noise ratios predicted from model andmeasured for PDCM communication systems operating onsubject one and two66
IV Summary of exponential coefficients and measured correlations used in this study ..... 69

## ACKNOWLEDGEMENT

I would like to express my gratitude for the encouragement and assistance I received from my research supervisor, Dr. G.B. Anderson I would also like to thank Messrs. M. Koombes and J. Stuber whose technical assistance made this work possible, Mr. H.H. Black for the image reproductions used in this thesis, and Miss N. Duggan for typing the thesis.

I also acknowledge the financial assistance received from the National Research Council of Canada through Grant NRC A-7994.

## I. INTRODUCTION

### 1.1 Motivation of Study

It is a well known fact that picture images contain: a large amount of redundancy and therefore any channel used to transmit these images must necessarily operate well below the maximum capacity for that channel. Therefore some form of source encoding should be possible which would reduce the bandwidth requirement for transmitting a source and thereby make more efficient use of the communication channel. This concept has motivated a wealth of experiemnts studying possible source encoding techniques for non-time varying or single frame images. The major reason for the lack of experimentation with time varying images have been; 1) the added complexity of analysis when a third dimension is added to the situation, 2) the great difficulty in obtaining good quality experimental source images, and 3) the already high cost, In terms of both time and expense, of single frame processing is multiplied when a larger number of frames are used.

With the development of more sophisticated technology; most notably faster, more reliable, and less expensive computing facilities; it has recently become more practical to consider extending known image processing techniques to treat time varying images. However, to date very little quantitative work has been carried out in this area. Most work that has been reported has emphasized subjective comparison of results obtained from using various ad hoc redundency reduction techniques.

For these reasons, the motivation for the work in this thesis has been to develop an experimental computer based system capable of handling motion pictures and provide an appropriate data base for studying time varying images, as well as begin a more methodological study of
time varying image coding techniques.

### 1.2 Review of Related Work

When any physical phenomona is to be studied, it is usually convenient to use a model, involving only a few essential parameters, to describe that phenomona. When a model which adequately describes the phenomona has been chosen, analysis of that phenomona becomes more tractible. When the physical phenomona is to be modeled as a random process, the mathematical description of that process will be a statistical one.

Kretmer [1] was one of the first to measure some of the statistics of a video signal produced by a television camera. These statistics included the first order probability distribution of the amplitude of the video signal, the first order probability distribution of the error amplitude resulting from using a linear prediction scheme, and the autocorrelation function of a typical signal. More significantly, Kretzmer was one of the first to measure the autocorrelation between two adjacent frames for a time varying image.

Schreiber [2] has measured the second and third order probability distributions of the video signal amplitude as well as some measures of the second and third order entropies of typical picture images. Limb [3] has measured some first to fourth order entropies and concluded that most redundancy is associated with the second order probability density function for low detail images while higher order statistics are responsible for the redundancies in medium and high detail images. The time dimensions.was studied more seriously by Wallace [4] and by Seyler [5,6] who measured the first order probability distributions for television frame differences.

Measurement of some of the statistics of video images made it possible to develop analytically tractible models which were considered to be reasonable descriptions of these images. Franks [7] proposed a model for random picture images and derived expressions based on this model for the second order statistics. One of the expressions was the power spectral density which was found to be expressible as a product of three components each separately arising from the influence of point to point, line to line, and frame to frame correlation. This model was the stationary, wide sense Markov model which is commonly used today.

For his model, Franks assumed separability of the effects of the dimensions of the sequence. This necessarily produced a separable autocorrelation function. Kretzmer's work [1], however, suggested, at least in the spacial dimensions, that a nonseparable form might be more appropriate. This question will be discussed further in this thesis.

As has been previously mentioned, a wealth of literature exists on redundancy removal techniques for still or single frame images. Of these studies, the more recent ones have been most notably concerned with linear transformations and block quantization [8] - [11], and differential pulse-code modulation (DPCM) encoders [12] - [15].

In the area of redundancy removal techniques applied to time varying images, much less work has been done [16] - [19]. These works can be considered to be mainly ad hoc experiments since most researchers have applied purely subjective criterion for selecting their encoding methods and also for evaluating the results. Mounts [17], however, has applied a form of DPCM encoding whereby he used the sample in the immediately previous frame as the predicted value and then encoded the difference signal. In this thesis we determine the effect of coding
dimensionality and data model on DPCM coding efficiency for experimental data.

### 1.3 Scope of Thesis

The purpose of this study is to obtain a high quality data set which can be used for subsequent studies in image coding techniques and other communication system studies, to measure the autocorrelation function of selected time varying monochromatic images and to examine the ability of certain historical models to describe the autocorrelation function, and, finally, to measure the signal to noise ratio (SNR) of DPCM systems when different types of prediction schemes are used.

In chapter II the digitization system used to obtain a high quality data set is discussed. The system design requirements are presented, the evolution of the system into its final form is discussed, and the format of the digitized images as stored on computer magnetic tape is described.

In Chapter III a number of commonly used models for time varying images are discussed. The autocorrelation functions for two such images are measured and compared to the model descriptions in order to evaluate the models.

In Chapter IV several DPCM communication systems are examined in order to evaluate the effect of using models in determining theoretical system performances as compared to the measured performance with real data and to evaluate the effect of increased dimensionality on the system performance.

## II. THE IMAGE DIGITIZER

### 2.1 Introduction

In experimental studies it is necessary to have both an experimental system and also the material or data on which the experiments will be performed. If the picture material or data can be procured from an external source, then we need be concerned only with procuring or building an experimental system. If, as in the case of this study, the test data is not available in a form which can be used directly, then the preparation or interfacing of the picture material can become a significant part of the project. The main purpose of this chapter will be to discuss the experimental picture processing system developed to digitize and study time dependent monochromatic images.

A primary objective in designing a high quality digitizing system is to achieve uniform sampling. For example, it would be impossible to differentiate between motion caused by translation of an object in an image and motion between frames caused by translation of the sampling points during digitization of the time varying image data. Similarly, errors would occur from non-uniform sampling in the spacial dimensions.

Maintaining the uniformity of sampling in the spacial dimensions had already been provided for prior to the beginning of this project in that a digitizing system for single frame images had already been built. This system consisted of a precision flying spot scanner operated under the control of a Supernova minicomputer. The minicomputer controls the position of the spot on the screen of the display and its duration. This light spot is then focused on an image transparency
and the resulting modulated light signal collected in a photomultiplier tube. The video signal produced by the photomultiplier is integrated for the duration of the spot and this level is then digitized and transmitted to the minicomputer for storage on magnetic tape. The problem then remaining to obtain digitized time varying images was a matter of applying this single frame processing to a sequence of frames in such a way as to obtain precise interframe registration.

At first it was felt that the most simple solution might be to apply software registration techniques. However this would require benchmarks to be added to each frame so that translational and rotational errors could be detected. Then individual frames could be corrected to some reference orientation provided the errors were small and the size of the images digitized were larger than the desired image size. It would then also be required to sample the image at a rate sufficiently high to make interpolation errors negligible when the reference sample points fell between the points actually sampled. This, of course, would still not solve the redisplay problem. This feature would be necessary in future works involving subjective evaluation of results produced from applying image processing techniques. Redisplaying of the images would also require good quality interframe registration. In this case, the film would have to be held in a very accurate position while the film was being exposed. Realizing this need, it was proposed that it would be possible to use only one device for both scanning and redisplay purposes. Then software registration could be dispensed with entirely.

### 2.2 The Film Transport and Hardware

Photographs of the mechanical film transport designed to
provide high precision interframe registration and of the digitizing system are shown in figure 2.1. Lateral registration is performed by two sets of two rolling bearings located at either end of the image plane. The two bearings in each set are separated by exactly the width of the film. When the film passes between the bearings, the edges of the film ride on the bearings keeping the film exactly centred between them.

The film is kept flat in the image plane by making use of the natural curvature of the film. The film is inserted in the transport in such a way that the curvature would cause the film to flex out of the image plane toward the photomultiplier or back side of the transport. A smoothly polished pressure plate is then applied to this side of the film to keep it in the image plane: On the opposite side of the film to the pressure plate, the film rides on a set of rails situated at each edge of the film. The purpose of using rails on one side of the film is to reduce the force required to advance the film by reducing the friction.

The width of the transport body near the film aperture was dictated by space constraints imposed by the optics of the digitizing system. The transport was required to present an image to a plane between two lenses which had to be closely spaced to obtain the desired optical focus.

Two light tight shutters are provided as well as a base for subsequent mounting of solenoids to operate them. These shutters, in combination with a light tight lid, will make it possible to use unexposed film in the transport for redisplaying time varying images. On1y one shutter need be opened for redisplay purposes but both must be open to allow light to pass through while digitizing the image.

The film is driven through the image plane by using a sprocketed wheel, such is normally used in 35 mm projectors, to pull the



Figure 2.1B photograph of 35 mm film transport - top view



Figure 2.1D Photograph of image digitizing system - Close-up
film. The film is then drawn from the source reel (marked with an 's' in figure 2.1) which maintains back tension via a friction clutch; guided by two small spools into the image plane; and drawn across the film aperture onto the drive sprocket. From here the film is loaded onto a take-up spool (marked with a 'D' in figure 2.1) driven by a slipping drive belt. The slipping belt drive allows for the change in diameter of the take-up spool as more film is wound onto it and is also driven from the same drive shaft which drives the sprocket.

The driving power for the system is supplied by a small stepping motor which advances exactly fifteen degrees per step. This step size is further reduced through a high precision gear train to about 0.035 degrees rotation of the drive sprocket per step of the stepping motor. Since one full frame advancement requires ninety degrees of rotation of the drive sprocket, about 2800 steps are needed to advance one full frame. Note that this gives a possible interframe registration accuracy of about one tenth of a sampling interval when the image is sampled on a 256 by 256 grid.

After having built the apparatus as described above, a number of tests were performed to check for possible sources of error. It was discovered that the nature of the load on the motor was such that steps would be missed occasionally at some stepping rates and that the motor could only be started at low rates. It was also found that, once the motor had been started, the stepping rate could be advanced slowly to the maximum speed of the motor.

To overcome this problem, a shaft encoder was built and mounted on the stepping motor. This encoder and the associated digital hardware determines the direction and size of any motion of the motor and
increments or decrements a step counter accordingly. In this way it was possible to use computer control to obtain proper film advancement. The computer algorithm used for this is shown in figure 2.2. The step counter is first cleared (N10C 47 instruction on the Supernova). Then a number of pulses equal to the number of steps to be advanced is issued from the computer (N1OP 47). At the completion of the sequency of pulses, the number in the step counter is read into the computer (DIA -, 47; where - is the desired accumulator) and compared to the desired number of steps. If some steps have been missed, the step counter is again cleared and the number of missed steps are reissued. This process continues until no steps have been missed.

The second problem, that of having to start at a low stepping rate, is really a non-problem if the amount of time taken to advance the film is unimportant. .However, it was found that the time required at the highest stepping rate at which the motor would start was about 75 seconds per frame. This means that about six hours would be required for a three hundred frame film segment. For this reason, an acceleration function was programmed to accelerate the motor to optimum speed and then decelerate it again near the end of the advance. The deceleration was added in case overshoot was possible. With this ramping function the film advance time for three hundred frames was reduced to 1.5 hours.

### 2.3 Interframe Registration Technique

The mechanical system as it has been described thus far is more than adequate for obtaining the required interframe registration precision between any two adjacent frames of a film sequence. However, when the number of frames to be registered becomes large, small registration errors which are insignificant between two adjacent frames could


Figure 2.2 Flow diagram for film advance algorithm
accumulate, resulting in a large overall misregistration. For this reason it is desireable to have some references on the film so that, when the cummulative error begins to grow, it can be corrected.

Conventional motion picture equipment provides for frame registration while filming and projecting motion pictures. The references used are the perforations located along the edges of the film which draw the film through the camera on projector. As shown in figure 2.3, the motion picture industry standards [20] require a very accurate location of these perforations with respect to the film image.

The aperture size of the film transport was increased allowing the performations along one edge of the film to be scanned by the flying spot scanner. A sketch of the output video waveform thus generated is shown by the solid line in figure 2.4. The high level corresponds to light passing through the perforations "and the low level corresponds to light passing through the film. The spikes at the transitions can best be explained by light being scattered when striking the edges of the perforations.

Based on the characteristics of the output video waveform, a scheme was devised to determine the locations of the edges of the perforations, with a very low sensitivity to variations in flying spot intensity and opacity of the film. The algorithm used first determines the mean of the video waveform. Then all values of the waveform which are above the mean are set to a logical high level and all those below the mean are set to a logical low level, resulting in the square waveform shown by the dashed line in figure 2.4. Transitions from a logical high level to a logical low level and vice versa are taken to correspond to an edge of a perforation of the film.


| Dimensions | Inclies | Milinmeters |
| :---: | :---: | :---: |
| A Fihm width | $1.377 \pm 0.001$ | $34.975 \pm 0.105$ |
| 3 Perforation pitch | $0.1870 \pm 0.0005$ | $4.7 .30 \pm 0.6113$ |
| C. Perforation width | $0.1100 \pm 0.00009$ | $2.794+0.010$ |
| D Perforution height | $0.0750 \pm 0.000 .4$ | $1.981=0.010$ |
| E Edere to perlotation | $0.079 \pm 0.002$ | $2.01 \pm 0.05$ |
| $F$ Wiubh between pertorations | $0.969 \pm 0.602$ | $25.37 \pm 0.05$ |
| G. Perfonaton skewness | 0.001 12:0x | 0.03 t:1:x |
| I. 100 consectave perforation pieh intervals | $18.700 \pm 0.015$ | $4 \% .98=0.38$ |
| $R$ Ratias of perforator fillet | $0.020=0.001$ | $0.51 \pm 0.08$ |

Figure 2.3 Standard dimensions for 35 mm motion picture film


Figure 2.4 Sketch of video waveform from single scan along perforations of 35 mm film and the associated decision waveform generated.

