MODIFICATIONS TO HORN ANTENNAS WHICH REDUCE BEAMWIDTH AND BACK RADIATION

by

AHMAD SAFAAI-JAZI
B.A.Sc., Arya-Mehr University of Technology, 1971

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF APPLIED SCIENCE

in the Department of Electrical Engineering

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

June 1974
In presenting this thesis in partial fulfilment of the requirements for an advanced degree at the University of British Columbia, I agree that the Library shall make it freely available for reference and study. I further agree that permission for extensive copying of this thesis for scholarly purposes may be granted by the Head of my Department or by his representatives. It is understood that copying or publication of this thesis for financial gain shall not be allowed without my written permission.

Department of Electrical Engineering

The University of British Columbia
Vancouver 8, Canada

Date June 24, 1974
ABSTRACT

Modifications to horn antennas through choke flanges and parabolic cylinder reflectors are investigated. The latter are introduced to narrow the beamwidth, increase the gain and reduce the back radiation without increasing the horn length.

The geometrical diffraction theory concepts and the Kirchhoff method are employed in the calculation of E-plane patterns of modified horns. The Kirchhoff method is applied to determine the fields reflected from the chokes and parabolic reflectors in the forward region where the geometrical diffraction theory fails. Very close agreement is obtained between the computed and measured patterns in the front region. In the rear directions the general level of back radiation for choked horns and the approximate lobe structure for horns with reflectors are predicted.

Modification of horns by chokes results in a considerable reduction in the back radiation, but only a slight improvement in the beamwidth and the gain. Modification by parabolic reflectors, on the other hand, will result in a significant reduction in the beamwidth and substantial increase in the on-axis gain if the focal length of the reflectors is optimal. Considerably lower back radiation is also achieved.
# TABLE OF CONTENTS

ABSTRACT .................................................................................................................. ii

TABLE OF CONTENTS ........................................................................................... iii

LIST OF TABLES ....................................................................................................... iv

LIST OF ILLUSTRATIONS ....................................................................................... v

ACKNOWLEDGMENT .............................................................................................. vii

1. INTRODUCTION ................................................................................................. 1

2. RADIATION MECHANISMS AND E-PLANE PATTERNS OF HORN ANTENNAS ..................................................... 6
   2.1 Diffraction by a Wedge ................................................................................... 6
   2.2 Radiation Mechanisms .................................................................................. 7
   2.3 Formulation of Radiation Pattern ................................................................. 9

3. AN ANALYSIS FOR E-PLANE PATTERNS OF CHOKED HORN ANTENNAS ......................................................... 16
   3.1 Introduction .................................................................................................... 16
   3.2 A Nonisotropic Line Source in a Parallel-plate Waveguide ........................... 16
   3.3 Reflection from Choke Interior .................................................................... 18
   3.4 Diffracted Fields ......................................................................................... 22

4. A NEW MODIFICATION TO HORN ANTENNAS THROUGH PARABOLIC CYLINDER FLANGES ............................................ 26
   4.1 Introduction .................................................................................................... 26
   4.2 Reflection from Parabolic Flanges ................................................................ 27
   4.3 Diffracted Fields and Radiation Pattern ..................................................... 32
   4.4 Optimization of Focal Length .................................................................... 33

5. NUMERICAL AND EXPERIMENTAL RESULTS .................................................................................................. 36
   5.1 Comparison of Measured and Computed Patterns ..................................... 39
   5.2 Comparison of Patterns Before and After Modification ......................... 45
   5.3 Errors ............................................................................................................ 53

6. CONCLUSIONS .................................................................................................... 51

REFERENCES ......................................................................................................... 55

APPENDIX ............................................................................................................... 57


**LIST OF TABLES**

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Dimensions of Test Antennas</td>
<td>36</td>
</tr>
<tr>
<td>II</td>
<td>Optimal Focal Lengths for Test Horns</td>
<td>37</td>
</tr>
<tr>
<td>III</td>
<td>Dimensions of Chokes in the E-plane</td>
<td>39</td>
</tr>
<tr>
<td>IV</td>
<td>Measured and Computed Values of Beamwidths, Increases in the On-axis Gain and Back-to-Front Ratios</td>
<td>45</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Coordinates for a Line Source Near a Wedge</td>
<td>7</td>
</tr>
<tr>
<td>2.2</td>
<td>Coordinates for a Horn in the E-Plane</td>
<td>8</td>
</tr>
<tr>
<td>2.3</td>
<td>Radiation Mechanisms</td>
<td>9</td>
</tr>
<tr>
<td>2.4</td>
<td>Geometry of Images in the Lower Wall</td>
<td>12</td>
</tr>
<tr>
<td>3.1</td>
<td>A Line Source in a Parallel-Plate Waveguide</td>
<td>18</td>
</tr>
<tr>
<td>3.2</td>
<td>Geometry of a Choked Horn Antenna in the F-Plane</td>
<td>19</td>
</tr>
<tr>
<td>4.1</td>
<td>Geometry of a Horn Modified by Parabolic Flanges in the E-Plane</td>
<td>27</td>
</tr>
<tr>
<td>5.1</td>
<td>Diagram of a Pyramidal Horn Alone and with Choke and Parabolic Flanges</td>
<td>37</td>
</tr>
<tr>
<td>5.2</td>
<td>Antenna Range and Test Horns</td>
<td>38</td>
</tr>
<tr>
<td>5.3</td>
<td>E-Plane Patterns of Horn A with Chokes</td>
<td>40</td>
</tr>
<tr>
<td>5.4</td>
<td>E-Plane Patterns of Horn B with Chokes</td>
<td>41</td>
</tr>
<tr>
<td>5.5</td>
<td>E-Plane Patterns of Horn A with Parabolic Reflectors</td>
<td>43</td>
</tr>
<tr>
<td>5.6</td>
<td>E-Plane Patterns of Horn B with Parabolic Reflectors</td>
<td>44</td>
</tr>
<tr>
<td>5.7</td>
<td>A Comparison of Measured E-Plane Patterns of Horn A Before and After Modifications</td>
<td>46</td>
</tr>
<tr>
<td>5.8</td>
<td>A Comparison of Computed E-Plane Patterns of Horn A Before and After Modifications</td>
<td>47</td>
</tr>
<tr>
<td>5.9</td>
<td>Patterns of Horn A with Parabolic Reflectors at Several Wavelengths</td>
<td>49</td>
</tr>
<tr>
<td>5.10</td>
<td>Beamwidths of Horn A with Parabolic Reflectors</td>
<td>49</td>
</tr>
<tr>
<td>5.11</td>
<td>A Comparison of H-Plane Patterns of Horn A with parabolic Reflectors</td>
<td>50</td>
</tr>
<tr>
<td>A.1</td>
<td>Coordinates for a Line Source Near a Half Plane</td>
<td>57</td>
</tr>
</tbody>
</table>
A.2  Error in $I(Y)$ for $r < r_0$ ........................................ 60
A.3  Error in $\bar{I}(Y)$ for $r < r_0$ ................................. 61
ACKNOWLEDGMENT

I am thankful to Dr. E. V. Jull, my supervisor, for his interest, advice and guidance during the research work and writing this thesis.

The financial support from the University of British Columbia in the form of UBC graduate fellowship and from the National Research Council of Canada during the months of May and June is acknowledged.
1. INTRODUCTION

Rectangular horn antennas are widely used in microwave communications and radio astronomy and as primary feeds. Considerable attention has been paid to improve the radiation patterns of horns for lower side and back lobes, narrower beamwidth and higher gain. In a pyramidal horn antenna fed by a waveguide supporting the dominant TE_{10} mode the electric field distribution is uniform in the principal E-plane, whereas it is sinusoidally tapered in the H-plane. This, in other words, implies that the aperture edges parallel to the incident magnetic field, designated as E-plane edges, are illuminated much stronger than the edges parallel to the incident electric field, called H-plane edges. Indeed, the illumination of the H-plane edges is usually by rays first diffracted at horn-waveguide junction, while the E-plane edges are excited by the geometrical optics waves. Hence, the diffracted fields by the E-plane edges are stronger than those of the H-plane edges, are the actual source of back lobes of the E-plane pattern, and contribute significantly to the H-plane pattern. The last arises from rays obliquely incident upon the E-plane edges producing diffracted fields which can contribute to the H-plane pattern. Consequently any modification to pyramidal horn antennas which results in an improved radiation pattern and higher gain deals primarily with the E-plane pattern.

