THE EFFECT OF MAJOR STOCK DOWNTURNS ON EXECUTIVE STOCK OPTION CONTRACTS

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ABSTRACT

This dissertation analyzes the effect of a stock market downturn on executive compensation plans which include stock option contracts. A model is developed to determine sufficient conditions for which the optimal compensation contract exhibits characteristics of a fixed salary plus stock option. If a publicly known shift in the distribution of firm value occurs after contracting and before the agent takes his action, then it can be shown to be in the principal's interest to renegotiate the agent's contract. The resulting contract is again a fixed salary plus stock options with lower exercise prices than in the original contract. It is assumed that the shift in the distribution of firm value is a low probability event that is not contracted upon. To determine whether or not it is optimal to contract on a low probability event the set of original contract and rengotiated contract is compared to a contract that is complete with respect to the event. Benefits to complete contracting exist if the agent commits to stay after information about the event becomes available. However, if the agent can leave at any time, the principal may prefer, initially, not to contract on low probability events and simply renegotiate the contract if a low probability event occurs. Renegotiation can take the form of lowering the exercise price of outstanding stock options or adding a layer of options with a lower exercise price than existing outstanding options. Nonparametric tests on stock option grants in 1985 through 1988 indicate that the size of grants in 1987 and 1988 is significantly larger than in 1985 and 1986. These results support the prediction that stock options outstanding in 1987 were renegotiated following the stock crash in October 1987.

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Chapter 1

Introduction

This dissertation analyzes the effect of a stock market downturn on executive compensation plans which include stock option contracts. A model is developed in which the optimal compensation contract exhibits characteristics of a fixed salary plus stock option. If a publicly known shift in the distribution of firm value occurs after contracting and before the agent takes his action, then it can be shown to be in the principal's interest to renegotiate the agent's contract. The resulting contract is again a fixed salary — in fact, the same fixed salary as before — plus stock options with lower exercise prices than in the original contract. Renegotiation can take the form of lowering the exercise price of outstanding stock options or adding a layer of options with a lower exercise price than existing outstanding options. This prediction is tested on stock option grants in 1985 through 1988. The results support the prediction that stock options outstanding in 1987 were renegotiated following the stock crash in October 1987.

When stock markets crashed on October 19, 1987, executives' compensation was impacted in a major way because their compensation plans included stock options or other stock–price based incentive plans. These compensation losses or penalties occurred even

though they may have been managing their firms very well. Ideally, the executive should be rewarded or penalized according to the quality of the job he or she performs. The crash is an example of a market wide factor, outside the control of executives, which can affect executives' performance measure — the price of their companies' stocks. A natural question, therefore, arises: what changes were made, after the crash, to compensation plans that included stock–price based performance measures?

Stock option contracts have become a very popular and significant way of compensating executives. Table 1.1 presents the percent of firms with a stock option plan in May 1986 as estimated by the U.S. Conference Board. Clearly, these plans are used extensively. The amount of compensation received by executives from these plans is also quite large. Table 1.1 shows the median grant size[1] as a percent of fixed salary, and the median gain[2] on stock options exercised in 1985, both in dollars and as a percent of salary. Obviously, many executives' compensation depends highly on the capriciousness of the stock market.

Stock option contracts typically offer an executive the opportunity to purchase company stock at a set price (usually the market price at the time of granting) after a given time period has elapsed (usually one year or in given amounts each year for several years). There is usually no restriction on selling the stock so purchased. However, the option

[1]Grant size is the number of shares granted times the market price of the stock at the time of the grant.

[2]The gain at exercise is defined as the number of shares exercised times the difference between the current market price of the stock and the exercise price of the option. These gains could have accrued over several years.

Table 1.1: U.S. Conference Board Statistics

Industry	Percent of Firms with Stock Option Plans	Grant Size as a Percent of Salary	1985 Gain at Exercise	
			in Dollars	as a Percent of Salary
Commercial Banking	61	86	$108,000	58
Construction	56	71	NA	NA
Diversified Service	100	87	NA	NA
Insurance	45	54	$104,000	41
Manufacturing	82	88	$102,000	39
Retail Trade	73	90	$172,000	45
Utilities - Gas & Electric	24	79	NA	NA

itself cannot be traded and is forfeited if the executive leaves the firm. In some plans, the executive does not need to purchase the stock to receive the gains from holding the option. In these plans, stock appreciation rights entitle the executive to receive cash in the amount of the exercise gain for the option. Companies have granted stock appreciation rights in tandem with options to help executives avoid transaction costs if they do not wish to hold the stock after exercising the option.

The stock market crash is an example of publicly available information about noise in a performance measure. When an executive's compensation depends on the company's stock price, stock price is acting as a measure of the executive's performance. The executive has incentive to ensure that his or her actions will increase the price of the stock. However, a performance measure's usefulness in motivating an agent lies in its information about the agent's actions. Other information will also be useful if it provides

additional information about the agent's actions or the extent to which existing performance measures are influenced by events outside the agent's control. Since knowledge that there was a crash provides additional information about an event that affects the firm's stock price and that is outside the executive's control, an optimal complete contract will reflect that information. Alternatively, if the existing contract does not make adjustments in the performance measure for uncontrollable reductions due to the crash, it will be optimal to renegotiate the contract to reflect that reduction.

To analyze the effect of the crash, we must first determine whether the crash is an event that has been contracted upon. In an optimal contract, an event will be explicitly contracted upon if the event has a positive probability of occurrence, the event and its impact on the agent's performance is verifiable as well as observable, the cost of contracting is low, and the benefits are sufficiently high. It is reasonable to think of the crash as a positive but low probability event; stock market downturns of varying degrees have happened frequently in the past but large declines are very rare. The event itself is easily verified by reported changes in stock market indices. However, the verifiability of the impact of the event on the firm's market price (the performance measure) may be difficult. While the executive and the Board of Directors may be able to agree on the effect of the crash on the performance measure, the effect may not be verifiable and, hence, is unenforceable by a court. If the impact of the event is easily verified, then the cost of contracting on a particular event is likely to be small — the contract need only reflect a contingency for the event. However, if there are many low probability events, it

is likely to be costly to establish contingencies for all such events. Benefits to complete contracting exist if the agent will accept a lower expected cost compensation, for the same action, in a complete contract than in an incomplete contract. The complete contract can yield further benefits over the incomplete contract if a higher action is optimal under the complete contract. Since the incomplete contract is always feasible under the complete contract constraints, the complete contract must have a different optimal compensation contract than in the incomplete contract in order for there to be a benefit to the complete contract. As shown in Chapter 2, if the crash is predecision information and the agent can leave after the information becomes known, the optimal compensation contract is the same under a complete contract and an incomplete contract with renegotiation. Thus, although a crash is an observable event with a positive probability of occurrence, the impact of a crash on the performance measure may not be verifiable and the cost of contracting may be higher than any potential benefits. It is not clear, therefore, whether contracts will include a contingency for a crash. Note that this case is not symmetric. Upshifts are fundamentally different than downshifts. The general model of a contract complete with respect to a downshift is developed in Chapter 2.

Theory has little to tell us about the specific form compensation should take or how that form will change after a market downturn. Existing literature — see Holmström [18] and Grossman and Hart [12] — shows that it is optimal to impose risk on a risk-averse[3]

[3]Harris and Raviv [13] have shown that if the agent is risk-neutral, the principal will avoid moral hazard by selling the firm to the agent.

agent in order to motivate him or her when moral hazard is present. Conditions which ensure that the function is increasing in the payoff have also been developed. However, we do not have a set of general conditions that yield a compensation contract that takes the form of a fixed wage plus stock options. Hence, to obtain this form, specific distribution and/or utility functions must be assumed. In this study, the distribution function is assumed to be uniform. Chapter 3 demonstrates that this assumption is sufficient to obtain an optimal compensation function that takes the form of a fixed salary plus stock options.

In the following analysis, the crash is assumed to have the effect of shifting down the support in the distribution of outcomes given the agent's effort. That is, if the agent exerts the same effort in managing the firm, the expected outcome will be lower if there is a crash than if there is no crash. While a market wide downshift of this sort could impact market reservation wages, in this study, it is assumed that the reservation wage does not change. In Chapter 4, it is shown that, in an incomplete contract, the crash will induce the principal to renegotiate and the new optimal contract will have the same fixed salary but the 'bonuses' will be paid at lower firm values than in the original contract. The fixed salary is determined by the reservation wage while the contingent payment is determined by the support of the distribution of outcomes given the agent's effort.

The theory predicts that stock option contracts will be renegotiated after the crash. This implies that there will be more grants after the crash than before. A sample of 189 firms with grant data from 1985 to 1988 and 90 firms with grant data from 1986 to 1988

is used to test this hypothesis. The results of three nonparametric tests are significant and in the predicted direction. The firms are also categorized into three groups according to the magnitude of the return on the stock over the crash week and tested to determine if there are differences in the size of post crash grants between the three groups. These results are significant for only one of the two cross sectional tests. It may be that the data is too noisy and the groups too small to find a significant result in the second test. However, overall, the tests do support the existence of renegotiation in the form of higher post crash grants. These tests and results are presented in Chapter 5.

This dissertation does not take into account management ownership of company stock. Ownership of company stock provides a significant incentive for an executive to exert effort to improve the stock price. If such an executive also had stock options outstanding at the time of the crash, he or she may still have the enough incentive to work hard for the company despite the loss in wealth. A useful extension of this work would be to group firms by the magnitude of management ownership to determine if the post crash grants differ between the groups.

This dissertation also does not allow for alternative performance measures such as accounting profit. Most existing executive compensation contracts utilize both accounting numbers and stock price as performance measures. The model developed in the following chapters is a one period model. Thus, end of period firm value and period net income are essentially the same number. To extend the model, a second period could be added. The current cash flow (accounting net income) and the stock price could then differ and

compensation could be a function of either number. The market wide phenomenon would affect only the end of period value and not the current cash flows. This model would give some predictions about the nature of renegotiations in the one performance measure given the existence of an alternative measure that is unaffected by the crash.

Chapter 2

Complete Contracting

A complete contract is one in which the sharing of payoffs is defined for all possible events. A contract is said to be incomplete with regard to some possible future information if the contract is silent with respect to how that information will affect the sharing of the payoff. In this paper, the event which may or may not be contracted upon is a stock market downturn or crash. The contract is a compensation contract between a principal and an agent within the standard principal-agent model of Holmström [18].

The above definitions of complete and incomplete contracts are somewhat loose. In fact, most compensation contracts are in one sense complete. They are complete in the sense that they specify the manager's compensation for all possible events described in terms of the primary information (in this case, the market price of the firm's stock). On the other hand, the contracts are incomplete in that they make no provision for adjusting that compensation conditional on other information that might be relevant (e.g., whether a stock market crash was the cause of a low stock market price). This other information may be ignored because the event it describes has such a low probability of occurrence

that the cost of writing the event into the contract is larger than the benefit, or because both parties to the contract realize they can renegotiate if the event occurs. The contract developed in the model will be considered incomplete if the information is not specifically contracted on despite the fact that the incomplete contract does specify payments to the agent for all possible outcomes.

2.1 Assumptions

2.1.1 Risk

To model the problem, we must first make a number of assumptions. Shareholders, by virtue of their ability to diversify their wealth, are presumed to be risk neutral. Managers, on the other hand, are presumed to be strictly risk–averse. One reason for this risk-aversion assumption is that typically all of their human capital is tied to the fortunes of the company for which they work. The other reason is that, as Harris and Raviv [13] have shown, in a moral hazard model, if the manager is risk neutral, then the optimal contract is one in which the agent buys the firm from the principal and assumes all the risk. Therefore, if the firm is characterized by non-owner managers, a logical assumption is that the manager is risk-averse.

2.1.2 Utility

The agent's utility function is assumed to be additively separable into utility for consumption, $U(c)$, and disutility for effort, $V(a)$ (where c is compensation and a is effort). Risk aversion implies that the utility for consumption function is strictly concave — $U'(c) > 0$ and $U''(c) < 0$. The agent has strict aversion to work. His disutility, $V(a)$, is increasing and convex — $V' > 0$ and $V'' > 0$. As is standard in the principal-agent literature, the agent is assumed to choose an action in the principal's favour if the agent is indifferent between actions.

2.1.3 Action

It must also be true that the manager's actions cannot be monitored. If the action can be monitored, then the principal could pay a fixed salary if the desired action were taken and impose a penalty if any other action were taken. This fixed wage and penalty contract is a first-best contract. Hence, a contract with a variable payment will only be optimal when the manager must be motivated to take an action that the principal cannot know for certain has been taken. Alternatively, it may be the case that the optimal action is not known. In fact, the manager may have been hired to make a judgment as to the best action given the company's business and the economic environment. The principal may know what action has been taken but not whether there is a better action that could have been taken. As long as the manager's action cannot be either prespecified

or perfectly monitored, then the principal has no way to make compensation contingent on that performance and the optimal contract cannot be a fixed salary. Therefore, the action of the agent or manager is assumed to be unobservable.

The principal must find some signal, that is informative about the manager's action, upon which to base his compensation. The signal will not be perfectly informative because that is equivalent to perfectly monitoring the manager's actions. The price of company stock is such a signal. The price of the company's stock varies as cash flows are realized and as information about future cash flows is released. The manager's actions influence both the cash flows and the release of information. In addition, both the cash flows and information about future cash flows are influenced by factors uncontrollable by the manager. Stock price aggregates all the available information and, therefore, is a noisy or imperfect measure of the manager's performance.

Of course, basing the manager's compensation on the stock price of the firm will only motivate the manager to work hard if the manager's actions affect the stock price. If the manager's actions or effort cannot affect the stock price, then compensating the manager according to stock price changes imposes risk on the manager for no purpose. Therefore, it is assumed that the value of the firm, x, is a function of the agent's action and a random state of nature.

2.1.4 Labour Market

This is a partial equilibrium model in which the reservation utility, $\overline{U}$, is exogenous and unaffected by the downturn. The reservation utility is determined by the executive's expected utility from alternative employment opportunities. Obviously, a market wide phenomenon such as a stock market crash would have an affect on the labour market and reservation utilities. However, it is not clear whether the effect would be negative or positive. If the crash caused some firms to close down, there could be fewer opportunities for the existing pool of executives and reservation utility would drop. It is also possible that executives' talents are more critical in bad times and that their reservation utility could rise. In order to isolate the potential effect of the crash on compensation plans, the reservation utility is held constant.

2.1.5 Single Period

This is a single period model in the sense that output is only produced once and there is only one settling up. While a multi-period model would be better, it would also be much more complex. We seek to first understand what is happening in a simple setting and then add complexity if it will help our understanding. A single period model allows the assumption that current earnings and firm value are the same. End-of-period firm value, x, is assumed to be observable and verifiable at the end of the period. Hence, if there are N units of stock outstanding, then the end-of-period stock price is x/N. Since

the term, end-of-period firm value, is cumbersome, output will be used in its place. Also note that the value of output, x, is the value before the deduction for compensation. It is a gross value.

2.1.6 Market Crash

The end-of-period firm value is a function of firm-specific factors, such as production costs, managerial effort, and market factors, such as general economic conditions. The occurrence of a downshift, a market factor outside the control of the executive, is a random variable with binary support. The event, e, can either be no shift ($e = 0$) or a shift ($e = 1$) with the shift having a low probability of p. The conditional distribution of firm value given the agent's effort and the event is $f(x|a, e)$. Sufficient conditions are assumed such that the optimal compensation is monotonic in output. Under the first-order approach, sufficient conditions are that the conditional distribution satisfies the Monotone Likelihood Ratio Condition and the Spanning Condition, as shown in Grossman and Hart [12].[1] Later in the paper, there will be other conditions that ensure the optimal compensation is monotonic in output. Furthermore, the conditional no shift distribution is assumed to first order stochastic dominate the conditional shift distribution for the same effort. A complete contract, in this setting, would explicitly state the payments to be made in the case of no shift and how those payments may change in the case of a shift.

[1]See Jewitt [22] for alternative sufficient conditions.

2.2 Problem

The principal's problem is to maximize expected end-of-period firm value, net of compensation paid to the agent, subject to the agent receiving his reservation utility and subject to the agent having motivation to take the desired action. The problem can be classified into predecision or postdecision information depending on when the information about the event becomes known. Observe that the information about the event is common knowledge. Neither the principal nor the agent has any private information about this event.

2.2.1 Predecision Information — Problems A and B

With predecision information, the agent and principal agree on a contract and, before the agent chooses his action, the information about whether or not the event happened becomes known. When the agent chooses his action, he knows for certain which outcome distribution is applicable. Therefore, there are two sets of incentive compatibility constraints — one for each outcome distribution.

Problem A

The form of the reservation utility constraint depends on whether the agent can leave or not. If the agent can leave (Problem A), then the reservation utility constraint must be satisfied individually for each case. This means there will be two reservation utility

constraints. With two reservation utility and two incentive compatibility constraints, the problem is essentially the same as a precontract information problem in which the two cases are each treated as separate problems.

The problem can be stated in general as follows. Recall that, $f(x|a,0)$ and $f(x|a,1)$ represent the conditional outcome distributions given the agent's action under the no shift and shift cases respectively. Let p be the probability of the shift occurring, as perceived by the principal and the agent at the time of contracting. There can be two action choices — a_0 for the no shift case and a_1 for the shift case. To differentiate between payments in the two cases, let $C(x,0)$ and $C(x,1)$ be the payments for outcomes in the no shift and shift cases, respectively. The principal's problem is:

$$\text{Maximize } (a_0, a_1 \in \mathcal{A}, C(x,0), C(x,1))$$

$$(1-p)\int [x - C(x,0)]\, f(x|a_0,0)\, dx \;+\; p\int [x - C(x,1)]\, f(x|a_1,1)\, dx$$

Subject to:

$$\int U(C(x,0))\, f(x|a_0,0)dx - V(a_0) \;\geq\; \overline{U}$$

$$\int U(C(x,1))\, f(x|a_1,1)dx - V(a_1) \;\geq\; \overline{U}$$

and for all $\hat{a} \neq a_0$

$$\int U(C(x,0))f(x|a_0,0)dx - V(a_0) \;\geq\; \int U(C(x,0))f(x|\hat{a},0)dx - V(\hat{a})$$

and for all $\hat{a} \neq a_1$

$$\int U(C(x,1))f(x|a_1,1)dx - V(a_1) \;\geq\; \int U(C(x,1))f(x|\hat{a},1)dx - V(\hat{a})$$

Assume that the first order approach is applicable.[2] The incentive compatibility constraints can then be replaced by the following

$$\begin{aligned}\int U(C(x,0))f_a(x|a_0,0)dx - V'(a_0) &= 0\\ \int U(C(x,1))f_a(x|a_1,1)dx - V'(a_1) &= 0\end{aligned}$$

The solution to Problem A can then be characterized as follows (where λ_{1a} and λ_{2a} are the Lagrange multipliers on the reservation utility constraints and μ_{1a} and μ_{2a} are the Lagrange multipliers on the incentive compatibility constraints)

$$\begin{aligned}U'(C(x,0)) &= \left[\frac{\lambda_{1a}}{1-p} + \frac{\mu_{1a}}{(1-p)}\frac{f_a(x|a_0,0)}{f(x|a_0,0)}\right]^{-1}\\ U'(C(x,1)) &= \left[\frac{\lambda_{2a}}{p} + \frac{\mu_{2a}}{p}\frac{f_a(x|a_1,1)}{f(x|a_1,1)}\right]^{-1}\\ \frac{\mu_{1a}}{1-p} &= \frac{\int[x-C(x,0)]\,f_a(x|a_0,0)\,dx}{\int U(C(x,0))f_{aa}(x|a_0,0)dx - V''(a_0)}\\ \frac{\mu_{2a}}{p} &= \frac{\int[x-C(x,1)]\,f_a(x|a_1,1)\,dx}{\int U(C(x,1))f_{aa}(x|a_1,1)dx - V''(a_1)}\end{aligned}$$

Problem B

If the agent cannot leave (Problem B), then the agent must be compensated enough so that his expected utility over both events is at least equal to the reservation utility. This results in a single reservation utility constraint. The principal's problem is:

$$\text{Maximize } (a_0, a_1 \in \mathcal{A}, C(x,0), C(x,1))$$

$$(1-p)\int[x-C(x,0)]\,f(x|a_0,0)\,dx \;+\; p\int[x-C(x,1)]\,f(x|a_1,1)\,dx$$

[2]The first order approach is *not* appropriate for the model developed in Chapter 3. However, it is used here to illustrate the differences between the three problems.

Subject to:

$$(1-p)\left(\int U(C(x,0))\,f(x|a_0,0)dx - V(a_0)\right)$$
$$+p\left(\int U(C(x,1))\,f(x|a_1,1)dx - V(a_1)\right) \geq \overline{U}$$

and for all $\hat{a} \neq a_0$

$$\int U(C(x,0))\,f(x|a_0,0)dx \;-\; V(a_0) \;\geq\; \int U(C(x,0))\,f(x|\hat{a},0)dx \;-\; V(\hat{a})$$

and for all $\hat{a} \neq a_1$

$$\int U(C(x,1))\,f(x|a_1,1)dx \;-\; V(a_1) \;\geq\; \int U(C(x,1))\,f(x|\hat{a},1)dx \;-\; V(\hat{a})$$

Using the same approach as above, the solution can be characterized as follows, (where λ_b is the Lagrange multiplier on the reservation utility constraint and μ_{1b} and μ_{2b} are the Lagrange multipliers on the incentive compatibility constraints)

$$\begin{aligned}
U'(C(x,0)) &= \left[\lambda_b + \frac{\mu_{1b}}{(1-p)}\frac{f_a(x|a_0,0)}{f(x|a_0,0)}\right]^{-1} \\
U'(C(x,1)) &= \left[\lambda_b + \frac{\mu_{2b}}{p}\frac{f_a(x|a_1,1)}{f(x|a_1,1)}\right]^{-1} \\
\frac{\mu_{1b}}{1-p} &= \frac{\int[x-C(x,0)]\,f_a(x|a_0,0)\,dx}{\int U(C(x,0))f_{aa}(x|a_0,0)dx - V''(a_0)} \\
\frac{\mu_{2b}}{p} &= \frac{\int[x-C(x,1)]\,f_a(x|a_1,1)\,dx}{\int U(C(x,1))f_{aa}(x|a_1,1)dx - V''(a_1)}
\end{aligned}$$

Comparing the solutions of Problems A and B, we see that the solutions are the same when all of the following hold

$$\mu_{1a} = \mu_{1b} \qquad \mu_{2a} = \mu_{2b} \qquad \frac{\lambda_{1a}}{1-p} = \frac{\lambda_{2a}}{p} = \lambda_b$$

What is of interest are sufficient conditions for the above to hold and sufficient conditions for the above not to hold. In a later section, the problems are analyzed under a transformation of variables approach. There we see that, if the compensation contracts in Problem A differ only in a relabeling of output levels and the optimal action is unchanged after the shift, then the solution to Problem A is the same as the solution to Problem B. However, if the optimal action changes after the shift, then the solutions to Problems A and B differ even if they implement the same shift and no shift actions.

2.2.2 Postdecision Information — Problem C

With postdecision information, the agent is already committed to an action when information about the crash becomes known. Hence, the agent has only one action choice for both cases. The information is available for purposes of measuring the agent's performance but does not affect the agent's action decision. In this problem, there is one expected profit statement, one reservation utility constraint, and one set of incentive compatibility constraints.

$$\text{Maximize } (a \in \mathcal{A}, C(x,0), C(x,1))$$

$$(1-p)\int [x - C(x,0)]\, f(x|a,0)\, dx \;+\; p\int [x - C(x,1)]\, f(x|a,1)\, dx$$

Subject to:

$$(1-p)\left(\int U(C(x,0))\, f(x|a,0)dx - V(a)\right)$$
$$+p\left(\int U(C(x,1))\, f(x|a,1)dx - V(a)\right) \geq \overline{U}$$

and for all $\hat{a} \neq a$

$$(1-p)\left(\int U(C(x,0))\, f(x|a,0)dx - V(a)\right) + p\left(\int U(C(x,1))\, f(x|a,1)dx - V(a)\right)$$
$$\geq (1-p)\left(\int U(C(x,0))\, f(x|\hat{a},0)dx - V(\hat{a})\right)$$
$$+ p\left(\int U(C(x,1))\, f(x|\hat{a},1)dx - V(\hat{a})\right)$$

Using the same approach as in Problems A and B, the solution can be characterized as follows, (where λ_c is the Lagrange multiplier on the reservation utility constraint and μ_c is the Lagrange multiplier on the incentive compatibility constraint)

$$\begin{aligned} U'(C(x,0)) &= \left[\lambda_c + \mu_c \frac{f_a(x|a,0)}{f(x|a,0)}\right]^{-1} \\ U'(C(x,1)) &= \left[\lambda_c + \mu_c \frac{f_a(x|a,1)}{f(x|a,1)}\right]^{-1} \end{aligned}$$

$$\mu_c = \frac{(1-p)\int[x - C(x,0)]\, f_a(x|a,0)\, dx + p\int[x - C(x,1)]\, f_a(x|a,1)\, dx}{(1-p)\int U(C(x,0))f_{aa}(x|a,0)dx + p\int U(C(x,1)f_{aa}(x|a,1)dx - V''(a)}$$

2.3 Comparison of Solutions

Two types of shifts will be considered — an additive shift and a multiplicative shift. Let z be the factor in the production of output, x, over which the agent's action has some effect. The distribution of the factor, z, is $f(z|a)$ and higher efforts first order stochastic dominate lower efforts. Output, x, is a function of z and shift characteristics, α and δ, i.e., $x = g(z, \alpha, \delta)$. For the no shift case, output is equal to the agent's factor of

production, i.e., $g(z, \alpha, \delta) = z$. For an additive shift, shift output is lowered by a fixed amount, i.e., $g(z, \alpha, \delta) = z - \delta$, where $\delta > 0$. For a multiplicative shift, shift output is a fraction of null event output, i.e., $g(z, \alpha, \delta) = \alpha z$, where $\alpha < 1$. For both types of shift the no shift distribution first order stochastic dominates the downshift distribution.

The factor z is introduced to illustrate that the information content of the output x does not change with the shift. The output x represents the gross return to the principal and is important in determining which action the principal wishes the agent to implement. However, when it comes to determining the optimal incentive contract for motivating that action, the principal is only concerned with the informativeness of x about a. That informativeness is reflected entirely by z. Assuming that $g(\cdot)$ is invertible with respect to z, the shift outputs have the same *information content* about the agent's action as the no shift outputs. However, the output labels for the same performance differ.

Definition — Relabeling The shift compensation contract written on x is said to be a relabeling of the no shift compensation contract written on x when the payments are the same in both contracts for the same values of z. Thus, if $C(x|a, 0) = C(g^{-1}(x)|a, 1)$, then the shift contract differs from the the no shift contract only in a relabeling of output levels.

First, assume that z can be contracted upon. Since z is the factor which the agent can affect, z is the obvious choice of performance measure. The shift does not affect

z; the shift only affects x. Thus, the distribution of z is unaffected by the shift and $f(z|a,1) = f(z|a,0) = f(z|a)$. Although z is unaffected by the shift, the optimal shift case contract on z may differ from the optimal no shift case contract. The no shift and shift case compensation contracts are denoted $C(z|a,0)$ and $C(z|a,1)$, respectively. After the optimal contract(s) on z are characterized, the results can be interpreted in the situation in which only x is available as a performance measure.

2.3.1 General Contracts on z

When z and $g(z,\alpha,\delta)$ are substituted for x in the no shift and shift cases, respectively, into Problems A, B, and C, the problems are denoted A', B', and C', respectively.

Problem C' is the simplest one of the three because the only choice variables available to the principal are the compensation contracts. The agent cannot alter the action choice after learning of the shift. The solution to Problem C' is characterized as follows[3]

$$U'(C(z|a,0)) = \left[\lambda_c + \mu_c \frac{f_a(z|a)}{f(z|a)}\right]^{-1}$$
$$U'(C(z|a,1)) = \left[\lambda_c + \mu_c \frac{f_a(z|a)}{f(z|a)}\right]^{-1}$$

$$\mu_c = \frac{\int ((1-p)[z - C(z|a,0)] + (p)[g(z,\alpha,\delta) - C(z|a,1)])\, f_a(z|a)\, dz}{\int [(1-p)U(C(z|a,0)) + pU(C(z|a,1)]\, f_{aa}(z|a)dz - V''(a)}$$

[3]Recall that while the conditional distribution of z is not affected by the shift, the compensation can still be affected. Thus, compensation, $C(z|a,0)$ or $C(z|a,1)$, is a function of both the action and event, and the conditional distribution of z is only a function of a, $f(z|a)$.

Since the marginal utilities of the two compenation functions are equivalent, it must be that the optimal no shift case compensation contract on z is the same as the optimal shift case compensation contract on z regardless of the form of $g(z, \alpha, \delta)$. This occurs because the shift has no impact on the distribution of z given the action a and only one action choice is available. Because the information content, z, has not changed, there is no need to change the compensation contract.

However, z is not directly observable and the contract is written explicitly on the directly observable measure x. The transformations from x to z are made implicitly in the contract. While the contract written on z does not vary between the no shift and shift cases, the contracts written on x vary in order to capture the transformation within the contract. The compensation contracts differ only in a relabeling of the shift output levels. This is stated formally in the following proposition.

