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A STUDY OF THE ENERGY BALANCE OF A DOUGLAS-FIR FOREST

by

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ABSTRACT

This thesis is in the form of four self contained papers that report aspects of a study of the energy balance of a young Douglas-fir forest growing at the University of British Columbia Research Forest at Haney, B.C. The chapters discuss the experimental methods used and special instrumentation developed, the data analysis, and some attempts to understand the relationships observed by employing simple forest models and ideas on boundary layer equilibration processes.

Chapter 1. The psychrometric apparatus design for Bowen ratio determination reported by Sargeant and Tanner was modified and a new apparatus built. Modification of the intake design improved the symmetry and rigidity of the sensor mounting. Wet and dry bulb differences were measured with an error less than 0.01°C over a vertical distance of 1 meter. Continuous measurements of the Bowen ratio over a 7.8-meter Douglas-fir forest were made for 6 weeks. An example of the energy balance for the forest for 1 day using this equipment is reported.

Chapter 2. Daily evapotranspiration from a Douglas-fir forest was calculated using Webb's average Bowen ratio method. Webb's method is generalized to include the effects of the diabatic wind profile. Over a 17-day period characterized by light winds, the modified Webb

method agreed with the daily totals of half-hour energy balance calculations to within 1 1/2% on the average, while Webb's method overestimated by 26% on the average.

Chapter 3. Energy balance measurements of evapotranspiration from a young Douglas-fir forest are reported for a period of 18 days in July 1970 when soil water was not limiting. Peak daily evapotranspiration rates characteristically occurred two to three hours after solar noon and evapotranspiration showed a short-term independence from net radiation. This behaviour is interpreted as being a consequence of the large forest roughness. Daily evapotranspiration and net radiation were, however, well correlated. Values of surface diffusion resistance calculated from Monteith's combination formula are presented. Daytime values showed significant day-to-day differences and an attempt to define a potential evapotranspiration rate assuming a constant daytime surface resistance was not successful. Comparison of evapotranspiration measurements with a potential evaporation formula for wet surfaces developed by Priestley and Taylor suggests that evaporation of intercepted water proceeds 20% more rapidly than evapotranspiration from the non-wetted canopy.

Chapter 4. The process of modification of the Bowen ratio, with distance downwind of a change in surface wetness, is considered with the view to establishing the final equilibrium ratio of the fluxes of sensible and latent heat after advective effects become negligible. A method of generating, from the coupled equations for heat and vapour diffusion, two new diffusion equations in composite variables,

which may be solved independently, is developed.

The method leads to the conclusions that there is a basic difference between equilibration over land and over water. Also the rate of equilibration depends strongly on atmospheric mixing and hence surface roughness and atmospheric stability. It is concluded that, for terrestrial surfaces, the equilibrium evaporation rate is approximated by

$$LE = \left(\frac{s}{s + \gamma} \right) (R_n - G)$$

for 24 hour periods. This result is in accord with some recent experimental findings.

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CHAPTER III

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INTRODUCTION

This thesis is in four parts, each presented in the form of an independent paper. The papers were written at intervals during my investigations into the subject of forest energy balances as part of my doctoral studies at the University of British Columbia.

The material presented in the first two chapters has been published, with only minor differences, in the journal 'Boundary Layer Meteorology' under the senior authorship of Dr. T.A. Black. These sections are included in the thesis so that as much as possible of the energy balance research in which I participated should be presented under one cover. This earlier work also provides a background for the main presentation of the experimental results and interpretation presented in chapters three and four.

The most immediate problem facing a researcher intending to study forest energy balances is that of making measurements. Equipment of suitable design is not commercially available for measuring some of the most critical meteorological elements. For this reason, the first problem considered in this study is that of the design of suitable equipment to measure the very small gradients of temperature and humidity above the forest, needed to calculate the energy balance components by the Bowen ratio method. The success of this basic instrumental design work has provided opportunity to collect a large body of data of apparently high quality. As such it is the

basis of all of the work following. This work is reported in the first chapter.

Having collected the raw data in the field in July and August 1970, the next problem was to calculate the energy balance components. A problem encountered was that of estimating the heat storage rate in the canopy biomass and air column. An estimate of this quantity was made based on simple assumptions and an estimate of the canopy biomass heat capacity. An attempt was made to cross-check these calculations by comparing the totals of the fluxes calculated each half-hour, with those calculated by a 24-hour averaging method that did not include heat storage estimates. Very poor agreement was obtained in this comparison using an averaging method taken from the literature. The second chapter of this thesis is devoted to an examination of Bowen ratio averaging methods in order to resolve the discrepancy.

In the third chapter the main body of the experimental results is reported. An attempt is made to account for some regular patterns observed in the calculated energy balance components. A simple canopy model is used that treats the canopy as an idealized single leaf.

One interesting outcome is that the daily evapotranspiration correlated very well with net radiation even though results from the analysis using the single leaf model indicated that surface resistance changed significantly from day to day. According to the model the surface resistance was closely related to the total stomatal

resistance of the canopy and was, as such, a purely physiological variable. Net radiation is largely determined by solar radiation and so is an independent variable. The implication was that the vapour pressure deficit is an entirely dependent variable determined uniquely by stomatal resistance and net radiation.

Two explanations seemed possible. One was that the surface resistance was largely a disguised meteorological variable produced by using a grossly oversimplified model.

This possibility was investigated but not reported in this thesis. The conclusion was that, while the surface resistance is not a quantitatively exact measure of total parallel stomatal resistance of forest canopies, it is quite closely related and the changes in surface resistance noted did represent changes in tree physiological condition manifested as changes in stomatal resistance.

The other possibility was that the air moving over the forest was modified in such a way that a balance was reached between surface resistance and vapour pressure deficit of the air. This seemed to produce a unique average Bowen ratio which was temperature dependent, but independent of surface resistance. This idea was investigated and the resulting theoretical treatment is presented in Chapter four of this thesis.

CHAPTER 1

PSYCHROMETRIC APPARATUS FOR BOWEN RATIO DETERMINATION OVER FORESTS

PSYCHROMETRIC APPARATUS FOR BOWEN RATIO
DETERMINATION OVER FORESTS

Abstract

The psychrometric apparatus design for Bowen ratio determination reported by Sargeant and Tanner was modified and a new apparatus built. Modification of the intake design improved the symmetry and rigidity of the sensor mounting. Wet and dry bulb differences were measured with an error less than 0.01°C over a vertical distance of 1 meter. Continuous measurements of the Bowen ratio over a 7.8-meter Douglas-fir forest were made for 6 weeks. An example of the energy balance for the forest for 1 day using this equipment is reported.

1. Introduction

The West Coast forest environment imposes some stringent requirements on apparatus used to determine the Bowen ratio. Temperature and vapor pressure gradients are small compared with those over an agricultural crop due to increased scale. A large vertical displacement between sensors makes the fetch requirements for a satisfactory measurement prohibitively large. Thus an apparatus with small sensor separation and high accuracy is necessary. In addition, an apparatus is required that would give reliable results over an extended period for energy balance studies. This paper reports on the construction and

performance of a psychrometric apparatus designed to accomplish this. It was built to a design modified from that reported by Sargeant and Tanner (1967).

2. Sensors and Measurement Circuit

The temperature sensors were two matched pairs of 1N2326 germanium diodes. Principles of diode circuitry have been discussed by Sargeant (1965). The germanium diode was chosen because of its linear voltage-temperature characteristic and large temperature sensitivity of about $2.3 \text{ mV } ^\circ\text{C}^{-1}$. From 24 diodes 2 pairs were selected whose temperature sensitivities matched to 0.4% and 0.1% and whose junction voltages at 10°C were matched to 0.08 mV and 0.03 mV respectively. These closely matched pairs of diodes eliminated the need for balancing resistors in the bridge circuit (Figure 1). The voltage difference between terminals A and B of Figure 1 is proportional to the difference between the two dry bulb temperatures, $(T_{d1} - T_{d2} = \Delta T_d)$, while the voltage difference between C and D is proportional to the difference between the two wet bulb temperatures $(T_{w1} - T_{w2} = \Delta T_w)$. The voltage difference between E and A, B, C and D is linearly related to the corresponding wet or dry bulb temperature. A 4-wire bridge configuration was used to simplify the calculation of the absolute temperatures by eliminating any need to consider the lead resistance.

3. Sensing Heads

In field use other investigators have experienced difficulties in obtaining reliable long-term measurements using the psychrometric apparatus designed by Sargeant and Tanner (T.A. Black, personal communication). In attempting to improve the performance of the apparatus, particular attention was paid to improving the alignment, rigidity and durability of the sensing-head assembly. Attention was also paid to the improvement of measurement symmetry.

The sensing heads were made from 2.54-cm I.D. stainless steel tube with a 1.3-mm wall thickness (Figure 2). Slots 11.9 cm long and 1.1 cm deep were milled out of the tube 2.5 cm from the air-intake end. A stainless-steel trapdoor made from tube of the same size was made to fit each slot. Flanges were silver soldered on each end of the trapdoor to position it. A pair of diodes was mounted to each trapdoor using a thixotropic epoxy resin (3-M Scotchcast No. 10) so that their axes lay on the tube axis as shown in Figure 3. The wet-bulb was mounted 2.5 cm behind the dry-bulb diode. A 2.1-cm I.D. by 16-cm long acrylic tube attached to the stainless-steel tube was the water reservoir for the wet bulb. The rubber stopper in the end of the reservoir provided access for insertion of the wick. A boiled shoelace (sock type) served satisfactorily as a wick which passed from the reservoir to the wet bulb diode through a thin-wall stainless-steel tube silver soldered to the trapdoor. The sensing heads were covered with a layer of polyurethane thermal insulation 1.3 cm thick and wrapped with adhesive aluminized mylar for radiative shielding. The mylar also held the trapdoor in place.

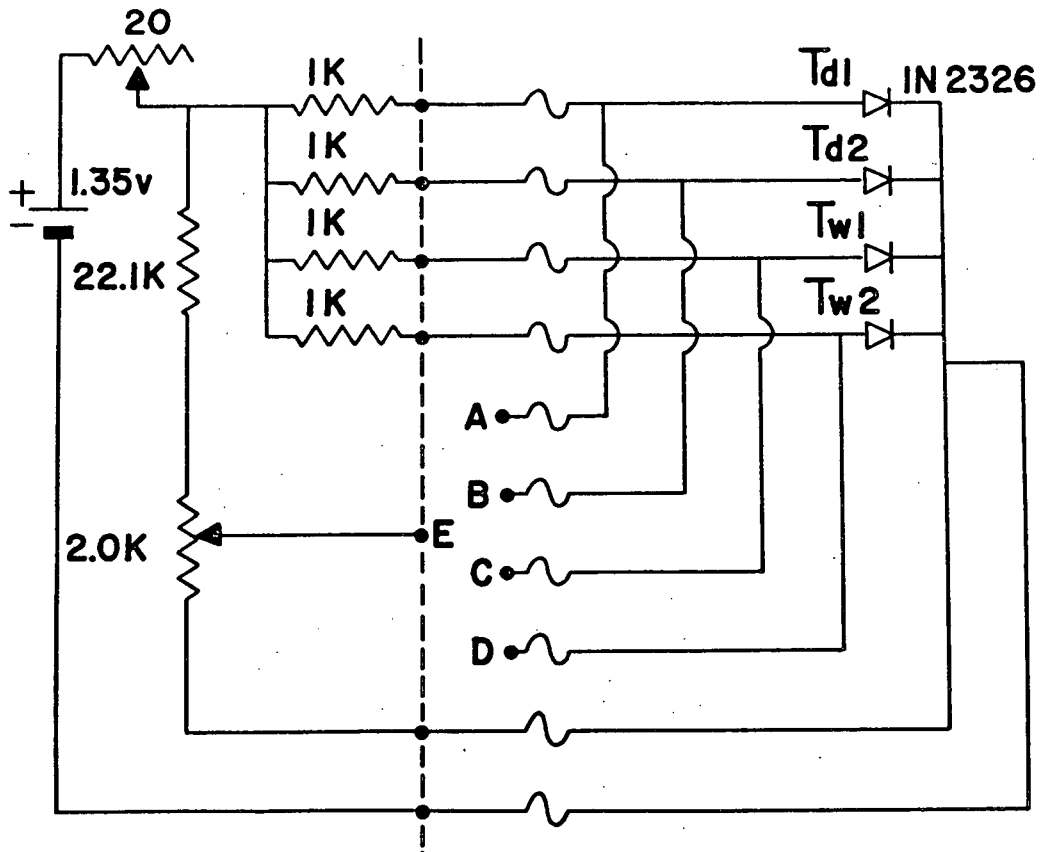


Figure 1 Measurement circuit used in the psychrometric apparatus for Bowen ratio determination. All fixed resistors are low temperature coefficient metal film type. The 1K resistors are matched to 0.2%.

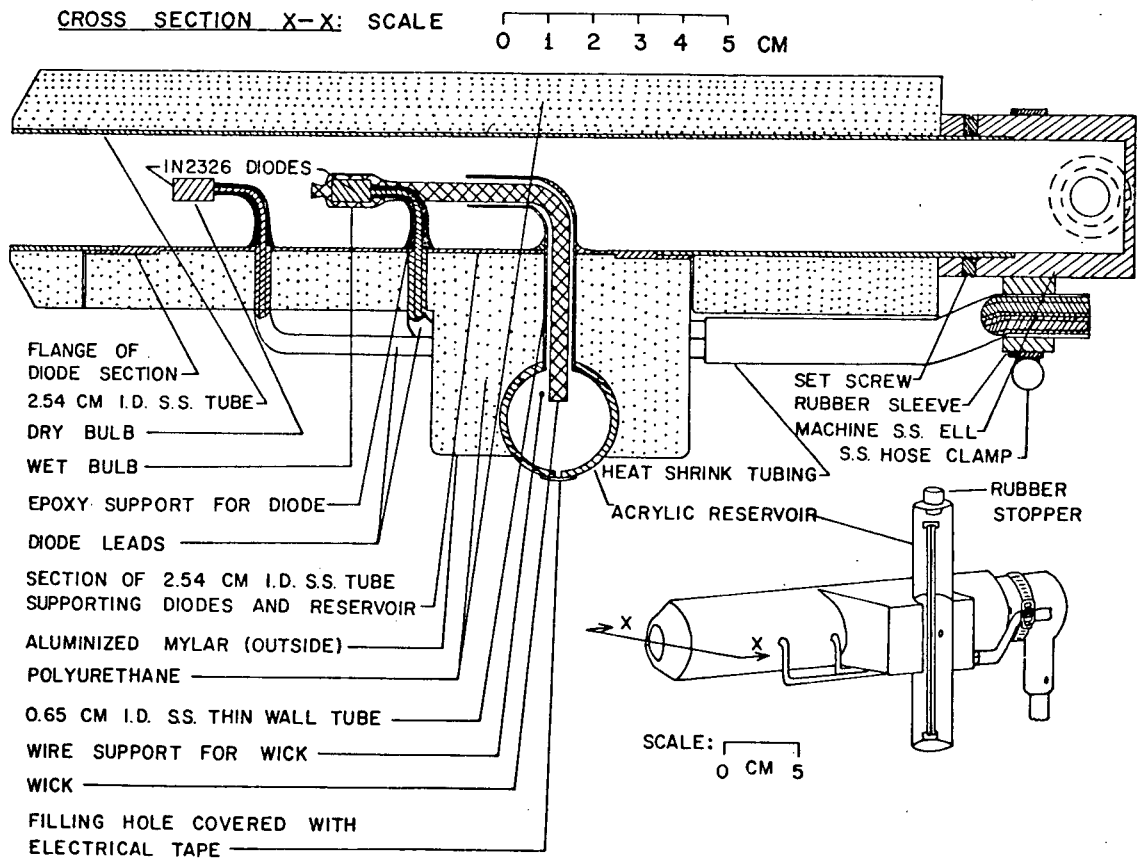


Figure 2 Sensing head design. The second head is a mirror image of the head illustrated.

