THE USE OF FIN-CORRUGATED PERIODIC SURFACES FOR THE REDUCTION OF INTERFERENCE FROM LARGE REFLECTING SURFACES

by

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Electrical Engineering

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Abstract

The use of periodic structures to reduce interference from large reflecting surfaces is proposed. Instrument landing system (ILS) interference from large hangars and terminal buildings is cited as a typical problem. An analytical and numerical investigation of an infinite fin-corrugated surface composed of infinitely thin fins of spacing $\lambda/2 < a < \lambda$ under TM polarized plane wave illumination is described. Specular reflection from this surface can be completely converted to backscatter in a direction opposite to the incident wave when the angle of incidence from the normal to the surface and the fin height are properly chosen. Experiments were performed at 35 and 37 GHz. using finite size fin-corrugated surfaces with fins of finite thickness under non-plane wave illumination and the results indicate that these surfaces behave essentially as predicted. In addition, the experimental surfaces remain completely effective for small oblique angles of incidence and have sufficient bandwidth for ILS applications.
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\( a \) = fin period
\( a' \) = fin spacing
\( a_0 \) = incident field amplitude coefficient in the overall problem
\( \overline{A}_0, \overline{A}_{-1}, \overline{A}_0, \overline{A}_1 \) = incident field amplitude coefficients in the component problems
\( b_0, b_1 \) = amplitude reflection coefficients, \( n=0 \) and \( n=-1 \) modes
\( a_1, a_2 \) = reflected field amplitude coefficient in the overall problem
\( b_0, b_1, B_0, B_1, B_{-1}, B_{1}, \overline{B}_0, \overline{B}_1 \) = reflected field amplitude coefficients in the component problems
\( c_n \) = transmitted field amplitude coefficient in the overall problem
\( C \) = constant
\( C_0, C_1, \overline{C}_0, \overline{C}_1, C_0, C_{-1}, \overline{C}_0, \overline{C}_{-1} \) = transmitted field amplitude coefficients in the component problems
\( d \) = fin height
\( f \) = frequency of the incident wave
\( g(s) \) = \( \bar{\phi}(0,s) \)
\( h \) = \( k_0 \sin \theta_i \)
\( k_0 \) = propagation constant of free space
\( K \) = constant
\( \lambda \) = transmission distance between horn and surface
\( m, n \) = mode numbers
\( \frac{P_{r0}}{P_i}, \frac{P_{r-1}}{P_i} \) = power reflection coefficients, \( n=0 \) and \( n=-1 \) modes
\( P(s) \) = special function

\( R_1, R_2, R_3, R_{4,m}, R_5, R_6, R_{7,m} \) = residues in Problem #3

\( R_m \) = surface resistivity of the fins

\( t \) = fin thickness

\( u^2 \) = \( k_0^2 + s^2 \)

\( x, y, z \) = space coordinates

\( Z_0 \) = impedance constant of free space

\( \alpha \) = attenuation coefficient of the \( n=1 \) mode in the fin region

\( \alpha_i \) = angle of rotation

\( \gamma_n \) = propagation coefficient of the \( n^{th} \) mode, \( n=\pm 0, 1, 2, \ldots, 0<z<d \)

\( \Gamma_n \) = propagation coefficient of the \( n^{th} \) mode, \( n=\pm 0, 1, 2, \ldots, z<0 \)

\( \Gamma(z) \) = gamma function

\( \theta_i \) = angle of incidence

\( \theta_{i_{op}} \) = optimum angle of incidence

\( \lambda \) = free space wavelength

\( \phi(x,z) \) = magnetic field component in the y-direction, \( H(x,z) \)

\( \hat{\phi}(x,s) \) = bilateral Laplace transform of \( \phi(x,z) \)

\( \omega \) = angular frequency
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Introduction

Instrument landing system (ILS) interference from large hangars and terminal buildings is a problem confronting many of today's crowded airport installations. Although some problems can be eliminated by placing the offending structures at non-critical angles to the runways, this becomes very difficult in a multi-runway system. It is suggested that the interference could be eliminated by placing a properly designed periodic surface on the reflecting structure so that all of the incident energy is scattered back in the direction of the ILS transmitter. In this way, no interfering reflections would be received along the runway.

It is known from the theory of diffraction gratings that a maximum amount of energy is transferred from the specularly reflected mode to one or more backscattered modes when the period of the surface, 'a', satisfies Bragg's Law, \( k_0 \sin \theta_1 = m \lambda \), where \( k_0 = \frac{2\pi}{\lambda} \), \( \lambda \) is the free space wavelength and \( \theta_1 \) is the angle of incidence from the normal to the surface as shown in figure 1.1. However, the range of periods over which only one backscattered mode is generated depends entirely on the surface.

![Figure 1.1 Periodic Surface Demonstrating Bragg's Law.](image)
A number of authors [1-6] have analysed the problem of plane wave incidence on a periodic surface consisting of an infinite set of semi-infinite parallel conducting plates. They found that a single backscattered mode, travelling in a direction opposite to the incident wave, is generated for some angles of incidence when the plates have a period in the range $\lambda/2 < a < \lambda$. This surface is not particularly useful in itself, but similar structures, consisting of an infinite set of parallel plates terminated by a perfectly-conducting plane, are of interest. These structures, sometimes referred to as comb gratings or fin-corrugated surfaces, have been rigorously analysed by Tseng [7] and Tseng, Hessel and Oliner [8] using a scattering matrix approach and an integral transform technique similar to that used by Collin [6, P.430] for semi-infinite plates. Tseng et al [8] have shown that, for TE polarization of the incident wave and the proper choice of corrugation depth, there is complete cancellation of specular reflection at an angle of incidence $\theta_i = \sin^{-1}(\lambda/2a)$. Later DeSanto [12,13], using a modified calculus of residues technique, confirmed these results and described similar results for TM polarization.

The problem of TM polarized plane wave scattering from a fin-corrugated structure with a modulated corrugation depth and fin spacing in the range $0 < a < \lambda/2$ has been analysed rigorously by Hessel and Hochstadt [9] using the above-mentioned scattering matrix and integral transform technique. While no numerical results were given, this structure may also be expected, with the proper choice of corrugation depths, to transfer all reflected power to the backscattered mode.

Perfectly conducting sinusoidal surfaces have been investigated by Zaki and Neureuther [10,11] using a numerical solution of the appro-
appropriate integral equations. They found, for both TE and TM polarization, the same scattering properties as observed by Tseng et al [8] for fin-corrugated surfaces. However, for TM polarization, the reduction of specular reflection seemed to occur over a much wider range of incident angles.

A generalization of the fin-corrugated surface, the rectangular groove or lamellar grating has been studied by Wirgin and Deleuil [14] and Wirgin [15,16] for both polarizations. Numerical results were obtained by truncating and then solving a set of simultaneous linear equations for the spectral order amplitudes. Numerical results at $\theta = 30^\circ$ exhibiting complete cancellation of the specularly reflected mode were confirmed by experiment. This structure has also been investigated by Hessel and Schmoys [17], who were interested in its application as a frequency sensitive mirror in a laser cavity.

A similar structure to the above, the triangular groove or echelette grating, has been used extensively in spectroscopy. However, this structure is very difficult to analyse rigorously and only a limited number of accurate numerical results have been obtained [18].

From practical considerations, it is apparent that the fin-corrugated periodic surface is the most promising in the ILS application as it is much simpler to construct than the other structures. In addition it can be analysed rigorously and investigated numerically without the costly inversion of matrices required by the lamellar surfaces. However, it is also apparent that the work to date has only been concerned with the idealized fin-corrugated surface, that is, the infinite surface composed of perfectly-conducting, infinitely thin, fins under plane wave illumination. No experimental work has been carried out on
more realizable surfaces. The purpose of this thesis is to provide such an experimental investigation using numerical results, from the analysis of idealized structures, as a guide in designing the surfaces.

In Chapter 2, a rigorous analysis is presented for the problem of TM polarized plane wave incidence on an infinite fin-corrugated surface composed of perfectly-conducting, infinitely thin fins. The theoretical approach is the same as that used by Tseng [7] and Tseng et al [8] for TE polarization and Hessel and Hochstadt [9] for the modulated fin-corrugated surface. TM polarization was chosen because this is the polarization used in ILS installations. An investigation of attenuation in the parallel fin region concludes the chapter.

In Chapter 3, the numerical results for some specific cases are presented and compared with those obtained by other workers. Some interesting relations between the fin height and fin spacing for optimum cancellation of specular reflection are included. The chapter is concluded with a discussion of the affects of attenuation.

In Chapter 4, the experimental results for four specific finite sized fin-corrugated surfaces are presented together with a procedure for predicting the behaviour of any fin-corrugated surface composed finitely thick fins.

Conclusions from the investigation are presented in Chapter 5.
2. Theoretical Analysis

2.1 Formulation of the Overall Problem

The fin-corrugated structure which shall be analysed is shown schematically in figure 2.1. It consists of an infinite set of perfectly-conducting parallel fins mounted on an infinite, perfectly-conducting plane. The fins, which are of height "d" and periodic spacing "a", shall be assumed infinitely thin. The notation and approach here follows that used by Collin [6, P.430] for a simpler structure.

Consider a y-invariant, TM polarized, TEM plane wave with components $H_y(x,z)$, $E_x(x,z)$ and $E_z(x,z)$ incident at an angle $\theta_\perp$ from the normal to the structure, as shown in figure 2.1. From Maxwell's equations

\begin{align}
E_x(x,z) &= \frac{jZ_0}{k_0} \frac{\partial \phi(x,z)}{\partial z} \quad (2.1) \\
E_z(x,z) &= -\frac{jZ_0}{k_0} \frac{\partial \phi(x,z)}{\partial x} \quad (2.2)
\end{align}

where

\begin{equation}
\phi(x,z) = H_y(x,z) \quad (2.3)
\end{equation}

and $Z_0$ and $k_0$ are the impedance and propagation constants, respectively, of free space. The magnetic field component is

\begin{equation}
\phi_\perp(x,z) = a_0 e^{-jhx - \Gamma_0 z} \quad z < 0 \quad (2.4)
\end{equation}

where

\begin{equation}
h = k_0 \sin \theta_\perp \quad (2.5)
\end{equation}

\begin{equation}
\Gamma_0 = jk_0 \cos \theta_\perp \quad (2.6)
\end{equation}
Figure 2.1 Fin-Corrugated Structure with Incident TEM Plane Wave.
and the time dependence $e^{j\omega t}$ has been omitted. Because of the discontinuity at the fin-air interface, a reflected and transmitted wave will result. The reflected wave will be composed of an infinite sum of $y$-invariant, TM polarized TEM modes whereas the transmitted wave will contain one TEM mode plus an infinite sum of similar, but higher order, TM modes. The transmitted wave, however, will be reflected by the terminating plane back toward the fin-air interface resulting in further reflection and transmission. Thus, the total reflected field above the interface will be governed by multiple reflections within the fin region.

As mentioned in Chapter 1, a procedure for analysing such a problem has been described by Tseng [7] and used successfully by Tseng, Hessel and Oliner [8] and Hessel and Hochstadt [9]. It consists essentially of representing the discontinuity at the fin-air interface by a scattering matrix, $S$, which relates, at the discontinuity, the amplitudes of the scattered modes to those of the incident modes. This allows the overall problem to be broken up into a finite number of component problems, the number depending on the number of propagating modes in the fin region, which can be analysed one at a time. The only assumption that must be made here is that the fin height, $d$, is large enough to prevent reflection of evanescent modes at the terminating plane. This procedure will form the basis of the theoretical analysis to follow.

2.1.1 Boundary and Edge Conditions

Because of the periodicity of the fin-corrugated structure, it is necessary to consider only a single period as depicted in figure 2.2. In the region $z < 0$, the electric and magnetic fields are periodic in $x$. 
Thus, from Floquet's Theorem [6, P.368] and equations (2.1) to (2.3),

\[
\frac{\partial \phi(x, z)}{\partial x} \bigg|_{x=ma} = e^{-jhma} \frac{\partial \phi(x, z)}{\partial x} \bigg|_{x=0} \quad m = 0, 1, 2, \ldots \quad z<0 \quad (2.7)
\]

\[
\phi(ma, z) = e^{-jma} \phi(0, z) \quad m = 0, 1, 2, \ldots \quad z<0 \quad (2.8)
\]

Also, in the region \(0 < z < d\), because the tangential component of the electric field is zero at the surface of a perfect-conductor,

\[
\frac{\partial \phi(x, z)}{\partial x} \bigg|_{x=ma} = 0 \quad m = 0, 1, 2, \ldots \quad 0 < z < d \quad (2.9)
\]

\[
\frac{\partial \phi(x, z)}{\partial z} \bigg|_{z=d} = 0 \quad \text{all } x \quad (2.10)
\]

These equations make up the boundary conditions of the overall problem.

![Figure 2.2 Single Period with Boundary Conditions.](image-url)
Notice that equation (2.9) allows the region of \( z \) in equation (2.7) to be extended to \(-\infty < z < d\). However, because the tangential component of the magnetic field is discontinuous across the conducting fin by an amount equal to the current on the fin, the region of equation (2.8) cannot likewise be extended.

The edge condition, or the behaviour of the field at the edge of the parallel fins, is also of importance. It can be shown as in Collin [6, P.18] that

\[
\phi(ma,z) \sim z^{1/2} \quad \text{as} \quad z \to o \quad m = \pm 0, 1, 2, \ldots
\]  

(2.11)
asymptotically, and hence the magnetic field component is finite in the neighbourhood of the fin edge. In addition, the radiation condition,

\[
\phi(x,z) \to 0 \quad \text{as} \quad z \to -\infty
\]  

(2.12)
must also be met in the solution of the overall problem.