Proposition 1 *In the post decision problem, the optimal no shift compensation contract is identical to the optimal shift case compensation contract except for a relabeling of output levels.*

In Problem A', if the no shift and shift actions are identical, then the compensation contracts differ only in a relabeling of output levels. This is seen by looking at the following minimum cost compensation problem.

$$\text{Minimize } (C(z|a_0, 0), C(z|a_1, 1))$$

$$(1-p)\int C(z|a_0, 0)\, f(z|a_0)\, dz \;+\; p\int C(z|a_1, 1)\, f(z|a_1)\, dz$$

Subject to:

$$\begin{aligned}
\int U(C(z|a_0,0))\, f(z|a_0)dz - V(a_0) &\geq \overline{U} \\
\int U(C(z|a_1,1))\, f(z|a_1)dz - V(a_1) &\geq \overline{U} \\
\int U(C(z|a_0,0)) f_a(z|a_0)dz - V'(a_0) &= 0 \\
\int U(C(z|a_1,1)) f_a(z|a_1)dz - V'(a_1) &= 0
\end{aligned}$$

If $a_0 = a_1 = a$, then the minimum cost contract is the same in both the no shift and shift contracts, i.e., $C(z|a,0) = C(z|a,1)$. This holds for any invertible function $g(z,\alpha,\delta)$. The contracts are the same because Problem A is equivalent to a precontract information problem. In a precontract information problem, there is essentially a separate problem for each case. Because the shift does not affect the conditional distribution, the two minimum cost compensation problems are the same and have the same minimum cost contracts.

The contracts written on x, however, differ. As in Problem C', the output levels in the shift compensation contract must be relabeled.

Proposition 2 *In Problem A, the minimum cost contracts, to implement a given action, differ only in a relabeling of output levels in the shift case contract.*

2.3.2 Additive Shift

In an additive shift, the expected output for any action is reduced by a fixed amount, i.e., $g(z,\alpha,\delta) = z - \delta$ and $E(x|a,1) = E(x|a,0) - \delta$.

The optimal action in the shift case of Problem A' is the same as the optimal action in the no shift, if the shift is additive. The optimal action choices are the actions at which the first derivatives of the objective function are equal to zero. That is, the actions which equate expected marginal output with expected marginal cost. The optimal no shift case action, a_0 solves

$$\frac{d}{da}\left(\int [z - C(z|a_0, 0)]\, f(z|a_0)\, dz\right) = 0$$

The optimal shift case action, a_1, solves

$$\frac{d}{da}\left(\int [z - C(z|a_1, 1)]\, f(z|a_1)\, dz\right) = 0$$

The two equations yield the same action level.

Proposition 3 *If the shift causes an additive shift in the conditional distribution of output, the no shift and shift case action choices are identical for Problem A.*

The action choice does not change with the shift because neither the marginal expected output nor the marginal expected cost are affected by an additive shift.

In Problems A' and B', the agent can alter his action choice after information about the event becomes known. To illustrate how the shift affects these choices, we need to look at how the specific shifts affect the first order conditions, which are characterized as follows.

Problem A':

$$U'(C(z|a_0, 0)) = \left[\frac{\lambda_{1a}}{1-p} + \frac{\mu_{1a}}{(1-p)}\frac{f_a(z|a_0)}{f(z|a_0)}\right]^{-1} \tag{2.1}$$

$$U'(C(z|a_1,1)) = \left[\frac{\lambda_{2a}}{p} + \frac{\mu_{2a}}{p}\frac{f_a(z|a_1)}{f(z|a_1)}\right]^{-1} \tag{2.2}$$

$$\frac{\mu_{1a}}{1-p} = \frac{\int[z - C(z|a_0,0)]\, f_a(z|a_0)\, dz}{\int U(C(z|a_0,0)) f_{aa}(z|a_0)dz - V''(a_0)} \tag{2.3}$$

$$\frac{\mu_{2a}}{p} = \frac{\int[g(z,\alpha,\delta) - C(z|a_1,1)]\, f_a(z|a_1)\, dz}{\int U(C(z|a_1,1)) f_{aa}(z|a_1)dz - V''(a_1)} \tag{2.4}$$

Problem B':

$$U'(C(z|a_0,0)) = \left[\lambda_b + \frac{\mu_{1b}}{(1-p)}\frac{f_a(z|a_0)}{f(z|a_0,0)}\right]^{-1} \tag{2.5}$$

$$U'(C(z|a_1,1)) = \left[\lambda_b + \frac{\mu_{2b}}{p}\frac{f_a(z|a_1)}{f(z|a_1)}\right]^{-1} \tag{2.6}$$

$$\frac{\mu_{1b}}{1-p} = \frac{\int[z - C(z|a_0,0)]\, f_a(z|a_0)\, dz}{\int U(C(z|a_0,0)) f_{aa}(z|a_0)dz - V''(a_0)} \tag{2.7}$$

$$\frac{\mu_{2b}}{p} = \frac{\int[g(z,\alpha,\delta) - C(z|a_1,1)]\, f_a(z|a_1)\, dz}{\int U(C(z|a_1,1)) f_{aa}(z|a_1)dz - V''(a_1)} \tag{2.8}$$

Note that the expected marginal utilities of compensation are equal for the no shift and shift cases in Problem B'. In Problem A', the expected marginal utilities of compensation are only equal when

$$\frac{\lambda_{1a}}{1-p} = \frac{\lambda_{2a}}{p} \tag{2.9}$$

In fact, equation (2.9) is a sufficient condition for the compensation functions in Problem B' to be the same as in Problem A'. This equation holds for the additive shift.

We know that, for Problem A', $C(z|a,0) = C(z|a,1)$ which implies that the marginal utilities are also equal. Thus, for a given action a, we know

$$\begin{aligned} \frac{\lambda_{1a}}{1-p} + \frac{\mu_{1a}}{(1-p)}\frac{f_a(z|a)}{f(z|a)} &= \frac{\lambda_{2a}}{p} + \frac{\mu_{2a}}{p}\frac{f_a(z|a)}{f(z|a)} \\ \Rightarrow \qquad \frac{\lambda_{1a}}{1-p} - \frac{\lambda_{2a}}{p} &= \left(\frac{\mu_{2a}}{p} - \frac{\mu_{1a}}{(1-p)}\right)\frac{f_a(z|a)}{f(z|a)} \end{aligned} \tag{2.10}$$

Substituting into equation (2.10) from equations (2.3) and (2.4), we see that the left-hand side of equation (2.10) is simplifies to

$$\left(\frac{\int[g(z,\alpha,\delta)-z]\,f_a(z|a)\,dz}{\int U(C(z|a,1))f_{aa}(z|a)dz - V''(a)}\right)\frac{f_a(z|a)}{f(z|a)} \tag{2.11}$$

Equation (2.11) is zero for the additive shift because

$$\int[g(z,\alpha,\delta)-C(z|a_1,1)]\,f_a(z|a_1)\,dz \;=\; \int[z-C(z|a_1,1)]\,f_a(z|a_1)\,dz$$

Since

$$\delta\int f_a(z|a)dz \;=\; 0$$

Thus, for the additive shift, the expected marginal utilities of compensation for the no shift and shift cases in Problem A' are equal. Thus, for the additive shift equations (2.1) to (2.4) are the same as equations (2.5) to (2.8). Hence, the optimal solution to Problem B' yields the same actions and compensation contracts as in Problem A'. Sufficient conditions for this are that there is relabeling for the contracts in Problem A' and the optimal action choice is the same for both the no shift and shift cases. Note that where the optimal action choice in Problem A' differs for the no shift and shift cases, equation (2.11) will be non-zero. Hence, sufficient conditions for the solutions to Problems A' and B' to differ, is that the opitmal action choices to differ in the no shift and shift cases of Problem A'.

Since the action choice does not change with the shift for Problems A' and B', the solutions to all three problems are the same.

The additive shift has no impact on the marginal output; the output is merely reduced by a fixed amount for all actions. This lack of economic impact on the marginal output implies that neither the timing of the information nor the commitment of the agent affects the contract. For both Problems A' and B', the no shift compensation contract written on x differs from the shift compensation contract in the same way as in Problem C' — a relabeling of the shift outputs to capture the information content, z.

Proposition 4 *If the shift causes an additive shift in the conditional distribution of output, all three problems yield the same solution — the no shift and shift case action choices are identical and the no shift and shift case compensation contracts only differ in a relabeling of the shift case output levels.*

2.3.3 Multiplicative Shift

With a multiplicative shift, the expected shift case output is a fraction of the expected no shift case output for the same action level, i.e., $g(z, \alpha, \delta) = \alpha z$ and $E(x|a, 1) = \alpha E(x|a, 0)$ where $\alpha < 1$.

In Problem A', the minimum cost contract to implement action a is always the same in both the no shift and shift case contracts. However, the action choice differs between the no shift and shift cases. The optimal actions, a_0 for the no shift case and a_1 for the shift case, solve, respectively,

$$\frac{d}{da}\left(\int [z - C(z|a_0, 0)]\, f(z|a_0)\, dz\right) = 0$$

$$\frac{d}{da}\left(\int [\alpha z - C(z|a_1, 1)]\, f(z|a_1)\, dz\right) = 0$$

One action cannot be the solution to both of these equations. In Problem A' with the multiplicative shift, the optimal no shift case action differs from the optimal shift case action even though the no shift and shift case minimum cost compensation contracts, for a given action level, are the same. Since the no shift action differs from the shift action in Problem A', the solution to Problem A' will differ from the solution to Problem C' when there is a multiplicative shift.

Proposition 5 *Assuming that both the no shift and shift case solutions are interior and that second order conditions are satisfied, a multiplicative downshift results in a lower shift case action than the no shift case action for Problem A'.*

Proof: See Appendix A.1

The marginal expected output is lower in the no shift case than in the shift case. If the optimal action is interior and second order conditions are satisfied, the action which equates marginal expected compensation to marginal expected output in the shift case will be lower than the optimal no shift case action.

Since the optimal action choices in Problem A' differ between the no shift and shift cases, the solution to Problem B' differs from the solution to Problem A'.

Proposition 6 *When there is a multiplicative downshift, the solution to Problem B' differs from the solution to Problem A'.*

Proof: See Appendix A.2

Since the solution to Problem A' is feasible in Problem B', a different set of optimal compensation contracts implies that the principal can meet the reservation utility with a lower cost set of compensation contracts than in Problem A'. The optimal compensation contracts in Problem B' give the agent a conditional expected utility higher than the reservation utility in one case and lower than the reservation utility in the other case. In the former case, the agent strictly prefers to leave after the information becomes known whereas, in the latter case, the agent strictly prefers to stay. Since the information about the shift causes a change in the optimal action, there is a corresponding change in the compensation to induce that action. This change, caused by the information, creates some variation in payments to the agent. By contracting before the information is known and committing to stay, the agent is able to share the risk of that information with the principal. The ex-ante contract has less variation in payments to the agent and hence, the agent demands a lower risk premium.

The multiplicative shift results in different solutions in all three problems. In Problem C, the compensation contracts are effectively the same for the no shift and shift cases. The only difference is that the shift case contract reflects an appropriate relabeling of the no shift case contract. In Problem A the optimal no shift and shift actions differ. Hence, the optimal compensation contracts in Problem A differ from those in Problem C. Finally, the optimal compensation contracts in Problem B differ from those in Problem A even if the same actions are implemented in the two problems.

2.4 Incomplete Contracting

A literature exists which examines the impact on the initial contract of an anticipated future opportunity to renegotiate. Some authors identify contracts that are renegotiation-proof. That is, renegotiation is allowed, but the contracting parties have no incentive to renegotiate. For example, Fudenberg and Tirole [10] consider a post-action renegotiation in a standard principal-agent model, and identify a renegotiation-proof menu of contracts. Demougin [8] considers pre-action renegotiation subsequent to the agent's acquisition of a private signal; conditions are identified under which an optimal contract exists without communication and, hence, this contract does not induce renegotiation. Green and Laffont [11] look at the problem of underinvestment when a contract can be renegotiated after one party makes an investment. They show that if the nature of the bargaining process is known at the time of contracting, then it is possible to specify an optimal contract that will induce fully efficient investment and noørenegotiation.

Other analyses identify situations in which it is optimal to renegotiate. For example, Huberman and Kahn [19] show it can be optimal for the initial contract to be designed to protect one party from undesirable behaviour by the other party, and then to renegotiate after the danger is past. Hart and Tirole [16] identify conditions under which a sequence of short-term contracts are equivalent to a long-term contract with renegotiation.

The analyses mentioned above generally ignore contracting and renegotiation costs. Hart and Moore [15], on the other hand, assume that contracts are incomplete because of

contracting costs. They consider how the contracting parties can cope with this incompleteness by specifying a mechanism for revising the terms of the initial contract. They demonstrate that the feasible set of revisions is severly limited by the parties ability to renegotiate the initial contract. The limitations depend on the means of communication available to the parties at the time of renegotiation. The following analysis differs in one important respect — there is no private information. The parties may prefer to renegotiate because some public information has altered the benefits of the contract, but the parties are not concerned with determining each other's private information.

If the incomplete contract represents optimal, and not myopic, behaviour by the principal and the agent, then we must demonstrate that the proposed initial contract is optimal given the anticipated renegotiation if a downshift occurs. This could be demonstrated through backward induction, but it is sufficient to demonstrate that the incomplete contract with renegotiation is equivalent to the optimal complete contract. This equivalency, in Problem A, depends crucially on our assumptions that only a downshift is possible, the occurence of the downshift is public knowledge, the agent's reservation utility does not change if the downshift occurs, and the agent can leave the firm after a downshift occurs. As demonstrated below, under these conditions, the agent's expected utility is less than his or her reservation utility, given a downshift and the complete no shift contract. Consequently, if the complete no shift contract is the intial incomplete contract, renegotiation after a downshift will result in the implementation of the complete downshift contract. Observe that the complete contract can be interpreted as a

renegotiation-proof contract.

Contracting and renegotiation costs are suppressed in the following analysis. If there were no renegotiation costs and upshifts were considered in Problem A, the principal could act strategically and write the inital contract using the upshift distribution. If the upshift does not occur then the principal would renegotiate, implying that renegotiation would almost always occur. If there are positive renegotiation costs and the probability of an upshift ocurring is sufficiently small, then the principal may prefer to write the initial contract using the most likely distribution, and then renegotiate after an upshift if the benefits from renegotiation are larger than the costs.

The parties to a contract always have the option of not contracting on specific events. The parties may choose not to contract on low probability events with the understanding that they will renegotiate if a low probability event occurs. If the same payments and effort levels can be induced with an incomplete contract as with a complete contract, then the incomplete contract would be prefered if the expected costs of renegotiating are less than the costs of contracting initially. Hence, the complete contract acts as a benchmark against which the incomplete contract can be compared. Thus, having solved for the complete contract we now seek to determine whether the solution to the incomplete contract is equivalent.

2.4.1 Problem A^i

First, let us consider the Problem A setting in which the agent is free to leave after knowledge about the downshift event becomes known. In an incomplete setting, denoted Problem A^i, the principal and agent agree upon an initial contract which does not explicitly state how the payment to the agent changes if a low probability event occurs. The principal and agent agree to renegotiate if the low probability event occurs. Before the agent chooses an action, knowledge about the event becomes known to both the principal and the agent. At this point, the agent has the opportunity to stay under the terms of the original contract, negotiate a new contract, or leave the firm. If the agent stays, then he or she chooses an action. Finally, x is produced and shared according to whatever contract is in effect.

Proposition 7 *Assume the complete no shift contract is used as the initial contract. If the incentive compatibility and reservation utility constraints bind under the initial contract, the agent prefers to leave after a downshift occurs unless the principal renegotiates.*

Proof: A montonically increasing compensation function and first order stochastic dominance of the no shift over the downshift distribution implies that the agent's expected utility of compensation for a given action is greater under the no shift than the downshift distribution. A binding reservation utility constraint implies that the expected utility under the original contract with the no shift distribution and the optimal action is equal to his or her reservation utility. For any other action under the original contract and the no

shift distribution, the expected utility is less than or equal to the expected utility with the optimal action and, therefore, is less than or equal to the reservation utility. Since the reservation utility is assumed to be unaffected by the downshift, the original contract does not give the agent his or her reservation utility with the downshift distribution and any action. If the principal does not renegotiate, the agent prefers to leave.

Q.E.D.

This result does not apply to upshifts. The original contract gives the agent, after an upshift, more than the original reservation utility. Unless the reservation utility has increased substantially, the agent strictly prefers to stay.

Both the principal and the agent anticipate that there will be renegotiation if a downshift occurs that was not contracted on. Therefore, the initial contract is negotiated using the conditional distribution of output that does not include the possibility of the low probability event. The distribution of interest is $f(x|a,0)$, the same distribution as that in the no shift case of Problem A. Although, the low probability event is ignored in the distribution used to determine the initial contract, the fact that the event can occur is not ignored. If the event occurs and the contract is renegotiated, the conditional distribution of output is then $f(x|a,1)$, the same distribution as that in the shift case of Problem A. The conditional distributions of output in Problem A^i are the same as in Problem A because the agent can leave after information about the event becomes known. In Problem A this implies that there are essentially two separate problems —

one for the no shift case and one for the shift case. The initial contract problem is a simple principal-agent problem where the principal chooses an action-compensation pair to maximize the principal's share conditional on the agent getting his or her reservation wage and the agent having incentive to choose the desired action.

Initial Contract — Problem A^i

$$\text{Maximize } (a_0 \in \mathcal{A}, C(x,0)) \qquad \int [x - C(x,0)]\, f(x|a_0,0)\, dx$$

Subject to:

$$\int U(C(x,0))\, f(x|a_0,0)dx - V(a_0) \;\geq\; \overline{U}$$

and for all $\hat{a} \neq a_0$

$$\int U(C(x,0))f(x|a_0,0)dx - V(a_0) \;\geq\; \int U(C(x,0))f(x|\hat{a},0)dx - V(\hat{a})$$

As before, assume that the first order approach is applicable. The solution to the initial contract in Problem A^i can then be characterized as follows (where λ_0 is the Lagrange multiplier on the reservation utility constraint and μ_0 is the Lagrange multipliers on the incentive compatibility constraint):

$$\begin{aligned} U'(C(x,0)) &= \left[\lambda_0 + \mu_0 \frac{f_a(x|a_0,0)}{f(x|a_0,0)}\right]^{-1} \\ \mu_0 &= \frac{\int [x - C(x,0)]\, f_a(x|a_0,0)\, dx}{\int U(C(x,0)) f_{aa}(x|a_0,0)dx - V''(a_0)} \end{aligned}$$

Note that $\mu_0 = \frac{\mu_{1a}}{1-p}$. When μ_0 is substituted into the marginal utility constraint, it is also clear that $\lambda_0 = \frac{\lambda_{1a}}{1-p}$. Hence, the minimum cost initial contract for Problem A^i is equivalent to the minimum cost no shift contract for Problem A.

Renegotiation — Problem A^i

The initial contract may affect the renegotiated contract. After the event occurs, the firm is still committed to the initial contract while the agent is not. Any renegotiated contract must leave the agent at least as well off as the initial contract would. In effect, there are two reservation utility constraints. One ensures the agent is willing to work and the other ensures the agent (weakly) prefers the renegotiated contract. As discussed earlier, in the event of a downshift, the agent receives less than the reservation utility from any action under the initial contract. Thus, in this case, the constraint that ensures the renegotiated contract is preferred to the initial contract is not binding. This constraint would be binding if there was an upshift.

Renegotiation will occur if the principal gains and the agent is no worse off. The principal's share under the renegotiated contract must be at least as much as his or her share under the initial contract given the shift distribution and the agent's preferred action. If the initial contract is such that the agent prefers to leave if there is no renegotiation, then a positive expected profit from the renegotiated contract is sufficient to ensure renegotiation. On the other hand, if the agent prefers to stay, then the renegotiated contract must yield a higher share to the principal than leaving the initial contract in place.

Shift Contract — Problem A^i

$$\text{Maximize } (a_1 \in \mathcal{A}, C(x,1)) \qquad \int [x - C(x,1)]\, f(x|a_1,1)\, dx$$

Subject to:

$$\int U(C(x,1))\, f(x|a_1,1)dx - V(a_1) \;\geq\; \overline{U}$$

and for all $\hat{a} \neq a_1$

$$\int U(C(x,1))f(x|a_1,1)dx - V(a_1) \;\geq\; \int U(C(x,1))f(x|\hat{a},1)dx - V(\hat{a})$$

and for all $\hat{a} \neq a_1$

$$\int U(C(x,1))f(x|a_1,1)dx - V(a_1) \;\geq\; \int U(C(x,0))f(x|\hat{a},1)dx - V(\hat{a})$$

The second constraint is the standard incentive compatibility constraint and the third constraint ensures the contract leaves the agent at least as well off as the initial contract under the shift distribution. The third constraint is not binding if, without renegotiation, the agent prefers to leave.

Following the same approach as above, the solution to the renegotiation contract can be characterized by (where λ_{Ai} is the Lagrange multiplier on the reservation utility constraint and μ_{Ai} is the Lagrange multipliers on the incentive compatibility constraint):

$$\begin{aligned} U'(C(x,1)) &= \left[\lambda_{Ai} + \mu_{Ai}\frac{f_a(x|a_1,1)}{f(x|a_1,1)}\right]^{-1} \\ \mu_{Ai} &= \frac{\int [x - C(x,1)]\, f_a(x|a_1,1)\, dx}{\int U(C(x,1))f_{aa}(x|a_1,1)dx - V''(a_1)} \end{aligned}$$

Note that $\mu_{Ai} = \frac{\mu_{2a}}{p}$. When μ_{Ai} is substituted into the marginal utility constraint, it is also clear that $\lambda_{Ai} = \frac{\lambda_{2a}}{p}$. Hence, the minimum cost renegotiated contract for Problem A^i is equivalent to the minimum cost shift contract for Problem A.

Since the minimum cost contracts in Problem A^i are the same as the minimum cost contracts in Problem A for a given action, there is no benefit to contracting on low probability events if the agent can leave after the event becomes known. Whether or not complete contracting is preferred to incomplete contracting depends on whether the expected cost of recontracting is greater than the actual cost of complete contracting. If the actual cost of recontracting is the same as the actual complete contracting cost, then it is preferable not to write a complete contract since the recontracting cost is only incurred when the event occurs. With no benefit to complete contracting, the actual cost of recontracting must be considerably greater than the cost of complete contracting for complete contracting to be preferred.

Note that recontracting differs from post information contracting because there is an enforceable contract in existence. The agent can enforce the payment schedule of the initial contract and any renegotiated contract must ensure the agent is at least as well off or the agent will not renegotiate. On the other hand, the principal cannot enforce the the agent's action choice because it is not observable. Thus, the contract adds a constraint to the recontracting problem that the contracting post information problem does not have. The resulting contracts are the same when the existing contract constraint is not binding. In the case of an upshift in the conditional distribution that first order stochastic

dominates the initial distribution, the agent's expected utility, for every action, is greater under the shift distribution than the no shift distribution. Under the initial contract, the agent received the reservation utility. Thus, under the shift distribution, the agent receives more than the reservation utility. In that case, the reservation utility constraint does not bind and the existing contract constraint does bind. Thus, for upshifts, the principal can reduce expected compensation by writing a complete contract. If the benefit is large enough, the principal will strictly prefer to contract specifically on possible stock price run-ups.

2.4.2 Problem B'

In Problem B', the initial incomplete contract cannot be the same as the no shift compensation in the complete contract, if the optimal no shift action differs from the shift action. In that case, the principal absorbs some of the risk of the shift and offers the agent a contract that gives him or her less than the reservation utility for one case and more for the other case. If the initial incomplete contract gives the agent less than his or her reservation utility if no shift occurs, then the agent will not accept the contract. The agent anticipates that, if a shift occurs, the principal will offer an new contract with the same expected utility as the initial contract. Thus, the agent prefers to go elsewhere and get his or her reservation utility. The principal will not offer the agent an initial contract in which the agent has an expected utility greater than his or her reservation utility because the principal anticipates that any renegotiation will give the agent that

higher expected utility because the initial contract is enforceable. Thus, it would appear that the incomplete contract will not be equivalent to the complete contract for Problem B.

If the initial incomplete contract in Problem B^i is the same as the initial contract in Problem A^i and a shift occurs, the renegotiated contract may differ from the renegotiated contract in Problem A^i. Since the agent cannot threaten to leave, the existing contract constraint binds and the reservation utility constraint does not bind. Recall that a downshift and the initial contract leaves the agent with an expected utility, for any action, that is less than his or her reservation utility. Hence, the agent's expected utility at the time of the initial contract is less than his or her reservation utility and the agent will not accept the contract. Thus, it is not clear what form an incomplete contract in Problem B^i will take except that it will differ from the complete contract in Problem B.

In this problem, the agent cannot threaten to leave. However, the action choice does give the agent some bargaining power. The principal prefers to renegotiate when there is a gain to giving the agent incentive to choose an action different than that the agent would choose given the initial contract and the shift distribution. Hence, the reservation utility constraint does not enter into the renegotiating problem. Instead, the existing contract constraint binds. If the initial contract is the same as in Problem A^i, the renegotiation problem is

$$\text{Maximize } (a_1 \in \mathcal{A}, C(x,1)) \qquad \int [x - C(x,1)]\, f(x|a_1,1)\, dx$$

Subject to:

and for all $\hat{a} \neq a_1$

$$\int U(C(x,1))f(x|a_1,1)dx - V(a_1) \;\geq\; \int U(C(x,0))f(x|\hat{a},1)dx - V(\hat{a})$$

and for all $\hat{a} \neq a_1$

$$\int U(C(x,1))f(x|a_1,1)dx - V(a_1) \;\geq\; \int U(C(x,1))f(x|\hat{a},1)dx - V(\hat{a})$$

Since the agent receives less than the reservation utility with the existing contract regardless of the action choice, the right-hand side of the first constraint, the existing contract constraint, is equal to a constant less than $\overline{U}$. The solution can be characterized by (where λ_{Bi} is the Lagrange multiplier on the existing contract constraint and μ_{Bi} is the Lagrange multiplier on the incentive compatibility constraint):

$$\begin{aligned} U'(C(x,1)) &= \left[\lambda_{Bi} + \mu_{Bi}\frac{f_a(x|a_1,1)}{f(x|a_1,1)}\right]^{-1} \\ \mu_{Bi} &= \frac{\int [x - C(x,1)]\, f_a(x|a_1,1)\, dx}{\int U(C(x,1))f_{aa}(x|a_1,1)dx - V''(a_1)} \end{aligned}$$

Note that $\mu_{Bi} = \frac{\mu_{1b}}{p}$ for a given $C(x,1)$. However, λ_{Bi} is not equal to λ_{Ai} because the reservation utility constraint in Problem A^i differs from the existing contract constraint in this problem. Thus, even if the initial contract in Problem B^i were the same as in Problem A^i, the renegotiated contracts would differ. This is because the agent can enforce the existing contract but cannot leave. The agent would anticipate this reduced bargaining position if a downshift occurs and would demand an increased risk premium in the initial contract to assume that risk. Hence, the initial contract in Problem B^i also

differs from the initial contract in Problem A^i.

2.4.3 Problem C^i

In the case where a downshift occurs after the agent chooses an action, there is the question of whether or the principal can commit not to renegotiate. If the principal cannot commit to a contract that will not be renegotiated, then problems could arise because the agent anticipates that the principal will offer a full insurance (fixed wage) contract and the agent, therefore, chooses the minimum action. Fudenberg and Tirole [10] have shown that there can be an optimal contract with an action other than the minimum action in the case where renegotiation can occur after the agent has chosen his or her action. In this case, the principal offers a menu of contracts that the agent chooses among after choosing an action. At the point at which the menu of contracts is offered, the problem is one of adverse selection where the agents's type is the action choice he or she made. The selection from the menu reveals the agent's type. The optimal solution cannot have the agent choose the high action with probability one because the agent would anticipate that the principal would offer the full insurance contract and there would be no incentive to choose the high action. Instead the optimal solution induces the agent to randomize over effort levels and the menu of contracts offered ranges from safe contracts for low effort workers to risky contracts for high effort workers. It is also possible for the agent to earn rents. Finally, they show that the principal can implement the same optimal solution by offering a single initial contract in which the agent earns the same rent. If the agent

earns rent, the optimal solution with renegotiation differs from the optimal solution with commitment not to renegotiate. For Problem C^i, it is assumed that commitment not to renegotiate is possible.[4]

If commitment not to renegotiate is possible, then incomplete contract is based on the prior distribution, conditional on the agent's action, $\tilde{f}(x|a) = \sum_{e \in \mathcal{E}} p(x|a,e)p(e)$ where $\mathcal{E}$ is the set of all possible events and $p(e)$ is the probability of event e occurring. This distribution takes into account all low probability events but ignores the fact that the events will be known expost. The contract is written on all possible values of the performance measure yet does not take into account one or more events that may affect the performance measure and provide incremental information about the agent's action. These events may be thought of as contributing to noise in the performance measure. A low probability event that causes a low firm value will result in a low compensation to the agent regardless of the action taken. The agent just draws a bad outcome. Because the agent assumes this extra risk, the agent will require an extra risk premium in the compensation contract. Note that the contract in Problem C^i is complete with regard to output x, and the fact that its probability is influenced by low probability events, yet the e are treated as noncontractible events even though they will be known prior to compensating the agent.

Problem C^i does not yield the same contracts as any of the other problems. The

[4]Of course, while Fudenberg and Tirole assume that it is not possible to commit to no renegotiation after the action is taken, they must assume that there cannot be a second round of renegotiation after the agent makes his or her choice from the menu.

problem structure for C^i is identical to that for the initial contract in Problem A^i with $C(x)$ and $\tilde{f}(x|a)$ replacing $C(x,0)$ and $f(x|a,0)$ respectively. The solution to Problem C^i is characterized as follows (where λ_{Ci} is the Lagrange multiplier on the reservation utility constraint and μ_{Ci} is the Lagrange multipliers on the incentive compatibility constraint):

$$\begin{aligned} U'(C(x)) &= \left[\lambda_{Ci} + \mu_{Ci}\frac{\tilde{f}_a(x|a)}{\tilde{f}(x|a)}\right]^{-1} \\ \mu_{Ci} &= \frac{\int [x - C(x)]\, \tilde{f}_a(x|a)\, dx}{\int U(C(x))\tilde{f}_{aa}(x|a)dx - V''(a)} \end{aligned}$$

The distribution $\tilde{f}(x|a) = \sum_{e\in\mathcal{E}} f(x|a,e)p(e)$ differs from both $f(x|a,0)$ and $f(x|a,1)$ as long as $p(e) > 0$ for more than one e. The minimum cost contract in Problem C^i therefore differs from the minimum cost contracts in both Problems A^i and B^i.

