Adaptive Resource Allocation for Multiuser OFDM-based Cognitive Radio Systems

by

Tao Qin

B. Eng., McMaster University, Canada, 2005

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF APPLIED SCIENCE
in
THE FACULTY OF GRADUATE STUDIES
(Electrical and Computer Engineering)

THE UNIVERSITY OF BRITISH COLUMBIA
April 2007
© Tao Qin, 2007
Abstract

Major challenges in the design of next generation wireless communication systems include harsh propagation environments and scarce resources such as power and spectrum. Cognitive radio (CR) is a promising concept for improving the utilization of scarce radio spectrum resources. Orthogonal frequency division multiplexing (OFDM) is regarded as a technology which is well-matched for CR systems. Dynamic resource allocation is an important task in such systems.

In this thesis, a novel fair multiuser resource allocation algorithm for OFDM CR systems is presented. Although not optimal, the algorithm has low computational complexity. The algorithm attempts to maximize the total transmit bit rate (system throughput) of a group of secondary (unlicensed or CR) users subject to (1) a total transmit power constraint for secondary users, (2) a maximum tolerable interference level which can be tolerated by primary (licensed) users. The algorithm is fair in the sense that it tries whenever possible to allocate bits to users who have not received their fair share of service. Simulation results show that the proposed algorithm achieves a performance close to optimal. The effect on system throughput of changing various system parameter values is also examined.

A novel cost minimization algorithm for multiuser OFDM cognitive radio systems is also proposed. The objective is to minimize a cost function which takes into account the interference power experienced by the primary user as well as the base station transmit power for secondary users given minimum bit rate requirements for each secondary user. It is found that the proposed algorithm provides a performance which is fairly close to optimal. The influence of a relative weight parameter on the base station (BS) transmit power for secondary users and the primary user interference power is also discussed.
Contents

Abstract ................................................................. ii

Contents ................................................................. iii

List of Tables ........................................................... vi

List of Figures .......................................................... viii

List of Symbols ........................................................ xii

List of Abbreviations .................................................. xv

Acknowledgements ...................................................... xvii

Dedication ............................................................... xviii

1 Introduction .......................................................... 1

1.1 Evolution of Wireless Communication Systems .............. 1

1.2 Motivation .......................................................... 2

1.3 Thesis Contributions ............................................. 4

1.4 Thesis Organization .............................................. 4

2 Preliminaries ......................................................... 6

2.1 Wireless Communication Channel ............................. 6
2.1.1 Signal Propagation in a Wireless Channel ................. 6
2.1.2 Large-Scale Path loss .................................. 7
2.1.3 Small-Scale Fading and Multipath ....................... 8

2.2 Orthogonal Frequency Division Multiplexing .................. 12
  2.2.1 Orthogonality ........................................... 12
  2.2.2 OFDM System ............................................ 13

2.3 Cognitive Radio .............................................. 14

2.4 Mutual Interference in OFDM-based Cognitive Radio System ... 16
  2.4.1 Interference Introduced by Secondary User Signal .......... 17
  2.4.2 Interference Introduced by Primary User Signal .......... 18

3 Fair Adaptive Resource Allocation ................................. 19
  3.1 Introduction .................................................. 19
  3.2 System Model ............................................... 20
  3.3 Proposed Algorithms ....................................... 22
    3.3.1 Basic Algorithm ...................................... 22
    3.3.2 Reduced Complexity Algorithm ......................... 25
  3.4 Simulation Results .......................................... 29

4 Cost Minimization Resource Allocation ............................ 45
  4.1 Introduction .................................................. 45
  4.2 System Model ............................................... 46
  4.3 Proposed Algorithm ........................................ 47
    4.3.1 Maximal Cost Reduction by a New Subcarrier .......... 48
    4.3.2 Proposed Resource Allocation Algorithm .............. 50
  4.4 Simulation Results .......................................... 51

5 Conclusions and Suggestions for Future Work ..................... 64
5.1 Contributions of the Thesis ........................................ 64
5.2 Future work .......................................................... 65

Bibliography .............................................................. 67
## List of Tables

3.1 Average values of number of bits, $b_k$, and power, $P_k$, loaded onto subcarrier $k$ as well as interference power, $I_k$, seen by the primary user due to the signal transmitted on subcarrier $k$ over 10000 channel realizations with $\mu_R = 1$  
3.2 Average (over 10000 allocation periods) values of number of bits, $b_k$, and power, $P_k$, loaded onto subcarrier $k$ as well as interference power, $I_k$, seen by the primary user due to the signal transmitted on subcarrier $k$ for constant channel power gains, $|h_{mk}|^2$ and $|g_k|^2$, equal to $4/\pi$.  
3.3 Power gains $|h_{mk}|^2$ and $|g_k|^2$ of subcarrier $k$.  
3.4 RC Algorithm results of $b_k$, $P_k$ and $I_k$ for same channel realization gains as in Table 3.3 with identical NBRW  
3.5 RC Algorithm results of $b_k$, $P_k$ and $I_k$ for same channel realization gains as in Table 3.3 with no NBRW  
3.6 Optimization software results of $b_k$, $P_k$ and $I_k$ for same channel realization gains as in Table 3.3 with no NBRW  
4.1 Average values of number of bits, $b_k$, and power, $P_k$, loaded onto subcarrier $k$ as well as interference power, $I_k$, seen by the primary user due to the signal transmitted on subcarrier $k$ and cost, $C_k$, on subcarrier $k$ over 10000 channel realizations with $\mu_R = 1$.
4.2 Values of number of bits, \( b_k \), and power, \( P_k \), loaded onto subcarrier \( k \) as well as interference power, \( I_k \), seen by the primary user due to the signal transmitted on subcarrier \( k \) for constant channel power gains, \( |h_{mk}|^2 \) and \( |g_k|^2 \), equal to \( 4/\pi \). ........................................ 58

4.3 Power gains \( |h_{mk}|^2 \) and \( |g_k|^2 \) of subcarrier \( k \) ........................................ 61

4.4 Cost Minimization algorithm result of \( b_k, P_k, I_k \) and \( C_k \) for same channel realization gains as in Table 4.3 ................................. 62
List of Figures

2.1 Generation and reception of OFDM signals. ........................................ 13
2.2 Block diagram of a multicarrier OFDM digital communication system
(taken from [25]). ..................................................................................... 14
2.3 Basic cognitive cycle (taken from [5]). ................................................. 16
2.4 Primary user band of width $W_p$ and secondary user sub-bands, each of
width $W_s$ (taken from [21]). ................................................................. 17

3.1 Total bit rate, $R_s$, of 4 secondary users versus maximum tolerable inter-
ference power, $I_{th}$, of the primary user with $P_{max} = 1$ W, $P_p = 5$ W,
$N_0 = 10^{-8}$ W/Hz and $\mu_R = 1$. ......................................................... 30

3.2 Total bit rate, $R_s$, of 4 secondary users versus maximum tolerable inter-
ference power, $I_{th}$, of the primary user with $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz,
$\mu_R = 1$ and identical nominal secondary user bit rate weights. .............. 31

3.3 Total bit rate, $R_s$, of 4 secondary users versus maximum tolerable inter-
ference power, $I_{th}$, of the primary user with $P_{max} = 1$ W, $N_0 = 10^{-8}$ W/Hz,
$\mu_R = 1$ and identical nominal secondary user bit rate weights. .............. 32
3.4 Total bit rate, $R_s$, of 4 secondary users versus maximum tolerable interference power, $I_{th}$, of the primary user with $P_{max} = 1$ W, $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz and identical nominal secondary user bit rate weights. 33

3.5 Total bit rate, $R_s$, of 4 secondary users versus maximum tolerable interference power, $I_{th}$, of the primary user with $P_{max} = 1$ W, $P_p = 5$ W, $\mu_R = 1$ and identical nominal secondary user bit rate weights. 33

3.6 Total bit rate, $R_s$, of secondary users versus maximum tolerable interference power, $I_{th}$, of the primary user with $P_{max} = 1$ W, $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz, $\mu_R = 1$ and identical nominal secondary user bit rate weights. 34

3.7 Total bit rate, $R_s$, of secondary users versus maximum tolerable interference power, $I_{th}$, of the primary user with $P_{max} = 1$ W, $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz, $\mu_R = 1$ and no nominal secondary user bit rate weights. 35

3.8 Average number of bits, $b_k$, loaded per subcarrier over 10000 channel realizations with $\mu_R = 1$ 36

3.9 Average power, $P_k$, loaded per subcarrier over 10000 channel realizations with $\mu_R = 1$ 37

3.10 Average interference, $I_k$, per subcarrier over 10000 channel realizations with $\mu_R = 1$ 37

3.11 Average number of bits, $b_k$, loaded per subcarrier over 10000 channel realizations for constant channel power gains equal to $4/\pi$. 39

3.12 Average power, $P_k$, loaded per subcarrier over 10000 channel realizations for constant channel power gains equal to $4/\pi$. 39

3.13 Average interference, $I_k$, per subcarrier over 10000 channel realizations for constant channel power gains equal to $4/\pi$. 40
3.14 Power gains $|h_{mk}|^2$ and $|g_k|^2$ of subcarrier $k$ .......................... 41
3.15 Interference factor, $IF_k$ and the interference power introduced by primary user's signal into subcarrier $k$ band at user $m$, $S_{mk}$ .......................... 42
3.16 RC Algorithm results of number of bits, $b_k$, loaded for same channel realization gains as in Table 3.3 with identical NBRW .................. 43

4.1 Total system cost, $C_{\text{total}}$, versus total required transmit bit rate, $R_s^{req}$, of 4 secondary users with uniform BRW and $\alpha = 250/251$. .......................... 53
4.2 Total system cost, $C_{\text{total}}$, versus total required transmit bit rate, $R_s^{req}$, of 4 secondary users with different BRW and $\alpha = 250/251$. .......................... 53
4.3 Total power required, $P_{\text{total}}$, by secondary users and interference, $I_{\text{total}}$, introduced to primary user versus $\alpha$ value with the total required bit rate for 4 secondary user $R_s^{req} = 18.75$ Mbps and uniform BRW. ... 54
4.4 Average number of bits, $b_k$, loaded per subcarrier over 10000 channel realizations with $\mu_R = 1$ .......................... 56
4.5 Average power, $P_k$, loaded per subcarrier over 10000 channel realizations with $\mu_R = 1$ .......................... 56
4.6 Average interference, $I_k$, per subcarrier over 10000 channel realizations with $\mu_R = 1$ .......................... 57
4.7 Average cost, $C_k$, per subcarrier over 10000 channel realizations with $\mu_R = 1$. .......................... 57
4.8 Number of bits, $b_k$, loaded per subcarrier for constant channel power gains equal to $4/\pi$. .......................... 58
4.9 Power, $P_k$, loaded per subcarrier for constant channel power gains equal to $4/\pi$. .......................... 59
4.10 Interference, $I_k$, per subcarrier for constant channel power gains equal to $4/\pi$. .......................... 59
4.11 Cost, $C_k$, per subcarrier for constant channel power gains equal to $4/\pi$. 60
4.12 Power gains $|h_{mk}|^2$ and $|g_k|^2$ of subcarrier $k$. 61
4.13 Interference factor, $IF_k$ and the interference power introduced by primary user’s signal into subcarrier $k$ band at user $m$, $S_{mk}$. 62
4.14 Cost Minimization Algorithm results of number of bits, $b_k$, loaded for same channel realization gains as in Table 4.3. 63
List of Symbols

\( A \) \hspace{1cm} \text{Peak amplitude of the dominant signal}
\( a_l(t) \) \hspace{1cm} \text{Signal amplitude for path} \ l
\( a_{mk} \) \hspace{1cm} \text{Subcarrier allocation indicator}
\( B_c \) \hspace{1cm} \text{Coherence bandwidth of channel}
\( B_i \) \hspace{1cm} \text{Total number of bits per symbol period allocated to user} \ i
\( B^\text{req}_m \) \hspace{1cm} \text{Minimum required number of bits per symbol period for user} \ m
\( B_s \) \hspace{1cm} \text{Bandwidth of transmitted signal}
\( c \) \hspace{1cm} \text{Velocity of light}
\( C_{\text{total}}(\alpha) \) \hspace{1cm} \text{Total cost}
\( d_k \) \hspace{1cm} \text{Spectral distance between subcarrier} \ k \hspace{1cm} \text{and the center frequency of the primary user band}
\( d_0 \) \hspace{1cm} \text{Reference distance}
\( f_c \) \hspace{1cm} \text{Carrier frequency}
\( f_{d,\text{max}} \) \hspace{1cm} \text{Maximum Doppler shift}
\( g_k \) \hspace{1cm} \text{Channel gain from the base station to the primary user for subcarrier} \ k
\( g_l(t) \) \hspace{1cm} \text{Channel gain for path} \ l
\( h_{mk} \) \hspace{1cm} \text{Subcarrier} \ k \hspace{1cm} \text{gain from the base station to user} \ m
\( h(t, \tau) \) \hspace{1cm} \text{Time-variant impulse response}
\( IF(d_k) \) \hspace{1cm} \text{Interference factor}
\( I_o(\cdot) \) Modified Bessel function of the first kind and zero-order

\( I_{th} \) Primary user's maximum tolerable interference power

\( I_{total} \) Total interference power seen by the primary user due to the signals destined for secondary users

\( K \) Number of subcarrier

\( L \) Number of resolvable paths

\( M \) Number of secondary users

\( n \) Path loss exponent

\( N_0 \) One-sided noise power spectral density

\( PL(d) \) Path loss

\( P_{max} \) Total secondary user power budget

\( P_{mk} \) Transmit power allocated to subcarrier \( k \) of user \( m \)

\( P_{total} \) Total transmit power for signals destined to secondary users

\( R_i \) Total bit rate of user \( i \)

\( R_{req}^m \) Minimum required bit rate of user \( m \)