2.1.2 General Solutions

The expressions for the magnetic fields above and below the fin-air interface of figure 2.1 must satisfy the reduced Helmholtz equation

\[
(\nabla^2 + k_0^2)\phi(x,z) = 0
\]  

(2.13)
subject to the conditions of section 2.1.1. The incident field expression, (2.1), and the expression for the total reflected field in the region \( z < 0 \),

\[
\phi_r(x,z) = \sum_{n=\infty}^{\infty} b_n e^{-j(h+\frac{2\pi n}{a})x+\Gamma_n z} \quad z < 0
\]  

(2.14)
satisfy the boundary conditions (2.7) and (2.8). However, for (2.13) to hold, it is necessary that

$$\Gamma_n^2 = \left(h + \frac{2n\pi}{a}\right)^2 - k_0^2 \quad n = \pm 0, 1, 2, \ldots$$
(2.15)

Similarly, the expression for the total field in the region $0 < z < d,

$$\phi_t(x,z) = \sum_{n=0}^{\infty} c_n \cos\left(\frac{n\pi x}{a}\right)e^{-\gamma_n z} \quad 0 < z < d,$$
(2.16)

satisfies (2.7), (2.8) and (2.13) if and only if

$$\gamma_n^2 = \left(\frac{n\pi}{a}\right)^2 - k_0^2 \quad n = +0, 1, 2, \ldots$$
(2.17)

Notice that if $\gamma_n^2 < 0$ for some $n$, $\gamma_n$ is imaginary and positive (if the positive root is chosen to satisfy the radiation condition, (2.12)) and the corresponding mode is propagating. However, if $\gamma_n^2 \geq 0$ for some $n$, $\gamma_n$ is real and positive and the corresponding mode is evanescent. A similar set of rules apply to equation (2.15). Thus, the number of propagating modes in the region $z < 0$ is determined by the values of $a$ and $h$, whereas the number in the region $0 < z < d$ depends only on $a$.

As stated in Chapter 1, Tseng, Hessel and Oliner [8] analysed the problem of a TE polarized plane wave incident on a fin-corrugated structure. They found, for this polarization, the power in the specularly reflected mode can be completely transferred to a backscattered mode travelling in a direction opposite to the incident wave if two propagating modes are in the fin region. More recent works by DeSanto [12,13] have confirmed these results and have shown identical results for TM polarization. Also, Hessel and Hochstadt [9] investigated a fin-corrugated structure with a modulated corrugation depth. They discovered,
for TM polarization and the general case of a uniform corrugation depth (as in figure 2.1), a backscattered mode is not excited if only one propagating mode is in the fin region. It was therefore decided, for the present study, that the \( n = 0 \) and \( n = 1 \) modes shall be assumed to propagate in the fin region. This assumption restricts the fin spacing to the range \( \lambda/2 < a \leq \lambda \), where \( \lambda \) is the free space wavelength, and the number of propagating modes in the air region to two; the \( n = 0 \) specularly reflected mode and the \( n = -1 \) backscattered mode. It should be noted, however, that the \( n = -1 \) mode may be evanescent for some angles of incidence.

2.2 Representation of the Fin-Air Discontinuity by a Scattering Matrix

The notation and approach here follows that used by Hessel and Hochstadt [9].

The fin-air discontinuity of the fin-corrugated structure may be represented as a four port scattering junction as shown in figure 2.3(a). Each port may be considered as a modal waveguide supporting one of the four propagating modes under consideration. Assume that each port is excited one at a time as shown in figures 2.3(b) and 2.3(c). The resulting four problems are termed the component problems and the \( A \)'s, \( B \)'s and \( C \)'s of figure 2.3(c) are the incident, reflected and transmitted field amplitude coefficients, respectively, corresponding to these problems. The matrix equation of the junction is of the form

\[
\mathbf{b} = S \cdot \mathbf{a}
\]

or

\[
(2.18)
\]
Figure 2.3 Four Port Scattering Junction Representation. (a) Overall Problem. (b) and (c) Component Problems showing the Field Amplitude Coefficients.
where $\mathbf{a}$ and $\mathbf{b}$ are column vectors whose components are the modal amplitudes associated with the component problems and $\mathbf{S}$ is a scattering matrix with elements consisting of the appropriate amplitude reflection and transmission coefficients.

Since the $n = 0$ mode is the only mode incident on the structure from $z < 0$,

$$a_2 = 0 \quad (2.20)$$

In addition, because of the boundary condition (2.10),

$$a_3 = e^{-2\gamma_0 d} b_3 \quad (2.21)$$

$$a_4 = e^{-2\gamma_1 d} b_4 \quad (2.22)$$

These conditions, together with the matrix equation (2.19), constitute a system of seven equations which may be solved uniquely in terms of the amplitude of the incident field, $a_1$. The expressions for the specular
amplitude reflection coefficient, $\frac{b_1}{a_1}$, and the backscatter amplitude reflection coefficient, $\frac{b_2}{a_1}$, which result are

$$\frac{b_1}{a_1} = \frac{B_0}{A_0} + \frac{C_0}{A_0} e^{-2\gamma_0 d} \Delta_0 + \frac{\bar{C}_0}{\bar{A}_1} e^{-2\gamma_1 d} \Delta_{-1}$$  \hspace{1cm} (2.23)

$$\frac{b_2}{a_1} = \frac{B_{-1}}{A_0} + \frac{C_{-1}}{A_0} e^{-2\gamma_0 d} \Delta_0 + \frac{\bar{C}_{-1}}{\bar{A}_1} e^{-2\gamma_1 d} \Delta_{-1}$$  \hspace{1cm} (2.24)

$$\Delta_0 = \frac{\bar{C}_0}{\bar{A}_0} \left[ 1 - \frac{\bar{B}_1}{\bar{A}_1} e^{-2\gamma_1 d} \right] + \frac{B_0 \bar{C}_1}{A_0 \bar{A}_1} e^{-2\gamma_1 d} \left[ 1 - \frac{B_1 e^{-2\gamma_0 d}}{A_0} \right]$$

$$- \frac{B_1 B_0 e^{-2(\gamma_0 + \gamma_1) d}}{A_0 \bar{A}_1}$$  \hspace{1cm} (2.25)

$$\Delta_{-1} = \frac{\bar{C}_1}{\bar{A}_0} \left[ 1 - \frac{B_0 e^{-2\gamma_0 d}}{A_0} \right] + \frac{B_{-1} C_0}{A_0 A_0} e^{-2\gamma_0 d} \left[ 1 - \frac{B_{-1} e^{-2\gamma_1 d}}{A_0} \right]$$

$$- \frac{B_{-1} B_0 e^{-2(\gamma_0 + \gamma_1) d}}{A_0 A_1}$$  \hspace{1cm} (2.26)

The corresponding power reflection coefficients are

$$\frac{P_{r_0}}{P_1} = \left| \frac{b_1}{a_1} \right|^2$$  \hspace{1cm} (2.27)

$$\frac{P_{r-1}}{P_1} = \Gamma_{-1} \left| \frac{b_2}{a_1} \right|^2$$  \hspace{1cm} (2.28)

where

$$\frac{P_{r_0}}{P_1} + \frac{P_{r-1}}{P_1} = 1$$  \hspace{1cm} (2.29)
due to the conservation of power in a lossless system.

2.3 Formulation of the Component Problems

Three of the four component problems represented by figure 2.3(c) are shown schematically in figure 2.4. Problem #2 has been omitted because, as equation (2.20) indicates, it is not needed in the solution of the overall problem. The boundary conditions for the remaining problems are the same as equations (2.7) to (2.9) except that the value d is now replaced by $+\infty$. The general solutions to these component problems, as listed below, satisfy these boundary conditions and Helmholtz's equation, (2.13), subject to equations (2.15) and (2.17).

**Problem # 1**

$z<0 \quad \phi(x,z) = \bar{A}_0 e^{-jhx - \Gamma_0 z} + \sum_{n=-\infty}^{\infty} \bar{B}_n e^{-j(h + \frac{2n\pi}{a})x + \Gamma_n z}$  \hspace{1cm} (2.30)

$z>0 \quad \phi(x,z) = \sum_{n=0}^{\infty} \bar{c}_n \cos\left(\frac{n\pi x}{a}\right) e^{-\gamma_n z}$  \hspace{1cm} (2.31)

**Problem # 3**

$z<0 \quad \phi(x,z) = \sum_{n=-\infty}^{\infty} \bar{c}_n e^{-j(h + \frac{2n\pi}{a})x + \Gamma_n z}$  \hspace{1cm} (2.32)

$z>0 \quad \phi(x,z) = A_0 e^{\gamma_0 z} + \sum_{n=0}^{\infty} B_n \cos\left(\frac{n\pi x}{a}\right) e^{-\gamma_n z}$  \hspace{1cm} (2.33)

**Problem # 4**

$z<0 \quad \phi(x,z) = \sum_{n=-\infty}^{\infty} \bar{c}_n e^{-j(h + \frac{2n\pi}{a})x + \Gamma_n z}$  \hspace{1cm} (2.34)
Figure 2.4 Component Problems. (a) Problem #1, TEM Mode Incident in Region z<0. (b) Problem #3, TEM Mode Incident in Region z>0. (c) Problem #4, TM \textsubscript{1} Mode Incident in Region z>0.
\[
\phi(x,z) = A_1 \cos(\frac{\pi x}{a}) e^{\gamma_1 z} + \sum_{n=0}^{\infty} B_n \cos(\frac{n \pi x}{a}) e^{-\gamma n z}
\] (2.35)

In the following sections, a formal solution for the amplitude coefficients of Problem #3 will be presented using an integral transform construction technique described by Collin [6, P.418] and used by Tseng, Hessel and Oliner [8] and Hessel and Hochstadt [9]. It will become apparent in section 2.6 that not all of the component problems need to be solved in detail. The integral transform expression obtained for Problem #3 can be transformed to suit the others, thus eliminating a great deal of unnecessary analysis.

2.4 The Transformed Problem

The initial step of the integral transform technique is to suppress the variable \(z\) by taking the bilateral Laplace transform of the field solution \(\phi(x,z)\). A solution for the transformed problem which satisfies the transformed boundary conditions and the transform of Helmholtz's equation is then constructed. In the final step, the solution for \(\phi(x,z)\) is found by inverting the transform and evaluating the inversion integral in terms of its residues. The desired amplitude coefficients can then be obtained by matching this solution with the original field expansion.

Let the bilateral Laplace transform of the total field \(\phi(x,z)\) be [6, P.433]

\[
\Phi(x,s) = \int_{-\infty}^{\infty} e^{-sz} \phi(x,z) \, dz
\] (2.36)

The inverse transform is then...
\begin{equation}
\phi(x,z) = \frac{1}{2\pi j} \int_{\Gamma} e^{sz} \phi(x,s) ds \tag{2.37}
\end{equation}

where \(\Gamma\) is a specially chosen contour in the complex s-plane running parallel to the imaginary s-axis through a region in which \(\tilde{\phi}(x,s)\) is analytic. It is shown in Appendix A that the region of analyticity for Problem \#3 is well defined. For convenience, a special function \(g(s)\) shall be defined as follows:

\begin{equation}
g(s) = \tilde{\phi}(o,s) = \int_{-\infty}^{\infty} e^{-sz} \phi(o,z) dz \tag{2.38}
\end{equation}

\textbf{2.4.1 Solution of the Transformed Problem}

The Laplace transform of the reduced Helmholtz's equation is

\begin{equation}
\frac{\partial^2 \tilde{\phi}(x,s)}{\partial x^2} + u^2 \tilde{\phi}(x,s) = 0 \tag{2.39}
\end{equation}

where

\begin{equation}
u^2 = k_o^2 + s^2 \tag{2.40}
\end{equation}

The transformed variable, \(\tilde{\phi}(x,s)\), of this differential equation must satisfy the following transformed boundary conditions obtained from equations (2.7), (2.8) and (2.9):

\begin{equation}
\frac{\partial \tilde{\phi}(x,s)}{\partial x} \bigg|_{x=ma} = e^{-jhma} \frac{\partial \tilde{\phi}(x,s)}{\partial x} \bigg|_{x=0} \quad m=\pm 0, 1, 2, \ldots \quad z<\infty \tag{2.41}
\end{equation}

\begin{equation}
\tilde{\phi}(ma,s) = e^{-jhma} \tilde{\phi}(o,s) \quad m=\pm 0, 1, 2, \ldots \quad z<0 \tag{2.42}
\end{equation}

\begin{equation}
\frac{\partial \tilde{\phi}(x,s)}{\partial x} \bigg|_{x=ma} = 0 \quad m=\pm 0, 1, 2, \ldots \quad z>0 \tag{2.43}
\end{equation}
A general solution for this transformed problem is of the form

\[ \phi(x,s) = A(s) \sin(ux) + B(s) \cos(ux) \]  

(2.44)

Substituting this expression into the boundary condition (2.41) with \( m=1 \), yields

\[ A(s) = \frac{B(s) \sin(ua)}{[\cos(ua) - e^{-jha}]} \]  

(2.45)

However, from equation (2.38)

\[ g(s) = \phi(0-,s) = B(s) \]  

(2.46)

and hence

\[ A(s) = \frac{g(s) \sin(ua)}{[\cos(ua) - e^{-jha}]} \]  

(2.47)

Therefore, from equation (2.44)

\[ \phi(x,s) = \frac{g(s) [\cos(u(a-x)) - e^{-jha} \cos(ux)]}{[\cos(ua) - e^{-jha}]} \]  

(2.48)

and the inverse transform, equation (2.37), is [6, P.434]

\[ \phi(x,z) = \frac{1}{2\pi j} \int_{\Gamma} e^{sz} g(s) [\cos(u(a-x)) - e^{-jha} \cos(ux)] \cos(ua) - e^{-jha}] \]  

(2.49)

where \( u^2 = k_o^2 + s^2 \).

In order to evaluate equation (2.49), a representation for \( g(s) \) must be obtained. This can be done as follows:

Substituting equation (2.49) into the boundary condition (2.9) with \( m=0 \), yields
\[
\int \frac{e^{sz}g(s)u \sin(ua)ds}{[\cos(ua) - e^{-jha}]} = 0 \quad z > 0 \quad (2.50)
\]

Similarly, substituting equation (2.49) into the boundary condition (2.8) with \(m = 1\), yields
\[
\int \frac{e^{sz}g(s)[\cos(ha) - \cos(ua)]ds}{[\cos(ua) - e^{-jha}]} = 0 \quad z < 0 \quad (2.51)
\]

Thus, it is apparent that the integrands of (2.50) and (2.51) are regular (that is, contain no singularities) for \(z > 0\) and \(z < 0\), respectively. Therefore, \(g(s)\) is a meromorphic function (contains only poles) and can be constructed from ratios of entire functions subject to (2.50) and (2.51). It is shown in Appendix B that a suitable construction for \(g(s)\) would be [6, P.435]

\[
g(s) = \frac{P(s) \{\cos(ua) - e^{-jha}\}}{(s-y_0)(s+y_0)(s-r_0) \left[ \frac{(s+\gamma_n)}{n\pi} \right] e^{-sa} \left[ \frac{(s-R_n)(s-R_n)}{n\pi} \right] \left[ \frac{sa}{a} \right]}
\]

where \(P(s)\) is an entire function yet to be determined.

