CONTRACT NEGOTIATION, INCOMPLETE
CONTRACTING, AND ASYMMETRIC INFORMATION
(ESSAYS IN MANAGERIAL ACCOUNTING RESEARCH)
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#### Abstract

This thesis contributes to the managerial accounting research literature. The methodology used is basically analytical modelling. Part I focuses on voluntary financial accounting disclosure. Following a detailed survey of the existing literature, an analytical model of an entry game with continua of types is provided to advance the results of prior research. By explicitly considering both a potential entrant and potential investors, this model incorporates two opposing forces that may influence an incumbent's decision to disclose or withhold private information. Various equilibria are characterized and discussed. Part II of the thesis focuses on firms' contractual relationships. The analyses extend traditional agency theory analysis to situations in which complete contracting is costly. Two models related to incomplete contracting are offered. One model analyzes the influence of contracting costs on a firm's contracting strategy in the context of the firm's internal transfer of goods and services. The results of this analysis provide insights and a new basis for the research of the transfer pricing issue. The second model deals with the incentive issues within organizations. The analysis focuses on the situations in which verifiable performance measures are unavailable. In the model, two kinds of incentives, namely, high-powered and low-powered incentives, are analyzed. We find that contract renewal based on observable (but non-verifiable information) can provide useful lowpowered incentives in an hierarchical organization in which employees build up human capital. This may provide useful insights into managerial accounting system design.


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## Chapter 1

## INTRODUCTION TO THE THESIS

This thesis contributes to the managerial accounting research literature. The analysis concentrates on two important elements of managerial accounting issues: information and contracts. The thesis consists of six chapters. This chapter (Chapter 1) serves as an introduction to the whole thesis. The remaining five chapters are grouped into two parts.

Part I, which consists of Chapters 2 and 3, focuses on voluntary financial accounting disclosure. As pointed out by Feltham [1984], in the process of operating a firm, management is likely to acquire considerable private information about the factors that affect the outcomes of the firm's activities. Some of that private information is ultimately revealed by mandatory and voluntary public reports, and some may be revealed by observed management actions. To understand the information content of accounting reports and the reporting choices made by management, we must understand the market forces that create incentives for management to acquire and then reveal or disguise private information. Therefore, a firm's behaviour in voluntarily disclosing its private information is an important issue in managerial and financial accounting research.

Many types of individuals are potentially interested in management's private information. These include cur-
rent and potential investors, creditors, suppliers, employees, customers, competitors, and regulators. Management has a primarily cooperative relationship with some, but with others the relationship is primarily noncooperative. Therefore, both cooperative and non-cooperative game theory provide analyses that are relevant to understanding management's choices. Furthermore, since there are many individuals and firms competing for the economy's resources, analyses that explicitly recognize the impact of competitive market forces are particularly relevant.

The analysis in Part I pursues the objectives mentioned above. Particularly, we summarize and advance prior research in this field. Chapter 2 is a detailed survey of the existing literature. It also serves as an introduction to Part I. Chapter 3 analyzes a formal model to extend the results of prior research. Our model depicts an entry game in which the incumbent is concerned about both the potential entry of an entrant and the valuation of his firm by potential investors, who will supply capital for his investment in his market. The incumbent has private information about the profitability of the product market and he may be uncertain about the set of beliefs that will induce the entrant to enter his market. By explicitly considering both a potential entrant and
potential investors, our model incorporates two opposing forces that may influence an incumbent's decision to disclose or withhold private information. Various equilibria are characterized and their nature is discussed. Our results provide possible explanations for observed voluntary disclosure choices made by firms.

Part II, which consists of Chapters 4, 5 and 6, contributes to our understanding of the role of managerial accounting in a firm's contractual relationships. The main purpose of our analysis is to extend traditional agency theory analysis to situations in which complete contracting is costly and, hence, contracts are frequently incomplete.

The basic concern of agency theory is with the "control and information relations" manifested in the search for the most preferred feasible contract between the principal and his agent. The agency contract delegates to the agent the responsibility to "manage" a portion of the firm's operations in return for compensation that is effectively a share of the firm's outcome. Depending on the sharing rule, the agent's compensation may be a fixed remuneration or a non-trivial function of the outcome or other information about his performance (thus, imposing compensation risk on the agent). The interests of the principal and the agent are likely to conflict since the
agent is assumed to maximize his own utility, and his choices may not maximize the principal's net profit. Thus, demand for information for contracting and monitoring purposes is raised. A managerial accounting system is designed to supply this information.

Agency theory provides a framework within which the differences between the objectives of the principal and the agent are incorporated as an integral part of the theory. In this way, managerial accounting research can analyze the important motivational aspects of various traditional accounting issues. Its considerable capacity for putting management accounting into a broader and more coherent context, for offering a more rigorous representation of management accounting concepts, and for clarifying important analytical as well as behavioral aspects of management accounting issues, has greatly advanced managerial accounting research in the last twenty years.

One key feature of the traditional agency theory is its emphasis on complete contracting. In other words, it is assumed (either implicitly or explicitly) that contracting costs are trivial. Hence, the results of such analyses are valid in a perfect world where contracting is costless, or in cases in which the impact of contracting costs is trivial relative to transaction gains. However, the usefulness of these results are limited when we deal
with cases in which transaction costs play a crucial role. In these cases, complete contracting is costly (or impossible) and, hence, contracts are always incomplete. Therefore, it is useful to extend agency theory to incorporate incomplete contracting theory and practice. The analyses in Part II provide this type of extension.

Chapter 4 is a brief introduction to incomplete contracting research. Chapter 5 analyzes a model in which contracting costs are explicitly considered in the context of internal transfers of goods and services. Our results show that contracting costs can have a significant impact on contracting strategy. Contract efficiency can be more precisely defined when the contracting costs are explicitly considered. This efficiency is reached through the maximization of the net trading gain, taking contracting costs into account. Particularly, when the trading gain is constant among various contracting strategies, then this efficiency can be reached by minimizing contracting costs. Our analysis also provides insights into managerial accounting research by relating our results to the transfer pricing policies used by firms. This gives insights into existing transfer pricing practice and provides a new basis for analyzing a firm's transfer pricing policies.

Chapter 6 considers another model dealing with incen-
tive issues within organizations where verifiable performance measures are unavailable. Our model analyzes two kinds of incentives, high-powered and low-powered (see the chapters for a detailed explanation). The former can be observed either in the market or in an organization, while the latter primarily exist in hierarchical organizations. Most employees in a firm are industrious not because they have contingent contracts, but because the contract renewal process provides low-powered incentives in longterm employment relations. Our results may provide insights into managerial accounting system design. The common ground of the analyses in this thesis is how economic agents deal with uncertainties and information asymmetries in exchange (of capital, goods, and services). Information and contracting are two fundamental aspects of this common problem. They are intimately connected, and sometimes, difficult to separate. Therefore, our analyses can be viewed as a theory for transactions from an accountants' perspective.

## References 1

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## PART I

## VOLUNTARY DISCLOSURE

## OF PRIVATE INFORMATION

## Chapter 2

## DISCRETIONARY DISCLOSURE

 RESEARCH IN THE 1980s:
## A SURVEY

### 2.1 Introduction

The decade of the 1980s represents a period during which accounting researchers paid more attention to firms' voluntary accounting disclosure than ever before. This represents a significant contribution for the following reasons. First, unless we understand firms' incentives to withhold their private information, we will not have a solid base for mandatory accounting disclosure regulations. Firms' voluntary disclosures may reveal all the information required by these regulations. Second, predicting managers' behavior in deciding when to withhold or disclose information can be useful in evaluating the consequences of alternative mandatory reporting procedures.

This chapter reviews the advances in this important accounting research area in the last decade. Our survey covers a number of published analytical and empirical papers and also a few unpublished working papers. We believe this chapter has value for the following reasons. First, it provides a summary of the main results of the extant literature. Second, we provide a clear classification of all the analytical models. This classification exhibits the similarities and differences among various models along several key dimensions. This may assist future research, particularly, when one attempts to build
new models. Third, the survey raises some questions about issues which may stimulate future research.

This chapter is organized as follows. Section 2.2 discusses the key dimensions along which various models differ. Sections 2.3 to 2.5 summarize two-player, oligopolistic, and three-player disclosure models, respectively. Section 2.6 summarizes a few signalling models that closely relate to disclosure models. Section 2.7 identifies some related empirical work.

### 2.2 Key Dimensions of Analytical Models

Analytical models of discretionary information disclosure differ along several key dimensions. The differences mainly result from various assumptions and model structures. First, models differ as to the number of players explicitly modelled. The simplest models have only two players, usually a firm versus its financial market or its potential rival, or sometimes, a seller versus a buyer. We shall refer to this group as twoplayer models. Recently, a few papers have analyzed more complicated models with three players -- a firm, a financial market, and an opponent. We shall refer to these papers as containing three-player models. There is another set of papers, mostly in the economics literature, that deal with the same issue in a setting of oligopolis-
tic games. Most of these papers assume that a finite number of firms (sometimes only two) compete in a product market playing Cournot or Bertrand games with asymmetric information. We shall refer to these papers as containing oligopolistic models.

The second crucial difference among models results from differences in assumptions about the cost associated with information transfer. Typically, there are two different assumptions about this cost. First, some models assume that the informed player(s) can make verifiable announcements regarding his private information, and the verification cost is negligible. In other words, it is possible for the informed player to communicate credibly with the uninformed players at a reasonable cost level. Second, some models assume that the information possessed by the informed player is unverifiable, i.e., the verification cost is prohibitively high. Hence, truthful information transfer through announcements is impossible. The use of an indirect mechanism, such as a contingent contract or an exogenous costly signal, is necessary to convincingly communicate players' private information to the uninformed players. We refer to a model with the first assumption as a disclosure model, and to a model with the second assumption as a signalling model. Since the arguments, techniques, and results in these two kinds of
models are quite different, we examine them in separate sections.

The following dimensions mainly relate to disclosure models, but some of them also apply to signalling models. Most disclosure papers assume that the manager of the firm can only make truthful announcements, i.e., they cannot lie. The motivation not to lie is not explicitly modelled. The justification for this assumption is based on the following arguments: (i) the market can costlessly verify or audit any of manager's claims; (ii) there is a threat of significant penalties if managers are "caught" misrepresenting their information; (iii) antitrust law and SEC regulations prohibit firms from making fraudulent disclosures; (iv) firms are concerned about their "reputations". However, although firms cannot lie, they can make incomplete disclosures. This is the third dimension which relates to different assumptions about the limitation on firm's disclosure strategy choices. We define "complete disclosure"' as a strategy by which a firm discloses all the information it holds at the moment of disclosure. If complete disclosure is required, then the manager has only

[^2]two choices: either to tell all he ${ }^{2}$ knows or to keep silent. In contrast, if a manager can make an "incomplete disclosure", then he can choose to truthfully disclose either a part of his information or a noisy representation of that information. For example, when a manager observes $(x, y)$, he can disclose either $x, y$, or $u=y+z$, where $z$ is a zero mean disturbance.

Fourth, models differ as to the assumed motivation of the managers. Most papers do not explicitly model manager incentives. Instead, they typically assume that the manager is exogenously motivated to maximize either the current market value of the firm, or its expected end-ofperiod cash flows to the initial equity holders (we will refer to this as the expected payoff). Some models have included both objectives, either by taking a weighted average of the two (Miller/Rock [1985]), or by treating the objective of the manager as private information (Dontoh [1990]). The reasons for these objectives are typically not discussed (except in Myers/Majluf [1984]).

Fifth, in those models in which the managers maximize the firm's expected payoffs, some completely ignore the capital market to focus entirely on the product market, usually in a stochastic oligopolistic setting. On the

[^3]other hand, others assume that the manager must obtain funds from the capital market and is therefore indirectly concerned with the firm's current market value.

Sixth, models differ as to whether disclosure can have a direct impact on the firm's end-of-period cash flow. Information is termed proprietary if it can have a direct impact, and non-proprietary if it does not. As Dye [1985a] points out, the former class includes not only information whose disclosure could alter a firm's future operating cash flows due to actions by competitors, but also information whose disclosure could generate regulatory actions, create potential legal liabilities, reduce consumer demand for its products, induce labour unions or other suppliers to renegotiate contracts, or cause revisions in the firm's credit standing. The information is, in the traditional sense, strategically valuable. The non-proprietary information includes information, such as annual earnings' forecasts, whose release would affect the prices of the firms' stocks, but not the distribution of the firms' future cash flows. Obviously, the characterization is a simplification to ease the analysis. Specifically, what constitutes non-proprietary information must be defined in reference to a particular set of expectations about a particular firm's future earnings. Seventh, models in which the information is propri-
etary differ as to how disclosure (or non-disclosure) impacts on the firm's end-of-period cash flows. In some models, the impact takes the form of a cost that is incurred if, and only if, disclosure takes place, independent of the information disclosed. Other models assume that the impact depends on the actions of an "opponent". The opponent can be a competitor or potential entrant in the firm's product market, a labour union, or a regulatory agency such as those concerned with taxation or utility rates. The key difference in this dimension is whether the disclosure cost is exogenously given or endogenously derived.

Eighth, models differ as to whether managers always have private information. If there is a positive probability of "no information", then "non-disclosure" can be the result of either "no information" or non-disclosure by an informed manager. This assumes, of course, that the manager cannot communicate that he lacks information.

Ninth, models differ as to whether the set of possible private signals is binary ("good" versus "bad") or a continuum. Finally, models differ as to whether managers voluntarily choose (and commit to) a disclosure policy prior to receiving their private information, or they choose to disclose or not disclose their private information after they know their information.

We organize the analytical papers we will summarize into four sections. Each section contains a table which exhibits the main characteristics of the models in its class using the related dimensions discussed above. Sections 2.3 through 2.5 summarize three groups of disclosure papers. Section 2.6 summarizes a few related signalling models.

### 2.3 Two-Player Disclosure Models

Table 2-1 exhibits the main characteristics of the papers in this class.

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Insert Table 2-1 here
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Grossman and Hart [1980] is one of the earliest papers dealing with the voluntary disclosure issue. They model a seller and a buyer of an item. The seller knows privately the quality of the item. They assume lying is illegal and there are no transaction or disclosure costs. Based on an adverse selection argument, they conclude that it will always be in the seller's interest to disclose the quality of the item voluntarily. The only equilibrium for their model is a full disclosure equilibrium. The intuition underlying this result is that when the seller withholds information, the buyer's suspicions about the quality of the item are so great that they discount its qual-
ity to the point that the seller is always better served to disclose what he knows. Such a single threat gives incentive to the seller to disclose except when his item has the lowest quality in the market.

Milgrom [1981] includes a persuasion game which is similar to Grossman/Hart [1980]. The differences follows. First, Milgrom assumes that the seller's signal may be multi-dimensional and he may conceal any dimension of his signal. Second, Milgrom uses sequential equilibrium concept, which is a more restrictive concept than a Nash equilibrium. However, since the basic assumptions about the disclosure cost are the same as Grossman/Hart, Milgrom reaches the same conclusion that in every sequential equilibrium, the seller uses a strategy of full disclosure.

Jovanovic [1982], like Grossman/Hart [1980], examines the disclosure of the quality of an item. The two players in the model are, again, a seller and a buyer. The question investigated is whether the free market offers sellers enough incentives to disclose information about the quality of their product. Jovanovic is the first to consider the impact of an exogenous cost for credibly and truthfully disclosing information. This cost ensures a partial disclosure equilibrium with one threshold value. The paper interprets the model in two ways. In the first, information and its disclosure yield only private gains --
they only lead to a redistribution of income among sellers. In the second, information raises welfare because it results in goods being traded from people who value them less to people who value them more. The paper concludes that, whether information is of purely private value or not, more than the socially optimal amount of disclosure takes place. Hence, in a world where false claims cannot occur, the free market offers ample incentives for disclosure.

The first paper published in an accounting journal on this topic is Verrecchia [1983]. A crucial contribution of this paper is its introduction of an exogenous proprietary cost -- the cost associated with disclosing information which may be proprietary in nature, and therefore may be potentially damaging. The existence of this cost induces uncertainty about the manager's private information when information is withheld. Investors are unsure whether a particular non-disclosure occurs because: the managers' private information is "bad news"; or (ii) the information is "good news", but not sufficiently good news to warrant incurring the proprietary cost. The market's inability to unambiguously interpret non-disclosure as "bad news" is sufficient to support a partial disclosure equilibrium with one threshold level. Below this threshold level the manager withholds his private informa-
tion -- a result consistent with Jovanovic's [1982] finding. Verrecchia also shows that as the proprietary cost increases, the threshold level of disclosure increases, i.e., the manager's incentive to withhold information increases and he discloses less information. This implies, in turn, that more competition results in less voluntary disclosure.

Verrecchia [1990] studies the same model as Verrecchia [1983] but focuses on how the quality of information available to a manager affects his incentives to disclose or withhold that information in the presence of external parties who have rational expectations about his motivation. The quality of the manager's information about the uncertain liquidating value of the risky asset is represented by the precision of the zero mean normal distributed random noise. Under the assumption that the market prices the risky asset at its expected value, the paper has following conclusions. First, the threshold level of disclosure decreases as the quality of manager's information increases, and increases as the quality of prior beliefs increases. Second, the probability of disclosure increases as the manager's information increases, and falls as the quality of prior beliefs increases. These results are consistent with Jung/Kwon [1988].

Dye [1985a] offers two different models to support a
partial disclosure equilibrium. The first model depends on the assumption that the market is unsure whether the manager is endowed with private information. Hence, if a manager withholds information, the investors cannot discern whether he has received information but chosen not to release it, or he has not received information. Of course, the manager is assumed to be incapable of making a credible announcement that he has not received new information. These assumptions result in a partial disclosure equilibrium similar to that of Verrecchia [1983]. The threshold level of disclosure decreases as the probability that manager receives information increases. When this probability approaches one, a full disclosure policy results.

A technical error in Dye's analysis is corrected by Jung/Kwon [1988]. The latter reexamines Dye's [1985a] model with the following amendment. In the absence of disclosure, they allow outside investors to revise their belief that managers have received no information, i.e., they use posterior probabilities instead of the unconditional probabilities used in Dye [1985a]. This correction enables them to resolve the problem of potential multiplicity of partial disclosure policies and, thereby, establish its uniqueness. They also provide two results in a comparative statics analysis. First, the threshold
level decreases as the probability that the manager has received information increases. If one believes this probability increases as time elapses, then this may explain why worse news is released later. Secondly, the threshold value increases as the market's prior belief becomes worse, in the sense of stochastic dominance. Hence, if one believes that, prior to information disclosure by the manager, investors independently acquire information about the firm's value, then this acquisition may trigger the release of private information which had previously been suppressed due to its unfavourableness but has now become favourable compared to the information that the market has independently acquired.

The second model in Dye [1985a] assumes that managers possess non-proprietary private information and there is a moral hazard problem between the manager and the firm's shareholders. Although the disclosure of a manager's information, by definition, will not alter the firm's earnings, it may alter the manager's compensation since the optimal agency contract depends on the firm's stock price, which will be affected by the firm's disclosure. Dye analyzes the optimal contracts and finds that investors and the manager always weakly prefer contracts which encourage the manager not to disclose his information. The adverse effects that disclosure may have on
both the owners and the manager of a firm result from the fact that the manager's contract depends on the firm's stock price. If the manager discloses his information, then the stock price contains more information about the firm's earnings, but it may not contain more information about the manager's actions. In addition, information that is useful for forecasting net income may be detrimental for contracting purposes. Therefore, policies which encourage management disclosure of private information may produce superior forecasts of the firm's earnings, but inferior measures of the manager's actions. In summary, disclosures may exacerbate agency problems between the manager and the shareholders.

Dye [1985b] focuses on the relation between mandatory reports and voluntary disclosure. The analysis of this issue depends critically on the firm's motivation for choosing among financial reporting techniques. One key assumption of this paper is that a firm's choice among reporting requirements is influenced by how that choice alters its ability to protect its proprietary information. To endogenize the proprietary cost of the firm's disclosure, Dye models an entry game with two players: a firm and its rival. The dissemination of the established firm's operating information will assist rivals in determining whether to enter the firm's market. If entry occurs, the
firm's future earnings will be reduced. The analysis of this entry game shows that, in equilibrium, the established firm is better off with more discretion in the choice of accounting techniques. If voluntary supplementary disclosures are considered, the firm is always weakly better off with less detailed reporting requirements. Dye concludes that, by imposing more detailed reporting requirements, accounting boards do not necessarily increase investors' knowledge of the firms' future earnings prospects. This result can occur for either of two reasons. First, mandatory and voluntary disclosures are sometimes substitutes, so that the "amount" of information produced by "more detailed" mandatory reports may be offset by a reduction in voluntary disclosures. Second, firms may be able to reveal information by their choice among accounting techniques, so that the mandatory use of a "more detailed", but uniform, accounting procedure may remove this potential source of information. Dye also provides conditions under which more detailed reporting requirements increase the amount of information publicly revealed about firms.

Dye [1986] analyzes disclosure policies adopted by managers endowed with both proprietary and non-proprietary information. This model explains selective disclosure of managerial information and the effects of changes in fi-
nancial reporting requirements on firms' voluntary disclosure policies. It establishes that increasing mandatory reporting requirements can increase the incentives for voluntary disclosure. To derive this result, Dye assumes the manager observes non-proprietary signal $x$ and proprietary signal $y$. Disclosure of $y$ incurs cost $c$. Disclosure of $x$ alone, although it is non-proprietary, still incurs cost $\underline{c}$ since it may reveal something about $y$. Under this cost structure, the optimal strategy of the manager has the following characteristics. First, with positive probability, the non-proprietary information is not disclosed. Second, a policy of absolutely no disclosure is typically not credible except where $c$ and $c$ are sufficiently large. Third, for each realization of $x$, no disclosure is preferred to full disclosure for a $y$ that is less than some threshold value. The reverse is true for other values of $y$. Fourth, good news is more likely to be disclosed or partially disclosed than bad news. Finally, the payoffs of no, partial, and full disclosure policies depend on the value of ( $x, y$ ). It is possible for each of these policies to be optimal. The intuition for these results is that disclosure of non-proprietary information may partially reveal manager's proprietary information. Hence, disclosure of good news may assuage investors' concerns regarding the firm's future earnings prospects
while, at the same time, worsening these prospects by divulging proprietary information.

## Summary of Two-Player Models

One obvious merit of a two-player model is its simplicity. By focusing on one dimension of the relation between the firm and its environment, the analyses of these models derive relatively simple disclosure strategies. Most of the papers in this group consider a game played by a firm and its investors. Both the manager and the investors of the firm are assumed to be rational. Under various pre-specified rules, the equilibrium of the game induces different disclosure strategies. The models we summarized in this section have shown that a firm's disclosure strategy may be influenced by various factors such as the market perception about the firm's private information, the costs incurred in information release, the quality of manager's private information, the manager's incentive contract, and alternative communication channels. Depending on the combination of factors considered in the model, the firm may choose full, partial, or non-disclosure strategies. A partial disclosure equilibrium derived in these models usually consists of a single threshold level. The manager discloses his private signal if it is above this level, and withholds if it is
below. In the extreme cases, the threshold level goes to infinity, either positive or negative, resulting in a full or non-disclosure equilibrium, respectively. The key element in deriving a partial disclosure equilibrium is the market's inability to infer manager's private information precisely when the manager does not disclose.

We can view a two-player model as a partial analysis of the entire problem. While this is an important step in obtaining a more complete understanding of the whole problem, a firm's environment is likely to be so complex that its management will face multi-dimensional influences. The manager's decisions must involve tradeoffs among the various considerations. Hence, a simple two-player model is not sufficient to obtain a full picture of the disclosure problem and, thus, more complicated models are required in further research.

### 2.4 Oligopolistic Models

Table 2-2 exhibits the main characteristics of the papers summarized in this section. The table shows that there is a significant distinction between the models in this group and those in the last section. That is, almost all the oligopolistic models deal with voluntary disclosure policy instead of voluntary signal disclosure. Hence, in this section, the central question is whether firms
have incentive to commit to a policy to pool their private information, i.e., to commit to disclosing their information.

The strong assumption that firms can commit to a disclosure policy through a commitment mechanism such as a trade association or mandated disclosure rules (enforced by auditors) is made by all papers in this category. For example, Kirby [1988] and others have suggested that trade associations may be a mechanism for committing to a given level of disclosure (although this does not preclude additional voluntary disclosure). Feltham/Gigler/Hughes [1990] examine line-of-business reporting and assume that one commits to a given level of audited information through "consistency" of reporting practice by the firm, and assume that voluntary disclosure is not credible. The viability of this assumption is an open question. For example, Darrough [1990] comments that even if firms prefer to commit ex ante to pooling information, their preferences regarding the disclosure of their information after they receive their signal may differ. The firms which receive unfavourable signals might attempt to add as much noise as possible if there is any room for choosing the level of information detail (i.e., the degree of aggregation). These firms may fail to comply with their commitment, and it may be costly to monitor and to penal-
ize firms for such opportunistic behaviour.

Insert Table 2-2 here

Novshek/Sonnenschein [1982] is one of the earliest papers that deal with information acquisition and release in a competitive market. They assume a linear demand function in a cournot duopoly model. There is uncertainty about the value of the quantity intercept. Firm i independently acquires $n_{i}$ information signals and allows $m_{i}$ of these signals to be used in a common pool to be made "available " to both firms. Under the notion of fulfilled expectations (or Bayesian Nash equilibrium), they show that the equilibrium expected profit for firm i is: (i) increasing in $n_{i}$; (ii) increasing in $n_{j}$ when there is some pooling but unaffected by $m_{j}$ if there is no pooling; (iii) decreasing in $m_{i}$ if firm $j$ retains some private information, but unaffected by $m_{i}$ if firm $j$ places all of its information in the common pool $\left(m_{j}=n_{j}\right)$; (iv) increasing in $m_{j}$ if $n_{j}>n_{i}$ or $2 m_{i}>n_{i}-n_{j}$. The last conclusion can be interpreted as a situation in which the opposing firm controls more information, or in which the firm's contribution to the pool is relatively large. In addition, they conclude that if firms can contract to pool information, then, when $n_{1}=n_{2}$, no pooling and full pooling are weakly undominated. When $n_{1}$ and $n_{2}$ are not equal, both of the
above equilibria are again undominated. Full pooling leads to the highest total profit, but no pooling leads to the highest profit for the firm with "control" over more observations.

Clarke [1983] investigates the incentives for firms to share private information about cost or demand in a stochastic market. For a n-firm oligopolistic model, the paper shows that in a full Bayes-Cournot equilibrium, there is no mutual incentive for all firms in an industry to share information. However, if cooperative quantity setting is possible, then there is always an incentive to share -- as long as a suitable profit distribution can be negotiated among the conspirators. Technically, Clarke assumes uncertain market variables may be parameterized by normal distributions so that the precise conditional expectations that characterize the equilibrium can be explicitly computed.

Vives [1984] analyzes two types of duopoly information equilibria, cournot and Bertrand. He allows the incentives for information sharing and its welfare consequences to depend on the type of competition, the nature of the goods (substitutes or complements), and the degree of product differentiation. The uncertainty comes from an unknown common price intercept of a linear demand function. Firm i's signal $s_{i}$ is an independent and unbiased
estimate of the intercept. If there is no sharing of private signals, then the firms have independent information. On the other hand, they have correlated information if they pool their signals. The paper focuses on selfenforcing pooling agreements. In the first stage of a two-stage game, firms commit their disclosure policies to an agency prior to the market data collection. At the second stage market research is conducted, the agency sends the pooled signals to the firms, and a Bayesian game is played. It shows that the two-stage game has a unique subgame perfect equilibrium in dominant strategies at the first stage. With substitutes it involves no pooling of information in Cournot competition and complete pooling in Bertrand competition. With complements the results reverse. The intuition behind these results follows. When the goods are substitutes with Cournot competition, an increase in the precision of the rival's information and an increase in the correlation of the signals have adverse effects on the expected profit of the firm. Therefore, not to share any information is a dominant strategy. On the other hand, in Bertrand competition the two factors mentioned above have positive effects on the expected profit of the firm. Hence, to put everything in the common pool is a dominant strategy. When the goods are complements all above arguments are reversed.

Fried [1984] examines the nature of the equilibrium solution to the duopoly problem under various incomplete information structures and incentives for information production and disclosure. The paper focuses on the game in which duopolists face the choice of making their cost functions known to their opponents. At the first stage of a two-stage game, firms make information production and disclosure decisions which are assumed to be enforceable and known. At the second stage, firms make quantity output decisions based upon the information available to them as a consequence of the first stage decision. The results of the analysis are: (i) that it will be in the best interest of each duopolist to produce information about himself; (ii) that both firms are better off when they disclose as opposed to the case where only one of them or neither of them discloses; (iii) that a firm is better off disclosing even if the competitor does not reciprocate; (iv) that if one firm cannot obtain its own cost function, the other firm might be better off disclosing his own cost function; (v) that in an environment that does not permit disclosure, one firm might be better off if the other firm goes ahead and produces information. These results can be explained by decomposing the information disclosed into two components: "firm-specific" and "common" cost information. Once the firms know their own cost functions, the
only information left to disclose is "firm-specific". Thus the effect of any disclosure will be confined to a "collusive" one, since its sole purpose will be to allow the opponents to make the necessary and mutually beneficial "counter" adjustments. Thus, even unilateral disclosure would be in the best interest of the duopolist. Li [1985] studies the incentives for information sharing among firms in an oligopolistic industry. The uncertainty is about either the demand function or the individual cost functions. The paper assumes that the private information that firms receive has equal accuracy and obeys a linear conditional expectation property. Different uncertainties, as mentioned above, are studied separately in two models that are all two-stage games. Similar to those in Vives [1984] and Fried [1984]. The paper derives pure-strategy Nash equilibria which are subgame-perfect under a symmetric information structure where firms receive private signals with equal precision. The results show that there is a systematic difference between the incentives to share common demand and private cost information. No information sharing is the unique equilibrium when an oligopolist faces stochastic demand that is common to all the firms. Conversely, complete information sharing is the unique equilibrium when the private costs are uncertain. These differences are due to
the distinct nature of the information: whether it has "private" value or "common" value. Knowledge of the demand has "common" value, while knowledge of the costs has "private" value.

Gal-or [1985] addresses the same issue as the other papers summarized above. Her model is very similar to Novshek/Sonnenschein [1982] and Clarke [1983]. The principal novelty of this paper is that (i) competitors have available a continuum of incomplete revelation strategies, similar to Li [1985], and (ii) various degrees of initial correlation among private signals are allowed and are the focus of this analysis. The conclusion is that in an oligopoly where firms observe signals about linear stochastic demand, private information is never revealed if firms behave as Nash competitors in setting output levels. This result is derived regardless of the degree of initial correlation among signals. This implies that no information sharing is the unique equilibrium regardless of whether firms can make inferences about the signals observed by others.

Gal-Or [1986] pursues the same objective as Vives [1984], i.e., to examine how incentives for two duopolists to share information hornestly depends on the nature of competition (Cournot or Bertrand), and the information structure. However, in Gal-Or [1986] the uncertainty is
about unknown private costs. The analysis of cournot equilibria is similar to Fried [1984], but the analysis of Bertrand equilibria shows that when the information is about unknown costs, firms have no incentives to pool information. With unknown private costs, the paper shows that sharing is a dominant strategy with cournot competition, and concealing is a dominant strategy with Bertrand competition. The intuition behind the results is that the pooling of private information about unknown costs has two effects on the firm. On the one hand, more accurate information about the rival's cost is available, and the strategies can be more accurately chosen so that the firm and its competitor's likelihood of mistakenly over-producing or under-producing is reduced. This has an unambiguous positive effect on the payoff of the firm. On the other hand, the pooling of the information reduces the correlation among the decision rules to expand or contract output. This may have a positive or negative effect depending upon the slope of the reaction functions of the firms. If they are downward sloping (Cournot competition) reduced correlation has a positive effect, and if they are upward sloping (Bertrand competition) reduced correlation has a negative effect.

Gal-Or [1986] extends Fried [1984]'s result to the situations in which each duopolist observes its own costs
with noise and may send noisy signals to its rival. One key assumption of the paper is that firms must commit themselves to a fixed amount of garbling prior to learning their signals. That is, firms are allowed to reveal their private information incompletely (partially) but the accuracy must be reported ex ante. Hence, it is crucial that the transmission of the information is conducted by an "outside agency". The results of this and prior papers imply that an industry will have incentive to create an "association" that collects and publicizes information depending on the nature of competition in the industry and the nature of the information structure. If firms compete in quantity the "association" will collect and publicize information about a parameter of the model that is different for each firm. If they compete in price it will collect and publicize information about a parameter of the model that is common to all firms.

Shapiro [1986] independently and concurrently studies the same issue as Fried [1984] and Gal-or [1986], which deal with the case of private information about costs. Shapiro analyzes oligopoly information exchange and firms' decisions to join a trade association that exchanges the cost information. His model includes the case of positive correlation between the firms' realized costs. In addition, it provides a complete analysis of the welfare
effects of information exchange. As in all other papers in this category, it assumes that the firms can verify each other's reports, and firms can commit to the disclosure policy chosen before the arrival of private information. Technically, it assumes a linear demand function, a linearity property for the conditional expectations, and identical distribution of costs. The results can be summarized as follows. The oligopolists all prefer an industry wide cost sharing agreement to no cost sharing, and strictly so if the correlation coefficient is strictly less than one. Such a cost sharing agreement also raises expected welfare, but it reduces expected consumer surplus. These conclusions hold in both symmetric and asymmetric situations. The paper also shows that complete information sharing is the unique coalitional outcome in the core of the membership game, where the firms correctly anticipate the expected profits they will earn for any given pattern of information exchange.

Kirby [1988] reexamines Clarke's [1983] Cournot oligopolistic model but uses a quadratic cost function. In contrast to much of the theoretical work on the incentives of Cournot oligopolists not to share information about market demand, she shows that firms may be better off sharing information than keeping it private. Furthermore, she shows that sharing information may constitute a

Nash equilibrium and always improves expected consumer surplus. The information-sharing arrangements examined in this paper are different from Novshek/Sonnenschein [1982], Gal-Or [1985], and Li [1985], where firms independently select the amount of their private information to be shared, and yet all receive the resulting aggregate. Kirby assumes the same rule as in Clarke [1983], where the trade association gathers the private signals of individual firms, aggregates the signals, and then disseminates the aggregate signal to each of the participating firms. The key result is that the benefit from sharing information depends upon the shape of the cost function. If the quadratic cost coefficient $d$ is sufficiently large, i.e., the cost function is sufficiently convex, then information sharing is Pareto preferred to a setting of private information and forms a Nash equilibrium. The intuition is that as d increases, marginal cost also increases, and "errors" in production become very costly. Hence, the value from sharing information increases.

Dontoh [1990] models a (n+1)-firm oligopoly in which one of these firms is endowed with private information about the stochastic demand parameter. The informed firm's incentive to disclose information depends upon its manager's objectives; there are two types of firms, both of which are consistent with the value maximization cri-
terion. Type A firms maximize current market value, while type $B$ firms maximize end-of-period payoff with no concern for how they are valued by market currently. The model assumes a firm's type is private information, i.e., the market cannot identify whether a firm is type A or type B. If an informed firm is type $A$, then it has incentive to disclose good news but withhold bad news. For an informed type $B$ firm, the disclosure behavior is reversed, i.e., it discloses bad news but withholds good news. Since the market cannot identify a firm's type, when the information is withheld, the market is also incapable of unambiguously inferring whether the information is good or bad news. This, as in Verrecchia [1983] and Dye [1985a], is sufficient to support a partial equilibrium. It is noteworthy that in this model, a firm's type has no influence either on any uninformed firm's strategy in the first stage of the game, or on all firms' output decision in the second stage.

Hughes/Kao [1990] study the equilibria that emerge under different disclosure regimes, thereby leading to predictions of how outputs, profits and levels of investment in R\&D vary across those regimes. In their model, R\&D is defined as an activity which results in an uncertain reduction in the marginal costs of producing a consumption good. The actual marginal costs are directly
observable only to the firm. They investigate the nature of oligopolistic equilibria when firms publicly report both R\&D spending and the realization of their marginal costs, compared with the benchmark regime in which firms report $R \& D$ spending only. In the symmetric case where firms are identical prior to marginal cost realizations, they find that if managers are risk neutral, then the equilibrium expected profit is higher under full disclosure than under partial disclosure. However, equilibrium levels of $R \& D$ spending are the same under both regimes. If managers are risk averse and their compensation is proportional to profits, then the equilibrium levels of R\&D spending are greater under the benchmark regime than under full disclosure. This is because the risk under full disclosure can only be ameliorated by reducing R\&D, whereas under partial disclosure the output decisions also afford an opportunity for risk reduction. Introducing asymmetries in cost uncertainty and the risk preferences of firm managers modify the results described above, but not in a major way. Given risk neutrality, the levels of equilibrium $R \& D$ spending are unaffected by an asymmetry in cost uncertainty, and are ordered similarly across disclosure regimes under both symmetric and asymmetric risk preferences. Furthermore, under the benchmark regime, a firm with a risk neutral manager enjoys an advantage over
a firm with a risk averse manager. The paper does not consider welfare issues, nor the efficiency of R\&D decisions.

Darrough [1990] analyzes the incentives of firms in a duopoly market to disclose private information, and examines how disclosure incentives are influenced by the nature of competition and private information. Both Cournot and Bertrand competition are investigated when firms receive private information either on demand or on costs. The first part of the paper unifies the extant results in the prior literature about the incentive to disclose voluntarily in an ex ante sense: firms select a disclosure policy before they receive a signal on the uncertainty parameter. The second part considers whether firms have incentives to disclose their private signals voluntarily after they receive these signals. In this ex post case, firms are endowed with private information before they make disclosure decisions. By combining these two parts, the paper interprets how market structures affect voluntary disclosure incentives. It also offers predictions about the type of equilibria that prevail in different market settings. The results of part 1 and 2 are best shown in Tables 2-3(1) and 2-3(2), which are quoted from the paper.

Based on the results mentioned above, in Bertrand competition with demand uncertainty and Cournot competition with cost uncertainty, firms would be willing to commit ex ante to the most accurate disclosure. They might elect to do so by way of trade associations or by committing to a mandated disclosure policy. Ex post, those firms with unfavourable signals find themselves with "bad luck". Had they known the realization, they would have acted differently. If firms do not ex ante commit to a disclosure policy, they try to hide unfavourable signals. However, their effort will likely be futile because a rational market will interpret the signal of "No disclosure" as one with unfavourable news, thereby starting the unravelling process. On the other hand, if the ex ante consensus is no disclosure, those firms with favorable signals may try to "signal" their information through other means of voluntary disclosure. Whether perfect revelation ensues depends upon what additional factors are considered in the model.

## Summary of Oligopolistic Models

This class of papers focuses on the incentives for firms to disclose private information in a competitive
environment. All the papers assume the information can be costlessly verified if disclosed. The models usually are a n-firm oligopoly or its simplification -- a two-firm duopoly. Manager's objective is always to maximize the expected profit of his firm. The incentives used to induce managers to pursue this objective are usually not considered. Most papers determine an optimal voluntary disclosure policy, i.e., firms can commit ex ante to their information sharing strategy before they observe their signals. Almost all models have continuous signal variables. The results show that the incentives to pool information depend on many factors. The most important ones are: the competitive environment (Cournot or Bertrand), the type of uncertainty (demand or cost), the nature of the products (substitutes or complements), and the form of the cost functions (linear or quadratic). The optimal information sharing policies may induce full sharing, no sharing, or partial sharing. The intuition behind the results is the twofold influence of information pooling on firms' profits. On the one hand, more accurate information about the rival is available, and the strategies can be more accurately chosen so that the likelihood that some firms under-produce or over-produce can be reduced. On the other hand, the pooling of information may increase or reduce the correlation among the decision
rules about the production quantities. The latter may have positive or negative effects on firms' profits depending on the various factors described above.

Most oligopoly disclosure models consider only a single relation between $a$ firm and its environment: competition in the product market. Such a model can be viewed as essentially a two-player model in which the rivals in the product market replace the investors in the financial market as the second player. The analysis of these models provide important contributions to our understanding of disclosure choice, but the contribution is at best a partial analysis of the larger problem with a diverse set of players.

Many topics in this area have not been exploited. For instance, most papers assume a linear cost function to enhance the tractability. However, as Kirby [1988] shows in her model, a quadratic cost function may reverse all the results derived from a linear cost function model if other elements of the model are not changed. Whether this is true in other models is still an open question. Also, most papers assume firms can commit to their strategies ex ante. As pointed by Darrough [1990], the ability of an oligopoly to enforce such commitment is doubtful. For example, if a firm commits ex ante to a non-disclosure policy, but ex post the firm receives a signal which it
would like to disclose, there is little an association can do to prohibit the firm from voluntarily disclosing its information by some indirect means. Hence, it is important to consider both ex ante and ex post incentives. The research in this aspect of disclosure choice has just started.

### 2.5 Three-Player Disclosure Models

Table 2-4 presents the main characteristics of the papers summarized in this section. As indicated by the title of this section, the models in this group consider games in which three players interact with each other.

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Insert Table 2-4 here
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Bhattacharya/Ritter [1983] is the first paper to consider the behavior of an informed firm facing conflicting effects of its private information release. An asymmetrically informed agent is motivated to communicate its privately known "good news" to the financial market but can do so only through channels or signals which directly convey information to competing agents. The private information is valuable in the research of both the informed firm and its competitors. Hence, the informed firm faces a tradeoff between (i) reducing the value of its informational advantage, and (ii) obtaining capital at
terms that reflect good news about its innovation prospects, thus lowering the ownership dilution suffered by the existing owners of the "research technology". In this way, the model connects the influence of information release on product market competition with its influence on financial market valuation. The proprietary cost of disclosure is, therefore, endogenized in the model. There are $N$ firms engaged in an R\&D "race" for a patentable invention, which has known private value V. This value accrues only to the first firm to succeed. since all uninformed entrants can be viewed as one opponent of the informed firm, the whole game can be viewed as having three players, with the financial market as the third player. ${ }^{3}$ The model allows the informed firm to decide the disclosure level after the firm receives its private information. Hence, the model is a typical voluntary signal incomplete disclosure setting. The cost of disclosure is endogenously determined by the number of entrants, which is a function of the private information and its disclosure level. All firms must obtain capital from the capital market through equity issues. Other

[^4]financial means are not considered. Disclosure by the informed firm affects the terms at which it can obtain capital. In making its disclosure decision, the informed firm also considers the adverse affect that the disclosure has on its conjectured intrinsic value. This value represents its belief regarding the discounted expected value of its invention payoff as a function of its disclosure, taking the impact of such disclosure on the number and testing rate of its competitors into account. The equilibrium provides partial disclosure, with the characteristic that the proportion of private knowledge disclosed declines as the private knowledge increases.

Lanen/Verrecchia [1987] analyze how the use of management accounting information to make operating decisions can imprecisely communicate that information when direct (precise) disclosure is costly. Their model consists of three players: the owner of a firm, the supplier of the technology, and an external party who potentially evaluates the firm's prospects. ${ }^{4}$ Their analysis focuses on the owner's tradeoff between an efficient operating decision and the disclosure of proprietary information.

[^5]Specifically, they consider how proprietary data, that is generated by the firm's internal accounting system in making a production decision, are imperfectly inferred by outsiders who observe the outcome of the decision process. They identify conditions under which optimal management decision making is altered by the existence of this potential indirect communication alternative to disclosure. This implies that when operating decisions depend on private information, the operating decision made may deviate from the efficient decision from an outsider's perspective. Thus, their analysis offers one rationale for firms pursuing policies such as retaining managers of below-average competence or obsolete technologies. When the role of the replacement decision as a communication mechanism is considered, these policies may in fact be optimal.

Darrough/Stoughton [1990] is a three-player model about an entry game. The basic idea of the model is similar to Bhattacharya/Ritter [1983]. The difference is that in the current paper, the opponent is a potential entrant. The informed firm trades off between the benefits and costs of the disclosure. The benefits of disclosing good news come from a higher financial market evaluation. The proprietary cost is due to the fact that the disclosure could compromise the incumbent's competi-
tive position by providing strategic information to potential competitors. The game modelled consists of two stages. At the first stage, the incumbent firm, as a monopolist in its industry, raises $k$ units of capital from the financial market by selling a portion of the firm. The terms of financing are influenced by the disclosure strategy, and the potential entrant's decision. If entry takes place, the second stage is a duopoly game. Otherwise, the incumbent is still in its monopoly position. The private signal is binary, i.e., good news versus bad news. Under a condition which amounts to entry deterrence being more important than financial valuation, they have identified three equilibria of their disclosure-entry game: (i) a disclosure equilibrium in which the incumbent discloses both good and bad news, which occurs when the prior belief is optimistic or the entry cost is relatively low; (ii) a non-disclosure equilibrium in which the incumbent discloses no information, which occurs when the prior belief is relatively pessimistic or the entry cost is relatively high; and (iii) a partial disclosure equilibrium in which only unfavourable bad news is disclosed. An implication of the model is that competition, through threat of entry, encourages voluntary disclosure -- a result that differs from Verrecchia's [1983] conclusion. This is because Darrough/Stoughton deal with pre-entry
competition. In their setting, when entry costs are low, entry is more likely to occur so that the motive for entry deterrence becomes dominant for an incumbent with good news. This will result in a full disclosure equilibrium. While Verrecchia [1983] deals with post-entry competition, and disclosure always serves to reduce the informed firm's competitive advantage. Thus, stronger competition will result in less voluntary disclosure.

Wagenhofer [1990] analyzes a model similar to Darrough/Stoughton [1990] but with a continuum of private signals. Knowledge of this signal is valuable to both the financial market and an opponent. The opponent decides to take a beneficial action only if the signal is sufficiently favourable. This action imposes proprietary costs on the disclosing firm. The firm does not raise capital from the capital market, but is assumed to maximize its current market price. With an additional assumption that the firm's market price is equal to the value of the signal, the paper derives the following equilibrium results: (i) a sequential equilibrium with full disclosure always exists; (ii) a partial disclosure equilibrium with two non-disclosure intervals may exist; and (iii) full non-disclosure is never a part of a sequential equilibrium. In addition, the paper points out that multiple equilibria may exist. Chapter 3 of this dissertation pursues the same topic
as the other papers in this category. ${ }^{5}$ The basic model is very similar to Wagenhofer [1990] except that in the latter model the incumbent requires no funds from the capital market and the manager seeks to maximize the current market value of the firm, while in the former model the amount of capital raised from the capital market is an important parameter and the manager seeks to maximize the expected end-of-period payoff. A key feature that distinguishes the analysis in Chapter 3 from other disclosure papers is that it introduces private entrant information. That is, the model in Chapter 3 allows for the possibility that the incumbent may not know the entrant's break-even point and, therefore, does not know what beliefs will induce the entrant to enter. The major impact of this change is to eliminate equilibria in which the incumbent firm partially discloses his information and the entrant plays a mixed strategy. In the model in which the entrant has private information, the entrant plays a pure strategy, whether the incumbent discloses his private information or not.

The model in Chapter 3 expresses both the monopolist's and the duopolists' profits as linear functions of the incumbent's information. However, it also demon-

[^6]strates that this is consistent with standard duopoly models in which firms have quadratic profit functions. The distributions used to describe the incumbent's information and the entrant's break-even point are quite general, but the paper focuses on two extreme cases: common knowledge versus a uniform distribution about the entrant's break-even point. Based on these assumptions, the analysis explicitly solves for all possible equilibria. The results show that partial disclosure equilibria exist when the firm has a relatively balanced concern for the responses of both markets. The most interesting aspect of their results is that there are two possible partial disclosure equilibria. PD-L equilibria are characterized by a capital market in which the non-disclosure firms have a lower market value than all disclosure firms. PD-H equilibria, on the other hand, are characterized by a capital market in which some disclosure firms have lower market values than the non-disclosure firms. Since the equilibria apply on a firm-by-firm basis, this result implies that, in equilibrium, we would expect to observe firms that choose to withhold information even though its release would increase their market value, while other firms disclose information even though withholding it would increase their market value.

Chapter 3 is the first analysis to identify situ-
ations in which a full disclosure equilibrium does not exist, and only a partial equilibrium prevails. In addition, it shows that when full and partial disclosure equilibria coexist, the full disclosure equilibrium will not be stable under a suitable refinement criterion. This implies that, under certain conditions, withholding information may be the only equilibrium strategy -- a result consistent with empirical observations.

## Summary of Three-Player Models

Obviously, three-payer models are more advanced than most two-player or oligopoly models. With regard to the scope of analysis, a three-player model considers two dimensions of the impact of disclosure while the other models usually consider only one dimension. The resulting equilibria for such a model are more complex, as intuitively would be the case in the real world. These models show that firms may withhold both good news and bad news. The market value of the non-disclosing firms may be higher or lower than disclosing firms. The key factor in the existence of various equilibria is the relative importance to the informed firm of diverse influences of disclosing information. The firm must tradeoff among multi-dimensional benefits and costs from its disclosure strategies. Verrecchia's [1990b] comments about Darrough/Stough-
ton [1990] may apply to most papers in this category. The innovation suggested in a three-player model is that the proprietary costs arising in a discretionary disclosure equilibrium can be endogenized by appealing to the notion of an entry game among firms in a product market whose degree of product differentiation is exogenously specified. This suggestion is novel in that it couples two unrelated areas of research, namely, discretionary disclosure by financial managers and entry games among firms, to produce equilibria where proprietary costs occur naturally. However, the extent of endogeneity is limited, because in all these models exogenous costs must exist to preclude full disclosure. For example, the need to raise an amount $k$ of capital to produce in the product market is just such a cost.

Verrecchia also raises the following concerns. First, the structure of the game is such that the possibility of full disclosure is never eliminated and, in fact, is supported by a variety of criteria. This is quite true in Darrough/Soughton [1990] and Wagenhofer [1990], but partially solved in Chapter 3 since it identifies situations in which there is no full disclosure equilibrium. Furthermore, if a partial disclosure equilibrium exists, full disclosure does not survive the Grossman/Perry stability criterion. However, even in

Chapter 3, a full disclosure equilibrium is still sustained in many situations. Second, the entry game may give exaggerated benefit to disclosing "bad news" in two ways: (1) it exaggerates the usefulness of "bad news" as a signal to discourage market entrants by suggesting that common information like sales data, would not already be known by potential competitors in the absence of disclosure; (2) it exaggerates the positive impact of disclosing "bad news" by ignoring the costs to managers associated with attempts to terminate their tenure in the wake of "bad news", either externally in the form of hostile takeovers or internally in the form of shareholder disapproval. The entry game exaggerates entry from without but ignores entry from within.

In addition to the above comments, the following two points are noteworthy. First, the multiplicity of equilibria is an unsolved problem in general. There may be a need to develop multi-period models, or to induce other arguments, such as reputation, to deal with such problems. Second, Bhattacharya/Ritter [1983], Darrough/ Stoughton [1990], and Chapter 3 consider only equity financing. There are other financial arrangements that firms can use to raise capital. Interesting issues of choice of financial structure can arise, since different financial contracts may involve differing disclosure "requirements".

Future research may endogenize the choice of financial arrangement to build more realistic models.

### 2.6 Signalling Models

As mentioned in Section 2.2, when truthful information transfer through announcements is impossible, firms may use indirect communication mechanisms to "signal" their private information. The research along this line forms another group of papers under the title of signalling models. Both disclosure models and signalling models focus on firms' behavior in releasing private information. Hence, many signalling papers are closely related to the disclosure models summarized in the prior sections. Generally, a signalling model consists of one or more uninformed players and one or more informed players. An uninformed player might be an insurer, an investor, an employer, or a customer, whereas an informed player might be an insuree, an entrepreneur, a manager, a worker, or a supplier. The asymmetric information may pertain to the likelihood the insuree will suffer a loss, the probability over the potential outcome from a firm's operations, the skill or productivity of the worker, or the quality of a product that is being sold. Hence, the uninformed player is typically a "buyer" and the informed player is a "seller" of some "good". The seller knows
more about the "quality" of the "good" than does the buyer. In each case there is a range of quality levels, which are usually referred to as "types". The key assumption of signalling models is that private information of the informed player regarding his type cannot be directly and credibly transferred to the uninformed players. In other words, the costs of information transfer or verification are prohibitively high, so that direct disclosure will not work. However, the "price" which the seller can obtain for his good will be influenced by the buyer's perception about the type of good being sold. The latter, in turn, is influenced by the information the buyer receives. There are two ways the seller can indirectly "signal" his type to the buyer: either through contingent contracts or exogenous costly signals.

These two ways of signalling are qualitatively different, but are not completely distinct or separate. Sometimes, exogenous signals may be included in a contingent contract. The fundamental principle of signalling is that an action taken by a relatively higher type seller will be more costly for a relatively lower type seller and, hence, the buyer can identify the true type of the seller through the seller's action.