In short horn antennas a significant amount of energy is diffracted by horn edges into the lateral and rear directions.
which results in substantial back radiation. The suppression of back lobes have long received considerable attention. LaGrone and Roberts [1] in an experimental investigation showed that quarter wavelength choke flanges on the aperture edges can substantially reduce back radiation. In fact, chokes are utilized to suppress current distribution on the exterior surfaces of the horn antenna. Consequently radiation in the rear directions, which is caused by these current distributions, is reduced. Lawrie and Peters [2] used pyramidal horn antennas in which the conducting E-plane walls are corrugated surfaces. The E-plane and H-plane patterns of a corrugated horn are nearly identical over the main beam if the horn has a square aperture. The radiation pattern may also be with significantly reduced side and back lobes, but the chief advantage of this modification is the resulting identical E and H-plane radiation beams. Furthermore, corrugated horns do not fall into the category of flanged horns. Koshy et al [3] studied the effect of flanges on the radiation patterns of aperture antennas. Their investigation accounts for focusing, broadening, and tilting the radiation beam.

In this thesis a new modification to horn antennas through parabolic cylinder reflectors is introduced. This modification accounts for narrowing the beamwidth, improving the gain and reducing the back radiation. Indeed, the fields diffracted by the E-plane edges are cylindrical waves with rays emanating from the edges; therefore, parabolic reflectors with foci at these edges can be utilized to reflect all rays in the rear directions into the forward direction. Consequently radiated power in the
forward direction increases, resulting in narrower beamwidth and higher gain. Furthermore, back radiation depends entirely on the fields diffracted by the reflector edges, these fields are at least doubly diffracted and weak. Hence, substantially lower back radiation is also anticipated.

The analysis of E-plane patterns of flanged horns can be based upon the Kirchhoff method in only the forward region. In the lateral and rear directions, however, this method fails entirely to predict the radiation pattern, because the aperture field distribution is assumed to be that of the incident fields and zero outside the aperture, and this is not a valid assumption in these regions. The geometrical diffraction theory concepts, on the other hand, have recently been used to successfully compute the radiation patterns of aperture antennas in all regions. It has also been used in other antenna and waveguide problems.

Initially, Keller [4] introduced his 'geometrical theory of diffraction' and later used it to study the diffraction by an aperture [5]. This theory, sometimes abbreviated as GTD, is strictly valid for the diffraction of incident waves of very short wavelengths. In 1963, Ohba [6] employed 'the geometrical method for diffraction' to compute the gain and the radiation pattern of a corner reflector antenna. In 1965, Russo et al [7] used the asymptotic solution for line source diffraction to analyse the E-plane patterns of horns. They refer to this diffraction of line source as 'edge diffraction theory'. Using 'edge diffraction theory', in 1966, Yu et al [8] presented a
comprehensive analysis for E-plane patterns of horns, taking into account multiple diffraction and reflection from horn interior. Yee et al [9] calculated the reflection coefficient of a parallel-plate waveguide using the geometrical diffraction theory. The gain of pyramidal horns and the reflection coefficient of a long E-plane sectoral horn have been recently derived by Jull [10], [11] who made use of the concepts of the geometrical theory of diffraction.

The Keller's geometrical theory of diffraction has singularities on the shadow boundaries and further encounters the difficulty of the calculation of the fields at caustics. The edge diffraction theory which rests on the solution to the far field of a line source near a conducting half plane (wedge), on the other hand, is free of singularities, but the difficulty of the evaluation of the fields at caustics still remains a challenging problem. In analyzing the radiation patterns of parabolic flanged horns we encounter a similar problem. Fortunately the Kirchhoff method is reliable for on-axis calculations. Consequently edge diffraction theory and the Kirchhoff method as an alternative for the calculation of on-axis fields of chokes and parabolic reflectors will be used in our analysis.

Most of the existing modifications to horn antennas have been studied experimentally. In this thesis attempts are made to analyze the E-plane patterns of flanged horns. The problem of principal plane radiation patterns can be reduced to a two-dimensional one if the antenna has separable aperture field
distributions. Rhodes [12] has experimentally demonstrated that the E-plane patterns for a horn with fixed flare angle in that plane and fed by a waveguide supporting the TE$_{10}$ mode, are essentially independent of the flare angle in the H-plane, assuming that the slant length of the horn is constant. Furthermore, the approximate aperture field distributions for a horn are known to be separable; therefore, the problem of the E-plane pattern can be regarded as two-dimensional.

In chapter 2 of this thesis diffraction by a perfectly conducting wedge is reviewed and radiation mechanisms for a horn are discussed. Then the E-plane far field radiation pattern is formulated. In chapter 3 the radiation pattern of a horn modified by choke flanges is studied. The effect of parabolic cylinder reflectors on the radiation pattern of a horn is investigated in chapter 4. The optimization of focal length is also verified. The measured and computed radiation patterns of modified horns are presented in chapter 5.
2. RADIATION MECHANISMS AND E-PLANE PATTERNS OF HORN ANTENNAS.

In this chapter the diffraction of an isotropic line source by a perfectly conducting wedge and the formulation of the E-plane far field radiation pattern of a horn are reviewed. This review forms the basis for the later chapters.

2.1 Diffraction by a Wedge.

As is well known, Sommerfeld [13] was the first to establish an exact solution for the diffraction of a plane electromagnetic wave normally incident upon the edge of a perfectly conducting half plane. Later, Pauli [14] derived an asymptotic formula for the diffraction of a plane wave by a wedge. In 1965, Russo et al [7] used Pauli's formulation in conjunction with the principal of reciprocity to obtain the far zone diffraction of an isotropic line source by a wedge. The diffraction by a half plane is contained therein as a particular case, when the wedge angle tends to zero. There is, however, an exact solution for the diffraction of a line source by a half plane [15], which reduces to a simple closed form for far fields [11]. In Fig. 2.1. the diffracted far field at \( r, \theta \) due to a line source at \( r_0, \theta_0 \) can be written as

\[
\psi^d_x(r, \theta) = \frac{\exp(-jkr)}{(kr)^{1/2}} \left[ v(r_0, \theta + \theta_0, n) \mp v(r_0, \theta - \theta_0, n) \right], \quad (2-1)
\]

\( \mp \) for \( E \) polarization,

where
\[ v(r, \alpha, n) = \exp\left[i(kr_0 \cos \alpha + \frac{\pi}{4})\right] \cdot \frac{(\frac{\pi}{2})^{1/2}}{n(\cos \frac{\pi}{n} - \cos \frac{\alpha}{n})} \]

\[ \cdot F \left((kr_0(1 + \cos \alpha))^{1/2}\right) + R_0 \]

with \( F(r) = \int_0^\infty \exp(-j\mu^2) d\mu \) the complex Fresnel integral. The far field of the line source in isolation is \( \psi_x = \exp(-jkr)/(kr) \) when \( kr \gg 1 \). In (2-1) and (2-2), \( r, \theta \) are polar coordinates with respect to the diffracting edge, \( k = \frac{2\pi}{\lambda} \) is the free-space propagation constant and \( \lambda \) is the wavelength, \( n \) is obtained by putting the wedge angle equal to \( (2-n)\pi \) and \( R_0 \) represents higher order terms negligible for \( kr_0 \) large and \( n \) close to 2.

Fig. 2.1. Coordinates for a line source near a conducting wedge.

### 2.2 Radiation Mechanisms

A magnetic line source at the apex of a corner reflector is an appropriate model for a two-dimensional E-plane horn antenna fed by a parallel-plate waveguide supporting the TEM mode. The reflector is formed by two conducting half planes intersecting
at an angle $2\theta_E$ as shown in Fig. 2.2. From the primary source at $S$, a uniform cylindrical wave is radiated in the region $-\theta_E < \theta < \theta_E$. This wave is called the geometrical optics wave. It excites edges A and B, giving rise to the first order diffraction. The resulting diffracted fields, designated as singly diffracted fields, are cylindrical waves emanating from the horn edges. The geometrical optics and singly diffracted rays are illustrated in Fig. 2.3(a). Each edge may now be considered as an induced line source illuminating the other one.

![Fig. 2.2. Coordinates for a horn antenna in the E-plane](image)

The illumination of edge A by a singly diffracted field from edge B results in second order diffraction. Likewise, the singly diffracted rays from edge A hit edge B producing another doubly diffracted field. The singly diffracted rays from both edges A and B further strike wedges S and W to give four more doubly diffracted fields. Fig. 2.3(b) shows only those of the lower wedges. The second order diffracted waves continue to establish third and higher order diffractions. It may be seen from (2-1) and (2-2) that the higher the order of diffraction, the weaker the intensity of the diffracted field.

Part of the diffracted rays from edges A and B enter the
The reflected rays may be treated by the method of images [8]. The number of images is determined by the flare angle of the horn. Further diffractions take place when some of the reflected rays strike back the horn edges.

