Essays on International Economics and Industrial Organization

by

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Abstract

This dissertation addresses two issues in international economics and one issue in industrial organization. The first two chapters use sticky-price intertemporal optimizing models with incomplete financial markets to analyze the dynamics of the current account after technology shocks and the effects of the optimal monetary policy on current account movements. The third chapter models the upgrade behavior of existing software users and new software users under two market structures. The first chapter studies a small open economy with two sectors. In a perfect foresight, rational expectation general equilibrium model, with sticky prices in the non-traded goods sector, the evolution of the current account following a positive technology shock is efficient even without time consistent optimal monetary policy. The second chapter extends the general equilibrium model to a two-country economy and analyzes the effects of the optimal monetary policy on current account dynamics. The welfare gain for home households from the individual optimal expansionary monetary policy mainly comes from the home country's terms of trade improvement when most firms price the export prices in buyer currency. The third chapter finds that, for the new software version, the software vendor should offer a price discount to existing users and charge a higher price to new users if software users are sufficiently heterogeneous. For the old software version, a price discount should be applied to existing users with a higher price to new users if the price of the co-existing new version is high.

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Summary

This dissertation consists of two essays in international economics and one essay in industrial organization. The first two essays use sticky-price intertemporal optimizing models with incomplete financial markets to analyze the dynamics of the current account after technology shocks and the effects of the optimal monetary policy on current account movements. The third chapter studies the upgrade behavior of existing software users and new software users under two market structures.

The first essay studies a small open economy with two sectors. In a perfect foresight, rational expectation general equilibrium model, with sticky prices in the non-traded goods sector, the current account responses to monetary shocks depend on the elasticity of substitution between consumption of non-traded and traded goods (the intratemporal effect) and risk aversion (the intertemporal consumption smoothing effect), the country's initial net foreign asset position, and the degree of monopolistic competition. In this analysis, the current account can go into either deficit or surplus in response to an expansionary monetary policy. With flexible traded good prices and only slow price adjustment in the non-traded goods sector, the current account reacts quite efficiently to technological shocks in a small open economy. The welfare gain for households from adopting optimal monetary policy in contrast to constant money growth rule is quantitatively small.

The second essay extends the general equilibrium model to a two-country economy and analyzes the effects of the optimal monetary policy on current account dynamics. With sluggish price adjustment among the firms, and with most firms pricing their exports in the buyers' currency, the home monetary authority will choose an expansionary monetary policy when facing a home technological improvement. The expansionary monetary policy, in turn, will improve the welfare of home households at the expense of the welfare of foreign households. The welfare gain for home house-
holds, from this optimal expansionary monetary policy, mainly comes from the home
country's terms of trade improvement. If a supranational monetary authority is to
choose an optimal monetary policy for both countries, the welfare gain from the
expansionary monetary policy for both home and foreign counties is quantitatively
small. In this environment, the current account response to the optimal monetary
policy of the supranational monetary authority will also be quantitatively small.

The third essay investigates the upgrade behavior of existing software users and
new software users under two market structures: Structure 1 - where both the old and
new versions coexist in the market; and Structure 2 - where only the new version
is available in the market. We find that keeping the old version in the market is
an effective tool to salvage the "disappearance" demand of software upgrade. The
"cannibalization" effect between the old version and new version, is different for
existing users, compared to new users. We explore the possibility of software vendors
utilizing price discriminations between existing users and new users as a method to
reap higher profit. We find that, for the new software version, the software vendor
should offer a price discount to existing users and charge a higher price to new users if
the users are sufficiently heterogeneous. For the old software version, a price discount
should be offered to existing users with a higher price for new users, if the price of
the co-existing new version is high.
Chapter 1

Current Account Dynamics and Optimal Monetary Policy in a Small-Open Economy

1.1 Introduction

An open economy can run current account imbalances, thus, enabling different directions of movement of consumption and production in the short-run. For the last two decades, most of the current account literature has emphasized the intertemporal approach.\(^1\) Emphasizing the importance of consumption smoothing and investment, this literature focuses on a country's intertemporal budget constraint: a current account deficit now means trade surpluses in the future. This line of research has its attractive feature that the analysis is based on optimizing models, where explicit preferences, technology and capital market access are present. Most papers, however, have focused on non-monetary environments or flexible-price economies. In such models, money is always neutral, so that there is no room to analyze the effects of monetary policies on the current account.

In recent years, researchers have been developing intertemporal optimizing sticky-price open economy macroeconomics models. The new wave of research was initiated by Obstfeld and Rogoff (1995a)'s \textit{redux} model.\(^2\) They introduced imperfect compe-

\(^1\)Please see Obstfeld and Rogoff (1995b) for a survey on this literature.
\(^2\)See Lane (2001b) for a comprehensive survey of the recent literature in this area.
tition and sticky prices into the dynamic general equilibrium macroeconomic framework. In these models, monetary shocks have real effects, thus providing a potential role for monetary policies. Obstfeld and Rogoff (1995a) show that with sticky prices and purchasing power parity (PPP), a positive home money shock generates a long-run improvement in the home current account in their two-country model. Betts and Devereux (2000a) also show that in a two-country world with pricing-to-market, a domestic monetary expansion will in general improve the home country's current account as long as not all goods are priced under local currency pricing.

In this paper, we propose a small open economy dynamic general equilibrium model with incomplete financial markets to analyze the dynamics of the current account after different shocks and the effects of the optimal monetary policy on current account. There are many factors that affect the current account dynamics. Lane (2001a) demonstrates that in the short-run the response of the current account to a monetary shock depends on the parameter values of the elasticity of substitution between the consumption of traded and non-traded goods and the risk aversion measure. Devereux (2000) studies the impact of a devaluation on the current account in a model with pricing-to-market and shows that the impact depends critically on the extent of pricing-to-market, and the intratemporal and intertemporal effects. Thoenissen (2003) claims that a key determinant of the current account dynamics is the initial net foreign asset position. Lombardo (2002) shows that the degree of competition qualitatively affects the current account response to nominal shocks in a two-country world.

We show that, in our model with sticky prices in the non-traded goods sector, the current account responses to monetary shocks depend on the values of the elasticity of substitution between consumption of non-traded and traded goods, the "intratemporal effect" and the risk aversion measure, the "intertemporal consumption smoothing effect", the country's initial net foreign asset position and the degree of monopolistic competition. In this analysis, the current account can go into either deficit or surplus in response to an expansionary monetary policy. Our results confirm the studies of the above-mentioned research.

Our main contribution is to show how the optimal monetary policy will affect
the current account dynamics. The optimal monetary policy maximizes the welfare of a representative agent, given frictions in the economic environment. We construct a small open general equilibrium model with two sets of frictions - costly price adjustments by imperfectly competitive non-traded goods firms and monopolistic competition. For evaluating the welfare of the representative agent, we have an explicit utility function and the dynamics of the utility of the agent. In our model, we investigate the passive optimal response of the monetary authority to a positive technology improvement in the traded good sector. We find that the optimal monetary policy has two different implications. With both frictions - costly price adjustments by imperfectly competitive non-traded goods firms and monopolistic competition, the optimal monetary authority takes the firms' price setting behavior as given. This gives a big incentive to the monetary authority to inflate and raise output and hence consumption - the discretionary result. The optimal monetary policy can expand the economy above the natural rate of output using an expansionary monetary policy. In this case, due to the monopolistic competition in the non-traded goods sector, the non-traded goods' output is sub-optimally low. Domestic output is demand-determined in the short-run because of the slow price adjustment in the non-traded goods sector. With slow price adjustments, the monetary expansion can boost current demand for the non-traded goods consumption above the natural level of output in the short-run. The optimal monetary policy can increase the household's welfare compared to the case in which the monetary authority simply responds to the unexpected technology improvement by a passive increase of money supply. The expansionary optimal monetary policy has a great impact on the initial current account response. In this case, the current account responses to the positive technology shock with the optimal monetary policy are very different from the current account responses without the optimal monetary policy.

In the discretionary case, although the monetary authority has substantial leverage over real activity in our model economy, the discretionary monetary policy is not time-consistent. The monetary authority cannot systematically move the small open economy away from the natural rate of output in the long-run. To have a time-consistent optimal monetary policy, the monetary authority has to follow a commit-
ment policy. Because people understand the monetary authority’s incentives in our perfect foresight model, the surprise monetary expansion and the resulting benefits cannot arise systematically. To have the time-consistent monetary policy, the monetary authority should not take advantage of the firm’s price setting behavior. In our model, we use firm subsidies for the non-traded goods sector to eliminate the monopoly distortion. The firms still set the prices over a markup, but get a lump-sum transfer from the government to compensate for the monopolistic power. After we get rid of the monopolistic competition distortion, the output is already optimal in the non-traded goods sector. We have flexible prices in the traded good sector. The only distortion in the economy is the sluggish price adjustment in the non-traded goods sector. With the technology improvement in the traded good sector, the real economy is expanding, an expansionary monetary policy by the monetary authority will be welfare improving theoretically. Nevertheless, in our model, as the technology improvement in the traded good sector is temporary, and the sluggish price adjustment is only in the non-traded goods sector, the welfare gain from the expansionary monetary increase in response to the technology shock, will be small. In this case, without the markup distortion in the small open economy, the optimal monetary policy will increase the money supply by a negligible amount and be time-consistent. The passive expansionary monetary policy, in fact, will decrease the household’s welfare since the economy is already at the long-run optimal output level. The welfare gain for the household, from adopting the optimal monetary policy, will be quantitatively small. The current account response to the optimal monetary policy is quantitatively small, too. Thus, in this case, once we remove the monopolistic distortion in the welfare analysis, the current account responds fairly efficiently to the unexpected technology improvement.

For the small open economy in our model, the time-consistent optimal monetary policy to a positive technology shock only improves the household’s welfare by a negligible amount. The time-consistent optimal monetary policy is to expand the economy by a quantitatively small amount. The response of the current account to the technology shock will not change quantitatively with the optimal monetary policy. In other words, the evolution of the current account to the technology shock
is already efficient, even without the optimal monetary policy. For a small open economy, the optimal monetary policy has an insignificant effect on the welfare of the economy and current account dynamics.

The detailed structure of the work is as follows: Section 1.2 presents the model. Section 1.3 presents the equilibrium of the model. Section 1.4 discusses the calibration and solution with flexible prices and then presents the more general case where the prices are sticky in the non-traded goods sector. Finally, Section 1.5 concludes the paper.

1.2 The Model

This paper assumes a perfect foresight small open economy with a representative household, firms and a domestic government. The economy has two different types of goods - a homogeneous traded good and differentiated non-traded goods. The non-traded goods sector is monopolistically competitive. The price of the traded good is covered by the law of one price.\(^3\)

1.2.1 Households

The economy is populated by a continuum of consumers/households of measure unity. The representative household is endowed with a certain amount of time, which is divided between leisure and work. She consumes two types of goods - traded and differentiated non-traded goods. The consumer can hold two types of nominal assets: non-interest bearing home money \(M\), a one-period noncontingent foreign debt \(D\) denominated in foreign currency. She gets income from labor income, profits from domestic firms and lump-sum government transfers and pays back interests on the foreign debt.

Preferences The lifetime utility of the home representative household is:

\[
\max \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{X}{1-\epsilon} \left( \frac{M_t}{P_t} \right)^{1-\epsilon} - \frac{\eta}{1+\psi} H_t^{1+\psi} \right]
\]

\(^3\)In the following, we normalize the foreign currency price of the traded good to unity and consider a flexible exchange rate regime so that the domestic currency price of the traded good and the nominal exchange rate are the same: \(P_t = \epsilon_t\), where \(\epsilon_t\) is the nominal exchange rate, defined as the number of domestic currency units per unit of foreign currency.
where $\beta \in (0, 1)$ is the discount rate. $\sigma$ is the inverse of intertemporal elasticity of aggregate consumption. The household’s instantaneous utility depends positively on consumption, $C_t$, and real money balances $M_t / P_t$, where $M_t$ is nominal balances held at the beginning of period $t$ and $P_t$ is a consumption based price index for period $t$. $H_t$ is the labor effort at time $t$.

The consumption index $C_t$ is a CES aggregate of traded and non-traded goods

$$C_t = \left[ a^\rho C_{Nt}^{\rho - 1} + (1 - a)^{\rho - 1} C_{Tt}^{\rho - 1} \right]^{\rho - 1}$$

where $\rho > 0$ is the constant elasticity of substitution between traded and non-traded goods. The non-traded good is in turn defined over the consumption of differentiated goods, so that

$$C_{Nt} = \left[ \int_0^1 C_{Nt}(i)^{1-\lambda} di \right]^{1-\lambda}$$

where $C_{Nt}$ is an index of consumption of the non-traded goods, $C_{Tt}$ is the consumption of traded good. There exists a continuum of home produced non-traded goods indexed by $i \in [0, 1]$ and a homogeneous traded good. All consumption goods are perishable. $C_{Nt}(i)$ denotes date $t$ consumption of the non-traded good $i$ and $\lambda > 1$ denotes the elasticity of substitution among non-traded goods.

The non-separability between traded and non-traded goods consumption means that shocks to the non-traded goods sector have spillover effects on the traded good consumption and hence the current account. For instance, in the case that traded good consumption rises together with non-traded goods consumption, a boom in the non-traded sector will cause an increase in demand for imports and a current account deficit.

The consumption price indices are defined as

$$P_t = \left[ a P_{Nt}^{1-\rho} + (1 - a) P_{Tt}^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

(1.2.2)

$$P_{Nt} = \left[ \int_0^1 P_{Nt}(i)^{1-\lambda} di \right]^{1-\lambda}$$

Assume the law of one price holds for the traded good, $P^*_t$ is the world price of the traded good and $e_t$ is the current exchange rate: $P_{Tt} = e_t P^*_t$. 

6
Optimal consumption behavior implies that the individual demands for non-traded and traded goods are given as:

\[ C_{Tt} = (1 - a)(\frac{P_{Tt}}{P_t})^{-\rho}C_t \quad C_{Nt} = a(\frac{P_{Nt}}{P_t})^{-\rho}C_t \]

\[ C_{Nt}(i) = \left[ \frac{P_{Nt}(i)}{P_{Nt}} \right]^{-\lambda}C_{Nt} \]

The household can provide different types of labor services. There exists a continuum of labor types, indexed by \( i \in [0, 1] \). Let \( H_t(i) \) denote the number of hours of type \( i \) labor. The variable \( H_t \) that appears in the utility function is defined as a sum of labor services in non-traded and traded goods sectors:

\[ H_t = \int_0^1 H_{Nt}(i) di + H_{Tt} \]

**Household Budget Constraint** The household can hold two financial assets: local money and nominal debts denominated in foreign currency. The debts have a maturity of one period. The household’s budget constraint in period \( t \) is:

\[ M_t + \epsilon_t(1 + i_{t-1})D_{t-1} + P_tC_t = M_{t-1} + \epsilon_tD_t + T_t + W_tL_t + \Pi_t \quad (1.2.3) \]

where \( D_t \) is the household’s stock of foreign currency debts that become due in period \( t \). \( i_{t-1} \) is the nominal interest rate on the foreign debt. The household owns the firms and \( \Pi_t \) is the profit from the traded and non-traded goods firms.

The representative household’s intertemporal consumption decisions and her demand for money are determined by maximizing the life-time utility specified in (1.2.1) subject to the restriction that the budget constraint (1.2.3) holds in all periods and for all states of the world. Ruling out the Ponzi schemes, we can get the following first-order conditions:

\[ \frac{W_t}{P_t} = \eta C_t^\sigma H_t^\phi \quad (1.2.4) \]

\[ 1 = \beta(1 + i_t)C_{t+1}^{\sigma} \quad P_t \quad C_{t+1}^{-\sigma} \quad P_{t+1} \quad \epsilon_{t+1} \quad \epsilon_t \quad (1.2.5) \]

\[ \chi \left( \frac{M_t}{P_t} \right)^{-\epsilon} = C_t^{-\sigma} \left[ 1 - \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right] \quad (1.2.6) \]

where equation (1.2.4) is the intratemporal optimal labor supply schedule, equation (1.2.5) is the intertemporal Euler condition and equation (1.2.6) is the optimal money demand schedule.
1.2.2 Firms and the Structure of Goods Markets

Two types of firms exist in the country: (i) producers of non-traded consumption goods; and (ii) producers of the traded consumption good. Domestic producers use domestic labor as the only input, and labor is immobile internationally. The period $t$ production functions of the firms producing non-traded good $i$ and traded good are concave and given by:

$$Y_{Nt}(i) = A_{Nt}H_{Nt}(i)^{\alpha} \quad Y_{Tt} = A_{Tt}H_{Tt}$$

where $Y_{Nt}$ and $Y_{Tt}$ are the firms' outputs and $\alpha < 1, \gamma < 1$, while $A_{Nt}$ and $A_{Tt}$ are period $t$ labor productivities. $A_{Nt}$ and $A_{Tt}$ are exogenous random variables.$^4$

There are competitive profit maximizing firms in the traded good sector. This implies that the price is equal to marginal cost:

$$W_t = \gamma P_{Tt} A_{Tt} H_{Tt}^{-1}$$

where $W_t$ is the wage rate. In the non-traded goods sector, each production firm has monopolistic market power, and sets its price as a markup over the marginal cost. If non-traded goods prices were perfectly flexible, then the profit maximizing decision for firm $i$ would imply:

$$\frac{\lambda}{\lambda - 1} W_t = \alpha P_{Nt}(i) A_{Nt} H_{Nt}(i)^{\alpha - 1}$$

The main objective of this paper is to address the current account dynamics with the optimal monetary policy. In this respect, we need to have price rigidity in our model. We assume that non-traded goods firms adjust their prices with an adjustment cost. We follow Sheshinski and Weiss (1977) and Rotemberg (1983) and introduce sluggish price adjustment by assuming that the non-traded good firm faces a resource cost that is quadratic in the inflation rate of the goods it produces:

$$\pi_{Nt}(i) = P_{Nt}(i) Y_{Nt}(i) - W_t H_{Nt}(i) - \frac{\delta}{2} \left[ \frac{P_{Nt}(i)}{P_{Nt-1}(i)} - 1 \right]^2 P_{Nt}(i) Y_{Nt}(i)$$

where price adjustment cost is $\frac{\delta}{2} \left[ \frac{P_{Nt}(i)}{P_{Nt-1}(i)} - 1 \right]^2 Y_{Nt}(i)$. The parameter $\delta$ measures the degree of price stickiness. The higher is $\delta$ the more sluggish is the adjustment of nominal prices. If $\delta = 0$, then prices are flexible.

$^4$In this small open economy model, $A_{Nt}$ and $A_{Tt}$ are assumed to be unity at steady-state.
The producer of non-traded good \(i\) maximizes

\[
\Pi_{Nt}(i) = \sum_{j=0}^{\infty} \Omega_{t,t+j} \pi_{Nt}(i)
\]

where

\[
\Omega_{t,t+j} = \beta^j \frac{C_{t+j}^{-\sigma}}{P_{t+j} C_t^{-\sigma}}
\]

is the pricing kernel used to value date \(t + j\) pay-offs. As firms are owned by the representative household, it is assumed that firms value future payoffs according to the household’s intertemporal marginal rate of substitution in consumption. The firms’ pricing decision is given by:

\[
\begin{aligned}
(1 - \chi)Y_{Nt}(i) + \frac{\lambda}{\alpha} \frac{H_{Nt}(i)}{P_{Nt}(i)} - \delta \left( \frac{P_{Nt}(i)}{P_{Nt-1}(i)^2} - \frac{1}{P_{Nt-1}(i)} \right) P_{Nt}(i) Y_{Nt}(i) \\
- \frac{\delta(1 - \chi)}{2} \left( \frac{P_{Nt}(i)}{P_{Nt-1}(i)} - 1 \right)^2 Y_{Nt}(i) \\
+ \frac{\beta \sigma}{C_{t+1}} \frac{P_{t}}{P_{t+1}} \left( \frac{P_{Nt+1}(i)^2}{P_{Nt}(i)^3} - \frac{P_{Nt+1}(i)}{P_{Nt}(i)^2} \right) P_{Nt+1}(i) Y_{Nt+1}(i) = 0
\end{aligned}
\]

With sticky prices in the non-traded goods sector, the price of good \(i\) equals the product of the shadow value of one extra unit of output (the marginal cost), times a markup. Symmetric non-traded goods firms make identical choices in equilibrium. The markup \(\Psi_{t}(i)\) depends on the output demand as well as on the impact of today’s pricing decision on today’s and tomorrow’s costs of adjusting the output price:

\[
\Psi_{t}(i) = \lambda Y_{Nt}(i) \left\{ (\lambda - 1)Y_{Nt}(i) + \delta \left[ \left( \frac{P_{Nt+1}(i)^2}{P_{Nt}(i)^3} - \frac{P_{Nt+1}(i)}{P_{Nt}(i)^2} \right) P_{Nt+1}(i) Y_{Nt+1}(i) \\
- \frac{\beta \sigma}{C_{t+1}} \frac{P_{t}}{P_{t+1}} \left( \frac{P_{Nt+1}(i)^2}{P_{Nt}(i)^3} - \frac{P_{Nt+1}(i)}{P_{Nt}(i)^2} \right) P_{Nt+1}(i) Y_{Nt+1}(i) \right] \right\}^{-1}
\]

If \(\delta = 0\), i.e., if prices are fully flexible, \(\Psi_{t}(i) = \frac{\lambda}{\chi - 1}\) is the familiar constant-elasticity markup. If \(\delta \neq 0\), price rigidity generates endogenous fluctuations of the markup. The markup \(\Psi_{t}(i)\) depends on \(P_{Nt+1}(i), P_{Nt}(i), P_{Nt-1}(i), Y_{Nt}(i)\) and \(Y_{Nt+1}(i)\). The non-traded goods firms react to \(\frac{P_{Nt+1}(i)}{P_{Nt+1}(i)}\) dynamics in their pricing decisions. Changes in monetary policy generate changes in \(\frac{P_{Nt+1}(i)}{P_{Nt+1}(i)}\) dynamics. Hence, they affect the non-traded goods prices and the markup. Through this channel, they generate different
dynamics of relative non-traded goods prices and the current account. If $\lambda$ approaches infinity, firms have no monopoly power, and the markup reduced to 1, the competitive level. Under perfect competition, the presence of a cost of adjusting the price level is irrelevant to the firms decision. Some degree of monopoly power is necessary for the nominal rigidity to matter.

1.2.3 Government

The government issues local currency, has no expenditures, and runs a balanced budget every period. The nominal lump-sum transfers of seignorage revenues $T_t$ are given by:

$$T_t = M_t - M_{t-1}$$

1.2.4 Debt Elastic Interest Rate

This set-up of our model implies incomplete asset markets. A well-known consequence is that this model will display non-stationary dynamics, so that linearizing the model around the initial steady-state could yield a poor approximation of the nonlinear model. As Schmitt-Grohe and Uribe (2003) point out, the unconditional variances of endogenous variables are infinite, even if exogenous shocks are bounded. In our rational expectation model, we use a nonlinear solution to avoid the approximation error of log-linearization, for evaluating welfare and also to accommodate large shocks. We follow Schmitt-Grohe and Uribe (2001) and Bergin (2006) and impose a "premium" on the asset return which is proportional to the outstanding stock of foreign debts.

