Four Essays on Supply Chain Management

by

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Abstract

This thesis, which consists of four separate essays, employs decentralized newsvendor models to address some critical problems in the context of decentralized retail supply chains.

Essay 1 examines the effectiveness of returns policies in a decentralized newsvendor model, in which a manufacturer sells a product to an independent retailer facing uncertain demand and the retail price is endogenously determined by the retailer. This model will be referred to as the PD-newsvendor model.

Essay 2 investigates the effect of sequential commitment in the decentralized PD-newsvendor model with buybacks. Sequential commitment allows the self-profit maximizing parties to commit to the contract parameters (e.g., wholesale price, retail price, buyback price and order quantity) sequentially and alternately, and we investigate its effect on the equilibrium profits of the channel and its members.

Essay 3 analyzes the effect of price and order postponement in the PD-newsvendor model, possibly with a buyback option. Such postponement strategies can be used by a retailer by delaying his operational decisions (order quantity and retail price) until after demand uncertainty is observed.

Essay 4 considers a supply chain wherein an assembler buys complementary components (or products) from n suppliers, assembles the n components into a final product, and sells it at a fixed retail price over a single selling season. We analyze two contracting systems between the assembler and the suppliers: push and pull. In the push system, the suppliers initiate the process by offering their wholesale prices to the assembler, and the assembler then orders from the suppliers well in advance of the selling season. In the pull system, the assembler first sets the wholesale prices for the different suppliers, and then the suppliers decide how much to produce and bear all of the inventory risk. In both systems, suppliers can form alliances among themselves or act independently.

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Chapter 1

Introduction

1.1 Motivation

A supply chain consists of all entities which combine together to produce and deliver goods to its customers. Thus, it may consist of the raw material suppliers, through factories, warehouses, distribution centres, retail stores to the end customers. Supply chain management is concerned with the flow of material, information and money among members in the channel. Since very few firms can manage the entire channel, most supply chains have a decentralized organizational structure. They consist of independently managed entities who pursue and implement strategies which maximize their own welfare. Thus, the decentralized structure of the channel may cause a misalignment among its members' objectives, which may reduce the effectiveness and performance of the channel.

In this thesis, we address various critical issues related to coordination, competition and cooperation in the context of a single-period decentralized retail supply chain (i.e., newsvendor problem), in which channel members make their own inventory and pricing decisions. For example, returns policies are commonly applied in many industries, such as the publishing industry and fashion wear, and they are known to coordinate the newsvendor problem when the retail price is exogenously given. In this thesis we analyze the effect and role of returns policies when the retail price is endogenously determined. We demonstrate that these policies result with a minimal improvement in channel efficiency, but, generally, shift expected profits from the retailer to the supplier. Additionally, for example, we introduce a sequential commitment approach, which allows members to commit to contract parameters sequentially and alternately, and our results can provide insight to channel members who engage in a negotiation process to decide upon the values of contract parameters. We also investigate the effect of price or order postponement in the newsvendor model. Postponement was extensively studied in the context of a centralized system, and we demonstrate that, in a decentralized system, though, in general, postponement increases the expected profits of the channel members, it could also decrease, for a low manufacturing cost, the expected profits of both members. Finally, we study issues of alliance formation among suppliers, and the effect of two models of contracting between them and a single assembler on the expected profits of the channel members and their incentives to increase their efficiency.

1.2 Basic Model Formulation and Some Notation

In this section we introduce the decentralized single-period newsvendor model, which is the basic model setup considered in this thesis. In this model, a manufacturer (M) sells a single product, possibly together with a buyback option, to an independent retailer (R) who sets an order quantity and a retail price that affects uncertain demand. Both M and R maximize their own profits. The decision sequence is as follows: M, who has unlimited production capacity and can produce the items at a fixed marginal cost c, is a Stackelberg leader. M initiates the process by offering a per unit constant (or linear) wholesale price w, at which items will be sold to R prior to the selling season, and a per unit constant (or linear) buyback rate b, at which she will buy back unsold items at the end of the selling season. In response to the proposed w and b, R commits to an order quantity Q prior to the selling season, and a per unit selling price p, at which to sell the items during the season. Thereafter, stochastic demand is realized. At the end of the season, R returns all unsold inventory to M, receiving a refund of b for each unit returned.[1.1]

Demand, X, that R faces is stochastic, and randomness in demand is price-independent and can be modeled either in an additive or multiplicative manner. More specifically, demand is modeled as $X = D(p) + \xi$ in the additive case (Mills (1959)), and as $X = D(p) \cdot \xi$ in the multiplicative case (Karlin and Carr (1962)), where $D(p)$ is the deterministic part of X which decreases in the retail price p, and ξ captures the random factor of the demand model, which could have either a continuous or discrete distribution and is defined on the interval $[L, U]$ with mean μ_ξ. To assure that positive demand is possible for some range of p, we require in this thesis that $D(p) + L \geq 0$, for some p, in the additive demand model, and $L \geq 0$ for the multiplicative demand model. If ξ follows a continuous distribution, let $F(\epsilon)$ and $f(\epsilon)$ be the distribution and density functions of ξ, respectively, where ϵ is a possible realization of ξ. $F(\epsilon) = 0$ for $\epsilon \in [0, L]$ and $F(\epsilon) = 1$ for

[1.1] The constant (or linear) buyback contracts are prevalent in many industries, e.g., books, newspapers, recordings, dairy products, etc., and indeed, they are considered to be one of the most popular contracts after wholesale price-only contracts, and typify manufacturer-distributor relations in many markets, see, e.g., Marvel and Peck (1995). In Chapters 2, 3 and 4 in this thesis, we investigate various issues in supply chain management based on the newsvendor problems under linear buyback contracts, and in Chapter 5, we study an assembly system under a wholesale price-only contract.

$\epsilon \in [U, \infty)$. See Petruzzi and Dada (1999) for a review of these two demand models.

We make the following assumptions in this thesis.

(I) Information on the demand distribution and production cost function is symmetric among all players[1.2], and all players are risk-neutral.

(II) Unmet demand is lost and there is no penalty cost for unsatisfied demand, and there is no salvage value of the unsold inventory, except for Chapter 2, wherein a discussion of the effect of a positive salvage value of the unsold inventory on the implementation of the returns policies is provided.

(III) For feasibility, the following relationships hold: (i) $c \leq w \leq p$ and (ii) $0 \leq b \leq w$. The order quantity will never be higher than the largest possible demand, i.e., $Q \leq UD(p)$, for the multiplicative demand model, and $Q \leq D(p) + U$, for the additive demand model.

(IV) For tractability, the lower bound L of the support of ξ is assumed to be zero, i.e., $L = 0$, in all chapters except in Chapter 4, wherein $L \geq 0$.

In this thesis, we will refer to the model described above, wherein the retail price p is determined endogenously by R, as the price-dependent (PD) newsvendor model. By contrast, when the retail price is exogenously fixed, the corresponding newsvendor model is called the price-independent (PI) newsvendor model. If M offers only a per unit constant wholesale price, then we refer to the contract between M and R as a wholesale price-only contract, and when a per unit constant buyback option is offered together with a wholesale price, we refer to the contract as a buyback contract (or returns policy). We will refer to the PD-newsvendor model with a multiplicative (respectively, additive) demand function as the multiplicative (respectively, additive) PD-newsvendor model.

In the multiplicative PD-newsvendor model with buybacks, in which all contract parameters (or decision variables) are chosen before demand uncertainty is resolved, we can express M's and R's expected profit functions as follows:

$$E\Pi_M = (w - c)Q - bE[Q - D(p)\xi]^+ \text{ and } E\Pi_R = (p - w)Q - (p - b)E[Q - D(p)\xi]^+, \qquad (1.1)$$

[1.2] Note that in the price-dependent newsvendor model, analyzed in Chapters 2, 3 and 4, the retailer can affect the value of expected demand by setting different values of the retail price, and in the price-independent assembly system covered in Chapter 5, the assembler assembles all complementary components and sells the finished product to the end customers. Thus, the assumption that the demand distribution and cost functions are known information to all players does not necessarily imply that the manufacturer can control the entire channel, or that s/he does not need the services of the retailer.

where $E[Q - D(p)\xi]^+ = QF(\frac{Q}{D(p)}) - \int_0^{\frac{Q}{D(p)}} D(p)\epsilon f(\epsilon|p)d\epsilon$ is the expected unsold inventory. The most commonly used types of the deterministic demand function in the Economics and Operations Management literature are: linear, $D(p) = 1-p$, exponential, $D(p) = e^{-p}$, and negative polynomial, $D(p) = p^{-q}$, where $q > 1$. The restriction $q > 1$ is used to ensure that R's optimal retail price will be upper bounded. For $D(p) = 1 - p$, we assume in this thesis that $c < 1$, since for $c = 1$, both M and R get a zero profit due to the fact that demand is zero. Note that the analysis in this thesis can be easily extended to more general deterministic demand functions: linear $D(p) = a(k - p)$, exponential $D(p) = ae^{-sp}$ and negative polynomial $D(p) = ap^{-q}$, where $q > 1$, for any positive values of a, k and s. For instance, if $D(p) = a(k - p)$, let $p = k \cdot p'$, $w = k \cdot w'$, $b = k \cdot b'$, $Q = ak \cdot Q'$ and $c = k \cdot c'$. Then, it is not difficult to verify that the expected profit functions of M and R, given by (1.1), can be transformed to: $E\Pi_M(w, b, p, Q, c) = ak^2 \cdot E\Pi'_M(w', b', p', Q', c')$ and $E\Pi_R(w, b, p, Q, c) = ak^2 \cdot E\Pi'_R(w', b', p', Q', c')$, where $E\Pi'_M$ and $E\Pi'_R$ are the expected profit functions of M and R, respectively, with respect to the expected demand function $D(p') = 1 - p'$ and the marginal manufacturing cost c'. Thus, the analysis in a model with decisions (w, b, p, Q), cost c and $D(p) = a(k - p)$ coincides with that in a model with decisions (w', b', p', Q'), cost c' and $D(p') = 1 - p'$. Note that due to this normalization, the performance of the multiplicative models with and without buybacks (Chapter 2), with and without sequential commitment (Chapter 3), with and without decision postponement with respect to demand (Chapter 4), and the integrated system is independent of individual values of c and k, but is dependent on $\frac{c}{k}$, which can be referred to as the *normalized marginal manufacturing cost*. Similarly, for $D(p) = ae^{-sp}$, the performance of this model is independent of individual values of c and s, and depends only on $s \cdot c$. Note further that due to this normalization, the mean of ξ can be normalized to $\mu_\xi = 1$ in the multiplicative model, which will be assumed to be the case in the multiplicative models in this thesis. For $\mu_\xi = 1$, the expected demand function coincides with the deterministic part in demand, i.e., $E(X) = D(p)$.

In the additive PD-newsvendor model, the corresponding M's and R's expected profit functions can be expressed as follows:

$$E\Pi_M = (w - c)Q - bE[Q - D(p) - \xi]^+ \text{ and } E\Pi_R = (p - w)Q - (p - b)E[Q - D(p) - \xi]^+. \quad (1.2)$$

In this thesis, we are almost exclusively concerned with the multiplicative model. The additive model is only studied in Chapter 4, wherein the objective is to analyze the effect of various postponement strategies in the PD-newsvendor model. In general, the analysis of the additive model is much more complex than that of the multiplicative model, and, therefore we only analyze in Chapter 4 the additive model without buybacks and with $X = \xi - p$ where $D(p) = -p$ is linear in p.

Similar to the multiplicative demand case, the analysis can be extended to a general deterministic linear demand function $D(p) = a(k-p)$ for any positive values of a and k. Note that the expected demand function in the additive model is equal to the sum of the deterministic demand and the mean of ξ, i.e., $E(X) = D(p) + \mu_\xi$.

The following notation will be used in this thesis.

Notation 1.2.1 *We will denote by* $(\cdot)^I$, $(\cdot)^*$ *and* $\hat{(\cdot)}^*$, *respectively, the equilibrium values in the integrated system, the system under a buyback contract and the system under a wholesale price-only contract.*

Finally, let us clarify the difference between the notions of "optimal" and "equilibrium" used in this thesis. Note that we consider in the four essays various multi-stage Stackelberg games and backward induction is used to solve these problems. We will use the term "optimal" to represent, e.g., a member's best response functions in the intermediate stages, and use the term "equilibrium" to stand for the final solutions after solving, completely, the Stackelberg game. For example, when we solve a four-stage Stackelberg game by using backward induction, the solutions in the intermediate stages, i.e., Stages 4, 3 and 2, are referred to as a member's "optimal" or "best" response functions, and the solution derived in Stage 1 is the "equilibrium" value of the game. In general, the solution derived, e.g., in Stage 1, is termed "equilibrium" since it is a product of vertical competition between the upstream manufacturer (or suppliers) and the downstream retailer (or assembler).

1.3 Summary of Major Findings and Contributions

This thesis consists of four separate essays which are concerned with various issues in a decentralized supply chain. The first essay, in Chapter 2, is concerned with linear returns policies between channel members. The second one, in Chapter 3, introduces a new decision-making approach, i.e., sequential commitment, which allows channel members to commit to their decision variables sequentially and alternately. The third essay, in Chapter 4, investigates the effect of postponing the downstream retailer's pricing and ordering decisions with respect to demand uncertainty, i.e., price and order postponement strategies, in a decentralized channel. The last essay, in Chapter 5, considers a decentralized assembly system wherein component suppliers can form alliances before they interact with the assembler.

The first essay studies the desirability of introducing buybacks and their effectiveness in the multiplicative price-dependent newsvendor model for three commonly used expected demand functions:

linear, negative polynomial and exponential, wherein the retail price is determined endogenously by the retailer. It provides a new insight as to why, returns policies are not more prevalent in practice. For a zero salvage value of unsold inventory, we demonstrate that in equilibrium, buybacks will be introduced for linear and exponential expected demand functions, but they are not introduced for a negative polynomial expected demand function. In those cases where buybacks are introduced, we show that their introduction has an insignificant effect on channel efficiency improvement. By contrast, their introduction may significantly increase the manufacturer's expected profit, and significantly decrease the retailer's expected profit. Thus, we suggest that in the absence of a positive salvage value, the introduction of buybacks to the price-dependent newsvendor model is probably not motivated by a desire to increase channel efficiency. Rather, it is more likely motivated by the significantly favorable, for the manufacturer, effect it has on the distribution of the channel profit.

It is found that with a zero salvage value, whenever buybacks are implemented in equilibrium, the wholesale price, channel profit allocation between the manufacturer and the retailer and channel efficiency coincide with those values in the corresponding deterministic model, wherein the deterministic demand function coincides with the expected demand function in the price-dependent newsvendor model. This finding implies that the introduction of buybacks improves the channel efficiency in the price-dependent newsvendor model up to the efficiency of the corresponding deterministic model.

The second essay introduces a sequential commitment approach for determining the values of contract parameters, and analyzes its effect on the PD-newsvendor model with buybacks. Our analysis reveals that the sequential commitment approach endogenizes the first mover decision. Indeed, while in the traditional approach (i.e., take-it-or-leave-it paradigm) it is arbitrarily assumed that one of the parties, usually, M, is the leader, in the sequential commitment approach, under certain conditions (e.g., uniform random component of demand and linear, exponential and negative polynomial expected demand functions), both M and R prefer that M will move first. Additionally, we show that the sequence $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ (which is referred to as Sequence 2 in Chapter 3), according to which M first offers a buyback rate b, R then commits to a retail price p, M then sets the value of w and R then orders Q, is the unique equilibrium sequence in the sense that both parties prefer that M will move first, and neither party can benefit by resequencing the order at which it commits to the contract parameters under its control.

We show that the introduction of sequential commitment to the PD-newsvendor model with buybacks can significantly improve M's and the channel expected profits, but it can also decrease

R's expected profit. For example, Sequence 2 under a uniform distribution of the random component of demand and a linear expected demand function always increases M's and the channel's expected profits, e.g., for $c = 0.9$, M's and the channel's expected profits are improved by 79.25% and 21.25%, respectively, and it always decreases R's expected profit, e.g., for $c = 0.9$, R's expected profit is deteriorated by 73.51%.

The third essay analyzes the impact of postponing the retailer's price and order decisions in the multiplicative and additive price-dependent newsvendor model until after demand uncertainty is resolved. We show that, in general, despite vertical competition and aside for some cases, the effect of either price or order postponement are quite beneficial for the channel and its members. As such, postponement could be viewed as a viable strategy to increase channel efficiency. Notwithstanding the usual benefits of postponement, it is clearly demonstrated in this essay that for some parameter values, e.g., when the manufacturing cost is relatively low in the multiplicative model, the effects of postponement in a decentralized system are qualitatively different than their effect in a centralized system. Indeed, both in the multiplicative and additive models under a wholesale price-only contract, price and order postponement can make the channel worse off, and in some instances, they could even make both the manufacturer and the retailer strictly worse off. In this regard, as far as we know, we are the first to provide examples wherein the expected value of perfect information in a competitive environment, modeled as a Stackelberg game, is negative.

We also demonstrate that in a decentralized setting, the party, i.e., the retailer, who initiates postponement, does not necessarily end up gaining the lion share of the increase in the expected profit. Finally, the results in the multiplicative model also quite clearly demonstrate that the effect of postponement depends on the type of contract. Specifically, with buyback options, neither price postponement nor order postponement affects the equilibrium wholesale price, profit allocation ratio and channel efficiency. However, without buybacks, such postponement strategies can significantly change the equilibrium values. In particular, as explained above, such strategies can make both the manufacturer and the retailer strictly worse off, which does not happen when a buyback option is offered.

The last essay studies a price-independent newsvendor model consisting of a single assembler who buys complementary components or products from n suppliers under two contracting systems: push and pull. In both systems, we investigate the stability of coalition structures in the suppliers' coalition formation game, and analyze the Stackelberg game between the assembler and the suppliers. We demonstrate that push and pull contracts, which allocate differently inventory risk among players, induce, in equilibrium, qualitatively different outcomes.

We show that in the push model, the expected profit of the assembler and the total expected profit of all suppliers are maximized when all suppliers join to form a single alliance in their negotiation with the assembler. Nevertheless, an equilibrium analysis which employs the Nash equilibrium concept reveals that, in equilibrium, all suppliers will deal independently with the assembler. However, when farsighted concepts are used to analyze alliance formation, it is shown that, under certain conditions, the alliance consisting of all suppliers will be formed. As mentioned earlier, such an alliance will maximize the assembler's expected profit and the total expected profit of all suppliers. It will also maximize consumer surplus.

Finally, we show that the assembler always prefers the pull system to the push system. However, the suppliers' preferences between these two systems depend on their own manufacturing costs. More specifically, suppliers with relatively lower manufacturing costs prefer push to pull since under pull, they are apparently not compensated enough for the risk they bear due to uncertain demand. On the other hand, suppliers with relatively higher manufacturing costs prefer pull to push since they are compensated proportionally to their cost. It is interesting to note that if all suppliers have the same manufacturing cost, then all suppliers prefer push to pull.

To summarize, the remainder of the thesis is as follows. Chapters 2, 3 and 4 study various issues in the PD-newsvendor model (with an endogenous retail price) with a single manufacturer and a single retailer. More specifically, in Chapter 2 we consider the effect of linear returns policies between M and R on the expected profits of the channel and its members, and their effect on the equilibrium values of decision variables. Chapter 3 introduces a sequential commitment approach, which allows channel members to commit to the decision variables under their control sequentially and alternately, investigates its effect on the equilibrium expected profits, and provides some insight to channel members who follow a bargaining process to determine the values of contract parameters. In Chapter 4 we examine various decision postponement strategies (i.e., price and order postponement) and investigate the benefits of managing information flow regarding stochastic demand. Chapter 5 studies two supply chain systems (push and pull) in the PI-newsvendor model (i.e., exogenous retail price) under a wholesale-price only contract, wherein there are n suppliers selling complementary components to an independent assembler. In this chapter, we investigate and compare the push and pull systems and examine the issue of alliance formation among suppliers in both systems.

Chapter 2

On the Effectiveness of Returns Policies in the Price-Dependent Newsvendor Model

2.1 Introduction

Manufacturers, whose products are subject to random demand, often accept returns of unsold goods for full or partial credit. For example, books, newspapers, recordings, CDs, dairy products, costume jewelry, fashion wear, computer products and peripherals, and perishable services, such as airline tickets and hotel rooms, are usually allowed to return to their source in North America for full or partial credit. In general, a supply chain composed of independent agents trying to maximize their own profits does not achieve channel coordination, see, e.g., Spengler (1950). Pasternack (1985) was the first to show that buybacks can coordinate the basic price-independent newsvendor model, wherein a manufacturer (M) offers a good to a retailer (R) for a constant wholesale price and a constant buyback rate (linear pricing), and R, who faces a fixed retail price and stochastic demand, needs to decide upon the optimal order quantity. Subsequently, other contracts, such as, e.g., quantity-flexibility (Tsay (1999)), sales-rebate (Taylor (2002a)), and revenue-sharing (Pasternack (2002), Cachon and Lariviere (2005)) have also been shown to be able to coordinate the basic newsvendor model. See also Lariviere (1999), Tsay et al. (1999) and Cachon (2004b) for some excellent reviews of coordination mechanisms for the basic newsvendor model and related models.

As noted by Kandel (1996), the price-dependent (PD) newsvendor model, wherein the retail price is determined endogenously by R, is considerably more complicated. However, Emmons and Gilbert (E&G) (1998) have shown that if the wholesale price is large enough, both M and R would benefit from the introduction of buybacks when the expected demand function is linear. It has been conjectured by Lariviere (1999), and it has been proved by Bernstein and Federgruen

(2005), that constant wholesale and buyback prices (i.e., independent of other decision variables) cannot, in general, lead to coordination in the PD-newsvendor model. By contrast, contracts which do not employ constant wholesale and buyback prices can induce coordination. Indeed, e.g., revenue-sharing contracts and the "linear price discount sharing" scheme, have been shown by Cachon and Lariviere (C&L) (2005) and by Bernstein and Federgruen (2005), respectively, that they could induce coordination in the PD-newsvendor model. We note, however, that as discussed by C&L, revenue-sharing contracts require the ability for M to verify *ex post* R's revenue, which may be costly, and as noted by Bernstein and Federgruen (2005), the "linear price discount sharing" scheme bears close resemblance to the traditional "bill back" or "count-recount" schemes, which, unfortunately, are reported to be disliked by retailers (see, e.g., Blattberg and Neslin (1990), Chapter 11).

Marvel and Peck's (M&P) (1995) model, which assumes constant wholesale and buyback prices and is somewhat different than the traditional supply chain model in the Operations Management (OM) literature, incorporates two types of uncertainty: One with respect to product valuation and the other concerning the number of customers arriving to the retail store. They show that uncertainty *only* about product valuation leads to manufacturers' preference for a wholesale price-only contract, whereas uncertainty *only* about the number of arrivals induces manufacturers to offer buybacks in their contracts. Thus, valuation uncertainty leads to theoretically opposite results than those derived for arrival uncertainty, which, as M&P suggest, explains why return good systems are not more wide spread than observed. Note that if there is only arrival uncertainty, then M&P's model essentially reduces to the basic price-independent newsvendor model wherein the selling price coincides with a representative customer's product valuation. If there is only product valuation uncertainty, then the equilibrium order quantity is either zero or equal to the known and fixed number of arriving customers.

In this chapter we study the PD-newsvendor model with constant wholesale and buyback prices, described in §1.2, which, as stated by M&P, typifies manufacturer-distributor relations in many markets. Our objective is *not* to investigate channel coordination. Rather, our aim is to investigate possible factors that affect the introduction of returns. Thus, we address queries, such as that by Lariviere (1999, Section 8.6, second paragraph), as to why constant wholesale price and buyback rate contracts are not more prevalent: "Given the apparent power of returns policies, it is not surprising that they are common in industries such as publishing. Indeed, one may wonder why they are not even more common. Relatively little work has examined this issue...".