\( R_s \) Total bit rate for all secondary users

\( R_{req}^s \) Total minimum bit rate for all secondary users

\( s_b(t) \) Baseband signal

\( S_{mk} \) Interference power introduced by the signal destined for the primary user into the subcarrier \( k \) band at user \( m \)

\( T_c \) Coherence time of channel

\( T_s \) Signal symbol duration

\( u_k \) Signal symbol for subcarrier \( k \)

\( v \) Velocity of mobile

\( W_p \) Primary user bandwidth

\( W_s \) Secondary user bandwidth

xiii
\( X_{\sigma} \) Gaussian distributed random variable

\( \alpha \) Relative importance of power versus interference

\( \lambda_m \) Nominal bit rate weight for user \( m \)

\( \sigma_r \) Root mean squared delay spread of channel

\( \tau_l(t) \) Propagation delay for path \( l \)

\( \varphi_l(t) \) Channel phase shift for path \( l \)

\( \Phi_{RR}(e^{j\omega}) \) Power spectral density of the primary user's signal
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>1G</td>
<td>First Generation</td>
</tr>
<tr>
<td>2G</td>
<td>Second Generation</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>B3G</td>
<td>Beyond Third Generation</td>
</tr>
<tr>
<td>BA</td>
<td>Basic algorithm</td>
</tr>
<tr>
<td>BER</td>
<td>Bit error rate</td>
</tr>
<tr>
<td>BRW</td>
<td>Bit rate weight</td>
</tr>
<tr>
<td>BS</td>
<td>Base station</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
</tr>
<tr>
<td>CR</td>
<td>Cognitive Radio</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel state information</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FIR</td>
<td>Finite impulse response</td>
</tr>
<tr>
<td>Ftp</td>
<td>File transfer protocol</td>
</tr>
<tr>
<td>GSM</td>
<td>Global System for Mobile Communications</td>
</tr>
<tr>
<td>Http</td>
<td>Hypertext transfer protocol</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
</tr>
<tr>
<td>IP</td>
<td>Integer programming</td>
</tr>
<tr>
<td>ISI</td>
<td>Intersymbol interference</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>LOS</td>
<td>Line of sight</td>
</tr>
<tr>
<td>MI</td>
<td>Minimum interference</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-input multiple-output</td>
</tr>
<tr>
<td>MP</td>
<td>Minimum power</td>
</tr>
<tr>
<td>NBRW</td>
<td>Nominal bit rate weight</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal frequency division multiplexing</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PSD</td>
<td>Power spectral density</td>
</tr>
<tr>
<td>RC</td>
<td>Reduced complexity</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>rms</td>
<td>Root mean squared</td>
</tr>
<tr>
<td>T − R</td>
<td>Transmitter-Receiver</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunications System</td>
</tr>
<tr>
<td>UWB</td>
<td>Ultra-wide band</td>
</tr>
<tr>
<td>VoIP</td>
<td>Voice over Internet Protocol</td>
</tr>
</tbody>
</table>
Acknowledgements

I would like to take this opportunity to convey my great appreciation to my supervisor, Dr. Cyril Leung, whose constructive and patient guidance, continuous encouragement and deep insight in the research area have helped me immeasurably throughout the course of my thesis research. This thesis would never have been written without his assistance.

I am deeply indebted to my parents, Suyan Yin and Ronghua Qin, for their endless love, constant support, immense encouragement and sacrifices over the years. I owe everything I have been able to accomplish to them.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under Grant OGP0001731 and by the UBC PMC-Sierra Professorship in Networking and Communications.

TAO QIN

THE UNIVERSITY OF BRITISH COLUMBIA

April 2007
To my parents
Chapter 1

Introduction

1.1 Evolution of Wireless Communication Systems

Guglielmo Marconi first demonstrated the feasibility of radio communication across the English channel in 1899. In the ensuing eighty years, wireless communications gradually enabled countless applications including radio and TV broadcasting, public safety and emergency services, etc. [1]. During the past decade, the wireless communications industry has been one of the fastest growing sectors of the economy worldwide. It is predicted that this trend will be further accentuated in the next several years [2]. We are currently in the midst of a new revolution in wireless communications with new exciting services and applications being contemplated. The age of comprehensive (any sender, any time, any place, any content, any recipient) personal communications is close at hand.

Three generations of cellular communication systems have so far been developed for public use and plans are underway for beyond third generation (B3G) systems [3]. First generation (1G) systems, based on analog technology, catered mostly to voice users. Second generation (2G) systems, as exemplified by Global System for Mobile Communications (GSM) and code division multiple access (CDMA) IS-95,
provided low rate data services in addition to voice services. Third generation (3G) systems offer significantly higher capacity and support variable data transmission rates. Indoor data rates up to 2 Mb/s and mobile data rates up to 144 Kb/s are possible. There are two key objectives for B3G systems. One is to provide data rates up to 100 Mb/s and 1 Gb/s in mobile and stationary environments respectively. The second is to develop handsets which can operate interchangeably with different networks (e.g., cellular, UMTS, and WiFi) and smoothly switch from one to another.

1.2 Motivation

The scarcity of spectrum resources and the widely time-varying nature of wireless channels represent two major obstacles in meeting the rapidly growing demand for wireless communications services. In contrast to the enormous bandwidth available in optical fibre communication systems, spectrum resources in wireless communication systems are quite limited. Furthermore, the wireless channel is an open medium which is subject to severe interference, fading and noise. It is thus important to develop strategies which can make efficient use of spectrum resources available in a wireless system.

The Federal Communications Commission (FCC) has published a report [4] which concludes that most of the licensed spectrum bands are largely under-utilized. Cognitive radio (CR) [5] has been proposed as a possible approach to improve spectrum utilization. The basic idea behind CR is an approach which enables unused spectrum segments in a target spectrum pool to be located and used by secondary (unlicensed) users without causing significant interference to the primary (licensed) users. Spectrum efficiency can be increased by allowing secondary users to access such unused bands at the right locations and opportune times [5].
Orthogonal frequency division multiplexing (OFDM) is regarded as a technology which is well-matched for CR systems due to its resistance to multipath intersymbol interference (ISI) and its flexibility in allocating resources among secondary users [6]. It is shown that OFDM is the good candidate for wide-band cognitive radio systems because it can easily produce signals that can fit different spectral masks.

In CR systems, the primary and secondary users will often simultaneously use adjacent frequency bands. It is therefore important to manage mutual interference problems which might arise. It is shown in [7] that there is mutual interference between primary users and OFDM based secondary users due to the non-orthogonality of their respective transmitted signals. The introduced mutual interference between the two categories of users depends on the transmitted power as well as the spectral distance between them.

The problem of power, bit and subcarrier loading for multiuser OFDM has been studied in [8–20]. However, the algorithms proposed in these papers are only applicable in conventional multiuser OFDM systems with only one type of users (e.g., secondary users). They do not consider mutual interference which may arise between primary and secondary users. In [21], the problem of bit and power loading in an OFDM-based CR system in which mutual interference is explicitly considered is studied assuming one secondary user. The objective is to maximize the throughput of the secondary user subject to a maximum interference power that can be tolerated by the primary user. In this thesis, the model in [21] is extended to the case of multiple secondary users in Chapter 3; fairness, secondary user power and integer bit loading constraints are also included. In Chapter 4, the problem of minimizing a certain cost function subject to minimum secondary user bit rate constraints is studied.
1.3 Thesis Contributions

The main contributions of this thesis are:

1. New optimization models for fair adaptive resource allocation and minimum cost resource allocation for multiuser OFDM CR systems are formulated.

2. A new fair multiuser resource allocation algorithm for OFDM CR systems is proposed. The algorithm attempts to maximize the total transmit bit rate of secondary users subject to a total transmit power constraint for secondary users and a maximum tolerable interference level which can be tolerated by primary users. The algorithm is fair in the sense that it tries to allocate bits to users who have not received their fair share of service as much as possible.

3. A new cost minimization algorithm for multiuser OFDM CR systems is presented. The objective is to minimize a cost function which takes into account the interference power experienced by the primary user as well as the base station transmit power for secondary users given minimum bit rate requirements for each secondary user.

4. The performances of the proposed algorithms are examined using computer simulations. The effects of changing various system parameters are studied.

1.4 Thesis Organization

The remainder of this thesis is organized as follows: In Chapter 2, some basic information about wireless communication channels, OFDM technology, CR and mutual interference in OFDM CR systems are reviewed. In Chapters 3 and 4, models for resource allocation are formulated and low-complexity, suboptimal algorithms are
proposed. The performances of the proposed algorithms are investigated using computer simulation. The main results and contributions of the thesis are summarized in Chapter 5 which also includes suggestions of topics for future study.
Chapter 2

Preliminaries

2.1 Wireless Communication Channel

The wireless channel places fundamental limitations on the performance of wireless communication systems. Due to multiple propagation paths, the received signal consists of multiple delayed and attenuated copies of the transmitted signal. In addition, the wireless channel is time varying due to the motion of the mobile users or the surroundings.

2.1.1 Signal Propagation in a Wireless Channel

There are three basic signal propagation mechanisms, namely reflection, diffraction and scattering [1]:

- Reflection occurs when the electromagnetic wave impinges upon an object of very large dimensions compared to the wavelength and/or with different electromagnetic properties. The wave is partially reflected and partially transmitted.

- Diffraction occurs due to the obstruction of the radio path between the transmitter and the receiver by a dense body with large dimensions relative to the
wavelength resulting in the formation of secondary waves behind the obstructing body.

- Scattering occurs when the medium has many objects with small dimensions relative to the wavelength and reflections from these objects cause the radio waves to diffuse or scatter in all directions.

Basically fading is the cumulative effect of the above mechanisms.

2.1.2 Large-Scale Path loss

Both theoretical and measurement-based propagation models suggest that the path loss increases as a power of the Transmitter-Receiver (T-R) distance [1], i.e.,

\[ \bar{PL}(d) \propto \left( \frac{d}{d_o} \right)^n \]  

(2.1)

or

\[ \bar{PL}(d)_{dB} = \bar{PL}(d_o)_{dB} + 10n \log \left( \frac{d}{d_o} \right) \]  

(2.2)

where \( n \) is the path loss exponent, \( d_o \) is a reference distance which is determined from measurements and \( d \) is the T-R separation distance. The bars in (2.1) and (2.2) denote the ensemble average of all possible path loss values for a given value of \( d \).

The model in (2.2) does not account for the fact that the surrounding environmental clutter may be vastly different at two different locations having the same T-R separation. Measurements have shown that at any value of \( d \), the path loss \( PL(d) \) at a particular location is random and distributed log-normally about the mean distance-dependent value \( \bar{PL}(d) \), i.e.

\[ PL(d)_{dB} = \bar{PL}(d_o)_{dB} + X_\sigma = \bar{PL}(d_o)_{dB} + 10n \log \left( \frac{d}{d_o} \right) + X_\sigma \]  

(2.3)

where \( X_\sigma \) is a zero-mean Gaussian distributed random variable (in dB) with standard deviation \( \sigma \) (also in dB). This phenomenon is referred to as log-normal shadowing.
2.1.3 Small-Scale Fading and Multipath

Small-scale fading refers to the rapid fluctuations in the amplitude, phase, or multipath delay of a received signal component over a short period of time or travel distance. Such fading is caused by the superposition of two or more versions of the transmitted signal which arrive at the receiver at slightly different times. Multipath in the radio channel creates small-scale fading effects. The three most important effects are [1,22]:

- Rapid changes in signal strength over a small travel distance or time interval.
- Random frequency modulation due to varying Doppler shifts on different multipath signals.
- Time dispersion (echoes) caused by multipath propagation delays.

Assume that $s_b(t)$ is the baseband signal and $f_c$ is the carrier frequency. The corresponding radio frequency (RF) signal transmitted over the wireless channel can be written as

$$s(t) = Re[s_b(t)e^{j2\pi f_ct}]$$

(2.4)

Let $a_l(t)$ and $\tau_l(t)$ denote the amplitude and the propagation delay for the $l^{th}$ path.

The received bandpass signal is given by

$$r(t) = \sum_l a_l(t)s(t-\tau_l(t)) = Re \left\{ \left[ \sum_l a_l(t)e^{-j2\pi f_c\tau_l(t)}s_b(t-\tau_l(t)) \right] e^{j2\pi f_ct} \right\}$$

(2.5)

It is apparent from (2.5) that the equivalent baseband signal is

$$r_b(t) = \sum_l a_l(t)e^{-j2\pi f_c\tau_l(t)}s_b(t-\tau_l(t))$$

(2.6)

It can be seen from (2.6) that the multipath channel can be regarded as a time-varying finite impulse response (FIR) system with output

$$r_b(t) = s_b(t) \otimes h(t, \tau)$$

(2.7)
where

\[ h(t, \tau) = \sum_l a_l(t) e^{-j2\pi f_c \eta(t)} \delta(\tau - \eta_l(t)) \]  

(2.8)

is the impulse response of the channel at time \( t \) due to an impulse input applied at time \( t - \tau \). According to the central limit theorem [22], the time-varying impulse response \( h(t, \tau) \) can be modeled as a complex-valued Gaussian random process in the \( t \) variable since the total number of multipaths is usually very large in most wireless communication systems.

When the modulated symbol duration is much greater than the largest path delay, all the paths cannot be resolved. In this case, all the frequency components in the transmitted signal bandwidth will go through almost the same random attenuation and phase shift. In this case the channel impulse response is expressed as

\[ h(t, \tau) = g(t) e^{j\varphi(t)} \delta(\tau) \]  

(2.9)

On the other hand, when the propagation delay is larger than the symbol duration, some of the multipaths can be resolved. In this case, the frequency components in the transmitted signal will undergo different attenuations and phase shifts along the different paths. The channel impulse response is expressed as

\[ h(t, \tau) = \sum_{l=1}^{L} g_l(t) e^{j\varphi_l(t)} \delta(\tau - \eta_l(t)) \]  

(2.10)

where \( L \) is the number of resolvable paths, \( g_l(t) \) denotes the gain and \( \varphi_l(t) \) denotes the phase shift for the \( l \)-th path.