Contingent contracts have both risk sharing and signalling dimensions. That is, such a contract transfers
both risk and information. Signalling models with contingent contracts may differ in several fundamental dimensions. Some models assume competitive buyers; i.e., there are a large number of buyers who will purchase the good at a competitive price, based on the buyers' knowledge of the quality of the good. In this case, the net trading surplus goes totally to the seller. Alternatively, some models assume a monopolistic buyer; i.e., there is a single buyer who seeks to obtain the "goods" as cheaply as possible, retaining a maximum share of surplus for himself. He must pay the supplier a price sufficient to induce him to sell the good; that "reservation" price is taken as a given. Models also differ as to "who moves first". Some analyses depict the market as functioning as if each buyer offers a menu of contracts to the suppliers, and each supplier chooses the best contract from among all those offered. The buyer is assumed to commit to the contracts he offers and he cannot change them after he has seen the suppliers' responses. Other analyses assume that each supplier offers a single contract to the buyers, who then have some response to make. For example, the contract may specify a price as well as other terms such as warranties, and the buyer merely accepts or rejects the contract. Many buyers may accept the same contract, or there may be a single good which goes to the first buyer
who accepts it. Alternatively, the seller may offer a contract without specifying a price and then accept the best price that is offered.

Signalling models with exogenous costly signals differ mainly as to the dimensions of the private information and the types of signals used. Early papers assume that this information is represented by one parameter so one signal can fulfill the signalling task. Later work assumes two dimensions of the private information, so two signals are necessary to do the job. These signals are assumed, as pointed above, to be costly and the costs are negatively correlated with the type.

There are a large number of papers involving signalling. A complete survey of this category is not the objective of this chapter. We primarily focus on those papers that deal with the issues of obtaining of capital from and sharing risks with the capital market, or communicating information to competitors in a product market. These papers are closely related to voluntary disclosure research because they provide an alternative way for firms to communicate with the market. Table 2-5 exhibits the main characteristics of some important papers that are selected based on the above criteria. The following is a summary of these papers.

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Insert Table 2-5 here
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The influences of information asymmetry on markets in which buyers are imperfectly informed about the quality of a collection of differentiated products that appear on the supply side of the market were analyzed first by Akerlof [1970]. Assuming that the asymmetrical information persists, Akerlof concludes that high quality sellers may withdraw their products from the market because their products cannot be distinguished and therefore are priced according to the average.

Spence [1973] [1976] deals with the other aspect of the same issue. He analyzes efforts by sellers to "tell" buyers about the products, and therefore, to change the initial asymmetric informational structure of the market. Spence defines these differentiating activities, as they pertain to information, as signalling from the seller's point of view. It also can be referred to as screening from the buyers' standpoint. Spence [1973] claims that education can signal productive potential if its costs are negatively correlated with that potential. Therefore, in general, better workers will acquire more education.

Akerlof and Spence's insightful findings were advanced by a series of excellent papers. Rothschild/ Stiglitz [1976] provides a seminal analysis of insurance
markets. Jaffee/Russell [1976] use signalling game to analyze a loan market. Salop/Salop [1976] focus on the labor market, analyzing the influence of unobservable employees' characteristics -- the probability of quitting the job. Akerlof [1976] presents four examples about the use of exogenous signals (indicators) to predict the behavior of economy and individuals to resolve information asymmetry problems.

Following the above antecedents, Leland/Pyle [1977] analyze signalling in financial markets. In financial markets, informational asymmetries are particularly pronounced. It is commonly recognized that entrepreneurs possess "insider" information about their own projects for which they seek financing. In their paper this information is modelled as private knowledge about the expected value of the risky project. It is also assumed that there is no credible way the entrepreneur can convey this information directly to other potential shareholders. However, the potential shareholders will respond to a signal by the entrepreneur regarding his evaluation of the expected value if they know that it is in the self-interest of the entrepreneur to send true signals. The signal analyzed is the retained ownership of the entrepreneur. The market perceives the expected value of the project to be a function of the signal, and the equilibrium valuation function
is explicitly derived. The properties of this function show that the greater the entrepreneur's willingness to take a personal stake in the project, the more investors are willing to pay for their share of it. These conclusions have been supported by empirical observations.

Milgrom/Roberts [1982] use a signalling model to explain limit pricing. They model an entry game in which neither the established firm nor the potential entrant is perfectly informed as to the other firm's unit cost. In such a situation, the pre-entry price may become a signal regarding the established firm's costs, which in turn are a determinant of the post-entry price and profits for the entrant. Thus, the relationship that a lower price (by signalling lower costs) tends to discourage entry emerges endogenously in equilibrium. Hence, limit-pricing can be an equilibrium behavior, with the established firm attempting to influence the entry decision by charging a pre-entry price which is below the simple monopoly level. However, since the entrant will, in equilibrium, recognize the incentives for limit-pricing, its expectations of the profitability of entry will not be consistently biased by the established firm's behavior. Then, depending on the particular equilibrium that is established and the parameters of the model, the probability of entry may fall short of, equal to, or even exceed what it would be if there
were complete information, and thus no limit pricing. Myers/Majluf [1984] develop an equilibrium model in which a manager's issue-invest decision may signal his private information about the firm's value. The model considers a firm that has assets-in-place with a valuable real investment opportunity. The firm has to issue common shares to raise part or all the cash required to undertake the investment project. If managers have insider information there must be some cases in which that information is so favorable that management, if it acts in the interest of the old shareholders, will refuse to issue shares even if it means passing up a good investment opportunity. Investors, aware of their relative ignorance, will reason that a decision not to issue shares signals "good news". The news conveyed by an issue is bad or at least less good. This affects the price investors are willing to pay for the issue, which in turn, affects the issue-invest decision. Under reasonable simplifying assumptions, the paper solves the equilibrium share price conditional on the issue-invest decision. Of course, it assumes rational investors and a rational firm which bases its issue-invest decision on the price it faces. The results can explain several aspects of corporate financing behavior, including the tendency to rely on internal sources of funds and to prefer debt to equity if external financing is required.

Miller/Rock [1985] extend the standard finance model of the firm's dividend/investment/financing decisions by allowing the firm's managers to know more than outside investors about the true state of the firm's current earnings. The extension endogenizes the effects of dividend announcements, which, in a world of rational expectations, serve as signals for the market to deduce the unobserved information about firm's current earnings. The cost of signalling that attribute to the market by increasing dividends is the foregone use of the funds in productive investment. This cost of signalling any specified level of earnings will be higher, the lower the level of earnings actually achieved.

Trueman [1986] is a signalling model which tries to explain why managers often release earnings forecasts prior to actual earnings announcements. It would appear that managers should at best be indifferent to such a release given that the actual earning will be disclosed at a future date. The paper argues that the forecast release gives investors a more favorable assessment of the manager's ability to anticipate economic environment changes, and to adjust production plans accordingly. Hence, in this model, a voluntary forecast release serves as a signal about the manager's talent, which is, thereby translated into a higher firm market value. In other
words, the manager's motivation to release his earnings forecast stems not from his desire to inform investors about his revised expectation for the period's earnings, but from his desire to inform them that he has received new information about the period's earnings. This means that the manager will be just as willing to release bad news as he is to release good news. In turn, this implies that the average price change at the time of forecast release will be positive.

Hughes [1986] extends Leland/Pyle's [1977] model to a bivariate signalling model. At issue is the information asymmetry between investors and the issuer of an initial public offering about the value of the security. To avoid market failure, a solution is proposed in which the issuer makes a disclosure about firm value that is verified by an investment banker. The investment banker implicitly enters into a contingent contract with investors which imposes a penalty if the ex post observable cash flow indicates fraudulent disclosure. The feature of the model is that the private information has two elements: the expected value and the variance of the future cash flows. To signal both elements, retained ownership as one signal is not enough. Hence the second signal, direct disclosure, is used to complete the task. The equilibrium is solved under the assumptions of exponential utility,
normal distribution, and an exogenously given disclosure cost function.

Grinblatt/Hwang [1988] pursue the same issue as Hughes [1986] except that in their model, the issuer uses retained ownership and underpricing as two signals to communicate his private information about the mean and the variance. The equilibrium valuation formula and underpricing are solved assuming a mean/variance utility function and bivariate value of the variance.

Gertner/Gibbons/Scharfstein [1988] develop a threeplayer signalling model, which analyzes an informed firm's choice of financial structure when the financing contract is observed not only by the capital market but also by a second uninformed party, such as a competing firm. The informed firm's gross profit is endogenous, because the second party's action depends on the transaction it observes between the informed firm and the capital market. The main result of this two-audience signalling model is that the "reasonable" capital-market equilibria maximize the ex ante expectation of the informed firm's endogenous gross product-market profits. In this sense the character of capital-market equilibrium is determined by the structure of the product-market. Thus, it may be misleading to analyze the firm's activities in the financial market separately from its activities in the product market. In
addition, the paper shows that, generically, either all the reasonable equilibria are separating or all the reasonable equilibria are pooling. This is in contrast to earlier work on the information content of financial structure and to more recent work on refinement in signalling games, both of which focus on separating equilibria. Hence the paper claims that full disclosure need not be reasonable equilibrium behaviour.

## Summary of signalling models

The representative signalling papers summarized above show the following. First, instead of the focus in most disclosure papers (that firms may have incentives to withhold their private information), signalling papers focus on the fact that firms may have incentives to communicate their private information to relevant parties. Interestingly, one firm's incentives to reveal its information may be created by another firm's incentives to withhold information. In order to separate a firm from another firm with "bad characteristics", the firm has incentives to reveal information about its true state. Second, when credible disclosure is impossible, firms may use indirect ways to signal their private information. Thus, signalling and disclosure are alternative methods for communicating private information. Third, signalling
is costly. Therefore, if credible direct communication is costless, then it will be used instead of costly signalling. However, if credible direct communication is costly, then signalling and direct disclosure are alternative costly communication devices. In that setting, identifying the communication device used.by a privately informed firm is a matter of identifying his equilibrium choice.

### 2.7 Empirical and Behavioral Research in Voluntary Disclosure

Direct tests of the results derived from the analytical disclosure models are rare because of the difficulty in determining when a manager is withholding information. In addition, even if one believes that a manager is withholding information, one cannot verify whether the undisclosed information is "good" news or "bad" news if the information is never disclosed. Hence, empirical analyses of managers' behaviour are largely restricted to the examination of the timing of the release mandated accounting reports, and the examination of market reaction to "missing" announcements that were expected by the market based on the firm's traditional disclosure behavior. The theoretical justification for a possible connection between withholding and delaying disclosure of information
is provided by Verrecchia [1983] and Jung/Kwon [1988]. Verrecchia [1983] suggests a generalization of his model to allow the proprietary cost to be a function of time. Jung/Kwon [1988] assume the probability that the manager has received information is an increasing function of time. Thus, the manager's decision to withhold information may change as time elapses. This change results in an observable delay of information disclosure.

The papers that examine the timing of firms' disclosures are important to voluntary disclosure research for the following reasons. First, if firms have incentives to withhold information, but disclosure of this information is mandated, then we may observe systematic delays in the firms' disclosures. For example, firms may withhold "bad" news as long as possible, until the due date of the reporting requirement. Second, if the disclosure is not mandatory, then an investigation of the relative timing and quantity of "good news" versus "bad news" may reveal something about firms' disclosure behavior. Third, examining the market's reaction to the timing of firms' reporting may provide evidence of the market's perception of firms' disclosure strategies. For example, such an examination may reveal whether the market interprets a "non-report" as a signal of forthcoming "bad" news. Pastena and Ronen [1979] empirically test the impli-
cations of the existence of a disincentive to produce and disseminate negative information. They do so by examining the extent of delay in the release of negative foreknowledge until the time such disclosures are forced on management as a result of the annual audit or through other media uncontrolled by management. They define information hardness as the probability of imminent disclosure by sources uncontrolled by management or as a result of an audit or both. The empirical results provide support for the hypothesis that: (i) managers act as if they attempt to delay the dissemination of negative information, relative to positive information; (ii) they act as if they disclose primarily soft positive information as contrasted with soft negative information; and (iii) they disclose negative information essentially only after such information becomes hard. They conclude that managers have sufficient discretion over the timing of the generation and dissemination of negative information so that there is a planned delay of negative soft information.

Kross [1982] explores whether a later than expected earnings disclosure is perceived as a sign of bad news by the capital market. The test is conducted after a determination is made for a sample of firms that bad news is reported later than good news. It finds that later earnings announcements have a higher probability of containing
bad news than do early announcements for the sample of firms tested. It also discovers that the shares of late reporting firms earn lower residual returns than early reporting firms during this period. These time effects are still evident when news effects are controlled.

Patell and Wolfson [1982] documented systematic patterns in the exact timing of announcements in relation to the hours of operation of the major stock exchanges. They test the "market wisdom" that good news is released during trading while bad news is held until after the market closes. The statistical analysis of earnings and dividend announcements yields results that are consistent with the conjecture that the likelihood of "bad news" disclosures increases after the close of trading for the day. The relative proportion of announcements of increased earnings or dividends was significantly higher during trading than after trading. The price changes were more likely to be positive for during-trading releases, while there was a marked shift toward negative price changes for after-trading announcements. This stock price response may contribute to an interpretation of the systematic timing behavior as an attempt to reduce the public exposure of unfavourable events.

Givoly and Palmon [1982] present evidence on the timeliness of annual earnings announcements in the United

States. They analyze its possible determinants and examine the relationship between the information content of the accounting report and its timeliness. Specifically, they find that announcements containing bad news tend to be delayed. Investigation of the relationship between company characteristics and timeliness indicates that size is inversely related and complexity of the audit is directly related to the reporting delay. However, the explanatory power of these variables is small.

Kross and Schroeder [1984] examine both the association between quarterly announcement timing (early or late) and the type of news (good or bad) reported, and the relationship between stock returns and timing around the earnings announcement date. The objective is to determine whether the association between announcement timing and stock returns persists after controlling for the sign and magnitude of the earnings forecast error and firm size. The results show that early quarterly earnings announcements (i) contain better news, and (ii) were associated with large abnormal returns relative to late announcements. These results hold independent all controllable effects mentioned above.

Chambers and Penman [1984] provide descriptive evidence on the relationship between timeliness of earnings reports and stock price behaviour surrounding their
release. They find some relationship between the time lag in reporting and return variability at the report date for reports of relatively small firms bearing good news. Timely interim reports of small firms which bring good news are associated with higher price reactions than are those with longer time lags. This is not observed for reports revealing bad news or reports for relatively large firms. They also find that when reports are published earlier than expected, they tend to have larger price effects than when they are published on time or later than expected. Unexpectedly early reports are characterized by good news, whereas unexpectedly late reports tend to bear bad news. When firms miss their expected reporting dates, the market interprets this as bad news.

Penman [1980] examines voluntary forecast disclosure to provide evidence relevant to the following two issues. The first issue deals with information content -- do voluntary earnings forecasts convey information to investors about the firms which publish them? The second issue deals with full disclosure -- does voluntary forecast disclosure result in the publication of only a subset of the earnings forecast information potentially available, and if so, what characterizes that subset? The results of the tests with respect to the information content issue indicate that corporate earnings forecasts, on average,
possess information relevant to the valuation of firms. With respect to the full disclosure issue, the tests indicate that the returns on sample securities of forecasting firms during the fiscal year in which the forecast is made are, on average, higher than those on the market as a whole, other things being held constant. It appears that firms with relatively poor earnings prospects and relatively low security returns do not reveal their relative position through an earnings forecast.

Leftwich, Watts, and Zimmerman [1981] investigate the economic incentives of managers to provide interim reports voluntarily. They analyze why corporations choose a particular reporting frequency for external purposes. They explore whether the monitoring process associated with issuing capital to parties outside the firm can explain why managers exceed minimum reporting requirements. Their results suggest that reporting frequency is connected with the choice of Stock Exchange, firm's reporting history, and firm's capital structure. However, the results are not strong.

In the behavioural accounting research literature, Gibbins, Richardson, and Waterhouse [1990] present interview data regarding various aspects of firm disclosure. Their informants viewed the output of the disclosure process as a set of components, including the particular
information disclosed and a variety of related management activities. This set of outputs is influenced by several variables, which they categorize as the firm's disclosure position and its antecedents, specific disclosure issues faced by the firm, external consultants and advisors, and structure. They theorize that firms develop a stable internal preference for the way in which disclosure is managed. Two dimensions of a firm's disclosure position are identified -- ritualism and opportunism. The former refers to a set of internal behavioral patterns characterized by a propensity toward uncritical adherence to prescribed norms. The latter refers to a propensity to seek firm-specific advantage in the disclosure of information. Opportunistic disclosure behaviour involves an attempt by the firm to closely manage the disclosure process, creating and taking advantage of opportunities as they arrive.

## Summary of Empirical Results:

As shown by the above summary, empirical research has provided only indirect evidence with respect to the results derived from analytical models. The empirical results show that managers withhold some information and disclose others. In particular, they can manipulate the timing of disclosures and thereby affect the impact of the information release. On the other hand, empirical evi-
dence suggests that the market is rational in interpreting the observed disclosure decision of firms. When the market evaluates any information disclosed, timeliness and other disclosure characteristics are taken into account. These results are consistent with most recent analytical findings.

However, the empirical research to date has provided limited information about the issue. There is an obvious imbalance between analytical modelling and empirical investigation.

Table 2-1: Two-player Disclosure Models

| Papers |  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Grossman/H. | [1980] | TCD | CMV | - | NDC | - | - | DT | VSD | FD |
| Milgrom | [1981] | TID | CMV | - | NDC | - | - | CT | VSD | FD |
| Jovanovic | [1982] | TCD | CMV | - | - | EXC | - | CT | VSD | PD |
| Verrecchia | [1983] | TCD | CMV |  | - | EXC | - | CT | VSD | PD |
| Dye (1) | [1985a] | TCD | CMV | - | NDC | - | NPI | CT | VSD | PD |
| (2) | [1985a] | TCD | U | - | - | ENC | - | CT | VSD | ND |
| Dye | [1985b] | TCD | CMV | - | - | ENC | - | CT | VDP | PD |
| Dye | [1986] | TID | CMV | - | - | EXC | - | CT | VSD | ALL |
| Jung/Kwon | [1988] | TCD | CMV | - | NDC | - | NPI | CT | VSD | PD |
| Verrecchia | [1990] | TCD | CMV | - | - | EXC | - | CT | VSD | PD |

(1) TCD: Truthful Complete Disclosure.

TID: Truthful Incomplete Disclosure.
(2) CMV: manager maximizes Current Market Value of the firm.
EPV: manager maximizes the End-of-Period Value of the initial shareholders' equity in the firm.
U: manager is Utility maximizer.
(3) CMF: manager must obtain Capital Market Funds.
(4) NDC: No Disclosure Costs (non-proprietary information).
(5) EXC: EXogenous Costs of proprietary information. ENC: ENdogenous Costs of proprietary information.
(6) NPI: positive probability of No Private Information.
(7) BT: Binary Types.

DT: Discrete Types.
CT: Continuum Types.
(8) VSD: Voluntary Signal Disclosure.

VDP: Voluntary Disclosure Policy.
(9) FD: Full Disclosure equilibrium.

ND: Non-Disclosure equilibrium.
PD: Partial Disclosure equilibrium.
ALL: FD/ND/PD

Table 2-2: Oligopolistic Models


Table 2-3(1): Ex Ante Equilibrium Strategies


Table 2-3(2): Ex Post Equilibrium Strategies

| Cournot |
| :--- |
| Hemand Bertrand <br> L: D H: D <br> Cost L: ND <br> H: D H: ND <br> L: ND L: D |

ND: Non-Disclosure
D: Disclosure
H: Firms with Favourable Information
L: Firms with Unfavourable Information
**********************************************************
Papers
(1) (2)
(3) (4)
(5) (6) (7)
(8) (9)

Bhatta./R. [1983] TID EPV CMF - ENC - DT VSD PD
Lanen/Ver. [1987] TCD U - - EXC - CT VSD PD Darro./St. [1990] TCD EPV CMF - ENC - BT VSD FD/PD/ND Wagenhofer [1990] TCD CMV - - ENC - CT VSD FD/PD Chapter 3 [1990] TCD EPV CMF - ENC - CT VSD FD/PD

(1) TCD: Truthful Complete Disclosure.

TID: Truthful Incomplete Disclosure.
(2) CMV: manager maximizes Current Market Value of the firm.
EPV: manager maximizes the End-of-Period Value of the initial shareholders' equity in the firm.
U: manager is Utility maximizer.
(3) CMF: manager must obtain Capital Market Funds.
(4) NDC: No Disclosure Costs (nonproprietary information).
(5) EXC: EXogenous Costs of proprietary information.

ENC: ENdogenous Costs of proprietary information.
(6) NPI: positive probability of No Private Information.
(7) BT: Binary Types.

DT: Discrete Types.
CT: Continuum Types.
(8) VSD: Voluntary Signal Disclosure.

VDP: Voluntary Disclosure Policy.
(9) FD: Full Disclosure equilibrium.

ND: Non-Disclosure equilibrium.
PD: Partial Disclosure equilibrium.

Table 2-5: Signalling Models
****************************************************************
Papers
(1) (2) (3) (4)
(5)

| Akerlof | $[1970]$ | - | $A$ | - | DT | - |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Spence | $[1973]$ | EXS | B | - | CT | EDUCATION |
| Rothsch./Stigl. | $[1976]$ | CON | C | UMF | BT | CONTRACT |
| Jaffee/Russell | $[1976]$ | CON | D | UMF | BT | CONTRACT |
| Salop/Salop | $[1976]$ | CON | E | UMF | BT | CONTRACT |
| Akerlof | $[1976]$ | EXS | F | - | BT | INDICATOR |
| Leland/Pyle | $[1977]$ | EXS | G | - | CT | RETAINED OWNERSHIP |
| Milgrom/Roberts | $[1982]$ | EXS | H | - | CT | PRE-ENTRY PRICE |
| Myers/Majluf | $[1984]$ | EXS | I | - | CT | INVEST/FIN. POLICY |
| Miller/Rock | $[1985]$ | EXS | J | - | CT | DIVIDEND POLICY |
| Hughes | $[1986]$ | EXS | K | - | CT | RETAIN OW/DIR. DIS. |
| Grinblatt/Hwang | $[1988]$ | EXS | K | - | BT | RETAIN OW/UND. PRI. |
| Gertn./Gib./Sch $[1988]$ | EXS | J | - | BT RETAIN OW/DEBT |  |  |

(1) EXS: EXogenous Signal(s)

CON: CONtingent contract
(2) Private Information

A: quality of product or service
B: productivity
C: risk level of insurance
D: default risk level for lending
E: probability of a employee quitting
F: effort level/ability
G: mean value of risky project
H: cost
I: value of asset-in-place and new project
J: expected cash flows
K: mean and variance of a risky project
(3) UMF: Uninformed player Move First

IMF: Informed player Move First
(4) BT: Binary signals

DT: Discrete signals
CT: Continuum signals
(5) Signals

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## Chapter 3

## VOLUNTARY FINANCIAL

## DISCLOSURE IN AN ENTRY GAME

## WITH CONTINUA OF TYPES

### 3.1 Introduction

As part of the management process, managers constantly acquire information about their firm's future profitability. To the extent that this information is not known by others, we refer to it as private management information. If the firm's ownership is publicly traded, then this information will ultimately become publicly known (or obsolete) as investors receive the firm's quarterly and annual financial reports. In addition, managers can voluntarily reveal their private information by issuing reports, such as management forecasts of future earnings. We observe that, from time to time, managers issue such reports, but the criteria they use in determining when to make them is not well understood.

A key characteristic of the reporting of private management information is that managers do not always report their information and that they reveal (or withhold) both "good" and "bad" news. Several recent papers provide models of managers' voluntary disclosure decisions. The models are typically constructed so that managers do not always disclose or withhold their information, despite rational behaviour by both the privately informed managers and interested parties external to the firm. In disclosure models, it is assumed that the managers decide whether or not to disclose their information,
but they do not lie if they choose to disclose. The motivation not to lie is not explicitly modeled, but is derived from either the assumed threat of significant penalties if managers are "caught" mis-representing their information or the assumed availability of a costless verification mechanism. Consequently, these models do not get into "signalling" issues. ${ }^{1}$

A detailed survey and classification of the existing literature in this research area is provided in the prior chapter. In this chapter, we provide a model to further develop the results of prior works. The main features and contributions of our model are as following.

Our model explicitly considers three players. We focus on the disclosure decision made by the privately informed manager of an "incumbent" firm (I) which is undertaking an investment in a product market for which it requires funds from the capital market. The consequences of his disclosure decision depend on the action of an opponent, termed the "entrant" (E), and the "capital market's" (M) valuation of the securities issued by the incumbent. Relating our model to the key dimensions we used to classify the analytical disclosure models in the

[^7]last chapter, we summarize our assumptions as follows:
(1) The manager seeks to maximize the expected end-of-period value of the initial shareholders' equity in $I$.
(2) The manager must obtain funds from $M$ and the "cost" of those funds depends on M's beliefs regarding the manager's private information and the action that $E$ will take.
(3) The information is proprietary in the sense that its disclosure can influence I's end-of-period cash flow.
(4) The impact of disclosure on I's end-ofperiod cash flow is not exogenous, but instead depends on the E's action.
(5) E employs a simple decision rule: E "enters" if, and only if, the expected level of I's private information (based on the information available to E) exceeds E's "break-even" point. "Entry" by E reduces I's end-of-period cash flow. There is no other explicit modelling of the product market.
(6) The probability that the manager has no private information is assumed to be zero. Alternatively, we can permit the probability of no private information to be positive, and then assume that the manager can costlessly communicate whether or not he has received information. That is, no information can be distinguished from nondisclosure.
(7) We consider a continuum of possible manager "types", i.e., a continuum of possible private information signals.
(8) The manager makes his disclosure decision after he observes his private information.

Our basic model is very similar to Wagenhofer [1990] except that in his model I requires no funds from the
capital market and the manager seeks to maximize the current market value of the firm. With respect to two dimensions, our model is the same as that of Darrough and Stoughton [1990], but we differ from their model in that they only consider binary private information -- the manager gets either "good" news or "bad" news. A key feature that distinguishes our analysis from both of these papers, and most other disclosure models, is that we introduce private entrant information. That is, we allow for the possibility that I may not know E's break-even point and, therefore, does not know what beliefs will induce E to enter. The major impact of this change is to eliminate equilibria in which I partially discloses his information and E plays a mixed strategy. In the model in which E has private information, $E$ plays a pure strategy, whether $I$ fully discloses his private information or not. Section 3.2 presents the basic elements of our disclosure model, including a statement of the sequence of events. Section 3.3 identifies the alternative disclosure policies that I might employ, characterizing his expected wealth as a function of his information for both the full disclosure and partial disclosure policies. This characterization is provided for three different assumptions with respect to I's knowledge about E's break-even point. Section 3.4 identifies the conditions under which full
disclosure equilibria exist, providing explicit characterization of these conditions in terms of the amount of capital required by $I$ and the costs incurred if $E$ enters I's market. Section 3.5 extends this characterization to the identification of the conditions under which partial disclosure equilibria exist. Section 3.6 discusses some of the issues that arise when there are multiple equilibria, and presents the implications of some refinements of the basic sequential equilibrium concept. Finally, section 3 . 7 provides some concluding remarks.

An interesting aspect of the equilibria identified in this paper is that there are two types of full disclosure equilibria and two types of partial disclosure equilibria. The full disclosure equilibria differ only in the way in which $M$ and $E$ will respond to non-disclosure. In one type, non-disclosure will induce $M$ to provide the desired capital at the least favourable terms possible. In the other type, non-disclosure will induce $E$ to enter I's market with probability one. The partial disclosure equilibria differ with respect to the relationship between the market value of the disclosing and non-disclosing firms. In the "low" case, the market value of the nondisclosing firms is less than all disclosing firms, whereas in the "high" case, there are some disclosing firms that have lower market values than the non-disclosing
firms.
The key factor in the existence of the various equilibria is the relative importance to I of under-valuation by $M$ versus entry by $E$. The importance of under-valuation increases as the amount of capital required increases, whereas the importance of entry increases as entry costs incurred by the incumbent ${ }^{2}$ increase.

The other interesting results pertain to the refinement of possible multiple equilibria. Under a simplified distribution assumption, we proved that both full disclosure and partial disclosure will not fail the Cho and Kreps' intuitive stability criterion if they do exist. However, a full disclosure equilibrium will fail Grossman and Perry's perfect equilibrium criterion if there also exists a partial disclosure equilibrium. Furthermore, a partial disclosure equilibrium will fail the Grossman and Perry criterion when there exists another partial equilibrium which dominates the first.

[^8]
### 3.2 The Basic Model

We focus on the disclosure decision made by the manager of an incumbent firm that is about to invest in a new market. The manager acts on behalf of the firm's current equity-holders and I is used to denote both the manager acting in that capacity and the current equityholders. I is assumed to seek to maximize the expected end-of-period cash flow of the firm. The focus on "expected cash flow" implies risk neutrality, which can be motivated by an assumption that the current equity-holders are well-diversified investors ${ }^{3}$ and the risks associated with I's decisions are diversifiable. I's focus on the end-of-period value, as opposed to current market value, also results from diversification. As demonstrated by Feltham and Christensen [1988], well-diversified investors in a large economy can achieve an efficient allocation of resources and consumption without knowing each manager's firm-specific information as long as the manager of each firm in an investor's portfolio acts so as to maximize the "true value" of investor's equity.

The basic sequence of events are depicted in Table 3-

[^9]1. In addition to the incumbent (I), we consider the capital market (M) and a potential entrant (E).

Insert Table 3-1 here

The planned investment requires $k$ dollars of capital, which must be obtained from the capital market (M). The end-of-period cash flow of the firm (I's payoff) is a random variable $\tilde{x}$, from which $I$ will compensate $M$ for the funds supplied. I's private information about $\tilde{x}$ is represented by a random variable $\tilde{\mu}$; the realized value $\mu$ is I's type. We assume that $\tilde{\mu}$ is denominated so that its cumulative distribution function, denoted $\Phi(\mu)$, is defined on the unit interval $[0,1]$.

I's expected payoff if the investment is not undertaken is $\pi^{0} \geq 0$. If the investment in the new market is undertaken, then the payoff will be influenced by whether E, a potential competitor, chooses to enter the same market. If $E$ does not enter $(e=0)$, then $I$ will be a monopolist and the cumulative distribution function for $\tilde{x}$ given that $I$ is type $\mu$ is $F(x \mid \mu, 0), x \in[0, \infty)$. On the other hand, if $E$ does enter ( $e=1$ ), then $I$ will be a duopolist and the cumulative distribution function for $\tilde{\mathbf{x}}$ given that I is type $\mu$ is $F(x \mid \mu, 1), x \in[0, \infty)$. The posterior means in
both cases are assumed to be linear functions of $\mu ;^{4}$ in particular,

$$
\begin{aligned}
& \pi(\mu, 0)=E[x / \mu, 0]-\int_{0}^{\infty} x d F(x / \mu, 0)-a \mu+b \\
& \pi(\mu, 1)=E[x \mu, 1]-\int_{0}^{\infty} x d F(x \mid \mu, 1)-c \mu+d
\end{aligned}
$$

We make the following assumptions with respect to the payoff parameters:
(A.1) $a>0$ and $a-c \equiv \delta \epsilon[0, a]$. The expected payoff is an increasing function of $\mu$ (e.g., a bigger $\mu$ indicates a more favourable market) and there is a non-negative variable entry cost, $\delta$.
(A.2) $d-k \geq \pi^{0}$ and $b-d \equiv \Delta \epsilon\left[0, b-k-\pi^{0}\right]$. Entry by $I$ is desirable even if $\mu=0$ and $E$ enters, and there is a non-negative fixed entry cost, $\Delta$.
(A.3) $\delta>0$ and/or $\Delta>0$. There is a strictly positive entry cost.

Two special cases are of particular interest in subsequent analysis:

Variable entry cost: $d=b$ and $a-c=\delta \epsilon(0, a]$.

[^10]Fixed entry cost: ${ }^{5} c=a$ and $b-d=\Delta \epsilon\left(0, b-k-\pi^{0}\right]$. These relationships are depicted in Figure 3-1.

```
Insert Figure 3-1 here
```

Whereas I will enter the market no matter what information he has, E's entry costs are assumed to be such that he will only enter if his expectation about I's type $\mu$ is at least as large as his break-even point, denoted $\gamma$. We can motivate this by assuming that E too maximizes his expected end-of-period cash flow and that his value is also an increasing linear function of $\mu$, where $\mu$ represents information about demand in the market of interest.

Let $Y$ denote $M$ and E's information about $\bar{\mu}$ at the time they make their decisions, let their posterior beliefs with respect to $\tilde{\mu}$ be represented by the cumulative distribution function $\Phi(\mu \mid y)$, and let their posterior expectation with respect to $\tilde{\mu}$ be denoted $v(y)$. Hence,

$$
\begin{equation*}
v(y)=\int_{0}^{1} \mu d \Phi(\mu \mid y) \tag{3.2.1}
\end{equation*}
$$

E's break-even point is a random variable $\tilde{\gamma}$ with a prior cumulative distribution function $G(\gamma)$ defined on the unit interval. E learns his break-even point ( $\bar{\gamma}=\gamma$ )

[^11]prior to making his entry decision; he enters with certainty if $v(y)>\gamma$ and does not enter if $v(y)<\gamma .^{6}$ We allow for the possibility that $E$ may play a mixed entry strategy if he is indifferent between entering and not entering. Hence, we represent his strategy as a function of this posterior expectation $v$ with respect to I's type and his own break-even point:
\[

e(v, \gamma) $$
\begin{cases}-1 & \text { if } v>\gamma  \tag{3.2.2}\\ \in[0,1] & \text { if } v=\gamma \\ -0 & \text { if } v<\gamma\end{cases}
$$
\]

I and M do not observe E's break-even point prior to making their decisions. However, they do observe the information $y$ that $E$ receives about I's type and, therefore, know his posterior expectation $v(y)$. Consequently, from I and m's perspective, given posterior expectation $v$ and E's strategy e(•), the probability that E will enter is

$$
\begin{equation*}
p(v)-\int_{0}^{1} e(v, \gamma) d G(\gamma) \tag{3.2.3}
\end{equation*}
$$

Observe that there are two reasons why I and M may be

[^12]uncertain about whether E will enter. First, they may be uncertain about his break-even point. Second, even if they know $\gamma$ (because $G(\gamma)$ is concentrated on a single mass point), $E$ may be indifferent between entering/not entering and be playing a mixed strategy. These two perspectives play an important role in our subsequent analysis. We refer to cases in which $G(\gamma)$ is concentrated at a single mass point $\bar{\gamma}$ as ones in which e's break-even point is common knowledge, ${ }^{7}$ and to cases in which $G^{\prime}(\gamma)>0 \forall$ $\gamma \epsilon(0,1)$ as ones in which E's break-even point is not common knowledge. Observe that in the latter case, mixed strategies are of no consequence and the probability that E will enter is
\[

$$
\begin{equation*}
p(v)=G(v) \tag{3.2.4}
\end{equation*}
$$

\]

That is, the probability of entry is equal to the probability that E's type is less than E's expectation about $\mu$ given $y$. (If E's break-even point is common knowledge, then (3.2.4) defines the maximum entry probability given expectation v.)
$M$ and $E$ have the same information $Y$ about I's type when they make their decisions. This information consists of two elements: a report (or "no report") made by I regarding his type and the contract $\alpha$ offered to M in

[^13]return for $k$ units of capital. Let $m$ represent the report (message) sent by $I$ regarding his type and let $M(\mu)=$ $\{\mu, \mathrm{n}\}$ represent the set of possible reports that can be sent by $I$ if he is type $\mu$, with $m=\mu$ representing disclosure of his type and $m=n$ representing "no report". Observe that we do not allow I to lie about his type. This can be motivated by assuming that either there is a costless verification mechanism and I chooses whether to use that mechanism or there are penalties imposed by antifraud laws and detection mechanisms that are sufficient to deter $I$ from lying. An implication of this assumption is that $v(\mu, \alpha)=\mu$ for all $\alpha$.

A key issue is the nature of the contract offered by I to $M$ in return for capital $k$. Following Darrough and Stoughton [1990], in most of our analysis we assume that I obtains its capital by offering $M$ equity in the firm. ${ }^{8}$ An obvious alternative would be to issue debt, particularly if the debt is riskless (i.e., $F(k \mid \mu, e)=0, \forall \mu \in[0,1]$, $e \in\{0,1\})$. If debt is risky, then much the same issues aries as occur with the issuance of equity.

The equity contract is represented by $\alpha$, the share of

[^14]I's payoff to be received by M. M can either accept (r=0) or reject $(r=1)$ the contract, ${ }^{9}$ and we assume that $M$ will only reject a contract if, based on information $Y$, $M$ believes that the contract has a negative net present value. ${ }^{10}$ Let $v(v, p)$ denote the expected end-of-period cash flow of the firm given that $M$ has expectation $v$ about I's type and believes that $E$ will enter with probability p, i.e.,

$$
\begin{equation*}
V(v, p)=p \cdot \pi(v, 1)+(1-p) \cdot \pi(v, 0) \tag{3.2.5}
\end{equation*}
$$

This value determines the minimum share of I's payoff that M will accept in return for capital $k$, given expectation $v$ and entry probability $p$. We represent that minimum share by

$$
\begin{equation*}
\alpha^{*}(v, p)=k / V(v, p) \tag{3.2.6}
\end{equation*}
$$

[^15]
### 3.3 I's Strategy Choice

I's disclosure strategy is represented by $N \subset[0,1]$, the set of signals $\mu$ that will not be disclosed. The set of signals $\mu$ that will be disclosed are denoted $\mathrm{D}=[0,1]$ $\backslash N$. There are three basic kinds of disclosure strategies:
(i) Full Disclosure (FD): $\quad \mathbf{D}=[0,1], N=\varnothing$;
(ii) Full Non-disclosure (FN): $\mathbf{D}=\varnothing, N=[0,1]$; and
(iii) Partial Disclosure (PD): $D \subset[0,1], N=[0,1]$ $\backslash D$, where both $D$ and $N$ have positive measure.

I's Expected End-of-period Wealth:
I's expected end-of-period wealth, given signal $\mu$, contract $\alpha$, market response $r$, and entry probability $p$, is

$$
\mathrm{W}(\mu, \alpha, r, p)=(1-r) \cdot(1-\alpha) \cdot \mathrm{V}(\mu, p)+r \cdot \pi^{0} \quad(3.3 .1)
$$

Observe that $W(\cdot)$ is an increasing linear function of $\mu$ for every $\alpha \in[0,1), r \in[0,1)$, and $p \in[0,1]$.

Recall that if I discloses his type $\mu$, then $M$ and E's expectation is $\nu(y)=\mu$. In this case the probability that $E$ will enter is $p=G(\mu)$ and sequential rationality implies that the best contract that $M$ will accept is $\alpha^{*}(\mu, G(\mu))$. Consequently, I's maximum expected wealth, given disclosure of $\mu$ and an accepted equity contract, can be represented as

$$
\begin{align*}
W_{D}(\mu) & =W\left(\mu, \alpha^{*}(\mu, G(\mu)), 0, G(\mu)\right) \\
& =[1-k / V(\mu, G(\mu))] \cdot V(\mu, G(\mu)) \\
& =V(\mu, G(\mu))-k \tag{3.3.2}
\end{align*}
$$

That is, I's expected wealth is equal to his expected payoff minus the cost of the capital invested.

Assume that if I does not disclose his information, then he will offer the least cost contract that $M$ would accept given $M$ and E's expectation with respect to I's type given non-disclosure. In that case, given non-disclosure expectation $v$, the contract offered and accepted is $\alpha^{*}(v, p(v))$ and type $\mu^{\prime}$ s expected wealth can be expressed as

$$
\begin{align*}
& W_{N}(\mu, v, p(v)) \\
= & W\left(\mu, \alpha^{*}(v, p(v)), 0, p(v)\right) \\
= & {[1-k / v(v, p(v))] \cdot V(\mu, p(v)) } \tag{3.3.3}
\end{align*}
$$

For any given expectation $v$ and entry probability $p(v), W_{N}$ is an increasing linear function of $\mu$.

I selects the disclosure choice that will maximize his expected end-of-period wealth. Equations (3.3.2) and (3.3.3) specify type $\mu$ 's expectations for disclosure and non-disclosure, respectively, given his beliefs about how $E$ and $M$ will respond to his choice. Later we examine the intervals over which $I$ will choose to disclose ( $W_{D}>W_{N}$ ) and not to disclose $\left(W_{D} \leq W_{N}\right)$ his type. However, first we
consider the nature of $W_{0}$ under specific distributional assumptions with respect to E's break-even point.

## I's Expected wealth from full disclosure:

In this analysis we assume that the prior beliefs about E's break-even point are characterized by a beta distribution on the unit interval, ${ }^{11}$ i.e.,

$$
G(\gamma)-\int_{0}^{\gamma} \beta_{0} \cdot t^{\beta_{1}-1} \cdot(1-t)^{\beta_{2}-1} d t \quad \forall \gamma \in(0,1)
$$

where $\beta_{0}$ is the normalizing constant and $\beta_{1}, \beta_{2}>0$ are exogenous parameters. The mean and variance of this distribution are

$$
\begin{gathered}
\bar{\gamma}=E[\tilde{\gamma}]-\frac{\beta_{1}}{\beta_{1}+\beta_{2}} \\
\operatorname{var}[\bar{\gamma}]-\frac{\beta_{1} \cdot \beta_{2}}{\left(\beta_{1}+\beta_{2}\right)^{2} \cdot\left(\beta_{1}+\beta_{2}+1\right)}
\end{gathered}
$$

We restrict our analysis to three special cases:
$\bar{\gamma}$ Common Knowledge: $\quad \beta_{1}=n \bar{\gamma}, \beta_{2}=\mathrm{n}(1-\bar{\gamma})$, and n $\overrightarrow{\tilde{\gamma}} \infty$, so that $\operatorname{var}[\tilde{\gamma}] \rightarrow 0$ and $\tilde{\gamma}=\bar{\gamma}$ with probability one.

Uniform Distribution: $\beta_{1}=\beta_{2}=1$, so that $G(\gamma)=$ $\gamma$ and $\bar{\gamma}=1 / 2$

Unimodel Distribution: $\beta_{1}, \beta_{2}>1$ (and finite). In the uniform and unimodel cases, $G^{\prime}(\gamma)>0 \forall \gamma \in(0,1)$,

[^16]and we refer to these as situations in which E's breakeven point is not common knowledge.

Figure 3-2 depicts $W_{D}$ for each of the three cases, and is represented by the dark line denoted "ABCD". Observe that $W_{D}$ is bounded below by $\pi(\mu, 1)-k$ and above by $\pi(\mu, 0)-k$. These are the expected net payoffs given that $E$ enters or does not enter, respectively. ${ }^{12}$

Observe that when E's break-even point $\bar{\gamma}$ is common knowledge, $W_{D}$ is a "Z-shaped" broken-line. It is equal to $\pi(\mu, 0)-k$ for $\mu \epsilon[0, \bar{\gamma})$, and then drops to $\pi(\mu, 1)-k$ for $\mu \epsilon(\bar{\gamma}, 1]$. The discrete drop is caused by the increase in E's entry probability from zero to one as the signal $\mu$ shifts from being less than E's break-even point to exceeding it.

In the two cases in which E's break-even point is not common knowledge, $W_{D}$ is strictly between the two bounds. In the uniform distribution case, $W_{D}$ is concave and, in Figure 3-2(b), "B" $=W_{D}\left(\mu^{*}\right)$ identifies the interior maximum. In the unimodel distribution case, $W_{D}$ is initially concave and then convex, producing an "S-shaped" curve.

[^17]In Figure 3-2 $(c)$, " $B "=W_{D}\left(\mu^{*}\right)$ is the local interior maximum and "C" $=W_{0}\left(\mu_{*}\right)$ is the local interior minimum. (Recall, from (3.2.4), that $\mathbf{p}(\mu)=G(\mu)$. )

Lemma 3.3.1: ${ }^{13}$ If $I$ and M's belief about $\tilde{\gamma}$ is a uniform distribution on ( 0,1 ), then $W_{0}$ is concave (strictly concave if $\delta>0$ ). If their belief is a beta distribution with $\beta_{1}, \beta_{2}>1$, then:
(a) There exists a type $\mu_{0} \epsilon(0,1)$ such that $W_{0}$ is strictly concave on the interval $\left(0, \mu_{0}\right)$ and strictly convex on the interval ( $\mu_{0}, 1$ ); and
(b) There exist types $\mu^{*}$ and $\mu_{*}$ such that $\mu^{*}$ $\epsilon\left(0, \mu_{0}\right)$ is a local interior maximum and $\mu_{*}$ $\epsilon\left(\mu_{0}, 1\right)$ is a local interior minimum.

## Characterization of Disclosure and Non-disclosure

## Sets:

The "Z" and "S" shapes of $W_{D}$ in the common knowledge and unimodel distribution cases (see Figure 3-2), and the linearity of $W_{N}$, implies that, in these cases, $W_{N}$ cannot intersect $W_{D}$ more than three times. Furthermore, the concavity of $W_{D}$ in the uniform distribution case and the linearity of $W_{N}$ implies that, in this case, $W_{N}$ cannot intersect $W_{D}$ more than twice.

[^18]> Lemma $3.3 .2:^{14}$ If $\tilde{\gamma}=\bar{\gamma}$ is common knowledge or $\tilde{\gamma}$ has a unimodel beta distribution, then $W_{N}$ intersects $W_{D}$ at most three times. If $\tilde{\gamma}$ has a uniform distribution, then $W_{N}$ intersects $W_{D}$ at most twice.

To illustrate this result, Figure 3-3 depicts a case in which $\tilde{\gamma}=\bar{\gamma}$ is common knowledge and the entry cost is variable. The dark line "UVWXYZ" represents the maximum of $W_{D}$ and $W_{N}$ at each $\mu$ if, and only if, $W_{D}(\mu)>W_{N}(\mu, v, p(v))$.

```
Insert Figure 3-3 here
```

An important implication of Lemma 3.3.2 is that, in the common knowledge and unimodel cases, if N and D are non-empty sets with positive measure, then $N$ and $D$ consist of intervals and the number of intervals in each set is no more than two. Furthermore, if both $N$ and $D$ consist of two intervals, then there exist three types $\mu_{1}<\mu_{2}<\mu_{3}$ at which $W_{N}$ intersects $W_{0}$. These types are such that: ${ }^{15}$

[^19]\[

$$
\begin{aligned}
& \mathrm{N}=\mathrm{N}_{1} \cup \mathrm{~N}_{2}, \text { with } \mathrm{N}_{1}-\left[0, \mu_{1}\right], \mathrm{N}_{2}=\left[\mu_{2}, \mu_{3}\right] \\
& \mathrm{D}=\mathrm{D}_{1} \cup \mathrm{D}_{2}, \text { with } \mathrm{D}_{1}=\left(\mu_{1}, \mu_{2}\right), \mathrm{D}_{2}=\left(\mu_{3}, 1\right] \quad \text { (3.3.4) }
\end{aligned}
$$
\]

In the common knowledge case, $\mu_{2}=\bar{\gamma}$, whereas in the unimodel distribution case

$$
0<\mu_{1}<\mu^{*}<\mu_{2}<\mu_{*}<\mu_{3}<1 .
$$

In the uniform distribution case, $N$ consists of two intervals and such that:

$$
\begin{gather*}
0<\grave{\mu}_{1}<\mu^{*}<\mu_{2}<1 \\
N=N_{1} \cup N_{2}, \text { with } N_{1}=\left[0, \mu_{1}\right], N_{2}-\left[\mu_{2}, 1\right] \\
D=\left(\mu_{1}, \mu_{2}\right) \tag{3.3.5}
\end{gather*}
$$

Observe that if a non-trivial non-disclosure region $N$ exists, then it always includes types close to zero. However, it does not necessarily follow that the disclosure region D always contains types close to one.

### 3.4 Full Disclosure Equilibria

The basic equilibrium concept used in this paper is that of a sequential equilibrium. ${ }^{16}$ In our disclosure game, a sequential equilibrium is represented by $\Gamma=$ ( $N, \alpha, r, e, v)$. The first element is I's disclosure policy (characterized by his non-disclosure set $N$ ). The second $(\alpha)$ is a function specifying the contract offered to $M$ by each type $\mu \in[0,1]$. The third (r) is a function specifying the probability with which $M$ will reject each possible contract given each possible report. The fourth (e) is E's entry probability given each possible break-even point and each possible report and contract from $I$. The fifth (v) is a function specifying $M$ and E's expectation about I's type given each possible report and contract from I.

Sequential equilibria must have sequentially rational strategies that are based on consistent beliefs. Consistency of beliefs implies that the posterior expectation $v$ is computed by Bayes' theorem if possible. Sequential rationality requires that $E$ enter if his posterior expectation $v$ is less than his break-even point and that $m$ accept a contract if $v$ and $\alpha$ are such that he expects to earn a positive profits. $I$, on the other hand, must have

[^20]no incentive to disclose his information if $\mu \in N$ and have no incentive to offer a contract other than $\alpha(\mu)$.

In this section we focus on full disclosure equilibria, i.e., $N=\varnothing$. The following lemma specifies the basic condition that must be specified for the existence of a full disclosure equilibrium.

Lemma 3.4.1: A full disclosure sequential equilibrium exists if, and only if, there is an expectation $v$ such that

$$
\begin{equation*}
W_{D}(\mu) \geq W_{N}(\mu, v, G(v)) \quad \forall \mu \in[0,1] \tag{3.4.1}
\end{equation*}
$$

where $G(v)$ is the maximum entry probability that is consistent with (3.2.2) and (3.2.3).

This lemma establishes that, if the requisite expectation $v$ exists, then a full disclosure equilibrium can be sustained by letting $M$ and $E$ hold expectation $v$ if $I$ does not disclose his information, no matter what contract he offers. Furthermore, the necessity part of the lemma allows us to identify full disclosure equilibria by considering a single non-disclosure expectation for all contracts.

We refer to a full disclosure equilibrium that is sustained by non-disclosure expectation $v$, satisfying (3.4.1), as an FD-v equilibrium.

Initially we consider two extreme cases in which full disclosure is the only sequential equilibrium. Later we
consider conditions under which the existence of full disclosure equilibria depend on the parameter values.

## Exogenous Entry Choice:

E's action is irrelevant to I's disclosure policy choice if E's probability of entry is independent of E's belief about $\mu$. This occurs, for example, if E's type is known to be equal to either zero or one. In the first case, $E$ will enter no matter what $I$ discloses and in the latter case $E$ will not enter no matter what I discloses. Observe that, in this setting, I would like to have $M$ hold as high an expectation of $\tilde{\mu}$ as possible, since this will give I the most favourable contract terms. However, as is well-known, in equilibrium it is not possible for $I$ to withhold information in an attempt to increase M's expectations. To see that this is the case, consider any measurable set $N \subset[0,1]$. If $M$ believes that $I$ is employing this strategy, then the best contract that 1 can obtain with non-disclosure is $\alpha^{*}(v, p)$, where

$$
\begin{equation*}
v=\bar{\mu}(N) \equiv \int_{N} \mu \mathrm{~d} \Phi(\mu) / \Phi(N) \tag{3.4.2}
\end{equation*}
$$

However, for every $\mu>v, \mu \in N, V(\mu, p)-k>W_{N}(\mu, v, p)$, for any exogenous probability of entry $p \in[0,1]$. That is, the better types in any non-disclosure "pool" always wants to let the market know that their firm is worth more than the
average member of that "pool".


#### Abstract

Proposition 3.4.2: ${ }^{17}$ If the probability of entry $p$ is independent of E 's beliefs, then the only sequential equilibrium is a FD-0 equilibrium, i.e., a full-disclosure equilibrium in which M holds belief $v=0$ if $I$ does not disclose.


## Capital Obtained By Issuing Riskless Debt:

If $I$ can issue riskless debt to obtain the required $k$ units of capital, then the current market value of his firm is immaterial to his disclosure decision. In this case, $I$ is only concerned with E's beliefs. In particular, $I$ would like $E$ to believe that $\mu$ is less than $\gamma$, so as to avoid the negative impact of E's entry into his market.

If E's type is not common knowledge, then $I$ is motivated to always reveal $\mu$ in order to minimize the probability that $E$ will enter. To see this, consider a measurable non-disclosure set $N \subset[0,1]$ and let $v=\bar{\mu}(N)$. Observe that the poorer types in the pool, i.e., all $\mu<$ $v, \mu \in N$, prefer to disclose their type because $G(\mu)<G(v)$.

[^21]
#### Abstract

Proposition 3.4.3: ${ }^{18}$ If $I$ can obtain his capital by issuing riskless debt and E's type is not common knowledge, then the only sequential equilibrium is an FD-1 equilibrium, i.e., a full disclosure equilibrium in which $E$ holds belief $v$ = 1 if $I$ does not disclose.