We investigate the effect of buybacks for three different expected demand functions: linear, negative polynomial and exponential. For a linear expected demand function and a uniformly distributed random component of the demand model, our PD-newsvendor model coincides with E&G's model. Our main results, for a zero salvage value, are:

(i) The manufacturer may elect not to offer buybacks. Indeed, buybacks are not introduced in equilibrium when the expected demand function is a negative polynomial function of the retail price.

(ii) If buybacks are introduced in equilibrium, they have a relatively insignificant effect on channel efficiency improvement.

(iii) By contrast, if buybacks are introduced in equilibrium, they could have a rather dramatic effect on profit distribution. They could significantly increase M's expected profit and significantly decrease R's expected profit. For example, for a linear expected demand function and a uniformly distributed random component of demand, the introduction of buybacks is shown to increase M's expected profit by 12.5% to 23.94% and to decrease R's expected profit by 15.62% to 20.63%.

(iv) Our analysis demonstrates that the introduction of buybacks in equilibrium induces higher wholesale price, retail price and retail inventories than those obtained under wholesale price-only contracts.

(v) In the PD-newsvendor model with buyback options, for a uniformly distributed random component of demand, the wholesale price, channel efficiency and profit distribution between M and R coincide with those in the corresponding model with deterministic demand.

It can also be shown that the introduction of a positive salvage value in the PD-newsvendor model may have a significant effect on the possible implementation of a returns policy. For example, for a positive and equal salvage value at M's and R's locations, buybacks are introduced for all three expected demand functions..

Our findings provide several answers to Lariviere's query as to why return good systems are not more common. Indeed, as it is the case for a negative polynomial expected demand function and a zero salvage value, a manufacturer may prefer not to offer buybacks in equilibrium. Further, if buybacks are introduced, their insignificant effect on channel efficiency would be further diminished by the additional costs that would arise in a return system, which are not accounted for by the model and which will not be incurred by a wholesale price-only contract (see related discussions in, e.g., Lariviere (1999) and Lariviere and Porteus (2001)). Thus, the introduction of buybacks by manufacturers in the PD-newsvendor model could be viewed by retailers, perhaps correctly, as an attempt to grab additional channel profit at their expense. To the extent possible, therefore, retailers would object to the introduction of return good systems.

The remainder of this chapter is organized as follows: §2.2 recalls the price-dependent (PD) newsvendor model, as introduced in §1.2 in Chapter 1. In §2.3 we analyze the PD-newsvendor model, as studied by E&G, wherein the expected demand function is linear in the retail price and the random component of demand is uniformly distributed. §2.4 extends the analysis to negative polynomial and exponential expected demand functions. In §2.5 we discuss an extension to more general demand distributions and the effect of a positive salvage value on the implementation of the returns policies, and we reveal a surprising relationship between the PD-newsvendor model with buybacks and the corresponding deterministic model. Conclusions and future research are provided in §2.6. All proofs in this chapter are presented in the appendix in §2.7.

2.2 Model Formulation

Consider the single-period PD-newsvendor model with buyback policies described in §1.2. It is assumed in this chapter that unsatisfied demand is lost, there is no penalty cost for lost sales[2.1], and that the salvage value of unsold inventory is zero for[2.2] both M and R. Recall that for feasibility, we assume: (i) $c \leq w \leq p$ and (ii) $0 \leq b \leq w$.

The stochastic demand, X, that R faces is assumed to be of a multiplicative form $X = D(p)\xi$, which is a commonly used model in the Economics and OM literature. $D(p)$ is the deterministic part of X, which decreases in the retail price p, and ξ ($\xi \geq 0$) is the random part of X. Recall from §1.2 that $F(\cdot)$ and $f(\cdot)$ are the distribution and density functions of ξ, respectively. The multiplicative demand model was initially proposed by Karlin and Carr (1962).

It would be interesting and challenging to extend our analysis to the additive demand model

[2.1]The zero penalty cost assumption is made mainly for tractability reasons. A positive penalty cost (or goodwill cost) of unmet demand (or lost sales) accounts for consumers' dissatisfaction and for potential business losses, especially in a multi-period setting. Generally speaking, incorporating a penalty cost for unsold inventory complicates the analysis significantly. When a goodwill cost is present in the PD-newsvendor model, closed-form expressions for equilibrium decisions and profits are not available for any of the expected demand functions considered in this chapter. Nevertheless, we have conducted a numerical investigation of the PD-newsvendor model for linear and negative polynomial expected demand functions. According to our findings, for a low goodwill cost, the results hold. That is, buybacks are introduced for the linear case but not for the negative polynomial expected demand case. However, for a high enough goodwill cost, buybacks are introduced in both cases.

[2.2]The implications of relaxing this assumption are considered in §2.5.2.

wherein $X = D(p) + \xi$. The additive model, which is also commonly used in the literature, would be an appropriate model wherein the variance of demand is unaffected by the expected demand level. By contrast, the multiplicative model is appropriate where the variance of demand increases with expected demand in a manner which leaves the coefficient of variation unaffected.

We note, however, that the additive model may lead to qualitatively different results than the multiplicative model (see, e.g., Mills (1959), Emmons and Gilbert (1998), Song et al. (2004), and, in particular, the excellent survey by Petruzzi and Dada (1999)). Moreover, it appears that it is less tractable than the multiplicative model (see, e.g., Padmanabhan and Png (1997), Wang et al. (2004) and §4.7 in Chapter 4 in this thesis). Indeed, even when ξ has a binary distribution and $D(p)$ is linear in p, it is difficult to derive a closed-form expression for, e.g., the equilibrium value of w, in the PD-newsvendor problem with an additive demand model.

Finally, let us note the main differences between the multiplicative and additive demand models and M&P's demand model. In the multiplicative and additive models, the stochastic demand is precisely the number of arrivals, which is a function of the retail price p. It can be assumed that *all* arriving customers in these models are familiar with the product, well informed about the retail price, and are interesting in buying it. If n customers were to arrive, then it would be optimal to order n items. However, in M&P's model, the number of arrivals is independent of the retail price. That is, the number of arriving customers is stochastically the same regardless whether the retail price is very high or very low. This could be interpreted as if the arriving customers are uninformed about the retail price. Once in the store, either all or none will buy the product, depending on whether a representative customer's product valuation exceeds the retail price. Thus, in M&P's model, if n customers were to arrive, by contrast with the multiplicative and additive demand models, it may be optimal to stock nothing.

In this chapter, we adopt E&G's assumption that the random part of demand, ξ, follows a uniform distribution[2.3] on the interval $[0, 2]$, i.e., $f(\epsilon) = 0.5$ on $[0, 2]$. Thus, $E(\xi) = 1$, and we can simplify M's and R's expected profit functions, given by (1.1), to:

$$E\Pi^M(w,b) = (w-c)Q - b\frac{Q^2}{4D(p)} \quad \text{and} \quad E\Pi^R(p,Q) = (p-w)Q - (p-b)\frac{Q^2}{4D(p)}. \tag{2.1}$$

We analyze in the next section the effect of buybacks in the PD-newsvendor model with a linear expected demand function. In §2.4, we extend our study to two other expected demand

[2.3]The analysis can be easily extended to a uniform distribution of ξ on $[0, U]$ for any $U > 0$. Due to the normalization of the deterministic demand function of X described in §1.2, it is known that the upper bound U will have no impact on the equilibrium values of w, b and p, and for the equilibrium values of Q, $E\Pi_M$ and $E\Pi_R$, we have the following relationship: $Q^*(U) = \frac{U}{2}Q^*(U=2)$, $E\Pi^*_M(U) = \frac{U}{2}E\Pi^*_M(U=2)$ and $E\Pi^*_R(U) = \frac{U}{2}E\Pi^*_R(U=2)$. See §2.5 where we briefly report on computational results with power and triangle distributions of ξ.

functions. Recall that, unless otherwise noted, we denote by $(\cdot)^I$, $(\cdot)^*$ and $\hat{(\cdot)}^*$ the equilibrium values in the integrated system, the system under a buyback contract and the system under a wholesale price-only contract, respectively.

2.3 Effect of Buybacks with Linear Expected Demand

We analyze in this section the effect of buybacks in the PD-newsvendor model wherein the expected demand function, $D(p)$, is linear of the form $D(p) = 1 - p$. Note that when $p = 1$, market demand is zero and both M and R gain zero expected profits. Thus, we assume that $p < 1$ in the sequel, except as otherwise noted, and $c \leq w \leq p < 1$. We further note that, for any retail price p, the highest demand from the end-customer market is $2D(p)$ since $\xi \leq 2$.

From (2.1) and for $D(p) = 1 - p$, M's and R's expected profit functions can be simplified to:

$$E\Pi_L^M = (w-c)Q - b\frac{Q^2}{4(1-p)} \text{ and } E\Pi_L^R = (p-w)Q - (p-b)\frac{Q^2}{4(1-p)}, \tag{2.2}$$

where the subscript "L" stands for "linear expected demand". The total expected channel profit, $E\Pi_L^{Total}$, is the sum of the expected profits of M and R.

According to R's expected profit function, given by (2.2), and for any given pair (w, b), E&G have shown that R's optimal retail price and order quantity are:

$$p_L^* = \frac{3b + 1 + \sqrt{(1+8w-9b)(1-b)}}{4} \text{ and } Q_L^* = \frac{2(1-p_L^*)(p_L^* - w)}{p_L^* - b}. \tag{2.3}$$

Taking R's reaction functions into account, M's expected profit function becomes:

$$E\Pi_L^M = (w-c)Q_L^* - b\frac{(Q_L^*)^2}{4(1-p_L^*)}. \tag{2.4}$$

Substituting $w = c$ and $b = 0$ into (2.3), we obtain the unique equilibrium values of p and Q in the corresponding integrated system[2.4]:

$$p_L^I = \frac{1+\sqrt{1+8c}}{4} \text{ and } Q_L^I = \frac{(3-\sqrt{1+8c})^2}{4}. \tag{2.5}$$

Substituting p_L^I and Q_L^I into the expected integrated channel profit function: $E\Pi_L^I = (p-c)Q - p\frac{Q^2}{4(1-p)}$, and simplifying gives:

$$E\Pi_L^I = \frac{(3-\sqrt{1+8c})^3(1+\sqrt{1+8c})}{64}. \tag{2.6}$$

E&G have shown that for all wholesale prices $w \in (w_T, 1)$, where w_T is a threshold value less than 1, both M and R are better off when M offers a positive buyback rate (i.e., $b > 0$).

[2.4]Note that $3 - \sqrt{1+8c} > 0$ since $c < 1$. Similarly for other expressions containing $3 - \sqrt{1+8c}$ in the sequel.

However, by contrast with E&G, we are able to find closed-form expressions for w_T, the equilibrium wholesale price, w_L^*, and equilibrium buyback rate, b_L^*. We further show that when the expected demand function is linear, as assumed by E&G, the efficiency[2.5] of the PD-newsvendor model with buybacks is precisely 75%, and that the increased efficiency due to the introduction of buybacks is insignificant, and bounded by 3.16%. By contrast, we demonstrate that the introduction of buybacks has a significant effect on the distribution of the channel profit between M and R. Explicitly, we prove that the introduction of buybacks increases M's expected profit by 12.5% to 23.94%, whereas, R's expected profit decreases by 15.62% to 20.63%.

We start by providing a closed-form expression for w_T.

Proposition 2.3.1 *For any wholesale price in the interval* $(w_T \equiv \frac{2+30c+3\sqrt{6(1+5c)}}{50}, 1)$*, there exists a buyback rate* $b > 0$*, at which both* M *and* R *earn higher expected profits than when* $b = 0$.

Observe that neither Proposition 2.3.1 nor Proposition 2 in E&G implies that buybacks are used in equilibrium. Rather, they merely assert that when the wholesale price is large enough, both M and R benefit from the introduction of buybacks. To prove that buybacks are used in equilibrium, we need Proposition 2.3.2 and Lemma 2.3.3 below. In Proposition 2.3.2, we derive an explicit expression for the equilibrium wholesale price, $\hat{w}_L^*$, in the PD-newsvendor model under a wholesale price-only contract, wherein M first commits to a wholesale price w, and then R commits to a retail price p and an order quantity Q.

Now, substituting $b = 0$ into the expected profit functions of M and R under a contract with buybacks, given by (2.2), we obtain M's and R's expected profit functions, $E\hat{\Pi}_L^M$ and $E\hat{\Pi}_L^R$, respectively, in a wholesale price-only contract:

$$E\hat{\Pi}_L^M = (w-c)Q \quad \text{and} \quad E\hat{\Pi}_L^R = (p-w)Q - p\frac{Q^2}{4(1-p)}. \tag{2.7}$$

Proposition 2.3.2 *In the PD-newsvendor model under a wholesale price-only contract, M's equilibrium wholesale price is:* $\hat{w}_L^* = \frac{5+32c+3\sqrt{17+64c}}{64}$.

The following relationship holds between w_T and $\hat{w}_L^*$.

Lemma 2.3.3 $w_T < \hat{w}_L^*$.

In view of Proposition 2.3.1 and Lemma 2.3.3, we have:

[2.5]The efficiency of a supply chain is defined as the ratio of the equilibrium channel profit to the corresponding integrated channel profit.

Corollary 2.3.4 *Buybacks are introduced in equilibrium in the PD-newsvendor model with a linear expected demand function.*

For $b = 0$ and knowing $\hat{w}_L^*$, we are able to calculate $\hat{p}_L^*$ and $\hat{Q}_L^*$ in a wholesale price-only contract by substituting $b = 0$ and $\hat{w}_L^*$ into p_L^* and Q_L^*, given in (2.3):

$$\hat{p}_L^* = \frac{7+\sqrt{17+64c}}{16} \text{ and } \hat{Q}_L^* = \frac{(9-\sqrt{17+64c})^2}{64}. \tag{2.8}$$

Substituting the resulting $\hat{p}_L^*$ and $\hat{Q}_L^*$ further into $E\hat{\Pi}_L^M$ and $E\hat{\Pi}_L^R$, given by (2.7), and simplifying provides us with M's and R's equilibrium expected profits in the PD-newsvendor model under a wholesale price-only contract:

$$E\hat{\Pi}_L^{M*} = T(3+\sqrt{17+64c}) \text{ and } E\hat{\Pi}_L^{R*} = T(\frac{7}{2}+\frac{1}{2}\sqrt{17+64c}), \tag{2.9}$$

and the equilibrium total expected channel profit under a wholesale price-only contract is:

$$E\hat{\Pi}_L^{Total*} = T(\frac{13}{2}+\frac{3}{2}\sqrt{17+64c}), \tag{2.10}$$

where[2.6] $T = \frac{(9-\sqrt{17+64c})^3}{8192}$.

Lemma 2.3.5 *In the PD-newsvendor model under a wholesale price-only contract, in equilibrium, the ratio of M's and R's expected profits is bounded between 1.28 and 1.5.*

We are now able to calculate the channel efficiency with wholesale price-only contracts.

Proposition 2.3.6 *The channel efficiency with a wholesale price-only contract is strictly increasing in c and is bounded between 71.84% and 74.07%.*

A possible explanation for the increased efficiency as a function of c is that as c increases, the range for w decreases since $c \le w \le p < 1$. Thus, an increase in c decreases the possibility for double marginalization. See also Chapter 4 in this thesis for a similar behavior of channel efficiency in decentralized systems under decision postponement.

In the PD-newsvendor model with buybacks, E&G had to resort to a numerical and graphical investigation to analyze the equilibrium expected profits of M, R and the overall channel as a function of w, for parameter values[2.7] $(c, a, k) = (1, -3, 5)$. Fortunately, we are able to derive closed-form expressions for the equilibrium values of w_L^* and b_L^*, and therefrom to derive explicit expressions for M's and R's equilibrium expected profits.

[2.6] Note that $9 - \sqrt{17+64c} > 0$ since $c < 1$. Similarly for other expressions which contain $9 - \sqrt{17+64c}$ in the sequel.

[2.7] E&G have considered a "general" linear expected demand function of the form $D(p) = a(k - p)$, see also §1.2.

Proposition 2.3.7 *In the PD-newsvendor model with buybacks, the equilibrium values of M's decision variables are:* $(w_L^* = \frac{1+c}{2}, b_L^* = \frac{1}{2})$*, and in equilibrium,*

$$E\Pi_L^{M*} = 2E\Pi_L^{R*} = \frac{(3-\sqrt{1+8c})^3(1+\sqrt{1+8c})}{128}. \tag{2.11}$$

Having the equilibrium wholesale and buyback prices, we are able to calculate R's equilibrium retail price and order quantity:

$$p_L^* = \frac{5+\sqrt{1+8c}}{8} \text{ and } Q_L^* = \frac{(3-\sqrt{1+8c})^2}{8}. \tag{2.12}$$

Further, having the equilibrium expected profits of M and R in contracts with buybacks, given by (2.11), and the integrated channel profit, given by (2.6), we derive the following conclusion.

Proposition 2.3.8 *The channel efficiency of the PD-newsvendor model with buybacks is 75%.*

Propositions 2.3.6 and 2.3.8 imply[2.8] that as compared to the wholesale price-only contract, the improvement in channel efficiency due to the introduction of buybacks is decreasing in c, and it is quite insignificant, at most 3.16% for $c = 0$. This result should be contrasted with the significant effect of buybacks on channel efficiency improvement in the basic newsvendor model, wherein the retail price is exogenously determined. Indeed, Lariviere and Porteus (2001) have studied the basic newsvendor model under a wholesale price-only contract, and they have shown, e.g., that the channel efficiency under such a contract is only 75% when demand follows a uniform distribution. But, as shown by Pasternack (1985), the channel can be perfectly coordinated when buybacks are introduced in the basic newsvendor model, which implies that buybacks can increase channel efficiency by 25% for uniformly distributed demand.

From the above discussion we conclude that channel efficiency improvement is unlikely to be the motivation behind the introduction of buybacks to the PD-newsvendor model. The following two propositions suggest another motivation for their introduction in this model.

Proposition 2.3.9 *In the PD-newsvendor model, the percentage improvement in M's equilibrium expected profit due to the introduction of buybacks is strictly decreasing in* c *and is bounded between 23.94%, for* $c = 0$*, and 12.5%, for* $c \to 1$.

Proposition 2.3.10 *In the PD-newsvendor model, the percentage deterioration of R's equilibrium expected profit due to the introduction of buybacks is strictly decreasing in* c *and is bounded between 20.63%, for* $c = 0$*, and 15.62%, for* $c \to 1$.

[2.8] Recall that the channel efficiency in the wholesale price-only contract is increasing in c.

A possible explanation for the decreased improvement in M's equilibrium expected profit (Proposition 2.3.9) and the decreased deterioration in R's equilibrium expected profit (Proposition 2.3.10), as a function of c, is similar to that given for Proposition 2.3.6. That is, as c increases, there is less room for M to manipulate w to improve her welfare.

In view of Propositions 2.3.9 and 2.3.10, we may conclude that a possible motivation for the introduction (by M) of buyback policies to the PD-newsvendor model is the significant and favorable, for M, effect it has on the distribution of the channel profit.

Propositions 2.3.9 and 2.3.10 are consistent with E&G's findings for the specific instance of the PD-newsvendor model they have studied, wherein $(c, a, k) = (1, -3, 5)$. Indeed, in their specific example, $c' = c/k = 1/5 = 0.2$, and there is an 18.92% increase in M's expected profit and a 19.26% decrease in R's expected profit, due to the introduction of buybacks.

Proposition 2.3.11 below reveals the relationships among the equilibrium wholesale and retail prices and the order (or production) quantities in supply contracts with and without buybacks and in the vertically integrated channel.

Proposition 2.3.11 *In the PD-newsvendor model:*

(i) $\hat{w}_L^* < w_L^*$,

(ii) $p_L^I < \hat{p}_L^* < p_L^*$ *and*

(iii) $\hat{Q}_L^* < Q_L^* < Q_L^I$.

It follows from Proposition 2.3.11 that, as expected, the integrated channel would be preferred by the end customers to a decentralized supply channel with or without buybacks, in the sense that it offers a lower retail price and makes a larger amount of the product available to customers. But, while the retail price with buybacks is strictly higher than that without buybacks, the quantity available for the end customers in a supply chain with buybacks is strictly larger than that without buybacks. We note that the results derived in Proposition 2.3.11 are consistent with those derived by M&P for their demand model.

2.4 Effect of Buybacks with Other Expected Demand Functions

In this section, we maintain the assumption that ξ follows a uniform distribution, and we investigate the robustness of our results, presented in §2.3, for other expected demand functions. Specifically, in §2.4.1 the expected demand function is a negative polynomial function of the retail price, and in §2.4.2, the expected demand function is exponential.

2.4.1 Negative polynomial expected demand

In this subsection, we study the PD-newsvendor model with a negative polynomial expected demand function of the retail price p, $D(p) = p^{-q}$, where $q > 1$ and $w \leq p < \infty$. The restriction $q > 1$ is used to ensure that R's optimal retail price will be finite. The analysis can be easily extended to a general $D(p) = ap^{-q}$, where $a > 0$.

According to (2.1), M's and R's expected profit functions, denoted as $E\Pi_N^M$ and $E\Pi_N^R$, in the PD-newsvendor model with $D(p) = p^{-q}$ and buyback options are:

$$E\Pi_N^M = (w-c)Q - b\frac{Q^2}{4p^{-q}} \text{ and } E\Pi_N^R = (p-w)Q - (p-b)\frac{Q^2}{4p^{-q}}, \tag{2.13}$$

where the subscript "N" stands for "negative polynomial demand". Let

$$p_N^* = \frac{qw + qb + w - 2b + J}{2(q-1)} \text{ and } Q_N^* = \frac{2(p_N^*)^{-q}(p_N^* - w)}{p_N^* - b}, \tag{2.14}$$

where $J \equiv \sqrt{(q+1)^2w^2 - 2(q^2-q+2)wb + (q-2)^2b^2}$.

Proposition 2.4.1 *In the PD-newsvendor model with buybacks and $D(p) = p^{-q}$, for any given (w,b), R's optimal reaction functions are given by (2.14).*

Substituting $w = c$ and $b = 0$ into p_N^* and Q_N^*, we obtain the unique equilibrium p_N^I and Q_N^I in the corresponding integrated system:

$$p_N^I = \frac{q+1}{q-1}c \text{ and } Q_N^I = \frac{4(q-1)^q}{(q+1)^{q+1}}c^{-q}.$$

Substituting p_N^I and Q_N^I into the integrated channel profit function: $E\Pi_N^I = (p-c)Q - p\frac{Q^2}{4p^{-q}}$, and simplifying gives:

$$E\Pi_N^I = \frac{4(q-1)^{q-1}}{(q+1)^{q+1}}c^{1-q}. \tag{2.15}$$

Having R's reaction functions, p_N^* and Q_N^*, given by (2.14), M's expected profit function becomes:

$$E\Pi_N^M = (w-c)Q_N^* - b\frac{(Q_N^*)^2}{4(p_N^*)^{-q}}. \tag{2.16}$$

Similar to the case when the expected demand function is linear (Proposition 2.3.1), we find a range of wholesale prices in which exercising buybacks can benefit both M and R.

