A FEEDLOT REPLACEMENT MODEL

by

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B.Sc.A., University of British Columbia, 1966

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

in the Department

of

Agricultural Economics

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

September, 1972
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Date September 1972
ABSTRACT

The purpose of the study was to develop a realistic method of determining the optimum replacement time of steers in a feedlot in which there are sequential feeding cycles.

There are two models for steers developed and discussed in this paper. The first model optimizes the length of stay in the feedlot assuming that feedlot capacity is the limiting constraint. The second model optimizes the length of stay in the feedlot when working capital is the limiting constraint.

A third model is developed to determine the optimum length of feeding time for market hogs in a feeding barn.

An evaluation of each model is made. It is concluded that the cattle models have a limited use because of the high variability in performance. The hog replacement model is quite valid and can be used in a modern hog feeding enterprise.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>v</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>vi</td>
</tr>
<tr>
<td>Chapter</td>
<td></td>
</tr>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. THEORY</td>
<td>3</td>
</tr>
<tr>
<td>III. FEEDLOT APPLICATIONS</td>
<td>9</td>
</tr>
<tr>
<td>IV. METHODOLOGY</td>
<td>11</td>
</tr>
<tr>
<td>Collection and Treatment of Data</td>
<td>11</td>
</tr>
<tr>
<td>Definition of Terms</td>
<td>13</td>
</tr>
<tr>
<td>The Replacement Models</td>
<td>14</td>
</tr>
<tr>
<td>Cattle Feedlot Replacement Model I</td>
<td>14</td>
</tr>
<tr>
<td>Cattle Feedlot Replacement Model II</td>
<td>16</td>
</tr>
<tr>
<td>Hog Replacement Model III</td>
<td>17</td>
</tr>
<tr>
<td>V. RESULTS AND DISCUSSION</td>
<td>19</td>
</tr>
<tr>
<td>VI. CONCLUSIONS</td>
<td>33</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>35</td>
</tr>
<tr>
<td>APPENDIX</td>
<td>38</td>
</tr>
<tr>
<td>Table</td>
<td>Page</td>
</tr>
<tr>
<td>-------</td>
<td>------</td>
</tr>
<tr>
<td>1. Weights and Feed Intake of Steers</td>
<td>39</td>
</tr>
<tr>
<td>2. Weights and Feed Intakes of Hogs</td>
<td>40</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Profit Maximization over Time</td>
<td>6</td>
</tr>
<tr>
<td>2.</td>
<td>Average Net Revenue and Marginal Net Revenue Per Animal Per Week - Model I</td>
<td>20</td>
</tr>
<tr>
<td>3.</td>
<td>Average Net Revenue and Marginal Net Revenue Per Dollar Per Week - Model II</td>
<td>21</td>
</tr>
<tr>
<td>4.</td>
<td>Effect of Incremental Costs on Optimum Replacement Time - Models I and II</td>
<td>22</td>
</tr>
<tr>
<td>5.</td>
<td>Effect of Price of Finished Steers on Optimum Replacement Time - Model I</td>
<td>23</td>
</tr>
<tr>
<td>6.</td>
<td>Effect of Price of Feed on Optimum Replacement Time - Model I</td>
<td>24</td>
</tr>
<tr>
<td>7.</td>
<td>Effect of Price of Feed on Optimum Replacement Time - Model II</td>
<td>25</td>
</tr>
<tr>
<td>8.</td>
<td>Average Net Revenue and Marginal Net Revenue per Unit of Floor Space Per Week - Model III - Hogs</td>
<td>26</td>
</tr>
<tr>
<td>10.</td>
<td>Effect of Price of Feed on Optimum Replacement Time - Model III - Hogs</td>
<td>28</td>
</tr>
<tr>
<td>11.</td>
<td>Growth Curve for Steers - Models I and II</td>
<td>41</td>
</tr>
<tr>
<td>12.</td>
<td>Feed Intake for Steers - Models I and II</td>
<td>42</td>
</tr>
<tr>
<td>13.</td>
<td>Growth Curve for Hogs</td>
<td>43</td>
</tr>
<tr>
<td>14.</td>
<td>Estimated Feed Intake of Hogs</td>
<td>44</td>
</tr>
</tbody>
</table>
ACKNOWLEDGEMENT

I wish to extend my warmest thanks to Dr. G.R. Winter and Dr. P.L. Arcus who spent many hours reading and advising me on this thesis. Their advice and encouragement was very much appreciated.