The nominal return of the debt is closely related to its world nominal interest rate $i_t^*$. 

$$i_t = i_t^* + \psi_2(e^{D_t-d} - 1) \tag{1.2.11}$$

where $d$ is a constant - the initial aggregate level of foreign debt. The nominal interest rate of the debt depends on the real holdings of the foreign debts in the entire home economy. This means that domestic households take $d$ as given when deciding on the optimal holding of the foreign debt. The term $\psi_2(e^{D_t-d} - 1)$ is a country-specific interest rate premium. As borrowers, consumers will be charged a premium over
the foreign interest rate; and as lenders, they will receive a remuneration that is lower than the foreign interest rate. Another way to describe this cost is to assume the existence of intermediaries in the foreign asset market (which are owned by foreign households) who can borrow from and lend to households of the rest of the world at the rate \(i^*_t\), but who can borrow from and lend to households of the home country at the rate \(i_t\). For characterizing the incomplete financial structure, we do not really need to introduce this additional cost. Nevertheless, this will be useful for pinning down a well-defined steady-state for consumption and assets. The primary motivation for including this term here is to remove the element of nonstationarity in the model. Introducing the risk premium term as a function of debts, ensures that wealth allocations in the long-run converge to a unique steady state. In practice the parameter \(\psi_2\) will be set at a very low level in calibrating the model.\(^5\)

1.3 Equilibrium

The Flexible-Price Equilibrium As all households and firms are identical, we can drop the subscript \(i\). We first consider the case when all prices are perfectly flexible. Combine equations (1.2.3) and (1.2.7), we can get

\[
(1 + i_{t-1})D_{t-1} = D_t + P_t^\epsilon[A_T H^\gamma_T - (1 - a)(P_{t1}/P_t)^{\rho}C_t]
\]

(1.3.12)

This is the market equilibrium condition. The labor working in either traded or non-traded goods sector will get the same wage rate,

\[
\frac{\lambda}{\lambda - 1} e_t P_t^\epsilon A_T H^\gamma_T = \alpha A_N H^\gamma_N P_N
\]

(1.3.13)

The equilibrium of this fully flexible price economy is a collection of eight sequences \((D_t, P_t, C_t, e_t, H_{Tt}, H_{Nt}, P_{Nt}, i_t)\) satisfying 8 equilibrium conditions. These include (1.3.12), (1.3.13), the three household first-order equations, the debt-elastic interest rate equation (1.2.11), the price index (1.2.2) and the non-traded goods market clear condition.

The steady-state Stationarity fails for an open economy whenever the equilibrium rate of aggregate per capita consumption growth is independent of the economy's

\(^5\)In our paper, this elasticity of the interest rate premium is set at \(7 \times 10^{-3}\).
aggregate per capita net foreign assets. The constant consumption in the steady-state does not determine a unique steady-state for net foreign assets. In our model, the debt elastic interest rate ensures the stationarity of the model. The cost \( \psi_2(e^{D_t - \bar{d}} - 1) \) captures that, when the economy-wide holdings of foreign-currency denominated debts are above (below) the steady-state level, \( \bar{d} \), individual agents receive less (more) than the gross rate of return. When the equilibrium aggregate level of foreign debts in the economy is equal to \( \bar{d} \), this cost will disappear. We denote steady-state levels of variables without a time subscript \( t \). The steady-state level of domestic interest rate \( i \) is equal to the world interest rate \( i^* = \frac{1 - \beta}{\beta} \). Given the initial steady-state level of net foreign currency debt \( D \), we can solve for the steady state level of the real variables.

The Sticky-Price Equilibrium For the sticky price equilibrium, we also have eight equations and now we have equation (1.2.9) instead of (1.3.13).

The current account with sticky prices may be computed as:

\[ CA_t = \frac{e_t(D_t - D_{t-1})}{P_t} = -\frac{e_{t-1}D_{t-1}}{P_t} + \frac{P_{t+1}}{P_t} (Y_t - C_t) \]

The current account is determined by the interest payment for past debt and net exports. The real exchange rate is:

\[ q_t = \frac{e_t P_t^*}{P_t} \]

### 1.4 Calibration and Solution

#### 1.4.1 Calibration

As this is a perfect foresight, rational expectation model, we can use a non-linear solution technique - multiple shooting method. This method is used to solve two-point boundary problems. Lipton et al. (1982) give a detailed discussion. This solution technique allows us to avoid the approximation errors from log-linearization and gives us a trajectory for the current account after a shock.

We follow the open economy macro literature in picking parameter values for our basic experiments. The inverse of the intertemporal elasticity of substitution is set at 2, following Backus, Kehoe, and Kydland (1995). The rate of time preference
is set at 0.01, so that the subjective discount factor is 0.99. The value of \( \eta \) is just a scale factor, so we set it arbitrarily to unity. The share of non-traded goods in the consumer price index \( a \) is set at 0.5, following the evidence cited in Cook and Devereux (2001) for Malaysia and Thailand. The inverse of the elasticity of labor supply is set to 0.5, so that \( \psi = 0.5 \). This is roughly following Rotemberg and Woodford (1998)\(^6\). The elasticity of substitution between non-traded and traded goods is set at 0.75 which follows directly from Ostry and Reinhart (1992). The elasticity of substitution between varieties of non-traded goods is \( \lambda \), and this governs the equilibrium markup of price over cost in the non-traded goods sector. We follow Rotemberg and Woodford (1998), where they set \( \lambda = 7.66 \) which implies an average mark-up of 15%. We assume that non-traded goods production is relatively labor intensive, with \( \alpha = 0.7 \), and traded goods is relatively non-labor intensive, with \( \gamma = 0.3 \). The consumption elasticity of money demand for household is equal to \( \frac{1}{\epsilon} \) in this model. According to Mankiw and Summers (1986), this variable is very close to unity and hence \( \epsilon \) is set to 1. For the price stickiness, we follow Ireland (2001) and choose the price adjustment cost parameter \( \delta = 77 \). Table 1.1 reports the baseline calibration assumptions.

In this paper, we also investigate whether or not initial holdings of foreign debts/assets will affect the current account responses to the various shocks. As documented by Lane and Milesi-Ferretti (2001), net foreign assets over GDP vary across countries and are different from zero. The authors argue that the level of net foreign assets is a key state variable and a crucial determinant of the benefits of international financial integration. The values of the steady-state net debt position, relative to consumption, set between -0.5 and 0.5, appears to be reasonable for OECD countries.

### 1.4.2 Impulse Response Analysis

We calibrate the model to explore how the current account dynamics will respond to a positive technology shock with and without optimal monetary policy. Before we can do that, however, we investigate how the current account responds to a

\(^6\)In Rotemberg and Woodford (1998), the inverse of the elasticity of labor supply is set to 0.47.
positive technology shock or an expansionary monetary shock. Recent research shows that the current account can go into either deficit or surplus when facing a positive technology shock or an expansionary monetary shock. Lee and Chinn (2006) show that the current account improves in response to temporary technology shocks for G7 countries. In this section, we demonstrate the factors that affect the current account dynamics.

**Current Account Dynamics with Flexible Prices**

With flexible prices, the monetary shocks will only affect the prices, and not the real variables in this economy. With real shocks, the prices will adjust instantly.

**Money Supply Shocks** With perfectly flexible prices, money is indeed neutral in our model. Under an unforeseen permanent money supply shock, the prices (domestic price and non-traded goods prices) instantly adjust and jump to the new equilibrium levels by the same magnitude, while the current account remains at zero at all times. The fact that the small open economy holds a positive or negative initial debt does not affect the results in this analysis.

**World Interest Rate Shocks** The initial debt holding in the small open economy will affect the interest payment and hence the real economy. With zero initial debt holding, upon a temporary 1% increase in the world interest rate, domestic currency depreciates, exports increase, and imports decrease. The current account will go into surplus upon the positive temporary world interest rate shock in period 1, and then go back to balance in the next period, since the shock is only temporary and prices adjust instantly. With a positive debt holding, the current account goes into surplus in period 1 upon the shock, then goes into a deficit in period 2, and then returns again to zero in period 3. Upon the shock, as the domestic interest rate increases, consumption decreases, households work more, and the traded good output increases. Households consume less and produce more of the traded good, so the current account improves right away. As the economy is holding debt, however, higher interests must be paid, this increased interest payment causes the current account to dive into a deficit in the next period. Since the shock is only temporary, everything returns to the initial equilibrium level again. Similarly, if the country holds foreign assets
initially, the current account improves right away in period 1, and will not go to zero in period 2, because of the increased interest income from holding foreign assets, though it will still go back to a balance, finally. See Figure 1.1 for the current account responses to a 1% temporary increase in the world interest rate.

**Technology Shocks** We also calibrate the current account dynamics to technology shocks in the traded good sector. The technology shock process takes the following form:

\[
\ln A_{Tt+1} = \mu_1 \ln A_{Tt} + \varepsilon_{Tt+1}
\]  

(1.4.14)

where \( \varepsilon_{Tt+1} \) is the technology shock in the traded good sector, \( \mu_1 = 0.95 \) (following the real business cycle literature). Upon a persistent technology shock, as the traded good's production is more efficient, the output of the traded good will increase.

From the households' first-order conditions, we get:

\[
\frac{C_{Tt+1}}{C_{Tt}} = \beta (1 + i_t) \left( \frac{e_t}{e_{t+1}} \right)^{\frac{1}{\sigma}} \left( \frac{P_{Ft}}{P_{Ft+1}} \right)^{\frac{\rho}{\sigma}}.
\]

This equation is the Euler Equation, governing the dynamic movement of consumption. The consumption growth depends on the sequence of relative prices - the so-called "consumption-based real interest rate effect". If the aggregate price level relative to the price of traded goods, is currently low compared to its future value, present consumption is encouraged over future consumption. Meanwhile, the relatively higher future traded goods price also encourages consumption substitution from traded goods to non-traded goods. The former effect dominates if the intertemporal elasticity of substitution \( \frac{1}{\sigma} \) is greater than the intratemporal elasticity of substitution \( \rho \), i.e., \( \frac{1}{\sigma} > \rho \), as the spillover between non-traded and traded consumption is positive. When \( \frac{1}{\sigma} < \rho \), however, the spillover between non-traded and traded consumption is negative, as the intratemporal substitution effect dominates the intertemporal substitution effect.

First, we show the current account response with zero initial net foreign assets. With an unexpected 1% temporary persistent technology shock in the traded good sector, the output of the traded good sector will increase as the production of the traded good is more efficient. As the real exchange rate appreciates, domestic consumption of the traded good also increases. The increase in the output of the traded
good is greater than the increase in the consumption of the traded good, so that the current account goes into surplus at the time of the shock. With our basic calibration, upon the 1% technology shock in the traded good sector, the current account demonstrates a 0.27% surplus in the current period GDP ratio. Lee and Chinn (2006) show that the current account improves in response to temporary technology shocks for G7 countries. Our results are consistent with this empirical result. As the technology shock is very persistent, the output of the traded good gradually decreases from the maximum level upon the impact. After 18 quarters, the consumption becomes greater than the production of the traded good, and the current account sinks into deficit, reaching a 0.039% GDP ratio deficit after 37 quarters.

With the technology shock, in our basic calibration $\sigma = 2$, i.e. $\frac{1}{\sigma} < \rho = 0.75$, the spillover between non-traded and traded consumption is negative. If we change $\sigma = 0.5$, so that $\frac{1}{\sigma} > \rho$, the non-traded consumption moves together with the traded consumption. In this case, the traded good consumption increases more than it would when $\sigma = 2$. Upon the shock, the current account still goes into surplus, but by a smaller amount in the basic case, i.e., by only 0.24% of the current GDP. Figure 1.2 shows the current account, traded and non-traded consumption responses to the traded sector technology shock with $\frac{1}{\sigma} < \rho$ and $\frac{1}{\sigma} > \rho$. The impulses are percentage deviations from the initial steady-states except for the current account responses. With a non-zero initial net foreign debt, the current account response to the technology shock is almost the same.

**Current Account Dynamics with Sticky Prices**

Lane (2001a) shows that monetary shocks empirically are a significant source of variation in the current account and can help explain the high volatility of the current account. Hence, we must also include monetary shocks in our model. The price stickiness is modeled by a price adjustment cost in the non-traded goods sector. The parameter $\delta$ measures the degree of price stickiness in the non-traded goods sector. In our basic calibration $\delta = 77$, 78% of the non-traded goods prices fully adjust within one year with a 1% one-time money supply shock. In this section, we first study the case where the country has zero initial net foreign assets. Then,
we study the case where the small open economy has positive/negative initial net foreign debt.

**Money Supply Shocks** For simplicity, we assume that money supply is characterized in terms of a money supply rule. The money supply rule is:

\[
\ln M_t = \ln M_{t-1} + \mu (\ln M_{t-1} - \ln M_{t-2}) + \epsilon_{Mt}
\]

where \(\mu = 0.5\), and \(\epsilon_{Mt}\) is the unexpected money supply shock.

This section shows that there are a few factors affect the current account response to a monetary shock. First, the intratemporal effect and risk aversion will affect how the current account responds to a monetary shock. With a 1% unexpected temporary money growth shock, money supply increases permanently. The surprise monetary expansion stimulates extra demand, and hence, the production of non-traded goods. When \(\frac{1}{\sigma} < \rho\), the spillover between non-traded consumption and traded consumption is negative, as the intratemporal substitution effect dominates the intertemporal substitution effect. Increased consumption of non-traded goods means that the traded good consumption decreases. Increased production of non-traded goods also crowds out the production of the traded good. As a result, the traded production decreases upon the shock. Both the consumption and production of the traded good decrease. In our basic calibration, the inverse elasticity of labor supply is 0.5, the production of traded good decreases by a smaller amount, so the current account improves upon the impact of the shock. If we change the value of \(\sigma\) so that \(\frac{1}{\sigma} > \rho\), the spillover between non-traded consumption and traded consumption is positive, increased consumption of non-traded goods also stimulates consumption of the traded good, since the elasticity of substitution between the consumption of non-traded and traded goods is low (relative to the intertemporal elasticity of substitution). At the same time, non-traded goods production crowds out the traded good production. So the current account goes into deficit. Figure 1.3 shows the current account responses to an unexpected 1% positive money growth shock with \(\sigma = 2\) and \(\sigma = 0.5\) when \(\rho = 0.75\). The magnitude of the current account response is comparable to Lane (2001a)'s result. This analysis further confirms that the elasticity of substitution between consumption of non-traded and traded goods (the
infratemporal effect) and risk aversion (the intertemporal consumption smoothing effect) will affect how the current account responds to shocks.

Second, the elasticity of labor supply also affects the initial response of the current account to a monetary shock. With the same monetary shock, we show how the inverse elasticity of labor supply $\psi$ affects the responses of the current account. When the inverse elasticity of labor supply increases, and the elasticity of labor supply decreases, households are willing to provide less labor with the shock. As the demand for non-traded goods increases, the labor supply for non-traded goods has to increase. The traded good output decreases more with the same shock. The current account will show less surplus upon the shock. Figure 1.4 gives the current account responses to the monetary shock with different elasticities of labor supply. We can see that with different elasticities of labor supply, the current account responds differently to the same monetary shock.

Third, Equation (1.2.10) shows that changes in monetary supply generate changes in $\frac{p_{Nt}(t)}{p_{Nt-1}(t)}$ dynamics, which will affect the markup and the current account. With monopolistic competition, the output of non-traded goods falls below the competitive level. At steady-state, as $\lambda$ increases, the markup of the non-traded goods' firms decreases. With less monopolistic competition, the non-traded goods' firms are more competitive and have a higher output of non-traded goods. The surprise monetary expansion shock will increase the output in the non-traded goods sector and tend to correct the monopolistic distortion. With a higher $\lambda$, as the equilibrium of outputs for the non-traded goods is already higher, the monetary expansion will cause the non-traded goods' outputs to increase less. So the traded good output will decrease less with a higher $\lambda$.

In our basic calibration, when $\frac{1}{\sigma} < \rho$, the increased consumption of non-traded goods is accompanied by decreased consumption of the traded good. As non-traded consumption increases less with a higher $\lambda$, the traded consumption also decreases less. Now, we have both consumption and production of the traded good decreasing less, with lower markup. Figure 1.5 demonstrates the current account responses to an unexpected 1% positive money growth shock with different levels of markup - 15%, 7% and 2%. With the lower markup, net exports are crowded out by the increased production of non-traded goods.
fall when \( \frac{1}{\sigma} < \rho \). This is the same result as was found by Lombardo (2002). We have shown, in equation (1.2.10), that the markup is necessary for the monetary shock to have an effect. With less markup distortion, the real economy will respond less. The markup matters quantitatively for the current account response to a monetary shock. The markup also works with the intratemporal substitution effect and the intertemporal substitution to affect the current account response. Figure 1.6 shows that with \( \frac{1}{\sigma} > \rho \), the current account improves with the lower markup. The reason for this result is that, as \( \frac{1}{\sigma} > \rho \), the traded consumption increases together with the non-traded consumption, and will increase less with a higher \( \lambda \). Thus, with the higher \( \lambda \) and the lower markup, the traded output decreases less and traded consumption increases less. Therefore, the current account improves.

In this section, we confirm that the intertemporal elasticity of substitution and the intratemporal elasticity of substitution, as well as the firm markup, affects the current account response to monetary shocks. We show that the elasticity of labor supply also affects the initial current account response. These factors not only affect the current account response independently, but also work together to affect the initial response of the current account. We need to keep these factors in mind when investigating whether or not these factors will have any effect on our optimal monetary policy analysis.

**Current Account Dynamics with Non-Zero Initial Net Foreign Asset** In this section, we explore how the non-zero initial net foreign asset will affect the current account dynamics with technology and monetary shocks. In the small open economy with flexible exchange rates, the domestic interest rate is decided by the world interest rate. Any feasible technology and monetary shock will not change the domestic interest rate sufficiently to alter quantitatively the dynamics of the current account response.

**Technology Shocks** Before examining the optimal monetary policy for the current account responses, we first must see what impact a technology shock will have on the current account dynamics. We still use the technology shock Equation (1.4.14) in the traded good sector. Figure 1.7 shows the current account response to an unexpected 1% increase of the traded good technology. Upon the technology shock, the traded
good sector will increase its production since the production becomes more efficient. The increase in output is greater than the increase in the consumption of the traded good, and the current account goes into surplus upon the impact of the shock. With our basic calibration, the magnitude of the current account response is 0.27% of the current period GDP upon the shock. During convergence, the current account has an over-adjustment deficit of up to 0.039% of the current GDP.

1.4.3 Current Account Dynamics with Optimal Monetary Policy

Discretionary Monetary Policy

In this section, we study optimal monetary policy with current account movements and see how the optimal monetary policy will affect the current account dynamics. We first study the case when the net foreign asset is initially zero, and explore the optimal monetary policy to a temporary positive technology shock in the traded good sector. The optimal monetary policy maximizes the welfare of the representative household in this small open economy. The technology shock process takes the form of (1.4.14). In our model, we assume that the monetary authority receives perfect information about the current state of the economy. So, the optimal active monetary policy is that the money supply responds instantly to the technology shock. Upon the unexpected positive technology shock in the traded good sector, the monetary authority immediately recognizes the temporary technology shock and sets the appropriate money supply to maximize the welfare of the household. The money supply rule is:

\[ \ln M_t = \ln M_{t-1} + \mu (\ln M_{t-1} - \ln M_{t-2}) + \epsilon_{Mt} \]

\[ \epsilon_{M1} = a_1 \epsilon_{T1} \]

and now, \( \mu = 0.5 \) and \( a_1 \) is the parameter that has to be determined optimally by the monetary authority.

In this economy, we have two kinds of distortions - nominal rigidity and monopolistic competition in the non-traded goods sector. With a 1% unexpected positive technology shock in the traded good sector, the sector increases its production since the production is more efficient now. The increased production of the traded good crowds out the non-traded goods production. As the real economy is expanding
with the positive technology improvement, intuitively, welfare will improve if the small open economy increases the monetary supply at the same time. Due to the monopoly distortion in the non-traded goods sector, the output of the non-traded goods is already sub-optimally low. The monetary authority takes pre-set prices as given, and can increase the money supply and choose the optimal money supply level to increase the welfare of the household. This unanticipated monetary expansion, upon the technology shock, leads to increases in real economic activities. As the price adjustment is slow in the non-traded goods sector, domestic output is demand-determined in the short-run. The surprise monetary expansion boosts the current demand for the non-traded goods and causes the outputs of the non-traded goods to increase above the sub-optimal level and even above the natural rate level of output. The increased amount of money also works with the technology shock, boosting the total employment above the steady-state level (natural rate level) in the short-run. Equivalently, this nominal shock increases the employment above the natural rate level and eliminates part of the existing distortions in the economy. This effect comes from two channels: 1) the increased money supply will enable the firm to adjust less to decrease the welfare loss from sticky prices; and 2) the increased money supply will minimize the monopolistic distortion from the non-traded goods sector. This result corresponds to the benefits from surprise inflation in Barro and Gordon (1983). The surprise monetary expansion gives rise to the discretionary monetary policy problem, as the monetary authority can exploit the firms' pricing decision when it makes the monetary policy. This gives us the no-commitment result. The benefit from this surprise monetary expansion, in this case, is an expansion of real economic activity. Because of the existing distortions in this economy, the economy starts off in a sub-optimally low equilibrium, and the expansion of the non-traded goods sector improves the household's welfare.

With the basic calibration, the optimal value of the magnitude of the money injection parameter $a_1$ is equal to 12.4. Upon the unforeseen 1% technology shock in the traded good sector, the monetary authority increases its money supply by 12.4%. This optimal monetary policy will increase the household's welfare by 0.0095% than with no monetary policy. If the monetary authority has just a passive 1% monetary
expansion to the 1% technology shock, the welfare of the household can still increase by a small amount 0.0017%, which is much lower than 0.0095%.

The optimal monetary policy also affects the current account responses and magnifies the initial current account/GDP responses upon the shocks, by 39.4%, according to our basic calibration. Figure 1.8 shows the impulse responses under sticky prices with the optimal monetary policy and a passive 1% money supply increase (basic calibration). As analyzed in the above section, the intratemporal effect and risk aversion, elasticity of labor supply, and the firms' markup affect the current account response to monetary shocks. We checked how these factors will affect the optimal monetary policy to the 1% technology shock, and found similar results, in that the expansionary monetary supply will increase the household's welfare. Nevertheless, the current account response to the optimal monetary policy does change with different factors. Figure 1.9 shows that with $\frac{1}{\sigma} > \rho$, the current account will actually go into deficit with the optimal monetary policy. In general, the optimal monetary policy welfare analysis can still be carried out with different parameters and the optimal monetary policy is seen to have a big impact on the current account dynamics. The optimal monetary policy can either amplify or dampen the initial responses of the current account. In this case, the current account does not respond efficiently to the technology improvement alone.

**Optimal Monetary Policy with Commitment**

In this discretionary regime, the monetary authority can print more money in response to the technology improvement and expand the money supply in surprise to the public. Because people understand the monetary authority’s incentives in our perfect foresight model, however, this type of surprise and the resulting benefits cannot arise systematically. The monetary authority cannot systematically move the small open economy away from the natural rate of output in the long-run. To have a time-consistent optimal monetary policy, the monetary authority must follow a commitment policy. For a time consistent monetary policy, the monetary authority should not be able to exploit the firms’ pricing behavior. In our welfare analysis, we use firm subsidies to eliminate the monopolistic distortion in the non-traded
goods sector. The non-traded goods firms still set their prices over a markup, but get a lump-sum subsidy from the government, each period, to compensate for the monopolistic distortion. Given the firm subsidy, the monetary authority's incentive to exploit the firms' pricing decisions is removed.

In the small open economy, after we use the firm subsidy to eliminate the markup distortion, the only distortion in the model is the sluggish price adjustment in the non-traded goods sector. With the technology improvement in the traded goods sector, the real economy is expanding, and theoretically, the monetary authority can increase the monetary supply to match the real economy expansion. The increase of money supply in response to the expansion of the real economy will be welfare improving, in theory. In our small open economy, however, the technology improvement is temporary, and the price stickiness is only in the non-traded goods sector, so the welfare gain from an expansionary monetary policy is very small in our model. Now, no monopolistic competition exists in the non-traded goods sector, and the benefit for the monetary authority to use the surprise money expansion disappears.

