ESSAYS IN PRODUCTION THEORY: EFFICIENCY MEASUREMENT AND COMPARATIVE STATICS

By

Maria Nimfa F. Mendoza

B. Sc. (Applied Mathematics) University of the Philippines

M. Sc. (Statistics) University of the Philippines

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

ECONOMICS

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

November 1989

In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the head of my department or by his or her representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Economics

The University of British Columbia
Vancouver, Canada

Date December 21, 1989

Abstract

Nonparametric linear programming tests for consistency with the hypotheses of technical efficiency and allocative efficiency for the general case of multiple output-multiple input technologies are developed in Part I. The tests are formulated relative to three kinds of technologies — convex, constant returns to scale and quasiconcave technologies. Violation indices as summary indicators of the distance of an inefficient observation from an efficient allocation are proposed. The consistent development of the violation indices across the technical efficiency and allocative efficiency tests allows us to obtain comparative measures of the degrees of technical inefficiency and pure allocative inefficiency. Constrained optimization tests applicable to cases where the producer is restricted to optimizing with respect to a subset of goods are also proposed. The latter tests yield the revealed preference-type inequalities commonly used as tests for consistency of observed data with profit maximizing or cost minimizing behavior as limiting cases. Computer programs for implementing the different tests and sample results are listed in the appendix.

In part II, an empirical comparison of nonparametric and parametric measures of technical progress for constant returns to scale technologies is performed using the Canadian input-output data for the period 1961–1980. The original data base was aggregated into four sectors and ten goods and the comparison was done for each sector. If we assume optimizing behavior on the part of the producers, we can reinterpret the violation indices yielded by the efficiency tests in part I as indicators of the shift in the production frontier. More precisely, the violation indices can be considered nonparametric chained indices of technical progress. The parametric measures of technical progress were obtained through econometric profit function estimation using the generalized McFadden flexible functional form with a quadratic spline model for technical progress proposed by Diewert and Wales (1989). Under the assumption of constant

returns, the index of technical change is defined in terms of the unit scale profit function which gives the per unit return to the normalizing good. The empirical results show that the parametric estimates of technical change display a much smoother behavior which can be attributed to the incorporation of stochastic disturbance terms in the estimation procedure and, more interestingly, track the long term trend in the nonparametric estimates.

Part III builds on the theory of minimum wages in international trade and is a theoretical essay in the tradition of analyzing the effects of factor market imperfections on resource allocation. The comparative static responses of the endogenous variables — output levels, employment levels of fixed-price factors with elastic supply and flexible prices of domestic resources — to marginal changes in the economy's exogenous variables — output prices, fixed factor prices and endowments of flexibly-priced domestic resources — are examined. The effect of a change in a fixed factor price on other flexible factor prices can be decomposed Slutsky-like into substitution and scale effects. A symmetry condition between fixed factor prices and flexible factor prices is obtained which clarifies the concepts of "substitutability" and "complementarity" between these two kinds of factors. As an illustration, the model is applied to the case of a devaluation in a two-sector small open economy with rigid wages and capital as specific factors. The empirical implementation of the general model for the Canadian economy is left to more able econometricians but a starting point can be the sectoral analysis performed in Part II.

Table of Contents

List of Tables

List of Figures

Acknowledgement

I thank my major adviser, Erwin Diewert, for his intellectual and personal generosity. Without his support, I would not have made it through my very rough sailing of graduate school. I am fortunate to have such a brilliant adviser and working with him has been one of my life's greatest pleasures.

I thank James Roumasset who has been a personal mentor and who made it possible for me to study at UBC. I am grateful to Terry Wales for the computer programs for the nonlinear optimization routines; Ken White for econometric help and, moreover, for pushing me though at times I resented it; and Ken Hendricks for reading the draft of my thesis and raising good questions in my departmental thesis defence. The meticulous review of my work by my external examiner, Ulrich Kohli of the University of Geneva, and his insightful comments and suggestions are deeply appreciated. Sincere thanks go to Alan and Esther Chung for providing office space, computer help, enlightening discussions and dinner invitations.

I thank my friends who have not given up on me, including those who constantly berated me to finish; and my family, especially my mother and sister Divina, for their emotional and financial support. This dissertation is dedicated to J.P. Mendoza; I am proud to be his daughter.

Overview

This dissertation is a collection of three essays in production theory. The major focus of this research is the development of measurement methods in production analysis rather than the evaluation of a specific economic policy issue. The first essay (part I) proposes nonparametric linear programming tests for consistency of observed data with the hypothesis of productive efficiency at the firm level. The empirical performance of a nonparametric measure of technical progress vis-à-vis a parametric measure of technical progress is examined in the second essay (part II) in an application to the Canadian input-output data. An efficiency test developed in part I is used to obtain the nonparametric measures of technical progress and in addition, econometric profit function estimation assuming an explicit functional form is carried out to yield the parametric measures of technical progress for four aggregated sectors covering the Canadian economy. Whereas the first two essays analyze the economic behavior of price-taking producers facing technological constraints, the third essay (part III) takes a more general view of the economy by linking the different sectors through a trade-theoretic approach to production theory. In this framework, the assumptions of price-taking behavior and technological constraints are still maintained for individual producers and the constraints on factor endowments for the domestic economy are explicitly taken into account. Specifically, part III looks at the effects of factor price rigidities on resource allocation.

The assumption of productive efficiency or optimizing behavior plays a central role in economics. For example, in duality theory, Hotelling's lemma and Shephard's lemma giving the derivative properties of profit functions and cost functions, respectively, derive from the assumption of optimizing behavior of producers. Given the technological constraints determining the feasible output-input combinations and the market constraints as reflected in the prices of goods, the producer is assumed to maximize some objective function. The production plan chosen by the producer is then technically efficient; that is, the plan is some point at the boundary of the firm's production possibilities set. Loosely speaking, technical efficiency implies that the firm is producing maximal output given input levels and minimizing input use given output levels. Moreover, if at the prevailing market prices, the producer is maximizing the value of the objective function describing its behavioral goal (profit maximization, cost minimization or some variant of restricted profit maximization), then the production plan is allocatively efficient. Since technical inefficiency entails some waste of resources and, consequently, an economic cost, then an allocatively efficient production plan must necessarily be technically efficient in some sense. Given the observed production plans and prices faced by firms, it would be of interest to test whether these data are consistent with the hypothesis of optimizing behavior.

The efficiency tests proposed in part I can be used to empirically test whether the regularity conditions often imposed in production theory hold. This study also defines violation indices as indicators of the degree of departure of an observed production plan from the maintained efficiency hypotheses. Since the art of economic theorizing requires some level of abstraction to get meaningful results, the pattern of violation indices may be explained by some unaccounted factors like technical change, differences in firm sizes and regulation costs and hence, may suggest to the researcher depending on prior beliefs and subjective interpretation modifications regarding model and variable specification. In this function, the efficiency tests can then serve as a tool for exploratory data analysis. In their straightforward use in efficiency studies of a cross-section of firms in an industry, the violation indices measure the economic loss due to inefficiency and as such can be of practical importance. Since the proposed efficiency tests also recover efficient allocations relative to a production plan rated inefficient, the efficiency tests can also be useful for prescriptive purposes in efficiency studies.

The major contributions of part I to the field of nonparametric efficiency measurement are: (1) the generalization of technical efficiency and allocative efficiency tests to handle multiple output-multiple input technologies; (2) the definition and consistent development across the efficiency tests of violation indices or efficiency loss measures which are invariant to units of scale in the measurement of goods; and (3) the formulation of constrained optimization or partial profit maximization tests applicable to cases where the producer is restricted to optimizing with respect to a subset of goods. There are three general sets of tests proposed: technical efficiency tests using quantity data only, constrained optimization or partial profit maximization tests using quantity data and a subset of price information, and unconstrained optimization or complete profit maximization tests using both quantity and price data. Each of the above sets of tests is developed relative to three kinds of technologies — convex, constant returns to scale and quasiconcave technologies.

A basic problem in empirical production analysis is that the underlying technology is generally unknown. In the nonparametric approach taken in this study, a deterministic production frontier is assumed. Based on observed quantity data, the approximation technique uses the mathematical tools of convex analysis. The regularity conditions imposed on the unknown technologies facilitate the description of the technologies using closed, convex sets exhibiting free disposal. The true production possibilities sets in the convex and convex cone case, and the true upper level sets in the quasiconcave case are approximated by the convex hulls of observed data. Thus, our approach is essentially an elaboration of the original pioneering works of Farrell (1957) and Farrell and Fieldhouse (1962).

In the context of multiple output technologies and due to the complications brought by the assumption of free disposability which plays an important role in the theory of efficiency measurement, it is necessary to give a precise definition of technical efficiency and introduce the use of an efficiency direction vector specifying the goods with respect to which we would like to measure the inefficiency. The definition of technical efficiency used in this study is conditional on the efficiency director vector chosen. The violation index for a given technical efficiency test gives the equiproportionate increase in outputs and decrease in inputs indexed in the efficiency direction vector needed for an observation to satisfy the technical efficiency hypothesis. Input based and output based measures can be obtained by specifying the efficiency direction vector appropriately. The violation indices for the technical efficiency tests are equiproportionate loss measures; the required adjustment indicated by the violation index may move an inefficient production plan in the interior of the production possibilities set to a free disposal region at the boundary of the production possibilities set.

For the allocative efficiency tests, a profit function or a restricted profit function is posited to describe the economic objective of the firm. The formulation of the constrained optimization tests encompass a wide range of behavioral descriptions among which are full profit maximization, revenue maximization, cost minimization and variable profit maximization. A set of reference goods is specified to obtain the violation index which measures the value of goods in this set lost due to allocative inefficiency. With a consistent specification of the efficiency direction vector in the technical efficiency test, the violation index in the allocative efficiency test can be decomposed into components due to technical inefficiency and due to pure allocative inefficiency. A LeChatelier principle proposition for measures of allocative inefficiency is obtained which says that given a fixed reference set of goods, the violation index cannot decrease as the number of goods (or prices) with respect to which the producer can optimize increases. This proposition is consistent with the notion that the more goods a producer can freely vary, the higher profits can be. In the limiting cases of full profit maximization, the constrained optimization tests involving the solution of linear programming problems yield violation indices which can alternatively be calculated using revealed preference-type inequalities.

Appendix A lists the modified efficiency tests incorporating the no technological regress assumption. Computer programs for implementing the different efficiency tests and sample results using the Canadian input-output data (described in appendix B) are given in appendix C. The interpretation of the empirical results in appendix C is meant to illustrate the concepts discussed in part I and hence, the violation indices are interpreted as measures of inefficiencies. This interpretation may be considered more applicable to cross-section data on firms rather than the kind of time series data used. Like those of statistical tests, the results of the efficiency tests can be used for several purposes depending on the economic problem under study, and their interpretation can be subject to some element of arbitrariness depending on extraneous information possessed by the researcher. It must be noted that the efficiency tests are developed under joint assumptions on the technology and the optimizing behavior of the producers.

If we assume optimizing behavior on the part of the producers, the violation indices obtained from the proposed efficiency tests in part I can be reinterpreted as indicators of the shift in the production frontier. In cross section data on firms' production plans, the violation indices can be used as multifactor productivity measures with smaller violation indices indicating better productivity performance. With time series data, the violation indices as shown in part II can be interpreted as chained indices of technical progress or productivity growth. Since productivity improvements mean greater effectiveness in utilizing resources to produce output goods, the efficiency gain can have an impact on economic agents, both producers or owners of factors of production and consumers, in terms of higher real incomes and standards of living. Interest in productivity measurement therefore has practical significance.

In part II, an empirical comparison of nonparametric and parametric measures of technical progress for constant returns to scale technologies is performed. The Canadian input-output data for the period 1961–1980 are used. The data are aggregated into four sectors: resources, export market-oriented manufacturing, domestic market-oriented manufacturing, and services sectors. Each sector is assumed to have a single output technology with output from the other sectors entering the production process as intermediate inputs. A ten-good model is used for the resources sector and a nine-good model for each of the three other sectors. The data is described in appendix B. Measures of technical progress are obtained for each sector.

With the assumption of constant returns, the index of technical change is defined in terms of the unit scale profit function which gives the per unit return to the scaling or normalizing good. The nonparametric measures of technical change are based on the violation indices calculated using the unconstrained optimization test for a convex conical technology developed in part I. For each sector, the pattern of shift in the production frontier indicated by the nonparametric index of technical progress is consistent with that yielded by the Divisia and Fisher productivity change indices. The parametric measures of technical change are obtained through econometric profit function estimation using the generalized McFadden flexible functional form with a quadratic spline model for technical progress proposed by Diewert and Wales (1989b). The sectoral profit function estimation procedure and results, together with some interpretation of the price elasticities of output supply and input demand, are discussed in Appendix D.

The empirical results show that the parametric estimates of technical change display a much smoother behavior which can be attributed to the incorporation of stochastic disturbance terms in the estimation procedure. However, the parametric estimates of technical change do track the long term trend in the nonparametric estimates. Both nonparametric and parametric methods use the same information: price and quantity data. The nonparametric method has the advantage of not imposing any functional form in characterizing the production technology and is computationally much simpler. The nonparametric estimates in this study, obtained without introducing stochastic disturbance terms, do not possess statistical properties needed for hypothesis testing in the context of statistical inference.

In part III, we move to a more general view of the production sector of the domestic economy by linking the different sectors and explicitly modeling the constraints on factor endowments in a small open economy model. The constrained GNP maximization approach of Neary (1985) and the fundamental matrix equation of production theory as developed by Diewert and Woodland (1977) and Diewert (1982) are adapted to analyze the effects of factor price rigidities on resource allocation. We investigate in what sense the Tobin-Houthakker conjecture in consumer theory that says a reduction in the ration of one good will increase the consumption of unrationed substitutes and decrease the demand for unrationed complements hold in production. In a trade-theoretic approach to production theory, the analogous question is how do rigid factor prices affect other flexible factor prices. The comparative static responses of the endogenous variables — output levels, employment levels of fixed-price factors with elastic supply and flexible prices of domestic resources — to marginal changes in the economy's exogeneous variables — output prices, fixed factor prices and endowments of flexibly-priced domestic resources —

are examined. The effect of a change in a fixed factor price on flexible factor prices can be decomposed into a pure substitution term and a scale effect term. Hence, factor intensities which determine the scale effect term, in addition to substitution possibilities among factors, play a role in determining this response of the flexible factor prices. However a similar Tobin-Houthakker conjecture occurs when we look at the symmetry condition between fixed factor prices and flexible factor prices and their respective employment levels and endowments. In this interpretation, the essense of "substitutability" and "complementarity" differs from that as reflected in the price elasticities of output supply and input demand obtained in the parametric sectoral analysis in part II.

As an illustration, the model is used to analyze the effects of a devaluation in a two-sector small open economy with rigid wages and capital goods as specific factors. This exercise highlights the mechanism by which currency devaluation works as an employment stimulation policy tool. In the presence of wage rigidity, the devaluation leads to a more than proportional increase in returns to the sector-specific capital that makes possible the output and national income effects. The effectiveness of the devaluation as an employment policy tool depends on the wage elasticity of labor demand in the economy which is a function of both substitution terms and factor intensities at the sectoral level.

To conclude this overview, we note some major assumptions and limitations of the approaches taken in this research. The competitive market framework have been utilized in all three essays. The prices we use are assumed to be outside the control of the individual producers. In the empirical comparison of nonparametric and parametric measures of technical progress in part II and the trade-theoretic model used to derive the comparative static results in part III, the sectoral technologies are assumed to exhibit constant returns to scale. These simplifying assumptions enable us to ignore the complications due to the possibility of market power of firms and the necessity of modeling market structure, entry and exit of firms and demand conditions arising from the consumption side of the economy. Also, the models used are static, and dynamic considerations such as adjustment costs, price expectations and uncertainty, etc. are not taken into account. The thrust of this research is efficiency which is concerned with the allocation of resources among alternative uses. Economic policies and undertakings must ultimately address the satisfaction of human needs and wants and this can involve equity and distributional concerns. The efficiency measurement methods can be useful, however, in evaluating possible efficiency-equity tradeoffs.

Part I

Nonparametric Measures of Technical and Allocative Efficiency

Chapter 1

Introduction

Nonparametric linear programming tests for consistency with the hypothesis of productive efficiency at the firm level for the general case of multiple output-multiple input technologies are developed in this study. Given the technology, prices and a behavioral description of the firm as embodied in its economic objective function, productive efficiency[1] at the firm level is defined as utilizing the optimal combination of outputs and inputs given the constraints faced by the firm. We consider three general technologies — convex, constant returns to scale and quasiconcave technologies. Productive efficiency subsumes technical efficiency which we define as producing at some point at the boundary of the production possibilies set for the convex and convex conical technologies and of the upper level set for a quasiconcave technology. Violation indices as summary indicators of the distance of an inefficient observation from an efficient allocation are proposed. Alternatively, the violation indices can be interpreted as measures of economic loss due to inefficiency. As is the case with most nonparametric programming approaches to efficiency analysis, productive efficiency is measured in the following tests relative to the best-practice technology. Hence, in addition to the violation indices, we can recover efficient allocations corresponding to an inefficient one. This feature of the tests can be useful for prescriptive purposes.

This study draws on the earlier works in nonparametric production analysis in economics and data envelopment analysis (DEA) in operations research. The pioneering works in nonparametric production analysis were done by Farrell (1957) and Farrell and Fieldhouse (1962) and latter important contributions were made by Afriat (1972), Hanoch and Rothschild (1972),

[1]Productive efficiency is alternatively referred to as economic efficiency or allocative efficiency.

Diewert and Parkan (1983), Varian (1984a) and Färe, Grosskopf and Lovell (1985).[2] An introductory survey to data envelopment analysis can be found in Charnes and Cooper (1985) and Charnes, Cooper, Golony and Seiford (1985).

The major contributions of this study to the field of nonparametric efficiency measurement are the generalization of technical efficiency and allocative efficiency tests to handle the multiple output-multiple input technologies; the definition and consistent development, across technical efficiency and allocative efficiency tests, of violation indices as measures of inefficiency; and the formulation of constrained optimization tests applicable to cases where the producer is restricted to optimizing with respect to a subset of goods. The symmetric treatment of output goods and input goods facilitates the handling of multiple output-multiple input technologies. The resulting violation indices, being equiproportionate loss measures, are invariant to the units of measurement of goods. As such, the violation indices are in the spirit of Debreu's (1951) concept of coefficient of resource utilization whose inverse measures the economic loss due to inefficient utilization of resources. The consistent development of the violation indices allows us to obtain comparative measures of the degrees of technical inefficiency and pure allocative inefficiency. The constrained optimization tests, using price and quantity data, turn out to be the more general formulations for testing for overall productive efficiency; they yield the revealed preference-type inequalities commonly used as tests for consistency of observed data with profit maximizing or cost minimizing behavior as limiting cases.

Empirically, the efficiency tests developed can be used for several purposes depending on the economic problem at hand, and the prior beliefs of and subjective interpretation of the results by the researcher. Since the tests are developed under joint assumptions on the technology and the optimizing behavior of the producer, the tests can be used in efficiency studies of firms (as is the focus in DEA) or in testing for the regularity conditions of production theory (as is

[2] A closely related approach is the estimation of frontier production functions and frontier cost functions. A survey of this approach can be found in Forsund, Lovell and Schmidt (1980) and an example of estimating a frontier cost function is given by Kopp and Diewert (1982). The programming approach involves an optimization per observation while the regression approach uses a single optimization over all observations. The statistical approach has the advantage of being capable of modelling stochastic disturbances outside the control of the firm. We do not pursue the stochastic approach in this thesis.

the focus in nonparametric production analysis). In the latter, the tests can be used as a data exploratory tool prior to parametric estimation, and hence serve as a complementary tool to statistical or econometric analysis. It is possible that the pattern of the calculated violation indices can be explained by some "left-out" variable or factor not taken into consideration. Examples are technical progress in time series data, firm sizes in cross-section data or simply an unaccounted factor like cost incurred to satisfy environmental regulations.

In chapter 2, we describe the three general technologies relative to which the various efficiency tests are developed; we also lay out how convex sets are nonparametrically constructed from observed data to approximate the unknown technologies. In the next three chapters (3, 4 and 5), technical efficiency is given a precise definition and the technical efficiency hypothesis, test and violation index for each of the three technologies are described. Chapter 6 introduces the concept of allocative efficiency and illustrates the decomposition of the violation index into its technical inefficiency and pure allocative inefficiency components. Chapters 7 to 10 describe the allocative efficiency tests assuming constrained optimizing behavior, that is, when the producer can optimize only with respect to a subset of goods. Chapter 11 shows that for unconstrained optimization or complete profit maximization, the linear programming tests reduce to a comparison of inqualities. Chapter 12 illustrates how the previous tests can be modifed to account for technical progress. We conclude in chapter 13 with a summary and a mention of some limitations of the study.

Chapter 2

Three Alternative Nonparametric Specifications of Technology

2.1 Description of technologies

In this study, measures of firm productive inefficiencies are developed relative to the technology of the firm under consideration. We posit three kinds of general technologies: convex, constant returns to scale and quasiconcave technologies. We consider a technology with N goods described by its production possibilities set, say T, containing all the feasible $z \in R^N$ vectors attainable by the given technology. We treat output goods and input goods symmetrically; outputs are measured positively and inputs are measured negatively. With respect to inputs and outputs, we require the following (innocuous) assumptions for feasibility and boundedness:

1. $z \equiv (z_1, z_2, \ldots, z_N) \in T$ and $z_i > 0$ implies there exists at least one j such that $z_j < 0$, that is, the net production of a positive amount of an output always requires the net use of another good as an input, and
2. outputs are finite for all finite inputs.

The following regularity conditions are imposed on the different technologies.

- Conditions I (for a convex technology): T is a subset of R^N (the N-dimensional Euclidean space) and is
  - (i) closed;
  - (ii) exhibits free disposal: $z \in T$, $z' \leq z \Rightarrow z' \in T$; and
  - (iii) convex.

- Conditions *II* (for a convex constant returns to scale technology): T is a subset of R^N and is

  (i) closed,

  (ii) exhibits free disposal,

  (iii) convex, and is

  (iv) a cone: $z \in T$, $\lambda \geq 0 \Rightarrow \lambda z \in T$.

We generally describe the technology by means of a production possibilities set. In some instances, it is helpful to describe the technology in terms of a production function. We first single out a particular good, say good n. We then describe the technology by a function f^n such that

$$z_n = f^n(z_1, z_2, \ldots, z_{n-1}, z_{n+1}, \ldots, z_N) \equiv f^n(z^n)$$

where z^n is the z vector with the nth component dropped and $f^n(z^n)$ denotes the maximum amount of net output of good n that can be produced given z^n. The interpretation of f^n depends on whether z_n is an output or input:

1. If good n is an output, then f^n is a production function; that is, $f^n(z^n)$ gives the maximal amount of output n that the technology can produce given amounts z_i of other outputs to produce and amounts $-z_j$ of inputs available where i denotes an output subscript and j denotes an input subscript.[1]

2. If good n is an input, then f^n is a factor requirements function; that is, $f^n(z^n)$ gives the negative of the minimum amount of input n required to produce outputs z_i given that other inputs $-z_j$ are available.

The regularity conditions for a quasiconcave technology will be defined over the individual good technology functions f^n, $n = 1, 2, \ldots, N$.

[1] If the vector z^n is not consistent with any net output for good n, define $f^n(z^n) = -\infty$.

- Conditions III (for a quasiconcave technology): The function f^n is a real-valued function of $N-1$ variables defined over a nonempty, closed, convex subset of R^{N-1} and is

  (i) continuous from above,

  (ii) nonincreasing, and

  (iii) quasiconcave.

In conditions III, part (i) means that the upper level sets $L(z_n) \equiv \{z^n : f^n(z^n) \geq z_n\}$ are closed; part (ii) means that higher production of other outputs and/or decrease in availability of other inputs cannot increase the output of good n if $z_n > 0$, or decrease the input requirements for good n if $z_n < 0$; and part (iii) implies that the upper level set $L(z_n)$ is convex. Note that the given regularity conditions describe the technologies by means of convex sets that are closed and exhibit free disposal. We exploit these convexity properties of the technologies through the use of the mathematical tools of convex analysis to derive various efficiency tests. Since efficient points must lie on the boundary of the relevant convex set, the continuity property or equivalently the closure property of the convex sets ensure the recovered efficient points do belong to the pertinent technology set. The free disposal property offers a theoretical justification for the technical feasibility of inefficient observations.

Among the three general technologies, the quasiconcave technology is least restrictive; individual technology functions f^n can have nonconvexities exhibiting flats or increasing returns to scale. A convex technology must have diminishing or constant returns to scale; individual technology functions f^n corresponding to a convex technology must be concave. A constant returns to scale technology is the most restrictive; individual technology functions f^n must be positively linearly homogeneous and concave. Also, a convex constant returns to scale technology implies convexity of its production possibilities set, and the latter implies quasiconcavity of the individual technology functions $f^n, n = 1, 2, \ldots, N$. The differences among the three general technologies are illustrated in figure 2.1 for a single output (z_2)-single input (z_1) case where $z_2 = f^2(z_1)$ and f^2 is a production function (describing the technology with respect to output z_2), $z_2 > 0$, $z_1 < 0$.

The production possibilities set T, a subset of R^N, is convex if and only if $z' \in T$, $z'' \in T$ and $0 \leq \lambda \leq 1$ implies $\lambda z' + (1-\lambda)z'' \in T$. If, in addition, T is closed under nonnegative scalar multiplication, that is, $z \in T$, $\lambda \geq 0$ implies $\lambda z \in T$, then T is a convex cone and exhibits constant returns to scale. The individual technology function f^n is quasiconcave if and only if its upper level sets $L(z_n) \equiv \{z^n : f^n(z^n) \geq z_n\}$ are convex. Corresponding regularity conditions on the individual technology functions f^n, $n = 1, 2, \ldots, N$ can be derived when the production possibilities set is either a convex set or a convex cone. Thus the following conditions on f^n are equivalent to our old conditions I and II on the production possibilities set T.

- Conditions I' (for a convex technology): The function f^n is a real-valued function of $N-1$ variables which is defined over a nonempty, closed convex subset of R^{N-1} and is

  (i) continuous,

  (ii) nonincreasing, and

  (iii) concave.

- Conditions II' (for a convex constant returns to scale technology): The function f^n is a real-valued function of $N-1$ variables which is defined over a nonempty, closed convex subset of R^{N-1} (which is a cone) and is

  (i) continuous,

  (ii) nonincreasing,

  (iii) concave, and

  (iv) positively linearly homogeneous.

Concavity of f^n is a more restrictive condition than quasiconcavity. Concavity rules out interior flats and increasing returns in the technology function. The function f^n defined over a convex subset of R^{N-1}, say C, is concave if and only if $z^{n1} \in C$, $z^{n2} \in C$, $0 \leq \lambda \leq 1$ implies

$$f^n(\lambda z^{n1} + (1-\lambda)z^{n2}) \geq \lambda f^n(z^{n1}) + (1-\lambda)f^n(z^{n2}).$$

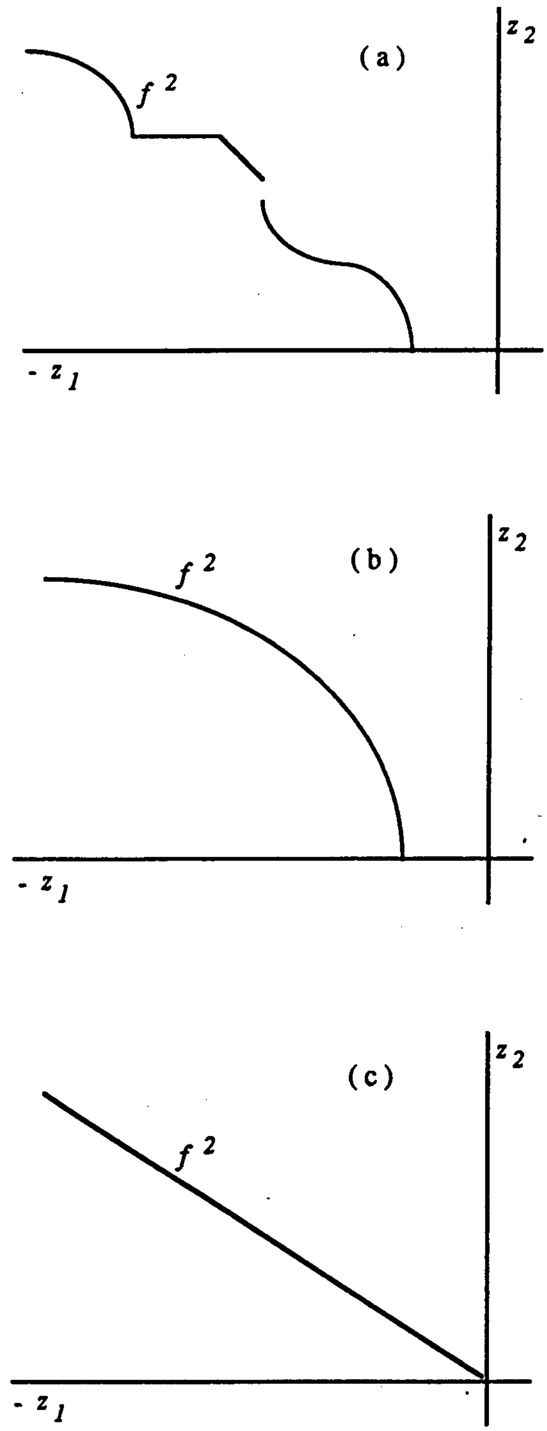

Figure 2.1: Illustration of the three classes of technologies in the single output-single input case: (a) quasiconcave technology, (b) convex technology, and (c) convex cone technology

Equivalently, the function f^n is concave if and only if its hypograph H^n defined by

$$H^n \equiv \{z : z \in R^N, f^n(z^n) \geq z_n\}$$

is convex. If , in addition, f^n is positively linearly homogeneous, that is, for every $z^n \in C$, $\lambda \geq 0$, we have $f^n(\lambda z^n) = \lambda f^n(\lambda z^n)$, then the hypograph of f^n is a convex cone. We have the following proposition relating conditions I' and II'' to conditions I and II, respectively.

- Proposition: The production possibilities set T satisfies conditions $I(II)$ if and only if for all goods n, $n = 1, 2, \ldots, N$, the individual technology functions f^n satisfy conditions I' (II''). Moreover, the hypographs H^n of f^n, $n = 1, 2, \ldots, N$ are identical and are given by T, that is, $H^n = T$ for all n, $n = 1, 2, \ldots, N$.

The regularity conditions on f^n given by conditions I' imply that the hypograph H^n of the function f^n is closed, exhibits free disposal and is convex. It is trivial to show that if $z \in T$, then $z \in H^n$; and if $z \in H^n$, then $z \in T$. Since this must hold for all goods n, then it follows that the hypographs H^n, $n = 1, 2, \ldots, N$ are all identical to the production possibilities set T described by conditions I. The first part of the proposition naturally follows. The equivalence of conditions II and II' for a constant returns to scale technology can be analogously shown.

In our subsequent discussion, we shall work mainly maintaining the assumptions of conditions I and II where the concept of a production possibilities set is used to describe the convex and constant returns to scale technologies; and with conditions III where the concept of a technology function is used to describe a quasiconcave technology. The preceding discussion will be helpful in showing that the asymmetric choice of a good in the multiple output case is equivalent to focusing on the particular production function that corresponds to that good.

2.2 Construction of approximating convex sets

The true technology is generally unknown and the information at hand is data on the amounts of the outputs produced and the inputs used in the production process. Suppose we have J observations; we denote by $z^1, z^2, \ldots, z^J$ the observed N-dimensional quantity vectors. The

J observations may pertain to a cross-section of production units whose underlying technology may be assumed relatively homogeneous or to a particular production unit at various time periods. A particular observation denoted by the vector $z^j \equiv (z_1^j, z_2^j, \ldots, z_N^j)$ has components $z_n^j > 0$ if good n is a net output, $z_n^j < 0$ if good n is a net input, and $z_n^j = 0$ if good n is neither produced nor used by the production unit j.

Given the quantity data $\{z^j : j = 1, 2, \ldots, J\}$ we approximate the relevant convex sets by constructing the free disposal convex hull of the observed data in a manner satisfying the given regularity conditions on the unknown technologies. For a convex technology satisfying conditions I, the production possibilities set T is approximated by the set $\hat{T}_1$ defined by

$$\hat{T}_1(z^1, z^2, \ldots, z^J) \equiv \{z : \textstyle\sum_{j=1}^J \lambda^j z^j \geq z, \sum_{j=1}^J \lambda^j = 1 , \lambda^j \geq 0, j = 1, 2, \ldots, J\}. \tag{2.1}$$

For a constant returns to scale technology, we construct the free disposal conic convex hull of the observed data to approximate the unknown production possibilities set T satisfying conditions II. In this case, the approximating convex set $\hat{T}_2$ will be

$$\hat{T}_2(z^1, z^2, \ldots, z^J) \equiv \{z : \textstyle\sum_{j=1}^J \lambda^j z^j \geq z, \lambda^j \geq 0, j = 1, 2, \ldots, J\}. \tag{2.2}$$

Since a convex cone is closed under addition and nonnegative scalar multiplication, the restriction $\sum_{j=1}^J \lambda^j = 1$ is dropped in (2.2). Figure 2.2 illustrates the construction of the N-dimensional free disposal convex hulls for the two-good case ($N = 2$) under the assumptions of conditions I and conditions II. Let the first good be an output and the second be an input.

For a quasiconcave technology, it is necessary to single out one good to play an asymmetric role. The $N-1$ dimensional convex set approximation to the upper level set corresponding to a particular level of good n, say z_n, is constructed in the following manner. Define the index set

$$I(z_n) \equiv \{j : z_n^j \geq z_n, j = 1, 2, \ldots, J\} \tag{2.3}$$

containing the observation indices whose values of good n is at least as large as z_n. If $I(z_n)$ is not empty, then the free disposal convex set approximation to $L(z_n) \equiv \{z^n : f^n(z^n) \geq z_n\}$ is

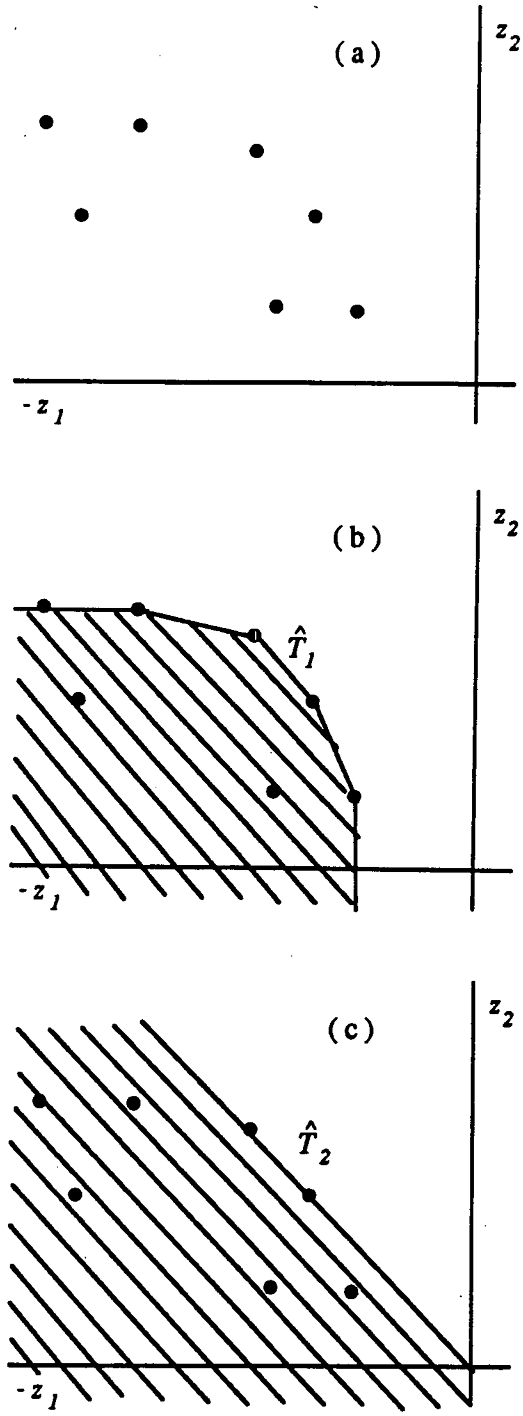

Figure 2.2: Construction of free disposal convex hulls under the assumptions of a convex technology and a constant returns to scale technology: (a) observed production plans, (b) free disposal convex hull, and (c) free disposal conic convex hull

given by $\hat{L}(z_n)$ defined as follows:

$$\hat{L}(z_n; z^1, z^2, \ldots, z^J) \equiv \{z^n : \sum_{j \in I_i^n} \lambda^j z^{nj} \geq z^n, \sum_{j \in I_i^n} \lambda^j = 1, \lambda^j \geq 0, j \in I(z_n)\} \qquad (2.4)$$

where z^{nj} is the vector z^j without its nth component, and z^n is a z vector without its nth component; that is, z^n is an $N-1$ dimensional vector. If the index set given in (2.3) has only one element, say $j*$, then the corresponding convex set $\hat{L}(z_n)$ defined in (2.4) reduces to

$$\hat{L}(z_n) = \{z^n : z^{nj*} \geq z^n\}$$

which is an $N-1$ dimensional orthant with the origin translated to z^{nj*}; the set $\hat{L}(z_n)$ is closed, convex and exhibits free disposal. If the index set $I(z_n)$ in (2.3) is empty, then the set $\hat{L}(z_n)$ is empty; trivially, the empty set is closed and convex.

Figure 2.3 illustrates the construction of the sets $\hat{L}(z_n)$ for a three good ($N = 3$) technology. Let the first good be an output ($z_1 \geq 0$) and the second and third goods be inputs ($z_2 \leq 0, z_3 \leq 0$). We construct upper level sets for the output good whose levels are denoted beside the observed points. For output level $z_1 = 9$, the constructed level set $\hat{L}(9)$ is the set of all input combinations (z_2, z_3) northeast of the only observed production plan with output $z_1 = 9$. By definition, the approximating level set for output level $z_1 = 8$ is also $\hat{L}(9)$ though no production plan is observed to produce an output level $z_1 = 8$. Since the quasiconcave technology function is assumed to be nonincreasing, then any input combination which can yield an output level of $z_1 = 9$ can also produce a lower output level of $z_1 = 8$. In this case, the production function f^1 would display a flat region. The constructed level set corresponding to an output level of $z_1 = 5$ is denoted by $\hat{L}(5)$. Note that the level sets corresponding to output levels $z_1 \geq 5$ are contained in $\hat{L}(5)$, that is, $\hat{L}(z_1) \subseteq \hat{L}(5)$ for $z_1 \geq 5$.

The sets $\hat{T}_1(z^1, z^2, \ldots, z^J)$ and $\hat{T}_2(z^1, z^2, \ldots, z^J)$ defined in (2.1) and (2.2), and $\hat{L}(z_n; z^1, z^2, \ldots, z^J)$ defined by (2.3) and (2.4) are, by construction, closed, convex and exhibit free disposal. Hence, these constructed sets also satisfy the regularity conditions we imposed on the unknown technologies. The sets have the additional properties of being polyhedral since they are generated by a finite number of points and of being the smallest convex sets containing the

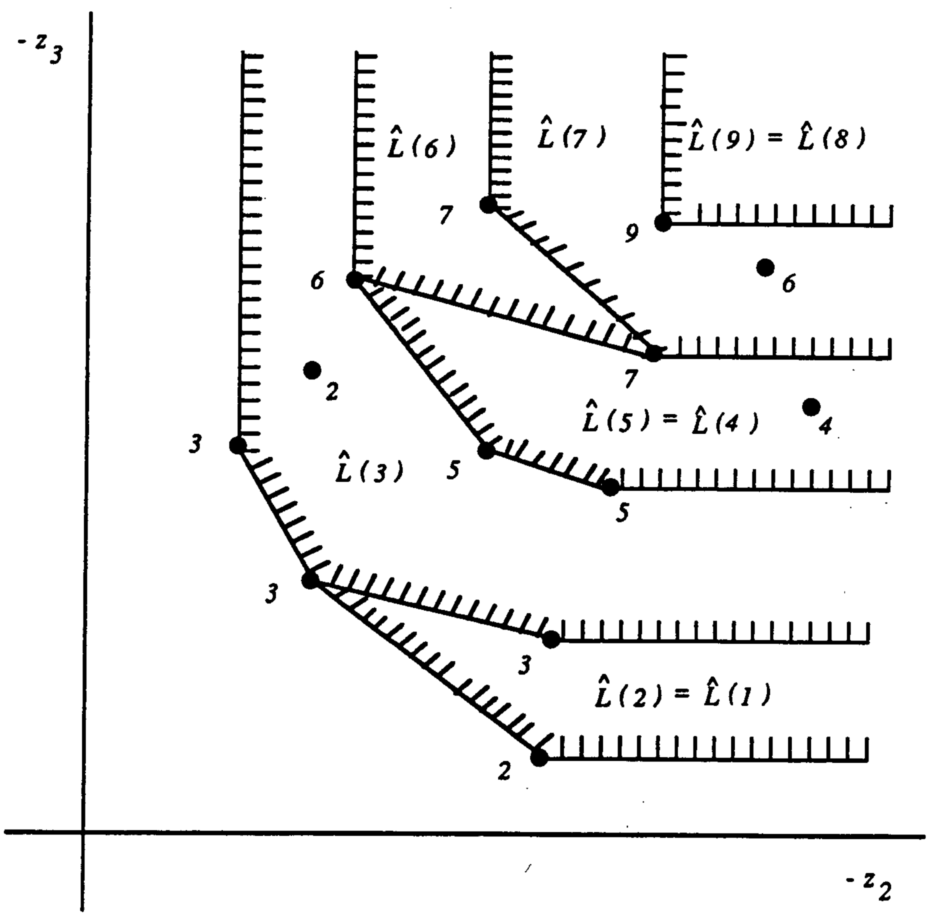

Figure 2.3: Construction of approximating level sets for a quasiconcave technology

observed data and satisfying their respective regularity conditions. Therefore, the boundaries of the constructed production possiblities set in the convex case, and of the upper level set in the quasiconcave case are piecewise linear. This implies that our approximation assumes a constant marginal rate of substitution or transformation of goods between observed (data) points on the boundary of the sets. The degree of substitution or transformation between goods may then be underestimated; a greater number of data points or observations on the frontier will yield a closer approximation to the actual degree of substitution. Since inefficiencies will be measured in terms of the distance of an observed point relative to a point on the boundary of the set, our methodology will err more on the side of underestimating the true value of inefficiency and towards acceptance of the efficiency hypothesis.

Chapter 3

The Measurement of Technical Inefficiency for a Convex Technology

3.1 Definition of technical efficiency

The concept of technical efficiency is based on maximizing the production of output goods given a vector of input goods or minimizing the levels of input use given target output levels. For a convex technology, the technically efficient allocations must lie on the boundary of its production possibilities set. The free disposal assumption lends a complication on how an observed allocation z^j lying on the boundary of the constructed convex hull $\hat{T}_1$ will be deemed technically efficient. This problem is addressed in the following discussion.

Consider figure 3.4 showing the constructed convex hull of some observed points generated by a single output (z_2)-single input (z_1) technology. An interior point, like c, has an open neighborhood around it contained in $\hat{T}_1$ and there are an infinite number of directions and paths to adjust the allocation c to the boundary of the constructed production possibilities set $\hat{T}_1$. Clearly, the point c is technically inefficient since more output z_2 could have been produced given the amount of input $-z_1$, or a lower level of input $(-z_1)$ could have been used to produce the given level of output z_2. There are numerous ways of measuring the degree of inefficiency of the allocation c. What is desired is an indicator of the relative distance of an inefficient allocation to a point on the boundary of the production possibilities set.

For the point c, we can measure the inefficiency by the relative amount of input wasted and hence, we can compare c with the point c^1. The ratio cc^1/cf gives the proportional decrease in input use needed for the efficient production of the given level of output cg. Alternatively, we can compare c with the point c^2 which gives the maximal amount of output that could be produced given the input level og. The corresponding inefficiency measure is c^2c/c^2g which gives

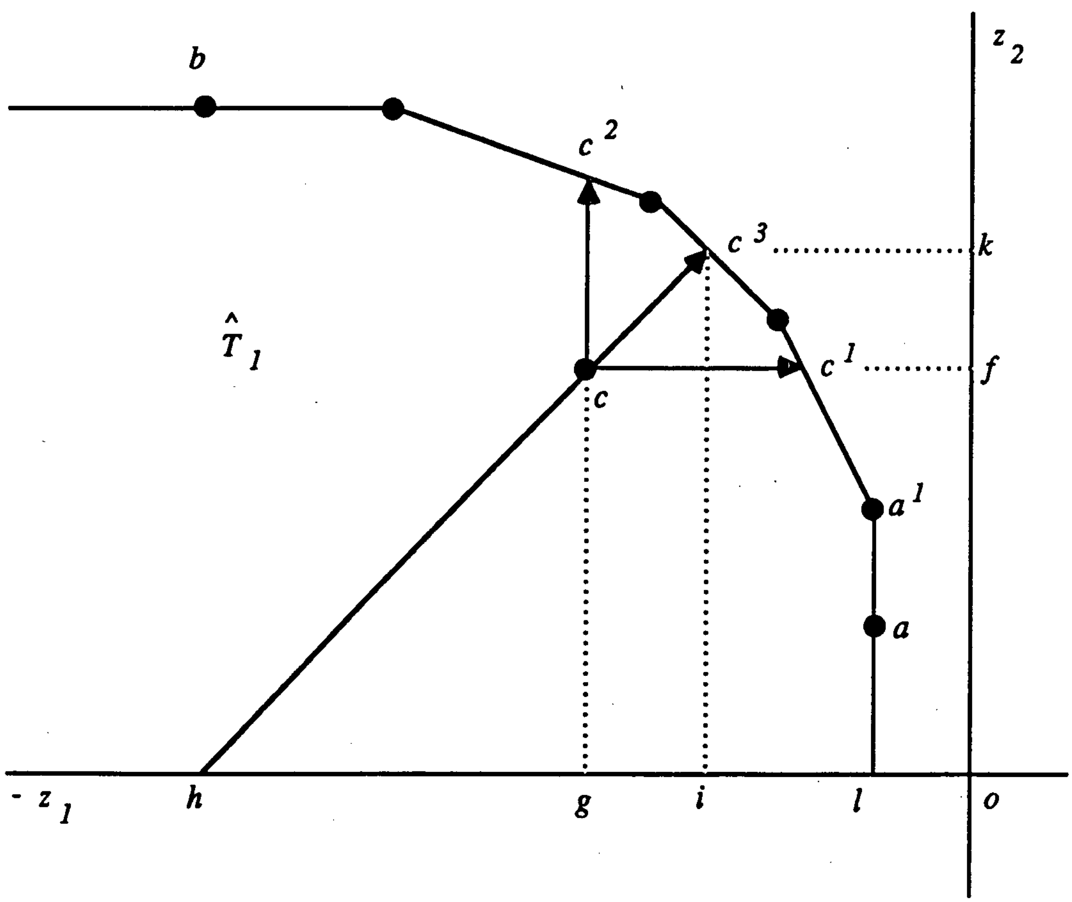

Figure 3.4: Convex hull of observations from a single output-single input technology, an example

the proportional increase in output needed for the efficient use of the given amount of input og. It is also possible to move the point c to the boundary through a simultaneous adjustment in output and input levels; the point c can be compared with c^3. In this case, an aggregate measure of the degree of adjustment in both output and input needed to make the allocation c technically efficient is obtained as follows.

We ask what proportional reduction in input utilization along with a proportional increase of the same magnitude in output is required to move the point c onto the efficient frontier. In order to obtain this measure of the inefficiency of observation c graphically, let hg equal the distance og along the input axis. Now draw a straight line joining h to c and extend this line until it hits the frontier at c^3. The inefficiency measure in this case is $ig/hg = kf/fo$. Note that this *composite* inefficiency measure is smaller than our earlier *input based* measure of inefficiency $c^1c/cf = c^1c/hg$ (since ig is less than c^1c), and it is also smaller than our earlier *output based* measure of inefficiency $c^2c/cg = c^2c/fo$ (since kf is less than c^2c). However, all three measures of inefficiency have the very important property of being invariant to scale changes in the units of measurement.

In the one output-one input case, the above three measures are the only ones we consider. Unfortunately, the three alternative measures can yield conflicting results. Consider the observed points a and b which lie on the free disposal sections of the boundary of the constructed production possibilities set. The production plan a uses minimal input for its given level of output but is output-inefficient; more output could have been produced using the given input level. If we set the direction for efficiency adjustment along the input axis, the inefficiency measure would be zero. If we set the adjustment direction in terms of the output good, we obtain a positive inefficiency loss measure $a^1a/al > 0$. There is no allocation strictly northeast of the point a, and so, if we measure inefficiency by specifying simultaneous positive adjustments in the input-output levels we obtain zero inefficiency. The production plan b which is output-efficient but input-inefficient can be analogously analyzed.

The foregoing discussion suggests that technical efficiency can only be defined conditional on the direction of efficiency adjustment desired. Let E be the subset of goods with respect to which we want to measure inefficiency. Define an N-dimensional vector γ such that its nonzero entries correspond to the goods in E. Corresponding to each good n, $n = 1, 2, \ldots, N$, we set γ_n equal to $0, -1$ or $+1$ in the following manner: if good n is an output, choose $\gamma_n = 0$ or $+1$; if good n is an input, choose $\gamma_n = 0$ or -1; if good n is sometimes a net output and sometimes a net input, choose $\gamma_n = 0$. Hereafter, we call the vector γ as an efficiency direction vector. Following are some examples which illustrate how this vector may be specified.

- case (i): to measure inefficiency by the amount of output n lost due to inefficiency or which could have been produced if the firm were efficient: set $\gamma_n = 1, \gamma_k = 0$ for $k \neq n$.

- case (ii): to measure inefficiency by the amount of input n wasted due to inefficiency or which could have been saved if the firm were efficient: set $\gamma_n = -1, \gamma_k = 0$ for $k \neq n$.

- case (iii): to measure inefficiency with respect to all outputs: set $\gamma_n = 1$ if good n is an output, and $\gamma_n = 0$ otherwise. This case may be applicable when we want to test whether a firm is producing maximal output, or when we test later for overall productive efficiency using price information as well and the firm's behavior can be characterized as revenue-maximizing.

- case (iv): to measure inefficiency with respect to all inputs: set $\gamma_n = -1$ if good n is an input, and $\gamma_n = 0$ otherwise. This case may be applicable when we want to test whether a firm, given its output levels, is using minimal inputs or when the firm's behavior can be characterized as cost-minimizing. The waste measure obtained in this case corresponds to Debreu's (1951) concept of coefficient of resource utilization.

- case (v): to measure inefficiency with respect to variable goods; firms may have some fixed output levels or fixed factors: set $\gamma_n = 1$ if good n is a variable output, $\gamma_n = -1$ if good n is a variable input, and $\gamma_n = 0$ if good n is a fixed output or input.

Let us now formally define technical efficiency for a production plan z. Suppose the production plan z is generated by a convex technology described by a production possibilities set T

satisfying conditions I. We define the production plan z to be weakly technically efficient for the technology set T if and only if z is a boundary point of T, that is, z belongs to the frontier of T and is not an interior point.

We also require a definition of technical efficiency with respect to the goods in the set E; we term this technical E-efficiency. The production plan z is technically E-efficient for T if and only if

1. $z \in T$, and
2. there is no $z' \in T$ such that $z' \equiv (z'_1, z'_2, \ldots, z'_N) \geq z \equiv (z_1, z_2, \ldots, z_N)$ and $z'_n > z_n$ for all $n \in E$.

Hence, by definition, the production plan z is technically E-efficient (or technically efficient conditional on the efficiency direction vector γ) if and only if there exists no alternative production plan $z \in T$ that is a strict Pareto improvement[1] to z with respect to the goods in set E. Note that if z is technically E-efficient for T, then z cannot be an interior point of T, and hence, the production plan z is, by necessity, weakly technically efficient.

3.2 Test for technical efficiency (test 1)

An objective of this study is to develop empirical measures of inefficiency. An indicator of the distance of an observed production plan relative to an efficient allocation on the boundary of the unknown production possibilities set is desired. Given the quantity data $\{z^j : j = 1, 2, \ldots, J\}$ we construct the convex hull of the data points to approximate the true technology set under the assumption that they were generated by an underlying technology satisfying conditions I. If an observation z^j is technically E-efficient, then by our definition, no strictly Pareto improving adjustments in the goods contained in the set E is feasible. If we let δ_j^* denote the violation index or loss measure for observation j, then we would like δ_j^* to equal zero in this case.

[1]The strong Pareto criterion is used for technical efficiency for a convex technology because the boundary of the constructed production possibilities set has free disposal regions; strict monotonicity does not hold globally at its boundary.

A technical efficiency test must be devised such that an observation z^i, given the data $\{z^j : j = 1, 2, \ldots, J\}$ and an efficiency direction vector γ, is uniquely determined to be either efficient or inefficient, but not both. We lay out our technical efficiency hypothesis for a convex technology and the proposed test for consistency of the data $\{z^j : j = 1, 2, \ldots, J\}$ with the hypothesis. In addition, a measure of inefficiency for observation i , denoted by δ_i^*, is proposed. As a violation index, δ_i^* is zero if observation i is consistent with the hypothesis, and positive, otherwise. As a measure of the degree of deviation of observation i from the maintained hypothesis, δ_i^* gives the magnitude of the proportional adjustment in the goods in the set E needed to make the observation i efficient; the necessary adjustment to the vector z^i is given by the $\delta_i^* \hat{\gamma} z^i$, where $\hat{\gamma}$ is the chosen efficiency direction vector diagonalized into an NxN matrix with the elements of γ running down the main diagonal and having zeroes elsewhere. We now formally state our first efficiency hypothesis as follows.

- Technical Efficiency Hypothesis I (for a convex technology): The data $\{z^j : j = 1, 2, \ldots, J\}$ are generated by an underlying technology satisfying conditions I and are technically E-efficient.

In order to determine whether a given set of data $\{z^j : j = 1, 2, \ldots, J\}$ is consistent with the technical efficiency hypothesis I, it turns out to be necessary to choose the efficiency direction vector γ so that the following condition is satisfied:

$$\gamma_n z_n^j > 0 \text{ for } n \in E, j = 1, 2, \ldots, J. \tag{3.1}$$

This condition implies that the choice of goods n belonging to the set E is restricted to goods consistently either a net output or a net input across all observations. If the data $\{z^j : j = 1, 2, \ldots, J\}$ and the efficiency direction vector γ satisfy condition (3.1), then the technical efficiency hypothesis I can be tested by solving J linear programming problems contained in Test 1 below.

- Test 1

1. For each observation i, $i = 1, 2, \ldots, J$, solve the following linear programming subproblem i:

$$\max_{\delta_i \geq 0, \lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{\delta_i : \textstyle\sum_{j=1}^{J} \lambda^j z^j \geq z^i + \delta_i \hat{\gamma} z^i, \sum_{j=1}^{J} \lambda^j = 1\} \equiv \delta_i^* \tag{3.2}$$

   where $\hat{\gamma}$ is the efficiency direction vector γ diagonalized into a matrix. The informational requirements to solve the linear programming problem (3.2) are the quantity data $\{z^j : j = 1, 2, \ldots, J\}$ and the efficiency direction vector γ.

2. Consider the following consistency condition: Suppose δ_i^* is the optimized objective function for the ith subproblem (3.2), $i = 1, \ldots, J$. If condition (3.1) holds and $\delta_i^* = 0$ for all i, $i = 1, \ldots, J$, then the data $\{z^j : j = 1, 2, \ldots, J\}$ are consistent with technical efficiency hypothesis I for a convex technology. If the condition (3.1) holds and $\delta_i^* > 0$ for some i, then observation z^i violates technical efficiency hypothesis I and the data $\{z^j : j = 1, 2, \ldots, J\}$ are not consistent with this hypothesis.

Caution is warranted in interpreting a violation of the hypothesis. Note that the linear programming test is a joint test for the regularity conditions on the unknown technology and technical efficiency. There are two major areas of possible use of the efficiency tests: efficiency studies of production units and testing for regularity conditions on production functions.[2] Researchers in the first area can interpret a violation as being due to technical inefficiency, i.e., a failure to produce maximal output and/or to use minimal inputs. On the other hand, researchers in the second area often assume that a convex technology exists, and so a violation may be interpreted as a failure of curvature conditions on the technology.

In the formulation of the linear programming problem (3.2), the search for a point on the boundary of $\hat{T}_1$ is restricted to Pareto improvements on z^i through the terms $\gamma_n z_n^i \geq 0$, $n = 1, 2, \ldots, N$ and $\delta_i \geq 0$. The set of feasible Pareto improving allocations, including those in the weak and strong Pareto sense, is restricted to points z^* such that z^* is on the boundary

[2]Allocative efficiency tests, later discussed, are more pertinent to testing for regularity conditions on profit and cost functions which assume optimizing behavior on the part of the producers.

of $\hat{T}_1$ and

$$z^* = z^i + \hat{r}\hat{\gamma}z^i \geq z^i \tag{3.3}$$

for some $r \geq 0_N$ and $\hat{r}$ is the r vector diagonalized into a matrix. For each $(z^*; z^i, \gamma)$, we define a distance indicator function as

$$\delta_i(z^*; z^i, \gamma) \equiv \min_n\{r_n : n \in E, z^* = z^i + \hat{r}\hat{\gamma}z^i, r \geq 0_N\} \geq 0. \tag{3.4}$$

There remains the problem of choosing a particular z^* among the set of feasible (strong and weak) Pareto improving production plans with respect to the goods in E. For example, in figure 3.4 we can see that if γ has no zero components, there are an infinite number of allocations on the boundary of $\hat{T}_1$, including c^1 and c^2, that are northeast of c. With our definition of technical E-efficiency, we would like δ_i to be positive, or equivalently, $r_n > 0$ for all $n \in E$, if ever there exists a feasible production plan z^* which is a strict Pareto improvement with respect to the goods in E. A search over all feasible z^* can be done such that we obtain δ_i^* defined by

$$\delta_i^* \equiv \max_{z^*}\{\delta_i(z^*; z^i, \gamma) : z^* \text{ is on the boundary of } \hat{T}_1 \text{ and } z^* = z^i + \hat{r}\hat{\gamma}z^i \text{ for some } r \geq 0_N\}. \tag{3.5}$$

Hence, $\delta_i^* > 0$ if the production plan z^i, conditional on γ, is technically inefficient, and $\delta_i^* = 0$ otherwise.

Since z^* belongs to $\hat{T}_1$, we can express z^* as $z^* = \sum_{j=1}^J \lambda^j z^j$ for some $\lambda^1 \geq 0, \ldots, \lambda^J \geq 0$ such that $\sum_{j=1}^J \lambda^j = 1$, and from (3.3) and (3.4), we have $z^* \geq z^i + \delta_i\hat{\gamma}z^i$. Therefore, the feasibility region in terms of $\lambda \equiv (\lambda^1, \lambda^2, \ldots, \lambda^J)^T$ can be expressed as

$$\{\lambda : \textstyle\sum_{j=1}^J \lambda^j z^j \geq z^i + \delta_i\hat{\gamma}z^i, \sum_{j=1}^J \lambda^j = 1, \lambda \geq 0_J\}. \tag{3.6}$$

To ensure z^* is on the boundary of $\hat{T}_1$, the linear programming formulation (3.2) jointly determines z^* and δ_i^*, or equivalently, λ^* and δ_i^*. If $z^* = \sum_{j=1}^J \lambda^{j*} z^j$ is not on the boundary of $\hat{T}_1$, then δ_i^* is not maximal since z^* would be in the interior of $\hat{T}_1$ and there would always exist for any γ a strictly Pareto-improving allocation. Hence, at the optimal solution to the subproblem (3.2), the allocation $z^* = \sum_{j=1}^J \lambda^{j*} z^j$ has to be at the boundary of $\hat{T}_1$.

To recapitulate, the optimized value δ_i^* serves both as an indicator of the consistency of a particular observation with the technical efficiency hypothesis and of the distance of the observed z^i from the boundary of its hypothesized production possibilities set given an efficiency direction. As a violation index, the magnitude of δ_i^* is a measure of the degree of departure of z^i from the maintained hypothesis. If δ_i^* is positive, the violation can be due either to inappropriate assumptions on the underlying technology or to pure inefficiency in the sense of failure to produce maximal outputs and utilize minimal inputs. In production efficiency studies where more weight is placed on the second factor, δ_i^* can be interpreted as a loss measure giving the equiproportionate amount of a specified bundle of goods wasted due to technical inefficiency. Since δ_i^* measures proportional changes, it is invariant to scale in the measurement of goods.

3.3 Some notes on the technical efficiency test

3.3.1 Specifying the efficiency direction vector γ

Two extreme examples of specifying the efficiency direction vector γ for the N-good model are discussed. First, let the vector γ have zero components except for a good n. Then the efficiency test reduces to the question: Under the regularity conditions for a convex technology, is z_n maximal for z^n? If good n is an output, technical E-efficiency requires that output z_n be the maximal amount of good n that the firm can produce given it must produce at least as much other outputs and has available inputs at levels $(z_1, z_2, \ldots, z_{n-1}, z_{n+1}, \ldots, z_N)$. If good n is an input, then technical E-efficiency requires the $-z_n$ to be the minimal amount of input n required to produce at output levels and other input availability indicated by $(z_1, z_2, \ldots, z_{n-1}, z_{n+1}, \ldots, z_N)$. In the one output ($y$)-one input ($x$) case, let $(z_1, z_2) = (y, -x)$. If $\gamma = (1, 0)^T$, then technical efficiency requires that output level y be maximal for x; if more output can be produced given x, then $(z_1, z_2) = (y, -x)$ is technically E-inefficient (or technically inefficient conditional on γ). If $\gamma = (0, -1)^T$, then technical efficiency requires x to be the minimal (z_2 be maximal) amount of input required to produce y; if y could have been produced with a lesser amount of input, then

$(z_1, z_2) = (y, -x)$ is technically E-inefficient.

At the other extreme, we can let γ have no zero components. In this case, the linear programming subproblem (3.2) reduces to testing whether z^i is on the boundary of $\hat{T}_1$. If $\delta_i^* > 0$, then z^i is in the interior of $\hat{T}_1$. Otherwise, if $\delta_i^* = 0$, then the production plan z^i is weakly technically efficient.

Whether an observation i is deemed technically efficient or not generally depends on the efficiency direction vector γ. If z^i is an interior point of $\hat{T}_1$, then z^i will always be judged to be technically inefficient independently of γ. If z^i is on the boundary of $\hat{T}_1$ and does not lie on any free disposal region of the boundary, then z^i will always be technically efficient independently of γ. If z^i is on the boundary of $\hat{T}_1$ but lies on a free disposal region at the boundary, then detecting a violation at observation i will be sensitive to the specification of the efficiency direction vector γ. The magnitude of δ_i^* will generally differ with γ since the latter determines the efficient point on the boundary of $\hat{T}_1$ with which z^i is compared. The ranking of observations i, $i = 1, 2, \ldots, J$ by δ_i^* will generally vary from one γ specification to another.

3.3.2 Complications due to the free disposability assumption

Solution of the linear programming problem (3.2) yields two possibly different technically E-efficient production plans relative to z^i; let us denote by z^* and z^{**} the allocations $z^* \equiv z^i + \delta_i^* \hat{\gamma} z^i$ and $z^{**} \equiv \sum_{j=1}^{J} \lambda^{j*} z^j$. If observation i is technically E-efficient (or technically efficient given the corresponding efficiency direction vector γ), then $\delta_i^* = 0$ and $z^* \equiv z^i + \delta_i^* \hat{\gamma} z^i = z^i$. With the inequality constraint in the linear programming problem (3.2), the vector $z^{**} \equiv \sum_{j=1}^{J} \lambda^{j*} z^j$ may be different from $z^* = z^i$. Since at the optimal solution, at least one of the constraints corresponding to the goods $n \in E$ must be binding (otherwise, δ_i^* is not maximal), then $z_n^{**} = z_n^* = z_n^i$ for at least a good $n \in E$. Hence, z^{**} does not offer z^i a strictly Pareto improving adjustment in the direction of γ and the minimum proportional adjustment needed for z^i to attain z^{**} is still given by $\delta_i^* = 0$. The divergence between z^{**} and z^* is a sufficient indicator that $z^* = z^i$ is in the free disposal region of some good n not necessarily in E. It is only sufficient because the solution of the linear programming problem (3.2) does not guarantee that $z^{**} = \sum_{j=1}^{J} \lambda^{j*} z^j$ be on the relative interior of and away from any free disposal region at the boundary of $\hat{T}_1$, and z^{**} may equal z^*. The free disposability occurs in the good n where $\sum_{j=1}^{J} \lambda^{j*} z_n^j > z_n^i$. The production plan z^{**}, in itself, is technically E-efficient.

Suppose observation i is technically E-inefficient, that is, $\delta_i^* > 0$. If we look at the subspace of $\hat{T}_1$ generated by conditioning on the values of z_n^i, $n \notin E$, then z^i will be an interior point in this subspace.[3] Hence, there will always exist an equiproportionate adjustment vector, given by $\delta_i^* \mid z_n^i \mid$ where $\delta_i^* > 0$ and $n \in E$, that will bring the observed point z^i to the boundary of this subspace. The point on the boundary of this subspace attained by the equiproportionate adjustment in the goods $n \in E$ is $z^* \equiv z^i + \delta_i^* \hat{\gamma} z^i$. The vector z^* is technically E-efficient and the relative distance of z^i is indicated by δ_i^*. The procedure of obtaining δ_i^* by maximizing the minimum of ratios $r_n \equiv (z_n^* - z_n^i)/ \mid z_n^i \mid$, $n \in E$ yields $\delta_i^* = r_n$ for all $n \in E$ and the technically efficient point generated is z^*. The optimized value δ_i^* can then be interpreted as an equiproportionate waste measure; it gives the equiproportionate increase in outputs and decrease in inputs belonging to E which could have been achieved if the firm were technically E-efficient.

Again, the point $z^{**} \equiv \sum_{j=1}^{J} \lambda^{j*} z^j$ may differ from $z^* \equiv z^i + \delta_i^* \hat{\gamma} z^i$ if z^* is in a free disposal region at the boundary of $\hat{T}_1$ for some good n not necessarily in E. The production plan z^{**} yields a strict Pareto improvement on z^i for the goods in E and is itself technically E-efficient. In contrast to z^*, the allocation z^{**} can be attained from z^i by adjustment in the goods $n \in E$ that is not necessarily equiproportionate. We can recover the adjustment ratios $r_n^{**} \equiv (z_n^{**} - z_n^i)/ \mid z_n^i \mid$, $n = 1, 2, \ldots, N$. The lower bound for the adjustment ratios of the goods in E is given by δ_i^*; hence, δ_i^* gives the minimum proportional increase in outputs and decrease in inputs belonging to E required for z^i to be technically E-efficient. If there is variation in r_n^{**}, the ratios can be indicative of sources and varying degrees of inefficiencies in the output-input mix in the production plan z^i. It must be cautioned though that the test procedure we have

[3] If the set E contains all N goods, then z^i lying on the free disposal region with respect to all N dimensions of $\hat{T}_1$ is equivalent to z^i being an interior point of $\hat{T}_1$.

does not guarantee that z^{**}, as well as z^*, not be in any of the free disposal regions of the boundary of $\hat{T}_1$, and $z^{**} = z^*$ is possible.

3.3.3 An area of further research

An avenue of further research is suggested by our analysis. The definition of technical efficiency can be weakened such that a production plan z^i is defined to be technically efficient conditional on the efficiency direction vector γ if and only if there exist no production plan $z' \in T$ that is a weak Pareto improvement on z^i with respect to the goods in E.[4] The linear programming problem (3.2) has to be reformulated such that the point on the boundary $z^{**} \equiv \sum_{j=1}^{J} \lambda^{j*} z^j$ does not lie on any free disposal region with respect to the goods in E. Thus, if z^i lies on a free disposal region at the boundary of $\hat{T}_1$ with respect to at least one good in E, then z^i given γ will be deemed technically inefficient. If all N goods are in E, such a reformulation of the test will pick any allocation in a free disposal region, whether in the interior of $\hat{T}_1$ or at the boundary, to be technically inefficient.

The suggested modifications may require redefinition of the violation index δ_i^* and the reformulation of the linear programming problem (3.2) may have to explicitly incorporate some form of maximizing the slack variables. If the search for a boundary point can ensure that with respect to the goods in E it does not lie on a free disposal region, then the divergence between $z^{**} \equiv \sum_{j=1}^{J} \lambda^{j*} z^j$ and the observed z^i may be useful in identifying and measuring varying degrees of input and output inefficiencies. In the field of operations research, a related approach termed data envelopment analysis or DEA (see, for example, Charnes and Cooper (1985) and Charnes, Cooper, Golany and Seiford (1985)) attempts to address the problem of correcting for free disposability at the boundary by the maximization of slack variables. Though the economic theory underlying their approach, for example in describing the technology they are analyzing, does not seem to be as rigorous as that found in the nonparametric analysis literature in economics, some insights may be gained from this body of work.

[4] We may still want to retain the concept of conditionality on the vector γ to handle fixed goods.

3.4 Dual interpretation

The dual to the linear programming problem (3.2) is

$$\min_{q \geq 0_N, \mu_i}\{-z^{iT}q + \mu_i : z^{iT}\hat{\gamma}q \geq 1, -z^{jT}q + \mu_i \geq 0, j = 1, 2, \ldots, J\}. \tag{3.7}$$

If we interpret the dual variables $q \geq 0_N$ as a vector of nonnegative prices, then $z^{iT}q$ is the profit obtained for the production plan z^i at prices q. Suppose $q^* \geq 0_N$ and μ_i^* solve problem (3.7). By the duality theorem in linear programming $\delta_i^* = -z^{iT}q^* + \mu_i^*$. If observation i satisfies our technical efficiency hypothesis, then $\delta_i^* = -z^{iT}q^* + \mu_i^* = 0$. Together with the constraints in problem (3.7), this implies

$$\mu_i^* = z^{iT}q^* \geq z^{jT}q^*, \; j = 1, 2, \ldots, J \tag{3.8}$$

and

$$z^{iT}\hat{\gamma}q^* \geq 1. \tag{3.9}$$

Hence, the observed quantity vector z^i solves the profit maximization problem $\pi(q^*)$ defined as

$$\pi(q^*) \equiv \max_z\{q^{*T}z : z \in \hat{T}_1\} \equiv \mu_i^* \tag{3.10}$$

where μ_i^* (unrestricted in sign) is the optimal profit at at prices q^* given the technological constraints. This condition is equivalent to z^i being a boundary point of $\hat{T}_1$. The price normalization (3.9) and assumption (3.1) imply that there exists at least one $n \in E$ such that $q_n^* > 0$, that is, at least one good n in E is not at free disposal. The above analysis derives from the supporting hyperplane theorem for convex sets.

A dual definition of technical efficiency for a convex technology is obtained.

- Definition. Given a convex technology set T satisfying conditions I, a production plan $z \in T$ is technically efficient relative to an efficiency direction vector γ or technically E-efficient where E is the set of coordinate axis indices corresponding to the nonzero components of γ, if and only if, there exists a semipositive price vector $q^* > 0_N$ such that $q_n^* > 0$ for at least one good n belonging to set E , and at prices q^*, the production plan z is a profit maximal choice.

The optimal price vector $q^* > 0_N$ can be related to Koopmans' efficiency prices. If γ has no zero component then a production plan z is technically efficient if and only if z is at the boundary of the production possiblities set. As Koopmans (1951, p.462) has noted, "the price concept established does not in any way presuppose the existence of a market or of exchanges of commodities between different owners". The technical efficiency prices q^* may differ greatly from actual market prices of the goods.

Suppose a violation of the technical efficiency hypothesis is detected at observation i, that is, $\delta_i^* = -z^{iT}q^* + \mu_i^* > 0$. Then at the optimal solution $q^* > 0_N$, μ_i^* unrestricted in sign, we have

$$\begin{aligned} \mu_i^* &> z^{iT}q^*; && (3.11)\\ \mu_i^* &\geq z^{jT}q^*,\ j = 1, 2, \ldots, J; \text{ and} && (3.12)\\ z^{iT}\hat{\gamma}q^* &= 1. && (3.13) \end{aligned}$$

Equation (3.13) follows from the complementary slackness condition in linear programming; since $\delta_i^* > 0$, then the constraint $z^{iT}\hat{\gamma}q \geq 1$ must be binding at q^*. Since one of the J inequalities in (3.12) must hold with strict equality, inequalities (3.11) and (3.12) imply

$$\mu_i^* = \max_j\{z^{jT}q^* : j = 1, 2, \ldots, J\} > z^{iT}q^*. \tag{3.14}$$

Then, the violation index δ_i^* can be expressed as

$$\begin{aligned} \delta_i^* &= (\max_j\{z^{jT}q^* : j = 1, 2, \ldots, J\} - z^{iT}q^*)/z^{iT}\hat{\gamma}q^* \\ &= \max_j\left\{\frac{(z^j - z^i)^T q^*}{z^{iT}\hat{\gamma}q^*} : j = 1, 2, \ldots, J\right\} && (3.15)\\ &> 0. \end{aligned}$$

Hence, the violation index δ_i^* in its dual interpretation measures the distance of the observation i from some profit maximal choice, say z^{j*}, $j* \in \{1, 2, \ldots, J\}$ in terms of the shortfall from the optimal profit μ_i^* with the goods in E as the reference goods. The profit maximal choice z^{j*} is a function of prices q^* and μ_i^* (dual to z^* and δ_i^*) jointly determined by the linear programming problem (3.7).

Also, by the complementary slackness condition in linear programming, it follows that

$$\lambda^{j*}(-z^{jT}q^* + \mu_i^*) = 0 \text{ for } j = 1, 2, \ldots, J. \tag{3.16}$$

If $\mu_i^* > z^{jT}q^*$ for an observation j at the optimal solution for subproblem i, then the corresponding dual variable must be zero, that is, $\lambda^{j*} = 0$. In this case, the particular observation j carries zero weight in the optimal convex combination $\sum_{j=1}^{J} \lambda^{j*} z^j$ which is a point on the boundary of $\hat{T}_1$ relative to which the current observation z^i is being compared. By the same reasoning, if $\delta_i^* > 0$ and $\lambda^{j*} > 0$ for an observation j at the optimal solution for subproblem i, then $z^{jT}q^* = \mu_i^*$ and observation j is relatively efficient to z^i and, if assumption (3.1) holds, is itself technically E-efficient. In the DEA literature, the observations j with corresponding $\lambda^{j*}s$ positive are termed "evaluators" of observation i. It is possible to obtain $\delta_i^* = 0$ and $\lambda^{j*} > 0$ for some $j \neq i$ and $z^j \neq z^i$; in this instance, observation i is likely to be in a free disposal region at the boundary of the production possibilities set and $q_n^* = 0$ for at least a good n. Nevertheless, the observations z^i and $z^j s$ with positive $\lambda^{j*}s$ must lie on the same supporting hyperplane with efficiency prices given by q^*, or equivalently, in DEA terminology, along the same facet of the convex hull.

By looking at the dual formulation, we can also infer why the efficiency test yields measures only of relative efficiency. At the optimal solution of the dual formulation (3.7), at least one of the constraints $-z^{jT}q^* + \mu_i^* \geq 0$, $j = 1, 2, \ldots, J$ must be binding for the objective function to be at a minimum. If the constraint is binding for some observation k, then $\mu_i^* = z^{kT}q^*$ and there exist prices $q^* > 0_N$ with $q_n^* > 0$ for at least one good n belonging to E such that z^k is a profit maximal production plan. By the dual definition of technical efficiency, observation k is technically E-efficient. With the assumption $\gamma_n z_n^k > 0$ for all $n \in E$, then the optimized value δ_k^* will be zero when the subproblem (3.2) is solved for observation k.[5] The result of the test at observation i establishes the possible existence of a price vector q^* which can support z^k as

[5]Note that $\delta_k^* = -z^{kT}q^* + \mu_k^*$ is homogeneous of degree one in q^* and μ_k^*. Hence, if at observation i the optimal price vector is $q_i^* > 0_N$ and $z^{kT}\hat{\gamma}q_i^* \geq 1$, then with the assumption that $\gamma_n z_n^k > 0$ for all $n \in E$, a price vector $q_k^* > 0_N$ (and μ_k^*), which is a scalar multiple of q_i^* (and μ_i^*), exist such that $z^{kT}\hat{\gamma}q_k^* \geq 1$ and δ_k^* remains zero.

an efficient allocation. Hence, given the data set $\{z^j : j = 1, 2, \ldots, J\}$, at least one observation will be rated technically efficient, that is, $\delta_j^* = 0$ for at least one j, $j = 1, 2, \ldots, J$. Intuitively, the above result follows from the fact that the convex hull of the observed data points is being used to approximate the production possibilites set. As such, inferences about efficiency are relative to the "best-practice" technology.

The primal formulation (3.2) of the efficiency test utilizes the concept of an "internal representation" of a convex set (see, for example, Rockafeller (1970)). Using Carathéodory's theorem, the points of a convex set can be expressed as a convex combination of a subset of points in the set. On the other hand, the dual formulation (3.7) uses the concept of an "external representation" of a convex set as the intersection of some collection of half-spaces. For the (polyhedral) convex hull of a set of points, only a finite number of points and a finite number of half-spaces is required. Whereas the primal formulation works on quantity space (through λ), the dual formulation works on the price space. If the linear programming subproblems are solved for the whole data set $\{z^i : i = 1, 2, \ldots, J\}$, then the primal formulation implicitly constructs an "inner approximation" or "inner bound" (Bazaraa and Shetty (1979, p.60), Diewert and Parkan (1983, p.139), Varian (1984b, p.60)) to the true convex production possibilities set. In the dual formulation, the observations which pass the test and their corresponding hyperplanes trace an "outer approximation" or "outer bound" to the unknown convex production possibilities set. The "inner approximation" and "outer approximation" generally differ but coincide at the observed points lying on the boundary of either constructed sets.

3.5 Special case: assuming concavity of an individual technology function

In some instances, the focus of interest is the technology with respect to a particular good. As mentioned earlier in the description of various technologies, the technology function f^n for a singled-out good n can be interpreted either as a production function if good n is an output, or as a factor requirement function if good n is an input. Assuming f^n is continuous, nonincreasing and concave, then technical efficiency requires z_n to be the maximal amount of net output of good n which can be produced given z^n, that is $z_n = f^n(z^n)$. If good n is an input, then technical efficiency requires $-z_n$ to be the minimal amount of input n required to produce outputs given availability of other inputs at levels indicated by $z^n \equiv (z_1, z_2, \ldots, z_{n-1}, z_{n+1}, \ldots, z_N)$.

The technical efficiency hypothesis for a concave technology function f^n and the corresponding efficiency test follow.

- Technical Efficiency Hypothesis I' (for a concave technology function f^n): The data $\{z^j : j = 1, 2, \ldots, J\}$ are generated by an underlying technology satisfying conditions I and are technically E-efficient where $E = \{n\}$ contains only the nth coordinate axis index.

The first part of the hypothesis implies that the technology function f^n satisfies conditions I'; the second part implies $z_n^j = f^n(z^{nj})$, $j = 1, 2, \ldots, J$. Note that the hypothesis assumes that the underlying production possibilites set is convex. To test the technical efficiency hypothesis I', perform test 1 using the linear programming formulation (3.2) with the efficiency direction vector specification: $\gamma_n = 1$ or -1, accordingly, if good n is an output or input; $\gamma_k = 0$ for $k \neq n$. If $\delta_i^* = 0$ for $i = 1, 2, \ldots, J$, then the data are consistent with the technical efficiency hypothesis I'.

As illustrated in figure 3.4, detecting a violation at a particular observation can be sensitive to the choice of the good n. If $E = \{1\}$, then the production plan a is technically efficient and b technically inefficient. If $E = \{2\}$, then we obtain opposite results. In some sense, both production plans are inefficient because more output could have been produced at a and less inputs could have been used at b. Both production plans a and b are at the boundary of the constructed production possibilites set but in the free disposal regions. Therefore, to test for simultaneous maximality of outputs and minimality of inputs, test 1 has to be performed for $n = 1, 2, \ldots, N$ with $E = \{n\}$ at a time. Observations which pass the N test then satisfy the DEA definition of efficiency (Charnes and Cooper, 1985, p.72):

"100 % efficiency is attained for any DMU (decision-making unit) only when

(a) none of its outputs can be increased without either

(i) increasing one or more of its inputs or

(ii) decreasing some of its outputs.

(b) none of its inputs can be decreased without either

(i) decreasing some of its outputs or

(ii) increasing some of its other inputs."

Next it is shown that the formulation of test 1 is a generalized version of the Diewert-Parkan (1983, p.141) efficiency test for a concave technology function f^n in a multiple output-multiple input context. The Diewert-Parkan test where a good n is asymmetrically chosen and using our notation is :

(i) For $i = 1, 2, \ldots, J$, solve the linear programming subproblem i:

$$\max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{\textstyle\sum_{j=1}^J \lambda^j z_n^j : \sum_{j=1}^J \lambda^j z^{nj} \geq z^{ni}, \sum_{j=1}^J \lambda^j = 1\} \equiv \check{f}^n(z^i). \tag{3.17}$$

(ii) If $z_n^i = \check{f}^n(z^i)$ for $i = 1, 2, \ldots, J$, then the data $\{z^j : j = 1, 2, \ldots, J\}$ are consistent with the technical efficiency hypothesis I' for some f^n satisfying conditions I'. If $z_n^i < \check{f}^n(z^i)$ for some i, then the data is not consistent with the technical efficiency hypothesis I' for any f^n satisfying conditions I'.

(iii) The violation index can be defined as $\Delta^i \equiv (\check{f}^n(z^i) - z_n^i) / \mid z_n^i \mid \geq 0$ which gives the proportional increase in z_n^i needed to put z^i on the efficient frontier.

Note that the Diewert-Parkan formulation (3.17) implicitly assumes that the underlying production possibilities set is convex. The linear programming subproblem (3.2) for the case $E = \{n\}$ can be rewritten as

$$\max_{\delta_i \geq 0, \lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{\delta_i : \textstyle\sum_{j=1}^J \lambda^j z^{nj} \geq z^{ni}, \sum_{j=1}^J \lambda^j z_n^j \geq z_n^i + \delta_i \gamma_n z_n^i,$$
$$\textstyle\sum_{j=1}^J \lambda^j = 1\} \equiv \delta_i^*. \tag{3.18}$$

At the optimal solution for (3.18), the constraint $\sum_{j=1}^J \lambda^{j*} z_n^j \geq z_n^i + \delta_i^* \gamma_n z_n^i$ has to be binding; otherwise, a higher value for δ_i^* can still be obtained. In the case of a single good being asymmetrically chosen, let us redefine δ_i as

$$\delta_i \equiv \frac{\sum_{j=1}^{J} \lambda^j z_n^j - z_n^i}{\gamma_n z_n^i} \tag{3.19}$$

and rewrite (3.18) as

$$\delta_i^* = \max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \left\{ \frac{\sum_{j=1}^{J} \lambda^j z_n^j - z_n^i}{\gamma_n z_n^i} : \sum_{j=1}^{J} \lambda^j z^{nj} \geq z^{ni}, \sum_{j=1}^{J} \lambda^j = 1 \right\}. \tag{3.20}$$

Since δ_i is a monotonic function of $\sum_{j=1}^{J} \lambda^j z_n^j$ for a fixed z_n^i, problem (3.18) reduces to

$$\max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{ \sum_{j=1}^{J} \lambda^j z_n^j : \sum_{j=1}^{J} \lambda^j z^{nj} \geq z^{ni}, \sum_{j=1}^{J} \lambda^j = 1 \} \tag{3.21}$$

which is the Diewert-Parkan linear programming subproblem (3.17). From definition (3.19), it is easy to see that at the optimal solution for the problems (3.17) and (3.18), our consistency conditions ($\delta_i^* = 0$ implies efficiency, $\delta_i^* > 0$ implies inefficiency) are equivalent to parts (ii) and (iii) of the Diewert-Parkan test with $\Delta^i = \delta_i^*$.

If the data $\{z^j : j = 1, 2, \ldots, J\}$ pass the technical efficiency test for a concave technology function f^n, then a nonparametric (piecewise linear) concave function f^{n*} satisfying conditions I' and defined by

$$f^{n*}(z^n) \equiv \max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{ \sum_{j=1}^{J} \lambda^j z_n^j : \sum_{j=1}^{J} \lambda^j z^{nj} \geq z^n, \sum_{j=1}^{J} \lambda^j = 1 \} \tag{3.22}$$

can be recovered. The function f^{n*} defined by (3.22) gives the boundary of the convex production possibilities set $\hat{T}_1$ along the nth dimension.

Chapter 4

The Measurement of Technical Inefficiency for a Convex Conical Technology

4.1 Definition of and test for technical efficiency (test 2)

A comparison of the regularity conditions I and II on the production possibilities sets shows that a constant returns to scale (CRS) technology is a special case of a convex technology. As such, the definition of technical efficiency for a convex technology must necessarily hold in the CRS case. A production plan z is technically E-efficient if and only if there exists no alternative plan $z' \in T$, where T is the production possibilities set, that is a strict Pareto improvement to z with respect to the goods in set E. Again, it is necessary to choose the goods in set E such that they all have nonzero quantity values, or equivalently, to specify the efficiency direction vector such that

$$\gamma_n z_n^j > 0 \text{ for } n \in E, j = 1, 2, \ldots, J. \tag{4.1}$$

If the data $\{z^j : j = 1, 2, \ldots, J\}$ and the efficiency direction vector γ satisfy condition (4.1), then the following technical efficiency hypothesis for a convex constant returns to scale technology can be tested using test 2 given below.

- Technical Efficiency Hypothesis II (for a convex conical technology): The data $\{z^j : j = 1, 2, \ldots, J\}$ are generated by an underlying technology satisfying conditions II and are technically E-efficient.

To test the above hypothesis, given the data $\{z^j : j = 1, 2, \ldots, J\}$ and the efficiency direction vector γ corresponding to set E, test 2 which involves solving J linear programming subproblems is performed.

- Test 2

1. For each observation i, $i = 1, 2, \ldots, J$, solve the following linear programming subproblem i:

$$\max_{\delta_i \geq 0, \lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{\delta_i : \textstyle\sum_{j=1}^{J} \lambda^j z^j \geq z^i + \delta_i \hat{\gamma} z^i\} \equiv \delta_i^* \tag{4.2}$$

where $\hat{\gamma}$ is the efficiency direction vector γ diagonalized into a matrix.

2. Consider the following consistency condition: If condition (4.1) holds and $\delta_i^* = 0$ for all i, $i = 1, \ldots, J$, then the data $\{z^j : j = 1, 2, \ldots, J\}$ are consistent with the technical efficiency hypothesis II for a convex constant returns to scale technology. If condition (4.1) holds and $\delta_i^* > 0$ for some i, then observation z^i violates technical efficiency hypothesis II and the data $\{z^j : j = 1, 2, \ldots, J\}$ are not consistent with this hypothesis.

Test 2 is more restrictive than test 1; the regularity conditions II require the production possibilities set to be a cone in addition to being convex. Geometrically, the technically efficient points for a CRS technology must lie along rays at the boundary of the production possibilities set and furthermore, these rays must pass through the origin. Given the same γ, an observation may pass the convexity test but fail the CRS test. An observation which passes test 2 must necessarily pass test 1. The constructed production possibilities set for a convex technology $\hat{T}_1$ is contained in the set $\hat{T}_2$ constructed under the assumption of a convex CRS technology. Since $\hat{T}_1 \subseteq \hat{T}_2$, as can be seen in figure 2.2, then it is possible that points on the boundary of $\hat{T}_1$ lie in the interior of $\hat{T}_2$. Hence, the violation index δ_i^* for observation i obtained with test 2 cannot be less than the value of δ_i^* obtained with test 1, given the same efficiency direction vector γ. We can expect at least as many observations violating the technical efficiency hypothesis of test 2 as against that of test 1.

The foregoing discussion suggests an indirect way to test for returns to scale, at least, the possible existence of decreasing returns as opposed to constant returns. Given a γ configuration, tests 1 and 2 for a convex technology and a CRS technology, respectively, can be performed. If the violation indices δ_i^* significantly increase going from test 1 to test 2, then the data can be interpreted as being more consistent with the convex technology assumption. How large an

observation no. (1)	(z_1, z_2, z_3) (2)	δ_i^* when $\gamma = (-1, 0, 0)^T$ (3)	δ_i^* when $\gamma = (-1, -1, 0)^T$ (4)	δ_i^* when $\gamma = (-1, -1, 1)^T$ (5)
1	(-1,-4,1)	0.000000	0.000000	0.000000
2	(-2,-2,1)	0.000000	0.000000	0.000000
3	(-4,-1,1)	0.000000	0.000000	0.000000
4	(-5,-1,1)	0.200000	0.000000	0.000000
5	(-3,-2,1)	0.333333	0.142857	0.076923
6	(-2,-3,1)	0.250000	0.142857	0.076923
7	(-9,-2,1)	0.777778	0.500000	0.333333

Table 4.1: Two input-constant output example for test 2

increase in δ_i^* is indicative of a departure from the CRS assumption is left, at this stage, to the subjective judgment of the researcher.

