Design of Structural Geometries for Large Telescope Enclosures

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Abstract

With the rapid advancements in telescope technology, the next generation telescopes will be on the order of 20 – 50 [m] in diameter. Since traditional telescopes have been substantially smaller, this huge increase in telescope size necessitates a study into new solutions for telescope enclosure geometry.

In an attempt to come up with new geometries for large telescope enclosures, a study was carried out on Platonic and Archimedean spheres. After careful consideration, the Platonic icosahedral sphere (a type of geodesic dome) and the Archimedean rhombicosidodecahedral sphere was selected for further analysis.

Sensitivity analysis was performed on the two selected configurations by varying the enclosure radius (R), and the member cross-section size (CSS), thickness (T), and type (CT). A 3-dimensional model of each case was generated in a finite element analysis program and the corresponding nodal deflections and member forces were obtained. After plotting the results, it was discovered that for both the rhombicosidodecahedron and icosahedron configuration, the optimal R is 25 [m]. The optimal CSS for the rhombicosidodecahedron configuration is 0.35 [m] while the optimal CSS for the icosahedron configuration is 0.3 [m]. The optimal T for both configurations is 0.02 [m]. In addition, the optimal CT for the rhombicosidodecahedron configuration is circular while the optimal CT for the icosahedron configuration is square. The two configurations were also compared against one another and
it was discovered that the icosahedron configuration generally performs better than the rhombicosidodecahedron configuration.

In addition to exploring new structural geometries for telescope enclosures, one must not forget all the expertise which has been put in older telescope enclosures. PhotoModeler is a photogrammetry software which allows its user to take pictures of an existing structure then generate 3-dimensional models using various functions in the software. Using this program, one can combine structural attributes from older telescope enclosures with new geometries to create a hybrid enclosure suitable for next generation telescopes.

Finally, as a supplementary to the structural geometries suggested in this report, several composite materials which may be suitable for use as cladding are also presented. These composite panels include: metal composites, polymer composites, and honeycomb core composites.
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List of Symbols and Abbreviations

\( a_R = \) resolution limit [radians]
\( \lambda = \) wavelength
\( \delta = \) angular deficiency
\( \phi = \) Golden Ratio
\( \rho = \) density of fluid
\( \nu = \) kinematic viscosity
\( A = \) projected area normal to direction of flow, lateral acceleration due to earthquake
\( C = \) center point of lens
\( C_a = \) shape factor (snow load calculation)
\( C_b = \) basic roof snow load factor (snow load calculation)
\( C_D = \) drag coefficient
\( C_e = \) exposure factor
\( C_g = \) gust factor
\( C_p = \) external pressure coefficient
\( C_s = \) slope factor (snow load calculation)
\( C_w = \) wind exposure factor (snow load calculation)
\( CSS = \) member cross-section size
\( CT = \) member cross-section type
\( D = \) aperture (width of the objective), object diameter
\( F = \) focal point of central rays
\( F' = \) focal points of edge rays
\( f = \) vortex shedding frequency
\( F_D = \) drag force
\( F_s = \) force due to factored snow and ice load
\( I_E = \) importance factor for earthquake
\( I_w = \) importance factor for wind load
\( I_s = \) importance factor for snow load (snow load calculation)
\( i = \) unit vector along X-axis
\( j = \text{unit vector along } Y\text{-axis} \)
\( \hat{k} = \text{unit vector along } Z\text{-axis} \)
\( M_v = \text{factor to account for higher mode effect on base shear} \)
\( \text{NBCC} = \text{National Building Code of Canada} \)
\( p = \text{number of sides in each face of a Platonic solid, external pressure caused by wind on structure} \)
\( q = \text{number of faces meeting at each vertex in a Platonic solid, reference velocity pressure} \)
\( R = \text{enclosure radius} \)
\( R_d = \text{ductility related force modification factor} \)
\( \text{Re} = \text{Reynolds number} \)
\( R_o = \text{overstrength related force modification factor} \)
\( S(T_a) = \text{spectral acceleration at time } T_a \text{ according to UHS} \)
\( S_r = 1 \text{ in 50 year rain load (snow load calculation)} \)
\( S_s = \text{snow and ice load} \)
\( S = \text{factored snow and ice load} \)
\( St = \text{Strouhal number} \)
\( T = \text{member cross-section thickness} \)
\( T_a = \text{fundamental lateral period of vibration of structure} \)
\( \text{UHS} = \text{Uniform hazard spectrum} \)
\( V = \text{wind velocity} \)
\( W = \text{wind load} \)
Glossary

**Angular Resolution**: the ability of an optical instrument to measure angular separations between points on an object.

**Boundary layer**: a thin layer attached to the boundary in which viscous effects are concentrated.

**Chromatic aberration**: caused by the lens having a different refraction index for different wavelengths of light, resulting in a fringe of color around an image.

**Coma**: a monochromatic aberration of a lens where a point source image cannot be focused, the resulting image has a comet-shaped appearance.

**Dual polyhedron**: pairs of polyhedra where the vertices of one correspond to the faces of its dual.

**F-ratio**: ratio of focal length to aperture.

**Focal length**: a measure of how an optical system focuses or diverges light.

**Golden Ratio ($\phi$)**: the golden ratio is obtained between two numbers if the ratio of the smaller number to the larger number is the same as the ratio of the larger number to the sum of the two numbers.

$$\phi := \frac{1 + \sqrt{5}}{2}$$

**Inviscid flow**: flow in which viscous effects are not significant.

**Laminar flow**: flow in which layers of fluid move smoothly over or alongside adjacent layers. Streamlines are smooth curves.

**Mesh**: an interwoven or intertwined structure with evenly spaced and uniform openings.
**Reynolds Number (Re):** a parameter which depicts a flow regime.

\[ Re := \frac{V D}{\nu} \]

Where
- \( V \) = velocity \([L/T]\)
- \( D \) = characteristic length \([L]\)
- \( \nu \) = kinematic viscosity \([L^2/T]\)

**Separated region:** region of re-circulating flow

**Spherical aberrations:** occurs when light rays hitting near the edges of a lens refract differently than light rays hitting the center

**Stagnation point:** a point where fluid comes to rest

**Steady flow:** quantities such as velocity, pressure, or density do not depend on time

**Strouhal number (St):** a dimensionless parameter which describes the shedding frequency of an object

**Turbulent flow:** a flow which does not exhibit laminar flow characteristics. Turbulent flow is also characterized by random particle motion and distortion superimposed on a general streaming motion.

**Uniform Hazard Spectrum:** spectral acceleration plot, for a given period, the spectral acceleration has the same probability of exceedance as the spectral acceleration at any other period on the plot

**Unsteady flow:** quantities vary with time

**Viscous flow:** flow in which viscous effects are significant

**Vortex shedding:** long blunt objects placed normal to fluid flow often exhibit vortex shedding. Occurs at \(40 < Re < 10000\) and accompanied by turbulence above \(Re = 300\)

**Vortices:** a spiral motion of fluid, a whirling mass of water

**Wake:** a region of velocity defect that grows due to diffusion
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1. Overview and Summary

The purpose of this thesis is to explore different approaches to enclosing large telescopes. With rapid advancements in telescope and aerospace technology, higher demands have been placed on the telescope housing structure, also known as the enclosure. Telescope enclosures must provide the telescope with minimal temperature gradients and protection from natural elements (wind, snow, rain...etc) while spanning up to 50 to 100 [m] in diameter. All of the above requirements necessitate an exploration into the beginnings of telescope enclosure design in an attempt to generate new geometric solutions for the problem of housing a large telescope. The following sections will include a brief description of the steps taken to generate new enclosure geometries and the analysis completed to validate each new design.

1.1. Literature Review

Preliminary research has already been performed in this area and observatories all over the world have been placing increasing emphasis on the design of next generation telescopes and enclosures. Mixter and Porter from the National Optical Astronomy Observatories in the United States have proposed a geodesic type, rotating 91 [m] diameter aluminum dome as one possible solution to enclosing large telescopes. The paper concludes that the design of an enclosure to house telescopes up to 50 [m] in diameter is possible using current construction methods. Other research in this area includes Andersen and Christensen’s paper on the upper limit of the size of telescope enclosures. Their paper concludes that it is entirely possible to design and construct enclosures capable of housing telescopes up to 100 [m] in diameter. Andersen and Christensen’s proposed design is 131 [m] in diameter and has an
1. Overview and Summary

octagonal bottom level and a pentagonal top. The papers alluded to in this section are a few examples of the numerous research efforts currently dedicated to the study of large telescope enclosures.

In addition to the research described above which is directly targeted at designing large telescope enclosures, there are studies in the mathematics field which aim at developing geometric algorithms to enclose large spaces. Otero et al. has published a paper describing methods of generating a spatial mesh or structure polyhedrons which approximates the shape of the ideal structure. This method, also known as computational geometry, demonstrates that 2-dimensional polyhedrons may be mapped onto a 3-dimensional surface systematically to obtain a dome like structure.

This report is based upon the ideologies of the papers described above, combining spatial mesh generation with structural analysis in an attempt to explore and propose new structural geometries for large telescope enclosures.

1.2. Overview

In order to improve upon a particular structure, one must understand its historical evolution. By doing so, one can retain a structure’s strengths while eradicating its weaknesses. As a result, the first section in this report provides the reader with a brief history of the evolution of telescopes and telescope enclosures. After providing the reader with basic background information, the next section explains the various methods of generating and obtaining enclosure geometries.
1. Overview and Summary

After extensive research into various geometries, two configurations were selected, the Platonic icosahedral sphere and the Archimedean rhombicosidodecahedral sphere. In order to evaluate the performance of these configurations at different conditions, different “cases” were developed. For each case, a different variable (enclosure radius, member cross-section size, member cross-section thickness, and member cross-section type) was changed. After developing these cases, a finite element program was used to analyze the structure under different loading conditions as suggested by provisions in the NBCC 2005. Nodal deflections and member forces were then obtained from the program and plotted.

This report also introduces a relatively new software called PhotoModeler which has the capabilities to generate 3-dimensional models from photographs. Using this program, one can assess the performance of existing telescope enclosures and combine its good qualities with potential designs in order to obtain a hybrid structure with “optimal” characteristics.

Finally, as a supplementary to the structural geometries proposed in this report, several composite materials are presented as potential options for cladding material.
2. The Historical Evolution of Telescopes and Telescope Enclosures

Before one ventures into the evolution and history of telescope enclosures, it is important to have a basic understanding of the telescope itself. By understanding how conventional telescopes function, enclosures can be designed to better suit the telescope's needs. The following sections are a brief summary of the evolution of telescopes.

2.1. History of Telescopes

The earliest form of the telescope can be found as far back as 3000 years ago ("Telescope", Wikipedia). A rock crystal lens found in the Nimrud palace complex in northern Iraq in 1980 suggests that the ancient Assyrians used it as a means of observing the heavens. However, scientists claim that due to the lens' low quality, it would have been an inadequate vision aid (Whitehouse 1999). The second appearance of early telescopes, also known as the Visby lenses, was in Gotlandia, Sweden. Found in an ancient Viking grave which dates back to the 10th century, the Visby lenses were thought to be used as magnifiers by the Vikings.

Dutch spectacle maker, Hans Lippershey is the first person documented for assembling the first telescope in 1608 ("Telescope", Wikipedia). Shortly after, Galileo Galilei, one of the most famous astronomer and physicist of all time, created his own telescope and employed it for astronomical purposes. Galileo's telescope consisted of a convex lens fastened to one extremity of a leaden tube and a concave lens on the other end. His telescope was the earliest form of refracting telescopes. Improvements to Galileo's telescope were made by German astronomer and mathematician, Johannes Kepler ("Telescopes", Encyclopaedia Britannica).
2. The Historical Evolution of Telescopes and Telescope Enclosures

Kepler suggested that the usage of two convex lens rather than a convex and a concave lens. However, it was William Gascoigne who first employed Kepler’s suggestions into an actual telescope. By the mid-17th century, Kepler’s telescope became widely used; although the sharpness was inferior to Galileo’s telescope, its wide field of vision made it a popular choice amongst astronomers of that time.

In 1666, Sir Isaac Newton discovered that different wavelengths of light had different refraction indices. Prior to Newton’s discovery, it was believed that errors in a refracting telescope are solely caused by spherical aberrations (described in section 2.2). After recognizing that chromatic aberration was the main cause of inaccuracy in refracting telescopes, Newton constructed the first reflecting telescope from an alloy of tin and copper. Although Newton is often thought to have constructed the first reflecting telescope, the idea of a reflecting telescope was actually proposed by Scottish mathematician and astronomer, James Gregory, in 1663, three years prior to Newton’s discovery of the refrangibility of light. The Gregorian telescope, aptly named after the Scottish astronomer, employed elliptical mirrors, thus preventing potential image distortion caused by spherical aberration (“Telescopes”, Encyclopaedia Britannica). Unfortunately, Gregory was not skilled in telescope construction; consequently, his ideas were not employed in an actual telescope until 1673.

A third type of telescope, the achromatic refracting telescope, was created in 1733 by Chester Moor Hall, an English mathematician (“Hall, Chester Moor”, Encyclopedia Britannica Online). Hall proposed that the different refrangibility of light could be corrected if a
2. The Historical Evolution of Telescopes and Telescope Enclosures

A combination of lenses formed by different glass was used. As a result, the achromatic refracting telescope was able to produce images free from color.

2.2. Types of Telescopes

Telescopes may be divided into two large categories: radio telescopes and optical telescopes. Each of these two types of telescopes will be discussed in the following sections with emphasis placed on the latter.

2.2.1. Radio Telescopes

Contrary to optical telescopes, radio telescopes do not reflect light rays; instead, radio telescopes measure the intensity of radio waves over a specified band of frequencies. Radio telescopes generally consist of a large parabolic reflector/dish which focuses incoming radiations emitted by extraterrestrial bodies onto an antenna, also known as the feed ("Astronomical Radio Telescopes", Kosmoi). The radio signal is filtered and amplified, then processed by a computer. The resulting output is usually an image which follows the shape of the radio emissions. By scanning a section of the sky multiple times, images generated from each scan may be pieced together to form a map of the sky. Because radio telescopes do not need light in order to detect the presence of an extraterrestrial body, it is able to "see" objects invisible to optical telescopes. However, similar to optical telescopes, the accuracy of an image obtained from a radio telescope is affected by the following factors:

1. Accuracy of reflecting surface (manufacturing irregularities)
2. Wind loading and thermal loading
3. Gravity induced deflection of dish
Deflection becomes an important issue when its magnitude exceeds a few percent of the operation wavelength. Since radio waves are much longer than optical light waves, an image of comparable resolution to an image obtained from optical telescopes require a much larger radio telescope or an array of radio telescopes working together. However, as the diameter of the dish increases, deflection problems are amplified. Hence, it is important to obtain a balance between magnification and accuracy.

2.2.2. Optical Telescopes

Optical telescopes function by employing a primary element (a mirror or a lens) to gather and focus light. An eyepiece is then used to magnify the object and produce a virtual image ("Telescopes", Encyclopaedia Britannica). There are several common parameters which describes a telescope, they are: angular resolution, focal length, and light gathering power. Each parameter characterizes a telescope’s capability and suitability for different purposes.

Angular resolution is the ability of an optical instrument to measure angular separations between points on an object. If turbulence in the atmosphere and telescope imperfection is ignored, angular resolution may be calculated using the following formula ("Optical Telescope", Wikipedia):

\[ \alpha_R := \frac{1.22 \lambda}{D} \]

where \( a_R = \text{resolution limit [radians]} \)

\( \lambda = \text{wavelength} \)

\( D = \text{aperture (width of the objective)} \)
The focal length is a measure of how an optical system focuses or diverges light. For a telescope, it specifically describes how wide an angle the telescope can measure with a given eyepiece. A shorter focal length yields greater magnification of an object while a longer focal length yields less magnification (refer to figure 2.1 below).

The f-ratio is another term often associated with the focal length. It is the ratio of focal length to the aperture. Hence a small f-ratio will indicate a wider field of view. The diameter of a telescope determines its light gathering power. Since area is proportional to the square of the radius, if the diameter of an objective lens is multiplied by two, its light gathering power will increase by four-folds.

There are two main types of optical telescopes: refracting (Galilean) and reflecting (Newtonian, Cassagrain). Over the years, a third type of optical telescope has emerged. This third type is a hybrid of the Cassegrain telescope and combines principles of refraction and reflection; hence, it uses a combination of lenses and mirrors to produce images. Each of the aforementioned optical telescopes will be described in the following sections.
2. The Historical Evolution of Telescopes and Telescope Enclosures

Refracting Telescopes -

Refracting telescopes function by refracting or bending light using lenses. Parallel light rays converge at a single focal point and non-parallel light rays converge at a focal plane (See Figure 2.2 below).