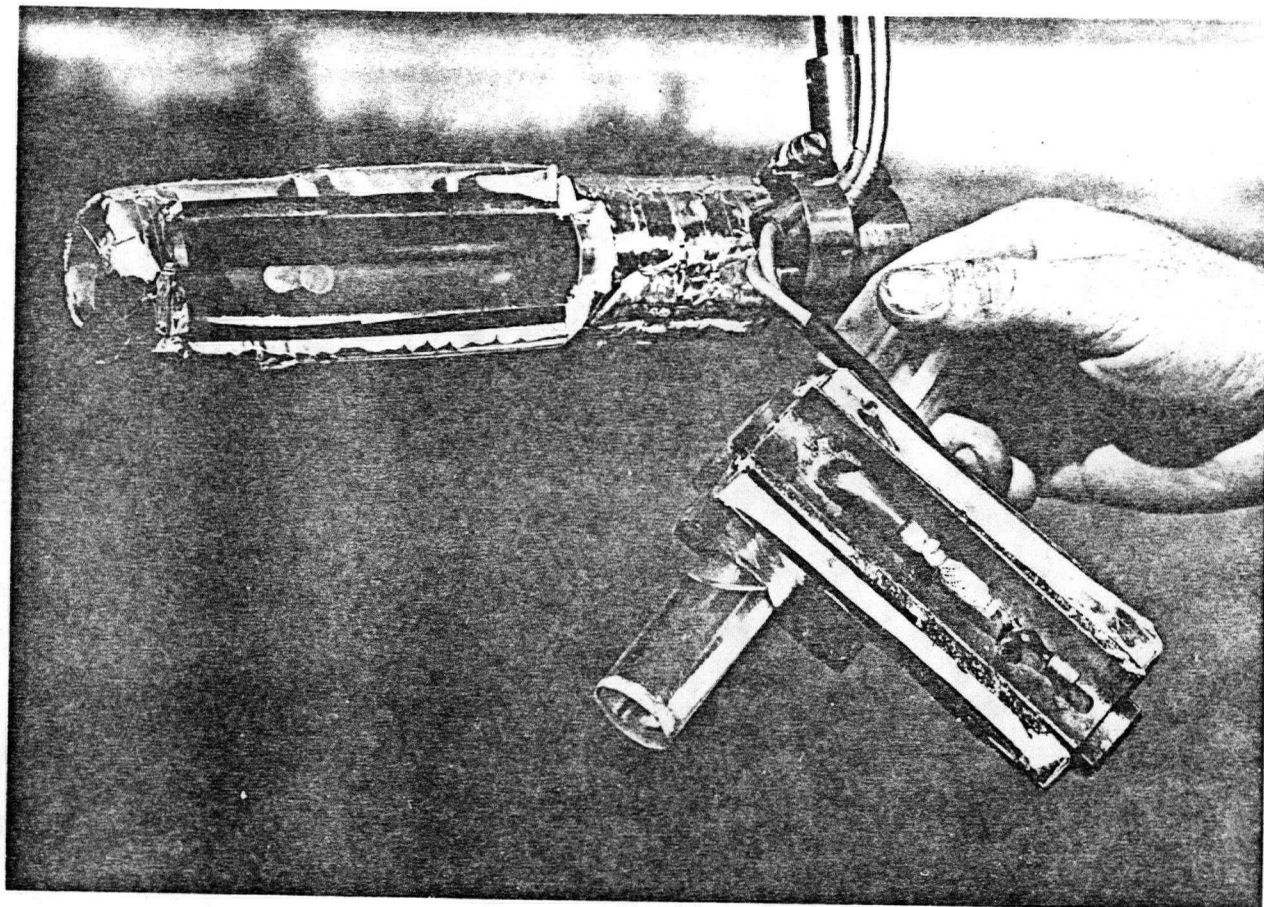


Figure 3 Sensing head with the trapdoor displayed to show the mounting of the diode sensors and wicking system.

It was found unnecessary to insulate the water reservoir. The reservoirs had to be refilled at intervals of 2 to 5 days.

The sensing heads were mounted on a rotating assembly (Figure 4) similar to that of Sargeant and Tanner. The 0.9-cm I.D. stainless steel tube which formed the vertical arms also served as air tubes for aspiration. In a more recent model air is drawn through the pipe that supports the assembly and through the bearing thus avoiding the flexible tube looped around the reversing motor in Figure 4. A Gast vacuum pump housed on the ground 10 m from the tower drew air through semirigid polyethylene tubing. The pump flow rate was adjusted to 170 l min^{-1} resulting in a ventilation velocity past the diodes of 300 cm sec^{-1} . It was found necessary to cushion the action of the rotating sensing assembly by using rubber cushions on the positioning stops. A stiff synthetic rubber linkage between the axially mounted reversing motor and the rotating assembly prevented damage to the motor.

4. Performance and Evaluation

The apparatus was in continuous operation from July 7 to August 28, 1970 above a 7.8-m high Douglas-fir forest. The equipment was used to determine Bowen ratios as part of a forest energy balance study during the summer.

In an attempt to optimize the height of the instrument placement with respect to fetch and surface heterogeneity requirements, the axis of the rotation of the assembly was positioned 8.6 m above the soil surface. With this choice of height, the measured gradients were not

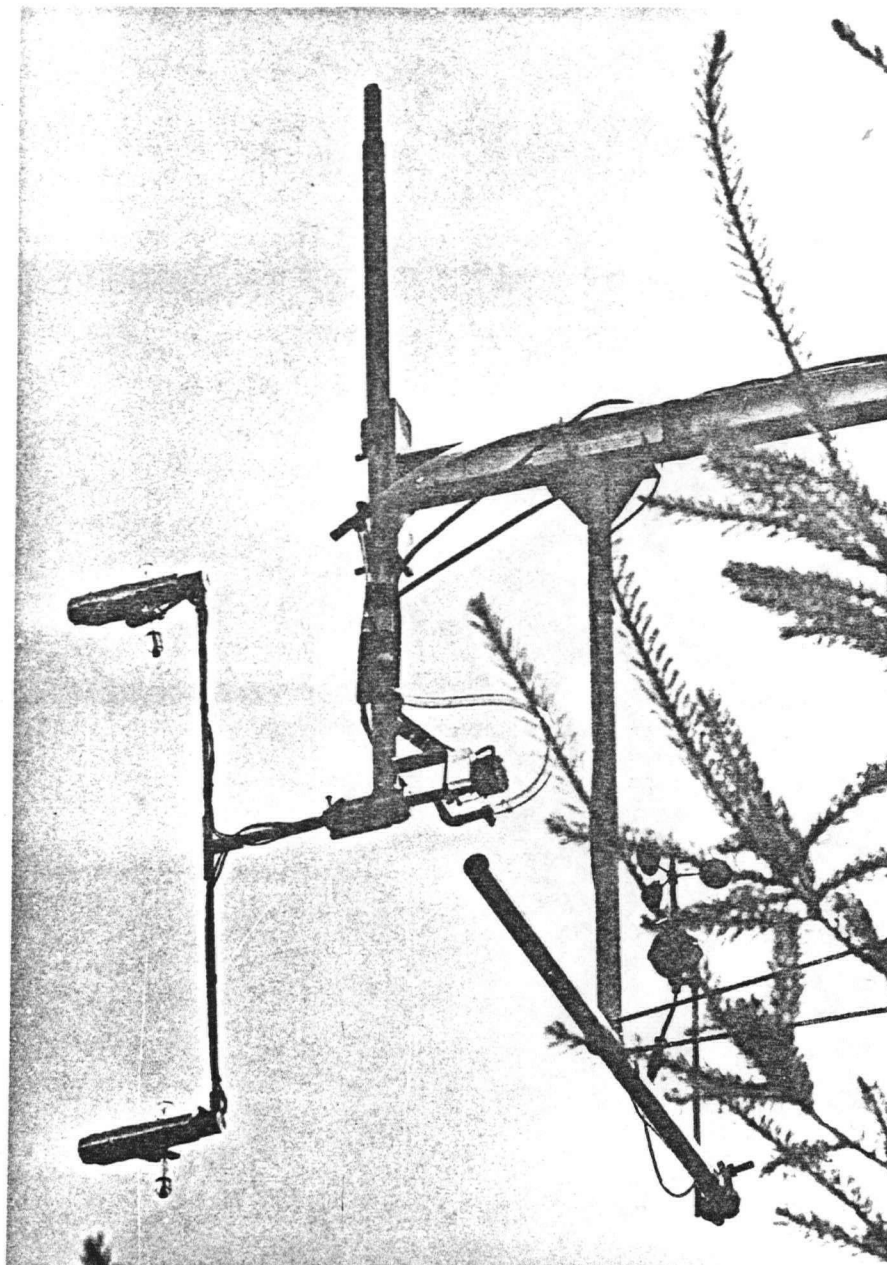


Figure 4 Apparatus in position above the Douglas-fir forest. Reversing motor and air tube for aspiration can be seen.

influenced noticeably by varying the position of the apparatus horizontally over the canopy.

The Bowen ratio defined as the ratio of the sensible to the latent heat flux is given by

$$\beta = \frac{\Delta T + \Gamma \Delta Z}{(s + \gamma) \Delta T_w - \gamma \Delta T} \quad (1)$$

where s is the slope of the saturation vapour pressure curve at T_w , γ is the psychrometric constant, $\Gamma (= -.01 \text{ } ^\circ\text{C/m})$ is the adiabatic lapse rate and ΔZ is the distance between the sensors. This requires the measurement of wet and dry bulb differences and absolute wet bulb temperatures.

The differences in wet and dry-bulb temperatures were recorded continuously on a 2-pen strip-chart recorder with both pens set with full-scale range of 5 mv throughout the experiment. No long-term zero drift was observed. Examples of daytime and nighttime signal traces are shown in Figure 5. During the day temperature differences could be read to 0.01 $^\circ\text{C}$ while in stable nighttime conditions differences of less than 0.005 $^\circ\text{C}$ could be resolved. Temperature differences over a vertical distance of 1 meter during the day never exceeded 0.4 $^\circ\text{C}$; at night they rarely exceeded 0.2 $^\circ\text{C}$. The wet and dry-bulb temperatures of one of the heads were recorded every 7 minutes on a separate recorder.

As can be seen in Figure 5 the time constant of the sensing heads was approximately 1 minute. For this reason only data recorded for the last 10 minutes of each 15-minute period were used in the Bowen ratio calculations. The difference between voltage averages for two of these consecutive 10-minutes periods were used to calculate half-hourly Bowen ratios. An example of a diurnal trend of wet and dry

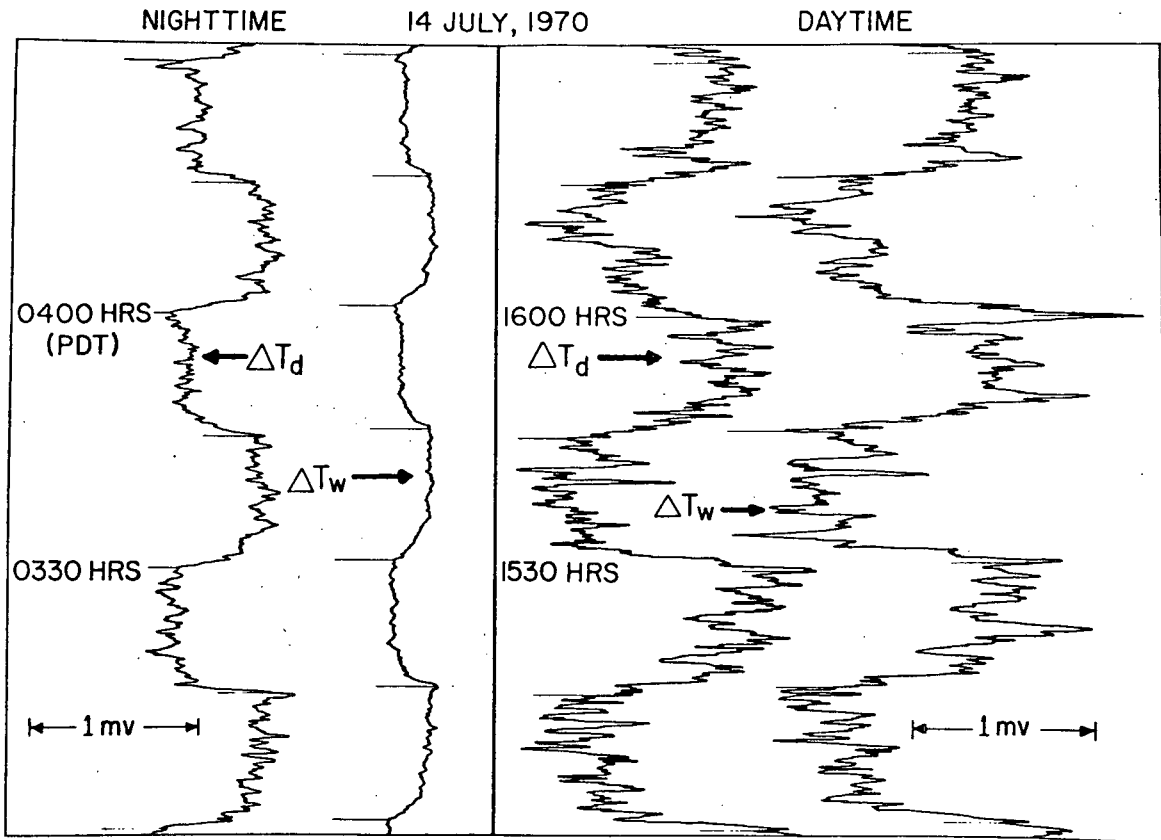


Figure 5 Typical traces of dry bulb and wet bulb differences over 1 meter (ΔT_d and ΔT_w) obtained with the psychrometric apparatus. Diode sensitivity is 2.25 mv C⁻¹.

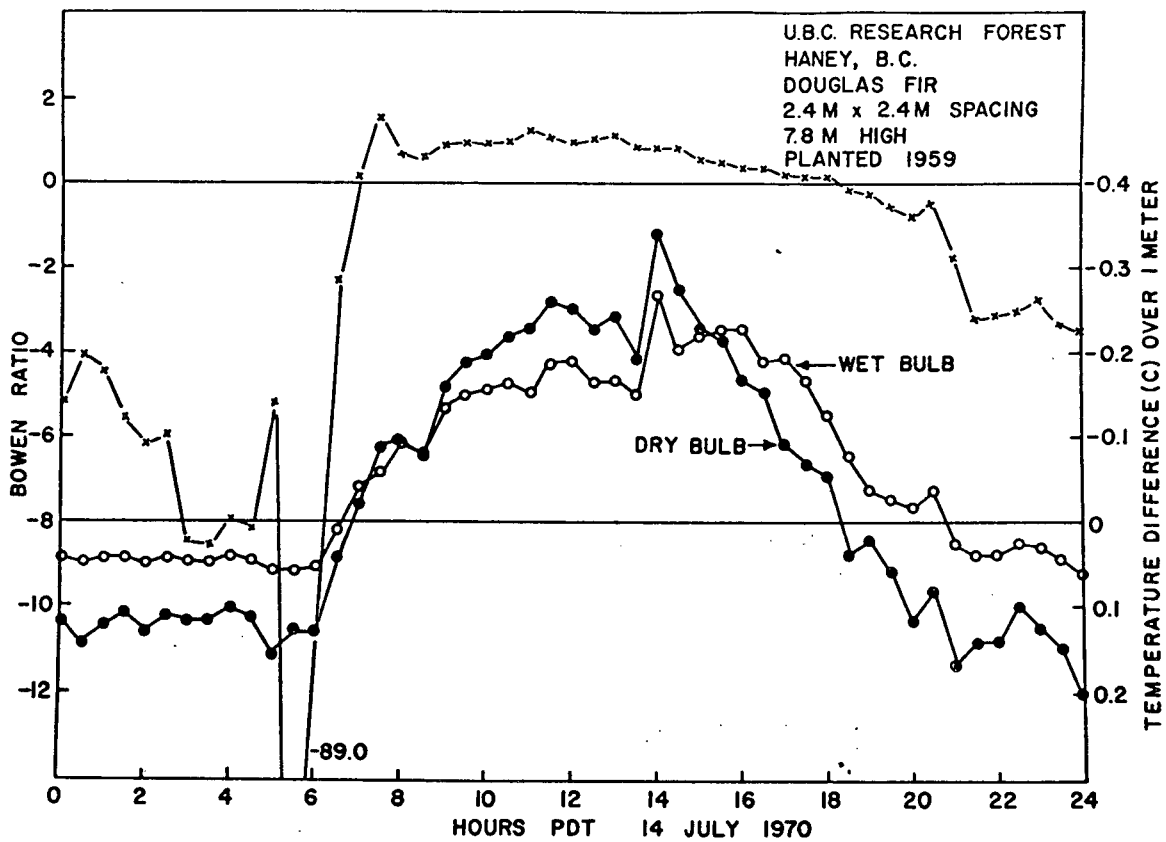


Figure 6 Example of the measured dry and wet bulb differences and the calculated Bowen ratios.