In cases where the number of goods $N \geq 3$, it is possible that at the optimal solution for subproblem (4.2), the points $\sum_{j=1}^{J} \lambda^{j*} z^j$ and $(z^i + \delta_i^* \gamma z^i)$ are different. This result can be similarly interpreted in terms of the complications brought about by the free disposability assumption as in the convex case. In this instance, the point $(z^i + \delta_i^* \hat{\gamma} z^i)$ lies on a free disposal region at the boundary of the conical convex hull of the observed data $\{z^j : j = 1, 2, \ldots, J\}$. To illustrate this possibility, we perform test 2 for a two-input (z_1,z_2), constant output (z_3) example taken from Chang and Guh (1988). The data and results of test 2 for different γ configurations are listed in table 4.1. Figure 4.5 illustrates the data; in three-dimensional space, the boundary of the conical convex hull of the observed data would consist of the points along the rays drawn from the origin and passing through the points at the boundary of the level set for $z_3 = 1$.

From column 5 of table 4.1, the results show that the observations which are at the boundary of the conic convex hull of the observed data are 1, 2, 3 and 4. The same observations are at the boundary of the level set corresponding to $z_3 = 1$ as verified by the results in column 4 and shown in figure 4.5. However, the test at $\gamma = (-1, -1, 0)^T$ indicates that observation 4 is at best weakly efficient; this is shown by the linear programming results for this particular observation. At $\gamma = (-1, -1, 0)^T$ and $i = 4$, the optimal solution of subproblem (4.2) yields $\delta_4^* = 0.000000$

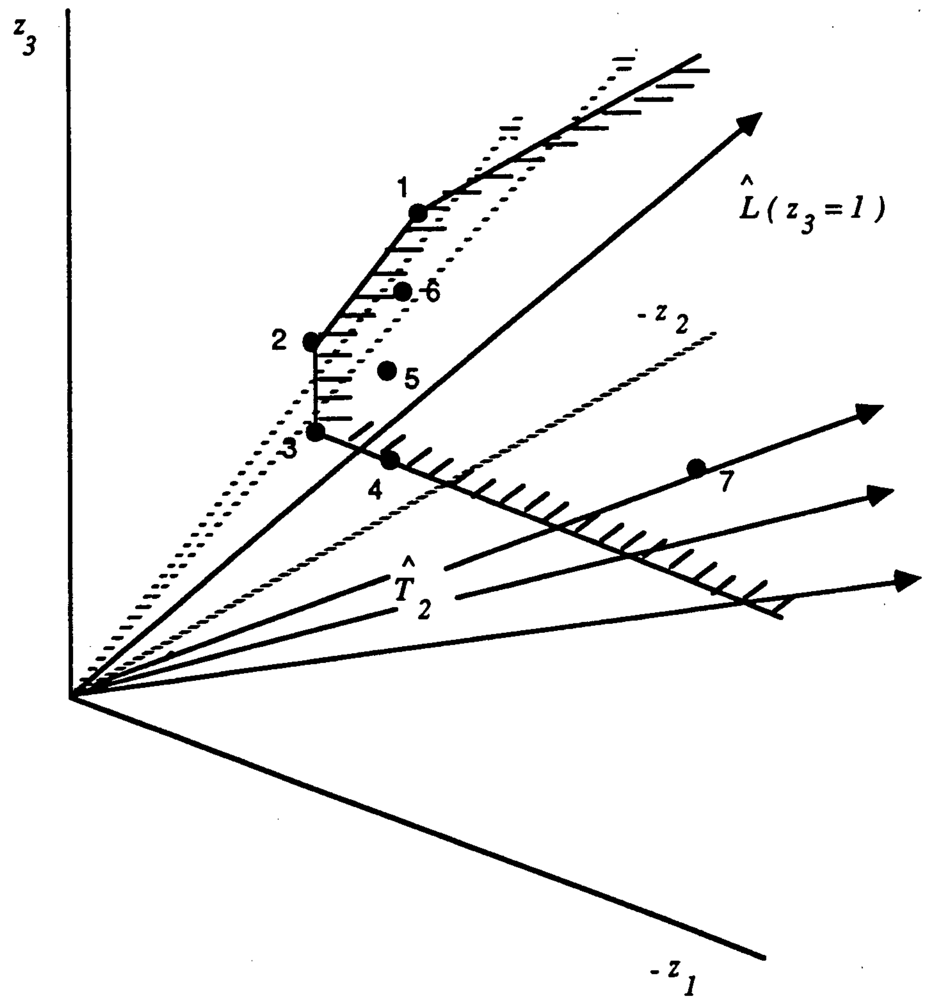

Figure 4.5: The conic convex hull for a two input-constant output example

and $\lambda^{3*} > 0$ with $\lambda^{j*} = 0$ for $j \neq 3$; hence, $z^* = z^i + \delta_i^* \hat{\gamma} z^i = z^4$ but $z^{**} = \sum_{j=1}^{J} \lambda^{j*} z^j = z^3$. Note that observation 3, not 4, enters the optimal basis for observation 4, as indicated by a positive λ^{3*}; the difference between $z^* = z^i + \delta_i^* \hat{\gamma} z^i$ and $z^{**} = \sum_{j=1}^{J} \lambda^{j*} z^j$ indicates that free disposability occurs with respect to good 1. If test 2 is performed with $\gamma = (-1, 0, 0)^T$, then we ask the question whether, assuming a CRS technology, input 1 is minimal given output level is at least at z_3 and $-z_2$ of input 2 is available. The earlier results on the weak efficiency of observation 4 are supported by the results of test 2 with $\gamma = (-1, 0, 0)^T$. If observation 4 is an interior point of the conic convex hull of the observed data, detecting a violation at observation 4 will be invariant to the specification of the efficiency direction vector γ.

4.2 Dual interpretation

The dual formulation to the linear programming subproblem (4.2) is given by

$$\min_{q \geq 0_N} \{-z^{iT} q : z^{iT} \hat{\gamma} q \geq 1, -z^{jT} q \geq 0, j = 1, 2, \ldots, J\} \tag{4.3}$$

and consists of a price normalization constraint and J homogeneous linear inequalities. Hence, the "outer approximation" to the production possibilities set is also a polyhedral convex cone. The dual variables in (4.3) can be similarly interpreted as in the convex case given by (3.7). Under the regularity conditions for a constant returns to scale technology, maximal profit μ_i^* is restricted to zero. A dual definition of technical efficiency for a CRS technology is obtained.

- Definition. Given a convex conical technology set T satifying conditions II, a production plan $z \in T$ is technically efficient relative to an efficiency direction vector γ or technically E-efficient where E is the set of coordinate axis indices corresponding to the nonzero components of γ, if and only if, there exists a semipositive price vector $q^* > 0_N$ such that $q_n^* > 0$ for at least one good n belonging to set E , and at prices q^*, the production plan z yields zero profit.

Observations which pass the CRS efficiency test can be interpreted as operating at a technically optimal scale in the sense that they have exploited possible increasing returns and decreasing returns in the technology. Even if the underlying technology is everywhere convex, we can interpret the observations passing test 2 as consistent with scale efficiency in the context of "most productive scale size" of Banker (1984); that is, observations that fail test 2, and hence are "scale-inefficient", can move to an efficient point by either a more than proportionate increase in output when inputs are increased, or a more than proportionate decrease in inputs when outputs are decreased. Alternatively, scale efficiency can be viewed as consistent with the zero-profit long-run competitive equilibrium and hence, is socially desirable (Färe, Grosskopf and Lovell, 1985) if some price mechanism can be implemented such that the optimal "efficiency" prices can prevail.

If an observation i fails test 2, that is, $\delta_i^* = -z^{iT}q^* > 0$, then the production plan z^i yields a loss at prices q^*. The following relationships hold as well:

$$0 > z^{iT}q^*; \tag{4.4}$$

$$0 \geq z^{jT}q^*, \; j = 1, 2, \ldots, J; \text{ and} \tag{4.5}$$

$$z^{iT}\hat{\gamma}q^* = 1. \tag{4.6}$$

Since at least one of the J inequalities in (4.5) must hold with strict equality, equations (4.4) and (4.6) imply

$$0 = \max_j\{z^{jT}q^* : j = 1, 2, \ldots, J\} > z^{iT}q^* = -\delta_i^*. \tag{4.7}$$

Then the violation index δ_i^* can be expressed as

$$\delta_i^* = \frac{0 - z^{iT}q^*}{z^{iT}\hat{\gamma}q^*} > 0. \tag{4.8}$$

In its dual interpretation, the violation index δ_i^* measures the distance of the observed z^i from the zero-profit allocation at prices q^* in terms of the loss incurred at z^i with the goods in E as reference goods. The relationship of the value of the current production plan $z^{iT}q^*$, the value of the basket of reference goods $z^{iT}\hat{\gamma}q^*$, and the violation index δ_i^* is more apparent in the following rearrangement of equation (4.8):

$$z^{iT}q^* + \delta_i^*(z^{iT}\hat{\gamma}q^*) = 0. \tag{4.9}$$

If γ is specified such that δ_i^* is an output based measure, then $z^{iT}\hat{\gamma}q^* = 1$ is the implicit revenue obtained with the production plan z^i if prices were q^*. Equation (4.8) in this case would reduce to

$$\delta_i^* = \frac{\text{“cost”}}{\text{“revenue”}} - 1 > 0 \tag{4.10}$$

where the cost and revenue concepts are only implicit in the sense that q^* are efficiency prices and not necessarily prevailing market prices. Equation (4.10) says δ_i^* is the excess of cost over revenue measured as a proportion of revenue. Alternatively, from (4.9), we can say that δ_i^* gives the equiproportionate increase in outputs needed to attain zero profit. For an input based δ_i^*, we can interpret $z^{iT}\hat{\gamma}q^* = 1$ as the cost incurred with production plan z^i if prices were q^*. In this case, equation (4.8) reduces to

$$\delta_i^* = \frac{\text{“cost”-“revenue”}}{\text{“cost”}} > 0 \tag{4.11}$$

which gives the excess of cost over revenue as a proportion of cost. Equivalently, an input based δ_i^* gives the equiproportionate reduction in inputs needed to attain zero profit. The question arises whether the equiproportionate adjustment $\delta_i^*\hat{\gamma}z^i$ is technically feasible. The answer is yes; the adjustment moves the observation z^i to a boundary point which necessarily belongs to the approximating convex cone. As discussed earlier, the obtained allocation $(z^i + \delta_i^*\hat{\gamma}z^i)$ may lie on a free disposal region at the boundary of the constructed production possibilities set.

At the optimal solution of subproblem i and if assumption (4.1) holds, the evaluators of observation i, which themselves are technically E-efficient, can be identified by positive $\lambda^{j*}s$. When an output variable is zero at z^i, it is possible to obtain optimal $\lambda^{j*}s$ to be all zero for observation i implying the shutdown point is the relatively efficient allocation given the efficiency direction vector γ. Since the free disposal conic convex hull of the observed data is being used to approximate the unknown technology, then given the data set $\{z^j : j = 1, 2, \ldots, J\}$, at least one observation j will be rated technically E-efficient with $\delta_i^* = 0$.

4.3 Special case: assuming linear homogeneity of an individual technology function

As in the convex technology case, corresponding conditions on the individual technology functions f^n, $n = 1, 2, \ldots, N$ when the production possibilities set is described by a convex cone satisfying conditions II can be obtained. The regularity conditions imposed on f^n are given by conditions II'; that is, the individual technology function f^n is continuous, nonincreasing, concave and positively linearly homogeneous. Since the convex conical technology is a special case of a convex technology, the definition and test for technical E-efficiency are analogous. Technical efficiency for a linearly homogeneous technology function f^n requires z_n be the maximal net output given the subvector of other goods z^n. Formally, the technical efficiency hypothesis and test for a linearly homogeneous function f^n are as follow.

- Technical Efficiency Hypothesis II' (for a linearly homogeneous concave technology function f^n): The data $\{z^j : j = 1, 2, \ldots, J\}$ are generated by an underlying technology satisfying conditions II and are technically E-efficient where $E = \{n\}$ contains only the nth coordinate axis index.

The first part of the hypothesis implies that the technology function f^n satisfies conditions II'; the second part implies $z_n^j = f^n(z^{nj})$, $j = 1, 2, \ldots, J$. To test the efficiency hypothesis II' for the data set $\{z^j : j = 1, 2, \ldots, J\}$, perform test 2 using the linear programming formulation (4.2) with the efficiency direction vector specification: $\gamma_n = 1$ or -1, accordingly, if good n is an output or input; $\gamma_k = 0$ for $k \neq n$. If $\delta_i^* = 0$ for $i = 1, 2, \ldots, J$, then the data are consistent with the technical efficiency hypothesis II'. If $\delta_i^* > 0$ for some i, then a violation of the technical efficiency hypothesis II' occurs at observation i. If the data set $\{z^j : j = 1, 2, \ldots, J\}$ passes the test, the recoverable nonparametric function f^{n*} satisfying conditions II' can be defined by

$$f^{n*}(z^n) \equiv \max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{\textstyle\sum_{j=1}^J \lambda^j z_n^j : \sum_{j=1}^J \lambda^j z^{nj} \geq z^n\}. \tag{4.12}$$

This test forces the technically efficient observations with $\delta_i^* = 0$ not to be on the free disposal section of the boundary of the conic convex hull at least with respect to the nth good.

If the above test is performed with respect to all goods, that is, $E = \{n\}$, $n = 1, 2, \ldots, N$, then observations which pass all N tests are boundary points not lying on any free disposal region of the conic convex hull of the observed data and hence, are DEA-efficient.

Chapter 5

The Measurement of Technical Inefficiency for a Quasiconcave Technology

5.1 Definition of and test for technical efficiency (test 3)

The efficiency tests for a quasiconcave technology require choosing a particular good n and focusing on its individual technology as described by the function f^n assumed to satisfy conditions III. The function f^n is assumed to be continuous from above, nonincreasing and quasiconcave. Geometrically, the definition of technical efficiency is analogous to that in the convex case. Whereas in the convex case technical efficiency is defined relative to the boundary of the production possibilities set in N-dimensional space, in the quasiconcave case technical efficiency is defined relative to the boundary of the upper level set in $N-1$ dimensional space corresponding to a particular level of the singled-out good n. Hence, technical efficiency in the latter is conditional on a truncated efficiency direction vector, denoted by γ^n, which is the full vector γ with the nth component deleted. Let us denote the corresponding set containing the indices of the coordinate axes of the goods with respect to which we want to measure inefficiency as E^n.

A formal definition of technical efficiency for a quasiconcave technology follows. If the technology function is f^n, define the upper level set $L(z_n) \equiv \{z^n : f^n(z^n) \geq z_n\}$. A production plan z, which has the nth component equal to z_n and the remaining components written as the vector z^n, is technically E^n-efficient if and only if there exists no alternative vector $\hat{z}^n$ belonging to the upper level set $L(z_n)$ which is a strict Pareto improvement on z^n with respect to the directions in E^n. If the production plan z is not technically E^n-efficient, then there exist $\hat{z}^n \in L(z_n)$ such that $\hat{z}_k > z_k$ for all $k \in E^n$ and $f(\hat{z}^n) \geq z_n$.

In the case of a concave technology function, the definition of technical efficiency requires

$z_n = f^n(z^n)$ and z_n to be maximal for z^n; passing the corresponding efficiency test does not preclude the possibility that z^n is not E^n-maximal for z_n. In figure 3.4, if we perform the convexity test (test 1) with the efficiency direction vector $\gamma = (0,1)^T$ or equivalently with $E = \{2\}$, then the observed point b passes the test implying output z_2 is maximal given input $-z_1$ at point b. For technical efficiency relative to a quasiconcave technology, we require z^n to be maximal for z_n at least with respect to the goods $n \in E^n$. In the one output (z_2)-one input (z_1) case and focusing on the production function f^2 where $z_2 = f^2(z_1)$, this definition requires the input level $-z_1$ to be minimal for the output level z_2. Given the observed points b and d at the same output level in figure 3.4, clearly the point b does not use minimal input for the given output level and hence, is technically E^2-inefficient where $E^2 = \{1\}$. The definition of technical efficiency for a quasiconcave technology in this study, after taking into account the sign convention for inputs, is consistent with the usual cost minimization concepts and tests which often assume a quasiconcave technology. This study offers a generalized version of the efficiency tests to handle multiple outputs and multiple inputs.

We now state our technical efficiency hypothesis and test for a quasiconcave technology function f^n.

- Technical Efficiency Hypothesis *III* (for a quasiconcave technology): The data $\{(z^j_n, z^{nj}) : j = 1,2,\ldots,J\}$ are generated by an underlying technology function f^n satisfying conditions *III* and are technically E^n-efficient.

To test the above hypothesis, given the data $\{z^j : j = 1,2,\ldots,J\}$ and the truncated efficiency direction vector γ^n, test 3 below is performed. It involves solving J linear programming subproblems where each problem corresponds to an observation. Since upper level sets are used to describe the quasiconcave technology, it is necessary to construct an index set containing the observations with values for z_n greater than or equal to its current value z^i_n. In constrast to tests 1 and 2 where the full range of observations $1,2,\ldots,J$ is used in each linear programming subproblem, only a subset of observations is used in each subproblem in test 3. Again, we shall choose the goods k indexed in the set E^n and the efficiency direction coefficients γ_k so that the following condition holds:

$$\gamma_k z_k^j > 0 \text{ for } k \in E^n,\ j = 1, 2, \ldots, J. \tag{5.1}$$

The technical efficiency test for a quasiconcave technology follows.

- Test 3

1. For each observation i, $i = 1, 2, \ldots, J$,

   (a) define an index set

   $$I_i^n \equiv \{j : z_n^j \geq z_n^i,\ j = 1, 2, \ldots, J\}, \text{ and} \tag{5.2}$$

   (b) solve the following linear programming subproblem i:

   $$\max_{\delta_i \geq 0, \lambda^j \geq 0, j \in I_i^n} \{\delta_i : \textstyle\sum_{j \in I_i^n} \lambda^j z^{nj} \geq z^{ni} + \delta_i \hat{\gamma}^n z^{ni},\ \sum_{j \in I_i^n} \lambda^j = 1\} \equiv \delta_i^* \tag{5.3}$$

   where γ^n, z^{ni}, z^{nj} are the original vectors with their nth row (corresponding to good n) deleted, and $\hat{\gamma}^n$ is the vector γ^n diagonalized into an $(N-1)x(N-1)$ matrix with zero off-diagonal elements.

2. Consider the following consistency condition: If condition (5.1) holds and $\delta_i^* = 0$ for all i, $i = 1, \ldots, J$, then the data $\{z^j : j = 1, 2, \ldots, J\}$ are consistent with the technical efficiency hypothesis III for a quasiconcave technology. If condition (5.1) holds and $\delta_i^* > 0$ for some i, then observation z^i violates the technical efficiency hypothesis III and the data $\{z^j : j = 1, 2, \ldots, J\}$ are not consistent with this hypothesis.

Next, we show that the consistency condition above necessarily follows from the technical efficiency hypothesis III. Suppose the technical efficiency hypothesis III holds. Then, for each z_n^i, $i = 1, 2, \ldots, J$, there exists a closed convex upper level set $L(z_n^i) \equiv \{z^n : f^n(z^n) \geq z_n^i\}$ which we approximate by the convex hull $\hat{L}(z_n^i; z^1, z^2, \ldots, z^J)$ as defined in (2.3) and (2.4). If $j \in I_i^n$, then $z_n^j \geq z_n^i$ and $f^n(z^{nj}) \geq f^n(z^{ni})$ using (5.2) and the technical efficiency hypothesis III. By the quasiconcavity of f^n, we obtain

$$f^n(z^{ni}) \leq f^n(\textstyle\sum_{j \in I_i^n} \lambda^{j*} z^{nj}) \tag{5.4}$$

where λ^{j*}, $j* \in I_i^n$ are optimal for subproblem (5.3). At the optimal solution to (5.3), we also have from the linear programming constraint

$$z^{ni} + \delta_i^* \hat{\gamma}^n z^{ni} \leq \textstyle\sum_{j \in I_i^n} \lambda^{j*} z^{nj}. \tag{5.5}$$

It follows from the nonincreasingness property of f^n that

$$f^n(\textstyle\sum_{j \in I_i^n} \lambda^{j*} z^{nj}) \leq f^n(z^{ni} + \delta_i^* \hat{\gamma}^n z^{ni}). \tag{5.6}$$

Equations (5.4) and (5.6) yield

$$f^n(z^{ni}) \leq f^n(\textstyle\sum_{j \in I_i^n} \lambda^{j*} z^{nj}) \leq f^n(z^{ni} + \delta_i^* \hat{\gamma}^n z^{ni}). \tag{5.7}$$

Suppose $\delta_i^* > 0$. Then, from the positivity of $\gamma_k z_k^i$ for $k \in E^n$, the following inequalities hold:

$$z_k^i < (1 + \delta_i^* \gamma_k) z_k^i \quad \text{for } k \in E^n, \tag{5.8}$$

which implies by the nonincreasingness property of f^n that

$$f^n(z^{ni}) \geq f^n(z^{ni} + \delta_i^* \hat{\gamma}^n z^{ni}). \tag{5.9}$$

There are two possible cases for equation (5.7):

case (i): $f^n(z^{ni}) < f^n(z^{ni} + \delta_i^* \hat{\gamma}^n z^{ni})$. Clearly, equation (5.9) leads to a contradiction and hence, we have a violation of the technical efficiency hypothesis III at observation i.

case (ii): $f^n(z^{ni}) = f^n(z^{ni} + \delta_i^* \hat{\gamma}^n z^{ni})$. By assumption (5.1), $\hat{\gamma}^n z^{ni} \neq 0_{N-1}$. Hence, if $\delta_i^* > 0$ and case (ii) holds, then z^{ni} is not technically E^n-efficient. The point $(z^{ni} + \delta_i^* \hat{\gamma}^n z^{ni})$ belongs to $\hat{L}(z_n^i; z^1, z^2, \ldots, z^J) \subseteq L(z_n^i)$ such that (5.8) holds; output(input) levels for goods indexed in E^n can be increased(decreased) and still yield the same level of z_n^i. Hence, a violation of the technical efficiency hypothesis III occurs at observation i.

Therefore, if technical efficiency hypothesis III holds, then $\delta_i^* = 0$ for $i = 1, 2, \ldots, J$.

Suppose $\delta_i^* = 0$ for all observations $i = 1, 2, \ldots, J$. Then the data $\{z^j : j = 1, 2, \ldots, J\}$ can be rationalized by the function $f^{n*}(z^n)$ defined by

$$\begin{aligned} f^{n*}(z^n) \quad &\equiv \quad \max_i \{z_n^i : i = 1, 2, \ldots, J \text{ and } i \text{ such that } \textstyle\sum_{j \in I_i^n} \lambda^j z^{nj} \geq z^n, \\ &\qquad \textstyle\sum_{j \in I_i^n} \lambda^j = 1,\ \lambda^j \geq 0,\ j \in I_i^n\} \end{aligned} \tag{5.10}$$

which satisfies condition *III*. Geometrically, if $\delta_i^* = 0$ then observation z^i must lie on the boundary of the level set $\hat{L}(z_n^i)$ and there is no alternative $z^n \in \hat{L}(z_n^i)$ that is a strict Pareto improvement to z^{ni} with respect to the goods indexed in E^n. Hence, $\delta_i^* = 0$ implies that observation i satisfies the definition of technical E^n-efficiency for a quasiconcave technology function f^n.

The function $f^{n*}(z^n)$ defined in (5.10) is illustrated for the single output (z_2)-single input (z_1) case in figure 5.6. Suppose the true production function satisfying conditions *III* is given by f^2 where $z_2 = f^2(z_1)$; the heavy dots denote the observed production plans. Based on these observed points, the production function that can be recovered as defined in (5.10) is given by f^{2*}, a step function.[1] Note that the function f^{2*} is continuous from above, nonincreasing and quasiconcave. Not all the points on the graph of f^{2*} are technically efficient; the definition of technical efficiency for a quasiconcave technology function reduces to $-z_1$ being minimal for output z_2. The point b is technically E^2-inefficient since $\hat{L}(z_2^b) = \hat{L}(z_2^a) = \{z_1 : -\infty < z_1 \leq c\}$ and $z_1^b = d$ is in the interior of this ray. Observations which pass test 3 though must necessarily lie on the boundary of the production possibilities set approximation to the quasiconcave technology.

If it is desired to redefine technical efficiency such that "frontier points" on the graph of f^{n*} are deemed technically efficient, then test 3 can be modified appropriately. This alternative definition, which Hanoch and Rothschild (1972) use, poses the question whether z_n is maximal for z^n that is, whether $z_n = f^n(z^n)$ given the quantity subvector z^n and f^n is a quasiconcave

[1] The production possibilities set corresponding to this step function is similar to the polyhedral production set used by Deprins, Simar and Tulkens (1984) in their third method of measuring technical efficiency based on the sole assumption of input and output disposability. This method corresponds to our case of measuring technical inefficiency assuming a quasiconcave technology. They describe the production possibilities set as "also polyhedral, but not necessarily a convex one (p.251)".

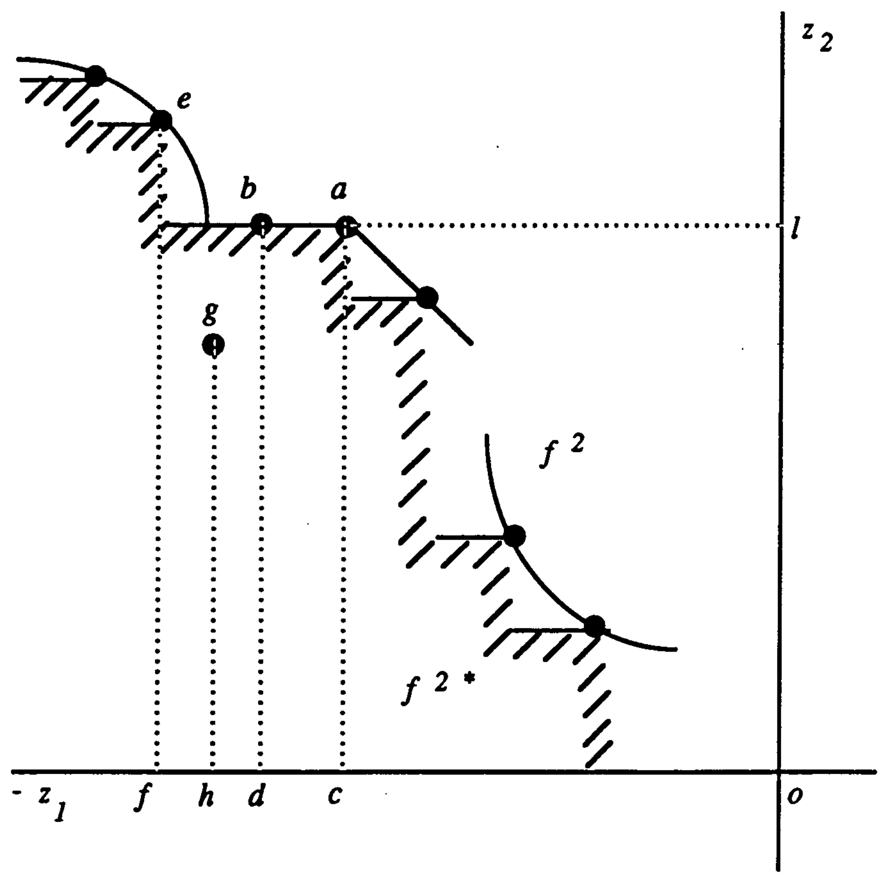

Figure 5.6: Technical efficiency for a quasiconcave technology function, single output-single input case

technology function satisfying conditions III. Let us rephrase the technical efficiency hypothesis for a quasiconcave technology as follows.

- Technical Efficiency Hypothesis III' (for a quasiconcave technology): The data $\{(z_n^j, z^{nj}) : j = 1,2,\ldots,J\}$ are generated by an underlying technology function f^n satisfying conditions III and $z_n^j = f^n(z^{nj})$ for all $j = 1,2,\ldots,J$.

The corresponding efficiency test is test 3 with the following modifications.

1. For each observation i, $i = 1,2,\ldots,J$,

   (a) redefine the index set I_i^n in (5.2) as

   $$\hat{I}_i^n \equiv \{j : z_n^j > z_n^i, j = 1,2,\ldots,J\}, \text{ and} \tag{5.11}$$

   (b) solve the following linear programming subproblem i which is (5.3) now defined over an unrestricted δ_i variable:

   $$\max_{\delta_i,\lambda^j\geq 0, j\,\in\, I_i^n}\{\delta_i : \textstyle\sum_{j\in I_i^n}\lambda^j z^{nj} \geq z^{ni} + \delta_i\hat{\gamma}^n z^{ni}, \sum_{j\in I_i^n}\lambda^j = 1\} \equiv \delta_i^*. \tag{5.12}$$

2. The consistency condition is changed to: If condition (5.1) holds and $\delta_i^* < 0$ for all i, $i = 1,\ldots,J$, then the data $\{z^j : j = 1,2,\ldots,J\}$ are consistent with the technical efficiency hypothesis III' for a quasiconcave technology. If condition (5.1) holds and $\delta_i^* \geq 0$ for some i, then observation z^i violates the technical efficiency hypothesis III' and the data $\{z^j : j = 1,2,\ldots,J\}$ are not consistent with this hypothesis.

The level set implicitly used in (5.12) will include subdata only on observations with values for z_n strictly greater than the current value z_n^i. With this revised definition of technical efficiency, an observation z^i passes the test if and only if the observed subvector z^{ni} is external to the level set constructed using the index set $\hat{I}_i^n$ defined in (5.11); in this case, $\delta_i^* < 0$. If observation i fails the test, then $\delta_i^* \geq 0$ gives the equiproportionate increase in output goods and decrease in input good contained in E^n needed to bring z^i to the efficient frontier. In figure 5.6, the point b passes the test since z_2^b is maximal for $z_1^b = d$; geometrically, the point d

is external to the ray $\{z_1 : -\infty < z_1 \leq f\}$. The point g is technically E^2-inefficient since $z_1^g = h$ is in the interior of the ray $\{z_1 : -\infty < z_1 \leq c\}$; the violation index will be $\delta_i^* = ch/oh$.

Alternatively, the technical efficiency hypothesis III' or Hanoch-Rothschild hypothesis can be tested without the aforementioned modifications by performing the original test 3 repeatedly ($N-1$ times). Single out a good k, $k \neq n$, at a time and specify the efficiency direction vector γ^k such that $E^k = \{n\}$, that is, γ^k has only one nonzero element which corresponds to good n. If the violation indices for all $N-1$ tests at observation i, $i = 1, 2, \ldots, J$, are zero, then the data are consistent with the technical efficiency hypothesis III' for a quasiconcave technology. If the violation index is positive for some observation i in one of the $N-1$ tests, then observation z^i violates the technical efficiency hypothesis III'.

For the rest of this study, we revert to our original definition of technical E^n-efficiency for a quasiconcave technology function f^n. Whether an observation is technically efficient or not is not invariant to the choice of the good n to play an asymmetric role. The sensitivity of the results arises if the observed vector z^i happens to lie on either vertical or horizontal flats at the boundary of the approximating production possibilities set. Note that under our regularity conditions I', II' and III, a quasiconcave function can have flats in the relative interior of its graph, while a concave function can only have flats at the global maximum of its graph. In figure 5.6, if we focus on the factor requirement function f^1 where $z_1 = f^1(z_2)$, then point b will be technically E^1-efficient (but E^2-inefficient) since $z_2^b = l$ is at the boundary of the level set $\{z_2 : 0 < z_2 \leq l\}$. Given the input level $-z_1^b$, the output level $z_2^b = l$ is maximal. In test 3, the magnitude of the violation index δ_i^* will depend on the choice of the good n and the efficiency direction vector γ^n.

At the optimal solution to subproblem (5.3), there can also be a divergence between the points $\sum_{j \in I_i^n} \lambda^{j*} z^{nj}$ and $(z^{ni} + \delta_i^* \hat{\gamma}^n z^{ni})$. Such a divergence can be interpreted analogously as in the convex and convex cone cases as due to $(z^{ni} + \delta_i^* \hat{\gamma}^n z^{ni})$ lying on the free disposal region at the boundary of the relevant convex hull. The underlying production set approximation for a quasiconcave technology, say $\hat{T}_3$, is smaller than those of the convex $(\hat{T}_1)$ and convex cone $(\hat{T}_2)$

technologies. In particular, $\hat{T}_3 \subseteq \hat{T}_1 \subseteq \hat{T}_2$. Across the three technical efficiency tests and given identical efficiency direction vectors such that $E^n = E$, the violation index δ_i^* cannot decrease as we go from test 3 (quasiconcavity test) to test 1 (convexity test) to test 2 (CRS test). Hence, if the value of δ_i^* significantly increases going from test 3 to test 1, for $E^n = E$, then the result can be indicative of possible increasing returns or flats in the true underlying technology; in short, a departure from the convexity assumption. Since conditions III for a quasiconcave technology are quite weak, violations of test 3 can be indicative more of a failure to maximize production of outputs or minimize input use.

Test 3 can be repeatedly performed with respect to each good n, $n = 1, 2, \ldots, N$. If an observation i passes all the N test, then observation i does not lie on any flat portion at the boundary of the production possibilities set approximation. In the empirical implementation of the test, the choice of the good n to play the asymmetric role can depend on the economic problem at hand. This arbitrariness holds true as well in the specification of the efficiency direction vector γ or γ^n. The researcher may want to focus on the production of a particular output. The objective of the analysis may be on the efficiency of some input, for example, labor. In some cases, output based measures or input based measures are desired. Nevertheless, the multiple output-multiple input framework offered in this study is amenable to a wide range of applications.

5.2 Dual interpretation

Another perspective on the technical efficiency concept and test for a quasiconcave technology function f^n can be gleaned from the dual to the primal linear programming subproblem (5.3):

$$\min_{q^n \geq 0_{N-1}, \mu_i} \{-z^{niT} q^n + \mu_i : z^{niT} \hat{\gamma}^n q^n \geq 1, \, -z^{njT} q^n + \mu_i \geq 0, \, j \in I_i^n\} \tag{5.13}$$

If we interpret the dual variables $q^n \geq 0_{N-1}$ as a vector of nonnegative prices, then $-z^{niT} q^n$ is the net cost of producing $z_{n.}^i$ at prices q^n. Therefore, if there exist prices q^{n*} such that $q_k^* > 0$

for at least one $k \in E^n$ and z^{ni} solves the following net cost minimization problem:

$$C(q^{n*}, z_n^i) \equiv \min_{z^n}\{-q^{n*T}z^n : f^{n*}(z^n) \geq z_n^i\} \equiv -\mu_i^* \tag{5.14}$$

where f^{n*} is defined by (5.10), then z^i is technically E^n-efficient for some f^n satisfying conditions *III*. The optimal net cost at prices q^{n*} is given by $-\mu_i^*$. A dual definition of technical efficiency for a quasiconcave technology function f^n is obtained.

- Definition. Given a quasiconcave technology function f^n satisfying conditions *III*, a production plan z is technically efficient relative to an efficiency direction vector γ^n, or technically E^n-efficient where E^n is the set of coordinate axis indices corresponding to the nonzero components of γ^n, if and only if, there exists a semipositive price vector $q^{n*} > 0_{N-1}$ such that $q_k^* > 0$ for at least one good k belonging to the set E^n, and at prices q^{n*} and net output level z_n, the subvector z^n is a cost minimal choice.

This definition uses the concept of Koopmans' efficiency prices and requires the possible existence of such prices, not necessarily prevailing market prices, to support a production plan.

Alternatively, as in the convex case, the violation index δ_i^* can be expressed as

$$\delta_i^* \equiv \max_j \left\{ \frac{(z^{nj} - z^{ni})^T q^{n*}}{z^{niT}\hat{\gamma}^n q^{n*}} : j \in I_i^n \right\}. \tag{5.15}$$

With our sign convention for inputs and outputs, the violation index δ_i^* measures the distance of the subvector z^{ni} from the cost minimizing allocation, say z^{nj*}, in terms of the excess "cost" incurred at z^{ni} relative to that at z^{nj*} with the goods in E^n as reference goods. The magnitude of δ_i^* gives the number of units of the basket of goods in E^n, evaluated at the shadow prices q^{n*}, lost due to inefficiency.

Chapter 6

The Nonparametric Measurement of Allocative Inefficiency

Allocative efficiency is sometimes referred to as economic efficiency. Given data on quantities and relevant prices, the behavioral goal of the firm as modeled by an objective function, and the firm's technological constraints, we would like to test whether the firm is using the optimal mix of outputs and/or inputs at the given prices. As will be shown, allocative efficiency subsumes technical efficiency. Two general cases are discussed: the first, when plant managers are not able to optimize with respect to some inputs or outputs; and the second, when complete and accurate price information is available and plant managers can freely vary all goods. The former can be interpreted as a case of constrained optimization; the latter, as unconstrained optimization. The former is also pertinent to short-run considerations when some factors are fixed; the latter can be associated with longer-run optimization when all inputs and outputs are variable.

The concept of allocative efficiency is illustrated for a two-good ($N = 2$) convex technology in figure 6.7. The observed production plans are denoted by heavy dots and $\hat{T}_1$ is the convex hull of the observed points. Let us focus on the point a which is in the interior of $\hat{T}_1$ and hence, is technically inefficient independently of γ, the efficiency direction vector. Suppose an output based violation index is desired and we set $\gamma = (0,1)^T$ or $E = \{2\}$. The proportional increase in output z_2 needed to bring a to the point b at the boundary of $\hat{T}_1$ is given by $\delta_a^* = ba/ae = gh/hf$. The slashed isoprofit line through the points a and b have slope $-q_1^*/q_2^*$ where q_1^* and q_2^* solve the dual linear programming subproblem (3.7).

Suppose producer a faces prices $p \equiv (p_1, p_2)^T$. Then, the profit-maximizing production plan, given prices p, is the point c where the isoprofit line P with slope $-p_1/p_2$ is tangent to $\hat{T}_1$.

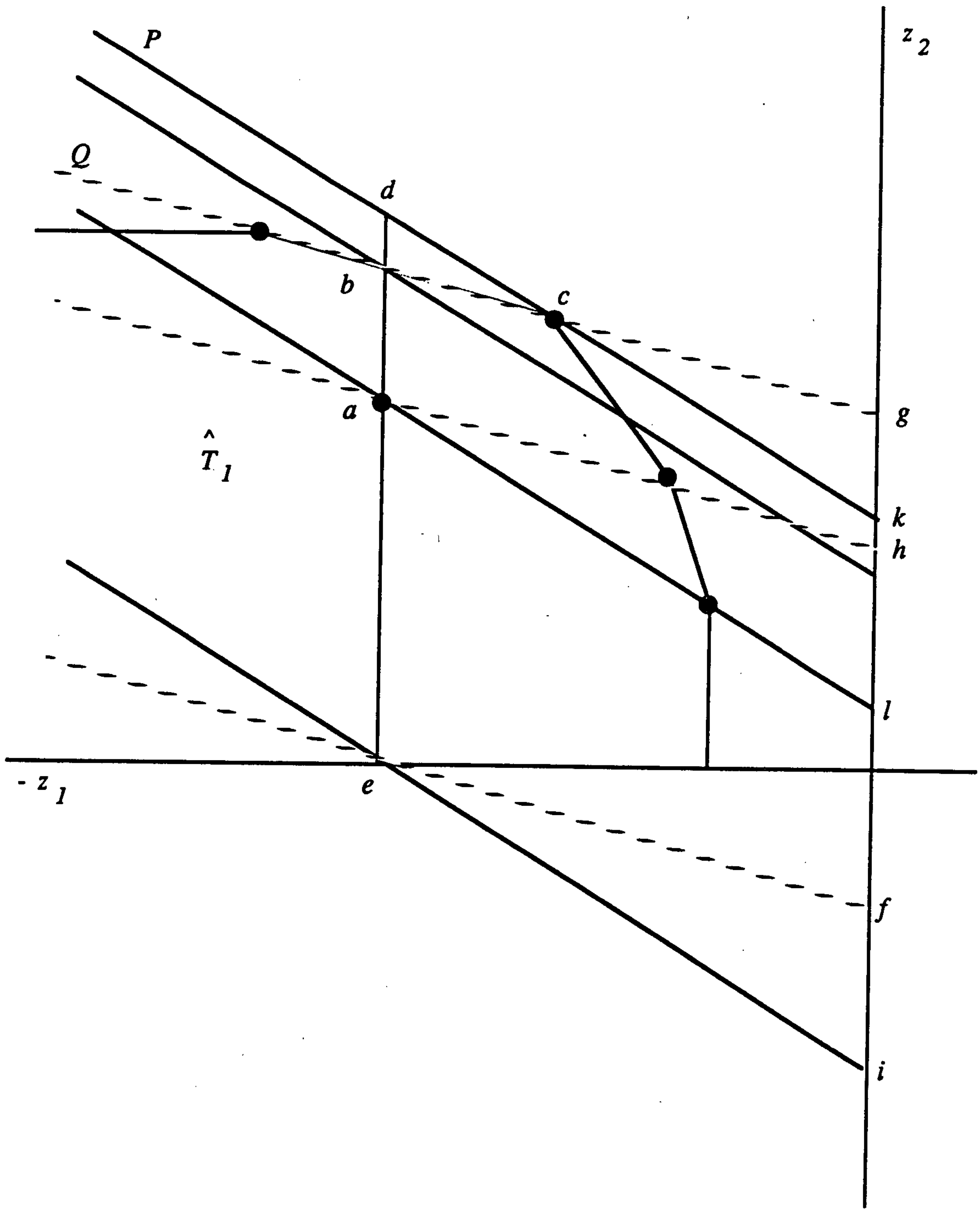

Figure 6.7: Allocative efficiency for a two-good convex technology

The point b is technically efficient but does not yield maximal profits at the given prices; the isoprofit line with slope $-p_1/p_2$ passing through b is lower than the line P. A gauge of the distance between the point a and the relatively (given the prices producer a faces) allocatively efficient production plan c is desired. Since the concept of allocative efficiency is price dependent, the violation index should also reflect the shortfall in profits at a relative to that in c. Another criterion we desire for the allocative efficiency violation index is consistency or comparability with the technical efficiency violation index δ_a^* defined earlier by (3.2) in chapter 3. To satisfy this criterion, we use the same goods indexed in the set E as the reference goods.

In figure 6.7, a measure of allocative inefficiency at a in terms of output loss is given by the vertical distance between the points a and d. The point d, though not technically feasible given $\hat{T}_1$, lies on the same isoprofit line as c. The resulting proportional loss measure, say ε_a^*, would be da/ae. The following relationship holds between the two violation indices:

$$\varepsilon_a^* = \frac{da}{ae} > \delta_a^* = \frac{ba}{ae} > 0.$$

The allocative efficiency violation index ε_a^* can be decomposed into its components in the following manner:

$$\varepsilon_a^* = \delta_a^* + (\varepsilon_a^* - \delta_a^*)$$

with δ_a^* due solely to technical inefficiency and $(\varepsilon_a^* - \delta_a^*)$ due solely to failure to respond efficiently to prices. The latter source of inefficiency we term as "pure" allocative inefficiency. A proportional increase in output at a by δ_a^* brings us to the boundary of $\hat{T}_1$ and hence, the output loss due to pure allocative inefficiency given by $(\varepsilon_a^* - \delta_a^*)z_2^a$ can be considered hypothetical. Therefore, for prescriptive purposes, an efficiency test has to be devised that will also yield the allocatively efficient point c.

In terms of the reference good z_2, the violation index ε_a^* is also equal to kl/li $(= da/ae)$. In this form, the index can be expressed as

$$\varepsilon_a^* = \frac{p^T c - p^T a}{p^T \hat{\gamma} a} \quad \left(= \frac{kl}{li} = \frac{p^T c - p^T a}{p^T a - p^T (a - \hat{\gamma} a)} \right)$$

which has as numerator the distance between the two parallel isoprofit lines with slope $-p_1/p_2$ passing through c and a, and as denominator the distance between the two parallel isoprofit line through a and e. Hence, the violation index ε_a^* gives the proportion of the value of the reference goods, $p^T\hat{\gamma}a = p_2a_2$, lost due to allocative inefficiency (both pure allocative inefficiency and technical inefficiency). At prices $p \equiv (p_1, p_2)^T$, actual profits at a must be increased by $\varepsilon_a^*(p_2a_2)$ to attain allocative efficiency (or economic efficiency), that is,

$$p^Tc = p^Ta + \varepsilon_a^*(p^T\hat{\gamma}a).$$

Graphically, it is apparent that

$$\varepsilon_a^* = \frac{p^Tc - p^Ta}{p^T\hat{\gamma}a} = \frac{kl}{li} > \delta_a^* = \frac{q^{*T}c - q^{*T}a}{q^{*T}\hat{\gamma}a} = \frac{gh}{hf}$$

where $q^{*T} \equiv (q_1^*, q_2^*)$ corresponds to the slope of the "efficiency price" line Q.

In the next chapters, the more general formulations of the allocative efficiency tests in the context of partial profit maximization are developed. Suppose plant managers are not able to optimize with respect to some inputs or outputs, or we simply do not have accurate price data for some goods. The constrained optimization tests are applicable in the short run when the firm has some fixed factors, such as capital equipment and structures. The behavior of the firm can be described as maximizing a restricted or variable profit function. The following tests are also helpful in evaluating the economic performance of nonprofit entities, regulated firms and government enterprises. Market prices may not exist for their output (which can be undesirable like pollution) or output levels may be mandated by law; even if output prices exist, they may be set beyond market forces. However, they are most likely competitive in the inputs market. An appropriate description of the behavior of such producers is cost minimization. Alternatively, some researchers have defined efficiency in the context of producing maximal output (which can be multi-valued) given a vector of inputs; the corresponding behavioral description for allocative efficiency testing might be revenue maximization. As well, the constrained optimization tests that follow offer a general framework wherein the unconstrained optimization or complete profit maximization tests can be derived as special cases. As in the analysis of technical efficiency, the allocative efficiency tests are developed relative to a postulated general technology assumed to satisfy the appropriate regularity conditions.

Suppose the firm can optimize with respect to K goods where $1 \leq K \leq N$. Let us denote this set of goods by S. Hence, the relevant prices for a particular firm, say firm i, are $p_n^i > 0$, $n \in S$. We also specify a set $E \subseteq S$ containing the desired reference goods with respect to which we want to measure the efficiency loss. We retain the assumption $| z_n^j | > 0$ for all $n \in E$ and $j = 1, 2, \ldots, J$. Let us denote by ε_i^S the violation index for the allocative efficiency hypothesis at observation i and given the prices of the goods in S. Hence, if $\varepsilon_i^S = 0$, then observation i is allocatively efficient with respect to the goods in set S; otherwise, $\varepsilon_i^S > 0$ implies observation i is allocatively inefficient with respect to the goods in S. The value of the reference goods is the price-weighted sum of outputs and inputs (indexed positively) in the set E. The violation index ε_i^S gives the proportion of this basket of goods or, more accurately, the value of this basket of goods given up due to economic inefficiency. In this sense, the violation index is also termed a loss measure.

Chapter 7

The Measurement of Allocative Inefficiency for a Convex Technology Assuming Partial Profit Maximization

7.1 Allocative efficiency test (test 4)

Suppose the true technology has a convex production possibilities set satisfying conditions I. Given quantity and price data, $\{z^j : j = 1, 2, \ldots, J\}$ where $z^j \equiv (z_1^j, z_2^j, \ldots, z_N^j)$ and $\{p_n^j : n \in S, j = 1, 2, \ldots, J\}$, we want to test whether the production plan z^i is constrained allocatively efficient at prices $p_n^i > 0$, $n \in S$. The efficiency test involves solving a linear programming problem given below for each observation i.

- Test 4

  1. For each observation i, $i = 1, 2, \ldots, J$, solve the following linear programming subproblem i and define the violation index ε_i^S by

$$\max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{\textstyle\sum_{n \in S} p_n^i (\sum_{j=1}^J \lambda^j z_n^j) : \sum_{j=1}^J \lambda^j z_n^j \geq z_n^i,\ n \notin S,\ \sum_{j=1}^J \lambda^j = 1\} \tag{7.1}$$

$$\equiv \textstyle\sum_{n \in S} p_n^i z_n^i + \varepsilon_i^S \sum_{n \in E} p_n^i |z_n^i|. \tag{7.2}$$

  Note that rewriting the optimized objective function in (7.1) as (7.2) defines the violation index for the ith observation.

  2. Consider the following consistency condition: If $\varepsilon_i^S = 0$ for all i, $i = 1, 2, \ldots, J$, then all J observations are allocatively efficient with respect to the goods in set S. If $\varepsilon_i^S > 0$, then observation i is not allocatively efficient with respect to the goods in set S.

Under the assumption of conditions I for the technology and given quantity data, a production possibilities set approximation is given by

$$\hat{T}_1(z^1, z^2, \ldots, z^J) \equiv \{z : \sum_{j=1}^J \lambda^j z^j \geq z, \sum_{j=1}^J \lambda^j = 1 \; , \lambda^j \geq 0, j = 1, 2, \ldots, J\}. \tag{7.3}$$

For any nonnegative price vector $p \geq 0_N$ and $z \in \hat{T}_1$, there exist $\lambda^j \geq 0$, $j = 1, \ldots, J$ such that $\sum_{j=1}^J \lambda^j = 1$ and

$$p^T(\sum_{j=1}^J \lambda^j z^j) \geq p^T z. \tag{7.4}$$

The unconstrained profit maximization problem for producer i facing a complete set of prices $p^i \geq 0_N$ can be formulated as

$$\begin{aligned} &\max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{p^{iT}(\sum_{j=1}^J \lambda^j z^j) : \sum_{j=1}^J \lambda^j = 1\} \\ &\equiv p^{iT} z^* \\ &\geq p^{iT} z^j, \; j = 1, 2, \ldots, J \end{aligned} \tag{7.5}$$

where $z^* \equiv \sum_{j=1}^J \lambda^{j*} z^j$ is the unconstrained allocatively efficient point and λ^{j*}, $j = 1, 2, \ldots, J$, solve the linear programming problem (7.5).

If producer i cannot optimize with respect to good n, $n \notin S$, then we add the constraint $\sum_{j=1}^J \lambda^j z_n^j \geq z_n^i$, $n \notin S$. This constraint implies that output must be at least as large as z_n^i if $z_n^i > 0$, or input must be at most equal to $-z_n^i$ if $z_n^i < 0$. This is just the free disposal assumption applied to good n, $n \notin S$. As will be seen more clearly in the saddlevalue problem approach to this constrained optimization problem, the prices for good n, $n \notin S$ are irrelevant to the firm. Hence, the constrained optimization problem for a convex technology can be formulated as given in (7.1). A violation index $\varepsilon_i^S \geq 0$ can also be obtained.

The optimal production plan at prices $\{p_n^i : n \in S\}$ is z^{S*} with components $\sum_{j=1}^J \lambda^{j*} z_n^j$ for $n \in S$ and z_n^i for $n \notin S$ where λ^{j*}, $j = 1, 2, \ldots, J$ solve (7.1). In contrast to the point $(z^i + \delta_i^* \hat{\gamma} z^i)$, the vector z^{S*} is not necessarily a Pareto improvement on z^i except possibly for the goods not in S. The optimal partial profit given S is $\sum_{n \in S} p_n^i z_n^{S*}$ while partial profit at the current production plan is $\sum_{n \in S} p_n^i z_n^i$. Their difference is given by $\varepsilon_i^S \sum_{n \in E} p_n^i |z_n^i|$. Whereas the adjustment $\delta_i^* \hat{\gamma} z^i$ is always technically feasible, the adjustment $\varepsilon_i^S |z_n^i|$ for all $n \in E$ is not necessarily feasible.

Note too that the problem formulation (7.1) uses firm-specific producer prices. In a cross-section of firms, actual prices faced by producers may vary from market prices due to, for example, transportation costs or tax distortions in the economy. In time series data, relative prices of goods may change.

The Lagrangian function corresponding to the left-hand side problem in (7.1) is

$$\phi(\lambda, q, \mu) \equiv \textstyle\sum_{n \in S} p_n^i (\sum_{j=1}^J \lambda^j z_n^j) + \sum_{n \notin S} q_n (\sum_{j=1}^J \lambda^j z_n^j - z_n^i) + \mu(1 - \sum_{j=1}^J \lambda^j) \tag{7.6}$$

where $\lambda \geq 0_J$, $q \geq 0_{N-K}$ and μ is unrestricted. The Kuhn-Tucker necessary optimality conditions corresponding to (7.6) are:

for $j = 1, 2, \ldots, j$:

$$\begin{aligned} \textstyle\sum_{n \in S} p_n^i z_n^j + \sum_{n \notin S} q_n^* z_n^j - \mu^* &\leq 0, \\ \lambda^{j*} &\geq 0, \\ \lambda^{j*}(\textstyle\sum_{n \in S} p_n^i z_n^j + \sum_{n \notin S} q_n^* z_n^j - \mu^*) &= 0; \end{aligned} \tag{7.7}$$

for $n \notin S$:

$$\begin{aligned} \textstyle\sum_{j=1}^J \lambda^{j*} z_n^j - z_n^i &\geq 0, \\ q_n^* &\geq 0, \\ q_n^*(\textstyle\sum_{j=1}^J \lambda^{j*} z_n^j - z_n^i) &= 0; \end{aligned} \tag{7.8}$$

and

$$1 - \textstyle\sum_{j=1}^J \lambda^{j*} = 0 \tag{7.9}$$

where $\lambda^* \geq 0_J$ solves (7.1), and $q^* \geq 0_{N-K}$ and μ^* unrestricted are Lagrange multipliers such that

$$\phi(\lambda, q^*, \mu^*) \leq \phi(\lambda^*, q^*, \mu^*) \leq \phi(\lambda^*, q, \mu)$$

for all $\lambda \geq 0_J$, $q \geq 0_{N-K}$ and μ unrestricted. The vector q^* can be interpreted as the shadow prices for the goods not in S and μ^* as the optimal shadow profit. At the optimal production plan z^{S*} the value of the shortfall in output and excess supply of inputs not in S at shadow prices q^* is zero. For convenience, let us term a production plan z that is allocatively efficient with respect to the goods in set S as allocatively S-efficient. Hence, the plan z^{S*} with components $\sum_{j=1}^{J} \lambda^{j*} z_n^j$ for $n \in S$ and z_n^i for $n \notin S$ is allocatively S-efficient.

7.2 Some results

7.2.1 Technical efficiency proposition

The allocatively S-efficient production plan z^{S*} is technically S-efficient and does not lie on any free disposal region with respect to the goods in S. This proposition implies that z^{S*} is on the boundary of the production possibilities set; the second part follows from the positivity of prices for goods in S. There exist prices $q^* \geq 0_N$ with $q_n^* = p_n^i > 0$ for $n \in S$ and $q_n^* \geq 0$ for $n \notin S$ such that z^{S*} is a profit maximal choice. By the dual definition of technical efficiency for a convex technology, the vector z^{S*} is technically S-efficient.

The proof of the proposition uses the saddlevalue problem approach discussed earlier. From the complementary slackness condition in (7.7) and summing across all $j's$, we obtain

$$\sum_{n \in S} p_n^i \left(\sum_{j=1}^{J} \lambda^{j*} z_n^j\right) + \sum_{n \notin S} q_n^* \left(\sum_{j=1}^{J} \lambda^{j*} z_n^j\right) - \mu^* \sum_{j=1}^{J} \lambda^j = 0$$

which implies

$$\sum_{n \in S} p_n^i \left(\sum_{j=1}^{J} \lambda^{j*} z_n^j\right) + \sum_{n \notin S} q_n^* z_n^i = \mu^* \tag{7.10}$$

using (7.9) and the complementary slackness condition in (7.8). It then follows that z^{S*} is technically E-efficient for $E = S$ and $E = \{n\}$ for any $n \in S$. Therefore, we obtain the proposition above.

7.2.2 Relative efficiency proposition

There exists at least one observation j ($j \in \{1, 2, \ldots, J\}$ and $j = i$ is possible) which is relatively efficient to observation i at given prices p_n^i, $n \in S$ yielding the same shadow profit level μ^*; this observation has the corresponding λ^{j*} positive, that is, $\lambda^{j*} > 0$. From (7.9), at least one of the $\lambda^{j*}s$ must be positive and from (7.10) and the complementary slackness condition in (7.7), the corresponding observation z^j yields the same shadow profit level as z^{S*} at prices p_n^i, $n \in S$ and q_n^*, $n \notin S$.

This proposition implies that observations with positive λ^{j*} lie on the same facet of the convex hull as z^{S*}. The facet is identified by μ^* and the prices p_n^i, $n \in S$ and q^*, $n \notin S$. In the DEA-terminology, these observations are evaluators of observation i.

7.2.3 Nonnegativity of ε_i^S

The violation index for allocative S-efficiency, ε_i^S, is nonnegative; that is, $\varepsilon_i^S \geq 0$. Note that $\lambda^i = 1$ and $\lambda^j = 0$ for $j \neq i$ is feasible for problem (7.1). Hence, z^i is feasible but not necessarily optimal for problem (7.1). It follows that

$$\textstyle\sum_{n \in S} p_n^i (\sum_{j=1}^{J} \lambda^{j*} z_n^j) \geq \sum_{n \in S} p_n^i z_n^i \tag{7.11}$$

and

$$\varepsilon_i^S \textstyle\sum_{n \in E} p_n^i \mid z_n^i \mid \geq 0.$$

By assumption, $p_n^i > 0$ and $\mid z_n^i \mid > 0$ for $n \in E$; thus, $\varepsilon_i^S \geq 0$.

If $\varepsilon_i^S = 0$, then (7.11) must hold with equality and z^i is allocatively S-efficient. If $\varepsilon_i^S > 0$, then z^i does not yield maximal partial profit with respect to the goods indexed in S.

7.2.4 Comparability of ε_i^S and δ_i^*

If the technical and allocative efficiency tests for a convex technology (tests 1 and 4, respectively) are performed for observation i with identical specifications of the reference set E such that $E \subseteq S$, then the resulting violation indices are comparable and moreover, $\varepsilon_i^S \geq \delta_i^*$. Suppose test 1 is performed; the resulting point $(z^i + \delta_i^* \hat{\gamma} z^i)$, with components $(z_n^i + \delta_i^* | z_n^i |)$ if $n \in E$ and z_n^i if $n \notin E$, is feasible for the partial profit maximization problem (7.1). The optimal $\lambda^j s$ for problem (3.2) obtained in test 1 are feasible but not necessarily optimal for problem (7.1) since $E \subseteq S$ implies $\bar{S} \subseteq \bar{E}$.[1] Hence,

$$\textstyle\sum_{n \in S} p_n^i (\sum_{j=1}^J \lambda^{j*} z_n^j) \geq \sum_{n \in S} p_n^i (z_n^i + \delta_i^* \gamma_n z_n^i)$$

where λ^{j*}, $j = 1, 2, \ldots, J$, solve (7.1). By definition from (7.1), the inequality can be rewritten as

$$\textstyle\sum_{n \in S} p_n^i z_n^i + \varepsilon_i^S \sum_{n \in E} p_n^i | z_n^i | \geq \sum_{n \in S} p_n^i z_n^i + \delta_i^* \sum_{n \in E} p_n^i | z_n^i |.$$

Since $\sum_{n \in E} p_n^i | z_n^i | > 0$, it follows that

$$\varepsilon_i^S \geq \delta_i^*. \tag{7.12}$$

With a consistent specification of the efficiency direction vector γ in test 1 and the set E in test 4, the allocative efficiency violation index can be decomposed into its components:

$$\varepsilon_i^S = \delta_i^* + (\varepsilon_i^S - \delta_i^*). \tag{7.13}$$

The term δ_i^* measures the waste due purely to technical inefficiency, or loosely defined, the failure to produce at some point on the boundary of the production possibilities set. The second term $(\varepsilon_i^S - \delta_i^*)$ can be attributed to pure allocative inefficiency, or the failure to respond efficiently to the prevailing prices of the goods in S.

The relative magnitude of the two terms may depend on the choice of goods in the set E. In empirical applications, the researcher must be guided by the economic problem under consideration. Extraneous information, other than price and quantity data, can be helpful in modeling the behavior of the firm.

7.2.5 Special cases: $K = 0, 1, N-1, N$

(1) $K = 0$. In the absence of market prices for goods, as the case may be in controlled economies, the objective function in the linear programming subproblem (7.1) becomes vacuous. Hence,

[1] The sets $\bar{S}$ and $\bar{E}$ are the complements of S and E, respectively.

the allocative efficiency test is not applicable in this instance. The producer would not face a price-dependent objective function to guide its behavior. As Koopmans (1951) has noted, the notion of efficiency in the context of such an economy reduces to that of technical efficiency, that is, producing at the boundary of the production possibilities set. The choice of production plan must still be subject to technological feasibility considerations. The appropriate test for efficiency in the case $K = 0$ is the technical efficiency test which uses quantity data only.

(2) $K = 1$. If there is only one good, say good n, with a relevant market price, then the allocative efficiency test 4 is equivalent to the technical efficiency test 1 with the reference set $E = \{n\}$. The revenue maximization or cost minimization problem, as the case may be depending on whether good n is an output or input, in (7.1) would entail merely the search for the maximal quantity of good n given the quantities of the other goods. Mathematically, the solution of the linear programming subproblem (7.1) is equivalent to the solution of the Diewert-Parkan technical efficiency formulation (3.17) which we have shown to test for the special case of technical E-efficiency where $E = \{n\}$. Both test 4 (with $K=1$) and test 1 with $E=\{n\}$ will yield identical values for their violation indices, that is, $\delta_i^* = \varepsilon_i^S$. Intuitively, this result follows because the existence of a market price for a good ensures that with respect to this good, an optimizing producer will not be at a free disposal region of the production possibilities set.

(3) $K = N-1$. If there is only one good not in S, then the partial profit maximization problem reduces to a net cost minimization problem if the excluded good is an output, or to a net revenue maximization problem if the excluded good is an input. The special case with $K = N-1$ may be applicable to instances wherein the firm has actually several fixed goods which can be aggregated to form a Leontief or Hicksian composite good. For a convex technology, the appropriate test for consistency with the net cost minimization (or net revenue maximization) hypothesis is still test 4.

The usual cost minimization tests found in the literature and which involve the simpler "revealed preference"-type inequalities (see, for example, Diewert and Parkan (1983) and Varian (1984a)) assume a quasiconcave technology. A similar test will be derived later for testing for unconstrained optimization under the assumption of a quasiconcave technology. If the true technology is convex, then a test using these inequalities may underestimate the degree of inefficiency. Intuitively, the production possibilities set approximation asssuming a quasiconcave technology is contained by that for a convex technology. Given the same efficiency direction, the distance of an observed point to the quasiconcave frontier cannot be larger than the distance to the boundary of the convex technology set.

More rigorously, in the linear programming formulation for a convex technology given in (7.1), the constraint for a good n, $n \notin S$ is $\sum_{j=1}^{J} \lambda^j z_n^j \geq z_n^i$ where $\sum_{j=1}^{J} \lambda^j = 1$ does not restrict the positive $\lambda^{j*}s$ to observations where $z_n^j \geq z_n^i$. In contrast, for a quasiconcave technology, an upper level set for z_n^i is constructed and the corresponding constraint is $\sum_{j \in I_i^n} \lambda^j z_n^j \geq z_n^i$ where $\sum_{j \in I_i^n} \lambda^j = 1$.

(4) $K = N$. When $K = N$, plant managers can freely vary all goods and complete and accurate price data are available. In this case, the producer has an unconstrained optimization problem. (Implicitly, the producer still remains bounded by technological feasibility considerations.) The test for full profit maximization, assuming a convex technology, is test 4 with $S \equiv \{1, 2, \ldots, N\}$. As will be shown in a later chapter, the test in this case reduces to a comparison of inequalities; the resulting test we number as test 7. Both tests, either solving the linear programming problem (7.1) or comparing inequalities, yield identical values for the violation index ε_i^S where S contains all the N goods.

7.2.6 Restricting μ^* to be nonnegative

As can be seen from the Lagrangian function (7.6), the shadow profit μ^* can be negative, zero or positive for a convex technology. Hence, negative profits or losses can be consistent with profit maximization for a convex technology. If the shadow profit μ^* is restricted to be nonnegative, then this is equivalent to having $\sum_{j=1}^{J} \lambda^j \leq 1$ or to assuming that $0_N \in T$. The production possibilities set approximation is given by $\hat{T}$ defined by

$$\hat{T}(z^1, z^2, \ldots, z^J) \equiv \{z : \textstyle\sum_{j=1}^{J} \lambda^j z^j \geq z, \sum_{j=1}^{J} \lambda^j \leq 1, \lambda^j \geq 0, j = 1, 2, \ldots, J\}. \qquad (7.14)$$

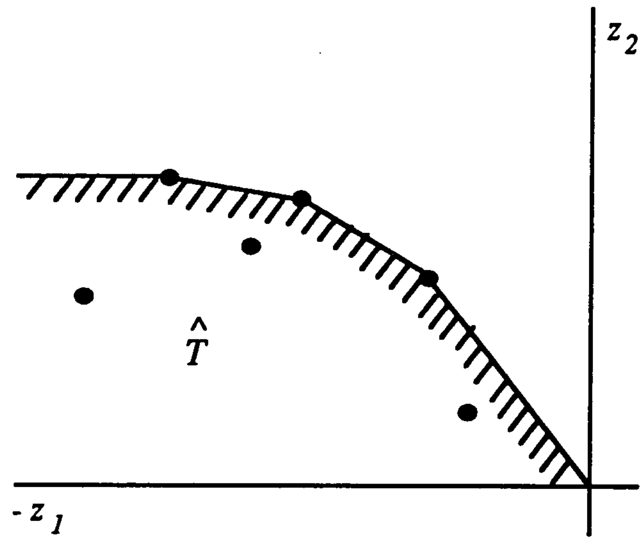

Figure 7.8: Production possibilities set $\hat{T}$ with $\mu^* \geq 0$

As illustrated in figure 7.8 for a two-good technology, the technology described by $\hat{T}$ in (7.14) is assumed to be conical near the origin and display decreasing returns elsewhere. Färe, Grosskopf and Lovell (1985, p.181) describe $\hat{T}$ in (7.14) as the smallest closed star-like set containing the observed $z^j s$; only radial contraction to the origin is allowed.

The test for allocative S-efficiency assuming a convex technology and restricting shadow profits to be nonnegative can be done in two ways. Perform test 4 with the linear programming problem (7.1) modified to either

$$\max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{\textstyle\sum_{n \in S} p_n^i (\sum_{j=1}^J \lambda^j z_n^j) : \sum_{j=1}^J \lambda^j z_n^j \geq z_n^i,\ n \notin S,\ \sum_{j=1}^J \lambda^j \leq 1\} \tag{7.15}$$

or

$$\max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{\textstyle\sum_{n \in S} p_n^i (\sum_{j=0}^J \lambda^j z_n^j) : \sum_{j=0}^J \lambda^j z_n^j \geq z_n^i,\ n \notin S,\ \sum_{j=0}^J \lambda^j = 1\} \tag{7.16}$$

where $z^0 \equiv 0_N$. Problem (7.15) explicitly includes the restriction $\sum_{j=1}^J \lambda^j \leq 1$ while problem (7.16) includes the origin in the convex hull. The violation index ε_i^S will have the same value using either formulation.

Chapter 8

A LeChatelier Principle for Measures of Allocative Inefficiency

As the number of goods K with respect to which the producer can optimize increases, the difference between the firm's optimized objective function value and its actual realized value cannot decrease. Given a fixed reference set E, the violation index ε_i^S defined in (7.1) cannot decrease as the number of goods in S increases. This result is consistent with the conventional wisdom that the unrestricted or long-run profit is always at least as large as the restricted or short-run profit.

We prove the proposition by induction. Suppose we increase by one the number of elements in set S containing the goods with respect to which producer i can optimize; denote the new set by S'. Corresponding to the set S', the number of constraints in (7.1) is reduced by one. Let us rewrite problem (7.1) for these two cases:

$$\begin{aligned} g^K(\lambda^*) &\equiv \textstyle\sum_{n\in S} p_n^i (\sum_{j=1}^J \lambda^{j*} z_n^j) & (8.1) \\ &\equiv \max_{\lambda^1\geq 0,\ldots,\lambda^J\geq 0} \{\textstyle\sum_{n\in S} p_n^i (\sum_{j=1}^J \lambda^j z_n^j) : \sum_{j=1}^J \lambda^j z_n^j \geq z_n^i,\ n\notin S,\ \sum_{j=1}^J \lambda^j = 1\}; \\ g^{K+1}(\lambda^{**}) &\equiv \textstyle\sum_{n\in S'} p_n^i (\sum_{j=1}^J \lambda^{j**} z_n^j) & (8.2) \\ &\equiv \max_{\lambda^1\geq 0,\ldots,\lambda^J\geq 0} \{\textstyle\sum_{n\in S'} p_n^i (\sum_{j=1}^J \lambda^j z_n^j) : \sum_{j=1}^J \lambda^j z_n^j \geq z_n^i,\ n\notin S',\ \sum_{j=1}^J \lambda^j = 1\}. \end{aligned}$$

It can be shown that

$$g^{K+1}(\lambda^{**}) - \textstyle\sum_{n\in S'} p_n^i z_n^i \;\geq\; g^K(\lambda^*) - \sum_{n\in S} p_n^i z_n^i. \tag{8.3}$$

The proof is as follows:

$$\begin{aligned} g^{K+1}(\lambda^{**}) - \textstyle\sum_{n\in S'} p_n^i z_n^i &\equiv \textstyle\sum_{n\in S'} p_n^i (\sum_{j=1}^J \lambda^{j**} z_n^j) - \sum_{n\in S'} p_n^i z_n^i \\ &\geq \textstyle\sum_{n\in S'} p_n^i (\sum_{j=1}^J \lambda^{j*} z_n^j) - \sum_{n\in S'} p_n^i z_n^i \end{aligned}$$

since λ^{j*} is feasible but not necessarily optimal

for problem (8.2);

$$\begin{aligned}
&= \textstyle\sum_{n\in S} p_n^i(\sum_{j=1}^J \lambda^{j*} z_n^j) + p_{K+1}^i(\sum_{j=1}^J \lambda^{j*} z_{K+1}^j) \\
&\quad - \textstyle\sum_{n\in S} p_n^i z_n^i - p_{K+1}^i z_{K+1}^i
\end{aligned}$$

where p_{K+1}^i, z_{K+1}^i are the price and quantity of the

$(K+1)$th good added to the set S to obtain S'

$$\begin{aligned}
&\equiv g^K(\lambda^*) - \textstyle\sum_{n\in S} p_n^i z_n^i + p_{K+1}^i(\sum_{j=1}^J \lambda^{j*} z_{K+1}^j - z_{K+1}^i) \\
&\geq g^K(\lambda^*) - \textstyle\sum_{n\in S} p_n^i z_n^i
\end{aligned}$$

since $p_{K+1}^i > 0$ by assumption and

$(\sum_{j=1}^J \lambda^{j*} z_{K+1}^j - z_{K+1}^i) \geq 0$ from (8.1) and $n = (K+1) \notin S$.

Thus, we obtain inequality (8.3). If we divide both sides of (8.3) by the value of the reference goods $\sum_{n\in E} p_n^i |z_n^i|$, then we have $\varepsilon_i^{S'} \geq \varepsilon_i^S$ where $\varepsilon_i^{S'}$ and ε_i^S are the violation indices corresponding to sets S' and S, respectively.

Chapter 9

The Measurement of Allocative Inefficiency for a Convex Conical Technology Assuming Partial Profit Maximization

9.1 Allocative efficiency test (test 5)

Suppose the true technology has a production possibilities set that is a convex cone and satisfies conditions II. Let $1 \leq K \leq N-1$, that is, the set S is nonempty and contains at most $N-1$ goods. The test for allocative S-efficiency of the production plans z^i for prices $p_n^i > 0$, $n \in S$ and $i = 1, 2, \ldots, J$ is given below.

- Test 5

1. For each observation i, $i = 1, 2, \ldots, J$, solve the following linear programming subproblem i and define the violation index ε_i^S by

$$\max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{\textstyle\sum_{n \in S} p_n^i (\sum_{j=1}^J \lambda^j z_n^j) : \sum_{j=1}^J \lambda^j z_n^j \geq z_n^i,\ n \notin S\} \tag{9.1}$$

$$\equiv \textstyle\sum_{n \in S} p_n^i z_n^i + \varepsilon_i^S \sum_{n \in E} p_n^i |z_n^i|. \tag{9.2}$$

   Note that rewriting the optimized objective function in (9.1) as (9.2) defines the violation index for the ith observation.

2. Consider the following consistency condition: If $\varepsilon_i^S = 0$ for all i, $i = 1, 2, \ldots, J$, then all J observations are allocatively efficient with respect to the goods in set S. If $\varepsilon_i^S > 0$, then observation i is not allocatively efficient with respect to the goods in set S.

The production possibilities set approximation for a constant returns to scale technology satisfying conditions II is $\hat{T}_2$ defined as

$$\hat{T}_2(z^1, z^2, \ldots, z^J) \equiv \{z : \textstyle\sum_{j=1}^{J} \lambda^j z^j \geq z,\ \lambda^j \geq 0,\ j = 1, 2, \ldots, J\}. \tag{9.3}$$

Since any $z \in \hat{T}_2$ can be scaled up or down along a ray through the origin and still be in $\hat{T}_2$, output levels are unbounded in $\hat{T}_2$. In contrast, output levels are bounded from above in the set $\hat{T}_1$ defined by (7.3) for a convex technology satisfying the regularity conditions I. This difference can be seen in figure 2.2.

With constant returns, if a producer can make positive profits, he can theoretically scale up the production plan and obtain infinite profits. On the other hand, profits are bounded from below since $z = 0_N$ is in its production possibilities set. Since we observe only finite quantity vectors, the meaningful profit-maximizing choice for a firm having a constant returns to scale technology is a zero profit allocation. However, if the firm has found such a zero profit production plan, the producer can scale his operations up or down by a nonnegative factor and be indifferent to the scale of operations. We therefore further assume that the firm arbitrarily fixes the level of at least one good to determine the scale of its operations. Hence, the restriction $K \leq N - 1$.

The corresponding Lagrangian function is

$$\phi(\lambda, q) \equiv \textstyle\sum_{n\in S} p_n^i (\sum_{j=1}^{J} \lambda^j z_n^j) + \sum_{n\notin S} q_n (\sum_{j=1}^{J} \lambda^j z_n^j - z_n^i) \tag{9.4}$$

and the necessary Kuhn-Tucker optimality conditions are

for $j = 1, 2, \ldots, j$:

$$\begin{aligned} \textstyle\sum_{n\in S} p_n^i z_n^j + \sum_{n\notin S} q_n^* z_n^j &\leq 0, \\ \lambda^{j*} &\geq 0, \\ \lambda^{j*}(\textstyle\sum_{n\in S} p_n^i z_n^j + \sum_{n\notin S} q_n^* z_n^j) &= 0; \text{ and} \end{aligned} \tag{9.5}$$

for $n \notin S$:

$$\textstyle\sum_{j=1}^{J} \lambda^{j*} z_n^j - z_n^i \geq 0,$$

$$\begin{aligned} q_n^* &\geq 0, \\ q_n^*(\textstyle\sum_{j=1}^{J} \lambda^{j*} z_n^j - z_n^i) &= 0. \end{aligned} \tag{9.6}$$

Again, the Lagrange multiplier $q_n^* \geq 0$, $n \in S$ can be interpreted as the shadow price of good n. If good n is an input and is in excess supply, then $q_n^* = 0$. If good n is an output more of which could have been produced, then $q_n^* = 0$. If $q_n^* > 0$, the converse holds. With a convex cone technology, the optimal shadow profit μ^* which appears in (7.7) is restricted to zero. Note that in (7.6), μ is the Lagrange multiplier attached to the restriction $\sum_{j=1}^{J} \lambda^j = 1$ which is dropped in (9.1).

The solution to the linear programming problem (9.1) can be similarly interpreted as in the convex case. Where the convex cone results differ will be discussed next. There are three possible solution configurations for (9.1). They are: (i) $\lambda^* > 0_J$ and finite, that is, λ^* has a finite positive value for at least one j; (ii) $\lambda^* = 0_J$; or (iii) λ^* is unbounded. If $\varepsilon_i^S > 0$, then the allocatively S-efficient production plan at the given prices is z^{S*} with components $\sum_{j=1}^{J} \lambda^{j*} z_n^j$ for $n \in S$ and z_n^i for $n \notin S$. After correcting for free disposability in the goods not in S, an allocation z^{**} where $z^{**} \equiv \sum_{j=1}^{J} \lambda^{j*} z^j$ can be obtained. As elaborated in the discussion of the technical efficiency tests, the linear programming formulation we have does not guaranteee detection of free disposability along all N dimensions. The first two cases yield a finite z^{S*}. The first case results in nontrivial allocations for $z^{S*}$and z^{**}. The second case yields $z^{**} = 0_N$ or the shutdown point. The third case implies that at prices p_n^i, $n \in S$, the shadow profit is infinite. Having an input not in S ensures a finite solution. If all the goods not in S are output goods, it is possible, though it does not necessarily follow, that λ^* would be unbounded.

9.2 Some results

The relative efficiency proposition for testing for allocative S-efficiency assuming the technology is described by a convex cone satisfying conditions II is as follows. Assuming a finite optimal solution to the linear programming problem (9.1), either (i) there exist at least one observation j

($j \in \{1, 2, \ldots, J\}$ and $j = i$ is possible) which is relatively efficient to observation i at given prices $p_n^i > 0$, $n \in S$ yielding a zero shadow profit;[1] or (ii) the optimal choice for the producer, given the prices $p_n^i > 0$, $n \in S$, is the shutdown point. Case (i) obtains when λ^* is finite and nonzero, that is $\lambda^* > 0_N$. The proof uses the complementary slackness condition in (9.5); the geometric interpretation is analogous to that in the convex case. Case (ii) obtains when the optimal solution λ^* is the zero vector, that is, $\lambda^* = 0_J$.

Assuming a finite optimal solution to the linear programming solution (9.1), the technical efficiency and LeChatelier principle propositions and the results on the nonnegativity of ε_i^S and the comparability of ε_i^S and δ_i^* hold as well in the constant returns to scale case as in the convex case.

When a firm having a constant returns to scale technology can optimize with respect to all goods but one, that is, $K = N - 1$, we consider the producer as having an unconstrained allocative efficiency problem. As discussed earlier, at least one good is held fixed to determine the scale of operations. Test 5 can be performed to test for allocative efficiency. An alternative test (test 8), involving a comparison of inequalities, is given in a later chapter. At $K = N - 1$, both tests yield identical values for ε_i^S.

Let $(\varepsilon_i^S)_{convex}$ and $(\varepsilon_i^S)_{CRS}$ be the violation indices obtained froms tests 4 and 5, respectively, for a given set S of size K where $1 \leq K \leq N - 1$; prices $p_n^i > 0$, $n \in S$; and a fixed reference set E. It can be shown that the violation index obtained under the convex conical technology assumption cannot be smaller than that obtained under the convex technology assumption, that is,

$$(\varepsilon_i^S)_{CRS} \geq (\varepsilon_i^S)_{convex}. \tag{9.7}$$

The proof is simple. Suppose $\hat{\lambda}^j$, $j = 1, 2, \ldots, J$ solve (7.1) and λ^{j*}, $j = 1, 2, \ldots, J$ solve (9.1). Then, the $\hat{\lambda}^j s$ are feasible but not necessarily optimal for problem (9.1). Hence,

$$\textstyle\sum_{n \in S} p_n^i (\sum_{j=1}^J \lambda^{j*} z_n^j) \geq \sum_{n \in S} p_n^i (\sum_{j=1}^J \hat{\lambda}^j z_n^j), \tag{9.8}$$

[1] This observation has the corresponding λ^{j*} positive ($\lambda^{j*} > 0$).

that is, the optimal partial profit assuming a constant returns technology cannot be smaller than the optimal partial profit assuming a convex technology. Since the right-hand side terms of (7.1) and (9.1) are identical except for the violation indices, the result follows. Intuitively, this result arises because the convex production possibilities set approximation $\hat{T}_1$ is contained in the convex conical production possibilities set approximation $\hat{T}_2$.

Chapter 10

The Measurement of Allocative Inefficiency for a Quasiconcave Technology Assuming Partial Profit Maximization

10.1 Allocative efficiency test (test 6)

Suppose the true technology has nonconvexities in its production possibilities set. We assume that the individual technologies can be described by quasiconcave functions satisfying conditions *III*. Single out a good n with respect to whose technology we would like to test for allocative efficiency. Let the set S^n contain the K goods (where $1 \leq K \leq N-1$) with respect to which the producer can optimize. We assume S^n does not contain good n. Given prices $p^i_m > 0$, $m \in S^n$ and quantity data $\{z^j : j = 1, 2, \ldots, J\}$, we would like to test whether the production plans z^j, $j = 1, 2, \ldots, J$ are allocatively S^n-efficient, that is, the $z^j s$ are allocatively efficient with respect to the goods in the set S^n. Specify a set $E \subseteq S^n$ containing the desired reference goods. The efficiency test follows.

- Test 6

  1. For each observation i, $i = 1, 2, \ldots, J$,

     (a) define an index set

$$I^n_i \equiv \{j : z^j_n \geq z^i_n,\ j = 1, 2, \ldots, J\}, \text{ and} \tag{10.1}$$

     (b) solve the following linear programming subproblem i and define the violation index ε^S_i by

$$\max_{\lambda^j \geq 0, j \in I^n_i}\{\textstyle\sum_{m \in S^n} p^i_m (\sum_{j \in I^n_i} \lambda^j z^j_m) : \sum_{j \in I^n_i} \lambda^j z^j_m \geq z^i_m,\ m \notin S^n, \\ \textstyle\sum_{j \in I^n_i} \lambda^j = 1\} \tag{10.2}$$

$$\equiv \sum_{m\in S^n} p^i_m z^i_m + \varepsilon^S_i \sum_{m\in E} p^i_m | z^i_m |. \tag{10.3}$$

Note that rewriting the optimized objective function in (10.2) as (10.3) defines the violation index for the ith observation.

2. Consider the following consistency condition: If $\varepsilon^S_i = 0$ for all i, $i = 1, 2, \ldots, J$, then all J observations are allocatively efficient with respect to the goods in set S^n. If $\varepsilon^S_i > 0$, then observation i is not allocatively efficient with respect to the goods in set S^n.

The regularity conditions III for a technology function f^n imply the existence of closed convex upper level sets. The level set approximation for z^i_n is given by

$$\hat{L}(z^i_n; z^1, z^2, \ldots, z^J) \equiv \{z^n : \sum_{j\in I^n_i} \lambda^j z^{nj} \geq z^n, \sum_{j\in I^n_i} \lambda^j = 1, \lambda^j \geq 0, j \in I^n_i)\} \tag{10.4}$$

where I^n_i is defined in (10.1). For any nonnegative price vector $p^n \geq 0_{N-1}$ and $z^n \in \hat{L}(z^i_n)$, there exist $\lambda^j \geq 0$, $j \in I^n_i$ such that $\sum_{j\in I^n_i} \lambda^j = 1$ and

$$p^{nT}(\sum_{j\in I^n_i} \lambda^j z^{nj}) \geq p^{nT} z^n. \tag{10.5}$$

The unconstrained optimization problem for producer i facing prices $p^{ni} \geq 0_{N-1}$ can be formulated as

$$\begin{aligned} & \max_{\lambda^j \geq 0, j\in I^n_i} \{p^{niT}(\sum_{j\in I^n_i} \lambda^j z^{nj}) : \sum_{j\in I^n_i} \lambda^j = 1, j \in I^n_i\} \qquad (10.6) \\ & \equiv p^{niT} z^{n*} \\ & \geq p^{niT} z^{nj}, \ j \in I^n_i, \end{aligned}$$

where $z^{n*} \equiv \sum_{j\in I^n_i} \lambda^{j*} z^{nj}$ and λ^{j*}, $j* \in I^n_i$ solve (10.6).

If $z^i_n > 0$, or good n is an output, then subproblem (10.6) can be interpreted as a net cost minimization problem, that is, finding $z^n \in \hat{L}(z^i_n)$ such that the net cost of producing z^i_n is minimal given prices p^{ni}. If $z^i_n < 0$, or good n is an input, then subproblem (10.6) can be seen as one of maximizing the net return to this input. With either interpretation, the objective function is a partial or restricted profit function.

If the firm is unable to optimize with respect to some goods, then additional constraints are introduced to subproblem (10.6). The objective function, which remains a partial profit function, is appropriately modifed as well. Formulation (10.2) then obtains for the constrained optimization problem. Since $\hat{T}_1$ in (7.3) and $\hat{L}$ in (10.4) have the same property of being closed convex sets, the results of test 6 can be interpreted analogously as those of test 4 for a convex technology.

10.2 Some results

Keeping in mind that the analysis uses level sets in $N-1$ dimensional space, we obtain technical efficiency, relative efficiency and LeChatelier principle propositions similar to those in test 4 for a convex technology. The results on the nonnegativity of ε_i^S and the comparability of ε_i^S and δ_i^* hold where the violation indices are obtained under the quasiconcavity assumption. An alternative test (test 9) for the limiting case $K = N-1$ or unconstrained optimization for a quasiconcave technology is presented in the next chapter.

If, for notational simplicity, we define $S^n = S$, then it follows that, for given prices $p_m^i > 0$, $m \in S$ and a fixed reference set E,

$$(\varepsilon_i^S)_{CRS} \geq (\varepsilon_i^S)_{convex} \geq (\varepsilon_i^S)_{quasiconcave} \qquad (10.7)$$

where the ε_i^S are obtained from tests 5, 4 and 6, respectively. Suppose $\tilde{\lambda}^j$, $j \in I_i^n$ are optimal for problem (10.2). Then, $\tilde{\lambda}^j$, $j \in I_i^n$ and $\lambda^j = 0$, $j \notin I_i^n$ are feasible but not necessarily optimal for problem (7.1). The second inequality in (10.7) follows. The first inequality in (10.7) is given in (9.7). The values of the partial profit functions will be related in the following manner:

$$\textstyle\sum_{m\in S} p_m^i (\sum_{j=1}^J \lambda^{j*} z_m^j) \geq \sum_{m\in S} p_m^i (\sum_{j=1}^J \hat{\lambda}^j z_m^j) \geq \sum_{m\in S} p_m^i (\sum_{j\in I_i^n} \tilde{\lambda}^j z_m^j) \qquad (10.8)$$

where the $\lambda^{j*}s$ and $\hat{\lambda}^j s$ are optimal for the constant returns to scale problem (9.1) and the convex problem (7.1), respectively.

The allocative efficiency tests 4, 5 and 6, as well as tests 7, 8 and 9 to be presented in the next chapter, are developed under the joint hypotheses of the regularity conditions on the technology and of efficiency. A positive violation index can be attributed to either of two sources. If the researcher gives lesser weight to inefficiency, then significant differences in the values of the violation indices in (10.7) can be interpreted in the context of the curvature conditions on the technology.

Chapter 11

The Measurement of Allocative Inefficiency Assuming Complete Profit Maximization

11.1 The convex technology case (test 7)

Suppose complete and accurate price data are available and firms can freely vary all goods. The following tests for unconstrained allocative efficiency are limiting cases of the corresponding tests for constrained optimization. The saddlevalue problem approach is used to derive the complete profit maximization tests; they can alternatively be shown using revealed preference type arguments (see Diewert and Parkan (1983, pp.150–152), Hanoch and Rothschild (1972), Varian (1984a, pp.57–64; 1984b)). The unconstrained optimization tests and their corresponding constrained optimization versions yield identical values for the violation indices for a fixed set of reference goods. The following tests are simpler to implement since they do not require solving linear programming problems and involve only a comparison of inequalities.

The informational requirements are identical for the constrained and unconstrained optimization tests. Price and quantity data and the specification of a set E of reference goods are required. Using the notation in the technical efficiency tests, the set E corresponds to an N-dimensional vector γ with nonzero components corresponding to the goods in E. Let us denote by ε_i^* the violation index at observation i given the price vector $p^i \equiv (p_1^i, p_2^i, \ldots, p_N^i)^T \gg 0_N$. If $\varepsilon_i^* = 0$, then observation i is (unconstrained) allocatively efficient relative to the technology posited; $\varepsilon_i^* > 0$ implies otherwise. In this chapter, we give the formulas for the violation index ε_i^* which is ε_i^S when the size of the set S is at its limiting value.

To test for allocative efficiency assuming the underlying technology satisfies conditions I, the violation index ε_i^* is defined in test 7 given below.

- Test 7. Suppose $K = N$, the number of elements in set S. Calculate the violation index ε_i^* defined as

$$\varepsilon_i^* \equiv \max_j \left\{ \frac{p^{iT}(z^j - z^i)}{p^{iT}\hat{\gamma} z^i} : j = 1, 2, \ldots, J \right\}. \tag{11.1}$$

The proof of the equivalence of test 7 and test 4 where $K = N$ is given below.