Proposition 2.4.2 *In the PD-newsvendor model with $D(p) = p^{-q}$, for any wholesale price w, where $w > w_T^N \equiv \frac{qc}{q-1}$, exercising a buyback option benefits both M and R.*

M's equilibrium values of decision variables, (w_N^*, b_N^*), and the equilibrium expected profits of M and R are as follows:

Proposition 2.4.3 *In the PD-newsvendor model with buybacks and $D(p) = p^{-q}$, the equilibrium values of M's decision variables are: $(w_N^* = \frac{qc}{q-1}, b_N^* = 0)$, and, in equilibrium,*

$$E\Pi_N^{M*} = \frac{4(q-1)^{2q-1}}{c^{q-1}q^q(q+1)^{q+1}}, \quad E\Pi_N^{R*} = \frac{4(q-1)^{2q-2}}{c^{q-1}q^{q-1}(q+1)^{q+1}} \text{ and } \frac{E\Pi_N^{M*}}{E\Pi_N^{R*}} = \frac{q-1}{q}. \tag{2.17}$$

Proposition 2.4.3 implies that when the expected demand function is a negative polynomial of the retail price, M elects *not* to offer a buyback option in equilibrium. Thus, as suggested earlier, Proposition 2.4.2, which proves the existence of a range of wholesale prices at which both M and R would benefit from the implementation of buybacks, is not sufficient for the introduction of buybacks in equilibrium. Rather, a sufficient condition for the introduction of buybacks is that the equilibrium wholesale price, for a wholesale price-only contract, falls in the interval of wholesale prices at which both M and R would benefit from buybacks. Indeed, in the linear case, this wholesale price falls in that interval (Lemma 2.3.3), and thus, in equilibrium, buybacks are used. However, for the negative polynomial demand case, this wholesale price is not in that interval (Propositions 2.4.2 and 2.4.3), and, indeed, in equilibrium, buybacks are not used.

Proposition 2.4.3 should be compared with the result derived by M&P, according to which M would always prefer to offer buybacks in equilibrium in the presence of uncertainty only with respect to the number of arrivals. However, this specific result appears to be implied by their model. Indeed, when the uncertainty is only with respect to the number of arrivals, M and R know with certainty the customers' valuation of the product. By requesting a wholesale price equal to the customers' valuation, M, in M&P's model, is able to secure the entire channel profit by implementing a complete consignment contract (full return for full credit). In fact, as noted in §1.2, M&P's model with only arrival uncertainty essentially coincides with the price-independent newsvendor model wherein the retail price is exogenously fixed. Thus, M&P's result that full-credit buybacks are offered when there is only arrival uncertainty is consistent with the literature on channel coordination through buybacks in the price-independent newsvendor model (Pasternack (1985), Kandel (1996)), wherein M is able to secure the entire channel profit by setting $w = b = p$. Observe that in the presence of uncertainty both with respect to valuation and arrivals, but when there is not very much arrival uncertainty, M may not offer buybacks in M&P's model (M&P (1995)).

Having the equilibrium expected profits of M and R, given by (2.17), and the integrated channel profit, given by (2.15), we can derive the channel efficiency with (or without) buybacks.

Proposition 2.4.4 *The channel efficiency of the PD-newsvendor model with (or without) buybacks and with $D(p) = p^{-q}$, is $\frac{(q-1)^{q-1}(2q-1)}{q^q}$, where $q > 1$, and it is strictly decreasing in q.*

Note that price elasticity of a negative polynomial expected demand $D(p) = p^{-q}$ is $-\frac{dD(p)/dp}{D(p)/p} = q$ (> 1). Thus, the larger q is, the more sensitive customers are to changes in the retail price, and the more severe are the effects of double marginalization. This may explain the decrease of channel efficiency as a function of q.

Similar to the linear expected demand case, one can show that the following relationships hold among the equilibrium values of decision variables for an integrated firm and a decentralized channel with a negative polynomial expected demand function: (i) $p_N^I < p_N^*$, and (ii) $Q_N^* < Q_N^I$. That is, the integrated firm charges a lower retail price and produces a larger quantity than the decentralized system.

2.4.2 Exponential expected demand

We consider in this subsection the PD-newsvendor model with an exponential expected demand function, i.e., $D(p) = e^{-p}$, where $w \leq p < \infty$.

For $D(p) = e^{-p}$, M's and R's expected profit functions with buybacks, given by (2.1), reduce to:

$$E\Pi_E^M = (w-c)Q - b\frac{bQ^2}{4e^{-p}} \text{ and } E\Pi_E^R = (p-w)Q - (p-b)\frac{Q^2}{4e^{-p}}, \tag{2.18}$$

where the subscript "E" stands for "exponential expected demand". Similar to the negative polynomial expected demand case (Proposition 2.4.1), according to R's expected profit function, given in (2.18), it can be shown that R's reaction functions for any given w and b are:

$$p_E^* = \frac{w+b+1+\sqrt{(w-b)^2+6(w-b)+1}}{2} \text{ and } Q_E^* = \frac{2e^{-p_E^*}(p_E^*-w)}{p_E^*-b}. \tag{2.19}$$

Substituting $w = c$ and $b = 0$ into p_E^* and Q_E^*, we obtain the unique equilibrium p_E^I and Q_E^I in the corresponding integrated system: $p_E^I = \frac{c+1+H}{2}$ and $Q_E^I = \frac{2e^{-p_E^I}(p_E^I-c)}{p_E^I}$, where $H \equiv \sqrt{c^2+6c+1}$. Substituting p_E^I and Q_E^I into the corresponding integrated channel profit function: $E\Pi_E^I = (p-c)Q - p\frac{Q^2}{4e^{-p}}$, and simplifying gives:

$$E\Pi_E^I = \frac{(c+3-H)(-c+1+H)}{4}e^{-\frac{c+1+H}{2}}. \tag{2.20}$$

Let us first consider the model under a wholesale price-only contract. Substituting $b = 0$ into R's reaction functions p_E^* and Q_E^* for contracts with buybacks, given by (2.19), and simplifying provides us with R's reaction functions in the wholesale price-only contract:

$$\hat{p}_E^* = \frac{w+1+Z}{2} \text{ and } \hat{Q}_E^* = e^{-\frac{w+1+Z}{2}}(w+3-Z), \tag{2.21}$$

where $Z = \sqrt{w^2+6w+1}$. By substituting the resulting $\hat{Q}_E^*$ further into M's expected profit function under a wholesale price-only contract, $E\hat{\Pi}_E^M = (w-c)Q$, we obtain:

$$E\hat{\Pi}_E^M = (w-c)(w+3-Z)e^{-\frac{w+1+Z}{2}}. \tag{2.22}$$

Proposition 2.4.5 *In the PD-newsvendor model under a wholesale price-only contract with $D(p) = e^{-p}$, the equilibrium wholesale price, $\hat{w}_E^*(\geq c)$, is the unique solution to the nonlinear equation:*

$$(w+5)(w-c)+(w-c-2)\sqrt{w^2+6w+1} = 0. \tag{2.23}$$

Using Proposition 2.4.5, we are able to show:

Lemma 2.4.6 *In the PD-newsvendor model under a wholesale price-only contract with $D(p) = e^{-p}$, in equilibrium, M's expected profit is strictly smaller than R's expected profit.*

We are also able to obtain explicit expressions for the equilibrium wholesale price, w_E^*, and buyback rate, b_E^*, in the PD-newsvendor model with buybacks and exponential expected demand.

Proposition 2.4.7 *In the PD-newsvendor model with buybacks and $D(p) = e^{-p}$, M's expected profit is globally maximized at $(w_E^* = 1+c, b_E^* = 1)$, and in equilibrium,*

$$E\Pi_E^{M*} = E\Pi_E^{R*} = \frac{(c+3-H)(-c+1+H)}{4}e^{-\frac{3+c+H}{2}}, \tag{2.24}$$

where $H = \sqrt{c^2+6c+1}$.

Having derived the equilibrium expected profits of M and R, given by (2.24), and the integrated channel profit, given by (2.20), we are able to calculate the channel efficiency of the PD-newsvendor model with buybacks.

Proposition 2.4.8 *The channel efficiency of the PD-newsvendor model with buybacks and $D(p) = e^{-p}$ is $\frac{2}{e} \approx 73.58\%$.*

By Proposition 2.4.5, the equilibrium wholesale price under a price-only contract, $\hat{w}_E^*$, is implicitly given by (2.23), and it seems unlikely that a closed-form expression for $\hat{w}_E^*$, as a function of c, can be found. Fortunately though using Maple 6 we are able to solve for $\hat{w}_E^*$ for any given

	Percentage (%) improvement			Equilibrium decision variables									
				Integrated		Buybacks				No buybacks			
c	M's profit	R's profit	Channel efficiency	p_E^I	Q_E^I	w_E^*	b_E^*	p_E^*	Q_E^*	$\hat{w}_E^*$	$\hat{p}_E^*$	$\hat{Q}_E^*$
0.0	11.14	-4.33	2.02	1.00	0.7358	1.00	1.00	2.00	0.2707	0.56	1.86	0.2175
0.1	9.22	-4.16	1.51	1.18	0.5602	1.10	1.00	2.18	0.2061	0.69	2.03	0.1741
0.5	5.54	-3.23	0.70	1.78	0.2424	1.50	1.00	2.78	0.0892	1.16	2.61	0.0816
1.0	3.65	-2.43	0.38	2.41	0.1048	2.00	1.00	3.41	0.0385	1.72	3.25	0.0365
2.0	2.06	-1.54	0.17	3.56	0.0249	3.00	1.00	4.56	0.0092	2.79	4.42	0.0089
5.0	0.74	-0.63	0.04	6.74	0.0006	6.00	1.00	7.74	0.0002	5.87	7.64	0.0002

Table 2.1: Supply chain performance due to buybacks with $D(p) = e^{-p}$

value of c. Indeed, in Table 2.1 above we present the equilibrium values of the decision variables for the integrated channel, and of the channel with and without buybacks, as well as the effect of buybacks on the equilibrium expected profits of M and R and the channel efficiency. Recall that by Proposition 2.4.8, the channel efficiency under a buyback contract is $\frac{2}{e} \approx 73.58\%$.

Based on Table 2.1, we can make the following observations.

Observation 2.4.9 *The percentage increase in channel efficiency due to buybacks is decreasing in c, and is maximized at $c = 0$ for which it is* 2.02%.

Observation 2.4.10 *In equilibrium, due to the introduction of buybacks, M's expected profit increases at a decreasing rate in c and R's expected profit decreases at a decreasing rate in c.*

Table 2.1 and Observations 2.4.9 and 2.4.10 imply that for an exponential expected demand function, the introduction of buybacks in equilibrium has an insignificant effect on channel efficiency. However, by contrast, it may have a relatively large and favorable, for M, effect on the distribution of the channel profit. For example, when $c = 0$, M's expected profit increases by 11.14%, while R's expected profit decreases by 4.33%. These results are consistent with those obtained in the linear expected demand case.

Further based on Table 2.1 we can make the following observation.

Observation 2.4.11 *From Table 2.1:*

(i) $\hat{w}_E^* < w_E^*$,

(ii) $p_E^I < \hat{p}_E^* < p_E^*$ *and*

(iii) $\hat{Q}_E^* < Q_E^* < Q_E^I$.

Observation 2.4.11 is consistent with the corresponding results derived in §2.3 and §2.4.1 for the linear and negative polynomial expected demand functions.

2.5 Discussion and Extensions

In this section we reveal a close relationship between the PD-newsvendor model with buybacks and the corresponding deterministic model, discuss extensions for other distributions of ξ and examine the effect of introducing a positive salvage value for unsold inventory.

2.5.1 The PD-newsvendor model and the corresponding deterministic model

In the deterministic model, M, in Stage 1, offers a wholesale price w to R, who then determines the selling price p in Stage 2, which induces demand $D(p)$ that coincides with the expected demand function in the newsvendor model. Obviously in the deterministic case, R would order a quantity that is exactly equal to the deterministic demand. Thus, no buybacks are necessary in the deterministic model. Let us take the linear demand as an example and analyze the deterministic model with $D(p) = 1 - p$.

Example 2.5.1 Deterministic model with $D(p) = 1 - p$.

In Stage 2, for any given w, R determines p to maximize $\hat{\Pi}_R(D) = (p-w)D(p)$, where $D(p) = 1-p$. Clearly, $\hat{\Pi}_R$ is concave in p, which gives $\hat{p}^*(D) = \frac{1+w}{2}$. Taking R's reaction function $p^*(D)$ into consideration, M's profit in Stage 1 becomes: $\hat{\Pi}_M = (w-c)D(p) = \frac{1}{2}(w-c)(1-w)$, which is, again, concave in w. Thus, $\hat{w}^*(D) = \frac{1+c}{2}$, and accordingly, $\hat{p}^*(D) = \frac{3+c}{4}$ and $\hat{\Pi}^*_M(D) = 2\hat{\Pi}^*_R(D) = \frac{(1-c)^2}{8}$. For the integrated channel in the deterministic model, we substitute $w = c$ into $\hat{\Pi}_R = (p-w)(1-p)$. Then, $p^I(D) = \frac{1+c}{2}$ and the corresponding integrated channel profit is $\Pi^I(D) = \frac{(1-c)^2}{4}$.

A similar analysis can be carried out for the other two expected demand functions. The results are summarized[2.9] in Table 2.2 in the following page. These results will be further recalled in Table 4.1 in Chapter 4 and we will further refer to them in §4.5 and §4.6 in that chapter.

[2.9] Recall that c in Table 2.2 is the normalized marginal manufacturing cost due to the normalization of the deterministic part in the demand function described in §1.2. More specially, for the linear $D(p) = a(k-p)$, c represents $\frac{c}{k}$, and for the exponential $D(p) = ae^{-sp}$, c represents $s \cdot c$.

$D(p)$	$\hat{\Pi}_M^*$	$\hat{\Pi}_R^*$	Profit distribution	$\hat{w}^*$	$\hat{p}^*$	$\hat{Q}^*$	Π^I	p^I	Q^I	Channel efficiency
$1-p$	$\frac{(1-c)^2}{8}$	$\frac{(1-c)^2}{16}$	$2:1$	$\frac{1+c}{2}$	$\frac{3+c}{4}$	$\frac{1-c}{4}$	$\frac{(1-c)^2}{4}$	$\frac{1+c}{2}$	$\frac{1-c}{2}$	75%
e^{-p}	e^{-c-2}	e^{-c-2}	$1:1$	$1+c$	$2+c$	e^{-2-c}	e^{-c-1}	$1+c$	e^{-1-c}	$\frac{2}{e} \approx 73.58\%$
p^{-q}	$\frac{(q-1)^{2q-1}}{q^{2q}c^{q-1}}$	$\frac{(q-1)^{2q-2}}{q^{2q-1}c^{q-1}}$	$q-1:q$	$\frac{qc}{q-1}$	$\frac{cq^2}{(q-1)^2}$	$(\frac{q}{q-1})^{-2q}c^{-q}$	$\frac{(q-1)^{q-1}}{q^q c^{q-1}}$	$\frac{qc}{q-1}$	$(\frac{q}{c(q-1)})^{-q}$	$\frac{(2q-1)(q-1)^{q-1}}{q^q}$

Table 2.2: Equilibrium values in the deterministic model

The results derived for the PD-newsvendor model with a uniform ξ, presented in §2.3 and §2.4, and those given in Table 2.2 reveal a remarkable connection between the multiplicative PD-newsvendor model with buybacks and the corresponding deterministic model, which are summarized in Theorem 2.5.2 below.

Theorem 2.5.2 *In the PD-newsvendor model with a uniform ξ and buyback options, when the expected demand function is either linear, negative polynomial or exponential, the wholesale price, the channel efficiency and the profit distribution between M and R coincide with those in the corresponding deterministic model.*

In fact, to ascertain the robustness of the results derived for a uniform ξ, we have carried out a numerical investigation for two families of demand distributions of ξ: power distributions with a non-negative exponent ($f(\epsilon) = \gamma(\epsilon)^t$, $t \in [0, \infty)$) and triangle distributions on the interval $[r, 2-r]$ for any $r \in [0, 1)$. The numerical results are presented in Tables 2A.2, 2A.3, 2A.4 and 2A.5 in the appendix in §2.7. Moreover, for a power demand distribution, for any $t \in [0, \infty)$, Chapter 4 in this thesis further derives implicit solutions for the equilibrium values of the contract parameters and expected profits in the PD-newsvendor model with linear expected demand. The numerical study, whose results are presented in Tables 2A.2, 2A.3, 2A.4 and 2A.5 in the appendix, reveals that the results derived analytically for a uniform ξ are quite robust. More explicitly, for the power and triangle families of distributions of ξ:

(i) In equilibrium, buybacks are introduced for linear and exponential expected demand functions, while they are not used for a negative polynomial expected demand function.

(ii) The increase in channel efficiency due to buybacks is relatively insignificant, if at all.

(iii) Buybacks essentially shift the channel profit from R to M.

(iv) Buybacks increase the equilibrium retail price and inventory level.

Thus, based on the results obtained for the PD-newsvendor model for power and triangle distributions of ξ, we can make the following observation:

Observation 2.5.3 *In the PD-newsvendor model with buyback options, for power and triangle distributions of ξ and linear, negative polynomial and exponential expected demand functions:*

(i) The equilibrium wholesale and buyback prices are independent of the distribution of ξ.

(ii) *The channel profit distribution between M and R and the channel efficiency are independent of the distribution of ξ. Further, for linear and exponential expected demand functions, the profit distribution and channel efficiency are independent of the model parameters, (i.e., (c, a, k) for the model with $D(p) = a(k - p)$ and (c, a, s) for $D(p) = ae^{-s \cdot p}$.*

Observation 2.5.3 suggests that for an arbitrary distribution of ξ (> 0) and $D(p) = a(k-p)$, w_L^* and b_L^* can be derived from Proposition 2.3.7, $E\Pi_L^{M*} = 2E\Pi_L^{R*}$ and the channel efficiency is 75% for any (c, a, k); for $D(p) = p^{-q}$, w_N^* and b_N^* can be derived from Proposition 2.4.3, $E\Pi_N^{M*} = \frac{q-1}{q} E\Pi_N^{M*}$ and the channel efficiency is $\frac{(q-1)^{q-1}(2q-1)}{q^q}$, and for $D(p) = ae^{-s \cdot p}$, w_E^* and b_E^* can be derived from Proposition 2.4.7, $E\Pi_E^{M*} = E\Pi_E^{R*}$ and the channel efficiency is $\frac{2}{e} \approx 73.68\%$ for any (c, a, s).

Thus, in view of the above results we make the following conjecture:

Conjecture 2.5.4 *In the PD-newsvendor model with buyback options:*

(i) *For a general distribution of $\xi(\geq 0)$, the wholesale price, the channel efficiency and the profit distribution between M and R coincide with those in the corresponding deterministic model.*

(ii) *The buyback rate is independent of the distribution of ξ.*

Conjecture 2.5.4 implies[2.10] that the addition of buybacks to a wholesale price-only contract model increases the channel efficiency up to the efficiency of the corresponding deterministic model. This explains why buybacks are not implemented in the negative polynomial expected demand case, wherein the channel efficiency under a wholesale price-only contract coincides with the efficiency of the corresponding deterministic model.

Naturally, Conjecture 2.5.4, which was verified by Song et al. (2004) for our three expected demand functions, implies a significant reduction in the computational burden associated with solving the PD-newsvendor model with a buyback option. Indeed, the equilibrium wholesale price, efficiency and profit allocation are derived from the corresponding deterministic model. The increase in efficiency due to buybacks is available once the efficiency of the wholesale price-only contract is found, and the equilibrium buyback rate for an arbitrary ξ can be found by solving the model for a simple form of ξ, such as, e.g., a uniform ξ.

[2.10] After the completion of essentially the current version of this chapter, and motivated by essentially the current and previous versions of this chapter, Song, Ray and Li (2004) have managed to verify Conjecture 2.5.4 for a ξ which has the IFR (increasing failure rate) property and for linear, negative polynomial and exponential expected demand functions.

2.5.2 Positive salvage value of unsold inventory

A zero salvage value for both M and R was assumed for unsold inventory. There are cases, however, where some salvage value can be generated from unsold inventory. In this subsection, we will briefly consider the effect of a positive salvage value on the possible implementation of buybacks in equilibrium for a uniformly distributed ξ.

We denote by S_M (respectively, S_R) the salvage value at M's (respectively, R's) location, and we will consider the following cases: (1) $S_M = S_R = S$, (2) $S_M > S_R$ and (3) $S_M < S_R$. It is reasonable to assume that $\max(S_M, S_R) < c$ to avoid a situation of producing for salvaging. We briefly discuss the three cases below.[2.11]

(1) $S_M = S_R = S$. In this case, without loss of generality, we can assume[2.12] that $b \geq S$. Then, M's and R's expected profit functions can be written as:

$$E\Pi_M = (w - c)Q - b\frac{Q^2}{4D(p)} + S\frac{Q^2}{4D(p)} \quad \text{and} \quad E\Pi_R = (p - w)Q - (p - b)\frac{Q^2}{4D(p)}, \tag{2.25}$$

where R's expected profit function coincides with his expected profit function in the case of no salvage value.

Following the same steps as in the model with a zero salvage value, one can show that M's equilibrium values of decision variables are:

- ($w_L^* = \frac{1+c}{2}, b_L^* = \frac{1+S}{2}$) for a linear expected demand function $D(p) = 1 - p$;
- ($w_N^* = \frac{q}{q-1}c, b_N^* = \frac{q}{q-1}S$) for a negative polynomial expected demand function $D(p) = p^{-q}$, and
- ($w_E^* = 1 + c, b_E^* = 1 + S$) for an exponential expected demand function $D(p) = e^{-p}$.

Thus, the introduction of a positive salvage value at M's and R's locations in the PD-newsvendor model with buybacks does not affect the equilibrium wholesale price, and it can be further shown that it has no impact on channel efficiency and the profit distribution between M and R. Thus, Theorem 2.5.2 holds for a positive salvage value, where $S_M = S_R$. However, by contrast with the case of a zero salvage value, buybacks are implemented for a negative polynomial expected demand function when $S_M = S_R > 0$. Apparently, the introduction of a positive salvage value is enough to make a buyback option attractive for M.

[2.11] The detailed analysis is available upon request.

[2.12] For $b < S$, there are clearly no returns. For $b = S$, there is no difference between returns and no returns, and considering the extra costs that are possibly associated with returns, we can assume no returns. For $b > S$, actual returns may take place.

(2) $S_M > S_R$. As compared to Case (1), a higher salvage value for M would provide her with an additional incentive to buy back unsold inventory. Indeed, returns are introduced in all three expected demand cases.

(3) $S_M < S_R$. If R has an advantage salvaging unsold inventory, no returns may occur for all three expected demand functions. Indeed, if $S_R - S_M$ is large enough, M will prefer $b = 0$ to $b > S_R$.

The results above imply that the existence of a positive salvage value (and perhaps other costs associated with a returns policy) may have a significant effect on the possible implementation of a returns policy. We note, however, that our results are consistent with those presented in Kandel (1996) for the basic price-independent newsvendor model. Specifically, as noted in Kandel (1996), milk and flowers seem to have different returns policies. Unsold milk is usually returned to the milk processing plants, while unsold flowers are often disposed at the retail store by price discounting. The allowance for returns of unsold milk is due to the fact that milk processing plants (i.e., M) can use it to produce other dairy items, while a grocery store (i.e., R) does not have such a capability. On the other hand, it is more economic for a flower retailer to sell unsold flowers at a discount price than to return them to the flower suppliers. See further Kandel (1996) for other industrial examples, e.g., apparels and produce, where different returns policies are implemented for unsold items due to differences in salvage values.

Finally, in addition to the form of expected demand function and the salvage value of unsold inventory, factors, such as transportation cost or new product introduction consideration could also affect returns policies. For example, in the textbook publishing industry, publishers are willing, sometime even trying hard, to buy back used or unsold textbooks in order to promote and increase revenues from a new edition of the textbook.

2.6 Conclusions and Further Research

We have studied in this chapter the PD-newsvendor problem with a multiplicative probabilistic demand model. We have investigated the desirability of introducing buybacks and their effect on the equilibrium values of decision variables, channel efficiency and profit distribution for three commonly used expected demand functions: linear, negative polynomial and exponential. Initially, we have assumed a zero salvage value. For this case, we have demonstrated that in equilibrium, buybacks will be introduced for linear and exponential expected demand functions, but they are not introduced for a negative polynomial expected demand function. In those cases where buybacks are introduced, we have shown that their introduction has an insignificant effect on channel efficiency improvement. By contrast, their introduction in those cases may significantly increase M's expected profit, and significantly decrease R's expected profit. Thus, we suggest that in the absence of the salvage value, the introduction of buybacks to the PD-newsvendor model is probably not motivated by a desire to increase channel efficiency. Rather, it is more likely motivated by the significantly favorable, for M, effect it has on the distribution of the channel profit. These results partially explain why returns policies are not more common.