I also thank Dr. R.G. Peterson and Dr. J.D. Boyd, who gladly served as members of the committee and made many invaluable suggestions on the writing of the thesis.
Chapter I

INTRODUCTION

Today's feedlots are part of one of the most rapidly changing industries in North America. In a few short years the livestock feeding industry has evolved from the once familiar 100 head farm feedlot to highly intensified feeding enterprises with capacities ranging upwards to 100,000 head.

These changes have been the result of the development of high concentrate feeding programs which have permitted almost total mechanization of the feeding operation.

The increased mechanization has greatly increased the overhead and capital costs. In order to make more efficient use of the fixed costs, feedlot operators are finding it necessary to operate on a continuous basis throughout the year. In this situation the feedlot operator is faced with the problem of when to sell each group of cattle and replace them with another group. In a continuous operation, where each group of cattle is replaced immediately by a new group, the operator is concerned with maximizing average profit per unit of time as opposed to maximizing profit:per animal, which is the criterion applied where a single group of cattle is fed until marketed and then not replaced immediately. The continuous operation has an opportunity cost of time whereas in the single lot, time has no opportunity cost.
because the feedlot will be sitting idle the remainder of the year.

To date there have been two approaches to the replacement problem in the feedlot. The most commonly used approach is one in which the steer is fed until the time when maximum price per pound or maximum grade is obtained. Ranta (13) demonstrated that this was false economy and that the steer in fact should be shipped at a lighter weight when the marginal cost per pound of gain equalled the selling price. Faris (7) suggested that the marginal cost of gain in the steer should include an opportunity cost for time as is demonstrated by Trant and Winder (14) for broiler production.

This paper will expand on the theory proposed by Faris (7) to include changes in animal quality and yield with the length of the feeding period.

Two models are proposed. The first model assumes that feedlot capacity is fixed and is the limiting constraint. The second model illustrates the difference in replacement pattern when working capital rather than feedlot capacity is assumed to be the limiting constraint.
Chapter II

THEORY

The purpose of a replacement model is to maximize net revenue or profit. Profit can be maximized according to two criteria. Firstly it can be maximized per production period. This is illustrated by Dillon (5) in his timeless analysis of unconstrained profit maximization. He showed that unconstrained profit maximization in a perfectly competitive market occurred when each variable input was used at the level such that \( P_y \cdot MP_i = P_i \), \( i = 1, 2, \ldots, n \),

where \( P_y \) = price of output or product
\( MP_i \) = marginal product of input
\( P_i \) = price of input
\( n \) = number of inputs.

The most general statement of this timeless profit maximizing condition is that for best operating conditions the Marginal Revenue or Marginal Value Product of \( X_i \), given by \( d(P_\cdot Y)/dX_i \), must equal the marginal cost of \( X_i \), given by \( d(P \cdot X_i)/dX_i \).

Thus \( d(P_\cdot Y)/dX_i = d(P \cdot X_i) \cdot dX_i \) where \( Y \) = amount of output and \( X_i \) = amount of input.

In other words, this last increment of \( X_i \) must pay for itself. In extrapolating this to the feedlot, profit maximization will occur when the cost of an extra pound of gain equals the selling price per pound.
The second criteria for profit maximization is to maximize profit per unit of time. Again Dillon (5) demonstrates that the same general principle of equating marginal revenue and marginal cost must prevail for profit maximization over time. The only difference is that marginal cost over time must allow for time opportunity cost and time preference effects as well as the direct marginal costs of inputs. Profit maximization over time involves more complex analysis than in the timeless case. To maintain as much simplicity as possible while outlining the relevant principles, Dillon (5) first considers a simple response process operating over time but in the absence of time preference, i.e., a zero interest rate is assumed. He then adds the time dimensions. This analysis spotlights the effect of the time opportunity costs of inputs.