When the process of diffraction and reflection is completed, the far field radiation pattern of the horn can be obtained by superimposing the geometrical optics and various diffracted and reflected fields as described in the following section.

2.3 Formulation of Radiation Pattern

In this formulation (2-2) is employed to determine the far diffracted fields. The cylindrical wave propagation factor $\exp(-j kr)/(kr)^{1/2}$ to the far field is omitted, because only the angular dependence is of interest. The symmetry property about the Z axis, Fig. 2.2, is further used to simplify the problem.
by considering only the upper half region \(0 \leq \theta \leq \pi\). All the first subscripts refer to the points at which diffraction occurs, while the second subscripts refer to the points of origin of incident rays. The superscripts indicate the order of diffraction. This problem was originally formulated by Yu et al [8], which is modified here by omitting some unrealistic terms.

Referring to Fig. 2.2, the geometrical optics wave from the primary source at \(S\) is a uniform cylindrical wave, normalized to unity

\[ H_S(\theta) = 1, \quad 0 \leq \theta < \theta_E. \]  

(2-3)

Edges A and B are illuminated by the cylindrical wave from \(S\) with zero incident angle. Excluding the waves diffracted into the horn, the singly diffracted fields from these edges in the far zone can be written as

\[ H_{AS}^{(1)}(\theta) = v(2E, \pi - \theta_E + \theta, n), \quad 0 \leq \theta \leq \pi, \]  

(2-4)

where \(n=2\). By symmetry,

\[ H_{AS}^{(1)}(-\theta) = 0 \leq \theta < \frac{\pi}{2} \]  

\[ H_{BS}^{(1)}(\theta) = \{ H_{AS}^{(1)}(2\pi - \theta), \quad \pi - \theta_E < \theta \leq \pi \} \]  

(2-5)

The diffracted waves reflected by the horn walls may be
viewed as cylindrical waves emanating from the edge images. Fig. 2.4 shows the geometry of the images in the lower wall. Insofar as the radiation pattern in the upper half region is concerned, the image waves from the lower wall should be taken into account. The last image in the upper wall may also contribute to the radiation pattern. The image waves from the lower wall are obtained by substituting $\theta$ in (2-4) by $-2i\theta_R - \theta$, giving

$$H_{L1}^{(1)}(\theta) = H^{(1)}_{AS}(-2i\theta_R - \theta), \quad \frac{\pi}{2} - (i+1)\theta_R < \theta < \frac{\pi}{2} - i\theta_R; \quad i = 1, 2, \ldots, h,$$

(2-6)

$h$ being the largest integer $\leq \frac{\pi}{2\theta_R}$.

Similarly, the image wave from the upper wall corresponding to the last image is

$$H_{U1}^{(1)}(\theta) = H^{(1)}_{AS}(-2h\theta_R + \theta), \quad 0 \leq \theta < (h+1)\theta_R - \frac{\pi}{2}.$$

(2-7)

The subscripts $L$ and $U$ indicate that the image waves are from the lower and the upper walls respectively. When the ratio $\pi/2\theta_R$ is not an integer the valid region for the last image in the lower wall should be modified to

$$0 \leq \theta < \pi - (2h+1)\theta_R.$$

(2-8)

We now examine the second order diffractions. Edge A is illuminated by the singly diffracted field of an intensity of
\[ H^{(1)}_{BS} \left( \frac{\pi}{2} \right) \text{ from edge B. Since the diffracted waves are slowly varying functions in the neighbourhood of any particular angle, it is a reasonable assumption that the illumination of edge A is by a uniform cylindrical wave from B. Under this assumption, the second order diffracted field from edge A is} \]

\[ H^{(2)}(\theta) = H^{(1)}_{BS} \left( \frac{\pi}{2} \right) \left[ v(b, \frac{\pi}{2} + \theta, n) + v(b, \frac{3\pi}{2} - 2\theta_E + \theta, n) \right], \quad 0 \leq \theta \leq \pi \quad (2-9) \]

Edge A is further illuminated by the rays from the images in the lower wall, giving rise to a set of second order diffracted fields given by

\[ H^{(2)}_{AI}(\theta) = H^{(1)}_{Li} \left( \frac{\pi}{2} - i\theta_E \right) \left[ v(l_i, \frac{\pi}{2} - i\theta_E + \theta, n) + v(l_i, \frac{\pi}{2} - (i+2)\theta_E + \theta, n) \right], \quad 0 \leq \theta \leq \pi \quad (2-10) \]

where

\[ l_i = z_{i-1} \cos \theta_E + z_o \cos i\theta_E, \quad i = 1, 2, \ldots, h-1. \]

Summing the second order diffracted fields from edge A gives

---

Fig. 2.4. Geometry of images in the lower wall.
The second order diffracted fields from B and the second order image waves can be obtained from (2-11) by substituting $\theta$ by appropriate arguments like those in (2-5), (2-6) and (2-7).

The waveguide-horn junction forms two pairs of wedges of angles of $\pi + \theta_E$ on the horn exterior and interior respectively. The doubly diffracted fields due to these wedges do not contribute significantly to the far field radiation pattern. Moreover, evaluation of these fields by (2-2) is not valid because the arguments $n = 1 \pm \frac{\theta_E}{\pi}$ to be used in (2-2) are not close enough to 2 and therefore the remainders $\tilde{R}'s$ are unlikely to be negligible. Yu et al [8], however, treated these wedges as if they were of angles of $2\theta_E$ and $2(\pi - \theta_E)$. We believe this is not a valid assumption. It is preferable to omit these fields in the calculation of radiation pattern.

It should be evident now how the higher order diffracted fields can be determined. At each stage the intensity of the illuminating ray is calculated. Then the solution to the diffraction of a line source by a wedge (or half plane) is employed. In doing so, the total far diffracted field from edge A is derived as
\[ H_A(\theta) = H_A^{(1)}(\theta) + \sum_{m=2}^{M} H_A^{(m)}(\theta) + h^{-1} \sum_{i=1}^{H_A} H_A^{(m)}(\theta) , \] (2-12)

where \( M \) indicates the order of the diffracted fields to be taken into account, and

\[ H_A^{(m)}(\theta) = C_{AB}^{(m-1)} [v(b, \frac{\pi}{2} + \theta, n) + v(b, \frac{3\pi}{2} - 2\theta_E + \theta, n)] , \] (2-13a)

\[ H_A^{(m)}(\theta) = C_{AI}^{(m-1)} [v(\xi_1, \frac{\pi}{2} + 1\theta_E + \theta, n) + v(\xi_1, \frac{3\pi}{2} - (i+2)\theta_E + \theta, n)] , \] (2-13b)

with

\[ C_{AB}^{(m)} = H_A^{(m)} (-\frac{\pi}{2}) , \] (2-14a)

\[ C_{AI}^{(m)} = H_A^{(m)} (-\frac{\pi}{2} - 1\theta_E) . \] (2-14b)

It should be noticed that for \( m=1 \), \( H_A(\theta) = H_A^{(1)}(\theta) \).

By symmetry, the total diffracted field from edge B is derived as

\[ H_B(\theta) = \begin{cases} H_A(-\theta) & 0 \leq \theta < \frac{\pi}{2} , \\ H_A(2\pi-\theta) & \pi-\theta_E < \theta \leq \pi . \end{cases} \] (2-15)

Similarly, the total diffracted fields reflected by the lower
wall due to the \( i \) th image is

\[
H_{Li}(\theta) = H_A(-2i\theta_E - \theta), \quad \frac{\pi}{2} - (i+1)\theta_E < \theta < \frac{\pi}{2} - i\theta_E ,
\]  

(2-16)

and that of the last image in the upper wall is

\[
H_{Uh}(\theta) = H_A(-2h\theta_E + \theta), \quad 0 < \theta < (h+1)\theta_E - \frac{\pi}{2} .
\]  

(2-17)

The boundaries of the last image in the lower wall should follow (2-8).