The existing distortion in the economy is then only the slow price adjustment in the non-traded goods sector. The monetary authority will not be able to exploit the non-traded goods firms' pricing decisions, which amounts to a commitment monetary policy. To see whether or not the monetary policy can increase the household's welfare with only one distortion - price rigidity, we use firm subsidies to get rid of the markup distortion in the non-traded goods sector. We assume that the government runs balanced budgets. The government taxes non-traded goods firm revenues at a rate that compensates for monopoly power in the zero-inflation steady-state, and removes the markup over the marginal cost charged by firms in a flexible-price world. The output of the non-traded good firm, $P_{Nt}(i)Y_{Nt}(i)$, is taxed at a tax rate of $\tau$. The tax rate is determined by $1 - \tau = \frac{1}{1 - \lambda}$, which gives $\tau = -\frac{1}{1 - \lambda}$. Because $\lambda > 1$, the tax rate is negative, firms receive a subsidy on their revenues and pay lump-sum taxes $T_{f_t} = \tau P_{Nt}(i)Y_{Nt}(i)$.

The profits of the non-traded good firm becomes

$$\pi_{Nt}(i) = (1 - \tau)P_{Nt}(i)Y_{Nt}(i) + T_{f_t} - W_tH_{Nt}(i) - \frac{\delta}{2} \left[ \frac{P_{Nt}(i)}{P_{Nt-1}(i)} - 1 \right]^2 P_{Nt}(i)Y_{Nt}(i)$$

In this case, no monopolistic distortion exists in the non-traded goods sector. The
non-traded outputs are already at the optimal level, so the monetary authority does not have an incentive to generate a surprise monetary expansion. The optimal monetary policy only increases the household's welfare by a negligible amount. With only price rigidity distortion in this economy, the optimal level of the money injection parameter \( a_1 \) is equal to 0.01. Upon the unforeseen 1% technology shock in the traded good sector, the monetary authority can increase its money supply by 0.01%. This optimal monetary policy will increase the household's welfare by a negligible amount. The optimal monetary policy also affects the current account response, but the policy has a small impact on the initial current account/GDP responses at the time of the shock. Figure 1.10 shows the current account responses with the optimal monetary policy under the basic calibration. The intratemporal effect and risk aversion, elasticity of labor supply, and the firm markup do not affect our welfare result.

As the optimal monetary policy now is basically "do-nothing quantitatively", the optimal monetary policy has a minimum impact on the current account dynamics.

The current account responds to the unexpected technology improvement quite efficiently, even without the optimal monetary policy. Thus, when we remove the monopolistic markup distortion in our economy, we find that, theoretically, a role still exists for the optimal monetary policy in current account adjustment. Quantitatively, however, this role is very small. Thus, we conclude that, at least in relation to productivity-driven movements in the current account, the current account adjustment is quite efficient in the absence of compensating monetary policy. From a practical point of view, this suggests that little reason exists to argue that central banks should try to adjust exchange rates or interest rates to affect net exports or the current account in small open economies.

We investigated whether or not the non-zero initial net foreign asset will affect our results here. With either initial debt or asset, we get the same result for our welfare analysis. We also explored the case when we have sticky prices in the non-traded goods sector and local currency pricing, and the same result was found.
1.4.4 Current Account Dynamics with Interest Rate Rule

In this section, we investigate the current account dynamics with a popular monetary policy - an interest rate rule. Here, we describe the current account responses and welfare comparison under several interest rate rules that stem from the work of Taylor (1993). In Chari, Kehoe, and McGrattan (2002), the authors show that any interest rate rule can be interpreted as a money growth rule and vice versa. In practice, however, some simple interest rate rule can be a very good approximation of the monetary policy. Thus, we consider the implications of the interest rate rules as being similar to the rules studied by Taylor (1993) and Clarida, Gali, and Gertler (2000).

Following the recent literature (Woodford 2003 and Clarida et al. 2000), in abstracting from the details of the monetary mechanism, we simply assume that the monetary authority is committed to a domestic targeting rule with interest rate smoothing. In particular, we assume that nominal interest rates \( i_t \) are a function of lagged nominal interest rates, inflation rates, and exchange rate changes, according to:

\[
(1 + i_t) = (1 + i_{t-1})^{\rho_r} \left[ (1 + i^*) \left( \frac{P_t}{P_{t-1}} \right)^{\gamma_s} \right]^{(1-\rho_r)} \left( \frac{S_t}{S^*} \right)^{\gamma_r} e^{\epsilon_{rt}}
\]

where \( \frac{P_t}{P_{t-1}} \) is the inflation from \( t-1 \) to \( t \); and \( i^* \) is the steady-state world interest rate. The parameter \( \gamma_s \) represents the degree of exchange rate flexibility or the coefficient of exchange rate intervention. As long as \( \gamma_s > 0 \), a determinate equilibrium value is present for the nominal exchange rate. \( \gamma_s \) is exogenously given and measures the preference of policy makers. When \( \gamma_s \) approaches zero, it represents a flexible exchange rate regime. \( \rho_r \) measures the degree of interest rate smoothing, and \( \epsilon_{rt} \) is a normally distributed, mean-zero shock. We set \( \rho_r = 0.79 \) and \( \gamma_r = 2.15 \) following Clarida, Gali, and Gertler (2000). In this small open economy, no capital control means that we cannot have autonomous monetary policies with the fixed exchange rate. Nevertheless, we can still analyze the cases having different levels of flexibility in the exchange rate movements.

With the non-linear solution method, it is accurate to compare the welfare results under different interest rules. It is also possible to incorporate large interest rate
shocks for our welfare analysis.

**Technology Shocks** When we use the Taylor rule in our model, the positive technology shock generates similar responses for the current account, consumption, and output as was the case with the money supply rule. With our basic calibration, the magnitude of the current account response is 0.55% of the current period GDP upon the shock. Figure 1.11 shows the responses to an unexpected 1% increase of the traded good sector technology.

**Interest Rate Monetary Policy Shocks** Figure 1.12 shows the current account dynamics to an unexpected 50% negative interest rate policy shock (The nominal interest rate decreases from 0.01 to 0.005). With a lower interest rate, the returns to production decline and the output decreases. With this looser monetary policy shock, consumption increases and the current account goes into deficit. For the welfare analysis, we find that with lower $\gamma_s$ (more flexibility in the exchange rate) in the interest rate rules, the household’s welfare increases with the expansionary monetary policy. The current account responds less with the more flexible exchange rate since some of the shock is absorbed by the changes in prices. A higher weight on inflation (higher $\gamma_p = 2.15$) in the interest rate rule will also generate higher welfare with the expansionary monetary policy shock.

Figure 1.13 shows the current account responses to unexpected 50% and 200% positive interest rate policy shocks, respectively (The nominal interest rate increases from 0.01 to 0.015, and to 0.03). We also explore the cases with firm subsidies and non-zero initial debt levels, and obtain similar results.

### 1.5 Conclusions

In this paper, we show that the current account dynamics depend on the parameter values of intertemporal and intratemporal elasticities of substitution, the elasticity of labor supply, the degree of monopolistic competition, the initial net foreign asset position, and the types of shocks. With flexible traded good prices and only slow price adjustment in the non-traded goods sector, the current account reacts quite efficiently to technological shocks in the small open economy. The welfare gain for households from adopting optimal monetary policy, in contrast to having
a constant money growth rule, is quantitatively negligible for zero/non-zero initial
debt/asset positions. For a small open economy, the optimal monetary policy has
an insignificant effect on the welfare of the economy and current account dynamics.
Table 1.1: Baseline Calibration Parameters for Chapter 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Economic Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0.5</td>
<td>Share of non-traded goods in the consumer price index</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.7</td>
<td>Labor share in non-traded goods production</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.3</td>
<td>Labor share in traded goods production</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.75</td>
<td>Elasticity of substitution between non-traded and traded goods</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2</td>
<td>Inverse of the intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>7.66</td>
<td>Elasticity of substitution between varieties of non-traded goods</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.5</td>
<td>Inverse of the elasticity of labor supply</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>1</td>
<td>Inverse elasticity of money demand for household</td>
</tr>
</tbody>
</table>

Figure 1.1: A 1% Temporary World Interest Rate Increase with Different Initial Net Debt Levels (Flexible Prices)
Figure 1.2: A Positive 1% Technology Shock in the Traded Good Sector with Different Intertemporal Elasticities of Substitution (Flexible Prices)
Figure 1.3: An Unexpected 1% Money Supply Increase with Different Intertemporal Elasticities of Substitution (Sticky Prices)
Figure 1.4: An Unexpected 1% Money Supply Increase with Different Elasticities of Labor Supply [$\sigma = 2$, $\rho = 0.75$] (Sticky Prices)
Figure 1.5: An Unexpected 1% Money Supply Increase with Different Firm Markups $[\sigma = 2, \rho = 0.75]$ (Sticky Prices)
Figure 1.6: An Unexpected 1% Money Supply Increase with Different Firm Markups
[\sigma = 0.5, \rho = 0.75] (Sticky Prices)
Figure 1.7: A 1% Positive Technology Shock in the Traded Good Sector with Basic Calibration (Sticky Prices)
Figure 1.8: Optimal Monetary Policy With No Firm Subsidy \([\sigma = 2, \rho = 0.75]\)
Figure 1.9: Optimal Monetary Policy With No Firm Subsidy \( \sigma = 0.5, \rho = 0.75 \)
Figure 1.10: Optimal Monetary Policy With Firm Subsidies (Basic Calibration)
Figure 1.11: A Positive 1% Technology Shock in the Traded Good Sector with Interest Rate Rule (Sticky Prices).
Figure 1.12: An Unexpected 50% Interest Rate Decrease (Sticky Prices)
Figure 1.13: An Unexpected 50%/200% Interest Rate Increase (Sticky Prices)
Chapter 2

Current Account Dynamics and Optimal Monetary Policy in a Two-Country Economy

2.1 Introduction

The US current account deficit to GDP ratio reached an unprecedented level of 5.7% in 2005. With the US being a large country in the global economy, this huge current account deficit is attracting new attentions in studies of the current account. The sustainability of the US current account has been discussed substantially in the literature.\(^1\) To study the dynamics of a large country such as the US, however, we need a two-country economy model.

Whether monetary policies can have any impact on the current account movements is also an important research topic. Lane (2001a) shows that monetary shocks are a significant source of variation in the external account, and a surprise monetary expansion can generate a persistent external account surplus. As monetary shocks can have a great impact on current account dynamics, a role may exist for the monetary authority in regards to the current account dynamics. Extensive research has been done on the open macroeconomics literature on optimal monetary policies with country-specific productivity shocks. A notable feature in this literature is the lack of study on how the optimal monetary policies will affect the current account movements. If some part of the current account movements can be attributed to

---

\(^1\)Please refer to Clarida (2006) for an overview, and Engel and Rogers (2006) and Obstfeld and Rogoff (2005) for different views.
the optimal monetary policies by the monetary authority, we would not need to be concerned about the current account balances/movements.

To address the question of how the optimal monetary policy will affect the current account movements, we introduce the current account into a standard two-country general equilibrium model with sticky prices. As the current account plays a crucial role in the transmission of shocks, according to Obstfeld and Rogoff (1995a), it is important to take the current account dynamics into consideration when studying monetary policies. Thus, in this research, we use sluggish price-adjustment and monopolistic competition to introduce money into the model and study the impact of monetary policy on current account movements. As this is a two-country model, we can focus on international spillovers and macroeconomic interdependence in the world economy. Moreover, we use a non-linear solution method, which allows for a rigorous welfare analysis, by providing an index of social welfare - the expected utility of the representative agent in Home and Foreign countries.

In the literature, two lines of research are related to this paper. The first line of research uses new open economy macroeconomic models, following Obstfeld and Rogoff (1995a), to study the factors affecting the current account movements with monetary shocks. These research studies show that the initial current account response to an expansionary monetary shock depends on the values of the elasticity of substitution between the consumption of traded and non-traded goods (the "infratemporal effect") and the risk aversion measure (the "intertemporal consumption smoothing effect"), the degree of monopolistic competition, and the degree of local currency pricing (LCP). These current account responses are different from the small open economy model of the previous chapter. In a small open economy model, Lane (2001a) shows that the spillover between traded and non-traded consumption is always negatively correlated when the "infratemporal effect" dominates. In our two-country model, however, the spillover between traded and non-traded consumption is negatively correlated when the "infratemporal effect" dominates with a technology shock, but is mostly positively correlated with an expansionary monetary shock. The difference with the monetary shock from the small open economy model is mainly from the relative future traded goods price changes in the economy.
As in Lombardo (2002), we also show that in the two-country economy, changes in the monetary supply will generate changes in the goods price dynamics. This will affect the markup, and the current account will respond to the monetary shock in different ways when the markups are different. Betts and Devereux (2000a) show that, in a two-country world, monetary policy can have different impacts on the current account with pricing-to-market. We show how the price setting in home or foreign currencies will affect the current account dynamics. As found by Devereux (2000), when all export prices are set in the producers' currency, the current account response is dominated by the "expenditure-switching" effects of the exchange rate change. When all export goods are priced under pricing-to-market, the response of the current account depends on the "intertemporal consumption smoothing" effect.

The second line of research explores the optimal monetary policy response to domestic technology shocks in an open economy. Obstfeld and Rogoff (2002) argue that a monetary policy, in which the monetary authorities respond solely to their domestic shocks, delivers the best possible outcome. Corsetti and Pesenti (2004) show that, with local currency pricing, a common monetary policy is the optimal choice for all countries. Devereux and Engel (2003) find that optimal monetary policy requires a fixed exchange rate under LCP pricing. Thus, home and foreign monetary authorities respond identically to country-specific productivity shocks. If prices are set in the currency of producers, and the pass-through from exchange rate to consumer prices is complete, then the flexible exchange rate is a central part of the optimal monetary policy. Nonetheless, none of the above studies in the optimal monetary policy literature analyze the impact of the optimal monetary policy on the current account responses.

Our main contribution is the welfare analysis for the optimal monetary policies in this two-country economy, and the investigation of how the optimal monetary policies will affect the current account movements in the home country. Engel and Rogers (2006) show that the US may have had more favorable technology improvements than the rest of the world in recent years, which may contribute to the current account deficit. In our two-country model, therefore, we allow the optimal monetary policy to respond to an unexpected technology improvement shock in the home traded goods
sector.

In our welfare analysis, we consider the case where most of the firms use local currency pricing. The exchange rate pass-through is low. This has been shown empirically by Engel (1993), Engel and Rogers (1996), Engel (1999), and many others. We find that, with sticky prices and monopolistic competition, given a monetary supply rule, the no-commitment home individual optimal monetary policy in response to an unexpected positive home technology shock in the home country traded goods sector, is to increase the money supply for the home country when foreign monetary policy is absent. This optimal monetary policy will increase the home household’s welfare at the expense of the foreign household’s welfare with the terms of trade improving for the home country. The foreign individual optimal monetary policy is to increase the foreign money supply when the home monetary policy is absent. These two individual optimal monetary policies can improve the implementing country’s welfare through three channels - by matching the expansion of the real economy; exploiting the firms’ pricing decisions with markup distortions; and with favorable terms of trade changes. We also find that the optimal monetary policies of a supranational monetary authority is to increase the money supply in both home and foreign countries symmetrically and by a large amount. With low exchange rate pass-through, the best course of action for the supranational monetary authority is to avoid any asymmetric policy response to the asymmetric unexpected positive home technology shock. The real exchange rate is stable, the terms of trade between the home and foreign country do not change, and welfare improvement is symmetric for both countries with the optimal supranational monetary policy. These optimal monetary policies have a large impact on the current account initial responses to the home technology improvement.

We also show that, in the case of no monopolistic competition, with only sticky prices, a supranational authority’s optimal monetary policy is to increase the home and foreign money supply by a quantitatively small amount. A home policy maker, however, will still choose to increase home money supply to improve home welfare at the expense of the foreign country. With the home monetary injection, the home price level will increase, which will cause the home country to have more favorable
terms of trade. As for the foreign consumer, with a higher home price level, the foreign consumer will consume less home produced goods and work more to satisfy the increased demand from the home consumer. Thus, the foreign consumer will consume less and work more, and end up with lower welfare. The individual optimal monetary policies can improve the implementing country’s welfare through two channels - by matching the expansion of the real economy, and through favorable terms of trade changes. The current account still responds to the individual optimal monetary policies and the response may be substantial, in contrast to what was shown in the previous chapter. The supranational optimal monetary policy, however, only requires small monetary injections to the economy. As no markup distortion is present, the supranational optimal monetary policy is time-consistent. In this environment, the current account response to the optimal monetary policy of the supranational monetary authority will be quantitatively small.

The detailed structure of the work is as follows. The two-country model is presented in Section 2.2. Section 2.3 discusses the calibration, and Section 2.4 presents the current account dynamics to shocks. Section 2.5 studies the current account dynamics in response to the optimal monetary policies. Finally, Section 2.6 concludes the chapter.

2.2 Structure of the Model

2.2.1 General Features

This section reviews the main building blocks of the model. To capture the dynamics of the current account, we set up a perfect foresight two-country general equilibrium model with incomplete financial markets. The countries are denoted as home and foreign. Each country has a representative household, firms, a monetary authority and a domestic government. There are two sectors in each country - non-traded goods sector and traded goods sector. Foreign variables are marked by an "∗", and by an f subscript when necessary.
2.2.2 Market Structure

There are two types of goods in the economy: non-traded and traded goods. Final traded goods in this economy \((Y_{Tt})\) are produced by aggregating over a continuum of home goods indexed by \(i \in [0,1]\) along with aggregating over imported foreign goods indexed by \(i \in [0,1]\). The aggregation technology for producing final traded goods is:

\[
Y_{Tt} = \left[ b^\theta Y_{Tht}^{\frac{\theta}{\theta - 1}} + (1 - b)^\theta Y_{Tft}^{\frac{\theta}{\theta - 1}} \right]^{\frac{1}{\theta - 1}}
\]

where \(\theta > 0\) is the constant elasticity of substitution between home and foreign produced traded goods and \(b\) is the share of home traded goods in the traded goods price index. \(Y_{Tht}\) is an index of the home produced traded goods, \(Y_{Tft}\) is the home imported of the foreign produced traded goods. The non-traded good, home and foreign produced traded goods are in turn defined as:

\[
Y_{Nt} = \left[ \int_0^1 Y_{Nt(i)}^{1-\frac{1}{\lambda}} di \right]^{\frac{1}{1-\lambda}}
\]

\[
Y_{Tht} = \left[ \int_0^1 y_{Tht(i)}^{1-\frac{1}{\lambda}} di \right]^{\frac{1}{1-\lambda}}
\]

\[
Y_{Tft} = \left[ \int_0^1 y_{Tft(i)}^{1-\frac{1}{\lambda}} di \right]^{\frac{1}{1-\lambda}}
\]

The continuum of home produced non-traded goods are indexed by \(i \in [0,1]\). All goods are perishable. \(Y_{Nt(i)}\) \((y_{Tht(i)}, y_{Tft(i)})\) denotes date \(t\) non-traded (home produced, foreign produced traded) good \(i\) and \(\lambda > 1\) denotes the elasticity of substitution among the differentiated goods.

Final traded goods producers behave competitively, maximizing profit each period:

\[
Max \ \Pi_{Tt} = P_{Tt} Y_{Tt} - P_{Tht} Y_{Yht} - P_{Tft} Y_{Yft},
\]

where \(P_{Tt}\) is the overall price index of the final traded good, \(P_{Tht}\) is the price index of home goods and \(P_{Tft}\) is the price index of foreign goods, all denominated in the home currency. The price indices are defined as

\[
P_{Tt} = \left[ b P_{Tht}^{1-\theta} + (1 - b) P_{Tft}^{1-\theta} \right]^{\frac{1}{1-\theta}}
\]
It is assumed that fraction \( s \) of firms exhibit local currency pricing (pricing-to-market PTM), i.e., they set the price of goods in the currency of the buyer. And the remaining fraction \( 1 - s \) of firms exhibit producer currency pricing. \( p_{T_t} \) is the home currency prices, while \( p_{T_t}^* \) is the foreign currency price.

\[
P_{N_t} = \left[ \int_0^1 P_{N_t}(i)^{1-\lambda} di \right]^{\frac{1}{1-\lambda}}
\]

\( p_{Tht}(i) \) represents home currency price of the home produced good.

\[
P_{Tft} = \left[ \int_0^s p_{Tft}(i)^{1-\lambda} di + \int_s^1 (e_t p_{Tft}^*(i))^{1-\lambda} di \right]^{\frac{1}{1-\lambda}}
\]

where \( p_{Tft}(i) \) is home currency price of a foreign PTM good \( i \), while \( p_{Tft}^*(i) \) is foreign currency price of a foreign non-PTM good. The exchange rate (home unit cost of foreign currency) is given by \( e_t \). And the price index of home exports may be expressed by:

\[
P_{Tht}^* = \left[ \int_0^s p_{Tht}^*(i)^{1-\lambda} di + \int_s^1 (\frac{1}{e_t} p_{Tht}(i))^{1-\lambda} di \right]^{\frac{1}{1-\lambda}}
\]

where \( p_{Tht}^*(i) \) is foreign currency price of a home PTM good \( i \), while \( p_{Tht}(i) \) is home currency price of a home non-PTM good.