The time dependent analysis is as follows. Consider a time dependent response process

\[ y = F(X_1, X_2, \ldots, X_n) \]  
\[ X_i = F_i(t), \quad i = 1, 2, \ldots, n \]

where \( y \) = amount of output
\( X_i \) = amount of input
\( t \) = time or length of response period with the output being harvested at the end of the variable response period \( t \). The process is to be repeated sequentially over time.

Assuming no input or output price changes over time, no input constraints, a fixed capacity or scale of enterprise in the short run, and no time preference, best operating
conditions for each run of the response period must be identical, and maximization of total profit over a number of runs must be equivalent to maximizing profit per unit of time. Denoting profit per unit of time by $\Pi^*$ and the cost of the fixed inputs or the fixed set-up cost per response period (i.e., incremental costs) by $F$, we have the unconstrained objective function

$$\Pi^* = (P_Y Y - \sum_{i=1}^{n} P_i X_i - F)/t,$$

where $Y$ = amount of output

$X_i$ = amount of input

$P_Y$ = price of output

$P_i$ = price of input $X_i$

t = length of response period.

While the fixed input cost $F$ is fixed for each run of the response process, the ratio $F/t$ is not constant for $t$ variable. Hence, unlike the timeless analysis, fixed input costs must be included in the time dependent objective function.

Maximum profit per unit of time implies $d\Pi^*/dt$ of equation (3) equal to zero. Thus setting $d\Pi^*/dt$ equal to zero, and rearranging the resulting equation, we must have

$$P_Y (dY/dt) - \sum_{i=1}^{n} P_i (dx_i/dt) = (P_Y Y - \sum_{i=1}^{n} P_i X_i - F)/t$$

for maximum $\Pi^*$. More simply if we denote by:

$R$: output gain or revenue per response period $P_Y Y$

$C$: input loss or cost per response period $\sum_{i=1}^{n} P_i X_i + F$

then equation (4) can be written as

$$dR/dt - dC/dt = \Pi^*.$$
In words, equations (4) and (5) imply that in the absence of time preference the marginal profit per unit of time (LHS) must equal the average profit per unit of time (RHS) if maximum profit per unit of time is to be achieved assuming \( \mathcal{T}^* \) convex.

Second order conditions, i.e., if \( d' \mathcal{T}/dt \) is negative, will prove that \( \mathcal{T}^* \) is convex and thus we have a maximum.

![Figure 1. Profit Maximization over Time](image)

The profit maximizing criterion of equations (4) or (5) is illustrated in Figure 1. Profits or net revenue \( (P_y Y - \sum P_i X_i - F) \) as a function of length of the response period is shown by the curve \( \phi AB \). Maximum profit from a single response period occurs at B with a response period of length OH. In contrast, maximum profit per unit of time occurs at A where the slope of the profit curve equals maximum average profit and the response period is only of length OG. Thus equation (5) is satisfied at A. As Figure 1 indicates, best operating conditions for a sequence of response processes implies each response period should be shorter
than if the response process were only carried through once. The logic of this is that as inputs are used beyond $A$, marginal profit per unit of time is less than the maximum average profit per unit of time that could be obtained by using these inputs in the next response period. Only if harvesting of output occurs at $A$ of Figure 1 will the marginal value product be equal to marginal cost over time.

In terms of variable inputs, the conditions for maximum profit per unit of time is that $\frac{d \pi^*}{d x_i}$ be zero. Thus differentiating equation (3) we must have

$$P_y \left( \frac{dY}{dx_i} \right) = P_i + \left( \frac{dt}{dx_i} \right) (P_Y - \sum_{i=1}^{n} P_i x_i - F)/t$$

(6)

or, akin to equation (5)

$$\frac{dR}{dx_i} = \frac{dC}{dx_i} + \left( \frac{dt}{dx_i} \right) \pi^*.$$  

(7)

