YIELD-DENSITY RESPONSES IN MONOCULTURES AND MIXTURES

OF BEANS (*PHASEOLUS VULGARIS* L.)

AND BEETS (*BETA VULGARIS* L.)

by

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90

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ABSTRACT

Interference among neighbouring plants, often due to competition for limited resources, is central to subjects such as yield-density relationships, intercropping, self-thinning in dense plant stands and low reproductive yield in certain crops. An experiment was conducted to investigate plant interference in associated populations of beans (*Phaseolus vulgaris* L.) and beets (*Beta vulgaris* L.). Plants of the two species were grown at different total densities and at different mixture proportions in a randomized complete block design. Several analytical procedures were used to interpret and define treatment effects.

The analysis of variance indicated that yield was significantly reduced with either increasing total population density or increasing bean proportions in mixtures. The interactions of total population density and mixture proportions were only occasionally significant.

Parameters of non-linear models used to define yield-density relationships indicated that beans were the superior competitor, both against themselves and against beets. The model parameters were also used to determine differential yield responses on total dry weight, leaf dry weight, leaf number and leaf area in the bean-beet mixtures. Yield advantage was observed in leaf dry weight and leaf number when model parameters were used in calculating land equivalent ratios whereas total dry weight and leaf area showed yield disadvantage. Using observed values to calculate land equivalent ratios indicated yield advantage in all four variables.

Plant size inequalities, as determined by the Gini coefficient tended to decrease in beet monocultures with increasing population density. In

monocultures of beans and in the bean-beet mixtures, plant size distri-bution was not systematically changed by density and mixture treatments.

Yield component analysis indicated that the variation in total yield due, to either population density or mixture treatments increased with age; the variation due to the population density by mixture proportions interaction remained relatively constant throughout the growing season. Leaf number per plant was the yield component which was most frequently a significant source of yield variation both in the forward and backward yield component analysis.

Plant growth analysis indicated that leaf area ratio and specific leaf weight were higher at higher population densities and at higher bean proportions. Harvest index decreased with increasing population density and with increasing proportions of the competing species in beets. Absolute growth, relative growth and unit leaf rates increased with time and declined after reaching a peak at about 68 days after planting. Both the lowest population density of 16 plants m^{-2} and the mixture treatment with the least proportion of beans had the greatest increase in absolute growth, relative growth and unit leaf rates.

Allometric relationships between total plant dry weight and any secondary measure per plant were influenced in different ways by density and mixture treatments and by time of harvest. The composition of models also varied considerably. The interpretation of plant interference, therefore is strongly influenced by the choice of plant characteristics which are measured, and by the time of measurement.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDICES

SYMBOLS AND TERMINOLOGY

Symbols	Page (First Appearance)	Definition
α	29	Allometric coefficient
a	17	Intercept in yield-density models
AGR	103	Absolute growth rate
ANOVA	36	Analysis of variance
β	29	Allometric exponent
b	17	Regression coefficient in yield-density models
BN	47	Branch number
c	21	Regression coefficient in yield-density models
CP	196	Mallows CP statistic
δ	30	Non-allometric parameter
D	22	Total mixture population density
d.f.	47	Degrees of freedom
DR	40	Root diameter
DR/WL	40	Root diameter per leaf dry weight
e	29	Base of natural logarithms
E	103	Unit leaf rate
ε'	29	Residual variation in yield in allometric models
F	103	Leaf area ratio
FWPD	47	Pod fresh weight
FWR	58	Root fresh weight
G	39	Gini coefficient
γ	30	Non-allometric parameter
G'	39	Unbiased Gini coefficient

Symbols	Page (First Appearance)	Definition
H	103	Harvest index
HN	45	Harvest number
i, j	17	Subscripts indicating species. Subscript i denotes the test species. Subscript j, when present, denotes the companion species
k	30	Subscript indicating k^{th} allometric and non allometric exponent
LA	39	Leaf area
LA/LN	40	Leaf area per leaf number
LER	22	Land equivalent ratio for a mixture
LL	55	Live leaf number
LN	39	Leaf number
$\log_{10}$	37	Logarithms base 10
$\log_e$ (same as ln)	29	Natural logarithm
LWR	103	Leaf weight ratio
MPN	55	Marketable pod number
n	39	Sample size in Gini coefficient computation
p, q	39	Subscripts denoting p^{th} and q^{th} yield per plant in Gini coefficient computation
PN	39	Pod number
PN/WST	40	Pod number per stem dry weight
R	103	Relative growth rate
R^2	140	Coefficient of determination
$\mathbf{s}$	39	Grand mean yield per plant of a variable in Gini coefficient computation
s	39	Yield per plant in Gini coefficient computation

Symbols	Page (First Appearance)	Definition
SLA	103	Specific leaf area
Φ, Φ'	18	Exponents controlling the form of the yield-density models
SN	55	Seed number
T	47	Plant height
TDP	36	Two dimensional partioning
UPN	55	Unmarketable pod number
W	47	Total dry weight (total shoot dry weight for beans, and total shoot and storage root dry weight for beets)
WL	39	Leaf dry weight
WL/LA	40	Leaf dry weight per leaf area
WMPD	55	Marketable pod dry weight
WP	58	Petiole dry weight
WPD	40	Pod dry weight
WPD/PN	40	Pod dry weight per pod number
WR	40	Root dry weight
WR/DR	40	Root dry weight per root diameter
WS	55	Seed dry weight
WST	39	Stem dry weight
WST/WL	40	Stem dry weight per leaf dry weight
WUPD	55	Unmarketable pod dry weight
ξ	30	Non-allometric parameter
X	17	Population density of a species (plants per land area)
y	17	Mean yield of the test species per plant
Y	17	Mean yield of the test species per unit land area
z	29	Yield of secondary measure of a plant in allometric relationships

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1. INTRODUCTION

Interference among neighbouring plants, often due to competition for limited resources, is central to subjects such as yield-density relationships, intercropping, mortality in dense plant stands, and low reproductive yields in crops. Despite its importance, the nature and mechanisms of competition among plants are not well understood. There are many reasons for this lack of understanding. Among them are the inconsistent usage of, or different meanings attributed to, the term competition when it is used in relation to plants. Also, the experimental designs used to study competition have often had shortcomings, and methods of analyzing results from competition experiments have needed improvement. Further complications arise from the potentially complex nature of competitive relationships, because different environmental resources might be the cause of competition at different times during growth, and because non-competitive interferences (e.g. allelochemical interference) may take place among associated plants.

The research described in this thesis pertains to the interference between intercropped bean (*Phaseolus vulgaris* L.) and beet (*Beta vulgaris* L.) plants. In addition to considering the effects of interference on final crop yields, this work was intended to advance our knowledge of the mutual influences of intercropped species upon each other. For this reason, experiments were undertaken to detail the timing and sites within plants of their responses to species population densities and mixture proportions, and to detail the effects of such treatments on the distribution of plant size among members of each species. Specific objectives of the research were:

(i)   to develop mathematical relationships defining the response of shoot biomass, and certain other measures of plant growth, to population densities of beans (*Phaseolus vulgaris* L.) and beets (*Beta vulgaris*) in monocultures and mixtures;

(ii)   to use those mathematical relationships to determine whether intercrops of bean and beet plants are more productive than corresponding monocultures (*i.e.* to quantify the 'differential yield response' of the intercrops);

(iii)   to assess how different population densities and mixture proportions affect the frequency distribution of plant size within each species;

(iv)   to evaluate the effects of population density treatments on quantitative relationships among different measures of growth (*i.e.* allometric relationships) in each species;

(v)   to analyze the dynamics of plant growth, using procedures of plant growth analysis to assess the performance of plants and plant parts, as it was affected by experimental treatments; and

(vi)   to determine the contributions of morphological yield components to variation in final agricultural yield, and to quantify the effects of population density and mixture treatments on those components.

2. LITERATURE REVIEW

2.1 The Crops Used in This Study

2.1.1 Beans

Out of about 150 species in the genus *Phaseolus*, the snap or common bean, *Phaseolus vulgaris* L., is the species most widely cultivated (Yamaguchi 1983, Singh 1989). A member of the Leguminosae, the plant is an annual which reproduces solely by seed. The plant can be a determinate (bush) type or indeterminate (climbing or pole) type (Peirce 1987). After the primary leaves, the plant produces alternate trifoliate leaves.

P. vulgaris may self-pollinate, but there is large variation in plant characteristics within and among cultivars (Singh 1989). Pods are long and narrow, varying in length from 8 to 20 cm, and are 1 to 1.5 cm wide. The number of seeds in a filled pod also varies between 4 and 12 depending on cultivar and conditions during growth. Also varying are characteristics such as seed length (0.7 to 1.5 cm), seed weight (< 15 to > 60 g/100 seeds), seed form (globular to kidney shaped), and seed coat color (white, yellow, greenish pink, reddish purple, brown or black). Seed color can be solid, striped or mottled (Singh 1989). "String" (*i.e.* fibrous pod tissue) formation is controlled genetically, and is influenced by cultivar and temperature (Drijfhout 1978).

P. vulgaris is a warm season crop which is thought to have originated in central America (Gepts *et al.* 1988). It was distributed to other parts of the world soon after European contact with central America (Ware and McCollum 1975). Now it is grown in temperate zones during the warm months, and in tropical and subtropical farming regions (Wallace

1978). *P. vulgaris* is a mesophyte, requiring about 500-800 mm of water per growing season for optimum growth. In areas with low rainfall, irrigation is recommended for higher yields (Ware and McCollum 1975). The optimum temperature for germination is about 30C. Germination is greatly reduced above 35C, and below 10C germination does not occur (Peirce 1987). Vegetatively, the plant grows best between 21 and 26.7C. High temperatures during flowering can cause embryo abortion (Ware and McCollum 1975).

P. *vulgaris* is usually planted in rows with intra-row spacing varying between 5 and 10 cm and inter-row spacing of 45 to 90 cm (Lorenz and Maynard 1988). Population densities therefore, are usually in the range of 11 to 39 plants per square meter. A planting depth of 2 to 5 cm is typical, and well aerated soils with good drainage are recommended (Pierce 1987). Beans form a symbiotic association with nitrogen fixing bacteria, and such soil conditions are excellent for nitrogen fixation, thus reducing beans requirement for nitrogen fertilizer.

P. *vulgaris* is mostly grown for fleshy pods and immature seeds (Peirce 1987). In some parts of the world, the tender shoots are used as a pot herb (Singh 1989). Pod harvesting occurs between about 45 and 90 days after planting (Shoemaker 1953). In a bush type, frequent harvesting is recommended in order to enhance greater pod formation before the plant reaches maturity (Yamaguchi 1983).

P. *vulgaris* can be susceptible to a number of fungal diseases (e.g. anthracnose (*Colletotrichum lindemuthianum* Sacc. & Magn.), angular leaf spot (*Isariopsis griseola* Sacc.), rust (*Uromyces phaseoli typica* Arth.), ascochyta leaf spot (*Ascochyta phaseolorum* Sacc.), downy mildew (*Phytophthora phaseoli* Thaxt.), viral diseases (e.g. bean curly top virus,

bean dwarf mosaic virus, bean yellow mosaic virus, bean golden mosaic virus) and bacterial diseases (e.g. common bacterial blight (*Xanthomonas phaseoli* E.F. Smith), halo blight (*Pseudomonas phaseolicola* Burk.). Insect pests, such as bean leaf beetle (*Cerotoma trifurcata*), can also be a major problem.

2.1.2 Beets

Beets, *Beta vulgaris* L., belong to the family Chenopodiaceae, the goosefoot family. The species *B. vulgaris* has four types: sugar beet, table beet, fodder beet and swiss chard (Whitney and Duffus 1986). In this thesis, beets or *B. vulgaris* will refer to only the table beet type. The plant originated from the Mediterranean region of north Africa, Europe, and west Asia (Ware and McCollum 1975). Beets have been cultivated since at least the third century, AD.

B. vulgaris is a biennial, and is grown in cultivation for storage roots and tops (Peirce 1987). The plant has a short and platelike stem, the crown. The leaves are simple in form and are arranged on the crown in a closed spiral (Ware and McCollum 1975). The color of the leaves may vary from dark red to light green (Ware and McCollum 1975). The "seed", which is in fact a fruit, may contain 2 to 6 true seeds (Shoemaker 1953, Ware and McCollum 1975). A monogerm type in which each fruit has only one seed, has also been released (Peirce 1987). The storage root is the result of swelling of the hypocotyl plus a small portion of the tap root. The swelling is caused by growth of several concentric vascular cambia which are visible as 'rings' when the roots are sectioned. Storage roots are usually red in color, but golden cultivars are not uncommon. The red color in *B. vulgaris* is due to betacyanin pigment, but roots also contain a yellow pigment,

betaxanthin (Peirce 1987). The main root system is a taproot that can grow to a depth of 3 m. A few lateral roots also tend to develop at the base of the swollen edible structure (Peirce 1987).

B. vulgaris is a cool season crop. It can tolerate both cold and hot temperatures, but the plant cannot withstand severe freezing (Shoemaker 1953). Germination occurs at soil temperatures between 10 and 29C, while exposure for 14 days or more to temperatures between 4 and 10C induces bolting at the expense of fleshy root development (Peirce 1987). Flower induction is also accelerated if the plants are exposed to long days (Peirce 1987). Beets can thrive on a range of soils but for optimum growth, slightly acid soils with pH between 6.0 and 7.0 are recommended (Shoemaker 1953, McCollum 1975). The water requirement for *B. vulgaris* varies depending on soil type, but adequate soil moisture during the entire growing season is required to maintain tender root tissues (Shoemaker 1953).

B. vulgaris is normally planted at a depth of 1.5 cm to 2.5 cm in rows 30 to 75 cm apart which are thinned after crop emergence to achieve a plant spacing of 5 to 10 cm within rows (Lorenz and Maynard 1988). This corresponds to population densities in the range of 13 to 67 plants per square meter. Fertilizer requirements vary with soil type and fertility. If soils are deficient in boron, application of boric acid at a rate of 9 to 36 kilograms per hectare is recommended to prevent internal black spot.

Harvesting of *B. vulgaris* depends on its intended use. Early in the growing season, the thinnings can be used as greens, or the crop can be harvested for bunching purposes when the roots are between 3 and 4 cm in diameter. The 'baby beet' is harvested when the roots are between 4 and 5 cm diameter. Roots are harvested for pickling and canning when

they are about 7 cm in diameter. The mature harvesting stage is when roots are between 7 and 10 cm in diameter. These roots have their shoots removed, and may be stored for several months after harvest (Shoemaker 1953). While small sized beet storage roots are marketed intact, larger sizes are used for sliced or diced products (Peirce 1987).

B. vulgaris is rarely attacked by disease but can have cercospora leaf spot (*Cercospora beticola* Sacc.) phoma leaf spot (*Phoma betae*), downy mildew (*Peronospora schachtii*) (Ruppel 1986a), and leaf curly top, a virus disease transmitted by beet leafhoppers (*Circulifer tenellus* Baker) (Ruppel 1986b). Insect pests such as beet leafhopper, webworm (*Loxostege sticticalis* L.), spinach leaf miner (*Pegomya hyoseyami* Panzer), cutworms and wireworms can be a problem too (Shoemaker 1953). Lack of boron may also cause black pitting, surface cankers, heart rot, or dry rot (Shoemaker 1953).

2.2 Crop Mixtures

Systems of agricultural plant production include both monospecific plant associations, referred to as monocultures, and associations of different plant genotypes, referred to as mixtures. Trenbath (1974) suggested that combinations of different cultivars, or different age classes of the same cultivar, also can be considered to be crop mixtures.

In agriculture, the use of monocultures offers certain advantages for crop management, including ease of mechanization, and relative simplicity of pest control practices (Beets 1982). Species mixtures, however, have the potential to exploit a greater range of environmental resources than can be utilized by a single species. Mixed pastures and forages are of worldwide importance, and are the best example of the use of crop mixtures in

developed countries. Otherwise, the use of crop mixtures is now prominent mainly in tropical and subtropical regions (Kass 1978). This type of agriculture has persisted in those regions because of advantages such as better utilization of environmental resources (Baker and Norman 1975), greater yield stability in variable environments (Beets 1982), reduced soil erosion (by rapidly providing vegetative cover, Beets 1982, Gomez and Gomez 1983), greater tolerance to disease and pests (where the severity of attack is proportional to host plant population density, Andrews and Kassam 1976), easier pest control in some crop mixtures (Andrews and Kassam 1976), better weed control, and sometimes better labour utilization (Baker and Norman 1975). It should be noted that intercropping may have disadvantages, such as: yield reduction due to adverse competition and allelopathic effects, and complexities in management, especially in cases where a high level of mechanization is essential (Willey 1979a & b).

Two systems exist for growing crop mixtures: mixed cropping and intercropping. Both involve the simultaneous growing of two or more crop species on the same piece of land (Mead 1979, Yunusa 1989). In intercropping, plants are grown in rows, and various arrangements of the species within and between the rows are possible. Mixed cropping, however, entails less organization; it involves species randomly mixed within rows or broadcasted together (Mead 1979). In some cases the terms intercropping and mixed cropping have been used interchangeably (Willey 1979a & b). The distinction between them may be significant, however, because the spatial arrangement of coexisting species within mixtures is sometimes of primary importance in determining mixture performance (Andrews 1972, Yunusa 1989, Mead 1979).

Plants in mixed cropping or intercropping systems need not be sown or harvested at the same time, but must grow together for a significant part of their growing period (Willey 1979a & b, Ofori and Stern 1987). Several types of mixed cropping/intercropping exist: row intercropping, i.e. the growing of mixtures in rows (Andrews and Kassam 1976); strip cropping, i.e. growing of mixtures in alternating strips or blocks on the same piece of land (Beets 1982, Trenbath 1974); relay intercropping, i.e. the coexisting species are not sown or harvested at the same time (Andrews and Kassam 1976); and patch intercropping, i.e. the growing of mixtures in patches (Papendick et al. 1976).

Crop mixtures are believed to have the potential to yield more than monocultures on an equivalent land area basis (Trenbath 1974, Andrews and Kassam 1976). This could be due to reduced interspecific competition compared to intraspecific competition, or it could be a result of the fuller exploitation of environmental resources due to niche differentiation among different species (Trenbath 1974). As a result, many different plant species have been grown in mixtures. Common mixture combinations include maize (*Zea mays*) associated with some members of the Leguminosae family including *Phaseolus vulgaris* (Harwood and Price 1976), maize with millet (*Setaria italica*) or cassava (*Manihot esculenta*) (Harwood and Price 1976), many forage/legume crop associations (Drolsom and Smith 1976), and sorghum (*Sorghum spp.*) in mixture with a legume or with millet (Kassam and Stockinger 1973, Norman 1974). Tree/tree and tree/annual crop associations are also common (Harwood and Price 1976). Other than the binary mixtures mentioned above, multi-species crop mixtures are also used (Norman 1974). Previous research on bean/beet intercropping, however, seems to be absent.

2.3 Plant Interference

2.3.1 Nature of Interference

Competition among organisms has been an important issue in biology. Darwinian expressions such as "struggle for existence" and "survival of the fittest" highlight the importance of competition and other types of interactions among organisms. Competition is important in both plant and animal associations, but it has proved difficult to advance a generally accepted definition for plant competition (Clements *et al.* 1929, Bleasdale 1960, Grime 1979, Begon and Mortimer 1981).

Grime (1979) emphasized the importance of environmental resources, defining competition as: "the tendency of neighbouring plants to utilize the same quantum of light, ion of mineral nutrient, molecule of water or volume of space." On the other hand, Bleasdale (1960) indicated that competition between two plants occurs when "the growth of either or both plants is reduced or modified as compared with their growth or form in isolation."

Harper (1961) criticized the use of the term competition to describe the overall aspects of interactions among neighbours, and suggested that the term should be abandoned and be replaced by other terms. Grime (1977), however, argued that competition is too useful a word and competition for resources is too important a mechanism to be discarded. Hall (1974a) pointed out that competition tends to be used to describe ecological and agronomic processes in a rather loose manner. Silvertown (1987) divided the different versions of definitions of the word competition into two categories: those that define the interaction among species on the basis of the mechanisms involved (e.g. Grime 1979), and those that define

the interaction in terms of the outcome between two competing species (e.g. Begon and Mortimer 1981, Bleasdale 1960).

Partly due to the difficulty in providing an acceptable definition for competition, some workers (Harper 1961, Hall 1974a & b, Trenbath 1974, Penney 1986) have preferred to use the term interference. The definition of interference was given by Harper (1961) as: "all responses of an individual plant or plant species to its total environment as this is modified by the presence and/or growth of other individuals." Thus, interference is a comprehensive term which encompasses both competitive and noncompetitive interactions among neighbours. A plant might interfere with its neighbours through competition for resources or in other ways, e.g. through allelochemistry or through influences on herbivores or pests. Harper's (1961) definition permits the possibility of beneficial interference, such as the promotion of pollinators, or sheltering of a plant from environmental stresses. Harper's (1961) concept of interference has two main advantages: (i) it is a term that does not imply a specific mechanism by which neighbours affect a plant's growth, and (ii) it directs attention to plant responses, which are the means by which the effects of neighbours can be assessed experimentally. Different mechanisms of interference may occur together and interact (Trenbath 1976, Harper 1977). Intraspecific interference can occur in monocultures, and among members of the same species within mixed crop associations. Interspecific interference occurs between plants of different species in mixtures.

Although competition for resources is only one potential component of interference, there is reason to believe that competition may often be of considerable importance. Many agronomic studies, for example, have

demonstrated strong improvements in crop growth with additional resource supply when population densities are high.

The effectiveness with which a particular plant competes for essential growth resources depends on a number of factors such as plant population density (Wiener 1984), plant arrangement (Yunusa 1989) and the ability of the particular plant or plant species to access resources in its vicinity (Watkinson 1980, 1984). At very low population density, plants may be too widely spaced to compete for resources and may grow as if they are in isolation (Trenbath 1974). This state can be approximated in a young crop, before plant size is sufficient to cause interference. As growth continues, however, expanding root and shoot systems lead to interference, and possibly competition for resources, among neighbours.

Despite competition and other detrimental components of interference among neighbours, coexistence among plant species is a common phenomenon in both natural and agricultural plant communities. Grubb (1977) has reviewed the different mechanisms by which plants coexist. Variation in competitive ability with age (Watt 1955), balanced mixtures (Marshall and Jain 1969), differences in life forms (Turkington 1975), phenological separation (Bratton 1976), and local variations in the environment (Thomas and Dale 1976) are some of the mechanisms known to play a part in plant coexistence.

2.3.2 Some Experimental Approaches to the Study of Plant Interference

Much of our present knowledge concerning plant interference has come from agricultural cropping systems and laboratory studies. This is because agricultural systems offer several experimental advantages

compared to natural plant communities: they contain simple plant populations, have a quick turnover through the use of annual crops, and they allow appropriate control treatments to be used. Also, in agricultural systems, the supply of resources can be partly controlled and/or managed, the populations under study are relatively uniform, the environment can be defined, and other experimental circumstances such as plant population density, plant arrangement, and timing of association are under some control of the experimenter (Radosevich and Holt 1984, Radosevich 1987, Snaydon 1980). Moreover, relevance to agricultural yield is another motivation for studying interference in agricultural cropping systems. There are limitations, however, of using agricultural systems for studies of interference. These include: the simplicity of agricultural studies limits the direct application of their results to complex natural situations; agricultural research has concentrated on annuals, biennials and other short lived perennials, and this may not relate well to long term natural associations; the scale of agricultural experiments is limited in extent; and, agricultural experiments have concentrated on yield rather than other biologically important outputs.

Many different approaches to studying plant interference and competition have been developed. Techniques of neighbourhood analysis have been used by Levin and Kerster (1971), Bella (1971), Trenbath and Harper (1973), Yeaton and Cody (1976), Mack and Harper (1979) Ford and Diggle (1981), Weiner (1984), and Cannell et al. (1984). These techniques take into account the importance of the pattern and arrangement of individuals in a population. This is important because it has been shown that the ability of an individual plant to exploit its environment depends on its position within the area defined by its neighbours, its time of

emergence, stage of development and its size relative to its neighbours (Ross and Harper 1972, Ford 1975).

The focus of neighbourhood analysis is on the individual plant and its immediate surroundings, and it can be applied in relatively complex circumstances. It is insufficient, however, simply to define a plant's neighbourhood. The quality of the target plant must also be considered because it is the balance between the target plant and its neighbours that will determine the fate of a competitive interaction.

Other approaches concentrate more on the collective performance of plant populations. Two experimental forms which commonly have been used to investigate interference in crop systems are the replacement series (de Wit 1960) and the additive series (Donald 1963). In binary replacement series experiments, proportions of two species in mixture are varied, but total density is held constant. In additive experiments, a constant density of one species is established with a variety of densities of another species.

The additive series has been useful for the study of weed-crop associations because it can assess how different weed population densities affect a crop at fixed population density (Dew 1972, Cousens 1985). It is a simple approach, but the effects of total population density and weed population density on the crop are confounded since both factors change together.

Replacement series have been used widely in competition studies. Results are often presented graphically as replacement diagrams in which the yield of each species is plotted against its proportion in mixture. Replacement diagrams have been used to indicate the stronger competitor and the degree of niche overlap between species (Khan *et al.* 1975). Replacement series have been used to study competition for specific

nutrients (Hall 1974b). The replacement series approach is not preferred in crop-weed studies because high weed proportions are not usual in crops. A drawback with replacement series is that the choice of total population density is arbitrary and could condition species performance, especially when species of different sizes are mixed. Results from replacement series have proved difficult to interpret. This is because the performance of species in mixtures has been interpreted on the basis of their performance in monocultures at the arbitrary chosen density. Jolliffe *et al.* (1984) criticized the original method of interpreting replacement series, developed by de Wit (1960), which is no longer used.

Some studies have incorporated both replacement and additive series together by repeating a replacement series experiment at a range of total population densities, a structure termed as an addition series by Spitters (1983). Addition series experiments embody the exploration of a range of proportions and densities of the mixed species, and they are favoured by contemporary researchers (Radosevich 1987, Radosevich and Roush 1990, Rejmanek *et al.* 1989).

Some other experimental forms used to study interference have controlled the spatial arrangement of plants. These include the honeycomb layout in which a test plant is surrounded by six equidistant plants (Martin 1973,). The neighbouring plants could either be of the same species or different species. Nelder (1962) and Bleasdale (1967) described experimental arrangements in which plant spacing was systematically varied, although these have not been used widely in recent years.

In his 1979 review, Mead (1979) indicated that new approaches need to be developed for this field of study. Replacement and additive series, as well as systematic designs, are forms of yield-density experiments. During

the past decade, general yield-density studies have proved to be useful in investigating interference, as discussed in the following section.

2.3.3 Yield-density Relationships

Defining the relationships that exist between population density and yield has been of great concern to plant scientists (Kira *et al.* 1956, Shinozaki and Kira 1956, Holliday 1960, Bleasdale and Nelder 1960, Bleasdale and Thompson 1966, Mead 1966, Gillis and Ratkowsky 1978, Vandermeer 1984). In addition to the agronomic need to define appropriate densities for crop production, mathematical models of yield-density relationships can be used to express plant interference.

Models of yield-density relationships can be divided into two groups: those that describe plants in monocultures and those that describe plants growing in mixtures. Willey and Heath (1969) thoroughly reviewed early attempts to construct yield-density models, which form the basis of current models. Yield-density models are of greatest value when their parameters possess meaning relevant to the biology of plant growth and interference.

2.3.4 Monoculture Models

Monoculture yield-density data have often been successfully described using mathematical equations (Kira *et al.* 1953, Holliday 1960, Mutsaers 1989). Many of these yield-density models have proved to be asymptotic in that an increase in population density leads to an increase in yield per unit land area until an upper limit is reached at high population densities. Typical of asymptotic relationships are data of total shoot biomass or other measures of vegetative parts. Parabolic yield-

density relationships have also been encountered, particularly with data from reproductive yield such as grain or seed. It is preferable to model the relationship between population density (X) and yield per plant (y), rather than yield per land area (Y), because the latter combines dependent and independent variables (Y = yX).

Many yield density models were reviewed by Willey and Heath (1969). Among these are reciprocal yield density equations which can describe both asymptotic and parabolic yield-density relationships. These models seem to have been widely accepted because they can be derived from basic concepts of interference (Jolliffe 1988), they offer a potentially powerful approach to data interpretation, and they contain parameters which seem to have biological relevance (Jolliffe 1988, Rejmanek *et al.* 1989).

Reciprocal yield-density models were first applied empirically by Kira *et al.* (1953) and Shinozaki and Kira (1956). A simple functional relationship between mean shoot dry weight per plant and population density was expressed as:

$$y_i^{-1} = a_i + b_{ii}X_i \qquad (2.1)$$

where y_i is the mean yield per plant and X_i is plant population density. Parameter **a** expresses the reciprocal mean yield of an isolated plant while the parameter **b** expresses the strength of intraspecific competition. This equation can only describe an asymptotic yield-density relationship.

Holliday (1960) also developed a reciprocal model and extended the above relationship to include parabolic yield-density responses by adding a quadratic term.

$$y_i^{-1} = a_i + b_{ii}X_i + b_{ii}'X^2 \tag{2.2}$$

Again parameter **a** represents the inverse of mean yield of an isolated plant and b_{ii} and b_{ii}' are expressing intraspecific interference. When $b_{ii} = 0$, then there is an asymptotic relationship; when b_{ii} is greater than zero the relationship is parabolic.

Bleasdale and Nelder (1960) modified Kira *et al.*'s (1953) equation to give a more general fit to yield-density data:

$$y_i^{-\Phi} = a_i + b_{ii}X_i^{\Phi'} \tag{2.3}$$

the ratio of Φ and Φ' determine the form of the yield-density relationship. The relationship is parabolic when Φ is less than Φ', and it is asymptotic when $\Phi = \Phi'$. Since the ratio Φ to Φ' is the main factor influencing Bleasdale and Nelder's equation, Bleasdale (1966) simplified the equation by setting Φ' equal to unity. Thus the equation becomes:

$$y_i^{-\Phi} = a_i + b_{ii}X_i \quad \text{or} \quad (y_i = (a_i + b_{ii}X_i)^{-1/\Phi}) \tag{2.4}$$

Vandermeer (1984) argued that Bleasdale (1966) could equally have suggested setting Φ equal to unity in which case the equation:

$$y_i^{-1} = a_i + b_{ii}X_i^{\Phi'} \quad \text{or} \quad (y_i = (a_i + b_{ii}X_i^{\Phi'})^{-1}) \tag{2.5}$$

emerges, which according to Vandermeer (1984) has a satisfying biological interpretation. The parameter b_{ii} relates to the area and intensity of the competitive interaction, while Φ' relates to the rate at which the intensity of

competition decays as a function of interplant distance. He achieved this by altering the assumption of equal competition within a specified region for the classical yield-density relationship. He replaced this assumption with that of variable competition.

Another reciprocal model equivalent to equation 2.4 was put forward by Watkinson (1980, 1984):

$$y_i = y_{max}(1 + a_{ii}'X_i)^{-b'_i} \tag{2.6}$$

where $a_{ii}' = a_{ii}^{-1}b_i$, $b_i' = \Phi^{-1}$, and $y_{max} = a_i^{-1/\Phi}$. Watkinson (1980, 1984) attached biological meanings to model parameters: parameter y_{max} is the yield of an isolated plant, a_{ii}' is the population density at which interference among neighbouring plants begins to be present, and b' is a measure of the efficiency of resource acquisition from the area surrounding the plant.

Bleasdale and Nelder's (1960) equation (equation 2.4) has been widely used in analyzing yield-density data (Gillis and Ratkowsky 1975, 1978), but it has been found to produce biased estimations in cases where the data were non-normally distributed (Gillis and Ratkowsky 1978). It has also been criticized for being difficult to give a simple biological interpretation of parameters **a** and **b** when the model describes a parabolic relationship (Vandermeer 1984, Watkinson 1980). In an asymptotic situation, as population density approaches zero, the value of y approaches 1/**a**; thus, the reciprocal of **a** can be used as a measure of a species' genetic potential in a certain environment. Similarly, as the population density approaches infinity, yield per plant approaches the asymptotic value of 1/**b**, and the inverse of **b** can be used as a measure of

environmental potential. Gillis and Ratkowsky (1978) indicated that in the parabolic relationship where Φ is less than 1, the biological meaning for the parameters **a** and **b** are confounded with the effects of Φ. They pointed out that $\mathbf{a}^{-(1/\Phi)}$ measures genetic potential in a parabolic situation. Gillis and Ratkowsky (1978) reparameterized Bleasdale and Nelder's (1960) equation. A simple biological interpretation to all of their new parameters, however, is not clear.

2.3.5 Mixture Models

Wright (1981) and Spitters (1983) extended the inverse monoculture yield density model to a two species system. They expressed a pair of equations as:

$$y_{ij}^{-1} = \mathbf{a}_i + \mathbf{b}_{ii}X_i + \mathbf{b}_{ij}X_j \qquad (2.7)$$

$$y_{ji}^{-1} = \mathbf{a}_j + \mathbf{b}_{jj}X_j + \mathbf{b}_{ji}X_i \qquad (2.8)$$

where the first subscript corresponds to the species whose biomass is represented as the dependent variable (*i.e.* the test species) and the second subscript identifies the associated species. The coefficients $\mathbf{b}_{ii}$ and $\mathbf{b}_{jj}$ measure effects of intraspecific competition. The coefficients $\mathbf{b}_{ij}$ and $\mathbf{b}_{ji}$ measure the effects of the associated species on the test species. Thus, the coefficients formally separate intra- and interspecific competition.

Watkinson's (1981) reparameterized equation was also extended to include a binary mixture situation (Firbank and Watkinson 1985) that corresponds to:

$$y_{ij}^{-\Phi_i} = \mathbf{a}_i + \mathbf{b}_{ii}X_i + \mathbf{b}_{ij}X_j \qquad (2.9)$$

$$y_{ji}^{-\Phi_j} = a_j + b_{jj}X_j + b_{ji}X_i \qquad (2.10)$$

Jolliffe (1988) indicated that the model could potentially be extended to include interaction terms and higher order polynominals, for example

$$y_{ij}^{-\Phi_i} = a_i + b_{ii}X_i + b_{ij}X_j + c_{ii}X_iX_i + c_{ij}X_iX_j + c_{jj}X_jX_j \qquad (2.11)$$

Potentially, these same models can be extended to multiple species systems where more than two species are intercropped though so far few experiments with more than two species have been done (Jolliffe 1988, Rejmanek *et al.* 1989).

The above equations have been fitted by normal multiple regression procedures. When population density treatments X_i and X_j are correlated, Jolliffe (1988) proposed an alternative approach of fitting the regression in stages. Jolliffe (1988) also showed the applicability of such models to the interpretation of differential yield responses.

2.4 Differential Yield Responses

Overyielding in intercrops in relation to the corresponding monocultures is of central importance in mixed cropping. Several indices to determine the performance of crop mixtures have been suggested and were reviewed by Potdar (1986). Jolliffe (1988) demonstrated the use of yield-density models for the interpretation of differential yield responses.

Land equivalent ratio (LER) is a useful index of the combined performance of species in binary mixtures (Willey and Osiru 1972). This index is calculated from:

$$LER = (Y_{ij}/Y_{ii}) + (Y_{ji}/Y_{jj}) \qquad\qquad (2.12)$$

where Y represents yield per unit land area, the first subscript designates the species providing the data for Y and the second subscript indicates the companion species. *i.e.* Y_{ij} indicates the yield per unit land area of species i grown in mixture with species j. Similarly Y_{jj} is the yield per unit land area of species j in monoculture. In this evaluation, the mixtures and monocultures are assessed at the same total population density. There are three possible outcomes from a mixed crop (Willey 1979a & b): mutual inhibition, (LER < 1) mutual cooperation (LER > 1) and mutual compensation (LER = 1).

In equation 2.12 Y_{ii}, Y_{ij}, Y_{jj} and Y_{ji} are expressed on a per unit land area basis. Since $Y_{ii} = y_{ii}X_i$, equation 2.4 can be converted to a unit land area basis as:

$$Y_{ii} = y_{ii}X_i = X_i(a_i + b_{ii}X_i)^{-1/\Phi_i} \qquad\qquad (2.13)$$

for monoculture yield per unit land area of species i. Similarly,

$$Y_{ij} = y_{ij}X_i = X_i(a_i + b_{ii}X_i + b_{ij}X_j)^{-1/\Phi_i} \qquad\qquad (2.14)$$

expresses yield of species i per unit land area in the presence of species j. The same relationships can be formed for species j to express Y_{jj} and Y_{ji}.

Let $D = X_i + X_j$, *i.e.* D represents the total population density of the mixture. Then:

$$X_i = D - X_j \qquad (2.15)$$

It follows that (Jolliffe 1988):

$$Y_{ij} = y_{ij}X_i = X_i(\mathbf{a}_i + \mathbf{b}_{ii}(D - X_j) + \mathbf{b}_{ij}X_j)^{-1/\Phi_i}$$
$$= X_i(\mathbf{a}_i + \mathbf{b}_{ii}D - (\mathbf{b}_{ii} - \mathbf{b}_{ij})X_j)^{-1/\Phi_i} \qquad (2.16)$$

Similarly:

$$Y_{ji} = y_{ji}X_j = X_j(\mathbf{a}_j + \mathbf{b}_{jj}D - (\mathbf{b}_{jj} - \mathbf{b}_{ji})X_i)^{-1/\Phi_j} \qquad (2.17)$$

Combined mixture yield will therefore be modelled by the sum of equation 2.16 and 2.17. From equation 2.13 monoculture yield per unit land area at $X_i = D$ is given by

$$Y_{ii} = X_i/(\mathbf{a}_{ii} + \mathbf{b}_{ii}D)^{-1/\Phi_i} \qquad (2.18)$$

Comparison with equation 2.16 indicates that the differential yield response for one of the species is controlled by the difference: $\mathbf{b}_{ii} - \mathbf{b}_{ij}$. If intraspecific interference is greater than interspecific interference, *i.e.* $\mathbf{b}_{ii} - \mathbf{b}_{ij}$ is positive, then this is subtracted from $(\mathbf{a}_i + \mathbf{b}_{ii}D)$, which reduces the denominator of equation 2.14, hence increasing yield per unit land area.

2.5 Size Hierarchies

Plant populations often contain a few large individuals and many suppressed small individuals (Weiner 1984), *i.e.* the size distribution is positively skewed. This can develop in even-aged monoculture populations

which begin with a normal frequency distribution of seedlings. In such a skewed distribution, a few dominant individuals represent most of the biomass, while the numerous suppressed individuals contribute only a small portion of the biomass (Stern 1965, Weiner 1985, Weiner and Thomas 1986). Weiner and Solbrig (1984) have referred to such distributions as size hierarchies or size inequalities. Variations in size have ecological and evolutionary significance because small individuals are likely to suffer density-dependent mortality while large plants are more likely to contribute to future generations (Harper 1977). Also, size variations have commercial implications in cases where uniformity of produce is desirable.

Plant size inequalities are the result of variation in plant growth rates which are caused by competition (Rejmanek et al. 1989) and other factors, such as: genetic variation (Bonan 1988), seed size, order of seedling emergence, age differences (Ross and Harper 1972), environmental non-uniformity (Hara 1984a & b), neighbourhood effects (Hara 1984a & b), parasites, herbivores, pathogens and interactions among these factors (Weiner and Thomas 1986). Competition seems to be a major contributor, and efforts have been made to classify its effects. Two types of effects have been distinguished: symmetric effects in which competition is thought to act on all individuals in proportion to their sizes, thus reducing variation in growth rate and size inequalities (Weiner 1985, Connolly 1986, Weiner and Thomas 1986); and asymmetric effects where differences in growth rate are amplified by disproportionate sharing of available resources so that large plants utilize more resources and deprive the small individuals (Ford 1975, Bonan 1988, Weiner 1985).

The importance of competition in causing size hierarchies was demonstrated by Bonan (1988). He found that size structure was partly due to spatial distribution and availability of resources within a stand. On the other hand Weiner (1985), working with *Lolium* and *Trifolium*, found that spatial pattern was of minor importance in causing inequalities. Weiner (1985) also found that size inequalities always increased with increasing population density. In a mixture of *Lolium* and *Trifolium*, he found that the dominant species, *Lolium*, exhibited less size inequality than in monocultures, whereas the subordinate species, *Trifolium*, had greater size inequality in mixtures. The inequalities were greater for reproductive dry matter than for overall shoot weight, and these findings represent an asymmetric effect. Thus, competition is one-sided, operating more strongly on small than on large individuals (Weiner and Thomas 1986). In Weiner's (1985) study, interference also caused decreased mean plant mass, increased relative variation in plant mass and increased concentration of mass within a small fraction of the population.

2.6 Plant Growth Analysis

If interference is the response of plants to their neighbours (Harper 1961), then progress in understanding interference may occur through the documentation of those responses. Measurement of final crop yield or seasonal production can indicate the agronomic results of interference. Inferences concerning processes of interference, however, could be formed from knowledge of the timing and sites of responses within plants. Methods of plant growth analysis offer one means to provide such knowledge.

Formal methods of plant growth analysis date from the early work of Gregory (1918), Blackman (1919), Briggs *et al.* (1920a, 1920b) and Engeldow and Wadham (1923). Modern reviews of the subject were done by Evans (1972), Causton and Venus (1981) Hunt (1982) and Fraser and Eaton (1983). Methods of growth analysis are an aid in the quantitative interpretation of growth variation. One advantage they possess is their requirement for simple input data which can be collected in field experiments, such as leaf areas and component dry weights.

Several variants of plant growth analysis exist. Conventional plant growth analysis involves growth indices such as relative growth rate, leaf area ratio and unit leaf rate. Those indices are obtained from a series of observations of leaf areas and dry weights during the course of growth. The 'classical' form of conventional plant growth analysis computes growth indices from observations made at pairs of harvests; the 'functional' approach to conventional plant growth analysis computes growth indices from growth curves fitted to data from a set of harvests (Hunt 1982). Conventional plant growth analysis is helpful in assessing how plant performance is dependent on growth rates, persistence of growth, and dry matter partitioning (Jolliffe *et al.* 1982). Much attention has been given to methods of fitting growth curves; polynomials, splined polynomials and the Richards function are growth functions commonly used today (Hunt 1982, Causton and Venus 1981).

Sub-organismal demographic analysis is a second major variant of plant growth analysis. It was introduced by Bazzaz and Harper (1977), and it applies demographic concepts to the population of components which exist within individual plants. Issues such as the appearance ('births'), abundance, disappearance ('deaths'), lifetimes, functional

histories ('fates') and prominence of plant components are assessed. Sub-organismal demographic analysis has not yet been widely applied to plants but is becoming widely used by zoologists working with sessile organisms, and it is most useful for plants which produce large numbers of similar components (Hunt and Bazzaz 1980).

In a statistical sense, 'yield' can be used to indicate any particular dependent variable. In the agronomic sense, which will be the predominant usage in this thesis, 'yield' denotes some particular output of plant growth, such as seed or fruit. The third major variant of plant growth analysis, yield component analysis, takes yield to be the mathematical product of a set of yield components (Engeldow and Wadham 1923). Yield components are in turn formed as ratios from measures of morphological constituents of the plant (Fraser and Eaton 1983). Thus, yield component analysis is concerned with the contributions of yield components to variation in yield, and the relationships among yield components.

Yield component analysis has been used extensively in improving the grain yield in rice (Matsushima *et al.* 1964 Matsushima 1966, 1976, 1980, Ishizuka 1971, Yoshida 1972, Yoshida and Parao 1972, Cock and Yoshida 1973, Yoshida 1973a & b, Murata and Matsushima 1974, Murayama 1979). Complex relationships can exist among yield components (Siefker and Hancock 1986). Matsushima (1966) reported that the components can act in parallel, in opposition, or sometimes may control each other, thereby compensating for either increases or decreases in other components. Many statistical procedures have been used to explore such relationships (Fraser and Eaton 1983). A relatively new procedure, the analysis of yield component relationships by two dimensional partitioning

(TDP), combines multiple regression procedures with the analysis of variance (Eaton *et al.* 1986). Other workers (Siefker and Hancok 1986, Hancock *et al.* 1984, 1983) have analysed yield component relationships using path coefficient analysis (Li 1956). This procedure involves standardization of the regression coefficients so that the degree of influence of the independent variable on the dependent variable is unrelated to physical units.

Thus, conventional plant growth analysis, suborganismal demographic analysis, and yield component analysis treat different aspects of plant growth. The three approaches were developed independently, but in recent years they have been linked with each other and can be considered to form different branches of the general field of plant growth analysis (Jolliffe and Courtney 1984). Hunt (1980) and Hunt and Bazzaz (1981) showed that parallel analyses could be performed at the sub-organismal level using demography and conventional growth analysis. Jolliffe *et al.* (1982) linked conventional plant growth analysis and yield component analysis. Further connections among all three approaches were demonstrated by Jolliffe and Courtney (1984).

The application of plant growth analysis in studying yield-density relationships in mixtures has been minimal. Roush and Radosevich (1985) used plant growth analysis to characterize the competitiveness of four annual weed species. Few other workers have exploited these analytical procedures in intercropping studies (Potdar 1986, Jolliffe *et al.* 1988).

2.7 Allometry

Allometric relationships can be defined as quantitative relationships that exist among different features of an organism as growth proceeds

(Jolliffe *et al.* 1988). Mathematical models have been used to describe such allometric relationships, particularly the power function for bivariate allometry popularized by Huxley (1932):

$$y = \alpha z^{\beta} \tag{2.19}$$

where y and z are two measures of an organism, or part of an organism, parameter α is the allometric coefficient and parameter β is the allometric exponent.

It should be noted that α and β express the proportionality between y and z, as can be seen by dividing the above equation by z. Also, the power function is an empirical relationship, although the allometric exponent is related to the relative growth rates of y and z (Whitehead and Myerscough 1962). Hence, concepts of allometry are relevant to plant growth analysis. Parameter α is dependent on the arbitrary choice of scale of measurement for z since it is the value of y when z equals 1.0.

The power function equation is linearized by transforming it to $\log_e$ scale:

$$\log_e(y) = \log_e(\alpha) + \beta\log_e(z) + \log_e(\varepsilon) \tag{2.20}$$

where $\log_e(\varepsilon)$ has been added to account for residual variation in $\log_e(y)$ not accounted for by $\log_e(\alpha)$, β and $\log_e(z)$. Jolliffe *et al.* (1988) expanded this model to detail allometric responses to experimental treatments *i.e.* experimental treatments can affect $\log_e(y)$ through allometric adjustments, via changes in α and/or β, or through non-allometric adjustments, via ε.

For two experimental treatments (e.g. X_i and X_j), their effect on α, β and ε can be expressed as:

$$\log_e(\alpha) = \log_e(\alpha_0) + \delta_1\log_e(\alpha_1 X_i) + \delta_2\log_e(\alpha_2 X_j) + \delta_3\log_e(\alpha_3 X_i X_j) \qquad (2.21)$$

$$\beta = \beta_0 + \beta_1 X_i + \beta_2 X_j + \beta_3 X_i X_j \qquad (2.22)$$

$$\log_e(\varepsilon) = \log_e(\varepsilon_0) + \xi_1\log_e(\varepsilon_1 X_i) + \xi_2\log_e(\varepsilon_2 X_j) + \xi_3\log_e(\varepsilon_3 X_i X_j) \qquad (2.23)$$

The treatment effects are expressed through values of δ_k, β_k ξ_k where $k > 0$. Terms that are difficult to separate can be grouped as follows:

$$\log_e(\alpha') = \log_e(\alpha_0) + \delta_1\log_e(\alpha_1) + \delta_2\log_e(\alpha_2) + \delta_3\log_e(\alpha_3) \qquad (2.24)$$

$$\log_e(\varepsilon') = \log_e(\varepsilon_0) + \xi_1\log_e(\varepsilon_1) + \xi_2\log_e(\varepsilon_2) + \xi_3\log_e(\varepsilon_3) \qquad (2.25)$$

$$\gamma_k = \delta_k + \xi_k \text{ (where } k = 1, 2, \text{ or } 3) \qquad (2.26)$$

Treatment effects on terms having β_k have been separated from other treatment effects which have been expressed by terms containg γ_k. Thus equation 2.20 expands to:

$$\log_e(y) = \log_e(\alpha') + \beta_0\log_e(z) + \beta_1 X_i\log_e(z) + \beta_2 X_j\log_e(z) + \beta_3 X_i X_j\log_e(z) +$$
$$\gamma_1\log_e(X_i) + \gamma_2\log_e(X_j) + \gamma_3\log_e(X_i X_j) + \log_e(\varepsilon') \qquad (2.27)$$

In their work with orchardgrass (*Dacytylis glomerata* L.) and timothy (*Phleum pratense* L.), Jolliffe *et al.* (1988) found that population density

treatments changed allometric exponents. This adjustment changed as growth proceeded and each species responded differently.

2.8 Objectives of the Thesis

Interference among associated plants, in monocultures and mixtures of different plant species is an active area of interest and study to both agronomists and ecologists. Methodologies for measuring interference and for accounting for yield responses have advanced rapidly during the past decade.

As specified in the introduction, this study was done to measure interference in defined populations of beans and beets, and to explore how interference arose by detailing the timing and sites of plant responses. These crops were chosen because: (1) both crops grow well in British Columbia, (2) recommended plant spacings for both crops overlap, (3) both crops mature at about the same time, (4) they provide an interesting contrast between a crop whose yield depends on above ground reproductive development (bean) and one which depends on vegetative growth (beet), and (5) they provide a contrast between a nitrogen fixing crop (bean) and a non-fixing crop (beet). This study has the potential, therefore to address questions about plant interference in terms of both practical consideration of growing beans and beets, and the academic purpose of trying to better understand interference in simple plant associations.

3. MATERIALS AND METHODS

3.1 Experimental Layout and Crop Production Procedures

Field experiments were performed at the Totem Park Field Station of the University of British Columbia, Vancouver, Canada in the summers of 1984 and 1987 on a sandy loam soil with pH 6.1. The 1984 experiment was designed and conducted by Potdar (1986). Two plant species, *Phaseolus vulgaris* L. cv 'Topcrop' and *Beta vulgaris* L. cv 'Ruby Queen' were used in both years. The experimental design was a randomized complete block with 20 treatments randomized within each block. The 20 treatments were made up of 4 planting densities (66, 50, 33, 16 plants m^{-2}) of each species, and 5 mixtures at each density. The mixture proportions varied between 0 and 100% in uniform steps. Table 3.1 shows the treatment combinations in both replacement series (de Wit 1960) and additive series (Donald 1963) present within each block. In 1984, there were 3 blocks, and in 1987 there were 2 blocks with treatments replicated twice in each block.

In 1984, individual plots were 14.6 m long and 3.2 m wide, while in 1987 plots varied in size between 3.2 m and 14.6 m in length and were all 3.2 m wide. The less dense plots were longer than the highly populated plots as shown in Table 3.2. Before seeding, the experimental area was sprayed with Dazomet (BASF Basamid Granular 90%) at a rate of 400 kg/ha in 1983 and with glyphosate at the rate of 5 L/ha in 1987 for weed control. In both years, the experimental areas were also fertilized before seeding with ammonium phosphate (11:50:0) at the rate of 224 kg/ha. Seeding was done by hand between May 30 and June 3 in 1984 and between June 1 and June 4 in 1987. Both beans and beets were planted at 0.45 m between row spacing in both years with varying within row

Table 3.1   Treatment combinations of beans and beets

Total Population Density (plants m^{-2})	Mixture Proportions				
	0:4	1:3	2:2	3:1	4:0
	Species Population Densities (plants m^{-2})				
16	00:16*	04:12	08:08	12:04	16:00
33	00:33	08:25	16:16	25:08	33:00
50	00:50	12:37	25:25	37:12	50:00
66	00:66	16:50	33:33	50:16	66:00

*Species population densities, Beans:Beets.

Table 3.2   Population densities and plot sizes in the 1987 experiment

Plants m^{-2}	m of Row Length per Plant*	Plot Size m
4.17	0.600	3.2 x 14.6
8.34	0.300	3.2 x 10.1
12.50	0.200	3.2 x 10.1
16.67	0.150	3.2 x 3.2
25.00	0.100	3.2 x 3.2
33.33	0.075	3.2 x 3.2
37.50	0.067	3.2 x 3.2
50.00	0.050	3.2 x 3.2
66.67	0.038	3.2 x 3.2

*Interrow spacing was 0.45 m in all plots

spacing depending on plant density and mixture proportion (Table 3.2). To facilitate planting, the same seeding depth (2 cm) was used for both beets and beans. Thinning was done when shoots were about 5 cm tall. In both years, experimental areas were weeded by hand throughout the growing period. Occasional irrigation was done as required to supplement natural rainfall.

3.2 Harvests and Primary Data Collection

In 1984, six harvests, performed at 40, 51, 63, 69, 75, and 92 days after planting were done for both beans and beets. A seventh harvest was included at 107 days after planting for beets. In 1987, only one harvest at the end of the season was done for both plant species: beans were harvested between August 21 and August 23, and beets were harvested between August 27 and August 31. In 1984, 5 plants per species per harvest were collected from each plot, and in 1987 15 plants were taken. For beans, shoots were cut to ground level, while for beets, the shoots and major roots were pulled from the ground. The harvested material was taken to the laboratory where measurements were begun before plant material wilted. Heights of individual plants for each species were determined using a ruler. Measurements from the base of the stem to the apex of the longest leaf were recorded. Each plant was then subdivided into components: leaves, stem, pods, and flowers for beans, and leaves, petioles and storage roots for beets. Live and dead leaves were counted, and the area of live leaves per plant was measured using an LI-COR LI-3000 leaf area meter. Pods were graded into marketable and unmarketable grades, as described in section 3.3, and the numbers in each category were recorded. Fresh weights per plant of marketable pods

and storage root were recorded for beans and beets respectively. In beets, storage root diameters were also determined. Dry weights per plant of all components were obtained after drying the material in a forced air oven at 75C. Duration of drying was 4 days, except for beet storage roots which were left for 7 days or more until constant weights were approached. After obtaining the bean pod dry weights, seeds were removed and their numbers and dry weights were determined.