Suppose $K = N$. Then the linear programming subproblem i in (7.1) reduces to

$$\begin{aligned} &\max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{ p^{iT} (\textstyle\sum_{j=1}^J \lambda^j z^j) : \sum_{j=1}^J \lambda^j = 1\} \\ &\equiv p^{iT} z^i + \varepsilon_i^S \textstyle\sum_{n \in E} p_n^i | z_n^i | \end{aligned} \tag{11.2}$$

where the set E is a subset of the N goods chosen as the reference goods. The Kuhn-Tucker necessary optimality conditions are

for $j = 1, 2, \ldots, j$:

$$\begin{aligned} p^{iT} z^j - \mu^* &\leq 0, \\ \lambda^{j*} &\geq 0, \\ \lambda^{j*}(p^{iT} z^j - \mu^*) &= 0; \end{aligned} \tag{11.3}$$

and

$$1 - \textstyle\sum_{j=1}^J \lambda^{j*} = 0. \tag{11.4}$$

From (11.3) and (11.4), we obtain

$$\max_j \{ p^{iT} z^j : j = 1, 2, \ldots, J\} = \mu^*. \tag{11.5}$$

Equation (11.4) ensures equality for at least one j. Using the complementary slackness condition in (11.3) and summing across all $j's$, we have

$$\begin{aligned} p^{iT} (\textstyle\sum_{j=1}^J \lambda^{j*} z^j) &\equiv p^{iT} z^i + \varepsilon_i^S \textstyle\sum_{n \in E} p_n^i | z_n^i | \text{ using (11.2)} \\ &= \mu^* \text{ using (11.3)} \\ &= \max_j \{ p^{iT} z^j : j = 1, 2, \ldots, J\} \text{ using (11.5)}. \end{aligned} \tag{11.6}$$

An expression for ε_i^S, using (11.6), is then

$$\begin{aligned} \varepsilon_i^S &= \frac{\max_j\{p^{iT}z^j : j = 1,2,\ldots,J\} - p^{iT}z^i}{\sum_{n\in E} p_n^i |z_n^i|} \\ &= \max_j \left\{ \frac{p^{iT}(z^j - z^i)}{p^{iT}\hat{\gamma}z^i} : j = 1,2,\ldots,J \right\}. \end{aligned} \tag{11.7}$$

If we denote by ε_i^* the value of ε_i^S when $K = N$, then (11.1) follows.

Suppose $j*$ solves (11.1). Then, at prices p^i, a profit maximal production plan is z^{j*}. The vector z^{j*} is not necessarily a Pareto improvement on z^i, that is, it does not follow that $z^{j*} \geq z^i$. By moving from z^i to z^{j*}, (full) profits can be increased by $\varepsilon_i^*(p^{iT}\hat{\gamma}z^i)$, which is the value lost due to inefficiency. The possibility of free disposability lends a complication in the analysis of technical efficiency and constrained optimization. If a full range of (nonzero) market prices exist and the firm can optimize with respect to all goods, then the allocatively efficient points will not lie on any free disposal section of the production possibilities set. Note the similarity between ε_i^* in (11.1) and δ_i^* in (3.15). For a convex technology and given a fixed set E of reference goods, the following relationship among the different violation indices holds:

$$\delta_i^* \leq \varepsilon_i^S \leq \varepsilon_i^* \tag{11.8}$$

where E is contained in S. The first inequality is given in (7.12) and the second follows from the LeChatelier principle proposition. Inequality (11.8) implies that profits can be higher the greater the number of goods the producer can vary and the lesser the restrictions on the direction of adjustment. A decomposition of the violation index ε_i^* is possible:

$$\varepsilon_i^* = \delta_i^* + (\varepsilon_i^S - \delta_i^*) + (\varepsilon_i^* - \varepsilon_i^S). \tag{11.9}$$

Equation (11.9) can be interpreted accordingly; the last term (ε_i^*-ε_i^S) measures the waste due to failure to optimize with respect to the goods not in S.

Suppose $E = S$ contains all the input goods. Then, the violation index δ_i^* gives the proportional reduction in inputs needed to bring z^i to a point on the boundary of the production possibilities set. The magnitude of ε_i^S gives the proportional reduction in cost needed for z^i to be consistent with cost minimizing behavior given input prices and current output production levels. Minimal cost can be obtained by being more technically efficient and by altering the input mix; this may entail increasing the use of some inputs and decreasing other inputs. Even if the firm is cost minimizing, its output mix may not yield maximal profits at prevailing prices. By being able to vary all goods, inputs and outputs, the possible increase in profits is equivalent to savings of ε_i^* of the initial cost incurred by the firm.

The proposed tests can then be applicable to efficiency studies for an industry with regulated firms, and private firms subject to less regulations and whose output prices may be more reflective of market conditions. The behavior of regulated firms can often be described as cost minimizing while that of private firms as profit maximizing. Though the existence of public or regulated enterprises can be justified by other economic and social objectives, it is still enlightening to have a measure of productive efficiency losses in weighing the tradeoffs among possibly conflicting objectives of the firm. The analysis also indicates a reason why government-controlled or state-run firms or any firm, for that matter, insulated from some market price mechanism, may be at a disadvantage when forced to compete in a world market economy.

If the ratio of profits to cost or revenue is small, as is the usual case in empirical applications, the magnitude of the violation index ε_i^* in (11.1) obtained with the set E containing all the input goods will not differ much from that obtained with the set E containing all the output goods. As profit approaches zero, the ratio of the input based ε_i^* to the output based ε_i^* approaches 1.0. Generally, the magnitude of the violation indices proposed in this study, as well as the ranking of observations by the violation index, will depend on the goods chosen to be in the reference set E.

11.2 The convex conical technology case (test 8)

For the unconstrained optimization test assuming a constant returns to scale technology satisfying conditions II, a good n is chosen to determine the scale of operations of the firm. We pick the normalizing good n such that either $z_n^j > 0$ for all j, $j = 1, 2, \ldots, J$, that is, good n

is consistently an output, or $z_n^j < 0$ for all j, $j = 1, 2, \ldots, J$, that is, good n is consistently an input. The violation index ε_i^* is calculated as described in test 8.

- Test 8

  1. Choose a normalizing good, say good n.
  2. Define the normalized truncated vectors $\bar{z}^{nj}$, $j = 1, 2, \ldots, J$ where

$$\bar{z}^{nj} \equiv \frac{z^{nj}}{|\, z_n^j \,|}, \tag{11.10}$$

     where $|\, z_n^j \,|$ is the absolute value of the normalizing good at observation j, and the zero vector

$$z^{n0} \equiv 0_{N-1}. \tag{11.11}$$

  3. Calculate the violation index ε_i^* as

$$\varepsilon_i^* \equiv \max_j \left\{ \frac{p^{niT}(\bar{z}^{nj} - \bar{z}^{ni})}{\sum_{m \in E} p_m^i |\, \bar{z}_m^i \,|} : j = 0, 1, \ldots, J \right\} \tag{11.12}$$

     if either (i) good n is an input, or (ii) good n is an output and $\max_j \{p^{niT} \bar{z}^{nj} : j = 0, 1, \ldots, J\} \leq 0$. Otherwise, set ε_i^* to be unbounded.

The proof of the equivalence of test 8 and test 5 when $K = N - 1$ is given below.

Suppose $K = N - 1$. Then the linear programming subproblem (9.1) reduces to

$$\begin{aligned} &\max_{\lambda^1 \geq 0, \ldots, \lambda^J \geq 0} \{p^{niT}(\textstyle\sum_{j=1}^J \lambda^j z^{nj}) : \sum_{j=1}^J \lambda^j z_n^j \geq z_n^i\} \\ &\equiv p^{niT} z^{ni} + \varepsilon_i^S \textstyle\sum_{m \in E} p_m^i |\, z_m^i \,| \end{aligned} \tag{11.13}$$

where the set E is a subset of the $N - 1$ goods in S and good n is held fixed. The Kuhn-Tucker necessary optimality conditions are

for $j = 1, 2, \ldots, j$:

$$\begin{aligned} p^{niT} z^{nj} + q_n^* z_n^j &\leq 0, \\ \lambda^{j*} &\geq 0, \\ \lambda^{j*}(p^{niT} z^{nj} + q_n^* z_n^j) &= 0; \end{aligned} \tag{11.14}$$

and

$$\begin{aligned} \textstyle\sum_{j=1}^{J} \lambda^{j*} z_n^j - z_n^i &\geq 0, \\ q_n^* &\geq 0, \\ q_n^*(\textstyle\sum_{j=1}^{J} \lambda^{j*} z_n^j - z_n^i) &= 0. \end{aligned} \tag{11.15}$$

From equation (11.14), we obtain

$$0 \geq \max_j\{p^{niT} z^{nj} + q_n^* z_n^j : j = 1, 2, \ldots, J\}. \tag{11.16}$$

Let us assume a finite optimal solution to (11.13) exists. If a $\lambda^{j*} > 0$, then there exists at least one j, $j \in \{1, 2, \ldots, J\}$, such that equality holds. There is no $\sum_{j=1}^{J} \lambda^j = 1$ restriction so the optimal λ^* can be the zero vector. Hence, (11.16) can be rewritten as

$$0 = \max_j\{p^{niT} z^{nj} + q_n^* z_n^j : j = 0, 1, 2, \ldots, J\}. \tag{11.17}$$

where $z^{n0} \equiv 0_{N-1}$.

Using (11.14) and (11.15), we obtain the relation

$$p^{niT}(\textstyle\sum_{j=1}^{J} \lambda^{j*} z^{nj}) + q_n^* z_n^i = 0. \tag{11.18}$$

By construction in (11.13), the above equation can be reexpressed as

$$p^{niT} z^{ni} + \varepsilon_i^S \textstyle\sum_{m \in E} p_m^i |z_m^i| + q_n^* z_n^i = 0. \tag{11.19}$$

It is harmless to divide each term in (11.19) by $|z_n^i| > 0$ and each of the elements inside the brackets in (11.17) by their respective $|z_n^j| > 0$.

If good n is an input, then equating (11.17) and (11.19) after the normalization of variables yields

$$\begin{aligned} 0 &= p^{niT} \bar{z}^{ni} + \varepsilon_i^S \textstyle\sum_{m \in E} p_m^i |\bar{z}_m^i| - q_n^* \\ &= \max_j\{p^{niT} \bar{z}^{nj} - q_n^* : j = 0, 1, \ldots, J\} \end{aligned} \tag{11.20}$$

where $\max_j\{p^{niT}\bar{z}^{nj} : j = 0,1,\ldots,J\} \geq 0$ since the zero vector is a feasible choice and where normalized values are denoted by $\bar{z}_m \equiv z_m/z_n$. Hence, there will always exist a $q_n^* \geq 0$ such that (11.20) holds. Hence, the violation index ε_i^S can be calculated as

$$\varepsilon_i^S \equiv \max_j \left\{ \frac{p^{niT}(\bar{z}^{nj} - \bar{z}^{ni})}{\sum_{m\in E} p_m^i |\bar{z}_m^i|} : j = 0,1,\ldots,J \right\}. \tag{11.21}$$

If good n is an output and a finite solution to (11.13) exists, then equating (11.17) and (11.19) yields

$$\begin{aligned} 0 &= p^{niT}\bar{z}^{ni} + \varepsilon_i^S \textstyle\sum_{m\in E} p_m^i |\bar{z}_m^i| + q_n^* \\ &= \max_j\{p^{niT}\bar{z}^{nj} + q_n^* : j = 0,1,\ldots,J\} \end{aligned} \tag{11.22}$$

where

$$\max_j\{p^{niT}\bar{z}^{nj} : j = 0,1,\ldots,J\} \leq 0. \tag{11.23}$$

If (11.23) does not hold, then infinite profits can be made by firm i by scaling up production. In this case, the optimal solution to (11.13) is unbounded and there exists no $q_n^* \geq 0$ such that (11.22) holds. Hence, if (11.23) does not hold, set ε_i^S to be unbounded since the maximal profit is unbounded. Otherwise, the violation index ε_i^S, after algebraic manipulation of (11.22), is also given by (11.21). If we let $\varepsilon_i^* \equiv \varepsilon_i^S$ when $K = N-1$, then (11.12) obtains.

The violation index ε_i^* above is similar in form to the violation index for the unconstrained optimization test in the convex case given by (11.1). In contrast, the calculation of the violation index ε_i^* in test 8 uses normalized variables, includes the zero vector, and can possibly have an unbounded solution. The results of test 8 can be similarly interpreted.

Instead of full profit maximization, the analysis for the constant returns case is done in terms of normalized profit maximization. Assuming a finite solution to the corresponding linear programming problem, let $j*$ solve (11.12). From equations (11.20) and (11.22), the following equations are obtained:

$$q_n^* z_n^{j*} = \max_j\{p^{niT}z^{nj} : j = 0,1,\ldots,J\} = p^{niT}z^{nj*} \tag{11.24}$$

for good n an input, and

$$-\ q_n^* z_n^{j*} = \max_j\{p^{niT} z^{nj} : j = 0, 1, \ldots, J\} = p^{niT} z^{nj*} \tag{11.25}$$

for good n an output. Therefore, the normalized profit maximization problem is equivalent to finding a production plan such that, consistent with the zero profit condition, there exists a shadow price for good n such that good n receives the maximal return if it is an input, or the cost of producing good n is minimal if it is an output.

Any scalar multiple of z^{j*} belongs to the production possibilities set and yields the same zero shadow profit. Assuming z^{j*} is not a zero vector, one such allocation is $\hat{z}^j \equiv z^{j*}(z_n^i / z_n^{j*})$ which is the efficient vector yielded by the linear program in test 5. Identical values for the violation indices are obtained from tests 5 and 8 , and $j*$ enters the optimal basis with $\lambda^{j*} > 0$ in test 5. The partial profit values in test 5 differ from the normalized profit values in test 8 by a factor of $|\, z_n^i \,|$.

With full price data on $p \equiv (p_1, \ldots, p_N)^T \gg 0_N$, the usual profit maximization test for a constant returns to scale technology is a joint test of the profit maximization conditions for a convex technology and the zero profit condition (see Diewert and Parkan (1984, pp.151-152), Varian (1984a, p.586)). That is, z^i is consistent with profit maximization if and only if

$$p^{iT} z^i \geq p^{iT} z^j,\ j = 1, 2, \ldots, J; \tag{11.26}$$

and

$$p^{iT} z^i = 0. \tag{11.27}$$

Real world data usually fails the zero profit condition (11.27) unless the data were manipulated to impose it. Even if accounting methods ensure this condition, there is a problem of the meaningfulness of the constructed prices. Zero profit does not imply that the allocation is profit maximal at the given prices. Also, it is difficult to obtain a violation index that captures simultaneously departures from the profit maximization and zero profit conditions, as we have done in this study. However, the cost of constructing our efficiency measure is that we ignore price information on one good. This may be innocuous if we can accept the idea that in most production processes, there is at least one factor or good, which can be intangible, for which a market price does not exist. The returns to this good is imputed to the good chosen to determine the scale of operations, or what we term here as the normalizing good.

Some theoretical models assume constant returns to scale. Empirical implementation of these models then requires that relations (11.26) and (11.27) hold, or are not significantly violated inorder to estimate a profit function using the N prices. If the violation indices obtained in test 8 are small, then at most two additional goods, one an input and the other an output can be introduced into the model. Let $p \equiv (p_1, \ldots, p_N)^T$, $z \equiv (z_1, \ldots, z_N)^T$ and assume good $N+1$ is an input or $z_{N+1} \leq 0$, and good $N+2$ an output or $z_{N+2} \geq 0$. Let the price and quantity of these goods at observation j be defined in the following manner:

If $p^{jT} z^j > 0$, then let $z^j_{N+1} = -1$, $p^j_{N+1} = p^{jT} z^j$, and $z^j_{N+2} = 0$.

If $p^{jT} z^j < 0$, then let $z^j_{N+2} = 1$, $p^j_{N+1} = -p^{jT} z^j$, and $z^j_{N+1} = 0$.

If $p^{jT} z^j = 0$, then let $z^j_{N+1} = z^j_{N+2} = 0$.

Instead of the normalizing good in test 8 absorbing the losses or profits, we let good n enter the profit function estimation at its market price and let a hypothetical output good absorb the losses and a hypothetical input good absorb the profits. Alternatively, the profit function estimation can be done without introducing hypothetical goods by redefining the price of the omitted nth good (the normalizing good) such that the zero profit condition (11.27) holds.

11.3 The quasiconcave technology case (test 9)

Assuming the technology function f^n satisfies conditions III, the violation index ε^*_i measuring unconstrained allocative inefficiency is defined in test 9.

- Test 9

1. Single out a good n with respect to whose technology we would like to perform the test.

2. Define the index set I_i^n as

$$I_i^n \equiv \{j : z_n^j \geq z_n^i, j = 1, 2, \ldots, J\}. \tag{11.28}$$

3. Calculate the violation index ε_i^* as

$$\varepsilon_i^* \equiv \max_j \left\{ \frac{p^{niT}(z^{nj} - z^{ni})}{p^{niT}\hat{\gamma}^n z^{ni}} : j \in I_i^n \right\}. \tag{11.29}$$

The proof of the equivalence of test 9 and test 6 when $K = N-1$ is analogous to that of the corresponding tests under convexity (tests 7 and 4) and hence, is omitted. The detection of allocative inefficiency at a particular observation can depend on the good singled out. This sensitivity of the result arises because a quasiconcave function can have "flats". An example is the case $z_n^j > z_n^i$, $z^{nj} = z^{ni}$ for some $j \in I_i^n$, z^{ni} is cost minimal for z_n^i, and $p^i = p^j$. The linear programming problem (10.2) for unconstrained optimization with a quasiconcave technology reduces to a net cost minimization problem if good n is an output, and to a net revenue maximization problem if good n is an input.

The magnitude of ε_i^* in (11.29) for a quasiconcave technology is not comparable with the ε_i^* in (11.1) for a convex technology. If we single out a good n and let S be the set of all goods except good n (implying $K = N-1$), then comparable indices and their ranking are given by the following inequality relation (assuming a fixed reference set $E \subseteq S$):

$$(\varepsilon_i^*)_{CRS} \geq (\varepsilon_i^S)_{convex} \geq (\varepsilon_i^*)_{quasiconcave} \tag{11.30}$$

where $(\varepsilon_i^*)_{CRS}$, $(\varepsilon_i^S)_{convex}$ and $(\varepsilon_i^*)_{quasiconcave}$ are obtained using tests 8, 4 and 9, respectively. Inequality (11.30) follows from (10.7).

The unconstrained optimization test for a quasiconcave technology function f^n can be performed with respect to each good n, $n = 1, 2, \ldots, N$. If an observation i passes all N tests, then producer i with production plan z^i is responding efficiently to the full set of prices

$p^i \equiv (p^i_1, p^i_2, \ldots, p^i_N)^T \gg 0_N$. Unlike in the convex case, recourse is made to singling out a good at a time and performing N separate tests because of possible nonconvexities in the production possibilities set. The general technology can exhibit increasing returns and hence, the usual tangency condition between the price hyperplane and the production possibilities set for profit maximization does not hold, as does in the convex case.

In the allocative efficiency tests, both for constrained and unconstrained optimization, producers are assumed to be price takers with respect to the K goods belonging to the set S. A quasiconcave technology can have increasing returns which may be inconsistent with pure competition. If a firm exerts some degree of market power over prices of two or more of the N goods, the appropriate allocative efficiency test is test 6 which involves solving linear programming problems. More stringent tests on whether the price and quantity data are consistent with some form of market structure other than pure competition can be devised. Varian (1984a) offers a test for consistency with the hypothesis of profit maximization under monopolistic behavior. Since there is no general theory of imperfect competition, specific price setting rules have to be posited. Factors like strategic variables (price or quantity), behavior (cooperative or noncooperative), firm entry and exit conditions, demand conditions, etc. can be taken into account.

A decomposition of ε_i^* in (11.29) into its components due to technical inefficiency (δ_i^*) and pure allocative inefficiency due to failure to optimize, given the relevant prices, with respect to goods in S^n ($\varepsilon_i^S - \delta_i^*$) and with respect to goods not in S^n ($\varepsilon_i^* - \varepsilon_i^S$) can be made. The comparability of ε_i^* in (11.29) as a measure of allocative efficiency with δ_i^* in (5.3) for technical efficiency for a quasiconcave technology is apparent in the dual expression for δ_i^* in (7.12).

Chapter 12

The Efficiency Tests with the No Technological Regress Assumption

With time series data such that the observation index j corresponds to time, violations of the preceding efficiency hypotheses can be due to technological progress. Over time, the production possibilities set of the firm can be growing due to increasing effectiveness in the use of production inputs as a result of research and development, innovation, etc. or due to increasing availability of resources such as capital, labor or raw materials. Hence, if an observation j is deemed relatively efficient to observation i, $j > i$, it is possible that the production vector z^j is not feasible at period i. It may then be desired to incorporate the no technological regress assumption with the efficiency tests.

To deal with technical change, we follow the Diewert-Parkan (1983, pp.153–157) method of comparing the current observation only with the earlier observations. Suppose there is no technological regress, that is, $T^j \subseteq T^i$ if $j < i$. To illustrate how the efficiency tests can be modified to take into account the added assumption of no technological regress, we show the revised technical efficiency test for a convex technology.

First, we formally state the technical efficiency and no technological regress hypothesis.

- Technical Efficiency and No Technological Regress Hypothesis I (for a convex technology): The data $\{z^j : j = 1, 2, \ldots, J\}$ are generated by an underlying technology satisfying conditions I and are technically E-efficient. Furthermore, the production possibilities sets at periods i and j, $j < i$, are such that $T^j \subseteq T^i$.

Again, we choose the goods in E such that the efficiency direction vector γ satisfies condition (3.1). To test the above hypothesis, given quantity data and the efficiency direction vector, we perform the following test.

- Test 1′

1. For each observation i, $i = 1, 2, \ldots, J$, solve the following linear programming subproblem i:

$$\max_{\delta_i \geq 0, \lambda^1 \geq 0, \ldots, \lambda^i \geq 0} \{\delta_i : \sum_{j=1}^{i} \lambda^j z^j \geq z^i + \delta_i \hat{\gamma} z^i, \sum_{j=1}^{i} \lambda^j = 1\} \equiv \delta_i^* \qquad (12.1)$$

where $\hat{\gamma}$ is the efficiency direction vector γ diagonalized into a matrix.

2. Consider the following consistency condition: Suppose δ_i^* is the optimized objective function for the ith subproblem (12.1), $i = 1, \ldots, J$. If condition (3.1) holds and $\delta_i^* = 0$ for all i, $i = 1, \ldots, J$, then the data $\{z^j : j = 1, 2, \ldots, J\}$ are consistent with the technical efficiency and no technological regress hypothesis I for a convex technology. If the condition (3.1) holds and $\delta_i^* > 0$ for some i, then observation z^i violates the technical efficiency and no technological regress hypothesis I and the data $\{z^j : j = 1, 2, \ldots, J\}$ are not consistent with this hypothesis.

In contrast to test 1, the linear programming subproblem (12.1) constructs the convex hull of only the first i data points $\{z^1, z^2, \ldots, z^i\}$ instead of the whole set of J observations. The modified test is then less restrictive.

Since modifying the rest of the efficiency tests to account for technical change is a straightforward exercise, the list of changes in the various efficiency tests are relegated to appendix A. The same trick can be adapted for cross-section data when it is desired to divide the set of firms into subgroups for purposes of comparison. Productivity may be a function of the size of the firm as measured by output or installed capital. Depending on the purpose of the study, the researcher may want to take into account, if present, the disparity in firm sizes.

Chapter 13

Conclusion

In summary, nonparametric linear programming tests for consistency with the hypotheses of technical efficiency and allocative efficiency are proposed. The tests are formulated relative to three kinds of general technologies — convex, constant returns to scale and quasiconcave technologies. For the technical efficiency tests (tests 1, 2 and 3), the informational requirements are quantity data on production plans and an efficiency direction vector which corresponds to the subset of goods with respect to which inefficiency is to be measured. Relative to the general technology posited, technical efficiency is given a precise definition wherein being a boundary point of the production possibilities set is a necessary but not sufficient condition. The efficiency tests yield violation indices δ_i^*, invariant to scale in the measurement of goods, which give the equiproportionate adjustment in goods, as specified in the efficiency direction vector, needed for a particular observation to satisfy the hypotheses. Alternatively, the violation indices can be interpreted as measures of the loss or waste due to failure to maximize production of outputs or minimize the use of inputs. Dual interpretations of the technical efficiency definitions and tests are also presented to link the analysis to Koopmans' notion of efficiency prices and the concepts of profit maximization and cost minimization.

With price and quantity data, tests for allocative efficiency are proposed. Given a behavioral description of the firm as embodied by an objective function and the technological constraints under which the firm is operating, allocative efficiency entails the use of the optimal mix of outputs and inputs. For unconstrained optimization, the tests (7, 8 and 9) reduce to a comparison of inequalities. For a convex technology, a production plan z^i is allocatively efficient if

$$p^{iT}z^i \geq p^{iT}z^j, \; j = 1, 2, \ldots, J;$$

that is, if at prices $p^i \gg 0_N$, the production plan z^i yields the largest profit when compared to z^j, $j = 1, 2, \ldots, J$. For a constant returns technology, z^i is allocatively efficient, given z^i_n, if there exists a shadow price $q^*_n \geq 0$ for good n such that

$$0 = p^{niT} z^{ni} + q^*_n z^i_n \geq p^{niT} z^{nj} + q^*_n z^j_n,\ j = 0, 1, 2, \ldots, J$$

where z^0 is the shutdown point; that is, (z^{ni}, z^i_n) yields the maximal profit of zero at prices (p^{niT}, q^*_n). For a quasiconcave technology, z^i is allocatively efficient, given z^i_n, if

$$p^{niT} z^{ni} \geq p^{niT} z^{nj},\ j \in I^n_i;$$

that is, z^{ni} yields maximal net revenue for z^i_n among the alternative allocations in the upper level set of z^i_n. In the unconstrained optimization tests, the objective functions of the linear programming problems take the form of an unrestricted profit function in the convex case, a normalized profit function (whose value is the shadow price of the normalizing good) in the constant returns case, and a net revenue (or negative net cost function) in the quasiconcave case.

The unconstrained optimization tests are special cases of the more general linear programming tests for constrained optimization. If some goods are fixed or their competitive prices do not exist, then the constrained optimization tests are given by tests 4, 5 and 6. The objective functions of these linear programming problems are partial profit functions, restricted versions of the unconstrained forms. The allocatively efficient allocations are technically efficient in the sense that they lie on the boundary of the relevant convex set, production possibilities set or upper level set as the case may be, and with respect to the goods with positive market prices, do not lie on the free disposal region. The violation indices ε^*_i for unconstrained optimization and ε^S_i for constrained optimization give the proportion of the value of goods in the reference set lost due to allocative inefficiency. The waste is measured relative to the optimal value of the objective function.

A number of future areas of research are indicated by the limitations of the foregoing analysis. The violation indices proposed in this study have been interpreted so far to be measures of the degree of violation of the maintained efficiency hypotheses. A deterministic approach has been taken and the possibility of measurement errors ignored. The researcher may have prior reasons to suspect data errors and the efficiency test can be alternatively used to explore how "good" or "bad" the data is before parametric estimation of production, cost or profit functions. To address the problem of measurement error, Hanoch and Rothschild (1972) suggest some ad hoc ways to modify the tests and Varian (1985) offers a procedure in the context of statistical hypothesis testing.

The assumption of nonincreasingness or free disposal in our regularity conditions for the various technologies rules out the possibility of congestion. Firms may produce multiple outputs, some of which like pollution may be undesirable. In the framework of this study, undesirable output goods, if measurable, can be treated as negative outputs and hence would enter the tests as inputs. If indeed congestion is present, our technical efficiency and constrained efficiency tests would tend to overestimate the true violation index since the constructed convex set would be larger than the true production possibilities set. A revision of the technical efficiency tests to satisfy the DEA definition of efficiency would also yield an "unbiased" violation index since the chosen efficient points would not be in any free disposal region of the boundary of the constructed convex set. If market prices do not exist for the "undesirable outputs", then the constrained optimization tests can be performed. If (positive) market prices do exist, then these prices ensure that the allocative efficiency tests would yield the optimal amounts of the undesirable outputs. For further studies on nonparametric efficiency analysis with congestion, the reader is referred to Färe, Grosskopf and Lovell (1985) and Färe, Grosskopf, Lovell and Pasurka (1989).[1]

In the real world too we may observe negative profits or firms producing at a loss. This can sometimes be explained by dynamic considerations such as expectations, strategic behavior and adjustment processes (such as entry and exit of firms). The basically static approach adopted in this study abstracts from these complications. In the tradition of nonparametric

[1] The above authors also use hyperbolic efficiency measures in contrast to the equiproportionate adjustment measures defined by the violation indices in this study.

production analysis in economics, the framework in this study can be extended to testing for other regularity conditions on the technology, such as homotheticity and separability, and to forecasting behavior. These extensions may require solving quadratic or nonlinear programming problems instead of linear programming problems.

Part II

Nonparametric and Parametric Measures of Technical Progress: An Application to the Canadian Input-Output Data

Chapter 14

Introduction

Nonparametric and parametric measures of technical progress are compared in this study. The nonparametric measures of technical progress are obtained by a reinterpretation of the violation indices yielded by the efficiency tests of part I. In this context, we assume optimizing behavior on the part of the producers and the violation indices as indicators of the shift in the production frontier. The parametric measures of technical progress are obtained through econometric profit function estimation explicitly incorporating technical change variables. The time variable is used as a proxy for the technical change variables and is introduced additively through a quadratic spline subfunction developed by Diewert and Wales (1989b). The symmetric generalized McFadden functional form, also introduced by Diewert and Wales (1987), is used in the profit function estimation. For convenience (though unnecessary), constant returns to scale is assumed.

Annual Canadian input-output data for the period 1961–1980 are used. Measures of technical progress are estimated for four aggregated sectors:

I. resources sector;

II. manufacturing sector, export market-oriented;

III. manufacturing sector, domestic market-oriented; and the

IV. services sector.

We assume the sectors have single output technologies. The resources sector (I) has ten output and input goods; they are

(1) resource goods (from sector I),
(2) manufactured goods (from sector II),
(3) manufactured goods (from sector III),
(4) service goods (from sector IV),
(5) imports,
(6) labor,
(7) inventories,
(8) machinery and equipment,
(9) structures, and
(10) land.

For the resources sector (I), the output good is good 1 and the other nine goods are all input goods. The other three sectors (II, III and IV) each has nine output and input goods; since the quantity data on structures and land for these sectors are proportional in the original database, these two goods have been aggregated as a Leontief composite good which we have termed as "structures" (good 9) in the current data set. The capital rental prices for the capital inputs (goods 7–10) are calculated using internal rates of return. A detailed description of the construction of the data set for the empirical estimation performed in this study can be found in appendix B.

To calculate the nonparametric measures of technical progress, the allocative efficiency test for a convex conical technology assuming complete profit maximization (test 8 in part I) was performed for each of the four sectors with structures as the normalizing good. For each sector, a unit scale profit function was also estimated with structures as the scale variable. The measure of technical progress is then defined as the marginal change in the return to this factor (structures) due to the passage of a time period divided by the value of some basket of reference goods.

For a particular sector, let there be $N+1$ goods ($N+1 = 10$ for sector I, and $N+1 = 9$ for sectors II, III and IV). Let the $(N+1)$th good be the normalizing good or the good chosen to be the scale variable. In the allocative efficiency test and the profit function estimation performed in this study, the price of this good is not used. However, we use quantity data on this good; denote the quantity of this good by z_{N+1}. For the quantity data, outputs are indexed positively and inputs are indexed negatively. Denote the price vector of the first N goods at period t as $p^t \equiv (p_1^t, p_2^t, \ldots, p_N^t)^T \gg 0_N$ and the quantity vector at period t as $z^t \equiv (z_1^t, z_2^t, \ldots, z_N^t)^T$. Let $\pi(p^t, t)$ be the period t unit scale profit function. If we denote the period t unit scale production possibilities set containing the feasible z vectors when $| z_{N+1} | = 1$[1] by $\mathcal{S}^t$, then the unit scale profit function $\pi(p^t, t)$ is defined as

$$\pi(p^t, t) \equiv \max_z \{p^T z : \; z \in \mathcal{S}^t\}. \tag{14.1}$$

Under the assumption of constant returns to scale and finite production plans, the optimal profit defined over the whole set of $N+1$ goods is zero. Hence, if an input is chosen as the scale variable, then the unit scale profit function defined by (14.1) gives the per unit return to the normalizing or scaling input. If a capital good which can be considered a fixed factor is chosen as the scale variable, then the unit scale profit function in (14.1) can be interpreted as a short-run or restricted or variable (unit scale) profit function defined over the first N variable goods.

Mathematically, we define an index of technical progress at period t, say Δ^t, as

$$\Delta^t \equiv \frac{\frac{\partial \pi(p^t, t)}{\partial t}}{\sum_{m \in E} p_m^t \mid \bar{z}_m^t \mid} \tag{14.2}$$

where E is the set containing the indices of the chosen reference goods and $m \neq N+1$, and $\bar{z}_m^t \equiv z_m^t / \mid z_{N+1}^t \mid$ where $\mid z_{N+1}^t \mid$ is the absolute value of the quantity of good $N+1$, the normalizing good, at period t. Note that the numerator in (14.2) can be positive, zero or negative while the denominator is always positive. Hence, $\Delta^t > 0$ implies technical progress at period t and $\Delta^t < 0$ implies technical regress at period t.

[1]Since a convex cone is closed under addition and nonnegative scalar multiplication, then the projection of the production possibilities set in $(N+1)$-dimensional space into the unit scale production possibilities set in N-dimensional space is harmless.

The numerator in equation (14.2) is the marginal change in the per unit return to the normalizing (or scaling) good $N+1$. If we multiply the numerator and denominator by $|\, z^t_{N+1} \,|$, we obtain

$$\Delta^t \equiv \frac{\frac{\partial \pi(p^t,t)|z^t_{N+1}|}{\partial t}}{\sum_{m \in E} p^t_m \,|\, z^t_m \,|}. \tag{14.3}$$

The numerator in (14.3) gives the marginal change in the return to the normalizing good $N+1$ at period t due to the passage of a time period. Alternatively, the numerator in (14.3) can be interpreted as the change in the value of outputs minus the change in the value of inputs other than the normalizing good due to the passage of a time period. The denominator in (14.3) has unscaled quantity variables.

For the empirical exercise, we focus on the private domestic production sector and the economy's output goods 1–4 are chosen as the reference goods. At the sectoral level, some of these goods enter as intermediate inputs. If $\Delta^t > 0$, then the index of technical progress gives the proportional increase in the sector's contribution to the economy's value of output goods available for final demand (including exports) due to technical change occuring in that sector.

The index of technical change defined by equation (14.2) uses price information. If we define a multifactor productivity measure as the ratio of a quantity index of output goods to a quantity index of input goods, then to measure technical change it is imperative to net out relative price effects as reflected in the substitution of goods. Both the nonparametric approach using the allocative efficiency tests developed in part I and the parametric approach using either profit or cost function econometric estimation are able to isolate these relative price effects. In the efficiency tests, a deterministic approach is taken and the residual unexplained component of the multifactor productivity measure is all attributed to technical change. Hence, we can expect the estimates of the index of technical progress obtained using the nonparametric approach to be quite erratic or to display a greater degree of variability.

On the other hand, the stochastic approach requires the specification of the distribution of the disturbance terms. In the profit function estimation performed in this study, the usual error structure — errors are distributed normally with zero mean and covariance structure Σ — for simultaneous equations is assumed. The quadratic spline subfunction used in modeling technical change attempts to be less restrictive and have greater capability than the simple quadratic subfunction in capturing the ups and downs of the rates of technical change. However, compared to the nonparametric efficiency tests, we can expect the profit function estimates of the index of technical progress to display a smoother behavior. The empirical exercise that follows aims to investigate the comparative behavior of the estimates of the index of technical progress obtained under the nonparametric and parametric approaches.

Chapter 15

A Nonparametric Approach to Measuring Technical Progress

The dual representations of production technologies — either through profit or cost functions — assume optimizing behavior on the part of the producers. As was noted in part I, the efficiency tests developed are based on the joint hypotheses on the technology and the behavior of the producer. Hence, a positive violation index can be due to either of two sources. For the sectoral aggregated data we are using, the attribution of positive violation indices to failure of the producers to respond optimally may be suspect. We can interpret the positive violation indices as due to violations of the technology assumptions; in particular, that no technical change is occuring and the true underlying technology remains constant from time period to another. If indeed technological change is occuring, then the production possibilities set may be expanding or shrinking over time.

Since we assume constant returns to scale, the allocative efficiency test assuming complete profit maximization for a convex cone technology (test 8 in part I) was performed for each of the four sectors.[1] Neither the shutdown point nor an unbounded solution (since an input, structures, was used as the normalizing good) was obtained as the optimal allocation for any observation; hence, the allocative efficiency unconstrained optimization violation index ε_t^* (see equation (11.12), part I) can be expressed as

$$\varepsilon_t^* \equiv \max_j \left\{ \frac{p^{tT}(\bar{z}^j - \bar{z}^t)}{\sum_{m \in E} p_m^t \mid \bar{z}_m^t \mid}, \; j = 1, 2, \ldots, T \right\} \tag{15.1}$$

where $\bar{z}^j \equiv (\bar{z}_1^j, \bar{z}_2^j, \ldots, \bar{z}_N^j)^T \equiv z^j / \mid z_{N+1}^j \mid$, and $T \equiv$ maximum time index. The violation indices using test 8 in part I for the four sectors are listed in table 15.2 and are plotted in

[1]Test 8 was also performed for the data set using capital rental prices based on an exogenous bond rate. The results even in magnitudes of the violation indices in all four sectors do not differ significantly from those reported here for the data set using capital rental prices based on internal rates of return.

	violation index ε_t^* using test 8			
year,t	sector I	sector II	sector III	sector IV
1961	0.10542	0.15834	0.14341	0.13116
1962	0.05616	0.12018	0.10799	0.10106
1963	0.02377	0.10041	0.10086	0.08981
1964	0.02448	0.08306	0.08327	0.06938
1965	0.02328	0.07228	0.07604	0.05787
1966	0.00000	0.08498	0.07670	0.04788
1967	0.06135	0.08632	0.08497	0.04601
1968	0.03270	0.06943	0.07018	0.03466
1969	0.01889	0.04745	0.06054	0.03407
1970	0.03312	0.07279	0.06436	0.02983
1971	0.03515	0.06656	0.05363	0.02982
1972	0.03247	0.05035	0.03759	0.02076
1973	0.00000	0.03097	0.01830	0.01434
1974	0.04741	0.02919	0.02238	0.01648
1975	0.10403	0.04823	0.04690	0.01560
1976	0.10567	0.02656	0.02105	0.00425
1977	0.12384	0.00984	0.00738	0.01277
1978	0.15356	0.00000	0.00318	0.01393
1979	0.16163	0.01241	0.00000	0.00596
1980	0.18950	0.02804	0.00096	0.00000

Table 15.2: Test 8 violation indices ε_t^* for the four sectors

figures 15.9–15.12.

The violation indices ε_t^* can be interpreted as chained indices of technical change. Let us denote the nonparametric estimate of the period t index of technical progress defined in equation (14.2) (or equation (14.3)) by $\hat{\Delta}^t$. The nonparametric index of technical progress $\hat{\Delta}^t$ can then be defined as

$$\begin{aligned} \hat{\Delta}^t &\equiv \frac{(1+\varepsilon_{t-1}^*)}{(1+\varepsilon_t^*)} - 1 \\ &= \frac{\varepsilon_{t-1}^* - \varepsilon_t^*}{(1+\varepsilon_t^*)}. \end{aligned} \tag{15.2}$$

The values of $\hat{\Delta}^t$ for the four sectors for the period 1962–1980 are listed in table 15.3 and are plotted in figures 15.13–15.16.

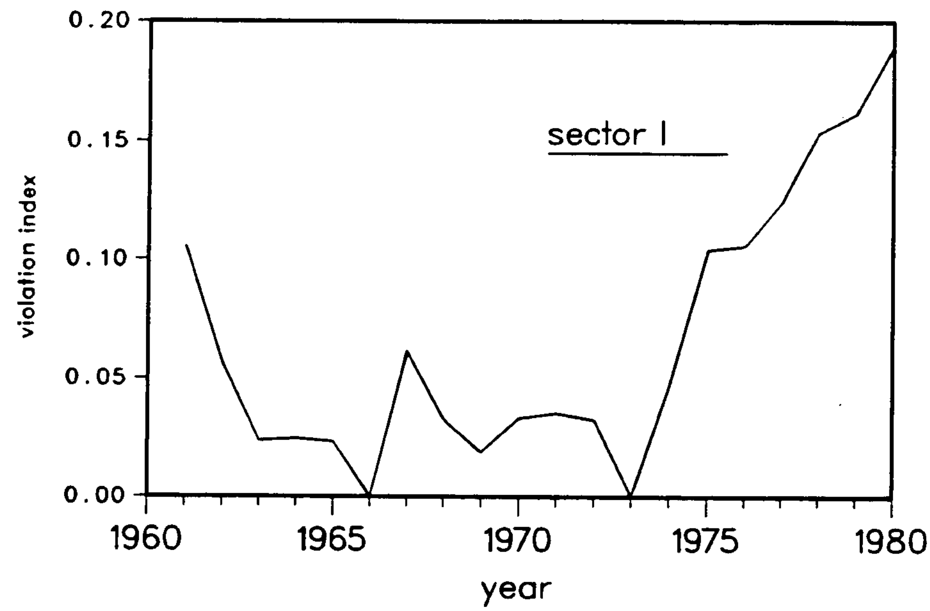

Figure 15.9: Test 8 violation indices ε_t^*, sector I

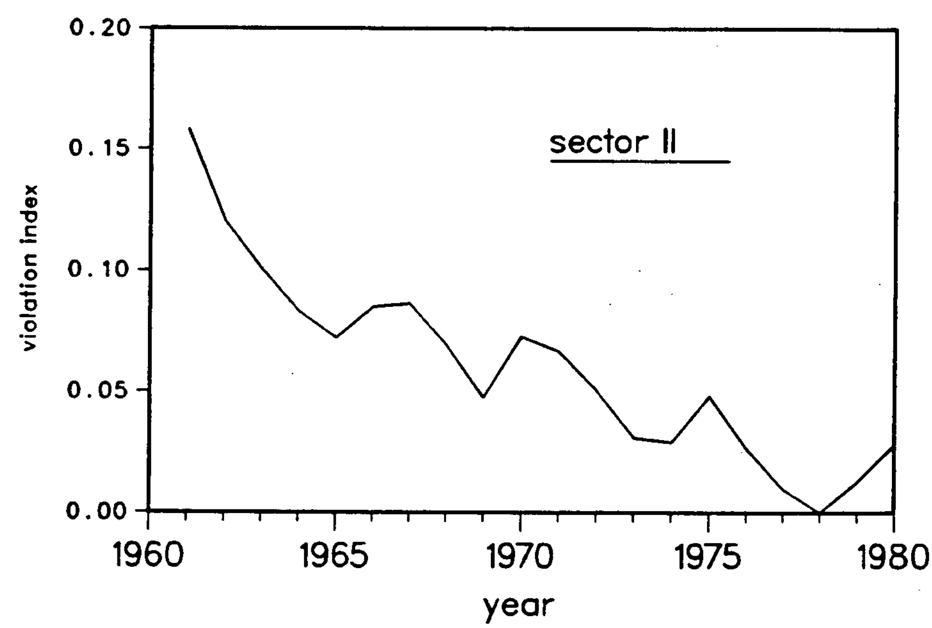

Figure 15.10: Test 8 violation indices ε_t^*, sector II

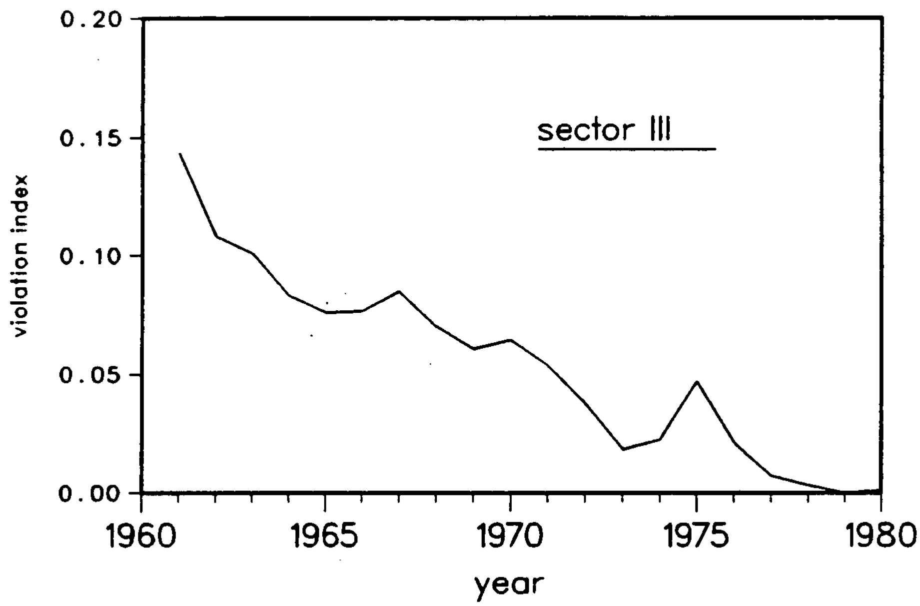

Figure 15.11: Test 8 violation indices ε_t^*, sector III

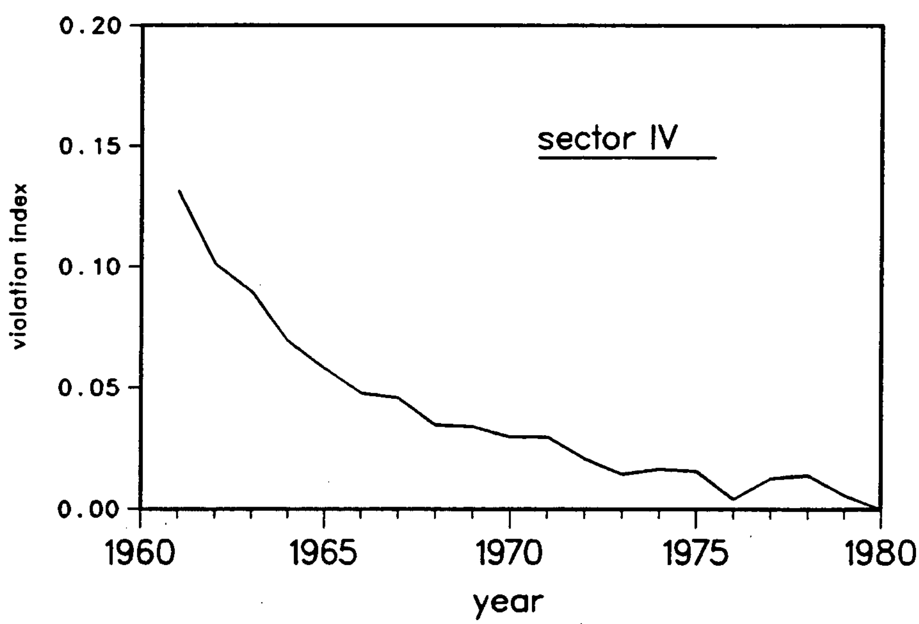

Figure 15.12: Test 8 violation indices ε_t^*, sector IV

year, t	nonparametric index $\hat{\Delta}^t$			
	sector I	sector II	sector III	sector IV
1962	0.04664	0.03406	0.03196	0.02733
1963	0.03164	0.01797	0.00648	0.01033
1964	-0.00069	0.01602	0.01624	0.01910
1965	0.00118	0.01006	0.00672	0.01088
1966	0.02328	-0.01171	-0.00061	0.00953
1967	-0.05780	-0.00123	-0.00762	0.00178
1968	0.02774	0.01579	0.01383	0.01097
1969	0.01355	0.02099	0.00909	0.00057
1970	-0.01377	-0.02362	-0.00359	0.00412
1971	-0.00196	0.00584	0.01019	0.00002
1972	0.00259	0.01543	0.01545	0.00888
1973	0.03247	0.01880	0.01894	0.00633
1974	-0.04526	0.00172	-0.00399	-0.00211
1975	-0.05128	-0.01817	-0.02341	0.00087
1976	-0.00149	0.02111	0.02532	0.01130
1977	-0.01617	0.01656	0.01357	-0.00841
1978	-0.02576	0.00984	0.00418	-0.00115
1979	-0.00695	-0.01225	0.00318	0.00793
1980	-0.02343	-0.01521	-0.00096	0.00596

Table 15.3: Nonparametric indices of technical progress $\hat{\Delta}^t$ for the four sectors

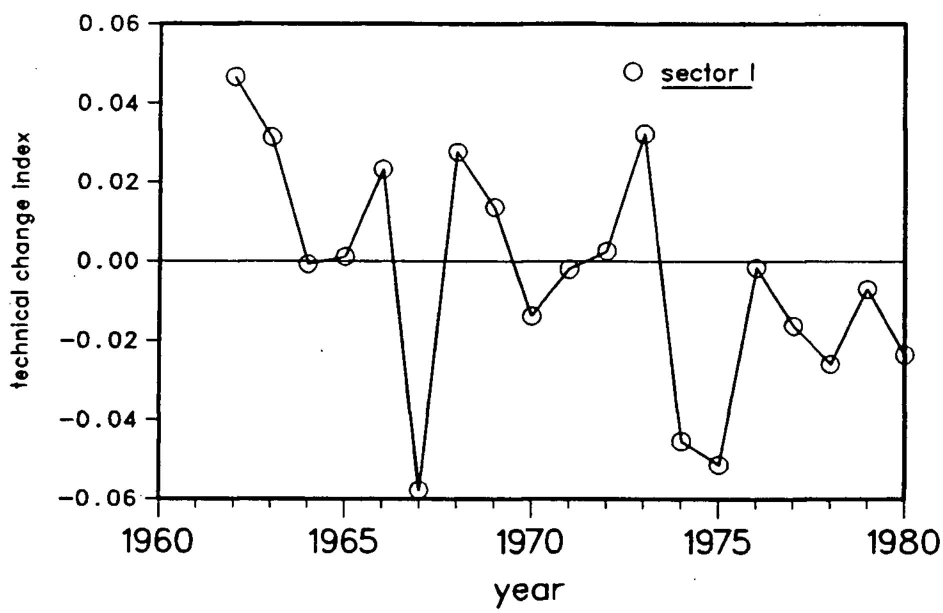

Figure 15.13: Nonparametric indices $\hat{\Delta}^t$, sector I

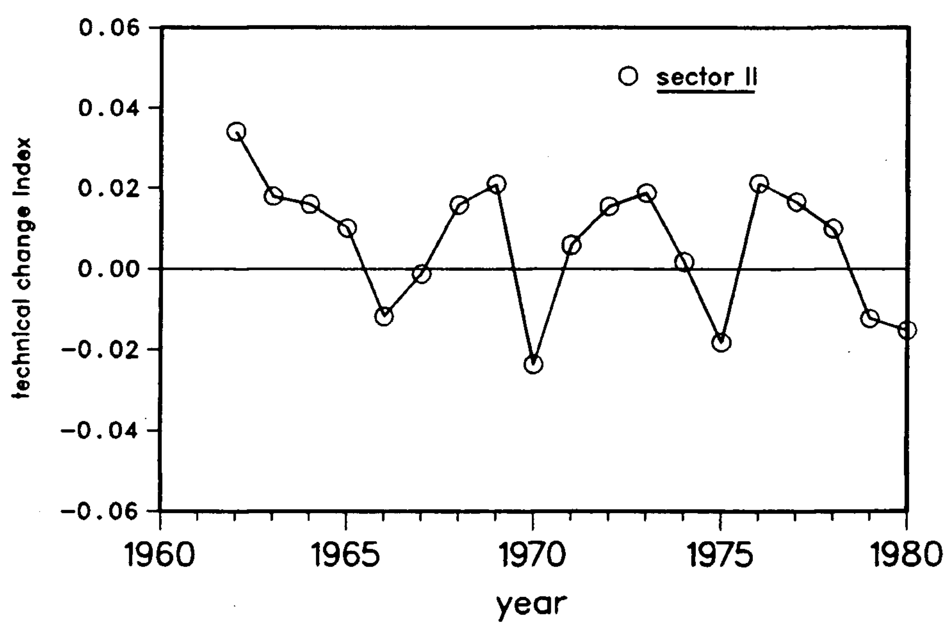

Figure 15.14: Nonparametric indices $\hat{\Delta}^t$, sector II

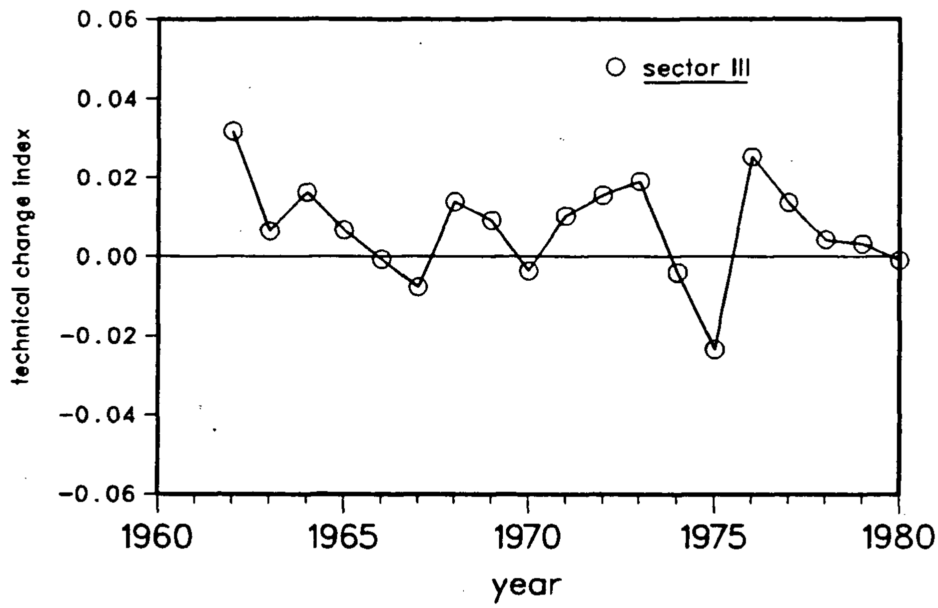

Figure 15.15: Nonparametric indices $\hat{\Delta}^t$, sector III

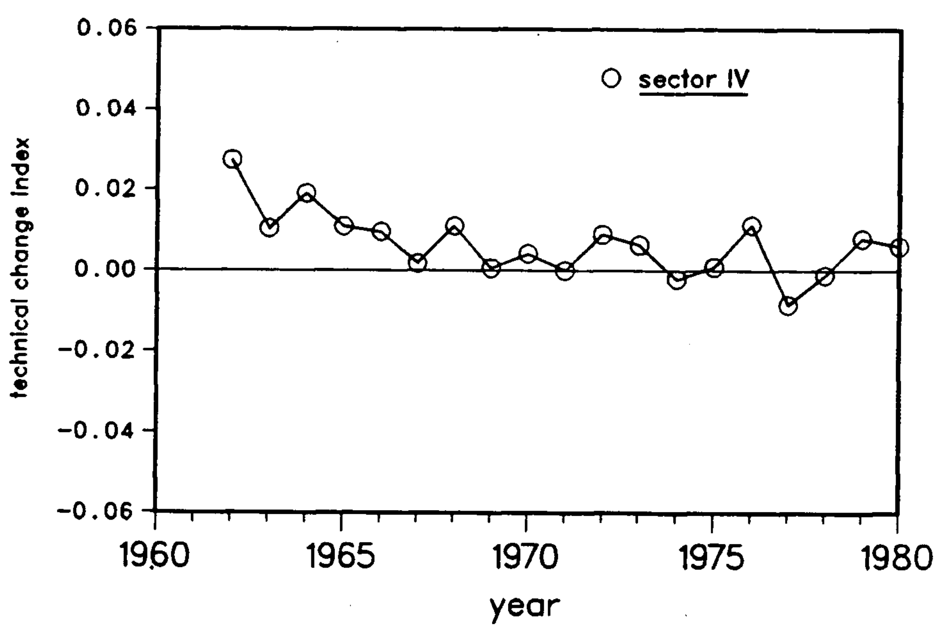

Figure 15.16: Nonparametric indices $\hat{\Delta}^t$, sector IV

The results obtained using the nonparametric index $\hat{\Delta}^t$ indicate that technological regress occured during the following years in the given sectors:

for sector I:	1964, 1967, 1970, 1971, 1974–1980;
for sector II:	1966, 1967, 1970, 1975, 1979, 1980;
for sector III:	1966, 1967, 1970, 1974, 1975, 1980; and
for sector IV:	1974, 1977, 1978.

These findings are given support by the corresponding efficiency test incorporating the no technological regress assumption, test 8′ in appendix A, which was also performed for each of the four sectors. The results are given in table 15.4; the values of ε_t^* are given only for the years violating the efficiency and no technological regress assumption of test 8′. Almost the same set of years is discerned by the nonparametric index $\hat{\Delta}^t$ (obtained by unchaining the violation indices ε_t^* from test 8) and the efficiency and no technological regress test 8′ to be periods where technological regress occured. For most years where the results conflict either the nonparametric index $\hat{\Delta}^t$ is small or the violation index ε_t^* for test 8′ is small.

Next we compare the nonparametric index of technical progress $\hat{\Delta}^t$ with total factor productivity measures based on the index number approach. Divisia and Fisher indices of productivity change were calculated for each of the four sectors of the Canadian economy. In the economic approach to index numbers based on the assumption of optimizing behavior on the part of the producers, Caves, Christensen and Diewert (1982) and Diewert (1989b) have shown that the Divisia (or Törnqvist or translog) and the Fisher productivity change indices, being exact for some flexible functional form, are superlative. The Fisher ideal quantity index has the added property of being the only function that satisfies some twenty tests considered desirable in the axiomatic approach to index number theory (Diewert, 1989b). The econometric computer package SHAZAM by K.J. White (1987) was used to compute the productivity indices.

For each sector, chained output and input Divisia and Fisher ideal quantity indices are obtained for each year t. The output quantity indices are defined over the four output goods (goods 1–4) with quantities of intermediate inputs indexed negatively. The input quantity

year, t	violation index ε_t^*	observation relatively efficient to t
sector I:		
1964	0.001795	1963
1967	0.061346	1966
1968	0.032703	1966
1969	0.015616	1966
1970	0.031200	1966
1971	0.016859	1966
1972	0.009744	1966
1974	0.047410	1973
1975	0.104030	1966
1976	0.105673	1966
1977	0.123843	1966
1978	0.153557	1966
1979	0.161633	1966
1980	0.189503	1966
sector II:		
1966	0.007065	1965
1967	0.003043	1965
1970	0.021252	1969
1971	0.009695	1969
1975	0.012834	1974
1979	0.012405	1978
1980	0.028041	1978
sector III:		
1967	0.000555	1966
1974	0.003566	1973
1975	0.025523	1973
1980	0.000963	1979
sector IV:		
1977	0.009799	1976
1978	0.011078	1976
1979	0.001898	1976

Table 15.4: Test 8′ violation indices ε_t^* for the four sectors

indices (with input quantities now indexed positively) are defined over the primary inputs: goods 5–10 for the resources sector and goods 5–9 for the other sectors. Output growth and input growth indices for year t are calculated by dividing the current year t's quantity index by its lagged value. A productivity change index is then defined as the ratio of the output growth index to the input growth index minus one; that is,

$$\tau^t \equiv \frac{\text{output growth index}}{\text{input growth index}} - 1 \tag{15.3}$$

where τ^t is the productivity change index for year t. If $\tau^t > 0$, then there occurs a productivity improvement at period t or an outward shift of the production frontier from year $t-1$ to year t. If $\tau^t \leq 0$, then no productivity improvement occurs at period t.

Let us denote by τ_D^t the Divisia productivity change index for year t and by τ_F^t the Fisher productivity change index for year t. The values of τ_D^t and τ_F^t for the different sectors are listed in tables 15.5–15.6. As reflected too in figures 15.17–15.20 where their curves coincide, the Divisia and Fisher productivity indices are almost identical. The magnitude of the nonparametric index $\hat{\Delta}^t$ is not expected to be directly comparable to the magnitudes of the productivity indices τ_D^t and τ_F^t since the technical progress index Δ^t (equation (14.2)) and productivity index τ^t (equation (15.3)) are defined differently. The interpretation of $\hat{\Delta}^t$, τ_D^t and τ_F^t as measures of the shift in the production frontier (or its dual representations) rests on the assumption of optimizing behavior on the part of the producers. The consistency of the patterns of behavior of the three indices, as illustrated in figures 15.17–15.20, lends support to the reinterpretation of the violation indices ε_t^* in (15.1) as chained indices of technical progress in the context of the time series data used.

	Divisia productivity change index τ_D^t			
year, t	sector I	sector II	sector III	sector IV
1962	0.11498	0.04388	0.06869	0.03527
1963	0.06345	0.03174	0.01168	0.01737
1964	-0.00307	0.02517	0.03573	0.02765
1965	0.00475	0.02290	0.02055	0.01813
1966	0.04396	-0.01614	0.00786	0.02019
1967	-0.09872	0.00678	-0.00185	0.00997
1968	0.04993	0.03716	0.04377	0.02325
1969	0.02422	0.04411	0.02634	0.00492
1970	-0.02681	-0.04532	0.00216	0.00878
1971	0.00658	0.02272	0.03059	0.00635
1972	0.00992	0.02952	0.03502	0.01614
1973	0.06629	0.04119	0.04242	0.01274
1974	-0.07655	0.01089	-0.00846	-0.00055
1975	-0.06721	-0.02821	-0.04545	0.00428
1976	-0.00233	0.04215	0.06297	0.01766
1977	-0.01805	0.03516	0.03131	-0.01253
1978	-0.04178	0.01991	0.00926	-0.00095
1979	0.01316	-0.02534	0.00726	0.01171
1980	-0.01039	-0.03163	-0.00282	0.00875

Table 15.5: Divisia productivity change indices τ_D^t for the four sectors

	Fisher productivity change index τ_F^t			
year,t	sector I	sector II	sector III	sector IV
1962	0.11499	0.04388	0.06871	0.03575
1963	0.06345	0.03173	0.01168	0.01737
1964	-0.00307	0.02517	0.03573	0.02765
1965	0.00475	0.02290	0.02055	0.01813
1966	0.04397	-0.01614	0.00787	0.02018
1967	-0.09868	0.00677	-0.00186	0.00996
1968	0.04994	0.03715	0.04377	0.02324
1969	0.02423	0.04411	0.02634	0.00492
1970	-0.02682	-0.04533	0.00214	0.00878
1971	0.00639	0.02268	0.03057	0.00569
1972	0.00991	0.02951	0.03503	0.01614
1973	0.06692	0.04119	0.04263	0.01274
1974	-0.07661	0.01095	-0.00842	-0.00053
1975	-0.06723	-0.02828	-0.04553	0.00427
1976	-0.00232	0.04214	0.06297	0.01765
1977	-0.01805	0.03516	0.03130	-0.01252
1978	-0.04178	0.01991	0.00926	-0.00093
1979	0.01317	-0.02534	0.00727	0.01173
1980	-0.01035	-0.03164	-0.00283	0.00874

Table 15.6: Fisher productivity change indices τ_F^t for the four sectors

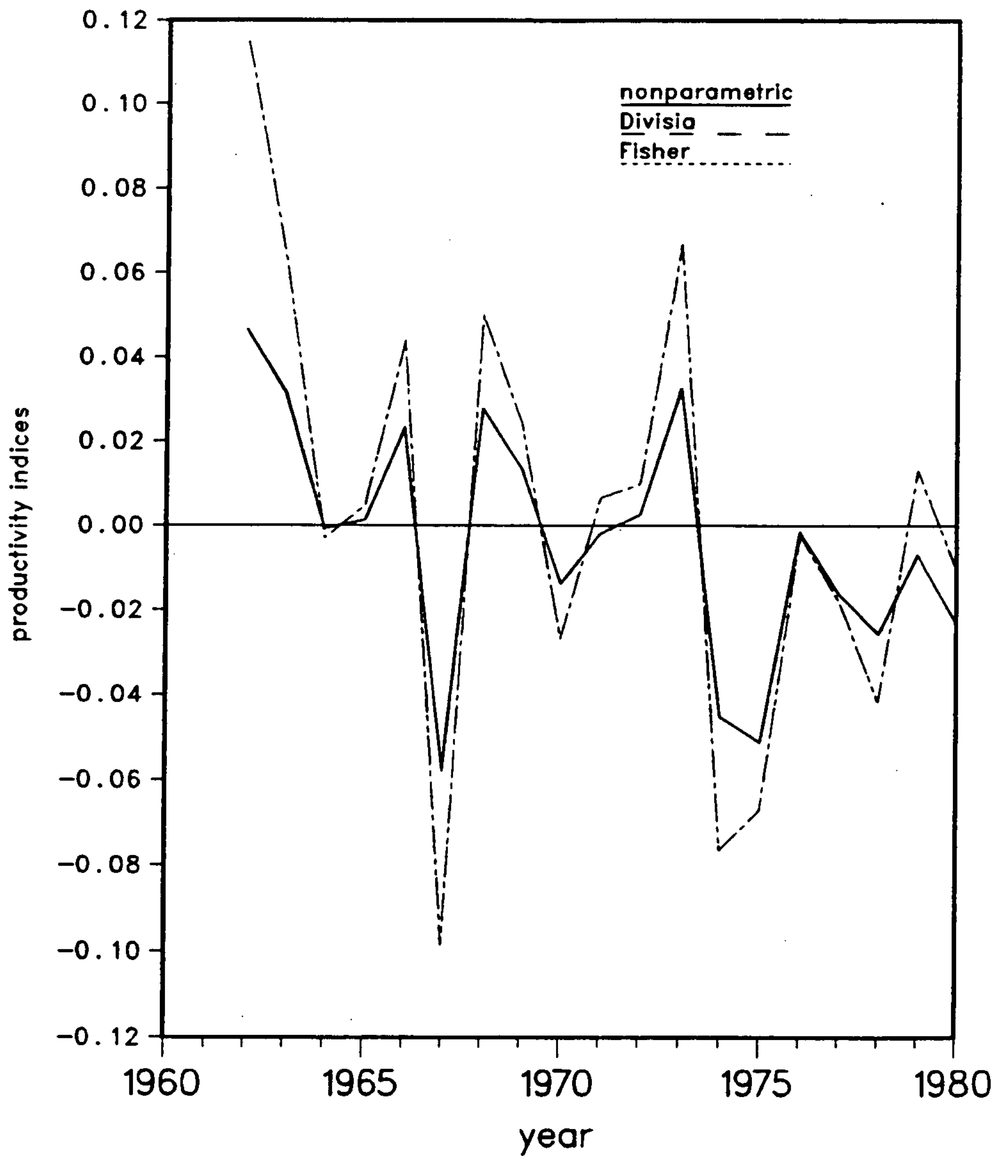

Figure 15.17: Comparative behavior of $\hat{\Delta}^t$, τ_D^t and τ_F^t, sector I

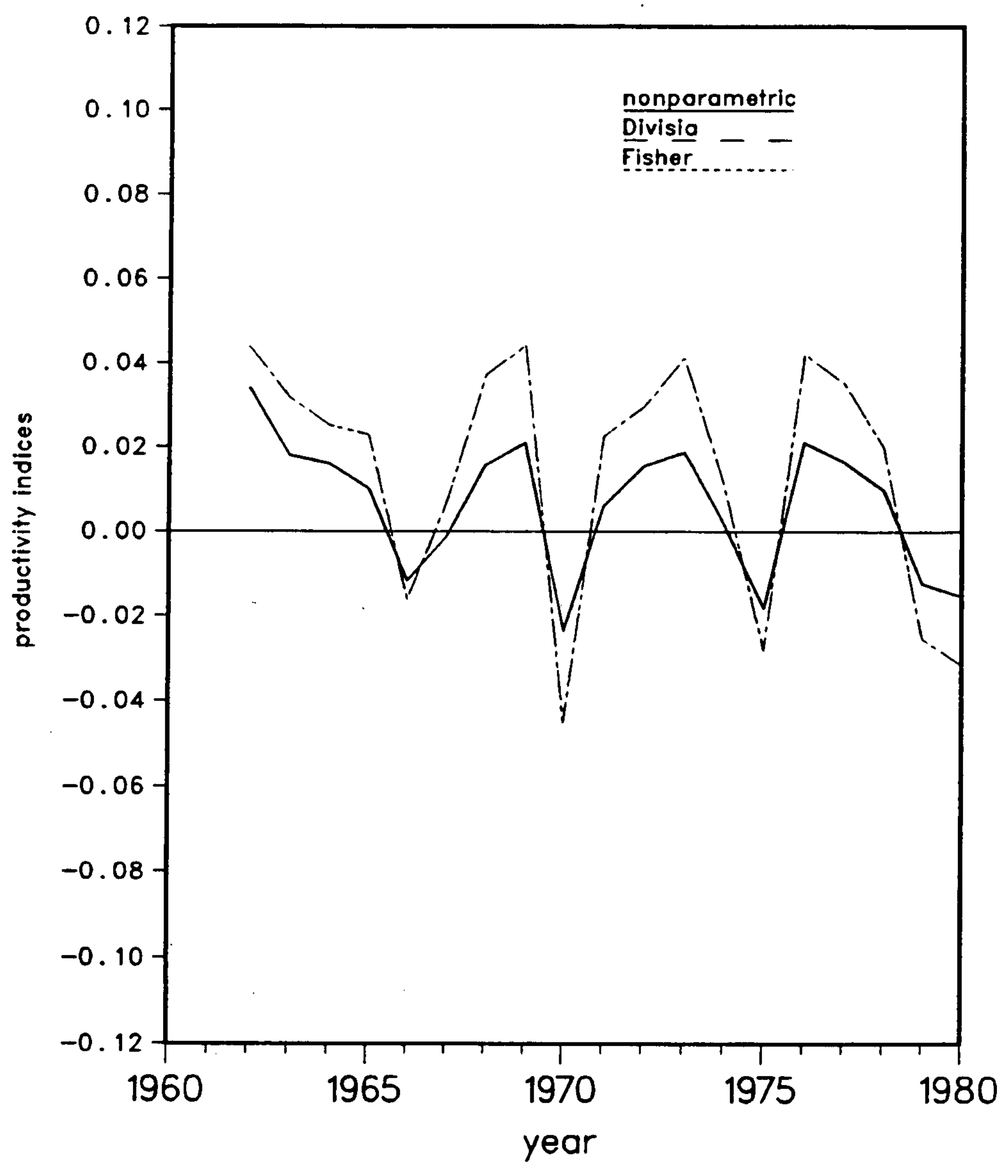

Figure 15.18: Comparative behavior of $\hat{\Delta}^t$, τ_D^t and τ_F^t, sector II

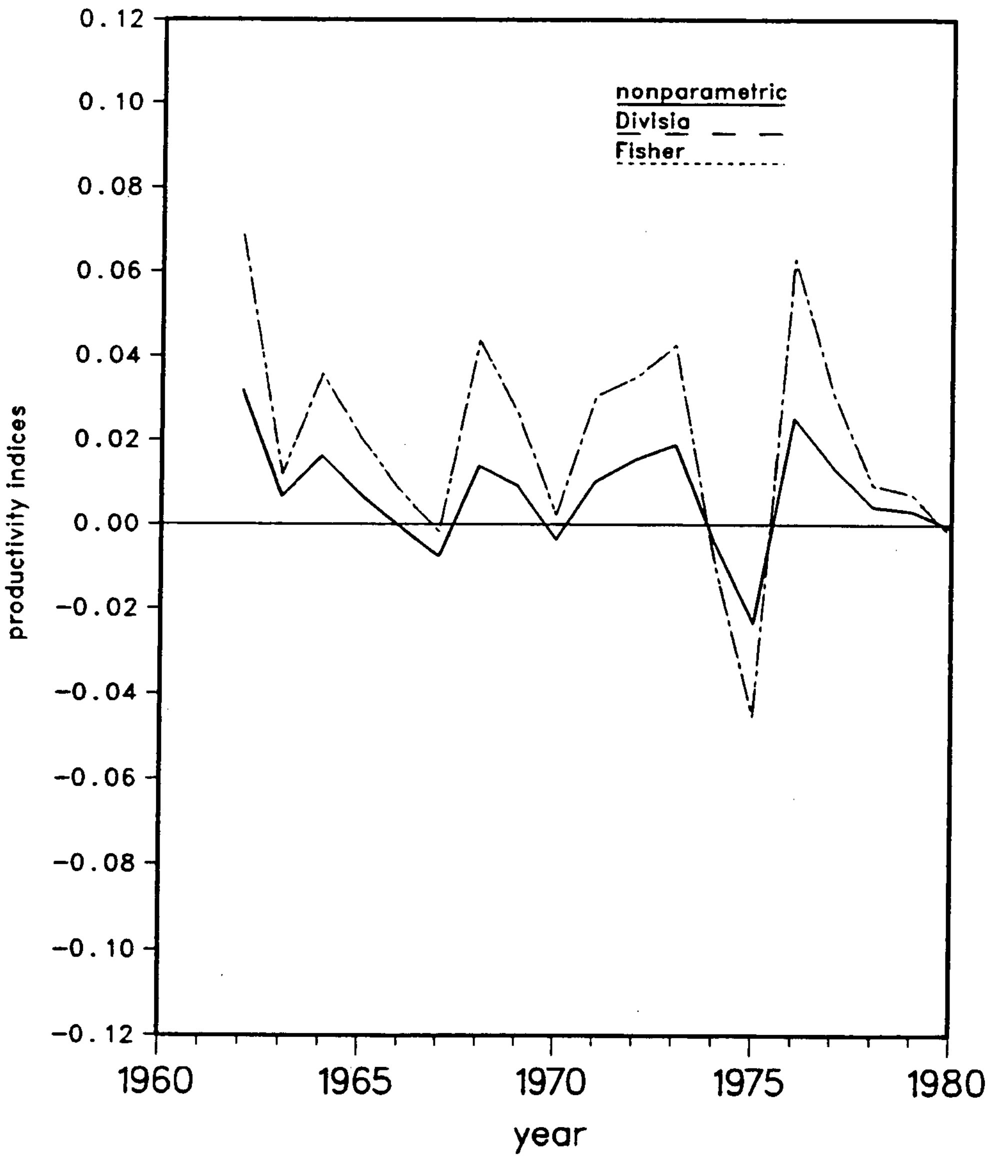

Figure 15.19: Comparative behavior of $\hat{\Delta}^t$, τ_D^t and τ_F^t, sector III

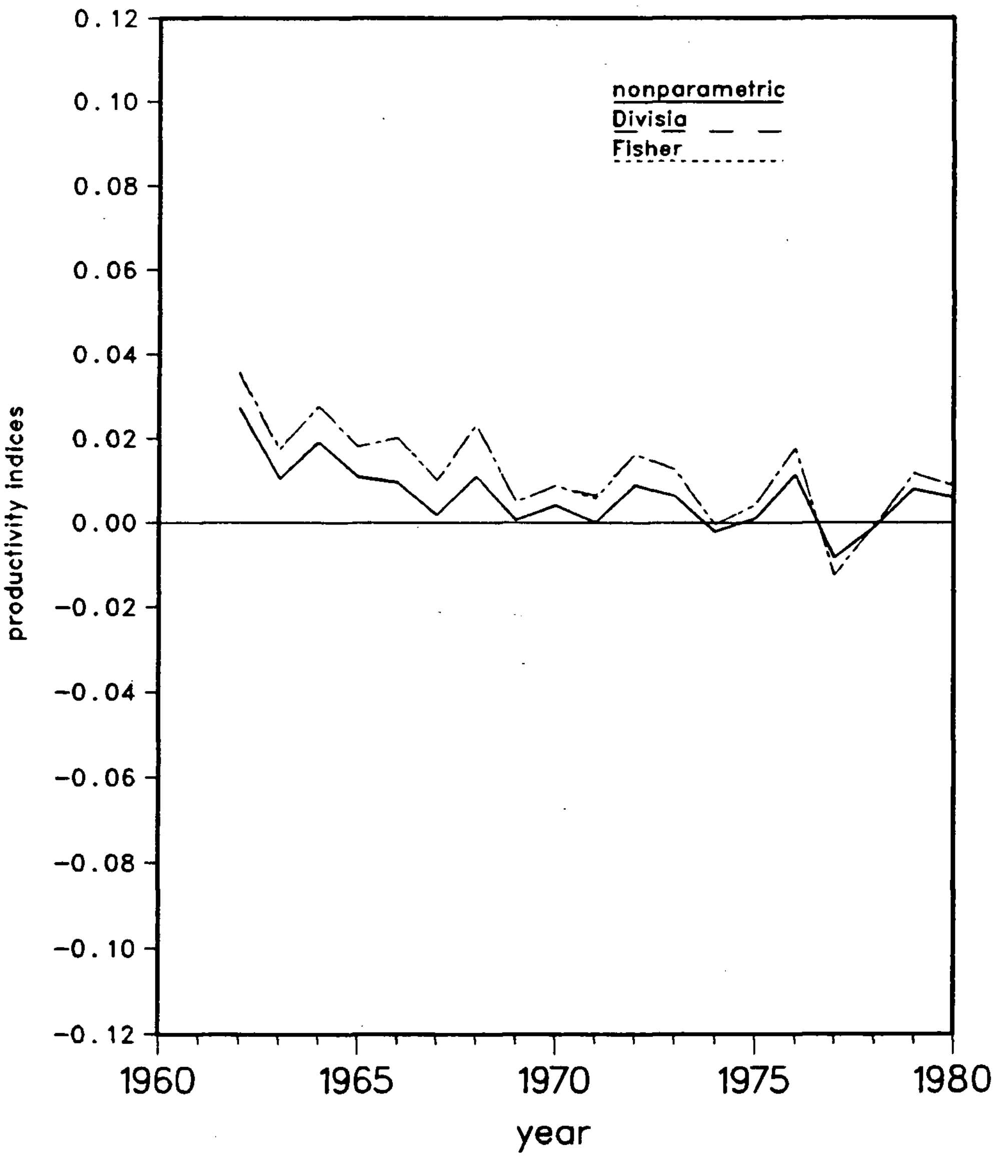

Figure 15.20: Comparative behavior of $\hat{\Delta}^t$, τ_D^t and τ_F^t, sector IV

Chapter 16

A Parametric Approach to Measuring Technical Progress

In the parametric approach to estimating the index of technical progress defined by equation (14.2), a functional form is posited for the unit scale profit function. The symmetric generalized McFadden flexible functional form with a quadratic spline model for technical progress, proposed by Diewert and Wales (1989b), is used. This model for the unit scale profit function has the property of being TP (technical progress) flexible. The unit scale profit function takes the following form:

$$\pi(p,t) \equiv h(p) + d(p,t) \tag{16.1}$$

where

$$h(p) \equiv p^T b^1 + \frac{1}{2}\left(\frac{p^T B p}{\alpha^T p}\right) \tag{16.2}$$

and

$$d(p,t) \equiv \begin{cases} p^T b^2 t + \frac{1}{2} p^T b^3 t^2 & \text{for } t \leq t_1; \\ \\ p^T b^2 t + \frac{1}{2} p^T b^3 t_1^2 + p^T b^3 (t - t_1) t_1 & \\ \qquad + \frac{1}{2} p^T b^4 (t - t_1)^2 & \text{for } t_1 \leq t \leq t_2; \\ \\ p^T b^2 t + \frac{1}{2} p^T b^3 t_1^2 + p^T b^3 (t_2 - t_1) t_1 & \\ \qquad + \frac{1}{2} p^T b^4 (t_2 - t_1)^2 + p^T b^3 (t - t_2) t_1 & \\ \qquad + p^T b^4 (t - t_2)(t_2 - t_1) + \frac{1}{2} p^T b^5 (t - t_2)^2 & \text{for } t_2 \leq t \leq T. \end{cases} \tag{16.3}$$

The exogenous parameters of the model are $\alpha \equiv (\alpha_1, \alpha_2, \ldots, \alpha_N)^T \geq 0_N$ in (16.2) and the break points in the quadratic spline model t_1 and t_2 in (16.3). The endogenous parameters are the N-dimensional vectors b^1, b^2, b^3, b^4 and b^5, and the NxN matrix B. Convexity of the profit function in prices is imposed by restricting the B matrix in (16.2) to satisfy the restrictions: $B = B^T$ and B is positive semidefinite. For identification, a reference vector p^* is chosen such that $Bp^* = 0_N$.

The Diewert-Wales (1989b) method of specifying the α vector was followed. The price vector for 1971 was arbitrarily chosen as p^*. A system of N equations corresponding to output supply and input demand equations for each sector was estimated by maximum likelihood estimation using nonlinear optimization routines. To aid convergence, the semiflexible estimation technique (Diewert and Wales, 1988) wherein the rank of the B matrix is restricted was used. For each sector, several runs of the model using different break points t_1 and t_2 were performed to search for those which yield higher values of the likelihood function. Table 16.7 presents the R^2 and log likelihood values, the number of parameters estimated, the rank of the B matrix and the values of t_1 and t_2 for the final models selected for the four sectors.[1]

Denote the parametric estimator of the index of technical progress (given in (14.2)) obtained using the functional form defined by equations (16.1)–(16.3) as $\tilde{\Delta}^t$. The numerator in this estimator is then

$$
\begin{aligned}
\frac{\partial \pi(p,t)}{\partial t} &= \frac{\partial d(p,t)}{\partial t} \\
&= \begin{cases} p^T b^2 + p^T b^3 t & \text{for } t \le t_1; \\ p^T b^2 + p^T b^3 t_1 + p^T b^4 (t - t_1) & \text{for } t_1 \le t \le t_2; \\ p^T b^2 + p^T b^3 t_1 + p^T b^4 (t_2 - t_1) + p^T b^5 (t - t_2) & \text{for } t_2 \le t \le T. \end{cases}
\end{aligned}
\tag{16.4}
$$

The economy's output goods 1–4 are used as the reference goods in the denominator of (14.2). The values of $\tilde{\Delta}^t$ for the four sectors are listed in table 16.8 and are plotted in figures 16.21–16.24.

[1]A more detailed description of the estimation procedure and lists of the parameter estimates and price elasticities of output supply and input demand, together with some interpretation, can be found in appendix D.

	equation	R^2 values* sector 1	2	3	4
1	resource goods (I)	0.956	0.910	0.984	0.977
2	manufactured goods (II)	0.966	0.979	0.959	0.968
3	manufactured goods (III)	0.990	0.993	0.980	0.987
4	service goods (IV)	0.995	0.985	0.993	0.998
5	imports	0.928	0.988	0.962	0.988
6	labor	0.888	0.911	0.889	0.993
7	inventories	0.957	0.970	0.982	0.875
8	machinery and equipment	0.997	0.999	0.998	1.000
9	land	0.999			
	log likelihood	512.133	381.420	357.068	277.785
	number of parameters	66	65	65	65
	rank of B	3	5	5	5
	t_1, t_2	1967,1975	1965,1975	1967,1975	1970,1976

*The R^2 values are calculated as one minus the ratio of the variance of the residuals to the variance of the output supply or input demand variable.

Table 16.7: Model specification, R^2 and log likelihood values for the four sectors

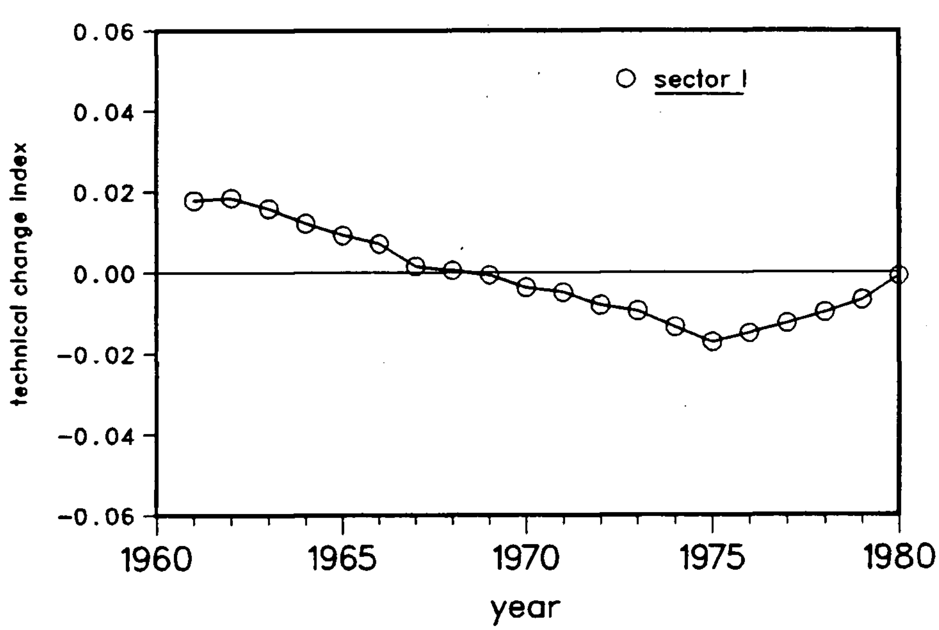

Figure 16.21: Parametric indices $\tilde{\Delta}^t$, sector I

	parametric index $\tilde{\Delta}^t$			
year, t	sector I	sector II	sector III	sector IV
1961	0.01782	0.02746	0.01516	0.01793
1962	0.01838	0.01842	0.01229	0.01631
1963	0.01576	0.01202	0.01102	0.01498
1964	0.01208	0.00656	0.00958	0.01333
1965	0.00919	0.00233	0.00896	0.01197
1966	0.00712	0.00370	0.00883	0.01086
1967	0.00159	0.00469	0.00926	0.01010
1968	0.00054	0.00539	0.00930	0.00923
1969	-0.00057	0.00576	0.00901	0.00830
1970	-0.00366	0.00705	0.00902	0.00733
1971	-0.00483	0.00853	0.00899	0.00633
1972	-0.00797	0.00890	0.00820	0.00537
1973	-0.00939	0.00928	0.00714	0.00442
1974	-0.01337	0.00982	0.00625	0.00319
1975	-0.01720	0.01074	0.00629	0.00201
1976	-0.01490	0.00677	0.00605	0.00074
1977	-0.01241	0.00294	0.00548	0.00181
1978	-0.00973	-0.00051	0.00516	0.00291
1979	-0.00671	-0.00427	0.00480	0.00418
1980	-0.00087	-0.00797	0.00367	0.00557

Table 16.8: Parametric indices of technical progress $\tilde{\Delta}^t$ for the four sectors

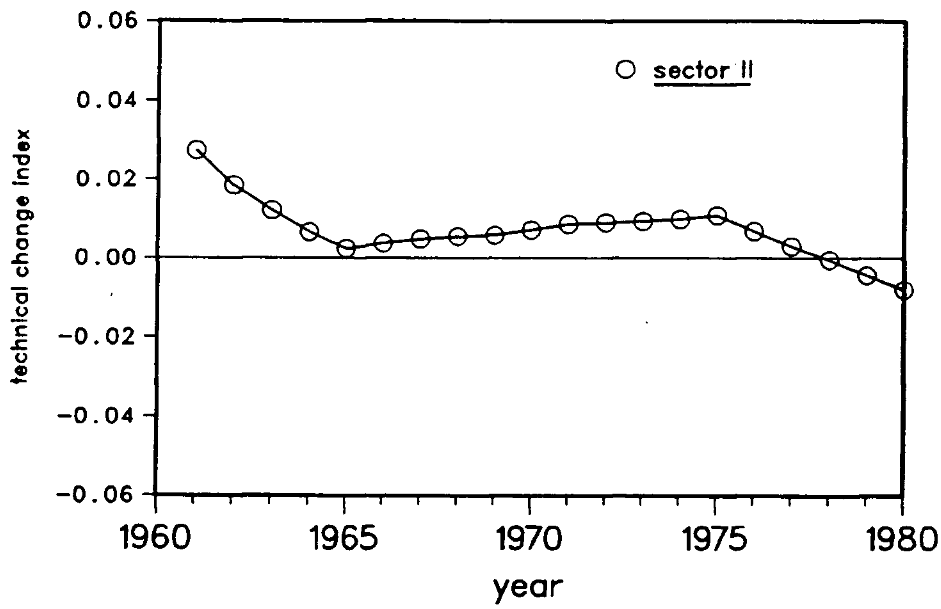

Figure 16.22: Parametric indices $\tilde{\Delta}^t$, sector II

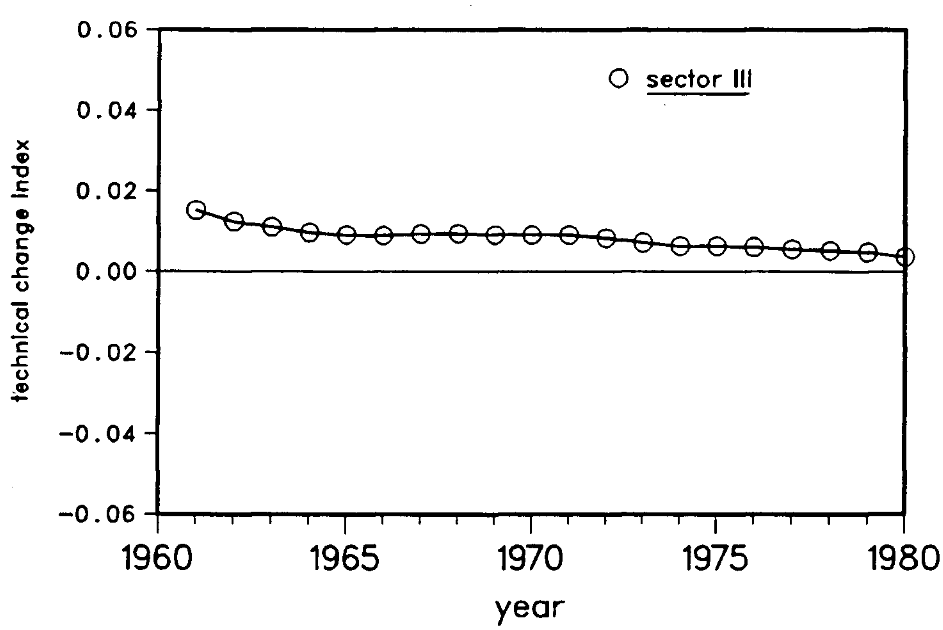

Figure 16.23: Parametric indices $\tilde{\Delta}^t$, sector III

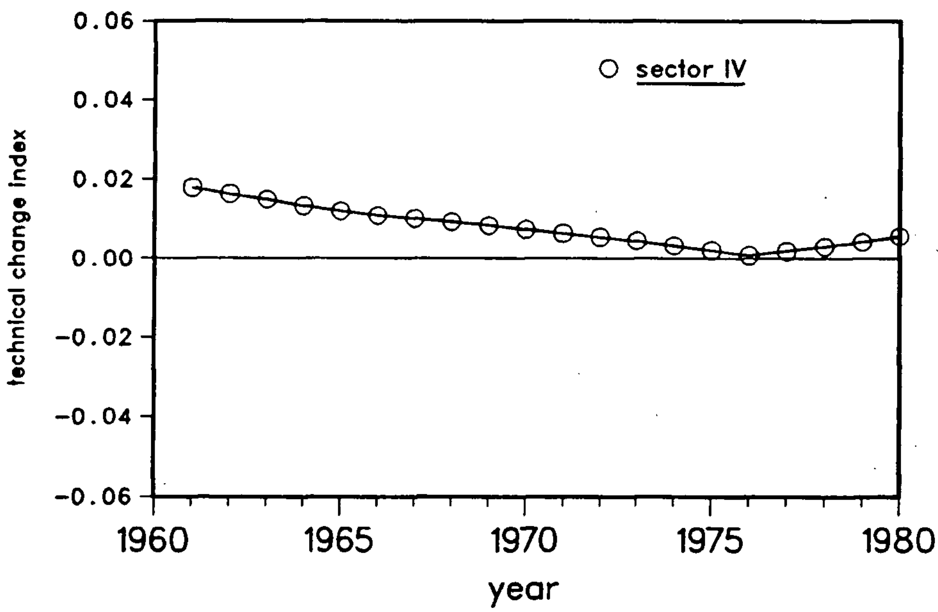

Figure 16.24: Parametric indices $\tilde{\Delta}^t$, sector IV

Chapter 17

A Comparison of Nonparametric and Parametric Estimates of Technical Progress

The comparative behavior of the nonparametric measure of technical progress $\hat{\Delta}^t$ and the parametric measure of technical progress $\tilde{\Delta}^t$ for the four aggregated private domestic sectors of the Canadian economy for the period 1961–1980 is illustrated in figures 17.25–17.28. Since the parametric approach incorporates a stochastic disturbance term in modeling the unit scale profit function, the estimates of the parametric index $\tilde{\Delta}^t$ display a much smoother behavior. The interesting feature of figures 17.25–17.28 though is that the parametric index $\tilde{\Delta}^t$ seems to track the long term trend in the nonparametric index $\hat{\Delta}^t$. Both approaches then seem to isolate relative price effects similarly. While the nonparametric approach attributes the residual all to technical change, the parametric approach takes into account the possibility of random noise in the empirical relationship.