The algorithm described above was implemented digitally. Sampling was done at a much higher rate than that used to sample the film images in order to provide more than sufficient interframe registration accuracy. Using this technique it was found that the location of an edge could be determined repeatedly over a large range of film segments and with scanning intensities varying from saturation to very low levels.

In order to process a section of film, the algorithm shown in figure 2.5 is used. Before any processing takes place, the position of the first perforation is determined as an origin position. Subsequent frames are then properly registered by aligning their perforations with the original perforation position.

### 2.4 Automatic Intensity Control for the Flying Spot Scanner

Once the interframe registration techniques described above had been developed, it was possible to maintain uniform sampling in all three image dimensions. There only remained the problem of guaranteeing consistent quantization with time.

Sources of error in quantization could arise due to time drifts in the electronic hardware. To determine the possible causes and the degree of drift over time, experiments were conducted using a single frame placed in the digitization aperture and then scanned repeatedly. Numerous parameters were measured for both short term and long term variations. The only variation detectable was in the intensity of the beam spot of the flying spot scanner. This variation was not detectable on a frame by frame basis, but a gradual decrease of intensity, was measured. In 3.5 hours the intensity of the beam decreased from normal digitizing intensity to zero.


Figure 2.5 Flow diagram for a general film processing. algorithm.

The beam intensity is directly measurable as a result of the addition of the perforation detection scheme discussed earlier. Since the location of the perforation is known, it is possible to scan a number of points in the perforation. The digitized values thus obtained can be averaged to obtain a measure of the beam intensity. Having measured the beam intensity, the computer can transmit a digital number to a digital/analog converter which controls the gain of the video amplifier of the flying spot scanner. The computer algorithm used in controlling the beam intensity is shown in figure 2.6 . The computer hunts for a prespecified intensity, increasing the beam intensity when it is too low and decreasing it when it is too high. When the correct intensity is found, the algorithr is terminated.

### 2.5 The Digitized Images

The complete flow diagram for the image digitizing algorithm is shown in figure 2.7. This algorithm incorporates all the features of the system described previously in this chapter. All the subroutines written to implement the algorithm are included in the appendix. Care must be exercised if modifying calls to routines since two different calling sequences are employed. Some routines are written in fortran and calls to and from them use the normal fortran calling sequence in Data General's DOS fortran package. Assembler routines called from fortran routines and assembler routines calling fortran routines will use the DOS sequence for these calls. However linkages between two assembler routines use normal accumulator parameter passing techniques and not the DOS sequence. These facts are well documented in the routines.

For subsequent experimentation, digitized images consisting


Figure 2.6 Flow diagram of beam intensity control algorithm


Figure 2.7 Flow diagram for the Image Digitizing Algorithm
of 256 by 256 eight bit samples per frame were stored on magnetic computer tape compatible with the IBM system. The information is stored in two eight bit image samples per word with the first byte as the odd numbered sample and the second byte as the even numbered sample. These words are stored in blocks of 2048 words corresponding to sixteen lines from the image. Sixteen blocks therefore make up a single frame. Successive frames are delimited by end of file markers with an additional end of file being placed before the first frame. No end of file was placed after the last frame but this can easily be added for situations where it is needed.

## III. MODELS FOR IMAGES

### 3.1 Introduction

In this chapter, we consider the question of statistical modelling of monochromatic time varying images. A measure is presented for comparing the validity of certain popular models which assume data stationarity. Non-stationary models would probably describe the physcial phenomona more closely. However the criterion of analytic tractability makes the choice of a stationary model imperative.

In section 3.2 , the models of interest are presented and discussed relative to their analytic tractability. The ability of these models to represent picture data is then evaluated as a result of experiments on selected test data.

### 3.2 Stationary Models for Monochromatic Time Varying Images

A monochromatic time varying image constitutes a three dimensional source which can be specified by its grey level $u(x, y, t)$ at each of the coordinates ( $x, y, t$ ) where $x$ and $y$ are the spacial coordinates and $t$ is the time coordinate. Sampling and quantization of the continuous source function $u(x, y, t)$ generates a three dimensional field of discrete samples $u^{\prime}\left(x_{i}, y_{j}, t_{k}\right)$. An ensemble of such images can then be modelled by interpreting $u^{\prime}\left(x_{i}, y_{j}, t_{k}\right)$ as a random field.

For simplicity it can be assumed that the random field is a zero mean field. This can always be made true by determining the mean intensity of the field and then subtracting this mean from every point in the field. That is if

$$
\begin{equation*}
\bar{u}=E\left\{u^{\prime}\left(x_{i}, y_{j}, t_{k}\right)\right\} \tag{3.2.1}
\end{equation*}
$$

where $E\{\cdot\}$ denotes the expectation operator, then a zero mean field $u^{\prime}{ }_{o}\left(x_{i}, y_{j}, t_{k}\right)$ can be generated as

$$
\begin{equation*}
u^{\prime}{ }_{o}\left(x_{i}, y_{j}, t_{k}\right)=u^{\prime}\left(x_{i}, y_{j}, t_{k}\right)-\bar{u} \tag{3.2.2}
\end{equation*}
$$

Previous experiments [7] have led to the adoption of the GaussMarkov field as the most widely used model for pictorial data. This stationary model is specified in terms of its autocorrelation function as

$$
\begin{equation*}
R\left(x, x^{\prime}, y, y^{\prime}, t, t^{\prime}\right)=\sigma^{2}\left\{\exp \left[-\alpha\left|x-x^{\prime}\right|-\beta\left|y-y^{\prime}\right|-\gamma\left|t-t^{\prime}\right|\right]\right\} \tag{3.2.3}
\end{equation*}
$$

where $\sigma^{2}$ is the signal power of the source and $\alpha, \beta$ and $\gamma$ are parameters. Defining $\Delta x=x-x^{\prime}, \Delta y=y-y^{\prime}$, and $\Delta t=t-t^{\prime}$, (3.2.3) can be rewritten as

$$
\begin{equation*}
R(\Delta x, \Delta y, \Delta t)=\sigma^{2}\{\exp [-\alpha|\Delta x|-\beta|\Delta y|-\gamma|\Delta t|]\} \tag{3.2.4}
\end{equation*}
$$

In what follows, we normalize our experimental data so ..that our interest lies with

$$
\begin{equation*}
R_{o}(\Delta x, \Delta y, \Delta t)=\frac{R(\Delta x, \Delta y, \Delta t)}{\sigma^{2}}=\exp [-\alpha|\Delta x|-\beta|\Delta y|-\gamma|\Delta t|] \tag{3.2.5}
\end{equation*}
$$

The primary attraction of the Gauss-Markov field model is the analytic simplicity arising from the separability of the autocorrelation function into a product of one dimensional autocorrelation functions
where

$$
\begin{equation*}
R_{0}(\Delta x, \Delta y, \Delta t)=R x_{0}(\Delta x) R y_{0}(\Delta y) R t_{0}(\Delta t) \tag{3.2.6}
\end{equation*}
$$

$$
\begin{aligned}
\operatorname{Rx}_{0}(\Delta x) & =\exp [-\alpha|\Delta x|] \\
\operatorname{Ry}_{0}(\Delta y) & =\exp [-\beta|\Delta y|] \\
\operatorname{Rt}_{0}(\Delta t) & =\exp [-\gamma|\Delta t|]
\end{aligned}
$$

In addition, other analytic simplifications can result from the condition that all statistical information about a sample point is contained in
adjacent previous samples.
Consider what separability means for a field in two spacial
dimensions

$$
\begin{equation*}
R_{0}(\Delta x, \Delta y)=R x_{0}(\Delta x) R y_{0}(\Delta y) \tag{3.2.7}
\end{equation*}
$$

Assume a homogeneous field where $\mathrm{Rx}_{\mathrm{o}}(\Delta)=\mathrm{Ry}_{\mathrm{o}}(\Delta)$, then

$$
R_{0}(\Delta, \Delta)=\left[\operatorname{Rx}_{0}(\Delta)\right]^{2}=\left[\operatorname{Ry}_{0}(\Delta)\right]^{2}
$$

or

$$
R_{0}(\Delta, \Delta)=\operatorname{Rx}_{0}(2 \Delta)=\operatorname{Ry}_{0}(2 \Delta)
$$

From this it can be seen that, for this model, the autocorrelation function decreases more rapidly with distance along the spacial diagonal than along either of the spacial axes. Intuitively this does not seem reasonable since the orientation of the spacial axes with respect to the image ts somewhat arbitrary. Also, previous work [15] has shown the correlation along the spacial diagonal as predicted by this model to be much lower than the value measured for experimental images. Kretzmer [1] also supports this arguement by finding that, in general, there was no preferred direction of correlation for the images be studied.

On the basis of these arguements, another stationary model in two spacial dimensions is obvious with

$$
\begin{equation*}
R_{0}(\Delta x, \Delta y)=\exp \left[-\left(\alpha^{2}(\Delta x)^{2}+\beta^{2}(\Delta y)^{2}\right)^{1 / 2}\right] \tag{3.2.8}
\end{equation*}
$$

Note that for $\mathrm{x}=0$ or $\mathrm{y}=0$, this model reduces to the same form as the Gauss-Markov field. This means that the parameters $\alpha$ and $\beta$ are identifiable with the $\alpha$ and $\beta$ of the Gauss-Markov field model. Equation (3.2.8) may be extended to model a field generated by a time varying
image. In this case we have

$$
\begin{equation*}
R_{o}(\Delta x, \Delta y, \Delta t)=\exp \left[-\left(\alpha^{2}(\Delta x)^{2}+\beta^{2}(\Delta y)^{2}+\gamma^{2}(\Delta t)^{2}\right)^{1 / 2}\right] \tag{3.2.9}
\end{equation*}
$$

The remainder of this chapter is devoted to the study of the models proposed by (3.2.5) and (3.2.9). The objective in what follows will be to obtain a measure of the closeness of fit of these models to selected experimental data.

### 3.3 Closeness of Fit Criterion

One aim of this thesis is to measure the accuracy of certain popular models in describing image data. In order to measure a model's closeness of fit to experimental data, a meaningful measure must first be chosen. The measure chosen here is the mean squared error (MSE) between the experimentally determined autocorrelation function and the mode 1 with parameters $\alpha, \beta$ arrd" $\gamma$ "adjusted to 'provide anleast squares fit to the experimental data. A more compact measure is defined by

$$
\begin{equation*}
D=-10 \log _{10}(\mathrm{MSE}) \tag{3.3.1}
\end{equation*}
$$

For example, if the MSE is 0.0001 , the measure is $D=40.0$. From (3.3.1) it can be seen that $D_{1}>D_{2}$ when $M S E_{1}<\operatorname{MSE}_{2}$.

## $3: 4$ The Source Images

Two professionally produced 35 millimeter film strips were chosen as subjects for experimental computations. Figure 3.1 contains selected frames from these strips together with the subject number and a number corresponding to the location of the frame in the film strip. A large amount of degradation has occurred in the reproduction of these pictures for this thesis. These subjects were chosen subjectively on the basis of the amount of spacial detail and type of motion exhibited.

subject one, frame 1

subject one, frame 34

subject one, frame 68

subject one, frame 18

subject one, frame 51

subject one, frame 84

Figure 3.1A Photographs of selected frames from experimental image film strips

subject one, frame 100

subject two, frame 18

subject two, frame 51

subject two, frame 1

subject 2 , frame 34

subject two, frame 68

Figure 3.1B Photographs of selected frames from experimental image film strips

subject two, frame 84

subject two, frame 118

subject two, frame 151

subject two, frame 101

subject two, frame 134


Figure 3.1C Photographs of selected frames from experimental image film strips

subject two, frame 184

subject two, frame 218

subject two, frame 251

subject two, frame 201

subject two, frame 234

subject two, frame 268

Figure 3.1D Photographs of selected frames from experimental image film strips


Figure 3.1E Photographs of selected frames from experimental image film strips

It is believed the subjects chosen represent a good variation on subject detail and motion. Subject one and two consist of 100 and 300 frames, respectively, at 25 frames per second.

### 3.4.1 Subject One

Subject one was chosen primarily because of the specialized nature of the motion involved. Only a small portion of the image undergoes motion and this motion is executed quite quickly. Also, for the first half of the sequence, this motion has a periodic acceleration and deceleration. The background, which remains stationary throughout the sequency of frames, consists of a stock exchange board on which a grid has been drawn containing names and numbers. The lettering and grid are of high spacial detail and the board spaces are of low spacial detail.

Within"the "first ten frames, a woman's head and shoulders pass in front of the camera, but outside the depth of field and therefore out of focus. This constitutes a low spacial detail, moving image. For the remainder of the sequence, very little of the head remains visible, while a hand erases some of the numbers on the board and then enters new numbers in their place.

The erasure of the board takes place from frame fifteen to frame fifty-one. This section is temporally quite periodic in nature. Figure 3.2 demonstrates this periodicity. This diagram is a plot of the average temporal correlation at a time lag of one frame as measured along the sequence of frames. The rapid variation in the function of figure 3.2 from frame fifteen to frame fifty-one corresponds to rapid hand motion. The dashed line in figure 3.2 is the mean temporal correlation at a time lag of one frame over the entire 100 frame film sequence.


Figure 3.2 Average temporal correlation at a time lag of one frame versus the frame number of the first frame for subject one

### 3.4.2 Subject Two

Subject two was chosen because of its wide variety of content. Both the background and the foreground vary with time and contain varying amounts of spacial detail. This image also contains varying rates of motion, from rapid to almost stationary. The background consists of both high detail and low detail areas and changes as the camera is panned to follow the foreground objects.