All the fields described above are zero outside the defined regions. The far zone radiation pattern is now obtained by superimposing all the relevant terms and considering a common phase reference, say edge \( A \).

\[
H_{tot}(\theta) = H_S(\theta) \ Y_{AS} + H_A(\theta) + H_B(\theta) \ Y_{AB} + \sum_{i=1}^{h} H_{Ai} \ Y_{Ai} 
+ H_{Uh}(\theta) \ Y_{AB} \ Y_{Bh}
\]  

(2-18)

Where the local phase factors referred to edge \( A \) are

\[
Y_{AS} = \exp[-jk\theta_E \cos(\theta-\theta_E)] ,
\]

\[
Y_{AB} = \exp[-jk b \sin \theta] ,
\]

\[
Y_{Ai} = \exp[-jkli \sin(i\theta_E + \theta)] ,
\]

and

\[
Y_{Bh} = \exp[-jk \theta_h \sin (h\theta_E - \theta)] .
\]  

(2-19)
3. AN ANALYSIS FOR E-PLANE PATTERNS OF CHOKED HORN ANTENNAS

3.1 Introduction

The problem to be studied in this chapter is the E-plane far-field radiation pattern of a pyramidal horn antenna modified by choke flanges. LaGrone and Roberts [1] experimentally studied this problem, but no attempt was made to theoretically investigate the effect of chokes on the radiation pattern. The geometrical method of diffraction is a promising method for the treatment of a choked horn. Difficulty, however, arises when the on-axis fields reflected from the chokes are to be determined. The Kirchhoff theory results, on the other hand, are known to be satisfactory on the beam axis. Consequently both methods are used alternately in an overall analysis of the chokes.

In what follows we first review the field produced by a line source in a parallel-plate waveguide. Then the field reflected from the choke interior is determined. At the end the fields diffracted at the antenna edges and the total field are calculated.

3.2 A Nonisotropic Line Source in a Parallel-Plate Waveguide

Yee et al [9] have studied the behaviour of a line source in a parallel-plate waveguide. They converted the sum of the fields on the multiply reflected rays into modal form. Here we content ourselves with their results for a particular case; when the line source produces a transverse magnetic field. The
far field at $r, \theta$ that the source would produce in free space can be expressed as

$$H_o(r, \theta) = \frac{\exp(-j kr)}{(kr)^{1/2}} u(\theta),$$

(3-1)

where $r, \theta$ are polar coordinates from the source and $k$ is the free-space propagation constant. Let the source be at $y=y_o, z=0$ as shown in Fig. 3.1. Then, according to Yee et al, the field at $y, z$ in the guide is

$$H_x(y, z) = \left(\frac{n}{2}\right)^{1/2} \exp(-j \frac{n}{4}) \sum_{m=-\infty}^{\infty} e_m (K_m d)^{-1} \left[u(\theta_m) \exp(j m \pi y_o/d) + u(-\theta_m) \exp(-j m \pi y_o/d)\right] \exp(-j K_m |z|) \cos(m \pi y/d),$$

(3-2)

where $e_m, K_m$ and $\theta_m$ are constants given by

$$e_0 = 1, \quad e_m = 2; \quad m \neq 0,$$

$$K_m = \left[k^2 - \left(\frac{m \pi}{d}\right)^2\right]^{1/2},$$

$$\sin \theta_m = \frac{m \pi}{kd}, \quad \cos \theta_m = (\sgnz) \left(\frac{K_m}{k}\right).$$

(3-3)

When the line source is put on either conducting wall, $H_x(y, z)$ is obtained by setting $u(-\theta_m)=0$, in (3-2), [9].
Fig. 3.1. Coordinates for a magnetic line source in a parallel-plate waveguide.

3.3 Reflection from Choke Interior

A two-dimensional choke may be viewed as an open-ended parallel-plate waveguide short circuited at one end. The geometry of a choked horn antenna is illustrated in Fig. 3.2. Considering the upper choke, edges A and A' may be regarded as nonisotropic line sources which excite $\text{TM}_{om}$ modes in the choke. The excited modes are reflected back after travelling the choke depth. Noticing that both line sources A and A' are on the conducting walls and using (3-2), the reflected TM waves can be written as

$$
H_x^r(y,z) = \left(\frac{\pi}{2}\right)^{1/2} \exp\left(-j\frac{\pi}{4}\right) \sum_{m=0}^{\infty} \epsilon_m(k_md)^{-1} \left[H_A^C(\pi - \theta_m)ight]
+ H_A^C(-\pi + \theta_m) \exp(-j\pi m) \cdot \exp[-jK_m(z + 2d')]
\cdot \cos \left[\frac{m\pi}{d} (y - \frac{b}{2})\right], \quad z \leq 0.
$$

(3-4)

where $y, z$ denote rectangular coordinates from the centre of the aperture, $H_A^C$ and $H_A^C$, to be determined in the next step, represent the diffracted fields from edges A and A' in which the cylindrical wave propagation factor, i.e., $\exp(-jkr)/(kr)^{1/2}$ is omitted. The superscript c indicates that these fields belong to
a choked horn. From (3-3) $\theta_m$ is obtained as

$$\theta_m = \sin^{-1} \left( \frac{m\pi}{kd} \right), \quad 0 \leq \theta_m \leq \frac{\pi}{2}. \quad (3-5)$$

The modes for which $k < \frac{m\pi}{d}$, or equivalently $K_m$ is imaginary, are nonpropagating modes vanishing rapidly if the frequency is not nearly equal to the cut-off frequency for the first evanescent mode. In addition, the idea of using a choke as a high impedance element is usually applicable to only single mode operation. Thus, we consider only the first term of the series in (3-4) which corresponds to a TEM wave.

The reflected TEM wave has a shadow boundary in the forward direction; therefore, we employ the Kirchhoff method for the calculation of far field in the forward region. Considering both chokes, the aperture field distribution is symmetrical about the Z axis. Using the Kirchhoff solution in a two-dimensional problem [16], the far field of the chokes in isolation is

---

Fig. 3.2. Geometry of a choked horn antenna in the E-plane.
\[ H_1^C(\theta) = \frac{k}{(2\pi)^{1/2}} \exp(-j\frac{\pi}{4}) g(\sin\theta), \]  \hspace{1cm} (3-6) \\

where

\[ g(\sin\theta) = \frac{b}{2} + d \int_{b/2}^{b} H^r_x(y,0) \cdot \cos(k_1 y) \, dy, \quad k_1 = k \sin\theta. \]  \hspace{1cm} (3-7) \\

From (3-4) and (3-5) for \( m=0 \),

\[ H^r_x(y,0) = \left(\frac{\pi}{2}\right)^{1/2} (kd)^{-1} [H_A^C(\pi) + H_A^C(-\pi)] \exp[-j(\frac{\pi}{4} + 2kd')] \]  \hspace{1cm} (3-8) \\

Upon using (3-7) and (3-8) in (3-6) we obtain

\[ H_1^C(\theta) = (kd \sin\theta)^{-1} [H_A^C(\pi) + H_A^C(-\pi)] \cdot \left\{ \sin[k(b/2 + d)\sin\theta] \right. \\
\left. - \sin(\frac{kb}{2} \sin\theta) \right\} \exp(-j2kd'). \]  \hspace{1cm} (3-9) \\

At \( \theta = 0 \), \[ H_1^C(0) = [H_A^C(\pi) + H_A^C(-\pi)] \exp(-j2kd') \]  \hspace{1cm} (3-10) \\

For large angles \( \theta \), the far field of the chokes can be well approximated by the singly diffracted fields due to the reflected TEM wave. Invoking the diffraction of a plane wave by a perfectly conducting wedge \([14]\) and using (3-8), these singly diffracted fields are derived as
\[
H_{1A}^C(\theta) = \frac{H_x(y, 0) \exp(-j\frac{\pi}{4}) \sin \frac{\pi}{n}}{(2\pi)^{1/2} n(\cos \frac{\pi}{n} - \cos \frac{\pi - \theta}{n})},
\] (3-11a)

\[
H_{1A'}^C(\theta) = \frac{H_x(y, 0) \exp(-j\frac{\pi}{4})}{(8\pi)^{1/2} \sin \frac{\theta}{2}},
\] (3-11b)

where \( n = 2 - \frac{6\theta}{\pi} \) and \( \theta_E \) is the angle of wedge A. By symmetry, the corresponding fields for the lower choke are

\[
H_{1B}^C(\theta) = H_{1A}^C(-\theta), \quad 0 \leq \theta < \frac{\pi}{2},
\] (3-12a)

\[
H_{1B'}^C(\theta) = H_{1A'}^C(-\theta), \quad 0 \leq \theta < \frac{\pi}{2}.
\] (3-12b)

Taking edge A as a common phase reference, the total far field scattered by the chokes is

\[
H^C(\theta) = \begin{cases} 
H^C_{1A}(\theta) \cdot Y_{A0}, & 0 \leq \theta \leq \theta_1, \\
H^C_{1A}(\theta) + H^C_{1A'}(\theta) Y_{AA'}, & 0 \leq \theta < \theta_2, \\
H^C_{1B}(\theta) Y_{AB} + H^C_{1B'}(\theta) Y_{AB'}, & \theta_2 \leq \theta < \frac{\pi}{2}, \\
H^C_{1A'}(\theta) \cdot Y_{AA'}, & \frac{\pi}{2} \leq \theta \leq \pi.
\end{cases}
\] (3-13)

where
\[ Y_{A0} = \exp[-j \frac{k}{2} b \sin \theta] , \]
\[ Y_{AA'} = \exp[-j kd \sin \theta] , \]
\[ Y_{AB} = \exp[-j k (b+d) \sin \theta] . \]

(3-14)

are the phase factors referred to \( A \) and \( Y_{AB} \) is given by (2-19).