It is interesting to note that whenever buybacks are implemented, in equilibrium, in the PD-newsvendor model, the wholesale price, profit distribution between M and R and channel efficiency coincide with those values in the corresponding deterministic model. Since a return system involves costs not incorporated in this model (see, e.g., Lariviere (1999) and Lariviere and Porteus (2001)), buybacks will not be introduced when a wholesale price-only contract is relatively efficient. Indeed, as we have shown, buybacks are not introduced in the negative polynomial expected demand function case with a zero salvage value, wherein the channel efficiency under a wholesale price-only contract coincides with that in the deterministic model.

However, we have also shown that the existence of a positive salvage value may have a significant effect on the introduction of buybacks. For example, for a positive and equal salvage value at M's and R's locations, buybacks will be introduced for all three expected demand functions. Thus, if the salvage value at M is positive and larger than that at R, M has an additional incentive to introduce buybacks. These results may explain why some industries implement a return system, and are consistent with the related discussion in Kandel (1996).

Several natural extensions of our results could be pursued. For example, it would be useful to study other expected demand functions, and it would be interesting to extend the analysis to the PD-newsvendor model with an additive demand model. As suggested earlier, however, (see also Emmons and Gilbert (1998), Mills (1959), and Petruzzi and Dada (1999)), the additive model could produce results which are different from those derived in the multiplicative demand model. Indeed, as the following example demonstrates, Conjecture 2.5.4 is not valid for the additive model.

Example 2.6.1 The PD-newsvendor model with buybacks and $X = \xi + 1 - p$, where ξ follows a uniform distribution with $f(\epsilon) = 0.5$ on $[0, 2]$.

In this additive model, M's and R's expected profit functions, given by (1.2) can be simplified to:

$$E\Pi_M = (w-c)(z+1-p) - \frac{1}{4}bz^2 \text{ and } E\Pi_R = (p-w)(z+1-p) - \frac{1}{4}(p-b)z^2 \qquad (2.26)$$

where $z = Q - D(p)$, which is the "stocking factor", which was introduced in Petruzzi and Dada (1999).

Proposition 2.6.2 *In the additive model with buybacks, for a uniformly distributed $\xi \in [0,2]$ and for $D(p) = 1 - p$, the equilibrium value of $z^* \in [0,2]$ is the unique solution to:*

$$5z + 6 - 9z^2 - 6c - 4cz + 2z^3 = 0.$$

Accordingly, the values of the other equilibrium values of the decision variables are:

$$p^* = -\frac{(z^*)^2}{8} + \frac{5z^*}{8} + \frac{3}{4} + \frac{c}{4}, \quad w^* = 2p^* - z^* - 1 + \frac{(z^*)^2}{4} \text{ and } b^* = \frac{-4z^* - 4 + 4p^* + 2z^*p^* + (z^*)^2}{2z^*}.$$

Substituting the equilibrium values of decision variables, given in Proposition 2.6.2, into M's and R's expected profit functions, given by (2.26), we can calculate the equilibrium expected profits of M and R. These values, as well as the equilibrium values of the decision variables in this model as a function of c, are presented in Table 2A.6 in the appendix in §2.7.

In order to calculate channel efficiency in the additive model with buybacks, we need to consider the integrated channel. Proposition 2.6.3 below presents the equilibrium values of decision variables and expected profit in the integrated channel.

Proposition 2.6.3 *In the integrated channel with an additive demand model, $X = \xi + 1 - p$, where $\xi \in [0,2]$ follows a uniform distribution, the equilibrium value of $z^I \in [0,2]$ is the unique solution to:*

$$z^3 - 6z^2 + 4(1-c)z + 8(1-c) = 0,$$

and, accordingly, $p^I = \frac{4(z^I+1+c)-(z^I)^2}{8}$ and $E\Pi^I = (p^I - c)(z^I + 1 - p^I) - \frac{1}{4}p^I(z^I)^2$.

The right most column in Table 2A.6 in the appendix displays the channel efficiency of the additive model under buybacks as a function of c.

Proposition 2.6.2 and Table 2A.6 immediately imply that Conjecture 2.5.4 in the multiplicative model is not valid for the additive model. Indeed, it is not difficult to show that the equilibrium wholesale price in the additive model, given by Proposition 2.6.2, is not equal to the equilibrium wholesale price, $w^* = \frac{2+c}{2}$, in the deterministic model with demand $E(X) = 1 + D(p) = 2 - p$, which can be easily derived from Table 2.2 by using, reversely, the normalization of the expected demand function described in §1.2, i.e., from $D(p) = 1 - p$ to $D(p) = a(k - p)$. Secondly, from Table 2A.6, it is clear that channel efficiency and profit distribution between M and R depend on the value of c. Thus, they do not coincide with those values in the deterministic model, wherein channel efficiency is 75% and profit distribution between M and R is $2:1$, irrespective of the value of c.

2.7 Appendix

Proof of Proposition 2.3.1. As pointed out by E&G, for a given w, $E\Pi_L^R$ is non-decreasing in b. Thus, the proof will follow if we are able to show that $\frac{\partial E\Pi_L^M}{\partial b}(b=0) > 0$ for any $w \in (w_T, 1)$. For convenience, we present derivatives of p_L^* and Q_L^* with respect to w and b in Table 2A.1 below. Recall that p_L^* and Q_L^* are given by (2.3). The derivation of the partial derivative expressions is straightforward.

Expressions corresponding to p_L^*	Expressions corresponding to Q_L^*
$p_L^* = \frac{1}{4}(1+3b+\sqrt{(1+8w-9b)(1-b)})$	$Q_L^* = 4+2w-6p_L^*$
$\frac{\partial p_L^*}{\partial w} = \frac{1-b}{\sqrt{(1+8w-9b)(1-b)}}$	$\frac{\partial Q_L^*}{\partial w} = 2 - \frac{6(1-b)}{\sqrt{(1+8w-9b)(1-b)}}$
$\frac{\partial p_L^*}{\partial b} = -\frac{5+4w-9b-3\sqrt{(1+8w-9b)(1-b)}}{4\sqrt{(1+8w-9b)(1-b)}}$	$\frac{\partial Q_L^*}{\partial b} = -6\frac{\partial p_L^*}{\partial b} = \frac{3(5+4w-9b-3\sqrt{(1+8w-9b)(1-b)})}{2\sqrt{(1+8w-9b)(1-b)}}$

Table 2A.1: Summary of some partial derivatives

It follows from (2.4) that

$$\frac{\partial E\Pi_L^M}{\partial b} = (w-c)\frac{\partial Q_L^*}{\partial b} - \frac{1}{4}Q_L^*(1-p_L^*)^{-2}[(1-p_L^*)(Q_L^*+2b\frac{\partial Q_L^*}{\partial b}) + bQ_L^*\frac{\partial p_L^*}{\partial b}]. \tag{2A.1}$$

By evaluating p_L^*, Q_L^*, $\frac{\partial p_L^*}{\partial b}$ and $\frac{\partial Q_L^*}{\partial b}$, given in Table 2A.1, at $b=0$, substituting them and $b=0$ into (2A.1) and simplifying, we obtain that: $\frac{\partial E\Pi_L^M}{\partial b}(b=0) = \frac{(-3+\sqrt{1+8w})^2}{16\sqrt{1+8w}}(20w+1-12c-3\sqrt{1+8w})$.

Let $A_1(w) \equiv 20w+1-12c-3\sqrt{1+8w}$. $\frac{\partial E\Pi_M}{\partial b}(b=0) > 0$ if and only if $A_1(w) > 0$. Note that $A_1(w)$ is convex in w and it has two stationary points, w_1 and w_2, where $w_1 = \frac{2+30c-3\sqrt{6(1+5c)}}{50} < c < \frac{2+30c+3\sqrt{6(1+5c)}}{50} = w_2 < 1$. The last two strict inequalities follow since $c < 1$. Let $w_T = w_2$. Since $A_1(w) > 0$ when $w > w_T$, we have verified that for any $w \in (w_T, 1)$, $\frac{\partial E\Pi_L^M}{\partial b}(b=0) > 0$. □

Proof of Proposition 2.3.2. By substituting $b=0$ into (2.3), we derive R's reaction functions in a wholesale price-only contract: $\hat{p}_L^* = \frac{1+\sqrt{1+8w}}{4}$ and $\hat{Q}_L^* = \frac{(3-\sqrt{1+8w})^2}{4}$. Upon their substitution in M's expected profit function, given by (2.7), and simplifying, we obtain: $E\hat{\Pi}_L^M = \frac{(w-c)(3-\sqrt{1+8w})^2}{4}$. The first derivative of $E\hat{\Pi}_L^M$ with respect to w is: $\frac{dE\hat{\Pi}_L^M}{dw} = \frac{(3-\sqrt{1+8w})}{4\sqrt{1+8w}}(-16w+3\sqrt{1+8w}-1+8c)$.

Let $A_2(w) \equiv -16w + 3\sqrt{1+8w} - 1 + 8c$. Then, since $c \leq w < 1$, $\frac{dA_2(w)}{dw} = -16 + \frac{12}{\sqrt{1+8w}} < 0$. Thus, $A_2(w)$ is strictly decreasing in w. Let $\bar{w} = \frac{5+32c+3\sqrt{17+64c}}{64}$. It follows from the definition of $A_2(w)$ that $A_2(w) > 0$ for $c \leq w < \bar{w}$, $A_2(w) < 0$ for $\bar{w} < w < 1$ and $A_2(w) = 0$ for $w = \bar{w}$. Since $\frac{dE\hat{\Pi}_L^M}{dw} = \frac{(3-\sqrt{1+8w})}{4\sqrt{1+8w}} A_2(w)$ and $\frac{(3-\sqrt{1+8w})}{4\sqrt{1+8w}} > 0$, we conclude that $\frac{dE\hat{\Pi}_L^M}{dw} > 0$ for $c \leq w < \bar{w}$, $\frac{dE\hat{\Pi}_L^M}{dw} < 0$ for $\bar{w} < w < 1$ and $\frac{dE\hat{\Pi}_L^M}{dw} = 0$ for $w = \bar{w}$. Therefore, $E\hat{\Pi}_L^M$ is pseudo-concave in $w \in [c, 1)$ and is uniquely maximized at $\hat{w}_L^* = \bar{w} = \frac{5+32c+3\sqrt{17+64c}}{64}$. □

Proof of Lemma 2.3.3. Simply compare $w_T = \frac{2+30c+3\sqrt{6(1+5c)}}{50}$ and $\hat{w}_L^* = \frac{5+32c+3\sqrt{17+64c}}{64}$. □

Proof of Lemma 2.3.5. From (2.9) the ratio, F, of M's and R's equilibrium expected profits can be simplified to $F \equiv \frac{E\hat{\Pi}_L^{M*}}{E\hat{\Pi}_L^{R*}} = \frac{6+2\sqrt{17+64c}}{7+\sqrt{17+64c}}$, which is strictly increasing in c. Thus, the ratio of M's and R's equilibrium expected profits, F, is bounded between $F(c=0) \approx 1.28$ and $F(c \to 1) = 1.5$. □

Proof of Proposition 2.3.6. By (2.6) and (2.10), the efficiency of a wholesale price-only contract is:

$$\frac{E\hat{\Pi}_L^{Total*}}{E\Pi_L^I} * 100\% = \frac{(9-\sqrt{17+64c})^3(13+3\sqrt{17+64c})}{256(3-\sqrt{1+8c})^3(1+\sqrt{1+8c})} = \frac{2(13+3\sqrt{17+64c})(3+\sqrt{1+8c})^3}{(1+\sqrt{1+8c})(9+\sqrt{17+64c})^3} * 100\%,$$

which increases in c, and thus it is bounded between 71.84% for $c = 0$ and 74.07% for $c \to 1$. □

Proof of Proposition 2.3.7. It follows from (2.4) that

$$\frac{\partial E\Pi_L^M}{\partial w} = Q_L^* + (w-c)\frac{\partial Q_L^*}{\partial w} - \frac{1}{4}bQ_L^*(1-p_L^*)^{-2}[2(1-p_L^*)\frac{\partial Q_L^*}{\partial w} + Q_L^*\frac{\partial p_L^*}{\partial w}]. \quad (2A.2)$$

Substituting p_L^*, Q_L^*, $\frac{\partial p_L^*}{\partial w}$ and $\frac{\partial Q_L^*}{\partial w}$, given in Table 2A.1, into (2A.2), and simplifying gives us: $\frac{\partial E\Pi_L^M}{\partial w} = \frac{(A-3B)}{4AB}[(A-3B)(AB+3b)+8(w-c)B]$, where $A \equiv \sqrt{1+8w-9b}$ and $B \equiv \sqrt{1-b}$. Since $b \leq w < 1$, $A - 3B < 0$. Thus, the first-order condition of $E\Pi_L^M$ with respect to w implies:

$$(A-3B)(AB+3b) + 8B(w-c) = 0. \quad (2A.3)$$

Similarly, by substituting p_L^*, Q_L^*, $\frac{\partial p_L^*}{\partial b}$ and $\frac{\partial Q_L^*}{\partial b}$, given in Table 2A.1, into (2A.1), and simplifying, the first-order condition of $E\Pi_L^M$ with respect to b yields:

$$(A-3B)[2B(AB+6b) + b(A-3B)] + 24B^2(w-c) = 0. \quad (2A.4)$$

Solving (2A.3) and (2A.4) reveals that $(w_L^* = \frac{1+c}{2}, b_L^* = \frac{1}{2})$ is the unique stationary point of M's expected profit function, and the Hessian matrix at this stationary point is:

$$\begin{vmatrix} \frac{\partial^2 E\Pi_M}{\partial w^2} & \frac{\partial^2 E\Pi_M}{\partial w \partial b} \\ \frac{\partial^2 E\Pi_M}{\partial b \partial w} & \frac{\partial^2 E\Pi_M}{\partial b^2} \end{vmatrix} = \frac{4}{z(-3+z)^3} \begin{vmatrix} Mww & Mwb \\ Mbw & Mbb. \end{vmatrix},$$

where $z \equiv \sqrt{1+8c}$, $Mww \equiv 8c^3z - 72c^3 + 648c^2 - 24zc^2 - 462zc + 1350c + 261 - 251z$, $Mbb \equiv 8(8c^2 + 56c - 12zc + 17 - 15z)$ and $Mwb = Mbw \equiv 6(-60c^2 + 4zc^2 - 150c + 46zc - 33 + 31z)$.

Since $c \in [0,1)$, it is not difficult to verify that $z < 3$, $Mww > 0$, $Mbb > 0$ and $Mwb = Mbw < 0$. Furthermore, we have $Mww \cdot Mbb - Mwb \cdot Mbw > 0$, which implies that the Hessian matrix at $(w^* = \frac{1+c}{2}, b^* = \frac{1}{2})$ is negative definite. Thus, this point is the global maximizer of M's problem. Accordingly, we have: $E\Pi_L^{M*} = 2E\Pi_L^{R*} = \frac{(3-\sqrt{1+8c})^3(1+\sqrt{1+8c})}{128}$. □

Proof of Proposition 2.3.9. From (2.9) and (2.11), after some simplifications, the percentage improvement of M's equilibrium expected profit due to the introduction of buybacks reduces to:

$$\frac{E\Pi_L^{M*} - E\hat{\Pi}_L^{M*}}{E\hat{\Pi}_L^{M*}} * 100\% = (\frac{(1+\sqrt{1+8c})(9+\sqrt{17+64c})^3}{8(3+\sqrt{17+64c})(3+\sqrt{1+8c})^3} - 1) * 100\%, \tag{2A.5}$$

which decreases in c, and thus it is bounded between 12.5% for $c \to 1$ and 23.94% for $c = 0$. □

Proof of Proposition 2.3.10. From (2.9) and (2.11), after some simplifications, the percentage deterioration of R's equilibrium expected profit due to the introduction of buybacks reduces to:

$$\frac{E\Pi_L^{R*} - E\hat{\Pi}_L^{R*}}{E\hat{\Pi}_L^{R*}} * 100\% = (\frac{(1+\sqrt{1+8c})(9+\sqrt{17+64c})^3}{8(7+\sqrt{17+64c})(3+\sqrt{1+8c})^3} - 1) * 100\%, \tag{2A.6}$$

which increases in c, and thus it is bounded between -20.63% for $c = 0$ and -15.62% for $c \to 1$. □

Proof of Proposition 2.3.11. (i) $\hat{w}_L^* < w_L^*$. By Propositions 2.3.2 and 2.3.7, we know that $\hat{w}_L^* = \frac{5+32c+3\sqrt{17+64c}}{64}$ and $w_L^* = \frac{1+c}{2}$. Thus, we have $w_L^* - \hat{w}_L^* = \frac{3}{64}(9 - \sqrt{17+64c}) > 0$ since $c < 1$.

(ii) $p_L^I < \hat{p}_L^* < p_L^*$. By (2.5) and (2.8), we have $\hat{p}_L^* - p_L^I = \frac{7+\sqrt{17+64c}}{16} - \frac{1+\sqrt{1+8c}}{4}$. We simplify it to $\frac{1}{16}(3 + \sqrt{17+64c} - 4\sqrt{1+8c})$, which can be shown to be positive for any $c < 1$. Thus, $p_L^I < \hat{p}_L^*$. Similarly, by (2.8) and (2.12), we have $p_L^* - \hat{p}_L^* = \frac{5+\sqrt{1+8c}}{8} - \frac{7+\sqrt{17+64c}}{16} = \frac{1}{16}(3 + 2\sqrt{1+8c} - \sqrt{17+64c})$. It can be shown that $\hat{p}_L^* < p_L^*$ for any $c < 1$. Thus, $p_L^I < \hat{p}_L^* < p_L^*$. Similarly for the proof of (iii). □

Proof of Proposition 2.4.1. For any given w, b and p, $E\Pi_N^R$, given in (2.13), is concave in Q. Thus, at optimality, $Q_N^* = \frac{2p^{-q}(p-w)}{p-b}$. By substituting Q_N^* into $E\Pi_N^R$, we obtain $E\Pi_N^R = \frac{p^{-q}(p-w)^2}{p-b}$. By employing the same proof method as that used in Proposition 2.3.2 to prove that $E\hat{\Pi}_L^M$ is pseudo-concave in w, we can prove that $E\Pi_N^R$ is pseudo-concave in p. Thus, $E\Pi_N^R$ is uniquely maximized at $p_N^* = \frac{qw+qb+w-2b+J}{2(q-1)}$, where $J \equiv \sqrt{(q+1)^2w^2 - 2(q^2-q+2)wb + (q-2)^2b^2}$. □

Proof of Proposition 2.4.2. As in the proof of Proposition 2.3.1, for any w, R's expected profit is non-decreasing in b. Thus, the proof will follow if we can show that $WT \equiv \frac{\partial E\Pi_N^M}{\partial b}(b = 0) > 0$ for any $w > w_T^N$, where $E\Pi_N^M$ is given by (2.16). By using Maple 6, we get that $WT = -\frac{4(q-1)^q}{w^{q+1}(q+1)^{q+2}}(-w(q-1) + qc)$, where $-\frac{4(q-1)^q}{w^{q+1}(q+1)^{q+2}} < 0$ since $q > 1$. Thus, $WT > 0$ for any $w > \frac{qc}{q-1}$. □

Proof of Proposition 2.4.3. Using Maple 6 to solve, simultaneously, the first-order conditions of $E\Pi_N^M$ with respect to w and b, i.e., $\frac{\partial E\Pi_N^M}{\partial w} = 0$ and $\frac{\partial E\Pi_N^M}{\partial b} = 0$, gives a unique solution $(w_N^* =$

$\frac{qc}{q-1}, b_N^* = 0$). (For brevity, we don't present the intermediate results here.) The Hessian matrix at this stationary point is:

$$\left| \begin{array}{cc} \frac{\partial^2 E\Pi_M}{\partial w^2} & \frac{\partial^2 E\Pi_M}{\partial w \partial b} \\ \frac{\partial^2 E\Pi_M}{\partial b \partial w} & \frac{\partial^2 E\Pi_M}{\partial b^2} \end{array} \right| = \left| \begin{array}{cc} Mww = -4[\frac{(q-1)^2}{cq(q+1)}]^{q+1} & Mwb \\ Mbw & Mbb = -\frac{8(q-1)^{2q+2}}{c^{q+1}q^q(q+1)^{q+4}} \end{array} \right|,$$

where $Mwb = Mbw \equiv \frac{4(q-1)^{2q+2}}{(cq)^{q+1}(q+1)^{q+2}}$. Since $q > 1$, we have $Mww \cdot Mbb - Mwb \cdot Mbw > 0$, which implies that the Hessian matrix at the unique stationary point is negative definite. Thus, the unique stationary point ($w^* = \frac{qc}{q-1}, b^* = 0$) is the global maximizer of M's problem in Stage 1. Upon substitution of this point, and the corresponding p_N^* and Q_N^*, given by (2.14), into M's and R's expected profit functions, given by (2.13), we obtain that in the PD-newsvendor model with a negative polynomial expected demand function, $E\Pi_N^{M*} = \frac{4(q-1)^{2q-1}}{c^{q-1}q^q(q+1)^{q+1}}$, $E\Pi_N^{R*} = \frac{4(q-1)^{2q-2}}{c^{q-1}q^{q-1}(q+1)^{q+1}}$, and $\frac{E\Pi_N^{M*}}{E\Pi_N^{R*}} = \frac{q-1}{q}$. □

Proof of Proposition 2.4.5. By (2.22), we have $\frac{\partial E\hat{\Pi}_E^M}{\partial w} = \frac{e^{-\frac{w+1+Z}{2}}(w+3-Z)}{2Z}B(w)$, where $B(w) \equiv -wZ + cZ + 2Z - w^2 + wc - 5w + 5c$ and $Z = \sqrt{w^2 + 6w + 1}$. Note that $B(w)$ can be transformed to: $B(w) = (-w + c + 2)\sqrt{w^2 + 6w + 1} - (w + 5)(w - c)$, which is concave in w since $\frac{\partial^2 B(w)}{\partial^2 w} < 0$. Since $B(w = c) > 0$ and $B(w) < 0$ for w large enough, there exists a unique $\bar{w}$ such that $B(w) > 0$ for $c \leq w < \bar{w}$, $B(w) < 0$ for $\bar{w} < w$ and $B(w) = 0$ for $w = \bar{w}$. Since $\frac{\partial E\hat{\Pi}_E^M}{\partial w} = \frac{e^{-\frac{w+1+Z}{2}}(w+3-Z)}{2Z}B(w)$ and $\frac{e^{-\frac{w+1+Z}{2}}(w+3-Z)}{2Z} > 0$, we conclude that $\frac{\partial E\hat{\Pi}_E^M}{\partial w} > 0$ for $c \leq w < \bar{w}$, $\frac{\partial E\hat{\Pi}_E^M}{\partial w} < 0$ for $\bar{w} < w$ and $\frac{\partial E\hat{\Pi}_E^M}{\partial w} = 0$ for $w = \bar{w}$. Therefore, $E\hat{\Pi}_E^M$ is pseduo-concave in $w \in [c, \infty)$ and uniquely maximized at $\hat{w}_E^*$, where $\hat{w}_E^*$ is the unique solution to $B(w) = 0$ such that $\hat{w}_E^* \geq c$. □

Proof of Lemma 2.4.6. Substituting $b = 0$ into M's and R's expected profit functions under a wholesale price-only contract, given by (2.18), gives $E\hat{\Pi}_E^M = (w - c)Q$ and $E\hat{\Pi}_E^R = (p - w)Q - \frac{pQ^2}{4e^{-p}}$. To prove $E\hat{\Pi}_E^{M*} < E\hat{\Pi}_E^{R*}$, we need to show that in equilibrium, $(w - c)Q < (p - w)Q - \frac{pQ^2}{4e^{-p}}$, i.e., $0 < Q < \frac{4e^{-p}(p+c-2w)}{p}$. By substituting $\hat{p}_E^*$, given by (2.21), into $\bar{Q} \equiv \frac{4e^{-p}(p+c-2w)}{p}$, and simplifying, we obtain: $\bar{Q} = \frac{4e^{-\frac{w+1+Z}{2}}(-3w+1+Z+2c)}{w+1+Z}$, where $Z = \sqrt{w^2 + 6w + 1}$. Thus, we need to compare, at $w = \hat{w}_E^*$, the equilibrium order quantity in the wholesale price-only contract, $\hat{Q}_E^*$, given by (2.21), and $\bar{Q}$, where $\hat{w}_E^*$ is the equilibrium wholesale price, which satisfies (2.23) in Proposition 2.4.5. Now, at $w = \hat{w}_E^*$, $\hat{Q}_E^* < \bar{Q}$ if $(w + 3 - Z)(w + 1 + Z) < 4(-3w + 1 + Z + 2c)$, which holds if and only if $5w-4c-1 < Z$. By Proposition 2.4.5, in equilibrium, $c = \frac{1+3w^2+18w-(w+5)Z}{2(6+w)}$, and upon substituting c into $5w-4c-1 < Z$ and simplifying, we obtain that $\hat{Q}_E^* < \bar{Q}$ if $(4+w)\sqrt{w^2 + 6w + 1} < w^2+7w+8$, which holds for any $w (\geq c)$. Thus, we conclude that in equilibrium, $\hat{Q}_E^* < \bar{Q}$, which completes the proof of Lemma 2.4.6. □

Proof of Proposition 2.4.7. Having derived R's reaction functions, p_E^* and Q_E^*, given by (2.19), M's expected profit function becomes: $E\Pi_E^M = (w-c)Q_E^* - \frac{b(Q_E^*)^2}{4e^{-p_E^*}}$.