The motion consists of two men walking up a set of stairs into the view of the camera which is mounted on the landing. Once the men reach the landing, one of them pauses as the other continues around him, at the edge of the film, and then proceeds up a second flight of stairs. Once the second man has passed out of view of the camera, there is a period of very little motion in which the man on the landing casually surveys his surroundings. This is a asegment of very high correlation, often reaching 0.99. The man on the landing then turns and rapidly mounts the second flight of stairs as the camera pans to follow him, thus completing the entire film sequence.

Again these details can be observed in figure 3.3 which is a plot of the average temporal correlation at a lag of one as measured along the 300 frame film segment. The dashed line indicates the mean temporal correlation for a lag of one frame over the entire film segment.

### 3.5 Measured Mode1 Accuracy

To evaluate the models, the autocorrelation functions of the experimental film subjects were computed along five directions in the field;
a) along the x spacial axis



Figure 3.3 Average temporal correlation at a lag of one frame versus the frame number of the first frame for subject two.
b) along the $y$ spacial axis
c) along the spacial diagonal
d) along the time ( $t$ ) axis
e) along the diagonal in the $x-t$ plane.

Then exponential curves were fitted, by a least squares fit method, to each of the computed correlation functions to determine $\alpha, \beta$ and $\gamma$ for the models. The measurement of the autocorrelation functions was performed on the Supernova system while the least squares fit was computed on the IBM $360 / 67$ system.

Defining the sample interval in each of the $x, y$ and $t$ directions of the field to be unity, the desired autocorrelation functions could be computed by evaluating

$$
\begin{align*}
& \rho(k)=\int \frac{1}{\sigma^{2}} \cdot \frac{1}{(L-k) M N} \sum_{\ell=1}^{L-k} \sum_{m=1}^{M} \sum_{n=1}^{N} u^{\prime}{ }_{0}(\ell, m, n) u^{\prime}{ }_{o}(\ell+k, m, n)  \tag{3.5.6a}\\
& \text { for the } \mathrm{x} \text { direction } \\
& \frac{1}{\sigma^{2}} \frac{1}{L(M-k) N} \sum_{l=1}^{L} \sum_{m=1}^{M-k} \sum_{n=1}^{N} u^{\prime}{ }_{o}(l, m, n) u^{\prime}{ }_{o}(l, m+k, n)  \tag{3.5.6b}\\
& \text { for the } \mathrm{y} \text { direction } \\
& \begin{array}{c}
\frac{1}{\sigma^{2}} \frac{1}{(L-k)(M-k) N} \sum_{\ell=1}^{L-k} \sum_{m=1}^{M-k} \sum_{n=1}^{N} u_{o}^{\prime}(\ell, m, n) u^{\prime}{ }_{o}(\ell+k, m+k, n) \\
\text { for the spacial diagonal }
\end{array} \tag{3.5.6c}
\end{align*}
$$


for the time direction

$$
\begin{equation*}
\frac{1}{\sigma^{2}} \frac{1}{(L-k) M(N-k)} \sum_{\ell=1}^{L-k} \sum_{m=1}^{M} \sum_{n=1}^{N-k} u^{\prime}{ }_{0}(\ell, m, n) u^{\prime}{ }_{0}(\ell+k, m, n+k) \tag{3.5.6e}
\end{equation*}
$$

for the $x-t$ diagonal
where

$$
\begin{equation*}
\sigma^{2}=\frac{1}{L M N} \sum_{\ell=1}^{L} \sum_{m=1}^{M} \sum_{n=1}^{N} u^{\prime}{ }_{o}(\ell, m n) u^{\prime}{ }_{o}(\ell, m, n) \tag{3.5.6f}
\end{equation*}
$$

and $L=256, \quad M=256, \quad N=$ number of frames.
However, about 64 hours of continuous computing time would be required to evaluate only a single point of one of these functions for the 100 frame subject. The same evaluation performed on the 300 frame subject would take 192 hours. If fifteen points on each function in (3.5.6) were to be evaluated for both subjects, the total continuous computing time required would be about 192,000 hours or 800 days.

Since this amount of computation time is prohibitive, it was decided to compute the desired correlation data by averaging over a subset of the field. (3.5.6) was modified to reflect this fact with $\rho(k)=\left(\frac{1}{\sigma^{2}} \frac{1}{L_{1} M_{0} N} \sum_{\ell=1}^{L i} \sum_{m=1}^{M} \sum_{n=1}^{N} \dot{u}_{0}^{\prime}{ }_{0}\left(\ell^{\prime \prime}, m^{\prime}, n^{\prime}\right) u^{\prime}{ }_{0}\left(l^{\prime}+k, m^{\prime}, n^{\prime}\right)\right.$ for the x direction

$$
\frac{1}{\sigma^{2}} \frac{1}{L_{o} M_{1} N} \sum_{\ell=1}^{L_{o}^{o}} \sum_{m=1}^{M} \sum_{n=1}^{N} u_{o}^{\prime}{ }_{o}^{\left.\left(\ell^{\prime}, m^{\prime}, n\right) u^{\prime}{ }_{o}\left(\ell^{\prime}, m^{\prime}+k, n\right)\right)}
$$

for the y direction

$$
\begin{aligned}
\frac{1}{\sigma^{2}} \frac{1}{L_{1} 1_{1} N} & \sum_{\ell=1}^{L} \sum_{m=1}^{M} \sum_{n=1}^{N} u^{\prime}{ }_{o}\left(\ell^{\prime}, m^{\prime}, n\right) u^{\prime}{ }_{o}\left(\ell^{\prime}+k, m^{\prime}+k, n\right) \\
& \text { for the spacial diagonal }
\end{aligned}
$$

$$
\frac{1}{\sigma^{2}} \frac{1}{L_{0} M_{0}(N-k)} \sum_{\ell=1}^{L} \sum_{m=1}^{M} \sum_{n=1}^{M} u^{N-k} u^{\prime}\left(\ell^{\prime}, m^{\prime}, n\right) u^{\prime}{ }_{o}\left(\ell^{\prime}, m^{\prime}, n+k\right)
$$

for the time direction

$$
\begin{aligned}
& \frac{1}{\sigma^{2}} \frac{1}{L_{1} M_{0}(N-k)} \sum_{\ell=1}^{L} \sum_{m=1}^{M} \sum_{n=1}^{N-k} u_{0}^{\prime}\left(\ell^{\prime}, m^{\prime}, n\right) u_{o}^{\prime}\left(\ell^{\prime}+k, m^{\prime}, n^{\prime}+k\right) \\
& \text { for the } x-t \text { diagonal }
\end{aligned}
$$

where

$$
\begin{align*}
& \sigma^{2}=\frac{1}{L_{0} M_{o} N} \sum_{\ell=1}^{L} \sum_{m=1}^{M} \sum_{n=1}^{N} u^{\prime}{ }_{o}^{\prime}\left(\ell^{\prime}, m^{\prime}, n\right) u_{o}^{\prime}\left(\ell^{\prime}, m^{\prime}, n\right)  \tag{3.5.7f}\\
& \ell^{\prime}=8 \ell-7, m^{\prime}=8 m-7, L_{0}=32, M_{0}=32, L_{1}=\lceil 1 / 8(256-k)\rceil \text {, } \\
& \text { and } M_{1}=\lceil 1 / 8(256-k)\rceil \text { where }\lceil\cdot\rceil \text { denotes "nearest larger integer". The } \\
& \text { computing time required to evaluate (3.5.7) is about } 300 \text { hours; a much } \\
& \text { more realistic figure. }
\end{align*}
$$

Using (3.5.7), the autocorrelation function was computed along each of the five chosen directions for both subjects. This was repeated for the $x$ direction for subject one using two other subsets of the field, yielding results which differed numerically by less than one percent, thus verifying the soundness of this modified approach. The computed autocorrelation functions for both subjects along each of the directions chosen are shown in figure 3.4. The $x^{\prime}$ s denote the computed correlation values and the solid line is the least squares fit to those points of an exponential. Remember that in one dimension (3.2.5) and (3.2.9) reduce to the same function. The values of the exponents obtained by fitting exponentials to the data points are tabulated in table $I$ along with the mean squared errors for these fitted curves.

It is difficult to compare these results to those obtained by other researchers due to the differences in the picture data analyzed. The exponential coefficients are a function of spacial detail and degree of motion of the image, the sampling rate in each of the dimensions, the number of levels of quantization, and the amount of noise in the digitized image. However, these results are consistent in form with previous results [1, 7, 9, 15].

(a)

(b)

Figure 3.4A Correlation in $x$-direction for (a) subject one (b) subject two

(a)

(b)

Figure 3.4B Correlation in y-direction for (a) subject one (b) subject two



Figure 3.4C Correlation in time direction for (a) subject one (b) subject two


(b)

Figure 3.4D Correlation in $x-y$ direction for (a) subject one (b) subject two



Figure 3.4E Correlation in $x-t$ direction foro (a) subject one (b) subject two

| SUBJECT | DIRECTION | EXPONENT | $\begin{gathered} -10 \log _{10} \\ (\mathrm{MSE}) \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| ```Subject 1 (100 frames)``` | x | 0.2297 | 20.63 |
|  | y | 0.1921 | 16.49 |
|  | t. | 0.0519 | 18.90 |
|  | xy | 0.3051 | 19.99 |
|  | xt | 0.3793 | 23.09 |
|  | x | 0.0551 | 25.89 |
|  | y | 0.1128 | 29.73 |
| $\begin{array}{r} 2 \\ (300 \end{array}$ | t | 0.0610 | 20.57 |
|  | xy | 0.1171 | 29.67 |
| $\begin{aligned} & (300 \\ & \text { frames) } \end{aligned}$ | xt | 0.1024 | 21.46 |

TABLE I Exponential constants for least square fitted curves and the resulting error measure.

As can be seen from figure 3.4, the computed autocorrelation function initially falls off more rapidly than the fitted exponential model and less rapidly later. This difficulty in fitting an exponential to the data suggests that some model other than an exponential would be more accurate in describing pictorial data.

Having computed the autocorrelation function of the two subjects and fitted exponential curves to the computed data, it is a straightforward matter to evaluate the proposed models as to the closeness of fit to the experimental data. Using the curves fitted to the computed data along the $x, y$ and $t$ axes, together with the proposed models, the correlation along the $x-y$ and $x-t$ diagonals $c a n$ be predicted. These predicted diagonal autocorrelation functions are given in figure 3.5 for both subjects along with the actual computed diagonal correlation functions., Measures of closeness.af. fit for the two models and both subjects are summarized in table IIa.

From table IIa it can be seen that the model which most closely matches the experiment data for both diagonal directions is difficult to determine. What table IIa indicates is that a compromise model between (3.2.5) and (3.2.9) is appropriate, namely

$$
\begin{equation*}
R_{0}(\Delta x, \Delta y, \Delta t)=\exp \left[-\left(\alpha^{2}(\Delta x)^{2}+\beta^{2}(\Delta y)^{2}\right)^{1 / 2}-\gamma|\Delta t|\right] \tag{3.5.8}
\end{equation*}
$$

Now defining

$$
\begin{align*}
\mathrm{Rt}_{0}(\Delta t) & =\exp [-\gamma|\Delta t|]  \tag{3.5.9a}\\
\mathrm{Rx}_{0}(\Delta x) & =\exp [-\alpha|\Delta x|]  \tag{3.5.9b}\\
\mathrm{Ry}_{0}(\Delta y) & =\exp [-\beta|\Delta y|] \tag{3.5.9c}
\end{align*}
$$

(3.5.8) can be rewritten as

$$
\begin{equation*}
R_{0}(\Delta x, \Delta y, \Delta t)=R t_{0}(\Delta t) \exp \left[-\left(\left[\ln \left(\operatorname{Rx}_{0}(\Delta x)\right)\right]^{2}+\left[\ln \left(\operatorname{Ry}_{0}(\Delta y)\right)\right]^{2}\right)^{1 / 2}\right] \tag{3.5.10}
\end{equation*}
$$


(a)

(b)

Figure 3.5A Predicted and computed diagonal autocorrelation functions for (a) Subject one (b) Subject two

(a)


Figure 3.5B Predicted and computed diagonal autocorrelation functions for (a) Subject one (b) subject two

|  |  | Separable | NonSeparable |
| :---: | :---: | :---: | :---: |
| Subject | x, t | 24.92 | 20.48 |
| (100 frames) | x, y | 24.61 | 48.55 |
| Subject | $\mathrm{x}, \mathrm{t}$ | 28.54 | 23.42 |
| (300 frames) | $\mathrm{x}, \mathrm{y}$ | 19.58 | 33.84 |

(a)

| Subject | Separable | Non- <br> Separable |  |
| :--- | :---: | :---: | :---: |
| Subject <br> 1 <br> $(100$ frames $)$ | $x, y$ | 24.98 | 23.37 |
| Subject <br> 2 | $x, t$ | 18.23 | 24.53 |
| $(300$ frames $)$ | $x, y$ | 23.17 | 27.02 |

(b)

Table II Model accuracy ( $D=-10 \log _{10}$ (MSE)) in predicting diagonal autocorrelation based on (a) exponentially fitted curves (b) computed data points
and similarly (3.2.9) and (3.2.5) can be rewritten, respectively, as $R_{0}(\Delta x, \Delta y, \Delta t)=\exp \left[-\left(\left[\ln \left(R x_{0}(\Delta x)\right)\right]^{2}+\left[\ln \left(R y_{o}(\Delta y)\right)\right]^{2}+\left[\ln \left(R t_{0}(\Delta t)\right)\right]^{2}\right)^{1 / 2}\right]$ (3.5.11)

$$
\begin{equation*}
R_{0}(\Delta x, \Delta y, \Delta t)=R_{0}(\Delta x) R y_{o}(\Delta y) R t_{0}(\Delta t) \tag{3.5.12}
\end{equation*}
$$

Now consider substituting the computed experimental values for the autocorrelation function for $R x_{0}(\Delta x), R y_{0}(\Delta y)$ and $R t_{0}(\Delta t)$ in equations (3.5.10), (3.5.11) and (3.5.12). Through this substitution and computation we determine whether or not the experimentally measured autocorrelation takes a separable or non-separable form.