3.3 Grading Procedure for Bean Pods

Pods were graded into marketable and unmarketable categories by using holes cut through a Plexiglas template as guidelines. Pods from individual plants were sieved through holes of different diameters, and the following grades were obtained:

Grade I - Pods which passed through 6.6 mm diameter hole (< 6.6 mm)

Grade II - Pods which passed through 9.1 mm diameter hole (6.6-9.05 mm)

Grade III- Pods which do not pass through 9.1 mm diameter hole (>9.1 mm)

These grades were condensed from Canadian government standards. The pod size range for the above grades are as follows:

Range	Average	Sieve Size
5.8-7.4 mm	6.6 mm	2
7.4-8.4 mm	7.9 mm	3
8.4-9.7 mm	9.1 mm	4
9.7-10.7 mm	10.2 mm	5
>10.2 mm	>10.2 mm	6

Sieve sizes 2 to 4 are considered to be marketable pods in British Columbia.

3.4 Analytical Procedures

3.4.1 An Overview of the Data Analysis

In outline, the data analysis included several steps:

(i) Analysis of variance (ANOVA), which was carried out to determine the occurrence of significant effects due to density and mixture treatments.

(ii) Overall yield was analyzed next using non-linear regression models developed to describe yield-density relationships in both monocultures and mixtures. The models were then used to interpret competitive performance of beets and beans as well as the differential yield responses of the mixtures.

(iii) The Gini coefficient (Weiner and Solbrig 1984) was computed on the 1987 data to determine the degree of size inequality among individuals of different characteristics within treatments.

(iv) The contributions of yield components to total yield variation, and the relationships among yield components, were determined using two dimensional partitioning (TDP), a procedure involving a combination of the analysis of variance and stepwise multiple regression analysis.

(v) Conventional plant growth analysis was done on the 1984 data to detect the timing and sites of treatment effects and the responses of physiologically relevant measures of plant performance.

(vi) The effects of population density and mixture proportions treatments on quantitative relationships between different measures of plant growth were explored by allometric analysis.

Through these procedures, the overall effects of experimental treatments on each species are defined, and some of the relationships underlying yield of each species are detailed.

3.4.2 Analysis of Variance

A partitioned Layard's homogeneity of variance test was done on the raw data before conducting an analysis of variance. Based on the homogeneity of variance test results, the data were transformed to a $\log_{10}$ scale, and the analysis of variance was then done on the transformed data to test for significant differences among treatments.

3.4.3 Yield-density Relationships

3.4.3.1 Intraspecific Interaction

In the first stage in developing these relationships, total dry weight, leaf dry weight, leaf area and live leaf number data for each species in pure stand (monoculture) were analyzed by fitting a simple reciprocal model (equivalent to equation 2.4):

$$(y_{ii})^{-\Phi_i} = a_i + b_{ii}X_i \qquad (3.1)$$

The P:9R BMDP statistical program (Dixon 1985) was used to fit the monoculture regressions.

3.4.3.2 Interspecific Interaction

Parameter estimates obtained in the monoculture yield-density regressions were used in a second stage to determine interspecific

interference in the bean-beet mixtures (Jolliffe 1988). This additional, interspecific, interference is measured by the mixture yield-density model:

$$(y_{ij})^{-\Phi_i} = a_i + b_{ii}X_i + b_{ij}X_j \tag{3.2}$$

This model is an extension of the monoculture model (3.1). In the mixture yield-density model, a_i, b_{ii} and Φ_i were fixed at values obtained in the monoculture yield-density model using the monoculture yield data. Values for b_{ij} were then obtained from a linear regression of residuals obtained after fitting the monoculture model, $i.e$ the difference between $y_{ij}^{-\Phi}$ and the value of $y_{ii}^{-\Phi}$ predicted from model 3.1 was regressed against X_j, with no constant being formed in that regression. The assumption used in this second stage in developing the model for y_{ij} is that there will be no difference between the yield in mixtures, $y_{ij}^{-\Phi}$, and the yield of that species in monocultures, $y_{ii}^{-\Phi}$, when interspecific interference, b_{ij}, is zero.

3.4.4 Differential Yield Response

The predicted combined yield per unit land area for bean-beet mixtures for total dry weight, leaf dry weight, leaf number and leaf area at harvest 6 was described by using the equation (Jolliffe 1988):

$$LER = \{(X_i(a_i + b_{ii}D - (b_{ii} - b_{ij})X_j)^{-1/\Phi_i})/(D(a_{ii} + b_{ii}D)^{-1/\Phi_i}\} + \{X_j(a_j + b_{jj}D - (b_{jj} - b_{ji})X_i)^{-1/\Phi_j})/D(a_{jj} + b_{jj}D)^{-1/\Phi_j} \tag{3.3}$$

LER stands for land equivalent ratio, and D represents total population density in mixtures $i.e.$ $(X_i + X_j)$. The predicted values of LER were compared with the observed values which were calculated from equation:

$$LER = (Y_{ij}/Y_{ii}) + (Y_{ji}/Y_{jj}) \qquad (3.4)$$

which is identical to equation 2.12

3.4.5 Plant Hierarchies

Plant inequalities were determined using the Gini coefficient (Weiner and Solbrig 1984), a measure of inequality. This was calculated using the equation:

$$G = \sum_{p=1}^{n} \sum_{q=1}^{n} |s_p - s_q| / 2n^2 \mathbf{s} \qquad (3.5)$$

where G, the Gini coefficient is the arithmetic average of the absolute values of the differences between all possible pairs of individuals, s represents the size of individuals in the sample, $\mathbf{s}$ is the sample mean, p and q are subscripts denoting all pairs of individual observations and n is the sample size. Since the calculated G for a small sample is a biased estimator of the population's G, sample G's were multiplied by $n/(n-1)$ to give unbiased estimates of the population Gini coefficient G'. Standard error estimates were obtained from a bootstrapping procedure (Efron 1981, 1982).

3.4.6 Yield Component Analysis

Primary variates for beans were leaf number (LN), leaf area (LA), leaf dry weight (WL), stem dry weight (WST), pod number (PN), and pod dry weight (WPD). For beets, the primary variates were LN, LA, WL, root diameter (DR), and root dry weight (WR). Ratios of the primary variates i.e. yield components, were constructed, after arranging the variates in an

assumed chronological sequence of development. The sequence was based on the order of development of the components during plant growth *i.e.* leaves appear and expand in area, then they accumulate dry matter which is translocated to the stem and pods in beans or to the storage roots in beets. The overall yield component models were as follows:

Beans: Y = LN X (LA/LN) X (WL/LA) X (WST/WL) X (PN/WST) X
        (WPD/PN)   (3.6)

where Y = WPD.

Beets: Y = LN X (LA/LN) X (WL/LA) X (DR/WL) X (WR/DR)   (3.7)

where Y = WR.

To measure the contribution of each yield component to variation in yield, and to assess the effects of treatments on these variates, two dimensional partitioning was done on the data. This procedure involves a combination of stepwise multiple regression analysis and analysis of variance (Eaton *et al.* 1986). Both the forward analysis, in which the components are entered into the regression the way they appear in equations 3.6 and 3.7, and the backwards analysis, in which the components are entered into the regression in the reverse sequence, were performed on the data.

3.4.7 Plant Growth Analysis

Plant growth analysis was done to measure the quantitative performance of plants, or plant parts, as affected by treatments during the entire growing period. Plant growth curves were fitted to the 1984 data over time for leaf area per plant, leaf dry weight per plant, leaf area ratio, leaf weight ratio, and harvest index using a cubic spline regression technique (Jolliffe and Courtney 1984). Growth indices commonly used in functional plant growth analysis were then computed from the fitted curves. This study involved 16 combinations of population density and mixture proportions (Table 3.1). In order to simplify the presentation of the results of the growth analysis, a subset of the overall data was selected for intensive analysis. Because interference among neighbouring plants was assumed to be most intense at high population densities, the data from the highest population density of 66 plants m^{-2} were analysed to determine growth responses to mixture proportions. Also the 2:2 mixture treatment (Table 3.1) represents equality of opportunity for interference between species and that mixture was selected for analysis to determine growth responses to population density.

3.4.8 Plant Allometric Relationships

The 1984 and 1987 data were fitted separately to the model:

$$\log_e(y_i) = \log_e(\alpha') + \beta_0\log_e(z_i) + \beta_1X_i\log_e(z_i) + \beta_2X_j\log_e(z_i) + \beta_3X_iX_j\log_e(z_i) + \gamma_1\log_e(X_i) + \gamma_2\log_e(X_j) + \gamma_3\log_e(X_iX_j) + \log_e(\varepsilon') \quad (3.8)$$

to determine the effect of population densities and mixture proportions on allometric relationships. In the model, y_i represents shoot dry weight per

plant and z_i represents a secondary plant variable being evaluated in relation to shoot dry weight. In beans z_i was taken to be leaf area, leaf dry weight, seed number, live leaf number, total pod dry weight, unmarketable pod dry weight, plant height, marketable pod dry weight, or marketable pod fresh weight. Similarly in beets, z_i represented leaf area, leaf dry weight, petiole dry weight, live leaf number, storage root diameter, storage root fresh weight or storage root dry weight. As before, the X_i represents the population density of the test species while X_j represents the population density of the competing species. A best subset multiple regression analysis using the P:9R BMDP statistical package (Dixon 1985) was used in the analysis as described by Jolliffe *et al.* (1988).

4. RESULTS

4.1 An Overview of the Results

Presentation of the results will be as follows: Visual observations made on plants during growth will be described first followed by analysis of variance (ANOVA) results on all primary variables. Yield-density relationships and differential yield responses will be presented next. Yield component analysis, plant growth analysis and allometric relationships will be reported last.

4.2 Visual Observations

Plants started to emerge about 10 days to 21 days after planting. Beans were early to emerge, while beets were slow. Generally, germination was very good for beans, while in beets the sowing of more than 3 seeds per hill helped to produce the required planting densities, except in one replicate of the 66 plants m^{-2} monoculture of the 1987 beets where fewer plants grew than intended. The possible reason for the slow and poor germination in beets could have been due to a deeper seeding depth (about 2 cm) than was warranted by the small seed size.

Visually, both beans and beets seemed to be healthy throughout the growing seasons. Plants were noticeably smaller in the high population density treatments. Bean stalks from the less dense treatments were thicker than those from densely populated plots, which will be evident in the ANOVA results for bean stem dry weight presented in section 4.3.2.

4.3 Analysis of Variance

4.3.1 Homogeneity of Variance Test

Homogeneity of variance is one of the assumptions for many statistical tests including ANOVA. This was tested for all primary variables and ratio indices using the partitioned Layard's homogeneity of variance test. The raw data indicated heteroscedasticity at the 5% level of significance for both the 1984 and 1987 data (Table 4.1). After transformation of the data to a $\log_{10}$ scale, the homogeneity of variance assumption was largely satisfied as most variables had homogeneous variances at the 5% level of significance for both crop species in both years (Table 4.2).

4.3.2 General Results from the Analysis of Variance

As detailed below, the results for both species both in 1984 and 1987 indicated a significant reduction in yield per plant of all primary variables with increasing total population density. The effects due to increasing mixture proportions of the competing species were also a decrease in yield per plant of the test species in beets and an increase in yield per plant with increasing mixture proportions of the competing species in beans. The effects due to interactions between population density and mixture proportions were infrequently significant.

4.3.2.1 Beans 1984

In the 1984 study, the results of the ANOVA for beans varied from harvest date to harvest date and from variable to variable (Table 4.3). Data from the last 5 harvests were analyzed for all variables. Pod number, pod fresh weight and pod dry weight data from the first harvest,

Table 4.1   Summary of homogeneity of variance test of the raw data; percentage of variates homogeneous at the 5% level of significance as influenced by experimental treatments

Source of variation	1984 Harvest Dates						1987
	HN1[†]	HN2	HN3	HN4	HN5	HN6	
BEANS							
Population density (D)	70	33	53	43	10	55	85
Mixture proportions (M)	50	70	58	75	58	60	85
D x M	70	73	48	80	95	100	72
Overall mean	**63**	**88**	**53**	**66**	**66**	**72**	**81**

Source of variation	1984 Harvest Dates							1987
	HN1[†]	HN2	HN3	HN4	HN5	HN6	HN7	
BEETS								
Population density (D)	60	38	55	45	53	48	55	75
Mixture proportions (M)	50	25	10	28	63	45	23	25
D x M	40	68	65	70	100	83	63	25
Overall mean	**50**	**44**	**43**	**48**	**72**	**59**	**47**	**42**

[†]Harvest number (1-6 in beans and 1-7 in beets)

Table 4.2   Summary of homogeneity of variance test of the transformed data (log_{10} scale); percentage of variates homogenous at the 5% level of signifince as influenced by experimental treatments

Source of variation	1984 Harvest Dates						1987
	HN1[†]	HN2	HN3	HN4	HN5	HN6	
BEANS							
Population density (D)	80	87	88	91	28	91	100
Mixture proportions (M)	90	66	62	82	34	74	100
D x M	90	82	71	91	100	100	75
Overall mean	**87**	**78**	**74**	**88**	**54**	**88**	**92**

Source of variation	1984 Harvest Dates							1987
	HN1[†]	HN2	HN3	HN4	HN5	HN6	HN7	
BEETS								
Population density (D)	85	72	84	88	100	82	100	100
Mixture proportions (M)	90	42	47	59	84	36	47	100
D x M	95	60	54	60	84	71	82	75
Overall mean	**90**	**58**	**62**	**69**	**89**	**63**	**76**	**92**

[†]Harvest number (1-6 in beans and 1-7 in beets)

Table 4.3   Analysis of variance results for the 1984 bean data: Variance ratios for the effects of population density and mixture proportions on primary variables tested at different stages of growth

Age at Harvest (days)	Source of Variation	d.f.	T	LN	BN	PN	Variables FWPD	LA	WL	WST	WPD	W
40	Blocks	2	6*	4*	4*	-	-	3	5*	4*	-	5*
(HN 1)†	Density	3	1	2	2	-	-	5**	7**	1	-	4**
	Mixture	3	2	2	3	-	-	1	3**	2	-	2
	D x M	9	1	1	1	-	-	1	1	1	-	1
	Exp. Err.	30	3**	2*	1	-	-	3**	2*	2**	-	2**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-	-
51	Blocks	2	4*	1	2	3	2	3	2	1	2	2
(HN 2)	Density	3	0	23**	11**	0	0	17**	33**	15**	0	24**
	Mixture	3	2	4*	2	0	1	1	5**	1	1	3
	D x M	9	1	1	1	1	1	1	2	2	1	2
	Exp. Err.	30	4**	1	2*	2**	2*	3**	2*	2**	1	2**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-	-
63	Blocks	2	1	2	4*	3	21**	2	1	1	23**	1
(HN 3)	Density	3	0	39**	13**	17**	10**	19**	31**	15**	7**	22**
	Mixture	3	2	14**	6**	5**	2	3	10**	3*	1	6**
	D x M	9	0	1	0	0	1	0.	1	1	2	1
	Exp. Err.	30	6**	1	1	1	1	4**	2**	3**	1	2
	Samp. Err.	192	-	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-	-

†Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

Table 4.3 (cont'd)   Analysis of variance results for the 1984 bean data: Variance ratios for the effects of population density and mixture proportions on primary variables tested at different stages of growth

Age at Harvest (days)	Source of Variation	d.f.	T	LN	BN	PN	Variables FWPD	LA	WL	WST	WPD	W
69	Blocks	2	2	1	5*	4*	9**	4*	1	1	17**	1
(HN 4)	Density	3	3	54**	28**	36**	60**	18**	32**	10**	75**	17**
	Mixture	3	0	34**	21**	22**	45**	10**	21**	7**	47**	11**
	D x M	9	0	3*	1	2	2*	1	1	1	3*	1
	Exp. Err.	30	8	1	1	1	0	3**	2	4**	0	3**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-	-
75	Blocks	2	1	4*	5*	3	2	1	2	3	1	3
(HN 5)	Density	3	0	78**	29**	53**	25**	35**	49**	18**	64**	27**
	Mixture	3	4	35**	12**	17**	19**	9**	19**	4**	19**	9**
	D x M	9	1	4**	3**	2*	1	2	3**	2	5**	2*
	Exp. Err.	30	5	0	1	1	2	2	2*	3**	1	2**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-	-
92	Blocks	2	0	3	0	1	2	1	1	0	0	0
(HN 6)	Density	3	1	47**	28**	53**	50**	20**	31**	25**	45**	13**
	Mixture	3	1	26**	8**	18**	24**	10**	18**	6**	18**	12**
	D x M	9	0	2	1	1	1	0	1	1	1	1
	Exp. Err.	30	5	1	1	2**	1	2	3	3	2**	3**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-	-

*Significant at P = 0.05
**Significant at P = 0.01

done 40 days after planting, were not analyzed as the majority of observations for these variables were zero.

The ANOVA indicated significant treatment responses for most variables tested. Total population density was found to significantly affect total dry weight, leaf dry weight, and leaf area at all harvest dates. From harvest 2 to harvest 6, all variables were significantly affected by population density except plant height which was significantly affected only at harvest 4. Branch number was also not significantly affected by population density at harvest 2 (Table 4.3). When density effects were significant, increasing population density reduced the mean yield per plant of the variable in question (Fig. 4.1).

The effects due to mixture proportions were significant much later in the growing season on most of the variables, except for leaf dry weight which was influenced at all harvest dates. Leaf number was significantly affected from harvest 2 onwards, pod number and stem dry weight from harvest 3 and pod dry weight, pod fresh weight and branch number from harvest 4 onwards. Plant height was never significantly affected by mixture proportions throughout the study (Table 4.3). In all significantly affected cases, increasing mixture proportions of the competing species increased yield per plant (Fig. 4.2).

Unlike the main effects, population density by mixture proportions interactions were seldom significant. Only three variables, leaf number, pod fresh weight and pod dry weight were significantly affected at harvest 4 and six variables, leaf number, branch number, pod number, leaf dry weight, pod dry weight and total dry weight at harvest 5 (Table 4.3). The trend was a decrease in yield per plant with increasing total population density and increasing mixture proportions of beets. The effect of
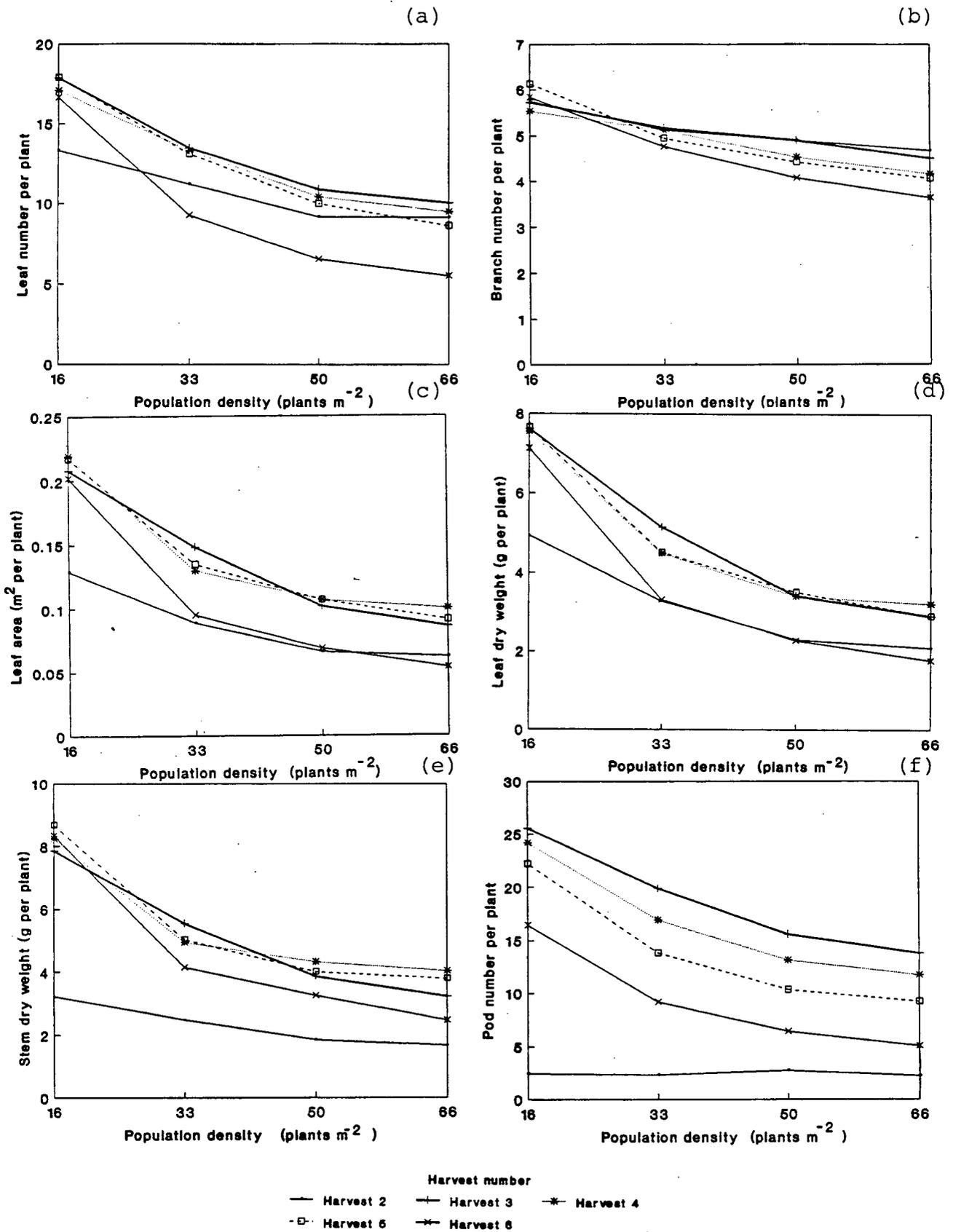
Fig. 4.1 The effect of population density on bean yield variables at different stages of growth (1984 experiment)

(g)

(h)

(i)

Harvest number

—•— Harvest 2   —+— Harvest 3   —*— Harvest 4
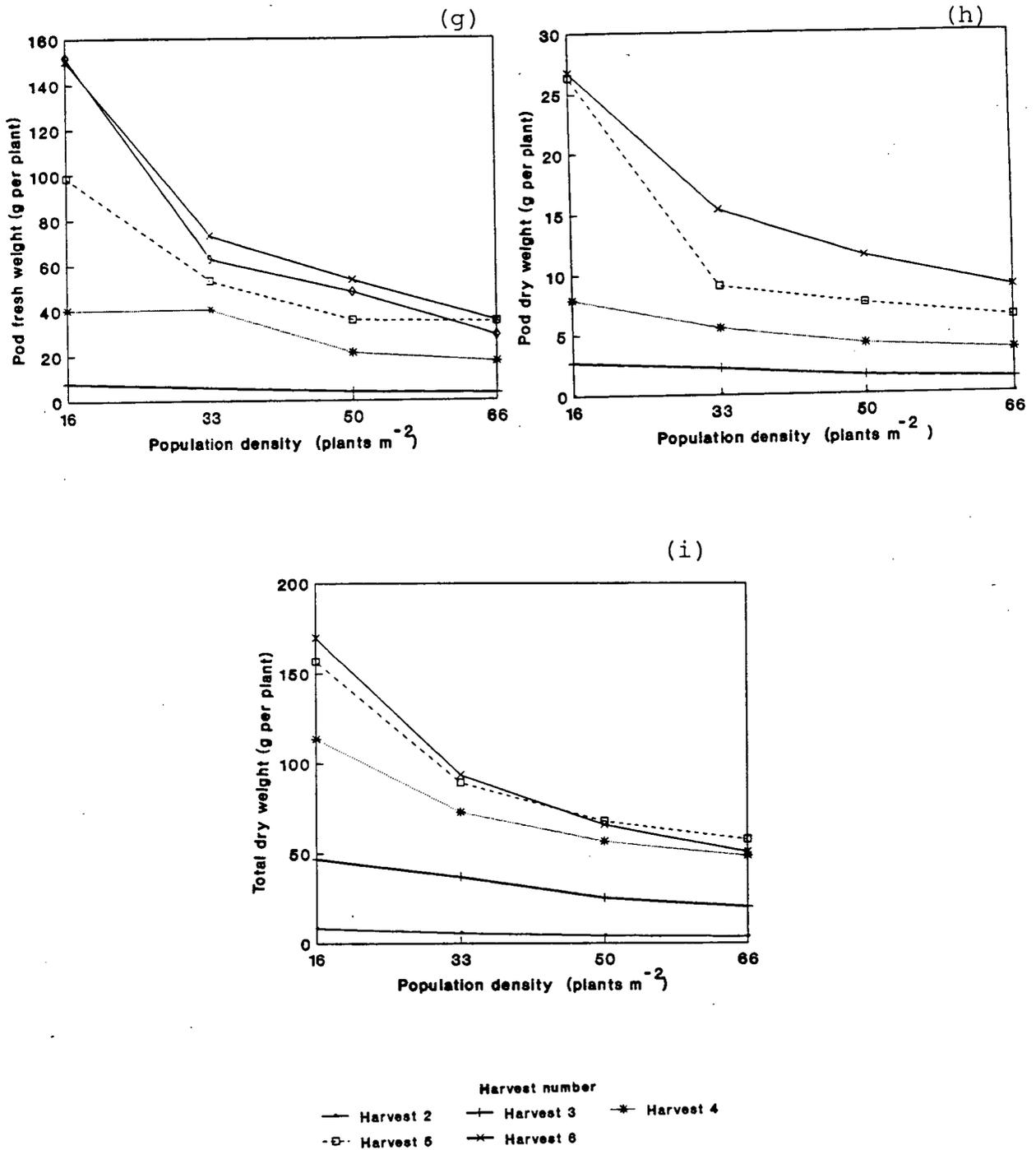
- -□- - Harvest 5   —×— Harvest 6

Fig. 4.1 (cont'd)   The effect of population density on bean yield variables at different stages of growth (1984 experiment)

Fig. 4.2   The effect of mixture proportions on bean yield variables at different stages of growth (1984 experiment)

53
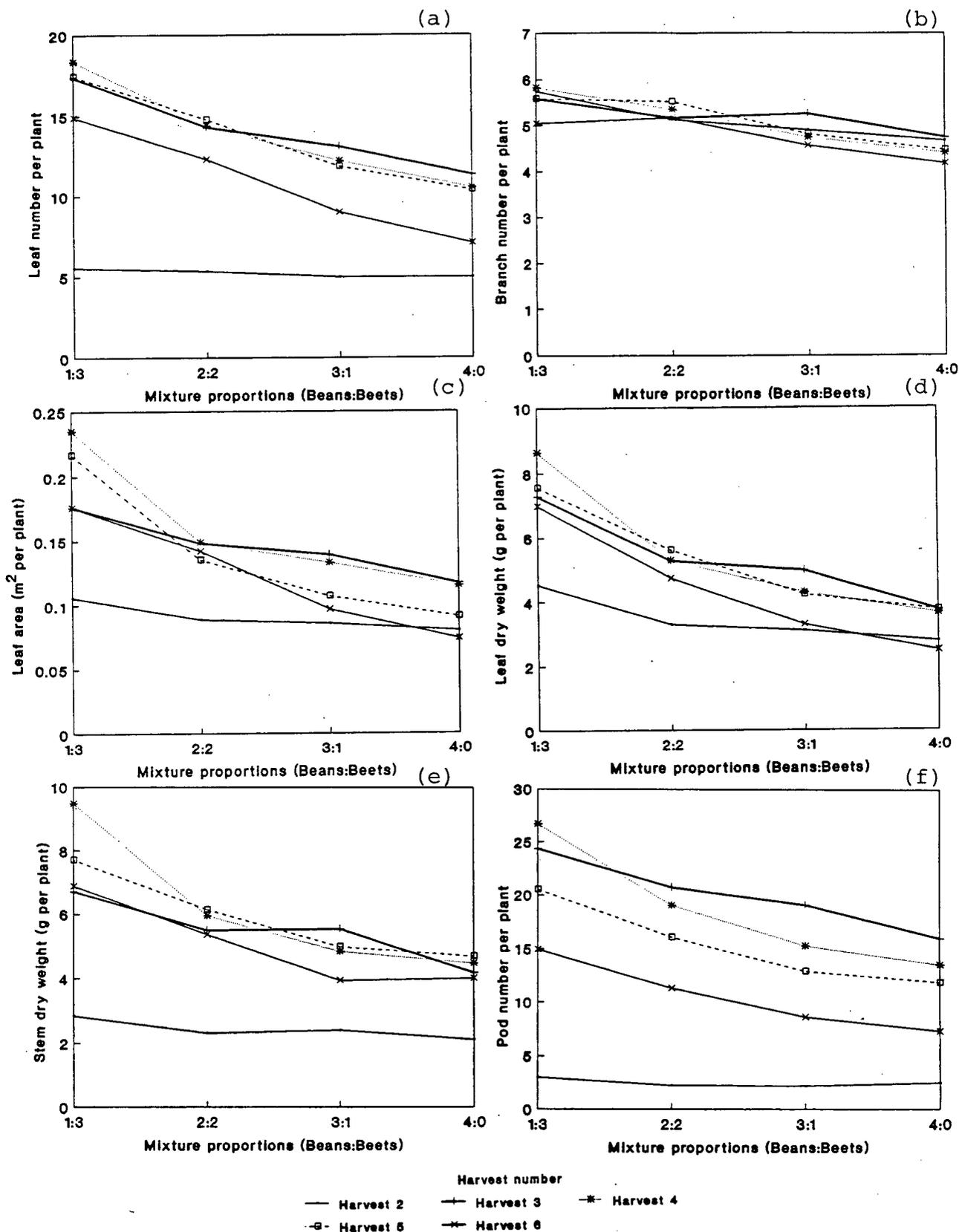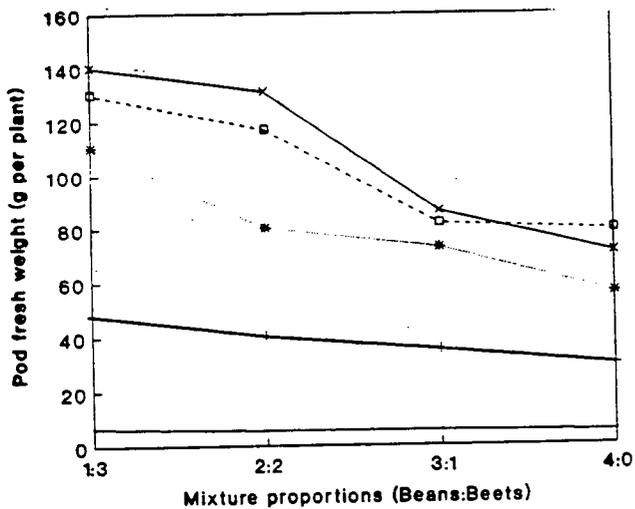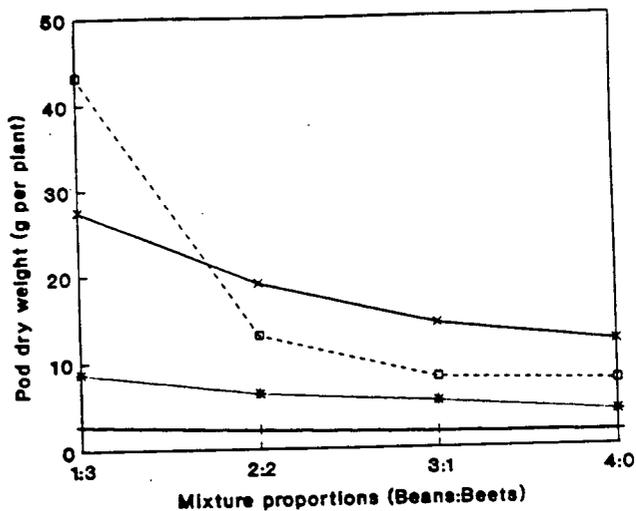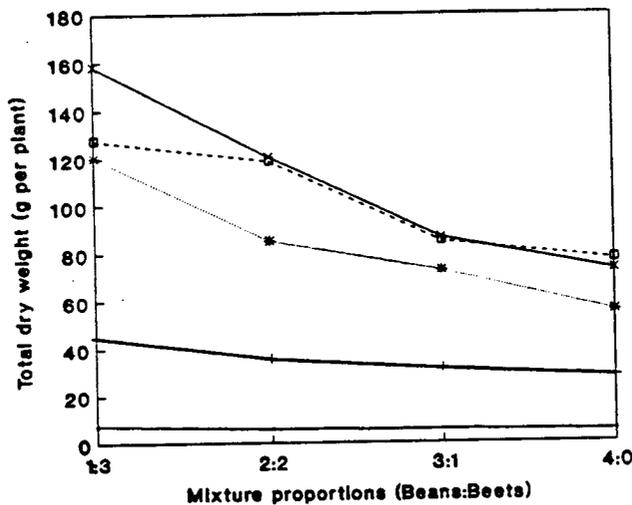
Fig. 4.2 (cont'd)   The effect of mixture proportions on bean yield variables at
different stages of growth (1984 experiment)

increasing mixture proportions of beets seemed to be greater at higher than at lower population densities (Appendix 8.1). Block effects were significant only early in the growing season (Table 4.3).

4.3.2.2 Beans 1987

The ANOVA results for the 1987 experiment showed significant population density effects on number of seeds, seed dry weight, marketable pod fresh and dry weights, and marketable pod number (Table 4.4). In all these variables, the trend was an increase in yield per plant from the first population density level of 16 plants m^{-2} to the second population density level of 33 plants m^{-2}, followed by a decrease at the third and fourth density levels of 50 plants m^{-2} and 66 plants m^{-2} respectively (Fig. 4.3). Other variables (live leaf number, dead leaf number, pod number, leaf area, leaf dry weight, stem dry weight, unmarketable pod dry weight, and total dry weight) did not show significant effects due to population density, but also tended to have similar responses (data not shown). This pattern would suggest that at the lowest population densities there was little or no interference. Above the second density level, plant interference tended to result in a reduction in mean yield of the variables tested. Large variations observed among individuals also made it difficult to detect significant differences between treatments. The same problem could account for lack of significant differences due to mixture proportions in any of the variables tested (Table 4.4). Total population density by mixture proportions interactions also were not statistically significant at the 5% level.

Table 4.4 Analysis of variance results for the 1987 bean data: Variance ratios for the effects of population density and mixture proportions on primary variables of beans tested at the final harvest.

Source of Variation	d.f.	LL	MPN	UPN	FWPD	WS	SN	LA	WL	WST	WMPD	WUDP	W
Blocks	1	0	0	8**	0	2	3	0	0	2	0	17**	0
Density	3	3	3	1	4*	5**	4*	2	2	1	4*	0	3
Mixture	3	2	1	1	2	2	2	1	1	0	2	1	1
D x M	9	0	0	0	1	1	2	0	0	0	1	1	0
Exp. Err.	15	4**	5**	4**	4**	2*	1	5**	7**	3**	4**	2*	6**
Samp. Err.	32	6**	5**	2**	4**	2**	2**	6**	5**	7**	4**	2*	6**
Sub Samp. Err.	896	-	-	-	-	-	-	-	-	-	-	-	-
Total	959	-	-	-	-	-	-	-	-	-	-	-	-

*Significant at P = 0.05
**Significant at P = 0.01

56
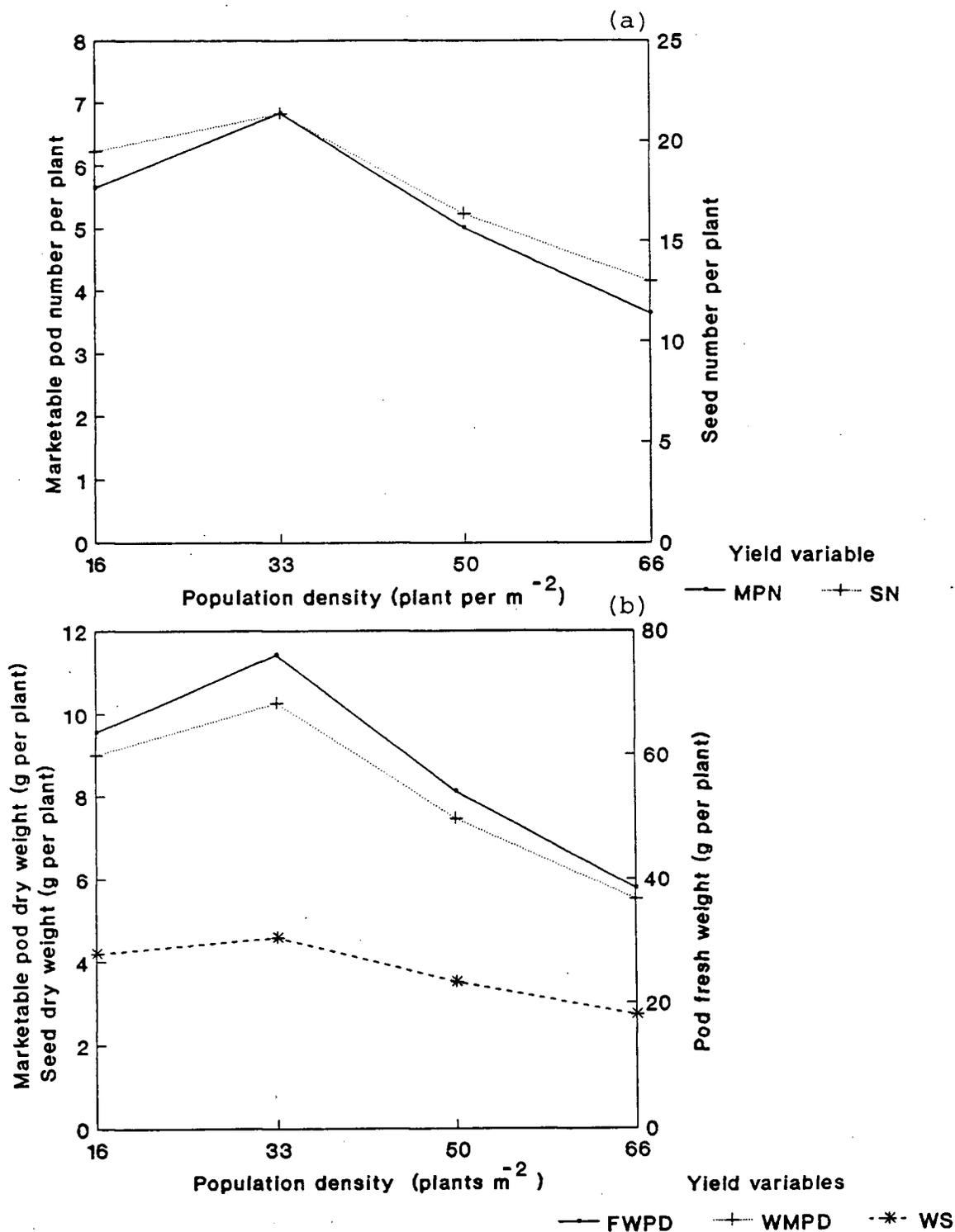
Fig. 4.3   The effect of population density on bean yield variables (1987 experiment)

(a)   Marketable pod   number and seed number
(b)   Pod fresh weight, marketable pod dry weight and seed dry weight

4.3.2.3 Beets 1984

Compared to the bean data for 1984, the beet data were much more consistent from harvest date to harvest date and from variable to variable. As in beans, only some of the variables (plant height, leaf number, leaf dry weight, petiole dry weight, leaf area and total dry weight) were analyzed for the first harvest done 40 days after planting. Root diameter, root fresh weight and root dry weight were not analysed as the storage roots had not yet begun to enlarge.

Other than at harvest 1, where no significant differences due to population density were detected in any of the variables tested, results for harvests 2 to 7 showed significant effects due to total population density on all variables except plant height at harvest 6 (Table 4.5). Mixture proportions exerted significant effects on all variables tested, except on plant height, at harvest 2 and 3, leaf area at harvests 4, 5 and 6, and petiole dry weight at harvest 6. The total population density by mixture proportions interactions were significant only for leaf number and leaf dry weight at harvest 2 (Table 4.5).

The responses in all these harvests were reductions in yield per plant with increasing total population density (Fig. 4.4). Similar trends were also observed due to increasing mixture proportions of beans (Fig. 4.5) except for beet plant height where an increase in height due to increasing bean proportions was observed (Fig. 4.5i). As for beans, block effects on the beet results were only significant in a few cases (Table 4.5).

4.3.2.4 Beets 1987

The ANOVA for the 1987 results for beets indicated significant effects of both population density and mixture proportions on all primary

Table 4.5   Analysis of variance for the 1984 beet data: Variance ratios for effects of population density and mixture proportions of primary variables tested at different stages of growth

Age at Harvest	Source of Variation	d.f.	Variables								
			T	LN	DR	FWR	LA	W	WL	WP	WR
40	Blocks	2	1	1	-	-	0	0	0	0	-
(HN 1)†	Density	3	0	0	-	-	0	1	1	1	-
	Mixture	3	0	2	-	-	1	1	2	1	-
	D x M	9	1	1	-	-	1	1	1	1	-
	Exp. Err.	30	7**	4**	-	-	6**	7**	7**	7**	-
	Samp. Err.	192	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-
51	Blocks	2	4*	0	0	1	2	1	1	1	0
(HN 2)	Density	3	7**	15**	8**	15**	11**	7**	12**	9**	3*
	Mixture	3	1	10**	11**	12**	6**	10**	15**	5*	8**
	D x M	9	1	3*	2	1	2	1	2*	1	1
	Exp. Err.	30	3**	3**	3**	2*	3**	3**	2**	3**	4**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-
63	Blocks	2	1	0	0	0	1	0	0	0	1
(HN 3)	Density	3	6**	15**	15**	16**	15**	17**	18**	15**	19**
	Mixture	3	0	12**	17**	18**	5**	15**	16**	7**	23**
	D x M	9	0	1	1	0	0	1	1	0	1
	Exp. Err.	30	7**	3**	2**	3**	4**	3**	4**	4**	3**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-

†Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

Table 4.5 (cont'd)   Analysis of variance for the 1984 beet data: Variance ratios for effects of population density and mixture proportions of primary variables tested at different stages of growth

Age at Harvest	Source of Variation	d.f.	T	LN	DR	FWR	LA	W	WL	WP	WR
69	Blocks	2	2	2	4*	2	1	1	1	0	2
(HN 4)	Density	3	1	8**	3*	5**	4**	5**	6**	3*	5**
	Mixture	3	2	8**	5**	10**	3*	9**	10**	5**	13**
	D x M	9	0	1	1	1	0	1	0	0	0
	Exp. Err.	30	7**	3**	4**	5**	7**	5**	6**	7**	4**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-
75	Blocks	2	2	1	1	0	2	0	0	0	0
(HN 5)	Density	3	2	9**	11**	12**	6**	10**	10**	5**	13**
	Mixture	3	0	7**	14**	14**	2	11**	10**	3*	17**
	D x M	9	1	0	0	0	0	0	0	0	1
	Exp. Err.	30	12**	6**	7**	7**	12**	8**	9**	10**	7**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-
92	Blocks	2	3*	0	1	1	3	1	2	2	0
(HN 6)	Density	3	2	10**	8	10**	6**	8**	8**	4**	10**
	Mixture	3	1	9**	13	15**	2	10**	9**	2	16**
	D x M	9	0	0	0	0	0	0	0	0	0
	Exp. Err.	30	10**	3**	6	5**	10**	7**	9**	9**	5**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-
107	Blocks	2	9**	4*	1	1	5	2	3	4*	1
(HN 7)	Density	3	3**	6**	11	10**	8**	10**	12**	7**	10**
	Mixture	3	2	7**	23**	23**	4**	17**	15**	4**	23**
	D x M	9	2	1	1	1	1	0	1	1	1
	Exp. Err.	30	6**	3	3**	3**	5**	3**	4**	3**	3**
	Samp. Err.	192	-	-	-	-	-	-	-	-	-
	Total	239	-	-	-	-	-	-	-	-	-
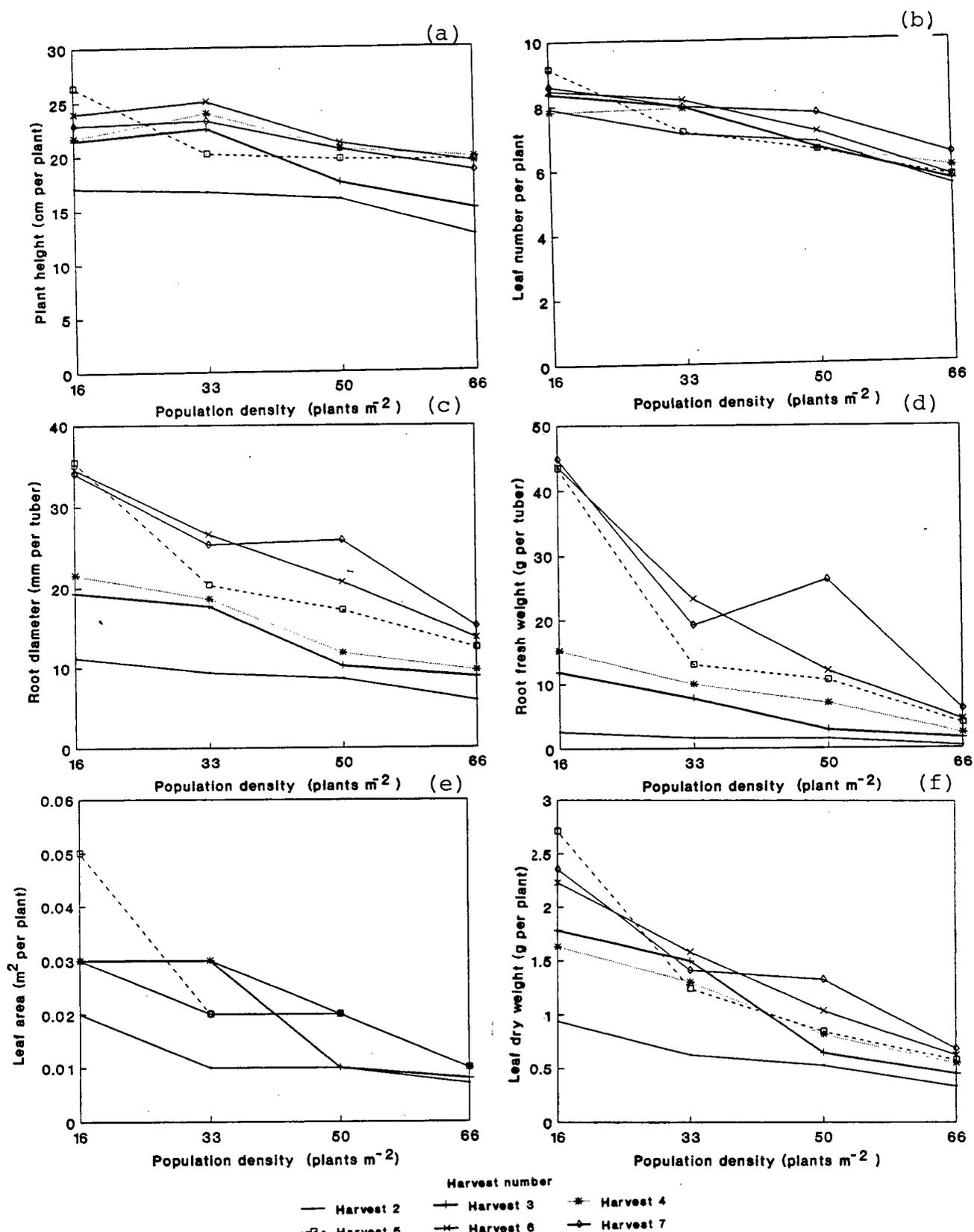
*Significant at P = 0.05
**Significant at P = 0.01

Fig. 4.4   The effect of population density on beet yield variables at different stages of growth (1984 experiment)

(g)

(h)

(i)

Harvest number

— Harvest 2   —+— Harvest 3   —*— Harvest 4
-ᴑ- Harvest 5   —×— Harvest 6   —◇— Harvest 7
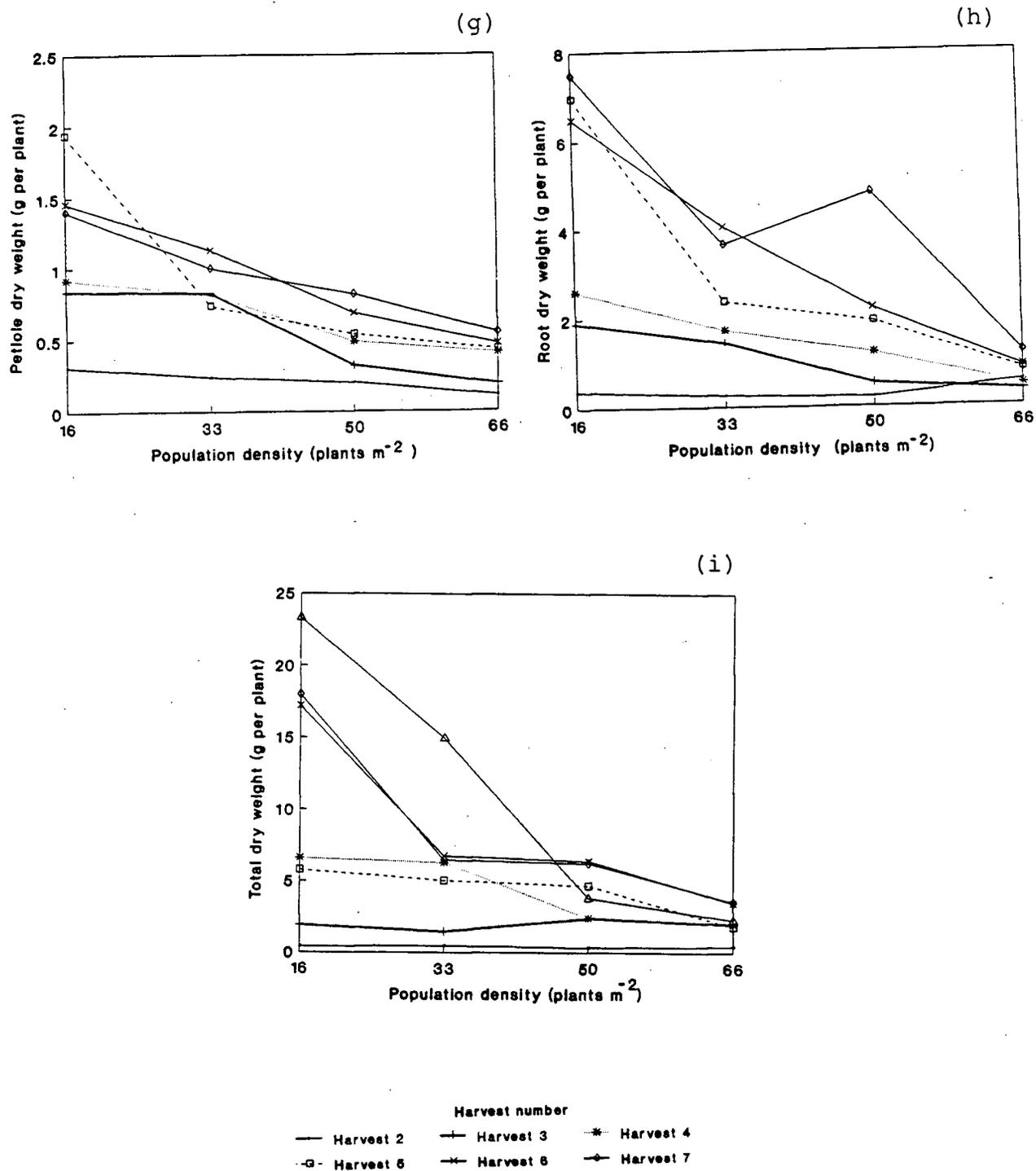
Fig. 4.4 (cont'd)   The effect of population density on beet yield variables at different stages of growth (1984 experiment)
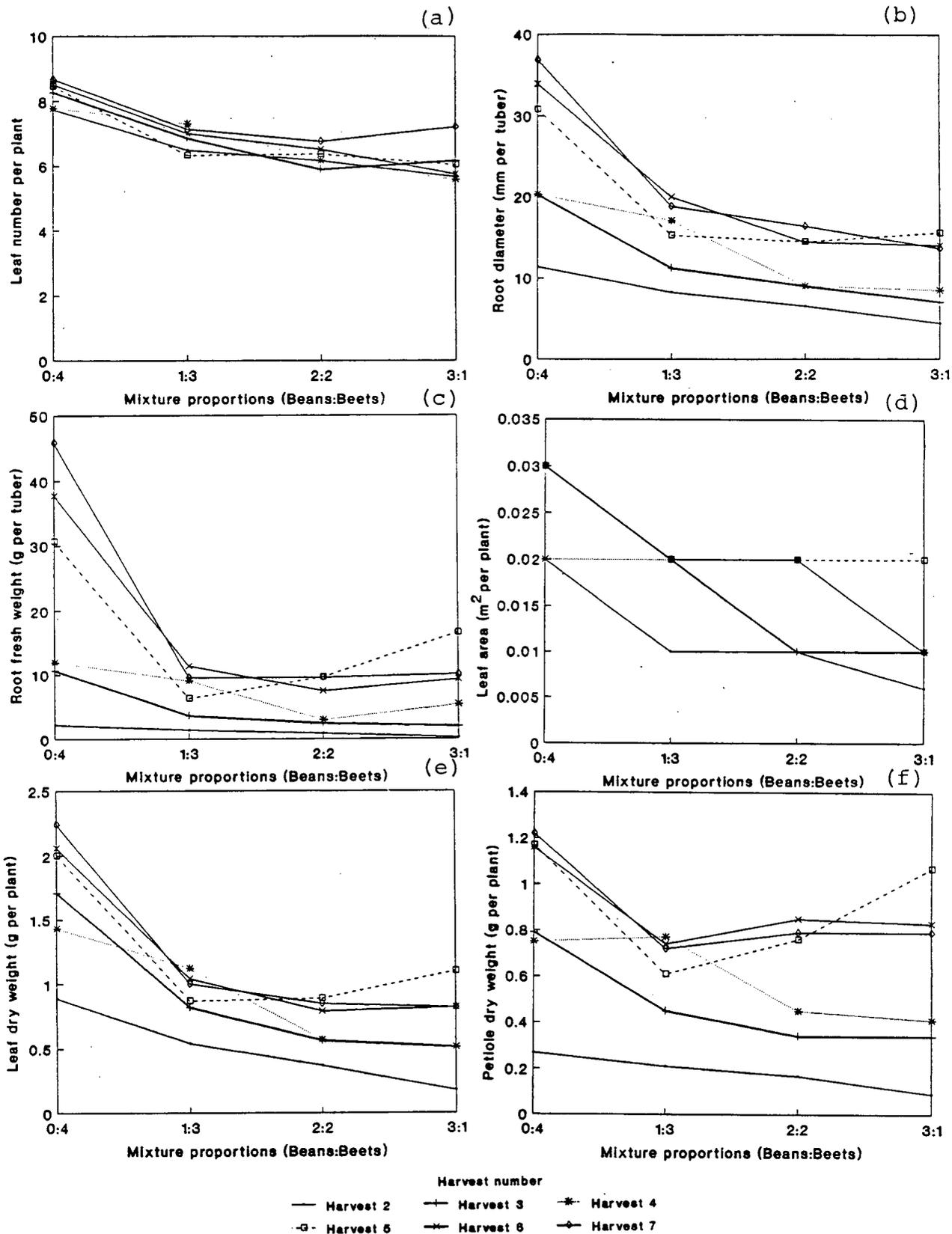
Fig. 4.5   The effect of mixture proportions on beet yield variables at different stages of growth (1984 experiment)
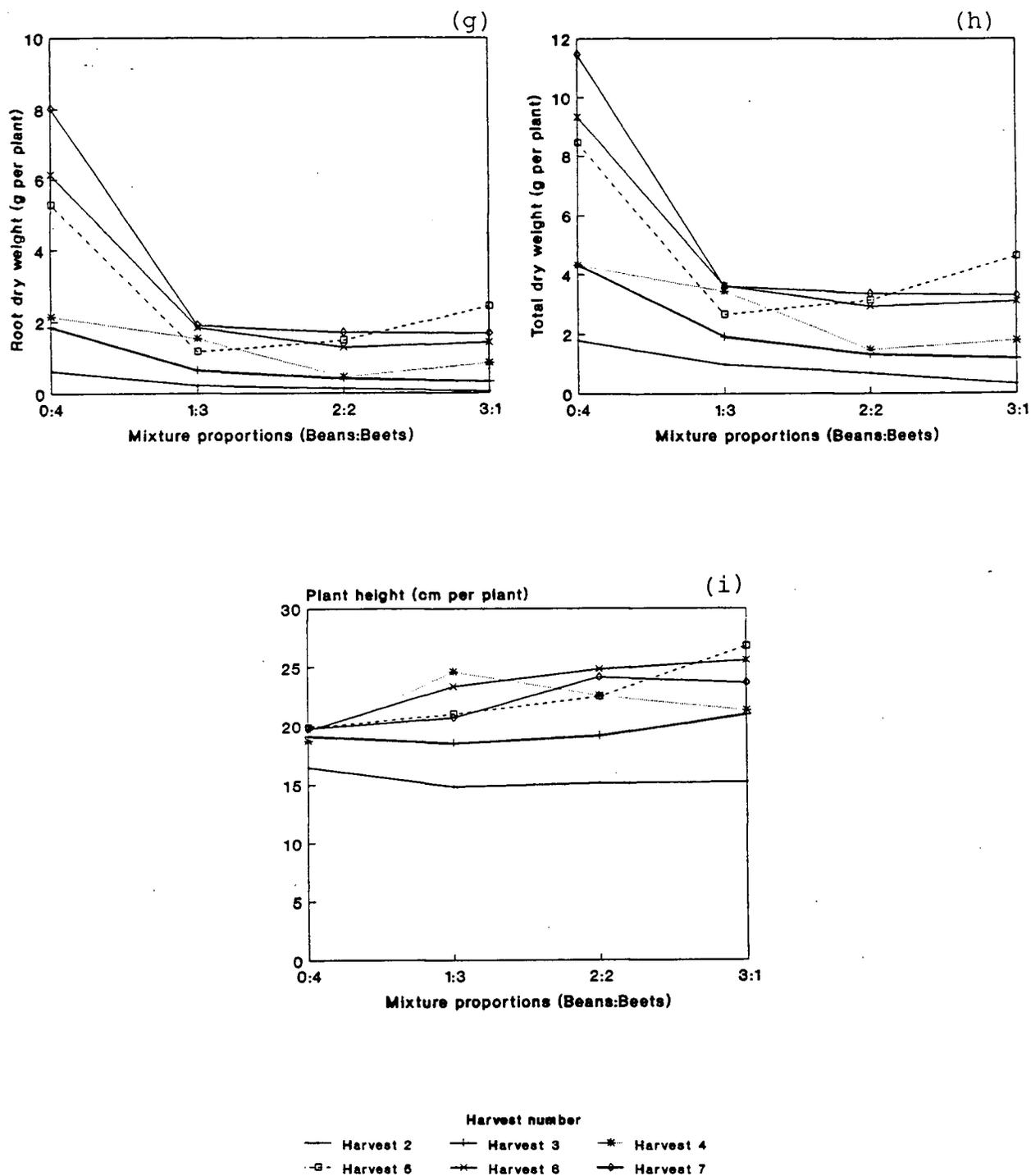
Fig. 4.5 (cont'd) The effect of mixture proportions on beet yield variables at different stages of growth (1984 experiment)

variables tested (Table 4.6). Increasing plant population density generally reduced total dry weight, number of live leaves, tuber fresh weight, leaf dry weight, petiole dry weight and root dry weight per plant (Fig. 4.6). Similarly, increasing the mixture proportions of beans reduced total dry weight, number of live leaves, tuber diameter, leaf area, tuber fresh weight, leaf dry weight, petiole dry weight and root dry weight per plant (Fig. 4.7). The population density by mixture proportions interaction was not statistically significant at the 5% level of significance (Table 4.6).