![Kepler's Telescope](image1)

Objective Lens (Convex)

Eyepiece Lens

Eye

Kepler's Telescope

![Galileo's Telescope](image2)

Objective Lens (Convex)

Eyepiece Lens (Concave)

Eye

Galileo's Telescope

Figure 2.2 Refracting Telescopes

Refracting telescopes are prone to residual **chromatic** and **spherical aberrations**. Chromatic aberration is caused by the lens having a different refraction index for different wavelengths of light, resulting in a fringe of color around an image. There are two types of chromatic aberrations, longitudinal and transverse. Longitudinal chromatic aberration occurs when different wavelengths are focused at different focal lengths and transverse chromatic aberration occurs when the wavelengths focus at different positions on the focal plane. However, chromatic aberration can be minimized by using two lenses of different chemical composition; thus reducing the amount of chromatic aberration over a certain range of wavelengths (Nave 2000).
Spherical aberration occurs when light rays hitting near the edges of a lens refract differently than light rays hitting the center (See Figure 2.3 below). The result is a blurred image due to failure of the light rays to focus at a common point (Hendersen 2004). Spherical aberration may be minimized by using a system of convex and concave lenses or using aspheric lenses.

![Figure 2.3 Spherical Aberration](image)

\[ C = \text{Center Point of Lens} \quad F = \text{Focal Point of Central Rays} \]

\[ F' = \text{Focal Point of Edge Rays} \]

Over the years, refracting telescopes have become less popular as a choice for research telescopes due to its numerous problems. As mentioned above, refracting telescopes are prone to chromatic and spherical aberrations. In addition, the volume of the lens used in a refracting telescope must be entirely free of defects or air bubbles. This can be difficult to achieve when the lens exceeds 30 - 50 [cm] in diameter. Even if it were possible to manufacture perfects lenses, it would be difficult to provide support for the lens so that it would not sag due to gravity. Since support can only be provided around the perimeter of the
2. The Historical Evolution of Telescopes and Telescope Enclosures

lens, this is a major issue for large lenses. On the other hand, the requirements for reflecting telescopes are less stringent (more details on reflecting telescopes will be presented in the following section). For reflecting telescopes, only one side of the mirror needs to be perfect and support can be administered on the entire backside of the mirror.

Reflecting Telescope -

Reflecting telescopes employ a system of curved and plane mirrors to produce an image ("Reflecting Telescopes", Wikipedia). The reflecting telescope was originally created by Sir Isaac Newton to overcome chromatic aberration which is a large problem in refracting telescopes. Because the image is reflected rather than refracted, the problem of different color having different refraction indices is eliminated.

In a reflecting telescope, the image is magnified and reflected by the primary mirror(s) then reflected by a secondary mirror to a focal plane where the image can be viewed. Although chromatic aberration is absent in reflecting telescopes, other types of image distortion such as spherical aberration and coma exists. If spherical mirrors are used, spherical aberration may become an issue for the telescope. However, this problem can be eliminated by adding a corrector lens or using a different shaped mirror. Parabolic mirrors are sometimes used to replace spherical mirrors to prevent spherical aberration. Unfortunately, parabolic mirrors are often associated with another type of image distortion – coma. Coma occurs when light rays from a point source (a star) are not parallel to the optical axis. Similar to spherical aberration, light rays which are further away from the optical axis are not reflected to the same point as central rays, resulting in a wedge-shaped image (see figure2.4 below).
There are several different types of reflecting telescope designs (see figure 2.5 below).

The Newtonian focus consists of a paraboloid primary mirror and a flat secondary mirror which reflects the light rays to a focal plane located on the side of the telescope. This type of design is simple and inexpensive; hence, it is most popular amongst amateur astronomers. One drawback of the Newtonian focus design is that the secondary mirror is supported by struts which reduce the quality of the viewed image.
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Similar to the Newtonian focus, the Cassegrain focus uses a paraboloid primary mirror; however, it employs a hyperboloid secondary mirror which reflects the light through an opening in the primary mirror. The secondary mirror is usually supported by a clear glass plate which seals the telescope tube, eliminating the problem of image obstruction affecting all Newtonian telescopes.

The Ritchey-Chretien focus has the same mirror configuration as the Cassegrain focus, the only difference is that the primary mirror is hyperbolic rather than parabolic. This simple change results in the elimination of coma and spherical aberration.

Cassegrain-Hybrid telescopes -

The Schmidt-Cassegrain focus and the Maksutov-Cassegrain focus are two common types of hybrid telescopes whose principles are based on the Cassegrain focus. Both telescopes have a similar mirror configuration as the classical Cassegrain focus (see figure 2.6 below). Spherical aberration is eradicated in the Schmidt-Cassegrain focus where a Schmidt corrector plate is added behind the secondary mirror. The Schmidt corrector plate is formed by producing a vacuum environment on one side of the plate and grinding the plate such that spherical aberration created by the primary mirror is abolished. Lastly, the Maksutov-Cassegrain focus is similar to a Schmidt-Cassegrain focus except that all lenses are spherical in shape. In addition, the secondary mirror is commonly a mirrored section of the corrector lens which results in less image obstruction.
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2.3. Evolution of Telescope Enclosures

Modern research telescopes have become increasingly expensive and powerful over the last decades. However, these telescopes are only capable of generating high quality images under very specific conditions; hence the need for an enclosure. In addition to sheltering expensive equipments, telescope enclosures protect the telescope from natural elements (rain, snow, dust, and wind) and temperature gradients. With the size of modern research telescopes reaching diameters of 30 – 50 [m], the size of telescope enclosures have increased substantially as well. In order to understand the implication of this sudden increase in size demands, it is important to first become familiar with the different types of telescope enclosures which exist today.

Figure 2.6 Cassegrain-Hybrid Telescopes
The requirements for a typical telescope enclosure are as follows (Quatti et al. 2000):

- Minimize enclosed volume and surface area (infrastructure cost usually proportional to developed surface)
- Protect telescope from solar degradation, snow, rain, dust, and wind
- Eliminate wind disturbance to the telescope during viewing
- Regulate ambient temperature within the enclosure to match external temperatures

In accordance with the first requirement, it is easy to understand why most telescope enclosures are spherical in shape. Spheres have a lesser volume and surface area than any cubic or prismatic solids. In addition, since telescopes usually rotate in a circular path, using a spherical enclosure would be the most space efficient approach (see figure 2.7 below).

One of the most conventional telescope enclosure designs is the rotating circular dome design with shutters which open laterally. The dome shaped enclosure allows for a complete range of motion for the telescope inside. Although this design is economical to build, it is better suited for smaller telescopes (Desroches 2003). As the size of the telescope increases, the enclosure size must increase accordingly. Since this dome shaped design consists of an “orange-peel” like structure, stability issues may pose as a problem when the diameter of the enclosure becomes sufficiently large.
A small change was later made to the aforementioned design to give the opening more freedom. Instead of having the shutters opening laterally, the shutters now slide over the enclosure vertically (see figure 2.8 below).

1 Courtesy of the Gemini Observatory Website <http://www.gemini.edu/>
2. The Historical Evolution of Telescopes and Telescope Enclosures

Although this design allows for more viewing freedom, it is mechanically more difficult and hence expensive to construct.

As the telescope enclosure evolved, another small change was made to improve its performance, the addition of arch girders (see figure 2.9 below).

![Arch Girder Design](image)

Figure 2.9 Arch Girder Design

Arch girders provide an overall increase in strength for the enclosure; thus allowing the construction of larger telescopes. The girders provide good resistance to distributed loads (wind, snow, dead load) but offer little advantage when faced with concentrated loads (crane, shutter, wind screen) (AURA et al. 2002). Although the arch girders contribute to an increase in strength for the enclosure, they lead to a break in surface continuity which may result in wind stability (vortex shedding) issues.

As telescopes became larger, the need for an enclosure which is easier to transport and construct arose. This led to variations from the classical dome-shutter design. The Calotte or
2. The Historical Evolution of Telescopes and Telescope Enclosures

“two-axis” system (see Figure 2.10 below) retains the domed shape structure of the classical design but deviates in its opening configuration.

![Calotte Telescope Enclosure Configuration](image)

**Figure 2.10 Calotte Telescope Enclosure Configuration**

The absence of protruding arch girders minimizes the effects of wind forces and eliminates the problem of uneven weight distribution on the ring girder. However, there are also several problems associated with this design. Due to its small opening, ventilation is often an issue with the Calotte design. Another challenge is the design of the support structure for the subsection which rotates at a significant angle to the horizontal. Nevertheless, the Calotte design has the potential of housing larger telescopes than the conventional arch girder design due to limitations in arch girder sizes.
Geodesic domes are another method of housing telescopes. Made up of relatively small members which are connected together to form a space frame, geodesic domes are lighter and hence more economical to build and construct than conventional shell structures. Though most telescope enclosures are spherical in shape, more recent telescope enclosures have taken on different geometries such as faceted, rectangular, and cylindrical. With all three geometries, straight sections can be used for structural framing; thus simplifying the overall design and construction process. Although these geometries yield a smaller enclosed volume, they make up for it in the cost savings achieved. These new designs however, are not without fault. Abrupt changes in surface geometry (edges and corners) result in higher wind loading demands. In addition, higher roof loads are also expected due to accumulation of snow and ice on the flat roof.

Figure 2.11 Cylindrical Telescope Enclosure Configuration
2. The Historical Evolution of Telescopes and Telescope Enclosures

Figure 2.12 Faceted Telescope Enclosure Configuration

Figure 2.13 Rectangular Telescope Enclosure Configuration
3. Methods of Generating Enclosure Geometries

There are currently many softwares available with capabilities to generate meshes on 3-dimensional surfaces.

3.1. Platonic and Archimedean Solids

Triangles – the basic building block of all polygons. It is no surprise that triangles are considered the strongest polygon. Thousands of years ago, the ancient Egyptians built triangular pyramids which still stand tall today. Across the ocean in South America, the ancient Aztecs and Mayan civilization also built massive pyramids which used to be one of the tallest buildings in America (see Figure 3.1).

Figure 3.1 Mayan Pyramid located in the ancient city of Uxmal

3. Methods of Generating Enclosure Geometries

It is no coincidence that these separate civilizations based their architecture on triangular shapes. Triangles are stable and easy to build compared to rectangular frames (squares) and arches (circles). For this reason, it is also found in the modern world in the form of trusses.

The two structural configurations suggested in this report is based on a repeated pattern of triangles arranged such that a spherical 3-dimensional structure is achieved. Both configurations are based on simple polyhedral (Platonic or Archimedean) solids projected onto the surface of a circumscribing sphere. The following sections will provide background information on both Platonic and Archimedean solids.

3.1.1. Platonic Solids

Platonic solids consist of five regular polyhedrons and date back to the ancient Greeks (Neubert 2003). Although evidence suggests that the Neolithic people of Scotland discovered these polyhedrons long before the Greeks, they were not fully documented until the time of Plato from which they are named after.
The Platonic solids represent all the regular polyhedrons which exist. Platonic solids are made up of a single repeated shape; hence its popularity in construction due to its simple elegance. In other words, a regular polyhedron must satisfy the following criterions ("Platonic Solids", Wikipedia):

1. All its faces are congruent regular polygons, and
2. The same number of faces meet at each vertices

Due to these characteristics, each Platonic solid can be described by a Schl"afli symbol, \(\{p, q\}\). The \(p\) represents the number of sides in each face and \(q\) represents the number of faces meeting at each vertex.

\[\text{Figure 3.2 Platonic Solids}^3\]
3. Methods of Generating Enclosure Geometries

Table 3.1 Properties of Platonic Solids

<table>
<thead>
<tr>
<th>Polyhedron</th>
<th>Faces</th>
<th>Edges</th>
<th>Vertices</th>
<th>Schlafli symbol</th>
<th>Vertex Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrahedron</td>
<td>4</td>
<td>6</td>
<td>4</td>
<td>{3, 3}</td>
<td>3.3.3</td>
</tr>
<tr>
<td>Cube</td>
<td>6</td>
<td>12</td>
<td>8</td>
<td>{4, 3}</td>
<td>4.4.4</td>
</tr>
<tr>
<td>Octahedron</td>
<td>8</td>
<td>12</td>
<td>6</td>
<td>{3, 4}</td>
<td>3.3.3.3</td>
</tr>
<tr>
<td>Dodecahedron</td>
<td>12</td>
<td>30</td>
<td>20</td>
<td>{5, 3}</td>
<td>5.5.5</td>
</tr>
<tr>
<td>Icosahedron</td>
<td>20</td>
<td>30</td>
<td>12</td>
<td>{3, 5}</td>
<td>3.3.3.3.3</td>
</tr>
</tbody>
</table>

From Table 3.1 above, one can see that a tetrahedron, which is made up of triangles has the Schlafli symbol \{3,3\}. This is because each face is made up of triangles which has 3 sides, and at each vertex, 3 triangles meet.

The vertex configuration represents the polyhedron’s vertex figure. The number conveys the number of edges in each face. Again, using the tetrahedron as an example, one can see that the vertex configuration is 3.3.3; this means that at each vertex, there are 3 triangles and that each triangle has 3 edges.

In order to construct 3-dimensional models of Platonic solids, it is important to first introduce several pertinent angles. Each of these angles will be introduced below (“Platonic Solids”, Wikipedia):

**Dihedral angle, \( \theta \) – interior angle between two face planes**

**Equation 3.1 Dihedral Angle Expression**

\[
\sin\left(\frac{\theta}{2}\right) = \frac{\cos\left(\frac{\pi}{q}\right)}{\sin\left(\frac{\pi}{p}\right)}
\]

where \( p \) and \( q \) are variables in the Schlafli symbol
Angular deficiency, \( \delta \) – difference between the sum of the face angles at a vertex and \( 2\pi \)

Equation 3.2 Angular Deficiency

\[
\delta := 2\pi - q \cdot \pi \left( 1 - \frac{2}{p} \right)
\]

The following table summarizes the dihedral angles and angular deficiency for each Platonic solid.

Table 3.2 Summary of dihedral angle and angular deficiency for each Platonic solid

<table>
<thead>
<tr>
<th>Polyhedron</th>
<th>Dihedral angle ((\theta))</th>
<th>(\tan \frac{\theta}{2})</th>
<th>Defect ((\delta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrahedron</td>
<td>70.53°</td>
<td>(\frac{1}{\sqrt{2}})</td>
<td>(\pi)</td>
</tr>
<tr>
<td>Cube</td>
<td>90°</td>
<td>1</td>
<td>(\frac{\pi}{2})</td>
</tr>
<tr>
<td>Octahedron</td>
<td>109.47°</td>
<td>(\sqrt{2})</td>
<td>(\frac{2\pi}{3})</td>
</tr>
<tr>
<td>Dodecahedron</td>
<td>116.56°</td>
<td>(\phi)</td>
<td>(\frac{\pi}{5})</td>
</tr>
<tr>
<td>Icosahedron</td>
<td>138.19°</td>
<td>(\phi^2)</td>
<td>(\frac{\pi}{3})</td>
</tr>
</tbody>
</table>

The \( \phi \) symbol in the table is the Golden Ratio. The golden ratio is actually a number (1.61803...) and has been associated with the architecture of various ancient civilizations (Egyptians and Greeks). In simple English, the golden ratio is obtained between two numbers if the ratio of the smaller number to the larger number is the same as the ratio of the larger number to the sum of the two numbers (“Golden Ratio”, Wikipedia). If two such numbers exist their ratio will be equal to 1.61803...
3. Methods of Generating Enclosure Geometries

Equation 3.3 Golden Ratio

\[ \phi := \frac{1 + \sqrt{5}}{2} \]

The reason Platonic solids were chosen as a starting point for the generation of structural geometries was its relationship to spheres. Each Platonic solid is related to 3 spheres: the circumscribed sphere, the inscribed sphere, and the midsphere. The circumscribed sphere passes through the vertices of the solid while the inscribed sphere is tangent to the center of each face. The midsphere is tangent to the midpoint of each edge. The circumscribed sphere is of most interest for the purpose of this report because all intermediate points are projected onto the circumscribing sphere. This will be further explained in Section 4.

3.1.2. Archimedean Solids

Archimedean solids are semi-regular convex polyhedrons and are named after the famous Greek mathematician and philosopher, Archimedes. While the Platonic solids described above consist of one type of polygon meeting at each vertex, Archimedean solids have more than one type of polygon meeting at each vertex. Archimedean solids are more complex to construct but can better conform to spheres than Platonic solids; thus its selection for this report.
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There are 5 types of Archimedean solids (Weisstein 2004) (see Figure 3.3 below):

1. Truncation of Platonic solids
2. Quasi-regular polyhedrons
3. Great Rhombi Archimedians
4. Small Rhombi Archimedians
5. Snub Archimedians

The first class is obtained by taking the Platonic solids and cutting off all the vertices to produce a new solid; the resulting solid has more than one type of polygon face. Quasi-regular polyhedrons consist of 2 different types of polygonal faces. In addition, each type of polygon is completely surrounded by the other type of polygon. The Great Rhombi Archimedians are also called the truncated quasi-regular polyhedrons; this is misleading because truncation and distortion of quasi-regular polyhedrons is required to yield the great rhombi Archimedians. Both the great rhombi Archimedians and small rhombi Archimedians have several faces which lie on the same plane as their dual polyhedron; thus its prefix “rhombi”. The snub Archimedean are chiral solids; this means that it has a left handed and right handed form. Due to its anti-symmetry, it is a poor candidate for spherical development compared to the other Archimedean solids.
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**Truncated Platonic Solids**
- Truncated Tetrahedron
- Truncated Cube
- Truncated Dodecahedron
- Truncated Octahedron
- Truncated Icosahedron

**Quasi-Regular Polyhedrons**
- Cuboctahedron
- Icosidodecahedron

**Small Rhombi Archimedeans**
- Rhombicosidodecahedron
- Rhombicuboctahedron

**Great Rhombi Archimedeans**
- Snub Cube
- Snub Dodecahedron

**Snub Archimedeans**
- Snub Cube
- Snub Dodecahedron

Figure 3.3 Archimedean Solid Classifications

Archimedean solids can also be described by its vertex configuration. One can see that because Archimedean solids have more than one type of face, the vertex configuration consists of different numbers. It is important to keep in mind that vertex configurations are

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3. Methods of Generating Enclosure Geometries

written in a clockwise direction around a vertex. Hence, 3.4.5.4 is the same as 4.5.4.3 but is
different from 3.5.4.4.