When there is no line-of-sight (LOS), \( g_l(t) \) is Rayleigh distributed with probability density function (pdf)

\[ p(g) = \begin{cases} \frac{g}{\sigma^2} e^{-\frac{g^2}{2\sigma^2}}, & 0 \leq g \leq \infty \\ 0, & g < 0 \end{cases} \]  

(2.11)

where \( \sigma^2 \) is the time-average power of the received signal before envelope detection.
When there is a direct path (i.e., LOS case), $g_i(t)$ is Ricean distributed with pdf

$$p(g) = \begin{cases} \frac{A_g}{\sigma^2} e^{-\frac{g^2 + A^2}{2\sigma^2}} I_0 \left( \frac{A_g}{\sigma^2} \right), & 0 \leq g \leq \infty \\ 0, & g < 0 \end{cases}$$ (2.12)

where $A \geq 0$ denotes the peak amplitude of the dominant signal and $I_0(\cdot)$ is the modified Bessel function of the first kind and zero-order.

The delay spread and coherence bandwidth are parameters that describe the time dispersive nature of the channel in a local area. The mean excess delay is the first moment of the power delay profile and is defined to be

$$\bar{\tau} = \frac{\sum_i a_i^2 \tau_i}{\sum_i a_i^2}$$ (2.13)

The root mean squared (rms) delay spread is the square root of the second central moment of the power delay profile and is defined to be

$$\sigma_{\tau} = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2}$$ (2.14)

where

$$\bar{\tau}^2 = \frac{\sum_i a_i^2 \tau_i^2}{\sum_i a_i^2}$$ (2.15)

The coherent bandwidth is a range of frequencies over which two frequency components have significant correlation. If the coherence bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.9, then the coherence bandwidth of the channel is approximately [23]

$$B_c \approx \frac{1}{50\sigma_{\tau}}$$ (2.16)

Doppler spread and coherence time are parameters which describe the time varying nature of the channel in a small-scale region. If we assume that the channel is
wide sense stationary, the Doppler power spectrum of a mobile channel for an omni-directional mobile antenna and the received plane wave with uniformly distributed arrival angle can be given by

$$\phi_D(f_d) = \frac{\sigma^2}{\pi f_{d,\text{max}} \sqrt{1 - \left(\frac{f_d}{f_{d,\text{max}}}\right)^2}}$$

(2.17)

where $f_{d,\text{max}}$ is the maximum Doppler shift given by

$$f_{d,\text{max}} = \frac{v}{c} f_c$$

(2.18)

where $v$ is the velocity of the mobile and $c$ is the velocity of light.

The coherence time, $T_c$, is the time domain dual of the maximum Doppler spread, $f_{d,\text{max}}$, and is used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain. The Doppler spread and coherence time are inversely proportional to one another, i.e.,

$$T_c \approx \frac{1}{f_{d,\text{max}}}.$$ 

(2.19)

If the channel has a constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal, the received signal will undergo flat fading. In other words, a signal undergoes flat fading if $B_s << B_c$ where $B_s$ is the bandwidth of transmitted signal and $B_c$ is the coherence bandwidth of the channel. If the channel has a constant gain and linear phase response over a bandwidth that is smaller than the bandwidth of transmitted signal, i.e., $B_s > B_c$ and $T_s < \sigma_T$, the received signal will undergo frequency selective fading. When this occurs, the received signal includes multiple versions of the transmitted waveform, therefore the received signal is distorted and ISI will be induced by the channel.

If the channel impulse response varies rapidly within the symbol duration, i.e., the coherence time of the channel is smaller than the symbol period of the transmitted signal, the channel is referred to as fast fading. That is, a signal undergoes fast fading if $T_s > T_c$. If $T_s << T_c$, the channel is said to be slow fading.
2.2 Orthogonal Frequency Division Multiplexing

High-data-rate communications are limited not only by noise but often more significantly by the intersymbol interference (ISI) due to the time dispersive nature of wireless channels. Generally, the effects of ISI are negligible as long as the delay spread is significantly shorter than the duration of one transmitted symbol. OFDM [24] has been considered as a very promising solution for supporting high-data-rate transmission in future broadband wireless communication systems due to its resistance to ISI. The basic idea of OFDM is to divide the available spectrum into several subcarriers so that the information symbols are transmitted in parallel on the subcarriers over the wireless channel. This allows us to design a system supporting high data rates while maintaining symbol durations much longer than the delay time of channel. By doing so, each subcarrier experiences almost a flat fading, and the effects of the multipath channels are reduced [25,26].

2.2.1 Orthogonality

Let $K$ and $T_s$ denote the number of subcarriers and the the signal symbol duration respectively. Let $u_k$, $k = 0, 1, \ldots, K - 1$ denote the signal symbol for each subcarrier $k$ and $f_0$ and $f_k = f_0 + k/T_s$ denote the carrier frequencies of subcarrier 0 and $k$ subcarrier respectively. We assume that the transmitted signal, $s(t)$, is $\text{rect}(t) = 1$, $|t| \leq T_s/2$. The baseband representation, $s_b(t)$, of the transmitted signal can be expressed as

$$s_b(t) = \begin{cases} \sum_{k=0}^{K-1} u_k \cdot \text{rect}(t - \frac{T_s}{2}) \cdot \exp\left[j2\pi \frac{k}{T_s}(t)\right], & 0 \leq t \leq T_s \\ 0, & t > T_s \end{cases}$$

A block diagram which shows the generation of an OFDM signal and the recovery of the data symbols is shown in Fig. 2.1.
The subcarriers are orthogonal since

\[
\frac{1}{T_s} \int_0^{T_s} \exp(j2\pi f_k t) \cdot \exp(-j2\pi f_n t) dt = \begin{cases} 
1, & k = n \\
0, & k \neq n 
\end{cases}
\]  

(2.21)

The data symbol on subcarrier \( k \) can be recovered as follows:

\[
\hat{u}_k = \frac{1}{T_s} \int_{t_s}^{T_s + t_s} \exp \left[ -j2\pi \frac{k}{T_s} (t - t_s) \right] \cdot \sum_{n=0}^{K-1} u_n \cdot \exp \left[ j2\pi \frac{n}{T_s} (t - t_s) \right] dt \\
= \frac{1}{T_s} \sum_{n=0}^{K-1} u_n \int_{t_s}^{T_s + t_s} \exp \left[ -j2\pi \frac{n-k}{T_s} (t - t_s) \right] dt = u_k
\]  

(2.22)

As we can see from (2.22), the data symbol, \( u_k \), on subcarrier \( k \) can be retrieved since the result of integral for other subcarriers is zero. Thus orthogonality is established.

### 2.2.2 OFDM System

The OFDM baseband signal can be generated using an Inverse Discrete Fourier Transform (IDFT). Let \( K \) samples are taken at times \( t = nT_s/K, n = 0, 1, \ldots K - 1 \) in (2.20) yielding

\[
s_n = s \left( \frac{nT_s}{K} \right) = \sum_{k=0}^{K-1} u_k e^{j2\pi kn/K}, \quad n = 0, 1, \ldots, K - 1.
\]  

(2.23)
Since (2.23) is the IDFT of $u_k$. Therefore, the data symbols, $u_k$, can be retrieved at the receiver by using a DFT, i.e.

$$u_k = \sum_{n=0}^{K-1} s_n e^{-j2\pi kn/K}, \quad i = 0, 1, \ldots, K - 1. \quad (2.24)$$

The Fast Fourier Transform (FFT) is used in practice as it is computationally more efficient than the DFT.

A block diagram of an OFDM system is shown in Fig. 2.2. As shown, cyclic prefix is usually inserted in order to avoid ISI [25].

![Block diagram of a multicarrier OFDM digital communication system](taken from [25]).

### 2.3 Cognitive Radio

As high performance wireless data networks and services are widely deployed, the scarcity of spectrum resources will represent a serious impediment. It is reported in [4] that most of the licensed spectrum bands are poorly utilized, resulting in spectrum holes [5]:

"A spectrum hole is a band of frequencies assigned to a primary user, but, at a particular time and specific geographic location, the band is not being utilized by that user."
Spectrum utilization can be improved significantly by allowing a secondary (un-licensed) user to access spectrum holes [5]. CR [27,28], based on the concept of software-defined radio, has been proposed as a means to improve spectrum efficiency by exploiting spectrum holes. Cognitive radio is a spectrum sharing technology as is ultra-wide band (UWB). The key difference is that while the UWB signal spectrum overlaps with that of primary user signals, a CR signal spectrum resides mainly in the spectrum holes. As a result, a CR device may transmit at high signal powers for long range communication applications such as broadband wireless access in ways that cause no harmful interference to the primary users.

There are three key concepts in CR [6]:

• Sensing - The ability to identify spectrum holes.

• Flexibility - The ability to easily change transmit signal frequency and spectrum shape to fit spectrum holes.

• Non-interference - Transmission to secondary users should not cause harmful interference to primary users.

Through interaction with the RF environment, a CR system performs the following tasks [5]:

1. Radio-scene analysis, which involves:
   • estimation of interference “temperature” to ensure no harmful interference to primary user.
   • detection of spectrum holes.

2. Channel identification, which involves:
   • estimation of channel-state information (CSI);
• prediction of channel capacity for use by the transmitter.

3. Transmit-power control and dynamic spectrum management.

Task 1) and 2) are carried out in the receiver, and task 3) is carried out in the transmitter. Through interaction with the RF environment, these three tasks form a cognitive cycle which is shown in a basic form in Fig. 2.3.

Figure 2.3: Basic cognitive cycle (taken from [5]).

2.4 Mutual Interference in OFDM-based Cognitive Radio System

CR provides a novel approach for improving the utilization of radio spectrum. It is suggested in [6] that OFDM is an ideal physical layer candidate for wide-band CR systems. However, if the primary users employ non-OFDM based signaling (e.g.,
single-carrier CDMA), there may be mutual interference between the primary and secondary users due to the non-orthogonality of their respective transmit signals [7].

In [21], a model for the downlink of an OFDM CR system with one base station (BS), one primary and one secondary user is proposed as shown in Fig. 2.4. The primary user and the secondary user share adjacent frequency bands as shown in Fig. 2.4. The primary user band, of width $W_p$ Hz, is surrounded on each side by $K/2$ subcarriers with each subcarrier occupying a band of width $W_s$ Hz. The $K$ subcarriers are used for transmission to the secondary user using OFDM. As the BS can transmit simultaneously to both primary and secondary user, the primary user’s signal causes interference to the secondary user and vice-versa [7,21]. In this thesis, we consider an extension of the model to $M$ multiple secondary users.

![Figure 2.4: Primary user band of width $W_p$ and secondary user sub-bands, each of width $W_s$ (taken from [21]).](image)

**2.4.1 Interference Introduced by Secondary User Signal**

This interference is caused by the side lobes of the OFDM signal. The transmit signal on each subcarrier is a rectangular non-return-to-zero (NRZ) signal. The power spectral density (PSD) of the $k^{th}$ subcarrier signal is modeled as [7]

$$
\Phi_k(f) = P_k T_s \left( \frac{\sin \pi f T_s}{\pi f T_s} \right)^2,
$$

(2.25)
where $P_k$ is the transmit power of the $k^{th}$ subcarrier signal and $T_s$ is the symbol duration which consists of the sum of useful symbol duration and guard interval. The interference power introduced by this signal into the primary user's band is

$$I_k(d_k, P_k) = \int_{d_k-W_s/2}^{d_k+W_s/2} |g_k|^2 \Phi_k(f) \, df = P_k \text{IF}_k,$$

(2.26)

where $g_k$ is the channel gain from the BS to the primary user for subcarrier $k$, $d_k$ is the spectral distance between subcarrier $k$ and the center frequency of the primary user band and $\text{IF}_k = T_s \int_{d_k-W_s/2}^{d_k+W_s/2} \left( |g_k|^2 \frac{\sin \pi f T_s}{\pi f T_s} \right)^2 \, df$ denotes the interference factor for subcarrier $k$.

### 2.4.2 Interference Introduced by Primary User Signal

The interference power introduced by the signal destined for the primary user, hereafter referred to as the primary user's signal, into the subcarrier $k$ band at user $m$ is

$$S_{mk}(d_k) = \int_{d_k-W_s/2}^{d_k+W_s/2} |h_{mk}|^2 \Phi_{RR}(e^{jw}) \, dw,$$

(2.27)

where $h_{mk}$ is the subcarrier $k$ gain from the BS to user $m$ and $\Phi_{RR}(e^{jw})$ is the PSD of the primary user's signal.
Chapter 3

Fair Adaptive Resource Allocation

3.1 Introduction

In multimedia wireless communication, many applications do not have a prescribed bit transmission rate, e.g., best effort services (Hypertext transfer protocol (Http), Email, File transfer protocol (Ftp), etc.). For these services, the objective of resource allocation is to maximize the system throughput while ensuring some level of fairness among users. This chapter focuses on fair resource allocation in a multiuser OFDM based cognitive radio system.

Fair scheduling for OFDM systems have been studied in [10,13] assuming that the power is equally distributed among the subcarriers but power and bit allocation on each subcarrier have not been investigated. A power loading algorithm for a fixed subcarrier allocation was studied in [17]. Joint subcarrier and power allocation to maximize the transmitted bit rate with proportional fairness was investigated in [16,19] assuming infinite granularity in bit rate. In [20], a proposed algorithm was to maximize the system throughput with total power budget and integer bit constraint while achieving proportional fairness among users.

