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AN EXPLORATORY STUDY OF THE EFFECT OF CO-OPERATIVE GROUP LEARNING, INVOLVING TUTORING, ON THE ACHIEVEMENT AND ATTITUDES OF GRADE EIGHT PUPILS IN NEW MATHEMATICS

by

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B.Ed., University of British Columbia, 1961

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF EDUCATION

in the Department
of
EDUCATION

We accept this thesis as conforming to the
required standard

THE UNIVERSITY OF BRITISH COLUMBIA
September, 1972

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ABSTRACT

An exploratory investigation into the effect of co-operative group learning, involving tutoring, on the achievement and attitudes of 174 grade eight pupils in new mathematics is described. Three volunteer teachers and six volunteered mathematics classes were involved. Five hypotheses concerning test performance and one concerning attitudes were advanced. Using the scores obtained in the mathematics sections of the Stanford Achievement Test (Advanced) and an entering behaviour test of prior mathematics learning, to establish similarity of the groups, instruction was carried out over a period of twelve weeks. A retention test was given two months later. Attitude scores from data collected by Semantic Differential before and after the experiment were analysed using a model for multidimensional analysis of Semantic Differential attitude data (McKie and Foster, 1972). Achievement in algebra learning and retention reached the .05 level of statistical significance, favouring the experimental group. No differences for treatment occurred for instructor effects, instructor by group interaction or attitudes at the .05 level of statistical significance. Conclusions for further research and practice are drawn.

Approved: _____

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ACKNOWLEDGMENTS

The writer wishes to express his sincere appreciation for the generous help and guidance given by Dr. Denis C. Rodgers, advisor for the thesis, and for the willing assistance given by Dr. Stephen F. Foster, Dr. Robert Conry, Dr. James Sherrill, Dr. Douglas McKie, Dr. Harold Ratzlaff, Dr. Arthur H. Elliott and Dr. Roland Gray in the preparation of the thesis.

In addition, the writer wishes to express his thanks to the principal, Mr. W. L. Bazeley for his full co-operation, and to the Department of Research and Services of the Vancouver Public School System, particularly Dr. N. Ellis and Mr. A. Moodie, for the encouragement offered and for providing standardized testing materials. The co-operation and efforts of the volunteer teachers and their respective grade eight mathematics classes are also gratefully acknowledged.

The writer also wishes to give special thanks to Dr. Robert Conry, Dr. Douglas McKie and Mr. Robert David Meldrum M.Sc. for their untiring work on the computer programs.

Finally, sincere appreciation is offered to my wife, Doris, without whose constant encouragement, assistance, personal sacrifice and devotion to typing, the writing of this thesis would have been more difficult.

CHAPTER 1

INTRODUCTION

Thelen (1968) has stated that perhaps the most compelling reason for using students as tutors is to change the social-psychological climate of the school from individual competitiveness to concern for each other. Tutoring through the development of processess of co-operative inquiry involving students performing responsible tasks on behalf of themselves and others in the school, seems to be a useful innovative practice which may improve achievement in mathematics and develop more positive attitudes toward mathematics. Thelen (1970, P. 19) suggests:

> As common purpose develops, cross-cultural, cross-generational, and authority barriers to communication are reduced and tolerable heterogeneity increased. As processes of co-operative inquiry develop, the social-psychological 'climate' of the school begins to change from competitiveness to concern for each other, and the anxiety which distorts children's view of each other and themselves is reduced.

During the past ten years in widely separated parts of the U.S. a number of schools have experimented with students teaching each other. Some of the more important programs were "Student Team Action", Fleming (1969), in Portland, Oregon; "Cadet Teacher Certificate", Rogers (1967), in Overland Park, Kansas; "High School Friend Program", Konikow (undated), Downers Grove, Chicago. These innovative programs arose spontaneously and simultaneously in many parts of the country. Thelen believes many of them have arisen because of what he

likes to call "caring". He cites the following experiment conducted by Rosenthal and Jacobson and reported in The Scientific American, as straightforward evidence that caring really matters:

> A relatively non-verbal intelligence test, that was purported to predict imminent 'blooming' or intellectual growth, was administered in a west coast elementary school. In each of the 18 classes (an average, below average and above average track in each of the six grades) about 20% of the children were reported to their teachers as being likely to show unusual intellectual gains in the coming year. Actually, the names had been picked by means of a table of random numbers. The children were re-tested eight months later, the tests were scored by the investigators, and the change in IQ for each child was computed. As Rosenthal and Jacobson first reported in Psychological Reports in 1966, for the school as a whole the supposed 'bloomers' showed a mean gain of 12.2 points compared with 8.4 for the control group. The effect of expectations was greater in the lower grades than in the upper grades. The 'bloomers' in the first and second grades gained respectively 15.4 and 9.5 points more than the control children. The effect was more striking (and more independent of age) on the 'reasoning' than on the 'vocabulary' portions of the test; it was about the same, regardless of 'track' level.
>
> Teachers were also asked at the end of the year to describe their pupils. They characterized the 'bloomers' as having a better chance of becoming successful; as being significantly more interesting, curious and happy, and as somewhat more appealing, adjusted and affectionate. Curiously, those control-group children who gained in IQ were not rated this favorably by their teachers; in fact, the more the undesignated children - particularly the slow-track control children - gained in IQ, the more they were regarded as being less well adjusted, interesting

> and affectionate. Rosenthal and Jacobson point out that their findings, which have been supported in subsequent studies, may bear importantly on current efforts to improve the education of children in city slums. (Scientific American Nov. 1967 P.54)

Thelen (1968) commenting on Rosenthal's experiment says:

> Old hands at this racket have known for years that the old put-down crack 'all you're doing is capitalizing on the Hawthorne Research', is exactly what we are trying to do. The Hawthorne effect is the sense of being cared for, and it's the main effect. The other things we do are just to produce the Hawthorne effect. (Thelen 1968, P. 8).

Thelen believes that the climate of a classroom can be changed through the development of the norm of concern for each other. He wants to substitute processes of co-operative inquiry for the anxious competitiveness which he maintains presently distorts the children's perceptions of each other. Thelen considers that in the helping relationship knowledge is the currency of interaction. He has stated that tutoring can increase by a large factor the amount of teaching going on in a classroom. Instruction could be individualized on a one-to-one tutorial basis. He states that tutoring can provide remedial resources pinpointed to the pupils when they most need help. "It is one thing to schedule opportunity for youngsters to get help, but what about having students available who can help in a crisis when the teacher has to keep on with the class?" (Thelen 1968, P. 12). Finally he says one of the things that got him started on tutoring in the first place is the possibility of using tutorial activity as a way to develop

the child's own insight into the teaching-learning process so that he can co-operate more effectively with his own teachers in meaningful learning activities. That is, it might contribute to the objective of the child's learning how to learn.

Statement of the Problem

Specifically, this study will attempt to investigate the effect of co-operative group learning, involving tutoring, on the achievement and attitudes of grade eight pupils in algebra. It is designed to provide data which may be helpful in suggesting answers to following questions.

(1) Does a co-operative form of group learning, in which students help each other by tutoring, result in better learning as indicated by higher achievement scores and greater retention of the materials learned than occurs in a traditional classroom?

(2) Does co-operative group learning bring about a positive change of attitude toward the learning of mathematics?

Definition of terms

The term co-operative group learning, as used in this study, means having two students, who have pretest scores above the class mean tutor and provide help for two students, who have scores lower than the class mean. Both pairs will have their desks pulled together to form a foursome, which will work together. The help provided will be mainly in the form of tutoring.

Tutoring, as referred to in this study, means having the

student who understands how to do the work, show the student who doesn't know. The intent is that there will be a one-to-one relationship in which the stronger pupil will be the tutor and the weaker pupil the tutee.

The term attitude, as used in this study, is defined as a learned, emotionally toned predisposition to react in a consistent way, favourably or unfavourably toward mathematics.

Algebra, as referred to in this study, means Unit Four: Algebra in Introduction to Mathematics (Brumfiel, Eicholz and Shanks 1962, 175-240).

Computation, as used in this study, is presumed to measure proficiency in computational skills related to addition, subtraction, multiplication and division. The four operations are extended to include computation with fractions, solution of a number sentence, and solution of per cent examples.

Concepts, as used in this study, is presumed to measure the understanding of place value, Roman numerals, operational terms, the meaning of fractions, number series, number names, estimation, average, number sentences, per cent, decimal fractions, common denominator, rounding whole numbers, decimal and common fraction equivalents, reduction of fractions and geometric terms. It also measures knowledge of formulas, properties of operations, prime numbers and understanding of non-decimal bases.

Applications, as used in this study, is presumed to

measure reasoning with problems taken from life experiences. The pupil is required to apply his mathematical knowledge and ability to think mathematically in practical situations which concern area, volume, ratio , graphs, tables, scales, per cent, business transactions, averages, problems with circles and other geometric figures and the selection of mathematical models for problems.

CHAPTER TWO

REVIEW OF THE LITERATURE

Most of the literature on tutoring comes in the form of articles, letters or anecdotal reports written on the various tutoring projects which have sprung up spontaneously in many parts of the U.S. Very few studies, with the exception of those in reading, have statistically analysed any form of empirical data. Not a single study involving tutoring in mathematics was found in which empirical data had even been collected. Most of the studies seem to have been written under Thelen's assumption that tutoring 'works'. 'Educators, almost to a man, feel that tutoring works. I can think of no other innovation which has been so consistently perceived as successful' (Thelen 1970, P.17). Many of the studies involved older tutors working with much younger students and as a result cannot be directly compared with the present study. More than half of the available literature is contained in articles, papers and working essays written by Thelen (1968). Many of these are published in The Thelen Collection available from the University of Chicago.

Achievement

Schoeler and Pearson (1970) analyzed the changes occurring in 115 below average readers enrolled in a Milwaukee community-sponsored Voluntary Reading Tutoring Program for periods ranging from 1 to 4.6 months. An attempt was made to assess improvements

in reading competence as well as in attitude towards reading. Before-and-after reading competence was assessed using the Spache Diagnostic Reading Scales. All eight parts of the Spache Diagnostic Reading Scales showed above average gain during the average 3.1 month testing period as shown in Table I.

TABLE I

Results of the Spache Diagnostic Reading Scales

Expected average gain	3.1 months
Testing period	3.1 months

Test Section	Months Mean Gain	95 Percent Confidence Interval *
Word Recognition	5.2	4.2 - 6.2
Oral Reading	3.6	3.7 - 4.5
Consonant Sounds	8.9	7.3 -10.5
Vowel Sounds	4.3	4.2 - 4.4
Consonant Blends	6.8	5.2 - 8.4
Common Syllables	5.4	4.4 - 6.4
Blending	4.4	3.3 - 5.5
Letter Sounds	9.2	7.2 -11.2

*.Computation adopted from Ferguson (1966) P.157-158
Note - Table I reproduced from The Reading Teacher, 23: April 1970, 628.