Table 3.3 Archimedean Solids Properties

<table>
<thead>
<tr>
<th>Name</th>
<th>Faces</th>
<th>Edges</th>
<th>Vertices</th>
<th>Vertex Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truncated Tetrahedron</td>
<td>8</td>
<td>18</td>
<td>12</td>
<td>3.6.6</td>
</tr>
<tr>
<td>Truncated Cube</td>
<td>14</td>
<td>36</td>
<td>24</td>
<td>3.8.8</td>
</tr>
<tr>
<td>or Truncated Hexahedron</td>
<td>14</td>
<td>36</td>
<td>24</td>
<td>4.6.6</td>
</tr>
<tr>
<td>Truncated Octahedron</td>
<td>14</td>
<td>36</td>
<td>24</td>
<td>4.6.6</td>
</tr>
<tr>
<td>Truncated Dodecahedron</td>
<td>32</td>
<td>90</td>
<td>60</td>
<td>3.10.10</td>
</tr>
<tr>
<td>Truncated Icosahedron</td>
<td>32</td>
<td>90</td>
<td>60</td>
<td>5.6.6</td>
</tr>
<tr>
<td>Cuboctahedron</td>
<td>14</td>
<td>24</td>
<td>12</td>
<td>3.4.3.4</td>
</tr>
<tr>
<td>Icosidodecahedron</td>
<td>32</td>
<td>60</td>
<td>30</td>
<td>3.5.3.5</td>
</tr>
<tr>
<td>Rhombicuboctahedron or Small</td>
<td>26</td>
<td>48</td>
<td>24</td>
<td>3.4.4.4</td>
</tr>
<tr>
<td>Rhombicuboctahedron</td>
<td>26</td>
<td>48</td>
<td>24</td>
<td>3.4.4.4</td>
</tr>
<tr>
<td>Rhombicosidodecahedron or</td>
<td>62</td>
<td>120</td>
<td>60</td>
<td>3.4.5.4</td>
</tr>
<tr>
<td>Small Rhombicosidodecahedron</td>
<td>62</td>
<td>120</td>
<td>60</td>
<td>3.4.5.4</td>
</tr>
<tr>
<td>Truncated Cuboctahedron or</td>
<td>26</td>
<td>72</td>
<td>48</td>
<td>4.6.8</td>
</tr>
<tr>
<td>Great Rhombicuboctahedron</td>
<td>26</td>
<td>72</td>
<td>48</td>
<td>4.6.8</td>
</tr>
<tr>
<td>Truncated Icosidodecahedron</td>
<td>62</td>
<td>180</td>
<td>120</td>
<td>4.6.10</td>
</tr>
<tr>
<td>or Great Rhombicosidodecahedron</td>
<td>62</td>
<td>180</td>
<td>120</td>
<td>4.6.10</td>
</tr>
<tr>
<td>Snub Cube</td>
<td>38</td>
<td>60</td>
<td>24</td>
<td>3.3.3.3.4</td>
</tr>
<tr>
<td>Snub Dodecahedron</td>
<td>92</td>
<td>150</td>
<td>60</td>
<td>3.3.3.3.5</td>
</tr>
</tbody>
</table>
3. Methods of Generating Enclosure Geometries

3.2. Geodesic domes

The word geodesic means the shortest distance between two or more points in space. Geodesic domes are made up of geodesics which are arranged in a triangular pattern, the resulting structure approximately emulates a spherical structure. The orientation of the triangles is usually obtained by triangulation of regular Platonic solids. Since triangles are the most stable polygon and spheres possess the highest enclosed volume to surface area ratio, the geodesic dome is an extremely economical option for enclosing large volumes.

The first geodesic dome was the Zeiss optics company’s planetarium dome which was constructed in Jena, Germany in 1922 (Weisstein 2005). However, it wasn’t until the 1940s that the idea of structural geodesic dome was developed and patented by R. Buckminster Fuller. Fuller experimented with the idea of combining creative geometry with structures in an attempt to address the post World War II housing situation. Although the use of geodesic domes as housing structures never became popular, it did become a popular choice for industrial and public buildings due to its low construction cost.
3. Methods of Generating Enclosure Geometries

Perhaps the most famous geodesic dome is the Montreal Biosphere which served as the America Pavilion during the World Exhibition Expo 1967 (see Figure 3.4 above). This geodesic dome was designed by Fuller and stands 62 [m] tall. Other examples of large geodesic structures include the 216 [m] Fantasy Entertainment Complex in Kyosho Isle, Japan; the 160 [m] Superior Dome in Northern Michigan University; the 162 [m] Tacoma Dome in Tacoma, Washington; and the 153 [m] Walkup Skydome in Northern Arizona University.

3.2.1. Advantages and Disadvantages of Geodesic Domes

Although Fuller had originally developed the geodesic dome to address the potential “housing shortage” he foresaw after the Second World War, geodesic domes are actually

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5 Courtesy of Wikipedia, <http://en.wikipedia.org/wiki/Montreal_Biosph%C3%A8re>
3. Methods of Generating Enclosure Geometries

more suitable for commercial and industrial use. This is because geodesic domes are difficult to design compared to conventional housing structures. Most residential homes are designed and built by builders rather than engineers. The complex response of spherical structures to wind and eccentric loading make it necessary for geodesic domes to be designed by a certified engineer. Additionally, compared to traditional houses, domes are difficult to partition and furnish. In order to fit properly, cabinries and windows will have to be custom made; thus increasing construction cost.

However, geodesic domes are still ideal structures for commercial and industrial use. Its light weight components make it easy to construct, even without the use of a crane. Due to its arch-like properties, domes are extremely strong; in fact, the larger the dome, the stronger it is. Its aerodynamic shape also enhances its performance under wind loads (wind loading effects will be further discussed in section 5). As mentioned above, another advantage of the dome is that is has the highest enclosed volume to surface area ratio. Since cladding costs are a major component of building budgets, minimizing surface area will result in lower cladding costs; hence lowering overall project costs.

3.2.2. Geodesic Dome Classification

Geodesic domes are usually formed by triangulation of Platonic solids. Each surface of the Platonic solid is divided into one or more triangle. The vertices of each triangle are then projected onto the surface of a circumscribing sphere to produce a spherical structure. Common Platonic solids used in geodesic triangulation include the icosahedron, octahedron, and the tetrahedron (Yves 2001)
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Triangulation orientation and density determines how closely geodesic domes approximate a sphere. As expected, the more divisions there are on each face, the rounder the dome. At the core of each geodesic dome is a Platonic solid. Since each Platonic solid has a circumscribing sphere, creating a more detailed dome is just a matter of dividing each face into smaller triangles then projecting the vertices out onto the circumscribing sphere.

The way in which each face is divided can be described by the following geodesic dome classification table (Yves 2001). Class I divisions are parallel to the edges of the faces, Class II divisions are perpendicular to the edges of the faces, and Class III divisions are at a random angle to the edges of the faces.

<table>
<thead>
<tr>
<th>Class</th>
<th>Description</th>
<th>Picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Divisions are edge to edge</td>
<td><img src="image1.png" alt="Picture" /></td>
</tr>
<tr>
<td>II</td>
<td>Divisions are vertex to edge</td>
<td><img src="image2.png" alt="Picture" /></td>
</tr>
<tr>
<td>III</td>
<td>Divisions are at incremental angles</td>
<td><img src="image3.png" alt="Picture" /></td>
</tr>
</tbody>
</table>

Geodesic domes are also distinguished by frequencies. The frequency number refers to the number of times each edge is divided. For example, in a geodesic dome of frequency 2, each edge is divided into two sections. Below are several examples of different geodesic domes:
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3.3. Spherical tiling programs

There are currently hundreds of spherical tiling programs available on the market. Although some of these programs were developed solely for research purposes, many have become commercial software which provides aid in various disciplines. Applications of these software range from earth science studies, to structural analysis, to mathematical research. The following paragraphs briefly describe several spherical tiling programs which were found as shareware on the internet:

**Gmsh**

Gmsh is a 3-dimensional finite element grid generator. It also has a built-in CAD engine and post-processor. Gmsh was developed as a meshing tool to solve academic problems with parametric input. The following figure is a screen capture from Gmsh. Models in Gmsh are created by defining points, lines, surfaces, and volumes. After the geometry is defined, one can generate a surface mesh with a simple click of a button. Gmsh reads .geo files.
3. Methods of Generating Enclosure Geometries

Makros-A

![Screen capture from Makros-A](image)

Figure 3.6 Screen capture from Gmsh

Makros-A is another meshing software available on the internet. It has the capability to read AutoCAD files, which make it an extremely efficient and easy to use program. Similar to
Gmsh above, once the geometry is defined, a surface mesh is automatically generated according to user input parameters.

NetGEN

NetGEN is an automatic 3-dimensional tetrahedral mesh generator. Compared to the two programs above, NetGEN allows the user to have more control over the shape, size, and density of the generated mesh. This program also contains modules for mesh optimization and hierarchical mesh refinement. NetGEN reads .geo, .stl, and .stlb files.
4. Geometric Solutions to the Enclosure Problem

After exploring various methods of generating telescope enclosure geometries, it appeared that the best candidates were Platonic solids and Archimedean solids. Platonic and Archimedean solids are symmetric (except for the two chiral Archimedean solids) and relatively easy to model given the vertex equations; hence, they make good candidates for telescope enclosure. According to the National Building Code (NBCC) 2005, the response of regular structures is easier to predict than irregular structures; thus, it is generally recommended that designs be kept as symmetric and simple as possible.

From the five Platonic solids, it is apparent that the icosahedron would be the best choice for geodesic dome generation due to its spherical likeness (compared to the other Platonic solids) prior to any triangulation. As for the Archimedean solids, it can easily be seen that the rhombicosidodecahedron and the snub dodecahedron possess the most spherical likeness. However, due to the unsymmetrical nature of the snub dodecahedron, the rhombicosidodecahedron was the obvious choice.

4.1. Generation of Icosahedron Geodesic Dome

The first step to generating a geodesic dome is to determine its core solid, in our case, the icosahedron. There are a total of 12 vertices in the icosahedron and their coordinates can be described as follows ("Icosahedron", The Physics Encyclopaedia):
4. Geometric Solutions to the Enclosure Problem

\[(0, +1, +\Phi) \quad (0, +1, -\Phi) \quad (0, -1, +\Phi) \quad (0, -1, -\Phi)\]
\[(+1, +\Phi, 0) \quad (+1, -\Phi, 0) \quad (-1, +\Phi, 0) \quad (-1, -\Phi, 0)\]
\[(+\Phi, 0, +1) \quad (+\Phi, 0, -1) \quad (-\Phi, 0, +1) \quad (-\Phi, 0, -1)\]

Figure 4.1 Coordinates of Vertices for Icosahedron

Again, the \(\phi\) symbol above is the golden ratio. By multiplying each coordinate by a factor, one can obtain icosahedrons of any sizes.

Example.

Let \(r = \) radius of circumscribing sphere
\(a = \) radius of circumscribing sphere (unfactored)
\(\phi = \) golden ratio

The variable "a" can easily solved by the simple distance equation.

**Equation 4.1 Distance Equation**

\[a := \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}\]

Then, using the coordinate \((0, +1, +\phi)\) as an example:

\[a := \sqrt{(0 - 0)^2 + (1 - 0)^2 + (\phi - 0)^2}\]

\[a := \sqrt{1 + \phi^2}\]

If a radius of \(r = 10 \text{ [m]}\) is required, then the above coordinate may be multiplied by the factor \(r/a\) to obtain new coordinates. The new coordinates for this case will be:
4. Geometric Solutions to the Enclosure Problem

\[ x_{\text{new}} := \frac{0 - r}{a} \quad \quad z_{\text{new}} := \frac{r}{a} \]

\[ x_{\text{new}} = 0 \quad \quad z_{\text{new}} = 8.507 \]

\[ y_{\text{new}} := 1 - \frac{r}{a} \]

\[ y_{\text{new}} = 5.257 \]

A macro was developed in excel such that with the input of the desired radius, all corresponding vertices for the particular solid would be calculated and displayed. Having the coordinates in excel is not very useful unless it is somehow able to be converted into a model or a drawing. Having a drawing in .dwg or .dxf format allows the solid to be imported into numerous structural analysis and modeling programs such as SAP2000 and SolidWorks. To address this need, a simple script was written by Mike Gedig from AMEC Dynamic Structures such that text files can be imported as objects into AutoCAD. This program allows the import of points, lines, texts, polylines, extruded regions, and many other objects. However, for the purpose of this research, only the import of points, lines, and texts was needed. The syntax for the import of points, lines, and texts are as follows:

<table>
<thead>
<tr>
<th>Description</th>
<th>Command</th>
<th>Data</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>points</td>
<td>p</td>
<td>x, y, z</td>
<td>p, 0, 0, 0</td>
</tr>
<tr>
<td>lines</td>
<td>l</td>
<td>x1, y1, z1, x2, y2, z2</td>
<td>l, 0, 0, 0, 1, 1, 1</td>
</tr>
<tr>
<td>text</td>
<td>t</td>
<td>txt, x, y, z, height, rotation (deg), justif. (0-L, 1-C, 2-R)</td>
<td>t, node1, 1.2, 1.3, 2.5, 2, 0, 0</td>
</tr>
</tbody>
</table>
4. Geometric Solutions to the Enclosure Problem

Figure 4.2 and Figure 4.3 below are the results of importing the coordinates generated in Excel into AutoCAD.

![Figure 4.2 Points, lines, and texts for icosahedron imported from Excel to AutoCAD](image)

![Figure 4.3 Points, lines, and texts for rhombicosidodecahedron imported from Excel to AutoCAD](image)

After plotting the coordinates, it is now necessary to plot lines between these coordinates. Since there are no indication as to which coordinates should be joined by a line, a simple
macro was written to automatically calculate the distance between a particular coordinate and all the other coordinates. By comparing the values of these distances, one can determine which two coordinates are joined by a line. From basic geometry, one can easily see that the edges of a Platonic solid also represent the shortest distance between any 2 vertices on the solid. Hence, a line should be inserted when the distance between a particular point and another point is a minimum.

The geodesic dome chosen for this project was the Class I 3 frequency (CI-3V) dome. Obtaining the coordinates of all the intermediate points was relatively simple. Since the coordinates of the vertices of the great triangle is known, coordinates of intermediate points (points 4, 5, 6, 7, 8, and 9) were found using simple division (see Figure 4.4 below). The central point (point 10) was found by applying the midpoint equation between points 5 and 8.

![Figure 4.4 Icosahedron Face Numbering System](image)

After obtaining the coordinates of all the intermediate points, the last step was to project the points onto the circumscribing sphere. This was done using vector mathematics.
Let point \( P \) be an intermediate point on the face of the icosahedron and let point \( Q \) be point \( P \) projected onto the circumscribing sphere. In vector mathematics, each point in space can be described by a direction and magnitude relative to the origin. In other words, since the coordinates of the intermediate points are known, one simply has to determine the unit vector then multiply it by the magnitude (the radius of the circumscribing sphere in our case).

The unit vectors, \( \hat{i}, \hat{j}, \) and \( \hat{k} \) can be found using the following equations:

Using the symbols introduced in Figure 4.5 above.

**Equation 4.2 Unit Vector for X-axis**

\[
\hat{i} := \frac{\Delta x}{r}
\]

**Equation 4.3 Unit Vector for Y-axis**

\[
\hat{j} := \frac{\Delta y}{r}
\]
4. Geometric Solutions to the Enclosure Problem

Equation 4.4 Unit Vector for Z-axis

\[
\hat{z} := \frac{\Delta z}{r}
\]

Relative to the global origin, the new set of coordinates \((y, y, z)\) can be found by multiplying the unit vector by the radius of the circumscribing sphere \((R)\) desired.

Equation 4.5 Projected Coordinates

\[
\begin{align*}
    x_{\text{projected}} &:= \hat{i} \times R \\
    y_{\text{projected}} &:= \hat{j} \times R \\
    z_{\text{projected}} &:= \hat{k} \times R
\end{align*}
\]

<table>
<thead>
<tr>
<th>Intermediate Point</th>
<th>(i)-vector</th>
<th>(j)-vector</th>
<th>(k)-vector</th>
<th>Projected Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-28.34, 8.76, 31.70)</td>
<td>-0.65</td>
<td>0.20</td>
<td>0.73</td>
<td>(-32.65, 10.09, 36.50)</td>
</tr>
</tbody>
</table>

By using the equations specified above, one can project all intermediate points onto the circumscribing sphere. Table 4.2 above summarizes the results for a single point. By repeating the process for all other intermediate points, the geodesic sphere below can be obtained.