The above-mentioned algorithms can be applied in conventional multiuser OFDM
systems with only one type of users (e.g., secondary users). They do not consider mutual interference which may arise between primary and secondary users. In contrast, the mutual interference is considered in [21] for the case of one secondary user and the throughput of the secondary user is maximized subject to a maximum interference power that can be tolerated by the primary user. We now propose and evaluate a low complexity resource allocation algorithm to maximize the total transmit bit rate in a multiuser OFDM-based CR system with secondary user power, fairness, integer bit loading constraints as well as a maximum interference power that can be tolerated by the primary user. The algorithm first allocates bits to users to ensure fairness, and then uses a greedy approach for subcarrier, power and bit loading allocations.

3.2 System Model

We consider the resource allocation and mutual interference model described in Section 2.4. We consider resource allocation on the downlink of an OFDM CR radio system in which a base station (BS) serves one primary and $M$ secondary users. The primary user and the secondary users share adjacent frequency bands as shown in Fig. 2.4. The primary user band, of width $W_p$ Hz, is surrounded on each side by $K/2$ subcarriers with each subcarrier occupying a band of width $W_a$ Hz. The subcarriers are used for transmission to the secondary users using OFDM. As the BS can transmit simultaneously to both primary and secondary users, the primary user’s signal can cause interference to the secondary users and vice-versa [7,21]. It is assumed that the channel is slowly time-varying and that the BS has perfect channel state information.

Let $P_{mk}$ denote the transmit power allocated to subcarrier $k$ of user $m$. Following [11], the maximum number of bits in a symbol transmitted on this subcarrier is set
where \( \lfloor \cdot \rfloor \) denotes the floor function, \( N_0 \) is the one-sided noise PSD and \( S_{mk} \) is given by (2.27). The parameter \( \Gamma \) in (3.1) depends on required target bit error rate (BER); \( \Gamma \) increases as the target BER is decreased. For convenience, the term \( \Gamma \) is set to unity in the remainder of this thesis.

Let \( a_{mk} \in \{0, 1\} \) be a subcarrier allocation indicator, i.e., \( a_{mk} = 1 \) if and only if subcarrier \( k \) is allocated to user \( m \). It is assumed that each subcarrier can be used for transmission to at most one user at any given time.

Let the total interference power seen by the primary user due to the signals destined for secondary users be

\[
I_{\text{total}} = \sum_{m=1}^{M} \sum_{k=1}^{K} a_{mk} P_{mk} I F_k, \tag{3.2}
\]

and the total transmit power for signals destined to secondary users be

\[
P_{\text{total}} = \sum_{m=1}^{M} \sum_{k=1}^{K} a_{mk} P_{mk}. \tag{3.3}
\]

Our objective is to maximize the total bit rate for secondary users subject to total transmit power, fairness and primary user interference constraints. Specifically, the optimization problem is expressed as follows:

\[
\max W_s \sum_{m=1}^{M} \sum_{k=1}^{K} a_{mk} b_{mk} \tag{3.4}
\]

subject to

\[
a_{mk} \in \{0, 1\}, \forall m, k \tag{3.5}
\]

\[
\sum_{m=1}^{M} a_{mk} \leq 1, \forall k \tag{3.6}
\]

\[
P_{mk} \geq 0, \forall m, k \tag{3.7}
\]

\[
P_{\text{total}} \leq P_{\text{max}}, \tag{3.8}
\]

\[
I_{\text{total}} \leq I_{\text{th}}. \tag{3.9}
\]
where $P_{\text{max}}$ is the total secondary user power budget and $I_{th}$ is the primary user's maximum tolerable interference power. Inequality (3.6) follows from the assumption that a subcarrier can be allocated to at most one user. Inequalities (3.8) and (3.9) correspond to the power and interference constraints respectively. The nominal bit rate weight (NBRW) for user $m$ is denoted by $\lambda_m$ so that $\lambda_m / \sum_{i=1}^{M} \lambda_i$ is the fraction of the total secondary user bits loaded that is to be fairly allocated to user $m$. It is also convenient to denote the total number of bits per symbol period allocated to user $i$ by $B_i \triangleq \sum_{k=1}^{K} b_{ik}$ and define the total bit rate, $R_i$, of user $i$ as $R_i \triangleq W_s B_i$. The total bit rate for all secondary users is $R_s \triangleq \sum_{m=1}^{M} R_m$.

### 3.3 Proposed Algorithms

An optimal solution to the integer programming problem in (3.4) is computationally complex and thus not suitable for wireless communications systems in which channel conditions are constantly changing. In Section 3.3.1, a low-complexity algorithm is proposed based on the basic algorithm described in Section 3.3.2.

#### 3.3.1 Basic Algorithm

The basic algorithm (BA) is based on a greedy approach: it successively assigns bits, one at a time, to the user and subcarrier requiring the smallest incremental increase in power or producing the smallest incremental increase in interference. For convenience, we denote these two subcarriers by $k_P$ and $k_I$ respectively.

From (3.1), the incremental power required for transmitting one bit to user $m$ on subcarrier $k$ is given by

$$\Delta P_{mk} = \frac{N_0 W_s + S_{mk} 2^{b_{mk}}}{|h_{mk}|^2},$$

(3.10)
From (2.26) and (3.10), the incremental interference power generated by such a transmission to the primary user is

$$\Delta I_{mk} = \Delta P_{mk} IF_k.$$  \hspace{1cm} (3.11)

We associate the process of loading bits with a tree structure. At each node, up to two branches can be spawned corresponding to the subcarriers $k_P$ and $k_I$. At each level of the tree, one bit is allocated. The result of the algorithm is a path in the tree whose length gives the total number of bits loaded; the path also specifies the subcarrier, bit and power loading for each secondary user.

A pseudo-code description of the basic algorithm is given below.

*** Basic Algorithm (BA) ***

1. Initialization (root node)
   
   (a) Set $P = 0$, $I = 0$.
   
   (b) Set $B_m = 0$ for $m \in \{1, 2, \ldots, M\}$.
   
   (c) Set $b_{mk} = 0$ and calculate $\Delta P_{mk}$ as in (3.10) and $\Delta I_{mk}$ as in (3.11), for $m \in \{1, 2, \ldots, M\}$ and $k \in \{1, 2, \ldots, K\}$.

2. While (there are new nodes) for each node
   
   (a) Determine $m^* = \arg \min_m B_m/\lambda_m$; ties are first broken in decreasing order of $\lambda$, then randomly $^1$.
   
   (b) Determine $k_P = \arg \min_k \Delta P_{m^* k}$.
   
   (c) Determine $k_I = \arg \min_k \Delta I_{m^* k}$.
   
   (d) If $(P + \Delta P_{m^* k_P} \leq P_{\max})$ and $(I + \Delta P_{m^* k_P} IF_{k_P} \leq I_{th})$,

   create a (new) child node with parameter values obtained by updating the corresponding parameter values of the parent node as follows:

   $^1$The throughput could be improved by taking into account the gains of subcarrier and their spectrum distances to the primary user band when breaking ties.
\[ B_{m^*} = B_{m^*} + 1, \quad b_{m^*k_p} = b_{m^*k_p} + 1, \]
\[ P = P + \Delta P_{m^*k_p}, \quad I = I + \Delta I_{m^*k_p} IF_{k_p}, \]
calculate \( \Delta P_{m^*k_p} \) as in (3.10) and \( \Delta I_{m^*k_p} \) as in (3.11),
set \( \Delta P_{mk_p} = \infty, \Delta I_{mk_p} = \infty, \forall m \neq m^*. \)
Otherwise, no child node is created.

(e) If \( (I + \Delta I_{m^*k_1} \leq I_{th}) \) and \( (P + \Delta I_{m^*k_1} IF_{k_1} \leq P_{\text{max}}) \),
create a (new) child node with parameter values obtained by updating the
parameter values of the parent node as follows:
\[ B_{m^*} = B_{m^*} + 1, \quad b_{m^*k_1} = b_{m^*k_1} + 1, \]
\[ I = I + \Delta I_{m^*k_1}, \quad P = P + \Delta I_{m^*k_1} IF_{k_1}, \]
calculate \( \Delta P_{m^*k_1} \) as in (3.10) and \( \Delta I_{m^*k_1} \) as in (3.11),
set \( \Delta P_{mk_1} = \infty, \Delta I_{mk_1} = \infty, \forall m \neq m^*. \)
Otherwise, no child node is created.

(f) If at a given level of the tree, no child node is created for user \( m^* \), i.e., no
bit loading is possible given the power and interference constraints, then
set \( m^* \) to be the user with the next higher value of \( B_m/\lambda_m \) and go to step
2b). Stop if all users have been considered.

The number, \( \sum_{m=1}^M B_m \), of bits allocated to secondary users is given by the length of
the path produced by the algorithm.

In step 1) the parameter values for the root node are set. In step 2), the user \( m^* \)
who has so far received the least (normalized) service, i.e., \( m^* = \arg \min_m B_m/\lambda_m \) is
identified for bit allocation; the two subcarriers\(^2\) for that user which either requires
the least power or generates the least interference for transmitting one bit are then
determined. As long as the power and interference constraints are not violated, one
\(^2\)It is possible that these two subcarriers are the same, in which case at most one child node
would be created.
or two child nodes are created and their parameter values are obtained. If a user is not able to make use of available power resources, these are then made available to the user with the next higher value of $B_m/\lambda_m$.

The basic algorithm yields a bit loading solution which is relatively close to optimal. However, its computational complexity is $O(2^{\text{num\_bits}})$, where $\text{num\_bits}$ denotes the total number of bits loaded. We next describe an algorithm with a much lower computational complexity.

### 3.3.2 Reduced Complexity Algorithm

In the reduced complexity (RC) algorithm, the main idea is to develop a measure for the relative importance of the power needed to transmit to secondary users versus the interference power introduced to the primary user. This is then used to determine whether to select either subcarrier $k_P$ or subcarrier $k_I$ in the basic algorithm, thereby avoiding the creation of more than one child node in the tree.

A minimum power (MP) algorithm is used to determine the interference power, $I_{MP}$, introduced into the primary user's band if at each bit loading, we choose the subcarrier which minimizes the incremental power needed for the selected secondary user.

*** Algorithm MP ***

1. Step 1 - Initialization
   
   (a) Set $P = 0$, $I_{MP} = 0$.
   
   (b) Set $B_m = 0$ for $m \in \{1,2,\ldots,M\}$.
   
   (c) Set $b_{mk} = 0$ and calculate $\Delta P_{mk}$ as in (3.10), for $m \in \{1,2,\ldots,M\}$ and $k \in \{1,2,\ldots,K\}$.

2. Step 2
(a) Determine \( m^* = \arg\min_m B_m/\lambda_m \); ties are first broken in decreasing order of \( \lambda \), then randomly.

(b) Determine \( k_P = \arg\min_k \Delta P_{m^*k} \).

(c) If \( (P + \Delta P_{m^*k_P} \leq P_{max}) \), perform the following updates:

\[
B_{m^*} = B_{m^*} + 1, \quad b_{m^*k_P} = b_{m^*k_P} + 1,
\]

\[
P = P + \Delta P_{m^*k_P}, \quad I_{MP} = I_{MP} + \Delta P_{m^*k_P} \quad IF_{k_P},
\]

calculate \( \Delta P_{m^*k_P} \) as in (3.10),

set \( \Delta P_{mk_P} = \infty, \forall m \neq m^* \),

and go to step 2a).

(d) If \( (P + \Delta P_{m^*k_P} > P_{max}) \), then set \( m^* \) to be the user with the next higher value of \( B_m/\lambda_m \) and go to step b). Stop if all users have been considered.

Similarly, a minimum interference (MI) algorithm is used to determine the total power, \( P_{MI} \), required for transmitting to the secondary users if at each bit loading, we choose the subcarrier which minimizes the incremental interference power introduced into the primary user's band.

*** Algorithm MI ***

1. Step 1 - Initialization

(a) Set \( P_{MI} = 0, I = 0 \).

(b) Set \( B_m = 0 \) for \( m \in \{1, 2, \ldots, M\} \).

(c) Set \( b_{mk} = 0 \) and calculate \( \Delta I_{mk} \) as in (3.11), for \( m \in \{1, 2, \ldots, M\} \) and \( k \in \{1, 2, \ldots, K\} \).

2. Step 2

(a) Determine \( m^* = \arg\min_m B_m/\lambda_m \); ties are first broken in decreasing order of \( \lambda \), then randomly.
(b) Determine \( k_I = \arg \min_k \Delta I_{m^* k}. \)

(c) If \( I + \Delta I_{m^* k_I} \leq I_{th} \), perform the following updates:

\[
B_{m^*} = B_{m^*} + 1, \quad b_{m^* k_I} = b_{m^* k_I} + 1, \\
I = I + \Delta I_{m^* k_I}, \quad P_{MI} = P_{MI} + \Delta I_{m^* k_I}/IF_{k_I}, \\
\text{calculate } \Delta I_{m^* k_I} \text{ as in (3.11)}, \\
\text{set } \Delta I_{mk_I} = \infty, \forall m \neq m^*, \\
\text{and go to step 2a).}
\]

(d) If \( I + \Delta I_{m^* k_I} > I_{th} \), then set \( m^* \) to be the user with the next higher value of \( B_m/\lambda_m \) and go to step b). Stop if all users have been considered.

The relative importance of the secondary user power and primary user interference is measured using

\[
VP = \frac{P_{MI} - P_{max}}{P_{max}}, \quad (3.12)
\]

and

\[
VI = \frac{I_{MP} - I_{th}}{I_{th}}, \quad (3.13)
\]

respectively. Note that \( VP \) is negative when \( P_{max} > P_{MI} \) and \( VI \) is negative when \( I_{th} > I_{MP} \).

The RC algorithm uses \( VI \) and \( VP \) as follows:

*** Algorithm RC ***

1. Step 1 - Initialization

(a) Set \( P = 0, I = 0. \)

(b) Set \( B_m = 0 \) for \( m \in \{1, 2, \ldots, M\}. \)

(c) Set \( b_{mk} = 0 \) and calculate \( \Delta P_{mk} \) as in (3.10) and \( \Delta I_{mk} \) as in (3.11), for \( m \in \{1, 2, \ldots, M\} \) and \( k \in \{1, 2, \ldots, K\}. \)

2. Step 2
(a) Determine \( m^* = \arg \min_m B_m/\lambda_m \); ties are first broken in decreasing order of \( \lambda \), then randomly.