4.3.3 Summary of Analysis of Variance Results

In summary, both population density and mixture proportions significantly affected yield of almost all variables in both species in 1984 and in beets in 1987. The response was a reduction in yield per plant with increasing population density for both species and with increasing proportions of the competing species in beets. In beans, increasing mixture proportions of beets increased yield per plant. Population density by mixture proportions interactions were rarely significant.

4.4 Yield-density Relationships

Non-linear models were used to define yield-density relationships in both monocultures and mixtures using equations 3.1 and 3.2 respectively. For each species, total dry weight, leaf number, leaf weight and leaf area were considered as the dependent variables in separate models.

Table 4.6   Analysis of variance results for the 1987 beet data: Variance ratios for the effects of population density and mixture proportions on primary variables tested at the final harvest

Source of Variation	d.f.	LL	DR	FWR	LA	WL	WP	WR	W
					Variables				
Blocks	1	2	0	0	0	0	0	0	0
Density	3	13**	7**	13**	8**	13**	7**	11**	3*
Mixture	3	23**	15**	21**	9**	21**	7**	21**	5*
D x M	9	1	1	1	1	1	2	1	1
Exp. Err.	15	1	1	1	1	1	1	1	2
Samp. Err.	32	2**	4**	4**	4**	3**	4**	4**	4**
Sub Samp. Err.	192	-	-	-	-	-	-	-	-
Total	255	-	-	-	-	-	-	-	-

*Significant at P = 0.05
**Significant at P = 0.01
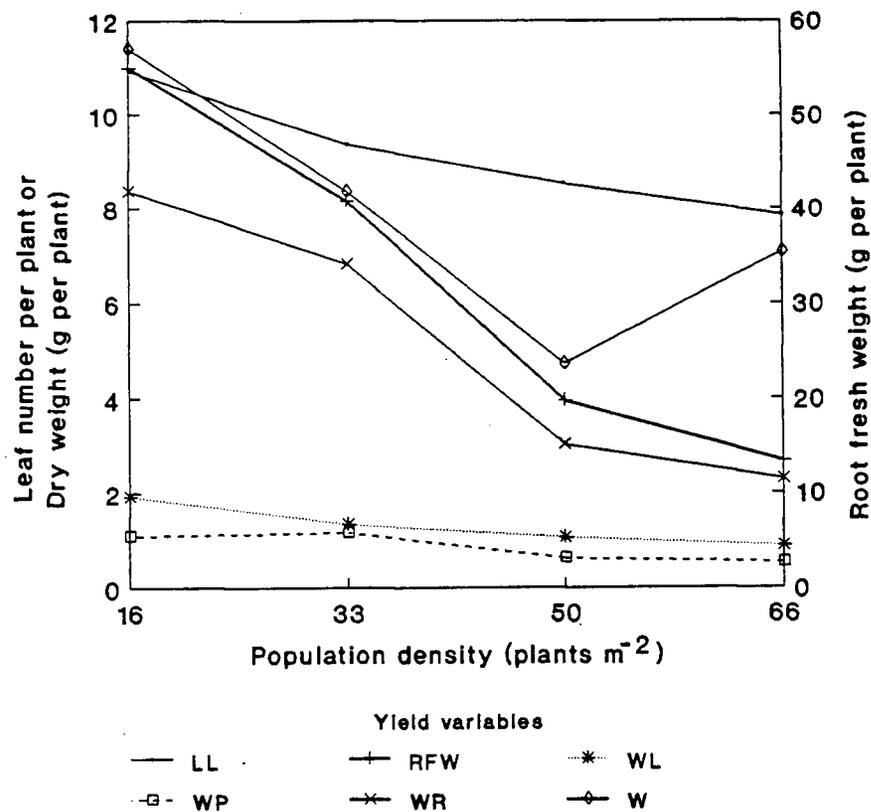
Fig 4.6   The effect of population density on beet live leaf number, total dry weight, root fresh weight, leaf dry weight, petiole dry weight and root dry weight (1987 experiment)
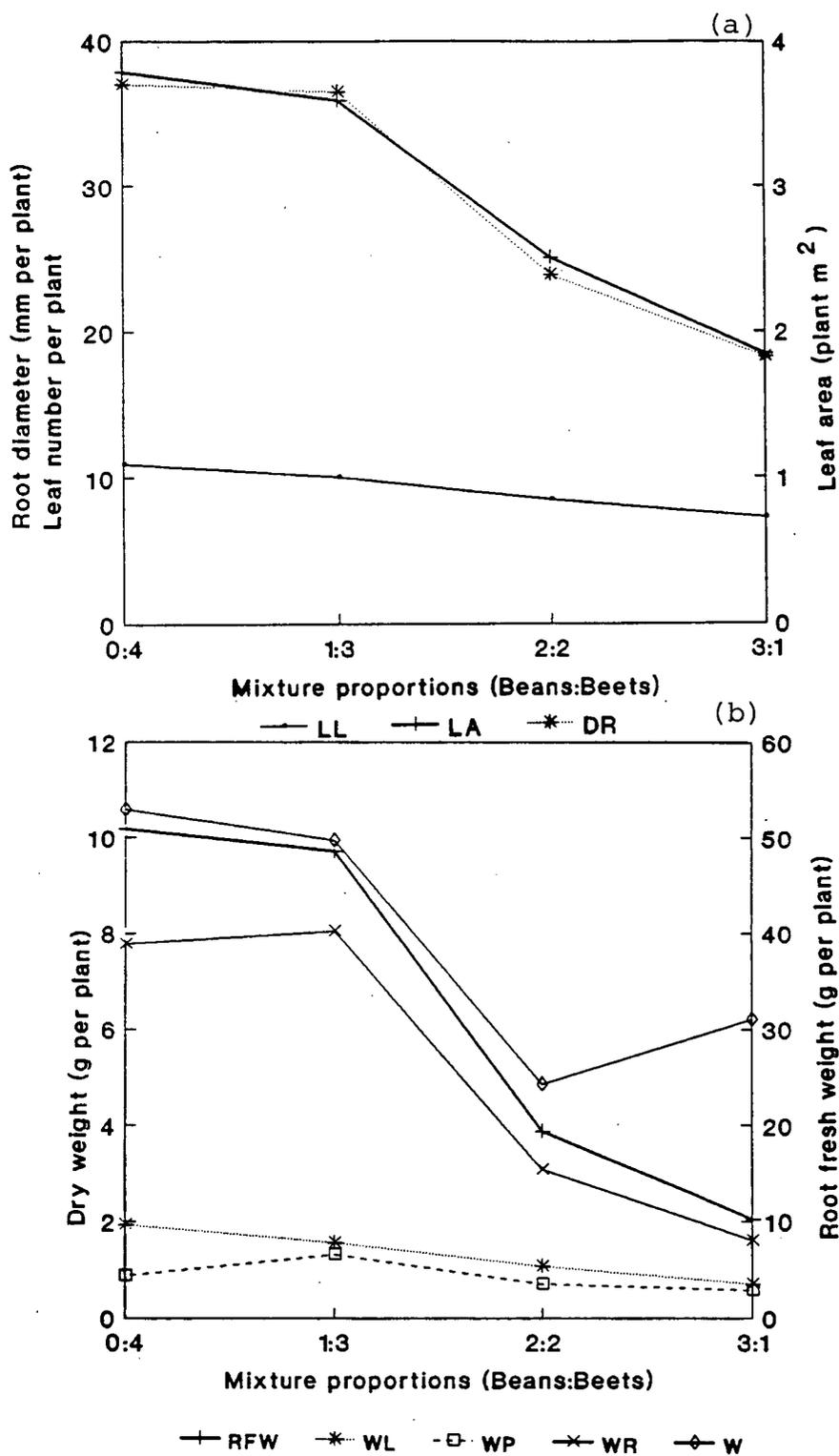
Fig. 4.7   The effect of mixture proportions on beet yield variables
(1987 experiment)

(a) Live leaf number, leaf area and root diameter
(b) Root fresh weight, leaf dry weight, petiole dry weight, root
dry weight and total dry weight

4.4.1 Yield-density Regressions

Table 4.7 shows the parameters and statistics while Table 4.8 shows standard deviations and error mean squares of the reciprocal yield-density model 3.2. As indicated earlier, parameters Φ, **a** and **b** serve to express different aspects of species performance and interrelationships. Parameter b_{ii} expresses responsiveness of y to intraspecific interference and parameter b_{ij} is a measure of plant responsiveness to interspecific interference. Hence, the ratio b_{ii}/b_{ij} estimates the relative influence of intra- to interspecific interference. Parameter **a** which expresses the reciprocal mean yield of an isolated plant as scaled by the parameter Φ was found to be negative in some cases (Table 4.7) implying negative reciprocal mean yield. This could partly be due to the fact that the model best describes interference at high plant population densities and thus would not provide a good estimate of yield of an isolated plant. Under the conditions of this experiment, the results indicate that beans were the stronger competitor, both against themselves and against beets (Table 4.7).

The exponent, $(-\Phi)$, which may be related to the acquisition and utilization of resources within the space accessible by a plant (Watkinson 1984), was negative in all cases; large variations in Φ among harvests were noticed in both species. This variation might be a result of the shifts in plant development or changes in the relative importance of competition at different stages of plant development.

4.4.2 Summary of Yield-density Relationships Results

Interference among associated plants of beans and beets grown in monocultures and mixtures were found to be quite complex. The

Table 4.7a Estimates of parameter values for the response of total dry weight per plant to population densities*

Age at Harvest (days)	Estimates of Model Parameter Values				
	Φ_i	a_i	b_{ii}	b_{ij}	b_{ii}/b_{ij}
BEANS					
40	-0.630	0.0426	0.0041	0.0036	1.11
51	-2.35	0.352	0.00431	0.0030	1.43
63	-8.58	0.599	0.0021	0.0017	1.24
69	-0.544	-0.00028	0.00003	0.000013	2.31
75	-0.784	-0.00097	0.00016	0.000082	1.95
92	-0.819	-0.00184	0.00024	0.000095	2.53
BEETS					
40	-0.559	3.99	0.0081	5.77	0.001
51	-0.286	0.132	0.00001	342.000	<0.001
63	-51.2	0.956	0.00042	0.002	0.21
69	-23.8	0.908	0.00098	0.003	0.28
75	-0.525	0.0148	0.0012	0.851	0.001
92	-6.40	0.576	0.00037	0.014	0.26
107	-32.10	0.904	0.0006	0.003	0.20

*Equation (3.2): $(y_{ij})^{-\Phi i} = a_i + b_{ii}X_i + b_{ij}X_j$

Table 4.7b   Estimates of parameter values for the response of leaf dry weight per plant to
population densities*

Age at Harvest (days)	Estimates of Model Parameter Values				
	Φ_i	a_i	b_{ii}	b_{ij}	b_{ii}/b_{ij}
BEANS					
40	-0.64	-0.0064	0.0123	0.0099	1.24
51	-1.69	0.2772	0.0076	0.0050	1.52
63	-2.47	0.3880	0.0055	0.0020	2.75
69	-0.41	-0.0429	0.0031	0.0010	3.26
75	-0.89	-0.0012	0.0070	0.0273	0.26
92	-1.25	-0.0551	0.0181	0.0039	4.64
BEETS					
40	-0.30	49.294	0.3272	5563.000	0.0001
51	-0.40	-5.663	0.0326	94.290	0.0003
63	-18.32	0.9354	0.0010	0.0051	0.2000
69	-0.54	0.4070	0.0028	1.919	0.001
75	-0.46	-0.1863	0.0145	7.207	0.002
92	-2.17	0.4326	0.0079	0.0481	0.16
107	-0.54	-0.045	0.0083	0.7358	0.01

*Equation (3.2): $(y_{ij})^{-\Phi i} = a_i + b_{ii}X_i + b_{ij}X_j$

Table 4.7c   Estimates of parameter values for the response of leaf number per plant to population densities[*]

Age at Harvest (days)	Estimates of Model Parameter Values				
	Φ_i	a_i	b_{ii}	b_{ij}	b_{ii}/b_{ij}
BEANS					
40	-0.21	0.00002	0.00004	0.00002	0.20
51	-0.27	-0.00003	0.00001	0.00001	0.70
63	-1.06	0.0522	0.0013	0.0005	2.60
69	-0.63	0.0016	0.00064	0.0001	6.40
75	-1.51	0.1081	0.0029	0.0007	4.14
92	-1.62	0.0873	0.0064	0.0013	4.92
BEETS					
40	-0.24	0.0006	0.000001	0.0001	0.01
51	-0.65	0.0377	0.000123	0.0017	0.07
63	-0.56	0.0117	0.0003	0.0016	0.19
69	-0.61	0.0291	0.00012	0.0015	0.08
75	-0.27	0.00001	0.00001	0.0002	0.05
92	-2.02	0.3357	0.0005	0.0032	0.16
107	-2.41	0.3865	0.00005	0.0029	0.02

*Equation (3.2): $(y_{ij})^{-\Phi i} = a_i + b_{ii}X_i + b_{ij}X_j$

Table 4.7d Estimates of parameter values for the response of leaf area per plant to population densities*

Age at Harvest (days)	Estimates of Model Parameter Values				
	Φ_i	a_i	b_{ii}	b_{ij}	b_{ii}/b_{ij}
BEANS					
40	-1.12	-552.600	67.76	-44.431	-1.52
51	-0.95	3.650	0.3126	0.303	1.03
63	-20.99	1.067	0.0011	0.0006	1.67
69	-0.26	-8535.000	534.9000	463.813	1.15
75	-0.84	0.7503	0.3548	0.2141	1.66
92	-1.22	-0.5917	0.2984	0.2187	1.36
BEETS					
40	-0.91	300.55	0.3648	-12.382	-0.03
51	-0.60	483.76	13.48	1233.04	0.01
63	-10.50	1.3107	0.0027	0.0101	0.27
69	-2.47	3.409	0.0345	0.1285	0.27
75	-0.39	-11533.000	770.100	153967.000	0.01
92	-0.73	24.653	5.065	73.964	0.16
107	-0.41	-2480.000	289.900	48920.100	0.01

*Equation (3.2): $(y_{ij})^{-\Phi i} = a_i + b_{ii}X_i + b_{ij}X_j$

Table 4.8a   Standard deviations and error mean squares for the response of total dry weight per plant to population densities

Age at harvest	STAGE 1 Asymptotic standard deviations			RMS	STAGE 2 Std. error	RMS
	Φ_i	a_i	b_{ii}		b_{ij}	
BEANS						
40	0.545	0.055	1.09	3.09	0.001	0.043
51	0.699	0.003	4.93	4.17	0.0005	0.014
63	3.896	0.023	449.72	444.6	0.0003	0.004
69	4.998	0.199	0.384	0.076	0.000001	0.000002
75	0.949	0.045	0.49	0.1310	0.0001	0.0003
92	1.164	0.054	0.492	0.124	0.0004	0.0008
BEETS						
40	1885.84	5.85	178.62	0.131	1.26	100027.9
51	1107.30	2.61	425.83	0.004	118.99	885655352
63	12.08	0.11	83.50	0.002	0.0001	0.0005
69	19.08	0.13	183.59	0.002	0.0001	9000.07
75	0.047	0.005	0.693	84.37	0.187	2196.35
92	3.26	0.055	34.41	0.003	0.165	187.22
107	5.94	0.072	103.11	0.014	0.123	2459.05

Table 4.8b   Standard deviations and error mean squares for the response of leaf dry weight per plant to population densities

Age at harvest	STAGE 1 Asymptotic standard deviations			RMS	STAGE 2 Std. error	RMS
	Φ_i	a_i	b_{ii}		b_{ij}	
BEANS						
40	0.49	0.008	9.21	1.42	0.002	0.28
51	0.52	0.004	2.10	1.30	0.001	0.030
63	0.76	0.005	4.78	2.81	0.0004	0.012
69	0.55	0.002	2.22	1.40	0.0003	0.006
75	0.62	0.001	4.96	1.28	0.061	230.62
92	0.61	0.003	0.53	3.73	0.001	3.40
BEETS						
40	11140.68	240.81	6.42	5.64	1674.86	-
51	0.07	0.001	8.67	2.71	44.58	124330936.07
63	2.87	0.048	621.77	1.28	0.0002	0.0034
69	10.85	0.029	7.85	1.37	0.294	5407.66
75	1.11	0.014	5.85	2.33	1.84	211728.66
92	0.36	0.004	0.98	2.92	0.004	0.77
107	0.25	0.061	0.90	2.36	0.140	1234.73

Table 4.8c Standard deviations and error mean squares for the response of leaf number per plant to population densities

Age at harvest	STAGE 1 Asymptotic standard deviations			RMS	STAGE 2 Std. error	RMS
	Φ_i	a_i	b_{ii}		b_{ij}	
BEANS						
40	122.21	3.27	0.46	0.001	0.000004	0.000001
51	8.09	2.33	0.09	0.002	0.000002	0.0000
63	4.18	0.033	0.08	0.001	0.0001	0.001
69	7.44	0.125	0.053	0.001	0.0001	0.0002
75	9.02	0.176	1.76	0.001	0.0002	0.002
92	8.15	0.137	1.59	0.002	0.0004	0.011
BEETS						
40	1568.65	5.68	0.065	0.003	0.00002	0.00002
51	8.52	3.19	0.179	0.002	0.0001	0.002
63	1.89	0.25	60.120	0.002	0.0001	0.001
69	3.05	0.13	5.63	0.002	0.0001	0.001
75	2.19	0.11	0.88	0.001	0.00003	0.0001
92	4.07	0.08	0.76	0.001	0.0002	0.004
107	1.09	0.081	0.94	0.023	0.0003	0.004

Table 4.8d   Standard deviations and error mean squares for the response of leaf area per plant to population densities

Age at harvest	STAGE 1 Asymptotic standard deviations			RMS	STAGE 2 Std. error	RMS
	Φ_i	a_i	b_{ii}		b_{ij}	
BEANS						
40	6200.28	631.05	0.48	0.0007	4.23	1134141.99
51	20.37	1.007	0.83	0.0808	0.04	100.09
63	1.51	0.019	332.75	0.002	0.0001	0.001
69	13948.79	3703.62	0.18	0.002	177.84	197845779
75	11.36	0.83	0.57	0.002	0.037	86.82
92	10.11	0.98	1.29	0.003	0.022	29.93
BEETS						
40	742688.53	1033.13	401.77	0.00001	0.69	29369.34
51	23588.49	599.63	4.31	0.0001	262.79	4320283383
63	7.53	0.06	172.63	0.0004	0.001	0.21
69	38.78	0.66	10.68	0.0003	0.011	8.27
75	38807.85	8558.93	0.45	0.0007	42231.2	10562564.91
92	405.91	35.12	0.87	0.0005	14.92	13922821.8
107	40576.54	4259.63	0.611	0.0005	15669.8	Too large

mathematical relationships as defined by non-linear inverse models have indicated strong intra- and interspecific interferences. In this study, the model parameters consistently suggested that beans were better competitors both in monoculture populations and in mixtures.

4.5 Differential Yield Responses of Mixtures

The predicted combined yield per land area for bean-beet mixtures was evaluated using equation 3.3 while the observed combined yield was done using equation 3.4 at 92 days after planting. The predicted land equivalent ratio (LER) for total dry weight indicated yield disadvantage while the observed figures indicated yield advantages for total population density of 66 plants per m^{-2} (Fig. 4.8). Similar results were obtained when other total population densities of 33 and 50 plants m^{-2} were considered (data not shown), but the total population density of 16 plants per m^{-2} predicted overyielding while the observed results indicated yield disadvantage (Fig 4.9).

The predicted and the observed LER for leaf dry weight and leaf number at population density of 66 plants m^{-2} were found to be greater than 1 indicating yield advantage in these two variables (Figs. 4.10 and 4.11). The other three total population densities of 16, 33 and 50 plants m^{-2} also indicated yield advantage (data not shown).

In case of leaf area, the predicted land equivalent ratio at total population density of 66 plants m^{-2} indicated yield disadvantage while the observed figures indicated yield advantage (Fig. 4.12). Other total population densities also indicated similar results (data not shown). Marketable yield land equivalent ratios were also calculated but model 3.4 was not totally successful in predicting combined yield per land area
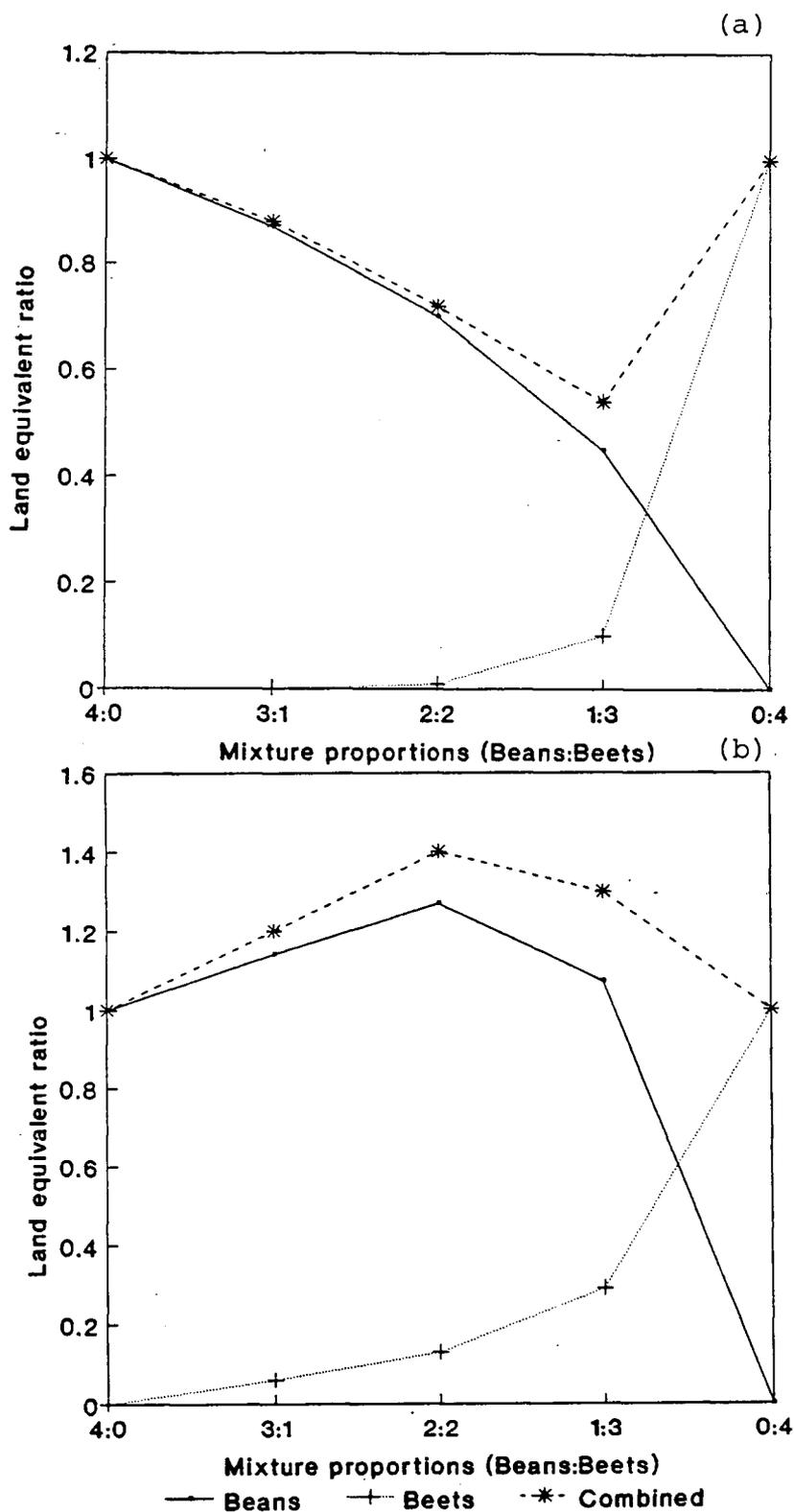
Fig. 4.8   Land equivalent ratio for total dry weight per unit land area at total population density of 66 plants m⁻² (1984 experiment)

(a) Predicted
(b) Observed
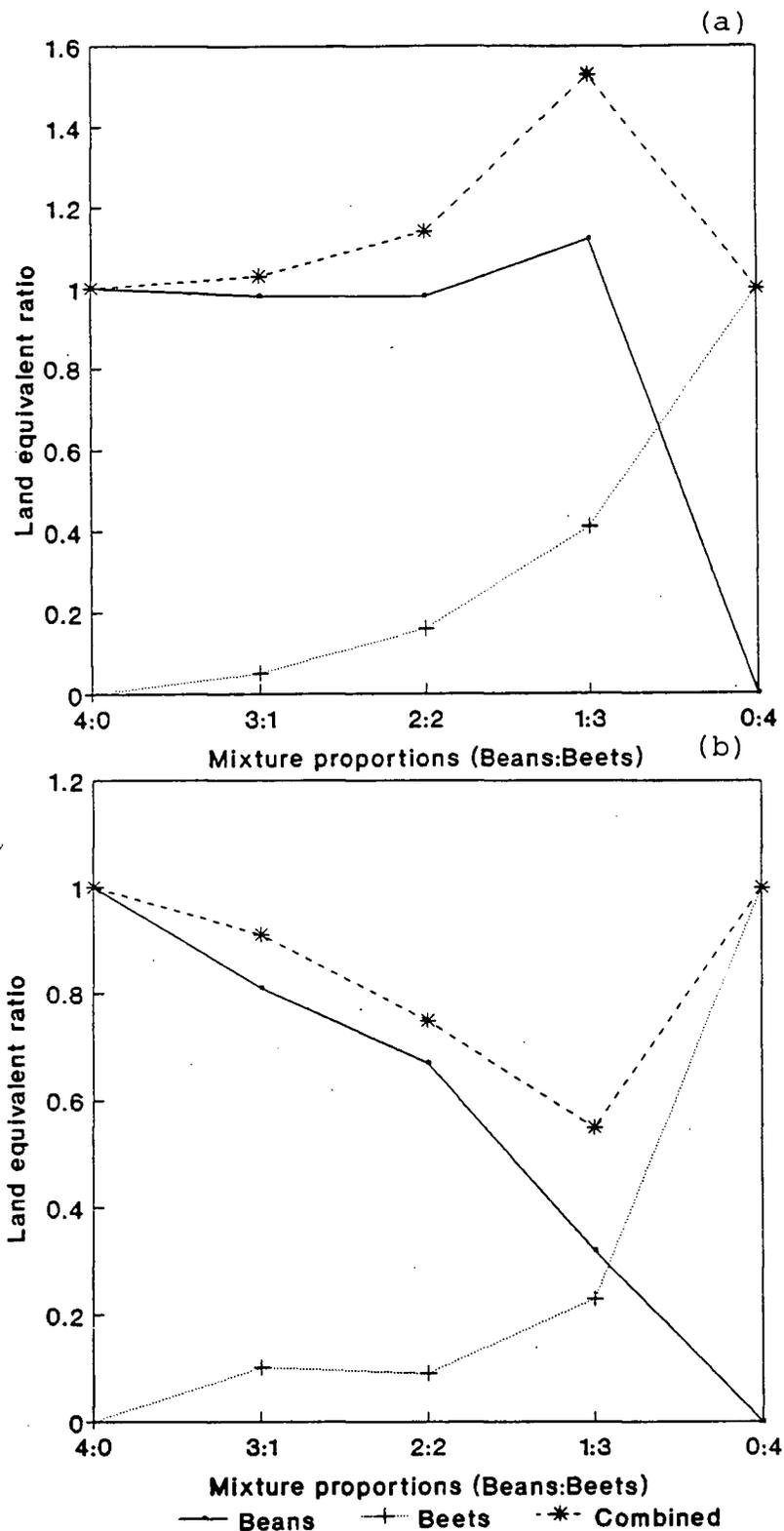
Fig. 4.9   Land equivalent ratio for total dry weight per unit land area at total population density of 16 plants m^{-2} (1984 experiment)

(a) Predicted
(b) Observed
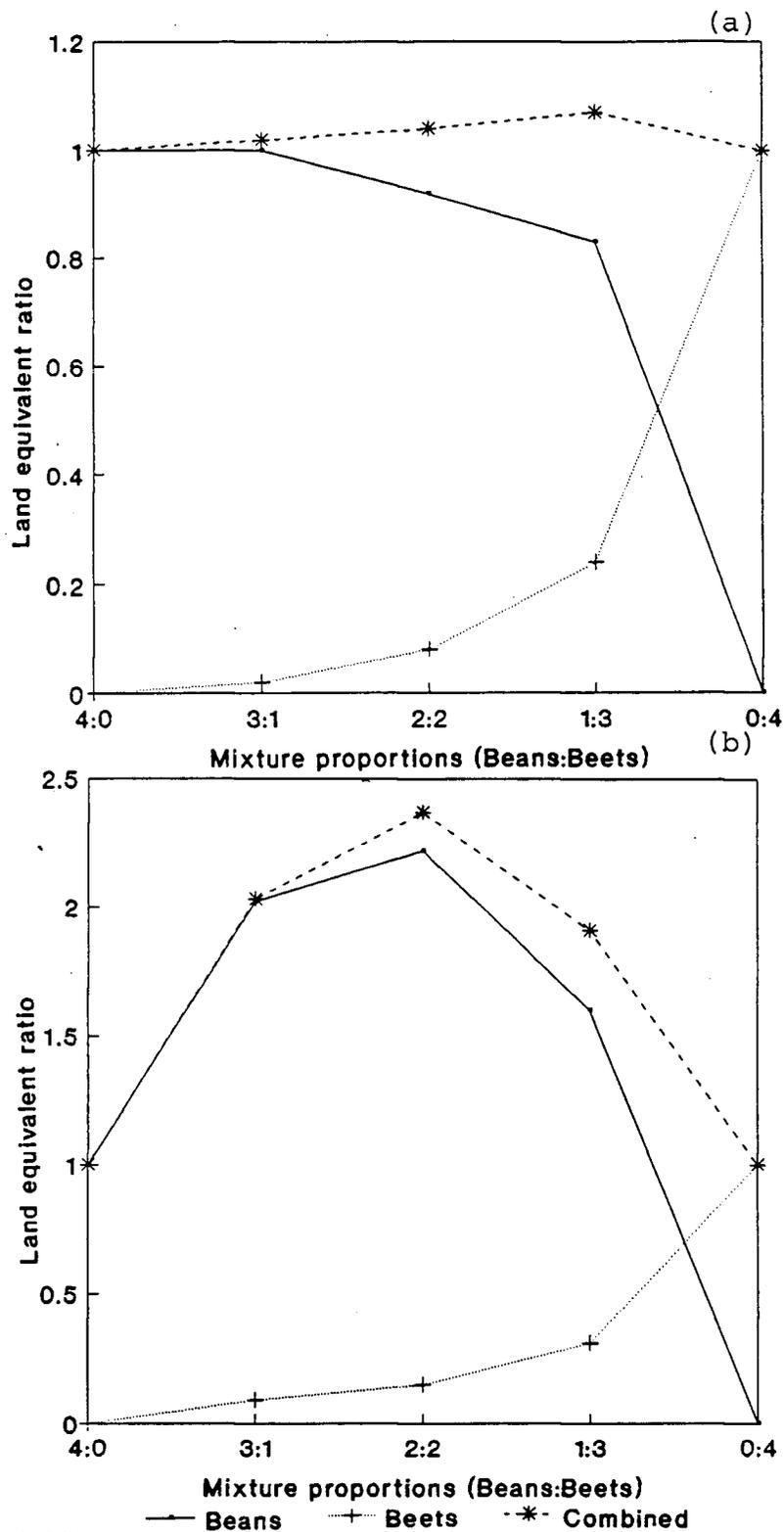
Fig. 4.10   Land equivalent ratio for leaf dry weight per unit land area at total population density of 66 plants m^{-2} (1984 experiment)

(a) Predicted
(b) Observed
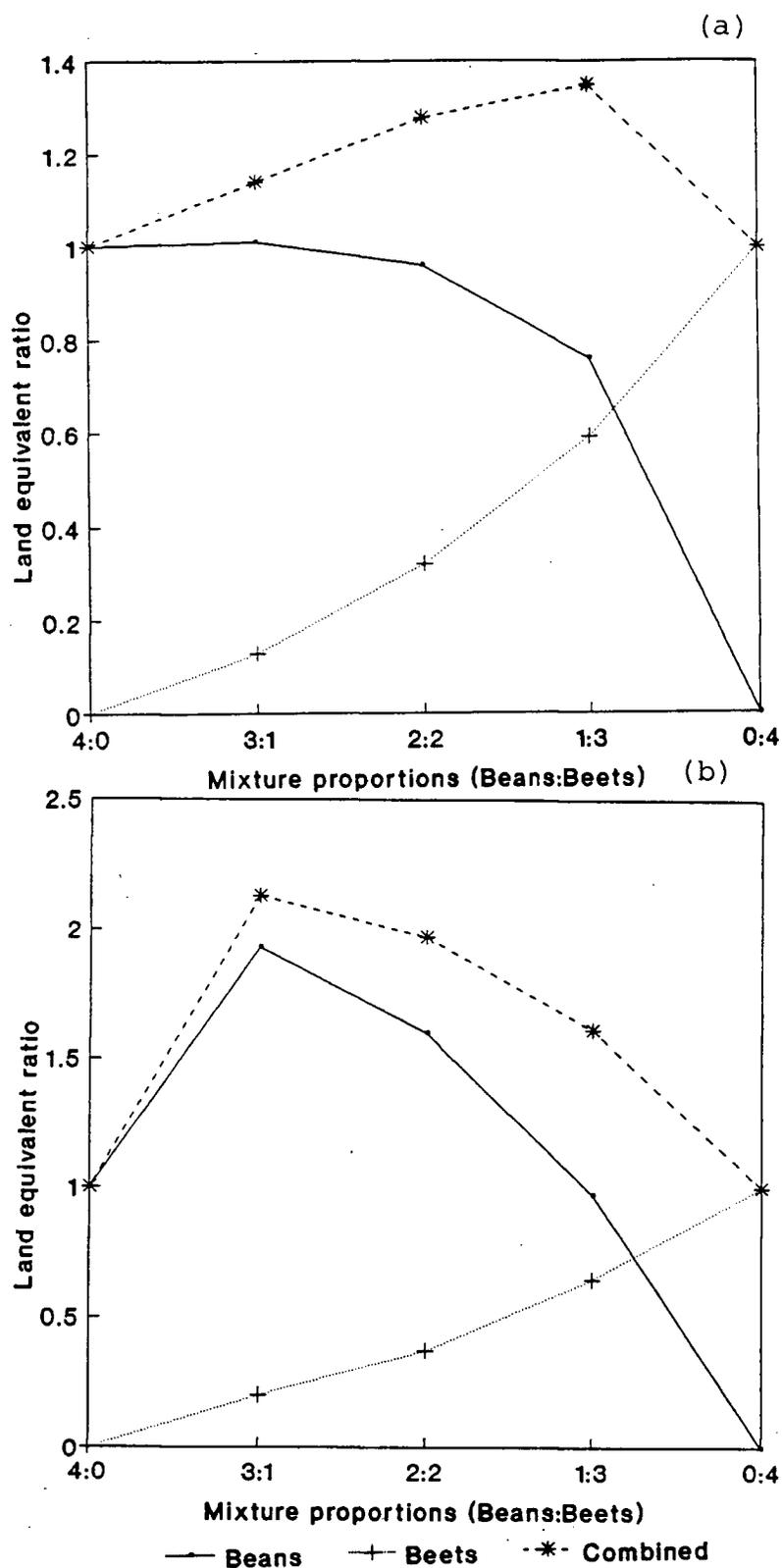
Fig. 4.11   Land equivalent ratio for leaf number per unit land area at total population density of 66 plants m^{-2} (1984 experiment)

(a) Predicted
(b) Observed
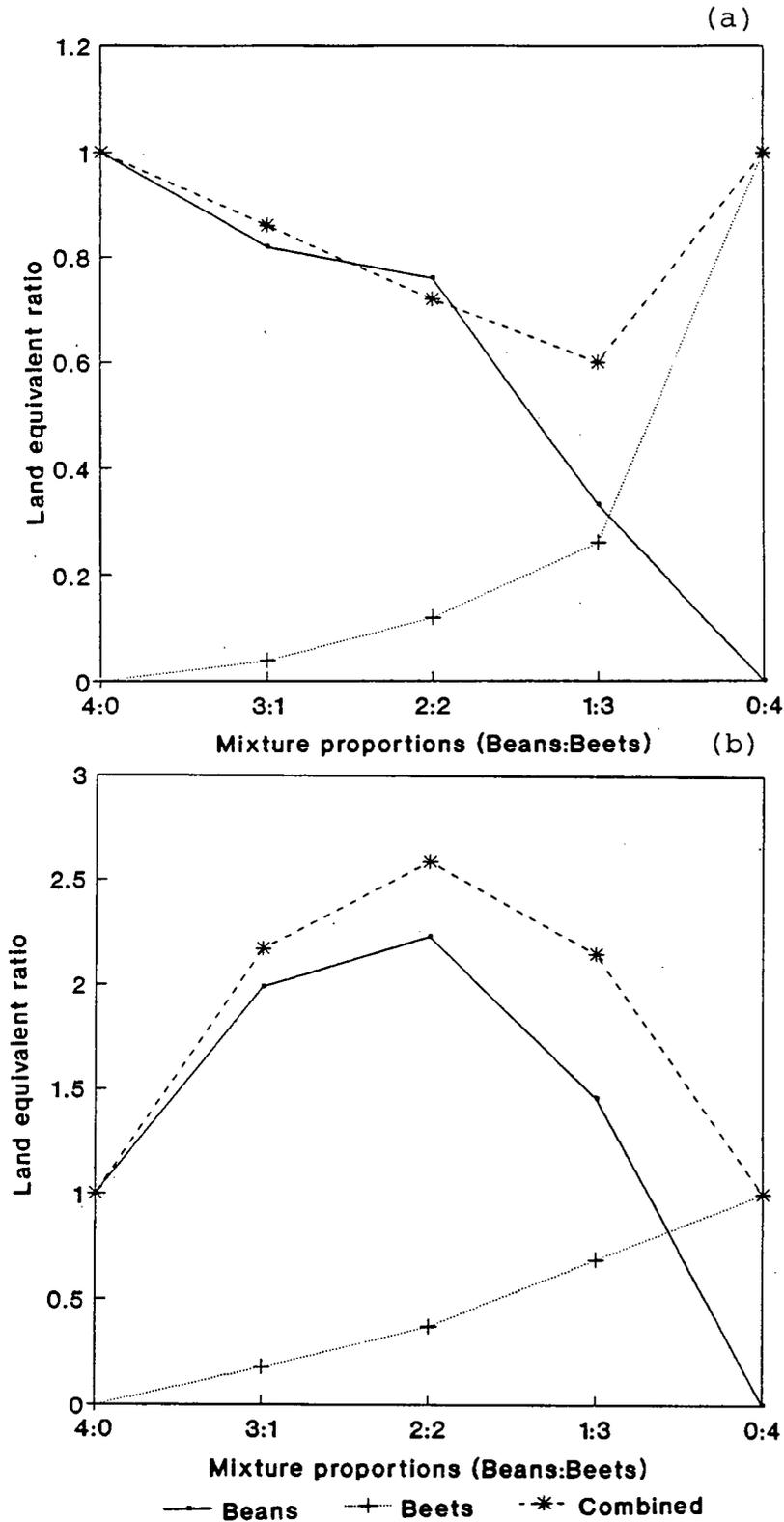
Fig. 4.12   Land equivalent ratio for leaf area per unit land area at total
population density of 66 plants m^{-2} (1984 experiment)

(a) Predicted
(b) Observed

in this variable. The problem was in beets where although the non-linear regressions converge, the parameters estimated were associated with error mean squares and coefficient of variation of over 1000. Thus the parameter estimates obtained could not produce reasonable LER when fitted to equation 3.4, but observed LER were greater than 1 indicating yield advantage at 66 plants m^{-2} (Fig. 4.13) and at other population densities as well (data not shown).

4.5.1 Summary of Differential Yield Responses Results

Applying the mathematical models to determine differential yield responses indicated yield advantage for leaf number and leaf dry weight and a yield disadvantage for total dry weight and leaf area in mixtures as compared to their corresponding monocultures while observed values indicated yield advantage in all the variables tasted.

4.6 Size Hierarchies

Size inequality among individuals in bean and beet populations was evaluated using the Gini coefficient. The Gini coefficient has a minimum value of 0, when all individuals are equal, and it has a maximum of 1.0 in an infinite population in which all individuals except one have a value of zero. Mathematically, the Gini coefficient is calculated from equation 3.5 and graphically, after ranking individuals according to biomass in ascending order, the cumulative percentage of biomass is plotted against the cumulative percentage of the population. A perfect equality will produce a diagonal line from the origin to the right corner (Appendix 8.2). The degree of deviation from the diagonal line, the Lorenz curve, is a measure of inequality. It can be expressed as the ratio
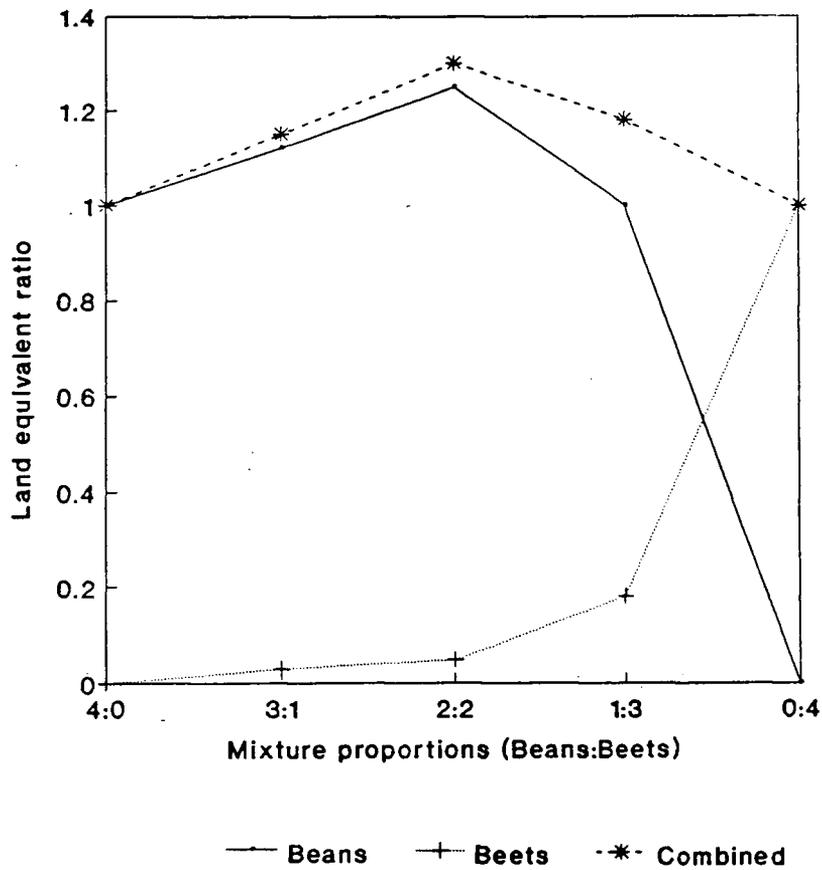
Fig. 4.13   Observed land equivalent ratio for marketable yield per unit land area at total population density of 66 plants m^{-2} (1984 experiment)

of the area between the diagonal line and the curve over the triangular area below the diagonal. This ratio is the Gini coefficient (Weiner and Solbrig 1984).

Tables 4.9 and Appendix 8.3 contain the results for the plant size distribution in beans while Table 4.10 and Appendix 8.4 contain those for beets. Size distribution of almost all the bean variables (Tables 4.9 and Appendix 8.3) seemed not to have followed any particular pattern in response to treatments except in the case of leaf number where in monoculture populations a steady increase in Gini coefficients with increasing population density was observed (Table 4.9c). When the mixture proportions were pooled, no pattern was again observed, but pooling densities consistently indicated that the bean monoculture treatment had the highest G' (unbaised Gini coefficient) in all variables except seed number and seed weight, and the 3:1 bean:beet ratio had the lowest G' except for marketable pod number, unmarketable pod number, seed number and seed weight. The ranking of the yield variables in order of increasing G', was as follows: leaf number (0.269) < total dry weight (0.291) < pod fresh weight (0.307) < stem dry weight (0.320) < marketable pod number (0.324) < marketable pod dry weight (0.336) < leaf area (0.338) < leaf dry weight (0.373) = seed dry weight (0.373) < seed number (0.374) < unmarketable pod number (0.384) < unmarketable pod weight (0.549) (Table 4.11a). When 95% confidence intervals were obtained from the bootstrapping procedure, it was found that leaf number had significantly lower size inequality compared to all variables tested. Total dry weight was not significantly different from pod dry weight which was in turn not significantly different from stem dry weight and marketable pod number. Stem dry weight was not significantly

Table 4.9a   Gini coefficients for total dry weight distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions 3:1	2:2	1:3	0:4	Mixture Pooled
16	G'	0.165	0.241	0.290	0.205	-	0.286
33	G'	0.275	0.185	0.202	0.173	-	0.259
50	G'	0.218	0.208	0.172	0.255	-	0.262
66	G'	0.286	0.190	0.236	0.239	-	0.344
Density Pooled	G'	0.287	0.208	0.242	0.237	-	0.291†

Table 4.9b   Gini coefficients for leaf dry weight distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions 3:1	2:2	1:3	0:4	Mixture Pooled
16	G'	0.232	0.321	0.366	0.250	-	0.380
33	G'	0.395	0.285	0.228	0.220	-	0.399
50	G'	0.273	0.284	0.208	0.359	-	0.342
66	G'	0.363	0.234	0.315	0.363	-	0.419
Density Pooled	G'	0.363	0.286	0.299	0.309	-	0.373†

*Beans:Beets
†Gini coefficient for the whole data set

Table 4.9c   Gini coefficients for leaf number distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	Mixture Proportions					Mixture Pooled
		4:0*	3:1	2:2	1:3	0:4	
16	G'	0.177	0.218	0.236	0.281	-	0.272
33	G'	0.190	0.169	0.218	0.361	-	0.211
50	G'	0.222	0.171	0.165	0.270	-	0.245
66	G'	0.323	0.133	0.259	0.231	-	0.333
Density Pooled	G'	0.281	0.175	0.231	0.257	-	0.269[†]

Table 4.9d   Gini coefficients for leaf area distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	Mixture Proportions					Mixture Pooled
		4:0*	3:1	2:2	1:3	0:4	
16	G'	0.208	0.316	0.336	0.272	-	0.338
33	G'	0.349	0.256	0.243	0.216	-	0.300
50	G'	0.227	0.299	0.186	0.333	-	0.316
66	G'	0.379	0.232	0.326	0.298	-	0.389
Density Pooled	G'	0.331	0.280	0.289	0.289	-	0.338[†]

*Beans:Beets

[†]Gini coefficient for the whole data set

Table 4.10a   Gini coefficients for total dry weight distribution of beets grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions 3:1	2:2	1:3	0:4	Mixture Pooled
16	G'	-	0.394	0.505	0.385	0.606	0.519
33	G'	-	0.397	0.595	0.448	0.343	0.548
50	G'	-	0.426	0.463	0.529	0.431	0.525
66	G'	-	0.530	0.499	0.772	0.376	0.757
Density Pooled	G'	-	0.472	0.604	0.800	0.500	0.625†

Table 4.10b   Gini coefficients for leaf dry weight distribution of beets grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions 3:1	2:2	1:3	0:4	Mixture Pooled
16	G'	-	0.315	0.410	0.316	0.409	0.388
33	G'	-	0.337	0.460	0.331	0.263	0.434
50	G'	-	0.300	0.340	0.430	0.321	0.398
66	G'	-	0.383	0.326	0.487	0.305	0.427
Density Pooled	G'	-	0.365	0.470	0.440	0.356	0.438†

*Beans:Beets
†Gini coefficient for the whole data set

Table 4.10c   Gini coefficients for live leaves distribution of beets grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions				Mixture Pooled
			3:1	2:2	1:3	0:4	
16	G'	-	0.141	0.152	0.172	0.219	0.179
33	G'	-	0.135	0.167	0.110	0.135	0.184
50	G'	-	0.128	0.143	0.150	0.125	0.163
66	G'	-	0.138	0.132	0.176	0.120	0.164
Density Pooled	G'	-	0.154	0.177	0.181	0.163	0.185†

Table 4.10d   Gini coefficients for leaf area distribution of beets grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions				Mixture Pooled
			3:1	2:2	1:3	0:4	
16	G'	-	0.308	0.362	0.296	0.501	0.391
33	G'	-	0.272	0.405	0.294	0.237	0.384
50	G'	-	0.293	0.324	0.391	0.363	0.384
66	G'	-	0.348	0.330	0.443	0.271	0.365
Density Pooled	G'	-	0.333	0.434	0.403	0.401	0.413†

*Beans:Beets
†Gini coefficient for the whole data set

different from marketable pod number, marketable pod dry weight, and leaf area. No significant differences were observed between leaf dry weight, seed dry weight, seed number and unmarketable pod number, but unmarketable pod dry weight had significantly the highest G' among all variables tested (Table 4.11a).