The LHS of equations (6) and (7) is the marginal value product of $x_i$. The RHS, the marginal cost is the sum of two parts. The first of these is the cost of a unit of $x_i$ without regard to time. We will call this the direct marginal cost of $x_i$. The second part is the time opportunity cost of a unit of $x_i$. It consists of the maximum average profit per unit of time possible in the next response period, $(P_Y - \sum_{i=1}^{n} P_i x_i - F)/t$, multiplied by the time, $\frac{dt}{dx_i}$, required to utilize a unit of $x_i$. Compared to the timeless analysis, the effect of introducing time is to increase the marginal cost $x_i$ by the amount $\left( \frac{dt}{dx_i} \right) \pi^*$. In turn, this implies that the best operating conditions over time involve lower levels of the variable inputs than are implied by the timeless analysis.
While the above analysis has been couched in terms of the fairly simple case of a sequence of response periods based on equations (1) and (2), the same general principle applies in more complicated cases without time preference. Thus should physical relations or prices be different in the next response period, the role still remains that response in the present period should be harvested when, per unit of time, its marginal net revenue is equal to the maximum average net revenue expected from the next response period. Concomitantly, variable inputs should be used to the point where their marginal revenue is equal to their overall marginal cost inclusive of time opportunity cost based on future revenue possibilities. The difficulty, of course, lies in making a good estimate of revenue possibilities from the next run of the response process.
Chapter III

FEEDLOT APPLICATIONS

In applying this theory to the feedlot it is assumed that the feedlot is operating on a continuous basis throughout the year. That is, whenever any animals are sold, they are replaced immediately by another group of feeders. This theory can be applied to a feedlot that only feeds a single lot of cattle in a year, however it must be kept in mind that the time opportunity cost or the average profit of the next immediate response run is zero. Thus they are fed until the Marginal Revenue Product equals the Marginal Cost irrespective of the time opportunity cost. This time will be different from the time for the continuous operation.

The first model (Model I) for cattle assumes that the production from a feedlot is limited by an optimum or fixed capacity of the lot rather than being limited by capital or feed supplies. The second model (Model II) assumes that working capital is the limiting factor and maximizes profit per dollar invested rather than per animal unit of feedlot capacity.

By taking into consideration these assumptions it is possible to conceptualize the similarity between the cattle in a feedlot and the machines in a factory or other productive process. The cattle can be visualized as rela-
tively long-lived machines with a specific function as time progresses. However, unlike machines, feedlot cattle have an appreciating value with time as compared to the depreciating value of machines with time. The cattle appreciate in value with time because they are constantly gaining weight as well as increasing in value per pound as a result of an increasing yield and grade with time on feed. The increasing value per pound will reach a maximum beyond which the price is reduced because either the carcass is too heavy or too fat or both. This dissimilarity does not affect the operation of a basic replacement model. The basic difference will be that the feedlot model will attempt to maximize average net revenue per unit of time while the equipment model attempts to minimize average cost per unit of time. The theory applies equally well to the hog feeder barn as it does to the cattle feedlot except that the hogs will require increasing floor space as time progresses. Beef cattle also have an increasing space requirement but it has not been considered for the beef model. Increasing space requirements are considered in the hog replacement model (Model III).
Chapter IV

METHODOLOGY

Collection and Treatment of Data

The data used for the feed intake curves and growth curves of models I and II was collected from some beef cattle nutrition experiments by Freding (10). In the Freding experiments, thirty Hereford steer calves were divided into six groups. They had an average starting weight of 535 lbs. and remained on feed until they reached 1000 lbs. at which point they were slaughtered. Weekly animal weights and feed intakes were recorded.

The animal weights and feed intakes were fitted to several polynomial regression curves using time as the independent variable, \( W = f(t, t^2 \ldots) \) \( F = g(t, t^2 \ldots \text{ etc.}) \). The highest degree polynomial to have a significant (\( P \leq .05 \)) increase in fit was chosen.

It should be noted at this point that the curves are not meant to represent a general population but only the individual animals that were tested. Likewise the replacement model results do not represent the optimum replacement pattern for a general type, only the group tested.

The price function \( P_y(t) = f(t) \) (this is not price speculation) is an assumed curve based on an experiment by Zinn et. al. (17) which related the grade and yield of beef.
carcasses to time on feed for a group of cattle similar to the ones used in models I and II.

Weights were assigned to prices in relation to the grade and yield at specific intervals to determine average price at those time intervals. The average prices were then fitted to a third degree polynomial in order to determine the price function \( P(t) \). In determining the effect of a change in price on the optimum replacement time, the price functions were assumed to be parallel for varying prices of choice steers, i.e., the price of good steers was in constant proportion to the price of choice steers.