To illustrate how the parametric approach smoothens the behavior of the index of technical progress, the residuals[1] for the nine equations of the parametric model for sector I are plotted against the graphs of the technical progress indices $\hat{\Delta}^t$ and $\tilde{\Delta}^t$ in figures 17.29–17.37. We choose sector I, the resources sector, because it displays the most volatility in the nonparametric measure $\hat{\Delta}^t$, and in the profit function estimation had the most computational difficulties.[2] The magnitude of $\hat{\Delta}^t$ (or ε_t^*) may be due partly to allocative inefficiency; there has been a much rapid rise in prices in this sector relative to the rest of the economy and the violation indices ε_t^* may reflect problems in adjusting to price changes. The equations correspond to

[1] The residuals are the differences between actual and predicted values of output supply or input demand, not unit scale output supply or input demand. See Appendix D, equation (D.11) for the specification of the stochastic disturbance term.

[2] Convergence of the nonlinear optimization routines was not obtained with models using one break point for the quadratic spline representation of technical progress; even with two break points, the sector I model required the most trials of different values for t_1 and t_2 since false convergence was often encountered.

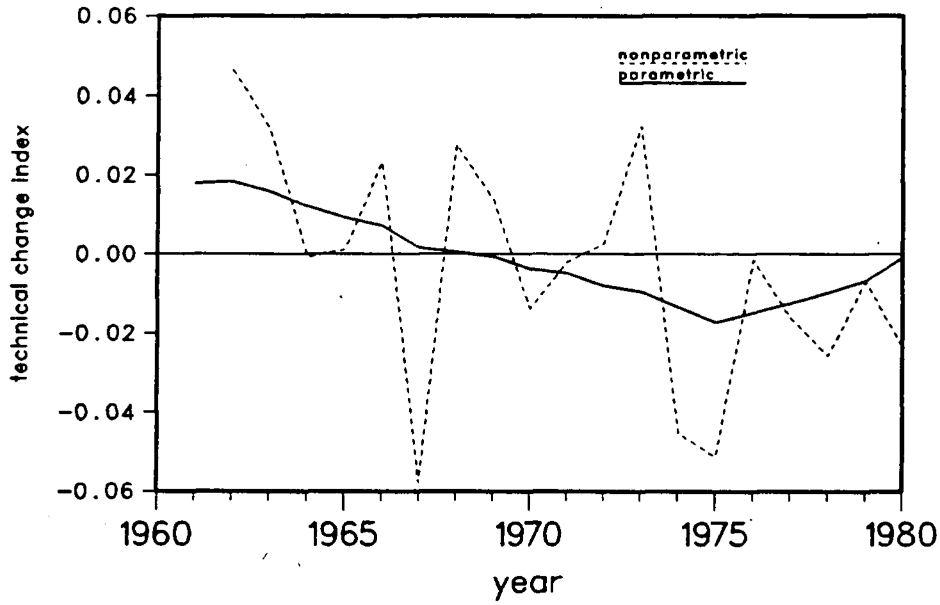

Figure 17.25: Comparative behavior of $\hat{\Delta}^t$ and $\tilde{\Delta}^t$, sector I

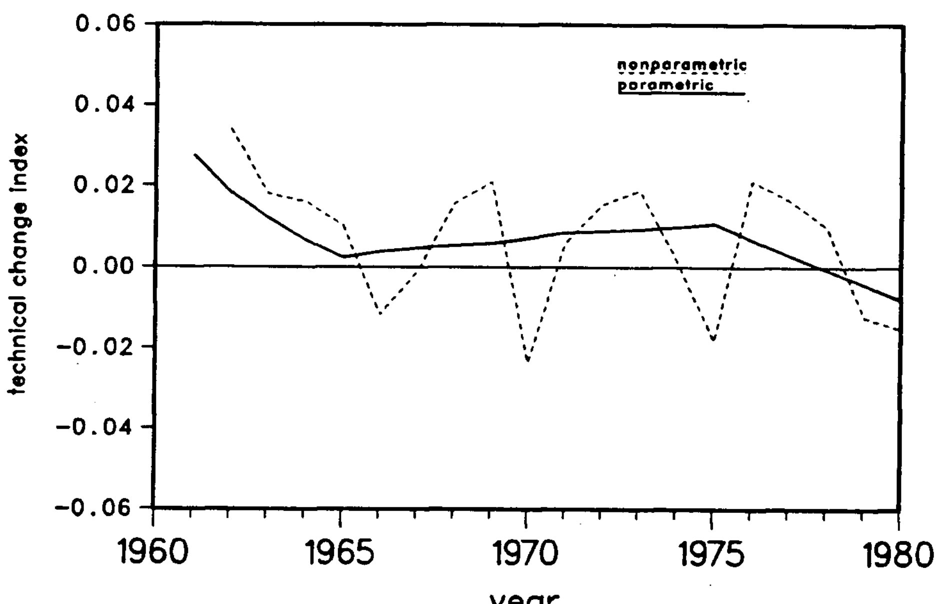

Figure 17.26: Comparative behavior of $\hat{\Delta}^t$ and $\tilde{\Delta}^t$, sector II

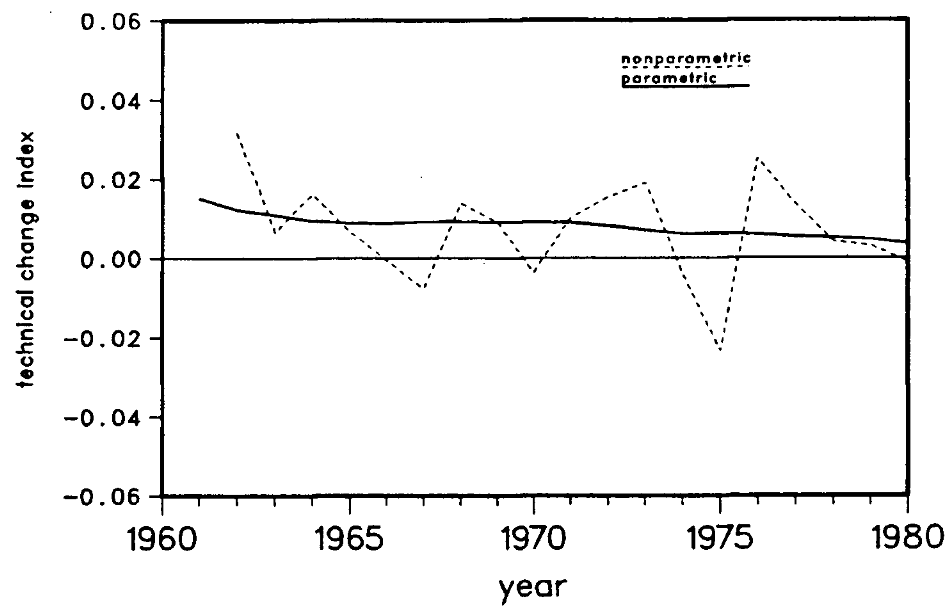

Figure 17.27: Comparative behavior of $\hat{\Delta}^t$ and $\tilde{\Delta}^t$, sector III

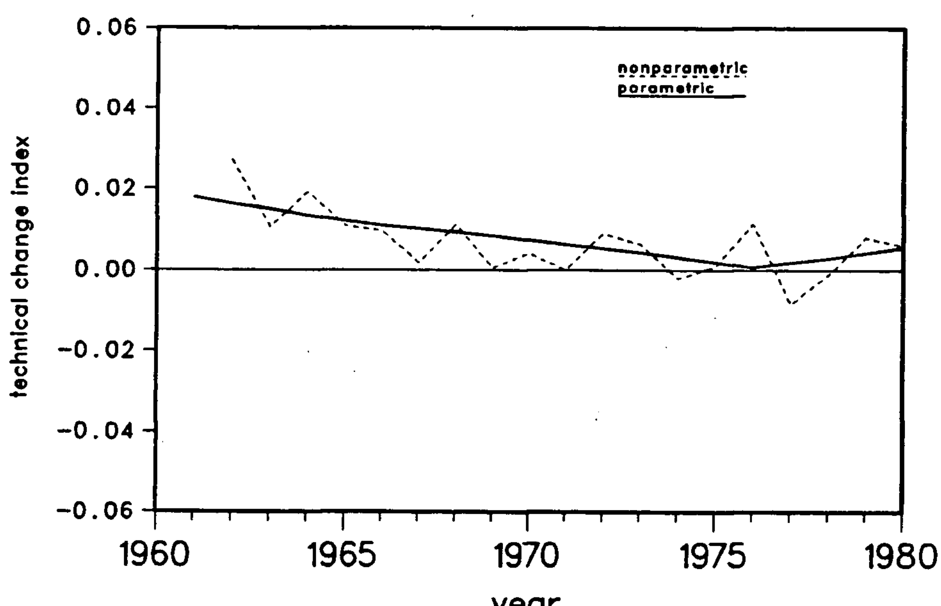

Figure 17.28: Comparative behavior of $\hat{\Delta}^t$ and $\tilde{\Delta}^t$, sector IV

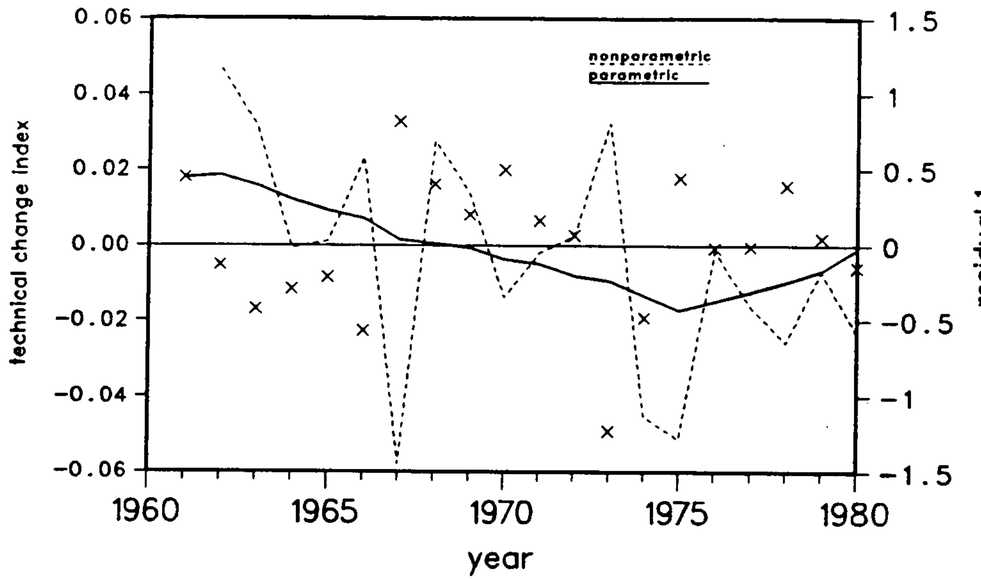

Figure 17.29: Residual plot for equation 1, sector I

output supply and input demand equations for the nine goods listed in table 16.7. Note that for each equation, the magnitude of the residuals from the parametric estimation is likely to be higher at time periods where the peaks and troughs of the nonparametric index $\hat{\Delta}^t$ occur or where the difference between the values of the indices $\hat{\Delta}^t$ and $\tilde{\Delta}^t$ is higher.

An advantage of the nonparametric approach is that no functional form is imposed on the data. Another is its low computational cost. The nonparametric index $\hat{\Delta}^t$ is based on the efficiency test violation indices ε_t^* which require merely a comparison of inequalities; even if linear programming problems have to be solved, these are computationally much easier to solve compared to the nonlinear optimization needed in profit or cost function estimation. A disadvantage of the nonparametric approach is its possible sensitivity to extreme observations and measurement error. Also, since no specification about stochastic disturbance terms is made, the nonparametric estimates have no statistical properties and hence cannot be subjected to the usual statistical hypothesis testing.[3]

[3]See Varian (1985) though on how a pseudo-statistical test can be performed to take into account measurement

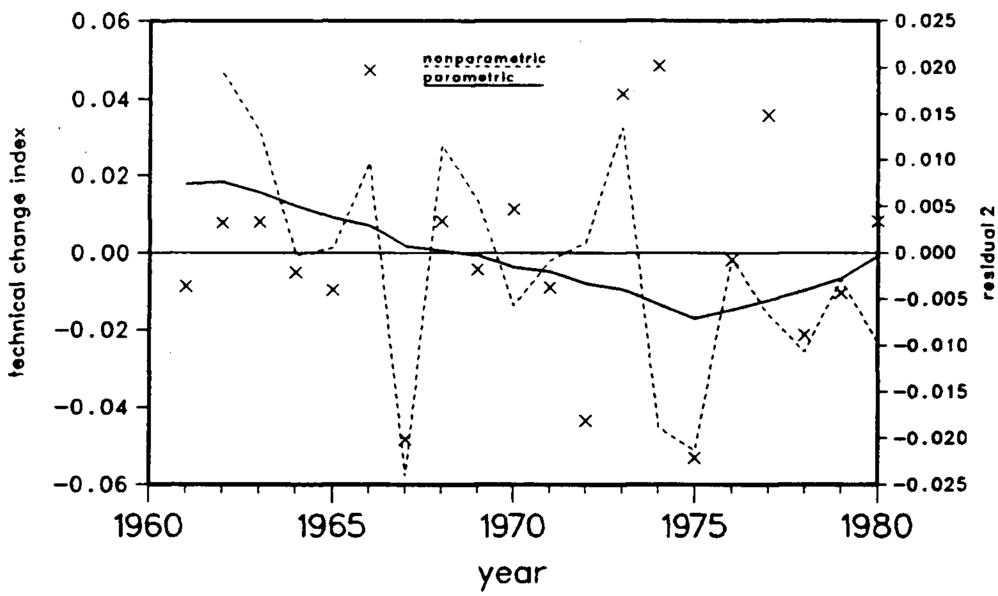

Figure 17.30: Residual plot for equation 2, sector I

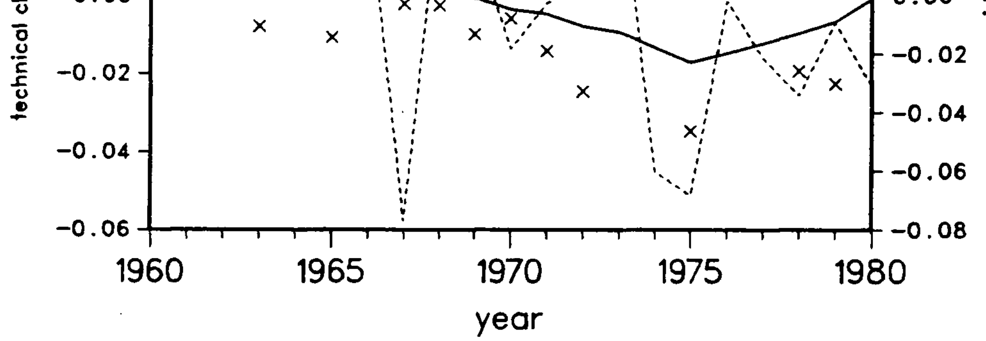

Figure 17.31: Residual plot for equation 3, sector I

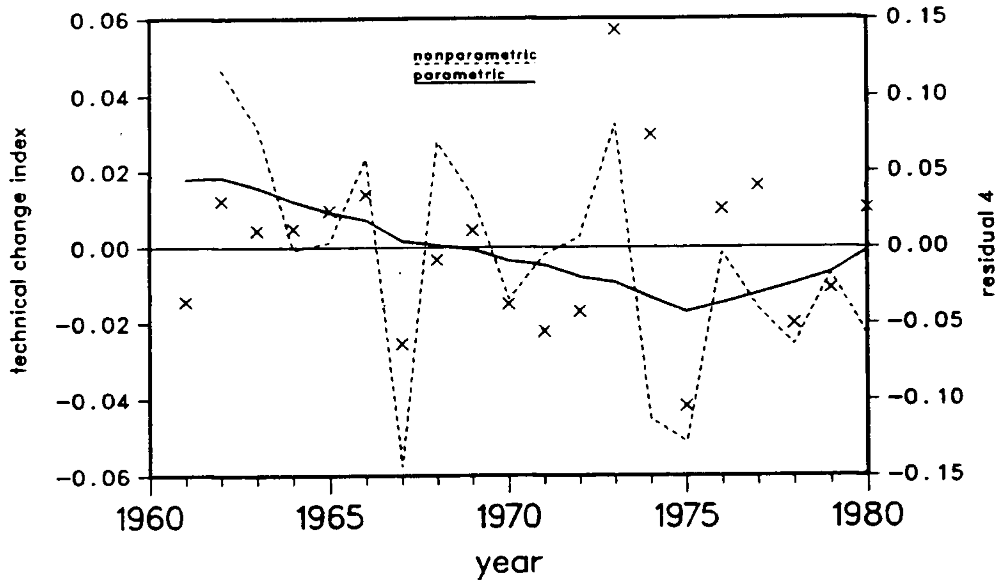

Figure 17.32: Residual plot for equation 4, sector I

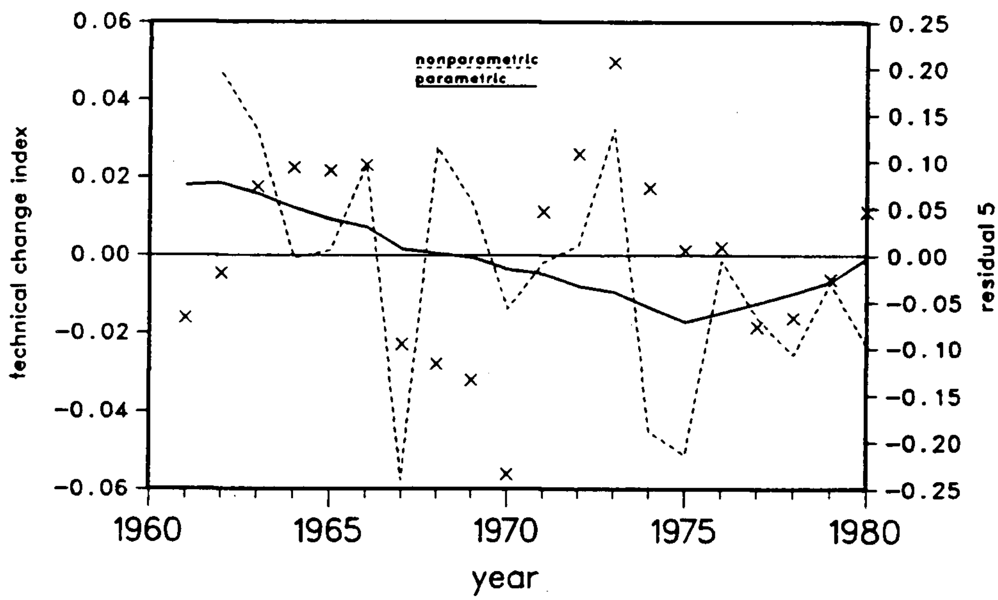

Figure 17.33: Residual plot for equation 5, sector I

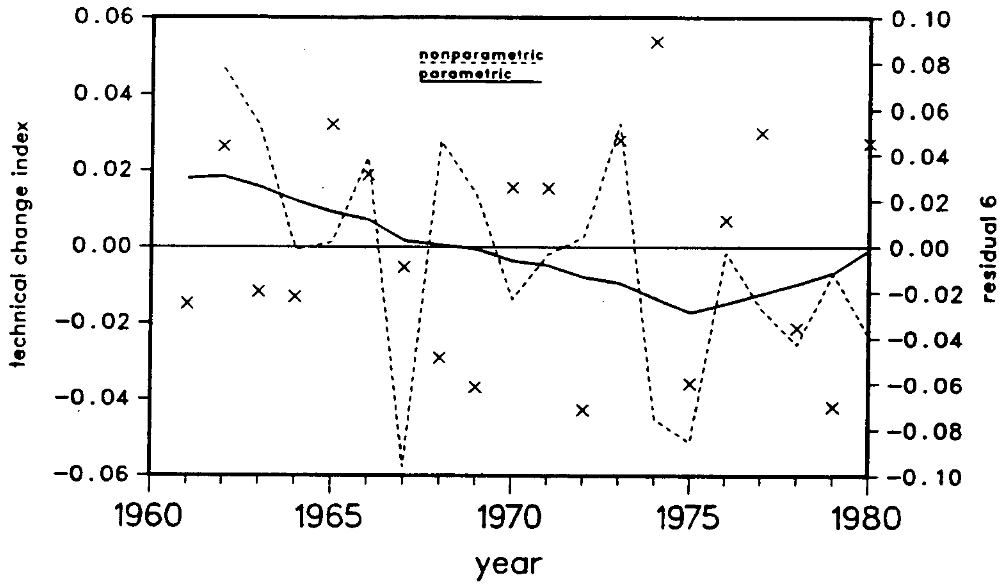

Figure 17.34: Residual plot for equation 6, sector I

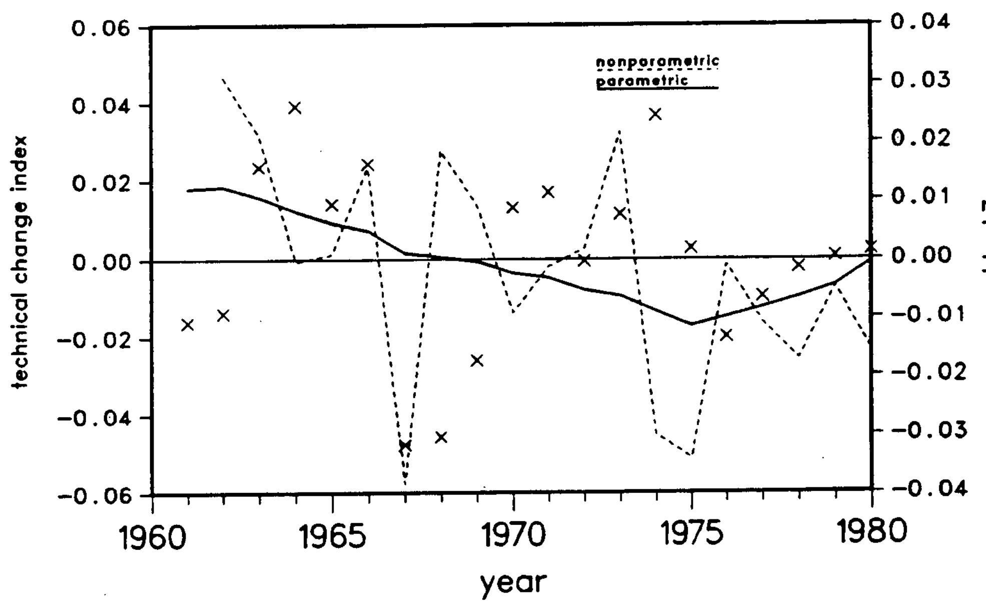

Figure 17.35: Residual plot for equation 7, sector I

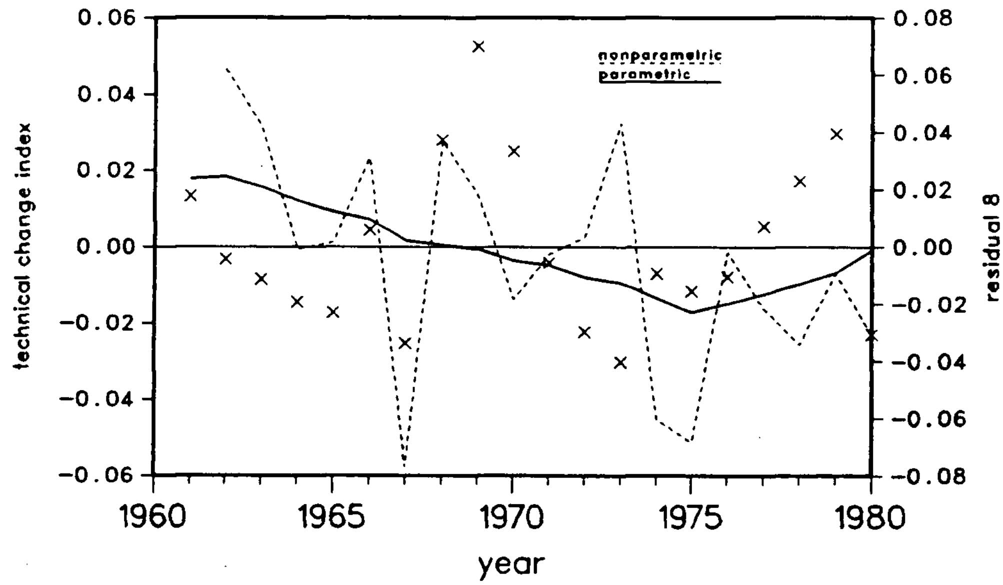

Figure 17.36: Residual plot for equation 8, sector I

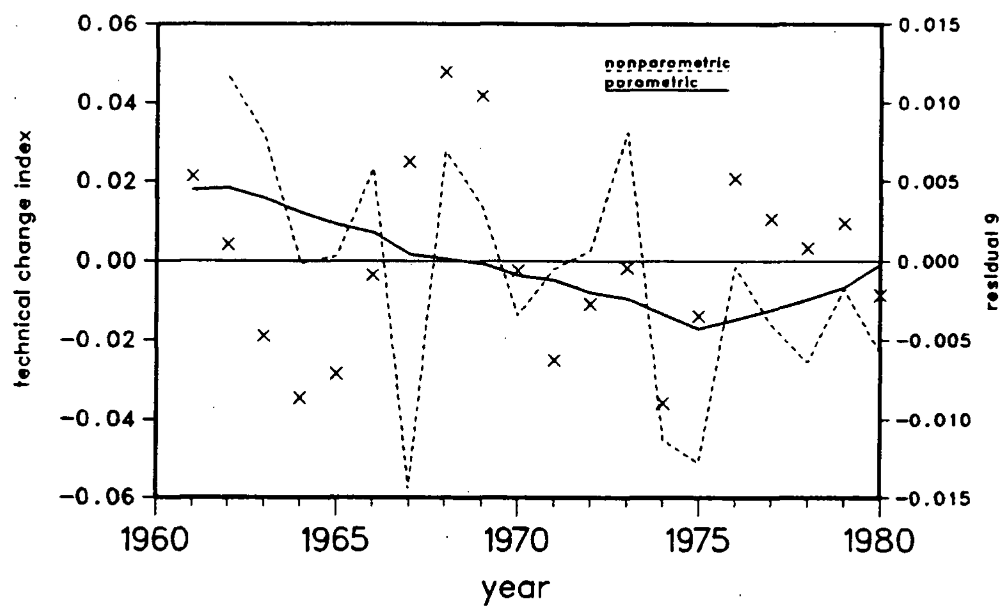

Figure 17.37: Residual plot for equation 9, sector I

A two-stage procedure is suggested as a possible computational alternative to getting a parametric estimate of the index of technical progress Δ^t. First, obtain the nonparametric indices $\hat{\Delta}^t$ and then fit a functional form such as (16.4) on the obtained $\hat{\Delta}^t$ (after adjusting for the denominator in Δ^t). The parametric estimation part would then involve a fewer number of parameters and hence would be less computationally burdensome.

The foregoing discussion has focused on the techniques of measuring technical progress. As an aside, it may be worthwhile to comment on the technical regress findings for the resources sector. The Divisia indices of output growth and input growth for the resources sector are graphed in figure 17.38. To obtain these Divisia growth indices, a Divisia quantity index of the first four goods, the output goods of the production sector of the economy (with the quantities of intermediate inputs indexed negatively) and a Divisia quantity index of the primary inputs (goods 5–10) were first constructed per year. The Divisia growth index is then the ratio of the current year quantity index to the previous year quantity index. Note that, as seen in figure 17.38, the rate of input growth in the resources sector has been quite steady over the years while the output growth index displays much greater fluctuations. This is in marked contrast to the manufacturing sectors (II and III) and the services sector (IV) where the patterns of the Divisia indices of output and input growth follow each other quite closely. As mentioned earlier, the resources sector experienced a considerable output price increase in the 1970s. The low price elasticities for this sector, discussed in appendix D, may be indicative of a low degree of flexibility in this sector. However, this is at most a partial explanation since the divergence of the patterns of output and input growth is also observed in the 1960s.

Other explanations have been offered in the literature. One is that to model producer behavior for the resources sector which is subject to output price volatility it may be necessary to incorporate intertemporal dynamics such as price expectations and uncertainty. Another is that the observed technological regress is due to resource depletion and an appropriate model for measuring multifactor productivity should correct for the quality of the resource stock (see

---

errors.

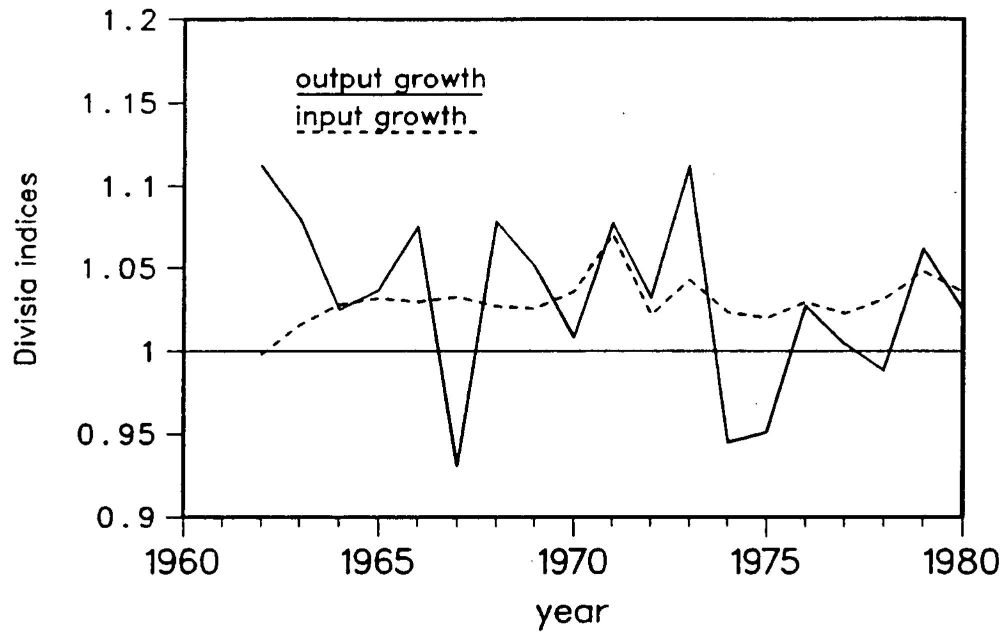

Figure 17.38: Divisia indices of output and input growth in the resources sector (I)

for example, Lasserre and Ouellette (1988)). Related to the resource depletion hypothesis, more of the other inputs may then be required to maintain production levels in this sector as the quality of the resource stock deteriorates. Also, exploration and development of new resources have to be undertaken which entails employment of nonproduction workers. There can also be a considerable lag between exploration and development, which may require large outlays of capital investment, and actual production. Because of the nature of the capital goods in this sector, it may be difficult to adjust factor inputs in the short run. Adjusting the capital data for capacity utilization may address this latter problem. Whether taking into account the phenomenon of resource depletion in measuring technical change in the resources sector requires shifting into a dynamic model of producer behavior and more than data adjustment necessitates further investigation.

Part III

Comparative Statics for a Small Open Economy in the Presence of Factor Price Rigidity

Chapter 18

The Model

18.1 Introduction

In the theory of consumer behavior under conditions of quantity constraints, it has been shown by Latham (1980) and Neary and Roberts (1980) that the Tobin-Houthakker conjecture that a reduction in the ration of one good will increase the consumption of unrationed substitutes and decrease the demand for unrationed complements may not hold. The derivatives of the rationed demand functions can be decomposed Slutsky-like into income and substitution effects. The income effect arises from the divergence between the individual's shadow or demand price for the rationed good (termed "virtual price" by Neary and Roberts) and its actual price.

In the context of a trade-theoretic approach to production theory, an analogous question can be posed. In a small open economy, how do factor price rigidities affect resource allocation? In particular, how do changes in these fixed or rigid factor prices affect other flexible factor prices? The role of factor substitution in the response of flexible factor prices to exogenous changes in fixed factor prices has to be examined.

It can be shown that since a fix-price factor acts as a negative output in a constrained gross national product (GNP) maximization problem, a Stolper-Samuelson type generalization can be obtained. If the domestic resources are used as inputs to production, as in the usual case treated in international trade theory, an increase in a fixed factor price leads to at least one flexible factor price decreasing. The response of flexible factor prices can be decomposed into a pure substitution term and a scale effect term. Furthermore, if we have local factor price equalization in the sense that factor prices are independent of changes in their endowments, then substitution possibilities among fix-price factors and flexible-price factors, even if they exist, will not affect the response of the flexible factor prices to changes in the fixed factor prices. In this case, there is solely a scale effect.

In this study, the constrained GNP function is used to derive the comparative statics for the production sector of a small open economy with both fixed and flexible factor prices. An illustration of the model through a simple analysis of the effect of a devaluation with rigid wages in a two-sector specific-factors model is presented. We conclude with a discussion of how the model can be empirically estimated, the associated problems in its econometric implementation, and the possible policy implications that can be derived.

18.2 Model formulation, a constrained GNP problem

In contrast to flex-price domestic resources with given endowments and inelastic supply, fix-price domestic resources will have a variable or elastic supply. In the standard full-employment general equilibrium models, factors with excess supply have a zero price. In the real world, these zero prices are rarely, if ever, observed. Hence, the framework to be used in the following model hopes to get around this theoretical deficiency and implicitly assumes that unemployment of domestic resources is due to factor price rigidities. Specifically, the fixed prices of these domestic resources are set above their market clearing levels.[1]

The constrained GNP maximization approach adopted here follows that of Neary (1985). In his comparative statics exercises, Neary compared a small open economy with some fix-price productive factors having perfectly elastic supply to a hypothetical unconstrained economy (with all factor prices flexible) with the endowments of factors set at their utilization levels in the constrained economy. Some important results are: the change in the employment levels

[1]For a domestic resource for which the economy has a given endowment, it is important to make this assumption so that its supply remains elastic. In contrast, imports used as intermediate inputs to production in a small open economy can be considered a fix-price factor but need not be constrained in this manner; its supply will always be elastic.

The model to be used is in the genre of the "fixprice method" proposed by Hicks (1965). This formulation does not imply that "fixed prices" remain the same from period to period. In the sense that prices do not necessarily adjust in response to demand and supply disequilibrium in the short run, price determination becomes exogenous to the model. Signalling of excess demand or excess supply for price adjustment over the longer run is not precluded. As Hicks has pointed out, though the fixprice method tends to be macroeconomics-oriented, it is important to decipher the workings of individual markets.

of the fix-price factors due to a marginal increase in their prices is the inverse of the change in those factor prices induced by a marginal change in their endowments in the corresponding flex-price economy; factor price rigidities lead to an increase in the economy's price-output responsiveness and a reduction in the responsiveness of the remaining flexible factor prices to changes in their endowments; and an equiproportionate decrease in the fixed factor prices will raise the value of gross national product (assuming factor immobility across countries).

A more direct approach to the comparative statics exercises for this constrained GNP maximization problem is the use of the fundamental matrix equation of production theory as developed by Diewert and Woodland (1977) and Diewert (1982). This approach is more amenable to empirical implementation and does not require direct comparison between constrained and unconstrained economies. The model also allows joint production and intermediate inputs. The development of the model relates the behavior of individual firms or industries to the aggregate economy's maximization problem.

Let there be M constant returns to scale sectors with industry scales $z \equiv (z_1, z_2, \ldots, z_M)^T \geq 0_M$ producing K net outputs of internationally traded goods $y \equiv (y_1, y_2, \ldots, y_K)^T$ with given world prices $p \equiv (p_1, p_2, \ldots, p_K)^T \gg 0_K$. There are $Q + N$ domestic resources of which the first Q factors $l \equiv (l_1, l_2, \ldots, l_Q)^T$ have fixed prices $r \equiv (r_1, r_2, \ldots, r_Q)^T \gg 0_Q$ and the last N factors have initial endowments $v \equiv (v_1, v_2, \ldots, v_N)^T$ with associated prices $w \equiv (w_1, w_2, \ldots, w_N)^T \gg 0_N$. The exogenous variables are the world prices of internationally traded goods p, the fixed prices of the variable-supply factors r, and the endowments of the flex-price factors v. The endogenous variables are the industry scale variables z, the employment levels of the fix-price factors l, the prices of inelastically supplied factors w, and the net outputs of internationally traded goods y.

Let the feasible set of net outputs for the mth sector be described by $T^m \equiv \{z_m C^m : z_m \geq 0\}$ where C^m is a nonempty, closed, convex set of feasible net outputs when industry m operates at unit scale. At the industry level, the output and input prices are parametric. Let inputs be indexed with a negative sign and outputs with a positive sign. Thus, the unit scale profit function for sector m can be defined as

$$\pi^m(p,r,w) \equiv \max_{a,b,c}\{p\cdot c - r\cdot b - w\cdot a:\ (c,-b,-a)\in \mathcal{C}^m\}, \tag{18.1}$$

where, for convenience, we have defined the vectors a, b and c to be nonnegative. The unit scale production possbilities set $\mathcal{C}^m$ can be recovered as

$$\mathcal{C}^m \equiv \{(c,-b,-a):\ p\cdot c - r\cdot b - w\cdot a \leq \pi^m(p,r,w) \text{ for every } p \gg 0_K, r \gg 0_Q, w \gg 0_N\}. \tag{18.2}$$

If π^m is differentiable, we obtain, by Hotellings's lemma, the optimal unit scale input-output coefficients:

$$\begin{aligned} c^m(p,r,w) &= \nabla_p \pi^m(p,r,w), &(18.3)\\ -b^m(p,r,w) &= \nabla_r \pi^m(p,r,w), &(18.4)\\ -a^m(p,r,w) &= \nabla_w \pi^m(p,r,w). &(18.5)\end{aligned}$$

The industry profit function π^m is nondecreasing in p, nonincreasing in r and w, homogeneous of degree one, convex and continuous in (p,r,w) for $p \gg 0_K, r \gg 0_Q, w \gg 0_N$.

Given world prices p, fixed factor prices r and resource endowments v, the economy's aggregate profit maximization problem is

$$\begin{aligned} &\max_{(c^m,-b^m,-a^m)\in\mathcal{C}^m, m=1,2,\ldots,M} \max_{z\geq 0_M}\{\textstyle\sum_{m=1}^M (p\cdot c^m - r\cdot b^m)z_m :\\ &\qquad\qquad \textstyle\sum_{m=1}^M a^m z_m \leq v\} &(18.6)\\ &= \max_{(c^m,-b^m,-a^m)\in\mathcal{C}^m, m=1,2,\ldots,M}[\max_z\{\textstyle\sum_{m=1}^M (p\cdot c^m - r\cdot b^m)z_m :\\ &\qquad\qquad v - \textstyle\sum_{m=1}^M a^m z_m \geq 0_N,\ z\geq 0_M\}] &(18.7)\\ &= \max_{(c^m,-b^m,-a^m)\in\mathcal{C}^m, m=1,2,\ldots,M}[\max_{z\geq 0_M}\min_{w\geq 0_N}\{\textstyle\sum_{m=1}^M (p\cdot c^m - r\cdot b^m)z_m\\ &\qquad\qquad + w\cdot(v - \textstyle\sum_{m=1}^M a^m z_m)\}] &(18.8)\\ &= \max_{z\geq 0_M}\min_{w\geq 0_N}\{\textstyle\sum_{m=1}^M \pi^m(p,r,w)z_m + w\cdot v\} &(18.9)\\ &\equiv \pi(p,r,v), &(18.10)\end{aligned}$$

where we have used the saddlepoint theorem for linear programs to transform the constrained maximization problem in (18.7) to the saddlepoint problem in (18.8), and definition (18.1) to go from (18.8) to (18.9). The pure dual form of the economy's profit maximization problem is

$$\pi(p,r,v) = \min_{w \geq 0_N}\{w \cdot v : \ \pi^m(p,r,v) \leq 0, \ m = 1,2,\ldots .M\}. \tag{18.11}$$

The economy's profit maximization problem $\pi(p,r,v)$ can be considered the constrained (or restricted) GNP function. It yields the value of aggregate output minus the cost of fix-price factors.[2] The value of $\pi(p,r,v)$ would also be the sum of payments to flex-price domestic resources. In contrast, the usual unconstrained GNP function gives the value of output which equals the sum of all factor payments in the economy.

The model, so far, has been discussed in the context of the Q fix-price factors as domestic resources. However, the elements of the variable factor supply vector $l \equiv (l_1, l_2, \ldots, l_Q)^T$ can be imports used as inputs in the production of internationally traded goods. With the small country assumption, the prices of imports are exogenous. As well, price controls on nontraded output (produced goods) can be modeled by lumping these goods with internationally traded goods. Note that the small country assumption plays an important role in the dichotomy of goods and factors. For ease of interpretation in subsequent discussion, the vector l will be treated as fix-price domestic resources used as inputs in the production of internationally traded goods y.

18.3 First-order conditions

Equation (18.9) is a concave programming problem. Suppose $z^* \gg 0_M$, $w^* \gg 0_N$ solve (18.9) when the exogenous variables are set at p^*,r^*,v^*. The first-order conditions for an interior solution yield the zero-profit conditions and flex-price resource exhaustion conditions:

$$\begin{aligned} \pi^m(p^*,r^*,w^*) &= 0, \ m = 1,2,\ldots,M; & (18.12) \\ \textstyle\sum_{m=1}^{M} \nabla_w \pi^m(p^*,r^*,w^*) z_m^* + v^* &= 0_N. & (18.13) \end{aligned}$$

[2]The constrained GNP function $\pi(p,r,v)$ corresponds to Neary's factor-price constrained revenue function where fix-price factor employment levels are interpreted as "negative outputs" sold at their fixed prices.

Define the matrices of optimal input-output coefficients using (18.3)–(18.5) as

$$\begin{aligned} A &\equiv [a^{1*}, a^{2*}, \ldots, a^{M*}] \\ &\equiv [-\nabla_w \pi^1(p^*, r^*, w^*), -\nabla_w \pi^2(p^*, r^*, w^*), \ldots, -\nabla_w \pi^M(p^*, r^*, w^*)]; \end{aligned} \tag{18.14}$$

$$\begin{aligned} B &\equiv [b^{1*}, b^{2*}, \ldots, b^{M*}] \\ &\equiv [-\nabla_r \pi^1(p^*, r^*, w^*), -\nabla_r \pi^2(p^*, r^*, w^*), \ldots, -\nabla_r \pi^M(p^*, r^*, w^*)]; \end{aligned} \tag{18.15}$$

$$\begin{aligned} C &\equiv [c^{1*}, c^{2*}, \ldots, c^{M*}] \\ &\equiv [\nabla_p \pi^1(p^*, r^*, w^*), \nabla_p \pi^2(p^*, r^*, w^*), \ldots, \nabla_p \pi^M(p^*, r^*, w^*)] \end{aligned} \tag{18.16}$$

where A, B and C are of dimensions NxM, QxM and KxM, respectively. The first-order conditions (18.12) and (18.13) can be rewritten in matrix form, with the use of equations (18.1) and (18.14)–(18.16), as

$$p^{*T}C - r^{*T}B - w^{*T}A = 0_M^T, \tag{18.17}$$

$$Az^* = v^*. \tag{18.18}$$

The economy's net output vector y and employment levels for fix-price factors l are given by

$$y(p^*, r^*, v^*) \equiv \textstyle\sum_{m=1}^{M} \nabla_p \pi^m(p^*, r^*, w(p^*, r^*, v^*)) z_m(p^*, r^*, v^*), \tag{18.19}$$

$$l(p^*, r^*, v^*) \equiv -\textstyle\sum_{m=1}^{M} \nabla_r \pi^m(p^*, r^*, w(p^*, r^*, v^*)) z_m(p^*, r^*, v^*), \tag{18.20}$$

or in matrix notation,

$$y^* = Cz^*, \tag{18.21}$$

$$l^* = Bz^*. \tag{18.22}$$

18.4 Fundamental matrix equations

If we assume that the unit profit functions π^m are twice continuously differentiable, total differentiation of (18.12), (18.13) and (18.20) yields the following equations relating the endogenous variables w, z and l to the exogenous variables v, p and r:

$$\begin{bmatrix} -S_{ww} & A & 0_{NxQ} \\ A^T & 0_{MxM} & 0_{MxQ} \\ -S_{rw} & B & -I_Q \end{bmatrix} \begin{bmatrix} dw \\ dz \\ dl \end{bmatrix} = \begin{bmatrix} I_N & S_{wp} & S_{wr} \\ 0_{MxN} & C^T & -B^T \\ 0_{QxN} & S_{rp} & S_{rr} \end{bmatrix} \begin{bmatrix} dv \\ dp \\ dr \end{bmatrix} \tag{18.23}$$

where

$$S_{ij} \equiv \textstyle\sum_{m=1}^{M} \nabla_{ij}^2 \pi^m(p^*, r^*, w^*) z_m^* \tag{18.24}$$

are the aggregate substitution matrices. Since the industry unit scale profit functions are convex in (p, r, w), the symmetric matrices S_{pp}, S_{rr} and S_{ww} are positive semidefinite. The linear homogeneity of π^m implies the following:

$$S_{pp}p^* + S_{pr}r^* + S_{pw}w^* = 0_K; \tag{18.25}$$

$$S_{rp}p^* + S_{rr}r^* + S_{rw}w^* = 0_Q; \tag{18.26}$$

$$S_{wp}p^* + S_{wr}r^* + S_{ww}w^* = 0_N. \tag{18.27}$$

To obtain the comparative static responses of w, z and l to changes in v, p and r, we apply the implicit function theorem. The inverse of the $(N + M + Q)$-dimension square matrix in the left hand side of (18.23) is required. Define the submatrix G as

$$G \equiv \begin{bmatrix} -S_{ww} & A \\ A^T & 0_{MxM} \end{bmatrix}. \tag{18.28}$$

Assume that the NxM matrix A has rank $M \leq N$, that is, the number of flex-price resources N is at least as great as the number of industries M.[3] Furthermore, assume that the matrix $S_{ww} + AA^T$ is positive definite.[4] The positive semidefiniteness of S_{ww}, the rank condition on A, and the positive definiteness of $S_{ww} + AA^T$ are necessary and sufficient conditions for the existence of the inverse of G. Define this matrix to be

$$G^{-1} \equiv \begin{bmatrix} -S_{ww} & A \\ A^T & 0_{MxM} \end{bmatrix}^{-1} \equiv \begin{bmatrix} D & E \\ E^T & F \end{bmatrix}. \tag{18.29}$$

[3] Since the rank of a matrix is the number of linearly independent rows or columns in that matrix, the rank condition on A, $\text{rank}(A) = M$, requires distinct industry or sectoral classification.

[4] The positive definiteness of $S_{ww} + AA^T$ implies that $S_{ww} + AA^T$ has a positive determinant and hence is of rank N. Since AA^T is of rank M, the flex-price factor substiution matrix S_{ww} has rank at least equal to $N - M$.

The properties of G^{-1} are developed in the appendix of the work by Diewert and Woodland (1977).

If we denote the $Qx(N+M)$ matrix H as $H \equiv [S_{rw}, -B]$, then the inverse of the left hand side matrix in (18.23) can be expressed as

$$\begin{bmatrix} G & 0_{(N+M)xQ} \\ -H & -I_Q \end{bmatrix}^{-1} = \begin{bmatrix} G^{-1} & 0_{(N+M)xQ} \\ -HG^{-1} & -I_Q \end{bmatrix}. \tag{18.30}$$

Hence, we obtain the comparative static response matrix

$$\begin{aligned}
&\begin{bmatrix} \nabla_v w & \nabla_p w & \nabla_r w \\ \nabla_v z & \nabla_p z & \nabla_r z \\ \nabla_v l & \nabla_p l & \nabla_r l \end{bmatrix} \\
&= \begin{bmatrix} G^{-1} & 0_{(N+M)xQ} \\ -HG^{-1} & -I_Q \end{bmatrix} \begin{bmatrix} I_N & S_{wp} & S_{wr} \\ 0_{MxN} & C^T & -B^T \\ 0_{QxN} & S_{rp} & S_{rr} \end{bmatrix} \\
&= \begin{bmatrix} D & E & 0_{NxQ} \\ E^T & F & 0_{MxQ} \\ -(S_{rw}D - BE^T) & -(S_{rw}E - BF) & -I_Q \end{bmatrix} \begin{bmatrix} I_N & S_{wp} & S_{wr} \\ 0_{MxN} & C^T & -B^T \\ 0_{QxN} & S_{rp} & S_{rr} \end{bmatrix} \\
&= \begin{bmatrix} D & DS_{wp} + EC^T & DS_{wr} - EB^T \\ E^T & E^T S_{wp} + FC^T & E^T S_{wr} - FB^T \\ -(S_{rw}D - BE^T) & \begin{array}{c} -[(S_{rw}D - BE^T)S_{wp} \\ +(S_{rw}E - BF)C^T \\ +S_{rp}] \end{array} & \begin{array}{c} -(S_{rw}D - BE^T)S_{wr} \\ +(S_{rw}E - BF)B^T \\ -S_{rr} \end{array} \end{bmatrix}
\end{aligned} \tag{18.31}$$

To interpret the above equations, it is helpful to list down the conditions satisfied by the matrices D, E and F that appear in (18.29). As given in the appendix of Diewert and Woodland (1977), these are:

$$-S_{ww}D + AE^T = I_N \tag{18.32}$$

$$A^T D = 0_{MxN} \tag{18.33}$$

$$A^T E = I_M \tag{18.34}$$

$$-S_{ww} E + AF = 0_{NxM} \tag{18.35}$$

$$D = D^T \tag{18.36}$$

$$F = F^T \tag{18.37}$$

$$F = E^T S_{ww} E \tag{18.38}$$

where D is a negative semidefinite matrix and F is a positive semidefinite matrix.

Chapter 19

Comparative Static Results

1. The matrices $\nabla_v w = D$ and $\nabla_v z = E^T$ have the usual interpretation as given in Diewert and Woodland (1977) with N as the number of flex-price domestic resources. Local factor price equalization obtains when the number of industries is equal to the number of flex-price domestic resources, that is, $M = N$. In this case, the matrix A is square of dimension N and rank N, its inverse A^{-1} exists and from (18.33), D becomes a zero matrix. This yields $\nabla_v w \equiv \left[\frac{\partial w}{\partial v}\right] = 0_{NxN}$ which means flexible factor prices are independent of small changes in endowments.[1]

Also, if $M = N$, the domestic resources exhaustion equation (18.18) can be rewritten as $z^* = A^{-1}v^*$, meaning the industry scales are completely determined by the input coefficient matrix A and the initial endowment vector v^*. In this case, the fix-price factors, having an elastic supply, do not seem to impose constraints on the scale of production. Hence, $\nabla_v z \equiv \left[\frac{\partial z}{\partial v}\right] = A^{-1}$.

2. Flexible factor prices w are now homogeneous of degree one in (p, r). The homogeneity property of $w(p, r, v)$ can be seen as follows:

$$
\begin{aligned}
[\nabla_p w]\, p^* + [\nabla_r w]\, r^* &= (DS_{wp} + EC^T)p^* + (DS_{wr} - EB^T)r^* \text{ from (18.31)} \\
&= D(S_{wp}p^* + S_{wr}r^*) + E(C^T p^* - B^T r^*) \\
&= -DS_{ww}w^* + EA^T w^* \text{ using (18.27) and (18.17)} \\
&= (-DS_{ww} + EA^T)w^* \\
&= I_N w^* \text{ using (18.32)} \\
&= w^*.
\end{aligned}
$$

[1]Generally, even if $M \neq N$, it has been shown by Neary (1985) that the response of flexible factor prices to changes in endowments is less negative relative to the unconstrained case where all domestic resources have flexible prices. In this sense, the economy has a greater tendency towards "local" factor price equalization.

We then have

$$[\nabla_p w]\, p^* + [\nabla_r w]\, r^* \;=\; w^*. \tag{19.1}$$

For $\lambda > 0$, $w(\lambda p, \lambda r, v) = \lambda w(p, r, v)$. A proportional increase in internationally traded goods prices (such as through currency depreciation) does not necessarily lead to a proportional increase in flexible factor prices if there are fix-price factors. With respect to the endowment vector v, the flexible factor prices still retain the property of homogeneity of degree zero, that is, for $\lambda > 0$, $w(p, r, \lambda v) = w(p, r, v)$.

3. Industry scale functions $z(p, r, v)$ are no longer homogeneous of degree zero in world prices p. We have

$$[\nabla_p z]\, p^* \;\equiv\; \left[\frac{\partial z}{\partial p}\right] p^* \;=\; -\,[\nabla_r z]\, r^*. \tag{19.2}$$

Equivalently, industry scales are homogeneous of degree zero in (p, r). For $\lambda > 0$, $z(\lambda p, \lambda r, v) = z(p, r, v)$. Intuitively, since a proportional change in prices p and r lead to a proportional change in w (as given in result 2), relative prices in the economy do not change. Hence, there is no real effect and industry scales remain unchanged.

Industry scales remain homogeneous of degree one in v, that is, $z(p, r, \lambda v) \;=\; \lambda z(p, r, v)$ for $\lambda > 0$. Equivalently, $[\nabla_v z]\, v^* \;\equiv\; \left[\frac{\partial z}{\partial v}\right] v^* \;=\; z^*$. Since the fix-price factors are in elastic supply, the industry scales or production levels are determined or limited by the exhaustion of inelastically supplied (flex-price and fully employed) factors.

4. Rybczynski-type and Stolper-Samuelson type generalizations still obtain in the analysis of $\nabla_v z$ and $\nabla_p w$.

5. a) The response of flexible factor prices to marginal changes in the fixed factor prices is given by

$$\begin{aligned}
\nabla_r w \equiv \left[\frac{\partial w}{\partial r}\right] \;&=\; D S_{wr} - E B^T \\
&=\; \left[\frac{\partial w}{\partial v}\right] S_{wr} - \left[\frac{\partial z}{\partial v}\right]^T B^T \text{ using (18.31).}
\end{aligned} \tag{19.3}$$

A Slutsky-type decomposition of the total effect can be made: the first term measures the pure substitution effect between fix-price factors and flex-price factors, and the second term measures the scale effect. The scale effect can be explained as follows: changing, say, a fixed factor price leads to a change in the employment level of that elastically supplied factor such that its value of marginal product equals its new fixed price; therefore, the vector of domestic resources available for productive activities changes.

The first term in the right hand side of (19.3) will be zero if either $S_{wr} = 0_{NxQ}$ or $D = 0_{NxN}$. In the first case, there is no substitution between fix-price factors and flex-price factors or equivalently, there is no change induced in the input coefficient matrix A by a change in fixed factor prices. In the second case, the number of industries is equal to the number of flex-price factors ($M = N$), the flexible factor prices w are completely determined by world output prices p and local factor price equalization holds; changes in the matrix A will have no effect on prices w. In either of these cases, the total effect $\left[\frac{\partial w}{\partial r}\right]$ will be due purely to changes in scale.

b) Stolper-Samuelson type generalizations obtain with the fixed factor prices r_q acting like a world output price p_k and the fix-price factor behaving like a "negative" internationally traded good. If we premultiply equation (19.3) by A^T, we obtain

$$\begin{aligned} A^T\left[\frac{\partial w}{\partial r}\right] &= A^T D S_{wr} - A^T E B^T \\ &= -B^T \text{ using (18.33) and (18.34)}\,. \end{aligned} \tag{19.4}$$

Transposing, we have

$$\left[\frac{\partial w}{\partial r}\right]^T A = -B. \tag{19.5}$$

If the matrix A is nonnegative and B contains at least one positive element in the qth row, that is, all flex-price domestic resources are inputs or nonproduced and at least one industry uses the qth fix-price factor as an input, then the qth row of $\left[\frac{\partial w}{\partial r}\right]^T$:

$$\left[\frac{\partial w_1}{\partial r_q}, \frac{\partial w_2}{\partial r_q}, \ldots, \frac{\partial w_N}{\partial r_q}\right] \tag{19.6}$$

has at least one negative element. If r_q increases, at least one flexible factor price will decrease.

A weakness of the fundamental matrix approach, probably due to its generality, is that it does not yield more specific results on which flex-price factor will gain or lose. Let us try the special case of $K = M = 2$ internationally traded goods (assuming a single output for each industry), $N = 2$ flex-price domestic resources and $Q = 1$ fix-price domestic resource. Since we have $M = N$, the matrix D is a zero matrix and the response of flexible factor prices to marginal changes in fixed factor prices would be solely a scale effect. Substitution possibilities among domestic resources will not affect w; hence, we would suspect relative factor intensities to determine the price responses. For this special case, equation (19.3) reduces to

$$\begin{aligned}
\nabla_r w \equiv \begin{bmatrix} \frac{\partial w_1}{\partial r_1} \\ \frac{\partial w_2}{\partial r_1} \end{bmatrix} &= -EB^T \text{ since } D = 0_{2x2} \\
&= -(A^{-1})^T B^T \text{ using (18.34)} \\
&= -(BA^{-1})^T \text{ where } B = [b_1^1, b_1^2] \text{ and } A = \begin{bmatrix} a_1^1 & a_1^2 \\ a_2^1 & a_2^2 \end{bmatrix} \\
&= \frac{1}{\Delta} \begin{bmatrix} a_2^1 b_1^2 - a_2^2 b_1^1 \\ a_1^2 b_1^1 - a_1^1 b_1^2 \end{bmatrix} \text{ where } \Delta \equiv |A| = a_1^1 a_2^2 - a_1^2 a_2^1.
\end{aligned} \tag{19.7}$$

Let us assume that the matrices A and B are positive, that is, all domestic resources are used as inputs. From the previous analysis, at least one flexible factor price will decrease. In (19.7), we can sign the following quantities:

$$\begin{aligned}
\Delta \gtrless 0 &\quad \text{if} \quad \frac{a_1^1}{a_1^2} \gtrless \frac{a_2^1}{a_2^2} \\
a_2^1 b_1^2 - a_2^2 b_1^1 \gtrless 0 &\quad \text{if} \quad \frac{a_2^1}{a_2^2} \gtrless \frac{b_1^1}{b_1^2} \\
a_1^2 b_1^1 - a_1^1 b_1^2 \gtrless 0 &\quad \text{if} \quad \frac{b_1^1}{b_1^2} \gtrless \frac{a_1^1}{a_1^2}.
\end{aligned} \tag{19.8}$$

The six possible factor intensity configurations and the resulting effects on the flexible factor prices are given in table 19.9. An examination of the table indicates that the factor that varies the most in its input coefficient ratio from that of the fix-price factor tends to gain with an increase in the fixed factor price. Vice-versa, the more similar the flex-price factor in the manner it is allocated to the different productive activities (as measured by relative factor intensities), the greater the tendency for this factor to decrease in price with an increase in the fixed factor price.

factor intensities	$\Delta \equiv \mid A \mid$	$\frac{\partial w_1}{\partial r_1}\left(=-\frac{\partial l_1}{\partial v_1}\right)$	$\frac{\partial w_2}{\partial r_1}\left(=-\frac{\partial l_1}{\partial v_2}\right)$
$\frac{a_1^1}{a_1^2} > \frac{a_2^1}{a_2^2} > \frac{b_1^1}{b_1^2}$	+	+	-
$\frac{a_1^1}{a_1^2} > \frac{b_1^1}{b_1^2} > \frac{a_2^1}{a_2^2}$	+	-	-
$\frac{a_2^1}{a_2^2} > \frac{a_1^1}{a_1^2} > \frac{b_1^1}{b_1^2}$	-	-	+
$\frac{a_2^1}{a_2^2} > \frac{b_1^1}{b_1^2} > \frac{a_1^1}{a_1^2}$	-	-	-
$\frac{b_1^1}{b_1^2} > \frac{a_1^1}{a_1^2} > \frac{a_2^1}{a_2^2}$	+	-	+
$\frac{b_1^1}{b_1^2} > \frac{a_2^1}{a_2^2} > \frac{a_1^1}{a_1^2}$	-	+	-

Table 19.9: Factor intensity configurations and resulting effects on the flexible factor prices

c) From equations (19.3) and (18.31), we obtain the following symmetry condition:

$$\begin{aligned}
\nabla_r w &= DS_{wr} - EB^T \\
&= [S_{rw}D - BE^T]^T \text{ where } S_{rw} = S_{wr}^T \\
&= -[\nabla_v l]^T. \qquad (19.9)
\end{aligned}$$

This implies $\frac{\partial w_n}{\partial r_q} = -\frac{\partial l_q}{\partial v_n}$ or the change in the nth flexible factor price due to a marginal increase in the qth fixed factor price is equal in magnitude but opposite in direction to the change in the employment level of the qth fix-price factor due to a marginal increase in the endowment of the nth flex-price resource. Suppose we redefine factor complementarity and substitutability in terms of $\nabla_v l$: the nth flex-price factor and qth fix-price factor are complements if $\frac{\partial l_q}{\partial v_n} > 0$, that is, an increase in the endowment of the nth flex-price factor induces a higher employment level for the fix-price factor; otherwise, if $\frac{\partial l_q}{\partial v_n} < 0$, then they are substitutes. In this context, we can get a result that is analogous to the Tobin-Houthakker conjecture: an exogenous increase in the fixed price of a factor will reduce the price of its complements and increase the price of its substitutes.[2]

[2]This interpretation is merely another description of factor price behavior and begs the question of explaining

Note that the symmetry condition in equation (19.9) holds in general whereas the corresponding symmetry between flex-price factor and output variables as described in equation (19.10) below holds only under more restrictive conditions. The possibility of intermediate inputs and joint production complicates the relationship. The presence of intermediate inputs is innocuous if the local factor price equalization condition holds, that is, if factor prices are independent of changes in endowments; this nullifies any effect of input substitution on factor prices. Specifically, we obtain the symmetry condition

$$\nabla_p w = [\nabla_v z]^T = [\nabla_v y]^T \tag{19.10}$$

only if $C = I_K$ and $K = M$ (single output technologies)[3] and either $S_{wp} = 0_{NxK}$ (no intermediate inputs) or $M = N$ (local factor price equalization) hold.

6. The analysis of the matrix $\nabla_v l = -(S_{rw}D - BE^T)$ yields a Rybczynski-type result: if A is nonnegative and B contains at least one positive element in the qth row, that is, all flex-price resources are inputs or nonproduced and at least one industry uses the qth fix-price factor, then the qth row of $\nabla_v l$:

$$\left[\frac{\partial l_q}{\partial v_1}, \frac{\partial l_q}{\partial v_2}, \cdots, \frac{\partial l_q}{\partial v_N}\right] \tag{19.11}$$

has at least one positive element. In this case, there exists at least one flex-price factor whose increase in endowment leads to greater utilization of the variable supply factor q.[4] This result is obtained by substituting (19.9) into (19.5) to yield

$$\left[\frac{\partial l}{\partial v}\right] A = B. \tag{19.12}$$

Again, the total effect can be decomposed Slutsky-like into two sources: a pure substitution effect and a scale effect. In the context of marginal changes in the endowment vector v, the condition $S_{rw} = 0_{QxN}$ implies that the B matrix does not change with respect to changes in

---

the mechanism behind price changes. What is of interest are the underlying determinants of factor price behavior, as we have attempted to identify in isolating substitution and scale effects.

[3]With single output technologies, that is $C = I_K$ and $K = M$, we have the equality of the scale and output variables: $z_m = y_m$.

[4]If each fix-price factor is used by at least one industry, then in the modified sense of substitution as defined in result 5c, each fix-price factor has at least one complement among the flex-price factors.

flexible prices w; the scale effect is due to a change in the vector of domestic resources available for productive activities. For the special case discussed in result 5b where $K = M = N = 2$ and $Q = 1$, the signs of the elements of $\nabla_v l$:

$$\nabla_v l \equiv \left[\frac{\partial l}{\partial v}\right] = \left[\frac{\partial l_1}{\partial v_1}, \frac{\partial l_1}{\partial v_2}\right] = \left[-\frac{\partial w_1}{\partial r_1}, -\frac{\partial w_1}{\partial r_2}\right] \tag{19.13}$$

can be obtained from the previous table by using the symmetry condition (19.9). The total effect is solely a scale effect. Note that $\frac{\partial l_1}{\partial v_n}$ will be negative (substitutes in the modified sense of result 5c) if their associated factor intensity ratios are not adjacent to each other (in the inequalities); otherwise, it is positive.

7. The response of industry scales z to changes in the fixed factor prices r is given by

$$\nabla_r z \equiv \left[\frac{\partial z}{\partial r}\right] = E^T S_{wr} - FB^T. \tag{19.14}$$

This equation can again be interpreted as a Slutsky-type decomposition. The first term accounts for the effect of substitution between fix-price factors and flex-price factors. The second term corresponds to the scale effect arising from a change in the utilization of the fix-price factors.

If there is no substitution between fix-price factors and flex-price factors ($S_{wr} = 0_{NxQ}$), as in the case when fix-price factors enter production in fixed coefficients, then $\left[\frac{\partial z}{\partial r}\right] = -FB^T$ where F is an MxM positive semidefinite matrix. Premultiplying by B, we obtain

$$B\left[\frac{\partial z}{\partial r}\right] = -BFB^T, \tag{19.15}$$

a negative semidefinite matrix which implies

$$\textstyle\sum_{m=1}^{M} b_q^m \frac{\partial z_m}{\partial r_q} \leq 0, \; q = 1, 2, \ldots, Q \text{ and} \tag{19.16}$$

$$\textstyle\sum_{m=1}^{M} b_i^m \frac{\partial z_m}{\partial r_j} = \sum_{m=1}^{M} b_j^m \frac{\partial z_m}{\partial r_i}, \; i, j = 1, 2, \ldots, Q. \tag{19.17}$$

8. The change in the employment levels of fix-price factors due to a change in world prices p is given by

$$\begin{aligned}\nabla_p l \equiv \left[\frac{\partial l}{\partial p}\right] &= -(S_{rw}D - BE^T)S_{wp} - (S_{rw}E - BF)C^T - S_{rp} \\ &= -S_{rp} + \left[\frac{\partial l}{\partial v}\right] S_{wp}, - \left[\frac{\partial z}{\partial r}\right]^T C^T \text{ using (18.31)}. \end{aligned} \tag{19.18}$$

The first two terms on the right hand side measure the pure substitution effects: -S_{rp}, the direct effect due to substitution between internationally traded goods and fix-price factors and $\left[\frac{\partial l}{\partial v}\right] S_{wp}$, the indirect effect due to substitution between internationally traded goods and flex-price factors. The third term can be interpreted as a scale effect. There is no clear-cut result on the definiteness property of the matrix $\nabla_p l$. Equation (19.18) can be rewritten as

$$\nabla_p l \;=\; -S_{rp} - [S_{rw},\ -B] \begin{bmatrix} D & E \\ E^T & F \end{bmatrix} \begin{bmatrix} S_{wp} \\ C^T \end{bmatrix}. \tag{19.19}$$

Let us consider the usual case of no joint production and no intermediate inputs: $M = K$, $C = I_K$, $S_{wp} = 0_{NxK}$, $S_{rp} = 0_{QxK}$. Equation (19.18) reduces to

$$\begin{aligned} \nabla_p l &= -(S_{rw}E - BF) \\ &= -[\nabla_r z]^T \text{ using (19.14)} \\ &= -[\nabla_r y]^T \text{ since } y = z \text{ when } M = K \text{ and } C = I_K. \end{aligned} \tag{19.20}$$

In this case, we obtain the symmetry condition

$$\frac{\partial l_q}{\partial p_k} = -\frac{\partial z_k}{\partial r_q} = -\frac{\partial y_k}{\partial r_q}. \tag{19.21}$$

If we further assume $S_{rw} = 0_{QxN}$ as discussed in result 7, postmultiplication of (19.20) by B^T yields

$$\left[\frac{\partial l}{\partial p}\right] B^T = BFB^T = -\left[B\left[\frac{\partial y}{\partial r}\right]\right]^T, \tag{19.22}$$

a positive semidefinite matrix.

9. The response of fix-price factor employment to changes in their price is given by

$$\begin{aligned} \nabla_r l \equiv \left[\frac{\partial l}{\partial r}\right] &= -(S_{rw}D - BE^T)S_{wr} + (S_{rw}E - BF)B^T - S_{rr} \\ &= -S_{rr} + \left[\frac{\partial l}{\partial v}\right] S_{wr} + \left[\frac{\partial z}{\partial r}\right] B^T \text{ using (18.31)} \\ &= -S_{rr} + \left[\frac{\partial w}{\partial r}\right] S_{wr} + \left[\frac{\partial z}{\partial r}\right] B^T \text{ using (19.9)} \\ &= -S_{rr} + S_{rw}\left[\frac{\partial w}{\partial r}\right] + B\left[\frac{\partial z}{\partial r}\right]. \end{aligned} \tag{19.23}$$

The total effect can be decomposed into a direct substitution effect among fix-price factors, an indirect substitution effect between flex-price factor and fix-price factors and a scale effect. Equation (19.23) can be rewritten as

$$\nabla_r l \;=\; -[S_{rw}D - BE^T,\; I_Q] \begin{bmatrix} S_{ww} & S_{wr} \\ S_{rw} & S_{rr} \end{bmatrix} \begin{bmatrix} DS_{wr} - EB^T \\ I_Q \end{bmatrix} \tag{19.24}$$

where the symmetric matrix of aggregate substitution terms in the right hand side is positive semidefinite. Hence, the matrix $\nabla_v l$ is negative semidefinite. This implies that $\frac{\partial l_q}{\partial r_q} \leq 0$; an increase in the fixed price of a factor reduces or at most does not change the employment level of that factor. If $S_{wr} = 0_{NxQ}$ as discussed in result 7, then equation (19.23) reduces to

$$\nabla_r l \;=\; -S_{rr} - BFB^T \tag{19.25}$$

where both matrices, S_{rr} and BFB^T, are positive semidefinite.

It can also be shown that l is homogeneous of degree zero in (p, r), that is, $l(\lambda p, \lambda r, v) = l(p, r, v)$ for $\lambda > 0$. Hence, $\nabla_p l p^* + \nabla_r l r^* = 0_Q$.

10. The usual comparative static results apply to $\nabla_p y$ and $\nabla_v y$. In the response of output to changes in fixed factor prices, we get

$$\nabla_r y \equiv \left[\frac{\partial y}{\partial r}\right] \;=\; \textstyle\sum_{m=1}^{M} \nabla^2_{pr}\pi^m z_m \;+\; \sum_{m=1}^{M} \nabla^2_{pw}\pi^m z_m \left[\frac{\partial w}{\partial r}\right] \;+\; \sum_{m=1}^{M} \nabla_p \pi^m z_m \left[\frac{\partial z}{\partial r}\right] \tag{19.26}$$

obtained by differentiating equation (18.19). Equation (19.26) can be rewritten as

$$\nabla_r y \;=\; S_{pr} + S_{pw}(DS_{wr} - EB^T) + C(E^T S_{wr} - FB^T). \tag{19.27}$$

A definiteness property for $\nabla_r y$ cannot be established. The terms on the right hand side of equation (19.27) can be interpreted accordingly.

11. The homogeneity properties of w, z, l and y and the definiteness properties of $\nabla_p y$ and $\nabla_r l$ can also be established from the definition and convexity properties of the constrained GNP function $\pi(p, r, v)$ as given in equations (18.6) and (18.11).

Chapter 20

An Illustration: Devaluation with Rigid Wages in a Specific-Factors Model

In recent years, there has been a resurgence in interest on the type of labor unemployment generated by real wage rigidity rather than by Keynesian aggregate demand deficiency. Currency devaluation is an alternative employment stimulation policy. The effects of a devaluation are examined in the context of a two-sector specific factors model.[1] It has been argued by Neary (1978) that capital sectoral specificity need not necessarily have only a short run interpretation. In the medium run or long run, capital equipment is generally not physically transferred from one sector to another and capital stock adjustments take the form of varying investments relative to depreciation.

Suppose we have two single-output sectors ($M = K = 2$) with output y_1 and y_2 and associated world prices p_1 and p_2. Each sector uses a sector-specific input, say capital; imports and labor as fix-price factors; and output from the other sector as an intermediate input. Let l_1 and l_2 be the total quantity of imports and labor, respectively, utilized by the economy; let r_1 and r_2 be their exogenous prices. The endowments of sector-specific capital are v_1 for sector 1 and v_2 for sector 2; their endogenous prices are w_1 and w_2, respectively. The configurations of the unit input-output coefficient matrices A, B and C and of the substitution matrix S_{ww} are as follow:

$$A \equiv \begin{bmatrix} a_1^1 & 0 \\ 0 & a_2^2 \end{bmatrix}; \quad B \equiv \begin{bmatrix} b_1^1 & b_1^2 \\ b_2^1 & b_2^2 \end{bmatrix}; \quad C \equiv \begin{bmatrix} 1 & -c_1^2 \\ -c_2^1 & 1 \end{bmatrix};$$

[1]Early analysis of the specific-factors model were done by Samuelson (1971), Jones (1971), Mayer (1974), Mussa (1974) and Neary (1978). Samuelson (1971) shows that free trade will tend to partial, though not complete, factor price equalization between countries. With factor specificity, anti-Rybczynski and anti-Stolper-Samuelson outcomes explain factor owners' responses to trade policies (such as workers supporting protection for capital-intensive industries and opposing liberal immigration policies) as short-run reactions.

$$S_{ww} \equiv \begin{bmatrix} S_{w_1w_1} & 0 \\ 0 & S_{w_2w_2} \end{bmatrix}, \tag{20.1}$$

where $a_i^j > 0$, $b_i^j > 0$, $c_i^j \geq 0$, $(i, j = 1, 2)$ are scalars. A devaluation can be modelled as a proportional increase in output prices (p_1, p_2) and import price (r_1) such that

$$\frac{dp_1}{p_1} = \frac{dp_2}{p_2} = \frac{dr_1}{r_1} = \lambda > 0; \ \frac{dr_2}{r_2} = 0; \ dv_1 = dv_2 = 0. \tag{20.2}$$

We assume that wages do not change and the capital stocks remain constant. The effects of a devaluation on capital returns, output, employment, imports and national income are discussed next.

1. Returns to sector specific capital. The changes in capital returns w_1 and w_2 due to the devaluation are given by

$$\begin{aligned} dw_1 &= \left(\frac{p_1 - p_2c_2^1 - r_1b_1^1}{a_1^1}\right)\lambda = \left(\frac{r_2b_2^1 + w_1a_1^1}{a_1^1}\right)\lambda = \left(\frac{r_2b_2^1}{a_1^1} + w_1\right)\lambda > \lambda w_1 > 0; \\ dw_2 &= \left(\frac{p_2 - p_1c_1^2 - r_1b_1^2}{a_2^2}\right)\lambda = \left(\frac{r_2b_2^2 + w_2a_2^2}{a_2^2}\right)\lambda = \left(\frac{r_2b_2^2}{a_2^2} + w_2\right)\lambda > \lambda w_2 > 0. \end{aligned} \tag{20.3}$$

In full employment models, a devaluation has a neutral effect since domestic factor prices increase proportionally. In contrast, we now obtain a more than proportional increase in capital returns in both sectors since the specific factors absorb the gain that could have accrued to labor under full employment. Hence, the higher the value-added content in a sector, the greater the gains to capital owners in that sector arising from the devaluation. Export promotion by currency devaluation will not be as beneficial to capital owners in sectors with low value added, such as those in assembly of completely-knocked-down parts.

2. Output. The nonproportional increase in capital returns (w_1 and w_2) makes possible the output effects given by

$$\begin{aligned} dy_1 &= \left[-S_{w_1r_2}r_2 + S_{w_1w_1}\left(\frac{p_1 - p_2c_2^1 - r_1b_1^1 - w_1a_1^1}{a_1^1}\right)\right]\frac{\lambda}{a_1^1} \\ &= \left[-S_{w_1r_2}r_2 + S_{w_1w_1}\left(\frac{r_2b_2^1}{a_1^1}\right)\right]\frac{\lambda}{a_1^1}; \end{aligned}$$

$$
\begin{aligned}
dy_2 &= \left[-S_{w_2r_2}r_2 + S_{w_2w_2}\left(\frac{p_2 - p_1c_1^2 - r_1b_1^2 - w_2a_2^2}{a_2^2}\right)\right]\frac{\lambda}{a_2^2} \\
&= \left[-S_{w_2r_2}r_2 + S_{w_2w_2}\left(\frac{r_2b_2^2}{a_2^2}\right)\right]\frac{\lambda}{a_2^2}.
\end{aligned} \tag{20.4}
$$

By the positive semidefiniteness of the substitution matrix S_{ww}, its elements $S_{w_1w_1}$ and $S_{w_2w_2}$ are nonnegative. The output of a particular sector will increase if labor and capital in that sector are substitutes; that is, $S_{w_1r_2} < 0$ for sector 1 and $S_{w_2r_2} < 0$ for sector 2. Labor and capital are not necessarily substitutes in the presence of intermediate inputs and imports. However, it is still possible to obtain a positive output effect if the degree of complementarity between labor and capital is sufficiently small.

Note that the output effects in equations (20.4) can be rewritten as

$$
\begin{aligned}
dy_1 &= \left[\frac{d}{dw_1}\left(\frac{b_2^1}{a_1^1}\right)\right](\lambda r_2); \\
dy_2 &= \left[\frac{d}{dw_2}\left(\frac{b_2^2}{a_2^2}\right)\right](\lambda r_2).
\end{aligned} \tag{20.5}
$$

Hence, as long as the labor-capital ratio in a sector increases, the devaluation leads to an expansion of that sector. The reason why the change in output can be expressed as a function of wages r_2 can be seen in the following discussion.

3. Employment. From the comparative statics exercise (result 9), we know that the employment levels of fix-price factors l are homogeneous of degree zero in the exogenous prices (p, r); that is, $[\nabla_p l]\, p^* + [\nabla_r l]\, r^* = 0_Q$. For our example here, we then have

$$
\begin{bmatrix} \frac{\partial l_1}{\partial p_1} & \frac{\partial l_1}{\partial p_2} \\ \frac{\partial l_2}{\partial p_1} & \frac{\partial l_2}{\partial p_2} \end{bmatrix}
\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}
+
\begin{bmatrix} \frac{\partial l_1}{\partial r_1} & \frac{\partial l_1}{\partial r_2} \\ \frac{\partial l_2}{\partial r_1} & \frac{\partial l_2}{\partial r_2} \end{bmatrix}
\begin{bmatrix} r_1 \\ r_2 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{20.6}
$$

Together with the negative semidefiniteness of the matrix $\nabla_r l$, the homogeneity property can be used to derive the change in employment dl_2:

$$
dl_2 = \frac{\partial l_2}{\partial p_1}dp_1 + \frac{\partial l_2}{\partial p_2}dp_2 + \frac{\partial l_2}{\partial r_1}dr_1
$$

$$
\begin{aligned}
&= \lambda\left(\frac{\partial l_2}{\partial p_1}p_1 + \frac{\partial l_2}{\partial p_2}p_2 + \frac{\partial l_2}{\partial r_1}r_1\right) \\
&= -\frac{\partial l_2}{\partial r_2}(\lambda r_2) \\
&\geq 0. \qquad (20.7)
\end{aligned}
$$

Hence, labor employment increases or at the least does not change with a devaluation. The proportional change in employment depends on the wage elasticity of labor demand, as can be seen in the expression

$$\frac{dl_2}{l_2} = -\left(\frac{\partial l_2}{\partial r_2}\frac{r_2}{l_2}\right)\lambda. \qquad (20.8)$$

The above derivation also shows that a devaluation of $\lambda > 0$ is equivalent to a proportional decrease in the wage rate of λ:

$$dp_1 = dp_2 = dr_1 = 0;\ \frac{dr_2}{r_2} = -\lambda;\ dv_1 = dv_2 = 0. \qquad (20.9)$$

This suggests how devaluation is a policy instrument for reducing real wages and increasing employment. Empirical estimation of the wage elasticity of labor demand will indicate whether exchange rate adjustments are an effective employment policy tool.

4. Imports. Similarly, the change in quantity of imports due to the devaluation is given by

$$dl_1 = -\frac{\partial l_1}{\partial r_2}(\lambda r_2) \qquad (20.10)$$

where

$$
\begin{aligned}
\frac{\partial l_1}{\partial r_2} &= -S_{r_1 r_2} + \left(S_{w_1 r_2}\frac{b_1^1}{a_1^1} + S_{w_2 r_2}\frac{b_1^2}{a_2^2}\right) \\
&\quad + \left[\left(S_{w_1 r_1} - S_{w_1 w_1}\frac{b_1^1}{a_1^1}\right)\frac{b_2^1}{a_1^1} + \left(S_{w_2 r_1} - S_{w_2 w_2}\frac{b_1^2}{a_2^2}\right)\frac{b_2^2}{a_2^2}\right]. \qquad (20.11)
\end{aligned}
$$

A priori, we cannot sign the direction of change in imports dl_1. It will be opposite in sign to the wage elasticity of imports. Let us analyze the change in imports due to a wage rate change $\left(\frac{\partial l_1}{\partial r_2}\right)$. The import response can be decomposed into three components of which the latter two can be attributed to changes in capital returns (w_1, w_2) induced by the wage rate change.

The first component as measured by the term $-S_{r_1 r_2}$ is the direct effect due to the substitution between labor and imports. The second component $S_{w_1 r_2}\frac{b_1^1}{a_1^1} + S_{w_2 r_2}\frac{b_1^2}{a_2^2}$ is the indirect effect due to substitution between labor and capital in the two sectors which leads to a change in the import-capital ratios $\left(\frac{b_1^1}{a_1^1}, \frac{b_1^2}{a_2^2}\right)$. The third component is the scale effect due to a change in labor employment. This last component can be reexpressed as

$$\left(S_{w_1 r_1} - S_{w_1 w_1}\frac{b_1^1}{a_1^1}\right)\frac{b_2^1}{a_1^1} + \left(S_{w_2 r_1} - S_{w_2 w_2}\frac{b_1^2}{a_2^2}\right)\frac{b_2^2}{a_2^2}$$
$$= \left[\frac{d}{dw_1}\left(\frac{b_1^1}{a_1^1}\right)\right] b_2^1 + \left[\frac{d}{dw_2}\left(\frac{b_1^2}{a_2^2}\right)\right] b_2^2. \tag{20.12}$$

Let us assume that labor and imports are substitutes ($S_{r_1 r_2} < 0$) and labor and capital are substitutes ($S_{w_1 r_2} < 0$ and $S_{w_2 r_2} < 0$, which imply both sectors expand). Then the first component in equation (20.11) is positive while the other two components are negative. Hence the direct substitution effect has to predominate the combined indirect substitution and scale effects for imports to increase with an increase in wages (or decrease with a devaluation).

The preceding discussion on the mechanism behind the import response to wage changes is an illustration of the LeChatelier principle (see, for example, Samuelson (1960)). The wage elasticity of imports in the case of rigid wages (unemployment) will be different from that in a flexible-wage (full employment) scenario. The change induced in capital returns in the rigid wage case underlies the difference.

5. National income. Ignoring international factor payments and abstracting from the absence of indirect taxes, we can express the gross national (or domestic) product GNP net of capital depreciation as the sum of domestic factor payments or national income, NI:

$$NI \equiv r_2 l_2 + w_1 v_1 + w_2 v_2. \tag{20.13}$$

The change in national income due to the devaluation is given by

$$d(NI) = r_2 dl_2 + v_1 dw_1 + v_2 dw_2 > 0 \tag{20.14}$$

which is positive. Hence, national income increases because employment increases or at the least does not fall ($dl_2 \geq 0$, equation (20.7)) and capital returns increase ($dw_1 > 0$, $dw_2 > 0$, equation (20.3)).

The devaluation resulting in an increase in national income is not surprising. Initially, we have a rigid wage above the market clearing level. The devaluation reduces the degree of distortion in the factor markets caused by wage rigidity. It is equivalent to a reduction in the real wage. Even if the wage elasticity of labor demand is low (and hence, the devaluation does not lead to a significant increase in employment), gains still accrue to capital owners, particularly in sectors with high value added. Empirical estimation will yield information on the magnitude of the income distribution repercussions of devaluation.

We can exploit the symmetry condition between prices and quantities of fix-price and flex-price factors ($\frac{\partial w_n}{\partial r_q} = -\frac{\partial l_q}{\partial v_n}$, equation (19.9)) to predict the employment effects of investment in our specific-factors model. We have shown that a devaluation is equivalent to a decrease in wages and that it leads to increases in returns to capital in both sectors. Hence, we have for any sector m, $\frac{\partial w_n}{\partial r_2} < 0$; that is, a wage increase will decrease capital returns. By the symmetry condition, we have $\frac{\partial l_2}{\partial v_m} > 0$; that is, capital investment will increase labor employment. In terms of magnitudes, the greater the gains of a devaluation to capital owners (or equivalently, the greater the loss to them due to a wage increase and which is likely to occur in higher value-added sectors), the greater the employment effects of investment.

The specific-factors model in this section can be generalized to any $M = K > 2$ sectors and the comparative statics results of a devaluation obtained will remain robust. A two-sector model was used to highlight sectoral impacts of a devaluation. Note that the substitutability between intermediate inputs (other than imports) have not been so far mentioned in the discussion of our specific-factors model. The model has been rigged, by having an equal number of sectors and flex-price factors (capital), to yield "local" factor price equalization; that is, competitive prices of domestic primary factors are independent of changes in their endowments. Technically, we would have $M = N$, the number of sectors equal to the number of flex-price factors. As noted in our comparative statics result 1, we have the matrix $\nabla_v w = D = 0_{NxN}$. Hence, the substitutability between intermediate goods and flex-price factors would not affect the factor prices w. In equation (18.31), the first term in the expression $\nabla_p w = DS_{wp} + EC^T$ is a zero matrix.