It is shown in Appendix C that the exponential factors and the terms \(\left( \frac{n\pi}{a} \right)\) and \(\left( \frac{2n\pi}{a} \right)^2\) in the denominator of (2.52) are used to ensure the uniform convergence of the infinite products. [6, P.435]

2.4.2 Determination of the Final Integral Representation

In order to find a suitable function \(P(s)\), it is first necessary to determine the asymptotic behaviour of \(g(s)\). This is done in Appendix D where it is shown that \(g(s)\) is asymptotic to \(s^{-3/2}\) as \(|s| \to \infty\).
If \( P(s) \) is now chosen so that \( g(s) \) has algebraic growth at infinity and is asymptotic to \( s^{-3/2} \) as \( |s| \to \infty \), it is assured that \( \phi(x,z) \) has the correct behaviour at the fin edges, equation (2.11). It is shown in Appendix E that \( P(s) \) is of the form

\[
P(s) = C e^{-\frac{sa}{2\pi} \ln 2}
\]

where \( C \) is a constant.

Substituting (2.53) into equation (2.52) and the result into equation (2.49) yields the final integral representation of the total field,

\[
\phi(x,z) = \frac{C}{2\pi j} \int_{-j\infty}^{j\infty} \frac{sz^{-\frac{sa}{2\pi} \ln 2}}{(s-\gamma_0)(s+\gamma_0)(s-\Gamma_0)} \left\{ \frac{e^{-jha \cos(ux)}}{e^{\frac{n\pi}{a}}} \right\} ds
\]

where \( u^2 = k_0^2 + s^2 \). The pole plot for this problem is shown in figure 2.5.

![Pole Plot of the Integral Representation, Equation (2.54), Problem #3.](image)
2.5 Solution of the Field Amplitude Coefficients by the Method of Residues

The final step in the analysis of Problem 3 is the evaluation of equation (2.54) using the method of residues. Once this is done, the result can be matched with the original field expansion for \( \phi(x,z) \) to obtain the desired field amplitude coefficients.

For \( z>0 \), the integration contour of figure 2.5 is closed in the LHP. Therefore

\[
\phi(x,z) = C \sum \text{Residues} \tag{2.55}
\]

If the residues at \( s=+Y_0, s=-Y_0, s=±Y_1 \) and \( s=±Y_n \) are denoted by \( R_1, R_2, R_3 \) and \( R_{4,m} \), respectively, where \( m=±2,3,... \), then

\[
R_1 = \frac{-Y_0^a}{e^{\frac{\pi}{2} \ln 2}} (1-e^{-jha}) e^{\gamma_0 z} \int_{1}^{\infty} \frac{(\gamma_n + Y_0)}{(n\pi/\varepsilon a)} e^{\frac{\gamma_0 a}{n\pi}} \int_{1}^{\infty} \frac{(\Gamma_n - Y_0)(\Gamma_n - Y_0)}{(2n\pi/\varepsilon a)^2} e^{\frac{\gamma_0 a}{n\pi}} \tag{2.56}
\]

\[
R_2 = \frac{Y_0^a}{e^{\frac{\pi}{2} \ln 2}} (1-e^{-jha}) e^{\gamma_0 z} \int_{1}^{\infty} \frac{(\gamma_n - Y_0)}{(n\pi/\varepsilon a)} e^{\frac{\gamma_0 a}{n\pi}} \int_{1}^{\infty} \frac{(\Gamma_n + Y_0)(\Gamma_n + Y_0)}{(2n\pi/\varepsilon a)^2} e^{\frac{\gamma_0 a}{n\pi}} \tag{2.57}
\]

\[
R_3 = \frac{Y_1^a}{e^{\pi (1-n2-1)}} (1+e^{-jha}) \left( \frac{\pi}{\varepsilon a} \right) \cos(\frac{n\pi}{\varepsilon a}) e^{\gamma_1 z} \int_{1}^{\infty} \frac{(\gamma_n - Y_1)}{(n\pi/\varepsilon a)} e^{\frac{\gamma_1 a}{n\pi}} \int_{1}^{\infty} \frac{(\Gamma_n + Y_1)(\Gamma_n + Y_1)}{(2n\pi/\varepsilon a)^2} e^{\frac{\gamma_1 a}{n\pi}} \tag{2.58}
\]
\[
R_{4,m} = \frac{\gamma_m}{\pi \left( \ln 2 - \frac{1}{m} \right)} e^{-j \alpha} \left( \frac{m \pi}{a} \right) \frac{\cos(m \pi x/a)}{a} e^{-\gamma_m z} \\
(\gamma_m + \alpha)^2 \gamma_0^2 \gamma_m \int_1^\infty \frac{\gamma_m}{(\gamma_m - \gamma_m)} \int_1^\infty \frac{\gamma_m}{(\gamma_m + \alpha)} \frac{\gamma_m}{(\gamma_m - \alpha)} \frac{\gamma_m}{(\gamma_m + \alpha)} e^{-\gamma_m z}
\]

(2.59)

where \( \gamma_m^2 = \left( \frac{m \pi}{a} \right)^2 - k_0^2 \). Therefore, from equation (2.55),

\[
\phi(x,z) = C(R_1 + R_2 + R_3 + \sum_{m=2}^\infty R_{4,m}).
\]

(2.60)

However, the original field expansion for \( \phi(x,z) \) in the region \( z > 0 \), equation (2.33), may be rewritten in the form

\[
\phi(x,z) = A_0 e^{\gamma_0 z} + B_0 e^{-\gamma_0 z} + B_1 \cos\left( \frac{\pi x}{a} \right) e^{-\gamma_1 z}
\]

\[
+ \sum_{n=2}^\infty B_n \cos\left( \frac{\pi x}{a} \right) e^{-\gamma_n z}
\]

(2.61)

Matching terms in this expansion with equation (2.60) yields the following amplitude coefficients:

\[
A_0 = CR_1 e^{-\gamma_0 z}
\]

(2.62)

\[
B_0 = CR_2 e^{\gamma_0 z}
\]

(2.63)

\[
B_1 = \frac{CR_3 e^{\gamma_1 z}}{\cos\left( \frac{\pi x}{a} \right)}
\]

(2.64)

\[
B_m = \frac{CR_m e^{\gamma_m z}}{\cos\left( \frac{m \pi x}{a} \right)} \quad m = 2, 3, \ldots
\]

(2.65)

For \( z \to \infty \), the integration contour of figure 2.5 is closed in
the RHP and

\[ \phi(x,z) = -C \sum \text{Residues} \]  

(2.66)

If the residues at \( s=\Gamma_0 \), \( s=\Gamma_{-1} \) and \( s=\Gamma_m \) are denoted by \( R_5 \), \( R_6 \) and \( R_{7,m} \), respectively, where \( m=\pm 1, \pm 2, \pm 3, \ldots \), then

\[ R_5 = \frac{-\Gamma_0 a}{\pi \ln 2} \sin(\alpha a) \frac{-j \alpha x + \Gamma_0 z}{e^{\Gamma_0 z}} \]

\[ \left( \frac{\Gamma_0^2 - \gamma_0^2}{\Gamma_0^2 - \gamma_0^2} \right) \sum_{n=1}^{\infty} \frac{(-\text{e}^{\Gamma_0 z})}{(\frac{n \pi}{a})} e^{-\frac{2n \pi}{a}} \]

(2.67)

\[ R_6 = \frac{-\Gamma_{-1} a}{\pi (\ln 2 + 1)} \sin(\alpha a) \frac{2 \pi a}{e^{\Gamma_{-1} z}} \]

\[ \left( \frac{\Gamma_0^2 - \gamma_{-1}^2}{\Gamma_{-1} - \gamma_0^2} \right) \sum_{n=1}^{\infty} \frac{(-\text{e}^{\Gamma_{-1} z})}{(\frac{n \pi}{a})} e^{\frac{2n \pi}{a}} \]

(2.68)

\[ R_{7,m} = \frac{-\Gamma_m a}{\pi (\ln 2 + \frac{1}{m})} \sin(\alpha a) \frac{2m \pi}{e^{\Gamma_m z}} \]

\[ \left( \frac{\Gamma_0^2 - \gamma_m^2}{\Gamma_m - \gamma_m^2} \right) \sum_{n=1}^{\infty} \frac{(-\text{e}^{\Gamma_m z})}{(\frac{n \pi}{a})} e^{\frac{2n \pi}{a}} \]

(2.69)

where \( \gamma_m^2 = (h + \frac{2m \pi}{a})^2 - k_0^2 \). Therefore, from equation (2.66),

\[ \phi(x,z) = -C (R_5 + R_6 + \sum_{m=\pm \infty}^{\infty} R_{7,m}) \]

(2.70)

\( m \neq 0, -1 \)
However, the original field expansion for \( \phi(x,z) \) in the region \( z<0 \), equation (2.32), may be rewritten in the form

\[
\phi(x,z) = C_0 e^{-jhx+\Gamma_0 z} + C_{-1} e^{-j(h-\frac{2\pi}{a})x+\Gamma_{-1} z} + \sum_{n=-\infty}^{\infty} C_n e^{-j(h+\frac{2n\pi}{a})x+\Gamma_n z}
\]

(2.71)

Matching terms in this expansion with equation (2.70) yields the remainder of the amplitude coefficients.

\[
C_0 = -CR_5 e^{jhx-\Gamma_0 z}
\]

(2.72)

\[
C_{-1} = -CR_6 e^{j(h-\frac{2\pi}{a})x-\Gamma_{-1} z}
\]

(2.73)

\[
C_m = -CR_7 e^{j(h+\frac{2m\pi}{a})x-\Gamma_m z} \quad m = +1, \pm 2, 3, \ldots
\]

(2.74)

2.6 Solution of the Remaining Component Problems

As stated in section 2.3, it is not necessary to analyse all of the component problems in detail. The integral transform expression and pole plot obtained for Problem # 3 can be easily transformed to suit the others simply by removing the pole corresponding to the incident TEM mode and adding the pole which corresponds to the incident mode of the problem to be analysed.

For example, in Problem # 4, the incident mode in the region \( z>0 \) is a TM_1 mode. Therefore, the incident pole would be located at \( s=+\gamma_1 \) on the complex s-plane, as depicted in the pole plot of figure 2.6 and the integral representation corresponding to this problem would be
Figure 2.6 Pole Plot of the Integral Representation, Equation (2.75), Problem #4.

Figure 2.7 Pole Plot of the Integral Representation, Equation (2.76), Problem #1.
as follows

**Problem # 4**

\[
\phi(x,z) = \frac{C}{2\pi j} \int_{-j\infty}^{j\infty} \frac{sz - sa \ln 2}{e^{\pi u} \left( \cos(u(a-x)) - e^{-jha} \cos(ux) \right)} \left( \frac{s+\gamma_0}{s-\Gamma_0} \right) \frac{1}{(n\pi)^2} e^{-\frac{n\pi}{a}} \left( \frac{s+\gamma}{2n\pi} \right) e^{\frac{n\pi}{a}} ds
\]

(2.75)

However, in Problem # 1, the incident mode in the region $z<0$ is a TEM mode. In this case, the incident pole would be located at $s=-r_0$ on the complex $s$-plane, as shown in the pole plot of figure 2.7, and the integral representation would be as given in equation (2.76).

**Problem # 1**

\[
\phi(x,z) = \frac{C}{2\pi j} \int_{-j\infty}^{j\infty} \frac{sz - sa \ln 2}{e^{\pi u} \left( \cos(u(a-x)) - e^{-jha} \cos(ux) \right)} \left( \frac{s+\gamma_0}{s-\Gamma_0} \right) \frac{1}{(n\pi)^2} e^{-\frac{n\pi}{a}} \left( \frac{s+\gamma}{2n\pi} \right) e^{\frac{n\pi}{a}} ds
\]

(2.76)

The field amplitude coefficients for Problem # 4:

\[\bar{A}_1, \bar{B}_1, \bar{C}_0, \text{ and } \bar{C}_1\]  \hspace{1cm} (2.77)

and for Problem # 1:

\[\bar{A}_0, \bar{B}_0, \bar{B}_1, \bar{C}_0, \text{ and } \bar{C}_1\]  \hspace{1cm} (2.78)

can now be computed using the method of residues and coefficient matching technique described in section 2.5. Once this is complete, the
final component reflection and transmission coefficients for all three problems may be determined.

2.7 Determination of the Component Reflection and Transmission Coefficients

The final component reflection and transmission coefficients of equation (2.19) are determined by taking the proper ratios of the field amplitude coefficients obtained in sections 2.5 and 2.6. The results are tabulated in tables I and II. It should be clear from the analysis carried out in Appendix C that the infinite products contained in these coefficients are uniformly convergent.

Substituting equations (2.79) to (2.90) of tables I and II into equations (2.25) and (2.26) and the results into equations (2.23) and (2.24) yields the overall amplitude reflection coefficients of the n=0 reflected and n=-1 backscattered modes. Substituting these coefficients into equations (2.27) and (2.28) yields the final power reflection coefficients of the overall problem.