If E's type is common knowledge ( $\bar{\gamma}=\bar{\gamma}$ ), then $I$ will reveal his private information if $\mu<\bar{\gamma}$ and $v>\bar{\gamma}$ and will not disclose it if $\mu>\bar{\gamma}$ and $v<\bar{\gamma}$. Consequently, any equilibrium disclosure policy must be such that either: (i) $N \subset[\bar{\gamma}, 1]$ or (ii) $[\bar{\gamma}, 1] \subset N$ and $\bar{\mu}(N) \leq \bar{\gamma}$. In (i), any disclosure strategy such that $N \subset[\bar{\gamma}, 1]$ is an equilibrium strategy -- non-disclosure induces $E$ to enter and all types in the non-disclosure set are indifferent between disclosure and non-disclosure. This equilibrium always exists when E's type is common knowledge and I can issue riskless debt. In (ii), if $\mu \in[\bar{\gamma}, 1]$, then $I$ hides his good news through non-disclosure, and non-disclosure does not induce E to enter because I also does not disclose for sufficient types worse than $\bar{\gamma}$. This equilibrium exists if, and only if, $\bar{\gamma} \geq \bar{\mu}$ (the a priori mean of I's type). A full disclosure equilibrium exists in this setting provided that $v \geq \bar{\gamma}$.

[^22]Proposition 3.4.4: If $I$ can obtain his capital by issuing riskless debt and E's type is common knowledge, with $\bar{\gamma} \in(0,1)$, then,
(a) FD-v equilibria, $\forall v \in[\bar{\gamma}, 1]$, always exist.
(b) An FN (full non-disclosure) equilibrium exists if, and only if, $\bar{\gamma} \geq \bar{\mu}$;
(c) PD (partial disclosure) equilibria in which $\mathrm{N} \subset[\bar{\gamma}, 1]$ always exist, and PD equilibria in which $[\bar{\gamma}, 1] \subset \mathbb{N}$ exist if, and only if $\bar{\gamma}$ $\geq \bar{\mu}$.

## simultaneous Concern for Undervaluation and Entry:

The preceding analysis establishes that an FD-0 equilibrium exists if $I$ is only concerned with how $M$ values his firm ( $I$ fully discloses his information in order to avoid undervaluation). On the other hand, an FD1 equilibrium exists if $I$ is only concerned with avoiding entry by $E$ (I fully discloses his information in order to minimize the probability of entry by E). We now consider situations in which $I$ is concerned with both undervaluation by $M$ and entry by E. This is ensured by assuming that $I$ must issue equity to $M$ in order to obtain the desired capital and either E's type is not common knowledge or it is common knowledge with $\bar{\gamma} \epsilon(0,1)$. Full disclosure equilibria can exist in these contexts, with the form depending on whether undervaluation by m or entry by E is I's dominant concern.

The following proposition provides a precise charac-
terization of the conditions under which various full disclosure equilibria exist when E's break-even point is either uniformly distributed or is common knowledge.

Proposition 3.4.5: If E's break-even point $\tilde{\gamma}$ is uniformly distributed (or common knowledge at $\bar{\gamma}$ ), then one of the three following possibilities hold:
(a) an FD-0 equilibrium exists if $K_{2}<d$ and $k$ $\epsilon\left[K_{2}, \mathrm{~d}\right]$;
(b) an FD-1 (or FD- $\bar{\gamma}$ ) equilibrium exists if $K_{1}$ $>0$ and $k \in\left[0, K_{1}\right] \cap[0, d]$;
(c) an FD equilibrium does not exist if $K_{1}<K_{2}$ and $k \in\left(K_{1}, K_{2}\right) \cap[0, d]$.

In the uniform distribution case

$$
\mathrm{K}_{1}=\frac{\Delta}{c}[c+d] \quad \mathrm{K}_{2}=\frac{b}{a}[\delta+\Delta]
$$

and in the common knowledge case

$$
\mathrm{K}_{1}=\frac{\Delta}{c \bar{\gamma}}[\overline{\bar{\gamma}}+d] \quad \mathrm{K}_{2}=\frac{b}{a \bar{\gamma}}[\delta \bar{\gamma}+\Delta]
$$

To obtain greater insight into the above proposition we consider the two special cases introduced in section 3.2: the variable entry cost case in which $\Delta=b-d=0$ and $\delta=a-c>0$; and the fixed entry cost case in which $\delta=a-$ $c=0$ and $\Delta=b-d>0$. For a given basic value $b$, we can now consider the impact on disclosure of three elements of the model: maximum undervaluation (a), cost of entry ( $\delta$ or $\Delta)$, and the amount of capital required (k). The following
depicts the relationship between these elements and the existence of full disclosure equilibria when $\tilde{\gamma}$ is uniformly distributed.

Variable Entry Cost ( $\mathrm{d}=\mathrm{b}$ and $\delta \epsilon(0, \mathrm{a})$ ):
No FD equilibrium exists if $k \in .\left(0, K_{2}\right)$
An $F D-0$ equilibrium exists if $k \in\left[K_{2}, d\right]$, where

$$
\mathrm{K}_{2}=\frac{\delta}{a} b
$$

Small Fixed Entry Cost $(c=a$ and $\Delta \in(0, \Delta)$, where $\bar{\Delta}=a b /(a+b)):$

An FD-1 equilibrium exists if $k \in\left(0, K_{1}\right]$
An $F D-0$ equilibrium exists if $k \in\left[K_{2}, d\right]$, where

$$
K_{1}=\frac{\Delta}{a}[b+(a-\Delta)] \quad K_{2}=\frac{\Delta}{a} b<K_{1}
$$

Large Fixed Entry Cost ( $c=a$ and $\Delta \epsilon(\Delta, b))$ :
An FD-1 equilibrium exists if $k \in(0, d]$

The above relationships for alternative levels of $k$ and $\delta$ or $\Delta$ are depicted in Figure 3-4. In the variable entry cost case, a full disclosure equilibrium exists, based on the threat of under-valuation (FD-0), if large amounts of capital are required. However, if small amounts of capital are required, then there is no full disclosure equilibrium -- the threat of undervaluation is
not sufficiently strong and full disclosure cannot be sustained by a threat of entry because low types face very small entry costs. As depicted in Figure 3-4(a), the minimum capital requirement necessary for the existence of an FD-O equilibrium is an increasing function of the variable entry cost.

## Insert Figure 3-4 here

In the fixed entry cost case, a full disclosure equilibrium based on the threat of entry (FD-1) always exists for at least small amounts of capital. In fact, if the entry cost is large, then an FD-1 equilibrium exists for all capital requirements. On the other hand, if the entry cost is small and the capital requirements are large, then we again have a situation in which there is a full disclosure equilibrium based on the threat of undervaluation (FD-0). In any event, as illustrated in Figure 3-4(b), at least one full disclosure equilibrium always exists in the fixed entry cost case. The minimum capital requirement for existence of an FD-0 equilibrium is again an increasing function of the entry cost (for small $\Delta$ ).

In the two special cases considered above, the range of capital requirements over which an FD-0 equilibrium exists is independent of whether e's type is common knowledge or not. More generally, in the variable entry cost
case with common knowledge of E's type, the characterization of existence of full disclosure equili-bria is precisely the same as above. In the fixed entry cost case with common knowledge of $E$ 's type, we replace a with $a \bar{\gamma}$ in the characterization of the range of capital for which an FD- $\bar{\gamma}$ equilibrium (instead of an FD-1 equili-brium) is viable.

Now consider a beta distribution with $\beta_{1}, \beta_{2}>1$. From Lemma 3.3.2 we know that, under these conditions, there exists a type $\mu_{0} \epsilon(0,1)$ such that $W_{0}$ is convex on ( $\mu_{0}, 1$ ) and has a local interior minimum at $\mu_{*} \epsilon\left(\mu_{0}, 1\right)$. We also have the following result.

Lemma 3.4.6: If $G(\gamma)$ is a beta distribution with $\beta_{1}, \beta_{2}>1$, then there exists a unique type $v \in$ ( $\mu_{*}, 1$ ) such that, at $\mu=v$,

$$
\frac{d W_{D}(\mu)}{d \mu}=\frac{d W_{N}(\mu, v, G(v))}{d \mu}
$$

Using this result we obtain the following characterization of full disclosure equilibria.

Proposition 3.4.7: If $G(\gamma)$ is a beta distribution with $\beta_{1}, \beta_{2}>1$, then, for $v$ satisfying the conditions of Lemma 3.4.6, one of the following three possibilities must hold:
(a) an FD-0 equilibrium exists if $W_{N}(\mu, 0,0) \leq$ $\mathrm{W}_{\mathrm{D}}(\mu) \forall \mu \in\left(\mu_{*}, 1\right)$;
(b) an FD-v equilibrium exists if $W_{N}(0, v, G(v))$ $\leq W_{0}(0)$ :
(c) an FD equilibrium does not exist if neither (a) nor (b) hold.

It is difficult to determine the parameter values that produce these results because $v$ is endogenously determined by those values. However, the characterization is similar to the case in which E's type is common knowledge. In the variable entry cost case we again have the situation in which there is never an FD-v equilibrium and an $\mathrm{FD}-0$ equilibrium can be sustained if $k \in[b \delta / a, d]$.

### 3.5 Partial Disclosure Equilibria

The analysis in the preceding section identifies conditions under which full disclosure equilibria exist. This section identifies conditions under which partial equilibria exist, and examines their basic characteristics. A partial disclosure equilibrium always exists if there is no full disclosure equilibrium and, for some parameter values, there can be both full and partial disclosure equilibrium. The following section discusses some of the issues that arise when there are multiple equilibria.

General Characterization of Efficient Partial Disclosure Equilibria:

A sequential equilibrium $\Gamma=(\mathbf{N}, \boldsymbol{\alpha}, \mathbf{r}, \mathbf{e}, \boldsymbol{v})$ is termed a partial disclosure (PD) equilibrium if both $N$ and $D=I \backslash N$ are measurable subsets of $I$. The previous section has established that a partial equilibrium can only exist if $I$ faces simultaneous threats of under-valuation by $M$ and entry by $E$. Hence, in this section we assume that I must obtain his desired capital $k$ by issuing equity to $M$ and that E's break-even point is either not common knowledge or is common knowledge at $\bar{\gamma} \epsilon(0,1)$.

In any sequential equilibrium, type $\mu \in I$ will offer
the optimal contract $\alpha^{*}(v, p(v))$ if he discloses his type. ${ }^{19}$ In this section, we consider only those partial disclosure equilibria in which $I$ offers optimal contract $\alpha^{*}(v, p(v))$ if $\mu \in N$, where $v=\mu(N)$ and $p(v)$ is consistent with (3.2.2) and (3.2.3). That is, we consider only those equilibria in which all types who do not disclose their private information, offer (and obtain acceptance) of the optimal contract given $M$ and $E ' s$ beliefs.

Lemma 3.3.2, and the associated discussion and figures, establishes that any partial disclosure equilibrium cran be characterized by the points at which $W_{N}$ intersects $W_{0}$. (See figure 3-3 for an illustration of the following result.)

```
Lemma 3.5.1: If \(\Gamma=(N, \alpha, r, e, v)\) is a partial
disclosur equilibrium in which \(\alpha(\mu)=\alpha^{0}=\alpha^{*}\left(v^{0}\right.\),
\(\left.p\left(v^{0}\right)\right), \forall \mu \in N\), where \(v^{0}=v\left(n, \alpha^{0}\right)=\bar{\mu}(N)\), then
(generically) there exist three points \(0<\mu_{1}<\)
\(\mu_{2}<\mu_{3} \leq 1\) such that: \({ }^{20}\)
(a) \(N=N_{1} \cup N_{2}\), where \(N_{1}=\left[0, \mu_{1}\right]\) and \(N_{2}=\left[\mu_{2}\right.\), \(\mu_{3}\) ].
(b) Either \(\mu_{1}=v^{0}\) or \(\mu_{2}=v^{0}\).
```

${ }^{19}$ This follows from sub-game perfection, since there is only one type that can provide report $\mu$ and $M$ will accept the contract if he knows it is offered by $\mu$.
${ }^{20}$ There are parameter values for which $\mu_{1}=\mu_{2}$, but any perturbation of those values will result in $\mu_{4}<\mu_{2}$.

A key feature of the non-disclosure set $\mathbf{N}$ is that it always contains a set $N_{1}$ of "bad" types ( $\mu$ close to zero) plus another set $N_{2}$ of "better" types. $N_{1}$ consists of types who choose non-disclosure because so doing decreases their expected capital costs (due to over-valuation) more than it increases their expected entry costs. $N_{2}$, on the other hand, consists of types who choose non-disclosure because so doing decreases their expected entry costs and thereby also decreases their expected capital costs, even though they are subsequently undervalued. Except in nongeneric cases, the disclosure set $D$ always contains a set of "intermediate" types $D_{1}=\left(\mu_{1}, \mu_{2}\right)$ and may contain a set of "high" types $D_{2}=\left(\mu_{3}, 1\right]$, if $\mu_{3}<1$. This implies that, unlike in Verrecchia [1983] and Dye [1985], the disclosure policy is not characterized by a single threshold that divides the non-disclosure and disclosure sets. The characterization obtained here is similar to that obtained by Wagenhofer [1990].

A second key feature of the non-disclosure set $N$ is that the posterior non-disclosure expectation $v^{0}$ can be either below or above the types in disclosure set $D_{1}$. We refer to partial disclosure equilibria in which $v^{0}=\mu_{1}$ as PD-L equilibria, and those in which $v^{0}=\mu_{2}$ as $\mathrm{PD}-\mathrm{H}$ equi-
libria. ${ }^{21}$ Observe that in a PD-L equilibrium, all types who disclose their information receive a higher market price than those who choose non-disclosure. On the other hand, in a PD-H equilibrium, at least some types who disclose their information receive a lower market price than those who choose non-disclosure. ${ }^{22}$

## Characterization of the Uniform Distribution Case:

In our characterization of partial disclosure equilibria, we identify the range of capital levels over which the two types of partial disclosure equilibria exist. These ranges are closed intervals contained in the set [ $0, d$ ] and, hence, we can represent them as follows:
$\mathrm{K}-\mathrm{H}=\left[\underline{k}_{1}, \overline{\mathrm{k}}_{1}\right]=$ the set of capital requirements for which a PD-H equilibrium exists
$\mathrm{K}-\mathrm{L}=\left[\underline{\mathrm{k}}_{2}, \overline{\mathrm{k}}_{2}\right]=$ the set of capital requirements for which a PD-L equilibrium exists.

To obtain this characterization we must make an explicit assumption about the prior beliefs regarding I's type. For this purpose, we assume that $\tilde{\mu}$ is uniformly distribut-

[^23]ed, i.e., $\Phi(\mu)=\mu$. We first consider the case in which $E$ is also uniformly distributed.

Proposition 3.5.2: If both I's type and E's break-even point are uniformly distributed, then the bounds on the sets $K-L$ and $K-H$ have the follow-ing characteristics (where $K_{1}$ and $K_{2}$ are defined in Proposition 3.4.5):
(a) $K-H=\left[\underline{k}_{1}, \overline{\mathrm{~K}}_{1}\right] \supset\left[\min \left\{\mathrm{K}, \mathrm{K}_{1}\right\}, \max \left\{\mathrm{K}, \mathrm{K}_{1}\right\}\right] \cap[0, \mathrm{~d}] ;$
(b) $K-L=\left[\underline{k}_{2}, \bar{k}_{2}\right] \supset\left[\min \left\{\kappa, K_{2}\right\}, \max \left\{\kappa, K_{2}\right\}\right] \cap[0, d] ;$ where

$$
k=\left[\frac{1}{2} \delta+\Delta\right] \cdot\left[\frac{1}{2}+\frac{b-\Delta / 2}{a-\delta / 2}\right]
$$

A key implication of this proposition is that a partial disclosure equilibrium exists for any value of $k$ for which there is no full disclosure equilibrium. In particular, if $K_{1}<K_{2} \leq d$, then the proposition implies that $\left[K_{1}, K_{2}\right] \subset K-H \cup K-L$.

Another key implication is that both full and partial disclosure equilibria exist for some parameter values. for example, if $K_{2}<K_{1} \leq d$, then $\left[K_{2}, K_{1}\right] \subset K-H \cup K-L$, implying that at least one partial disclosure equilibrium as well as both an FD-0 and an FD-1 equilibrium exist if $k \epsilon$ $\left[K_{2}, K_{1}\right]$.

Appendix 3.B provides additional details on the characterization of $\mathrm{K}-\mathrm{L}$ and $\mathrm{K}-\mathrm{H}$. We summarize and illustrate that characterization for the variable and fixed
entry cost cases. Recall, from section 3.4, that variable entry costs and uniformly distributed break-even points produce a case in which $K_{1}=0$ (there are no FD-1 equilibria) and $K_{2}=\delta \cdot b / a$. Hence, $K_{1}<K_{2} \leq d$, and we obtain the following characterization:

$$
\begin{aligned}
& \mathrm{k}_{1}-0<\kappa \leq \overline{\mathrm{k}}_{1} \\
& \mathrm{k}_{2} \leq \min \left\{\mathrm{k}, \mathrm{~K}_{2}\right\} \leq \max \left\{\kappa, \mathrm{K}_{2}\right\}<\overline{\mathrm{k}}_{2}
\end{aligned}
$$

where

$$
k-\left[\frac{1}{2} \delta\right] \cdot\left[\frac{1}{2}+\frac{b}{a-\delta / 2}\right]
$$

Figure 3-5(a) depicts the relationship between $K_{2}, k$, and $\overline{\mathrm{k}}_{2}$ as $\delta$ increases from zero to a. All three are increasing functions of $\delta$ and, in this numerical example, $\underline{k}_{2}=$ $\min \left\{K, K_{2}\right\} .^{23} \operatorname{PD}-H$ equilibria exist if the capital requirements are small, and the allowable capital requirement increases as the entry cost increases. Partial disclosure equilibria do not exist if the capital requirements are large and the entry costs are small (see Figure 3-4 for the conditions under which full disclosure equilibria exist), but PD-L equilibria exist if capital requirements are not too large relative to the entry

[^24]costs. Finally, both PD-L and PD-H equilibria exist if both the capital requirements and the entry costs are large.

Insert Figure 3-5 here

Recall from Section 3.4, that fixed entry costs and uniformly distributed break-even points produce a case in which $K_{1}>K_{2}=\Delta \cdot b / a$, implying that full disclosure equilibria always exists. If $\delta=0$ and $a>\Delta$ (as in Figure 35), then we obtain the following characterization of the partial disclosure equilibria:

$$
k_{2}-K_{2}<\bar{k}_{2}-\kappa-k_{1}<\mathrm{K}_{1}-\bar{k}_{1}
$$

where

$$
k=\Delta \cdot\left[\frac{1}{2}+\frac{b-\Delta / 2}{a}\right]
$$

Figure 3-5(b) depicts the relationship between $K_{1}, K_{2}$, and $k$ for alternative values of $\Delta$. Observe that there are no partial disclosure equilibria if the capital requirements are either large or small. Only a narrow band of capital requirements can result in partial disclosure, and both the upper and lower bounds on that band increase as $\Delta$ increases.

## Characterization of the Common Knowledge Case:

We again assume that I's type is uniformly distributed, but now consider the case in which E's break-even point is common knowledge at $\bar{\gamma} \in(0,1)$. In this setting, $M$ and E's non-disclosure posterior expectation $v^{0}$ is such that in a PD-L equilibrium, $\mu_{1}=v^{0}=\bar{\gamma}$. Furthermore, E's probability of entry given that he observes nondisclosure, denoted $e^{0}$, is equal to zero in a $P D-L$ equilibrium, but is between zero and one in a PD-H equilibrium. That is, $\mathrm{PD}-\mathrm{H}$ equilibria are always such that E is indifferent between entry and no entry if he observes non-disclosure, and he plays a mixed strategy, in which he enters with probability $e^{0}$ if $I$ chooses non-disclosure and offers contract $\alpha^{0}=\alpha^{*}\left(v^{0}, e^{0}\right)$. This mixed strategy is set at the level that will induce $I$ to choose non-disclosure if, and only if, $\mu \in \mathbb{N}$.

Proposition 3.5.3: If I's type is uniformly distributed and E's break-even point is common knowledge at $\bar{\gamma} \epsilon(0,1)$, then the bounds on $K-L$ and $\mathrm{K}-\mathrm{H}$ have the following characteristics (where $\mathrm{K}_{1}$ and $K_{2}$ are as specified in Proposition 3.4.5):
(a) If $\bar{\gamma}<\bar{\mu}=1 / 2$, then

$$
\begin{aligned}
K-H= & {\left[\underline{k}_{1}, \overline{\mathrm{~K}}_{1}\right] \supset\left[\min \left\{\kappa, K_{1}\right\}, \max \left\{\kappa, K_{1}\right\}\right] \cap[0, d] } \\
K-L= & {\left[\underline{k}_{2}, \overline{\mathrm{~K}}_{2}\right] \supset\left[\mathrm{K}_{2}, \mathrm{~K}\right] \cap[0, \mathrm{~d}], \text { where } } \\
& \kappa=[2 \delta \bar{\gamma}+\Delta]\left[1+\frac{b}{a \bar{\gamma}}\right]>K_{2}
\end{aligned}
$$

(b) If $\bar{\gamma}>\bar{\mu}=1 / 2$, then ${ }^{24}$

$$
\begin{aligned}
& \mathrm{K}-\mathrm{H}=\left[0, \overline{\mathrm{k}}_{1}\right] \supset\left[0, \mathrm{~K}_{1}\right] \cap[0, d] \\
& \mathrm{K}-\mathrm{L}-[0, \stackrel{\mathrm{k}}{2}] \supset\left[0, \mathrm{~K}_{2}\right] \cap[0, d]
\end{aligned}
$$

Observe that this proposition has the same two key implications as Proposition 3.5.2 (which considers the case in which $\tilde{\gamma}$ is uniformly distributed): First, a partial disclosure equilibrium exists whenever a full disclosure equilibrium does not exist, i.e., if $K_{1}<K_{2}$, then $\left[K_{1}, K_{2}\right] \subset K-L \cup K-H$. Second, both full and partial disclosure equilibria exist for some parameter values, e.g., if $K_{1}>K_{2}$, then $\left[K_{2}, K_{1}\right] \subset K-L \cup K-H$. Further observe that if $E$ has a high break-even point ( $\bar{\gamma}>\bar{\mu}$ ), then both types of partial disclosure equilibria exist for small capital levels, but neither may exist for small capital levels if $E$ has a low break-even point ( $\bar{\gamma}<\bar{\mu}$ ).

To provide additional insight into these results we again consider the variable and fixed entry cost cases. In the variable entry cost setting $(\Delta=0), K_{1}=0<K_{2}=$ $\delta \cdot b / a$ and we obtain the following characterization of $\mathrm{K}-\mathrm{L}$ and $\mathrm{K}-\mathrm{H}$ for $\bar{\gamma}<\bar{\mu}$ :

$$
\underline{\mathrm{k}}_{1}=0<\underline{\mathrm{k}}_{2} \leq \mathrm{K}_{2}<\mathrm{k} \leq \overline{\mathrm{k}}_{1}, \overline{\mathrm{k}}_{2}
$$

[^25]where
$$
\kappa=\frac{2 \cdot \delta}{a} \cdot[a \cdot \bar{\gamma}+b]
$$

Numerical examples indicate that $\underline{k}_{2}=\mathrm{K}_{2}$ and $\mathrm{k}=\overline{\mathrm{k}}_{1}<\overline{\mathrm{k}}_{2}$. The values for $\mathrm{k}_{2}=\mathrm{K}_{2}$ and $\overline{\mathrm{k}}_{1} \approx \overline{\mathrm{k}}_{2}$ are depicted in Figure 3-6(a) for $\bar{\gamma}=1 / 3$ and values of $\delta$ ranging from zero to a. Observe that $\mathrm{PD}-\mathrm{H}$ equilibria always exist unless the capital requirement is large and the entry cost is small, and that there is a non-trivial region over which both PDL and $\mathrm{PD}-\mathrm{H}$ equilibria exist. (In the latter region, FD-0 equilibria also exist.)

## Insert Figure 3-6 here

If $\bar{\gamma}>\bar{\mu}$ in the variable entry cost case, then $\underline{k}_{1}=$ $\underline{k}_{2}=0<\overline{\mathrm{k}}_{1}, \overline{\mathrm{k}}_{2}$. The values of $\overline{\mathrm{k}}_{1}$ and $\overline{\mathrm{k}}_{2}$ are depicted in Figure 3-6(b) for the $\bar{\gamma}=2 / 3$. In this setting, $P D-L$ equilibria always exist unless the capital requirement is large and the entry cost is small, and there is a nontrivial region over which both $P D-L$ and $P D-H$ equilibria exist. In 3-6(b) the overlap of $\mathrm{K}-\mathrm{H}$ and $\mathrm{K}-\mathrm{L}$ occurs for small capital requirements, whereas in 3-6(a) the overlap occurs for intermediate capital levels.

The characterization changes considerably when the entry cost is fixed. Figure 3-7(a) presents an example in which $\bar{\gamma}=1 / 3$, while Figure 3-7(b) presents the same
example except that $\bar{\gamma}=2 / 3$. Figure 3-7(a) is similar to the uniform distribution case in that there is only a limited range of capital requirements and entry cost values over which PD equilibria exist. Furthermore, within that range there is considerable overlap between $K$ L and $\mathrm{K}-\mathrm{H}$. In Figure 3-7(b), on the other hand, both PD-L and $P D-H$ equilibria exist unless the entry cost is small and the capital requirement is large. Hence, given fixed entry costs, there is much more opportunity for partial disclosure equilibria to exist if it is common knowledge that E has a high break-even point instead of a low breakeven point.

## Insert Figure 3-7 here

The complexity of the case in which beliefs about $\tilde{\gamma}$ are strictly unimodel makes it difficult to provide a precise characterization of the conditions under which PD$L$ or PD-H equilibria exist. The unimodel distribution lies between the two extremes of the uniform distribution and common knowledge cases, and will be very similar to the case of common knowledge at $\bar{\gamma}$ if $G^{\prime}(\gamma)$ is highly peaked. The key difference between the common knowledge and unimodel distribution cases is that, in the latter case, mixed strategies are not required to sustain PD-H equilibria. If $G^{\prime}(\gamma)$ is highly peaked, then the probabil-
ity of entry can be significantly modified by slightly shifting the non-disclosure expectation in the vicinity of the mean $\bar{\gamma}$. This is essentially the same as exogenously shifting the entry probability e when $\bar{\gamma}$ is common knowledge and equal to the non-disclosure mean.

These similarities suggest (see Figure 3-6 and 3-7) that, with variable entry costs, there is a broad range of capital requirements and entry cost values for which partial disclosure equilibria exist, whereas with fixed entry costs, there is only a narrow range of capital requirements and entry cost values for which these equilibria exist. Whether it is PD-L or PD-H equilibria that exist, particularly in the variable cost case, depends significantly on whether the mean of E's break-even point is greater than or less than the mean of I's type.

Finally, observe that partial disclosure equilibria never exist if the entry costs are small and the capital requirements are large. In that setting, the only equilibrium that exists is a full disclosure equilibrium in which M assigns the lowest possible value to any firm that does not disclose its type.

### 3.6 Multiple Equilibria and Their Refinements

The preceding analysis establishes that in our disclosure model there are parameter values for which there is a single disclosure equilibrium (either full or partial) and there are other parameter values for which there are multiple equilibria. If the multiple equilibria are all full disclosure equilibria, then they all provide $I$ with the same expected wealth -- only the belief held to sustain the equilibrium differs and, in equilibrium, that the out-of-equilibrium strategy never has to be carried out. However, substantive issues arise when there are both full and partial disclosure equilibria or multiple partial disclosure equilibria. We explore these issues more fully in this section.

First, observe that all $\mu \epsilon I$ weakly prefer a partial disclosure equilibrium over a full disclosure equilibrium, and all $\mu \in N$ strictly prefer non-disclosure (except $\mu_{1}, \mu_{2}$, and $\mu_{3}$ ). On the other hand, E has a strict ex ante preference for a full disclosure equilibrium, while $M$ is indifferent (he always receives the expected market return). Full disclosure equilibria are sustained by either the under-valuation of $M$ or entry by $E$ if $I$ chooses the out-of-equilibrium action of not disclosing his type. In our discussion of full disclosure equilibria we explicitly identified the out-of-equilibrium belief (expecta-
tion) held by $M$ and $E$. The issue here is whether those beliefs are plausible.

## Cho and Kreps Intuitive criterion

Cho and Kreps [1987] provide an "intuitive criterion" that is a necessary condition for the stability ${ }^{25}$ of an equilibrium in a signalling game. The general thrust of their criterion is to permit threats to sustain an equilibrium if among the types that could weakly benefit from a favourable response to the out-of-equilibrium action, there is at least one type to which the proposed threat would be an optimal response. Hence, in an FD-v equilibrium, the strategy for $M$ and $E$ to hold expectation $v$, given non-disclosure and any contract $\alpha$, can be justified provided that $\mu=v$ is among the set of types who would at least weakly benefit from a more favourable expectation. We shall prove first in this section that full disclosure and partial disclosure equilibria satisfy this criterion. The following notation is used to adapt the cho and Kreps [1987] intuitive criterion to our setting.

$$
\begin{aligned}
\mathrm{W}^{*}(\mu)= & \text { equilibrium expected wealth for type } \mu . \\
Q(T)= & \text { the set of possible expectations that can } \\
& \text { be obtained by varying probability func- } \\
& \text { tions defined over the set of types } T \subset I,
\end{aligned}
$$

[^26]\[

$$
\begin{aligned}
& \text { i.e., the smallest interval in }[0,1] \text { that } \\
& \text { contains T. } \\
& G^{*}(v)=\text { the minimum probability of entry if } E \text { holds } \\
& \text { expectation } v \text {. } \\
& \alpha^{*}\left(v, G^{*}(v)\right)=\text { the minimum contract } M \text { will accept given } M \\
& \text { and E's belief and entry probability to be } \\
& G^{*}(v) \text {. } \\
& r^{*}(\alpha, v, p)= \begin{cases}0 & \text { if } \alpha \geq \alpha^{*}(v, p) \\
1 & \text { if } \alpha<\alpha^{*}(v, p)\end{cases} \\
& v^{*}(\alpha)=\text { the cutoff point of } M \text { and E's belief given } \\
& \alpha \text {, i.e., } v^{*} \text { separates the regions of expec- } \\
& \text { tations in which } \mathbf{r}^{*}=0 \text { and } \mathbf{r}^{*}=1 \text {. } \\
& A(\alpha)=\text { The accept region for contract } \alpha \text {. } \\
& R(\alpha)=\text { The reject region for contract } \alpha \text {. } \\
& \mathrm{W}^{\dagger}(\mu, \alpha)=\max \quad \mathrm{W}\left(\mu, \alpha, \mathrm{r}^{*}\left(\alpha, v, \mathrm{G}^{*}(v)\right), \mathrm{G}^{*}(v)\right) \\
& v \in[0,1] \\
& \text { expected wealth for } \mu \text { from contract } \alpha \text { given } \\
& \text { the most favourable possible responses from } \\
& M \text { and } E \text {. } \\
& \mathrm{T}^{*}(\alpha)=\left\{\mu \quad \mid \mathrm{W}^{*}(\mu) \leq \mathrm{W}^{\dagger}(\mu, \alpha)\right\} \\
& \text { the set of types that weakly prefer } \alpha \text { if it } \\
& \text { would induce } M \text { and } E \text { to respond favourably. }
\end{aligned}
$$
\]

Definition 3.6.1: A sequential equilibrium $\Gamma=$ ( $N, \alpha, r, e, v$ ) fails the CK-criterion if, for any out-of-equilibrium contract $\alpha, \mathrm{T}^{\star}(\alpha) \neq \varnothing$ and there is some type $\mu^{\prime} \in \mathrm{T}^{*}(\alpha)$ such that

$$
W^{*}\left(\mu^{\prime}\right)<\min _{v \in Q\left(T^{*}(\alpha)\right)} W\left(\mu^{\prime}, \alpha, r^{*}(\alpha, v, G(v)), G(v)\right)
$$

Proposition 3.6.1: Assume that E's break-even point is either uniformly distributed or common knowledge. If a full disclosure equilibrium exists, then it does not fail the CK-criterion.

The preceding proposition has established that full disclosure equilibria satisfy the CK-criterion. The next proposition establishes that partial disclosure equilibria also satisfy this criterion.

> Proposition 3.6.2: Assume that I's type is uniformly distributed and E's break-even point is either uniformly distributed or common knowledge. If a partial disclosure equilibrium exists, then it does not fail CK-criterion.

Observe that if $T^{*}(\alpha)$ is non-empty, then there is at least one type that would weakly prefer to offer $\alpha$ if $m$ and $E$ would respond favourably to that contract. However, the proof establishes that none of these types would offer this contract if $M$ and $E$ responded unfavourably, even though their response must be based on an expectation that recognizes that only the types in $T^{*}(\alpha)$ could conceivably be taking this out-of-equilibrium action. In particular, in an FD-O equilibrium, if $\alpha$ is desirable to any type $\mu \in I$ given a favourable response, then $\alpha$ is also preferred by type $\mu=0$. Consequently, under the $C K$-criterion, $M$ and $E$ are "justified" in holding expectation $v=0$ when they
observe any out-of-equilibrium contract, and the existence of an FD-O equilibrium assures us that all $\mu \in I$ will at least weakly prefer to disclose their type rather than have $M$ and $E$ respond to $\alpha$ on the basis of expectation $v=$ 0. A similar result holds if an FD-1 equilibrium exist and an FD-0 equilibrium does not exist. If both FD-1 and FD-O equilibrium exist, then there are some $\alpha$ for which $T^{*}(\alpha) \neq \varnothing$ and $T_{2}=\varnothing$. Hence, the belief $v=0$ is credible for all $\alpha$, whereas $v=1$ is not.

## Perfect Sequential Equilibria

Grossman and Perry [1986] provide an alternative equilibrium refinement. They do not allow $M$ and $E$ to use "conservative" beliefs in determining their responses to an out-of-equilibrium contract offered by I. Instead, GP require $M$ and $E$ to respond on the basis of their prior beliefs with respect to all types $\mu \in I$ that would benefit from the out-of-equilibrium contract if they responded on the basis of those beliefs. We adapt Grossman and Perry's refinement, with some modification, to our setting. ${ }^{26}$

[^27]Definition 3.6.2: A sequential equilibrium $\Gamma=$ ( $N, \alpha, r, e, v$ ) fails the GP-criterion if for any out-of-equilibrium contract $\alpha, T^{*}(\alpha) \neq \varnothing$ and there exists a measurable set $T \subset T^{*}(\alpha)$ such that for $v=\bar{\mu}(T)$ and some $p \in\left[G(v), G^{*}(v)\right]$ :
(a) $\alpha \geq \alpha^{*}(v, p)$
(b) $W^{*}(\mu) \leq W(\mu, \alpha, 0, p) \quad \forall \mu \in T$
(c) $W^{*}(\mu) \geq W(\mu, \alpha, 0, \mathrm{p}) \quad \forall \mu \in I \backslash T$

Clearly, if both $F D$ and $P D$ equilibria exist, then the FD equilibrium fails the GP-criterion since $\alpha=\alpha^{0}$ and $T=$ N constitute the basis for failure. On the other hand, the lack of a PD equilibrium implies that an FD equilibrium does not fail the GP-criterion, since failure implies the existence of a PD equilibrium.

Proposition 3.6.3: An FD equilibrium fails the GP-criterion if, and only if, there also exists a PD equilibrium.

The final issue is whether a PD-equilibrium satisfies the GP-criterion. Note that if there exist two PD-equilibria A and B simultaneously, then following conclusions are mutually exclusive: (i) A dominates B; (ii) A is dominated by $B$; (iii) $A$ and $B$ are non-comparable. Clearly, if both PD-L and PD-H equilibria exist and one Pareto dominates the other with respect $\mu \epsilon I$, then the Pareto dominated equilibrium fails the GP-criterion. For a single or

Pareto dominant PD equilibrium, or when two PD equilibria are Pareto non-comparable. We have

Proposition 3.6.4: A PD equilibrium fails the GP-criterion if, and only if, there exists another PD equilibrium which dominates it.

### 3.7 Concluding Remarks

We have explored the extent to which a firm will disclose its private information in a context in which the firm is concerned about the response to that information (or its non-disclosure) by both the capital market and competitors in the firm's product market. In particular, we assume that the firm's information can be ordered such that it would prefer to reveal good news to the capital market and bad news to product market. Full disclosure will definitely occur if only one of these markets is of concern to the firm, or if the response of one market clearly dominates the other. However, partial disclosure equilibria exist when the firm has a relatively balanced concern for the responses of both markets.

- The firm's interest in the capital market is assumed to arise from the desire to obtain capital at the most favourable terms possible. We have assumed that the capital investment is desirable no matter what information the firm has and no matter what response occurs in the
product market. Furthermore, we have assumed that the firm must issue equity to obtain that capital. Obvious extensions to the current analysis would be to consider the impact of issuing risky debt instead of equity and to allow the range of information to be such that the project is undesirable for some lower range of signals.

Appendix 3.A provides a model of competition in a product market that can be represented by the linear functions we have used. To obtain the desired linearity, we assume that the firms face a common price uncertainty that is a decreasing function of aggregate production and that the competing firms have identical expected variable costs. An obvious extension of our analysis would be to explore the impact of alternative product market assumptions, e.g., the firms face different expected variable costs or compete on price (i.e., a Bertrand equilibrium instead of a Cournot equilibrium).

Perhaps the most interesting aspect of our results is that there are two possible partial disclosure equilibria. PD-L equilibria are characterized by a capital market in which the non-disclosure firms have a lower market value than all disclosure firms. PD-H equilibria, on the other hand, are characterized by a capital market in which some disclosure firms have lower market values than non-disclosure firms. Since the equilibria apply on a firm-by-
firm basis, this result implies that, in equilibrium, we would expect to empirically observe firms that choose to withhold information even though its release would increase their market value, while other firms disclose information even though withholding it would increase their market value.

Neither type of partial disclosure equilibria exist if capital requirements are large and entry costs are small. They also do not exist if capital requirements are small, the entry cost is large and fixed, and $E$ will not enter unless he receives bad news (i.e. $\bar{\gamma}<\bar{\mu}$ ). However, PD-H equilibria exist for small capital requirements if the entry cost is variable or if $E$ will not enter unless he receives good news (i.e., $\bar{\gamma}>\bar{\mu}$ ). Furthermore, PD-L equilibria exist for only a narrow band of capital requirements and entry cost values, except when the entry cost is fixed and E will not enter unless he receives good news.

Tables

| $t_{0}$ | I, $M$, and $E$ hold homogeneous prior beliefs $\Phi, F$, and $G$ with respect to I's type ( $\tilde{\mu}$ ), I's end-of-period value (payoff $\tilde{x})$, and E's break-even point ( $\tilde{\gamma}$ ). |
| :---: | :---: |
| $t_{1}$ | I learns his type ( $\tilde{\mu}=\mu$ ), which gives him private information about his payoff ( $\tilde{x}$ ) |
| $t_{2}$ | I chooses between publicly disclosing (m $=\mu$ ) or not disclosing ( $m=n$ ) his type (private information). |
| $t_{3}$ | I offers $M$ a contract, which specifies the share ( $\alpha$ ) of I's payoff that is to be given $M$ in return for $k$ units of capital. |
| $t_{4}$ | $M$ and E form a posterior expectation (v $=v(m, \alpha)$ ) with respect to I's type ( $\mu$ ) given I's report (m) and the contract $(\alpha)$ he has offered. |
| $t_{5}$ | $M$ assesses the value of the firm ( $V=$ $V(v, p(v))$ and accepts the contract if $\alpha \cdot \mathrm{V} \geq \mathrm{k}$, or rejects it if $\alpha \cdot \mathrm{V}<\mathrm{k}$. |
| $t_{6}$ | E learns his break-even point ( $\tilde{\gamma}=\gamma$ ) and enters with probability one ( $\mathrm{e}=1$ ) if it is less than his expectation with respect to I's type $(\gamma<v)$ or enters with probability zero ( $e=0$ ) if it is greater $(\gamma>v)$. E can choose to enter with a probability between zero and one if $\boldsymbol{\gamma}=\boldsymbol{v}$. |
| $t_{7}$ | I and $M$ share the realized payoff ( $\tilde{X}=$ $x$ ): $I$ receives $(1-\alpha) \cdot x$ and $M$ receives $\alpha \cdot x$. |

Table 3-1: Sequence of events

## Figures

Figure 3-1: Expected Outcomes.


## Figure 3-2: Expected End-of-Period Wealth <br> Under Full Disclosure.

(a) E's Breakeven Point is Common Knowledge

(b) E's Breakeven Point is Uniformly Distributed

(c) E's Breakeven Point has a Unimodel Distribution


Figure 3-3: Disclosure Versus Non-disclosure E's Breakeven Point is Common Knowledge


Figure 3-4: Capital Requirement/Entry Cost Conditions under which Full Disclosure Equilibria Exist
E's Breakeven Point is Uniformly Distributed
(a) Variable Entry Cost
(b) Fixed Entry Cost

Figure 3-5: Capital Requirement/Entry Cost Conditions under which Partial Disclosure Equilibria Exist
E's Breakeven Point is Uniformly Distributed


Figure 3-6: Capital Requirement/Entry Cost Conditions under which Partial Disclosure Equilibria Exist
Variable Entry Cost/Breakeven Point Common Knowledge


Figure 3-7: Capital Requirement/Entry Cost Conditions under which Partial Disclosure Equilibria Exist

## Fixed Entry Cost/Breakeven Point Common Knowledge

$k$
$b$
b $\quad$ (a) $\bar{\gamma}=1 / 3<\bar{\mu}=1 / 2$
No PD Equilibria


Figure 3-8: End-of-Period Value Curve and Contract Curve


Figure 3-9: FD Contract Curves with Different Parameter Values


$\mathrm{N}=\mathrm{k} /(\mathrm{c}+\mathrm{d})$

Figure 3-10: Comparison of Equilibria


## Appendix 3.A: Payoffs in a Cournot Equilibrium Entry Game with Demand Uncertainty

Assume that $I$ is entering a market in which the expected selling price is a linear function of the total output to be sold in that market. In particular, assume that the production quantity is scaled such that the selling price is equal to $\xi-Q$, where $\xi$ is an uncertain demand parameter and $Q$ is the total amount produced in that market. I has private information with respect to $\boldsymbol{\xi}$; let $m$ denote his posterior expectation given that information and let $[\underline{m}, \bar{m}]$ denote the set of possible values of $m$.

The production quantities for $I$ and $E$ are denoted $q$ and $q_{e}$, respectively. Hence, $Q=q$ if $E$ does not enter and $Q=q+q_{e}$ if $E$ does enter. The expected production costs for $I$ and $E$ are $\zeta \bullet q+k$ and $\zeta \bullet q_{e}+k_{e}$, respectively, where $k$ and $k_{e}$ are investments that must be made at the start of the period. Observe that the expected unit variable production cost $\zeta$ is assumed to be the same for both producers. There is no private knowledge with respect to the variable production costs, although $E$ may have private knowledge of $k_{e}$.

If $E$ does not enter, then $I$ is a monopolist in the market. His expected end-of-period payoff (excluding the investment k) is

$$
\pi(m, q)=[m-q] \bullet q-\zeta \bullet q
$$

The optimal production quantity (differentiate $\pi$ with respect to $q$ and set $\pi^{\prime}$ equal to zero) is $q^{*}=[m-\zeta] / 2$ and the optimal expected payoff is

$$
\pi^{*}(m)=\pi\left(m, q^{*}\right)=[m-\zeta]^{2} / 4
$$

This can be translated into a linear model if we represent I's private information as

$$
\mu=\frac{\pi^{*}(m)-\pi^{*}(m)}{\pi^{*}(\bar{m})-\pi^{*}(m)}
$$

and define the payoff function parameters as

$$
a=\left[\pi^{*}(\bar{m})-\pi^{*}(m)\right] \quad b=\pi^{*}(m)
$$

Assume that if E invests $\mathrm{k}_{\mathrm{e}}$ and enters the market, then he will learn $m$ before he selects $q_{e}$. That is, even if he does not know $m$ when he makes his investment decision, he will know it when he makes his production decisions under duopoly. Therefore, when I and E make their production decisions under duopoly, their expected payoffs are known to be

$$
\begin{gathered}
\pi\left(m, q, q_{e}\right)=\left[m-\left(q+q_{e}\right)\right] \cdot q-\zeta \cdot q \\
\pi_{\theta}\left(m, q, q_{\theta}\right)=\left[m-\left(q+q_{\theta}\right)\right] \cdot q_{\theta}-\zeta \cdot q_{\theta}
\end{gathered}
$$

Differentiating $\pi$ with respect to $q$ and $\pi_{e}$ with respect to $q_{e}$ provides the following first-order conditions:

$$
\begin{aligned}
q & =\frac{1}{2} \cdot\left(m-\zeta-q_{e}\right) \\
q_{e} & =\frac{1}{2} \cdot(m-\zeta-q)
\end{aligned}
$$

Solving these two equations in two unknowns provides the following equilibrium expected payoffs and production quantities:

$$
\begin{aligned}
\pi^{\dagger}(m) & \equiv \pi\left(m, q^{\dagger}, q_{e}^{\dagger}\right)-[m-\zeta]^{2} / 9 & q^{\dagger}-(m-\zeta) / 3 \\
\pi_{e}^{\dagger}(m) & \equiv \pi_{e}\left(m, q^{\dagger}, q_{e}^{\dagger}\right)-[m-\zeta]^{2} / 9 & q_{e}^{\dagger}-(m-\zeta) / 3
\end{aligned}
$$

Observe that $\pi^{\dagger}(m)=(4 / 9) \cdot \pi^{*}(m)$. Hence, we can express I's expected payoff, given entry, as a linear function of $\mu$, the previously defined representation of private information. In particular, the payoff function parameters are

$$
c=\pi^{\dagger}(m)-\pi^{\dagger}(\bar{m}) \quad d=\pi^{\dagger}(m)
$$

Now consider E's break-even point. Given information $Y$, he will enter if $E\left[\pi_{e}^{+}(m) \mid Y\right] \geq k_{e}$ and we can express $E$ 's expected profit given $m$ as a linear function of $\mu$ :

$$
\pi_{e}^{\dagger}(m)=\pi^{\dagger}(m)=c^{\bullet} \mu+d
$$

where $\mu, \mathrm{c}$, and d are as defined above. Consequently, E will enter the market if

$$
c^{\bullet} E[\mu \mid y]+d \geq k_{\theta}
$$

This implies that E's break-even point is $\gamma=\left[k_{e}-d\right] / c$. The preceding demonstrates that the linear payoff functions used in this paper can be viewed as representing a firm competing in a product market in which the initial entrant (I) has private information about the uncertain intercept of a linear price function. The linear representation of the expected monopoly profit is quite general - we can use that representation in any setting in which I's expected payoff is a strictly increasing function of some scalar representation of his private information (with finite bounds on the set of possible information). However, it does not necessarily follow that I's and E's duopolistic expected profits are linear functions of that same representation of private information. For example, it is crucial in the above model that $I$ and $E$ have the same expected variable per unit production cost. Differences in their expected costs will result in $\pi^{\dagger}$ and $\pi_{e}{ }^{\dagger}$ being nonlinear functions of $\mu$. That would complicate the analysis of disclosure equilibria, but whether it would change the qualitative results is an open question.

## Appendix 3.B: Proofs

## Proof of Lemma 3.3.1

Since $\mathrm{E}^{\prime}$ 's break-even point is not common knowledge,

$$
\begin{align*}
W_{D}(\mu) & -V(\mu, G(\mu))-k \\
& =G(\mu) \pi(\mu, 1)+(1-G(\mu)) \pi(\mu, 0)-k \\
& =a \mu+b-[\delta \mu+\Delta] G(\mu)-k  \tag{3.B.1}\\
W_{D^{\prime}}(\mu) & -a-8 G(\mu)-[\delta \mu+\Delta] G^{\prime}(\mu) \tag{3.B.2}
\end{align*}
$$

In the uniform distribution case, $G(\mu)=\mu$ and $G^{\prime}(\mu)=1$. Hence,

$$
W_{D} \prime \prime(\mu)=-2 \delta \leq 0
$$

which establishes that $W_{D}$ is concave, and strictly concave if $\delta>0$. Furthermore, if $\delta>0$, then setting (3.B.2) equal to zero establishes that $W_{D}$ has an interior maximum at $\mu^{*}$ $=(a-\Delta) /(2 \delta)$ if $a \epsilon(\Delta, 2 \delta+\Delta)$. If $a \leq \Delta$, then $\mu^{*}=0$, and if $a-\Delta \geq 2 \delta$ then $\mu^{*}=1$.

In the unimodel case,

$$
\begin{aligned}
G(\mu) & =\int_{0}^{\mu} \beta_{0} t^{\beta_{1}-1}(1-t)^{\beta_{2}-1} d t \\
G^{\prime}(\mu) & =\beta_{0} \mu^{\beta_{1}-1}(1-\mu)^{\beta_{2}-1} \\
G^{\prime \prime}(\mu) & =\beta_{0}\left[\left(\beta_{1}-1\right) \mu^{\beta_{1}-2}(1-\mu)^{\beta_{2}-1}-\left(\beta_{2}-1\right) \mu^{\beta_{1}-1}(1-\mu)^{\beta_{2}-2}\right] \\
& =G^{\prime}(\mu) \frac{\left[\left(\beta_{1}-1\right)-\left(\beta_{1}+\beta_{2}-2\right) \mu\right]}{\mu(1-\mu)}
\end{aligned}
$$

Hence,

$$
\begin{aligned}
W_{D}^{\prime \prime}(\mu) & =-2 \delta G^{\prime}(\mu)-[\delta \mu+\Delta] G^{\prime \prime}(\mu) \\
& =-G^{\prime}(\mu) \frac{B(\mu)}{\mu(1-\mu)}
\end{aligned}
$$

where

$$
B(\mu)=-\delta\left(\beta_{1}+\beta_{2}\right) \mu^{2}+\left[\delta\left(\beta_{1}+1\right)-\Delta\left(\beta_{1}+\beta_{2}-2\right)\right] \mu+\Delta\left(\beta_{1}-1\right)
$$

Given that $\beta_{1}, \beta_{2}>1$, we have

$$
\begin{aligned}
\mathrm{B}(0) & =\Delta\left(\beta_{1}-1\right) \geq 0 \\
\mathrm{~B}(1) & =-\delta\left(\beta_{1}+\beta_{2}\right)+\left[\delta\left(\beta_{1}+1\right)-\Delta\left(\beta_{1}+\beta_{2}-2\right)\right]+\Delta\left(\beta_{1}-1\right) \\
& =-(\delta+\Delta)\left(\beta_{2}-1\right)<0
\end{aligned}
$$

Therefore, $B(\mu)$ has at least one root in $[0,1]$. To show that $B(\mu)$ has only one root in $[0,1]$, observe that

$$
B^{\prime}(\mu)=-2 \delta\left(\beta_{1}+\beta_{2}\right) \mu+\delta\left(\beta_{1}+1\right)-\Delta\left(\beta_{1}+\beta_{2}-2\right)
$$

Let $\mu^{\dagger}$ satisfy $B^{\prime}\left(\mu^{\dagger}\right)=0$. If $\mu^{\dagger} \notin[0,1]$, then $B(\mu)$ is monotone in $[0,1]$, implying that its root in $[0,1]$ is unique. If $\mu^{\dagger} \epsilon[0,1]$, then from $B^{\prime \prime}(\mu)=-2 \delta \cdot\left(\beta_{1}+\beta_{2}\right) \leq 0$, we know that $\mathrm{B}(\mu)$ is non-decreasing in $\left[0, \mu^{\dagger}\right]$ and is decreasing in $\left[\mu^{\dagger}, 1\right]$, implying that $B(\mu)$ has a unique root in $\left[\mu^{\dagger}, 1\right]$.

Let $\mu_{0}$ denote the unique root of $B(\mu)$ in $[0,1]$. We then have $\mathrm{W}_{\mathrm{D}}{ }^{\prime \prime}(\mu)<0$ if $\mu \epsilon\left[0, \mu_{0}\right)$ and $\mathrm{W}_{\mathrm{D}}{ }^{\prime \prime}(\mu)>0$ if $\mu \epsilon$ ( $\left.\mu_{0}, 1\right]$. These inequalities imply the conclusions in (a). Based on (a), it is straightforward to show (b) by noting
that $W_{D}{ }^{\prime}(\mu)$ is positive at both $\mu=0$ and $\mu=1$. $W_{D}$ must reach its maximum and minimum at some interior $\mu^{*}$ and $\mu_{\star}$, respectively.
Q.E.D.

Proof of Lemma 3.4.1
A full disclosure sequential equilibrium exists if $M$ and E's consistent and sequentially rational response to non-disclosure and any contract $\alpha$ are such that $I$ weakly prefers disclosure for all types $\mu \in I$. To sustain full disclosure equilibria given non-disclosure expectation $v$, we let $p(v)=G(v)$. This is always the case if E's breakeven point is common knowledge.

To prove the "if" part of the lemma, let $M$ and $E$ hold the stated expectation $v$ for non-disclosure and all contracts $\alpha$. E will enter with probability $G(v)$ and $M$ will accept the contract if, and only if, $\alpha \geq \alpha^{*}(v, G(v))$. Hence, under the stated conditions, I will weakly prefer disclosure for all $\mu \in I$.