Figure 4.6 Transformation of Icosahedron to Geodesic Sphere
4. Geometric Solutions to the Enclosure Problem

4.2. Generation of Rhombicosidodecahedron Dome

Generation of the rhombicosidodecahedron dome is similar to the generation of the icosahedron geodesic dome. In fact, generation of the rhombicosidodecahedron dome is simpler in that its faces do not have to be divided into frequencies and classes then projected.

Coordinates of vertices for the rhombicosidodecahedron dome have been derived and are as follows ("Rhombicosidodecahedron", Wikipedia):

\[
\begin{align*}
(+1, +1, +\phi^3) & \quad (+1, +1, -\phi^3) & \quad (+1, -1, +\phi^3) & \quad (-1, +1, +\phi^3) \\
(+1, -1, -\phi^3) & \quad (-1, -1, +\phi^3) & \quad (-1, +1, -\phi^3) & \quad (-1, -1, -\phi^3) \\
(+\phi^3, +1, +1) & \quad (+\phi^3, +1, -1) & \quad (+\phi^3, -1, +1) & \quad (-\phi^3, +1, +1) \\
(+\phi^3, -1, -1) & \quad (-\phi^3, -1, +1) & \quad (-\phi^3, +1, -1) & \quad (-\phi^3, -1, -1) \\
(+1, +\phi^3, +1) & \quad (+1, +\phi^3, -1) & \quad (+1, -\phi^3, +1) & \quad (-1, +\phi^3, +1) \\
(+1, -\phi^3, -1) & \quad (-1, -\phi^3, -1) & \quad (-1, +\phi^3, -1) & \quad (-1, -\phi^3, -1) \\
(+\phi^2, +\phi, +2\phi) & \quad (+\phi^2, +\phi, -2\phi) & \quad (+\phi^2, -\phi, +2\phi) & \quad (-\phi^2, +\phi, +2\phi) \\
(+\phi^2, -\phi, -2\phi) & \quad (-\phi^2, -\phi, +2\phi) & \quad (-\phi^2, +\phi, -2\phi) & \quad (+\phi^2, -\phi, -2\phi) \\
(+2\phi, +\phi^2, +\phi) & \quad (+2\phi, +\phi^2, -\phi) & \quad (+2\phi, -\phi^2, +\phi) & \quad (-2\phi, +\phi^2, +\phi) \\
(+2\phi, -\phi^2, -\phi) & \quad (-2\phi, -\phi^2, +\phi) & \quad (-2\phi, +\phi^2, -\phi) & \quad (+\phi^2, -\phi^2, -\phi) \\
(+\phi, +2\phi, +\phi^2) & \quad (+\phi, +2\phi, -\phi^2) & \quad (+\phi, -2\phi, +\phi^2) & \quad (-\phi, +2\phi, +\phi^2) \\
(+\phi, -2\phi, -\phi^2) & \quad (-\phi, -2\phi, +\phi^2) & \quad (-\phi, +2\phi, -\phi^2) & \quad (+\phi, -2\phi, -\phi^2) \\
(+2\phi, 0, +\phi^2) & \quad (+2\phi, 0, -\phi^2) & \quad (-2\phi, 0, +\phi^2) & \quad (-2\phi, 0, -\phi^2) \\
(+\phi^2, +2\phi, 0) & \quad (+\phi^2, -2\phi, 0) & \quad (-\phi^2, +2\phi, 0) & \quad (-\phi^2, -2\phi, 0) \\
(0, +\phi^2, +2\phi) & \quad (0, +\phi^2, -2\phi) & \quad (0, -\phi^2, +2\phi) & \quad (0, -\phi^2, -2\phi)
\end{align*}
\]

Figure 4.7 Coordinates for Vertices of Rhombicosidodecahedron

Again, after plotting the vertices of the rhombicosidodecahedron, the next step is to determine which two points should be joined by a line. The macro written for the icosahedron was used for the rhombicosidodecahedron. Unlike the icosahedron, the rhombicosidodecahedron does not consist of triangles only; in fact, it is made up of triangles, pentagons, and squares. By triangulating all the non-triangular faces on the
4. Geometric Solutions to the Enclosure Problem

rhombicosidodecahedron and projecting the corresponding vertices onto the circumscribing sphere, one can create a stronger and more stable structure (see Figure 4.8 below).

![Triangulation of Rhombicosidodecahedron](image)

**Figure 4.8 Triangulation of Rhombicosidodecahedron**

For a fixed circumscribing sphere radius, both the icosahedron geodesic sphere and the rhombicosidodecahedron have line segments of approximately the same length. It is important for the two configurations to have member lengths which are approximately the same when comparing their performance against one another.

Now that the model geometries have been established, one can perform finite-element-analysis on the models to determine which may be a better choice given a set of boundary conditions. The load factor formulation and model properties of the suggested models will be explored in the next section.
5. Formulation of Model and Loads for Finite Element Analysis

Prior to analysis, it is important to first establish the model properties and loads of a structure. Hence, this section will explain the assumptions and model parameters considered for the two FEM models. The software chosen for the finite element analysis was called ANSYS. In addition, the basis from which the load patterns and load magnitudes were devised will also be explained.

5.1. Case Definition and Load Combination

In order to analyze the sensitivity of the two telescope enclosure configurations to changes in model parameters, a number of different “cases” were established (see Table 5.1 below).

<table>
<thead>
<tr>
<th>Case</th>
<th>Variable</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Radius</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>Radius</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>Radius</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>Radius</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>Cross-section size</td>
<td>0.45</td>
</tr>
<tr>
<td>6</td>
<td>Cross-section size</td>
<td>0.3</td>
</tr>
<tr>
<td>7</td>
<td>Cross-section size</td>
<td>0.35</td>
</tr>
<tr>
<td>8</td>
<td>Thickness</td>
<td>25</td>
</tr>
<tr>
<td>9</td>
<td>Thickness</td>
<td>25</td>
</tr>
<tr>
<td>10</td>
<td>Thickness</td>
<td>25</td>
</tr>
<tr>
<td>11</td>
<td>Cross-section size</td>
<td>Dia. 0.4</td>
</tr>
</tbody>
</table>

The radius of the enclosure (R), cross-section size (CSS) of members, cross-section thickness (T) of members, and cross-section type of members were varied and compared to the “control” case.
5. Formulation of Model and Loads for Finite Element Analysis

### Table 5.2 Control Case

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>50 m</td>
</tr>
<tr>
<td>Cross-section size</td>
<td>0.4 m</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.05</td>
</tr>
<tr>
<td>Cross-section</td>
<td>Square</td>
</tr>
</tbody>
</table>

For each case, the values of a variable were varied while the values of the other variables were kept constant at the control value specified above. For example, the variable of interest in Case 1 to 4 is enclosure radius. The value of the enclosure radius was varied from 20 to 50 [m] at intervals of 10 [m] while other variables were kept at a constant value.

In addition to the different case definitions, different load combinations were also considered.

### Table 5.3 Load Combinations for Ultimate Limit States

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Loads</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.25D + 1.5S</td>
</tr>
<tr>
<td>2</td>
<td>1.25D + 1.4W</td>
</tr>
<tr>
<td>3</td>
<td>1.0D + 1.0E</td>
</tr>
</tbody>
</table>

5.2. Type of Structure

The type of structure assumed for the analysis of the two proposed telescope enclosure configuration was a 3-dimensional frame. By definition, a 3-dimensional frame consists of members that are connected rigidly (Ghali et al. 2003). Based on this definition, the element **Beam44** was chosen from the element library in ANSYS. **Beam44** is a 3-dimensional Elastic Tapered Unsymmetric Beam. It is a uniaxial element with tension, compression, torsion, and bending capabilities (ANSYS Help File 2005). Although the cross-section used in the analysis is uniform for the length of the member, the cross-section size changes for different
5. Formulation of Model and Loads for Finite Element Analysis

cases specified above. Hence, the command “SECTYPE” and “SECDATA” was used to specify customized cross-sections. Beam44 has 6 degrees of freedom at each node, translation along the x, y, and z axis and rotation about the x, y, and z axis. The X-axis is oriented along the element and the Y and Z-axis are perpendicular to the cross-section of the element.

5.3. Material Properties

The material assumed for the model was 300W steel with the following properties:

Table 5.4 Member Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Shear Modulus</td>
<td>70 GPa</td>
</tr>
<tr>
<td>Density</td>
<td>7800 kg/m³</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>0.25</td>
</tr>
</tbody>
</table>

5.4. Dead and Live Load

The only dead load contribution in this structure is the self weight of the members. No equipment, partitions, or other building materials were considered in the calculation of the dead load. There are no live loads on this structure; thus, live loads were not included as one of the load combinations above.
5. Formulation of Model and Loads for Finite Element Analysis

5.5. Snow and Ice Loads

Situated on top of high mountain ranges and in remote locations, telescope enclosures are often subjected to extreme conditions of snow and ice loading.

Table 5.5 Snow and Ice Loading as provided by AMEC Dynamic Structures

<table>
<thead>
<tr>
<th>Snow Load</th>
<th>150 kg/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ice Load</td>
<td>68 kg/m²</td>
</tr>
</tbody>
</table>

The snow and ice loads given above are unfactored loads. Hence, these values must be factored according to the provisions specified by clause 4.1.6.2(1) in the NBCC 2005. The corresponding calculations are as follows:

$$S = I_s \left[ S_s \left( C_b \cdot C_w \cdot C_s \cdot C_a \right) + S_r \right]$$

$$S = 1.484 \text{kPa}$$
In order to transform the above numbers into average keypoint forces, the average cladding area was calculated for each configuration and case. For the rhombicosidodecahedron, there were three different types of triangle sizes. Going back to the base shape, the rhombicosidodecahedron consists of triangles, squares, and pentagons. These three shapes were further subdivided to obtain the existing telescope enclosure configuration. Hence, there are 3 different types of triangles, one from each shape.

Figure 5.1 Different Triangles on the Rhombicosidodecahedron Configuration
Sample calculation for snow and ice loading on rhombicosidodecahedron, Case 1

Triangle 1

\[
\begin{align*}
H_j & := 5.7826 \text{m} \\
B_j & := 5.6339 \text{m} \\
\text{Area}_1 & := \frac{1}{2} B_1 \cdot H_1 \\
\text{Area}_1 & = 16.289 \text{m}^2
\end{align*}
\]

Triangle 2

\[
\begin{align*}
\theta_2 & := \arccos \left( \frac{\frac{1}{2} B_2}{a_2} \right) \\
B_2 & := 5.7438 \text{m} \\
a_2 & := 5.6339 \text{m}
\end{align*}
\]
5. Formulation of Model and Loads for Finite Element Analysis

\[ \theta_2 = 59.353 \text{deg} \]

\[ H_2 := \tan \left( \frac{\theta_2}{2} \right) \cdot B_2 \]

\[ H_2 = 4.847 \text{m} \]

\[ \text{Area}_2 := \frac{1}{2} B_2 H_2 \]

\[ \text{Area}_2 = 13.92 \text{m}^2 \]

Triangle 3

\[ B_3 := 5.6339 \text{m} \]

\[ a_3 := 4.9619 \text{m} \]

\[ \theta_3 := \arccos \left( \frac{\frac{1}{2} \cdot B_3}{a_3} \right) \]

\[ \theta_3 = 55.409 \text{deg} \]

\[ H_3 := \tan \left( \theta_3 \right) \cdot \frac{1}{2} \cdot B_3 \]

\[ H_3 = 4.085 \text{m} \]

\[ \text{Diagram of Triangle 3} \]
5. Formulation of Model and Loads for Finite Element Analysis

\[
\text{Area}_3 := \frac{1}{2} B_3 H_3
\]

\[
\text{Area}_3 = 11.507 \text{m}^2
\]

Average area for Rhombicosidodecahedro

\[
\text{Area}_R := \frac{1}{3} (\text{Area}_1 + \text{Area}_2 + \text{Area}_3)
\]

\[
\text{Area}_R = 13.905 \text{m}^2
\]

For simplification purposes, an average area of the three triangles specified above was taken as the tributary area for the snow and ice load.

**Snow and Ice Load**

\[
F_s := \text{Area}_R \times S
\]

\[
F_s = 20.635 \text{kN}
\]

According to the above calculation, an average force of 20.64 kN is applied on each triangle.

Since each triangle has 3 vertices, the equivalent keypoint load would be \(20.64 \text{kN}/3 = 6.88 \text{kN}\). However, keypoints are shared by adjacent triangles. On average, a keypoint is shared by 6 triangles; as a result, the actual keypoint load is \(6.88 \text{kN} \times 6 = 41.28 \text{kN}\).

5.6. Wind Loads

Wind loads are extremely important for large structures because lift and drag forces are proportional to the cross-section of the structure. Hence, the larger the structure, the larger the load on the structure. Since the structure of interest in this report has a spherical shape,
flows past spheres or bluff bodies are of special interest and will be elaborated on in the following section.

Fluid flows are generally divided into two different types, *inviscid flow* and *viscous flow*. In inviscid flow, the effects of viscosity are generally negligible whereas in viscous flows, viscous effects may not be ignored. External flows are a good example of inviscid flow because viscous effects are limited to act within the thin *boundary layer*.

Referring to Figure 5.2 below, one can see that the boundary layer around a bluff body separates at the largest cross section due to positive pressure gradient. At the front center of the sphere is a *stagnation point* where the velocity of the fluid is, theoretically, zero. The boundary layer may be classified as either *laminar* or *turbulent*. In the laminar boundary layer, there is no significant mixing of fluid particles; as a result, this type of flow is highly time dependent. Turbulent flows are characterized by randomness; this means that parameters such as velocity and pressure vary with time and space (Potter 2002).

When a spherical structure is subjected to wind loading, surface friction drag is relatively small, compared to pressure drag which results from the large area at the rear of a body subjected to a reduced pressure in the *wake region*. The wake is a region with a growing velocity defect as shown in Figure 5.2 below. It is different from the *separated region* which is a region of re-circulating flow. The amount of drag induced on an immersed body is proportional to the cross-sectional area of the wake, which, in turn is influenced by how
streamlined the body is. For example, a streamlined body will induce less drag than a bluff body of similar dimensions simply due to the shape of the body.

The point of separation of an immersed body is affected by several parameters including geometry, pressure gradient, and most important of all, the Reynolds number. When the Reynolds number is small (<1), the flow is considered to be viscous and the boundary layer extends to infinity. At a Reynolds number of 1 to 10, a true boundary layer forms and remains laminar along the entire surface of the body, separating at the rear and creating a narrow turbulent wake. For a Reynolds number of 10 to 60, separation points migrate towards the front of the body and create a larger wake. When the Reynolds number is between 60 and 140, symmetrical vortices begin to develop and grow behind the laminar

---

6 Courtesy of Mechanics of Fluids by Potter and Wiggert
separation points. Once the vortices reach a certain size, it will break away and move downstream. These vortices take turn breaking away from the body and create a lifting force in the process. Since the vortices are on opposite ends of the body, the lifting force generated is of equal magnitude but opposite direction. As a result, the body begins vibrating at the vortex shedding frequency. If this frequency coincides with the natural frequency, the applied force may be magnified due to resonance.

The vortex shedding frequency may be described by the Strouhal Number (St) which is obtained by dimensional analysis. See Equation 5.1 below.

**Equation 5.1 The Strouhal Number**

\[
St = \frac{fD}{V}
\]

where

- \(St\) = Strouhal Number [dimensionless]
- \(f\) = vortex shedding frequency \([T^{-1}]\)
- \(D\) = diameter \([L]\)
- \(V\) = wind velocity \([L/T]\)

When the Reynolds number (Re) is between 300 and 10,000, the Strouhal number is constant at approximately 0.21 (Potter 2002). This means that within this Re range, the vortex shedding frequency increases linearly with velocity. However, as the Re increases past 5x10^4, the vortex shedding frequency increases rapidly and becomes unpredictable.
5. Formulation of Model and Loads for Finite Element Analysis

Since it is known that the Strouhal number is approximately 0.21 for Re < 10,000. One can approximate the vortex shedding frequency and compare this value against the natural vibration frequency of the structure. To ensure that the assumption of St = 0.21 is valid, one must ensure that the Re < 10,000. This is shown below:

*Rhombicosidodecahedron and Icosahedron, Case 1*

Assume temperature ≈ 20°C.

\[
V := 78 \frac{m}{s} \quad \text{Average Wind Velocity}
\]

\[
D := 50m \quad \text{Diameter of Structure}
\]

\[
u := 1.51 \frac{m^2}{s} \quad \text{Kinematic Viscoity of Air}
\]

\[
Re := \frac{V \cdot D}{\nu}
\]

\[
Re = 2.583 \times 10^3
\]

Since the Re is indeed less than 10,000, the assumption of St = 0.21 is valid.

\[
St := f \cdot \frac{D}{V} \quad \text{St} := 0.21
\]

\[
f := St \cdot \frac{V}{D}
\]

\[
f = 0.164Hz
\]

From the above equations, the vortex shedding frequency is calculated as 0.164 Hz. Now, one can compare this value to the natural vibration frequency of the structure to determine
whether vortex shedding effects will result in resonance of the structure. Due to the complexity of the structure, it can be deduced that the enclosure will have many modes of vibration. However, the only vibration mode of interest for this particular scenario is the mode with a frequency close to the St frequency calculated above. As a result, only the first 12 modes of the enclosure will be required.

The two configurations above, icosahedron and rhombicosidodecahedron, were analyzed in SAP and the following data was obtained for case 1 of each configuration.