Note also that this minimum cost contract differs from the contracts in Problem C. The minimum cost contract in Problem C^i is constant across events whereas the minimum cost contract in Problem C varies across events whenever the likelihood ratios for the events are not equal. Since a constant contract across events is feasible but not optimal for Problem C, it must be that the minimum cost contracts in Problem C have a lower expected cost than a contract that is constant across events. The difference in the expected cost of the contracts is the risk premium demanded by the agent to assume the risk of the event. The principal prefers the incomplete contract to the complete contract when the cost of contracting on all possible events is greater than the risk premium demanded by the agent.

2.5 Summary

This chapter analyzes compensation contracts, in a simple moral hazard model, that are either complete or incomplete with respect to a low probability event. The event causes a downward shift in the distribution of firm output conditional on the agent's action. The implication of this is that firm output has the same information content about the agent's actions in both the no shift and shift cases. However, a compensation contract written on output alone does not capture all the information available about the agent's action. Thus, optimal compensation contracts are written on both the event and firm output.

Three separate problems are addressed: Problem A — information about the event becomes known before the agent chooses an action and the agent can leave; Problem B — information about the event becomes known before the agent chooses an action and the agent is committed to stay; and Problem C — information becomes known after the agent has chosen an action.

In all three problems, the expected cost of contracting is lower for the incomplete contract than for the complete contract because the contracting cost for the low probability event is only incurred in the incomplete contract if the event occurs. However, in the incomplete contract, there may be renegotiation costs. These "contracting costs" must be compared to any differences in expected compensation to determine whether complete contracting is preferred to incomplete contracting.

In Problem A, there is no interaction between the no shift and shift contracts and the two cases — no shift or shift — can be treated as separate problems. The shift contract only differs from the no shift contract in a relabeling of output levels to ensure that the shift output levels reflect the same information content as the no shift output levels. Aside from the relabeling issue, the principal does not care either about which case is applicable or what pair of actions is to be implemented. Only the action to be implemented in a particular case is needed to determine the contract for that case. With this independence between the actions in the two cases, the cases can be treated as separate problems. With a downshift, the incomplete contracts with renegotiation are identical to the complete contracts. There is no benefit to complete contracting.

This is not true for an upshift. In that case, with an incomplete contract, the agent's expected utility under the shift and original contract will be greater than the original reservation utility. Hence, if the principal chooses to renegotiate after an upshift, he must offer a contract that provides the agent with as much expected utility as the old contract, which is greater than the agent would have received under a complete contract. In any event, with incomplete contracting, the agent earns rents if an upshift occurs.[5] Thus, there may be a benefit to complete contracting on upshifts. It is interesting to note that few executive contracts are complete with respect to upshifts. See Chapter 5 for some

[5]The earning of rents with an upshift assumes that the incomplete contract is such that there is no renegotiation in the no shift case. As was mentioned earlier, if there were no renegotiation costs it would be optimal to offer the optimal upshift contract as the initial contract and then renegotiate if there is no shift. Of course, this is unlikely to optimal if there are contracting and renegotiation costs and the upshift is a low probability event.

examples of compensation contracts that specify the schedule of payments to be made after market shifts.

In contrast, for Problem B, there are may be benefits to complete contracting. The optimal complete contract can only be determined when the *pair* of actions is known. The complete contract to implement the no shift action a_0 depends on both a_0 and the shift action a_1, whereas the the incomplete contract to implement action a_0 depends only on a_0. Because the agent cannot leave after information about the event becomes known, the principal can offer a complete contract that protects the agent from the some of the risk of the shift. The agent strictly prefers to leave if there is no downshift and strictly prefers to stay if there is a downshift. Thus, the principal can absorb the risk of the shift and reduce expected compensation by writing a complete contract if the agent commits to stay. It is not clear what form an incomplete contract will take in Problem B'. However, that contract is not equivalent to the complete contract.

In Problem C, there is a benefit to complete contracting. In this problem, we assume there is no renegotiation. With the possibility of renegotiation, Fudenberg and Tirole [10] have shown that a contract that offers a menu of contracts after the action choice is made is renegotiation-proof, if the agent randomizes over the action choice. Otherwise, the agent anticipates that the principal will offer a fixed wage at renegotiation and, therefore, chooses the minimum action.[6] The incomplete contract without renegotiation

[6]In a multi-period setting, reputation effects can serve to prevent such a situation. The principal's reputation from previous renegotiations affects both the agent's decision to accept an incomplete contract with renegotiation and the agent's action choice. In this case only the action choice is significant.

treats the low probability event as noise in the conditional distribution of output. The agent is paid only according to firm output and receives, with a relatively high probability, a low payment for all levels of effort whenever there is a downshift. The agent demands a risk premium in the incomplete contract over the complete contract. Thus, the complete contract has a lower expected compensation than the incomplete contract in Problem C.

Problem A, where the agent is free to leave after information about the event becomes known, is the only problem in which there is no difference in expected compensation between the complete and incomplete contract. In Problem A, the choice between complete and incomplete contracts rests on the relative costs of contracting on low probability events versus the costs of renegotiating.

Renegotiation and contracting costs are not explicitly modeled in the analysis. However, the analysis implies that the principal will always choose to renegotiate after a downshift if the initial contract does not explicitly specify the payments that are to be made when a downshift occurs. This result is independent of the size of the downshift. Since we do not appear to observe renegotiation after small downshifts, this suggests that there are renegotiating costs and that these costs make renegotiation uneconomical unless the shifts are large. The increased profit to the principal from motivating the agent to work harder under the renegotiated contract must be sufficiently large to overcome the renegotiation costs.

The analysis that follows assumes an incomplete contract as in Problem A^i. However, the results will also hold for a complete contract in Problem A in which there is predecision

information and the agent can leave.

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Chapter 3

Pre-Crash Contract

The pre-crash model is a standard principal-agent moral hazard model with the output uniformly distributed over a range determined by the agent's effort. Without assuming a specific distribution, very little can be said about the optimal contract other than what conditions are necessary for the agent's payment to be increasing in output. The use of the uniform distribution allows the determination of an explicit solution for the compensation contract. The optimal payment to the agent is in the form of a fixed salary plus a bonus contract. The bonus is only payable after a specific output level is reached and, then, increases as output increases. Hence, the optimal contract, in this model, is very similar to a fixed salary plus stock options contract.

The contract is incomplete in the sense that the possibility of a crash is ignored. The crash is perceived as a low probability event which, if it occurs, may result in renegotiation of the contract. Thus, the model is addressing the no shift case in Problems A and A^i from the previous chapter.

3.1 Assumptions

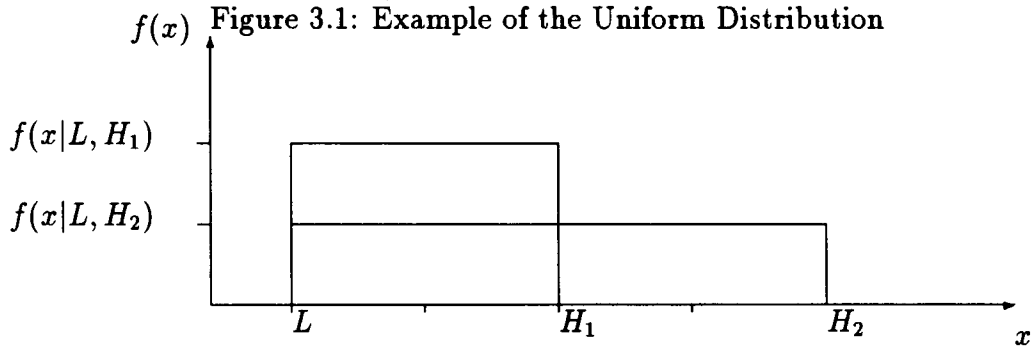
3.1.1 Uniform Distribution

The gross end-of-period market value of the firm, denoted output x, is assumed to be uniformly distributed over the range $(\underline{b}(e), \overline{b}(a,e))$. The effort, a, determines the upper end of the range of value, $\overline{b}(a,e)$, and the total range, denoted $\Delta(a,e) = \overline{b}(a,e) - \underline{b}(e)$. The event, e, can affect either or both ends of the range. The uniform distribution is characterized by a constant density for all possible outcomes within a given range; for a range (L, H), the density function is $\frac{1}{H-L}$; the expected value is $\frac{H+L}{2}$; and the variance is $\frac{(H-L)^2}{12}$. If the range is increased, the probability of any given event included in both ranges, will decrease. See figure 3.1 for an illustration of the probability density function $f(x)$ for two different ranges — (L, H_1) and (L, H_2) where $H_2 - L = 2 \times (H_1 - L)$. For this model, the expected outcome and variance, given the agent's action, are, respectively,

$$\overline{x}(a,e) = E(x|a,e) = \frac{\overline{b}(a,e) + \underline{b}(e)}{2} \qquad \text{and} \qquad E((x-\overline{x})^2) = \frac{\Delta(a,e)^2}{12}$$

It is well known that a first-best solution is possible with moving support on the lower end of the range. The principal merely needs to have a large enough penalty for any outcomes within the lower undesired range so that the agent will strictly prefer to choose the desired action. To avoid this situation, the low end is anchored at $\underline{b}(e)$. The low end is assumed to be strictly greater than 0. In fact, $\underline{b}(e)$ is assumed to be large enough to ensure the principal can always pay the agent. Since all actions have a positive

Figure 3.1: Example of the Uniform Distribution



probability of yielding outcomes close to $\underline{b}(e)$, the lower bound must be greater than the payment to the agent for that outcome. Thus, bankruptcy is not possible.

The upper end of the range is an increasing function of effort. Thus, higher effort increases the range of output while it decreases the probability of any particular output interval. Of course, the probability of any output interval that was not in the original range will increase because the probability of that interval was zero previously. In other words, while bad outcomes are possible with any effort, good outcomes are only possible if sufficient effort has been taken. Thus, as effort increases, the probability of bad outcomes will decrease while good outcomes become possible. Given a particular output level, the principal can know with certainty that effort levels, below the one required to produce a range with that level of output as the upper end, were <u>not</u> taken. There is, therefore, moving support on the upper end of the range. This type of moving support does <u>not</u> provide for a first-best solution. Only moving support on the low end allows for the use of sufficient penalties to drive a first-best solution.

The set of possible actions, $\mathcal{A}$, is assumed to be an interval on the real line with a lower

bound, $\underline{a}$. This lower bound can be thought of as the effort level the agent is prepared to exert for a fixed salary. This effort level is relatively costless to the agent in terms of disutility. Alternatively, this level of effort can be thought of as the level immediately above the level for which the principal can fire the agent for lack of performance.

3.1.2 Timeline

The timeframe is as follows: [1]

$t = 0$ The principal and agent agree on a contract $C(x|a, 0)$ and an action a.

$t = 1$ The agent chooses an action.

$t = 2$ The output, x, is produced and shared according to the contract $C(x)$.

3.2 Problem

In this problem, both the principal and agent ignore the possibility of a crash and do not contract on that possibility. The principal maximizes expected output, net of compensation payments, by choosing the lowest cost compensation contract, $C(x|a, 0)$, that induces the agent to choose the desired action a. The agent must get his reservation utility or he will not work. Without loss of generality the reservation utility can be assumed to be zero. Also, the agent will maximize his expected utility through his choice

[1] As this model depicts the no shift case, 0 is substituted for e.

of action. The problem can be stated as follows:

$$\underset{(a \in \mathcal{A}, C(x|a,0))}{\text{Maximize}} \quad \int_{\underline{b}(0)}^{\bar{b}(a,0)} [x - C(x|a,0)]\, f(x|a,0)\, dx$$

subject to:

$$\int_{\underline{b}(0)}^{\bar{b}(a,0)} U(C(x|a,0))\, f(x|a,0)\, dx - V(a) \;\geq\; 0$$

and for all $\hat{a} \neq a$

$$\int_{\underline{b}(0)}^{\bar{b}(a,0)} U(C(x|a,0))\, f(x|a,0)\, dx - V(a) \;\geq\; \int_{\underline{b}(0)}^{\bar{b}(\hat{a})} U(C(x|a,0))\, f(x|\hat{a},0)\, dx - V(\hat{a})$$

Despite considering a continuous set of actions, we cannot use the first-order approach[2] to determine the optimal contract in the above problem. The first-order approach essentially assumes that the incentive constraint is only binding for the next lower action. However, with the uniform distribution, the incentive compatibility constraints are binding for all lower actions. The problem arises due to the moving support on the upper bound and the constant likelihood ratios in the areas of common support for two different actions. In the first-order characterization of an optimal contract, a constant likelihood ratio leads to a fixed wage which gives no incentive to choose the higher effort level. Hence, the solution violates the incentive compatibility constraint. Thus, another approach must be used to find the optimal compensation contract.

To solve for the optimal contract for a continuous action set, we first obtain the minimum cost contract for each action in a discrete action set. Then, the limit is taken

[2]See Jewitt [22], Grossman and Hart [12] and Rogerson [31] for conditions under which the first order approach is valid.

as the interval between adjacent actions goes to zero. The final step is to identify the action that maximizes the difference between the expected output and the expected contract cost.

3.3 Solution to the Discrete Case

The action set $\mathcal{A} = \{a_1, \ldots, a_N\}$ is ordered such that $\bar{b}(a_i, 0) < \bar{b}(a_j, 0)$ if $i < j$. The principal's initial problem is to solve the following problem for each $a \in \mathcal{A}$:

$$\underset{C(x|a,0)}{\text{Minimize}} \quad \int_{\underline{b}(0)}^{\bar{b}(a,0)} C(x|a,0)\, f(x|a,0)\, dx$$

Subject to:

$$\int_{\underline{b}(0)}^{\bar{b}(a,0)} U(C(x|a,0))\, f(x|a,0)\, dx - V(a) \;\geq\; 0$$

and for all $\hat{a} \neq a$

$$\int_{\underline{b}(0)}^{\bar{b}(a,0)} U(C(x|a,0))\, f(x|a,0)\, dx - V(a) \;\geq\; \int_{\underline{b}(0)}^{\bar{b}(\hat{a},0)} U(C(x|\hat{a},0))\, f(x|\hat{a},0)\, dx - V(\hat{a})$$

The following proposition identifies some interesting characteristics of the solution to this problem.

Proposition 8 *The optimal contract to implement a_n is such that:*[3]

(a) payments are constant on each interval $(\bar{b}(a_{i-1}, 0), \bar{b}(a_i, 0))$, $i = 1, \ldots, n$,

(b) the reservation utility constraint is binding and

(c) all lower incentive action constraints are binding.

[3] Let $\bar{b}(a_0) = \underline{b}(0)$

The proofs for parts (a), (b), and (c) are in appendices A.3, A.4, and A.7, respectively.

The principal does not vary the payments to the agent within any interval because the likelihood ratios do not vary within that interval. Another way to think of this is that the information about actions does not vary within the interval due to the constant probabilities. If $C(x|a,0)$ varies for $x \in (\bar{b}(a_{i-1},0), \bar{b}(a_i,0))$, then unnecessary risk is being imposed on the agent, for which he must be compensated.

The principal need only consider the incentive compatibility constraints for lower actions. To ensure that the agent takes the desired action, the contract must not make him better off if he takes another action. The principal can ignore higher actions because the agent's disutility of effort is increasing in the action taken. Thus, if the maximum payment is for the highest outcome possible when the desired action is taken, then the principal will be certain that a higher action will not be taken.[4] The agent will not incur more disutility for the same payment schedule. However, the principal does need to consider all the lower actions. In essence, the optimal contract will make the agent indifferent among all actions less than and including the principal's desired action. The agent will then choose the action that the principal desires.[5]

The principal must consider all lower incentive compatibility constraints. To see why this occurs, consider a feasible contract in which one of these constraints was not binding.

[4]Higher actions reduce the probability that the agent receives low payments and increase the probability that higher payments are received. If there is a cap on the maximum payment, the disutility of increased effort, over the desired effort level, will more than offset the increased expected utility from the higher expected payments.

[5]This assumption is innocuous because the principal can make the agent strictly prefer the desired action by making the payment for the highest desired outcome range ε more.

With this contract, the payment for one interval is less than the binding payment and the payment for a higher interval is more than the binding payment, such that the reservation utility is still binding. Because the agent is risk averse, a less costly feasible contract that yields the same expected utility can be constructed by reducing the variance in the payments. This follows from a lemma similar to Jensen's inequality. In this case, there is a set of two points with the same expected value but a higher expected utility to the agent as two outer points.

We can now solve for the minimum cost contract to implement action a_n. From proposition 8(a) we know that the optimal contract takes the form (where the k_i are constants):

$$C(x|a_n, 0) = \begin{cases} k_1 & \text{if } \underline{b}(0) \leq x \leq \bar{b}(a_1, 0) \\ k_2 & \text{if } \bar{b}(a_1, 0) \leq x \leq \bar{b}(a_2, 0) \\ \vdots & \\ k_n & \text{if } \bar{b}(a_{n-1}, 0) \leq x \leq \bar{b}(a_N, 0) \end{cases}$$

From proposition 8(c) we know that all the the lower incentive constraints are binding. Therefore, the constraints can be written as follows:

For all $i = 2, \ldots, n$:

$$U(k_i) \;=\; U(k_1) - V(a_1) + \left[\frac{\Delta(a_i, 0)V(a_i) - \Delta(a_{i-1}, 0)V(a_{i-1})}{\Delta(a_i, 0) - \Delta(a_{i-1}, 0)}\right] \tag{3.12}$$

From proposition 8(b) we know that the reservation utility constraint is also binding. This implies that:

$$U(k_1) = V(a_1) \tag{3.13}$$

It is convenient to introduce h, the inverse function of U. That is, $h(w) = c$ if $U(c) = w$. Because U is concave, h will be convex. Utilizing h permits an explicit representation of the compensation function. Equation (3.13) can be substituted into equation (3.12) and the expressions for the k_i's can now be written:

$$\text{For all } i = 2, \ldots, n: \qquad k_i = h\left(\frac{\Delta(a_i, 0)V(a_i) - \Delta(a_{i-1}, 0)V(a_{i-1})}{\Delta(a_i, 0) - \Delta(a_{i-1}, 0)}\right)$$
$$\text{and} \qquad k_1 = h(V(a_1))$$

Thus, the optimal contract to implement action a_n is:

$$C(x|a_n, 0) = \begin{cases} h(V(a_1)) & \text{if } \underline{b}(0) < x \leq \overline{b}(a_1, 0) \\ h\left(\frac{\Delta(a_2,0)V(a_2) - \Delta(a_1,0)V(a_1)}{\Delta(a_2,0) - \Delta(a_1,0)}\right) & \text{if } \overline{b}(a_1, 0) < x \leq \overline{b}(a_2, 0) \\ \vdots & \\ h\left(\frac{\Delta(a_n,0)V(a_n) - \Delta(a_{n-1},0)V(a_{n-1})}{\Delta(a_n,0) - \Delta(a_{n-1},0)}\right) & \text{if } \overline{b}(a_{n-1}, 0) < x \leq \overline{b}(a_N, 0) \end{cases} \tag{3.14}$$

Note that this contract only differs from contracts to implement other actions in the output level at which compensation becomes constant. The payment for any interval $(\overline{b}(a_{i-1}, 0), \overline{b}(a_i, 0))$ is the same in contracts $C(x|a_j, 0)$ and $C(x|a_k, 0)$ if $i < j$ and $i < k$.

3.4 The Continuous Case

3.4.1 The Optimal Contract

To determine the compensation contract for a continuous action set, $\mathcal{A} = (\underline{a}, \overline{a})$, the limit is taken as the interval between actions goes to 0 in the contract for the discrete case. Let k_a be the payment for the discrete interval $(\overline{b}(a - \epsilon, 0), \overline{b}(a, 0))$.

To determine the limit as ϵ goes to zero, we assume that $\Delta(a,0)$ is a linear function of a. For a linear function, $\Delta(a-\epsilon,0) = \Delta(a,0) - \Delta'(a,0)\epsilon$.

$$k_a = h\left(\frac{\Delta(a,0)V(a) - \Delta(a-\epsilon,0)V(a-\epsilon)}{\Delta(a,0) - \Delta(a-\epsilon,0)}\right)$$

$$\begin{aligned}
\lim_{\epsilon\to 0} k_a &= \lim_{\epsilon\to 0} h\left(\frac{\Delta(a,0)[V(a)-V(a-\epsilon)] + \Delta'(a,0)\epsilon V(a-\epsilon)}{\Delta'(a,0)\epsilon}\right) \\
&= h\left(\lim_{\epsilon\to 0}\left[\frac{\Delta(a,0)[V(a)-V(a-\epsilon)] + \Delta'(a,0)\epsilon V(a-\epsilon)}{\Delta'(a,0)\epsilon}\right]\right) \\
&= h\left(\frac{\Delta(a,0)}{\Delta'(a,0)}\left[\lim_{\epsilon\to 0}\frac{V(a)-V(a-\epsilon)}{\epsilon}\right] + \lim_{\epsilon\to 0} V(a-\epsilon)\right) \\
&= h\left(\frac{\Delta(a,0)}{\Delta'(a,0)}V'(a) + V(a)\right)
\end{aligned}$$

To express k_a in terms of x, a must be expressed as a function of x. Let $r(x,0) = a$ be the inverse of $\bar{b}(a,0) = x$. Thus, k_a as a function of x is:

$$k_a = h\left(\frac{x - \underline{b}(0)}{\Delta'(a,0)}V'(r(x,0)) + V(r(x,0))\right)$$

Thus, the least cost contract to implement $a^\star$ is:

$$C(x|a^\star,0) = \begin{cases} h(V(\underline{a})) & x \in [\underline{b}(0), \bar{b}(\underline{a},0)] \\ h\left(\frac{x-\underline{b}(0)}{\Delta'(a,0)}V'(r(x,0)) + V(r(x,0))\right) & x \in (\bar{b}(\underline{a},0), \bar{b}(a^\star,0)] \\ h\left(\frac{\Delta(a^\star,0)}{\Delta'(a,0)}V'(a^\star) + V(a^\star)\right) & x \in (\bar{b}(a^\star), \bar{b}(\bar{a})] \end{cases} \tag{3.15}$$

This compensation contract is in the form of a fixed salary plus a bonus with a cap on maximum compensation. The fixed salary is the amount $h(V(\underline{a}))$ payable if outputs in the range $[\underline{b}(0), \bar{b}(\underline{a},0)]$ are observed. The bonus is the additional amount paid if a higher output is observed. This bonus is only applied after a given output level, $\bar{b}(\underline{a},0)$,

is observed and increases with output to a given level of output, $\bar{b}(a^\star, 0)$. The cap is determined by the effort exerted which in turn determines the entire range of possible output levels. Thus, within the range of possible outputs, this contract is the same form as a fixed salary plus bonus compensation contract without a cap.

3.4.2 Examples

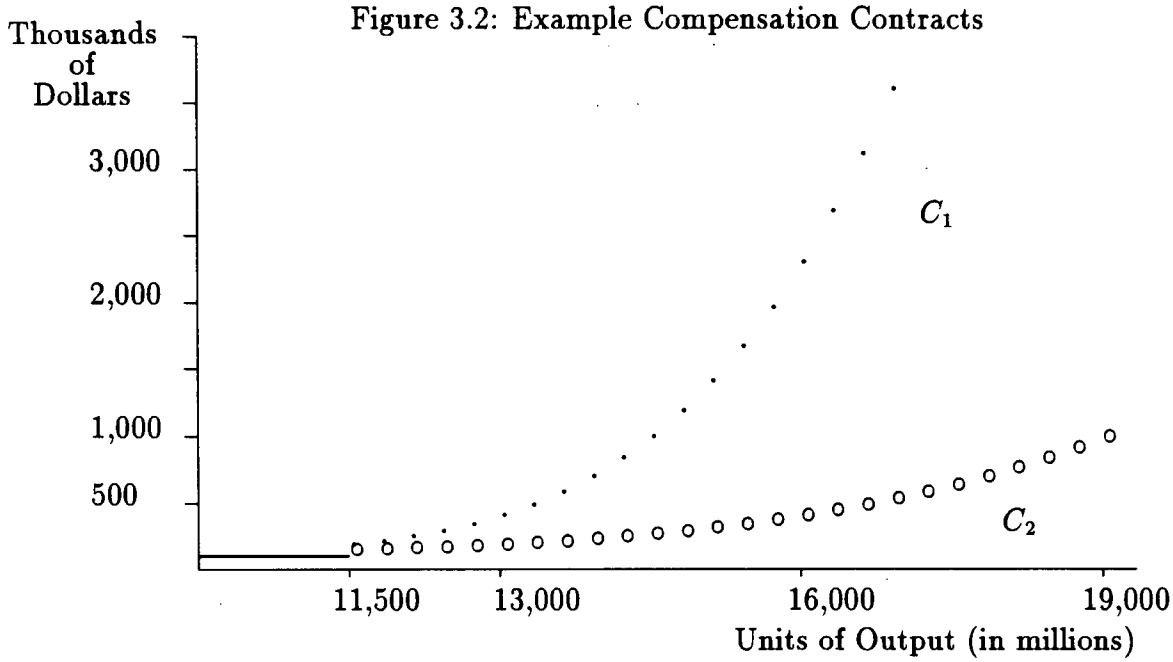
At this point, an example and a graph will aid in visualizing the optimal contract. Let the agent have a square root utility function, which implies that the inverse utility function is a square function, i.e., $U(c) = \sqrt{c}$ and $h(w) = w^2$. Let the agent have a squared disutility function. Thus, $V(a) = \gamma a^2 + 300$ and $V'(a) = 2\gamma a$.[6] Finally, let $\underline{b}(0) = 10m$ (where m is millions) and $\bar{b}(a, 0)$ be $6000a + 10m$. This implies $r(x, 0) = \frac{x-10m}{6000}$, $\Delta(a, 0) = 6000a$, and $\Delta'(a, 0) = 6000$. Hence, the least costly contract to implement an action a is:

$$C(x|a,0) = \begin{cases} (\gamma \underline{a}^2 + 300)^2 & x \in [10m, 10m + 6000\underline{a}] \\ \left(\frac{3\gamma(x-10m)^2}{6000^2} + 300\right)^2 & x \in (10m + 6000\underline{a}, 10m + 6000a] \\ (3\gamma a^2 + 300)^2 & x \in (10m + 6000a, 10m + 6000\overline{a}] \end{cases}$$

In Figure 3.2, the graph of the function $C(x|a, 0)$ above is presented for $\mathcal{A} = [250, 2000]$ and $\gamma = 50^{-2}$ (compensation C_1) and $\gamma = 100^{-2}$ (compensation C_2). The horizontal axis is units of output in thousands and the vertical axis is compensation in thousands of

[6]This disutility function ensures that the compensation payments are such that there is a significant basic wage. A reservation utility of 300 and a disutility function $V(a) = \gamma a^2$ would have the same effect. The fixed amount serves to shift the intercept of the compensation function upwards. A reservation utility other than zero could also be incorporated into the utility for consumption function. Because both the utility for consumption and the disutility for effort functions are not restricted to any specific functional forms, the assumption that the reservation utility equals zero is innocuous.

Figure 3.2: Example Compensation Contracts



dollars. The fixed salaries, in this example, are $105,625 and $93,789, for C_1 and C_2, respectively. which are payable for any output up to and including 11,500,000 units of output. For outputs above 11,500,000 units, there is also a bonus. At an output of 16,000,000 units the salary and bonus for C_1 would be $105,625 and $2,144,375 respectively for a total of $2,250,000. At an output of 16,000,000 units the salary and bonus for C_2 would be $93,789 and $266,211 respectively for a total of $360,000. This graph illustrates that the optimal contract is a fixed salary plus bonus contract.

3.4.3 Stock Option Contract

Figure 3.3 shows the graph of a fixed salary plus stock option contract. The horizontal axis is firm value before compensation is paid and the vertical axis is dollars of

compensation. If an executive has a fixed salary of \$100,000 plus an option on 1,000 shares at an exercise price of \$10.00 (where there are 1,000,000 shares outstanding), the compensation, as a function of firm value before compensation is paid, would be $Comp_1$ in Figure 3.3. This curve is linear in the bonus portion whereas the curve in Figure 3.2 is convex. However, if the executive has several option contracts at different exercise prices, the compensation contract would look like $Comp_2$. If the number of shares also increases as the exercise price increases in the different options, the compensation contract would look like $Comp_3$. Table 3.1 contains the option contracts used to determine $Comp_1$, $Comp_2$, and $Comp_3$. Obviously, when an executive has several layers of outstanding options, the compensation is a convex function of gross firm value. The degree of convexity depends on the number of different exercise prices and the number of shares in each option contract. We can, therefore, conclude that the optimal compensation contract in the model developed here resembles the fixed salary plus stock options contract seen in practice - especially those compensation contracts with several options at different prices.[7]

3.5 Optimal Choice of Action

The principal has determined the least costly feasible contract to implement each action. The next step is to choose the most profitable action given the contract required

[7]Layers of outstanding options typically occur because option grants have been made yearly and the manager has chosen not to exercise. I am not aware of any contracts in which layers of options are granted at the same time. Thus, the layers of options appear to arise from an optimization problem with constraints from previous contracts rather than a pure current period optimization problem.