2.7.1 Reflection and Transmission Coefficients at the Optimum Angle of Incidence

The condition,

\[ k_0 a \sin \theta_{i_{op}} = \pi \]  

relates the angle of incidence giving the maximum reduction in specular reflection (the optimum angle of incidence, \( \theta_{i_{op}} \)) to the periodicity of the corrugated surface (the fin spacing, a). Substituting this value of \( \sin \theta_1 \) into the expression for \( h \), equation (2.5), gives

\[ h_{op} = \frac{\pi}{a} \]  

(2.92)
\[
\frac{B}{A_0} = -(2\gamma_0)\pi \frac{(\gamma_0 - \gamma_0)}{(\gamma_0 + \gamma_0)} \sum_{n=1}^{\infty} \frac{(\gamma_n - \gamma_0)(\Gamma_{n} + \Gamma_{0})(\Gamma_{-n} + \Gamma_{0})}{(\gamma_0 + \gamma_0)(\gamma_{-n} + \gamma_{0})(\gamma_{n} + \gamma_{0})(\Gamma_{-n} + \Gamma_{0})}
\]
(2.79)

\[
\frac{B_{-1}}{A_0} = -(\Gamma_{0} + \Gamma_{-1})\pi \frac{2\gamma_0}{\gamma_0 + \gamma_0} \sum_{n=1}^{\infty} \frac{2\gamma_0}{(\gamma_0 - \gamma_0)(\gamma_0 + \gamma_0)} \left( \frac{(\gamma_n - \gamma_0)(\Gamma_{n} + \Gamma_{0})(\Gamma_{-n} + \Gamma_{0})}{(\gamma_0 + \gamma_0)(\gamma_{-n} + \gamma_{0})(\gamma_{n} + \gamma_{0})(\Gamma_{-n} + \Gamma_{0})} \right)
\]
(2.80)

\[
\frac{C_{0}}{A_0} = j e \frac{(1 - e^{-j\hbar})}{\hbar^2 \sin(h\alpha)} \sum_{n=1}^{\infty} \frac{(\gamma_n - \gamma_0)(\Gamma_{n} + \Gamma_{0})(\Gamma_{-n} + \Gamma_{0})}{(\gamma_0 - \gamma_0)(\gamma_{-n} + \gamma_{0})(\gamma_{n} + \gamma_{0})(\Gamma_{-n} + \Gamma_{0})}
\]
(2.81)

\[
\frac{C_{1}}{A_0} = j e \frac{(1 + e^{-j\hbar})}{\sin(h\alpha)(\gamma_0 + \gamma_1)(\gamma_0 - \gamma_1)(\gamma_{1} + \gamma_{1})(\gamma_{1} - \gamma_{1})} \sum_{n=1}^{\infty} \frac{(\gamma_n - \gamma_0)(\Gamma_{n} + \Gamma_{0})(\Gamma_{-n} + \Gamma_{0})}{(\gamma_0 - \gamma_0)(\gamma_{-n} + \gamma_{0})(\gamma_{n} + \gamma_{0})(\Gamma_{-n} + \Gamma_{0})}
\]
(2.82)

\[
\frac{B_{0}}{A_0} = -2(\gamma_0)\pi \frac{(\gamma_0 - \gamma_0)}{(\gamma_0 + \gamma_0)} \sum_{n=1}^{\infty} \frac{(\gamma_n + \gamma_0)(\Gamma_{n} - \gamma_0)(\Gamma_{-n} - \gamma_0)}{(\gamma_0 + \gamma_0)(\gamma_{-n} + \gamma_{0})(\gamma_{n} + \gamma_{0})(\Gamma_{-n} + \gamma_{0})}
\]
(2.83)

\[
\frac{B_{1}}{A_0} = e \frac{(1 - e^{-j\hbar})}{(1 + e^{-j\hbar})(\gamma_0 - \gamma_1)(\gamma_0 + \gamma_1)(\gamma_{1} + \gamma_{1})(\gamma_{1} - \gamma_{1})} \sum_{n=1}^{\infty} \frac{(\gamma_n + \gamma_0)(\Gamma_{n} - \gamma_0)(\Gamma_{-n} - \gamma_0)}{(\gamma_0 - \gamma_0)(\gamma_{-n} + \gamma_{0})(\gamma_{n} + \gamma_{0})(\Gamma_{-n} + \gamma_{0})}
\]
(2.84)

Table I Amplitude Reflection and Transmission Coefficients, Sub-Optimum Case.
\[
\frac{C_0}{A_0} = \frac{e^{(\gamma_0-\Gamma_0)^2 \ln 2}}{h^2 (1-e^{-jha})} \sin(ha)(2\gamma_0)(\Gamma_0-\gamma_0) \sum_{n=1}^{\infty} \frac{(\gamma_n+\gamma_0)(\Gamma_n-\gamma_0)(\Gamma_n-\gamma_0)}{(\gamma_n+\Gamma_0)(\Gamma_n-\Gamma_0)(\Gamma_n-\Gamma_0)} \\
\]

\[
\frac{C_{-1}}{A_0} = \frac{e^{(\gamma_0-\Gamma_0)^2 \ln 2}}{h^2 (1-e^{-jha})} \sin(ha)(2\gamma_0)(\gamma_1+\gamma_0)(\Gamma_1-\gamma_0) \sum_{n=1}^{\infty} \frac{(\gamma_n+\gamma_0)(\gamma_n-\gamma_0)(\Gamma_n-\gamma_0)}{(\gamma_n+\Gamma_1)(\Gamma_n-\Gamma_1)(\Gamma_n-\Gamma_1)} \\
\]

\[
\frac{P_0}{A_1} = \frac{e^{(\gamma_0+\gamma_1)^2 \ln 2}}{(1+e^{-jha})(\gamma_0+\gamma_0)} \sum_{n=1}^{\infty} \frac{(\gamma_n+\gamma_1)(\gamma_n-\gamma_1)(\Gamma_n-\gamma_1)}{(\gamma_n-\gamma_0)(\gamma_n+\gamma_0)(\Gamma_n+\gamma_0)} \\
\]

\[
\frac{P_1}{A_1} = -e^{(\gamma_1+\gamma_1)^2 \ln 2} \frac{(\gamma_0+\gamma_1)(\Gamma_0-\gamma_1)(\Gamma_1-\gamma_1)(\Gamma_1+\gamma_1)}{(\gamma_0-\gamma_1)(\Gamma_0+\gamma_1)(\Gamma_1+\gamma_1)(\Gamma_1+\gamma_1)} \sum_{n=1}^{\infty} \frac{(\gamma_n+\gamma_1)(\Gamma_n-\gamma_1)(\Gamma_n-\gamma_1)}{(\gamma_n-\gamma_1)(\Gamma_n+\gamma_1)(\Gamma_n+\gamma_1)} \\
\]

\[
\frac{\bar{C}_0}{A_1} = \frac{-e^{(\gamma_1-\Gamma_0)^2 \ln 2}}{(1+e^{-jha})(\gamma_0+\gamma_0)} \sum_{n=1}^{\infty} \frac{(\gamma_n+\gamma_1)(\gamma_n-\gamma_1)(\Gamma_n-\gamma_1)}{(\gamma_n+\Gamma_0)(\Gamma_n-\Gamma_0)(\Gamma_n-\Gamma_0)} \\
\]

\[
\frac{\bar{C}_{-1}}{A_1} = \frac{-e^{(\gamma_1-\Gamma_0)^2 \ln 2}}{(1+e^{-jha})(\gamma_1+\Gamma_1)(\gamma_0+\Gamma_1)(\Gamma_0-\Gamma_1)(\Gamma_1-\Gamma_1)} \sum_{n=1}^{\infty} \frac{(\gamma_n+\gamma_1)(\Gamma_n-\gamma_1)(\Gamma_n-\gamma_1)}{(\gamma_n+\Gamma_1)(\Gamma_n-\Gamma_1)(\Gamma_n-\Gamma_1)} \\
\]

Table II Amplitude Reflection and Transmission Coefficients, Sub-Optimum Case.
\[ \frac{B_0}{A_0} = -\left(\frac{2\Gamma_0}{\pi}\right) e^{a_1n^2} \frac{(\gamma_0 - \Gamma_0)(\Gamma_1 + \Gamma_0)(\Gamma_1 - \Gamma_0)}{2(\gamma_0 + \Gamma_0)(\gamma_1 + \Gamma_0)(\Gamma_1 - \Gamma_0)} \sum_{n=0}^{\infty} \frac{(\gamma_n - \Gamma_0)(\Gamma_n + \Gamma_0)(\Gamma_n - \Gamma_0)}{(\gamma_n + \Gamma_0)(\Gamma_n - \Gamma_0)(\Gamma_n - \Gamma_0)} \] (2.98)

\[ \frac{B_{-1}}{A_0} = e^{a_1n^2} \frac{(\Gamma_0 + \Gamma_{-1})(\gamma_0 - \Gamma_0)(\Gamma_1 + \Gamma_0)}{2(\gamma_0 + \Gamma_0)(\gamma_1 + \Gamma_0)(\Gamma_1 - \Gamma_0)} \sum_{n=0}^{\infty} \frac{(\gamma_n - \Gamma_0)(\Gamma_n + \Gamma_0)(\Gamma_n - \Gamma_0)}{(\gamma_n + \Gamma_0)(\Gamma_n - \Gamma_0)(\Gamma_n - \Gamma_0)} \] (2.99)

\[ \frac{C_0}{A_0} = je^{a_1n^2} \frac{(4\Gamma_0)(\gamma_0 - \Gamma_0)(\Gamma_1 + \Gamma_0)(\Gamma_1 - \Gamma_0)}{2(\pi\gamma_1)(\gamma_1 - \gamma_0)(\Gamma_1 + \gamma_0)(\Gamma_1 - \gamma_0)} \sum_{n=0}^{\infty} \frac{(\gamma_n - \Gamma_0)(\Gamma_n + \Gamma_0)(\Gamma_n - \Gamma_0)}{(\gamma_n + \gamma_0)(\Gamma_n + \gamma_0)(\Gamma_n + \gamma_0)} \] (2.100)

\[ \frac{C_1}{A_0} = \frac{(2\Gamma_0)(\gamma_0 - \Gamma_0)(\Gamma_1 + \Gamma_0)(\Gamma_1 - \Gamma_0)}{(\Gamma_0 + \gamma_1)(\gamma_0 - \Gamma_1)(\Gamma_1 + \gamma_1)(\Gamma_1 - \Gamma_1)} \sum_{n=0}^{\infty} \frac{(\gamma_n - \Gamma_0)(\Gamma_n + \Gamma_0)(\Gamma_n - \Gamma_0)}{(\gamma_n + \Gamma_1)(\Gamma_n + \Gamma_1)(\Gamma_n + \Gamma_1)} \] (2.101)

\[ \frac{B_0}{A_0} = \text{Same as equation (2.81), Table I} \] (2.102)

\[ \frac{B_1}{A_0} = (0+j0) \] (2.103)

Table III Amplitude Reflection and Transmission Coefficients, Optimum Case.
\[
\frac{c_0}{A_0} = \frac{(\gamma_0 - \Gamma_0)^2}{\pi^2} \frac{a_0}{2} \frac{\ln 2}{2} \frac{(a^2 \gamma_0 \gamma_1)(\gamma_1 + \gamma_0)(\Gamma_0 - \gamma_0)(\Gamma_1 - \gamma_0)(\gamma_n + \gamma_0)(\Gamma_n - \gamma_0)(\gamma_n - \gamma_0)}{2 \pi^3 (\gamma_1 + \gamma_0)(\Gamma_1 - \gamma_0)} \left[ \left( \gamma_n + \gamma_0 \right) \left( \gamma_n + \gamma_0 \right) \left( \gamma_n + \gamma_0 \right) \right] \quad (2.104)
\]

\[
\frac{c_{-1}}{A_0} = \frac{(-1)^2}{\pi^2} \frac{a_0}{2} \frac{\ln 2}{2} \frac{(a^2 \gamma_0 \gamma_1)(\gamma_1 + \gamma_0)(\Gamma_0 - \gamma_0)(\Gamma_1 - \gamma_0)(\gamma_n + \gamma_0)(\Gamma_n - \gamma_0)(\gamma_n - \gamma_0)}{2 \pi^3 (\gamma_1 + \gamma_0)(\Gamma_1 - \gamma_0)} \left[ \left( \gamma_n + \gamma_0 \right) \left( \gamma_n + \gamma_0 \right) \left( \gamma_n + \gamma_0 \right) \right] \quad (2.105)
\]

\[
\frac{\bar{b}_0}{A_0} = (0 + j0) \quad (2.106)
\]

\[
\frac{\bar{b}_1}{A_1} = (0 + j0) \quad (2.107)
\]

\[
\frac{\bar{c}_0}{A_1} = \frac{(\gamma_1)(\gamma_0 + \gamma_1)(\Gamma_1 - \gamma_1)}{\pi^2} \frac{\ln 2}{2} \frac{(a^2 \gamma_0 \gamma_1)(\gamma_1 + \gamma_0)(\Gamma_0 - \gamma_0)(\Gamma_1 - \gamma_0)(\gamma_n + \gamma_0)(\Gamma_n - \gamma_0)(\gamma_n - \gamma_1)}{2 \pi^3 (\gamma_1 + \gamma_0)(\Gamma_1 - \gamma_0)} \left[ \left( \gamma_n + \gamma_1 \right) \left( \gamma_n + \gamma_1 \right) \left( \gamma_n + \gamma_1 \right) \right] \quad (2.108)
\]

\[
\frac{\bar{c}_{-1}}{A_1} = \frac{(-1)^2}{\pi^2} \frac{a_0}{2} \frac{\ln 2}{2} \frac{(a^2 \gamma_0 \gamma_1)(\gamma_1 + \gamma_0)(\Gamma_0 - \gamma_0)(\Gamma_1 - \gamma_0)(\gamma_n + \gamma_0)(\Gamma_n - \gamma_0)(\gamma_n - \gamma_1)}{2 \pi^3 (\gamma_1 + \gamma_0)(\Gamma_1 - \gamma_0)} \left[ \left( \gamma_n + \gamma_1 \right) \left( \gamma_n + \gamma_1 \right) \left( \gamma_n + \gamma_1 \right) \right] \quad (2.109)
\]

Table IV Amplitude Reflection and Transmission Coefficients, Optimum Case.
However, as \( h \to \frac{\pi}{a} \), several factors in the reflection and transmission coefficients of tables I and II approach zero. Specifically,

\[
\lim_{h \to \frac{\pi}{a}} \sin(ha) = 0
\]  
(2.93)

\[
\lim_{h \to \frac{\pi}{a}} (1+e^{-jha}) = 0
\]  
(2.94)

\[
\lim_{h \to \frac{\pi}{a}} (\gamma_1-\Gamma_0) = 0
\]  
(2.95)

\[
\lim_{h \to \frac{\pi}{a}} (\gamma_1-\Gamma_{-1}) = 0
\]  
(2.96)

\[
\lim_{h \to \frac{\pi}{a}} (\Gamma_{-1}-\Gamma_0) = 0
\]  
(2.97)

Therefore, coefficients which contain one or more of these factors in their denominator are singular at the optimum angle of incidence and it is necessary to evaluate their limit as \( h \to \frac{\pi}{a} \). The coefficients obtained, valid only at the angle \( \theta_{1op} = \sin^{-1}(\lambda/2a) \), are displayed in tables III and IV.