Figure 3.6 is a presentation of the diagonal autocorrelation functions derived using (3.5.10), (3.5.11) and (3.5.12), and the experimentally computed autocorrelation along the two diagonal directions for both subjects. The corresponding closeness of fit measures are given in table IIb. From table IIb it can be seen that the model which most closely matches the experimental data for both diagonal directions is given by (3.5.8).

(a)


Figure 3.6A Predicted and computed diagonal autocorrelation functions for (a) subject one (b) subject two



Figure 3.6B Predicted and computed diagonal functions for (a) subject one (b) subject two

## IV. THE DPCM COMMUNICATION SYSTEM

### 4.1 Introduction

In this chapter, we determine the effect of the error in using the Gauss-Markov model to describe the source data on estimates of the system performance for a commonly used type of communication system. Due to the assumption of stationarity of the source data, the estimate of the system performance could be poor when the source data is non-stationary. Estimates of the performance of a DPCM communication system using both the optimum, linear, one-dimensional predictor and the optimum, linear, two-dimensional predictor are obtained for selected test data. These estimates are then compared to the computed system performances for that source data.

```
4.2 Sys:tem Model
```

A general model for a DPCM communication system is shown in figure 4.1. The input samples $x_{i}$ are obtained by three dimensional low pass filtering, sampling, and fine quantization of a monochromatic time varying image $u(x, y, t)$. Image sampling is performed by raster scanning successive frames of the image as described in Chapter 2. The bandwidth of the low pass filtering is determined by the width of the spot and the optics of the scanning system.

$$
\text { A linear predictor is used to form a predicted value } z_{i}
$$

which is compared to the input sample $x_{i}$. The error $e_{i}$ is then quantized, coded, and transmitted over the digital channel. The predicted value $z_{i}$ is determined from the previous samples via an equation of the form

$$
\begin{equation*}
z_{i}=\sum_{j=1}^{N} \alpha_{i} y_{i-j} \tag{4.2.1}
\end{equation*}
$$



Figure 4.1 A DPCM communication system
where $y_{i}=x_{i}+q_{i}$ and $q_{i}=S_{i}-e_{i}$, the quantization error [15]. In the receiver, a similar predicted value $\hat{z}_{i}$ is added to the received signal to form the output sample $\hat{x}_{i}=r_{i}+\hat{z}_{i}$, where

$$
\begin{equation*}
\hat{z}_{i}=\sum_{j=1}^{N} \alpha_{j} \hat{x}_{i-j} \tag{4.2.2}
\end{equation*}
$$

In what follows, we consider two situations: optimum, onedimensional, linear prediction based on the immediately previous sample and optimum, two-dimensional, linear prediction based on the nearest three previous samples. By optimum linear prediction is meant the minimization of $E\left\{e_{i}{ }^{2}\right\}$ by the proper choice of the coefficients in (4.2.1). If quantization error is negligible, this criterion is equivalent to minimizing

$$
\begin{equation*}
\sigma e^{2}=E\left\{\left(x_{i}-z_{i}\right)^{2}\right\} \tag{4.2.3}
\end{equation*}
$$

where $z_{i}$ is now given by

$$
\begin{equation*}
z_{i}=\sum_{j=1}^{N} \alpha_{j} x_{i-j} \tag{4.2.4}
\end{equation*}
$$

$\sigma e^{2}$ is called the prediction error variance.
A more convenient form of (4.2.3) for comparing system performance, results from normalizing by $\sigma x^{2}=E\left\{x_{i}{ }^{2}\right\}$;

$$
\begin{equation*}
\frac{\sigma e^{2}}{\sigma x^{2}}=\frac{E\left\{\left(x_{i}-z_{i}\right)^{2}\right\}}{E\left\{x_{i}{ }^{2}\right\}} \tag{4.2.5}
\end{equation*}
$$

Performance of these linear prediction schemes will be measured in terms of the signal-to-noise ratio given by

$$
\begin{equation*}
\mathrm{SNR}=10 \log _{10}\left(\frac{\sigma \mathrm{e}^{2}}{\sigma \mathrm{x}^{2}}\right)^{-1} \tag{4.2.6}
\end{equation*}
$$

### 4.3 Optimum Linear One-Dimensional Prediction

If the input sequence $x_{i}$ is generated from a one dimensional field, then the optimum linear predictor is found by finding the $\alpha$ which minimizes

$$
\begin{align*}
\frac{\sigma e^{2}}{\sigma x^{2}} & =\frac{E\left\{\left(x_{i}-\alpha x_{i-1}\right)^{2}\right\}}{E\left\{\left(x_{i}\right)^{2}\right\}} \\
& =\left(1-2 \alpha \rho+\alpha^{2}\right) \tag{4.3.1}
\end{align*}
$$

where

$$
\begin{equation*}
\rho=\frac{E\left\{x_{i} x_{i-1}\right\}}{E\left\{x_{i}^{2}\right\}}=\frac{R(1)}{\sigma x^{2}} \tag{4.3.2}
\end{equation*}
$$

It follows from (4.3.1) that the optimum prediction coefficient is $\alpha=\rho$, resulting in

$$
\begin{equation*}
\left(\frac{\sigma e^{2}}{\sigma x^{2}}\right)_{\min }=\left(1-\rho^{2}\right) \tag{4.3.3}
\end{equation*}
$$

Although $\alpha=\rho$ yields an optimum linear predictor for onedimensional sources, it is of interest to determine the increase in prediction error variance resulting from using a non-optimum prediction coefficient. Let this non-optimum prediction coefficient be $\alpha$ ' and let $\Delta=\alpha^{\prime}-\rho$ represent the deviation from the optimum prediction coefficient. Then the increase in prediction error variance can be shown to be

$$
\begin{equation*}
\left(\frac{\sigma e^{2}}{\sigma x^{2}}\right)_{\alpha=\alpha^{\prime}}-\left(\frac{\sigma e^{2}}{\sigma x^{2}}\right)_{\alpha=\rho}=\Delta^{2} \tag{4.3.4}
\end{equation*}
$$

If $|\Delta|=0.1$, then the ratio ( $\sigma \mathrm{e}^{2} / \sigma \mathrm{x}^{2}$ ) changes by only 0.01 . This implies that the prediction error variance is relatively insensitive to variations in the prediction coefficient. This is also reflected in the flatness of the curves in figure 4.2 which plot $\left(\sigma e^{2} / \sigma \mathrm{x}^{2}\right)$ versus the prediction


Figure $4.2 \sigma e^{2} / \sigma x^{2}$ versus prediction coefficient for one dimensional linear prediction
coefficient.
Figure 4.3 is composed of curves relating the signal to noise ratio in decibels to $\alpha$ for different values of $\rho$. Although the flatness of the curves varies with $\rho$, these curves exhibit relative insensitivity to $\Delta$ near the optimum $\alpha$.

The degradation in signal to noise ratio from the optimum can be shown to be

$$
\begin{align*}
& 10 \log _{10}\left(\frac{\sigma \mathrm{e}^{2}}{\sigma \mathrm{~m}^{2}}\right)_{\alpha=d i}^{-1}-10 \log _{10}\left(\frac{\sigma \mathrm{e}^{2}}{\sigma \mathrm{~m}^{2}}\right)_{\alpha=\rho}^{-1} \\
& \quad=10 \log _{10}\left(\frac{1-\rho^{2}}{1-\rho^{2}+\Delta^{2}}\right) \tag{4.3.5}
\end{align*}
$$

Figure 4.4 is a presentation of the plots of (4.3.5) for different values of $\rho$. We note that irrespective of $\rho$, the system performance is not greatly effected by using prediction coefficients in a range near the otpimum value. For example, if $\rho$ is 0.9 , then picking $\alpha$ in the range 0.8 to 1.0 will produce a degradation less than 0.23 decibels, or about five percent.

A previous study [15] by K.Y. Chang has examined the change in system performance when the correlation of the input data is varied while $\alpha$ is kept constant. However consideration should be made of the fact that the optimum performance also varies with $\rho$. Thus, where Chang shows a decrease of performance of $13 \%$ for $\alpha=0.89$ and $\rho$ decreased to 0.8 , the optimum performance would also have decreased. From figure 4.4 it can be seen that the degradation from optimum performance is 0.12 decibels or about three percent.

Two sets of experiments were performed to verify the general
nature of the curves shown in figures 4.2 and 4.3 . As can be seen from
OIDH 35 ION OL
7UNOIS

these figures, there is good correspondence between the experimentally determined results and the analytic results.

For the points indicated by an $x$ in figures 4.2 and 4.3 , the one-dimensional data was generated from subject one using successive points in the x-direction. The measured correlation between adjacent samples for this data is 0.9188 . For the points in these figures indicated by a $\Delta$, the one-dimensional data was generated from subject one using successive points in the time direction. The measured correlation between adjacent samples for this data is 0.7501 .

### 4.4 The Optimum Linear Two-Dimensional Predictor

For an input sequence to the DPCM communication system consisting of samples from a two-dimensional field $f(i, j)$, we consider the optimum linear predictor using the nearest three previous samples to predict the next sample. The coefficients in this predictor; $\alpha_{1}, \alpha_{2}$, $\alpha_{3}$; are chosen to minimize

$$
\begin{equation*}
\frac{\sigma e^{2}}{\sigma f^{2}}=\frac{E\left\{\left[f(i, j)-\alpha_{1} f(i-1, j)-\alpha_{2} f(i, j-1)-\alpha_{3} f(i-1, j-1)\right]^{2}\right\}}{E\left\{[f(i, j)]^{2}\right\}} \tag{4.4.1}
\end{equation*}
$$

Defining $\quad \rho_{1}=\frac{E\{f(i, j) f(i-1, j)\}}{E\left\{[f(i, j)]^{2}\right\}}$,
$\rho_{2}=\frac{E\{f(i, j) f(i, j-1)\}}{E\left\{[f(i, j)]^{2}\right\}}$,

$$
\begin{equation*}
\rho_{3}=\frac{E\{f(i, j) f(i-1, j-1)\}}{E\left\{[f(i, j)]^{2}\right\}}, \tag{4.4.2c}
\end{equation*}
$$

(4.3.1) can be rewritten in the form

$$
\begin{gather*}
\frac{\sigma e_{4}^{2}}{\sigma f^{2}}=1+\alpha_{1}^{2}+\alpha_{2}^{2}+\alpha_{3}^{2}-2 \alpha_{1} \rho_{1}-2 \alpha_{2} \rho_{2}-2 \alpha_{3} \rho_{3}+2 \alpha_{1} \alpha_{2} \rho_{3}+2 \alpha_{1} \alpha_{3} \rho_{2}+ \\
2 \alpha_{2} \alpha_{3} \rho_{1} \tag{4.4.3}
\end{gather*}
$$

It follows that the $\alpha_{1}, \alpha_{2}$ and $\alpha_{3}$ which minimize (4.4.3) are given by

$$
\begin{align*}
& \alpha_{1}=\frac{\rho_{1}^{3}-\rho_{1}-\rho_{1} \rho_{2}^{2}+2 \rho_{2} \rho_{3}-\rho_{1} \rho_{3}^{2}}{\rho_{1}^{2}+\rho_{2}^{2}+\rho_{3}^{2}-2_{1}^{\rho} \rho_{2} \rho_{3}-1}  \tag{4.4.4a}\\
& \alpha_{2}=\frac{\rho_{2}^{2}-\rho_{2}-\rho_{1}^{2} \rho_{2}+2 \rho_{1} \rho_{3}-\rho_{2} \rho_{3}^{2}}{\rho_{1}^{2}+\rho_{2}^{2}+\rho_{3}^{2}-2 \rho_{1} \rho_{2} \rho_{3}-1}  \tag{4.4.4b}\\
& \alpha_{3}=\frac{\rho_{3}^{2}-\rho_{3}-\rho_{1}^{2} \rho_{3}+2 \rho_{1} \rho_{2}-\rho_{2}^{2} \rho_{3}}{\rho_{1}^{2}+\rho_{2}^{2}+\rho_{3}^{2}-2 \rho_{1} \rho_{2} \rho_{3}-1} \tag{4.4.4c}
\end{align*}
$$

Continuation of the analysis at this point becomes unweildly. For this reason, a further restriction is usually placed on the model; the autocorrelation function is assumed separable. With this added restriction, $\rho_{3}=\rho_{1} \rho_{2}$ and (4.4.4) reduce to

$$
\begin{equation*}
\alpha_{1}=\rho_{1} ; \quad \alpha_{2}=\rho_{2} ; \quad \alpha_{3}=-\rho_{1} \rho_{2} \tag{4.4.5}
\end{equation*}
$$

Substituting into equation (4.4.3), the minimized ratio becomes

$$
\begin{equation*}
\left(\frac{\sigma e^{2}}{\sigma f^{2}}\right)_{\min }=\left(1-\rho_{1}^{2}\right) \cdot\left(1-\rho_{2}^{2}\right) \tag{4.4.6}
\end{equation*}
$$

Now, as in section 4.2.1, we consider the effect on performance of the DPCM system when any or all of the prediction coefficients; $\alpha_{1}, \alpha_{2}$ and $\alpha_{3}$; are not the optimum coefficients. Let the non-optimum coefficients be $\alpha^{\prime}{ }_{1}, \alpha^{\prime}{ }_{2}$ and $\alpha^{\prime}{ }_{3}$ and define $\Delta_{1}=\alpha_{1}^{\prime}-\rho_{1}, \Delta_{2}=\alpha_{2}^{\prime}-\rho_{2}, \Delta_{3}=\alpha_{3}^{\prime}+$ $\rho_{1} \rho_{2}$ as the deviation from the optimal coefficients. The increase in prediction error variance is given by

$$
\begin{align*}
& \left(\frac{\sigma e^{2}}{\sigma f^{2}}\right)_{\alpha_{i}=\alpha_{i}^{\prime}}-\left(\frac{\sigma e^{2}}{\sigma f^{2}}\right)_{\left(\alpha_{i}\right) o p t .} \\
& \quad=\Delta_{1}^{2}+\Delta_{2}^{2}+\Delta_{3}^{2}+2 \rho_{1} \Delta_{2} \Delta_{3}+2 \rho_{1} \Delta_{1} \Delta_{3}+2 \rho_{1} \rho_{2} \Delta_{1} \Delta_{2} \tag{4.4.7}
\end{align*}
$$

Note that if any two of the deviations are zero, this equation reduces to (4.3.4). However, we also note that the performance is much more sensitive to changes of the coefficients. For example, if $\rho_{1}=\rho_{2}=0.9$, $\alpha_{1}$ and $\alpha_{2}$ are the optimum prediction coefficients, and $\alpha_{3}$ is varied in the range 0.8 to 1.0 , then the system performance will be degraded as much as 28 percent.