In beet monocultures, all the variables tested showed a general decline in the Gini coefficients with increasing population density. In all cases, the highest Gini coefficient was obtained at a population density of 16 plants m^{-2}, while the lowest value was at 33 plants m^{-2} with the other two density treatments lying in between (Table 4.10 and Appendix 8.4). The beet plants grown in mixture with beans showed no clear pattern in their Gini coefficients among treatments.

Whereas pooling the densities did not show any pattern in G', when mixture proportions were pooled, there was a general increase in G' with increasing population density for all variables except leaf area and number of dead leaves. For beets, the G' results were as follows in an ascending order: leaf number (0.185) < root diameter (0.350) < leaf area (0.413) < dead leaf number (0.436) < leaf weight (0.438) < petiole dry weight (0.449) < total dry weight (0.625) < root dry weight (0.641) < root fresh weight (0.661) (Table 4.11b). Based upon the confidence interval obtained from the bootstrapping procedure, leaf number had significantly the lowest while root diameter had the second lowest size inequality compared to the other variables tested. Leaf area and number of dead leaves did not differ significantly from each other so were number of dead leaves, leaf dry weight and petiole dry weight. Total dry weight, root dry weight and root fresh weight had significantly the highest Gini coefficients indicating high size inequality (Table 4.11b).

Table 4.11a   Gini coefficients for bean yield variables in ascending order

Variable	Gini coefficient (G')	95% confidence interval
LN	0.269a*	0.257 - 0.281
W	0.291b	0.282 - 0.300
FWPD	0.307bc	0.296 - 0.318
WST	0.320cd	0.312 - 0.328
MPN	0.324cd	0.314 - 0.334
WMPD	0.336d	0.325 - 0.347
LA	0.338d	0.328 - 0.348
WL	0.373e	0.361 - 0.385
WS	0.373e	0.362 - 0.384
SN	0.374e	0.365 - 0.383
UPN	0.384e	0.375 - 0.393
WUPD	0.549f	0.542 - 0.556

Table 4.11b   Gini coefficients for beet yield variables in ascending order

Variable	Gini coefficient (G')	95% confidence interval
LN	0.185a	0.181 - 0.189
DR	0.350b	0.338 - 0.362
LA	0.413c	0.404 - 0.422
DLN	0.436cd	0.421 - 0.451
WL	0.438d	0.427 - 0.449
WP	0.449d	0.439 - 0.459
W	0.625e	0.598 - 0.652
WR	0.641e	0.627 - 0.655
FWR	0.661e	0.647 - 0.675

*Gini coefficients with the same letter are not significantly different from each other based upon 95% confidence interval (P <0.05).

4.6.1 Summary of Size Distribution Results

In summary, high Gini coefficients were generally obtained in most variables in both species. Responses due to treatments were not detected but in beets, a general decline in Gini coefficients with increasing population density was observed. Comparisons within species but between variables indicated that leaf number had the lowest Gini coefficient in both species but comparisons between species seemed to indicate that beets had higher G's than beans.

4.7 Yield Component Analysis

The contribution of yield components to the total yield variation, and the relationships among yield components were determined. The two dimensional partitioning (TDP) procedure (Eaton *et al.* 1985) was used in the analysis.

4.7.1 Beans 1984

The data for the 1984 bean experiment collected at 40 days from planting were not analyzed as most of the plants at this stage had no pod dry weight, the component considered to be yield. The results for later harvest dates indicated that some effects of treatment were significant from 63 days through to 92 days after planting (Table 4.12). The effects of population density on total yield variation rose from 0% at 51 days to 42% at the final harvest. For mixture proportions, the percent of total yield variation rose from 1% at 51 days from planting to 16% at the final harvest. The population density by mixture proportions interactions were significant, but remained relatively constant at about 5% throughout the growing period (Table 4.12).

Table 4.12a   Two dimensional partitioning of yield variation in beans: 1984 data (forward analysis)

Age at Harvest (days)	Source of Variation	d.f.	LN	LA/LN	WL/LA	WST/WL	PN/WST	WPD/PN	Sum of Product	Yield WPD
51	Blocks	2	0	0	0	0	3	0	1	5*
(HN 2)[†]	Density	3	5**	0	0	1	3	0	-8	0
	Mixture	3	1	0	0	0	1	0	-1	1
	D x M	9	1	0	0	0	4*	0	-1	5*
	Exp. Err.	30	2*	0	0	1	16**	0	2	22**
	Samp. Err.	192	10**	0	0	4**	45**	2	7	67**
	Total	239	18**	0	0	6**	73**	3	-	100
63	Blocks	2	0	0	0	0	1	6**	10	17**
(HN 3)	Density	3	8**	0	0	0	0	1	-2	8**
	Mixture	3	3	0	0	1	0	0	-3	1
	D x M	9	1	0	0	0	1	4*	-1	5*
	Exp. Err.	30	2	1	0	1	5**	6**	-5	11**
	Samp. Err.	192	12**	1	0	1	20**	22**	2	58**
	Total	239	27**	3**	0	3**	27**	39**	-	100
69	Blocks	2	0	0	0	0	0	1	2	4
(HN 4)	Density	3	20**	0	0	0	0	0	6	26**
	Mixture	3	13**	0	0	0	0	0	4	17**
	D x M	9	3	0	0	0	0	1	-1	3
	Exp. Err.	30	4**	2**	0	0	0	4	-6	4**
	Samp. Err.	192	27**	3**	1	0	2	16**	-3	46**
	Total	239	66**	7**	2	0	2	23**	-	100
75	Blocks	2	1	1	0	0	0	0	-2	0
(HN 5)	Density	3	21**	0	0	0	0	0	15	37**
	Mixture	3	10**	0	0	0	0	1	0	11**
	D x M	9	3	1	0	0	0	2	3	9**
	Exp. Err.	30	3**	3	0	0	1	7**	-7	6**
	Samp. Err.	192	22**	4**	1	0	6**	13**	-8	38**
	Total	239	60**	8**	1	0	8**	22**	-	100
92	Blocks	2	0	0	0	0	0	0	-1	0
(HN 6)	Density	3	8**	1	2	1	0	0	30	42**
	Mixture	3	5**	0	2	0	0	0	9	16**
	D x M	9	2	1	0	1	1	0	-2	3
	Exp. Err.	30	4**	9**	2	2	2	2	-10	9**
	Samp. Err.	192	16**	13**	9**	5**	5**	8**	-25	30**
	Total	239	35**	24**	15**	8**	8**	10**	-	100

[†]Harvest number (1-6 for beans and 1-7 for beets)

Note: Numbers within the table are expressed as a percentage of total sum of squares for yield at each harvest.

*Significant at P = 0.05
**Significant at P = 0.01

Table 4.12b Two dimensional partitioning of yield variation in beans: 1984 data (backward analysis)

Age at Harvest (days)	Source of Variation	d.f.	Yield Components WPD/PN	PN/WST	WST/WL	WL/LA	LA/LN	LN	Sum of Product	Yield WPD
51	Blocks	2	3	0	0	0	0	0	1	5*
(HN 2)†	Density	3	1	1	0	0	1	0	-2	0
	Mixture	3	1	0	0	0	0	0	0	1
	D x M	9	3	1	0	0	0	0	0	5*
	Exp. Err.	30	16**	3**	0	0	1	0	2	22**
	Samp. Err.	192	58**	10**	0	0	1	1	-2	68**
	Total	239	81**	15**	0	1	3**	1	-	100
63	Blocks	2	21**	0	0	0	0	1	-5	17**
(HN 3)	Density	3	2	0	0	0	1	0	5	8**
	Mixture	3	0	0	0	1	0	0	0	1
	D x M	9	7**	0	0	0	0	0	-2	5*
	Exp. Err.	30	12**	0	0	1	1	1	-4	11**
	Samp. Err.	192	46**	0	0	1	2**	3**	6	58**
	Total	239	88**	0	0	3**	4**	4**	-	100
69	Blocks	2	1	0	0	0	1	0	2	4
(HN 4)	Density	3	1	0	0	0	4*	4*	17	26**
	Mixture	3	2	0	0	0	2	3	10	17**
	D x M	9	2	0	0	0	1	2	-1	3
	Exp. Err.	30	8**	0	0	0	6**	3	-13	4
	Samp. Err.	192	29**	0	0	1	13**	18**	-15	46**
	Total	239	43**	0	0	2**	26**	29**	-	100
75	Blocks	2	1	0	0	0	1	1	-3	0
(HN 5)	Density	3	1	0	0	0	2	9**	24	37**
	Mixture	3	1	0	0	0	0	3	6	11**
	D x M	9	3	0	0	1	1	2	2	9**
	Exp. Err.	30	8**	0	0	2	5**	4**	-13	6**
	Samp. Err.	192	17**	0	1	9**	10**	18**	-17	38**
	Total	239	31**	0	1	13**	19**	36**	-	100
92	Blocks	2	0	0	0	0	0	0	-1	0
(HN 6)	Density	3	0	0	0	1	3	12**	26	42**
	Mixture	3	0	0	1	1	1	1	13	16**
	D x M	9	0	0	0	0	2	2	-1	3
	Exp. Err.	30	2**	0	1	1	9**	5**	-8	9**
	Samp. Err.	192	10**	0	3	5**	15**	25**	-29	30**
	Total	239	13**	1	6**	8**	29**	44**	-	100

†Harvest number (1-6 for beans and 1-7 for beets)

Note: Numbers within the table are expressed as a percentage of total sum of squares for yield at each harvest.

*Significant at P = 0.05

**Significant at P = 0.01

Block effects were larger in the early stages of growth and declined to 0% in the later stages. The experimental error was significant, indicating large variations among plants within the same treatment.

In the forward analysis, the yield components were included in the stepwise regression in chronological order of their development. This analysis indicated that treatments had strong effects on yield late in the growing season (Table 4.12a)

The yield components, LN, LA/LN, PN/WST and WPD/PN made significant contributions to total yield variation at all harvest dates, except LA/LN did not make a significant contribution at harvest 2 or PN/WST at harvest 4. Other yield components, namely WST/WL and WL/LA, were significant at harvests, 2, 3 and harvest 6 respectively (Table 4.12a). The yield component LN was almost always the major component source of the treatment effects. It was affected by population density throughout the growing season. Mixture proportions effected LN starting at harvest 4, and population density by mixture proportions interactions were not significantly effective. The component PN/WST was a source of treatment effects at harvest 2, and WPD/PN contributed significantly at harvest 3. Both of those components were significantly affected by population density by mixture proportions interactions. Other yield components contributed to yield variation, especially at the final harvest, but not through direct treatment effects.

In the backward analysis, yield components were fitted in the stepwise regression equation in the inverse order of their chronological development. The analysis indicated that WPD/PN and LA/LN always had significant contributions to total yield at all harvest dates (Table 4.12b). The yield component PN/WST had a significant effect only at the

second, harvest in contrast to WST/WL which made significant contributions to yield variation only at the last harvest. WL/LA and LN made significant contributions to yield variation at all harvests except at 51 days from planting. Significant effects due to population density were observed for LA/LN at 69 days and for LN at 69, 75 and 92 days. Significant effects due to population density by mixture proportions interactions, were observed with the yield component WPD/PN at 63 after planting

4.7.2 Beans 1987

In the 1987 study, where only one final harvest was done at about 83 days from planting, the TDP for bean indicated that yield was significantly affected by population density, mixture proportions and population density by mixture proportions interactions (Table 4.13). Due to large variations observed among plants within the same treatment, none of the components showed significant effects due to treatments. Overall, the yield components LN, WL/LA, PN/WST and WPD/PN made significant contributions to yield variation in the forward analysis, and the yield components WPD/PN, PN/WST, LA/LN and LN had significant effects in the backward analysis (Table 4.13a and b).

4.7.3 Beets 1984

Similar to the bean results for the 1984 growing season, the beet data collected at 40 days from planting were not analyzed because the storage root had not began to grow. The results obtained in the forward analysis at later harvests indicated that both population density and mixture proportions treatments had significant effects at harvest dates

Table 4.13a   Two dimensional partitioning of yield variation in beans: 1987 data (forward analysis)

Source of Variation	d.f.	Yield Components						Sum of Product	Yield WPD
		LN	LA/LN	WL/LA	WST/WL	PN/WST	WPD/PN		
Blocks	1	0	0	0	0	1	0	0	1
Density	3	1	0	0	0	1	0	3	4*
Mixture	3	0	0	0	0	0	0	2	3
D x M	9	0	0	0	0	2	0	2	4*
Exp. Err.	15	1	0	1	0	2	0	4	7**
Samp. Err.	32	1	0	0	0	3	0	2	6**
Sub Samp. Err.	896	5**	0	3	0	72**	5**	-11	74**
Total	959	8**	0	4**	0	81**	6**	-	100

Table 4.13b   Two dimensional partitioning of yield variation in beans: 1987 data (backward analysis)

Source of Variation	d.f.	Yield Components						Sum of Product	Yield WPD
		WPD/PN	PN/WST	WST/WL	WL/LA	LA/LN	LN		
Blocks	1	0	2	0	0	0	0	-1	1
Density	3	0	2	0	0	0	0	2	4*
Mixture	3	0	1	0	0	0	0	1	3
D x M	9	0	2	0	0	0	0	1	4*
Exp. Err.	15	0	2	0	0	1	1	3	8**
Samp. Err.	32	0	5**	0	0	1	1	-1	6**
Sub Samp. Err.	896	7**	62**	0	0	3**	7**	-5	74**
Total	959	8**	77**	0	0	5**	10**	-	100

Note: Numbers within the table are expressed as a percentage of total sum of squares for yield at each harvest.

*Significant at P = 0.05
**Significant at P = 0.01

(Table. 4.14a). Population density by mixture interactions were significant except at 63 and 92 days after planting. The effect of population density rose from 8% at 51 days from planting to 33% at both 75 and 92, days and then declined to 15% at 107 days after planting. The mixture effect also rose from 20% at 51 days from planting to 34% at 107 days from planting. The population density by mixture proportions interaction remained below 10% throughout the growing period (Table 4.14a and b). Blocks did not make any significant contributions to variation in yield for beets.

The forward analysis (Table 4.14a) indicated that the yield components LN, LA/LN, and WL/LA made significant contributions to yield variation at all harvest dates. The yield component DR/WL was significant at all harvests except at 69 days from planting. WR/DR was significant earlier in the growing season, but was not significant at both 92 and 107 days from planting. At all harvest dates, the main source of treatment effects was LN which was significantly affected by both population density and mixture proportions. The population density by mixture proportions interaction was also significant at 51 and 69 days from planting.

In the backward analysis (Table 4.14b), the yield component WR/DR accounted for almost all the yield variation. DR/WL also had a significant contribution at 69 days from planting and LA/LN contributed significantly to yield variation at 51, 63, and 69 days from planting. The source of treatment effects was WR/DR, with population density and mixture proportions affecting that component significantly at all harvests from 51 days onwards. Population density by mixture proportions

Table 4.14a   Two dimensional partitioning of yield variation in beets: 1984 data (forward analysis)

Age at Harvest (days)	Source of Variation	d.f.	LN	Yield Components				Sum of Product	Yield WR
				LA/LN	WL/LA	DR/WL	WR/DR		
51 (HN 2)[†]	Blocks	2	0	1	0	0	0	-1	1
	Density	3	14**	0	0	0	1	-7	8**
	Mixture	3	9**	·0	1	0	0	9	20**
	D x M	9	7**	0	0	1	1	9	18**
	Exp. Err.	30	9**	2**	1	1	6**	3	23**
	Samp. Err.	192	23**	10**	2**	3**	6**	-4	39
	Total	239	62**	14**	5**	5**	14**	-	100
63 (HN 3)	Blocks	2	0	0	0	0	0	-1	1
	Density	3	16**	1	0	0	0	6	24**
	Mixture	3	13**	0	4	0	0	12	29**
	D x M	9	2**	1	1	0	0	-1	3
	Exp. Err.	30	11**	2**	3**	2**	1	-5	13**
	Samp. Err.	192	24**	4**	7**	4**	4**	-12	31**
	Total	239	66**	7**	15**	7**	6**	-	100
69 (HN 4)	Blocks	2	2	0	2	0	0	0	3
	Density	3	9**	0	1	0	0	1	11**
	Mixture	3	10**	1	6**	0	0	12	29**
	D x M	9	4**	1	1	0	1	0	5*
	Exp. Err.	30	12**	4**	1	0	2**	3	22**
	Samp. Err.	192	28**	5**	6**	1	5**	-15	30**
	Total	239	64**	10**	17**	1	8**	-	100
75 (HN 5)	Blocks	2	1	0	1	0	0	-3	1
	Density	3	16**	0	0	0	0	8	24**
	Mixture	3	13**	0	4**	0	0	14	33**
	D x M	9	3	1	1	0	0	1	7**
	Exp. Err.	30	18**	2**	3**	1	1	-6	19**
	Samp. Err.	192	18**	2**	5**	4**	2**	-14	17**
	Total	239	70**	6**	15**	6**	3**	-	100
92 (HN 6)	Blocks	2	0	2	0	1	0	-2	1
	Density	3	12**	0	0	0	0	7	20**
	Mixture	3	11**	0	5**	1	0	15	33**
	D x M	9	1	1	2	0	0	-2	2
	Exp. Err.	30	12**	4**	3**	2**	1	-2	20**
	Samp. Err.	192	23**	5**	6**	7**	1	-16	25**
	Total	239	59**	13**	16**	10**	2	-	100
107 (HN 7)	Blocks	2	3	1	1	0	0	-4	1
	Density	3	6**	1	1	0	0	7	15**
	Mixture	3	7**	0	4	1	0	20	34**
	D x M	9	3	1	1	1	0	-2	5*
	Exp. Err.	30	11**	3**	1	2**	0	-3	15**
	Samp. Err.	192	26**	5**	3**	8**	1	-17	30**
	Total	239	57**	10**	11**	13**	2	-	100

[†]Harvest number (1-6 for beans and 1-7 for beets)

Note: Numbers within the table are expressed as a percentage of total sum of squares for yield at each harvest.

*Significant at P = 0.05

**Significant at P = 0.01

Table 4.14b   Two dimensional partitioning of yield variation in beets: 1984 data (backward analysis)

Age at Harvest (days)	Source of Variation	d.f.	WR/DR	Yield Components DDR/WL	WL/LA	LA/LN	LN	Sum of Product	Yield WR
51	Blocks	2	1	0	0	0	0	0	1
(HN 2)†	Density	3	4*	0	0	0	0	4	8**
	Mixture	3	15**	0	0	0	0	5	20**
	D x M	9	7**	0	0	0	0	2	9**
	Exp. Err.	30	29**	0	0	1	0	-7	23**
	Samp. Err.	192	37**	1	0	3**	1	-2	39**
	Total	239	92**	1	0	5**	1	-	100
63	Blocks	2	1	0	0	0	0	0	1
(HN 3)	Density	3	22**	0	0	0	0	2	24**
	Mixture	3	28**	0	0	0	0	0	29**
	D x M	9	3	0	0	0	0	0	3
	Exp. Err.	30	12**	0	0	1	0	0	13**
	Samp. Err.	192	30**	0	0	2**	1	-2	31**
	Total	239	96**	0	0	3**	1	-	100
69	Blocks	2	0	1	0	0	0	1	3
(HN 4)	Density	3	6**	1	0	1	0	4	11**
	Mixture	3	18**	1	0	0	0	9	29**
	D x M	9	6**	1	0	1	0	-2	5*
	Exp. Err.	30	19**	5**	0	3**	0	-5	22**
	Samp. Err.	192	23**	9**	0	4**	1	-7	30**
	Total	239	72**	18**	0	9**	1	-	100
75	Blocks	2	0	0	0	0	0	0	1
(HN 5)	Density	3	22**	0	0	0	0	-2	24**
	Mixture	3	34**	0	0	0	0	-1	33**
	D x M	9	7**	0	0	0	0	0	7**
	Exp. Err.	30	18**	0	0	0	0	1	19**
	Samp. Err.	192	17**	0	0	1	1	-1	17**
	Total	239	98**	0	0	1	1	-	100
92	Blocks	2	0	1	0	0	0	0	1
(HN 6)	Density	3	19**	0	0	0	0	0	20**
	Mixture	3	33**	0	0	0	0	-1	33**
	D x M	9	3	0	0	0	0	-1	2
	Exp. Err.	30	18**	0	0	0	0	1	20**
	Samp. Err.	192	25**	0	0	0	0	-3	25**
	Total	239	98**	0	0	1	1	-	100
107	Blocks	2	1	0	0	0	0	0	1
(HN 7)	Density	3	14**	0	0	0	0	1	15**
	Mixture	3	33**	0	0	0	0	1	34**
	D x M	9	4*	0	0	0	0	1	5*
	Exp. Err.	30	15**	0	0	0	0	-1	15**
	Samp. Err.	192	31**	0	0	0	0	-2	30**
	Total	239	99**	0	0	0	1	-	100

†Harvest number (1-6 for beans and 1-7 for beets)

Note: Numbers within the table are expressed as a percentage of total sum of squares for yield at each harvest.

*Significant at P = 0.05

**Significant at P = 0.01

interactions was also significant at 51, 69, 75 and 107 days from planting.

4.7.4 Beets 1987

The forward analysis performed on beet data from the final harvest done at about 90 days after planting in 1987 showed that yield was influenced by population density, mixture proportions and population density by mixture proportions interactions. The last line in Table 4.15a indicates that all five yield components made significant contributions to variation in yield. The source of treatment effects was LN, which was significantly affected by population density,and mixture proportions. Blocks did not make any significant contributions to variation in total yield.

In the backward analysis, the yield components WR/DR and LA/LN made significant contributions to variation in yield. The treatment effects came from WR/DR which was significantly affected by population density, mixture proportions and their interactions (Table 4.15b).

4.7.5 Summary of Yield Component Analysis Results

Treatments were found to affect final agricultural yield of both beans and beets later in the growing season in 1984. Treatment effects were more drastic for main effects than their interactions in both years. In both the forward and backwards analysis, the yield component that entered into the equation first contributed the most to total yield variation. The yield component LN was generally found to be the source of treatment effects.

Table 4.15a   Two dimensional partitioning of yield variation in beets: 1987 data (forward analysis)

Source of Variation	d.f.	Yield Components LN	LA/LN	WL/LA	DR/WL	WR/DR	Sum of Product	Yield WR
Blocks	1	0	0	0	0	0	0	0
Density	3	9**	0	0	0	0	5	15**
Mixture	3	16**	0	0	0	0	10	27**
D x M	9	3	1	0	1	1	0	5**
Exp. Err.	15	3**	1	0	0	1	0	7**
Samp. Err.	32	8**	4**	1	1	1	4	18**
Sub Samp. Err.	192	25**	10**	3**	4**	5**	-18	28**
Total	255	64**	16**	4**	6**	9**	-	100

Table 4.15b   Two dimensional partitioning of yield variation in beets: 1987 data (backward analysis)

Source of Variation	d.f.	Yield Components WR/DR	DDR/WL	WL/LA	LA/LN	LN	Sum of Product	Yield WR
Blocks	1	0	0	0	0	0	0	0
Density	3	14**	0	0	0	0	1	15**
Mixture	3	23**	0	0	0	0	4	27**
D x M	9	5*	0	0	0	0	-1	5*
Exp. Err.	15	5**	0	0	0	0	1	7**
Samp. Err.	32	15**	0	0	1	0	2	18**
Sub Samp. Err.	192	30**	1	0	3**	1	-6	28**
Total	255	93**	1	0	4**	2	-	100

Note: Numbers within the table are expressed as a percentage of total sum of squares for yield at each harvest.

*Significant at P = 0.05

**Significant at P = 0.01

4.8 Plant Growth Analysis

Conventional plant growth analysis was used to determine the quantitative effect of experimental treatments over time using growth indices but before discussing plant growth analysis results, ANOVA results for ratio indices (leaf area ratio (F), specific leaf area (SLA), leaf weight ratio (LWR) and harvest index (H)) will be dealt with first. Analysis of variance results for total dry weight, leaf dry weight, and leaf area were already stated in section 4.2.2. Regression results for the primary variables (total dry weight, leaf dry weight and leaf area) will then be described, followed by those for ratio indices (leaf area ratio, specific leaf area, leaf weight ratio, and harvest index). Results from indices computed from first derivatives (absolute growth rate (AGR), relative growth rate (R) and unit leaf rate(E)) will be discussed last. Since fitted curves were not done on the 1987 data, where only one harvest was performed on each species, only growth curves for the 1984 data will be presented.

4.8.1 Beans 1984: Growth Indices (Leaf Area Ratio, Specific Leaf Area, Leaf Weight Ratio and Harvest Index): Analysis of Variance Results

Analysis of variance done on growth indices for beans in 1984 indicated that leaf area ratio and harvest index were not significantly affected by treatments at any of the harvests except for leaf area ratio at harvests 5 and 1 where population density and mixture proportions had a significant effect respectively. On the other hand, specific leaf area responded significantly different to both treatments at all harvest dates (Table 4.16). Specific leaf area increased with increasing population

Table 4.16   Analysis of variance results for the 1984 bean data: Variance ratios for the effects of population density and mixture proportions on growth indices tested at different stages of growth

Age at Harvest (days)	Source of Variation	d.f.	F	Variables SLA	LWR	H
40 (HN 1)[†]	Blocks	2	2.83**	3.12	1.13	-
	Density	3	1.86	3.17*	24.12**	-
	Mixture	3	3.78*	9.90**	6.42**	-
	D x M	9	1.26	1.17	2.28*	-
	Exp. Err.	30	2.11**	1.68**	1.68**	-
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
51 (HN 2)	Blocks	2	7.51**	16.02**	2.71	2.31
	Density	3	1.26	5.66**	8.17**	0.24
	Mixture	3	1.68	7.72**	6.86**	0.46
	D x M	9	0.36	0.36	0.60	0.81
	Exp. Err.	30	3.44**	3.87**	2.87**	1.34
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
63 (HN 3)	Blocks	2	11.14**	10.79**	19.02**	22.32**
	Density	3	0.77	3.05*	1.14	0.15
	Mixture	3	0.36	13.14**	0.61	0.45
	D x M	9	1.57	0.39	1.52	1.70
	Exp. Err.	30	1.74**	3.55**	2.02**	1.25
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
69 (HN 4)	Blocks	2	0.24	17.09**	7.23**	2.98
	Density	3	1.97	3.46*	1.64	1.93
	Mixture	3	1.68	5.54**	1.21	1.52
	D x M	9	0.63	0.67	0.58	0.54
	Exp. Err.	30	2.26**	1.50*	2.06**	2.31**
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
75 (HN 5)	Blocks	2	0.11	8.33**	0.68	0.43
	Density	3	2.93*	7.67**	0.57	0.33
	Mixture	3	2.21	12.38**	0.96	0.83
	D x M	9	1.41	0.89	0.81	0.80
	Exp. Err.	30	2.51**	2.27**	2.74**	4.68**
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
92 (HN 6)	Blocks	2	3.30*	1.44	0.85	1.54
	Density	3	1.30	4.33**	1.97	0.51
	Mixture	3	0.18	4.29**	3.34*	0.03
	D x M	9	0.29	1.24	0.91	0.94
	Exp. Err.	30	0.02	1.24	0.13	0.49
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

density and with decreasing beet proportions in mixtures (Fig 4.14). Leaf weight ratio was not affected by treatments except at harvests 1 and 2 where both population density and mixture proportions significantly reduced leaf weight ratio with increasing population density and decreasing beets proportions in mixtures (Fig. 4.15). Leaf weight ratio was also significantly reduced with decreasing beets proportions at harvest 6 (Fig. 4.15b). No differences were observed due to population density by mixture proportions interactions in any of the indices throughout the growing period.

4.8.2 Beans 1987: Growth Indices (Leaf Area Ratio, Specific Leaf Area, Leaf Weight Ratio and Harvest Index): Analysis of Variance Results

Leaf area ratio, specific leaf area and leaf weight ratio results of beans in 1987 did not show any significant response to both main treatment factors (Table 4.17). Whereas no particular trends were noticed for leaf area ratio and leaf weight ratio, specific leaf area tended to be lowest for plants grown at 33 plants m^{-2} and was highest for plants grown at 66 plants m^{-2}. Plants at population density of 50 plants m^{-2} treatment had the second highest while those at 16 plants m^{-2} had the second lowest (data not shown). Regarding mixture proportions treatments, the 3:1 bean:beet ratio tended to have the lowest leaf area ratio and it increased with increasing beets proportions (data not shown).

Similarly, the ANOVA results for harvest index indicated that both population density and mixture proportions did not significantly affect the proportions of the marketable yield (Table 4.17). Qualitatively, there was a non-significant increase from 51% at 16 plants m^{-2} to 53% at 33
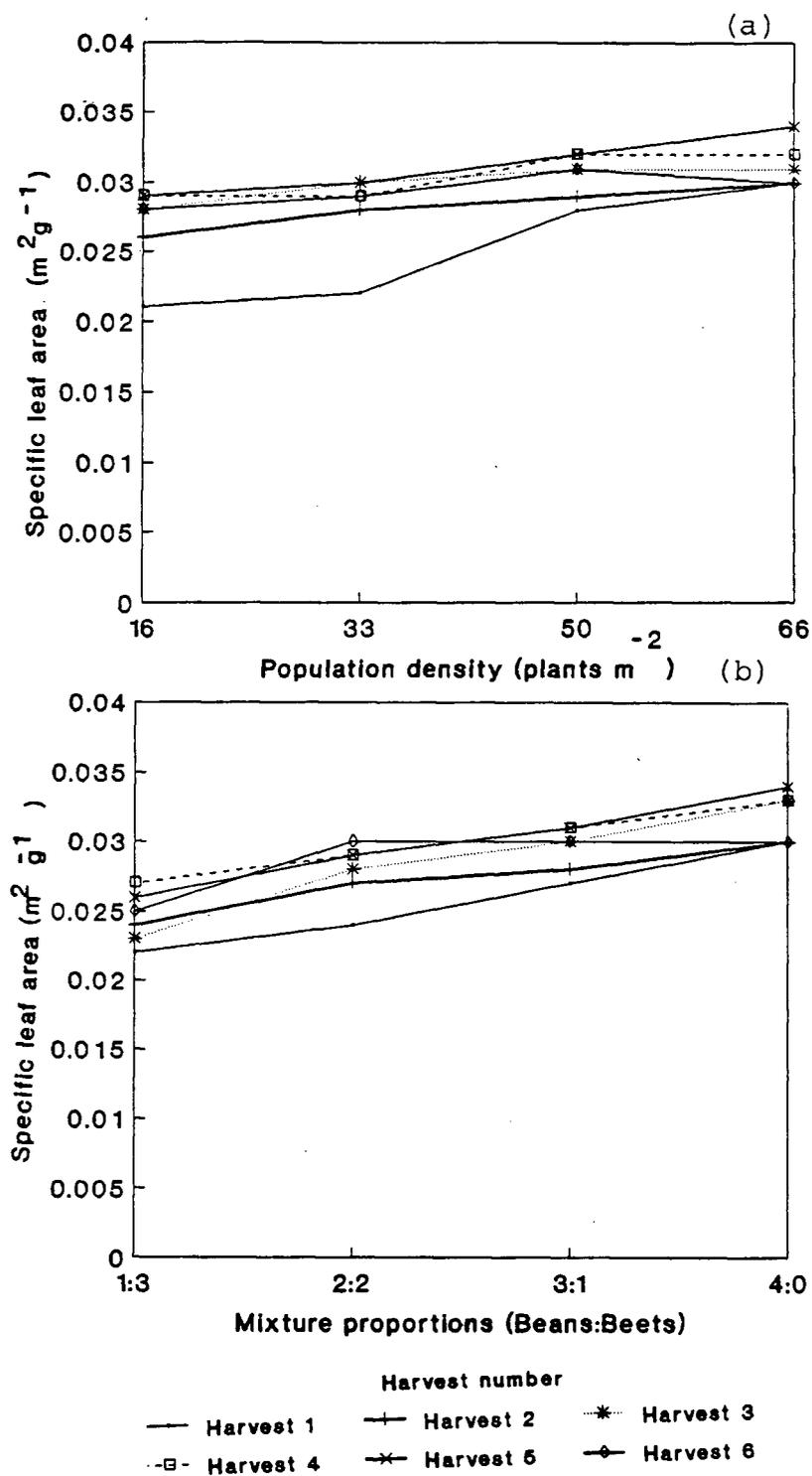
Fig. 4.14   The effect of population density and mixture proportions on bean specific leaf area at different stages of growth (1984 experiment)

(a) Population density
(b) Mixture proportions
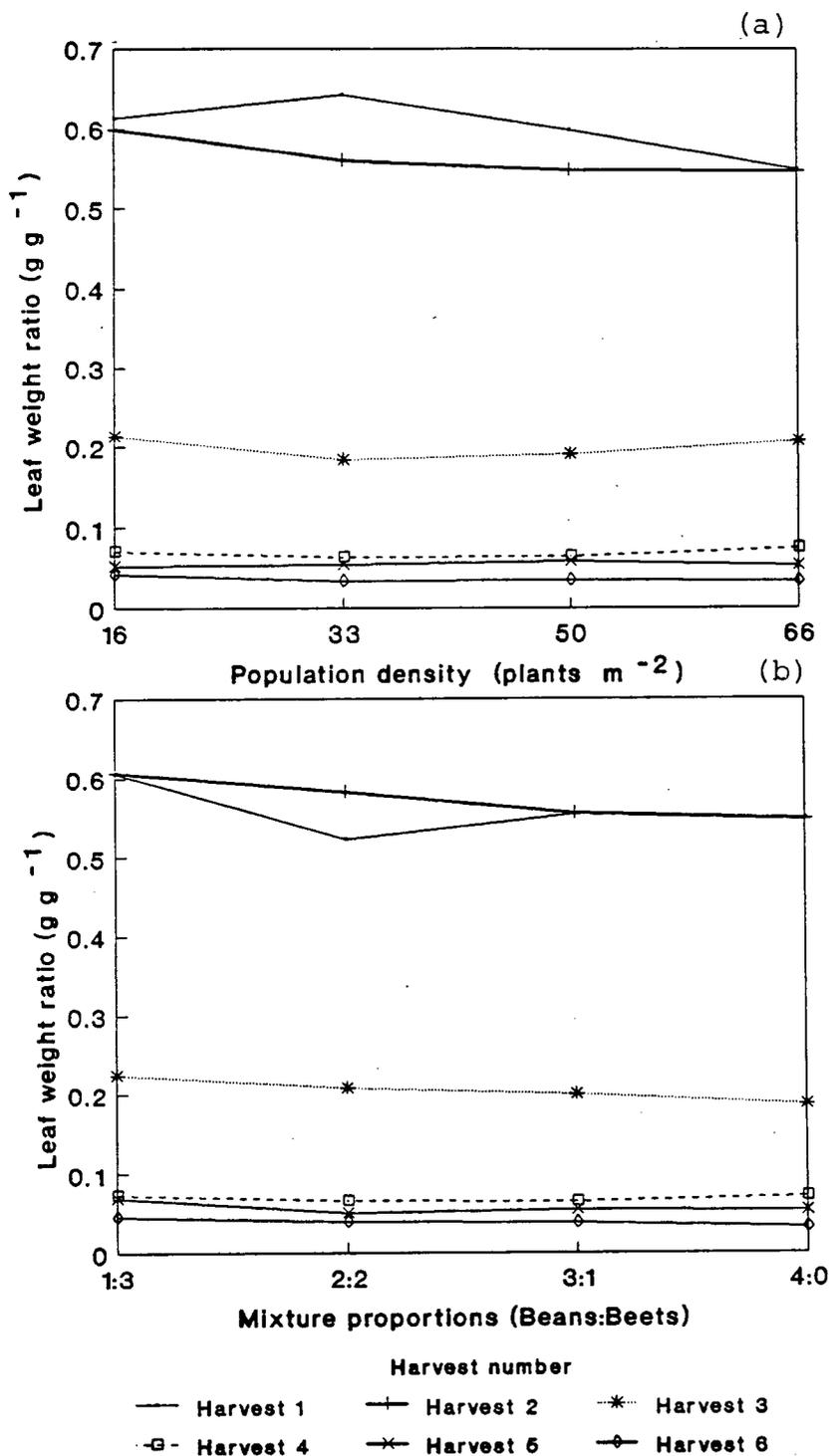
Fig. 4.15   The effect of population density and mixture proportions on bean leaf weight ratio at different stages of growth (1984 experiment)

(a) Population density
(b) Mixture proportions

Table 4.17   Analysis of variance results for the 1987 bean data: Variance ratios for the effects of population density and mixture proportions on growth indices of beans tested at the final harvest

Source of Variation	d.f.	F	Variables SLA	LWR	H
Blocks	1	0.31	3.38	0.32	9.08**
Density	3	0.27	2.46	1.38	0.74
Mixture	3	1.06	1.56	0.85	0.18
D x M	9	0.56	0.26	0.33	1.07
Exp. Err.	15	1.52	2.15	4.93**	1.34
Samp. Err.	32	2.20**	2.40**	1.48**	1.92
Sub Samp. Err.	896	-	-	-	-
Total	959	-	-	-	-

*Significant at P = 0.05

**Significant at P = 0.01

plants m^{-2} followed by a decline to 50% at 50 plants m^{-2}, and a further decrease to 48% at 66 plants m^{-2}. A similar (non-significant) trend due to increasing beet proportions was also observed. An increase in marketable yield proportions from 51% at 4:0 to 53% at 3:1 followed by a decrease to 51% and 49% at 2:2 and 1:3 bean:beet proportions respectively.

4.8.3 Beets 1984: Growth Indices (Leaf Area Ratio, Specific Leaf Area, Leaf Weight Ratio and Harvest Index): Analysis of Variance Results

Leaf area ratio, was significantly affected by both population density and mixture proportions at all harvest dates except at harvest 2 where only mixture proportions treatment effects were significant. The population density by mixture proportions interactions were significant at harvest 7 (Table 4.18). Leaf area ratio was found to increase with increasing population density and with increasing proportions of beans, the competing species (Fig. 4.16). This trend was easily seen at harvest 7 where the population density by mixture proportions interactions were detected. Leaf area ratio was highest at high population density and at high bean proportions in mixtures (Fig. 4.16c).

Specific leaf area was significantly affected by both treatments at all harvest dates except at harvest 2 and 3 where only effects due to mixture proportions were significant. Interactions between population density and mixture proportions were found to be significant later in the growing season from harvest 4 to harvest 7 (Table 4.18). Specific leaf area increased with increasing population density and with increasing beans proportions in mixtures (Fig. 4.17a and b). At higher densities,

Table 4.18   Analysis of variance for the 1984 beet data: Variance ratios for effects of population density and mixture proportions on growth indices tested at different stages of growth

Age at Harvest (days)	Source of Variation	d.f.	F	Variables SLA	LWR	H
40 (HN 1)[†]	Blocks	2	0.48	0.29	1.13	-
	Density	3	5.98**	7.14**	24.12**	-
	Mixture	3	4.36**	10.64**	6.42**	-
	D x M	9	1.87	2.22	2.23	-
	Exp. Err.	30	5.41**	5.85**	1.69*	-
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
51 (HN 2)	Blocks	2	0.42	5.54**	1.89	1.07
	Density	3	1.29	2.75	0.79	0.20
	Mixture	3	11.63**	24.61**	0.30	2.54
	D x M	9	0.92	0.80	0.52	0.43
	Exp. Err.	30	5.00**	2.93**	9.66**	3.98
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
63 (HN 3)	Blocks	2	3.78	3.27*	1.28	5.21**
	Density	3	3.60*	1.76	2.78	5.46**
	Mixture	3	32.27**	28.22**	3.13*	14.02**
	D x M	9	1.67	1.21	0.63	0.82
	Exp. Err.	30	1.63*	2.01	2.39**	2.92**
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
69 (HN 4)	Blocks	2	12.56**	26.15**	0.61	5.87**
	Density	3	4.36**	8.66**	0.70	3.13*
	Mixture	3	41.07**	81.88**	3.50*	16.91**
	D x M	9	1.90	2.25	1.13	1.48
	Exp. Err.	30	2.22	1.72*	2.81**	3.68**
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
75 (HN 5)	Blocks	2	0.84	12.49**	2.82	0.06
	Density	3	11.02**	8.06**	6.61**	9.66**
	Mixture	3	44.12**	54.37**	9.32**	22.81**
	D x M	9	1.67	3.41**	0.65	1.77
	Exp. Err.	30	3.77**	3.41**	3.63**	5.61**
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
92 (HN 6)	Blocks	2	1.49	2.98	0.03	0.83
	Density	3	7.11**	2.33	6.21**	7.78**
	Mixture	3	39.81**	36.13**	10.07**	23.07**
	D x M	9	0.81	2.17*	0.85	0.89
	Exp. Err.	30	3.71**	4.08**	2.13	3.09**
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-
107 (HN 7)	Blocks	2	7.38**	12.50**	0.95	1.87
	Density	3	8.52**	11.84**	3.06*	5.53**
	Mixture	3	55.77**	61.95**	11.56**	21.64**
	D x M	9	2.85**	3.51**	1.40	1.50
	Exp. Err.	30	1.63*	2.14**	1.69	2.66**
	Samp. Err.	192	-	-	-	-
	Total	239	-	-	-	-

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01
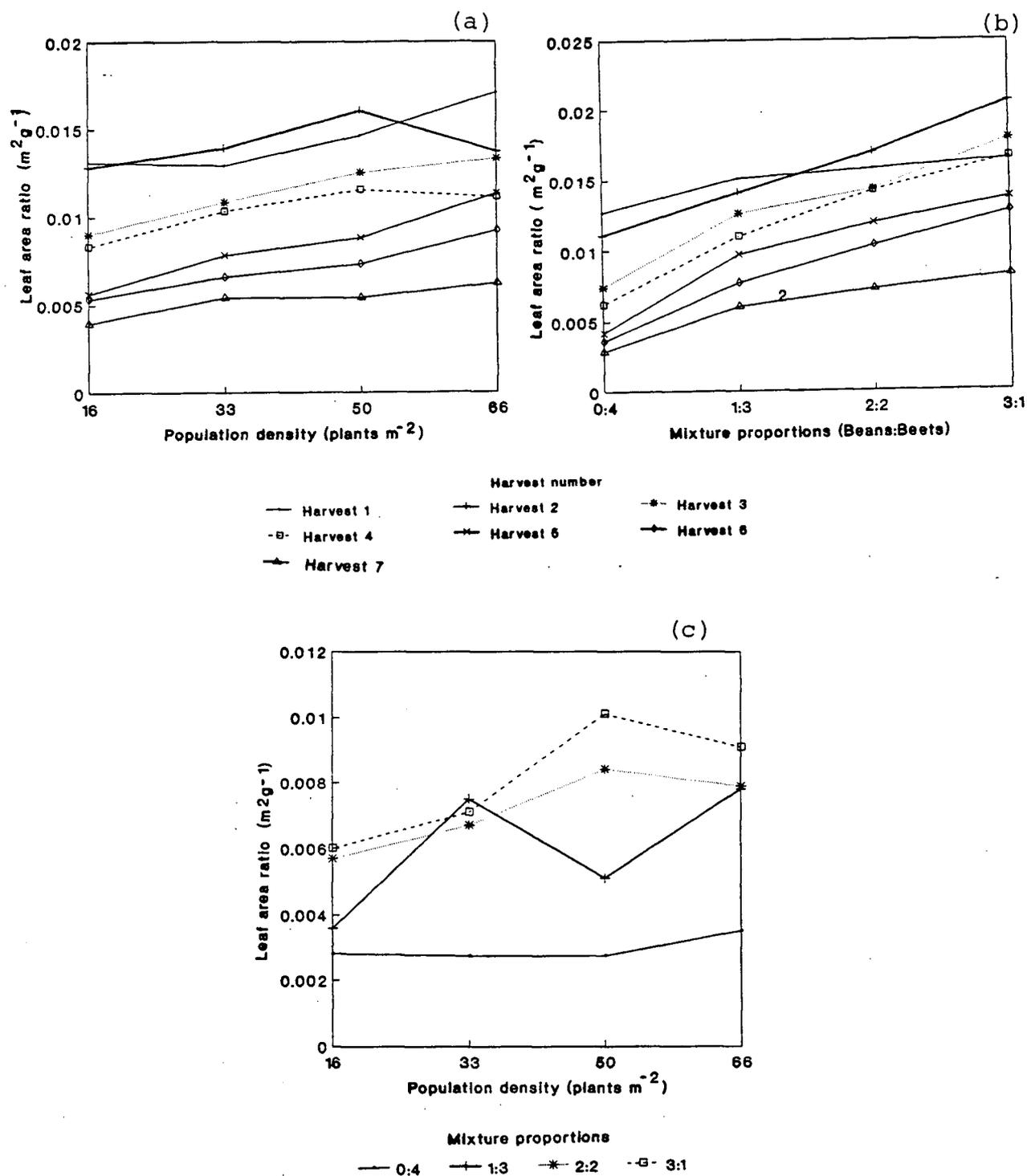
Fig. 4.16   The effect of population density, mixture proportions and population density by mixture proportions interaction on bean leaf area ratio at different stages of growth (1984 experiment)

(a) Population density
(b) Mixture proportions
(c) Population density by mixture proportions interaction
    (at 92 days from planting)
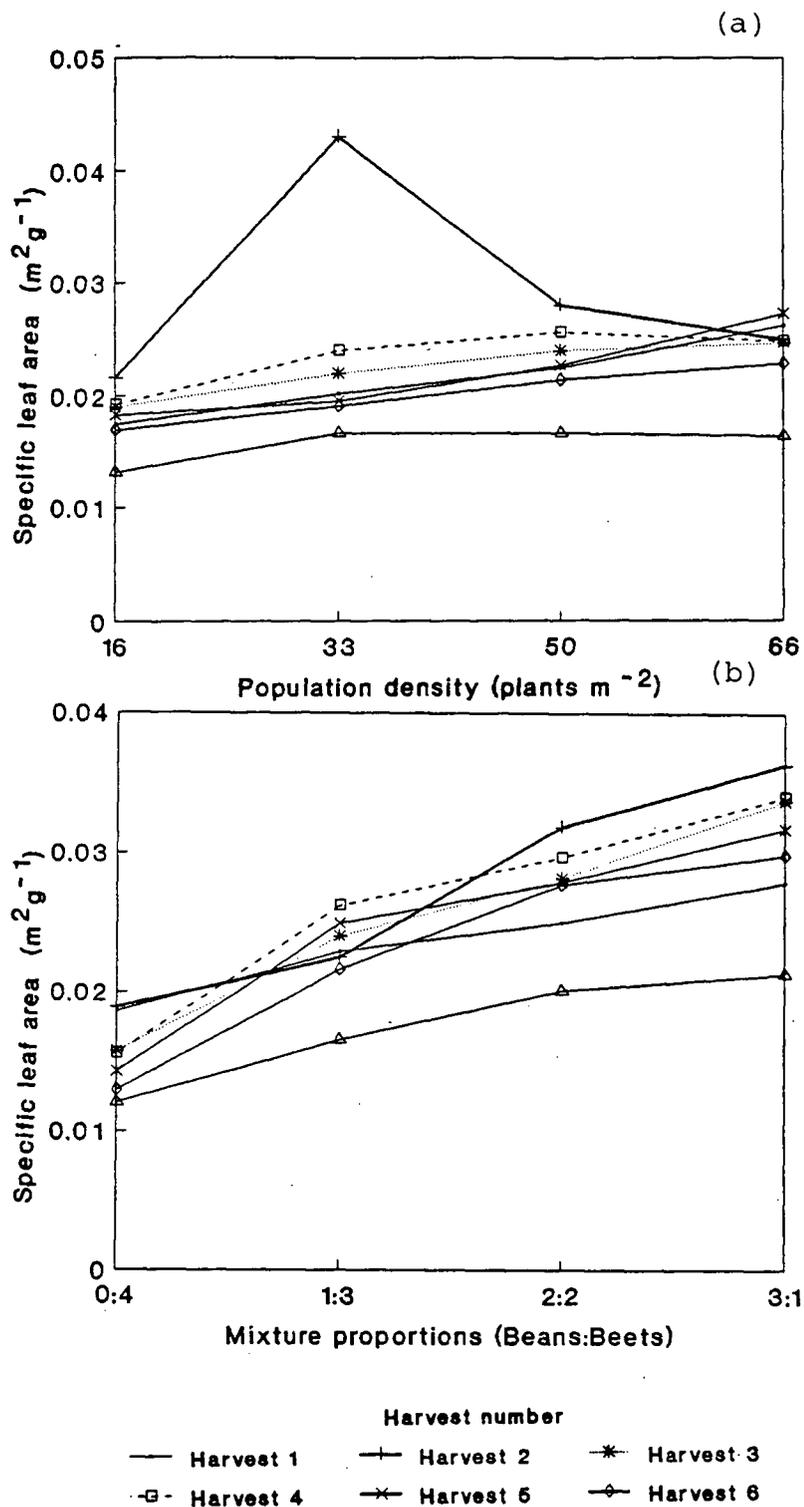
Fig. 4.17   The effect of population density and mixture proportions on beet specific leaf area at different stages of growth (1984 experiment)

(a) Population density
(b) Mixture proportions

the increase in specific leaf area was more with increasing bean proportions than at lower densities (Fig. 4.18).

Leaf weight ratio was also significantly affected by treatments later in the growing season from harvest 3 to harvest 7 though at harvest 4 only mixture proportions were significant (Table 4.18). Leaf weight ratio increased with increasing population density and with increasing beans proportions in mixtures (Fig. 4.19). The population density by mixture interactions were not significant at any time during the experiment.