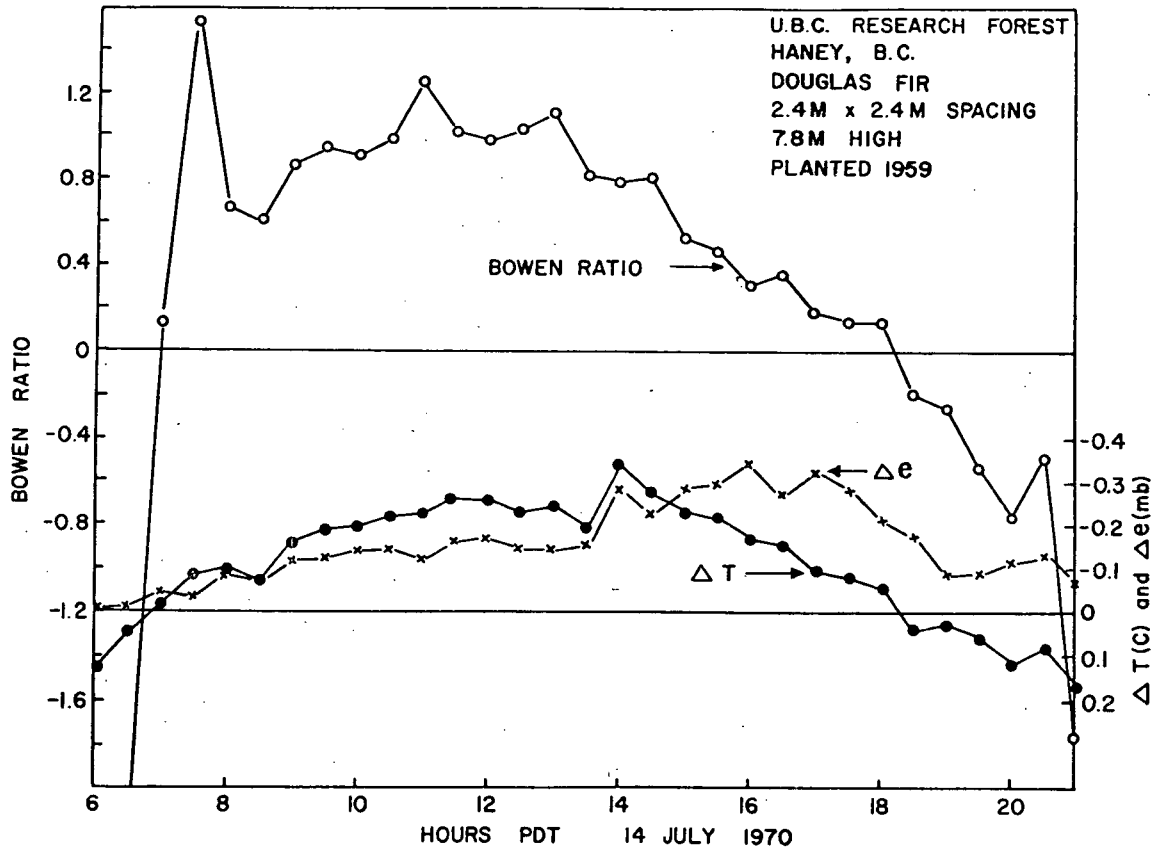


Figure 7 Daytime Bowen ratios on an expanded scale. The vapour pressure differences over 1 meter are calculated from the wet and dry bulb differences shown in Figure 6 and measured absolute wet bulb temperatures.

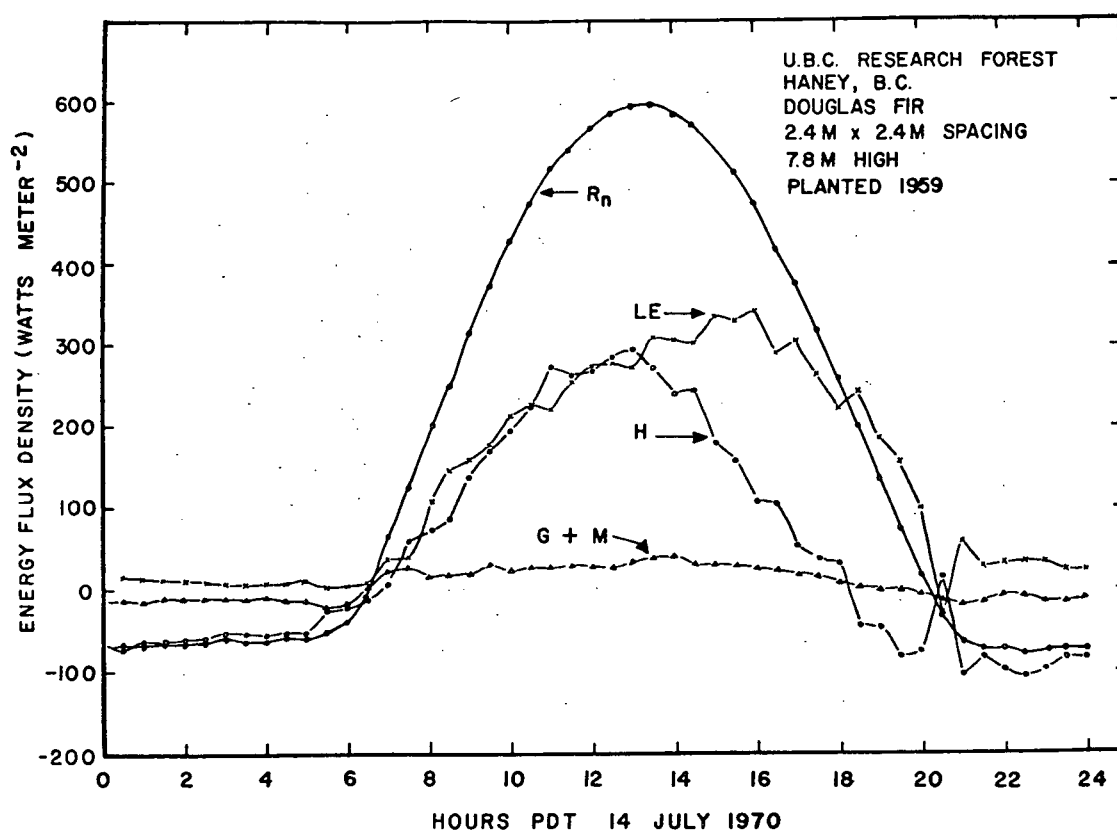


Figure 8 Energy balance diagram for July 14, 1970. The anomalous behaviour of the calculated sensible and latent heat flux densities of 2030 h is due to a Bowen ratio near -1.

bulb temperature differences over 1 m measured with the apparatus and the resulting Bowen ratio trend are shown in Figure 6. The vapour pressure and temperature differences are shown together with the Bowen ratio on an amplified scale in Figure 7 for daytime hours when most of the evapotranspiration occurs. Satisfactory results from the Bowen-ratio apparatus were obtained for all analyzed data. Somewhat unstable nighttime Bowen ratios are unimportant since there is little available energy at night for evaporation. Although there were considerable fluctuations in the vapour pressure and temperature differences, the Bowen ratios were generally reasonable. A large Bowen ratio value was regularly observed at about 0730 during the experiment. This is apparently because shortly after sunrise the sensible heat flux increases more rapidly than the evaporation rate.

Net radiation flux density, R_n , soil heat flux density, G and canopy energy storage rate, M were measured for each half hour. These measurements together with the Bowen ratio were used to calculate the latent heat flux density, LE for each half hour using the well known energy balance/Bowen ratio equation

$$LE = (R_n - G - M)/(1 + \beta) \quad (2)$$

The results for July 14, 1970 are shown in Figure 8. Tanner and Fuchs (1970) give an analysis of errors in the psychrometric determination of the Bowen ratio and the subsequent calculation of the evaporation rate using (2). They show that

$$\delta E/E = [(\delta R_n + \delta G + \delta M)/|R_n - G - M|] + |\beta|\psi \quad (3)$$

where

$$\psi = \delta\Delta T_d / \Delta T_d + \delta\Delta T_w / \Delta T_w + \delta s / (s + \gamma) \quad (4)$$

and the δ 's indicate the absolute errors of various parameters.

The relative errors in ΔT_d and ΔT_w were each usually less than 5% during the daytime hours. The last term in (4) which depends upon the accuracy of the measurement of the absolute wet bulb temperature was less than 1%. The relative error in the evaporation rate becomes unacceptably large for large Bowen ratios. During the afternoon hours when evaporation was found to be greatest, Bowen ratios were generally found to be less than 0.8. The morning Bowen ratios rarely exceeded 1.4. Uncertainties in this half hourly Bowen ratio determination caused uncertainties in the total daily evaporation estimates of less than 10%.

The reliable long-term performance and acceptably small error have enabled this data to be used in water and energy balance studies on the experimental site (Black and McNaughton, 1972).

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CHAPTER 2
AVERAGE BOWEN-RATIO METHODS OF CALCULATING
EVAPOTRANSPIRATION APPLIED TO A
DOUGLAS-FIR FOREST

AVERAGE BOWEN-RATIO METHODS OF CALCULATING
EVAPOTRANSPIRATION APPLIED TO A
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Abstract

Daily evapotranspiration from a Douglas-fir forest was calculated using Webb's average Bowen ratio method. Webb's method is generalized to include the effects of the diabatic wind profile. Over a 17-day period characterized by light winds, the modified Webb method agreed with the daily totals of half-hour energy balance calculations to within 1 1/2% on the average, while Webb's method overestimated by 26% on the average.

1. Introduction

The accurate measurement of evapotranspiration from forests still remains one of the real challenges of micrometeorology. Federer (1970) gives an excellent discussion of the methods that have been used and the problems encountered. The energy balance/Bowen ratio method has been shown to give reliable measurements in many agricultural situations (Tanner, 1968). The method is difficult to apply exactly to forests because of the relatively large heat capacity of the canopy volume. Where daily values of evapotranspiration are required a method such as Webb's average Bowen-ratio method (Webb, 1964) which eliminates the need to consider short-

term energy storage changes is of obvious utility.

This paper reports the results of applying Webb's average Bowen ratio method to data collected at the University of British Columbia Research Forest over a 7.8-m high Douglas fir plantation in the summer of 1970. The measurements were taken during conditions of light winds. Webb's method gave poor agreement with the results from the complete energy balance. A diabatic correction to Webb's method based on the KEYPS diabatic profile model (Lumley and Panofsky, 1964) is developed and applied to the data.

2. Symbols and Units

D	Zero plane displacement (m)
G	Soil heat flux density (W m^{-2})
H	Sensible heat flux density (W m^{-2})
K_v	Eddy diffusivity for vapour ($\text{m}^2 \text{sec}^{-1}$)
K_h	Eddy diffusivity for heat ($\text{m}^2 \text{sec}^{-1}$)
K_m	Eddy diffusivity for momentum ($\text{m}^2 \text{sec}^{-1}$)
L	Latent heat of vapourization of water (J kg^{-1})
LE	Latent heat flux density (evapotranspiration) (W m^{-2})
M	Rate of heat storage in the canopy volume on an area basis. This includes sensible heat storage in trees and air, and latent heat storage in air (W m^{-2})
P	Rate of photosynthetic energy storage on area basis (W m^{-2})
Ri	Richardson number (dimensionless)
Rn	Net radiation flux density (W m^{-2})

R_v	Gas constant for water vapour ($\text{mb m}^3 \text{ kg}^{-1} \text{ K}^{-1}$)
T	Temperature (K)
c_p	Specific heat of moist air ($\text{J kg}^{-1} \text{ K}^{-1}$)
c_t	Specific heat of the aerial parts of the tree ($\text{J kg}^{-1} \text{ K}^{-1}$)
e	Vapour pressure (mb)
g	Acceleration of gravity (m sec^{-2})
h	Canopy height (m)
k	von Karman constant (0.428)(dimensionless)
t	Time (sec)
u	Wind velocity (m sec^{-1})
u_*	Friction velocity (m sec^{-1})
z	Height from the soil surface (m)
z_0	Roughness length (m)
β	Bowen ratio (dimensionless)
γ	Psychrometric constant ($\text{mb } ^\circ\text{C}^{-1}$)
ρ	Density of moist air (kg m^{-3})
ρ_t	Density of aerial parts of the tree (kg m^{-3})
ϕ	Diabatic influence function defined by $\partial u / \partial x = (u_*/kz)\phi$ (dimensionless)
Φ	Integrated diabatic influence function (dimensionless)

3. Theory

The energy balance of a forest volume for two-dimensional exchange is given by

$$\begin{aligned}
R_n - \int_0^z \frac{\partial}{\partial x} (\rho u c_p T) dz - \int_0^z \frac{\partial}{\partial x} (Le u / R_v T) dz = LE + H + G + P \\
+ \frac{\partial}{\partial t} \int_0^z (\rho_t c_t T) dz + \frac{\partial}{\partial t} \int_0^z (\rho c_p T) dz + \frac{\partial}{\partial t} \int_0^z (Le / R_v T) dz
\end{aligned} \quad (1)$$

The last two terms on the left side of (1) are the horizontal divergence of sensible and latent heat and for a forest of considerable horizontal extent, these terms are negligible. The last three terms on the right side of (1) are the rate of change of heat stored in the trees, sensible heat stored in the air of the canopy volume, and latent heat in the air of the canopy volume. Defining the Bowen ratio, $\beta = H/LE$ and neglecting P , we may write the latent heat flux density as

$$LE = \frac{R_n - G - M}{1 + \beta} \quad (2)$$

where M is the sum of the three canopy heat storage terms in (1). Now if (1) is averaged over a period of, say, one day and an average Bowen ratio is defined as $\bar{\beta} = \bar{H}/\bar{LE}$, where the overbar refers to the average value, then

$$\bar{LE} = \frac{\bar{R_n} - \bar{G} - \bar{M}}{1 + \bar{\beta}} \quad (3)$$

which reduces to

$$\bar{LE} = \frac{\bar{R_n}}{1 + \bar{\beta}} \quad (4)$$

if \bar{G} and \bar{M} are approximately zero.

Now since the eddy diffusion equations for sensible and latent heat flux densities are

$$H = -\rho c_p K_h (\Delta T / \Delta z) \quad (5)$$

and

$$LE = -(L/R_v T) K_v (\Delta e / \Delta z) \quad (6)$$

the average Bowen ratio $\bar{\beta}$ is written as

$$\bar{\beta} = \frac{\overline{\rho c_p K_h (\Delta T / \Delta z)}}{\overline{(L/R_v T) K_v (\Delta e / \Delta z)}} = \gamma \frac{\overline{K_h \Delta T}}{\overline{K_v \Delta e}}, \quad (7)$$

where $\gamma = \rho c_p R_v T / L \approx 0.66 \text{ mb/K}$ is the psychrometric constant.

Using the relations, $K_h = \bar{K}_h + K'_h$, $K_v = \bar{K}_v + K'_v$, $\Delta T = \bar{\Delta T} + \Delta T'$ and $\Delta e = \bar{\Delta e} + \Delta e'$, where the prime refers to the deviation from the average, (7) becomes

$$\bar{\beta} = \gamma \frac{\bar{K}_h \bar{\Delta T} + \overline{K'_h \Delta T'}}{\bar{K}_v \bar{\Delta e} + \overline{K'_v \Delta e'}} \quad (8)$$

Now setting $\bar{\beta}_{\text{raw}} = \gamma \bar{\Delta T} / \bar{\Delta e}$ and assuming $K_v = K_h = \lambda K_m$, where λ is a constant, then

$$\bar{\beta} = \bar{\beta}_{\text{raw}} \frac{1 + r_T}{1 + r_e} , \quad (9a)$$

where

$$r_T = \overline{K_m' \Delta T'} / \overline{K_m \Delta T} \quad (9b)$$

and

$$r_e = \overline{K_m' \Delta e'} / \overline{K_m \Delta e} . \quad (9c)$$

Webb assumes that K_h and K_v are proportional to wind speed. This amounts to assuming that neutral stability conditions prevail. Equations (9b) and (9c) can then be written as

$$r_T = \overline{u' \Delta T'} / \overline{u \Delta T} \quad (10a)$$

and

$$r_e = \overline{u' \Delta e'} / \overline{u \Delta T} . \quad (10b)$$

In the KEYPS diabatic wind profile model (Lumley and Panofsky, 1964) the momentum diffusivity may be expressed as

$$K_m = k u_* (z - D) / \phi \quad (11)$$

The friction velocity, u_* is determined from the diabatic wind profile expression

$$u = \frac{u_*}{k} \left[\ln \left(\frac{z - D}{z_0} \right) + \Phi \right] . \quad (12)$$

The diabatic influence functions, Φ and ϕ , are related as follows

$$\phi = (1 - 18 Ri)^{-1/4} \quad (13)$$

and

$$\Phi = \int_{D+z_0}^z (\phi - 1)(z - D)^{-1} dz , \quad (14)$$

where the Richardson number, Ri is given to a close approximation by

$$Ri = \frac{g}{T} \frac{(T_2 - T_1)}{(u_2 - u_1)^2} \ln \left(\frac{z_2 - D}{z_1 - D} \right) (z - D) , \quad (15)$$

where T is the mean of T_1 and T_2 . Lettau has solved (13) and (14) to give

$$\Phi = \Phi(Ri) \quad (16)$$

which is given in Table 1. With knowledge of the wind profile, (11), (12), (13), (15) and (16) can be solved to give K_m , which can be substituted into (9) to give an exact value of $\bar{\beta}$.