### 4.5 Signal-to-Noise Ratio Measurement

The signal-to-noise ratio for various DPCM communication systems was measured for subjects one and two using the Supernova minicomputer. In so doing, the mean values were first subtracted from the digitized samples representing the subjects. Refering to figure 4.5 which shows the seven nearest previous samples to sample $x_{i o}$, the prediction schemes tested were.

1. Previous Sample ( $x_{i 1}$ )
2. Previous frame ( $\mathrm{x}_{\mathrm{i} 3}$ )
3. Nearest three previous samples in $x-y$ plane

$$
\left(x_{i 1}, x_{i 2}, x_{i 4}\right)
$$

4. Nearest three previous samples in $x-t$ plane

$$
\left(x_{i 1}, x_{i 3}, x_{i 6}\right)
$$

It has been shown [15] that, for a Gauss-Markov field, the nearest neighbor prediction schemes we are looking at are optimum for all schemes using as many previous elements as desired for prediction.


Figure 4.5 $\begin{aligned} & \text { Nearest previous samples for } \mathrm{x}_{\mathrm{i} 0} \text { used for } \\ & \text { prediction }\end{aligned}$

For schemes one and two, the signal-to-noise ratio was determined by first measuring the prediction error variance

$$
\begin{equation*}
\sigma e^{2}=\frac{1}{N} \sum_{i=1}^{N}\left(x_{i o}-\alpha x_{i j}\right)^{2} \tag{4.5.1}
\end{equation*}
$$

where N is the number of image samples involved and

$$
x_{i j}= \begin{cases}x_{i 1} & \text { for scheme one } \\ x_{i 3} & \text { for scheme two }\end{cases}
$$

The optimum prediction coefficient results when $\alpha=\rho$, where $\rho$ is the average correlation between two adjacent samples. We can determine $\rho$ either by direct measurement or from the exponential model, i.e.

$$
\begin{equation*}
\rho=\exp [\varepsilon] \tag{4.5.2}
\end{equation*}
$$

where $\varepsilon=\alpha$ for scheme one and $\varepsilon=\gamma$ for scheme two. Measurements were made using both prediction coefficients based on the measurement of $\rho$ and on (4.5.2). The signal-to-noise ratio was then determined for each case as

$$
\operatorname{SNR}=-10 \log _{10}\left(\sigma \mathrm{e}^{2} / \sigma^{2}\right),
$$

where

$$
\begin{equation*}
\sigma^{2}=\frac{1}{N} \sum_{i=1}^{N}\left(x_{i o}\right)^{2} . \tag{4.5.4}
\end{equation*}
$$

The results of these measurements are sumarized in table
III. Included in the table is the predicted signal-to-noise ratio computed as

$$
\begin{equation*}
S N R_{p}=-10 \log _{10}\left(1-\rho^{2}\right), \tag{4.5.5}
\end{equation*}
$$

where $\rho$ again takes on a value obtained from direct measurement and from (4.5.2). Comparison of the results for schemes one and two leads to the following conclusion: the correspondence between predicted and measured

| Subject | $\searrow$ |  | scheme 1 $\left(x_{i 1}\right)$ | scheme 2 $\left(x_{i 2}\right)$ | $\begin{aligned} & \text { scheme } 3 \\ & \left(x_{i 1}, x_{i 2},\right. \\ & \left.x_{14}\right) \end{aligned}$ | $\left(\begin{array}{c} \text { scheme } 4 \\ \left(x_{i 1}, x_{i 3},\right. \\ \left.x_{i 6}\right) \end{array}\right.$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Subject } \\ & 1 \\ & \text { (100 frames) } \end{aligned}$ | $\begin{array}{r} \text { from } \\ (4.5 .2) \end{array}$ | predicted | 4.34 | 10.04 | 9.28 | 14.38 |
|  |  | measured | 6.66 | 3.08 | 9.51 | 12.84 |
|  | measured | predicted | 8.10 | 3.59 | 10.01 | 11.69 |
|  |  | measured | 7.40 | 3.34 | 10.55 | 14.01 |
| ```Subject ``` | from$(4.5 .2)$ | predicted | 9.87 | 9.39 | 16.80 | 19.26 |
|  |  | measured | 13.54 | 4.41 | 16.88 | 14.65 |
|  | me as ured | predicted | 14.20 | 4.62 | 18.09 | 18.82 |
|  |  | measured | 13.89 | 4.61 | 17.01 | 15.26 |

TABLE III Signal-to-noise ratios predicted from model and measured for DPCM communication systems operating on subjects one and two
signal-to-noise ratios is poor when the least-squares fitted model is used to determine $\rho$, whereas the correspondence is very good when the measured correlation is used. The poor correspondence can be attributed to the poor fit of the exponential model to the first point of the autocorrelation function curve (see figure 3.4). From (4.5.5) it can be seen that a small variation in $\rho$ will cause a large change in predicted signal to noise ratio. The system performance, however, is improved only slightly be using the measured correlation coefficient instead of the model correlation coefficient. This is consistent with the analysis of section 4.3.

For prediction schemes three and four, the experimental prediction error variance can be determined by evaluating

$$
\begin{equation*}
\sigma e^{2}=\frac{1}{N} \sum_{i=1}^{N}\left(x_{i o}-\alpha_{1} x_{i \cdot 1}-\alpha_{2} x_{i k}-\alpha_{3} x_{i \ell}\right)^{2}, \tag{4.5.6}
\end{equation*}
$$

where $\begin{aligned} x_{i k} & =\left\{\begin{array}{ll}x_{i 2} & \text { for scheme three } \\ x_{i \ell} & = \begin{cases}x_{i 4} & \text { for scheme four } \\ x_{i 6} & \text { for scheme three }\end{cases} \end{array} \text { for } \begin{array}{ll} & \end{array} \text { four four }\right.\end{aligned}$
Assuming a separable autocorrelation function, the optimum predictor results when $\alpha_{1}=\rho_{1}, \alpha_{2}=\rho_{2}$ and $\alpha_{3}=-\rho_{1} \rho_{2}$ where $\rho_{1}$ is the correlation coefficient in the $x$-direction and $\rho_{2}$ is the correlation coefficient in the $y$-direction for scheme three and the time direction for scheme four. As before, $\rho_{1}$ and $\rho_{2}$ can be obtained both by direct measurement and from the exponential model.

The measured signal-tomoise ratios are summarized in table
III as well, along with the predicted signal-to-noise ratio determined as

$$
\begin{equation*}
\operatorname{SNR}_{\mathrm{p}}=-10 \log _{10}\left(1-\rho_{1}^{2}\right)\left(i-\rho_{2}^{2}\right) \tag{4.5.7}
\end{equation*}
$$

It can be seen that using the measured correlation coefficients again produces the best results. There is good correspondence between the predicted and measured signal-to-noise ratios even when the correlation coefficients were determined from least-squares fitted models. This is due to a compensating effect. As can be seen from figure 3.4, the measured $x$-direction correlation coefficient is greater than the value determined from the exponential model whereas this situation is reversed for both the $y$-direction and the time-dimension correlation coefficients. This means that the first term in parenthesis in (4.5.7) is decreased while the second term is increased, producing the mentioned compensation.

Theoretical analysis based on the model of a Gauss-Markov field shows that the signal to noise ratio for two-dimensional prediction is the sum of the signal-to-noise ratios for one-dimensional prediction in each of the two orthogonal directions. From table III it can be seen that, for subject one, the measured signal to noise ratio for two-dimensional prediction was about three decibels greater than the sum of the measured signal-to-noise ratios for one-dimensional predictions in the two orthogonal directions. For subject two this twodimensional prediction performance was three decibels lower than the sum. This is probably due to the fact that the Gauss-Markov model poorly describes subject two in the $x-t$ plane, whereas the model more closely matches the data for subject one.

Table IV is a summary of the exponential coefficients and the measured correlations for the two subjects used in this study.

| SUBJECT | DIRECTION | $\varepsilon$ | $\rho$ |
| :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Subject } \\ 1 \\ \text { (100 frames) } \end{gathered}$ | x | 0.2297 | 0.9188 |
|  | y | 0.1921 | 0.5960 |
|  | t | 0.0519 | 0.7501 |
|  | xy | 0.3051 | 0.5587 |
|  | xt | 0.3793 | 0.6891 |
| $\begin{aligned} & \text { Subject } \\ & 2 \\ & \text { (300 frames) } \end{aligned}$ | x | 0.0551 | 0.9811 |
|  | y | 0.1128 | 0.8516 |
|  | t | 0.0610 | 0.8094 |
|  | xy | 0.1171 | 0.8498 |
|  | xt | 0.1024 | 0.7955 |

TABLE IV Summary of exponential coefficients and measured correlations used in this study

## v. CONCLUSION

The results obtained from this study are summarized in the following.

1) A system has been designed and implemented which can be used to process and study time varying images
2) Using this system, experimental images have been digitized and stored on computer magnetic tape. This data has been stored on tape compatable with that used by larger computing installations making this data relatively installation independent and therefore readily available for further experiments.
3) Statistics for two motion picture images were computed and compared to the statistics predicted by previously used models. For these images it was found that an autacorrelation function of the form $R(\Delta x, \Delta y, \Delta t)=\exp \left[-\left(\alpha^{2}(\Delta x)^{2}+\beta^{2}(\Delta y)^{2}\right)^{1 / 2}-\gamma|\Delta t|\right]$ was most accurate in predicting these statistics.
4) The effect of using the Gauss-Markov mode1 in designing a DPCM communication system on the performance of that system in terms of signal-to-noise ratio was determined by comparing predicted and measured signal-to-noise ratios for the system.

## APPENDIX

Supernova Support Routines for Digitizer
The following subroutines are used to digitize picture images located on 35 mm film using the system described in Chapter Two. In order to use them, the main routine SCANN must be called from a fortran routine as follows:

CALL SCANN
STOP
END
This will ensure porper initialization of the fortran calling sequence.
A11 subroutines are compiled and assembled separately. Once the above fortran program has been compiled and assembled, it must be loaded along with the following subroutines and the fortran library FORT.LB using the supernova relocatable loader.

Qges ACOUT


THIS EUSOMUTIHE IS DESEGUED TE GUTFUT
；A 16 git mogit ong onl format on the ti
GCe met bontain the bove to ee


ACDUT
9COUT
ZRTH ；GAVE RETURU
B． 450 ：GATE 40＇S
AB．

日． EE
Q．Ef1
2.2 .520

1．
g．ç ；SUESEGUEHT THEEE EIT
－

日． B ：EAEH EIT
2． $2, \mathrm{sec}$
6． 0
BE
inope

8 E 1
LODFI
E．C4G ：ELAHK EHAEACTER

Grace ：REGTOEE AE＇S
1． AE 1
2． ACZ
ERTN ；RETUEN
；CONETRHTS

EN刀

agge ERGHT

```
                                    ; conethats
```

| 6064\% 650.60 | ETH: | 0 |
| :---: | :---: | :---: |
| 90645 Enabet | Acg: | 9 |
| E0nctabercsu | RE1: | E |
| aberfemater | +6: | 0 |
| Qug-it bepets | 348: | 346 |
| geges: 50960 c | Br | 9 |
| 66gtereachey | Bmes | $\underline{\square}$ |
| 606\%3 06eced | AETi: | 609 |

 ; bHEGA THE ETATUE DF A weITE COMHANA
 TAFE: URIT.