Two patterns are generated, one in the region \( 0 \leq \theta \leq \theta_1 , \theta_1 \) being a small angle, from the Kirchhoff result. A second pattern is generated in the region \( \theta_2 < \theta \leq \pi , \theta_2 < \theta_1 \), from the geometrical diffraction theory result. The two patterns then overlap in the region \( \theta_2 < \theta < \theta_1 \) giving a complete pattern in the region \( 0 \leq \theta \leq \pi \).

3.4 Diffracted Fields

In chapter 2 we calculated \( H_A(\theta) \) as being the total field diffracted by the upper edge of an unmodified horn. \( H_A \) is implicitly a function of \( n \) through \( v(r_0, a, n) \). This can be seen from (2-4), (2-12) and (2-13). Before modification \( n=2 \), while after modification \( n=2 - \frac{\theta_E}{\pi} \), where \( \theta_E \) is one half of the flare angle. For convenience we define \( \tilde{H}_A(\theta) \) to be obtained from \( H_A(\theta) \) by using \( n = 2 - \frac{\theta_E}{\pi} \) in it.

Considering the upper choke, a ray from edge \( A \) in the direction \( \frac{\pi}{2} \) hits edge \( A' \). The field intensity on this ray is \( \tilde{H}_A(\frac{\pi}{2}) \). By using (2-1) and (2-2) the field diffracted at \( A' \)
can be written as

\[ H_{A'}^{C}(\theta) = \bar{H}_{A}(\frac{\pi}{2}) \left[ v(d, \frac{\pi}{2} + \theta, 2) + v(d, \frac{3\pi}{2} + \theta, 2) \right], \quad 0 \leq \theta \leq \pi. \quad (3-15) \]

The diffracted rays from edge A' hit back edge A furnishing a second order interaction. To calculate the resulting interaction field we should notice that edge A' is on the shadow boundary of the rays from A. It has been shown that [9] the diffracted field on a shadow boundary is the asymptotic approximation of one-half the geometrical optics field. Furthermore, this field appears to emanate from the source or its image and not from the diffracting edge. The second order interaction field is now accordingly obtained.

\[ H_{A'A}^{C}(\theta) = C_{1} \left[ v(d, \frac{\pi}{2} - \theta, n) + v(d, \frac{3\pi}{2} - \theta, n) \right] + C_{2} \left[ v(2d, \frac{\pi}{2} - \theta, n) + v(2d, \frac{3\pi}{2} - \theta, n) \right], \quad 0 \leq \theta < \frac{\pi}{2}, \quad (3-16) \]

With

\[ C_{1} = \bar{H}_{A}(\frac{\pi}{2}) \cdot v(d, 0, 2) \]
\[ C_{2} = \frac{1}{2} \bar{H}_{A}(\frac{\pi}{2}) \]

The triply and higher order interaction fields are neglected. Inclusion of these fields does not generally yield a better approximation. Bowman [17], who compared the exact solution for scattering by an open-ended parallel-plate
waveguide with the corresponding result derived by means of the geometrical diffraction theory method, found discrepancies in the contributions from the triply and higher order interactions.

The total diffracted field from edge A is now obtained.

\[ H_A^C(0) = H_A(0) + H_{AA}(0) , \quad 0 \leq \theta < \frac{\pi}{2} . \]  

(3-18)

By symmetry, the fields diffracted by wedge B and edge B' are

\[ H_B^C(0) = H_A^C(-\theta) , \quad 0 \leq \theta < \frac{\pi}{2} , \]  

(3-19)

\[ H_B'(0) = H_A^C(-\theta) , \quad 0 \leq \theta < \frac{\pi}{2} . \]  

(3-20)

It should be noticed that the boundaries in (3-15) to (3-20) exclude the fields diffracted into the horn and the chokes. The geometrical optics field is equal to that for the horn without chokes; i.e,

\[ H_S^C(0) = 1 , \quad 0 \leq \theta < \theta_E . \]  

(3-21)

The diffracted fields reflected from horn interior are obtained from (2-16) and (2-17) in which \( H_A(0) \) is substituted by \( \tilde{H}_A(0) \); i.e,

\[ H_{Li}^C(0) = \tilde{H}_A(-2i\theta_E - \theta) , \]  

(3-22a)

\[ H_{Uh}^C(0) = \tilde{H}_A(-2\theta_E + \theta) . \]  

(3-22b)
The boundaries in (3-22) are the same as those in (2-16) and (2-17).

Considering the upper half region $0 \leq \theta \leq \pi$, the total far field is obtained by superimposing all the relevant terms described above with the local phase factors referred to A.

$$H^C_{tot}(\theta) = H^C_S(\theta) \cdot Y_{AS} + H^C_A(\theta) + H^C_B(\theta) Y_{AB} + H^C_A(\theta) \cdot Y_{AA'}$$

$$+ H^C_B(\theta) Y_{AB'} + \sum_{i=1}^{h} H^C_{Li}(\theta) Y_{Ai} + H^C_U(\theta) Y_{AB} \cdot Y_{Bh}$$

$$+ H^C(\theta).$$

(3-23)

Where $Y_{AS}$, $Y_{AB}$, $Y_{Ai}$ and $Y_{Bh}$ are given by (2-19) and $Y_{AA'}$ and $Y_{AB'}$ by (3-14).
4. A NEW MODIFICATION TO HORN ANTENNAS THROUGH PARABOLIC CYLINDER FLANGES

4.1 Introduction

Studies of horn antennas have revealed that the principal source of back radiation is the fields caused by edge diffraction in the E-plane. Moreover, in the E-plane these fields have cylindrical wave structure with rays originating from the edges. Therefore cylindrical parabolic flanges may be used to reflect all diffracted rays in the rear directions into the forward direction. In fact, a cylindrical parabolic reflector has the property of converting a cylindrical wave radiated by a line source at the focus into a plane wave at the aperture. Consequently radiated power in the forward region increases resulting in a narrower beamwidth. In addition, the illumination of a flange edge is by rays first diffracted at the horn edge; therefore, lower back radiation is also expected.

In the analysis of the E-plane far field radiation pattern of a horn modified by parabolic cylinder flanges we employ the Kirchhoff method for the calculation of far reflected fields from the flanges, and the geometrical method of diffraction for the calculation of the fields diffracted at the antenna edges. In the following steps, reflection from parabolic flanges is first studied. Then by using some of the equations derived in the previous chapters the total far field is determined. Optimization of the focal length is verified at the end.
4.2 Reflection from Parabolic Flanges

The geometry of a horn antenna modified by parabolic cylinder flanges is shown in Fig. 4.1. Considering the upper flange, edge A may be regarded as a primary feed at the focus producing a nonisotropic cylindrical wave to illuminate the aperture. For small angles $\theta$, the far reflected field is obtained by employing the Kirchhoff method; i.e. determining the Fourier transform of the aperture field distribution. Before reflection the field on a ray may be written as

$$H_x^1 (\rho, \phi) = \frac{\exp(-jk\rho)}{(k\rho)^{1/2}} H_A^p (\phi)$$

(4-1)

where $H_A^p (\phi)$, to be determined in Sec. 4.3, represents the radiation pattern of line source A. The superscript in (4-1) and elsewhere means that the field is for a horn modified by parabolic flanges. After reflection the rays are collimated and no more spreading takes place out to the aperture plane. The
reflection coefficient is +1, for the line source A produces a transverse magnetic field and the reflector is assumed to be perfectly conducting. The distance from the focus to a point at the aperture plane accounts for the total phase change along the ray path ARY in Fig. 4.1. When the focus is in the aperture plane this distance is equal to 2f, with f the focal length of the reflector. The field on the reflected ray at the aperture plane can now be written.

\[
\hat{H}_x^R(\rho,\phi) = \frac{\exp(-12\pi f)}{(k\rho)^{1/2}} H_A^P(\phi) \quad (4-2)
\]

Considering both flanges, the aperture field distribution is symmetrical about the Z axis in Fig. 4.1. Using the Kirchhoff solution in a two-dimensional problem with electric field having tangential components [16], the far reflected field from the flanges is determined as

\[
H_1^P(\theta) = \frac{k \exp(-\pi/4)}{(2\pi)^{1/2}} q(\sin\theta) \quad (4-3)
\]

where

\[
q(\sin\theta) = 2\int_0^{b/2} \hat{H}_x^R(\rho,\phi) \cos(k_1 y) \, dy, \quad k_1 = k \sin\theta \quad (4-4)
\]

The cylindrical wave propagation factor \( \exp(-j kr)/(kr)^{1/2} \) has been omitted in (4-3). Also, \( \rho,\phi \) are polar coordinates which are used in the calculation of the aperture field distribution, whereas \( r, \theta \) denote polar coordinates at the far field.