Table 5.6 SAP output of vibration frequencies for first 12 modes (Rhombicosidodecahedron)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Period [s]</th>
<th>Frequency [cyc/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.396102</td>
<td>2.524602</td>
</tr>
<tr>
<td>2</td>
<td>0.381929</td>
<td>2.618288</td>
</tr>
<tr>
<td>3</td>
<td>0.143666</td>
<td>6.960589</td>
</tr>
<tr>
<td>4</td>
<td>0.118604</td>
<td>8.431419</td>
</tr>
<tr>
<td>5</td>
<td>0.08025</td>
<td>12.46106</td>
</tr>
<tr>
<td>6</td>
<td>0.078851</td>
<td>12.68215</td>
</tr>
<tr>
<td>7</td>
<td>0.071842</td>
<td>13.91943</td>
</tr>
<tr>
<td>8</td>
<td>0.071589</td>
<td>13.96863</td>
</tr>
<tr>
<td>9</td>
<td>0.067944</td>
<td>14.718</td>
</tr>
<tr>
<td>10</td>
<td>0.063863</td>
<td>15.65852</td>
</tr>
<tr>
<td>11</td>
<td>0.063422</td>
<td>15.7674</td>
</tr>
<tr>
<td>12</td>
<td>0.062995</td>
<td>15.87428</td>
</tr>
</tbody>
</table>
5. Formulation of Model and Loads for Finite Element Analysis

Figure 5.3 Deformed Shape (Mode 1) of Rhombicosidodecahedron

Table 5.7 SAP output of vibration frequencies for first 12 modes (Icosahedron)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Period [s]</th>
<th>Frequency [cyc/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.572764</td>
<td>1.7459</td>
</tr>
<tr>
<td>2</td>
<td>0.546453</td>
<td>1.83</td>
</tr>
<tr>
<td>3</td>
<td>0.169415</td>
<td>5.9027</td>
</tr>
<tr>
<td>4</td>
<td>0.143683</td>
<td>6.9597</td>
</tr>
<tr>
<td>5</td>
<td>0.085347</td>
<td>11.717</td>
</tr>
<tr>
<td>6</td>
<td>0.082647</td>
<td>12.1</td>
</tr>
<tr>
<td>7</td>
<td>0.07876</td>
<td>12.697</td>
</tr>
<tr>
<td>8</td>
<td>0.07801</td>
<td>12.819</td>
</tr>
<tr>
<td>9</td>
<td>0.075862</td>
<td>13.182</td>
</tr>
<tr>
<td>10</td>
<td>0.075439</td>
<td>13.256</td>
</tr>
<tr>
<td>11</td>
<td>0.070794</td>
<td>14.125</td>
</tr>
<tr>
<td>12</td>
<td>0.070434</td>
<td>14.198</td>
</tr>
</tbody>
</table>
Looking at the vortex shedding frequency and natural frequency, one can conclude that resonance effects are negligible. For the other cases proposed with a smaller diameter, vibration frequencies are larger and even further away from the St frequency of 0.16 Hz. Achieving a vortex shedding frequency of 0.16 Hz is nearly impossible because the St number remains relatively constant after Re reaches 300. The vortex shedding frequency does not rise rapidly until Re reaches $5 \times 10^4$. For a structure with a diameter of 50 m, this would be equivalent to an average wind velocity of approximately 700 m/s. Hence, it is safe to say that vortex shedding would not be an issue for this particular telescope enclosure.
The term drag may be defined as “the force the flow exerts on the body in the direction of flow”. For the purpose of this report, one can visualize it as the force exerted on the structure by a lateral acting force, such as wind. The calculation of the force exerted on the enclosure by wind may be calculated using the following equation.

**Equation 5.2 Drag Force**

\[ F_D = \frac{1}{2} C_D \rho V^2 A \]

where

- \( C_D \) = drag coefficient [dimensionless]
- \( \rho \) = density of fluid [M/L^3]
- \( V \) = velocity of fluid [L/T]
- \( A \) = projected area normal to direction of flow [L^2]

The only variables above which depend on the actual structure are the projected area, \( A \) and the drag coefficient, \( C_D \). The projected area can be easily calculated using simple geometry; however, the drag coefficient requires more thought and understanding of basic fluid dynamics principles.
From Figure 5.6 above, one can see that at $10^3 < \text{Re} < 2 \times 10^5$, the drag coefficient remains relatively constant for both smooth spheres and cylinders. It is also important to note that at $\text{Re} \approx 2 \times 10^5$, boundary layer for smooth surface blunt bodies undergoes transition to a turbulent state and pushes separation back, resulting in a decrease in drag. Comparing the smooth bodies against rough bodies, one can also see that surface roughness results in a slight drop in drag at $5 \times 10^4 < \text{Re} < 2 \times 10^5$ (as shown by the dashed line above).

From the calculations and Figure 5.6 above, one can conclude that a drag coefficient of approximately 0.7 is acceptable.
5. Formulation of Model and Loads for Finite Element Analysis

Sample calculation for reference wind velocity, Case 1

**Wind Load**

\[ D := 50 \text{m} \quad U := 1.51 \frac{\text{m}^2}{\text{s}} \quad V := 78 \frac{\text{m}}{\text{s}} \quad \text{Standard wind velocity value used at AMEC Dynamic Structures} \]

\[ H := 45.253 \text{m} \]

\[ A := D \cdot H \]

\[ A = 2.263 \times 10^3 \text{m}^2 \]

\[ Re := \frac{V \cdot D}{u} \]

\[ Re = 2.583 \times 10^3 \]

\[ C_D := 0.7 \quad \text{At} \ Re = 2583 \]

\[ q := \frac{1}{2} C_D \rho \cdot V^2 \]

\[ q = 2.564 \text{kPa} \quad \text{Reference velocity pressure} \]

After obtaining the reference velocity pressure, one can use the NBCC 2005 code to factor this value to obtain an appropriate wind loading on the structure of interest. The specified external pressure caused by wind on the structure may be calculated using the following equation:
5. Formulation of Model and Loads for Finite Element Analysis

Equation 5.3 External Pressure Caused by Wind on Structure

\[ p := I_w \cdot q \cdot C_e \cdot C_g \cdot C_p \]  

Where

- \( I_w \) = Importance factor for wind load
- \( q \) = reference velocity pressure (calculated above)
- \( C_e \) = exposure factor
- \( C_g \) = gust factor
- \( C_p \) = external pressure coefficient

All the values are specified in the code with the exception of \( q \) which was calculated above and \( C_p \) which is described in the commentary section of the code. The external pressure coefficient is a dimensionless ratio of wind-induced pressures on a building to the dynamic pressure of the wind speed at a particular reference height. For simplification purposes, it was assumed that the wind loading acts parallel to the ground surface. Under this assumption, the value of the external pressure coefficient is 1.0.

Sample calculation for wind loading according to NBCC 2005, Case 1

Importance factor for wind load

\( I_w := 1 \)

Exposure factor

\[ C_e := \left( \frac{h}{10} \right)^0.2 \quad C_e = 1.352 \]

Gust factor

\( C_g := 2 \)

External pressure coefficient

\( C_p := 1 \)

\[ p := I_w \cdot q \cdot C_e \cdot C_g \cdot C_p \]

\[ p = 6.935 \text{kPa} \]
The external pressure calculated above was multiplied by the average projected area of the structure. This yields the average force exerted on the enclosure. By dividing this average force by the number of keypoints within that projected area, the average force exerted on each keypoint may be obtained.

\[
A := D \cdot H
\]

\[
N_{kp} := 259
\]

\[
W := p \frac{A}{N_{kp}}
\]

\[
W = 60.585 \text{kN}
\]

A force of 60.6 kN is exerted on each keypoint in direct contact with lateral wind forces.
5. Formulation of Model and Loads for Finite Element Analysis

5.7. Earthquake Loads

In order to achieve unobstructed viewing, telescope enclosures are almost always situated on top of remote and high mountain ranges. If these mountain ranges are located near volcanoes or active seismic locations, then the telescope enclosure must be designed to resist seismic forces.

To simulate reality as much as possible the earthquake load specifications in this project were obtained from the NBCC 2005 code. The appropriate site characteristics and location must be considered in the selection of spectral acceleration values; however, for the purposes of this report, the **Uniform Hazard Spectrum** (UHS) for Vancouver was used (see Figure 5.8 below).

![NBCC 2005 UHS (Vancouver, Class A Site)](image)

**Figure 5.8 Uniform Hazard Spectrum for Vancouver**

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5. Formulation of Model and Loads for Finite Element Analysis

Since it was assumed that the telescope would be situated on top of a mountain range, a Class A site was used for analysis. Class A sites consist of hard rock and have an average shear wave velocity of greater than 1500 [m/s]. The Vancouver UHS above was adjusted to reflect a Class A site. This adjustment is done by multiplying spectral accelerations at \( T_a = 0.2, 0.5, 1.0, \) and \( 2.0 \) [s] by corresponding \( F_a \) (acceleration-based site coefficient) and \( F_v \) (velocity-based site coefficient) values. These values may be found from Table 4.1.8.4B and Table 4.1.8.4A from NBCC 2005. After finding these values, intermediate spectral acceleration points were linearly interpolated to obtain the above graph.

The base shears suggested by NBCC 2005 was calculated as a function of the weight of the building to obtain lateral accelerations.

Equation 5.4 below illustrates how the lateral accelerations for each case were obtained.

**Equation 5.4 Minimum Lateral Acceleration due to Earthquake**

\[
A := \frac{S(T_a)M_vI_E}{R_dR_o} \quad \text{[NBCC 2005 4.1.8.11(2)]}
\]

where  
- \( S(T_a) \) = spectral acceleration at time \( T_a \)  
- \( T_a \) = fundamental lateral period of vibration of structure  
- \( M_v \) = factor to account for higher mode effect on base shear  
- \( I_E \) = earthquake importance factor for structure  
- \( R_d \) = ductility related force modification factor  
- \( R_o \) = overstrength related force modification factor
5. Formulation of Model and Loads for Finite Element Analysis

In order to calculate the lateral acceleration specified above, the fundamental lateral period of vibration for each case had to be obtained. This was done using SAP2000 and the results are presented again for convenience:

Table 5.8 Fundamental Lateral Period of Vibration for Rhombicosidodecahedron

<table>
<thead>
<tr>
<th>Case</th>
<th>Fundamental Period, $T_a$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.396</td>
</tr>
<tr>
<td>2</td>
<td>0.334</td>
</tr>
<tr>
<td>3</td>
<td>0.214</td>
</tr>
<tr>
<td>4</td>
<td>0.139</td>
</tr>
<tr>
<td>5</td>
<td>0.395</td>
</tr>
<tr>
<td>6</td>
<td>0.399</td>
</tr>
<tr>
<td>7</td>
<td>0.398</td>
</tr>
<tr>
<td>8</td>
<td>0.396</td>
</tr>
<tr>
<td>9</td>
<td>0.396</td>
</tr>
<tr>
<td>10</td>
<td>0.395</td>
</tr>
<tr>
<td>11</td>
<td>0.397</td>
</tr>
</tbody>
</table>

Table 5.9 Fundamental Lateral Period of Vibration for Icosahedron

<table>
<thead>
<tr>
<th>Case</th>
<th>Fundamental Period, $T_a$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.573</td>
</tr>
<tr>
<td>2</td>
<td>0.455</td>
</tr>
<tr>
<td>3</td>
<td>0.337</td>
</tr>
<tr>
<td>4</td>
<td>0.218</td>
</tr>
<tr>
<td>5</td>
<td>0.571</td>
</tr>
<tr>
<td>6</td>
<td>0.577</td>
</tr>
<tr>
<td>7</td>
<td>0.575</td>
</tr>
<tr>
<td>8</td>
<td>0.572</td>
</tr>
<tr>
<td>9</td>
<td>0.572</td>
</tr>
<tr>
<td>10</td>
<td>0.572</td>
</tr>
<tr>
<td>11</td>
<td>0.574</td>
</tr>
</tbody>
</table>
5. Formulation of Model and Loads for Finite Element Analysis

After obtaining the fundamental lateral period of vibration, the spectral accelerations for each particular case was obtained from the Vancouver UHS. One can see that the stiffer a structure is, the higher the acceleration. As a result, when comparing Case 1 to 4, one can see that the enclosure with the smallest radius (Case 4), due to its shorter and stiffer members, would be subjected to the highest earthquake load.

The $M_v$ factor accounts for higher mode effects of the structure. According to the NBCC 2005 code, $M_v$ factors are found in Table 4.1.8.11 and are dependent on the type of lateral resisting system of the structure. The ratio of $S_a(0.0)/S_a(2.0)$ usually indicates the approximate location of the site; the west coast of Canada usually has a value of $< 8.0$ while the east coast of Canada usually has a value of $> 8.0$. For the purpose of this analysis, it was decided that a moment resisting frame system would be chosen as the lateral force resisting system. Hence, according to Table 5.10 below, the value of $M_v$ is 1.0.

<table>
<thead>
<tr>
<th>$S_a(0.2)/S_a(2.0)$</th>
<th>Type of Lateral Resisting System</th>
<th>$M_v$ for $T_a \leq 1.0$</th>
<th>$M_v$ for $T_a \geq 2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 8.0</td>
<td>Moment resisting frames or &quot;coupled walls&quot;</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Braced frames</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Wall, wall-frame systems, other systems</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>&gt; 8.0</td>
<td>Moment resisting frames or &quot;coupled walls&quot;</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Braced frames</td>
<td>1.0</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Wall, wall-frame systems, other systems</td>
<td>1.0</td>
<td>2.5</td>
</tr>
</tbody>
</table>
The $I_E$ factor accounts for how important the structure is. The structure under consideration in this analysis is extremely expensive and costly to repair; hence it has been classified under the high importance category with a value of 1.3. Values for the $I_E$ factor may be obtained from Table 4.1.8.5 in the NBCC 2005.

In order to account for the inelastic properties of a structure, NBCC divides the base shear by a force reduction factor ($R_d$) and a system overstrength factor ($R_o$). The force reduction factor is related to the amount of ductility capacity in the structure while the overstrength factor accounts for dependable forms of overstrength (cross-section size rounded up, strain hardening...etc). For this analysis, an elastic beam was chosen; as a result, both $R_d$ and $R_o$ have values of 1.0.

**Sample Calculations for Lateral Acceleration**

*Rhombicosidodecahedron, Case 1*

\[ T_a := 0.396 \text{s} \]

At $T_a = 0.396\text{s}$, \[ S(T_a) := 0.56g \]

\[ M_v := 1.0 \quad I_E := 1.3 \]

\[ R_d := 1.0 \quad R_o := 1.0 \]

\[ A := \frac{(S(T_a) \cdot M_v \cdot I_E)}{R_d \cdot R_o} \]

\[ A = 0.728g \]

*Icosahedron, Case 1*
5. Formulation of Model and Loads for Finite Element Analysis

\[ T_a := 0.573s \]

At \( T_a = 0.573s \),
\[ S(T_a) := 0.41g \]

\[ M_v := 1.0 \quad I_E := 1.3 \]

\[ R_d := 1.0 \quad R_o := 1.0 \]

\[ A := \frac{(S(T_a) \cdot M_v \cdot I_E)}{R_d \cdot R_o} \]

\[ A = 0.533g \]

**Table 5.11 Linear Acceleration for Rhombicosidodecahedron**

<table>
<thead>
<tr>
<th>Case</th>
<th>Linear Acceleration (g's)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.73</td>
</tr>
<tr>
<td>2</td>
<td>0.80</td>
</tr>
<tr>
<td>3</td>
<td>0.96</td>
</tr>
<tr>
<td>4</td>
<td>0.98</td>
</tr>
<tr>
<td>5</td>
<td>0.73</td>
</tr>
<tr>
<td>6</td>
<td>0.73</td>
</tr>
<tr>
<td>7</td>
<td>0.73</td>
</tr>
<tr>
<td>8</td>
<td>0.73</td>
</tr>
<tr>
<td>9</td>
<td>0.73</td>
</tr>
<tr>
<td>10</td>
<td>0.73</td>
</tr>
<tr>
<td>11</td>
<td>0.73</td>
</tr>
</tbody>
</table>

**Table 5.12 Linear Acceleration for Icosahedron**

<table>
<thead>
<tr>
<th>Case</th>
<th>Linear Acceleration (g's)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.53</td>
</tr>
<tr>
<td>2</td>
<td>0.65</td>
</tr>
<tr>
<td>3</td>
<td>0.80</td>
</tr>
<tr>
<td>4</td>
<td>0.96</td>
</tr>
<tr>
<td>5</td>
<td>0.53</td>
</tr>
<tr>
<td>6</td>
<td>0.53</td>
</tr>
<tr>
<td>7</td>
<td>0.53</td>
</tr>
<tr>
<td>8</td>
<td>0.53</td>
</tr>
<tr>
<td>9</td>
<td>0.53</td>
</tr>
<tr>
<td>10</td>
<td>0.53</td>
</tr>
<tr>
<td>11</td>
<td>0.53</td>
</tr>
</tbody>
</table>
5. Formulation of Model and Loads for Finite Element Analysis

Table 5.11 and Table 5.12 above summarize the linear acceleration calculated for each of the cases specified in section 5.1.

5.8. Summary of Loads

The following tables are summaries of the loads applied on each structure.