2.7.2 Attenuation of the \( n=1 \) Mode in the Parallel Fin Region

For applications such as the ILS problem it is necessary to consider situations in which the angles of incidence are very close to grazing. However, as equation (2.91) indicates, this requires the use
of surfaces with fin spacings very close to \( \frac{\lambda}{2} \), the cutoff fin spacing for the \( n=1 \) mode in the fin region. Attenuation of a waveguide mode is large close to cutoff and it may not always be possible to consider \( \gamma_1 \) as pure imaginary. Consequently, it was considered worthwhile to determine the attenuation coefficient, \( \alpha \), of the \( n=1 \) mode in the fin region.

Using a well-known perturbation technique [6, P.183] it can be shown that the attenuation of the \( n=1 \) mode per unit length in the fin region is

\[
\alpha = \frac{2R_m}{Z_0 a (1 - \left(\frac{\lambda}{2a}\right)^2)^{1/2}} \text{ nepers/ unit length}
\]

where \( R_m \) is the surface resistivity of the fins. If a resistivity for copper is assumed, equation (2.110) becomes

\[
\alpha = \frac{2.3965 \times 10^{-5}}{a \left[ \lambda (1 - \left(\frac{\lambda}{2a}\right)) \right]^{1/2}} \text{ nepers/ meter}
\]

where \( \lambda \) and \( a \) are in meters. Therefore, the true propagation coefficient is

\[
\gamma_1' = \alpha + \gamma_1
\]

where \( \gamma_1 \) is given by equation (2.17). Equation (2.111) is known to predict too high an attenuation as the cutoff spacing is approached and can therefore be regarded as an upper limit under these conditions.
3. Numerical Results

3.1 Introduction

An examination of the final expressions for the overall reflection coefficients will reveal that they are functions of three parameters; 1) the fin spacing, \( a \) (or optimum angle of incidence, \( \theta_{\text{op}} \)), 2) the fin height, \( d \) and 3) the angle of incidence, \( \theta_1 \). In most applications, such as the ILS problem mentioned in Chapter 1, the optimum angle of incidence would be known. Therefore, it was decided that the coefficients would be computed as functions of \( d \) and \( \theta_1 \) at fixed \( a \).

The major source of error in the numerical computations was the evaluation of the infinite products. The Krummer transformation method described by Tseng et al [8] and the remainder approximation technique described by DeSanto [12,13] were not used. Instead, a fixed 100 products were taken and shown to give two to three decimal accuracy at a very reasonable cost. Increasing the number of products to 500 gave, on the average, differences of less than 0.5% in the magnitudes of the power reflection coefficients and less than 0.05% in the relative phases of the amplitude reflection coefficients while increasing the cost by nearly four times.

The results displayed in this chapter were computed assuming no attenuation of the modes in the fin region. A discussion of the effects of attenuation of the \( n=1 \) mode is given in section 3.5.

3.2 Relative Power as a Function of Fin Height

In figures 3.1(a) and (b), the relative power (power reflection coefficients) of the specularly reflected (\( n=0 \)) and backscattered (\( n=-1 \))
Figure 3.1 Relative Power of the $n=0$ and $n=-1$ Modes vs. Fin Height (no attenuation)
(a) $a=0.578 \lambda \theta_{i_{op}}=59.99^\circ$ 100% reduction at $d=0.559 \lambda=0.968a$
(b) $a=0.506 \lambda \theta_{i_{op}}=81.24^\circ$ 100% reduction at $d=0.501 \lambda=0.989a$
modes are plotted with respect to fin height, d, for incident angles 60° and 81.2°. Because power is conserved, a perfect transfer takes place between the two modes and the sum of their reflection coefficients is unity for any value of d. Similar results have been obtained by Zaki and Neureuther [10,11] for sinusoidal surfaces and by DeSanto [12, 13] for fin-corrugated surfaces. Note, in each plot there is a point of 100% power transfer to the n=-1 mode. This is referred to by the above-mentioned authors as the "Brewster-angle effect." It was found that every fin spacing has at least one such point for d<\lambda and many more at larger fin heights.

Figure 3.1(b) has two interesting features. First, there is a rapid interchange of power with fin height between the n=0 and n=-1 modes at about d=0.55\lambda. This phenomenon is referred to as a "Wood S-anomaly" [8,10-13]. Second, the power in the n=0 mode is less than 0.1 for 0.15\lambda<d<0.5\lambda. This latter feature could prove extremely useful, since it allows the design of near optimum surfaces using fin heights much smaller than the fin spacings and without severe tolerances.

In figure 3.2, the n=0 and n=-1 power reflection coefficients and the relative phases of the corresponding amplitude reflection coefficients, are plotted as functions of fin height for a case in which the fin spacing is very large. These plots were made to compare with those of DeSanto [13,fig.5]. The two are essentially identical\(^1\). Figure 3.2 demonstrates the same periodicities with d and the same correlations between the power reflection coefficients and the derivatives of the respective relative phases as described by DeSanto [12,13].

The expressions used to generate the plots of figures 3.1 and 3.2 are not valid near d=0 because they do not take into account the DeSanto assumes time dependence e^{-i\omega t}.
Figure 3.2 Relative Power and Relative Phase of the $n=0$ and $n=-1$ Modes vs. Fin Height (no attenuation).

$a=0.834\lambda$ \hspace{1cm} $\theta_{i_{\text{op}}} = 36.84^\circ$ \hspace{1cm} 100% reduction at $d=1.21\lambda=1.45a$
contributions from the evanescent modes reflected from the terminating plane. However, for TM polarization, it is known that when $d=0$, the power reflection coefficient of the $m=0$ mode is unity and the phase of the respective amplitude reflection coefficient is zero.

3.3 Relative Power as a Function of Incident Angle

In figures 3.3(a) and (b), the relative power of the $m=0$ and $m=-1$ modes are plotted with respect to the angle of incidence, $\theta_i$. The fin spacings used are those of figures 3.1(a) and (b), respectively, and the fin heights correspond to the 100% transfer points of those plots. Similar results have been described by Tseng [7] and Tseng et al [8] for a fin-corrugated surface with TE polarization and by Zaki and Neureuther [10,11] for a sinusoidal surface with both polarizations. The 100% transfer points in figures 3.3(a) and (b) are located at the optimum angles of incidence given by equation (2.91). Because these angles are known as Bragg-angles in crystal structures, the 100% transfer points are sometimes referred to as "Bragg-angle anomalies". The widths of these anomalies are directly dependent on the periodicities of their respective surface with small widths corresponding to small periods. This behaviour is emphasised in figures 3.4(a) and (b). The variation of the relative phase of the $m=0$ and $m=-1$ amplitude coefficients with $\theta_i$ is also given in the plots.

It was stated in section 2.1.2 that the $m=-1$ mode in the air region may be either propagating or evanescent, depending on the angle of incidence. The angles where these transformations occur are clear in figures 3.3 and 3.4. The corresponding points on the $m=0$ curves are referred to as "Rayleigh anomalies" [8,10-13].
Figure 3.3 Relative Power of the $n=0$ and $n=-1$ Modes vs. Angle of Incidence (no attenuation).

(a) $a=0.578 \lambda \quad \theta_{i_{op}}=59.99^\circ \quad d=0.559 \lambda \quad 100\%$ reduction

(b) $a=0.506 \lambda \quad \theta_{i_{op}}=81.24^\circ \quad d=0.501 \lambda \quad 100\%$ reduction
Figure 3.4 Relative Power and Relative Phase of the $n=0$ and $n=-1$ Modes vs. Angle of Incidence (no attenuation).
(a) $a=0.578\lambda \theta_{i_{op}}=59.99^\circ \ d=0.559\lambda$ 100% reduction, (b) $a=0.506\lambda \theta_{i_{op}}=81.24^\circ \ d=0.501\lambda$ 100% reduction.
3.4 Optimum Fin Height as a Function of Fin Spacing

In figures 3.5(a) and (b), the fin height which gives 100% transfer of power to the \( n=-1 \) mode is plotted as a function of the fin spacing (or optimum incidence angle).

For angles of incidence in the range \( 50^\circ < \theta_{\text{op}} < 90^\circ \), the optimum fin height is about equal to the fin spacing. This is an important feature since most ILS reflection problems have incident angles in this range. It will become apparent in Chapter 4 that these plots may also be used to predict the optimum performance of surfaces with fins of finite thickness.

3.5 Attenuation of the \( n=1 \) Mode in the Fin Region

The perturbation solution for the attenuation of the \( n=1 \) mode in the fin region, as determined in section 2.7.2, was introduced into the analysis and plots similar to those of figures 3.1-3.5 were made. As expected, there were no significant alterations of the results for \( a>0.53\lambda \). However, small changes were noticed at near grazing angles and these deserve some mention.

In the plots of relative power with fin height, it was found, as expected, that the total power in the two modes was less than unity and decreased with increasing fin height. However, the positions of 100% power transfer to the \( n=-1 \) mode remained unaltered. In the plots of relative power with incident angle, it was found that the relative power of the \( n=0 \) mode decreased slightly (from unity) with increasing angle of incidence, over the range where the \( n=-1 \) mode was evanescent. Small changes in the shape of the Bragg-angle anomaly were also detected, but there was no noticeable change in the optimum angle. It should be noted that the above changes were extremely small, in most cases, less than 0.1%. There-
Figure 3.5 Optimum Fin Height vs. Fin Spacing, (a) $d/\lambda$, (b) $d/a$
fore, it was concluded that attenuation is not a significant factor in the design of a fin-corrugated surface.
4. Experimental Results

4.1 Introduction

The numerical results of Chapter 3 are based on the rigorous solution of the problem of plane wave incidence on an infinite, fin-corrugated surface composed of perfectly-conducting, infinitely thin fins. Therefore, as far as this problem is concerned, the results do not require experimental verification. However, as mentioned in Chapter 1, the purpose of the study was to get some idea of how to predict the behaviour of more realizable surfaces. In particular, it is desirable to know how the behaviour of the idealized surface is affected by:

1) finite size
2) imperfect conductivity
3) finitely thin fins
4) non-plane wave illumination
5) slightly oblique illumination

Therefore, some experimental results for finite surfaces were needed to compare with the results of Chapter 3.

It was decided that the finite surfaces would be modelled after a typical ILS problem, since problems of this nature were the main motivation for the study. A model frequency of 35 GHz ($\lambda = 8.566$ mm.) was chosen so the ILS dimensions would be reduced to a reasonable scale. This is a scale factor of 1:318 for a 110 MHz ($\lambda = 2.726$ m.) ILS frequency. The measurements of section 4.3.2 were made at 37 GHz ($\lambda = 8.103$ mm.) for reasons explained in that section.

4.2 Experimental Arrangement

Four experimental surfaces were made in all. The fin spacings $a'$, fin periods $a$, fin heights $d$ and overall dimensions are given with a profile in figure 4.1. The procedure for choosing $a$, $a'$ and $d$ will be given in the next section. The fin thickness $t$ was chosen at the time
Plate 3A  $a = 0.526 \text{ cm.} \quad d = 0.447 \text{ cm.} \quad 26.32 \text{ cm.} \times 11.25 \text{ cm.}$
Plate 3B  $a = 0.526 \text{ cm.} \quad d = 0.480 \text{ cm.} \quad 50 \text{ grooves}$
Plate 1A  $a = 0.434 \text{ cm.} \quad d = 0.432 \text{ cm.} \quad 26.93 \text{ cm.} \times 11.25 \text{ cm.}$
Plate 1B  $a = 0.434 \text{ cm.} \quad d = 0.173 \text{ cm.} \quad 62 \text{ grooves}$

Figure 4.1  A TM Polarized Plane Wave Incident on a Fin-Corrugated Surface with $t=0.028 \text{ cm.} \pm 0.002 \text{ cm.}$
of the design to be $\lambda/30$, or $t=0.028$ cm., about the smallest thickness possible in the milling process used. The overall dimensions correspond to a typical hangar wall size of $31\lambda$ by $13\lambda$. One surface was milled on each side of two 2.54 cm. thick brass plates. A photograph of one of the plates is given in figure 4.2. The cover in (a) is a brass reference plate.

In the ILS system on runway 14/32 at Toronto International Airport, an interfering hangar is located at $\theta_i=81.25^\circ$ about 3 km. from the transmitter, about half the range necessary for plane wave illumination of the entire hangar surface. This corresponds to a distance of about 10 m. at 35 GHz. Since distances of this size were not available indoors, a range with dimensions shown in figure 4.3(a) was used. This reduced range is a more severe test of the behaviour of the surfaces under non-plane wave illumination. A photograph of the actual range used is shown in figure 4.4. Identical pyramidal horns with 25 dB gain and E-plane 3 dB beamwidths of $9^\circ$ were used for transmitting and receiving when $\theta_i<70^\circ$. Direct transmission between the horns was blocked by an absorber suspended between the horns (see figure 4.4). For measurements near grazing incidence, a paraboloidal reflector transmitting antenna with $2^\circ$ beamwidth was used.

A diagram of the experimental circuit is shown in figure 4.3(b). The receiving antenna crystal current reading with the surface exposed is returned to the reading obtained with the surface covered by the reference plate by adjusting the precision variable attenuator at the transmitter. The reduction in attenuation is the difference in attenuator settings to an accuracy of about $\pm0.1$ dB. The klystron output is monitored continuously to detect output level changes during the measurements.
Figure 4.2 Plates 1A and 1B. (a) With Reference Plate. (b) Without Reference Plate.
Figure 4.3 Experimental Arrangement. (a) The Experimental Range (see Table V, Appendix G, for the transmission distances, l) (b) The Experimental Circuit

Plates 1A,1B  \( h = 0.88 \text{m} \)  \( l = 2.27 \text{m} \) at \( \theta_i = 69.0^\circ \)
Plates 3A,3B  \( h = 0.54 \text{m} \)  \( l = 2.61 \text{m} \) at \( \theta_i = 55.5^\circ \)
Figure 4.4 Experimental Range for Plates 3A and 3B. Transmitting Horn in Foreground.
Figure 4.5 Plate 3A on Mounting Platform, $\alpha_1 = 0$. 
To investigate the effects of oblique illumination, the surfaces were mounted on a rotatable platform. A photograph of Plate 3A mounted on the platform with angle of rotation $\alpha_l = 0$ is shown in figure 4.5. The perpendicular metal plate attached to the end of the surface is used to simulate the side of the hangar and prevent transmission underneath the surface.

