TWO MODELS OF DYNAMIC INPUT DEMAND:

ESTIMATES WITH CANADIAN MANUFACTURING DATA

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ABSTRACT

Over the past decade there has been a number of innovations in the estimation of input demand equations. In particular, ways of incorporating the hypothesis of rational expectations into empirical models of the firm have been developed and improved upon.

This research agenda was perhaps inspired by the Lucas critique of econometric policy evaluation, which suggested that econometric models which did not explicitly take account of how expectations of the future affect current behaviour would give misleading results regarding the possible effects of various government policies. Lucas specifically directed part of his critique at empirical models of business investment, which had been used previously in the assessment of tax policies designed to affect investment.

This thesis has a dual purpose. First, two distinct models of input demand are estimated with Canadian manufacturing data. Each of the models incorporates to some degree the hypothesis of rational expectations, but the specifications of technology differ. Neither of these models, to our knowledge, has been estimated with Canadian data. We are interested in whether either model explains well the behaviour of the Canadian manufacturing sector, and in how the results compare with the (few) U.S. applications of this type of model.

The second purpose is to use the results of these models in simulations to assess the effect of changes to the after-tax

rental rate of capital on investment and employment in manufacturing. While there have been studies in Canada (and elsewhere) that attempt to calculate the effects of various tax policies on investment, most studies were done prior to the innovation of techniques in estimating models with rational expectations. This thesis is able to examine the effects of a particular change while remaining immune to the Lucas critique. If the modelling of expectations is correct, this could not only improve the reliability of the estimates, but also give some indication of the empirical importance of the Lucas critique.

The results can be summarized as follows. The two models give very different estimates of price elasticities of demand for capital and labour, even though they are similar in many respects and are estimated with a common data set. It is also the case that their estimates of the effects of temporary and permanent changes to the rental rate are different. Adjusting the reduced form parameters of the input demand equations to account for changes in tax policy regimes alters the results to a significant degree, suggesting that the explicit modelling of expectations matters in an empirically relevant sense. However, these effects are in opposite directions for the two models considered here. All this suggests that more research is required into the relationship between expectations of future policy and investment behaviour.

TABLE OF CONTENTS

LIST OF TABLES

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CHAPTER 1

Introduction

Over the past decade there has been a number of innovations in the estimation of input demand equations. In particular, ways of incorporating the hypothesis of rational expectations into empirical models of the firm have been developed and improved upon.

This research agenda was perhaps inspired by the Lucas (1976) critique of econometric policy evaluation, which suggested that econometric models which did not explicitly take account of how expectations of the future affect current behaviour would give misleading results regarding the possible effects of various government policies. Lucas specifically directed part of his critique at empirical models of business investment, which had been used previously in the assessment of tax policies designed to affect investment.

This thesis has a dual purpose. First, two distinct models of input demand are estimated with Canadian manufacturing data. Each of the models incorporates to some degree the hypothesis of rational expectations, but the specifications of technology differ. Neither of these models, to our knowledge, has been estimated with Canadian data. We are interested in whether either model explains well the behaviour of the Canadian manufacturing sector, and in how the results compare with the (few) U.S. applications of this type of model.

The second purpose is to use the results of these models to assess the effect of the rental rate of capital in Canada on investment and employment in manufacturing. While there have been several studies in Canada (and elsewhere) that attempt to calculate the effects of various tax policies on investment, most studies were done prior to the innovation of techniques in estimating models with rational expectations. This thesis will be able to examine the effects of temporary and permanent changes in the rental rate while remaining immune to the Lucas critique. If the modelling of the expectations process is correct, this could not only improve the reliability of the estimates, but also give some indication of the empirical importance of the Lucas critique.

In the remainder of this chapter, sections 1.1 through 1.4 give a brief history of modelling input demands and investment, and sketch out the origins of the models estimated in this thesis. Section 1.5 contains a short survey of research on the question of the effects of tax policy on investment, and shows how the method of policy analysis we have adopted differs from those used in previous studies.

1.1 The Neoclassical Model of Investment with Distributed Lags

The essence of neoclassical theories of investment is that a firm will choose its path of capital accumulation, and of other inputs, with the objective of maximizing the present discounted value of the income stream generated by the firm. Key secondary assumptions (see Jorgenson (1967, p.136)) are that the firm purchases all inputs, including capital goods, in competitive markets, that the firm sells output in a competitive market, and that the contribution of capital to output is through a flow of services proportional to the stock of capital.

It can be presumed that the firm purchases capital goods to use itself, or that the firm leases capital goods from a separate owner. In either case, the cost of using a unit of capital for one time period, called the user cost of capital or the rental rate of capital, is the same, and is a key determinant of the optimal capital stock for the firm. Standard microeconomic theory of the firm suggests that, if the marginal product of capital is decreasing with the level of capital input, and if the firm may obtain any capital stock it wishes at market prices, firms will desire a level of capital input such that the marginal product of capital is equal to the user cost.

In the absence of corporate taxes or subsidies, the user cost of one dollar's worth of capital for one year is equal to the interest rate plus the depreciation rate on capital minus any capital gains resulting from a change in the price of capital goods over the year (see Jorgenson (1967, p.143) or Boadway (1980, p.253)). Corporate taxation, together with tax exemptions for depreciation and interest, yields a more complicated expression for the user cost of capital; this is described later in Chapter 2.

It has long been accepted that while the neoclassical model of the firm can give us a theory of the demand for capital, this is not the same thing as a theory of investment. Consider an often cited extract from Haavelmo (1960, p.216):

> What we should reject is the naive reasoning that there is a demand schedule for investment which could be derived from a classical scheme of producers' behavior in maximizing profit. The demand for *investment* cannot simply be derived from the demand for *capital*. Demand for a finite addition to the stock of capital can lead to *any rate* of investment, from almost zero to infinity, depending on the additional hypothesis we introduce regarding the speed of reaction of capital-users. I think that the sooner this naive, and unfounded, theory of the demand-for-investment schedule is abandoned, the sooner we shall have a chance of making some real progress in constructing more powerful theories to deal with the capricious short-run variations in the rate of private investment. (Emphasis in original).

This view has not been universally accepted however. Bliss (1975, p. 304) points out that if we have determined the demand for capital at time t and at time t+1 then we have also determined the rate of net investment over that time interval. What Haavelmo seems to be saying, suggests Bliss, is that the demand for net investment will not necessarily be equal to the change in the levels of capital stock that equates marginal product to user cost from one period to the next, because for various reasons (described below) firms will not always choose those levels of capital. In any case, Haavelmo, when discussing the 'demand for investment' is referring to what firms actually do, whereas when discussing the 'demand for capital' he is referring to the levels of capital firms would choose in a world of no time lags on the delivery of capital goods and no adjustment costs with respect to the levels of capital input.

Jorgenson's solution to the problem posed by Haavelmo is as follows. Let K_t^* be the level of the capital stock at time t that satisfies the condition of marginal product of capital equals the user cost. Assume that each period new investment projects are initiated such that the backlog of uncompleted projects is equal to the difference between K_t^* and the actual stock of capital K_t. Let $w(L)$ be a power series in the lag operator L, which describes the rate at which investment projects in progress become completed, and which is given. If δ is the depreciation rate, and gross investment in any period equals investment in new projects plus replacement investment, then gross investment at time t is given by

$$I_t = w(L)[K_t^* - K_{t-1}^*] + \delta K_t \qquad (1.1)$$

(Jorgenson (1963, pp.249-51)). The demand for capital is thus transformed into a 'demand for investment' by the application of a rational distributed lag process (see Jorgenson (1966) for an analysis of the properties of such a process).

Jorgenson (1963) and Hall and Jorgenson (1967) estimate the parameters of investment functions like (1.1), and from these are able to infer the response of investment with respect to changes in user costs and output prices. Eisner and Nadiri (1968) are critical of Hall and Jorgenson's reliance on the Cobb-Douglas specification of technology, when the evidence suggests that something more general is called for, and of their method of choosing an appropriate lag structure, since it turns out that results are highly dependent on the restrictions placed on $w(L)$.

Gaudet, May, and McFetridge (1976) apply the neoclassical model with a distributed lag to Canadian manufacturing data (1952 to 1973), using a CES production function (which is recommended by Eisner and Nadiri). They suggest that the results of the model could be useful in the analysis of tax policies, since they find that the user cost of capital has a statistically significant effect on investment.

How the modelling of investment has changed from the method of Jorgenson over the past two decades is the subject of the next three sections.

1.2 The Neoclassical Model of Investment with Adjustment Costs

The concept of adjustment costs was incorporated into the neoclassical theory of investment because, in the words of Lucas (1967b, p.78):

> ...many students of investment behavior have recognized the incongruity of developing a rigorous economic theory of the determination of [K_t^*] and then combining this with an *ad hoc* theory of adjustment.

When we speak of adjustment costs we mean that in the production function output depends not only upon the levels of inputs, but also on the rate at which these levels are changing. In particular, the faster the levels of inputs are changing, the lower will be the rate of output, *ceteris paribus*.

Gould (1968), Lucas (1967a, 1967b), Mortensen (1973), Schramm (1970), and Treadway (1969, 1970, 1971) all specify adjustment costs as convex. That is, the cost to the firm of changing its level of capital input in any period is increasing with the absolute value of the change and is increasing at an increasing rate. All of the studies mentioned above base adjustment costs on the rate of net investment, with the exception of Gould, who uses gross investment, and Lucas (1967a), who uses gross investment as a proportion of capital stock.

Since it is assumed in these studies that capital markets are competitive, the rationale given for adjustment costs is that when there is net investment output is less, given the stocks of capital and labour, than it would be in the absence of net investment. This is because some labour and capital must be devoted to installing, and learning how to work with, the new levels of inputs (see Nickell (1978, Chapter 3)).

With adjustment costs incorporated into the neoclassical model - i.e. it remains the case that firms seek to maximize the present discounted value of cash flow, and that they buy inputs and sell outputs in competitive markets - the resulting demand for net investment has the same form as was obtained by Jorgenson.

That is, net investment is proportional to the difference between the 'target stock' of capital K_t^* and actual capital K_t. This reduced form is called the flexible accelerator. The difference between this result and Jorgenson's is that the adjustment cost model gives a net investment function that is the result of *equilibrium* capital accumulation, whereas Jorgenson's is essentially *disequilibrium* (see Bliss (1975, p. 305)). In the adjustment cost model the movement towards K_t^* is not immediate because it would not be profit maximizing to do so. In Jorgenson's distributed lag model movement towards K_t^* is not immediate because there are time lags involved in completing capital projects that cannot be avoided no matter what price the firm is willing to pay.[1]

In either the distributed lag or the adjustment cost model information about future prices is valuable to the firm. If the existing level of capital input at any time constrains the firm in its problem of maximizing the present discounted value of future cash flow, then the choice of net investment in the current period will determine the constraint at the beginning of the next period. If prices are changing in some way over time, the way they are

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[1] See Almon (1965) for an empirical study of such unavoidable time lags between capital appropriations and expenditures. The notion of unavoidable delays between initiating a capital project and the capital being available for production services reappears in the "time-to-build" model of Kydland and Prescott (1982).

changing will affect future target capital stocks and therefore current net investment. The innovation to empirical models of investment which followed the introduction of adjustment costs was the introduction of rational expectations.

1.3 Rational Expectations and the Lucas Critique

Rational expectations as currently applied to economic models dates from Muth (1961). He describes the idea as follows:

> ...expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the "objective" probability distributions of outcomes).

In practice, the information set postulated by modellers includes (1) the structure of the model itself, and (2) all past values of relevant variables.

An implication of the assumption of rational expectations is that forecast errors of firms should be random and uncorrelated to any variables in the information set or to past values of errors, since any such correlation would be useful information to the firm that should be incorporated into the subjective probability distribution of outcomes. This is why rational expectations really amounts to a consistency condition; except for a random error term the expectations of firms should be consistent with the outcomes of the model which incorporates the firms.

We have mentioned earlier that corporate tax policy influences the user cost of capital, and that in the presence of adjustment costs firms will want to make forecasts of user costs to help determine the optimal level of net investment for the current period. This means that under rational expectations the relationship between the rate of net investment and the current user cost of capital will vary according to expectations of future user costs. Yet Hall and Jorgenson (1967) treat the relation between investment and user costs as though it were stable.

The method of Hall and Jorgenson for estimating the effects of tax policy on investment was as follows. First, derive a relation between the user cost and the optimal capital stock from a static perspective, K_t^*. Then estimate the distributed lag that relates investment to lagged values of K_t^*. Then ask how K^* would have been different had user costs been different, say because of some hypothetical alternative tax policy. Then, *given the distributed lag function which was estimated using actual data,* ask how the path of investment would have been different.

The critique of econometric policy evaluation by Lucas (1976) is based on the idea that reduced form parameters of an econometric model, say the distributed lag function $w(L)$ for example, will not be invariant to changes in policy. Thus policy analysts should distinguish between structural and reduced form parameters of models (a warning made as early as 1953 by Marschak). Structural parameters, for the purposes of policy analysis, are those which are invariant to changes in the policy regime, where by policy regime we mean the process which guides year to year changes to the tax structure. Since in a model of investment where expectations of future rental rates matter the reduced form of the investment equation will change with changes in the policy regime, the results of Hall and Jorgenson on the effects of tax policy on investment are not reliable.

The research agenda suggested by the Lucas critique was to devise means of estimating models which included rational expectations and which could identify the structural parameters (in models of the firm these are usually taken to be the technology and adjustment cost parameters). With the structural parameters estimated the modeller could determine how reduced form parameters would change for different policy regimes, and could more accurately assess the impacts of various policies.

Although much of this research was focussed on the estimation of complete macroeconomic models[2], we direct our attention to techniques of estimating models of input demand in the presence of adjustment costs, called dynamic models of input demand.

A firm which seeks to maximize the discounted stream of returns from production subject to the constraints of existing levels of inputs, adjustment costs to changing these levels of inputs, and market prices for inputs and output which evolve over time in a way beyond its control, is faced with a problem of

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[2] See Chow (1983, Chapter 11), Taylor (1979), or Wallis (1980) for example.

optimal control.[3] The first order conditions for the solution to the firm's maximization problem include the Euler equations and the transversality conditions. The technique referred to as "limited-information estimation" involves estimating the parameters of the model by estimating the Euler equations directly. Kennan (1979) describes how the Euler equations might be estimated efficiently.[4] Applications of this method to dynamic models of input demand are described in the following section.

Under the technique known as "full-information estimation" the model is solved for all the first-order conditions and then estimated. There are two methods of achieving this. Hansen and Sargent (1980, 1981, 1982) solve for the firm's optimal decision rule for input demands in the current period as a function of lagged values of input levels and future expected prices. They then use results of prediction theory to express future expected prices in terms of current information (past observations of variables useful in forecasting relevant prices and knowledge of the model which determines these variables). This gives an input demand equation which can be estimated since the right-hand side variables - lagged input levels and the current information set - are all observed variables. Technology parameters of the firm and

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[3] A standard reference is Chow (1975). For a treatment which incorporates into the optimal control problem recent developments in rational expectations see Sargent (1987, Chapter 1).

[4] Also see Hansen (1982) and Hansen and Singleton (1982).

parameters which are used in the projection of current information into expected future prices are estimated directly. Simultaneously, the model used to forecast future prices is also estimated. There will be cross-equation restrictions in this simultaneous estimation, and tests of the cross-equation restrictions amount to a joint test of the model of the firm and of the hypothesis of rational expectations. Lucas and Sargent (1981, p. xvii) refer to the cross-equation restrictions as a "hallmark" of rational expectations models; the firm's decision rule for input demand is explicitly related to the model used in forecasting prices.

Chow (1980a, 1981) takes a somewhat different approach, although the estimation remains full-information and does not yield results different from Hansen and Sargent. Chow uses the parameterization of standard optimal control theory, where the model of the movement of input and output prices is incorporated into the "transition equation", which represents the dynamic constraint for the firm in its optimization problem. When the parameters of the system are estimated, it remains true that there are cross-parameter restrictions testing the joint hypothesis of the model of the firm and rational expectations.

Full-information methods have so far only been applied to problems where the firm's objective function can be described in linear or quadratic terms. This leads to linear input demand functions, at least in the reduced form parameters, although the cross-equation restrictions are non-linear and extremely complex even for models with only two inputs. Linear-quadratic objective functions also allow the modeller to invoke the principle of "certainty-equivalence"; the solution to the firm's optimal control problem in the uncertain world is the same as it would be had the firm perfect foresight.