The lowest mean gain was 3.6 months in oral reading. When confidence intervals were calculated at the 95 percent level of confidence, only the oral reading score did not meet the criterion. In all categories pupils achieved at a better than average rate despite the fact that they, up to that point in their school careers, had been underachievers in reading. Subjective assessments of attitude change were achieved through

questionnaires administered to teachers, parents, pupils and student tutors. Evidence of improvement was found in pupils' attitudes toward reading, school and themselves. Although evidence about attitude building is not conclusive because of its presently subjective nature, statistically significant gains in attitude at the .05 level were obtained. Only 3 of the original 115 pupils involved did not wish to continue in the program.

Frager and Stern (1970) conducted a study in Los Angeles involving 48 sixth grade tutors, both high and low achievers, and 48 kindergarten children, who had been tested and found in need of remedial work. Tutors received two modes of counselling on tutoring procedures before assuming their duties.

The kindergarten children were divided into three treatment groups: children taught by tutors who had received counselling by method one, children taught by tutors counselled by method two, and a third group used as a control. Within each of the two experimental treatments, half the children were taught by tutors who had made high scores and half by those who made low scores on the Stanford Achievement Test. The language-readiness program used by the tutors in working with the kindergarten children was the McNeil ABC Learning Activities (1966).

> Using the criterion test provided with the McNeil ABC Learning Activities as a pretest and posttest measure, it was demonstrated that the kindergarten children who received tutoring, whether by the first or second method, were superior to the children who

> did not receive tutoring ($t = 6.3$ and 8.0, $p < .001$, respectively). Not only did the children show gains in learning, but they also looked forward to the tutoring sessions and absenteeism became virtually non-existent. (Frager and Stern 1970, P. 405).

In a program developed in the Oneida Consolidated School District (New York), Bell, Garlock and Colella (1961) made a definite attempt to provide a greater measure of individualized instruction and student involvement in the learning process. Each of thirty-four elementary students who had learning difficulties was provided with a volunteer high school tutor. No special selection procedure was used for the tutors except that they must have shown some competence in their tutorial subject. At the conclusion of the program, questionnaires were mailed to the elementary teachers who made referrals, to the student tutors and to the parents of the children who had been tutored. The questionnaires asked for comments on the tutoring program, on benefits to the students and tutors and on changes observed in pupils due to participation in the program. Bell, Garlock and Colella reported the questionnaire results as follows:

> There were slight variations in the wording of the questionnaires depending on whether they were addressed to parents, tutors, or teachers. All of the parents who responded were favorably impressed by the program. All reported that they had noticed an improvement in the child's performance in the tutorial area. The parents

expressed their belief that the high school students would acquire feelings of accomplishment and self-confidence as a result of their role in the program. Parents also reported that the tutors would learn to be more patient and would acquire a greater sense of responsibility.

Some of the opinions expressed by the high school students were similar to those expressed by the parents. The students indicated that the greatest satisfaction they had derived from the program was that of having helped others. They also felt that they were of some value in improving the academic status of their charges. Those that are prospective teachers reported increased enthusiasm about a career in teaching. Some who were undecided as to the grade level on which they wanted to teach found this experience helpful in making this choice.

The teachers, in their responses to the questionnaires, noted particularly the influence of the program on the high school student. They indicated that these students were sincere, conscientious, faithful, and punctual. The teachers also indicated that the elementary pupils were very responsive to the enthusiasm of the high school students. The younger children knew the day of the tutoring session, and were eager in their anticipation of the arrival of their tutors. The changes in the elementary pupils themselves were two-fold: First of all, the classroom teachers stated that the pupils being tutored showed considerable improvement in their academic performance, particularly in rote-type operations, e.g., multiplication, division. Secondly, the pupils involved had more positive attitudes towards school and their studies. (Bell, Garlock and Colella 1961, P.243).

Attitudes

Jackson and Strattner (1964) have suggested that"a person

with a positive attitude toward a subject learns more readily than does a person with a negative attitude..."(P.524) Montessori (1967) has been quoted as having stated that experience showed us that children had only a slight interest in arithmetic in comparison with the enthusiasm which they had for written language. Wilson (1969) has stated that before the advent of progressive education, arithmetic caused more school failures than any other subject. Even if this is no longer true Aiken (1970), who has done considerable work in the field, has suggested that attitudes about mathematics probably have not changed in recent years.

Given the widespread belief that children do, indeed, have a stable, measurable attitude toward the learning of mathematics, and that this attitude has some effect upon their immediate achievement in the subject, continued investigation in this area is justifiable. A search of the literature on attitude testing, however, quickly indicates a scarcity of relevant, non-cognitive instruments developed and validated for a junior-high school population. The reasons for this dearth may be that it is extremely difficult to construct items for attitude measurement that are comprehensible and yet not transparent to young children.

The Semantic Differential, Osgood, Suci and Tannenbaum (1957), which is a combination of controlled association and scaling procedure seems to have the most potential for this

investigation, because it possesses the required properties of simplicity of format, and content; opaqueness regarding responses and sensitivity to the degrees of attitudinal intensity. Accordingly a general model for multidimensional amalysis of semantic differential data constructed by McKie and Foster (1972) will be used.

Dutton (1968) found that (1) children have ambivalent feelings toward arithmetic, liking some aspects and disliking others (2) younger children have more positive attitudes toward the new mathematics than do older children. He suggested that the findings be used as a basis for additional research on atttitude development and as clues for improved instructional practices in the teaching of new mathematics.

Neale (1969) discussed attitudinal objectives and attitude toward learning mathematics. The subjects were 215 grade six pupils in eight classrooms in suburban elementary schools in Minnesota. He found correlations between achievement and attitudes were low - (.2 to .4) and that increasingly unfavourable attitudes developed as students went through school.

Anttonen (1969) examined the relationships between mathematics attitude and mathematics achievement over a six year period from the late elementary to the late secondary school level. The subjects were 607 students from an above average socio-economic suburb of St. Paul, Minnesota. Using a .05 level of significance the results showed a significant positive

correlation of .305 between elementary attitude scores and secondary attitude scores. Significant positive correlations of .2 to .4 at the .05 level of significance also existed between all measures of attitude and achievement.

Retention of Learning

Word and Davis (1938) showed that principles, generalizations and applications of principles were remembered much better over periods of time as opposed to factual material. Similarly Anderson (1949) revealed that knowledge of number facts learned with understanding were retained more effectively than when the facts were learned in a mechanical, rote fashion.

Bruce and Freeman (1942) studied the rates of forgetting for materials with varying degrees of meaningfulness. They found that initial loss was very rapid for both nonsense-syllables and for poetry. After thirty days, however, the group that had learned an equivalent amount of poetry retained about twice as much of the material learned.

Underwood (1968) maintains that retention is primarily and perhaps only, related to the degree of original learning.

> Thus it is clear that a list of five 3-letter words, such as CAT, PEN, BUS, FAR, AND ELK, would be learned more rapidly than a list of five non-words, such as RZL, DBQ, HFG, BJX, and PCR. Technically, we say the words have a higher level of meaningfulness than do the non-words, but if both sets are learned to an equal degree, they will be equally remembered 24 hours later. The critical point is the

> "equal degree of learning," because degree of learning is the one variable that does influence the amount of forgetting (Underwood, 1968 P. 538).

Concerning individual differences in rate of learning and retention he writes:

> A slow-learning student, therefore, may take much longer to achieve a level of learning attained by a rapid-learning student, but given the equivalent degree of learning, forgetting does not differ. (Underwood 1968, P. 539).

CHAPTER THREE

DESIGN

The present study is designed to investigate the effect of co-operative group learning, involving tutoring, on the achievement and attitudes of grade eight pupils in new mathematics. This chapter outlines the procedures to be used in carrying out the investigation. Specific attention is given to the topics: sampling procedure, experimental design, materials used, administration and scoring of tests, treatments, analyses of data and the hypotheses stated in the null form.

The experiment will be carried out from December 14, 1970 to April 2, 1971. Pretests will be administered during the first week and posttests during the final week. A retention test will be given two months after the final posttest.

Sampling Procedure

The population for the study will consist of all the grade eight students (N=311) in a Vancouver, British Columbia high school of 1700 pupils. Three of the four mathematics teachers at the grade eight level have volunteered. The volunteers teach three, three and two classes respectively; however, the teachers having three classes will have one of their classes randomly eliminated from the experiment. Thus six grade eight classes will remain. Of the two classes being

taught by each teacher, the class which receives the experimental treatment will be determined by a coin flip. The fourth mathematics teacher at the grade eight level is not available to participate in the study.

Teacher 1 is single, in her twenties and has a B.A. honours in English. She has been teaching for four years. Teacher 2 is married and has a B.A. in Political Science and Economics and an M.A. in History. He has been teaching for twelve years. Teacher 3 is married and has a B.Ed., an M.Ed. and a Ph.D. in Educational Psychology. He has been teaching for ten years. All three teachers have mathematics majors and have taught mathematics regularly during their teaching careers. They are all enthusiastic to participate in the experiment.

The teachers were assigned their classes in September; each class consists of pupils placed in it in a non-systematic manner by computer. The computer assigns pupils to classes according to the number already in the class. The class with the fewest members automatically receives the next pupil. Final class sizes are 29, 32, 34, 33 and 36 pupils. The reason for the variations in class size is that not all students assigned to each class in June return to enrol in September.

For the purposes of data analyses, test scores will be retained for only twenty-nine members of each class. The reason for this is to maintain an orthogonal design. In this study the lowest class enrollment is twenty-nine. To

get each cell down to twenty-nine, score cards will be removed at random from the classes with more than twenty-nine. Those students whose cards are removed will still "participate" in the experiment although their scores will not be analysed.

Design and Procedures

Data will be collected from 174 pupils in six grade eight classes, with each teacher teaching two classes. The design will be as shown in Table 2.

Table 2

Tabular Representation of Design

	Treatment	
Teacher	Experimental (1)	Control (2)
1	n=29 (29)	n=29 (32)
2	n=29 (32)	n=29 (34)
3	n=29 (33)	n=29 (36)
	N=87 (94)	N=87 (102)

Note: Real class sizes in parentheses.

Materials

The objective evaluation program for this study will involve five pretests, five posttests and a retention test to be administered according to the date schedule shown in Table 3 and Table 4.