In Fig. 4.1. The equation of the upper parabola may be
described as

\[ \rho = \frac{2f}{1-\cos \phi} \quad (4-5) \]

We further have

\[ y = \rho \sin \phi + \frac{b}{2} \quad (4-6) \]

From (4-5) and (4-6) we obtain

\[ \rho = \frac{f}{\sin \frac{\phi}{2}} \quad (4-7) \]

\[ y = 2f \cot \frac{\phi}{2} + \frac{b}{2} \quad (4-8) \]

Using (4-7) in (4-2) yields

\[ \hat{H}_x^r (\rho, \phi) = \frac{\exp(-j2kf)}{(k\rho)^{1/2}} \sin \frac{\phi}{2} H_A^p (\phi) \quad (4-9) \]

From (4-2), (4-3), (4-8) and (4-9) we obtain

\[ H_1^p (\theta) = 2 \left( \frac{f}{\lambda} \right)^{1/2} \exp[-j(2kf - \frac{\pi}{4})] \int_{-\pi/2}^{\pi/2} H_A^p (\phi) \cos[k(2f \cdot \cot \frac{\phi}{2} + \frac{b}{2}) \sin \theta] \, d\phi / \sin \frac{\phi}{2} \quad (4-10) \]

The integral in (4-10) is numerically evaluated.

For large angles \( \theta \), the far field radiated from the flanges is approximated by the interaction fields between the horn edges and the reflector edges. In order to calculate these
fields we further approximate each reflector by a half-plane tangent to it at the edge. Considering the upper reflector, a ray from edge A in the direction \( \frac{\pi}{2} \) hits edge A'. The intensity of illumination is \( H_A^P \left( \frac{\pi}{2} \right) \). Using (2-1) and (2-2) and noticing that the angle of incidence is \( \frac{\pi}{4} \), the field diffracted at A' is determined as

\[
H_A^P (\theta) = H_A^P \left( \frac{\pi}{2} \right) \left[ \nu(2f, \frac{\pi}{2} + \theta, 2) + \nu(2f, \pi + \theta, 2) \right].
\]  

(4-11)

The diffracted rays from A' subsequently hit edge A producing a second order interaction field given by

\[
H_{AA'}^P (\theta) = H_{A'}^P \left( \frac{\pi}{2} \right) \left[ \nu(2f, \frac{\pi}{2} - \theta, n) + \nu(2f, \frac{3\pi}{2} - \theta, n) \right],
\]  

(4-12)

\[ n = 2 - \frac{\theta \pi}{\pi}. \]

The triply and higher order interaction fields are neglected. By symmetry, the corresponding fields diffracted by the lower edges are

\[
H_A^P (-\theta), \quad 0 \leq \theta < \frac{\pi}{2},
\]

\[
H_B^P (\theta) =
\]

\[
H_A^P (2\pi - \theta), \quad \frac{3\pi}{4} < \theta \leq \pi.
\]  

(4-13)
\[
H_{BB}^P(\theta) = H_{AA'}^P(-\theta) . \tag{4-14}
\]

The total field contributed by the parabolic reflectors to the radiation pattern can now be obtained by considering a common phase reference, say edge A.

\[
H^P(\theta) = \begin{cases} 
H_1^P(\theta)Y_{AO} & , 0 \leq \theta \leq \theta_1 , \\
H_A^P(\theta)Y_{AA'} + H_B^P(\theta)Y_{AB'} + Y_{AB} & , \theta_2 \leq \theta < \pi/2 , \\
H_A^P(\theta)Y_{AA'} & , \pi/2 \leq \theta \leq 3\pi/4 , \\
H_A^P(\theta)Y_{AA'} + H_B^P(\theta)Y_{AB'} & , 3\pi/4 \leq \theta \leq \pi . \end{cases} \tag{4-15}
\]

\(Y_{AO} \), \(Y_{AA'}\), and \(Y_{AB'}\) are obtained from (3-14) in which \(d\) is replaced by \(2f\).

The far field radiation pattern is obtained by first including the Kirchhoff result in the region \(0 \leq \theta \leq \theta_1\) and then the geometrical diffraction theory result in the region \(\theta_2 \leq \theta \leq \pi\), \(\theta_2 < \theta_1\). The two corresponding patterns overlap in the region \(\theta_2 < \theta < \theta_1\) resulting in a complete pattern.
4.3 Diffracted Fields and Radiation Pattern

Excluding the fields diffracted into the horn and into the reflectors, the diffracted fields from edges A and B are written as

\[ H_{A}^{P}(\theta) = \bar{H}_{A}(\theta) , \quad 0 \leq \theta < \frac{\pi}{2} , \]  

\[ H_{B}^{P}(\theta) = \bar{H}_{A}(-\theta) , \quad 0 \leq \theta < \frac{\pi}{2} , \] 

where \( \bar{H}_{A} \) was determined in Sec. 3.4. The geometrical optics field \( H_{S}^{P} \) and the diffracted fields reflected from horn interior \( H_{Li}^{P} \) and \( H_{Uh}^{P} \) are equal to those for a choked horn given by (3-21) and (3-22). 

The total far field in the upper half region \( 0 \leq \theta \leq \pi \) can now be obtained.

\[ H_{\text{tot}}^{P}(\theta) = H_{S}^{P}(\theta)Y_{AS} + H_{A}^{P}(\theta) + H_{B}^{P}(\theta)Y_{AB} + \sum_{i=1}^{h} H_{Li}^{P}(\theta)Y_{Ai} \]

\[ + H_{Uh}^{P}(\theta) \cdot Y_{AB} \cdot Y_{Bh} + H^{P}(\theta) . \]  

Where \( Y_{AS} \), \( Y_{AB} \), \( Y_{Ai} \), and \( Y_{Bh} \) are the local phase factors given by (2-19).
4.4 Optimization of Focal Length

Parabolic reflectors with different focal lengths may be utilized for modification of a given horn antenna. All reflectors are confocal, but only some of them appear to significantly increase the on-axis power. These have focal lengths such that maximum on-axis power is achieved. Accordingly the on-axis far field of a horn in isolation and that of its parabolic flanges should be in phase.

The on-axis far field of the horn is well approximated by the sum of the geometrical optics field and the singly diffracted fields. The on-axis geometrical optics field referred to edge A is

\[ H^G = \exp(-j k_\lambda \cos \theta_E). \]  

(4-19)

From (2-4) and (2-5) the total on-axis singly diffracted field is

\[ H^{\text{sing}} = 2v(\xi_E, \pi - \theta_E, n) , n = 2 - \frac{\theta_E}{\pi}. \]  

(4-20)

The on-axis field reflected from the flanges is obtained by setting \( \theta = 0 \) in (4-10), yielding

\[ H^P = 2(\xi / \lambda)^{1/2} \exp[-j(2k_f - \pi/4)] \int_{\pi/2}^\pi \frac{H_A^P(\phi)}{\sin \frac{\phi}{2}} d\phi \]  

(4-21)

We must have
\[ |H_g + H_{\text{sing}}| = |H_P| + 2N\pi, \quad N = 0, 1, \ldots \quad (4-22) \]

In order to evaluate \( |H_P| \), we may approximate \( H_A^P \) by the singly diffracted field from edge \( A \) as follows:

\[ H_A^P(\phi) = \nu(\ell_E, \pi - \theta_E + \phi, n). \quad (4-23) \]

Using the asymptotic form of Fresnel integral, i.e.,

\[ \int_0^\infty \exp(-j\mu^2) d\mu \approx \exp(-j\tau^2)/(2j\tau), \quad \tau \gg 1, \]

and applying (2-2), \( H_A^P \) reduces to

\[ H_A^P(\phi) = \exp[-j(kE_{\ell_E} + \pi/4)] Q(\phi) \quad (4-24) \]

where

\[ Q(\phi) = \frac{\sin \frac{\pi}{n}}{\left(2\pi k_{\text{E}}E_{\ell_E}\right)^{1/2} n (\cos \frac{\pi}{n} - \cos \frac{\pi - \theta_E + \phi}{n})}. \]