Comparisons of the two methods as applied to dynamic models of input demand are made by West (1986) and Prucha and Nadiri (1986). West compares limited- and full-information methods numerically, and finds that the full-information method lowers standard errors only slightly, and that its parameter estimates tend to be more biased than limited-information estimates when the model is misspecified. Monte Carlo comparisons of the two techniques by Prucha and Nadiri, on the other hand, find "considerable gains in statistical efficiency" (p.209) from using full-information methods.

1.4 Modelling Investment with Rational Expectations

Full-information estimation of a dynamic model of input demand is usually carried out with the assumption of a linear-quadratic objective function for the firm.[5] If y_t is a scalar output and x_t is a column-vector of inputs then the usual form of the production function, excluding adjustment costs is

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[5]See Epstein and Yatchew (1985), Hansen and Sargent (1980, 1981), Meese (1980), Sargent (1978), and West (1986) for examples.

$$y_t = a'x_t + x_t'Ax_t/2 \tag{1.2}$$

where a is a vector and A is a symmetric and negative definite matrix. Convex adjustment costs are also specified as quadratic, and could be represented by

$$(x_t - x_{t-1})'B(x_t - x_{t-1})/2 \tag{1.3}$$

where B is a symmetric matrix.

The theoretical results of such a model of the firm under rational expectations are described by Lucas and Prescott (1971), who are primarily concerned with the equilibrium price of capital, and by Sargent (1979, Chapter 14, and 1981) who considers in particular the optimal decision rule for the firm.

The first empirical use of this model is Sargent (1978), who considers the demand for labour, both straight-time and over-time, when there are adjustment costs present. Sargent takes wages as being exogenous with respect to labour demand (this assumption is justified on the basis of causality tests with which Sargent begins the paper). Kennan (1988), on the other hand, estimates a model using Sargent's firm together with endogenous labour supply, where labour suppliers also have linear-quadratic objective functions.[6]

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[6] Kennan's model is drawn from Sargent (1979, Chapter 16). See Nickell (1986) for a complete survey of dynamic models of labour

Meese (1980) uses full-information techniques to estimate the demand for capital and labour by U.S. manufacturing (using quarterly data from 1947 to 1974) according to a linear-quadratic model. After justifying the modelling of user costs and wages as exogenous with respect to input levels, he simultaneously estimates a four equation model: one equation each for demand for labour and demand for capital, and a bivariate autoregressive model of user costs and wages. The hypothesis of rational expectations imposes restrictions between parameters of the former two equations and the latter. The restrictions are highly non-linear. Meese remarks (pp.149-50):

> The estimation of the constrained version of the model...is a difficult task. Few software routines are capable of estimating a model of such complexity...

Estimation is carried out by appending a "concentrated likelihood function" with a penalty function, where the penalty function weights the various restrictions of the model. A likelihood ratio test of the model, comparing the restricted version to what amounts to an unrestricted four equation vector autoregression, rejects the theoretical restrictions at any significance level greater than 2%.

Epstein and Yatchew (1985) take the theoretical model used by Meese, and find a reparameterization of the estimating equations that somewhat simplifies the estimation of the restricted model.

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demand.

It is this simplified version that is estimated, without amendment, with Canadian data in this thesis in Chapter 3, so a full discussion of the Epstein and Yatchew method is deferred until later.

We now turn our attention to alternative methods of estimating dynamic models of input demand with rational expectations.

A model of Tobin's (1969) has generated a method of modelling investment known as "q-theory". Tobin's q is the ratio of the nominal market value of a firm to the nominal value of the firm's capital stock evaluated at replacement cost. A value of q greater than one should lead to positive net investment, since the value of the new capital will be greater than its cost. The rate of investment, assuming there are convex adjustment costs, should then be positively related to the current value of q. It is assumed that the value of the firm's equity captures the market's expectations about the future value of capital.

Hayashi (1982) makes two important observations. First, to model investment we should use "marginal q" rather than "average q" as the explanatory variable, where the former is the marginal change in the market value of the firm for an additional unit of capital divided by the price of a unit of capital, and the latter is the total market value of the firm divided by the total value of the capital stock at replacement values. Second, a q-theory model using marginal q and the neoclassical model with adjustment costs and rational expectations are equivalent theories[7].

Q-theory models of investment have been estimated by Hayashi, Abel (1980), Summers (1981), and McKibbin and Siegloff (1988). A problem with empirical application of q-theory is that:

> ...(with) the use of stock market valuation to infer investor perception of physical investment opportunities...the information must be taken in toto. But the information relevant to investment may be overshadowed by the volatility of the "noise" in stock market fluctuations. (Bosworth commenting on Summers (1981), p. 130).

A further problem, mentioned by Abel (1980, p. 77) is that if we rely on variations in the stock market to explain fluctuations in investment, it "begs the question, since it does not explain what [factors determine values in] the stock market".

A second alternative method to modelling dynamic input demand is that referred to earlier as the "limited-information" method, where the Euler equations of the firm's optimal control problem are estimated directly. Although some useful information is sacrificed when this technique is used, there is the advantage that we need not restrict ourselves to technologies that are linear-quadratic. This allows the possible use of production functions that more closely fit the facts.

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[7]Hayashi credits Lucas and Prescott (1971) for this insight, although they did not put it in these terms.

Pindyck and Rotemberg (1983a, b) apply limited-information techniques to a model with a translog restricted cost function, quadratic adjustment costs, and rational expectations. The model is estimated with annual U.S. manufacturing data (1948-71 for (1983a) and 1949-76 for (1983b)). Shapiro (1986a) uses the same techniques, but with a Cobb-Douglas production function, for quarterly U.S. manufacturing data from 1955-80. Kokkelenberg and Bischoff (1986) use a polynomial approximation to a short-run variable cost function on quarterly U.S. manufacturing data from 1959 to 1977. While there are no studies of this type, of which we are aware, that apply to Canadian data, Carmichael, Mohnen, and Vigeant (1989) apply a translog variable cost function to Quebec manufacturing data (annual 1962-83). Their tests fail to reject the restrictions imposed by the model. It is interesting to note here their results for the elasticity of capital with respect to user costs, since this statistic will also be estimated in this thesis. They find the "impact elasticity" to be - 0.098 and the long-run elasticity (for a shock to user costs that is permanent and immediately recognized as permanent) to be -0.271.

The final approach to estimating dynamic input demands with rational expectations is from Epstein and Denny (1983). A flexible functional form is chosen for the value function of a firm's variable cost minimization problem, where adjustment costs are present. A limited specification of expectations is allowed; it is supposed that real input prices follow first-order autoregressive processes.[8] While this may not be consistent with "fully rational" expectations, in that there may be other information available useful for forecasting input prices, it closely approximates what is usually specified in practice in rational expectations models anyway. A more complete discussion of this model is found in Chapter 4 of this thesis.

Neither the linear-quadratic rational expectations model of dynamic input demand, nor the model of Epstein and Denny, have been estimated, to our knowledge, with Canadian aggregate data.[9] One of the two principal contributions of this thesis is to estimate, and compare, these two methods of estimating input demands, using Canadian manufacturing data.

1.5 Tax Policy And Investment

Since the introduction of neoclassical theories of investment, which provided a link between user costs of capital, and therefore corporate tax policy, to investment, researchers have been using these models to consider the effects of various tax policies on investment.

Hall and Jorgenson (1967) is frequently cited as the seminal

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[8] Morrison and Berndt (1981) estimate a static expectations version of a model similar to Epstein and Denny's.

[9] Bernstein (1986) estimates a static expectations version of Epstein and Denny's model with the pooled data of some Canadian firms engaged in research and development.

article in this field. The number of studies on U.S. tax policy and investment since Hall and Jorgenson is immense; a survey of neoclassical models of investment and tax policy in the U.S. is given by Chirinko (1986, 1987). Chirinko and Eisner (1983) compare the empirical predictions of a number of U.S. macroeconomic models regarding tax policy and investment.

Here we will confine the discussion to Canadian studies of investment, which will later be compared to the results obtained in this thesis.

Bird (1980) classifies the various studies that have been done in Canada by three types: survey, econometric, and quasi-empirical. The two major surveys on tax policy and investment in Canada are Helliwell (1966) and the Tax Measures Review Committee (1975). Helliwell considers the behaviour of 70 large firms, 35 of which are in manufacturing, the others deal in resources or services. These firms are taken from those which were interviewed by the Royal Commission on Banking and which were also sent questionnaires by the Royal Commission on Taxation. Two tax initiatives are examined. One is a 1961 change to depreciation allowances, which allowed depreciation at double the normal rates in the year an asset was purchased, with normal rates of depreciation in following years. This provision lasted until January 1, 1964. Summarizing the results of the surveys, Helliwell (1968, p. 128) says the measure was not "...thought by firms to have had a noticeable influence on their investment expenditures". The other initiative is a 1963 proposal to allow 50% straight-line depreciation on machinery and equipment for capital purchased in the two years commencing June 14, 1963 by firms which are either 25% Canadian owned and controlled or are involved in manufacturing and processing in designated areas of slow growth. The measure was to expire in June 1965, although it was later extended to the end of 1966. Helliwell (1966, pp. 170-72) provides a number of responses by managers which in general suggest that the main impact of this incentive was to change the timing of investment projects. For example, one company spokesman said:

> Although we wouldn't undertake a project because of the accelerated depreciation, we probably will order our equipment early to allow us to take whatever advantage is obtainable.

The Tax Measures Review Committee was especially interested in the effects on firms of the investment tax incentives introduced in the 1972 federal budget. These incentives consisted of accelerated depreciation allowances and lower corporate tax rates in the manufacturing sector. With 1,288 firms responding to their survey, they found:

> ...83 per cent of the respondents anticipated that the tax measures would have some positive impact on their operations. ...Increased investment expenditures as a result of the tax measures were anticipated by 47 per cent of the respondents...[10]

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[10] Tax Measures Review Committee (1975, p. 9).

The positive responses obtained are surprising given the survey results of Helliwell, and as we shall see later are not consistent with empirical studies. May (1979, p. 73) claims "The findings of the Committee were greeted with a good deal of skepticism by both the professional and political communities". It is well known that economists are often skeptical regarding any results of surveys.

By quasi-empirical studies of investment Bird (1980, p. 42) has in mind research that "...uses numbers, but in a much less formal way than in the econometric studies". Three examples are Hyndman (1974), Harman and Johnson (1978), and Johnson and Scarth (1979).

Hyndman's concern is the effects of the 1972 corporate tax changes for manufacturing firms that were also the focus of the Tax Measures Review Committee study discussed earlier. Hyndman does not actually calculate the effects on investment, but rather assesses the effects of the tax changes on the costs of production, leaving the reader to infer what the overall effects might be. He says the 1972 changes lowered user costs of machinery and equipment in manufacturing by at most 20%, which he claims increases the price of final output relative to costs by about 3%.

Harman and Johnson estimate the elasticity of investment with respect to user costs of capital using a model first suggested by Coen (1971), in which investment depends on new orders, cash flow, past investment, and the user cost relative to wages. The results are used to calculate the impacts of various investment incentives from the 1963 federal budget to the 1972 budget. Since the only figures reported are the present value of induced investment from the budget initiatives it is difficult to infer the relevant elasticities.

Johnson and Scarth, like Harman and Johnson, place much emphasis on the ratio of induced investment to the level of the "tax expenditure" by the government. Hall and Jorgenson's (1967) model of investment is used to calculate the effects of an investment tax credit and a lowering of the corporate tax rate. The model of investment itself is not actually estimated, but rather parameter values are imposed, so the impact of investment incentives on investment is given *a priori*.

Turning to the final category of research on investment behaviour, econometric studies on Canada are Wilson (1967), McFetridge and May (1976), and Braithwaite (1983).

Wilson's econometric model of investment assumes putty-clay capital[11] and a distributed lag linking completed investment projects to capital appropriations. At any time the optimal capital stock depends on user costs of capital, availability of corporate funds, and output. A range of reduced forms involving the aforementioned variables are estimated. The long run elasticity of investment to the interest rate is estimated as -0.67. Similar models estimated with U.S. data found somewhat

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[11] Capital is putty-clay in the particular sense that factor proportions are fixed for completed projects, but not for "backlogged" projects (Wilson (1967, p. 36)).

lower elasticities: Jorgenson (1963) obtained -0.38 and Bischoff (1971) found -0.23. Wilson also found, as did the U.S. studies, that the peak investment response to an interest rate shock was one year following the shock (the model is quarterly).

McFetridge and May estimate a model of investment similar to Jorgenson's, with investment being log-linear in output and relative input prices and depending on an *ad hoc* lag structure. The impact elasticity of capital stock with respect to the user cost of capital, a_1 in their notation, is -0.08, and the long run elasticity, π_1, is -0.43.[12] The results are used to analyse the accelerated depreciation changes of 1972. Their estimate of the extra gross investment induced by the change in tax policy is only about one-quarter the estimate of the Tax Measures Review Committee referred to earlier.[13]

Braithwaite embeds his model of investment in the Economic Council of Canada's macroeconomic model CANDIDE 2.0. Investment is modelled as depending on distributed lags of the value of output relative to the user cost of capital, and levels of the capital stock; i.e. no production function is explicitly described. The

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[12] In Chapters 3 and 4 of this thesis the elasticity of capital with respect to user cost is estimated and can be compared directly to McFetridge and May's results.

[13] May (1979) remarks that the Tax Measures Review Committee claimed larger effects of tax policy on investment than *any* empirical study.

estimates of his investment equation are difficult to interpret. On the other hand, a simulation of Braithwaite's which will be of interest to us is an increase in the investment tax credit. He considers a permanent increase, in 1980, by a factor of 1.8 in the investment tax credit. Since in 1980 the base rate of the investment tax credit in Canada was 7%, the experiment involves increasing the base rate to 12.6%. The effect on investment in machinery and equipment in manufacturing, relative to the base case, expressed in terms of millions of 1971 dollars is:[14]

year	1980	1981	1982	1983	1984	1985
change	+30	+82	+126	+134	+125	+68

In 1983, where the effect on investment peaks, the effect is 3.2% of the base case gross investment.

Each of the studies described in this section take quite different approaches to estimating the effects of tax policy on investment. Bird (1980, p. 46) remarks "anyone trying to discern the effects of incentives on investment from the studies reviewed above must feel as though he has wandered into the Tower of Babel". Yet with the exception of the Tax Measures Review Committee there does seem to be some consensus that the demand for capital is fairly inelastic with respect to the user cost.

Feldstein (1982) eloquently states the case for considering a

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[14]Braithwaite (1983, p. 67).

wide range of theoretical models of investment when attempting to ask any question, and for looking for results which seem to be invariant to model selection. In this thesis two models of input demand will be estimated. Each of the models assumes firms face adjustment costs when changing input levels. For each of the models we obtain estimates under alternative *a priori* specifications of static and non-static expectations.

It is clear that the models used here will not exhaust all possible ways of calculating the effects of the rental rate on investment, but it is hoped that the results will complement the other studies described above. We will be interested in whether rational expectations models of input demand with adjustment costs generate elasticities very different from those found by others.

The case can also be made that the models used here represent something of an improvement over previous studies; techniques designed to improve the way input demand models are estimated, which have only very recently been developed, are put to use.

In Chapter 2, the data to be used in estimation is described in detail.

The linear-quadratic model is described in detail and estimated in Chapter 3. Some analysis of the time-series properties of the data is undertaken to ensure that the data are compatible with the theoretical model to which they will be applied. The simplified estimation procedure of Epstein and Yatchew (1985) is used. The method of doing policy analysis with the model is described, and some simulations are carried out which consider the effects of changes in user costs on investment.

In Chapter 4 the model of Epstein and Denny (1983) is described and estimated. Simulations similar to those in Chapter 3 are done for the purposes of comparison, and provide an indication of the dependence of policy simulation results on the particular model of dynamic input demand chosen.

The results of these three chapters are useful for three reasons. First, they provide us with more information on the usefulness of such models and whether there are distinct advantages in using one type of specification rather than another. Chapter 4 contains an application of Davidson and MacKinnon's (1981) "P test", which, for each of the two models, evaluates the model where the other model is taken as the alternative hypothesis.Second, they will provide some estimates of the effects of the rental rate of capital on investment, which can then be compared to previous studies. Third, we will obtain some empirical evidence on the dependence of the reduced form parameters of input demand equations on expectations. Alan Blinder has remarked:[15]

> The Lucas critique may be correct, but I have seen no persuasive evidence in any sphere to indicate that it is empirically important. The empirical case is yet to be made. The big question is whether changes in policy regimes cause large changes in coefficients. Maybe they cause just very tiny changes.

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[15] In Klamer (1983, p. 166).

We will provide evidence on the question of the degree to which changes in the 'tax policy regime' affect the reduced form coefficients of input demand equations.

Chapter 5 concludes the thesis. The results, and the comparisons with previous studies, will be summarized, and possible future research will be described.