Table 3

Table of Pretests and Dates of Administration

Pretests	Date
A. Stanford Achievement (Form W. Advanced, 1964)	
(1) Arithmetic Computation	Dec. 14, 70
(2) Arithmetic Concepts	Dec. 15, 70
(3) Arithmetic Applications	Dec. 16, 70
B. Vancouver School Board Mathematics Grade 8 December Test	Dec. 17, 70
C. McKie-Foster Attitude Measurement Scale (1972)	Dec. 18, 70

Table 4

Table of Posttests and Dates of Administration

Posttests	
A. Stanford Achievement (Form X. Advanced, 1964)	
(1) Arithmetic Computation	March 29, 71
(2) Arithmetic Concepts	March 30, 71
(3) Arithmetic Applications	March 31, 71
B. Algebra Test (Form A)	April, 1, 71
C. McKie-Foster Attitude Measurement Scale (1972)	April 2, 71
Retention Test	
D. Algebra Test (Form B)	June 7, 71

Achievement Pretests

The Stanford Achievement Test (Advanced Battery) is primarily designed for testing from the beginning of Grade 7 to the end of Grade 9. The Advanced Battery contains eight tests and the three of these which deal with grades 7 to 9 Arithmetic will be used, namely: Arithmetic Computation, Arithmetic Concepts and Arithmetic Applications. Four forms of this battery W, X, Y and Z are available and the W and X forms will be used as pretests and posttests respectively. According to the information supplied by the authors, Kelley, Gardner, Madden and Rudman, (1964, P.3) these tests are "matched for content and difficulty;" they represent "equally good measures of their respective subjects and yield directly comparable results." The pretest (Form W), used mainly as a control variable and in conjunction with the Vancouver School Board Grade Eight test, which has been given yearly at Christmas from 1959 to 1965, forms the basis for ranking the students in the experimental group. The board test, in conjunction with the three Stanford tests, will be used primarily as an entering behaviour test to establish the similarity of the six classes. The board test is presumed to measure the competencies required to begin the study of algebra and will be used as a statistical control for the study.

Achievement Posttests

The Stanford Posttests (Form X) will be used to measure the mathematical development of the classes involved in the experiment. The algebra posttest and the retention test will be constructed by the experimenter from existing parallel forms of the Vancouver School Board Grade Eight June mathematics tests. These forms have been used yearly by the School Board in all its schools form 1959 through 1968. Only the questions which deal with the algebra material to be studied will be used. The two forms A and B thus constructed, containing forty-one questions each, will be pilot-tested by administering them on successive days to the experimenter's Grade Nine pupils. Means, standard deviations and intercorrelations for the pilot test are shown in Table 5.

Table 5

Table of Means, Standard Deviations and Intercorrelation for Algebra A Test and Algebra B Retention Test. (Pilot Test)

Form	Mean	S.D	Corr.
A (Posttest)	30.33	6.52	
B (Retention)	32.11	6.02	.89

The inter-test correlation is .89 which is a high correlation and the experimenter assumes both these tests are measuring essentially the same algebra material. The

slight rise in the mean average may be attributable to practice effect.

Attitude Measurement

To collect data on attitudes toward mathematical concepts, a general attitude towards mathematics dimension will be derived from a model for the multidimensional analysis of attitude data obtained by the Semantic Differential, Osgood, Suci and Tannenbaum (1957). The model to be used will be adapted from work done by McKie and Foster (1972). Ten mathematical concepts; Mathematics text book; Operations with whole numbers; Percentage; Operations with fractions; Arithmetic in bases other than ten; Solving equations; Union and intersection of sets; Solving problems; Algebra and Negative numbers will be rated on each of six bipolar, seven-point adjective scales namely; good-bad; beautiful-ugly; colorful-colorless; valuable-worthless; enjoyable-distasteful; happy-sad.

Development and Pilot Test of the Attitude Measuring Instrument.

The ten particular concepts were chosen by the experimenter because they were prominent in the students' recent mathematical experiences and related to the introduction of algebra to the students. The six bipolar adjectives used, were believed to intercorrelate highly while loading on a single evaluative factor. Ten concepts dealing with attitudes toward non-mathematical school-related topics were alternated

with the ten concepts dealing with attitudes toward mathematical concepts to help prevent a "mental set".

The attitude measurement scale was pilot tested by administering it to ninety grade eight students who were not involved in the experiment. An evaluative factor was identified by means of factor analysis. Four of the six bipolar adjectives namely; good-bad, valuable-worthless, enjoyable-distasteful, happy-sad, proved to be evaluative. An evaluation score was obtained on each concept for each person. To test whether it was justified to treat the ten mathematics concepts as a cohesive homogeneous (global) cluster, a factor analysis was performed on the Persons X Concepts matrix of the evaluation scores. Results of this analysis indicated that all ten concepts loaded on a global factor (cluster). Therefore a global attitude score for each pupil was obtained by summing scores for each of the ten concepts over the four evaluative bipolar adjectives.

Administration and Scoring of Tests.

All teachers will score the tests they administer. Accuracy of scoring will be checked when the experimenter re-checks marks and totals. Scores will then be tabulated for analysis. All the pretests and posttests will be time limit tests. The Stanford tests will be administered in accordance with the instructions and time limits given in the instructional manual. The time limits for the board and

algebra tests will be provided as matters of administrative convenience rather than for the purpose of placing any premium upon speed of work. The time limits will be calculated to give practically all pupils sufficient time to attempt all questions. The intent is that the tests will be power tests and not speed tests.

In the eight week period between the administration of the algebra posttest and the algebra retention test no algebra will be taught thus ensuring that the pupils will get no formal practice in these skills. During this period the students will begin work on the next section of their text which deals almost entirely with geometry.

Treatments

The control group students will be seated alphabetically and individually in a five-row style class room. Those students who have visual or auditory problems will be identified and seated near the front of the room. All students will be required to work individually, desks not moved. Each teacher will present the lesson material in his normal teaching manner and in the sequence in which it occurs in the text. He will continue to use the style of presentation already familiar to his students. When help is required by a student in the control group, the teacher will administer it at his desk or at the student's desk whichever is more convenient. If the teacher feels a problem has occurred, which is causing

difficulty for many of the students, he will solicit the attention of the entire class, go to the blackboard and explain fully whatever seems to be causing the difficulty.

The experimental group will be divided into foursomes as far as possible. No attempt will be made to form any special groupings according to sex. Two groups of five will be necessary because two of the three experimental class sizes are not multiples of four. Each foursome will be comprised of two pairs established as follows. In each experimental class the students will be ranked highest to lowest according to the total sum of raw scores each obtains on the four pretests. To ensure that each pair has a high-scoring pupil and a corresponding low-scoring pupil the pairs will be formed as follows: (1, 17), (2, 18), (3, 19) -- (16, 32). Each pair chosen will be listed on a slip and placed in a container. Two pairs will be pulled from the container in sequence to form a foursome. Swapping of pairs will be permitted if some students are not happy with their original placement. The composition of the group will still be by pairs, so that the original idea of having two weak students and two strong students per group will be maintained.

During the experiment each foursome or group will sit together and work together. The only time they will be seated individually will be when tests are being administered. The teacher's presentation of lesson material will be the same as for his control group except for the seating arrangement of

the pupils. When the teaching part of the lesson is over, the teacher will encourage each group to work together with the two stronger pupils providing help for the two weaker pupils. Pupils will be encouraged to discuss problems with each other in their own groups. A pupil who understands will be expected to provide help in his group for the pupil who is having difficulty. On occasions when members of one group have grasped a concept more quickly than members of another group, they will be encouraged to help the members of the latter group.

When help is required by students in the experimental group the teacher will administer assistance at his desk or at the group's station whichever is more convenient. If the same difficulty seems to be occurring for a number of students the teacher will bring the problem to the attention of all groups and explain it to the entire class.

During the course of the experiment the experimenter will visit each of the six classrooms at least once a week to see that instructions are being followed, and to check on each teacher's rate of progress in covering the material. The fact that the experiment will be conducted in the experimenter's home school will allow him to maintain tight controls on the investigation. Teachers will keep notes on how each experimental group functions, making note of any peculiar or unusual occurrences.

The experimenter will, if possible, obtain mental test

scores for pupils, but this may not be possible as I.Q. tests have not been administered by the Vancouver School Board as a matter of policy since 1968.

Data Analyses

Means, standard deviations and intercorrelations will be obtained for the pretest, posttest and retention achievement scores and the attitude measurement prescores and postscores. Analyses of variance or covariance, where required, will be performed on data from all the achievement tests and attitude measurements. Statistical significance levels of .05 and beyond will be accepted as evidence against the null hypotheses.

Hypotheses

The hypotheses for this study, stated in the null form will be as follows:

Hypothesis One

No significant difference between the experimental group and the control group will occur for learning achievement in computation.

Hypothesis Two

No significant difference between the experimental group and the control group will occur for learning achievement in concepts.

Hypothesis Three

No significant difference between the experimental group and the control group will occur for learning achievement

in applications.

Hypothesis Four

No significant difference between the experimental group and the control group will occur for learning achievement in algebra.

Hypothesis Five

No significant difference between the experimental group and the control group will occur for retention of learning in algebra.

Hypothesis Six

No significant difference between the experimental group and the control group will occur in attitudes towards mathematics.

CHAPTER FOUR

RESULTS

Results are based on the experimenter's classroom observations, teachers' anecdotal records and pupils' test scores. Initially a summary is given of the experimenter's notes relative to the classrooms in which the study was conducted. This is followed by a summary of the notes and remarks submitted by the co-operating teachers. Summaries of statistical analyses follow these summary statements.

Assessment of Classroom Situations

Classroom atmosphere and organization varied considerably among the six participating classrooms. The researcher visited each classroom weekly to check on progress and to see that the study was being carried on as planned. Although all the teachers tended to use a "questioning" technique in approaching their lessons, the researcher felt that Teacher 2 tended to use more of a "lecture" approach. Boredom or disinterest on the part of the students or teachers was not evident to the researcher during his visits. Interest level seemed to be high throughout the classes.

An attempt was made to secure mental test scores for the subjects, but these scores proved difficult to obtain. The small number of scores which were obtained were not usable as they were from different tests and they had been

administered at different ages. Hence the groups could be compared only on the basis of pretest scores.

At the start of the experiment some pupils in Teacher 1's experimental class which, according to that teacher, was a very friendly one, were made unhappy by being placed in the groups the experimenter had assigned to them. They wanted their "democratic" right to set up new groups within the limitation of two strong and two weak students using the pairs already selected. This was done after some discussion, and the experiment continued as planned. It was felt that making this change would remove any possible lack of co-operation on their part.