HLL ETPGRS REE GHECKED AHD WHESE FOSEIE:E FEMGITE ATTEASTS AFE MADE.
 EUFFES TO EE BAITTE ACI fUGT EOHTGIA THE WORI GIZE GF THE EUTFEE
:
;
TITL CHET
EHT GHWRT
ERTH GEKIF, ACOUT, BRITE
. HEEL
$\therefore$ TKTM


| 日GE\％－Hnet |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 65646 trates | ． 14 P | l．ate | ：YES |  |
| 0064\％－2tan | L1A | 19．TiT1 | ：Untitumat | EPEUE |
| 日gesermitur stop： | \＆F | EHBET |  |  |
|  | STETM |  |  |  |
| － 6 gese bustabu | ETH |  |  |  |
| 6095\％ 606468 | ME |  |  |  |

：OQHETGHTS

```
gagesmeverz riz: 12
00555%06015 E15: I5
```



```
0日65%17%G7% ABRT: BEITE
```



```
000E1 G00g%e neg: . G
```



```
ggeg3*60060日 ¢ce: 0
```



```
0日QES'b60日G日 E:TRY: G
```






60672. 845516
66973'04757
06574847640
60675. $65210 \%$

6gari"genaz?
601684651111

6日16e'teg1日5
60163'65112e
96164'64752z
$09105 \cdot 626655$
60166 6261日5
E日107.65416\%
06116.041525
66111 652111
69112.g47516
$06113 \cdot 620124$
0日114•E42522
6 6115* 44651
$06116 \cdot 94101$
06115: 6 ge165
60120'042960

```
MO 1.GM1 SFACE EFWEFEE
64122'06827
00123.0ET022
10124'030425
64125*87122
0日120'86SE2z
0日127'60日%77
6日130'0502z2
66121* 66017%
00132:000735
6@1z=62441%
```

INTIS
moc
LJA
DOAS
EKPTH
JHP
HIOL
IHTEN
LIH
LIA

1，EM1 ；GFACE REWEREE
1．MTA
2． 046
二 Tita
ATA ；WAIT FOR TAFE
．-1
ATA
日，EETEY ；RETFi＝$\because ?$
$\because \mathrm{Cl} \mathrm{E}$

; constarls

```
0604 6mbleT
```









```
    60zat!20gmb
50zegreg101
gezag606105
GEcz% bed10E
05239 6445:4
gaz51 64244
60za-35gize
062%3*日4,50%
E0254642065
6e23E'052105
g925e'642045
0620% 0-6440
6日240684E17
00241/651122
09242'042502
00243'652g40
06244%40516
00245-2cemba
```



```
062:7.0106412
6Ec5ergegu40
06251/620日40
6@z5e-6gz131
06253'050195
00254.650日47
64255/854447
0日256'520124
60257.647446
0625日'641517
062E1* 647124
00262.044516
06453*6525日5
60264*020117
90203·日51646
g⿴巳ee'gこ351E
0日267'823446
0日270.052117
@0271"日20110
06272'640514
06275*852040
E6274* 82g645
06275'020144
0日2F6'0000000
0627F.g@g.6日'T&T4: +1
```



```
G630日'0日E412
08361'02804日
00302"0206440
60363 日42516
60304 6. 6204日
06305.647506
```

```
BGES CHMET
    06565'g%9124
    00367640506
    00515`0%2446
    06311* 042510
    003:ご暞517
    06313'052518
    0534*5%2185
    60515*051105
    603:5.042046
```



```
    60326045%17
    60521"05z516
    b0zaz'052046
    00323'047165
    6032%*053440
    603e5'6Ez161
    6032605610%
    00327'020161
    6@359,647104
    61%31•6080106
```



```
    T&T <<E><ב>DHAELE TG WELTE DH TAFE - EXESUTI
    603S3*0egale
    60334•05E51E
    00.35*日cusce
    00356'04z165
    06357620124
    EE346.04744日
    00341•05352z
    60342'044524
    B@Zc%"042446
    06344*84516
    00345'02012%
    60346'046520
    ब6347:042446
    0625日.62044日
    00551*94253日
    40352'042565
    40353'852524
    60354.044517
    60355'047646
    6035E'952105
    G635%'651115
    00360'04451=
    60.351*-40524
    00362*日42564
    00363'606400
```



```
O THIS SUEROHTINE HAS EEEH MESTCHED TO METEG
C OF THE SS MM FILN
C
C
c
C
C
r
C
C
C
C
c
C
C
C
C
C
L
c
C
O
C
C
C
C
O
C
C
c
C LTMEP IS CALLEI' 'TG SGAH THE FERFORAMTGHE
c
C
C
C
    SUEROUTiHE EDGE (IENGi, IEDED)
    HIMENEIOH IYE(1024)
c
C *** ECAH FERFORATIOHS
    CALL LIHEF (IYE`
c
C *** SET THREGBHOLI AT MEAN
C
    S=0.日
    00 20日 I=1.1024
    S=S+IYECI)
    2go continue
        ITHP=Sハ1924.6日
c
c *** FIHIL LEAMIHG EDGE OF PERFGRATIOH
c
    no 201 I=1.1024
    J=I
    IF (IYE(I).GE.JTHR) GQ TO zQ2
    201 COHTINHE
        STOF.'NO EDGE FOUND"
c
C ****. FIND TRAILIHG EDIGE OF FERFOROTIOH
C
&
    202 10 203 I=,1,1024
```

```
        I= J
        IF (IYE{I}.LT.ITHE) GO TO -G4
    zge coltatue
        GT0F 'H0 ETGE FOumy"
    2G4 IF (IEIN:.ig.g) G0 TO 20S
r
C.f:* EET ELGEIT TO MIFFEPENGE
C
        IEMCD=1-IEMa1
        RETUFW
C +:% SET EDREI TQ EMGE LOCATION
2gS IEmGi=,
        RETUEH
```

日Qs $51+6$


|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Bra | 17，Tirs | Mrsember AtM MEre\％ |
|  | 129 | 1．बातए |  |
| Eased coso | ．9\％ | GAEk？ |  |
| 日cosergos | LI！ | E． BCO ； | FESGEE AE＇ |
|  | 119 | ？，mL 1 |  |
| Gesestas | ：T0 | $\because$ ACz |  |
|  | ． AF | aruy | FETGEQ |
| ． |  | ；Cubstmbts |  |
| G6tsen aboter ETH： | 9 |  |  |
|  | 8 |  |  |
| gexam betema nc： | 0 |  |  |
|  | 5 |  |  |
|  | － |  |  |
|  | 7 |  | $\bigcirc$ |
|  | $\because$ |  |  |
|  | 8 |  |  |
| Q6G\％ETE11E CH： | 116 |  |  |
|  | 6 |  |  |
|  | $\square$ |  |  |
|  | －\％St |  |  |
|  | GTrum |  | ． |
|  | F\％G\％ |  |  |
|  | FGEF |  | ． |
|  | W世1\％ |  |  |
|  | cever |  |  |
|  | H0RE | ， |  |
|  | 大日SiA |  |  |
|  | OETHT |  |  |
|  | $+1$ <br> ．TxT | $\leqslant 15<12\rangle$ | FJLM FOSITIGUJHG\％ |
| 69113 9648 |  |  |  |
|  |  | $\cdots$－ | －．．． |
| g91．5＇日2egtc |  |  |  |
|  |  |  | ． |
| QS117685175 | ． | ： |  |
| 6G120．62， |  |  | ．．． |
| E912194753 |  | ．． |  |
| 8612\％日4， |  |  |  |
|  |  |  |  |
| 69124 047313 |  |  | － |
| 69125＊64367 |  |  | ． |
|  |  |  | － |
|  | $\begin{aligned} & +1 \\ & . T \times T \end{aligned}$ | $\langle\langle 15\rangle\langle 12\rangle$ | EHTEF COMAMPTIS |
| 69136 6esatz |  |  |  |
|  |  |  |  |
| 日413298ctat |  |  | ， |
| 29133026910 |  |  |  |
| 60124＊20848 |  |  | － |
| 0513642515 |  |  |  |
|  |  |  | －． |
| 601\％ 151040 |  |  |  |
| 日日1 40－6\％isit |  |  |  |
| 6E141＊ 6 ¢6t5 |  | － |  |
| 2049＊9世5］6 |  |  |  |
| cotaztectas |  |  |  |

```
Q@g flpg%
```




```
    TMT <15<<&>
```



```
    40ygegeg+12
    B014%'40%erg
```



```
    5,5j1'560440
```



```
    Egry"!matm
    00054.06:%11
```



```
    BE10E Breing
    6%15%-601%3
    got6g'tugjes
    E%151 6500%:
```



```
    00t6g'30190%
    g@tev'By101:
    8016c* E%T10%
    g@15E-gro10%
    geteretegi06
    00170'0s1.0ig
    80171'646506
    017% बesc%5
    6क1%*EOTGO
    6日17G"62gnas
```



```
        T&T <<S><ME> HOW HAHM TIHESG
    0%176'066412
    0日177'600545
```



```
    6ала1'6"कठ%る
    0mece'bemeta
    0620,64as.6
    65604'06354?
    0日cesegusut
    Gmक6'04713!
    g0agresere%
    &0216,0445!5
    6g%11'E455z3
```



```
    [1213'020040
    G0214*020%45
    00215:02064%
    gn215+650640
    g021:*e%cca
    grcag'gegecs
```







```
        .T&T <<1こ〉く12》
        FOSITIONING DOME. ANG WGA...
    gn225+000412
    gezer'neबकरक
```



```
    0223!-324246
    G0c5e"bemert
    0123%'450117
```

[^0]0日G1 FHFOS
．TITL EHT EXTH
．ESTI
HEEL
．06060：640514 FHETE：STA

E6062． 054510 FMP0S：
06603646510
$06094 \cdot 644510$
60605 650516
0606E• 920475
06067＂966日a2
$06010 \cdot 670906$

66012•日20472
06013．606062
6GEI4＇gTEETG
66区15＇606460
60616．620467
06017．日6e9ez
04629．079604 68021＊006409 $00622 \cdot 6 \times 6474$ － 0 日23• 124474 06024． 030474 60625＇142460 60日26＂14E460 66027＇940823 $06930 \cdot 644624$ $06031 \cdot 020471$
 60033＇624472

STA
ETA
GTA

## ETA

LIHA
．Srstm
．FCHAR
Ji4F

## LIA

 SYETM．FEHAE
JHP
LIA EYSTM
．FCHAR
． HF P
LIM

LDA
ZUE
SUS
ETA
ETA
LDA
STA
LIA
b．ACNT ；FLIT FULSE
$\begin{array}{lll}\text { LIAA } & 1, \text { GIDELY；} & \text { COBHT AHI } \\ \text { LDA } & 2, C .1 & \text { DELAT ADIS }\end{array}$
© 3
ZRTH ；SAVE RETURH
日．ACG ；SRVE AC＇S
1． $\mathrm{ACL}_{1}$
2．ACZ
g．CIE ；QUTPUT AH＇$s$＇
0.015

Q．cs

2． 6.
2． 0
It Auta－
I HCREMERT
LOCATIENS
INITIALIEE
COUHTEE
（1）TCHT
1．IELAY；
IHITIALIzE DELAO
；THIS SUEROUTIHE HAS EEEA DESIGHEI TO GIVAHCE 3 E WM FILM EY OHE FEAHE

A Colli for fitsta eete encel to the
；IHDEX GOCATIOH OF THE EDGE DF THE FERECRATIUNE

A BALL TO FHFGS ATVHNCES THE FILM TO THE HEST FBAME ：

FETEF is Chlleg TO GIIt EDREESTIOH GTEFE DUE TG FOEITIDMAL EREDR

ETGE IG GALLED TO DETEFMIHE THE ERROR： IH FOGITIOA EY DETEOTIHS THE LOGATIOH
 TO THE ISTEX LOGATIOA EEDEI＂

ERGHT is GAlGED TG ADUUST THE ERIGHTHEGS DF THE FLYAHG EFOT ECAHAER
；
；
；
FMrog
FHFOS FMSTA
FETEF，ETGE ERGHT
FCAL

Ma，EDGE：

TO DEHOTE EHD OF SCAN

DELAT AIMEESSES

| Q602 FMFOS |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | ator | 47 |  |  |
| goncse | LDA | 日． 223 |  | IHITIALIEE FULSE |
| 日日bestgobucs | ETH | 日．FCHT |  | COMHT HHT |
|  | LIM | B．日 B |  | melat |
|  | AIOF | 4 |  | IEGUE PULEE |
|  | 8ue | G． 1 |  | IMITIALIzE |
| 60942＇044484 | EイA | 1．DELA |  | IteLA＇ |
| 606439144E | ES2 | MELA |  | IELAY |
|  | ＋19 | $\because 1$ |  | LOQF |
|  | 582 | PCHT |  | Last fulse？ |
|  | 测 | Longe |  | WQ－COHTIAUE |
|  | HE | TGMT |  | FFAtE in FOSITIOH？ |
| 06056 605765 | MiP | LogF 1 |  | H0－Cohtinue |
| 66651.859447 | DIA | 日，47 |  | GETAEA FULEE EGURT |
| 60652＇924455 | LDA | 1．HFLS |  | HHT EREDRE ？ |
| 60653 165406 | 345 | 日． 1 |  |  |
| 60654＇125113 | MロツL | 1．1．540 |  |  |
| gense 066434 | 180 | BaFST |  | ＇GE－GQ CQEREDT |
|  | ISt | B．FCAL |  | DETEEHINE EDISE IIFFEREHCE |
| 68659＇18787 | EmGE |  |  |  |
| 60660．06066e | $z$ |  |  |  |
| 66061＇006119＇ | EMGE1 |  |  |  |
| gence＇0091z1． | ETGED |  |  |  |
| 日acesene4436 | LIP | 1，ETGEI | ： | HEGATIVE ？ |
| 66064＇1251：2 | movit | 1．1．825 |  |  |
| 606Es 909405 | JdF | mode | ； | YES |
| 61066 125965 | Mide | 1．1．5HR | ； | 2ERO ？ |
| 6096？ 696463 | J HF | Iotic |  | YES |
| 6egre geedel | 15F | GAFET |  | GO EGERECT |
| 60671 906765 | MF | LODF 3 |  | EECHEEK FOSITIOH |
| agere＇aces 14 DONE： | LIA | Q．CH | ； | OUTFUT AH＇H＇ |
| 65663＇06062 | STSTM |  | j | To wenote end of hove |
| 06674＇97648日 | ．FCHAE |  |  |  |
| 日6075＊06E406 | MF |  |  |  |
| Qe97E．g日e． 11 | ISE | ghert | ： | Andugt eeighthess |
| － 06067 ＇020414 | LIIA | 6．ACD |  | EESTORE AC＇S |
| E0169．024414 | Lina | 1，HC1 |  |  |
| $06161 \cdot 636414$ | LIA， | 2．ALC |  |  |
| 00162．602416 | STP | geth | ； | EETURH |
|  |  | ；COHET |  |  |
| 40163．006012 612： | 12 |  |  |  |
| 06164606015015 | 15 |  |  |  |
|  | 123 |  |  |  |
| 6016E 606115 Ont | 115 |  |  | － |
| Q6167＇17377\％askt： | EFGHT |  |  |  |
| Q日118．6日G日ge ERGE1： | 9 |  |  | ． |
| 6日111．17T37 AFET： | FGTEF |  |  |  |
| $06112 \mathrm{Dag60日} \mathrm{ETH:}$ | $\square$ |  |  |  |
|  | 0 |  |  |  |
| 00114＇60日609 AC1： | $\square$ |  |  |  |
| 06115．000980 Acz： | $\underline{\square}$ |  |  |  |
| 06116．60513日＇ACNT： | CNT |  |  | ．． |
| 0日117．060151＊nDELY： | ILAY |  |  |  |
| 0日12日 6nderi bi： | 1 |  |  |  |
|  | $\square$ |  |  |  |
| grase＇gnobel Guitt： | 21 |  |  | ． |
| 0日1z3＇360日e日 TCHT： | 日 |  |  |  |