4.3 Results and Discussion

The experimental results which are presented in graph form in this section are presented in tabular form in Appendix F.

4.3.1 Plates 3A and 3B

It was decided that the first two surfaces (Plates 3A and 3B) should have a fin spacing $a'$ corresponding to an optimum angle of incidence near 60°, since measurements at this angle were relatively easy to make. Therefore, the fin spacing was chosen to be $a' = 0.581\lambda$ ($\theta_{i_{op}} = 59.4^\circ$) at 35 GHz. Adding the fin thickness gives a fin period of $a = 0.614\lambda$ ($\theta_{i_{op}} = 54.5^\circ$). The relative power vs. fin height plots of these values are given in figure 4.6. It is obvious that there is a great deal of difference between the two curves, but since the most effective value of $d$ is determined by the waveguide properties, the $a'$ curve was used to determine the fin heights; Plate 3A (optimum): $d = 0.561\lambda$, Plate 3B (sub-optimum): $d = 0.522\lambda$.

Experimental measurements of the relative power of the $n=0$ mode were taken over a range of angles $45^\circ \leq \theta_i \leq 70^\circ$ using the procedure outlined in section 4.2. The results for the surface of Plate 3A are shown in figures 4.7(a) and (b). A reduction of 23.4 dB or 99.54% is achieved at an angle of 55.5°. There is at least 10 dB reduction over
Figure 4.6 Relative Power of the $n=0$ Mode vs. Fin Height, Plates 3A and 3B (with attenuation)
(solid curve): $a'=0.581\lambda$ $\theta_{i_{\text{op}}}=59.35^\circ$ 100% reduction at $d=0.562\lambda=0.967a$
(broken curve): $a=0.614\lambda$ $\theta_{i_{\text{op}}}=54.55^\circ$ 100% reduction at $d=0.598\lambda=0.973a$
Plate 3A: $d=0.560\lambda$ Plate 3B: $d=0.522\lambda$
Figure 4.7 Relative Power of the n=0 Mode vs. Angle of Incidence, Plate 3A (with attenuation).
(a) and (b) Experimental: $a' = 0.581 \lambda$, $a = 0.614 \lambda$, $d = 0.560 \lambda$ at $f=35$ GHz. $\theta_{i} = 55.5^\circ$
- 23.4 dB or 99.54% reduction
(b) (solid curve): $a' = 0.581 \lambda$, $\theta_{i} = 59.35^\circ$, $d = 0.560 \lambda$ 99.97% reduction
(broken curve): $a = 0.614 \lambda$, $\theta_{i} = 54.55^\circ$, $d = 0.560 \lambda$ 93.49% reduction
Figure 4.8 Predicted Relative Power of the n=0 Mode vs. Angle of Incidence, Plate 3A (with attenuation).

\[ \text{a}=0.614\lambda \quad \theta_{i \text{op}} = 54.55^\circ \quad \text{d} = 0.595\lambda \quad 99.97\% \text{ reduction} \]
the range $48^\circ \leq \theta_i \leq 64^\circ$. The computed relative power vs. $\theta_i$ plots for $a'$ (solid curve) and $a$ (broken curve) with $d=0.561\lambda$ are also shown in figure 4.7(b). It seems that the maximum amount of reduction given by the experimental surface is determined by the proper fin spacing - fin height combination $(a',d)$ while the angle of incidence at which this maximum reduction occurs is determined solely by the fin period $a$. This phenomenon suggested the following procedure for predicting the relative power vs. $\theta_i$ curve of any fin-corrugated surface composed of finitely thick fins.

Suppose 100% reduction of the power in the $n=0$ mode is required at $\theta_i$ degrees. This angle determines the fin period $a$. Subtracting the appropriate fin thickness yields the fin spacing $a'$. With this value of $a'$, the fin height $d$ is determined from figure 3.5(a). Thus, the surface would be constructed with parameters $(a, a', d)$. Now, to predict the relative power vs. $\theta_i$ curve for this surface, the fin period $a$ is used with the plot of figure 3.5(a) to determine the "adjusted fin height", $d_a$. These parameters $(a, d_a)$ when used in the theoretical program give the required curve.

This procedure was carried out on the parameters of the surface of Plate 3A yielding $a=0.614\lambda$, $d_a=0.595\lambda$. The results are shown in figure 4.8. The optimum reduction predicted is 11.9 dB more than observed, but this is only a difference of 0.43% in relative power. The 1° displacement of the measured curve is attributed to experimental error.

The experimental results for the sub-optimum surface of Plate 3B are shown in figures 4.9(a) and (b). A reduction of 15.4 dB or 97.10% is achieved at an angle of 55.5°. There is at least 10 dB reduction over the range $49^\circ \leq \theta_i \leq 64^\circ$. Notice that the same situation occurs in figure 4.9(b) as it did in 4.7(b), except that now the experimental curve has
Figure 4.9 Relative Power of the n=0 Mode vs. Angle of Incidence, Plate 3B (with attenuation).

(a) and (b) Experimental: $a'=0.581 \lambda$, $a=0.614 \lambda$, $d=0.522 \lambda$ at $f=35$ GHz. $\theta_1=55.5^\circ$

15.4 dB or 97.10% reduction

(b) (solid curve): $a'=0.581 \lambda$, $\theta_{i_{0p}}=59.35^\circ$, $d=0.522 \lambda$ 88.85% reduction

(broken curve): $a=0.614 \lambda$, $\theta_{i_{0p}}=54.55^\circ$, $d=0.522 \lambda$ 75.94% reduction
Figure 4.10 Predicted Relative Power of the n=0 Mode vs. Angle of Incidence, Plate 3B (with attenuation).

\( a = 0.614 \lambda \ \theta_{i_0} = 54.55^\circ \ \text{d}_a = 0.549 \lambda \) 88.84% reduction
Figure 4.11 Relative Power of the $n=0$ Mode vs. Angle of Rotation, Plate 3A, Sub-Optimum Incidence $\theta_1=61^\circ$, $f=35$ GHz. (solid curve): Reduction Due to the Fins only, (dashed curve): Reduction Due to Plate Orientation only.
Figure 4.12 Relative Power of the n=0 Mode vs. Frequency of the Incident Wave, Plate 3A, (with attenuation)

Experimental: $a' = 0.581\lambda$, $a = 0.614\lambda$, $d = 0.560\lambda$ at $f = 35$ GHz., $\theta_i = \theta_{i_{op}} = 55.5^\circ$

Predicted (broken curve): $a = 0.614\lambda$, $d = 0.595\lambda$ at $f = 35$ GHz., $\theta_i = \theta_{i_{op}} = 54.55^\circ$
more reduction than the a' curve. This indicates that the prediction procedure outlined above is inaccurate except for the case with 100% reduction. The predicted curve, computed using a sub-optimum curve similar to that of figure 3.5(a), is shown in figure 4.10.

Figure 4.11 shows a plot of the relative power of the n=0 mode as a function of the angle of rotation $\alpha_1$ for Plate 3A at an angle of incidence $\theta_1=61^\circ$ (sub-optimum angle). The dashed curve indicates the power reflected from the reference plate as it is rotated. The solid curve shows the reduction of power due to the fins alone. Apparently, the surface remains effective for oblique incidence over the range $-10^\circ<\alpha_1<10^\circ$. As ILS glide path angles are about 2 1/2° the surface should be essentially as effective in reducing interference along the glide path as in the horizontal plane.

The experimental plot of the relative power of the n=0 mode vs. frequency of the incident wave for the surface of Plate 3A at a fixed angle of $\theta_{i_{op}}=55.5^\circ$ is shown in figure 4.12. As the 4 MHz band over which the ILS systems operate scales to 1.3 GHz., there would be no bandwidth limitation for this surface. Figure 4.12 also contains the predicted curve (broken curve) found using the predicted parameters of figure 4.8.

4.3.2 Plates 1A and 1B

The second two surfaces (Plates 1A and 1B) were chosen to have a fin period a, at 35 GHz. corresponding to an optimum angle of incidence near 80°. However, it was discovered in the preliminary experimental measurements that, because of the finite thickness of the fins, the fin spacing a' was well below the cutoff spacing $\lambda/2$ and that the surfaces would give no reduction. Therefore, the frequency of the
Figure 4.13 Relative Power of the n=0 Mode vs. Fin Height, Plates 1A and 1B (with attenuation)
(solid curve): $a' = 0.502 \lambda$ $\theta_{\text{op}} = 85.42^\circ$ 100% reduction at $d = 0.498 \lambda = 0.992a$
(broken curve): $a = 0.536 \lambda$ $\theta_{\text{op}} = 68.88^\circ$ 100% reduction at $d = 0.523 \lambda = 0.975a$
Plate 1A: $d = 0.533 \lambda$ Plate 1B: $d = 0.213 \lambda$
Figure 4.14 Relative Power of the n=0 Mode vs. Angle of Incidence, Plate 1B (with attenuation).

(a) and (b) Experimental: $a' = 0.502\lambda$ $a = 0.536\lambda$ $d = 0.213\lambda$ at $f = 37$ GHz. $\theta_{1op} = 69.0^\circ$

21.7 dB or 99.33% reduction

(b) (solid curve): $a' = 0.502\lambda$ $\theta_{1op} = 85.42^\circ$ $d = 0.213\lambda$ 98.33% reduction

(broken curve): $a = 0.536\lambda$ $\theta_{1op} = 68.88^\circ$ $d = 0.213\lambda$ 70.69% reduction
Figure 4.15 Predicted Relative Power of the n=0 Mode vs. Angle of Incidence, Plate 1B (with attenuation).

\[ a = 0.536\lambda, \quad \theta_{i} = 68.88^\circ, \quad d = 0.510\lambda, \quad 98.34\% \text{ reduction} \]
incident wave was increased to 37 GHz, so that \( a' = 0.502\lambda \) 
\( (\theta_{i_{\text{op}}} = 85.42^\circ) \) and \( a = 0.536\lambda \) (\( \theta_{i_{\text{op}}} = 68.88^\circ \)). The relative power vs. fin height plots of these values are given in figure 4.13. Because the fin thickness is comparable to the fin spacing the two curves are very different. The fin heights were chosen from the \( a' \) curve as before, but because of the frequency change to 37 GHz., they both became sub-optimum; Plate 1A: \( d = 0.533\lambda \), Plate 1B: \( d = 0.213\lambda \).

The experimental results for the surface of Plate 1B are shown in figures 4.14(a) and (b). A reduction of 21.7 dB or 99.33\% is achieved at an angle of 69\°. There is at least 10 dB reduction over the range 62\° < \( \theta_i \) < 79\°. Notice the great difference between the \( a \) and \( a' \) theoretical curves of figure 4.14(b). This indicates that it would be impossible to have complete reduction at angles \( \theta_{i_{\text{op}}} > 75^\circ \) from any surface unless the fins are made thinner than \( t \) or the surface is modelled at a lower frequency. Measurements taken at the optimum angle of incidence for various values of \( \alpha_i \) indicated that the surface of Plate 1B remains effective for oblique incidence over a range of angles \(-10^\circ < \alpha_i < 10^\circ\).

The prediction procedure of section 4.3.1 was carried out on the parameters of this surface yielding \( a = 0.536\lambda \), \( d_a = 0.510\lambda \). The results are shown in figure 4.15. The optimum reduction predicted is 3.9 dB less than observed, a difference of 0.99\% in relative power.

The experimental results for the surface of Plate 1A are shown in figure 4.16. The reduction was much less than expected with no maximum reduction observed at 69\°, the angle of optimum reduction according to the period. Also, there was a variation in the results at each angle and so only the average reduction could be plotted. It is believed that these irregularities in behaviour were caused by a Wood S-anomaly lying on
Figure 4.16 Relative Power of the n=0 Mode vs. Angle of Incidence, Plate 1A.
Experimental: $a'=0.502\lambda$, $a=0.536\lambda$, $d=0.533\lambda$ at $f=37$ GHz.
$\theta_{i_{op}}=69.0^\circ$
Figure 4.17 Relative Power of the n=0 Mode vs. Frequency of the Incident Wave, Plate 1B (with attenuation)

Experimental: \(a' = 0.502\lambda\), \(a = 0.536\lambda\), \(d = 0.213\lambda\) at \(f = 37\) GHz, \(\theta_1 = \theta_{1_{op}} = 69.0^\circ\), \(f = 36.88\) GHz.

Predicted (broken curve): \(a = 0.536\lambda\), \(d_a = 0.510\lambda\) at \(f = 37\) GHz, \(\theta_1 = \theta_{1_{op}} = 68.88^\circ\)
or very near the value chosen as the fin height for this surface (see figure 4.13). Small variations in the klystron frequency change $d/\lambda$ at the anomaly and give large variations in the reflection coefficient.

The experimental plot of the relative power of the $n=0$ mode vs. frequency of the incident wave for the surface of Plate 1B at fixed angle $\theta_{1\text{op}}=69^\circ$ is shown in figure 4.17. A frequency of 37 GHz. was the highest attainable with the klystron source used. The frequency $f_c=36.88$ GHz. is the cutoff frequency (corresponding to the cutoff fin spacing) of the $n=1$ mode in the fin region. This surface with short fins seems to operate very effectively over a range of frequencies below cutoff. Figure 4.17 also contains the predicted curve (broken curve) found using the predicted parameters of figure 4.15.
5. Conclusions

In this study, a rigorous analysis of plane wave scattering from an idealized fin-corrugated periodic surface was presented and numerical results from it compared with experimental results obtained for finite surfaces under non-plane wave illumination. Summarized below are the conclusions drawn:

1) An optimum fin-corrugated surface demonstrating complete cancellation of specular reflection can be designed for any fin period in the range $\lambda/2 < a < \lambda$. For angles of incidence in the range $50^\circ < \theta_1 < 90^\circ$, the usual range for most applications, the optimum fin height for these surfaces does not need to be any larger than the fin period.

2) For situations in which the angle of incidence is near grazing ($\theta_1 > 80^\circ$), reductions in specular reflection of 90% or greater are possible with fin heights much shorter than the fin periods and without severe tolerances on the fin heights.

3) The angular width of the Bragg-angle anomaly demonstrating complete cancellation of specular reflection decreases with the period of the surface. However, it is still sufficiently wide for most applications, even near grazing.