<table>
<thead>
<tr>
<th>Case</th>
<th>Load</th>
<th>Unfactored Load</th>
<th>Load Factor</th>
<th>Factored Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 5 - 11</td>
<td>Dead (m/s²)</td>
<td>9.81</td>
<td>1.25</td>
<td>12.26</td>
</tr>
<tr>
<td></td>
<td>Snow/Ice (N/keypoint)</td>
<td>41270</td>
<td>1.50</td>
<td>61905</td>
</tr>
<tr>
<td></td>
<td>Wind (N/keypoint)</td>
<td>60580</td>
<td>1.40</td>
<td>84812</td>
</tr>
<tr>
<td></td>
<td>Earthquake (m/s²)</td>
<td>7.11</td>
<td>1.00</td>
<td>7.11</td>
</tr>
<tr>
<td>2</td>
<td>Dead (m/s²)</td>
<td>9.81</td>
<td>1.25</td>
<td>12.26</td>
</tr>
<tr>
<td></td>
<td>Snow/Ice (N/keypoint)</td>
<td>26274</td>
<td>1.50</td>
<td>39411</td>
</tr>
<tr>
<td></td>
<td>Wind (N/keypoint)</td>
<td>37080</td>
<td>1.40</td>
<td>51912</td>
</tr>
<tr>
<td></td>
<td>Earthquake (m/s²)</td>
<td>7.89</td>
<td>1.00</td>
<td>7.89</td>
</tr>
<tr>
<td>3</td>
<td>Dead (m/s²)</td>
<td>9.81</td>
<td>1.25</td>
<td>12.26</td>
</tr>
<tr>
<td></td>
<td>Snow/Ice (N/keypoint)</td>
<td>14780</td>
<td>1.50</td>
<td>22170</td>
</tr>
<tr>
<td></td>
<td>Wind (N/keypoint)</td>
<td>22510</td>
<td>1.40</td>
<td>31514</td>
</tr>
<tr>
<td></td>
<td>Earthquake (m/s²)</td>
<td>9.46</td>
<td>1.00</td>
<td>9.46</td>
</tr>
<tr>
<td>4</td>
<td>Dead (m/s²)</td>
<td>9.81</td>
<td>1.25</td>
<td>12.26</td>
</tr>
<tr>
<td></td>
<td>Snow/Ice (N/keypoint)</td>
<td>6618</td>
<td>1.50</td>
<td>9927</td>
</tr>
<tr>
<td></td>
<td>Wind (N/keypoint)</td>
<td>9224</td>
<td>1.40</td>
<td>12914</td>
</tr>
<tr>
<td></td>
<td>Earthquake (m/s²)</td>
<td>9.59</td>
<td>1.00</td>
<td>9.59</td>
</tr>
</tbody>
</table>
### Table 5.14 Summary of Loads on Icosahedron

<table>
<thead>
<tr>
<th>Case</th>
<th>Load</th>
<th>Unfactored Load</th>
<th>Load Factor</th>
<th>Factored Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 5 - 11</td>
<td>Dead (m/s²)</td>
<td>9.81</td>
<td>1.25</td>
<td>12.26</td>
</tr>
<tr>
<td></td>
<td>Snow/Ice (N/keypoint)</td>
<td>80766</td>
<td>1.50</td>
<td>121149</td>
</tr>
<tr>
<td></td>
<td>Wind (N/keypoint)</td>
<td>61380</td>
<td>1.40</td>
<td>85932</td>
</tr>
<tr>
<td></td>
<td>Earthquake (m/s²)</td>
<td>5.20</td>
<td>1.00</td>
<td>5.20</td>
</tr>
<tr>
<td>2</td>
<td>Dead (m/s²)</td>
<td>9.81</td>
<td>1.25</td>
<td>12.26</td>
</tr>
<tr>
<td></td>
<td>Snow/Ice (N/keypoint)</td>
<td>60010</td>
<td>1.50</td>
<td>90015</td>
</tr>
<tr>
<td></td>
<td>Wind (N/keypoint)</td>
<td>38080</td>
<td>1.40</td>
<td>53312</td>
</tr>
<tr>
<td></td>
<td>Earthquake (m/s²)</td>
<td>6.33</td>
<td>1.00</td>
<td>6.33</td>
</tr>
<tr>
<td>3</td>
<td>Dead (m/s²)</td>
<td>9.81</td>
<td>1.25</td>
<td>12.26</td>
</tr>
<tr>
<td></td>
<td>Snow/Ice (N/keypoint)</td>
<td>33750</td>
<td>1.50</td>
<td>50625</td>
</tr>
<tr>
<td></td>
<td>Wind (N/keypoint)</td>
<td>23110</td>
<td>1.40</td>
<td>32354</td>
</tr>
<tr>
<td></td>
<td>Earthquake (m/s²)</td>
<td>7.89</td>
<td>1.00</td>
<td>7.89</td>
</tr>
<tr>
<td>4</td>
<td>Dead (m/s²)</td>
<td>9.81</td>
<td>1.25</td>
<td>12.26</td>
</tr>
<tr>
<td></td>
<td>Snow/Ice (N/keypoint)</td>
<td>11902</td>
<td>1.50</td>
<td>17853</td>
</tr>
<tr>
<td></td>
<td>Wind (N/keypoint)</td>
<td>9472</td>
<td>1.40</td>
<td>13261</td>
</tr>
<tr>
<td></td>
<td>Earthquake (m/s²)</td>
<td>9.46</td>
<td>1.00</td>
<td>9.46</td>
</tr>
</tbody>
</table>
6. Finite Element Analysis Results

After the different cases and load combinations were defined, the models were analyzed using ANSYS. For each of the case defined, the model was subjected to three different load combinations (dead + snow & ice, dead + wind, and dead + earthquake). Hence, a total of 66 models were analyzed. In order to allow for a more efficient process, input script files were written such that for each difference case, only a small variable had to be change and not the entire model.

The results which are of interest to this report are the maximum deflection at each node and the maximum structural force and moment of each element. Another parameter which was of interest was the structural mass of the model. This was considered an important factor in comparing the models against one another because huge savings in mass may result in more lenient deflection tolerances.

Table 6.1 Summary of Masses for Rhombicosidodecahedron Structure

<table>
<thead>
<tr>
<th>Case</th>
<th>Mass (1000 kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2423.54</td>
</tr>
<tr>
<td>2</td>
<td>1938.83</td>
</tr>
<tr>
<td>3</td>
<td>1454.12</td>
</tr>
<tr>
<td>4</td>
<td>969.42</td>
</tr>
<tr>
<td>5</td>
<td>2769.76</td>
</tr>
<tr>
<td>6</td>
<td>1731.1</td>
</tr>
<tr>
<td>7</td>
<td>2077.32</td>
</tr>
<tr>
<td>8</td>
<td>1994.23</td>
</tr>
<tr>
<td>9</td>
<td>1537.22</td>
</tr>
<tr>
<td>10</td>
<td>1052.51</td>
</tr>
<tr>
<td>11</td>
<td>1903.44</td>
</tr>
</tbody>
</table>
Table 6.2 Summary of Masses for Icosahedron Structure

<table>
<thead>
<tr>
<th>Case</th>
<th>Mass (1000 kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1844.24</td>
</tr>
<tr>
<td>2</td>
<td>1475.39</td>
</tr>
<tr>
<td>3</td>
<td>1106.54</td>
</tr>
<tr>
<td>4</td>
<td>737.7</td>
</tr>
<tr>
<td>5</td>
<td>2107.7</td>
</tr>
<tr>
<td>6</td>
<td>1317.31</td>
</tr>
<tr>
<td>7</td>
<td>1580.78</td>
</tr>
<tr>
<td>8</td>
<td>1517.55</td>
</tr>
<tr>
<td>9</td>
<td>1169.78</td>
</tr>
<tr>
<td>10</td>
<td>800.93</td>
</tr>
<tr>
<td>11</td>
<td>1448.46</td>
</tr>
</tbody>
</table>

Table 6.1 and Table 6.2 above summarize the mass of each structure for each particular case. From the above values, one can see that the rhombicosidodecahedron is approximately 24% heavier than the icosahedron option. This is because the rhombicosidodecahedron structure has more members and divisions. The member lengths in the icosahedron are relatively longer and hence more prone to buckling and other types of failure. If configuration selection was strictly based on nodal deflection, element forces, and element moments, the rhombicosidodecahedron would be the optimal choice. However, once the above masses were considered in the decision making process, the results were radically different.

6.1. Optimal Cases

The results obtained from each case were plotted on graphs in order to compare them against one another. An optimal case would have minimal nodal deflection and structural element force and moment. With deflection, it is obvious that smaller structures will have shorter member lengths; if cross-section size and thickness is kept constant, then the structure will experience less deflection than larger structures. However, one must keep in mind that a smaller structure, although lighter in weight, will have a smaller enclosed area. As a result, it
6. Finite Element Analysis Results

is important to find a balance point by minimizing nodal deflection and element forces while maximizing enclosed area.

The following graphs are average results from analysis of the different cases of rhombicosidodecahedron and icosahedron under different loading combinations. Please note that the series in each graph is an average value of the three load combinations specified in section 5.1. Each graph represents a different response parameter ($U_x$, $U_y$...etc). Within each graph are three series; each series represent changes in the response parameter when a certain variable (radius of enclosure, cross-section size/type) is changed. For graphs which reflect each load combinations separately, please refer to Appendix A.

**Deflection (Rhombicosidodecahedron)**

One can see from the graphs below that as the radius of the telescope enclosure increases in size, the maximum nodal deflection increases proportionally. After a radius of approximately 20 [m], there is a sharp increase in the slope of the curve, this means that the rhombicosidodecahedron is perhaps better suited for a radius of less than 20 [m]. Comparing the change in response due to changes in cross-section size and thickness, it is apparent that a change in thickness results in a more drastic change in maximum deflection. After reviewing the changes in mass of the structure at different cross-section size and thickness, it can be concluded that increasing the cross-section thickness to obtain a lower deflection is a better option than increasing the cross-section size in terms of weight.
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**Average X-deflection**

![Graph showing Average X-deflection for Rhombicosidodecahedron](image)

- **Figure 6.1** Average X-deflection for Rhombicosidodecahedron

**Average Y-deflection**

![Graph showing Average Y-deflection for Rhombicosidodecahedron](image)

- **Figure 6.2** Average Y-deflection for Rhombicosidodecahedron
6. Finite Element Analysis Results

**Average Z-deflection**

![Graph of Average Z-deflection](image)

**Figure 6.3 Average Z-deflection for Rhombicosidodecahedron**

**Average Structural Force (X)**

![Graph of Average Structural Force (X)](image)

**Figure 6.4 Average Structural Force (X) for Rhombicosidodecahedron**
Finite Element Analysis Results

Figure 6.5 Average Structural Force (Y) for Rhombicosidodecahedron

Figure 6.6 Average Structural Force (Z) for Rhombicosidodecahedron
6. Finite Element Analysis Results

**Structural Force (Rhombicosidodecahedron)**

For all three structural force graphs, there is a sharp increase in element force between a radius of 15 [m] and 20 [m] followed by a decrease after a radius of 20 [m]. This increase is especially significant in the axial (\(F_x\)) component of the element force. For the major and minor shear forces (\(F_y\) and \(F_z\)), the increase is less significant. Both the axial and shear forces retain an approximately linear shape when subjected to changes in the cross-section size and thickness. Looking at the axial force graph (Figure 6.4 above), one can see that changing the cross-section thickness has more impact than changing the cross-section size due to the flat slope of the cross-section size curve. However, in terms of major and minor shear forces Figure 6.5 and Figure 6.6), changing the cross-section size or thickness will have similar impact on the magnitude of the force.

![Average Structural Moment (X)](image)

**Figure 6.7 Average Structural Moment (X) for Rhombicosidodecahedron**
6. Finite Element Analysis Results

**Figure 6.8** Average Structural Moment (Y) for Rhombicosidodecahedron

**Figure 6.9** Average Structural Moment (Z) for Rhombicosidodecahedron
6. Finite Element Analysis Results

**Structural Moment (Rhombicosidodecahedron)**

The structural moments display a similar trend as the structural forces above. Both torsional and bending moments display a linear trend when the cross-section size and thickness is altered. Similar to the graphs for structural forces, the torsional and bending moments experience a sudden increase in magnitude between a radius of 15 [m] and 20 [m] with a decrease after 20 [m].

**Deflection (Icosahedron)**

The trends in the deflection vs. radius for the icosahedron appear to be quite different from that of the rhombicosidodecahedron above. However, please keep in mind that these graphs are obtained by taking the *average* of the response from different load combinations. The plateau behaviour seen below is characteristic of the second load combination (dead + wind). This trend was less significant in the rhombicosidodecahedron configuration above. What this implies is that for the icosahedron configuration, savings in deflection will not be obtained by decreasing the radius size within the range of 15 – 20 [m].
6. Finite Element Analysis Results

**Average X-deflection**

Figure 6.10 Average X-deflection for Icosahedron

**Average Y-deflection**

Figure 6.11 Average Y-deflection for Icosahedron

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6. Finite Element Analysis Results

![Average Z-deflection Graph](image)

**Figure 6.12 Average Z-deflection for Icosahedron**

*Structural Force (Icosahedron)*

Similar to the deflection, the element axial and shear forces experience a plateau between a radius of 15 [m] to 20 [m]. However, there is an increasing trend (as the radius increases) with the axial force while the shear forces display a decreasing trend. From the sudden increase in axial force at a radius of 25 [m], one may postulate that the structure has buckled (failed); however, one must bear in mind that the 25 [m] radius is the “control case” and also appears in the cross-section size and thickness series. The cross-section size and thickness series do not display any abrupt increases; hence the possibility of the control case failing is unlikely.
6. Finite Element Analysis Results

**Average Structural Force (X)**

![Graph showing the average structural force (X) for different cross-section sizes and thicknesses.](image)

**Figure 6.13 Average Structural Force (X) for Icosahedron**

**Average Structural Force (Y)**

![Graph showing the average structural force (Y) for different cross-section sizes and thicknesses.](image)

**Figure 6.14 Average Structural Force (Y) for Icosahedron**
6. Finite Element Analysis Results

**Figure 6.15 Average Structural Force (Z) for Icosahedron**

**Structural Moment (Icosahedron)**

Compared to the rhombicosidodecahedron, the icosahedron configuration had a smaller range in torsional and bending moment magnitude when the radius was changed. In addition, the average torsional and bending moment in the icosahedron did not experience the sudden spike in magnitude at a radius of 20 [m] as observed in the rhombicosidodecahedron configuration. This means that the icosahedron configuration is more stable and less influenced by the radius of the structure. The moment vs. cross-section size/thickness plots displayed a similar trend as the rhombicosidodecahedron plots, increasing linearly with increasing size or thickness.
6. Finite Element Analysis Results

**Average Structural Moment (X)**

![Graph of Average Structural Moment (X) for Icosahedron](image1)

**Average Structural Moment (Y)**

![Graph of Average Structural Moment (Y) for Icosahedron](image2)

Figure 6.16 Average Structural Moment (X) for Icosahedron

Figure 6.17 Average Structural Moment (Y) for Icosahedron
6. Finite Element Analysis Results

![Average Structural Moment (Z)](chart)

**Figure 6.18 Average Structural Moment (Z) for Icosahedron**

The following table summarizes the results of optimal cases for each variable and load combination with the consideration of mass included in the calculation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Load Combo</th>
<th>Rhombicosidodecahedron</th>
<th>Icosahedron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal Radius (m)</td>
<td>1, 2, 3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Optimal Cross-section Size (m)</td>
<td>1, 2, 3</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>Optimal Cross-section Thickness (m)</td>
<td>1, 2, 3</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Optimal Cross-section Type</td>
<td>1, 2, 3</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>11</td>
<td>1</td>
</tr>
</tbody>
</table>
Referring to Table 6.3 above, one can see that for both the rhombicosidodecahedron and the icosahedron configuration, case 1 (25 [m] radius) is the optimal case when choosing the optimal radius. The optimal cross-section size is case 6 (0.3 [m] cross-section size) when load combinations 1, 2, and 3 were considered separately. However, when the average of the three load combination results was considered, case 7 (0.35 [m] cross-section size) had a more optimal cross-section size than case 6 for the rhombicosidodecahedron configuration. The optimal cross-section thickness is case 10 (0.02 [m] cross-section thickness) for both configurations. Comparing the results of using a square cross-section vs. a circular cross-section, it appeared that the circular cross-section yielded more optimal results. The above decisions were arrived at using the following formulas:

**Table 6.4 Optimization Formulas**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>( \frac{P \cdot m}{V} )</td>
</tr>
<tr>
<td>Cross-section size</td>
<td>( P \cdot m \cdot \text{CSS} )</td>
</tr>
<tr>
<td>Cross-section thickness</td>
<td>( P \cdot m \cdot t )</td>
</tr>
<tr>
<td>Cross-section shape</td>
<td>( P \cdot \pi )</td>
</tr>
</tbody>
</table>

where  
\[ P = \text{parameter (} U_x, U_y, U_z, F_x, F_y, F_z, M_x, M_y, M_z \text{)} \]  
\( m = \text{mass of structure} \)  
\( V = \text{volume of enclosed structure} \)  
\( \text{CSS} = \text{cross-section size} \)  
\( t = \text{cross-section thickness} \)

Each of the parameter or results obtained from analysis was first multiplied by the mass of the corresponding structure in order to factor in the importance of mass. Mass is an
important issue in construction and the lighter a structure is, the more cost effective it will be to construct. However, a lighter structure is often smaller and smaller structures will always have smaller deflections and element forces. As a result, an ideal structure will have a radius of zero [m]. Clearly, one cannot have such a structure; hence, the enclosed volume must also be taken into consideration. In the selection of cases in Table 6.3, the case with the lowest value from each variable group (ie. Case 1-4, case 1, 5, 6, 7, Case 1, 8, 9, 10, and Case 1, 11) was deemed the “best” case for that particular parameter. The case which is deemed the “best” case for the most parameter is then selected as the optimal case.