An ANOVA for harvest index was also not done for the beet data from the harvest done at 40 days after planting since most plants had not developed storage roots by that time. The results from harvest 2 indicated a significant effect due to mixture proportions while those of other harvest dates indicated significant responses due to both population density and mixture proportions (Table 4.18). The trend was a decrease in marketable yield proportions with increasing population density (Fig. 4.20a). The largest difference between the lowest and the highest population densities was observed at harvest 5 where a 24% reduction in marketable yield was recorded across the range of densities. In mixtures, however, the decrease in marketable yield proportions was observed with increasing proportions of beans (Fig. 4.20b). The population density by mixture proportions interactions did not show significant responses at any of the harvests (Table 4.18).
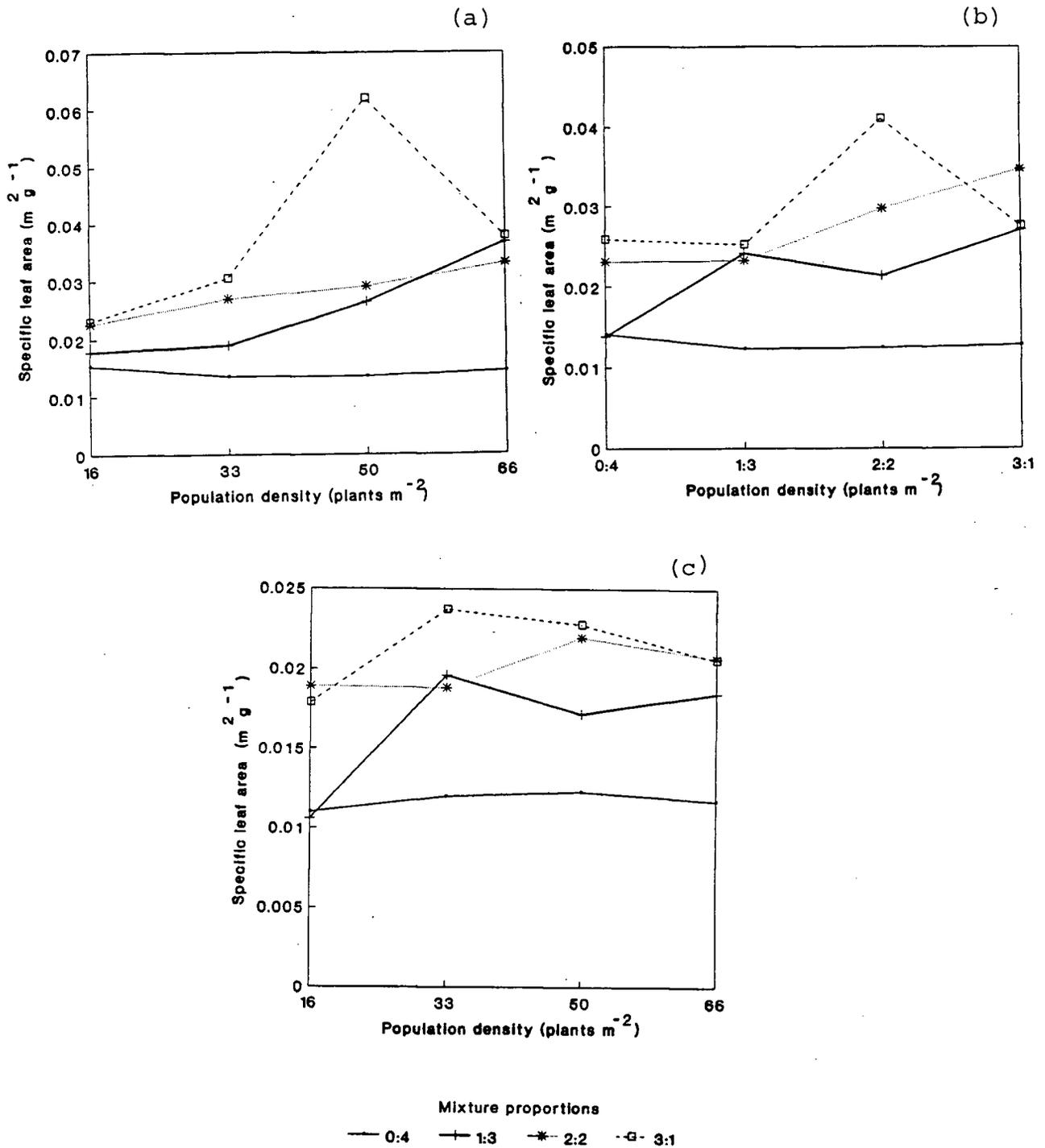
Fig. 4.18   The effect of population density and mixture proportions
interactions on beet specific leaf area (1984 experiment)

(a) Harvest 5
(b) Harvest 6
(c) Harvest 7

(a)

(b)

Harvest number

—— Harvest 1   —+— Harvest 2   —*— Harvest 3

-▫- Harvest 4   —✕— Harvest 5   —◇— Harvest 6

—△— Harvest 7
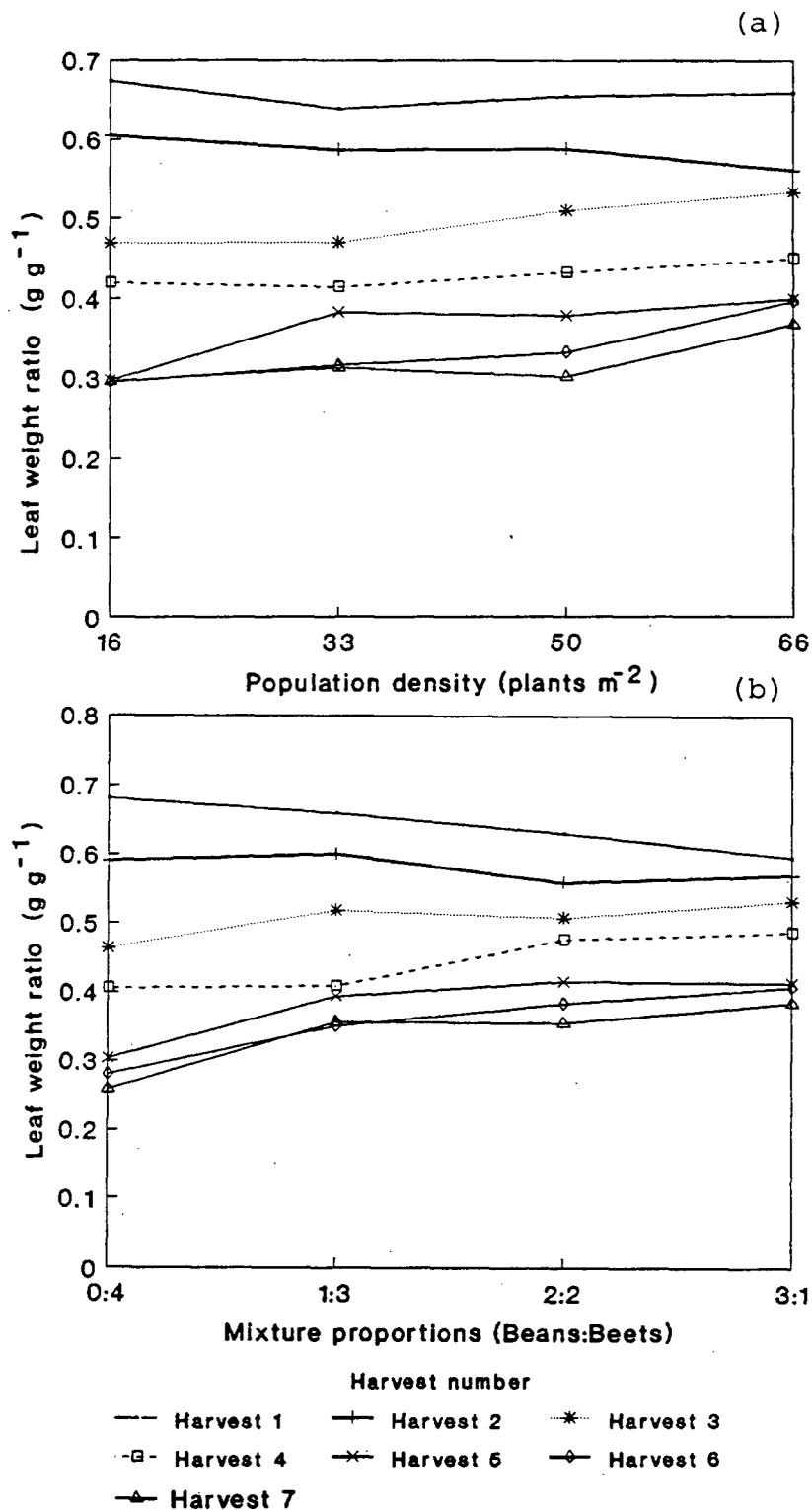
Fig. 4.19   The effect of population density and mixture proportions on beet leaf weight ratio at different stages of growth (1984 experiment)

(a) Population density
(b) Mixture proportions
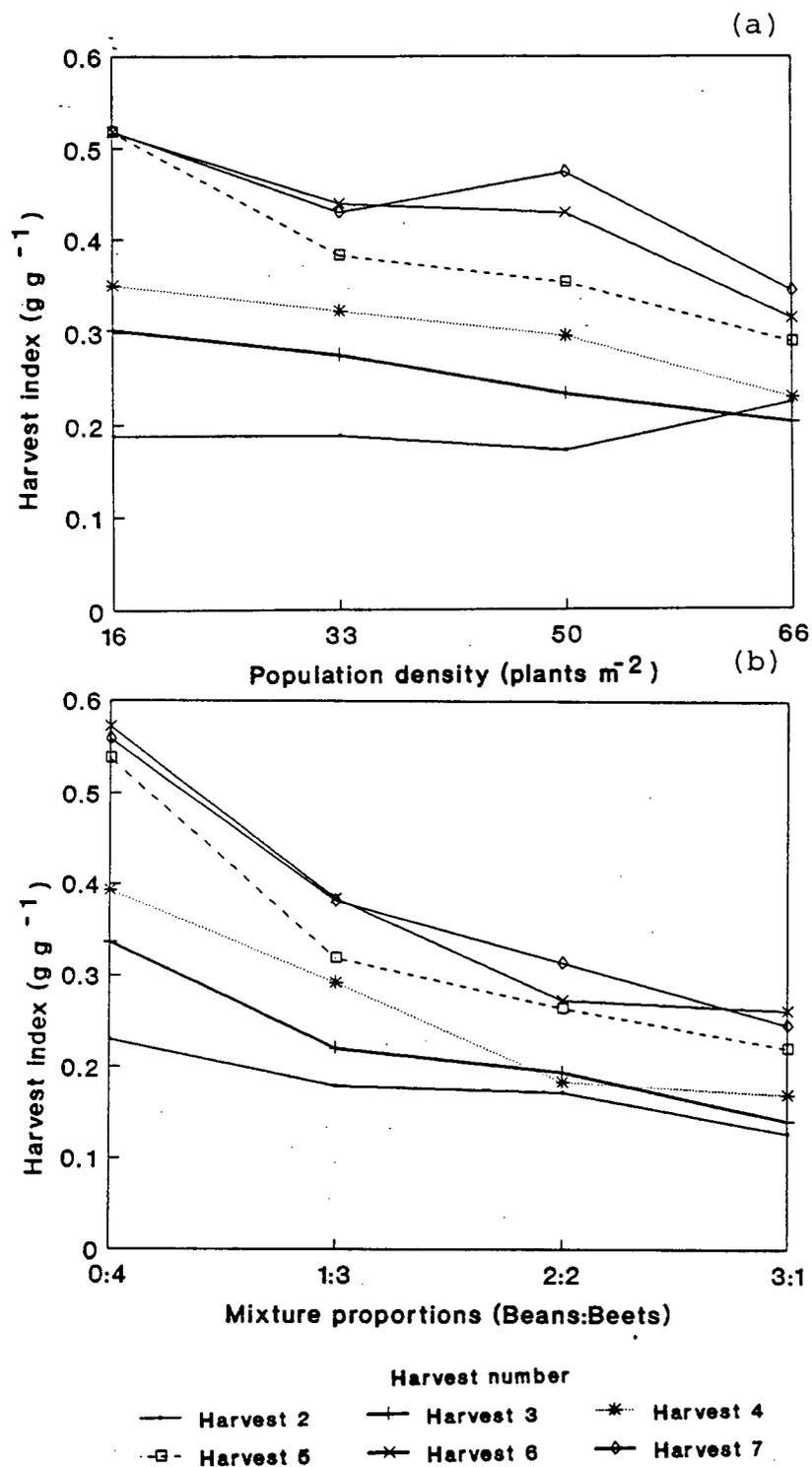
Fig. 4.20   The effect of population density and mixture proportions on beet harvest index at different growth stages (1984 experiment)

(a) Population density
(b) Mixture proportions

4.8.4 Beets 1987: Growth Indices (Leaf Area Ratio, Specific Leaf Area, Leaf Weight Ratio and Harvest Index): Analysis of Variance Results

The response of leaf area ratio, and specific leaf area was not significantly different between treatments. Leaf weight ratio was also not significantly affected by population density but the effect due to mixture proportions was highly significant (Table 4.19). The mixture proportion 1:3 bean:beet ratio had the lowest leaf weight ratio while 0:4 had the second lowest followed by 2:2. The mixture proportion 3:1 had the highest leaf weight ratio of the 4 treatments (Fig.4.21). Leaf area ratio though not significant, tended to increase with increasing population density and with increasing proportions of beans in bean:beet mixture treatments. Leaf area ratio tended to be high for the population density of 33 plants m^{-2} but had no clear pattern in mixture proportions (data not shown). The population density by mixture proportions interaction was neither significant nor did it follow any particular pattern.

Population density significantly affected harvest index in beets (Table 4.19). The percentage increase in marketable yield proportion was from 62% at 16 plants m^{-2} to 112% at 33 plants m^{-2}. Both high population density treatments of 50 and 66 plants m^{-2} had lower proportions of 51% and 45% respectively. Mixture effects were not significant but a similar trend to the beans 1987 data was observed (i.e. 64% at 0:4, 117% at 3:1, 50% at 2:2, and 39% at 1:3 mixture proportions of beans:beets). The population density by mixture proportions interaction was not significant, but quantitative declines in marketable yield proportion with increasing total population density and bean proportions in the mixtures were observed (data not shown).

Table 4.19   Analysis of variance results for the 1987 beet data: Variance ratios for the effects of population density and mixture proportions on growth indices tested at the final harvest

Source of Variation	d.f.	F	Variables SLA	LWR	H
Blocks	1	2.58	0.61	0.20	0.20
Density	3	2.58	1.15	0.53	3.72*
Mixture	3	1.01	3.05	8.19**	1.22
D x M	9	0.71	0.48	0.76	2.16
Exp. Err.	15	0.90	1.29	0.68	0.58
Samp. Err.	32	0.95	2.13**	1.76	1.89**
Sub Samp. Err.	192	-	-	-	-
Total	255	-	-	-	-

*Significant at P = 0.05
**Significant at P = 0.01
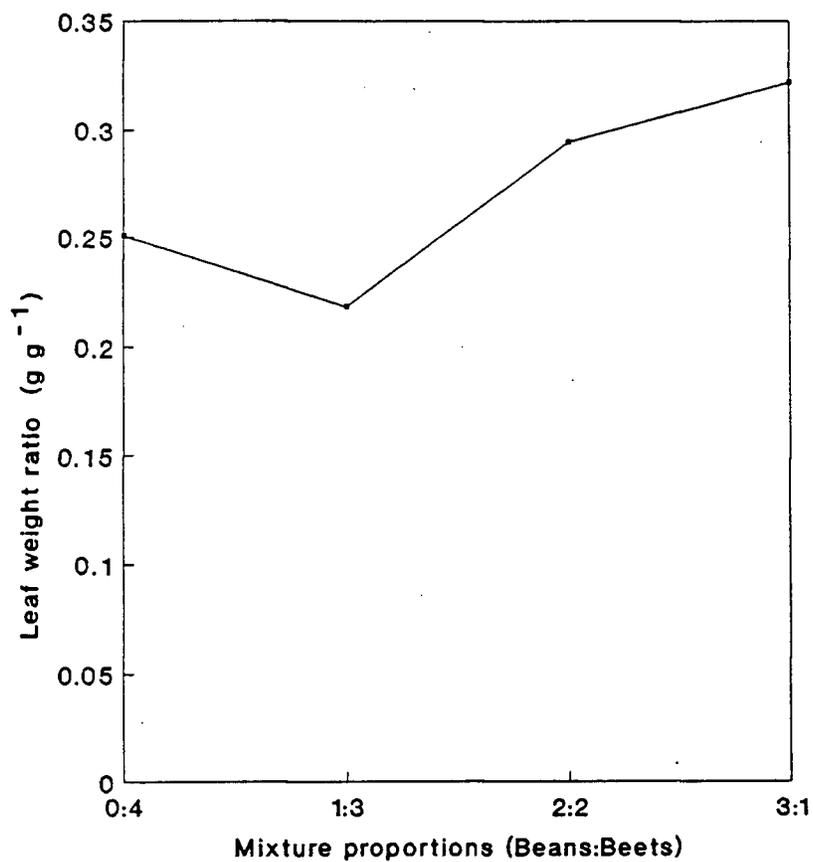
Fig. 4.21   The effect of mixture proportions on beet leaf weight ratio
              (1987 experiment)

4.8.5 Beans 1984: Primary Variables (Total Dry Weight, Leaf Dry Weight and Leaf Area): Regression Results

Regressions for total dry weight indicated an increase in total dry weight per plant with time for all mixture proportions at the total population density treatment of 66 plants m^{-2}. The increase was highest at mixture proportion 1:3 bean:beet ratio and decreased with increasing mixture proportions of beans. This response was evident after 51 days from planting though at the final harvest (92 days from planting), a slight decline was observed (Fig. 4.22a). Leaf dry weight and leaf area per plant underwent an early response to different mixture proportions at 66 plants m^{-2} population density. The 1:3 bean:beet mixture proportion had again the highest increase in leaf dry weight and leaf area per plant. In the other mixture proportions treatments, the increase in leaf dry weight and leaf area per plant decreased with increasing beans proportions. The decline in leaf dry weight and leaf area was also observed at the final harvest (Fig 4.22b and c).

The effect of increasing plant population density on total dry weight at 2:2 mixture proportions was evident after 51 days from planting whereas leaf dry weight and leaf area responded to treatments early in the growing period. The response curves were similar to those stated above. Total dry weight, leaf dry weight and leaf area per plant were seen to increase with time. The increase was highest at 16 plant m^{-2} and decreased with increasing population density. These changes remained in effect throughout growth though a slight decline in yield at the final harvest was noticed at all population densities in all three variables tested (Fig. 4.23). This slight decrease in yield per plant seen at
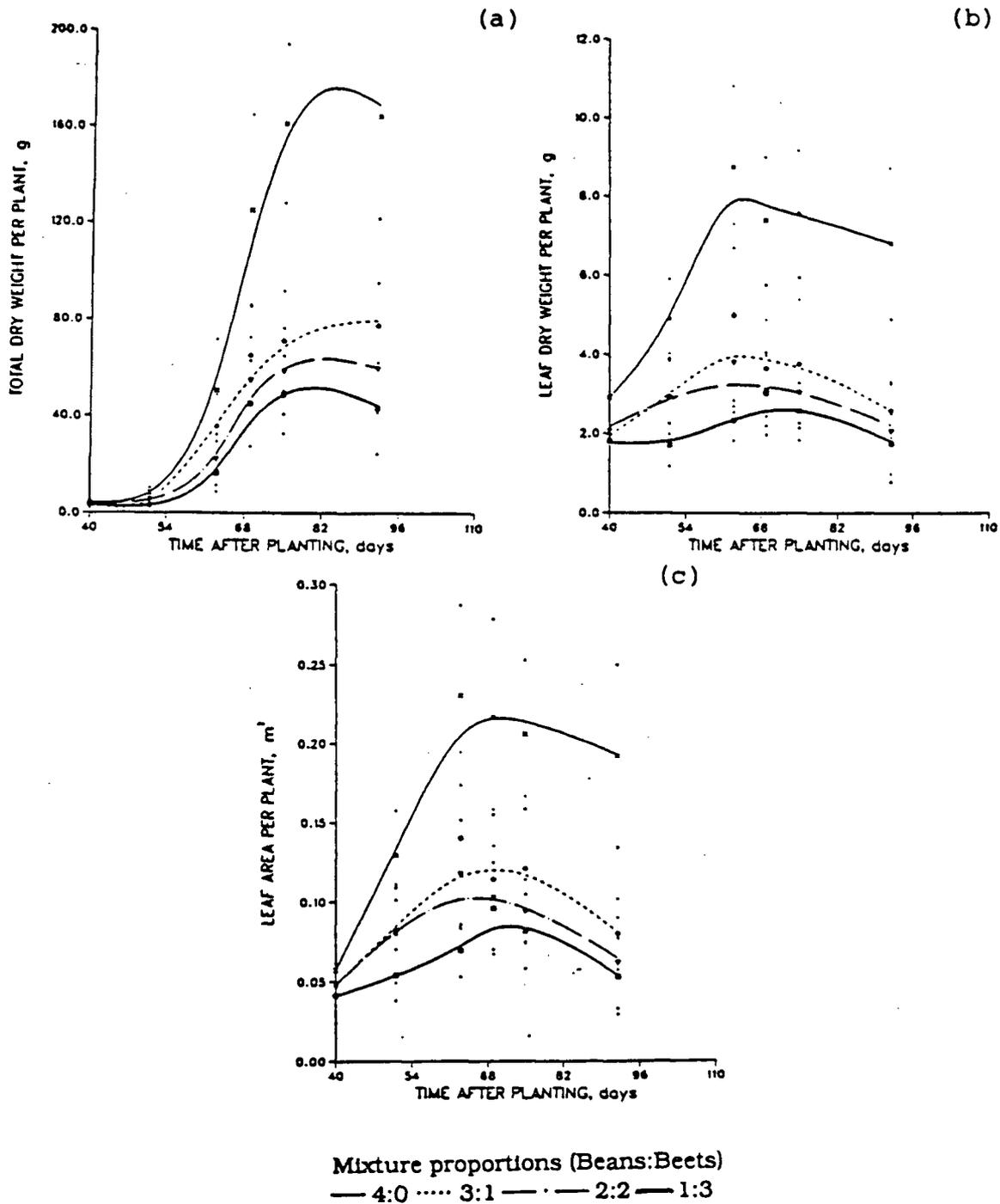
Fig 4.22   Changes in total dry weight, leaf dry weight and leaf area
per plant in beans during growth resulting from increasing
mixture proportions of beets at total population density of
66 plants m⁻² (1984 experiment)

(a) Total dry weight per plant
(b) Leaf dry weight per plant
(c) Leaf area per plant

(a) Total dry weight per plant

(b) Leaf dry weight per plant

(c) Leaf area per plant

Population density (plants m⁻²)
—— 16 ····· 33 — · — 50 ——— 66
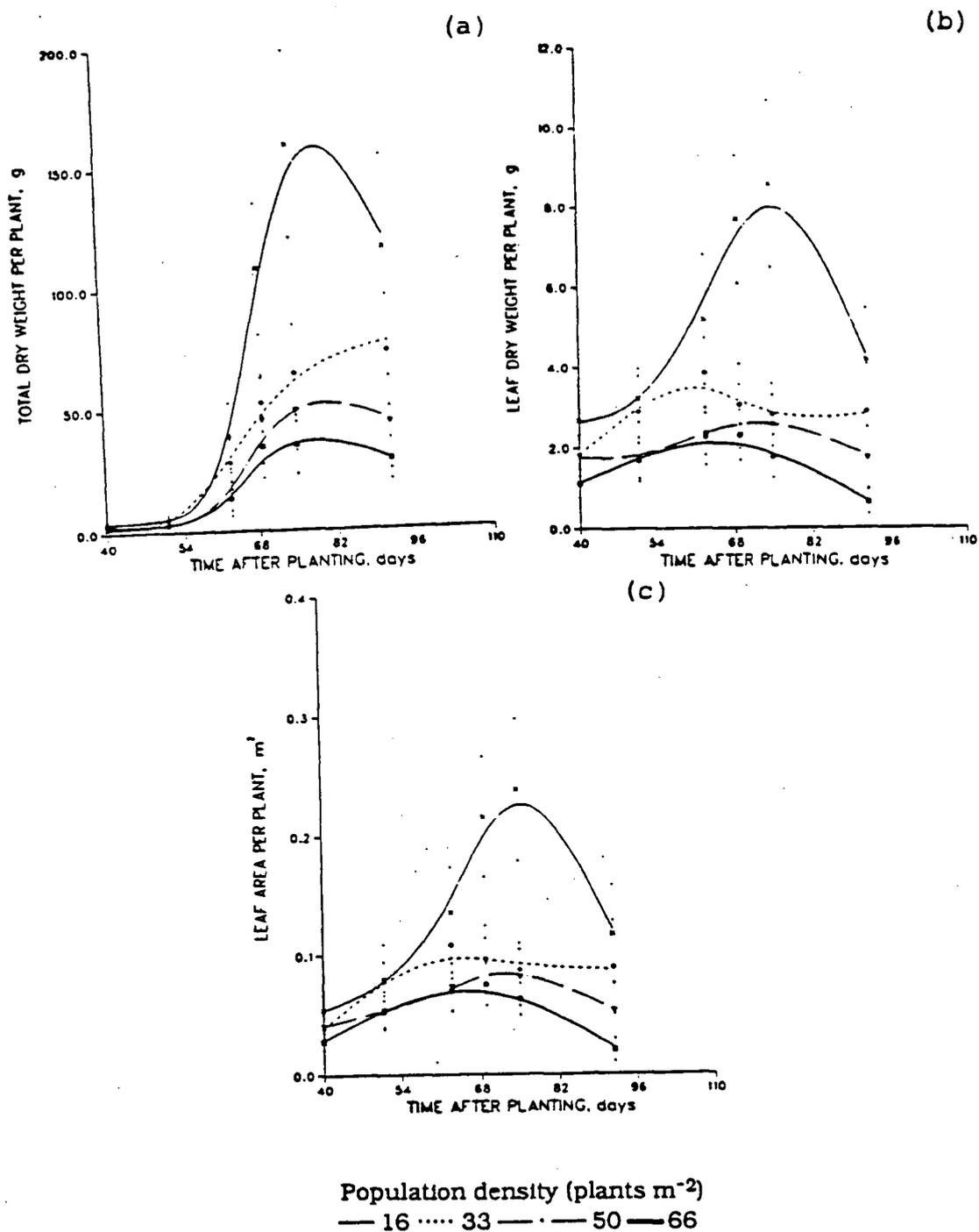
Fig. 4.23   Changes in total dry weight, leaf dry weight and leaf area per plant in beans during growth resulting from increasing total population density at 2:2 bean:beet mixture proportion (1984 experiment)

(a) Total dry weight per plant
(b) Leaf dry weight per plant
(c) Leaf area per plant

the final harvest could be due to over fitting of the data by plant growth analysis procedure.

4.8.6 Beets 1984: Primary Variables (Total Dry Weight, Leaf Dry Weight and Leaf Area): Regression Results

Fitted growth curves for total dry weight for beets were similar to those of beans. The effect due to increasing bean mixture proportions at 66 plants m^{-2} started to show at 51 days from planting and remained in effect throughout the growing season. Treatment effects on leaf dry weight were earlier than on total dry weight. In both yield variables, the increase in yield decreased with increasing proportions of beans. Leaf area did not seem to be very drastically affected by treatments (Fig. 4.24).

The effect of increasing population density beyond 16 plants m^{-2} at 2:2 mixture proportion did not cause drastic increases in yield throughout the growing season. A big increase in total dry weight per plant took place only at the lowest population density of 16 plants m^{-2}. A similar pattern was also observed for leaf dry weight and leaf area per plant (Fig 4.25).

4.8.7 Beans 1984: Growth Indices (Leaf Area Ratio, Leaf Weight Ratio, Harvest Index and Specific Leaf Area): Regression Results

Leaf area ratio, leaf weight ratio and harvest index did not seem to be affected by different mixture proportions when examined at the total population density of 66 plants m^{-2} (Fig. 4.26a-c). A general decline in leaf area ratio and leaf weight ratio was observed with time while harvest index generally increased. Although a general increase in specific leaf

Mixture proportions (Beans:Beets)
—— 0:4 ····· 1:3 —— · —— 2:2 ——— 3:1
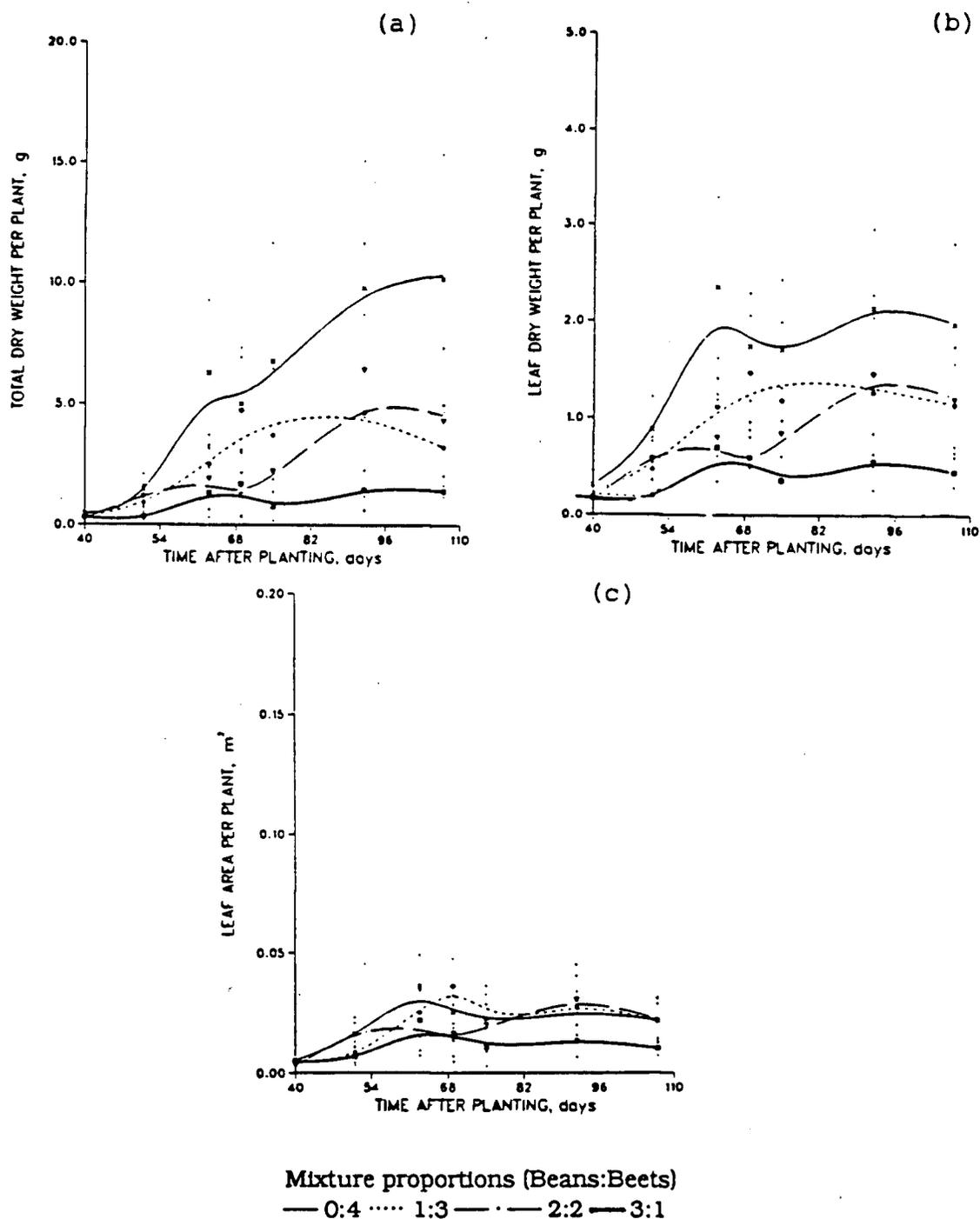
Fig. 4.24   Changes in total dry weight, leaf dry weight and leaf area
per plant in beets during growth resulting from increasing
mixture proportions of beans at total population density of
66 plants m^{-2} (1984 experiment)

(a) Total dry weight per plant
(b) Leaf dry weight per plant
(c) Leaf area per plant
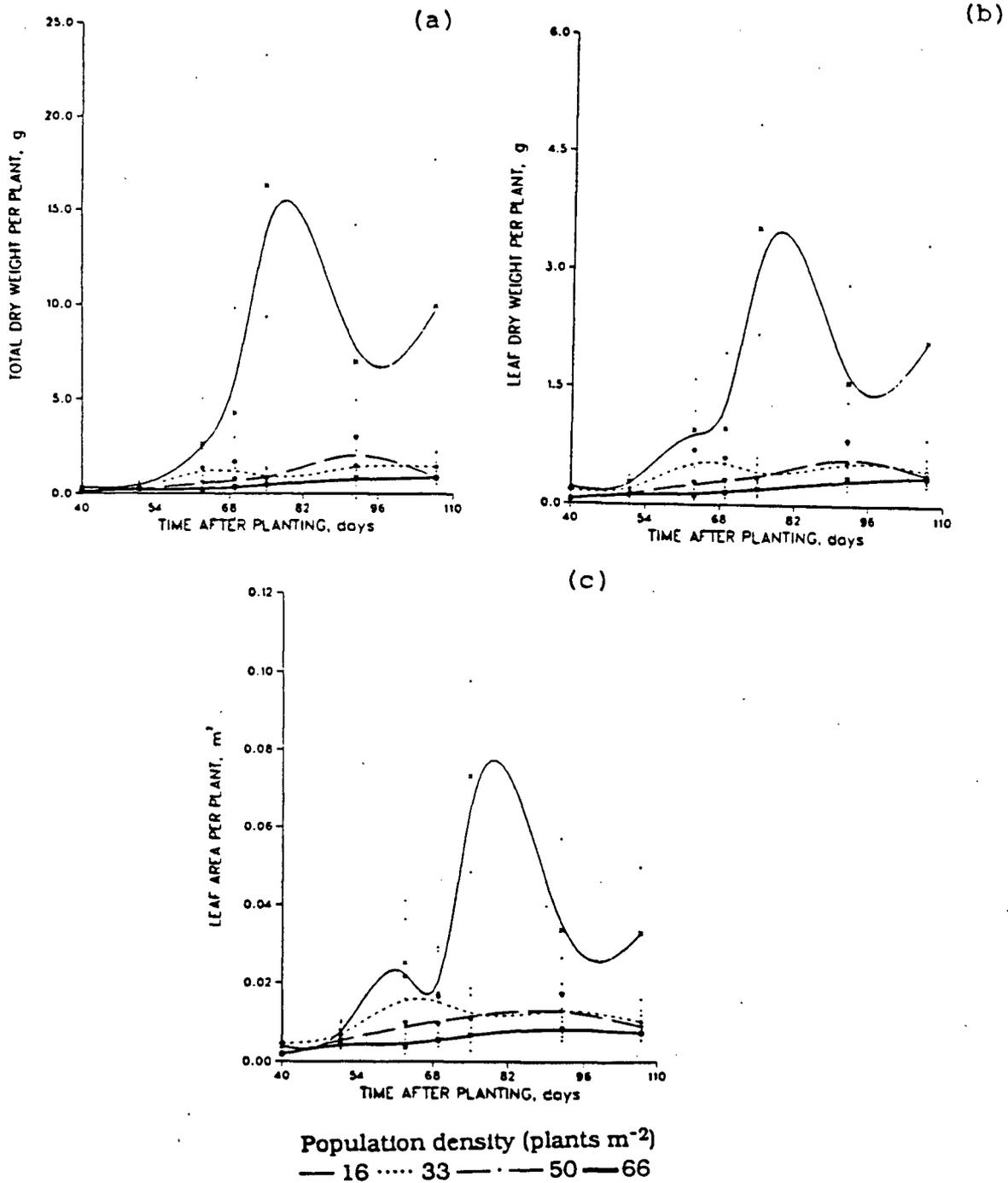
Fig. 4.25   Changes in total dry weight, leaf dry weight and leaf area
per plant in beets during growth resulting from increasing
total population density at 2:2 bean:beet mixture proportion
(1984 experiment)

(a) Total dry weight per plant
(b) Leaf dry weight per plant
(c) Leaf area per plant
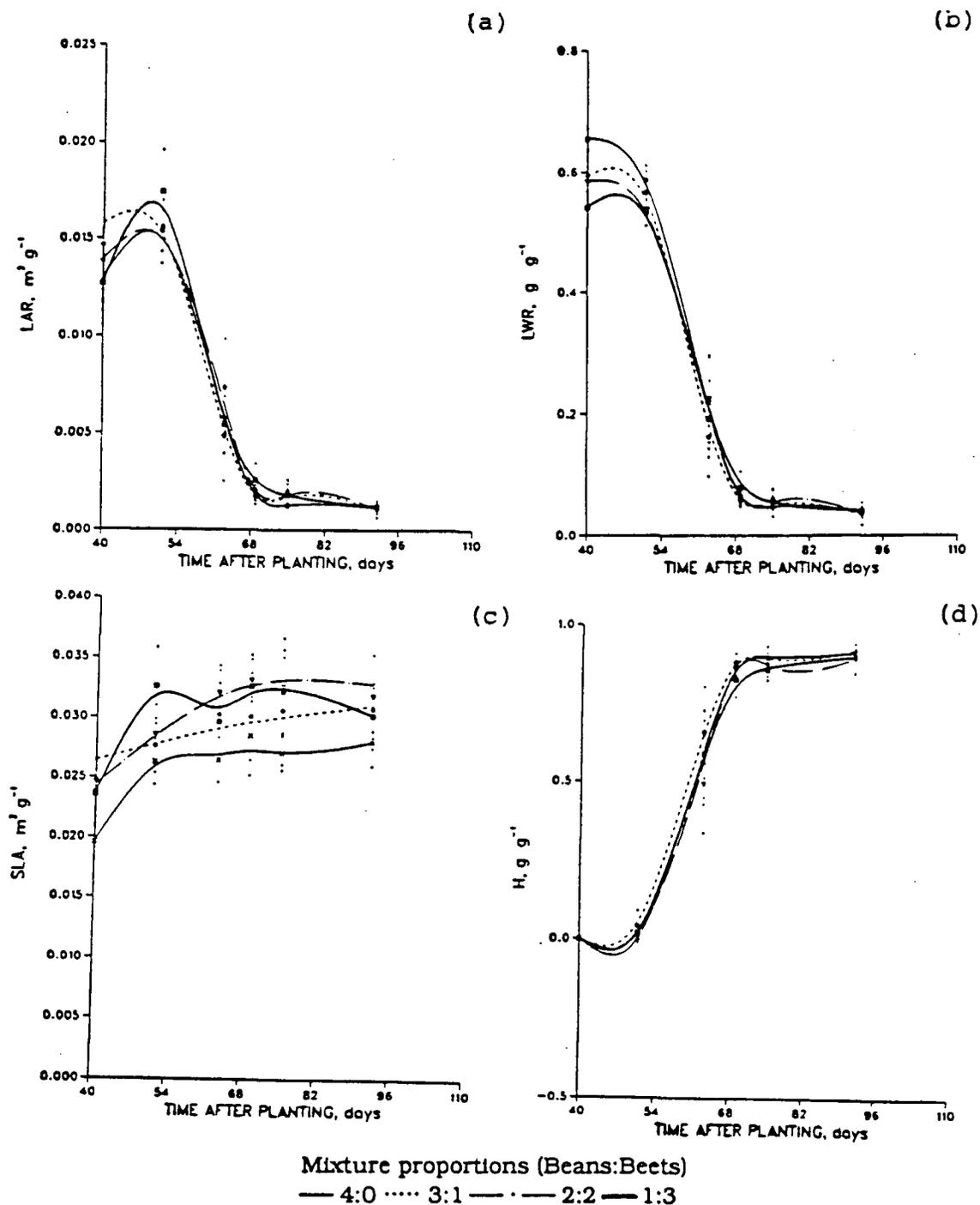
Fig. 4.26 Changes in leaf area ratio, leaf weight ratio, specific leaf area and harvest index per plant in beans during growth resulting from increasing mixture proportions of beets at total population density of 66 plants m⁻² (1984 experiment)

(a) Leaf area ratio per plant
(b) Leaf weight ratio per plant
(c) Specific leaf area per plant
(d) Harvest index per plant

area was also observed with time, the changes in mixture proportions did affect this index (Fig. 4.26d). The effect was in the reverse order of what has been described for the three primary variables in sections 4.3.2.1 and 4.3.2.3. The increase in specific leaf area decreased with decreasing beets proportions. These results are similar to those of changing total population density at 2:2 mixture proportions (Fig. 4.27a-d).

4.8.8 Beets 1984: Growth Indices (Leaf Area Ratio, Leaf Weight Ratio, Harvest Index and Specific Leaf Area): Regression Results

In beets, leaf area ratio, leaf weight ratio, harvest index and specific leaf area were all affected by changing the mixture proportions at 66 plants m^{-2} total population density treatment. Leaf area ratio, specific leaf weight, and leaf weight ratio, were reduced with time but the extent of reduction differed with mixture proportions treatment. The decrease was highest for the mixture proportion treatment with more bean and decreased with decreasing beans proportions (Fig. 4.28a-c). The increase in harvest index also differed with different mixture proportions within the 66 plants m^{-2} population density treatment. The increase decreased with increasing bean proportions (Fig. 4.28d). The results due to increasing population density at the 2:2 mixture proportions treatment were similar to those described above. Leaf area ratio, specific leaf area and leaf weight ratio decreased with time although a lot of fluctuations within treatments were seen (Fig. 4.29a-c). The increase in harvest index with time was also not systematic between treatments (Fig 4.29d).
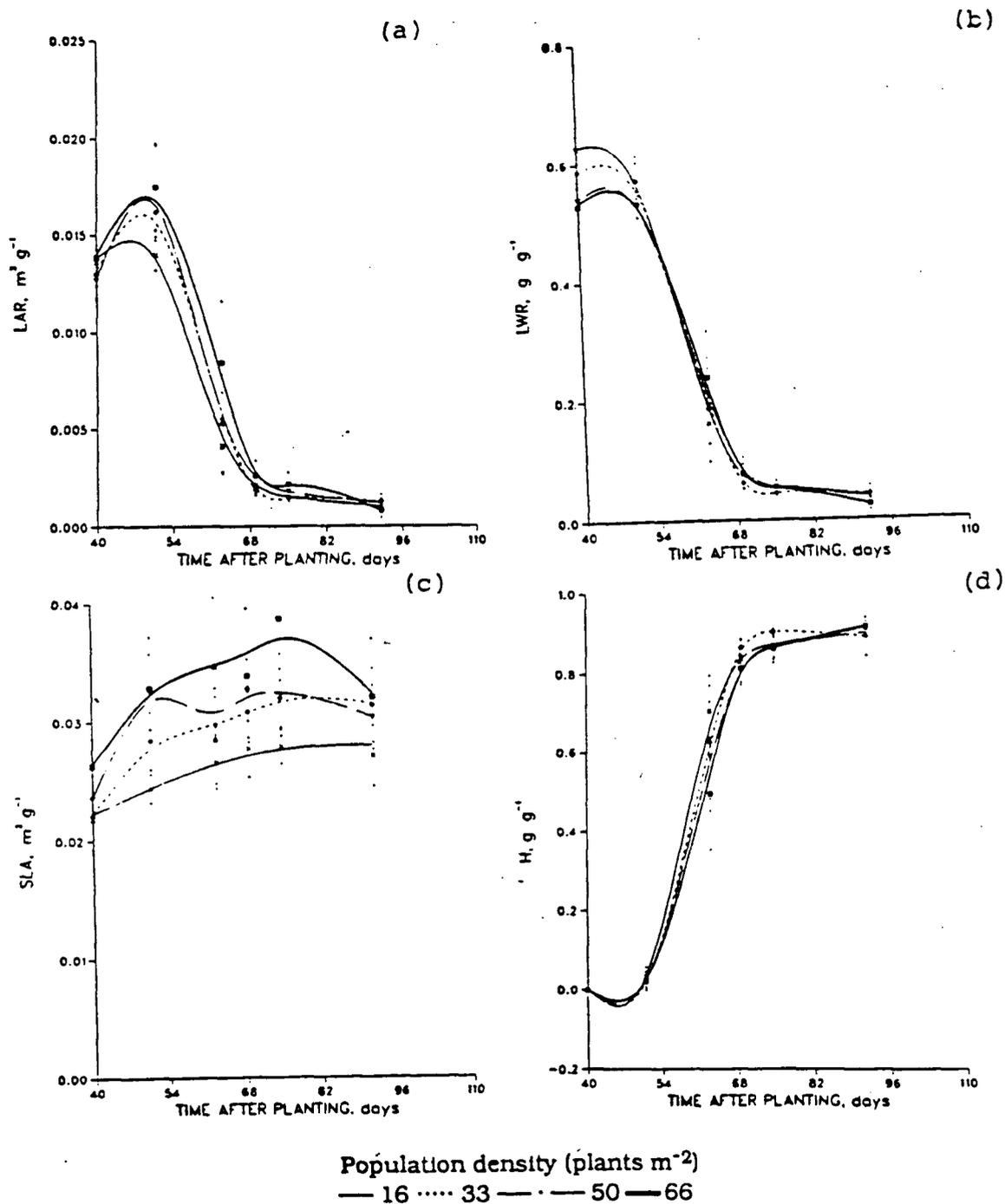
Fig. 4.27   Changes in leaf area ratio, leaf weight ratio, specific leaf area and harvest index per plant in beans during growth resulting from increasing total population density at 2:2 bean:beet mixture proportion (1984 experiment)

(a)   Leaf area ratio per plant
(b)   Leaf weight ratio per plant
(c)   Specific leaf area per plant
(d)   Harvest index per plant

Mixture proportions (Beans:Beets)
—— 0:4 ····· 1:3 —— · —— 2:2 —— 3:1
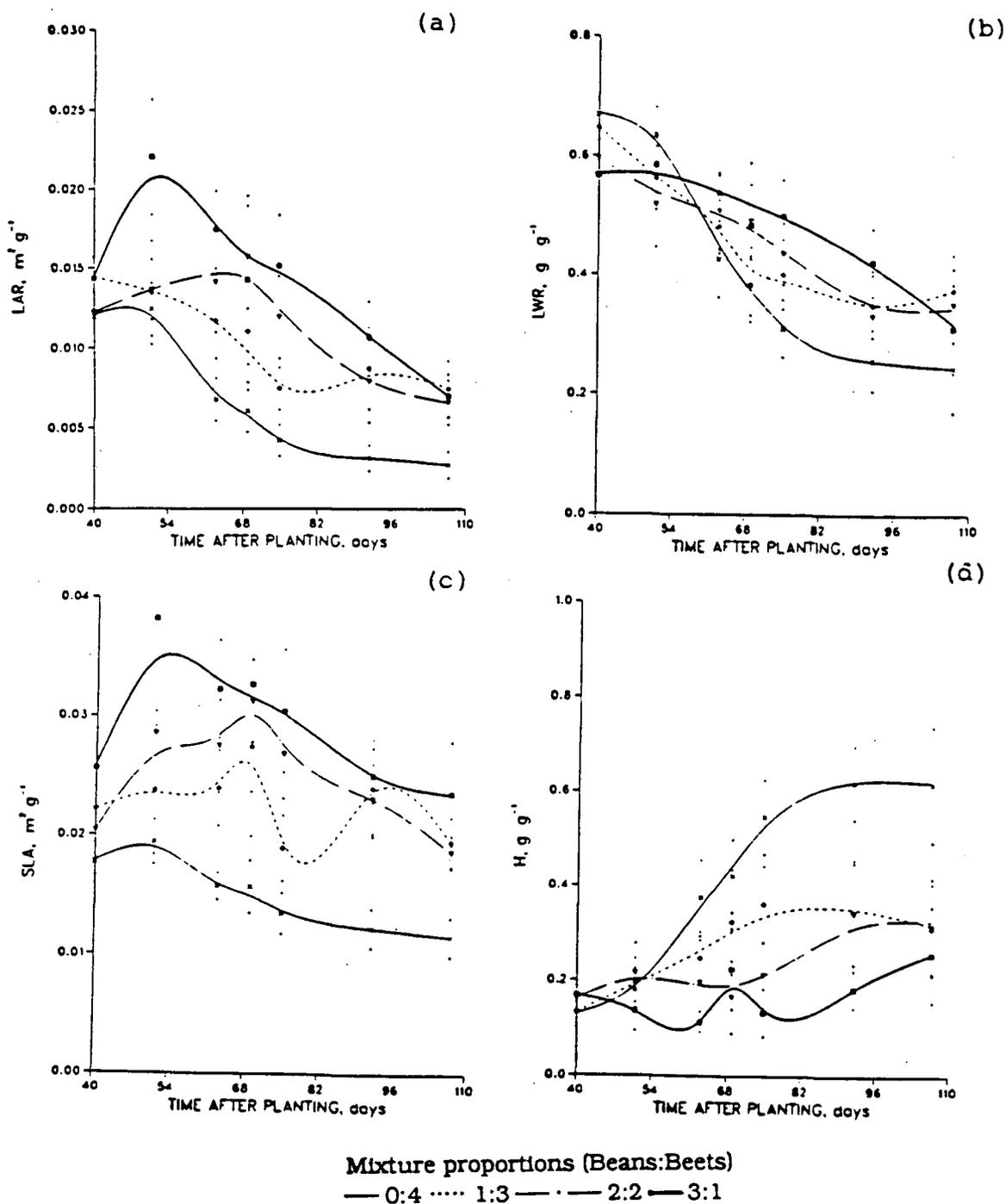
Fig. 4.28   Changes in leaf area ratio, leaf weight ratio, specific leaf area and harvest index per plant in beets during growth resulting from increasing mixture proportions of beans at total population density of 66 plants m⁻² (1984 experiment)

(a) Leaf area ratio per plant
(b) Leaf weight ratio per plant
(c) Specific leaf area per plant
(d) Harvest index per plant
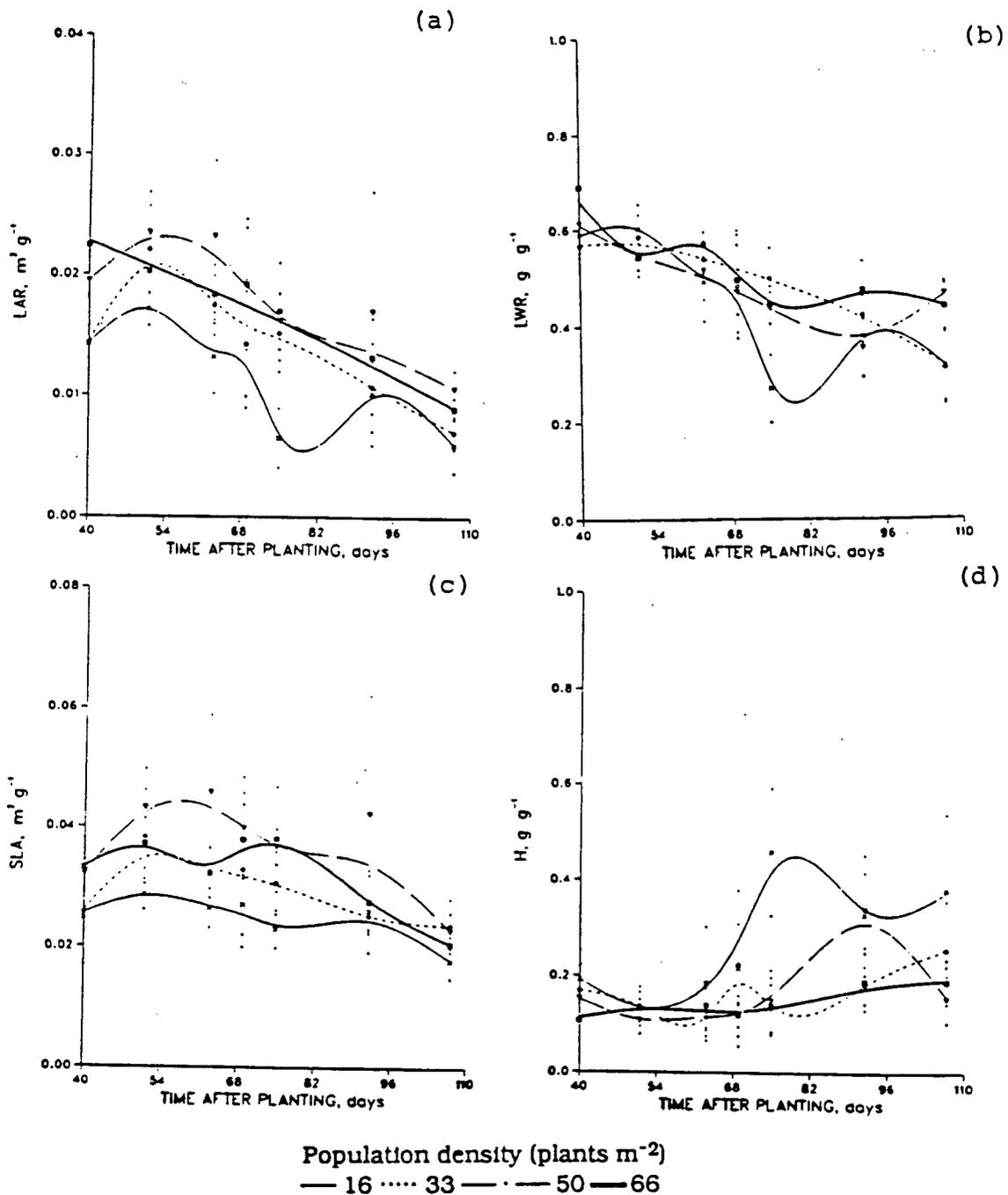
Fig. 4.29   Changes in leaf area ratio, leaf weight ratio, specific leaf area and harvest index per plant in beets during growth resulting from increasing total population density at 2:2 bean:beet mixture proportion (1984 experiment)

(a) Leaf area ratio per plant
(b) Leaf weight ratio per plant
(c) Specific leaf area per plant
(d) Harvest index per plant

4.8.9 Beans 1984: Growth Indices (Absolute Growth Rate, Relative Growth Rate and Unit Leaf Rate): Regression Results

All three growth indices (absolute growth rate, relative growth rate and unit leaf rate) were influenced by treatments. They all increased with time and reached a maximum in the middle of the growing season and thereafter declined. For absolute growth rate and unit leaf rate, the mixture proportion 1:3 bean:beet ratio had the most effect whereas the differences among other treatments was not so drastic (Fig. 4.30a and c). For relative growth rate, all treatments seemed to have the same effect (Fig 4.30b). The effect of increasing population density at the 2:2 mixture proportions treatments had similar responses as described above (Fig. 4.31a-c).