(b) Determine \( k_P = \arg \min_k \Delta P_m \).

(c) Determine \( k_I = \arg \min_k \Delta I_m \).

(d) Compute \( X = \frac{V I(\Delta P_m \Delta I_m/k_P - \Delta I_m/k_I)}{\Delta I_m/k_I} \) and \( Y = \frac{V P(\Delta I_m/k_I - \Delta P_m/k_P)}{\Delta P_m/k_P} \).

(e) If \( X \geq Y \), set \( k^* = k_I \); otherwise set \( k^* = k_P \).

(f) If \((P + \Delta P_m k^* \leq P_{\text{max}}) \) and \((I + \Delta I_m k^* \leq I_{\text{th}})\),
perform the following updates:
\[
B_m = B_m + 1, \quad b_{m,k^*} = b_{m,k^*} + 1,
\]
\[
I = I + \Delta I_m k^*, \quad P = P + \Delta P_m k^*,
\]
calculate \( \Delta P_{m,k^*} \) as in (3.10) and \( \Delta I_{m,k^*} \) as in (3.11),
set \( \Delta P_{mk^*} = \infty, \quad \Delta I_{mk^*} = \infty \), \( \forall m \neq m^* \),
and go to step 2a).

(g) If \((P + \Delta P_m k^* > P_{\text{max}}) \) or \((I + \Delta I_m k^* > I_{\text{th}})\),
then set \( m^* \) to be the user with the next higher value of \( B_m/\lambda_m \) and go to step b). Stop if all users have been considered.

The number, \( B \), of bits allocated to secondary users is given by \( \sum_{m=1}^{M} B_m \).

The complexity of the RC algorithm is \( O(\text{num.\ bits} \times K) \), where \( \text{num.\ bits} \) denotes the total number of bits loaded. This is much lower than the \( O(2^{\text{num.\ bits}}) \) complexity for the basic algorithm.

In the case when no NBRWs are specified, algorithms BA, MP, MI and RC can still be used to determine \( R_s \) with the following slight modifications. For example, in algorithm RC steps 2a) and 2g) are omitted; steps 2b), 2c), 2d) and 2e) are changed to
2b): Determine \((m_P,k_P) = \arg \min_{m,k} \Delta P_{mk}\),
2c): Determine \((m_I, k_I) = \arg \min_{m, k} \Delta I_{mk},\)

2d): Compute \(X = \frac{V(l(\Delta P_{n_p} - \Delta I_{mk}))}{\Delta I_{mk}}\) and \(Y = \frac{V(l(\Delta I_{mk} - \Delta P_{m_{pp}}))}{\Delta P_{m_{pp}}}.\)

2e): If \(X \geq Y,\) set \((m^*, k^*) = (m_I, k_I);\) otherwise set \((m^*, k^*) = (m_p, k_p).\)

### 3.4 Simulation Results

In this section, average bit rate results for the RC algorithm, obtained using computer simulations, are provided. Results for the basic algorithm are not available due to the long computational times required.

We consider a system with one primary and \(M\) secondary users. The secondary user band is 5 MHz wide and consists of 16 subcarriers, each with a bandwidth, \(W_s,\) of 0.3125 MHz. The primary user bandwidth is \(W_p = W_s\) and the symbol duration is \(T_s = 4 \mu s.\) It is assumed that the subcarrier gains \(h_{mk}\) and \(g_k,\) for \(m \in \{1, 2, \ldots, M\}, k \in \{1, 2, \ldots, K\}\) are outcomes of independent, identically distributed (i.i.d.) Rayleigh random variables (rv's) with means \(\mu_R.\) The noise is assumed to be additive white Gaussian noise (AWGN) with PSD, \(N_0.\) The PSD, \(\Phi_{RR}(e^{j\omega}),\) of the signal transmitted to the primary user is assumed to be that of an elliptically filtered white noise process. The total secondary user bit rate results are obtained by averaging over 10,000 channel realizations.

In order to evaluate the performance of the RC algorithm, an integer programming optimization software package was used. However, the long computational time allowed only 20 channel realizations to be used. It was found that among the 20 channel realizations, the total bit rate obtained using the RC algorithm was 6.4% lower than the optimal solution in the worst case. Averaged over the 20 channel realizations, the difference was 3.5%.

Fig. 3.1 shows the total bit rate, \(R_s,\) of 4 the secondary users as a function of the maximum tolerable interference power level, \(I_{th},\) of the primary user for different
nominal bit rate weights (NBRW) with $P_{max} = 1$ W, the transmit power, $P_p$, to the primary user equal to 5 W, $N_0 = 10^{-8}$ W/Hz and $\mu_R = 1$. As to be expected, it can be seen that $R_s$ increases with $I_{th}$ and that the highest bit rate is obtained when there is no NBRW are specified. For the case when there is no NBRW requirement, $R_s$ is obtained using a slight modification of the RC algorithm. As the bit rate requirements for users become less uniform, $R_s$ decreases due to an effective decrease in diversity.

![Figure 3.1](image_url)

Figure 3.1: Total bit rate, $R_s$, of 4 secondary users versus maximum tolerable interference power, $I_{th}$, of the primary user with $P_{max} = 1$ W, $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz and $\mu_R = 1$.

Fig. 3.2 shows the total bit rate, $R_s$, of the 4 secondary users as a function of the maximum tolerable interference power level, $I_{th}$, of the primary user for different values of $P_{max}$ with the transmit power, $P_p$, to the primary user set to 5 W, $N_0 = 10^{-8}$ W/Hz, $\mu_R = 1$ and identical nominal secondary user bit rate weights. As to be expected, $R_s$ increases with $P_{max}$. As $I_{th}$ decreases, the curves for different $P_{max}$ values tend to merge. The reason is that the system becomes interference limited and the power available for transmission to secondary users is no longer a limiting factor.
It can also be seen that for a given value of $P_{\text{max}}$, $R_s$ reaches a limit as $I_{th}$ increases. This is because beyond a certain $I_{th}$ value, the system is no longer limited by the interference power that the primary user can tolerate.

Figure 3.2: Total bit rate, $R_s$, of 4 secondary users versus maximum tolerable interference power, $I_{th}$, of the primary user with $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz, $\mu_R = 1$ and identical nominal secondary user bit rate weights.

Fig. 3.3 shows the total bit rate, $R_s$, of the 4 secondary users as a function of the maximum tolerable interference power level, $I_{th}$, of the primary user for different values of the transmit power, $P_p$, to the primary user with $P_{\text{max}} = 1$ W, $N_0 = 10^{-8}$ W/Hz, $\mu_R = 1$ and identical nominal secondary user bit rate weights. It can be seen that $R_s$ decreases with $P_p$. When the BS transmits at a higher power level to the primary user, the secondary users experience more interference, resulting in a decrease in $R_s$.

Fig. 3.4 shows the total bit rate, $R_s$, of the 4 secondary users as a function of the maximum tolerable interference power level, $I_{th}$, of the primary user for different values of the fading mean, $\mu_R$ with $P_{\text{max}} = 1$ W, $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz and
identical nominal secondary user bit rate weights. It can be seen that \( R_s \) increases with \( \mu_R \).

Fig. 3.5 shows the total bit rate, \( R_s \), of the 4 secondary users as a function of the maximum tolerable interference power level, \( I_{th} \), of the primary user for different values of the AWGN PSD, \( N_0 \) with \( P_{\text{max}} = 1 \text{ W} \), \( P_p = 5 \text{ W} \) and identical nominal secondary user bit rate weights. It can be seen that \( R_s \) decreases with \( N_0 \). When the noise level is higher, the secondary users experience more noise, resulting in a degradation in \( R_s \).

Fig. 3.6 shows the total bit rate, \( R_s \), of secondary users versus the maximum tolerable interference power, \( I_{th} \), of the primary user for different number, \( M \), of secondary users with \( P_{\text{max}} = 1 \text{ W} \), \( P_p = 5 \text{ W} \), \( N_0 = 10^{-8} \text{ W/Hz} \), \( \mu_R = 1 \) and identical nominal secondary user bit rate weights. It can be seen that for small values of \( I_{th} \), \( R_s \) increase with \( M \). For larger values of \( I_{th} \), \( R_s \) seems to peak for \( M = 6 \) and
Figure 3.4: Total bit rate, $R_s$, of 4 secondary users versus maximum tolerable interference power, $I_{th}$, of the primary user with $P_{max} = 1$ W, $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz and identical nominal secondary user bit rate weights.

Figure 3.5: Total bit rate, $R_s$, of 4 secondary users versus maximum tolerable interference power, $I_{th}$, of the primary user with $P_{max} = 1$ W, $P_p = 5$ W, $\mu_R = 1$ and identical nominal secondary user bit rate weights.
decreases as $M$ is increased further. This is due to the fact that one subcarrier can be used by at most one user, and fairness constraints may result in subcarriers with poor channel gains being allocated to same users. Fig. 3.7 shows that with no fairness constraints, $R_s$ increases with $M$ as a result of a larger diversity gain.

![Figure 3.6: Total bit rate, $R_s$, of secondary users versus maximum tolerable interference power, $I_{th}$, of the primary user with $P_{max} = 1$ W, $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz, $\mu_R = 1$ and identical nominal secondary user bit rate weights.]

For the next set of simulation results, the parameter values are set as follows: $I_{th} = 5 \times 10^{-3}$ W, $P_{max} = 1$ W, $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz and identical nominal secondary user bit rate weights, NBRW = 1 : 1 : 1 : 1 are used. Let $b_k$, $P_k$ and $I_k$ denote respectively the number of bits loaded on subcarrier $k$ due to the signal transmitted on subcarrier $k$.

Table 3.1 lists the average value of $b_k$, $P_k$ and $I_k$ as a function of subcarrier $k$. The average values are obtained using 10000 channel realizations with $\mu_R = 1$. The results from Table 3.1 are also plotted in Fig. 3.8 to Fig. 3.10. It can be seen that relatively more bits and power are loaded onto the subcarriers which are relatively
Figure 3.7: Total bit rate, $R_s$, of secondary users versus maximum tolerable interference power, $I_{th}$, of the primary user with $P_{\text{max}} = 1$ W, $P_p = 5$ W, $N_0 = 10^{-8}$ W/Hz, $\mu_R = 1$ and no nominal secondary user bit rate weights.

farther away from the primary user band. This is because a subcarrier which is close to the primary band introduces more interference for the same loaded power. Table 3.1 also shows that $\sum_k P_k < P_{\text{max}}$ and $\sum_k I_k < I_{th}$ as required.

Table 3.2 lists the average value of $b_k$, $P_k$ and $I_k$ as a function of subcarrier $k$ assuming constant channel power gains equal to $4/\pi$. The results from Table 3.2 are also plotted in Fig. 3.11 to Fig. 3.13. An average over 10000 allocation periods is used since ties among users with equal fairness priorities are broken by a random selection in the RC algorithm. It can be seen that total number of bits loaded is similar to that in Table 3.1 for the nonconstant user gains.

Simulation results for one sample channel realization also provide insight into the RC algorithm. The subcarrier power gains $|h_{mk}|^2$ and $|g_k|^2$ are listed in Table 3.3 and shown in Fig. 3.14. The interference factor, $IF_k$ and the interference power introduced by primary user's signal into subcarrier $k$ band at user $m$, $S_{mk}$, are computed from
Table 3.1: Average values of number of bits, $b_k$, and power, $P_k$, loaded onto subcarrier $k$ as well as interference power, $I_k$, seen by the primary user due to the signal transmitted on subcarrier $k$ over 10000 channel realizations with $\mu_R = 1$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$b_k$</th>
<th>$P_k$</th>
<th>$I_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.4503</td>
<td>8.85E-02</td>
<td>6.38E-05</td>
</tr>
<tr>
<td>2</td>
<td>4.2572</td>
<td>8.31E-02</td>
<td>6.54E-05</td>
</tr>
<tr>
<td>3</td>
<td>4.0747</td>
<td>7.80E-02</td>
<td>7.04E-05</td>
</tr>
<tr>
<td>4</td>
<td>3.7871</td>
<td>6.66E-02</td>
<td>1.06E-04</td>
</tr>
<tr>
<td>5</td>
<td>3.4739</td>
<td>5.75E-02</td>
<td>1.82E-04</td>
</tr>
<tr>
<td>6</td>
<td>3.1539</td>
<td>5.21E-02</td>
<td>2.74E-04</td>
</tr>
<tr>
<td>7</td>
<td>2.7398</td>
<td>4.68E-02</td>
<td>4.30E-04</td>
</tr>
<tr>
<td>8</td>
<td>0.3128</td>
<td>1.57E-02</td>
<td>2.94E-04</td>
</tr>
<tr>
<td>9</td>
<td>0.3109</td>
<td>1.55E-02</td>
<td>2.96E-04</td>
</tr>
<tr>
<td>10</td>
<td>2.7402</td>
<td>4.70E-02</td>
<td>4.19E-04</td>
</tr>
<tr>
<td>11</td>
<td>3.1734</td>
<td>5.29E-02</td>
<td>2.69E-04</td>
</tr>
<tr>
<td>12</td>
<td>3.4619</td>
<td>5.68E-02</td>
<td>1.84E-04</td>
</tr>
<tr>
<td>13</td>
<td>3.8073</td>
<td>6.76E-02</td>
<td>1.05E-04</td>
</tr>
<tr>
<td>14</td>
<td>4.0995</td>
<td>7.96E-02</td>
<td>7.00E-05</td>
</tr>
<tr>
<td>15</td>
<td>4.2806</td>
<td>8.48E-02</td>
<td>6.67E-05</td>
</tr>
<tr>
<td>16</td>
<td>4.4606</td>
<td>8.92E-02</td>
<td>6.36E-05</td>
</tr>
<tr>
<td>Total</td>
<td>52.5841</td>
<td>0.982</td>
<td>2.96E-03</td>
</tr>
</tbody>
</table>