CHAPTER 2

The Data

In this chapter all data that will be used in estimating the input demand models in Chapters 3 and 4 are described.

2.1 Wages and Rental Rates

Table 2.1 lists the data used in this thesis. The nominal rental rate for capital is from the Economic Council of Canada CANDIDE 2.0 Database, and is described in detail by Braithwaite (1983).

The following derivation of the implicit rental rate of capital is taken from Boadway (1980). Imagine a perfectly competitive firm which uses capital, k, to produce output, y. Call the marginal product of capital MPK and the purchase price of a unit of capital q. According to the neo-classical theory of investment in the absence of any adjustment lags or adjustment costs in capital stock, the firm will purchase units of capital up to the point where the price of a unit of capital is equal to its net-of-tax present discounted value of marginal revenue product:

$$q_t = \int_t^{\infty} p_s MPK_s (1-u_s) e^{-(R+\delta)(s-t)} ds + q_t u_t Z_t (1-ITC_t) + q_t ITC_t \tag{2.1}$$

where p_s is the price of output at time s, u is the profits tax rate, R is the interest rate, δ is the depreciation rate on capital, Z is the present discounted value of deductions allowed for depreciation and interest costs, and ITC is the investment tax credit rate. Note that it is presumed here, as is actually the case in Canada, that the amount of capital eligible for depreciation and interest allowances is reduced by the amount of the investment tax credit. Differentiating equation (2.1) with respect to t gives:

$$\dot{q}_t=(R+\delta)q_t(1-u_tZ_t(1-ITC_t))-ITC_t-(1-u_t)p_tMPK_t+\dot{q}_tu_tZ_t(1-ITC_t)+\dot{q}_tITC_t$$

where $\dot{q}_t$ is the time derivative of q_t. Solving for MPK_t gives:

$$MPK_t = ((R + \delta)q_t - \dot{q}_t)(1 - u_tZ_t)(1 - ITC_t)/((1 - u_t)p_t) \tag{2.2}$$

The right hand side of this equation is the real implicit rental rate of capital, which is the net-of-tax cost of using a unit of capital for one time period. At each point in time the desired capital stock is the level where the marginal product of capital equals the real implicit rental rate, hereafter denoted r_t.

In the series for the *nominal* rental rate in Canadian manufacturing given in Table 2.1, the investment goods price index q_t is set equal to 1 in 1971. The Economic Council of Canada provides data for the nominal rental rate on capital; this series will be deflated by the manufacturing sector output price index p_t, also listed in Table 2.1, to generate the real series r_t for use in the model of Chapter 3, and the series will be deflated by the manufacturing sector material input price index m_t when used in the model of Chapter 4. Since p_t is normalized to equal 1 in

1971, r_t is then the after-tax annual cost in 1971 manufacturing output dollars of using one 1971 dollar's worth of machinery and equipment.

Braithewaite (1983, pp 9-15) describes the data in great detail, so only a brief description is given here. We note at the outset that the rental rate data can only approximate the true rental rate, due to the many complexities of the tax system, and due to our ignoring some of the possible effects on firm behaviour of the corporate tax, while taking full account of others.

The interest rate R is a weighted average of the expected equity cost of capital and the after-tax bond rate, the weights being determined by the historical equity share of total capital.[1]

The corporate tax rate u is the "effective" tax rate, obtained by dividing total taxes paid by net taxable income. Ideally, in trying to evaluate the effects of the real rental rate on investment, we would want to use a measure of the *marginal* effective tax rate rather than the *average* rate. This data was unavailable to us; for some recent research on how effective marginal tax rates might be calculated see Boadway (1987) or Boadway, Bruce, and Mintz (1987).

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[1]There exists a body of research on the effects of changes in the corporate tax system on the financing decisions of firms, but it is beyond the scope of this thesis to attempt to incorporate any such effects in our data. See Auerbach (1983) for a survey of this research.

Equation (2.2) describing the rental rate of capital contains the assumption that there is full loss-offsetting. In fact in Canada loss-offsetting is imperfect. Mintz (1988) provides some estimates of how this imperfection might affect effective tax rates in Canada.

The investment tax credit (ITC) was introduced in the federal government budget of 1975. The terms of the ITC are described in detail by Timbrell (1975). Boadway and Kitchen (1984, p.146) describe the changes to the ITC up to 1984.

Briefly, the ITC is a tax credit on gross investment in structures and machinery and equipment (note that in this thesis capital will refer simply to machinery and equipment). The measure of the value of capital stock used for depreciation allowances is reduced by the amount of the investment tax credit. When introduced, the base rate of the ITC was five per cent. At the end of 1978 the base rate was increased to seven per cent, and it remained at this level through 1984.

In equation (2.1) the effective real rental rate on capital is reduced by exactly the ITC rate; if r_t is the real rental rate before the ITC is introduced, it is $r_t(1 - ITC)$ after the ITC is introduced. This is the result of assuming that the purchase price of capital goods, q, is determined in a competitive international market, of which the Canadian manufacturing sector is but a small part. If we imagine that capital is actually rented or leased, then we could describe the model as assuming that the lessee bears the burden of the corporate tax and receives the benefits of the investment tax credit. This has been the working assumption in other studies of taxation and investment; see, for example, the general equilibrium analysis in Hamilton and Whalley (1989, esp. pp.383-4). Note that we ignore the treatment by foreign governments of corporate income earned in Canada by multi-national firms. This simplifies the construction of the real rental rate series, but this simplification is perhaps justifiable; see Hartman (1985) for an explanation of why the foreign tax rates can be irrelevant to the investment decision of a multi-national firm.

The nominal wage data are also from the CANDIDE 2.0 Database, and are average hourly earnings in manufacturing. In Chapter 3 this series is deflated by the manufacturing output price index p_t, so that w_t is average hourly earnings in 1971 manufacturing output dollars.

The output price index p_t and the material input price index m_t are constructed from Statistics Canada's Input-Output data (annual catalogues 15-201E and 15-202E), prices being implied and revealed by dividing the current dollar statistics with the constant dollar statistics. Prices for each sector are weighted in the index by the sector's share of total manufacturing output in that year.

2.2 Capital and Labour Inputs

Capital and labour inputs for manufacturing are listed in Table 2.1. Capital, k_t, is machinery and equipment in Canadian manufacturing as given by the Economic Council of Canada, and is measured in terms of millions of 1971 dollars' worth. Braithewaite (1983, pp. 15-16) describes how it is calculated. Labour, l_t, is measured in terms of millions of manhours, and is also taken from the Economic Council of Canada's database. Note that the data are constructed such that $r_t k_t$, annual expenditures on capital services, and $w_t l_t$, annual expenditures on labour services, are both measured in terms of millions of 1971 dollars (note that the deflator is a manufacturing sector output price index).

2.3 Output

The real output of the manufacturing sector, y_t, is from Statistics Canada's **Input-Output Statistics of the Canadian Economy**, various issues from 1961 to 1984. Since k_t and l_t are capital and labour in the "manufacturing sector", output is taken from the total output of the manufacturing sector, and does not simply represent manufacturing goods. Inspection of the Input-Output tables reveals that the manufacturing sector produces some non-manufactured goods (e.g. some services) and that some other sectors produce manufactured goods (e.g. the agriculture and forestry sectors produce some manufactured goods). Here we make the definition of output consistent with the definition of inputs. Note also that the implicit output price index is derived from the output of the manufacturing sector, and not from the output of manufactured goods produced by the entire economy.

TABLE 2.1
Data

Year	p	y	w^N	l	r^N	k	m
1961	0.8315	29649.6	2.18	2789	.113	9049.3	0.7978
1962	0.8393	32324.9	2.24	2903	.121	9300.3	0.8130
1963	0.8486	34734.9	2.30	3010	.121	9531.0	0.8254
1964	0.8588	37951.9	2.36	3201	.114	10207.5	0.8400
1965	0.8714	41292.6	2.63	3184	.119	11055.8	0.8596
1966	0.8926	44149.2	2.78	3377	.122	12183.2	0.8866
1967	0.9104	45235.4	2.99	3367	.132	12954.7	0.9067
1968	0.9254	48126.1	3.23	3328	.138	13327.5	0.9240
1969	0.9524	51338.9	3.41	3445	.144	13907.3	0.9574
1970	0.9792	50617.6	3.65	3370	.149	14742.6	0.9834
1971	1.0000	53479.1	3.91	3346	.143	15336.4	1.0000
1972	1.0417	57571.9	4.18	3466	.158	15811.3	1.0478
1973	1.1449	62669.9	4.57	3630	.135	16696.2	1.1694
1974	1.3677	65105.2	5.27	3702	.148	17813.4	1.3894
1975	1.5331	61241.8	6.13	3513	.163	18715.3	1.5391
1976	1.6209	65056.3	6.92	3599	.164	19404.4	1.6136
1977	1.7471	66613.1	7.62	3564	.190	20004.6	1.7349
1978	1.9204	70069.5	8.10	3702	.211	20376.6	1.9069
1979	2.1896	73236.8	8.81	3877	.236	21013.3	2.1880
1980	2.4646	71942.4	9.55	3920	.274	21982.7	2.4243
1981	2.7481	72967.5	11.01	3911	.344	23286.0	2.6130
1982	2.9396	65851.7	12.31	3517	.356	23805.9	2.7331
1983	3.0411	69292.3	13.21	3484	.362	23698.8	2.7886
1984	3.1385	76700.9	13.49	3621	.424	23603.4	2.9028

p is a price index of goods and services produced by the Canadian manufacturing sector, y is the quantity of such goods and services measured in millions of 1971 dollars, w^N is average hourly earnings in manufacturing measured in current dollars, l is manhours of labour in manufacturing measured in millions, r^N is the implicit rental rate of machinery and equipment in manufacturing measured in the current dollar cost of renting one 1971 dollar's worth of capital, k is the stock of machinery and equipment in manufacturing measured in millions of 1971 dollar's worth, and m is a price index of material inputs used by the Canadian manufacturing sector.

The source for p, m, and y is Statistics Canada, **Input-Output Statistics of the Canadian Economy** (various issues), and the source for all other data is the CANDIDE 2.0 databank of the Economic Council of Canada.

CHAPTER 3

The Linear Quadratic Model

In this chapter a model of input demand is specified and is estimated with the data from the Canadian manufacturing sector described in Chapter 2. Before the model is described, it is necessary to examine some of the time series properties of the data, since the model requires that the data satisfy certain conditions.

3.1 A Time Series Analysis of the Data

We now examine some of the time series properties of the wage and rental rate data. We ask in turn (i) whether the two time series are stationary, and (ii) whether the wage and rental rate are exogenous with respect to the levels of input demands. The motivation for this examination is that when the model of input demand is later estimated we will need some way to represent how firms might have formed expectations of future input prices. One possible method is to assume that a linear time series model of input prices can represent the way firms made expectations.

We deflate the nominal rental rate and nominal wage by the manufacturing output price index given in Table 2.1.

Assume for now (the assumption is justified below) that the rental rate and wage each follow a first-order autoregressive process, which we write as

$$r_t = v_1 + \theta_{11} r_{t-1} + \varepsilon_{1t} \tag{3.1}$$

$$w_t = v_2 + \theta_{22} w_{t-1} + \varepsilon_{2t} \tag{3.2}$$

where ε_{it}, i = 1, 2 are random errors.

Hansen and Sargent (1981, p.136) establish the fact that a necessary condition for convergence of the firm's input decision rule in the rational expectations linear quadratic model to be estimated in this chapter is $|\theta_{11}| < (1+R)^{.5}$ and $|\theta_{22}| < (1+R)^{.5}$ where R is the discount rate. In this section more stringent restrictions are tested: $|\theta_{11}| < 1$ and $|\theta_{22}| < 1$. In other words, it will be determined whether r_t and w_t can be considered stationary processes.

From OLS estimates of (3.1) and (3.2) we obtain $\theta_{11} = .8271$ (.1222) and $\theta_{22} = .9211$ (.0473) where standard errors are in parentheses. Fuller (1976) demonstrates that the estimates of standard errors of θ_{11} and θ_{22} do not have standard distributions under the null hypotheses $\theta_{11} = 1$ and $\theta_{22} = 1$, so the estimates above cannot be used to establish whether θ_{11} and θ_{22} are "significantly different" from 1.

An appropriate test for $\theta_{11} < 1$ and $\theta_{22} < 1$ is described by Dickey and Fuller (1979). Nelson and Plosser (1982) use this test to examine whether U.S. aggregate output is stationary around a trend. The following equations are estimated by ordinary least squares:

$$(r_t - r_{t-1}) = \alpha_{10} + \alpha_{11} r_{t-1} + \alpha_{12}(r_{t-1} - r_{t-2}) + u_{1t} \tag{3.3}$$

$$(w_t - w_{t-1}) = \alpha_{20} + \alpha_{21} w_{t-1} + \alpha_{22}(w_{t-1} - w_{t-2}) + u_{2t} \tag{3.4}$$

Enough lagged values of the dependent variables of (3.3) and (3.4) are included on the right hand sides of these equations until the error terms are white-noise; one lag turned out to be sufficient for these series. OLS estimates of (3.3) and (3.4) are:

$$r_t - r_{t-1} = .024(.017) - .191(.131)r_{t-1} + .044(.238)(r_{t-1} - r_{t-2})$$

$$w_t - w_{t-1} = .376(.183) - .090(.049)w_{t-1} + .381(.197)(w_{t-1} - w_{t-2})$$

where standard errors are in parentheses.

The test of stationarity asks whether estimates of α_{11} and α_{21} are negative and significantly different from zero; if so, the null hypothesis of non-stationarity is rejected. The test statistic is $(n - p)\alpha_{i1}(1 - \alpha_{i2})^{-1}$ where i = 1,2 for r and w respectively, n is the number of observations (22) and p is the number of right hand side variables in the regression (3). So for r this test statistic equals -3.788 and for w it equals -2.751. Fuller (1976, p. 371) provides the distribution of this statistic (in his notation the test statistic is distributed as $n(\rho_\mu - 1)$). From his Table 8.5.1 we find that the null hypothesis of non-stationarity is not rejected: the 0.1 significance level is approximately -10.2.[1]

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[1] West (1988) in his discussion of "near random-walk behavior" points out that with only a small number of observations, as we have here, it will be unlikely that non-stationarity can be rejected as a possibility even if the true value of the lagged

The only study of which I am aware that examines the stationarity of real wages in Canada is the preliminary results given by Sigurdson and Stewart (1990); they suggest that real wages in Canada follow a random walk. Altonji and Ashenfelter (1980) are unable to reject the hypothesis that the real average hourly wage for the entire U.S. economy follows a random walk. Sargent (1978), Meese (1980), and Epstein and Yatchew (1985) are all empirical models of dynamic factor demand estimated with U.S. manufacturing sector data; none of these studies contains an explicit test of stationarity for real input prices, although stationarity is imposed by detrending the data.

We now consider the properties of the rental rate series in more detail.

The result of OLS estimation is

$$r_t = \underset{(.0159)}{.0222} + \underset{(.1222)}{.8271}r_{t-1}$$

with standard errors in parentheses. The Durbin h statistic is .1609, suggesting no serial correlation of the error terms. Applying Gujarati's (1978, p. 246) "runs test", we find 12 positive and 11 negative residuals (for the annual sample 1962 to 1984) and 11 runs. The 5% critical values for positive and negative serial correlation are 7 runs and 18 runs, respectively, so this test provides some further evidence for no serial

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term parameter is as low as 0.8.

correlation.

The software SHAZAM (White 1978) provides the researcher with 7 different heteroskedasticity test statistics, all involving an examination of the relationship between estimated residuals (or some transformation of them) and the independent variables, or the predicted values of the regression (or some transformation of them). All 7 test statistics are χ^2 with 1 degree of freedom. The maximum statistic of the set of seven for the rental rate equation is 1.740, so one is led to assume homoskedasticity.

We now consider tests of structural breaks in the series. Harvey (1981, pp. 151-4) discusses how one might analyse the cumulative sum and cumulative sum of squares of recursive residuals. Harvey's t-test on recursive residuals (see his equation (2.10), p. 156) helps identify whether the recursive residuals tend to be the same sign. The t-statistic for our forward recursive estimation is -0.6084 and for our backward recursive estimation is +0.4285, each with 20 degrees of freedom. The plots of the cumulative sum of squares also present no evidence of misspecification.