Referring to Table 6.4 above, one can see the optimization formula for “radius” is $P*m/V$. The goal was to minimize the magnitude of the parameter (deflection and element forces) and mass while maximizing the enclosed volume; this is achieved when minimizing the formula.

The optimization formula for the cross-section size variable is $P*m*CSS$. This formula aims at minimizing the magnitude of the parameter, mass, and cross-section size. It is known that smaller cross-sections have larger deflections but lesser element forces. Hence, a balance point must be reached. By multiplying the optimization formula by the cross-section size, one is essentially compensating for its deficiencies. For example, a structure with a smaller element size will have a larger deflection than a structure with a larger element size. The large deflection will be scaled down when it is multiplied by the small value of the cross-section size. The small deflection in the large element structure will be scaled up when it is multiplied by the large value of the cross-section size. One problem with this method is that
6. Finite Element Analysis Results

the user must carefully consider the correlation between cross-section size and deflection/element forces. This will be further discussed in section 9.

The optimization formula for cross-section thickness is $P \cdot m \cdot t$. Similar to the two optimization formulas above, both the parameter and mass magnitude is minimized in this formula. The cross-section thickness works in a similar manner as the cross-section size. A smaller thickness will result in larger deflections but smaller element forces. Hence, by multiplying the parameters by the cross-section thickness, an optimized point may be reached.
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6.2. Optimal Configuration

Table 6.5 Comparison of Rhombicosidodecahedron and Icosahedron Configurations for Different Cases

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Load Combo</th>
<th>Optimal Choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius (m)</td>
<td>50</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>1, 2, 3</td>
<td>R</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>I</td>
</tr>
<tr>
<td>Cross-section Size (m)</td>
<td>0.4</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>0.45</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>0.35</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>R</td>
</tr>
<tr>
<td>Radius (m)</td>
<td>0.05</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>0.03</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td>Cross-section Type</td>
<td>Square</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td>Circle</td>
<td>1, 2, 3</td>
<td>I</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>I</td>
</tr>
</tbody>
</table>

After comparing the individual cases within each configuration, the performance of each configuration was compared against one another. Table 6.5 above summarizes the optimal configuration for each individual case. It is evident that for a majority of the cases, the icosahedron configuration is the optimal case. This is determined by comparing the value given by the above optimization formulas. For each parameter (i.e. $U_x$), the configuration with the lower optimized value was given a point. At the end, the configuration with the
6. Finite Element Analysis Results

most points for each individual case was deemed the “optimal” configuration, yielding the above table.
7. Use of Photogrammetry to Assess Existing Structures

Photogrammetry, as its name implies, is the combination of photography and metrology. Using photogrammetry, one can obtain accurate 3-dimensional coordinates of any given object or structure from a set of 2-dimensional photographs.

Similar to most conventional surveying techniques, photogrammetry is based on the principles of triangulation. Triangulation is a technique used to determine the distance between two points or the relative position of two or more points in space. In photogrammetry, by taking two or more photographs of an object, points which are visible in both photographs may be triangulated to obtain their relative positions.

Photogrammetry can be easily applied to the assessment of existing structures. Building plans and specifications for structures may be difficult to obtain at times, especially when the structure in question is an older building. Existing land surveying tools involve the use of cumbersome and expensive equipment; hence, photogrammetry may be an ideal alternative in such cases.

In the design of new telescope enclosures, a company often takes one of two paths:

1. Recycle a previous design
2. Generate an entirely new design
7. Use of Photogrammetry to Assess Existing Structures

However, there are currently many interesting and innovative telescope enclosure designs available and it would be quite wasteful to not examine and learn the benefits of each.

If one could study the pros and cons of existing telescope enclosure designs, then a “super” or “hybrid” enclosure encompassing the advantages of several conventional designs may be created. This process may be aided by the use of photogrammetry. With photogrammetry, 3-dimensional representations of different existing telescope enclosures may be created quickly and efficiently. Once these models are loaded into any CAD programs, merging and combining individual features from each design will be quite simple.

7.1. PhotoModeler Pro

There is a software developed by Eos Systems Inc. called “PhotoModeler Pro” which aids the user in extracting measurements and 3-dimensional models from photographs. Operation of the software is relatively simple and the only equipment required is a camera and a computer.

Similar to any photogrammetry software available on the market, the accuracy and precision of the resulting model is dependent on the quality of the camera used. The generation of 3-dimensional models in PhotoModeler is based on 3 principles (PhotoModeler User Manual 2004):

1. PhotoModeler assumes that a ray of light coming from some point through the focal node of the lens of a camera and hitting the film can be described by a straight line.

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2. By knowing where the camera was at the time of exposure, PhotoModeler can use the location of where the ray of light hit the film to calculate the equation of that ray of light in three dimensions.

3. Each point that is to be measured is imaged in at least two photographs, and preferable in three of more. These points are used to compute the light ray positions and their intersections for determining the positions in 3-dimensional space.

The above assumptions are based on a perfect case scenario and may be affected by the following factors:

Air effects
Air effects are only a concern for aerial photos since air may cause distortion in photographs. Objects which are less than 1000 [m] in size are usually not prone to air distortion problems.

Lens Distortion
Lens distortion is virtually unavoidable because all lens are prone to some degree of distortion. However, PhotoModeler has the capability to compensate for lens distortion through a calibration procedure; as a result, regular consumer cameras may be used to obtain fairly accurate measurements.

Imperfect Imaging and Point Location
Both of these problems may be alleviated by using digital cameras. PhotoModeler assumes that the imaging media (the photograph) is perfectly flat. Photographs produced by film
cameras need to be developed then scanned into the computer for processing. This procedure may cause the photograph to bend or the image to blur. Another problem which may cause imperfect imaging is the limits of resolution. Objects which are smaller require a higher resolution camera to obtain accurate measurements. Imperfect point location refers to the uncertainty in the distance between the imaging medium and the camera lens and body. In digital cameras, this is fixed and is not a problem; however, in film cameras, this distance may be difficult to obtain.

Changes in Camera Characteristics

The last factor which affects measurement accuracy is the changes in the internals of the camera between photographs. For a single measurement project, the focal length, lens distortion characteristics, and lens position must remain constant in order for the software to calculate the position of the camera and generate 3-dimensional coordinates. Hence, the user must refrain from using the zoom feature or changing the camera lens.

Once a suitable camera is selected for a certain project, the camera must be calibrated for PhotoModeler to obtain its focal length, imaging scale, image center, and lens distortion. This information is used to generate relationships between points on a photograph and the location of 3-dimensional points. The procedures for calibrating a camera for PhotoModeler are as follows:
7. Use of Photogrammetry to Assess Existing Structures

1. Take pictures of the calibration grid below.

![Figure 7.1 Calibration Grid for PhotoModeler]

2. Take 2 photos from each side (total of 8), one with the camera in regular orientation then another with the camera rotated 90° CW.

3. Load photos into PhotoModeler by clicking on New Project → Calibration Project

4. Execute calibration then check for adequacy of calibration.

Table 7.1 Camera Calibration Adequacy Check

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Max. Suggested Value</th>
<th>Actual Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Marking Residual</td>
<td>1</td>
<td>0.322</td>
</tr>
<tr>
<td>Max RMS Residual</td>
<td>0.5</td>
<td>0.151</td>
</tr>
<tr>
<td>Total Error</td>
<td>0.02</td>
<td>0.012</td>
</tr>
</tbody>
</table>

7 Courtesy of PhotoModeler User Manual
5. Check marking residual display to ensure there are no patterns in the error as this will indicate that the camera lens has high distortion. Common patterns to look for include: error bars pointing in one direction, point towards the center, or pointing away from the center.

Creating Simple Drawings

1. Take photographs of the object. For higher accuracy, 2 or more pictures should be taken. In addition, photograph should be taken at an angle greater than 90° from one other. The accuracy of the resulting model will depend on the number of photographs taken; it is generally recommended that each point on the object appear on 3 or more photographs.

2. Start a new project. Load pictures into program.

3. Mark points on all pictures. Link pictures by clicking on points which appear in both pictures then click on the "quick reference" button.

4. Once 8 or more points are referenced, click on the process button.

5. Mark out the lines and curves on the object then reference the lines and curves from each picture.

6. Add texture and surface where needed.
Example 1.
For relatively simple and symmetrical objects such as the water bottle below, two photographs may be sufficient. The water bottle was placed on the calibration grid in order to obtain more reference points. Once the photographs are loaded into the program, the user marks points and lines which appear in more than one photograph. These points and lines are then referenced between the photographs so that the software will recognize that they are the same. In addition, there are other features in PhotoModeler such as automatic cylinder and surface generation which aid in the generation of the 3-dimensional model. For example, when generating cylinders one only needs to mark four points on the surface of the cylinder. The only requirement is that the first two points must be on one edge and the second two points on the other edge.

Figure 7.3 Marking Points on a Cylinder

Figure 7.4 below shows the process of referencing curves and lines which appear in both photographs.
7. Use of Photogrammetry to Assess Existing Structures

Figure 7.4. Referencing curves and lines from pictures

Figure 7.5. 3-dimensional view of generated bottle with texture added
7. Use of Photogrammetry to Assess Existing Structures

After referencing the appropriate points and lines, a wire model can be viewed in the 3-D viewer. PhotoModeler also has the capability to add texture and skins to the model in order to enhance its realistic qualities. Figure 7.5 above is a model of the water bottle with the texture function. This 3-dimensional creation is an almost exact replica of the real object and only takes a minutes to generate.

Figure 7.6. Model exported into AutoCAD
Example 2.

Example 1 above demonstrated PhotoModeler's ability to model simple symmetrical objects. However, PhotoModeler can also be used to model complicated structures with hidden lines and details. The following example is a Lego® structure with hidden corners and lines.

![Lego Structure](image)

Figure 7.7 Lego Structure

Since the structure consists of four base columns overlayed by horizontal members, some edges are not entirely visible. However, by assuming that the edges are straight, one can approximate its location by locating the endpoints of the lines. As long as the endpoints are visible in 2 or more photographs, a line can be drawn between the points to represent the hidden line. This is useful when modeling smaller objects where it is difficult to photograph every hidden detail.

Another important point shown by this example is the difference a high quality camera makes on the accuracy of the resulting 3-dimensional model. The model in Figure 7.8 below
7. Use of Photogrammetry to Assess Existing Structures

was created from photographs taken by a 5 mega-pixel camera. Since the object is small, the user must zoom in several times to mark each reference point. However, the image obtained by a mediocre camera will become blurry when the zoom exceeds 400%. The model in Figure 7.9 was created from photographs taken by an 8 mega-pixel camera. Compared to Figure 7.8, the model in Figure 7.9 is clearly more accurate. This is because the image remained focused even when the zoom exceeded 400%, allowing the user to accurately mark the location of reference points.

Figure 7.8 3-dimensional model of Lego structure (Poor quality camera)
7. Use of Photogrammetry to Assess Existing Structures

Figure 7.9 3-dimensional model of Lego structure (Higher quality camera)
8. Cladding Options for Enclosure

After giving suggestions for the structural geometry of a telescope enclosure, it would only make sense to dedicate a small section in this report to cladding options for the enclosure.

Traditionally, the type of cladding selected for a telescope enclosure is mainly governed by the shape of the underlying superstructure. Cylindrical enclosures often utilize corrugated steel or aluminum cladding (see Figure 8.1 below).

Steel plate skins are often welded onto spherical enclosures with curved steel ribs. This is also known as the “orange-peel” approach mentioned in previous sections (please refer to Figure 8.1).

Figure 8.1 The Subaru Telescope in Hawaii

8 Courtesy of the Subaru Telescope Website <http://www.subarutelescope.org/>
8. Cladding Options for Enclosure

Figure 8.2. For geodesic domes, steel or aluminum triangular plates secured by external batten strips are often used (see Figure 8.3 below).

Figure 8.2 The Gemini Telescope in Hawaii

Figure 8.3 The Hobby-Eberly Telescope in Texas

9 Courtesy of Andrew Stephens <http://www.astro.psu.edu/users/lwr/>
8. Cladding Options for Enclosure

As seen in the above telescope enclosures, traditionally, steel and aluminum panels have been the material of choice when it comes to cladding options. However, with the considerable advances in material engineering, the use of composite materials as cladding may become more popular in the future. The following paragraphs describe several composite panels which may be suitable for use as enclosure cladding options.

*Alucobond by Alcan*

Alucobond is an aluminum composite material which consists of two sheets of smooth 0.02” thick aluminum sheets thermo-bonded to a polyethylene core in a continuous process. This product may also be pre-finished with a premium Kynar® coating in a spectrum of colors to add to the aesthetics of any building. Alucobond is used as external cladding on office buildings, hospitals, convention centers, hotels, airports, and many other types of buildings.

*Versapanel by Centria*

The Versapanel is a structural panel with G90 galvanized steel skins and a poured-in-place core of urethane modified isocyanurate foam for insulation values up to R32. The galvanized steel skins provide structural strength. However, Versapannels should not be expected to provide diaphragm strength for buildings. In addition to its light weight and easy installation, the panel also provides superior weather-tightness.

*Meteon by Trespa*

Meteon is a flat panel with thermosetting resin skins homogeneously reinforced with wood fibers. All Meteon panels are manufactured under high pressure and temperatures and is
resistant to UV radiation in sunlight. The high modulus of elasticity, tensile, and flexural strength means that Meteon panels are highly impact resistant and durable.

**R-Span Foamed-in-place Panel by Metecno**

Similar to the Versapanel above, the R-Span foamed-in-place panel features galvanized steel skins. The core material is foamed-in-place polyurethane produced by expanding blended chemical components between the steel skins to obtain a completely monolithic sandwich panel. R-span panels are lightweight and high strength, providing a more economical alternative to expensive plate materials.

**Honeycomb Cores by Plascore**

Plascore provides a wide range of cladding panels with a variety of outer skin material such as aluminum, steel, aramid fibers, or thermoplastic. Each skin material is matched with a different core material to provide strength and durability. Table 8.1 below summarizes the attributes of different Plascore panels. Plascore panels are used in aerospace, marine, architectural, construction and many other industries.
### Table 8.1 Summary of Panel Attributes

<table>
<thead>
<tr>
<th>Skin Material</th>
<th>Core Material</th>
<th>Attributes</th>
</tr>
</thead>
</table>
| Aluminum      | Aluminum Alloy Foil | • Elevated use temperatures  
• High thermal conductivity  
• Flame resistant  
• Excellent moisture and corrosion resistance  
• Fungi Resistance  
• Low weight/High Strength |
| Galvanized Steel | Annealed SS304 | • Excellent moisture and corrosion resistance  
• Flame resistance  
• Fungi resistance  
• Elevated use temperatures  
• High thermal conductivity  
• Flame resistant  
• Excellent moisture and corrosion resistance  
• Fungi Resistance  
• Low weight/High Strength |
| Aramid Fibers | PN1 & PN2 Nomex | • Fire resistant/self extinguishing (PN2 only)  
• High strength to weight ratio  
• Corrosion resistant  
• Excellent dielectric properties  
• Thermally insulating  
• High toughness  
• Excellent creep and fatigue performance  
• Good thermal stability  
• Densities as low as 1.5 pcf (PN1) and 2.0 pcf (PN2)  
• Over expanded cell configuration suitable for forming simple curves  
• Compatible with most adhesives used in sandwich composites  
• Up to 40% lighter than comparable Nomex® honeycomb  
• Extremely high strength to weight ratio  
• Excellent thermal and moisture stability  
• Improved shear strength and modulus  
• Conforms to stringent smoke, toxicity and flammability standards  
• High toughness  
• Good thermal forming response |
| Thermoplastic | Polycarbonate | • Excellent dielectric properties  
• Good thermal and electric insulator  
• Conductive grades available  
• Fire resistant  
• Corrosion resistant  
• Fungi resistant  
• Use temperatures below 200°F  
• Small cell sizes at high densities  
• Available transparent and in colors  
• High strength to weight ratio  
• Corrosion, fungi, rot, chemical and moisture resistant  
• Sound and vibration dampening  
• Energy absorbing  
• Thermoformable  
• Temperature use to 180°F  
• Recyclable |
|              | Polypropylene | • Excellent dielectric properties  
• Good thermal and electric insulator  
• Conductive grades available  
• Fire resistant  
• Corrosion resistant  
• Fungi resistant  
• Use temperatures below 200°F  
• Small cell sizes at high densities  
• Available transparent and in colors  
• High strength to weight ratio  
• Corrosion, fungi, rot, chemical and moisture resistant  
• Sound and vibration dampening  
• Energy absorbing  
• Thermoformable  
• Temperature use to 180°F  
• Recyclable |