The mechanism described in this section can be considered the impact effect of a devaluation on resource allocation. There would be a general tendency towards these effects as long as there is a less than proportionate increase in wages. In reality, a devaluation can trigger domestic inflationary pressures and the final outcome with respect to employment of a devaluation will depend on the sensitivity and speed of real wage adjustments to exchange rate changes.

Chapter 21

Towards Econometric Implementation

21.1 Policy implications

Aside from making explicit the mechanism by which exogenous shocks, such as changes in output and import prices or increases in capital stock, generate general equilibrium effects on resource allocation, econometric implementation of the model yields empirical information useful to policy makers. With wage rigidity, one important estimate that can be derived is the wage elasticity of labor demand; if this turns out to be small, then wage flexibility policies may not be very effective in promoting employment. A substantial decrease in real wages would be needed to induce greater employment. In this case, devaluation as an employment-stimulating policy tool would not make much difference though it can still have income distribution repercussions. Also, a low elasticity of employment with respect to real wages is favorable to labor unions which can extract wage premiums at little cost in terms of employment.

If unemployment is due to real wage rigidity, then labor employment will not be a function solely of wages. The relative importance of the determinants of employment such as wages, output and import prices and capital stock changes can be evaluated. With the decline of the dollar in the United States from 1985 to 1987, there has been a revival of and increased employment in the manufacturing industries, particularly the export sectors. Some have conjectured that it was not as much the weak dollar but the capital stock readjustments (accelerated scrapping, modernization and retooling of the factories, etc.) that took place which contributed more to the competitive turnaround of the U.S. economy.

Since imports are inputs to production as well, changes in their prices will affect the competitive returns to domestic primary factors. With the small open economy assumption, imports have exogenous prices; hence, the behavior of (endogenous) prices of domestic primary factors in response to changes in import prices is also described by our "revised" Tobin-Houthakker conjecture (the decomposition of the response to substitution and scale effects and the clarification of the "substitutability" and "complementarity" definitions). The relationship between imports and returns to durables or capital, and imports and labor employment can be examined. In the period 1961–1980, the relative share in value terms of imports in production increased while that of labor decreased in Canada.

Information on the price elasticities of output can be useful in studying the effects of free trade on Canada. A problem is estimating the magnitude of the expected decrease in prices of traded goods arising from a tariff reduction; hence, empirical estimation of resultant output changes is not a straighforward exercise. Actual prices reflect not only explicit tariffs and domestic taxes but also the price effect of other market distortions, trade (quotas, standards and content requirements, etc.) as well as nontrade (marketing boards, interprovincial trade restrictions, etc.). For example, Canadian food processors say that free trade will not significantly decrease the prices of eggs, poultry, milk and the like which are subject to marketing boards. Some of them argue that currency devaluation will be more beneficial to them than a reduction of tariffs, since under the latter, their competitivenesss vis-à-vis the U.S. market is hampered by the relatively higher prices of their input commodities or raw materials. Therefore, the price effect of a tariff reduction cannot be assessed independently of other nontariff distortions in the economy. Another complicating factor is the nonuniformity in application of tariffs. In Canada, tariff rates are governed by several multilateral trade agreements; hence, for the same class of goods different rates may apply depending on the country of origin of the imports.[1]

A weakness of the modeling approach taken in this study is the naive way of introducing factor price rigidity. Though there can be varying causes, models endogenizing price rigidity for a domestic resource is a possible area of future research. Also, social welfare considerations, other than factor income distribution, are ignored. Nevertheless, this study can generate partial

[1]Rates may differ according to most-favored nation tariff, general preferential tariff, British preferential tariff, U.K. (United Kingdom) and Iceland tariff, etc.

information useful in assessing equity-efficiency tradeoffs of government policies impinging on the production sector of the economy.

21.2 The shadow price of a fix-price factor

The theoretical framework discussed assumes that the fix-price production factors have elastic supplies. Generally, in disequilibrium, there can be excess demand for these factors. Labor shortages can occur. Quotas or foreign exchange controls can limit the inflow of imported raw materials. In this case, the production side of the economy is rationed and gives rise to the divergence of the shadow prices of these factors from their market prices (or fixed prices). The model can be reformulated to account for this possibility. Given an endowment $\bar{l}$ of the fix-price factors,[2] the economy's aggregate profit maximization problem becomes

$$\begin{aligned}
&\max_{(c^m,-b^m,-a^m)\in\mathcal{C}^m, m=1,2,\ldots,M} \max_{z\geq 0_M} \{\textstyle\sum_{m=1}^M (p\cdot c^m - r\cdot b^m) z_m : \\
&\qquad\qquad \textstyle\sum_{m=1}^M a^m z_m \leq v,\ \sum_{m=1}^M b^m z_m \leq \bar{l}\} \\
&= \max_{(c^m,-b^m,-a^m)\in\mathcal{C}^m, m=1,2,\ldots,M} [\max_z \{\textstyle\sum_{m=1}^M (p\cdot c^m - r\cdot b^m) z_m : \\
&\qquad\qquad v - \textstyle\sum_{m=1}^M a^m z_m \geq 0_N,\ \bar{l} - \sum_{m=1}^M b^m z_m \geq 0_Q,\ z \geq 0_M\}] \\
&= \max_{(c^m,-b^m,-a^m)\in\mathcal{C}^m, m=1,2,\ldots,M} [\max_{z\geq 0_M} \min_{w\geq 0_N, \sigma\geq 0_Q} \{\textstyle\sum_{m=1}^M (p\cdot c^m - r\cdot b^m) z_m \\
&\qquad\qquad + w\cdot(v - \textstyle\sum_{m=1}^M a^m z_m) + \sigma\cdot(\bar{l} - \sum_{m=1}^M b^m z_m)\}] \\
&\qquad\qquad \text{using the saddlepoint theorem for} \\
&\qquad\qquad \text{linear programs,} \\
&= \max_{z\geq 0_M} \min_{w\geq 0_N, \sigma\geq 0_Q} \{\textstyle\sum_{m=1}^M \pi^m(p,r,w) z_m + w\cdot v + \sigma\cdot\bar{l}\} \text{ using definition (18.1),} \\
&\equiv \tilde{\pi}(p,r,v,\bar{l}) \\
&= \min_{w\geq 0_N, \sigma\geq 0_Q} \{w\cdot v + \sigma\cdot\bar{l} :\ \pi^m(p, r+\sigma, w) \leq 0,\ m = 1,2,\ldots,M\}.
\end{aligned}$$

[2] For labor, the assumption of fixed endowment is a simplification. This assumes that each worker offers a fixed number of hours of work. It may be argued that the economy is not endowed with a certain amount of labor but has an endogenous supply of this factor. That is, the workers have given time endowments and preferences defined over leisure and consumption goods. Hence, a more complete treatment of labor would simultaneously consider the production side and consumption side of the economy.

This maximization problem $\tilde{\pi}(p, r, v, \bar{l})$ explicitly incorporates the constraint on the fix-price factors. Corresponding to these constraints, a new vector of Lagrange multipliers $\sigma \geq 0_Q$ is introduced. These Lagrange multipliers σ can be thought of as the wedge between the shadow price or virtual price of the fix-price factors and their market (or fixed) prices. This interpretation becomes clearer upon examination of the first-order conditions and complementary slackness conditions. Suppose $z^* \gg 0_M$, $w^* \gg 0_N$, $\sigma^* \geq 0_Q$ solve the above problem when the exogenous variables are set at p^*, r^*, v^*, $\bar{l}^*$. Then the following conditions hold:

$$\begin{aligned}
\pi^m(p^*, r^* + \sigma^*, w^*) &= 0,\ m = 1, 2, \ldots, M; \\
\textstyle\sum_{m=1}^{M} \nabla_w \pi^m(p^*, r^* + \sigma^*, w^*) z_m^* + v^* &= 0_N; \\
\textstyle\sum_{m=1}^{M} \nabla_{r'} \pi^m(p^*, r^* + \sigma^*, w^*) z_m^* + \bar{l}^* &\geq 0_Q \text{ where } r' \equiv r + \sigma; \\
\sigma^* \cdot [\textstyle\sum_{m=1}^{M} \nabla_{r'} \pi^m(p^*, r^* + \sigma^*, w^*) z_m^* + \bar{l}^*] &= 0.
\end{aligned}$$

Note that the nonnegativity of the Lagrange multipliers σ, the feasibility constraint and complementary condition on the fix-price factors imply that for each of these factors q, $q = 1, 2, \ldots, Q$,

$$\sigma_q^* \left[\sum_{m=1}^{M} \frac{\partial \pi^m(p^*, r^* + \sigma^*, w^*)}{\partial r_q'} z_m^* + \bar{l}_q^* \right] = 0.$$

By Hotelling's lemma,

$$-\tilde{b}_q^{m*} \equiv \frac{\partial \pi^m(p^*, r^* + \sigma^*, w^*)}{\partial r_q'}$$

is the optimal unit scale input requirement of fix-price factor q in sector m when faced with the price $r^* + \sigma^*$. The above condition can be rewritten as

$$\sigma_q^* \left(\bar{l}_q^* - \sum_{m=1}^{M} \tilde{b}_q^{m*} z_m^* \right) = 0,\ q = 1, 2, \ldots, Q.$$

For each fix-price factor, there are three possible cases.

Case 1. $\sigma_q^* = 0$ and $\bar{l}_q^* - \sum_{m=1}^{M} \tilde{b}_q^{m*} z_m^* = 0$. In this case, the price of factor q is set at its market-clearing level and $r'^* = r^*$. There is no divergence between its shadow price and its market price. The fix-price equilibrium is a Walrasian equilibrium. Walrasian demand and supply of this factor are equal.

Case 2. $\sigma_q^* = 0$ and $\bar{l}_q^* - \sum_{m=1}^{M} \tilde{b}_q^{m*} z_m^* > 0$. In this case, there is excess supply of the fix-price factor and for the producer, the market price it faces is the fixed price r_q^*. The resource constraint on this factor is ineffective and can be ignored as was done in the theoretical framework discussed in the text. This is the case for involuntary unemployment in the labor market. The producers or demanders of labor are not rationed but the consumers or suppliers of labor are compelled to consume more leisure than they desire. From the viewpoint of the whole economy, the divergence of the shadow price of labor from the prevailing wage arises from the consumption side of the economy. If the wage is rigid at too high a level, the quantity constraint on the amount of leisure consumed/labor supplied imposed on the suppliers of labor (given constant time endowments) comes from the production side of the economy. The comparative statics for the production side of an economy in this regime was carried out in the text.

Case 3. $\sigma_q^* > 0$ and $\bar{l}_q^* - \sum_{m=1}^{M} \tilde{b}_q^{m*} z_m^* = 0$. In this case, there is excess demand for the fix-price factor at the fixed price r_q^* and its shadow price is above this fixed price by the amount $\sigma_q^* > 0$ implicitly defined by the equation

$$-\sum_{m=1}^{M} \frac{\partial \pi^m(p^*, r^* + \sigma^*, w^*)}{\partial r_q'} z_m^* = \bar{l}_q^*.$$

This expression for the virtual price $r_q'^* = r_q^* + \sigma_q^*$ is a standard result in the microeconomic literature on quantity rationing.[3] The virtual price or shadow price is the price that would induce the unconstrained producers to employ $\bar{l}_q^*$ of factor q. This virtual price is not necessarily what will prevail in a Walrasian equilibrium because the flexible factor prices w^* can have different configurations in these two regimes.

Empirical implementation of the comparative statics for a production economy in this regime is more involved because of the unobserved Lagrange multipliers σ_q^*. Whereas in case 2, the sectoral profit functions depend only on observables z^*, p^*, r^* and w^*, in this case the shadow price

[3]See Latham (1980, pp.310-311), Neary and Roberts (1980, p.30), Deaton (1981, p.59), Lee and Pitt (1986, p.1238), and Lee (1986, p.301), among others.

wedge σ_q^* is a latent variable.[4] Virtual price functions and rationed demand functions corresponding to some functional forms have been analytically derived by Neary and Roberts (1980) for the linear expenditure system and by Deaton (1981) for an extended version of the linear expenditure system and a rationed almost ideal demand system. The empirical findings of the latter study indicate that treating housing as a ration explains much of the inhomogeneity observed with unrationed demand functions obtained in earlier studies.

Theoretically, a comparison between the observed market price and the virtual price determines the classification of sample observations into periods of excess supply or excess demand. This procedure is equivalent to determining whether σ_q^* is zero or positive. A theoretical work on the use of virtual prices in the specification of inverse demand and supply functions as an alternative to the conventional econometric disequilibirium models with supply and demand functions and min conditions can be found in Lee (1986). This new approach has the advantage of more computationally tractable likelihood functions without necessarily the multiple integrals of the usual multimarket disequilibrium models. However, other problems in likelihood maximization endemic to disequilibrium models remain. Among them are: coherency conditions for nonlinear systems, multiple local maxima, and unboundedness of the likelihood function when some variances of the disturbances approach zero.[5]

From the above discussion, it can be inferred that the comparative statics discussed in the text should be performed only for periods where there is excess supply of the fix-price factors. However, information from sectoral profit functions estimated from time series data are inputs to the comparative static response matrix. Hence, ideally, for the periods when there is excess demand for the fix-price factor, its shadow price should be used instead of the observed market price. Otherwise, an error in variables is made which may bias the parameter estimates for the sectoral profit functions.

[4] This shadow price wedge corresponds to the virtual tax in exchange economies considered by Cornielje and van der Laan (1986).

[5] Survey articles on econometric disequilibrium models can be found in Quandt (1982) and Maddala (1983, 1986).

21.3 Econometric estimation

The major interest in this study is the development of a trade-theoretic approach in the analysis of resource allocation in the presence of unemployment due to real wage rigidity. Factor market imperfections, in general, have aroused interest among trade theorists. The study of real wage rigidity, in particular, has been couched in terms of the theory of minimum wage rates in international trade. In contrast to the large country $2x2$ model used by Brecher (1974a, 1974b) in his pioneering works, this study uses the small country assumption; otherwise, the complications due to terms of trade effects have to be addressed. Additionally, the restriction that the number of flexibly priced factors be at least as great as the number of industries is imposed to avoid specialization in production. Later works in this area were done by Dixit (1978), Schweinberger (1978), Muellbauer and Winter (1980), Neary (1985) and Flug and Galor (1986). The essence of "minimum wages" in trade theory is quite different from that in labor economics; it has to do more with the degree of flexibility, especially downwards, of labor prices relative to other factor prices.

Recent developments in fix-price models can help enrich the "theory of minimum wage rates" in trade literature. However, fix-price theory, as well as the theory of minimum wage rates, is subject to the major criticism of being unable to explain price rigidities and hence, its policy prescriptions are suspect. Without a theory of price determination (for the fix-price good), it has weak predictive power. The implicit contract theory and efficiency wage theory in labor economics are attempts to explain the dual phenomenon of rigid wages and unemployment. Appeal is sometimes made to the existence of socioeconomic and institutional factors, such as labor unions, minimum wages and unemployment insurance, to explain wage rigidity. It seems there can be a multitude of causes of inertia in wage adjustment, some of which are sector-specific.

In his 1979 presidential address to the American Economic Association, Solow (1980) argues that the absence of an acceptable theory to explain the failure of the labor market to clear does not necessarily negate the premise that wages do not move flexibly to clear the market. Rejecting studies that explain the then existing unemployment as voluntary leisure resulting from intertemporal substitution of future work with perceived higher discounted value of earnings, he counters:

> It is thus legitimate to wonder why the unemployed do not feel themselves to be engaged in voluntary intertemporal substitution, and why they queue up in such numbers when legitimate jobs of their usual kind are offered during a recession. (Solow, 1980, p.7)

His hunch is the real wage elasticity of labor demand is low[6] and with a low elasticity of labor demand, together with other institutional factors, sellers of labor services are likely to resist downward wage adjustments. Moreover, the labor market cannot be modeled just like any goods market, say the buying and selling of cloth. The labor market responds to both pecuniary and nonpecuniary elements.

Unemployment and real wage rigidity is a contentious issue which this thesis does not aim to resolve. Empirical analysis is left to more able econometricians. At most, some suggestions and possible problems are outlined here.

A review of published empirical works on testing for excess supply of labor using disequilibrium models and on estimation of natural rates of unemployment indicates that there could have been excess demand for labor in Canada and the United States during the years 1965–1969 and 1972–1974. In the years of excess demand for labor, the production sector is rationed and the producer shadow price of labor diverges from the market wage. Hence, in the sectoral profit estimation the shadow price of labor should be used instead of the market wage for these years. Furthermore, the comparative statics exercise outlined in this study should not be performed

[6] A survey of empirical studies on aggregate employment-wage elasticity can be found in Hamermesh (1986). Estimates of the magnitude of the aggregate long-run constant-output labor demand elasticity for developed countries for the period 1950 onwards lie in the range 0.15–0.50. The estimates are based on the Hicksian formula for the own-wage elasticity of labor demand at constant output, η_{ll}, which for the two-factor case is given by $\eta_{ll} = -(1 - s)\sigma < 0$ where s is the labor share in total revenue and σ is the elasticity of substitution between capital and labor.

for these years.

The theoretical framework discussed can be applied to annual input-output data. The first step is the estimation of sectoral unit scale profit functions $\pi^m(p, r, w)$, $m = 1, 2, \ldots, M$ as defined in equation (18.1). Flexible functional forms providing a second-order approximation with constant returns to scale and technical change variables can be used in specifying the sectoral profit functions π^m. The dimension of the estimation model will be limited by the number of observations at hand. Hence, sectoral aggregation may be necessary. With semiflexible estimation (Diewert and Wales, 1988), the number of goods in the model can be increased.

The theoretical model has assumed constant returns to scale and competitive behavior. Imposing constant returns when otherwise may bias the parameter estimation of sectoral profit functions. In the absence of a general theory of imperfect competition, one has to posit particular market structures and pricing behaviors to handle increasing returns and imperfect competition (see, for example, Helpman and Krugman (1985)). It is recognized that scale economies are a potential source of gains from trade liberalizaiton. For Canada, cost of protection estimates range from 0.5 to 2 percent of GNP under the competitive constant returns assumption; incorporating possible scale economies yields estimates of 8 to 12 percent (Harris, 1984).

Price distortions are another source of economic waste. By using actual prices of inputs and outputs faced by the producer, the inefficiency due to non-Pareto optimality of prices can be captured. Let us redefine the price vector as

$$h \equiv (p_1, p_2, \ldots, p_K, r_1, r_2, \ldots, r_Q, w_1, w_2, \ldots, w_N)^T$$

and the quantity vector as

$$x \equiv (y_1, y_2, \ldots, y_K, -l_1, -l_2, \ldots, -l_Q, -v_1, -v_2, \ldots, -v_N)^T.$$

Let τ^m be a $K + Q + N$ vector of price wedges faced by sector m. Then the actual price faced by sector m is

$$h^m \equiv h + \tau^m.$$

For estimation, the sectoral profit function π^m should take the more general definition

$$\pi^m(h^m) \equiv \max_{x^m}\{h^m \cdot x^m : x^m \equiv (y^m, -l^m, -v^m)^T \in \mathcal{S}^m\}$$

where the elements of x^m are indexed positively(negatively) for outputs(inputs) and $\mathcal{S}^m$ is the constant returns to scale production possibilities set of sector m.

Other than labor, imports can be considered a fix-price factor. Our small country assumption implies that import prices are exogenous. Appelbaum and Kohli (1979) have tested the small open economy hypothesis with respect to Canada-United States trade. Their study indicates that Canada does not exert significant power in its imports market but does in its export markets.

The results of the sectoral profit function estimation can then be used to calculate the comparative static response matrix as given in equation (18.31). For purposes of interpretation, this matrix has to be reexpressed in elasticity form taking into account the price differentials among sectors for the same goods.

Bibliography

[1] Afriat, S.N., 1972, Efficiency estimation of production functions, International Economic Review 13, 568–598.

[2] Appelbaum, E. and U.R. Kohli, 1979, Canada-United States trade: tests for the small-open-economy hypothesis, Canadian Journal of Economics 12, 1–14.

[3] Avriel, Mordecai, 1976, Nonlinear programming analysis and methods (Prentice-Hall, New Jersey).

[4] Banker, R., 1984, Estimating most productive scale size using data envelopment analysis, European Journal of Operational Research 17, 35–44.

[5] Bazaraa, M.S. and C.M. Shetty, 1979, Nonlinear programming: theory and algorithms (Wiley, New York).

[6] Bowlin, W.F., A. Charnes, W.W. Cooper and H.D. Sherman, 1985, Data envelopment analysis and regression approaches to efficiency estimation and evaluation, Annals of Operations Research 2, 113–138.

[7] Brecher, R.A., 1974a, Minimum wage rates and the pure theory of international trade, Quarterly Journal of Economics 88, 98–116.

[8] Brecher, R.A., 1974b, Optimal commercial policy for a minimum-wage economy, Journal of International Economics 4, 139–149.

[9] Cas, A., 1984, New measures of multifactor productivity and structural change for Canadian industries and economy, Unpublished paper (Productivity Unit, Input-Output Division, Statistics Canada, Ottawa).

[10] Caves, D.W., L.R. Christensen and W.E. Diewert, 1982, The economic theory of index numbers and the measurement of input, output and productivity, Econometrica 50, 1393–1414.

[11] Chang, Kuo-Ping and Yeah-Yuh Guh, 1988, Linear production functions and the data envelopment analysis, Unpublished paper (Department of Economics and Department of Industrial Engineering, National Tsing Hua University, Taiwan, R.O.C.).

[12] Charnes, A. and W.W. Cooper, 1985, Preface to topics in data envelopment analysis, Annals of Operations Research 2, 59–94.

[13] Charnes, A., W.W. Cooper, B. Golany and L. Seiford, 1985, Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions, Journal of Econometrics 30, 91–107.

[14] Cornielje, O.J.C. and G. van der Laan, 1986, The computation of quantity-constrained equilibria by virtual taxes, Economics Letters 22, 1–6.

[15] Deaton, A., 1981, Theoretical and empirical approaches to consumer demand under rationing, in: A. Deaton, ed., Essays in the theory and measurement of consumer behavior in honor of Sir Richard Stone (Cambridge University Press, Cambridge) 55–72.

[16] Debreu, G., 1951, The coefficient of resource utilization, Econometrica 19, 273–292.

[17] Deprins, Dominique, Léopold Simar and Henry Tulkens, 1984, Measuring labor-efficiency in post offices, in: M. Marchand, P. Pestieau and H. Tulkens, eds., The performance of public enterprises: concepts and measurement (North Holland, Amsterdam) 243–267.

[18] Diewert, W.E., 1976, Exact and superlative index numbers, Journal of Econometrics 4, 115–145.

[19] Diewert, W.E., 1980a, Aggregation problems in the measurement of capital, in: D. Usher, ed., The measurement of capital (University of Chicago Press, Chicago) 433–528.

[20] Diewert, W.E., 1980b, Capital and the theory of productivity measurement, American Economic Review 70, 260–267.

[21] Diewert, W.Erwin, 1982, A fundamental matrix equation of production theory with application to the theory of international trade, Department of Economics Discussion Paper No. 82–17 (University of British Columbia, Canada).

[22] Diewert, W.E., 1989a, The measurement of productivity, Department of Economics Discussion Paper No. 89–04 (University of British Columbia, Canada).

[23] Diewert, W.E., 1989b, Fisher ideal output, input and productivity indexes revisited, Department of Economics Discussion Paper No. 89–07 (University of British Columbia, Canada).

[24] Diewert, W.Erwin and Celik Parkan, 1983, Linear programming tests of regularity conditions for production functions, in: W. Eichorn, K. Henn, K. Neumann, R. Shephard, eds., Quantitative studies on production and prices (Physica-Verlag) 131–158.

[25] Diewert, W.E. and T.J. Wales, 1987, Flexible functional forms and global curvature conditions, Econometrica 55, 43–68.

[26] Diewert, W.E. and T.J. Wales, 1988, A normalized quadratic semiflexible functional form, Journal of Econometrics 37, 327–342.

[27] Diewert, W.E. and T.J. Wales, 1989a, Linear and quadratic spline models for consumer demand functions, Department of Economics Discussion Paper No. 89–16 (University of British Columbia, Canada).

[28] Diewert, W.E. and T.J. Wales, 1989b, Quadratic spline models for producer demand and supply functions, Preliminary paper (Department of Economics, University of British Columbia, Canada).

[29] Diewert, W.E. and A.D. Woodland, 1977, Frank Knight's theorem in linear programming revisited, Econometrica 45, 375–398.

[30] Dixit, A., 1978, The balance of trade in a model of temporary equilibrium with rationing, Review of Economic Studies 45, 393–404.

[31] Färe, Rolf, Shawna Grosskopf and C.A. Knox Lovell, 1985, The measurement of efficiency of production (Kluwer-Nijhoff Publishing, MA).

[32] Färe, Rolf, Shawna Grosskopf, C.A.K. Lovell and Carl Pasurka, 1989, Multilateral productivity comparisons when some outputs are undesirable: a nonparametric approach, Review of Economics and Statistics 71, 90–98.

[33] Farrell, M.J., 1957, The measurement of productive efficiency, Journal of the Royal Statistical Society, Series A 120, 251–281.

[34] Farrell, M.J. and M. Fieldhouse, 1962, Estimating efficient production functions under increasing returns to scale, Journal of the Royal Statistical Society, Series A 125, 252–267.

[35] Flug, K. and O. Galor, 1986, Minimum wage in a general equilibrium model of international trade and human capital, International Economic Review 27, 149–164.

[36] Forsund, Finn R., C.A. Knox Lovell and Peter Schmidt, 1980, A survey of frontier production functions and of their relationship to efficiency measurement, Journal of Econometrics 13, 5–25.

[37] Fortin, Nicole M., 1988, Evaluating the aggregation biases in a production economy: a stochastic approach, Ph.D. thesis (University of British Columbia, Canada).

[38] Hamermesh, D.S., 1986, The demand for labor in the long run, in : O. Ashenfelter and R. Layard, eds., Handbook of labor economics (North Holland, Amsterdam) 429–471.

[39] Hanoch, Giora and Michael Rothschild, 1972, Testing the assumptions of production theory: a nonparametric approach, Journal of Political Economy 80, 256–275.

[40] Harris, R., 1984, Applied general equilibrium analysis of small open economies with scale economies and imperfect competition, American Economic Review 74, 1016–1032.

[41] Helpman, E. and P.R. Krugman, 1985, Market structure and foreign trade: increasing returns, imperfect competition, and the international economy (MIT Press, Massachusetts).

[42] Hicks, J., 1965, Capital and growth (Oxford University Press, London).

[43] Jones, R.W., 1971, A three-factor model in theory, trade and history, in: J.N. Bhagwati et al., eds., Trade, balance of payments and growth: Essays in honor of C.P. Kindleberger (North Holland, Amsterdam) 3–21.

[44] Jorgenson, Dale, Frank Gollop and Barbara Fraumeni, 1987, Productivity and U.S. economic growth (Harvard University Press, MA).

[45] Jorgenson, D.W. and Z. Griliches, 1967, The Explanation of productivity change, Review of Economic Studies 34, 249–283.

[46] Koopmans, T.C., 1951, Efficient allocation of resources, Econometrica 19, 455–465.

[47] Koopmans, T.C., 1953, Activity analysis and its applications, American Economic Review 43, 406–414.

[48] Koopmans, T.C., 1957, Three essays on the state of economic science (McGraw-Hill, New York).

[49] Kopp, Raymond J. and W.Erwin Diewert, 1982, The decomposition of frontier cost function deviations into measures of technical and allocative efficiency, Journal of Econometrica 19, 219–231.

[50] Kuhn, H.W. and A.W. Tucker, 1950, Nonlinear programming, in: Jerzy Neyman, ed., Proceedings of the Second Berkeley Symposium on Mathematical Statistics (Berkeley, CA) 481–492.

[51] Lasserre, Pierre and Pierre Ouellette, 1988, On measuring and comparing total factor productivities in extractive and non-extractive sectors, Canadian Journal of Economics 21, 826–834.

[52] Latham, R., 1980, Quantity constrained demand functions, Econometrica 48, 307–313.

[53] Lee, L.F., 1986, The specification of multimarket disequilibrium econometric models, Journal of Econometrics 32, 297–332.

[54] Lee, L.F. and M.M. Pitt, 1986, Microeconomic demand systems with binding nonnegativity constraints: the dual approach, Econometrica 54, 1237–1242.

[55] Maddala, G.S., 1983, Limited-dependent and qualitative variables in econometrics (Cambridge University Press, Cambridge).

[56] Maddala, G.S., 1986, Disequilibrium, self-selection and switching models, in: Z. Griliches and M.D. Intriligator, eds., Handbook of econometrics 3 (Elsevier Science Publishers, Amsterdam) 1633–1688.

[57] Mayer, W., 1974, Short-run and long-run equilibrium for a small open economy, Journal of Political Economy 82, 955–967.

[58] McLeod, Young and Weir Company Ltd., 1961–1980, 40 bond yield averages (Toronto, Canada).

[59] Muellbauer, J. and D. Winter, 1980, Unemployment, employment and exports in British manufacturing, a non-clearing market approach, European Economic Review 13, 383–409.

[60] Mussa, M., 1974, Tariffs and the distribution of income: the importance of factor specificity, substitutability, and intensity in the short and long run, Journal of Political Economy 82, 1191–1203.

[61] Neary, J.P., 1978, Short-run capital specificity and the pure theory of international trade, Economic Journal 88, 488–510.

[62] Neary, J.P., 1985, International factor mobility, minimum wage rates, and factor-price equalization: a synthesis, Quarterly Journal of Economics 100, 551–570.

[63] Neary, J.P. and K.H.S. Roberts, 1980, The theory of household behavior under rationing, European Economic Review 13, 25–42.

[64] Ostensoe, L.A., 1986, Production analysis for applied general equilibrium modelling, Honors thesis (Department of Economics, University of British Columbia, Canada).

[65] Quandt, R.E., 1982, Econometric disequilibrium models, Econometric Reviews 1, 1–63.

[66] Rockafeller, R.T., 1970, Convex analysis (Princeton University Press, New Jersey).

[67] Samuelson, P.A., 1960, An extension of the LeChatelier principle, Econometrica 28, 368–379.

[68] Samuelson, P.A., 1971, Ohlin was right, Swedish Journal of Economics 73, 365–384.

[69] Schweinberger, A.G., 1978, Employment subsidies and the theory of minimum wage rates in general equilibrium, Quarterly Journal of Economics 92, 361–374.

[70] Solow, R.M., 1980, On theories of unemployment, American Economic Review 70, 1–11.

[71] Solow, R.M., 1986, Unemployment: getting the questions right, Economica 53, S23–S34.

[72] Statistics Canada, 1984, The input-output structure of the Canadian economy in constant prices 1971–1980, Catalogue 15-202E (Minister of Supply and Services, Ottawa).

[73] Statistics Canada, 1987, Fixed capital flows and stocks (historical) 1936–1983, Catalogue 13-568 (Minister of Supply and Services, Ottawa).

[74] Varian, Hal R., 1984a, The nonparametric approach to production analysis, Econometrica 52, 579–597.

[75] Varian, Hal R., 1984b, Microeconomic analysis (W.W. Norton, New York).

[76] Varian, Hal R., 1985, Non-parametric analysis of optimizing behavior with measurement error, Journal of Econometrics 30, 445–458.

[77] White, K.J., 1987, SHAZAM: A general computer program for econometric methods (version 5), American Statistician 41, 80.

Appendix A

Incorporating the No Technological Regress Assumption with the Efficiency Tests

Listed below are the modifications in the various efficiency tests needed to incorporate the no technological regress assumption. The changes involve merely a change in the range of the observation indices used in constructing the relevant convex sets.

- Test 1′. Technical Efficiency and No Technological Regress Test for a Convex Technology:

  Modify the linear programming subproblem (3.2) to

  $$\max_{\delta_i \geq 0, \lambda^1 \geq 0, \ldots, \lambda^i \geq 0} \{\delta_i : \textstyle\sum_{j=1}^{i} \lambda^j z^j \geq z^i + \delta_i \hat{\gamma} z^i, \ \textstyle\sum_{j=1}^{i} \lambda^j = 1\} \equiv \delta_i^*.$$

- Test 2′. Technical Efficiency and No Technological Regress Test for a Convex Conical Technology:

  Modify the linear programming subproblem (4.2) to

  $$\max_{\delta_i \geq 0, \lambda^1 \geq 0, \ldots, \lambda^i \geq 0} \{\delta_i : \textstyle\sum_{j=1}^{i} \lambda^j z^j \geq z^i + \delta_i \hat{\gamma} z^i\} \equiv \delta_i^*.$$

- Test 3′. Technical Efficiency and No Technological Regress Test for a Quasiconcave Technology:

  Modify the index set defined by (5.2) to

  $$I_i^n \equiv \{j : z_n^j \geq z_n^i, \ j = 1, 2, \ldots, i\}.$$

- Test 4′. Allocative Efficiency and No Technological Regress Test for a Convex Technology Assuming Partial Profit Maximization:

Modify the linear programming subproblem (7.1) to

$$\max_{\lambda^1\geq 0,\ldots,\lambda^i\geq 0}\{\textstyle\sum_{n\in S}p_n^i(\sum_{j=1}^i\lambda^j z_n^j) : \sum_{j=1}^i\lambda^j z_n^j \geq z_n^i,\ n\notin S,\ \sum_{j=1}^i\lambda^j = 1\}$$
$$\equiv \textstyle\sum_{n\in S}p_n^i z_n^i + \varepsilon_i^S\sum_{n\in E}p_n^i|z_n^i|.$$

- Test 5′. Allocative Efficiency and No Technological Regress Test for a Convex Conical Technology Assuming Partial Profit Maximization:

  Modify the linear programming subproblem (9.1) to

$$\max_{\lambda^1\geq 0,\ldots,\lambda^i\geq 0}\{\textstyle\sum_{n\in S}p_n^i(\sum_{j=1}^i\lambda^j z_n^j) : \sum_{j=1}^i\lambda^j z_n^j \geq z_n^i,\ n\notin S\}$$
$$\equiv \textstyle\sum_{n\in S}p_n^i z_n^i + \varepsilon_i^S\sum_{n\in E}p_n^i|z_n^i|.$$

- Test 6′. Allocative Efficiency and No Technological Regress Test for a Quasiconcave Technology Assuming Partial Profit Maximization:

  Modify the index set defined by (10.1) to

$$I_i^n \equiv \{j : z_n^j \geq z_n^i,\ j = 1,2,\ldots,i\}.$$

- Test 7′. Allocative Efficiency and No Technological Regress Test for a Convex Technology Assuming Complete Profit Maximization:

  Redefine the violation index defined in (11.1) to

$$\varepsilon_i^* \equiv \max_j\left\{\frac{p^{iT}(z^j - z^i)}{p^{iT}\hat{\gamma}z^i} : j = 1,2,\ldots,i\right\}.$$

- Test 8′. Allocative Efficiency and No Technological Regress Test for a Convex Conical Technology Assuming Complete Profit Maximization:

  Redefine the violation index defined in (11.12) to

$$\varepsilon_i^* \equiv \max_j\left\{\frac{p^{niT}(\bar{z}^{nj} - \bar{z}^{ni})}{\sum_{m\in E}p_m^i|\bar{z}_m^i|} : j = 0,1,\ldots,i\right\}.$$

- Test 9′. Allocative Efficiency and No Technological Regress Test for a Quasiconcave Technology Assuming Complete Profit Maximization:

  Modify the index set defined by (11.28) to

$$I_i^n \equiv \{j : z_n^j \geq z_n^i,\ j = 1, 2, \ldots, i\}.$$

In each test, if the violation index (δ_i^*, ε_i^S or ε_i^*) is zero for all observations i, $i = 1, 2, \ldots, J$, then the data are consistent with the efficiency hypothesis being tested. Otherwise, if the violation index is positive for some observation i, then a violation of the efficiency hypothesis occurs at this observation.

Appendix B

The Canadian Input-Output Data, 1961–1980

B.1 The data base

The original data base used in this study is the 1961–1980 annual Canadian input-output accounts compiled by Cas (1984) for use in preliminary studies on multifactor productivity measures for the Input-Output Division of Statistics Canada. This data base draws from both published and unpublished data sources at Statistics Canada. It is generally comparable to published Canadian input-output tables at the "M" level of aggregation. Whereas the latter gives commodity by industry transaction accounts, the present data base has been modified into an industry by industry classification. It covers the domestic private industrial sectors for which the profit-maximizing model is applicable though some industries, for example, broadcasting and rail transportation, are heavily subsidized by the government. Dummy industries, used for routing goods whose precise commodity content are unknown, and which appear in published accounts are not included in the present data base. Due to problems obtaining data on its capital stock, the "post office" industry was also removed from this data set.

Input-output data for 37 sectors of the Canadian economy are given. For each sector, there are 57 data items: 5 output variables, 37 intermediate input variables, 9 primary input variables and 6 financial variables. These data items are listed in table B.10. Each variable is given in values of current and constant (1961 or 1971) dollars from which implicit price deflators can be derived; constant dollar values can then be used as quantities.

The output variables are classified according to end use. Intermediate inputs pertain only to purchases of current output of domestic industries by domestic industries. Primary inputs are termed "primary" in the sense that the origin of these goods are external in time or boundaries

outputs:	
1. gross output	4. exports
2. intermediate output	5. re-exports
5. final demand	
intermediate inputs from sectors:	
1. agriculture and fishing	20. nonmetallic mineral products
2. forestry	21. petroleum and coal products
3. mines, quarries and oil wells	22. chemical and chemical products
4. food and beverages	23. miscellaneous manufacturing
5. tobacco products	24. construction
6. rubber and plastic products	25. air transporation, other utilities and transportation
7. leather	
8. textiles	26. railway transportation and telegraph
9. knitting mills	27. water transportation
10. clothing	28. motor transportation
11. woods	29. urban and suburban transportation
12. furniture and fixtures	30. storage
13. paper and allied industry	31. broadcasting
14. printing, publishing and allied industries	32. telephone
15. primary metals	33. electric power
16. metal fabricating	34. gas distribution
17. machinery	35. trade
18. transportation equipment	36. finance, insurance and real estate
19. electrical products	37. commercial services
primary inputs:	
1. imports, competitive	6. finished inventories
2. imports, noncompetitive	7. machinery and equipment (M&E)
3. government goods	8. structures (S)
4. labor	9. land
5. raw inventories	
financial variables:	
1. commodity indirect taxes	4. royalties
2. subsidies	5. capital consumption allowance, M&E
3. other indirect taxes	6. capital consumption allowance, S

Table B.10: Data items for each of the 37 sectors relative to current production of the 37 domestic business sectors; hence, the inclusion of imports and government goods in this group of inputs. In contrast to published Canadian input-output accounts where imports are included in the use and final demand matrices, the present data base has netted out this component of the intermediate input matrices. The land data series was constructed by Cas, Diewert and Ostensoe (1986). The "financial" variables are used to adjust prices of outputs and factors so as to reflect actual prices received or paid by the producers.

B.2 Data manipulations at the sectoral level

All prices and quantities obtained are normalized such that 1961 prices equal 1.0 and expenditures or nominal values (current dollar values in \$1000 Canadian) are preserved.

B.2.1 Output and intermediate input prices and quantities

Each of the 37 sectors is assumed to produce a single output with market prices p_i, $i = 1, 2, \ldots, 37$. These prices p_i are obtained as implicit price deflators calculated as the ratio of current dollar value to constant dollar value of gross output:

$$p_i \equiv \frac{\text{current dollar value of gross output of sector } i}{\text{constant dollar value of gross output of sector } i}, \; i = 1, 2, \ldots, 37.$$

For price consistency, the prices of intermediate inputs are also set equal to their corresponding output prices. That is, if we let p_{ij} be the price of a unit of good i used in the production of good j, then $p_{ij} = p_i$, $i, j = 1, 2, \ldots, 37$. The corresponding quantities of intermediate inputs are adjusted accordingly. Let q_{ij} be the quantity of good i used to produce good j; then

$$q_{ij} \equiv \frac{\text{current dollar value of input } i \text{ used in sector } j}{p_i}.$$

For the estimation of profit functions, the concept of net output is used. Hence, gross output quantities have to be adjusted by subtracting the quantity of intermediate inputs coming from the same sector. Denote the net output by sector i as q_i; then

$$q_i \equiv (\text{constant dollar value of gross output of sector } i) - q_{ii}.$$

Own input prices and quantities are set to zero: $p_{ii} = q_{ii} = 0$.

B.2.2 Missing observations for 1980 intermediate inputs

Prior to doing data manipulations on the data base, a check for missing observations was done. There are 23 missing observations for current dollar (as well as constant dollar) values for intermediate inputs in 1980. Due to differences in commodity by industry configurations and the netting out of imports in the use matrix for the present data base, cross-checking with published input-output accounts is precluded. Hence, an ad hoc procedure was used to fill in the missing observations.

For five sectors (3, 15, 16, 22, 23) in 1980, there are significant differences between the value of the intermediate output variable and the sum of the values of the good used as intermediate inputs (as given by $\sum_{j=1}^{37} p_{ij}q_{ij}$, $i = 3, 15, 16, 22, 23$). These differences are listed in table B.11. Except for miscellaneous manufacturing, the figures indicate that a substantial proportion, 8% to 23%, of intermediate output may have been unallocated to various users. For miscellaneous manufacturing, the 1980 value of intermediate output was revised so that the ratio of intermediate output to gross output is identical to that of 1979. To impute values for the missing observations, the share of the using sector j of the value of intermediate output of sector i ($p_{ij}q_{ij}$/value of intermediate output of sector i) in 1979 is assumed to carry over to 1980. The 1979 values and the 1980 imputed values for the missing ovservations are listed in table B.12. Once the imputed current dollar values for the missing observations for the intermediate inputs are obtained, the relevant price and quantity variables, p_{ij} and q_{ij}, can easily be calculated as outlined in the previous section.

B.2.3 Imports and inventories

The attribution of imports and inventories to different categories can be considered questionable. Whereas all sectors have positive competitive import values for all years 1961–1980, only eight sectors $(4, 6, 8, 20, 22, 23, 35, 37)$ have positive noncompeting import values for all years. Eleven other sectors $(3, 7, 9, 10, 13–16, 18, 19, 21)$ have positive noncompeting values for one to nineteen years. The other 18 sectors have zero noncompeting imports for all years. Of the total value

sector i		value of intermediate output* (1)	value of the good used as intermediate inputs* (2)	difference (1)-(2) (3)	percentage discrepancy (3)/(1) (4)
3	mines, etc.	17554720	16121327	1433393	8.17%
15	primary metals	9798036	8392645	1405391	14.34%
16	metal fabricating	8150517	6282122	1868395	22.92%
22	chemical, etc.	6995275	6149460	845815	12.09%
23	misc. manufacturing	1066220	1074116	-7896	-0.74%

$*$ Figures are in current dollars.

Table B.11: Inconsistencies in the value of intermediate output and the sum of values of the good used as intermediate inputs, 1980

of imports, both competitive and noncompetitive used by the whole economy (the 37 sectors), noncompeting imports account only for 2.3% to 6.1%.

Except for three sectors (2, 3, 37), all sectors have positive values for raw inventories input. The absence of raw inventories in forestry and in the mines, quarries and oil wells sectors is disturbing. The resource and manufacturing sectors (1–23) have positive finished inventories. The rest — construction, transportation, communication, utilities and services sectors (24–37) — have zero finished inventories. Of the group with both positive values for raw and finished inventories (sectors 1, 4–23), all have identical data series for the years 1961–1970; for agriculture and fishing the implicit price deflators for the two data items are the same for all years. For the whole economy, raw inventories as a proportion of the value of total inventories has decreased from 68% in 1961 to 55% in 1980; the share of finished inventories has increased from 32% in 1961 to 45% in 1980.

In the light of the above data deficiencies, it has been decided to merge imports and inventories into single categories. The price and quantity variables for imports, p_M and q_M, were calculated as follows:

$$p_M \equiv \frac{N_{CM} + N_{XM}}{R_{CM} + R_{XM}},$$
$$q_M \equiv R_{CM} + R_{XM}$$

	originating sector i		using sector j	1979 value for intermediate input $p_{ij}q_{ij}$	1980 imputed value for intermediate input $p_{ij}q_{ij}$
3	mines,	3	mines,etc.	880393	1008678
	quarries,	18	transportation equipment	29951	34315
	and oil wells	22	chemical,etc.	340748	390400
			total	1251092	1433393
15	primary	3	mines,etc.	185221	157207
	metals	18	transportation equipment	1342057	1139073
		22	chemical,etc.	128555	109111
			total	1655833	1405391
16	metal	3	mines,etc.	119078	128917
	fabricating	15	primary metals	176037	190582
		16	metal fabricating	811038	878050
		19	electrical products	160451	173708
		22	chemical,etc.	125582	135958
		23	misc. manufacturing	50303	54459
		35	trade	158132	171198
		37	commercial services	125179	135522
			total	1725800	1868395
22	chemical and	15	primary metals	122519	133806
	chemical	18	transportation equipment	108047	118000
	products	19	electrical products	143078	156258
		23	misc. manufacturing	112959	123365
		35	trade	71816	78432
		37	commercial services	216051	235954
			total	774470	845815
23	miscellaneous	3	mines,etc.	20440	22102
	manufacturing	18	transportation equipment	44715	48351
		22	chemical,etc.	28486	30803
			total	93641	101256*

* This figure has been revised as described in the text and hence differs from the entry in column 3 of table B.11.

Table B.12: Imputed values for missing observations for 1980 intermediate inputs where

$$
\begin{aligned}
N_{CM} &\equiv \text{current dollar or nominal value of competitive imports,} \\
N_{XM} &\equiv \text{current dollar or nominal value of noncompetitive imports,} \\
R_{CM} &\equiv \text{constant dollar or real value of competitive imports, and} \\
R_{XM} &\equiv \text{constant dollar or real value of noncompetitive imports.}
\end{aligned}
$$

The price and quantity variables for inventories, p_I and q_I, were calculated analogously:

$$
\begin{aligned}
p_I &\equiv \frac{N_{RI} + N_{FI}}{R_{RI} + R_{FI}}, \\
q_I &\equiv R_{RI} + R_{FI}
\end{aligned}
$$

where

$$
\begin{aligned}
N_{RI} &\equiv \text{current dollar or nominal value of raw inventories,} \\
N_{FI} &\equiv \text{current dollar or nominal value of finished inventories,} \\
R_{RI} &\equiv \text{constant dollar or real value of raw inventories, and} \\
R_{FI} &\equiv \text{constant dollar or real value of finished inventories.}
\end{aligned}
$$

B.2.4 Tax adjustment of output and input prices

The relevant output and factor prices for profit function estimation are "the output prices that reflect the revenue actually received by the firm and the input prices that reflect the actual cost paid by the firm for the use of the inputs in the production process" (Diewert, 1980a, p.479). Output prices should reflect payments, after subsidies and royalties, to the producer, that is,

$$
\begin{aligned}
p_i^* &\equiv p_i + s_i - ry_i \\
&= p_i \left(\frac{N_i^*}{N_i} \right)
\end{aligned}
$$

where

$p_i^* \equiv$ tax adjusted output price,

$p_i \equiv$ market output price (initially obtained as implicit price deflator),

$s_i \equiv$ per unit subsidy receipts,

$ry_i \equiv$ per unit royalty payments,

$N_i \equiv$ current dollar value of gross output,

$N_i^* \equiv$ current dollar value of gross output plus subsidy minus royalty, and

$\frac{N_i^*}{N_i} \equiv$ proportion of the market price actually received by the producer.

All sectors received some form of subsidy for at least some years. The data reflect royalty payments from only five sectors: forestry; mines, quarries and oil wells; woods; electric power; and trade (the latter for only 1976–1980). Note that we might have expected an industry like petroleum and coal products to have positive royalty payments. The data on royalty payments can be considered questionable. The way the national accounts are constructed, royalties on natural resources are classified as an industry under the finance, insurance and real estate sector. Ideally, these royalty payments should be considered as taxes levied on the use of government-owned natural resources or as rental prices for a separate capital aggregate for land or a natural resource factor of production.

Given the data, we list in table B.13 the sectors whose producer prices p_i^* deviate, on the average, by more than 1% from their market price p_i. Hence, for broadcasting, an additional 48 cents was given by the government as subsidy for every dollar received in the marketplace. It must be noted though that the subsidy rate has been consistently decreasing over the years but still remained relatively high compared to other sectors. On the other hand, seven to eight cents of every dollar received by the producer in forestry and the mining sectors went back to the government as royalties. Prior to 1974, there was hardly any price distortion as measured by $\frac{N_i^*}{N_i}$ in the petroleum and coal products sector; thereafter, the subsidy ranged from a minimum of 5 cents for every dollar in 1978 to 26 cents for every dollar in 1975 and in 1980. A similar

sectors		average $\frac{N^*}{N}$	minimum $\frac{N^*}{N}$, (year)	maximum $\frac{N^*}{N}$, (year)
1	agriculture and fishing	1.036	1.008 (1963)	1.064 (1975)
2	forestry	0.931	0.896 (1979)	0.957 (1976)
3	mines, etc.	0.925	0.831 (1978)	0.976 (1961)
21	petroleum, etc.	1.054	*	*
26	rail transportation, etc.	1.082	1.035 (1971)	1.163 (1975)
27	water transportation	1.019	1.006 (1974)	1.039 (1977)
31	broadcasting	1.478	1.355 (1980)	1.564 (1961)
34	gas distribution	1.042	*	*

* See text for interpretation for the petroleum and coal products sector and the gas distribution sector.

Table B.13: Average proportion of market price received by the producers in sectors with the most output price distortions

increase in subsidy rates occured in the gas distribution sector. Pre-1974, the subsidies were negligible; the subsidy rate was 5 cents for every dollar in 1974 and increased to a maximum of 14 cents for every dollar thereafter. The increase in subsidy rates to the petroleum and coal products and gas distribution sectors can be interpreted as policy responses to the oil price shock that occured in 1974–1975.

The tax-adjusted output prices p_i^* we obtained are the relevant prices we seek for profit function estimation. However, we must be cautious in interpreting the value of $\frac{N_i^*}{N_i}$ as an indicator of relative sectoral price distortions. Keeping in mind the data imperfections, we interpret these numbers as measuring the effect of explicit royalties and subsidies per se on actual producer revenue. There are other price and nonprice distortions or policy instruments which may have more significant effects on producer prices, that is, directly on our "unadjusted" output prices p_i.

Margins arise between the value received by the producer to cover his cost of production and the value paid by the purchaser. At the producer side, we have to adjust for subsidies and royalties. Trade, transportation and delivery margins have already been recorded as output of the relevant sectors in the original data base; hence, the initial unadjusted output prices p_i are those levied at the final stage of production. As well, these trade, transportation and delivery margins appear separately as intermediate inputs in the accounts of the purchasing sector. Another margin that has to be accounted for is the value of commodity indirect taxes paid by the using sectors for their intermediate inputs.[1]

Since the data base gives only the total value of commodity indirect taxes paid by each sector, an ad hoc assumption has to be made on how to distribute them over the relevant inputs. We assume an equal commodity tax rate for all intermediate inputs and imports in a particular sector. Let t_j denote the commodity tax rate in sector j. We define t_j as the ratio of the value of commodity indirect taxes in sector j to the sum of the values of intermediate inputs from other sectors and the value of imports:

$$t_j \equiv \frac{\text{value of commodity indirect taxes paid by sector } j}{\sum_{i \neq j} p_{ij} q_{ij} \; + \; p_M q_M}.$$

Then, the tax-adjusted input prices p^*_{ij}, $j = 1, 2, \ldots, 37$ and p^*_M are

$$\begin{aligned} p^*_{ij} &\equiv p_{ij}(1 + t_j) \equiv p_i(1 + t_j), \\ p^*_M &\equiv p_M(1 + t_j). \end{aligned}$$

With this tax imputation, the total cost to the purchasing sector, say sector j, for its intermediate inputs from other domestic sectors and imports is equal to the value of the inputs assessed at the initial prices p_{ij}, p_M plus the value of commodity indirect taxes paid by the purchasing sector:

$$\begin{aligned} \textstyle\sum_{i=1}^{37} p^*_{ij} q_{ij} + p^*_M q_M &= (\textstyle\sum_{i=1}^{37} p_{ij} q_{ij} + p_M q_M) \\ &\qquad + (\text{value of commodity indirect taxes paid by sector } j). \end{aligned}$$

The commodity tax rates t_j were lowest in the manufacturing sectors (4–23, except 20) and electric power industry; it was usually less than 1% to about 1.5% in these sectors. The rates, ranging from 4% to 18%, were generally higher in the construction and air, motor and

[1]Commodity indirect taxes include federal taxes (excise duties, excise taxes and federal sales taxes, oil export charge, petroleum levy, Canadian ownership charge, natural gas and gas liquids tax, and air transport tax), provincial taxes (amusement tax, fuel tax, profits of liquor commissions, liquor gallonage tax, Quebec Hydro levy, and sales tax on gas, electricity, telephone and telegraph), local amusement tax, other provincial and local sales taxes, and import duties.

urban transportation sectors. The rest of the industries, usually resource and service sectors, have rates averaging 3% to 6%. In reality, commodity taxes are commodity-specific; hence, the sectoral differences in our calculated effective commodity tax rates t_j can be explained by differences in sectoral input requirements.

B.2.5 Capital rental prices

Definition of capital service and capital rental price

The capital input variables are taken to be the durable goods: inventories, machinery and equipment, structures and land ($k = 1, 2, 3$ and 4, respectively). The data base gives values of capital stock in current and constant dollars. For a particular capital good, say k, asset price p_k^K and quantity q_k^K were obtained as

$$\begin{aligned} p_k^K &\equiv \frac{\text{current dollar value of capital stock } k}{\text{constant dollar value of capital stock } k}, \\ q_k^K &\equiv \text{constant dollar value of capital stock } k. \end{aligned}$$

For profit function estimation, the relevant price for a durable input is the capital rental price or user cost of a flow of capital services used in the production process. As noted by Jorgenson and Griliches (1967), though asset prices may be considered the discounted value of all future capital services across capital goods, they are not proportional to their service prices (what we terms here as rental price or user cost) because different capital goods can have different economic and physical depreciation rates and asset price appreciation rates (capital gains or losses).

Denote the rental price for capital good k as p_k^{K*}. Following Diewert (1980a), we define the rental price p_k^{K*} before corporate income tax as

$$p_k^{K*} \equiv \frac{r_k p_k^K + (1 + r_k)\tau_k p_k^K + \delta_k \hat{p}_k - (\hat{p}_k - p_k^K)}{1 + r_k}$$

where

$$p_k^{K*} \equiv \text{rental price for capital good } k,$$

$p_k^K \equiv$ asset price (or purchase price) for capital good k,

$\hat{p}_k \equiv$ next period's expected asset price for capital good k,

$r_k \equiv$ rate of return for capital good k,

$\tau_k \equiv$ property tax rate for capital good k, and

$\delta_k \equiv$ combined depreciation and obsolescence rate for capital good k.

Reiterating Jorgenson and Griliches' point, we can see from the above formula that asset prices and rental prices are proportional to each other (and can be econometrically safe to be substituted for each other) only under some restrictive conditions. One case is when there are static expectations ($\hat{p}_k = p_k^K$) and the discount, property tax and depreciation rates (r_k, τ_k, δ_k) are identical across all capital goods and across all sectors. Ideally, the rental prices must be adjusted to reflect the actual cost to the producers after corporate income taxes. Due to problems in matching financial data on corporate income taxes collected on a company basis and input-output data collected on an establishment basis, this adjustment cannot be carried out in this study.[2] This implies that the rental prices we obtain may not be capturing the distortionary effect of policies such as investment tax credits and accelerated depreciation tax rules.

Previous attempts at incorporating nonstatic expectations of future asset prices by Ostensoe (1986) and Fortin (1988) yielded negative and erratic capital rental prices. Drawing from these studies but still acknowledging that the problem of modelling capital gains still remain, we henceforth assume static expectations for future asset prices: $\hat{p}_k = p_k^K$. We also assume that in a sector with more than one type of capital, the rate of return r_k is equal across asset types. The reason for this assumption was expressed by Jorgenson, Gollop and Fraumeni (1987, p.145):

> It is with respect to each asset's nominal rate of return that economic agents choose the optimal mix of capital stocks, altering the composition of capital input until all

[2] An example of the incorporation of corporate tax liabilities in the calculation of capital rental prices using United States data can be found in Jorgenson, Gollop and Fraumeni (1987, p.128).

nominal rates of return are equalized across asset classes. Consequently, measured property compensation should be allocated among assets on the basis of equality of the nominal rate of return for all assets.

With these two assumptions, the capital rental formula reduces to

$$p_k^{K*} = \left[\frac{r + (1+r)\tau_k + \delta_k}{1+r}\right] p_k^K$$

where r is the rate of return or discount rate identical across asset types in a particular sector.

Implicitly, we assume that capital service flows are proportional to capital stocks, that is, the rate of capital utilization is constant over time. Hence, we take the variable q_k^K as the relevant quantity measure for our profit function estimation. The problem of estimating capital stocks and rental prices with consistent patterns of relative efficiency is discussed by Jorgenson, Gollop and Fraumeni (1987).

Calculation of capital rental prices

For the empirical implementation of the capital rental price formula, we further assume that inventories of raw materials and finished goods and machinery and equipment capital stocks are not levied property taxes and that inventories and land have zero depreciation rates. In short, we assume $\tau_1 = \tau_2 = 0$ and $\delta_1 = \delta_4 = 0$.

A detailed description of the data on the capital series on machinery and equipment, and structures can be found in the Statistics Canada Catalogue 13-568: Fixed Capital Flows and Stocks (Historical), 1936–1983 compiled by the Construction Division. Our data on machinery and equipment include capital items charged to operating expenses; structures include both building and engineering construction. The current and constant dollar value of capital stocks are mid-year net stock values based on the difference between the cumulative value of past gross investment and cumulative depreciation. The depreciation or capital consumption allowance data are also mid-year values and are based on the straight-line depreciation rule.

Since the quantity variable q_k^K has already netted out depreciation for the same year, we would like a quantity variable, say q_k^{K*}, gross of depreciation. We define the adjusted variable as

$$q_k^{K*} \equiv \begin{cases} q_k^K + (\text{capital consumption allowance in constant dollars} \\ \qquad \text{for capital good } k),\ k = 2,3 \\ q_k^K,\ k = 1,4. \end{cases}$$

This correction leads to lower depreciation and property tax rates especially for sectors having assets whose specified service lives are shorter. For example, machinery and equipment in forestry, construction, and air and motor transportation sectors have service lives of 10 years or less whereas in other sectors the service lives range from 15 to 35 years.

Our data on other indirect taxes, which are not commodity-specific, are assumed to comprise mainly of property taxes levied on fixed capital: structures and land. The property tax rates τ_k are defined as the ratio of the value of other indirect taxes paid to the value of structures and land in a given sector:

$$\tau_k \equiv \begin{cases} \dfrac{\text{other indirect taxes in current dollar paid by sector}}{\sum_{k=3}^{4} p_k^K q_k^{K*}},\ k = 3,4 \\ 0,\ k = 1,2 \end{cases}$$

where $p_k^K q_k^{K*} \equiv$ current dollar value of structures and land in given sector, respectively.

The twenty-year average of the property tax rates for most sectors ranges from 1% to 5%. It was slightly higher (7–10%) in knitting mills, clothing, furniture and fixtures, and miscellaneous manufacturing. It was less than 1% for the petroleum and coal products, electric power and transportation sectors except for motor transportation. The property tax rates τ_k obtained for the construction (24) and motor transportation (28) industries are substantially higher, averaging 25% and 34%, respectively. These values may indicate some data anomalies pertaining to these sectors. As noted by Fortin (1988), the taxes for these two industries may include payments for permits and licenses. If this is the case, then the asset prices of fixed capital in these sectors have to be adjusted to reflect these payments; in an efficient market we would expect these payments to be capitalized in the purchase price.

The ratio of the capital consumption allowance to the value of capital stock yields the effective depreciation rates. We define the rates δ_k as

$$\delta_k \equiv \begin{cases} \dfrac{\text{capital consumption allowance in current dollars for capital good } k}{p_k^K q_k^{K*}}, & k = 2,3 \\ 0, \; k = 1,4. \end{cases}$$

The capital stock and consumption allowance data for machinery and equipment, and structures are based on the straight-line depreciation rule. We would then expect the depreciation rates δ_k to be inversely related to the service life specified for that asset located in a particular sector.

The estimates for the depreciation rates are quite stable over the years. The depreciation rates for machinery and equipment are lowest (about 4%) in the electric power and gas distribution sectors where these assets, excluding capital items charged to operating expenses, have the longest specified service life of 35 years. Most sectors have depreciation rates from 5% to 10%. Some sectors (1, 2, 6, 7, 23–25, 28, 29, 37) where most machinery and equipment have service lives of 15 years or less have average depreciation rates between 12% and 16%.

Having longer service lives of at least 20 years to at most 75 years, structures have lower depreciation rates. The sectoral averages for the depreciation rates of structures range from 2% to 7%, with most clustering in the 3–5% range.

It is theoretically unclear which discount rate r is to be used. In practice, either an internal rate of return or an exogenous bond rate is used. Diewert (1980a) argues that the relevant interest rate should reflect the firm's actual borrowing and lending rates. The choice of an exogenous bond rate can be justified on the grounds that firms make their decisions based on ex ante rates of return and the exogenous bond rates would be more reflective of the opportunity cost of capital. In the same manner that equality of rates of return determine the optimal mix of capital goods in a particular sector, then the discount rate should also be identical across sectors if there is relatively smooth capital adjustment in the economy. Additionally, calculated internal rates of return may be negative and tend to be more erratic compared to an exogenous bond rate.

The use of sector-specific internal rates of return is consistent with the assumption of constant returns to scale technologies; it enables us to impose the equality of value of output and factor payments. It seems, too, with static expectations, the sectoral reallocation of capital may respond to ex post user costs. As in the labor market where sectoral wage dispersion does not seem to be an unreasonable phenomenon, the rates of return for capital assets can vary across sectors. The choice of the relevant discount rate may be related to the problem that the industries operate in a world subject to risks and uncertainty and are in a constant flux of adjustment.

To see how both rates perform empirically, we calculate sector-specific internal rates of return and compare them to an exogenous bond rate. First, for each sector i, $i = 1, 2, \ldots, 37$, we calcualate the surplus value S_i which is the net value added by capital assets after variable costs:

$$\begin{aligned} S_i &\equiv \text{surplus value for sector } i \\ &= p_i^* q_i - \textstyle\sum_{j=1}^{37} p_{ji}^* q_{ji} - p_M^* q_M - p_G q_G - p_L q_L \end{aligned}$$

where

$$\begin{aligned} p_G &\equiv \text{unit price of government goods used in sector } i \\ &= \frac{\text{current dollar value of government goods used in sector } i}{\text{constant dollar value of government goods used in sector } i}, \\ q_G &\equiv \text{quantity of government goods used in sector } i \\ &= \text{constant dollar value of government goods used in sector } i, \\ p_L &\equiv \text{unit price of labor input in sector } i \\ &= \frac{\text{current dollar value of labor input in sector } i}{\text{constant dollar value of labor input in sector } i}, \text{ and} \\ q_L &\equiv \text{quantity of labor input in sector } i \\ &= \text{constant dollar value of labor input in sector } i. \end{aligned}$$

We then equate capital income to the surplus value:

$$S_i = \textstyle\sum_{k=1}^{4} p_k^{K*} q_k^{K*}.$$

Given our rental price formula for p_k^{K*}, we can solve for the internal rate of return for sector i, say r_i, as

$$r_i = \frac{S_i - P_i - D_i}{A_i + P_i - S_i}$$

where

$$\begin{aligned}
P_i &\equiv \text{property taxes on capital assets paid by sector } i \\
&= \textstyle\sum_{k=1}^{4} \tau_k p_k^K q_k^{K*} \\
&= \text{other indirect taxes paid by sector } i, \\
D_i &\equiv \text{value of depreciation of capital assets in sector } i \\
&= \textstyle\sum_{k=1}^{4} \delta_k p_k^K q_k^{K*} \\
&= \text{capital consumption allowances in sector } i, \text{ and} \\
A_i &\equiv \text{value of capital assets in sector } i \\
&= \textstyle\sum_{k=1}^{4} p_k^K q_k^{K*}.
\end{aligned}$$

The sectoral averages and standard deviations of internal rates of return over the period 1961–1980 are listed in table B.14. Beforehand, let us examine some inconsistencies in the calculated internal rates of return. The rail, water and urban transportation sectors (26, 27, 29) have either near zero or negative internal rates of return. In these sectors, the surplus value which we equate to capital income is too small and in the years with negative returns cannot cover property taxes and depreciation costs.[3] For the urban transportation sector, the surplus was negative for 1972 to 1980. For selected years, negative internal rates of return were also obtained for forestry (1975, 1976), primary metals (1976) and petroleum and coal products (1969, 1976, 1978). In these years, there was a considerable drop in their surplus values. Since the negative internal rates of return were generally small in magnitude[4] except for the urban transportation sector, for subsequent empirical estimation we set these rates to zero. The negative internal rates of return may, after all, reflect inefficiencies in these sectors or slow adjustment to external shocks.

[3] From our earlier discussion, the property tax rates and depreciation rates for these sectors seem reasonable.
[4] Magnitudes were usually less than 0.009.

	sector	internal rates of return average*	standard deviation	proportion of economy's capital stock, average*
1	Agr & Fish	0.105517	0.032098	0.070233
2	Forest	0.052536	0.028257	0.005529
3	Mines,etc	0.141935	0.034892	0.062376
4	Food & Bev	0.192317	0.013683	0.022483
5	Tobacco	0.189888	0.045612	0.002066
6	Rub & Plas	0.184638	0.078319	0.003381
7	Leather	0.083946	0.039032	0.000903
8	Textile	0.087968	0.026585	0.006552
9	Knitting	0.145049	0.060775	0.001012
10	Clothing	0.308656	0.096094	0.001858
11	Woods	0.110226	0.080586	0.007315
12	Furn & Fix	0.239743	0.051399	0.001424
13	Paper,etc	0.084048	0.048859	0.023639
14	Printing	0.235046	0.050968	0.004458
15	Pri Metals	0.057638	0.028322	0.022070
16	Metal Fab	0.208812	0.045787	0.009212
17	Machinery	0.244392	0.081077	0.005495
18	Transport	0.163221	0.062689	0.013601
19	Elec Prod	0.208085	0.051838	0.007770
20	Non Metal	0.130039	0.033322	0.007547
21	Pet & Coal	0.023998	0.029733	0.012484
22	Chemical	0.095060	0.040507	0.017164
23	Misc Manuf	0.210496	0.053974	0.002618
24	Construct	1.658639	2.451802	0.009476
25	Air Tran	0.032037	0.007964	0.034234
26	Rail Tran	-0.002551	0.006324	0.038919
27	Water Tran	0.004475	0.012749	0.010452
28	Motor Tran	1.348575	0.397836	0.002959
29	Urban Tran	-0.057397	0.045573	0.004450
30	Storage	0.062577	0.027002	0.003574
31	Broadcast	0.174301	0.050592	0.001809
32	Telephone	0.069288	0.009257	0.036151
33	Elec power	0.026961	0.006564	0.099529
34	Gas distr	0.090609	0.017178	0.007848
35	Trade	0.180829	0.038916	0.057177
36	F.I.R.E.	0.051077	0.008432	0.358857
37	Comm serv	0.358631	0.081720	0.023375

∗ Averages are over 20 years.

Table B.14: Sectoral 20-year average internal rates of return

A more serious cause for concern are the high internal rates of return obtained for construction (24) and motor transportation (28). They were generally greater than 0.50, and greater than 1.0 for 1973–1980 in the construction sector. For the motor transportation sector, they range from 0.74 to 2.01. These results may be indicative of data problems for these sectors; recall from our earlier discussion that the property tax rates for these sectors are suspiciously high, averaging 25% and 34%, respectively. It is possible that there is undervaluation of capital assets in these sectors.

A possible source of error is in the accounting of leased or rented components of a sector's capital stock which are recorded as a primary input in the finance, insurance and real estate sector. This would not pose a problem in our gross output profit function estimation if these leased capital is properly recorded as intermediate inputs in the using sectors. Table B.14 also gives the 20-year average proportion of the economy's value of capital stock for the different sectors. The entries in the last column are the means of annual proportions of the total capital stock as given by $A_i/\sum_{i=1}^{37} A_i$. The bulk of the economy's capital stock is in the finance, insurance and real estate sector which accounts for 33% to 40% of the total.[5] The construction sector is relatively large compared to other domestic sectors; the value of its output is about 10% of the sum of the value of output of all sectors $(p_{24}^* q_{24}/\sum_{i=1}^{37} p_i^* q_i)$.[6] But, as seen in table B.14, it has only about 1% of the economy's total capital stock.[7] A significant proportion of its capital input must be recorded as primary input in the finance, insurance and real estate sector.

The same data error must be underlying the high internal rates of return in the motor transportation sector, though this is not as immediately clear. This sector is smaller with its output accounting for about 2% of the sum of the value of output of all sectors $(p_{28}^* q_{28}/\sum_{i=1}^{37} p_i^* q_i)$

[5]It must also be noted that the finance, insurance and real estate sector includes "owner occupied dwellings" as a sub-industry. Since the purpose of this study is the modelling of production technologies of profit-maximizing business sectors of the economy, the inclusion of owner occupied dwellings which can be considered a consumer durable can be questionable.

[6]The denominator $\sum_{i=1}^{37} p_i^* q_i$ is a gross output concept and hence does not give the private gross domestic product which nets out intermediate inputs, imports and government goods.

[7]The output of the construction industry in the data base includes the output of construction labor in other industries; all inputs, except for the capital stock, have been properly adjusted. Hence, the capital stock of sectors engaged in own-account construction activities will tend to be overestimated. The extent to which this possible source of error contributes to the low capital stock data recorded in the construction industry is unknown.

but its recorded total capital stock is only about 0.3% of the economy's total stock. Nevertheless, a closer examination of the capital stock data in the construction and motor transportation sectors is warranted. For subsequent empirical estimation, the internal rates of return for these two sectors are set at the economy-wide averages, as discussed below.

Ignoring the aforementioned anomalies in the calculated internal rates of return, we note that within sectors the estimated rates are erratic in the sense that there generally are no consistent increasing or declining trends except for a few sectors. Internal rates of return were 10% or more in the 1960's, and less than 10% in the 1970's in the chemical and gas distribution sectors (22, 34). In the petroleum and coal products and commercial services sectors (21, 37), there is a declining trend in internal rates of return which is more marked in the latter sector. From table B.14, we can infer that the higher the mean sectoral internal rate of return, the variability of rates in that sector tends to be greater. The food and beverages sector (4) and finance, insurance and real estate sector (36), probably due to its huge capital stock, have quite stable internal rates of return. Probably due to government regulation, the internal rates of return has very low variability in the air transportation, telephone and electric power industries (25, 32, 33).

To facilitate a more direct comparison with an external bond rate, the annual average and variance of sector-specific internal rates of return, weighted by the sector's proportion of the economy's total capital stock value, were calculated. We define the annual weighted average and variance of internal rates of return across sectors, say $\bar{r}_i$ and $V(r_i)$, respectively, as

$$\begin{aligned} \bar{r}_i &\equiv \textstyle\sum_{i=1}^{37} r_i \left(\frac{A_i}{\sum_{j=1}^{37} A_j} \right), \\ V(r_i) &\equiv \textstyle\sum_{i=1}^{37} (r_i - \bar{r}_i)^2 \left(\frac{A_i}{\sum_{j=1}^{37} A_j} \right). \end{aligned}$$

Note that as discussed earlier, negative internal rates of return have been set to zero, and for the construction and motor transportation sectors, we have set $r_{24} = r_{28} = \bar{r}_i$. The results are given in table B.15. For an external bond rate, following Fortin (1988), we use the annual average of McLeod, Young and Weir's monthly 10 industrial bond yield average (also listed in

	internal rates of return		
year	weighted average	weighted standard deviation	McLeod, Young and Weir's 10 industrial bond yield, average
1961	0.083871	0.069202	0.0550
1962	0.091580	0.071394	0.0544
1963	0.094619	0.071858	0.0537
1964	0.097758	0.077412	0.0549
1965	0.096656	0.079609	0.0563
1966	0.094136	0.074457	0.0643
1967	0.083479	0.071633	0.0703
1968	0.089054	0.074986	0.0787
1969	0.087531	0.075099	0.0866
1970	0.079076	0.065497	0.0922
1971	0.080637	0.073217	0.0840
1972	0.083002	0.079482	0.0831
1973	0.093489	0.090409	0.0842
1974	0.087360	0.084356	0.1001
1975	0.074704	0.071649	0.1073
1976	0.071822	0.068574	0.1058
1977	0.066134	0.057062	0.0972
1978	0.068757	0.061839	0.0996
1979	0.075095	0.072375	0.1074
1980	0.074180	0.068493	0.1311

Table B.15: A comparison of calculated internal rates of return and the McLeod, Young and Weir's 10 industrial bond yield average

table B.15). The behavior of the two discount rates is illustrated in figure B.39.

The two series exhibit different time trends; while internal rates of return show a general decline, the exogenous bond rate has a strong upward trend. The external bond rate, being a nominal interest rate, follows closely the behavior of inflation rates. Theoretically, anticipated inflation should be accounted for by the capital gains term in our general rental price formula. With our static expectations assumption, the rental prices we would obtain using the external bond rates will be biased upwards particularly in the 1970's decade of high inflation rates. This fact highlights the importance of modelling and incorporating asset price expectations in our capital rental price formula. On the other hand, the use of static expectations when capital markets are relatively efficient may be justifiable in calculating internal rates of return that

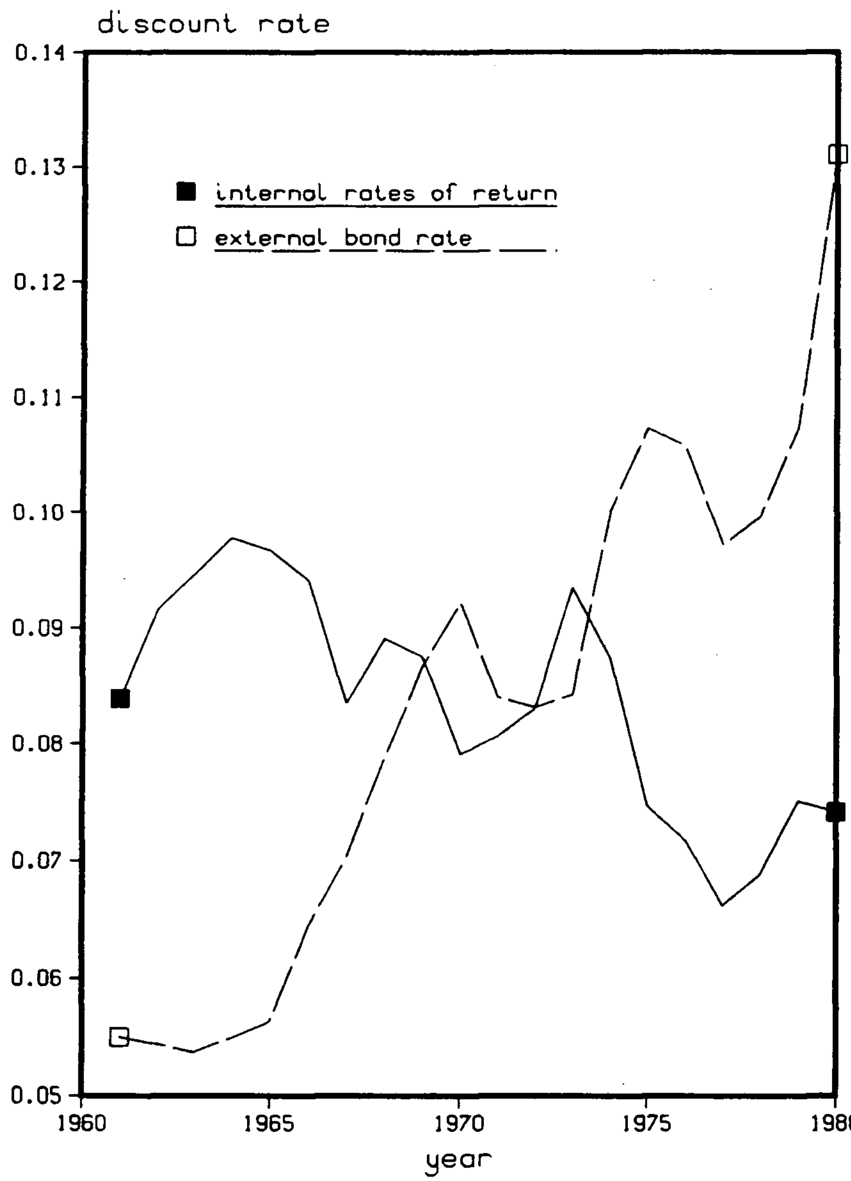

Figure B.39: The behavior of the weighted average of the internal rates of return and the McLeod, Young and Weir's 10 industrial bond yield average yield reasonable indicators of returns to equities. Figure B.39 also shows that the volatility of internal rates of return, in the aggregate, is not unreasonable. The range of its mean is even narrower than the range of the external bond rate.

As discussed above, the choice of an internal rate of return or an exogenous bond rate can have different implications on the rental price behavior. Profit function estimation, particualrly using full information maximum likelihood methods, can be sensitive to misspecification. For these reasons, two capital rental price series were constructed using the two discount rates.

B.2.6 List of retained variables

After doing the data manipulations on the original data base, we retain only a subset of variables prior to goods and sector aggregation. The list of retained price and quantity variables for each of the 37 sectors is as follows: For $i = 1, 2, \ldots, 37$:

1. net output: p_i^*, q_i
2. intermediate inputs: p_{ji}^*, q_{ji}, $j = 1, 2, \ldots, 37$
3. primary inputs:
   i) imports: p_M^*, q_M
   ii) labor: p_L^*, q_L
   iii) capital services:
      a) inventories: p_1^{K*}, q_1^{K*}
      b) machinery and equipment: p_2^{K*}, q_2^{K*}
      c) structures: p_3^{K*}, q_3^{K*}
      d) land: p_4^{K*}, q_4^{K*}

In subsequent econometric estimation of profit functions, intermediate inputs and primary inputs are treated symmetrically as goods having prices exogenous to the sector. In this manner, we do not impose separability restrictions between intermediate inputs and primary inputs which are implicitly assumed with value-added production or profit functions. A graphical plot of prices of output, imports and labor for each sector shows that the price of labor services has increased the most over the twenty-year period. For each sector, we have two sets of capital rental price series using different discount rates: an internal rate of return and an exogenous bond rate.

A discussion of some of the output variables in the original data base and government goods dropped in our condensed list is inorder. For each sector, the sum of intermediate output and final demand equals gross output. Intermediate output is purchased by domestic producers as input to production. Final demand is output that ultimately goes for consumption, investment, government purchases and exports. To obtain the gross private domestic product, we add the final demand outputs and subtract the imports of the 37 domestic business sectors. It is the sum of sectoral value added, in our case, returns to labor and capital sevices, that is termed "domestic product" in the usual national accounting framework.

Government produced goods and services[8] used as inputs in production comprise less than 1% of the value of primary inputs for the whole economy. As such, its omission can be deemed harmless.

B.3 Aggregation

Under the assumption of single-output industries, four sectors or output goods are specified. To aggregate the original 37 sectors into four general industries, a heuristic approach was taken. Theoretically, aggregation of goods is justified on the basis of either Hicksian separability where restrictions are on the domain of producer prices or functional separability where restrictions are imposed on the production technologies. Testing for functional separability is a separate study in itself and hence is precluded in our empirical analysis. Though the groupings of the 37 sectors were guided as well by traditional industry classifications, an examination of the plots of their output prices (see figures B.40–B.46) indicates that within group price variation

[8] These government goods include postal services and the like but exclude output of other crown corporations (e.g. Air Canada, Via Rail, CN Rail, CBC) which are recorded as output of the relevant business sectors.

is less than that between groups. Hence, our sector aggregation can be justified by appealing to Hicks aggregation theorem that says goods whose prices move proportionally act as a composite good.

The aggregate industries and their subsectors are:

I. resources sector
   - (1) agriculture and fishing
   - (2) forestry
   - (3) mines, quarries and oil wells
   - (21) petroleum and coal products

II. manufacturing, export-oriented sector[9]
   - (11) woods
   - (13) paper and allied industry
   - (15) primary metals
   - (17) machinery
   - (18) transportation equipment

III. manufacturing, domestic market-oriented sector
   - (4) food and beverages
   - (5) tobacco products
   - (6) rubber and plastic products
   - (7) leather
   - (8) textiles
   - (9) knitting mills
   - (10) clothing
   - (12) furniture and fixtures
   - (14) printing, publishing and allied industries
   - (16) metal fabricating
   - (19) electrical products
   - (20) nonmetallic mineral products
   - (22) chemical and chemical products
   - (23) miscellaneous manufacturing

IV. services sector
   - (24) construction
   - (25) air transportation, other utilities and transportation
   - (26) railway transportation and telegraph

[9]Information from the output variables classified according to end use in the original data set was used to select these subsectors.