Finally we consider the sets of sequential Chow tests and Goldfeld and Quandt tests for structural break. With 23 observations the Chow tests have 2 degrees of freedom in the numerator and 19 in the denominator. The 10% critical value for the F-statistic is 2.61. This is exceeded at one point in the sample, specifically between 1972 and 1973.[2] Examining the plot of

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[2] Since we are testing for structural break without asking *a priori*

the residuals of the AR(1) regression we find the largest residual in terms of absolute value occurs in 1973, the first full year accelerated depreciation allowances were in effect (see Boadway and Kitchen (1984 pp. 128-9) for details). The Goldfeld and Quandt (1973) test for structural break, which is based on the ratio of the sum of squared residuals from regressions using the sample before a break and after, does not yield a test statistic (like the Chow test, an F-statistic) at any possible break point that exceeds the 10% critical value.

Of the Chow tests, the Goldfeld and Quandt tests, and various tests on the pattern of recursive residuals, only the Chow test gives any evidence of structural break. Casual observation of the data and of the residuals of the AR(1) estimation does not find any obvious break in the time series.

There are two implications of this. First, we will be able to use our entire 1961 to 1984 data set when estimating the linear-quadratic rational expectations model, which requires that the input prices follow stationary processes (we examine the wage later). We need not estimate for different "regimes".

Second, it suggests that perhaps one should be wary of interpreting any change in corporate tax rules - say the introduction of accelerated depreciation allowances, or a change in the rate of the investment tax credit, or a change in the profits tax rate - as a change in the "policy regime". Sims (1982,

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where it might occur, it is not surprising that at least one Chow test is significant at the 10% level.

p.108) writes:

> ...it is a mistake to think that decisions about policy can only be described, or even often be described, as choice among permanent rules of behavior for the policy authorities. A policy action is better portrayed as implementation of a fixed or slowly changing rule.

This is a possible way to think about corporate tax policy in Canada. Suppose that the rule the government is following is to stabilize to some degree the real rental rate of capital. If, because of the design of the corporate income tax, high inflation causes the real rental rate to rise beyond levels which the government thinks appropriate, special tax credits and allowances may be introduced to offset the harmful effects of inflation. Seen from this angle, the introduction of the ITC might not represent a "regime change" at all, but rather is simply a manifestation of a rule that was already in place.

This generates problems for those who wish to examine the effects of one aspect of tax policy, for example the investment tax credit, in particular. If one is using a model where firms' expectations are presumed rational, how can one specify expectations for the counter-factual policy of no investment tax credit? Indeed, how does one specify the counter-factual policy in the model? Should one assume that the other parts of the tax system remain unchanged, then one is, as a counter-factual, considering what would have been a change in regime.

This problem is examined further below, where estimated models are used in simulations for some counter-factual time series of the rental rate.

We now examine the stationarity properties of the real wage rate.

The result of OLS estimation is

$$w_t = .3613 + .9211w_{t-1}$$
$$(.1750) \quad (.0473)$$

with standard errors in parentheses. The Durbin h statistic is 1.8012, suggesting there might be serial correlation in the residuals. There are 13 positive, and 10 negative, residuals, with 9 runs. The 5% critical values for serial correlation are 7 and 18 (see Gujarati (1978, pp. 440-1)), so there is some evidence against serial correlation as well. Casual observation of a plot of the residuals yields no clear evidence for or against serial correlation.

The values of the 7 χ^2 statistics for heteroskedasticity given by SHAZAM range from 0.280 to 2.437. The 10% critical value with one degree of freedom is 2.706, so there is no strong evidence of heteroskedasticity.

Regarding the recursive residuals, the cumulative sum of squares yields no casual evidence of misspecification. Harvey's t-test of the cumulative residuals (1981, p. 156) yields a statistic of -0.953 for the forward recursive residuals, which does not lead one to suspect misspecification, but a statistic of -2.967 for the backward recursive residuals. This does suggest some sort of misspecification, but the plot of residuals gives no clear indication where any structural change in the series might have taken place.

The Chow test statistic has an F distribution with (2, 19) degrees of freedom. The highest statistic is obtained when the sample is divided between 1964 and 1965, where it is 2.554, but this is less than the 5% significance level of 3.52. Goldfeld and Quandt tests similarly give no evidence of structural break.

A casual look at the data in Table 2.1 reveals that the real wage increased over the first part of the sample but seemed to level off thereafter. The OLS estimates given above suggest that *if* the series is stationary it has an estimated mean of 4.58. Given a 1961 value of 2.62, if one used the estimates we obtained of the parameters of the AR(1) regression, one would predict an increase in the wage over some time followed by a levelling off, which perhaps provides the intuition behind why there seems to be no discernable pattern in the residuals. A time trend added to equation (3.2) proved to be insignificant, with a t-statistic of only 0.548.

Note that in the models we estimate in Chapters 3 and 4, we will assume firms form expectations using these simple autoregressive processes,[3] so the residuals of these regressions translate in the models into forecast errors by firms. If expectations are to be described as rational in the model, there should be no information embodied in the residuals, and it is for this reason we have examined the properties of the residuals of the rental rate and wage equations in such depth.

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[3]This is standard practice in empirical applications of these models; see all the papers referred to in Chapter 1.4.

Modelling input demands in the manufacturing sector is simplified if it can be assumed in the model that real wages are not caused (in the Granger (1969) - Sims (1972) sense) by real rental rates on capital, the demand for capital, or the demand for labour, and that real rental rates are not caused by real wages, the demand for capital, or the demand for labour. This assumption greatly simplifies the specification of the firms' input decision rules, because it means that there is no feedback from the firms' input decisions to input prices. The assumption is used by Sargent(1978), Meese (1980), and Epstein and Yatchew (1985) among others to obtain a tractable solution. The technique used here (and in the above mentioned papers by Meese and Epstein and Yatchew) for testing the exogeneity of input prices is from Geweke (1978).

We begin by estimating the vector autoregression

$$S_t = a + A_1 S_{t-1} + A_2 S_{t-2} + bt + \varepsilon_t \tag{3.5}$$

where $S_t = (k_t, l_t, r_t, w_t)'$, k is capital, l is labour, r is the real rental rate of capital, w is the wage, a and b are 4×1 vectors of parameters, A_1 and A_2 are 4×4 matrices of parameters, t is time, and ε_t is a vector of errors, serially uncorrelated but perhaps correlated across equations.

We consider two alternatives to unrestricted estimation of (3.5):(i) that the lagged k and l terms have zero coefficients in the r and w equations, and (ii) that the lagged r and w terms have zero coefficients in the k and l equations.

The evidence is that there is stronger causality from lagged prices to current input levels than there is from lagged input levels to current prices. The likelihood ratio test statistic for null hypothesis (i) is 18.225 and the Wald test statistic is 23.329 (each of which is distributed χ^2 with 8 degrees of freedom). The likelihood ratio statistic for null hypothesis (ii) is 21.630 and the Wald test statistic is 27.224.

With the small sample we are using we cannot say with any confidence whether the causality is statistically significant. Using the likelihood ratio and Wald test statistics as given above leads one to reject the exogeneity of any of the variables in question. But Epstein and Yatchew suggest modifying the likelihood ratio statistics in a way suggested by Nelson and Schwert (1982) by multiplying the statistics by (T - K)/T where T is the sample size and K is the number of parameters in the unrestricted model (2.7). This is meant to correct for the problem of using large sample theory to examine a small-sample model. Since T = 22 and K = 40, the amended statistic is meaningless for our purposes, or perhaps warns us that with this small a sample we simply cannot say anything, if the Nelson and Schwert correction is appropriate.

3.2 The Model

The model is called linear quadratic because the quadratic specification of both the adjustment costs and the output function leads to input demands which are linear in real input prices. An assumption of the model is that firms form expectations of future input prices rationally. Rational expectations are defined as expectations formed as a result of using available information efficiently. More specifically it means individuals and firms in the model make use of (a) past observations of variables and (b) knowledge of the structure of the economic model in forming expectations. In this model of input demand rational expectations are incorporated by assuming that firms know past values of real input prices, and that an VAR(1) specification can be used to represent the "model" firms use to make forecasts. See Chapter 1 for a survey of models of this type that have been analysed and estimated.

In this chapter the estimation technique is taken from Epstein and Yatchew (1985). The assumptions and functional form used in their paper are not different from other linear quadratic rational expectations models of input demand, but the parameterization of the estimating equations is different. Its advantage is that the cross-equation restrictions that are implied in rational expectations models of input demand are more simply specified than in other parameterizations using essentially identical models (e.g. Hansen and Sargent (1980, 1981) on one hand, or Chow (1980b, 1981, 1983) on the other). For ease of reference Epstein and Yatchew's notation is used.

A firm produces output y with inputs capital k and labour l. Define the vector $x_t = (k_t \; l_t)'$, and the production function is

$$y_t = a'x_t + x_t'Ax_t/2 + (x_t - x_{t-1})'B(x_t - x_{t-1})/2 + S^1(t), \tag{3.6}$$

where a is a 2×1 vector of parameters, A and B are each 2×2 matrices of parameters, and $S^1(t)$ is a scalar time trend meant to capture changes in technology. Matrix A is symmetric and negative definite and B is diagonal and negative definite. Hansen and

Sargent (1981) and Chow (1980b, 1981) allow non-zero off-diagonal terms in the adjustment cost matrix B, but this carries the cost of complicating the solution of the model considerably.

Costs of adjusting input levels are captured by the term in (3.6) involving B. They are called "internal adjustment costs" because the costs are expressed in terms of lost output (see Treadway (1969, p.229)). Note that adjustment costs depend on net changes in input levels. In Gould (1968) adjustment costs are based on gross investment, while in Lucas (1967a) they are based on the percentage change in input levels. One could imagine that in the model (3.1) adjustment costs are capturing the disruption involved in changing the level of any input, so that no adjustment costs arise from purely replacement investment.

The real rental rate of capital is r_t and the real wage is w_t, and the vector of input prices is written $z_t = (r_t \; w_t)'$. At time $t = 0$ a firm chooses a rule for setting x_t to solve the problem

$$\max E_0 \sum_{t=0}^{\infty} \rho^t \, [a'x_t + x_t'Ax_t/2 + (x_t - x_{t-1})'B(x_t - x_{t-1})/2 + S^1(t) - z_t'x_t] \tag{3.7}$$

subject to x_{-1} given, where $\rho = (1 + R)^{-1}$ and R is a constant rate of discount. Each time period the firm recalculates the solution to the problem, making use of any new information.

Input prices follow the process

$$z_t = v + \theta z_{t-1} + \varepsilon_t, \tag{3.8}$$

where the 2x1 vector v and the 2x2 matrix θ are parameters and ε_t is a random error term.

An advantage of the linear quadratic specification is that the problem (3.7) can be solved under the assumption that the firm has perfect foresight; i.e. certainty equivalence applies. The solution has the form

$$x_t^* - x_{t-1} = M(x_{t-1} - \bar{x}_t), \tag{3.9}$$

where x_t^* is the optimal decision at time t, $\bar{x}_t$ is the "target level" of x at time t, and M is the "adjustment matrix" (Epstein and Yatchew (1985, pp. 239-40)). The matrix M solves the equation

$$M^2 - (1 + R)B^{-1}AM - RM - B^{-1}A(1 + R) = 0, \tag{3.10}$$

and $\bar{x}_t$ is given by

$$\bar{x}_t = A^{-1}(J_t - a), \tag{3.11}$$

where

$$J_t = D \sum_{s=t}^{\infty} (I + D)^{-(s - t + 1)} E_t z_s, \tag{3.12}$$

$$D = AB^{-1}(1 + R) + \hat{R} - M', \tag{3.13}$$

where $\hat{R}$ is a 2x2 diagonal matrix with every diagonal entry equal to R, and I is a 2x2 identity matrix.

The vector J_t is a weighted average of current and expected future input prices. If expectations are static, say that $E_t z_s = \bar{z}$ for all $s = t,\ldots,\infty$, then $J_t = \bar{z}$. With rational expectations, however, J_t is clearly going to depend somehow on current prices and on the parameters of the model used to forecast future prices, namely v and θ. The solution for J_t given by Epstein and Yatchew (p. 241) is as follows:

$$J_t = \alpha + \beta z_t, \tag{3.14}$$

where α and β are defined by

$$v = \beta^{-1} D\alpha \tag{3.15}$$

and

$$\theta = \beta^{-1}((I + D)\beta - D) \tag{3.16}$$

and D is as defined in (3.13).[4]

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[4]In the following chapter the input price autoregressions are assumed to be independent from one another; i.e. θ is assumed to be a diagonal matrix. While one might think this could be a useful simplifying assumption in this model, in fact it would greatly complicate matters, adding a number of restrictions to the estimation. We do not know how much difference in the results of

As a final step in deriving the estimating equations, define $P = BM$. The simplified parameterization of Epstein and Yatchew refered to earlier will define the input demand equations in terms of a, B, P, and J_t. Equation (3.10) can be solved for A as

$$A = P/(1 + R) - B + B(I + M)^{-1}. \tag{3.17}$$

This represents parameter restrictions on the solution to (3.7). Other restrictions derived in Lucas (1967b) are

$$M \text{ has 2 real eigenvalues between } -1 \text{ and } 0 \tag{3.18}$$

and

$$P \text{ is symmetric and positive definite.} \tag{3.19}$$

Writing the solution of the model in the form in which it is to be estimated we have

$$x_t = (I + B^{-1}P)x_{t-1} - B^{-1}PA^{-1}(J_t - a) + u_t \tag{3.20}$$

and

$$z_t = v + \theta z_{t-1} + \varepsilon_t, \tag{3.21}$$

---

the models of this and the following chapter are due to this different treatment of the evolution of input prices.

where A is as defined in (3.17). The parameters to be estimated are B, P, a, α, and β. Technology is completely described by B, P, and a, while α and β are the parameters relating v and θ to input demands. In the estimation v and θ are expressed in terms of α and β. Restriction (3.17) is imposed writing (3.20) using B, P, and a as the only technology parameters. Restriction (3.19) is half-imposed, as P is confined to be symmetric but is not confined to be positive definite, and restriction (3.18) is not imposed (but is satisfied by the data in any case, as we see below). Error terms u_t are meant to reflect "random errors of optimization and errors in the data" (Epstein and Yatchew p. 243). In principle they should be independent of the residuals ε_t from (3.21), but this restriction is not imposed.

Before the estimates are presented, it is interesting to see how Lucas' (1976) critique of econometric policy evaluation applies to the problem of input demands. A change in the policy governing rental rates on capital (e.g. a change in corporate tax policy) would change the parameters of v and θ. This in turn, through (3.16) and (3.14), changes the parameters α and β, which changes the relation between J and current input prices, which changes the parameters relating input prices to input demands. In sum, the reduced form parameters of the input demand equations change when v and θ change. Lucas warns policy analysts to realize that this change occurs.

3.3 Estimates

Epstein and Yatchew assume that firms can observe this period's input prices before having to decide this period's input levels. Chow (1980b, 1981), in his formulation of an otherwise identical model, assumes firms must choose period t input levels based only on observations of period t-1 (and earlier) prices. Since here we are not certain about the validity of the model, both specifications will be estimated.

Tables 3.1 and 3.2 show the unrestricted estimates of the reduced form of the 4 equation model (3.20) and (3.21), under the assumption that firms choose period t input levels after period t prices become known and under the assumption that the period t input levels must be chosen before period t prices become known, respectively. The unrestricted models are estimated with SHAZAM's (White (1978)) three-stage least squares.

No parameter signs change across the two sets of estimates, although magnitudes change slightly. Surprisingly, in both tables the coefficient on wages in the labour equation is positive, although not significantly so.

Restricted estimates of the models are given in Tables 3.3 and 3.4. In the tables the following notation is used:

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \quad B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \quad a = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix}$$

The models are estimated with the non-linear maximum-likelihood option of SHAZAM. The data is annual Canadian manufacturing from 1962 to 1984. Starting values for the maximization process were chosen by first estimating a static expectations version of the model. A selection of different starting values in the neighbourhood of the final solution all converged on the estimates shown.

Table 3.3 gives estimates under the assumption that firms are able to observe current input prices before setting input levels. The implied reduced-form parameters are very different from the unrestricted estimates of Table 3.1. In particular, the demands for capital and labour are each completely inelastic with respect to both input prices; the input demand equations reduce to a simple bi-variate autoregression with constant terms. This specification then leaves no possibility for any simulations of interest regarding changes to the rental rate.

Table 3.4 gives estimates under the assumption that firms must choose any year's input levels before prices are observed. Comparing these results to Table 3.3, we find in Table 3.4 that the demand for capital does respond to rental rates, although the impact elasticity is only approximately -.004, and it does not in Table 3.3, but otherwise the reduced forms are the same between the two cases.

The estimates of structural parameters are very different in magnitude across the two specifications, although all are of the same sign across the two tables. But since M, the adjustment matrix, equals $B^{-1}P$, the absolute values of B and P are not going to be well identified.