Using (4-24) in (4-21), we obtain

\[ H^P = 2\left(\frac{f}{\lambda}\right)^{1/2} \exp[-j(2k_f + k_{\ell_E})] \int_0^\pi Q(\phi) \frac{d\phi}{\sin \frac{\phi}{2}}. \quad (4-25) \]

In \( \frac{\pi}{2} \leq \phi \leq \pi \), \( Q(\phi)/\sin \frac{\phi}{2} \) is always positive, therefore the integral in (4-25) is positive. Hence, from (4-22) the optimal focal length is
\[ f = N \frac{\lambda}{2} - \left( \frac{x}{2} + \frac{\beta}{\pi} \cdot \frac{\lambda}{4} \right) , \quad (4-26) \]

where \( \beta = \sqrt{H^6 + H_s^{\text{sing}}} \) is numerically evaluated, and \( N \) is chosen for convenience, with the limitation that small reflectors will act less effectively. The asymptotic form of Fresnel integral is not recommended in the calculation of \( \beta \), because shadow boundaries lie too close to the on-axis direction.
5. NUMERICAL AND EXPERIMENTAL RESULTS

Modifications to horn antennas through choke flanges and parabolic cylinder reflectors were theoretically studied in chapters 3 and 4. In this chapter numerical and experimental results are compared and the effects of these modifications on the beamwidth, gain and back radiation of a horn are verified.

To compute the radiation patterns given by (2-18), (3-23) and (4-18), these equations were programmed. Computed patterns include diffracted fields of orders as high as three. The Kirchhoff method in the $|\theta| < 25^\circ$ region and the geometrical method of diffraction in the $|\theta| > 15^\circ$ region were used in the computation of modified patterns.

Two rectangular pyramidal horn antennas, horn A and horn B, were constructed for the experimental studies. The dimensions of these horns are given in Table I. The horns were fed by a waveguide with interior dimensions 1.016 and 2.286 cm. The diagram of the antennas are shown in Fig. 5.1. The operating frequency was 9.0 GHz for horn A and 8.5 GHz for horn B. To

<table>
<thead>
<tr>
<th>Horn</th>
<th>$l_E$(cm)</th>
<th>$l_H$(cm)</th>
<th>$a$(cm)</th>
<th>$b$(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9.6</td>
<td>11.1</td>
<td>9.0</td>
<td>7.8</td>
</tr>
<tr>
<td>B</td>
<td>8.9</td>
<td>10.5</td>
<td>7.6</td>
<td>6.0</td>
</tr>
</tbody>
</table>
Fig. 5.1. (a) Pyramidal horn dimensions (b) Choked horn dimensions in the E-plane (c) Parabolic flanged horn dimensions in the E-plane

design the parabolic reflectors, we first calculated a series of optimal focal lengths at the operating frequencies using (4-26). These are given in Table II. The reflectors then were

<table>
<thead>
<tr>
<th>Horn</th>
<th>f(cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.84</td>
</tr>
<tr>
<td>B</td>
<td>1.36</td>
</tr>
</tbody>
</table>

Constructed with a focal length of 6.10 cm for horn A and 4.90 cm for horn B. The chokes were designed to have widths less than 1/2 wavelength and depths equal to \( \frac{1}{4} + \frac{N}{2} \lambda \); N being an integer. The dimensions of the chokes in the E-plane are given in Table III. The choke depths correspond to \( N=1 \), \( \lambda_A = 3.33 \text{ cm} \) and \( \lambda_B = 3.53 \text{ cm} \). The parabolic reflectors and the chokes have the same widths as the horns in the H-plane.

The far field radiation patterns were measured using the horns as receiving antennas. The antenna range and the test horns are shown in Fig. 5.2.
Fig. 5.2. (a) antenna range (b), horn with chokes, (c) horn with parabolic cylinder flanges.
In what follows first computed and measured patterns are compared. Then a comparison of patterns with and without the modifications is presented. An error investigation for the computed patterns is given at the end.

5.1 Comparison of Measured and Computed Patterns

a. Horn with chokes

A comparison of computed and measured patterns of horn A with chokes is shown in Fig. 5.3. There is close agreement over most of the angular range in the front region, but discontinuities at the transition boundaries $\theta = \pm 90^\circ$. Moreover, the fine structure of the pattern in the rear directions is not represented, but the general level of back radiation is predicted. The back-to-front ratio as an index of the back radiation is obtained with sufficient accuracy. Discontinuities in the computed pattern may be eliminated by including triply and higher order interaction fields between the choke edges. Inclusion of these fields, as discussed in sec. 3.4, does not generally improve the radiation pattern. A better approximation might be obtained by including the fields diffracted at the exterior corners of the chokes. To a first approximation these fields are of small intensity and neglected. A similar comparison of computed and measured patterns for horn B with

<table>
<thead>
<tr>
<th>Table III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horn</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
</tbody>
</table>
Fig. 5.3. E-plane patterns of horn A with chokes at $\lambda = 3.33$ cm.
--- measured, ------ computed.
Fig. 5-4. E-plane patterns of horn B with chokes at λ=3.53 cm.

---

I
choke is shown in Fig. 5.4. The same conclusions drawn for horn A remain true, but there is generally more error in the predicted pattern of this horn compared with that for horn A, because it has smaller dimensions.

b. Horn with parabolic reflectors

A comparison of computed and measured patterns of horn A with parabolic reflectors. Fig. 5.5, shows a good agreement over most of the range. The computed pattern, however, deviates from the measured one in the region $|\theta| > 120^\circ$, largely because the parabolic reflectors were approximated by half planes in the calculation of back radiation. Also, undesired reflections produced by the mount system partly account for this deviation. Nevertheless, the approximate lobe structure in the rear directions and particularly the back-to-front ratio can be predicted with reasonable accuracy. There are also small discontinuities at $\theta = \pm 90^\circ$ because triply and higher-order interaction fields between the reflector edges were neglected. A similar comparison of computed and measured patterns of horn B with reflectors is shown in Fig. 5.6. The asymmetry in the measured pattern of this horn is mainly due to the asymmetry in the construction of the horn.
Fig. 5.5. E-plane patterns of horn A with parabolic reflectors at \( \lambda = 3.33 \) cm. --- measured, ---- computed
Fig. 5.6. E-plane patterns of horn B with parabolic reflectors. At \( \theta = 3.53 \) cm, measured --- computed.
5.2 Comparison of Patterns Before and After Modifications

The measured patterns of horn A alone, with chokes and with parabolic reflectors are compared in Fig. 5.7. Fig. 5.8. compares the corresponding computed patterns. Measured and computed values of beamwidths, increases in the on-axis gain and back-to-front ratios are given in Table IV. It is evident that modification by chokes results in a substantial reduction in the back radiation, but improvement in the beamwidth and the on-axis gain is insignificant. LaGrone and Roberts [1] reached the same conclusions in their experimental investigation of choked horns, but they achieved a greater reduction in the back radiation. The amount of this reduction depends mainly on the choke width. This dimension was not given. Therefore, no explanation can be given for this difference.

Table IV

<table>
<thead>
<tr>
<th>Measured</th>
<th>Beamwidth (Degrees)</th>
<th>Back-to-front ratio (dB)</th>
<th>On-axis gain increase (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horn A</td>
<td>23.0</td>
<td>-23.2</td>
<td>-</td>
</tr>
<tr>
<td>Horn A with chokes</td>
<td>21.0</td>
<td>-29.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Horn A with reflectors</td>
<td>9.5</td>
<td>-35.6</td>
<td>1.9</td>
</tr>
<tr>
<td>Horn B</td>
<td>30.0</td>
<td>-24.7</td>
<td>-</td>
</tr>
<tr>
<td>Horn B with chokes</td>
<td>27.0</td>
<td>-29.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Horn B with reflectors</td>
<td>12.1</td>
<td>-34.9</td>
<td>2.1</td>
</tr>
</tbody>
</table>

1 In reply to a letter requesting the size of the choke, they wrote: "This dimension is not critical. It depends entirely on the amount of power being radiated. This power usually is much too small to cause a voltage arc over."
Fig. 5.7. A comparison of measured E-plane patterns of horn A before and after modifications at $\lambda = 3.33$ cm.

- horn alone,
- horn with chokes,
- horn with parabolic reflectors
Fig. 5.8. A comparison of computed E-plane patterns of horn A before and after modifications at $\lambda = 3.33$ cm.