Using Maple 6 to solve, simultaneously, the first-order conditions of $E\Pi_E^M$ with respect to w and b, i.e., $\frac{\partial E\Pi_E^M}{\partial w} = 0$ and $\frac{\partial E\Pi_E^M}{\partial b} = 0$, gives a unique stationary point $(w_E^* = 1+c, b_E^* = 1)$. The Hessian matrix (for brevity, again, we don't present the intermediate results) at this point is:

$$\begin{vmatrix} \frac{\partial^2 E\Pi_M}{\partial w^2} & \frac{\partial^2 E\Pi_M}{\partial w \partial b} \\ \frac{\partial^2 E\Pi_M}{\partial b \partial w} & \frac{\partial^2 E\Pi_M}{\partial b^2} \end{vmatrix} = \begin{vmatrix} Mww & Mwb \\ Mbw & Mbb \end{vmatrix},$$

where

$$Mww \equiv -\frac{16\beta(2c^4 + 18c^3 + 2c^3H + 12c^2H + 42c^2 + 14cH + 23c + 3 + 3H)}{H(1+c+H)^4},$$

$$Mbb \equiv -\frac{8c^3 + 11c^2 - c^2H + 8cH + 17c + 3 + 3H)\beta}{H(1+c+H)^4},$$

$$Mwb = Mbw = \frac{16(2c^3 + 2c^2H + 15c^2 + 7cH + 13c + 2 + 2H)\beta}{H(1+c+H)^4},$$

where $\beta = e^{-\frac{3+c+H}{2}}$, and, as we recall, $H = \sqrt{c^2+6c+1}$. One can verify that since $\beta > 0$ and $Mww < 0$, $Mbb < 0$ and $Mww \cdot Mbb - Mwb \cdot Mbw > 0$, which implies that the Hessian matrix at the unique stationary point is negative definite, and M's expected function is globally maximized at $(w^* = 1+c, b^* = 1)$. Thus, the equilibrium expected profits of M and R are: $E\Pi_E^{M*} = E\Pi_E^{R*} = \frac{(c+3-H)(-c+1+H)}{4} e^{-\frac{3+c+H}{2}}$, where $H = \sqrt{c^2+6c+1}$. □

Proof of Proposition 2.6.2. Recall that backward induction is used to solve the two stage Stackelberg game.

Stage 2: Given (w,b), R chooses (p,z) to maximize $E\Pi_R = (p-w)(z+1-p) - \frac{(p-b)z^2}{4}$, as displayed in (2.26), which is concave in z for any given p. Thus, $E\Pi_R$ is maximized at $z^* = \frac{2(p-w)}{p-b}$, and accordingly, $E\Pi_R(p) = (p-w)(1-p) + \frac{(p-w)^2}{p-b}$, which is unimodal in p. Therefore, we can conclude that there exists a unique interior solution (p^*, z^*), which satisfies the first-order conditions: $\frac{\partial E\Pi_R}{\partial z} = 0$ and $\frac{\partial E\Pi_R}{\partial p} = 0$, which gives:

$$w = 2p - z - 1 + \frac{z^2}{4} \text{ and } b = \frac{-4z - 4 + 4p + 2zp + z^2}{2z}. \tag{2A.7}$$

Stage 1: In Stage 1, we work with (p,z) for M's problem instead of with (w,b). Substituting (w,b), given by (2A.7), into M's expected profit function, $E\Pi_M = (w-c)(z+1-p) - \frac{bz^2}{4}$, and simplifying gives $E\Pi_M = -2p^2 + p(\frac{5}{2}z + 3 - \frac{z^2}{2} + c) - \frac{z^2}{4} - \frac{3z}{2} - 1 + \frac{z^3}{8} - cz - c$, which is clearly concave in p and uniquely maximized at $p^*(z) = -\frac{z^2}{8} + \frac{5z}{8} + \frac{3}{4} + \frac{c}{4}$ for any given z. Thus, M's expected profit function reduces to $E\Pi_M(z) = \frac{1}{32}(z^4 - 6z^3 + 5z^2 - 4cz^2 + 12z - 12cz + 4 - 8c + 4c^2)$, which can be easily shown to be unimodal in z. Thus, the first-order condition gives us the equilibrium value of z^*, which satisfies: $5z + 6 - 9z^2 - 6c - 4cz + 2z^3 = 0$. $\square$

Proof of Proposition 2.6.3. Similar to the analysis in Stage 1 in the proof of Proposition 2.6.2, the expected profit in the integrated channel, $E\Pi_I = (p - c)(z + 1 - p) - \frac{1}{4}pz^2$, is unimodal in (p, z). Thus, Substituting $w = c$ and $b = 0$ into the first-order conditions, given by (2A.7), we can derive the equilibrium value of the stocking factor, $z^I \in [0, 2]$, is the unique solution to $G(z) = z^3 - 6z^2 + 4(1 - c)z + 8(1 - c) = 0$, and the equilibrium value of the retail price is $p^I = \frac{4(z^I+1+c)-(z^I)^2}{8}$. $\square$

	Percentage (%) improvement in equilibrium values due to buybacks											
	$t = 0$			$t = 1$			$t = 2$			$t = 4$		
c	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency
0.00	23.94	-20.78	3.10	15.49	-14.51	2.46	11.42	-10.84	2.09	7.48	-7.45	1.47
0.10	20.91	-19.90	2.44	13.76	-13.82	2.04	10.20	-10.71	1.63	6.71	-7.25	1.19
0.20	18.93	-19.27	1.99	12.46	-13.30	1.71	9.25	-10.32	1.36	6.10	-7.03	0.98
0.40	16.34	-18.11	1.49	10.63	-12.71	1.16	7.84	-9.63	0.97	5.15	-6.35	0.75
0.80	13.47	-16.74	0.91	8.35	-11.03	0.75	6.06	-8.23	0.61	3.91	-5.46	0.44
0.90	12.83	-15.91	0.95	7.96	-10.72	0.69	5.74	-8.12	0.50	3.69	-5.33	0.37

Table 2A.2: Improvement in the equilibrium values of M's and R's expected profits and channel efficiency for $f(\epsilon) = \gamma \cdot (\epsilon)^t$ and $D(p) = 1 - p$

	Percentage (%) improvement in equilibrium values due to buybacks											
	$t=0$			$t=1$			$t=2$			$t=4$		
c	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency
0.00	11.14	-4.32	2.03	7.08	-2.58	1.46	5.19	-1.83	1.13	3.38	-1.10	0.79
0.10	9.22	-4.10	1.53	6.05	-2.46	1.17	4.48	-1.78	0.91	2.94	-1.11	0.64
0.50	5.54	-3.33	0.66	3.76	-2.04	0.57	2.82	-1.42	0.48	1.87	-0.91	0.34
1.00	3.65	-2.40	0.39	2.46	-1.57	0.30	1.84	-1.13	0.25	1.22	-0.66	0.20
2.00	2.06	-1.50	0.18	1.35	-0.92	0.15	0.99	-0.70	0.10	0.65	-0.44	0.08
5.00	0.74	-0.66	0.03	0.45	-0.32	0.05	0.32	-0.30	0.01	0.20	-0.18	0.01

Table 2A.3: Improvement in the equilibrium values of M's and R's expected profits and channel efficiency for $f(\epsilon) = \gamma \cdot (\epsilon)^t$ and $D(p) = e^{-p}$

	Percentage (%) improvement in equilibrium values due to buybacks														
	$r = 0$			$r = 0.25$			$r = 0.5$			$r = 0.75$			$r = 0.9$		
c	$E\Pi^*_M$	$E\Pi^*_R$	Channel efficiency	$E\Pi^*_M$	$E\Pi^*_R$	Channel efficiency	$E\Pi^*_M$	$E\Pi^*_R$	Channel efficiency	$E\Pi^*_M$	$E\Pi^*_R$	Channel efficiency	$E\Pi^*_M$	$E\Pi^*_R$	Channel efficiency
0.00	22.49	-9.40	6.59	18.21	-8.00	5.53	12.92	-5.66	4.22	6.73	-2.69	2.46	2.73	-0.97	1.08
0.10	15.77	-12.42	3.26	12.97	-10.31	2.87	9.26	-7.12	2.32	4.82	-3.42	1.41	1.95	-1.29	0.63
0.20	12.90	-13.05	1.96	10.62	-10.64	1.83	7.51	-7.14	1.57	3.85	-3.43	0.97	1.54	-1.29	0.43
0.40	10.61	-12.35	1.28	8.44	-9.90	1.16	5.67	-6.12	1.05	2.75	-2.73	0.64	1.07	-1.18	0.23
0.80	8.35	-11.03	0.75	5.36	-6.82	0.72	2.29	-3.70	0.46	1.18	-1.60	0.17	0.38	-0.54	0.05
0.90	7.93	-10.72	0.69	4.29	-5.59	0.58	2.13	-2.88	0.30	0.84	-1.18	0.12	0.30	-0.49	0.03

Table 2A.4: Improvement in the equilibrium values of M's and R's expected profits and channel efficiency for a triangle distribution of ξ and $D(p) = 1 - p$

	Percentage (%) improvement in equilibrium values due to buybacks														
	$r = 0$			$r = 0.25$			$r = 0.5$			$r = 0.75$			$r = 0.9$		
c	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency	$E\Pi_M^*$	$E\Pi_R^*$	Channel efficiency
0.00	12.55	-0.58	3.89	10.39	-0.41	3.31	7.52	0.06	2.59	4.00	0.67	1.66	1.64	0.37	0.73
0.10	9.14	-1.47	2.53	7.60	-0.79	2.31	5.52	-0.12	1.88	2.94	0.15	1.10	1.21	0.08	0.47
0.50	4.26	-1.46	0.96	3.54	-1.33	0.76	2.56	-0.90	0.58	1.36	-0.40	0.35	0.56	-0.13	0.16
1.00	2.47	-1.31	0.40	2.08	-0.98	0.39	1.50	-0.82	0.24	0.78	-0.35	0.16	0.32	-0.18	0.05
2.00	1.35	-0.95	0.14	1.09	-0.75	0.12	0.74	-0.62	0.04	0.37	-0.21	0.05	0.14	-0.08	0.02
5.00	0.45	-0.23	0.08	0.33	-0.25	0.03	0.20	-0.15	0.02	0.09	-0.04	0.02	0.03	-0.08	0.00

Table 2A.5: Improvement in the equilibrium values of M's and R's expected profits and channel efficiency for a triangle distribution of ξ and $D(p) = e^{-p}$

c	w^*	b^*	p^*	z^*	Q^*	$E\Pi_M^*$	$E\Pi_R^*$	Ratio	p^I	z^I	Q^I	$E\Pi^I$	Efficiency
0.00	0.88	0.70	1.41	1.50	1.09	0.56	0.18	3.09	1.00	2.00	2.00	1.00	74.71%
0.10	0.89	0.66	1.40	1.36	0.97	0.46	0.14	3.19	1.05	1.81	1.76	0.81	74.64%
0.20	0.91	0.62	1.38	1.24	0.86	0.37	0.11	3.31	1.08	1.63	1.55	0.65	74.56%
0.30	0.93	0.58	1.37	1.12	0.75	0.29	0.08	3.46	1.11	1.46	1.35	0.50	74.46%
0.40	0.95	0.55	1.35	1.00	0.65	0.22	0.06	3.67	1.14	1.30	1.16	0.38	74.32%
0.50	0.97	0.52	1.33	0.88	0.55	0.16	0.04	3.97	1.16	1.14	0.98	0.27	74.12%
0.60	0.99	0.49	1.31	0.77	0.46	0.11	0.02	4.47	1.17	0.97	0.80	0.18	73.82%
0.70	1.01	0.46	1.28	0.65	0.37	0.07	0.01	5.48	1.17	0.80	0.63	0.11	73.30%
0.80	1.03	0.43	1.24	0.52	0.28	0.04	0.004	8.88	1.16	0.63	0.46	0.05	72.21%

Table 2A.6: Equilibrium values in the additive model with buybacks and $D(p) = 1 - p$

Chapter 3

On Sequential Commitment in the Price-Dependent Newsvendor Model

3.1 Introduction

It is essentially exclusively assumed in the vast literature on the decentralized newsvendor model that the decision makers commit to the values of the decision variables under their control *simultaneously.* For example, in the price-dependent (PD) newsvendor model with buybacks (e.g., Kandel (1996), Emmons and Gilbert (1998)), the manufacturer (M), assumed to be the Stackelberg leader, initiates the process by offering a take-it-or-leave-it contract, in which M decides (after taking the retailer's reaction into account), simultaneously, upon the values of a per unit wholesale price w and a per unit buyback rate b, at which she will buy excess items back at the end of the selling season. The retailer (R) then decides upon a per unit retail price p and an order quantity Q. Similarly, when one uses a sales-rebate scheme (Taylor (2002a)) to coordinate the channel in a PD-newsvendor model, M simultaneously decides upon w, a per unit rebate r ($r > 0$), which will be given above a fixed threshold T, and then R decides upon p and Q.

Our objective in this chapter is to introduce a sequential commitment approach for determining the contract parameters and to analyze its effect on the efficiency of the supply chain and the fortunes of its members. By sequential commitment approach we mean that the parties can decide upon the values of the decision variables sequentially and alternately, and they can also decide upon the order at which they commit to the values of the decision variables under their control. For example, consider the PD-newsvendor model with buybacks, in which M controls w and b and R controls p and Q. Then, we would like to study the effect of an approach wherein, for example, M, after setting up the value of w, expects R to either commit to a retail price p, or to an order quantity Q, or both, before she commits to a buyback rate b. After R sets the value of either p or

Q, or both, M commits to a buyback rate b, and then, R sets the value of the remaining decision variable under his control. We assume that all commitments by M and R to values of contract parameters under their control are credible and verifiable.

As compared to the traditional approach, in which contracting follows the take-it-or-leave-it paradigm, the sequential commitment approach introduces more flexibility to contracting. Indeed, while the traditional approach does not incorporate any element of bargaining or negotiation between M and R, the sequential approach captures some aspects of a bargaining process by which the values of contract parameters (e.g., wholesale price, buyback rate or order quantity) are determined. As such, our sequential commitment approach is in the spirit of contributions by, e.g., Nagarajan and Bassok (2002), and Iyer and Vilas-Boas (2003), wherein the interaction between M and R, for the purpose of determining the values of contract parameters, is modeled as the Nash bargaining problem (1950). Such an approach to model contracting between the parties should be viewed more realistic than the traditional approach. Indeed, to quote from Nagarajan and Bassok (2002): "Anecdotal evidence and articles in the academic literature have overwhelmingly indicated that relationships between agents in a supply chain are characterized by bargaining over terms of the trade. Sellers and buyers often negotiate price, quantity, delivery schedules, etc."

To gain an insight into the effect of such a sequential commitment approach, we study in this chapter its effect on the PD-newsvendor model under linear buyback contracts when M controls w and b and R controls p and Q. Linear buyback contracts, wherein the wholesale and buyback prices are constant and independent of the values of other decision variables, e.g., retail price, are very popular. Indeed, they are prevalent in several industries, and are considered to be one of the most popular contracts after wholesale price-only contracts, see, e.g., Marvel and Peck (1995).

We investigate two different power structures regarding the first mover in the channel. More specifically, we study not only the case where M is the Stackelberg leader but also the case in which R plays the Stackelberg leader role. We note, however, that being the Stackelberg leader does not necessarily imply being more powerful. Indeed, a more powerful firm could, in principle, force the other firm to move first if it is advantageous for it to do so.

The literature on supply chains with buybacks and coordination is quite extensive. In general, a supply chain composed of independent agents trying to maximize their own profits does not achieve channel coordination, see, e.g., Spengler (1950). Pasternack (1985) was the first to show that buybacks can coordinate the basic price-independent newsvendor model, wherein the retail price p is fixed exogenously, and Padmanabhan and Png (1995) have discussed and analyzed the benefits and costs of accepting returns from retailers. Subsequently, other contracts, such as, e.g., quantity-flexibility (Tsay (1999)), sales-rebate (Taylor (2002)), and revenue-sharing (Pasternack (1999), Cachon and Lariviere (2005)), have also been shown to be able to coordinate the basic newsvendor model. See also Lariviere (1999), Tsay et al. (1999) and Cachon (2004b) for some excellent reviews of coordination mechanisms for the basic newsvendor model and related models.

As noted by Kandel (1996), the price-dependent (PD) newsvendor model, wherein the retail price is determined endogenously by R, is considerably more complicated. Emmons and Gilbert (1998) have studied the PD-newsvendor model in which the expected demand function is linear and the random component of demand follows a uniform distribution, and they have shown that if the wholesale price is large enough, both M and R would benefit from the introduction of buybacks. Chapter 2 in this thesis have shown that in the PD-newsvendor model studied by Emmons and Gilbert (1998), wherein, e.g., expected demand is a linear function of the retail price, the efficiency of the supply chain with buybacks is precisely 75%, and channel efficiency improvement due to the introduction of buybacks is quite insignificant and is upper bounded by 3.16%. It has been conjectured by Kandel (1996) and Lariviere (1999), and it has been proven by Bernstein and Federgruen (2005) that constant wholesale and buyback prices (i.e., independent of other decision variables) alone cannot, in general, lead to coordination in the PD-newsvendor model. By contrast, contracts which do not employ constant wholesale and buyback prices can induce coordination. For example, revenue-sharing contracts and the "linear price discount sharing" scheme, have been shown by Cachon and Lariviere (2005) and by Bernstein and Federgruen (2005), respectively, that they could induce coordination in the PD-newsvendor model.

Our objective in this chapter is *not* to investigate channel coordination. Rather, our aim is to investigate the effect of sequential commitment in the PD-newsvendor model, bearing in mind that such an approach introduces more flexibility to the possible interaction between M and R, and that it can provide useful information to the parties who use bargaining to determine the values of contract parameters.

Our main results are:

(I) For a uniformly distributed random component of demand, and linear, exponential and negative polynomial expected demand functions:

  (i) The decision as to who will be the first mover has been endogenized. That is, R would rather have M move first, and M would be pleased to do so.

  (ii) Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$, wherein M first offers a buyback rate b, R then determines the retail price p, M subsequently decides upon the wholesale price w, and finally, R sets Q, is the unique equilibrium sequence. That is, R does not want to be the first mover, and neither party would like to resequence the order at which it has committed to the contract parameters under its control.

(II) Sequential commitment in the PD-newsvendor model can significantly increase (respectively, decrease) M's (respectively, R's) expected profit. For example, when the random component of demand follows a uniform distribution and for a linear expected demand function, Sequence 2 can increase (respectively, decrease) M's (respectively, R's) equilibrium expected profit by 79.25% (respectively, 73.51%) when the marginal manufacturing cost is 0.9.

(III) By contrast with the negligible effect of buybacks in the traditional PD-newsvendor model, buybacks, coupled with sequential commitment, can have a significant impact on channel efficiency. For example, for a uniformly distributed random part in the demand model and for a linear expected demand function, Sequence 2 can increase channel efficiency from 10.90%, for a zero manufacturing cost, to 21.25% when the manufacturing cost is 0.9.

Finally, we note that our sequential commitment approach can be viewed as decision postponement. That is, M and R delay their decisions about the values of decision variables under their control until the counterpart commits to a decision variable under their control. However, in the various postponement strategies, which were extensively studied in the Operations Research/Operations Management literature (e.g., Lee and Tang (1997), Aviv and Federgruen (2001), van Mieghem and Dada (1999), and Chapter 4 in this thesis), the decision makers delay some operational decisions (e.g., production or pricing) until additional information, usually about demand, is obtained. In our sequential commitment approach, all decisions are made before demand uncertainty is resolved.

The remainder of the chapter is organized as follows. In §3.2 we recall the traditional PD-newsvendor model with buybacks, as introduced in §1.2 in Chapter 1. In §3.3 we study the effect of sequential commitment in this model, assuming that the expected demand function is linear in the retail price. For some of the results derived in §3.3, the random portion of demand can have a general distribution, while for other results, as will be noted, we have to assume that the random part of demand is uniformly distributed. We extend the analysis to other expected demand functions, namely, exponential and negative polynomial, and to a power distribution of the random component of demand in §3.4. Conclusions and further research are discussed in §3.5. All proofs in this chapter are presented in the appendix in §3.6.

3.2 The PD-newsvendor Model with Buybacks

Consider the single-period price-dependent (PD) newsvendor model with buyback options described in Section 1.2, wherein a manufacturer (M) sells a single product to an independent retailer (R) facing stochastic demand from the end-customer market. R must commit to a per unit retail price for the entire selling season and an order quantity in advance of the selling season. The decision sequence is as follows. M, who has unlimited production capacity and can produce the items at a fixed marginal cost c, is a Stackelberg leader. M initiates the process by offering a per unit constant (or linear) wholesale price w, at which items will be sold to R prior to the selling season, and a per unit constant (or linear) buyback rate b, at which she will buy back the unsold items at the end of the selling season. In response to the proposed w and b, R commits to an order quantity Q prior to the selling season, and a per unit selling price p, at which to sell the items during the season. Thereafter, demand uncertainty is resolved. At the end of the season, R returns all unsold inventory to M, receiving a refund of b for each unit returned. It is assumed in this chapter that unsatisfied demand is lost, there is no penalty cost for lost sales, and that the salvage value of unsold inventory is zero for both M and R. Recall that, for feasibility, we always assume: (i) $c \le w \le p$ and (ii) $0 \le b \le w$.

In this chapter, we consider a multiplicative demand model, $X = D(p)\xi$, where $D(p)$ is the deterministic part of X, which decreases in the retail price p, and $\xi \in [0, U]$ captures the random factor of the demand model, and is retail price independent. Let $F(\cdot)$ and $f(\cdot)$ be the distribution and density functions of ξ, respectively.[3.1] For the multiplicative demand model, recall from (1.1) that M's and R's expected profit functions can be expressed as follows:

$$E\Pi_M(w,b) = (w-c)Q - bE[Q-X]^+ \text{ and } E\Pi_R(p,Q) = (p-w)Q - (p-b)E[Q-X]^+, \quad (3.1)$$

where $X = D(p)\xi$ and $E[Q-X]^+ = QF(\frac{Q}{D(p)}) - \int_0^{\frac{Q}{D(p)}} D(p)\epsilon f(\epsilon)d\epsilon$ is the expected unsold inventory. Since $\xi \le U$, we always assume that $Q \le UD(p)$.