Since the model of Table 3.4 is the only one which could conceivably be of interest to one examining the effects of rental rates on investment, the rest of this chapter will focus on this model.

The restriction (3.18), that M have 2 real eigenvalues between 0 and -1, is satisfied, although it was not imposed in the estimation. Since M is given by $B^{-1}P$, the estimated value of M is

$$\hat{M} = \begin{bmatrix} -.1144 & 1.9576 \\ -.0003 & -.1883 \end{bmatrix}$$

and the eigenvalues of $\hat{M}$ are -.1237 and -.1790.

Restrictions which are not satisfied are that P be positive definite and that B be negative definite. In particular B_2 is positive, which is the wrong sign; it suggests negative costs to adjusting the labour input. Also, consider the matrix A, from the production function, which should be negative definite to ensure constant or decreasing returns to scale. Its implied value has been given above; the positive element in column 2 row 2 implies an increasing marginal product of labour.

Virtually all aspects of the "labour side" of this model fail. Adjustment costs are the wrong sign, and the marginal product of labour at various data points indicate a negative marginal product that is increasing. Yet on the "capital side", adjustment costs are positive, and the marginal product of capital is found to be positive and decreasing.

In the reduced form the result is a labour demand that is completely inelastic with respect to both input prices and to the capital stock, and is actually just an AR(1) process with a constant term, where the mean of the process is estimated at 3654.7. So the significant effect of labour input on capital demand in the capital equation is simply a term capturing this AR process.

It is interesting to note that Epstein and Yatchew's results of this same 4 equation model with rational expectations, using U.S. annual manufacturing data from 1948 to 1977, are very similar to the results obtained here (see their Table 5, p. 249). In both cases estimated signs are $P_1 > 0$, P_2 and $P_3 < 0$ (so neither case satisfies the restriction P positive definite), and $B_1 < 0$ and $B_2 > 0$ (so both cases have B_2 being the wrong sign).

The marginal significance of the test of the cross equation restrictions is found by taking the difference in the log of the determinant of the sigma matrices, multiplying this by the number of observations (23), and comparing this test statistic with the χ^2 distribution with 3 degrees of freedom. The marginal significance of the restrictions is only 2.53%, which would suggest rejection of the model by the data.

The impact elasticities of input demands, as of 1975, are:

	short run elasticity with respect to:	
	r	w
capital	-.004	+.004
labour	+.000	+.000

Long run elasticities are found by applying equation (3.11), which describes how steady state demands change with respect to a change in J_t. A permanent change in input prices would be represented by a change in J_t, which is a weighted index of current and expected future input prices. The change in the target values of inputs at time t, given by $\bar{x}_t$, with respect to a change in J_t is given by our estimate of A^{-1}. From the estimates given in Table 3.4 the implied estimate of A is

$$\hat{A} = \begin{bmatrix} -.00005997 & .002329 \\ .002329 & .9458 \end{bmatrix}.$$

At 1975 capital and labour input levels are 18715.3 and 3513.0, respectively, and the rental rate and the wage rate are .106 and 4.00. In that same year the elements of J_t corresponding the rental rate and the wage are .113 and 4.134 (from equation (3.14) and the estimates of α and β given in Table 3.4). Applying equation (3.11) we obtain the target levels of capital and labour in 1975 of 24925.5 and 3613.5. So at 1975 the long run elasticities of input demand, where this means the relative change in target input levels per relative change in J_t, are:

	long run elasticity with respect to:	
	r	w
capital	-.069	+.006
labour	+.001	+.001

As long as M is a stable matrix, i.e. has 2 real eigenvalues between -1 and 0, and our estimate *is* a stable matrix, then the method of calculating long-run elasticities by examining the relative change in target values is the same as we would find if we looked at the relative change in long run *actual* values of inputs. Consider the following. If

$$x_t = (I + M)x_{t-1} + MA^{-1}a - MA^{-1}J_t, \tag{3.22}$$

then

$$\begin{aligned} x_{t+s} = {} & (I + M)^{s+1}x_{t-1} + (I + (I + M) + (I + M)^2 + \dots + \\ & (I + M)^s)MA^{-1}a - MA^{-1}J_{t+s} - (I + M)MA^{-1}J_{t+s-1} - \\ & \dots - (I + M)^s MA^{-1}J_t. \end{aligned} \tag{3.23}$$

If we change all J_τ, $\tau = t, t+1, \dots, \infty$, by ΔJ, then the change in x_{t+s} is

$$\Delta x_{t+s} = -(I + (I + M) + (I + M)^2 + \dots + (I + M)^s)MA^{-1}\Delta J, \tag{3.24}$$

and as $s \to \infty$, if M is stable, this converges to

$$\Delta x_{t+s} = -(I - (I + M))^{-1}MA^{-1}\Delta J = A^{-1}\Delta J. \tag{3.25}$$

So A^{-1} gives us the change in the target input levels *and* the change in the actual levels if M is a stable matrix.

It is interesting to note how different are the estimates of long run elasticities if we ignore our specification of expectations and simply take the reduced form parameters of input demand as given for any expectations. Write the reduced form as

$$x_t = (I + M)x_{t-1} + \Gamma z_{t-1} + c. \tag{3.26}$$

Ignoring expectations, a change in current and future input prices by Δz would lead to a change in x_{t+s} of

$$\Delta x_{t+s} = (I + (I + M) + (I + M)^2 + \ldots + (I + M)^s)\Gamma\Delta z, \tag{3.27}$$

and as $s \to \infty$, if M is stable this converges to

$$\Delta x_{t+s} = -M^{-1}\Gamma\Delta z. \tag{3.28}$$

Taking the reduced form estimates of M and Γ from Table 3.4 the estimated long run elasticities are

	long run elasticity with respect to:	
	r	w
capital	-.035	+.039
labour	+.001	+.001

The estimate of the long run elasticity of capital stock with respect to the rental rate, when estimated considering only the reduced form of the model, is only about one half the size of the estimate when we account for expectations as specified in equation (3.21). We cannot measure whether this difference in the estimates of long run elasticities is significant in an econometric sense, since we have no measure of standard errors. The long run elasticity of capital with respect to the rental rate that we double when treating expectations as rational is a very small number (in elasticity terms); whether we should regard this difference in estimates as quantitatively important is perhaps answered in the following section of this chapter. There simulations of the model are run under both methods; where reduced forms are changed to account for a new regime, and where they are not.

Because both capital and labour are slow to adjust to their target levels, it is useful to know what the medium run elasticities are. The matrix $(I + (I + M) + (I + M)^2 + \ldots)$ is slow to converge to $-M^{-1}$. From our estimate of M of

$$\hat{M} = \begin{bmatrix} -.1144 & 1.9576 \\ -.0003 & -.1883 \end{bmatrix}$$

we find that $(I + (I + M) + (I + M)^2 + \ldots + (I + M)^s)$ equals

$$\begin{bmatrix} 4.5158 & 19.3905 \\ -.0030 & 3.7838 \end{bmatrix} \quad \text{when } s = 5,$$

$$\begin{bmatrix} 6.4027 & 44.0446 \\ & \\ -.0068 & 4.7400 \end{bmatrix} \text{ when } s = 10,$$

and that

$$-\hat{M}^{-1} = \begin{bmatrix} 8.5093 & 88.4639 \\ & \\ -.0136 & 5.1697 \end{bmatrix},$$

which indicates that convergence to long run levels is very slow. Based on these calculations, the five- and ten-year elasticities, expressed in terms of target levels so that they are more easily compared to the long run elasticities given above, are

	five-year elasticity		ten-year elasticity	
	r	w	r	w
capital	-.037	+.002	-.052	+.004
labour	+.000	+.001	+.001	+.001

We also estimated the model with static expectations. This is achieved by setting J_t, which is like an index of current and expected future input prices, equal to z_{t-1} (or, equivalently, by setting $\alpha = 0$ and β equal to the identity matrix). The two equations of (3.20) are then estimated. Estimates of structural parameters under static expectations are similar to those for dynamic expectations listed in Table 3.4; no signs change, and relative magnitudes are roughly the same. Static expectations estimates are:

P_1	$.14155\times10^{-3}$ (.00010)	P_2	$-.22744\times10^{-2}$ (.00173)
P_3	-2.6673 (.92691)	B_1	$-.11599\times10^{-2}$ (.00086)
B_2	12.536 (9.1424)	a_1	-3.5094 (2.9434)
a_2	-3471.0 (1.4765)		

This implies a reduced form for input demands of:

$$k_t = -3664.545 + .878k_{t-1} + 1.961l_{t-1} - 3725.996r_{t-1} + 2.711w_{t-1}$$

$$l_t = 770.645 - 0k_{t-1} + .787l_{t-1} + 2.711r_{t-1} + .219w_{t-1}$$

which is similar to those obtained with dynamic expectations (compare with Table 3.4), although the estimate of the elasticity of capital demand with respect to the rental rate is much greater.

The likelihood ratio statistic for the 2 equation static expectations model against the unrestricted 2 equation model is 8.670, which is distributed χ^2 with 3 degrees of freedom. The marginal significance of the restrictions is then 3.36%.

Finally, we turn to estimates of the production function, given in equation (3.1). Unrestricted estimation yields

$$y_t = \underset{(126090)}{29417} - \underset{(4.5320)}{2.0855}k_t + \underset{(91.362)}{2.9297}l_t - \underset{(.00004)}{.0001}k_t^2 - \underset{(.0166)}{.0020}l_t^2 + \underset{(.0016)}{.1322}k_t l_t$$

$$+ \underset{(.0012)}{.0001}(k_t - k_{t-1})^2 - \underset{(.0118)}{.0026}(l_t - l_{t-1})^2 + \underset{(815.16)}{3238.8}t.$$

The Wald χ^2 statistic for the joint test of both adjustment terms being zero is .0747 with 2 degrees of freedom, suggesting insignificance. The signs on the coefficients lend further doubt on the usefulness of this model. Restricted estimation of the output equation together with the four equation model of input demands and input prices yielded results with such a poor fit of the data that they are not worth reporting.

3.4 Simulations

Table 3.5 gives the results of a number of simulations made using the parameter estimates of the 4 equation restricted linear-quadratic model presented in Table 3.4. Although the merits of this model as an explanation of the data have been found to be dubious, the simulations at least illustrate the principles behind doing policy analysis with a rational expectations model, and illustrate the empirical significance of how expectations are specified.

For all the simulations in Table 3.5, we imagine someone in 1975 making long-range forecasts of the capital stock (since labour in these estimates seems to simply follow a predetermined path, the forecasts of labour are not recorded in the table; all simulations lead to a forecast value of labour in 1984 of 3603, while its actual value turned out to be 3621).

Column A lists the actual data. Column B lists a forecast of capital made in 1975, using the model of Table 3.4, and gives the standard errors of the forecasts. The standard errors were found using the method given by Judge et. al. (1988, pp. 764-67) for calculating the variance of forecasts with VAR(1) systems.

The simulation in Column C is the result of a negative shock to the rental rate on capital in 1975 by a factor of 10%. This lowers r in 1975 from .1060 to .0954. The underlying time-series parameters of r_t are left unchanged. This shock has two effects on the path of capital. First, the future rental rate depends on its past values, so even though the time-series parameters are unchanged, there will be some persistent effects on rental rates from this one-time shock. Second, since capital demand responds to the shock to the rental rate in 1976, and capital demand depends on its own past values, there will be further persistent effects. But given the stationarity of rental rates, and the fact that the adjustment matrix M is "stable" (two real eigenvalues between 0 and -1), the effect of this shock in the very long-term tends asymptotically to zero.

Comparing Columns B and C we find the impact effect, in 1976, is to increase the capital stock by .04% over what it otherwise would have been. By 1984, the effect of the shock is a capital stock only .02% greater than what it otherwise would have been. Eventually the effects of the shock die out completely.

The simulation in Column D is somewhat unusual. Here in 1975 there is a permanent decrease in rental rates of 10%, but the path of wages is left unchanged. This is achieved by lowering the 1975 value of the rental rate by 10% directly, as we did in simulation C, and in changing the bivariate autoregressive process of the rental rate and wages from

$$z_t = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} z_{t-1}$$

to

$$z_t = \begin{bmatrix} .9v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \theta_{11} & .9\theta_{12} \\ 1.11\theta_{21} & \theta_{22} \end{bmatrix} z_{t-1}.$$

What is unusual about the simulation is not that there is a permanent shift in the path of rental rates, but rather that we assume firms are unaware that the shift in the path has taken place. They observe rental rates correctly, but they do not realize the change in regime; in their minds each year brings a surprisingly low rental rate.

This simulation is presented as a contrast to the one in Column E, which has the same permanent lowering of rental rates as simulation D, but which presumes that firms do realize (immediately) that there has been a change in regime, although they had not anticipated this change at all, and reset their input demand rules accordingly.

Estimating the revised input demand equations proceeds as follows. First, the change in the path of rental rates has involved a change in the values of the parameters v and θ (see equation (3.21)). According to (3.20), input demands depend on input prices through the structural parameters B, P, and A, which do not change with the changes in v and θ, and through the parameters in J_t, which will change with the changes in v and θ. The relationship between J_t and v and θ is given in equations (3.14), (3.15), and (3.16). With the new values of v and θ, new values of α and β are implied. The new values of α and β are

$$\alpha = \begin{bmatrix} .0995 \\ .6430 \end{bmatrix} \qquad \beta = \begin{bmatrix} .3965 & -.0094 \\ -.5909 & .8836 \end{bmatrix} .$$

Comparing these values of α and β to Table 3.4 we see, as we would expect, no radical changes.

The second step is to incorporate the new α and β into the input demand equations. The new input demand equations will in the reduced form have different coefficients on the input price terms, and the constant terms will also change. The reduced form parameters relating current input demand to the previous year's levels do not change, as they depend only on the structural parameters P and B. The new reduced form of the system is

$$\begin{bmatrix} k_t \\ l_t \\ r_t \\ w_t \end{bmatrix} = \begin{bmatrix} .886 & 1.958 & -720.808 & 19.167 \\ 0 & .812 & .837 & .148 \\ 0 & 0 & .670 & -.006 \\ 0 & 0 & 1.587 & .981 \end{bmatrix} \begin{bmatrix} k_{t-1} \\ l_{t-1} \\ r_{t-1} \\ w_{t-1} \end{bmatrix} + \begin{bmatrix} -4206.004 \\ 687.044 \\ .060 \\ -.039 \end{bmatrix},$$

and the simulation in Column E is based on this system.

The change in the input demand equations has a substantial effect on the results. Compare Columns B, D, and E. Columns D and E involve the same lowering of rental rates. For 1976 simulation D gives a capital stock .04% higher than it would otherwise have been, but simulation E has a capital stock that is .12% higher than it would otherwise have been. The values for 1980 are D: .18% higher and E: .48% higher. For 1984 we have D: .26% and E: .67%.

Even though in this model capital is quite inelastic with respect to rental rates, we find that accounting for the changes in input demand rules that should take place if the change in the rental rate path is recognized by the firm and incorporated into their input demand rules leads to a difference in the predicted effects of the rental rate change by a factor of around 2 or 3. While one might interpret this result as suggesting that how we specify the expectations process can have large effects, we must keep in mind that this particular model was rejected by the data, and that we should not form general conclusions based on the results of this chapter.

3.5 Conclusions

The linear quadratic rational expectations model of input demand, estimated with Canadian manufacturing data, is found wanting in many respects. Some restrictions implied by the model were accepted by the data, others were not. The model generated a demand for capital equation close to that obtained by unrestricted regression, although the restrictions reduced the elasticity of capital with respect to rental rates. But a demand for labour that is perfectly inelastic with respect to both input prices must be somewhat suspect. Also, the adjustment costs for labour were of the wrong sign. In the following chapter we estimate an alternative model of dynamic input demand, and compare the performance of the two models.

TABLE 3.1

Estimate of Unrestricted Linear-Quadratic Model where Input Demands Depend on Current Input Prices.

	Dependent variable k_t	Dependent variable l_t
k_{t-1}	.89175 (.02120)[a]	-.01311 (.01080)
l_{t-1}	1.8416 (.24982)	.67065 (.12288)
r_t	-5814.4 (3724.3)	-2387.2 (2347.6)
w_t	-93.470 (179.06)	167.51 (107.49)
constant	-2852.3 (1035.1)	1066.6 (625.19)
standard error	173.08	115.97

	Dependent variable r_t	Dependent variable w_t
r_{t-1}	.71315 (.13746)	.95233 (1.8451)
w_{t-1}	-.00588 (.00411)	.93761 (.05524)
constant	.05832 (.02916)	.17842 (.39143)
standard error	.00908	.12193

$$\text{sigma}^{b} = \begin{bmatrix} 29958. & & & \\ 8869.3 & 13449. & & \\ .03047 & .10051 & .00008 & \\ -.36046 & -9.4383 & .00021 & .01487 \end{bmatrix}$$

log of determinant of sigma = 5.0322

a Standard errors of parameter estimates in parentheses.
b sigma is the variance-covariance matrix of the 4 equation system where the order of the equations is, by dependent variable, k, l, r, w.