- horn alone,
- horn with chokes,
- horn with parabolic reflectors.
A comparison of the pattern of the horn with and without parabolic reflectors shows a significant reduction in the beamwidth and back radiation. A considerable increase in the on-axis gain was also achieved. Improvement in the beamwidth, however, depends on the size of the reflectors and the frequency of operation. It is evident from (4-10) that the larger the focal length, the greater the intensity of the reflected fields, but maximum increase in the on-axis gain is achieved only at frequencies for which the focal length is optimal. Fig. 5.9. Compares the radiation beams at several frequencies for horn A with reflectors. The narrowest beam is obtained at 9.0 GHz. At 10.0 GHz the main beam is split into two lobes. This splitting is because the on-axis field from the reflectors is nearly 180° out of phase with the on-axis field of the horn alone. The variation of beamwidth with frequency is illustrated in Fig. 5.10.

The reduction of beamwidth may be viewed as a shortening effect of the horn length. In other words, the length of a horn modified by parabolic appendages is comparable with that of a longer unmodified horn of equal beamwidth. For instance, the standard gain horn with a slant length of 32.0 cm and a flare angle of 13.02° has a beamwidth of 12.5° at 9.0 GHz, while the beamwidth of horn A with reflectors is 11°.

Next we examine the effect of parabolic appendages on the H-plane pattern. A comparison of measured H-plane patterns of horn A before and after modification is shown in Fig. 5.11. There is a slight reduction of beamwidth and sidelobe levels, but
Fig. 5.9. Patterns of horn A with parabolic reflectors

Fig. 5.10. Beamwidth of horn A with parabolic reflectors.
Fig. 5.11. A comparison of H-plane patterns of horn A with parabolic reflectors.
considerable reduction in the back-to-front ratio. This shows that the \(E\)-plane fields of a horn affect the \(H\)-plane pattern especially in the rear directions.

### 5.3 Errors

The solution for the far zone diffraction of a magnetic line source by a perfectly conducting half plane (wedge) described in sec. 2.1 has been widely used as a canonical solution in the analysis of \(E\)-plane patterns of horn antennas. This solution given by (2-2) was used in the calculation of radiation patterns of horns modified by chokes or parabolic cylinder reflectors. (2-2) is exact when the diffracting object is a half plane. Therefore, singly diffracted fields in a corner reflector as a model for pyramidal horns can be calculated exactly. In modified horns, however, the approximate solution for diffraction by a wedge is required and there is some error in the singly diffracted fields. The amount of the error depends on the wedge angle, the distance between the source and the diffracting edge and the angle of observation. Using (2-2) in the calculation of interaction fields results in further errors due to:

1. Diffracted fields in a two-dimensional problem are nonisotropic cylindrical waves, whereas they are assumed isotropic in the calculation of the interaction fields. This assumption then results in errors in these fields. It is evident that the higher the order of diffraction the more nonuniformity in the diffracted fields and consequently the greater the resulting error. However, this error is not very large in
angular regions not close to the shadow boundaries.

2. Diffraction coefficients used in the calculation of interaction fields are strictly valid for the far field, while the ray path between the two interacting edges is usually only a few wavelengths. An error investigation in Appendix reveals that if the aperture width to slant length ratio in a horn is only slightly less than unity, the interaction fields can be calculated by (2-2) with sufficient accuracy. In general, the larger the electrical dimensions of a horn, the more accurate the predicted pattern.

In addition to these errors, the waveguide feed and the mount system may have a noticeable effect on the radiation pattern especially in the rear directions. The difference between the predicted and measured back-to-front ratios of horn A, Table IV, is largely due to the mount system and the waveguide feed.
6. CONCLUSIONS

The geometrical method of diffraction along with the Kirchhoff method was employed in analyzing the E-plane far field radiation patterns of pyramidal horns modified by choke flanges and by parabolic cylinder reflectors. Comparison of computed and measured patterns of the choked horns showed good agreement in the overall lobe structure in the front region. In the rear directions the general lobe level was predicted. It was observed that modification by chokes resulted in a substantial reduction in the back radiation, but only a slight improvement in the beamwidth and the gain.

The modification of horns through parabolic cylinder reflectors was introduced in order to narrow the beamwidth, improve the gain and reduce the back radiation. Computed and measured patterns of the horns modified by parabolic reflectors were in a very close agreement over most of the angular range. As was observed this modification can result in a significant reduction in the beamwidth and a considerable increase in the on-axis gain. Substantially lower back radiation was also achieved. The chief advantage of this modification in comparison with the modification by chokes is the resulting narrower beamwidth.

Consequently a short horn with substantial directivity has been achieved. The antenna is light and simple to construct. It has a narrower beamwidth than ordinary horns. This compact horn may find useful application in many situations where
conventional reflectors cannot be easily used and where conventional horns are inconveniently long.
REFERENCES


14. W. Pauli, "On asymptotic series for functions in the


AN ERROR INVESTIGATION OF APPROXIMATE SOLUTIONS FOR DIFFRACTION
OF A LINE SOURCE BY A HALF-PLANE

The exact solution for the diffraction of a line source near a perfectly conducting half-plane, shown in Fig. A.1., is given in [16,ch.11] as outlined below

\[ \psi_x^d = I(\theta + \theta_o) \pm I(\theta - \theta_o), \pm \text{ for } E \text{ polarization,} \]

with

\[ I(\gamma) = \pm \left( \frac{2}{\pi} \right)^{1/2} \exp[j\left(\frac{\pi}{4} - kR\right)] \int_0^\infty \frac{\exp(-\mu^2)}{[k(R_1 - R)]^{1/2} (\mu^2 + 2kR)^{1/2}} \, d\mu \]

\[ \pm \text{ for } \cos \xi > 0, \quad (A-1) \]

and \( R_1 = r + r_o \).

\( \psi_x^d \) denotes the diffracted field at point P and k is the free-space propagation constant. In the subsequent analysis we

![Fig. A.1 Coordinates for line source T near a half-plane](image_url)
consider only \( I(\gamma) \) for \( 0 < \gamma < \pi \)

Although (A-1) is exact it can not be conveniently used in the analysis of E-plane patterns of horn antennas. There are, however, useful approximations as described below.

If \( kR_1 \gg 1 \), \( \mu \) can be replaced by its lower limit in the nonexponential term of the integrand of (A-1) to yield

\[
I_1(\gamma) = \frac{\left(\frac{2}{\pi}\right)^{1/2}}{[k(R_1+R)]^{1/2}} \exp\left[i\left(\frac{\pi}{4} - kR\right)\right] \int_0^\infty \frac{\exp(-\mu^2) d\mu}{[k(R_1+R)]^{1/2}}. \tag{A-2}
\]

If further \( R \gg R_o, R_1 \) and \( R \) in (A-2) may be replaced by \( R_1 = R - r_o \), \( R = r - r_o \cos \gamma \) in the nonexponential terms and by \( R_1 = r + r_o \), \( R = r - r_o \cos \gamma \) in the exponential terms, yielding

\[
I(\gamma) = \frac{\exp(-jkr)}{(kr)^{1/2}} \cdot \frac{\exp\left[i\left(\frac{\pi}{4} + kr_o \cos \gamma\right)\right]}{(\frac{\pi}{2})^{1/2}} \int_0^\infty \frac{\exp(-\mu^2) d\mu}{[kr_o (1 + \cos \gamma)]^{1/2}}. \tag{A-3}
\]

(A-3) is identical with (2-2) if \( n=2 \) is used in it.

Although (A-3) is subject to the constraint \( R \gg R_o \), in some situations it is used without having satisfied this constraint. For instance, doubly diffracted fields in a horn are calculated by employing (A-3), while the aperture width is usually smaller than the slant length. To investigate the errors in \( I(\gamma) \) two cases are considered as follows:
a. \( r > r_0 \), which favours the constraint \( r > r_0 \). The phase and the amplitude errors defined as \( |\mathbf{I} - \mathbf{I}| \) and \(-20\log_{10}|\mathbf{I}|\) respectively are plotted versus \( r \) for several values of \( r \) and \( r_0 \), Fig. A.2. It is observed that for \( r_0 > 0.5\lambda \) and \( \gamma < 120^\circ \) the errors are well below 6.5° and 1.5 dB. The maximum errors are less than 7 and 3 dB.

b. \( r < r_0 \), the corresponding phase and amplitude error characteristics are shown in Fig. A.3. As is expected larger errors result. It is seen that for \( r > 0.5\lambda \) and \( \gamma < 90^\circ \) these errors are less than 6° and 1 dB. Consequently if \( r > 0.5\lambda \) is not \( \ll r_0 \) and is not close to 180°; i.e., the region of concern is not close to shadow boundaries, the resulting errors are negligible.
Fig. A.2. Error in $\tilde{r}(\gamma)$ for $\gamma < \Gamma$. 
Fig. A.3. Error in $\bar{I}(\gamma)$ for $r<r_0$. 