To prove the "only if" part of the lemma, assume that the condition does not hold, and yet there is a full disclosure equilibrium. This implies that for each contract $\alpha \epsilon[0,1]$ that there exists an expectation $v$ such that either $\alpha<\alpha^{*}(v, G(v))$ or $W_{0}(\mu) \geq(1-\alpha) \cdot V(v, G(v)) \forall \mu \in I$, i.e., either $M$ will reject $\alpha$ or $I$ will prefer disclosure to acceptance of $\alpha$ (given $p(v)=G(v)$ ). Consider the
expectation that produces the largest minimum acceptable ownership equity, i.e.,

$$
v^{0} \in \max _{v \in[0,1]} \alpha^{*}(v, G(v))
$$

and let $\alpha^{0}=\alpha^{*}\left(v^{0}, G\left(v^{0}\right)\right)$. By assumption, $W_{D}<W_{N}\left(\mu, v^{0}\right.$, $G\left(v^{0}\right)$ ) for some $\mu>0$, but the existence of a full disclosure equilibrium implies that there exists some expectation $\nu^{\dagger} \neq v^{0}$ such that $\alpha^{0} \geq \alpha^{*}\left(v^{\dagger}, G\left(v^{\dagger}\right)\right)$ and $W_{D}(\mu) \leq(1-$ $\left.\alpha^{0}\right) \cdot \mathrm{V}\left(\mu, \mathrm{G}\left(v^{\dagger}\right)\right) \forall \mu \in \mathrm{I}$. However, that contradicts the assumption because

$$
W_{N}\left(\mu, \nu^{t}, G\left(\nu^{t}\right)\right) \geq\left(1-\alpha^{0}\right) v\left(\mu, G\left(\nu^{t}\right)\right) \geq W_{D}(\mu) \quad \forall \mu \in I
$$

and, hence, $\mathbf{v}^{\dagger}$ satisfies condition (3.4.1).
Q.E.D.

## Proof of Proposition 3.4.4

As above, let $p(v)=G(v)$, the maximum entry probability given expectation $v$.
(a) If I obtains his capital by issuing riskless debt, then

$$
\begin{array}{r}
W_{D}(\mu)-G(\mu) \pi(\mu, 1)+(1-G(\mu)) \pi(\mu, 0)-k \\
W_{N}(\mu, v, G(v))-G(v) \pi(\mu, 1)+(1-G(v)) \pi(\mu, 0)-k
\end{array}
$$

For any $v \geq \bar{\gamma}$, we have $G(v)=1 \geq G(\mu) \forall \mu \in I$, so that we always have

$$
W_{D}(\mu) \geq W_{N}(\mu, v, G(v)) \quad \forall \mu \in I
$$

Therefore, a full disclosure equilibrium always exists in which $N=\varnothing$ and $v \in[\bar{\gamma}, 1]$ for all contracts offered with non-disclosure.
(b) If $\bar{\gamma} \geq \bar{\mu}$, under full non-disclosure, $v=\bar{\mu} \leq \bar{\gamma}$ so that $G(v)=0 \leq G(\mu) \forall \mu \in I$. Hence,

$$
W_{D}(\mu) \leq W_{N}(\mu, v, G(v)) \quad \forall \mu \in I
$$

That is, a full non-disclosure equilibrium exists in which $\mathbf{N}=I$ and with $v=\bar{\mu}$ for all contracts offered with nondisclosure.

On the other hand, if $\bar{\gamma}<\bar{\mu}$, then under full nondisclosure, $v=\bar{\mu}>\bar{\gamma}$ so that $G(v)=1 \geq G(\mu) \quad \forall \mu \in I$. However, for any $\mu \epsilon(\bar{\gamma}, \bar{\mu}), G(\mu)=0$ implies that disclosure is better than non-disclosure for 1 . This contradicts the definition of an FN-equilibrium.
(c) If $N \subset[\bar{\gamma}, 1]$ and $v \in[\bar{\gamma}, 1]$, then $G(\mu)=G(v)=1 \forall$ $\mu \in \mathbf{N}$ and

$$
W_{D}(\mu)=W_{N}(\mu, v, G(v)) \quad \forall \mu \in N
$$

Therefore, a partial disclosure equilibrium $\Gamma$ exists in which $N \subset[\bar{\gamma}, 1]$ and $v \in[\bar{\gamma}, 1]$ for all contracts offered with non-disclosure.

The sufficiency part of the last condition can be shown by construction. If $\bar{\gamma} \geq \bar{\mu}$, then we can find $\mu^{\dagger} \subset$ $[0, \bar{\mu}]$ such that if $N=\left[\mu^{\dagger}, 1\right], \bar{\mu} \leq v<\bar{\gamma}$ and $G(v)=0 . A$ partial disclosure equilibrium $\Gamma$ then exists in which $N=$
$\left[\mu^{\dagger}, 1\right]$ and $v \in[0, \bar{\gamma})$.
The necessity part can be shown by contradiction. If $\bar{\gamma}<\bar{\mu}$, let $N$ be any partial disclosure equilibrium strategy of $I$ such that $[\bar{\mu}, 1] \subset \mathbb{N}$. Then we have $v \geq \bar{\mu} \geq \bar{\gamma}$, so $G(v)=1$. However, $N$ is not an optimal strategy for $\mu \in \mathbb{N} \backslash[\bar{\gamma}, 1]$ since $G(\mu)=0 \forall \mu \epsilon \mathbb{N} \backslash[\bar{\gamma}, 1]$. This contradicts our assumption, so we must have $\bar{\gamma} \geq \bar{\mu}$.
Q.E.D.

## Proof of Proposition 3.4 .5

(a) An FD-0 equilibrium exists if, and only if,

$$
\mathrm{W}_{\mathrm{D}}(\mu) \geq \mathrm{W}_{\mathrm{N}}(\mu, 0,0)=\left(1-\frac{k_{\mathrm{i}}}{b}\right)(a \mu+b) \quad \forall \mu \in I
$$

In the uniform distribution case, $G(\gamma)=\gamma$ and

$$
\mathrm{W}_{\mathrm{D}}(\mu)-a \mu+b-\mu(\delta \mu+\Delta)-k
$$

The concavity of $W_{D}$ (see Lemma 3.3.1) and $W_{D}(0)=W_{N}(0,0$, 0), imply that

$$
\begin{aligned}
W_{D}(\mu) & \geq W_{N}(\mu, 0,0) \quad \forall \mu \in[0,1] \\
\Leftrightarrow \quad W_{D}(1) & \geq W_{N}(1,0,0) \\
\Leftrightarrow \quad c+d-k & \geq\left(1-\frac{k}{b}\right)(a+b) \\
\Leftrightarrow \quad k & \geq \frac{b}{a}[8+\Delta]=K_{2}
\end{aligned}
$$

In the case where $\tilde{\gamma}=\bar{\gamma}$ is common knowledge, since $W_{D}$ is "Z-shaped" and $W_{D}(0)=W_{N}(0,0,0)$, it follows that

$$
\begin{aligned}
W_{D}(\mu) & \geq W_{N}(\mu, 0,0) \quad \forall \mu \in I \\
W_{D}(\bar{\gamma}) & \geq W_{N}(\bar{\gamma}, 0,0) \\
\Leftrightarrow c \bar{\gamma}+d-k & \geq\left(1-\frac{k}{b}\right)(a \bar{\gamma}+b) \\
\Leftrightarrow \quad k & \geq \frac{b}{a \bar{\gamma}}[8 \bar{\gamma}+\Delta]=K_{2}
\end{aligned}
$$

(b) If $v=1$, then $G(v)=1$ and

$$
\mathrm{W}_{\mathrm{N}}(\mu, 1,1)=\left(1-\frac{k}{c+d}\right)(c \mu+d)
$$

In the uniform distribution case, the concavity of $W_{D}$ and $W_{D}(1)=W_{N}(1,1,1)$ imply that

$$
\begin{aligned}
& W_{\mathrm{D}}(\mu) \\
& \qquad \quad \mathrm{W}_{\mathrm{N}}(\mu, 1,1) \quad \forall \mu \in I \\
& \Leftrightarrow \quad b-k \geq\left(1-\frac{k}{c+d}\right) d \\
& \quad b W_{\mathrm{N}}(0,1,1) \\
& k \leq \frac{\Delta}{c}[c+d]=\mathrm{K}_{1}
\end{aligned}
$$

In the common knowledge case, $G(\bar{\gamma})=1$ and

$$
W_{N}(\mu, \bar{\gamma}, 1)=\left(1-\frac{k}{c^{\bullet} \bar{\gamma}+d}\right) \cdot\left(c^{\bullet} \mu+d\right)
$$

The "Z-shape" of $W_{D}, W_{D}(\bar{\gamma})=W_{N}(\bar{\gamma}, \bar{\gamma}, 1)$, and $W_{N}(\mu, \bar{\gamma}, 1)<c$ $=W_{D}^{\prime}(\mu)$ imply that

$$
\begin{aligned}
& W_{D}(\mu) \geq W_{N}(\mu, \bar{\gamma}, 1) \quad \forall \mu \in I \\
& \Leftrightarrow \quad W_{D}(0) \geq W_{N}(0, \bar{\gamma}, 1) \\
& \Leftrightarrow \quad b-k \geq\left(1-\frac{k}{c \bar{\gamma}+d}\right) d \\
& \leftrightarrow \quad k \leq \frac{\Delta}{c \bar{\gamma}}[c \bar{\gamma}+d]=K_{1}
\end{aligned}
$$

(c) The existence of an FD equilibriụm if $K_{2} \leq K_{1}$ is obvious. If $K_{1}<K_{2}$ and $k \in\left(K_{1}, K_{2}\right)$, then it is obvious from the proofs of (a) and (b) that there can be no FD-0, FD-1, or $\mathrm{FD}-\bar{\gamma}$ equilibria. To demonstrate that there is no other non-disclosure expectation $v$ that can sustain a full disclosure equilibrium, assume that an FD-v equilibrium exists for $v \in(0,1)$.

In the uniform distribution case, $W_{D}$ is strictly concave if $\delta>0$. The contradiction then follows immediately from the fact that $W_{N}$ must either intersect $W_{D}$ at $\mu$ $=v$ or, if they are tangent at $\mu=v, W_{N}>W_{D} \forall \mu \in I ; \mu \neq v$. If $W_{D}$ is linear (i.e., $\delta=0$ ), we cannot have $W_{D} \geq W_{N} \forall \mu \in I$ because

$$
W_{D}{ }^{\prime}=a-\Delta \neq W_{N} \prime=(1-k /(a v+b-\Delta v)) \cdot a \forall v \in(0,1)
$$

given that $k<K_{2}$.
In the common knowledge case, the contradiction follows from the fact that $W_{D}$ and $W_{N}$ must intersect at $\mu=$ $v$. To see this, observe that, if $\mu \epsilon(0, \bar{\gamma})$, then $W_{D}{ }^{\prime}=a$ and $W_{N}^{\prime}=(1-k / V(v, 0)) \cdot a<a$ and, if $v \epsilon(\bar{\gamma}, 1]$, then $W_{D}{ }^{\prime}=$ $c$ and $W_{N}{ }^{\prime}=(1-k / V(v, 1)) \cdot c<c$.
Q.E.D.

Proof of Lemma 3.4 .6
From Lemma 3.3.1 we have that $d W_{0} / d \mu$ (specified by (3.B.2)) is continuous and increasing at all $\mu \in\left(\mu_{*}, 1\right)$, with

$$
\left.\frac{d W_{D}}{d \mu}\right|_{\mu-\mu,}=\left.0 \quad \frac{d W_{D}}{d \mu}\right|_{\mu-1}=C
$$

$W_{N}$ is defined in (3.3.3), with $p(v)=G(v)$ in the unimodel distribution case. Observe that

$$
\begin{align*}
\frac{d W_{N}}{d \mu}= & \left(1-\frac{k}{V(v, G(v))}\right)[a-\delta G(v)]>0 \\
\frac{d^{2} W_{N}}{d \mu d v}= & -\frac{k \cdot d V(v, G(v)) / d v}{[V(v, G(v))]^{2}} \bullet[a-\delta G(v)] \\
& -\left(1-\frac{k}{V(v, G(v))}\right) \delta G^{\prime}(v)
\end{align*}
$$

since

$$
\frac{d V(v, G(v))}{d v}=\frac{d W_{D}(v)}{d \mu}>0 \quad \forall v \in\left[\mu_{*}, 1\right]
$$

(3.B.4) implies that $d W_{N} / d \mu$ is decreasing for $v \in\left[\mu_{*}, 1\right]$. This, plus the fact that

$$
\frac{d W_{N}(\mu, 1,1)}{d \mu}=\left(1-\frac{k}{c+d}\right) c<c
$$

implies that there must be a unique $v \in\left(\mu_{*}, 1\right)$ such that

$$
\left.\frac{d W_{D}}{d \mu}\right|_{\mu-v}=\left.\frac{d W_{N}}{d \mu}\right|_{\mu-v}
$$

Q.E.D.

## Proof of Proposition 3.4.7

(a) It is obvious that an FD-0 equilibrium exists if this condition is satisfied.
(b) The definition of $v$ and the convexity of $W_{D}$ on [ $\left.\mu_{0}, 1\right]$ implies that $W_{D} \geq W_{N} \forall \mu \in\left[\mu_{0}, 1\right]$. The convexity of $W_{D}$ on $\left[0, \mu_{0}\right]$ and the fact that $W_{D}>W_{N}$ at $\mu=\mu_{0}$ implies that $W_{D}$ $\geq W_{N}$ on $\left[0, \mu_{0}\right]$ if it holds at $\mu=0$.
(c) For any other expectation $v, W_{N}$ intersects $W_{D}$. This follows from the fact that $W_{D}$ is concave on $\left[0, \mu_{0}\right]$ and tangency would imply that $W_{N}>W_{D} \forall \mu \epsilon\left[0, \mu_{0}\right], \mu \neq v$, and in the proof of Lemma 3.4 .6 we demonstrate that the expectation defined there is the only point at which $W_{N}$ is tangent to the convex portion of $W_{D}$.
Q.E.D.

Proof of Lemma 3.5.1
(a) Lemma 3.3.2 establishes that $W_{N}$ intersects $W_{D}$ at most three times. Observe that $d W_{D} / d \mu=a>d W_{N} / d \mu$ at $\mu=0$. Therefore, either $W_{N}>W_{D}$ at $\mu=0$ or $W_{N}$ intersects $W_{D}$ at most twice with $W_{N}>W_{D}$ for $\mu \epsilon\left(\mu_{1}, \mu_{2}\right)$. The latter would contradict our proposition, but it is impossible since in that case $v \in\left(\mu_{1}, \mu_{2}\right)$ and we know that $W_{D}(v)=$ $W_{N}(v, v, G(v))$.
(b) From the definition of $W_{N}$ and $W_{D}$, we observe that
$W_{N}(v, v, p(v))=W_{D}(v)$ if $p(v)=G(v)$. This latter condition always holds if E's break-even point is not common knowledge. Hence, $v$ must equal one of the three possible intersection points $\mu_{1}, \mu_{2}$, or $\mu_{3}$, and it cannot be $\mu_{3}$ since $N$ contains no values above $\mu_{3}$.

Q.E.D.

Proof of Proposition 3.5 .2
Lemma 3.3.2 establishes that $W_{N}$ intersects $W_{D}$ at most twice. Recall that $W_{N}(v, v, G(v))=W_{D}(v)$. If $v$ is a tangency point, then we have full non-disclosure, which can only occur in the "knife-edge case" in which $W_{D}$ achieves its maximum at $\mu^{*}=1 / 2$. It is obvious that $v$ cannot be the only intersection point, since that would imply that all $\mu \in N$ lie above or below the mean of $N$, which is an impossibility. Therefore, in a partial disclosure equilibrium, $W_{N}$ must intersect $W_{D}$ exactly twice and $N=$ $\left[0, \mu_{1}\right] \cup\left[\mu_{2}, 1\right]$, with $\mu_{1}<\mu_{2}$.

Since $v$ is the expectation over $N$, we have

$$
\begin{align*}
v & =\left[\int_{0}^{\mu_{1}} \mu d \mu+\int_{\mu_{2}}^{1} \mu d \mu\right] \cdot \frac{1}{\mu_{1}+1-\mu_{2}} \\
& =\frac{1}{2} \frac{\mu_{1}^{2}+1-\mu_{2}^{2}}{\mu_{1}+1-\mu_{2}} \tag{3.B.5}
\end{align*}
$$

Recall that $v=\mu_{1}$ in a PD-L equilibrium and $v=\mu_{2}$ in a PD-H equilibrium. We now combine these two conditions with the fact that $W_{N}=W_{D}$ at $\mu_{1}$ and $\mu_{2}$ to specify the conditions that must be satisfied in the two types of PD equilibria. Since $W_{N}$ always equals $W_{D}$ at $\mu=v$, the key conditions are that $W_{N}$ and $W_{D}$ are also equal at $\mu_{2}$ in a PDL equilibrium and at $\mu_{1}$ in a PD-H equilibrium.

```
PD-L equilibria:
Substitute \(\mu_{1}=v\) into (3.B.5) and solve for \(\mu_{2}\) :
\(\mu_{2}=v+[1-2 v]^{\frac{1}{2}} \quad \forall v \in\left[0, \frac{1}{2}\right]\)
```

The condition that $W_{D}=W_{N}$ at $\mu_{2}$ provides the following necessary condition for the existence of a PD-L equilibrium with expectation $v \in[0,1 / 2]$ :

$$
\begin{align*}
& a \mu_{2}+b-\left[\delta \mu_{2}+\Delta\right] \mu_{2}-k \\
= & \left(1-\frac{k}{a v+b-[\delta v+\Delta] v}\right)\left(a \mu_{2}+b-\left[\delta \mu_{2}+\Delta\right] v\right) \tag{3.B.7}
\end{align*}
$$

Solve (3.B.7) for $k$ :

$$
\begin{equation*}
k=\left[\delta \mu_{2}+\Delta\right]\left[v+\frac{b-\Delta v}{a-\delta v}\right] \tag{3.B.8}
\end{equation*}
$$

We now determine the values of $k \in[0, d]$ for which there exists a $v \in[0,1 / 2]$ such that condition (3.B.8) is satisfied with $\mu_{2}$ defined by condition (3.B.6). Observe that $k$ is a continuous function of $v$ and that the following values of $k$ hold for the two extreme values of $v$ :

$$
\begin{aligned}
& v=0 \quad \Rightarrow k-\frac{b}{a}[\delta+\Delta]=K_{2} \\
& v=1 / 2 \Rightarrow k-\left[\frac{1}{2} \delta+\Delta\right]\left[\frac{1}{2}+\frac{b-\Delta / 2}{a-\delta / 2}\right]=k
\end{aligned}
$$

Therefore, condition (3.B.8) holds for all values of $k$ between $k$ and $K_{2}$, if we ignore the boundary conditions, and that implies condition (a) of the proposition when we introduce the requirement that $k \in[0, d]$.

PD-H Equilibria:
Substitute $\mu_{2}=v$ into (3.B.5) and solve for $\mu_{1}$ :

$$
\begin{equation*}
\mu_{1}=v-[2 v-1]^{\frac{1}{2}} \quad \forall v \in\left[\frac{1}{2}, 1\right] \tag{3.B.9}
\end{equation*}
$$

Replace $\mu_{2}$ with $\mu_{1}$ in (3.B.7) to specify the condition that $W_{N}=W_{D}$ at $\mu_{1}$, and make the same substitution in (3.B.8) to specify $k$ as a function of $\mu_{1}$ and $v$ :

$$
\begin{equation*}
k=\left[\delta \mu_{1}+\Delta\right] \cdot\left[v+\frac{b-\Delta v}{a-\delta v}\right] \tag{3.B.10}
\end{equation*}
$$

Evaluating (3.B.10) at $v=1 / 2$ and $v=1$ establishes that (3.B.10) can hold for all values of $k$ between $k$ and $K_{1}$, if we ignore the boundary conditions, and that implies condition (b) of the proposition when we introduce the requirement that $k \in[0, d]$.

Further characterization of Partial Disclosure Equilibria Given that E's Break-even Point is Uniformly Distributed

## PD-L Equilibria:

Differentiate (3.B.8) with respect to $v:$

$$
\begin{aligned}
\frac{d k}{d v}= & {\left[\delta\left(v+(1-2 v)^{\frac{1}{2}}\right)+\Delta\right]\left[1-\frac{a \Delta-b \delta}{(a-\delta v)^{2}}\right] } \\
& +\left[\delta\left(1-(1-2 v)^{-\frac{1}{2}}\right)\right]\left[v+\frac{b-\Delta v}{a-\delta v}\right]
\end{aligned}
$$

The second term is always negative since $(1-2 v)^{-1 / 2}>1 \forall$ $v \in(0,1 / 2)$, and the sign of the first term depends on the sign of $(a-\delta v)^{2}-(a \Delta-b \delta)$. Furthermore,

$$
\begin{aligned}
& \left.\frac{d k}{d v}\right|_{v-0}=(\delta+\Delta)\left(1-\frac{a \Delta-b \delta}{a^{2}}\right) \begin{cases}<0 & \text { if } a^{2}<a \Delta-b \delta \\
>0 & \text { if } a^{2}>a \Delta-b \delta\end{cases} \\
& \left.\frac{d k}{d v}\right|_{v-\frac{1}{2}}=\left(\frac{1}{2} \delta+\Delta\right)\left(1-\frac{a \Delta-b \Delta}{\left(a-\frac{1}{2} \delta\right)^{2}}\right)+ \begin{cases}0 & \text { if } \delta=0 \\
-\infty & \text { if } \delta>0\end{cases}
\end{aligned}
$$

Let $\left[k_{\star}, k^{*}\right]$ represent the range of $k$ values for which there exists a $v \in[0,1 / 2]$ such that condition (3.B.8) is satisfied, if we ignore the upper bound $d$. If $\delta=0$, then $d k / d v=\Delta[a-\Delta] / a$, implying that $k$ and $K_{2}$ are the bounds, with their relative magnitudes depending on the sign of a - $\Delta$ :
(L.1) If $\delta=0$ and $a>\Delta$, then $k_{*}=K_{2}<k=k^{*}$
(L.2) If $\delta=0$ and $a<\Delta$, then $k_{*}=k<K_{2}=k^{*}$ If $\delta>0$ and $a^{2}<a \Delta-b \delta$, then $d k / d v<0 \forall v \in(0,1 / 2)$, which implies:
(L.3) If $\delta>0$ and $a^{2}<a \Delta-b \delta$, then $k_{*}=k<K_{2}=k^{*}$ On the other hand, if $\delta>0$ and $a^{2}>a \Delta-b \delta$, then both $K_{2}$ $(v=0)$ and $k(v=1 / 2)$ are local minima. This implies that there exists a global maximum $\mathrm{k}^{*}>\max \left\{\mathrm{K}_{2}, \mathrm{~K}\right\}$. Determining whether $\min \left\{K_{2}, k\right\}$ is a global minimum is complex and, hence, we summarize this case as follows:
(L.4) If $\delta>0, a^{2}>a \Delta-b \delta$, then $k_{*} \leq \min \left\{K_{2}, k\right\} \leq \max \left\{K_{2}\right.$, $\kappa$ \} $<\mathrm{k}^{*}$. To determine $\underline{\mathrm{k}}_{2}$ and $\overline{\mathrm{k}}_{2}$, recognizing the requirement that $k \in[0, d]$, we let $\underline{k}_{2}=\min \left(k_{*}, d\right)$ and $\bar{k}_{2}=\min \left\{k^{*}\right.$, d). If $\underline{k}_{2}=\bar{k}_{2}=d$, then a PD-L equilibrium does not exist.

## PD-H Equilibria:

Using essentially the same approach as in the PD-L case provides the following characterization of the range
of $k$ values [ $k_{\dagger}, k^{\dagger}$ ] for which there exists a $v \in[1 / 2,1]$
such that condition (3.B.10) is satisfied:

$$
\begin{aligned}
& \text { (H.1) If } \delta=0 \text { and } a>\Delta \text {, then } k_{t}=k<K_{1}=k^{\dagger} . \\
& \text { (H.2) If } \delta=0 \text { and } a<\Delta \text {, then } k_{t}=K_{1}<k=k^{\dagger} . \\
& \text { (H.3) If } \delta>0 \text { and }(a-\delta / 2)^{2}<a \Delta-b \delta, \text { then } \\
& k_{t}=K_{1}<k=k^{\dagger} . \\
& \text { (H.4) If } \delta>0 \text { and }(a-\delta)^{2}>a \Delta-b \delta, \text { then } \\
& k_{\dagger}<\min \left\{K_{1}, k\right) \leq \max \left(K_{1}, k\right\} \leq k^{\dagger} .
\end{aligned}
$$

We then obtain the desired bounds by letting $\underline{k}_{1}=\min \left\{k_{+}\right.$, d\} and $\bar{k}_{1}=\min \left\{\mathrm{k}^{\dagger}, \mathrm{d}\right\}$.
Q.E.D.

Proof of Proposition 3.5.3
Lemma 3.3.2 establishes that $W_{N}$ intersects $W_{D}$ at most three times. It is straightforward to prove that, in a common knowledge partial disclosure equilibrium, $W_{N}$ intersects $W_{D}$ either twice, at $0<\mu_{1}<\mu_{2}=\bar{\gamma}$, or three times, at $0<\mu_{1}<\mu_{2}=\bar{\gamma}<\mu_{3}<1$. The expectation for the two-intersection-point case is:

$$
\begin{align*}
v & =\left[\int_{0}^{\mu_{1}} \mu d \mu+\int_{\mu_{2}}^{\mu_{3}} \mu d \mu\right] \cdot \frac{1}{\mu_{1}+\mu_{3}-\mu_{2}} \\
& =\frac{1}{2} \cdot \frac{\mu_{1}^{2}+\mu_{3}^{2}-\mu_{2}^{2}}{\mu_{1}+\mu_{3}-\mu_{2}} \tag{3.B.11}
\end{align*}
$$

In examining partial disclosure equilibria in this setting it is useful to separately examine the cases in which $\bar{\gamma}<1 / 2$ and $\bar{\gamma}>1 / 2$.

PD-L Equilibria for $\bar{\gamma}<1 / 2:$
In a PD-L equilibrium with $\bar{\gamma}<1 / 2$, there must be three intersection points (since including $[\bar{\gamma}, 1] \subset \mathbf{N}$ would result in an expectation greater than $\bar{\gamma}$ ). Substitute $\mu_{1}=$ $v$ and $\mu_{2}=\bar{\gamma}$ into (3.B.11) and solve for $\mu_{3}$ :

$$
\begin{equation*}
\mu_{3}-v+\left[v^{2}+(\bar{\gamma}-v)^{2}\right]^{\frac{1}{2}} \tag{3.B.12}
\end{equation*}
$$

The requirement that $W_{D}$ equals $W_{N}$ at $\mu_{3}$ specifies that (recall that the entry probability is zero under $W_{N}$, but is equal to one at $\mu_{3}$ under $W_{D}$ ):

$$
\begin{equation*}
c \mu_{3}+d-k-\left(1-\frac{k}{a v+b}\right)\left(a \mu_{3}+b\right) \tag{3.B.13}
\end{equation*}
$$

Solve (3.B.13) for k:

$$
\begin{equation*}
k=\left[8 \mu_{3}+\Delta\right] \frac{a v+b}{a\left(\mu_{3}-v\right)} \tag{3.B.14}
\end{equation*}
$$

Observe that (3.B.12) and (3.B.14) are continuous functions of $v$ and that

$$
\begin{array}{ll}
v=0 \Rightarrow \mu_{3}=\bar{\gamma}, & k=K_{2}=\frac{b}{a \bar{\gamma}}[\delta \bar{\gamma}+\Delta] \\
v=\bar{\gamma} \Rightarrow \mu_{3}=2 \bar{\gamma}, & k-K=[2 \delta \bar{\gamma}+\Delta]\left[1+\frac{b}{a \bar{\gamma}}\right]>K_{2}
\end{array}
$$

Therefore, condition (3.B.14) holds for all $k \in\left[K_{2}, k\right]$.

## PD-H Equilibria for $\overline{\mathbf{\gamma}}<1 / 2$ :

With $v=\bar{\gamma}=\mu_{2}$, we express $\mu_{3}$ as a function of $\mu_{1}$ :

$$
\begin{equation*}
\mu_{3}=\bar{\gamma}+\left[2 \mu_{1}\left(\bar{\gamma}-\mu_{1}\right)\right]^{\frac{1}{2}} \tag{3.B.15}
\end{equation*}
$$

Given $v=\bar{\gamma}$ and the entry probability $e \in[0,1]$, the expected wealth under non-disclosure is

$$
W_{N}(\mu, \bar{\gamma}, e)=\left(1-\frac{k}{a \bar{\gamma}+b-e[\delta \bar{\gamma}+\Delta]}\right)(a \mu+b-e[\delta \mu+\Delta])
$$

Set $W_{N}$ equal to $W_{D}$ at $\mu_{1}<\bar{\gamma}$ and $\mu_{3}>\bar{\gamma}$ :

$$
\begin{align*}
& a \mu_{1}+b-k-W_{N}\left(\mu_{1}, v, e\right)  \tag{3.B.16}\\
& c \mu_{3}+d-k-W_{N}\left(\mu_{3}, v, e\right) \tag{3.B.17}
\end{align*}
$$

Solve (3.B.16) for $k$, for a given $\mu_{1}$ and $e$,

$$
\begin{equation*}
k=\frac{e\left(\delta \mu_{1}+\Delta\right)[(a-\delta e) \bar{\gamma}+(b-\Delta e)]}{\left(\bar{\gamma}-\mu_{1}\right)(a-\delta e)} \tag{3.B.18}
\end{equation*}
$$

Solve (3.B.17) for $k$, set the result equal to (3.B.18), and then solve for $e$, for a given $\mu_{1}$ and $\mu_{3}$,

$$
\begin{equation*}
e=\frac{\left(\bar{\gamma}-\mu_{1}\right)\left(\delta \mu_{3}+\Delta\right)}{\left(\bar{\gamma}-\mu_{1}\right)\left(\delta \mu_{3}+\Delta\right)+\left(\mu_{3}-\bar{\gamma}\right)\left(\delta \mu_{1}+\Delta\right)} \tag{3.B.19}
\end{equation*}
$$

A PD-H equilibrium exists for capital level $k$ if there exists an intersection point $\mu_{1} \epsilon[0, \bar{\gamma}]$ that induces an intersection point $\mu_{3}$ based on (3.B.15) and an entry probability e based on (3.B.19) such that (3.B.18) holds. Observe that (3.B.15), (3.B.18) and (3.B.19) are all continuous functions of $\mu_{1}$ and that:

$$
\begin{aligned}
& \mu_{1}=0 \Rightarrow \mu_{3}=\bar{\gamma}, \quad e-1, \quad \text { and } k-K_{1}=\frac{\Delta}{c \bar{\gamma}}[c \bar{\gamma}+d] \\
& \mu_{1}-\bar{\gamma} \Rightarrow \mu_{3}-2 \bar{\gamma}, \quad e-0, \quad \text { and } k-k=[2 \delta \bar{\gamma}+\Delta]\left[1+\frac{b}{a \bar{\gamma}}\right]
\end{aligned}
$$

Therefore, condition (3.B.18) holds for all values of $k$ between K and $\mathrm{K}_{1}$.

## PD-L Equilibria for $\overline{\mathbf{\gamma}}>1 / 2:$

In a PD-L equilibrium with $\bar{\gamma}>1 / 2, W_{N}$ can intersect $W_{D}$ either twice ( $\mu_{3}=1$ ) or three times ( $\mu_{3}<1$ ). Solve (3.B.11) to obtain $v=\mu_{1}$ as a function of $\bar{\gamma}=\mu_{2}$ and $\mu_{3}$ :

$$
\begin{equation*}
v=\left[2 \mu_{3}\left(\mu_{3}-\bar{\gamma}\right)\right]^{\frac{1}{2}}-\left(\mu_{3}-\bar{\gamma}\right) \tag{3.B.20}
\end{equation*}
$$

$W_{N}=W_{D}$ at $\mu_{3}<1$ is again characterized by (3.B.13) and solving for $k$ provides (3.B.14). Evaluate (3.B.14) and (3.B.20) at $\mu_{3}=\bar{\gamma}$ :

$$
\mu_{3}-\bar{\gamma} \Rightarrow v=0, k=K_{2}=\frac{b}{a \bar{\gamma}}[\delta \bar{\gamma}+\Delta]
$$

At $\mu_{3}=1$ we only require that $W_{N}$ be greater than or equal to $W_{D}$ (a corner solution), i.e., (3.B.13) is an inequality for $\mu_{3}=1$. Restating (3.B.14) to reflect this inequality provides the following condition for $\mu_{3}=1$ :

$$
\begin{equation*}
k \leq k_{2}=[\delta+\Delta] \frac{a v_{1}+b}{a\left(1-v_{1}\right)}>k_{2} \tag{3.B.21}
\end{equation*}
$$

where

$$
v_{1}=[2(1-\bar{\gamma})]^{\frac{1}{2}}-(1-\bar{\gamma})
$$

$\left(K_{2}>K_{2}\right.$ follows from $\delta+\Delta>\delta \bar{\gamma}+\Delta, a v_{1}+b>b$, and $a \cdot\left(1-v_{1}\right)<$ $\mathrm{a} \bar{\gamma})$. Therefore, a PD-L equilibrium exists for all $\mathrm{k} \epsilon$ $\left[0, k_{2}\right] \cap[0, d]$.

PD-H Equilibria for $\overline{\boldsymbol{\gamma}}>1 / 2$ :
With $v=\bar{\gamma}=\mu_{2}$, we express $\mu_{1}$ as a function of $\mu_{3}$ :

$$
\begin{equation*}
\mu_{1}-\bar{\gamma}-\left[\mu_{3}\left(2 \bar{\gamma}-\mu_{3}\right)\right]^{\frac{1}{2}} \tag{3.B.22}
\end{equation*}
$$

$W_{N}=W_{D}$ at $\mu_{1}<\bar{\gamma}$ is again characterized by (3.B.16) and (3.B.18) restates condition (3.B.16) in terms of k. If $\mu_{3}$ $<1$, then (3.B.19) states the implication of (3.B.17) and (3.B.18) for e. Evaluate (3.B.22), (3.B.19), and (3.B.18) at $\mu_{3}=\bar{\gamma}$ :

$$
\mu_{3}=v-\bar{\gamma} \Rightarrow \mu_{1}=0, e-1, k=K_{1}-\frac{\Delta}{c \bar{\gamma}}[c \bar{\gamma}+d]
$$

At $\mu_{3}=1$, (3.B.22) implies that $\mu_{1}=\bar{\gamma}-[2 \bar{\gamma}-1]^{1 / 2}$. Let $e_{1}$ and $k_{1}$ represent the corresponding solutions to (3.B .19) and (3.B.18).

Observe that (3.B.17) can be restated as an inequality if $\mu_{3}=1$, since this can be a corner solution with $W_{D}$ $\leq W_{N}:$

$$
\begin{equation*}
c+d-k \leq W_{N}(1, \bar{\gamma}, e) \tag{3.B.23}
\end{equation*}
$$

Combining (3.B.18) with (3.B.23) results in the restatement of (3.B.19) as an inequality: $e \leq e_{1}$.

A PD-H equilibrium, with $\mu_{3}=1$, exists for capital level $k$ if there exists an entry probability e $\epsilon\left[0, e_{1}\right]$ that satisfies (3.B.18), where $\mu_{1}$ is defined by (3.B.22). Observe that $k$ is a continuous function of $e$ with $k=0$ if $e=0$ and $k=k_{1}$ if $e=e_{1}$. Consequently, considering both $\mu_{3}<1$ and $\mu_{3}=1$, PD-H equilibria exist for all $k \in$ $\left[0, \max \left\{K_{1}, K_{1}\right\}\right] \cap[0, d]$.

## Proof of Proposition 3.6.1

We provide a detailed proof for the uniform distribution case and then make brief comments on a similar proof for the common knowledge case.

Uniform distribution case: If $\tilde{\gamma}$ is uniformly distributed, then $G^{*}(\mu)=G(\mu)=\mu$. The proof for this case consists of four lemmata.

First, some knowledge about $\alpha^{*}(v, v)$ and $V(v, v)$ is useful. Figure 3-8 depicts these curves in the uniform distribution case. 3-8(a) is the $V$ curve (similar to Figure 3-2(b) but increased in height by an amount k). 3$8(b)$ is the $\alpha^{*}$ curve, representing the equilibrium contract offered by all $\mu$ in an FD equilibrium. 3-8(c) represents the equilibrium contract line in a $\mathrm{PD}-v_{1}$ equilibrium. Note that in the last case, for all $\mu \in \mathbb{N}=$ $\left[0, t_{1}\right] \cup\left[t_{2}, 1\right], I$ offers one contract $\alpha^{*}\left(v_{1}\right)$ and $M$ accepts it.

## Insert Figure 3-8 here

Observe that $\mathrm{T}^{*}(\alpha)$ is empty if $\alpha$ is too small to be accepted by $M$, even if $M$ and $E$ 's expectation $v$ maximizes $V(v, v)$. Let $\alpha_{1}$ represents the smallest $\alpha$ that would be accepted by $M$, which equals $k / V\left(v^{*}, v^{*}\right)$, where $v^{*}$ is the value of $v$ which maximizes

$$
v(v, v)=a v+b-v[\delta v+\Delta]
$$

Let $\alpha_{2}$ represents the smallest $\alpha$ that would be accepted by all $M$ no matter what beliefs $M$ holds. The value of $\alpha_{1}$ and $\alpha_{2}$ depends on the shape of $V$, which, in turn, is determined by the parameter values. We can classify all different situations into four cases which are mutually exclusive. The corresponding $\alpha^{*}(v, v)$ curves are depicted in Figure 39.
(a) $\Delta<c-\delta$, i.e., b $<c+d$ and $a-\Delta>2 \delta$. In this case, the V -curve is monotone increasing and, hence, the $\alpha^{*}$-curve is monotone de-creasing. Thus, $\alpha_{1}=k / c+d$ and $\alpha_{2}=$ k/b.
(b) $c-\delta<\Delta<c$, i.e., $b<c+d$ and $a-\Delta \in[0$, 2 $\delta$ ]. In this case, the $V$-curve has an interior maximum and a minimum at $v=0$. Hence,

$$
\begin{equation*}
\alpha_{1}=\frac{k}{\frac{(a-\Delta)^{2}}{4 \delta}+b} \tag{3.B.24}
\end{equation*}
$$

and $\alpha_{2}=\mathrm{k} / \mathrm{b}$.
(c) $c<\Delta<a$, i.e., $b>c+d$ and $a-\Delta \epsilon[0,2 \delta]$. In this case, the $v$-curve has an interior maximum and a minimum at $v=1$. Hence, the $\alpha_{1}$ is the same as (B.24) but $\alpha_{2}=k /(C+d)$.
(d) $\Delta>a$, i.e., b $>c+d$ and $a-\Delta<0$. The $V-$ curve is monotone increasing and the $\alpha^{*}-$ curve is monotone decreasing. Hence, $\alpha_{1}=$ $\mathrm{k} / \mathrm{b}$ and $\alpha_{2}=k /(c+d)$.

Insert Figure 3-9 here

Lemma 3.6.1: In all cases (a)-(d), $T^{*}(\alpha)=\varnothing$ if $\alpha<\alpha_{1}$.

Given any contract $\alpha$ such that $\alpha_{1} \leq \alpha \leq \alpha_{2}$, there always exist beliefs that will induce the market to accept the contract, and other beliefs that will induce the market to reject the contract. That is, both the sets $A(\alpha)$ and $R(\alpha)$ will be non-empty. Let $v_{1}$ and $v_{2}$ denote the market beliefs which separate $A(\alpha)$ and $R(\alpha)$, which are the solutions to $\alpha^{*}\left(v^{\prime \prime}, v\right)=\alpha$ :

$$
\left.\begin{array}{rl}
v_{1}(\alpha)= & \max \{0
\end{array}\right) \frac{1}{2 \delta}\{(a-\Delta)] \begin{aligned}
& \left.\left.-\left[(a-\Delta)^{2}+4 \delta\left(b-\frac{k}{\alpha}\right)\right]^{\frac{1}{2}}\right\}\right\} \\
v_{2}(\alpha)=\min \{1, & \frac{1}{2 \delta}\{(a-\Delta) \\
& \left.\left.+\left[(a-\Delta)^{2}+4 \delta\left(b-\frac{k}{\alpha}\right)\right]^{\frac{1}{2}}\right\}\right\}
\end{aligned}
$$

We then have following lemma.

Lemma 3.6.2: Given $\alpha_{1} \leq \alpha \leq \alpha_{2}$, the values of $v_{1}$ and $v_{2}$, depending on the different cases, are
(a) when $\Delta<c-d, v_{1}>0$ and $v_{2}=1$ for all $\alpha$;
(b) when $\Delta \epsilon[c-d, c], v_{1}>0$ for all $\alpha, v_{2}=1$ if $\alpha \epsilon\left[k /(c+d), \alpha_{2}=k / b\right], v_{2}<1$ if $\alpha \epsilon\left[\alpha_{1}\right.$, $\mathrm{k} /(\mathrm{c}+\mathrm{d})]$;
(c) when $\Delta \in[c, a], v_{2}<1$ for all $\alpha_{1} v_{1}=0$ if $\alpha \epsilon\left[k / b, \alpha_{2}=k /(c+d)\right]$, and $v_{1}>0$ if $\alpha \epsilon$ $\left[\alpha_{1}, k / b\right]$;
(d) when $\Delta>a, v_{1}=0$, and $v_{2}<1$ for all $\alpha$.

Observe that the sets

$$
\begin{aligned}
& A(\alpha)=\{v \mid r(\alpha, v, v)=0\}=\left[v_{1}, v_{2}\right] \\
& R(\alpha)=\{v \mid r(\alpha, v, v)=1\}=\left[0, v_{1}\right] \cup\left[v_{2}, 1\right]
\end{aligned}
$$

and in the cases where $v_{1}=0$ or $v_{2}=1$, one of these intervals is empty. For a given contract $\alpha$, the optimal expectation that $M$ and $E$ can hold (from I's perspective) is the smallest expectation that will induce $M$ to accept the contract. Using the results from Lemma 6.2, the following lemma specifies the optimal expectation for each of the four cases.

Lemma 3.6.3: The optimal expectation that $M$ and $E$ can hold in response to contract $\alpha$, for each of the four cases, is (a) $v_{1}>0$; (b) $v_{1}>0$; (c) $v_{1}>0$ if $\alpha \epsilon\left[\alpha_{1}, k / b\right], v_{2}<1$ if $\alpha \epsilon[\mathrm{k} / \mathrm{b}$; $k /(c+d)]$; and (d) $v_{2}<1$.

Based on this knowledge, the following lemma can be proved.

Lemma 3.6.4: If $T^{*}(\alpha) \neq \varnothing$, then $T^{*}(\alpha)=T_{1} \cup T_{2}$, where
(i) $T_{1}=\left[0, t_{1}\right]$ and $t_{1}>0$ if an $F D-0$ equilibrium exists;
(ii) $T_{2}=\left[t_{2}, 1\right]$ and $t_{2}<1$ if an FD-1 equilibrium exists and an FD-0 equilibrium does not exist.

Proof of Lemma 3.6.4: The general characterization of
$T^{*}(\alpha)$ follows from the fact that $W^{*}(\mu)=W_{D}(\mu)$ is concave and $W^{\dagger}(\mu, \alpha)$ is a linear function of $\mu$. To demonstrate this linearity, first observe that $\mathrm{W}^{\dagger}$ is a linear function of $\mu$ if $r$ and $p$ are fixed. Next observe that the smallest expectation $v$ such that $r^{*}(\alpha, v, v)=0$ is the optimal expectation for all $\mu \in I$. That is, the desired expectation is the smallest value of $v$ satisfying

$$
\alpha \geq \alpha^{*}(v, v)=\frac{k}{a v+b-v \bullet[\delta v+\Delta]}
$$

which implies $v=v_{1}$ or $v_{2}$ according to lemma 3.6.3.
Since $\mathrm{T}^{*}(\alpha) \neq \varnothing, \mathrm{W}^{*}$ and $\mathrm{W}^{\dagger}$ must intersect at least once $\left(W^{\dagger}\right.$ cannot exceed $W^{*}$ for all $\mu \in I$ since that would require $\alpha<\alpha_{1}$ ). Given the above expectation, an intersection between $W^{*}$ and $W^{\dagger}$ satisfies

$$
a t+b-t \cdot[\delta t+\Delta]-k=(1-\alpha) \cdot[a t+b-v \cdot(\delta t+\Delta)] \text { (3.B.27) }
$$

For the moment ignore any bounds on $t$ and let $t_{1}<t_{2}$ be the two roots to this quadratic equation.

If $v>0$, then $t_{1}=v$, implying that $t_{1}>0$. Hence, to prove condition (i) we need only prove that $v=0$ is impossible if $T^{*}(\alpha) \neq \varnothing$ and an $\mathrm{FD}-0$ equilibrium exists. (a.1) $\mathrm{FD}-0 \Leftrightarrow \mathrm{k} \geq \mathrm{K}_{2}=(\delta+\Delta) \cdot \mathrm{b} / \mathrm{a} \Leftrightarrow \mathrm{k} / \mathrm{b} \geq(\delta+\Delta+\mathrm{k}) /(\mathrm{a}+\mathrm{b})$. (a.2) $\mathrm{T}^{*}(\alpha)=\varnothing$ if $\mathrm{W}^{*}>\mathrm{W}^{\dagger}, \forall t \in I$. Assume $v=0$, this holds if

$$
a t+b-t \cdot[\delta t+\Delta]-k>(1-\alpha) \cdot[a t+b]
$$

$$
\Leftrightarrow \quad \alpha>\max \{\mathrm{k} / \mathrm{b},(\delta+\Delta+\mathrm{k}) /(\mathrm{a}+\mathrm{b})\}
$$

Hence, from (a.1), if an FD-0 equilibrium exists, $T^{*}(\alpha) \neq$ $\otimes$ implies $\alpha<\alpha_{2}=k / b$. This, in turn, implies that cases (c) and (d) in which $v_{1}=0$ will not occur. Hence $v_{1}>0$ implying $t_{1}>0$.

For a proof of (ii), note that when an FD-0 equilibrium does not exist and $\mathrm{T}^{*}(\alpha) \neq \varnothing$, this implies that either case (c) or (d), in which $k / b<\alpha<k /(c+d)$, holds. Note that the optimal expectation is now $v_{2}$ ! (Because the acceptance set $A(\alpha)=\left[0, v_{2}\right]$, the biggest $v$ that will accept $\alpha$ appears at $v_{2}$ instead of 0 ). Hence it is easy to show that

$$
\alpha<k /(c+d) \Leftrightarrow v_{2}<1 \Leftrightarrow t_{2}<1
$$

Hence an FD-1 equilibrium exists.

## Common Knowledge Case:

If $\tilde{\gamma}=\bar{\gamma}$ is common knowledge, then $\mathrm{G}^{*}(v)=0 \forall \mu \epsilon[0$, $\bar{\gamma}]$ and $G^{*}(\mu)=1 \forall \mu \epsilon(\bar{\gamma}, 1]$. The proof of the proposition is similar to the proof for the uniform distribution case and is not given here.
Q.E.D.

## Proof of Proposition 3.6.2

In a PD equilibrium, $W^{*}$ is the maximum of $W_{D}$ and $W_{N}$. Lemma 3.6.4 again applies. In the uniform distribution case it can be shown that if, $\mathrm{T}^{*}(\alpha) \neq \varnothing$, then either $v=0$
or $v=1$ is a credible threat, i.e., at least one of these expectations is such that it would result in the rejection of $\alpha$ and belongs to $\mathrm{T}^{*}(\alpha)$. Similarly, in the common knowledge case it can be shown that, if $T^{*}(\alpha) \neq \varnothing$, then either $v=0$ or $v=\bar{\gamma}$ is a credible threat.
Q.E.D.

## Proof of Proposition $\mathbf{3 . 6 . 3}$

We provide a proof based on the uniform distribution. The proof for other distributions is more complicated. The "if" part is straightforward as discussed above. The "only if" part can be shown in two steps. First, given an out-of-equilibrium contract $\alpha$, assume $M$ and $E$ hold belief $\nu(\alpha)=v_{1}(\alpha)$. Based on $v_{1}, \alpha$ will be accepted and $T^{*}(\alpha)=$ $\left[0, t_{1}\right] \cup\left[t_{2}, 1\right]$. If there exists a $T \subset T^{*}(\alpha)$ such that $v_{1}=$ $\bar{\mu}(T)$, then $T$ forms a PD equilibrium, a contradiction.

Second, consider the case in which $M$ and $E$ hold beliefs $v^{\prime}(\alpha)<v_{1}(\alpha)$. Let the equilibrium contract corresponding to $v^{\prime}$ be $\alpha^{\prime}$. Since $v^{\prime}<v_{1}$, we have $\alpha^{\prime}$ > $\alpha^{*}$. From Figure 6-8, it can be seen that $A\left(\alpha^{\prime}\right) \supset A\left(\alpha^{*}\right)$, i.e., we have $T^{*}\left(\alpha^{\prime}\right) \subset T^{*}\left(\alpha^{*}\right)$ since $T^{*}(\alpha)=I / A(\alpha)$. If $T \subset$ $T^{*}\left(\alpha^{\prime}\right)$, then $T \subset T^{*}\left(\alpha^{*}\right)$ and $T$ forms a $P D$ equilibrium. This again results in a contradiction.
Q.E.D.

Proof of Proposition 3.6.4
The "if" part is obvious. For the "only if" part, let us assume there does not exist another dominant PD equilibrium. Let the equilibrium non-disclosure contract be $\alpha^{0}$, and let $M$ and E's belief in the equilibrium be $v^{0}=$ $v\left(\alpha^{0}\right)$. For some out-of-equilibrium contract $\alpha_{1}<\alpha^{0}$, if M and $E$ respond with $v_{1}=v\left(\alpha_{1}\right)$, then it must be that $v_{1}>$ $v^{0}$. Otherwise, the contract $\alpha_{1}$ will be rejected. The $\mathrm{W}^{\dagger}\left(\alpha_{1}\right)$ curve cannot lie above $\mathrm{W}^{\dagger}\left(\alpha^{0}\right)$ otherwise $\mathrm{W}^{\dagger}\left(\alpha_{1}\right)$ becomes a dominant PD-equilibrium. This implies that they must cross, and $\mathrm{T}^{*}\left(\alpha_{1}\right)$ can be either $\left[0, v_{1}\right]$ or $\left[v_{1}, 1\right]$. (See Figure 3-10.) However, one interval cannot form a PD equilibrium so that a $T$ satisfying the GP-criterion condition will not exist. Hence, a single PD equilibrium will not fail the GP-criterion.

```
Insert Figure 3-10 here
```

For $v_{1}>v\left(\alpha_{1}\right)$, i.e., $\alpha>\alpha^{*}(v, v)$, the arguments are similar.
Q.E.D.