TABLE 3.2

Estimate of Unrestricted Linear-Quadratic Model where Input Demands Depend on Lagged Input Prices.

	Dependent variable k_t	Dependent variable l_t
k_{t-1}	.89080 (.02172)[a]	-.01246 (.01049)
l_{t-1}	1.8431 (.24106)	.65938 (.11636)
r_{t-1}	-4295.1 (3073.8)	-1542.7 (1677.4)
w_{t-1}	-47.842 (154.29)	170.79 (78.072)
constant	-3210.4 (860.60)	986.56 (447.35)
standard error	181.02	102.49

	Dependent variable r_t	Dependent variable w_t
r_{t-1}	.71315 (.13746)	.95233 (1.8451)
w_{t-1}	-.00588 (.00411)	.93761 (.05524)
constant	.05832 (.02916)	.17842 (.39143)
standard error	.00908	.12193

$$\text{sigma}^{b} = \begin{bmatrix} 32769. & & & \\ 9783.5 & 10504. & & \\ -.46505 & -.06417 & .00008 & \\ -2.9321 & -9.4383 & .00021 & .01487 \end{bmatrix}$$

log of determinant of sigma = 5.0314

a Standard errors of parameter estimates in parentheses.
b sigma is the variance-covariance matrix of the 4 equation system where the order of the equations is, by dependent variable, k, l, r, w.

TABLE 3.3

Estimate of Restricted Linear-Quadratic Model where Input Demands depend on Current Input Prices

Technological Parameters			
P_1	18.449 (2001.7)[a]	P_2	-320.94 (34821.)
P_3	-65630. (7120600.)	B_1	-166.35 (18048.)
B_2	298440. (32379000.)	a_1	-522700. (56711000.)
a_2	-87360000. (9478100000.)		

Parameters of J			
α_1	.11469 (.02207)	α_2	.52768 (.40223)
β_{11}	.37882 (.08566)	β_{12}	-.01073 (.00454)
β_{21}	-.82642 (1.6725)	β_{22}	.92099 (.07531)

$$\text{sigma}^{b} = \begin{bmatrix} 36060. & & & \\ 10874. & 13923. & & \\ -.50141 & -.10572 & .00008 & \\ -2.9683 & -9.5567 & .00020 & .01568 \end{bmatrix}$$

log of determinant of sigma = 5.4362
marginal significance of restrictions = .0257

Restricted estimates of reduced form

$$k_t = -4207.088 + .889k_{t-1} + 1.929l_{t-1} - .010r_t + 0w_t$$
$$l_t = 810.933 - .001k_{t-1} + .780l_{t-1} + 0r_t + 0w_t$$
$$r_t = .071 + .652r_{t-1} - .007w_{t-1}$$
$$w_t = -.084 + 1.382r_{t-1} + .994w_{t-1}$$

a Standard errors of parameter estimates in parentheses.
b sigma is the variance-covariance matrix of the 4 equation system where the order of the equations is, by dependent variable, k, l, r, w.

TABLE 3.4

Estimate of Restricted Linear-Quadratic Model where Input Demands depend on Lagged Input Prices

Technological Parameters			
P_1	$.28307\times10^{-3}$ (.01858)[a]	P_2	$-.48440\times10^{-2}$ (.32678)
P_3	-2.9801 (38.030)	B_1	$-.24745\times10^{-2}$ (.16848)
B_2	15.830 (197.64)	a_1	-6.8069 (464.49)
a_2	-3470.9 (46354)		

Parameters of J			
α_1	.11043 (.12705)	α_2	.68720 (2.2692)
β_{11}	.39786 (.75508)	β_{12}	-.00993 (.01935)
β_{21}	-.79212 (3.0780)	β_{22}	.88276 (.51155)

$$\text{sigma}^{b} = \begin{bmatrix} 35224. & & & \\ 10487. & 13861. & & \\ -.46782 & -.10919 & .00008 & \\ -2.7036 & -9.2650 & .00021 & .01530 \end{bmatrix}$$

log of determinant of sigma = 5.4367
marginal significance of restrictions = .0253

Restricted estimates of reduced form

$$k_t = -4225.810 + .886k_{t-1} + 1.958l_{t-1} - 723.758r_{t-1} + 20.140w_{t-1}$$
$$l_t = 687.078 - 0k_{t-1} + .812l_{t-1} + .801r_{t-1} + .147w_{t-1}$$
$$r_t = .067 + .670r_{t-1} - .007w_{t-1}$$
$$w_t = -.039 + 1.428r_{t-1} + .981w_{t-1}$$

a Standard errors of parameter estimates in parentheses.
b sigma is the variance-covariance matrix of the 4 equation system where the order of the equations is, by dependent variable, k, l, r, w.

TABLE 3.5

Forecast values of the capital stock under various conditions.

	Simulation					
Year	A	B		C	D	E
1975	18715.3	18715.3	(-)	18715.3	18715.3	18715.3
1976	19404.4	19229.2	(187.7)	19236.9	19236.9	19253.2
1977	20004.6	19721.8	(390.5)	19733.4	19737.6	19768.1
1978	20376.6	20189.1	(616.9)	20202.2	20212.7	20255.8
1979	21013.3	20628.6	(839.0)	20641.7	20659.7	20713.7
1980	21982.7	21039.1	(1044.5)	21051.2	21076.9	21140.7
1981	23286.0	21420.1	(1228.7)	21430.7	21464.1	21536.3
1982	23805.9	21772.0	(1390.3)	21780.8	21821.4	21901.0
1983	23698.8	22095.4	(1529.8)	22102.5	22149.6	22235.7
1984	23603.4	22391.6	(1648.9)	22396.9	22450.0	22541.7

Description of Simulations

A: Actual data for capital stock (machinery and equipment).

B: Simulated forecast using linear quadratic model, with reduced form estimates given in Table 3.4, starting at 1975, standard errors in parentheses.

C: Simulated forecast, using the model of B, with a one-off negative shock to the real rental rate of capital in 1975 of 10%.

D: Simulated forecast, using the model of B, with a permanent lowering of the path of the real rental rate by 10%, beginning in 1975, where firms do not realize there has been a change in regime.

E: Simulated forecast, with a permanent lowering of the path of the real rental rate by 10%, beginning in 1975, where the reduced form parameters of the linear quadratic model have been adjusted to reflect the change in the path of rental rates (i.e. where firms do realize there has been a change in regime).

CHAPTER 4

Epstein and Denny's Model

An alternative model of input demand to that described and estimated in Chapter 3, the model of Epstein and Denny (1983), is estimated in this chapter, using the same data from the Canadian manufacturing sector from 1962 to 1984.

A value function is said to have a flexible functional form when it provides a second order approximation to an arbitrary function that is consistent with the underlying economic theory.[1] In this chapter the value of the firm's cost minimization problem is described by a flexible functional form. Input demand functions are derived for the case of firms having static expectations of input prices and the case of firms' forecasts of input prices being described by first order autoregressive processes.

The flexible functional form model of this chapter is similar to the linear quadratic model of the previous chapter in a number of respects. In both models firms use capital and labour to produce a single output, real input prices are exogenous to the manufacturing sector, and there are internal, convex costs of adjusting the levels of inputs. In both models we assume firms forecast future input prices rationally, where these rational forecasts are approximated in the estimation with those generated by first order autoregressions (although a difference between the models is that the linear quadratic model allows expectations to be modelled as higher order autoregressions if desired, whereas

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[1] See Diewert (1974, p. 133) or Epstein (1981, p. 87).

the model in this chapter does not).

One difference between the two models is that with a flexible functional form, by definition, the only restrictions placed on the technology are those required by the assumption of profit maximization (or cost minimization). This applies both to the technology of "gross output" (i.e. output before adjustment costs have been subtracted) and to the specification of the adjustment costs themselves. For example, in the linear quadratic model adjustment costs were assumed to be quadratic and additive, whereas in the flexible functional form, the more general specification that costs be increasing and convex with respect to changes in inputs, and not necessarily additively separable, is used.

The other major difference is that in the model of this chapter firms take the level of output as given at any point in time, while in the linear quadratic model output was endogenous. To our knowledge there are no existing models of the firm that have been empirically implemented where output is endogenous, expectations are rational, there are adjustment costs, and the functional form is flexible.[2] We see below that treating output as given greatly improves the fit of this model, relative to that of the linear quadratic model.

Finally, the model of this chapter implicitly allows for the

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[2]This would certainly be a worthwhile project. Epstein (1981) provides the theoretical model for the case of static expectations.

contribution of a material input, while the linear quadratic model had only the two inputs, labour and capital.

4.1 The Model

The specification is taken directly from Epstein and Denny (1983) without alteration. As in Chapter 3 denote output at time t y_t, inputs $x_t = (k_t \ l_t)'$, real input prices $z_t = (r_t \ w_t)'$, and the constant discount rate R. Input prices in this chapter are deflated by the materials price index m, given in Table 2.1, so z_t in this chapter is very close to, but not identical to, z_t from Chapter 3. Both capital and labour are quasi-fixed. Define a purely variable input, materials, v_t, to whose price we will normalize the quasi-fixed input prices. The technology of a firm is given by

$$y_t = F(v_t, x_t, x_t - x_{t-1}). \tag{4.1}$$

The variable cost function is given by

$$C(y_t, x_t, x_t - x_{t-1}) = \min_{v_t} \{v_t : y_t \leq F(v_t, x_t, x_t - x_{t-1})\}. \tag{4.2}$$

We assume for now that the firm expects current input prices and output to remain constant, although this will later be relaxed. Under static expectations we set the firm's problem at time 0 as that of choosing a time path of input levels in order to minimize over an infinite horizon the present discounted value of future costs:

$$\min_{\dot{x}_t} \int_0^{\infty} e^{-Rt} \, [C(y, x_t, \dot{x}_t) + z_t' x_t] dt \tag{4.3}$$

subject to $\dot{x}_t = x_t - x_{t-1}$, x_0 given, and $x_t > 0$ for all t.

Each time period expectations of input prices and output are revised, and the solution to problem (4.3) is recalculated.

Let $V(x_0, y, z)$ be the value of the problem (4.3). We note here that below technology will be defined in terms of the form of V. If we define V_{zx} and V_z as 2x2 and 2x1 matrices of partial derivatives, respectively, then the optimal decision rule for adjusting x, as derived by Epstein (1981), is

$$\dot{x}_t^*(x_t, y, z) = V_{zx}^{-1} (x_t, y, z)[RV_z'(x_t, y, z) - x_t]. \tag{4.4}$$

Epstein and Denny (1983, pp. 651-2) list the properties V must satisfy if C is to satisfy certain regularity conditions.[3] A particular specification of V that satisfies those properties is

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[3]There are six conditions: C must be positive, C must be increasing in y and $\dot{x}$ and decreasing in x, C must be convex in $\dot{x}$, a unique solution to problem (4.3) must exist for each (x_0, y, z), the unique solution must have a unique steady state input level $\bar{x}$ that is globally stable, and for any $(x_0, y, \dot{x}^*)$ there exists a vector of input prices z^* such that $\dot{x}^*$ is the optimal policy at time 0 in problem (4.3) given (x_0, y, z^*).

$$V(x_t, y, z) = [z' \ 1]\begin{bmatrix} \Phi & \phi \\ \phi' & b \end{bmatrix}\begin{bmatrix} z \\ 1 \end{bmatrix} y/2 + (z'\Psi^{-1} + A_x')x_t$$
$$+ R^{-1}(z'\Psi^{-1}\lambda + h) + Q_x'x_t/y + x_t'Q_{xx}x_t/2y \qquad (4.5)$$

where Φ, Ψ, and Q_{xx} are each 2x2 matrices of parameters, ϕ, A_x, λ, and Q_x are each 2x1 vectors of parameters, and b and h are scalar parameters.

Combining the solution (4.4) of the dynamic problem (4.3) with the flexible functional form (4.5), the optimal rule for $\dot{x}_t$ is

$$\dot{x}_t^* = M(x_{t-1} - \bar{x}), \qquad (4.6)$$

where

$$M = \hat{R} - \Psi, \qquad (4.7)$$

$$\bar{x}(y, z) = -(\hat{R} - \Psi)^{-1}\{R\Psi[\Phi z + \phi]y + \lambda\}, \qquad (4.8)$$

and where $\hat{R}$ is (as in Chapter 3) a diagonal matrix with each diagonal element equal to R. The vector $\bar{x}$ represents the steady state, or target, demands for the quasi-fixed factors, and is a function of the level of output and of input prices.

The optimal rule (4.6) has a reduced form identical to that which arises in the linear quadratic model (see equation (3.9)). The structural parameters underlying M are clearly different across the two models, however.

The restrictions which V must satisfy to be consistent with cost minimization, and which will be tested in the estimation of the model are

$$\Phi \text{ is symmetric} \tag{4.9}$$

$$M \text{ has 2 real eigenvalues between -1 and 0.} \tag{4.10}$$

Condition (4.10) was also imposed on the linear quadratic model (see condition (3.13)).

The input demand functions implied by (4.6), (4.7), and (4.8) will be estimated. A final step before estimating the model will be to incorporate technical change by changing (4.1) to

$$y_t = e^{\gamma t}(m_t, x_t, x_t - x_{t-1}), \tag{4.11}$$

where γ represents the exponential rate of technological change. In the analysis above substitute $y_t e^{-\gamma t}$ for y_t. Defining the following reduced form parameters using the same notation as Epstein and Denny,

$$E = R\Psi\Phi, \tag{4.12}$$

$$G = R\Psi\phi, \tag{4.13}$$

the estimating equation for this static expectations case is

$$x_t/y_t = (I + M)x_{t-1}/y_t + [Ez_t + G](1 + \gamma)^{-t} + \lambda/y_t + u_t, \tag{4.14}$$

where u_t is a random error vector. Epstein and Denny use the assumption that changes in x affect y only after a one period lag, so y_t is predetermined in (4.14). This allows the use of standard econometric techniques. Below, when the model is estimated with non-static expectations, we consider as alternatives (i) input demands depending on lagged output, and (ii) input demands depending on predicted current output where the prediction is made in the preceding time period. We assume input prices are exogenous, although we found in Chapter 3 that with this data set this assumption may be suspect. However, especially when we work with non-static expectations in this model, it is an assumption which must be made for purposes of estimation.

4.2 Estimates

Equation (4.14) is estimated with the non-linear maximum likelihood option of SHAZAM.

Table 4.1 gives the results of estimation of (4.14) with the restriction (4.9) imposed. The notation is

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix}, \ \Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}, \ \phi = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \ \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}.$$

The single restriction imposed on the estimates, which is $\Phi_{12} = \Phi_{21}$, is rejected. The other restriction imposed by the model is that M be a stable adjustment matrix. Since the choice of a value of the discount rate, R, does not affect the reduced form of the model, we follow Epstein and Denny by setting R = .07. Then the estimated value of M is

$$\hat{M} = \begin{bmatrix} -.1212 & 1.5230 \\ .0420 & -.9026 \end{bmatrix},$$

which has eigenvalues of -.047 and -.977, satisfying restriction (4.10).

Epstein and Denny (p. 659) interpret the elements of M as follows. If labour is at its steady state value, 12% of the adjustment of the capital stock towards its steady state level occurs in one year. If capital is at its steady state level, 90% of the adjustment in labour occurs in one year. Using U.S. manufacturing data for the annual observations 1947 to 1976 Epstein and Denny obtain the identical adjustment parameters of 12% and 90%. In the linear quadratic model of Chapter 3, the respective rates of adjustment were 11% and 21%. So for some reason the linear quadratic specification predicts a much slower rate of adjustment for labour than does the more flexible specification, but the two models each predict the same speed of adjustment for capital.

The reduced form parameters implied by the restricted estimation are

$$\hat{E} = \begin{bmatrix} -.2012 & .0030 \\ -.0140 & -.0092 \end{bmatrix}, \quad \hat{G} = \begin{bmatrix} .0206 \\ .0899 \end{bmatrix}, \quad \text{and } \hat{\lambda} = \begin{bmatrix} -2890.4 \\ 319.82 \end{bmatrix}.$$

The diagonal elements of E are the own-price coefficients for the capital/output and labour/output ratios, so their negative sign is expected.