Given the aggregation functions above, demand will be allocated between home and foreign goods according to:

\[
Y_{Tft} = (1 - b)(\frac{p_{Tft}}{P_{Tt}})^{-\theta} Y_{Tt} \quad Y_{Tht} = b(\frac{P_{Tht}}{P_{Tt}})^{-\theta} Y_{Tt}
\]

with individual demands for non-traded and traded goods are given as:

\[
Y_{Nt}(i) = \left[ \frac{p_{Nt}(i)}{P_{Nt}} \right]^{-\lambda} Y_{Nt}
\]

\[
y_{Tht}(i) = \left[ \frac{p_{Tht}(i)}{P_{Tht}} \right]^{-\lambda} Y_{Tht}
\]

\[
y_{Tft}(i) = \left[ \frac{p_{Tft}(i)}{P_{Tft}} \right]^{-\lambda} Y_{Tft} \quad \text{for } i = 0, \ldots, s
\]

\[
y_{Tft}(i) = \left[ \frac{e_t p_{Tft}^*(i)}{P_{Tft}} \right]^{-\lambda} Y_{Tft} \quad \text{for } i = s, \ldots, 1
\]

Analogous conditions apply to the foreign country.
2.2.3 Firm Behavior

The firms hire labor $H_t$ at the nominal wage rate $W_t$, and labor is immobile internationally. Resetting prices is assumed to be costly because of quadratic menu costs. The period $t$ production functions of the firms producing non-traded good and traded good $i$ are concave and given by:

$$Y_{Nt}(i) = A_{Nt} H_{Nt}(i)$$
$$Y_{Tt}(i) = A_{Tt} H_{Tt}(i)$$

where $Y_{Nt}$ and $Y_{Tt}$ are the firms' outputs and $\alpha < 1, \gamma < 1$, while $A_{Nt}$ and $A_{Tt}$ are period $t$ labor productivities. $A_{Nt}$ and $A_{Tt}$ are exogenous random variables and subject to shocks.\(^2\)

In the non-traded goods sector, with sticky prices, firms choose $p_{Nt}(i)$ to

$$Max \pi_{Nt}(i) = p_{Nt}(i)Y_{Nt}(i) - W_t H_{Nt}(i) - \frac{\delta}{2} \left[ \frac{p_{Nt}(i)}{p_{Nt-1}(i)} - 1 \right]^2 p_{Nt}(i) Y_{Nt}(i)$$

where price adjustment cost is $\frac{\delta}{2} \left[ \frac{p_{Nt}(i)}{p_{Nt-1}(i)} - 1 \right]^2 Y_{Nt}(i)$. The parameter $\delta$ measures the degree of price stickiness. The higher is $\delta$, the more sluggish is the adjustment of nominal prices. If $\delta = 0$, then prices are flexible. The producer of non-traded good $i$ maximizes

$$\Pi_{Nt}(i) = \sum_{j=0}^{\infty} \Omega_{t,t+j} \pi_{Nt}(i)$$

where

$$\Omega_{t,t+j} = \beta^j \frac{C_{t+j}^{1-\sigma}}{P_{t+j} C_t^{1-\sigma}}$$

is the pricing kernel used to value date $t + j$ pay-offs. As firms are owned by the representative household, it is assumed that firms value future payoffs according to the household's intertemporal marginal rate of substitution in consumption. The firms' pricing decision is given by:

$$(1 - \lambda)Y_{Nt}(i) + \frac{\lambda}{\alpha} W_t H_{Nt}(i) - \delta \left\{ \frac{p_{Nt}(i)}{p_{Nt-1}(i)} - \frac{1 - \lambda}{2} \left( \frac{p_{Nt}(i)}{p_{Nt-1}(i)} - 1 \right) \right\}$$

in this perfect foresight model, $A_{Nt}$ and $A_{Tt}$ are assumed to be unity at steady-state.
In the domestically produced traded goods sector, the pricing-to-market firms choose $p_{Th(i)}$ and $p^*_i$ to

$$
Max \quad \pi_T(i) = p_{Th(i)}y_{Th(i)} + e_{i}p^*_i y_{Th(i)}* - W_{T}H_{T}(i) \\
- \frac{\delta}{2} \left[ p_{Th-1}(i) - 1 \right]^2 p_{Th(i)}y_{Th(i)} - \frac{\delta}{2} \left[ p^*_{Th-1}(i) - 1 \right]^2 p^*_i y_{Th(i)}*
$$

Non-PTM firms choose $p_{Th(0)}$ and $p_{Th(i)}$ to

$$
Max \quad \pi_T(i) = p_{Th(i)}y_{Th(i)} + e_{i}p_{Th(i)} y_{Th(i)}* - W_{T}H_{T}(i) \\
- \frac{\delta}{2} \left[ p_{Th-1}(i) - 1 \right]^2 p_{Th(i)}y_{Th(i)} - \frac{\delta}{2} \left[ p_{Th-1}(i) - 1 \right]^2 p_{Th(i)}y_{Th(i)}*
$$

The optimal price setting rule for domestic sales of all home firms ($i = 0, ..., 1$) is:

$$
(1 - \lambda) y_{Th(i)} + \frac{\lambda}{\gamma} W_{T}H_{T}(i)^{1-\gamma} y_{Th(i)} - \delta \left( \frac{p_{Th(i)}}{p_{Th-1}(i)} - 1 \right) p_{Th(i)}y_{Th(i)}*
$$

The optimal price setting rule for exports for PTM-firms ($i = 0, ..., s$) is:

$$
(1 - \lambda) e_{i}y^*_{Th(i)} + \frac{\lambda}{\gamma} W_{T}H_{T}(i)^{1-\gamma} y^*_{Th(i)} - \delta e_{i} \left( \frac{p^*_{Th(i)}}{p^*_{Th-1}(i)} - 1 \right) p^*_{Th(i)}y^*_{Th(i)}
$$

The optimal price setting rule for exports for non PTM-firms ($i = s, ..., 1$) is:

$$
(1 - \lambda) e_{i}y^*_{Th(i)} + \frac{\lambda}{\gamma} W_{T}H_{T}(i)^{1-\gamma} y^*_{Th(i)} - \delta e_{i} \left( \frac{p^*_{Th(i)}}{p^*_{Th-1}(i)} - 1 \right) p^*_{Th(i)}y^*_{Th(i)}
$$

If prices are perfectly flexible, the price-setting is a markup over the marginal cost. However, in the presence of price adjustment costs, price-setting deviates from the simple markup with additional forward-looking and backward-looking terms. Price stickiness generated through adjustment costs allows all firms to reset prices if the costs of price stickiness become large.

### 2.2.4 Household Behavior

The two countries are symmetric. The “Home” country is modeled directly, and the “Foreign” country can be derived using the same method. The home economy
is populated by a continuum of consumers/households of measure unity. The representative household is endowed with a certain amount of time, which is divided between leisure and work. She consumes two types of goods - traded and non-traded goods. The household can hold three nominal financial assets: non-interest bearing home money $M$, and two types of non-contingent bonds, one denominated in home currency, $B_h$, paying returns $i$, and the other denominated in foreign currency, $B_f$, paying returns $i^*$. She gets income from labor income $W$, profits from domestic firms and lump-sum government transfers, and pays back interest on the one-year bonds. The household derives utility from consumption $C_t$, and supplying labor $H_t$ lowers utility. For simplicity, real money balances $M_t^p$ are also introduced in the utility function. Preferences are additively separable in these three arguments.

The lifetime utility of the home representative household is:

$$
\max \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi}{1-\epsilon} \left( \frac{M_t}{P_t} \right)^{1-\epsilon} - \frac{\eta}{1+\psi} H_t^{1+\psi} \right]\right\} \quad (2.2.1)
$$

where $\beta \in (0, 1)$ is the discount rate. $\sigma$ is the inverse of intertemporal elasticity of aggregate consumption. $\epsilon$ is the inverse of consumption elasticity of money demand for households. $\psi$ is the inverse of the elasticity of labor supply. Supplying labor lowers utility. $\chi$ and $\eta$ are scale factors.

The consumption index $C_t$ is a CES aggregate of traded and non-traded goods:

$$
C_t = \left[ a^\rho C_{N_t}^{\frac{a-1}{\rho}} + (1-a)^\rho C_{T_t}^{\frac{b-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}
$$

where $\rho > 0$ is the constant elasticity of substitution between traded and non-traded goods and $a$ is the share of non-traded goods in the consumer price index. $C_{N_t}$ is an index of consumption of the non-traded goods, $C_{T_t}$ is the consumption of the traded goods.

The household can provide different types of labor services. There exists a continuum of labor types, indexed by $i \in [0, 1]$. Let $H_t(i)$ denote the number of hours of type $i$ labor. The variable $H_t$ that appears in the utility function is defined as a sum of labor services in non-traded and traded goods sectors:

$$
H_t = \int_0^1 (H_{N_t}(h) + H_{T_t}(h))dh
$$
Analogous conditions apply to the foreign country.

The household’s budget constraint in period $t$ is:

$$B_{ht} + e_t B_{ft} + \frac{\phi_d}{2} e_t (B_{ft} - \bar{B}_f)^2 + M_t + P_t C_t = (1 + i_{t-1})B_{ht-1} + e_t(1 + i_{t-1}^*)B_{ft-1} + M_{t-1} + T_t + W_t H_t + \Pi_t$$  \hspace{1cm} (2.2.2)

where $\frac{\phi_d}{2} e_t (B_{ft} - \bar{B}_f)^2$ is the household’s convex costs of adjusting foreign currency asset holding. $B_{ht}$ and $B_{ft}$ are stock of home and foreign currency bond holdings that become due in period $t$. $\bar{B}_f$ is the long-run level of foreign currency bond holding, and $\phi_d$ is a constant parameter defining the portfolio adjustment cost function. $i_{t-1}$ and $i_{t-1}^*$ are the nominal interest rate on the home and foreign bonds. The household owns the firms and $\Pi_t$ is the profit from the firms and $T_t$ is the government lump-sum transfer. The domestic households take $\bar{B}_f$ as given when deciding on the optimal holding of the foreign assets. The term $\frac{\phi_d}{2} e_t (B_{ft} - \bar{B}_f)^2$ can also represent a country-specific interest rate premium. Introducing the risk premium term as a function of debts forces wealth allocations in the long run to return to their initial distribution. Such a term is common device in macro models.\(^3\) In practice, the parameter $\phi_d$ will be set at a very low level in calibration of the model.

The household problem implies the following optimality conditions. First, households will smooth consumption across time periods according to:

$$1 = \beta(1 + i_t) \frac{C_{t+1}^{-\sigma}}{P_{t+1}} \frac{P_t}{C_t^{-\sigma}}$$  \hspace{1cm} (2.2.3)

Households prefer marginal utilities to be constant across time periods, unless a rate of return on saving exceeding their time preference induces them to lower consumption today relative to the future. Second, household’s optimal money demand schedule is:

$$\chi \left( \frac{M_t}{P_t} \right)^{-\gamma} = C_t^{-\gamma} \left[ 1 - \beta \frac{C_{t+1}^{\gamma}}{P_{t+1}^{\gamma}} \frac{P_t}{C_t^{\gamma}} \right]$$  \hspace{1cm} (2.2.4)

Third, optimal portfolio choices imply the interest rate parity condition:

$$1 = \frac{e_{t+1}}{e_t} \frac{1 + i_t^*}{(1 + i_t)(1 + \phi_d(B_{ft} - \bar{B}_f))}$$  \hspace{1cm} (2.2.5)

Fourth, households supply labor to the point that the marginal disutility of labor equals its marginal product:

\[ \frac{W_t}{P_t} = \eta C_t^\sigma H_t^\psi \]  

(2.2.6)

The home government issues the local currency, has no expenditures, and runs a balanced budget every period. The nominal lump-sum transfers from seigniorage revenues are then given by:

\[ T_t = M_t - M_{t-1} \]

We consolidate the public and private sectors to determine the resource constraint for the home economy. The current account may be computed in this model as:

\[ CA_t = \frac{(B_{ht} - B_{ht-1}) + e_t(B_{ft} - B_{ft-1})}{P_t} \]

\[ = \frac{i_{t-1}B_{ht-1} + e_t i_{t-1}B_{ft-1}}{P_t} + \frac{e_t P_t^* Y_{Th}^* - P_{Tft} Y_{Tft}}{P_t} \]  

(2.2.7)

The current account is determined by the interest payment for past debt and net exports. The real exchange rate is: \( q_t = \frac{e_t P_t^*}{P_t} \). Hence, an increase (decrease) in the real exchange rate represents a real depreciation (appreciation).

The home goods market clearing conditions are: domestically produced non-traded goods are all consumed domestically, domestically produced traded goods are consumed by both domestic and foreign consumers:

\[ Y_{Tt} = y_{Tht} + y_{Tht}^* \]  

(2.2.8)

The home bonds market clearing conditions are:

\[ B_{ht} + B_{ht}^* = 0 \]  

(2.2.9)

Equilibrium is a set of 33 equations determining 33 sequences: \( B_{ht}, B_{ft}, B_{ht}^*, B_{ft}^*, e_t, i_t, i_t^*, W_t, W_t^*, C_t, C_t^*, H_{Nt}, H_{Nt}^*, H_{Tt}, H_{Tt}^*, Y_{Th}, Y_{Tht}, Y_{Tht}^*, Y_{Tht}^*, Y_{Tft}, Y_{Tft}^*, P_t, P_t^* \) and all the prices for the individual traded and non-traded goods. The 33 equilibrium conditions are: the definition of demand conditions for non-traded goods and home/foreign produced traded goods, the overall price index, the home consumed traded goods
price index (2.2.1), the price index for imports, the four price setting rules for domestic firms, the money demand condition (2.2.4), labor supply condition (2.2.6), the Euler equation (2.2.3), the interest parity condition (2.2.5), market clearing conditions for goods (2.2.8) and bonds (2.2.9), along with foreign counterparts for each of the above condition and the balance of payments constraint (2.2.7).

The equilibrium conditions are all used in the form of non-linear equations. We use all the equations to solve for trajectories of the 33 sequences. In our two-country model, the foreign bond holding adjustment cost ensures the stationarity of the model. The cost \( \frac{1}{2} e_t (B_f - \bar{B}_f)^2 \) captures the situation when the economy-wide holdings of foreign-currency denominated assets are different from the steady-state level, \( \bar{B}_f \), individual agents have to pay extra cost. When the equilibrium aggregate level of foreign debts in the economy is equal to \( \bar{B}_f \), this cost will disappear. This yields unique steady-state levels of real variables.

2.3 Baseline Calibration

We use a non-linear solution technique to evaluate the welfare of the representative household. The solved model is also used to extract impulse responses subject to technology and monetary shocks. Lipton et al. (1982) give a detailed discussion.

Our calibration serves to illustrate the properties of the model and is not intended to match any particular pair of economies. We assume that Home and Foreign are two countries equal in size. The elasticity of substitution between non-traded and traded goods is set at 0.75, which follows directly from Ostry and Reinhart (1992). The elasticity of substitution between home and foreign traded goods is set to 2, following Obstfeld and Rogoff (2005). The share of local pricing firms \( s \) is set to 0.9, according to estimates of Bergin (2006). To see how the local currency pricing will affect our welfare analysis, we also set the share of local pricing firms \( s \) to 0.1 for high exchange rate pass-through. We set the rate of time preference at 0.01, so that the subjective discount factor is 0.99. We follow Rotemberg and Woodford (1998) in setting the inverse of the elasticity of labor supply to 0.5, so that \( \psi = 0.5 \). The degree of monopolistic competition is also taken from Rotemberg and Woodford (1998), where they set \( \lambda = 7.66 \), which implies an average markup of
15%. We follow the open economy macro literature in choosing parameter values for our basic experiments. The inverse of the intertemporal elasticity of substitution $\sigma$ is set at 2, following Backus, Kehoe, and Kydland (1995). The value of $\eta$ is a scale factor for labor supply, so we set it to 2.5. The share of non-traded goods in the consumer price index $a$ is set at 0.5. The share of home goods in the traded goods aggregator, $b$, is set at 0.7, reflecting the 30% share of imports in GDP for the home country. We assume that non-traded goods production is relatively labor intensive, with $\alpha = 0.7$, and traded goods is relatively non-labor intensive, with $\gamma = 0.3$. The consumption elasticity of money demand for the household is equal to $\frac{1}{\varepsilon}$ in this model. According to Mankiw and Summers (1986), this variable is very close to unity and hence, $\varepsilon$ is set to 1. For the price stickiness, the price adjustment cost is set at $\delta = 20$, which implies that 87% of the price has adjusted 4 periods after a monetary shock. Bond adjustment cost, $\psi_d = 0.007$, following the estimate of Schmitt-Grohe and Uribe (2003), and is necessary in to negate the unit root associated with the incompleteness of the asset markets. For the initial debt/asset position, we set it to zero first and then change it to non-zero to see whether the non-zero net foreign debt/asset will affect the current account dynamics. Table 2.1 reports the baseline calibration assumptions.

We calibrated two types of shocks: shocks to the traded goods sector and shocks to the monetary supply. The technology shock process takes the following form:

$$\ln A_{Tt+1} = \mu_1 \ln A_{Tt} + \epsilon_{Tt+1}$$  \hspace{1cm} (2.3.10)

where $\epsilon_{Tt+1}$ is the technology shock in the traded good sector, $\mu_1 = 0.42$ measures the persistence of the technology shock (following Schmitt-Grohe and Uribe (2003)). For simplicity, we assume that money supply is characterized in terms of a money supply rule. The money supply rule is:

$$\ln M_t = \ln M_{t-1} + \mu (\ln M_{t-1} - \ln M_{t-2}) + \epsilon_{Mt}$$  \hspace{1cm} (2.3.11)

where $\mu = 0.5$ measures the persistence of the monetary shock, and $\epsilon_{Mt}$ is the unexpected money supply shock.
2.4 Impulse Response Analysis for the Current Account

In this two-country model, we explore the factors which may affect the qualitative responses of the current account to shocks. We have: 1) the intertemporal elasticity of substitution \((\frac{1}{\sigma})\), which governs whether or not the household wants to consume more today or tomorrow; 2) the elasticity of substitution between traded and non-traded goods \((\rho)\), which shows whether or not the consumer prefers more traded or non-traded goods; 3) the elasticity of substitution between home and foreign traded goods \((\theta)\), which demonstrates whether or not the consumer prefers home or foreign goods; 4) the elasticity of substitution between varieties of differentiated goods \((\lambda)\), an elasticity that reveals the equilibrium markup for the firms; and 5) the share of firms that set the price of goods in the currency of the buyer \(s\).

The effects of intertemporal elasticity of substitution and the elasticity of substitution between traded and non-traded goods are different from the small open economy case. The elasticity of substitution between home and foreign traded goods \((\theta)\), and the local currency pricing cannot be well discussed in a small open economy framework. In our welfare analysis, we also investigate whether or not these factors have any welfare implication.

2.4.1 Intertemporal and Intratemporal Elasticity of Substitution

Driskill (2001) illustrates that, in a small endowment economy with non-traded goods, the key effects for transmission of shocks to the current account depends on the relative price of traded and non-traded goods. He also shows that whether or not the trade balance effect is dampened depends on the sign of the cross-partial derivatives with respect to traded and non-traded goods of the utility function. Lane (2001a) also shows that, in a small open economy, the intertemporal elasticity of substitution \((\frac{1}{\sigma})\) and the elasticity of substitution between traded and non-traded goods \((\rho)\) affect the current account responses qualitatively. In the small open economy, the price of traded goods is the nominal exchange rate, while in our two-country economy, the price of traded goods is a CES aggregation of the price of home produced and
imported foreign produced traded goods. In the two-country world, the effect of the existence of non-traded goods on current account is not the same as in the small open economy.

From the household’s first-order conditions, we can get:

\[
\frac{C_{T_{t+1}}}{C_{T_t}} = \beta(1 + i_t)\left[\frac{P_{t+1}}{P_{t+1}}\right]^{\frac{1}{\sigma}} \left[\frac{P_{t+1}}{P_{t+1}}\right]^{\frac{1}{\sigma}} (2.4.12)
\]

The consumption growth depends on the sequence of relative prices, \((\frac{P_{t+1}}{P_{t+1}})^{\frac{1}{\sigma}}\) and the 2nd term \(\left[\frac{P_{t+1}}{P_{t+1}}\right]^{\frac{1}{\sigma}}\). In Lane (2001a) and Driskill (2001), the consumption growth only depends on the second term \(\left[\frac{P_{t+1}}{P_{t+1}}\right]^{\frac{1}{\sigma}}\). For this term, this is the price ratio of current and future aggregate price level relative to the price of traded goods. If \(\frac{P_{t+1}}{P_{t+1}}\) is currently low compared to its future value, this encourages present consumption over future consumption, i.e. \(C_{T_t}\) will tend to increase. Meanwhile, the relatively higher future traded goods price also encourages consumption substitution from traded goods to non-traded goods, i.e. \(C_{T_t}\) will decrease. The former effect dominates if the intertemporal elasticity of substitution \(\frac{1}{\sigma}\) is greater than the infratemporal elasticity of substitution \(\rho\), i.e. \(\frac{1}{\sigma} > \rho\). Considering the above effects, we can see that non-traded and traded goods consumption is highly positively correlated when \(\frac{1}{\sigma} > \rho\). We also have the extra term \((\frac{P_{t+1}}{P_{t+1}})^{\frac{1}{\sigma}}\) also affecting the consumption growth.

As \(\sigma\) increases, the intertemporal elasticity of substitution declines, agents are less willing to smooth consumption across states of nature and time. As a result, relative consumption moves by less than the real exchange rate, so that the current account improves more. Figure 2.1 shows the result when \(\frac{1}{\sigma} < \rho\) and \(\frac{1}{\sigma} > \rho\) with a 1% positive technology shock in the home traded goods sector. The technology shock follows equation (2.3.10). With higher \(\sigma\), the current account responds less to the shock due to the agents’ unwillingness to smooth consumption across time. With the positive technology shock, home traded goods’ production is more efficient, the real prices of the home produced traded goods has to decrease to clear the market. The relative price changes lead to a negative correlation between traded and non-traded goods consumption when \(\frac{1}{\sigma} < \rho\) and a positive correlation when \(\frac{1}{\sigma} > \rho\). The impulses
are percentage deviations from the initial steady-states except for the current account responses.

In the case of an expansionary monetary shock, the household will use more money to buy consumption goods, and both traded and non-traded goods consumption will increase. The monetary shock follows equation (2.3.11). Now, the correlations between the traded and non-traded goods consumption are both positive. As the domestic currency is depreciating, the traded goods consumption increase will be less when \( \sigma \) is high, as the household is less willing to smooth consumption across time. Figure 2.2 shows that when \( \frac{1}{\sigma} < \rho \), the current account improves following the monetary shock. When \( \sigma \) is low, the household is willing to smooth consumption, with higher money holding, and will increase its traded goods consumption, so the current account deteriorates.

In this two-country model, the non-traded goods and traded goods consumption are not always negatively correlated when \( \frac{1}{\sigma} < \rho \). In the small open economy, the non-traded goods and traded goods consumption are always negatively correlated when \( \frac{1}{\sigma} < \rho \). This is explained by the relative future traded goods prices playing one more role in the traded goods consumption growth in the two-country world.

2.4.2 Home and Foreign Goods Elasticity of Substitution

In a model without non-traded goods, the cross-country substitutability \( \theta \) captures the sensitivity of the consumption allocation between home and foreign, with respect to the terms of trade.\(^4\) This shows whether goods produced in different countries are poor or close substitutes. When \( \theta > 1 \), home and foreign produced goods are close substitutes in consumption, and a positive correlation exists between the terms of trade and the current account. When the relative price of imported goods increases, households substitute away from imported towards home produced goods, and with other things remaining constant, the current account improves. While \( \theta < 1 \), home and foreign produced goods are poor substitutes, the increase of the imported goods' relative price will cause the current account to worsen. Empirical evidence suggests that values of \( \theta \) are above unity. Hence, in the following analysis,
we focus on the case where $\theta > 1$.

2.4.3 Local Currency Pricing

We have assumed that a fraction $s$ of firms use local currency pricing when pricing their exports. Devereux (2000) shows that, when all export prices are set in the producers’ currency, the current account responses are dominated by the “expenditure-switching” effects of exchange rate change. When export prices are all set by pricing-to-market, the response of the current account depends on the intertemporal “consumption smoothing” effect.

In the Mundell-Fleming model, the trade balance will improve if the Marshall-Lerner conditions hold with a currency devaluation. This is an “atemporal condition” that hinges on the elasticities of demand for home and foreign traded goods. As we have shown, the intertemporal elasticity of substitution ($\frac{1}{\theta}$) and the cross-country substitutability $\theta$ affect the current account responses through the intertemporal “consumption smoothing” considerations and the “atemporal elasticity” effect. When all export prices are set in the producers’ currencies, the “expenditure-switching” effects of the exchange rate change will dominate the response of the current account. On the other hand, when all export prices are set in the buyers’ currencies, the “expenditure-switching” effect is irrelevant, since exchange rate movements will not affect the consumption prices. As the consumption prices do not change, the Marshall-Lerner conditions are also irrelevant. Now, the impact of shocks on the current account movements depends on the intertemporal consumption smoothing in consumer preferences. To demonstrate this point, we change our basic calibration to $\theta = 0.75$.

In our calibration, we use more realistic pricing fractions. We have high local currency pricing, with 90% of the firms pricing their exports in the foreign currency $s = 0.9$; and low local currency pricing, with 10% of the firms pricing their exports in the foreign currency $s = 0.1$. Now, the impact of a monetary expansionary shock on the current account is determined by the relative strength of the “atemporal elasticity” of substitution between traded goods and the “intertemporal elasticity” of substitution between future and current consumption. The monetary shock follows
equation (2.3.11).

When both the foreign and the home country are symmetric ($b = 0.5$), and the elasticity of substitution between home and foreign produced traded goods is low, $\theta = 0.75$, the higher fraction $s = 0.9$ of firms using local currency pricing will cause the current account to respond more (0.1% of GDP), compared to the situation with a low degree of PTM ($s = 0.1$), which will cause a smaller (0.005% of GDP) current account improvement. With higher $s$, the consumption smoothing effect dominates, and the expansionary monetary shock will cause the current account to improve more. Figure 2.3 shows the current account responses with different levels of local currency pricing.