Assessment of Teachers' Written Comments

The general reaction of the teachers was that the pupils in both experimental and control classes enjoyed the work and that most lessons had gone well and smoothly. The lesson material as presented in the text seemed to have been adequate and easily followed by both teachers and students. The logical and sequential development of the study unit was helpful to the teachers in the preparation of their lessons. Although no formal daily schedule was set the three teachers seemed to be about the same spot in the text, when the experimenter made his weekly check. A detailed account of the teachers' comments on group work is given in Appendix A.

Teacher 3 was absent for the sixth and seventh weeks of the experiment. With the co-operation of the principal and the

Vancouver School Board a suitable substitute teacher was employed to take over. The experimenter met with this teacher and fully explained all relevant details of the experiment. During the two weeks this substitute teacher was employed the experimenter made daily visits to this classroom, thus ensuring that the experiment could continue as planned.

Results of statistical analyses

Table 6 presents the means, standard deviations and intercorrelations for the Stanford Computation, Concepts, Applications, Board and Attitudes pretests for all students. Table 7 presents the means, standard deviations and intercorrelations for the Stanford Computation, Concepts, Applications, Algebra A and Attitudes posttests. Intercorrelations among the Stanford Pretests range from .70 to .73 and among the Stanford posttests from .61 to .74. The Board pretest correlates with the Stanford pretests in the range of .61 to .73, and the Algebra A posttest correlates with the Stanford posttests in the range of .64 to .72. The correlations between attitudes and all achievement tests are all positive and range from .11 to .21.

Computation. In order to compare the experimentals and controls with regard to computation, Table 8 presents the analysis of variance for the Stanford Computation pretest (Form W). There is a significant difference (α = .05) between the experimental and the control treatment groups. The experimental mean is 23.54 and the control mean is 20.62; therefore the experimental group was superior to the control group in computation before the experiment began.

TABLE 6

Summary Table of Pretest Means, Standard Deviations and Intercorrelations for Stanford (Form W), Board and Attitude Measures (N=174).*

Variable	Mean	S.D.
Computation	22.08	8.22
Concepts	24.95	6.98
Applications	18.37	4.79
Board	47.39	18.43
Attitudes	42.35	10.40

Correlations **

	Comp.	Con.	App.	Board
Computation	--			
Concepts	73	--		
Applications	70	70	--	
Board	73	72	61	
Attitudes	19	16	15	16

* For significance @ α = .05 and d/f=173, r=.148
** Decimals omitted.

TABLE 7

Summary Table of Posttest Means, Standard Deviations and Intercorrelations for Stanford (Form X), Algebra and Attitude Measures (N=174).*

Variable	Mean	S.D.
Computation	23.75	9.29
Concepts	24.75	7.17
Applications	19.56	5.67
Algebra (Form A)	27.55	7.08
Attitudes	42.83	11.92

Correlations **

	Comp.	Con.	App.	Algebra
Computation	--			
Concepts	71	--		
Applications	61	74	--	
Algebra (Form A)	67	72	64	--
Attitudes	17	18	11	21

* For significance @ α = .05 and d/f=173, r=.148
** Decimals omitted.

There is no significant instructor effect and there is no significant interaction effect between treatment and instructor.

Comparing the groups in terms of computation, after the experiment, Table 9 presents an analysis of covariance for the Stanford Computation posttest using the Stanford Computation pretest scores as the covariate. There is a significant instructor effect placing the teachers in the following order (1,3,2) with respective means of 25.34, 23.96 and 21.93. This means that the differential characteristics of these teachers were important causing variance in the dependent variable which is the Stanford Computation posttest. There is no significant difference ($\alpha = .05$) for treatment or interaction between treatment and instructor.

Concepts. In terms of concepts mastery Tables 10 and 11 present analyses of variance results for the Stanford Concepts pretests and posttests. There is no significant difference ($\alpha = .05$) for treatment, instructor, or interaction between instructor and treatment in either variable.

Applications. Comparing the groups in terms of applications before the experiment, Table 12 presents an analysis of variance for the Stanford Applications (Form W) pretest. There is no significant difference ($\alpha = .05$) for treatment, instructor or interaction between instructor and treatment. Thus, for applications the groups were similar at the start of the experiment.

TABLE 8

Summary Analysis of Variance Table for Computation Pretest: Stanford Achievement (Form W, Advanced) (N=174).

Source	df	SS	MS	F	P
Treatment	1	370.78	370.78	5.62	.02*
Instructor	2	185.53	92.76	1.41	.25
Treat. x Instr.	2	52.91	26.45	0.40	.68
Error	168	11078.00	65.94		
Total	173	11687.00			

	Experimental	Control
Treatment means	23.54	20.62

*Significant @ α = .05

TABLE 9

Summary Analysis of Co-variance Table for Computation Post-test: Stanford Achievement (Form X, Advanced) (N=174).

Source	df	SS	MS	F	P
Treatment	1	18.94	18.94	.72	.40
Instructor	2	337.30	168.65	6.40	.002**
Treat. x Inst.	2	14.29	7.14	.27	.77
Error	167	4403.60	26.37		
Total	172	4774.13			

	1	2	3
Instructor Means	25.34	21.93	23.96

**Significant @ α = .01

TABLE 10

Summary Analysis of Variance Table for Concepts Pretest: Stanford Achievement (Form W, Advanced) (N=174).

Source	df	SS	MS	F	P
Treatment	1	178.02	178.02	3.69	.054
Instructor	2	30.01	15.01	.31	.74
Treat. X Instr.	2	115.25	57.63	1.19	.31
Error	168	8110.30	48.28		
Total	173	8433.60			

TABLE 11

Summary Analysis of Variance Table for Concepts Posttest: Stanford Achievement (Form X, Advanced) (N=174).

Source	df	SS	MS	F	P
Treatment	1	143.47	143.47	2.76	.09
Instructor	2	5.53	2.76	.05	.94
Treat. x Inst.	2	38.22	19.11	.37	.70
Error	168	8717.70	51.89		
Total	173	8904.90			

TABLE 12

Summary Analysis of Variance Table for Applications Pre-test: Stanford Achievement (Form W, Advanced) (N=174).

Source	df	SS	MS	F	P
Treatment	1	21.39	21.39	.93	.34
Instructor	2	22.70	11.35	.49	.62
Treat. x Instr.	2	52.15	26.08	1.13	.33
Error	168	3872.50	23.05		
Total	173	3968.70			

TABLE 13

Summary Analysis of Variance Table for Applications Post-test: Stanford Achievement (Form X, Advanced) (N=174).

Source	df	SS	MS	F	P
Treatment	1	148.97	148.97	4.70	.03*
Instructor	2	2.49	1.25	.04	.95
Treat. x Inst.	2	73.60	36.80	1.16	.32
Error	168	5325.90	31.70		
Total	173	5550.90			

	Experimental	Control
Treatment means	20.48	18.63

* Significant @ α = .05

Comparing the groups in terms of applications, after the experimental period, Table 13 presents an analysis of variance for the Stanford Applications (Form X) posttest. There is a significant difference for treatment (α = .05). The experimental group performed better on the posttest than the control group. There is no significant difference for instructor or interaction between treatment and instructor.

Grade Eight Mathematics. Comparing the groups in terms of the amount of mathematics learning which had occurred in grade eight mathematics in three months, Table 14 presents an analysis of variance for the Board pretest, which is an entering behaviour test of prior mathematics learning from September to December. There is no significant difference (α = .05) for treatment or interaction meaning that the experimental and control groups are similar at the beginning of the experiment. There is a significant difference (α = .05) for instructor. According to the New Duncan Multiple Range Test (Halm and Le, 1972) there are two homogeneous subsets (2,3) and (3,1). Thus, instructors 1 and 2 differed significantly. The differential characteristics of these two teachers were systematically related to variance the Board pretest.

TABLE 14

Summary Analysis of Variance Table for Board (Dec.) Pretest. (N=174).

Source	df	SS	MS	F	P
Treatment	1	1018.60	1018.60	3.12	.08
Instructor	2	2444.80	1222.40	3.74	.03*
Treat. x Instr.	2	385.74	192.87	.59	.56
Error	168	54920.00	326.98		
Total	173	58769.00			
		1	2	3	
Instructor means		51.33	42.34	48.48	

* Significant @ α = .05

Algebra. In terms of algebra mastery, Table 15 presents an analysis of covariance for the Algebra A posttest using the Board pretest scores as the covariate. There is a significant difference (α = .05) for treatment; thus the experimental group did significantly better than the control group on the Algebra A posttest. There is also a significant difference for instructor. According to the Duncan New Multiple Range Test (1972) there is one homogeneous subset (1,2), meaning that Teachers 1 and 3 and Teachers 2 and 3 differ significantly. Thus, the differential characteristics of these teachers were systematically related to variance in the Algebra A posttest.

TABLE 15

Summary Analysis of Covariance Table for Algebra (Form A) Posttest (N=174).

Source	df	SS	MS	F	P
Treatment	1	100.37	100.37	5.31	.02*
Instructor	2	230.41	115.21	6.09	.003**
Treat. x Instr.	2	19.63	9.82	.52	.60
Error	167	3158.10	18.91		
Total	172	3508.51			

	Experimental	Control
Treatment means	28.31	26.77

	1	2	3
Instructor means	28.84	27.75	26.04

* Significant @ $\alpha = .05$
** Significant @ $\alpha = .01$

TABLE 16

Summary Analysis of Covariance Table for Algebra Retention (Form B). (N=174).

Source	df	SS	MS	F	P
Treatment	1	231.52	23.52	9.75	.002**
Instructor	2	85.47	42.73	1.80	.17
Treat. x Instr.	2	28.23	14.11	.59	.56
Error	167	3966.40	23.75		
Total	172	4311.62			

	Experimental	Control
Treatment means	27.39	25.06

** Significant @ $\alpha = .01$

Retention. Comparing the groups in terms of retention of algebra after a period of two months, Table 16 presents an analysis of covariance for the Algebra B Retention test, using the Board pretest scores as the covariate. There is a significant difference (α = .05) for treatment. The experimental group did significantly better than the control group on the retention test. No significant difference for instructor or interaction between treatment and instructor exists for the retention test.