4) The relative power vs. $\theta_1$ curve for any optimum, finite sized, fin-corrugated surface composed of finitely thick fins can be predicted using a procedure outlined in section 4.3.1. However, this procedure is not accurate for sub-optimum surfaces.

5) The finite size used for these surfaces, which was a scaled
down size of a typical hangar wall in the ILS problem, had essentially no effect on the performance of the surface. Reductions of nearly 100% were achieved with 50 to 60 corrugations.

6) Non-plane wave illumination of these fin-corrugated surfaces seemed to have very little effect on their performances. The experimental range used in this study was about 1/3 the length of an actual ILS range and hence was a very severe test for the surfaces.

7) The experimental fin-corrugated surfaces remained completely effective for angles of rotation in the range $-10 < \alpha_i < 10$. Since ILS glide path angles are about 2 1/2°, these surfaces should be essentially as effective in reducing interference along the glide path as in the horizontal plane.

8) The experimental surfaces used in this study were not very frequency sensitive. The bandwidths over which they operated effectively were much larger than required when scaled to the ILS frequency range. The frequency sensitivity of the experimental surfaces was predictable using the parameters obtained in the prediction procedure of section 4.3.1.

9) The attenuation of the $n=1$ mode in the fin region seems to have little effect on the performance of the theoretical surfaces, even when the frequency is close to cutoff. In addition, it was found in the experimental investigation that the finite surface with short fins would operated very effectively over a range of frequencies below cutoff.

10) With an experimental frequency of 35 GHz. and the fin thickness used in this study, it is impossible to construct
a surface which will be optimum at an angle of incidence greater than 75°, since in these cases, the fin spacing would be too far below cutoff. It is believed that the value of thickness used in this study was the thinnest possible at 35 GHz. This seriously limits any near grazing investigations at this frequency.
Appendix A. Determination of the Region of Analyticity

The following is a method described by Collin [6, P.433] in which the region of analyticity of the transform function $\hat{\phi}(x,s)$ is determined:

Let $\hat{\phi}(x,s)$ be represented as the sum of two functions; $\hat{\phi}_+(x,s)$, which is analytic in the right half of the complex $s$-plane (RHP) and $\hat{\phi}_-(x,s)$, which is analytic in the left half of the complex $s$-plane (LHP). Then

$$\hat{\phi}(x,s) = \int_{-\infty}^{\infty} e^{-sz} \phi(x,z) dz + \int_{0}^{\infty} e^{-sz} \phi(x,z) dz \quad (A.1)$$

Now consider Problem # 3. In the region $z>0$, equation (2.33) may be written as

$$\phi(x,z) = A_0 e^{Y_0 z} + B_0 e^{-Y_0 z} + B_1 \cos\left(\frac{2\pi}{a}\right) e^{-Y_1 z} \quad (A.2)$$

since the evanescent modes die out within a very short distance of the fin-air interface. Let the total field $\phi(x,z)$ possess a very small amount of loss so that

$$Y_0 = Y_0^{\perp} + jY_0^{\parallel} \quad (A.3)$$

and

$$Y_1 = Y_1^{\perp} + jY_1^{\parallel} \quad (A.4)$$

where $Y_0^{\perp} \ll Y_0^{\parallel}$, $Y_1^{\perp} \ll Y_1^{\parallel}$ and $Y_0^{\perp}$, $Y_0^{\parallel}$, $Y_1^{\perp}$ and $Y_1^{\parallel}$ are real and positive. Substituting (A.2) into the expression for $\hat{\phi}_+(x,s)$ and integrating, it is obvious the result can only be finite if $s_1 > Y_0^{\perp}$, $s_1 \geq -Y_0^{\perp}$, $s_1 \geq -Y_1^{\perp}$ where $s = s_1 + js_2$. Therefore, $\hat{\phi}_+(x,s)$ is analytic in the RHP if and only if
Im \( s \geq \frac{\gamma_0}{2} \) and \( \text{Im} \, s \geq -\gamma_1 \) \hspace{1cm} (A.5)

Similarly, in the region \( z < 0 \), equation (2.32) may be written as

\[
\phi(x,z) = C_0 e^{-jhx + \Gamma_0 z} + C_{-1} e^{-j(h - \frac{2\pi}{\alpha})x + \Gamma_{-1} z} \]  \hspace{1cm} (A.6)

and it may be assumed that

\[
\Gamma_0 = \Gamma_0 + j\Gamma_{10} \] \hspace{1cm} (A.7)

and

\[
\Gamma_{-1} = \Gamma_{-1} + j\Gamma_{-11} \] \hspace{1cm} (A.8)

where \( \Gamma_{0} \ll \Gamma_{10} \), \( \Gamma_{-1} \ll \Gamma_{-11} \) and \( \Gamma_0, \Gamma_{10}, \Gamma_{-1}, \Gamma_{-11} \) are real and positive. Substituting (A.6) into the expression for \( \tilde{\phi}_-(x,s) \) and integrating, it follows that \( \tilde{\phi}_-(x,s) \) is analytic in the LHP if and only if

\[
\text{Im} \, s < \Gamma_0 \quad \text{and} \quad \text{Im} \, s < \Gamma_{-1} \] \hspace{1cm} (A.9)

Therefore, if it is assumed that \( \gamma_0 \ll \gamma_1 \ll \Gamma_0 \ll \Gamma_{-1} \), the strip of common analyticity for \( \tilde{\phi}_+(x,s) \) and \( \tilde{\phi}_-(x,s) \) is as shown in figure A.1 and contains the integration contour \( \Gamma \).

![Figure A.1. The Strip of Common Analyticity.](image-url)
Appendix B. Determination of $g(s)$

It was shown in section 2.4.1 that $g(s)$ is a meromorphic function, that is, a function which contains only poles. The following is a method described by Collin [6, P.434] in which $g(s)$ is constructed from ratios of entire functions subject to the constraints (2.50) and (2.51):

For $z>0$, the integrand of equation (2.50) is regular if $g(s)/[\cos(ua)-e^{-jha}]$ cancels the zeros of $[u \sin(ua)]$ corresponding to $u = \frac{n\pi}{a}$ for $n=+0,1,2,...$ (B.1)

Substituting (B.1) into (2.40) and comparing with (2.17) it is seen that

$$s = \pm \gamma_n$$ for $n=+0,1,2,...$ (B.2)

For $n=0$, $s_1+js_2 = \pm \gamma_0 \pm j\gamma_0 \, 11$ (B.3)

or $s_1 = \pm \gamma_0 \, 1$ $s_2 = \pm \gamma_0 \, 11$ (B.4)

where $\gamma_0 \, 1 < \gamma_0 \, 11$ as before. Similarly, for $n=1$,

$s_1 = \pm \gamma_1 \, 1$ $s_2 = \pm \gamma_1 \, 11$ (B.5)

where $\gamma_1 \, 1 < \gamma_1 \, 11$, and, for $n=+2,3,...$,

$s_1 = \pm \gamma_n$ (B.6)

since $\gamma_n$ is pure real for $n \neq 0,1$. However, because $z>0$ corresponds to the LHP and the $n=0$ mode is the only mode incident in the fin region, the zeros of $[u \sin(ua)]$ are
\[ \begin{align*}
  n=0 & \quad s = \pm \gamma_0 \frac{1}{1} + j \gamma_0 \frac{11}{1} \\
  n=+1 & \quad s = -\gamma_1 \frac{1}{1} - j \gamma_1 \frac{11}{1} \\
  n=+2,3,\ldots & \quad s = -\gamma_n
\end{align*} \] (B.7)

These zeros are plotted in figure B.1.

For \( z<0 \), the integrand of (2.51) is regular if \( g(s)/[\cos(ua) - e^{-jha}] \) cancels the zeros of \( [\cos(ua) - \cos(ha)] \) corresponding to

\[ u = h + \frac{2n\pi}{a}, \quad n = 0,1,2,\ldots \] (B.8)

Substituting (B.8) into (2.40) and comparing with (2.15) it can be shown that

\[ s = \pm \Gamma_n \quad \quad n = \pm 0,1,2,\ldots \] (B.9)

For \( n=0 \),

\[ s_1 + js_2 = \pm \Gamma_0 \frac{1}{1} + j \Gamma_0 \frac{11}{1} \] (B.10)

or

\[ s_1 = \pm \Gamma_0 \frac{1}{1} \quad \quad s_2 = \pm \Gamma_0 \frac{11}{1} \] (B.11)

where \( \Gamma_0 \frac{1}{1} \ll \Gamma_0 \frac{11}{1} \) as before. Similarly, for \( n=-1 \),

\[ s_1 = \pm \Gamma_{-1} \frac{1}{1} \quad \quad s_2 = \pm \Gamma_{-1} \frac{11}{1} \] (B.12)

where \( \Gamma_{-1} \frac{1}{1} \ll \Gamma_{-1} \frac{11}{1} \), and, for \( n = +1,+2,3,\ldots \),

\[ s_1 = \pm \Gamma_n \] (B.13)

since \( \Gamma_n \) is pure real for \( n \neq 0,-1 \). However, because \( z<0 \) corresponds to the RHP, the zeros of \( [\cos(ua) - \cos(ha)] \) are
Figure B.1 Zero Plot of \( u \sin(ua) \) and \( \cos(ha) - \cos(ua) \) with losses.

Figure B.2 Pole Plot of \( g(s)/[\cos(ua) - e^{-jha}] \)
\[
\begin{align*}
n=0 & \quad s = +\Gamma_0^1 + j\Gamma_0^{11} \\
n= -1 & \quad s = +\Gamma_{-1}^1 + j\Gamma_{-1}^{11} \\
n= +1, 2, 3, \ldots & \quad s = +\Gamma_n
\end{align*}
\]  

(B.14)

These zeros are also plotted in figure B.1.

Now assume that the losses \(\gamma_0^1, \gamma_1^1, \Gamma_0^1\) and \(\Gamma_{-1}^1\) tend to zero. The zero plot of figure B.1 then transforms to the zero plot of figure B.2. Since the poles of \(g(s)/[\cos(ua) - e^{-jha}]\) must cancel the zeros of \([u \sin(ua)]\) and \([\cos(ha) - \cos(ua)]\), figure B.2 is therefore the pole plot of that function. It follows that a suitable construction for \(g(s)\) would be

\[
g(s) = \frac{P(s) \{\cos(ua) - e^{-jha}\}}{(s-\gamma_0)(s+\gamma_0)(s-\Gamma_0^1) \prod_{1}^{\infty} \left( \frac{(s+\gamma_n^1)}{(n\pi/a)} e^{-\frac{sa}{n\pi}} \right) \prod_{1}^{\infty} \left( \frac{(s-\Gamma_n^1)(s-\Gamma_{-n}^1)}{(2n\pi/a)^2} e^{\frac{sa}{n\pi}} \right)}
\]

(B.15)

where \(P(s)\) is an entire function yet to be determined. It will be shown in Appendix C that the exponential factors and the terms \(\left(\frac{n\pi}{a}\right)\) and \(\left(\frac{2n\pi}{a}\right)^2\) in the denominator of (B.15) are used to ensure the uniform convergence of the infinite products.
Appendix C. Convergence of the Infinite Products in \( g(s) \)

From equations (2.17) and (2.15), it is obvious that, for large \( n \), \( \gamma_n \to \left(\frac{n \pi}{a}\right) \), \( \Gamma_n \to \left(\frac{2n \pi}{a} + h\right) \) and \( \Gamma_n \to \left(\frac{2n \pi}{a} - h\right) \). Therefore,

\[
\prod_{n=1}^{\infty} \left(1 + \frac{z}{n} \right) e^{\frac{-z n}{n \pi}} = \prod_{n=1}^{\infty} \left(1 + \frac{sa}{n \pi} \right) e^{\frac{-sa}{n \pi}} \tag{C.1}
\]

and

\[
\prod_{n=1}^{\infty} \left(1 - \frac{z}{n} \right) e^{\frac{z n}{2n \pi}} = \prod_{n=1}^{\infty} \left(1 - \frac{(s-h)a}{2n \pi} \right) e^{\frac{(s-h)a}{2n \pi}} \left(1 - \frac{(s+h)a}{2n \pi} \right) e^{\frac{(s+h)a}{2n \pi}} \tag{C.2}
\]

However, the right hand sides of (C.1) and (C.2) are of the same form as the Euler-Mascheroni expression

\[
\prod_{n=1}^{\infty} \left(1 + \frac{z}{n} \right) e^{\frac{-z n}{z \Gamma(z)}} = \frac{e^{-\gamma z}}{z \Gamma(z)} \quad \gamma = \text{constant} \tag{C.3}
\]

which is finite for \(|z| < \infty\). Therefore, the infinite products of \( g(s) \), equation (B.15) of Appendix B, are uniformly convergent.
Appendix D. Asymptotic Behaviour of \( g(s) \)

In section 2.1.1, it was stated that the total field \( \psi(x,z) \) is asymptotic to \( z^{1/2} \) in the neighbourhood of a fin edge. That is,

\[
\psi(ma,z) = A(ma)z^{1/2} + \text{higher order terms} \quad \text{as} \quad z \to 0 \quad (D.1)
\]

where \( m = \pm 0, 1, 2, \ldots \). Substituting this expression into equation (2.38) gives

\[
g(s) = \int_{-\infty}^{\infty} e^{-sz} [A(o)z^{1/2} + \text{higher order terms}]dz \quad (D.2)
\]

Watson's Lemma guarantees that if this result is integrated term by term, the asymptotic expansion of \( g(s) \) as \( |s| \to \infty \) will result.

Consider the first term only. The integrand has a branch point at \( z=0 \) and so if the branch cut is chosen as shown in figure D.1 and the integral closed in the upper half plane, by Jordan's Lemma

\[
g(s) = \int_{-\infty}^{\infty} e^{-sz}A(o)z^{1/2}dz = -2A(o) \cdot \int_{0}^{\infty} e^{-sz}z^{1/2}dz \quad (D.3)
\]

Making the substitution \( t = -jz \) and using the integral representation for the gamma function, it is found that

\[
g(s) = -2A(o)\Gamma\left(\frac{3}{2}\right)s^{-3/2} \quad \text{as} \quad |s| \to \infty \quad (D.4)
\]

Thus, \( g(s) \) is asymptotic to \( s^{-3/2} \) as \( |s| \to \infty \).
Figure D.1  $z$-Plane Representation Showing Branch Cut.

\[ z = z_1 + jz_2 \]
Appendix E. Determination of P(s) in g(s)

It was stated in section 2.4.2 that P(s) must be chosen so that g(s) has algebraic growth at infinity and is asymptotic to \( s^{-3/2} \) as \( |s| \to \infty \). The following approach to this problem is similar to that used by Collin [6, P.436].