Model III for hogs uses data from Purdue by Foster (9). There were eight hogs per group and four groups with an average starting weight of 105 lbs. Growth and feed intake on time curves were fitted by using polynomial regression. The data contained only five observations. Consequently the curves may not accurately predict the actual response of these hogs.

The grade index in the Foster data, based on back fat thickness in relation to body weight, did not appear to change significantly throughout the feeding period. Thus it was assumed for this group of hogs that the grade index and price function would remain constant throughout the feeding period.
Definition of Terms in Models I and II

Models I and II use certain terms, which, for convenience, are summarized below.

\[ \Pi = \text{Net Revenue}. \]  
Net revenue refers to the gross value received for the sale of the product less the value of the variable costs of production and the incremental costs or fixed input costs per production run. Overhead costs such as capital expenses on feedlot facilities are not included as it is these facilities for which we are trying to find an opportunity cost.

\[ \eta^* = \text{Average Net Revenue Per Unit of Time}. \]  
This is equivalent to the Net Revenue (\( \Pi \)) divided by the length of time in the production period (\( t + t_s \)).

\[ \Pi_E = \text{Expected Net Revenue of Subsequent Production Period}. \]  
This is calculated using expected response curves and selling prices of the next group of animals. A method of incorporating price forecasts could be developed but has not received consideration here. The subscript E denotes expected functions.

\[ P_Y = R = \text{Value of the finished animal} = 0.96 \cdot P(t) \cdot W(t) \]  
where \( P(t) \) - price of slaughter cattle as a function of time in the feedlot

\[ W(t) \]  
weight of the animal as a function of time in the feedlot

0.96 allows for a 4% shrink.
\[ Z P_i X_i = \text{Variable costs} = PF \cdot F(t) + i K(t). \]

where, \( PF \) = price of feed

\( F(t) \) = Feed intake as a function of time

\( i \) = current interest rate on working capital

\( K(t) \) = working capital required

\[ = \int_{t=0}^{n} (PF \cdot F(t) + F) dt. \]

\( I.C. = F = \text{Incremental costs:} \) Incremental costs in this model are defined as those costs incurred while marketing the present lot of cattle and subsequently replacing them with another lot of cattle. This includes:

- cost of the replacement animal;
- average veterinary costs;
- selling costs (commission, trucking);
- and average death loss.

\( t \) = length of time on full feed expressed in weeks.

\( t_s \) = starting time. This is the time from which the animal arrives in the feedlot until it is on full feed.

**The Replacement Models**

Three replacement models have been considered.

I. **Cattle Feedlot Replacement Model I.** This model will maximize the average net revenue or profit per unit of time. In an example, if a feedlot has a 1000 head capacity, its maximum throughput per year is 1000 head times 52 weeks in a year or 52,000 animal-weeks per year. This model will then maximize the dollar return per animal-week.

**Assumptions:**

1. Feedlot capacity is fixed and is the limiting constraint
influencing the number of cattle fed in the feedlot at any given time.

2. Sufficient working capital is available at a given price.
   In this model the price is assumed to be 8% per annum.

3. Sufficient feed supplies are available at a given price.

4. Space requirements do not change as the animal grows larger.

5. This feedlot is an insignificant part of the industry and thus does not influence the price of inputs or outputs.

6. The growth curve of a given group of cattle can be identified. In this model the growth curve was taken to be: \( W(t) = 536.35 + 23.49t \) where \( w(t) = \) body weight in pounds and \( t = \) length of time on feed in weeks. (From Freding (10), see Table I, page 39).

7. The feed intake curve of the group of cattle can be identified. For this model the feed intake was taken to be: \( F(t) = 5.91 + 90.33t + 1.92t^2 \) where \( F(t) \) equals total feed intake in pounds and \( t = \) length of time on feed in weeks. (From Freding (10), see Table I, page 39).

8. The relationship of grade and yield to time on feed can be identified. For this model the price function was taken to be: \( P(t) = 26.02 - .046t + .01t^2 \) where \( P(t) = \) price per pound liveweight and \( t = \) length of time on feed. (See ref. p. 11).