According to another calibration, $b = 0.7$, the home country has a larger share in the traded goods consumption. Again, lower $s$ means a lower consumption smoothing effect. With the home country’s higher share in the consumption, lower $s$ means the “consumption-switching” effect dominates, and the current account improves more with $s = 0.1$. Figure 2.4 shows the current account responses with different levels of local currency pricing.

2.4.4 Markup

Changes in monetary supply generate changes in $\frac{P_a(i)}{P_{a-1(i)}}$ dynamics, and this effect exists for all prices. These changes will affect the markup and the current account. With monopolistic competition, the output falls below the competitive level. At steady-state, as $\lambda$ increases, the markup decreases. With less monopolistic competition, the firms are more competitive and have higher output. The surprise monetary expansion shock will increase the output and tend to correct the monopolistic distortion. In the case of our basic calibration, with $\frac{1}{\sigma} < \rho$, the current account improves upon the monetary shock. With a higher $\lambda$, the monopolistic distortion will be less, and the output level at the steady-state will be less distorted, to indicate the convergence to high output level without the monopolistic competition. The output expanding effect of the monetary shock will be less effective with less monopolistic distortion. The home traded goods consumption will increase less with less monopolistic distortion, which will be reflected by either more exports or less imports. Under
the specification $\frac{1}{\sigma} < \rho$, with higher $\lambda$, the positive current account response to the monetary shock will become less significant. Figure 2.5 demonstrates the current account responses to an unexpected 1% positive money growth shock with different levels of markup. With the lower markup, the net exports in the current account (2.2.7) deteriorates when $\frac{1}{\sigma} < \rho$. This is the same result as found by Lombardo (2002).

Nevertheless, Figure 2.6 shows that with $\frac{1}{\sigma} > \rho$, the current account improves with the lower markup. This can be explained by $\frac{1}{\sigma} > \rho$, as the home traded consumption increases, it will increase less with a higher $\lambda$. Now, with the higher $\lambda$ and lower markup, the home country’s imports increase less, and therefore, the current account improves. From Figure 2.5 and Figure 2.6, we can see that the current account responds by less to the monetary shock with less monopolistic competition.

2.4.5 Non-Zero Initial Debt/Asset Positions

As documented by Lane and Milesi-Ferretti (2001), net foreign assets over GDP vary across countries and are different from zero. In this section, we investigate whether or not the initial net debt/asset position will affect our current account analysis. We find that, with either a net debt or an asset position, the result impulse responses are very much the same as in the case of no initial debt/asset. This is because neither the temporary technology shock in the traded goods sector nor the temporary money growth shock will affect the interest rate enough for the interest payment to matter in this two-country economy.

2.5 Optimal Monetary Policies

In this section, we use the two-country model to study the optimal monetary policy for each country and the optimal monetary policy for a supranational monetary authority. We still use the technology shock as in Equation (2.3.10) in the home traded goods sector. Engel and Rogers (2006) show that the US may have had more favorable technology improvements than the rest of the world, in recent years, which may contribute to the current account deficit. So, in our two-country model, we have the optimal monetary policy to respond to the home unexpected technology improve-
merit shock in the traded goods sector. We study the case where the net foreign asset is initially zero, then we explore whether or not the net debt/asset holdings will affect our welfare results. We assume that the monetary authority receives perfect information about the current state of the economy. The optimal active monetary policy is thus money supply responding instantly to the technology shock. Upon the unexpected technology shock in the home traded goods sector, the monetary authority immediately realizes what the temporary technology shock is, and sets an appropriate money supply to maximize the welfare of the household. The money supply rule is the same as in Equation (2.3.11). Now, with both home and foreign monetary authorities, different objectives can exist for the optimal monetary policy. 1) The home monetary authority can choose its optimal monetary policy to maximize the home household’s welfare, taking the foreign monetary authority’s current policy as given. 2) The foreign monetary authority can choose its optimal monetary policy to maximize the foreign household’s welfare, taking the home monetary authority’s current policy as given. 3) We assume that a supranational monetary authority is in place.\textsuperscript{5} It can choose the optimal monetary policy for both home and foreign monetary supply, to maximize the home and foreign household’s welfare. In our model, no interaction occurs between the home and foreign monetary authorities, and they each take the other’s current policy as given.

2.5.1 Optimal Monetary Policies With Markup Distortion

In this economy, the shock is an unexpected technology improvement in the home traded goods sector, which is an asymmetric technology shock. When markup distortion is present in the economy, the monetary authorities have perfect information on the way in which firms set their prices. So, with the technology shock, the monetary authority can take advantage of the price setting behavior of the firms. Due to the monopolistic competition in both the traded and non-traded goods sectors, output is sub-optimally low. Domestic output is demand-determined in the short-run because of the slow price adjustment. With slow price adjustments, the monetary expansion can boost the current demand for the non-traded goods consumption above the nat-

\textsuperscript{5}We will study the case of the Nash game in future research.
ural level of output in the short-run. This is the no-commitment monetary policy, as discussed in detail for the small open economy model.

Next, we focus on the welfare implications of the three different optimal monetary policies.

1) The home monetary authority chooses its optimal monetary policy to maximize the home household’s welfare, taking the foreign monetary authority’s current policy as given. In our model, no interaction occurs between home and foreign monetary authorities. The home monetary authority can use expansionary monetary policy to match the real expansion of the economy. With our basic calibration, 90% of firms price their exports in the buyers’ currency, and when the foreign monetary authority does not respond to the home technology shock, the optimal monetary policy for the home country is to increase the home money supply by 15%, which will improve the home household’s welfare by nearly 0.05%. With this home monetary expansion, the home currency will depreciate and the home country’s terms of trade will improve. With most firms using LCP, the exports are mostly in foreign currency, and the exchange rate depreciation will cause the export prices in the domestic currency to increase; but since the imports are mostly priced in home currency, the depreciation will not change the home import prices. Thus, the home country’s terms of trade will improve. Part of the home country’s welfare improvement comes from its terms of trade improvement. This optimal monetary policy will decrease the foreign household’s welfare by 0.04%. We can see that the total welfare of both home and foreign countries is still improving with this policy, and the total welfare of both home and foreign increases by 0.004%. The total welfare improvement comes from two channels. First, the home real economy is expanding with the technology improvement, and the matching with home money expansion is efficient. Second, with the markup, the monetary authority can expand the economy by pushing the output above the natural rate level. This will improve the household’s welfare.

2) The foreign monetary authority chooses its optimal monetary policy to maximize the foreign household’s welfare, taking the home monetary authority’s current policy as given. When the home monetary authority is not responding to the technology shock, the foreign monetary authority can increase its monetary supply by 15% to
maximize the foreign household’s welfare by nearly 0.05%. As in case 1), the home welfare is decreased by 0.04%, while the total welfare improves by 0.004%. The home welfare is lower because of the unfavorable terms of trade. With the asymmetric home technology shock, we can see that the home and foreign monetary authorities can have the same response.

3) A supranational monetary authority chooses the optimal monetary policy for both home and foreign monetary supplies to maximize the home and foreign household’s welfare. The home money supply should increase by 14% and, at the same time, the foreign monetary authority increases its money supply by the same amount, 14%. Then, the welfare of home and foreign countries each increases by 0.007% and the total welfare of both countries improves by 0.007%.

In our basic calibration, the exchange rate pass-through is low: 90% of the firms set their exports in the buyers’ currency. In this case, a supranational monetary authority has a symmetric monetary policy for both home and foreign counties. Thus, the real exchange rate is very stable. Corsetti and Pesenti (2004) show that the use of exchange rate for stabilization purposes would entail excessive welfare costs, in the form of higher import prices and lower purchasing power across countries. The best course of action for the supranational monetary authority is to use symmetric policy to respond to the asymmetric home technology shock.

Figure 2.7 shows the current account responses to an unexpected 1% increase of the home traded goods technology with different optimal monetary policies. Upon the technology shock, the traded goods sector will increase its production since it will be more efficient. With our basic calibration, the magnitude of the current account response is 0.08% of the current period GDP, at the time of the technology shock. With home or foreign optimal monetary policy, the current account has big movements. The individual home or foreign optimal monetary policy tends to inject a lot of money into the economy. As Betts and Devereux (2000b) show with LCP, an unanticipated home monetary expansion will increase the home welfare and decrease the foreign welfare. The welfare improvement for the individual country mainly comes from the terms of trade improvement following the injection of money. With the injection of money, the current account has excessive movement as the result
of the individual country’s optimal monetary policy. This suggests a coordination gain from having a social planner. The social planner’s optimal monetary policy eliminates the terms of trade effect caused by the welfare improvement for a single country, and the supranational monetary authority will maximize the welfare of both countries. The symmetric optimal monetary policy from the social planner also has a big impact on the current account movements. The welfare improvement for the social planner’s optimal monetary policy comes from matching the expanding real economy and the markup distortion. This supranational optimal monetary policy also has a certain impact on the movements of the current account.

2.5.2 Optimal Monetary Policies With No Markup Distortion

As the firm markup enables the monetary authorities to exploit the firms’ pricing decision, the monetary authorities can expand the economy above its long-run output level. This policy is not time-consistent. In this section, we use firm subsidies to eliminate the markup distortion. After eliminating the markup distortion, the monetary authorities use time-consistent monetary policies. The three different optimal monetary policies are as follows:

1) The home monetary authority can use expansionary monetary policy to match the real expansion of the economy. The optimal monetary policy for home is to increase the home money supply by 15%, which will improve the home household’s welfare by nearly 0.03%. With this home monetary expansion, the home currency will depreciate and the home country’s terms of trade will improve with most of the firms using LCP. This optimal monetary policy will decrease the foreign household’s welfare by 0.04%. Moreover, the total welfare of both the home and the foreign countries decreases by 0.0002%. The total welfare deteriorates. Now without markup distortion, the monetary authority cannot exploit the firms’ pricing decisions, and the home optimal monetary policy causes an excess movement in the exchange rate, which is a welfare cost in the economy.

2) The foreign monetary authority chooses to increase its monetary supply by 15%

\footnote{Please see Chapter 1 for a detailed discussion about firm subsidy.}
to maximize the foreign household’s welfare. This will increase the foreign household’s welfare by nearly 0.03%, and the home welfare is decreased by 0.04%, while total welfare deteriorates by 0.0002%. The home welfare is lower because of the unfavorable terms of trade. With the asymmetric home technology shock, we can see that the home and foreign monetary authorities can still have the same response.

3) A supranational monetary authority chooses the following optimal monetary policy: the home money supply should increase by 0.3% and, at the same time, the foreign monetary authority increases its money supply by 0.2%. Then, the welfare of the home country increases by 0.0004% and the welfare of the foreign country decreases by 0.0004%, while the total welfare of both countries improves by 0.000003%. The welfare gain from this optimal monetary policy comes from the matching of the real economy expansion.

With no markup distortion, the individual country’s optimal monetary policy is actually total welfare deteriorating. This confirms the results of Corsetti and Pesenti (2004) in that the use of exchange rate for stabilization purposes would entail excessive welfare costs. The optimal monetary policy of the individual home country and the foreign country is still welfare improving for the single country, as it mainly comes from the favorable terms of trade change because of the LCP. Figure 2.8 shows the initial response of the current account to a 1% home traded goods sector technology improvement, with and without the optimal monetary policies. As the home monetary expansion is welfare improving for a single country, the optimal monetary policy for the home country is to inject lots of money into the home country to get a favorable welfare improvement. The individual optimal monetary policy still has a great impact on the current account dynamics. With no markup distortion, however, the supranational authority’s optimal monetary policy is time consistent. It only requires a small monetary injection in both countries to match the real economy expansion. Now, the current account responses to the home technology shock with no monetary policy is almost the same as with an optimal monetary policy chosen by a supranational monetary authority. The current account response to the home technology shock is fairly efficient even without any monetary policy from the social planner’s view.
2.5.3 Discussions

We also explore the case where the technology shock is not limited to the traded goods sector. With technology shocks in both the traded and non-traded goods sectors, we get similar result for our welfare analysis. As documented by Lane and Milesi-Ferretti (2001), net foreign assets over GDP vary across countries and are different from zero. In this section, we investigate whether or not the initial net debt/asset position will affect our welfare analysis. The result is the same as in the case of no initial debt/asset. We find that, with either a net debt or an asset position, the welfare improvement is still quantitatively small from the supranational optimal monetary policy with no markup distortions to a positive home traded good sector technology shock.

2.6 Conclusions

In this paper, we show that the current account dynamics depends on the degree of intratemporal and intertemporal elasticities of substitution, the degree of monopolistic competition, and the degree of local currency pricing. With monopolistic competition, surprise monetary expansion responding to an unexpected technology shock, will increase the home household’s welfare at the expense of the foreign country’s welfare, with most of the firms using LCP. We also study the case where a time-consistent optimal monetary policy is used. With only one distortion - sticky prices, for a supranational monetary authority, the current account response to the optimal monetary policy is quantitatively small. For a home monetary authority, however, a home monetary injection at the time of the technology shock can still improve the home household’s welfare with most firms pricing their exports in the buyers’ currency.
Table 2.1: Baseline Calibration for Chapter 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2</td>
<td>$a$</td>
<td>0.5</td>
<td>$\rho$</td>
<td>0.75</td>
<td>$\delta$</td>
<td>20</td>
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<tr>
<td>$\gamma$</td>
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<td>$b$</td>
<td>0.7</td>
<td>$\theta$</td>
<td>2</td>
<td>$\lambda$</td>
<td>7.66</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.7</td>
<td>$\psi$</td>
<td>0.5</td>
<td>$\epsilon$</td>
<td>1</td>
<td>$\chi$</td>
<td>0.005</td>
</tr>
<tr>
<td>$\psi_d$</td>
<td>0.007</td>
<td>$\eta$</td>
<td>2.5</td>
<td>$s$</td>
<td>0.9</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Figure 2.1: An Unexpected Positive 1% Technology Shock in the Traded Good Sector with Different Consumption Smoothing
Figure 2.2: An Unexpected Positive 1\% Monetary Shock with Different Consumption Smoothing
Figure 2.3: An Unexpected Positive 1% Monetary Shock with Different Exchange Rate Pass-Through \( [b = 0.5; \theta = 0.75] \)
Figure 2.4: An Unexpected Positive 1% Monetary Shock with Different Exchange Rate Pass-Through \( b = 0.7; \ \theta = 0.75 \)
Figure 2.5: An Unexpected Positive 1% Monetary Shock With Different Markups [$\sigma = 2$]
Figure 2.6: An Unexpected Positive 1% Monetary Shock With Different Markups [$\sigma = 0.5$]
Figure 2.7: Home 1% Technology Shock With and Without the Optimal Monetary Policy
Figure 2.8: Home 1% Technology Shock With and Without the Optimal Monetary Policy With Firm Subsidies
Chapter 3

Software Upgrade, Consumer Behavior and Software Vendor’s Choice

3.1 Introduction

In the last two decades, the software industry has grown rapidly. Prud’homme and Yu (2002) show that total software investments increased by almost 1400% from 1981 to 2001. With the rapid growth of the software industry, decisions about software upgrade have become increasingly important to software vendors. The main reasons for upgrading software, from the viewpoint of the software vendor, are as follows:

1) Software is a durable good. Without any new functionality, existing consumers will not buy the software again. So, “planned obsolescence”, where the current software is made deliberately out-of-date due to the release of a new version, can help the software vendor to extract more surplus from existing consumers;

2) As the market demand evolves, consumers need additional software functions. Through upgrades, Ng (2001) shows that the software vendor can not only better serve the existing market but also stimulate new demand;

3) If operating system software is upgraded, Marinoso (2001) shows that application software may need to be upgraded to ensure compatibility; and,

4) The rapid growth of information technology results in faster CPU, and larger RAM and hard disks, all of which supply more flexibility for new generations of software.
Some constraints affect the software vendor, in relation to software upgrades. First, upgrading software requires an extra investment and a great effort. Second, if the old version is not withdrawn from the market after the new version is introduced, the old software competes with the new, and can draw potential consumers away from the new version. We call this a “cannibalization problem”. If the old version is retired, the software vendor may lose those consumers who do not care about the enhanced features of the new version and would prefer using the less expensive older version. Third, while such upgrades usually include many worthwhile features, software users are beginning to resist the steady stream of upgrades. For example, Qiu (2002) estimates that there had been still millions consumers were still using Windows 3.X and Windows 95, even after Windows XP was released. Software users complain that upgrading software is expensive as they need to spend time and money to learn how to install and use the new versions. Specifically, Bulkeley (1990) finds that the compatibility problem of new versions of software forces users to upgrade their hardware, which can be even more expensive than the upgraded software. Fudenberg and Tirole (1998) argue that some consumers may choose to leapfrog the new version. Consequently, the “disappearance market” tends to grow (i.e., a group of people choose to withdraw from the market) as software users become less sensitive to technological progress.

Obviously, for the software vendor, the problem of maximizing profit or market share under the above conditions or environment is a big issue. To achieve a maximization goal, the software vendor needs to know how consumers respond when the vendor releases a new version of software onto the market. Accordingly, the vendor needs to make the right decision on several issues involving software upgrades:

1) While the new version is released, should the old version still be kept in the marketplace? If so, for how long?

2) Should price discrimination be undertaken, and how?

Several earlier papers have investigated software upgrades from the technical viewpoint or from the view of social welfare.\(^1\) Another stream of software research

\(^1\) Please see Xie and Hong (1998), Xie and Hong (1999), Shinohara et al. (1997), Ellison and Fudenberg (2000) and Fudenberg and Tirole (1998).
focused on software editions (static versioning), that is, delivering several versions with different quality levels (or feature sets) such as a professional edition, a student edition and a home edition at the same time. To the best of our knowledge, to date no research has been conducted to fully and systematically investigate the relationship between consumer behavior and software upgrading decisions for the software vendor. This study contributes to the literature by exploring this untapped area. We model the consumer behavior of software users facing software upgrading. Based on the behavior, we investigate the software vendors’ choices on quality levels, pricing strategies, customer services, and market structures.

We study the behavior of software users under two market structures: Structure 1 - where both the old and new versions coexist in the market; and Structure 2 - where only the new version is available in the market. We find that keeping the old version in the market is an effective tool to reduce the “disappearance” of existing users. The “cannibalization” effect between the old version and new version, for the existing users, is different from that for new users. This finding is non-intuitive since we believed that the new version could be substituted by the old version for both existing and new users. We also find that, when the price of the old version is low enough, the total market share under market structure 1 is greater than the market share under market structure 2. Nevertheless, even if the market share under market structure 1 is greater than that under market structure 2, the total profit under market structure 1 is not necessarily greater than that under market structure 2. We show the conditions under which the software vendor should choose market structure 1 to maximize the total profit. In addition, one important finding in this study is that, as software users become more heterogeneous, the difference between the two market structures diminishes, in terms of market share and profit to the software vendor. Given the lack of existing research on the choice of market structure for the software vendor, our work is directed towards filling the gap and providing some insights for the software vendors’ optimal upgrade choices. We argue that, for a new version, the software vendor can charge a lower price to existing users and a higher price to new users, under either of two market structures, providing that the

\(^2\text{Please see Varian (1997), Raghunathan (2000) and Haruvy and Prasad (1998).}\)
software consumers are sufficiently heterogeneous. Furthermore, if the price of the new version is quite high, the software vendor can charge a lower price to existing users and a higher price to new users for the old version. This finding is different from previous studies including Yang (1997b), where the condition to implement discriminated prices to existing users and new users is based on the relative quality of the new version compared to the old version.

The software vendor has some advantages in conducting upgrades, compared to physical durable goods (such as hardware) producers for two key reasons. First, software is an information good and its marginal cost is almost zero. Second, unlike the secondhand markets for other durable goods, Fudenberg and Tirole (1998) show that the secondhand market for software is restricted by difficulty in preventing duplication, and thus, copyright infringement. Legitimately, only the software vendor has the right to sell the old version in the market. The prohibition of a secondhand market for software is motivated by the difficulty in preventing duplication and thus copyright infringement in the presence of such a market. Or equivalently, the software vendor is able to completely control the secondhand market of his products. On the contrary, as hardware users can also sell their used items, the hardware vendor is not the only supplier of the old-model hardware in the market.

In the next section we provide a review of relevant research. In Section 3.3, we present our formal models of both existing users' and new users' behaviors under two different market structures. The optimal choices of the software vendor are analyzed in Section 3.4, which is followed by our discussion of results in Section 3.5. The concluding remarks and summary are arranged in Section 3.6.

3.2 Literature Review

Many durable goods, such as textbooks, automobiles, and electronics have well-established secondhand markets that can drain revenue from the vendor who is trying to sell new products. Much prior research has been devoted to the impact of used-good markets on durable product sales of the monopolist. These studies show, for example, that a monopolist can be effective in reducing competition from such used-

\footnote{Please see Miller (1974), Liebowitz (1992), Berkovec (1985) and Rust (1986).}
good markets through “planned obsolescence” (like revised textbook editions). An alternative to “planned obsolescence” for the monopolist is to eliminate the competition from product durability by refusing to sell the product and only renting or leasing it as shown in Bulow (1982). Coase (1972) points out that a durable goods monopoly that sells products has less market power than a monopoly that rents the goods (known as Coase Conjecture). Shy (1995) states that “a monopoly selling a durable good earns a lower profit than a renting monopoly”.

Extensive literature can be found that debates the relationship between the degree of a firm’s monopoly power and the quality or durability it chooses to release each time.4 Kleiman and Ophir (1966) and Levhari and Srinivasan (1969) show that a monopolist has an incentive to release lower quality products, in comparison to what can be seen in a competitive market structure. Nevertheless, Swan (1970a) and Swan (1970b) argue that such “planned obsolescence” is never actually optimal. Swan demonstrates that the monopolist can maximize profits by setting a durability level equal to the competitive level and raising prices. This result is known as “Swan’s independence result” and has proven to be the impetus for several subsequent debates.5 A recent series of papers by Wilhelm and various co-authors focus on the upgrading of contents and the timing of high-tech products (hardware such as desktops and laptops).6

Software is a durable good that can potentially be used infinitely without physical erosion. Typically, durability of a product greatly affects the frequency of repeated purchase by consumers. Software users have no incentive to re-purchase the software. Upgrades or revisions, however, may be effective ways to make existing software “obsolete” and provide an incentive for current users to purchase the updated version. Existing literature about software upgrade can be divided into three categories. The first considers software upgrades and release decisions from the technical perspective.7 Papers in this category mainly discuss the tradeoff between new software

4Please see a survey by Schmalensee (1979).
5See, for example, Miller (1974), Kihlstrom and Levhari (1977), Spence (1975) and Liebowitz (1992).
6Please see Wilhelm and Xu (2002), Wilhelm et al. (2003), Damodaran and Wilhelm (2003b) and Damodaran and Wilhelm (2003a).
7Please see Xie and Hong (1998), Xie and Hong (1999) and Shinohara et al. (1997).
reliability (time required to check software error and failure) and the software's release time in the market. These papers do not consider the demand side or market factors. The second category of research papers considers software upgrades from the perspective of welfare economics. Ellison and Fudenberg (2000) examine two reasons explaining why a monopolist has an incentive to introduce more upgrades than is socially optimal, when upgraded software is backwardly compatible. Fudenberg and Tirole (1998) study conditions under which a monopolist can choose to offer upgrade discounts to existing consumers. In Fudenberg and Tirole (1998), only one new update is present, and customers will not "leadfrog" the new update if the marginal cost of production is zero. The third category of research papers is centered on software editions or static versioning. Varian (1997), Raghunathan (2000) and Haruvy and Prasad (1998) conduct studies on delivering several versions with different quality levels or feature sets, such as a professional edition, a student edition, and a home edition, simultaneously. The purpose of static software versioning is mainly to segment the market and to take advantage of the sub-markets by price and quality discrimination. In addition, Hui and Tam (2002) investigate the optimal levels of software functionality in a duopoly market and Yang (1997a) discusses the optimal upgrade time in software provision with network effects. In Sahin and Zahedi (2001), by using the consumer satisfaction index, the authors build a Markov Decision Process (MDP) model to identify optimal and near-optimal policies among the choices: warranty, maintenance, and upgrade of software systems to software vendors.