| Q日a3 FMFOS |  |  |
| :---: | :---: | :---: |
| 6512＋ 5 ¢0t5 | PCHT： | 9 |
| 60125． 44830 | nelay： | 44634 |
| 60120．6scesa | ロロしゃ： | 0 |
| 00127． 60565 | HPLS： | 5860 |
| 06136＂E6922： | CHT： | 2： |
| 00131606050 |  | 519 |
| 60132． $0^{66674}$ |  | 74 |
|  |  | 121 |
| 60134＇0691\％1 |  | 151 |
| 60135＇6062c5 |  | 225 |
| 60i36＇6日ecse |  | 250 |
|  |  | 727 |
| 6iglab＇001136 |  | 1136 |
| 09141＊006T27 |  | T27 |
| 60142＂600e50 |  | 25日 |
| 日6143＇609225 |  | 225 |
| 06144：600151 |  | 151 |
| 06145609121 |  | 121 |
| Q6146． $\mathrm{EQ日ET4}$ |  | －4 |
| E6147＇600056 |  | 59 |
| 60156960024 |  | 24 |
| 69151＇E09727 | ILAY： | T27 |
| 60152609116 |  | 116 |
| 60153．068932 | － | 32 |
| 66159．600914 |  | 14 |
| 0015596008E |  | $E$ |
| G615c＇benegz |  | 3 |
| 66159＇gergez |  | 2 |
|  |  | 1 |
| 96161／06E日69 |  | 0 |
|  |  | －1 |
| 0116317766 |  | －2 |
| 06164＇177775 |  | －3 |
| 09165．17772 |  | －6 |
| 6016E 177764 |  | －14 |
| 96167＇177746 |  | －32 |
| 6E170－176EEE |  | $-116$ |
| 66171／177651 |  | －727 |

0601 FETEF

900E1 540432 ETA


GQE日4＊125112 LDOPA：HOUL
 GQE日E 125054
 G5B15：GU42J ITME：LTA
 E6012． $9 \mathrm{EG42} \quad$ LIA 60612•6日2417 JMF 64E14＂E44422 $3706 T$ ： EG615124cta 90616 6ED247
日月620 620417 80621－15648日
日曰62\％＇ 51677 ब6ロこ4＂225404 の6G25 gggo 69日2E＂06日447
 E6536＂185460 09631～60975
．HEEL
；THIS EUEFGUTIHE HAS EEEH TESIEHEJ TO
 HUMEEF OF ETEFS

THE IEGIFEI HMNEEF MUGT EE IH ACi
$\begin{array}{ll}\text { TITL FSTEF } \\ . \text { EHT } & \text { FSTEF }\end{array}$

ETH ZBETH ；SAUE FETUEH
9．ACG ：EHVE AB＇E
1，AE：
シ HO
i． $1, \Xi 己 C$ ：NEGHTIUE ？
ITIHE ；YES
1，1．Gご：EEPG？
ETAFT ；HO
Q． HE ：FESTGEE HC ： S
1． HE 1
－AEZ
GETH ：EETUFH
1．STEF
1,1 ；HEG．STEF EGUHTEF
47 ；ELEAF EDItTEE
$4 \overline{7}$ ；ISSUE FULEE
日．HELAG
日． 6
G．E．SZE ：MELAY LOTF＇
$-1$
1． 1.62 SF ；EHOUTH FULEES？
LODF ； 1 HO
日． $4 \vec{r}$ ；DETAIH EULHT
1，STEFG
Q． 1 ；HEW EOUHT
LDOFA．；EACK AGHIN
；COHSTARTS

|  | TITL－ <br> EHT <br> HEEL | ```THIS gumgutinE has Emen IESIGHEI TO ACOEFT HULT:FLE MHGIT BEDTHAL INTEGERE FBOM TT A&TH GGNQEET TG EIHAEG INTEGEPS JHFUT IS TERHIHATEI OA FEGOGHITIOH```   ```EITE MTLL EE TFUHGHTEL DH THE LEF? HEGATIUE HOHEEES HAvE HGT EEEH GOHSTDEES RETHARG THE EIHARG vALUE IH HEQ GTHMH GTHUM``` |
| :---: | :---: | :---: |
| 60696．034436 GTHMm： | STA | 3．RTH ：GAvE EETIEN |
| 60661． 640432 | STA | 9．ACb |
| 6egee＇g44452 | STH | 1． HCL |
| 6ebes＇mstaza | ETA | 2．ACz |
| 60884＊ 5246 | cue | 2．2 |
| 日gege bbebez Lo口f： | ．STSM | ；GETALH HIGIT |
| 06006．0ET406 | －GChame |  |
| 60667．069tco | MFP |  |
| 60616．E日core | ．Stetm |  |
| 6061．1．6T6060 | ．．．PCHAE |  |
| 60612．66640 | Jitr |  |
|  | LIIA | 1．Scif ；AEQVE LOUER LIMET |
| 66014＇106533 | EuEL ${ }_{\text {\％}}$ | 9．1．SHE |
| 60615．664413 | JPP | IOHE ；HO－RETURH |
| 69016．624422 | LIA | 1．CTz ；EELOU UFPEE LIMIT？ |
| G0817．12c5i3 | SUE： | 1． 1.640 |
| 06926．0694ig | Jif | IIAHE ；HU－EETURH |
| 00621．024．te0 | LIAA | 1，EIT ；AECII TGEINAE：＇才 |
| 6062e＇123c80 | ANI | 1． 9 |
| 60623＇1456eg | Mov | 2．1 ；MULTIFLY HUmEEE EY TEH |
| 06124＇153120 | ADIEL | 2， |
| 60925：133120 | ADIIZL | 1．2 |
| 6n6es＇1130日g | HDD | Q．2 ；AIIM MIGT |
| 06027＇6E0T5E | INF | LOOF |
| G0日3g＇begabz inde： | LDA | E．ACE |
| 00631．024453 | LIA | 1，ACI |
| 6e932＇ 6 cc 4 4 | MFF | －6TH |


| 9033＇29659日 | ACb： | 6 |
| :---: | :---: | :---: |
| 60634＇606gev | AC1： | Q |
|  | ACE： | $\underline{0}$ |
| 6日635＇680606 | ETid： | 9 |
| 日6039＇0gegst | Cs？： | 57 |
| 06040．606日72 | E7E： | 72 |
| 06041 ¢00017 | C17： | 17 |

```
C THIS gUERGHTIME HAS REEH TESIGHEO TO
G mETERMIME THE AWEFMBE SPOT FFIGHTMESS
r. a= THE FLGING SFOT ECGMDEE
C THE INTENGITO IE :AEASUEEN THEOMGH THE
c FILM ferfotatiohS bHEN FOSEIELE
C IF THERE :S IHSUFEIGIEHT FERFGRATION
```



```
O THE IHTEHGITY IS GEAGLPEM OH THE
O FIEST 4O FGMTS GF THE FEFGOATIOH ECAH
G THE METHGE UEET GILL EE mHGGEJAHT
O TEOUGHOUT THE EET OF FFBHES SG&HAEL
- SINGE THE FEFFDEATION IS MOWED TO
C THE SAME LDCHTIGA WITH EVEFG ADQARCE
C CHLLEN FEOM ASEEMDLEE BG:
C JEE F.FCAL
            IHTHE
        2
        IEngl
        IHTE
        WHERE EDGEI IS THE IHJEX LOCATIDH
        OF THE FTLH FERPORATIDH AHD TNTHS
        IS THE AVERAGE INTENEMTY MEAGURED
        IUE TO THE DYHAMIG ETORAGE ALLOCATIOH
        UEEH EY THE SMPEEHOQA FORTEAN .
```



```
        FOR SUESERUENT EUERGUTiHE CALLS
        THEREFORE IEOG: MuST BE STORET IH
        AN fSSEMELEE ROUTINE OR IH THE GOHmOH
        AFEA
    SUEROUTIHE IHTHS <IEDG1, IHFS>
    BINEMEION I'TEG1E24%
    call LIHEF (IME)
    S=0.0
    J=IEIG1-60
    K=IFIIG1-20
C
c
        NO 200 I=1.K
        S=S+IYB(1)
    20g continue
r
c
    201 10 202 I=1,41
        S=S+IYE(I)
    2az cuntINUE
C
    203 INTS=5/40.00
        RETUEH
        END
```

```
0BO1 LINEP
    177E11 
bbgegi FS.=1
EgQgergemegi Fs.
Gggal GEGg61.fl IHEF:
60日G2"62g42z
6066?'06eg44
06064*663644
```



```
B6nge"028431
06667'106400
60616"024425
60411.644425
60日1こ'066%7?
04T
0日G14'06,0644
60615'gegr7%
```



```
E0617.024415
60620'133400
60日21'051400
06822"175409
00523'0c4413
16024"030414
06625*147600
00026'644410
06027:101454
06936'060763
06831'666944
06632,}36017
ggeg3'g660日a%
|E
LDA
LIE
noc
LDA
I.VA
HEG
1DA
STH
TITL
ENT
; THIS GUEROUTIHE HAS FEEH IESIGHEN TO
; THIS GUEROUTIHE HAS FEEH IESIGHEN TO
EMTIL
; THIS GUEROUTIHE HAS FEEH IESIGHEN TO
; THIS GUEROUTIHE HAS FEEH IESIGHEN TO
; THIS GUEROUTIHE HAS FEEH IESIGHEN TO
; THIS GUEROUTIHE HAS FEEH IESIGHEN TO
; THIS GUEROUTIHE HAS FEEH IESIGHEN TO
; THIS GUEROUTIHE HAS FEEH IESIGHEN TO
; THIS GUEROUTIHE HAS FEEH IESIGHEN TO
; THIS GUEROUTIHE HAS FEEH IESIGHEN TO
. HEEL
G6日ge begegi
SR G.EFYL ; ENTEF
CHA
E.03777
0,44 ; ISSUF 'i TG SEAUNEF
G.4.4 ; ISEUE 2 TO SCAHHER
3, \, 3 : EUFFEE ADDEESS
G,FFLIN ; INITIALIZE PGIHT
G.a ; EOHHTEE
1.0777 ; X-CDGEmINATE
1.%%
1.44 ; ISEHE% CDORIIAATE
BKRIN 44 ; WAIT FOR
&Fin
-1 ; SCANHER
MFF - -1 ; SGANHER
IIH 2.44 ; GETAIN YALUE
LIAA 1,OST77 ; MASK TG
H4TH
1.2 ; E'EWEN EITS
STA 
ETA 
GTA 
IN
2.INOS ; EOORIINGTE
2.1 ; AHI
1.XP ; STORE
GIID
STH
IHC
1MF
G,g.sZR ; mOHE LINE ?
CONT ; NO- COHTINNUE
44
B.FEET ; RETURH
; conetante
```






```
0日e3E.908,00 %%: 6
```

0日e3E.908,00 %%: 6
gage7.g日cteg ;Fl.IH: z0eg

```
gage7.g日cteg ;Fl.IH: z0eg
```




G00: LIHEF
EHD
gage SGHA

039691

TKTH
TITL GCAH
EHT EGAN
 E\＆TL FOHL
HEEL

```
; THIS SUGFOUTIHE HGS REEH MEEIGHEN TG
; goAN a gemulute Of fabace
```



```
GCAH
FCHL
1
; romermats
```




日Gega begezenmas: ACz


gages sgaseriz: TKTE



QEEIZ17T7TF AFSTH: FBETA
ब6E13.1777 REET: EलGH?