4.8.10 Beets 1984: Growth Indices (Absolute Growth Rate, Relative Growth Rate and Unit Leaf Rate): Regression Results

Fig. 4.32a-c shows the effect of changing mixture proportions at 66 plants m^{-2} population density treatment. No clear pattern was observed on treatment effect with time. Due partly to overfitting of the data, Fig. 4.32a-c indicate that all three indices fluctuated so much during growth (Fig. 4.32a-c). Similar patterns were seen for different total population densities at mixture proportion 2:2 treatment except the fluctuations were drastic only for 16 plants m^{-2} population density (Fig. 4.33).

(a)

(b)

(c)

Mixture proportions (Beans:Beets)
—— 4:0 ····· 3:1 —— · —— 2:2 —— 1:3
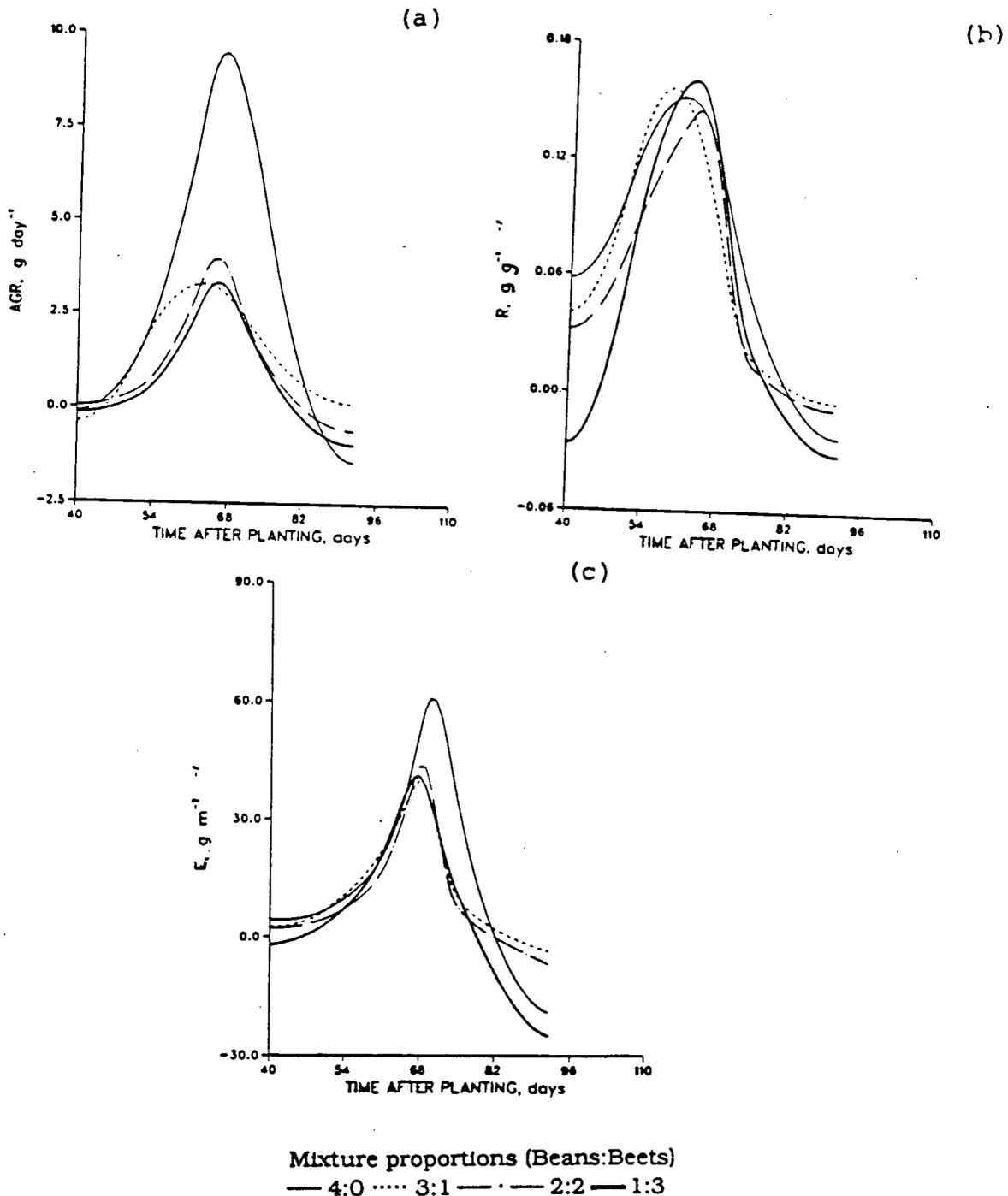
Fig. 4.30   Changes in absolute growth rate, relative growth rate and unit leaf rate per plant in beans during growth resulting from increasing mixture proportions of beets at total population density of 66 plants m⁻² (1984 experiment)

(a) Absolute growth rate per plant
(b) Relative growth rate per plant
(c) Unit leaf rate per plant

Population density (plants m⁻²)
— 16 ⋯⋯ 33 — · — 50 — 66
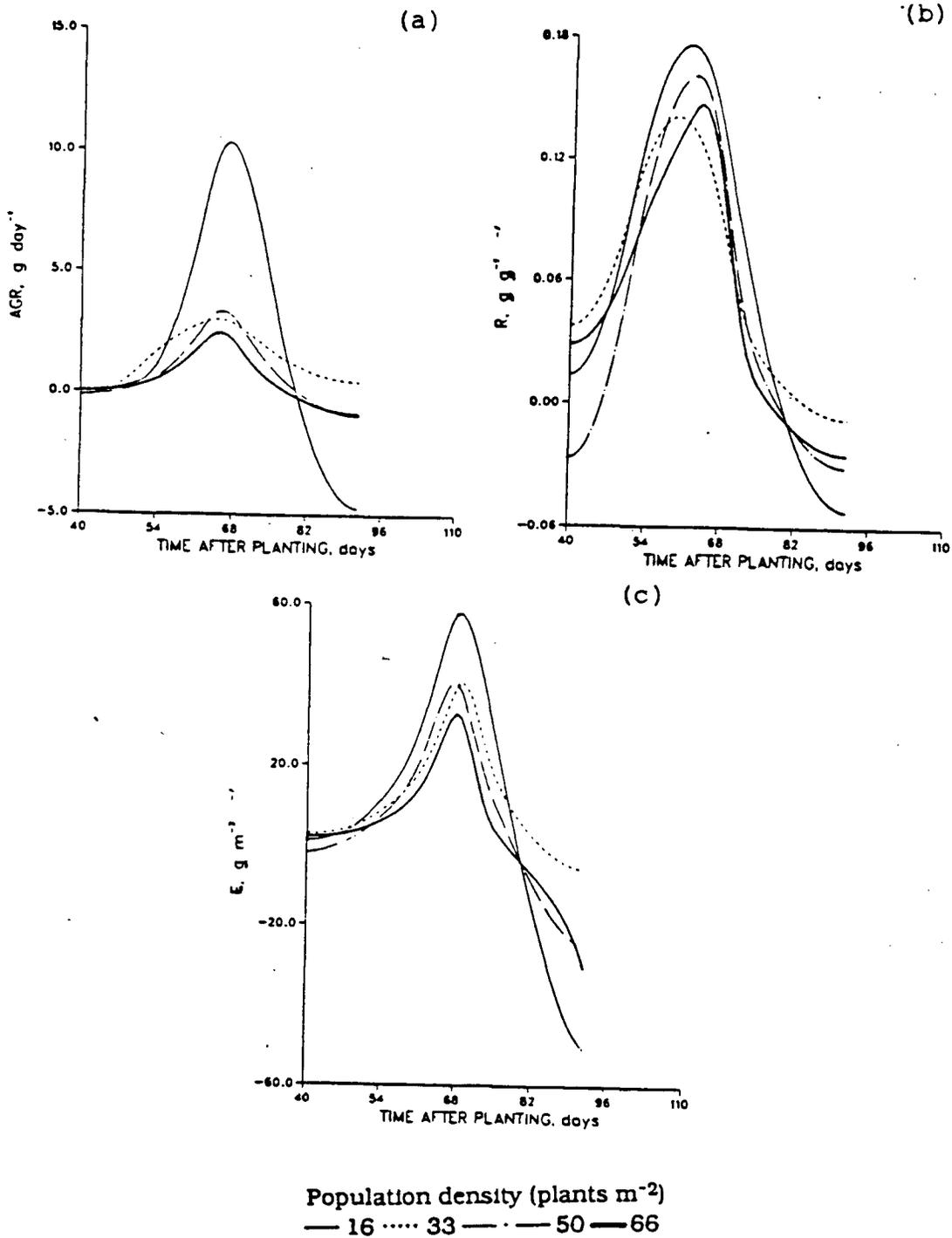
Fig. 4.31   Changes in absolute growth rate, relative growth rate and
unit leaf rate per plant in beans during growth resulting
from increasing total population density at 2:2 bean:beet
mixture proportion (1984 experiment)

(a) Absolute growth rate per plant
(b) Relative growth rate per plant
(c) Unit leaf rate per plant
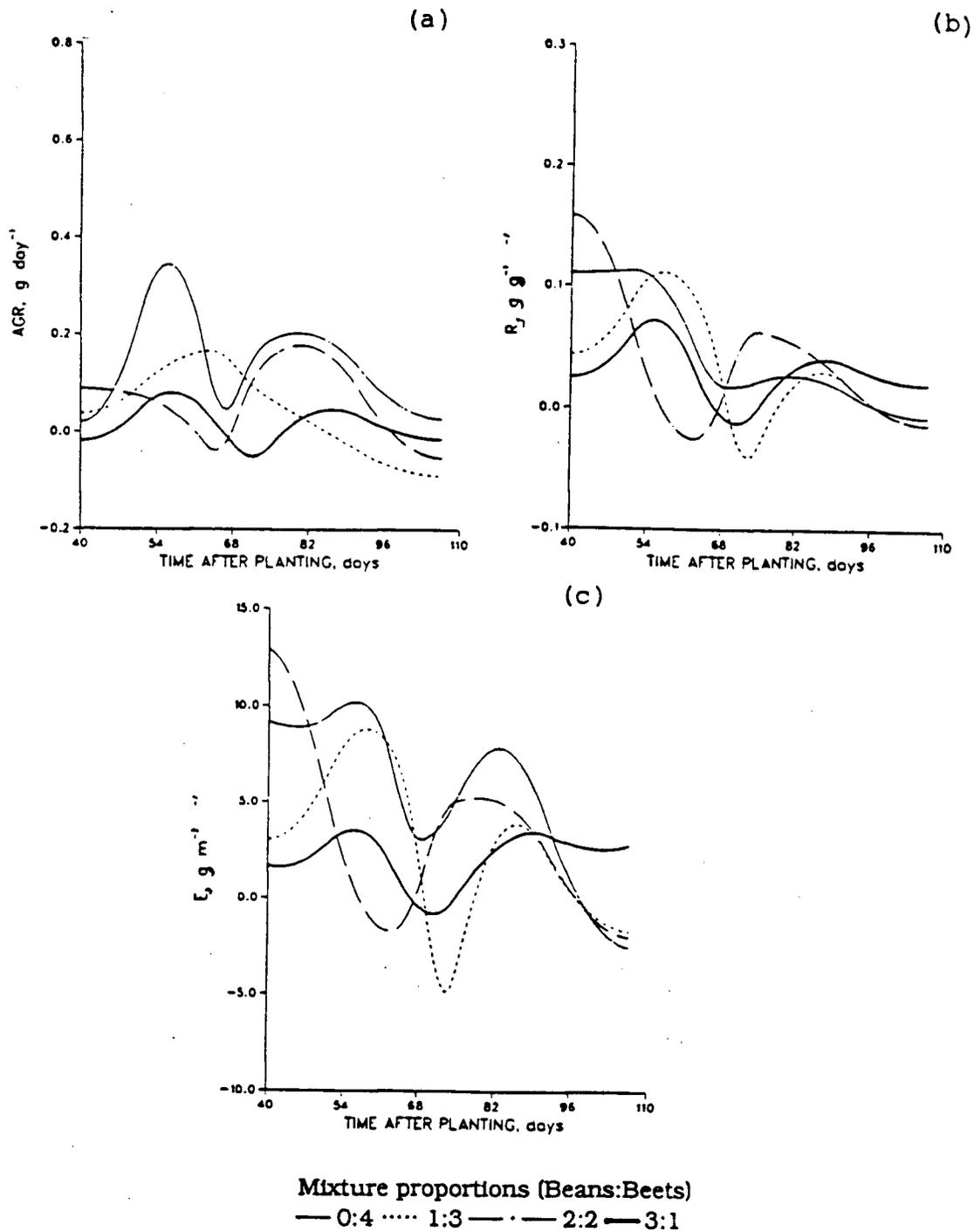
Fig. 4.32   Changes in absolute growth rate, relative growth rate and unit leaf rate per plant in beets during growth resulting from increasing mixture proportions of beans at total population density of 66 plants m^{-2} (1984 experiment)

(a) Absolute growth rate per plant
(b) Relative growth rate per plant
(c) Unit leaf rate per plant

Population density (plants m⁻²)
—— 16 ····· 33 ——·—— 50 ——66
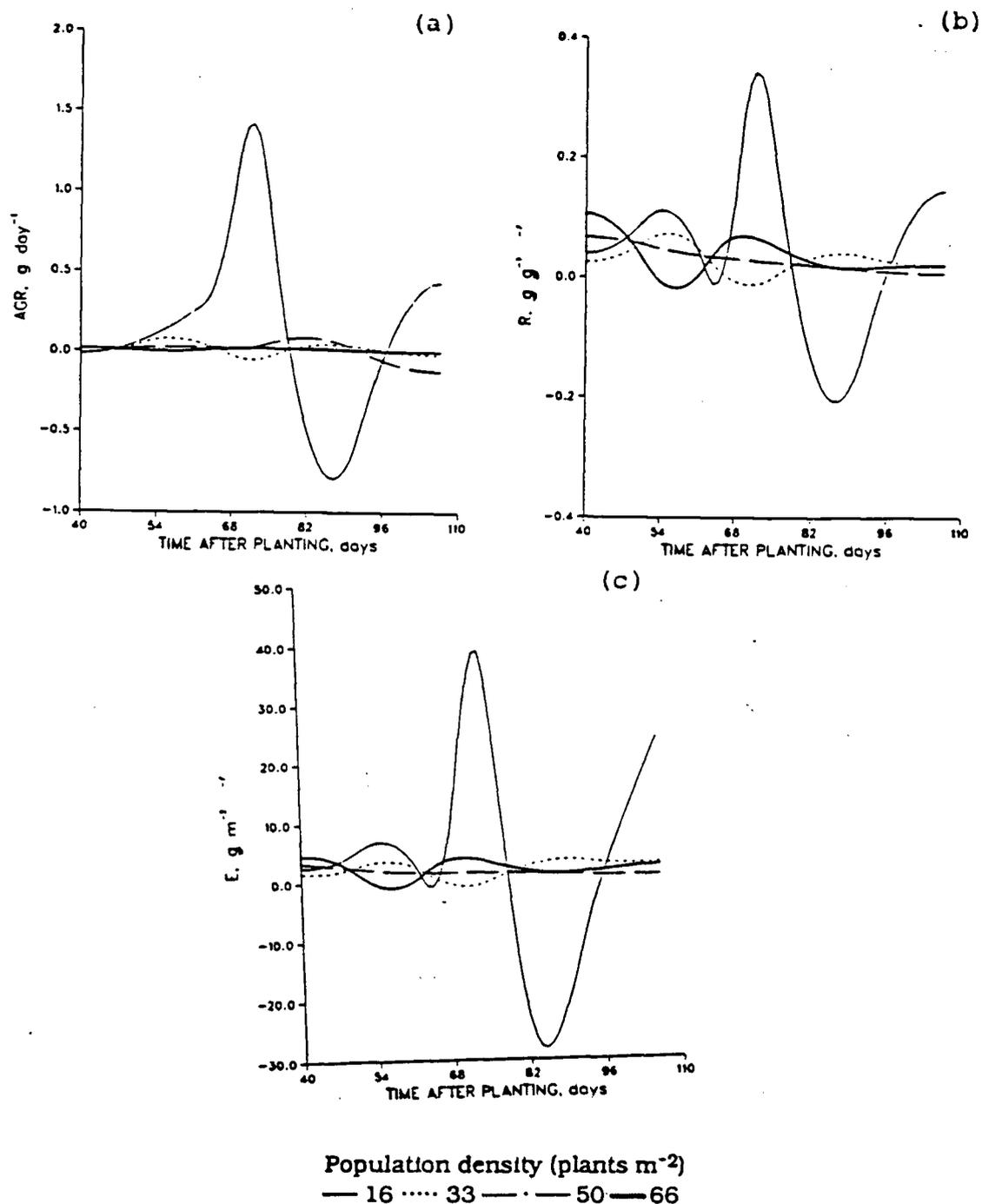
Fig. 4.33   Changes in absolute growth rate, relative growth rate and unit leaf rate per plant in beets during growth resulting from increasing total population density at 2:2 bean:beet mixture proportion (1984 experiment)

(a) Absolute growth rate per plant
(b) Relative growth rate per plant
(c) Unit leaf rate per plant

4.8.11 Summary of Plant Growth Analysis Results

Specific leaf area was found to increase with increasing population density in both species and with increasing mixture proportions of the competing species in beets. In beans, increasing mixture proportions of the competing species reduced specific leaf area. Leaf area ratio and specific leaf weight were significantly higher at higher population densities in beets monocultures and at high bean proportions in mixtures but generally not significant for beans although both ratios tended to increase with increasing densities in bean monocultures and with decreasing beets proportions in mixtures. Derived ratios increased with time and declined after reaching a peak at about 68 days. Both the lowest population density of 16 plants m^{-2} in both beans and beets, and the mixture proportions treatment with the least proportion of the competing species in beets had the greatest increase in absolute growth, relative growth and unit leaf rates. In beans, the mixture proportion with the highest proportion of beets had the greatest increasing in derived ratios. This pattern was also observed in the primary variables; total dry weight, leaf dry weight, leaf number and leaf area.

4.9 Plant Allometric Relationships

The relationships that exist between one part of the organism and another, or between part of the organism and the whole, are allometric relationships. The effect of treatments on allometric relationships were analysed using equation 3.8, and the results for each species are described in the following sections.

4.9.1 Beans 1984

Parameters and statistics for the allometric relationship of $\log_e$ shoot dry weight (W) ($=y_i$) against either $\log_e$ plant height (T), leaf number (LN), leaf area (LA), leaf dry weight (WL), stem dry weight (WST), branch number (BN), pod number (PN), pod fresh weight (FWPD), or pod dry weight (WPD) (z_i) for the 1984 bean data are contained in Appendix 8.5, and Table 4.20 has the summary of the standard partial regression coefficients. The size and sign of the standard partial regression coefficient can be used to indicate the relative magnitude and direction of the relationships between W and independent variables. Significant terms containing $\log_e(z)$, i.e. those containing parameter β, are indications of significant allometric relationships with $\log_e(W)$. Significant terms containing parameter α indicate treatment effects on $\log_e(W)$ which are non-allometric. Generally, the allometric relationships between $\log_e(y_i)$ and $\log_e(z_i)$ variables were influenced by different factors both between different z_i variables and between harvests within each z_i. The models as determined by the best subset regressions varied considerably. Results from some harvest dates had as many as 7 terms (including the allometric parameter α) explaining the variation in $\log_e(W)$ while the results in other harvest dates had as few as two terms, the allometric parameter α and the independent variable containing the β_0 exponent. The latter case indicates that treatments had no effect in influencing the allometric relationship between $\log_e(W)$ and $\log_e(z_i)$ variable (e.g. Appendix 8.5.2 the harvest at 63 days from planting). In situations where the allometric relationship was dependant on more than the 2 terms mentioned above, treatment effects explain the presence of the other terms in those models. The treatments could have either

Table 4.20   Summary of the allometric analysis for the 1984 bean data. Standard partial regression coefficients for allometric relationships of secondary variables with ln W (= y)

Age at harvest (days)	Potential independent variable	Parameter	Secondary variables (= z)								
			T	LN	LA	WL	WST	BN	PN	FWPD	WPD
40 (HN 1)†	Intercept	$\ln(\alpha')$	-4.032**	-0.166**	0.059**	-0.039**	-0.152**	-0.178**	-0.120**	-	-
	$\ln(z)$	β_0	0.849**	0.055**	0.017**	0.010**	0.139**	0.019**	0.083**	-	-
	$X_i\ln(z)$	β_1	0.424**	-	-	-	-	0.0003*	-0.001**	-	-
	$X_j\ln(z)$	β_2	-	-	-	-	-	-	-0.003**	-	-
	$X_iX_j\ln(z)$	β_3	-0.106	-	-	-	-	-	0.0001**	-	-
	$\ln(X_i)$	γ_1	-0.588**	-0.035**	-	-0.010**	-0.013**	-0.046*	-	-	-
	$\ln(X_j)$	γ_2	-1.251**	-0.071**	0.007**	0.004	0.006**	0.007**	-	-	-
	$\ln(X_iX_j)$	γ_3	0.977**	0.093**	-	-	-	-	-	-	-
51 (HN 2)	Intercept	$\ln(\alpha')$	-3.372**	-0.153**	0.060**	-0.039**	-0.062**	-0.196**	-0.250*	0.232**	-0.427**
	$\ln(z)$	β_0	0.692**	0.078**	0.028**	0.010**	0.016**	0.024**	0.140**	0.001*	0.187**
	$X_i\ln(z)$	β_1	-0.130	-0.001**	0.0002**	-	-	-	-0.001**	0.003**	0.004
	$X_j\ln(z)$	β_2	-	-	-	-	-	-	-0.001**	-	-
	$X_iX_j\ln(z)$	β_3	0.139	-	-	-	-	-	-	-	-
	$\ln(X_i)$	γ_1	-0.560**	-	-	-0.009**	-0.014**	-0.019**	-	-0.073**	-
	$\ln(X_j)$	γ_2	-0.178*	-0.064*	-	-	-	-	-	-0.155**	-
	$\ln(X_iX_j)$	γ_3	-	0.084*	-	-	-	-	-	0.178**	-
63 (HN 3)	Intercept	$\ln(\alpha')$	-0.168	-0.245**	0.370**	0.206**	0.236**	-0.447**	0.36**	-0.181	-0.136**
	$\ln(z)$	β_0	0.337**	0.097**	0.100**	0.028**	0.032**	0.063**	0.200**	0.055**	0.038**
	$X_i\ln(z)$	β_1	-0.530**	-	0.002*	-	-	-	-0.002**	-0.001	0.001**
	$X_j\ln(z)$	β_2	-	-	-	-	-	-	-	-	-
	$X_iX_j\ln(z)$	β_3	-	-	-	-	-	-	-	-	-
	$\ln(X_i)$	γ_1	-	-	0.120*	-0.043**	-0.049**	-	-	-	-
	$\ln(X_j)$	γ_2	-	-	-	-0.081*	-0.093**	-	-0.036	-	-
	$\ln(X_iX_j)$	γ_3	-	-	-	-0.106*	0.121**	-	-	-0.031	-

†Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Table 4.20 (cont'd)   Summary of the allometric analysis for the 1984 bean data. Standard partial regression coefficients for allometric relationships of secondary variables with ln W (= y)

Age at harvest (days)	Potential independent variable	Parameter	Secondary variables (= z)								
			T	LN	LA	WL	WST	BN	PN	FWPD	WPD
69 (HN 4)	Intercept	$\ln(\alpha')$	0.609**	0.311**	0.103**	-0.248	-0.132**	-0.319**	0.367**	0.106**	-0.056
	$\ln(z)$	β_0	0.135**	0.074**	0.057**	0.034**	0.033**	0.041**	0.124**	0.040**	0.008
	$X_i \ln(z)$	β_1	-0.001**	-	0.001**	-	-	-0.0002**	-	-	-
	$X_j \ln(z)$	β_2	-	-	0.0001*	-	-	-	0.005*	-	-
	$X_i X_j \ln(z)$	β_3	-	-	-	0.00002*	-	-	-	-0.00004**	-
	$\ln(X_i)$	γ_1	-0.127*	-0.483**	-	-0.039**	-	-	-0.064**	-	0.009
	$\ln(X_j)$	γ_2	-	-	-	-	0.069**	-	-0.207**	-	-0.015
	$\ln(X_i X_j)$	γ_3	-	-	0.040*	0.340*	-0.093**	-	0.209**	0.036**	0.019
75 (HN 5)	Intercept	$\ln(\alpha')$	0.905**	0.440**	0.099**	-0.254**	-0.143**	-0.368**	0.141	0.111**	-0.082
	$\ln(z)$	β_0	0.144**	0.086**	0.060**	0.036**	0.033**	0.047**	0.078	0.035**	0.009
	$X_i \ln(z)$	β_1	0.002	-	0.001**	-	-	-0.0003**	-0.001	-0.001**	-0.0002
	$X_j \ln(z)$	β_2	0.003*	0.003**	-	-	-	-	-	-	-
	$X_i X_j \ln(z)$	β_3	-	-	-	-	-	-	-	-	-
	$\ln(X_i)$	γ_1	-0.220**	-0.077**	-	-0.037**	-	-	-	-	0.022
	$\ln(X_j)$	γ_2	-0.306*	-0.211**	-	-	-	-	-	-	-
	$\ln(X_i X_j)$	γ_3	0.305*	0.216**	-	-	-	-	-0.027	-	-
92 (HN 6)	Intercept	$\ln(\alpha')$	0.438**	0.095**	0.117**	-0.435**	-0.176**	0.221**	0.485**	0.175**	-0.025**
	$\ln(z)$	β_0	0.097**	0.037**	0.035**	0.053**	0.032**	0.084**	0.141**	0.033**	0.004**
	$X_i \ln(z)$	β_1	-0.001*	-0.001**	-	-	-0.001*	-0.0003**	-0.003**	-	0.0001**
	$X_j \ln(z)$	β_2	-	-	-	-0.001	-0.001**	-	-	-	0.0002*
	$X_i X_j \ln(z)$	β_3	-	-	0.00002**	0.00004**	0.00004**	-	-	-	-0.0001**
	$\ln(X_i)$	γ_1	0.108**	-	-0.050**	-0.076**	-	-	-0.129	-0.037**	--
	$\ln(X_j)$	γ_2	-	-	-	-	-	-	-	-	-0.022
	$\ln(X_i X_j)$	γ_3	-	-	-	-	-0.041**	-0.023**	-	-	0.023*

*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

influenced the allometry via those terms with β_1, β_2 and β_3, and/or the treatments could have influenced the non-allometric variation (γ_k terms) which account for some of the variation due to $\log_e(\varepsilon)$ in equation 3.8 and/or the treatment might have well influenced the allometric parameter α. It is possible that some of the treatment effects on the allometric parameter α could be allocated to the γ_k terms and not $\log_e(\alpha)$ so that a significant γ_k term would either be due to direct treatment effects on $\log_e(W)$ not related to allometry and/or may express treatment effects on the parameter α. The parameters β_k and γ_k acquire the units of measurement of the related independent variable.

Except for a few cases (e.g. $\log_e(LA)$ for harvests at 40, 63 and 69 days from planting), population density, mixture proportions or population density by mixture proportions interaction generally reduced $\log_e(W)$ (Appendix 8.5). All terms containing β_0 were positive and most of them were significant showing a direct allometric relationship aside from the effects of treatments. Significant interactions between treatments and the allometric exponents β_1, β_2, and β_3 were also detected in most regressions.

Some of the independent terms in the models did not make significant contributions to explaining the variation in $\log_e(W)$ but were included in the models because of the best subset regressions technique used (Appendix 8.5). The removal of the non-significant terms from the models may cause a non significant reduction in R^2, but this may invalidate the best subset regression.

The direction of response of the allometric parameters α, β_k, and γ_k on the allometric relationships followed no particular pattern. A factor would respond positively in one harvest and negatively in other harvest

within the same z_i variable. Similarly, no obvious pattern was observed between z_i variables at the same harvest date.

The coefficients of determination (R^2) which resulted from fitting equation 3.8 also varied from harvest date to harvest date within each z_i variable. In almost all cases, the data harvested at 63 days from planting had the lowest R^2 value except in FWPD and WPD where the plants harvested at 51 days from planting had the lowest R^2 values. In most relationships the R^2 values were quite high, ranging between 0.70 to 0.99, but R^2 values of as low as 0.29 were observed. The higher R^2 values indicate an obvious allometric relationship (Appendix 8.5).

4.9.2 Beans 1987

As described by model 3.8, the results for the allometric relationships that exist between $\log_e(y_i)$ and $\log_e(z_i)$ are shown in Appendix 8.6, while Table 4.21 contains the standard partial regression coefficients. Similar to the 1984 bean data, each $\log_e(z_i)$ variable related to $\log_e(y_i)$ had a different model. The number of independent variables explaining the variation in $\log_e(W)$ varied from 5 to 7, again counting the allometric parameter α. All relationships indicated a strong treatment effect, either on the terms containing the allometric exponent β_k or the terms containing non-allometric coefficients γ_k. The allometric parameter α could also have been influenced by treatment factors which can be expressed in the γ_k terms. Similar to the 1984 bean data was also the presence of non-significant terms in some models because of the best subset regression technique used in the analysis.

Unlike the 1984 bean data, some pattern was observed in the direction of response of the allometric exponent β_0 in that it was found to

Table 4.21 Summary of the allometric analysis for the 1984 bean data. Standard partial regression coefficients for allometric relationships of secondary variables with ln W (= y)

Potential independent variable	Parameter	Secondary variables (= z)										
		LN	LA	WL	WST	MPN	UPN	FWPD	WUPD	WMPD	SN	WS
Intercept	$\ln(\alpha')$	-0.305	2.910**	3.539**	4.042**	2.234**	4.745**	-1.387**	6.505**	1.804**	1.450**	4.613**
$\ln(z)$	β_0	0.817**	0.817**	1.004**	0.781**	0.961**	0.489**	0.937**	0.164**	0.927**	0.729**	0.610**
$X_i\ln(z)$	β_1	-0.256**	-	-0.111**	-	-0.115**	-0.097	-0.228**	0.161**	-0.116**	-0.382**	-
$X_j\ln(z)$	β_2	-0.217**	-	-0.159**	-	-0.125**	-	-0.152**	-	-0.105**	-0.305**	-
$X_iX_j\ln(z)$	β_3	-0.116*	-0.123**	0.090**	-0.071*	-0.073	-0.183**	-0.142**	-0.075	-0.143**	-0.149**	-0.238**
$\ln(X_i)$	γ_1	0.269**	-0.073*	-	-0.171**	0.087**	-0.175**	0.280**	-0.324**	0.127**	0.318**	-0.125**
$\ln(X_j)$	γ_2	0.202**	-0.451**	-	-0.849**	-	-1.152**	0.225**	-1.560**	-	-	-0.869**
$\ln(X_iX_j)$	γ_3	-	0.491**	-	0.825**	0.130**	1.144**	-	1.367**	0.186**	0.306**	0.937**

*Significant at P = 0.05
**Significant at P = 0.01

be positive in all the z_i variables tested indicating a positive influence on the direct allometric relationship between $\log_e(W)$ and $\log_e(z_i)$ variable in question. The parameter α was also positive in all cases except in $\log_e(FWPD)$ and $\log_e(LN)$ z_i variables. The allometric exponents β_1, β_2 and β_3, and the non-allometric parameters γ_1, γ_2, and γ_3 did not follow any pattern. These parameters were found to respond positively in one z_i variable and negatively in another.

The R^2 values obtained also differed among z_i variables. In general, the R^2 values were lower compared to those for the 1984 bean data. Most z_i variables had R^2 values ranging between 0.58 to 0.79 except variables UPN, WUP, SN and WS who had R^2 values below 0.40. Although significant at 5% level of significance, these low R^2 values do not indicate an obvious allometric relationships between $\log_e(W)$ and the corresponding $\log_e(z_i)$ variables.

4.9.3 Beets 1984

The allometric relationships between $\log_e(y_i)$ and $\log_e(z_i)$ varied greatly depending on the $\log_e(z_i)$ variable being evaluated in relation to $\log_e(y_i)$ and time at harvest (Table 4.22, and Appendix 8.7). Thus, a different model for each $\log_e(y_i)$ at each harvest date within and between $\log_e(z_i)$ was the common occurrence. The largest model in the 1984 beets data had all 8 terms accounting for the variation in $\log_e(W)$ while the smallest models had 3 terms indicating treatment effects in all cases. Based on the same reasoning as in the bean data, the non-significant terms were retained in the equations.

Just as in the bean data, the treatments could have influenced the allometry by influencing the allometric parameter α, the allometric terms

Table 4.22   Summary of the allometric analysis for the 1984 beet data. Standard partial regression coefficients for allometric relationships of secondary variables with ln W (= y)

Age at harvest (days)	Potential independent variable	Parameter	Secondary variable (= z)							
			T	LN	LA	WL	WP	DR	WR	FWR
40	Intercept	$\ln(\alpha')$	-7.136**	-8.419**	5.211**	-0.444**	-2.037**	-4.623**	-1.870**	-
(HN 1)†	$\ln(z)$	β_0	0.886**	0.885**	0.923**	0.986**	0.932**	1.005**	0.964**	-
	$X_i \ln(z)$	β_1	-0.115**	-0.398*	-	-	0.087**	-0.162**	-	-
	$X_j \ln(z)$	β_2	-0.337**	-0.672**	-	-	-	-	-0.058*	-
	$X_i X_j \ln(z)$	β_3	–	-	0.150**	-0.017	-	-0.038	-	-
	$\ln(X_i)$	γ_1	–	0.439*	-	-	0.080**	0.175**	-0.050*	-
	$\ln(X_j)$	γ_2	–	1.446**	-	-0.132**	0.318**	-	-	-
	$\ln(X_i X_j)$	γ_3	–	-0.969*	-	0.083**	-0.240**	-	-	-
51	Intercept	$\ln(\alpha')$	-5.372**	-5.377**	5.200**	-0.202**	-1.358**	-2.150**	-1.316**	-0.389**
(HN 2)	$\ln(z)$	β_0	0.769**	0.818**	1.025**	1.099**	1.139**	0.840**	0.912**	0.859**
	$X_i \ln(z)$	β_1	–	-	-0.240**	-0.199**	-0.143**	0.174**	0.133**	-0.089
	$X_j \ln(z)$	β_2	-0.289**	-0.126**	-0.161**	-0.100**	-0.149**	-	0.103	-
	$X_i X_j \ln(z)$	β_3	-	-	-	0.144**	0.180**	-	-0.210	0.170**
	$\ln(X_i)$	γ_1	-	0.051	-0.205**	-0.051**	-0.039*	-1.171**	0.004*	0.079**
	$\ln(X_j)$	γ_2	-0.112	-	-0.776**	-0.168**	0.143**	-0.050	-	0.335**
	$\ln(X_i X_j)$	γ_3	-	–	0.538**	0.122**	-	-	-	-0.365**
63	Intercept	$\ln(\alpha')$	-6.058**	-5.122**	5.209**	-0.500**	-1.230**	-1.995**	-0.814**	-0.003
(HN 3)	$\ln(z)$	β_0	0.755**	0.810**	0.970**	0.960**	1.030**	0.917**	1.006**	0.943**
	$X_i \ln(z)$	β_1	-0.311**	-0.241**	-0.274**	-0.059**	-0.069*	-0.107**	-	-0.071*
	$X_j \ln(z)$	β_2	-0.298**	-	-0.195**	-	-	0.167**	-0.145**	-
	$X_i X_j \ln(z)$	β_3	-0.174*	-0.265**	-	0.096**	0.093**	-	-	0.113**
	$\ln(X_i)$	γ_1	0.337**	0.256**	-0.271**	-	-	-	-	-
	$\ln(X_j)$	γ_2	–	–	-0.835**	-0.018*	0.252**	-0.442**	-0.330**	-
	$\ln(X_i X_j)$	γ_3	-0.135*	-	0.465**	-	-0.127	0.184**	0.234**	-

†Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

Table 4.22 (cont'd) Summary of the allometric analysis for the 1984 beet data. Standard partial regression coefficients for allometric relationships of secondary variables with ln W (= y)

Age at harvest (days)	Potential independent variable	Parameter	Secondary variable (= z)							
			T	LN	LA	WL	WP	DR	WR	FWR
69 (HN 4)	Intercept	$\ln(\alpha')$	-4.631**	-3.898**	5.589**	-0.404**	-1.443**	-3.729**	-0.730**	-0.014
	$\ln(z)$	β_0	0.724**	0.775**	0.960**	0.921**	0.940**	1.320**	0.959**	0.935**
	$X_i\ln(z)$	β_1	–	-0.123*	-0.179**	–	–	-0.265*	–	–
	$X_j\ln(z)$	β_2	-0.207**	-0.212*	–	–	0.138**	-0.074**	–	0.062
	$X_iX_j\ln(z)$	β_3	-0.110	-0.195**	-0.173**	0.042**	–	–	-0.058**	–
	$\ln(X_i)$	γ_1	–	–	-0.200**	-0.047**	0.027	0.279*	–	–
	$\ln(X_j)$	γ_2	-0.366**	-0.188	-0.378**	-0.173**	0.182**	1.365**	-1.101**	–
	$\ln(X_iX_j)$	γ_3	–	–	–	0.107*	–	-0.687**	–	–
75 (HN 5)	Intercept	$\ln(\alpha')$	-3.728**	-2.663**	4.427**	-0.678**	-2.107**	-3.076**	-0.190	-0.182**
	$\ln(z)$	β_0	0.667**	0.748**	0.786**	0.943**	1.142**	1.066**	0.815**	1.012**
	$X_i\ln(z)$	β_1	-0.137**	–	–	-0.045**	-0.153**	-0.105**	0.078**	–
	$X_j\ln(z)$	β_2	–	-0.171**	–	0.081**	–	-0.118*	0.073*	–
	$X_iX_j\ln(z)$	β_3	-0.146**	–	–	–	0.147**	0.071	0.078*	–
	$\ln(X_i)$	γ_1	–	-0.085	-0.050**	–	0.146**	0.082	-0.065**	–
	$\ln(X_j)$	γ_2	-1.007	–	-0.337**	–	0.660**	0.406**	-0.409**	0.048**
	$\ln(X_iX_j)$	γ_3	0.408**	-0.095	–	-0.035*	-0.396**	-0.329**	0.263**	–
92 (HN 6)	Intercept	$\ln(\alpha')$	-2.984**	-2.244**	4.785**	-0.553**	-1.367**	-2.492**	-0.608**	-1.130**
	$\ln(z)$	β_0	0.643**	0.628**	0.812**	0.877**	0.896**	0.978**	0.955**	0.958**
	$X_i\ln(z)$	β_1	-0.209**	-0.163**	0.044	–	–	-0.050**	–	–
	$X_j\ln(z)$	β_2	-0.223**	0.345*	–	0.090**	0.205**	0.038	-0.037*	0.080**
	$X_iX_j\ln(z)$	β_3	–	-0.144	-0.088**	–	–	–	–	-0.042
	$\ln(X_i)$	γ_1	–	–	–	-0.032*	–	–	–	–
	$\ln(X_j)$	γ_2	-1.694**	-1.248**	-0.389**	-0.071**	0.187**	–	-0.080**	–
	$\ln(X_iX_j)$	γ_3	-0.738**	0.712**	–	–	–	-0.058*	–	–
107 (HN 7)	Intercept	$\ln(\alpha')$	-3.133**	-3.817**	5.499**	-0.335**	-0.500**	-3.680**	-0.601**	-0.032
	$\ln(z)$	β_0	0.599**	0.761**	0.819**	0.881**	0.906**	1.096**	0.900**	1.055**
	$X_i\ln(z)$	β_1	-0.127**	-0.304**	–	–	–	-0.142*	–	–
	$X_j\ln(z)$	β_2	–	-0.222**	–	–	0.172**	-0.179**	–	-0.061**
	$X_iX_j\ln(z)$	β_3	–	–	-0.114**	0.052**	–	–	–	–
	$\ln(X_i)$	γ_1	–	0.206*	-0.061*	-0.092**	–	0.141*	–	-0.003
	$\ln(X_j)$	γ_2	-1.011**	–	-0.417**	-0.273**	0.407**	0.686**	-0.251**	0.079**
	$\ln(X_iX_j)$	γ_3	0.278	-0.178**	–	0.190*	-0.236*	-0.464*	0.145*	0.145*

*Significant at P = 0.05
**Significant at P = 0.01

containing the β_k exponent with the exception of the β_0 term, and/or the non-allometric terms containing the γ_k parameters. The effect of the population density of beets in monocultures or of beans and beet-bean interactions in mixtures did not follow any pattern.

Some specific patterns in the direction of response were observed in the case of the allometric parameter α. This parameter was found to be negative in all harvest dates in all the z_i variables except in the case of $\log_e(LA)$ where it was positive at all harvest dates. This would indicate a negative influence of the allometric parameter α in the other z_i variables and a positive influence on the relationship between $\log_e(W)$ and $\log_e(LA)$. The allometric exponent β_0 also followed a particular pattern in that it was found be have a positive influence on all the allometric relationships determined for the 1984 beets data. The other β_k and all the γ_k terms did not follow any particular pattern.

The R^2 values for almost all relationships in the 1984 beets data were above 0.70 indicating strong allometric relationships between $\log_e(W)$ and $\log_e(z_i)$ variables. A lower R^2 value of 0.58 was observed for the allometric relationship between $\log_e(W)$ and $\log_e(LN)$ at 40 days from planting. Lower R^2 values indicate weaker allometric relationships.

4.9.4 Beets 1987

Appendix 8.8 contain the parameters and statistics while Table 4.23 has the standard partial regression coefficients for the 1987 beets data. Similar to the other cases above, the models have been found to be different for each $\log_e(z_i)$ variable being tested in relation to $\log_e(y_i)$. The largest models had 8 independent terms explaining the variation in $\log_e(W)$ and the smallest models had 4 independent terms indicating

Table 4.23   Summary of the allometric analysis for the 1987 beet data. Standard partial regression coefficients for allometric relationships of secondary variables with ln W (=y)

Potential independent variable	Parameter	Secondary variables (= z)						
		LN	LA	WL	WP	DR	FWR	WR
Intercept	$\ln(\alpha')$	-0.620	0.029	1.073**	2.086**	-1.442**	-1.494**	0.315**
$\ln(z)$	β_0	0.564**	0.825**	0.889**	0.924**	0.786**	1.098**	1.075**
$X_i\ln(z)$	β_1	0.340**	-	-	-0.157**	0.163**	-0.186**	-0.118**
$X_j\ln(z)$	β_2	-	0.003**	0.281**	0.165**	-0.172**	-	0.152**
$X_iX_j\ln(z)$	β_3	0.248**	-0.176**	-0.375**	-0.294**	0.155**	-0.113**	-0.246**
$\ln(X_i)$	γ_1	-0.521**	0.079**	-	-0.069**	-0.235**	0.247**	0.067**
$\ln(X_j)$	γ_2	-0.249**	-	-	0.321**	-	0.097**	-0.171**
$\ln(X_iX_j)$	γ_3	-	-0.062**	-	-0.510**	-	-	0.187**

**Significant at P = 0.01

treatment effects in all cases. Non-significant terms were again retained in the models.

Just as in the 1984 beets data, the treatments influenced the allometric relationship either through the allometric parameter α and/or through the β_k terms excluding the β_0 term and/or through the non allometric terms containing the γ_k parameters. The presence of the terms in the models did not follow any particular pattern.

The allometric exponent β_0 influenced the allometric relationships positively in all the $\log_e(z_i)$ variables, whereas the other parameters were inconsistent in their response. They would respond positively in one $\log_e(z_i)$ variable and negatively in another variable, thus presenting no particular pattern

The R^2 values obtained were all about 0.70 and above, except in the $\log_e(W)$ vs $\log_e(LN)$ relationship where a lower R^2 value of 0.58 was observed. Again a lower R^2 value indicates a weaker relationship.

4.9.5 Summary of Allometry Results

Allometric relationships in each species were found to be quite complex. The allometric relationships between $\log_e(W)$ and $\log_e(z)$ were influenced by different factors within the same z_i variable at different harvest dates in both species in 1984 and were influenced by different factors among z_i variables in both species in both years. There was no pattern in the direction of responses but most variables indicated strong relationships as indicated by high R^2 values. Strong treatment influences were also detected for each species in both years.

5. DISCUSSION

5.1 An Overview

Interference among neighbouring plants in monocultures and mixtures of different plant species has been widely examined, although this seems to be the first study to look at bean-beet associations. The general aims of this study were to quantify the interference between beans and beets, and to reach some understanding about the nature of those interferences by detailing the timing and sites within plants of interference effects. Thus, different analytical approaches were applied to the data to meet these specific aims.

5.2 Visual Observations

Growth and development of both beans and beets was monitored throughout the growing season. Early in the growing season, beets were found to be quite slow to germinate. This could have been due to the excessive seeding depth in relation to seed size. Visually, plants in both monocultures and mixtures appeared to be quite healthy throughout the growing season. In both species, plants from densely seeded plots looked thinner and were taller than plants in less dense plots. The analysis of variance (ANOVA) results also indicated a reduction in per plant yield with increasing plant population density for most of the variables tested in both species in both years. The reductions in per plant yield reported in this study have commonly been observed by other researchers who have studied plant interference (e.g. Gaye 1990, Weiner 1984, Potdar 1986, Carmi 1986,).

5.3 Analysis of Variance

In 1984, treatment effects (*i.e.* population density and mixture proportions) on most variables were large later in the growing season. For total dry weight, leaf dry weight and leaf area in beans the effects due to population density were detected as early as 40 days after planting, and for leaf dry weight the effects due to mixture proportions were also detected early during growth. In beets, the effects due to mixture proportions on all variables except plant height were also detected early in the growing season. In 1987, there were fewer significant treatment effects especially in beans. This could have been due to the large variations among plants within the same treatment. In both 1984 and 1987, effects due to population density and mixture proportions interactions were not common. Hence, in both species and both years there were significant and diverse responses to treatments, and these responses were further explored by several lines of data analysis.

5.4 Yield-density Relationships

Plants grown at low population densities grow as if they are in isolation, and thus experience no interference from each other (Spitters 1983, Jolliffe *et al.* 1984). Interference among neighbouring plants intensifies with increasing proximity (Firbank and Watkinson 1985). Non-linear inverse models were used in this study to define yield-density responses because they have a biological basis (Jolliffe 1988) and also because their parameters serve as useful indices of plant interference. Thus, they offer a potentially powerful approach to data interpretation.

Four variables, total dry weight, leaf dry weight, leaf number and leaf area all of which were expressed on a per plant basis, were evaluated

in both beans and beets. Comparisons of the yield-density models formed for the different species and variables are made difficult by the effect of parameter Φ in scaling the yield variate. However, in this study all the models indicated that beans were better competitors both in monocultures and mixtures. This is indicated by the ratio b_{ii}/b_{ij} (Table 4.7), which was always greater than 1.0 for bean plants responding to themselves and beets, and less than 1.0 for beet plants responding to themselves and beans. Given the error in estimating parameters b_{ii} and b_{ij}, the competitive balances between beans and beets were approximately the same for all four variables and all harvests.

Inverse yield-density models have been found to be inadequate for certain yield variables, such as shoot height (Jolliffe 1988). Gillis and Ratkowsky (1978) also found the Bleasdale-Nelder equation to produce biased parameter estimates, especially in positively skewed data distributions. Large variations in the data could also impair the adequacy of the models, and these data were quite variable as could be seen from the high Gini coefficient values (indicators of size heterogeneity) for both monoculture and mixture data. The partitioned Layard homogeneity of variance test, and the significant F test between subsamples within a treatment were other indications of high variation in the data. Gaye (1990) also found the Bleasdale-Nelder model unsatisfactory for describing yield for culled and undersized graded fruit of bell pepper (*Capsium sativum*) and attributed the difficulty to the small proportions of these grades relative to total fruit yield. This could not have been a contributing factor in this present study as all four variables chosen were major components of the total yield.

5.5 Differential Yield Responses of Mixtures

The differential yield response of mixtures is based on a comparison of performance of crop mixtures in relation to their corresponding monocultures; overyielding in mixtures is the central issue in mixed cropping. Yield-density models (Jolliffe 1988) and direct computation of land equivalent ratio (LER) from the data were used to evaluate differential yield responses. When observed yields were used in calculating LER, results for both beans and beets usually indicated overyielding for the four variables (*i.e.* total dry weight, leaf dry weight, leaf number and leaf area) at all total population densities used in this study. The exception to this was with total dry weight at the total population density of 16 plants m^{-2}, where a yield disadvantage was obtained. The results obtained at 16 plants m^{-2} total population density might not be representative of interference effects, as the plants may not interfere strongly at low total population densities.

When equation 3.3 was used to predict LER, two variables, leaf weight and leaf number indicated yield advantage, while total dry weight and leaf area indicated yield disadvantage. Two causes for the discrepancy between the observed and the modelled results for LER of leaf area and total dry weight can be advanced. First, the yield-density models were developed using data from all the population densities; they might not necessarily predict the results at a particular density successfully, given the high variability in the observations. Second, the models were developed for yield per plant, while differential yields are on a per land area basis. Multiplication of yield per plant by population density to produce yield per land area ($Y = yX$) magnifies errors of estimation. This could explain also why model 3.3 could not predict LER

for marketable yield in beets. The observed LER for all the four variables ranged between 1.0 and 2.5.

5.6 Size Hierarchies

Plant size distribution, as measured by the Gini coefficient, was found to be highly and positively skewed for most variables at all four planting densities used in this study. That is, the results agree with the statement that plant populations often contain a few large individuals and many suppressed small neighbours (Weiner 1984). The results indicate that interference among bean, among beets and between bean and beet plants was "one-sided" (*i.e.* in a pairwise interaction there seemed to be a winner and a loser) (Weiner 1986, Weiner and Thomas 1986). Systematic changes in Gini coefficients in relation to experimental treatments appeared to occur in a few cases (e.g. increased Gini coefficient with increasing population density for leaf number of bean plants in monocultures). These patterns occurred more commonly in beets than in beans, but it is difficult to test the statistical significance of such apparent trends. Bonan (1988) found that the development of size hierarchies depends on a complex interaction of factors such as growth rate, spatial distribution and the degree to which competition for resources is symmetric or asymmetric. Pooling the mixture proportions to study the effect of population density did not reveal any pattern, but when population densities were pooled to evaluate the effect of mixture proportions, it was consistently found that the monoculture bean treatments had higher Gini coefficients than mixed treatments. This may reflect that in beans intraspecific competition was more intense than interspecific competition.

In beet monoculture treatments, the 33 plant m^{-2} population density treatment had the lowest Gini coefficient indicating less variation in plant size. Under normal circumstances, one would expect the lowest population density treatment to have the lowest Gini coefficient because of less or no interference among neighbouring plants (Weiner 1986). Whereas pooling population density treatments did not show any pattern, mixture proportions pooled indicated an increase in Gini coefficients with increasing population density. These findings are also in agreement with those of Wiener (1985), and Wiener and Thomas (1986) who found an increase in the Gini coefficient with increasing population densities. Rice (1990), working with two *Erodium* species, also found an increase in reproductive hierarchies with increasing population density.

The high Gini coefficients for the whole data set for many variables are a clear indication of large variations in the data. Beets were more variable than beans, judging from the Gini coefficients values. Increasing the sample size by pooling population densities and mixture proportions tended to produce clearer treatment effects suggesting that a sample size of more than 60 individuals might have led to a clearer perception of treatment effects.

5.7 Yield Component Analysis

Three approaches were used to analyze treatment effects on growth: yield component analysis, conventional plant growth analysis and allometric analysis. These procedures were intended to examine relationships underlying the overall growth responses among different plant attributes as they are influenced by experimental treatments.

Yield component analysis is a procedure that expresses yield as the product of a set of morphological yield components. The procedure has been used extensively in improving grain yield in rice (e.g. Matsushima 1966, 1980). The two dimensional partitioning (TDP) technique attempts to identify how the variation in yield is associated with variation in yield components, and how yield component variation depends on experimental sources of variation (Eaton *et al.* 1985). The yield components are entered in the regression both in the forward manner (chronological sequence) and in the reverse order *(i.e.* the backward analysis).

The effects of treatments on the yield and on individual yield components were more prominent later in the growing season. This is not surprising as interference among neighbouring plants is intensified as plants grow bigger. Expanding shoot and root systems most likely lead to competition as the resources start to be limiting (Trenbath 1974). Yield component analysis indicated that the source of treatment effects was in most cases the yield component leaf number (LN), regardless of whether the forward or backward TDP analysis was done. Both population density and mixture proportions treatments, and their interaction, affected the yield components, though not necessarily at the same harvest time. Eaton *et al.* (1985) and Jolliffe *et al.* (1990) also found strong treatment effects on some yield components.

Yield component analysis was therefore able to single out the yield components which were directly and indirectly affected by treatments and also quantified the contribution of each yield component to the total yield. The contribution of each yield component varied among harvests and differed depending on whether the forward or backward analysis was

used. In both cases the yield component that entered the regression first seemed to have contributed the most to the total yield variation. In part, this may be because the first component entering the model has the opportunity to account for all of the variation in yield; later components can only account for residual variation not explained by earlier ones in the model. Significant yield components which enter the model last can be considered to act directly on yield variation, since effects of all other components in the model have been taken into account. For example, this was the case with the yield component LN in the backwards analysis in beans at harvests 4 and 6 (Table 4.12b)

5.8 Plant Growth Analysis

Conventional plant growth analysis is a time based analytical model which uses growth indices such as relative growth rate and leaf area ratio to express the presence of assimilatory structures and their performance as growth proceeds. Fitted growth curves are used to compute the growth indices and to indicate the timing and extent of changes in plant performance during growth.