Figure 3.8: Average number of bits, $b_k$, loaded per subcarrier over 10000 channel realizations with $\mu_R = 1$
Figure 3.9: Average power, $P_k$, loaded per subcarrier over 10000 channel realizations with $\mu_R = 1$

Figure 3.10: Average interference, $I_k$, per subcarrier over 10000 channel realizations with $\mu_R = 1$
Table 3.2: Average (over 10000 allocation periods) values of number of bits, $b_k$, and power, $P_k$, loaded onto subcarrier $k$ as well as interference power, $I_k$, seen by the primary user due to the signal transmitted on subcarrier $k$ for constant channel power gains, $|h_{mk}|^2$ and $|g_k|^2$, equal to $4/\pi$.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$b_k$</th>
<th>$P_k$</th>
<th>$I_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.79054</td>
<td>1.23E-01</td>
<td>1.18E-04</td>
</tr>
<tr>
<td>2</td>
<td>4.38795</td>
<td>9.62E-02</td>
<td>1.04E-04</td>
</tr>
<tr>
<td>3</td>
<td>4.15695</td>
<td>8.56E-02</td>
<td>1.01E-04</td>
</tr>
<tr>
<td>4</td>
<td>3.44091</td>
<td>5.38E-02</td>
<td>1.09E-04</td>
</tr>
<tr>
<td>5</td>
<td>3.43027</td>
<td>5.86E-02</td>
<td>2.28E-04</td>
</tr>
<tr>
<td>6</td>
<td>2.971</td>
<td>4.41E-02</td>
<td>2.71E-04</td>
</tr>
<tr>
<td>7</td>
<td>2.43961</td>
<td>3.71E-02</td>
<td>4.01E-04</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>2.44809</td>
<td>3.73E-02</td>
<td>4.01E-04</td>
</tr>
<tr>
<td>11</td>
<td>2.92612</td>
<td>4.29E-02</td>
<td>2.64E-04</td>
</tr>
<tr>
<td>12</td>
<td>3.43067</td>
<td>5.87E-02</td>
<td>2.28E-04</td>
</tr>
<tr>
<td>13</td>
<td>3.43807</td>
<td>5.37E-02</td>
<td>1.09E-04</td>
</tr>
<tr>
<td>14</td>
<td>4.15655</td>
<td>8.56E-02</td>
<td>1.01E-04</td>
</tr>
<tr>
<td>15</td>
<td>4.38348</td>
<td>9.58E-02</td>
<td>1.03E-04</td>
</tr>
<tr>
<td>16</td>
<td>4.79117</td>
<td>1.23E-01</td>
<td>1.18E-04</td>
</tr>
<tr>
<td>Total</td>
<td>51.19</td>
<td>0.9952</td>
<td>2.66E-03</td>
</tr>
</tbody>
</table>
Figure 3.11: Average number of bits, $b_k$, loaded per subcarrier over 10000 channel realizations for constant channel power gains equal to $4/\pi$.

Figure 3.12: Average power, $P_k$, loaded per subcarrier over 10000 channel realizations for constant channel power gains equal to $4/\pi$. 
Figure 3.13: Average interference, $I_k$, per subcarrier over 10000 channel realizations for constant channel power gains equal to $4/\pi$.

(2.26) and (2.27) and shown in Fig. 3.15. Table 3.4 lists RC Algorithm results of $b_k$, $P_k$ and $I_k$ for same channel realization gains as in Table 3.3 with identical NBRW and Fig. 3.16 shows bits loaded, $b_k$, by the RC algorithm for one channel realization with identical NBRW given the channel gains as in Table 3.3. It shows that the subcarriers which are relatively far from the primary user band and which have higher channel gains and lower interference powers are loaded with more bits.

Tables 3.5, 3.6 list results obtained using the RC algorithm and an integer programming (IP) software tool respectively for the same channel realization as in Table 3.3 but with no NBRW. It can be seen that the IP tool allows a total of 55 bits to be loaded compared to 53 for the RC algorithm.
Table 3.3: Power gains $|h_{mk}|^2$ and $|g_k|^2$ of subcarrier $k$

|   | $|h_{mk}|^2$ | $|g_k|^2$ |
|---|-------------|-------------|
| $k$ | $m = 1$ | $m = 2$ | $m = 3$ | $m = 4$ |
| 1  | 3.2696     | 1.1201     | 0.0285 | 0.0285 | 1.066666667 |
| 2  | 1.4643     | 0.3143     | 0.0335 | 0.0335 | 0.705882353 |
| 3  | 3.6124     | 0.4277     | 0.3962 | 0.3962 | 0.537634409 |
| 4  | 0.6835     | 0.5574     | 0.3517 | 0.3517 | 2.64516129  |
| 5  | 0.8245     | 0.2532     | 0.0348 | 0.0348 | 0.9375      |
| 6  | 1.7045     | 0.9063     | 0.4533 | 0.4533 | 2.945121951 |
| 7  | 0.1013     | 0.1931     | 0.3499 | 0.3499 | 1.705882353 |
| 8  | 0.2559     | 1.4854     | 0.3177 | 0.3177 | 2.945121951 |
| 9  | 2.7717     | 2.5351     | 1.5518 | 1.5518 | 2.221544715 |
| 10 | 0.1593     | 0.052      | 0.0216 | 0.0216 | 0.41764706  |
| 11 | 0.2714     | 0.7534     | 0.6052 | 0.6052 | 0.354166667 |
| 12 | 0.3617     | 0.5237     | 0.2586 | 0.2586 | 1.419354839 |
| 13 | 2.4509     | 1.2138     | 0.4888 | 0.4888 | 3.125       |
| 14 | 1.0655     | 0.3891     | 3.3885 | 3.3885 | 0.860215054 |
| 15 | 1.1155     | 0.4223     | 0.8137 | 0.8137 | 1.058823529 |
| 16 | 0.4893     | 1.7369     | 2.3656 | 2.3656 | 1.2         |

Figure 3.14: Power gains $|h_{mk}|^2$ and $|g_k|^2$ of subcarrier $k$
Figur 3.15: Interference factor, $IF_k$ and the interference power introduced by primary user's signal into subcarrier $k$ band at user $m$, $S_{mk}$

Table 3.4: RC Algorithm results of $b_k$, $P_k$ and $I_k$ for same channel realization gains as in Table 3.3 with identical NBRW

<table>
<thead>
<tr>
<th>$k$</th>
<th>user #</th>
<th>$b_k$</th>
<th>$P_k$</th>
<th>$I_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>4</td>
<td>4.16E-02</td>
<td>3.32E-05</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>4</td>
<td>6.28E-02</td>
<td>3.77E-05</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>5</td>
<td>9.81E-02</td>
<td>4.91E-05</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3.28E-02</td>
<td>4.93E-05</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
<td>2.08E-02</td>
<td>1.71E-04</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>4</td>
<td>1.11E-01</td>
<td>1.33E-04</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>2</td>
<td>4.28E-02</td>
<td>6.21E-04</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>1</td>
<td>1.01E-02</td>
<td>3.54E-05</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>4</td>
<td>1.37E-01</td>
<td>2.32E-04</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>2</td>
<td>1.15E-02</td>
<td>5.07E-05</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>4</td>
<td>7.84E-02</td>
<td>3.92E-04</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>6</td>
<td>2.03E-01</td>
<td>1.62E-04</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>3</td>
<td>3.39E-02</td>
<td>3.05E-05</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>5</td>
<td>1.12E-01</td>
<td>1.00E-04</td>
</tr>
<tr>
<td>Total</td>
<td>4</td>
<td>49</td>
<td>0.995</td>
<td>0.00210</td>
</tr>
</tbody>
</table>
Figure 3.16: RC Algorithm results of number of bits, $b_k$, loaded for same channel realization gains as in Table 3.3 with identical NBRW

Table 3.5: RC Algorithm results of $b_k$, $P_k$ and $I_k$ for same channel realization gains as in Table 3.3 with no NBRW

<table>
<thead>
<tr>
<th>$k$</th>
<th>user #</th>
<th>$b_k$</th>
<th>$P_k$</th>
<th>$I_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>8.54E-02</td>
<td>6.83E-05</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>5</td>
<td>1.30E-01</td>
<td>7.78E-05</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>6</td>
<td>1.99E-01</td>
<td>9.97E-05</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3.28E-02</td>
<td>4.93E-05</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>3</td>
<td>3.31E-02</td>
<td>2.72E-04</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>3</td>
<td>4.05E-02</td>
<td>4.86E-05</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>2</td>
<td>2.39E-02</td>
<td>3.47E-04</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>2</td>
<td>3.04E-02</td>
<td>1.06E-04</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>2</td>
<td>2.43E-02</td>
<td>4.13E-05</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>4</td>
<td>5.76E-02</td>
<td>2.53E-04</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>4</td>
<td>5.89E-02</td>
<td>2.94E-04</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>5</td>
<td>9.99E-02</td>
<td>7.99E-05</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>4</td>
<td>7.26E-02</td>
<td>6.54E-05</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>5</td>
<td>9.68E-02</td>
<td>8.71E-05</td>
</tr>
<tr>
<td>Total</td>
<td>4</td>
<td>53</td>
<td>0.985</td>
<td>0.00189</td>
</tr>
</tbody>
</table>
Table 3.6: Optimization software results of $b_k$, $P_k$ and $I_k$ for same channel realization gains as in Table 3.3 with no NBRW

<table>
<thead>
<tr>
<th>$k$</th>
<th>user #</th>
<th>$b_k$</th>
<th>$P_k$</th>
<th>$I_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>5</td>
<td>8.54E-02</td>
<td>6.83E-05</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>5</td>
<td>1.30E-01</td>
<td>7.78E-05</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>4</td>
<td>4.75E-02</td>
<td>2.37E-05</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>3.05E-02</td>
<td>4.58E-05</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>3</td>
<td>3.31E-02</td>
<td>2.72E-04</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>4</td>
<td>8.93E-02</td>
<td>1.07E-04</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>3</td>
<td>5.59E-02</td>
<td>8.10E-04</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>3</td>
<td>7.08E-02</td>
<td>2.48E-04</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>3</td>
<td>6.38E-02</td>
<td>1.08E-04</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>4</td>
<td>5.76E-02</td>
<td>2.53E-04</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>4</td>
<td>5.89E-02</td>
<td>2.94E-04</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>5</td>
<td>9.99E-02</td>
<td>7.99E-05</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>4</td>
<td>7.26E-02</td>
<td>6.54E-05</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>5</td>
<td>9.68E-02</td>
<td>8.71E-05</td>
</tr>
<tr>
<td>Total</td>
<td>4</td>
<td>55</td>
<td>0.992</td>
<td>0.00254</td>
</tr>
</tbody>
</table>
Chapter 4

Cost Minimization Resource Allocation

4.1 Introduction

In Chapter 3, we discussed fair resource allocation for applications which do not require a prescribed bit transmission rate in a multiuser OFDM based CR system. However, certain applications such as Voice over Internet Protocol (VoIP) and multimedia video require a minimum bit rate guarantees for users admitted into the system. In this chapter we study resource allocation for such applications in a multiuser OFDM based CR system.

In [8], the authors consider OFDM transmission in a multiuser environment and formulate a problem of minimizing the overall transmit power by adaptively assigning subcarriers to the users along with the number of bits and power level for each subcarrier. An algorithm is proposed which finds a suboptimal solution by allowing the number of loaded bits to be a real number. Since two Lagrange multipliers have to be found at same time, the complexity of the algorithm is $O(K^4)$ where $K$ is number of subcarriers. The algorithms proposed in [9,14] model the problem of minimizing
the total power as a nonlinear optimization problem with integer variable constraint. In [11, 12], a minimization of the total power is attempted by approximately allocating equal power to each subcarrier. In [15], an algorithm based on greedy single user subcarrier and bit allocation algorithm to allocate the subcarriers and number of bits to each user. If a subcarrier is desired by more than one user, the algorithm assigns the subcarrier to a user appropriately so that total transmit power is minimized. In [18], an algorithm is proposed based on the concept of marginal utility to solve the problem with a better performance than [15] at approximately the same computational complexity.

The above-mentioned algorithms can be applied in conventional multiuser OFDM systems with only one type of users (e.g., secondary users) as they do not consider mutual interference which may arise between primary and secondary users. In contrast, the mutual interference is considered in [21] for the case of one secondary user and the throughput of the secondary user is maximized subject to a maximum interference power that can be tolerated by the primary user. In this chapter, a resource allocation algorithm is proposed which minimizes a cost function which takes into account interference experienced by a primary user as well as power required to transmit to secondary users in a multiuser OFDM-based CR system with secondary user bit rate constraints. The algorithm iteratively allocates subcarriers in such a way as to reduce cost.