Taking $\lambda = 1$ and substituting (12) into (11) and then into (6) and integrating gives the aerodynamic equation for calculating the

TABLE I

Dependence of the Integrated Diabatic Influence Function Φ on
the Richardson Number, Ri (From Lettau, 1962)*

Ri	Φ	Ri	Φ
0.0555	3.716	-0.15	-0.367
0.0554	2.343	-0.20	-0.436
0.0553	1.905	-0.30	-0.552
0.0552	1.660	-0.40	-0.639
0.0551	1.496	-0.50	-0.712
0.055	1.375	-0.60	-0.775
0.054	0.870	-0.70	-0.831
0.053	0.859	-0.80	-0.880
0.052	0.731	-0.90	-0.926
0.051	0.647	-1.00	-0.967
0.050	0.584	-1.10	-1.006
0.040	0.308	-1.20	-1.041
0.030	0.188	-1.30	-1.074
0.020	0.109	-1.40	-1.105
0.010	0.049	-1.50	-1.134
0.000	0.000	-1.60	-1.162
-0.020	-0.077	-1.70	-1.188
-0.040	-0.140	-1.80	-1.213
-0.060	-0.192	-1.90	-1.237
-0.100	-0.283	-2.00	-1.261

* This table is reproduced in full since in the iterative methods used for this paper, it is important to have a well behaved function defined beyond the range of solutions found. There is now evidence to suggest that the table is not accurate for very unstable conditions and for stable conditions particularly for $Ri > .03$ (Pruitt et al., 1971; Webb, 1970).

latent heat flux density.

$$LE = - \frac{k^2(L/R_v T)(e_2 - e_1)(u_2 - u_1)}{(\ln[(z_2 - D)/(z_1 - D)])^2 \phi^2} \quad (17)$$

4. Methods and Results

4.1 Method for Calculating K_m

Measurements of the temperature difference, $(T_1 - T_2)$ between the heights z_1 and z_2 , the average air temperature, T , and the wind speed, u_3 , at height z_3 substituted into (12), (13), (15) and (16), result in the following set of equations.

$$u_3 = \frac{u_*}{k} \left[\ln \left(\frac{z_3 - D}{z_0} \right) + \phi_3 \right] \quad (18)$$

$$u_2 - u_1 = \frac{u_*}{k} \left[\ln \left(\frac{z_2 - D}{z_1 - D} \right) + \phi_2 - \phi_1 \right] \quad (19)$$

$$Ri_1 = \left[\frac{g}{T} \frac{(T_2 - T_1)}{(u_2 - u_1)^2} \ln \left(\frac{z_2 - D}{z_1 - D} \right) \right] (z_1 - D) \quad (20)$$

$$Ri_2 = \left[\frac{g}{T} \frac{(T_2 - T_1)}{(u_2 - u_1)^2} \ln \left(\frac{z_2 - D}{z_1 - D} \right) \right] (z_2 - D) \quad (21)$$

$$Ri_3 = \left[\frac{g}{T} \frac{(T_2 - T_1)}{(u_2 - u_1)^2} \ln \left(\frac{z_2 - D}{z_1 - D} \right) \right] (z_3 - D) \quad (22)$$

$$\Phi_1 = \Phi_1 (Ri_1) \quad (23)$$

$$\Phi_2 = \Phi_2 (Ri_2) \quad (24)$$

$$\Phi_3 = \Phi_3 (Ri_3) \quad (25)$$

This forms a set of 8 equations in 10 unknowns, u_* , $(u_2 - u_1)$, Ri_1 , Ri_2 , Ri_3 , Φ_1 , Φ_2 , Φ_3 , z_0 and D .

The roughness length, z_0 , can be calculated from the regression equation of Szeicz et al., (1969)

$$\log_{10} z_0 = \log_{10} h - 0.98 \quad (26)$$

while the zero plane displacement, D , can be calculated from the regression equation of Stanhill (1969)

$$\log_{10} D = 0.9793 \log_{10} h - 0.1942 \quad (27)$$

where h is the height of the canopy. Thus (18) to (25) become a set of 8 equations in 8 unknowns which are solvable by iteration. From the solution for each half-hour period, u_* and ϕ were used to calculate K_m from (11). The modified Webb method uses these values of K_m with (4),

(9a), (9b) and (9c) to calculate daily evapotranspiration.

4.2 Comparison of Evapotranspiration Estimates

The data collected from the Douglas-fir plantation allowed calculation of averages of R_n , $T_2 - T_1$, $e_2 - e_1$, and T for each half-hour period in the interval July 8 to July 24, 1970. Temperature and vapour pressure measurements were made at 8.1 m (z_1) and 9.1 m (z_2) above the soil surface with a reversible psychrometric apparatus (Black and McNaughton, 1971). Integrated wind speeds were recorded with a Casella sensitive anemometer at 8.75 m (z_3) above the soil surface at irregular intervals, about 20 readings per day, 3/4 of which were taken in the daylight hours. Estimates of mean wind speed for each half hour were obtained by interpolation. Fetch was between 200 and 400 meters for the most frequent wind directions. Beyond the plantation boundary was forest of similar type though with differences in slope and roughness. These data were used to calculate total evapotranspiration for each day using (4) with the uncorrected average Bowen ratio $\bar{\beta}_{raw}$, by the Webb method using (4), (9a), (10a) and (10b), and setting $K_h = K_v$, and also by the Webb method modified for diabatic conditions using (4), (9a), (9b) and (9c). These values are tabulated in columns (2), (3) and (4) of Table II. Calculations of evapotranspiration over half-hour periods using the total energy balance, equation (1), were also made and the daily totals are tabulated in column (1) of Table II. The two horizontal divergence terms and the photosynthetic flux density in (1) are assumed negligible. Errors in the total energy balance values

due to difficulties in estimating canopy volume storages and soil heat flux are estimated to be less than 5%.

The Webb method produces better estimates of evapotranspiration than those calculated using the uncorrected average Bowen ratio in all but three cases but the improvement is not great. This reflects the absence of any marked correlation between the wind speed and the gradients of temperature and vapour pressure. In particular, the wind speed showed no definite diurnal trend as did the computed values of K_m (due to the influence of stability in the latter case). Values from the Webb method are consistently too large, being, on the average, 26% greater than those calculated from the total energy balance (Figure 1). Values from the modified Webb method agree with the energy balance values much more closely, differing by less than 1 1/2% on the average (Figure 1). The evapotranspiration estimate provided by the modified Webb method is a significant improvement over the Webb method estimate even on the day with highest wind and smallest range of calculated Richardson number (July 12). On that day, Ri had extreme values of -0.105 and 0.041 with averages for unstable and stable periods of -0.063 and 0.027, respectively.

As a byproduct of the analysis, (17) can be used to compute evapotranspiration. The results are shown in Table II, column 6 and Figure 1 for the 17-day period. There is an average overestimate of 34% compared to the energy balance values. This is not surprising since the aerodynamic method results are very sensitive to the assumed values of z_0 and D . This approximate agreement indicates that the wind profile solutions described in this paper are realistic.

TABLE II

Daily Estimates of Evapotranspiration (mm) from an 11-Year Old, 7.8-Meter High Douglas Fir Plantation at U.B.C. Research Forest, Haney, B.C. in the Summer of 1970. $\bar{G}_5/(1+\bar{\beta})$ Must Be Subtracted From the Modified Webb Method Estimates to Correct for Net Daily Soil Heat Flux at the 5-cm Depth

Date	Half-hour Energy Balance	$\bar{Rn}(1+\bar{\beta}_{raw})$	Webb Method	Modified Webb Method	$\bar{G}_5/(1+\bar{\beta})$	Aerodynamic Method	Mean Wind Speed m/s
1970		Col.2	Col.3	Col.4	Col.5	Col.6	Col.7
July 8	4.34	5.43	5.14	4.28	0.09	5.53	1.35
9	4.39	5.20	5.04	4.16	0.01	6.37	1.26
10	4.03	5.03	4.91	3.90	-0.03	5.89	1.44
11	4.20	5.09	4.90	4.14	0.01	5.84	1.41
12	4.15	5.05	4.97	4.28	0.03	5.54	1.70
13	4.08	5.22	5.14	4.12	0.02	4.98	1.61
14	4.65	5.55	6.29	4.74	0.07	6.37	1.55
15	4.80	6.15	7.38	5.04	0.21	6.71	1.53
16	2.40	3.67	3.21	2.38	0.01	3.26	1.53
17	3.84	5.05	4.55	3.71	-0.01	5.20	1.17
18	4.29	5.09	5.05	4.05	0.13	5.71	1.30
19	3.61	4.67	4.60	3.59	0.10	5.22	1.30
20	3.10	4.53	3.90	2.76	0.02	3.43	1.15
21	1.46	1.69	1.83	1.26	-0.21	1.95	1.13
22	3.18	4.01	3.79	3.17	-0.03	4.07	1.08
23	2.56	3.53	3.47	2.66	-0.02	2.95	1.36
24	1.75	2.97	2.35	1.74	-0.06	2.45	1.13
TOTAL	60.83	77.93	76.52	59.98	0.34	81.47	

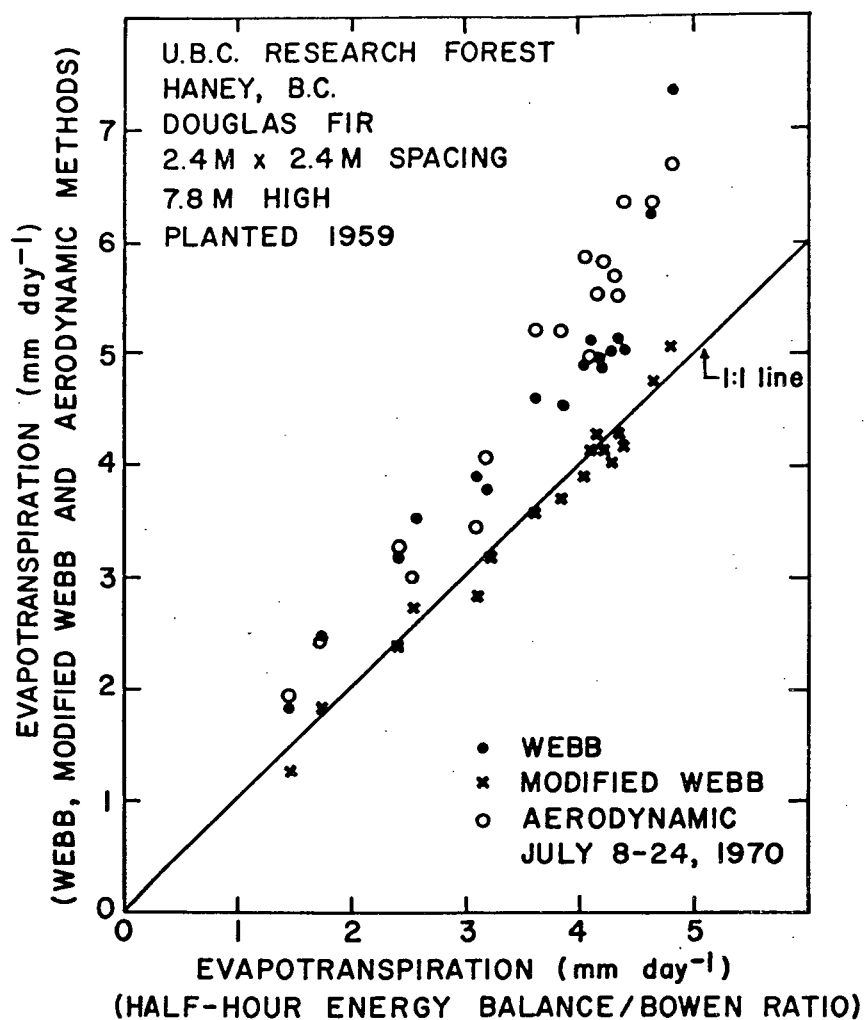


Figure 1 Daily evapotranspiration from a Douglas-fir forest calculated by the Webb, modified Webb and aerodynamic methods compared with half-hour energy balance/Bowen ratio measurements.

5. Discussion

The reason for the failure of the Webb method was the strongly diabatic wind profiles associated with low wind speeds during the measurement period (see Column 7, Table II). The Richardson numbers calculated from the synthetic wind profiles varied between +0.048 and -0.29. Under these conditions the modified Webb method agreed well with the energy balance measurements. Since the Webb method is a special case of the modified Webb method for neutral conditions, it would be expected that the two methods would give similar results for windy conditions. Webb (1960) suggests that approximate wind speed estimates are adequate for calculations of evapotranspiration. For the modified Webb method a sensitive anemometer is required because the effects of stability are greater for light winds.

The exact canopy height, h , is somewhat ill defined. To examine the effect of the uncertainty in h on the values computed by the modified Webb method, values were recomputed using $h = 8.5$ m instead of the best estimate of 7.8 m. This increased the values of z_0 and D , calculated from (26) and (27), from 0.82 m to 0.89 m, and 4.8 m to 5.2 m, respectively. The evapotranspiration was increased by 4% on the average. The method is therefore rather insensitive to uncertainties in h . The values of z_0 and D calculated from the regression equations, though satisfactory here, may not be suitable in all cases. These geometric constants depend upon such canopy characteristics as tree spacing and canopy shape as well as height.

The soil may act as a significant heat source or sink. To test the assumption that \bar{G} may be neglected, values of the total heat

flux downward calculated from measurements made with two heat flux plates at a depth of 5 cm were made for each day. The correction due to this term, $\bar{G}_5/(1+\bar{\beta})$, is tabulated in Column 5, Table II. Over the 17-day period it reduces the total estimate of evapotranspiration by 0.34 mm and is not significant for the period of measurement though it may be significant for periods when the soil is rapidly heating or cooling as in spring or fall. An important feature of the Webb method is that correction for heat storages can be made by making the calculation for longer averaging periods and making an energy storage estimate before and after the measurement period. In the present case, soil temperature profile measurements would be convenient. The heat storage capacities of the top 5 cm of soil and the canopy volume are small and changes in daily heat storages were negligible for the measured changes in temperatures.

Although alternative assumptions about the ratio of K_h/K_m could be made and alternate formulations of diabatic wind profile theory could be used, it appears that the present method gives adequate results within the range of stability encountered.

In this paper, the results of the half-hour energy balance have been used as a standard. A water balance was performed independently on the site (Willington, 1971) using measurements of root-zone water storage change and drainage. For the 15-day period from July 8 to July 22, the evapotranspiration was calculated to be 54 ± 9 mm. This compared favourably with the energy balance estimate of 56.5 mm.

6. Conclusion

The Webb method overestimated the evapotranspiration from a young Douglas-fir plantation when winds were light. The modified Webb method, however, provided accurate evapotranspiration estimates. Both methods require only the net radiation and wind above the canopy, the temperature and vapour pressure differences between two levels (both above the canopy), and empirical values of the zero plane displacement and the roughness length that can be taken from published regression equations.

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CHAPTER 3

A STUDY OF EVAPOTRANSPIRATION FROM A DOUGLAS-FIR FOREST USING THE ENERGY BALANCE APPROACH

A STUDY OF EVAPOTRANSPIRATION FROM A DOUGLAS-FIR FOREST
USING THE ENERGY BALANCE APPROACH

Abstract

Energy balance measurements of evapotranspiration from a young Douglas-fir forest are reported for a period of 18 days in July 1970 when soil water was not limiting. Peak daily evapotranspiration rates characteristically occurred two to three hours after solar noon and evapotranspiration showed a short-term independence from net radiation. This behaviour is interpreted as being a consequence of the large forest roughness. Daily evapotranspiration and net radiation were, however, well correlated. Values of surface diffusion resistance calculated from Monteith's combination formula are presented. Daytime values showed significant day-to-day differences and an attempt to define a potential evapotranspiration rate assuming a constant daytime surface resistance was not successful. Comparison of evapotranspiration measurements with a potential evaporation formula for wet surfaces developed by Priestley and Taylor suggests that evaporation of intercepted water proceeds 20% more rapidly than evapotranspiration from the non-wetted canopy.

1. Introduction

In July and August, 1970, energy balance/Bowen ratio measurements of evapotranspiration from a young Douglas-fir forest were made as part of a hydrologic balance experiment on a site in the University of British Columbia Research Forest.

Examination of the energy balances computed from the measurements reveals several clear patterns in the results. It is the purpose of this paper to present the results for the initial 18 day period of the experiment and to discuss these patterns and some of the implications of the results. As a basis for interpretation of the results, Monteith's canopy transpiration model has been used for part of the analysis (Monteith, 1965).

2. Symbols and Units

D	zero plane displacement (m)
E	evapotranspiration rate ($\text{kg m}^{-2} \text{s}^{-1}$)
E_0	free evaporation rate ($\text{kg m}^{-2} \text{s}^{-1}$)
E_{75}	total 24-hour evapotranspiration assuming daytime $r_s = 75 \text{ s m}^{-1}$ (mm)
G	soil heat flux density (W m^{-2})
I	gross interception loss (mm)
H	sensible heat flux density (W m^{-2})
L	latent heat of vaporization of water (J kg^{-1})
M	rate of heat storage in the canopy volume on an area basis (W m^{-2})

P	potential evapotranspiration (mm)
R _n	net radiation flux density (W m^{-2})
T	temperature (C)
c _p	specific heat of moist air ($\text{J kg}^{-1} \text{K}^{-1}$)
e ₀ [*]	saturation water vapor pressure at the canopy 'surface' temperature (mb)
e ₀	water vapor pressure at the canopy 'surface' (mb)
e _z	water vapor pressure at height z (mb)
e _z [*]	saturation water vapor pressure at height z (mb)
r _a	aerodynamic diffusion resistance at height z. Assumed equal to u_z/u_*^2 (s m^{-1})
r _s	surface diffusion resistance (s m^{-1})
s	slope of the saturation water vapor pressure curve (mb K^{-1})
u _z	wind velocity at height z (m s^{-1})
u _*	friction velocity (m s^{-1})
z	height above soil surface (m)
z ₀	roughness length (m)
ψ _m	soil water matric potential (bar)
α	empirical coefficient (dimensionless)
β	Bowen-ratio ($\equiv H/LE$) (dimensionless)
γ	psychrometric constant ($= 0.66 \text{ mb K}^{-1}$)
ρ	density of moist air (kg m^{-3})

In equations (9) and (10), Table 1 and Figures 3 and 4 the energy flux density terms have been integrated over 24 hour periods to give convenient practical units of mm equivalent depth of water evaporated.