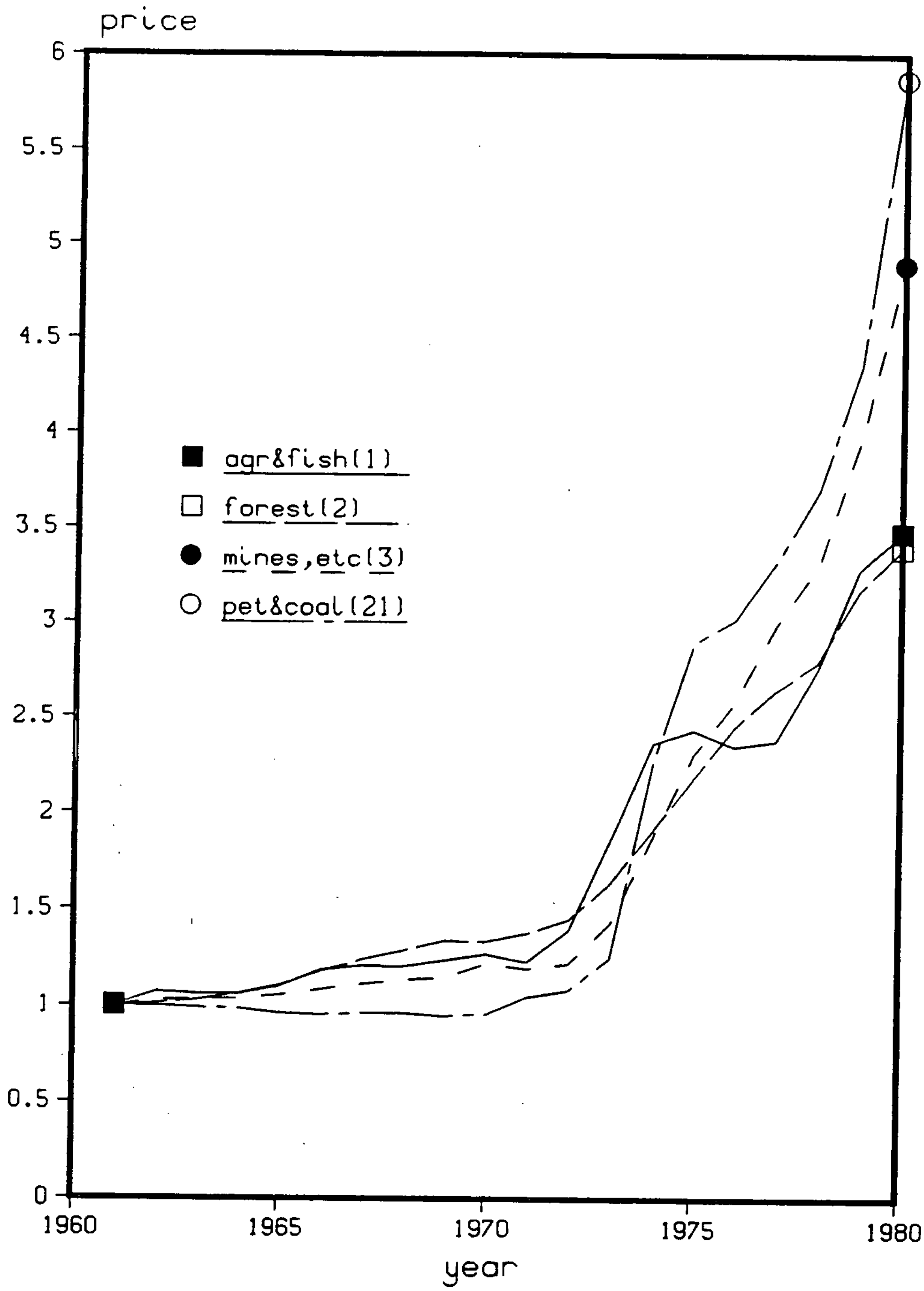

Figure B.40: Output prices for the resources sectors (I)

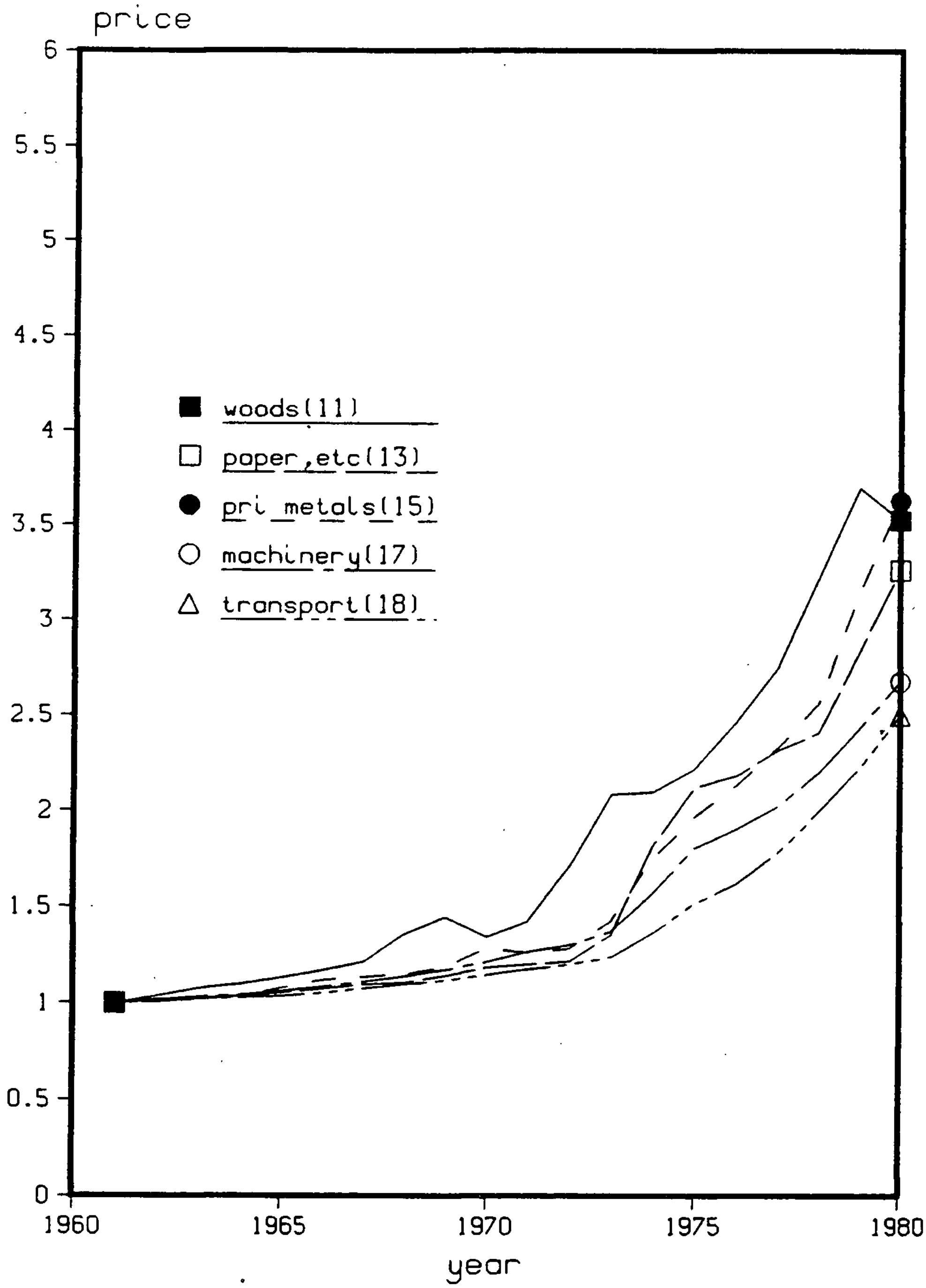

Figure B.41: Output prices for the export market-oriented manufacturing sectors (II)

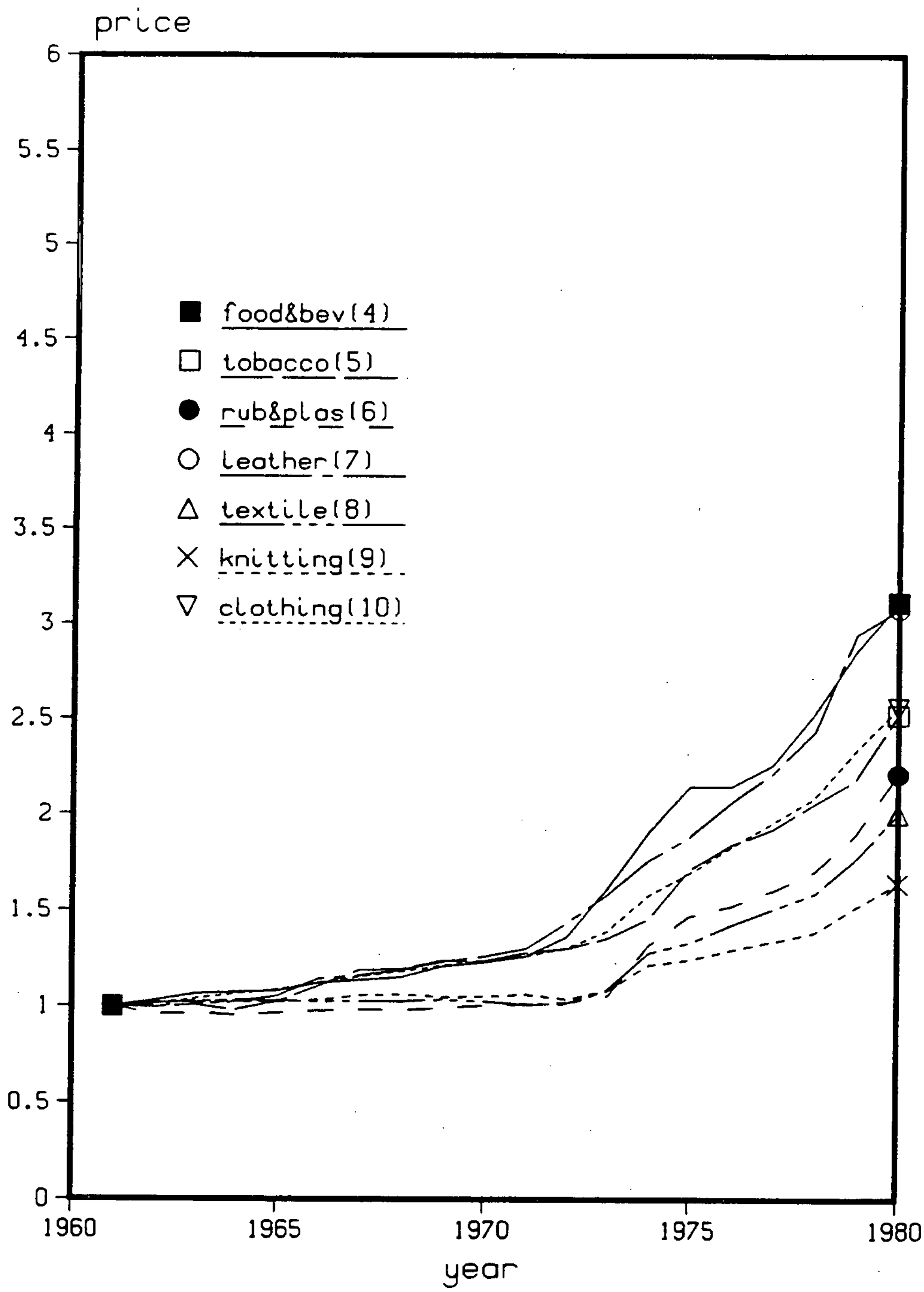

Figure B.42: Output prices for the domestic market-oriented manufacturing sectors (III), subgroup 1

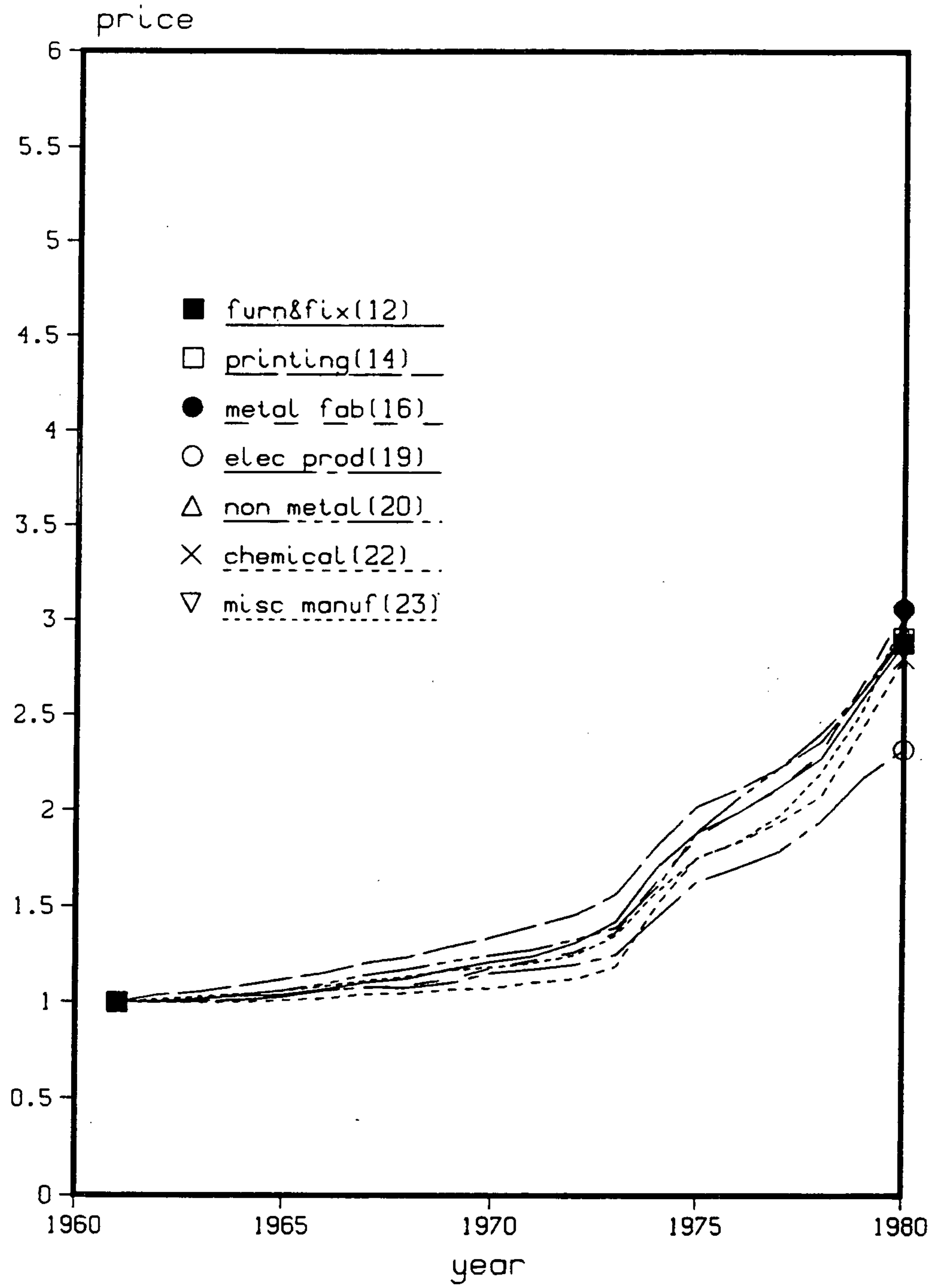

Figure B.43: Output prices for the domestic market-oriented manufacturing sectors (III), subgroup 2

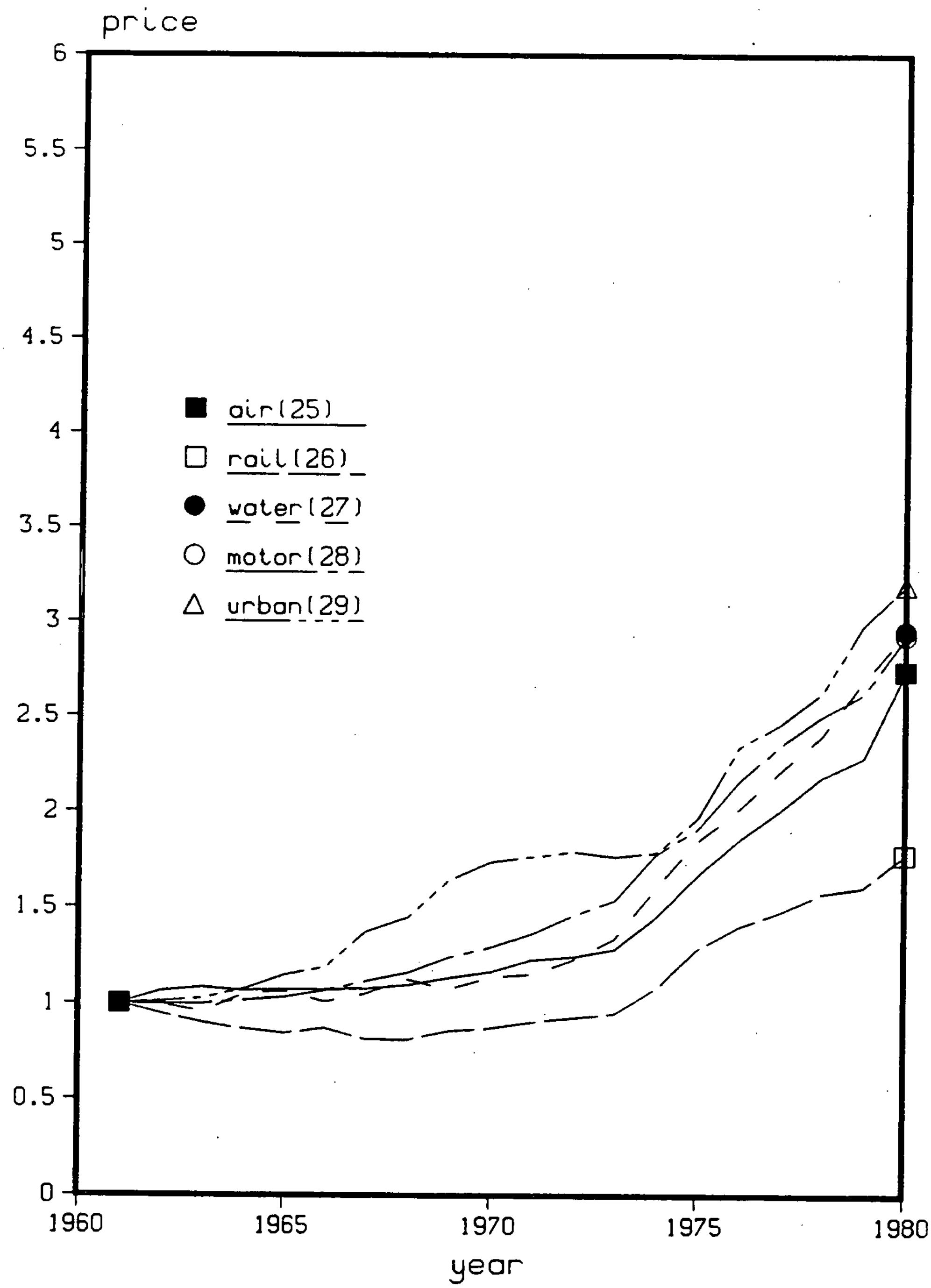

Figure B.44: Output prices for the services sectors (IV), transportation subgroup

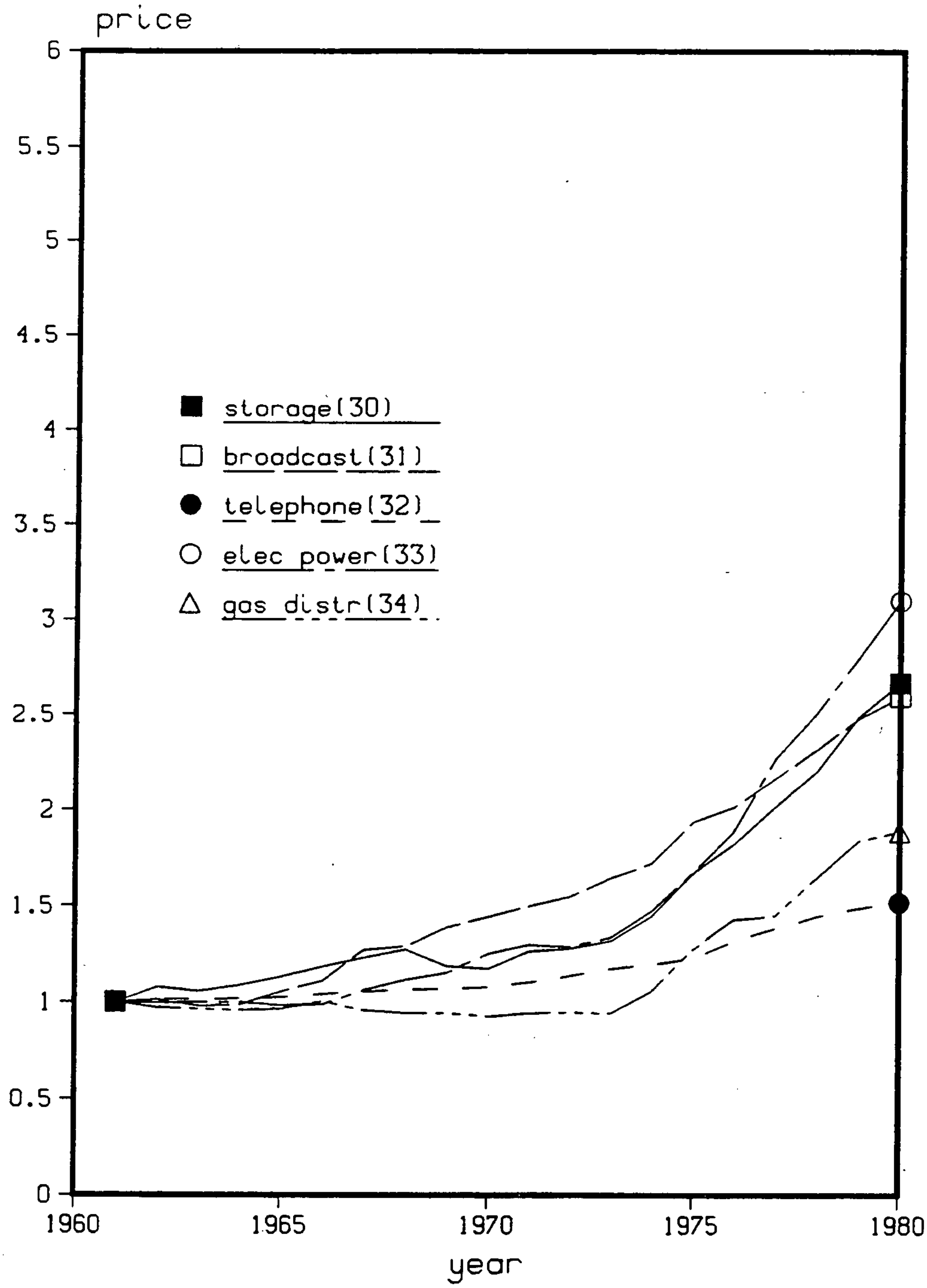

Figure B.45: Output prices for the services sectors (IV), communications and utilities subgroup

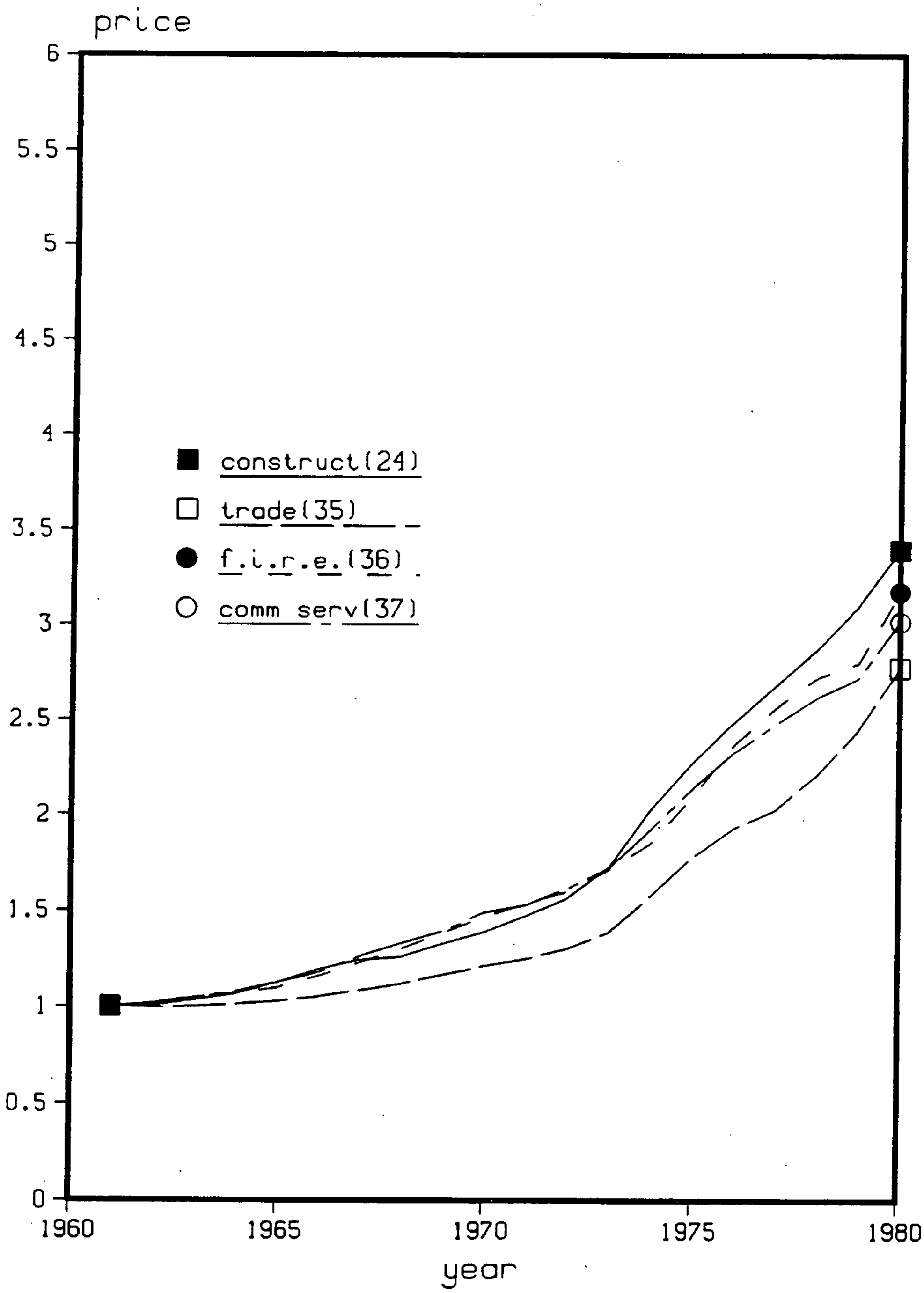

Figure B.46: Output prices for the services sectors (IV), construction, trade, f.i.r.e., and commercial services subgroup

(27) water transportation
(28) motor transportation
(29) urban transportation
(30) storage
(31) broadcasting
(32) telephones
(33) electric power
(34) gas distribution
(35) trade
(36) finance, insurance and real estate
(37) commercial services

Aggregation was carried out in two stages: first, at the sectoral level across goods (intermediate inputs) and second, across sectors to yield our aggregate industries. Prices were obtained as chained Divisia (Törnqvist or translog) indices; quantities were obtained residually such that expenditures are preserved.[10] The Törnqvist discrete approximation to the continuous Divisia index is superlative; it is exact for the flexible translog aggregator function. Being a superlative index, it has also been shown to possess approximate consistency-in-aggregation property (Diewert, 1980a).

For each aggregated sector, we have two sets of capital data based on different discount rates. Rental prices based on internal rates of return are more erratic and generally lower than those based on the external bond rates. This difference is more marked for inventories and land which have either zero property tax or depreciation rates. There was considerable variation in internal rates of return among the original 37 sectors; on the other hand the same external bond rate was applied to all the sectors. Thus, it is not unexpected that rental prices obtained using the external bond rates have a smoother behavior over time and even for a particular capital good, the trends of rental prices among the different aggregated sectors are almost identical.

Due to the manner by which prices and quantities were obtained in our aggregation procedure, the quantities of capital input also differ according to the discount rate used. At the level of 37-sector disaggregation, the quantities are identical across the two capital data sets

[10]The econometrics computer program SHAZAM by K.J. White (1987) was used to compute the Divisia indices.

I.	resources sector
II.	manufacturing sector, export market-oriented
III.	manufacturing sector, domestic market-oriented
IV.	services sector

Table B.16: List of aggregated sectors

which differ only in the rental prices. The implied capital service bill (rental price x quantity of capital input) is therefore conditional on the discount rate as well. Since in the aggregation procedure the capital service bill or expenditure is preserved, different rental prices would then yield different values for the quantity. The above discussion underscores the possibility that our profit function estimation can be sensitive to the specification of the capital variables.

Due to the manner by which the land data was constructed by Cas, Diewert and Ostensoe (1986), the land quantity data except for the agriculture and fishing sector at the 37-sector level of disaggregation are in fixed proportions to the quantity of structures (approximately 0.22 to 0.26). Hence, after the sectoral aggregation to four general industries was performed, structures and land were aggregated into a Leontief composite good for the manufacturing and services sectors (II, III and IV) in the following manner:

$$\begin{aligned} (\text{new})\ q_3^{K*} &\equiv q_3^{K*} + q_4^{K*}, \\ (\text{new})\ p_3^{K*} &\equiv \frac{p_3^{K*}q_3^{K*} + p_4^{K*}q_4^{K*}}{q_3^{K*} + q_4^{K*}} \end{aligned}$$

where the right hand side variables are as described in the list of retained variables at the sectoral level, and the new price variable is obtained such that the value of expenditure is preserved. Hereon, for convenience, we shall refer to the Leontief aggregate of structures and land in the aggregated sectors II, III and IV as simply structures. For the resources sector (I), we retain separate variables for structures and land.

In summary, we have four aggregated industries (table B.16) and ten goods (table B.17). For each sector, the first four goods are either a net output or an intermediate input. The last

(1)	resource goods (from sector I)
(2)	manufactured goods (from sector II)
(3)	manufactured goods (from sector III)
(4)	service goods (from sector IV)
(5)	imports
(6)	labor
(7)	inventories
(8)	machinery and equipment
(9)	structures
(10)	land

Table B.17: List of goods

six goods are primary inputs. The land price and quantity data are effectively set to zero for the manufacturing and services sectors (II, III and IV). For subsequent empirical analysis, quantities of net outputs are indexed positively and quantities of net inputs are indexed negatively. For scaling purposes, the quantities are further divided by a factor of 10^6; hence, the value of expenditure which can be obtained by multiplying the relevant price and quantity variables are in units of billion Canadian (current) dollars. Prices are normalized such that 1961 prices are equal to 1.0.

B.4 The constructed data sets

The constructed sectoral accounts with the capital rental prices based on internal rates of return are listed below. The constructed capital data series based on an exogenous bond rate are also listed separately for each of the four sectors.

Table B.18: The data for sector I, the resources sector

Sector I: Resources						
	net output		resources		exp-manuf	
year	quantity	price	quantity	price	quantity	price
1961	6.388490	1.000000	0.000000	0.000000	0.108087	1.000000
1962	7.052499	1.037613	0.000000	0.000000	0.111440	1.014777
1963	7.549368	1.036749	0.000000	0.000000	0.121395	1.029922
1964	7.826248	1.038113	0.000000	0.000000	0.137002	1.045107
1965	8.172144	1.061550	0.000000	0.000000	0.151658	1.067568
1966	8.801898	1.114239	0.000000	0.000000	0.174431	1.094045
1967	8.426695	1.139288	0.000000	0.000000	0.179564	1.118140
1968	8.951132	1.148554	0.000000	0.000000	0.190346	1.146075
1969	9.282653	1.167532	0.000000	0.000000	0.180709	1.181067
1970	9.401631	1.206057	0.000000	0.000000	0.196889	1.225520
1971	9.970642	1.201196	0.000000	0.000000	0.195240	1.252829
1972	10.329735	1.287679	0.000000	0.000000	0.189899	1.288753
1973	11.729394	1.586959	0.000000	0.000000	0.223233	1.378919
1974	11.345067	2.163120	0.000000	0.000000	0.235692	1.608616
1975	10.910657	2.492335	0.000000	0.000000	0.221426	1.811454
1976	11.313760	2.580402	0.000000	0.000000	0.233222	1.937699
1977	11.483928	2.795015	0.000000	0.000000	0.255290	2.085647
1978	11.481984	3.121217	0.000000	0.000000	0.252023	2.273325
1979	12.255849	3.699945	0.000000	0.000000	0.282798	2.600041
1980	12.640729	4.411511	0.000000	0.000000	0.271277	2.923751
	dom-manuf		services		imports	
year	quantity	price	quantity	price	quantity	price
1961	0.629793	1.000000	1.025314	1.000000	0.651119	1.000000
1962	0.668620	1.009155	1.126051	1.009202	0.577858	1.233829
1963	0.695051	1.026388	1.172576	1.022710	0.589565	1.268691
1964	0.743054	1.038356	1.248063	1.038945	0.591657	1.309422
1965	0.777908	1.050673	1.335080	1.062627	0.604773	1.320841
1966	0.872301	1.083464	1.405787	1.095994	0.645203	1.370586
1967	0.898229	1.099892	1.443741	1.140893	0.641647	1.370002
1968	0.897794	1.114265	1.494201	1.183047	0.705593	1.379412
1969	0.886964	1.153143	1.525226	1.236926	0.771333	1.356426
1970	0.908480	1.175219	1.551686	1.297293	0.797622	1.385346
1971	0.912856	1.208091	1.603493	1.350673	1.133501	1.144341
1972	0.940359	1.267690	1.703694	1.408225	1.269822	1.189303
1973	1.165134	1.411643	2.012527	1.492519	1.438189	1.357069
1974	1.182529	1.692004	2.103291	1.673627	1.378212	2.893564
1975	1.162251	1.901332	2.122066	1.872227	1.372453	3.429959
1976	1.210588	1.959881	2.295185	2.079581	1.342869	3.556358
1977	1.221552	2.066206	2.432256	2.244244	1.257737	3.964575
1978	1.237485	2.260185	2.545984	2.407110	1.261508	4.336803
1979	1.305133	2.562756	2.784181	2.551099	1.299690	5.532236
1980	1.360189	2.873773	2.973667	2.872205	1.233598	8.194973

Table B.18 (continued)

Sector I: Resources						
	labor		inventories		cap-M&E	
year	quantity	price	quantity	price	quantity	price
1961	1.526986	1.000000	0.257658	1.000000	0.974477	1.000000
1962	1.537920	1.155035	0.236531	1.673956	0.967728	1.158633
1963	1.488821	1.216110	0.250657	1.857383	0.979927	1.231795
1964	1.478606	1.204332	0.270415	1.611608	1.008983	1.230761
1965	1.504062	1.295214	0.259018	1.690091	1.045854	1.252861
1966	1.474743	1.493066	0.249352	2.217558	1.096940	1.344356
1967	1.464122	1.496646	0.256565	1.430552	1.166594	1.154664
1968	1.400979	1.630398	0.252045	1.541310	1.229138	1.184329
1969	1.349746	1.808434	0.260671	1.573871	1.271160	1.195877
1970	1.371695	1.906155	0.294792	1.308617	1.295795	1.164084
1971	1.353306	2.095066	0.301009	1.253800	1.320508	1.155752
1972	1.293386	2.299937	0.276501	1.483027	1.371592	1.224451
1973	1.360092	2.907178	0.196503	3.510525	1.457358	1.620626
1974	1.414700	3.291578	0.169916	5.003183	1.563316	1.942379
1975	1.382598	3.631992	0.150509	5.085024	1.690316	2.068841
1976	1.384431	4.020559	0.153134	4.018019	1.841283	1.998528
1977	1.399334	4.408647	0.166939	3.427975	2.004779	2.001755
1978	1.408160	4.879079	0.145943	4.297119	2.126939	2.279105
1979	1.472186	5.181565	0.120251	5.529375	2.238255	2.777311
1980	1.513216	5.845244	0.105367	6.126964	2.303775	3.146931

	cap-S		land	
year	quantity	price	quantity	price
1961	1.051897	1.000000	0.157218	1.000000
1962	1.130091	1.062735	0.169682	1.055788
1963	1.200532	1.120759	0.182722	1.123692
1964	1.274241	1.127154	0.198536	1.180153
1965	1.356734	1.116906	0.217224	1.180537
1966	1.451109	1.175998	0.238384	1.249582
1967	1.553798	1.080166	0.261559	1.125995
1968	1.651811	1.117851	0.281879	1.200644
1969	1.747637	1.125036	0.296819	1.146463
1970	1.847144	1.116097	0.306257	1.098177
1971	1.954259	1.079182	0.325562	0.948264
1972	2.049686	1.207907	0.353363	1.127055
1973	2.131857	1.843159	0.371761	2.022788
1974	2.221274	2.269770	0.398255	2.624213
1975	2.319290	2.309336	0.446428	2.804449
1976	2.427455	2.314483	0.497528	2.728358
1977	2.532432	2.485781	0.545635	2.763854
1978	2.664501	2.597678	0.615000	2.875689
1979	2.812549	3.439241	0.705863	3.891534
1980	3.075338	3.859592	0.818887	4.578141

Table B.19: The data for sector II, the manufacturing sector (export market-oriented)

Sector II: Manufacturing, Export Market-Oriented						
	net output		resources		exp-manuf	
year	quantity	price	quantity	price	quantity	price
1961	7.089804	1.000000	1.341709	1.000000	0.000000	0.000000
1962	7.756834	1.015587	1.383495	1.022700	0.000000	0.000000
1963	8.462628	1.027039	1.428701	1.038229	0.000000	0.000000
1964	9.485412	1.041505	1.553258	1.056743	0.000000	0.000000
1965	10.567053	1.062243	1.648344	1.086871	0.000000	0.000000
1966	11.235968	1.089246	1.665766	1.128543	0.000000	0.000000
1967	11.725869	1.112759	1.743833	1.171423	0.000000	0.000000
1968	12.831917	1.141234	1.838889	1.211578	0.000000	0.000000
1969	13.987605	1.176512	1.950108	1.249376	0.000000	0.000000
1970	13.567631	1.214419	2.099769	1.271914	0.000000	0.000000
1971	14.234698	1.236027	1.979726	1.291096	0.000000	0.000000
1972	15.544878	1.284886	2.112818	1.353306	0.000000	0.000000
1973	17.314678	1.399653	2.337245	1.586489	0.000000	0.000000
1974	18.041398	1.632355	2.185011	2.031106	0.000000	0.000000
1975	16.492716	1.835973	1.692250	2.414300	0.000000	0.000000
1976	17.809943	1.956790	1.850811	2.710048	0.000000	0.000000
1977	18.486470	2.128178	1.765179	3.038581	0.000000	0.000000
1978	19.701988	2.346463	1.779699	3.392082	0.000000	0.000000
1979	20.284628	2.700821	1.934461	3.966979	0.000000	0.000000
1980	19.557889	3.002165	2.001963	4.513225	0.000000	0.000000
	dom-manuf		services		imports	
year	quantity	price	quantity	price	quantity	price
1961	0.568828	1.000000	0.906523	1.000000	0.977387	1.000000
1962	0.671022	1.000327	0.982276	0.997330	1.081217	1.081267
1963	0.770343	1.005779	1.068108	0.997897	1.228443	1.094631
1964	0.882642	1.017522	1.197216	1.007568	1.468789	1.113372
1965	0.999005	1.035669	1.345723	1.023561	1.708937	1.125834
1966	1.085627	1.055387	1.483687	1.046020	1.919632	1.156208
1967	1.115533	1.075190	1.577666	1.084100	2.007965	1.200146
1968	1.207253	1.083365	1.683154	1.123447	2.459309	1.219124
1969	1.335115	1.110239	1.798719	1.173175	2.691873	1.254088
1970	1.275334	1.142737	1.799884	1.231258	2.595263	1.294618
1971	1.388669	1.168522	1.840433	1.276767	3.044751	1.240952
1972	1.427852	1.199486	1.991768	1.328342	3.485332	1.269917
1973	1.559194	1.276929	2.147521	1.402412	3.963157	1.341706
1974	1.590350	1.533877	2.340701	1.566645	4.313688	1.558066
1975	1.498918	1.735936	2.259968	1.755429	4.081083	1.865408
1976	1.579308	1.837759	2.389705	1.942656	4.369601	1.947160
1977	1.592931	1.945536	2.477600	2.095455	4.568010	2.194062
1978	1.704987	2.099040	2.688620	2.256407	4.882679	2.507265
1979	1.712680	2.384405	2.940197	2.413208	5.078234	2.807190
1980	1.725187	2.714167	3.015241	2.700301	4.482947	3.141109

Table B.19 (continued)

Sector II: Manufacturing, Export Market-Oriented						
	labor		inventories		cap-M&E	
year	quantity	price	quantity	price	quantity	price
1961	2.119785	1.000000	0.139830	1.000000	0.617652	1.000000
1962	2.241733	1.020438	0.140142	1.285324	0.635518	1.109476
1963	2.347976	1.058602	0.154408	1.443560	0.660243	1.207313
1964	2.516343	1.097354	0.167456	1.510000	0.702184	1.307317
1965	2.686661	1.154747	0.190346	1.619491	0.763447	1.380739
1966	2.815390	1.241190	0.219303	1.338196	0.839970	1.232211
1967	2.828762	1.340864	0.232885	1.328888	0.913000	1.071073
1968	2.809657	1.456240	0.242194	1.570020	0.954506	1.127606
1969	2.890357	1.557259	0.258157	1.817067	0.991791	1.289792
1970	2.799147	1.686883	0.267666	1.008121	1.056741	1.044481
1971	2.764914	1.814393	0.219643	1.245930	1.121957	1.111256
1972	2.906985	1.953254	0.218953	1.575181	1.164555	1.238862
1973	3.072781	2.140171	0.240360	2.229533	1.214828	1.588370
1974	3.126569	2.484959	0.269613	2.151007	1.280471	1.866524
1975	2.927767	2.840254	0.280483	1.621759	1.340268	1.569852
1976	3.007986	3.310478	0.252980	1.712065	1.382314	1.602402
1977	3.027801	3.643374	0.268715	1.921505	1.416345	1.894035
1978	3.172290	3.878281	0.289846	2.705790	1.446666	2.368748
1979	3.266791	4.213563	0.321056	3.601053	1.480658	2.986773
1980	3.259662	4.738964	0.356681	2.351243	1.547863	2.812982

	cap-S	
year	quantity	price
1961	0.414444	1.000000
1962	0.420784	1.107176
1963	0.430535	1.190106
1964	0.448799	1.248288
1965	0.478177	1.308641
1966	0.517172	1.181744
1967	0.553215	1.079364
1968	0.574461	1.152858
1969	0.594162	1.334347
1970	0.622544	1.071653
1971	0.645474	1.147545
1972	0.658373	1.296258
1973	0.675827	1.706880
1974	0.701327	2.043999
1975	0.727206	1.550815
1976	0.744526	1.611738
1977	0.761717	1.864861
1978	0.779613	2.275004
1979	0.800990	2.884777
1980	0.834024	2.530481

Table B.20: The data for sector III, the manufacturing sector (domestic market-oriented)

Sector III: Manufacturing, Domestic Market-Oriented						
	net output		resources		exp-manuf	
year	quantity	price	quantity	price	quantity	price
1961	12.037386	1.000000	1.828422	1.000000	1.169241	1.000000
1962	12.987668	1.013297	1.863090	1.062329	1.259862	1.018057
1963	13.605353	1.030103	1.946189	1.057284	1.351255	1.022763
1964	14.735647	1.040056	2.098514	1.052316	1.447883	1.041157
1965	15.823795	1.053652	2.166830	1.080163	1.584562	1.069105
1966	16.931363	1.085651	2.257699	1.144946	1.737049	1.101336
1967	17.241228	1.107805	2.354801	1.158415	1.728370	1.122732
1968	18.071040	1.121121	2.395088	1.163200	1.781939	1.141395
1969	19.055620	1.159219	2.439193	1.197145	1.832271	1.180501
1970	19.055881	1.187352	2.512004	1.218478	1.865613	1.243472
1971	20.057231	1.216044	2.691736	1.191584	1.934796	1.244704
1972	21.429801	1.271673	2.766351	1.325867	2.094266	1.274907
1973	22.864381	1.402400	2.621073	1.758798	2.232044	1.418403
1974	23.424968	1.658326	2.449706	2.228840	2.285692	1.766253
1975	22.408561	1.862847	2.654289	2.335712	2.098421	2.002042
1976	23.881598	1.935665	2.796468	2.317120	2.177060	2.132037
1977	24.314540	2.048225	2.987332	2.410752	2.128802	2.299330
1978	25.561322	2.236829	3.123728	2.795465	2.219740	2.477568
1979	26.803966	2.528423	3.172526	3.299165	2.236875	2.966844
1980	26.585478	2.823797	3.133351	3.620208	2.132916	3.394111
	dom-manuf		services		imports	
year	quantity	price	quantity	price	quantity	price
1961	0.000000	0.000000	1.751402	1.000000	1.612199	1.000000
1962	0.000000	0.000000	1.881884	1.004487	1.600771	1.079881
1963	0.000000	0.000000	1.993663	1.009156	1.663543	1.141548
1964	0.000000	0.000000	2.130058	1.020611	1.822574	1.140279
1965	0.000000	0.000000	2.283580	1.041980	1.976544	1.114379
1966	0.000000	0.000000	2.436646	1.070147	2.180249	1.126500
1967	0.000000	0.000000	2.480078	1.110855	2.205295	1.125105
1968	0.000000	0.000000	2.539887	1.151481	2.313007	1.124609
1969	0.000000	0.000000	2.674857	1.201688	2.560571	1.165566
1970	0.000000	0.000000	2.682302	1.258386	2.459276	1.203446
1971	0.000000	0.000000	2.754310	1.305960	2.814057	1.120774
1972	0.000000	0.000000	2.916878	1.359511	3.130511	1.159296
1973	0.000000	0.000000	3.086557	1.436884	3.467886	1.314222
1974	0.000000	0.000000	3.245139	1.598958	3.841500	1.666344
1975	0.000000	0.000000	3.245954	1.788455	3.381120	1.800503
1976	0.000000	0.000000	3.307732	1.977327	3.637338	1.818480
1977	0.000000	0.000000	3.361782	2.123620	3.547960	2.030523
1978	0.000000	0.000000	3.578964	2.280534	3.843679	2.238825
1979	0.000000	0.000000	3.861903	2.428958	4.244077	2.634669
1980	0.000000	0.000000	3.952335	2.717233	4.146493	3.210584

Table B.20 (continued)

Sector III: Manufacturing, Domestic Market-Oriented						
	labor		inventories		cap-M&E	
year	quantity	price	quantity	price	quantity	price
1961	3.866455	1.000000	0.390087	1.000000	0.790091	1.000000
1962	4.007279	1.028578	0.402435	1.215633	0.816207	1.159242
1963	4.094212	1.062301	0.424859	1.252342	0.842033	1.208093
1964	4.265746	1.099703	0.437419	1.384666	0.875287	1.375407
1965	4.471031	1.148536	0.466149	1.384827	0.928024	1.442232
1966	4.658253	1.235270	0.505314	1.318669	1.001514	1.400918
1967	4.656392	1.323675	0.528980	1.190127	1.075553	1.274247
1968	4.634462	1.409570	0.542721	1.342566	1.133056	1.336079
1969	4.683603	1.514101	0.565547	1.410297	1.184919	1.401385
1970	4.556537	1.626044	0.586728	1.196164	1.238159	1.281890
1971	4.532065	1.743428	0.500339	1.503670	1.287277	1.469134
1972	4.611433	1.882876	0.520267	1.685027	1.332858	1.570679
1973	4.764969	2.058962	0.565381	1.946266	1.395390	1.737622
1974	4.833621	2.380140	0.628511	2.131463	1.475864	1.856973
1975	4.752283	2.722542	0.644599	2.040733	1.555075	1.920806
1976	4.723326	3.150614	0.623038	2.102718	1.626614	2.005039
1977	4.565358	3.472681	0.619148	2.108469	1.688481	2.118139
1978	4.685567	3.684217	0.645320	2.455621	1.734585	2.483261
1979	4.815412	4.019026	0.687416	2.948763	1.782644	2.890741
1980	4.775804	4.572550	0.707758	2.674895	1.813132	2.874828

	cap-S	
year	quantity	price
1961	0.606574	1.000000
1962	0.623151	1.119752
1963	0.642046	1.152039
1964	0.665170	1.245956
1965	0.699341	1.294947
1966	0.746907	1.287254
1967	0.791364	1.253040
1968	0.828119	1.322063
1969	0.866487	1.395417
1970	0.906805	1.283205
1971	0.940722	1.517903
1972	0.963127	1.670885
1973	0.988821	1.921457
1974	1.025331	2.057440
1975	1.062938	2.019534
1976	1.092270	2.220401
1977	1.124662	2.255490
1978	1.161963	2.505986
1979	1.199084	2.862710
1980	1.228090	2.727067

Table B.21: The data for sector IV, the services sector

Sector IV: Services						
	net output		resources		exp-manuf	
year	quantity	price	quantity	price	quantity	price
1961	25.447810	1.000000	0.875907	1.000000	1.280840	1.000000
1962	26.676252	1.004829	0.852661	1.034028	1.264061	1.032044
1963	27.940061	1.020665	0.894182	1.033111	1.352863	1.053248
1964	30.025948	1.038074	0.955285	1.039502	1.470477	1.088668
1965	32.214360	1.067979	1.007009	1.047827	1.492395	1.122017
1966	34.453836	1.114736	1.156012	1.072106	1.569492	1.156154
1967	35.558786	1.161715	1.104906	1.092991	1.635840	1.189471
1968	36.930822	1.201863	1.086179	1.104841	1.653321	1.249679
1969	38.535515	1.261842	1.136589	1.115011	1.671295	1.306078
1970	39.557438	1.325342	1.163318	1.139757	1.707698	1.314224
1971	42.250994	1.381803	1.196356	1.174812	1.870303	1.348444
1972	44.775514	1.445799	1.329988	1.234390	1.898392	1.466019
1973	47.911077	1.550809	1.423878	1.476695	1.815247	1.670257
1974	50.275018	1.749947	1.364078	2.142910	1.838916	1.890350
1975	51.862215	1.968710	1.365785	2.468426	1.794571	2.053709
1976	54.699236	2.165522	1.432804	2.757956	1.974918	2.221193
1977	56.292097	2.326202	1.600500	3.123400	1.985405	2.423828
1978	58.398141	2.498532	1.746025	3.571692	1.953228	2.693428
1979	61.554562	2.653385	1.879707	4.067483	1.997412	3.118340
1980	63.767157	2.967861	1.937400	4.809709	1.980460	3.317524
	dom-manuf		services		imports	
year	quantity	price	quantity	price	quantity	price
1961	2.850217	1.000000	0.000000	0.000000	1.032118	1.000000
1962	3.080031	1.014433	0.000000	0.000000	0.717764	1.517208
1963	3.233912	1.027262	0.000000	0.000000	0.705851	1.577892
1964	3.475665	1.055144	0.000000	0.000000	0.793994	1.688200
1965	3.763027	1.080331	0.000000	0.000000	0.863286	1.723218
1966	3.986007	1.111411	0.000000	0.000000	0.978423	1.743120
1967	4.027994	1.140687	0.000000	0.000000	1.031074	1.761693
1968	4.169141	1.156044	0.000000	0.000000	1.082051	1.723657
1969	4.249828	1.195202	0.000000	0.000000	1.219990	1.752065
1970	4.355433	1.235274	0.000000	0.000000	1.214786	1.851175
1971	4.839141	1.267082	0.000000	0.000000	1.970859	1.306229
1972	4.978960	1.317792	0.000000	0.000000	2.170307	1.355255
1973	5.249498	1.425952	0.000000	0.000000	2.380077	1.458526
1974	5.233725	1.687207	0.000000	0.000000	2.505475	1.710694
1975	5.225492	1.882823	0.000000	0.000000	2.399229	1.913355
1976	5.499952	1.989473	0.000000	0.000000	2.511302	2.011226
1977	5.665069	2.107197	0.000000	0.000000	2.689136	2.232756
1978	5.783386	2.275023	0.000000	0.000000	2.819863	2.482589
1979	5.950791	2.557222	0.000000	0.000000	3.057310	2.776580
1980	5.940092	2.861909	0.000000	0.000000	3.139288	3.070752

Table B.21 (continued)

Sector IV: Services

	labor		inventories		cap-M&E	
year	quantity	price	quantity	price	quantity	price
1961	11.234121	1.000000	0.499994	1.000000	1.460116	1.000000
1962	11.619290	1.023880	0.520299	0.956480	1.518303	1.040389
1963	11.895067	1.063034	0.548213	1.006311	1.574607	1.073694
1964	12.366934	1.102978	0.561931	1.110753	1.647954	1.127365
1965	13.090155	1.160666	0.572043	1.094808	1.751854	1.145944
1966	13.535069	1.253518	0.597625	1.199915	1.895576	1.154594
1967	13.665839	1.351107	0.581371	1.217706	2.070180	1.137151
1968	13.666817	1.451743	0.533125	1.311124	2.246863	1.162876
1969	14.123674	1.569194	0.505476	1.391735	2.436544	1.166433
1970	14.110210	1.688741	0.539809	1.495045	2.631226	1.205674
1971	14.462436	1.837723	0.442178	1.630938	2.814189	1.244395
1972	15.067784	1.968243	0.361624	1.937491	3.046717	1.294961
1973	15.971162	2.166338	0.372176	2.175214	3.373524	1.347917
1974	16.929037	2.450203	0.348171	2.383500	3.764153	1.428437
1975	17.256626	2.816291	0.447577	2.540867	4.158640	1.503969
1976	17.533465	3.171177	0.488874	2.730098	4.533461	1.561888
1977	17.944677	3.394450	0.486190	2.087035	4.868069	1.644222
1978	18.519227	3.573324	0.401134	2.866397	5.194545	1.879136
1979	19.221697	3.748176	0.345426	3.959155	5.583736	2.077162
1980	19.717578	4.296999	0.359586	4.338340	6.015682	2.224588

	cap-S	
year	quantity	price
1961	5.825608	1.000000
1962	6.049437	1.007619
1963	6.285030	1.026021
1964	6.558808	1.064720
1965	6.882067	1.112967
1966	7.235363	1.180646
1967	7.569433	1.226004
1968	7.890240	1.304001
1969	8.234933	1.348724
1970	8.581558	1.422943
1971	8.969297	1.513358
1972	9.443831	1.585623
1973	9.988914	1.652258
1974	10.582015	1.747954
1975	11.195258	1.933909
1976	11.826278	2.243971
1977	12.535674	2.385420
1978	13.231093	2.540867
1979	13.870268	2.628307
1980	14.377347	2.957135

Sector I: Resources						
	inventories		cap-M&E		cap-S	
year	quantity	price	quantity	price	quantity	price
1961	0.227087	1.000000	0.848760	1.000000	0.734308	1.000000
1962	0.203089	1.047303	0.839489	1.036271	0.781798	1.009704
1963	0.215036	1.030674	0.848161	1.061460	0.826633	1.042815
1964	0.232245	1.048629	0.872371	1.094989	0.872262	1.056381
1965	0.222952	1.100948	0.904941	1.125648	0.922479	1.108032
1966	0.215263	1.320082	0.949056	1.182138	0.982136	1.235355
1967	0.221975	1.455095	1.008377	1.208545	1.047189	1.348057
1968	0.217291	1.626530	1.061347	1.266151	1.108862	1.454962
1969	0.227130	1.826663	1.096548	1.348249	1.170762	1.624284
1970	0.252924	1.972613	1.115754	1.425845	1.236791	1.736279
1971	0.251413	1.777211	1.135326	1.420743	1.309196	1.833680
1972	0.231957	1.941780	1.179486	1.464467	1.375157	1.951953
1973	0.169166	2.569619	1.254671	1.518346	1.432420	2.161825
1974	0.151474	3.849088	1.350557	1.819539	1.495302	2.694249
1975	0.142889	4.440838	1.462862	2.133749	1.565270	3.124458
1976	0.149687	4.590336	1.591711	2.271819	1.635938	3.494099
1977	0.156511	4.553407	1.729738	2.358169	1.702191	3.698859
1978	0.142418	5.391131	1.834384	2.665340	1.785242	4.015207
1979	0.120502	6.779880	1.931452	3.125832	1.874581	4.624146
1980	0.113386	9.064909	1.997021	3.862775	2.026919	5.466408

	land	
year	quantity	price
1961	0.086268	1.000000
1962	0.091847	1.030529
1963	0.098377	1.075770
1964	0.106453	1.122775
1965	0.115995	1.229504
1966	0.126522	1.443417
1967	0.137918	1.654847
1968	0.148022	1.882855
1969	0.155267	2.087160
1970	0.159798	2.211249
1971	0.170117	2.246751
1972	0.184393	2.462951
1973	0.194197	2.783838
1974	0.207830	3.715461
1975	0.231935	4.717755
1976	0.252724	5.459365
1977	0.276813	5.479471
1978	0.310122	5.968442
1979	0.354174	6.871396
1980	0.413645	9.166418

Table B.22: Capital data series based on an exogenous bond rate, sector I

Sector II: Manufacturing, Export Market-Oriented						
	inventories		cap-M&E		cap-S	
year	quantity	price	quantity	price	quantity	price
1961	0.076313	1.000000	0.472856	1.000000	0.310777	1.000000
1962	0.076596	1.004770	0.486746	1.026605	0.315613	1.021858
1963	0.083965	1.005430	0.505983	1.064638	0.322750	1.047083
1964	0.091586	1.041521	0.537510	1.139774	0.335343	1.083789
1965	0.102387	1.088788	0.583267	1.217316	0.356255	1.140548
1966	0.115252	1.265452	0.641311	1.313045	0.384495	1.248588
1967	0.123113	1.406697	0.697485	1.329949	0.410196	1.339978
1968	0.128381	1.612187	0.727795	1.388745	0.425820	1.426479
1969	0.134569	1.818726	0.752919	1.513458	0.440558	1.571085
1970	0.138769	1.973151	0.800110	1.616918	0.461652	1.698275
1971	0.121653	1.852294	0.852064	1.603145	0.480921	1.703257
1972	0.122501	1.914694	0.888178	1.656954	0.493828	1.789545
1973	0.129907	2.111803	0.920078	1.775663	0.505614	1.927482
1974	0.145378	2.909187	0.965346	2.209229	0.522270	2.381916
1975	0.152757	3.511676	1.013320	2.636296	0.541596	2.741109
1976	0.149906	3.781411	1.046277	2.814122	0.554417	3.019335
1977	0.149504	3.792476	1.070925	3.003033	0.567320	3.114732
1978	0.157234	4.239261	1.088803	3.415912	0.579464	3.376527
1979	0.172016	5.207380	1.107604	3.978079	0.591812	3.828683
1980	0.182643	6.869955	1.153381	4.950691	0.614269	4.641175

Table B.23: Capital data series based on an exogenous bond rate, sector II

Sector III: Manufacturing, Domestic Market-Oriented						
	inventories		cap-M&E		cap-S	
year	quantity	price	quantity	price	quantity	price
1961	0.164678	1.000000	0.516827	1.000000	0.383919	1.000000
1962	0.169764	0.995672	0.532623	1.042179	0.393531	1.021812
1963	0.178758	0.995883	0.548827	1.074446	0.404951	1.037866
1964	0.184200	1.024868	0.570569	1.164916	0.419491	1.069006
1965	0.195846	1.065092	0.605757	1.249673	0.441394	1.139481
1966	0.211290	1.243829	0.654497	1.355496	0.471838	1.248308
1967	0.222683	1.380210	0.703041	1.364515	0.500381	1.343386
1968	0.228521	1.546356	0.740182	1.424216	0.524520	1.421186
1969	0.238340	1.738964	0.772203	1.543129	0.549957	1.553902
1970	0.248481	1.886586	0.804503	1.647465	0.576534	1.679964
1971	0.207857	1.770582	0.833562	1.620201	0.598815	1.701685
1972	0.215227	1.811295	0.860768	1.644342	0.613483	1.786588
1973	0.232139	2.001246	0.899922	1.731891	0.629924	1.917496
1974	0.256425	2.846886	0.953438	2.132474	0.653947	2.353066
1975	0.263251	3.321801	1.009977	2.533978	0.681459	2.705036
1976	0.254863	3.429188	1.065924	2.652419	0.707355	2.987202
1977	0.253324	3.368375	1.116926	2.829180	0.736231	3.091905
1978	0.264671	3.731241	1.156359	3.237520	0.768397	3.359088
1979	0.280701	4.586677	1.194850	3.750944	0.796496	3.801051
1980	0.288731	6.133201	1.219294	4.650824	0.815957	4.618980

Table B.24: Capital data series based on an exogenous bond rate, sector III

Sector IV: Services						
	inventories		cap-M&E		cap-S	
year	quantity	price	quantity	price	quantity	price
1961	0.233352	1.000000	1.315189	1.000000	5.552619	1.000000
1962	0.241952	0.991364	1.352735	1.041860	5.762240	1.009823
1963	0.253990	0.986217	1.386427	1.064014	5.975821	1.024876
1964	0.260214	1.019418	1.433827	1.099282	6.225290	1.071754
1965	0.264663	1.063537	1.510808	1.129523	6.515181	1.142834
1966	0.276367	1.240609	1.622925	1.201201	6.818234	1.304128
1967	0.270340	1.392451	1.760261	1.245401	7.112892	1.450180
1968	0.249144	1.592179	1.900871	1.307811	7.413251	1.594196
1969	0.237284	1.804711	2.047023	1.395441	7.737688	1.786155
1970	0.251856	1.989671	2.192816	1.492537	8.064885	1.923506
1971	0.211834	1.888489	2.328460	1.478687	8.427107	1.921061
1972	0.176606	1.948844	2.487258	1.509143	8.866944	2.043214
1973	0.179386	2.108799	2.706953	1.568481	9.371714	2.286863
1974	0.168302	2.795965	2.975054	1.885795	9.907648	2.955057
1975	0.209924	3.354079	3.245710	2.193212	10.440952	3.495112
1976	0.230993	3.610360	3.506838	2.273766	10.999748	3.866969
1977	0.226609	3.540756	3.741300	2.405548	11.656002	3.980235
1978	0.190276	3.957381	3.969147	2.757107	12.316026	4.319935
1979	0.163974	4.672649	4.234024	3.122160	12.922131	4.878309
1980	0.172828	6.299277	4.522550	3.705302	13.400917	5.987700

Table B.25: Capital data series based on an exogenous bond rate, sector IV

Appendix C

Computer Programs for Implementing the Nonparametric Efficiency Tests

C.1 Introduction to the computer programs

Implementation of the nonparametric efficiency tests given in part I is illustrated by the following sample FORTRAN programs. The technical efficiency tests (tests 1–3) and the allocative efficiency tests (tests 4–6) require the solution of linear programming problems. The user-callable subprogram LIPSUB originally written by Dennis O'Reilly of the University Computing Services of the University of British Columbia is used. There are three subroutines:

1. TESTQ — performs the technical efficiency tests 1, 2 and 3 which require quantity data;
2. TESTPQC — performs the allocative efficiency tests 4, 5 and 6 assuming partial profit maximization and which require price and quantity data; and
3. TESTPQ — performs the allocative efficiency tests 7, 8 and 9 assuming complete profit maximization and which require price and quantity data.

The variant of the efficiency test incorporating the no technological regress assumption (listed in appendix A) can be performed by specifying the integer variable JTECH appropriately in the main calling program; set JTECH=1 if the no technological regress assumption is desired to be incorporated in the efficiency test, or set JTECH=0, otherwise. For the allocative efficiency tests 4 and 7 which assume a convex technology, if it is desired to restrict the optimal profits to be nonnegative (as described in subsection 7.2.6), then either set JPOSP=1 or JPMU=1 in the main calling program. Altogether, the above three subroutines are capable of performing twenty-two kinds of efficiency tests. The subroutines need not be modified by the user and all data inputting can be made in the main calling programs.

Sample main programs calling the subroutines are also given. Since the main calling programs will be specific to the data being processed and the efficiency tests desired, for illustrative purposes, we use the data on sector I, the resources sector.[1] Note that for the resources sector (I), there are 20 observations (1961–1980) and 10 goods (see table B.17). The sample main programs are in the files:

1. MAINQ1 — calls the subroutine TESTQ,
2. MAINPQC1 — calls the subroutine TESTPQC, and
3. MAINPQ1 — calls the subroutine TESTPQ.

Variables whose values have to be specified by the user are noted in the listings.

For allocating space, the dimensioning is done through the PARAMETER option; the user has to specify the value of the following variables:

JOBS	=	number of observations	(J),
JGDS	=	number of goods	(N),
JGOB	=	number of observations + number of goods	$(J+N)$,
JOBS2	=	number of observations + 2	$(J+2)$,
JGDS2	=	number of goods + 2	$(N+2)$, and
JGOB2	=	number of observations + number of goods + 2	$(J+N+2)$.

For the tests requiring the solution of linear programming problems, a detailed version of the results which includes the linear programming tableaus and solutions can be printed out by specifying JDET=1.[2]

The quantity variable is stored in the variable Z(n,j) where the first dimension refers to the goods index and the second dimension refers to the observation index. The price variable is stored in the variable P(n,j) and follows the same dimensioning convention as the quantity variable. If only the technical efficiency tests (1–3) are to be performed, the user need not enter

[1]See appendix B for a detailed description and listing of the data.

[2]This can be helpful in debugging when a problem is encountered in the solution of the linear programming problem. For most purposes, JDET=0 will suffice.

the price data. The efficiency direction vector γ is stored in the variables JDIR($\cdot$) of dimension JGDS. The user has to specify the values of the individual components JDIR(n) as +1, -1 or 0. For tests 4–6 which assume partial profit maximization, the set S containing the indices of goods with respect to which the producer can optimize and the set E containing the indices of the reference goods are specified through the variables JSET($\cdot$) and JNUM($\cdot$)[3], respectively, both of dimension JGDS. Let JSET(n)=1 if good n is in S; set JNUM(n)=1 if good n is in E.

For tests 3, 6 and 9 which assume a quasiconcave technology, a good with respect to whose technology the test is to be performed has to be singled out; this good is specified by JFNC=n where n is the index of the good chosen to play an asymmetric role. For test 8 which assumes a convex cone technology and complete profit maximization, the normalizing good is specified by JNORM=n where n is the index of the good chosen. The desired efficiency test is performed by specifying JTEST=t where t is the number (listed in table C.26) corresponding to the efficiency test and calling the appropriate subroutine. A summary of the different efficiency tests, their informational requirements and options are listed in table C.26. Listings of the sample main calling programs and subroutines follow. Sample results using the data on the resources sector (I) are also given.

C.2 Listings of computer programs

C.2.1 Subroutines TESTQ, TESTPQC and TESTPQ

Listing of subroutine TESTQ

```
      SUBROUTINE TESTQ(Z,JGDS,JGDS2,JOBS,JOBS2,JGOB2,JDIR,
     1                 TABLO,JVIN,JVOUT,OPTIM,TOL,
     2                 X,JPLAM,ZP,ZDIF,ZPDIF,JVIO,VVIO,JPASS,JGEZ,ZW,
     3                 JFRD,JPASSW,JPASSS)
C*********************************************************************
C
C         TEST 1.   TECHNICAL EFFICIENCY TEST
C                   QUANTITY DATA
C                   CONVEX TECHNOLOGY
C
C         TEST 2.   TECHNICAL EFFICIENCY TEST
C                   QUANTITY DATA
C                   CONSTANT RETURNS TO SCALE TECHNOLOGY
C
```

[3]In the sample computer programs below, the reference goods are called "numeraire" goods.

test and subroutine called	informational requirements	options
1. Test 1: TESTQ, JTEST=1	JOBS, JOBS2, JGDS, JGDS2, JGOB2, Z(n,j), JDIR(n)	JDET=0/1 JTECH=0/1
2. Test 2: TESTQ, JTEST=2	JOBS, JOBS2, JGDS, JGDS2, JGOB2, Z(n,j), JDIR(n)	JDET=0/1 JTECH=0/1
3. Test 3: TESTQ, JTEST=3	JOBS, JOBS2, JGDS, JGDS2, JGOB2, Z(n,j), JDIR(n), JFNC=n	JDET=0/1 JTECH=0/1
4. Test 4: TESTPQC, JTEST=4	JOBS, JOBS2, JGDS, JGDS2, JGOB2, Z(n,j), P(n,j), JSET(n), JNUM(n)	JDET=0/1 JTECH=0/1 JPOSP=0/1 or JPMU=0/1
5. Test 5: TESTPQC, JTEST=5	JOBS, JOBS2, JGDS, JGDS2, JGOB2, Z(n,j), P(n,j), JSET(n), JNUM(n)	JDET=0/1 JTECH=0/1
6. Test 6: TESTPQC, JTEST=6	JOBS, JOBS2, JGDS, JGDS2, JGOB2, Z(n,j), P(n,j), JSET(n), JNUM(n), JFNC=n	JDET=0/1 JTECH=0/1
7. Test 7: TESTPQ, JTEST=7	JOBS, JOBS2, JGDS, JGDS2, JGOB, Z(n,j), P(n,j), JDIR(n)	JTECH=0/1 JPOSP=0/1 or JPMU=0/1
8. Test 8: TESTPQ, JTEST=8	JOBS, JOBS2, JGDS, JGDS2, JGOB, Z(n,j), P(n,j), JDIR(n), JNORM=n	JTECH=0/1
9. Test 9: TESTPQ, JTEST=9	JOBS, JOBS2, JGDS, JGDS2, JGOB, Z(n,j), P(n,j), JDIR(n), JFNC=n	JTECH=0/1

Table C.26: Summary of the different efficiency tests, informational requirements and options

```
C          TEST 3.   TECHNICAL EFFICIENCY TEST
C                    QUANTITY DATA
C                    QUASICONCAVE TECHNOLOGY
C
C                    JTECH = 0  ASSUMES NO TECHNICAL CHANGE
C                          = 1  ASSUMES NO TECHNOLOGICAL REGRESS
C
C*********************************************************************
      IMPLICIT REAL*8(B-I,K-Z),CHARACTER*12(A)
      DIMENSION Z(JGDS,JOBS),JDIR(JGDS),TABLO(JGDS2,JOBS2),
     1          JVIN(JGDS2),JVOUT(JOBS2),OPTIM(JOBS),
     2          X(JGOB2),JPLAM(JOBS),ZP(JGDS),ZDIF(JGDS),ZPDIF(JGDS),
     3          JVIO(JOBS),VVIO(JOBS),JPASS(JOBS),JGEZ(JOBS),
     4          ZW(JGDS),JFRD(JOBS),JPASSW(JOBS),JPASSS(JOBS)
      CHARACTER AOFLOW*8
      DATA AOFLOW/'********'/
      COMMON /CONST/ CZERO,JDET,JTEST,JFNC,JNORM,JTECH
C
C          SWITCH JTEST INDICES
C
      IF (JTEST .EQ. 3) THEN
         JTEST = 1
C        (QUASICONCAVE CASE)
         GO TO 700
      END IF
      IF (JTEST .EQ. 1) THEN
         JTEST = 2
C        (CONVEX CASE)
         GO TO 700
      END IF
      IF (JTEST .EQ. 2) THEN
         JTEST = 3
C        (CONSTANT RETURNS TO SCALE CASE)
         GO TO 700
      END IF
  700 CONTINUE
C
C          PRINT TEST HEADING
C
      IF (JTEST .EQ. 1) THEN
         WRITE(6,420) JFNC
      ELSE IF (JTEST .EQ. 2) THEN
         WRITE(6,400)
      ELSE
         WRITE(6,410)
      END IF
      IF (JTECH .EQ. 0) THEN
         WRITE(6,422)
      ELSE
         WRITE(6,424)
      END IF
  400 FORMAT('1','TEST 1.  TECHNICAL EFFICIENCY TEST'/
     1       ' ','         QUANTITY DATA'/
     2       ' ','         CONVEX TECHNOLOGY'//)
  410 FORMAT('1','TEST 2.  TECHNICAL EFFICIENCY TEST'/
     1       ' ','         QUANTITY DATA'/
     2       ' ','         CONVEX CONE TECHNOLOGY'/
     3       ' ','         (CONSTANT RETURNS TO SCALE)'//)
  420 FORMAT('1','TEST 3.  TECHNICAL EFFICIENCY TEST'/
     1       ' ','         QUANTITY DATA'/
     2       ' ','         QUASICONCAVE TECHNOLOGY'/
     3       '0','  NOTE:  TEST PERFORMED WITH RESPECT TO'/
     4       ' ','         TECHNOLOGY OF GOOD N=',I4//)
  422 FORMAT(' ',9X,'( TEST ASSUMES NO TECHNICAL CHANGE )'//)
  424 FORMAT(' ',9X,'( TEST ASSUMES NO TECHNOLOGICAL REGRESS )'//)
```

```
C
C          SOLVE LP FOR JI=1,JOBS
C
      DO 5 JI = 1,JOBS
C
C          INITIALIZE VALUES FOR LP SUBROUTINE
C
      JCONST = JGDS + 1
      JEQUAL = 1
      JMAXIM = 1
      JFOBJ = 0
      JFRHS = 0
      IF (JTEST .EQ. 3) THEN
         JCONST = JGDS
         JEQUAL = 0
      END IF
C
C          ZERO TABLEAU
C
      DO 10 J1=1,JGDS2
         DO 15 J2=1,JOBS2
           TABLO(J1,J2) = 0.DO
   15    CONTINUE
   10 CONTINUE
C
C          SET UP TABLEAU
C
      JU = JOBS
      IF (JTECH .EQ. 1) JU=JI
C
C              SET UP CONSTRAINTS AND RHS (TEST 1)
C
      IF (JTEST .EQ. 1) THEN
      JCONST = JGDS
      J = 0
      DO 90 JJ = 1,JU
         IF (Z(JFNC,JJ) .GE. Z(JFNC,JI)) THEN
            J = J+1
            JGEZ(J) = JJ
         END IF
   90 CONTINUE
      JCGEZ = J
      DO 91 JJ = 1,JCGEZ
         JR = 1
         DO 92 J1 = 1,JGDS
            IF (J1 .EQ. JFNC) GO TO 92
            JR = JR+1
            TABLO(JR,JJ) = -Z(J1,JGEZ(JJ))
   92    CONTINUE
         TABLO(JGDS+1,JJ) = 1.DO
   91 CONTINUE
      JNVARS = JCGEZ+1
      JV1 = JNVARS+1
      JR = 1
      DO 93 J1 = 1,JGDS
         IF (J1 .EQ. JFNC) GO TO 93
            JR = JR+1
            TABLO(JR,JNVARS) = JDIR(J1)*Z(J1,JI)
            TABLO(JR,JV1) = -Z(J1,JI)
   93 CONTINUE
      TABLO(JGDS+1,JV1) = 1.DO
      ELSE
C              SET UP CONSTRAINTS AND RHS (TESTS 2,3)
C
      DO 20 J2=1,JU
         DO 22 J1=1,JGDS
            TABLO(J1+1,J2) = -Z(J1,J2)
   22    CONTINUE
```

```
         TABLO(JGDS2,J2) = 1.DO
   20 CONTINUE
      JNVARS = JOBS+1
      IF (JTECH .EQ. 1) JNVARS=JI+1
      JV1 = JNVARS+1
      DO 30 J1=1,JGDS
         JR = J1+1
         TABLO(JR,JNVARS) = JDIR(J1)*Z(J1,JI)
         TABLO(JR,JV1) = -Z(J1,JI)
   30 CONTINUE
      TABLO(JGDS2,JV1) = 1.DO
      END IF
C
C              SET UP OBJECTIVE FUNCTION
C
      TABLO(1,JNVARS) = 1.DO
C
C           SUM OF LP VARIABLES
C
      JLPV = JNVARS + JCONST
C
C           PRINT OUT INPUT TO LP SUBROUTINE
C
      IF (JDET .EQ. 0) GO TO 1234
      WRITE(6,100)JI,JCONST,JNVARS,JEQUAL,TOL
  100 FORMAT('1','OBSERVATION I=',I4/
     1       ' ',5X,'NUMBER OF CONSTRAINTS=',I4/
     2       ' ',5X,'NUMBER OF VARIABLES=',I4/
     3       ' ',5X,'NUMBER OF EQUALITY CONSTRAINTS=',I4/
     4       ' ',5X,'TOLERANCE=',F10.6//)
      WRITE(6,105)
  105 FORMAT(' ','TABLEAU IS:')
      IF (JTEST .EQ. 1) THEN
        CALL DPRMAT(TABLO,JGDS2,JOBS2,JGDS+1,JV1,1,1,
     1                   JGDS2,1)
      ELSE IF (JTEST .EQ. 2) THEN
        CALL DPRMAT(TABLO,JGDS2,JOBS2,JGDS2,JV1,1,1,JGDS2,1)
      ELSE
        CALL DPRMAT(TABLO,JGDS2,JOBS2,JGDS+1,JV1,1,1,
     1                   JGDS2,1)
      END IF
 1234 CONTINUE
C
C          CALL LIPSUB SUBROUTINE
C
      CALL LIPSUB(TABLO,JGDS2,JCONST,JNVARS,JEQUAL,JMAXIM,
     1            JFOBJ,JFRHS,TOL,JVIN,JVOUT,BBOBJ,UBOBJ,
     2            BBRHS,UBRHS,*999)
      JNP1 = JNVARS+1
      OPTIM(JI) = TABLO(1,JNP1)
      IF (JMAXIM .NE. 1) OPTIM(JI) = -OPTIM(JI)
C
      IF (JDET .EQ. 0) GO TO 2345
         WRITE(6,200)OPTIM(JI)
         WRITE(6,300) (JVIN(J),TABLO(J,JNP1),J=2,JCONST+1)
 2345 CONTINUE
C
C          CHECK FOR NEGATIVE VALUES IN THE OPTIMAL BASIS
C
      JWARN = 0
      DO 60 J = 2,JCONST+1
         IF (TABLO(J,JNP1) .LT. -CZERO) JWARN=1
   60 CONTINUE
      IF (JWARN .EQ. 1) WRITE(6,888)JI
C
```

```
C          PRINT SUMMARY FOR OBSERVATION I
C
C          CALCULATE WEAKLY EFFICIENT VECTOR ZW
C             ( SOME GOODS, BUT NOT ALL, MAY HAVE
C               ZERO SHADOW PRICES )
C
      DO 65 JN = 1,JGDS
         ZW(JN) = (1.D0 + (OPTIM(JI)*JDIR(JN))) * Z(JN,JI)
   65 CONTINUE
      DO 70 J = 1,JLPV
         X(J) = 0.D0
   70 CONTINUE
      DO 72 J = 2,JCONST+1
         X(JVIN(J)) = TABLO(J,JNP1)
   72 CONTINUE
C
C          COUNT POSITIVE LAMBDA'S
C
      JPL = 0
      IF (JTEST .EQ. 1) JU = JCGEZ
      DO 74 J = 1,JU
         IF (X(J) .GT. CZERO) THEN
            JPL = JPL+1
            JPLAM(JPL) = J
         END IF
   74 CONTINUE
C
C          CALCULATE EFFICIENT VECTOR ZP
C             ( ZP IS (IDEALLY) PARETO EFFICIENT, ALL GOODS
C               HAVE POSITIVE SHADOW PRICES BUT PROCEDURE DOES
C               NOT GUARANTEE THAT ZP CALCULATED IS SUCH )
C          AND DETERMINE IF EFFICIENT VECTOR ZW
C          IS IN FREE DISPOSAL SECTION OF THE
C          TECHNOLOGY SET
      JFRD(JI) = 0
C
      IF ( JTEST .EQ. 1) THEN
         DO 94 JG = 1,JGDS
            IF (JG .EQ. JFNC) THEN
               ZP(JG) = Z(JG,JI)
            ELSE
               ZP(JG) = 0.D0
               DO 95 JJ = 1,JCGEZ
                  ZP(JG) = ZP(JG) + (X(JJ)*Z(JG,JGEZ(JJ)))
   95          CONTINUE
            END IF
         ZDIF(JG) = ZP(JG) - Z(JG,JI)
         IF (ABS(Z(JG,JI)) .GT. CZERO) THEN
            ZPDIF(JG) = ZDIF(JG)/Z(JG,JI)
         ELSE
            ZPDIF(JG) = 9999.999D0
         END IF
         IF ((ABS(ZPDIF(JG)) .GT. CZERO) .AND. (JG .NE. JFNC))
     1      JFRD(JI) = 1
   94 CONTINUE
      ELSE
      DO 76 JG = 1,JGDS
         ZP(JG) = 0.D0
         DO 78 JJ = 1,JU
            ZP(JG) = ZP(JG) + (X(JJ)*Z(JG,JJ))
   78    CONTINUE
         ZDIF(JG) = ZP(JG) - Z(JG,JI)
         IF (ABS(Z(JG,JI)) .GT. CZERO) THEN
            ZPDIF(JG) = ZDIF(JG)/Z(JG,JI)
         ELSE
            ZPDIF(JG) = 9999.999D0
```

```
          END IF
          IF (ABS(ZPDIF(JG)) .GT. CZERO) JFRD(JI) = 1
   76 CONTINUE
       END IF
       IF (JTEST .EQ. 1) THEN
        WRITE(6,500) JI,OPTIM(JI),(JGEZ(JPLAM(J)),J=1,JPL)
       ELSE
        IF (JPL .GT. 0) THEN
           WRITE(6,500) JI,OPTIM(JI),(JPLAM(J),J=1,JPL)
        ELSE
           WRITE(6,501) JI,OPTIM(JI)
        END IF
       END IF
       WRITE(6,510)
       IF (JTEST .EQ. 1) THEN
          DO 96 J = 1,JGDS
             IF (J .EQ. JFNC) GO TO 96
               IF (ZPDIF(J) .GT. 9000.DO) THEN
                  WRITE(6,521) J,JDIR(J),ZW(J),ZP(J),Z(J,JI),
    1                          ZDIF(J),AOFLOW
               ELSE
                  WRITE(6,520) J,JDIR(J),ZW(J),ZP(J),Z(J,JI),
    1                          ZDIF(J),ZPDIF(J)
               END IF
   96     CONTINUE
          WRITE(6,525) JFNC,Z(JFNC,JI)
       ELSE
          DO 97 J = 1,JGDS
               IF (ZPDIF(J) .GT. 9000.DO) THEN
                  WRITE(6,521) J,JDIR(J),ZW(J),ZP(J),Z(J,JI),
    1                          ZDIF(J),AOFLOW
               ELSE
                  WRITE(6,520) J,JDIR(J),ZW(J),ZP(J),Z(J,JI),
    1                          ZDIF(J),ZPDIF(J)
               END IF
   97     CONTINUE
       END IF
       IF (OPTIM(JI) .GT. CZERO) THEN
          WRITE(6,550) JI
       ELSE IF (JFRD(JI) .EQ. 1) THEN
          WRITE(6,555) JI
       ELSE
          WRITE(6,560) JI
       END IF
  550 FORMAT('-','HENCE, OBSERVATION I=',I4,
    1        ' IS TECHNICALLY E-INEFFICIENT.')
  555 FORMAT('-','HENCE, OBSERVATION I=',I4,
    1        ' IS WEAKLY TECHNICALLY E-EFFICIENT.')
  560 FORMAT('-','HENCE, OBSERVATION I=',I4,
    1        ' IS TECHNICALLY E-EFFICIENT.')
       GO TO 5
  999 WRITE(6,900) JI
    5 CONTINUE
C
C           PRINT SUMMARY FOR DATAFILE
C
       WRITE(6,600)
       JNF = 0
       JNP = 0
       JMAX = 0
       VMAX = 0.DO
       JNPW = 0
       JNPS = 0
       DO 80 JJ = 1,JOBS
          IF (OPTIM(JJ) .GT. CZERO) THEN
```

```
          JNF = JNF+1
          JVIO(JNF) = JJ
          VVIO(JNF) = OPTIM(JJ)
          IF (OPTIM(JJ) .GT. VMAX) THEN
             JMAX = JJ
             VMAX = OPTIM(JJ)
          END IF
       ELSE
          JNP = JNP+1
          JPASS(JNP) = JJ
            IF (JFRD(JJ) .EQ. 1) THEN
               JNPW = JNPW+1
               JPASSW(JNPW) = JJ
            ELSE
               JNPS = JNPS+1
               JPASSS(JNPS) = JJ
            END IF
       END IF
 80 CONTINUE
    IF (JNF .GT. 0) THEN
       WRITE(6,610) (JVIO(J),VVIO(J),J=1,JNF)
       WRITE(6,615) JNF,VMAX,JMAX
    END IF
    WRITE(6,620) JOBS,JNP,JNF
    IF (JNP .GT. 0) THEN
       IF (JNPS .EQ. JOBS) THEN
          WRITE(6,625)
       ELSE
          WRITE(6,630)
          IF (JNPW .GT. 0)
   1         WRITE(6,632) JNPW,(JPASSW(J),J=1,JNPW)
          IF (JNPS .GT. 0)
   1         WRITE(6,634) JNPS,(JPASSS(J),J=1,JNPS)
       END IF
    END IF
    IF (JNF .GT. 0) THEN
       WRITE(6,640)
    ELSE
       WRITE(6,650)
    END IF
600 FORMAT('1','SUMMARY FOR DATAFILE:'//)
610 FORMAT(' ',5X,'VIOLATIONS ARE AT OBSERVATIONS:'/
   1       (' ',15X,'I=',I4,',  DELTA*=',F10.6))
615 FORMAT('0',10X,'TOTAL NUMBER OF VIOLATIONS FOR TEST IS ',I4/
   1       ' ',10X,'MAXIMUM DELTA*=',F10.6,' AT OBSERVATION I=',
   2              I4,'.'//)
620 FORMAT(' ',5X,'OUT OF ',I4,' OBSERVATIONS, ',I4,
   1              ' PASS AND ',I4,' FAIL THE TEST.')
625 FORMAT(' ',10X,'ALL OBSERVATIONS ARE ',
   1          'TECHNICALLY E-EFFICIENT.')
630 FORMAT(' ',10X,'THE OBSERVATIONS CONSISTENT WITH THE',
   1              ' HYPOTHESIS ARE:')
632 FORMAT(' ',15X,'(',I4,') WEAKLY TECHNICALLY E-EFFICIENT',
   1          ' OBSERVATIONS:'/(' ',20X,'I=',I4))
634 FORMAT(' ',15X,'(',I4,') TECHNICALLY E-EFFICIENT',
   1          ' OBSERVATIONS:'/(' ',20X,'I=',I4))
640 FORMAT('0',5X,'CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT'/
   1       ' ',5X,'             WITH THE HYPOTHESIS.')
650 FORMAT('0',5X,'CONCLUSION:  OVERALL, THE DATA IS CONSISTENT'/
   1       ' ',5X,'             WITH THE HYPOTHESIS.')
    RETURN
200 FORMAT(/,' ',5X,'OPTIMUM=',G20.12/)
300 FORMAT('0',5X,'PRIMAL SOLUTION:'/
```

```
     1        ' ',7X,'VARIABLE',8X,'VALUE'/
     2       (' ',8X,I5,7X,G20.12))
  500 FORMAT('2','SUMMARY FOR OBSERVATION I=',I4,':'//
     1        ' ',5X,'DELTA*=',F10.6/
     2        ' ',5X,'POSITIVE LAMBDA''S: ',I4,9(1X,I4)/
     3       (' ',5X,'                    ',I4,9(1X,I4)))
  501 FORMAT('2','SUMMARY FOR OBSERVATION I=',I4,':'//
     1        ' ',5X,'DELTA*=',F10.6/
     2        ' ',5X,'POSITIVE LAMBDA''S:    0')
  510 FORMAT('0','GOODS',2X,'EFFICIENCY',4X,'WEAKLY EFF.',5X,
     1             'STRONGLY EFF.',5X,
     2             'VECTOR Z(I)',7X,'DIFFERENCE',4X,
     3             'PROPORTIONAL'/
     4        ' ',1X,'NO.',2X,'DIR. VECTOR',5X,'VECTOR Z*',
     5             7X,'VECTOR Z**',
     6            26X,'Z**-Z(I)',6X,'DIFFERENCE'/
     7        ' ',86X,'(Z**-Z(I))/Z(I)'//)
  520 FORMAT(' ',I4,7X,I2,4X,4G17.5,4X,F8.5)
  521 FORMAT(' ',I4,7X,I2,4X,4G17.5,4X,A8)
  525 FORMAT('0',I4,49X,G15.5)
  888 FORMAT('0',5X,'WARNING:  THERE IS AT LEAST ONE NEGATIVE'/
     1        ' ',5X,'          VALUE IN THE OPTIMAL BASIS FOR THE'/
     2        ' ',5X,'          LP PROBLEM AT OBSERVATION I=',I4, '.'/
     3        ' ',5X,'          DATA SCALING PROBLEMS MAY EXIST.'/)
  900 FORMAT('0',5X,'PROBLEM ENCOUNTERED IN LP SUBROUTINE'/
     1        ' ',5X,'AT OBSERVATION I=',I4,'.'/)
      END
```