4.2 System Model

The system model is identical to that described in Section 3.2. Recall that the total interference power seen by the primary user due to the signals destined for secondary users is $I_{\text{total}}$ (see Eq. (3.2)) and the total transmit power for signals destined to secondary users is $P_{\text{total}}$ (see Eq. (3.3)).
Our objective is to minimize a weighted sum of interference and power subject to a minimum required bit rate for each secondary user. More specifically, the optimization problem is expressed as follows:

\[
\min_{b_{mk}, P_{mk}} C_{\text{total}}(\alpha) \triangleq [\alpha I_{\text{total}} + (1 - \alpha) P_{\text{total}}]
\]

subject to

\[
a_{mk} \in \{0, 1\}, \forall m, k \tag{4.2}
\]

\[
\sum_{m=1}^{M} a_{mk} \leq 1, \forall k \tag{4.3}
\]

\[
b_{mk} \in \mathbb{Z}, \forall m, k \tag{4.4}
\]

\[
P_{mk} \geq 0, \forall m, k \tag{4.5}
\]

\[
\sum_{k=1}^{K} a_{mk} b_{mk} \geq B_{m}^{\text{req}}, \forall m \tag{4.6}
\]

where \(B_{m}^{\text{req}}\) denotes the minimum required number of bits per symbol period for user \(m\). It is also convenient to define the minimum required bit rate, \(R_{m}^{\text{req}}\), of user \(m\) as \(R_{m}^{\text{req}} \triangleq W_{m} B_{m}^{\text{req}}\). The total minimum bit rate for all secondary users is \(R_{s}^{\text{req}} \triangleq \sum_{m=1}^{M} R_{m}^{\text{req}}\). Inequality (4.3) follows from the assumption that a subcarrier can be allocated to at most one user, whereas (4.6) guarantees a minimum bit rate for each secondary user. The term \(\alpha \in [0, 1]\) is a parameter which indicates the relative importance of power and interference. Increasing \(\alpha\) implies that interference reduction is relatively more important than power savings.

### 4.3 Proposed Algorithm

An optimal solution to the optimization problem in (4.1) is computationally complex and may not suitable for wireless communications systems in which channel conditions are constantly changing. In this section, a suboptimal algorithm with reduced computational load is proposed. Its performance is discussed in Section 4.4.
4.3.1 Maximal Cost Reduction by a New Subcarrier

In [18], the marginal utility of a new subcarrier $j$ for user $m$ refers to the maximal power reduction possible if subcarrier $j$ is assigned to user $m$. We extend this concept and define the marginal cost reduction as the maximal reduction in $C_{\text{total}}(\alpha)$ possible if subcarrier $j$ is assigned to user $m$.

From (3.1), the power savings gained by reducing one bit on subcarrier $k$ for user $m$ is given by

$$\Delta P_{mk}^- = \frac{N_0 W_s + S_{mk} \gamma_{mk}^{-1}}{|h_{mk}|^2},$$

(4.7)

and from (2.26), the resulting interference power reduction to the primary user is

$$\Delta I_{mk}^- = \Delta P_{mk}^- IF_k.$$  

(4.8)

From (3.1), the increase in BS transmit power needed to support one additional bit on subcarrier $k$ for user $m$ is given by

$$\Delta P_{mk}^+ = \frac{N_0 W_s + S_{mk} \gamma_{mk}^+}{|h_{mk}|^2},$$

(4.9)

and from (2.26), the resulting increase in interference power to the primary user is

$$\Delta I_{mk}^+ = \Delta P_{mk}^+ IF_k.$$  

(4.10)

Let $L_m$ denote the number of subcarriers already allocated to user $m$. Let $b_{ml}$, $P_{ml}$ and $I_{ml}$ denote the current number of bits, transmit power and interference power for the $l^{th}$ ($1 \leq l \leq L_m$) subcarrier of user $m$ respectively. The proposed algorithm reassigns bits of user $m$ to a (new) available subcarrier only if such a reassignment results in a reduction in the cost $C_{\text{total}}(\alpha)$.

The marginal cost reduction achievable by transferring bits from existing subcarriers of user $m$ to an available (unused) subcarrier $j$ can be calculated as follows:

*** Algorithm I ***
1. Step 1 - Initialization

(a) For each subcarrier $l_m (1 \leq l \leq L_m)$ currently used by user $m$, calculate the power reduction possible $\Delta P_{ml}^-$ and $\Delta I_{ml}^-$ while reducing one bit from subcarrier $l$ as in (4.7) and (4.8) respectively.

The cost reduction is $\Delta C_{ml}^- = \alpha \Delta I_{ml}^- + (1 - \alpha) \Delta P_{ml}^-$. 

(b) Set $b_{mj} = 0$, $\Delta C_{total}^{mj} = 0$, calculate $\Delta P_{mj}^+$ and $\Delta I_{mj}^+$ for increasing one bit on the subcarrier $j$ of user $m$ as in (4.9) and (4.10) respectively.

The cost is $\Delta C_{mj}^+ = \alpha \Delta I_{mj}^+ + (1 - \alpha) \Delta P_{mj}^+$. 

2. Step 2

(a) Determine $l^* = \arg \max_l \Delta C_{ml}^-$ for $l \in \{1, 2, \ldots, L_m\}$. 

(b) If $\Delta C_{ml}^* \leq \Delta C_{mj}^+$, go to step 3. Otherwise, transfer one bit from subcarrier $l^*$ of user $m$ to subcarrier $j$ (which is available) and by performing following updates:

$$b_{ml^*} = b_{ml^*} - 1, \quad b_{mj} = b_{mj} + 1,$$

$$\Delta C_{total}^{mj} = \Delta C_{total}^{mj} + (\Delta C_{ml^*}^- - \Delta C_{mj}^+),$$

calculate $\Delta P_{ml^*}^-$ as in (4.7) and $\Delta I_{ml^*}^-$ as in (4.8),

set $\Delta C_{ml^*}^- = \alpha \Delta I_{ml^*}^- + (1 - \alpha) \Delta P_{ml^*}^-$,

calculate $\Delta P_{mj}^+$ as in (4.9) and $\Delta I_{mj}^+$ as in (4.10),

set $\Delta C_{mj}^+ = \alpha \Delta I_{mj}^+ + (1 - \alpha) \Delta P_{mj}^+$,

and go to step 2a).

3. Step 3

The marginal cost reduction of subcarrier $j$ to user $m$ is $\Delta C_{total}^{mj}$ and $b_{ml}$ is the loaded bits on subcarrier $l$ of user $m$, for $l \in \{1, 2, \ldots, L_m + 1\}$. 

49
4.3.2 Proposed Resource Allocation Algorithm

The proposed resource allocation algorithm is based on the marginal cost reduction of a subcarrier. The goal is to minimize the total cost $C_{\text{total}}(\alpha)$ while satisfying the minimum bit rate requirements of the secondary users. Towards this end, a subcarrier with largest marginal cost reduction is allocated to the corresponding user until there is no further cost reduction possible.

*** Algorithm II ***

1. If $M > K$, declare no feasible solution and stop.
   Otherwise go on.

2. Initialization
   (a) Set $b_{mk} = 0$, for $m \in \{1, 2, \ldots, M\}$ and $k \in \{1, 2, \ldots, K\}$.
   (b) Set $U = \{1, \ldots, K\}$.
   (c) $V_{mk} = \frac{(N_0W_S + S_{mk})kF_k}{|P_{mk}|^2} + (1 - \alpha)\frac{(N_0W_S + S_{mk})}{|P_{mk}|^2}$, for $m \in \{1, 2, \ldots, M\}$ and $k \in \{1, 2, \ldots, K\}$.

3. While $\min_{mk} V_{mk} < \infty$
   (a) Determine $(m^*, k^*) = \arg \min_{mk} V_{mk}$.
   (b) Set $b_{m^*k^*} = B_{m^*}^{\text{req}}$,
      $$U = U - \{k^*\}.$$  
   (c) Set $V_{m^*k} = \infty$, for $k \in \{1, 2, \ldots, K\}$,
      $V_{mk^*} = \infty$, for $m \in \{1, 2, \ldots, M\}$.

4. Set $\Delta C_{\text{total}}^{mk} = 0, \forall m, k \notin U$.
   Run Algorithm I to compute $\Delta C_{\text{total}}^{mk}, \forall m, k \in U$. 

50
5. While \( \max_{mk} \Delta C_{mk}^{total} > 0 \)

(a) Determine \((m^*, k^*) = \arg \max_{mk} \Delta C_{mk}^{total}\), for \(m \in \{1, 2, \ldots, M\}\) and \(k \in \{1, 2, \ldots, K\}\).

(b) Run Algorithm I to update \(b_{m^* k^*}\), for \(k \in U\).

(c) Set \(U = U - \{k^*\}\), \(\Delta C_{total}^{mk^*} = 0\), for \(m \in \{1, 2, \ldots, M\}\).

(d) Run Algorithm I to compute \(\Delta C_{mk}^{total}\), for \(m \in \{1, 2, \ldots, M\}\) and \(k \in U\).

6. Compute \(I_{total}, P_{total}\) as in (3.3) and (3.2),

compute \(\alpha I_{total} + (1 - \alpha) P_{total}\).

In the algorithm, \(U\) represents the set of currently available (unused) subcarriers
and \(V_{mk}\) is the cost if one bit is loaded for user \(m\) on a currently empty subcarrier
\(k\). In step 3), exactly one subcarrier is assigned to each user with a non-zero bit rate
requirement and all the bits of any given user are loaded onto its assigned subcarrier.
In step 5), a (user, unassigned subcarrier) pair with maximal cost reduction is selected
and some of the bits loaded onto currently assigned subcarriers of the user will be
redistributed using Algorithm I. This step is repeated until no further reduction in
\(C_{total}(\alpha)\) is possible.

4.4 Simulation Results

In this section, average total cost, \(C_{total}(\alpha)\), results for the proposed algorithm, ob­
tained using computer simulations, are provided. The performance of the algorithm
relative to the optimal solution is also discussed.

We consider a system with one primary and \(M\) secondary users. The secondary
users have identical minimum bit rate weight (BRW) requirements, i.e., \(R_{m}^{req} =
R_{s}^{req}/M\), unless otherwise specified. The secondary user band is 5 MHz wide and
consists of 16 subcarriers, each with a bandwidth, $W_s$, of 0.3125 MHz. The primary user bandwidth is $W_p = W_s$ and the symbol duration is $T_s = 4 \mu s$. It is assumed that the subcarrier gains $h_{mk}$ and $g_k$, for $m \in \{1,2,\ldots,M\}, k \in \{1,2,\ldots,K\}$ are outcomes of independent, identically distributed (i.i.d.) Rayleigh random variables (rv’s) with means equal to 1. The additive white Gaussian noise (AWGN) PSD, $N_0$, is set to $10^{-8}$ W/Hz. The PSD, $\Phi_{RR}(e^{j\omega})$, of the signal transmitted to the primary user is assumed to be that of an elliptically filtered white noise process. The $C_{total}(\alpha)$ results are obtained by averaging over 10,000 channel realizations.

In Fig. 4.1, the average cost function $C_{total}(\alpha)$ value obtained using the proposed algorithm is plotted as a function of the total minimum bit rate, $R_{s}^{req}$, requirement for 4 secondary users with $\alpha = 250/251$. As expected, it can be seen that $C_{total}(\alpha)$ increases with $R_{s}^{req}$.

In order to assess the performance of the proposed algorithm, an integer programming optimization software package was used to solve the problem in (4.1). However, the long computational time allowed only 10 channel realizations to be used. It was found that among the 10 channel realizations, the total cost obtained using the proposed algorithm was 3.5% higher than the optimal solution in the worst case. Averaged over the 10 channel realizations, the difference was 2.2%.

Fig. 4.2 shows the average cost function $C_{total}(\alpha)$ value as a function of the total minimum bit rate, $R_{s}^{req}$, requirement for 4 secondary users with $\alpha = 250/251$ and different BRW for secondary users. As to be expected, it can be seen that $C_{total}$ increases with $R_{s}^{req}$ and that the lowest cost is obtained when BRW is uniform. As the bit rate requirements for users become less uniform, $C_{total}$ increases due to an effective decrease in diversity.

Fig. 4.3 shows the BS transmit power, $P_{total}$, for secondary users and the interference power, $I_{total}$, experienced by the primary user as a function of $\alpha$ assuming that...
Figure 4.1: Total system cost, $C_{total}$, versus total required transmit bit rate, $R_{req}$, of 4 secondary users with uniform BRW and $\alpha = 250/251$.

Figure 4.2: Total system cost, $C_{total}$, versus total required transmit bit rate, $R_{req}$, of 4 secondary users with different BRW and $\alpha = 250/251$. 
the total bit rate, $R_{s}^{eq}$, required by 4 secondary users is 18.75 Mbps. As $\alpha$ increases, so does $P_{total}$ whereas $I_{total}$ decreases.

Figure 4.3: Total power required, $P_{total}$, by secondary users and interference, $I_{total}$, introduced to primary user versus $\alpha$ value with the total required bit rate for 4 secondary user $R_{s}^{eq} = 18.75$ Mbps and uniform BRW.

For the following simulations results, a total minimum bit rate, $R_{s}^{eq} = 18.75$ Mbps, requirement for 4 secondary users with $\alpha = 250/251$ and identical minimum bit rate weight (BRW) requirements are assumed. Let $b_k$, $P_k$, $I_k$ and $C_k$ denote the number of bits loaded, required transmit power for secondary user, interference power experienced by the primary user due to secondary user signal and cost for subcarrier $k$ respectively.

Table 4.1 and Figs. 4.4 to 4.7 show that average value of $b_k$, $P_k$, $I_k$ and $C_k$ over 10000 channel realizations. As expected, the average number of bits, $b_k$, loaded and the average power, $P_k$, on subcarrier $k$, increase with the spectrum distance to the primary user band. The reason is that a subcarrier which is far away from the primary user band will introduce less interference to the primary user than one closer to the
primary user band if both subcarriers have equal allocated power.