Attitudes. Comparing the groups on attitudes toward mathematics before the experiment began, Table 17 presents an analysis of variance for the summated attitude prescores. There is no significant difference (α = .05) for treatment, which means that both the experimental and control groups are similar. There is a significant difference for instructor. According to the Duncan New Multiple Range Test (1972) there is one homogeneous subset, (3,1). Thus Teacher 2 differs from Teachers 1 and 3. The differential characteristics of these teachers inasmuch as Teacher 2 was more authoritarian than Teachers 1 and 3, were systematically related to variance in the summated attitude prescore. There is also a significant interaction difference between treatment and instructor. There is one homogeneous subset of five classes (6,4,3,1,2). Teacher 2's control group is significantly different from the other five classes and has the highest mean score 49.22.

TABLE 17

Summary Analysis of Variance for Summated Attitude Prescores. (N=174).

Source	df	SS	MS	F	P
Treatment	1	21.97	21.97	.22	.64
Instructor	2	1349.70	674.87	6.83	.002**
Treat. x Instr.	2	640.31	320.16	3.24	.04*
Error	168	16619.00	98.87		
Total	173	18622.00			

	1	2	3
Instructor means	41.07	46.16	39.68

Treat.x Instr.

Instructor	1		2		3	
Treatment	E	C	E	C	E	C
Means	42.23	39.91	43.10	49.22	40.51	38.84

* Significant @ α = .05
** Significant @ α = .01

TABLE 18

Summary Analysis of Covariance Table for Summated Attitude Postscores. (N=174).

Source	df	SS	MS	F	P
Treatment	1	321.67	213.67	2.62	.10
Instructor	2	84.60	42.30	.52	.60
Treat. x Instr.	2	189.02	94.51	1.16	.32
Error	167	13640.00	81.68		
Total	172	14127.00			

Comparing the groups with regard to attitudes towards mathematics after the three month treatment period, Table 18 presents an analysis of covariance for the summated attitude postscores using the summated attitude prescores as covariate. There is no significant difference (α = .05) for treatment, for instructor or for interaction between instructor and treatment. According to the Duncan New Multiple Range Test (1972) there is one homogeneous subset for instructor (3,1,2) and one homogeneous subset for interaction (6,4,2,3,5,1).

In summary, significant differences existed for computation before the start of the experiment. An analysis of covariance performed on the computation postscores, using the computation prescores as covariate, showed that no significant difference occurred for treatment (α = .05). Significant differences however, favouring the experimental group occurred in the Applications and Algebra A posttests and Algebra B retention test. No significant differences for treatment, for instructor or for interaction between instructor and treatment occurred for attitudes.

CHAPTER FIVE

SUMMARY AND CONCLUSIONS

This study was designed to investigate the effect of co-operative group learning on the achievement and attitudes of grade eight pupils in the algebra section of new mathematics. Three volunteer teachers each taught both an experimental and a control class. Four teacher - administered pretests were given to establish the similarity of the groups. Four post-tests and a retention test were given to establish the relative effectiveness of the respective learning approaches in affecting achievement and to measure the mathematical development of the classes. An attitudinal survey was administered before and after the experimental period to determine attitude toward mathematics scores for each student before and after the treatment. Analyses of variance or covariance when necessary were employed to statistically test the mean group differences for significance of both pretests and posttests. The .05 level of statistical significance was used throughout.

Summary of Findings

(1) Learning achievement for computation was not significantly enchanced by the co-operative group learning method. On the computation pretest the experimental group did significantly better at .05 level of statistical significance. When this initial difference was statistically removed by an analysis of covariance, using the computation prescores as covariate,

no significant difference for treatment was found. As a result of this finding the null hypothesis was accepted for Hypothesis One.

(2) Learning achievement for concepts was not significantly enhanced by the co-operative group learning method. On the concepts pretest there was no significant difference between the experimental and the control groups. No significant difference at the .05 level of statistical significance occurred for treatment on the posttest. As a result of the posttest finding the null hypothesis was accepted for Hypothesis Two.

(3) Learning achievement for applications was significantly enhanced by the co-operative group learning method. No significant difference between the experimental and the control group existed before the treatment period. On the applications posttest a significant difference ($\alpha < .03$) occurred in favour of the experimental group. Because of this posttest finding the null hypothesis was rejected for Hypothesis Three.

(4) Learning achievement for algebra was significantly enhanced by the co-operative group learning method. A significant difference ($\alpha < .02$) occurred for treatment in the Algebra A posttest. As a result of this finding the null hypothesis was rejected for Hypothesis Four.

(5) Retention of learning was significantly enhanced by the co-operative group learning method. A significant difference ($\alpha < .002$) occurred for treatment in the Algebra B retention

test. The null hypothesis was therefore rejected for Hypothesis Five.

(6) Attitudes towards mathematics were not significantly enhanced by the co-operative group method. No significant difference at the .05 level of statistical significance was found for treatment at the conclusion of the experiment. The null hypothesis was therefore accepted for Hypothesis Six.

Conclusions and Discussion

The experiment was concerned with the amount of learning and retention which occurred during a period of twelve weeks in the study of algebra as presented in Unit Four of Introduction to Mathematics (Brumfiel, Eicholz and Shanks, 1962).

The Algebra A posttest and the Algebra B retention test were the two tests which directly measured the amount of learning and retention which took place. Both of these tests showed empirically that the experimental group did better than the control group, well beyond the .05 significance criterion, which had been set for the experiment. The results of both tests in the opinion of the experimenter lend support to the use of co-operative group learning, involving tutoring, in the study of introductory algebra.

Regarding the Stanford Achievement (Form X, Advanced) post-test results, only in the case of applications was there a significant difference favouring the experimental group. The

three standardized posttests were given to find out whether or not mathematical skills were being maintained during the course of the experiment. It is the opinion of the experimenter that the significant difference which showed up in the applications posttest, occurred primarily, because there was a similarity between the types of questions that appeared on the applications test and the algebra material studied during the treatment period.

Attitudes towards mathematics, as measured, were not significantly enhanced by the co-operative group learning method. The experimenter felt that maybe the summated global score used to represent a general attitudinal factor was not a true indicator of a student's attitude towards mathematics. From an inspection of the group means and the large pretest instructor effect it seems plausible to argue that teachers make a big difference in students' attitudes and that one of the things one might try to do, is to fit students into classes taught by the type of teacher who best fits that student's particular needs.

From the teachers' comments on group work it seems that for Teachers I and 3 who were less structured in their teaching than Teacher 2, any grouping which had three boys and a girl or three girls and a boy, did not work well as a unit. With Teacher 2 this type of grouping did not cause as many problems. Perhaps the students in Teacher 2's class were more concerned

about following instructions since they were in a more structured classroom. Teacher 2 didn't feel that it made much difference whether a group was unevenly mixed. Teacher I was more worried about personality clashes which arose in groups that had a three and one format. As a result of these experiences it would seem plausible in future group work to form only groups of all boys, all girls or of two boys and two girls. A sociogram conducted before any groupings were made would probably help to eliminate many of the clashes, which occurred mostly with the three and one groupings.

Limitations

This study was limited to the use of volunteered grade eight mathematics classes. Insofar as the teachers and pupils of these classes were representative of a larger population of grade eight mathematics teachers and grade eight students, generalizations may be made of the results of this study. At best one should generalize with caution. Generalization of results to other than grade eight mathematics classes is not warranted. In fact the only generalization possible is to learning situations in which Unit Four (Algebra) of Introduction to Mathematics (Brumfield, Eicholz andShanks, 1962) is studied.

The three-month teaching period which occurred before the experiment began may have introduced teacher-student interaction contaminants which could not easily be eliminated or overlooked. This may have been one of the causes of the

significant difference for instructor which occurred in the attitudes and board pretests and the Algebra A posttest. The stylistic teaching differences of the three teachers especially Teacher 2 may have caused the variance for instructor in these three dependent variables.

It seems possible since Teacher 1's groups were allowed to rearrange their pairs to form foursomes on the basis of friendships and existing loyalties that this made Teacher 1's group work harder. It would be interesting to find out what would happen in an experiment using two experimental groups and one control group. One of two experimental groups would be allowed to work according to their own choice of pairs or based on sociogram results and the other experimental group would be set up by the experimenter, using pretest scores. It is also difficult to assess what effect the two week period taught by the substitute teacher, when Teacher 3 was ill, had on the results. The substitute teacher was very competent and as far as the experimenter could determine only the initial day of the two weeks was seriously affected while this teacher was being oriented to the experiment situation and there were some necessary interruptions in both the experimental and control classes.

It is also difficult to assess the effect of giving four mathematics tests to students prior to having them complete an attitudes towards mathematics measurement scale, which in the

opinion of the experimenter proved to be too long. It may have been that the attitudes were affected by the previous tests.

Other limitations were that intact classes had to be used and that the experiment was conducted in only one school.

Further Research

A replication and extension of this research with random assignment of experimental subjects to treatment groups or an increased number of classes per treatment group would tend to make the results more reliable by increasing sample size. Along with this, a more rigid equating of subjects, teachers and other important variables would result in a better controlled investigation.

The study of the usefulness of co-operative group learning needs to be extended to the teaching of subject matter other than mathematics. It may prove more useful for the teaching of course materials other than those involved in this study.

A beginning has been made in attempting to show the effect of co-operative group learning in the classroom teaching situation. The method has lent some support to claims for the effectiveness of this type of approach. Further research is needed to verify this result and to extend the investigation beyond the area of grade eight mathematics.

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APPENDIX A

Teachers' Comments on Group work.

Teacher 1 reported on her groups as follows:

Group 1: Four boys who enjoyed working together. One boy seemed to increase both his interest and his marks in this group.

Group 11: Four girls who worked extremely well together although one girl may have gone down in marks, as the type of person who needed structured teaching. Another girl rose to an "A" under the helpful guidance of her two best friends.

Group 111: Four girls - some discipline problems here which interfered with learning from each other.

Group 1V: Five girls - worked well - one girl benefitted the most. Lots of giggling at times in this group.

Group V: Three girls, one boy. This group disintegrated. The boy worked on his own. One girl was usually absent. The remaining two girls did work together.

Group V1: Four boys. Broke into two groups each having a weak and a strong student. The groups did not help each other.

Group V11: Four girls. Worked well together. The two strong students helped the two weak ones.

Teacher 1's overall impression was favourable. Except for the personality clashes, which seemed to affect groups 5 and 6, the pupils seemed to enjoy and profit from helping each other.

Teacher 2's Comments.

Group 1: Three boys and one girl. Worked well as a group. One boy was rather slap-happy and gossipy. The girl was the leader.

Group 11: Three boys and one girl. The girl tended to be on her own. One of the boys was a chatterbox.

Group 111: Three boys and one girl. Good cooperation at all times in this group.

Group 1V: Two boys and two girls. One boy absent often. The other boy and the two girls worked well together.

Group V: Three girls and one boy. The boy was a very weak student. All three girls helped him.