3. Monteith's Equation

Penman (1948) has shown that energy balance and aerodynamic equations for evaporation can be combined to give an expression for evaporation from extensive wet surfaces in terms of easily measurable parameters. Businger (1956) has developed a rational expression based on measured wind profiles for the original empirical wind function.

The accuracy of the resulting combination formula for calculating evaporation from wet surfaces with small roughness lengths has been established by the experimental work of Van Bavel (1966). In the modern resistance terminology this expression can be written as

$$E_0 = \frac{1}{L} \left(\frac{s}{s + \gamma} \right) (Rn - G - M) + \frac{\rho c_p (e_z^* - e_z)}{(s + \gamma) r_a L} \quad (1)$$

where the aerodynamic resistance, r_a , is given by

$$r_a = \frac{u_z}{u_*^2} \quad (2)$$

assuming similarity between the transfer coefficients for momentum, heat and water vapor and taking the virtual sink for momentum to be also a virtual source for heat and water vapor. The quantity E_0 , defined by (1), will be called the free evaporation rate in this paper.

Monteith (1965) introduced the effect of diffusive resistance to vapor flow of the stomata of the vegetative canopy by considering the canopy as a single extensive isothermal leaf. In his model transpiration from this 'leaf' is expressed as

$$E = \frac{\rho c_p}{L \gamma} \frac{(e_0^* - e_0)}{r_s} \quad (3)$$

where the subscript '0' refers to values at the canopy 'surface' obtained by extrapolating the temperature and humidity profiles down to $(z_0 + D)$. Following a similar procedure to Penman (1948), Monteith derived the expression for transpiration

$$E = \frac{E_0}{1 + \left(\frac{\gamma}{s + \gamma} \right) \frac{r_s}{r_a}} \quad (4)$$

where the surface resistance, r_s , is formally identified as the resistance of all the stomata of the leaves of the canopy acting in parallel.

Total evapotranspiration can be considered as entirely transpiration with small error for forests when intercepted water is not present since soil evaporation has been found to be small by many workers (e.g. Rutter, 1966).

4. Surface and Aerodynamic Resistances

The one-layer model is a considerable simplification of real plant canopies and has attracted strong criticism (Philip, 1963, 1966; Tanner, 1963; Tanner and Fuchs, 1968). These criticisms arise from the observation that the simple one-layer model ignores leaf boundary layer diffusion resistances and aerodynamic diffusion resistances between different levels of the canopy. An examination shows

that transpiration from each individual leaf surface can be summed to produce an expression for transpiration from the whole canopy that is in the form of (3) only under the assumptions that these resistances are indeed negligible. In general, the surface resistance cannot be rigorously identified with the stomatal resistance of all of the leaf surfaces acting in parallel and such interpretation must be justified by examination of the assumptions for each canopy studied.

From the results of Rutter (1967) for a Scots pine plantation, it can be seen that stomatal resistances were large compared to the aerodynamic and boundary layer resistances within the canopy. There is some expectation, therefore, that surface resistance may be closely related to stomatal resistance in that case and, by implication, also in the present analysis.

Good wind profile data for calculation of r_a is frequently unavailable. For this reason a consideration of the errors introduced into the values of E calculated from (4) caused by an error of estimate in r_a is appropriate here.

An error formula can be derived by differentiating (4) with respect to r_a and then integrating over the range of error from r_a to $r_a + \Delta r_a$. Thus

$$\frac{\Delta E}{E} = \frac{(s \beta - \gamma) \Delta r_a}{(s + \gamma)(r_a + \Delta r_a) + r_s \gamma} \quad (5)$$

where the Bowen ratio, β , has been introduced for convenience. It can be seen that E is independent of r_a when $\beta = \gamma/s$ as has been noted previously by Monteith (1965).

Anticipating the results of this paper, for a forest we can take $r_a \approx 5 \text{ s m}^{-1}$, $r_s \approx 75 \text{ s m}^{-1}$, $\beta \approx 1$ and assuming $T = 18^\circ\text{C}$, $s = 1.29 \text{ mb K}^{-1}$. An overestimate of r_a by 50% introduces only a 2 1/2% overestimate in the calculated value of E . Further, if we let r_a approach zero, then (4) becomes

$$E = \frac{\rho c_p}{L \gamma r_s} (e_z^* - e_z) \quad (6)$$

with only 5% underestimate. This result indicates that a forest transpiring in accordance with (4) is little affected by wind speed and that radiation is important indirectly through its effect on stomatal resistance and on temperature and therefore vapor pressure deficit. Both stomatal resistance and vapor pressure deficit are expected to show a slow response to changes in radiation and therefore (6) predicts poor correlation between changes in transpiration and radiation over periods of less than a few hours. In addition (6) suggests that diurnal trends in evapotranspiration rate will tend to follow the trend of atmospheric vapor pressure deficit if the daytime trend of stomatal resistance is not too large. Stewart and Thom (1973) have also examined forest transpiration and have independently derived a relationship (their equation (32)) that is easily shown to be equivalent to (6).

Hinckley and Scott (1971) have found no significant correlation between solar radiation and sap velocity in Douglas-fir trees under conditions of high atmospheric demand. Measurements of transpiration by Parker (1957) made by quick-weighing excised leaves of open grown oak and white

pine, show no short term response to changes in sunlight measured with a pyrheliometer. Comparisons of transpiration estimated from sap velocity, and wind speed and light intensity made by Ladefoged (1963) showed little correlation and also supports this conclusion. Ladefoged found, on the other hand, a dependence on relative humidity. Inspection of his Figures 10-13 suggests a strong correlation between vapor pressure deficit and transpiration in agreement with (6).

In this paper r_s is computed from Monteith's equation (4). The effect of an error in r_a on the value of r_s calculated from (4) can be found by transforming (4) to make r_s the subject of the equation and then differentiating with respect to r_a . Integration of the resulting expression over the range of error from r_a to $r_a + \Delta r_a$ then gives

$$\Delta r_s = \left(\frac{\beta s}{\gamma} - 1 \right) \Delta r_a \quad (7)$$

Substituting the values above, (7) shows that a 50% error in r_a introduces only 3% error into the value of r_s .

Cowan (1968) and Thom (1972) have examined the assumption of similarity of the aerodynamic diffusion resistance for momentum and those for heat and vapor. Both investigators consider that the assumption of similarity may not be appropriate for exchange within canopies. Stewart and Thom (1973) have included a discussion of this for a forest. However, in view of the insensitivity of forest transpiration and surface resistance values calculated from (4) to errors in r_a , this aspect will not be examined here. In calculating r_s for forests from (4) the major uncertainty will usually be caused by errors in the measured values of transpiration.

5. Experimental Site

The experimental site was located in a Douglas-fir plantation growing on a glacial outwash terrace in the U.B.C. Research Forest at Haney, British Columbia, at an altitude of 250 m. The plantation was of a fairly uniform height of about 7.8 m with some differences in stand spacing in the southern section. Fetch was between 200 and 400 m in all directions from NNE through E to SW, but was limited in the remaining directions by a river cutting adjacent to the site. The plantation area sloped down at about 5% towards the southwest. Beyond the plantation, about 200 m to the east, was older regrowth forest about 25 m tall on a slope of about 10% grading upwards to the east. Land in all directions was predominantly forest covered for more than two kilometers. It is unlikely that advected energy had any significant effect on the evaporation processes near the experimental tower.

Measurements of net radiation, wind speed at 8.75 m and wet and dry bulb temperatures at 8.1 m and 9.1 m (Black and McNaughton, 1971) were made with instruments supported above the canopy on a meteorological tower. A thermometer and dewcell were located in a small slatted screen at 3 m above the forest floor. Two soil heat flux plates were placed at a depth of 5 cm in the soil and the mean temperature of the surface 5 cm layer determined with two integrating thermometers. Samples of the top 5 cm of the soil were collected daily for determination of the heat capacity of the layer. A tensiometer-transducer system was used to monitor soil water potential at depths from 30 to 150 cm at 30-cm intervals. The tree root zone extended to a depth of 60 cm. Visual

observation of a wind vane at the top of the tower indicated that wind was predominantly from the southerly quarter during the daytime. At night a valley wind system produced northeasterly winds.

During the month of June preceding the experiment, 9.4 cm of rain was recorded at a permanent weather station about 400 m from the experimental site. Of this total, 6 cm was recorded in the period June 26-29. No rain fell between June 29 and the commencement of detailed measurements.

Sites with satisfactory fetch for micrometeorological measurements are rare in the mountainous Canadian West Coast region. In spite of the non-ideal conditions for the present study, it is felt that the older regrowth forest beyond the plantation boundary should have had very similar surface temperature and fluxes of heat and vapor and that little adjustment of the boundary layer properties would have been necessary.

6. Results and Discussion

6.1 Energy balance

From the data collected in the period July 8-25, 1970, average values of the terms of the familiar energy balance equation

$$R_n = H + LE + G + M \quad (8)$$

were evaluated for each half hour. Photosynthetic energy storage rate and horizontal energy flux divergence terms were assumed negligible.

Soil heat flux was calculated by correcting the 5-cm flux plate reading for heat storage changes in the top 5-cm layer. It was not

possible to evaluate the canopy volume heat storage rate term, M , with precision and values used in this analysis are estimated from the air temperature and vapor pressure within and above the canopy and estimates of biomass volume and heat capacity of the trees. In the results the quantity $(R_n - G - M)$ usually decreased to zero in the evening before the wet bulb gradient diminished to zero, indicating that the rate of heat released by the canopy was larger than the calculated value at that time. A method used to estimate the error in the daily values of evapotranspiration due to uncertainty in the values of G and M suggests that errors in the daily values from this source are negligible (Black and McNaughton, 1972).

The Bowen ratio technique was used to partition the available energy between H and LE . An example of the energy balance for one day from this period has been presented previously (Black and McNaughton, 1971). When the Bowen-ratio is near -1 large errors in the computed fluxes may occur. In this study values of the Bowen ratio in the range $-1.5 < \beta < 0.5$ were rare except for two or three values about 1800 PST each afternoon when total energy exchange was small. Little error has been introduced in smoothing values at these times by eye before making the daily (24 hour) totals of evapotranspiration reported in this paper. Results for selected days presented in Figures 1 and 2 of this paper are unsmoothed.

Summation of possible error from all sources except that introduced by ignoring photosynthesis and horizontal divergence terms, indicates that individual values of E through most of the day have an

uncertainty of 20%. However, consistency of the results indicates that random errors are usually less than this. Possible systematic error in net radiation measurement due to calibration ($2\frac{1}{2}\%$) and sampling (3%)(Federer, 1968) and error introduced by ignoring photosynthetic energy storage ($< 5\%$) do not affect the comparisons made in this paper. Errors due to horizontal flux divergence are not amenable to this type of analysis but do not appear to be large. It is expected that the 24 hour totals of evapotranspiration are accurate to within 15%.

Energy balance results were found to show a consistent pattern with two distinctive features that differentiate them from typical balances for low agricultural crops. Short term fluctuations in radiation did not produce corresponding proportional changes in the latent heat flux. An example of this is shown in Figure 1 for the partly cloudy day, July 23. This pattern may be contrasted with, for example, an energy balance of irrigated alfalfa-brome grass on a partly cloudy day measured by Tanner and Pelton (1960) where changes in net radiation and latent heat flux are strongly coupled. Secondly, peak evapotranspiration rates consistently occurred two to three hours after solar noon. This behaviour is shown in the energy balances presented for July 8, 10, 15 and 19 in Figure 2. Gay (1972) has reported a near identical pattern for a clear July day energy balance of a taller Douglas-fir forest at Cedar River, Washington, U.S.A. Fritschen (1973) has made lysimetric measurements of evapotranspiration from a single Douglas-fir tree in early May 1972 on the same site as used by Gay. His results also show that the daily evapotranspiration maxima occur several hours after the

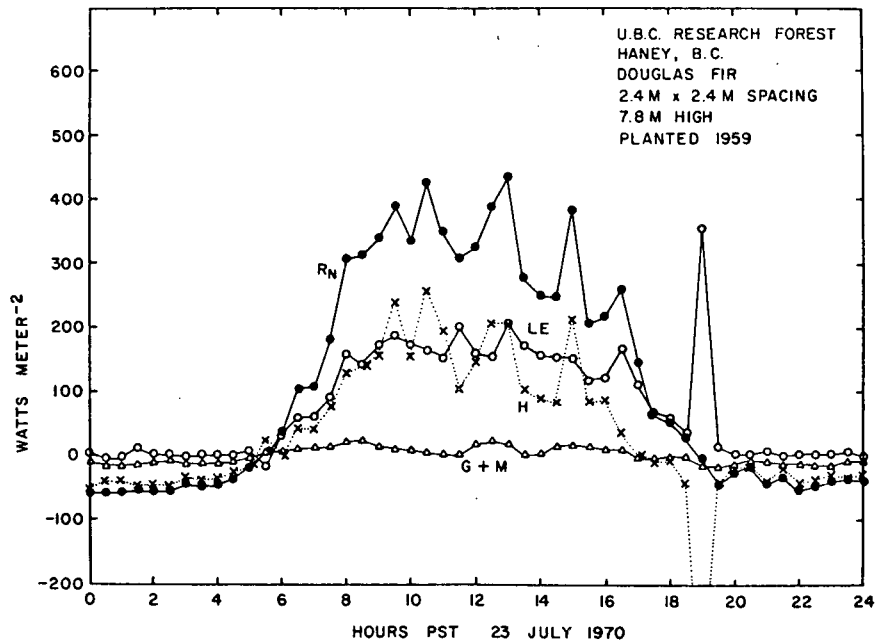


Figure 1 Energy balance diagram for a day with variable net radiation. The latent heat flux density is largely independent of the short term changes in net radiation. The Bowen ratio at 1900 PST was -0.97.

Figure 2 (A)

Daytime trends of meteorological parameters and the derived surface diffusion resistance of a 7.8-m high Douglas-fir forest at Haney, B.C. The free evaporation rate is shown as the equivalent latent heat flux density. Erratic values of the sensible and latent heat flux and consequently the surface resistance near 1800 PST are due to Bowen ratios near -1 at those times.

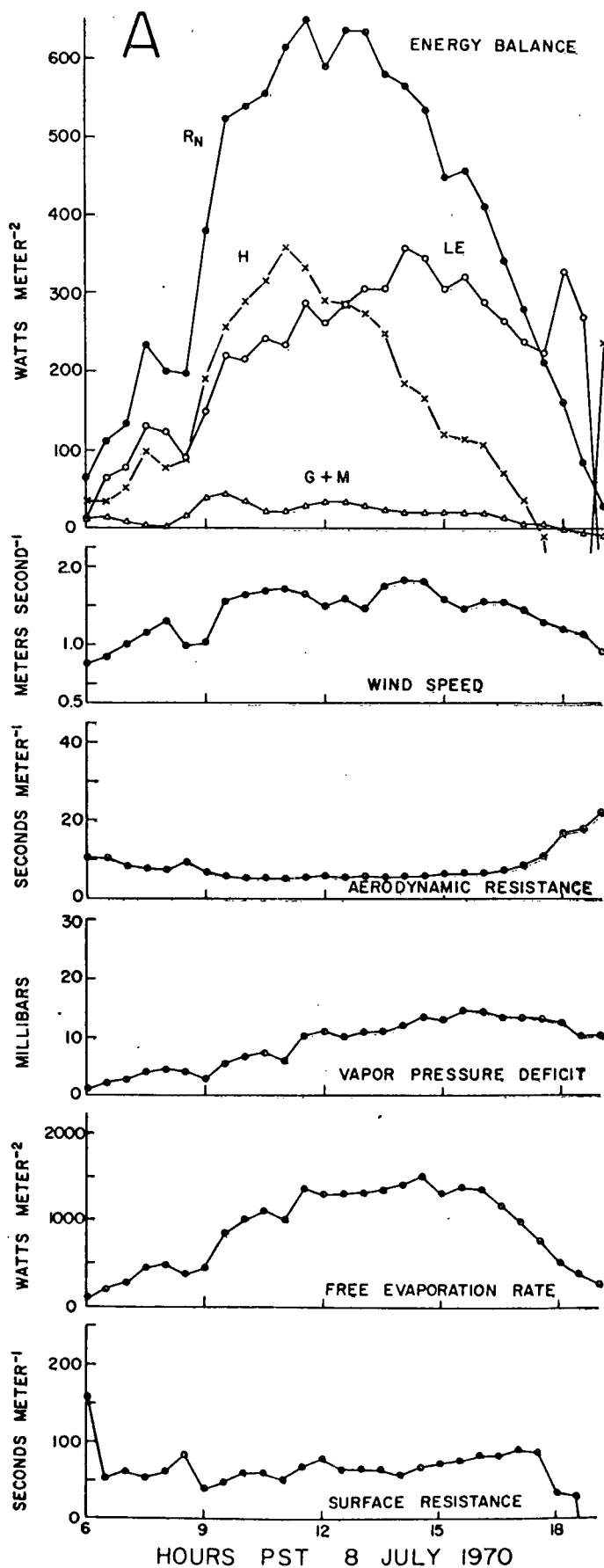


Figure 2 (B)

Daytime trends of meteorological parameters and the derived surface diffusion resistance of a 7.8-m high Douglas-fir forest at Haney, B.C. The free evaporation rate is shown as the equivalent latent heat flux density. Erratic values of the sensible and latent heat flux and consequently the surface resistance near 1800 PST are due to Bowen ratios near -1 at those times.