9. The expected response (growth, feed intake and grade) can be estimated for the subsequent group of cattle.
For this trial they were assumed to be identical to the previous group (6 - 8 above).

10. The expected cost of feeders, price of feed and price of slaughter cattle can be predicted for the subsequent lot.

As shown in Chapter II profit maximization will occur when the length of feeding period (t) is such that the marginal net revenue of the present group of cattle per unit of time is equal to the expected average net revenue per unit of time for subsequent group.

Thus for Model I the optimum time in the feedlot is when $d \Pi/dt = \Pi^*$

where $\Pi = .96P(t)W(t) - PF\cdot F(t) - i\cdot K(t) - F$

and $\Pi^* = \Pi/(t + t_s)$ where $t_s$ is the starting time. $\Pi^*$ is expressed in cents per unit of feedlot capacity per week.

II. Cattle Feedlot Replacement Model II. This model will maximize the profit per dollar of working capital invested, per unit of time. It is given that the working capital is the limiting constraint in this model rather than feedlot capacity as in the previous model. All other assumptions are the same as for Model I.

In this model II, profit will be maximized at length of feeding time (t) such that;

$d \Pi/dt = \Pi^*$,

where $\Pi = (.96P(t)W(t) - PF\cdot F(t) - F)/K(t)$
and \( \Pi = \frac{\Pi}{(t + t_s)} \) where \( t_s \) is starting time. \( \Pi^* \) is expressed in cents per cent of working capital per week.

III. Hog Replacement Model III. This model maximizes profit per unit of time. Since the floor space requirements vary with the size of the hog, we are maximizing profits per unit of floor space per week instead of per animal per week as in Model I.

Assumptions:
1. Floor space of the feeding barn is fixed and is the limiting constraint.
2. Floor space requirements were assumed to double from the start of the feeding period until the maximum weight was obtained, thus \( s(t) = 1 + t/14 \) where \( s(t) \) = space requirements and \( t = \) length of feeding period in weeks, max./4 weeks.
3. Maximum weight of hog is taken to be 240 lbs. live or 180 lbs. dressed.
4. Sufficient working capital is available at a given price. For this model the price of capital is taken at 8% per annum.
5. Sufficient feed supplies are available at a given price.
6. The growth curve for a given group of hogs can be identified. For this model the growth curve was taken to be \( W(t) = 105.65 + 12.31t - .15t^2 \), (From Foster (9) see Table 2, page 40.) where \( W(t) \) = weight of the hog in lbs., and \( t = \) length of feeding period in weeks.
7. The relationship of grade and yield to time on feed can be identified. For this trial the hog grade index was taken as a constant 103 throughout the feeding period.

8. The feed intake curve was taken to be: $F(t) = 3.3 + 32.5t + .60t^2$ (from Foster (9), see Table 2 page 40) where $F(t)$ = total feed intake in lbs. and $t$ = length of feeding period in weeks.

9. The expected response curves of subsequent groups of hogs do not change.

10. The incremental costs are a fixed price proportional to the current dressed weight slaughter price. For this model incremental costs were taken to be; $F = .70PH + \$15.00$. There are several types of contracts for the purchase of feeder hogs. This one is based on 70% of the dressed weight price. The $\$15.00$ allows for the heavy weight of these hogs at the start of the feeding period.

In Model III, profits will be maximized at a length of feeding time ($t$) such that

$$d\Pi/dt = \Pi^*$$

where $\Pi = 103PH \cdot W(t) - PF \cdot F(t) - F - i \cdot K(t)$

and $PH =$ price of hogs in cents per lb.

$PF =$ price of feed in cents per lb.

$K(t) =$ working capital required $= \int_{0}^{n} (PF \cdot F(t) + F) dt$

$\Pi^* = \Pi/t$

$\Pi^*$ will be expressed in cents per unit of floor space per week.
Chapter V

RESULTS AND DISCUSSION

The results shown in the following figures were obtained by substituting various assumed costs and prices for the independent variables in the equations of Chapter IV and solving the equations for the dependent variables.

ANR (Average Net Revenue) is the $\pi^*$ referred to in the preceding pages. MNR (Marginal Net Revenue) is $d\pi/dt$.