Group V1: Two boys and two girls. Worked well together.

Group V11: Two boys and two girls. One boy was absent often. The remaining three worked well together.

Group V111: Two boys and two girls. Good cooperation and interaction.

Teacher 2 reported that all his groups, seemed to work well together. He didn't feel that it made much difference that the groups were mixed. He thought that his good students suffered some. He also said that the pupils liked to sit

together.

Teacher 3's Comments

Group A: Two girls and two boys; generally working as two pairs; some group interaction.

Group B: Similar to group A.

Group C: Four boys; good group interaction and group work.

Group D: Three girls and one boy; boy appeared to be accepted, but tended to work alone; three girls showed good interaction and group work.

Group E: Three girls and one boy; boy generally isolated and tended to work alone; girls made little effort to include him; boy a weak student to begin with.

Group F: Four girls; good interaction and group work.

Group G: Two boys and two girls. Similar to Groups A and B.

Group H: Three boys and two girls; good interaction with one girl accepted and participating in group work. The second girl spent most of her time with Group G or working on her own.

APPENDIX B

*NOTE: ALL TESTS AND SURVEY SHEETS HAVE BEEN REDUCED IN SIZE FROM ORIGINALS TO FIT AN $8\frac{1}{2}$ x 11 FORMAT.

SURVEY

INSTRUCTIONS

The purpose of this survey is to find out how you rate some of the words or concepts you may have come across in Social Studies, Mathematics and Science.

For Example:

SCIENCE TEXTBOOK THIS YEAR

1) good	____ :	x :	____ :	____ :	____ :	____ :	____	bad
2) beautiful	x :	____ :	____ :	____ :	____ :	____ :	____	ugly

The above example means that this student considers this year's textbook is quite good and that he likes it.

IMPORTANT

1) Work rapidly - give your first impressions - do not dally over any item.
2) Try to make each judgement a separate and independent one.
3) Place your X in a space as shown above - NOT BETWEEN SPACES.

MY SCIENCE TEXT BOOK THIS YEAR

1) good	____ :	____ :	____ :	____ :	____ :	____ :	____	bad
2) beautiful	____ :	____ :	____ :	____ :	____ :	____ :	____	ugly
3) valuable	____ :	____ :	____ :	____ :	____ :	____ :	____	worthless
4) enjoyable	____ :	____ :	____ :	____ :	____ :	____ :	____	distasteful
5) colorful	____ :	____ :	____ :	____ :	____ :	____ :	____	colorless
6) happy	____ :	____ :	____ :	____ :	____ :	____ :	____	sad

MATHEMATICS TEXT BOOK

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distasteful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

SOCIAL STUDIES TEXT BOOK

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distasteful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

OPERATIONS WITH WHOLE NUMBERS

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distasteful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

SOCIAL STUDIES MAP WORK

1) good	____:	____:	____:	____:	____:	____:	____ bad
2) beautiful	____:	____:	____:	____:	____:	____:	____ ugly
3) valuable	____:	____:	____:	____:	____:	____:	____ worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____ distastefu
5) colorful	____:	____:	____:	____:	____:	____:	____ colorless
6) happy	____:	____:	____:	____:	____:	____:	____ sad

PERCENTAGE

1) good	____:	____:	____:	____:	____:	____:	____ bad
2) beautiful	____:	____:	____:	____:	____:	____:	____ ugly
3) valuable	____:	____:	____:	____:	____:	____:	____ worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____ distastefu
5) colorful	____:	____:	____:	____:	____:	____:	____ colorless
6) happy	____:	____:	____:	____:	____:	____:	____ sad

SCIENCE LAB WORK

1) good	____:	____:	____:	____:	____:	____:	____ bad
2) beautiful	____:	____:	____:	____:	____:	____:	____ ugly
3) valuable	____:	____:	____:	____:	____:	____:	____ worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____ distastefu
5) colorful	____:	____:	____:	____:	____:	____:	____ colorless
6) happy	____:	____:	____:	____:	____:	____:	____ sad

OPERATIONS WITH FRACTIONS

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distastefu
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

SOCIAL STUDIES READING

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distastefu
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

ARITHMETIC IN BASES OTHER THAN TEN

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distasteful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

... continued

SCIENCE READING

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distastefu
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

SOLVING EQUATIONS

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distastefu
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

SCHOOL

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distastefu
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

UNION AND INTERSECTION OF SETS

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distasteful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

LETTER GRADES

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distasteful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

SOLVING PROBLEMS

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distasteful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

... continued

ESSAY WRITING

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distasteful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

ALGEBRA

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distasteful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

ASSIGNED HOMEWORK

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distasteful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless
6) happy	____:	____:	____:	____:	____:	____:	____	sad

NEGATIVE NUMBERS

1) good	____:	____:	____:	____:	____:	____:	____	bad
2) beautiful	____:	____:	____:	____:	____:	____:	____	ugly
3) valuable	____:	____:	____:	____:	____:	____:	____	worthless
4) enjoyable	____:	____:	____:	____:	____:	____:	____	distastful
5) colorful	____:	____:	____:	____:	____:	____:	____	colorless

APPENDIX C

MATHEMATICS 8

DECEMBER

NAME _____

CLASS # _____

READ EACH QUESTION CAREFULLY BUT DO NOT SPEND TOO MUCH TIME ON ANY YOU DO NOT UNDERSTAND. NOTE CAREFULLY THE FORM REQUIRED FOR THE ANSWER AND PLACE IT IN THE SPACE PROVIDED. THERE IS ONE MARK FOR EACH ANSWER.

SECTION "A"

WRITE THE LETTER CORRESPONDING TO THE CORRECT ANSWER IN THE SPACE PROVIDED FOR ON THE RIGHT.

1) Which statement is true?

a) $5^4 = 4 + 4 + 4 + 4 + 4$
b) $5^4 = 4 . 4 . 4 . 4 . 4$
c) $4^3 = 4 . 4 . 4$
d) $2^3 = 2 . 3$
e) none of these

1= c

2) 42 written as the product of prime factors is:

a) 1 x 6 x 7
b) 37 + 5
c) 3 x 14
d) 2 x 3 x 7
e) all are correct

2= d

3) In what base are the numerals written if 2 x 2 = 10?

a) base two
b) base three
c) base four
d) base five
e) none of these

3= c

4) Which numeral represents the largest number?

a) 15_{ten}
b) 23_{four}
c) 111_{two}
d) 20_{seven}
e) 3_{twelve}

4= a

5) The sum of two odd numbers is always in the set of:

a) even numbers
b) odd numbers
c) multiples of three
d) prime numbers
e) squared numbers

5= a

6) Every natural number has <u>at least</u> the following divisors:

a) zero and one
b) zero and itself
c) one and itself
d) itself and two
e) none of these

6= c

7) Which of the following is not a prime?

a) 7
b) 41
c) 73
d) 87
e) 97

7= d

8) The set of all factors of 12 is:

a) (1,2,3,4,8,12)
b) (1,2,3,4,6,12)
c) (1,2,3,4,6)
d) (2,3,4,6)
e) (2,2,3)

8= b

9) Which is the complete factorization of 36?

a) 4 x 9
b) 2 x 3 x 6
c) 3 x 12
d) 2 x 2 x 3 x 3
e) 13 + 23

9= d

10) The operations which are commutative are:

a) addition and subtraction
b) addition and multiplication
c) subtraction and division
d) addition only
e) all four operations

10= b

11) An example of a natural number is:

a) ½ b) 3.06 c) 5/4 d) 3

11= d

12) An example of a repeating decimal is:

a) 5.75
b) 1.121121112...
c) $\overline{11}$
d) $\sqrt{2}$
e) none of these

12= a

13) An example of a prime number is:

a) 0
b) 1
c) 6
d) 35
e) none of these

13= e

14) The LCM of 12 and 24 is:

a) 2
b) 12
c) 24
d) 36
e) 48

14= c

15) The greatest common divisor of 7 and 8 is:

a) 0
b) 1
c) 7
d) 8
e) 56

15= b

16) Which of the following numerals is incorrect?

a) 16_{ten}
b) 26_{eight}
c) 14_{four}
d) 12_{three}
e) 100_{two}

16= c

17) The number written in base two is an even number if:

a) it ends in zero only
b) it ends in 1
c) it ends in zero or 2
d) the sum of the digits in an even number
e) there is no simple rule

17= a

18) If 6 is divided by zero the answer is:

a) 6
b) 0
c) 1
d) any number
e) it is impossible

18= e

S E C T I O N "B"

TRUE OR FALSE

READ EACH STATEMENT CAREFULLY. IF IT IS TRUE, WRITE THE WORD "TRUE" IN THE SPACE AT THE RIGHT. IF IT IS FALSE WRITE THE WORD "FALSE".

1) 10^4 means 10 x 10 x 10 x 10 — 1) TRUE

2) In the symbol 5^3 the exponent is 3 — 2) TRUE

3) In base five, the numeral before 40 is 39 — 3) FALSE

4) The numeral 8 represents the same number in base ten as it does in base four. — 4) FALSE

5) In base four we use only the symbols 0, 1, 2, 3, 4. — 5) FALSE

6) When we carry, in addition, the value of the carried digit depends upon the base. — 6) TRUE

7) Rational numbers are numbers which can be represented as fractions — 7= TRUE

8) There is a one - to - one correspondence between the natural numbers less than ten and the whole numbers less than ten. — 8= FALSE

9) Every natural number is a rational number. — 9= TRUE

10) The product of any whole number and zero is that whole number — 10= FALSE

11) $4^3 \neq 3^4$ — 11= TRUE

12) $5 < (6 + 5) \cdot 0$ — 12= FALSE

13) $16^2 < 16 \cdot 15$ — 13= FALSE

14) 6 base seven has the same value as 6 base ten — 14= TRUE

15) The Egyptian number system did not use place value — 15= TRUE

16) We inherit our numerals from the Arabs — 16= TRUE

17) Every rational number can be represented as a repeating decimal — 17= TRUE

S E C T I O N " C "

WRITE YOUR ANSWERS IN THE SPACES PROVIDED AT THE RIGHT

1) Write the following numerals in expanded notation. (example: ^{23}four = 2 x 4 + 3).

a) 3812_{ten}= b) 233_{four}= c) 10101_{two}=

(1)(a) $3 \cdot 1000 + 8 \cdot 100 + 1 \cdot 10 + 2$
(b) $2 \cdot 4^2 + 3 \cdot 4 + 3$
(c) $1 \cdot 2^5 + 1 \cdot 2^3 + 1$

2) Change to common fraction in lowest terms

a) 0.666...
b) 0.33...
c) 0.111...
d) 0.0202...