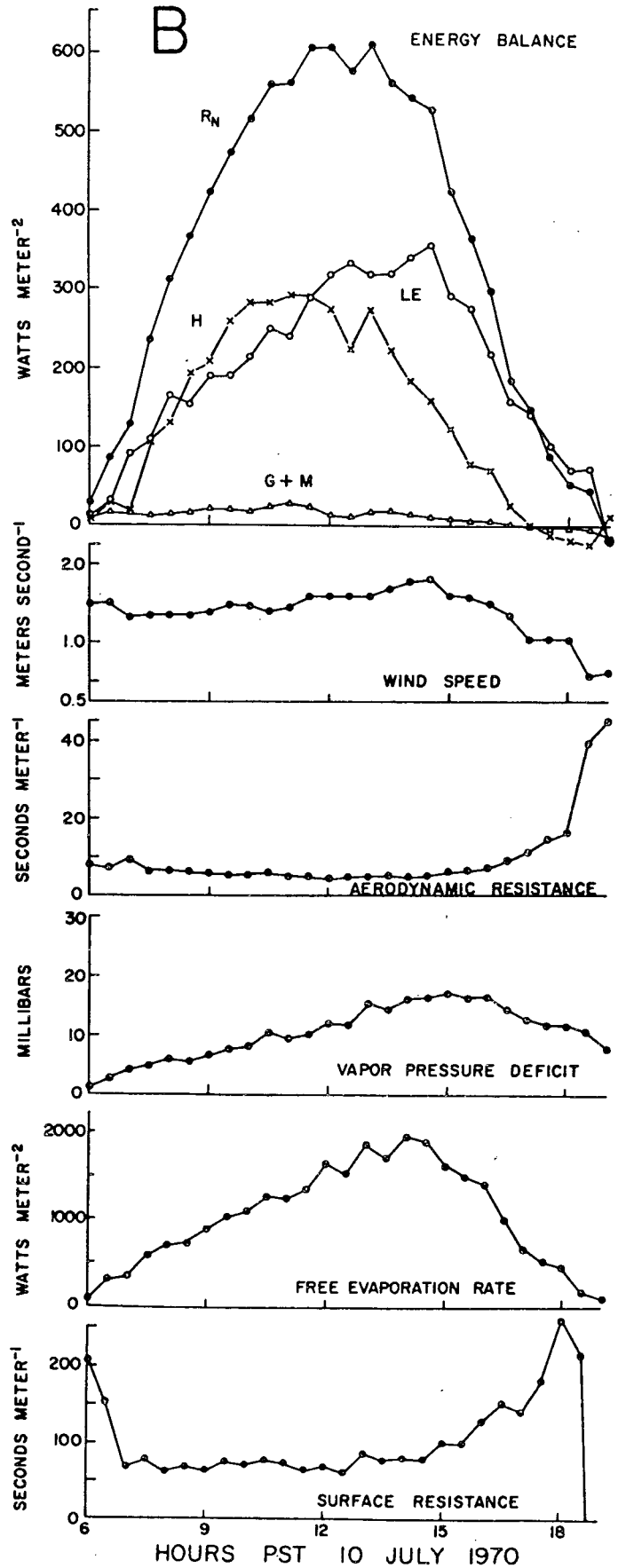


Figure 2 (C)

Daytime trends of meteorological parameters and the derived surface diffusion resistance of a 7.8-m high Douglas-fir forest at Haney, B.C. The free evaporation rate is shown as the equivalent latent heat flux density. Erratic values of the sensible and latent heat flux and consequently the surface resistance near 1800 PST are due to Bowen ratios near -1 at those times.

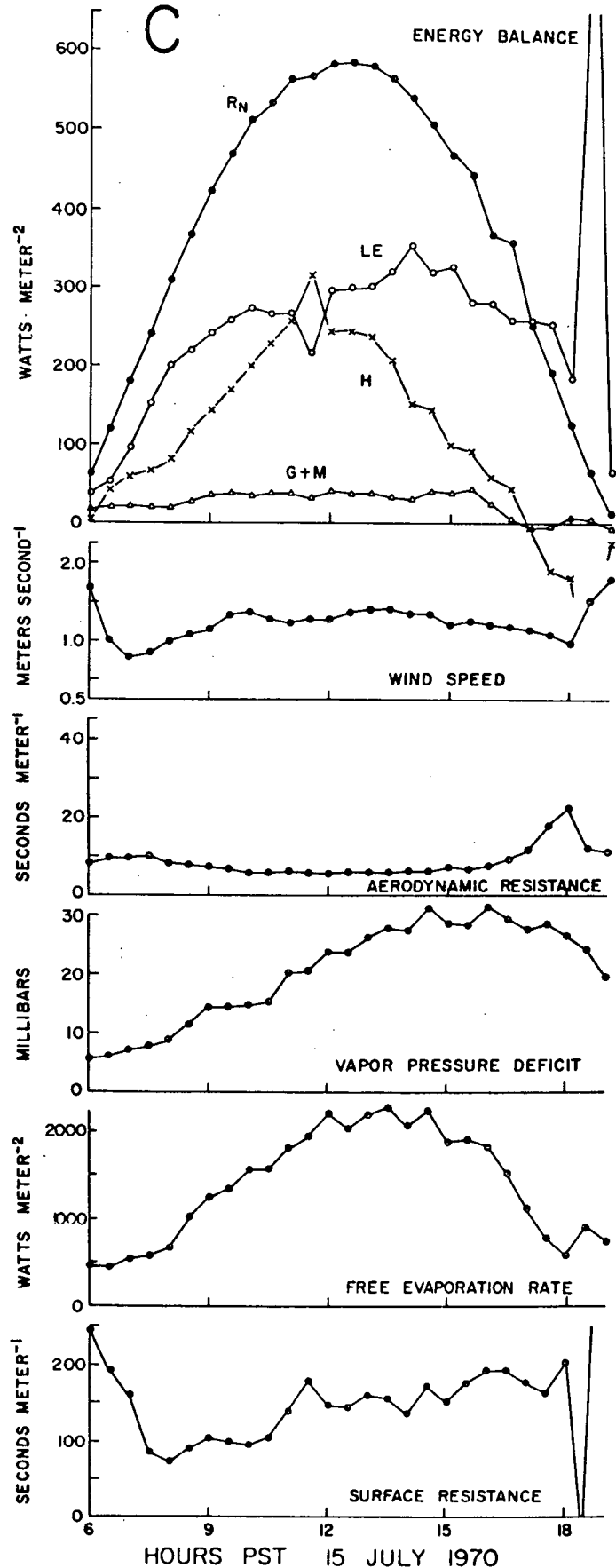
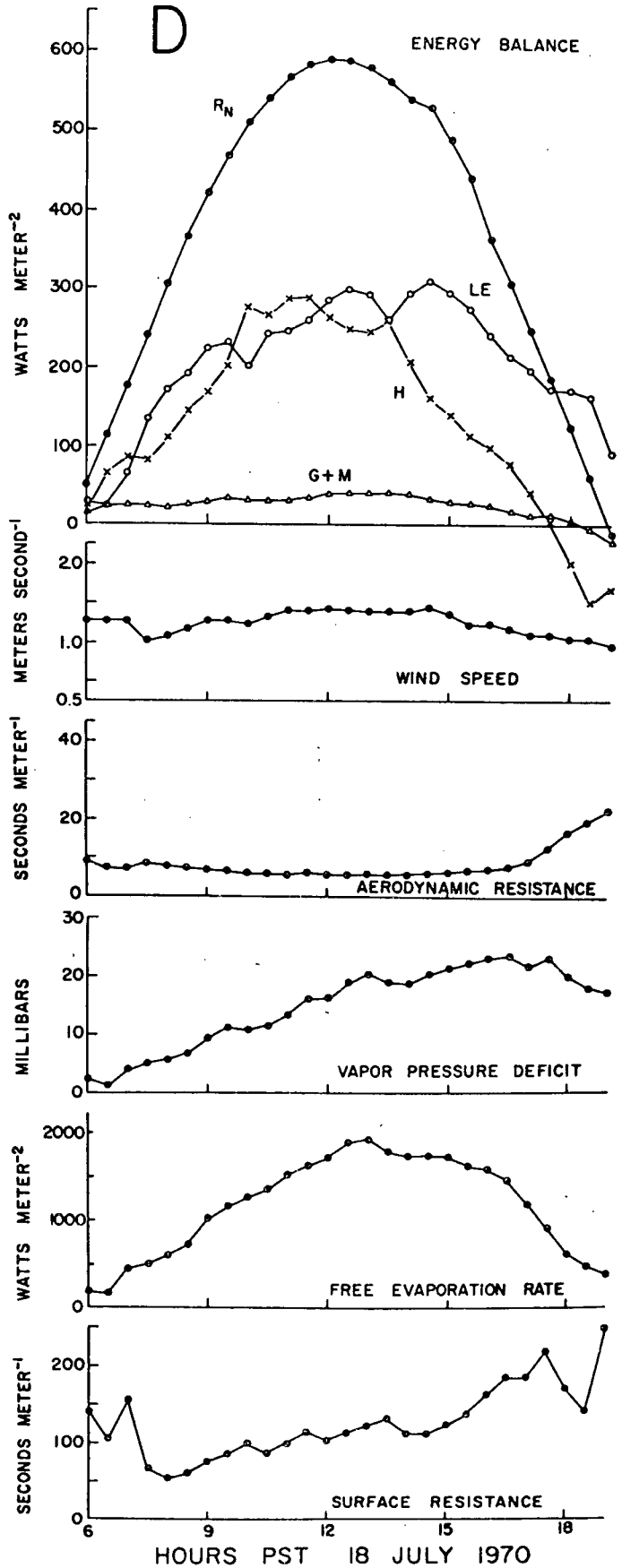


Figure 2 (D)

Daytime trends of meteorological parameters and the derived surface diffusion resistance of a 7.8-m high Douglas-fir forest at Haney, B.C. The free evaporation rate is shown as the equivalent latent heat flux density. Erratic values of the sensible and latent heat flux and consequently the surface resistance near 1800 PST are due to Bowen ratios near -1 at those times.



net radiation maxima. These results show that forest evapotranspiration is not directly 'driven' by net radiation in accordance with the approximation of Monteith's equation for rough surfaces (Equation 6).

6.2 Surface resistance

Taking the results of the measurements and calculations described above, only r_a remained to be evaluated in order to solve (4) for r_s .

Wind profiles suitable for use in calculating aerodynamic resistance, r_a , were not measured. Wind profiles have been estimated from wind at one height and temperature gradient measurements using diabatic profile theory and assumed values of surface roughness and displacement height (Black and McNaughton, 1972). Using similarity of heat, vapor and momentum transport, these profiles successfully predicted the diurnal variation of the eddy diffusivity and were used in that paper to compute latent heat flux with about 34% overestimation. Values of the friction velocity and wind speed at 8.6 m were calculated from the same synthetic profiles and used to calculate r_a from (2). These values are probably systematically underestimated by about 30%.

Daytime trends of wind speed and aerodynamic resistance calculated from the synthetic profiles assuming constant z_0 and D are shown for the four sample days in Figure 2. Aerodynamic resistance exhibited a marked diurnal pattern with a definite decrease and increase at about 0600 and 1730 PST respectively when reversals in the sensible heat flux occurred. Daytime values of r_a usually were between 5.5 and 7.5 s m^{-1} and showed less variation than the wind speed. Szeicz et al.

(1969) have noted a similar small variation in daytime values of r_a in their analysis of wind profiles measured above a 27-m high Norway-spruce forest in Germany.

Using (7) to estimate the error in r_s , calculated from (4), due to uncertainty in r_a shows that, in a worst case daytime situation, r_s derives an uncertainty of $\pm 8 \text{ s m}^{-1}$ from this source. This uncertainty is small compared with the diurnal and day-to-day differences in the calculated values of r_s .

Typical daytime r_s trends are shown in Figure 2. Nighttime and late afternoon values, when the Bowen-ratio was near -1, were erratic and frequently negative. Daytime trends for each of the days of July 8-13 generally followed the pattern observed for July 10 (Figure 2b). Of the 13 days following July 13, 6 had surface resistances notably higher than for the early period, two had notably lower values and the remainder had similar values. On the six days with higher surface resistances, the early morning values were about 60 s m^{-1} as observed for the days prior to July 13, but increased more rapidly during the day. This behaviour is illustrated in Figure 2 (c and d) for July 15 and 18 respectively.

Interpreting this as an increase in stomatal resistance on those days indicates that there was water stress induced stomatal closure. The result is unexpected since there was a plentiful supply of water in the root zone held at relatively high soil water matric potentials. Soil water matric potentials at the 60-cm depth are shown for 0600 PST each day in Table 1. The wetting front from the heavy

rainfall on July 25 had not reached 60 cm at the time of the recording.

On July 21 very light rain fell for about two hours commencing at 0930 PST and the weather station recorded 0.01 in (0.25 mm) of precipitation. The rain was almost totally intercepted by the canopy and the forest floor was not noticeably wetted. Surface resistance values immediately fell to about 15 s m^{-1} and remained near this value for 3 1/2 hours. During this period of lower surface resistances, energy balances results show that 0.31 mm of water evaporated. The period of low surface resistance, therefore, corresponded to the period during which intercepted water was probably present on the foliage. July 25 was a day of continuous rain and surface resistance values were close to zero all day.

Elsewhere in the coastal Douglas-fir region, Phillips (1967) has studied the daily trends of stomatal resistance of a dominant Douglas-fir tree in a closed stand at La Grande, Washington, U.S.A. Using a pressure infiltration technique Phillips found daily trends of stomatal resistance consistent with those shown in Figure 2. Phillips also noted that smaller stomatal resistances were observed on days of lower vapor pressure deficit as indicated by hygrothermograph measurements at ground level. If the surface resistance is simply the stomatal resistance of all of the leaves acting in parallel, and a leaf area index of about 10 is assumed for the U.B.C. forest, then a typical stomatal resistance for mornings of about 700 m^{-1} is indicated. Phillips' results show 150 s m^{-1} as a typical morning value.

As noted previously, the simple canopy model expressed by (4) should be treated with caution. Nevertheless, it does seem that the

day-to-day differences in surface resistance are unlikely to be artifacts of the model, since wind speed and net radiation were quite similar on days when resistances were significantly different as illustrated by the sample days in Figure 2. Furthermore, the trends of r_s are generally consistent with Phillips' measurements of stomatal resistance made on similar trees.

It seems likely, therefore, that the observed short term independence of the measured forest evapotranspiration and net radiation is a result of the roughness of the forest canopy and that the meteorological factor most directly controlling forest evapotranspiration is the vapor pressure deficit. This hypothesis should be readily testable by use of the more sophisticated multilayered canopy model of Waggoner et al. (1969), in which aerodynamic resistance above the canopy, net radiation, vapor pressure deficit and stomatal resistance parameters can all be manipulated independently.

6.3 Potential evapotranspiration estimates

Since water was plentiful in the root zone throughout the period July 7-25, actual forest evapotranspiration should have been at the potential rate. The estimates of evapotranspiration from the U.B.C. forest can therefore be used to test several approaches to the estimation of potential evapotranspiration.

The formula for the free evaporation rate (1), has two additive terms which may be called for convenience, the energy term and the convective term. Accurate determination of E_0 is relatively easy for low agricultural crops since the energy term predominates and the aero-

dynamic resistance can be determined with sufficient accuracy since the convective term is in the nature of a correction. For the U.B.C. forest the convective term was as much as seven times greater than the energy term due to small r_a values and consequently the uncertainty in r_a leads to a probable overestimate in E_0 of about 25%. Both vapor pressure deficit and E_0 are presented for the sample days in Figure 2. The free evaporation rate has been expressed as the latent heat flux density equivalent (LE_0) so that the measured evapotranspiration and free evaporation rates can be compared directly.

In spite of the uncertainty in E_0 , a comparison of the measured evapotranspiration rate with the free evaporation rate is instructive. Values of E and E_0 for the period July 8-25 are given in Table 2. It can be seen that E_0 is many times greater than E on all days but July 25. This, plus the difficulty of accurately measuring r_a and therefore of calculating E_0 , indicates that E_0 has little practical significance as a potential evapotranspiration measure for forests.