Listing of subroutine TESTPQC

```
      SUBROUTINE TSTPQC(Z,P,PI,JGDS,JGDS2,JOBS,JOBS2,JGOB2,
     1             TABLO,JVIN,JVOUT,OPTIM,TOL,
     2             EPS,ZP,ZDIF,ZPDIF,X,JPLAM,
     3             JVIO,VVIO,JPASS,JGEZ,JSET,JNUM,JINS,JNOTS)
*******************************************************************
C
C         TEST 4.   CONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                   PRICE AND QUANTITY DATA
C                   CONVEX TECHNOLOGY
C
C         TEST 5.   CONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                   PRICE AND QUANTITY DATA
C                   CONSTANT RETURNS TO SCALE TECHNOLOGY
C
C         TEST 6.   CONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                   PRICE AND QUANTITY DATA
C                   QUASICONCAVE TECHNOLOGY
C
C                   JTECH = 0  ASSUMES NO TECHNICAL CHANGE
C                         = 1  ASSUMES NO TECHNOLOGICAL REGRESS
C
C******************************************************************
      IMPLICIT REAL*8(B-I,K-Z),CHARACTER*12(A)
      DIMENSION Z(JGDS,0:JOBS),P(JGDS,0:JOBS),PI(0:JOBS,0:JOBS),
     1          TABLO(JGDS2,JOBS2),JVIN(JGDS2),JVOUT(JOBS2),
     2          OPTIM(JOBS),EPS(JOBS),ZP(JGDS),ZDIF(JGDS),ZPDIF(JGDS),
     3          X(0:JGOB2),JPLAM(JOBS),
     4          JVIO(JOBS),VVIO(JOBS),JPASS(JOBS),JGEZ(JOBS),
     5          JSET(JGDS),JNUM(JGDS),JINS(JGDS),JNOTS(JGDS)
      COMMON /CONST/ CZERO,JDET,JTEST,JFNC,JNORM,JTECH,JPOSP,JPMU
```

```
      DATA AVMAX/' UNBOUNDED'/
C
C          PRINT TEST HEADING
C
      DO 1 JN = 1,JGDS
           Z(JN,0) = 0.D0
           P(JN,0) = 0.D0
    1 CONTINUE
      IF (JTEST .EQ. 6) THEN
         WRITE(6,420) JFNC
      ELSE IF (JTEST .EQ. 4) THEN
         WRITE(6,400)
      ELSE
         WRITE(6,410)
      END IF
      IF (JTEST .EQ. 4) THEN
         IF ((JPOSP .EQ. 0) .AND. (JPMU .EQ. 0)) THEN
            WRITE(6,430)
         ELSE
            WRITE(6,435)
         END IF
      END IF
      IF (JTECH .EQ. 0) THEN
         WRITE(6,422)
      ELSE
         WRITE(6,424)
      END IF
  400 FORMAT('1','TEST 4.  CONSTRAINED ALLOCATIVE EFFICIENCY TEST'/
     1       ' ','         PRICE AND QUANTITY DATA'/
     2       ' ','         CONVEX TECHNOLOGY'//)
  410 FORMAT('1','TEST 5.  CONSTRAINED ALLOCATIVE EFFICIENCY TEST'/
     1       ' ','         PRICE AND QUANTITY DATA'/
     2       ' ','         CONVEX CONE TECHNOLOGY'/
     3       ' ','         (CONSTANT RETURNS TO SCALE)'//)
  420 FORMAT('1','TEST 6.  CONSTRAINED ALLOCATIVE EFFICIENCY TEST'/
     1       ' ','         PRICE AND QUANTITY DATA'/
     2       ' ','         QUASICONCAVE TECHNOLOGY'/
     3       '0',' NOTE:   TEST PERFORMED WITH RESPECT TO'/
     4       ' ','         TECHNOLOGY OF GOOD N=',I4//)
  422 FORMAT(' ',9X,'( TEST ASSUMES NO TECHNICAL CHANGE )'//)
  424 FORMAT(' ',9X,'( TEST ASSUMES NO TECHNOLOGICAL REGRESS )'//)
  430 FORMAT(' ',9X,'( OPTIMAL PROFITS UNRESTRICTED IN SIGN )'//)
  435 FORMAT(' ',9X,'( OPTIMAL PROFITS RESTRICTED ',
     1              'TO BE NONNEGATIVE )'//)
C
C          SOLVE LP FOR JI=1,JOBS
C
C**
      DO 5 JI = 1,JOBS
C**
C
C          INITIALIZE VALUES FOR LP SUBROUTINE
C
      JMAXIM = 1
      JFOBJ = 0
      JFRHS = 0
C
C          ZERO TABLEAU
C
      DO 10 J1=1,JGDS2
         DO 15 J2=1,JOBS2
            TABLO(J1,J2) = 0.D0
   15    CONTINUE
   10 CONTINUE
```

```
C
C          SET UP TABLEAU
C
      JU = JOBS
      IF (JTECH .EQ. 1) JU=JI
      JL = 1
      JADJ = 0
C
C          COUNT GOODS IN S AND GOODS NOT IN S
C
      J1 = 0
      J2 = 0
      DO 18 J = 1,JGDS
         IF ((JTEST .EQ. 6) .AND. (J .EQ. JFNC)) GO TO 18
         IF (JSET(J) .EQ. 1) THEN
            J1 = J1+1
            JINS(J1) = J
         ELSE
            J2 = J2+1
            JNOTS(J2) = J
         END IF
   18 CONTINUE
      JCINS = J1
      JCNOTS = J2
C
      IF (JCINS .EQ. 0) THEN
         WRITE(6,810)
         RETURN
      END IF
  810 FORMAT(' ','CONSTRAINED OPTIMIZATION TEST CANNOT'/
     1       ' ','BE PERFORMED;  THE SET S CONTAINING'/
     2       ' ','THE GOODS WITH RESPECT TO WHICH THE'/
     3       ' ','PRODUCER CAN OPTIMIZE HAS NO ELEMENTS.')
C
C              TO CHECK IF THERE IS AT LEAST ONE GOOD
C              NOT IN S (TEST 5 (CRS) ONLY)
C
      IF ((JTEST .EQ. 5) .AND. (JCNOTS .EQ. 0)) THEN
         WRITE(6,800)
         RETURN
      END IF
  800 FORMAT('1','CONSTRAINED OPTIMIZATION TEST FOR A CRS'/
     1       ' ','TECHNOLOGY CANNOT BE PERFORMED;  THERE '/
     2       ' ','ARE NO CONSTRAINTS IN THE LP PROBLEM.')
C
C              SET UP CONSTRAINTS AND RHS (TEST 6)
C
      IF (JTEST .EQ. 6) THEN
         JEQUAL = 1
         J = 0
         DO 30 JJ = 1,JU
            IF (Z(JFNC,JJ) .GE. Z(JFNC,JI)) THEN
               J = J+1
               JGEZ(J) = JJ
            END IF
   30    CONTINUE
         JCGEZ = J
         JNVARS = JCGEZ
         JV1 = JNVARS+1
         JR = 1
         DO 32 J1 = 1,JCNOTS
            JR = JR+1
            DO 34 J2 = 1,JCGEZ
               TABLO(JR,J2) = -Z(JNOTS(J1),JGEZ(J2))
   34       CONTINUE
            TABLO(JR,JV1) = -Z(JNOTS(J1),JI)
   32    CONTINUE
```

```
          JR = JR+1
          DO 36 J2 = 1,JV1
             TABLO(JR,J2) = 1.D0
   36     CONTINUE
          JCONST = JR-1
C
C               SET UP OBJECTIVE FUNCTION (TEST 6)
C
          DO 38 J2 = 1,JCGEZ
           DO 39 J1 = 1,JCINS
            TABLO(1,J2) = TABLO(1,J2) +
     1                    (P(JINS(J1),JI)*Z(JINS(J1),JGEZ(J2)))
   39      CONTINUE
   38     CONTINUE
        ELSE
C
C               SET UP CONSTRAINTS AND RHS (TEST 4,5)
C
        JNVARS = JU
        JV1 = JNVARS+1
        JCONST = JCNOTS +1
        JEQUAL = 1
CC*
        IF ((JTEST .EQ. 4) .AND. (JPMU .EQ. 1)) JEQUAL = 0
CC*
        IF (JTEST .EQ. 5) THEN
           JCONST = JCNOTS
           JEQUAL = 0
        END IF
C
        IF ((JTEST .EQ. 4) .AND. (JPOSP .EQ. 1)) THEN
           JNVARS = JU+1
           JV1 = JNVARS+1
           JL = 0
           JADJ = 1
        END IF
C
        JR = 1
        DO 20 J1 = 1,JCNOTS
           JR = JR+1
           DO 22 J2 = JL,JU
              J2P = J2
              IF (JADJ .EQ. 1) J2P = J2+1
              TABLO(JR,J2P) = -Z(JNOTS(J1),J2)
   22      CONTINUE
           TABLO(JR,JV1) = -Z(JNOTS(J1),JI)
   20   CONTINUE
        IF (JTEST .EQ. 4) THEN
           JR = JR+1
           DO 24 J2 = 1,JV1
              TABLO(JR,J2) = 1.D0
   24      CONTINUE
        END IF
C
C               SET UP OBJECTIVE FUNCTION (TESTS 4,5)
C
        DO 26 J2 = JL,JU
          J2P = J2
          IF (JADJ .EQ. 1) J2P = J2+1
          DO 29 J1 = 1,JCINS
           TABLO(1,J2P) = TABLO(1,J2P) + (P(JINS(J1),JI)*Z(JINS(J1),J2))
   29     CONTINUE
   26   CONTINUE
       END IF
C
C           SUM OF LP VARIABLES
C
       JLPV = JNVARS + JCONST
```

```
C
C         PRINT OUT INPUT TO LP SUBROUTINE
C
      IF (JDET .EQ. 0) GO TO 1234
      WRITE(6,100)JI,JCONST,JNVARS,JEQUAL,TOL
  100 FORMAT('1','OBSERVATION I=',I4/
     1        ' ',5X,'NUMBER OF CONSTRAINTS=',I4/
     2        ' ',5X,'NUMBER OF VARIABLES=',I4/
     3        ' ',5X,'NUMBER OF EQUALITY CONSTRAINTS=',I4/
     4        ' ',5X,'TOLERANCE=',F10.6//)
      WRITE(6,105)
  105 FORMAT(' ','TABLEAU IS:')
      CALL DPRMAT(TABLO,JGDS2,JOBS2,JR,JV1,1,1,JGDS2,1)
 1234 CONTINUE
C
C        CHECK IF JEQUAL < JNVARS
C
      JNOLP = 0
      IF (JNVARS .EQ. JEQUAL) THEN
         JNOLP = 1
         X(1) = 1.D0
         GO TO 82
      END IF
C
C        CALL LIPSUB SUBROUTINE
C
      CALL LIPSUB(TABLO,JGDS2,JCONST,JNVARS,JEQUAL,JMAXIM,
     1            JFOBJ,JFRHS,TOL,JVIN,JVOUT,BBOBJ,UBOBJ,
     2            BBRHS,UBRHS,*999)
      JNP1 = JNVARS+1
      OPTIM(JI) = TABLO(1,JNP1)
      IF (JMAXIM .NE. 1) OPTIM(JI) = -OPTIM(JI)
C
      IF (JDET .EQ. 0) GO TO 2345
         WRITE(6,200)OPTIM(JI)
         WRITE(6,300) (JVIN(J),TABLO(J,JNP1),J=2,JCONST+1)
 2345 CONTINUE
C
C        CHECK FOR NEGATIVE VALUES IN THE OPTIMAL BASIS
C
      JWARN = 0
      DO 60 J = 2,JCONST+1
         IF (TABLO(J,JNP1) .LT. -CZERO) JWARN=1
   60 CONTINUE
      IF (JWARN .EQ. 1) WRITE(6,888)JI
C
C         CALCULATE ACTUAL PARTIAL PROFIT, VALUE OF NUMERAIRE
C         GOODS, LOSS MEASURE
C
   82 PI(JI,JI) = 0.D0
      VNUM = 0.D0
      DO 40 J1=1,JCINS
        PI(JI,JI) = PI(JI,JI) + (P(JINS(J1),JI)*Z(JINS(J1),JI))
   40 CONTINUE
      DO 42 J1 = 1,JGDS
        IF (JNUM(J1) .EQ. 1)
     1      VNUM = VNUM + (P(J1,JI)*ABS(Z(J1,JI)))
   42 CONTINUE
C
C         CHECK FOR ZERO VALUES OF VNUM
C
      IF (ABS(VNUM) .LE. CZERO) THEN
         EPS(JI) = 999.999999D0
         WRITE(6,700) JI
         GO TO 5
```

```
      END IF
  700 FORMAT('2','WARNING:  FOR OBSERVATION I=',I4,', THE VALUE'/
     1       ' ',           OF NUMERAIRE GOODS IS ZERO;  HENCE,'/
     2       ' ',           EPSILON(S) IS UNDEFINED.')
C
C          CALCULATE LOSS MEASURE EPS(JI)
C
      IF (JNOLP .EQ. 1) THEN
         OPTIM(JI) = PI(JI,JI)
         EPS(JI) = 0.DO
         GO TO 84
      END IF
      EPS(JI) = (OPTIM(JI) - PI(JI,JI)) / VNUM
C
C          PRINT SUMMARY FOR OBSERVATION I
C
C          COUNT POSITIVE LAMBDA'S
C
      DO 70 J = 1,JLPV
         X(J) = 0.DO
   70 CONTINUE
      DO 72 J = 2,JCONST+1
         X(JVIN(J)) = TABLO(J,JNP1)
   72 CONTINUE
C
      IF (JADJ .EQ. 1) THEN
         DO 71 J = 1,JLPV
            X(J-1) = X(J)
  71     CONTINUE
      END IF
C
   84 JPL = 0
      JPLAM(1) = 0
      IF (JTEST .EQ. 6) JU = JCGEZ
      DO 74 J = JL,JU
         IF (X(J) .GT. CZERO) THEN
            JPL = JPL+1
            JPLAM(JPL) = J
         END IF
   74 CONTINUE
C
C          CALCULATE EFFICIENT VECTOR ZP
C
      IF ( JTEST .EQ. 6) THEN
         DO 94 JG = 1,JGDS
            IF (JG .EQ. JFNC) THEN
               ZP(JG) = Z(JG,JI)
            ELSE
               ZP(JG) = 0.DO
               DO 95 JJ = 1,JCGEZ
                  ZP(JG) = ZP(JG) + (X(JJ)*Z(JG,JGEZ(JJ)))
   95          CONTINUE
            END IF
         ZDIF(JG) = ZP(JG) - Z(JG,JI)
         ZPDIF(JG) = ZDIF(JG)/Z(JG,JI)
   94 CONTINUE
      ELSE
      DO 76 JG = 1,JGDS
         ZP(JG) = 0.DO
         DO 78 JJ = JL,JU
            ZP(JG) = ZP(JG) + (X(JJ)*Z(JG,JJ))
   78    CONTINUE
         ZDIF(JG) = ZP(JG) - Z(JG,JI)
         ZPDIF(JG) = ZDIF(JG)/Z(JG,JI)
   76 CONTINUE
      END IF
```

```
C
C          PRINT RESULTS FOR OBSERVATION I
C
      IF (JTEST .EQ. 6) THEN
       WRITE(6,500) JI,EPS(JI),(JGEZ(JPLAM(J)),J=1,JPL)
      ELSE
       WRITE(6,500) JI,EPS(JI),(JPLAM(J),J=1,JPL)
      END IF
      WRITE(6,510)
      WRITE(6,505) JCINS
      WRITE(6,520)(JINS(J),JNUM(JINS(J)),P(JINS(J),JI),ZP(JINS(J)),
     1              Z(JINS(J),JI),ZDIF(JINS(J)),
     2              ZPDIF(JINS(J)),J=1,JCINS)
      IF (JCNOTS .GT. 0) THEN
         WRITE(6,515) JCNOTS
         WRITE(6,525) (JNOTS(J),ZP(JNOTS(J)),Z(JNOTS(J),JI),
     1                ZDIF(JNOTS(J)),ZPDIF(JNOTS(J)),J=1,JCNOTS)
      END IF
      IF (JTEST .EQ. 6) WRITE(6,517) JFNC,Z(JFNC,JI)
      WRITE(6,530) VNUM,PI(JI,JI),OPTIM(JI)
      IF (EPS(JI) .GT. CZERO) THEN
         WRITE(6,550) JI
      ELSE
         WRITE(6,555) JI
      END IF
      GO TO 5
  999 WRITE(6,900) JI
      EPS(JI) = 999.999999D0
    5 CONTINUE
C
C          PRINT SUMMARY FOR DATAFILE
C
      WRITE(6,600)
      JNF = 0
      JNP = 0
      JMAX = 0
      VMAX = 0.D0
      DO 80 JJ = 1,JOBS
         IF (EPS(JJ) .GT. CZERO) THEN
            JNF = JNF+1
            JVIO(JNF) = JJ
            VVIO(JNF) = EPS(JJ)
            IF (EPS(JJ) .GT. VMAX) THEN
               JMAX = JJ
               VMAX = EPS(JJ)
            END IF
         ELSE
            JNP = JNP+1
            JPASS(JNP) = JJ
         END IF
   80 CONTINUE
      IF (JNF .GT. 0) THEN
         WRITE(6,610)
         JBOU = 0
         DO 88 J=1,JNF
            IF (VVIO(J) .GT. 999.D0) THEN
               JBOU = 1
               WRITE(6,612) JVIO(J),AVMAX
            ELSE
               WRITE(6,614) JVIO(J),VVIO(J)
            END IF
   88    CONTINUE
         IF (JBOU .EQ. 1) THEN
            WRITE(6,617) JNF,JMAX
         ELSE
            WRITE(6,615) JNF,VMAX,JMAX
```

```
          END IF
       END IF
       WRITE(6,620) JOBS,JNP,JNF
       IF ((JNP .GT. 0) .AND. (JNP .LT. JOBS))
     1    WRITE(6,630) (JPASS(J),J=1,JNP)
       IF (JNF .GT. 0) THEN
          WRITE(6,640)
       ELSE
          WRITE(6,650)
       END IF
       RETURN
   200 FORMAT(/,' ',5X,'OPTIMUM=',G20.12/)
   300 FORMAT('0',5X,'PRIMAL SOLUTION:'/
     1        ' ',7X,'VARIABLE',8X,'VALUE'/
     2       (' ',8X,I5,7X,G20.12))
   500 FORMAT('2','SUMMARY FOR OBSERVATION I=',I4,':'//
     1        ' ',5X,'EPSILON(S)=',F10.6/
     2        ' ',5X,'POSITIVE LAMBDA''S: ',I4,9(1X,I4)/
     3       (' ',5X,'                    ',I4,9(1X,I4)))
   505 FORMAT(' ','(',I4,') GOODS IN S:')
   515 FORMAT('0','(',I4,') GOODS NOT IN S:')
   517 FORMAT('0',I4,47X,G17.5)
   510 FORMAT('0','GOODS',3X,'NUMERAIRE',4X,'PRICE VECTOR',5X,
     1            'ALLOC. EFF.',6X,
     2            'VECTOR Z(I)',7X,'DIFFERENCE',4X,
     3            'PROPORTIONAL'/
     4        ' ',1X,'NO.',1X,'(=>EFFICIENCY',7X,'P(I)',
     5           10X,'VECTOR Z*',
     6           27X,'Z*-Z(I)',6X,'DIFFERENCE'/
     7        ' ',5X,' DIR. VECTOR)',68X,
     8                 '(Z*-Z(I))/Z(I)'//)
   520 FORMAT(' ',I4,7X,I2,4X,4G17.5,4X,F8.5)
   525 FORMAT(' ',I4,30X,3G17.5,4X,F8.5)
   530 FORMAT('0',5X,'VALUE OF NUMERAIRE GOODS=',G17.7/
     1 ' ',5X,'PARTIAL PROFIT AT CURRENT PRODUCTION PLAN=',G17.7/
     2 ' ',5X,'PARTIAL PROFIT AT ALLOCATIVELY EFFICIENT ',
     3                 'PRODUCTION PLAN=',G17.7)
   550 FORMAT('-','HENCE, OBSERVATION I=',I4,
     1        ' IS ALLOCATIVELY INEFFICIENT.')
   555 FORMAT('-','HENCE, OBSERVATION I=',I4,
     1        ' IS (CONSTRAINED) ALLOCATIVELY EFFICIENT.')
   600 FORMAT('1','SUMMARY FOR DATAFILE:'//)
   610 FORMAT(' ',5X,'VIOLATIONS ARE AT OBSERVATIONS:')
   612 FORMAT(' ',15X,'I=',I4,',  EPSILON(S)=',A10)
   614 FORMAT(' ',15X,'I=',I4,',  EPSILON(S)=',F10.6)
   615 FORMAT('0',10X,'TOTAL NUMBER OF VIOLATIONS FOR TEST IS ',I4/
     1        ' ',10X,'MAXIMUM EPSILON(S)=',F10.6,' AT OBSERVATION I=',
     2                 I4,'.'//)
   617 FORMAT('0',10X,'TOTAL NUMBER OF VIOLATIONS FOR TEST IS ',I4/
     1        ' ',10X,'MAXIMUM EPSILON(S)= UNBOUNDED AT OBSERVATION I=',
     2                 I4,'.'//)
   620 FORMAT(' ',5X,'OUT OF ',I4,' OBSERVATIONS, ',I4,
     1                 ' PASS AND ',I4,' FAIL THE TEST.')
   630 FORMAT(' ',10X,'THE OBSERVATIONS CONSISTENT WITH THE',
     1                 ' HYPOTHESIS ARE:'/
     2        (' ',15X,'I=',I4))
   640 FORMAT('0',5X,'CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT'/
     1        ' ',5X,'             WITH THE HYPOTHESIS.')
   650 FORMAT('0',5X,'CONCLUSION:  OVERALL, THE DATA IS CONSISTENT'/
     1        ' ',5X,'             WITH THE HYPOTHESIS.')
```

```
  888 FORMAT('0',5X,'WARNING:  THERE IS AT LEAST ONE NEGATIVE'/
     1       ' ',5X,'          VALUE IN THE OPTIMAL BASIS FOR THE'/
     2       ' ',5X,'          LP PROBLEM AT OBSERVATION I=',I4, '.'/
     3       ' ',5X,'          DATA SCALING PROBLEMS MAY EXIST.'/)
  900 FORMAT('2','SUMMARY FOR OBSERVATION I=',I4,':'//
     1       ' ',5X,'PROBLEM ENCOUNTERED IN LP SUBROUTINE.'/
     2       ' ',5X,'THERE IS NO FINITE SOLUTION TO THE LP'/
     3       ' ',5X,'PROBLEM;  HENCE, THE OBJECTIVE FUNCTION IS'/
     4       ' ',5X,'UNBOUNDED.  EPSILON(S) IS UNBOUNDED.')
      END
```

Listing of subroutine TESTPQ

```
      SUBROUTINE TESTPQ(Z,P,PI,JDIR,JGDS,JOBS,EPS,ZDIF,ZPDIF,
     1                  JVIO,VVIO,JPASS,ZN,JEFF,JBEST)
**********************************************************************
C
C          TEST 7.   UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                    PRICE AND QUANTITY DATA
C                    CONVEX TECHNOLOGY
C
C          TEST 8.   UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                    PRICE AND QUANTITY DATA
C                    CONSTANT RETURNS TO SCALE TECHNOLOGY
C
C          TEST 9.   UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                    PRICE AND QUANTITY DATA
C                    QUASICONCAVE TECHNOLOGY
C
C                    JTECH = 0  ASSUMES NO TECHNICAL CHANGE
C                          = 1  ASSUMES NO TECHNOLOGICAL REGRESS
C
C*********************************************************************
      IMPLICIT REAL*8(B-I,K-Z),CHARACTER*12(A)
      DIMENSION Z(JGDS,0:JOBS),P(JGDS,0:JOBS),JDIR(JGDS),
     1          PI(0:JOBS,0:JOBS),
     2          EPS(JOBS),ZDIF(JGDS),ZPDIF(JGDS),
     3          JVIO(JOBS),VVIO(JOBS),JPASS(JOBS),ZN(JGDS,JOBS),
     4          JEFF(JOBS),JBEST(JOBS)
      COMMON /CONST/ CZERO,JTEST,JFNC,JTECH,JNORM,JPOSP,JPMU
      DATA AVMAX/' UNBOUNDED'/
C
C          PRINT TEST HEADING
C
      DO 1 JN = 1,JGDS
          Z(JN,0) = 0.D0
          P(JN,0) = 0.D0
    1 CONTINUE
      IF (JTEST .EQ. 9) THEN
         WRITE(6,420) JFNC
      ELSE IF (JTEST .EQ. 7) THEN
         WRITE(6,400)
      ELSE
         WRITE(6,410) JNORM
      END IF
      IF (JTEST .EQ. 7) THEN
         IF (JPOSP .EQ. 0) THEN
            WRITE(6,430)
         ELSE
            WRITE(6,435)
         END IF
      END IF
```

```
      IF (JTECH .EQ. 0) THEN
         WRITE(6,422)
      ELSE
         WRITE(6,424)
      END IF
  400 FORMAT('1','TEST 7.  UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST'/
     1       ' ','         PRICE AND QUANTITY DATA'/
     2       ' ','         CONVEX TECHNOLOGY'//)
  410 FORMAT('1','TEST 8.  UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST'/
     1       ' ','         PRICE AND QUANTITY DATA'/
     2       ' ','         CONVEX CONE TECHNOLOGY'/
     3       ' ','         (CONSTANT RETURNS TO SCALE)'/
     4       '0',' NOTE:  TEST PERFORMED WITH NORMALIZING'/
     5       ' ','         GOOD N=',I4//)
  420 FORMAT('1','TEST 9.  UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST'/
     1       ' ','         PRICE AND QUANTITY DATA'/
     2       ' ','         QUASICONCAVE TECHNOLOGY'/
     3       '0',' NOTE:  TEST PERFORMED WITH RESPECT TO'/
     4       ' ','         TECHNOLOGY OF GOOD N=',I4//)
  422 FORMAT(' ',9X,'( TEST ASSUMES NO TECHNICAL CHANGE )'//)
  424 FORMAT(' ',9X,'( TEST ASSUMES NO TECHNOLOGICAL REGRESS )'//)
  430 FORMAT(' ',9X,'( OPTIMAL PROFITS UNRESTRICTED IN SIGN )'//)
  435 FORMAT(' ',9X,'( OPTIMAL PROFITS RESTRICTED ',
     1               'TO BE NONNEGATIVE )'//)
      DO 30 JN = 1,JGDS
         ZN(JN,0) = 0.D0
         DO 32 JJ = 1,JOBS
            IF (JTEST .EQ. 8) THEN
               ZN(JN,JJ) = Z(JN,JJ)/(ABS(Z(JNORM,JJ)))
            ELSE
               ZN(JN,JJ) = Z(JN,JJ)
            END IF
   32    CONTINUE
   30 CONTINUE
C
C         CALCULATE PI(I,J), PROFIT AT PRICES I, QUANTITIES J
C
CC*      WRITE(6,'(''1'')')
      DO 10 JI=1,JOBS
         DO 12 JJ=0,JOBS
            PI(JI,JJ) = 0.D0
            DO 14 JN=1,JGDS
               IF ((JTEST .EQ. 8) .AND. (JN .EQ. JNORM)) GO TO 14
               IF ((JTEST .EQ. 9) .AND. (JN .EQ. JFNC)) GO TO 14
               PI(JI,JJ) = PI(JI,JJ) + (P(JN,JI)*ZN(JN,JJ))
   14       CONTINUE
CC*     PRINT *,'I(PRICES)=',JI,'J(QUANT)=',JJ,'PI(I,J)=',PI(JI,JJ)
   12    CONTINUE
   10 CONTINUE
C
C         CHECK INEQUALITY FOR JI=1,JOBS
C
      DO 5 JI=1,JOBS
C
C         INITIALIZE VALUES FOR OBSERVATION JI
C
       EPS(JI) = 0.D0
       JMAX = JI
       PIMAX = PI(JI,JI)
C
C         CHECK INEQUALITIES
C
      JL = 1
```

```
      IF ((JTEST .EQ. 7) .AND. (JPOSP .EQ. 1)) JL=0
      IF ((JTEST .EQ. 8) .AND. (Z(JNORM,JI) .LT. 0.D0)) JL=0
      JU = JOBS
      IF (JTECH .EQ. 1) JU = JI
      DO 20 JJ= JL,JU
         IF (JTEST .EQ. 9) THEN
            IF (Z(JFNC,JJ) .LT. Z(JFNC,JI)) GO TO 20
         END IF
         IF (PI(JI,JJ) .GT. PIMAX) THEN
            JMAX = JJ
            PIMAX = PI(JI,JJ)
         END IF
   20 CONTINUE
C
C           CALCULATE VALUE OF NUMERAIRE GOODS
C
      VNUM = 0.D0
      DO 22 JN= 1,JGDS
         VNUM = VNUM + (P(JN,JI)*JDIR(JN)*ZN(JN,JI))
   22 CONTINUE
C
C           CHECK FOR ZERO VALUES OF VNUM
C
      IF (ABS(VNUM) .LE. CZERO) THEN
         EPS(JI) = 999.999999
         JEFF(JI) = 9999
         WRITE(6,700) JI
         GO TO 5
      END IF
  700 FORMAT('2','WARNING:  FOR OBSERVATION I=',I4,', THE VALUE'/
     1       ' ','           OF NUMERAIRE GOODS IS ZERO;  HENCE,'/
     2       ' ','           EPSILON* IS UNDEFINED.')
C
C           CALCULATE LOSS MEASURE EPS(JI)
C

      JEFF(JI) = JMAX
      EPS(JI) = (PI(JI,JMAX)-PI(JI,JI))/VNUM
      DO 24 JN=1,JGDS
         ZDIF(JN) = Z(JN,JMAX) - Z(JN,JI)
         ZPDIF(JN) = ZDIF(JN)/Z(JN,JI)
   24 CONTINUE
      JUNB = 0
      IF ((JTEST .EQ. 8) .AND. (Z(JNORM,JI) .GT. 0.D0)
     1    .AND. (PI(JI,JMAX) .GT. CZERO)) THEN
              JUNB = 1
              EPS(JI) = 999.999999D0
      END IF
C
C           PRINT SUMMARY FOR OBSERVATION I
C
   50 IF ((JTEST .EQ. 8) .AND. (JUNB .EQ. 1)) THEN
         WRITE(6,501) JI,AVMAX,JMAX
      ELSE
         WRITE(6,500) JI,EPS(JI),JMAX
      END IF
      WRITE(6,510) JMAX
      IF (JTEST .EQ. 8) THEN
         DO 60 J = 1,JGDS
            IF (J .EQ. JNORM) GO TO 60
            WRITE(6,520) J,JDIR(J),P(J,JI),Z(J,JMAX),Z(J,JI),
     1                   ZDIF(J),ZPDIF(J)
   60    CONTINUE
         WRITE(6,522) JNORM,Z(JNORM,JMAX),Z(JNORM,JI),
     1                   ZDIF(JNORM),ZPDIF(JNORM)
```

```
         WRITE(6,532) VNUM,PI(JI,JI),PI(JI,JMAX)
      ELSE IF (JTEST .EQ. 9) THEN
         DO 65 J = 1,JGDS
            IF (J .EQ. JFNC) GO TO 65
            WRITE(6,520) J,JDIR(J),P(J,JI),Z(J,JMAX),Z(J,JI),
     1                   ZDIF(J),ZPDIF(J)
 65      CONTINUE
         WRITE(6,522) JFNC,Z(JFNC,JMAX),Z(JFNC,JI),
     1                ZDIF(JFNC),ZPDIF(JFNC)
         WRITE(6,534) VNUM,PI(JI,JI),PI(JI,JMAX)
      ELSE
         WRITE(6,520) (J,JDIR(J),P(J,JI),Z(J,JMAX),Z(J,JI),
     1                ZDIF(J),ZPDIF(J),J=1,JGDS)
         WRITE(6,530) VNUM,PI(JI,JI),PI(JI,JMAX)
      END IF
      IF (EPS(JI) .GT. CZERO) THEN
         WRITE(6,550) JI
         IF ((JTEST .EQ. 8) .AND. (JUNB .EQ. 1)) WRITE(6,551)
      ELSE
         WRITE(6,555) JI
      END IF
    5 CONTINUE
C
C           PRINT SUMMARY FOR DATAFILE
C
      WRITE(6,600)
      JNF = 0
      JNP = 0
      JMAX = 0
      VMAX = 0.D0
      DO 80 JJ = 1,JOBS
         IF (EPS(JJ) .GT. CZERO) THEN
            JNF = JNF+1
            JVIO(JNF) = JJ
            VVIO(JNF) = EPS(JJ)
            JBEST(JNF) = JEFF(JJ)
            IF (EPS(JJ) .GT. VMAX) THEN
               JMAX = JJ
               VMAX = EPS(JJ)
            END IF
         ELSE
            JNP = JNP+1
            JPASS(JNP) = JJ
         END IF
 80   CONTINUE
      IF (JNF .GT. 0) THEN
         WRITE(6,609)
         JBOU = 0
         DO 81 J = 1,JNF
            IF ((JTEST .EQ. 8) .AND. (VVIO(J) .GT. 999.D0)) THEN
              JBOU = 1
              WRITE(6,611) JVIO(J),AVMAX
            ELSE
              WRITE(6,610) JVIO(J),VVIO(J),JBEST(J)
            END IF
 81      CONTINUE
         IF (JBOU .EQ. 1) THEN
            WRITE(6,616) JNF,AVMAX,JMAX
         ELSE
            WRITE(6,615) JNF,VMAX,JMAX
         END IF
      END IF
      WRITE(6,620) JOBS,JNP,JNF
      IF ((JNP .GT. 0) .AND. (JNP .LT. JOBS))
     1    WRITE(6,630) (JPASS(J),J=1,JNP)
      IF (JNF .GT. 0) THEN
```

```
         WRITE(6,640)
      ELSE
         WRITE(6,650)
      END IF
      RETURN
  500 FORMAT('2','SUMMARY FOR OBSERVATION I=',I4,':'//
     1       ' ',5X,'EPSILON*=',F10.6/
     2       ' ',5X,'OBSERVATION RELATIVELY EFFICIENT TO I:  J=',I4)
  501 FORMAT('2','SUMMARY FOR OBSERVATION I=',I4,':'//
     1       ' ',5X,'EPSILON*=',A10/
     2       ' ',5X,'OBSERVATION RELATIVELY EFFICIENT TO I:  J=',I4)
  510 FORMAT('0','GOODS',2X,'EFFICIENCY',4X,'PRICE VECTOR',5X,
     1          'ALLOC. EFF.',6X,
     2          'VECTOR Z(I)',7X,'DIFFERENCE',4X,
     3          'PROPORTIONAL'/
     4       ' ',1X,'NO.',2X,'DIR. VECTOR',8X,'P(I)',
     5         10X,'VECTOR Z*',
     6         27X,'Z*-Z(I)',6X,'DIFFERENCE'/
     7       ' ',5X,'(=>NUMERAIRE)',19X,'(Z(J),J=',I4,')',
     8          36X,'(Z*-Z(I))/Z(I)'//)
  520 FORMAT(' ',I4,7X,I2,4X,4G17.5,4X,F8.5)
  522 FORMAT('0',I4,30X,3G17.5,4X,F8.5)
  530 FORMAT('0',5X,'VALUE OF NUMERAIRE GOODS=',G17.7/
     1       ' ',5X,'PROFIT AT CURRENT PRODUCTION PLAN=',G17.7/
     2       ' ',5X,'PROFIT AT ALLOCATIVELY EFFICIENT ',
     3              'PRODUCTION PLAN=',G17.7)
  532 FORMAT('0',5X,'VALUE OF (NORMALIZED) NUMERAIRE GOODS=',G17.7/
     1       ' ',5X,'NORMALIZED PROFIT AT ',
     2              'CURRENT PRODUCTION PLAN=',G17.7/
     3       ' ',5X,'NORMALIZED PROFIT AT ALLOCATIVELY EFFICIENT ',
     4              'PRODUCTION PLAN=',G17.7)
  534 FORMAT('0',5X,'VALUE OF NUMERAIRE GOODS=',G17.7/
     1       ' ',5X,'RESTRICTED PROFIT AT ',
     2              'CURRENT PRODUCTION PLAN=',G17.7/
     3       ' ',5X,'RESTRICTED PROFIT AT ALLOCATIVELY EFFICIENT ',
     4              'PRODUCTION PLAN=',G17.7)
  550 FORMAT('-','HENCE, OBSERVATION I=',I4,
     1       ' IS ALLOCATIVELY INEFFICIENT.')
  551 FORMAT(' ','        MAXIMAL PROFITS IS UNBOUNDED.')
  555 FORMAT('-','HENCE, OBSERVATION I=',I4,
     1       ' IS ALLOCATIVELY EFFICIENT.')
  600 FORMAT('1','SUMMARY FOR DATAFILE:'//)
  609 FORMAT(' ',5X,'VIOLATIONS ARE AT OBSERVATIONS:')
  610 FORMAT(' ',15X,'I=',I4,',  EPSILON*=',F10.6,
     1             ',  J=',I4,' (ALLOCATION RELATIVELY EFFICIENT TO I)')
  611 FORMAT(' ',15X,'I=',I4,',  EPSILON*=',A10)
  615 FORMAT('0',10X,'TOTAL NUMBER OF VIOLATIONS FOR TEST IS ',I4/
     1       ' ',10X,'MAXIMUM EPSILON*=',F10.6,' AT OBSERVATION I=',
     2             I4,'.'//)
  616 FORMAT('0',10X,'TOTAL NUMBER OF VIOLATIONS FOR TEST IS ',I4/
     1       ' ',10X,'MAXIMUM EPSILON*=',A10,' AT OBSERVATION I=',
     2             I4,'.'//)
  620 FORMAT(' ',5X,'OUT OF ',I4,' OBSERVATIONS, ',I4,
     1              ' PASS AND ',I4,' FAIL THE TEST.')
  630 FORMAT(' ',10X,'THE OBSERVATIONS CONSISTENT WITH THE',
     1             ' HYPOTHESIS ARE:'/
     2       (' ',15X,'I=',I4))
  640 FORMAT('0',5X,'CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT'/
     1       ' ',5X,'             WITH THE HYPOTHESIS.')
  650 FORMAT('0',5X,'CONCLUSION:  OVERALL, THE DATA IS CONSISTENT'/
```

```
     1        ' ',5X,'                WITH THE HYPOTHESIS.')
      END
```

C.2.2 Sample main calling programs

Listing of MAINQ1

```
C*********************************************************************
C
C                MAIN PROGRAM   SCARDS=MAINQ1
C
C*********************************************************************
      IMPLICIT REAL*8(B-I,K-Z),CHARACTER*12(A)
C
C          NEXT LINE IS USER-SPECIFIED
C
      PARAMETER (JOBS=20,JGDS=10,JOBS2=22,JGDS2=12,JGOB2=32)
C
C
C
      DIMENSION Z(JGDS,JOBS),P(JGDS,JOBS),JDIR(JGDS),
     1          TABLO(JGDS2,JOBS2),JVIN(JGDS2),JVOUT(JOBS2),OPTIM(JOBS),
     2          X(JGOB2),JPLAM(JOBS),ZP(JGDS),ZDIF(JGDS),ZPDIF(JGDS),
     3          JVIO(JOBS),VVIO(JOBS),JPASS(JOBS),JGEZ(JOBS),
     4          ZW(JGDS),JFRD(JOBS),JPASSW(JOBS),JPASSS(JOBS)
      DIMENSION AFLIN(6)
      COMMON /CONST/ CZERO,JDET,JTEST,JFNC,JNORM,JTECH
C
C          SET VIOLATION INDICES .LT. CZERO TO ZERO
C
      CZERO = 1.0D-8
C
C          PRINT SHORT/DETAILED VERSION OF RESULTS
C                 (USER-SPECIFIED)
C                     JDET = 0  SHORT
C                          = 1  LONG (INCLUDES LP TABLEAUS)
C
      JDET = 0
C
C          READ IN DATA (USER-SPECIFIED)
C
      READ(11,5) AFLIN
    5 FORMAT(6A12)
      PRINT *,AFLIN
      DO 10 JY = 1,JOBS
         READ(11) (Z(JN,JY),JN=1,JGDS),(P(JN,JY),JN=1,JGDS)
   10 CONTINUE
C
C         ZERO EFFICIENCY DIRECTION VECTOR JDIR(JN), JN=1,JGDS
C
      DO 20 JN=1,JGDS
         JDIR(JN)=0
   20 CONTINUE
C
C          SPECIFY NONZERO COMPONENTS OF EFFICIENCY DIRECTION JDIR
C             (USER-SPECIFIED)
C
CC    JDIR(1) =  1
CC    JDIR(2) = -1
CC    JDIR(3) = -1
CC    JDIR(4) = -1
      JDIR(5) = -1
      JDIR(6) = -1
```

```
      JDIR(7) = -1
      JDIR(8) = -1
      JDIR(9) = -1
      JDIR(10)= -1
C
C          SPECIFY WRT WHICH GOOD N TECHNOLOGY
C          USER WOULD LIKE TO PERFORM TEST
C             ( JFNC IS USER-SPECIFIED)
C
      JFNC = 1
C
C          TOL = TOLERANCE LEVEL,  NUMBERS SMALLER THAN TOL IN
C                ABSOLUTE VALUE ARE CONSIDERED ZERO IN LP
C                OPTIMIZATION
C
      TOL = 0.DO
C
C
C
C          TEST 1.   TECHNICAL EFFICIENCY TEST
C                    QUANTITY DATA
C                    CONVEX TECHNOLOGY
C                    ( TEST ASSUMES NO TECHNICAL CHANGE,
C                      JTECH = 0 )
C
      JTECH = 0
      JTEST = 1
      CALL TESTQ(Z,JGDS,JGDS2,JOBS,JOBS2,JGOB2,JDIR,
     1            TABLO,JVIN,JVOUT,OPTIM,TOL,
     2            X,JPLAM,ZP,ZDIF,ZPDIF,JVIO,VVIO,JPASS,JGEZ,ZW,
     3            JFRD,JPASSW,JPASSS)
C
C
C
C          TEST 1.   TECHNICAL EFFICIENCY TEST
C                    QUANTITY DATA
C                    CONVEX TECHNOLOGY
C                    ( TEST ASSUMES NO TECHNOLOGICAL REGRESS,
C                      JTECH = 1 )
C
      JTECH = 1
      JTEST = 1
      CALL TESTQ(Z,JGDS,JGDS2,JOBS,JOBS2,JGOB2,JDIR,
     1            TABLO,JVIN,JVOUT,OPTIM,TOL,
     2            X,JPLAM,ZP,ZDIF,ZPDIF,JVIO,VVIO,JPASS,JGEZ,ZW,
     3            JFRD,JPASSW,JPASSS)
C
C
C
C          TEST 2.   TECHNICAL EFFICIENCY TEST
C                    QUANTITY DATA
C                    CONSTANT RETURNS TO SCALE TECHNOLOGY
C                    ( TEST ASSUMES NO TECHNICAL CHANGE,
C                      JTECH = 0 )
C
      JTECH = 0
      JTEST = 2
      CALL TESTQ(Z,JGDS,JGDS2,JOBS,JOBS2,JGOB2,JDIR,
     1            TABLO,JVIN,JVOUT,OPTIM,TOL,
     2            X,JPLAM,ZP,ZDIF,ZPDIF,JVIO,VVIO,JPASS,JGEZ,ZW,
     3            JFRD,JPASSW,JPASSS)
C
C
C
C          TEST 2.   TECHNICAL EFFICIENCY TEST
C                    QUANTITY DATA
C                    CONSTANT RETURNS TO SCALE TECHNOLOGY
C                    ( TEST ASSUMES NO TECHNOLOGICAL REGRESS,
```

```
C                       JTECH = 1 )
C
      JTECH = 1
      JTEST = 2
      CALL TESTQ(Z,JGDS,JGDS2,JOBS,JOBS2,JGOB2,JDIR,
     1           TABLO,JVIN,JVOUT,OPTIM,TOL,
     2           X,JPLAM,ZP,ZDIF,ZPDIF,JVIO,VVIO,JPASS,JGEZ,ZW,
     3           JFRD,JPASSW,JPASSS)
C
C
C
C         TEST 3.    TECHNICAL EFFICIENCY TEST
C                    QUANTITY DATA
C                    QUASICONCAVE TECHNOLOGY
C                    ( TEST ASSUMES NO TECHNICAL CHANGE,
C                      JTECH = 0 )
C
C         SPECIFY WRT WHICH GOOD N TECHNOLOGY
C         USER WOULD LIKE TO PERFORM TEST
C            ( JFNC IS USER-SPECIFIED)
C
      JTECH = 0
      JTEST = 3
      CALL TESTQ(Z,JGDS,JGDS2,JOBS,JOBS2,JGOB2,JDIR,
     1           TABLO,JVIN,JVOUT,OPTIM,TOL,
     2           X,JPLAM,ZP,ZDIF,ZPDIF,JVIO,VVIO,JPASS,JGEZ,ZW,
     3           JFRD,JPASSW,JPASSS)
C
C
C
C         TEST 3.    TECHNICAL EFFICIENCY TEST
C                    QUANTITY DATA
C                    QUASICONCAVE TECHNOLOGY
C                    ( TEST ASSUMES NO TECHNOLOGICAL REGRESS,
C                      JTECH = 1 )
C
C         SPECIFY WRT WHICH GOOD N TECHNOLOGY
C         USER WOULD LIKE TO PERFORM TEST
C            ( JFNC IS USER-SPECIFIED)
C
      JTECH = 1
      JTEST = 3
      CALL TESTQ(Z,JGDS,JGDS2,JOBS,JOBS2,JGOB2,JDIR,
     1           TABLO,JVIN,JVOUT,OPTIM,TOL,
     2           X,JPLAM,ZP,ZDIF,ZPDIF,JVIO,VVIO,JPASS,JGEZ,ZW,
     3           JFRD,JPASSW,JPASSS)
C
C
C
      STOP
      END
```

Listing of MAINPQC1

```
C*********************************************************************
C
C               MAIN PROGRAM   SCARDS=MAINPQC1
C
C*********************************************************************
      IMPLICIT REAL*8(B-I,K-Z),CHARACTER*12(A)
C
C         NEXT LINE IS USER-SPECIFIED
C
      PARAMETER (JOBS=20,JGDS=10,JOBS2=22,JGDS2=12,JGOB2=32)
C
```

```
C
C
      DIMENSION Z(JGDS,0:JOBS),P(JGDS,0:JOBS),JDIR(JGDS),
     1          PI(0:JOBS,0:JOBS),
     2          TABLO(JGDS2,JOBS2),JVIN(JGDS2),JVOUT(JOBS2),OPTIM(JOBS),
     3          X(0:JGOB2),JPLAM(JOBS),ZP(JGDS),ZDIF(JGDS),ZPDIF(JGDS),
     4          JVIO(JOBS),VVIO(JOBS),JPASS(JOBS),JGEZ(JOBS),
     5          ZW(JGDS),JFRD(JOBS),JPASSW(JOBS),JPASSS(JOBS),
     6          EPS(JOBS),ZN(JGDS,JOBS),JEFF(JOBS),JBEST(JOBS),
     7          JSET(JGDS),JNUM(JGDS),JINS(JGDS),JNOTS(JGDS)
      DIMENSION AFLIN(6)
      COMMON /CONST/ CZERO,JDET,JTEST,JFNC,JNORM,JTECH,JPOSP,JPMU
C
C          SET VIOLATION INDICES .LT. CZERO TO ZERO
C
      CZERO = 1.0D-8
C
C          PRINT SHORT/DETAILED VERSION OF RESULTS
C                (USER-SPECIFIED)
C                    JDET = 0  SHORT
C                         = 1  LONG (INCLUDES LP TABLEAUS)
C
      JDET = 0
C
C          READ IN DATA (USER-SPECIFIED)
C
      READ(11,5) AFLIN
    5 FORMAT(6A12)
      PRINT *,AFLIN
      DO 10 JY = 1,JOBS
         READ(11) (Z(JN,JY),JN=1,JGDS),(P(JN,JY),JN=1,JGDS)
   10 CONTINUE
C
C         ZERO EFFICIENCY DIRECTION VECTOR JDIR(JN), JN=1,JGDS
C              ELEMENTS OF SET S JSET(JN), JN=1,JGDS
C              INDICES OF NUMERAIRE GOODS JNUM(JN), JN=1,JGDS
C
      DO 20 JN=1,JGDS
         JDIR(JN)=0
         JSET(JN)=0
         JNUM(JN)=0
   20 CONTINUE
C
C          SPECIFY ELEMENTS OF SET S (CONTAINS GOODS WRT TO
C             WHICH PRODUCER CAN OPTIMIZE)
C             (USER-SPECIFIED)
C
CC    JSET(1) = 1
CC    JSET(2) = 1
CC    JSET(3) = 1
CC    JSET(4) = 1
      JSET(5) = 1
      JSET(6) = 1
      JSET(7) = 1
      JSET(8) = 1
      JSET(9) = 1
      JSET(10)= 1
C
C          SPECIFY NUMERAIRE GOODS
C             (USER-SPECIFIED)
C
CC    JNUM(1) = 1
CC    JNUM(2) = 1
CC    JNUM(3) = 1
CC    JNUM(4) = 1
```

```
      JNUM(5) = 1
      JNUM(6) = 1
      JNUM(7) = 1
      JNUM(8) = 1
      JNUM(9) = 1
      JNUM(10)= 1
C
      JFNC = 1
C
      JPOSP = 0
      JPMU = 0
C
C          OPTION FOR TESTS 4&7 (CONVEX TECHNOLOGY):
C            IF OPTIMAL PROFITS ARE RESTRICTED TO BE
C            NONNEGATIVE, EITHER
C                1)  SET JPOSP = 1 (INCLUDE THE Z(0)=0 VECTOR)
C            OR  2)  SET JPMU = 1 (SUM OF LAMBDA'S .LE. 1)
C            (USER-SPECIFIED, CHOOSE ONLY ONE OPTION)
C
CCC   JPOSP = 1
C                 STRICTLY OR
CCC   JPMU = 1
      IF (JPMU .EQ. 1) JPOSP = 0
C
C          IF TESTS FOR QUASICONCAVITY ARE TO BE PERFORMED,
C          SPECIFY A GOOD N, JFNC=N
C              (USER-SPECIFIED)
C
CCC   JFNC = OUTPUT
C
C          TOL = TOLERANCE LEVEL,  NUMBERS SMALLER THAN TOL IN
C                ABSOLUTE VALUE ARE CONSIDERED ZERO IN LP
C                OPTIMIZATION
C
      TOL = 0.D0
C
C
C
C          TEST 4.    CONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                     PRICE AND QUANTITY DATA
C                     CONVEX TECHNOLOGY
C                     ( TEST ASSUMES NO TECHNICAL CHANGE,
C                       JTECH = 0 )
C
      JTECH = 0
      JTEST = 4
      CALL TSTPQC(Z,P,PI,JGDS,JGDS2,JOBS,JOBS2,JGOB2,
     1            TABLO,JVIN,JVOUT,OPTIM,TOL,
     2            EPS,ZP,ZDIF,ZPDIF,X,JPLAM,
     3            JVIO,VVIO,JPASS,JGEZ,JSET,JNUM,JINS,JNOTS)
C
C
C
C          TEST 4.    CONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                     PRICE AND QUANTITY DATA
C                     CONVEX TECHNOLOGY
C                     ( TEST ASSUMES NO TECHNOLOGICAL REGRESS,
C                       JTECH = 1 )
C
      JTECH = 1
      JTEST = 4
      CALL TSTPQC(Z,P,PI,JGDS,JGDS2,JOBS,JOBS2,JGOB2,
     1            TABLO,JVIN,JVOUT,OPTIM,TOL,
     2            EPS,ZP,ZDIF,ZPDIF,X,JPLAM,
     3            JVIO,VVIO,JPASS,JGEZ,JSET,JNUM,JINS,JNOTS)
C
C
```

```
C
C           TEST 5.    CONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                      PRICE AND QUANTITY DATA
C                      CONSTANT RETURNS TO SCALE TECHNOLOGY
C                      ( TEST ASSUMES NO TECHNICAL CHANGE,
C                        JTECH = 0 )
C
      JTECH = 0
      JTEST = 5
      CALL TSTPQC(Z,P,PI,JGDS,JGDS2,JOBS,JOBS2,JGOB2,
     1            TABLO,JVIN,JVOUT,OPTIM,TOL,
     2            EPS,ZP,ZDIF,ZPDIF,X,JPLAM,
     3            JVIO,VVIO,JPASS,JGEZ,JSET,JNUM,JINS,JNOTS)
C
C
C
C           TEST 5.    CONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                      PRICE AND QUANTITY DATA
C                      CONSTANT RETURNS TO SCALE TECHNOLOGY
C                      ( TEST ASSUMES NO TECHNOLOGICAL REGRESS,
C                        JTECH = 1 )
C
      JTECH = 1
      JTEST = 5
      CALL TSTPQC(Z,P,PI,JGDS,JGDS2,JOBS,JOBS2,JGOB2,
     1            TABLO,JVIN,JVOUT,OPTIM,TOL,
     2            EPS,ZP,ZDIF,ZPDIF,X,JPLAM,
     3            JVIO,VVIO,JPASS,JGEZ,JSET,JNUM,JINS,JNOTS)
C
C
C
C           TEST 6.    CONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                      PRICE AND QUANTITY DATA
C                      QUASICONCAVE TECHNOLOGY
C                      ( TEST ASSUMES NO TECHNICAL CHANGE,
C                        JTECH = 0 )
C
C           SPECIFY WRT WHICH GOOD N TECHNOLOGY
C           USER WOULD LIKE TO PERFORM TEST
C              ( JFNC IS USER-SPECIFIED)
C
CCC   JFNC = OUTPUT
      JTECH = 0
      JTEST = 6
      CALL TSTPQC(Z,P,PI,JGDS,JGDS2,JOBS,JOBS2,JGOB2,
     1            TABLO,JVIN,JVOUT,OPTIM,TOL,
     2            EPS,ZP,ZDIF,ZPDIF,X,JPLAM,
     3            JVIO,VVIO,JPASS,JGEZ,JSET,JNUM,JINS,JNOTS)
C
C
C
C           TEST 6.    CONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                      PRICE AND QUANTITY DATA
C                      QUASICONCAVE TECHNOLOGY
C                      ( TEST ASSUMES NO TECHNOLOGICAL REGRESS,
C                        JTECH = 1 )
C
C           SPECIFY WRT WHICH GOOD N TECHNOLOGY
C           USER WOULD LIKE TO PERFORM TEST
C              ( JFNC IS USER-SPECIFIED)
C
CCC   JFNC = OUTPUT
      JTECH = 1
      JTEST = 6
      CALL TSTPQC(Z,P,PI,JGDS,JGDS2,JOBS,JOBS2,JGOB2,
     1            TABLO,JVIN,JVOUT,OPTIM,TOL,
     2            EPS,ZP,ZDIF,ZPDIF,X,JPLAM,
```

```
     3            JVIO,VVIO,JPASS,JGEZ,JSET,JNUM,JINS,JNOTS)
C
C
C
  999 STOP
      END
```

Listing of MAINPQ1

```
C*********************************************************************
C
C                 MAIN PROGRAM   SCARDS=MAINPQ1
C
C*********************************************************************
      IMPLICIT REAL*8(B-I,K-Z),CHARACTER*12(A)
C
C          NEXT LINE IS USER-SPECIFIED
C
      PARAMETER (JOBS=20,JGDS=10,JOBS2=22,JGDS2=12,JGOB=30)
C
C
C
      DIMENSION Z(JGDS,0:JOBS),P(JGDS,0:JOBS),JDIR(JGDS),
     1          PI(0:JOBS,0:JOBS),
     2          TABLO(JGDS2,JOBS2),JVIN(JGDS2),JVOUT(JOBS2),OPTIM(JOBS),
     3          X(JGOB),JPLAM(JOBS),ZP(JGDS),ZDIF(JGDS),ZPDIF(JGDS),
     4          JVIO(JOBS),VVIO(JOBS),JPASS(JOBS),JGEZ(JOBS),
     5          ZW(JGDS),JFRD(JOBS),JPASSW(JOBS),JPASSS(JOBS),
     6          EPS(JOBS),ZN(JGDS,JOBS),JEFF(JOBS),JBEST(JOBS)
      DIMENSION AFLIN(6)
      COMMON /CONST/ CZERO,JTEST,JFNC,JTECH,JNORM,JPOSP,JPMU
C
C          SET VIOLATION INDICES .LT. CZERO TO ZERO
C
      CZERO = 1.0D-8
C
C          READ IN DATA (USER-SPECIFIED)
C
      READ(11,5) AFLIN
    5 FORMAT(6A12)
      PRINT *,AFLIN
      DO 10 JY = 1,JOBS
         READ(11) (Z(JN,JY),JN=1,JGDS),(P(JN,JY),JN=1,JGDS)
   10 CONTINUE
C
C         ZERO EFFICIENCY DIRECTION VECTOR JDIR(JN), JN=1,JGDS
C
      DO 20 JN=1,JGDS
         JDIR(JN)=0
   20 CONTINUE
C
C          SPECIFY NONZERO COMPONENTS OF EFFICIENCY DIRECTION JDIR
C             (USER-SPECIFIED)
C
CC    JDIR(1) =  1
CC    JDIR(2) = -1
CC    JDIR(3) = -1
CC    JDIR(4) = -1
      JDIR(5) = -1
      JDIR(6) = -1
      JDIR(7) = -1
      JDIR(8) = -1
      JDIR(9) = -1
```

```
      JDIR(10)= -1
C
C          JNORM = OUTPUT FOR TEST 8
C          JFNC = OUTPUT FOR TEST 9
C
      JNORM = 1
      JFNC  = 1
C
      JPOSP = 0
C
C          OPTION FOR TESTS 4&7 (CONVEX TECHNOLOGY):
C            IF OPTIMAL PROFITS ARE RESTRICTED TO BE
C            NONNEGATIVE (INCLUDE THE Z(0)=0 VECTOR),
C            SET JPOSP = 1 (USER-SPECIFIED)
CCC   JPOSP = 1
C          TOL = TOLERANCE LEVEL,  NUMBERS SMALLER THAN TOL IN
C                ABSOLUTE VALUE ARE CONSIDERED ZERO IN LP
C                OPTIMIZATION
C
      TOL = 0.D0
C
C
C
C          TEST 7.    UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                     PRICE AND QUANTITY DATA
C                     CONVEX TECHNOLOGY
C                     ( TEST ASSUMES NO TECHNICAL CHANGE,
C                       JTECH = 0 )
C
      JTECH = 0
      JTEST = 7
      CALL TESTPQ(Z,P,PI,JDIR,JGDS,JOBS,EPS,ZDIF,ZPDIF,
     1            JVIO,VVIO,JPASS,ZN,JEFF,JBEST)
C
C
C
C          TEST 7.    UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                     PRICE AND QUANTITY DATA
C                     CONVEX TECHNOLOGY
C                     ( TEST ASSUMES NO TECHNOLOGICAL REGRESS,
C                       JTECH = 1 )
C
      JTECH = 1
      JTEST = 7
      CALL TESTPQ(Z,P,PI,JDIR,JGDS,JOBS,EPS,ZDIF,ZPDIF,
     1            JVIO,VVIO,JPASS,ZN,JEFF,JBEST)
C
C
C
C          PRIOR TO ACTUAL TEST 8.
C
C          NORMALIZING GOOD N=JNORM (USER SPECIFIED)
C
CCC   JNORM = OUTPUT
C
C          TO TEST IF |Z(JNORM,.)| > 0
C
      JWARN = 0
      DO 30 J = 1,JOBS
         IF (ABS(Z(JNORM,J)) .LT. CZERO) JWARN=1
   30 CONTINUE
      IF (JWARN .EQ. 1) THEN
         WRITE(6,300)
  300 FORMAT('1','TEST 8.  UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST'/
     1       ' ','         USING PRICE AND QUANTITY DATA'/
     2       ' ','         FOR A CONSTANT RETURNS TO SCALE'/
     3       ' ','         TECHNOLOGY CANNOT BE PERFORMED DUE TO'/
     4       ' ','         ZERO VALUES FOR THE NORMALIZING VARIABLE.')
```

```
          GO TO 999
       END IF
C          TEST 8.    UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                     PRICE AND QUANTITY DATA
C                     CONSTANT RETURNS TO SCALE TECHNOLOGY
C                     ( TEST ASSUMES NO TECHNICAL CHANGE,
C                       JTECH = 0 )
C
       JTECH = 0
       JTEST = 8
       CALL TESTPQ(Z,P,PI,JDIR,JGDS,JOBS,EPS,ZDIF,ZPDIF,
     1             JVIO,VVIO,JPASS,ZN,JEFF,JBEST)
C
C
C
C          TEST 8.    UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                     PRICE AND QUANTITY DATA
C                     CONSTANT RETURNS TO SCALE TECHNOLOGY
C                     ( TEST ASSUMES NO TECHNOLOGICAL REGRESS,
C                       JTECH = 1 )
C
       JTECH = 1
       JTEST = 8
       CALL TESTPQ(Z,P,PI,JDIR,JGDS,JOBS,EPS,ZDIF,ZPDIF,
     1             JVIO,VVIO,JPASS,ZN,JEFF,JBEST)
C
C
C
C          TEST 9.    UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                     PRICE AND QUANTITY DATA
C                     QUASICONCAVE TECHNOLOGY
C                     ( TEST ASSUMES NO TECHNICAL CHANGE,
C                       JTECH = 0 )
C
C          SPECIFY WRT WHICH GOOD N TECHNOLOGY
C          USER WOULD LIKE TO PERFORM TEST
C             ( JFNC IS USER-SPECIFIED)
C
CCC    JFNC = OUTPUT
       JTECH = 0
       JTEST = 9
       CALL TESTPQ(Z,P,PI,JDIR,JGDS,JOBS,EPS,ZDIF,ZPDIF,
     1             JVIO,VVIO,JPASS,ZN,JEFF,JBEST)
C
C
C
C          TEST 9.    UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
C                     PRICE AND QUANTITY DATA
C                     QUASICONCAVE TECHNOLOGY
C                     ( TEST ASSUMES NO TECHNOLOGICAL REGRESS,
C                       JTECH = 1 )
C
C          SPECIFY WRT WHICH GOOD N TECHNOLOGY
C          USER WOULD LIKE TO PERFORM TEST
C             ( JFNC IS USER-SPECIFIED)
C
CCC    JFNC = OUTPUT
       JTECH = 1
       JTEST = 9
       CALL TESTPQ(Z,P,PI,JDIR,JGDS,JOBS,EPS,ZDIF,ZPDIF,
     1             JVIO,VVIO,JPASS,ZN,JEFF,JBEST)
C
C
C
  999 STOP
      END
```

C.3 Sample results using sector I (resources) data

In this section, sample results for the efficiency tests 1–9 discussed in part I and their no technological regress variants listed in appendix A are given. For expository purposes, the results for a given test are interpreted on the assumption that the conditions on the technology hold and violations of the hypothesis are due to inefficiency.[4] Data on the resources sector (sector I) using capital rental prices based on internal rates of return are used. The data have 20 observations ($J = 20$) with each observation having price and quantity data on 10 goods ($N = 10$). For illustration, only the results on the individual observation for 1967 and summary results for the datafile are given for each test. In the following computer printouts, the observation numbers $I = 1, 2, \ldots, 20$ refer to the years $1961, 1962, \ldots, 1980$, respectively. The goods numbers $1, 2, \ldots, 10$ pertain to the corresponding goods as listed in table B.17. Quantity data enter the subroutines with outputs indexed positively and inputs indexed negatively.

The violation indices are input-based measures of inefficiencies. For the technical efficiency tests (1–3 and 1′–3′) and the unconstrained optimization tests (7–9 and 7′–9′), the nonzero components of the efficiency direction vector, γ or γ^n, correspond to the primary input goods. In the constrained optimization tests (4–6 and 4′–6′), the reference goods indexed in the set E are the same primary input goods. Hence, for the technical efficiency tests, the violation indices give the proportion of the primary goods wasted due to failure to produce at some point on the boundary of the production possibilities set. In other words, the same level of output goods in the economy could have been produced with δ_i^* less of the primary inputs utilized. For the allocative efficiency tests, the violation indices ε_i^S or ε_i^* give the proportion of the value of the primary inputs wasted due to productive inefficiency.

For tests 3 (3′), 6 (6′) and 9 (9′) which assume a quasiconcave technology, the efficiency tests are performed with respect to the production function f^1 of good 1, the output good for the resources sector.

[4]This interpretation may be more appropriate for cross-section firm data rather than for sectoral aggregated data. As shown in part II of this dissertation, the violation indices obtained for our sectoral input-output data can be interpreted as measures of technical progress.

C.3.1 Technical efficiency test results

The value of the violation index δ_i^* for observation i is given by 'DELTA*'. The following line lists the observation numbers j with the corresponding optimal primal variable λ^{j*} positive, that is, $\lambda^{j*} > 0$. For the convex and the convex cone technology cases, the technically E-efficient allocations relative to z^i are $z^* \equiv z^i + \delta_i^* \hat{\gamma} z^i$ and $z^{**} \equiv \sum_j \lambda^{j*} z^j$. The vector z^* can be attained from z^i through an equiproportionate adjustment in the goods indexed in the set E (corresponding to the goods with nonzero components in the efficiency direction vector γ). On the other hand, the vector z^{**} may require different proportional adjustments in the goods not necessarily in the set E. For example, in test 1 (assuming no technical change) below, the vectors z^{**} and z^* differ for the 1967 observation. This divergence indicates that with respect to some goods, the production plan z^* is at a free disposal region at the boundary of the constructed convex production possibilities set. For this observation, there is free disposability with respect to goods 2–4 and 8–10 at z^*. Note that good 1 is the output good and the last column indicates that δ_i^* is the minimum proportional decrease in all inputs (intermediate and primary) required to attain z^{**}; in this case, it makes sense to talk of varying degrees of inefficiencies in input use. The computer results for the quasiconcave technology case (tests 3 and 3′) can be analogously interpreted in the context of level sets.

The results show that test 2 (the convex cone case) is most restrictive with more observations violating the hypothesis and larger values for the violation indices. Test 3 (the quasiconcave case) is the weakest. Incorporating the no technological regress assumption into an efficiency test weakens the test. As can be seen in a comparison of the results of tests 2 and 2′, the range of observations with positive λ^{j*} for 1967 is restricted to $j = 1, 2, \ldots, 7$ when the no technological regress assumption is incorporated into the efficiency test. Hence, the violation index obtained in test 2′ cannot be larger than that of test 2.

Assuming no technical change:

```
TEST 1.  TECHNICAL EFFICIENCY TEST
         QUANTITY DATA
         CONVEX TECHNOLOGY

         ( TEST ASSUMES NO TECHNICAL CHANGE )
```

SUMMARY FOR OBSERVATION I= 7:

DELTA*= 0.003502
POSITIVE LAMBDA'S: 3 4 6 8

GOODS NO.	EFFICIENCY DIR. VECTOR	WEAKLY EFF. VECTOR Z*	STRONGLY EFF. VECTOR Z**	VECTOR Z(I)	DIFFERENCE Z**-Z(I)	PROPORTIONAL DIFFERENCE (Z**-Z(I))/Z(I)
1	0	8.4267	8.4267	8.4267	-0.55511E-14	-0.00000
2	0	-0.17956	-0.16196	-0.17956	0.17600E-01	-0.09801
3	0	-0.89823	-0.82260	-0.89823	0.75633E-01	-0.08420
4	0	-1.4437	-1.3574	-1.4437	0.86357E-01	-0.05981
5	-1	-0.63940	-0.63940	-0.64165	0.22473E-02	-0.00350
6	-1	-1.4590	-1.4590	-1.4641	0.51279E-02	-0.00350
7	-1	-0.25567	-0.25567	-0.25656	0.89859E-03	-0.00350
8	-1	-1.1625	-1.0924	-1.1666	0.74150E-01	-0.06356
9	-1	-1.5484	-1.4238	-1.5538	0.13005	-0.08370
10	-1	-0.26064	-0.23189	-0.26156	0.29671E-01	-0.11344

HENCE, OBSERVATION I= 7 IS TECHNICALLY E-INEFFICIENT.

```
SUMMARY FOR DATAFILE:

     VIOLATIONS ARE AT OBSERVATIONS:
               I=    7,  DELTA*=  0.003502

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS     1
          MAXIMUM DELTA*=  0.003502 AT OBSERVATION I=    7.


     OUT OF    20 OBSERVATIONS,    19 PASS AND     1 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               (   19) TECHNICALLY E-EFFICIENT OBSERVATIONS:
                    I=    1
                    I=    2
                    I=    3
                    I=    4
                    I=    5
                    I=    6
                    I=    8
                    I=    9
                    I=   10
                    I=   11
                    I=   12
                    I=   13
                    I=   14
                    I=   15
                    I=   16
                    I=   17
                    I=   18
                    I=   19
                    I=   20

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

TEST 2. TECHNICAL EFFICIENCY TEST
QUANTITY DATA
CONVEX CONE TECHNOLOGY
(CONSTANT RETURNS TO SCALE)

(TEST ASSUMES NO TECHNICAL CHANGE)

SUMMARY FOR OBSERVATION I= 7:

DELTA*= 0.036498
POSITIVE LAMBDA'S: 6 9

GOODS NO.	EFFICIENCY DIR. VECTOR	WEAKLY EFF. VECTOR Z*	STRONGLY EFF. VECTOR Z**	VECTOR Z(I)	DIFFERENCE Z**-Z(I)	PROPORTIONAL DIFFERENCE (Z**-Z(I))/Z(I)
1	0	8.4267	8.4267	8.4267	-0.11102E-14	-0.00000
2	0	-0.17956	-0.16698	-0.17956	0.12588E-01	-0.07010
3	0	-0.89823	-0.83493	-0.89823	0.63303E-01	-0.07048
4	0	-1.4437	-1.3461	-1.4437	0.97632E-01	-0.06762
5	-1	-0.61823	-0.61823	-0.64165	0.23419E-01	-0.03650
6	-1	-1.4107	-1.4107	-1.4641	0.53438E-01	-0.03650
7	-1	-0.24720	-0.23871	-0.25656	0.17855E-01	-0.06959
8	-1	-1.1240	-1.0508	-1.1666	0.11575	-0.09922
9	-1	-1.4971	-1.3905	-1.5538	0.16328	-0.10509
10	-1	-0.25201	-0.22849	-0.26156	0.33074E-01	-0.12645

HENCE, OBSERVATION I= 7 IS TECHNICALLY E-INEFFICIENT.

```
SUMMARY FOR DATAFILE:


     VIOLATIONS ARE AT OBSERVATIONS:
               I=    1,  DELTA*=  0.022340
               I=    7,  DELTA*=  0.036498
               I=    8,  DELTA*=  0.004188
               I=   10,  DELTA*=  0.008617
               I=   16,  DELTA*=  0.012313
               I=   17,  DELTA*=  0.014452
               I=   18,  DELTA*=  0.030742

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS    7
          MAXIMUM DELTA*=  0.036498 AT OBSERVATION I=   7.


     OUT OF   20 OBSERVATIONS,   13 PASS AND    7 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               (   13) TECHNICALLY E-EFFICIENT OBSERVATIONS:
                    I=    2
                    I=    3
                    I=    4
                    I=    5
                    I=    6
                    I=    9
                    I=   11
                    I=   12
                    I=   13
                    I=   14
                    I=   15
                    I=   19
                    I=   20

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

```
TEST 3.   TECHNICAL EFFICIENCY TEST
          QUANTITY DATA
          QUASICONCAVE TECHNOLOGY

  NOTE:   TEST PERFORMED WITH RESPECT TO
          TECHNOLOGY OF GOOD N=    1

          ( TEST ASSUMES NO TECHNICAL CHANGE )


SUMMARY FOR OBSERVATION I=    7:

     DELTA*=  0.000000
     POSITIVE LAMBDA'S:    7
```

GOODS NO.	EFFICIENCY DIR. VECTOR	WEAKLY EFF. VECTOR Z*	STRONGLY EFF. VECTOR Z**	VECTOR Z(I)	DIFFERENCE Z**-Z(I)	PROPORTIONAL DIFFERENCE (Z**-Z(I))/Z(I)
2	0	-0.17956	-0.17956	-0.17956	0.13878E-16	-0.00000
3	0	-0.89823	-0.89823	-0.89823	0.27756E-16	-0.00000
4	0	-1.4437	-1.4437	-1.4437	0.44409E-15	-0.00000
5	-1	-0.64165	-0.64165	-0.64165	0.27756E-16	-0.00000
6	-1	-1.4641	-1.4641	-1.4641	0.44409E-15	-0.00000
7	-1	-0.25656	-0.25656	-0.25656	0.27756E-16	-0.00000
8	-1	-1.1666	-1.1666	-1.1666	0.44409E-15	-0.00000
9	-1	-1.5538	-1.5538	-1.5538	0.44409E-15	-0.00000
10	-1	-0.26156	-0.26156	-0.26156	0.27756E-16	-0.00000
1				8.4267		

```
HENCE, OBSERVATION I=    7 IS TECHNICALLY E-EFFICIENT.


SUMMARY FOR DATAFILE:

     OUT OF   20 OBSERVATIONS,    20 PASS AND    0 FAIL THE TEST.
          ALL OBSERVATIONS ARE TECHNICALLY E-EFFICIENT.

     CONCLUSION:  OVERALL, THE DATA IS CONSISTENT
                  WITH THE HYPOTHESIS.
```

Assuming no technological regress:

```
TEST 1.  TECHNICAL EFFICIENCY TEST
         QUANTITY DATA
         CONVEX TECHNOLOGY

         ( TEST ASSUMES NO TECHNOLOGICAL REGRESS )



SUMMARY FOR OBSERVATION I=   7:

     DELTA*=  0.000000
     POSITIVE LAMBDA'S:    7
```

GOODS NO.	EFFICIENCY DIR. VECTOR	WEAKLY EFF. VECTOR Z*	STRONGLY EFF. VECTOR Z**	VECTOR Z(I)	DIFFERENCE Z**-Z(I)	PROPORTIONAL DIFFERENCE (Z**-Z(I))/Z(I)
1	0	8.4267	8.4267	8.4267	0.26645E-14	0.00000
2	0	-0.17956	-0.17956	-0.17956	-0.27756E-16	0.00000
3	0	-0.89823	-0.89823	-0.89823	-0.24980E-15	0.00000
4	0	-1.4437	-1.4437	-1.4437	-0.22204E-15	0.00000
5	-1	-0.64165	-0.64165	-0.64165	-0.22204E-15	0.00000
6	-1	-1.4641	-1.4641	-1.4641	-0.22204E-15	0.00000
7	-1	-0.25656	-0.25656	-0.25656	-0.97145E-16	0.00000
8	-1	-1.1666	-1.1666	-1.1666	0.00000E+00	0.00000
9	-1	-1.5538	-1.5538	-1.5538	-0.22204E-15	0.00000
10	-1	-0.26156	-0.26156	-0.26156	-0.55511E-16	0.00000

```
HENCE, OBSERVATION I=   7 IS TECHNICALLY E-EFFICIENT.



SUMMARY FOR DATAFILE:

     OUT OF   20 OBSERVATIONS,   20 PASS AND    0 FAIL THE TEST.
          ALL OBSERVATIONS ARE TECHNICALLY E-EFFICIENT.

     CONCLUSION:  OVERALL, THE DATA IS CONSISTENT
                  WITH THE HYPOTHESIS.
```

```
TEST 2.   TECHNICAL EFFICIENCY TEST
          QUANTITY DATA
          CONVEX CONE TECHNOLOGY
          (CONSTANT RETURNS TO SCALE)

          ( TEST ASSUMES NO TECHNOLOGICAL REGRESS )
```

SUMMARY FOR OBSERVATION I=   7:

DELTA*= 0.035683
POSITIVE LAMBDA'S: 6

GOODS NO.	EFFICIENCY DIR. VECTOR	WEAKLY EFF. VECTOR Z*	STRONGLY EFF. VECTOR Z**	VECTOR Z(I)	DIFFERENCE Z**-Z(I)	PROPORTIONAL DIFFERENCE (Z**-Z(I))/Z(I)
1	0	8.4267	8.4267	8.4267	-0.88818E-15	-0.00000
2	0	-0.17956	-0.16700	-0.17956	0.12569E-01	-0.07000
3	0	-0.89823	-0.83512	-0.89823	0.63111E-01	-0.07026
4	0	-1.4437	-1.3459	-1.4437	0.97880E-01	-0.06780
5	-1	-0.61875	-0.61770	-0.64165	0.23947E-01	-0.03732
6	-1	-1.4119	-1.4119	-1.4641	0.52244E-01	-0.03568
7	-1	-0.24741	-0.23872	-0.25656	0.17842E-01	-0.06954
8	-1	-1.1250	-1.0502	-1.1666	0.11641	-0.09979
9	-1	-1.4984	-1.3893	-1.5538	0.16455	-0.10590
10	-1	-0.25223	-0.22822	-0.26156	0.33337E-01	-0.12746

HENCE, OBSERVATION I=   7 IS TECHNICALLY E-INEFFICIENT.

```
SUMMARY FOR DATAFILE:

     VIOLATIONS ARE AT OBSERVATIONS:
               I=    7,  DELTA*=  0.035683
               I=   10,  DELTA*=  0.003389

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS    2
          MAXIMUM DELTA*=  0.035683 AT OBSERVATION I=   7.

     OUT OF   20 OBSERVATIONS,   18 PASS AND    2 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               (   18) TECHNICALLY E-EFFICIENT OBSERVATIONS:
                    I=    1
                    I=    2
                    I=    3
                    I=    4
                    I=    5
                    I=    6
                    I=    8
                    I=    9
                    I=   11
                    I=   12
                    I=   13
                    I=   14
                    I=   15
                    I=   16
                    I=   17
                    I=   18
                    I=   19
                    I=   20

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

```
TEST 3.  TECHNICAL EFFICIENCY TEST
         QUANTITY DATA
         QUASICONCAVE TECHNOLOGY

  NOTE:  TEST PERFORMED WITH RESPECT TO
         TECHNOLOGY OF GOOD N=    1

         ( TEST ASSUMES NO TECHNOLOGICAL REGRESS )


SUMMARY FOR OBSERVATION I=    7:

     DELTA*=  0.000000
     POSITIVE LAMBDA'S:     7
```

GOODS NO.	EFFICIENCY DIR. VECTOR	WEAKLY EFF. VECTOR Z*	STRONGLY EFF. VECTOR Z**	VECTOR Z(I)	DIFFERENCE Z**-Z(I)	PROPORTIONAL DIFFERENCE (Z**-Z(I))/Z(I)
2	0	-0.17956	-0.17956	-0.17956	0.13878E-16	-0.00000
3	0	-0.89823	-0.89823	-0.89823	0.27756E-16	-0.00000
4	0	-1.4437	-1.4437	-1.4437	0.22204E-15	-0.00000
5	-1	-0.64165	-0.64165	-0.64165	0.13878E-16	-0.00000
6	-1	-1.4641	-1.4641	-1.4641	0.22204E-15	-0.00000
7	-1	-0.25656	-0.25656	-0.25656	0.13878E-16	-0.00000
8	-1	-1.1666	-1.1666	-1.1666	0.22204E-15	-0.00000
9	-1	-1.5538	-1.5538	-1.5538	0.22204E-15	-0.00000
10	-1	-0.26156	-0.26156	-0.26156	0.13878E-16	-0.00000
1				8.4267		

```
HENCE, OBSERVATION I=   7 IS TECHNICALLY E-EFFICIENT.


SUMMARY FOR DATAFILE:

     OUT OF   20 OBSERVATIONS,   20 PASS AND    0 FAIL THE TEST.
          ALL OBSERVATIONS ARE TECHNICALLY E-EFFICIENT.

     CONCLUSION:  OVERALL, THE DATA IS CONSISTENT
                  WITH THE HYPOTHESIS.
```

C.3.2 Allocative efficiency test results (assuming partial profit maximization)

The goods indexed in the set S, containing the goods with respect to which the producer can optimize, are all the primary inputs. Therefore, the objective functions in the linear programming problems in the constrained optimization tests are negative cost functions. That is, the efficiency tests reduce to testing whether the resources sector, given the same output levels in the economy (or the same output level of good 1 and the same levels of intermediate input use of goods 2–4 by sector I), is minimizing cost with respect to the primary inputs. To minimize cost, the resources sector should use the optimal mix of the primary inputs (pure allocative efficiency) and produce at the boundary of the production possibilities set (technical efficiency).

The value of the violation index ε_i^S for observation i is given by 'EPSILON(S)'. In the convex technology case, the vector $z^* \equiv \sum_j \lambda^{j*} z^j$ differs from z^{S*} (defined in chapter 7, section 7.1) with respect only to the goods not in S. The other values given at the bottom of the results for the 1967 observation are:

VALUE OF NUMERAIRE GOODS $\equiv \sum_{n\in E} p_n^i |z_n^i|$,

PARTIAL PROFIT AT CURRENT PRODUCTION PLAN $\equiv \sum_{n\in S} p_n^i z_n^i$, and

PARTIAL PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN
$\equiv \sum_{n\in S} p_n^i z_n^{S*} \equiv \sum_{n\in S} p_n^i (\sum_j \lambda^{j*} z_n^j)$.

Hence, the economic loss due to failure to cost minimize with respect to the primary inputs for the resources sector in 1967, assuming the technology assumptions of test 4 hold, is

$$\varepsilon_i^S(\textstyle\sum_{n\in E} p_n^i |z_n^i|) = 0.053762(6.757257) = 0.363284$$

or approximately \$363 million (1967 Canadian dollars). The computer printouts for tests 5 (5′) and 6 (6′) can be similarly interpreted.

Note too that the sets S and E are identical in the following examples of the constrained optimization tests. The obtained violation indices can be decomposed, as suggested in part I and using the results of the earlier technical efficiency tests, into their components due to technical inefficiency and due to "pure" allocative inefficiency with respect to the goods in S (in this case, failure to use the optimal relative quantities of primary inputs). As a consequence also of the LeChatelier principle proposition shown in chapter 8, we can expect as least as many violations in the constrained optimization tests as in the corresponding technical efficiency test; we can also expect the magnitude of the violation index ε_i^S to be at least as large as the corresponding δ_i^* at a given observation.

Assuming no technical change:

TEST 4. CONSTRAINED ALLOCATIVE EFFICIENCY TEST
PRICE AND QUANTITY DATA
CONVEX TECHNOLOGY

(OPTIMAL PROFITS UNRESTRICTED IN SIGN)

(TEST ASSUMES NO TECHNICAL CHANGE)

SUMMARY FOR OBSERVATION I= 7:

EPSILON(S)= 0.053762
POSITIVE LAMBDA'S: 3 6

GOODS NO.	NUMERAIRE (=>EFFICIENCY DIR. VECTOR)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z*	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
(6) GOODS IN S:						
5	1	1.3700	-0.62854	-0.64165	0.13110E-01	-0.02043
6	1	1.4966	-1.4790	-1.4641	-0.14838E-01	0.01013
7	1	1.4306	-0.24974	-0.25656	0.68217E-02	-0.02659
8	1	1.1547	-1.0619	-1.1666	0.10471	-0.08975
9	1	1.0802	-1.3760	-1.5538	0.17775	-0.11440
10	1	1.1260	-0.22171	-0.26156	0.39849E-01	-0.15235
(4) GOODS NOT IN S:						
1			8.4267	8.4267	0.66613E-15	0.00000
2			-0.15854	-0.17956	0.21021E-01	-0.11706
3			-0.81921	-0.89823	0.79024E-01	-0.08798
4			-1.3359	-1.4437	0.10781	-0.07468

VALUE OF NUMERAIRE GOODS= 6.757257
PARTIAL PROFIT AT CURRENT PRODUCTION PLAN= -6.757257
PARTIAL PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN= -6.393974

HENCE, OBSERVATION I= 7 IS ALLOCATIVELY INEFFICIENT.

```
SUMMARY FOR DATAFILE:


     VIOLATIONS ARE AT OBSERVATIONS:
               I=   4,  EPSILON(S)=  0.006623
               I=   5,  EPSILON(S)=  0.006601
               I=   7,  EPSILON(S)=  0.053762
               I=   8,  EPSILON(S)=  0.039736
               I=  10,  EPSILON(S)=  0.030358
               I=  14,  EPSILON(S)=  0.048832
               I=  15,  EPSILON(S)=  0.100414
               I=  16,  EPSILON(S)=  0.095388
               I=  17,  EPSILON(S)=  0.100564
               I=  18,  EPSILON(S)=  0.125844
               I=  19,  EPSILON(S)=  0.042557

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS    11
          MAXIMUM EPSILON(S)=  0.125844 AT OBSERVATION I=  18.


     OUT OF   20 OBSERVATIONS,    9 PASS AND   11 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=   1
               I=   2
               I=   3
               I=   6
               I=   9
               I=  11
               I=  12
               I=  13
               I=  20

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

TEST 5. CONSTRAINED ALLOCATIVE EFFICIENCY TEST
PRICE AND QUANTITY DATA
CONVEX CONE TECHNOLOGY
(CONSTANT RETURNS TO SCALE)

(TEST ASSUMES NO TECHNICAL CHANGE)

SUMMARY FOR OBSERVATION I= 7:

EPSILON(S)= 0.075883
POSITIVE LAMBDA'S: 6 13

GOODS NO.	NUMERAIRE (=>EFFICIENCY DIR. VECTOR)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z*	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
(6) GOODS IN S:						
5	1	1.3700	-1.0245	-0.64165	-0.38282	0.59662
6	1	1.4966	-0.98630	-1.4641	0.47783	-0.32636
7	1	1.4306	-0.14323	-0.25656	0.11333	-0.44174
8	1	1.1547	-1.0471	-1.1666	0.11952	-0.10246
9	1	1.0802	-1.5286	-1.5538	0.25220E-01	-0.01623
10	1	1.1260	-0.26626	-0.26156	-0.47038E-02	0.01798
(4) GOODS NOT IN S:						
1			8.4267	8.4267	-0.66813E-15	-0.00000
2			-0.16052	-0.17956	0.19048E-01	-0.10608
3			-0.83702	-0.89823	0.61208E-01	-0.06814
4			-1.4437	-1.4437	0.00000E+00	0.00000

VALUE OF NUMERAIRE GOODS= 6.757257
PARTIAL PROFIT AT CURRENT PRODUCTION PLAN= -6.757257
PARTIAL PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN= -6.244499

HENCE, OBSERVATION I= 7 IS ALLOCATIVELY INEFFICIENT.

```
SUMMARY FOR DATAFILE:

     VIOLATIONS ARE AT OBSERVATIONS:
               I=    1,  EPSILON(S)=  0.151631
               I=    4,  EPSILON(S)=  0.015050
               I=    5,  EPSILON(S)=  0.006943
               I=    7,  EPSILON(S)=  0.075883
               I=    8,  EPSILON(S)=  0.046363
               I=    9,  EPSILON(S)=  0.004676
               I=   10,  EPSILON(S)=  0.034297
               I=   14,  EPSILON(S)=  0.051025
               I=   15,  EPSILON(S)=  0.103515
               I=   16,  EPSILON(S)=  0.096987
               I=   17,  EPSILON(S)=  0.101117
               I=   18,  EPSILON(S)=  0.126724
               I=   19,  EPSILON(S)=  0.120364
               I=   20,  EPSILON(S)=  0.134988

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS    14
          MAXIMUM EPSILON(S)=  0.151631 AT OBSERVATION I=    1.

     OUT OF   20 OBSERVATIONS,    6 PASS AND   14 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=    2
               I=    3
               I=    6
               I=   11
               I=   12
               I=   13

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

```
TEST 6.  CONSTRAINED ALLOCATIVE EFFICIENCY TEST
         PRICE AND QUANTITY DATA
         QUASICONCAVE TECHNOLOGY

  NOTE:  TEST PERFORMED WITH RESPECT TO
         TECHNOLOGY OF GOOD N=    1

         ( TEST ASSUMES NO TECHNICAL CHANGE )


SUMMARY FOR OBSERVATION I=   7:

     EPSILON(S)=  0.030633
     POSITIVE LAMBDA'S:    6
```

GOODS NO.	NUMERAIRE (=>EFFICIENCY DIR. VECTOR)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z*	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
(6) GOODS IN S:						
5	1	1.3700	-0.64520	-0.64165	-0.35565E-02	0.00554
6	1	1.4966	-1.4747	-1.4641	-0.10621E-01	0.00725
7	1	1.4306	-0.24935	-0.25656	0.72123E-02	-0.02811
8	1	1.1547	-1.0969	-1.1666	0.69654E-01	-0.05971
9	1	1.0802	-1.4511	-1.5538	0.10269	-0.06609
10	1	1.1260	-0.23838	-0.26156	0.23176E-01	-0.08861
(3) GOODS NOT IN S:						
2			-0.17443	-0.17956	0.51333E-02	-0.02859
3			-0.87230	-0.89823	0.25927E-01	-0.02887
4			-1.4058	-1.4437	0.37955E-01	-0.02629
1				8.4267		

```
     VALUE OF NUMERAIRE GOODS=      6.757257
     PARTIAL PROFIT AT CURRENT PRODUCTION PLAN=     -6.757257
     PARTIAL PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN=     -6.550264

HENCE, OBSERVATION I=   7 IS ALLOCATIVELY INEFFICIENT.
```

```
SUMMARY FOR DATAFILE:

     VIOLATIONS ARE AT OBSERVATIONS:
               I=   7,  EPSILON(S)=  0.030633
               I=  14,  EPSILON(S)=  0.018878
               I=  16,  EPSILON(S)=  0.063813
               I=  17,  EPSILON(S)=  0.081904
               I=  18,  EPSILON(S)=  0.107907

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS    5
          MAXIMUM EPSILON(S)=  0.107907 AT OBSERVATION I=  18.

     OUT OF   20 OBSERVATIONS,   15 PASS AND    5 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=    1
               I=    2
               I=    3
               I=    4
               I=    5
               I=    6
               I=    8
               I=    9
               I=   10
               I=   11
               I=   12
               I=   13
               I=   15
               I=   19
               I=   20

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

Assuming no technological regress:

```
TEST 4.  CONSTRAINED ALLOCATIVE EFFICIENCY TEST
         PRICE AND QUANTITY DATA
         CONVEX TECHNOLOGY

         ( OPTIMAL PROFITS UNRESTRICTED IN SIGN )

         ( TEST ASSUMES NO TECHNOLOGICAL REGRESS )


SUMMARY FOR OBSERVATION I=   7:

     EPSILON(S)=  0.053762
     POSITIVE LAMBDA'S:    3    6

GOODS   NUMERAIRE     PRICE VECTOR     ALLOC. EFF.      VECTOR Z(I)       DIFFERENCE     PROPORTIONAL
 NO. (=>EFFICIENCY       P(I)           VECTOR Z*                          Z*-Z(I)        DIFFERENCE
      DIR. VECTOR)                                                                      (Z*-Z(I))/Z(I)

(    6) GOODS IN S:
   5        1            1.3700          -0.62854          -0.64165        0.13110E-01     -0.02043
   6        1            1.4966           -1.4790           -1.4641       -0.14838E-01      0.01013
   7        1            1.4306          -0.24974          -0.25656        0.68217E-02     -0.02659
   8        1            1.1547           -1.0619           -1.1666        0.10471         -0.08975
   9        1            1.0802           -1.3760           -1.5538        0.17775         -0.11440
  10        1            1.1260          -0.22171          -0.26156        0.39849E-01     -0.15235

(    4) GOODS NOT IN S:
   1                                      8.4267            8.4267         0.66613E-15      0.00000
   2                                     -0.15854          -0.17956        0.21021E-01     -0.11706
   3                                     -0.81921          -0.89823        0.79024E-01     -0.08798
   4                                      -1.3359           -1.4437        0.10781         -0.07468

     VALUE OF NUMERAIRE GOODS=      6.757257
     PARTIAL PROFIT AT CURRENT PRODUCTION PLAN=     -6.757257
     PARTIAL PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN=     -6.393974

HENCE, OBSERVATION I=   7 IS ALLOCATIVELY INEFFICIENT.
```

```
SUMMARY FOR DATAFILE:


     VIOLATIONS ARE AT OBSERVATIONS:
               I=   7,  EPSILON(S)=  0.053762
               I=  14,  EPSILON(S)=  0.048832
               I=  15,  EPSILON(S)=  0.100414
               I=  16,  EPSILON(S)=  0.095388
               I=  17,  EPSILON(S)=  0.100564
               I=  18,  EPSILON(S)=  0.125844

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS    6
          MAXIMUM EPSILON(S)=  0.125844 AT OBSERVATION I=  18.


     OUT OF   20 OBSERVATIONS,   14 PASS AND    6 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=   1
               I=   2
               I=   3
               I=   4
               I=   5
               I=   6
               I=   8
               I=   9
               I=  10
               I=  11
               I=  12
               I=  13
               I=  19
               I=  20

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

TEST 5. CONSTRAINED ALLOCATIVE EFFICIENCY TEST
PRICE AND QUANTITY DATA
CONVEX CONE TECHNOLOGY
(CONSTANT RETURNS TO SCALE)

(TEST ASSUMES NO TECHNOLOGICAL REGRESS)

SUMMARY FOR OBSERVATION I= 7:

EPSILON(S)= 0.071954
POSITIVE LAMBDA'S: 6

GOODS NO.	NUMERAIRE (=>EFFICIENCY DIR. VECTOR)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z*	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
(6) GOODS IN S:						
5	1	1.3700	-0.61770	-0.64165	0.23947E-01	-0.03732
6	1	1.4966	-1.4119	-1.4641	0.52244E-01	-0.03568
7	1	1.4306	-0.23872	-0.25656	0.17842E-01	-0.06954
8	1	1.1547	-1.0502	-1.1666	0.11641	-0.09979
9	1	1.0802	-1.3893	-1.5538	0.16455	-0.10590
10	1	1.1260	-0.22822	-0.26156	0.33337E-01	-0.12746
(4) GOODS NOT IN S:						
1			8.4267	8.4267	-0.66613E-15	-0.00000
2			-0.16700	-0.17956	0.12569E-01	-0.07000
3			-0.83512	-0.89823	0.63111E-01	-0.07026
4			-1.3459	-1.4437	0.97880E-01	-0.06780

VALUE OF NUMERAIRE GOODS= 6.757257
PARTIAL PROFIT AT CURRENT PRODUCTION PLAN= -6.757257
PARTIAL PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN= -6.271043

HENCE, OBSERVATION I= 7 IS ALLOCATIVELY INEFFICIENT.

```
SUMMARY FOR DATAFILE:


     VIOLATIONS ARE AT OBSERVATIONS:
               I=    7,  EPSILON(S)=  0.071954
               I=    8,  EPSILON(S)=  0.038918
               I=   10,  EPSILON(S)=  0.027706
               I=   14,  EPSILON(S)=  0.051025
               I=   15,  EPSILON(S)=  0.103515
               I=   16,  EPSILON(S)=  0.096987
               I=   17,  EPSILON(S)=  0.101117
               I=   18,  EPSILON(S)=  0.126724
               I=   19,  EPSILON(S)=  0.120364
               I=   20,  EPSILON(S)=  0.134988

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS    10
          MAXIMUM EPSILON(S)=  0.134988 AT OBSERVATION I=  20.