Appendix 3.C: Notation Used in the Paper

| I | -- | Incumbent |
| :---: | :---: | :---: |
| M | -- | Capital Market |
| E | -- | Entrant. |
| $\tilde{\mathbf{x}}$ | -- | $\begin{aligned} & \text { I's end-of-period value (random) } \\ & \text { w.c.d.f. } F(x \mid \cdot) \end{aligned}$ |
| $\tilde{\mu}$ | -- | I's type, w.c.d.f. $\Phi(\mu)$ |
| $\bar{\mu}$ | -- | The prior mean of $\tilde{\mu}$ |
| $\mu$ | -- | The realized value of $\tilde{\mu}$, I's private information. |
| k | -- | The amount of investment needed. |
| $\pi^{0}$ | -- | I's payoff if the project is turn down. |
| e | -- | Entry probability of E. : |
| $e(v, \gamma)$ | -- | E's strategy given E's belief $v$ and type $\gamma$. |
| $p(v)$ | -- | I and M's belief about E's entry probability. |
| $\pi(\mu, 0)=a \mu+b$ | -- | I's expected end-of-period value given $\mu$ and $e=0$. |
| $\pi(\mu, 1)=c \mu+d$ | -- | I's expected end-of-period value given $\mu$ and $e=1$. |
| $\delta=a-c$ | -- | Variable entry cost. |
| $\Delta=b-d$ | -- | Fixed entry cost. |
| $\bar{\gamma}$ | -- | E's type w.c.d.f. G(\%). |
| $\gamma$ | -- | Realized value of $\tilde{\gamma}$, E's private information. |
| $\bar{\gamma}$ | -- | Expected value of $\tilde{\gamma}$ or common knowledge of E 's type. |
| $M(\mu)=\{\mu, \mathrm{n}\}$ | - | Set of possible reports type $\mu \in I$ can send. |
| $\mathrm{m} \in \mathrm{M}(\mu)$ | -- | Message sent by type $\mu \in I$. |
| Y | -- | $M$ and E's information about $\tilde{\mu}$. |
| $\Phi(\mu \mid Y)$ | -- | M and E's posterior beliefs about $\tilde{\mu}$ given $Y$. |
| $\bar{\mu}(Y)$ | -- | $M$ and E's posterior expectation about $\bar{\mu}$ given $Y$. |
| $r$ | -- | The probability with which M rejects the contract $\alpha$. |
| $\alpha$ | -- | Share of I's payoff to be received by M. |


| $\begin{aligned} & \alpha^{*}(v, p) \\ & =k / v(v, p) \end{aligned}$ |  | Minimum share M will accept given $v$ and p . |
| :---: | :---: | :---: |
| $V(v, p)$ |  | Firm's market price given belief $v$ and p. |
| $\mathbf{N}$ | -- | Non-disclosure region, i.e., I's strategy. |
| D | -- | Disclosure region. |
| $W(\mu, \alpha, r, p)$ | -- | I's expected end-of-period wealth. |
| $W_{D}(\mu)$ | -- | I's expected end-of-period wealth under disclosure. |
| $W_{N}(\mu, v, P(v))$ | -- | I's expected end-of-period wealth under non-disclosure. |
| $\mathrm{K}_{1}, \mathrm{~K}_{2}$ | -- | Boundaries of $k$ for a FD equilibrium. |
| $\underline{\mathrm{k}}_{1}, \overline{\mathrm{k}}_{1}, \underline{\mathrm{k}}_{2}, \overline{\mathrm{k}}_{2}, \mathrm{k}$ | -- | Boundaries of $k$ for a PD equilibrium. |
| $\mathrm{K}-\mathrm{H}=\left[\underline{\mathrm{k}}_{1}, \overline{\mathrm{k}}_{1}\right]$ |  | The set of $k$ for which a PD-H equilibrium exists. |
| $\mathrm{K}-\mathrm{L}=\left[\underline{\mathrm{k}}_{2}, \overline{\mathrm{k}}_{2}\right]$ | -- | The set of $k$ for which a PD-L equilibrium exists. |
| $\mu_{1}, \mu_{2}, \mu_{3}$ | -- | Intersections of $W_{D}$ and $W_{N}$. |
| $\beta_{0}, \beta_{1}, \beta_{2}$ | -- | Parameters of the beta distribution. |
| FD-v | -- | Full disclosure equilibrium sustained by non-disclosure belief $v$. |
| PD-L | -- | Partial disclosure equilibrium with $v$ $=\mu_{\text {}}$. |
| $\mathrm{PD}-\mathrm{H}$ | -- | Partial disclosure equilibrium with $v$ $=\mu_{2}$. |

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## PART II

## INCOMPLETE CONTRACTING

## Chapter 4

## INTRODUCTION TO PART II

Fifty years ago, Coase [1937] began to deal with some key questions that neoclassical economic theory had ignored. What is a firm? What factors determine a firm's size? What are the costs and benefits of integration? Coase's answers are based on his fundamental insights that: (1) some kinds of economic activities are too costly to coordinate by using the market price system; (2) markets and firms are alternative ways of organizing economic exchanges; and (3) uncertainty and opportunism increase the cost of using the price system.

In recent years, Coase's work has significantly influenced the development of research in the theory of organizations. Following Coase, Williamson [1975] [1985] has developed the concept of "transaction cost" and the theory of "transaction cost economics". Williamson [1975] summaries his model as follows:

The general approach to economic organization employed here can be summarized compactly as follows: (1) Markets and firms are alternative instruments for completing a related set of transactions; (2) whether a set of transactions ought to be executed across markets or within a firm depends on the relative efficiency of each mode; (3) the costs of writing and executing complex contracts across a market vary with the characteristics of the human decision makers who are involved with the transaction on the one hand, and the objective properties of the market on the other; and (4) although the human and environmental factors that impede exchanges between firms (across a market) manifest themselves somewhat differently within the firm, the same set of factors apply to both.

For Williamson [1985], the human characteristics that most affect the governance choice are bounded rationality and opportunism. The environmental characteristics that determine the choice between market and hierarchical governance are uncertainty, complexity, and small numbers. The existence of a small number of parties to an exchange increases the likelihood of opportunistic behaviour, while the existence of uncertainty and complexity makes bounded rationality operative, thus making it impossible for parties to an exchange to anticipate all possible future states in their relations. In such a setting, the costs of writing and enforcing a contract assuring all parties to an exchange of an outcome that all would deem as acceptable are high enough to be prohibitive. In this situation, a market would be an inefficient means of carrying forward a transaction, and it will be replaced by a hierarchy.

One of the criticisms of Williamson's work, as pointed by Kreps [1984], is that he is less convincing in his arguments that transacting through a hierarchical organization will lessen transaction costs. Although Williamson explicitly recognizes that transacting through a hierarchical framework will also incur transaction costs, he does not provide enough analysis about how they would differ from market mediated transaction costs. The
arguments as to why empirical observations show that hierarchical transactions usually incur lower transaction costs needs to be developed further.

Alchian and Demsetz [1972] also stress a type of transaction specific investment in developing their explanation of why firms exist. For Alchian and Demsetz, the reason for the existence of firms can ultimately be found in what they call team production. Team production exists when the collective output of a group of individuals is greater than the sum of the output of each of them separately, and where it is simultaneously difficult to discover each individual's contribution to the group's output. Individuals in these teams have firm specific skills -- skills whose value is greater in combination with other members of the particular team than in other exchange contexts.

However, the existence of productive teams does not, by itself, result in hierarchical forms of organizations. Teamwork only requires cooperation; it does not necessarily require an organizational hierarchy.

Alchian and Demsetz argue that the classical firm emerges because of the eventual need to monitor, or meter, the individuals that make up a productive team. Because it is difficult in a team to determine the individual contributions of each member, each member has an incentive
to shirk. The possibility of shirking will deter highoutput individuals from joining the team and may discourage customers and capital investors as well. One way to reduce shirking is to provide for monitoring of the performance of each team member. However, because the monitor also has incentive to shirk, other mechanisms to provide incentives for the monitor are necessary. One key structure for such incentives is so called property rights. The structure of property rights can reduce the likelihood of shirking by a monitor by causing him to bear the costs of such behaviour. These property rights include: (1) the right to the residual productivity of the team beyond that which is necessary to keep the team operating, (2) the right to observe' the productive input of individuals on the team, (3) the right to monitor all contracts with sources of input into the team, and (4) the right to sell these rights. These property rights define the ownership of the firm. The owner of the firm has strong incentives not to shirk his monitoring responsibilities, since by doing so, he would not be maximizing his personal wealth.

Klein, Crawford, and Alchian [1978] explore one particular cost for using the market system -- the possibility of post-contractual opportunistic behaviour. The particular circumstance they emphasize is the presence of
appropriable specialized quasi-rents. After a specific investment is made and such quasi-rents are created, the possibility of opportunistic behaviour is very real. There are two possible ways to solve the problem: vertical integration or contracts. As assets become more specific and more appropriable quasi-rents are created (and therefore the possible gains from opportunistic behaviour increase), the costs of contracting will generally increase more than the costs of vertical integration. Hence, ceteris paribus, we are more likely to observe vertical integration.

Following the work of Coase [1937], Williamson [1975], Alchian and Demsetz [1972], Klein, Crawford, and Alchian [1978], Grossman and Hart [1986] [1987] develop a theory of integration. They emphasize the benefits and the costs of "control" in response to situations in which there are difficulties in writing or enforcing complete contracts. Contractual rights can be of two types: specific rights and residual rights. When it is too costly for one party to specify a long list of the particular rights it desires over another party's assets, it may be optimal for that party to purchase all the rights except those specifically mentioned in the contract. Ownership is the purchase of these residual rights.

For Grossman and Hart, integration is defined in
terms of the ownership of assets so that at issue is when one firm will desire to acquire the assets of another firm. If one party acquires the rights of control, then that must reduce the rights of control of the other party. There are potential costs associated with removing control from those who manage productive activities. Therefore, it is desirable for firm 1 to integrate firm 2 only when firm l's control increases the productivity of its management more than the loss of control decreases the productivity of firm 2's management.

A basic idea in all the papers discussed above is that transaction costs are incurred in writing contracts. In a world where it is costless to write contracts, the parties engaging in a transaction would write a complete contract which specifies precisely each party's obligations in every conceivable state of the world. Under a complete contract, there is no need for contract renegotiation because everything would be anticipated in advance. Nor would any disputes ever occur since a third party could (costlessly) determine whether one of the parties had breached the contract, and would impose an appropriate penalty. Ownership is irrelevant under complete contracting even if information asymmetries exist.

However, the assumption of no transaction costs is unrealistic. Instead, in practice, transaction costs are
pervasive and unavoidable. A consequence of these costs is that contracts are always incomplete. An incomplete contract specifies some obligations of the contracting parties in some states but not in others. Furthermore, contracts are sometimes incomplete even though they specify the obligations of the contracting parties in all
states. The incompeletenss in that context arises because the obligations are held constant across some states for which it would be "optimal" to specify different obligations for different states. This is particularly the case for low probability events, such as the October 1987 stock market crash which Saly [1991] is examining. This results in the possibility that the parties will either be forced or be motivated to renegotiate the contract. In addition, disputes may occur either because of different interpretations of the contract, or due to opportunistic behaviour. The nature of incomplete contracting is analyzed by Hart and Moore [1988]. They focus on the cost of writing a contingent clause in a sufficiently clear and unambiguous way that it can be enforced. If the state variable is complex, this cost may be prohibitively high. Thus, the parties may end up writing an incomplete initial contract, and revising the contract once the state is realized. Given rational expectations by the parties, the possibility of contract renegotiation will affect the form of the
original contract. In addition, it may be in the parties' interests to constrain, in the initial contract, the final outcome of the renegotiation process. In other words, the parties' problem is to design an optimal revision game to be played once the state is realized.

Hart and Moore [1990] contribute further to the theory of property rights. They extend the idea that ownership is the control of assets to a multi-asset, multi-individual economy and study how changes in ownership affect the incentives of non-owners of assets (employees) as well as the incentives of owner-managers. One of the key assumptions in their analysis is that each individual's contribution is given by his Shapley value. The optimal assignment of assets based on their setting is such that an agent is more likely to own an asset if his action is sensitive to whether he has access to the asset and is important in the generation of surplus, or if he is a crucial trading partner for others whose actions are sensitive to whether they have access to the asset and are important in the generation of surplus. These results help us to understand the boundaries of the firm.

These recent advances in the organization theory should have a great influence on managerial accounting research. Accounting plays an information supporting role in a firm's economic activities. An accounting system can
be optimized only if we understand the firm's strategy in organizing transactions, either intra-firm or inter-firm. This part of the thesis contributes to incomplete contracting research from the perspective of managerial accounting. The main purpose is to extend the traditional contracting theory analysis of accounting.issues by incorporating a broader set of contracting strategies. The key issues we focus on are the same as those pursued by Grossman, Hart, and many others in the economics field. What are the benefits of "organizing transactions within the firm?" How can incomplete contracting work within the firm? The distinctive nature of our analyses, relative to the existing economics literature, is that we examine the problem from an accounting perspective and emphasize the implication of these new theories to managerial accounting research.

Each of the following two chapters provides an analytical model related to incomplete contracting. Chapter 5 analyzes contracting strategy by explicitly considering contracting costs. We show how contracting efficiency can be more precisely defined if we explicitly consider contracting costs, and how incomplete contracting can improve contracting efficiency through minimizing contracting costs. The results in this chapter provide new insights for the issue of transfer pricing for intra-firm transac-
tions. Basically, we believe that the transfer prices should be chosen based on the criterion of minimizing transaction costs.

Chapter 6 focuses on the incentive issue within organizations. Agency theory usually assumes complete contracting and, hence, provides so called high-powered incentives for the agents (see the chapter for a detailed definition). This is consistent with some employee levels in a firm, particularly top management. However, most employees' compensation is not based on explicit contingent contracts. Instead, they are motivated not to shirk by low-powered incentives. These incentives are provided by the anticipation of future contract renewal. In the model, employment contracts are incomplete and short-term. However, the employees' specific human assets provide incentives to maintain long-term employment relations. It is the sharing of the ex post gains from the employment relation that provides incentives for the employee and the firm to invest. More accurately, both economic agents are motivated by the anticipated sharing of expected future gains where those expectations will be influenced by the observed, but non-contractible, human asset levels.

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## Chapter 5

# INCOMPLETE CONTRACTING FOR 

## ECONOMIZING CONTRACTING

## COSTS

### 5.1 Introduction

As early as 1937, Coase pointed out that some kinds of economic activities are too costly to coordinate by using the price system (Coase [1937]). He claimed that markets and firms are alternative ways of organizing economic exchange. These perceptions led him to focus explicitly on contracting processes. Coase's insights were advanced by Williamson [1975] [1985]. Williamson focused on "transaction costs" which are, by Arrow [1974], the costs of running the economic system.

Coase and Williamson's work should be extremely important to managerial accounting research. Managerial accounting facilitates intra-firm transactions, which, by Coase's view, are substitutes for market transactions. Williamson argues that the main reason for a transaction to occur within a firm is that it is more costly to effect the transaction as a market exchange between two independent units than to incorporate both units within a single firm. What transaction and organizational characteristics allow a firm to provide these advantages? What implications do these organizational characteristics have for managerial accounting research? These questions have not been explored in accounting research to any significant extent.

Although the existence of transaction costs has been
recognized in accounting research, most of the literature to date assumes that these costs are trivial. Therefore, they are either ignored or combined with traditional production costs. Sometimes, transaction costs are relatively small with respect to transaction gains and, hence, ignoring them will have little impact on the analysis. However, there exist situations in which these costs are non-trivial, and play a crucial role in the contracting process. In these cases, ignoring these costs will cause serious bias in the results. For example, if contracting costs are ignored, then a bias toward complicated comprehensive contracting is present. This is certainly misguided if complete contracting is very costly (perhaps impossible).

The above issues motivate us to explicitly consider transaction costs. This is particularly important when we examine a firm's behaviour in organizing intra-firm transactions. A firm's managerial accounting system that reflects the firm's strategies and governance structure may be influenced by such considerations.

We explicitly consider contracting costs in a simple intra-firm transaction model. The main objective is to show the influence of contracting costs on contracting strategies. We show how contracting efficiency can be more precisely defined if we explicitly consider contract-
ing costs, and how incomplete contracting can improve contracting efficiency through minimizing contracting costs. Thus we extend traditional contracting theory, which emphasizes complete contracting, to a broader contracting strategy space. In short, taking contracting costs into account, incomplete contracting may be optimal in many cases. This is obviously consistent with empirical observations.

We use the concepts "transaction costs" and "contracting costs" inter-changeably. Observe that, since "transaction cost economics" takes a contract as the main transaction instrument, the transaction relation is contractual.

The main contributions of this paper are as follows. First, through a systematic analysis of different contracting processes for an intra-firm transaction, we establish conditions under which complete, null, or incomplete contracting may be optimal. To reach these results, we provide a formal definition for contracting costs. Second, if the incomplete contract is optimal, we provide a criterion for choosing the optimal governance structure and ex ante contract prices. In particular, we identify conditions under which a dual pricing contract can improve contract efficiency. Third, our results provide a possible explanation for observed transfer pricing policies in
managing repeated transfers of products within an organization. Specifically, we observe that firms establish "transfer pricing policies" to govern a series of future transactions. ${ }^{1}$ Although the prices specified by these policies may be renegotiable, most future transactions will be based on these pricing policies. .In particular, relatively few firms use bargaining between divisions as a means of determining transfer prices for repeated transactions. ${ }^{2}$ Our results show that when the firm faces uncertainty and information asymmetry, an incomplete contract in which a pair of prices is specified ex ante will minimize contracting costs so that the firm's net cash flows will be maximized. Therefore, setting a transfer pricing policy is an optimal way to govern repeated intra-firm transactions.

After this introductory section, the paper is organized as follows. Section 5.2 discusses contracting cost concepts. A detailed definition of contracting costs is

[^28]provided based on Williamson's point of view. Section 5.3 presents basic model elements. Section 5.4 analyzes contracting strategies in settings with verifiable information. Section 5.5 analyzes contracting strategies in settings with unverifiable information. Section 5.6 extends the results in Section 5.5 to settings with large ex post bargaining costs, and introduces bargaining under asymmetric information. In section 5.7 we discuss the implications of our results to the transfer pricing problem. Section 5.8 is a brief conclusion.

### 5.2 Contracting Costs

As with most other economic activities, the contracting process incurs costs. In some cases, contracting costs are small relative to the gains resulting from the transactions so that they can be ignored without causing serious bias in the analysis. In other cases, they are non-trivial so that ignoring them will result in misguided conclusions.

Unlike production costs or information costs, which have been extensively studied, there is little literature on the topic of contracting costs. Dye [1985] assumes that the cost of writing a contract is an increasing function of the number of contingencies in the contract. This is "analytically convenient", but is certainly overly
simplified.
We follow Williamson [1985], treating contracting costs as the major form of transaction costs. There are both ex ante and ex post transaction costs. The ex ante costs include the costs of drafting, negotiating, and safeguarding an agreement. The ex post costs include (i) the maladaption ${ }^{3}$ costs incurred when transactions drift out of contract alignment; (ii) the haggling costs incurred if bilateral efforts are made to correct ex post misalignments; (iii) the set up and running costs associated with the governance structure to which disputes are referred; and (iv) the bonding costs of effecting secure commitments. A key feature of the above definition is that the contracting process is viewed as a whole. Writing a contract is only one step in this whole process. This point is particularly important if the interaction of ex ante and ex post contracting costs is recognized.

To make the definition more precise, we classify contracting costs into the following categories:
(a) Specification Costs

Specification costs are the resources spent to clar-

[^29]ify the characteristics of the uncertain event. This clarification is extremely important for a contract provision since an equivocal specification will not only increase the difficulties of verification in later execution, but also creates room for opportunistic behavior by either contracting party so that dispute settlement costs may be incurred with larger probability. Depending on the nature of the uncertain events, this cost can range from trivial to extremely high. Some events have very simple scale measurements, such as net income, stock prices, procument quantities, or ages of insurants, which are very easy to specify in an agreement. In some cases, one mathematical formula can cover all events, even if the number of events is infinite. In such cases the cost of one event specification can be treated as trivial. In such a case, only the total specification cost has meaning. In the other extreme, some events have very complex characteristics. For instance, the quality of a product or service, the management effort required to establish a well functioning division, or a manager's non-pecuniary private benefit that does not show up in a firm's accounts. All are very difficult to specify, or the specification can be done only with high cost.

In intra-firm contracting, contract specifications are often based on accounting numbers. Hence, the costs
to establish and run an accounting system can be viewed as specification costs for intra-firm contracting.
(b) Ex Post Verification Costs

During contract execution, verification of a particular event realization is a necessary step in identifying the appropriate provision to be executed. Furthermore, verification is necessary for a third party ruling should a dispute between the two contracting parties arise.

Verification costs are intimately correlated with the specification costs. On the one hand, if an event is easy to specify, then its verification is also easy. This implies a positive correlation between verification and specification costs. On the other hand, high quality specification will simplify the verification, which implies a negative correlation between verification and specification costs. For simplicity, we pool the above two categories together. However, in doing so it must be stressed that the specification costs always occur ex ante, while the verification costs occur ex post. Therefore, when we count these costs, it is necessary to specify the time point at which we account for them. We can have an expected cost that is equal to the total specification costs plus the expected verification costs, or a realized cost that is equal to the total specification
costs plus the verification costs for a particular realized event. The ex ante expected value and the ex post realized value are identical only in the special case where we assume that the verification costs are constant for all events.
(c) Ex ante negotiation or bargaining costs These costs include the resources spent to find a mutually acceptable agreement. If the transaction will provide positive gains, then both parties have incentives to reach an agreement. However, since the division of those gains is negotiable, the parties must engage in bargaining to arrive at an agreement. Bargaining costs include not only the resources directly used in the process, but also the opportunity costs born by each party due to the delay of the transaction induced by the bargaining process. These costs depend crucially on the information structure. Bargaining when the parties have asymmetric information incurs particularly high costs. If contracting is ex ante complete, or there is a commitment to non-renegotiation and the specification of the provisions is perfect (so that there will not be any disputes in contract execution), then the above three categories cover all the contracting costs. However, in general, contracts are incomplete, renegotiation is
allowed, and disputes can arise in executing the contract. Therefore, the following costs may arise.
(d) Ex Post Renegotiation or Bargaining Costs Renegotiation and ex post bargaining is never desirable in classical comprehensive contracting since all relevant future contingencies are fixed in advance through an ex ante complete contract. However, renegotiation is desirable under incomplete contracting for the following reasons. First, due to the incompleteness of the initial contract, there may exist opportunities for improvements through renegotiation that would benefit both parties. The more incomplete the ex ante contract, the more opportunities there exist for such improvements. Second, there may exist maladaption problems since not every appropriate adaptation can be foreseen in advance; some may not be clear until the event materializes. Third, the parties may interpret an agreement differently due to an equivocal specification, a missing provision, or opportunistic strategies. Hence, ex post bargaining can be viewed as the complement of ex ante bargaining.
(e) Dispute Settlement Costs

Renegotiation and bargaining is often the first step in resolving disputes raised at the time of contract
execution. If this step is successful, then the dispute settlement costs include only the renegotiation costs. However, if this step fails, then a third-party ruling is necessary. Dispute settlement costs are mainly determined by the governance structure. Williamson [1985] points out that different governance structures provide different types of third party rulings. In a market structure, the court and legal system provide third party rulings which are expensive and often ineffective. The high settlement costs include the resources both parties must spend in the trial process. Neoclassical law emphasizes the role of an arbitrator in assisting the parties to resolve disputes. It has advantage of being more flexible. Compared to court rulings, an arbitration process lowers settlement costs and raises settlement effectiveness. Finally, in an integrated organization structure, the hierarchical power in an organization provides effective and low cost dispute settlements. Any dispute occurring in the organization can be settled by a higher ranking authority and the top management officer holds the right of final judgment. This is the most significant feature of integrated organizations.

In summary, contracting costs include mainly specification and verification costs, ex ante and ex post bargaining costs, and dispute settlement costs. In the model
we present in following sections, the focus is on intrafirm transactions. Based on the above arguments, the dispute settlement costs in an organization are low. Hence, for simplicity, we ignore dispute settlement costs and focus on the other costs incurred in the contracting process.

### 5.3 Basic Model Elements

We consider a transaction between a buying division (B) and a selling division (S). To simplify, we assume that $s$ and $B$ are risk-neutral, and that $s$ supplies $B$ with either one unit or zero units of a product at $t_{2}$. Let $c$ be the "transfer cost" that $s$ bears in supplying one unit to $B$ and let $v$ be the "transfer value" of the transaction to B if the product is supplied by s. Precisely, "transfer cost" is defined to be the net decrease in division s's operating cash flows if the product is transferred to division B, and "transfer value" is defined to be the net increase in division B's operating cash flows if the product is received from division s. A more detailed discussion of the determination of $v$ and $c$ is provided in a later section.

Implicitly, there is a central management that is the final claimant to all income and expenses of the two divisions, including the compensation paid to $S$ and $B$. We
consider the transaction from the perspective of the central management, who want to motivate division managers to make optimal trading decisions and minimize transaction costs so that the firm's net cash flow can be maximized.

From the central management's perspective, at the time of the transfer decision is made, it is optimal to make the transfer if, and only if, $v>c+T C_{2}$, where $T_{2}$ is the incremental future transaction costs required to accomplish the transfer. Note that any transaction costs $T C_{1}$ incurred prior to that point in time are irrelevant (they are sunk costs) relative to the decision made at that point at time. However, transaction costs incurred at earlier stages in the process are relevant to determining the operating policies that are implemented. Hence, these earlier transaction costs $T C_{1}$ may influence those "ex post" costs $\mathrm{TC}_{2}$. This implies that contracting costs, ex ante and ex post, are interactive.

We assume that the managers of divisions $s$ and $B$ are motivated to maximize their expected divisional profits. Their objectives may not align with central management's objective. This implies that when $S$ and $B$ are free to make independent decisions, these decisions may conflict with each other, and may deviate from the efficient rules. The incentive contracts for motivating $\mathbf{S}$ and B's managers are not explicitly modelled in following analysis. We
focus on the factors that influence divisional profits. The key issue is to determine the contracting strategy that the central management should implement to efficiently manage the transactions between $s$ and $B$.

Note that we are mainly interested in the transaction between $\mathbf{B}$ and $\mathbf{s}$ in this paper. Hence, "trading occurs" means that the product is transferred from $\mathbf{s}$ to $\mathbf{B ,}$ and "no trading" means $B$ does not trade internally with $S$. Observe that whether a trade occurs or not, $B$ and $s$ must also determine their optimal actions with respect to other activities in their divisions. For example, whether a trade occurs may influence the exchanges $B$ and $s$ make with the market.

Assume at $t_{1}$, that both $c$ and $v$ are uncertain and randomly distributed on the intervals $\mathrm{C}=[\mathrm{c}, \overline{\mathrm{c}}]$ and $\mathrm{V}=$ [ $\underline{v}, \overline{\mathrm{v}}], ~ r e s p e c t i v e l y . ~ T h e ~ j o i n t ~ p r o b a b i l i t y ~ d e n s i t y ~ f u n c-~-~$ tion for $v$ and $c$ is $f(v, c)$, which is assumed to be common knowledge at $t_{1}$. The values of $v$ and $c$ will be realized at $t_{2}$ and the transaction will proceed based on the contract (if there is one), and $s$ and $\mathrm{B}^{\prime}$ s trading decisions.

The same model is analyzed in a few other papers. For example, Grossman and Hart [1987] use this model to analyze the issue of vertical integration under the assumptions that: (i) complete contracting is impossible, due to the unverifiability of $v$ and $c$, so that the con-
tract written at $t_{1}$ is contingent only on the trading quantity; (ii) the contract can assign control to either one of the parties; and (iii) no ex post renegotiation and bargaining is allowed.

We reconsider this model in a different environment. In our setting, renegotiation is always passible. Specifically, we explicitly consider contracting costs in different regimes. We find that, under the criterion of minimizing contracting costs, not only do we replicate most of Grossman and Hart [1987]'s results, but we also show that incomplete contracting is optimal in most cases. This, in turn, provides a rationale for the use of ex ante transfer pricing policies for repeated intra-firm transactions within organizations.

We shall consider three different regimes with respect to the nature of information about $v$ and $c$.
(1) Regime 1. Contracting with verifiable information: $v$ and $c$ are assumed to be a verifiable components of accounting reports and, hence, complete contracting is possible.
(2) Regime 2. Contracting with unverifiable information: $v$ and $c$ are assumed observable but unverifiable and, hence, they are ex ante non-contractible but ex post contractible.
(3) Regime 3. Contracting with high ex post bargaining costs: This represents the cases where private information exists in the ex post bargaining process.

Regime 1 represents the case of complete contracting which is the focus of classical contracting theory. Regime 2 is the case analyzed by most of the existing incomplete contracting literature. It represents the case in which $v$ and $c$ are not formally reported by any accounting systems, but perhaps they can be ascertained by managers based on informal information that exists in the organization. The information that managers use to make these assessments is assumed to be publicly available to both $\mathbf{S}$ and $\mathbf{B}$, so that consensus about the realized values of $v$ and $c$ are easy to reach. In contrast, in Regime 3, managers may make their assessments based on private information. This fact, plus managers' opportunistic behaviour, will very likely create difficulties in reaching an agreement in the bargaining process. Thus, the contracting costs may be very significant.

We analyze these three regimes separately in the following sections. In all cases, the trading quantity $q$ $=1$ or 0 is assumed to be verifiable.

### 5.4 Contracting with verifiable information

Assume $v$ and $c$ are verifiable. Observe that in this setting all uncertainty is exogenous; there are no agency problems. Furthermore, both parties are assumed to be risk-neutral so that risk sharing has no value in improv-
ing the contracting efficiency. Full trading efficiency can be achieved either through ex ante complete contracting, or ex post bargaining. The only difference between the two alternatives is the influence of transaction costs. Different contracting procedures to split the trading gains between parties, given each party's bargaining power, may result in different net cash flows for the central management.
(a) Alternative 1: Ex ante complete contracting
$B$ and $s$ can contract at $t_{1}$ and make the contract directly contingent on $v$ and $c$. A contract can explicitly specify the efficient trading rule, such as:
(i) If $v>C$, then $s$ supplies one unit to $B$, i.e., $q=1$, and $B$ pays $\delta p_{1}(v, c)$;
(ii) If $v<c$, then no trade occurs between $s$ and $B$, i.e., $q=0$, and $B$ pays $s p_{0} .^{4}$

Independent of $p_{1}$ and $p_{0}, 5$ the full expected trading gain, gross of the total expected contracting costs TC ${ }^{P}$,

[^30]\[

$$
\begin{equation*}
W^{p}=\iint_{v>c}(v-c) f(v, c) d v d c \tag{5.4.1}
\end{equation*}
$$

\]

can be realized. In Figure 5-1, the possible realizations of all combinations of $v$ and $c$ are represented by the rectangle CIKE which is separated by the line $v=c$ into two regions. The region CIAJE represents all positive trading gain events, while the region $A K J$ represents all negative trading gain events. ${ }^{6}$ The efficient trading rules require that trading occurs in the former but does not occur in the latter.

## Insert Figure 5-1 here

From the central manager's point of view, given the efficient trading rules specified in (i) and (ii), it does not matter what prices $p_{1}$ and $p_{0}$ are set, because the net cash flows to him are $W^{p}-T C^{p}$, independent of these transfer prices. Furthermore, it also does not matter whether $B$ or $s$ bear the contracting costs, since these costs will not influence trading decisions and the central management will be the residual claimant. However, the values of $p_{1}$ and $p_{0}$ will determine the allocation of the trading gains between the divisional managers. If they

[^31]are set properly, the divisions' incentives to make efficient trading decisions can be aligned with central management's objectives. The bargaining power of each party is exogenously given and, hence, these prices can be determined in two steps. First, given $p_{0}, v$, and $c, p_{1}$ is chosen such that
\[

\left\{$$
\begin{array}{l}
v-p_{1} \geq-p_{0}  \tag{5.4.2}\\
p_{1}-c \geq p_{0}
\end{array}
$$ \quad c \leq p_{1}-p_{0} \leq v\right.
\]

These inequalities imply that the relative price $P(v, c)=$ $p_{1}-p_{0}$ is set such that trading is preferred by both $s$ and $B$ whenever $v>c$. In addition, $P$ depends on each party's bargaining power. For example, given B's bargaining power $\alpha$, for each pair of $v$ and $c(v>c), ~ a ~ N a s h$ bargaining solution can be found by solving ${ }^{7}$

$$
\begin{equation*}
\max _{P}(v-P)^{\alpha} \cdot(P-c)^{1-\alpha} \tag{5.4.3}
\end{equation*}
$$

The resulting price is $P=\alpha \cdot c+(1-\alpha) \cdot v=v-\alpha \cdot(v-c)$. Using this price to evaluate the expected gains of $B$ and $s$, we have,

[^32]\[

$$
\begin{aligned}
E G^{\mathrm{B}} & -\iint_{v>c}\left(v-p_{1}\right) \cdot f(v, c) \cdot \mathrm{d} v \mathrm{~d} c+\iint_{v<c}-p_{0} \bullet f(v, c) \cdot \mathrm{d} v \mathrm{~d} c \\
& -\iint_{v>c}\left(v-P-p_{0}\right) \cdot f(v, c) \cdot d v d c+\iint_{v<c}-p_{0} \cdot f(v, c) \cdot d v d c \\
& =\alpha \cdot \iint_{v>c}(v-c) \cdot f(v, c) \cdot d v d c-p_{0} \\
& =\alpha \bullet W^{p}-p_{0}
\end{aligned}
$$
\]

$$
\begin{aligned}
\mathrm{EG} \mathrm{~s} & =\iint_{v>c}\left(p_{1}-c\right) \cdot f(v, c) \cdot \mathrm{dvd} c+\iint_{v<c} p_{0} \bullet f(v, c) \cdot \mathrm{d} v \mathrm{~d} c \\
& =\iint_{v>c}\left(P+p_{0}-c\right) \cdot f(v, c) \cdot d v d c+\iint_{v<c} p_{0} \cdot f(v, c) \cdot d v d c \\
& =(1-\alpha) \iint_{v>c}(v-c) \cdot f(v, c) \cdot d v d c+p_{0} \\
& =(1-\alpha) \cdot W^{p}+p_{0}
\end{aligned}
$$

Second, let $G_{0}{ }^{B}$ and $G_{0}{ }^{s}$ denote the status quo positions of $B$ and $s$ before they come to contract. Without loss of generality, we assume $G_{0}{ }^{B}+G_{0}{ }^{s}=0$; then $p_{0}$ is chosen to solve

$$
\begin{aligned}
& \operatorname{Max}_{p_{0}}\left(\mathrm{EG}^{\mathrm{B}}-\mathrm{G}_{0}^{\mathrm{B}}\right)^{\alpha} \bullet\left(\mathrm{EG}^{\mathrm{S}}-\mathrm{G}_{0}^{\mathrm{S}}\right)^{1-\alpha} \\
& \Leftrightarrow \quad \operatorname{Max}\left(\alpha \cdot W^{p}-p_{0}-\mathrm{G}_{0}^{\mathrm{B}}\right)^{\alpha} \bullet\left[(1-\alpha) \mathrm{W}^{\mathrm{P}}+p_{0}-\mathrm{G}_{0}^{\mathrm{S}}\right]^{1-\alpha} \\
& p_{0} \quad \\
& p_{0}=\mathrm{G}_{0}^{\mathrm{S}}--\mathrm{G}_{0}^{\mathrm{B}}
\end{aligned}
$$

This implies that each division's net gain is determined by its bargaining power. Note that the role of $p_{0}$ in contracting is to adjust each party's post-contract position based on its status quo position and its bargaining power. The contracting costs of a complete ex ante contract
are evaluated as follows. First, two necessary steps in an ex ante contract are the specification of all possible events and the verification of the realized event. In this example, ex ante specification can be implemented by specifying accounting procedures and measurements, while verification may be accomplished by the firm's accountants or internal auditors. Depending on the nature of $v$ and $c$, the costs involved in these procedures may vary considerably. We denote these costs as CSV, which consists of the ex ante specification costs and the expected value of the ex post verification costs. Second, contract negotiation will involve costs. For a complete contract, all negotiations occur ex ante. We denote these negotiation costs by $T C N=\mathrm{CN}_{0}+T C N_{1}$, where $\mathrm{CN}_{0}$ is the cost of bargaining with respect to price $p_{0}$, while $\mathrm{TCN}_{1}$ is the total cost of bargaining with respect to $p_{1}(v, c)$, for all possible ( $v, c$ ) such that $v>c$. Hence, the total cost for a complete ex ante contract can be expressed as

$$
\begin{equation*}
T C^{P}=C S V+T C N \tag{5.4.4}
\end{equation*}
$$

(b) Alternative 2: "Null contract" and ex post bargaining. $B$ and $s$ can choose a "null contract" in their $t_{1}$ contract, which merely ensures a basic trading relation at $t_{2}$. This can be done through specifying a "no trade" pay-
ment $p_{0}$, which is similar to $p_{0}$ in the ex ante contracting case, and reflects the bargaining power and status quo positions of $s$ and $B$ at $t_{1} \cdot{ }^{8}$ Then, they must bargain ex post for a price $p_{1}(v, c)$ after $(v, c)$ is realized at $t_{2}$, if they want trading to occur. In this model, ex post bargaining of $p_{1}$ is the same as the first step bargaining of the ex ante complete contract since we assume that between $t_{1}$ and $t_{2}$ nothing happens to influence $s$ and B's bargaining positions. The same Nash bargaining solution as in the ex ante contracting case applies to the ex post bargaining process.

However, there do exist differences in both the trading behaviour and contracting cost aspects of these two alternatives. For comparison, let us assume, for each pair of $v$ and $c$, that the ex post bargaining cost for $p_{1}(v, c)$ is a constant $C N_{1}$, and is less than or equal to the ex ante total bargaining costs $\mathrm{TCN}_{1} .{ }^{9}$ Now consider

[^33]the components of the expected contracting costs for the null contract. First, the costs to specify and verify the events (i.e. v and c), which are incurred for a complete contract, are not incurred for a null contract. Second, the negotiation costs $\mathrm{CN}_{0}$ for a price $\mathrm{p}_{0}$ are the same for both contracts. Third, ex post bargaining. costs will be incurred with the probability that trading occurs. Thus, we can express the expected contracting costs for the null contract as
\[

$$
\begin{align*}
\mathrm{ETC}^{0} & -\mathrm{CN}_{0} \\
& +\mathrm{CN}_{1} \cdot \text { prob}\{\text { Ex post bargaining occurs }\} \tag{5.4.5}
\end{align*}
$$
\]

Observe that, taking into account the ex post bargaining cost, the efficient trading decision is different than in the ex ante contracting case. Specifically, when the realizations of $v$ and $c$ are such that

$$
0<v-c<C N_{1}
$$

then initiating the ex post bargaining process to seek a gain smaller then the bargaining costs incurred would not benefit the central management. Hence, the efficient trading region is characterized by

$$
\begin{equation*}
q-1 \text { if, and only if, } v-c-\mathrm{CN}_{1}>0 \tag{5.4.6}
\end{equation*}
$$

In Figure 5-2, the efficient ex post bargaining region is represented by CIMNE, which is obtained by eliminating a parallel band from the original positive trading region. The length of MA is equal to the ex post bargaining costs $\mathrm{CN}_{1}$.

```
Insert Figure 5-2 here
```

The expected trading gains are

$$
W^{0}-\iint_{v c+\mathrm{CN}_{1}}(v-c) \cdot f(v, c) \cdot d v d c
$$

It is obvious that $W^{0}<W^{p}$ and the difference between $W^{p}$ and $\mathrm{W}^{0}$ depends on $\mathrm{CN}_{1}$. Particularly, it is obvious that $W^{P}-W^{0}$ is an increasing function of $C N_{1}$ and if $C N_{1}=0$, $W^{p}-W^{0}=0$. On the other hand, the difference between $T^{p}$ and ETC ${ }^{0}$ is

$$
\begin{aligned}
\Delta \mathrm{TC} & =\mathrm{TC} \\
& \mathrm{P}-\mathrm{ETC}^{0} \\
& =\mathrm{CSV}+\mathrm{TCN}_{1}-\mathrm{CN}_{1} \cdot \operatorname{prob}\{\text { Area CIMNE }\}
\end{aligned}
$$

which, in general, is positive and increasing in $\mathrm{CN}_{1}$ if we assume $\mathrm{CN}_{1}=\mathrm{TCN}_{1} .{ }^{10}$ Thus, we can prove the following proposition.

[^34]Proposition 5.1: Assume contracting costs CSV is non-trivial, and ex post bargaining costs $\mathrm{CN}_{1}$ and ex ante total bargaining costs $\mathrm{TCN}_{1}$ are the same, then there exists a threshold value $\mathrm{CN}_{1}{ }^{+}$ such that when $\mathrm{CN}_{1}<\mathrm{CN}_{1}{ }^{+}$,

$$
\begin{equation*}
W^{0}-E T C^{0}>W^{P}-T C^{P} \tag{5.4.9}
\end{equation*}
$$

Proof: (see appendix). ${ }^{11}$
A key assumption in the above comparison is that, under ex ante complete contracting, there is no way to avoid specification and verification costs for those ( $v, c$ ) in which trade should not occur (either because $v<c$ or $v<c+C S V)$. Proposition 5.1 says that, when the ex post bargaining costs are relatively small, the efficiency of a complete contract is less than a "null" contract. The improvement mainly results from the savings in specification and verification costs. However, when the ex post bargaining costs are relatively high, the improvement may disappear due to a serious reduction in the trading region and the resulting gross trading gain. This result is not obtained by Grossman and Hart [1987] because they ignore contracting costs. The fact that the trading region is influenced by the ex post bargaining costs shows the distinct nature of ex ante contracting and ex post bargaining. Ex ante contracting costs are incurred before

[^35]the realization of uncertain events, while ex post bargaining occurs after the realization of the events. The differences in the efficient trading regions, i.e., the marginal region MAJN in the Figure 5-2, will be called the marginal adjustment and denoted as $M$.

If the central management can control the trading decisions made by divisions, then who will bear the ex post bargaining costs again does not matter. However, if this is not the case, then there may be incentive issues. For example, in the next section, we consider the case in which $v$ and $c$ are unverifiable to the central management. Then, if divisions do not bear these costs, $s$ and $B$ will have incentive to trade in $M$ even if the net benefit is negative. To align this divergence, it is better for the central management to allocate these costs to divisions. When $\mathrm{CN}_{1}$ is borne by the divisions, they will make trading decisions consistent with the efficient trading rules.

The above analysis demonstrates that even when contracting information is verifiable, complete ex ante contracting may not be economically efficient if the contracting costs are non-trivial. This is consistent with the observation that incomplete contracts are extensively used in the real world.

### 5.5 Contracting with unverifiable information

Assume that $v$ and $c$ are observable by $S$ and $B$, but unverifiable by the central management. This is the extreme case of Regime 1 in which the verification costs are prohibitively high so that CSV is very large. Hence, ex ante complete contracting is impossible. However, if the ex post bargaining costs are small relative to $v$ and c, then the null contract we analyzed in the last section can be applied to fulfill an efficient trading result. In this section, we assume $\mathrm{CN}_{1}$ is small relative to both v and $c$ and other contracting costs such as CSV. We shall show that, in this setting, the null contract can be improved by an incomplete ex ante contract.

An alternative to a null contract is to make the ex ante contract contingent on available information that is easy to specify and verify, e.g., the quantity $q$ to be traded. Such a contract may be inefficient ex post but the inefficiency can be corrected through ex post bargaining. One commonly used example is to specify a trading price $p_{1}$ independent of the uncertain events, in addition to a null contract:

$$
\text { B pays } s p_{1} \text { if } q=1 \text {, and } p_{0} \text { if } q=0
$$

In addition, the contract should specify some rule to
govern the trading decision. Examples of the latter are: ${ }^{12}$
(a) Independent trading relationship or non-integration (NI): Each party can decide whether to trade or not. This results in $q$ $=1$ if, and only if, both parties are willing to trade.
(b) Buyer control (BC): B has the power to determine whether there is trading or not. This results in $q=1$ if, and only if, $B$ is willing to trade.
(C) Seller control (SC): $s$ has the power to determine whether there is trading or not. This results in $q=1$ if, and only if, $s$ is willing to trade.

These rules, in combination with the parties' ex post decisions, determine the quantity traded. As we shall see, this will reduce ex post bargaining costs. We first use the non- integration relationship to illustrate the nature and cost of such a contract. The rule NI and a pair of prices $p_{0}$ and $p_{1}$ determine the following trading rule:

$$
\begin{equation*}
q=1 \text { if, and only if, } c \leq P \leq V \tag{5.5.1}
\end{equation*}
$$

where $P=p_{1}-p_{0}$.

Insert Figure 5-3 here

[^36]In Figure 5-3, the price $P$ divides all possible events into six regions and only the events in region $I$ satisfy the conditions given by (5.5.1). Therefore, without contract renegotiation, trading will only occur in region I ( $B C D F$ ), where $B$ pays 8 price $p_{1}$ for the transfer of the product. Transfer will not occur in all other regions because either $v<P$ or $P<c$ or both, so $B$ pays $p_{0}$ to $s$ for no transfer. Thus the contract covers all possible states. The costs of this ex ante contract can be evaluated as follows. The costs for specifying and verifying $v$ and $c$ are not incurred. The costs to specify a pair of prices $p_{1}$ and $p_{0}$ are negligible. The costs to bargain over a pair of prices is equivalent to bargaining for $p_{0}$ in the null contract case. The prices are chosen such that the ex ante post-contract positions are adjusted to reflect each division's bargaining position and power. (A detailed discussion appears later). Hence, it is reasonable to assume that these costs equal $\mathrm{CN}_{0}$. That is, the ex ante cost of this contract is essentially the same as for a null contract.

However, the trading result for this contract is not efficient because there are states, represented by region II (DFJE) and III (AFBI), in which the trading gain is positive and trading should occur, but it will not occur
under this contract. For instance, in III where $c<v<$ $p_{1}-p_{0}, s$ is willing to supply because $p_{1}-c>p_{0}$, but $B$ is unwilling to buy because $v-p_{1}<-p_{0}$. Based on this, Grossman and Hart [1987] conclude that this incomplete contract will result in an efficiency loss in areas II and III. This is certainly correct if renegotiation is not allowed. In our setting, however, much of this inefficiency can be corrected through renegotiating the price. Consider, say, a state in III is realized, so that trading will not occur because $B$ is unwilling to buy, as mentioned above. This results in an opportunity loss for $s$. Since both $B$ and $S$ can observe $c$ and $v$, they can renegotiate a new price $\hat{p}_{1}$ which is lower than $p_{1}$ such that $B$ is willing to buy under the new price $\hat{p}_{1}$. For example, let $\mathrm{CN}^{\mathrm{S}}$ and $\mathrm{CN}^{\mathrm{B}}$, with $\mathrm{CN}^{\mathrm{S}}+\mathrm{CN}^{\mathrm{B}}=\mathrm{CN}_{1}$, represent s and $\mathrm{B}^{\prime}$ s shares of the ex post bargaining costs, respectively. Then, any new price $\hat{\mathbf{p}}_{1}$ such that

$$
\begin{equation*}
c+\mathrm{CN}^{\mathrm{s}} \leq \hat{p}_{1}-p_{0} \leq v-\mathrm{CN}^{\mathrm{B}} \tag{5.5.2}
\end{equation*}
$$

will induce $B$ to buy because now $v-C N^{B}-\hat{p}_{1} \geq-p_{0}$ and $s$ is still willing to supply because $\hat{p}_{1}-c-C^{s} \geq p_{0}$. This new contract induces trading and both parties are better off than not trading under the initial contract. Again, the new price $\hat{p}_{1}$ depends on the bargaining power $B$ and $s$ have in the renegotiation. Since the allocation of the
gain is not an important issue in the following analysis, we consider any bargaining result that satisfies (5.5.2). Our only concern is that $B$ and $s$ have an incentive to reach an agreement on $\hat{\mathbf{p}}_{1}$ through bargaining so that trading will occur and efficiency will be regained in region III.

In the above discussion, we have taken into account the ex post bargaining cost $\mathrm{CN}_{1}$. For the reason given in prior discussion, we assume the ex post bargaining costs are born by $B$ and $s$. Therefore, ex post bargaining will not occur in the whole region III, nor will transfers. For events where $v-c<\mathrm{CN}_{1}$, the benefit net of contracting costs is negative, so that corrective action is not worthwhile. Therefore, part of the marginal region $M$ must be eliminated from II and III. The marginal adjustment of the region III is the overlapping region of $M$ and III. We shall denote it $M_{\text {III }}$ and denote the other marginal adjustment regions in the same way. In Figure 5-4, $M_{\text {III }}$ is represented by the parallel band inside region III.

Insert Figure 5-4 here

Ex post bargaining incurs cost $\mathrm{CN}_{1}$, and the probability of bargaining is represented by the probability of region II and III, subject to marginal adjustments. Hence, where $\mathrm{ETC}^{\mathrm{NI}}$ represents the expected contracting costs of

$$
\begin{equation*}
\mathrm{ETC}^{\mathrm{NI}}=\mathrm{CN}_{0}+\mathrm{CN}_{1} \bullet p r o b\left\{I I+I I I-M_{I I}-M_{I I I}\right\} \tag{5.5.3}
\end{equation*}
$$

the incomplete contract with the NI governance rule. Comparing (5.5.3) with (5.4.5), it is obvious that ETC ${ }^{\mathrm{NI}}<$ ETC ${ }^{0}$ because the null contract requires ex post bargaining in areas $I+I I+I I I-M_{1}-M_{H}-M_{H I I}$. In words, the NI-contract is less expensive than the null contract because of the reduction in the ex post bargaining costs (resulting from increased details in the ex ante contract). Furthermore, the resulting ex ante trading gain is also a little larger than $W_{0}$ because in $M_{1}$ (marginal region in $I$, which is a small triangular region in Figure 5-4), trading will occur without ex post bargaining. Therefore, the net benefit of the NI-contract is greater than for a null contract.

Proposition 5.2: ${ }^{13}$ An optimal ex ante incomplete contract, in which a pair of prices and an NI governance rule are specified, will result in a higher net gain for the central management, i.e.

$$
\begin{equation*}
\mathrm{W}^{\mathrm{NI}}-\mathrm{ETC}^{\mathrm{NI}}>\mathrm{W}^{0}-\mathrm{ETC}^{0} \tag{5.5.4}
\end{equation*}
$$

Cost evaluation (5.5.3) provides a different criterion for choosing $p_{1}$ and $p_{0}$. Grossman and Hart [1987] claim that $B$ and $s$ will choose $P=p_{1}-p_{0}$ to maximize the

[^37]expected trading gain
\[

$$
\begin{equation*}
\max _{P} W^{\mathrm{NI}}=\max _{P} \iint_{v>P}(v-c) \cdot f(v, c) \cdot d v d c \tag{5.5.5}
\end{equation*}
$$

\]

We suggest that $P$ be chosen to maximize the no bargain trading region $I$ ( $B C D F)$,

$$
\begin{equation*}
\max _{P} p r o b\{I\}-\max _{P} \iint_{v>p} f(v, c) \cdot d v d c \tag{5:5.6}
\end{equation*}
$$

or, equivalently, to minimize the bargaining probability of (II + III), excluding the marginal region $\left(M_{I I}+M_{I I I}\right)$ in these areas,

$$
\begin{equation*}
\min _{P} p r o b\left\{I I+I I I-M_{I I}-M_{I I I}\right\} \tag{5.5.7}
\end{equation*}
$$

The intuition here is clear. Since the trading gains can be guaranteed, economizing contracting costs is a reasonable criterion for setting ex ante contracts. Given (5.5.7), the ante bargaining process with respect to $p_{1}$ and $p_{0}$ can be simplified to bargaining with respect to $p_{0}$ only. Since, once $p_{0}$ is determined, $P$ can be obtained from (5.5.7), we immediately have $p_{1}=P+p_{0}$.

Now let us evaluate the contracting costs for the trading relationship under buyer and seller control governance rules. A buyer control rule, in combination with a pair of prices $p_{1}$ and $p_{0}$, induces a trading rule

$$
\begin{equation*}
q=1 \text { if, and only if, } v>P \tag{5.5.8}
\end{equation*}
$$

In Figure 5-4, we can see that the trading region $\{I+I I+V\}$ (BCEG) satisfies (5.5.8), so that trading will occur in this region without contract renegotiation. Compared with an NI-contract, this contract results in efficient trade in region $I$ and $I I$ but introduces inefficiency in region V. Since in region $I I, v>c>p_{1}-p_{0}, s$ is unwilling to supply the product since $p_{1}-c<p_{0}$, while $B$ is willing to buy since $v-p_{1}>-p_{0}$. Now $B$ need not ask for renegotiation, but can simply use his control to force $s$ to supply even if $s$ will incur a loss. This result is Pareto efficient and, hence, there is no renegotiation that can be raised by $\mathbf{s}$ that will be accepted by $B$.

The story for an event in region $V$ is totally different. Since $c>v>p_{1}-p_{0}, B$ will decide to trade for a gain of $v-P$ even though $s$ will incur a loss of $c-P$. However, now $s$ can propose a new price $\hat{\mathrm{p}}_{0}$ such that

$$
\begin{equation*}
v-\mathrm{CN}_{1}^{\mathrm{B}} \leq p_{1}-\hat{p}_{0} \leq C+\mathrm{CN}_{1}^{\mathrm{S}} \tag{5.5.9}
\end{equation*}
$$

i.e., $s$ can reduce the no trade price $p_{0}$ to $\hat{p}_{0}$ inducing $B$ to make a no trading decision. Under (5.5.9) B prefers no trade since $v-C N^{B}-p_{1} \leq-\hat{p}_{0}$ and $s$ is better off by not trading since $\hat{p}_{0} \geq p_{1}-c-C N s$. So the new contract makes both parties better off and will be accepted. As a consequence, most of the inefficiency in region $V$ is corrected.

Again the correction is incomplete due to marginal adjustment $\mathrm{M}_{\mathrm{v}}$.

The inefficiency in III can be corrected in the same way as in case NI. This implies that the probability of ex post bargaining under BC is determined by (III + V -$\left.M_{111}-M_{v}\right)$,

$$
\mathrm{ETC}^{\mathrm{BC}}-\mathrm{CN}_{0}+\mathrm{CN}_{1} \cdot p r o b\left\{I I I+V-M_{I I I}-M_{V}\right\} \text { (5.5.10) }
$$

Furthermore, the ex ante prices $p_{1}$ and $p_{0}$ under $B C$ should be chosen to minimize prob\{III $\left.+V-M_{1 I I}-M_{V}\right\}$ instead of prob\{II + III - $\left.M_{I I}-M_{1 I I}\right\}$ in case NI. Similarly, an analysis for the seller control case will result in

$$
\begin{equation*}
\mathrm{ETC}^{\mathrm{SC}}=\mathrm{CN}_{0}+\mathrm{CN}_{1} \bullet p r o b\left\{I I+I V-M_{I I}-M_{I I I}\right\} \tag{5.5.11}
\end{equation*}
$$

and the ex ante prices $p_{1}$ and $p_{0}$ under $S C$ should be chosen to minimize $\operatorname{prob}\left\{I I+I V-M_{I I}-M_{I V}\right\}$. Similar to Proposition 5.3, we have following proposition.