On July 25 rain fell continuously commencing at 0130 PST. The canopy was already wet from a light shower at about 1800 PST the previous evening. The assumption of a saturated surface in the derivation of (1) was satisfied. Error in the calculation of E_0 was small on July 25 since all but two of the half-hour values of the vapor pressure deficit were less than 0.5 mb and the energy term predominated. The net radiometer hemispheres were wet throughout the day so the long wave components of the measurement may have been attenuated. With the prevailing low cloud base, long wave balance is thought to have been small and further, any errors introduced into R_n entered almost equally

TABLE 1

Summary of Daily Evapotranspiration Data from a Douglas Fir Forest
at the U.B.C. Research Forest, Haney B.C.

1970 July	E* (mm)	E ₀ * (mm)	E ₇₅ * (mm)	$(\frac{S}{S+Y})(R_N-G)^*$ (mm)	R _N * (mm)	T** (C)	RAIN (mm)	-ψ _m *** (bars)
8	4.34	19.15	4.07	4.34	6.73	18.3		0.053
9	4.39	20.70	4.15	4.15	6.46	16.4		0.062
10	4.03	22.59	4.50	3.74	5.93	15.5		0.064
11	4.20	23.91	4.40	4.19	6.71	15.6		0.069
12	4.15	25.27	4.26	4.10	6.51	16.3		0.074
13	4.08	22.33	3.98	4.09	6.53	15.9		0.080
14	4.65	26.77	5.45	4.16	6.45	18.4		0.089
15	4.80	33.04	7.66	4.22	6.29	22.0		0.098
16	2.40	18.57	3.11	2.10	3.20	17.3		0.110
17	3.84	18.89	3.74	4.07	6.45	15.9		0.119
18	4.29	26.64	5.88	4.13	6.33	19.9		0.132
19	3.61	22.21	4.94	3.50	5.31	19.8		0.148
20	3.10	13.06	2.97	2.51	3.90	16.9		0.168
21	1.46	7.29	1.16	1.01	1.47	12.0	0.25	0.189
22	3.18	14.52	2.89	3.05	4.99	14.2		0.203
23	2.56	15.52	3.23	2.59	4.19	15.0		0.230
24	1.75	9.38	1.96	1.64	2.63	13.6	0.5	0.261
25	1.49	1.56	.43	1.26	2.05	12.3	33.5	0.297

* 24 hour totals expressed as equivalent depths.

** 24 hour arithmetic mean of half hourly temperatures above the forest.

*** Measurement at 60-cm depth in the soil at 0600 PST.

into the calculation of E and E_0 . The close agreement between E and E_0 suggests that (1) can be used to calculate evaporation from wet forests. However, the agreement is far better than expected in view of the small temperature gradients and hence large possible errors in Bowen ratio determinations used to calculate E . This result should not be considered definitive.

An alternative approach to the estimation of potential evapotranspiration that has been used is to attempt to determine representative values of the surface resistance for various surfaces (Szeicz and Long, 1969) so that these may be used in equation (4) to calculate evapotranspiration. We have therefore examined our results to see if a single daytime value of r_s could be found that would represent the typical behaviour of the forest when it was not short of water. Such a value, it was hoped, would represent the forest with the normal number of stomata open to a normal degree for stress free conditions.

Since the rise in surface resistance was noticed after July 13, only the first 6 days were used to determine the value. It was found that values of evapotranspiration for the first 6 days plus three later days could be calculated to within 10% by rather arbitrarily setting r_s to 75 s m^{-1} in the daytime period from 0700 to 1800 PST and 500 s m^{-1} for the remaining hours. Daily values of evapotranspiration calculated by this method, denoted by the symbol E_{75} , are plotted against the measured 24-hour totals of evapotranspiration in Figure 3. Two points lie significantly below the line and are for days when rain fell. Seven points fell significantly above the line. As a potential evapotranspiration

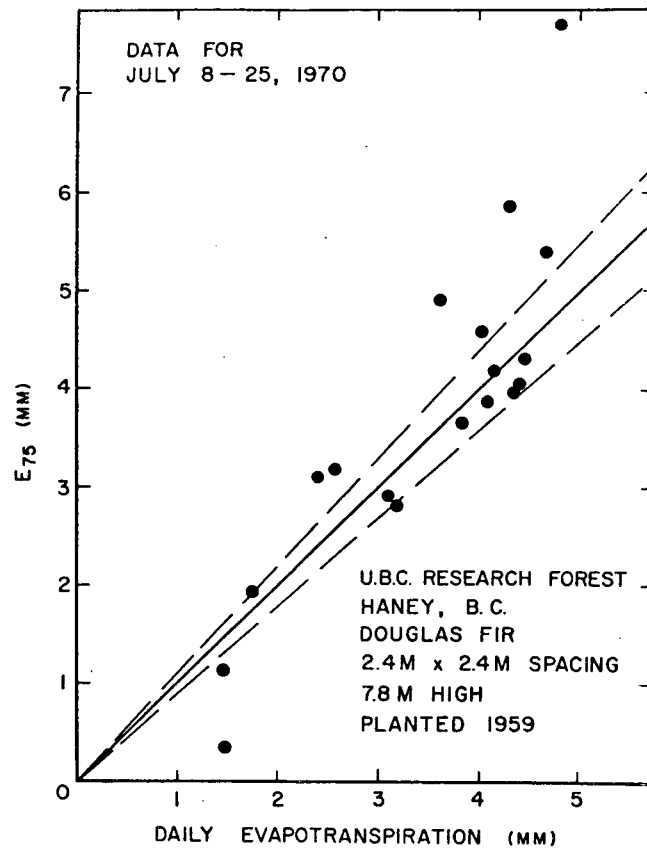


Figure 3 Comparison of calculated 24-hour evapotranspiration values assuming constant daytime surface resistance of 75 s m^{-1} with measured values. Broken lines represent 10% deviation from the 1:1 line.

measure, E_{75} may have some merit but on half of the days the difference between E_{75} and E was greater than 10% and on July 15 was 40%, even though there was a plentiful supply of water in the root zone throughout the period.

Where advected energy is not an important source of energy for evapotranspiration and soil water is adequate a close relationship between net radiation and daily evapotranspiration is expected. Examination of the data from the U.B.C. forest reveals that daily evapotranspiration and net radiation were highly correlated. Energy used in daily evapotranspiration was generally about two thirds of net radiation (Table 1).

Before such a correlation can be used with confidence as a basis for estimation of evapotranspiration, further work must be done to determine the influence of other factors, such as tree phenology and presence of intercepted water on the canopy, on the partitioning of the available energy between sensible and latent heat fluxes. Examination of the U.B.C. forest data allows us to make some estimate of the importance of interception. In the absence of advection, the maximum possible rate of evapotranspiration is given by $(R_n - G)$. With typically small G values for the forest, this implies that the maximum possible rate of evaporation of intercepted water after general rain could not be more than 50% faster than the observed rates.

Perhaps a more realistic estimate of the influence of intercepted water can be obtained by comparison of the result from the dry transpiring canopy with a relationship found by Priestley and Taylor (1972). They have surveyed several experiments over surfaces where

surface resistance was expected to be negligibly small and E_0 was expected to be equal to the actual evaporation rate. They examined both terrestrial and oceanic data from locations where the effects of advection were expected to be negligible. They used the data to determine a coefficient α to satisfy the equation

$$E = \frac{\alpha}{L} \left(\frac{s}{s + \gamma} \right) (R_n - G) \quad (9)$$

where the energy flux density terms are here 24-hour integrals and s is calculated from the mean surface temperature. The best value of α was found to be 1.26.

Measured evapotranspiration rates from the U.B.C. forest site plotted against $\frac{1}{L} \left(\frac{s}{s + \gamma} \right) (R_n - G)$ are shown in Figure 4. Neglecting July 21 and 25, on which rain fell, a value for α of 1.05 is found.

Considering the possible errors in determining each of these values of α , they are not necessarily significantly different. However, it does seem probable that evaporation from the U.B.C. forest, when wet, would proceed at a rate 20% faster than the expected evapotranspiration from the same canopy when well supplied with water but not wet. On July 25, α was 1.18 which is close to the value found by Priestley and Taylor for wet surfaces and tends to support the validity of this comparison. Data from July 21 is not suitable for use in this comparison since the high value of α for this day ($\alpha = 1.45$) is a result of a clear sky before dawn making the 24 hour radiation total unrepresentative of the daytime period. During the period when the canopy was wet on this day, E was about three quarters of R_n .

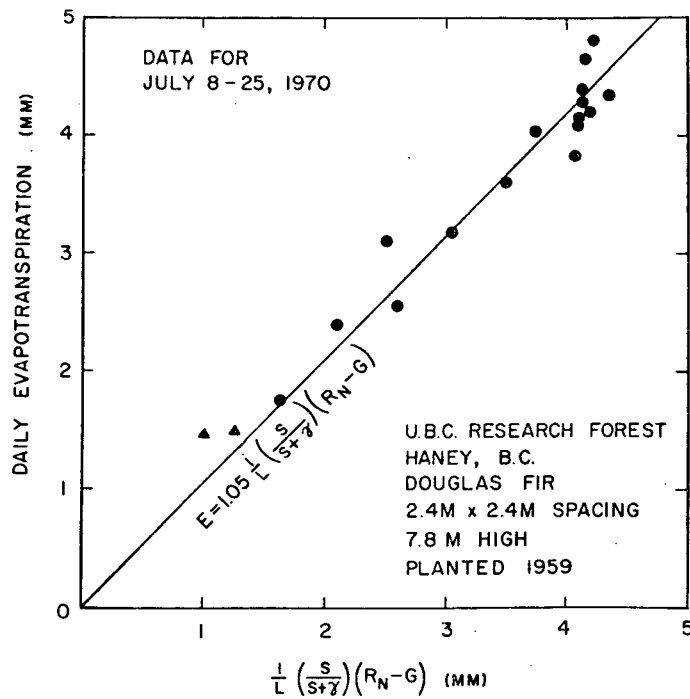


Figure 4 Measured daily values of evapotranspiration compared with $\frac{1}{L} \left(\frac{s}{s + \gamma} \right) (R_n - G)$. Data for July 21 and July 25, when rain fell, are shown as triangles and have not been used to determine the line of best fit. The available energy, $R_n - G$ is expressed as an equivalent depth of water evaporated.

An estimate of the importance of interception losses by the forest can now be made. Gross interception loss (I) is the amount of precipitation onto the forest that is caught by the canopy and re-evaporates without reaching the ground. However, because the leaves are wet, transpiration is at the same time suppressed and not all of the gross interception loss represents a loss in the hydrologic sense. The net interception loss is the difference between gross interception loss and the reduction in transpiration caused by the presence on the intercepted water. From the above result, only 17% of the gross loss can be considered to be net loss.

Perhaps the best potential evapotranspiration relationship that can be suggested from the present study is

$$P = \frac{1.05}{L} \left(\frac{s}{s + \gamma} \right) (R_n - G) + 0.17 I \quad (10)$$

7. Conclusions

Measurements of evapotranspiration made at the U.B.C. Research Forest showed that peak evapotranspiration rates occurred 2 to 3 hours after solar noon and that evapotranspiration was not strongly affected by short term changes in net radiation. On the basis of a simplified analysis using Monteith's combination canopy model, it is hypothesized that this is a direct effect of the large canopy roughness and that vapor pressure deficit is the dominant meteorological factor directly controlling forest evapotranspiration.

On a daily basis, measurements of net radiation and evapotranspiration were highly correlated. By comparison of the U.B.C. forest results with a relationship developed by Priestley and Taylor to predict evaporation from wet surfaces, it is tentatively inferred that evaporation of intercepted water proceeds 20% more rapidly than transpiration from the non wetted forest, and that therefore only 17% of gross interception losses at the U.B.C. forest site are to be considered as net interception losses.

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CHAPTER 4

EQUILIBRIUM EVAPORATION

EQUILIBRIUM EVAPORATION

Abstract

The process of modification of the Bowen ratio, with distance downwind of a change in surface wetness, is considered with the view to establishing the final equilibrium ratio of the fluxes of sensible and latent heat after advective effects become negligible. A method of generating, from the coupled equations for heat and vapour diffusion, two new diffusion equations in composite variables, which may be solved independently, is developed.

The method leads to the conclusions that there is a basic difference between equilibration over land and over water. Also the rate of equilibration depends strongly on atmospheric mixing and hence surface roughness and atmospheric stability. It is concluded that, for terrestrial surfaces, the equilibrium evaporation rate is approximated by

$$LE = \left(\frac{S}{S + \gamma} \right) (R_n - G)$$

for 24 hour periods. This result is in accord with some recent experimental findings.

1. Introduction

In the last three years there have appeared a number of papers which show measured daily latent heat flux and the quantity $s(R_n - G)/(s + \gamma)$ to be almost equal, where these and other symbols have the meanings given in the list of symbols. These measurements have been made over such diverse surfaces as wheat "under severe drying conditions" (Denmead and McIlroy, 1970), beans (Davies, 1971), arctic tundra (Rouse and Stewart, 1972), sweet corn (Wilson and Rouse, 1972) and Douglas-fir forest (McNaughton and Black, 1973). Though the expression $s(R_n - G)/(s + \gamma)$ has been referred to as the "equilibrium" evaporation rate this appellation has not been justified theoretically. (cf. Priestley and Taylor, 1972).

The data for the Douglas-fir forest have been analysed using a simplified canopy model due to Monteith (1965) (McNaughton and Black, 1973). The forest "surface resistance" was found to have varied significantly from day to day. This implied that the actual stomatal resistance of the foliage changed significantly. A further analysis of the same data using the more sophisticated layered canopy model of Waggoner et al. (1968), also suggested significant day-to-day differences in stomatal resistances (McNaughton, unpublished results). In view of this evidence of changes in forest behaviour, it is difficult to believe that the result

$$LE = \left(\frac{s}{s + \gamma} \right) (R_n - G) \quad (1)$$

could have occurred due to a fortuitous balancing of the influences of advected energy and decreasing water availability at the surface as suggested by Denmead and McIlroy (1970). It therefore seems probable that the result (1) is due to a well organized process of atmospheric equilibration with the underlying surface.

In principle it should be possible to study this process by considering the evolution of the fluxes of sensible and latent heat with distance downwind of a change in surface wetness. However, the available solutions to the problem of simultaneous diffusion of heat and vapour do not yield satisfactory results for large distances downwind. The theory of Philip (1959) and Rider *et al.* (1963) leads to an expression for the evolution of the Bowen ratio

$$\beta = \beta' - \frac{\epsilon(1 + \beta')^2}{(Rn-G)x^{1/9} + \epsilon(1+\beta')} \quad (2)$$

where β' is the initial upwind Bowen ratio and ϵ is a constant set by the upwind surface temperature and humidity conditions (Rider and Philip, 1960). For very large x , the Bowen ratio returns to its original upwind value. Common sense dictates, on the other hand, that initial conditions should eventually be unimportant and the Bowen ratio should ultimately depend on the properties of the downwind surface alone.

It is clear then that the theory of advection must be re-examined in order to study the Bowen ratio adjustment at large distances downwind. This is the purpose of this paper.

2. Symbols and Units

E	Evapotranspiration rate	$(\text{kg m}^{-2} \text{ s}^{-1})$
H	Sensible heat flux density	(W m^{-2})
K	Effective diffusivity for heat and vapour	$(\text{m}^2 \text{ s}^{-1})$
L	Latent heat of vapourization of water	(J kg^{-1})
R _n	Net radiation flux density	(W m^{-2})
a	Constant in equation (11), (12)	(mb K^{-1})
c _p	Specific heat of moist air	$(\text{J kg}^{-1} \text{ K}^{-1})$
e	Vapour pressure	(mb)
e [*]	Saturation vapour pressure	(mb)
p	Constant in (21)	$(\text{W m}^{-2} \text{ mb}^{-1})$
q	Constant in (21)	$(\text{W m}^{-2} \text{ mb}^{-1})$
r _s	Surface diffusion resistance	(s m^{-1})
u	Wind speed	(m s^{-1})
x	Horizontal distance downwind of a change in surface wetness	(m)
z	Height above ground	(m)
Λ	Defined by equation (13)	(W m^{-2})
Ψ	Defined by equation (11)	$(\text{W mb m}^{-2} \text{ K}^{-1})$
α	Constant in equation (45)	(dimensionless)
β	Bowen ratio	(dimensionless)
γ	Psychrometric constant	(mb K^{-1})
ε	Constant in equation (1)	$(\text{W m}^{-1} \text{ }^{8/9})$
θ	Absolute potential temperature	(K)
λ	Defined by equation (14)	
ν	Defined by equation (17)	

ρ	Density of moist air	(kg m ⁻³)
σ	Stephan-Boltzmann constant	(W m ⁻² K ⁻⁴)
ψ	Defined by equation (12)	(mb)

Subscripts

h	For sensible heat transport
m	Evaluated at the mean surface temperature
v	For vapour transport
x	Evaluated at the surface and at distance x downwind of the wetness change

3. Theory

Two-dimensional, steady state, atmospheric diffusion equations for sensible and latent heat are

$$\rho c_p u \frac{\partial \theta}{\partial x} = - \frac{\partial H}{\partial z} + \frac{\partial R_n}{\partial z} \quad (3)$$

where

$$H = -\rho c_p K_h \frac{\partial \theta}{\partial z} \quad (4)$$

and

$$\rho c_p u \frac{\partial e}{\partial x} = - \gamma \frac{\partial LE}{\partial z} \quad (5)$$

where

$$LE = - \frac{\rho c_p}{\gamma} K_v \frac{\partial e}{\partial z} \quad (6)$$

and the effects of downwind diffusion and horizontal radiation divergence are assumed negligible. Equations (4) and (6) are defining equations for the effective diffusivities for heat and water vapour.