If the random component, ξ, follows a power distribution on the interval[3.2] $[0, 2]$, then the density function of ξ is $f(\epsilon) = \gamma(\epsilon)^t$ for $t \in [0, \infty)$. To ensure $F(\xi = 2) = 1$, $\gamma = (t+1)2^{-(t+1)}$. Under a power distribution of ξ, we can simplify M's and R's expected profit functions, given by (3.1), to:

$$E\Pi_M(w,b) = (w-c)Q - \frac{\gamma \cdot bQ^{t+2}}{D(p)^{t+1}(t+1)(t+2)} \text{ and } E\Pi_R = (p-w)Q - \frac{\gamma \cdot (p-b)Q^{t+2}}{D(p)^{t+1}(t+1)(t+2)}. \quad (3.2)$$

[3.1] Please refer to Section 2.2 in Chapter 2 for a discussion related to the additive demand model.

[3.2] Recall from Footnote 2.3 in Chapter 2 that the analysis can be easily extended to an interval $[0, U]$ for any $U > 0$.

In the next section we study the effect of sequential commitment in the PD-newsvendor model with buybacks, assuming that the expected demand function is linear in the retail price[3.3], i.e., $D(p) = 1 - p$. For $D(p) = 1 - p$, we recall from §1.2 in Chapter 1 that $c < 1$, since for $c = 1$, both M and R get a zero profit due to the fact that demand is zero in this case.

3.3 Sequential Commitment in the PD-newsvendor Model

In this section, we introduce the sequential commitment approach and study its effect in the PD-newsvendor model with buybacks, wherein M controls (w, b) and R controls (p, Q). By contrast with the traditional approach, wherein M simultaneously offers w and b, and R, subsequently, commits to p and Q, in our sequential commitment approach, M and R can "sequence" their decision variables. Thus, M and R can commit to the decision variables under their control sequentially and alternately, as specified in the sequel.

First, we introduce some new definitions. We will refer to the PD-newsvendor model with buybacks, wherein M controls (w, b) and R controls (p, Q), as the *traditional* PD-newsvendor model, and to the usual ordering of decisions in the traditional PD-newsvendor model, wherein M first determines w and b, and R, subsequently, decides upon p and Q, as the traditional sequence, to be denoted as $\underline{M{:}w, b;\ R{:}p, Q}$. We will refer further to each possible ordering of decisions in the traditional PD-newsvendor model, resulting from sequential commitment, as a sequence, or, a sequencing instance. In general, the notation of a sequence corresponds to the order at which decisions are being made. Thus, for example, the sequence denoted as $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ corresponds to a sequential commitment where M, in Stage 1, offers b, then R, in Stage 2, decides on p, M then, in Stage 3, requests w, and finally, in Stage 4, R determines the order quantity Q. Similarly, the sequence $\underline{M{:}w;\ R{:}p, Q;\ M{:}b}$ corresponds to the case where M, in Stage 1, decides on w, R then, in Stage 2, decides simultaneously on p and Q, and finally, in Stage 3, M determines b.

Backward induction is used to solve these multi-stage Stackelberg games. In §3.3.1 we consider the case when M is the Stackelberg leader, and the effect of sequential commitment when M is the leader is discussed in §3.3.2. The case when R is the leader is analyzed in §3.3.3, and in §3.3.4 we investigate the equilibrium sequence(s).

[3.3]Recall from §1.2 in Chapter 1 that the analysis can be easily extended to a general linear expected demand function $D(p) = a(k - p)$ for any $a > 0$ and $k > 0$.

3.3.1 The manufacturer is the leader

Recall that in the PD-newsvendor model, M controls the wholesale price w and the buyback rate b. There are, in total, 7 sequencing instances resulting from the sequential commitment approach when M is the leader. We next consider each one of them separately.

The traditional sequence: $\underline{M{:}w, b;\ R{:}p, Q}$. Emmons and Gilbert (1998) have studied the traditional sequence with a linear expected demand function and a uniform distribution of ξ. For a uniform ξ, Chapter 2 in this thesis have derived closed-form expressions for the equilibrium values of the decision variables and expected profits, for linear, exponential and negative polynomial expected demand functions. Song et al. (2004) have extended these results to a ξ whose distribution has the increasing failure rate (IFR) property (i.e., $\frac{f(\epsilon)}{1-F(\epsilon)}$ is non-decreasing in ϵ). Let us recall their results for $D(p) = 1 - p$:

$$w^* = \frac{1}{2}(1+c),\ \ b^* = \frac{1}{2},\ \ p^* = \frac{1}{2}[1 + \frac{z^* - \Lambda(z^*)}{z^* - \Lambda(z^*) + \int_0^{z^*} \epsilon f(\epsilon) d\epsilon}],\ \ Q^* = (1-p^*)z^*,\ \text{ and} \tag{3.3}$$

$$E\Pi_M^* = 2E\Pi_R^* = \frac{1-p^*}{2}[z^* - \Lambda(z^*) - cz^*], \tag{3.4}$$

where z^* is the unique solution to $(1-F(z)) - c(1 + \frac{\int_0^z \epsilon f(\epsilon)d\epsilon}{z - \Lambda(z)}) = 0$, and, $\Lambda(z) = zF(z) - \int_0^z \epsilon f(\epsilon) d\epsilon$.

For example, when ξ has a uniform distribution on $[0, 2]$ (i.e., $t = 0$ in the power distribution), which has been analyzed in Chapter 2 in this thesis, the equilibrium values of decision variables and expected profits of M and R can be simplified to:

$$w^* = \frac{1+c}{2},\ \ b^* = \frac{1}{2},\ \ p^* = \frac{5+\sqrt{1+8c}}{8},\ \ Q^* = \frac{(3-\sqrt{1+8c})^2}{8},\ \text{ and} \tag{3.5}$$

$$E\Pi_M^* = 2E\Pi_R^* = \frac{(3-\sqrt{1+8c})^3(1+\sqrt{1+8c})}{128}, \tag{3.6}$$

where, as we recall, $c < 1$. The equilibrium values of decision variables and expected profits, as a function of c, under a uniformly distributed ξ are presented in the top block in Table 3A.1 in the appendix in §3.6. Note that in all tables in the appendix, including Tables 3A.2, 3A.3 and 3A.4, numbers are presented in scientific format.

Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$. According to the notation previously introduced, in Sequence 1, M initiates the process by proposing a wholesale price w in Stage 1. R then commits to a retail price p in Stage 2, M offers a buyback rate b in Stage 3, and, finally, R commits to ordering Q from M in Stage 4. Figure 3.1 in the following page represents the timeline in this sequence.

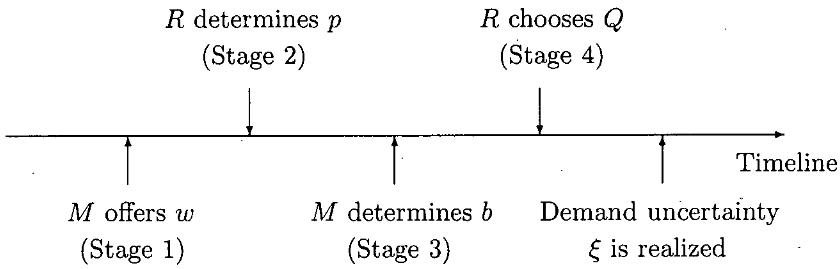

Figure 3.1: The timeline in Sequence 1: $M{:}w$; $R{:}p$; $M{:}b$; $R{:}Q$

Assumption 3.3.1 *Agents in the supply chain are willing to select an action which improves the performance of the supply chain (i.e., benefit their partners) as long as they are not adversely affected by such an action.*

Note that in Sequence 1, the decision on Q is made after the retail price p is set. Thus, choosing Q is equivalent to choosing z, where $z = \frac{Q}{D(p)}$ is the "stocking factor" introduced by Petruzzi and Dada (1999). See their paper for more details about the advantages of this transformation. For a uniformly distributed ξ, we are able to derive implicit expressions of the equilibrium values in Sequence 1.

Proposition 3.3.2 *In Sequence 1: $M{:}w$; $R{:}p$; $M{:}b$; $R{:}Q$, with a uniformly distributed $\xi \in [0,2]$ and $D(p) = 1-p$, the equilibrium value of the retail price $p^* \in [\frac{1+\sqrt{1+8c}}{4}, 1)$ is the unique solution to:*

$$(1-p)(p-c)\frac{dH_1(p)}{dp} + (1+c-2p)H_1(p) = 0, \tag{3.7}$$

where $H_1(p) = \frac{(\sqrt{2p^2-p-c}+\sqrt{p-c})^2}{p}$. Accordingly, the equilibrium values of the other decision variables are:

$$w^* = c + \sqrt{(2(p^*)^2 - p^* - c)(p^* - c)}, \quad b^* = \frac{p^*(3w^* - p^* - 2c)}{w^* + p^* - 2c}, \quad \text{and} \quad Q^* = \frac{(w^* + p^* - 2c)(1 - p^*)}{p^*},$$

and the equilibrium expected profits are:

$$E\Pi_M^* = \frac{(1-p^*)(w^* + p^* - 2c)^2}{4p^*} \quad \text{and} \quad E\Pi_R = \frac{(1-p^*)(p^* - w^*)(w^* + p^* - 2c)}{2p^*}.$$

The equilibrium values of the decision variables and expected profits in Sequence 1 for a uniformly distributed ξ, as a function of c, are presented in the top block in Table 3A.2 in the appendix.

Sequence 2: $M{:}b$; $R{:}p$; $M{:}w$; $R{:}Q$. For a uniformly distributed ξ, we have derived implicit expressions of the equilibrium values of decision variables and expected profits in this sequence.

Proposition 3.3.3 *In Sequence 2: M:b; R:p; M:w; R:Q, with a uniformly distributed* $\xi \in [0,2]$ *and* $D(p) = 1-p$*, the equilibrium value of the retail price* $p^* \in [\frac{1+\sqrt{1+8c}}{4}, 1)$ *is the unique solution to:*

$$(p-c)\frac{dH_2(p)}{dp} + H_2(p) = 0, \tag{3.8}$$

where $H_2(p) = \frac{-6p^2+5p+4pc-3c+B_2(p)}{p}$ *and* $B_2(p) = \sqrt{(2p^2-p-c)(-6p^2+7p+8pc-9c)}$*. Accordingly, the equilibrium values of the other decision variables are:*

$$b^* = \frac{-3(2(p^*)^2 - p^* - c) + B_2(p^*)}{2(2+c-3p^*)}, \quad Q^* = \frac{2(p^*-c)(1-p^*)}{(2p^*-b^*)}, \quad \text{and } w^* = p^* - \frac{(p^*-b^*)(p^*-c)}{2p^*-b},$$

and the equilibrium values of the expected profits are:

$$E\Pi_M^* = \frac{(p^*-c)H_2(p^*)}{8} \quad \text{and} \quad E\Pi_R^* = \frac{(1-p^*)(p^*-b^*)(p^*-c)^2}{(2p^*-b^*)^2}.$$

The equilibrium values of the decision variables and expected profits in Sequence 2 for a uniformly distributed ξ, as a function of c, are displayed in the top block in Table 3A.3 in the appendix.

Note that contracting which follows the sequence M:w; R:p, Q would result with the traditional wholesale price-only contract. Thus, we will refer to this sequence as the *wholesale price-only contract sequence.* Further, we will say that two sequences *coincide* if their corresponding equilibrium values of decision variables and expected profits are equal.

Next, we consider the following two sequences: Sequence 3: M:w; R:p, Q; M:b and Sequence 4: M:w; R:Q; M:b; R:p.

Sequence 3: *M:w; R:p, Q; M:b*. This sequence is a three-stage problem. In Stage 3, given (w, p, Q), M chooses b to maximize her expected profit in (3.1). Clearly, for any distribution of ξ and any form of $D(p)$, M would offer a zero buyback rate, i.e., $b^* = 0$. Thus, Sequence 3 reduces to the wholesale price-only contract sequence: M:w; R:p, Q. We immediately have:

Proposition 3.3.4 *For an arbitrary distribution of* ξ *and for an arbitrary expected demand function* $D(p)$*, Sequence 3: M:w; R:p, Q; M:b coincides with the wholesale price-only contract sequence.*

Sequence 4: *M:w; R:Q; M:b; R:p*. As shown by Proposition 3.3.5 below, Sequences 4 coincides, under some conditions, with Sequence 3 and the wholesale price-only contract sequence.

Proposition 3.3.5 *For a power distribution of* $\xi \in [0,2]$ *with* $f(\epsilon) = \gamma(\epsilon)^t$ *(where* $t > 0$ *and* $\gamma = (t+1)2^{-(t+1)}$*) and when* $D(p)$ *is a decreasing function of* p *and satisfies* $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$*, Sequence 4: M:w; R:Q; M:b; R:p coincides both with Sequence 3 and with the wholesale price-only contract sequence.*

Note that the condition $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$ in Proposition 3.3.5 is satisfied by three commonly used expected demand functions in the Operations Management and Economics literature: linear, $D(p) = 1 - p$, exponential, $D(p) = e^{-p}$, and negative polynomial, $D(p) = p^{-q}$.

Let us next study the remaining two sequences when M is the leader, which are shown in Propositions 3.3.6 and 3.3.7 below to result with R getting a zero profit.

Sequence 5: <u>M:b; R:p, Q; M:w</u>. In this three-stage sequence, in Stage 3, for any given (b, p, Q), M would choose w as large as possible, since her expected profit function, given by (3.1), is strictly increasing in w as long as $Q > 0$. Thus, $w^* = p$, which implies that R's expected profit function, given by (3.1), is not positive. R, therefore, should select $p^* = b$ if $b \geq c$ or $Q^* = 0$, as in both cases his expected profit is zero. By Assumption 3.3.1, R would select $p^* = b$ if $b \geq c$, which results with a consignment contract, in which M attains the total expected profit of the integrated channel and R gets a zero profit. M, in Stage 1, will definitely set $b \geq c$ to realize the expected profit of the integrated channel since, otherwise, she will get a zero profit. Thus, we have the following result.

Proposition 3.3.6 *For an arbitrary distribution of ξ and for an arbitrary expected demand function $D(p)$, in Sequence 5: <u>M:b; R:p, Q; M:w</u>, M gets the expected profit of the integrated channel and R gets a zero profit.*

A similar result is derived for Sequence 6 in the following proposition.

Proposition 3.3.7 *For an arbitrary distribution of ξ and for an arbitrary expected demand function $D(p)$, in Sequence 6: <u>M:b; R:Q; M:w; R:p</u>, M gets the entire expected channel profit, which is strictly less than the expected profit of the integrated channel, and R gets a zero profit.*

3.3.2 The effect of sequential commitment when M is the leader

Having analyzed all possible sequencing instances when M is the Stackelberg leader, we are able to compare the equilibrium profits of M, R and the channel for the different sequencing instances resulting from the sequential commitment approach, and to investigate the effect of such an approach on the equilibrium values of the decision variables and expected profits.

Proposition 3.3.8 *For a uniformly distributed ξ and $D(p) = 1 - p$,*

$$E\Pi_M^*(S2) > E\Pi_M^*(S1) > E\Pi_M^*(TS),$$

where "S1", "S2" and "TS" stand for "Sequence 1", "Sequence 2" and "the traditional sequence", respectively.

Tables 3A.1, 3A.2 and 3A.3 in the appendix, respectively, present the equilibrium values of the decision variables and expected profits of M, R and the channel in the traditional sequence: $\underline{M{:}w, b;\ R{:}p, Q}$, Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$ and Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ under a uniform distribution of ξ and $D(p) = 1 - p$. By comparing these values, we can immediately make the following observations:

Observation 3.3.9 *For a uniformly distributed ξ and for $D(p) = 1 - p$:*

(i) $E\Pi^*_M(S2) > E\Pi^*_M(S1) > E\Pi^*_M(TS)$.

(ii) $E\Pi^*_{M+R}(S2) > E\Pi^*_{M+R}(S1) > E\Pi^*_{M+R}(TS)$.

(iii) $E\Pi^*_R(TS) > E\Pi^*_R(S1) > E\Pi^*_R(S2)$.

(iv) $p^*(TS) > p^*(S1) > p^*(S2)$.

(v) $w^*(S2) > w^*(S1)$.

(vi) $b^*(S2) > b^*(S1)$ *and*

(vii) $Q^*(S2) > Q^*(S1)$,

where $E\Pi^*_{M+R}$ *stands for the equilibrium value of the expected channel profit.*

Note that Observation 3.3.9 (i) is consistent with Proposition 3.3.8, and implies that in these three sequences, in equilibrium, M will attain the highest expected profit in Sequence 2 and the lowest expected profit in the traditional sequence. Observation 3.3.9 (ii) suggests that both Sequence 1 and Sequence 2 increase channel efficiency, which, as we recall from Chapter 2 in this thesis, is 75% for the traditional sequence. However, it appears that they also adversely affect R's expected profit, as is evident from R's equilibrium expected profits in these three sequences presented in Tables 3A.1, 3A.2 and 3A.3 in the appendix. Indeed, from Tables 3A.1 and 3A.2, under a uniform ξ and for $c = 0.9$, as compared to the traditional sequence, Sequence 2 improves M's equilibrium expected profit and channel efficiency by 79.25% and 21.25%, respectively, and decreases R's equilibrium expected profit by 73.51%.

In general, in Sequences 1 and 2, M is delaying one of her two decisions in order to affect R's choice of retail price and order quantity in a manner beneficial to her. Apparently, it is more effective for M to delay her decision on the wholesale price w and offer a more generous buyback rate b, rather than delay her decision on b and offer a relatively low w.

By Propositions 3.3.4 and 3.3.5, Sequence 3: $\underline{M{:}w;\ R{:}p, Q,\ M{:}b}$ (for any distribution of ξ and for any form of $D(p)$) and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ (for a power distribution of ξ and for either a linear, exponential or negative polynomial $D(p)$) lead to a wholesale price-only contract.

By Propositions 2.3.6, 2.3.8, 2.3.9 and 2.3.10 in Chapter 2 in this thesis, we immediately have the following conclusion:

Corollary 3.3.10 *For a uniformly distributed ξ and for $D(p) = 1-p$, as compared to the traditional sequence, Sequence 3: $\underline{M{:}w;\ R{:}p, Q,\ M{:}b}$ and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ lead to a lower expected profit for M, a lower expected channel profit and a higher expected profit for R.*

Recall that in Sequence 5: $\underline{M{:}b;\ R{:}p, Q;\ M{:}w}$, M attains the expected profit of the integrated channel while R gets nothing. Thus, from M's point of view, Sequence 5 dominates all other sequences when she is the leader. However, we note that this sequence may not be reasonable because it forces R to commit, e.g., to an order quantity before the wholesale price is set. Similarly, in Sequence 6: $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$, M gets the entire expected channel profit and R gets nothing. But, again, it may be difficult for M to enforce Sequence 6 for the same reason that Sequence 5 may not be enforceable.

3.3.3 The retailer is the leader

The findings in the previous subsections raise a natural question: what can R achieve using the sequential commitment approach if he is the Stackelberg leader? Thus, we consider in this subsection a "power structure" in which R is the leader (see, e.g., Choi (1991), Trivedi (1998), Wang and Gerchak (2003), Gerchak and Wang (2004), Chapter 5 in this thesis, and Cachon (2004a) for other models in which R can act as the Stackelberg leader). Now, similar to the case when M is the leader, when R moves first, sequential commitment induces a total of seven sequences in the PD-newsvendor model wherein R controls p and Q and M controls w and b.

Let us first consider the sequence $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$, which will be referred to as Sequence 7. Following our general notation, in this four-stage sequence, R initiates the process by committing to a retail price p in Stage 1. In Stage 2, M sets her wholesale price w, R then commits to a quantity Q in Stage 3, and finally, M offers a buyback rate b. For a uniformly distributed ξ and for $D(p) = 1 - p$, we have the following result.

Proposition 3.3.11 *In Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$, for a uniformly distributed ξ and for $D(p) = 1 - p$, the equilibrium values of the decision variables are:*

$$p^* = \frac{1+\sqrt{1+8c}}{4}, \quad w^* = \frac{(1+\sqrt{1+8c})^2}{16}, \quad b^* = 0, \quad \textit{and} \quad Q^* = \frac{(3-\sqrt{1+8c})^2}{8}, \tag{3.9}$$

and the equilibrium expected profits of M and R are:

$$E\Pi_M^* = 2E\Pi_R^* = \frac{(3-\sqrt{1+8c})^3(1-\sqrt{1+8c})}{128}. \tag{3.10}$$

By comparing the equilibrium values in Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$ with those displayed in (3.5) and (3.6) for the traditional sequence: $\underline{M{:}w,b;\ R{:}p,Q}$, we immediately have the following conclusion.

Corollary 3.3.12 *For a uniformly distributed ξ and for $D(p) = 1 - p$:*

(i) $E\Pi^*_M(S7) = 2E\Pi^*_R(S7) = E\Pi^*_M(TS) = 2E\Pi^*_R(TS)$.

(ii) $Q^*(S7) = Q^*(TS)$.

(iii) $p^*(S7) < p^*(TS)$.

(iv) $w^*(S7) < w^*(TS)$ *and*

(v) $b^*(S7)(= 0) < b^*(TS)$,

where "TS" and "S7" stand for "the traditional sequence" and "Sequence 7", respectively.

Thus, as compared to the traditional sequence, wherein M is the leader and buybacks are implemented, Sequence 7, wherein R moves first, achieves the same equilibrium expected profits for channel members with the same equilibrium quantity, but lower wholesale and retail prices and without an implementation of a buyback policy.

The equilibrium values in the sequence $\underline{R{:}Q;\ M{:}w,b;\ R{:}p}$, which will be referred to as Sequence 8, are presented in Proposition 3.3.13 below.

Proposition 3.3.13 *For a power distribution of $\xi \in [0,2]$ with $f(\epsilon) = \gamma(\epsilon)^t$ (where $t > 0$ and $\gamma = (t+1)2^{-(t+1)}$) and when $D(p)$ is a decreasing function of p and satisfies $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$, Sequence 8: $\underline{R{:}Q;\ M{:}w,b;\ R{:}p}$, in equilibrium, results with both M and R realizing a zero profit.*

Similar to Proposition 3.3.5, the condition on $D(p)$ in Proposition 3.3.13: $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$, is satisfied by linear, exponential and negative polynomial expected demand functions.

Let us consider another sequence: $\underline{R{:}p;\ M{:}w,b;\ R{:}Q}$. In this sequence, in Stage 3, for any distribution of $\xi \in [0, U]$ and any form of $D(p)$, R's expected profit function, given by (3.1), is concave in Q and the optimal order quantity Q^* satisfies $\frac{dE\Pi_R}{dQ} = 0$, which leads to $p - w = (p-b)F(Q)$. In Stage 2, we work with (b, Q) instead of (w, b) for M's problem. Thus, M's expected profit function in Stage 2 becomes: $E\Pi_M = (p-c-(p-b)F(Q))Q - bE[Q-X]^+$, which can be easily shown to be increasing in b for any given (p, Q). Thus, $b^* = p$, which results with $w^* = p = b^*$, $E\Pi_R = 0$ for any p, and $E\Pi_M = (p-c)Q - pE[Q-X]^+$, which coincides with the expected profit function in the integrated channel. Since R's profit is always zero in Stage 1 regardless of the value of p, by Assumption 3.3.1, (p, Q) is set to maximize M's expected profit function, and we conclude that in equilibrium, M realizes the expected profit of the integrated channel and R gets a zero profit.