The impact elasticities generated by the static expectations model are, calculated at 1975 levels, as follows:

	short run elasticity with respect to:		
	r	w	y
capital	-.048	+.027	+.026
labour	-.018	-.439	+.615

The short run elasticities are very similar to those obtained by Epstein and Denny (1983, p. 661), in particular the own-price elasticities (although our estimate is for a labour demand more elastic with respect to the wage than they obtain) and the output elasticities.

To calculate the long run elasticities for this static expectations model we note that steady state input levels are given by

$$\bar{x}(y, z) = - M^{-1}\{[Ez + G]ye^{-\gamma t} + \lambda\} \quad (4.15)$$

(obtained by rewriting (4.8) in terms of reduced form parameters). As we did in Chapter 3 we will take the long run elasticity to mean the relative change in the target levels of inputs given a relative change in input prices or in output. Since we have found that M is a stable matrix, by the same reasoning in Chapter 3 this method of calculating long-run elasticities gives the same results as if we considered the long run change in actual values. Consider again the 1975 levels of capital and labour: 18715.3 and 3513.0.

Applying equation (4.15) we find the target levels of those two inputs at 1975 to be 35209.3 and 4384.7; each of these values is larger than the maximum levels of inputs observed in our 1961 to 1984 sample. This is somewhat surprising, since in the linear quadratic model of Chapter 3 we found target levels (at 1975 at least) much closer to actual levels.[4]

The long run elasticities, calculated at 1975, are:

	long run elasticity with respect to:		
	r	w	y
capital	-0.561	-1.181	+2.327
labour	-0.225	-0.831	+1.415

These values are all much larger in absolute terms than those found by Epstein and Denny. Note that the elasticity of capital demand with respect to the wage changes sign from the short to the

[4] Nickell (1985) points out that the optimal strategy for the firm in models of input demand with adjustment costs will not necessarily involve "asymptotically closing the gap between his choice variable and its optimal target value...given discounting it is not simply worth incurring the additional adjustment costs necessary to catch up completely with the [perhaps] growing target" (p. 121).

long run. This is due to two factors: the large coefficient (1.523) for capital with respect to lagged labour demand, and the elasticity of labour demand with respect to the wage. While the impact effect of a change in the wage is a movement of capital in the same direction, there is a large response of labour in the opposite direction to that of the wage change. After one period this change in labour demand has a substantial effect on capital demand, reversing the original effect of the change in the wage on capital. The large coefficient relating the demand for capital to lagged labour demand also appeared in the linear quadratic model in Chapter 3, both in the restricted and unrestricted estimation. The elasticity of capital with respect to the wage in that model was positive in both the short run and the long run. The sign did not change because labour was completely inelastic with respect to the wage.

Compared to the estimates we obtained in Chapter 3 with the linear quadratic model, we find with the flexible functional form a demand for capital that is much more elastic with respect to the rental rate, and a demand for labour that is, unlike in Chapter 3, responsive to changes in the wage and in the expected direction. Although identical data are used to estimate the two models, it is not clear exactly what difference in the models is responsible for the substantial difference in estimated elasticities.

A surprising result is that the elasticity of the demand for capital with respect to the wage is greater than its elasticity with respect to the rental rate. Morrison and Berndt (1981, p. 352) also obtain this result, albeit in a model where only capital is treated as a quasi-fixed factor.

The output elasticities suggest implausible decreasing returns to scale, casting some doubt on the reliability of the other estimated elasticities. Epstein and Denny also found output elasticities for both factors to be greater than one in the long run, although not to the degree obtained here.

As in the model of Chapter 3, adjustment to target levels is slow, so it is again instructive to calculate medium run elasticities. The slow convergence to the long run response with this model is for the same reasons as in Chapter 3, that is that the matrix $(I + (I + M) + (I + M)^2 + \ldots)$ is slow to converge to $-M^{-1}$. The five- and ten-year elasticities are

five-year elasticity with respect to:

	r	w	y
capital	-0.139	-0.238	+0.502
labour	-0.059	-0.459	+0.696

ten-year elasticity with respect to:

	r	w	y
capital	-0.228	-0.437	+0.888
labour	-0.094	-0.538	+0.849

We see that for capital, even after 10 years neither the elasticities with respect to prices nor the output elasticity have reached one half of their long run values. Even labour demand, which some researchers treat as a *variable* input, is remarkably (perhaps implausibly) slow to adjust to its long run level here. The lesson is that long run elasticities must be interpreted with great care, not only in this study, but in others as well; for example, Epstein and Denny's estimate of the adjustment matrix M is similar to ours, and so we would expect that their long run elasticities are also somewhat misleading.

Carmichael, Mohnen, and Vigeant's (1989) estimates of a translog variable cost function with annual Quebec manufacturing data yield a short run elasticity of capital with respect to the rental rate of -0.098, which is similar to our cost function estimate, but their corresponding long run elasticity is -0.271, substantially less than ours (although quite close to Epstein and Denny's U.S. manufacturing estimate). Their short run and long run elasticities of labour demand with respect to the wage are -0.118 and -2.354, respectively; their estimate of the impact elasticity is smaller than ours but their long run elasticity is substantially larger. Their short run and long run elasticities of capital with respect to output are +0.038 and -0.035, and of labour with respect to output are +2.339 and +1.713.

Two U.S. studies of cost functions where there are adjustment costs are Pindyck and Rotemberg (1983a) and Kokkelenberg and Bischoff (1986). Pindyck and Rotemberg, who report only the long run elasticities, find capital and labour have output elasticities of +1.476 and +1.031, and own-price elasticities of -2.927 and -0.784, respectively. Kokkelenberg and Bischoff find much lower elasticities: capital and labour have long run output elasticities of + 0.780 and +0.150, and long run own-price elasticities of -0.005 and -0.130, respectively. They claim these are "of reasonable magnitude" (p.429).

Taking all of the above studies together we find, first, that even over a fairly restricted class of models there is wide disagreement on the magnitudes of elasticities, and second, that our estimates of short run and long run own-price elasticities fall within the range established by previous studies, but that our estimates of input demand elasticity with respect to output are much higher than other studies, especially the long run elasticities. Epstein and Denny, and Pindyck and Rotemberg, also find evidence of decreasing returns to scale, but not to the same degree as our estimates.

4.3 The Model with Non-Static Expectations

Now we change the model somewhat by assuming firms do not expect output levels and input prices to remain constant, but rather form expectations by a process that we can represent with simple first order autoregressions. Recall from Chapter 3 that if the variable to be forecast follows a first order process, only one observation of the variable, plus estimates of the time series parameters of the variable, are required to make forecasts. Therefore the reduced form of input demands when expectations are formed this way will not be different from the static expectations formulation, although the interpretation of the reduced form will be different. Such will be the case here; the reduced form (4.14) will be reestimated, but the structural parameters will have different estimated values. Correct identification of the structural parameters is important if policy analysis using the results of the estimation is to be immune from the Lucas critique.

We model the evolution of output levels and input prices as follows:

$$\log (y_t) = \alpha + \log (y_{t-1}), \tag{4.16}$$

$$r_t = \mu_1(\exp(\theta_{11}) - 1)/\theta_{11} + \exp(\theta_{11})r_{t-1}, \tag{4.17}$$

$$w_t = \mu_2(\exp(\theta_{22}) - 1)/\theta_{22} + \exp(\theta_{22})w_{t-1}, \tag{4.18}$$

where $\exp(\omega)$ denotes e^{ω} and where the θ terms are not to be confused with their meaning in Chapter 3. This is an unusual method of describing first order AR processes, but it allows Epstein and Denny to incorporate the AR parameters into the input demand equations in a relatively straightforward way.

Equation (4.14) can now be reestimated, but with the reduced form parameters now being defined by

$$M = \Psi[(\hat{R} - \theta)\Psi^{-1} - I], \tag{4.19}$$

$$E = (R - \alpha)\Psi\Phi - \Psi(\Phi\theta + \theta\Phi), \tag{4.20}$$

$$G = \Psi[(\hat{R} - \hat{\alpha} - \theta)\phi - \Phi\mu], \tag{4.21}$$

$$\lambda \text{ replaced by } R^{-1}\Psi(\hat{R} - \theta)\Psi^{-1}\Lambda, \tag{4.21}$$

where I is the identity matrix, θ is 2×2 with θ_{11} and θ_{22} along the diagonal and zeroes off the diagonal, $\hat{\alpha}$ is 2×2 diagonal with each diagonal element equal to α, μ is 2×1 consisting of μ_1 and

μ_2, and Λ is 2x1 consisting of parameters Λ_1 and Λ_2 (Epstein and Denny (1983, p. 664)). Note that if θ, α, and μ all equal 0, i.e. if expectations are static, E, G, M, and λ return to their previous form.

Attempts to estimate (4.14) simultaneously with (4.16), (4.17), and (4.18) were not successful, in particular because estimates were always of an unstable M matrix, which means long run elasticities would not be well defined, unless the discount rate R was absurdly high. So we followed the method of Epstein and Denny by estimating (4.16), (4.17), and (4.18) separately, and then taking these parameters as given when it came to re-estimating (4.14), which was estimated with the non-linear maximum likelihood option of SHAZAM.[5] Note that care must therefore be taken when considering the estimates of standard errors of the model.

It was also necessary to place restrictions on Ψ so that we would have a stable M. Since it was not feasible to restrict M to be stable in any general sense in the estimation, we tried restricting Ψ to the particular values which would give us the reduced form for M that we obtained in the restricted estimation of the static expectations model.

Results of the 5 equations, with the restriction $\Phi_{12} = \Phi_{21}$ imposed, and with Ψ restricted to

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[5] This method is suggested by Wallis (1980).

$$\Psi = \begin{bmatrix} .4434 & -1.2883 \\ -.0716 & 1.0040 \end{bmatrix},$$

are given in Table 4.2. The likelihood statistic of the restrictions is 3.606, distributed χ^2 with 5 degrees of freedom, so they are not rejected.

The reduced form estimate of M is the same as for the static expectations case given above. Other reduced form parameters implied by the Table 4.2 estimates are

$$\hat{E} = \begin{bmatrix} -.1702 & +.0011 \\ +.0029 & -.0093 \end{bmatrix}, \quad \hat{G} = \begin{bmatrix} .0292 \\ .0756 \end{bmatrix},$$

and the vector which replaces λ from the static expectations case, as defined by (4.21) is

$$R^{-1}\Psi(\hat{R} - \theta)\Psi^{-1}\Lambda = \begin{bmatrix} -3149.991 \\ 703.127 \end{bmatrix}.$$

The implied values of impact elasticities, taken at 1975, are:

	short run elasticity with respect to:		
	r	w	y
capital	-.044	+.011	+.039
labour	+.004	-.486	+.510

The own-price elasticities are barely different from the static expectations estimates above.

Finding the long run elasticities in the non-static expectations case is somewhat complicated, since we must calculate how the reduced form parameters E and G change when input prices or the level of output have their expected future paths permanently shifted.

The long run elasticity with respect to output turns out to be not so difficult to calculate. From equation (4.16) above we see that a permanent proportional increase in y_t leaves the underlying exponential growth rate α unchanged. So the reduced form parameters of input demand are unaffected (although input demands *are* affected by the *level* change in output), and we can refer to equation (4.15), which describes the target input levels, to find the long run elasticities.

Regarding long run price elasticities, a long run proportional change in input prices would involve a change in the μ parameters from equations (4.17) and (4.18). This change would affect the reduced form parameters of the vector G; see equation (4.21). A change in μ changes G by a factor of $-\Psi\Phi$. Estimates of Ψ and Φ are given in Table 4.2. It turns out for our estimates (and for Epstein and Denny's) that μ and G will move in the same direction. This is surprising. For estimates where Ψ and Φ have the expected signs (so that own price elasticities are negative and the adjustment matrix M is stable) we obtain the result that adjusting reduced form parameters to take account of a change in expected future prices actually lowers the effects of the price change. For example, we will see in simulations below that a permanent fall in rental rates has a larger positive effect on input demands if we do not adjust the reduced form parameters than if we do make the (correct) adjustment. The converse was the case in the linear quadratic model of Chapter 3; the intuition behind the perverse result of the model of this chapter is not obvious.

From equation (4.15) and the estimates of reduced form parameters from this section we calculate target levels of capital and labour in 1975 of 35459.4 and 4416.4. These are nearly identical to the target level estimates from the static expectations model, and are higher than the actual levels at that time by factors of about 2 and 1.5 for capital and labour respectively. The long run elasticities are:

	long run elasticity with respect to:		
	r	w	y
capital	-0.178	-0.971	+2.096
labour	-0.055	-0.545	+1.232

Each of these elasticities is smaller in absolute value than those obtained from the static expectations model.

Once again, we should look at the medium run elasticities, since the adjustment to long run is very slow. The five- and ten-year elasticities are

	five-year elasticity with respect to:		
	r	w	y
capital	-0.047	-0.216	+0.459
labour	-0.003	-0.248	+0.587

	ten-year elasticity with respect to:		
	r	w	y
capital	-0.075	-0.376	+0.805
labour	-0.014	-0.311	+0.724

It is interesting to note the estimates of long run elasticities if we do not account for the change in the reduced form parameter G that should occur with a change in the long run path of prices. The output elasticities are unaffected, because the reduced form parameters of the input demands do not change with a permanent shift of a given proportion to output (although they would change if the rate of growth of output permanently changed). With G unadjusted the estimates of long run elasticities, obtained from the reduced forms, are

	long run elasticity (G unadjusted) with respect to:		
	r	w	y
capital	-0.453	-1.508	+2.104
labour	-0.166	-0.993	+1.237

For reasons noted above these are larger in absolute value than when G is adjusted to account for changed expectations.

We also tried estimating the non-static expectations model replacing y_t in the input demand equations with y_{t-1} and with $(1 + \alpha)y_{t-1}$ (which could be a proxy for expected y_t). There are no clear theoretical reasons for choosing one specification over another here. The logs of the likelihood functions from using y_t, y_{t-1}, and $(1 - \alpha)y_{t-1}$ are 236.359, 217.694, and 219.556, respectively. There are no changes in the signs of the structural parameters.

Finally, we attempted estimating Epstein and Denny's model imposing the short run elasticities we obtained in the linear quadratic model of Chapter 3. We are interested in whether this restriction substantially lowers the goodness of fit of the model.

In the model of this chapter, own price elasticities are determined by our estimate of E, which in turn is determined by the structural parameter Φ. We seek only to restrict the short run elasticity (we cannot simultaneously restrict the long run elasticity, since that would entail a complicated restriction on the adjustment process). Recall that in Chapter 3 we obtained elasticity of capital demand with respect to the rental rate and the wage of -.004 and +.004 respectively, and a labour demand inelastic with respect to both the rental rate and the wage. These are the short run elasticities we will impose here. Note that in Chapter 3 the rental rate and wage were deflated by the output price index, rather than the materials price index, so the comparison is not precise.

Given our imposed values of R, α, Ψ, and θ, we impose a value of Φ of

$$\Phi = \begin{bmatrix} -.0618 & +.0028 \\ -.0072 & +.0033 \end{bmatrix},$$

in order to obtain a value for E of

$$E = \begin{bmatrix} -.0157 & +.0004 \\ 0 & 0 \end{bmatrix},$$

and the elasticities given above.

When the system is estimated, the only free parameters are γ, ϕ, and Λ; five parameters in all. There are eight restricted parameters. The unrestricted log of the likelihood function is 238.16, and the restricted result is 188.84, so this restriction is clearly rejected. Since so many aspects of the linear quadratic model results were unsatisfactory, we are not altogether surprised that the elasticities obtained with that model are rejected by the model of this chapter.

This model with non-static expectations will now be used for simulations comparable to those done in Chapter 3 with the alternative model.

4.4 Simulations with Non-Static Expectations

Table 4.3 records the results of simulations which are analogous to those in Table 3.5 from the linear quadratic model. Column A lists actual data for capital in the manufacturing sector. Column B gives a long term forecast one might have made in 1975 using the flexible functional form model with non-static expectations as estimated above. At 1975, output in 1984 is forecast at 88170.8 while the actual value was 76700.9, the rental rate is forecast at .128 while the actual values were .130 in 1983 and .146 in 1984, and the wage is forecast at 4.43 while the actual value was 4.65. Labour is forecast to be 4062.1 in 1984 while the actual value was 3621.0. We see in the table that the capital stock throughout the 1980s would have been slightly over-estimated with this model.

In simulation C there is a 10% negative shock to the rental rate in 1975, but it is temporary in that the underlying parameters of the time series process of the rental rate, μ_1 and θ_{11}, are left unchanged. By 1984 the effects of the shock on the rental rate itself has almost completely dissipated: the 1984 base case (column B) forecast rental rate is .12775, and the one-off shock case (column C) forecast rental rate is .12621. But capital and labour demand depend heavily on their previous values, and even at 1984 there are significant lingering effects from the 1975 shock. Capital in 1984 is 1.10% higher than it otherwise would have been and labour is 4074.6, which is 0.31% higher than it otherwise would have been. Recall that the identical shock in the linear quadratic model left 1984 capital only 0.02% higher than the base case, and labour was completely unaffected by the shock.