Table 4.1: Average values of number of bits, $b_k$, and power, $P_k$, loaded onto subcarrier $k$ as well as interference power, $I_k$, seen by the primary user due to the signal transmitted on subcarrier $k$ and cost, $C_k$, on subcarrier $k$ over 10000 channel realizations with $\mu_R = 1$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$b_k$</th>
<th>$P_k$</th>
<th>$I_k$</th>
<th>$C_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.2196</td>
<td>1.38E-01</td>
<td>1.12E-04</td>
<td>6.60E-04</td>
</tr>
<tr>
<td>2</td>
<td>5.0489</td>
<td>1.34E-01</td>
<td>1.19E-04</td>
<td>6.53E-04</td>
</tr>
<tr>
<td>3</td>
<td>4.8841</td>
<td>1.31E-01</td>
<td>1.27E-04</td>
<td>6.46E-04</td>
</tr>
<tr>
<td>4</td>
<td>4.5499</td>
<td>1.14E-01</td>
<td>1.76E-04</td>
<td>6.30E-04</td>
</tr>
<tr>
<td>5</td>
<td>4.0237</td>
<td>9.25E-02</td>
<td>2.41E-04</td>
<td>6.08E-04</td>
</tr>
<tr>
<td>6</td>
<td>3.4717</td>
<td>7.56E-02</td>
<td>2.72E-04</td>
<td>5.72E-04</td>
</tr>
<tr>
<td>7</td>
<td>2.6925</td>
<td>5.49E-02</td>
<td>2.92E-04</td>
<td>5.09E-04</td>
</tr>
<tr>
<td>8</td>
<td>0.1193</td>
<td>6.31E-03</td>
<td>2.05E-05</td>
<td>4.55E-05</td>
</tr>
<tr>
<td>9</td>
<td>0.1083</td>
<td>5.80E-03</td>
<td>1.83E-05</td>
<td>4.14E-05</td>
</tr>
<tr>
<td>10</td>
<td>2.6841</td>
<td>5.45E-02</td>
<td>2.93E-04</td>
<td>5.09E-04</td>
</tr>
<tr>
<td>11</td>
<td>3.4752</td>
<td>7.60E-02</td>
<td>2.74E-04</td>
<td>5.75E-04</td>
</tr>
<tr>
<td>12</td>
<td>4.0055</td>
<td>9.16E-02</td>
<td>2.42E-04</td>
<td>6.06E-04</td>
</tr>
<tr>
<td>13</td>
<td>4.5544</td>
<td>1.15E-01</td>
<td>1.76E-04</td>
<td>6.34E-04</td>
</tr>
<tr>
<td>14</td>
<td>4.8942</td>
<td>1.30E-01</td>
<td>1.28E-04</td>
<td>6.47E-04</td>
</tr>
<tr>
<td>15</td>
<td>5.0521</td>
<td>1.34E-01</td>
<td>1.21E-04</td>
<td>6.55E-04</td>
</tr>
<tr>
<td>16</td>
<td>5.2165</td>
<td>1.38E-01</td>
<td>1.11E-04</td>
<td>6.59E-04</td>
</tr>
<tr>
<td>Total</td>
<td>60</td>
<td>1.491</td>
<td>0.00272</td>
<td>0.00865</td>
</tr>
</tbody>
</table>

Table 4.2 and Figs. 4.8 to 4.11 shows the number of loaded bits, $b_k$, for a constant channel gain case. As expected, $b_k$ increase with the spectrum distance to the primary user band. Since there is no diversity of channel gains, the allocation for subcarriers will depend on the spectrum distance to the primary user band. Note that Fig. 4.8 shows that subcarriers 13 and 14 carrying 5 bits each; since subcarrier 13 is closer to the primary user band than subcarrier 14, the power on subcarrier 13 is higher than that of subcarrier 14 as can be seen in Fig. 4.9.

Simulation results for one sample channel realization also provide insight into the cost minimization algorithm. The subcarrier power gains $|h_{mk}|^2$ and $|g_k|^2$ are listed in Table 4.3 and shown in Fig. 4.12. The interference factor, $IF_k$ and the interference
Figure 4.4: Average number of bits, $b_k$, loaded per subcarrier over 10000 channel realizations with $\mu_R = 1$

Figure 4.5: Average power, $P_k$, loaded per subcarrier over 10000 channel realizations with $\mu_R = 1$
Figure 4.6: Average interference, $I_k$, per subcarrier over 10000 channel realizations with $\mu_R = 1$

Figure 4.7: Average cost, $C_k$, per subcarrier over 10000 channel realizations with $\mu_R = 1$. 

57
Table 4.2: Values of number of bits, $b_k$, and power, $P_k$, loaded onto subcarrier $k$ as well as interference power, $I_k$, seen by the primary user due to the signal transmitted on subcarrier $k$ for constant channel power gains, $|h_{mk}|^2$ and $|g_k|^2$, equal to $4/\pi$.

<table>
<thead>
<tr>
<th>$k$</th>
<th>user#</th>
<th>$b_k$</th>
<th>$P_k$</th>
<th>$I_k$</th>
<th>$C_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>6</td>
<td>2.68E-01</td>
<td>2.56E-04</td>
<td>1.32E-03</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>5</td>
<td>1.40E-01</td>
<td>1.51E-04</td>
<td>7.06E-04</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>5</td>
<td>1.47E-01</td>
<td>1.74E-04</td>
<td>7.60E-04</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>5</td>
<td>1.58E-01</td>
<td>3.21E-04</td>
<td>9.50E-04</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>4</td>
<td>8.41E-02</td>
<td>3.27E-04</td>
<td>6.61E-04</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>3</td>
<td>4.48E-02</td>
<td>2.76E-04</td>
<td>4.53E-04</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>2</td>
<td>2.34E-02</td>
<td>2.53E-04</td>
<td>3.45E-04</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>2</td>
<td>2.34E-02</td>
<td>2.53E-04</td>
<td>3.45E-04</td>
</tr>
<tr>
<td>11</td>
<td>4</td>
<td>3</td>
<td>4.48E-02</td>
<td>2.76E-04</td>
<td>4.53E-04</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>4</td>
<td>8.41E-02</td>
<td>3.27E-04</td>
<td>6.61E-04</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>5</td>
<td>1.58E-01</td>
<td>3.21E-04</td>
<td>9.50E-04</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>5</td>
<td>1.47E-01</td>
<td>1.74E-04</td>
<td>7.60E-04</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>5</td>
<td>1.40E-01</td>
<td>1.51E-04</td>
<td>7.06E-04</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>6</td>
<td>2.68E-01</td>
<td>2.56E-04</td>
<td>1.32E-03</td>
</tr>
<tr>
<td>Total</td>
<td>4</td>
<td>60</td>
<td>1.731</td>
<td>0.00352</td>
<td>0.0104</td>
</tr>
</tbody>
</table>

Figure 4.8: Number of bits, $b_k$, loaded per subcarrier for constant channel power gains equal to $4/\pi$.  

58
Figure 4.9: Power, $P_k$, loaded per subcarrier for constant channel power gains equal to $4/\pi$.

Figure 4.10: Interference, $I_k$, per subcarrier for constant channel power gains equal to $4/\pi$. 
Figure 4.11: Cost, $C_k$, per subcarrier for constant channel power gains equal to $4/\pi$.

Power introduced by primary user’s signal into subcarrier $k$ band at user $m$, $S_{mk}$, are shown in Fig. 4.13. Table 4.4 lists cost minimization algorithm results of $b_k$, $P_k$, $I_k$ and $C_k$ for same channel realization gains as in Table 4.3 and Fig. 4.14 shows the number of bits, $b_k$, loaded for same channel realization given in Table 4.3. It can be observed that more bits are allocated to subcarriers with higher channel gains and more distance to the primary user band.
Table 4.3: Power gains $|h_{mk}|^2$ and $|g_k|^2$ of subcarrier $k$

| $k$ | $m = 1$ | $m = 2$ | $m = 3$ | $m = 4$ | $|g_k|^2$ |
|-----|--------|--------|--------|--------|--------|
| 1   | 0.7978 | 1.0967 | 1.105  | 1.9659 | 1.733333333 |
| 2   | 0.3041 | 1.5026 | 1.4111 | 4.8599 | 0.470588235 |
| 3   | 0.6567 | 6.792  | 0.4001 | 1.908  | 0.967741935 |
| 4   | 0.5636 | 0.5215 | 0.6929 | 0.0849 | 0.1875 |
| 5   | 1.1499 | 1.2774 | 0.3847 | 0.0576 | 5.258064516 |
| 6   | 0.6636 | 0.2516 | 1.3921 | 0.7334 | 0.583333333 |
| 7   | 0.6688 | 1.5011 | 0.479  | 0.4795 | 1.376470588 |
| 8   | 1.4466 | 2.7116 | 0.0526 | 1.6703 | 4.776422764 |
| 9   | 0.2655 | 0.9286 | 3.4747 | 1.2998 | 0.193089431 |
| 10  | 2.1545 | 0.8001 | 0.0334 | 0.9012 | 2.552941176 |
| 11  | 1.991  | 2.6844 | 0.6031 | 1.2403 | 0.041666667 |
| 12  | 1.215  | 0.5384 | 0.9744 | 0.3345 | 1.09674194 |
| 13  | 2.2001 | 1.2498 | 0.1159 | 1.8746 | 0.3125 |
| 14  | 2.2413 | 1.8125 | 0.8786 | 0.1137 | 0.752688172 |
| 15  | 2.4599 | 0.0369 | 1.5414 | 0.316  | 2.235294118 |
| 16  | 0.5244 | 1.4083 | 0.87   | 1.7632 | 0.666666667 |

Figure 4.12: Power gains $|h_{mk}|^2$ and $|g_k|^2$ of subcarrier $k$
Figure 4.13: Interference factor, $IF_k$ and the interference power introduced by primary user's signal into subcarrier $k$ band at user $m$, $S_{mk}$

Table 4.4: Cost Minimization algorithm result of $b_k$, $P_k$, $I_k$ and $C_k$ for same channel realization gains as in Table 4.3

<table>
<thead>
<tr>
<th>$k$</th>
<th>user #</th>
<th>$b_k$</th>
<th>$P_k$</th>
<th>$I_k$</th>
<th>$C_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>5</td>
<td>1.05E-01</td>
<td>1.37E-04</td>
<td>5.55E-04</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>6</td>
<td>1.70E-01</td>
<td>6.79E-05</td>
<td>7.44E-04</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>6</td>
<td>1.74E-01</td>
<td>1.56E-04</td>
<td>8.49E-04</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>5</td>
<td>2.22E-01</td>
<td>6.66E-05</td>
<td>9.51E-04</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>1.68E-02</td>
<td>2.74E-04</td>
<td>3.39E-04</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2</td>
<td>4.91E-02</td>
<td>1.38E-04</td>
<td>3.33E-04</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>2</td>
<td>3.56E-02</td>
<td>4.17E-04</td>
<td>5.57E-04</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>2</td>
<td>2.65E-02</td>
<td>5.74E-04</td>
<td>6.77E-04</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>5</td>
<td>1.59E-01</td>
<td>3.17E-05</td>
<td>6.63E-04</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>4</td>
<td>8.58E-02</td>
<td>2.92E-04</td>
<td>6.33E-04</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>5</td>
<td>1.26E-01</td>
<td>6.31E-05</td>
<td>5.66E-04</td>
</tr>
<tr>
<td>14</td>
<td>1</td>
<td>6</td>
<td>2.33E-01</td>
<td>1.63E-04</td>
<td>1.09E-03</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>5</td>
<td>1.26E-01</td>
<td>2.40E-04</td>
<td>7.43E-04</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>5</td>
<td>1.67E-01</td>
<td>8.36E-05</td>
<td>7.49E-04</td>
</tr>
<tr>
<td>Total</td>
<td>4</td>
<td>60</td>
<td>1.695</td>
<td>0.00270</td>
<td>0.00945</td>
</tr>
</tbody>
</table>
Figure 4.14: Cost Minimization Algorithm results of number of bits, $b_k$, loaded for same channel realization gains as in Table 4.3
Chapter 5

Conclusions and Suggestions for Future Work

The problem of dynamic loading of subcarriers, power, and bits in a multiuser OFDM cognitive radio system was studied. In this chapter, we summarize the main contributions of this thesis and provide some suggestions for further study.

5.1 Contributions of the Thesis

1. New optimization models for fair adaptive resource allocation and minimum cost resource allocation for multiuser OFDM CR systems were formulated.

2. A low complexity (RC) algorithm for allocating resources on the downlink of a multiuser OFDM CR system in which there is no minimum user bit rate requirements was proposed. The objective is to maximize the total bit rate, $R_s$, of secondary users given constraints on the total power available to the BS for transmission to secondary users and on the interference that the primary user can tolerate. The algorithm is fair in the sense that it tries to allocate bits to users who have not received their fair share of service as much as possible. It is
found that the algorithm has a much lower computational complexity than an optimal algorithm but still provides a $R_s$ performance which is close to optimal. The effect of changing various system parameter values on the performance of the algorithm was also studied.

3. A sub-optimal algorithm for allocating resources on the downlink of a multiuser OFDM CR system in which there is a set of minimum user bit rate requirements was proposed. The objective is to minimize a cost function which takes into account the interference power, $I_{total}$, experienced by the primary user as well as the base station transmit power, $P_{total}$, for secondary users, given minimum bit rate requirements for each secondary user. It is found that the proposed algorithm provides a performance which is fairly close to optimal. The influence of the relative weight parameter, $\alpha$, on $I_{total}$ and $P_{total}$ was also discussed.

5.2 Future work

- The two algorithms proposed in this thesis are based on the assumptions that the wireless channel does not change much during each resource allocation time and perfect channel information is available. In practice, channel estimation errors are inevitable. In corporation of imperfect channel estimation and/or fast fading would represent an interesting extension to the present work.

- In this thesis we proposed a resource allocation algorithm for a multiuser OFDM CR system in which there is no minimum user bit rate requirements and one for a multiuser OFDM CR system in which there is a set of minimum user bit rate requirements. In future wireless communication system, many different types of applicants will be supported. The design of resource allocation algorithms in such systems is an important issue.
• It is widely recognized that multiple-input multiple-output (MIMO) antenna architectures can increase the spectral efficiency of wireless communication systems [29]. OFDM can also be combined with MIMO antenna at the transmitter and receiver to increase the diversity gain and to enhance the system capacity [30]. It would be useful to study the benefits of MIMO in the design of adaptive resource allocation algorithms for OFDM based CR systems.
Bibliography