The ANOVA showed that the primary variables from which the growth indices were computed, were significantly affected by treatments. Similarly, ANOVA performed on the growth indices (leaf area ratio, leaf weight ratio, specific leaf weight) and the results usually indicated strong response of these indices to both population density and mixture proportions. In both beans and beets, specific leaf area increased with increasing population density and in beets with increasing density of the competing species in mixtures. For beans increasing mixture proportions of beets reduced specific leaf area. This could reflect a

mechanism by plants which serve to enhance the commitment of dry matter produced to leaf production as a plastic response to overcrowding. Whereas leaf area ratio and specific leaf weight did not generally respond to treatments in beans, both leaf area ratio and specific leaf weight were found to increase with increasing population density in beets monocultures, and with increasing proportions of beans in mixtures.

The proportion of marketable yield produced within plants (i.e. the harvest index) responded significantly to treatments in beets, but not in beans. An increase in population density or in the proportion of the competing species in mixtures caused a decrease in the proportion of beet storage root to total dry weight. This response is again a direct influence of interference in plant communities. The non-significant response of beans to treatment effects could be again due to large variations in the data set making the experimental error too large to be able to detect any significant differences.

Growth curves were generated from a subset of the overall results, involving the 66 plants m^{-2} population density and the 1:1 bean beet mixture proportions. Thus, the effects of increasing mixture proportions of the competing species was evaluated at 66 plants m^{-2} total population density, and the effects of increasing population density was evaluated at mixture proportion of 1:1 bean:beet ratio. The choice of conditions for evaluating plant responses by conventional plant growth analysis was arbitrary but it was based on the belief that interference among plants is intense at high population density and the 1:1 bean:beet ratio would give each species an equal opportunity to cause interference. Fitted curves

were not done on the 1987 data as only a final harvest was done during that year.

Fitted growth curves for the 1984 data showed that absolute growth rate, relative growth rate and unit leaf rate increased with time and reached a maximum at about 68 days from planting before declining. The monoculture beets or the mixture proportion with the most beets in beans had the highest of all three growth rates. Similarly the 16 plants m^{-2} density treatment had the highest absolute, relative and unit leaf growth rates. Thus, both increasing population density at 1:1 bean:beet mixture proportions in and increasing mixture proportions of beans at 66 plants m^{-2}, reduced absolute growth rate, relative growth rate and unit leaf rate of the test species. This is due to increasing intra- and interspecific interference with increasing total population density and increasing mixture proportions of beans.

5.9 Plant Allometric Relationships

Quantitative relationships that exist among different features of an organism as growth proceeds are referred to as allometric relationships (Jolliffe *et al.* 1988). The procedure of Jolliffe *et al.* (1988) was used to assess the effects of population density and mixture proportions on yield. The variation in yield (y) was partitioned to direct treatment effects, allometry on z, treatment effects with z, treatment effects on allometry with z, and residual variation. Since the allometric power equation (2.19) is equivalent to $y/z = \alpha z^{\beta-1}$, this approach can thus be used to analyze the effects of treatments on plant proportions.

The results obtained indicated that allometric relationships between $\log_e(y_i)$ and $\log_e(z_i)$ variables were influenced by different factors.

The effects of treatments were evident in situations where the variation in $\log_e(y_l)$ was explained by more than two terms including the allometric parameter α and the independent variable containing the β_0 exponent. In most cases, it was found that treatments influenced the allometric relationship either through the allometric coefficient a or through the allometric exponent β_k, $k>0$ or the non-allometric adjustments expressed through with the γ_k terms. The γ_k terms explain the direct effects of treatment on y, independent of allometry with z (Jolliffe *et al.* 1988).

The relationship between W and LA in beans indicated a strong positive contribution from $\log_e$(LA). The other allometric relationships were not consistent in the manner they responded. It is not clear why the direction of response of most of these parameters did not follow any particular pattern. One would be made to think that this was due to large variations which were observed in the data or it could be that the nature of response varies depending on the environmental factors not accounted for during the experiment. Such complexes in allometric responses were also shown by Jolliffe *et al.* (1988). The keeping of non-significant terms in the model when the best subset regression technique is used is necessary or the method may not be valid.

Few linear allometric relationships, in which only b_0 was significant, were observed in this study. Examples include W vs LN (Appendix 8.6.2), W vs LA (Appendix 8.6.3), W vs WL (Appendix 8.6.4). Linear allometries were mostly noted in beans at 40, 63 and 69 days from planting for LN, at 40 days for LA, at 40, 51, 63, and 75 days for WL. In beets only W vs LA (Appendix 8.8.3) at 75 days, W vs DR (Appendix 8.8.8) at 75 days, and W vs WR (Appendix 8.8.7) at 107 days

from planting were linear. All other relationships were curvelinear (i.e. at least one term containing β_1, β_2 or β_3 was significant).

High R^2 values in the allometric relationships are a good indication of strong relationships. Some of the relationships were significant but had very low R^2 values. The strength of these relationships is questionable. Significance could be a result of the many degrees of freedom which were available for the denominator component of the variance ratios tested.

5.10 A Summary of the Discussion

In summary, the yield responses of beans and beets to both population density and mixture proportions in both 1984 and 1987 were generally detected to be significant regardless of the analytical procedure used to access the effects. Each method was able to demonstrate additional information to the general finding of significant treatment effects on yield although in some cases discrepancies between the different analytical methods were detected.

Using the ANOVA indicated that high population densities reduced total plant yield in either species whether grown in monocultures or mixtures. Yield reduction was a result of intensified interference among plants with increasing population density and mixture proportions of bean. In 1984, the effects were seen to take place later in the growing season usually after 51 days from planting and onwards in both species, though variables like total dry weight, leaf dry weight and leaf area in beans due to population density and for leaf dry weight the effects due to mixture proportions were also detected early during growth. In beets, the effects due to mixture proportions on all variables except plant height

were also detected at 40 days after planting. Yield-density models were able to show which of the two species was a good competitor both in monocultures and in mixtures. Yield-density models thus showed that beans were better competitors when grown in monocultures and mixtures. The analysis of differential yield responses on the other hand were able to show that growing beans and beets together might be beneficial as the observed LER indicated yield advantage in mixtures as compared to their corresponding monocultures in all the variables tested. Yield component analysis was able to show that interference is a major source of yield variation. This procedure was able to single out the variables which were directly or indirectly affected by treatments. Both the forward and the backward analysis indicated that leaf number (LN) was the component which was directly influenced by treatments in both species. Just as the ANOVA, this analysis also indicated that in 1984, the effects of treatments were significant later in the growing season. Treatment effects were difficult to determine in the 1987 data because of the large variation between plants within treatments. These large variations among plants were demonstrated by high Gini coefficient values which were obtained when size frequency distribution was determined. The Gini coefficient was also able to show which yield components were more variable compared to others.

Similar to ANOVA and yield component analysis results, conventional plant growth analysis results also indicated that yield reduced with increasing population density and with increasing mixture proportions of beans. The reduction in yield was also evident 51 days after planting and onwards indicating intense plant interference as

plants grew larger. Both the efficiency and extent of assimilatory systems were affected by interference in beets.

The allometric relationship between $\log_e(W)$ (total yield) and yield components ($\log_e(z)$) were accessed and it was found that treatments affected the allometry. The effects due to treatment were as early as 40 days after planting. The direction of response was difficult to determine as no pattern was obtained but it seems that each plant part was affected differently by the different treatments.

6. CONCLUSIONS

1. Visually, plants in high population densities were smaller than those in less dense treatments. These observations were confirmed by the analysis of variance (ANOVA) which indicated that per plant yield in almost all variables was reduced with increasing population density and with increasing mixture proportions of beans.

2. Interference among associated plants of beans and beets was found to be quite complex. Non-linear inverse yield-density models, have indicated strong intra- and interspecific interferences. In this study, the model parameters consistently suggested that beans were better competitors than beets, both in monocultures and in mixtures.

3. Land equivalent ratios (LER) computed from observed yield values usually showed yield advantage for all variables. Applying the yield-density models to determine the differential yield responses indicated yield advantages for leaf number and leaf dry weight, and yield disadvantages for total dry weight and leaf area in mixtures as compared to their corresponding monocultures.

4. The effect of population density and mixture proportions on the frequency distribution of plant size, as expressed by the Gini coefficient, did not have obvious trends in beans monocultures and in beans or beets mixtures. In monocultures of beets, the plants were found to differ in size more at the lowest density of 16 plants m^{-2} and was least variable at 33 plants m^{-2}. The two population densities of 50 and 66 plants m^{-2}

were intermediate. This pattern was found to be true in all the yield variables tested. Pooling mixture proportions to study the effect of population density indicated a general increase in the Gini coefficient with increasing population density. When the yield variables were compared within species, leaf number was found to be least variable in either species but a comparison between species indicated that beets yield variables had higher Gini coefficient values than beans.

5. Treatments were found to affect agricultural yield of both beans and beets later in the growing season regardless of whether the forward and backward yield component analysis was done. Main effects were stronger than their interactions. In both the forward and backward analysis, the yield component that entered into the equation first contributed most to total yield variation. The yield component leaf number (LN) was often found to be the source of treatment effects.

6. Specific leaf area was found to increase with increasing population density and with increasing mixture proportion of beans in both beans and beets as time progressed. Leaf area ratio and specific leaf weight were significantly higher at higher population densities in beets monocultures and at high bean proportions in mixtures but generally not significant for beans although both ratios tended to increase with increasing densities in bean monocultures and with increasing beans proportions in mixtures. Derived ratios increased with time and declined after reaching a peak at about 68 days. Both the lowest population density of 16 plants m^{-2} and the mixture treatment with the least proportion of beans had the greatest increase in absolute

growth, relative growth and unit leaf rates.   This pattern was also observed in the primary variables; total dry weight, leaf dry weight, leaf number and leaf area.

7.   Bivariate allometric relationships in each species were also found to be quite complex.   Strong treatment influences were detected for each species and influences varied at different harvest dates.   Different allometric relationships and different treatment effects on allometry, were determined.

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8. APPENDICES

Appendix 8.1.1.1
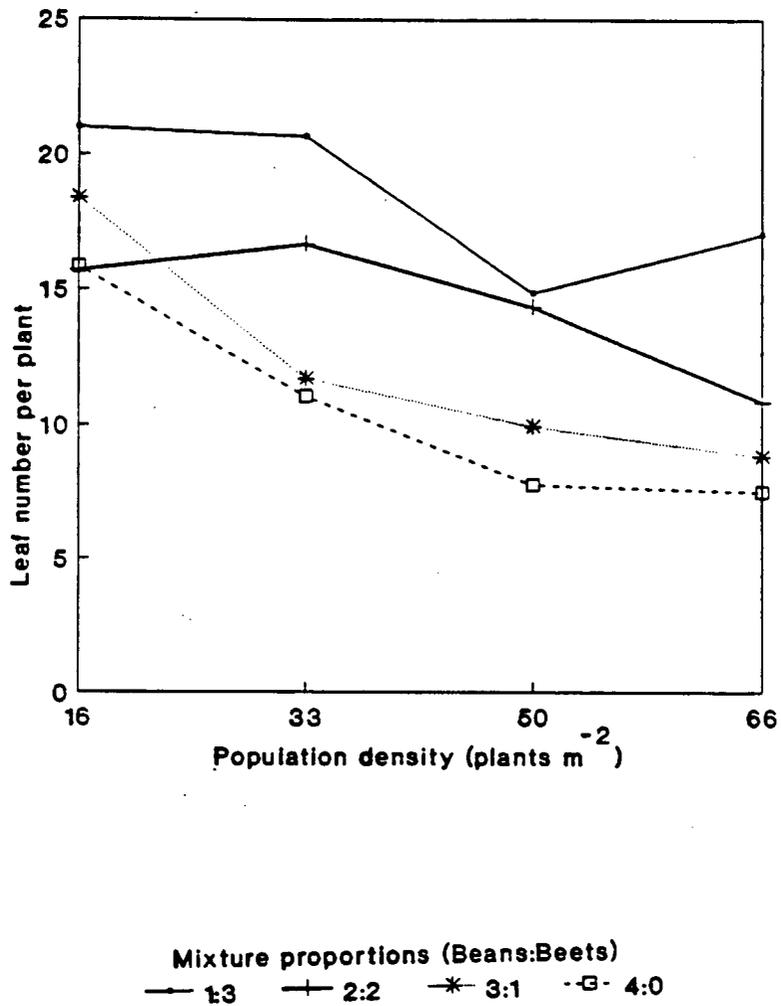
Mixture proportions (Beans:Beets)
—— 1:3   —+— 2:2   —*— 3:1   ·-❑-· 4:0

The effect of population density and mixture proportions interactions on bean leaf number harvested at 69 days after planting (1984 experiment)
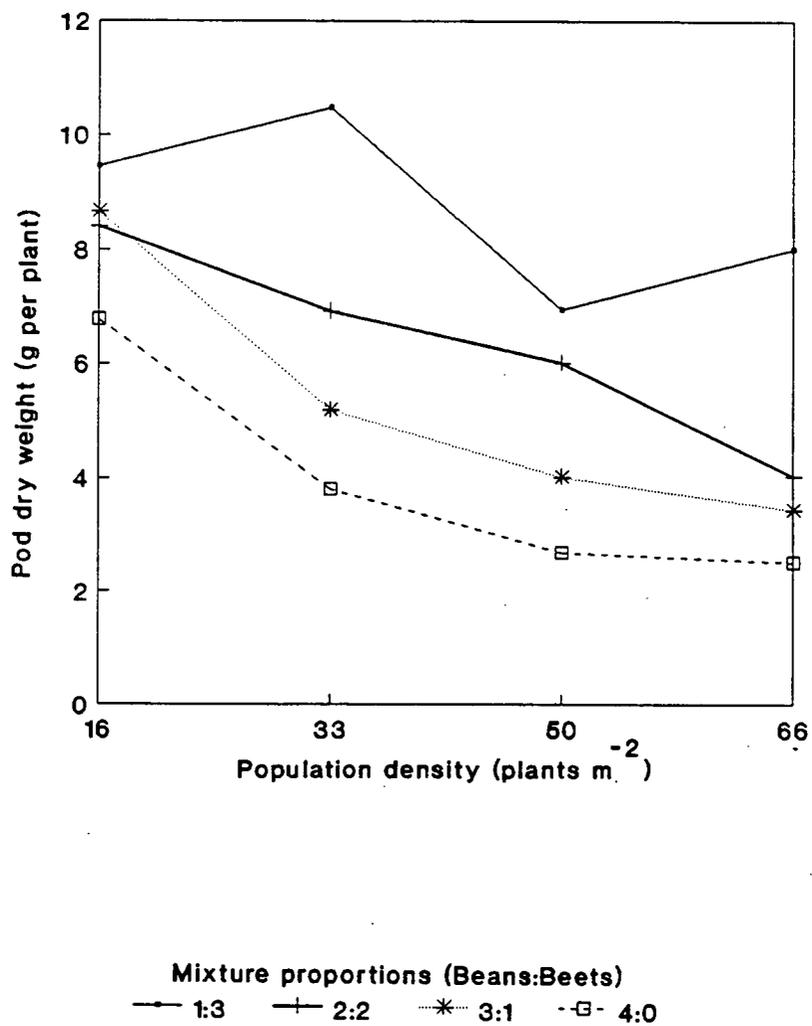
Appendix 8.1.1.2

The effect of population density and mixture proportions interactions on bean pod dry weight harvested at 69 days after planting (1984 experiment)

Appendix 8.1.1.3
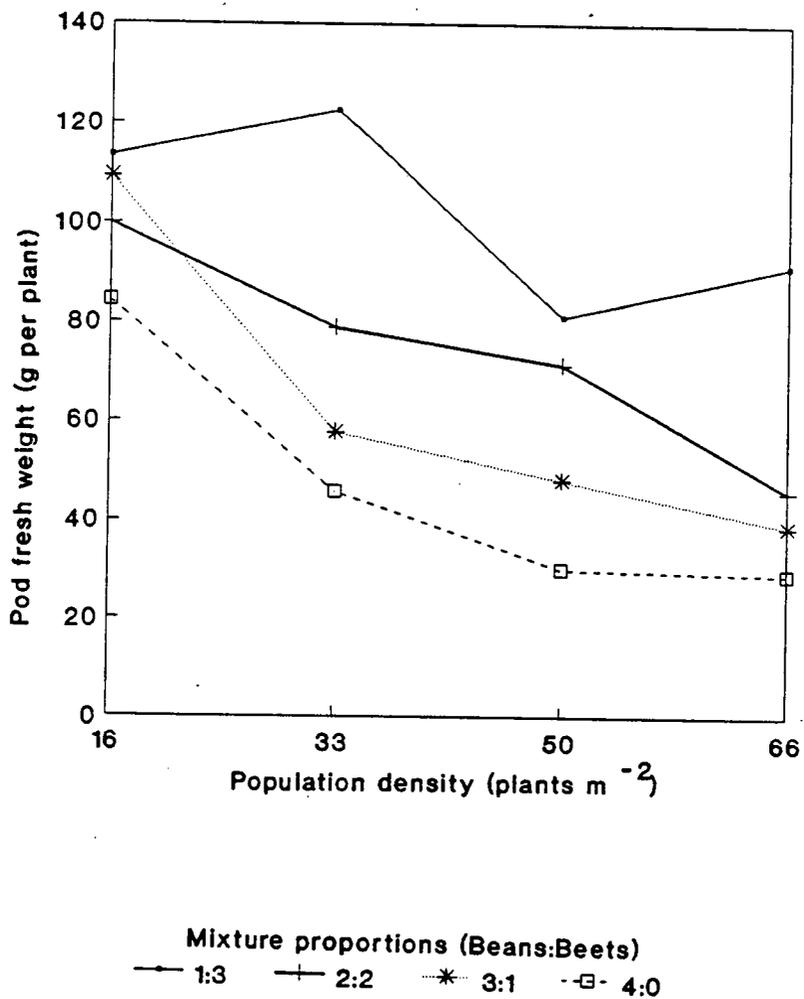
The effect of population density and mixture proportions interactions on bean pod fresh weight harvested at 69 days after planting (1984 experiment)

Appendix 8.1.2.1
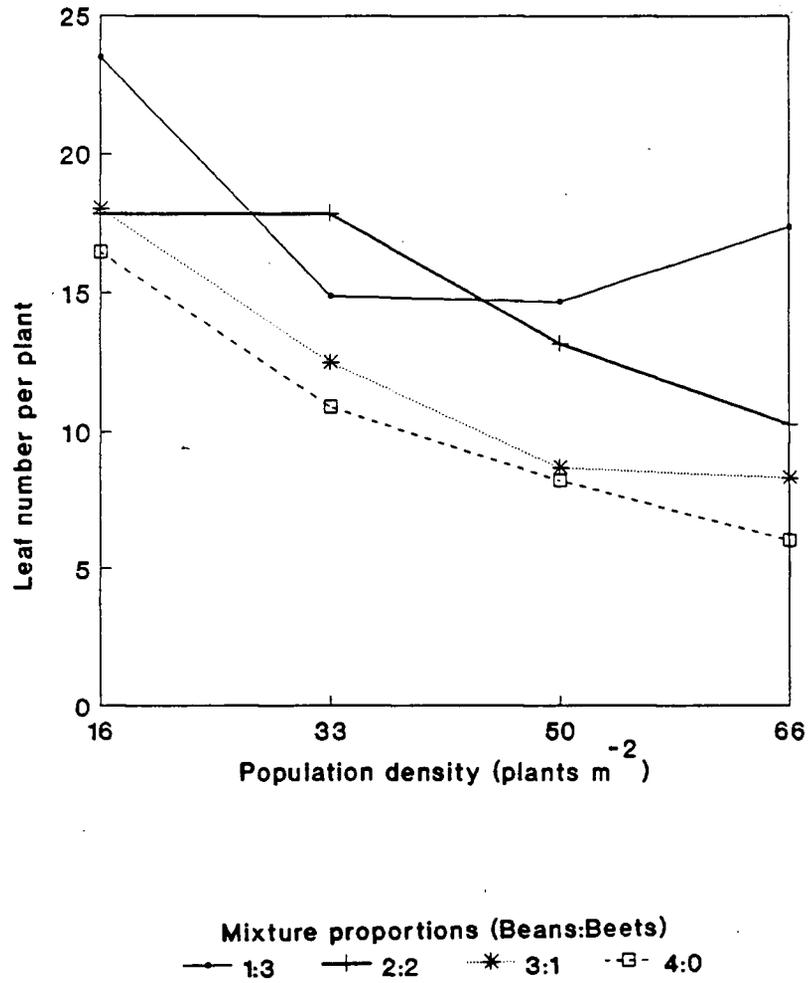
The effect of population density and mixture proportions interactions on bean leaf number harvested at 75 days after planting (1984 experiment)

Appendix 8.1.2.2
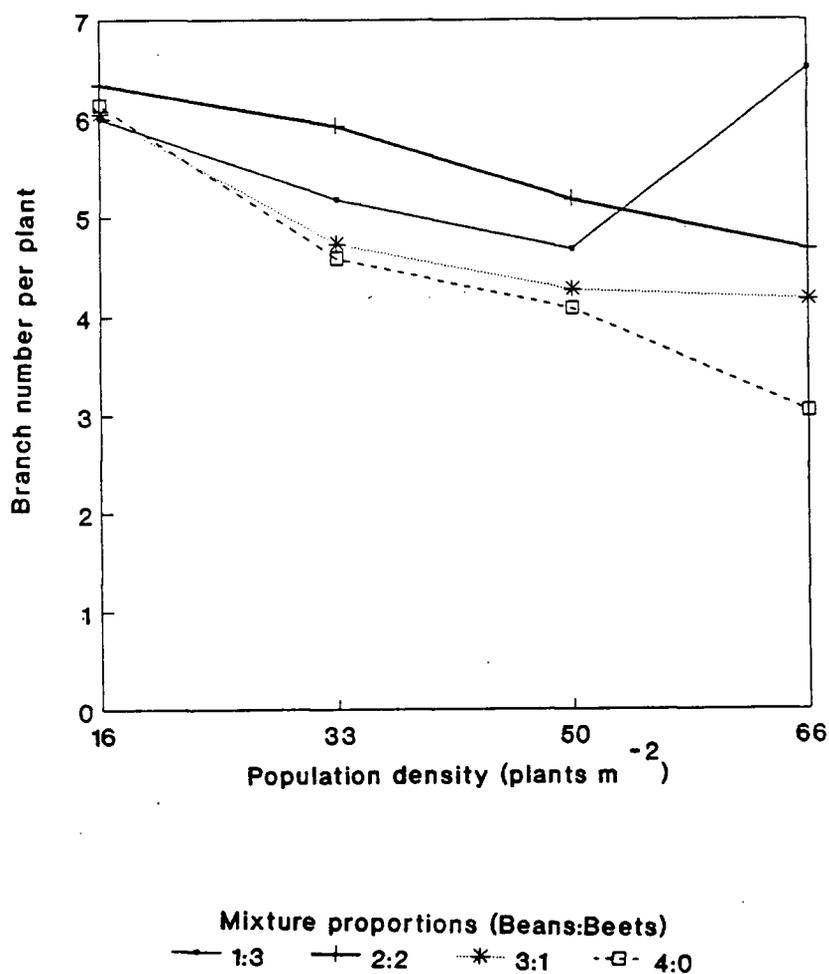
The effect of population density and mixture proportions interactions on bean branch number harvested at 75 days after planting (1984 experiment)

Appendix 8.1.2.3

Mixture proportions (Beans:Beets)
1:3   2:2   3:1   4:0
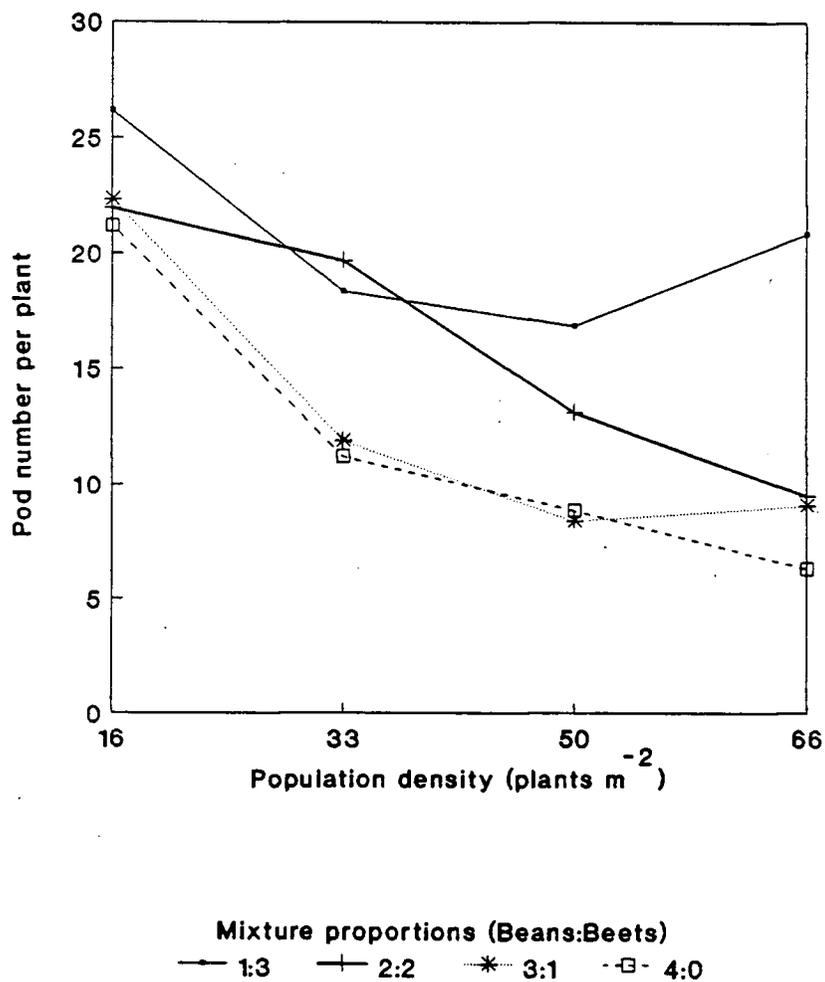
The effect of population density and mixture proportions interactions on bean pod number harvested at 75 days after planting (1984 experiment)

Appendix 8.1.2.4
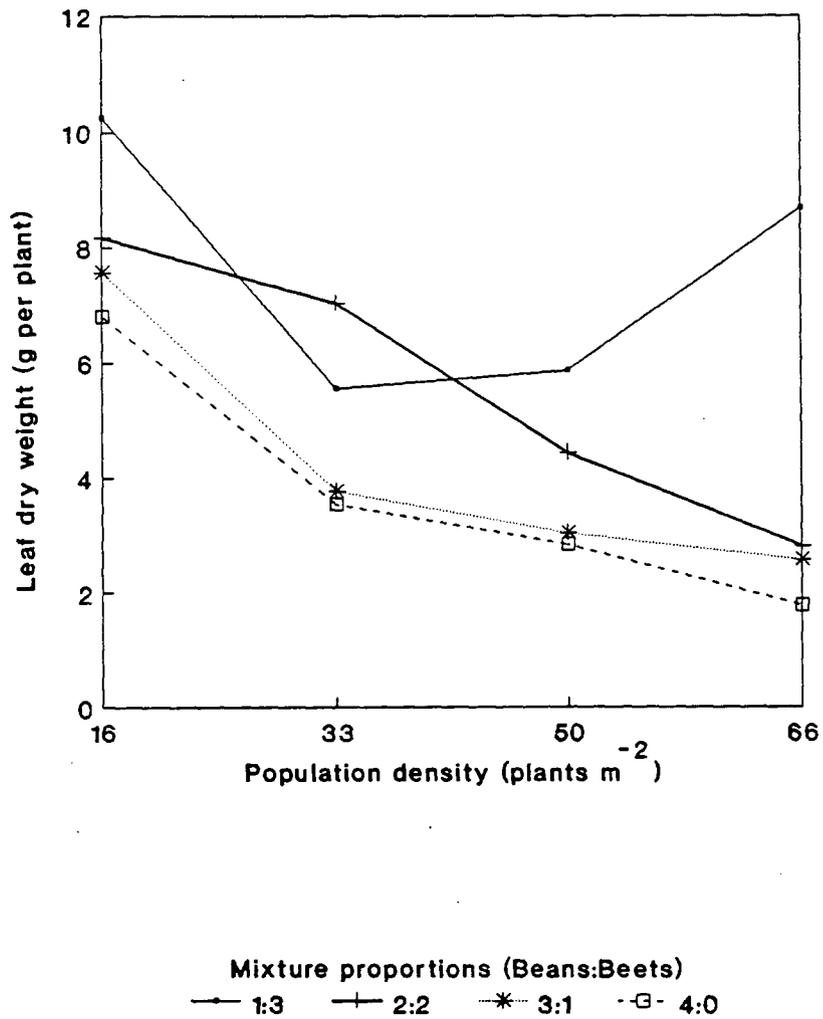
The effect of population density and mixture proportions interactions on bean leaf dry weight harvested at 75 days after planting (1984 experiment)

Appendix 8.1.2.5
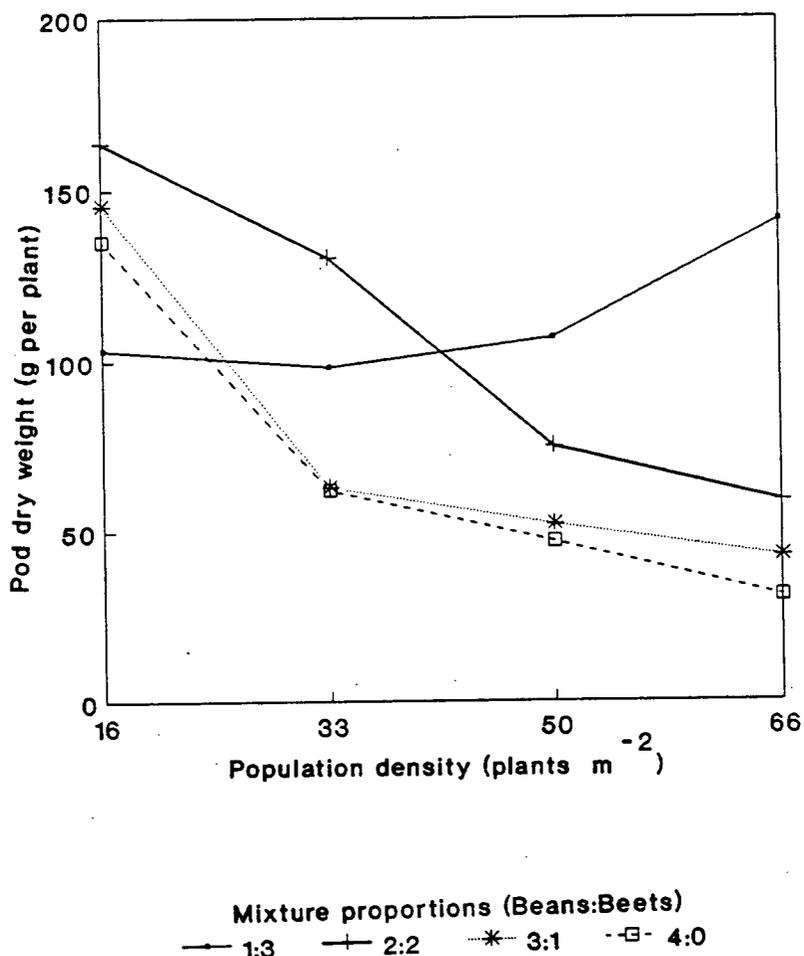
The effect of population density and mixture proportions interactions on bean pod dry weight harvested at 75 days after planting (1984 experiment)

Appendix 8.1.2.6
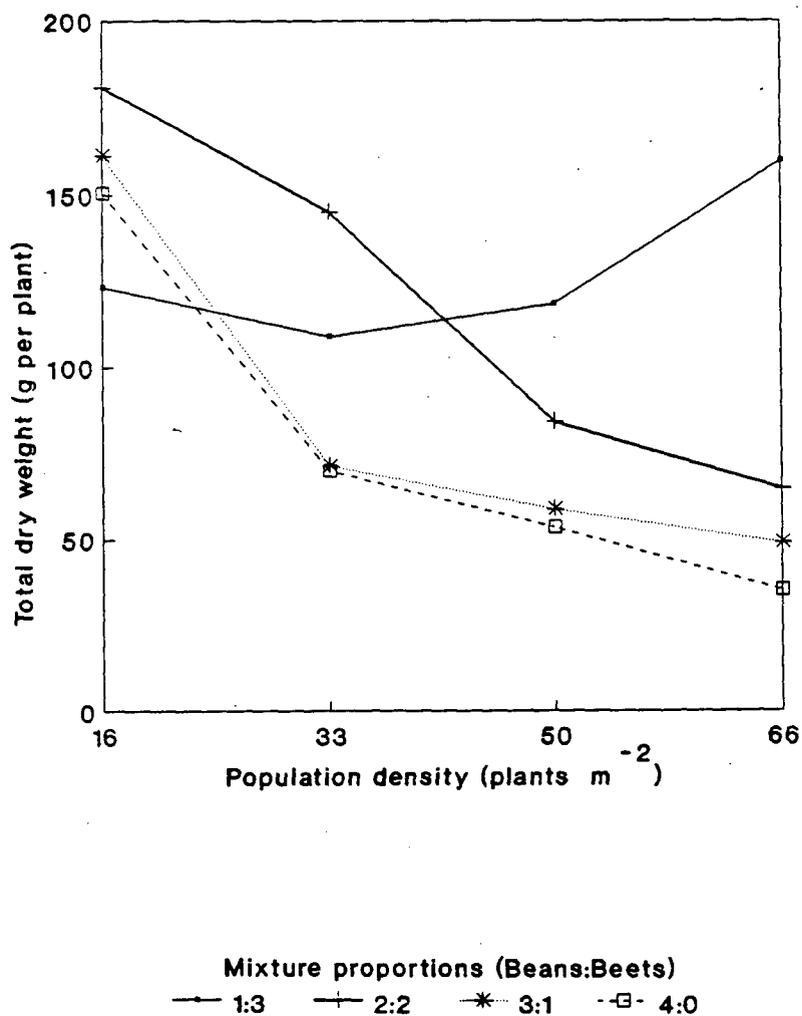
The effect of population density and mixture proportions interactions on bean total dry weight harvested at 75 days after planting (1984 experiment)

Appendix 8.2
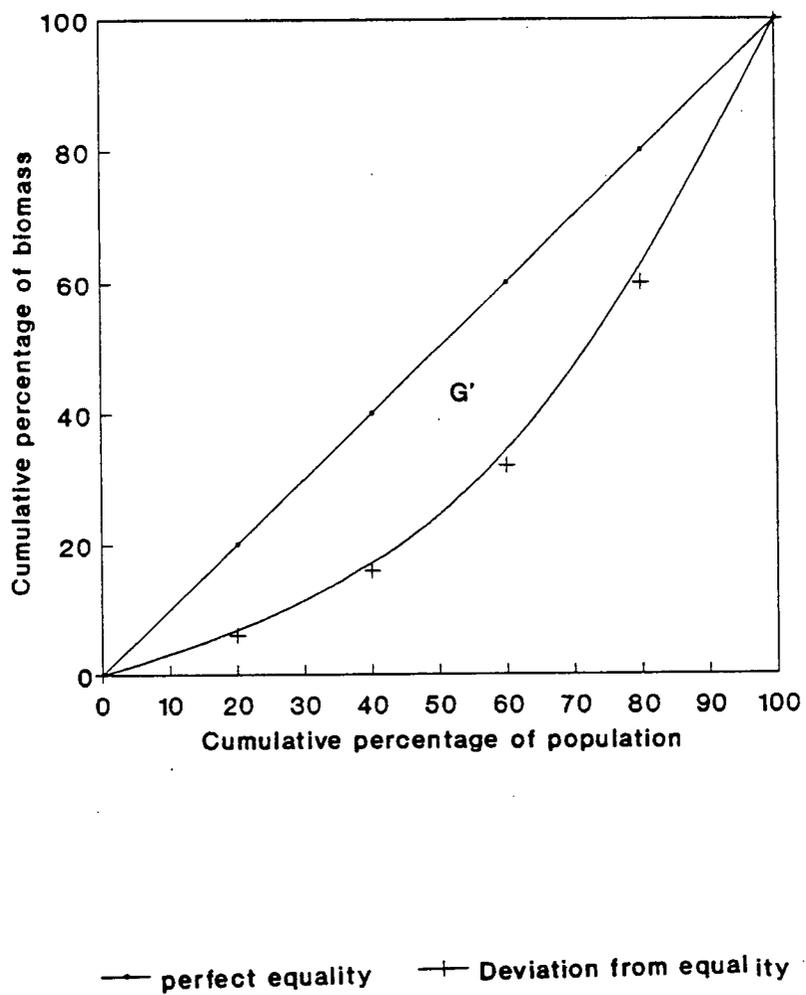
Lorenz curve showing size inequality in a yield variable (hypothetical data)

Appendix 8.3.1   Gini coefficients for stem dry weight distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions 3:1	2:2	1:3	0:4	Mixture Pooled
16	G'	0.220	0.339	0.376	0.185	-	0.324
33	G'	0.409	0.261	0.257	0.232	-	0.310
50	G'	0.242	0.307	0.199	0.299	-	0.309
66	G'	0.289	0.369	0.304	0.324	-	0.327
Density Pooled	G'	0.308	0.301	0.288	0.264	-	0.320†

Appendix 8.3.2   Gini coefficients for marketable pod number distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions 3:1	2:2	1:3	0:4	Mixture Pooled
16	G'	0.198	0.270	0.319	0.276	-	0.313
33	G'	0.232	0.234	0.220	0.206	-	0.277
50	G'	0.278	0.263	0.204	0.309	-	0.301
66	G'	0.350	0.221	0.278	0.285	-	0.395
Density Pooled	G'	0.308	0.249	0.278	0.293	-	0.324†

*Beans:Beets
†Gini coefficient for the whole data set

Appendix 8.3.3   Gini coefficients for unmarketable pod number distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	Mixture Proportions					Mixture Pooled
		4:0*	3:1	2:2	1:3	0:4	
16	G'	0.353	0.368	0.352	0.408	-	0.453
33	G'	0.339	0.356	0.380	0.286	-	0.369
50	G'	0.319	0.320	0.303	0.333	-	0.356
66	G'	0.326	0.304	0.259	0.305	-	0.339
Density Pooled	G'	0.386	0.407	0.336	0.346	-	0.384†

Appendix 8.3.4   Gini coefficients for pod fresh weight distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	Mixture Proportions					Mixture Pooled
		4:0*	3:1	2:2	1:3	0:4	
16	G'	0.235	0.308	0.310	0.271	-	0.324
33	G'	0.244	0.314	0.267	0.212	-	0.299
50	G'	0.276	0.297	0.256	0.321	-	0.311
66	G'	0.400	0.365	0.289	0.290	-	0.421
Density Pooled	G'	0.326	0.259	0.295	0.307	-	0.307†

*Beans:Beets
†Gini coefficient for the whole data set

Appendix 8.3.5   Gini coefficients for marketable pod dry weight distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions 3:1	2:2	1:3	0:4	Mixture Pooled
16	G'	0.214	0.289	0.354	0.283	-	0.326
33	G'	0.227	0.241	0.251	0.225	-	0.288
50	G'	0.292	0.258	0.225	0.322	-	0.302
66	G'	0.417	0.252	0.246	0.300	-	0.420
Density Pooled	G'	0.338	0.267	0.290	0.313	-	0.336†

Appendix 8.3.6   Gini coefficients for unmarketable pod dry weight distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions 3:1	2:2	1:3	0:4	Mixture Pooled
16	G'	0.570	0.517	0.510	0.615	-	0.567
33	G'	0.588	0.558	0.557	0.521	-	0.556
50	G'	0.579	0.515	0.497	0.548	-	0.537
66	G'	0.549	0.476	0.462	0.525	-	0.503
Density Pooled	G'	0.571	0.521	0.510	0.549	-	0.549†

*Beans:Beets
†Gini coefficient for the whole data set

Appendix 8.3.7   Gini coefficients for seed number distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	Mixture Proportions					Mixture Pooled
		4:0*	3:1	2:2	1:3	0:4	
16	G'	0.391	0.484	0.417	0.417	-	0.371
33	G'	0.349	0.385	0.425	0.413	-	0.320
50	G'	0.373	0.424	0.332	0.414	-	0.348
66	G'	0.432	0.491	0.344	0.477	-	0.450
Density Pooled	G'	0.354	0.360	0.335	0.372	-	0.374[†]

Appendix 8.3.8   Gini coefficients for seed dry weight distribution of beans grown in monocultures and mixtures

Total Population Density	Statistic	Mixture Proportions					Mixture Pooled
		4:0*	3:1	2:2	1:3	0:4	
16	G'	0.254	0.406	0.382	0.336	-	0.377
33	G'	0.249	0.301	0.284	0.294	-	0.318
50	G'	0.335	0.320	0.316	0.368	-	0.346
66	G'	0.463	0.305	0.291	0.417	-	0.456
Density Pooled	G'	0.379	0.342	0.333	0.381	-	0.373[†]

*Beans:Beets
[†]Gini coefficient of the whole data set

Appendix 8.4.1 Gini coefficients for dead leaves distribution of beets grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions 3:1	2:2	1:3	0:4	Mixture Pooled
16	G'	-	0.256	0.397	0.374	0.460	0.379
33	G'	-	0.323	0.527	0.553	0.305	0.449
50	G'	-	0.459	0.547	0.460	0.390	0.478
66	G'	-	0.385	0.391	0.601	0.379	0.436
Density Pooled	G'	-	0.360	0.471	0.496	0.393	0.436†

Appendix 8.4.2 Gini coefficients for petiole dry weight distribution of beets grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions 3:1	2:2	1:3	0:4	Mixture Pooled
16	G'	-	0.315	0.411	0.328	0.519	0.410
33	G'	-	0.485	0.488	0.363	0.265	0.464
50	G'	-	0.303	0.421	0.475	0.390	0.418
66	G'	-	0.453	0.358	0.521	0.318	0.429
Density Pooled	G'	-	0.413	0.475	0.450	0.406	0.449†

*Beans:Beets
†Gini coefficient for the whole data set

Appendix 8.4.3   Gini coefficients for root diameter distribution of beets grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions				Mixture Pooled
			3:1	2:2	1:3	0:4	
16	G'	-	0.207	0.356	0.264	0.358	0.313
33	G'	-	0.204	0.398	0.273	0.156	0.342
50	G'	-	0.222	0.303	0.332	0.290	0.336
66	G'	-	0.310	0.318	0.382	0.216	0.336
Density Pooled	G'	-	0.260	0.400	0.365	0.282	0.350†

Appendix 8.4.4   Gini coefficients for root fresh weight distribution of beets grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions				Mixture Pooled
			3:1	2:2	1:3	0:4	
16	G'	-	0.458	0.527	0.474	0.732	0.613
33	G'	-	0.464	0.717	0.588	0.389	0.625
50	G'	-	0.509	0.584	0.658	0.526	0.630
66	G'	-	0.611	0.627	0.671	0.456	0.618
Density Pooled	G'	-	0.546	0.694	0.654	0.622	0.661†

*Beans:Beets
†Gini coefficient for the whole data set

Appendix 8.4.5   Gini coefficients for root dry weight distribution of beets grown in monocultures and mixtures

Total Population Density	Statistic	4:0*	Mixture Proportions				Mixture Pooled
			3:1	2:2	1:3	0:4	
16	G'	-	0.438	0.558	0.460	0.670	0.582
33	G'	-	0.479	0.689	0.585	0.389	0.625
50	G'	-	0.498	0.569	0.642	0.499	0.615
66	G'	-	0.612	0.621	0.671	0.440	0.619
Density Pooled	G'	-	0.540	0.688	0.643	0.567	0.641†

*Beans:Beets
†Gini coefficient for the whole data set

Appendix 8.5.1   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln T in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-2.398**	-4.032	6.32	0.85	0.05	6,233
(HN 1)[†]	$\ln(T)$	β_0	1.824**	0.849				
	$X_i \ln(T)$	β_1	0.007**	0.424				
	$X_j \ln(T)$	β_2	-	-				
	$X_i X_j \ln(T)$	β_3	-0.0001	-0.106				
	$\ln(X_i)$	γ_1	-0.520**	-0.588				
	$\ln(X_j)$	γ_2	-0.531**	-1.251				
	$\ln(X_i X_j)$	γ_3	-0.561**	0.977				
51	Intercept	$\ln(\alpha')$	-2.014**	-3.372	4.86	0.78	0.81	5,234
(HN 2)	$\ln(T)$	β_0	1.596**	0.692				
	$X_i \ln(T)$	β_1	-0.001	-0.130				
	$X_j \ln(T)$	β_2	-	-				
	$X_i X_j \ln(T)$	β_3	0.0001	0.139				
	$\ln(X_i)$	γ_1	-0.496**	-0.560				
	$\ln(X_j)$	γ_2	-0.076*	-0.178				
	$\ln(X_i X_j)$	γ_3	-	-				
63	Intercept	$\ln(\alpha')$	-0.168	-0.168	2.48	0.29	0.54	2,237
(HN 3)	$\ln(T)$	β_0	1.157**	0.337				
	$X_i \ln(T)$	β_1	-0.007**	-0.530				
	$X_j \ln(T)$	β_2	-	-				
	$X_i X_j \ln(T)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.1 (cont'd) Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln T in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	2.431**	0.609	4.20	0.54	0.21	3,236
(HN 4)	$\ln(T)$	β_0	0.909**	0.135				
	$X_i \ln(T)$	β_1	-0.005**	-0.001				
	$X_j \ln(T)$	β_2	-	-				
	$X_i X_j(T)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.314*	-0.127				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				
75	Intercept	$\ln(\alpha')$	4.574**	0.905	6.02	0.59	0.21	6,233
(HN 5)	$\ln(T)$	β_0	0.663**	0.144				
	$X_i \ln(T)$	β_1	-0.004	0.002				
	$X_j \ln(T)$	β_2	0.008*	0.003				
	$X_i X_j(T)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.654**	-0.220				
	$\ln(X_j)$	γ_2	-0.761*	-0.306				
	$\ln(X_i X_j)$	γ_3	0.694*	0.305				
92	Intercept	$\ln(\alpha')$	2.047**	0.438	2.80	0.77	0.15	3,236
(HN 6)	$\ln(T)$	β_0	1.375**	0.097				
	$X_i \ln(T)$	β_1	-0.003*	-0.001				
	$X_j \ln(T)$	β_2	-	-				
	$X_i X_j(T)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.705**	0.108				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				

*Significant at $P = 0.05$
**Significant at $P = 0.01$
Note: ln is synonymous for $\log_e$

Appendix 8.5.2   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln LN in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-0.670**	-0.166	5.47	0.75	0.09	4,235
(HN 1)[†]	$\ln(LN)$	β_0	1.305**	0.055				
	$X_i\ln(LN)$	β_1	-	-				
	$X_j\ln(LN)$	β_2	-	-				
	$X_iX_j\ln(LN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.193**	-0.035				
	$\ln(X_j)$	γ_2	-0.214**	-0.071				
	$\ln(X_iX_j)$	γ_3	0.293**	0.093				
51	Intercept	$\ln(\alpha')$	-1.687**	-0.153	2.78	0.77	0.08	4,235
(HN 2)	$\ln(LN)$	β_0	1.499**	0.078				
	$X_i\ln(LN)$	β_1	-0.003**	-0.001				
	$X_j\ln(LN)$	β_2	-	-				
	$X_iX_j\ln(LN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.162*	-0.064				
	$\ln(X_iX_j)$	γ_3	0.186*	0.084				
63	Intercept	$\ln(\alpha')$	-0.779**	-0.245	0.54	0.53	0.37	1,238
(HN 3)	$\ln(LN)$	β_0	1.576**	0.097				
	$X_i\ln(LN)$	β_1	-	-				
	$X_j\ln(LN)$	β_2	-	-				
	$X_iX_j\ln(LN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.2 (cont'd)   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln LN in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	1.894**	0.311	2.15	0.72	0.13	2,237
(HN 4)	$\ln(LN)$	β_0	1.124**	0.074				
	$X_i \ln(LN)$	β_1	-	-				
	$X_j \ln(LN)$	β_2	-	-				
	$X_i X_j (LN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.173**	-0.483				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				
75	Intercept	$\ln(\alpha')$	4.112**	0.440	5.57	0.70	0.16	5,234
(HN 5)	$\ln(LN)$	β_0	0.767**	0.086				
	$X_i \ln(LN)$	β_1	-	-				
	$X_j \ln(LN)$	β_2	0.010**	0.003				
	$X_i X_j (LN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.481**	-0.077				
	$\ln(X_j)$	γ_2	-0.856**	-0.211				
	$\ln(X_i X_j)$	γ_3	0.825**	0.216				
92	Intercept	$\ln(\alpha')$	3.139**	0.095	4.23	0.69	0.20	2,235
(HN 6)	$\ln(LN)$	β_0	0.809**	0.037				
	$X_i \ln(LN)$	β_1	-0.009**	-0.001				
	$X_j \ln(LN)$	β_2	-	-				
	$X_i X_j (LN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				

*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.3   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln LA in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	4.200**	0.059	3.91	0.94	0.02	2,237
(HN 1)[†]	$\ln(LA)$	β_0	0.980**	0.017				
	$X_i\ln(LA)$	β_1	-	-				
	$X_j\ln(LA)$	β_2	-	-				
	$X_iX_j\ln(LA)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	0.030**	0.007				
	$\ln(X_iX_j)$	γ_3	-	-				
51	Intercept	$\ln(\alpha')$	4.148**	0.060	5.46	0.92	0.03	2,237
(HN 2)	$\ln(LA)$	β_0	0.970**	0.028				
	$X_i\ln(LA)$	β_1	0.001**	0.0002				
	$X_j\ln(LA)$	β_2	-	-				
	$X_iX_j\ln(LA)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
63	Intercept	$\ln(\alpha')$	4.820**	0.370	5.21	0.60	0.31	3,236
(HN 3)	$\ln(LA)$	β_0	1.010**	0.100				
	$X_i\ln(LA)$	β_1	0.004*	0.002				
	$X_j\ln(LA)$	β_2	-	-				
	$X_iX_j\ln(LA)$	β_3	-	-				
	$\ln(X_i)$	γ_1	0.260*	0.120				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.3 (cont'd)   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln LA in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	5.720**	0.103	4.56	0.76	0.11	4,235
(HN 4)	ln(LA)	β_0	0.674**	0.057				
	$X_i\ln(LA)$	β_1	0.003**	0.001				
	$X_j\ln(LA)$	β_2	0.0001*	0.0001				
	$X_iX_j(LA)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	0.089*	0.040				
75	Intercept	$\ln(\alpha')$	6.173**	0.099	6.89	0.79	0.11	2,237
(HN 5)	ln(LA)	β_0	0.709**	0.060				
	$X_i\ln(LA)$	β_1	0.005**	0.001				
	$X_j\ln(LA)$	β_2	-	-				
	$X_iX_j(LA)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
92	Intercept	$\ln(\alpha')$	6.780**	0.117	1.87	0.80	0.13	3,234
(HN 6)	ln(LA)	β_0	0.549**	0.035				
	$X_i\ln(LA)$	β_1	-	-				
	$X_j\ln(LA)$	β_2	-	-				
	$X_iX_j(LA)$	β_3	0.0001**	0.00002				
	$\ln(X_i)$	γ_1	-0.310**	-0.050				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				

*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.4   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WL in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-0.252**	-0.039	0.46	0.98	0.09	3,236
(HN 1)[†]	ln(WL)	β_0	1.041**	0.010				
	$X_i$ln(WL)	β_1	-	-				
	$X_j$ln(WL)	β_2	-	-				
	$X_iX_j$ln(WL)	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.093**	-0.010				
	$\ln(X_j)$	γ_2	0.007	0.004				
	$\ln(X_iX_j)$	γ_3	-	-				
51	Intercept	$\ln(\alpha')$	-0.129**	-0.039	1.62	0.98	0.08	2,237
(HN 2)	ln(WL)	β_0	0.935**	0.010				
	$X_i$ln(WL)	β_1	-	-				
	$X_j$ln(WL)	β_2	-	-				
	$X_iX_j$ln(WL)	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.108**	-0.009				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
63	Intercept	$\ln(\alpha')$	1.533**	0.206	3.03	0.72	0.12	4,235
(HN 3)	ln(WL)	β_0	1.413**	0.028				
	$X_i$ln(WL)	β_1	-	-				
	$X_j$ln(WL)	β_2	-	-				
	$X_iX_j$ln(WL)	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.450**	-0.043				
	$\ln(X_j)$	γ_2	-0.163*	-0.081				
	$\ln(X_iX_j)$	γ_3	0.207*	0.106				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at $P = 0.05$
**Significant at $P = 0.01$
Note: ln is synonymous for $\log_e$