     OUT OF    20 OBSERVATIONS,    10 PASS AND    10 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=    1
               I=    2
               I=    3
               I=    4
               I=    5
               I=    6
               I=    9
               I=   11
               I=   12
               I=   13

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

TEST 6. CONSTRAINED ALLOCATIVE EFFICIENCY TEST
PRICE AND QUANTITY DATA
QUASICONCAVE TECHNOLOGY

NOTE: TEST PERFORMED WITH RESPECT TO
TECHNOLOGY OF GOOD N= 1

(TEST ASSUMES NO TECHNOLOGICAL REGRESS)

SUMMARY FOR OBSERVATION I= 7:

EPSILON(S)= 0.030633
POSITIVE LAMBDA'S: 6

GOODS NO.	NUMERAIRE (=>EFFICIENCY DIR. VECTOR)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z*	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
(6) GOODS IN S:						
5	1	1.3700	-0.64520	-0.64165	-0.35565E-02	0.00554
6	1	1.4966	-1.4747	-1.4641	-0.10621E-01	0.00725
7	1	1.4306	-0.24935	-0.25656	0.72123E-02	-0.02811
8	1	1.1547	-1.0969	-1.1666	0.69654E-01	-0.05971
9	1	1.0802	-1.4511	-1.5538	0.10269	-0.06609
10	1	1.1260	-0.23838	-0.26156	0.23176E-01	-0.08861
(3) GOODS NOT IN S:						
2			-0.17443	-0.17956	0.51333E-02	-0.02859
3			-0.87230	-0.89823	0.25927E-01	-0.02887
4			-1.4058	-1.4437	0.37955E-01	-0.02629
1				8.4267		

VALUE OF NUMERAIRE GOODS= 6.757257
PARTIAL PROFIT AT CURRENT PRODUCTION PLAN= -6.757257
PARTIAL PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN= -6.550264

HENCE, OBSERVATION I= 7 IS ALLOCATIVELY INEFFICIENT.

```
SUMMARY FOR DATAFILE:


     VIOLATIONS ARE AT OBSERVATIONS:
               I=   7,  EPSILON(S)=  0.030633
               I=  14,  EPSILON(S)=  0.018878
               I=  16,  EPSILON(S)=  0.063813
               I=  17,  EPSILON(S)=  0.081904
               I=  18,  EPSILON(S)=  0.107907

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS    5
          MAXIMUM EPSILON(S)=  0.107907 AT OBSERVATION I=  18.


     OUT OF   20 OBSERVATIONS,    15 PASS AND    5 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=    1
               I=    2
               I=    3
               I=    4
               I=    5
               I=    6
               I=    8
               I=    9
               I=   10
               I=   11
               I=   12
               I=   13
               I=   15
               I=   19
               I=   20

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

C.3.3 Allocative efficiency test results (assuming complete profit maximization)

The value of the violation index ε_i^* for observation i is given by 'EPSILON*'. For test 7 (the convex technology case), the observation relatively efficient to observation i listed following the violation index is the observation j which solves the maximization problem (11.1) used in calculating the violation index. The values at the bottom of the results for the 1967 observation are:

VALUE OF NUMERAIRE GOODS $\equiv p^{iT}\hat{\gamma}z^i$,

PROFIT AT CURRENT PRODUCTION PLAN $\equiv p^{iT}z^i$, and

PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN $\equiv p^{iT}z^{j*}$.

where $j*$ is the observation j described above. Thus, if the technology assumptions of test 7 hold, then in 1967 the resources sector could have increased profits by

$$\varepsilon_i^*(p^{iT}\hat{\gamma}z^i) = 0.123946(6.757257) = 0.837535$$

or by $837 million (1967 Canadian dollars). Note that the previous results of test 4 indicate that $363 million is the loss due to failure to cost minimize with respect to the primary inputs; in test 4, the economy's output goods were restricted to levels at least as large as those observed in 1967. Since profit maximization entails both revenue maximization and cost minimization, the levels of goods (1–4) produced and used by the resources sector may not be profit maximal. Thus, we obtain in our example a higher economic loss ($\varepsilon_i^* > \varepsilon_i^S$) in test 7.

For test 8 which assumes a convex cone technology, we have at the bottom of the results for the 1967 observation the following:

VALUE OF (NORMALIZED) NUMERAIRE GOODS $\equiv \sum_{m\in E} p_m^i |\bar{z}_m^i|$,

NORMALIZED PROFIT AT CURRENT PRODUCTION PLAN $\equiv p^{niT}\bar{z}^{ni}$, and

NORMALIZED PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN $\equiv p^{niT}\bar{z}^{nj*}$

where $j*$ solves the maximization problem (11.12) and is listed in the line following the violation index. Since the above numbers are values of normalized variables, they can be interpreted as values per unit of the normalizing good. In the following example, the normalizing good is good 1, the output of the resources sector. Hence, the value of $p^{niT}\bar{z}^{ni}$ is the negative of the cost of per unit output (good 1) using the 1967 production plan. The optimal per unit output cost, at 1967 prices, is given by $p^{niT}\bar{z}^{nj*}$. If the normalizing good chosen is an input, then the allocative efficiency test 8 reduces to testing whether this input is receiving the maximal per unit return.

In test 8 (and 8′), if the corresponding linear programming problem has an unbounded solution, then the computer program duly notes this by setting 'EPSILON*=UNBOUNDED' and stating maximal profits are unbounded. The observation j printed in the line following the violation index is the observation which solves the problem (11.12).

In test 9 which assumes a quasiconcave technology, the restricted profit at the current production plan is $p^{niT}z^{ni}$ and the restricted profit at the allocatively efficient production plan is $p^{niT}z^{nj*}$, $j* \in I_i^n$ where $j*$ solves the maximization problem (11.29). Since the test is performed with respect to the production function of good 1 (the output good for the resources sector), the restricted profit is the (negative) cost of producing good 1 at levels at least as large as that in 1967 and at prices prevailing in 1967. If the singled-out good n with respect to which the test is performed is an input, then the restricted profit is the revenue or factor payment to good n.

The results of tests 7′, 8′ and 9′ incorporating the no technological regress assumption can be similarly interpreted.

Assuming no technical change:

```
TEST 7.  UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
         PRICE AND QUANTITY DATA
         CONVEX TECHNOLOGY

         ( OPTIMAL PROFITS UNRESTRICTED IN SIGN )

         ( TEST ASSUMES NO TECHNICAL CHANGE )


SUMMARY FOR OBSERVATION I=    7:

     EPSILON*=  0.123946
     OBSERVATION RELATIVELY EFFICIENT TO I:   J=  13
```

GOODS NO.	EFFICIENCY DIR. VECTOR (=>NUMERAIRE)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z* (Z(J),J= 13)	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
1	0	1.1393	11.729	8.4267	3.3027	0.39193
2	0	1.1181	-0.22323	-0.17956	-0.43669E-01	0.24319
3	0	1.0999	-1.1651	-0.89823	-0.26691	0.29715
4	0	1.1409	-2.0125	-1.4437	-0.56879	0.39397
5	-1	1.3700	-1.4382	-0.64165	-0.79654	1.24140
6	-1	1.4966	-1.3601	-1.4641	0.10403	-0.07105
7	-1	1.4306	-0.19650	-0.25656	0.60062E-01	-0.23410
8	-1	1.1547	-1.4574	-1.1666	-0.29076	0.24924
9	-1	1.0802	-2.1319	-1.5538	-0.57808	0.37203
10	-1	1.1260	-0.37176	-0.26156	-0.11020	0.42133

```
     VALUE OF NUMERAIRE GOODS=      6.757257
     PROFIT AT CURRENT PRODUCTION PLAN=     0.7290000E-02
     PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN=     0.8448278

HENCE, OBSERVATION I=    7 IS ALLOCATIVELY INEFFICIENT.
```

```
SUMMARY FOR DATAFILE:

     VIOLATIONS ARE AT OBSERVATIONS:
               I=   1,  EPSILON*=  0.295885,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   2,  EPSILON*=  0.131678,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   3,  EPSILON*=  0.049280,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   4,  EPSILON*=  0.049036,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   5,  EPSILON*=  0.044107,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   7,  EPSILON*=  0.123946,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   8,  EPSILON*=  0.064378,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   9,  EPSILON*=  0.041402,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  10,  EPSILON*=  0.070049,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  11,  EPSILON*=  0.070360,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  12,  EPSILON*=  0.061329,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  14,  EPSILON*=  0.074347,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  15,  EPSILON*=  0.144833,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  16,  EPSILON*=  0.145652,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  17,  EPSILON*=  0.158844,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  18,  EPSILON*=  0.194124,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  19,  EPSILON*=  0.168155,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  20,  EPSILON*=  0.151956,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS   18
          MAXIMUM EPSILON*=  0.295885 AT OBSERVATION I=   1.


     OUT OF   20 OBSERVATIONS,    2 PASS AND   18 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=   6
               I=  13

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

TEST 8. UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
PRICE AND QUANTITY DATA
CONVEX CONE TECHNOLOGY
(CONSTANT RETURNS TO SCALE)

NOTE: TEST PERFORMED WITH NORMALIZING
GOOD N= 1

(TEST ASSUMES NO TECHNICAL CHANGE)

SUMMARY FOR OBSERVATION I= 7:

EPSILON*= 0.100833
OBSERVATION RELATIVELY EFFICIENT TO I: J= 6

GOODS NO.	EFFICIENCY DIR. VECTOR (=>NUMERAIRE)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z* (Z(J),J= 6)	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
2	0	1.1181	-0.17443	-0.17956	0.51333E-02	-0.02859
3	0	1.0999	-0.87230	-0.89823	0.25927E-01	-0.02887
4	0	1.1409	-1.4058	-1.4437	0.37955E-01	-0.02629
5	-1	1.3700	-0.64520	-0.64165	-0.35565E-02	0.00554
6	-1	1.4966	-1.4747	-1.4641	-0.10621E-01	0.00725
7	-1	1.4306	-0.24935	-0.25656	0.72123E-02	-0.02811
8	-1	1.1547	-1.0969	-1.1666	0.69654E-01	-0.05971
9	-1	1.0802	-1.4511	-1.5538	0.10269	-0.06609
10	-1	1.1260	-0.23838	-0.26156	0.23176E-01	-0.08861
1			8.8019	8.4267	0.37520	0.04453

VALUE OF (NORMALIZED) NUMERAIRE GOODS= 0.8018870
NORMALIZED PROFIT AT CURRENT PRODUCTION PLAN= -1.138423
NORMALIZED PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN= -1.057566

HENCE, OBSERVATION I= 7 IS ALLOCATIVELY INEFFICIENT.

```
SUMMARY FOR DATAFILE:

     VIOLATIONS ARE AT OBSERVATIONS:
               I=   1,  EPSILON*=  0.186264,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   2,  EPSILON*=  0.099175,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   3,  EPSILON*=  0.042091,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   4,  EPSILON*=  0.043423,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   5,  EPSILON*=  0.040832,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   7,  EPSILON*=  0.100833,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   8,  EPSILON*=  0.053078,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   9,  EPSILON*=  0.033010,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  10,  EPSILON*=  0.056018,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  11,  EPSILON*=  0.059636,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  12,  EPSILON*=  0.053798,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  14,  EPSILON*=  0.071871,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  15,  EPSILON*=  0.134691,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  16,  EPSILON*=  0.140517,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  17,  EPSILON*=  0.159294,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  18,  EPSILON*=  0.190162,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  19,  EPSILON*=  0.192021,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  20,  EPSILON*=  0.208261,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)

        TOTAL NUMBER OF VIOLATIONS FOR TEST IS   18
        MAXIMUM EPSILON*=  0.208261 AT OBSERVATION I=  20.

     OUT OF   20 OBSERVATIONS,    2 PASS AND   18 FAIL THE TEST.
        THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=   6
               I=  13

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

```
TEST 9.  UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
         PRICE AND QUANTITY DATA
         QUASICONCAVE TECHNOLOGY

  NOTE:  TEST PERFORMED WITH RESPECT TO
         TECHNOLOGY OF GOOD N=    1

         ( TEST ASSUMES NO TECHNICAL CHANGE )



SUMMARY FOR OBSERVATION I=    7:

     EPSILON*=  0.042111
     OBSERVATION RELATIVELY EFFICIENT TO I:   J=    6
```

GOODS NO.	EFFICIENCY DIR. VECTOR (=>NUMERAIRE)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z* (Z(J),J= 6)	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
2	0	1.1181	-0.17443	-0.17956	0.51333E-02	-0.02859
3	0	1.0999	-0.87230	-0.89823	0.25927E-01	-0.02887
4	0	1.1409	-1.4058	-1.4437	0.37955E-01	-0.02629
5	-1	1.3700	-0.64520	-0.64165	-0.35565E-02	0.00554
6	-1	1.4966	-1.4747	-1.4641	-0.10621E-01	0.00725
7	-1	1.4306	-0.24935	-0.25656	0.72123E-02	-0.02811
8	-1	1.1547	-1.0969	-1.1666	0.69654E-01	-0.05971
9	-1	1.0802	-1.4511	-1.5538	0.10269	-0.06609
10	-1	1.1260	-0.23838	-0.26156	0.23176E-01	-0.08861
1			8.8019	8.4267	0.37520	0.04453

```
     VALUE OF NUMERAIRE GOODS=      6.757257
     RESTRICTED PROFIT AT CURRENT PRODUCTION PLAN=     -9.593144
     RESTRICTED PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN=    -9.308592

HENCE, OBSERVATION I=    7 IS ALLOCATIVELY INEFFICIENT.
```

```
SUMMARY FOR DATAFILE:

     VIOLATIONS ARE AT OBSERVATIONS:
               I=   7,  EPSILON*=  0.042111,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  14,  EPSILON*=  0.029694,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  15,  EPSILON*=  0.045774,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  16,  EPSILON*=  0.096028,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  17,  EPSILON*=  0.129704,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  18,  EPSILON*=  0.164990,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS    6
          MAXIMUM EPSILON*=  0.164990 AT OBSERVATION I=  18.

     OUT OF   20 OBSERVATIONS,   14 PASS AND    6 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=   1
               I=   2
               I=   3
               I=   4
               I=   5
               I=   6
               I=   8
               I=   9
               I=  10
               I=  11
               I=  12
               I=  13
               I=  19
               I=  20

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

Assuming no technological regress:

```
TEST 7.  UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
         PRICE AND QUANTITY DATA
         CONVEX TECHNOLOGY

         ( OPTIMAL PROFITS UNRESTRICTED IN SIGN )

         ( TEST ASSUMES NO TECHNOLOGICAL REGRESS )


SUMMARY FOR OBSERVATION I=   7:

     EPSILON*=  0.105371
     OBSERVATION RELATIVELY EFFICIENT TO I:  J=   6
```

GOODS NO.	EFFICIENCY DIR. VECTOR (=>NUMERAIRE)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z* (Z(J),J= 6)	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
1	0	1.1393	8.8019	8.4267	0.37520	0.04453
2	0	1.1181	-0.17443	-0.17956	0.51333E-02	-0.02859
3	0	1.0999	-0.87230	-0.89823	0.25927E-01	-0.02887
4	0	1.1409	-1.4058	-1.4437	0.37955E-01	-0.02629
5	-1	1.3700	-0.64520	-0.64165	-0.35565E-02	0.00554
6	-1	1.4966	-1.4747	-1.4641	-0.10621E-01	0.00725
7	-1	1.4306	-0.24935	-0.25656	0.72123E-02	-0.02811
8	-1	1.1547	-1.0969	-1.1666	0.69654E-01	-0.05971
9	-1	1.0802	-1.4511	-1.5538	0.10269	-0.06609
10	-1	1.1260	-0.23838	-0.26156	0.23176E-01	-0.08861

```
     VALUE OF NUMERAIRE GOODS=     6.757257
     PROFIT AT CURRENT PRODUCTION PLAN=    0.7290000E-02
     PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN=    0.7193083

HENCE, OBSERVATION I=   7 IS ALLOCATIVELY INEFFICIENT.
```

```
SUMMARY FOR DATAFILE:

     VIOLATIONS ARE AT OBSERVATIONS:
               I=   4,  EPSILON*=  0.002869,  J=   3 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   7,  EPSILON*=  0.105371,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   8,  EPSILON*=  0.052174,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   9,  EPSILON*=  0.023635,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  10,  EPSILON*=  0.044713,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  11,  EPSILON*=  0.022639,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  12,  EPSILON*=  0.011990,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  14,  EPSILON*=  0.074347,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  15,  EPSILON*=  0.144833,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  16,  EPSILON*=  0.145652,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  17,  EPSILON*=  0.158844,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  18,  EPSILON*=  0.194124,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  19,  EPSILON*=  0.168155,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  20,  EPSILON*=  0.151956,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS   14
          MAXIMUM EPSILON*=  0.194124 AT OBSERVATION I=  18.


     OUT OF   20 OBSERVATIONS,    6 PASS AND   14 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=   1
               I=   2
               I=   3
               I=   5
               I=   6
               I=  13

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

TEST 8. UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
PRICE AND QUANTITY DATA
CONVEX CONE TECHNOLOGY
(CONSTANT RETURNS TO SCALE)

NOTE: TEST PERFORMED WITH NORMALIZING
GOOD N= 1

(TEST ASSUMES NO TECHNOLOGICAL REGRESS)

SUMMARY FOR OBSERVATION I= 7:

EPSILON*= 0.100833
OBSERVATION RELATIVELY EFFICIENT TO I: J= 6

GOODS NO.	EFFICIENCY DIR. VECTOR (=>NUMERAIRE)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z* (Z(J),J= 6)	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
2	0	1.1181	-0.17443	-0.17956	0.51333E-02	-0.02859
3	0	1.0999	-0.87230	-0.89823	0.25927E-01	-0.02887
4	0	1.1409	-1.4058	-1.4437	0.37955E-01	-0.02629
5	-1	1.3700	-0.64520	-0.64165	-0.35565E-02	0.00554
6	-1	1.4966	-1.4747	-1.4641	-0.10621E-01	0.00725
7	-1	1.4306	-0.24935	-0.25656	0.72123E-02	-0.02811
8	-1	1.1547	-1.0969	-1.1666	0.69654E-01	-0.05971
9	-1	1.0802	-1.4511	-1.5538	0.10269	-0.06609
10	-1	1.1260	-0.23838	-0.26156	0.23176E-01	-0.08861
1			8.8019	8.4267	0.37520	0.04453

VALUE OF (NORMALIZED) NUMERAIRE GOODS= 0.8018870
NORMALIZED PROFIT AT CURRENT PRODUCTION PLAN= -1.138423
NORMALIZED PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN= -1.057566

HENCE, OBSERVATION I= 7 IS ALLOCATIVELY INEFFICIENT.

```
SUMMARY FOR DATAFILE:


     VIOLATIONS ARE AT OBSERVATIONS:
               I=   4,  EPSILON*=  0.003033,  J=   3 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   7,  EPSILON*=  0.100833,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   8,  EPSILON*=  0.053078,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=   9,  EPSILON*=  0.024862,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  10,  EPSILON*=  0.047804,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  11,  EPSILON*=  0.025799,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  12,  EPSILON*=  0.014382,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  14,  EPSILON*=  0.071871,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  15,  EPSILON*=  0.134691,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  16,  EPSILON*=  0.140517,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  17,  EPSILON*=  0.159284,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  18,  EPSILON*=  0.190162,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  19,  EPSILON*=  0.192021,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  20,  EPSILON*=  0.208261,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS   14
          MAXIMUM EPSILON*=  0.208261 AT OBSERVATION I=  20.


     OUT OF   20 OBSERVATIONS,    6 PASS AND   14 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=   1
               I=   2
               I=   3
               I=   5
               I=   6
               I=  13

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

```
TEST 9.  UNCONSTRAINED ALLOCATIVE EFFICIENCY TEST
         PRICE AND QUANTITY DATA
         QUASICONCAVE TECHNOLOGY

  NOTE:  TEST PERFORMED WITH RESPECT TO
         TECHNOLOGY OF GOOD N=    1

         ( TEST ASSUMES NO TECHNOLOGICAL REGRESS )


SUMMARY FOR OBSERVATION I=    7:

     EPSILON*=  0.042111
     OBSERVATION RELATIVELY EFFICIENT TO I:  J=   6
```

GOODS NO.	EFFICIENCY DIR. VECTOR (=>NUMERAIRE)	PRICE VECTOR P(I)	ALLOC. EFF. VECTOR Z* (Z(J),J= 6)	VECTOR Z(I)	DIFFERENCE Z*-Z(I)	PROPORTIONAL DIFFERENCE (Z*-Z(I))/Z(I)
2	0	1.1181	-0.17443	-0.17956	0.51333E-02	-0.02859
3	0	1.0999	-0.87230	-0.89823	0.25927E-01	-0.02887
4	0	1.1409	-1.4058	-1.4437	0.37955E-01	-0.02629
5	-1	1.3700	-0.64520	-0.64165	-0.35565E-02	0.00554
6	-1	1.4966	-1.4747	-1.4641	-0.10621E-01	0.00725
7	-1	1.4306	-0.24935	-0.25656	0.72123E-02	-0.02811
8	-1	1.1547	-1.0969	-1.1666	0.69654E-01	-0.05971
9	-1	1.0802	-1.4511	-1.5538	0.10269	-0.06609
10	-1	1.1260	-0.23838	-0.26156	0.23176E-01	-0.08861
1			8.8019	8.4267	0.37520	0.04453

```
     VALUE OF NUMERAIRE GOODS=      6.757257
     RESTRICTED PROFIT AT CURRENT PRODUCTION PLAN=     -9.593144
     RESTRICTED PROFIT AT ALLOCATIVELY EFFICIENT PRODUCTION PLAN=     -9.308592

HENCE, OBSERVATION I=    7 IS ALLOCATIVELY INEFFICIENT.
```

```
SUMMARY FOR DATAFILE:

     VIOLATIONS ARE AT OBSERVATIONS:
               I=   7,  EPSILON*=  0.042111,  J=   6 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  14,  EPSILON*=  0.029694,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  15,  EPSILON*=  0.045774,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  16,  EPSILON*=  0.096028,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  17,  EPSILON*=  0.129704,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)
               I=  18,  EPSILON*=  0.164990,  J=  13 (ALLOCATION RELATIVELY EFFICIENT TO I)

          TOTAL NUMBER OF VIOLATIONS FOR TEST IS    6
          MAXIMUM EPSILON*=  0.164990 AT OBSERVATION I=  18.


     OUT OF   20 OBSERVATIONS,    14 PASS AND    6 FAIL THE TEST.
          THE OBSERVATIONS CONSISTENT WITH THE HYPOTHESIS ARE:
               I=    1
               I=    2
               I=    3
               I=    4
               I=    5
               I=    6
               I=    8
               I=    9
               I=   10
               I=   11
               I=   12
               I=   13
               I=   19
               I=   20

     CONCLUSION:  OVERALL, THE DATA IS NOT CONSISTENT
                  WITH THE HYPOTHESIS.
```

Appendix D

Results of the Sectoral Profit Function Estimation

The symmetric generalized McFadden flexible functional form with a quadratic spline model for technical progress, proposed by Diewert and Wales (1989b), was used to estimate the unit scale profit function. The model then has the following functional form:

$$\pi(p,t) \equiv h(p) + d(p,t) \tag{D.1}$$

where

$$h(p) \equiv p^T b^1 + \frac{1}{2}\left(\frac{p^T B p}{\alpha^T p}\right) \tag{D.2}$$

and

$$d(p,t) \equiv \begin{cases} p^T b^2 t + \frac{1}{2} p^T b^3 t^2 & \text{for } t \le t_1; \\[2ex] p^T b^2 t + \frac{1}{2} p^T b^3 t_1^2 + p^T b^3 (t - t_1) t_1 & \\ \qquad + \frac{1}{2} p^T b^4 (t - t_1)^2 & \text{for } t_1 \le t \le t_2; \\[2ex] p^T b^2 t + \frac{1}{2} p^T b^3 t_1^2 + p^T b^3 (t_2 - t_1) t_1 & \\ \qquad + \frac{1}{2} p^T b^4 (t_2 - t_1)^2 + p^T b^3 (t - t_2) t_1 & \\ \qquad + p^T b^4 (t - t_2)(t_2 - t_1) + \frac{1}{2} p^T b^5 (t - t_2)^2 & \text{for } t_2 \le t \le T. \end{cases} \tag{D.3}$$

The values of the exogenous parameters of the model: $\alpha \equiv (\alpha_1, \alpha_2, \ldots, \alpha_N)^T \ge 0_N$ in (D.2) and the break points t_1 and t_2 in (D.3) for the estimated models are listed in table D.27.

The exogenous parameters α_n, $n = 1, 2, \ldots, N$ were calculated as follows:

1. Let $t*$ be the time index of the reference price vector p^* chosen to satisfy the restriction

$Bp^* = 0_N$. Then, define transformed price and quantity variables

$$\tilde{p}_n^t \equiv \frac{p_n^t}{p_n^{t*}} \text{ and} \tag{D.4}$$

$$\tilde{z}_n^t \equiv z_n^t(p_n^{t*}) \tag{D.5}$$

for all goods n, $n = 1, 2, \ldots, N$ and time periods t, $t = 1, 2, \ldots, T$.

2. For each good n, $n = 1, 2, \ldots, N$ define the mean value of the quantity variables in (D.5) over the T time periods as

$$\check{\tilde{z}}_n \equiv \frac{\sum_{t=1}^{T} \tilde{z}_n^t}{T}. \tag{D.6}$$

3. For each good n, $n = 1, 2, \ldots, N$ set α_n as

$$\alpha_n \equiv \frac{|\check{\tilde{z}}_n|}{\sum_{k=1}^{N} |\check{\tilde{z}}_k|} \left(= \frac{|p_n^{t*} \check{z}_n|}{\sum_{k=1}^{N} |p_k^{t*} \check{z}_k|} \right) \tag{D.7}$$

where $\check{z}_k$ is the mean of the unadjusted quantity variable. Hence, α_n is a relative measure of the value share of good n in production. If the relative value share of a good n is small, its corresponding α_n will also be small. Note too that $\alpha^T p^* = 1$ when p^* is set equal to the transformed price vector defined by (D.4).

For the empirical work undertaken in this study, the reference year chosen for each sector is $t* = 1971$; therefore, by equation (D.4), the transformed prices $\tilde{p}_n^{1971} = 1.0$ for all n or $p^* = 1_N$.

For each sector, several runs of the model using different values for the break points t_1 and t_2 were performed to search for those which yield higher values of the likelihood function. The size of the model and time and computer budget constraints precluded a full search over all possible t_1 and t_2 combinations. Initially, one-break models obtained by setting $t_2 = T$ and with the rank of the B matrix restricted to 5 were estimated. There were convergence difficulties encountered for the resources sector (sector I). For the other three sectoral profit functions, this form of the model involves 57 parameters. The one-break models with the t_1 specification yielding the higher values for the likelihood function were reestimated with a maximal rank specification for B (rank(B)$=N-1$) which has an additional 3 parameters.

parameter			sector I	sector II	sector III	sector IV
n	good n	α				
1	sector I goods	α_1	0.533	0.069	0.064	0.014
2	sector II goods	α_2	0.011	0.514	0.049	0.022
3	sector III goods	α_3	0.054	0.044	0.514	0.056
4	sector IV goods	α_4	0.110	0.071	0.078	0.573
5	imports	α_5	0.051	0.110	0.067	0.022
6	labor	α_6	0.136	0.150	0.169	0.268
7	inventories	α_7	0.012	0.009	0.018	0.007
8	machinery and equip.	α_8	0.076	0.035	0.040	0.038
9	land	α_9	0.016			
break points		t_1, t_2	1967,1975	1965,1975	1967,1975	1970,1976

Table D.27: Values of the exogenous parameters of the estimated models

The log likelihood values for the maximal-rank one-break models, given the same t_1 from the earlier estimated models, did not change (up to the units decimal place). Hence, the results indicate that semiflexible estimation, which requires fewer parameters, will not greatly alter the results as opposed to the maximal rank models. Reverting to a rank of 5 specification for the B matrix, the two-break models were at first estimated using the t_1 values selected for the one-break models as one of the break points. The log likelihood values increased in the order of 10 to 40 which can be considered significant, using the likelihood ratio test and considering that we added 8 more parameters. Therefore, the two-break models seem to be superior to the one-break models. Other combinations of t_1 and t_2 values were tried to obtain higher values of the likelihood function. For the resources sector which has $N+1 = 10$ goods, convergence was attained with the two-break model but there seems to be difficulty in estimation when the number of parameters is over 70; hence, the rank of the B matrix was restricted to 3.

The maximum likelihood estimation was performed using the transformed prices $\tilde{p}_n^t$ and quantities $\tilde{z}_n^t$ defined in equations (D.4) and (D.5) for $n = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$. The observations for the years $1961, 1962, \ldots, 1980$ are indexed consecutively by $t = 1, 2, \ldots, 20$. For the normalizing good (structures), good $N + 1$, the quantity variable was set to unity at the period $t*$ by using the transformed variable

$$\tilde{z}_{N+1}^{t} \equiv \frac{z_{N+1}^{t}}{z_{N+1}^{t*}} \tag{D.8}$$

for $t = 1, 2, \ldots, T$. Output quantities are indexed positively and input quantities are indexed negatively.

For each sector, a system of N unit scale output supply and factor demand equations can be obtained using Hotelling's lemma. The system of equations is given by

$$\frac{\tilde{z}^{t}}{|\,\tilde{z}_{N+1}^{t}\,|} = \nabla_{p}\pi(\tilde{p}^{t}, t). \tag{D.9}$$

The functional form assumed for the unit scale profit function π is defined by equations (D.1)–(D.3); the scalar $\tilde{z}_{N+1}^{t}$ and the components of the N-dimensional vectors $\tilde{z}^{t} \equiv (\tilde{z}_{1}^{t}, \tilde{z}_{2}^{t}, \ldots, \tilde{z}_{N}^{t})^{T}$ and $\tilde{p}^{t} \equiv (\tilde{p}_{1}^{t}, \tilde{p}_{2}^{t}, \ldots, \tilde{p}_{N}^{t})^{T}$ are defined by equations (D.8), (D.5) and (D.4), respectively. Since the unit scale profit function is convex in prices, then the matrix B in (D.2) is restricted to be a symmetric positive semidefinite matrix by setting

$$B = AA^{T} \tag{D.10}$$

where $A = [a_{ij}]$ is a lower triangular matrix. Since restriction (D.10) is imposed, the system of equations given in (D.9) becomes nonlinear in the parameters. For identification, the restriction $Bp^{*} = 0_{N}$ was imposed. To aid convergence of the nonlinear optimization routines, the semiflexible estimation technique where the rank of the matrix B is restricted to less than $N-1$ was used. With semiflexible estimation, fewer parameters need to be estimated but second-order derivatives can still attain arbitrary values. The actual estimation model used in this study took the form

$$\tilde{z}^{t} = \left[\nabla_{p}\pi(\tilde{p}^{t}, t)\right]\,|\,\tilde{z}_{N+1}^{t}\,| \;+ e \tag{D.11}$$

where we denote the stochastic disturbance term appended to the model in (D.11) by $e = (e_1, e_2, \ldots, e_N)^{T}$. We assume the e's are serially independent N-variate normal with mean zero and covariance Σ which is constant over time. Full information maximum likelihood estimation for nonlinear simultaneous equations using quasi-Newton algorithms was performed.

The parameter estimates and their asymptotic standard errors and t-values for each of the four sectors are listed in tables D.28–D.31. The reported estimates (and their standard errors) of the technical progress parameters — b^2, b^3, b^4 and b^5 — are scaled up by a factor of 100.[1] The standard errors are the square roots of the diagonal elements in the estimated covariance matrix, the inverse of the Hessian matrix of (minus) the log likelihood function. The t-values are computed by dividing the parameter estimates by their corresponding standard errors. Tables D.32–D.35 list the own price elasticities over the 20-year period and the cross-price elasticities for 1971. Five-year interval cross-price elasticities for the different sectors are presented in tables D.36–D.39. The price elasticities ϵ_{ij}, $i, j = 1, 2, \ldots, N$, are defined as

$$\epsilon_{ij} \equiv \frac{\partial \mid z_i \mid}{\partial p_j} \frac{p_j}{\mid \hat{z}_i \mid} = \frac{\partial \pi(p,t)}{\partial p_i \partial p_j} \frac{p_j}{\hat{z}_i} = \frac{\partial h(p)}{\partial p_i \partial p_j} \frac{p_j}{\hat{z}_i} \tag{D.12}$$

where $\mid \hat{z}_i \mid$ is the predicted value of z_i and where $h(p)$ is given in (D.2). By the homogeneity property of the profit function, $[\nabla_{pp}\pi]p = 0_N$; therefore, the row sums of the cross-price elasticities equal 1.0. Row and column labels $(1), (2), \ldots, (9)$ refer to the following goods, respectively:

(1) resource goods (from sector I),
(2) manufactured goods (from sector II),
(3) manufactured goods (from sector III),
(4) service goods (from sector IV),
(5) imports,
(6) labor,
(7) inventories,
(8) machinery and equipment, and
(9) land.

From tables D.32–D.35, an examination of the own price elasticities ϵ_{ii} across the four sectors suggests that elasticities are generally higher in the manufacturing sectors relative to the resources and services sectors. In particular, for the export market-oriented manufacturing

[1] The actual value of the estimate can then be obtained by dividing the reported values by 100.

sector, the own price elasticities for the intermediate inputs are generally higher. Since the corresponding cross-price elasticities would tend to be higher in magnitude with a greater value of the own price elasticity, the results can be indicative of greater flexibility in the manufacturing sectors. The manufacturing sectors, especially the export market-oriented sector, are in a better position to adjust in terms of additional output to changes in relative input prices.

In terms of growth theory, it would be of interest to explore the relationship between the elasticities which measure the degree of substitutability and complementarity among goods and the rates of technical progress in the four sectors. For example, the resources sector has been subjected to a dramatic increase in its output price in the 1970s (see Appendix B) and displays greater fluctuations in the obtained nonparametric rates of technical progress and a decline towards technological regress in the parametric measure of technical progress (see Part II). A closer examination of the Divisia indices for input growth and output growth for this sector indicates that the rate of input growth has been quite steady while the rate of output growth shows wilder fluctuations. The low degree of substitutability among the inputs, as seen in table D.32, can be a partial explanation of the obtained measures of technological progress. It is also often hypothesized in the literature that the services sector has an inherently low productivity growth; this can be related to the low degree of substitutability among the goods in this sector.

The own price elasticities of output supply are higher in the manufacturing sectors; they are in the range 1.6–2.3 but declining over the years in the export market-oriented sector and in the range 1.7–1.9 and increasing over the years in the domestic market-oriented sector. For the resources and services sector, output supply is generally inelastic with respect to its own price. However, the own price elasticity of output supply for the resources sector was increasing towards 1.0, from a value of 0.7 in 1961, over the years. For the services sector, it has been relatively stable in the range 0.21–0.27.

Demand for imports is generally more elastic with respect to its own price compared to the demand for labor with respect to its own price in all the sectors. However, except for the resources sector, the own price elasticity of import demand shows a declining trend. In contrast, the own price elasticity of labor demand, though less that 1.0 in all the sectors, shows an increasing trend. The own price elasticity of labor demand is relatively higher in the manufacturing sectors (0.4–0.9) and quite low in the resources sector (0.1–0.5) and the services sector (0.1–0.2). Hence, as can be seen too in the tables of cross-price elasticities (see rows corresponding to good (6)), employment effects of relative price changes will tend to be small in the services sector compared to the employment effects in the other sectors of the economy.

As expected, the capital goods which verge on being fixed factors display smaller own price elasticites relative to other inputs. Inventories, which can be more easily adjusted than machinery and equipment or land, has generally higher own price elasticities, particularly for the resources sector in the 1970s. Land in the resources sector, being in almost fixed supply, has very low own price elasticities. The own price elasticity of the demand for machinery and equipment is slightly higher in the resources sector compared to the other sectors.

The tables of cross-price elasticities ϵ_{ij} (tables D.32–D.39) show the degrees of substitutability and complementarity among the goods. We define two goods to be substitutes if the cross partial derivatives of the profit function with respect to the prices of the two goods is negative. In the resources sector (I), all the input goods are substitutes with respect to the output good. Looking at row (2) for sector II and row (3) for sector III in the tables of cross-price elasticites, we see the same relationship between inputs and the output good, that is, an increase in the input prices leads to a decline in the output level. In contrast to the other sectors, the services sector has positive cross price elasticities, as seen in row (4) of the tables of cross price elasticities for sector IV, for its output and the intermediate inputs from the resources sector and the capital good inventories. The magnitude of these positive cross-price elasticities are small though. The low and positive cross price elasticity of output supply with respect to the price of intermediate inputs from the resources sector may explain why the services sector has been relatively immune, in terms of output levels, to the price increases of resource goods in the 1970s.

We next note the pairs of substitute inputs in the different sectors. If two inputs are substitutes, then an increase in the price of one of the inputs, keeping all other prices constant, increases the demand for the other input. In the resources sector (see tables of cross-price elasticities for sector I), the pairs of substitute inputs are:

(3) and (7) — intermediate inputs from sector III and inventories,

(3) and (9) — intermediate input from sector III and land,

(4) and (9) — intermediate input from sector IV and land,

(6) and (8) — labor and machinery and equipment, and

(8) and (9) — machinery and equipment and land.

The obtained elasticities are however small. Note that land seems to be substitutable with several other inputs. As land which really proxies for resource stock becomes scarce or depleted, we would expect the producer to use other inputs more intensively to extract output from the resource stock. The elasticities indicate a small degree of substitutability between machinery and equipment and labor; hence, even if capital subsidies may have a positive employment effect, the increase in labor employment due to a lower capital rental price for machinery and equipment may be small unless the capital subsidies lower the rental price substantially. In 1980, a decrease of 1% in the rental price of machinery and equipment would lead to only a 0.08% increase in labor employment in the resources sector.

For sector II, the export market-oriented manufacturing sector, the following input pairs turn out as substitutes:

(1) and (3) — intermediate inputs from sectors I and III,

(1) and (6) — intermediate inputs from sector I and labor,

(1) and (7) — intermediate inputs from sector I and inventories,

(3) and (4) — intermediate inputs from sectors III and IV,

(3) and (8) — intermediate inputs from sector III and machinery and equipment,

(5) and (7) — imports and inventories, and

(7) and (8) — inventories and machinery and equipment.

The positive cross-price elasticities, particularly for the intermediate inputs, are generally higher in this sector compared to the resources and services sectors. Therefore, it can be inferred that there is greater potential for input substitution in this sector. The cross-price elasticities for the pairs (5) and (7), and (7) and (8) are small; in particular, for imports (5) and inventories (7), their cross price elasticities are negative in the early years and can generally be considered near zero.

The domestic market-oriented manufacturing sector, sector III, displays to a lesser extent compared to the export market-oriented sector some degree of substitutability between the following inputs:

(1) and (2) — intermediate inputs from sectors I and II,

(1) and (5) — intermediate inputs from sector I and imports,

(1) and (6) — intermediate inputs from sector I and labor,

(1) and (7) — intermediate inputs from sector I and inventories, and

(4) and (7) — intermediate inputs from sector IV and inventories.

Note that intermediate inputs of resource goods tend to be substitutable with a number of other inputs. This may have moderated the effects of the rapid rise of resource prices in the 1970s on this sector.

Though the cross-price elasticities are small, many pairs of inputs are turning up as substitutes in the services sector. These input pairs are:

(1) and (2) — intermediate inputs from sectors I and II,

(1) and (6) — intermediate inputs from sector I and labor,

(1) and (7) — intermediate inputs from sector I and inventories,

(1) and (8) — intermediate inputs from sector I and machinery and equipment,

(2) and (7) — intermediate inputs from sector II and inventories,

(2) and (8) — intermediate inputs from sector II and machinery and equipment,

(3) and (6) — intermediate inputs from sector III and labor,

(3) and (7) — intermediate inputs from sector III and inventories,

(3) and (8) — intermediate inputs from sector III and machinery and equipment,

(5) and (6) — imports and labor,

(5) and (7) — imports and inventories,

(5) and (8) — imports and machinery and equipment, and

(6) and (7) — labor and inventories.

The substitutability of labor and the intermediate inputs of resource goods from sector I could have also contributed to buffering the services sector from the negative employment (and output) consequences of the rapid rise in resource prices in the 1970s.

The above interpretation of the price elasticities are partial equilibrium in nature in the sense that it has been carried out only at the sectoral level where under the assumption of competitive markets, prices are assumed to be exogenous to the producer and the effects of a change in the price of a good on the prices of other goods are ignored. In a traditional trade-theoretic small open economy model of the economy linking the different sectors, the prices of domestic resources are treated as endogenous to the economy and the corresponding endowments of these domestic resources as exogenous variables. Part III of this dissertation offers a theoretical framework, the empirical implementation of which is left undone, to measure the comparative static response of the economy's endogenous variables — output levels, employment levels of fixed-price factors and flexible prices of domestic resources — to marginal changes in the economy's exogenous variables — output prices, fixed factor prices and endowments of flexibly-priced domestic resources. Such a general equilibrium approach can capture the effects of sectoral reallocation of resources and of sectoral variation in producer prices due to taxes and subsidies.

For example, for the sectoral analysis of the price elasticities, wages or the price of labor is assumed exogenous. In a general equilibrium framework if wages are considered flexible, then the employment effect due to changes in the exogenous variables such as output price changes or capital growth would entail merely a reallocation of the labor force among the different sectors; the effect on the overall employment level for the economy's production sector is zero because wages will adjust to ensure full employment. On the other hand, if wages are rigid and the supply of labor remains elastic, there can be a change in the overall employment level for the economy in addition to the sectoral reallocation that can occur. Generally, the comparative static response of the endogenous variables to marginal changes in the exogenous variables can be decomposed Slutsky-like into substitution and scale effects. Hence, the obtained price elasticities from the sectoral profit function estimation are just a partial description of the production sector of the economy. In growth accounting or measuring aggregate productivity, it may then be necessary to move to a general equilibrium approach for a more complete model of the production side of the economy. However, sectoral analysis remains an integral part of or at least a starting point for this approach.

Table D.28: Parameter estimates, standard errors and t-values, sector I

Sector I: Resources			
parameter	estimate	standard error (asymptotic)	t-value (asymptotic)
b^1			
b^1_1	13.799006	0.827440	16.676750
b^1_2	-0.094647	0.044012	-2.150480
b^1_3	-1.258935	0.101966	-12.346630
b^1_4	-2.218970	0.125610	-17.665570
b^1_5	-1.505089	0.176363	-8.534035
b^1_6	-5.787323	0.142188	-40.701840
b^1_7	-0.624200	0.022746	-27.441970
b^1_8	-2.109089	0.072935	-28.917240
b^1_9	-0.206752	0.005341	-38.708710
A			
a_{11}	3.286856	0.321766	10.215050
a_{21}	-0.244728	0.046339	-5.281282
a_{31}	-0.506251	0.118429	-4.274710
a_{41}	-0.778740	0.108409	-7.183325
a_{51}	-0.595962	0.056171	-10.609720
a_{61}	-0.666742	0.155798	-4.279539
a_{71}	-0.074340	0.042467	-1.750519
a_{81}	-0.418550	0.159305	-2.627350
a_{22}	-0.073879	0.072734	-1.015744
a_{32}	0.463805	0.179051	2.590355
a_{42}	0.114735	0.135048	0.849589
a_{52}	-0.061035	0.030047	-2.031323
a_{62}	-0.612971	0.320180	-1.914458
a_{72}	-0.165158	0.127568	-1.294669
a_{82}	0.439017	0.321775	1.364361
a_{33}	-0.020895	0.346730	-0.060262
a_{43}	0.099870	0.099222	1.006536
a_{53}	-0.000269	0.112486	-0.002392
a_{63}	0.529142	0.398849	1.326674
a_{73}	-0.218552	0.096419	-2.266688
a_{83}	-0.438923	0.270604	-1.622013

Table D.28 (continued)

Sector I: Resources			
parameter	estimate	standard error (asymptotic)	t-value (asymptotic)
b^2			
b^2_1	20.340799	33.431060	0.608440
b^2_2	-5.898559	1.177927	-5.007578
b^2_3	-9.149633	2.621344	-3.490435
b^2_4	-14.567232	4.591132	-3.172906
b^2_5	26.149877	6.848966	3.818077
b^2_6	35.263812	4.169841	8.456872
b^2_7	2.100519	0.845487	2.484389
b^2_8	-1.910438	2.849728	-0.670393
b^2_9	-1.965330	0.062321	-31.535750
b^3			
b^3_1	-7.134038	6.118050	-1.166064
b^3_2	0.929281	0.228558	4.065836
b^3_3	2.207236	0.518083	4.260395
b^3_4	3.234346	0.840145	3.849746
b^3_5	-5.341682	1.242165	-4.300298
b^3_6	-1.359469	0.713450	-1.905484
b^3_7	0.032681	0.160953	0.203048
b^3_8	1.733370	0.520742	3.328652
b^3_9	0.238763	0.012965	18.415550
b^4			
b^4_1	-1.883992	2.637624	-0.714276
b^4_2	0.090897	0.107499	0.845559
b^4_3	-0.190608	0.218444	-0.872570
b^4_4	-1.225994	0.379086	-3.234080
b^4_5	2.274203	0.539119	4.218366
b^4_6	-2.345498	0.379104	-6.186952
b^4_7	0.121752	0.103464	1.176748
b^4_8	-1.530660	0.281130	-5.444660
b^4_9	-0.217965	0.031984	-6.814838
b^5			
b^5_1	3.039584	5.614411	0.541390
b^5_2	-0.233545	0.236010	-0.989554
b^5_3	-0.446838	0.621889	-0.718518
b^5_4	0.231474	0.794469	0.291357
b^5_5	0.791551	1.206412	0.656119
b^5_6	0.645616	0.630591	1.023826
b^5_7	-0.466562	0.199491	-2.338764
b^5_8	0.791816	0.540652	1.464556
b^5_9	-0.314715	0.079027	-3.982366

Table D.29: Parameter estimates, standard errors and t-values, sector II

Sector II: Manufacturing, Export Market-Oriented			
parameter	estimate	standard error (asymptotic)	t-value (asymptotic)
b^1			
b_1^1	-2.582182	0.287752	-8.973637
b_2^1	7.629525	0.814035	9.372475
b_3^1	-0.567492	0.162557	-3.491039
b_4^1	-0.781366	0.194586	-4.015528
b_5^1	-0.001625	0.251405	-0.006462
b_6^1	-3.834844	0.319021	-12.020680
b_7^1	-0.175671	0.037645	-4.666511
b_8^1	-0.923145	0.019620	-47.050700
A			
a_{11}	1.655250	0.196655	8.417023
a_{21}	-1.750806	0.661433	-2.646988
a_{31}	-0.556929	0.142866	-3.898251
a_{41}	0.262824	0.189390	1.387743
a_{51}	0.849280	0.197018	4.310669
a_{61}	-0.330582	0.295279	-1.119559
a_{71}	-0.195123	0.036128	-5.400851
a_{22}	5.506777	0.262682	20.963670
a_{32}	-0.598850	0.142967	-4.188734
a_{42}	-0.753136	0.130206	-5.784184
a_{52}	-2.005486	0.166316	-12.058270
a_{62}	-1.958408	0.203998	-9.600114
a_{72}	-0.093459	0.042068	-2.221615
a_{33}	-1.262175	0.124537	-10.134900
a_{43}	1.063532	0.178741	5.950114
a_{53}	-0.225747	0.102218	-2.208487
a_{63}	0.165269	0.137032	1.206063
a_{73}	0.027141	0.019320	1.404779
a_{44}	-0.473024	0.248447	-1.903926
a_{54}	0.335029	0.269936	1.241139
a_{64}	0.038151	0.286182	0.133311
a_{74}	-0.075376	0.060743	-1.240903
a_{55}	0.003132	1.144062	0.002737
a_{65}	-0.002796	1.013147	-0.002759
a_{75}	-0.000894	0.323141	-0.002767

Table D.29 (continued)

Sector II: Manufacturing, Export Market-Oriented			
parameter	estimate	standard error (asymptotic)	t-value (asymptotic)
b^2			
b_1^2	-11.604543	12.655320	-0.916970
b_2^2	275.324850	11.588510	23.758430
b_3^2	-27.664699	5.494059	-5.035384
b_4^2	-37.610446	4.670205	-8.053276
b_5^2	-64.603093	9.020864	-7.161519
b_6^2	-91.974909	7.328957	-12.549520
b_7^2	-2.309978	1.150459	-2.007876
b_8^2	-5.070982	0.508327	-9.975835
b^3			
b_1^3	2.718063	2.969757	0.915248
b_2^3	-47.139093	3.542087	-13.308280
b_3^3	4.237387	1.228671	3.448755
b_4^3	5.812041	1.009672	5.756365
b_5^3	8.187019	2.167449	3.777260
b_6^3	20.984322	1.875887	11.186340
b_7^3	0.108092	0.267714	0.403758
b_8^3	0.570583	0.126702	4.503349
b^4			
b_1^4	-0.290378	0.683839	-0.424629
b_2^4	3.074872	2.286611	1.344729
b_3^4	1.087119	0.288349	3.770146
b_4^4	-0.093692	0.355381	-0.263639
b_5^4	-1.593152	0.623517	-2.555107
b_6^4	-1.116796	0.660685	-1.690360
b_7^4	0.442621	0.061552	7.190951
b_8^4	-0.061131	0.048046	-1.272350
b^5			
b_1^5	-3.008172	1.601283	-1.878601
b_2^5	-26.776933	7.123452	-3.758983
b_3^5	0.185781	0.649921	0.285852
b_4^5	1.002010	1.122139	0.892946
b_5^5	16.242363	1.838392	8.835091
b_6^5	3.468409	1.878409	1.846461
b_7^5	-0.825516	0.190162	-4.341108
b_8^5	0.959071	0.125465	7.644138

Table D.30: Parameter estimates, standard errors and t-values, sector III

Sector III: Manufacturing, Domestic Market-Oriented			
parameter	estimate	standard error (asymptotic)	t-value (asymptotic)
b^1			
b_1^1	-3.322721	0.239550	-13.870670
b_2^1	-1.500219	0.176741	-8.488239
b_3^1	16.919832	1.273659	13.284430
b_4^1	-2.635995	0.298254	-8.838074
b_5^1	-1.367133	0.319178	-4.283297
b_6^1	-8.245282	0.522735	-15.773360
b_7^1	-0.594410	0.147615	-4.026752
b_8^1	-1.468335	0.133692	-10.983000
A			
a_{11}	1.096568	0.134312	8.164330
a_{21}	-0.134455	0.207034	-0.649431
a_{31}	-0.824493	1.410008	-0.584743
a_{41}	0.198039	0.377527	0.524570
a_{51}	-0.011911	0.332377	-0.035835
a_{61}	-0.117676	0.550047	-0.213938
a_{71}	-0.345585	0.193487	-1.786086
a_{22}	0.904617	0.148623	6.086655
a_{32}	-6.078417	1.154577	-5.264625
a_{42}	0.633608	0.339589	1.865808
a_{52}	2.170306	0.287607	7.546085
a_{62}	2.130666	0.544107	3.915897
a_{72}	-0.008462	0.217924	-0.038828
a_{33}	-2.568172	1.885418	-1.362123
a_{43}	0.921985	0.306376	3.009329
a_{53}	0.220493	1.222544	0.180356
a_{63}	0.972877	0.890977	1.091923
a_{73}	0.354448	0.302668	1.171078
a_{44}	0.814931	0.314691	2.589624
a_{54}	-0.074592	0.574468	-0.129846
a_{64}	-0.318221	0.181506	-1.753223
a_{74}	-0.336136	0.228753	-1.469432
a_{55}	0.507081	0.435560	1.164203
a_{65}	0.088481	0.175135	0.505217
a_{75}	-0.061200	0.360530	-0.169750

Table D.30 (continued)

Sector III: Manufacturing, Domestic Market-Oriented			
parameter	estimate	standard error (asymptotic)	t-value (asymptotic)
b^2			
b_1^2	-11.070679	5.165415	-2.143231
b_2^2	-27.898475	3.143094	-8.876119
b_3^2	201.439450	18.928080	10.642360
b_4^2	-29.767489	2.833704	-10.504800
b_5^2	-23.083506	7.436062	-3.104265
b_6^2	-64.116532	7.420050	-8.640984
b_7^2	-8.533584	1.739235	-4.906517
b_8^2	-8.487185	1.734876	-4.892100
b^3			
b_1^3	2.747288	0.949014	2.894887
b_2^3	3.997967	0.582058	6.868678
b_3^3	-27.635982	4.189496	-6.596494
b_4^3	4.383823	0.694869	6.308847
b_5^3	1.205501	1.425000	0.845966
b_6^3	14.363389	1.318002	10.897850
b_7^3	1.439613	0.354211	4.064281
b_8^3	1.036993	0.213732	4.851843
b^4			
b_1^4	-1.230061	0.367373	-3.348261
b_2^4	-0.331166	0.403924	-0.819872
b_3^4	5.875659	3.250754	1.807476
b_4^4	-0.984166	0.369839	-2.661068
b_5^4	-0.537429	0.875302	-0.613992
b_6^4	-4.449707	0.747030	-5.956532
b_7^4	-0.085350	0.143799	-0.593541
b_8^4	-0.521291	0.106588	-4.890705
b^5			
b_1^5	0.017332	0.987729	0.017548
b_2^5	0.156246	1.159823	0.134716
b_3^5	4.737317	10.085370	0.469721
b_4^5	-1.590329	1.216161	-1.307663
b_5^5	-2.827169	2.526988	-1.118790
b_6^5	-1.924429	2.297103	-0.837764
b_7^5	-0.002854	0.470645	-0.006065
b_8^5	1.457042	0.354564	4.109392

Table D.31: Parameter estimates, standard errors and t-values, sector IV

Sector IV: Services			
parameter	estimate	standard error (asymptotic)	t-value (asymptotic)
b^1			
b_1^1	-2.046595	0.174851	-11.704820
b_2^1	-2.162369	0.322279	-6.709610
b_3^1	-6.097963	0.379425	-16.071570
b_4^1	53.229182	1.492589	35.662310
b_5^1	-2.404431	0.197531	-12.172430
b_6^1	-30.945265	1.320193	-23.439960
b_7^1	-1.182908	0.144762	-8.171427
b_8^1	-2.726253	0.103621	-26.309930
A			
a_{11}	-0.736051	0.108259	-6.799003
a_{21}	0.285793	0.227262	1.257550
a_{31}	-0.736665	0.373461	-1.972536
a_{41}	-0.964165	1.161019	-0.830447
a_{51}	-0.064099	0.165806	-0.386590
a_{61}	1.675580	0.854913	1.959942
a_{71}	0.127689	0.121532	1.050664
a_{22}	-1.253453	0.199214	-6.291977
a_{32}	-0.794715	0.544978	-1.458251
a_{42}	3.064562	1.001811	3.059023
a_{52}	-0.374866	0.132440	-2.830461
a_{62}	-1.041441	0.960164	-1.084650
a_{72}	0.047832	0.095831	0.499127
a_{33}	0.574061	0.287264	1.998374
a_{43}	-1.284578	0.672928	-1.908938
a_{53}	0.107470	0.597442	0.179883
a_{63}	-0.181678	0.781197	-0.232563
a_{73}	0.032806	0.346793	0.094598
a_{44}	-0.895283	0.857428	-1.044150
a_{54}	1.097282	0.601043	1.825629
a_{64}	-0.033783	1.429727	-0.023629
a_{74}	-0.246516	0.532554	-0.462894
a_{55}	-0.397483	1.591915	-0.249689
a_{65}	0.735647	0.979069	0.751375
a_{75}	-0.372666	0.460289	-0.809634

Table D.31 (continued)

Sector IV: Services			
parameter	estimate	standard error (asymptotic)	t-value (asymptotic)
b^2			
b_1^2	2.498398	3.713159	0.672850
b_2^2	-17.168179	4.080740	-4.207124
b_3^2	-17.930061	7.443717	-2.408751
b_4^2	140.092390	26.159090	5.355401
b_5^2	-1.748013	6.251669	-0.279607
b_6^2	26.506863	17.949280	1.476765
b_7^2	-0.227954	3.475307	-0.065592
b_8^2	0.558538	1.752088	0.318784
b^3			
b_1^3	0.494163	0.554107	0.891818
b_2^3	2.512109	0.721581	3.481396
b_3^3	3.205970	1.093766	2.931131
b_4^3	-14.920833	3.995621	-3.734297
b_5^3	-0.093894	0.983450	-0.095475
b_6^3	1.293060	2.490229	0.519253
b_7^3	0.854487	0.558848	1.529014
b_8^3	-1.398014	0.253352	-5.518068
b^4			
b_1^4	-2.398159	0.687961	-3.485895
b_2^4	-0.255665	0.971271	-0.263227
b_3^4	-0.893248	1.554534	-0.574608
b_4^4	-12.396742	6.840618	-1.812225
b_5^4	0.858210	1.289008	0.665792
b_6^4	5.873732	4.566607	1.286235
b_7^4	-1.017362	0.737588	-1.379310
b_8^4	1.470164	0.377277	3.896776
b^5			
b_1^5	-0.173813	1.227695	-0.141577
b_2^5	-1.012060	1.415244	-0.715114
b_3^5	-1.551109	2.802733	-0.553427
b_4^5	24.873926	14.402620	1.727042
b_5^5	-3.733729	2.378537	-1.569758
b_6^5	-8.905718	7.735176	-1.151327
b_7^5	1.341349	1.283622	1.044972
b_8^5	-1.593550	0.593514	-2.684939

Table D.32: Own price elasticities and 1971 cross-price elasticities, sector I

Sector I: Resources									
own price elasticities, ϵ_{ii}									
year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1961	0.679	-0.268	-0.361	-0.249	-0.284	-0.127	-0.133	-0.297	-0.074
1962	0.744	-0.272	-0.337	-0.228	-0.402	-0.144	-0.239	-0.330	-0.070
1963	0.778	-0.261	-0.336	-0.222	-0.479	-0.158	-0.281	-0.353	-0.070
1964	0.787	-0.240	-0.340	-0.223	-0.538	-0.166	-0.248	-0.360	-0.070
1965	0.794	-0.228	-0.336	-0.222	-0.545	-0.185	-0.268	-0.368	-0.067
1966	0.819	-0.228	-0.329	-0.218	-0.529	-0.215	-0.372	-0.388	-0.065
1967	0.782	-0.206	-0.340	-0.234	-0.452	-0.231	-0.222	-0.341	-0.059
1968	0.824	-0.227	-0.356	-0.245	-0.419	-0.264	-0.252	-0.362	-0.061
1969	0.863	-0.247	-0.380	-0.257	-0.382	-0.306	-0.267	-0.373	-0.056
1970	0.874	-0.250	-0.392	-0.268	-0.358	-0.335	-0.220	-0.362	-0.051
1971	0.890	-0.263	-0.421	-0.283	-0.287	-0.392	-0.220	-0.366	-0.042
1972	0.897	-0.258	-0.419	-0.278	-0.273	-0.425	-0.268	-0.364	-0.045
1973	0.874	-0.226	-0.355	-0.236	-0.248	-0.452	-0.755	-0.385	-0.064
1974	0.901	-0.200	-0.324	-0.199	-0.404	-0.401	-1.069	-0.348	-0.059
1975	0.901	-0.192	-0.328	-0.195	-0.430	-0.400	-1.018	-0.319	-0.053
1976	0.946	-0.212	-0.344	-0.208	-0.459	-0.448	-0.748	-0.290	-0.047
1977	0.968	-0.213	-0.347	-0.208	-0.504	-0.475	-0.595	-0.264	-0.041
1978	0.980	-0.207	-0.346	-0.199	-0.528	-0.487	-0.824	-0.271	-0.036
1979	0.984	-0.195	-0.337	-0.179	-0.613	-0.451	-1.167	-0.284	-0.039
1980	1.100	-0.211	-0.339	-0.169	-0.901	-0.458	-1.357	-0.275	-0.036
1971 cross-price elasticities, ϵ_{ij}									
i\j	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1)	0.890	-0.066	-0.137	-0.211	-0.161	-0.181	-0.020	-0.113	-0.000
(2)	3.239	-0.263	-0.361	-0.733	-0.605	-0.839	-0.122	-0.282	-0.033
(3)	1.483	-0.080	-0.421	-0.397	-0.244	-0.038	0.031	-0.379	0.043
(4)	1.152	-0.082	-0.200	-0.283	-0.206	-0.226	-0.008	-0.150	0.003
(5)	1.566	-0.120	-0.219	-0.365	-0.287	-0.347	-0.044	-0.178	-0.006
(6)	0.780	-0.074	-0.015	-0.179	-0.155	-0.392	-0.013	0.079	-0.033
(7)	0.667	-0.083	0.094	-0.047	-0.149	-0.096	-0.220	-0.149	-0.018
(8)	0.898	-0.046	-0.277	-0.217	-0.145	0.145	-0.036	-0.366	0.044
(9)	0.016	-0.026	0.155	0.019	-0.023	-0.290	-0.021	0.213	-0.042

Note: Column labels (1), ..., (9) refer to the following goods, respectively:

(1) resource goods (from sector I),
(2) manufactured goods (from sector II),
(3) manufactured goods (from sector III),
(4) service goods (from sector IV),
(5) imports,
(6) labor,
(7) inventories,
(8) machinery and equipment, and
(9) land.

Table D.33: Own price elasticities and 1971 cross-price elasticities, sector II

Sector II: Manufacturing, Export Market-Oriented								
own price elasticities, ϵ_{ii}								
year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1961	-1.020	2.310	-2.433	-1.186	-2.885	-0.524	-0.213	-0.115
1962	-1.002	2.165	-2.058	-1.053	-2.748	-0.485	-0.254	-0.120
1963	-0.973	1.988	-1.791	-0.946	-2.322	-0.470	-0.268	-0.125
1964	-0.964	1.893	-1.622	-0.883	-2.080	-0.468	-0.261	-0.129
1965	-0.963	1.844	-1.528	-0.846	-1.891	-0.485	-0.262	-0.130
1966	-1.015	1.842	-1.496	-0.812	-1.801	-0.521	-0.197	-0.111
1967	-1.049	1.896	-1.515	-0.815	-1.793	-0.569	-0.188	-0.093
1968	-1.035	1.902	-1.442	-0.823	-1.675	-0.620	-0.221	-0.093
1969	-1.012	1.902	-1.393	-0.830	-1.572	-0.660	-0.254	-0.100
1970	-1.059	1.893	-1.403	-0.811	-1.485	-0.707	-0.132	-0.077
1971	-0.994	1.811	-1.401	-0.808	-1.253	-0.759	-0.175	-0.079
1972	-0.974	1.777	-1.375	-0.796	-1.141	-0.801	-0.230	-0.082
1973	-1.062	1.691	-1.260	-0.746	-1.004	-0.799	-0.302	-0.094
1974	-1.295	1.639	-1.290	-0.668	-0.935	-0.781	-0.229	-0.092
1975	-1.565	1.699	-1.377	-0.644	-0.976	-0.805	-0.149	-0.068
1976	-1.672	1.762	-1.373	-0.676	-0.931	-0.904	-0.159	-0.063
1977	-1.783	1.810	-1.320	-0.665	-0.962	-0.935	-0.164	-0.067
1978	-1.769	1.813	-1.288	-0.635	-1.001	-0.920	-0.208	-0.075
1979	-1.719	1.696	-1.262	-0.554	-0.959	-0.856	-0.226	-0.084
1980	-1.815	1.779	-1.349	-0.542	-1.060	-0.891	-0.119	-0.072
1971 cross-price elasticities, ϵ_{ij}								
$i\backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1)	-0.994	1.052	0.335	-0.158	-0.510	0.199	0.117	-0.040
(2)	-0.157	1.811	-0.126	-0.250	-0.680	-0.554	-0.009	-0.035
(3)	0.571	1.439	-1.401	0.643	-0.628	-0.711	-0.081	0.168
(4)	-0.176	1.869	0.421	-0.808	-0.542	-0.627	-0.034	-0.103
(5)	-0.359	3.200	-0.259	-0.341	-1.253	-0.925	0.002	-0.066
(6)	0.105	1.950	-0.219	-0.295	-0.692	-0.759	-0.048	-0.041
(7)	1.062	0.569	-0.429	-0.275	0.032	-0.819	-0.175	0.035
(8)	-0.088	0.522	0.217	-0.204	-0.206	-0.171	0.009	-0.079

Note: Column labels (1), ..., (8) refer to the following goods, respectively:

(1) resource goods (from sector I),
(2) manufactured goods (from sector II),
(3) manufactured goods (from sector III),
(4) service goods (from sector IV),
(5) imports,
(6) labor,
(7) inventories, and
(8) machinery and equipment.

Table D.34: Own price elasticities and 1971 cross-price elasticities, sector III

Sector III: Manufacturing, Domestic Market-Oriented								
own price elasticities, ϵ_{ii}								
year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1961	-0.388	-0.398	1.799	-0.566	-2.029	-0.435	-0.349	-0.196
1962	-0.393	-0.383	1.834	-0.536	-2.326	-0.427	-0.409	-0.220
1963	-0.377	-0.362	1.814	-0.509	-2.460	-0.425	-0.401	-0.220
1964	-0.364	-0.347	1.774	-0.491	-2.323	-0.430	-0.444	-0.244
1965	-0.370	-0.335	1.709	-0.482	-2.060	-0.441	-0.427	-0.248
1966	-0.389	-0.327	1.680	-0.474	-1.925	-0.466	-0.380	-0.229
1967	-0.395	-0.326	1.677	-0.486	-1.826	-0.505	-0.331	-0.200
1968	-0.393	-0.331	1.733	-0.503	-1.812	-0.552	-0.394	-0.207
1969	-0.395	-0.336	1.782	-0.512	-1.822	-0.596	-0.414	-0.208
1970	-0.400	-0.355	1.852	-0.534	-1.909	-0.651	-0.332	-0.182
1971	-0.376	-0.335	1.766	-0.530	-1.514	-0.691	-0.461	-0.202
1972	-0.404	-0.325	1.764	-0.522	-1.436	-0.727	-0.507	-0.203
1973	-0.507	-0.321	1.744	-0.488	-1.404	-0.724	-0.493	-0.198
1974	-0.558	-0.342	1.761	-0.451	-1.517	-0.721	-0.427	-0.174
1975	-0.516	-0.341	1.685	-0.442	-1.358	-0.735	-0.365	-0.156
1976	-0.472	-0.362	1.803	-0.476	-1.399	-0.843	-0.388	-0.153
1977	-0.454	-0.387	1.923	-0.483	-1.576	-0.905	-0.374	-0.150
1978	-0.487	-0.370	1.830	-0.456	-1.416	-0.863	-0.394	-0.160
1979	-0.509	-0.392	1.793	-0.411	-1.387	-0.827	-0.416	-0.165
1980	-0.496	-0.428	1.910	-0.407	-1.600	-0.866	-0.324	-0.147
1971 cross-price elasticities, ϵ_{ij}								
$i\backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1)	-0.376	0.046	0.283	-0.068	0.004	0.040	0.119	-0.048
(2)	0.059	-0.335	2.158	-0.219	-0.787	-0.778	-0.016	-0.082
(3)	-0.036	-0.215	1.766	-0.255	-0.549	-0.613	-0.023	-0.075
(4)	-0.059	-0.148	1.731	-0.530	-0.411	-0.533	0.006	-0.056
(5)	0.004	-0.592	4.146	-0.457	-1.514	-1.480	-0.017	-0.088
(6)	0.016	-0.240	1.892	-0.242	-0.605	-0.691	-0.058	-0.072
(7)	0.483	-0.049	0.732	0.027	-0.074	-0.598	-0.461	-0.059
(8)	-0.081	-0.108	0.986	-0.108	-0.154	-0.309	-0.024	-0.202

Note: Column labels (1), ..., (8) refer to the following goods, respectively:

(1) resource goods (from sector I),
(2) manufactured goods (from sector II),
(3) manufactured goods (from sector III),
(4) service goods (from sector IV),
(5) imports,
(6) labor,
(7) inventories, and
(8) machinery and equipment.

Table D.35: Own price elasticities and 1971 cross-price elasticities, sector IV

Sector IV: Services								
own price elasticities, ϵ_{ii}								
year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1961	-0.419	-0.665	-0.287	0.231	-0.755	-0.130	-0.161	-0.367
1962	-0.433	-0.711	-0.285	0.264	-1.750	-0.131	-0.142	-0.371
1963	-0.422	-0.694	-0.277	0.263	-1.814	-0.133	-0.148	-0.373
1964	-0.414	-0.700	-0.275	0.266	-2.032	-0.135	-0.164	-0.380
1965	-0.409	-0.692	-0.270	0.263	-1.944	-0.138	-0.160	-0.366
1966	-0.402	-0.675	-0.263	0.255	-1.716	-0.142	-0.176	-0.343
1967	-0.399	-0.668	-0.258	0.250	-1.543	-0.146	-0.179	-0.312
1968	-0.386	-0.689	-0.252	0.245	-1.312	-0.150	-0.201	-0.297
1969	-0.375	-0.703	-0.250	0.242	-1.192	-0.153	-0.219	-0.273
1970	-0.372	-0.681	-0.248	0.239	-1.165	-0.158	-0.246	-0.258
1971	-0.364	-0.649	-0.244	0.214	-0.573	-0.164	-0.313	-0.248
1972	-0.361	-0.716	-0.246	0.218	-0.558	-0.168	-0.408	-0.237
1973	-0.409	-0.825	-0.253	0.225	-0.559	-0.173	-0.461	-0.219
1974	-0.601	-0.856	-0.277	0.226	-0.599	-0.182	-0.443	-0.194
1975	-0.605	-0.845	-0.278	0.225	-0.592	-0.191	-0.433	-0.177
1976	-0.578	-0.847	-0.269	0.223	-0.552	-0.200	-0.444	-0.164
1977	-0.609	-0.898	-0.273	0.230	-0.593	-0.206	-0.290	-0.158
1978	-0.632	-0.952	-0.280	0.232	-0.616	-0.210	-0.429	-0.165
1979	-0.660	-1.147	-0.305	0.244	-0.653	-0.212	-0.683	-0.167
1980	-0.677	-1.064	-0.306	0.237	-0.607	-0.222	-0.769	-0.154
1971 cross-price elasticities, ϵ_{ij}								
$i \backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1)	-0.364	0.141	-0.365	-0.477	-0.032	0.829	0.063	0.204
(2)	0.083	-0.649	-0.309	1.618	-0.177	-0.701	0.009	0.127
(3)	-0.088	-0.128	-0.244	0.400	-0.066	0.083	0.018	0.025
(4)	0.012	-0.069	-0.041	0.214	-0.037	-0.076	0.003	-0.006
(5)	-0.018	-0.170	-0.154	0.833	-0.573	0.025	0.055	0.002
(6)	0.045	-0.066	0.019	0.167	0.002	-0.164	0.004	-0.008
(7)	0.134	0.033	0.162	-0.288	0.207	0.154	-0.313	-0.088
(8)	0.087	0.093	0.043	0.101	0.002	-0.060	-0.018	-0.248

Note: Column labels (1), ..., (8) refer to the following goods, respectively:

(1) resource goods (from sector I),
(2) manufactured goods (from sector II),
(3) manufactured goods (from sector III),
(4) service goods (from sector IV),
(5) imports,
(6) labor,
(7) inventories, and
(8) machinery and equipment.

Table D.36: Five-year interval cross-price elasticities, sector I

Sector I: Resources									
cross-price elasticities, ϵ_{ij}									
$i \backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
year 1961:									
(1)	0.679	-0.051	-0.119	-0.159	-0.140	-0.083	-0.017	-0.110	-0.000
(2)	3.131	-0.268	-0.395	-0.724	-0.687	-0.543	-0.126	-0.333	-0.054
(3)	1.282	-0.070	-0.361	-0.316	-0.231	-0.035	0.024	-0.336	0.044
(4)	1.020	-0.076	-0.188	-0.249	-0.211	-0.139	-0.009	-0.146	-0.002
(5)	1.334	-0.107	-0.203	-0.313	-0.284	-0.200	-0.039	-0.175	-0.012
(6)	0.365	-0.039	-0.014	-0.095	-0.092	-0.127	-0.008	0.035	-0.024
(7)	0.428	-0.053	0.056	-0.035	-0.105	-0.044	-0.133	-0.098	-0.016
(8)	0.773	-0.039	-0.219	-0.160	-0.129	0.056	-0.027	-0.297	0.042
(9)	0.002	-0.039	0.180	-0.012	-0.055	-0.242	-0.028	0.269	-0.074
year 1965:									
(1)	0.794	-0.056	-0.122	-0.165	-0.186	-0.104	-0.031	-0.133	0.001
(2)	2.887	-0.228	-0.317	-0.588	-0.710	-0.536	-0.164	-0.301	-0.043
(3)	1.243	-0.063	-0.336	-0.287	-0.259	-0.024	0.043	-0.367	0.050
(4)	0.999	-0.069	-0.170	-0.222	-0.236	-0.147	-0.006	-0.150	0.001
(5)	2.270	-0.169	-0.309	-0.474	-0.545	-0.365	-0.091	-0.302	-0.015
(6)	0.460	-0.046	-0.011	-0.107	-0.132	-0.185	-0.010	0.064	-0.032
(7)	0.611	-0.064	0.084	-0.021	-0.149	-0.047	-0.268	-0.129	-0.019
(8)	0.843	-0.037	-0.228	-0.157	-0.157	0.092	-0.041	-0.368	0.054
(9)	-0.044	-0.027	0.155	0.006	-0.038	-0.230	-0.030	0.274	-0.067
year 1970:									
(1)	0.874	-0.063	-0.130	-0.197	-0.191	-0.159	-0.021	-0.112	-0.000
(2)	3.164	-0.250	-0.341	-0.684	-0.712	-0.743	-0.124	-0.274	-0.037
(3)	1.433	-0.075	-0.392	-0.366	-0.283	-0.033	0.031	-0.364	0.048
(4)	1.139	-0.079	-0.192	-0.268	-0.245	-0.204	-0.007	-0.147	0.003
(5)	1.625	-0.121	-0.219	-0.362	-0.358	-0.327	-0.046	-0.184	-0.007
(6)	0.725	-0.068	-0.014	-0.161	-0.175	-0.335	-0.012	0.075	-0.036
(7)	0.659	-0.078	0.090	-0.040	-0.171	-0.081	-0.220	-0.141	-0.019
(8)	0.902	-0.044	-0.266	-0.204	-0.173	0.132	-0.036	-0.362	0.050
(9)	0.009	-0.026	0.154	0.017	-0.029	-0.273	-0.022	0.220	-0.051

Table D.36 (continued)

Sector I: Resources									
cross-price elasticities, ϵ_{ij}									
$i\backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
year 1975:									
(1)	0.901	-0.050	-0.108	-0.153	-0.254	-0.171	-0.054	-0.107	-0.004
(2)	3.267	-0.192	-0.301	-0.529	-0.927	-0.752	-0.244	-0.271	-0.051
(3)	1.327	-0.057	-0.328	-0.284	-0.369	-0.066	0.059	-0.338	0.055
(4)	1.043	-0.056	-0.157	-0.195	-0.297	-0.196	-0.009	-0.135	0.002
(5)	1.521	-0.086	-0.179	-0.260	-0.430	-0.308	-0.081	-0.167	-0.010
(6)	0.940	-0.064	-0.029	-0.157	-0.282	-0.400	-0.021	0.068	-0.055
(7)	2.005	-0.139	0.178	-0.051	-0.500	-0.140	-1.018	-0.287	-0.047
(8)	0.855	-0.033	-0.219	-0.158	-0.222	0.099	-0.062	-0.319	0.059
(9)	0.087	-0.018	0.099	0.006	-0.037	-0.221	-0.028	0.165	-0.053
year 1980:									
(1)	1.100	-0.056	-0.113	-0.162	-0.419	-0.193	-0.042	-0.110	-0.005
(2)	3.941	-0.211	-0.310	-0.552	-1.507	-0.820	-0.202	-0.282	-0.056
(3)	1.613	-0.063	-0.339	-0.297	-0.601	-0.068	0.046	-0.353	0.062
(4)	1.057	-0.051	-0.135	-0.169	-0.402	-0.176	-0.008	-0.118	0.002
(5)	2.365	-0.121	-0.237	-0.349	-0.901	-0.430	-0.088	-0.225	-0.014
(6)	1.225	-0.074	-0.030	-0.171	-0.483	-0.458	-0.021	0.075	-0.063
(7)	3.663	-0.249	0.280	-0.110	-1.348	-0.281	-1.357	-0.508	-0.091
(8)	0.829	-0.030	-0.186	-0.137	-0.300	0.089	-0.044	-0.275	0.054
(9)	0.071	-0.012	0.063	0.005	-0.036	-0.146	-0.015	0.105	-0.036

Note: Column labels (1), ..., (9) refer to the following goods, respectively:

(1) resource goods (from sector I),
(2) manufactured goods (from sector II),
(3) manufactured goods (from sector III),
(4) service goods (from sector IV),
(5) imports,
(6) labor,
(7) inventories,
(8) machinery and equipment, and
(9) land.

Table D.37: Five-year interval cross-price elasticities, sector II

Sector II: Manufacturing, Export Market-Oriented								
cross-price elasticities, ϵ_{ij}								
$i\backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
year 1961:								
(1)	-1.020	1.215	0.373	-0.181	-0.579	0.121	0.123	-0.050
(2)	-0.231	2.310	-0.193	-0.338	-0.944	-0.529	-0.013	-0.062
(3)	0.875	2.381	-2.433	0.970	-1.097	-0.851	-0.135	0.291
(4)	-0.271	2.651	0.617	-1.186	-0.876	-0.695	-0.055	-0.186
(5)	-0.799	6.846	-0.646	-0.810	-2.885	-1.506	-0.005	-0.195
(6)	0.077	1.769	-0.231	-0.296	-0.694	-0.524	-0.047	-0.054
(7)	1.204	0.664	-0.565	-0.360	-0.034	-0.728	-0.213	0.033
(8)	-0.110	0.708	0.269	-0.270	-0.306	-0.184	0.007	-0.115
year 1965:								
(1)	-0.963	1.083	0.337	-0.154	-0.550	0.132	0.174	-0.058
(2)	-0.173	1.844	-0.141	-0.249	-0.767	-0.440	-0.015	-0.059
(3)	0.582	1.519	-1.528	0.615	-0.727	-0.579	-0.131	0.249
(4)	-0.200	2.024	0.464	-0.846	-0.666	-0.542	-0.060	-0.173
(5)	-0.505	4.396	-0.387	-0.470	-1.891	-0.997	-0.000	-0.146
(6)	0.076	1.588	-0.194	-0.241	-0.628	-0.485	-0.062	-0.055
(7)	1.009	0.544	-0.442	-0.268	-0.001	-0.621	-0.262	0.042
(8)	-0.098	0.626	0.244	-0.225	-0.269	-0.161	0.012	-0.130
year 1970:								
(1)	-1.059	1.159	0.359	-0.163	-0.570	0.209	0.103	-0.038
(2)	-0.169	1.893	-0.130	-0.254	-0.754	-0.543	-0.007	-0.035
(3)	0.581	1.444	-1.403	0.633	-0.669	-0.680	-0.068	0.162
(4)	-0.177	1.890	0.426	-0.811	-0.587	-0.610	-0.029	-0.102
(5)	-0.396	3.590	-0.287	-0.375	-1.485	-0.979	0.002	-0.070
(6)	0.106	1.897	-0.214	-0.286	-0.718	-0.707	-0.039	-0.039
(7)	0.966	0.474	-0.391	-0.252	0.024	-0.717	-0.132	0.028
(8)	-0.085	0.532	0.221	-0.207	-0.223	-0.169	0.007	-0.077

Table D.37 (continued)

Sector II: Manufacturing, Export Market-Oriented								
cross-price elasticities, ϵ_{ij}								
$i\backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
year 1975:								
(1)	-1.565	1.490	0.444	-0.167	-0.617	0.319	0.133	-0.038
(2)	-0.197	1.699	-0.110	-0.207	-0.631	-0.520	-0.005	-0.029
(3)	0.722	1.351	-1.377	0.580	-0.615	-0.744	-0.072	0.155
(4)	-0.169	1.592	0.361	-0.644	-0.459	-0.570	-0.027	-0.084
(5)	-0.328	2.544	-0.201	-0.241	-0.976	-0.753	0.001	-0.046
(6)	0.156	1.917	-0.223	-0.274	-0.688	-0.805	-0.044	-0.039
87)	1.267	0.379	-0.420	-0.257	0.015	-0.859	-0.149	0.024
(8)	-0.078	0.456	0.197	-0.171	-0.177	-0.165	0.005	-0.068
year 1980:								
(1)	-1.815	1.662	0.448	-0.145	-0.621	0.380	0.123	-0.032
(2)	-0.245	1.779	-0.101	-0.198	-0.668	-0.532	-0.003	-0.032
(3)	0.849	1.290	-1.349	0.549	-0.648	-0.794	-0.067	0.169
(4)	-0.154	1.427	0.308	-0.542	-0.415	-0.521	-0.023	-0.081
(5)	-0.377	2.744	-0.207	-0.237	-1.060	-0.813	0.000	-0.050
(6)	0.215	2.035	-0.236	-0.277	-0.756	-0.891	-0.044	-0.045
(7)	1.272	0.217	-0.365	-0.222	0.001	-0.802	-0.119	0.017
(8)	-0.066	0.442	0.183	-0.156	-0.169	-0.165	0.003	-0.072

Note: Column labels (1), ..., (8) refer to the following goods, respectively:

(1) resource goods (from sector I),
(2) manufactured goods (from sector II),
(3) manufactured goods (from sector III),
(4) service goods (from sector IV),
(5) imports,
(6) labor,
(7) inventories, and
(8) machinery and equipment.

Table D.38: Five-year interval cross-price elasticities, sector III

Sector III: Manufacturing, Domestic Market-Oriented								
cross-price elasticities, ϵ_{ij}								
$i\backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
year 1961:								
(1)	-0.388	0.036	0.396	-0.076	-0.024	0.006	0.094	-0.044
(2)	0.054	-0.398	2.430	-0.266	-1.026	-0.678	-0.022	-0.091
(3)	-0.058	-0.237	1.799	-0.270	-0.660	-0.483	-0.020	-0.072
(4)	-0.077	-0.180	1.878	-0.566	-0.534	-0.457	-0.002	-0.062
(5)	-0.025	-0.727	4.797	-0.559	-2.029	-1.319	-0.029	-0.109
(6)	0.003	-0.208	1.523	-0.208	-0.572	-0.435	-0.044	-0.059
(7)	0.434	-0.069	0.619	-0.011	-0.124	-0.440	-0.349	-0.058
(8)	-0.100	-0.138	1.122	-0.139	-0.233	-0.288	-0.029	-0.196
year 1965:								
(1)	-0.370	0.040	0.314	-0.065	-0.006	0.021	0.118	-0.052
(2)	0.057	-0.335	2.119	-0.211	-0.908	-0.604	-0.020	-0.097
(3)	-0.045	-0.215	1.709	-0.236	-0.631	-0.470	-0.024	-0.088
(4)	-0.064	-0.150	1.644	-0.482	-0.472	-0.412	0.002	-0.066
(5)	-0.007	-0.701	4.788	-0.514	-2.060	-1.352	-0.028	-0.127
(6)	0.010	-0.198	1.517	-0.190	-0.574	-0.441	-0.052	-0.070
(7)	0.430	-0.051	0.614	0.007	-0.093	-0.416	-0.427	-0.063
(8)	-0.091	-0.121	1.084	-0.117	-0.207	-0.269	-0.031	-0.248
year 1970:								
(1)	-0.400	0.052	0.299	-0.065	0.013	0.047	0.098	-0.043
(2)	0.069	-0.355	2.263	-0.222	-0.896	-0.769	-0.014	-0.076
(3)	-0.040	-0.229	1.852	-0.260	-0.631	-0.606	-0.017	-0.068
(4)	-0.060	-0.153	1.767	-0.534	-0.455	-0.517	0.003	-0.051
(5)	0.014	-0.693	4.822	-0.513	-1.909	-1.615	-0.017	-0.089
(6)	0.020	-0.241	1.873	-0.235	-0.653	-0.651	-0.048	-0.064
(7)	0.437	-0.049	0.570	0.015	-0.074	-0.517	-0.332	-0.051
(8)	-0.083	-0.110	0.967	-0.107	-0.165	-0.297	-0.022	-0.182

Table D.38 (continued)

Sector III: Manufacturing, Domestic Market-Oriented								
	cross-price elasticities, ϵ_{ij}							
$i \backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
year 1975:								
(1)	-0.516	0.060	0.359	-0.061	0.018	0.066	0.116	-0.042
(2)	0.083	-0.341	2.101	-0.190	-0.799	-0.770	-0.016	-0.068
(3)	-0.050	-0.213	1.685	-0.213	-0.546	-0.587	-0.016	-0.059
(4)	-0.063	-0.144	1.593	-0.442	-0.397	-0.505	0.002	-0.045
(5)	0.016	-0.532	3.587	-0.348	-1.358	-1.288	-0.015	-0.063
(6)	0.031	-0.262	1.972	-0.227	-0.658	-0.735	-0.056	-0.065
(7)	0.540	-0.053	0.556	0.010	-0.075	-0.563	-0.365	-0.050
(8)	-0.086	-0.102	0.880	-0.088	-0.142	-0.284	-0.022	-0.156
year 1980:								
(1)	-0.496	0.079	0.240	-0.047	0.061	0.106	0.094	-0.037
(2)	0.127	-0.428	2.583	-0.199	-1.067	-0.935	-0.012	-0.069
(3)	-0.037	-0.247	1.910	-0.214	-0.680	-0.664	-0.012	-0.056
(4)	-0.051	-0.134	1.503	-0.407	-0.400	-0.479	0.004	-0.036
(5)	0.054	-0.584	3.890	-0.326	-1.600	-1.378	-0.007	-0.049
(6)	0.057	-0.311	2.316	-0.238	-0.839	-0.866	-0.054	-0.064
(7)	0.569	-0.045	0.480	0.020	-0.049	-0.602	-0.324	-0.048
(8)	-0.081	-0.095	0.802	-0.074	-0.122	-0.265	-0.018	-0.147

Note: Column labels (1), ..., (8) refer to the following goods, respectively:
- (1) resource goods (from sector I),
- (2) manufactured goods (from sector II),
- (3) manufactured goods (from sector III),
- (4) service goods (from sector IV),
- (5) imports,
- (6) labor,
- (7) inventories, and
- (8) machinery and equipment.

Table D.39: Five-year interval cross-price elasticities, sector IV

Sector IV: Services								
cross-price elasticities, ϵ_{ij}								
$i \backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
year 1961:								
(1)	-0.419	0.148	-0.375	-0.298	-0.025	0.675	0.055	0.239
(2)	0.098	-0.665	-0.338	1.515	-0.187	-0.562	0.006	0.133
(3)	-0.109	-0.148	-0.287	0.494	-0.074	0.074	0.017	0.033
(4)	0.010	-0.077	-0.057	0.231	-0.047	-0.056	0.004	-0.007
(5)	-0.019	-0.221	-0.200	1.101	-0.755	0.030	0.058	0.006
(6)	0.052	-0.065	0.019	0.128	0.003	-0.130	0.002	-0.010
(7)	0.099	0.017	0.108	-0.190	0.133	0.056	-0.161	-0.061
(8)	0.143	0.120	0.068	0.132	0.005	-0.079	-0.021	-0.367
year 1965:								
(1)	-0.409	0.157	-0.378	-0.362	-0.026	0.712	0.055	0.251
(2)	0.099	-0.692	-0.330	1.613	-0.282	-0.568	0.007	0.152
(3)	-0.101	-0.141	-0.270	0.468	-0.095	0.087	0.016	0.036
(4)	0.011	-0.080	-0.054	0.263	-0.087	-0.051	0.005	-0.006
(5)	-0.020	-0.354	-0.279	2.219	-1.944	0.230	0.102	0.047
(6)	0.050	-0.064	0.023	0.116	0.021	-0.138	0.002	-0.010
(7)	0.091	0.019	0.102	-0.243	0.215	0.043	-0.160	-0.066
(8)	0.129	0.125	0.069	0.105	0.031	-0.072	-0.021	-0.366
year 1970:								
(1)	-0.372	0.149	-0.370	-0.462	-0.035	0.818	0.061	0.211
(2)	0.089	-0.681	-0.319	1.708	-0.260	-0.681	0.009	0.136
(3)	-0.089	-0.130	-0.248	0.421	-0.088	0.089	0.018	0.027
(4)	0.012	-0.072	-0.044	0.239	-0.066	-0.067	0.004	-0.005
(5)	-0.019	-0.233	-0.194	1.385	-1.165	0.132	0.075	0.018
(6)	0.045	-0.063	0.021	0.147	0.014	-0.158	0.003	-0.008
(7)	0.110	0.027	0.131	-0.297	0.256	0.095	-0.246	-0.077
(8)	0.091	0.099	0.048	0.088	0.014	-0.064	-0.018	-0.258

Table D.39 (continued)

Sector IV: Services								
cross-price elasticities, ϵ_{ij}								
$i \backslash j$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
year 1975:								
(1)	-0.605	0.179	-0.420	-0.458	-0.032	1.058	0.079	0.199
(2)	0.155	-0.845	-0.384	2.042	-0.220	-0.894	0.013	0.133
(3)	-0.141	-0.149	-0.278	0.481	-0.074	0.116	0.023	0.023
(4)	0.015	-0.077	-0.047	0.225	-0.039	-0.079	0.004	-0.003
(5)	-0.023	-0.181	-0.156	0.856	-0.592	0.035	0.060	0.001
(6)	0.073	-0.071	0.024	0.167	0.003	-0.191	0.004	-0.010
(7)	0.252	0.046	0.212	-0.422	0.267	0.179	-0.433	-0.101
(8)	0.107	0.082	0.037	0.042	0.001	-0.074	-0.017	-0.177
year 1980:								
(1)	-0.677	0.172	-0.360	-0.332	-0.026	0.970	0.080	0.174
(2)	0.242	-1.064	-0.449	2.431	-0.273	-1.057	0.017	0.154
(3)	-0.198	-0.175	-0.306	0.565	-0.084	0.143	0.029	0.026
(4)	0.016	-0.086	-0.051	0.237	-0.044	-0.076	0.005	-0.001
(5)	-0.025	-0.185	-0.147	0.852	-0.607	0.045	0.066	0.001
(6)	0.108	-0.084	0.029	0.171	0.005	-0.222	0.004	-0.012
(7)	0.515	0.078	0.337	-0.704	0.447	0.255	-0.769	-0.159
(8)	0.123	0.077	0.033	0.017	0.001	-0.078	-0.017	-0.154

Note: Column labels (1), ..., (8) refer to the following goods, respectively:

(1) resource goods (from sector I),
(2) manufactured goods (from sector II),
(3) manufactured goods (from sector III),
(4) service goods (from sector IV),
(5) imports,
(6) labor,
(7) inventories, and
(8) machinery and equipment.