As far as we know, no existing research offers insights into the software vendor's decision on software upgrade, based on the behavior of software users. Our work is related to Wilhelm and Xu (2002)'s work, but is different in several aspects. First, Wilhelm and Xu focus on the upgrade of computer hardware, which involves product design engineering, process design engineering, product planning and supply chain, and outsourcing. Our work focuses on software upgrading, and the production processes and operations for software differ from those of hardware products. Second, Wilhelm and Xu assume no cannibalization problem in the upgrading process, while we investigate the cannibalization problem in the software market.
3.3 Modeling Software Users' Behavior for Software Upgrade

When the software vendor releases a new software version, the hope is that both existing users and new users will buy the product. Thus, we analyze both existing users and new users, under the following two market structures: Structure 1 - both the old and the new versions coexist in the market, and Structure 2 - only the new version is available in the market. We perform our analysis for two sequential software upgrades. We call it the 2x2x2 model, where the first 2 refers to two categories of software users (existing and new), the second 2 refers to two market structures, and the third 2 refers to two sequential software upgrades.

3.3.1 Existing Users' Upgrade Demand Under Structure 1

We assume that the software user has the following utility function: \( U = U(Q, C, M) \) subject to the budget constraint \( Y = M + P \), where \( Q > 0 \) is the software quality which is measured by software features or functionalities; \( C > 0 \) are the other gains from using the software such as customer services (e.g., technical supports), and are functions of aggregate sales (e.g., positive network externalities); \( M > 0 \) is the consumption cost (value of the consumption) of all other commodities except the software; \( Y > 0 \) is the income; and \( P > 0 \) is the software price. We set up three time periods 0, 1, and 2 and the software vendor will release the new versions at period 1 and period 2, with strictly increasing quality, respectively. We assume that for any software version, during the period when it is just released \( C = C' \); in the next period \( C = wC' \), where \( 0 < w < 1 \); and after the next period, as the old version has compatibility problems, the software vendor reduces or stops customer services offered for the old version. We take \( w \) as a decision variable to the software vendor in our model (see the summary description of notations used in the Appendix). We assume that the software vendor only keeps the previous version as an old version, when he releases a new version under Structure 1. So, in periods 1 and 2, one new version and one old version are co-existing in the market. To simplify our analysis, we only investigate the existing users who are using the initial version at period 0. We assume that the information for the upgrade plan is perfectly known to the existing
software users, or that the existing users can perfectly forecast the software vendor's upgrade plan. Software users try to make an optimal decision about whether or not they want to upgrade the software and at which period(s). Since the fact that the user has the same or different income in each period does not affect the analytical results, we simply assume that the user has the same income \( Y \) in each of the three periods and the time discount rate is 0. See Figure 3.1 for the software releasing sequence under Structure 1.

At period 0, the existing user uses the initial software, of quality \( Q_0 \), which is greater than 0. At period 1, the software vendor releases the new version of software, which is of quality \( Q_1 = \alpha Q_0 \), where \( \alpha > 1 \). At period 2, the software vendor releases another new version of quality \( Q_2 = \beta Q_1 = \beta (\alpha Q_0) \), where \( \beta > 1 \).

To simplify our analysis and solve the inequalities afterwards, we assume \( \beta > 2 - \frac{1}{\alpha} \), i.e. \( Q_2 - Q_1 > Q_1 - Q_0 \).

The existing software user, at period 1, has five options when responding to the software upgrade plan:
1) no upgrade at all;
2) only upgrade to \( Q_1 \) at period 1;
3) only upgrade to \( Q_2 \) at period 2;
4) upgrade to \( Q_1 \) at period 1 and upgrade to \( Q_2 \) at period 2; or,
5) only upgrade to \( Q_1 \) at period 2.

**Definitions:** For existing software users, "Disappearance" demand refers to those consumers who finally choose Option 1, denoted as \( D^{00} \). "Opportunist" demand refers to the consumers who finally choose Option 2, denoted as \( D^{10} \). "Leapfrog" demand refers to the consumers who finally choose Option 3, denoted as \( D^{02} \). "Loyal" demand refers to the consumers who finally choose Option 4, denoted as \( D^{12} \). "Lagged" demand refers to the consumers who finally choose Option 5, denoted as \( D^{m} \).

Now let us look at the software user's utility at different periods. Following the utility function in Wilson (1980) and Akerlof (1970), we can write one software user's general utility in any period as \( U = M + ntQ + nC \) subject to the budget

\[ \text{Under the condition } \beta \leq 2 - \frac{1}{\alpha}, \text{ we can derive similar results as those under } \beta > 2 - \frac{1}{\alpha}. \text{ For simplicity, we only demonstrate the results under the condition } \beta > 2 - \frac{1}{\alpha}. \]

\[ \text{We choose the utility function from Wilson and Akerlof because the software user has similar choice options as described in them.} \]
constraint $Y = M +nP$, where $n$ is an indicator to show whether to buy the software ($n = 1$) or not ($n = 0$); $t$ is the utility index to measure the differences among the existing users. We assume $t \in [t_0, t_n]$ with $t_0 \geq 0$ and $t$ has the density function of $h(t)$. Therefore $H(t) = \int_{t_0}^{t} h(x)dx$ is the number of buyers with utility index less than $t$. If we substitute $M$ into the objective function, we get the new objective function without a constraint: $U = Y - nP + ntQ + nC$. Suppose the buyer chooses to buy the software $Q$, i.e., $n = 1$, his utility is $U = Y - P + tQ + C$; suppose the buyer chooses not to buy, i.e., $n = 0$, his utility is $U = Y$. Therefore, the buyer chooses to buy the software $Q$ if and only if his utility under $n = 1$ is greater than his utility under $n = 0$, i.e., $t > \frac{P-C}{Q}$. So, the total demand for the software is $D = \int_{t_0}^{t_n} h(x)dx$.\(^{10}\) In similar manner, we can write the existing user’s utility in period 1 as

$$U_1 = M + ntQ + (1-n)tQ_0 + (1-n)tQ_0 + (1-n)tQ_0 + (1-n)tQ_0 + (1-n)wC' + nC'$$

subject to $Y = M +nP$, where $n$ is an indicator variable to show whether the user upgrades his software ($n = 1$) or not ($n = 0$) at period 1, $P_1$ is the price for the new version $Q_1$ at period 1 (Later on, we use the notation $P_2$ for the price for version $Q_2$ at period 2, and $P_3$ for the price of the old version $Q_1$ at period 2). Substituting the constraint into the objective function, we get

$$U_1 = Y + nt(aQ_0) - nP_1 + (1-n)tQ_0 + (1-n)tQ_0 + (1-n)tQ_0 + (1-n)tQ_0 + (1-n)tQ_0 + (1-n)wC' + nC'.$$

From this utility function, if the user decides to upgrade to $Q_1$, i.e., $n = 1$, his utility in period 1 is $U_1 = Y + aQ_0 - P_1 + C'$. If he decides not to upgrade, i.e., $n = 0$, his utility in period 1 is $U_1 = Y + tQ_0 + wC'$. Note that the existing user has no incentive to buy the old version of $Q_0$ in period 1 because he has already had it. In the same fashion, we can derive the user’s utility in period 2, $U_2$, depending on whether he decides to upgrade and which version he chooses to upgrade in period 2. The total utility in the three periods 0, 1 and 2 under the five options is defined as $U^i = U_0 + U_1 + U_2$, $i = 1, 2, 3, 4, 5$. Without considering a discount rate, the total utilities during periods 0, 1 and 2 under the five options are described in Table 3.1.\(^{10}\)

\(^{10}\)In this study, we only study the case where upgrades prices are not endogenous.
Now, let us look at various demands for software upgrade. From the utility formulas in Table 3.1, the existing user finally chooses not to upgrade at either of period 1 or period 2 under the condition: \( U^1 > U^2, U^1 > U^3, \alpha > U^4 \) and \( \beta > U^5 \). From this condition, we get the following constraints for \( t \) to be satisfied simultaneously:

\[
\frac{P_1 - C'}{2(\alpha - 1)Q_0}, t < \frac{P_2 - (1 + w)C'}{Q_0(\alpha \beta - 1)}, t < \frac{-2C'' + P_1 + P_2}{Q_0(\alpha \beta - 2)} \text{ and } t < \frac{P_3 - C'w}{Q_0(\alpha - 1)}.
\]

Define

\[
t_u = \min \left( \frac{P_2 - (1 + w)C'}{Q_0\left(\frac{P_1 - C'}{2(\alpha - 1)Q_0}, \alpha \beta - 1\right)}, \frac{P_2 - (1 + w)C'}{Q_0(\alpha \beta - 1)}, \frac{-2C'' + P_1 + P_2}{Q_0(\alpha \beta - 2)}, \frac{P_3 - C'w}{Q_0(\alpha - 1)} \right) \text{ and } t_d = t_0.
\]

So, if \( t_u > t_d \), the total number of existing users who do not upgrade at either of period 1 or period 2 is \( D = \int_{t_d}^{t_u} h(x) dx \). In the same fashion, we get the various other demands for existing users. To simplify our analysis, we assume \( h(t) \) has a uniform density on the support \([0, t_n] \) and normalize \( C' \) and \( Q_0 \) into 1's. Table 3.2 shows the various demands, and the associated \( t_u \)'s and \( t_d \)'s.

Note that how the various demands, \( D^{00}, D^{10}, D^{02}, D^{12} \) and \( D^{01} \), co-exist in the market depends on the values of \( \alpha, \beta, w, P_1, P_2 \) and \( P_3 \). In the real world, these five demands will, in all likelihood, co-exist in the market. We assume that the five demands co-exist to conform to the reality in our analytical framework.\(^{11}\)

**Proposition 1** If all of the five demands of existing users: "disappearance", "opportunistic", "leapfrog", "loyal" and "lagged" demands, i.e., \( D^{00}, D^{10}, D^{02}, D^{12} \) and \( D^{01} \), co-exist in the market, then the five demands partition the support of \( t \) as Figure 3.2.

**Proof:** \( D^{00}, D^{10}, D^{02}, D^{12} \) and \( D^{01} \) co-exist in the market implies that \( D^{00}, D^{10}, D^{02}, D^{12} \) and \( D^{01} \) partition the whole market depends on the values of \( \alpha, \beta, w, P_1, P_2 \) and \( P_3 \). In the real world, these five demands will, in all likelihood, co-exist in the market. We assume that the five demands co-exist to conform to the reality in our analytical framework.\(^{11}\)

\(^{11}\)We tried numerical calibrations that under a set of parameter values, the five demands do co-exist.
The importance of proposition 1 is that, if the five demands co-exist, we can draw the demand partition pattern as in Figure 3.2, and can see how each demand varies as some parameters are changed. Note that the order of the different demands on the support of \( t \) from 0 to \( t_n \) are \( D_{00}, D_{01}, D_{10}, D_{02}, D_{12} \).

In order to investigate the role of the old version, we then assume the old version \( Q_1 \) is withdrawn from the market. Given the other conditions are unchanged, the distribution of various demands without the old version \( Q_1 \) at period 2 is described as Figure 3.3. Since \( \frac{P_2 - w}{P_3 - w} \) in Figure 3.2 is less than \( \frac{P_1 - 1}{2(\alpha - 1)} \) in Figure 3.3 as

\[ t^u = \min \left( \frac{P_1 - 1}{2(\alpha - 1)}, \frac{P_2 - 1 - w}{\alpha - 1}, \frac{-2 + P_1 + P_2}{\alpha + \alpha \beta - 2}, \frac{P_3 - w}{\alpha - 1} \right) = \frac{P_3 - w}{\alpha - 1} \]

for in Figure 3.2, obviously the “disappearance” market \( D_{00} \) is smaller under structure 1 than under structure 2. On the other hand, \( \frac{P_1 - 1}{2(\alpha - 1)} \) in Figure 3.3 is less than \( \frac{w - 1 - P_3 + P_1}{\alpha - 1} \) in Figure 3.2, implies that the “opportunistic” demand under structure 1 becomes smaller due to the appearance of the old version. Figure 3.2 shows that some existing users with low \( t \) will choose to upgrade his version in a “lagged” manner from \( Q_0 \) to \( Q_1 \) at period 2 instead of in an “opportunistic” manner, and some existing users with the lower \( t \), who would disappear from the market, now upgrade their software from \( Q_0 \) to \( Q_1 \) at period 2. The “lagged” demand is determined by \( \alpha, P_1, P_3 \) and \( w \). This finding indicates that keeping the old version with the new version in the market can attract some of the consumers who initially had no intention to upgrade at all, and thus salvage some of the “disappearance” demand. Usually, the software vendor would like to salvage the “disappearance” demand as much as possible. From Figure 3.2, we get the following proposition.

**Proposition 2** If \( P_3 \leq w \), the software vendor can salvage the whole “disappearance” demand.

**Proof:** In order to eliminate \( D_{00} \), we need \( t^d \geq t^u \) for \( D_{00} \), that is, \( 0 \geq \frac{P_3 - w}{\alpha - 1} \), which implies \( P_3 \leq w \). QED

This proposition tells us that if the software vendor wants all the “disappearance” demand to upgrade their software, he can adjust \( P_3 \) and \( w \) separately or jointly to
satisfy the condition $P_3 \leq w$. For example, one strategy would be to decrease $P_3$ at least to some level below $w$. Or the software vendor can increase $w$ greater than $P_3$. In either of these ways, some of existing users, who initially had no intention of buying any upgraded versions, now choose to buy version $Q_1$ at period 2. In addition, the co-existence of “lagged” demand implies $t^u \geq t^d$ for $D^{01}$, that is $\frac{w-1-P_3+P_4}{a-1} > \frac{P_4-w}{a-1}$ in Figure 3.2. Re-arranging this inequality, we get $P_3 < \frac{P_4+2w-1}{2}$. This tells us that if the price of version $Q_1$ in period 1 is decreased, that is, lower $P_1$, then the software vendor has to further decrease the price of such a version in period 2, that is, lowering $P_3$ in order to salvage the “disappearance” market. The reason for this is that $\frac{\partial P_3}{\partial P_1} = \frac{1}{2} > 0$ if we set $P_3 = \frac{P_4+2w-1}{2}$.

In most cases, the software vendor not only wants to salvage the “disappearance” demand, but also wants more “loyal” consumers among existing users, that is, he wants more existing users to upgrade in both periods 1 and 2. The following proposition provides some insights into how to increase the proportion of “loyal” consumers among existing users.

**Proposition 3** Decreasing the price of version $Q_1$ in period 1, decreasing the customer services in the second period, or increasing the quality of version $Q_1$ in period 1, will help to increase the proportion of “loyal” consumers.

**Proof:** In order to increase the interval for $D_{12}$, we need to decrease $\frac{P_1+w-1}{a-1}$, which suggests to either lower $P_1$ or $w$, or increasing $\alpha$, or doing them jointly. QED

Lowering $P_1$ has multiple effects on the demands of existing users. The overall effects of $P_1$ is such that decreasing $P_1$ will increase the demands $D^{10}$ and $D^{12}$, but decrease the demand $D^{01}$ and $D^{02}$. Lowering $P_1$ leads to more “loyal” consumers is reasonable since some existing users, who are willing to upgrade to $Q_2$ at period 2, are now also willing to upgrade to $Q_1$ at period 1 as the version $Q_1$ is cheaper. At the same time, some existing users who would have been willing to upgrade to $Q_2$ at period 2, now choose to upgrade to $Q_1$ at period 1 instead. This causes the shrinkage of $D^{02}$, and implies that there is a positive cross price elasticity between version $Q_1$ and version $Q_2$. Increasing the quality of version $Q_1$ at period 1 has similar effects as lowering $P_1$. The reason that decreasing $w$ can increase the “loyal”
demand is that, as the software users cannot get more customer services, this forces some existing users who initially only buy \( Q_2 \) in period 2 to buy \( Q_1 \) in period 1 as well. However, from Figure 3.2 we can see that while decreasing \( w \) increases “loyal” demand, it will increase the disappearance market as a side-effect. Therefore, if the software vendor cares about both “loyal” demands and “disappearance” demands, decreasing \( P_1 \) should be a better choice than decreasing \( w \) as decreasing \( P_1 \) will lead to a larger “loyal” demand, but will not lead to a larger “disappearance” demand.

**Comparative Static Analysis.**

It is helpful to understand how a change in the value of one parameter affects various upgrade demands. Our comparative statics results are listed in Table 3.3. The impact of lowering the price of version \( Q_1 \) at period 1 can be summarized as follows: It will increase the “opportunist” and “loyal” demands, and decrease the “leapfrog” and “lagged” demands, which is consistent with Proposition 3. But, changing \( P_1 \) does not affect the “disappearance” demand. Lowering the price of version \( Q_2 \) at period 2 has no impact on the “disappearance” demand, the “loyal” demand or the “lagged” demand. It will decrease the “opportunist” demand, increase the “leapfrog” demand. If we combine the impact of \( P_1 \) and \( P_2 \) together, we can find that decreasing the relative price of version \( Q_2 \) at period 2 with respect to the price of version \( Q_1 \) at period 1, that is, \( R = \frac{P_2}{P_1} \), will lead to decreasing the “opportunist” demand and increasing the “leapfrog” demand. The proof is outlined as follows:

\[
\frac{\partial D^{10}}{\partial R} = \frac{\partial D^{10}}{\partial P_2} - \frac{\partial D^{10}}{\partial P_1} \quad \text{and} \quad \frac{\partial D^{02}}{\partial R} = \frac{\partial D^{02}}{\partial P_2} - \frac{\partial D^{02}}{\partial P_1}.
\]

We know

\[
\frac{\partial D^{10}}{\partial P_2} > 0, \quad \frac{\partial D^{10}}{\partial P_1} < 0, \quad \frac{\partial D^{02}}{\partial P_2} < 0 \quad \text{and} \quad \frac{\partial D^{02}}{\partial P_1} > 0,
\]

therefore,

\[
\frac{\partial D^{10}}{\partial R} > 0 \quad \text{and} \quad \frac{\partial D^{02}}{\partial R} < 0.
\]

This result shows that there is a substitution effect between versions \( Q_1 \) at period 1 and \( Q_2 \) at period 2, which is referred to as the “horizontal” cannibalization in the existing users market.

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The price of version \( Q_1 \) as an old version in period 2 has some impacts on the "disappearance" demand, the "lagged" demand and "opportunist" demand. Lowering the price will decrease the "disappearance" demand and "opportunist" demand, and increase the "lagged" demand. This is consistent with Proposition 1. In addition, we have the following proposition.

**Proposition 4** Keeping the old version of software with the new version will only lead to the "cannibalization" effect on the "opportunist" demand of the existing users.

**Proof:** From Figure 3.2 and Figure 3.3, \( D_{02} \) and \( D_{12} \) do not change with/without the old version. From Table 3.3, we can see only \( D_{10} \) is the function of \( P_3 \). Neither \( D_{02} \) nor \( D_{12} \) is the function of \( P_3 \). These suggest that the appearance of the old version \( Q_1 \) has no influence on the demands \( D_{02} \) nor \( D_{12} \). QED

This proposition shows that keeping the old version and changing its price will only affect the purchase choices of the "opportunist" demand. This finding is non-intuitive as we believed all the existing users including \( D_{02} \) and \( D_{12} \) should be affected by the appearance of the old version \( Q_1 \) in period 2. The implication of this proposition is that if the software vendor does not care that some "opportunist" users become "lagged" users, keeping the old version with the new version is a better choice because the old version has no negative impact on the new version, but can salvage some "disappearance" demand. The reason that the appearance of version \( Q_1 \) as old version in period 2 has no effect on the demands \( D_{02} \) is that if the existing user decided not to buy \( Q_1 \) at period 1, he will not buy it at period 2 when it has become a little out-of-date. For \( D_{12} \), if the existing user decided to buy the version \( Q_1 \) at period 1, it is senseless for him to buy the old version at period 2 again as he will buy the new version \( Q_2 \).

The impact of lowering the quality level of version \( Q_1 \) can be briefly described as follow: the "disappearance" and "lagged" markets become larger and the "loyal" market becomes smaller, the "opportunist" demand becomes smaller, and the "leapfrog" demand becomes larger. The impact of lowering the quality level of version \( Q_2 \) is that the "opportunist" demand becomes larger, and the "leapfrog" demand becomes smaller. We can see that lowering the software quality has the equivalent
effects as increasing the software price has. In addition, the impact of the quality level of version $Q_1$ is relatively comprehensive, and the quality levels of the two versions $Q_1$ and $Q_2$ have some substitution effects between the “opportunistic” demand and the “leapfrog” demand.

3.3.2 Existing Users’ Upgrade Demand Under Structure 2

If the software vendor chooses not to keep any old version along with the new version in the market, the software releasing sequence can be described as Figure 3.4. Generally, the existing user has four upgrade choices, which are described as the first four options in Table 3.1. From the utility formula in Table 3.1, the existing user chooses not to upgrade at either of period 1 or period 2 under the condition: $U_1 > U_2$, $U_1 > U_3$ and $U_1 > U_4$. From this condition, we get the following constraints for $t$ that need to be satisfied simultaneously:

$$t < \frac{P_1 - c'}{2(\alpha - 1)Q_0}, \quad t < \frac{P_2 - (1 + w)C'}{Q_0(\alpha\beta - 1)}, \quad t < \frac{-2C' + P_1 + P_2}{Q_0(\alpha + \alpha\beta - 2)}$$

Define

$$t^u = \min\left(\frac{P_1 - c'}{2(\alpha - 1)Q_0}, \frac{P_2 - (1 + w)C'}{Q_0(\alpha\beta - 1)}, \frac{-2C' + P_1 + P_2}{Q_0(\alpha + \alpha\beta - 2)}\right)$$

and $t^d = t_0$. So, if $t^u > t^d$, the total number of existing users who do not upgrade at period 1 or period 2 is $D = \int_{t^d}^{t^u} h(x)dx$. In the same fashion, various demands for existing users can be figured out.

Because the existing users do not have the option of upgrading their software to $Q_1$ in period 2, we only have the following demands: $D^{00}$, $D^{01}$, $D^{02}$ and $D^{12}$ in Structure 2. If we assume that all these demands co-exist in the market, and $t$ is uniformly distributed in $[0, t_n]$ and normalize $C'$ and $Q_0$ into 1's, we get the partition of these demands as shown in Figure 3.3. So, we can see that most of the analysis about the behavior of existing users under the Structure 1 is still valid under the Structure 2, except that the “lagged” demand disappears, which means that the software vendor has lost one tool for reducing the “disappearance” market.
3.3.3 New Consumer’s Demand of Software Versions

We need to analyze the new user under the two market structures. There are two streams of new consumers in our 2x2x2 model, one in period 1 and another one in period 2. To simplify our analysis, let us only analyze the stream in period 2 as both streams share the same behavioral characteristics.

Case One: New User’s Demand under Structure 1

In this case, at period 2 the quality of old version is $Q_1$ and the quality of new version is $Q_2$. The price of the old version is $P_3$ and the price of new version is $P_2$ as previously. The utility function of the new user can be described as

$$U = M + tn(2 - n)Q_1 + n(2 - n)wC + t \frac{n(n - 1)}{2} Q_2 + n(n - 1)C$$

subject to

$$Y = M + n(2 - n)P_3 + \frac{n(n - 1)}{2} P_2.$$ 

Note that $n$ is an indicator variable with values of 0, 1 and 2. The new consumer has three options in the software market: buy the old version, buy the new version and not buy any versions at all. For the utility function, $n = 0$ means that the new consumer does not buy any version, $n = 1$ means he buys the old version and $n = 2$ means he buys the new version. Table 3.4 shows the three options to the new user.