Price and cost assumptions were as follows: $P(t)$, price of choice finished steers, $\$28/cwt.$, (Figures 2, 3, 6 and 7); $PF$, price of feed, $2.5\$/lb.$, (Figures 2-6); $PF$, price of feed, $3.0\$/lb.$, (Figures 8-9); $IC = F$, incremental costs $\$150/head$, (Figures 5-7); $PH$, price of hogs $\$24/cwt.$, (Figures 9 and 10).
Figure 2. Average Net Revenue and Marginal Net Revenue Per Animal Per Week - Model I
Figure 3. Average Net Revenue and Marginal Net Revenue Per Dollar Per Week - Model II
Figure 4. Effect of Incremental Costs on Optimum Replacement Time—Models I and II
Figure 5. Effect of Price of Finished Steers on Optimum Replacement Time — Model I
Figure 6. Effect of Price of Feed on Optimum Replacement Time - Model I
Figure 7. Effect of Price of Feed on Optimum Replacement Time—Model II
Figure 8. Average Net Revenue and Marginal Net Revenue per Unit of Floor Space Per Week—Model III—Hogs
Figure 9. Effect of Price of Hogs on Optimum Replacement Time - Model III - Hogs
Figure 10. Effect of Price of Feed on Optimum Replacement Time - Model III - Hogs
Discussion

The results of the steer models (Figures 2-7) indicate that incremental cost is the largest single factor in influencing the optimum time in the feedlot. Incremental cost, as earlier described, is influenced greatly by the price of the feeder steer which fluctuates considerably.

In Model I (Figure 4), a fluctuation in incremental cost of $10.00 will vary the optimum replacement time by about two weeks. This represents less than a $2.00 per cwt. change in the price of a feeder steer, which is not an unusual variation. A two week variation in feeding time can vary the average net revenue from $.05 to $.10 per week or $2.60 to $5.20 per animal unit per year (Figure 3).

Optimum replacement time is inversely proportional to the price of feed (Figure 6). Thus when the price of feed rises the optimum time in the feedlot decreases. However the influence of the price of feed is relatively small. It would require a $10.00 per ton fluctuation in price of feed to vary the optimum replacement time by one week.

Optimum time in the feedlot varies inversely with the price of slaughter steers (Figure 5). A $2.00 per hundred weight or a 6% rise in the price of slaughter steers without an accompanying increase in the price of feeder steers will result in a lessening of optimum time in the feedlot of 3 weeks or 12%. This is the opposite of what one would expect where a single lot of cattle is involved, where as the price of slaughter steers rises one expects the feeding time to
increase since marginal cost would be less than the new price and thus would have to continue to rise for an additional period of time.

Model II which maximizes the profit per dollar-day of working capital maximizes profit at a slightly reduced time in the feedlot. It can be noted from Figure 4 that at low incremental costs the optimum time on feed is as much as 5 weeks less for Model II than for Model I. This can result in an increased profit of $.0005 per dollar invested in working capital per week which is approximately $6.50 per animal unit per year. As incremental costs increase and profits decrease the optimum feeding time of Model II approaches that of Model I. This is because as profits decrease the opportunity costs of the capital will decrease until it reaches 8% per annum and then the two feeding times will be equal. The same holds true for the price of feed. An increase in price of feed will decrease profits thus delaying the optimum feeding time from what we would expect from Model I. This results in the more modified effect of the price of feed on optimum time in the feedlot than in Model I as can be seen in Figures 6 and 7.

In the hog model, the average weekly profit does not decrease as fast after optimum replacement time as with beef for the particular group of hogs tested. By keeping the hogs (Figure 8) an extra three weeks beyond the optimum time the average net revenue decreases approximately 1.8 cents per week per hog which amounts to $.90 per hog-space per year.
The price of feed has a small effect on the optimum replacement time (Figure 10). A change of $20.00 per ton (33%) will only alter the optimum replacement time by approximately one week. A rise in the price of feed means a longer feeding time.

The price of hogs has much greater effect on the optimum replacement time (Figure 9). Since the price of hogs is much more variable than the price of feed it can be concluded that this is the most important variable. A $6.00 per cwt. change in price (28%) will alter the optimum replacement time by approximately 3 weeks. An increase in the price of hogs results in a decrease in time on feed. This, again, is contrary to the marginal cost theory by which one would expect the optimum length of feeding period to vary directly with the price of hogs.