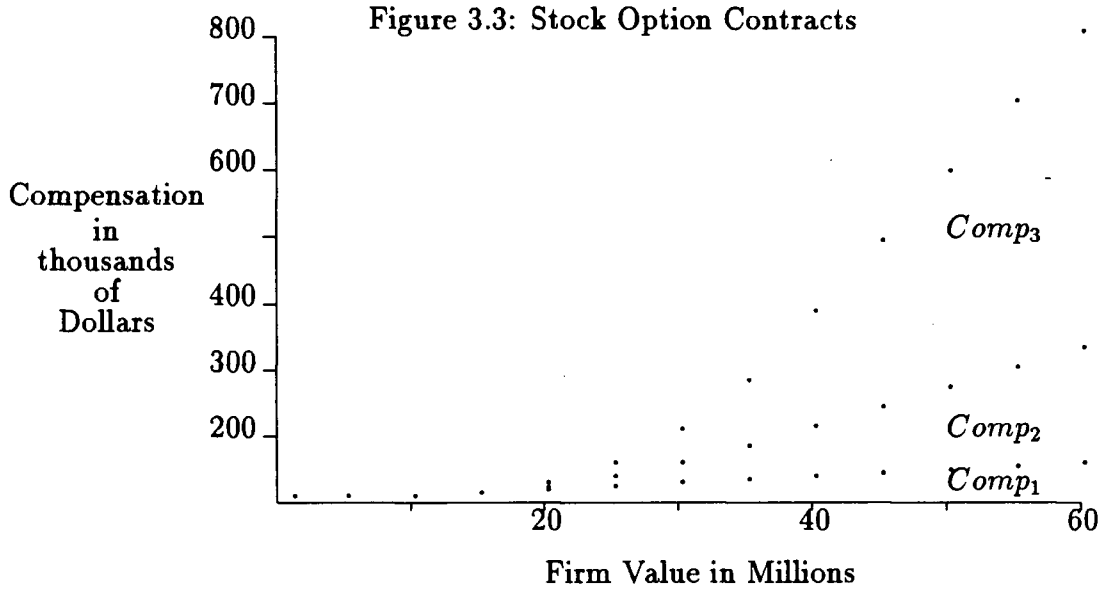


Table 3.1: Option Contracts used in Figure 3.3

Payment Type	$Comp_1$	$Comp_2$	$Comp_3$
Fixed Salary	\$100,000	\$100,000	\$100,000
Options Contract(s)	1000 @ \$10	1000 @ \$10	1000 @ \$10
(# of shares @ exercise price)		1000 @ \$15	2000 @ \$15
		1000 @ \$20	3000 @ \$20
		1000 @ \$25	4000 @ \$25
		1000 @ \$30	5000 @ \$30
		1000 @ \$35	6000 @ \$35

to implement that action. The problem can be stated as follows (where $E\Pi$ is the principal's expected share, denoted expected profit, which is equal to expected output less expected compensation paid to the agent):

$$Max\,(a \in \mathcal{A}) \qquad E\Pi\,(x|a,0) = \int_{\underline{b}(0)}^{\bar{b}(a,0)} [x - C(x|a,0)]\, f(x|a,0)\, dx$$

For all $a > \underline{a}$, $E\Pi\,(x|a,0)$ can be restated as follows:

$$\begin{aligned} E\Pi\,(x|a,0) &= \frac{\bar{b}(a,0) + \underline{b}(0)}{2} - \frac{\Delta(\underline{a},0)h(V(\underline{a}))}{\Delta(a,0)} \\ &\quad - \frac{1}{\Delta(a,0)} \int_{\bar{b}(\underline{a},0)}^{\bar{b}(a,0)} h\left(\frac{y - \underline{b}(0)}{\Delta'(a,0)} V'(r(y,0)) + V(r(y,0))\right) dy \end{aligned}$$

3.5.1 Characterization of an Interior Solution

The optimal action, denoted $a^\star$, is an interior solution to the above problem if $a^\star \in (\underline{a}, \bar{a})$. In that case, $a^\star$ is characterized by differentiating $E\Pi(x|a,0)$ and setting it equal to zero, i.e., $E\Pi'(x|a^\star,0) = 0$. Alternatively, we can view $a^\star$ as an action which equates marginal expected output to the marginal expected compensation cost, i.e., $\frac{\Delta'(a^\star,0)}{2} = EC'(x|a^\star,0)$.[8] The marginal expected net payoff (recalling that $\Delta''(a,0) = 0$ due to linearity of $\Delta(a,0)$) is:

$$\begin{aligned} &E\Pi'(x|a,0) \\ &= \frac{\Delta'(a,0)}{2} + \frac{\Delta'(a,0)\Delta(\underline{a},0)\, h(V(\underline{a}))}{\Delta(a,0)^2} - \frac{\Delta'(a,0)}{\Delta(a,0)} h\left(\frac{\Delta(a,0)}{\Delta'(a,0)} V'(a) + V(a)\right) \\ &\quad + \frac{\Delta'(a,0)}{\Delta(a,0)^2} \int_{\bar{b}(\underline{a},0)}^{\bar{b}(a,0)} h\left(\frac{y - \underline{b}(0)}{\Delta'(a,0)} V'(r(y,0)) + V(r(y,0))\right) dy \end{aligned}$$

[8] Note that $\underline{b}(0)$ is unaffected by a. Hence, $\bar{b}'(a,0) = \Delta'(a,0)$.

$$= \frac{\Delta'(a,0)}{\Delta(a,0)}\left[\frac{\Delta(a,0)}{2} + EC(x|a,0) - h\left(\frac{\Delta(a,0)}{\Delta'(a,0)}V'(a) + V(a)\right)\right] \tag{3.16}$$

Hence, determining $a^\star$ such that $E\Pi'(x|a^\star,0) = 0$ is equivalent to determining an $a^\star$ that solves

$$\frac{\Delta(a^\star,0)}{2} = h\left(\frac{\Delta(a^\star,0)}{\Delta'(a,0)}V'(a^\star) + V(a^\star)\right) - EC(x|a^\star,0) \tag{3.17}$$

To be an optimum, $a^\star$ must satisfy both first-order condition (3.17) and the expected net profit function must be concave at $a^\star$. Hence, a sufficient condition for $a^\star$ to be an optimum is that $E\Pi'(x|a^\star,0) = 0$ and $E\Pi''(x|a,0) < 0$ for all $a \in (\underline{a}, \overline{a})$. Observe that

$$\begin{aligned}
&E\Pi''(x|a,0) \\
&= -\frac{2\Delta'(a,0)^2}{\Delta(a,0)^3}\Delta(\underline{a},0)\,h(V(\underline{a})) \\
&\quad -\frac{2\Delta'(a,0)^2}{\Delta(a,0)^3}\int_{\underline{b}(\underline{a},0)}^{\overline{b}(a,0)} h\left(\frac{(x-\underline{b}(0))}{\Delta'(a,0)}V'(r(x,0)) + V(r(x,0))\right)dx \\
&\quad + \frac{2\Delta'(a,0)^2}{\Delta(a,0)^2}\,h\left(\frac{\Delta(a,0)}{\Delta'(a,0)}V'(a) + V(a)\right) \\
&\quad - \frac{\Delta'(a,0)}{\Delta(a,0)}\left[2V'(a) + \frac{\Delta(a,0)}{\Delta'(a,0)}V''(a)\right]h'\left(\frac{\Delta(a,0)}{\Delta'(a,0)}V'(a) + V(a)\right)
\end{aligned}$$

Thus, the expected net profit function is concave only if:

$$\frac{d}{da}h\left(\frac{\Delta(a,0)}{\Delta'(a,0)}V'(a) + V(a)\right) > \frac{2\Delta'(a,0)}{\Delta(a,0)}\left[h\left(\frac{\Delta(a,0)}{\Delta'(a,0)}V'(a) + V(a)\right) - EC(x|a,0)\right]$$

3.5.2 Corner Solution

The preceding analysis assumed that the solution $a^\star$ is interior. However, it is possible that the optimal action choice is a corner solution, $\underline{a}$ or $\overline{a}$. The condition under

which $\underline{a}$ is not optimal can be determined by setting $E\Pi'(x|a = \underline{a}, 0) > 0$. Hence, using equation (3.16), $\underline{a}$ is not optimal if, and only if

$$\frac{\Delta(\underline{a},0)}{2} > h\left(\frac{\Delta(\underline{a},0)}{\Delta'(a,0)}V'(\underline{a}) + V(\underline{a})\right) - h(V(\underline{a})) \tag{3.18}$$

The condition under which $\overline{a}$ is not optimal can be determined by setting $E\Pi'(x|a = \overline{a}, 0) < 0$. Using equation (3.16), $\overline{a}$ is not optimal if, and only if

$$\frac{\Delta(\overline{a},0)}{2} < h\left(\frac{\Delta(\overline{a},0)}{\Delta'(a,0)}V'(\overline{a}) + V(\overline{a})\right) - h(V(\overline{a})) \tag{3.19}$$

3.5.3 Example

The examples from section 3.4.2 can be continued to illustrate how the principal will choose the optimal action.

The expected compensation to the agent and the expected net profit to the principal, for action a, are:

$$\begin{aligned} EC(x|a,0) &= \gamma^2\left(\frac{9a^5 - 4\underline{a}^5}{5a}\right) + 600\gamma a^2 + 300^2 \\ E\Pi(x|a,0) &= \frac{\overline{b}(a,0) + \underline{b}(0)}{2} - \gamma^2\left(\frac{9a^5 - 4\underline{a}^5}{5a}\right) - 600\gamma a^2 - 300^2 \end{aligned} \tag{3.20}$$

As expected $EC(x|a,0)$ is convex in a,[9] and $E\Pi(x|a,0)$ is therefore concave in a.

[9] $EC(x|a,0)$ is convex if $EC''(x|a,0)$ is positive.

$$\begin{aligned} EC'(x|a,0) &= \gamma^2\left(\frac{36a^5 + 4\underline{a}^5}{5a^2}\right) + 1200\gamma a > 0 \\ \text{and} \quad EC''(x|a,0) &= \gamma^2\left(\frac{108a^5 - 8\underline{a}^5}{5a^3}\right) + 1200\gamma > 0 \end{aligned}$$

The solution is interior when:

$$8\gamma^2\underline{a}^3 + 1200\gamma\underline{a} < \frac{6000}{2} < \gamma^2\left(\frac{36\overline{a}^5 + 4\underline{a}^5}{5\overline{a}^2}\right) + 1200\gamma\overline{a}$$

To solve for the optimal action, $a^\star$, the first derivative of $E\Pi$ with respect to a is set equal to 0.

$$E\Pi'(x|a,0) = \frac{6000}{2} - \gamma^2\left(\frac{36a^5 + 4\underline{a}^5}{5a^2}\right) - 1200\gamma a$$

$$\Longrightarrow \qquad E\Pi'(x|a^\star,0) = 0 \quad \text{if} \quad \frac{6000}{2} = \gamma^2\left(\frac{36a^{\star^5} + 4\underline{a}^5}{5a^{\star^2}}\right) + 1200\gamma a^\star \tag{3.21}$$

Figure 3.4 illustrates the optimal action choice for this example where, as in the previous graph, $\gamma = 50^{-2}$, $\mathcal{A} = [250, 2000]$, $V(a) = \gamma a^2 + 300$, $\overline{b}(a,0) = 6000a + 10m$ and, for $\gamma = 50^{-2}$. The horizontal axis is action choice. The vertical axis is dollars.

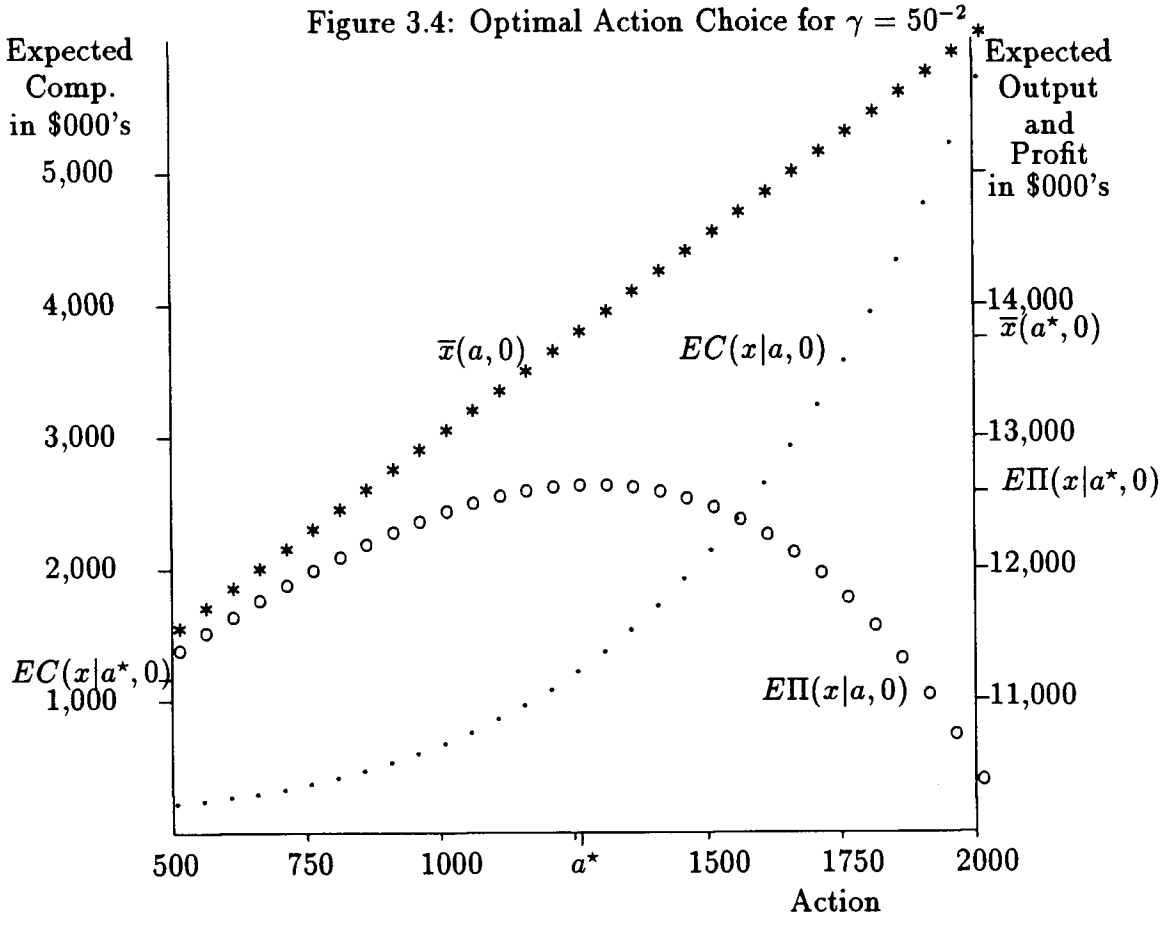
For C_1, the optimal action choice is interior. Equation (3.19) holds for $a = 2000$.

$$\begin{aligned}\gamma^2\left(\frac{36\overline{a}^5 + 4\underline{a}^5}{5\overline{a}^2}\right) + 1200\gamma\overline{a} &= \left(\frac{36(2000)^5 + 4(250)^5}{5(50)^4(2000)^2}\right) + 1200(50)^{-2}(2000) \\ &= 10,176.031 \\ &> \frac{6000}{2}\end{aligned}$$

However, for C_2, the optimal action choice is a corner solution, the maximum effort. For $a = 2000$, equation (3.19) does not hold.

$$\begin{aligned}\gamma^2\left(\frac{36\overline{a}^5 + 4\underline{a}^5}{5\overline{a}^2}\right) + 1200\gamma\overline{a} &= \left(\frac{36(2000)^5 + 4(250)^5}{5(100)^4(2000)^2}\right) + 1200(100)^{-2}(2000) \\ &= 816.002 \\ &< \frac{6000}{2}\end{aligned}$$

Figure 3.4: Optimal Action Choice for $\gamma = 50^{-2}$



This illustrates that the optimal action is decreasing in γ. This can be seen by totally differentiating equation (3.21) and holding $\underline{a}$ constant:

$$\frac{d\,a^\star}{d\,\gamma} = -\left(\frac{2\gamma a^\star(36a^{\star^5} + 4\underline{a}^5) + 6000a^{\star^4}}{\gamma^2(108a^{\star^5} - 8\underline{a}^5) + 6000\gamma a^{\star^3}}\right) < 0$$

An increase in γ makes each action more costly to the agent in terms of disutility and the agent demands more dollar compensation to ensure the reservation utility is still met. However, the higher dollar compensation moves the agent higher up on his utility for compensation curve. Because the agent has decreasing marginal utility for compensation, he demands more money for increased effort. That is, he experiences a wealth effect and the marginal cost to the principal is increased. Thus, the marginal cost will increase for increases in γ and the action for which marginal cost is equal to $\frac{\Delta'(a,0)}{2}$ will decrease.

Note also that the optimal action is decreasing in $\underline{a}$. As above, this can be seen by totally differentiating equation (3.21) and holding γ constant.

$$\frac{d\,a^\star}{d\,\underline{a}} = -\left(\frac{(5a^\star)(4\gamma^2\underline{a}^4)}{\gamma^2(108a^{\star^5} - 8\underline{a}^5) + 6000\gamma a^{\star^3}}\right) < 0$$

As $\underline{a}$ increases, the output range for which a fixed salary is paid increases and the fixed salary itself also increases. As with increases in γ, this increases the expected marginal cost of each action because the agent is at a higher point on his utility for compensation curve.

3.6 Summary

The optimal compensation function has the form of a fixed salary and a bonus that increases with firm value. This was shown to be similar to a fixed salary plus layers of stock options. The optimal action choice was also characterized. An example was used to show that the optimal action decreases as the disutility for effort increases and as the baseline action increases.

This form of the optimal contract is not unique to the uniform distribution. Convex compensation functions are optimal with other distributional assumptions. For example, in a problem in which the distribution function has a linear likelihood ratio and the utility function is a square root function, the optimal compensation function is also convex. A piecewise linear approximation of such a convex function would also resemble a fixed salary plus stock options contract. The uniform distribution gives the advantage of a precise characterization of the threshold point at which a bonus first becomes payable.

Chapter 4

Stock Market Crash

The crash is modeled as an event which shifts downward the conditional distribution of firm value. This event has a low probability of occurrence and is not contracted upon due to contracting costs. The event occurs prior to the agent choosing his action. Given the new distribution and the original contract, the agent will strictly prefer to take the baseline action. If the new distribution is still profitable enough that the principal prefers a higher action, then the principal will renegotiate. The renegotiated contract is similar to the original contract; the fixed salary is the same and the bonus is increasing in firm value. However, the agent receives a bonus at a lower firm value than in the original contract.

4.1 More Assumptions

This is an incomplete contract with respect to the event of a market crash. The crash is assumed to be one of many low probability events that are not included in the explicit compensation contract. The reasons for this incomplete contract include the cost of contracting on all low probability events and the difficulty of verifying the impact of

these events in court.

The event results in a downward shift of the conditional distribution of firm value. Let the post-crash range of possible outcomes be $(\underline{b}(1), \bar{b}(a,1))$ where $\bar{b}(a,1) \leq \bar{b}(a,0)$, $\underline{b}(1) \leq \underline{b}(0)$, and one of the bounds is strictly less than the pre-crash bound. The conditional distribution of firm value after a crash is $f(x|a,1)$.

It is also assumed that the agent can leave at any time. This assumption is crucial for the incomplete contract to be identical to the complete contract. Remember that whenever the action choice is affected by the event, the complete contract in which the agent precommits to stay differs from the complete contract in which the agent can leave. However, in this model, the agent never has incentive to leave because he can always receive his reservation utility by choosing the baseline effort. The reservation constraint in the renegotiation problem is determined by the existing contract *and* the expected utility the agent can receive from alternate employment. If the crash caused a decrease in the market reservation utility, the agent can still enforce the contract and receive the fixed salary even though his expected utility is higher than the market reservation utility. The enforceability of the original contract, as it stands, affects the reservation utility. If the crash caused an increase in reservation utility, then the market would determine the constraint in the renegotiation problem. With the assumption of no change in reservation utility, the fixed salary is the same in the original and the renegotiated contract. If the reservation utility increased, the fixed salary would also increase. Note also that an increase in the market reservation utility would always lead to renegotiation.

4.2 Timeline

The principal and agent agree on a contract $C(x|a^\star, 0)$ and an action $a^\star$ based upon common knowledge of the distribution $f(x|a, 0) = \frac{1}{\Delta(a,0)}$. Before the agent takes the action, a market downturn occurs. Both the agent and principal have new information that the distribution is now $f(x|a, 1) = \frac{1}{\Delta(a,1)}$. While the contract can be enforced, the agent is free to leave or to take whatever action he chooses. The principal will only choose to renegotiate the contract if he or she can benefit by inducing an action other than the one the agent would wish to take under the old contract. Thus, the contract may be renegotiated before the agent chooses the action. The agent implements his or her preferred action. Finally, the agent and the principal both observe the outcome and share the profits as agreed in the contract.

The new timeline is:

$t = 0$ The principal and agent agree on a contract $C(x|a, 0)$ and an action $a^\star$ based on $f(x|a, 0) = \frac{1}{\Delta(a,0)}$.

$t = 1$ A market crash occurs. The new distribution is $f(x|a, 1) = \frac{1}{\Delta(a,1)}$.

$t = 2$ The principal and agent MAY recontract and agree on a new contract $C(x|a, 1)$ and action $\hat{a}$.

$t = 3$ The agent chooses his preferred action.

$t = 4$ The output, x, is produced and shared according to whichever contract is in force.

The occurrence of the crash, therefore, creates the possibility of renegotiation between the agent and the principal. This renegotiation will only occur if renegotiating Pareto dominates not renegotiating. Both the agent and the principal can enforce the original contract's payment schedule. However, the principal cannot observe the agent's effort and, therefore, cannot enforce the effort part of the contract. Since, renegotiation will only occur if the agent and the principal will both at least weakly benefit, one of the two must be strictly better off and the other must at least do as well. To determine if there will be renegotiation, we first determine the action the agent will prefer under the original contract.

Proposition 9 *If the contract is not renegotiated, the agent will prefer the baseline effort.*

Proof: See Appendix A.8

If the principal does not renegotiate, the agent can only get his reservation utility by taking action $\underline{a}$. Therefore, if the contract is not renegotiated the agent will take the lowest possible action and get only the fixed salary. The bonus is no longer worth the effort necessary to achieve it.

4.3 New Optimal Contract

To determine whether the principal will wish to renegotiate, we first determine what the renegotiated contract is and then whether the principal will be better off with it. Because the renegotiated contract will give the agent his reservation utility for the new

optimal action, he will be indifferent between the two contracts. Hence, if the profit to the principal is greater under the renegotiated contract, then renegotiation will take place.

The least cost contract for action $\hat{a}$ with distribution $f(x|a,1)$ is:

$$C(x|\hat{a},1) = \begin{cases} h(V(\underline{a})) & x \in [\underline{b}(1), \bar{b}(\underline{a},1)] \\ h\left(\frac{x-\underline{b}(1)}{\Delta'(a,1)}V'(r(x,1)) + V(r(x,1))\right) & x \in (\bar{b}(\underline{a},1), \bar{b}(\hat{a},1)] \\ h\left(\frac{\Delta(\hat{a},1)}{\Delta'(a,1)}V'(\hat{a}) + V(\hat{a})\right) & x \in (\bar{b}(\hat{a},1), \bar{b}(\overline{a})] \end{cases} \tag{4.22}$$

We now consider three types of shifts. In the multiplicative shift, the upper and lower bounds of output are a fixed fraction of their pre-crash counterparts. In the additive shift, all crash output levels are a fixed amount less than their pre-crash counterparts. Finally, in the lower bound shift, the lower bound is a fixed amount less than the pre-crash lower bound but the upper bounds conditional on actions do not change.

4.3.1 Multiplicative Shift

A multiplicative shift appeared in Chapter 2. All shift output levels are the same fraction of the non shift output levels, i.e., $\bar{b}(a,1) = \alpha\bar{b}(a,0)$ and $\underline{b}(1) = \alpha\underline{b}(0)$ for $0 < \alpha < 1$. Thus, $\Delta(a,1) = \alpha\Delta(a,0)$ and $\Delta'(a,1) = \alpha\Delta'(a,0)$. The shift expected output is just α times the non shift expected output. The variance is α^2 times the non shift variance. Thus, both expected output and variance are reduced by a multiplicative

shift. The least cost contract for action $\hat{a}$, if the shift is multiplicative, is:

$$C(x|\hat{a},1) = \begin{cases} h(V(\underline{a})) & x \in [\alpha\underline{b}(0), \alpha\bar{b}(\underline{a})] \\ h\left(\frac{x-\alpha\underline{b}(0)}{\alpha\Delta'(a,1)}V'(r(x,1)) + V(r(x,1))\right) & x \in (\alpha\bar{b}(\underline{a}), \alpha\bar{b}(\hat{a})] \\ h\left(\frac{\Delta(\hat{a})}{\Delta'(a,1)}V'(\hat{a}) + V(\hat{a})\right) & x \in (\alpha\bar{b}(\hat{a}), \bar{b}(\bar{a})] \end{cases}$$

The expected cost of this contract is:

$$\begin{aligned} EC(x|\hat{a},1) &= \frac{\Delta(\underline{a},0)h(V(\underline{a}))}{\Delta(\hat{a},0)} \\ &\quad + \frac{1}{\Delta(\hat{a},0)} \int_{\alpha\bar{b}(\underline{a},0)}^{\alpha\bar{b}(a,0)} h\left(\frac{(x-\alpha\underline{b}(0))}{\alpha\Delta'(a,0)}V'(r(x,1)) + V(r(x,1))\right) dx \end{aligned}$$

Observe that the fixed salary and the maximum payment are the same as in the pre-crash contract. In fact, the expected minimum cost compensation in the crash contract is the same as the expected minimum cost compensation in the pre-crash contract for the same action.

Proposition 10 *If the crash causes a multiplicative shift, then the expected cost of a given action choice is the same for both the pre-crash and post-crash contract.*

See Appendix A.9 for a proof.

The output levels in the post-crash case can be viewed as the output levels in the pre-crash case with new labels. The information content of the relabeled output levels in the post-crash case is the same as the original output levels in the pre-crash case. There is no reason to change the payments to the agent if the action choice is not changed.

In Chapter 2, the first order approach was used to show that, without precommitment, the no shift and shift case optimal contracts for a given action only differ in the relabeling

of outputs in the case 2 contract. Although the analysis in this chapter does not utilize the first order approach, the results are the same. Since the payments in the contracts are the same for equivalent information and a multiplicative shift does not affect the probability distribution of the payments, the expected costs are the same. The probability distribution of payments for any action will be identical under the two contracts because the ratios of output ranges for pairs of actions do not change with the new distribution. In the discrete case, this can be seen clearly because the probability of being in the range $(\bar{b}(a_{i-1},1)\,,\,\bar{b}(a_i,1))$ given action a_n was taken is just

$$\begin{aligned}\frac{\bar{b}(a_i,1)-\bar{b}(a_{i-1},1)}{\bar{b}(a_n,1)} &= \frac{\alpha[\bar{b}(a_i,0)-\bar{b}(a_{i-1},0)]}{\alpha\bar{b}(a_n,0)}\\ &= \frac{\bar{b}(a_i,0)-\bar{b}(a_{i-1},0)}{\bar{b}(a_n,0)}\\ &= Prob(x\in(\bar{b}(a_{i-1},0)\,,\,\bar{b}(a_i,0))|a=a_n)\end{aligned}$$

4.3.2 Additive Shift

As in Chapter 2, the additive shift causes the output to be reduced by a fixed amount for all actions. That is, $\bar{b}(a,1)=\bar{b}(a,0)-\delta$ for all a and $\underline{b}(1)=\underline{b}(0)-\delta$. Thus, the total range does not change, i.e., $\Delta(a,1)=\Delta(a,0)$. The expected output will be reduced by the fixed amount δ. The variance remains the same as the pre-crash variance. The least cost contract for action $\hat{a}$, if there is an additive shift, is:

$$C(x|\hat{a},1)=\begin{cases} h(V(\underline{a})) & x\in[\underline{b}(0)-\delta,\bar{b}(\underline{a})-\delta]\\ h\left(\frac{x-\underline{b}(0)+\delta}{\Delta'(a)}V'(r(x,1))\;+\;V(r(x,1))\right) & x\in(\bar{b}(\underline{a})-\delta,\bar{b}(\hat{a})-\delta]\\ h\left(\frac{\Delta(\hat{a})}{\Delta'(a)}V'(\hat{a})+V(\hat{a})\right) & x\in(\bar{b}(\hat{a})-\delta,\bar{b}(\bar{a})]\end{cases}$$

The expected cost of this contract is:

$$\begin{aligned} EC(x|\hat{a},1) &= \frac{\Delta(\underline{a},0)h(V(\underline{a}))}{\Delta(\hat{a},0)} \\ &+ \frac{1}{\Delta(\hat{a},0)} \int_{\bar{b}(\underline{a},0)-\delta}^{\bar{b}(a,0)-\delta} h\left(\frac{(x-\underline{b}(0)+\delta)}{\Delta'(a,0)} V'(r(x,1)) + V(r(x,1))\right) dx \end{aligned}$$

As in the multiplicative shift, the fixed salary and maximum payment are the same as in the pre-crash contract and the expected compensation is the same in both contracts for the same action.

Proposition 11 *If the crash causes an additive shift, then the expected cost of a given action choice is the same for both the pre-crash and post-crash contract.*

The proof for the additive shift is similar to the proof for the multiplicative shift and requires only a transformation of variables. See Appendix A.10 for the proof.

The additive shift does not change the probability distribution of payments. The information content of the post-crash outputs is the same as the pre-crash output levels except for the relabeling. Hence, as with the multiplicative shift, the expected compensation for a given action does not change.