2. a) 2/3
b) 1/3
c) 1/9
d) 2/99

3) Factor completely: 275 — 3) 5·5·11

4) Carry out the operations indicated.

a) base four: 3+3=
b) base twelve: 5+8=
c) base four: 3.3=
d) base two: 111+1011=
e) base seven: 513.100=

4. a) 12
b) 11
c) 21
d) 10010
e) 51300

5) If a, b and c represent any rational numbers, complete the following statement of the distributive principle. — 5) $a \cdot b + a \cdot c$

a) $a \cdot (b + c) =$

6) Write the number 36 as the sum of two prime numbers — 6) 7+29

7) Express the fraction ¼ as a "decimal" in base twelve — 7) .3

8) Work out the following problems in your head if you can. In any case do as little writing as possible.

a) 3 (1/3· 2/5)
b) 1/6+(3/5+5/6)
c) 5(3/8· 8/5)
d) 3/4+0/5
e) 6(5/8+0/4)
f) 7·1/3+7·2/3

8. a) 2/5
b) 1 3/5
c) 3
d) 3/4
e) 3 3/4
f) 7

9) 1839 = _____$_6$ 9) 12303

10) a) $3/4 \div 3/4$ b) $5/8 \div 15/16$ 10. a) 1
c) $\frac{12}{1/3}$ d) $\frac{1/3 - \frac{1}{4}}{\frac{1}{2} - 1/3}$ b) 2/3
c) 36
d) 1/2

11) a) Write all the divisors of 30 11. a) 1, 2, 3, 5, 6, 10, 15, 30

b) Write all the divisors of 75 b) 1, 3, 5, 15, 25, 75.

c) Write the greatest common divisor of 30 and 75 c) 15

d) Write one other common divisor of 30 and 75 d) 5

12) a) What is the least common multiple of 8 and 12? 12. a) 24
b) Name one other common multiple of 8 and 12. b) 48

13) In the following questions use the abbreviations given for the basic principles listed below:

E.U.	Existence and uniqueness principle
C.P.	Commutative principle
A.P.	Associative principle
D.P.	Distributive principle
M. one	Multiplication property of one
M. zero	Multiplication property of zero
A. zero	Addition property of zero

INDICATE WHICH PRINCIPLE IS USED IN EACH QUESTION IN THE SPACE PROVIDED FOR , ON THE RIGHT.

a) $(6\cdot 3)\cdot 5=6\cdot(3\cdot 5)$ b) $5+6=11$ 13. a) A.P.
c) $a\cdot(b+c)=a.b+a\cdot c$ d) $9\cdot(8-8)=0$ b) E.U.
e) $1/3\cdot 4/5+2/3\cdot 4/5=(1/3+2/3)\cdot 4/5$ f) $x\cdot 1=x$ c) D.P.
g) $2/3+(3/5+1/3)=(3/5+1/3)+2/3$ d) M ZERO
e) D. P.
f) M. ONE
g) C.P.

14) The following sets are formed by dividing numbers by 3.

$\overline{0} = (0,3,6,9, \ldots)$

$\overline{1} = (1,4,7,10, \ldots)$

$\overline{2} = (2,5,8,11,\ldots)$

a) Why do we call the last set $\overline{2}$?

14. a) IT HAS REMAINDER OF 2 WHEN DIVIDED BY 3.

15) What is the one-hundredth number in the set $\overline{0}$ for modulo 8?

a) 792

16) CARRY OUT THE OPERATIONS INDICATED AND WRITE YOUR ANSWER IN THE SIMPLEST FORM IN THE COLUMN AT THE RIGHT.

a) $(3+2)\times 9$
b) $2/5\times 0/4$
c) $5+4\times 3$
d) $12\div 4+2$
e) $18\div(9-6)$
f) $\frac{7+9}{4}$
g) 3^2
h) $(\frac{1}{2})^3$
i) $3/7+(2/5+4/7)$
j) $9\cdot(5/8\cdot 1/9)$
k) $\frac{0}{3}\cdot(\frac{1}{4}+3/4)$
l) $5\cdot(1/5+0/5)$

16. a) 45
b) 0
c) 17
d) 5
e) 6
f) 4
g) 9
h) 1/8
i) 1 2/5
j) 5/8
k) 0
l) 1

Algebra A Post-Test

Read each statement carefully. If the statement is true, write the word "True" in the space provided. If it is false, write the word "False".

1. The set of integers does not contain zero. 1. FALSE
2. There is no last number in the set of whole numbers. 2. TRUE
3. The empty set is a subset of every set. 3. TRUE
4. The solution set of a sentence contains all the replacement values which give true statements. 4. TRUE
5. A proper subset can be the same as the given set. 5. FALSE
6. Every set has at least one member. 6. FALSE

Multiple Choice

INSTRUCTIONS: Write the letter corresponding to the correct answer in the space at the right of each question. USE CAPITAL LETTERS FOR ANSWERS!!

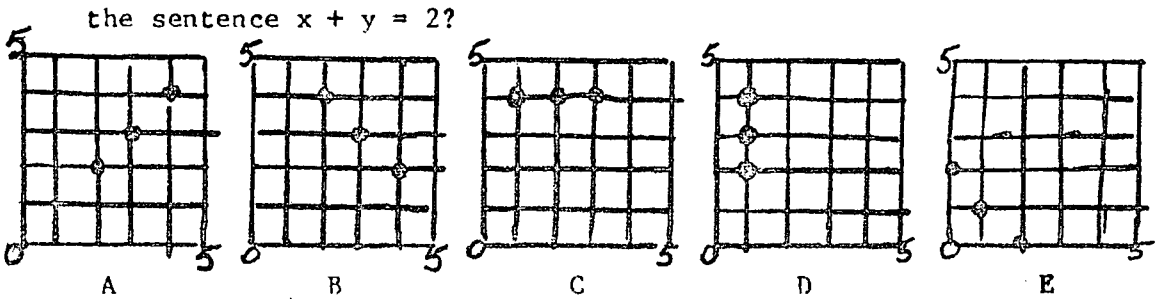
7. What operation is the same as subtracting negative 7?
   A. subtracting 7 B. adding 7 C. adding 0
   D. subtracting 0 E. adding (-7) 7. B

8. S = {0, 1, (-1), 2, (-2), . . . 10, (-10)}
   A number in the solution set for 3 [d + (-7)] = 0 is
   A. 7 B. 0 C. 1
   D. 3 E. (-7) 8. A

9. S = {1,2,3,4,5,6,7,8,9}
   A number pair in the solution set for $x^2 + y^2 = 25$ is
   A. (0,5) B. (5,5) C. (3,4)
   D. (1,5) E. (-4,3) 9. C

10. S = {0,1,2,3,4,5,6,7}
    Find the solution set for the sentence x<6 and x>4
    A. {0,1,2,3,7} B. {4,5,6} C. {5,6,7}
    D. {3,7} E. {5} 10. E

11. S = {0,1,2,3,4,5}
    Which of the graphs contains the solution set for the sentence x + y = 2?



11. E

12. The inverse of dividing by 2/7 is
    A. dividing by 2/7 B. multiplying by 7/2
    C. subtracting by 7/2 D. subtracting 2/7
    E. multiplying by 2/7 12. E

Select the sentence which best represents the stated situation in questions 13, 14 and 15.

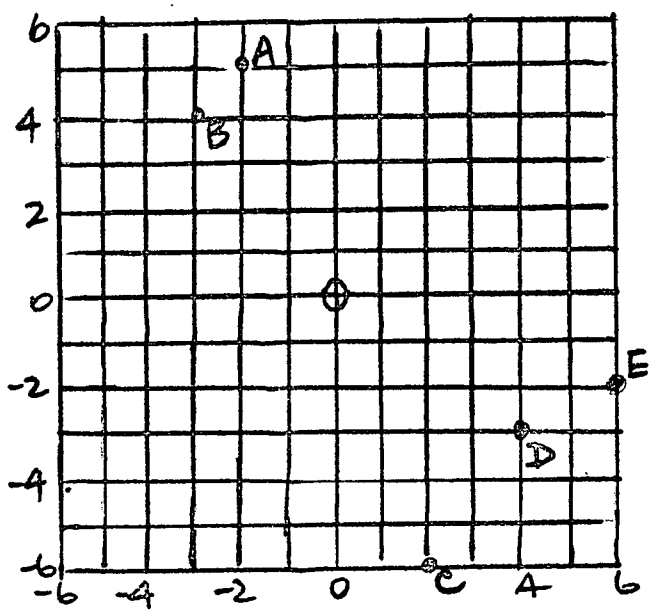
13. The sum of 3 times a number and 5 is 77.
A. $y^3 + 5 = 77$ B. $3 \cdot y + 15 = 77$ C. $3 \cdot y + 5 = 77$
D. $3 \cdot y = 77 + 5$ E. $3(y + 5) = 77$ 13. C

14. The sum of two consecutive odd numbers is 196.
A. $x + (x + 1) = 196$ B. $x + (x + 2) = 196$
C. $x \cdot (x + 1) = 196$ D. $x + 2 = 196$ E. $x + y = 196$ 14. B

15. Bobby weighs 15 pounds more than Alan. Together they weigh 98 pounds. Find the weight of each boy. If x represents Bobby's weight, which is correct?
A. $x + (x + 15) = 98$ B. $x + 15 = 98$ C. $x + (x - 15) = 98$
D. $2x = 98 - 15$ E. $x - x + 15 = 98$ 15. C

16. The sum of the ages of two children is 11 years. In five years one child will be twice as old as the other. How old are they now? Select the number pair.
A. (7,4) B. (6,5) C. (9,2)
D. (10,1) E. ($5\frac{1}{2}$, $5\frac{1}{2}$) 16. C

17. Name the point, A, B, C, D, or E, <u>not</u> in the solution set of the equation $xy = (-12)$



17. A

18. The sum of two numbers is 27. One-third of the smaller number is the same as one-sixth of the larger. Find the numbers. 18. 9, 18

19. The population of a certain city is 5,000 more than double what is was 50 years ago. The population now is 88,000. What was the population 50 years ago? 19. 41,500

20. In a certain class 3 out of 7 pupils were girls. Of the girls 2 out of 3 were married two years after they left school.
(a) Write a number pair comparing the number of girls in the class with the number of boys. 20. (3, 4)

In a certain grade eight class, 3 out of 5 pupils were girls. Of the boys, 5 out of 8 completed grade twelve.