There is only one value in the Expected Net Revenue Function that differs from the Net Revenue Function; that is the selling price of the hogs for which it is reasonable to expect a change within the production period. With more accurate market outlook information it would be quite possible to include a probabilistic expected price function within the model.

Since the replacement hogs are quite often of similar breeding to the sample group of hogs it can be assumed that they will have similar response curves. Also the incremental costs, being a fixed proportion of the selling price, eliminates the other differences between the Net Revenue Function
and the Expected Net Revenue Function. Environmental influences on the performance of the hogs are minimized by the controlled environment housing in which the hogs are now being raised.
Chapter VI

CONCLUSIONS

Two cattle feedlot replacement models have been presented. These will determine the optimum time at which a group of steers under a given set of circumstances should be sold from a feedlot. These models depend on being able to predict the performance of both the present and subsequent lots of cattle which is often a difficult task. Factors such as: varying environmental backgrounds prior to entering the feedlot; seasonal availability of feeders; mongrelization of the cow herds through the influx of exotic breeds resulting in an unpredictable genetic performance of the feeders; and exposure to varying weather conditions in the feedlot; make the feedlot performance of any group of animals highly unpredictable thus making the models of limited value.

The use of these models might be more reasonable in the southern United States, where backgrounding of cattle is becoming more common. With backgrounding, which is simply the preparation of the animals for an intensive feeding situation, it may be possible to obtain a standard type from a known previous environment and thus the response curves can be more easily predictable. Seasonal variation in supply of feeders is also reduced.
Model II is perhaps, a more realistic model, yet it is of limited value, for many of the Prairie feedlots since the lots are easily expandable at a low cost and are often not stocked to capacity. In the Fraser Valley of British Columbia or other areas with confined housing the cost of expansion is much greater and thus feedlot capacity is more often the limiting constraint making Model I more applicable.

The hog replacement model appears to have a definite value in the decision making process of tomorrow's hog producer. The present method of shipping the hog when it nears the maximum weight for Canadian Grading Standards is not a profit maximizing procedure. It has been shown here that profit can be increased by shipping earlier for this trial group of hogs. The optimum shipping time will vary for each type of hog and other varying circumstances. The unpredictable variables of the cattle models have been minimized or excluded in the hog model leaving a practical workable replacement model.
BIBLIOGRAPHY


### Table 1. Weights and Feed Intake of Steers

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>Weight (W(t))</th>
<th>Total Feed Intake/Steer (F(t))</th>
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Feed Intake: $F(t) = 5.91 + 90.33t + 1.92t^2$, \( r^2 = 0.999 \)  

Weight: $W(t) = 536.35 + 23.49t$, \( r^2 = 0.999 \)

Source: Freding (10)
Table 2. Weights and Feed Intakes of Hogs

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<th>Time (t)</th>
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<th>Feed Intake F(t)</th>
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</table>

Feed Intake \( F(t) = 3.1 + 33.4t + 0.52t^2 \) \( r^2 = .996 \) 

Weight: \( W(t) = 105.61 + 12.44t + 0.16t^2 \) \( r^2 = .998 \)

Source: Foster et. al (9)
Figure 11. Growth Curve for Steers - Models I and II,

\[ W(t) = 536.35 + 25.49t \]

\( H_0 \) of nonlinearity rejected at \( P \leq 0.05 \)

Source: Freding (10)
Figure 12. Feed Intake for Steers - Models I and II

\[ F(t) = 5.91 + 90.33t + 1.90t^2 \]

\( H_0 \) of linearity rejected at \( P \leq .05 \)

Source: Freding (10)
Figure 13. Growth Curve of Hogs

\[ W(t) = 3.3 + 33.5t + 0.60t^2 \]

\( H_0 \) of linearity rejected at \( P \leq 0.05 \)

Source: Foster (9)
Figure 14: Estimated Feed Intake of Hogs

\[ F(t) = 3.3 + 32.5t + .60t^2 \]

H\textsubscript{0} of linearity rejected at \( P \leq .05 \)

Source: Foster (9)