4.3.3 Lower Bound Shift

In a lower bound shift, the lower bound shifts by δ but the upper bounds, determined by the agent's effort remain unchanged, i.e., $\underline{b}(1) = \underline{b}(0) - \delta$ and $\bar{b}(a,1) = \bar{b}(a,0)$ for all a. Observe that $\Delta'(a,0) = \bar{b}'(a,0)$. Hence, $\Delta'(a,0) = \Delta'(a,1)$. Note also that if

$\bar{b}(a,1) = \bar{b}(a,0)$, then $r(x,1) = r(x,0)$. The expected output is reduced by $\frac{\delta}{2}$ and the variance is increased.

$$\begin{aligned} Var(x|a,1) &= \frac{(\Delta(a,0)+\delta)^2}{12} \\ &= Var(x|a,0) + \frac{\delta^2 + 2\delta\Delta(a,0)}{12} \\ &> Var(x|a,0) \end{aligned}$$

The least cost contract to implement action $\hat{a}$, if there is a lower bound shift, is:

$$C(x|\hat{a},1) = \begin{cases} h(V(\underline{a})) & x \in [\underline{b}(0)-\delta, \bar{b}(\underline{a})] \\ h\left(\frac{x-\underline{b}(0)+\delta}{\Delta'(a,0)}V'(r(x,0)) + V(r(x,0))\right) & x \in (\bar{b}(\underline{a}), \bar{b}(\hat{a})] \\ h\left(\frac{\Delta(\hat{a})+\delta}{\Delta'(a,0)}V'(\hat{a}) + V(\hat{a})\right) & x \in (\bar{b}(\hat{a}, \bar{b}(\overline{a})] \end{cases}$$

The expected cost of this contract is:

$$\begin{aligned} EC(x|\hat{a},1) &= \frac{(\Delta(\underline{a},0)+\delta)h(V(\underline{a}))}{\Delta(\hat{a},0)+\delta} \\ &+ \frac{1}{\Delta(\hat{a},0)+\delta}\int_{\bar{b}(\underline{a},0)}^{\bar{b}(\hat{a},0)} h\left(\frac{(x-\underline{b}(0)+\delta)}{\Delta'(\hat{a},0)}V'(r(x,0)) + V(r(x,0))\right) dx \end{aligned}$$

In the lower bound shift, the fixed salary remains the same as in the pre-crash contract and the bonus becomes payable at the same output level as in the pre-crash contract. However, the bonus is higher than in the pre-crash contract. The shift increases the probability of output being in the range $[\underline{b}(0)-\delta, \bar{b}(\underline{a})]$. This reduces the probability of receiving a bonus. The bonus needs to be larger in order to leave the agent as well off. The agent demands an increased risk premium to take on a more variable payment schedule. This implies that the expected compensation of a given action is higher in

the post-crash contract than in the pre-crash contract. While this is difficult to prove in general, it does hold in the example below.

4.3.4 Example

The effect on compensation of the three shifts can be illustrated using the example from Section 3.4.2. Remember that $U(c) = \sqrt{c}$, $h(w) = w^2$, $\overline{b}(a) = 6000a + 10m$, $\underline{b}(0) = 10m$, and $V(a) = \gamma a^2 + 300$. For the multiplicative shift, $\overline{b}(a,1) = \alpha(6000a + 10m)$, $\underline{b}(1) = \alpha(10m)$, and $r(x,1) = \frac{x-\alpha(10m)}{6000\alpha}$. For an additive shift, $\overline{b}(a,1) = 6000a + 10m - \delta$, $\underline{b}(1) = 10m - \delta$, and $r(x,1) = \frac{x-10m+\delta}{6000}$. For the lower bound shift, $\overline{b}(a,1) = 6000a + 10m$, $\underline{b}(1) = 10m - \delta$, and $r(x,1) = \frac{x-10m}{6000}$.

Under the multiplicative shift, the optimal compensation contract to implement a given action, a', is:

$$C(x|a',1) = \begin{cases} (\gamma \underline{a}^2 + 300)^2 & x \in [\alpha(10m)\,,\, \alpha(6000\underline{a} + 10m)] \\ \left(\frac{3\gamma(x-\alpha(10m))^2}{(6000\alpha)^2} + 300\right)^2 & x \in (\alpha(6000\underline{a} + 10m)\,,\, \alpha(6000a' + 10m)] \\ (3\gamma a'^2 + 300)^2 & x \in (\alpha(6000a' + 10m)\,,\, \alpha(6000\overline{a} + 10m)] \end{cases}$$

Under the additive shift, the optimal compensation contract to implement a given action, a', is:

$$C(x|a',1) = \begin{cases} (\gamma \underline{a}^2 + 300)^2 & x \in [10m - \delta\,,\, 6000\underline{a} + 10m - \delta] \\ \left(\frac{3\gamma(x-10m+\delta)^2}{6000^2} + 300\right)^2 & x \in (6000\underline{a} + 10m - \delta\,,\, 6000a' + 10m - \delta] \\ (3\gamma a'^2 + 300)^2 & x \in (6000a' + 10m - \delta\,,\, 6000\overline{a} + 10m - \delta] \end{cases}$$

Under the lower bound shift, the optimal compensation contract to implement action a is:

$C(x|a',1) =$

$$\begin{cases} (\gamma\underline{a}^2 + 300)^2 & x \in [10m - \delta\,,\, 6000\underline{a} + 10m] \\ \left(\frac{3\gamma(x-(10m-\delta))^2}{6000^2} + \frac{2\gamma\delta(x-10m)}{6000^2} + 300\right)^2 & x \in (6000\underline{a} + 10m\,,\, 6000a' + 10m] \\ \left(3\gamma a'^2 + \frac{2\gamma\delta a'}{6000} + 300\right)^2 & x \in (6000a' + 10m\,,\, 6000\overline{a} + 10m] \end{cases}$$

The expected cost of this contract is:

$EC(x|a',1) =$

$$\begin{aligned} & \frac{(6000\underline{a} + \delta)(\gamma\underline{a}^2 + 300)^2}{6000a' + \delta} \\ & + \frac{1}{6000a' + \delta}\int_{6000\underline{a}+10m}^{6000a'+10m} \left(\frac{3\gamma(x - 10m + \delta)^2}{6000^2} + \frac{2\gamma\delta(x - 10m)}{6000^2} + 300\right)^2 \\ = \; & \gamma^2\left(\frac{9a'^5 - 4\underline{a}^5}{5a'}\right) + 600a'^2 + 300^2 \\ & + \frac{\gamma^2\delta}{6000a' + \delta}\left(\frac{6a'^4}{5} - 2\underline{a}^4 + \frac{4\underline{a}^5}{5a'} + \frac{\delta(a'^3 - \underline{a}^3)}{4500}\right) \\ = \; & EC(x|a) + \frac{\gamma^2\delta}{6000a' + \delta}\left(\frac{6a'^4}{5} - 2\underline{a}^4 + \frac{4\underline{a}^5}{5a'} + \frac{\delta(a'^3 - \underline{a}^3)}{4500}\right) \end{aligned}$$

The last term is positive and hence, the expected cost of the lower bound shift contract is greater than the expected cost of the pre-crash contract. The difference in expected cost is due to the increased variability of payments to the agent after the crash. The post-crash contract has a higher probability of paying only the fixed salary because there is a larger region in which there is no information about the agent's action. Consequently, there is a smaller probability of receiving the bonus. The agent demands a higher risk premium to take on this riskier income stream.

Figures 4.1 and 4.2 illustrate the relabeling concept. Figure 4.1 shows the maximum compensation that can be attained with each possible effort level. This is based

on $C(x|a,1)$ and the inverse function $r(x,1)$, which specifies the minimum effort level required to achieve output level x. In contrast, Figure 4.2 shows compensation as a function of the output level achieved. In Figure 4.1, the maximum compensation for a given effort level are the same as C_1 from the pre-crash example for both the multiplicative and additive shifts. On the other hand,, as shown in Figure 4.2, compensation as a function of output differ for both types of shifts. This illustrates that, while the information content of output about effort may not change, compensation as a function of output can change because the output upper bound (or label) for a given effort level differs.

For the lower bound shift, Figure 4.1 illustrates that the post-crash compensation differs from the pre-crash compensation by more than mere relabeling. Both graphs show differences in pre-crash and post-crash compensation. Thus, the information content of output about effort changes after a lower bound shift. Post-crash compensation as a function of output for the additive and lower bound shifts are very similar as can be seen in Figure 4.2. Remember, that the lower bound shift example used the same change in the lower bound as in the additive shift example. Hence, the only difference between the two shifts is that the upper bound changes for the additive shift but not for the lower bound shift. Note that the break point is lower for the additive shift than for the lower bound shift. This is because the lower bound shift does not affect the upper bound of the baseline effort whereas the additive shift does.

Figure 4.1: Maximum Post-Crash Compensation Attainable from a Given Level of Effort

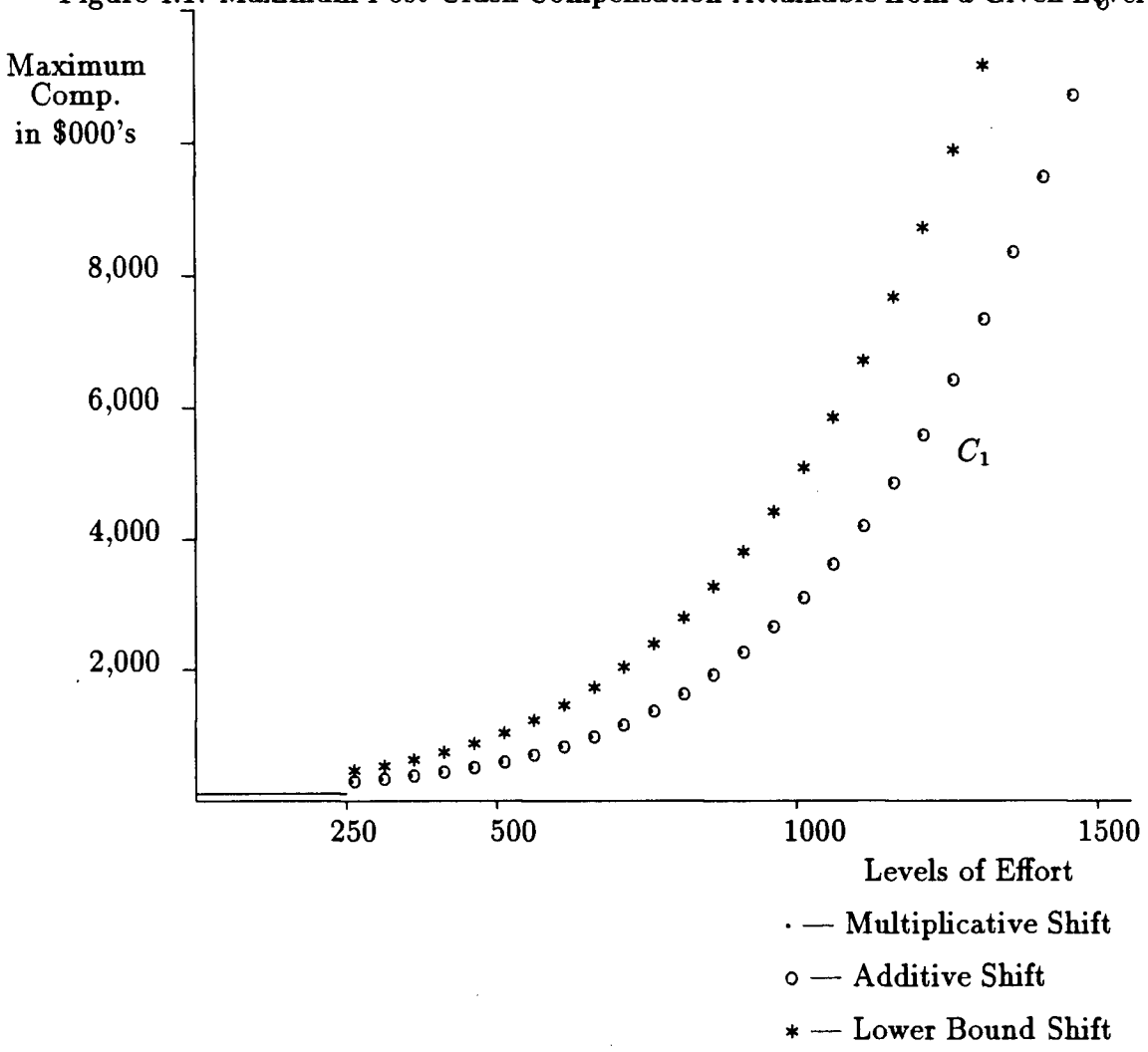
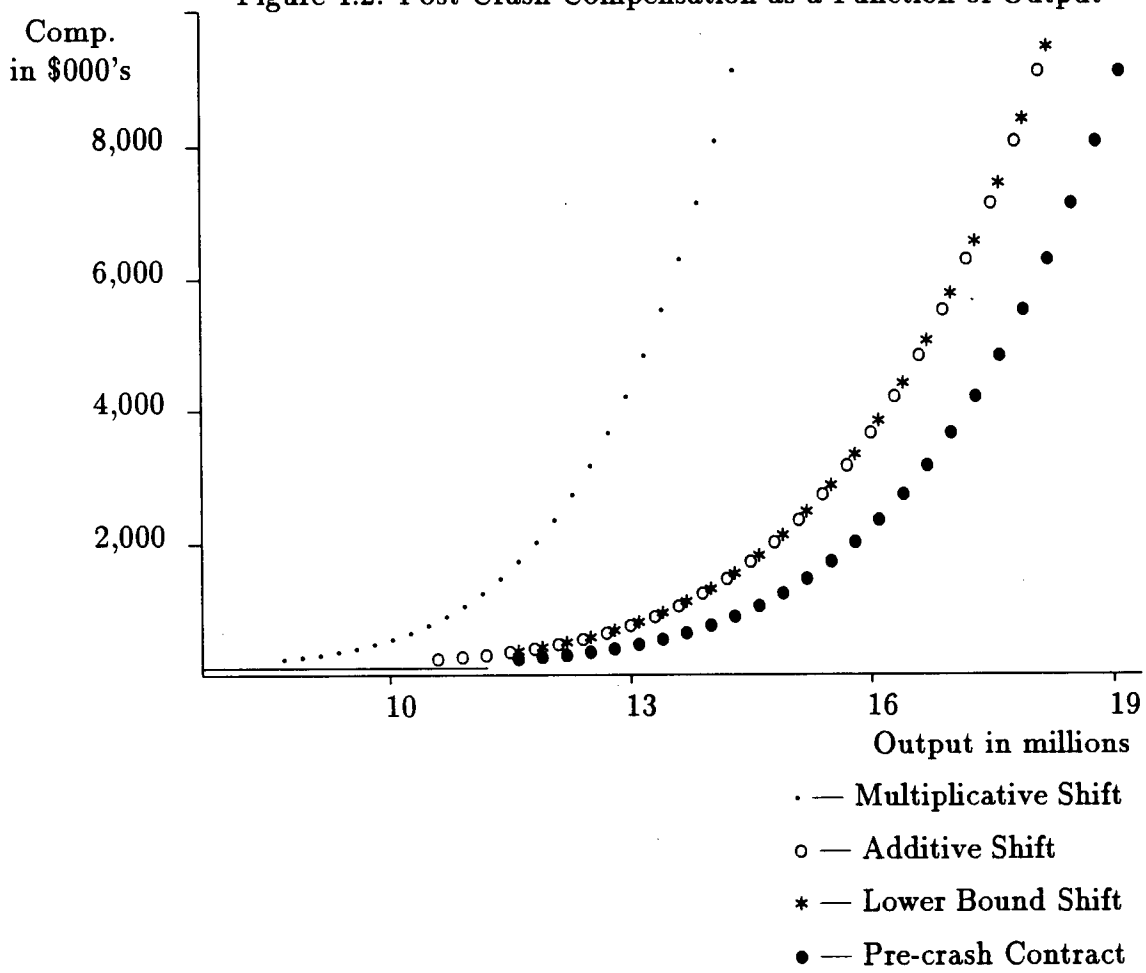


Figure 4.2: Post-Crash Compensation as a Function of Output



4.4 Optimal Action with New Contract

The principal now chooses the best action given the new contract $C(x|a,1)$. As in the original problem the principal maximizes expected profit. Note that

$$E\Pi(x|a,1) = \frac{\overline{b}(a,1)+\underline{b}(1)}{2} - EC(x|a,1) \tag{4.23}$$

$$E\Pi'(x|a,1) = \frac{\Delta'(a,1)}{\Delta(a,1)}\left[\frac{\Delta(a,1)}{2} + EC(x|a,1) - h\left(\frac{\Delta(a,1)}{\Delta'(a,1)}V'(a)) + V(a)\right)\right] \tag{4.24}$$

The optimal action choice under the new contract, $\hat{a}$, can be characterized by solving $E\Pi'(x|\hat{a},1) = 0$, i.e.,

$$\frac{\Delta(\hat{a},1)}{2} = h\left(\frac{\Delta(\hat{a},1)}{\Delta'(\hat{a},1)}V'(\hat{a}) + V(\hat{a})\right) - EC(x|\hat{a},1) \tag{4.25}$$

Proposition 12 *If the shift is additive, the optimal action is the same in both the post-crash contract and the pre-crash contract. If the shift is multiplicative, then the optimal action is lower in the post-crash contract than in the pre-crash contract.*

Proof: See Appendix A.11.

As in Chapter 2, the additive shift does not change the marginal expected compensation cost or the marginal expected outcome. Hence, the optimal action choice is the same pre- and post-crash.

The multiplicative shift does affect the marginal expected outcome. Since the marginal expected outcome is lower after the crash, the optimal action is also lower after the crash.

With a lower bound shift, the bonus part of the compensation is higher for a lower outcome level. The bonus section of the compensation is therefore more convex. The marginal expected compensation is higher while the marginal expected outcome remains unchanged. Thus, we expect the optimal action to be lower in the post-crash contract than in the pre-crash contract. Again the general proof of this is difficult. However, it is true for the example that the optimal action in the post-crash contract is lower than in the pre-crash contract.

4.5 Corner Solution and Renegotiation

Just as in the pre-crash case, the conditions under which a corner solution is not optimal occur when marginal expected profit at $\underline{a}$ is strictly greater than zero. Substituting for $a = \underline{a}$ in equation (4.24):

$$E\Pi'(x|a=\underline{a},1) = \frac{\Delta'(\underline{a},1)}{\Delta(\underline{a},1)}\left(\frac{\Delta(\underline{a},1)}{2} - \left[h\left(\frac{\Delta(\underline{a},1)}{\Delta'(\underline{a},1)}V'(\underline{a}) + V(\underline{a})\right) - h(V(\underline{a}))\right]\right)$$

Therefore $E\Pi'(x|a=\underline{a}) > 0$ if and only if:

$$\frac{\Delta(\underline{a},1)}{2} > h\left(\frac{\Delta(\underline{a},1)}{\Delta'(\underline{a},1)}V'(\underline{a}) + V(\underline{a})\right) - h(V(\underline{a})) \tag{4.26}$$

With an additive shift, a corner solution is optimal post-crash only if the corner solution is optimal pre-crash. If the shift is multiplicative, an optimal corner solution is more likely post-crash than pre-crash.

Proposition 13 *If the shift is multiplicative and the corner solution was optimal pre-crash, then the corner solution will also be optimal post-crash. It can also be the case that an interior solution is optimal in the pre-crash case while the corner solution is optimal in the post-crash case.*

The proof is in Appendix A.12.

In the case of a multiplicative shift, the expected cost of a post-crash contract is the same as for the pre-crash contract. However, the benefits to the principal are less in the post-crash contract. If the crash causes a large enough downward shift, it may no longer be profitable to pay a bonus for effort above the baseline effort level.

In the case of a lower bound shift, we expect that if the corner solution is optimal pre-crash, then the corner solution would be optimal post-crash. It can also be the case that an interior solution is optimal in the pre-crash case while the corner solution is optimal in the post-crash case. This would follow if the optimal action in the post-crash contract is is lower than in the pre-crash contract.

The principal will always weakly prefer to renegotiate if there are no costs to renegotiation. Without renegotiation, the agent will be induced to taking the minimum action. However, the payment for the baseline effort does not differ between the original contract and the renegotiated contract. Hence, the principal will be indifferent between $C(x|a,0)$ and $C(x|a,1)$ if the baseline effort is optimal in the renegotiated contract. If, however, $\underline{a}$ is not optimal in the post-crash contract, the principal will strictly prefer to renegotiate

with the agent.

4.6 Summary

A stock market downturn has been modeled as a low probability event that shifts down the conditional distribution of firm value. When the event occurs before the agent chooses his action, the original contract does not provide the agent with any incentives to exert effort. Hence, if the firm is still profitable enough that the principal wishes the agent to exert effort above the baseline effort level, the principal must renegotiate the contract.

If the principal does not renegotiate, the agent will take the baseline effort and receive his reservation utility. If the principal does renegotiate, the new contract $C(x|a,1)$ will give the same fixed salary, $h(V(\underline{a}))$, as $C(x|a,0)$. With an additive or multiplicative shift the bonus will be payable at a lower output level than that under $C(x|a,0)$. In stock option terms, the renegotiated contract will have a lower exercise price than the original contract. The exercise price of the lowest option is determined by the lowest outcome that can only be possible if some effort above the baseline effort level has been exerted. Because that outcome level is shifted down by the downturn, the exercise price will decrease in both types of shift.

The renegotiated contract under a multiplicative shift was shown to have the same expected cost as the original contract. The same information about actions is provided by crash outputs as was provided by non-crash outputs; the crash outputs are just relabeled

non-crash outputs. Because the proportion of supports for the different actions does not change, the bonus needed to induce the desired action $\hat{a}$ will not change. However, the point at which the outcome can only have come from $\hat{a}$ shifts down. Thus, the exercise price for the stock option shifts down but the potential bonus does not change.

Renegotiation will depend on how large the shift in profit distribution is and how profitable the firm was to begin with. Only in the case where the firm's profit becomes marginal after the event will the principal prefer the baseline effort. Because the reservation utility and disutility for action do not change, renegotiation will depend on whether the new output levels are profitable with the original disutility for action and reservation utility.

The question of whether the shift in the distribution is permanent or not has not been addressed. In a single period model there can only be one type of shift — a permanent one. Since there is only one settling up a temporary shift would not enter the model. To model a temporary shift would require a multi-period model.

This model is also not applicable to market-wide upturns. In a downturn, it is clear that the agent will wish to renegotiate. Hence, this model only needed to show when the principal will also wish to renegotiate. For an upturn, the tables are turned. In an upturn, the agent can receive a bonus without exerting effort. Effectively, the agent's reservation utility will shift upward and any new contract would be constrained to meet the new reservation utility. The agent will experience a positive wealth effect and the marginal expected cost of effort will shift upward. The principal cannot avoid this because the

original contract is enforceable and the agent will only accept a new contract that makes him as well off as the original contract. In fact, one of the often voiced criticisms of stock option contracts is that, when the stock market is booming, executives are rewarded for doing nothing. In a stock option contract, there is no ceiling or cap on the maximum bonus. A cap, such as the one in this model, would serve to reduce the effect of generally increasing stock prices. Renegotiation will occur if the principal wishes to give incentives to implement a different action than the one the original contract induces in the new situation.

The results are relatively robust. As mentioned in Chapter 3, convex compensation functions are optimal for other distributions. One can envision that a crash shifts the conditional distribution to the left such that for a given action the shift distribution first order stochastic dominates the no shift distribution. In that case, the optimal compensation will also shift to the left. A piecewise linear approximation of the new optimal compensation function would yield a increasing bonus beginning at a lower level of output — the same result as for the uniform distribution. Thus, the results would appear not to be due to the distributional assumption but instead driven by the convexity of the optimal compensation function.

Chapter 5

Empirical Tests

5.1 Introduction

This chapter presents empirical tests of whether executive stock options were renegotiated after stock prices fell in October 1987. The drop in stock prices was clearly a market wide phenomenon and its impact on the stock price of any one firm was largely outside of the influence of its executive. The outstanding options of many executives lost value and, with that loss in value, some of the incentive content of their compensation was lost as well. In fact many press articles, at that time, noted that some options were "underwater" (the current stock price had fallen below the exercise price of the options). The prediction from the previous chapter is that these conditions prompted a renegotiation, the consequence of which was to alter the compensation contract in force at the time. This chapter begins with a discussion of the existing empirical literature regarding executive compensation. A description of executive stock option contracts follows. Next, there is a statement of the hypothesis followed by a discussion of sources of potential bias and a discussion of the sample selection. Finally, the tests and results are presented.

5.2 Major Antecedents

Executive compensation has become a very controversial topic. Many in the popular press[1] have argued that CEO's are too well paid when they appear to be doing a poor job. Some have argued that the level of compensation should be regulated to ensure there is a cap to executive pay regardless of performance[2]. In the academic literature, Jensen and Murphy [21] have argued that CEO's are not well enough paid in comparison to the gains in firm value under their leadership. They have found that, on average, if a firm's shareholder wealth increases by $1000.00 the change in all pay and stock related wealth of the executive is an increase of only $3.25, with $2.50 of that due to changes in wealth related to stock ownership. As Jensen and Murphy [20] point out in another article, "It's not how much you pay but how" you pay that counts.

These arguments about the appropriate level of compensation ultimately depend on what performance measure is used. The choice of performance measure affects the form of compensation which in turn affects the level. Use of an accounting performance measure leads to compensation in the form of bonuses or performance units whereas use of a stock price performance measure leads to compensation in the form of options or restricted stock. The implicit assumption in Jensen and Murphy's work is that stock price is the appropriate performance measure because shareholders wish executives to maximize shareholder value and, if the executive's wealth changes are tied to shareholder wealth

[1]See Loomis [28], Crystal [5] and [6] for example.
[2]See Drucker [9].

changes, then the executive will have the appropriate incentives to maximize shareholder wealth. The implication is that such an executive will behave as if he or she were an owner-manager. This compensation plan may not be optimal. Since the stock price is affected by both market wide information and firm specific information, the stock price could drop at a time when the executive was taking actions to maximize shareholder wealth. In that case, there is too much noise in the chosen performance measure, stock price. The optimal compensation function will reward the executive for taking actions that maximize shareholder wealth and penalize the executive who is not taking actions to maximize shareholder wealth. Economic theory (principal-agent theory in particular) has shown that the optimal performance measure depends on the information the measure contains about the agent's action. This implies that, while the agent may be induced to maximize firm value, the value of the firm will not necessarily be the optimal performance measure.

Much of the empirical research into compensation has noted the lack of correlation between compensation and the market value of the firm. Early work by Lewellen [26], Lewellen and Huntsman [27] and Masson [29] determined that the level of compensation was more highly correlated with the size of the firm or sales than with the performance of the stock price of the firm. Murphy [30] did find a significant positive relationship between executive pay and corporate performance but, as Jensen and Murphy [21] point out, the magnitude of the relationship is very small.

Murphy regressed individual items of compensation (salary, bonus, deferred compensation, the value of options granted and the total compensation) on a stock index based on the rate of return realized by the shareholders of the firm. Both the dependent variable and the independent variables are deflated by the consumer price index to represent 1983-constant dollars. Finally, these variables are converted into logarithms. The regression coefficients are, therefore, interpreted as the effect of current real stock market returns on percentage changes in real compensation. Two separate regressions were performed — one in which the coefficients were allowed to differ for each executive and one in which the average compensation was regressed on the average performance index with the coefficients constrained to be the same for all executives. The results for the averages regression are inconclusive. However, the results for the individual regressions are clear and significant. While the coefficients on the total compensation variable and the individual compensation items variables are all significant, Murphy found a negative correlation between corporate performance and the ex-ante value of stock options granted. Murphy conjectured that "boards of directors are more likely to award options during low-performance years and will often reissue previously-granted options at a lower exercise price". [30, page 31] In a sense, it is this conjecture which the model in the preceding chapter seeks to explain. One reason why boards of directors might renegotiate these contracts is that the measure of low performance (i.e. lower stock prices) may be due to a factor or factors beyond the executive's control, such as the market crash of October

1987. The model predicts that companies, whose executive compensation contracts include options that kick in after a given threshold in the stock price is reached, will seek to renegotiate those contracts if there is an unanticipated downshift in the support of the distribution of the stock price. The model assumes that the downshift is outside of the executive's control and, hence, lower realizations of stock price need not be an indication of poor performance on the executive's part. In the renegotiated contract, the threshold is lowered. Empirically speaking, this corresponds to canceling and reissuing options at a lower exercise price than in the original contract. The empirical tests that follow examine whether firms reissue previously granted options at a lower price or otherwise renegotiate the option package after the October 1987 market-wide downturn in stock prices.

5.3 Description of Executive Stock Options Contracts

There are two forms of stock based compensation that the model is applicable to — stock options and stock appreciation rights. Stock options are typically issued with an exercise price equal to the current market price of the stock on the day of the grant and with a duration of 10 years. The options are usually not exercisable for one or more years. The options are nontransferable and are forfeited if the executive voluntarily leaves the firm. Stock appreciation rights are subject to the same restrictions as options and are usually issued in tandem with stock options, such that either the option or the right can be exercised but not both. The stock appreciation right entitles the executive to the cash equivalent of the difference between the current market value of the stock and the

exercise price. Although, stock options are nontransferable, the executive is free to sell the stock purchased through the exercise of a stock option at any time after a six month holding period.[3]

While stock options and stock appreciation rights both use the increase in stock price over the exercise price as the performance measure, the exercise of options is economically different from the exercise of stock appreciation rights. Firstly, the executive is exposed to stock price changes on exercised options for at least six months.[4] Other differences are tax effects, transaction costs and stock price impacts.

Stock options and stock appreciation rights differ in their effect on taxes payable by both the company and the executive if the options are incentive stock options. The company is entitled to a tax deduction for the amount of cash received by the executive at the time of exercise of a stock appreciation right while the gain is classified as income to the executive for tax purposes. In contrast the entire gain for an incentive stock option is taxed as capital gains at the time of disposal of the stock. There are no taxes payable at exercise by the executive and no tax deduction to the company. However, only a relatively small number of options, for each individual, qualify as incentive stock options. If the options are nonqualified (all options which do not qualify for the favourable tax

[3]If the company uses previously unissued shares to satisfy the options, then those shares may be subject to the 2 year holding period under S.E.C. Rule 144. It is not clear how companies, in general, issue shares to satisfy exercised options since there is no comment in the proxy statement about whether stocks were issued or how those stocks were issued. It is assumed that the executive is not, in general, subject to the 2 year holding period.

[4]During the time period of the sample, the six month rule was in effect for stock options. Interpretation of the rule regarding stock options changed in 1989 such that the time from granting of the option was considered in the six month time period.

treatment granted incentive stock options), the gain at the date of exercise is taxed as income at the time of exercise while the remaining gain is taxed as capital gains at the time of disposition of the stock. Thus, nonqualified options result in the same tax consequences at the time of exercise as do stock appreciation rights. The tax differences between most stock options and stock appreciation rights are mainly due to the fact that the exercise of a stock option requires stock to be purchased and sold.