The new consumer buys the old version if and only if $U_1 > U_0$ and $U_1 > U_2$, which suggests

$$t > \frac{P_3 - wC'}{\alpha Q_0} \text{ and } t < \frac{P_2 - P_3 + wC' - C'}{Q_0 \alpha (\beta - 1)}.$$ 

Define

$$t^d = \frac{P_3 - wC'}{\alpha Q_0} \text{ and } t^u = \frac{P_2 - P_3 + wC' - C'}{Q_0 \alpha (\beta - 1)}.$$ 

Therefore the total demand for the old version is $D_{old}^2 = \int_{t^d}^{t^u} h(x)dx$.$^{12}$ The new consumer chooses to buy the new version if and only if $U_2 > U_0$ and $U_2 > U_1$, which

$^{12}$In this section, we use $D_{old}^2$ to refer to the new users' demand for the old version $Q_1$, $D_{new}^2$ to refer to the new users' demand for the new version $Q_2$, and $D_{dis}^2$ to refer to the "disappearance" demand.
suggests
\[ t > \frac{P_2 - C'}{\alpha \beta Q_0} \text{ and } t > \frac{P_2 - P_3 + wC' - C'}{Q_0 \alpha (\beta - 1)}. \]

Define
\[ t^d = \max\left(\frac{P_2 - C'}{\alpha \beta Q_0}, \frac{P_2 - P_3 + wC' - C'}{Q_0 \alpha (\beta - 1)}\right), \]

and get \( D_{\text{new}}^2 = \int_{t^d}^{t^u} h(x)dx. \) By the same token, the new consumer does not buy any version if and only if \( U_0 > U_1 \) and \( U_0 > U_2 \), which suggests
\[ t < \frac{P_3 - wC'}{\alpha Q_0} \text{ and } t < \frac{P_2 - C'}{\alpha Q_0}. \]

Define
\[ t^u = \min\left(\frac{P_3 - wC'}{\alpha Q_0}, \frac{P_2 - C'}{\alpha Q_0}\right) \]

and get \( D_{\text{dis}}^2 = \int_{t^u}^{t^u} h(x)dx. \) Note that for the above \( \max \) and \( \min \) functions, if
\[ Q_0(\beta P_3 - P_2) + C' - \beta wC' > 0, \quad t^u = t^d = \frac{P_2 - C'}{\alpha \beta Q_0}. \]

If
\[ Q_0(\beta P_3 - P_2) + C' - \beta wC' < 0, \quad t^u = \frac{P_3 - wC'}{\alpha Q_0} \text{ and } t^d = \frac{P_2 - P_3 + wC' - C'}{Q_0 \alpha (\beta - 1)}. \]

**Proposition 5** If \( Q_0(\beta P_3 - P_2) + C' - \beta wC' > 0 \), there are no new consumers willing to buy the old version even if both the old and the new versions co-exist in the market. In this case, the market with two versions co-existing (i.e., Structure 1) is identical to the market with only the new version (i.e., Structure 2) to the new user.

**Proof:** If \( Q_0(\beta P_3 - P_2) + C' - \beta wC' > 0 \), the \( t \) value of the consumer who will buy the new version is between \( \left(\frac{P_2 - C'}{\alpha \beta Q_0}, t_0\right) \), and the \( t \) value of the consumer who will not buy any versions is between \( (t_0, \frac{P_2 - C'}{\alpha \beta Q_0}) \). It is easy to observe that these two intervals of \( t \) partition the entire support of \( t \). Therefore, no new consumers will buy the old version. QED

The condition \( Q_0(\beta P_3 - P_2) + C' - \beta wC' > 0 \), that is, \( (\beta P_3 - P_2) > \frac{\beta wC' - C'}{Q_0} \), suggests that \( P_3 \) is not significantly below \( P_2 \). If this happens, the consumer has few
incentives to buy the old version. If we draw the distribution of new consumers, we will get a figure identical to Figure 3.6 in Case Two in the next section. Under the condition $Q_0(\beta P_3 - P_2) + C' - \beta wC' > 0$ and the assumptions that $t$ has a uniform distribution on $[0, t_n]$, we normalize $C'$ and $Q_0$ into 1’s and figure out $D_{new}^2$ and $D_{dis}^2$ as follows:

$$D_{new}^2 = 1 - \frac{P_2 - 1}{t_n \alpha \beta} \text{ and } D_{dis}^2 = \frac{P_2 - 1}{t_n \alpha \beta},$$

which are the same as the demands in Case Two (see the next section). This proposition states that the new version release gives the vendor pressures to decrease the price of the old version. In other words, in order to induce some new consumer to buy the old version, the software vendor has to largely decrease the price of the old version.

Under the condition $Q_0(\beta P_3 - P_2) + C' - \beta wC' < 0$, which suggests that the price of the old version is significantly lower than the price of the new version, we have a different story: some new consumers choose to buy the old version; some choose to buy the new version; and some choose not to buy either of them. See Figure 3.5 for the distribution of the new consumers. Using the same assumptions that $t$ has a uniform distribution on $[0, t_n]$, and normalizing $C'$ and $Q_0$ into 1’s, we can figure out $D_{old}^2$, $D_{new}^2$ and $D_{dis}^2$ as follows:

$$D_{old}^2 = \frac{P_2 - \beta P_3 - 1 + \beta w}{t_n \alpha (\beta - 1)}, \quad D_{new}^2 = 1 - \frac{P_2 - P_3 + w - 1}{t_n \alpha (\beta - 1)} \text{ and } D_{dis}^2 = \frac{P_2 - w}{\alpha t_n}.$$

Case Two: New User’s Demand under Structure 2

In this case, the new consumer’s utility function can be described as the following:

$U = M + tnQ_2 + nC'$ subject to the budget constraint $Y = M + nP_2$, where $n$ is an indicator with the values of 0 and 1. The new consumer has two options: buy the version $Q_2$ or not, which are the options 1 and 3 in Table 3.4.

The new consumer will choose to buy the new version if and only if $U_1 > U_0$, which suggests $t > \frac{P_2 - C'}{\alpha \beta Q_0}$. Define $t^d = \frac{P_2 - C'}{\alpha \beta Q_0}$ and get $D_{new}^2 = \int_{t^d}^{t_0} H(x)dx$. The new consumers will not choose to buy the new version if and only if $U_1 < U_0$, which suggests $t < \frac{P_2 - C'}{\alpha \beta Q_0}$. Define $t^u = \frac{P_2 - C'}{\alpha \beta Q_0}$ and get $D_{dis}^2 = \int_{t^d}^{t^u} H(x)dx$. We can demonstrate the distribution of the new consumers in Figure 3.6. If we assume that
$t$ has a uniform distribution on $[0, t_n]$ and normalize $C'$ and $Q_0$ into 1's, we can figure out $D_{\text{new}}^2$ and $D_{\text{dis}}^2$ as follows:

$$D_{\text{new}}^2 = 1 - \frac{P_2 - 1}{t_n \alpha \beta} \quad \text{and} \quad D_{\text{dis}}^2 = \frac{P_2 - 1}{t_n \alpha \beta}.$$ 

The interesting observation for new users is that once some new users buy the old version in Structure 1, the total demand in Structure 1 must be greater than that in Structure 2. The intuition for this observation is that the old version at a low price will attract some new users who would initially have no intention to buy the new version at a high price. The proof is as follows. Given the condition

$$Q_0(\beta P_2 - P_2) + C' - \beta \alpha C' > 0,$$

which implies that some new users will buy the old version in Proposition 5, we can see $\frac{P_1 - \mu C'}{\alpha Q_0}$ in Figure 3.5 is less than $\frac{P_2 - C'}{\alpha \beta Q_0}$ in Figure 3.6, which proves the observation.

**Comparative Static Analysis**

Comparative static analysis can shed light on how demands change along with the changes of some relevant parameters. Table 3.5 shows us the main comparative static results for the new user. In Case One, as the table shows that under Structure 1, the demand for the old version is negatively related to the quality of the new version and, its own price, but is positively related to the price of the new version. This demonstrates that there is a cross substitution effect between the old version and the new version, called the “vertical” cannibalization. Conversely, the demand for the new version is positively related to its quality level and the price of the old version, but negatively related to its own price. The number of people who will not buy either of the versions is only positively related to the price of the old version since it is either the old version or no purchase. Figure 3.5 shows that the critical point between not buying at all and buying old version is $\frac{P_1 - \mu C'}{\alpha Q_0}$.

In Case Two, the demand for the new version is positively related to the quality level upgraded. The negative relation between the demand for the new version and the price of the new version, $P_2$, is straightforward. It follows that the number of new consumers who decide not to buy the new version, $D_{\text{dis}}^2$, is negatively related to the higher upgraded quality, $\beta$, and positively related to the price of the new version, $P_2$. 94
Another interesting question is that how the change of relative price of the old version with respect to the new version, that is $R = \frac{P_1}{P_3}$ affects the behavior of the new consumer. As the relative price of the old version to the new version becomes higher, the market for the old version become smaller, and the market for the new version become larger. The proof is straightforward:

$$\frac{\partial D_{\text{old}}^2}{\partial R} = \frac{\partial D_{\text{old}}^2}{\partial P_3} - \frac{\partial D_{\text{old}}^2}{\partial P_2} \quad \text{and} \quad \frac{\partial D_{\text{new}}^2}{\partial R} = \frac{\partial D_{\text{new}}^2}{\partial P_3} - \frac{\partial D_{\text{new}}^2}{\partial P_2}.$$ 

We know

$$\frac{\partial D_{\text{old}}^2}{\partial P_3} < 0, \quad \frac{\partial D_{\text{old}}^2}{\partial P_2} > 0, \quad \frac{\partial D_{\text{new}}^2}{\partial P_3} > 0 \quad \text{and} \quad \frac{\partial D_{\text{new}}^2}{\partial P_2} < 0,$$

therefore,

$$\frac{\partial D_{\text{old}}^2}{\partial R} < 0 \quad \text{and} \quad \frac{\partial D_{\text{new}}^2}{\partial R} > 0.$$

This result clearly shows there is a substitution effect between the old version and the new version, that is, the "vertical" cannibalization, when both versions co-exist in the market. The impact of the parameter $w$ is interesting. Increasing $w$ will increase the old version sale and salvage some disappearance demand, but decrease the new version sale. So, $w$ can be used as a potential tool to balance the sales from different categories of new users.

### 3.4 Choice of Software Vendor

Now, let us look at the choice of the software vendor for software upgrade. Quality levels, prices of different versions, customer services and market structures are the choice variables for the software vendor. Table 3.6 summarizes various demands under the two market structures.

It is obvious that there are associated upgrading costs to the software vendor. For the software vendor, Hu et al. (1998) show that this kind of costs is determined by the upgrading efforts. More specifically, in our analytical framework, among the parameters we analyzed, adjusting $P_1$, $P_2$ and $P_3$ will not incur any cost, but affect the market share or profit. However, while changing $\alpha$ or $\beta$ will affect both market share and profit, increasing $\alpha$ or $\beta$ implies investing more in software design and
development. The similar case is applied to $w$. That is, changing $w$ will affect both marketing share and profit, but increasing $w$ will incur some costs.

The software vendor might try to maximize either profit or market share (more discussion in the next section). We can see the optimization problem to the software vendor is a max-max problem. He firstly tries to maximize the profit or market share under each of the two market structures, then compares the two maximums to choose the higher one. We can write the software vendor's two basic objective functions as:

**Max: [(Max: The total market profit under Structure 1), (Max: The total market profit under Structure 2)]** or

**Max: [(Max: The total market share under Structure 1), (Max: The total market share under Structure 2)]**

Of course, the software vendor might have an objective to maximize total profit subject to the market share being not less than some level, or to maximize total market share subject to the market profit being not less than some level.

The total profit is composed of the profit from the existing users and the profit from the new users. The total market share is composed of the share from the existing users and the share from the new users. Note that the profit is equal to the revenue (the market share times the price) minus the associated upgrade cost (we assume that the upgrade cost is linear in the increased quality) and the associated customer service cost (we assume that the customer service cost is linear in the customer service level $C'$).

In our analytical framework, instead of implementing the two-step max-max optimizations, we compare the two market structures more straightforwardly.

We derive the following proposition about the market share.

**Proposition 6**  The total market share under market structure 1 is greater than the market share under market structure 2 if $P_3 < \max\left(\frac{P_i+2w-1}{2}, \frac{P_i+\beta w-1}{\beta}\right)$.

**Proof:** If we compare the total demand under Structure 1 with the total demand under Structure 2, we can see both structures share some same components. The different components for each of total demands are listed in Table 3.7.

For the existing users, the demand under Structure 1 is greater than the demand under Structure 2 if and only if $\frac{P_i+2w-1-2P_i}{2P_i(\alpha-1)} > 0$ which means $P_3 < \frac{P_i+2w-1}{2}$. On
the other hand, if $P_3 > \frac{P_3 + 2w - 1}{2\alpha}$, both market structures have the identical existing users because the “lagged” demand disappears as we analyzed after Proposition 2. Recall Proposition 5, to make Structure 2 different from Structure 1 to new users, we need $(\beta P_3 - P_1) + 1 - \beta w < 0$, i.e., $P_3 < \frac{P_3 + \beta w - 1}{\beta}$, which suggests the demand under Structure 1 dominates the demand under Structure 2 for new users. On the other hand, if $P_3 > \frac{P_3 + \beta w - 1}{\beta}$, both market structures are identical to the new users. Therefore, if $P_3 < \max \left( \frac{P_3 + 2w - 1}{2}, \frac{P_3 + \beta w - 1}{\beta} \right)$, the total demand under Structure 1 is greater than the total demand under Structure 2. QED

**Proposition 7** If the software vendor wants the maximum profit, he needs to choose market structure 1 if

$$\left(P_3 - \frac{P_1}{2}\right) \left(\frac{P_1 - 2P_3 + 2w - 1}{\alpha - 1}\right) > \left(P_2 - \beta P_3 - 1 + \beta w\right).$$

Otherwise, he needs to choose market structure 2.

**Proof:** Let us figure out the different profits for existing users between the two structures. Compared with Structure 2, Structure 1 has additional amount

$$\left(\frac{w - 1 + P_1 - P_3}{\alpha - 1} - \frac{P_3 - w}{\alpha - 1}\right) \frac{1}{t_n}$$

of “lagged” demand. On the other hand, Structure 2 has additional amount

$$\left(\frac{w - 1 + P_1 - P_3}{\alpha - 1} - \frac{P_1 - 1}{2(\alpha - 1)}\right) \frac{1}{t_n}$$

of “opportunist” demand. Whether the total profit from existing users under Structure 1 is greater than that under Structure 2 depends on whether

$$\left(\frac{P_2 - P_3 - w - 1}{\alpha(\beta - 1)} - \frac{P_3 - w}{\alpha}\right) \frac{1}{t_n} P_3 > \left(\frac{P_2 - P_3 - w - 1}{\alpha(\beta - 1)} - \frac{P_1 - 1}{2(\alpha - 1)}\right) \frac{1}{t_n} P_1.$$

Figure 3.7 shows the different demand for existing users under two market structures. Now let us look at the different profits for new users under the two market structures. In Figure 3.8, for new users the total support of $t$ is partitioned into three segments under Structure 1 and into two segments under Structure 2. Whether the total profit from new users under Structure 1 is greater than the total profit from new users under Structure 1 depends on whether

$$\left(\frac{P_2 - P_3 - w - 1}{\alpha(\beta - 1)} - \frac{P_3 - w}{\alpha}\right) \frac{1}{t_n} P_3 > \left(\frac{P_2 - P_3 - w - 1}{\alpha(\beta - 1)} - \frac{P_2 - w}{\alpha}\right) \frac{1}{t_n} P_2.$$
Therefore, the total profit from Structure 1 is greater than the total profit from Structure 2 if and only if

\[
\left(\frac{w - 1 + P_1 - P_3}{\alpha - 1} - \frac{P_3 - w}{\alpha - 1}\right) P_3 + \left(\frac{P_2 - P_3 + w - 1}{\alpha^2 (\beta - 1)} - \frac{P_2 - w}{\alpha^2}\right) P_3 > \\
\left(\frac{w - 1 + P_1 - P_3}{\alpha - 1} - \frac{P_1 - 1}{2(\alpha - 1)}\right) P_1 + \left(\frac{P_2 - P_3 + w - 1}{\alpha^2 (\beta - 1)} - \frac{P_2 - 1}{\alpha^2 \beta}\right) P_2.
\]

After some algebra, we get

\[
(P_3 - \frac{P_1}{2}) \left(\frac{P_1 - 2P_3 + 2w - 1}{\alpha - 1}\right) > (P_2 - \beta P_3) \left(\frac{P_2 - \beta P_3 - 1 + \beta w}{\beta - 1}\right).
\]

QED

This proposition shows that even if the market share under Structure 1 is greater than under Structure 2, the total profit under Structure 1 is not necessarily greater than that under Structure 2. \((P_3 - \frac{P_1}{2}) \left(\frac{P_1 - 2P_3 + 2w - 1}{\alpha - 1}\right)\) measures the profit gain for the old version from the existing market as the old version salvages some "disappearance" demands. However, since the old version has such a cannibalization effect to the new users, \((P_2 - \beta P_3) \left(\frac{P_2 - \beta P_3 - 1 + \beta w}{\beta - 1}\right)\) stands for the relative lost or opportunity cost when the old version attract some new users from using the new version to using the old version. Only under the condition that the profit gained from the old version is greater than the profit foregone due to the appearance of the old version, it is optimal for the software vendor to choose Structure 1. The ideal situation for the software vendor is that Structure 1 can bring both maximum total market share and maximum total profit. If not, he has to consider the tradeoff between the maximum total market share and the maximum total profit from each market structure. Let us write the difference between the left side and right side of the condition in Proposition 7 as:

\[
Diff = (P_3 - \frac{P_1}{2}) \left(\frac{P_1 - 2P_3 + 2w - 1}{\alpha - 1}\right) - (P_2 - \beta P_3) \left(\frac{P_2 - \beta P_3 - 1 + \beta w}{\beta - 1}\right).
\]

We can see that if \(w\) is reduced when the price of \(Q_2\) is relative higher than the price of \(Q_1\) in period 2, such the difference is becoming larger. That is, \(\frac{d(Diff)}{dw} < 0\) if

\[
(2P_3 - P_1)(\beta - 1) < (P_2 - P_3)\beta(\alpha - 1).
\]
This observation tells us if the software vendor sets the price for the new version is higher enough to satisfy the condition

\[(2P_3 - P_1)(\beta - 1) < (P_2 - P_3)\beta(\alpha - 1),\]

reducing the customer services, i.e., \(w\), will help him benefit from the market structure 1.

Another question is whether and how the software vendor can use price discrimination on existing users and new buyers. The significant difference between the existing users and new buyers is that the existing users can still use their software even when the new version shows up. The intuition is that, in order to induce the existing users to buy the new version, the software vendor should give them some price discounts.

**Proposition 8** The software vendor is able to use price discrimination on the existing user and the new user to reap higher profit from the new version and/or the old version. For the new version, the software vendor should charge a lower price to the existing user and a higher price to the new user under either of the two market structures if \(t_n\) is sufficiently large. For the old version, the software vendor should charge a lower price to the existing user and a higher price to the new user if the price of new version \(P_2\) is greater than \(\frac{2P_1 - \beta + 2}{\beta + 2}\). (See the proof in Appendix.)

This proposition tells us that the software vendor should charge a lower price for the new version to the existing user and a higher price to the new user if consumers are highly heterogeneous, i.e., \(t_n\) is sufficiently large. The proposition also tells us for the old version under Structure 1, the software vendor can charge a lower price to the existing user and a higher price to the new user if the price of new version is greater than some value. The previous study by Van Ackere and Reyniers (1995) states that by using trade-ins (some discounts) to existing buyers and introductory offers to new users, the monopolist can use third-degree price discrimination among consumers to reap higher profit. This study shows the conditions under which the software vendor should offer price discounts to existing users and charge higher prices to new users. If the conditions are not satisfied, such discriminated price policy is not optimal at
all. For example, suppose the software vendor decides to adopt discriminated prices of the new version to existing and new users. Suppose that $t_n$ is not sufficiently large, say, $t_n < \frac{2\beta^2 + 2\alpha - \beta - 1}{\alpha - 1}$ under market structure 1, which implies that the condition in Proposition 8 is not satisfied. Theoretically, the optimal price discrimination policy for the new version is that offering price discounts to new users and charge higher prices to existing users. If the software vendor still offer price discounts to existing users and charge higher prices to new users, this price policy is not optimal at all, and even worse than the uniform price policy. Our finding is also different from previous studies including Yang (1997b), where the condition to implement discriminated prices to existing users and new users is based on the relative new version quality to the old version.

The advantage that the software vendor can use for price discrimination is that it might be possible for him to distinguish the existing users from the new users by using some relevant information technology. One practice is that the software vendor supplies only additional software upgrade components for the existing users at a low price. In this way, such additional upgrade components are useless to the new users if they have no chance to firstly own the old version. Under this arrangement, as the software users have to self-report or self-select them as the existing users or the new users, the third-degree price discrimination is feasible and might be widely encouraged in the software industry.

### 3.5 Discussions and Limitations

1. **Market Share vs. Market Profit.** In our analytical framework, we layout two objectives for the software vendor: market share or market profit. The software vendor’s objective choice would depend on many issues including the following:

   a) From product life cycle (PLC) theory, during the introduction and growth stages, as the market share is the base for market profit, the software vendor should expand market share. As long as the products are more widely used and the market share becomes larger, the software vendor might switch to market profit.\[13\] The higher switching costs for software users due to sunk costs in learning to use current

---

\[13\]See Greenstein and Wade (1998) for PLC in the computer market.
software will help the software vendor to implement the switching of his objective from market share to profit. Taking profit as an ultimate objective is very important for the software vendor to achieve consistent successes, since more profit can guarantee more investment into new system development and software upgrade;

b) Another advantage of developing market share first, and reaping profit next, is that high market penetration is important for establishing a technology standard and for providing a platform for other application software. Once the software vendor’s product is a standard or a platform for other software in the market, the initial software upgrade will generate associated sequential upgrades of other software. Thus, the software vendor might be able to achieve the first-move advantage; and,

c) Regarding the competition in the software markets, even though we model the software vendor as a monopolist, in fact, there often are several software vendors with similar products following similar marketing strategies, which is especially common for non-leaders in the software market. For example, if a software vendor’s competitor is trying to deploy market expansion strategies, the vendor may have to follow similar strategies to maximize market share. To understand the detailed investigation of one software vendor’s response to other vendors with similar products, duopoly modeling is required, which is one of the potential research opportunities related to this study.

2. Strategic Customer Services. Our research shows that customer services are an effective strategic tool to help the software vendor achieve his objective. Nevertheless, when using \( w \), the software vendor should bear in mind some of the tradeoffs involved. For example, for existing users, decreasing \( w \) can increase the “loyal” demand, but will increase the disappearance market. In other case, we can see that decreasing \( P_1 \) should be a better choice as this will lead to larger “loyal” demands and will not lead to larger “disappearance” demands. For new users, increasing \( w \) will increase the old version sales, and salvage some disappearance demand, but will decrease the new version sales. In our model, for simplicity, we assume that the software vendor adopts the stationary customer services policy for any software version. In reality, however, the software vendor can use a more flexible non-stationary customer services policy. For example, the software vendor can adopt differentiated customer services for different versions in different periods, or implement service
discrimination for different software users.