---

10 Courtesy of www.plascore.com
## 8. Cladding Options for Enclosure

### Table 8.2 Summary of Cladding Material Properties

<table>
<thead>
<tr>
<th>Type</th>
<th>Company</th>
<th>Skin Material</th>
<th>Core Material</th>
<th>Thickness (inch)</th>
<th>Width</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal Composites</td>
<td>Centria</td>
<td>Aluminum</td>
<td>urethane modified isocyanurate</td>
<td>1.75</td>
<td>36&quot;</td>
<td>40' or greater</td>
</tr>
<tr>
<td></td>
<td>Alucobond</td>
<td>Aluminum</td>
<td>polyethylene</td>
<td>0.1182</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.1576</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.2364</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Galvanized Steel</td>
<td>Metecno, API</td>
<td>Aluminum</td>
<td>foamed in place urethane</td>
<td>2</td>
<td>42&quot;</td>
<td>10&quot; - 48&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td>4</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polymer Composites</td>
<td>Trespa</td>
<td>Thermosetting Resins</td>
<td>wood fibers</td>
<td>5/16</td>
<td>10'x5'</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3/8</td>
<td>8'x6'</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/2</td>
<td>12'x6'</td>
<td></td>
</tr>
<tr>
<td>Honeycomb Cores</td>
<td>Plascore</td>
<td>Aluminum</td>
<td>aluminum alloy foil</td>
<td></td>
<td>&lt;60&quot;</td>
<td>&lt;150&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stainless Steel</td>
<td>annealed SS304</td>
<td></td>
<td>&lt;48&quot;</td>
<td>&lt;96&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Aramid Fibers</td>
<td>DuPont Nomex paper©</td>
<td></td>
<td>&lt;60&quot;</td>
<td>&lt;120&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Kevlar® paper</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Thermoplastic</td>
<td>Polypropylene</td>
<td>0.25 - 12</td>
<td>&lt;72&quot;</td>
<td>&lt;600&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Polycarbonate</td>
<td>0.12 - 13</td>
<td>&lt;49&quot;</td>
<td>&lt;150&quot;</td>
</tr>
</tbody>
</table>

The above materials may be used as a reference or guide when determining the types of materials available on the market.
9. Conclusions and Recommendations

This report explored various geometrical solutions for enclosing next generation telescopes. Due to the rapid advancement in telescope technology, the size of telescopes may double or triple in size over the next 10 years. In order to ensure an efficient enclosure design for these next generation telescopes, studies must be done to explore new geometries and enclosing systems. The two configurations proposed and studied in this report is the Platonic icosahedral sphere and the Archimedean rhombicosidodecahedral sphere.

Sensitivity analysis was performed on the two selected configurations by varying the enclosure radius (R), and the member cross-section size (CSS), thickness (T), and type (CT). There were a total of 11 cases for each configuration with different R, CSS, T, and CT values. These cases were then subjected to three different load combinations (dead + snow, dead+ wind, and dead + earthquake) according to provisions from the NBCC 2005. After analysis was performed on these different cases, the results were summarized and the following conclusions were drawn:

1. Based on the average results of the three load combinations, the optimal radius for both the rhombicosidodecahedron and icosahedron configuration is 25 [m].

2. Based on the average results of the three load combinations, the optimal member cross-section size for the rhombicosidodecahedron configuration is 0.35 [m]. For the icosahedron configuration, it is 0.3 [m].

3. Based on the average results of the three load combinations, the optimal member cross-section thickness is 0.02 [m] for both configurations.
9. Conclusions and Recommendations

4. Based on the average results of the three load combinations, the optimal member cross-section type is circular for the rhombicosidodecahedron configuration. For the icosahedron configuration, the optimal member cross-section type is square.

5. The icosahedron configuration generally performs better in terms of deflection and member forces than the rhombicosidodecahedron configuration.

6. The increase in nodal deflection rises rapidly once the enclosure radius for the rhombicosidodecahedron configuration exceeds 20 [m]. However, when one takes into consideration the benefits of a larger enclosed volume, a larger enclosure may still be justified. For the icosahedron configuration, an enclosure radius of 15 – 20 [m] yields nodal deflections which are almost the same. Hence, a larger radius is almost always the better alternative.

7. For both configurations, member forces are generally less or rise less rapidly for larger values of R. For CSS and T, nodal deflections and member forces tend to increase approximately linearly with the x-axis.

Like any other research, there will always be room for improvements. The following are recommendations for further work:

1. The optimization formula used in this report is based on the assumption that mass, size, deflection, and member forces are of equivalent weight. However, this may not be the case in reality. For simplicity sake, it is assumed in this report that if one could decrease nodal deflections by 1 [mm], then a 1000 [kg] increase in weight would be acceptable. This assumed equivalence may not necessarily reflect reality. In order to address this issue, it is suggested that weight factors be incorporated into the optimization formula to increase its realistic virtue.
9. Conclusions and Recommendations

2. The PhotoModeler software is extremely useful in generating 3-dimensional models and combining the attributes of older buildings with newer designs. Hence, it would be interesting to actually utilize this software to create hybrid structures which possess the benefits of both old and new designs.

3. The two proposed configurations were fixed in terms of member subdivision. However, one must keep in mind that member subdivision is flexible and may be altered to obtain optimal member lengths. As a result, it is suggested that further work should be done to optimize the number of member subdivision in order to obtain a truly optimal structure.
10. Bibliography


10. Bibliography


PhotoModeler User Manual. EOS Systems Inc. pg. 147, 2004


10. Bibliography


A. Appendix A – Finite Element Analysis Results
A. Appendix A – Finite Element Analysis Results

A.1. Icosahedron Results

**X-Deflection vs. Radius of Enclosure**

![X-Deflection vs. Radius of Enclosure (Icosahedron)](image)

**Figure A. 1 X-Deflection vs. Radius of Enclosure (Icosahedron)**

**Y-Deflection vs Radius of Enclosure**

![Y-Deflection vs Radius of Enclosure (Icosahedron)](image)

**Figure A. 2 Y-Deflection vs. Radius of Enclosure (Icosahedron)**
A. Appendix A – Finite Element Analysis Results

Z-Deflection vs Radius of Enclosure

![Z-Deflection vs Radius of Enclosure](image)

Figure A. 3 Z-Deflection vs. Radius of Enclosure (Icosahedron)

Structural Forces vs. Radius of Enclosure

![Structural Forces vs. Radius of Enclosure](image)

Figure A. 4 Structural Force (X) vs. Radius of Enclosure (Icosahedron)
A. Appendix A – Finite Element Analysis Results

**Structural Force vs Radius of Enclosure**

![Graph showing structural force vs radius of enclosure](image)

- Load Combo 1 Fy
- Load Combo 2 Fy
- Load Combo 3 Fy
- Average Fy

Figure A. 5 Structural Force (Y) vs. Radius of Enclosure (Icosahedron)

**Structural Force vs Radius of Enclosure**

![Graph showing structural force vs radius of enclosure](image)

- Load Combo 1 Fz
- Load Combo 2 Fz
- Load Combo 3 Fz
- Average Fz

Figure A. 6 Structural Force (Z) vs. Radius of Enclosure (Icosahedron)
A. Appendix A – Finite Element Analysis Results

**Structural Moments vs. Radius of Enclosure**

![Graph showing structural moments vs. radius of enclosure](image)

*Figure A. 7 Structural Moment (X) vs. Radius of Enclosure (Icosahedron)*

**Structural Moment vs Radius of Enclosure**

![Graph showing structural moment vs. radius of enclosure](image)

*Figure A. 8 Structural Moment (Y) vs. Radius of Enclosure (Icosahedron)*
A. Appendix A – Finite Element Analysis Results

**Structural Moment vs Radius of Enclosure**

![Graph showing structural moment vs radius of enclosure](image)

- Load Combo 1 Mz  
- Load Combo 2 Mz  
- Load Combo 3 Mz  
- Average Mz

**Figure A. 9 Structural Moment (Z) vs. Radius of Enclosure (Icosahedron)**

**X-Deflection vs Cross Section Size**

![Graph showing x-deflection vs cross-section size](image)

- Load Combo 1 Ux  
- Load Combo 2 Ux  
- Load Combo 3 Ux  
- Average Ux

**Figure A. 10 X-Deflection vs. Cross-section size (Icosahedron)**
A. Appendix A – Finite Element Analysis Results

**Y-Deflection vs Cross-section Size**

![Y-Deflection vs Cross-section Size graph]

- Load Combo 1 Uy
- Load Combo 2 Uy
- Load Combo 3 Uy
- Average Uy

Figure A. 11 Y-Deflection vs. Cross-section size (Icosahedron)

**Z-Deflection vs Cross-section Size**

![Z-Deflection vs Cross-section Size graph]

- Load Combo 1 Uz
- Load Combo 2 Uz
- Load Combo 3 Uz
- Average Uz

Figure A. 12 Z-Deflection vs. Cross-section size (Icosahedron)
A. Appendix A – Finite Element Analysis Results

**Figure A. 13 Structural Force (X) vs. Cross-section size (Icosahedron)**

**Figure A. 14 Structural Force (Y) vs. Cross-section size (Icosahedron)**
A. Appendix A – Finite Element Analysis Results

**Structural Force vs Cross-section Size**

![Graph showing structural force vs cross-section size](image)

- Load Combo 1 $F_z$
- Load Combo 2 $F_z$
- Load Combo 3 $F_z$
- Average $F_z$

Figure A. 15 Structural Force (Z) vs. Cross-section size (Icosahedron)

**Structural Moments vs Cross-section Size**

![Graph showing structural moments vs cross-section size](image)

- Load Combo 1 $M_x$
- Load Combo 2 $M_x$
- Load Combo 3 $M_x$
- Average $M_x$

Figure A. 16 Structural Moment (X) vs. Cross-section size (Icosahedron)
A. Appendix A – Finite Element Analysis Results

**Structural Moment vs Cross-section Size**

![Graph showing the relationship between structural moment and cross-section size. The graph includes data for Load Combo 1, Load Combo 2, Load Combo 3, and average data.](image)

**Figure A.17 Structural Moment (Y) vs. Cross-section size (Icosahedron)**

**Structural Moment vs Cross-section Size**

![Graph showing the relationship between structural moment and cross-section size. The graph includes data for Load Combo 1, Load Combo 2, Load Combo 3, and average data.](image)

**Figure A.18 Structural Moment (Z) vs. Cross-section size (Icosahedron)**
A. Appendix A – Finite Element Analysis Results

Deflection vs Cross-section Thickness

Figure A. 19 X-Deflection vs. Cross-section Thickness (Icosahedron)

Y-Deflection vs Cross-section Thickness

Figure A. 20 Y-Deflection vs. Cross-section thickness (Icosahedron)
Z-Deflection vs Cross-section Thickness

![Z-Deflection vs Cross-section Thickness](image)

Figure A. 21 Z-Deflection vs. Cross-section Thickness (Icosahedron)

Structural Forces vs Cross-section Thickness

![Structural Forces vs Cross-section Thickness](image)

Figure A. 22 Structural Force (X) vs. Cross-section Thickness (Icosahedron)
A. Appendix A – Finite Element Analysis Results

**Structural Force vs Cross-section Thickness**

![Graph](image)

**Figure A. 23** Structural Force (Y) vs. Cross-section Thickness (Icosahedron)

**Structural Force vs Cross-section Thickness**

![Graph](image)

**Figure A. 24** Structural Force (Z) vs. Cross-section Thickness (Icosahedron)
Figure A. 25 Structural Moment (X) vs. Cross-section Thickness (Icosahedron)

Figure A. 26 Structural Moment (Y) vs. Cross-section Thickness (Icosahedron)
Figure A. 27 Structural Moment (Z) vs. Cross-section Thickness (Icosahedron)

Figure A. 28 X-Deflection vs. Enclosure Radius (Rhombicosidodecahedron)
A. Appendix A – Finite Element Analysis Results

**Y-Deflection vs Radius of Enclosure**

![Graph showing Y-deflection vs radius of enclosure.](image)

Figure A. 29 Y-Deflection vs. Radius of Enclosure (Rhombicosidodecahedron)

**Z-Deflection vs Radius of Enclosure**

![Graph showing Z-deflection vs radius of enclosure.](image)

Figure A. 30 Z-Deflection vs. Radius of Enclosure (Rhombicosidodecahedron)
A. Appendix A – Finite Element Analysis Results

**Structural Forces vs. Radius of Enclosure**

![Graph of structural forces vs. radius of enclosure with load combinations and average force](image)

Figure A. 31 Structural Force (X) vs. Radius of Enclosure (Rhombicosidodecahedron)

**Structural Force vs. Radius of Enclosure**

![Graph of structural forces vs. radius of enclosure with load combinations and average force](image)

Figure A. 32 Structural Force (Y) vs. Radius of Enclosure (Rhombicosidodecahedron)
A. Appendix A – Finite Element Analysis Results

**Structural Force vs Radius of Enclosure**

![Graph showing structural force vs. radius of enclosure with load combinations and average force.

Figure A. 33 Structural Force (Z) vs. Radius of Enclosure (Rhombicosidodecahedron)

**Structural Moments vs. Radius of Enclosure**

![Graph showing structural moment vs. radius of enclosure with load combinations and average moment.

Figure A. 34 Structural Moment (X) vs. Radius of Enclosure (Rhombicosidodecahedron)
A. Appendix A – Finite Element Analysis Results

**Structural Moment vs Radius of Enclosure**

![Graph showing structural moment vs radius of enclosure](image)

**Figure A. 35 Structural Moment (Y) vs. Radius of Enclosure (Rhombicosidodecahedron)**

**Structural Moment vs Radius of Enclosure**

![Graph showing structural moment vs radius of enclosure](image)

**Figure A. 36 Structural Moment (Z) vs. Radius of Enclosure (Rhombicosidodecahedron)**
A. Appendix A – Finite Element Analysis Results

**X-Deflection vs Cross Section Size**

![X-Deflection vs Cross Section Size](image)

Figure A. 37 X-Deflection vs. Cross Section Size (Rhombicosidodecahedron)

**Y-Deflection vs Cross-section Size**

![Y-Deflection vs Cross-section Size](image)

Figure A. 38 Y-Deflection vs. Cross-section size (Rhombicosidodecahedron)
A. Appendix A – Finite Element Analysis Results

**Z-Deflection vs Cross-section Size**

![Graph of Z-Deflection vs Cross-section Size](image)

Figure A. 39 Z-Deflection vs. Cross-section size (Rhombicosidodecahedron)

**Structural Forces vs Cross-section Size**

![Graph of Structural Forces vs Cross-section Size](image)

Figure A. 40 Structural Force (X) vs. Cross-section size (Rhombicosidodecahedron)
A. Appendix A – Finite Element Analysis Results

**Figure A. 41 Structural Force (Y) vs. Cross-section size (Rhombicosidodecahedron)**

**Figure A. 42 Structural Force (Z) vs. Cross-section size (Rhombicosidodecahedron)**
A. Appendix A – Finite Element Analysis Results

**Structural Moments vs Cross-section Size**

Figure A. 43 Structural Moment (X) vs. Cross-section size (Rhombicosidodecahedron)

**Structural Moment vs Cross-section Size**

Figure A. 44 Structural Moment (Y) vs. Cross-section Size (Rhombicosidodecahedron)
A. Appendix A – Finite Element Analysis Results

Structural Moment vs Cross-section Size

![Graph showing structural moment vs cross-section size.](image)

Figure A. 45 Structural Moment (Z) vs. Cross-section Size (Rhombicosidodecahedron)

X-Deflection vs Cross-section Thickness

![Graph showing X-deflection vs cross-section thickness.](image)

Figure A. 46 X-Deflection vs. Cross-section Thickness (Rhombicosidodecahedron)
A. Appendix A – Finite Element Analysis Results

**Y-Deflection vs Cross-section Thickness**

![Graph showing Y-Deflection vs Cross-section Thickness](image1)

*Figure A. 47 Y-Deflection vs. Cross-section Thickness (Rhombicosidodecahedron)*

**Z-Deflection vs Cross-section Thickness**

![Graph showing Z-Deflection vs Cross-section Thickness](image2)

*Figure A. 48 Z-Deflection vs. Cross-section Thickness (Rhombicosidodecahedron)*
A. Appendix A – Finite Element Analysis Results

**Figure A. 49 Structural Force (X) vs. Cross-section Thickness (Rhombicosidodecahedron)**

**Figure A. 50 Structural Force (Y) vs. Cross-section Thickness (Rhombicosidodecahedron)**
A. Appendix A – Finite Element Analysis Results

**Structural Force vs Cross-section Thickness**

![Graph showing structural force vs cross-section thickness](image)

**Figure A. 51 Structural Force (Z) vs. Cross-section Thickness (Rhombicosidodecahedron)**

**Structural Moments vs Cross-section Thickness**

![Graph showing structural moments vs cross-section thickness](image)

**Figure A. 52 Structural Moment (X) vs. Cross-section Thickness (Rhombicosidodecahedron)**
A. Appendix A – Finite Element Analysis Results

**Figure A. 53 Structural Moment (Y) vs. Cross-section Thickness (Rhombicosidodecahedron)**

**Figure A. 54 Structural Moment (Z) vs. Cross-section Thickness (Rhombicosidodecahedron)**