Q4ng B4erE STA
000tresteras E1A

46621:162460

60623' 0 ge771
96624 0.6667
69625'096061羊
6962e'17T77
60627. $9 \mathrm{gage2}$
ag030. $90611^{\circ}$
60921'406819'

6G1033606757

ब6535• 986756
G0036. $2 \mathrm{CQT4E}$
geg37'6日EEET
6ag4al acarte gentl:
frgean obeses
60442 606745
G6gas' 10240 g
06044. 142485
66045 3067\%


06050.006556
60951 - 2 ccsec
6065z'6E3644
gagseggegrt strem: INTIS


| CA！ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 901：5 048 62 |  | 97 | （1．EUF |  |
|  |  | 434 | \％ |  |
| 60159．6ctst |  | LDA | －METAT |  |
| 65158－6ce44 |  | Anot | ： 4 |  |
| 60153．50515 |  | 185 | Frictic 7 | ：HECt neIfe |
| 60554．02405 |  | LIA | 1．Enf | －Sump rinhters |
| 60155640464 |  | STA | B，Butr |  |
| 90156＊2さncg |  | 40\％ | 1．3 |  |
| 6310\％606418 |  | 1：\％ | ETE |  |
| 9616日＇063ez | $\because E S$ | ceprat | 円号 | ：Hat rige Eur weIte |
| 日giey getay |  | MP | －1 |  |
|  |  | sue | 1．： | AFJEST $=0$ |
| 64153＇04453 |  | TA | WMEST |  |
| 00164 609222 |  | HTEL | MTA |  |
| 60165－660244 |  | －Hive | 4 |  |
| Eqieg＇gegrit |  | IHTEH |  |  |
| 6atst＇0692？ | EITE： | HTTLS |  |  |
| 6日176．6日eve |  | WHE | Q．HTA | ：ISGUE gTart Lochtigh |
|  |  | ITA | 1．日TEtT | －IESuE wora Conlt |
| datre＇ograze |  | Iite | 1．HTA | ：to ita |
| 60173＇824471 |  | $\underline{\square 17}$ | 1． 060 | ：1Sgue licite |
| 60174 665122 |  | inds | j．ATH | ：Cutmmat |
| 09175＇924544 |  | LTA | －E．EVFF | ：EbHP |
| 6017E＇040443 |  | $\therefore$ TA | Q，EUF | ：EUFFEE |
| 0017\％121606 |  | nity | 1． 8 | ：．FOIHTERS |
| 60298．02444E |  | LIA | 1． 5 ca | －burfer counter |
| 60201＊ 64445 |  | STA | 1．EUFLL |  |
| 6a262．034454 |  | LiA | 3.61 | －FeOduce muta thee． |
| 60203＇152469 |  | Sue | 30 | ；ADMEESS |
| 90294＇840621 |  | STA | Q． 21 |  |
| 日e2es＇163Eam |  | H0I | 3．${ }^{1}$ |  |
|  | EEFLL： | DE2 | Wity | ：Labt LThes |
|  |  | 19F | Etlate | ：ha－Euntinue |
| 60210．965244 |  | HTDE | 44 | ：YES－CHEEK WRTTE |
|  |  | LIA | G．EUFF |  |
| 68212／024450 |  | LIA | 1，WICHT |  |
| 64213\％66459 |  | ，\％E | chemet |  |
|  |  | गG2 | FFBAE | ：LAET FRAME？ |
| 60215＇809457 |  | MP | ＋+7 | ：HO－MOyE TO MEST |
| 60215．026447 |  | LIA | E．T4 | ；mohe meschge |
| 60217＇606467 |  | 198 | Enbet |  |
| Gezegemegaig |  | LIA | Q．Ace | ：RESTORE AC＇S |
| 日6221 924419 |  | LIf | 1． ACL |  |
| 60222＇036410 |  | LEA | 2．902 |  |
| 60223＇6日2d94 |  | ImF | DETH | ：YES－EETURH |
| 602e4•60442 |  | ISR | E日Ftag | ；G？VE FILA |
| 60225 68EEE |  | HF | ETFET | －EESTHET EGMUHING |
| \％ |  |  | ：Coheth | AHTS |
| geves＇1アファア | BHET： | QPITE |  |  |
| 6日2er＇6909na | PTH： | $\square$ |  |  |
| 日6230．6genes | acb： | 9 |  |  |
| 062． 1 － 90980 | HC1： | E |  |  |
| 09232＇960969 | 902： | 9 |  |  |
| 09233 06906e | FFHHE： | 3 |  |  |
| －02st＇guspr | $\Xi$ | 37 |  |  |
|  | ce日： | 89 |  |  |

```
ब064 की%4
    बEz\sigmaE'gem000 FirgT: g
```







```
    0日244502+00 %% 2400
    Gge406g6e90 }
    gaz4ergegeg act ab
    g504%'060gge mPaL: 5
    00c5a`00,0e az: z
```



```
    geएz ifE4%g ra: - -000
    0日253 600g60 %: 0
    06254"E日E40. FPLIN: 400
    Ggres'gagage ri-IH: a
    agc5s.0日geby ri: 1
```







```
    @g264'906050 E56: 50
    8日265*509E4z*T4: TKT4
```



```
T&T1: .TBT <<S><IZ> SGAN IHITIALIZATIOH.
    09267. 366412
    0日2-2'日2g日40
    G6271.6cgetg
    60272.651593
    0日273'640516
    0027:'gEg111
    69275'047111
    gecregese111
    0日er7.040514
    0636日. 444532
    06361 646524
    963日2'044517
    60363'047606
```



```
    66304*g66412
    80395*620040
    06356'62064a
    06367'020040
    00318'020640
    00311*644117
    06312/653440
    00313'646565
    6日314*日47131
    60315* 320106
    0日S1E'E5116!
    06S17%64%585
    00320'05147%
    00221'820940
    6@Ses"62064%
    00323.920440
```

```
G0日e 3r-4,
    60%% ! 0%00+4
```



```
    gezer bgam!
                <1F.<1こ%
                            SCHN ELGIHS
```



```
    05333'650044
    60331'020540
    E@3こ2*GESg40
```




```
    G033%'64%5%%
    6en3s+6ealez
    05357.84त567
    04340,844516
    66%41'651466
```



```
    60342, 606412
    60343'020640
    64344'020640
```



```
    6034E.026040
    004476545%
    00550.64051E
    g6551'gede4
```



```
    60352'64240%
    64354'683407
    60355.063407
    0日35G'863400
```



```
        0G4日G日 EUFFE: .ELK q%G日
                            EHD
```



```
0002 E04bH
```



```
    0504%'02000日
```



```
                TYT <<S<<Iz> IS FILN FGSITIOHING IESIEED?
    g6egy'geg4y2
    geg5e'0261:!
    gat53'051*40
    0EES4'84315:
    gev5g'046115
    0日gse'6eg1%%
    9005%-64>5%%
    06462,g<4%2%
    06061+6445%
    G6日E2'g4%il:
    06063'04%:8%
    60654'020184
    getes*g425z3
    06666'84459%
    006G7'042594
    gGgre, B3,44%
    gegr1.'geget5
    gagre'gemg40
    4gg73'02gege
    00日74*ggegrexguESS: . +1
        TMT <<S><IZ> IS ERAHRIHG DESIEEE?
    Gng75.6%6412
    60g7e'626111
    g@ET7'0514,4,
    g@10g'05i5gz
    G0101:日ceste
    60162'647111
    06103'64516%
    06104'820464
    06105*642523
    6016E'0445z2
    06167'gcesb4
    66110.83%440
    40111'0egeta
    60112'getedg
    0113'620640
    801144020640
    60115'gegeme
    0日116'gegean
    60117'620g6e
    g@12g'egere1'guES4: .+1
            TMT <<15><12> mO YOU WIEH TO CONTIHUE?
    00121.0geti2
    0012z'026104
    0123.047440
    40124*654517
    0125'652446
    0112s.05s511
    0127'051515
    0日130'620124
    06131'04तध4#
    00132'041517
    00133'647124
    06134.644E15
    09135*05%565
```

```
00BE EG&RA
```






```
    0142*E206*5
    6#143*62@gBO
```



```
BEGद ThFGE
```




```
                .TXT <<S><IE> TAFE FOSITIONING.
    061g4PEEG412
    60105.Ecकe40
    0010E'6ल6E45
    60167'652161
    g611E"ESE165
    0111*020120
    60112'847523
    0@113'044524
    0日114%E44517
    0日115'g47111
    E19116'E47187
    60117'0gegeg
    0日120.g061z1'T%T2: .+1
                            TKT <<15><12> EHTER TAFE COMHANII:/
    00121'6064%2
    g@122'620046
    60123'626040
    01124'02E646
    00125'626046
    0.12E'g42516
    0612%'052105
    96130.65194日
    96131*652101
    00132'050105
    06133*0e日193
    01134'047515
    00135* 146501
    90156'547154
    g0137.635006
    00149.0日G141*TKT3: . +1
                            TMT
    g6141'gme41z
```


田－品日足
白

 $\frac{-1}{-7}$
$\underset{-1}{-1}$
$\underset{-1}{-1}$








 4
$\frac{4}{4}$
4


## 



```
0G44 TAFOS
    6%23E'842046
    002#4'042i!%
    60255'647105
```




```
    6024क"ब54c4日
    G6241* 648517
    00242.651155
    ब@c43'日5%44,
    g0e44, 02gere
    60245, Ecge4
```



```
    g@za?'62क5eg
```



GTA

66683' 050486
66684 946431
6060g' 922436 LODP
60606' 0186
$06607 \cdot 624525$
6egig 10子ch
66日11 12zT0
बegte'gesalz
gegiz'begege
6egintergese
60615 eederg
gatie'121065
Q6217"ゆystat
6日eç'606gez
6g日e1'6760¢g
6日022"606406
B6gez'becrez
gegea gevses EHI
gages'ge4cRE

69627'062481
ETA
STA
धTA
L.TA
152
LiA A
AHE
SUEG
IHF
STSTM
rehar
IMF
MOV 1. 日. SHR ; SECDHD ZERU?
diF
systim
EHM ; SES
3. ETH : SAVE EETUEH
TITLE BEITE
title meite
. EHT BRITE
HeEl
TA
Q
in
Q. ACG ; SHUE AC'S

1. AG 1
2. ar c
E. DIR ; STORE ADDEESS
G. EDAE ; OETAIH ThO CHARACTERS
Dine ; IHCREMEHT ADMRESE
3. C177: Mase
E. 1 ; SECOHTI IN AC 1
4. G, GUE : FIRST IH AEG-zERO?
ENII : YES
; HG-DUTFUT FIEST
THIS Sugenutine has eefr megignig ta
; OUTFUT A LIHE OF TEST TG THE TEGETOPE
    - UEinti the fume libetmotion
;
; AED MGGT EGATAIH THE LOEATIOM BE
; THE LIE TO EE MEITTEA
THE LIHE MUST EE WRITTEU JH TATH 1
; Frothat
;
:
. FCHER
Itre
JMF
JMF LOQF ; HEKT TUD
LIA G.aCg ; RESTORE AC'S
LIA $1 . \mathrm{ACL}_{1}$
LDA E:ADE
MF
GRTH ; RETURH
; conethnte

| 60959＇606cbe | ETH： | 5 |
| :---: | :---: | :---: |
| 06931＇606eta | rice： | $\square$ |
| 60632＇60460a | ACO 1 | $\square$ |
|  | ALE | 8 |
| 66esereberit | C175： | 177 |
|  | ITER： | E |

；conethnte

## REFERENCES

1. E.R. Kretzmer, "Statistics of television signals", Bell Syst. Tech. J., Vol. 31, July 1952, pp. 751-763.
2. W.F. Schreiber, "The Measurement of third order probability distributions of television signals", IRE Trans. Inform. Theory, Vol. IT-2, Sept. 1956, pp. 94-105.
3. J.O. Limb, "Entropy of quantized television signals", Proc. Inst. Elec. Eng. (London), Vol. 115, Jan. 1968, pp. 16-20.
4. P.R. Wallace, "Real-time measurement of element differences in television programs", Proc. IEEE (Lett), Vol. 54, Nov. 1966, pp. 15761577.
5. A.J. Seyler, "Real-time recording of television frame difference areas", Proc. IEEE (Lett.), Vo1. 51, March 1963, pp. 478-480.
6. A.J. Seyler, "Statistics of television frame differences", Proc. IEEE (Lett.), Vol. 53, Dec. 1965, pp. 2127-2128.
7. L.E. Franks, "A model for the random video process", Bell Syst. Tech. J., Apr. 1966, pp. 609-630.
8. W.K. Pratt, J. "Kane, and H.C."Andrews, "Hadomard transform image coding", Proc. IEEE, Vo1. 57, Jan. 1969, pp. 58-68.
9. A. Habibi and P.A. Wintz, "Image coding by linear transformation and block quantization", IEEE Trans. Commun. Technol., Vo1. COM-19, Feb. 1971, pp. 50-62.
10. G.B. Anderson and T.S. Huang, "Piecewise fourier transformation for picutre bandwidth compression", IEEE Trans. Commun. Technol., Vol. COM-19, Apr. 1971, pp. 133-140.
11. M. Tasto and P.A. Wintz, "Image coding by adaptive block quantization", IEEE Trans. Commun. Technol., Vol. COM-19, Dec. 1971, pp. 957-971.
12. A. Habibi, "Comparison of nth-order DPCM encoder with linear transformations and block quantization techniques", IEEE Trans. Commun. Techno1., Vo1. COM-19, Dec. 1971, Pp. 948-956.
13. J.B. O'Neal, Jr., "Predictive quantization systems (differential pulse code modulation) for the transmission of television signals", Bell Syst. Tech. J., Vol. 45, May/June 1966, pp. 689-721.
14. J.B. O'Neal, Jr., "A bound on signal-to-quantization noise ratios for digital encoding systems", Proc. IEEE, Vol. 55, March 1967, pp. 287-292.
15. K.Y. Chang, "Analysis and optimization of differential PCM systems operating on noisy communication channels", Ph.D. thesis, Oct. 1972, Dept. Elec. Eng., U.B.C.
16. R.C. Brainard, F.W. Mounts, B. Prasada, "Low resolution T.V.: subjective effects of frame repetition and picture replenishment", Bell Syst. Tech. J., Vo1. 46, Jan. 1967, pp. 261-271.
17. F.W. Mounts, "Frame-to-frame digital processing of TV pictures to remove redundancy", Symp. Picture Bandwidth Compression, MIT, Cambridge, April 1969, pp. 653-672.
18. J.E. Cunningham, "Frame-correction coding", Symp. Picture Bandwidth Compression, MIT, Cambridge, April 1969, pp. 623-652.
19. L.C. Wilkins, P.A. Wintz, "Bibliography on data compression, picture properties and picture coding", IEEE Trans. Inform. Theory, Vol. IT-17, March 1971, pp. 180-197.
20. H.M.R. Souto, The Technique of the Motion Picture Camera, Focal. Press Ltd., 1969.
21. N.G. Deryugin, "The power spectrum and autocorrelation of the television signal", Telecommunications, Vol. 7, 1957, pp. 1-12.

[^0]:    geed F 1 Pb 0gaz+651:5! arsarnae:日acosesuas gnest atros
     E404t-sesit Q6ene'rapat 6ヵ"tremer
    
    
     Qe2:7'951.0
    
     बataerararo
    
    