Stock appreciation rights and stock options also differ in transactions costs. Stock appreciation rights save transaction costs for the executive desiring cash. The exercise of a stock appreciation right entitles the executive to the full cash difference between the current market price and the exercise price. An executive with a stock option can exercise the option and subsequently sell the stock to gain cash. However, this transaction may require the executive to incur interest costs on money borrowed to finance the exercise and brokerage costs on the sale. The net cash received will be less than the cash from the exercise of a stock appreciation right with the same exercise price. On the other hand, stock options save transaction costs for the executive desiring to purchase stock. An executive with a stock appreciation right can use the cash from the exercise of the right to purchase stock. However, the executive would incur brokerage costs on the purchase. The exercise of a stock option is handled completely by the firm and there are no brokerage charges to the executive. For the most part stock appreciation rights are offered in tandem with stock options. Since the executive generally has the choice to exercise either a stock appreciation right or a stock option, the transaction costs do not

have to be incurred.

The requirement to actually purchase and ultimately sell the stock can lead to another difference between stock appreciation rights and stock options. In the finance literature, it has been shown that the sale of stock by insiders causes the price of the stock to fall. The exercise of a stock appreciation right may not have that effect. The existence of debt covenants on convertible debt may also impose a cost on the exercise of options rather than stock appreciation rights. If there are covenants restricting dilution of equity, these covenants may require payments to the holders of convertible debt when equity is diluted.[5] Hence, executives desiring cash from the exercise of their options have a further inducement to prefer stock appreciation rights.

For the purpose of this study stock options and stock appreciation rights are assumed to be equivalent.[6] The foremost reason is that companies do not always document the granting of, or the exercise of, stock appreciation rights separate from options. Also, since options and rights only differ after they are exercised and this study focuses on the granting of rights and options, the results should not be affected. Results could be affected if firms with executives who exercised options less than six months prior to the crash had incentive to compensate their executives for losses on their exercised options and that compensation took the form of increased stock option grants. The theory does not speak to this situation and it is not clear that this possible compensation would take

[5]There is evidence that firms issue new equity through underwriters rather than through rights issues despite the higher cost of underwriting new equity. This may be due to covenants, on convertible debt, that restrict dilution of equity.

[6]If a stock appreciation right is offered in tandem with an option, then only one option is counted.

the form of increased grants of options.

5.4 Statement of the Hypothesis

The theory predicts that stock options outstanding at the time of the crash will be renegotiated. This theory assumes that contracts are incomplete with respect to a crash. The reason for this focus is that the incomplete contract appears to approximate what exists in current business practise with respect to stock option plans. What form might a complete contract take? A complete contract would explicitly specify the schedule of payments to be made in the event of a crash. One example of a compensation contract that specifies this type of contingent payment is FMC Corp.'s contract with R. Mallott the Chairman and CEO. In 1988 executives who were granted options were also granted contingent performance awards which become payable in 1992 only if (i) the (Compensation) Committee determines that the 1988 options have little or no value, (ii) the participant has continued in the employment of the Company, and (iii) the performance objectives established by the Committee in 1988 are achieved. The contingent award is equal to the number of shares granted under option times approximately 75% of the then current value of the stock which for Mr. Mallott amounts to $975,000. Another example of an explicit event-contingent contract is a market-indexed stock option contract such as the one proposed by Mark Ubelhart [34]. He proposed that the option price be changed by a factor determined by the firm's beta and the overall movement in the market portfolio. Thus, a booming market would increase the exercise price of the option while a

crash would decrease the exercise price of the option. The executive would experience gains only when the stock price outperforms the market. It is interesting to note that contracts are generally not contingent on changes in the market. However, there could be an implicit agreement between the executive and the shareholders to implement a given contract after a crash. Such a contract would be implicitly complete with respect to a crash, yet it would be empirically indistinquishable from an incomplete contract with renegotiation. Since firms seldom write explicit contracts that are complete with respect to a crash and implicit contracts cannot be detected, the model developed earlier focuses on incomplete contracts.

Many firms grant stock options on a regular basis. In fact, executives often have layers of outstanding options at different exercise prices and exercise dates. This corresponds to the convexity of the optimal compensation in the model. What is at issue is the extent to which the crash prompted changes in grants beyond those which might be expected in the absence of a crash. The following is a general statement of the null and alternative hypotheses that are tested.

Hypothesis 1 (Null Hypothesis) *There was no change in executive stock option contracts as a consequence of the crash.*

Hypothesis 2 (Renegotiation Hypothesis) *Executive stock option contracts were renegotiated as a consequence of the crash.*

Renegotiations, predicted by the theory, can take two forms. Existing options could

be canceled and replacement options granted at a lower exercise price. Alternatively, new options could be granted at the lower exercise price without canceling any existing options. The latter renegotiation increases the convexity of the compensation function. Both forms or renegotiation result in increases in grants.

The renegotiated grants may appear as higher grants either in 1987 or in 1988. Firms tend to negotiate compensation arrangements at year end and all the firms in the sample have a December 31 year end. If a firm did not complete the negotiations until after January 1, 1988, the renegotiated grant appears with the 1988 grants.[7] If an option granted was renegotiated in 1987 or 1988, the firm may either report the transaction as such or report it implicitly as a grant and a cancellation in addition to other grants in 1987 or 1988. Since, in the end, the total grants for 1987 and 1988 include any renegotiated grants as well as options granted in the normal course of business, we expect to see higher grants in 1987 and/or 1988 than in years prior to the crash.

If a firm renegotiated in 1987, there will be higher than "normal" grants in 1987 but not in 1988. The reverse is true for firms who renegotiated in 1988. Thus, we have reason to expect some interdependencies between grants in 1987 and 1988 which may undermine tests comparing grants from one year with another year. Moreover, since renegotiation could take place in either 1987 or 1988, such individual year by year tests

[7]For example, BMC Industries states, "In January, 1988, the Board of Directors of the Company approved the replacement of options for 160,550 shares having an exercise price in excess of the then current market price with options having an exercise price equal to the then current market price". In another example, Arrow Electronics stated in their 1989 proxy that 85,000 of the 112,000 options granted in 1988 relate to 1987.

also lack power.[8] Testing whether the sum of 1987 and 1988 grants is drawn from a different distribution than the sum of 1985 and 1986 grants both increases the power of the tests as well as offsets the problem of serial correlation between 1987 and 1988.[9] Therefore, to operationalize the test for renegotiation, we compare the sum of 1987 and 1988 grants to the sum of 1985 and 1986 grants. The null hypothesis implies that these sums are drawn from the same distribution.[10] The renegotiation hypothesis implies that the sums of 1987 and 1988 grants are drawn from a distribution with a higher mean or median than the distribution from which the sums of 1985 and 1986 grants are drawn.

5.5 Sources of Potential Bias

One source of potential bias is that renegotiation of compensation may take the form of a switch from stock price based compensation to accounting number based compensation. The model does not address the issue of multiple performance measures. The only measure available in the model is the stock price. However, unlike stock based compensation, accounting number based compensation is unaffected by the crash. The relative importance of the two measures will depend on the incentive informativeness of each measure. Lambert and Larker [23] have shown that the relative noisiness of the two measures determines the weight each measure takes in the compensation function. The

[8]One could consider a vector test as a means of making full and simultaneous use of the data. Such a procedure is still potentially compromised by the interdependency just mentioned.

[9]The caveat to this is that there still could be serial correlation between 1986 and 1987.

[10]Certain tests relate only to the median whereas others consider a shift in distributions consistent with first-order stochastic dominance.

occurrence of the crash may also result in managers updating their prior probabilities of a crash. When the possibility of a crash has a higher likelihood of occurrence, accounting measures of performance may dominate stock based measures. Hence, the renegotiation could take the form of a move away from stock based measures to accounting measures. Since this would lead to reduced grants of options, this type of renegotiation introduces a conservative bias.

Another source of potential bias is the new U.S. tax laws that came into effect partially in 1987 and fully in 1988. These new tax rules effectively eliminate the difference between income and capital gains for tax purposes.[11] The tax changes only affect incentive stock options because incentive stock options are the only options for which the exercise gain can be taxed as capital gains if certain requirements are met.[12] Since the change eliminates the tax advantage of capital gains over income, incentive stock options will be less attractive after the tax change than before. This implies that firms may choose to renegotiate outstanding incentive stock options but grant fewer new incentive stock options than in previous years. Thus, any bias introduced by the tax change is conservative.

[11]From 1981 to 1986 the capital gains tax rate was 20% and the maximum personal income tax rate was 50%. 1987 is treated as a transitional year; the capital gains tax rate was 28% and the maximum personal income tax rate was 33%. Since 1988 the rates have been 28% for both capital gains and personal income. The maximum marginal corporate tax rate was 46% for 1983-87 and 34% after 1987.

[12]Gains from the exercise of stock appreciation rights are taxed as income at the time of exercise. For incentive stock options (qualified options), the entire gain is taxed as capital gains at the time of disposition of the stock purchased through the exercise of a stock option. For nonqualified options (all options which do not qualify for the favourable tax treatment granted incentive stock options), the gain at the date of exercise is taxed as income at the time of exercise while the remaining gain is taxed as capital gains at the time of disposition of the stock.

The entire gain for an incentive stock option ((disposal price - exercise price) $\times$ number of shares) is taxed at disposal at capital gains rates. There is no tax deduction to the company. In contrast, gains from nonqualified stock options are taxed at exercise and at disposal. The gain at exercise ((market price at time of exercise - exercise price) $\times$ number of shares exercised) is taxed as income to the executive and results in a tax deduction to the company at the time of exercise. The remaining gain (or loss) ((disposal price - market price at the time of exercise) $\times$ number of shares) is taxed at capital gains rates at the time of disposal. Post-1987 the tax rates for income or capital gains are the same. Hence, the only remaining advantage to the executive of an incentive stock option is the timing of tax payments. Despite the changes the incentive stock option continues not to result in any tax deduction to the company.

Since the tax changes result in incentive stock options becoming relatively less attractive after the change, there may be a switch in 1987 from incentive stock options to nonqualified options due to the tax change. Since the gain from incentive stock options is all taxed at the capital gains tax rate and that rate has increased, a firm may need to grant an increased number of incentive stock options to ensure the executive receives the same expected benefit. Thus, the tax effect on incentive stock options could create a potential bias in the results of the following tests. For nonqualified options and stock appreciation rights, the tax changes do not imply that increases in compensation are warranted. Quite the contrary, because the tax rates are generally lower, the executives will have more disposable income from the same compensation as before. Hence,

if the reservation utility has not changed, then the reservation utility can be met with a lower cost compensation contract. Any tax effect on nonqualified stock options and stock appreciation rights would serve to reduce the probability of finding any results.

5.6 Sample Selection

U.S. firms[13] must provide a reconciliation of the options outstanding through footnote disclosure in the financial statements. The footnotes were collected from Compustat's Corporate Text on CDROM.

As Compustat Corporate Text is the source of the footnote data, all firms must be on Compustat. There are 2486 firms on the Compustat tapes for 1987. Of these, 2311 are listed on the New York Stock Exchange or the American Stock Exchange and are, therefore, also on the tapes from the Center for Research in Security Prices (CRSP). The firms were also required to have a December 31 fiscal year end throughout the sample period.[14] This ensures that grant data is comparable from one period to the next. The December 31 fiscal year-end is the most common year-end and, therefore, will still result in a large sample while ensuring all firms have a common year-end. Because most firms analyze and potentially renegotiate compensation around their year-end, December 31 year-end firms are most likely to be reacting to the crash at the time of 1988 compensation

[13]American data is used because detailed information about stock options is available for American firms whereas Canadian firms are not required to report compensation information in as much detail.

[14]Firms were required to have no change in fiscal year end from 1982 through to the end of the sample period. While requiring the same fiscal year end back to 1982 may be overly restrictive, few firms are lost from the sample as fiscal year end changes are rare.

negotiations. Of the 2311 firms, 1183 of these have a December 31 year-end.

The Dialog Database was utilized to identify firms with existing stock option plans. The Dialog Database has a category called corporate exhibits which contains short references about types of compensation plans, law suits, pension plans and other corporate information mentioned in the 10K. Thus, a word search of corporate exhibits for all stock-based compensation plans will identify a potential population of firms with stock option plans. There were 610 firms with stock-based compensation plans, December 31 fiscal year ends, Compustat data, and listed on either the New York Stock Exchange or the American Stock Exchange.

Firms were further required to have complete CRSP returns for the sample time period.[15] In order to keep a common set of taxation rules for the management, 8 firms with head offices outside of the United States were excluded. A total of 488 firms survived all of the above screens.

After looking at the data, three data restrictions were imposed. First, the firm must have outstanding stock options and/or stock appreciation rights at the end of 1986, i.e. the firm must have a compensation plan that depends on the stock price before the market crash.[16] The model speaks to plans with existing stock options and not to any

[15]Complete CRSP returns were required for 1984 through 1988. The 1984 data restriction was included in anticipation of using the Black-Scholes [3] options pricing model to determine the value of options granted in 1985. However, this was abandoned. A small number of firms are lost due to this extra restriction.

[16]This exludes any firms that instituted stock option plans in 1987. A firm could have granted stock options early in 1987 and then renegotiated those options after the crash. Conversely, a firm could have first granted options in November or December 1987. Since this study only applies to the first type of firm and these two firms are indistinguishable empirically, all firms with a first stock option grant in

other compensation plans. Secondly, the firm must also provide complete and comparable information on grants. For any firms that merged with other firms or acquired other firms during the sample, only grant data that could be reconciled before and after the merger or acquisition was included. Finally, a firm must provide at least four years of grant data beginning in either 1985 or 1986.[17] Four years is needed to have at least two noncrash years to compare to 1987 and 1988. The final sample consists of 189 firms with grant data for 1985 through 1988 and 225 firms with grant data for 1986 through 1989. There are 135 firms in both of these sets.

5.7 Data Description

Tables 5.1 and 5.2 present some descriptive statistics of the grant data used in the tests that follow. As can be seen in Table 5.1 the grants are large in terms of the dollar value of stock subject to option. An option is the right to purchase one share of common stock at the exercise price. Grant size is defined as the number of options granted times the exercise price of the option. Since the exercise price is, in most cases, the market price of the stock on the day of the option grant, this represents the dollar value of the stock subject to these options. This is not the value of the option since the executive

1987 are excluded.

[17]Firms with data from 1986 through 1989 were included in the pooled analyses in order to increase sample size. While the predictions from the theory are clear when the comparison is between the years prior to the crash and the crash years, the predictions are not so clear for comparisons between 1987 or 1988 and 1989 or other years subsequent to the crash. The occurrence of the crash could cause firms and executives to update their prior probability assessments of the future possibility of a crash which may affect the grant process. Thus, the 1989 data is also used to provide some descriptive assessment of the possibility that the crash induced a shift away from stock-based performance measures.

must pay the exercise price to buy the shares. However, this does give an indication of the size of option grants being studied. The average grant size ranges from $7,790,000 to $13,500,000 while the median grant size ranges from $2,640,000 to $6,400,000. This dollar value of grants is presented for descriptive purposes only. All of the following tests utilize the number of options granted.

While the average and median grant size of 1989 grants is larger than any other year, the number of options granted in 1989 is not higher than the numbers granted in 1987 and 1988. This can be seen by inspection of Table 5.2. In addition, it is shown in Appendix B that the number of grants in 1989 is not significantly higher than the number of grants in 1987 or 1988.

Note also that the average and median grant size and scaled grants for both 1987 and 1988 are larger than the comparable numbers for 1985 and 1986. The following tests show that these differences are significant.

Table 5.1: Grant Size Description

Year	Total Number of Firms	Grant Size* in Thousands of Dollars			
		Average	Median	Range	
				Minimum	Maximum
1985	124	$ 7,790	$ 2,640	$ 30	$ 87,910
1986	201	10,070	4,100	6	156,000
1987	203	12,400	4,400	27	197,000
1988	186	11,600	4,500	4	120,000
1989	159	13,500	6,400	0	127,000

* Grant size is the number of shares granted subject to option times the exercise price of the option.

Table 5.2: Scaled Grant Description

Year	Grants Scaled by Outstanding Common Shares		Grants Scaled by Outstanding Options	
	Average	Median	Average	Median
1985	0.0096	0.00520	0.4516	0.2838
1986	0.0109	0.00620	0.4823	0.2602
1987	0.0154	0.00750	0.5276	0.2940
1988	0.0124	0.00750	0.4874	0.2857
1989	0.0113	0.00720	0.4389	0.2783

5.8 Tests

The hypothesis that the sums of 1987 and 1988 grants are drawn from a distribution with a higher median or mean than the distribution from which the sums of 1985 and 1986 grants are drawn can be tested using cross-sectional data (pooling across firms) or by using the firm as its own control (within firm tests).[18] There are three potential problems with aggregating grant data across firms: firm size differences, stock price differences, and differences in the relative importance of stock options to overall compensation. The size differential implies that when firms of different sizes have the same stock price the numbers of options granted are not comparable. It is well known that executives of larger firms have higher total compensation than executives in smaller firms. In addition, more executives may be included in an option plan in a large firm than in a small firm. The price differential occurs when firms of the same size have different stock prices. Hence,

[18]As discussed earlier, the two year sum test is more powerful than individual year by year tests. For the interested reader, a description of year by year tests is provided in Appendix B.

more options may be needed to provide the same level of compensation. Both size and price differences are eliminated if we compare the number options granted as a percent of outstanding common shares. The difference in relative importance of options to overall compensation, however, requires another approach. This is because, for a firm in which compensation from options is a high proportion of total compensation, the number of options granted will be larger relative to outstanding common shares than for a firm in which compensation from options is a small proportion of total compensation. On the other hand, a firm in which compensation from options is a high proportion of total compensation will also have a large number of options outstanding as well as large numbers of options granted, whereas a firm with less of a reliance on options will have relatively smaller numbers of options granted and outstanding. Thus, grants as a percent of outstanding options should be free of the relative importance of options to overall

compensation, making it a more suitable measure.[19,20] Hence, there are three data sets to consider — grants, grants as a percent of outstanding common shares, and grants as a percent of outstanding options.

In all, four nonparametric tests are performed — a sign test, a median test, a Wilcoxon-Mann-Whitney test and a Kolmogorov-Smirnov test.[21] Nonparametric tests were chosen because these tests do not require that the data be normally distributed. Because little is known about the process generating grants or about the resulting distribution of grants, it is desirable to have tests that do not rely on any particular distribution. Since the sample is not randomly selected, inferences from these tests are made relative to the population tested, i.e. inferences need not extend beyond the firms

[19]The number of options granted in 1985 or 1986 are divided by the balance of options outstanding as at January 1, 1985 whereas the number of options granted in 1987 or 1988 are divided by the balance of options outstanding as at January 1, 1987. An alternative would be to use a common divisor, the balance of options outstanding as at January 1, 1985. The number of options outstanding for year x is affected by the size of grants in year $x-1$ as well as the number of options exercised and canceled. Thus, using two different divisors to scale by outstanding options can introduce correlation into the ratios. On the other hand, using a common divisor to scale by outstanding options for both sums does not allow for changing circumstances from one year to the next. There is a general trend toward more firms using stock option plans in their compensation arrangements. If firms are also increasing the number of employees covered by these plans over the sample time period, then the number of options outstanding, for a given firm, could be increasing over time. For the firms in the sample, both the average and the median number of options outstanding in January 1985 is less than that in 1987 (an average of 1,151,820 in 1985 versus 1,307,699 in 1987 and a median of 576,929 in 1985 versus 697,550 in 1987). Using two different divisors introduces a conservative bias to the tests since the sums of 1987 and 1988 grants are, in general, divided by larger numbers than the sums of 1985 and 186 grants. Hence, the reported results for grants divided by outstanding options are for ratios using two different divisors. However, results for ratios using a common divisor are included in footnotes.

[20]It could be argued that scaling by the outstanding options dominates scaling by outstanding common shares. However, at a minimum, using both tests allows for some inference regarding whether the relative importance of options changed pursuant to the crash.

[21]Other nonparametric tests were performed. However, the other tests are either less powerful or are subject to specification concerns. The results of such tests are presented in Appendix B or in footnotes where applicable.

on which the tests were performed.[22] However, the sample consists of all firms with stock option plans for which sufficient data is available and, therefore, the results provide inferences of interest to this study.

5.8.1 Within Firm Data — Sign Test

A sign test is based on regarding each observation as an independent trial from identical Bernoulli distributions. In this case the observations are the sign of the difference, for each firm, between the sum of 1987 and 1988 grants and the sum of 1985 and 1986 grants. Under the null hypothesis there is an equal likelihood that grants increase or decrease one year over another. Thus, the probability that the grant for a firm in 1986 is greater than the grant for the same firm in 1985 is just one-half. As a crude check of this assumption, the number of firms with increases in grants in 1986 over grants in 1985 was checked. For 189 firms with grant data in both 1985 and 1986, 95 of those firms have larger grants in 1986 than in 1985, 87 have larger grants in 1985 than in 1986 and 7 have equal grants in both years. These numbers do not significantly differ from the numbers expected with a probability of .5 (see the pair 1985 with 1986 in Table B.1 in Appendix B).

The test statistic has a Binomial distribution for small samples. For samples larger

[22]Note that, in particular, the results need not extend to firms outside the universe of firms that are listed on either the New York Stock Exchange or the American Stock Exchange, have complete returns data, and have a December 31 year end since these firms were not randomly selected.

than 35 the normal approximation to the binomial distribution is used. This approximation becomes better when the correction for continuity is made. The correction reduces the difference between the observed number and the expected number by .5. Ties are dropped from the set. Hence, N is the number of paired observations with a nonzero difference. The distribution of the test statistic z is approximately standard normal.

$$z = \frac{(x \pm .5) - NP}{\sqrt{NPQ}}$$

(where x is the number of pluses, N is the total number of data points that are either plus or minus but not equal, P is the probability of a data point being a plus, and $Q = 1 - P$.)

In the sum of years sign test, the paired observations are the sum of options granted in 1985 and 1986 and the sum of options granted in 1987 and 1988. A comparison of the sum of options granted in 1986 and 1989 and the sum of options granted in 1987 and 1988 is also presented. However, this comparison is tenuous given the possibility that the crash not only induced renegotiations, but a structural change in compensation arrangements as well. The test results appear in Table 5.3. In both tests, there are significantly more firms with a higher sum of 1987 and 1988 grants than the sum of the off-crash year grants. This supports the hypothesis that firms renegotiated grants after the crash.

Table 5.3: Sign Test Comparing Two-year Sums of Grants

Difference of (1987 + 1988) minus	Number of firms with Negative Differences	Positive Differences	Total* Number of Firms	Expected Number of Positives	z (significance)
(1985 + 1986)	66	121	187	93.5	3.95 (.00005)
(1986 + 1989)	94	127	221	110.5	2.15 (.018)

* Firms in which the sum of grants are equal are excluded.

5.8.2 Data Pooled Across Firms — Median Tests

The median test is a procedure for testing whether two independent groups differ in central tendency. The two independent groups are pools of the two year sum of scaled grants where the two years are either 1985 and 1986 or 1987 and 1988. The tests are conducted for both grants scaled by outstanding common shares and grants scaled by outstanding options.[23]

The two groups to be compared are combined to determine the combined median. The counts of number of firms in which the two year sum of scaled grants is higher or lower than the combined median are cast into a 2 by 2 table as shown in Table 5.4. If both groups have the same median, we would expect each group to have about the same number of firms with grants above the combined median as below. The frequencies A and C are expected to be about equal and frequencies B and D are expected to be about

[23]The 1985 and 1986 grants are first divided by outstanding options as of January 1, 1985 while the 1987 and 1988 grants are divided by outstanding options as of January 1, 1987. For the two year sum, the ratio for 1985 is added to the ratio for 1986 for the same firm and likewise for the sum of the 1987 ratio and the 1988 ratio. The two year ratio sums are then pooled with the other firms' two year ratio sums. For grants scaled by outstanding common shares, grants are divided by the total of common shares outstanding at December 31 1989 (or 1988 if data is only available until 1988). Grant data was restated in terms of the common stock in the final year of data.

equal.

Table 5.4: Example of Median Test Table

Group	Number of scores		Combined
	Below Median	Above Median	
1	A	C	m
2	B	D	n
Total	A + B	C + D	N = m + n

The sampling distribution under the null hypothesis that the medians are equal is the hypergeometric distribution

$$P[C, D] = \frac{\binom{m}{C}\binom{n}{D}}{\binom{m+n}{C+D}}$$

For a sample where $N = m + n$ is larger than 20, the following χ^2 statistic corrected for continuity is used:

$$\chi^2 = \frac{N(|AD - BC| - N/2)^2}{(A+B)(C+D)(A+C)(B+D)},$$

with one degree of freedom. If several scores fall right on the combined median, then the groups are dichotomized into those scores that exceed the median and those that do not exceed the median.

Table 5.5 presents the test results for the median tests on grants scaled by outstanding common shares and grants scaled by outstanding options.[24] For grants scaled by

[24]The number of firms in the sample differ for the sum of 1985 and 1986 scaled grants and for the sum

outstanding common shares there are significantly more firms with the sum of 1987 and 1988 grants that are higher than the overall median two year grant. This implies that the median sum of 1987 and 1988 scaled grants is significantly higher than the median sum of 1985 and 1986 scaled grants. However, for grants scaled by outstanding options, there is no significant difference between the two year sums.[25] Recall that scaling grants by outstanding common shares utilizes a single common divisor while scaling by outstanding options utilizes two different divisors and that the 1987 divisor is, in general, larger than the 1985 divisor. The differing results for the two scaling methods could indicate that the sums of 1987 and 1988 grants are significantly larger than the sums of 1985 and 1986 grants but the difference is not significant when the 1987 and 1988 grants are deflated by the larger 1987 outstanding option balances.

5.8.3 Data Pooled Across Firms — Wilcoxon-Mann-Whitney Tests

The Wilcoxon-Mann-Whitney test tests whether two independent groups have been drawn from the same population. To apply the test, the observations from both groups are combined and a rank, in order of increasing size, is assigned to each observation. Observations that are tied are assigned the average rank of those tied observations. The sampling distribution of sum of ranks for the smallest group, W_x, rapidly approaches the

of 1987 and 1988 scaled grants because an all available data rule was used in these tests. Thus, only the sum of 1987 and 1988 scaled grants is included for firms with data from 1986 to 1988 whereas both two year sums are included for firms with data from 1985 to 1988.

[25]Recall that when grants are scaled by outstanding options, two different divisors are used for the different two year sums. When a common divisor is used to scale grants by outstanding options, there is a significant difference, at the ten percent level, in the predicted direction, between the sum of 1985 and 1986 scaled grants and the sum of 1987 and 1988 scaled grants.

Table 5.5: Median Test Comparing Two-year Sums of Grants

Grants scaled by Outstanding Common Shares

Two Year Sum	Number of Firms where the Two Year Sum is		Total Number of Firms	χ^2 (significance)
	Below Median	Above Median		
1985 + 1986	107	80	187	
1987 + 1988	125	151	276	
Total	232	231	463	6.35 (.02)

Grants scaled by Outstanding Options

Two Year Sum	Number of Firms where the Two Year Sum is		Total Number of Firms	χ^2 (significance)
	Below Median	Above Median		
1985 + 1986	90	83	173	
1987 + 1988	132	139	271	
Total	222	222	444	.34 (.60)

normal distribution as the two groups increase in size. The statistic, z, is asymptotically unit normally distributed.

$$z = \frac{W_x \pm .5 - m(N+1)/2}{\sqrt{[mn/N(N-1)]\left[(N^3 - N)/12 - \sum_{j=1}^{q}(t_j^3 - t_j)/12\right]}}$$

with m and n equal to the number of observations in the smallest and largest group respectively, $N = m + n$, q = the total number of ties, and t_j equal to the number of observations that are tied for each tie j. The value .5 is added if we wish to find probabilities in the left tail of the distribution and $-.5$ is added if we wish to find probabilities in the right tail of the distribution.

Table 5.6 presents the results of the Wilcoxon-Mann-Whitney tests for grants scaled

Table 5.6: Wilcoxon-Mann-Whitney Tests

For Grants Scaled by Outstanding	Two Year Sum				Adjustment For Ties
	1985 + 1986		1987 + 1988		
	Sum of Ranks	Number of Firms	Sum of Ranks	Number of Firms	
Options	29678.5	173	34224.5	184	144.0
	z^* (significance)		-1.32 (0.093)		
Common Shares	32345.5	187	37779.5	187	182.5
	z^* (significance)		-2.60 (0.005)		

* A negative difference implies that the sum of ranks for the sum of 1985 and 1986 grants is smaller than the sum of ranks for the sum of 1987 and 1988 grants.

by outstanding options and outstanding common shares. For both tests the sums of 1985 and 1986 grants are significantly smaller, at the ten percent level, than the sums of 1987 and 1988 grants.[26] Note that these results are stronger than the results of the median test in which there was no significant difference between the sums of 1985 and 1986 and the sums of 1987 and 1988 grants scaled by outstanding options. This difference in results is expected because the Wilcoxon-Mann-Whitney test is more powerful than the median test. The gain in statistical power is due to using both the sign of the difference from the median and a measure of the magnitude of that difference whereas the median test only employs the sign of the difference.

[26]Results for grants scaled by outstanding options when a single divisor is used are significant, in the predicted direction, at the one percent level.

5.8.4 Data Pooled Across Firms — Kolmogorov-Smirnov Tests

The Kolmogorov-Smirnov two-sample test determines whether the values of one sample are stochastically larger than the values of another independent sample. The test is sensitive to any kind of difference in the distributions from which the the two samples are drawn — differences in central tendency, in dispersion, in skewness, etc. To apply the test, the cumulative frequency distributions are determined for each sample using the same intervals for both distributions. The differences between the cumulative distributions are calculated for each interval. The test statistic

$$\chi^2 = \frac{4D^2(n_1 n_2)}{n_1 + n_2}$$

(where D is the largest difference in the predicted direction and n_1 and n_2 are the two sample sizes) is approximated by the chi-square distribution with two degrees of freedom.