Proposition 5.3: ${ }^{14}$ Both a BC-contract (a pair of prices and a $B$ control governance rule) and SC-contract (a pair of prices and an $S$ control governance rule) result in larger benefits net of contracting costs for the central management than does the null contract, i.e.,

$$
\begin{align*}
& W^{B C}-E T C^{B C}>W^{0}-E T C^{0}  \tag{5.5.12}\\
& W^{S C}-E T C  \tag{5.5.13}\\
& \\
& s C \\
& W^{0}-E T C^{0}
\end{align*}
$$

[^38]From (5.5.3), (5.5.10), and (5.5.11), and the discussion above, we conclude that different trading relationships result in almost the same trading gains but different contracting costs. Economizing contracting costs requires: (i) choosing a suitable trading relationship, i.e., appropriately assigning control between the two trading parties; and (ii) choosing suitable ex ante prices. We summarize the results in the following proposition.

Proposition 5.4: ${ }^{15}$ Depending on parameters $\bar{v}$, $\underline{v}, \bar{c}$, and $\underline{C}$, the advantage of different trading relationships NI, BC and SC can be ordered by their ex post bargaining probabilities

$$
\begin{align*}
& \operatorname{prob}\left\{I I^{N I}+I I I^{N I}-M_{I I}^{N I}-M_{I I I}^{N I}\right\} \\
& \operatorname{prob}\left\{I I I^{B C}+V^{B C}-M_{I I I}^{B C}-M_{I V}^{S C}\right\}  \tag{5.5.14}\\
& \operatorname{prob}\left\{I I^{S C}+I V^{S C}-M_{I I}^{S C}-M_{I V}^{S C}\right\}
\end{align*}
$$

where all probabilities are minimized by appropriately choosing ex ante contract prices $p_{0}$ and $p_{1}$. The best trading relationship corresponds to the smallest probability.

Proposition 5.4 provides the same results through contracting cost analysis as those obtained by Grossman and Hart [1987] through a trading gain comparison. The

[^39]calculation of probabilities in (5.5.14) is much simpler than the gains. In addition, our result can apply even if $v$ and $c$ are correlated. The correlation may influence the calculation of these probabilities, but the proposition itself will not be affected.

Our analysis has implicitly assumed that the probabilities of both the $v>c$ region and the $v<c$ region are strictly positive. If this is not the case, then some of the six regions created by the price $P$ will have zero probabilities. For these corner solutions, we have following proposition.

$$
\begin{aligned}
& \text { Proposition } 5.5: \text { If prob }\{v>c\}=1 \text {, then all } \\
& \text { probabilities in }(5.5 .14) \text { are zero. } \\
& \text { Proof: prob }\{v>c\}=1 \text { implies } v>\bar{v} \text {. Set } v>P \\
& >\bar{c} \text { under any rule, then all probabilities in } \\
& \text { (5.5.14) are zero. } \\
& \text { Q.E.D. }
\end{aligned}
$$

Clearly, when all ex post bargaining probabilities are zero, the governance rule can be selected arbitrarily. For the rest of the paper, we shall rule out this trivial case and assume $0<\operatorname{prob}\{\mathrm{v}>\mathrm{c}\}<1$, i.e., $\underline{\mathrm{v}}<\overline{\mathrm{c}}$ and $\overline{\mathrm{v}}>$ c. If we impose more restrictive distributional assumptions, we can directly calculate the probabilities in (5. 5.14). First, assume that $v$ and $c$ are independent of each other, i.e.,

$$
\begin{equation*}
f(v, c)=\phi(v) \bullet \Psi(c) \tag{5.5.15}
\end{equation*}
$$

where $\Phi$ and $\Psi$ are the respective cumulative distribution functions. The following proposition provides the optimal prices under various governance structure for this setting.

Proposition 5.6: Assume $0<\operatorname{prob}\{v>c\}<1, v$ and $c$ are independently distributed, and $\Phi(v)$ and $\Psi(c)$ are differentiable distributions with density functions $\phi(v)$ and $\psi(c)$. In addition, assume that the optimal $P=p_{1}-p_{0}$ which minimizes the ex post bargaining probabilities are interior solutions. Then $P$ is set to satisfy:
(i) under BC ,

$$
\begin{align*}
& \phi(P)\left[\int_{-\infty}^{P} \Psi(c) d c-\int_{P}^{+\infty} \Psi(c) d c\right] \\
+ & \Psi(P)\{[\Phi(P)-\Phi(P-C N)] \\
- & {[\Phi(P+C N)-\Phi(P)]\}=0 } \tag{5.5.16}
\end{align*}
$$

(ii) under SC,

$$
\begin{align*}
& \Psi(P)\left[\int_{-\infty}^{P} \phi(v) \mathrm{d} c-\int_{P}^{+\infty} \phi(v) \mathrm{d} c\right] \\
+ & \phi(P)\{[\Psi(P)-\Psi(P-C N)] \\
- & {[\Psi(P+C N)-\Psi(P)]\}=0 } \tag{5.5.17}
\end{align*}
$$

(iii) under NI,

$$
\begin{equation*}
\Psi(P) \bullet[1-\Phi(P)]=\Psi(P) \bullet \phi(P) \tag{5.5.18}
\end{equation*}
$$

If $P \notin \operatorname{VuC}(a \operatorname{corner}$ solution), $P$ is set to the boundary of Vuc.

One can provide a sharper characterization of the results of Proposition 5.6 under the BC and SC governance structures if more restrictive assumptions are made. For example, if $\mathrm{CN}_{1}$ is small relative to P , then we have

$$
[\Phi(P)-\Phi(P-C N)]-[\Phi(P+C N)-\Phi(P)] \sim 0(5.5 .19)
$$

Thus (5.5.16) becomes, assuming $\phi(\mathrm{v})>0 \forall \mathrm{v}$,

$$
\int_{-\infty}^{P} \Psi(c) d c \sim \int_{P}^{+\infty} \Psi(c) \mathrm{d} c \nleftarrow \Psi(P) \approx 1-\Psi(P)
$$

which implies that

$$
\Psi(P) \sim \frac{1}{2}
$$

This means that the solution of (5.5.16), to be denoted as $P^{B C}$, approximately equals the median of $\Psi(C)$. Similar arguments apply to $S C$, where $P^{S C}$ is approximated by the median of $\phi(v)$.

If $\phi(v)$ is uniformly distributed, then $\Phi(v)$ is linear in $v$ so that (5.5.19) becomes an exact equality. In this special case, under $B C, P^{B C}$ equals to the mean of $\Psi(C)$ and, under $S C, P^{S C}$ equals to the mean of $\phi(v)$. With this we have proved parts (i) and (ii) of the following corollary.

Corollary 5.7: If $v$ and $c$ are independently and uniformly distributed on $[\underline{v}, \overline{\mathrm{v}}]$ and $[\underline{c}, \bar{c}]$, respectively, then the optimal prices which minimize the ex post bargaining probabilities are:

$$
\begin{align*}
& \mathrm{P}^{\mathrm{BC}}=\mathrm{c}^{*}=(\underline{\mathrm{c}}+\overline{\mathrm{c}}) / 2 ;  \tag{i}\\
& \mathrm{P}^{\mathrm{SC}}=\mathrm{v}^{*}=(\underline{\mathrm{v}}+\overline{\mathrm{v}}) / 2 ;  \tag{ii}\\
& \mathrm{P}^{\mathrm{NI}}=(\underline{\mathrm{c}}+\overline{\mathrm{v}}) / 2 .
\end{align*}
$$

To provide additional intuition about the above analysis, Figure 5-5 depicts the different price settings for a given set of parameter values. We see from this figure that different prices result in different sizes of bargaining regions. In general, the ordering of the prices shown in this figure, i.e., $\mathrm{P}^{\mathrm{SC}}>\mathrm{P}^{\mathrm{NI}}>\mathrm{P}^{\mathrm{BC}}$, holds except when some prices are corner solutions.

Proposition 5.6 provides conditions that are satisfied by the optimal price settings. We have shown that if the distribution functions are differentiable, the optimal prices can be approximated by the median of the corresponding density functions under BC and SC. The precision of the approximation depends on the magnitude of the ex post bargaining cost $\mathrm{CN}_{1}$ and the shape of the distribution functions. The smaller is $\mathrm{CN}_{1}$ and the flatter is the distribution density function, the closer is
the optimal price $P$ to the median of the distribution function. Specifically, when the distribution is uniform, Corollary 5.7 shows that $P$ exactly equals the median (which equals the mean) of the distributions. Thus, the optimal price settings when $v$ and $c$ are independently distributed are completely characterized. .

Based on Corollary 5.7, it is straightforward to prove the following proposition.

Proposition 5.8: Assume that $v$ and $c$ are independently and uniformly distributed on $[\mathrm{v}, \overline{\mathrm{v}}]$ and [ $\mathrm{c}, \overline{\mathrm{c}}]$, respectively. Suppose also that $\overline{\bar{v}}>\mathbf{c}$ and $\underline{v}<\bar{c}$, i.e., $0<\operatorname{prob}\{\mathrm{v}>\mathrm{c}\}<1$. Then ${ }^{16}$
(a) if $\overline{\mathrm{v}}>\overline{\mathrm{C}}>\underline{\mathrm{c}}>\underline{\mathrm{v}}$, then BC is optimal and $P=c^{*}$;
(b) if $\overline{\mathbf{c}}>\overline{\mathbf{v}}>\underline{\mathbf{v}}>\mathbf{c}$, then SC is optimal and $\mathrm{P}=\mathrm{v}^{*}$;
(c) if $\overline{\mathrm{v}}>\overline{\mathrm{c}}>\underline{\mathrm{v}}>\mathrm{c}$, then BC and SC are better than NI,
(i) if $B C$ is optimal then $P=C^{*}$;
(ii) if $S C$ is optimal then $P=v^{*}$;
(d) if $\overline{\mathbf{c}}>\overline{\mathbf{v}}>\mathbf{c}>\mathbf{v}$, then NI is optimal and $P=(\underline{c}+\bar{v}) / 2$.

The different parameter cases are depicted in Figure 5-6. 5-6(a) represents the case in which the variation of $v$ is larger than the variation of $c$ and, hence, region III

[^40]and $V$ is relatively smaller than the other regions. This implies that the ex post bargaining probability will be smaller in the $B C$ case. In 5-6(b), the reverse is true, so SC is optimal. In $5-6(\mathrm{C}), \mathrm{IV}$ and V are relatively smaller than II and III, so that both BC and SC will be superior to NI. Which of BC and SC is optimal depends on the comparision of $\{I I+I V\}$ and $\{I I I+V\}$. For the figure depicted, $S C$ is better than BC. Finally, in 56(d), II and III are smaller than IV and V, so NI is optimal. Note that for simplicity, we do not mark off the marginal adjustments in Figure 5-6.

## Insert Figure 5-6 here

Proposition 5.8 characterizes the optimal governance structure in terms of the exogenous parameter sets $V$ and $C$ only. The results are similar to those reported in Proposition 3 of Grossman and Hart [1987]. However, by explicitly defining and minimizing the contracting costs and allowing ex post renegotiation, we expand the trading region to include almost all areas with positive trading gains, except a narrow band of marginal adjustment regions. Thus, we conclude that an incomplete contract which specifies a couple of prices and a governance structure can result in almost full production efficiency while at the same time minimizing total contracting costs.

When $v$ and $c$ are correlated with each other, an explicit characterization of the ex ante prices is difficult in general. However, since Proposition 5.4 still applies, numerical procedures can be used to identify the optimal governance structure and corresponding prices. A general procedure follows. First, solve the following problems for $P^{B C}, P^{S C}$, and $P^{N I}$, respectively.

$$
\begin{aligned}
& \max _{P^{B C}}\left[\iint_{\left.(c<v<P \cap v\rangle C+C N_{1}\right)}+\iint_{\left(P\langle v<c \in \cap c\rangle v+C N_{1}\right)}\right] f(v, c) d v d c \\
& \max _{P^{S C}}\left[\iint_{\left.(P<c<v \cap v\rangle C+C N_{1}\right)}+\iint_{\left.\{v<c<P \cap c\rangle v+C N_{1}\right)}\right] f(v, c) d v d C \\
& \max _{P^{N I}}\left[\iint_{\left.\{P<c<v \cap v\rangle C+C N_{2}\right)}+\iint_{\left.(c<v<P \cap v\rangle C+C N_{1}\right)}\right] f(v, c) d v d C
\end{aligned}
$$

Second, compare the maximum values of (5.5.20), and select the governance structure that corresponds to the minimum value.

Finally, an interesting question is whether there is another incomplete contract that is more efficient than those analyzed above. It seems quite unlikely if we restrict our analysis to the two contracting parties because a contract with a pair of prices has exhausted all available contractible information, i.e., the trading quantity. However, if the central management has a more extensive role in the contracting process than is assumed in the prior analyses, then, even if $v$ and $c$ are unverifiable, a more efficient contract may exist. We delay this
issue to the next section.

### 5.6 Contracting with Large Ex Post Bargaining Costs <br> In the last section, most of our analysis assumes

 small ex post bargaining costs. In fact, larger ex post bargaining costs strengthen the results of Proposition 5.2 and 5.3. In other words, an incomplete contract with a pair of prices and a governance rule is much better than a null contract when $\mathrm{CN}_{1}$ is large. To see this, let us go back to Figure 5-4. We can see that if $\mathrm{CN}_{1}$ becomes large, the marginal adjustment $M_{I}$ becomes significant. This implies that the increased trading gain resulting from the ex ante prices is increasing in $\mathrm{CN}_{1}$. Observe that the other influences of a large $\mathrm{CN}_{1}$ to both a null and an incomplete contract are the same. Thus the improvement of an ex ante incomplete contract relative to a null contract is more significant when the ex post bargaining costs are large.The case of small ex post bargaining costs corresponds to symmetric information situations. This is why we assume a symmetric information structure for both ex ante and ex post contracting in the last section. Hence, in that setting, the bargaining games, both ex ante and ex post, are ones of complete information, i.e., each party knows the other party's position with certainty. As shown
by Rubinstein [1982], while almost any outcome can be supported as a Nash equilibrium, there exists a unique subgame perfect equilibrium, in which one party, based on his bargaining position, makes an offer, which the other party immediately accepts. This implies that the bargaining costs involved in such games should be relatively low. The case in which ex post bargaining costs are large corresponds to situations in which information asymmetry exists. If either or both realizations of $v$ and $c$ are not publicly observable by both $B$ and 8 , then the two parties possess different information when they enter into ex post bargaining for price $P$. Thus, the bargaining games in these cases are ones with asymmetric information. For such a game, each party must acquire information about the other party's bargaining position during the bargaining process.

The literature on bargaining with imperfect information demonstrates that such bargaining may be very costly. For example, Grossman and Perry [1986a] analyze such a game in which two parties bargain over the price at which an item is to be sold. The seller's valuation is common knowledge but the buyer's valuation is known only to the buyer. Each party, in turn, makes an offer. The other party either accepts or responds with a counteroffer. As they bargain, their payoffs are discounted over time, so
that both have an incentive to come to an early agreement. They find that with asymmetric information, the sequential equilibrium concept does not result in a unique outcome for the game. Instead the concept puts very little restriction on how the parties divide the surplus from their trade or how long it takes to reach an agreement (i.e., how many offers and counteroffers are necessary to reach the equilibrium price). They show that the set of outcomes of the bargaining game can be greatly refined by the concept of perfect sequential equilibrium which is developed in Grossman and Perry [1986b]. Under some weak assumptions, they find that the game has a unique candidate perfect equilibrium which consists of an equilibrium price and a length of the bargaining time. The former is determined by the positions of both parties, i.e., their types or true valuations. The latter depends on the imperfectness of the seller's information about the buyer's type. The less informative is the seller's knowledge about the buyer's valuation, the more offers and counteroffers are necessary to reach an agreement. This implies that larger bargaining costs will be incurred and trading gains will be more heavily discounted.

Related to our analysis, from a contracting cost point of view, Grossman and Perry have shown that bargaining with asymmetric information may involve very high
bargaining costs. These costs include not only the resources used to make offers and counteroffers, but also the opportunity costs due to delaying or forgoing trades with positive gains. The following observation summarizes the above discussion and serves as a connection between the bargaining literature and our analysis.

Observation 5.9: When either or both realizations of $v$ and $c$ are not publicly observable by $S$ and $B$, ex post bargaining costs $C N_{1}$ will be larger than that in symmetric information cases.

Based on Observation 5.9, when ex post information asymmetry exists, a null contract which requires ex post bargaining for every event at which trading occurs must be very inefficient. On the one hand, when $\mathrm{CN}_{1}$ is large, the marginal adjustment region becomes so substantial that trading occurs only in a small part of the positive gain region. On the other hand, even if a trading occurs, the net gains will be much smaller because of the big bargaining costs. Hence the expected net total gains will be very low.

Observe that if ex ante complete contracting is impossible, the only choice left is incomplete contracting. Since we have assumed that 8 and $B$ have homogenous prior beliefs, when they come to bargain for an ex ante
contract, i.e., a pair of prices and a governance structure, there does not exist information asymmetry at that date. Hence, the ex ante contracting costs are the same as in the public information cases in the last section. Once an ex ante contract is agreed upon, ex post trading decisions only involve comparisons between the prices and each party's valuation of $v$ or $c$. Whether the valuations are public or private, as long as $v>P>c, t r a d i n g ~ w i l l$ always be the best choice for both parties. Trading will occur in region I without any influence of ex post information asymmetry. We summarize the above discussion in the following proposition.

> Proposition 5.10: ${ }^{17}$ When either or both realizations of $v$ and $c$ are not publicly observable by $S$ and $B$, an incomplete contract with a pair of prices $p_{1}$ and $p_{0}$ and a governance structure will be more efficient than a null contract.

The significance of Proposition 5.10 is that it extends the validity of the analysis in the last section to the asymmetric ex post information case. The criteria for choosing optimal governance structure and for setting of ex ante prices remain effective even if there is ex post private information. This is because these criteria

[^41]are derived by the principle of minimizing the ex post bargaining probability, which is valid whether ex post bargaining costs are small or large.

Now let us consider the question raised in the last section: are there any other incomplete contracts which would be more efficient than the one we have analyzed. The answer is a conditional YES. If some other mechanism can be introduced, contracting efficiency may be improved. Note that in prior analysis, we implicitly assumed that the contract must balance the transaction between $B$ and $s$, i.e., what B pays must be equal to what $s$ receives. This "single" pricing policy is necessary when no other mechanism can be used, and it also has the feature that it automatically results in a maximum trading region without introducing misincentives to trade in the negative gain region. This can be seen by noticing that the lines $v=P$ and $c=P$ intersect on the line $v=c$, which separates the positive and negative gains regions. Now we relax this restriction by assuming that the central management will permit the price received by $s$ to differ from that paid by B. This makes it possible for a "dual" pricing contract in which three prices ( $\mathrm{p}_{1}^{\mathrm{s}}, \mathrm{p}_{1}{ }^{\mathbf{B}}, \mathrm{p}_{0}$ ) can be set, along with a governance structure. We shall show that when ex post bargaining costs are high, a "dual" pricing contract may be more efficient than a "single" pricing
contract.
First note that it is never desirable to set $P^{s}<P^{B}$ (where $p^{s}=p_{1}{ }^{s}-p_{0}, P^{B}=p_{1}^{B}-p_{0}$ ) because such a contract is always dominated by a single price $P=P^{B}$. This can be seen in Figure $5-7$ where $P^{s 1}<P^{B}$, so the lines $C=P^{s 1}$ and $\mathrm{v}=\mathrm{P}^{\mathrm{B}}$ intersect at R , inside the positive gains region. The area I governed by this contract can always be expanded by increasing $\mathrm{P}^{s 1}$ to make the intersection reach the line $\mathrm{V}=\mathrm{c}, \mathrm{i} . \mathrm{e} ., \mathrm{P}^{\mathrm{S}}=\mathrm{P}^{\mathrm{B}}$. Therefore, a Pareto improvement can only occur when $\mathrm{P}^{\mathrm{S}}>\mathrm{P}^{\mathrm{B}}$.

We take the NI governance rule as an example. In Figure 5-8, $\mathrm{P}^{\mathrm{NI}}$ is the optimal price under NI governance. Let $P^{S}>P^{N I}>P^{B}$ and define a dual pricing contract ( $p_{1}{ }^{s}$, $\mathrm{p}_{1}{ }^{\mathrm{B}}, \mathrm{p}_{0}$ ) such that $\mathrm{P}^{\mathrm{S}}=\mathrm{p}_{1}{ }^{\mathrm{S}}-\mathrm{p}_{0}, \mathrm{P}^{\mathrm{B}}=\mathrm{p}_{1}{ }^{\mathrm{B}}-\mathrm{p}_{0}$. This means that if a trade occurs, $B$ pays $p_{1}{ }^{B}$ while $S$ receives $p_{1}{ }^{s}$. The difference $p_{1}{ }^{s}-p_{1}{ }^{B}$ is covered by the central management. As depicted in Figure 5-8, there are two differences between this contract and the NI-contract. First, region $I$ - $M_{I}$ is expanded to include the areas $B B^{*} M^{*} S$ and DTN*D*, which implies a reduction of the ex post bargaining probability and the expected bargaining costs. At the same time, $M_{I}$ is expanded from STF to include areas $M^{*}$ XFS and TFYN*, which represents an increase in expected trad-
ing gain without incurring bargaining costs. These benefits will be particularly significant when $\mathrm{CN}_{1}$ is large. Second, a dual pricing contract will always introduce misincentives to trade in the negative gains region. As shown in the Figure $5-8, \mathrm{~s}$ and B will trade in the triangle $X Y F^{*}$ which will result in negative gains. This inefficiency cannot be corrected by contract renegotiation, and represents a deadweight cost of the dual pricing contract. When the benefit of a dual pricing contract exceeds its deadweight cost, a Pareto improvement over a single pricing contract is realized.

Insert Figure 5-8 here

An optimal dual pricing contract can be found through choosing $P^{s}$ and $P^{B}$ to maximize the difference of the benefits and the costs. The detailed mathematics is very similar to what we have already provided in the last section and, therefore, will be omitted here.

### 5.7 Transfer Pricing for Economizing Contracting Costs

 Transfer pricing in decentralized organizations is a very important but difficult and frustrating topic. Many researchers have examined this topic from different directions, yet our understanding of it is far from complete. Eccles [1985] provides a detailed summary of the existingtheoretical and empirical literature. He also presents a clear picture of the transfer pricing problem from a practitioner's view point. The main contribution of his work is that he points out a direction for further research in this area. In particular, he claims that transfer pricing policy must depend on corporate strategy and administrative processes. Hence, no single policy is a solution for every situation. Prior analytical research in this area typically seeks to characterize the nature of an optimal transfer price under some specified set of conditions. The results are helpful, but are rather narrow in their scope.

Eccles' arguments imply that any transfer pricing model in which the organization strategy is ignored cannot capture the core of the problem. Hence, its explanatory power and assistance to practitioners must be limited. To view a transfer price as a simple variable in an organization's production decision, as in most economic theory and mathematical programming transfer pricing papers, overly simplifies the problem. Transfer pricing policy is a complex function of many variables. The most important one is corporate strategy.

Unfortunately, Eccles does not clearly state what he means by "corporate strategy". Based on transaction cost economics, corporate strategy can be interpreted as the
way by which a firm manages its various transactions. For example, a firm must decide whether a particular transaction should be conducted in the market or within the organization. If it is better to perform a transaction among its divisions, what is the most economical way to conduct it. From this perspective, transfer pricing itself is a part of the corporate strategy in dealing with intra-firm transactions. Hence, transfer pricing policy must be chosen to economize transaction costs.

A detailed analysis of transfer pricing is beyond the objective of this paper. However, it is worthwhile to point out that our basic model has provided a new basis for examining this complex topic. To support our claim, in this section we discuss how the basic results of our analysis can be used to model intra-firm transfers. We find that the firm's behaviour in setting ex ante prices endogenously derived in our model is quite consistent with Eccles' empirical evidence.

We start with an exploration of the possible nature of the transfer value and cost. For simplicity, we provide some examples only. Let $D_{1}$ represent the product $B$ wants $s$ to transfer, and assume that $B$ uses $D_{1}$ to produce $D_{2}$ for external sale. Most obviously, a division's valuation of the transfer is influenced by its operating conditions. The following examples describe three possible
conditions under which the selling division might be operating.
(a) If division 8 has excess capacity, then the transfer cost is equal to the incremental out-of-pocket cost of producing the transfer product $D_{1}$. The latter will often be the variable production costs.
(b) If division 8 can sell the transfer product $\mathrm{D}_{1}$ in the market (and will forego the sale if it is transferred), then the transfer cost is equal to the market price of $D_{1}$ minus any selling costs incurred on external sales but not incurred on internal transfers.
(c) If division 8 is operating at capacity and will forego the production and sale of another product $D_{3}$ if the original product $D_{1}$ is produced for transfer, then the transfer cost is equal to the incremental out-of-pocket cost of producing $D_{1}$ plus the revenue lost from the sale of $D_{3}$ minus the incremental out-of-pocket cost of producing and selling $D_{3}$ that is avoided.

The following examples describe three possible conditions under which Division $B$ might be operating:
(a) If division $B$ has excess capacity and lacks an alternative source of the transfer product $D_{1}$, then the transfer value is equal to the revenue from sale of $D_{2}$ minus the incremental out-of-pocket cost of processing $D_{1}$ to obtain and then sell $D_{2}$.
(b) If division $B$ can acquire $D_{1}$ from an external source if $D_{1}$ is not supplied by $S$, then the transfer value is equal to the external purchase price of $D_{1}$ plus any acquisition costs that would be avoided if acquired internally.
(c) If division $B$ is operating at capacity and

> would forego the production and sale of another product $D_{4}$ if it produced $D_{2}$, then the transfer value is equal to the revenue from sale of $D_{2}$, minus the incremental outof-pocket costs to process $D_{2}$, minus the $D_{4}$ revenue lost, and plus the incremental outof-pocket cost of producing and selling $D_{4}$ that is avoided.

From these examples, we can see that the valuations might have the following characteristics.
(a) Uncertainty -- Both valuations are influenced by the input and output prices and demands that are determined in the market. A key characteristic of the market is its uncertainty about demand and price. Therefore, the valuation of $v$ and $c$ for any future transfers must involve randomness.
(b) Unverifiability -- The valuations are based on observations of events that are difficult to document but which significantly influence the managers' expectations about the cost or value of the transfer. ${ }^{18}$ For example, a key element in determining the transfer value and cost is the division's opportunities to trade in the market, which is never reported by any accounting system. Therefore, it is very difficult to establish formal accounting procedures and measurement criterion to report these valu-

[^42]ations.
(c) Unobservability -- The valuations may depend on many events that are privately observed by the managers. In other words, the realizations of many events may not be publicly observable. For example, events influencing market demand, divisional capacity, out-of-pocket production costs, the demand for other products a division could produce, are all possible events that may be observed by a divisional manager only. Therefore, the valuations themselves may be private information.
(d) Complexity -- The valuations are based on multiple factors and many of these factors are multi-dimensional. For example, selling costs and acquisition costs include various expenses, some of which may not be directly measurable. Hence, the specification and verification of $v$ and $c$ may be difficult.

These valuation characteristics are consistent with the assumptions we made in this paper. They imply that ex ante complete contracting is very expensive or impossible for intra-firm transactions in most cases. Furthermore, ex post bargaining may be costly due to asymmetric information. Therefore, our conclusion that incomplete contracting is optimal in managing transactions with the characteristics mentioned above, indicates that incomplete contracting can be used to develop useful transfer pricing
models.
Our results can be directly applied to the special case in which no ex ante investments are necessary (e.g., S and B are well established divisions and capacities for producing the transfers are available) and transfers are repeated. To see this, assume that each period B will need one unit of the product $D_{1}$. Assume the transfer value of $D_{1}$ if it is supplied to $B$ by $s$ is $v_{t}$, and the transfer cost $S$ bears to supply it is $c_{t}$, as determined at the date $t . \quad v_{t}$ and $c_{t}$ are independent random variables with identical uniform distribution functions on $V$ and $C$, respectively.

Depending on the governance rule that is used, the one period contracting costs can be represented by (5.5. 3), (5.5.10), or (5.5.11). Let $r$ be the interest rate per period. The expected net present value of the total contracting costs for an infinite number of periods can be represented as ${ }^{19}$

$$
\begin{align*}
& \mathrm{ETC}^{\mathrm{BC}}=\mathrm{CN}_{0}+\frac{1+I}{I} \mathrm{CN}_{1} \bullet \text { prob }\left\{I I I+V-M_{I I I}-M_{V}\right\}  \tag{5.7.1}\\
& \mathrm{ETC}^{\mathrm{sC}}=\mathrm{CN}_{0}+\frac{1+I}{I} \mathrm{CN}_{1} \cdot \operatorname{prob}\left\{I I+I V-M_{I I}-M_{I V}\right\}  \tag{5.7.2}\\
& \mathrm{ETC}^{\mathrm{NI}}=\mathrm{CN}_{0}+\frac{1+I}{r} \mathrm{CN}_{1} \bullet \operatorname{prob}\left\{I I+I I I-M_{I I}-M_{I I I}\right\} \tag{5.7.3}
\end{align*}
$$

[^43]where $\mathrm{CN}_{0}$ is the ex ante negotiation cost for the price $p_{0}$ and $p_{1}$, and $\mathrm{CN}_{1}$ is the ex post bargaining cost for new price $\hat{\mathrm{p}}_{1}$ or $\hat{\mathrm{p}}_{0}$. As usual, we ignore the trivial costs to specify the ex ante contract. This implies that, in ex ante contracting, the optimal governance rule and ex ante prices should be set in the same way as in the one period case given by Proposition 5.3, 5.5 and 5.7. Thus, our model predicts that for repeated intra-firm transfers, the best ex ante contracting process is: (i) to choose a governance rule; (ii) to specify a price $p_{1}$ based on the governance rule selected; and (iii) to choose a price $p_{0}$ such that each division's ex ante expected gain is consistent with its status quo position and its bargaining power. After the transfer price is set and the divisions learn $v_{t}$ and $c_{t}$, they can decide whether to transfer the product or not, or propose to renegotiate the prices based on the value of $v_{t}$ and $c_{t}$ realized in each period. The transfer price should be consistent with the governance rule. The latter in turn, depends on the relative variations of $v$ and $c$. From Proposition 5.8, we know that
(a) When the variation of $v$ is bigger than that of $c$, then control should be assigned to $B$, and the corresponding transfer price is $p_{1}$ $=c^{*}+p_{0}$. That is, under BC, the optimal transfer price is determined by the mean transfer cost.
(b) When the variation of $c$ is bigger than that
of $v$, then control should be assigned to $s$, and the corresponding transfer price is $p_{1}$ $=\mathrm{v}^{*}+\mathrm{p}_{0}$. That is, under SC, the optimal transfer price is determined by the mean transfer value.
(c) When $\mathrm{v}^{*}>\mathrm{c}^{*}, \mathrm{BC}$ and SC are better than NI.
(d) When $\mathrm{v}^{*}<\mathrm{c}^{*}$, then it is optimal to let the two divisions share control. The transfer price should be set equal to $p_{1}=(\underline{c}+\bar{v}) / 2$ $+\mathrm{p}_{0}$, which satisfies
$$
v^{*}-\frac{\underline{v}+\bar{v}}{2}<\frac{c+\bar{v}}{2}<\frac{c+\bar{c}}{2}-c^{*}
$$

That is, the transfer price depends on both the transfer cost and the transfer value.

In summary, it says the transfer price should be set based on either the mean of B's transfer value or the mean of s's transfer cost or both. Eccles [1985] shows empirically that most transfer price policies belong to one of the following categories: (i) cost based pricing; (ii) market price based pricing; (iii) negotiated pricing; and (iv) dual pricing. We can show by examples that the optimal transfer pricing policy predicted by our model is consistent with his empirical observations. To see this, let us refer back to the examples about $\delta$ and B's operating conditions given in this section.

The three examples of s's operating conditions show that the transfer cost may be equal to: (a) the out-ofpocket production cost, which is often the variable production cost; (b) the market price less an adjustment for
savings on selling costs; or (c) the out-of-pocket costs (or variable production costs) plus an adjustment for the opportunity loss. (a) and (c) can be viewed as cost based pricing policies, while (b) can be viewed as a market price based policy. Therefore, if the uncertainty faced by $B$ is the main concern of the firm, the trading decision will be delegated to $B$, and either cost or market price based pricing may be observed. The three examples of B's operating conditions show that the transfer value may equal: (a) the market price of the final product less an adjustment for processing costs; (b) the market price of the transfer product plus an adjustment for savings due to not purchasing in the market; or (c) the market price less an adjustment for processing and opportunity loss. All three examples show that if the uncertainty faced by division 8 is the main concern of the firm, then the optimal governance rule is SC, and we will observe market price based policies only.

In cases in which NI is optimal, negotiated pricing policies are observed. In Section 5.6 , we show that when bargaining costs are large, dual pricing may be desirable. However, this ignores some other problems in dual pricing that are identified by Eccles [1985], such as the ambiguity it creates about the firm's strategy.

In summary, the above discussion shows the similar-
ities between the predictions of our model and empirical evidence. This is an indication that our analysis is on the right track.

### 5.8 Conclusion

We formally define contracting costs and incorporate them into our analysis. By explicitly considering contracting costs, we show that the contract efficiency concept can be made more precise. We find that the ex ante and ex post contracting costs have different influences on contracting results. Through contracting cost minimization, we show in our simple intra-firm transaction model, that incomplete contracting may be superior to complete contracting and null contracting. This result provides a foundation for incomplete contracting. In this way, our analysis extends traditional contracting theory to a broader contracting strategy space. Particularly, in our special setting, an incomplete contract, which specifies a pair of prices and a governance rule in advance, is superior relative to a complete contract (if it is available) or a null contract. An incomplete contract may optimize the firm's net cash flows in managing repeated intra-firm transactions. The assumptions for deriving these results are that: (i) the ex ante specification and ex post verification costs for the transfer cost $c$ and
tranfer value $v$ are high; and (ii) the ex post bargaining costs are high due to possible information asymmetries. These assumptions are consistent with the characteristics of typical intra-firm transactions. Thus, our results provide a rationale for observed transfer pricing policies for managing repeated intra-firm transactions. In addition, we characterize the optimal governance rule and ex ante prices. Under more restrictive assumptions, the prices are explicitly expressed as functions of the governance structure and the distribution parameters. Thus we extend the results of Grossman and Hart [1987] to the case in which contract renegotiation is allowed and contracting costs are explicitly considered.

Our results provide new insights into intra-firm transactions. Although we do not present a complete transfer pricing theory in this paper, hopefully we have provided a useful basis for further research into this important topic in management accounting.

Figure 5-1: Positive and Negative Gain Regions v without Contracting Costs.


Figure 5-2: Trading Region of a Null Contract with Ex Post Bargaining Costs.


Figure 5-3: Regions Divided by Ex Ante Prices.


I: $\mathbf{c}<\mathbf{P}<\mathbf{v}$ II: $\mathbf{P}<\mathbf{c}<\mathbf{v}$ III: $\mathbf{c}<\mathbf{v}<\mathbf{P}$
IV: $\mathbf{v}<\mathbf{c}<\mathbf{P} \quad$ V: $\mathbf{P}<\mathbf{v}<\mathbf{c} \quad$ VI: $\mathbf{v}<\mathbf{P}<\mathbf{c}$

Figure 5-4: Regions with Marginal Adjustments.


I: $\mathbf{c}<\mathbf{P}<\mathbf{v}$ II: $\mathbf{P}<\mathbf{c}<\mathbf{v}$ III: $\mathbf{c}<\mathbf{v}<\mathbf{P}$
IV: $\mathbf{v}<\mathrm{c}<\mathbf{P} \quad$ V: $\mathbf{P}<\mathrm{v}<\mathrm{c} \quad$ VI: $\mathrm{v}<\mathbf{P}<\mathrm{c}$

Figure 5-5: Ex Ante Prices Corresponding to


Figure 5-6: Different Parameter Categories.


BC or SC


Figure 5-7: Suboptimal Dual Pricing Contract.

$\mathbf{P}^{\mathbf{B}} \quad \mathbf{P}^{\mathbf{S}} \quad$ : A dual prices contract.

Figure 5-8: Optimal Dual Pricing Contract.


P: A singal price contract,
$P^{B} \quad P^{S} \quad$ : A dual prices contract.

## Appendix 5: Proofs

Proof of Proposition 5.1
When $\mathrm{c}<\mathrm{v}<\mathrm{c}+\mathrm{CN}_{1}$,

$$
\begin{equation*}
\Delta \mathrm{W}=\mathrm{W}^{\mathrm{p}}-\mathrm{W}^{0}-\iint_{c<v<c+C N_{1}}(v-c) \cdot f(v, c) \mathrm{d} v \mathrm{~d} c \tag{A5.1}
\end{equation*}
$$

is increasing in $\mathrm{CN}_{1}$. Assuming that $\mathrm{TCN}_{1}=\mathrm{CN}_{1}$,

$$
\begin{align*}
\Delta \mathrm{TC} & =\mathrm{CSV}+\mathrm{TCN}_{1}-\mathrm{CN}_{1} \bullet \text { prob }\{\bullet\} \\
& =\mathrm{CSV}+\mathrm{CN}_{1} \bullet(1-p r o b\{\bullet\}) \tag{A5.2}
\end{align*}
$$

Note that when $C N_{1}=0, \Delta W$ equals zero while $\Delta T C$ equals CSV, so that $\Delta W<\Delta T C$. If $\Delta W<\Delta T C$ for all $C N_{1}$, then (5.4.9) holds for $\mathrm{CN}_{1}<\mathrm{CN}_{1}{ }^{+}=\infty$. Otherwise, there is a value $\mathrm{CN}_{1}^{++}$such that $\Delta W>\Delta T C$. Then, since both $\Delta W$ and $\Delta T C$ are continuous functions of $C N_{1}$, there must be a value $\mathrm{CN}_{1}{ }^{+}$in $\left[\mathrm{O}, \mathrm{CN}_{1}{ }^{++}\right]$such that

$$
\begin{equation*}
\Delta \mathrm{W}=\mathrm{CSV}+\mathrm{CN}_{1}^{+} \cdot(1-p r o b\{\bullet\}) \tag{A5.3}
\end{equation*}
$$

This implies, for all $\mathrm{CN}_{1}<\mathrm{CN}_{1}{ }^{+}$, that (5.4.9) is true. Q.E.D.

Proof of Proposition 5.6
For simplicity, assume $v$ and $c$ are random variables in
$(-\infty,+\infty), 20$ and let $\mathrm{CN}_{1}=\mathrm{R}$.
(i) Under $B C, P$ is set to minimize $I I I+V-M_{I I I}^{B C}-$ $\mathrm{M}_{\mathrm{v}}{ }^{\mathrm{BC}}$,

$$
\begin{equation*}
\operatorname{Min}_{P}\left\{\int_{-\infty}^{P}\left[\int_{c+R}^{P} \phi(v) \mathrm{d} v\right] \Psi(c) \mathrm{d} c+\int_{P}^{+\infty}\left[\int_{P}^{c-R} \phi(v) \mathrm{d} v\right] \Psi(c) \mathrm{d} c\right\} \tag{A5.4}
\end{equation*}
$$

Note that the marginal adjustments have been reflected in the choices of the integral limits, and they have different signs in regions III and V. By direct calculation, (A5.4) is equal to

$$
\begin{aligned}
& \int_{-\infty}^{P}[\Phi(P)-\Phi(c+R)] \Psi(c) \mathrm{d} c+\int_{P}^{+\infty}[\Phi(c-R)-\Phi(P)] \Psi(c) \mathrm{d} c \\
= & \Phi(P) \int_{-\infty}^{P} \Psi(c) \mathrm{d} c-\int_{-\infty}^{P} \Phi(c+R) \Psi(c) \mathrm{d} c \\
+ & \int_{P}^{+\infty} \Phi(c-R) \Psi(c) \mathrm{d} c-\Phi(P) \int_{P}^{+\infty} \Psi(c) \mathrm{d} c
\end{aligned}
$$

The first-order condition with respect to $P$ is

$$
\begin{aligned}
& \Phi^{\prime}(P) \int_{-\infty}^{P} \Psi(C) \mathrm{d} c+\Phi(P) \Psi(P)-\Phi(P+R) \Psi(P) \\
& -\Phi(P-R) \Psi(P)-\Phi^{\prime}(P) \int_{P}^{+\infty} \Psi(C) \mathrm{d} c+\Phi(P) \Psi(P)-0 \text { (A5.5) }
\end{aligned}
$$

[^44]This is equivalent to (5.5.16).
(ii) Under $S C, P$ is set to minimize $I I+I V-M_{H}{ }^{\text {sc }}-$ $\mathrm{M}_{\mathrm{IV}}{ }^{\mathrm{sC}}$.

$$
\begin{equation*}
\underset{P}{\operatorname{Min}}\left\{\int_{-\infty}^{P}\left[\int_{v-R}^{P} \Psi(c) \mathrm{d} c\right] \phi(v) \mathrm{d} v+\int_{P}^{+\infty}\left[\int_{P}^{v+R} \Psi(c) \mathrm{d} c\right] \phi(v) \mathrm{d} v\right\} \tag{A5.6}
\end{equation*}
$$

The same procedure as (i) results in (5.5.17).
(iii) Under NI, P is set to minimize II + III $M_{I I}{ }^{N I}-M_{I I I}{ }^{N I}$

$$
\begin{equation*}
\underset{P}{\operatorname{Min}}\left\{\int_{-\infty}^{P}\left[\int_{-\infty}^{v-R} \Psi(c) \mathrm{d} c\right] \phi(v) \mathrm{d} v+\int_{P}^{+\infty}\left[\int_{P}^{v-R} \Psi(c) \mathrm{d} c\right] \phi(v) \mathrm{d} v\right\} \tag{A5.7}
\end{equation*}
$$

(A5.7) can be simplified as

$$
\begin{equation*}
\int_{-\infty}^{+\infty} \Psi(v-R) \phi(v) \mathrm{d} v-\Psi(P) \int_{P}^{+\infty} \phi(v) \mathrm{d} v \tag{A5.8}
\end{equation*}
$$

The first-order condition for (A5.8) is

$$
\begin{equation*}
-\Psi^{\prime}(P) \int_{P}^{+\infty} \phi(v) d v+\Psi(P) \phi(P)=0 \tag{A5.9}
\end{equation*}
$$

This is the same as (5.5.18).
Q.E.D.

## Proof of Corollary 5.7

Note that due to the assumption that the cumulative distribution functions are linear, (i) and (ii) are special cases of Proposition 5.6 (i) and (ii), with (5.5.19)
holding as an equality. For (iii), note that

$$
\Psi(c)=\frac{c-c}{\bar{c}-\varepsilon} \quad \Phi(v)=\frac{v-V}{\bar{V}-\underline{V}}
$$

Substituting into (5.5.18), we have

$$
\frac{1}{\bar{C}-c}\left[1-\frac{P-Y}{\bar{V}-Y}\right]-\frac{P-c}{\bar{c}-c} \cdot \frac{1}{\bar{V}-\underline{V}}
$$

Simplify to get (iii).
Q.E.D.

## Proof of the Proposition 5.8

The proof can be obtained by directly calculating all probabilities. We show the first one as an example. All other cases are similar. From Figure 5-6(a), for any P, the area for each region excluding the marginal adjustment, is

$$
\begin{aligned}
\operatorname{Area}\left(I I-M_{I I}\right) & =0.5(\bar{C}-P-R)^{2}+(\bar{C}-P)(\bar{V}-\bar{C})-0.5 R^{2} \\
\text { Area }\left(I I I-M_{I I I}\right) & =0.5(P-c-R)^{2} \\
\text { Area }\left(I V-M_{I V}\right) & =0.5(P-C-R)^{2}+(\bar{C}-V)(P-c)-0.5 R^{2} \\
\text { Area }\left(V-M_{V}\right) & =0.5(\bar{C}-P-R)^{2}
\end{aligned}
$$

Note that the marginal adjustment cannot make the last two terms in areas II and IV become negative, i.e., we always
have ${ }^{21}$

$$
\begin{align*}
& (\bar{C}-P)(\bar{v}-\bar{c})-0.5 R^{2} \geq 0  \tag{A5.10}\\
& (\bar{c}-\underline{F})(P-c)-0.5 R^{2} \geq 0 \tag{A5.11}
\end{align*}
$$

Under $B C$, the minimum Area (III $+V-M_{I I I}-M_{V}$ is reached at $P=c^{*}$. Hence

$$
A_{B C}=\operatorname{Min} \operatorname{Area}\left(I I I+V-M_{I I I}-M_{V}\right)=\frac{1}{4}(\bar{C}-c-2 R)^{2} .
$$

Under SC,

$$
\begin{align*}
A_{S C} & =A r e a\left(I I+I V-M_{I I}-M_{I V}\right) \\
& =A_{B C}+(\bar{C}-P)(\bar{V}-\bar{C})+(\varepsilon-\underline{\varepsilon})(P-\subset)-R^{2} \tag{A5.13}
\end{align*}
$$

The last three terms in (A5.13) are positive because of (A5.10) and (A5.11); therefore the minimum value of (A5. 13) when $P=\mathrm{V}^{*}$ must exceed (A5.12). This proves BC is better than SC.

Under NI,

$$
\begin{align*}
A_{N I} & =\text { Area }\left(I I+I I I-M_{I I}-M_{I I I}\right) \\
& =A_{B C}+(\bar{C}-P)(\bar{v}-\bar{C})-0.5 R^{2} \tag{A5.14}
\end{align*}
$$

The last two terms in (A5.14) are nonnegative from (A5. 10). Hence, $B C$ is better than NI. Q.E.D.

[^45]
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## Chapter 6

## CONTRACT RENEWAL

## AND LONG-TERM INCENTIVES

IN ORGANIZATIONS

### 6.1. Introduction

When a principal hires a risk- and work-averse agent, simultaneous achievement of the efficient allocation of risk and the efficient level of production is usually prevented by the agents' self-interested behaviour. It may be optimal for the principal to impose more than the efficient level of risk on the agent in order to improve the latter's motivation to produce. This is the central theme of principal-agent models that seek to provide a coherent and useful framework within which to examine managerial accounting procedures, and pose managerial accounting questions.

The primary construct utilized in agency theory has been the identification of complete contingent contracts for motivating economic agents. However, the "complete contracting" approach has distinct limitations in the insights that it can provide because it ignores contracting costs. Although the use of contingent contracts to provide high-powered incentives ${ }^{1}$ for agents has been found extensively in various financial and managerial accounting settings, most employee compensation is not

[^46]based on explicit contingent contracts. In most organizations, the decisions as to whom to promote and how to allocate bonuses and perquisites are, in practice, often left to the discretion of supervisors rather than completely specified in an employment contract. Even at the management level, managers are evaluated both by objective, quantitative factors and by subjective, qualitative factors since the latter are too difficult to be formalized in any accounting system.

The effectiveness of contingent contracts is hindered if the performance measures for a given agent are significantly influenced by factors beyond the agent's control or responsibility, including events controlled by other agents and uncertain events that are beyond any agents' control. In some situations, the accounting system can easily provide individual performance measures that primarily reflect the actions of an individual agent, thereby making contingent contracting effective. For example, if a firm is only concerned about the quantity produced by an agent and quantity data is reported by the accounting system, then a piece-rate can provide effective incentives for the worker. However, such systems are rare in large organizations. Managerial accounting systems mainly report data at an aggregate (e.g., divisional) level, but not the individual level. This is because the complexity
of most employees' tasks, particularly their cooperative nature, make it impossible to provide effective performance measures for each individual. This point has been illustrated in Alchian and Demsetz [1972] and Williamson [1985], using the manual freight example:

Two men jointly lift cargo into trucks. Solely by observing the total weight loaded per day, it is impossible to determine each person's marginal productivity ... The output is yielded by a team, by definition, and it is not a sum of separable outputs of each of its members.
Under the condition of technological non-separability, individual productivity cannot be assessed by measuring output. An assessment of inputs is needed. Thus, contingent contracts are less prevalent within organizations than at the senior management level.

The limited power of classical agency theory in examining contracting behaviour within organizations creates a demand for extending the theory to deal with a broader range of contracting strategies and managerial accounting system designs. This paper seeks to contribute to that extension. The model provided in this paper shows that a contingent contract is not the only means of providing incentives in organizations. Incomplete contracting with contract renewal can provide incentives for almost all employees in a hierarchical organization, particularly when a "hard" performance measure for an
employee is unavailable. Some special features of our model and the main contribution of this paper to the existing literature are described below.

First, we assume that most contracts made within an organization are incomplete contracts. The special nature of the hierarchy provides an environment in which contract renegotiation, renewal, and dispute resolution are much easier than in the market environment. Particularly, most employment contracts are short term contracts, and explicitly or implicitly specified as renewable. We shall show that this is a rational structure for creating low-powered incentives for agents who are employees in an organization. This is a significant extension of classical agency theory in which managerial accounting procedures are associated with complete contracting.

Second, although we observe the use of high-powered incentives at some levels in an organization, most employees appear to be motivated to work hard to contribute to the firm's operation for reasons other than short term benefits. This implies that high-powered incentive may not be the main force driving an employee's activities within an organization. Instead, the dominating concern of employees may be the long term benefits they perceive will follow from such behaviour. Even if the wages specified in short-term contracts are constant, employees will
have incentive to provide "high" level effort provided the expected future benefits offset the personal costs of that effort. This incentive is not viewed as high-powered because it is not created by current compensation that is explicitly contingent upon a pre-specified measure of the employee's performance. Instead, it is based on predicted consequences that depend upon equilibrium behaviour by both the firm's management and its employee.

Third, in most organizations, an employee is a subordinate to a (higher level) manager, his supervisor. A supervisor has authority over the subordinate, and is often responsible for providing a subjective evaluation of the subordinate's performance. Unlike much of the information provided by accounting systems, subjective judgments are unverifiable and subject to the supervisor's discretion. What is the role of such "soft" performance measure in organizations? Our model provides one aspect of the answer.

Fourth, the relationship between a firm and an employee is influenced by the human asset associated with the employee. Employment can be viewed as a transaction in which the firm acquires the services of a human asset from its owner, the employee. If human asset services are perfectly tradeable, then these services can be purchased in the spot market so that long-term relationships have no
value. However, long-term relationships are a significant aspect of the employment relation in most organizations and we examine those human asset characteristics that make such observed organization forms valuable. In addition, similar to a firm's other assets, human assets can be developed during an employment period. That development frequently requires investments from both the firm and the employee and we examine how those investments are influenced by the employment relation.

Our model views employment as a long term relationship governed by incomplete short term contracts. The reason the contracts are incomplete is that, for most employees, objective performance evaluation is unavailable. When negotiating and renewing the contract, the firm's management and the employee bargain over the gains resulting from human asset transactions. As long as the employee's bargaining power is positive, the transaction gains will be shared between the parties. This will create an incentive for the employee to work hard to build up his/her human asset. Since, in most cases, building up a human asset is correlated with the firm's profitability, this, in turn, provides incentives for the employee to contribute more to the organization.

Our model provides some interesting predictions. First, depending on the balance between the human asset
acquisition and decay rates, employee wages may or may not display downward rigid behaviour (Harris and Holmstrom [1982]), i.e. an employee's wage may or may not increase over his life time. Second, if the employment relation is sufficiently long-lasting, then the employee's incentive to work hard will be relatively stable. Third, the incentive to work hard will decline when the employment relation is close to termination. Finally, incentives are influenced by: the employee's bargaining power, the employee's human asset acquisition and decay rates, controls on the firm's capital investment, the firm's production technology, and the managerial accounting system. Changes in these elements can induce changes in incentives. This chapter is organized as follows. After this introductory section, Section 6.2 discusses two key concepts used in our model. Section 6.3 provides the model and analysis. Section 6.4 discusses the main predictions provided by our model.

### 6.2 Performance Measures and Human Assets (HA)

### 6.2.1 Hard and Soft Performance Measures

In hierarchical organizations, performance evaluation is very important in monitoring, controlling, and motivating employees. Agency models focus on complete contracting that is based on "hard" performance measures. Using Ijiri's [1971] classification, information is defined to be hard if it is constructed in such a way that it is difficult for people to disagree. In general, accounting systems provide relatively hard information. However, not all information available in an organization is hard. Some information may not be included in the accounting system but still may have value to management. For example, manager's subjective judgments are very important for making decisions. They are both imperfect and unverifiable and, hence, disputable. Based on traditional contracting theory, such soft information has no value (see Gjesdal [1981]). This seems inconsistent with empirical observations, i.e., we observe that subjective performance evaluation is widely used in monitoring and motivating employees in organizations. However, the existing literature has not provided an economic rationale for and explanation of the use of this information in governing contract relations.