Substituting the expressions (C.1) and (C.2) into equation (B.15), it is seen that g(s) differs by only a bounded function of s from \( g_1(s) \) where \( g_1(s) \) is given by

\[
g_1(s) = \frac{P(s) \left( \cos(ua) - e^{-jha} \right)}{s^3 \prod_{n=1}^{\infty} (1 + \frac{sa}{n\pi}) e^{n\pi} \prod_{n=1}^{\infty} (1 - \frac{(s-h)a}{2n\pi}) e^{2n\pi} \prod_{n=1}^{\infty} (1 - \frac{(s+h)a}{2n\pi}) e^{2n\pi}}
\]

(E.1)

Eliminating the infinite products in (E.1) using equation (C.3) with the proper value of \( z \) and noting that

\[
\Gamma(z) = \frac{\pi}{\sin(\pi z) \Gamma(1-z)}
\]

(E.2)

\( g_1(s) \) can be manipulated into the form

\[
g_1(s) = \frac{P(s) \left( \cos(ua) - e^{-jha} \right) (a\pi) \Gamma\left(\frac{sa}{\pi}\right)}{s^2 \sin\left(\frac{(s-h)a}{2}\right) \sin\left(\frac{(s+h)a}{2}\right) \Gamma\left(\frac{(s-h)a}{2\pi}\right) \Gamma\left(\frac{(s+h)a}{2\pi}\right)}
\]

(E.3)

Now, the asymptotic expansion of \( \Gamma(z) \) is

\[
\Gamma(z) = \sqrt{2\pi} e^{\ln z - 1/2} e^{-z+1} \quad |z| \to \infty
\]

(E.4)

If this expression is used to replace the \( \Gamma \) functions in equation (E.3)
and it is assumed that $h \ll s$ as $|s| \to \infty$, an asymptotic expansion for $g_1(s)$ of the form

$$g_1(s) = \frac{K P(s) \left( \cos(ua)-e^{-jha} \right) e^{\frac{sa}{\pi} \ln 2}}{s^{3/2} \sin(\frac{(s-h)a}{2}) \sin(\frac{(s+h)a}{2})} \quad |s| \to \infty$$

(E.5)

will result, where $K$ is a known constant. Therefore, for $g_1(s)$ and hence $g(s)$ to be asymptotic to $s^{-3/2}$, it is obvious that

$$P(s) = C e^{-\frac{sa}{\pi} \ln 2}$$

(E.6)

where $C$ is a constant.
Appendix F. Experimental Data

The following is a collection of the experimental data taken during the study. Most of this data appears in graphical form in Chapter 4.

<table>
<thead>
<tr>
<th>Plates 3A, 3B</th>
<th>Plates 1A, 1B</th>
</tr>
</thead>
<tbody>
<tr>
<td>[\theta_i] (degrees)</td>
<td>[\lambda] (m.)</td>
</tr>
<tr>
<td>70.0</td>
<td>2.25</td>
</tr>
<tr>
<td>67.5</td>
<td>2.29</td>
</tr>
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<td>65.0</td>
<td>2.34</td>
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<td>62.5</td>
<td>2.40</td>
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<tr>
<td>60.0</td>
<td>2.47</td>
</tr>
<tr>
<td>57.5</td>
<td>2.54</td>
</tr>
<tr>
<td>56.5</td>
<td>2.57</td>
</tr>
<tr>
<td>55.5</td>
<td>2.61</td>
</tr>
<tr>
<td>54.5</td>
<td>2.64</td>
</tr>
<tr>
<td>53.5</td>
<td>2.52</td>
</tr>
<tr>
<td>52.5</td>
<td>2.55</td>
</tr>
<tr>
<td>50.0</td>
<td>2.66</td>
</tr>
<tr>
<td>47.5</td>
<td>2.47</td>
</tr>
<tr>
<td>45.0</td>
<td>2.35</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table V Transmission Distance vs. Angle of Incidence (see figure 4.3(a))
Table VI  Relative Power of the $n=0$ Mode vs. Angle of Incidence, Plate 3A, $f=35$ GHz.

<table>
<thead>
<tr>
<th>$\theta_1$</th>
<th>$\alpha_1$</th>
<th>$-5.0^\circ$</th>
<th>$-2.5^\circ$</th>
<th>$0^\circ$</th>
<th>$+2.5^\circ$</th>
<th>$+5.0^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.0°</td>
<td>-6.9</td>
<td>-6.85</td>
<td>-6.75</td>
<td>-6.95</td>
<td>-7.1</td>
<td></td>
</tr>
<tr>
<td>67.5°</td>
<td>-8.0</td>
<td>-8.15</td>
<td>-7.9</td>
<td>-8.15</td>
<td>-8.2</td>
<td></td>
</tr>
<tr>
<td>65.0°</td>
<td>-9.95</td>
<td>-10.05</td>
<td>-9.8</td>
<td>-10.1</td>
<td>-10.0</td>
<td></td>
</tr>
<tr>
<td>62.5°</td>
<td>-12.3</td>
<td>-12.2</td>
<td>-11.95</td>
<td>-12.2</td>
<td>-12.2</td>
<td></td>
</tr>
<tr>
<td>60.0°</td>
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<td>-15.55</td>
<td>-15.15</td>
<td>-15.5</td>
<td>-15.4</td>
<td></td>
</tr>
<tr>
<td>57.5°</td>
<td>-20.4</td>
<td>-20.5</td>
<td>-20.0</td>
<td>-20.5</td>
<td>-20.3</td>
<td></td>
</tr>
<tr>
<td>56.5°</td>
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<td>-23.3</td>
<td>-23.0</td>
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<td>-22.8</td>
<td></td>
</tr>
<tr>
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<td>-23.4</td>
<td>-23.0</td>
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<tr>
<td>54.5°</td>
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<tr>
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<td>-19.7</td>
<td>-19.3</td>
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<tr>
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<td>-17.55</td>
<td>-16.75</td>
<td>-16.65</td>
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<tr>
<td>50.0°</td>
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<td>-11.9</td>
<td>-12.4</td>
<td>-11.9</td>
<td>-11.9</td>
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</tr>
<tr>
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<td>-9.1</td>
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<td>-8.75</td>
<td></td>
</tr>
<tr>
<td>45.0°</td>
<td>-6.05</td>
<td>-6.0</td>
<td>-6.35</td>
<td>-6.1</td>
<td>-6.05</td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>$-5.0^\circ$</td>
<td>$-2.5^\circ$</td>
<td>$0^\circ$</td>
<td>$+2.5^\circ$</td>
<td>$+5.0^\circ$</td>
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<tr>
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<td>------------</td>
<td>--------</td>
<td>------------</td>
<td>------------</td>
<td></td>
</tr>
<tr>
<td>70.0°</td>
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<td>-6.4</td>
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</tr>
<tr>
<td>67.5°</td>
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<td>-7.3</td>
<td>-7.4</td>
<td>-7.65</td>
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</tr>
<tr>
<td>65.0°</td>
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<td>-8.75</td>
<td>-8.55</td>
<td>-8.6</td>
<td>-9.0</td>
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<tr>
<td>62.5°</td>
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<td>-10.2</td>
<td>-10.2</td>
<td>-10.6</td>
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<tr>
<td>60.0°</td>
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<td>-12.1</td>
<td>-12.1</td>
<td>-12.2</td>
<td>-12.6</td>
<td></td>
</tr>
<tr>
<td>57.5°</td>
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<td>-14.2</td>
<td>-14.25</td>
<td>-14.2</td>
<td>-14.7</td>
<td></td>
</tr>
<tr>
<td>56.5°</td>
<td>-15.6</td>
<td>-15.0</td>
<td>-15.1</td>
<td>-15.0</td>
<td>-15.6</td>
<td></td>
</tr>
<tr>
<td>55.5°</td>
<td>-15.5</td>
<td>-15.2</td>
<td>-15.4</td>
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<tr>
<td>54.5°</td>
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<td>-14.8</td>
<td>-15.1</td>
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<td>-15.15</td>
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<tr>
<td>53.5°</td>
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<td>-14.4</td>
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<td>50.0°</td>
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<td>-10.9</td>
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<td>47.5°</td>
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<tr>
<td>45.0°</td>
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<td>-5.65</td>
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</table>

Table VII Relative Power of the n=0 Mode vs. Angle of Incidence, Plate 3B, $f=35$ GHz.
<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>RELATIVE POWER n=0 MODE (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.00</td>
<td>-9.6</td>
</tr>
<tr>
<td>33.20</td>
<td>-10.3</td>
</tr>
<tr>
<td>33.40</td>
<td>-10.75</td>
</tr>
<tr>
<td>33.60</td>
<td>-11.4</td>
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<tr>
<td>33.84</td>
<td>-12.45</td>
</tr>
<tr>
<td>34.00</td>
<td>-13.1</td>
</tr>
<tr>
<td>34.20</td>
<td>-14.55</td>
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<td>34.40</td>
<td>-16.20</td>
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<td>34.60</td>
<td>-18.40</td>
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<tr>
<td>35.20</td>
<td>-26.4</td>
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<tr>
<td>35.40</td>
<td>-30.6</td>
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<td>35.65</td>
<td>-31.8</td>
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<tr>
<td>35.80</td>
<td>-30.5</td>
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<tr>
<td>36.00</td>
<td>-27.8</td>
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<td>36.20</td>
<td>-25.5</td>
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<tr>
<td>36.40</td>
<td>-23.3</td>
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<tr>
<td>36.60</td>
<td>-21.5</td>
</tr>
<tr>
<td>36.80</td>
<td>-20.0</td>
</tr>
<tr>
<td>37.00</td>
<td>-18.7</td>
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</tbody>
</table>

Table VIII Relative Power of the n=0 Mode vs. Frequency of the Incident Wave, Plate 3A, $a_i=0^\circ$, $\theta_i=54.5^\circ$
<table>
<thead>
<tr>
<th>$\theta_1$ (degrees)</th>
<th>RELATIVE POWER n=0 MODE (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>79.0°</td>
<td>-10.15</td>
</tr>
<tr>
<td>78.0</td>
<td>-10.89</td>
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<td>77.0</td>
<td>-12.25</td>
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<td>76.0</td>
<td>-13.06</td>
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<td>75.0</td>
<td>-13.80</td>
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<td>74.0</td>
<td>-15.5</td>
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<tr>
<td>73.0</td>
<td>-17.4</td>
</tr>
<tr>
<td>72.0</td>
<td>-18.7</td>
</tr>
<tr>
<td>71.0</td>
<td>-20.75</td>
</tr>
<tr>
<td>70.0</td>
<td>-21.7</td>
</tr>
<tr>
<td>69.0</td>
<td>-21.7</td>
</tr>
<tr>
<td>($\alpha_1 = +2.5^\circ$)</td>
<td>-24.0</td>
</tr>
<tr>
<td>($\alpha_1 = +5.0^\circ$)</td>
<td>-25.8</td>
</tr>
<tr>
<td>($\alpha_1 = -2.5^\circ$)</td>
<td>-23.9</td>
</tr>
<tr>
<td>($\alpha_1 = -5.0^\circ$)</td>
<td>-25.5</td>
</tr>
<tr>
<td>68.0</td>
<td>-21.2</td>
</tr>
<tr>
<td>67.0</td>
<td>-19.9</td>
</tr>
<tr>
<td>66.0</td>
<td>-16.78</td>
</tr>
<tr>
<td>65.0</td>
<td>-14.5</td>
</tr>
<tr>
<td>64.0</td>
<td>-12.41</td>
</tr>
</tbody>
</table>

Table IX. Relative Power of the n=0 Mode vs. Angle of Incidence, Plate 1B, $\alpha_1=0^\circ$, f=37 GHz.
<table>
<thead>
<tr>
<th>$\theta_i$ (degrees)</th>
<th>AVERAGE RELATIVE POWER n=0 MODE (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>84.0</td>
<td>-0.478</td>
</tr>
<tr>
<td>83.0</td>
<td>-0.728</td>
</tr>
<tr>
<td><strong>82.5</strong></td>
<td>-0.763</td>
</tr>
<tr>
<td>82.0</td>
<td>-0.925</td>
</tr>
<tr>
<td>81.5</td>
<td>-0.717</td>
</tr>
<tr>
<td>81.0</td>
<td>-0.727</td>
</tr>
<tr>
<td>80.0</td>
<td>-0.81</td>
</tr>
<tr>
<td>79.0</td>
<td>-0.777</td>
</tr>
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<td>78.0</td>
<td>-0.64</td>
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<td>77.0</td>
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<td>-0.42</td>
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<tr>
<td>69.0</td>
<td>-0.383</td>
</tr>
<tr>
<td>68.0</td>
<td>-0.29</td>
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<tr>
<td>67.0</td>
<td>-0.283</td>
</tr>
<tr>
<td>66.0</td>
<td>-0.30</td>
</tr>
<tr>
<td>65.0</td>
<td>-0.30</td>
</tr>
<tr>
<td>64.0</td>
<td>-0.23</td>
</tr>
<tr>
<td>63.0</td>
<td>-0.263</td>
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</table>

Table X  Relative Power of the n=0 Mode vs. Angle of Incidence, Plate 1A, $\alpha_i=0^\circ$, f=37 GHz.
<table>
<thead>
<tr>
<th>FREQUENCY (GHz)</th>
<th>RELATIVE POWER n=0 MODE (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.35.8</td>
<td>-4.6</td>
</tr>
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<td>35.9</td>
<td>-6.4</td>
</tr>
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<td>36.0</td>
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<tr>
<td>36.4</td>
<td>-16.4</td>
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<tr>
<td>36.6</td>
<td>-22.7</td>
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<tr>
<td>36.8</td>
<td>-25.7</td>
</tr>
<tr>
<td>37.0</td>
<td>-21.7</td>
</tr>
</tbody>
</table>

Table XI  Relative Power of the n=0 Mode vs. Frequency of the Incident Wave, Plate 1B, $\alpha_i = 0^\circ$, $\theta_i = 69.0^\circ$
References


Historical