The renegotiation hypothesis implies that the sums of 1987 and 1988 scaled grants are stochastically larger than the sums of 1985 and 1986 scaled grants. Tables 5.7 and 5.8 present the Kolmogorov-Smirnov test results for grants scaled by outstanding common shares and grants scaled by outstanding options, respectively.[27] The results are similar to the results of the median test with the sums of 1987 and 1988 grants significantly greater than the sums of 1985 and 1986 grants when the grants are scaled by outstanding common shares and no significant difference when the grants are scaled by outstanding

[27]As was the case for the median tests, an all available data rule was used in these tests. Firms with data from 1986 to 1988 are included as well as firms with data from 1985 to 1988. Thus, the number of firms in the 1985 plus 1986 group differs from the number in the 1987 plus 1988 group.

Table 5.7: Kolmogorov-Smirnov Tests Comparing Two-year Sums of Grants

Grants scaled by Outstanding Common Shares

Intervals of Scaled Grants	Cumulative Distributions				Difference(*) Between the Distributions
	Sum 85 + 86		Sum 87 + 88		
	Count	f(x)	Count	f(x)	
≤ 0	7	.0374	8	.0290	.0084
$\leq .0022$	20	.1070	15	.0543	.0527
$\leq .0043$	36	.1925	28	.1014	.0911
$\leq .0073$	55	.2941	53	.1920	.1021
$\leq .0098$	72	.3850	76	.2754	.1096
$\leq .0118$	90	.4813	94	.3406	.1407
$\leq .0154$	108	.5775	126	.4565	.1210
$\leq .0195$	126	.6738	149	.5399	.1339
$\leq .0268$	144	.7701	188	.6812	.0889
$\leq .038$	162	.8663	224	.8116	.0547
$\leq .0551$	180	.9626	248	.8986	.0640
$\leq .2156$	187	1.000	274	.9928	.0072
$\leq .50$	187	1.000	276	1.000	0
χ^2(significance)					8.827 (.02)

* A positive difference implies that the cumulative distribution of the sum of 1985 and 1986 is larger than that of the sum of 1987 and 1988.

options.[28] The positive differences throughout for grants scaled by outstanding common shares indicate that the distribution of the sum of 1987 and 1988 scaled grants first-order stochastic dominate the distribution of the sum of 1985 and 1986 scaled grants.

[28]When a single divisor is used to scale grants by outstanding options, the Kolmogorov-Smirnov tests indicate a difference, that is significant at the five percent level, between the sum of 1985 and 1986 grants and the sum of 1987 and 1988 grants.

Table 5.8: Kolmogorov-Smirnov Tests Comparing Two-year Sums of Grants

Grants scaled by Outstanding Options

Intervals of Scaled Grants	Cumulative Distributions				Difference(*) Between the Distributions
	Sum 85 + 86		Sum 87 + 88		
	Count	f(x)	Count	f(x)	
≤ 0	6	.0347	8	.0295	.0052
$\leq .05$	13	.0751	9	.0332	.0419
$\leq .10$	17	.0983	18	.0664	.0319
$\leq .18$	24	.1387	20	.0738	.0649
$\leq .26$	32	.1850	36	.1328	.0522
$\leq .33$	41	.2370	49	.1808	.0562
$\leq .4$	52	.3006	69	.2546	.0460
$\leq .5$	71	.4104	99	.3653	.0451
$\leq .65$	94	.5434	134	.4945	.0489
$\leq .80$	118	.6821	170	.6273	.0548
$\leq .9$	125	.7225	183	.6753	.0472
≤ 1.0	130	.7514	190	.7011	.0503
≤ 1.5	149	.8613	235	.8672	-.0059
≤ 2.0	158	.9133	252	.9299	-.0166
≤ 3.0	167	.9653	259	.9557	.0096
Total	173	1.000	251	1.000	0
χ^2(significance)					1.779 (.50)

* A positive difference implies that the cumulative distribution of the sum of 1985 and 1986 is larger than that of the sum of 1987 and 1988.

5.8.5 Tests on Groupings by Crash Week Returns

It follows from the theory that firms are more likely to renegotiate grants if the crash had a larger rather than smaller impact on their future stock values. To test this aspect, firms were ranked according to the magnitude of their stock return over the crash week.[29] The firms were then grouped into thirds. The highest third is the group whose stock returns over the crash week were highest and, hence, the impact of the crash was lowest. We expect that these firms are less likely to renegotiate option plans than the firms whose stock returns over the crash week were the lowest.

For each grouping of crash week returns, two year sums of grants are compared, firm by firm, using the sign test discussed earlier. Table 5.9 presents the results. For all three groups, the sum of 1987 and 1988 grants is significantly greater, at the five percent level, than the sum of 1985 and 1986 grants. As predicted, the results are strongest for the lowest third and progressively weaker for the middle and highest thirds.

Table 5.10 shows, for each grouping of crash week returns, the results of the median tests on pooled scaled grants. For grants scaled by outstanding options, the results are as expected. For firms with the lowest crash week returns there is a significant difference in the two sets of two year sums of scaled grants. For all other firms, there is no significant difference.[30] However, the results are not as predicted for grants scaled

[29]Crash week returns are defined as the closing price October 23 minus closing price October 16 divided by closing price October 16. The magnitude of the weekly returns range from -.405 to +.125 with a median of -.182.

[30]When a single divisor is used to scale grants by outstanding options, the differences are in the predicted direction but no difference is significant.

by outstanding common shares. The only group for which the sum of 1987 and 1988 scaled grants is significantly larger than the sum of 1985 and 1986 scaled grants is the middle third of crash week returns. This difference in results could be due to a change in the relative importance of options in compensation. If there was an increase in the relative importance of options in compensation, then the number of options granted would be higher in 1987 and 1988 even if there was no reason to renegotiate. A scaling, such as outstanding common shares, that does not take into account the general increase in options outstanding would have more noise in the observations than a scaling, such as outstanding options, utilizing two different divisors. Hence, observations scaled by outstanding options would be more likely to indicate whether the increase in grants was due to the effects of the crash on the stock prices than observations scaled by outstanding common shares.

The results of the Wilcoxon-Mann-Whitney tests for each grouping of crash week returns are presented in Table 5.11. The results are as predicted for both types of scaling. For firms with the lowest crash week returns, the sums of 1987 and 1988 scaled grants are higher, at less than a five percent level of significance, than sums of 1985 and 1986 scaled grants.[31] There is also a difference, at less than a ten percent level of significance, in the two year sums of grants scaled by outstanding common shares for firms with crash week returns in the middle third. For all other firms, there is no significant difference

[31]When a single divisor is used to scale grants by outstanding options, the results are slightly stronger, in the same direction, as for grants scaled by outstanding options where two different divisors are used.

between the two sets of two year sum of scaled grants. This supports the prediction that firms are more likely to renegotiate options if the crash has a larger rather than smaller effect on their stock price.

The results of the Kolmogorov-Smirnov tests for each grouping of crash week returns are presented in Table 5.12. For grants scaled by outstanding options, the results are as predicted — the difference between the two year sums is significant only for firms with the lowest crash week returns.[32] For grants scaled by outstanding common shares, only for firms whose crash week returns are in the middle third is the difference between two year sums significant. These results are similar to the results of the median test above.

Thus, while the results of the sign test and the Wilcoxon-Mann-Whitney test are supportive of the hypothesis that firms are more likely to renegotiate if the crash had a larger rather than smaller impact on their future stock values, the results of the median and Kolmogorov-Smirnov tests are, generally, mixed.[33] Note that the sign test is the strongest of the three tests because grants are compared within the firm, i.e. the firm is used as its own control. The tests on pools of data may be contaminated by cross-sectional differences that are not controlled for in the measure employed. Partitioning the data set also reduces the degrees of freedom and, hence, the power of the test.

[32]When a single divisor is used to scale grants by outstanding options, the results are in the wrong direction; the results are a little stronger for firms with the highest crash week returns than for firms with the lowest crash week returns.

[33]Another test fails to find a relationship between the crash week stock returns and increases in post-crash grants. The Spearman rank-order correlation between stock returns over the crash week and the percentage increase in the sum of 1987 and 1988 grants over the sum of 1985 and 1986 grants for the same firm is -0.09 which is not significantly different from zero.

Table 5.9: Sign Test Comparing Two-year Sums of Grants

Rank of Stock Returns over Crash Week	Number of Firms where 1987 plus 1988 Grants minus 1985 plus 1986 Grants is Negative	is Positive	Total* Number of Firms	Expected Number of Positives	z (significance)
Lowest	18	44	62	31	3.18 (.0007)
Middle	23	39	62	31	1.91 (.028)
Highest	25	38	63	31.5	1.64 (.05)

* Firms in which the sum of grants are equal are excluded.

Table 5.10: Median Tests — Grouped by Third of Crash Week Returns

Grants per Outstanding Common Shares

Rank of Stock Returns over Crash Week	Two Year Sum of Grants	Number of Firms Below Median	Number of Firms Above Median	Total Number of Firms	χ^2 (significance)
Lowest	1985 + 1986	34	27	61	
	1987 + 1988	42	49	91	
Total		76	76	152	0.98 (.40)
Middle	1985 + 1986	36	23	59	
	1987 + 1988	39	52	91	
Total		75	75	150	4.02 (.05)
Highest	1985 + 1986	35	30	65	
	1987 + 1988	44	49	93	
Total		79	79	158	0.42 (.50)

Grants divided by Outstanding Options

Rank of Stock Returns over Crash Week	Two Year Sum of Grants	Number of Firms Below Median	Number of Firms Above Median	Total Number of Firms	χ^2 (significance)
Lowest	1985 + 1986	35	23	58	
	1987 + 1988	39	51	90	
Total		74	74	148	3.43 (.06)
Middle	1985 + 1986	25	30	55	
	1987 + 1988	47	42	89	
Total		72	72	144	0.47 (.50)
Highest	1985 + 1986	29	29	58	
	1987 + 1988	46	46	92	
Total		75	75	150	0.03 (.90)

Table 5.11: Wilcoxon-Mann-Whitney Tests — Grouped by Third of Crash Week Returns

Grants Scaled by Outstanding Common Shares

Rank of Stock Returns over Crash Week	Two Year Sum 1985 + 1986 Sum of Ranks	Number of Firms	1987 + 1988 Sum of Ranks	Number of Firms	Adjustment For Ties
Lowest	3472.0	62	4278.0	62	10.0
	z^* (significance)		-2.01 (0.022)		
Middle	3582.5	62	4167.5	62	5.0
	z^* (significance)		-1.46 (0.072)		
Highest	3798.0	63	4203.0	63	5.0
	z^* (significance)		-0.99 (0.161)		

Grants Scaled by Outstanding Options

Rank of Stock Returns over Crash Week	Two Year Sum 1985 + 1986 Sum of Ranks	Number of Firms	1987 + 1988 Sum of Ranks	Number of Firms	Adjustment For Ties
Lowest	3238.5	60	4142.5	61	5.0
	z^* (significance)		-2.18 (0.015)		
Middle	3432.5	57	3588.5	61	5.0
	z^* (significance)		-0.22 (0.413)		
Highest	3320.0	56	3701.0	62	5.0
	z^* (significance)		-1.32 (0.093)		

* A negative difference implies that the sum of ranks for the sum of 1985 and 1986 grants is smaller than the sum of ranks for the sum of 1987 and 1988 grants.

Table 5.12: Kolmogorov-Smirnov Tests — Grouped by Third of Crash Week Returns

Grants per Outstanding Common Shares

Rank of Stock Returns over Crash Week	Largest Difference in Cumulative Distributions(*)	χ^2 (significance)
Lowest	0.1653	4.01 (.15)
Middle	0.2222	7.14 (.05)
Highest	0.1075	1.78 (.50)

Grants divided by Outstanding Options

Rank of Stock Returns over Crash Week	Largest Difference in Cumulative Distributions(*)	χ^2 (significance)
Lowest	0.2277	7.39 (.03)
Middle	0.0807	0.89 (.60)
Highest	0.1009	1.46 (.50)

* A positive difference implies that the cumulative distribution of the sum of 1985 and 1986 is larger than that of the sum of 1987 and 1988.

5.8.6 Summary of Test Results

The results for all four classes of tests comparing the sum of 1987 and 1988 to the sum of 1985 and 1986 support the conclusion that grants in 1987 and 1988 are significantly greater than grants in 1985 and 1986, implying grants appear to have been renegotiated after the crash. The results are stronger when the full set of firms is used than when firms are partitioned according to the magnitude of their stock return over the crash week. The results are also stronger for the within firm tests than for the pooled cross-sectional tests. The results of the pooled cross-sectional tests on the full data set are stronger when grants are scaled by outstanding common shares than when grants are scaled by outstanding options. However, the latter scale is arguably the better scale since it attempts to control for the relative importance of options in compensation while the former scale does not. Thus, the difference in results between scaling by outstanding options and scaling by outstanding common shares indicates that the relative importance of options in compensation may have changed along with renegotiation per se. If, in the wake of the crash, there was a uniform reduction in the relative importance of options in compensation, then tests using data scaled by outstanding options would be weaker than tests using data scaled by outstanding common shares. However, as seen Table B.2 in Appendix B, there is some evidence to indicate that there has been no reduction in the relative importance of options in compensation. Thus, while the preponderance of evidence favours the hypothesis that the crash precipitated renegotiations of stock-based

compensation arrangements, there is need for caution in stating claims to this effect.

A natural extension to the empirical analysis would be to employ a cross-sectional regression to determine whether other factors may have prompted larger grants in 1987 and 1988. Each observation is the effect of an individual downshift. All these events occurred at the same time but the impact of the crash differed across firms. The dependant variable could be the percentage increase in the sum of 1987 and 1988 grants over the sum of 1985 and 1986 grants. The explanatory variables could include the impact of the crash, the existence of alternative incentive mechanisms and the relative importance of stock options to overall compensation. Clearly we would have to develop proxies for the explanatory variables. The magnitude of the stock return over the crash week is one proxy for the impact of the crash. This effectively assumes that the magnitude of the crash overwhelmed any firm specific events. An alternative proxy would be the firm's beta. Firms with high betas should be more affected by the crash than firms with low betas. The advantage of using beta is that firm specific events do not come into the measure. Since firms with a higher impact from the crash are more likely to renegotiate their stock option grants after the crash, we would expect to see a negative coefficient if the magnitude of the stock return was used as the proxy. If the beta was used as the proxy, we would expect to see a positive coefficient. If executives have sufficient wealth tied up in company stock, they still have incentive to work hard after the crash. Likewise if the executives have alternative forms of incentive compensation that are not affected by the crash, they may still have incentive to work hard. The percentage of inside stock

ownership could proxy for the proportion of wealth that executives have tied up in company stock. We would expect to see a negative coefficient on the percentage of inside stock ownership. Firms whose executives have small stockholdings have more need to renegotiate stock options than firms whose executives have large stockholdings. Data on other forms of incentive compensation is not readily available. Thus, the relative importance of stock options to overall compensation is not easily determined. The advantage of such a regression approach would be to gain some further refinement in tests of the theory by more effectively controlling for unwanted influences such as a differing impact of the crash across firms and a differing need across firms to renegotiate grants.

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Appendix A

Proofs

A.1 Proof of Proposition 5

The proof is by contradiction. First assume that $a_1 > a_0$. The action a_0 solves

$$\int [z - C(z|a_0, 0)]\, f_a(z|a_0)\, dz \;=\; 0$$

By assumption second order conditions are satisfied. Thus, for any action greater than a_0, the marginal net profit, for that action, must be negative, i.e.,

$$\begin{aligned} & \int [z - C(z|a_1, 1)]\, f_a(z|a_1)\, dz \;<\; 0 \\ \Longrightarrow \qquad & \int C(z|a_1, 1)\, f_a(z|a_1)\, dz \;>\; \int z\, f_a(z|a_1)\, dz \end{aligned}$$

However, a_1 satisfies

$$\begin{aligned} & \int [\alpha z - C(z|a_1, 1)]\, f_a(z|a_1)\, dz \;=\; 0 \\ \Longrightarrow \qquad & \int C(z|a_1, 1)\, f_a(z|a_1)\, dz \;=\; \int \alpha z\, f_a(z|a_1)\, dz \end{aligned}$$

By the assumption of the Monotone Likelihood Ratio Condition and the Spanning Condition, the compensation function is increasing in output. With higher efforts first-order stochastic dominating lower efforts and a function increasing in output, the left hand

side of the above equation is positive. Hence, the right hand side must also be positive. This implies

$$\begin{aligned}\int \alpha z\, f_a(z|a_1)\, dz &< \int z\, f_a(z|a_1)\, dz \\ &< \int C(z|a_1, 0)\, f_a(z|a_1)\, dz\end{aligned}$$

Remember that the minimum cost contracts in Problem A' are the same for a given action, i.e., $C(z|a_1, 0) = C(z|a_1, 1)$. Thus, we have a contradiction and a_1 must be less than a_0 for Problem A' when there is a multiplicative downshift.

A.2 Proof of Proposition 6

For the solution to Problem B' to be the same as the solution to Problem A',

$$\frac{\lambda_{1a}}{1-p} = \frac{\lambda_{2a}}{p} = \lambda_b \tag{A.27}$$

$C(z|a, 0) = C(z|a, 1)$ for $a_1 = a_0 = a$ implies that

$$\frac{\lambda_{1a}}{1-p} + \frac{\mu_{1a}}{(1-p)} \frac{f_a(z|a)}{f(z|a)} = \frac{\lambda_{2a}}{p} + \frac{\mu_{2a}}{p} \frac{f_a(z|a)}{f(z|a)} \tag{A.28}$$

Rearranging and substituting for μ_{1a} and μ_{2a}, we get

$$\frac{\lambda_{1a}}{1-p} - \frac{\lambda_{2a}}{p} = \frac{\int [g(z, \alpha, \delta) - z]\, f_a(z|a)\, dz}{\int U(C(z|a)) f_{aa}(z|a) dz - V''(a)} \tag{A.29}$$

For any $g(z, \alpha, \delta) = \alpha z$, the right hand side of equation (A.29) is strictly non-zero. Hence, equation (A.27) does not hold and the solutions to Problems A' and B' differ.

A.3 The optimal contract is constant on each interval $(\bar{b}(a_{i-1},0)\,,\,\bar{b}(a_i),0)$.

It is useful to let $C(x|a_n,0)$, a contract to implement action a_n, be defined as follows:

$$C(x|a_n,0) = \begin{cases} k_1 & \text{if } \underline{b}(0) < x \leq \bar{b}(a_1,0) \\ k_2 & \text{if } \bar{b}(a_1,0) < x \leq \bar{b}(a_2,0) \\ \vdots & \\ k_i & \text{if } \bar{b}(a_{i-1},0) < x \leq \bar{b}(a_i,0) \\ \vdots & \\ k_n & \text{if } \bar{b}(a_{n-1},0) < x \leq \bar{b}(a_N,0) \end{cases}$$

Let k_i be a variable payment on the interval $(\bar{b}(a_{i-1},0),\bar{b}(a_i,0))$ with an expected value of $\bar{k}_i$. Let contract $\overline{C}(x|a_n,0)$ be the same as $C(x|a_n,0)$ except that the payment on the interval $(\bar{b}(a_{i-1},0),\bar{b}(a_i,0))$ is $\bar{k}_i$. The expected cost of both contracts will be the same by design. However, the expected utility of $\overline{C}(x|a_n,0)$ will be greater than the expected utility of $C(x|a_n,0)$ by Jensen's inequality. In fact, a constant strictly less than $\bar{k}_i$ could be substituted for $\bar{k}_i$ in $\overline{C}(x|a_n,0)$ such that the expected utility of $\overline{C}(x|a_n,0)$ would be just equal to the expected utility of $C(x|a_n,0)$. The expected cost of $\overline{C}(x|a_n,0)$ would then be strictly less than the expected cost of $C(x|a_n,0)$. Note that this change would not violate either the reservation utility or the incentive compatibility constraints because the expected utility would not be changed. Since this is true for any i, it must be that every k_i will be a constant in the optimal contract.

A.4 The Reservation Utility Constraint is Binding

If a contract satisfies all the constraints while the reservation utility is not binding, then a less costly contract could be devised by taking ε away from the payment for the interval $(\underline{b}(0), \overline{b}(a_1, 0))$.

This change reduces the expected utility for lower actions by more than for the desired action because the probability of receiving an output in the interval $(\underline{b}(0), \overline{b}(a_1, 0))$ is greater for lesser actions. With the uniform distribution, all possible outcomes in the given range have equal probability of occurrence. Since higher actions have larger ranges of possible outcomes, the probability of each of those outcomes is less than the probability of any outcome for a lower action. Therefore, the weighting in the expected utility for the lowest interval, $(\underline{b}(0), \overline{b}(a_1, 0))$, will be highest for the lowest action and will be decreasing in the action. Taking ε away from the payment for the interval $(\underline{b}(0), \overline{b}(a_1, 0))$ will reduce the expected utility of the lowest action by the greatest amount and the expected utility of the desired action by the least amount.

If the incentive constraints were satisfied before this change, then they will still be satisfied after the change because the left-hand side of each incentive constraint has been reduced by more than the right-hand side. Because the reservation utility constraint was not binding, ε can be taken away from the payment for the interval $(\underline{b}(0), \overline{b}(a_1, 0))$ without violating the reservation utility constraint. Hence, a feasible less costly contract can be devised from a contract with a nonbinding reservation utility constraint. This can

be repeated until the new contract does have a binding reservation utility constraint.

A.5 Lemma

Given:

$$h(w) = k \text{ is convex} \tag{A.30}$$

$$b_1 < a_1 < a_2 < b_2 \tag{A.31}$$

$$\phi b_1 + (1-\phi)b_2 = \phi a_1 + (1-\phi)a_2 \tag{A.32}$$

Then

$$\phi h(a_1) + (1-\phi)h(a_2) < \phi h(b_1) + (1-\phi)h(b_2) \tag{A.33}$$

This lemma is very similar to Jensen's inequality except that there are two points between two other points instead of one point between two others. In Jensen's inequality, for a convex function, the function of the expected value of two points is strictly less than the expected value of the function of those two points. In this lemma the expected value of the function of the two internal points is strictly less than the expected value of the function of the two external points using the same weights that make the expected value of the internal points equal to the expected value of the external points.

To prove the lemma, an equality relationship is first shown between a_1 and the b's and then between a_2 and the b's drawing upon the fact that the a's lie between the b's. Then these relationships are substituted into equation (A.32). This allows ϕ to be expressed in terms of the relationship between the individual a's and the b's. Then convexity of $h(w)$

is used to get an inequality relationship between the individual a's and the b's. Summing the inequalities together and substituting for ϕ gives the final result.

Proof:

Relationship (A.31) implies that there exist δ_1 and δ_2 elements of $(0,1)$ such that

$$\delta_1 b_1 + (1-\delta_1)b_2 = a_1 \tag{A.34}$$

$$\delta_2 b_1 + (1-\delta_2)b_2 = a_2 \tag{A.35}$$

Substituting equations (A.34) and (A.35) into equation (A.32) we get

$$\phi b_1 + (1-\phi)b_2 = \phi[\delta_1 b_1 + (1-\delta_1)b_2] + (1-\phi)[\delta_2 b_1 + (1-\delta_2)b_2]$$

This simplifies to

$$(b_1 - b_2)[\phi(1-\delta_1) - \delta_2(1-\phi)] = 0$$

However, b_1 is strictly less than b_2 from relationship (A.31). Thus

$$\phi(1-\delta_1) - \delta_2(1-\phi) = 0$$

and

$$\phi = \frac{\delta_2}{1-(\delta_1-\delta_2)} \tag{A.36}$$

$$(1-\phi) = \frac{1-\delta_1}{1-(\delta_1-\delta_2)} \tag{A.37}$$

The convexity of $h(w)$ and equations (A.34) and (A.35) imply

$$h(a_1) < \delta_1 h(b_1) + (1-\delta_1)h(b_2) \tag{A.38}$$

$$h(a_2) < \delta_2 h(b_1) + (1-\delta_2)h(b_2) \tag{A.39}$$

Summing ϕ times equation (A.38) and $(1-\phi)$ times equation (A.39) gives

$$\begin{aligned}\phi h(a_1) + (1-\phi)h(a_2) &< \phi[\delta_1 h(b_1) + (1-\delta_1)h(b_2)] \\ &\quad +(1-\phi)[\delta_2 h(b_1) + (1-\delta_2)h(b_2)] \\ &= (\phi\delta_1 + (1-\phi)\delta_2)h(b_1) \\ &\quad +[\phi(1-\delta_1) + (1-\phi)(1-\delta_2)]h(b_2)\end{aligned} \tag{A.40}$$

If equations (A.36) and (A.37) are substituted for ϕ and $(1-\phi)$ into the expressions multiplying $h(b_1)$ and $h(b_2)$, those expressions become:

$$\begin{aligned}\phi\delta_1 + (1-\phi)\delta_2 &= \frac{\delta_2\delta_1 + (1-\delta_1)\delta_2}{1-(\delta_1-\delta_2)} \\ &= \frac{\delta_2}{1-(\delta_1-\delta_2)} \\ &= \phi\end{aligned} \tag{A.41}$$

and

$$\begin{aligned}\phi(1-\delta_1) + (1-\phi)(1-\delta_2) &= \frac{\delta_2(1-\delta_1) + (1-\delta_1)(1-\delta_2)}{1-(\delta_1-\delta_2)} \\ &= \frac{1-\delta_1}{1-(\delta_1-\delta_2)} \\ &= 1-\phi\end{aligned} \tag{A.42}$$

Substituting expressions (A.41) and (A.42) into equation (A.40) we get:

$$\phi h(a_1) + (1-\phi)h(a_2) < \phi h(b_1) + (1-\phi)h(b_2)$$

A.6 Example of a Feasible Contract

A feasible contract is one which satisfies all the incentive compatibility constraints with the reservation utility constraint binding.

Given a binding reservation utility and constant payments on the intervals, the incentive compatibility constraints for the contract to induce action a_n become:

$$\frac{\Delta(a_1,0)}{\Delta(a_i,0)}U(k_1)+\sum_{j=2}^{i}\frac{(\Delta(a_j,0)-\Delta(a_{j-1},0))}{\Delta(a_i,0)}U(k_j)\leq V_i \qquad \text{for all } a_i\epsilon\,(a_1,a_n) \tag{A.43}$$

where the k_j's are the payments specified by the compensation function $C(x|a_n,0)$.

Define $\varepsilon_i \geq 0$ as the amount which makes the left-hand side of equation (A.43) equal to the right-hand side. This amount can be thought of the amount by which the incentive constraint on a_i is <u>not</u> binding. Equation (A.43) can now be written as:

$$\frac{\Delta(a_1,0)}{\Delta(a_i,0)}U(k_1)+\sum_{j=2}^{i}\frac{(\Delta(a_j,0)-\Delta(a_{j-1},0))}{\Delta(a_i,0)}U(k_j)=V_i-\varepsilon_i \qquad \text{for all } a_i\,\epsilon\,(a_1,a_n)$$

Consider an arbitrary set of $\varepsilon_i \geq 0$, $i=1,\ldots,n$. Now solve for k_i by starting at k_1 and working up.

The incentive compatibility constraint for a_1 is

$$\frac{\Delta(a_1,0)}{\Delta(a_1,0)}U(k_1)=V_1-\varepsilon_1$$

$$\Longrightarrow \qquad U(k_1)=V_1-\varepsilon_1$$

For action a_2, the incentive compatibility constraint can now be written as:

$$\frac{\Delta(a_1,0)(V_1-\varepsilon_1)}{\Delta(a_2,0)}+\frac{\Delta(a_2,0)-\Delta(a_1,0)}{\Delta(a_2,0)}U(k_2) \;=\; V_2-\varepsilon_2$$

$$\Longrightarrow \qquad U(k_2) = \frac{\Delta(a_2,0)(V_2-\varepsilon_2)-\Delta(a_1,0)(V_1-\varepsilon_1)}{\Delta(a_2,0)-\Delta(a_1,0)}$$

For action a_3, the incentive compatibility constraint can now be written as:

$$\begin{aligned}\frac{\Delta(a_1,0)}{\Delta(a_3,0)}(V_1-\varepsilon_1) &+ \frac{\Delta(a_2,0)-\Delta(a_1,0)}{\Delta(a_3,0)}\left(\frac{\Delta(a_2,0)(V_2-\varepsilon_2)-\Delta(a_1,0)(V_1-\varepsilon_1)}{\Delta(a_2,0)-\Delta(a_1,0)}\right)\\ &+ \frac{\Delta(a_3,0)-\Delta(a_2,0)}{\Delta(a_3,0)}U(k_3) = V_3-\varepsilon_3\\ \Longrightarrow \qquad U(k_3) &= \frac{\Delta(a_3,0)(V_3-\varepsilon_3)-\Delta(a_2,0)(V_2-\varepsilon_2)}{\Delta(a_3,0)-\Delta(a_2,0)}\end{aligned}$$

Similarly, for action a_i we get:

$$\begin{aligned}U(k_i) &= \frac{\Delta(a_i,0)(V_i-\varepsilon_i)-\Delta(a_{i-1},0)(V_{i-1}-\varepsilon_{i-1})}{\Delta(a_i,0)-\Delta(a_{i-1},0)}\\ k_i &= h\left(\frac{\Delta(a_i,0)(V_i-\varepsilon_i)-\Delta(a_{i-1},0)(V_{i-1}-\varepsilon_{i-1})}{\Delta(a_i,0)-\Delta(a_{i-1},0)}\right)\end{aligned}$$

A feasible contract will therefore take the form:

$$C(x|a_n,0) = \begin{cases} h(V_1-\varepsilon_1) & \text{if } \underline{b}(0) < x \le \bar{b}(a_1,0)\\ h\left(\frac{\Delta(a_2,0)(V_2-\varepsilon_2)-\Delta(a_1,0)(V_1-\varepsilon_1)}{\Delta(a_2,0)-\Delta(a_1,0)}\right) & \text{if } \bar{b}(a_1,0) < x \le \bar{b}(a_2,0)\\ \vdots & \\ h\left(\frac{\Delta(a_i,0)(V_i-\varepsilon_i)-\Delta(a_{i-1},0)(V_{i-1}-\varepsilon_{i-1})}{\Delta(a_i,0)-\Delta(a_{i-1},0)}\right) & \text{if } \bar{b}(a_{i-1},0) < x \le \bar{b}(a_i,0)\\ \vdots & \\ h\left(\frac{\Delta(a_n,0)(V_n-\varepsilon_n)-\Delta(a_{n-1},0)(V_{n-1}-\varepsilon_{n-1})}{\Delta(a_n,0)-\Delta(a_{n-1},0)}\right) & \text{if } \bar{b}(a_{n-1},0) < x \le \bar{b}(a_n,0)\\ h\left(\frac{\Delta(a_n,0)(V_n-\varepsilon_n)-\Delta(a_{n-1},0)(V_{n-1}-\varepsilon_{n-1})}{\Delta(a_n,0)-\Delta(a_{n-1},0)}\right) & \text{if } \bar{b}(a_n,0) < x \le \bar{b}(a_N,0)\end{cases}$$

where $h(\cdot) = U^{-1}(\cdot)$ and $\varepsilon_i \ge 0$ for all i.