Consider another sequence: $\underline{R{:}Q;\ M{:}w;\ R{:}p;\ M{:}b}$ under general $\xi \in [0, U]$ and $D(p)$. In this sequence, M's choice of b is always zero, and thus, M's expected profit function becomes $E\Pi_M = (w-c)Q$. Whatever R's decision on p in Stage 3, M prefers a wholesale price w as large as possible in Stage 2, which, in turn, results with $w^* = D^{-1}(\frac{Q}{U})$, due to the assumption that $Q \leq UD(p) \leq UD(w)$. Such a choice for w^* will lead to $p^* = D^{-1}(\frac{Q}{U})$, and R has to choose $Q^* = 0$ in Stage 1, since otherwise, his expected profit is strictly negative. Therefore, in this sequence, both M and R get a zero profit.

In general, except for Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$, when R initiates the process by offering either a retail price p, or an order quantity Q, or both, his equilibrium expected profit is always zero. Proposition 3.3.14 below summarizes this result.

Proposition 3.3.14 *With exception of Sequence 7 and for a general distribution of ξ and a general form of $D(p)$, except for Sequence 8, wherein ξ is restricted to a power distribution and $D(p)$ is linear, exponential and negative polynomial, when R initiates the process by offering either a retail price p, or an order quantity Q, or both, his equilibrium expected profit is always zero. Among these six sequences, three induce a complete consignment contract in which M attains the total expected profit of the integrated channel, and in the other three sequences, M gets a zero profit.*

Corollary 3.3.12, Propositions 3.3.13 and 3.3.14 imply that for a uniform ξ and for $D(p) = 1-p$, R can never do strictly better than in the traditional sequence when he moves first.

3.3.4 The equilibrium sequence

Having considered all possible sequences, resulting from sequential commitment in the PD-newsvendor model, it is natural to investigate which, if any, of these sequences will emerge in equilibrium. For that purpose, let us assume that the first mover (either M or R) has been determined and M and R are free to decide, sequentially, upon the order in which they specify the values of the decision variables under their control. We consider the following two-stage Stackelberg game in order to find an equilibrium sequence.

Set of players: $\{M,\ R\}$.

Action: Each player chooses which decision variable(s) to decide upon in their first step.

Set of strategies available for M: $\{(w),\ (b),\ (w, b)\}$.

Set of strategies available for R: $\{(p),\ (Q),\ (p,Q)\}$.

Outcome: A sequencing instance.

Payoff: Equilibrium expected profits for M and R.

For example, if M is the leader, who initiates the process, then M could choose either w only, b only or both w and b in her first step (Stage 1), and then, R follows by deciding upon either p only, Q only or both p and Q in his first step (Stage 2). After the first two steps, the decision sequence is determined. We would then say that a sequential ordering of decision variables is an *equilibrium sequence* if neither M nor R can improve their expected profits by resequencing their decision variables. Naturally, if any party elects not to conclude a deal, both parties would realize a zero profit.

In the discussion below, it is assumed that if one of the two parties decides to specify, in any stage, only one of the two decision variables under his/her control, the other party is expected to commit to one or both of the decision variables under its control. Thus, e.g., R cannot insist that M should determine, simultaneously, the values of w and b before R determines the values of p and Q, and M cannot insist that R should specify both decision variables under his control after M has specified the value of one of the two decision variables under her control. Recall that being the Stackelberg leader does not necessarily imply being more powerful.

Now, assume that M is the first mover who offers R, initially, only a wholesale price w. Then, R can either choose a retail price p, or an order quantity Q, or both. To find what is best for R, we need to compare R's expected profit (i.e., payoff) in the following sequences: Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$, Sequence 3: $\underline{M{:}w;\ R{:}p,Q;\ M{:}b}$ and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$. According to Propositions 3.3.4 and 3.3.5, when ξ has a power distribution and $D(p)$ is either a linear, or exponential or negative polynomial function of p, Sequences 3 and 4 result with a wholesale price-only contract, and it follows from Proposition 3.3.15 below that R prefers Sequence 1 to Sequences 3 and 4.

Proposition 3.3.15 *For a power distribution of ξ and for any form of $D(p)$, for any given w, R's equilibrium expected profit derived from Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$ is strictly larger than his equilibrium expected profit in a wholesale price-only contract.*

By Propositions 3.3.4, 3.3.5 and 3.3.15, we can conclude that for a power distribution of ξ and when $D(p)$ is either a linear, exponential or negative polynomial function of p, when M offers, at the outset, a wholesale price w, R would then specify only the retail price p.

On the other hand, suppose M offers, initially, a buyback rate b, and consider Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$, Sequence 5: $\underline{M{:}b;\ R{:}p,Q;\ M{:}w}$ and Sequence 6: $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$. By Propositions 3.3.6 and 3.3.7, R's expected profit is always zero in Sequences 5 and 6, and according to the proof of Proposition 3.3.3, R realizes a strictly positive expected profit in Sequence 2 under a power distribution of ξ and $D(p) = 1 - p$. Thus, one can conclude that for a power distribution of ξ and for $D(p) = 1 - p$, the best response for R is, again, to commit in this stage only to a retail price p.

Thus, whatever M offers first, the best response for R is only to set his selling price p. Knowing this, M would only have to compare her performance in the traditional sequence: $\underline{M{:}w,b;\ R{:}p,Q}$, Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$ and Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$, to conclude that her preference, in case she moves first, is to set, in Stage 1, only the buyback rate, and we have:

Proposition 3.3.16 *When M is the Stackelberg leader in the PD-newsvendor model with buybacks, wherein ξ follows a uniform distribution and $D(p) = 1 - p$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence.*

Let us next consider the sequencing instances wherein R is the first mover. If R initiates the process by offering only a retail price p, then M would get the expected profit of the integrated channel by following, e.g., the sequence $\underline{R{:}p;\ M{:}w,b;\ R{:}Q}$. If R initiates the process by offering only Q, then, again, M would get the expected profit of the integrated channel by following the sequence $\underline{R{:}Q;\ M{:}b;\ R{:}p;\ M{:}w}$. When R initiates the process by offering both p and Q, M and R will both get a zero profit. Therefore, whatever R offers first when he is the first mover, his expected profit is always zero, and thus, he is indifferent between being the Stackelberg leader and not having any deal whatsoever with M. According to the proof of Proposition 3.3.3, R realizes a strictly positive expected profit in Sequence 2 under a power distribution of ξ and $D(p) = 1 - p$. Thus, our conclusion is therefore:

Proposition 3.3.17 *In the PD-newsvendor model under a power distribution of ξ and $D(p) = 1 - p$, R prefers not to be the leader and would rather have M move first.*

It follows from Propositions 3.3.16 and 3.3.17 that Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the only equilibrium sequential commitment instance in the PD-newsvendor model, and thus, in this case, the first mover is determined endogenously. We observe that this equilibrium sequence is neither the traditional sequence studied, e.g., by Emmons and Gilbert, nor a sequence in which w is proposed before b. In this sequence, M fares better and R fares worse than in the traditional sequence. However, in equilibrium, R is able to prevent M from grabbing the entire expected channel profit, which M would have achieved if she had the power to force R to adopt, e.g., Sequence 5: $\underline{M{:}b;\ R{:}p,Q;\ M{:}w}$.

Finally, it follows from Propositions 3.3.4, 3.3.5, and Corollary 3.3.10, that, for a uniform ξ and for $D(p) = 1 - p$, there are two sequences, i.e., Sequence 3: $\underline{M{:}w;\ R{:}p,Q;\ M{:}b}$ and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$, which coincide with the wholesale price-only contract, wherein R would be strictly better off than in the traditional sequence. However, in both sequences, M is the leader and she and the channel are strictly worse off. Thus, unless R can force M to move first, it may be impossible for him to implement these two sequences. Finally, it follows from Propositions 3.3.6 and 3.3.7, Observation 3.3.9 (iii), and Propositions 3.3.13 and 3.3.14, that in all other sequences, for a uniformly distributed ξ, R cannot improve his performance beyond that he can achieve in the traditional sequence, and he can even end up being worse off than in the traditional sequence.

3.4 Extensions and Discussions

We have studied in the previous section the effect of sequential commitment in the PD-newsvendor model with buybacks, in which, in some cases, it is assumed that the random component of demand, ξ, follows a uniform distribution and $D(p) = 1 - p$. In this section, we extend our analysis to more general demand distributions and other expected demand functions. More specifically, in §3.4.1 we extend the results to two other expected demand functions: exponential and negative polynomial, and in §3.4.2 we extend the results which have been obtained under the assumption that ξ is uniformly distributed to a power distribution. By Propositions 3.3.4, 3.3.6 and 3.3.7, Sequences 3, 5 and 6, respectively, were analyzed for a general ξ and a general $D(p)$, and by Proposition 3.3.5, Sequence 4 was studied for a power distribution of ξ and linear, exponential and negative polynomial expected demand functions. Similarly, by Proposition 3.3.14, all sequences in which R is the leader, except for Sequence 7 and Sequence 8, were analyzed for a general ξ and a general $D(p)$, and by Proposition 3.3.13, Sequence 8 was analyzed for a power distribution of ξ and linear, exponential and negative polynomial expected demand functions. Thus, to complete the analysis in this section, we only need to consider three sequences: Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ and Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$.

3.4.1 Extension to other expected demand functions

In this subsection, we extend the results derived in Section 3.3 for $D(p) = 1 - p$ to exponential, $D(p) = e^{-p}$, and negative polynomial, $D(p) = p^{-q}$, expected demand functions, where $q > 1$. Similar to the linear expected demand function case, the analysis can be easily extended to more general exponential, $D(p) = ae^{-sp}$, and negative polynomial, $D(p) = ap^{-q}$, functions, where $a > 0$, $s > 0$ and $q > 1$. The restriction that $q > 1$ is used to ensure that R's optimal retail price will be bounded. In this section, we focus on two major issues in this subsection: (i) M's (respectively, R's) equilibrium expected profit when M (respectively, R) moves first, and (ii) equilibrium sequence analysis. Note that in the traditional sequence, the equilibrium values of the decision variables and expected profits under a uniform ξ and an exponential or negative polynomial demand function are available in Chapter 2 in this thesis and Song et al. (2004).

Exponential expected demand function

We assume in this subsection that $D(p) = e^{-p}$, and ξ is uniformly distributed on $[0, 2]$. Let us first consider the case when M is the leader. Recall from Chapter 2 in this thesis and Song et al. (2004) that in the traditional sequence with an exponential expected demand function, the equilibrium values of the decision variables are:

$$w^* = 1 + c, \quad b^* = 1, \quad p^* = \frac{3 + c + H}{2}, \quad \text{and} \quad Q^* = (3 + c - H)e^{-\frac{3+c+H}{2}}, \tag{3.11}$$

where $H = \sqrt{c^2 + 6c + 1}$, and the equilibrium values of the expected profits are:

$$E\Pi_M^* = E\Pi_R^* = \frac{(3 + c - H)(1 - c + H)}{4} e^{-\frac{3+c+H}{2}}. \tag{3.12}$$

Similar to Proposition 3.3.8 in the linear expected demand function case, we have the following result for M's equilibrium expected profit with $D(p) = e^{-p}$:

Proposition 3.4.1 *For a uniformly distributed ξ and for $D(p) = e^{-p}$,*

$$E\Pi_M^*(S2) > E\Pi_M^*(S1) > E\Pi_M^*(TS),$$

where "S1", "S2" and "TS" stand for "Sequence 1", "Sequence 2" and "the traditional sequence", respectively.

Let us next compare Sequence 3: $\underline{M{:}w;\ R{:}p, Q;\ M{:}b}$, Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ and the traditional sequence: $\underline{M{:}w, b;\ R{:}p, Q}$, for a power distribution of ξ and $D(p) = e^{-p}$. Recall from

Propositions 3.3.4 and 3.3.5 that Sequences 3 and 4 coincide with the wholesale price-only contract sequence, and that it follows from Song et al. (2004) that the equilibrium buyback rate in the traditional sequence satisfies $b^* = 1 > 0$, which implies that M is strictly better off by offering a positive buyback price and her expected profit under buybacks is strictly larger than under a wholesale price-only contract. Thus, we have:

Corollary 3.4.2 *When ξ has a power distribution and $D(p) = e^{-p}$, in equilibrium, Sequence 3: $\underline{M{:}w;\ R{:}p,Q;\ M{:}b}$ and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ yield a lower expected profit for M than the traditional sequence.*

We are ready to discuss the equilibrium sequence when M moves first for a uniform distribution of ξ and $D(p) = e^{-p}$. Recall from the analysis in §3.3.4 that when M is the leader and initially offers a wholesale price w, R would specify only the retail price p. If M offers, initially, only a buyback rate b, then R can either choose a retail price p, or an order quantity Q, or both. Thus, we compare: Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$, Sequence 5: $\underline{M{:}b;\ R{:}p,Q;\ M{:}w}$ and Sequence 6: $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$. Recall from Propositions 3.3.6 and 3.3.7 that R's expected profit is always zero in Sequences 5 and 6, and according to the proof of Proposition 3.4.1, R realizes a strictly positive expected profit in Sequence 2. Thus, the best response for R is, again, to set his retail price p. Thus, by Proposition 3.4.1:

Corollary 3.4.3 *When M is the Stackelberg leader in the PD-newsvendor model with buybacks, wherein ξ follows a uniform distribution and $D(p) = e^{-p}$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence.*

Next, we consider the case when R is the leader. Since the analysis for all sequences when R is the leader, except for Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$, has been done for a general ξ and[3.4] a general $D(p)$, we only need to study Sequence 7 under a uniform ξ and $D(p) = e^{-p}$.

Proposition 3.4.4 *For a uniformly distributed ξ and for $D(p) = e^{-p}$,*

(i) $E\Pi_M^*(S7) = 2E\Pi_R^*(S7) = \frac{e}{2}E\Pi_M^*(TS) = \frac{e}{2}E\Pi_R^*(TS)$.

(ii) $Q^*(S7) = \frac{e}{2}Q^*(TS) > Q^*(TS)$.

(iii) $p^*(S7) = p^*(TS) - 1 < p^*(TS)$.

(iv) $w^*(S7) = w^*(TS) - \frac{3+c-\sqrt{c^2+6c+1}}{4} < w^*(TS)$ *and*

[3.4]Except for Sequence 8, for which, though, the result is valid for both exponential and negative polynomial expected demand functions.

(v) $b^*(S7)(=0) < b^*(TS)$,

where "TS" and "S7" stand for "the traditional sequence" and "Sequence 7", respectively.

Proposition 3.4.4 implies that the preference of M and R between the traditional sequence and Sequence 7 is not expected demand function invariant. Indeed, for example, by Proposition 3.4.4 (i), for a uniform ξ and $D(p) = e^{-p}$, M (respectively, R) realizes a strictly higher (respectively, lower) expected profit in Sequence 7 than in the traditional sequence. By contrast, for a uniform ξ and for $D(p) = 1 - p$, M's and R's expected profits in Sequence 7 coincide with their profits in the traditional sequence. Note, however, that in Sequence 7, for a uniform ξ, the ratio of M's and R's equilibrium expected profits coincides for both linear and exponential expected demand functions, and is equal to $2 : 1$.

Finally, we note that for both linear and exponential expected demand functions, whatever R offers initially when he is the leader, his expected profit is always zero. Thus, he is indifferent between being the Stackelberg leader or not having any deal whatsoever with M. However, according to the proof of Proposition 3.4.1, R realizes a strictly positive expected profit in Sequence 2. Thus, R prefers not to be the leader and would rather have M move first, and the unique equilibrium outcome is Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$.

Negative polynomial expected demand function

We assume in this subsection that $D(p) = p^{-q}$ with $q > 1$, and ξ follows a uniform distribution on $[0, 2]$. Recall from Chapter 2 in this thesis and Song et al. (2004) that in the traditional sequence with $D(p) = p^{-q}$, the equilibrium values of the decision variables are:

$$w^* = \frac{qc}{q-1}, \quad b^* = 0, \quad p^* = \frac{q(q+1)c}{(q-1)^2}, \quad \text{and} \quad Q^* = \frac{4(q-1)^{2q}}{(qc)^q(q+1)^{q+1}}, \tag{3.13}$$

and the equilibrium values of the expected profits are:

$$E\Pi_M^* = \frac{4(q-1)^{2q-1}}{c^{q-1}q^q(q+1)^{q+1}} \quad \text{and} \quad E\Pi_R^* = \frac{4(q-1)^{2q-2}}{(cq)^{q-1}(q+1)^{q+1}}. \tag{3.14}$$

As revealed by Proposition 3.4.5 below, M's preference among the traditional sequence and Sequences 1 and 2 is invariant to the type of expected demand function considered in this chapter.

Proposition 3.4.5 *For a uniformly distributed ξ and for $D(p) = p^{-q}$,*

$$E\Pi_M^*(S2) > E\Pi_M^*(S1) > E\Pi_M^*(TS),$$

where "S1", "S2" and "TS" stand for "Sequence 1", "Sequence 2" and "the traditional sequence", respectively.

Recall from Propositions 3.3.4 and 3.3.5 that for a power distribution of ξ and $D(p) = p^{-q}$, Sequence 3: $\underline{M{:}w;\ R{:}p, Q;\ M{:}b}$ and Sequence 4: $\underline{M{:}w;\ R{:}Q;\ M{:}b;\ R{:}p}$ coincide with the wholesale price-only contract sequence, and that it follows from Song et al. (2004) that the traditional sequence: $\underline{M{:}w, b;\ R{:}p, Q}$ also coincides with the wholesale price-only contract sequence. Thus, M's expected profit in Sequences 3 and 4 and the traditional sequence is identical.

Let us next seek the equilibrium sequence, assuming a uniform distribution of ξ and $D(p) = p^{-q}$. Consider the case when M is the leader. If M offers, initially, only a wholesale price w, then, recall from the analysis in §3.3.4 that R would specify only the retail price p. On the other hand, if M offers, initially, only a buyback rate b, then we need to compare: Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$, Sequence 5: $\underline{M{:}b;\ R{:}p, Q;\ M{:}w}$, and Sequence 6: $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$. Recall from Propositions 3.3.6 and 3.3.7 that R's expected profit is always zero in Sequences 5 and 6, and according to the proof of Proposition 3.4.5, R realizes a strictly positive expected profit in Sequence 2. Thus, R, again, would set only his retail price p when M initially offers b. Thus, by Proposition 3.4.5:

Corollary 3.4.6 *When M is the Stackelberg leader in the PD-newsvendor model with buybacks, wherein ξ follows a uniform distribution and $D(p) = p^{-q}$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence.*

Let us next compare Sequence 7 and the traditional sequence under a uniform ξ and $D(p) = p^{-q}$.

Proposition 3.4.7 *For a uniformly distributed ξ and for $D(p) = p^{-q}$,*

(i) $E\Pi_R^*(S7) < E\Pi_R^*(TS)$.

(ii) $E\Pi_M^*(S7) < E\Pi_M^*(TS)$ *for* $q \in (1,2)$, $E\Pi_M^*(S7) = E\Pi_M^*(TS)$ *for* $q = 2$ *and* $E\Pi_M^*(S7) > E\Pi_M^*(TS)$ *for* $q \in (2, \infty)$.

(iii) $E\Pi_M^*(S7) = 2E\Pi_R^*(S7)$.

(iv) $Q^*(S7) = \frac{1}{2}(\frac{q}{q-1})^q Q^*(TS) > Q^*(TS)$.

(v) $p^*(S7) = \frac{q-1}{q} p^*(TS) < p^*(TS)$.

(vi) $w^*(S7) = w^*(TS)$ *and*

(vii) $b^*(S7) = b^*(TS) = 0$.

Proposition 3.4.7 (ii) confirms that the preference of M and R between the traditional sequence and Sequence 7 depends on the form of the expected demand function. Indeed, for a uniform ξ and $D(p) = p^{-q}$, M would be strictly worse off in Sequence 7 ($1 < q < 2$), as compared to the traditional sequence. However, we note again that for $D(p) = p^{-q}$, as was the case for linear and exponential expected demand functions, when M is the Stackelberg leader, Sequence 2 is the unique equilibrium sequence, and R is indifferent between being the Stackelberg leader and not having any deal whatsoever with M. According to the proof of Proposition 3.4.5, R realizes a strictly positive expected profit in Sequence 2. Thus, R prefers not to be the leader and would rather have M move first, and the unique equilibrium sequence is Sequence 2: <u>M:b; R:p; M:w; R:Q</u>.

3.4.2 Extension to a power demand distribution

In this subsection we maintain the assumption that $D(p) = 1 - p$, whenever necessary, and extend the results derived for a uniformly distributed ξ to a more general power distribution. Note that by Song et al. (2004), the equilibrium values of the decision variables and expected profits are available for the traditional sequence with a power distribution of ξ. For comparison purposes, these values, as a function of c, are presented in Table 3A.1 in the appendix when the exponent t in the power distribution is equal to 1, 2 and 4. Let us next consider the following three sequences: Sequence 1: <u>M:w; R:p; M:b; R:Q</u>, Sequence 2: <u>M:b; R:p; M:w; R:Q</u> and Sequence 7: <u>R:p; M:w; R:Q; M:b</u>. For the proofs of Propositions 3.4.8, 3.4.9 and 3.4.10 below, please refer to the proofs of Propositions 3.3.2, 3.3.3 and 3.3.11, respectively, in the appendix.

Proposition 3.4.8 *In Sequence 1: <u>M:w; R:p; M:b; R:Q</u>, for a power distribution of $\xi \in [0, 2]$ with $f(\epsilon) = \gamma(\epsilon)^t$ (where $t > 0$ and $\gamma = (t+1)2^{-(t+1)}$), and for $D(p) = 1 - p$, the equilibrium value of the retail price p^* satisfies $\frac{dE\Pi_M(p)}{dp} = 0$, where $E\Pi_M(p) = \frac{\gamma(1-p)pz^{t+2}}{t+2}$, $z = [\frac{(t+1)[(t+1)(w(p)-c)+p-c]}{\gamma(t+2)p}]^{\frac{1}{t+1}}$, $w(p) = \frac{A+B}{2(t+1)(pt+1)}$, $A = -pt^2 + 2p^2t^2 + pt^2c + tc + 2ptc + 2c - pt + 2p^2t$, and $B = \sqrt{(t+2)(-ptc - c - p - pt + 2p^2 + 2p^2t)(-pt^2c - 2ptc - tc - 2c + 2p + pt - pt^2 + 2p^2t + 2p^2t^2)}$.*

It appears unlikely that closed-form expressions for the equilibrium values in Sequence 1 can be derived for any value of t. Thus, let us first consider the case of $t = 1$.

For $t = 1$, $w(p) = \frac{-2p+4p^2+3pc+3c+\sqrt{3(-pc-c-2p+4p^2)(-3pc-3c+2p+4p^2)}}{4(1+p)}$ (and since $w \in [c, p)$, we must have $p \geq \frac{2+c+\sqrt{c^2+20c+4}}{8}$), $z = \sqrt{\frac{2}{3\gamma}}\sqrt{\frac{2w+p-3c}{p}}$ and M's expected profit function becomes:

$$E\Pi_M = \frac{2}{9}\sqrt{\frac{2}{3\gamma}}(1-p)p(\frac{2w(p)+p-3c}{p})^{\frac{3}{2}}.$$

We use Matlab to search over $p \in [\frac{2+c+\sqrt{c^2+20c+4}}{8}, 1)$, to find the unique equilibrium retail price p^* which maximizes M's expected profit function, and accordingly, we can compute the equilibrium values of the other decision variables and expected profits of M and R. We have conducted a similar analysis for $t = 2$ and $t = 4$, but do not report the detailed analysis here. The equilibrium values in Sequence 1, as a function of c, for $t = 0$, 1, 2 and 4, are presented in Table 3A.2 in the appendix.

Let us next consider Sequence 2 under a power distribution of ξ.