In columns D and E are listed the forecast paths of the capital stock when in 1975 the rental rate is permanently lowered by 10% from its base case path. In column E the reduced form parameters have been altered as they should be given the change in regime, while in column D the base case reduced form is used. The path of rental rates is lowered permanently here by cutting the 1975 value of r by 10%, from .1106 to .0950, and cutting μ_1 by 10%, from .02819 to .02537. The reduced form of the rental rate path is changed from

$$r_t = .0254 + .8071r_{t-1}$$

to

$$r_t = .0228 + .7069r_{t-1}.$$

As explained above the change in μ changes the vector G, which is part of the reduced form of the input demand equation. G changes from

$$\hat{G} = \begin{bmatrix} .0292 \\ .0756 \end{bmatrix}$$

to

$$G = \begin{bmatrix} .0283 \\ .0756 \end{bmatrix}.$$

From equation (4.15), which describes the target levels of inputs, we see that the target levels $\bar{x}$ depend positively on G (all of the elements of $-M^{-1}$ are positive), so for given *current* levels of input prices the permanent reduction in future rental rates actually lowers the input targets. The result surprises because estimated own price elasticities are negative.

The change in the reduced form parameters has significant effects. Comparing the forecasts for 1984 levels, the capital stock for the case where reduced form parameters of the input demand equations are adjusted is 1.72% higher than the base case forecast, while when these parameters are not adjusted it is 3.12% higher than the base case. The difference is the same on the labour side; the column E (adjustment made) 1984 forecast is

4077.6, 0.38% higher than the base case, while the column D (adjustment not made) forecast is 4092.2, 0.74% higher than the base case.

4.5 Comparing the Performance of the Models

Both the linear quadratic rational expectations model of Chapter 3 and the flexible functional form model of this chapter seek to explain the manufacturing sector's demand for capital and labour. The models are non-nested; recall that in the linear quadratic model input prices were deflated with an output price index, whereas with the flexible functional form model the parameters of the value function of a restricted variable cost function required deflating quasi-fixed input prices with a variable input price index. Also, the rate of output is assumed to be determined exogenously in the latter, and appears as an explanatory variable in the input demand equations, while in the former output is endogenous and does not play a part in the estimation.

We now examine the relative performance of the two models with a test proposed by Davidson and MacKinnon (1981); specifically their "P test". Using their notation, let the input demand equations of the linear quadratic model of Chapter 3 be written in the form

$$x_t = f(X_t, \beta) + \varepsilon_{0t}, \tag{4.22}$$

and the input demand equations of the flexible functional form model be written in the form

$$x_t = g(Z_t, \gamma) + \varepsilon_{1t}, \tag{4.23}$$

where x_t is the vector of input demands (capital and labour), X_t and Z_t are the data used to estimate the two models, and β and γ are the parameters to be estimated.

Following Davidson and MacKinnon, define

$$\hat{f}_t = f(X_t, \hat{\beta}), \tag{4.24}$$

and

$$\hat{g}_t = g(Z_t, \hat{\gamma}), \tag{4.25}$$

where $\hat{\beta}$ and $\hat{\gamma}$ are the maximum likelihood estimates of the restricted dynamic expectations models, given in Tables 3.4 and 4.2 respectively.

We test the null hypothesis (4.22) against the alternative hypothesis (4.23) by estimating the regression

$$x_t - \hat{f}_t = \alpha(\hat{g}_t - \hat{f}_t) + X_t b + \varepsilon_t. \tag{4.26}$$

where b is a vector of unknown parameters and ε_t is an error term.If the null hypothesis is true, the true value of α is zero. If the alternative hypothesis is true, the estimate of α should converge asymptotically to one.

The estimate of α in (4.26) does not have an asymptotically valid standard error unless we also include on the right-hand side a vector representing the derivatives at each observation of the predicted values of the model with respect to its parameters: $\hat{F}_t$ in Davidson and MacKinnon's notation. If the model $f(X_t, \beta)$ is linear, then $\hat{F}_t$ is X_t. The linear quadratic model is linear, although with non-linear restrictions, so we include X_t in (4.26).

The elements of X_t are the right hand side variables of the linear quadratic model of Chapter 3: k_{t-1}, l_{t-1}, r_{t-1}, w_{t-1}, and a constant (note that r_t and w_t are deflated with an output price index in Chapter 3). The series $\hat{f}_t$ and $\hat{g}_t$ were generated from the estimates given in Tables 3.4 and 4.2. Equation (4.26) was estimated separately for capital and labour, by ordinary least squares.

For capital the estimate of α is 0.807, with a t-statistic of 1.419 (there are 17 degrees of freedom), and for the labour the estimate of α is 1.095, with a t-statistic of 9.934. This is strong evidence that we should reject the model (4.22), the linear quadratic model.

This does not mean we should necessarily accept the alternative model. To test the model of this chapter, the flexible functional form, we would have to construct the test so that the flexible functional form was the null hypothesis. We do the following regression:

$$x_t - \hat{g}_t = \alpha(\hat{f}_t - \hat{g}_t) + \hat{G}_t d + \nu_t, \tag{4.27}$$

where ν_t is an error term, d is a vector of parameters, and $\hat{G}_t$ represents the derivatives of the predicted values of $g(Z_t, \gamma)$

with respect to the parameters, namely $r_t y_t (1.01911)^{-t}$, $w_t y_t (1.01911)^{-t}$, $y_t (1.01911)^{-t}$, and a constant, in both the capital and labour equations, where 1.01911 is the estimate of γ from Table 4.2. Input prices r_t and w_t are here deflated by the materials input price index. Also included in the capital equation is $-t(1.01911)^{(-t-1)}(E_{11}r_t y_t + E_{12}w_t y_t + G_1 y_t)$, and in the labour equation $-t(1.01911)^{(-t-1)}(E_{21}r_t y_t + E_{22}w_t y_t + G_2 y_t)$, where these are the derivatives of the predicted values of the flexible functional form model with respect to γ, and where E_{ij} and G_i, i = 1, 2 are from the reduced form estimates given above.

For capital the estimate of α is 2.545, with a t-statistic of 0.297 (there are 15 degrees of freedom), and for the labour the estimate of α is -790.97, with a t-statistic of -1.408. The 5% critical value of the t-statistic is 1.753, so we fail to reject the flexible functional form in the presence of the linear quadratic model as an alternative hypothesis.

Finally, we ask whether what appears to be superior performance by Epstein and Denny's model relative to the linear quadratic model is due solely to the inclusion of output as an explanatory variable in the former but not the latter. We begin by regressing $\hat{g}_t$, the predicted values from the model of this chapter, on output and a constant. The results are, for capital

$$gk_t = -2825.4 + .344y_t, \tag{4.28}$$

and for labour

$$gl_t = 2475.5 + .018y_t. \tag{4.29}$$

We try Davidson and MacKinnon's P test of the linear quadratic model using the predicted values of the above equations as the alternative hypothesis, rather than $\hat{g}_t$. The results are that we still reject the null hypothesis of the linear quadratic model, even when set against simply that part of the predicted values of Epstein and Denny's model explained by current output. The estimate of α for the capital equation is .113 with a t-statistic of 2.346, and for labour α is 2.219, with a t-statistic of 7.954. This suggests that the inclusion of output in one model but not the other explains at least to some degree the different levels of performance of the models.

It is also the case, however, that the linear quadratic model is rejected by the Epstein and Denny model even when we discount for the effects of the inclusion of output as an explanatory variable. The estimated residuals of equations (4.28) and (4.29) could be thought of as the part of the demands for capital and labour explained by the "non-output variables" of the Epstein and Denny model. We performed regression (4.26) for capital and labour using the residuals from (4.28) and (4.29) in place of $\hat{g}_t$. The results were an estimate for α of .114 in the capital equation, with a t-statistic of 2.461, and .759 in the labour equation, with a t-statistic of 10.270. Again we reject the linear quadratic model.

4.6 Conclusions

The dynamic input demand model of Epstein and Denny (1983) has been estimated for both static expectations and for rational expectations, where by rational expectations we mean that the parameters describing the time series processes of input prices and output were incorporated into the input demand equations. In general the model performed well; a symmetry restriction was satisfied, and parameter estimates were of the expected sign. Elasticities were in the (large) range of estimates of other researchers, although estimates of output elasticities were implausibly high. In addition, according to the results of non-nested hypothesis tests between the model of this chapter and the linear quadratic model of Chapter 3, the linear quadratic model is rejected, but the model of this chapter is not.

We found a paradoxical result when it came to noting how target levels of input demand changed when the time series path of input prices changed. While with the linear quadratic model of Chapter 3 we found that ignoring the effect of the path of input prices on the reduced form parameters of input demand would lead one to *underestimate* capital's own price elasticity, in the model of this chapter ignoring this effect would cause one to *overestimate* the elasticity. The results of the models of Chapters 3 and 4 are so different with regards to estimated elasticities and to the changes in long run elasticities when we change the specification of expectation formation, that in the end we know little about the size or even the sign of the effects of heeding the Lucas critique in dynamic models of factor demand.

TABLE 4.1

Restricted Estimates of Flexible Functional Form Model with Static Expectations

Parameter Estimates			
Ψ_{11}	.19120 (.03325)[a]	Ψ_{12}	-1.5230 (.34998)
Ψ_{21}	-.04199 (.02292)	Ψ_{22}	.97260 (.07449)
λ_1	-2890.4 (560.18)	λ_2	319.82 (451.98)
ϕ_1	18.372 (11.877)	ϕ_2	2.1133 (1.1012)
Φ_{11}	-25.405 (15.504)	Φ_{12}	-1.3022 (1.2419)
Φ_{22}	-.19199 (.10463)	γ	.02566 (.00719)

$$\text{sigma}^{b} = \begin{bmatrix} .11811\times10^{-4} & \\ .13303\times10^{-5} & .55536\times10^{-6} \end{bmatrix}$$

log likelihood function = 234.4712

unrestricted[c] log likelihood function = 238.1624

marginal significance of restriction = .007

a standard errors of parameter estimates in parentheses.

b sigma is the variance-covariance matrix of the 2 equation system where the order of the equations is, by dependent variable, k/y, l/y.

c unrestricted estimate does not impose $\Phi_{12} = \Phi_{21}$.

TABLE 4.2

Restricted Estimates of Flexible Functional Form Model with Autoregressive Expectations.

Exogenous Parameters			
μ_1	.04456 (.01842)	μ_2	.32748 (.18897)
θ_{11}	-.34690 (.14197)	θ_{22}	-.06908 (.05060)
α	.04146 (.01018)		

Technology Parameters			
Λ_1	-856.05 (73.123)	Λ_2	235.89 (76.501)
ϕ_1	1.2184 (.25150)	ϕ_2	.76094 (.06895)
Φ_{11}	-1.0356 (.23971)	Φ_{12}	-.09908 (.03174)
Φ_{22}	-.06883 (.00835)	γ	.01911 (.00249)

$$\text{sigma}^{b} = \begin{bmatrix} .11497\times10^{-4} & \\ .15097\times10^{-5} & .55177\times10^{-6} \end{bmatrix}$$

log likelihood function = 236.3593

unrestricted[c] log likelihood function = 238.1624

marginal significance of restriction = .607

a standard errors of parameter estimates in parentheses.

b sigma is the variance-covariance matrix of the 2 equation system where the order of the equations is, by dependent variable, k/y, l/y.

c unrestricted estimate does not impose $\Phi_{12} = \Phi_{21}$, or the values for Ψ used to ensure a stable adjustment matrix.

TABLE 4.3

Forecast values of the capital stock under various conditions, using the flexible functional form model.

	Simulation				
Year	A	B	C	D	E
1975	18715.3	18862.2	18945.1	18945.1	18903.6
1976	19404.4	19611.9	19750.9	19771.2	19692.4
1977	20004.6	20325.8	20507.6	20562.5	20447.2
1978	20376.6	21018.1	21232.2	21332.9	21182.0
1979	21013.3	21696.4	21933.9	22089.4	21903.7
1980	21982.7	22366.7	22620.9	22838.0	22618.2
1981	23286.0	23034.5	23299.7	23584.0	23330.7
1982	23805.9	23704.5	23976.1	24331.7	24045.6
1983	23698.8	24380.8	24655.3	25085.3	24766.8
1984	23603.4	25066.8	25341.4	25848.2	25497.9

Description of Simulations

A: Actual data for capital stock (machinery and equipment).

B: Simulated forecast using flexible functional form model, with parameter estimates from Table 4.2, starting at 1975.

C: Simulated forecast, using the model of B, with a one-off negative shock to the real rental rate of capital in 1975 of 10%.

D: Simulated forecast, using the model of B, with a permanent lowering of the path of the real rental rate by 10%, beginning in 1975, where firms do not realize there has been a change in regime.

E: Simulated forecast, with a permanent lowering of the path of the real rental rate by 10%, beginning in 1975, where the reduced form parameters of the model have been adjusted to reflect the change in the path of rental rates (i.e. where firms do realize there has been a change in regime).

CHAPTER 5

Conclusion

In this thesis we have used data from Canadian manufacturing from 1961 to 1984 to estimate two distinct models of dynamic input demand, each of which to some degree incorporates the hypothesis of rational expectations. The alternative models generated quite different estimates of the elasticities of input demands with respect to their prices. Because of this, and because of other uncertainties about the most appropriate ways of modelling expectations about tax policy and other components of the rental cost of capital, no definitive conclusions could be reached on the effects of the rental rate of capital on investment.

There is wide disagreement, or at least uncertainty, about the importance of the rental cost of capital on business investment. We have here attempted to obtain results on this question using recently derived techniques for estimating dynamic models, but we have only succeeded in increasing our confusion on the question. Estimates of a linear quadratic rational expectations model, as specified in Epstein and Yatchew (1985), were of a demand for capital quite inelastic with respect to the rental rate. Estimates of a dynamic model using a flexible functional form, as derived by Epstein and Denny (1983), were of a capital demand with much greater elasticity with respect to the rental rate in the long run. When the models were compared using the P test of Davidson and MacKinnon (1981), Epstein and Denny's model seemed to outperform the linear quadratic model, although we were given some reason to believe that this might be due to the inclusion of output as an explanatory variable in the former but not in the latter.

When these two models were used to estimate the effects of a change in the time series path of the rental rate of capital, where the simulations were done in such a way as to avoid the Lucas critique of econometric policy evaluation, the flexible functional form model's prediction was of a much larger change to the medium and long term capital stock than the change predicted by the linear quadratic model. Each of these estimates in turn was quite different from the predictions of the corresponding elasticities from estimates of the two models with static expectations. Estimates of the effects of a change in the path of the rental rate where we ignored the warning of the Lucas critique gave an even *wider* range of estimates. We found that whether we adjusted input demand rules for various rental rate regimes (as Lucas would suggest we should do) made a large difference to our results, but the direction of the bias from not making the adjustments was ambiguous.

These results suggest that we still know very little about input demands when there are adjustment costs, and that there is much more to be done in this area. Listed below are some of the many questions that remain.

What are the effects on our results of using aggregate data to estimate a model of a "representative firm"? The model we estimate in Chapter 4 is at least potentially consistent with the aggregation of a number of firms, according to conditions for consistent aggregation set out in Blackorby and Schworm (1982), although we did not test whether these conditions were satisfied. Whether the linear quadratic model estimated in Chapter 3 is consistent with respect to aggregation is less clear. Geweke (1985) suggests that the biases from aggregation in such models might easily be as large as any biases from ignoring the warnings of the Lucas critique. It would be of interest to find whether this is true with actual data.

Can more interesting specifications of technology be developed? We mentioned above that a dynamic model of the firm with rational expectations, a flexible functional form, and endogenous output has never been estimated. Bernstein and Nadiri (1987) suggest that it would also be of interest to incorporate variable utilization rates in dynamic models, which could have the effect of endogenizing the depreciation rate of capital.

Finally, although this thesis has been purely a positive analysis of manufacturing in Canada and rental rates, we could in future attempt to use improved estimates of the private sector's response to tax policy to help answer some normative questions. Woodward (1974) and Kesselman, Williamson, and Berndt (1977) have questioned whether tax incentives for investment are the most appropriate device for dealing with unemployment problems. Our estimate of the linear quadratic model found labour demand totally inelastic with respect to the rental rate. It is difficult to apply our results from the model of Chapter 4 because there output was exogenously given, and clearly those who would subsidize investment with the goal of creating jobs are looking to the scale effects.

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