Appendix 8.5.4 (cont'd)   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WL in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	-0.417	-0.248	2.49	0.83	0.071	4,235
(HN 4)	$\ln(WL)$	β_0	0.654**	0.034				
	$X_i\ln(WL)$	β_1	-	-				
	$X_j\ln(WL)$	β_2	-	-				
	$X_iX_j(WL)$	β_3	0.0001*	0.00002				
	$\ln(X_i)$	γ_1	-0.268**	-0.039				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-0.074*	0.340				
75	Intercept	$\ln(\alpha')$	-1.096**	-0.254	-0.86	0.82	0.08	2,237
(HN 5)	$\ln(WL)$	β_0	0.707**	0.036				
	$X_i\ln(WL)$	β_1	-	-				
	$X_j\ln(WL)$	β_2	-	-				
	$X_iX_j(WL)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.190**	-0.037				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
92	Intercept	$\ln(\alpha')$	-1.668**	-0.435	2.45	0.82	0.18	4,233
(HN 6)	$\ln(WL)$	β_0	0.882**	0.053				
	$X_i\ln(WL)$	β_1	-	-				
	$X_j\ln(WL)$	β_2	-0.002	-0.001				
	$X_iX_j(WL)$	β_3	0.0001**	0.00004				
	$\ln(X_i)$	γ_1	-0.392**	-0.076				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				

*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.5   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln WST in
beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-0.387^{**}	-0.152	1.60	0.95	0.01	3,236
(HN 1)[†]	ln(WST)	β_0	1.945^{**}	0.139				
	$X_i\ln(WST)$	β_1	-	-				
	$X_j\ln(WST)$	β_2	-	-				
	$X_iX_j\ln(WST)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.152^{**}	-0.013				
	$\ln(X_j)$	γ_2	0.017^{**}	0.006				
	$\ln(X_iX_j)$	γ_3	-	-				
51	Intercept	$\ln(\alpha')$	-0.346^{**}	-0.062	1.68	0.96	0.01	2,237
(HN 2)	ln(WST)	β_0	1.058^{**}	0.016				
	$X_i\ln(WST)$	β_1	-	-				
	$X_j\ln(WST)$	β_2	-	-				
	$X_iX_j\ln(WST)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.114^{**}	-0.014				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
63	Intercept	$\ln(\alpha')$	1.011^{**}	0.236	4.28	0.65	0.15	4,235
(HN 3)	ln(WST)	β_0	1.479^{**}	0.032				
	$X_i\ln(WST)$	β_1	-	-				
	$X_j\ln(WST)$	β_2	-	-				
	$X_iX_j\ln(WST)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.330^{**}	-0.049				
	$\ln(X_j)$	γ_2	-0.286^{**}	-0.093				
	$\ln(X_iX_j)$	γ_3	0.356^{**}	0.121				

[†]Harvest number (1-6 in beans and 1-7 in beets)
[*]Significant at P = 0.05
[**]Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.5 (cont'd) Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WST in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	-1.515**	-0.132	1.38	0.73	0.11	3,236
(HN 4)	ln(WST)	β_0	0.752**	0.033				
	$X_i\ln(WST)$	β_1	-	-				
	$Xj\ln(WST)$	β_2	-	-				
	$X_iX_j(WST)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	0.269**	0.069				
	$\ln(X_iX_j)$	γ_3	-0.406**	-0.093				
75	Intercept	$\ln(\alpha')$	-1.788**	-0.143	0.00	0.69	0.13	1,237
(HN 5)	ln(WST)	β_0	0.760**	0.033				
	$X_i\ln(WST)$	β_1	-	-				
	$X_j\ln(WST)$	β_2	-	-				
	$X_iX_j(WST)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
92	Intercept	$\ln(\alpha')$	-2.472**	-0.176	7.66	0.84	0.10	5,234
(HN 6)	ln(WST)	β_0	0.922**	0.032				
	$X_i\ln(WST)$	β_1	-0.001*	-0.001				
	$X_j\ln(WST)$	β_2	-0.002**	-0.001				
	$X_iX_j(WST)$	β_3	0.0001**	0.00004				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-0.128**	-0.041				

*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.6   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln BN in
beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-4.589**	-0.178	2.06	0.94	0.02	4,235
(HN 1)[†]	$\ln(BN)$	β_0	0.970**	0.019				
	$X_i\ln(BN)$	β_1	0.001*	0.0003				
	$X_j\ln(BN)$	β_2	-	-				
	$X_iX_j\ln(BN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.102*	-0.046				
	$\ln(X_j)$	γ_2	0.035**	0.007				
	$\ln(X_iX_j)$	γ_3	-	-				
51	Intercept	$\ln(\alpha')$	-4.868**	-0.196	0.85	0.92	0.30	2,237
(HN 2)	$\ln(BN)$	β_0	1.998**	0.024				
	$X_i\ln(BN)$	β_1	-	-				
	$X_j\ln(BN)$	β_2	-	-				
	$X_iX_j\ln(BN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.055**	-0.019				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
63	Intercept	$\ln(\alpha')$	-5.018**	-0.447	-1.24	0.59	0.32	1,238
(HN 3)	$\ln(BN)$	β_0	1.158**	0.063				
	$X_i\ln(BN)$	β_1	-	-				
	$X_j\ln(BN)$	β_2	-	-				
	$X_iX_j\ln(BN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.6 (cont'd)   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln BN in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	-1.420^{**}	-0.319	1.87	0.76	0.11	2,237
(HN 4)	$\ln(BN)$	β_0	0.819^{**}	0.041				
	$X_i\ln(BN)$	β_1	-0.001^{**}	-0.0002				
	$X_j\ln(BN)$	β_2	-	-				
	$X_iX_j(BN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
75	Intercept	$\ln(\alpha')$	-1.751^{**}	-0.368	2.52	0.77	0.12	2,237
(HN 5)	$\ln(BN)$	β_0	0.900^{**}	0.047				
	$X_i\ln(BN)$	β_1	-0.001^{**}	-0.0003				
	$X_j\ln(BN)$	β_2	-	-				
	$X_iX_j(BN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
92	Intercept	$\ln(\alpha')$	0.739^{**}	0.221	2.40	0.80	0.12	3,234
(HN 6)	$\ln(BN)$	β_0	0.614^{**}	0.084				
	$X_i\ln(BN)$	β_1	-0.002^{**}	-0.0003				
	$X_j\ln(BN)$	β_2	-	-				
	$X_iX_j(BN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-0.061^{**}	-0.023				

*Significant at P = 0.05

**Significant at P = 0.01

Note: ln is synonymous for $\log_e$

Appendix 8.5.7   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln PN in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40 (HN 1)[†]	Intercept	$\ln(\alpha')$	-1.320**	-0.120	2.86	0.67	0.12	4,235
	$\ln(PN)$	β_0	1.780**	0.083				
	$X_i\ln(PN)$	β_1	-0.005**	-0.001				
	$X_j\ln(PN)$	β_2	-0.007**	-0.003				
	$X_iX_j\ln(PN)$	β_3	0.0003**	0.0001				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
51 (HN 2)	Intercept	$\ln(\alpha')$	-0.638*	-0.250	1.68	0.48	0.19	3,236
	$\ln(PN)$	β_0	1.645**	0.140				
	$X_i\ln(PN)$	β_1	-0.008**	-0.001				
	$X_j\ln(PN)$	β_2	-0.004**	-0.001				
	$X_iX_j\ln(PN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
63 (HN 3)	Intercept	$\ln(\alpha')$	1.190**	0.36	2.00	0.33	0.52	3,236
	$\ln(PN)$	β_0	1.630**	0.200				
	$X_i\ln(PN)$	β_1	-0.011**	-0.002				
	$X_j\ln(PN)$	β_2	-	-				
	$X_iX_j\ln(PN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.065	-0.036				
	$\ln(X_iX_j)$	γ_3	-	-				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.7 (cont'd) Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln PN in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	3.537**	0.367	4.73	0.68	0.15	5,234
(HN 4)	$\ln(PN)$	β_0	1.358**	0.124				
	$X_i \ln(PN)$	β_1	-	-				
	$X_j \ln(PN)$	β_2	0.013**	0.005				
	$X_i X_j(PN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.472**	-0.064				
	$\ln(X_j)$	γ_2	-0.622**	-0.207				
	$\ln(X_i X_j)$	γ_3	0.624**	0.209				
75	Intercept	$\ln(\alpha')$	2.800	0.141	4.76	0.66	0.17	3,236
(HN 5)	$\ln(PN)$	β_0	1.378	0.078				
	$X_i \ln(PN)$	β_1	-0.012	-0.001				
	$X_j \ln(PN)$	β_2	-	-				
	$X_i X_j(PN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-0.056	-0.027				
92	Intercept	$\ln(\alpha')$	3.209**	0.485	2.29	0.75	0.16	3,236
(HN 6)	$\ln(PN)$	β_0	1.431**	0.141				
	$X_i \ln(PN)$	β_1	-0.010**	-0.003				
	$X_j \ln(PN)$	β_2	-	-				
	$X_i X_j(PN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.186	-0.129				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				

*Sigificant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.8 Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln FWPD in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-	-	-	-	-	-
(HN 1)[†]	ln(FWPD)	β_0	-	-				
	$X_i$ln(FWPD)	β_1	-	-				
	$X_j$ln(FWPD)	β_2	-	-				
	$X_iX_j$ln(FWPD)	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
51	Intercept	$\ln(\alpha')$	3.905**	0.232	4.52	0.51	0.13	5,112
(HN 2)	ln(FWPD)	β_0	0.003*	0.001				
	$X_i$ln(FWPD)	β_1	0.009**	0.003				
	$X_j$ln(FWPD)	β_2	-	-				
	$X_iX_j$ln(FWPD)	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.676**	-0.073				
	$\ln(X_j)$	γ_2	-0.585**	-0.155				
	$\ln(X_iX_j)$	γ_3	0.646**	0.178				
63	Intercept	$\ln(\alpha')$	-0.205	-0.181	0.82	0.70	0.23	3,236
(HN 3)	ln(FWPD)	β_0	1.270**	0.055				
	$X_i$ln(FWPD)	β_1	-0.001	-0.001				
	$X_j$ln(FWPD)	β_2	-	-				
	$X_iX_j$ln(FWPD)	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-0.048	-0.031				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.8 (cont'd)   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln FWPD in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
60	Intercept	$\ln(\alpha')$	1.184**	0.106	5.28	0.77	0.11	3,236
(HN 4)	ln(FWPD)	β_0	1.058**	0.040				
	$X_i$ln(FWPD)	β_1	-	-				
	$X_j$ln(FWPD)	β_2	-	-				
	X_iX_j(FWPD)	β_3	-0.0001**	-0.00004				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	0.131**	0.036				
75	Intercept	$\ln(\alpha')$	2.090**	0.111	1.79	0.80	0.10	2,237
(HN 5)	ln(FWPD)	β_0	1.980**	0.035				
	$X_i$ln(FWPD)	β_1	-0.003**	-0.001				
	$X_j$ln(FWPD)	β_2	-	-				
	X_iX_j(FWPD)	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
92	Intercept	$\ln(\alpha')$	2.811**	0.175	-0.22	0.90	0.06	2,237
(HN 6)	ln(FWPD)	β_0	0.931**	0.033				
	$X_i$ln(FWPD)	β_1	-	-				
	$X_j$ln(FWPD)	β_2	-	-				
	X_iX_j(FWPD)	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.114**	-0.037				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				

*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.9   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln WPD in
beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-	-	-	-	-	-
(HN 1)[†]	ln(WPD)	β_0	-	-				
	$X_i \ln(WPD)$	β_1	-	-				
	$X_j \ln(WPD)$	β_2	-	-				
	$X_i X_j \ln(WPD)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				
51	Intercept	$\ln(\alpha')$	-3.325**	-0.427	0.24	0.11	1.01	2,115
(HN 2)	ln(WPD)	β_0	0.668**	0.187				
	$X_i \ln(WPD)$	β_1	0.007	0.004				
	$X_j \ln(WPD)$	β_2	-	-				
	$X_i X_j \ln(WPD)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				
63	Intercept	$\ln(\alpha')$	-2.700**	-0.136	2.33	0.87	0.27	2,237
(HN 3)	ln(WPD)	β_0	1.502**	0.038				
	$X_i \ln(WPD)$	β_1	0.005**	0.001				
	$X_j \ln(WPD)$	β_2	-	-				
	$X_i X_j \ln(WPD)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.5.9 (cont'd)   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WPD in beans - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	-0.524	-0.056	2.45	0.99	0.004	4,235
(HN 4)	$\ln(WPD)$	β_0	1.066	0.008				
	$X_i\ln(WPD)$	β_1	-	-				
	$X_j\ln(WPD)$	β_2	-	-				
	$X_iX_j(WPD)$	β_3	-	-				
	$\ln(X_i)$	γ_1	0.026	0.009				
	$\ln(X_j)$	γ_2	-0.026	-0.015				
	$\ln(X_iX_j)$	γ_3	0.039	0.019				
75	Intercept	$\ln(\alpha')$	-0.636	-0.082	0.59	0.99	0.004	3,236
(HN 5)	$\ln(WPD)$	β_0	1.071	0.009				
	$X_i\ln(WPD)$	β_1	-0.0003	-0.0002				
	$X_j\ln(WPD)$	β_2	-	-				
	$X_iX_j(WPD)$	β_3	-	-				
	$\ln(X_i)$	γ_1	0.076	0.022				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
92	Intercept	$\ln(\alpha')$	-0.155**	-0.025	6.25	0.99	0.002	6,233
(HN 6)	$\ln(WPD)$	β_0	1.008**	0.004				
	$X_i\ln(WPD)$	β_1	0.0002**	0.0001				
	$X_j\ln(WPD)$	β_2	0.001*	0.0002				
	$X_iX_j(WPD)$	β_3	-0.00001**	-0.0001				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.039	-0.022				
	$\ln(X_iX_j)$	γ_3	0.049*	0.023				

*Significant at P = 0.05
**Significant at P = 0.01
Note: ln is synonymous for $\log_e$

Appendix 8.6.1   Parameters and statistics for the best subset multiple regression model of the allometric relationship between ln W and ln LN in beans - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	-0.168	-0.305
$\ln(LN)$	β_0	0.799**	0.817
$X_i\ln(LN)$	β_1	-0.003**	-0.256
$X_j\ln(LN)$	β_2	-0.003**	-0.217
$X_iX_j\ln(LN)$	β_3	-0.0001*	-0.116
$\ln(X_i)$	γ_1	0.202**	0.269
$\ln(X_j)$	γ_2	0.082**	0.202
$\ln(X_iX_j)$	γ_3	-	-

Statistics

Mallows CP	6.15
R^2	0.58
Residue mean square	0.13
d.f.	6,950

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.6.2   Parameters and statistics for the best subset multiple regression
                   models of the allometric relationship between ln W and ln LA in
                   beans - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	1.601^{**}	2.910
$\ln(LA)$	β_0	0.668^{**}	0.817
$X_i \ln(LA)$	β_1	-	-
$X_j \ln(LA)$	β_2	-	-
$X_i X_j \ln(LA)$	β_3	-0.0001^{**}	-0.123
$\ln(X_i)$	γ_1	-0.055^{*}	-0.073
$\ln(X_j)$	γ_2	-0.183^{**}	-0.451
$\ln(X_i X_j)$	γ_3	0.287^{**}	0.491

Statistics

Mallows CP	5.26	
R^2	0.70	
Residue mean square	0.09	
d.f.	5,951	

*Significant at $P = 0.05$
**Significant at $P = 0.01$

Appendix 8.6.3   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln WL
in beans - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	1.960**	3.539
$\ln(WL)$	β_0	0.765**	1.004
$X_i\ln(WL)$	β_1	-0.003**	-0.111
$X_j\ln(WL)$	β_2	-0.005**	-0.159
$X_iX_j\ln(WL)$	β_3	0.0001**	0.090
$\ln(X_i)$	γ_1	-	-
$\ln(X_j)$	γ_2	-	-
$\ln(X_iX_j)$	γ_3	-	-

Statistics

Mallows CP	2.96
R^2	0.79
Residue mean square	0.06
d.f.	4,955

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.6.4   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln WST
in beans -1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	2.239**	4.042
$\ln(WST)$	β_0	0.737**	0.781
$X_i\ln(WST)$	β_1	-	-
$X_j\ln(WST)$	β_2	-	-
$X_iX_j\ln(WST)$	β_3	-0.0001*	-0.071
$\ln(X_i)$	γ_1	-0.129**	-0.171
$\ln(X_j)$	γ_2	-0.346**	-0.849
$\ln(X_iX_j)$	γ_3	0.486**	0.825

Statistics

Mallows CP	5.26
R^2	0.69
Residue mean square	0.10
d.f.	5,954

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.6.5   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln MPN
in beans - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	1.195**	2.234
ln(MPN)	β_0	0.865**	0.961
$X_i \ln(MPN)$	β_1	-0.002**	-0.115
$X_j \ln(MPN)$	β_2	-0.003**	-0.125
$X_i X_j \ln(MPN)$	β_3	-0.0001	-0.073
$\ln(X_i)$	γ_1	0.064**	0.087
$\ln(X_j)$	γ_2	-	-
$\ln(X_i X_j)$	γ_3	0.074**	0.130

Statistics

Mallows CP	6.36
R^2	0.75
Residue mean square	0.07
d.f.	6,930

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.6.6   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln UPN in beans - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	2.607**	4.745
$\ln(UPN)$	β_0	0.406**	0.489
$X_i \ln(UPN)$	β_1	-0.002	-0.097
$X_j \ln(UPN)$	β_2	-	-
$X_i X_j \ln(UPN)$	β_3	-0.0002**	-0.183
$\ln(X_i)$	γ_1	-0.132**	-0.175
$\ln(X_j)$	γ_2	-0.465**	-1.152
$\ln(X_i X_j)$	γ_3	0.667**	1.144

Statistics

Mallows CP	6.17
R^2	0.29
Residue mean square	0.47
d.f.	6,911

*Significant at $P = 0.01$
**Significant at $P = 0.01$

Appendix 8.6.7   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln FWPD
in beans - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	-0.740**	-1.387
$\ln(FWPD)$	β_0	0.749**	0.937
$X_i\ln(FWPD)$	β_1	-0.002**	-0.228
$X_j\ln(FWPD)$	β_2	-0.002**	-0.152
$X_iX_j\ln(FWPD)$	β_3	-0.0001**	-0.142
$\ln(X_i)$	γ_1	0.206**	0.280
$\ln(X_j)$	γ_2	0.089**	0.225
$\ln(X_iX_j)$	γ_3	-	-

Statistics

Mallows CP	7.11
R^2	0.77
Residue mean square	0.07
d.f.	6,930

*Significant at P = 0.05
**Significant at P = 0.02

Appendix 8.6.8   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WUPD in beans - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	3.576**	6.505
ln(WUPD)	β_0	0.075**	0.164
$X_i$ln(WUPD)	β_1	0.002**	0.161
$X_j$ln(WUPD)	β_2	-	-
$X_iX_j$ln(WUPD)	β_3	-0.0001	-0.075
$\ln(X_i)$	g_1	-0.243**	-0.324
$\ln(X_j)$	γ_2	-0.631**	-1.560
$\ln(X_iX_j)$	γ_3	0.797**	1.367

<u>Statistics</u>

Mallows CP	6.00
R^2	0.21
Residue mean square	0.24
d.f.	6,910

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.6.9   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln WMPD
in beans - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	0.965**	1.804
ln(WMPD)	β_0	0.750**	0.927
$X_i$ln(WMPD)	β_1	-0.002**	-0.116
$X_j$ln(WMPD)	β_2	-0.002**	-0.105
$X_iX_j$ln(WMPD)	β_3	-0.0001**	-0.143
$\ln(X_i)$	γ_1	0.093**	0.127
$\ln(X_j)$	γ_2	-	-
$\ln(X_iX_j)$	γ_3	0.106**	0.186

<u>Statistics</u>

Mallows CP	6.01
R^2	0.70
Residue mean square	0.09
d.f.	6,931

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.6.10   Parameters and statistics for the best subset multiple regression
                   models of the allometric relationship between ln W and ln SN
                   in beans - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	0.768**	1.450
$\ln(SN)$	β_0	0.539**	0.729
$X_i \ln(SN)$	β_1	-0.004**	-0.382
$X_j \ln(SN)$	β_2	-0.004**	-0.305
$X_i X_j \ln(SN)$	β_3	-0.0001*	-0.149
$\ln(X_i)$	γ_1	0.232**	0.318
$\ln(X_j)$	γ_2	-	-
$\ln(X_i X_j)$	γ_3	0.173**	0.306

Statistics

Mallows CP	6.96
R^2	0.35
Residue mean square	0.18
d.f.	6,903

*Significant at $P = 0.05$
**Significant at $P = 0.01$

Appendix 8.6.11   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln WS
in beans - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	2.443**	4.613
$\ln(WS)$	β_0	0.419**	0.610
$X_i \ln(WS)$	β_1	-	-
$X_j \ln(WS)$	β_2	-	-
$X_i X_j \ln(WS)$	β_3	-0.0003**	-0.238
$\ln(X_i)$	γ_1	-0.091**	-0.125
$\ln(X_j)$	g_2	-0.340**	-0.869
$\ln(X_i X_j)$	γ_3	0.529**	0.937

Statistics

Mallows CP	4.62
R^2	0.37
Residue mean square	0.18
d.f.	5,904

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.1   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln T in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40 (HN 1)[†]	Intercept	$\ln(\alpha')$	-6.571**	-7.136	4.41	0.79	0.18	3,235
	$\ln(T)$	β_0	2.374**	0.886				
	$X_i\ln(T)$	β_1	-0.002**	-0.115				
	$X_j\ln(T)$	β_2	-0.011**	-0.337				
	$X_iX_j\ln(T)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
51 (HN 2)	Intercept	$\ln(\alpha')$	-6.563**	-5.372	2.58	0.81	0.29	3,236
	$\ln(T)$	β_0	2.427**	0.769				
	$X_i\ln(T)$	β_1	-	-				
	$X_j\ln(T)$	β_2	-0.011**	-0.289				
	$X_iX_j\ln(T)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.098	-0.112				
	$\ln(X_iX_j)$	γ_3	-	-				
63 (HN 3)	Intercept	$\ln(\alpha')$	-8.356**	-6.058	6.03	0.83	0.33	6,233
	$\ln(T)$	β_0	2.715**	0.755				
	$X_i\ln(T)$	β_1	-0.009**	-0.311				
	$X_j\ln(T)$	β_2	-0.012**	-0.298				
	$X_iX_j\ln(T)$	β_3	-0.0003*	-0.174				
	$\ln(X_i)$	γ_1	0.689**	0.337				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-0.179*	-0.135				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.1 (cont'd)   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln T in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	-6.087**	-4.631	3.24	0.75	0.44	4,235
(HN 4)	$\ln(T)$	β_0	2.471**	0.724				
	$X_i \ln(T)$	β_1	-	-				
	$X_j \ln(T)$	β_2	-0.007**	-0.207				
	$X_i X_j(T)$	β_3	-0.0001	-0.110				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.344**	-0.366				
	$\ln(X_i X_j)$	γ_3	-	-				
75	Intercept	$\ln(\alpha')$	-5.389**	-3.728	4.36	0.85	0.32	5,234
(HN 5)	$\ln(T)$	β_0	2.548**	0.667				
	$X_i \ln(T)$	β_1	-0.004**	-0.137				
	$X_j \ln(T)$	β_2	-	-				
	$X_i X_j(T)$	β_3	-0.0002**	-0.146				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-1.0420	-1.007				
	$\ln(X_i X_j)$	γ_3	0.569**	0.408				
92	Intercept	$\ln(\alpha')$	-4.097**	-2.984	5.41	0.82	0.35	5,234
(HN 6)	$\ln(T)$	β_0	2.240**	0.643				
	$X_i \ln(T)$	β_1	-0.006**	-0.209				
	$X_j \ln(T)$	β_2	0.008**	0.223				
	$X_i X_j(T)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-1.665**	-1.694				
	$\ln(X_i X_j)$	γ_3	0.979**	0.738				
107	Intercept	$\ln(\alpha')$	-4.036**	-3.133	2.24	0.73	0.45	4,235
(HN 7)	$\ln(T)$	β_0	2.174**	0.599				
	$X_i \ln(T)$	β_1	-0.003**	-0.127				
	$X_j \ln(T)$	β_3	-	-				
	$X_i X_j \ln(T)$	β_4	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.932**	-1.011				
	$\ln(X_i X_j)$	γ_3	0.346	0.278				

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.2   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln LN in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-7.753**	-8.419	6.03	0.58	0.36	6,232
(HN 1)[†]	$\ln(LN)$	β_0	3.083**	0.885				
	$X_i\ln(LN)$	β_1	-0.011*	-0.398				
	$X_j\ln(LN)$	β_2	-0.033**	-0.672				
	$X_iX_j\ln(LN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	0.599*	0.439				
	$\ln(X_j)$	γ_2	0.957**	1.446				
	$\ln(X_iX_j)$	γ_3	-0.862*	-0.969				
51	Intercept	$\ln(\alpha')$	-6.569**	-5.377	2.64	0.76	0.37	3,236
(HN 2)	$\ln(LN)$	β_0	3.140**	0.818				
	$X_i\ln(LN)$	β_1	-	-				
	$X_j\ln(LN)$	β_2	-0.008**	-0.126				
	$X_iX_j\ln(LN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	0.093	0.051				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
63	Intercept	$\ln(\alpha')$	-7.065**	-5.122	5.79	0.80	0.39	4,235
(HN 3)	$\ln(LN)$	β_0	3.347**	0.810				
	$X_i\ln(LN)$	β_1	-0.009**	-0.241				
	$X_j\ln(LN)$	β_2	-	-				
	$X_iX_j\ln(LN)$	β_3	-0.001**	-0.265				
	$\ln(X_i)$	γ_1	0.524**	0.256				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.2 (cont'd)   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln LN in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69 (HN 4)	Intercept	$\ln(\alpha')$	-5.123**	-3.898	5.57	0.74	0.46	5,234
	$\ln(LN)$	β_0	3.219**	0.775				
	$X_i\ln(LN)$	β_1	-0.004*	-0.123				
	$X_j\ln(LN)$	β_2	-0.014*	-0.212				
	$X_iX_j(LN)$	β_3	-0.0005**	-0.195				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.177	-0.188				
	$\ln(X_iX_j)$	γ_3	-	-				
75 (HN 5)	Intercept	$\ln(\alpha')$	-3.851**	-2.663	3.77	0.81	0.63	4,235
	$\ln(LN)$	β_0	2.933**	0.748				
	$X_i\ln(LN)$	β_1	-	-				
	$X_j\ln(LN)$	β_2	-0.013**	-0.171				
	$X_iX_j(LN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.182	-0.085				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-0.132	-0.095				
92 (HN 6)	Intercept	$\ln(\alpha')$	-3.081**	-2.244	6.06	0.73	0.72	6,233
	$\ln(LN)$	β_0	2.569**	0.628				
	$X_i\ln(LN)$	β_1	-0.006**	-0.163				
	$X_j\ln(LN)$	β_2	0.023*	0.345				
	$X_iX_j(LN)$	β_3	-0.0004	-0.144				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-1.226**	-1.248				
	$\ln(X_iX_j)$	γ_3	0.944**	0.712				
107 (HN 7)	Intercept	$\ln(\alpha')$	-4.919**	-3.817	5.38	0.77	0.40	5,234
	$\ln(LN)$	β_0	2.980**	0.761				
	$X_i\ln(LN)$	β_1	-0.010**	-0.304				
	$X_j\ln(LN)$	β_2	-0.013**	-0.222				
	$X_iX_j\ln(LN)$	β_3	-	-				
	$\ln(X_i)$	γ_1	0.394*	0.206				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-0.221**	-0.178				

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.3   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln LA in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	4.798**	5.211	5.08	0.93	0.06	2,236
(HN 1)[†]	ln(LA)	β_0	1.075**	0.923				
	$X_i$ln(LA)	β_1	-	-				
	$X_j$ln(LA)	β_2	-	-				
	$X_iX_j$ln(LA)	β_3	0.0001**	0.150				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
51	Intercept	$\ln(\alpha')$	6.352**	5.200	6.55	0.94	0.09	6,233
(HN 2)	ln(LA)	β_0	1.231**	1.025				
	$X_i$ln(LA)	β_1	-0.003**	-0.240				
	$X_j$ln(LA)	β_2	-0.003**	-0.161				
	$X_iX_j$ln(LA)	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.372**	-0.205				
	$\ln(X_j)$	γ_2	-0.678**	-0.776				
	$\ln(X_iX_j)$	γ_3	0.635**	0.538				
63	Intercept	$\ln(\alpha')$	7.184**	5.209	6.08	0.91	0.17	6,233
(HN 3)	ln(LA)	β_0	1.222**	0.970				
	$X_i$ln(LA)	β_1	-0.004**	-0.274				
	$X_j$ln(LA)	β_2	-0.004**	-0.195				
	$X_iX_j$ln(LA)	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.554**	-0.271				
	$\ln(X_j)$	γ_2	-0.825**	-0.835				
	$\ln(X_iX_j)$	γ_3	0.619**	0.465				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.3 (cont'd) Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln LA in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	7.346**	5.589	7.16	0.93	0.13	5,234
(HN 4)	$\ln(LA)$	β_0	1.307**	0.960				
	$X_i \ln(LA)$	β_1	-0.003**	-0.179				
	$X_j \ln(LA)$	β_2	-	-				
	$X_i X_j (LA)$	β_3	-0.0001**	-0.173				
	$\ln(X_i)$	γ_1	-0.390**	-0.200				
	$\ln(X_j)$	γ_2	-0.356**	-0.378				
	$\ln(X_i X_j)$	γ_3	-	-				
75	Intercept	$\ln(\alpha')$	6.401**	4.427	0.40	0.94	0.13	3,236
(HN 5)	$\ln(LA)$	β_0	1.124**	0.786				
	$X_i \ln(LA)$	β_1	-	-				
	$X_j \ln(LA)$	β_2	-	-				
	$X_i X_j (LA)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.106**	-0.050				
	$\ln(X_j)$	γ_2	-0.348**	-0.337				
	$\ln(X_i X_j)$	γ_3	-	-				
92	Intercept	$\ln(\alpha')$	6.569**	4.785	2.73	0.92	0.15	4,235
(HN 6)	$\ln(LA)$	β_0	1.189**	0.812				
	$X_i \ln(LA)$	β_1	0.001	0.044				
	$X_j \ln(LA)$	β_2	-	-				
	$X_i X_j (LA)$	β_3	-0.0001**	-0.088				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.382**	-0.389				
	$\ln(X_i X_j)$	γ_3	-	-				
107	Intercept	$\ln(\alpha')$	7.085**	5.499	3.18	0.90	0.17	4,235
(HN 7)	$\ln(LA)$	β_0	1.186**	0.819				
	$X_i \ln(LA)$	β_1	-	-				
	$X_j \ln(LA)$	β_2	-	-				
	$X_i X_j \ln(LA)$	β_3	-0.0001**	-0.114				
	$\ln(X_i)$	γ_1	-0.117*	-0.061				
	$\ln(X_j)$	γ_2	-0.384**	-0.417				
	$\ln(X_i X_j)$	γ_3	-	-				

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.4   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WL in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-0.408**	-0.444	5.32	0.99	0.01	4,234
(HN 1)[†]	ln(WL)	β_0	0.984**	0.986				
	$X_i \ln(WL)$	β_1	-	-				
	$X_j \ln(WL)$	β_2	-	-				
	$X_i X_j \ln(WL)$	β_3	-0.00002	-0.017				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.087**	-0.132				
	$\ln(X_i X_j)$	γ_3	0.074**	0.083				
51	Intercept	$\ln(\alpha')$	-0.238**	-0.202	8.00	0.98	0.03	7,232
(HN 20	ln(WL)	β_0	1.057**	1.099				
	$X_i \ln(WL)$	β_1	-0.005**	-0.199				
	$X_j \ln(WL)$	β_2	-0.005**	-0.100				
	$X_i X_j \ln(WL)$	β_3	0.0002**	0.144				
	$\ln(X_i)$	γ_1	-0.089**	-0.051				
	$\ln(X_j)$	γ_2	-0.142**	-0.168				
	$\ln(X_i X_j)$	γ_3	0.138**	0.122				
63	Intercept	$\ln(\alpha')$	-0.625**	-0.500	2.19	0.99	0.02	4,235
(HN 3)	ln(WL)	β_0	0.869**	0.960				
	$X_i \ln(WL)$	β_1	-0.002**	-0.059				
	$X_j \ln(WL)$	β_2	-	-				
	$X_i X_j \ln(WL)$	β_3	0.0002**	0.096				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.016*	-0.018				
	$\ln(X_i X_j)$	γ_3	-	-				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.4 (cont'd) Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WL in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	-0.451**	-0.404	5.69	0.98	0.03	5,234
(HN 4)	ln(WL)	β_0	0.782**	0.921				
	$X_i\ln(WL)$	β_1	-	-				
	$X_j\ln(WL)$	β_2	-	-				
	$X_iX_j(WL)$	β_3	0.0001**	0.042				
	$\ln(X_i)$	γ_1	-0.077**	-0.047				
	$\ln(X_j)$	γ_2	-0.138**	-0.173				
	$\ln(X_iX_j)$	γ_3	0.116*	0.107				
75	Intercept	$\ln(\alpha')$	-0.804**	-0.678	3.42	0.98	0.03	4,235
(HN 5)	ln(WL)	β_0	0.774**	0.943				
	$X_i\ln(WL)$	β_1	-0.001**	-0.045				
	$X_j\ln(WL)$	β_2	0.004**	0.081				
	$X_iX_j(WL)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-0.040*	-0.035				
92	Intercept	$\ln(\alpha')$	-0.615**	-0.553	3.79	0.96	0.05	4,235
(HN 6)	ln(WL)	β_0	0.710**	0.877				
	$X_i\ln(WL)$	β_1	-	-				
	$X_j\ln(WL)$	β_2	0.004**	0.090				
	$X_iX_j(WL)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.052*	-0.032				
	$\ln(X_j)$	γ_2	-0.056**	-0.071				
	$\ln(X_iX_j)$	γ_3	-	-				
107	Intercept	$\ln(\alpha')$	-0.340**	-0.335	5.14	0.93	0.07	5,234
(HN 7)	ln(WL)	β_0	0.694**	0.881				
	$X_i\ln(WL)$	β_1	-	-				
	$X_j\ln(WL)$	β_2	-	-				
	$X_iX_j\ln(WL)$	β_3	0.0001**	0.052				
	$\ln(X_i)$	γ_1	-0.138**	-0.092				
	$\ln(X_j)$	γ_2	-0.199**	-0.273				
	$\ln(X_iX_j)$	γ_3	0.187*	0.190				

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.5   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WP in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-1.963**	-2.037	4.33	0.94	0.06	5,232
(HN 1)[†]	$\ln(WP)$	β_0	0.990**	0.932				
	$X_i \ln(WP)$	β_1	0.002**	0.087				
	$X_j \ln(WP)$	β_2	-	-				
	$X_i X_j \ln(WP)$	β_3	-	-				
	$\ln(X_i)$	γ_1	0.114**	0.080				
	$\ln(X_j)$	γ_2	0.221**	0.318				
	$\ln(X_i X_j)$	γ_3	-0.224**	-0.240				
51	Intercept	$\ln(\alpha')$	-1.538**	-1.358	7.30	0.94	0.08	6,233
(HN 2)	$\ln(WP)$	β_0	1.056**	1.139				
	$X_i \ln(WP)$	β_1	-0.003**	-0.143				
	$X_j \ln(WP)$	β_2	-0.007**	-0.149				
	$X_i X_j \ln(WP)$	β_3	0.0003**	0.180				
	$\ln(X_i)$	γ_1	-0.066*	-0.039				
	$\ln(X_j)$	γ_2	0.116**	0.143				
	$\ln(X_i X_j)$	γ_3	-	-				
63	Intercept	$\ln(\alpha')$	-1.545**	-1.230	4.45	0.94	0.10	5,234
(HN 3)	$\ln(WP)$	β_0	0.938**	1.030				
	$X_i \ln(WP)$	β_1	-0.002*	-0.069				
	$X_j \ln(WP)$	β_2	-	-				
	$X_i X_j \ln(WP)$	β_3	0.0002**	0.093				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	0.227**	0.252				
	$\ln(X_i X_j)$	γ_3	-0.155	-0.127				

[†]Harvest number (1-6 in beans and 1-7 in beets)

*Significant at P = 0.05

**Significant at P = 0.01

Appendix 8.7.5 (cont'd)   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WP in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	-1.654**	-1.443	3.20	0.93	0.09	4,235
(HN 4)	$\ln(WP)$	β_0	0.820**	0.940				
	$X_i \ln(WP)$	β_1	-	-				
	$X_j \ln(WP)$	β_2	0.007**	0.138				
	$X_i X_j (WP)$	β_3	-	-				
	$\ln(X_i)$	γ_1	0.046	0.027				
	$\ln(X_j)$	γ_2	0.149**	0.182				
	$\ln(X_i X_j)$	γ_3	-	-				
75	Intercept	$\ln(\alpha')$	-2.533**	-2.107	6.00	0.93	0.11	6,233
(HN 5)	$\ln(WP)$	β_0	0.949**	1.142				
	$X_i \ln(WP)$	β_1	-0.004**	-0.153				
	$X_j \ln(WP)$	β_2	-	-				
	$X_i X_j (WP)$	β_3	0.0003**	0.147				
	$\ln(X_i)$	γ_1	0.261**	0.146				
	$\ln(X_j)$	γ_2	0.568**	0.660				
	$\ln(X_i X_j)$	γ_3	-0.460**	-0.396				
92	Intercept	$\ln(\alpha')$	-1.544**	-1.367	3.00	0.91	0.112	3,236
(HN 6)	$\ln(WP)$	β_0	0.737**	0.896				
	$X_i \ln(WP)$	β_1	-	-				
	$X_j \ln(WP)$	β_2	0.010**	0.205				
	$X_i X_j (WP)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	0.151**	0.187				
	$\ln(X_i X_j)$	γ_3	-	-				
107	Intercept	$\ln(\alpha')$	-1.432**	-0.500	2.49	0.86	0.13	4,235
(HN 7)	$\ln(WP)$	β_0	0.672**	0.906				
	$X_i \ln(WP)$	β_1	-	-				
	$X_j \ln(WP)$	β_2	0.011**	0.172				
	$X_i X_j \ln(WP)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	0.278**	0.407				
	$\ln(X_i X_j)$	γ_3	-0.217*	-0.236				

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.6   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln DR in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-4.257**	-4.623	3.37	0.85	0.13	45,234
(HN 1) [†]	$\ln(DR)$	β_0	1.706**	1.005				
	$X_i\ln(DR)$	β_1	-0.005**	-0.162				
	$X_j\ln(DR)$	β_2	-	-				
	$X_iX_j\ln(DR)$	β_3	-0.0001	-0.038				
	$\ln(X_i)$	γ_1	0.239**	0.175				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
51	Intercept	$\ln(\alpha')$	-2.626**	-2.150	3.06	0.91	0.14	4,235
(HN 2)	$\ln(DR)$	β_0	1.490**	0.840				
	$X_i\ln(DR)$	β_1	0.005**	0.174				
	$X_j\ln(DR)$	β_2	-	-				
	$X_iX_j\ln(DR)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.310**	-1.171				
	$\ln(X_j)$	γ_2	-0.044	-0.050				
	$\ln(X_iX_j)$	γ_3	-	-				
63	Intercept	$\ln(\alpha')$	-2.752**	-1.995	5.41	0.95	0.09	5,234
(HN 3)	$\ln(DR)$	β_0	1.459**	0.917				
	$X_i\ln(DR)$	β_1	-0.003**	-0.107				
	$X_j\ln(DR)$	β_2	0.012**	0.167				
	$X_iX_j\ln(DR)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.436**	-0.442				
	$\ln(X_iX_j)$	γ_3	0.244**	0.184				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.6 (cont'd) Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln DR in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	-4.901**	-3.729	6.39	0.73	0.48	6,233
(HN 4)	ln(DR)	β_0	1.663**	1.320				
	$X_i \ln(DR)$	β_1	-0.006*	-0.265				
	$X_j \ln(DR)$	β_2	-0.050**	-0.074				
	$X_i X_j (DR)$	β_3	-	-				
	$\ln(X_i)$	γ_1	0.545*	0.279				
	$\ln(X_j)$	γ_2	1.284**	1.365				
	$\ln(X_i X_j)$	γ_3	-0.871**	-0.687				
75	Intercept	$\ln(\alpha')$	-4.447**	-3.076	8.00	0.96	0.08	7,232
(HN 5)	ln(DR)	β_0	1.767**	1.066				
	$X_i \ln(DR)$	β_1	-0.003**	-0.105				
	$X_j \ln(DR)$	β_2	-0.007*	-0.118				
	$X_i X_j (DR)$	β_3	0.0002	0.071				
	$\ln(X_i)$	γ_1	0.176	0.082				
	$\ln(X_j)$	γ_2	0.420**	0.406				
	$\ln(X_i X_j)$	γ_3	-0.459**	-0.329				
92	Intercept	$\ln(\alpha')$	-3.422**	-2.492	3.13	0.96	0.07	4,235
(HN 6)	ln(DR)	β_0	1.585**	0.978				
	$X_i \ln(DR)$	β_1	-0.001**	-0.050				
	$X_j \ln(DR)$	β_2	0.002	0.038				
	$X_i X_j (DR)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-0.077*	-0.058				
107	Intercept	$\ln(\alpha')$	-4.742**	-3.680	6.81	0.94	0.10	6,233
(HN 7)	ln(DR)	β_0	1.762**	1.096				
	$X_i \ln(DR)$	β_1	-0.003*	-0.142				
	$X_j \ln(DR)$	β_2	-0.009**	-0.179				
	$X_i X_j \ln(DR)$	β_3	-	-				
	$\ln(X_i)$	γ_1	0.270*	0.141				
	$\ln(X_j)$	γ_2	0.633**	0.686				
	$\ln(X_i X_j)$	γ_3	-0.577*	-0.464				

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.7   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WR in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40	Intercept	$\ln(\alpha')$	-1.821**	-1.870	1.38	0.87	0.13	3,234
(HN 1)[†]	$\ln(WR)$	β_0	1.034**	0.964				
	$X_i\ln(WR)$	β_1	-	-				
	$X_j\ln(WR)$	β_2	-0.002*	-0.058				
	$X_iX_j\ln(WR)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.072*	-0.050				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
51	Intercept	$\ln(\alpha')$	-2.050**	-1.316	9.93	0.92	0.19	5,234
(HN 2)	$\ln(WR)$	β_0	1.162**	0.912				
	$X_i\ln(WR)$	β_1	0.004**	0.133				
	$X_j\ln(WR)$	β_2	0.006	0.103				
	$X_iX_j\ln(WR)$	β_3	-0.0004	-0.210				
	$\ln(X_i)$	γ_1	0.102*	0.004				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_iX_j)$	γ_3	-	-				
63	Intercept	$\ln(\alpha')$	-1.542**	-0.814	5.27	0.94	0.21	4,235
(HN 3)	$\ln(WR)$	β_0	1.381**	1.006				
	$X_i\ln(WR)$	β_1	-	-				
	$X_j\ln(WR)$	β_2	-0.010**	-0.145				
	$X_iX_j\ln(WR)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.448**	-0.330				
	$\ln(X_iX_j)$	γ_3	0.427**	0.234				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.7 (cont'd) Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln WR in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	-1.394**	-0.730	2.93	0.95	0.17	3,236
(HN 4)	$\ln(WR)$	β_0	1.393**	0.959				
	$X_i\ln(WR)$	β_1	-	-				
	$X_j\ln(WR)$	β_2	-	-				
	$X_iX_j(WR)$	β_3	-0.0002**	-0.058				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.138**	-1.101				
	$\ln(X_iX_j)$	γ_3	-	-				
75	Intercept	$\ln(\alpha')$	-0.385	-0.190	8.00	0.97	0.14	7,232
(HN 5)	$\ln(WR)$	β_0	1.138**	0.815				
	$X_i\ln(WR)$	β_1	0.004**	0.078				
	$X_j\ln(WR)$	β_2	0.007*	0.073				
	$X_iX_j(WR)$	β_3	-0.0003*	0.078				
	$\ln(X_i)$	γ_1	-0.196**	-0.065				
	$\ln(X_j)$	γ_2	-0.591**	-0.409				
	$\ln(X_iX_j)$	γ_3	0.513**	0.263				
92	Intercept	$\ln(\alpha')$	-1.147**	-0.608	1.92	0.96	0.14	3,236
(HN 6)	$\ln(WR)$	β_0	1.312**	0.955				
	$X_i\ln(WR)$	β_1	-	-				
	$X_j\ln(WR)$	β_2	-0.003*	-0.037				
	$X_iX_j(WR)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.108**	-0.080				
	$\ln(X_iX_j)$	γ_3	-	-				
107	Intercept	$\ln(\alpha')$	-1.105**	-0.601	3.94	0.94	0.20	3,236
(HN 7)	$\ln(WR)$	β_0	1.284**	0.900				
	$X_i\ln(WR)$	β_1	-	-				
	$X_j\ln(WR)$	β_2	-	-				
	$X_iX_j\ln(WR)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-0.330**	-0.251				
	$\ln(X_iX_j)$	γ_3	0.257*	0.145				

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.8   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln FWR in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
40 (HN 1)[†]	Intercept	$\ln(\alpha')$	-	-	-	-	-	-
	$\ln(FWR)$	β_0	-	-				
	$X_i \ln(FWR)$	β_1	-	-				
	$X_j \ln(FWR)$	β_2	-	-				
	$X_i X_j \ln(FWR)$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				
51 (HN 2)	Intercept	$\ln(\alpha')$	-0.475**	-0.389	6.55	0.82	0.27	6,233
	$\ln(FWR)$	β_0	0.738**	0.859				
	$X_i \ln(FWR)$	β_1	-0.002	-0.089				
	$X_j \ln(FWR)$	β_2	-	-				
	$X_i X_j \ln(FWR)$	β_3	0.0003**	0.170				
	$\ln(X_i)$	γ_1	0.144**	0.079				
	$\ln(X_j)$	γ_2	0.293**	0.335				
	$\ln(X_i X_j)$	γ_3	-0.431**	-0.365				
63 (HN 3)	Intercept	$\ln(\alpha')$	-0.004	-0.003	3.16	0.93	0.13	3,234
	$\ln(FWR)$	β_0	0.712**	0.943				
	$X_i \ln(FWR)$	β_1	-0.002*	-0.071				
	$X_j \ln(FWR)$	β_2	-	-				
	$X_i X_j \ln(FWR)$	β_3	0.0002**	0.113				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				

[†]Harvest number (1-6 in beans and 1-7 in beets)
*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.7.8 (cont'd)   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln FWR in beets - 1984 experiment

Age at harvest (days)	Potential independent variable	Parameter	Regression coefficient	Std. partial regression coefficient	Mallows CP	R^2	RMS	d.f.
69	Intercept	$\ln(\alpha')$	-0.019	-0.014	0.93	0.96	0.07	2,237
(HN 4)	$\ln(\text{FWR})$	β_0	0.650**	0.935				
	$X_i \ln(\text{FWR})$	β_1	-	-				
	$X_j \ln(\text{FWR})$	β_2	0.003	0.062				
	$X_i X_j(\text{FWR})$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				
75	Intercept	$\ln(\alpha')$	-0.264**	-0.182	4.98	0.96	0.08	2,237
(HN 5)	$\ln(\text{FWR})$	β_0	0.727**	1.012				
	$X_i \ln(\text{FWR})$	β_1	-	-				
	$X_j \ln(\text{FWR})$	β_2	-	-				
	$X_i X_j(\text{FWR})$	β_3	-	-				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	0.049**	0.048				
	$\ln(X_i X_j)$	γ_3	-	-				
92	Intercept	$\ln(\alpha')$	-0.179**	-1.130	0.80	0.96	0.07	3,236
(HD 6)	$\ln(\text{FWR})$	β_0	0.690**	0.958				
	$X_i \ln(\text{FWR})$	β_1	-	-				
	$X_j \ln(\text{FWR})$	β_2	0.004**	0.080				
	$X_i X_j(\text{FWR})$	β_3	-0.0001	-0.042				
	$\ln(X_i)$	γ_1	-	-				
	$\ln(X_j)$	γ_2	-	-				
	$\ln(X_i X_j)$	γ_3	-	-				
107	Intercept	$\ln(\alpha')$	-0.041	-0.032	3.87	0.94	0.09	4,235
(HD 7)	$\ln(\text{FWR})$	β_0	0.723**	1.055				
	$X_i \ln(\text{FWR})$	β_1	-	-				
	$X_j \ln(\text{FWR})$	β_2	-0.004**	-0.061				
	$X_i X_j \ln(\text{FWR})$	β_3	-	-				
	$\ln(X_i)$	γ_1	-0.063	-0.003				
	$\ln(X_j)$	γ_2	0.073**	0.079				
	$\ln(X_i X_j)$	γ_3	0.257*	0.145				

*Significant at P = 0.05

**Significant at P = 0.01

Appendix 8.8.1   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln LN
in beets - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	-0.797	-0.620
$\ln(LN)$	β_0	2.083**	0.564
$X_i\ln(LN)$	β_1	0.011**	0.340
$X_j\ln(LN)$	β_2	-	-
$X_iX_{ftsj}\ln(LN)$	β_3	0.0005**	0.248
$\ln(X_i)$	γ_1	-0.911**	-0.521
$\ln(X_j)$	γ_2	-0.235**	-0.249
$\ln(X_iX_j)$	γ_3	-	-

Statistics

Mallows CP	4.07
R^2	0.56
Residue mean square	0.75
d.f.	5,942

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.8.2   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln LA
in beets - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	0.038	0.029
$\ln(LA)$	β_0	1.254**	0.825
$X_i \ln(LA)$	β_1	-	-
$X_j \ln(LA)$	β_2	0.013**	0.003
$X_i X_j \ln(LA)$	β_3	-0.0006**	-0.176
$\ln(X_i)$	γ_1	0.137**	0.079
$\ln(X_j)$	γ_2	-	-
$\ln(X_i X_j)$	γ_3	-0.085**	-0.062

Statistics

Mallows CP	7.24
R^2	0.70
Residue mean square	0.49
d.f.	5,943

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.8.3   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln WL
in beets - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	1.378**	1.073
ln(WL)	β_0	1.233**	0.889
$X_i\ln(WL)$	β_1	-	-
$X_j\ln(WL)$	β_2	0.020**	0.281
$X_iX_j\ln(WL)$	β_3	-0.001**	-0.375
$\ln(X_i)$	γ_1	-	-
$\ln(X_j)$	γ_2	-	-
$\ln(X_iX_j)$	γ_3	-	-

Statistics

Mallows CP	2.40
R^2	0.71
Residue mean square	0.47
d.f.	3,945

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.8.4   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln WP
in beets - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	2.677**	2.086
$\ln(WP)$	β_0	1.250**	0.924
$X_i \ln(WP)$	β_1	-0.006**	-0.157
$X_j \ln(WP)$	β_2	0.010**	0.165
$X_i X_j \ln(WP)$	β_3	-0.001**	-0.294
$\ln(X_i)$	γ_1	-0.121**	-0.069
$\ln(X_j)$	γ_2	0.302**	0.321
$\ln(X_i X_j)$	γ_3	-0.696**	-0.510

Statistics

Mallows CP	8.00
R^2	0.69
Residue mean square	0.52
d.f.	7,941

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.8.5   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln DR in beets - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	-2.852**	-1.443
ln(DR)	β_0	1.329**	0.786
$X_i \ln(DR)$	β_1	0.004**	0.163
$X_j \ln(DR)$	β_2	-0.005**	-0.172
$X_i X_j \ln(DR)$	β_3	-0.0002**	0.155
$\ln(X_i)$	γ_1	-0.411**	-0.235
$\ln(X_j)$	γ_2	-	-
$\ln(X_i X_j)$	γ_3	-	-

Statistics

Mallows CP	5.35
R^2	0.71
Residue mean square	0.48
d.f.	5,943

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.8.6   Parameters and statistics for the best subset multiple regression models of the allometric relationship between ln W and ln FWR in beets - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha')$	-1.918**	-1.494
ln(FWR)	β_0	0.857**	1.098
$X_i$ln(FWR)	β_1	-0.004**	-0.187
$X_j$ln(FWR)	β_2	-	-
$X_iX_j$ln(FWR)	β_3	-0.0002**	-0.113
$\ln(X_i)$	γ_1	0.432**	0.247
$\ln(X_j)$	γ_2	0.091**	0.097
$\ln(X_iX_j)$	γ_3	-	-

Statistics

Mallows CP	4.75
R^2	0.74
Residue mean square	0.42
d.f.	5,943

*Significant at P = 0.05
**Significant at P = 0.01

Appendix 8.8.7   Parameters and statistics for the best subset multiple regression
models of the allometric relationship between ln W and ln WR
in beets - 1987 experiment

Potential independent variable	Parameter	Regression coefficient	Standard partial regression coefficient
Intercept	$\ln(\alpha)$	0.405**	0.315
$\ln(WR)$	β_0	0.856**	1.075
$X_i\ln(WR)$	β_1	-0.003**	-0.118
$X_j\ln(WR)$	β_2	0.006**	0.152
$X_iX_j\ln(WR)$	β_3	-0.0004**	-0.246
$\ln(X_i)$	γ_1	0.118**	0.067
$\ln(X_j)$	γ_2	-0.166**	-0.171
$\ln(X_iX_j)$	γ_3	0.255**	0.187

Statistics

Mallows CP	8.00
R^2	0.78
Residue mean square	0.37
d.f.	7,941

*Significant at P = 0.05
**Significant at P = 0.01