Obviously, unlike evaluations that depend on formal
reported accounting information, which may be more objective and concrete, subjective judgment is "softer" in the sense that it is unverifiable. The quality of a subjective judgment, including its accuracy, speed, consistency, bias and acceptability, will be influenced by both the supervisor's ability and the information environment, i.e., the kind and accuracy of the information that is available about the subordinate's activities. In addition, a supervisor's opportunistic behaviour may induce moral hazard problems. To preclude this kind of behaviour from our analysis, which will focus on the employee's incentives, we shall assume that the supervisor is wellmotivated to make the evaluation on behalf of the firm's owners. That is, we shall not make a distinction between the supervisor who evaluates the employee's performance and the firm's owner who buys the service from the employee's human assets. Effectively, the firm obtains noncontractible information from the supervisor which evaluates the employee's performance. Hence, we shall not deal with the agency issue with respect to the supervisor.

### 6.2.2 Transferable and Non-Transferable Human Assets

It has long been recognized that a firm's value depends on both its tangible and intangible assets. Human assets are a very important part of a firm's intangible
assets. On the one hand, a firm's normal operation may be seriously influenced, or even discontinued, if it loses a significant component of its human assets. On the other hand, a firm's productivity can be significantly enhanced by effective development of its human assets.

Williamson [1985] classifies the components of human assets as either non-specific or specific. The first type consists of those employee skills that are valuable to a broad set of possible employers. If an employee only provides this kind of skill, then neither the firm nor the employee has a productive interest in maintaining a continuing employment relation. The firm can easily hire a substitute from the market, and the employee can move to alternative employment without loss of productive value. Furthermore, a firm's management will have no incentive to provide investments that develop these non-specific human assets unless there is an explicit contract or some other mechanism that will protect the firm's return on its investment. The second type, which we refer to as firmspecific, includes skills that have value only to a particular employer. This value is intimately associated with the employment relation. Once the employment relation is terminated, the value is lost to both the firm and its former employee. Thus, continuity of the employment relation can be valuable to both the firm and its
employee.
The key difference between non-specific and firmspecific human assets, in Williamson's classification, is the transferability of the assets. As Williamson points out, firm-specific human assets, including idiosyncratic technological experience and idiosyncratic organizational experience such as accounting and data processing conventions, internalization of other complex rules and procedures, etc., have little value to other firms. Hence, a market for such assets does not exist. Consequently, firm-specific human assets are non-transferable.

By definition, a non-specific human asset has value to other firms, so a market demand exists for it. However, the price at which a non-specific human asset can be transferred will be influenced by the observability of the characteristics that determine the asset's value. In particular, the market price of an asset is influenced by the market's ability to access the information that is available for evaluating it, and the price the market would pay for it if that information was common knowledge. If the market has perfect information about the asset, then like any other tradeable commodity, a transfer is not difficult. However, if the market has imperfect or no information, then the market price may not reflect its "true" value. In the extreme, a non-specific asset may
become essentially non-transferable because of lack of publicly observable information about the asset, and it may be significantly undervalued by the market.

In this chapter, we are mainly concerned with firmspecific human assets (FSHA). FSHA provide benefits for long-term employment relations and, as we shall show in following analysis, can result in the use of low-powered incentives for employees in organizations. However, since employees may hold FSHA and NSHA (Non-Specific Human Assets) concurrently, the trading of NSHA must be considered in the examination the trading of FSHA.

The development of human assets is a cumulative process which requires investments from both the firm and the employee. For example, training employees in new skills frequently involves contributions by the firm in terms of tuition fees or the time of skilled employees, whereas the employees being trained must provide personal time or effort. Furthermore, the development process often occurs jointly with daily production activities, i.e., the acquisition of human assets is often obtained directly or indirectly through operating experience. Therefore, human asset acquisition and daily production activities are often positively correlated, i.e., higher current productivity implies greater acquisition of human assets. Of course, the correlation between current pro-
ductivity and human asset acquisition may depend on the employees basic personal skill level.

Finally, employees will bargain over their share of the gains from human capital development when they recontract with the firm. The employees' bargaining power depends on the market condition and many other factors, such as the relative strength of the employee's union and the firm's management.

### 6.3 Model and Analysis

### 6.3.1 Basic Model Elements

We consider a situation in which a risk-neutral firm hires a risk- and effort-averse employee. The analysis covers $T$ (finite) periods, beginning at $t=0$ and ending at $t=T$ with period $t$ referring to interval [ $t-1, t$ ]. The employee's and the firm's utilities with respect to their net return from the employment relation are time-additive with the same time discount coefficient $\gamma,{ }^{2}$

$$
\begin{aligned}
& \text { Employee: } \mathrm{U}^{\mathrm{T}}\left(\tilde{w}_{1}, \ldots, \tilde{\mathrm{w}}_{T}\right)=\sum_{t-1}^{T} \gamma^{t-1} \cdot \mathrm{U}_{t}\left(\tilde{w}_{t}\right) \\
& \text { Firm: } \mathrm{V}^{\mathrm{T}}\left(\tilde{v}_{1}, \ldots .3 .1-1\right) \\
&
\end{aligned}
$$

[^47]where ${ }^{3}$
\[

$$
\begin{equation*}
\mathrm{U}_{t}\left(\tilde{w}_{t}\right)=\mathrm{E}\left[\tilde{w}_{t}\right]-\frac{r}{2} \operatorname{Var}\left[\tilde{w}_{t}\right] \tag{6.3.2}
\end{equation*}
$$

\]

and
$\tilde{w}_{t}=\quad a \quad$ random variable representing the employee's compensation for period t;
$\tilde{v}_{t}=\quad a \quad$ random variable representing the firm's benefit from the employment relation in period $t ;$
$r=$ risk aversion coefficient of the employee.
We view employment as a transaction between the firm and the employee in which the firm acquires the services provided by the employee's human asset. The firm contracts with an employee at the beginning of each period. After contracting, the employee provides his effort $e_{t}$ to the firm's operation with personal cost $C\left(e_{t}\right), e_{t} \in[0$, $+\infty)$, and

$$
\begin{equation*}
C(0)=0 \quad C^{\prime}\left(e_{t}\right)>0 \quad C^{\prime \prime}\left(e_{t}\right)>0 \tag{6.3.3}
\end{equation*}
$$

However, the employee's effort alone cannot create a productive outcome. The firm must provide investment to complement the employee's effort. While the nature of the firm's investment can take a variety of forms, including the provision of a good working environment and access to special production facilities, we assume that the firm's

[^48]investments can be represented by a single aggregate dollar amount.

If the firm invests capital $k_{t}$ (dollar amount) in a zero human asset employee who provides effort $e_{t}$, then the production output created by these inputs is a random variable $\tilde{X}_{t}$. The mean of $\tilde{X}_{t}$ is represented by the following Cobb-Douglas production function

$$
m_{t}\left(k_{t}, e_{t}\right)-\left(a+e_{t}\right)^{b} k_{t}^{1-b} \quad a>0,0<b<1 \quad \text { (6.3.4) }
$$

where $\underline{a}$ and $\underline{b}$ are constant parameters. (6.3.4) implies that as long as $\underline{k}_{t}>0$, even if the employee owns a zero human asset $H^{t-1}$ and provides zero effort, ${ }^{4} \tilde{X}_{t}$ will still have a positive mean $m_{t}$. The variance of $\tilde{X}_{t}$ is assumed constant over time and is denoted by $\sigma^{2}$. This implies that the variances of $\tilde{x}_{t}$ is independent of all $k_{t}$ and $e_{t}$. The employee's human asset decays at a rate of $1-\delta$ percent per period, but is increased by investments made in that asset each period. Let $H_{t}$ and $H^{t}$ be the human asset acquired in period $t$ and the human asset at time $t$, respectively; then

$$
\begin{equation*}
\mathrm{H}^{\mathrm{t}}-\sum_{j=0}^{t} \delta^{t-j} \mathrm{H}_{j} \quad 0<\delta \leq 1 \tag{6.3.5}
\end{equation*}
$$

[^49]In addition, let $A_{t}$ and $B_{t}$ denote firm-specific and nonspecific human assets in $H_{t}$ (FSHA and NSHA), respectively,

$$
\begin{equation*}
H_{t}-A_{t}+B_{t} \tag{6.3.6}
\end{equation*}
$$

and assume the same decay rate for $A_{t}$ and $B_{t}$. Hence, we have

$$
\begin{align*}
& A^{t}=\sum_{j=0}^{t} \delta^{t-j} A_{j}  \tag{6.3.7-1}\\
& B^{t}=\sum_{j=0}^{t} \delta^{t-j} B_{j}  \tag{6.3.7-2}\\
& H^{t}=A^{t}+B^{t} \tag{6.3.7-3}
\end{align*}
$$

A positive human asset $\mathrm{H}^{t-1}$ at the beginning of a period will additively enhance the production output, i.e., the total output will be $X_{t}+H^{t-1}$. In other words, in our model, human assets directly transfer into future productivity without influencing the productive return from the employee's effort $e_{t}$ or the firm's capital investment $k_{t}$. This assumption is for tractability only. In general, the productivity of both $e_{t}$ and $k_{t}$ may be influenced by $H^{t-1}$.

We assume that human asset acquisition is positively correlated with current operation output. This implies that a higher effort contribution will result in both higher current period productivity and higher human asset acquisition. Particularly, assume perfect linear rela-
tionships between $\tilde{\mathrm{A}}_{\mathrm{t}}\left(\tilde{\mathrm{B}}_{\mathrm{t}}\right)$ and $\tilde{X}_{t}{ }^{5}$

$$
\begin{align*}
& \tilde{A}_{t}=\phi_{t} \tilde{X}_{t}  \tag{6.3.8-1}\\
& \tilde{B}_{t}=\Psi_{t} \tilde{X}_{t}  \tag{6.3.8-2}\\
& \tilde{H}_{t}=\left(\phi_{t}+\Psi_{t}\right) \tilde{X}_{t}-h_{t} \tilde{X}_{t} \tag{6.3.8-3}
\end{align*}
$$

where $\phi_{t}$ and $\psi_{t}$ are constant coefficients. ${ }^{6}$ Therefore, in our model, inputs $e_{t}$ and $k_{t}$ contribute to both current and future productivity, as long as the employment relation between the firm and the employee continues. From (6.3.5) through (6.3.8), we have ${ }^{7}$

$$
\begin{equation*}
\tilde{H}^{t}=\sum_{j=0}^{t} \delta^{t-j} h_{j} \tilde{X}_{j} \tag{6.3.9}
\end{equation*}
$$

Throughout the following analysis, except for the benchmark case, we assume that the employee's effort cannot be perfectly observed by the firm through any monitoring device. For example, a supervisor who monitors the employee on behalf of the firm cannot observe the

[^50]employee's actions. This is consistent with the situation in which the employee's effort is complicated and multidimensional, so that its specification and verification are very difficult. Also, effort may not be a one-shot action, but a series of actions continuously provided during a long period. These characteristics of the employee's effort make a first-best efficient contract unattainable.

We consider two regimes for $X_{t}$ (the realization of $\tilde{X}_{t}$ ). In the first regime; we assume $X_{t}$ is a verifiable event and, hence, ex ante contractible. This represents the case where $X_{t}$ is reported by an accounting system, and is auditable. In the second regime, $X_{t}$ is an observable but unverifiable event and, hence, it is ex ante noncontractible but ex post contractible. This represents the more common case -- the individual's performance evaluation is not reported by the accounting system. Instead, the employee's performance is evaluated on the basis of his supervisor's subjective judgment, which is both unverifiable and imperfect. For simplicity, we also use $X_{t}$ to denote the subjective evaluation of the employee's performance.

Given the assumption of linear relationships between $\tilde{X}_{t}$ and $\tilde{H}_{t}\left(\tilde{A}_{t}, \tilde{B}_{t}\right)$, the realization of these variables have the same characteristics as $X_{t}$. That is, they are observ-
able, and they are verifiable only if $X_{t}$ is verifiable. We choose this setting to avoid the "learning problem" analyzed by Harris and Holmstrom [1982], so that we can concentrate on incentive issues.

The key feature of FSHA is its non-transferability. $A^{t}$ has a positive value if, and only if, the employment relation continues. Since $A^{t}$ has no value to other firms, there is no market in which to trade it. This implies that any trading of $A^{t}$ can only occur between the original employer and the employee. In contrast, NSHA is transferable. It has value to other firms, so there will be a market in which to trade $B^{t}$. Therefore, trading a human asset must include two parts that are governed by two different trading mechanisms. The trading of NSHA is governed by the market mechanism, while the trading of FSHA is governed by the bargaining processes within the firm.

### 6.3.2 Trading NSHA and FSHA

## Trading NSHA in the market

Consider a case in which an employee, who owns B units of a NSHA, chooses not to continue his employment with a firm but to go to the market. There are other employees seeking to sell their NSHAs, which are substitutable for the employee's asset. There are also
firms in the market trying to buy the services that can be provided by his NSHA. We shall not deal with the problem of how a firm chooses from among the alternative potential employees, but consider only the transaction of $B$ owned by this particular employee. A central question is how to price the asset to be traded. This will determine who captures the gain from the transaction.

First, we consider the case in which the value of $B$ is observable to all firms. In this case, the trading price of $B$, like any ordinary commodity, is determined by market demand and supply. A market equilibrium can be summarized by the implicit bargaining power of the employee, denoted $p_{m}(B) \in[0,1]$, which represents the percentage of NSHA value that can be obtained by the employee when contracting with a firm. In general, $p_{m}$ depends on the nature of the market for B. A stronger market demand relative to supply will result in a higher value of $p_{m}$. In the extreme, $p_{m}=1$ implies that the employee dominates the bargaining and captures all the gain from the transaction and leaves the firm with zero profit, while $p_{m}=0$ represents the other extreme. Given $p_{m}$, the price for $B$ simply equals $P=p_{m} B$. In the case in which the employing firm and the employee have private information about $B$ (which the remainder of the market does not have), the analysis would be more complex. This
information asymmetry would appear to weaken the employee's bargaining position, but without a more formal analysis it is impossible to assess its impact. For simplicity, we assume that all firms who are interested in hiring the employee have the same information about the NSHA as the original firm. Thus the market price for $B$ will be $p_{m}$ B.

## Trading FSHA between a firm and an employee

While trading NSHA is determined by a market mechanism, trading FSHA can only occur between a particular firm and its employee. The price the firm will pay for an FSHA with value $A$ is determined by the bargaining power the employee holds in contracting with the firm. Let $p(A)$ denote the bargaining power of the employee. We assume that $p(A)$ is independent of $p_{m}(B)$ and vice versa. The bargaining result is affected by: (i) the status quo position of each party when bargaining; (ii) the bargaining power each party holds; and (iii) the bargaining process. We focus on (i) and (ii), and assume that (iii) is a Nash bargaining process. Different bargaining processes may alter the particular bargaining result, but we believe that the general conclusions would not be altered.

In general, an employee holds both FSHA and NSHA when he comes to contract. Then the status quo position of the
employee is equal to the market price for his NSHA. Thus, we have following conclusion.

Proposition 6.1: Let $A$ and $B$ be an employee's FSHA and NSHA at the time of contracting.
Assume the employee's bargaining power is $p_{m}(B)$ in the market, and $p(A)$ in the firm. If $p(A)$ and $p_{m}(B)$ are independent with each other, then a Nash bargaining solution of the price that the firm will pay for $A$ and $B$ is $P=p_{m}(B) B+p(A) A$.

### 6.3.3 One-Period Model

We now examine a one-period model under the setting described above. Let $t=0$ denote the starting point and $t=1$ the ending point of the period. The event sequence is depicted in Figure 6-1.

| $\mathrm{t}=0$ |  | $\begin{array}{r} 1 \\ + \end{array}$ |
| :---: | :---: | :---: |
| Contract | Investment | Output |
| $\mathrm{w}_{1}$ | $e_{1}, k_{1}$ | $\mathrm{H}^{0}+\mathrm{X}_{1}$ |
| $\begin{aligned} & \mathrm{A}^{0} \geq 0 \\ & \mathrm{~B}^{0} \geq 0 \end{aligned}$ |  |  |

Figure 6-1: Event Sequence for One-Period Model

The employee begins the period with previously acquired human assets $A^{0}$ and $B^{0}$ (the source of that acquisition is not modelled in the one-period model). Observe that in a one-period model, human asset acquisition during the
period has no value to either the firm or the employee since the termination of all economic activity at $t=1$ is common knowledge. Hence, $A_{1}$ and $B_{1}$ are irrelevant to the analysis. We assume that $A^{0}$ and $B^{0}$ are observed by the firm before contracting. This implies that it is common knowledge that an employment relation will bring the firm an expected gain of $A^{0}+B^{0}$ in addition to the expected output resulting from the employee's current production effort. Our main concern is the case in which both $e_{1}$ and $X_{1}$ are non-contractible events, so that a contingent contract is infeasible. However, for comparison purposes, we first derive the usual first-best and second-best contracts in a Nash bargaining setting. We restrict our analysis to wage contracts that are linear functions of the realized output $\mathrm{X}_{1} .^{8}$ To simplify notation, in this section we drop the time subscripts from all variables. Also, since the trading prices for $A^{0}$ and $B^{0}$, which equal $p_{m} B^{0}+p^{0}$, have already been given in the above discussion, and are not influenced by incentives with respect to inputs $e$ and $k$, we exclude them from most formulas. For a complete expression, they should be added back where it is

[^51]necessary.
\[

$$
\begin{gather*}
\tilde{\mathrm{X}} \sim \mathrm{~N}\left(m, \sigma^{2}\right) \quad m=(a+e)^{b} k^{1-b}  \tag{6.3.10}\\
\tilde{\mathrm{~W}}(\tilde{\mathrm{X}})=\alpha \tilde{\mathrm{X}}+\beta \tag{6.3.11}
\end{gather*}
$$
\]

Given this contract $\tilde{W}(\tilde{X})$ (compensation from current inputs $k$ and e), the utilities of the employee and the firm (excluding the human asset price) are, respectively,

$$
\begin{aligned}
U(\tilde{w})=\alpha m+\beta-C(e)-\frac{1}{2} r \alpha^{2} \sigma^{2} & (6.3 .12-1) \\
\mathrm{V}(\tilde{\mathrm{X}}-\tilde{\mathrm{w}})=m-(\alpha m+\beta)-k & (6.3 .12-2)
\end{aligned}
$$

## The First-best Contract

Contrary to our assumption, let $e$ and $k$ be observable and verifiable by both parties so that they are contractible events. Since $e$ and $k$ do not influence the uncertainty about $\tilde{\mathrm{x}}, \mathrm{e}^{*}$ and $\mathrm{k}^{*}$ should be chosen to maximize

$$
m(k, e)-C(e)-k=(a+e)^{b} k^{1-b}-C(e)-k(6.3 .13)
$$

The first-order conditions for this maximization are

$$
\begin{array}{rlr}
b(a+e)^{b-1} k^{1-b} & =c^{\prime}(e) & (6.3 .14-1) \\
(1-b)(a+e)^{b} k^{-b} & =1 & (6.3 .14-2)
\end{array}
$$

This implies that $e^{*}$ is the unique solution of the equation

$$
\begin{equation*}
C^{\prime}\left(e^{*}\right)=b(1-b)^{\frac{1}{b}-1} \tag{6.3.15-1}
\end{equation*}
$$

and $\mathrm{k}^{*}$ is uniquely determined by

$$
\begin{equation*}
k^{*}=(1-b)^{\frac{1}{b}}\left(a+e^{*}\right) \tag{6.3.15-2}
\end{equation*}
$$

Contract coefficients $\alpha$ and $\beta$ must reflect the bargaining power of the two parties as well as their aversion to risk. Since the firm is assumed risk-neutral and the employee is risk averse, it is optimal to have the firm bear all the risk and pay the employee a flat wage. In other words, the first-best contract should set $\alpha=0$. Alternatively, $\alpha$ and $\beta$ can be found by solving the following problem. ${ }^{9}$
${ }^{9}$ Whether the bargaining power used in (6.3.16) should be $p_{m}$ or $p$ depends on the information the market holds about the production function and the employee's productivity. If the market has no such information so that pricing of the expected production is difficult, then the market influences on the bargaining of the expected production gain disappear. In such cases, bargaining power $p$ should be used here.

$$
\begin{align*}
& \underset{\alpha, \beta}{\operatorname{Max}} U^{D_{m}} V^{1-D_{m}}  \tag{6.3.16}\\
& \text { s.t. } e=e^{*} \quad k=k^{*}
\end{align*}
$$

The solution to this problem is summarized in the following lemma.

Lemma 6.2: The solution of (6.3.16) satisfies
(i) $\alpha=0$
(ii) $\begin{aligned} & \beta \text { is chosen such that } U=p_{m}(U+V) \text { and } \\ & V=\left(1-p_{m}\right)(U+V) \cdot 10\end{aligned}$

Note that in Lemma 6.2

$$
\begin{equation*}
U+V=m-k-C(e) \tag{6.3.17}
\end{equation*}
$$

which is the total gain from the transaction. Lemma 6.2 shows that the first-best solution provides efficient risk sharing, and the parties share the gain based on their bargaining power. Note that when $\alpha=0$, then $\beta=U$, and the wage of the employee is

$$
W=p_{m} \mathrm{~B}^{0}+p \mathrm{~A}^{0}+p_{m}(m-k-C(e))+C(e)
$$

[^52]Second-best contract when both $e$ and $k$ are non-con-

## tractible events

Now return to our assumption that $e$ is unobservable by the firm, but let $X$ be contractible. This may occur when the employee's contribution to the firm's income is formally reported by an accounting system. In that case, $X$ is verifiable to a third party who has authority to enforce the contract if there is a dispute when the contract is executed. This is a classical agency problem; but with a more general bargaining process. Note that the classical agency literature focuses on the efficiency aspects of contracting so that it can simply assume one contracting party holds all bargaining power without reducing the generality of its results. Hence, all gains from the transaction go to one party and the other party gets only its reservation utility, which is exogenously given. In this paper, we are dealing with an agency with a bargaining process; therefore, efficiency is not the only aspect we consider. Another important issue in our analysis is how the trading gains are allocated.

In the current case, we assume that both $e$ and $k$ are non-contractible events. The reason e may be non-contractible is the same as in the traditional agency theory. The reason $k$ may be non-contractible is that the firm's investment in a particular employee may be difficult to
separate from its other investments.
The optimal wage contract $\tilde{W}(\tilde{X})$ is the solution to the following problem: ${ }^{11}$

$$
\begin{array}{cll}
\{\alpha, \beta, k, e\} & U^{\mathcal{D}_{\mathrm{n}}} \mathrm{~V}^{1-\mathcal{P}_{\mathrm{n}}} & (6.3 .18-1) \\
\text { s.t. } & e \in \arg \max \mathrm{U} & (6.3 .18-2) \\
& k \in \arg \max \mathrm{~V} & (6.3 .18-3)
\end{array}
$$

where $U$ and $V$ are defined by (6.3.12). The solution to this problem is characterized in the following lemma.

Lemma 6.3: The second-best solution of problem (6. 3.18) when both $e$ and $k$ are non-contractible events has the following characteristics:
(1) Both e and $k$ are non-trivial functions of $\alpha$,
(6.3.70-1)

$$
k^{I I}=(1-\alpha)^{\frac{1}{b}}(1-b)^{\frac{1}{b}}\left(a+e^{I I}\right)
$$

(6.3.-19-2)

$$
c^{\prime}\left(e^{I I}\right)=\alpha(1-\alpha)^{\frac{1}{b}-1} b(1-b)^{\frac{1}{b}-1}
$$

(2) $U=p_{m}(U+\nabla), \quad \nabla=\left(1-p_{m}\right)(U+V) ;$ and
(3) $\alpha$ is determined by

$$
\begin{equation*}
\frac{d(U+V)}{d \alpha}=0 \tag{6.3.20}
\end{equation*}
$$

and $\alpha>0$, which deviates from the

[^53]
## efficient risk-sharing solution $\alpha=$ 0.12

Lemma 6.3 shows that when both $e$ and $k$ are non-contractible events, the second-best contract deviates from the efficient arrangement both in the investment levels and risk sharing. The former claim is based on the comparison of (6.3.19) with (6.3.15). Since $\alpha(1-\alpha)^{1 / b-1}<1$, $e^{I I}<e^{*}$ is obvious, and this, in turn, implies $k^{I I}<k^{*}$. The latter is based on the values of $\alpha$ derived in Lemmas 6.2 and 6.3. An efficient arrangement should have the firm to bear all the risk, i.e., $\alpha=0$, while in order to induce an $e^{I I}>0$, it must set $\alpha>0$. This result is established by classical agency theory, we merely provide a different setting.

## Second-best contract when $k$ is contractible

Now we consider the case in which input $k$ is contractible. This is possible, for example, if $k$ consists of separable investments that can be verified. The problem is the same as (6.3.18) except that constraint (6.3. 18-3) is eliminated. Now $k$ can be chosen to maximize $\mathrm{U}+\mathrm{V}$ rather than $V$. The solution of (6.3.18-1) subject to

[^54](6.3.18-2) is characterized in the following lemma.

## Lemma 6.4: The second-best solution of problem (6.3.18-1), when $k$ is a contractible event, has the following characteristics:

(1) Both e and $k$ are non-trivial functions of $\alpha_{p}$

$$
\begin{align*}
k^{I I} & =(1-b)^{\frac{1}{b}}\left(a+e^{I I}\right)  \tag{6.3.21-1}\\
C^{\prime}\left(e^{I I}\right) & =\alpha b(1-b)^{\frac{1}{b}-1} \tag{6.3.21-2}
\end{align*}
$$

(2) $\beta$ is chosen such that $U=p_{\square}(U+V), V$ $=\left(1-p_{m}\right)(U+\nabla)$;
(3) $\alpha$ is determined by (6.3.20), and $\alpha>0$ deviates from the efficient risk-sharing solution $\alpha=0$.

Lemma 6.4 shows that when one party's input is a contractible event and the other is not, then the secondbest contract still deviates from the efficient contract. This is obvious because of $\alpha<1$ so that $\mathrm{e}^{\mathrm{II}}<\mathrm{e}^{*}$ and $\mathrm{k}^{\text {II }}<$ $k^{*}$. However, the distortion of the investment level and risk sharing are different than when both $e$ and $k$ are noncontractible events. This can be seen from a comparison of (6.3.21) with (6.3.19). However, since the bargaining power assignments in these two cases are given exogenously (in general, they could be different), we cannot conclude whether each party or both will be better off. The only thing of which we are sure is that the total trading gain
$\mathrm{U}+\mathrm{V}$ is strictly larger when k is contractible than when $k$ is a non-contractible event.

## $X$ and $e$ are Non-contractible Events

Now we examine the case in which e is unobservable and $X$ is non-contractible. In this case, the wage for inducing an employment relation can only be a constant equal to the employee's asset price plus his share of the expected gain from production.

$$
W=p_{m} B^{0}+p A^{0}+p_{m}[m-C(e)-k]+C(e)(6.3 .22)
$$

where $e$ and $k$ are the equilibrium input levels. Given a constant wage, the employee has no incentive to provide an effort level higher than the minimum level e=0. Recognizing this, the firm chooses $k$ to maximize its expected utility given the employee's lowest effort input. Again, there is a difference between the case in which $k$ is contractible and the case in which $k$ is a non-contractible event. We summarize the results in Lemma 3.5.

Lemma 6.5: When $e$ and $x$ are non-contractible events, then the employee's wage can only be a constant and the expected effort level the employee will provide is zero. The capital investment level, independent of the contractibility of $k$, is given by

$$
\begin{equation*}
k^{0}=(1-b)^{\frac{1}{b}} a \tag{6.3.23}
\end{equation*}
$$

The intuition behind Lemma 6.5 follows. When $k$ is a contractible event, then the expected gain is ( $m(k, 0)$ k). The employee and the firm share this gain based on the bargaining power $\mathrm{p}_{\mathrm{m}}$ and hence, k can be chosen to maximize this gain. This results in (6.3.23). When $k$ is a non-contractible event, let $\hat{k}$ be the employee's ex ante belief about $k$ (in equilibrium, $\hat{\mathbf{k}}=\mathbf{k}$ ), then the perceived gain is $(\mathrm{m}(\hat{k}, 0)-\hat{k})$. Since the firm is the residual claimant, for any actual investment $k$, its actual gain is $(m(k, 0)-k)-p_{m}(m(\hat{k}, 0)-\hat{k})$. For any $\hat{k}$ set in the contract, a self-enforcing constraint for the firm is

$$
\begin{equation*}
k \in \operatorname{argmax} m(k, 0)-k \tag{6.3.24}
\end{equation*}
$$

which results in (6.3.23) again.

## Summary of the one-Period model

We derive the following conclusions from the above analysis. First, in a one-period model, a contingent contract (high-powered incentives) is essential if the agent is to be induced to provide more than the minimal effort level. This is the core of classical agency theory. The cost of this incentive is inefficient risk sharing -- the risk averse employee must bear risk that should be transferred to the firm when there is no incen-
tive problem. Second, high-powered incentives require verifiable performance measures. If such measures are available from the accounting system, then contingent compensation may be observed. For instance, we observe that a firm's top executives, including the top managers of its key responsibility centres (e.g., divisions), are frequently compensated on the basis of contracts that are contingent on firm or divisional financial performance measures reported by the accounting system. We also observe that employees operating at the lowest level of an organization may be paid on a piece-rate or sales commission basis if the accounting system monitors and reports individual production or sales information. In contrast, the accounting system does not report individual performance measures for many of the firm's employees and the firm typically compensates them with a wage that is set at the start of the period. This raises the question of whether these employees provide only a minimal level of effort and, if they do not, where do their incentives come from. A one-period model says YES to the first question and that is inconsistent with the real world observations. Therefore, we now consider multi-period models.

### 6.3.4 Two-Period Model

## Model Elements

We now extend the basic model to two periods. We assume that at $t=1$ the firm and the employee have an opportunity to sign a second period contract. If contract renewal is successful, then the employment relation continues. Otherwise, both the employee and the firm go to the market to find new partners. The event sequence is depicted in Figure 6-2.

| $\begin{aligned} & 0 \\ & + \end{aligned}$ |  | 1 |  | 2 |
| :---: | :---: | :---: | :---: | :---: |
|  |  | + |  | + |
| $\begin{aligned} & \text { Con } \\ & \mathrm{w}_{1} \end{aligned}$ | $\begin{aligned} & \text { Invest } \\ & e_{1} k_{1} \end{aligned}$ | Outcome $X_{1}$ |  |  |
|  |  | $\begin{gathered} \text { Contract } \\ w_{2} \end{gathered}$ | Invest $e_{2} k_{2}$ | Outcome $\mathrm{X}_{2}$ |
| $A^{0}$ | $A_{1}=\phi_{1} \mathrm{X}_{1}$ | $A^{1}=\delta A^{0}+A_{1}$ |  |  |
| $\mathrm{B}^{0}$ | $\mathrm{B}_{1}=\boldsymbol{\Psi}_{1} \mathrm{X}_{1}$ | $\mathrm{B}^{1}=\delta \mathrm{B}^{0}+\mathrm{B}_{1}$ |  |  |

Figure 6-2: Event Sequence for Two-Period Model

The key difference between a two-period model and a one-period model, in our setting, is that, along with the realization of $X_{1}$, human assets $A^{1}$ and $B^{1}$ are acquired before $t=1$. $A^{1}$ has value, if, and only if, the employment relation continues in the second period. On the other hand, $B^{1}$ has value in both the firm and the competitive market. Based on the discussion of subsection 6.3.2, we assume that the firm and the market have the same informa-
tion about $B^{1}$ so that the price for it will be $p_{m} B^{1}$.

## First-best benchmark contract for the two-period

## model

We first derive the efficient contract if all $e$ and $k$ are contractible events. This is done by considering the second period contract first. Since the economic activities will end at $t=2$, any human assets acquired in the second period are irrelevant. The first-best wage contract in the second period depends on whether the parties are able to commit to a multi-period contract (i.e., longterm contract). If they can, then $w_{2}$ will be independent of the realized value of $A^{1}$ and $B^{1}$-- the firm will bear all risk. However, if multi-period commitments (by either the firm and the employee) are not possible, then $w_{2}$ will be a random variable when viewed from the perspective of $t=0$. Since the variance of $\tilde{X}$ is due to an additive noise term, the riskiness of $w_{2}$ will be independent of the first-period input decisions. ${ }^{13}$ We shall assume in our analysis that long-term commitment is impossible (the justification will appear later). Hence, contracting in the second period is exactly the same as in the one-period model: $e_{2}=e^{*}$ and $k_{2}=k^{*}$ given by (6.3.15). Given $A^{1}$ and

[^55]$B^{1}$ realized at $t=1$, the employee's second period wage is equal to
\[

$$
\begin{aligned}
W_{2}\left(A^{1}, B^{1}\right) & -p_{m} B^{1}+p A^{1} \\
& +p_{m}\left[m_{2}^{*}-C\left(e^{*}\right)-k^{*}\right]+C\left(e^{*}\right)(6.3 .25)
\end{aligned}
$$
\]

which can be obtained through Lemma 6.2 with the following utilities

$$
\begin{align*}
& \mathrm{U}_{2}^{+}-\alpha_{2} m_{2}+\beta_{2}-\frac{I}{2} \alpha_{2}^{2} \sigma^{2}-\mathrm{C}\left(e_{2}\right)  \tag{6.3.26-1}\\
& \mathrm{V}_{2}^{+}=m_{2}-\left(\alpha_{2} m_{2}+\beta_{2}\right)-k_{2} \tag{6.3.26-2}
\end{align*}
$$

with $\alpha_{2}=0$ and $\beta_{2}=p_{m}\left[m_{2}^{*}-C\left(e^{*}\right)-k^{*}\right]$. Note that $U_{2}{ }^{+}$and $V_{2}^{+}$are not the total second period utilities but only the utilities derived from the second period inputs. The total utilities for the employee and the firm, evaluated at $t=1$, are $p_{m} B^{1}+p A^{1}+U_{2}^{+}$and $\left(1-p_{m}\right) B^{1}+(1-p) A^{1}+V_{2}^{+}$, respectively. The differences come from the human asset enhancement. Let the utilities resulting from the first period inputs (evaluted at $t=0$ ) be

$$
\begin{align*}
& \mathrm{U}_{1}^{+}=\alpha_{1} m_{1}+\beta_{1}-\frac{r}{2} \alpha_{1}^{2} \sigma^{2}-\mathrm{C}\left(e_{1}\right) \\
& \mathrm{V}_{1}^{+}=m_{1}-\left(\alpha_{1} m_{1}+\beta_{1}\right)-k_{1} \tag{6.3.27-2}
\end{align*}
$$

$$
(6.3 .27-1)
$$

where $e_{1}$ and $k_{1}$ are the equilibrium inputs. Then the two period expected utilities, evaluated at $t=0$, are

$$
\begin{align*}
\mathrm{U}^{2} & -p_{m} B^{0}+p A^{0}+U_{1}^{+} \\
& +\gamma\left(E\left[p_{m} B^{1}+p A^{1} \mid e_{1}, k_{1}\right]+U_{2}^{+}\right) \\
& -\frac{\gamma r}{2} V^{2}\left[p_{m} B^{1}+p A^{1}\right] \\
V^{2} & =\left(1-p_{m}\right) B^{0}+(1-p) A^{0}+V_{1}^{+} \\
& +\gamma\left(E\left[\left(1-p_{m}\right) B^{1}+(1-p) A^{1} \mid e_{1}, k_{1}\right]\right. \\
& \left.+V_{2}^{+}\right)
\end{align*}
$$

(6.3.28-2)

These expected utilities highlight the fact that the employee faces wage risk at $t=0$ and he cannot avoid this risk when contracts only hold for a single period. The risk comes from the negotiation at $t=1$, when the firm and the employee contract for $w_{2}$ based on the information they have at that point. Since $A^{1}$ and $B^{1}$ are random variables, $\mathrm{w}_{2}$ is also a random variable. However, this risk does not affect the bargaining that takes place at $t=0$ because the second period wage risk is independent of the choice of $e_{1}$ and $k_{1}$ (due to the additive structure of the production and utility functions).

Based on above discussion, the first period contract and inputs can be determined by maximizing the sum of the first period production and human asset acquisition. Assume the employee and the firm believe that the employment relation will continue in the second period. Consequently, $e_{1}$ and $k_{1}$ are chosen by solving following problem,

$$
\begin{align*}
& \operatorname{Max}_{1}, k_{1} m_{1}+\gamma E\left[\mathrm{H}_{1}\right]-k_{1}-\mathrm{C}\left(e_{1}\right) \\
\Leftrightarrow & \operatorname{Max}_{1}, k_{1}\left(1+\gamma h_{1}\right) m_{1}-k_{1}-\mathrm{C}\left(e_{1}\right) \tag{6.3.29}
\end{align*}
$$

where $h_{1}=\phi_{1}+\Psi_{1}$ is the human asset acquisition rate in the first period. The term $\gamma E\left[H_{1}\right]$ is the discounted expected value of the incremental production in the second period resulting from the first period's inputs. The solution to this problem is characterized by

$$
\begin{array}{rlr}
k_{1}^{*} & =\left(1+\gamma h_{1}\right)^{\frac{1}{b}}(1-b)^{\frac{1}{b}}\left(a+e_{1}^{*}\right) & (6.3 .30-1) \\
c^{\prime}\left(e_{1}^{*}\right) & =\left(1+\gamma h_{1}\right)^{\frac{1}{b}} b(1-b)^{\frac{1}{b}-1} & (6.3 .30-2)
\end{array}
$$

It is obvious that $k_{1}^{*}>k^{*}$, and $e_{1}^{*}>e^{*}$. The first period wage contract is the same as in a one-period model given in Lemma 6.2 except $k^{*}$ and $e^{*}$ should be replaced by $k_{1}^{*}$ and $e_{1}^{*}$ given by (6.3.30). That is, $w_{1}=p_{m} B^{0}+\mathrm{pA}^{0}+p_{m}\left(U_{1}^{+}+\right.$ $\left.V_{1}{ }^{+}\right)+C\left(e_{1}^{*}\right)$. Note that from (6.3.13)-(6.3.15), $\mathrm{k}^{*}$ and $e^{*}$ maximize $U+V$. Thus, from a single period point of view, $\mathrm{k}_{1}{ }^{*}$ and $\mathrm{e}_{1}^{*}$ are over-invested and $\mathrm{U}_{1}{ }^{+}+\mathrm{V}_{1}^{+}<\mathrm{U}+\mathrm{V}$. In addition, it is possible that the first period wage in a two-period model is less than in a one-period model, even if the employee's effort level is higher in the former case. The intuition for these differences is that in a two-period model, part of the first period investment is made because of the return that will be received in the
second period. That is, the payoffs from these investments, to both the firm and the employee, are deferred to the future period.

## second-best contracts when $k$ is a non-contractible

## event

If all $e$ and $k$ are non-contractible events, but $X_{i}$ (i=1,2) is contractible, then, classical agency contracts apply. As in the first-best case, we assume that only short-term (one-period) contracts are possible. Consequently, the second period contract is the same as in the one-period model and the first period contract must take into account the effects of human asset acquisition.

## Lemma 6.6: When both $e_{1}$ and $k_{1}$ are non-con-

 tractible events, the second-best first period contract in a two-period model is characterized by(1) both $e_{1}$ and $k_{1}$ are non-trivial functions of $\alpha$,

$$
\begin{align*}
k_{1}= & {\left[1-\alpha_{1}+\gamma\left(h_{1}-\theta_{1}\right)\right]^{\frac{1}{b}}(1-b)^{\frac{1}{b}} } \\
& \left(a+e_{1}\right) \tag{6.3.31-1}
\end{align*}
$$

$$
C^{\prime}\left(e_{1}\right)=\left(\alpha_{1}+\gamma \theta_{1}\right)\left[1-\alpha_{1}+\gamma\left(h_{1}-\theta_{1}\right)\right]^{\frac{1}{b}-1}
$$

$$
\begin{equation*}
b(1-b)^{\frac{1}{b}-1} \tag{6.3.31-2}
\end{equation*}
$$

where $\theta_{1}=\phi_{1} p+\phi_{1} p_{n}$ represents the employee's total bargaining power over his acquired human assets;
(2)

$$
\begin{aligned}
& U_{1}^{+}=p_{n}\left(U_{1}^{+}+V_{1}^{+}\right), \text {and } \nabla_{1}^{+}=\left(1-p_{m}\right)\left(U_{1}^{+}\right. \\
& \left.\nabla_{1}^{+}\right):
\end{aligned}
$$

(3) $\alpha$ is determined by (6.3.20), and $\alpha>0$ deviates from the efficient risk-sharing solution $\alpha=0$. Here,

$$
\begin{equation*}
\mathrm{U}_{1}^{+}+\mathrm{V}_{1}^{+}=\left(1+\gamma h_{1}\right) m_{1}-k_{1}-\mathrm{C}\left(e_{1}\right)-\frac{r}{2} \sigma^{2}\left(\alpha+\gamma \theta_{1}\right)^{2} \tag{6.3.32}
\end{equation*}
$$

is the total gain from contracting.

A comparison of the results of Lemma 6.6 with Lemma 6.3 shows that the bargaining power in the second period influences the input level in the first period of a twoperiod model. However, an analytical comparison of the levels of $k$ and $e$ in the two periods is complex because both $k$ and $e$ are non-trivial functions of $\alpha$, but the value of $\alpha$ is different in the two periods. The only solid conclusion we obtain here is that the second-best contract is different if the expected future benefits of human assets are taken into account.

The second-best contract when $k_{1}$ is a contractible

## event

If $k$ is verifiable, then as in a one-period model, the contract will be different from the contract given in Lemma 6.6. The only difference is that (6.3.31) is re-
placed by

$$
\begin{gathered}
k_{1}^{I I}=\left(1+\gamma h_{1}\right)^{\frac{1}{b}}(1-b)^{\frac{1}{b}}\left(a+e_{1}^{I I}\right) \quad(6.3 .33-1) \\
C^{\prime}\left(e_{1}^{I I}\right)=\left(\alpha+\gamma \theta_{1}\right)\left(1+\gamma h_{1}\right)^{\frac{1}{b}-1} b(1-b)^{\frac{1}{b}-1}(6.3 .33-2)
\end{gathered}
$$

where $k_{1}{ }^{11}$ is chosen to maximize the total gain rather than the firm's first period utility. In other words, when we solve (6.3.18), the constraint (6.3.18-3) should be taken off as in the one-period model. The differences between (6.3.33) and (6.3.31) are obvious although we cannot simply determine each party's preference over these two contracts for the reason given before. However, it is clear that contracting on $k$ does provide an opportunity to improve contracting efficiency.

## $X_{i}$ and $e_{i}$ are Non-contractible Events

If $X_{i}$ and $e_{i}$ are non-contractible events, then any ex ante contract can only be a constant. In this subsection, we examine the contracting behaviour in these cases.

## Second Period Contracting

In the above analysis, we assumed that the employment relation will be continued in the second period. Now we determine the conditions under which the firm and the
employee have an incentive to continue the employment relation given the opportunity to renew their contract at $t=1$. Observe that, given common knowledge that the employment relation will be terminated at $t=2$ and that the second period performance evaluation $X_{2}$ cannot enter into the revised wage contract $w_{2}$, the employee, as in a oneperiod model, has no incentive to provide effort $\mathbf{e}_{2}$ greater than zero. Thus the firm correspondingly invests $k_{2}=k^{0}$ which is specified by (6.3.23). The employee's current productivity (i.e., his incremental productivity from current effort) is identical to that of any other employee the firm can hire in the market (such employment will last for one period only). However, continuing the employment relation is strictly Pareto superior to terminating it as long as the firm-specific human asset $A^{1}$ is positive. This asset will increase the second period's productivity from $B^{1}+m^{0}$ to $A^{1}+B^{1}+m^{0},^{14}$ where $m^{0}=m\left(k^{0}\right.$, $0)$ is the current mean productivity created by the employee's lowest effort. Observe that even if the firm can contract with a new employee with the same $B^{1}$, it will lose its share of a valuable asset $A^{1}$ and repeat all the results of the one-period model in the second period. Similarly the employee will be better off if the employ-

[^56]ment relation continues because he can at most get $p_{m}\left(B^{1}+\right.$ $\mathrm{m}^{0}-\mathrm{k}^{0}$ ) in the market, while bargaining with his current employer permits him to obtain a share of the benefits from $A^{1}$ (as long as $p$ is positive). The above discussion implies that it is Pareto superior for the two parties to continue the employment relation rather than to terminate it.

Now consider what happens when the firm and the employee come to the bargaining table at $t=1$, with $A^{1}, B^{1}>$ 0. Observe that after $A^{1}$ and $B^{1}$ have been realized, the status quo position of the employee at $t=1$ is $p_{m}\left(B^{1}+m^{0}-\right.$ $k^{0}$ ), the sum of the price if he goes to the market with his NSHA $B^{1}$ and his share of expected production outcome. On the other hand, the status quo position of the firm is $\left(1-p_{m}\right)\left(B^{1}+m^{0}-k^{0}\right)$, the net return from hiring a new employee with $B^{1}$ from the market. Let $p$ represent the bargaining power the employee holds at the bargaining table. The following proposition summarizes the above discussion and the Nash bargaining equilibrium for the second period wage contract.

> Proposition 6.7: ${ }^{15}$ Assume that after the first-period, the firm and the employee have an opportunity to sign a second period employment

[^57]contract. Contracting is Pareto superior to not continuing the relation if, and only if, the firm specific human asset $A^{1}$ realized at the end of the first period is strictly positive and neither party has all of the bargaining power, i.e., $0<p<1$. Assume $A^{1}>0$ and $B^{1} \geq 0$, and the market price for $B^{1}$ is $p_{1} B^{1}$. Then, a Nash bargaining solution $W_{2}$ is given by
\[

$$
\begin{equation*}
w_{2}=p_{m}\left(B^{1}+m^{0}-k_{2}\right)+p A^{1} \tag{6.3.34}
\end{equation*}
$$

\]

where $k_{2}=k^{0}$ is given by Lemma 6.5.

Proposition 6.7 assures that continuing employment is an equilibrium strategy for both parties, and the equilibrium second period wage contract is simply a sharing of the expected gain based on the two parties' bargaining power. Thus, a long-term relationship need not be guaranteed by a long-term contract even if a long-term contract is available. This is particularly significant when the contracting is incomplete. As Williamson [1985] points out, for long term contracts executed under conditions of uncertainty, a complete specification of the contract is apt to be prohibitively costly, if not impossible. In addition, Macneil [1978] states:

Two common characteristics of long-term contracts are the existence of gaps in their planning and the presence of a range of processes and techniques used by contract planners to create flexibility in lieu of either leaving gaps or trying to plan rigidly.

Our results are also consistent with Alchian and Demsetz's
[1972] claim that
... neither the employee nor the employer is bound by any contractual obligations to continue their relationship. Long term contracts between employer and employee are not the essence of the organization we called a firm.

Therefore, although long term contracts offer the apparent advantage of reduced bargaining costs and long-term commitment, they may be too expensive due to other contracting costs. Based on Proposition 6.7, in our model, a short-term contract can provide incentives to maintain a long-term relationship and to provide more than minimal effort. Hence, when verifiable performance measures are unavailable, short-term employment contracts with contract renewal processes are an important incentive mechanism within organizations.

## First Period Investment Decisions

Under the Nash bargaining solution stated in (6.3
.34), the employee and the firm share the second period gain resulting from the first-period acquisition of human assets. The sharing of the gain from the NSHA and the expected output depends on market forces, whereas the sharing of the gain from the FSHA depends on the employee's bargaining power with the firm. The employee receives his market value plus $p$ percent of the gain from

FSHA. This share of the gain is a reward for continuing the employment relation. It also provides incentives for the employee to choose a first period effort level greater than the minimum effort. This is shown in the following proposition.

Proposition 6.8: Assume both e and $k$ are noncontractible events. If at $t=0$ the firm and the employee anticipate the second period's bargaining result, then the employee will have incentive to provide effort $e_{1}>0$ which is determined by

$$
c^{\prime}\left(e_{1}^{* *}\right)-\gamma \theta_{1}\left[1+\gamma\left(h_{1}-\theta_{1}\right)\right]^{\frac{1}{b}-1} b(1-b)^{\frac{1}{b}-1} \quad \text { 1) }
$$

Correspondingly, the firm will invest $\mathbf{k}_{\mathbf{1}}{ }^{* *}$ such that

$$
k_{1}^{* *}-\left[1+\gamma\left(h_{1}-\theta_{1}\right)\right]^{\frac{1}{b}}(1-b)^{\frac{1}{b}}\left(a+e_{1}^{* *}\right) \quad(6.3 .35-2)
$$

Proposition 6.8 has the following implications. First, anticipation of second period contract renewal provides incentives for the employee to provide more than minimal effort in the first period. This incentive is not obtained with short-term risk-bearing, but is stimulated by the anticipation of the benefits that will result from an ongoing employment relation. Since these benefits are not provided by an explicit contract, the incentives created are low-powered. Second, this incentive depends on: (i) $\gamma$-- the discount rate; (ii) $\phi_{1}$ and $\psi_{1}$-- the
impact of period 1 productivity on the increase in FSHA and NSHA; (iii) $p_{m}$ and $p$-- the employee's bargaining power in the market for his NSHA and the expected production output, and in the firm for his FSHA ; and (iv) b -a measure of the relative labour intensity of the production function.

For the case in which $k$ is a contractible event we have following parallel results.

Proposition 6.9: Assume $k$ is a contractible event. If at $t=0$ the firm and the employee anticipate the second period's bargaining result, then the employee will have incentive to provide effort $e_{1}{ }^{* *}>0$ which is determined by

$$
C^{\prime}\left(e_{1}^{* *}\right)=\gamma \theta_{1}\left(1+\gamma h_{1}\right)^{\frac{1}{b}-1} b(1-b)^{\frac{1}{b}-1} \quad(6.3 .36-1)
$$

Correspondingly, the firm will invest $k_{1}{ }^{* *}$ such that

$$
\begin{equation*}
k_{1}^{* *}=\left(1+\gamma h_{1}\right)^{\frac{1}{b}}(1-b)^{\frac{1}{b}}\left(a+e_{1}^{* *}\right) \tag{6.3.36-2}
\end{equation*}
$$

It is interesting to notice that the contractibility of $k$ has no impact in the final period (i.e., in the oneperiod model), but it does have an impact in the multiperiod model. This is because in the final period, with the agent receiving a fixed wage, the firm will make the "optimal" choice of $k$ since it receives all incremental benefits from that investment as well as bearing all
incremental costs. On the other hand, if $k$ is not contractible in a multi-period setting, while the firm will bear the entire incremental cost of increasing $k$ and will receive the entire current incremental benefits from that increase, it must share the future benefits (through the bargaining process). Thus the results of Proposition 6.5 are fundamentally different than for Proposition 6.8 and 6.9.

The first period wage contracts, corresponding to (6.3.36), are

$$
\begin{equation*}
w_{1}-p_{m} \mathrm{~B}^{0}+p \mathrm{~A}^{0}+p_{m}\left(m_{1}^{* *}-k_{1}^{* *}-\mathrm{C}\left(e_{1}^{* *}\right)\right)+\mathrm{C}\left(e_{1}^{* *}\right) \tag{6.3.37}
\end{equation*}
$$

where $m_{1}^{* *}=m_{1}\left(k_{1}{ }^{* *}, e_{1}^{* *}\right)$, and $\left(k_{1}^{* *}, e_{1}^{* *}\right)$ are given by either Propositions 6.8 or 6.9 .

The pattern of employees' wages over time is often an issue in the labour contract literature. For example, Harris and Holmstrom [1982] provide a long-term labour contract model in which worker's ability is assumed unknown. The firm learns about each worker's productivity by observing the worker's output over time. It is shown that, in equilibrium, a worker's wage never declines with age and increases only when the worker's market value increases above his current wage. In our two-period model, a comparison of (6.3.37) with (6.3.34) results in the following corollary.

Corollary 6.10: In a two-period model, the condition for the expected second period wage to exceed the first period wage is

$$
\begin{aligned}
& p_{m}\left[\psi_{1} m_{1}^{* *}-(1-8) \mathrm{B}^{0}\right]+p\left[\phi_{1} m_{1}^{* *}-(1-\delta) \mathrm{A}^{0}\right] \\
& >-p_{m}\left[\left(m^{0}-k^{0}\right)-\left(m_{1}^{* *}-k_{1}^{* *}\right)\right]+\left(1-p_{m}^{* *}\right) \mathrm{C}\left(e_{1}^{* *}\right) \quad(6.3 .38)
\end{aligned}
$$

Inequality (6.3.38) has a straight-forward interpretation. The left-hand-side of the inequality is the change in the employee's share of the benefits of his human assets, while the right-hand-side is his share of the reduction of the second period production gain due to the loss of incentives. Our conclusions differ from Harris and Holmstrom [1982] in that: (i) the second period wage will increase if, and only if, the increase in the human assets is greater than the reduction in production gain; (ii) the wage increase will not only be influenced by the worker's market value, which is only determined by his NSHA, but also by his FSHA, which has no market value; and (iii) the wage may decrease if the inequality is reversed. A wage increase can occur if human asset acquisition in the period is high, or the decay rate is low, or the production reduction in the second period is low. Any reverse of these conditions may cause a wage reduction. For example, a sports player's wage is likely to decrease sharply after his peak performance period because of a very high decay rate of his human assets. On the other
hand, we observe examples of senior employees' wages increasing not because their market values increase but because the specific human assets, such as the knowledge of the particular firm, is increasing.