21. Write a number pair comparing the number of girls in the class with the number of boys. 21. (3, 2)

22. If there were 40 students in the class, how many boys completed grade twelve? 22. 10

23. Set S has 8 elements and set T has 10. Set $S \cup T$ has only 15 elements. How many elements are in $S \cap T$? 23. 3

24. A man is 6 ft. tall. At a certain time of the day he casts a shadow of 4 ft. At the same time a tree casts a shadow of 20 ft. How high is the tree? 24. 30

Use the symbols >, <, and = to show the correct relation between the following number pairs. Write only the symbol in the space provided on the right. Do not change the order of the given numbers.

25. 5.687 _____ 5.6865 25. >

26. (-6) _____ 5 26. <

27. 4^6 _____ 16^3 27. =

Solve the following equations for set S = {0,1,2,3,4,5,6,7,8,9,10}.

28. 14/a = 2 28. {7}

29. x•9 = 0 29. {0}

30. 4(7 - k) = 16 30. {3}

Perform the following operations.

31. 8 - 11 + 7 = 31. {4}

32. (-6) • 3 = 32. {-18}

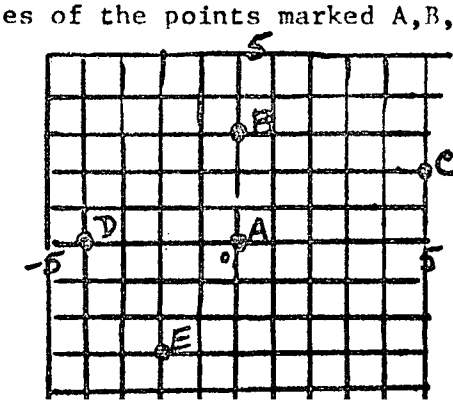
33. (-14) ÷ (-2) = 33. {7}

34. 8 - (-3) = 34. {11}

35. (-5) + (-4) = 35. {-9}

36. (-10) - 0 = 36. {-10}

Give the coordinates of the points marked A,B,C,D, and E.



37. A= (0, 0)

38. B= (0, 3)

39. C= (5, 2)

40. D= (-4, 0)

41. E= (-2, -3)

Algebra B Retention Test

Read each statement carefully. If the statement is true, write the word "True" in the space provided. If it is false write the word "False".

1. Subtraction is a commutative operation. 1. False
2. The empty set is a subset of each set. 2. True
3. The sum of an odd number and an even number is always an odd number. 3. True
4. Every natural number is a rational number. 4. True
5. 10^3 means 10 x 3. 5. False
6. If set S = {1,2,3,4,} and set T = {3,6,9} then $S \cup T$ = {3}. 6. False

Multiple Choice

INSTRUCTIONS: Write the letter corresponding to the correct answer in the space at the right of each question. USE CAPITAL LETTERS FOR ANSWERS!!

7. What operation is the same as subtracting negative 7?
A. subtracting 7 B. adding 7 C. adding 0
D. subtracting 0 E. adding (-7) 7. B

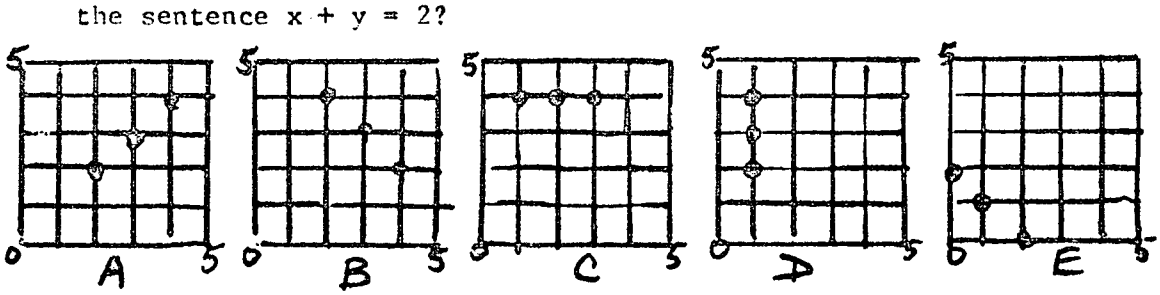
8. Which of the following is not a subset of the set {1,2,3,4,5}?
A. {1,3,5} B. {2,4} C. { }
D. {1,2,3,4,5} E. {1,3,5,7} 8. E

9. S = {0, 1,(-1), 2,(-2), . . . 10,(-10)}
A number in the solution set for 3 [d + (-7)] = 0 is
A. 7 B. 0 C. 1
D. 3 E. (-7) 9. A

10. S = {1,2,3,4,5,6,7,8,9}
A number pair in the solution set for $x^2 + y^2 = 25$ is
A. (0,5) B. (5,5) C. (3,4)
D. (1,5) E. (-4,3) 10. C

11. S = {0,1,2,3,4,5,6,7}
Find the solution set for the sentence x<6 and x>4
A. {0,1,2,3,7} B. {4,5,6} C. {5,6,7}
D. {3,7} E. {5} 11. E

12. S = {0,1,2,3,4,5}
Which of the graphs contains the solution set for the sentence x + y = 2?



12. E

Select the sentence which best represents the stated situation in questions 13, 14, and 15.

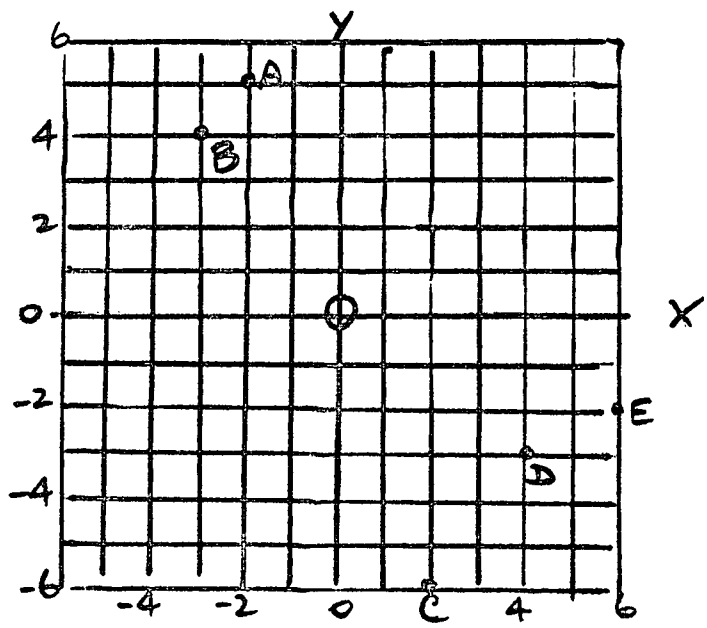
13. The difference between 3 times a number and 5 is 31.
A. $y^3 - 5 = 31$ B. $3 \cdot y - 15 = 31$ C. $3 \cdot y - 5 = 31$
D. $3 \cdot y = 31 - 5$ E. $3(y - 5) = 31$ 13. C

14. The sum of two consecutive odd numbers is 216.
A. $x + (x + 1) = 216$ B. $x + (x + 2) = 216$ C. $x \cdot (x + 1) = 216$
D. $x + 2 = 216$ E. $x + y = 216$ 14. B

15. Bobby weighs 11 pounds more than Alan. Together they weigh 141 pounds. Find the weight of each boy. If x represents Bobby's weight, which is correct?
A. $x + (x + 11) = 141$ B. $x + 11 = 141$ C. $x + (x - 11) = 141$
D. $2x = 141 - 11$ E. $x - x + 11 = 141$ 15. C

16. The sum of the ages of two children is 11 years. In five years one child will be twice as old as the other. How old are they now? Select the number pair.
A. (7,4) B. (6,5) C. (9,2)
D. (10,1) E. ($5\frac{1}{2}$,$5\frac{1}{2}$) 16. C

17. Name the point, A, B, C, D, or E, <u>not</u> in the solution set of the equation $xy = (-12)$.



17. A

18. The sum of two numbers is 20. One-half of the larger number is the same as two times the smaller number. Find the two numbers. 18. 4 , 16

19. My monthly salary is 75 dollars more than twice what it was ten years ago. I now earn 585 dollars a month. What was my salary ten years ago? 19. 255

20. Set S has 8 elements and set T has 10. Set $S \cup T$ has only 15 elements. How many elements are in $S \cap T$? 20. 3

21. A boy is 5 ft. tall. At a certain time of the day he casts a shadow of 3 ft. At the same time a building casts a shadow 21 st. How high is the building? 21. 35

Use the symbols <, >, and = to show the correct relation between the following number pairs. Write only the symbol in the space provided on the right. Do not change the order of the given numbers.

22. (a) 5.687 _____ 5.6865 — 22. (a) >
23. (b) (-6) _____ 5 — 23. (b) <
24. (c) 4^6 _____ 16^3 — 24. (c) =

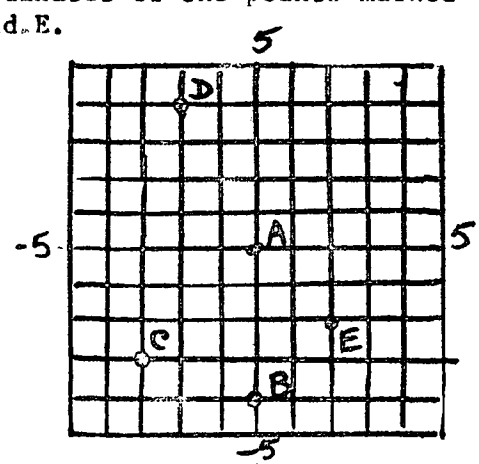
Solve the following equations for set S = {0,1,2,3,4,5,6,7,8,9,10}.

25. (a) 6a = 12 — 25. (a) {2}
26. (b) t/7 = 2 — 26. (b) {14}
27. (c) 24 = 3(5 + d) — 27. (c) {3}
28. (d) 1/3 a = 2 — 28. (d) {6}

Perform the following operations

29. (a) (-7) + 2 + 9 = — 29. (a) 4
30. (b) (-5) - 3 = — 30. (b) -8
31. (c) (-7) - (-3) = — 31. (c) -4
32. (d) 0 ÷ (-4) = — 32. (d) 0
33. (e) (-5) ÷ (-1) = — 33. (e) 5
34. (f) 0 - (-4) = — 34. (f) 4
35. (g) -4 - (+3) = — 35. (g) -7

Give the coordinates of the points marked A, B, C, D and E.



36. (a) A= (0,0)
37. (b) B= (0,-4)
38. (c) C= (-3,-3)
39. (d) D= (-2,4)
40. (e) E= (2,-2)

The inverse of dividing by 3/5 is
A. dividing by 3/5 B. multiplying by 5/3 C. multiplying by 3/5
D. subtracting 3/5 E. subtracting 5/3 41. C