3. **Perfect Information.** In our analytical framework, to simplify the analysis, we assume that the information for the upgrade plan is perfectly known to the existing software user, or that the existing user can perfectly forecast the upgrade plan of the software vendor. This assumption is based on the fact that, in reality, the pre-announcement or "vaporware" marketing strategy is widely used in the software industry. Hoxmeier (2000) finds that software vendors can use software preannouncements for a competitive purpose with few concerns about any negative impact on users, if the vendors deliver the promised features and functionality. Raghunathan (2000) shows that the software vendor can reap a higher profit when it pre-announces the future strategy. Even if the software vendors do not announce their upgrade plan in the next period, given the common knowledge that any software needs to be upgraded in the future, the software users should consider multi-periods software upgrades when they make a purchase decision. Instead of knowing the exact upgrade plan, they can figure out the expected values of various parameters involved in upgrading plans. Under this situation, the sophisticated Bayesian analysis might be required. However, as the upgrade parameters are controlled by the software vendor, our simplified two-step backward analysis (i.e., analyzing consumer behavior first, then investigating the vendor's choices) still shows some insights into the software vendor's optimal software upgrade choice.

4. **Hardware Upgrading vs Software Upgrading.** As a software upgrade might need an associated hardware upgrade, software users incur some associated indirect costs of hardware for their software upgrade plan. Or, the total costs to implement a software upgrade might be more than the cost of the new software. Except for the possibility that hardware users can sell used hardware, the consumer behavior for hardware upgrade is similar to the behavior for software upgrade. In our analytical framework, one computer user has similar upgrade options for hardware as described in Table 3.1. For simplicity, we do not include the associated hardware upgrade cost for a software upgrade in our framework as we believe that the consumer software

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upgrade behavior represents the consumer's hardware upgrade behavior. In any case, we should bear in mind that some differences exist between the hardware upgrade and software upgrade. First, software is an information good and its marginal cost is almost zero. Hardware does not share this property. Second, unlike the secondhand markets for hardware, the secondhand market for software is prohibited, due to the difficulty in preventing duplication, that could lead to copyright infringement as shown in Fudenberg and Tirole (1998). Legitimately, only the software vendor has the right to sell the old version in the market. Or equivalently, the software vendor is able to completely control the secondhand market for the products. For hardware, used hardware and older models of hardware are widely available in the markets. This implies that the hardware vendor losses some power to use used-goods or older models as strategic tools to achieve his objectives. On the other hand, we observe that the software upgrade has some barriers that hardware upgrade does not have. One important issue is that software vendor might incur losses from software piracy, which affect his upgrade plan. But, for hardware upgrade, no hardware piracy problem exists.

3.6 Conclusions

Software upgrade is an important issue to software vendors. This paper shows that several types of software upgrade demands exist in the software upgrade markets: "disappearance" demand, "opportunist" demand, "leapfrog" demand, "loyal" demand and "lagged" demand. Keeping an old version with a lower price is an effective way to salvage the "disappearance" demand. Keeping the old version with the new version will lead to different "cannibalization" effects for existing and new users. Decreasing the price of the new version in period 1, increasing the quality of the new version in period 1 or decreasing customer services will help to increase the proportion of "loyal" demand. If the price of the old version is not low enough, Structure 1 is identical to Structure 2 to new users. However, if the price of the old version is low enough, the total market share under Structure 1 is greater than the market share under Structure 2. So, if the software vendor wants to maximize his market share, lowering the price of the old version and choosing Structure 1 is
the right choice. However, even if the market share under Structure 1 is greater than that under Structure 2, the total profit under Structure 1 is not necessarily greater than under Structure 2. Proposition 7 tells us the condition under which the software vendor should choose Structure 1 to maximize the total profit. The ideal situation for the software vendor occurs when Structure 1 can bring both the maximum total market share and the maximum total profit. We find that as software users become heterogeneous, the difference in market share or profit between the two market structures are diminishing to the software vendor. The software vendor is able to use price discrimination on the existing user and the new user to reap a higher profit. We find that, for the new version, the software vendor should charge a lower price to the existing user and a higher price to the new user under either of the two market structures if consumers are sufficiently heterogeneous; for the old version, the software vendor should charge a lower price to the existing user and a higher price to the new user if the price of the new version is quite high. The software vendor gains an advantage by implementing price discrimination since, by using some information technology, the software vendor might distinguish the existing user from the new user, on the basis of the user's self-selection. Therefore, third-degree price discrimination should be encouraged in the software industry.
<table>
<thead>
<tr>
<th>Option</th>
<th>Behavior</th>
<th>Total Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No upgrade in two periods at all.</td>
<td>$U^1 = 3Y + 3tQ_0 + (1 + w)C$</td>
</tr>
<tr>
<td>2</td>
<td>Only upgrade to $Q_1$ at period 1.</td>
<td>$U^2 = 3Y + (1 + 2\alpha)tQ_0 + (2 + w)C - P_1$</td>
</tr>
<tr>
<td>3</td>
<td>Only upgrade to $Q_2$ at period 2.</td>
<td>$U^3 = 3Y + 2tQ_0 + (2 + 2w)C + \beta tQ_0 - P_2$</td>
</tr>
<tr>
<td>4</td>
<td>Upgrade to $Q_1$ in period 1 and upgrade to $Q_2$ in period 2.</td>
<td>$U^4 = 3Y + (1 + \alpha + \alpha\beta)tQ_0 + (3 + w)C - P_1 - P_2$</td>
</tr>
<tr>
<td>5</td>
<td>Only upgrade to $Q_1$ in period 2.</td>
<td>$U^5 = 3Y + 2tQ_0 + (1 + 2w)C + t\alpha Q_0 - P_3$</td>
</tr>
</tbody>
</table>
### Demand Conditions

<table>
<thead>
<tr>
<th>Demand Type</th>
<th>Conditions</th>
<th>( t^w ) and ( t^d )</th>
<th>Demand Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D^{00} )</td>
<td>No upgrade in two periods at all. ( U^1 &gt; U^2, U^3, U^4, U^5 )</td>
<td>( t^w = \min\left( \frac{P_1 - 1}{2(\alpha - 1)}, \frac{P_2 - 1 - w - 2 + P_1 + P_2}{\alpha \beta - 1}, \frac{P_2}{\alpha - 1} \right) )</td>
<td>( D' = \int h(x) , dx; ) ( D' = 0 ) if ( t^w &gt; t^d ), otherwise ( D' = 0 ) (( i=00,10,02,12 ) and ( 01 )).</td>
</tr>
<tr>
<td>( D^{10} )</td>
<td>Only upgrade to ( Q_1 ) at period 1. ( U^2 &gt; U^1, U^3, U^4, U^5 )</td>
<td>( t^w = \min\left( \frac{P_2 - w - P_1}{1 + \alpha \beta - 2 \alpha}, \frac{P_2}{\alpha \beta - 1} \right) )</td>
<td>( t^d = \max\left( \frac{P_1 - 1}{2(\alpha - 1)}, \frac{w - 1 - P_1 + P_2}{(\alpha - 1)} \right) )</td>
</tr>
<tr>
<td>( D^{02} )</td>
<td>Only upgrade to ( Q_2 ) at period 2. ( U^3 &gt; U^1, U^2, U^4, U^5 )</td>
<td>( t^w = \frac{P_2 + w - 1}{\alpha - 1} )</td>
<td>( t^d = \max\left( \frac{P_2 - 1 - w - P_1 + P_2}{\alpha \beta - 1}, \frac{P_2 - P_1}{1 + \alpha \beta - 2 \alpha}, \frac{P_2}{\alpha \beta - 1} \right) )</td>
</tr>
<tr>
<td>( D^{12} )</td>
<td>Upgrade to ( Q_1 ) in period 1 and upgrade to ( Q_2 ) in period 2. ( U^4 &gt; U^1, U^2, U^3, U^5 )</td>
<td>( t^w = t_0 )</td>
<td>( t^d = \max\left( \frac{P_1 + w - 1}{\alpha - 1}, \frac{P_1 - 1}{\alpha \beta - 1}, \frac{-2 + P_1 + P_2}{\alpha \beta - 2}, \frac{-2 + w - P_1 + P_2}{\alpha \beta - 1} \right) )</td>
</tr>
<tr>
<td>( D^{01} )</td>
<td>Only upgrade to ( Q_1 ) in period 2. ( U^5 &gt; U^1, U^2, U^3, U^4 )</td>
<td>( t^w = \min\left( \frac{w - 1 - P_1 + P_2 + P_3 - P_1 - 1}{(\alpha - 1)}, \frac{-2 + w - P_1 + P_2}{\alpha \beta - 2}, \frac{P_2}{\alpha \beta - 1} \right) )</td>
<td>( t^d = \frac{P_2}{\alpha - 1} )</td>
</tr>
</tbody>
</table>
Table 3.3: Comparative Static Analysis for Upgrade Demands of Existing Users

<table>
<thead>
<tr>
<th>Demand</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$\frac{P_2}{P_1}$</th>
<th>$P_3$</th>
<th>$w$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^{00} = \frac{P_1 - w}{t_s(\alpha - 1)}$</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
<td>$\downarrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
<td>N.A.</td>
</tr>
<tr>
<td>$D^{10} = \frac{\alpha w - P_1 + \alpha P_1 + \alpha^2 P_1 + 2 + \alpha^2 - 2 \alpha - \alpha^2 w + P_1 + \alpha^2 P_1 - 2 \alpha P_1 - \alpha P_1}{t_s(1 + \alpha^2 - 2 \alpha)(\alpha - 1)}$</td>
<td>$\uparrow$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
<td>$\downarrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>$D^{02} = \frac{-1 - \alpha \beta + 2 \alpha + \alpha \beta w - \alpha w + \alpha \beta P_1 - \alpha P_1 + P_1 - \alpha P_2}{t_s(1 + \alpha \beta - 2 \alpha)(\alpha - 1)}$</td>
<td>$\downarrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
<td>N.A.</td>
<td>$\downarrow$</td>
<td>$\uparrow$</td>
<td></td>
</tr>
<tr>
<td>$D^{12} = \frac{\alpha t_n - t_s + 1 - w - P_1}{t_s(\alpha - 1)}$</td>
<td>$\uparrow$</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
<td>$\uparrow$</td>
<td>$\downarrow$</td>
<td>N.A.</td>
</tr>
<tr>
<td>$D^{01} = \frac{1 + 2 \alpha P_1 - 2 P_1 + 2 w - 1}{t_s(\alpha - 1)}$</td>
<td>$\downarrow$</td>
<td>N.A.</td>
<td>N.A.</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
<td>N.A.</td>
</tr>
</tbody>
</table>
Table 3.4: New User’s Choice Under Structure 1

<table>
<thead>
<tr>
<th>Option</th>
<th>Behavior</th>
<th>Total Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No Purchase of Software (n=0)</td>
<td>( U_0 = Y )</td>
</tr>
<tr>
<td>2</td>
<td>Buy the Old Version (n=1)</td>
<td>( U_1 = Y - P_1 + t\alpha Q_0 + wC )</td>
</tr>
<tr>
<td>3</td>
<td>Buy the New Version (n=2)</td>
<td>( U_2 = Y - P_2 + t\alpha Q_0 + C )</td>
</tr>
</tbody>
</table>

Table 3.5: Comparative Static Analysis for Software Demands of New Consumers

<table>
<thead>
<tr>
<th>Demand</th>
<th>( \beta )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( \frac{P_3}{P_2} )</th>
<th>( w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case One: Both Old and New Versions Market</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_{old}^2 = \frac{P_2 - \beta P_1 - 1 + \beta w}{t_n \alpha (\beta - 1)} )</td>
<td>( \downarrow )</td>
<td>( \downarrow )</td>
<td>( \uparrow )</td>
<td>( \uparrow )</td>
<td>( \uparrow )</td>
<td></td>
</tr>
<tr>
<td>( D_{new}^2 = 1 - \frac{P_2 - P_1 + w - 1}{t_n \alpha (\beta - 1)} )</td>
<td>( \uparrow )</td>
<td>( \uparrow )</td>
<td>( \downarrow )</td>
<td>( \downarrow )</td>
<td>( \uparrow )</td>
<td></td>
</tr>
<tr>
<td>( D_{dis}^2 = \frac{P_1 - w}{\alpha t_n} )</td>
<td>( \uparrow )</td>
<td>( \uparrow )</td>
<td>N.A.</td>
<td>N.A.</td>
<td>( \downarrow )</td>
<td></td>
</tr>
<tr>
<td>Case Two: Single New Version Market</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( D_{new}^2 = 1 - \frac{P_2 - 1}{t_n \alpha \beta} )</td>
<td>( \uparrow )</td>
<td>N.A.</td>
<td>( \downarrow )</td>
<td>N.A.</td>
<td>N.A.</td>
<td></td>
</tr>
<tr>
<td>( D_{dis}^2 = \frac{P_2 - 1}{t_n \alpha \beta} )</td>
<td>( \downarrow )</td>
<td>N.A.</td>
<td>( \uparrow )</td>
<td>N.A.</td>
<td>N.A.</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.6: Types of Software Users in Simplified 2x2x2 model

<table>
<thead>
<tr>
<th>Market Structure</th>
<th>Type of Software Users</th>
<th>Types of Demands</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: Both Old and New Versions Coexist</td>
<td>Existing Users, New Users</td>
<td>( D_{00}^2, D_{10}^2, D_{02}^2, D_{12}^2, D_{01}^2 )</td>
</tr>
<tr>
<td>2: Only New Version Exists</td>
<td>Existing Users, New Users</td>
<td>( D_{00}^2, D_{10}^2, D_{02}^2, D_{12}^2 )</td>
</tr>
</tbody>
</table>
Table 3.7: Different Demands Under Two Market Structures

<table>
<thead>
<tr>
<th>The Difference between Structure 1 and Structure 2</th>
<th>For existing users</th>
<th>For new users</th>
</tr>
</thead>
<tbody>
<tr>
<td>P + 2w - 1 - 2P</td>
<td>2(a_2 (\alpha - 1))</td>
<td>P + w(\beta - \beta P - 1)</td>
</tr>
<tr>
<td>3P + Bw - BP</td>
<td></td>
<td>(t_\alpha \alpha)</td>
</tr>
</tbody>
</table>
Figure 3.1: Upgrade Software Releasing Sequence Under Structure 1

\[ Q_0, \quad Q_1 = \alpha Q_0, \quad Q_2 = \beta Q_1 = \beta(\alpha Q_0) \]

Figure 3.2: Distribution of Various Demands with Old Version

\[ D^{00}, \quad D^{01}, \quad D^{10}, \quad D^{02}, \quad D^{12} \]

\[ 0, \quad \frac{P_1 - w}{\alpha - 1}, \quad \frac{w - 1 - P_1 + P_1}{(\alpha - 1)}, \quad \frac{P_2 - w - P_1}{1 + \alpha \beta - 2\alpha}, \quad \frac{P_1 + w - 1}{\alpha - 1}, \quad t_n \]

Figure 3.3: Distribution of Various Demands without Old Version

\[ D^{00}, \quad D^{10}, \quad D^{02}, \quad D^{12} \]

\[ 0, \quad \frac{P_1 - 1}{2(\alpha - 1)}, \quad \frac{P_2 - w - P_1}{1 + \alpha \beta - 2\alpha}, \quad \frac{P_1 + w - 1}{\alpha - 1}, \quad t_n \]
Figure 3.4: Upgrade Software Releasing Sequence Under Structure 2

\[ Q_0 \quad Q_1 = \alpha Q_0 \quad Q_2 = \beta Q_1 = \beta(\alpha Q_0) \]

0 \quad 1 \quad 2

Figure 3.5: Distribution of the New Consumers Under Structure 1

\[ D_{\text{new}}^2 \quad D_{\text{old}}^2 \quad D_{\text{new}}^2 \]

\[ t_0 \quad \frac{P_i - wC}{\alpha Q_0} \quad \frac{P_i - P_i + wC - C}{Q_0, \alpha(\beta - 1)} \quad t_n \]

Figure 3.6: Distribution of the New Consumers Under Structure 2

\[ D_{\text{new}}^2 \quad D_{\text{new}}^2 \]

\[ t_0 \quad \frac{P_i - C}{\alpha \beta Q_0} \quad t_n \]
Figure 3.7: Demand of Existing Users

<table>
<thead>
<tr>
<th>Structure 1</th>
<th>Structure 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{P_1 - w}{\alpha} )</td>
<td>( \frac{w - 1 + R - P_1}{\alpha - 1} )</td>
</tr>
<tr>
<td>( \frac{R - 1}{2(\alpha - 1)} )</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.8: Demand of New Users

<table>
<thead>
<tr>
<th>Structure 1</th>
<th>Structure 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{P_1 - w}{\alpha} )</td>
<td>( \frac{P_2 - P_1 + w - 1}{\alpha(\beta - 1)} )</td>
</tr>
<tr>
<td>0</td>
<td>( \frac{P_2 - 1}{\alpha \beta} )</td>
</tr>
</tbody>
</table>

\( t_\pi \)
Bibliography


Appendix A

Appendices to Chapter 3

A.1 Proof of Propositions

Proof of Proposition 8:

First Step Let us prove that the total profit under price discrimination is greater than the total profit under a uniform price.

Approach One: With price discrimination, the software vendor maximizes the profit from existing users and maximizes the profit from new users by different prices respectively.

Approach Two: With a uniform price, the software vendor maximizes the sum of profits from both existing users and new users by the same price. We can see Approach Two is one special case of Approach One by adding a constraint that the price for existing users is equal to the price for new users. From the optimization theory, the optimal value of objective function will shrink if an additional non-redundant constraint is applied. Therefore, the total maximum profit under Approach One should be greater than (at least equal to) the total maximum profit under Approach Two.

Second Step For the same new version, let us show that the software vendor should charge a lower price for existing users and a higher price for new users if $t_n$ is sufficiently large, which means consumers are sufficiently heterogenous. Now the problem is that how the software vendor sets up the different prices of version $Q_2$ to existing users and new users when he maximizes two objective functions. The demands from existing users related to the price of version $Q_2$ is $D^{10}$, $D^{02}$ and $D^{12}$. From the first order condition to maximize the profit from existing users, we can solve the
optimal price level of version $Q_2$, $P_2^* = \frac{2P_1 + \alpha t_n - 2\alpha t_n + w + t_n}{2}$ under two market structures respectively. In the similar fashion, we can figure out the optimal price level of version $Q_2$ for new users under market structures 1 and 2. Please see Table A.2 for details. Recall $t_n$ is the maximum value for the support of $t$ and the variance of $t$ is $\frac{t_n^2}{12}$. A high $t_n$ value suggests the consumers are highly heterogenous. It is easy to check out the optimal price of version $Q_2$ for existing users is lower than new users if $t_n > \frac{2P_1 + 2w - P_0 - 1}{\sigma - 1}$ under Structure 1, and $t_n > \frac{P_0 + w - 1}{2\sigma - 1}$ under Structure 2.

Third Step Let us prove that for the old version, the software vendor should charge lower price to existing users and higher price to new users if the price of new version $P_2$ is greater than $\frac{\beta P_1 - \beta + 2}{2}$. Investigating the objective function under Structure 1, we can figure out the optimal price levels for existing users and new users listed in Table A.3. It is easy to check out that the optimal price of the old version to new users is higher than that to existing users (the left column is greater than the right column in Table A.3) if and only if $P_2 > \frac{\beta P_1 - \beta + 2}{2}$.

QED
Table A.1: Notations Used in the Order of Appearance

<table>
<thead>
<tr>
<th>Notation</th>
<th>Economic Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>Utility</td>
</tr>
<tr>
<td>$Q$</td>
<td>Software quality</td>
</tr>
<tr>
<td>$C$</td>
<td>Extra benefits from customer services etc</td>
</tr>
<tr>
<td>$M$</td>
<td>Consumption of other products except software</td>
</tr>
<tr>
<td>$Y$</td>
<td>Income</td>
</tr>
<tr>
<td>$P$</td>
<td>Software price</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>Initial software quality in period 0</td>
</tr>
<tr>
<td>$C'$</td>
<td>The extra benefits of $C$ available in the period when an upgraded software is released</td>
</tr>
<tr>
<td>$w$</td>
<td>Customer services level in the next period after an upgraded software is released</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>Software quality upgraded in period 1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Magnitude of quality upgrade in period 1</td>
</tr>
<tr>
<td>$Q_2$</td>
<td>Software quality upgraded in period 2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Magnitude of quality upgrade in period 2</td>
</tr>
<tr>
<td>$P_1$</td>
<td>Software price for $Q_1$ in period 1</td>
</tr>
<tr>
<td>$P_2$</td>
<td>Software price for $Q_2$ in period 2</td>
</tr>
<tr>
<td>$P_3$</td>
<td>Software price for $Q_3$ in period 2</td>
</tr>
<tr>
<td>$n$</td>
<td>Indicator variable for consumer upgrade options</td>
</tr>
<tr>
<td>$t$</td>
<td>Utility index</td>
</tr>
<tr>
<td>$t_0$</td>
<td>The lower boundary of $t$'s support</td>
</tr>
<tr>
<td>$t_u$</td>
<td>The upper boundary of $t$'s support</td>
</tr>
<tr>
<td>$h(t)$</td>
<td>The density function of $t$</td>
</tr>
<tr>
<td>$U_j$</td>
<td>Utility in Period $j$, $j=1,2$ and 3</td>
</tr>
<tr>
<td>$U^i$</td>
<td>Utility under consumer's option $i$, $i=1,2,3,4$ and 5</td>
</tr>
<tr>
<td>$D$</td>
<td>Software upgrade demand</td>
</tr>
<tr>
<td>$D^{00}$</td>
<td>No upgrade in periods 1 or period 2, i.e., “disappearance” demand</td>
</tr>
<tr>
<td>$D^{01}$</td>
<td>An upgrade to $Q_1$ at period 1 and no upgrade in period 2, i.e., “opportunist” demand</td>
</tr>
<tr>
<td>$D^{02}$</td>
<td>No upgrade in period 1 and an upgrade to $Q_2$ in period 2, i.e., “leapfrog” demand</td>
</tr>
<tr>
<td>$D^{12}$</td>
<td>An upgrade to $Q_1$ in period 1, and an upgrade to $Q_2$ in period 2, i.e., “loyal” demand</td>
</tr>
<tr>
<td>$D^{01}$</td>
<td>No upgrade in period 1, but an upgrade to $Q_1$ in period 2, i.e., “lagged” demand</td>
</tr>
<tr>
<td>$R$</td>
<td>$R = P_i / P_j$, the relative price $P_i$ with respect to $P_j$</td>
</tr>
<tr>
<td>$D_{new}$</td>
<td>New users demand to buy new version</td>
</tr>
<tr>
<td>$D_{old}$</td>
<td>New users demand to buy old version</td>
</tr>
<tr>
<td>$D_{disc}$</td>
<td>New users choose not to buy any versions</td>
</tr>
</tbody>
</table>
Table A.2: Optimal Prices of the New Version for Existing and New Users

<table>
<thead>
<tr>
<th>Structure</th>
<th>Existing Users</th>
<th>New Users</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_2^* = \frac{2P_i + \alpha \beta t_n - 2\alpha t_n + w + t_n}{2}$</td>
<td>$P_2^* = \frac{2P_i + \alpha \beta t_n - 2\alpha t_n + w + t_n}{2}$</td>
</tr>
<tr>
<td></td>
<td>$P_2^* = \frac{\alpha \beta t_n - \alpha t_n - w + 1 + P_1}{2}$</td>
<td>$P_2^* = \frac{\alpha \beta t_n - \alpha t_n - w + 1 + P_1}{2}$</td>
</tr>
</tbody>
</table>

Table A.3: The Optimal Prices of the Old Version to Different Users

<table>
<thead>
<tr>
<th>Structure</th>
<th>Existing Users</th>
<th>New Users</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_3^* = \frac{2w + P_i - 1}{4}$</td>
<td>$P_3^* = \frac{P_i - 1 + \beta w}{2\beta}$</td>
</tr>
</tbody>
</table>