A.7 All Lower Action Constraints are Binding

If the constraint on action a_i is binding, then, by definition, $\varepsilon_i = 0$. Note that ε_i only affects payments k_i and k_{i+1}.

Let $C(x|a_n, 0)$ be a feasible contract with $l < n$ constraints not binding; each nonbinding constraint defines an $\varepsilon_j > 0$. These nonbinding constraints can be any constraints except the reservation utility constraint. Define a new contract $\hat{C}(x|a_n, 0)$ as the same as $C(x|a_n, 0)$ except that one ε_j, which was strictly positive in $C(x|a_n, 0)$, is set equal to 0. Thus, for all $i < j$, $k_i = \hat{k}_i$.

For $i = j$ and for $i = j + 1$ we get

$$U(\hat{k}_j) = \frac{\Delta(a_j, 0)V_j - \Delta(a_{j-1}, 0)(V_{j-1} - \varepsilon_{j-1})}{\Delta(a_j, 0) - \Delta(a_{j-1}, 0)} \tag{A.44}$$

$$U(\hat{k}_{j+1}) = \frac{\Delta(a_{j+1}, 0)(V_{j+1} - \varepsilon_{j+1}) - \Delta(a_j, 0)V_j}{\Delta(a_{j+1}, 0) - \Delta(a_j, 0)} \tag{A.45}$$

$$U(k_j) - U(\hat{k}_j) = -\frac{\Delta(a_{j,0})\varepsilon_j}{\Delta(a_j, 0) - \Delta(a_{j-1}, 0)} \tag{A.46}$$

$$U(k_{j+1}) - U(\hat{k}_{j+1}) = \frac{\Delta(a_j, 0)\varepsilon_j}{\Delta(a_{j+1}, 0) - \Delta(a_j, 0)} \tag{A.47}$$

For all $i > j + 1$, $k_i = \hat{k}_i$.

Using equations (A.44) and (A.45) we can calculate the difference in expected utility of the two contracts. Since the only difference between the two contracts is the payments for the intervals $(\bar{b}(a_{j-1}, 0), \bar{b}(a_j, 0))$ and $(\bar{b}(a_j, 0), \bar{b}(a_{j+1}, 0))$

$$\begin{aligned} & EU(C(x|a_n, 0)) - EU(\hat{C}(x|a_n, 0)) \\ = & \frac{\Delta(a_j, 0) - \Delta(a_{j-1}, 0)}{\Delta(a_n, 0)}(U(k_j) - U(\hat{k}_j)) \\ & + \frac{\Delta(a_{j+1}, 0) - \Delta(a_j, 0)}{\Delta(a_n, 0)}(U(k_{j+1}) - U(\hat{k}_{j+1})) \\ = & \ 0 \end{aligned} \tag{A.48}$$

This is also due to the fact that both contracts just satisfy the reservation utility

constraint. Hence, both the expected utility of $C(x|a_n, 0)$ and the expected utility of $\hat{C}(x|a_n, 0)$ equal 0.

Equation (A.48) implies that

$$\phi U(k_j) + (1-\phi)U(k_{j+1}) = \phi U(\hat{k}_j) + (1-\phi)U(\hat{k}_{j+1}) \tag{A.49}$$

where

$$\phi = \frac{\Delta(a_j, 0) - \Delta(a_{j-1}, 0)}{\Delta(a_{j+1}, 0) - \Delta(a_{j-1}, 0)}$$

and equations (A.46) and (A.47) imply that

$$U(k_j) < U(\hat{k}_j) < U(\hat{k}_{j+1}) < U(k_{j+1}) \tag{A.50}$$

Given equations (A.49), (A.50) and $h(w) = k$ convex (where $w = U(k)$), we can apply the Lemma and get

$$\phi k_j + (1-\phi)k_{j+1} > \phi \hat{k}_j + (1-\phi)\hat{k}_{j+1} \tag{A.51}$$

Now we can use equation (A.51) when we compare the expected costs of the two contracts.

$$
\begin{aligned}
& E[C(x|a_n,0)] - E[\hat{C}(x|a_n,0)] \\
&= \frac{\Delta(a_j,0)-\Delta(a_{j+1},0)}{\Delta(a_n,0)}k_j + \frac{\Delta(a_{j+1},0)-\Delta(a_j,0)}{\Delta(a_n,0)}k_{j+1} \\
&\quad - \left(\frac{\Delta(a_j,0)-\Delta(a_{j+1},0)}{\Delta(a_n,0)}\hat{k}_j - \frac{\Delta(a_{j+1},0)+\Delta(a_j,0)}{\Delta(a_n,0)}\hat{k}_{j+1}\right) \\
&= \frac{\Delta(a_{j+1},0)-\Delta(a_{j-1},0)}{\Delta(a_n,0)}(\phi k_j + (1-\phi)k_{j+1}) \\
&\quad - \frac{\Delta(a_{j+1},0)-\Delta(a_{j-1},0)}{\Delta(a_n,0)}\left(\phi\hat{k}_j + (1-\phi)\hat{k}_{j+1}\right) \\
&> 0
\end{aligned}
$$

Thus, if a feasible contract has l nonbinding constraints, a less costly contract can be devised merely by making one of the nonbinding constraints binding. Since, l was any number of constraints less than n, a contract can be improved so long as $l > 0$. Thus, all incentive constraints will bind at the optimum.

A.8 Proof of Proposition 9

First assume that $a > \underline{a}$. Recall that $r(x,0) = a$ is the inverse of $\bar{b}(a,0) = x$. Similarly $r(x,1) = a$ is the inverse of $\bar{b}(a,1) = x$. The expected utility to the agent of action a given $C(x|a^\star,0)$ and the new distribution $f(x|a,1)$ is:

$$
\begin{aligned}
& E[U(C(x|a,0))] \\
&= \int_{\underline{b}(1)}^{\bar{b}(a,1)} \frac{U(C(x|a,0))}{\Delta(a,1)}\,dx \;-\; V(a) \\
&= \int_{\underline{b}(1)}^{\bar{b}(\underline{a},1)} \frac{V(\underline{a})}{\Delta(a,1)}\,dx \\
&\quad + \frac{1}{\Delta(a,1)}\int_{\bar{b}(\underline{a},1)}^{\bar{b}(a,1)} \left[\frac{x-\underline{b}(0)}{\Delta'(a,0)}V'(r(x,0)) + V(r(x,0))\right]\,dx \;-\; V(a)
\end{aligned}
$$

$$= \frac{(\bar{b}(\underline{a}) - \underline{b}(1))V(\underline{a})}{\Delta(a,1)} + \left(\frac{1}{\Delta(a,1)}\right)(x - \underline{b}(0))V(r(x,0))\Big|_{\bar{b}(\underline{a},1)}^{\bar{b}(a,1)} - V(a)$$

$$= \frac{(\underline{b}(0) - \underline{b}(1))V(\underline{a})}{\Delta(a,1)} + \frac{(\bar{b}(a,1) - \underline{b}(0))V(r(\bar{b}(a,1),0))}{\Delta(a,1)} - V(a) \qquad \text{(A.52)}$$

Because $\bar{b}(a,0)$ is increasing and linear in a, the inverse function $r(x,0)$ is also increasing and linear in x. If the event has a negative effect on the distribution, then $\bar{b}(a,1) \leq \bar{b}(a,0)$ and $\underline{b}(1) \leq \underline{b}(0)$ with at least one of the inequalities being a strict inequality. The former inequality implies that $r(\bar{b}(a,1),1) \leq r(\bar{b}(a,0),0)$ and $V(r(\bar{b}(a,1),1)) \leq V(r(\bar{b}(a,0),0)) = V(a)$. Thus, if $\underline{b}(0) - \underline{b}(1) > 0$, then equation (A.52) is strictly less than zero. If $\underline{b}(0) - \underline{b}(1) = 0$, then $\bar{b}(a,1) < \bar{b}(a,0)$, $V(r(\bar{b}(a,1),1)) < V(a)$, and equation (A.52) is strictly less than zero. Thus, for any $a > \underline{a}$ the agent's expected utility is strictly less than 0, his reservation utility. However, if the agent takes action $\underline{a}$, then $C(x|a^\star,0) = h(V(\underline{a}))$ and he will get his reservation utility.

A.9 Proof of Proposition 10

First note that, since the upper bound of the range of x is assumed to be linear in a, both $\bar{b}(a,0)$ and its inverse $r(x,0)$ can be written as

$$\bar{b}(a,0) = x = \Delta'(a,0)a + \underline{b}(0) \quad \text{and} \quad r(x,0) = a = \frac{x - \underline{b}(0)}{\Delta'(a,0)}$$

With the multiplicative shift $\bar{b}(a,1) = \alpha\bar{b}(a,0)$ and $\underline{b}(1) = \alpha\underline{b}(0)$. Hence,

$$\bar{b}(a,1) = x = \alpha(\Delta'(a,0)a + \underline{b}(0)) \quad \text{and} \quad r(x,1) = a = \frac{x/\alpha - \underline{b}(0)}{\Delta'(a,0)}$$

$$\longrightarrow r(x,1) \quad = \quad r(x/\alpha,0)$$

Similarly, $\bar{b}(a,1)$ and $r(x,1)$ can be written as

$$\bar{b}(a,1) = x = \Delta'(a,1)a + \underline{b}(1) \quad \text{and} \quad r(x,1) = a = \frac{x - \underline{b}(1)}{\Delta'(a,1)}$$

The expected compensation of any action $a > \underline{a}$ is:

$$\begin{aligned}
&EC(x|a,1) \\
&= \frac{\Delta(\underline{a},1)h(V(\underline{a})}{\Delta(a,1)} + \frac{1}{\Delta(a,1)} \int_{\bar{b}(\underline{a},1)}^{\bar{b}(a,1)} h\left(\frac{(x-\underline{b}(1))}{\Delta'(a,1)} V'(r(x,1)) + V(r(x,1))\right) dx \\
&= \frac{\alpha\Delta(\underline{a},0)h(V(\underline{a})}{\alpha\Delta(a,0)} \\
&\quad + \frac{1}{\alpha\Delta(a,0)} \int_{\alpha\bar{b}(\underline{a},0)}^{\alpha\bar{b}(a,0)} h\left(\frac{(x-\alpha\underline{b}(0))}{\alpha\Delta'(a,0)} V'(r(x/\alpha,0)) + V(r(x/\alpha,0))\right) dx \\
&= \frac{\Delta(\underline{a},0)h(V(\underline{a})}{\Delta(a,0)} + \frac{1}{\alpha\Delta(a,0)} \int_{\bar{b}(\underline{a},0)}^{\bar{b}(a,0)} h\left(\frac{(t-\underline{b}(0))}{\Delta'(a,0)} V'(r(t,0)) + V(r(t,0))\right) \alpha\, dt \\
&= EC(x|a,0)
\end{aligned}$$

A.10 Proof of Proposition 11

Because the additive shift causes the entire range to shift to the left by δ, the total range, $\Delta(a)$, does not change.

$$\begin{aligned}
&EC(x|a,1) \\
&= \frac{\Delta(\underline{a},1)h(V(\underline{a}))}{\Delta(a,1)} + \frac{1}{\Delta(a,1)} \int_{\bar{b}(\underline{a},1)}^{\bar{b}(a,1)} h\left(\frac{(x-\underline{b}(1))}{\Delta'(a,1)} V'(r(x,1)) + V(r(x,1))\right) dx \\
&= \frac{\Delta(\underline{a},0)h(V(\underline{a}))}{\Delta(a,0)} \\
&\quad + \frac{1}{\Delta(a,0)} \int_{\bar{b}(\underline{a},0)-\delta}^{\bar{b}(a,0)-\delta} h\left(\frac{(x-\underline{b}(0)+\delta)}{\Delta'(a,0)} V'(r(x,1)) + V(r(x,1))\right) dx
\end{aligned}$$

$$= \frac{\Delta(\underline{a},0)h(V(\underline{a}))}{\Delta(a,0)} + \frac{1}{\Delta(a,0)} \int_{\bar{b}(\underline{a},0)}^{\bar{b}(a,0)} h\left(\frac{(t-\underline{b}(0))}{\Delta'(a,0)}V'(r(t,0)) + V(r(t,0))\right) dt$$

$$= EC(x|a,0)$$

A.11 Proof of Proposition 12

For the additive shift, $\Delta(a,0) = \Delta(a,1)$ and $\Delta'(a,0) = \Delta'(a,1)$. Substituting into equation (4.25):

$$\frac{\Delta(\hat{a},0)}{2} = h\left(\frac{\Delta(\hat{a},0)}{\Delta'(\hat{a},0)}V'(\hat{a}) + V(\hat{a})\right) - EC(x|\hat{a},0) \tag{A.53}$$

Since equation (A.53) is equivalent to equation (3.17) which defines the optimal pre-event action, the optimal action is the same in both the pre-event and post-event contract.

For the multiplicative shift, $\alpha\Delta(a,0) = \Delta(a,1)$. Substituting into equation (4.25):

$$\frac{\alpha\Delta(\hat{a},0)}{2} = h\left(\frac{\Delta(\hat{a},0)}{\Delta'(\hat{a},0)}V'(\hat{a}) + V(\hat{a})\right) - EC(x|\hat{a},0) \tag{A.54}$$

First assume that, contrary to the proposition, $a^\star \leq \hat{a}$. If, as has been assumed, $E\Pi(x|a,0)$ is concave, then, for all actions greater than $a^\star$, $E\Pi'(x|a,0)$ must be negative. $E\Pi'(x|\hat{a},0) < 0$ implies:

$$\frac{\Delta(\hat{a},0)}{2} \leq h\left(\frac{\Delta(\hat{a},0)}{\Delta'(a,0)}V'(\hat{a}) + V(\hat{a})\right) - EC(x|\hat{a},0) \tag{A.55}$$

Equations (A.54) and (A.55) will both hold only when $\alpha \geq 1$ — a null or upward shift. This contradicts the assumption that $\alpha < 1$.

A.12 Proof of Proposition 13

In the case of a multiplicative shift, equation (4.26) becomes:

$$\frac{\alpha\Delta(\underline{a},0)}{2} > h\left(\frac{\Delta(\underline{a},0)}{\Delta'(\underline{a},0)}V'(\underline{a}) + V(\underline{a})\right) - h(V(\underline{a}))$$

If the corner solution is optimal in the no shift case, then from equation (3.18) it must be that:

$$\frac{\Delta(\underline{a},0)}{2} \leq h\left(\frac{\Delta(\underline{a},0)}{\Delta'(\underline{a},0)}V'(\underline{a}) + V(\underline{a})\right) - h(V(\underline{a}))$$

Since $\alpha\Delta(\underline{a},0) < \Delta(\underline{a},0)$, it must be that the baseline effort is optimal in the post-event contract also.

It is also possible that

$$\frac{\alpha\Delta(\underline{a},0)}{2} < h\left(\frac{\Delta(\underline{a},0)}{\Delta'(\underline{a},0)}V'(\underline{a}) + V(\underline{a})\right) - h(V(\underline{a})) < \frac{\Delta(\underline{a},0)}{2}$$

In that case, an interior solution is optimal in the pre-event contract while the corner solution is optimal in the post-event contract.

Appendix B

Year by Year Tests

The four sets of tests conducted with the sum of two years of grants can be duplicated comparing one year of grants with another. As mentioned earlier, these tests may be flawed by possible serial correlation and are generally weaker than the tests on two year sums. However, the year by year tests do provide some support for the renegotiation hypothesis.

In the year by year Sign Test, the paired observations are the number of options granted in the same company for two different years. The sample of firms is 189 firms with grant data for 1985 through 1988. The pairs are 1985 grants with 1986 grants, 1985 grants with 1987 grants, etc. The results are presented in Table B.1. When 1987 grants are compared to 1985 or 1986 grants, significantly more than half of the firms have higher grants in 1987. Likewise, when 1988 grants are compared to 1986 grants, significantly more than half of the firms have higher grants in 1988.

Table B.2 presents year by year sign tests on data from 1985 through 1989. The 135 firms with complete data for all five years are used in this set of tests. While there are not significantly more firms with 1989 grants higher than either 1987 or 1988, there

Table B.1: Sign Test — Year by Year Within Firm Comparison of Grants

Years being Compared		Number of firms with		Total (a)	Expected	z (b)
Year 1	Year 2	a Negative Difference	a Positive Difference	Number of Firms	Number of Positives	(significance)
1985	1986	95	87	182	91	-0.519 (.302)
1985	1987	110	74	184	92	-2.580 (.005)
1985	1988	97	81	178	89	-1.124 (.132)
1986	1987	107	77	184	92	-2.138 (.017)
1986	1988	104	76	180	90	-2.012 (.023)
1987	1988	89	90	179	89.5	0.000 (1.00)

(a) Firms in which the grants are equal in the comparing years are excluded.
(b) A negative value of z indicates that more firms have higher grants in year 2 than in year 1.

are significantly more firms with 1989 grants larger than either 1985 or 1986 grants. If, in the wake of the crash, firms moved toward alternative performance measures, 1989 grants would be less than 1985 or 1986 levels. Hence, these results do not support the hypothesis that there was a movement away from stock-based compensation after the crash.

Table B.2: Sign Test — Comparing 1989 Grants with Other Years

Years being Compared		Number of firms with		Total (a)	Expected	z (b)
		a Negative	a Positive	Number	Number of	(significance)
Year 1	Year 2	Difference	Difference	of Firms	Positives	
1985	1986	65	66	131	65.5	0.000 (1.0)
1985	1987	78	52	130	65	-2.193 (.014)
1985	1988	77	51	128	64	-2.210 (.014)
1985	1989	75	53	128	64	-1.856 (.032)
1986	1987	76	56	132	66	-1.654 (.049)
1986	1988	82	48	130	65	-2.894 (.002)
1986	1989	78	54	132	66	-2.002 (.023)
1987	1988	73	57	130	65	-1.316 (1.00)
1987	1989	69	61	130	65	-0.614 (.270)
1988	1989	66	64	130	65	-0.000 (1.00)

(a) Firms in which the grants are equal in the comparing years are excluded.
(b) A negative value of z indicates that more firms have higher grants in year 2 than in year 1.

An alternative test for the year by year tests is a joint test in which all four years are tested jointly for any difference between years. As mentioned earlier, this test may be flawed due to potential serial correlation. The Friedman two-way analysis of variance by ranks tests the null hypothesis that all four years have been drawn from the same population. For each firm, annual grants are ranked from 1 to 4 where the rank of 1 indicates the lowest grant in any of the four years. The test statistic

$$F_r = \frac{12\sum_{i=1}^{k} R_j^2 - 3N^2k(k+1)^2}{Nk(k+1) + \frac{Nk-T}{(k-1)}}$$

(where N is the number of firms, k is the number of years, R_j is the sum of ranks for year j and T is the sum of the cubes of the number of each set of ties) is distributed approximately χ^2 with degrees of freedom, $k - 1 = 3$. As can be seen in Table B.3, there is a significant difference, under the joint test, between the individual years. By inspection we can see both 1987 and 1988 have a higher sum of ranks than both 1985 and 1986. This supports the renegotiation hypothesis that grants are greater after the crash than before.

In the year by year median tests, scaled grants for all firms in the sample are pooled together by year. Tables B.4 and B.5 present the test results for the median tests on grants scaled by outstanding common shares and grants scaled by outstanding options, respectively.[1] For grants scaled by outstanding common shares, the median scaled 1985

[1]The number of firms in the sample differ for 1985 and all other years because an all available data rule was used in these tests. Thus, firms with data from 1986 to 1988 are included as well as firms with data from 1985 to 1988. The combined total number of firms is less than the total in the sum of years tests because the median observation is dropped when there are an odd number of firms in

grant is significantly less than either of the median scaled grants for 1987 or 1988. However, 1986 scaled grants are not significantly different than 1987 or 1988 scaled grants. For grants scaled by outstanding options, there are no significant differences between any pair of years.[2] In each comparison between pre-crash and post-crash years, the difference is in the correct direction. This is because there are more below median grants for the pre-crash year and more above median grants for the post-crash year. Finally, the median 1985 scaled grant is not significantly different from the median 1986 scaled grant and the median 1987 scaled grant is not significantly different from the 1988 scaled grant. Thus, the median tests on year by year scaled grants offer weak support, at best, for the hypothesis that 1987 and 1988 grants are higher than 1985 and 1986 grants.

In the year by year Kolmogorov-Smirnov Tests, sets of two years of pooled scaled grants are compared. As with the two year sum tests, a one-sided test is conducted and predictions of the direction of difference are required for each set of two years. While the renegotiation hypothesis gives clear predictions for the direction of expected differences in the 2 year sum comparisons and in the year by year comparisons of 1987 with 1985 and 1986 and of 1988 with 1985 and 1986, there are no predictions on directions of expected differences in the year by year comparisons of 1985 with 1986 and 1987 with 1988. In order for all the year by year tests to be one-sided, the highest absolute difference between

a group. The results for grants scaled by outstanding options are presented for the two divisor rule. Recall that observations for 1985 and 1986 are divided by outstanding options at January 1, 1985 whereas observations for 1987 and 1988 are divided by outstanding options at January 1, 1987. The all available data rule results in more observations for 1987 and 1988 than for 1985 and 1986.

[2]When a common divisor is used to scale grants by outstanding options, the median 1986 scaled grant is significantly less than the median scaled 1987 and 1988 grant.

the two distributions is used in the calculation of the test statistic for those pairs.[3]

Table B.6 presents the Kolmogorov-Smirnov test results for grants scaled by outstanding common shares and grants scaled by outstanding options.[4] For grants scaled by outstanding common shares, the 1987 ratios are significantly higher, at the one percent level, than the 1985 and 1986 ratios and the 1988 ratios are significantly higher, at the five percent level, than the 1985 ratios. While these results are supportive of the hypothesis that post-crash grants are higher than pre-crash grants, the significant differences between 1986 and 1985 ratios and between 1987 and 1988 ratios are not supportive. For grants scaled by outstanding options, the median 1987 scaled grant is significantly higher than the median 1985 and 1988 scaled grants. No other pairing has a significant difference. Thus, as with the median year by year tests, the Kolmogorov-Smirnov year by year tests offer weak support for the hypothesis that 1987 and 1988 grants are larger than 1985 and 1986 grants.

[3]While this choice increases the probability of concluding that there is a difference between the two distributions under comparison, finding a significant difference for those pairs does not lead to a rejection of the null hypothesis of no difference in annual grants. Instead, a significant difference between 1985 and 1986 grants or 1987 and 1988 grants could imply that grants are increasing over time or that there is too much noise in the granting process to find support for renegotiation. Thus, the choice of a one-sided test for the pairs for which there is no clear prediction for the direction of expected differences is conservative.

[4]As was the case for the median tests, an available data rule was used in these tests. Firms with data from 1986 to 1988 are included as well as firms with data from 1985 to 1988. Thus, the number of firms in the 1985 plus 1986 group differs from the number in the 1987 plus 1988 group.

Table B.3: Joint Test — Year by Year Within Firm Comparison of Grants

Year	Sum of Ranks	Ties	
		Type	Number
1985	437.5	Not tied	726
1986	442.5	Two tied	13
1987	501.5	Three tied	7
1988	488.5	Four tied	2
F_r			11.21
Level of Significance			(.02)

Table B.4: Median Tests — Year by Year

Grants scaled by Outstanding Common Shares

Year	Number of Firms with Scaled Grants		Total Number of Firms	χ^2 (significance)
	Below Median	Above Median		
1985	101	86	187	
1986	131	145	276	
Total	232	231	463	1.91 (.20)
1985	107	80	187	
1987	125	151	276	
Total	232	231	463	6.35 (.02)
1985	106	81	187	
1988	126	150	276	
Total	232	231	463	5.43 (.02)
1986	144	132	276	
1987	132	144	276	
Total	276	276	552	1.04 (.30)
1986	145	131	276	
1988	131	145	276	
Total	276	276	552	1.42 (.25)
1987	138	138	276	
1988	138	138	276	
Total	276	276	552	0.00 (1.00)

Table B.5: Median Tests — Year by Year

Grants scaled by Outstanding Options

Year	Number of Firms with Scaled Grants		Total Number of Firms	χ^2 (significance)
	Below Median	Above Median		
1985	84	89	173	
1986	89	84	173	
Total	173	173	346	0.18 (.70)
1985	91	82	173	
1987	132	139	271	
Total	223*	221	444	0.49 (.50)
1985	88	85	173	
1988	134	137	271	
Total	222	222	444	0.04 (.80)
1986	93	80	173	
1987	129	142	271	
Total	222	222	444	1.36 (.25)
1986	92	81	173	
1988	130	141	271	
Total	222	222	444	0.95 (.40)
1987	134	137	271	
1988	137	134	271	
Total	271	271	542	0.03 (.80)

* There are two observations exactly equal to the median.

Table B.7 presents the Wilcoxon-Mann-Whitney test results for grants scaled by outstanding common shares and grants scaled by outstanding options. For grants scaled by outstanding common shares, the results are in the predicted direction for all pairs except 1986 and 1988 where the difference in ranks is not significant. However, for grants scaled by outstanding options, there is no significant difference in ranks between any pair of years.[5] Thus, the year by year Wilcoxon-Mann-Whitney tests offer little support for the hypothesis that grants in 1987 and 1988 are larger than grants in 1985 and 1986.

[5]When a common divisor is used to scale grants by outstanding options, 1987 grants are significantly higher than both 1985 and 1986 grants. However, no other pair of years has a significant difference.

Table B.6: Kolmogorov-Smirnov Tests — Year by Year Comparisons

Grants scaled by Outstanding Common Shares

1985 Compared to	1986	1987	1988
Maximum Difference	.1129	.1528	.1288
χ^2	5.684	10.411	7.398
(significance)	(.10)	(.01)	(.05)
1986 Compared to		1987	1988
Maximum Difference		.1305	.0906
χ^2		9.401	4.531
(significance)		(.01)	(.30)
1987 Compared to			1988
Maximum Difference			-.1014
χ^2			5.676
(significance)			(.10)

Grants scaled by Outstanding Options

1985 Compared to	1986	1987	1988
Maximum Difference	.0693	.1199	.0400
χ^2	1.662	6.072	0.676
(significance)	(.50)	(.05)	(.75)
1986 Compared to		1987	1988
Maximum Difference		.0807	-.0656
χ^2		2.751	1.818
(significance)		(.30)	(.40)
1987 Compared to			1988
Maximum Difference			.0923
χ^2			4.617
(significance)			(.10)

A negative χ^2 indicates that at the largest point of difference between the two distributions, the lower year cumulative distribution is smaller than the higher year cumulative distribution.

Table B.7: Wilcoxon-Mann-Whitney Tests

Grants scaled by Outstanding Common Shares

Years Being Compared	Earlier Year Sum of Ranks	Earlier Year Number of Firms	Later Year Sum of Ranks	Later Year Number of Firms	Adjustment For Ties
1985 & 1986	41264.0	187	66152.0	276	6624.0
	z^* (significance)		-1.50 (0.130)		
1985 & 1987	38469.0	187	68947.0	276	6172.0
	z^* (significance)		-3.48 (0.001)		
1985 & 1988	40247.5	187	67168.5	276	17997.0
	z^* (significance)		-2.22 (0.035)		
1986 & 1987	71597.0	276	81031.0	276	3573.0
	z^* (significance)		-2.52 (0.017)		
1986 & 1988	73964.5	276	78663.5	276	12045.5
	z^* (significance)		-1.25 (0.183)		
1987 & 1988	78446.5	276	74181.5	276	11717.0
	z^* (significance)		+1.14 (0.208)		

Grants scaled by Outstanding Options

Years Being Compared	Earlier Year Sum of Ranks	Earlier Year Number of Firms	Later Year Sum of Ranks	Later Year Number of Firms	Adjustment For Ties
1985 & 1986	29462.0	173	30569.0	173	3276.0
	z^* (significance)		-0.60 (0.333)		
1985 & 1987	36350.5	173	62439.5	271	5336.0
	z^* (significance)		-1.63 (0.106)		
1985 & 1988	37788.0	173	61002.0	271	16259.0
	z^* (significance)		-0.40 (0.368)		
1986 & 1987	37045.0	173	61745.0	271	1833.5
	z^* (significance)		-1.10 (0.218)		
1986 & 1988	38533.5	173	60256.5	271	8112.5
	z^* (significance)		+0.32 (0.381)		
1987 & 1988	75649.5	271	71503.5	271	11719.0
	z^* (significance)		+1.14 (0.208)		

* A negative difference implies that the ranks for the earlier year are smaller than the ranks for the later year.