At the underlying surface, two boundary conditions must be satisfied. The first is the energy balance relationship

$$H_x + LE_x = Rn_x - G_x \quad (7)$$

The second boundary condition must specify the 'wetness' or the availability of water at the surface.

The simplest model of a vegetative canopy that has a reasonable physical basis is due to Monteith (1965). In this model the canopy is approximated by a single, extensive leaf with a surface resistance, r_s , formally identified as the total effect of all of the stomata of the leaves of the canopy, above unit area of ground, acting in parallel. Within the stomatal cavities of this 'leaf' the atmosphere is almost saturated. Evaporation from the leaf is described by

$$LE_x = \frac{\rho c_p}{\gamma r_s} [e^*(\theta_x) - e_x] \quad (8)$$

where the saturation vapour pressure has been written as a function of potential temperature rather than actual temperature on the

understanding that they are equal at the surface. Equation (8) is therefore taken to be the second lower boundary condition.

The essential feature of (8) in the following analysis is that the temperature and vapour pressure appear only in combination as the vapour pressure deficit. Since this is true of the single leaves that make up a vegetative canopy, it should be a very good approximation for complex real canopies provided θ_x and e_x are chosen judiciously from within the range of temperatures occurring within the canopy. Such a procedure may invalidate the assumption of transport similarity within the canopy. However, it will ultimately be the diffusivities in the free atmosphere that control the transport of heat and vapour and this dissimilarity within the canopy will not be of importance on a larger scale.

In this analysis the surface resistance will be assumed constant. This is not generally true since the stomatal resistances of plants depend on all of the factors that influence the internal plant water budget, including the evaporation rate itself. In this case r_s can be considered to be a function of the vapour pressure deficit (plus the other factors) rather than the evaporation since, by (8), the two are not independent. Extension of the theory to cover this possibility could be made but would increase complexity without adding to insight.

To complete the statement of the problem, initial profiles of θ and e must be known and the functional forms of the effective diffusivities, K_h and K_v , and the profile of wind speed, u , must be

specified. For the present discussion it is not necessary to specify these functions exactly except to assume that $K_h = K_v = K$ for all x and z and that R_n , u , and K are assumed to be known functions of x and z .

It follows from (3) - (6) that a new pair of variables ψ and Ψ , which are concentration-type and flux-type variables, respectively, satisfy an equation of the same (diffusion equation) form as previously,

$$\rho c_p u \frac{\partial \psi}{\partial x} = - \frac{\partial \Psi}{\partial z} \quad (9)$$

where

$$\Psi = - \rho c_p K_h \frac{\partial \psi}{\partial z} \quad (10)$$

if Ψ and ψ are defined by

$$\Psi = a(H - R_n + R_{n_x}) + \gamma LE \quad (11)$$

and

$$\psi = a \left[\theta + \frac{1}{\rho c_p} \int_0^z \frac{R_n(x, \xi) - R_{n_x}}{K(x, \xi)} d\xi \right] + e \quad (12)$$

respectively, and a is an arbitrary constant and ξ is a dummy height variable. For brevity of expression it is convenient to define

$$\Lambda = H - R_n + R_{n_x} , \quad (13)$$

and

$$\lambda = \theta + \frac{1}{\rho c_p} \int_0^z \frac{Rn(x, \xi) - Rn_x}{K(x, \xi)} d\xi \quad (14)$$

At the surface $\lambda = \theta$ and $\Lambda = H$.

If it is possible to select a value of a in such a way that the boundary condition to (9) and (10) can be written in terms of ψ and Ψ alone, then any such equation can be solved independently.

The boundary conditions (7) and (8) can be combined to give

$$aH_x + \gamma LE_x = a(Rn_x - G_x) - (a - \gamma) \frac{\rho c_p}{\gamma r_s} [e^*(\theta_x) - e_x] \quad (15)$$

The left hand side of (15) now corresponds to Ψ_x .

In general Rn_x and G_x are both dependent on surface temperature. Since ψ is a linear combination of λ and e , the next step is to introduce linear approximations for $Rn(\theta_x)$, $G(\theta_x)$ and $e^*(\theta_x)$. Thus

$$Rn(\theta_x) - G(\theta_x) \approx Rn(\theta_m) - G(\theta_m) + v(\theta_x - \theta_m) \quad (16)$$

where

$$v = \left. \frac{dRn}{d\theta} \right|_{\theta=\theta_m} - \left. \frac{dG}{d\theta} \right|_{\theta=\theta_m}, \quad (17)$$

and

$$e^*(\theta_x) \approx e^*(\theta_m) + s(\theta_x - \theta_m) \quad (18)$$

where

$$s = \left. \frac{de^*}{d\theta} \right|_{\theta=\theta_m} \quad (19)$$

and θ_m is a surface temperature central to the range of variation in the region of interest. Equation (15) now becomes

$$\begin{aligned} aH_x + \gamma LE_x = & a(Rn_m - G_m) - av\theta_m - (a - \gamma) \frac{\rho c_p}{\gamma r_s} (e_m^* - s\theta_m) \\ & + \theta_x [av - (a - \gamma) \frac{\rho c_p}{\gamma r_s}] \\ & + e_x [(a - \gamma) \frac{\rho c_p}{\gamma r_s}] \end{aligned} \quad (20)$$

The most general linear combination of ψ and Ψ is

$$\Psi_x = p\psi_x + q = pa\theta_x + pe_x + q. \quad (21)$$

Clearly q must be selected so that

$$q = a(Rn_m - G_m) - av\theta_m - (a - \gamma) \frac{\rho c_p}{\gamma r_s} (e_m^* - s\theta_m) \quad (22)$$

and p and a must satisfy

$$pa = av - (a - \gamma) \frac{\rho c_p}{\gamma r_s} s \quad (23)$$

and

$$p = (a - \gamma) \frac{\rho c_p}{\gamma r_s} . \quad (24)$$

The solution of (23) and (24) involves the solution of a quadratic equation for which there are always two real, distinct roots

$$a_{1,2} = \frac{1}{2} \left[\frac{\gamma r_s}{\rho c_p} + \gamma - s \pm \sqrt{\left(\frac{\gamma r_s}{\rho c_p} + \gamma - s \right)^2 + 4s\gamma} \right] \quad (25)$$

$$p_{1,2} = \frac{1}{2} \left[\frac{\gamma r_s}{\rho c_p} - \gamma - s \pm \sqrt{\left(\frac{\gamma r_s}{\rho c_p} + \gamma - s \right)^2 + 4s\gamma} \right] \frac{\rho c_p}{\gamma r_s} \quad (26)$$

except when $r_s = 0$ or $r_s = \infty$. The subscripts 1 and 2 are used to denote the first and second solutions corresponding to the positive and negative sign of the square root term respectively. With these values of a_1 and a_2 , two corresponding values of q can be found from (22).

The case where $r_s = \infty$ corresponds to dry desert and the heat flux problem is independent of any consideration of evaporation. For $r_s = 0$ two boundary conditions can still be found. One is constructed from the condition that the vapour pressure deficit is equal to zero while the other is found from the energy balance equation. This latter condition has been discovered previously by Rider et al. (1963) who, however, failed to realize that there were two possible ways of constructing the boundary condition. In fact the values of a_1 and a_2 calculated from (25) are appropriate even for $r_s = 0$.

Using the values of a_1 and a_2 found from (25), particular forms of the concentration-type and flux-type variables defined by (11) and (12) are defined, and can also be denoted by the subscripts 1 and 2. In this way the problem of finding the simultaneous solution of (3) - (6) subject to (7) and (8) is reduced to that of finding two independent solutions to the diffusion equations (9) and (10) in the composite variables $\Psi_{1,2}$ and $\psi_{1,2}$, with lower boundary conditions found as described. In principle, solutions can be found once the initial conditions and the functions K and u are specified. From these solutions, λ and e can be found since, from (12)

$$a_1 \lambda + e = \psi_1 \quad (27)$$

and

$$a_2 \lambda + e = \psi_2 \quad (28)$$

which leads to

$$\lambda = (\psi_1 - \psi_2)/(a_1 - a_2) \quad (29)$$

and

$$e = (a_2 \psi_1 - a_1 \psi_2)/(a_2 - a_1) \quad (30)$$

The fluxes of heat and vapour are then given by (4) and (6). Alternatively, (11) gives

$$a_1 \Lambda + \gamma LE = \Psi_1 \quad (31)$$

and

$$a_2 \Lambda + \gamma LE = \Psi_2 \quad (32)$$

giving

$$\Lambda = (\Psi_1 - \Psi_2)/(a_1 - a_2) \quad (33)$$

and

$$LE = (a_2 \Psi_1 - a_1 \Psi_2)/[\gamma (a_2 - a_1)] \quad (34)$$

The method of separating the diffusion equations presented to this point has a potential application to the problem of advection near a change in surface wetness such as in the case that has been studied by Rider et al. (1963). It can be seen that their assumption of constant surface temperature and vapour pressure downwind of the surface change is artificial and unnecessary. Perhaps a better correspondence between theory and experiment could have been obtained by retaining the assumptions of no radiation divergence, and of the power law forms of u and K , but allowing θ_x and e_x to vary downwind.

Vegetated surfaces have the property that usually the soil is well insulated and ground heat fluxes are small. It follows that $\frac{dG_x}{d\theta}$ will be positive but quite small. Rider et al. (1963) consider the dominant temperature effect on the radiative balance to be that on the outgoing long wave radiation. Far downwind of the wetness discontinuity, changes in temperature of the air overlying the surface will tend to modify this effect. Taking the emissivity of the surface to be nearly unity, which is true of most vegetative surfaces, and using Swinbank's (1963) empirical fourth power law to estimate the

downward long-wave radiation, gives

$$\frac{dRn_x}{d\theta} = -4\sigma\theta_x^3 + 4.78 \sigma\theta_x^3 \quad (35)$$

where the first term on the right gives the change in long-wave radiation emitted from the surface and the second term estimates the change in downward long-wave radiation from the air above. The variation of net radiation with surface temperature is thus given by

$$\frac{dRn_x}{d\theta} = + 0.78 \sigma\theta_x^3 \quad (36)$$

for sufficiently large distance downwind and a sufficiently deep adjusted layer. In his determination of the downward long wave radiation vs temperature correlation, Swinbank used data from situations with small vertical temperature gradients. Equation (36) will probably be appropriate for 24 hour averages.

For vegetated surfaces both $\frac{dRn_x}{d\theta}$ and $\frac{dG_x}{d\theta}$ are small and positive so that their difference should always be negligible. Therefore it is reasonable to take $v = 0$ which gives, from (25)

$$a_1 = \gamma \quad (37)$$

$$a_2 = -s \quad (38)$$

The Bowen ratio at the surface (where $\Lambda = H$) can be deduced from (33) and (34) and is given by

$$\beta = \frac{\gamma(\Psi_1 - \Psi_2)}{(s\Psi_1 + \gamma\Psi_2)} \quad (39)$$

It is also a consequence of setting $v = 0$ that Ψ_1 becomes equal to $\gamma(Rn_x - G_x)$ at the surface. The problem of finding the equilibrium evaporation rate now hinges on discovering the behaviour of Ψ_2 for very large x . With $v = 0$,

$$\Psi_2 = -s\Lambda + \gamma LE \quad (40)$$

and, since the lower boundary condition is of the radiation type, at large x

$$\frac{\partial \Psi_2}{\partial z} = -s \frac{\partial \Lambda}{\partial z} + \gamma \frac{\partial LE}{\partial z} = 0 \quad (41)$$

giving Ψ_2 as an undefined constant by integration. Now the air mass receiving the fluxes of vapour and 'heat' (Λ) is of finite extent so that, in the absence of condensation, accumulation of both quantities must occur within the profiles in proportion to the surface fluxes. In addition the profiles of λ and e must also be similar at large x since the transport of the two quantities is similar. Hence (41) implies

$$s\Lambda - \gamma LE = 0 \quad (42)$$

and a Bowen ratio at the surface given by

$$\beta = \frac{\gamma}{s} \quad (43)$$

which, with (7), leads to an expression for the equilibrium rate which is given by (1), in agreement with the results cited in the introduction.

When v is not negligible both Ψ_1 and Ψ_2 approach zero and both Λ and LE approach zero. The Bowen ratio is not then given by (43). In the extreme case of melting snow v becomes infinite and the heat and moisture flow equations are again independent, so that the Bowen ratio will depend entirely on initial conditions for all x . From the equilibrium Bowen ratio the equilibrium value of the surface vapour pressure deficit can also be calculated from (8) and is given by

$$e_x^* - e_x = \frac{s\gamma}{(s + \gamma)} \frac{r_s}{\rho c_p} (Rn_x - G_x) \quad (44)$$

Since the existence of flux divergence is central to the argument on the limiting behaviour of Ψ_2 , equilibrium is reached only in the sense that the Bowen ratio takes on an equilibrium value. The surface temperature and vapour pressure must continue to rise with distance. Inasmuch as the slope of the saturation vapour pressure curve is not a constant but a function of temperature, the equilibrium evaporation rate is more correctly a quasi-equilibrium rate that will change slowly as surface temperature changes slowly with distance.

4. Discussion

The practical significance of (1) must now be discussed. In the derivation, appeal is made to the fact that the airmass is finite. This is of course true, but if the air in a substantial fraction of the troposphere is involved then its heat capacity will be such that equilibrium will not be reached in any distance of practical interest.

The strength of the atmospheric 'feedback' depends on the divergence of the fluxes of heat and moisture which, in turn, depends on the depth of the interacting layer of air. The nature of the diffusivity-height relationship is then the determining factor in the rate of equilibration. In stable air, or when an unstable air layer is capped by an inversion, as is common in anticyclonic weather conditions, the diffusivity becomes quite small at some reasonably small height above the surface and the flux divergence in the lower layers of the atmosphere must be large. In these conditions equilibrium adjustment of the fluxes could be expected to occur in reasonably small distances, especially when the available energy ($Rn_x - G_x$) is large. Under unstable atmospheric conditions mixing in the atmosphere may be such that, for short periods, the atmosphere acts as a sink for heat and vapour. In this case equilibrium in the sense of (43) may not be approached sufficiently rapidly to be of practical importance.

The time dependent term in the diffusion equations has been consistently ignored throughout this analysis. As a result equation

(1) can not be expected to apply to any less than 24 hour averages. It is fortunate that the term $(\frac{S}{S + \gamma})$ is a slowly varying function of temperature since covariance of it and $(Rn_x - G_x)$ will be small and 24-hour average values of each of the parameters can be used in (1).

In view of the contradictory nature of the equilibrium evaporation expression (1) and that developed by Priestley and Taylor (1972) for equilibrium conditions over effectively wet surfaces, some discussion is required. Priestley and Taylor include a factor α in (1) to give

$$LE = \alpha \left(\frac{S}{S + \gamma} \right) (Rn - G) \quad (45)$$

and determine α to be 1.26 on the basis of experimental results from several sources. Two of the experimental results used are from oceanic measurements and another from a large lake. In these cases the effective surface diffusivity would be very large and γ not negligible. Hence they must be differentiated from the remainder of the data. Of the remaining data, one result is not significantly in conflict with (5). This is the result from Gurley for which $\alpha = 1.08$. In the data from the CSIRO lysimeter and the University of Wisconsin lysimeter, on 21 of 36 days and on 23 of 54 days, respectively, the energy used in evaporation exceeded net radiation. It is reasonable to assume that advection also occurred on some other days in each case and the test that evaporation should not exceed net radiation was not sensitive

enough to eliminate these. In the remaining result, obtained by profile methods at Wangara, a value of $\alpha = 1.33$ was found but the accuracy was low due to the smallness of the fluxes.

The equilibrium rate is of fundamental significance in two respects. It represents a modal evaporation rate and gives a general explanation for the well-known fact that the evaporation term is usually the more important of the two dissipation terms in the energy balance of natural surfaces.

Also, as already argued by Priestley and Taylor (1972), for a large enough area, advection effects must be negligible. Hence the equilibrium evaporation rate might, with justification, also be called the regional evaporation rate. For this reason the equilibrium rate should have significance as a climatological parameter for estimation of evaporation in much the same way as Thornthwaite's empirical potential evaporation equation is now commonly used. (Thornthwaite and Hare, 1965). It would have, in cases where the necessary data is available, the clear advantage that it has a sound, physical basis.

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