Finally, from (6.3.35-1), it is straightforward to determine the following comparative statistics.

```
Proposition 6.11: In a two-period model with non-contractible \(k\), the incentives created by the acquisition of human assets and second period contract renewal have the following properties. The employee's first-period effort is:
```

(1) increasing in the discount rate $\gamma$;
(2) increasing in $\phi_{1}$ and $\phi_{1}$
(3) increasing (decreasing) in $p$ and $p_{m}$ when

$$
\begin{equation*}
\left.\frac{\gamma \theta_{1}}{1+\gamma h_{1}}<( \rangle\right) b \tag{6.3.39}
\end{equation*}
$$

and reach their maximum when (6.3.39) holds as an equality:
(4) increasing (decreasing) in b if

$$
b>(<) 1-\frac{1}{R} \quad \text { where } R=1+\gamma\left(h_{1}-\theta_{1}\right) \quad(6.3 .40)
$$

and reaches its minimum when (6.3.40) holds as an equality.

The results of Proposition 6.11 can be interpreted as follows. First, the discount effects are obvious because the employee's investment has future benefits. A higher discount rate means higher returns on that investment,
which induces a larger incentive to invest. Second, the effects of the human asset acquisition rate are similar to the discount rate, higher acquisition rates result in higher returns on the effort invested.

Third, the effects of the bargaining power on the employee's incentive are more subtle as shown by (3). When the employee's bargaining power $p$ or $p_{m}$ are relatively small, the employee's incentive to provide effort increases as his bargaining power, either in the market or in the firm, increases. However, that incentive reaches its maximum at some threshold level. Above that level, when the employee has relatively strong bargaining power, the employee's incentive to provide effort decreases as his bargaining power increases. This is because there are two sides to the influence of an increase of the employee's bargaining power on the employee's ex post gain. On the one hand, the employee's share increases as his bargaining power increases. On the other hand, an increase on the employee's bargaining power implies a decrease on the firm's bargaining power. This, in turn, may reduce the firm's incentive to invest capital. The result is a reduction in the production output. The total influence is the sum of these two effects. When the former exceeds the latter, an increase in the employee's bargaining power will increase his effort level. In the reverse
case, a decrease in effort level occurs. In addition, the threshold level determined by (6.3.39) has an economic interpretation. On the right-hand-side of (6.3.39), b represents the sensitivity of production to the employee's effort input. On the left-hand-side, the numerator $\gamma \theta_{1}$ can be viewed as the employee's bargaining power over the output resulting from the inputs, while the denominator $1+\gamma h_{1}$ can be viewed as the return on his effort input. Hence, the left-hand-side of (6.3.39) is the "percentage" that the employee can capture from the output of his effort. When this "percentage" is less than the sensitivity $b$, the employee's incentives can be improved by increasing his bargaining power either in the market or in the firm. Some further implications of this result are discussed later.

Finally, the relative sensitivity of the outcome to labour and capital investments has an interesting effect. If the production technology is such that the outcome is relatively insensitive to the employee's effort level (i.e., b is close to zero), then increasing that sensitivity will reduce his incentive to work hard. On the other hand, if the outcome is relatively sensitive to the employee's effort level (i.e., b is close to one), then increasing that sensitivity will increase his incentive to work hard. The intuition behind this fact is again the
combination of two opposing effects. On the one hand, if the firm's capital is held constant, then the effort level always increases as $b$ increases. This can be seen by noticing that the factor $b(1-b)^{1 / b-1}$ is a increasing function of $b$ for $a l l b<1$. On the other hand, increasing $b$ reduces the firm's incentive to invest capital (because 1/b - 1 decreases in b). This, in turn, will reduce the employee's ex post gain and his incentives.

When $k$ is a contractible event, from (6.3.36-1), we can show that the results of Proposition 6.11(1) and (2) again hold. Condition (6.3.40) is still true but with $R=$ $1+\gamma h_{1}$. However the influences of $p$ and $p_{m}$ are different. It is obvious that $e_{1}$ is increasing in both $p$ and $p_{m}$. That means (6.3.39) is not true anymore.

From (6.3.35-2) and (6.3.36-2), we can see that the firm's capital investment choice is influenced by the same factors as those influencing the employee's effort choice, but in a more complicated way. Since the optimal capital investment is a linear function of the effort level, the effects of a particular parameter can be separated into two parts: a direct effect and an indirect effect through the change in the effort level. When the two effects go in the same direction, such as the influence of the discount rate or human asset acquisition rate, the conclusion is simple. However, if they go in opposite directions,
the conclusion depends on the value of the parameters. Basically, the incentive for the firm to invest capital has quite different characteristics than the employee's incentive to provide effort. To see this, observe that the employee's effort is an increasing function of both $p_{m}$ and $p$ when they are small. In particular, if $p_{m}=p=0$, then $\theta_{1}=0$ so that $e_{1}=0$, i.e., the low-powered incentives disappear if the employee loses all the bargaining power both in the market for NSHA and in the firm for FSHA. In contrast, when $p_{m}=p=1$ and $\theta_{1}=h_{1}$, then from (6.3.35-2) , $k_{1}=(1-b)^{1 / b}\left(a+e_{1}^{* *}\right)$, i.e., the firm's incentive to invest still exists although it is lower than the first-best level. The intuition for this difference is that the firm is the residual claimant. Although it loses its share of the human assets in the second period bargaining, it still receives the current production output. This is particularly clear when $k$ is contractible. In that case the firm's incentive is influenced only indirectly by the bargaining power through the employee's effort.

## Summary of the Two-Period Model

In our two-period model, human asset acquisition and anticipated second-period contract renewal create an incentive for the employee to work hard in the first
period. This incentive is not the classical high-powered type in that it is not created by contingent contracts that result in short term risk bearing. The low-powered incentives we have modelled are consistent with the incentives that appear to be in effect for many employees within most organizations. Furthermore, it provides a setting within which to examine the role of unverifiable performance evaluation in management accounting systems.

It is interesting to compare the results of different incentive mechanisms. We summarize the main results of the first period contracts in a two-period model in Table 6-1.

## Insert Table 6-1 here

A glance at Table 6-1 reveals the following conclusions. First, all incentive contracts, either highpowered or low powered, result in lower efficiency than that of the first-best bench-mark contract. This can be shown by observing that $\alpha<1$ and $\theta_{1}<h_{1}$, so that all the coefficients in these cells are less than the first-best coefficient. Second, for low-powered contracts, contracting on $k$ always creates stronger incentive than not contracting on $k$. The economic intuition for this result is that when $k$ is contractible, input distortion comes from the employee's effort only. Anticipating higher invest-
ment from the firm, the employee will expect a higher ex post gain and correspondingly provide higher effort. Finally, a comparison between the high- and the low-powered incentives shows that the high-powered contracts are characterized mainly by the risk-sharing coefficient $\alpha$, while low-powered contracts are determined only by the bargaining power $p$ and $p_{m}$ ( $p$ and $p_{m}$ influence high-powered incentives too). A more detailed comparison between the two mechanisms is left for future research.

### 6.3.5 Multi-Period Model

The results presented in the last subsection can be immediately extended to a multi-period model. Assume that there are $T$ periods with separating points ( $0,1,2, \ldots, T-$ 1, T). At each point $t=1,2, \cdots, T-1$, the firm and the employee can either terminate their relationship, or contract to extend the employment relation for one more period. The analysis can be started from the last period and traced back. The last period is exactly the same as in the two-period model: the employee negotiates a constant wage contract at $t=T-1$ that reflects his previously acquired human assets (and his bargaining power) and the fact that he will only provide the minimal level of effort, i.e., $e_{T}=0$. The firm rationally invests $k_{T}=k^{0}$. The second to the last period is the same as the first
period of a two-period model: the employee will invest $e_{T-1}>0$ because he predicts that he can capture a $p_{m}$ share of NSHA and the expected output and a $p$ share of FSHA in the last period contract bargaining. He will choose $e_{T-1}$ to maximize the difference between this bonus and his personal effort cost, taking the discount.rate into account. For any $t(0<t<T)$, we have following proposition.

> Proposition 6.12: Assume a, >0. There exist incentives for the firm and the employee to continue their employment relation. The employee has incentive to insert more than the minimal effort level in all periods except the last period. If the bargaining power is constant over time and $k$ is non-contractible, then (k, $\left.{ }_{\mathbf{t}}\right\}$ are characterized by.

$$
\begin{align*}
C^{\prime}\left(e_{t}\right)= & \gamma M(\gamma \delta, T-t) \theta_{t}^{\bullet} \\
& {\left[1+\gamma M(\gamma \delta, T-t)\left(h_{t}-\theta_{t}\right)\right]^{\frac{1}{b}-1} } \\
& b(1-b)^{\frac{1}{b}-1} \tag{6.3.41-1}
\end{align*}
$$

$$
\begin{align*}
k_{t}= & {\left[1+\gamma M(\gamma \delta, T-t)\left(h_{t}-\theta_{t}\right)\right]^{\frac{1}{b}} . } \\
& (1-b)^{\frac{1}{b}}\left(a+e_{t}\right) \tag{6.3.41-2}
\end{align*}
$$

where $h_{t}=\phi_{t}+\phi_{t}, \theta_{t}=p_{m} \phi_{t}+p \phi_{t}$, and

$$
\begin{equation*}
M(R, t)=1+R+\cdots+R^{t-1}=\frac{1-R^{t}}{1-R} \tag{6.3.42}
\end{equation*}
$$

For the case in which $k_{t}$ are contractible events, we have

Proposition 6.13: Under the same conditions as Proposition 6.12 , if $k$ is a contractible event, then (6.3.41) become

$$
\begin{align*}
C^{\prime}\left(e_{t}\right)= & \gamma M(\gamma \delta, T-t) \theta_{t} \\
& {\left[1+\gamma M(\gamma \delta, T-t) h_{t}\right]^{\frac{1}{b}-1} } \\
& b(1-b)^{\frac{1}{b}-1} \tag{6.3.43-1}
\end{align*}
$$

$k_{t}=\left[1+\gamma M(\gamma \delta, T-t) h_{t}\right]^{\frac{1}{b}}$.

$$
\begin{equation*}
(1-b)^{\frac{1}{b}}\left(a+e_{t}\right) \tag{6.3.43-2}
\end{equation*}
$$

It is easy to show that the first-best solution of a multi-period model is given by (similar to a two-period model)

$$
\begin{aligned}
k_{t}^{*}= & {\left[1+\gamma M(\gamma \delta, T-t) h_{t}\right]^{\frac{1}{b}} } \\
& (1-b)^{\frac{1}{b}}\left(a+e_{t}\right) \\
C^{\prime}\left(e_{t}\right)= & {\left[1+\gamma M(\gamma \delta, T-t) h_{t}\right]^{\frac{1}{b}} } \\
& b(1-b)^{\frac{1}{b}-1}
\end{aligned}
$$

which are the solutions to

$$
\operatorname{Max}_{k_{t}, e_{t}}\left[1+\gamma M(\gamma \delta, T-t) h_{t}\right] m_{t}-k_{t}-C\left(e_{t}\right)
$$

Comparing (6.3.41) or (6.3.43) with the first-best solutions, it is obvious that both of them are inferior to the first-best solutions. This conclusion holds even as $T$ goes to infinity, i.e., the employment relation is everlasting.

The incentives created by the anticipation of future contract renewal in a multi-period model have properties similar to those in a two-period model as described by Proposition 6.11. In addition, we have following proposition.

Proposition 6.14: In a multi-period model, if we assume that the bargaining power and human asset acquisition rates are constant over time, i.e., all $p_{n}, p, \phi_{t}, \phi_{t}, h_{t}$, and $\theta_{t}$ are constants, then:
(1) the employee's effort $e_{t}$ is decreasing in $t$;
(2) if the total employment period is long enough, i.e., $T$ is large enough, then the employee's effort levels are close to a constant over time except when the time is close to termination;
(3) the firm's investment is decreasing in $t$;
(4) if $T$ is large enough, the firm's investment levels are close to a constant over time except when the time is close to termination.

Proposition 6.14 provides some interesting results about the nature of employee incentives. First, the employee's effort level is, in general, decreasing over time. Of course this result is derived under our assumption that human asset acquisition is proportional to the current production output, but this should be also true when human asset acquisition is positively correlated with his effort level. The crucial point is that when an employee invests effort into a firm's operation, he will not only anticipate short term benefits but also long term return. When the number of future periods are decreasing, his expected return is decreasing which reduces his incentive to invest effort. This is consistent with the frequent observation that senior employees in an organization work less hard than most junior employees, although the former get higher salaries. Second, when the expected employment is long enough, an employee's incentives will be quite stable over time, except when he is close to retirement. This incentive is not guaranteed by an explicit incentive contingent contract, but is motivated by the firm's and the employee's rational expectation of future contract renewal. This result seems consistent with real world observations that most employee's contracts are not a contingent type, but most of them are quite industrious. Third, the firm's investment behaviour
is similar to the employees' incentives. In general, the firm invests less for an employee when he is close to retirement, but is stable if the expected employment is long enough.

Finally, we have the following generalization of Corollary 6.10 in a multi-period model.

Corollary 6.15: In a multi-period model, the condition for the wage increasing over time is

$$
\begin{align*}
& p_{m}\left[\Psi_{t} m_{t}-(1-\delta) B^{t}\right]+p\left[\phi_{t} m_{t}-(1-\delta) A^{\mathrm{t}}\right]> \\
& -p_{m}\left[\left(m_{t+1}-k_{t+1}\right)-\left(m_{t}-k_{t}\right)\right] \\
& -\left(1-p_{m}\right)\left[C\left(e_{k+1}\right)-C\left(e_{k}\right)\right] \tag{6.3.44}
\end{align*}
$$

The general interpretation of (6.3.44) is the same as for Corollary 6.10. When the period of employment is long enough, then based on Proposition 6.14 (2) and (4), the expected production $m_{t}$ is close to constant over time. This implies that the right-hand side of (6.3.44) is very close to zero. Hence, condition (6.3.44) will hold for any small amount of human asset acquisition, and the employee's wage will be bid up. This can explain why we frequently observe that employees' wages are increasing over the employment period. However, this is not always true. As commented before, if the employee's production reduction is large enough, his wage may decrease. This is different than the Harris and Holmstrom [1982] results.
6.4 Implications for Managerial Accounting system Designs

It has long been recognized that information provided by any accounting system is only a part of the information circulating in organizations. One particular function of an accounting system is to harden that information. The question of why firms choose to provide hard information has been the focus of accounting research over the last twenty years. One commonly accepted point of view is Gjesdal's [1981] insight that accounting information plays a key role in the stewardship process. Since shareholders of a firm usually delegate decision-making to managers, there is a demand for information about the manager's actions for control purposes. Control is modelled in most of the existing literature through an agency relationship. Following Ijiri's [1971] point of view that stewardship information should be as hard as possible, Gjesdal [1981] and many others claim that soft information has no value for stewardship processes.

Our results show that the claim that hardness is a necessary characteristic of stewardship information is only true for high-powered incentive mechanisms. Williamson [1985] points out that in hierarchical organizations, there may exist different mechanisms from those in the marketplace. In this paper, we formally modelled such a mechanism with respect to incentives. Our analysis shows
that there can exist two different kinds of incentives in organizations. On the one hand, high-powered incentives, characterized by explicit contingent contracts, depend crucially on hard accounting information. This is consistent with Gjesdal's insight and most of the existing incentive literature. On the other hand, low-powered incentives are initiated by rational expectations of future contract renewal. They make use of all available soft or hard information in the organization, perhaps providing a cheaper way for motivating employees. The merits of low-powered incentives relative to high-powered incentives can be summarized as following. First, it can make use of soft information so that the costs of hardening information are avoided. This economy in information costs may be significant. Second, since most employees' tasks and their consequences are multi-dimensional, the design of high-powered incentive contracts may be extremely difficult, if not impossible. In contrast, low-powered incentives depend on contract renewal, for which it is easier to subjectively consider a variety of information that pertains to the multi-dimensional factors that affect the firm's value.

Thus, we claim that soft information has value in providing incentives to employees within organizations. Then why do we observe high-powered incentives in firms,
particularly at some levels such as top management or divisional managers? The answer is: (i) for top managers in a firm, soft performance measures are not available due to monitoring difficulties; (ii) hard financial accounting data or inside auditable managerial accounting data are available at a relatively low cost; and (iii) given the appropriate hard information, contingent contracts provide more efficient and effective incentives. While top managers are most likely to be motivated by high-powered incentives, middle rank managers may face both kinds of incentives: some part of their compensation may be specified by contingent contracts that explicitly depend on available hard accounting data, while other parts of their compensation, such as base salary and promotion, are based on all available information about their performance.

Another implication of our results pertains to the design of managerial accounting systems. Financial accounting data are relatively hard because they are auditable by independent auditors based on GAAP. However, management accounting is an internally oriented system that need not conform to GAAP. Hence, managerial accounting systems may include both hard and soft information. On the one hand, if the information provided by the system will be used for explicit contracting purposes, such as a contract between the top management and a divisional
manager, then hardness is essential. This kind of information must be internally auditable. As mentioned above, if monitoring is difficult and imperfect, then this may be the only way to provide incentives for the divisional managers. On the other hand, if monitoring through the hierarchical structure is effective, then.hardening information is not essential even if the information is used for incentive purposes. This provides a criterion for determining the scope of internal auditing.

Our analysis can be extended in several directions. First, Proposition 6.11 shows that low-powered incentive may be influenced by various factors such as the employee human asset acquisition rate, the discount rate, or the bargaining power, etc. Some of these factors may be controllable within organizations. This suggests that it may be useful to endogenize various parameters in our model. The following are some possible examples.

Most obviously, an employee's bargaining power may be influenced by government regulations or various "internal regulations" of a corporation. The latter may be explicitly or implicitly determined by a firm's reputation or "corporate culture" (using Kreps' [1984] terminology). Note that in our model, bargaining power is the ability to capture the ex post trading gains in contract negotiation. Proposition 6.11 (2) shows that the employee's incentive
is increasing as his bargaining power increases when his bargaining power is small. In addition, there is an equilibrium in which the employee's incentive reaches its maximum $\left(p_{m}=p=b\right)$. There should exist a value of employee bargaining power between zero and $b$ such that the firm's net share of the gain is maximized. Given this fact, the firm may find some way to commit to giving the employee a particular level of bargaining power.

Other parameters that could be endogenized are the employee's human asset acquisition rates. In our model, these rates are exogenously given and independent of the firm's investment. It may be possible for the firm to choose not only the optimal investment level, but also the way it invests. That is, the firm can allocate resources to influence both the employees' human asset acquisition and his production output. This can be done by allowing $\phi_{t}$ and $\psi_{t}$ to be functions of the investment. For example, a firm may provide on-the-job training programs so that an employee can acquire more skill, either NSHA and FSHA, in a certain period. Then, for the same bargaining power, the employee's incentive will be strengthened in the following employment periods.

The second potential extension pertains to the market influence. More extensive modelling of the market power $\left(p_{m}\right)$ is an obvious area that should be explored. In our
model, $\mathrm{p}_{\mathrm{m}}$ is exogenously given. How the labor market actually operates and how it influences the bargaining within the firms are very important for fully understanding low-powered incentives.

Other interesting issues may be: (i) to build a model
to incorporate a learning process, i.e., to introduce uncertainties about an employee's human assets; (ii) to introduce asymmetric predecision information for connecting decision-making with low-powered incentives; (iii) to examine the influence of errors in the supervisor's subjective judgements on low-powered incentives; and (iv) most importantly, to examine the relation between various managerial accounting issues and low-powered incentives.

Tables
TABLE 6-1: Comparison of Incentives Created By Different Contracts

| CONTRACt | $e_{1}$ is characterized by $c^{\prime}\left(e_{1}\right)=\bigoplus b(1-b)^{\frac{1}{b}-1}$ <br> where $\theta$ is stated below for each of the contracts. | $k_{1}$ is specified by $k_{1}=\Theta(1-b)^{\frac{1}{b}}(a+e$ <br> where $\theta$ is stated below for each of the contracts. |
| :---: | :---: | :---: |
| FIRST-BEST (BENCH MARK CASE | $\left(1+\gamma h_{1}\right)^{\frac{1}{b}}$ | $\left(1+\gamma h_{1}\right)^{\frac{1}{b}}$ |
| HIGH- <br> POWERED <br> SECOND- <br> BEST <br> (k NONCON- <br> (RACTIBLE) | $\left(\alpha_{1}+\gamma \theta_{1}\right)\left[1-\alpha_{1}+\gamma\left(h_{1}-\theta_{1}\right)\right]^{\frac{1}{b}-1}$ | $\left[1-\alpha_{1}+\gamma\left(h_{1}-\theta_{1}\right)\right]^{\frac{1}{b}}$ |
| HIGH- <br> POWERED <br> SECOND- <br> BEST <br> (k CON- <br> TRACTIBLE) | $\left(\alpha_{1}+\gamma \theta_{1}\right)\left(1+\gamma h_{1}\right)^{\frac{1}{b}-1}$ | $\left(1+\gamma h_{1}\right)^{\frac{1}{b}}$ |
| LOW-POWERED (k NONCONtRACTIBLE) | $\gamma \theta_{1}\left[1+\gamma\left(h_{1}-\theta_{1}\right)\right]^{\frac{1}{b}-1}$ | $\left[1+\gamma\left(h_{1}-\theta_{1}\right)\right]^{\frac{1}{b}}$ |
| LOWPOWERED (k CONTRACTIBLE) | $\gamma \theta_{1}\left(1+\gamma h_{1}\right)^{\frac{1}{b}-1}$ | $\left(1+\gamma h_{1}\right)^{\frac{1}{b}}$ |

## Appendix 6

## Proof of Proposition 6.1

With $A$ and $B$, the employee's status quo position is $p_{m} B$ and the firm is $\left(1-p_{m}\right) B$. The Nash problem is

$$
\begin{aligned}
& \quad \operatorname{Max}\left(P-p_{m} \mathrm{~B}\right)^{p} \cdot\left[\mathrm{~A}+\mathrm{B}-P-\left(1-p_{m}\right) \mathrm{B}\right]^{1-p} \\
-\quad & P-p \mathrm{~A}+p_{m} \mathrm{~B}
\end{aligned}
$$

Q.E.D.

## Proof of Lemma 6.2

U and V are given by (6.3.12). Note that e and k are independent of $\alpha$ and $\beta$. Take the derivative of (6.3.16) with respect to $\beta$

$$
\begin{array}{ll} 
& p_{m} U^{p_{m}-1} V^{1-p_{m}}-\left(1-p_{m}\right) U^{p_{m}} V^{-p_{m}}=0 \\
& p_{m} V-\left(1-p_{m}\right) U=0  \tag{A6.1}\\
\& & U=p_{m}(U+V), V=\left(1-p_{m}\right)(U+V)
\end{array}
$$

This is result (ii). Take the derivative of (6.3.16) with respect to $\alpha$

$$
\begin{equation*}
p_{m} U^{p_{m}-1} V^{1-p_{m}}\left(m-r \alpha \sigma^{2}\right)+\left(1-p_{m}\right) U^{p_{m}} V^{-p_{m}}(-m)=0 \tag{A6.2}
\end{equation*}
$$

Using (A6.1), (A6.2) implies that $\alpha=0$, which is result (i).

Proof of Lemma 6.3
We first show (1). For any given constants $\alpha$ and $\beta$,

$$
\begin{align*}
& (6 \cdot 3 \cdot 18-2)-\alpha b(a+e)^{b-1} k^{1-b}-C^{\prime}(e)  \tag{A6.3}\\
& (6 \cdot 3 \cdot 18-3)-(1-\alpha)(1-b)(a+e)^{b} k^{-b}=1
\end{align*}
$$

(A6.4)
(6.3.19-1) follows directly from (A6.4). (6.3.19-2) is obtained by substituting (6.3.19-1) into (A6.3). Thus, both $e$ and $k$ are non-trivial functions of $\alpha$. This implies that the derivative of $U+V$ with respect to $\alpha$ will include non-zero terms $d e / d \alpha$ and $d k / d \alpha$. Hence, $\alpha$ will deviate from the first-best value which is the value when these derivatives are zero. The proof of (2) and (3) are the same as in Lemma 6.2. (A6.1) holds because both $e$ and $k$ are independent of $\beta$, while (A6.2) follows from the envelope theorem.

> Q.E.D.

Proof of Lemma 6.4
Again (2) and (3) do not depend on the particular forms of U and V so their proofs are the same as in Lemma 6.2.
(A6.3) is unchanged but (A6.4) must be changed to (6.3.14-
2) because $k$ is chosen to maximize $U+V$ rather than $V$.
(6.3.21) immediately follows from (A6.3) and (6.3.14-2).
Q.E.D.

Proof of Lemma 6.5
Given a constant wage, any non-zero effort increases the employee's personal cost but provides no benefits, so zero effort is his rational choice. Let $m=a^{b} k^{1-b}$ be the expected output given zero effort. Let $\hat{\mathfrak{k}}$ be the employee's belief about firm's investment and let k be the true investment. Then the perceived trading gain is $\hat{m}-\hat{k}$ (if $\hat{\mathbf{k}}$ is implemented, $\hat{\mathbf{m}}=a^{b} \hat{\mathrm{k}}^{1-b}$ ), and the bargaining results in $\hat{U}=p_{m}(\hat{m}-\hat{k})$ and $\hat{V}=\left(1-p_{m}\right)(\hat{\mathrm{m}}-\hat{k})$, where we ignore the risk premium since the choice of $e$ and $k$ will not influence the variances. If $k$ is contractible, then $\hat{k}=k$ will be implemented and $k$ is chosen to maximize ( $m-k$ ). However, if $k$ is not contractible, then the firm selects $k$ to maximize its true gain $V=m-k-p_{m}(\hat{m}-\hat{k})$, i.e., the firm claims the whole output and pays the contracted wage to the employee. This implies that for any given $\hat{k}$, the firm will choose $k$ to maximize $m-k$. Hence, independent of the contractibility of $k$, (6.3.23) is true.

> Q.E.D.

Proof of Lemma 6.6
The proof is very similar to the proof of Lemma 6.3.Q.E.D.

Proof of (6.3.33)
The proof is very similar to the proof of Lemma 6.4.Q.E.D.

## Proof of Proposition 6.7

When $A^{1}>0$, the fact that a continuing relation superior to termination is obvious. On the other hand, if $A^{1}=0$, then both the employee and the firm can go to the market to get the same gains, so a continuing relation has no positive value.

Lemma 6.5 shows that independent of the contractibility of $k$, in the last period, the firm will choose $k^{0}$ given by (6.3.23). Hence, at $t=1$, the status quo positions of each party are:

$$
\begin{align*}
\text { employee: } p_{m}\left(\mathrm{~B}^{1}+m^{0}-k^{0}\right)  \tag{A6.5}\\
\text { firm: }\left(1-p_{m}\right)\left(\mathrm{B}^{1}+m^{0}-k^{0}\right) \tag{A6.6}
\end{align*}
$$

(A6.5) means the employee can go to the market to sell his $B^{1}$ and his expected production output at the market price, while (A6.6) means the firm can also go to the market to buy $B^{1}$ and the same output at the market price. On the other hand, an agreement with wage $w_{2}$ will bring the two parties the following benefits:

$$
\begin{align*}
& \text { employee: } W_{2}  \tag{A6.7}\\
& \text { firm: } A^{1}+B^{1}+m^{0}-k^{0}-W_{2} \tag{A6.8}
\end{align*}
$$

The Nash bargaining solution is the solution to the following problem:

$$
\begin{align*}
\operatorname{Max}_{W_{2}} & {\left[w_{2}-p_{m}\left(B^{1}+m^{0}-k^{0}\right)\right] p_{0} } \\
& {\left[\mathrm{~A}^{1}-w_{2}+p_{m}\left(\mathrm{~B}^{1}+m^{0}-k^{0}\right)\right]^{1-p} } \tag{A6.9}
\end{align*}
$$

Taking the derivative of (A6.9) with respect to $\mathrm{w}_{2}$, and setting it equal to zero, provides (6.3.34). Q.E.D.

## Proof of Proposition 6.8:

Given (6.3.34), we know that in the second period contract the employee always receives a share of $H A$ equal to $\mathrm{pA}^{1}+\mathrm{p}_{\mathrm{m}}$ $B^{1}$. Thus the employee's total expected return from a positive effort input is the sum of two parts: a share of the current equilibrium production output $p_{m}\left[m_{1}-k_{1}-\right.$ $\left.C\left(e_{1}\right)\right]$ which is included in the first period wage, and a share of the acquisition of human assets which will be paid in the second period wage. Hence the employee's net benefits from $e_{1}$ is ${ }^{16}$

$$
\begin{align*}
& p_{m}\left(\hat{m}_{1}-\hat{k_{1}}-C\left(e_{1}\right)\right)+\gamma\left(p_{m} \mathrm{~B}_{1}+p \mathrm{~A}_{1}\right)+\mathrm{C}\left(\hat{e}_{1}\right)-\mathrm{C}\left(e_{1}\right)  \tag{AG.10}\\
= & p_{m} \hat{n}_{1}+\gamma \theta_{1} m_{1}-\mathrm{C}\left(e_{1}\right)+\left(1-p_{m}\right) \mathrm{C}\left(\hat{e}_{1}\right)-p_{m} \hat{k_{1}}
\end{align*}
$$

where $\hat{e}_{1}$ is the firm's perception about the employee's effort and $\hat{m}_{1}=m\left(\hat{k}_{1}, \hat{e}_{1}\right)$. For any given $\hat{\mathrm{k}}_{1}$ and $\hat{\mathrm{e}}_{1}$, the $\mathrm{e}_{1}$ that maximizes (A6.10) must satisfy

[^58]\[

$$
\begin{equation*}
\gamma \theta_{1} b\left(a+e_{1}\right)^{b-1} k_{1}^{1-b}-C^{\prime}\left(e_{1}\right) \tag{A6.11}
\end{equation*}
$$

\]

On the other hand, the firm's total expected return from any investment is equal to its claim $m_{1}$ less the payment to employee at $t=1$, i.e.,

$$
\begin{align*}
& m_{1}-\left[p_{m}\left(\hat{m}_{1}-\hat{k}_{1}-C\left(\hat{e}_{1}\right)\right)+C\left(\hat{e}_{1}\right)\right]-k_{1} \\
+ & \gamma\left[\left(1-p_{m}\right) B_{1}+(1-p) A_{1}\right] \\
= & {\left[1+\gamma\left(h_{1}-\theta_{1}\right)\right] m_{1}-k_{1}-p_{m}\left(\hat{m}_{1}-\hat{k}_{1}\right)-C\left(\hat{e}_{1}\right) } \tag{A6.12}
\end{align*}
$$

For any given $\hat{e}_{1}$ and $\hat{k}_{1}$, the $k_{1}$ which maximizes (A6.12) must satisfy

$$
\begin{equation*}
\left[1+\gamma\left(h_{1}-\theta_{1}\right)\right](1-b)\left(a+\hat{E}_{1}\right)^{b} k_{1}^{-b}=1 \tag{A6.13}
\end{equation*}
$$

Since both parties are rational, in equilibrium $\hat{k}_{\mathbf{q}}=\mathbf{k}_{\mathbf{1}}$ and $\hat{e}_{1}=e_{1}$, and (A6.11) and (A6.13) must be solved simultaneously. This can be done by solving $\mathrm{k}^{1}$ from (A6.13) to get (6.3.35-2) first, and then by substituting (6.3.35-2) into (A6.11) to solve for $C^{\prime}\left(e_{1}\right)$ to obtain (6.3.35-1).
Q.E.D.

## Proof of Proposition 6.9

If $k$ is a contractible event, then $\hat{k}_{1}=k_{1}$ and (A6.11) is unchanged. (A6.13) should be replaced by (6.3.30-1), i.e.,

$$
\begin{equation*}
k_{1}=\left(1+\gamma h_{1}\right)^{\frac{1}{b}}(1-b)^{\frac{1}{b}}\left(a+e_{1}\right) \tag{A6.14}
\end{equation*}
$$

This is (6.3.36-2). Substitute it into (A6.11) to get (6.3.36-1).
Q.E.D.

## Proof of Corollary 6.10

The results follow immediately from directly calculating $E\left[\tilde{w}_{2}\right]-w_{1}$. Q.E.D.

Proof of Proposition 6.11
Note that the left-hand side of (6.3.35-1) is an increasing function of $e_{1}$, so we need only show that the righthand side's derivative has the correct sign.
(1) Let

$$
\begin{aligned}
f & =\gamma \theta_{1}\left[1+\gamma\left(h_{1}-\theta_{1}\right)\right]^{\frac{1}{b}-1} \\
& =\gamma \theta_{1} R^{\frac{1}{b}-1}
\end{aligned}
$$

where $R=1+\gamma\left(h_{1}-\theta_{1}\right)$. Then

$$
\frac{d f}{d \gamma}=\theta_{1} R^{\frac{1}{b}-1}+\gamma \theta_{1}\left(\frac{1}{b}-1\right) R^{\frac{1}{b}-2}\left(h_{1}-\theta_{1}\right)>0
$$

Here we used the fact that $h_{1} \geq \theta_{1}$ and $R>0$.
(2) Let $f$ be the same as in (1).

$$
\frac{d f}{d \Psi_{1}}=\gamma R^{\frac{1}{b}-2}\left[p_{m} R+\gamma \theta_{1}\left(\frac{1}{b}-1\right)\left(1-p_{m}\right)\right]>0
$$

From the symmetry of $\phi_{1}$ and $\psi_{1}$, the same is true for $\phi_{1}$. (3) f is the same as in (1).

$$
\begin{aligned}
\frac{d f}{d p_{m}} & =\gamma \Psi_{1} R^{\frac{1}{b}-1}+\gamma \theta_{1}\left(\frac{1}{b}-1\right) R^{\frac{1}{b}-2}\left(-\gamma \Psi_{1}\right) \\
& =\gamma \psi_{1} R^{\frac{1}{b}-2}\left[1+\gamma h_{1}-\frac{1}{b} \gamma \theta_{1}\right] \\
& >(<) 0 \Leftrightarrow(6.3 .39)
\end{aligned}
$$

From the symmetry of $p$ and $p_{m}$ in $f$, we know (6.3.39) is also true for $p$.
(4) Let

$$
g=R^{\frac{1}{b}-1} b(1-b)^{\frac{1}{b}-1}
$$

Take the Log of both sides,

$$
\log g=\left(\frac{1}{b}-1\right) \log R+\log b+\left(\frac{1}{b}-1\right) \log (1-b)
$$

Then take the derivative to obtain

$$
\begin{aligned}
\frac{d g}{d b} & =-g \frac{1}{b^{2}} \log [R(1-b)]>(<) 0 \\
& \Leftrightarrow R(1-b)\langle( \rangle) 1 \\
& \Leftrightarrow b>\left(\langle ) 1-\frac{1}{R}\right.
\end{aligned}
$$

Q.E.D.

## Proof of Proposition 6.12

The employee receives a share of the current production to the extent it is included in his negotiated wage. Taking
his wage as fixed, the employee will select $e_{t}$ to maximize the stream of future benefits from his acquisition of human assets $A_{t}+B_{t}=\left(\phi_{t}+\Psi_{t}\right) X_{t}=h_{t} X_{t}$. Hence, his effort level choice is based on the net present value of the future benefits

$$
\begin{align*}
& \gamma\left(p A_{t}+p_{n t} B_{t}\right)+\gamma^{2} \delta\left(p A_{t}+p_{m t} B_{t}\right)+\ldots+ \\
+ & \gamma^{T-t} \delta^{T-t-1}\left(p A_{t}+p_{m} B_{t}\right)-C\left(e_{t}\right) \\
- & {\left[\gamma\left(1+\gamma \delta+\ldots+(\gamma \delta)^{T-t-1}\right) \theta_{t}\right] m_{t}-C\left(e_{t}\right) } \\
= & \gamma M(\gamma \delta, T-t) \cdot \theta_{t} m_{t}-C\left(e_{t}\right) \tag{A6.15}
\end{align*}
$$

where $M(\gamma \delta, T-t)$ is given as (6.3.42). The first-order condition of (A6.15) is

$$
\begin{equation*}
\gamma M(\gamma \delta, T-t) \theta_{t} b\left(a+e_{t}\right)^{b-1} k^{1-b}=C^{\prime}\left(e_{t}\right) \tag{A6.16}
\end{equation*}
$$

Similarly, the firm will select $k_{t}$ to maximize the net present value of investment $k_{t}$ (given that the employee's current wage has been fixed)

$$
\begin{align*}
& m_{t}+\sum_{j-t+1}^{T} \gamma^{j-t} \delta^{j-t-1}\left(h_{t}-\theta_{t}\right) m_{t}-k_{t} \\
= & {\left[1+\gamma M(\gamma \delta, T-t)\left(h_{t}-\theta_{t}\right)\right] m_{t}-k_{t} } \tag{A6.17}
\end{align*}
$$

The firm will choose $k_{t}$ such that

$$
\begin{equation*}
\left[1+\gamma M(\gamma \delta, T-t)\left(h_{t}-\theta_{t}\right)\right](1-b)\left(a+e_{t}\right)^{b} k_{t}^{-b}=1 \tag{A6.18}
\end{equation*}
$$

Solving (A6.16) and (A6.18) simultaneously, we obtain (6.3.41-1) and (6.3.41-2).
Q.E.D.

## Proof of Proposition 6.13

If $k$ is a contractible event, then $k_{t}$ should be chosen to maximize

$$
\begin{align*}
& m_{t}+\sum_{j-t+1}^{T} \gamma^{j-t} \delta^{j-t-1} h_{t} m_{t}-k_{t}  \tag{Аб.19}\\
& =\left[1+M(\gamma \delta, T-t) h_{t}\right] m_{t}-k_{t}
\end{align*}
$$

instead of (A6.17). Thus (A6.18) becomes

$$
\left[1+\gamma M(\gamma \delta, T-t) h_{t}\right](1-b)\left(a+e_{t}\right)^{b} k_{t}^{-b}=1 \quad \text { (A6.20) }
$$

Solving (A6.16) and (A6.20) simultaneously, we obtain (6.3.43-1) and (6.3.43-2).
Q.E.D.

## Proof of Proposition 6.14

Differentiating (6.3.41-1) and (6.3.41-2) with respect to $M$, and noting that $M(\gamma \delta, T-t)$ is decreasing in $t$ for fixed T, provides results (1) and (3). Further observe that as $T$ goes to infinity, $M$ approaches its limit $1 /(1-\gamma \delta)$. In addition, the convergence of $M$ is as fast as $(\gamma \delta)^{t}$. This implies that, for large $T-t, M$ is very close to a constant and, hence, $e_{t}$ and $k_{t}$ are close to constants that are characterized by

$$
\begin{aligned}
k_{t}^{\infty}= & {\left[1+\frac{\gamma\left(h_{t}-\theta_{t}\right)}{1-\gamma \delta}\right]^{\frac{1}{b}} } \\
& (1-b)^{\frac{1}{b}}\left(a+e_{t}\right) \\
C^{\prime}\left(e_{t}^{\infty}\right)= & \frac{\gamma \theta_{t}}{1-\gamma \delta}\left[1+\frac{\gamma\left(h_{t}-\theta_{t}\right)}{1-\gamma \delta}\right]^{\frac{1}{b}-1} \\
& b(1-b)^{\frac{1}{b}}
\end{aligned}
$$

Q.E.D.

## Proof of Corollary 6.15

The wage contract for period $t$ and $t+1$ are, respectively,

$$
\begin{align*}
\mathrm{w}_{t}= & p_{m} \mathrm{~B}^{\mathrm{t}}+p \mathrm{~A}^{\mathrm{t}} \\
& +p_{m}\left(m_{t}-k_{t}-\mathrm{C}\left(e_{t}\right)\right)+\mathrm{C}\left(e_{t}\right)  \tag{A6.21}\\
\mathrm{w}_{t+1}= & p_{m} \mathrm{~B}^{\mathrm{t+1}}+p \mathrm{~A}^{t+1} \\
& +p_{m}\left(m_{t+1}-k_{t+1}-C\left(e_{t+1}\right)\right)+C\left(e_{t+1}\right) \tag{A6.22}
\end{align*}
$$

The results follow.
Q.E.D.

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[^0]:    ${ }^{1}$ Page that the table is inserted (Page that the table appears).

[^1]:    ${ }^{2}$ Page that the figure is inserted (Page that the figure appears).

[^2]:    "Some papers call "complete" and "incomplete" disclosure as "full" and "partial" disclosure, respectively. We would like to save these terms for the types of equilibria that appear in the subsequent discussion.

[^3]:    ${ }^{2}$ In this dissertation, the pronoun "he" represents either "he" or "she".

[^4]:    ${ }^{3}$ The market in this model is not a fully active player in the game in that it does not play strategically. Instead, the market is quite passive and is only modelled to the extent of considering how it forms beliefs about the firm's cash flows. This point is common to all papers in this class, including the model in Chapter 3.

[^5]:    ${ }^{4}$ The role of the supplier is not explicitly modelled. The supplier merely serves as a player to whom the owner can commit to a replacement standard, which is an important determinant in the way in which the retention-replacement decision is viewed by external investors.

[^6]:    ${ }^{5}$ The analysis in chapter 3 is a more extensive presentation of the analysis contained in Feltham/Xie [1991].

[^7]:    ${ }^{1}$ There have been a number of models in which management actions, such as their choice of dividend policy or capital structure, are viewed as methods of providing outsiders with assurances that they are not lying.

[^8]:    ${ }^{2}$ In most of the entry game literature, "entry cost" refers to the cost incurred by the entrant if he chooses to enter a market. In this chapter, we use "entry cost" to refer to the reduction in profit incurred by the incumbent if the entrant enters.

[^9]:    ${ }^{3}$ Paul Fischer raises an interesting question with regard to the objectives of well-diversified investors if they hold shares in both firms I and $E$. We effectively assume the firms are owned by two different sets of well diversified investors and, hence, they have no incentive to motivate the managers' of the firms to collude.

[^10]:    ${ }^{4}$ Appendix 3.A describes a product market in which the selling price is a linear decreasing function of the total output supplied by $I$ and $E$ and $I$ has private information about the intercept of that price function. The expected profits are not linear functions of I's price information, but the appendix demonstrates how the linear model used in this paper can be interpreted as a representation of that market.

[^11]:    ${ }^{5}$ Darrough and Stoughton [1990] and Wagenhofer [1990] focus on this case.

[^12]:    ${ }^{6}$ We assume that the player's strategies in this threeperson game constitute a sequential equilibrium. Sequential rationality requires that $E$ select the action that maximizes his expected payoff given his beliefs at the time he takes his action.

[^13]:    ${ }^{7}$ That is, $G(\gamma)=0 \forall \gamma \in[0, \bar{\gamma})$ and $G(\gamma)=1 \forall \gamma \in[\bar{\gamma}, 1]$.

[^14]:    ${ }^{8}$ The assumption that $I$ obtains capital through only issuing equity removes the possibility of signalling through the choice of payoff function on the security, as is done in Brennan and Kraus [1987]. Of course, such signalling is not necessary in our analysis since it is assumed that direct disclosure is viable.

[^15]:    ${ }^{9}$ We must stress that the market is a player in the game, but it is not a strategic player. In other words, the market is not a fully active player in our game in that it does not play strategically. Instead, the market is quite passive and is only modelled to the extent of considering how it forms beliefs about the firm's cash flows.
    ${ }^{10}$ We could allow $M$ to play a mixed strategy if the expected net return is zero. However, I can always avoid this case by setting $\alpha$ slightly higher. It is sufficient for our analysis to allow only $E$ to play mixed strategies, and then only when his break-even point is common knowledge.

[^16]:    ${ }^{11}$ See $\operatorname{Degroot~[1970,~p.40].~}$

[^17]:    ${ }^{12}$ In this figure, $a=15, b=37, c=6, d=31$, and $k$ $=28$. In the unimodel distribution case, $\beta_{1}=5$ and $\beta_{2}=10$, implying that $\beta_{0}=10,010$.

[^18]:    ${ }^{13}$ See the appendix for a proof of this lemma, as well as the proofs for other lemmas and propositions.

[^19]:    ${ }^{14}$ This lemma can be proven rigorously, but the proof is tedious and we merely appeal to the reader's intuition given the shapes depicted in Figure 3-2.
    ${ }^{15}$ In general, $\mathbf{N}$ consists of two intervals, but $D$ can consist of a single interval, with $D_{2}$ empty. It is possible to have $D_{1}$ empty, but only in "knife-edge" cases that are not generic.

[^20]:    ${ }^{16}$ See Kreps and Wilson [1982]. We take some liberty in applying their concept since, technically, sequential equilibria are only defined for finite types and actions.

[^21]:    ${ }^{17}$ The preceding discussion sketches the proof of this result.

[^22]:    ${ }^{18}$ The preceding discussion sketches the proof of this result.

[^23]:    ${ }^{21}$ While the Wagenhofer [1990] model is similar to ours, it is sufficiently different and specialized that, unlike our results, FD equilibria always exist and PD-H equilibria never exist.
    ${ }^{22}$ Observe that, in a PD-H equilibrium, not all types in $D_{1}$ have a lower market value than the non-disclosure firms. However, because $W_{N}$ has a strictly positive slope, there are always types at the low end of $D_{1}$ that have a strictly lower market value than $V\left(v^{0}, p\left(v^{0}\right)\right)$.

[^24]:    ${ }^{23}$ We found this condition to hold in other numerical examples, but the complexity of expressions did not allow us to prove that it would always hold when $\Delta=0$.

[^25]:    ${ }^{24}$ The proof provides a more detailed characterization of $\mathrm{K}-\mathrm{H}$ and $\mathrm{K}-\mathrm{L}$. In particular, the proof establishes that $\overline{\mathrm{k}}_{2}$ is strictly greater than $K_{2}$ and that $\bar{k}_{1}$ may be greater than $\mathrm{K}_{1}$. We omit the additional detail from the text so that the proposition focuses on the key aspects of the result.

[^26]:    ${ }^{25}$ Stability is a refinement of Nash equilibria that has been proposed by Kohlberg and Mertens [1986].

[^27]:    ${ }^{26}$ It is an open question as to what entry probability we should use in the case in which E's break-even point is common knowledge and $v=\bar{\gamma}$. We have chosen to allow $p$ to vary between zero and one (in all other situations $G^{*}(v)=$ G(v)). This is consistent with the perspective that the common knowledge case is the limit of the unimodel distribution case, in which $G^{*}(v)=G(v)$ for all $v \in[0,1]$.

[^28]:    ${ }^{1}$ In the following analysis the central management sets a policy regarding which division has control of the transfer decision and the divisions negotiate ex ante prices. Although this seems inconsistent with some observations, we believe that the divisions play some role in the setting of these prices. It is unlikely that the central management can act alone in deciding the transfer policies. We leave this question open for further research.
    ${ }^{2}$ Atkinson [1987] found that only $7 \%$ of Canadian firms determine transfer prices on the basis of negotiation.

[^29]:    ${ }^{3}$ Williamson [1985] uses the term adaptation to describe the actions or processes the contracting parties take to adjust to environmental conditions. Similarly, the term maladaption means poor or inadquate adaptation.

[^30]:    ${ }^{4}$ The zero measure events $v=c$ are not important. When this occurs, the trading decision can be set arbitrarily.
    ${ }^{5} p_{0}$ can be thought of as the damages the buyer pays the seller or vice versa. It is a kind of penalty for a failure to complete the transaction.

[^31]:    ${ }^{6}$ The shapes of these regions depend on the parameter values of boundaries of $V$ and $C$. Particularly, they can be extended to infinity if either $v$ and $c$ is distributed on infinite intervals.

[^32]:    ${ }^{7}$ See Kalai [1985] for a detailed discussion.

[^33]:    ${ }^{8}$ There is an implicit assumption in our analysis, that a division cannot refuse to bargain at $t_{1}$-- they must bargain and the results depend on their exogenously specified bargaining power. $p_{0}$ ensures that the expected gains from future negotiations and trades (or no trades) result in expected gains consistent with their initial bargaining power. Hence, in general, $p_{0}$ may not be zero.
    ${ }^{9}$ In general, ex post bargaining costs for $p_{1}$ for one particular realization of ( $v, c$ ) may be less than the total costs of bargaining for all possible realizations of ( $\mathrm{v}, \mathrm{c}$ ). However, if $p_{1}$ can be expressed as a function of $v$ and $c$ (as well as $\alpha$ ), then the costs of bargaining may be independent of the number of realizations.

[^34]:    ${ }^{10}$ This is to simplify our discussion. In fact, as long as $\mathrm{TCN}_{1}$ and $\mathrm{CN}_{1}$ are positively correlated, i.e., TCN increases as $C N$, increases, $\triangle T C$ can be shown to be increasing in $\mathrm{CN}_{1}$.

[^35]:    ${ }^{11}$ All the proofs of propositions are presented in the appendix if they do not appear following the propositions.

[^36]:    ${ }^{12}$ Another case is one in which both $S$ and $B$ have no control of the trading decision. This implies that the central management must make the trading decision for the two divisions. That is in conflict with the concept of decentralized firms and, hence, we exclude this case from our analysis. Thanks to Rajiv Banker for comments on this point.

[^37]:    ${ }^{13}$ The preceding arguments sketch the proof of this proposition.

[^38]:    ${ }^{14}$ The preceding arguments sketch the proof of this proposition.

[^39]:    ${ }^{15}$ The preceding arguments sketch the proof of this proposition.

[^40]:    ${ }^{16}$ The following categories exhaust all non-trivial situations.

[^41]:    ${ }^{17}$ The preceding discussion sketches the proof of this proposition.

[^42]:    ${ }^{18}$ Keep in mind that $v$ and $c$ are expectations based on all the information available to the managers at the time they make their transfer decision.

[^43]:    ${ }^{19}$ Evaluated at $t=0$.

[^44]:    ${ }^{20}$ For bounded intervals, the results will not be changed, but the calculations are different.

[^45]:    ${ }^{21} \mathrm{We}$ assume R is ${ }^{\prime}$ 'small'' relative to other parameters. For ''big'' $R$, the calculation may differ but the conclusions
    will not be changed.

[^46]:    1 Williamson [1985] introduces the term "high powered incentives" to refer to the incentives created by the market price system. We use this term in a slightly different way. In this paper, the term "high powered incentives" refers to contracts which specify an explicit relationship between compensation and some measure of performance.

[^47]:    ${ }^{2}$ In general, the time discount rates may not be the same for the firm and the employee. We make this assumption for simplicity only.

[^48]:    ${ }^{3}$ The employee's disutility for effort is introduced later.

[^49]:    4 Zero effort does not mean no effort, but rather a normalized lowest effort level.

[^50]:    ${ }^{5}$ Imperfect relationships would create additional risk. Given the employee's special utility function, this will not influence the incentives to insert effort, provided the risk created by the imperfect measurement is independent of his effort level. For the case in which this risk depends on employee's effort, the analysis may be more complicated.
    ${ }^{6}$ This implies that $A_{t}, B_{t}$, and $H_{t}$ are random variables, since $X_{t}$ is a random variable.
    ${ }^{7}$ To simplify our analysis, we ignore the negative values of $H_{t}$, i.e., we assume the probability distributions are such that there is essentially zero probability of negative $H_{t}$ despite the use of a normal distribution.

[^51]:    ${ }^{8}$ In the following analyses there is no loss of generality in considering only linear contracts except when we consider the second-best contract. Since linear contracts may not be optimal in the second-best cases, there is a potential loss of generality in our results.

[^52]:    ${ }^{10}$ The direct additivity of these utility expressions follows from the fact that mean/variance utility functions have been used and the utility functions have been scaled such that an increase in the employee's compensation by $\$ 1$ increases $U$ by 1 unit and decreases $V$ by 1 unit. In general, it is meaningless to add utility functions since intercomparison of utilities across individuals is not acceptable with von Newmann/Morgenstern utility functions.

[^53]:    ${ }^{11}$ As pointed in a prior footnote, the optimal secondbest contract is not linear in general. Our restriction on a linear contract may have a potential loss of generality in this subsection.

[^54]:    ${ }^{12}$ Observe that $U+V$ is independent of $p_{m}$ and $p$, and so is $\alpha$. This implies that the bargaining process has no influence on equilibrium risk sharing.

[^55]:    ${ }^{13}$ This riskiness of $w_{2}$ will influence the employee's ex ante expected utility.

[^56]:    ${ }^{14}$ This assumes that the firm would hire another employee from the market with same NSHA.

[^57]:    ${ }^{15}$ The explicit recognition of the contracting costs could change this limiting result. That is, things change as the limits are approached if there are contracting costs.

[^58]:    ${ }^{16}$ The share of current output is based on the perceived expectation of the firm and the employee, whereas the HA at the end of the period depends on the their true expectation.