Proposition 3.4.9 *In Sequence 2: M:b; R:p; M:w; R:Q, for a power distribution of* $\xi \in [0,2]$ *with* $f(\epsilon) = \gamma(\epsilon)^t$ *(where* $t > 0$ *and* $\gamma = (t+1)2^{-(t+1)}$*), and for* $D(p) = 1-p$*, the equilibrium value of the retail price* p^* *satisfies* $\frac{dE\Pi_M(p)}{dp} = 0$*, where* $E\Pi_M(p) = \frac{t+1}{t+2}(1-p)(p-c)z$*,* $z = (\frac{(t+1)(p-c)}{\gamma((t+2)p-(t+1)b(p))})^{\frac{1}{t+1}}$*,* $b(p) = \frac{-v+\sqrt{v^2-4ug}}{2u}$*,* $u = -(t+1)(pt - tc - c - 2 + 3p)$*,* $v = -pt^2 + t^2c + 9p^2t + 3p^2t^2 - 3pt^2c - 5ptc + 6p^2 - 3p - 3c - 4pt$*, and* $g = -p(t+2)(-2ptc + tc - pt + 2p^2t - p + 2p^2 - c)$*.*

Similar to Sequence 1, it is difficult to derive closed-form expressions for the equilibrium values in Sequence 2 for any value of t. Thus, we consider below some specific values of t.

For $t = 1$, $b(p) = \frac{4p+c-9p^2+4pc+\sqrt{-15p^4+24p^3-34p^2c+24p^3c+8pc^2-8p^2+8pc+c^2-8p^2c^2}}{4(1+c-2p)}$ (and since $b(p) \in [0,p)$, we must have $p \geq \frac{1}{15}((6-\sqrt{6})(1+c) + \sqrt{3(7-2\sqrt{6})(2c^2+c+2\sqrt{6}c+2)}))$, $z = \sqrt{\frac{2(p-c)}{\gamma(3p-2b(p))}}$ and M's expected profit function becomes:

$$E\Pi_M = \frac{2}{3}\sqrt{\frac{2}{\gamma}}(1-p)(p-c)\sqrt{\frac{p-c}{3p-2b(p)}}.$$

Again, we use Matlab to search over $p \in [\frac{1}{15}((6-\sqrt{6})(1+c)+\sqrt{3(7-2\sqrt{6})(2c^2+c+2\sqrt{6}c+2)}), 1)$, to find the equilibrium retail price p^* which maximizes M's expected profit function, and accordingly, we can calculate the equilibrium values of the other decision variables and expected profits of M and R. Similarly, we have analyzed the cases for $t = 2$ and $t = 4$, but do not report the detailed analysis here. The equilibrium values in Sequence 2, as a function of c, for $t = 0$, 1, 2 and 4, are presented in Table 3A.3 in the appendix.

Finally, we examine Sequence 7 under a power distribution of ξ.

Proposition 3.4.10 *In Sequence 7: R:p; M:w; R:Q; M:b, for a power distribution of* $\xi \in [0,2]$ *with* $f(\epsilon) = \gamma(\epsilon)^t$ *(where* $t > 0$ *and* $\gamma = (t+1)2^{-(t+1)}$*), and for* $D(p) = 1-p$*: for* $c = 0$*, the equilibrium values of the decision variables are:* $z^* = (\frac{t+1}{\gamma(t+2)})^{\frac{1}{t+1}}$*,* $p^* = \frac{1}{2}$*,* $w^* = \frac{1}{2(t+2)}$ *and* $b^* = 0$*; and for* $c > 0$*,* $p^*(z) = \frac{c}{1-\frac{\gamma(t+2)}{t+1}z^{t+1}}$ *and the equilibrium value of the stocking factor* z^* *satisfies* $\frac{dE\Pi_R(z)}{dz} = 0$*, where* $E\Pi_R = \frac{\gamma c(t+1)[(t+1)(1-c)-\gamma(t+2)z^{t+1}]z^{t+2}}{(t+2)[(t+1)-\gamma(t+2)z^{t+1}]^2}$*.*

Similar to Sequences 1 and 2, it is difficult to derive closed-form expressions for the equilibrium values in Sequence 7 for any value of t. So let $c > 0$, and consider the case where $t = 1$.

For $t = 1$, $p(z) = \frac{c}{1-\frac{3\gamma}{2}z^2}$ (and since $p \in [c,1)$, we must have $z \leq \sqrt{\frac{2(1-c)}{3\gamma}}$), and R's expected profit function reduces to: $E\Pi_R = \frac{2\gamma c}{3}\frac{[2(1-c)-3\gamma z^2]z^3}{(2-3\gamma z^2)^2}$, which is unimodal in $z \in [0, \sqrt{\frac{2(1-c)}{3\gamma}}]$. We have used Matlab to search over $z \in [0, \sqrt{\frac{2(1-c)}{3\gamma}}]$, to find the equilibrium stocking factor z^* which maximizes R's expected profit function in Stage 1, and accordingly, we can calculate the equilibrium values of the other decision variables and expected profits of M and R. Similarly, we have analyzed the cases for $t = 2$ and $t = 4$, but do not report the detailed analysis in these two cases. The equilibrium values in Sequence 7, as a function of c, for $t = 0$, 1, 2 and 4, are presented in Table 3A.4 in the appendix.

Based on the numerical results derived for the traditional sequence and Sequences 1 and 2, which are displayed in Tables 3A.1, 3A.2 and 3A.3, respectively, in the appendix, we observe that:

Observation 3.4.11 *For a power distribution of ξ with $f(\epsilon) = \gamma(\epsilon)^t$ and $t = 1$, 2 and 4, and for $D(p) = 1 - p$:*

(i) $E\Pi^*_M(S2) > E\Pi^*_M(S1) > E\Pi^*_M(TS)$.

(ii) $E\Pi^*_{M+R}(S2) > E\Pi^*_{M+R}(S1) > E\Pi^*_{M+R}(TS)$.

(iii) $E\Pi^*_R(TS) > E\Pi^*_R(S2)$ *and* $E\Pi^*_R(S1) > E\Pi^*_R(S2)$.

(iv) $p^*(TS) > p^*(S1) > p^*(S2)$.

(v) $w^*(S2) > w^*(S1)$, *and*

(vi) $Q^*(S2) > Q^*(S1)$ *and* $Q^*(S2) > Q^*(TS)$,

where, as we recall, $E\Pi^*_{M+R}$ *stands for the equilibrium expected channel profit.*

Note that Observation 3.4.11 (i) is consistent with Proposition 3.3.8 and Observation 3.3.9 (i), according to which, in equilibrium, M attains the highest expected profit in Sequence 2 and the lowest expected profit in the traditional sequence. Note further that Observation 3.4.11 (ii) implies that both Sequences 1 and 2 improve channel efficiency, which is consistent with Observation 3.3.9 (ii). Observation 3.4.11 (iii) implies that R attains the lowest expected profit in Sequence 2, which is consistent with Observation 3.3.9 (iii). Based on R's equilibrium expected profit in the traditional sequence and Sequence 1, as displayed in Tables 3A.1 and 3A.2, respectively, for $t = 1$, 2 and 4, we observe that R's expected profit in Sequence 1 is larger (respectively, smaller) than in the traditional sequence when the manufacturing cost c is small (respectively, large). For example, when $t = 2$, as compared to the traditional sequence, there is an increase (respectively, decrease) of 1.65% (respectively, 14.15%) in R's expected profit for $c = 0$ (respectively, $c = 0.9$). This observation implies that R's preference between the traditional sequence and Sequence 1 is not demand distribution invariant. Indeed, for example, by Observation 3.3.9 (iii), for a uniform ξ and for $D(p) = 1 - p$, R always realizes a lower equilibrium expected profit in Sequence 1 than in the traditional sequence.

Let us now seek the equilibrium sequences when M moves first, assuming a power distribution and $D(p) = 1 - p$. If M is the leader who initially offers a wholesale price w in the first step, then, as we recall from the analysis in §3.3.4, R's best response is to commit only to a retail price p in his first step. On the other hand, if M initially offers a buyback rate b, then, we recall from the analysis in §3.3.4 that R, again, would prefer to set only his retail price p. Therefore, Observation 3.4.11 (i) immediately implies that for a power distribution with $t = 1$, 2 and 4 and for $D(p) = 1 - p$, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence.

By examining the equilibrium values in the traditional sequence and Sequence 7 in Tables 3A.1 and 3A.4, respectively, in the appendix, we can make the following observations:

Observation 3.4.12 *For a power distribution of ξ with $f(\epsilon) = \gamma(\epsilon)^t$ and $t = 1$, 2 and 4, and for $D(p) = 1 - p$:*

(i) $E\Pi^*_M(S7) > E\Pi^*_M(TS)$.

(ii) $E\Pi^*_{M+R}(S7) > E\Pi^*_{M+R}(TS)$.

(iii) $E\Pi^*_R(TS) > E\Pi^*_R(S7)$.

(iv) $w^*(TS) > w^*(S7)$.

(v) $p^*(TS) > p^*(S7)$, *and*

(vi) $Q^*(S7) > Q^*(TS)$.

Observation 3.4.12 (i), (ii) and (iii) imply that for a power distribution and $t = 1$, 2 and 4, in equilibrium, M and the channel (respectively, R) realize a higher (respectively, lower) expected profit in Sequence 7, with a larger order quantity, than in the traditional sequence. This result is different from that derived for the uniform distribution case, wherein M's and R's equilibrium expected profits and order quantity in the traditional sequence and Sequence 7 coincide. Observation 3.4.12 (iv) and (v) are consistent with Corollary 3.3.12 (iii) and (iv), respectively, according to which, Sequence 7 results with lower wholesale and retail prices than in the traditional sequence.

Finally, recall from §3.3.4 that when R is the first mover, he gets a zero profit, and that he realizes a strictly positive expected profit in Sequence 2. Thus, R would rather not be the first mover, and, as was the case for a uniform ξ, Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence for a power distributed ξ with $t = 1$, 2 and 4, and the first mover in these cases is determined endogenously.

3.5 Conclusions and Further Research

We have introduced in this chapter the sequential commitment approach for determining the values of contract parameters, and have analyzed its effect on the PD-newsvendor model with buybacks.

As argued earlier, compared to the traditional approach, the sequential commitment approach introduces additional flexibility to contracting between members in the supply chain. Indeed, while contracting according to the traditional approach follows the take-it-or-leave-it paradigm, the sequential commitment approach allows members, if they so desire, not to commit simultaneously to values of all contract parameters under their control. It also allows them to strategically sequence the order by which they commit to these values. As such, the sequential commitment approach is more in line with other approaches to model contracting in the supply chain (see, e.g., Nagarajan and Bassok (2002), and Iyer and Valis-Boas (2003)), and it can provide some insight, such as who should move first, or which contract parameter should be discussed first, or which pair of parameters should be discussed as a package (e.g., b and p), or which orders of the issues to be negotiated should be avoided since they may lead to an impasse, when the supply chain members engage in a negotiation process for determining the values of contract parameters.

Our analysis has revealed that the sequential commitment approach endogenizes the first mover decision. Indeed, while in the traditional approach it is arbitrarily assumed that one of the parties, usually M, is the leader, in the sequential commitment approach, under certain conditions (e.g., uniform ξ and linear, exponential and negative polynomial expected demand functions), both parties prefer that M will move first. Additionally, it was revealed that Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ is the unique equilibrium sequence in the sense that both parties prefer that M will move first, and neither party can benefit by resequencing the order at which it commits to contract parameters under its control.

We have further demonstrated that sequential commitment can have a significant effect on the supply chain performance and on the fortunes of its members. Indeed, sequential commitment can significantly increase M's expected profit, as compared to the traditional sequence. For example, based on Tables 3A.1, 3A.2 and 3A.3, for a uniform ξ, $D(p) = 1 - p$ and $c = 0.9$, Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$ and Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ improve M's equilibrium expected profit by 25.19% and 79.25%, respectively, as compared to the traditional sequence. In that respect we note that for an arbitrary distribution of ξ and an arbitrary form of $D(p)$, Sequence 5: $\underline{M{:}b;\ R{:}p, Q;\ M{:}w}$ and Sequence 6: $\underline{M{:}b;\ R{:}Q;\ M{:}w;\ R{:}p}$ result with M attaining the entire expected channel profit and R getting nothing. However, Sequences 5 and 6 require, e.g., that R commits to an order quantity before the wholesale price is set, and thus are not very realistic.

By contrast, sequential commitment could adversely affect significantly R's performance. For example, for a power distribution of ξ with $t = 0$ (uniform), and $t = 1$, 2 and 4, and for $D(p) = 1-p$,

R can never do better than in the traditional sequence when he moves first, and when M is the first mover, sequential commitment can significantly decrease R's equilibrium expected profit. For example, based on Tables 3A.1 and 3A.3, for a uniform ξ and for $D(p) = 1 - p$, R is always worse off in Sequence 2 than in the traditional sequence, and, e.g., Sequence 2 decreases R's equilibrium expected profit by 73.51% for $c = 0.9$.

We can further conclude from Tables 3A.1, 3A.2 and 3A.3 in the appendix that buybacks, coupled with sequential commitment, can increase significantly channel efficiency. For example, Sequence 2, for a uniform ξ and for $D(p) = 1 - p$, increases channel efficiency from 10.90%, for $c = 0$, to 21.25%, for $c = 0.9$. This result should be compared to the relatively insignificant effect of introducing buybacks in the PD-newsvendor model. Indeed, as it was shown in Chapter 2 in this thesis, for a uniform ξ and for $D(p) = 1 - p$, buybacks increase channel efficiency by at most 3.16%.

Finally, our results demonstrate that the sequential commitment approach could have a significant effect in the PD-newsvendor model, and it would be interesting to investigate the robustness of our results for different distributions of ξ, other than the power distribution, as well as for other expected demand functions. It would also be interesting to extend the sequential commitment approach to other Operations Management models as well as to the additive demand model (i.e., $X = D(p) + \xi$) of the PD-newsvendor problem. However, as suggested in §2.2 in Chapter 2, (see also Emmons and Gilbert (1998), Mills (1959), and Petruzzi and Dada (1999)), the additive model could produce results which are qualitatively different from those derived for the multiplicative demand model.

3.6 Appendix

Proof of Proposition 3.3.2. We use backward induction to solve Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$, which is a four-stage Stackelberg game, assuming, initially, that $\xi \in [0, 2]$ follows a general power distribution $f(\epsilon) = \gamma(\epsilon)^t$, where $\gamma = (t+1)2^{-(t+1)}$, and $D(p) = 1 - p$. Recall that the expected profit functions of M and R are given in (3.2).

Stage 4: Given (w, p, b), R chooses an order quantity Q to maximize his expected profit, given by (3.2). Note that choosing Q is equivalent to choosing z, where $z = \frac{Q}{D(p)}$. One can easily verify that $E\Pi_R(z)$ is concave in z. Thus, $\frac{dE\Pi_R(z)}{dz} = 0$ gives us the unique optimal z^*, which satisfies $bz^{t+1} = pz^{t+1} - \frac{t+1}{\gamma}(p - w)$, and R's expected profit function reduces to: $E\Pi_R = \frac{t+1}{t+2}(p - w)D(p)z^*$.

Stage 3: Given (w, p), M chooses her optimal b to maximize her expected profit, given by (3.2).

We work with z instead of b for M's problem (see Lariviere (1999)). Taking into account z^* from Stage 4, M's expected profit function becomes: $E\Pi_M = D(p)[(w-c)z + \frac{(p-w)z}{t+2} - \frac{\gamma p z^{t+2}}{(t+1)(t+2)}]$, which is concave in z. Thus, $\frac{dE\Pi_M(z)}{dz} = 0$ gives us the unique optimal $z^* = [\frac{t+1}{\gamma(t+2)}]^{\frac{1}{t+1}} \cdot [\frac{(t+1)(w-c)+p-c}{p}]^{\frac{1}{t+1}}$, and M's expected profit function reduces to: $E\Pi_M = \frac{\gamma}{t+2} D(p)p \cdot (z^*)^{t+2}$.

Stage 2: Given w and knowing z^*, R chooses p to maximize his expected profit function, which reduces to $E\Pi_R = \frac{t+1}{t+2} D(p)(p-w)z^*$, where $D(p) = 1-p$. The first-order condition (F.O.C.) yields $\frac{dE\Pi_R(w,p)}{dp} = A \cdot \frac{t+1}{\gamma(t+2)^2 p^2 (z^*)^t}$, where $A = (t+1)(1-2p+w)p[(t+1)(w-c)+p-c] + (1-p)(p-w)[c-(t+1)(w-c)]$. Since $A(p=w) > 0$ and $A(p=1) < 0$, we have $\frac{dE\Pi_R(p)}{dp}(p=w) > 0$ and $\frac{dE\Pi_R(p)}{dp}(p=1) < 0$, and the optimal retail price is an inner solution (i.e., $w < p^*(w) < 1$) which satisfies $\frac{dE\Pi_R(p)}{dp} = 0$, i.e., $A(p) = 0$.

Stage 1: We work with p instead of w for M's problem in Stage 1. Note that A can be written as a function of w as follows: $A(w) = (t+1)(1-2p+w)p[(t+1)(w-c)+p-c] + (1-p)(p-w)[c-(t+1)(w-c)]$, which is quadratic in w, and there is a unique $w^*(p) \in [c,p)$, $w^*(p) = \frac{-pt^2+2p^2t^2+pt^2c+tc+2ptc+2c-pt+2p^2t+\sqrt{(t+2)(-ptc-c-p-pt+2p^2+2p^2t)(-pt^2c-2ptc-tc-2c+2p+pt-pt^2+2p^2t+2p^2t^2)}}{2(t+1)(pt+1)}$, which satisfies $A(w^*(p)) = 0$. M's problem in Stage 1 is to choose p to maximize $E\Pi_M = \frac{\gamma}{t+2}(1-p)pz^{t+2}$, where $z = [\frac{t+1}{\gamma(t+2)}]^{\frac{1}{t+1}} \cdot [\frac{(t+1)(w^*(p)-c)+p-c}{p}]^{\frac{1}{t+1}}$.

To complete the proof of Proposition 3.3.2, we need to consider the case where ξ is uniformly distributed, i.e., $t=0$. For $t=0$, $w^*(p) = c + \sqrt{(p-c)(2p^2-p-c)}$ (and since $w \in [c,p)$, we must have $p \geq \frac{1+\sqrt{1+8c}}{4}$), $z^*(w,p) = \frac{w+p-2c}{2\gamma p}$ and M's expected profit function in Stage 1 becomes:

$$E\Pi_M = \frac{1}{8\gamma}\frac{1-p}{p}(w^*(p)-c+p-c)^2 = \frac{1}{8\gamma}(1-p)(p-c)H_1, \tag{3A.1}$$

where $H_1 = \frac{(\sqrt{2p^2-p-c}+\sqrt{p-c})^2}{p}$. One can verify that for $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$, $\frac{d[(1-p)(p-c)]}{dp} < 0$, $\frac{d^2[(1-p)(p-c)]}{dp^2} < 0$, $\frac{dH_1}{dp} > 0$ and $\frac{d^2H_1}{dp^2} < 0$. Thus, $E\Pi_M$ in (3A.1) is concave in p, and the F.O.C. yields the unique equilibrium retail price p^* in Sequence 1. Accordingly, it is easy to derive the equilibrium values of the other decision variables: $w^* = c + \sqrt{(2(p^*)^2 - p^* - c)(p^*-c)}$, $b^* = \frac{p^*(3w^*-p^*-2c)}{w^*+p^*-2c}$, $z^* = \frac{(w^*+p^*-2c)}{p^*}$ and $Q^* = (1-p^*)z^*$, and the expected profits: $E\Pi_M^* = \frac{(1-p^*)(w^*+p^*-2c)^2}{4p^*}$ and $E\Pi_R^* = \frac{(1-p^*)(p^*-w^*)(w^*+p^*-2c)}{2p^*}$, where we recall that $\gamma = \frac{1}{2}$ for $t=0$. □

Proof of Proposition 3.3.3. We analyze Sequence 2: *M:b*; *R:p*; *M:w*; *R:Q*, assuming, initially, that $\xi \in [0,2]$ has a power distribution $f(\epsilon) = \gamma(\epsilon)^t$, where $\gamma = (t+1)2^{-(t+1)}$, and $D(p) = 1-p$.

Stage 4: R's problems in Stage 4 in Sequences 2 and 1 coincide. Thus, the unique z^* satisfies $w = p - \frac{\gamma \cdot (p-b)z^{t+1}}{t+1}$, and R's expected profit function becomes: $E\Pi_R = \frac{\gamma}{t+2} D(p)(p-b)(z^*)^{t+2}$.

Stage 3: Knowing (b,p) and $z^*(b,p,w)$, we solve M's problem in Stage 3 by working with

z instead of w. M's expected profit function as a function of w becomes: $E\Pi_M = D(p)[(p - c)z - \frac{\gamma z^{t+2}}{(t+1)(t+2)}[(t+2)p - (t+1)b]]$, which is concave in z. Thus, $\frac{dE\Pi_M(z)}{dz} = 0$ yields $z^*(b,p) = (\frac{(t+1)(p-c)}{\gamma \cdot [(t+2)p-(t+1)b]})^{\frac{1}{t+1}}$, and M's expected profit function becomes: $E\Pi_M = \frac{t+1}{t+2}D(p)(p-c)z^*$.

Stage 2: Given b and taking into account $z^*(b,p)$ in Stage 3, R chooses p to maximize $E\Pi_R(p) = \frac{\gamma}{t+2}D(p)(p-b)(z^*(b,p))^2$, where $D(p) = 1-p$. The F.O.C. yields: $\frac{dE\Pi_R(p)}{dp} = \frac{A}{\gamma[(t+2)p-(t+1)b]^2 z^{t-1}}$, where $A = (t+1)(p-c)(1+b-2p)[(t+2)p-(t+1)b] + 2(1-p)(p-b)[(t+2)c-(t+1)b]$. Since $A(p = \max(b,c)) > 0$ and $A(p=1) < 0$, $\frac{dE\Pi_R(b,p)}{dp} > 0$ and $\frac{dE\Pi_R(b,p)}{dp} < 0$, we conclude that the optimal retail price is an inner solution (i.e., $\max(b,c) < p^*(b) < 1$) satisfying $\frac{dE\Pi_R(p)}{dp} = 0$, i.e., $A(p) = 0$. Note that $E\Pi_R = \frac{t+1}{t+2}(1-p)(p^*(b)-b)(z^*(b,p^*(b)))^{t+2}$, which is strictly positive since $p^*(b) > \max(b,c)$ and $z^*(b,p^*(b)) > 0$.

Stage 1: Again, we work with p instead of b for M's problem in Stage 1. Note that A can be written as a function of b as follows: $A(b) = (t+1)(p-c)(1+b-2p)[(t+2)p-(t+1)b] + 2(1-p)(p-b)[(t+2)c-(t+1)b] = u \cdot b^2 + v \cdot b + g = 0$, where $u = -(t+1)(pt - tc - c - 2 + 3p)$, $v = -pt^2 + t^2c + 9p^2t + 3p^2t^2 - 3pt^2c - 5ptc + 6p^2 - 3p - 3c - 4pt$ and $g = -p(t+2)(-2ptc + tc - pt + 2p^2t - p + 2p^2 - c)$, which is quadratic in b, and there is a unique $b^*(p) = \frac{-v+\sqrt{v^2-4ug}}{2u} \in [0,p)$, satisfying $A(b^*(p)) = 0$. M's problem in Stage 1 is to choose p to maximize $E\Pi_M = \frac{t+1}{t+2}(1-p)(p-c)z$, where $z = (\frac{(t+1)(p-c)}{\gamma((t+2)p-(t+1)b^*(p))})^{\frac{1}{t+1}}$.

To complete the proof of Proposition 3.3.3, we need to consider the case where ξ is uniformly distributed, i.e., $t = 0$. For $t = 0$, $b^*(p) = \frac{-3(2p^2-p-c)+\sqrt{(2p^2-p-c)(-6p^2+7p+8pc-9c)}}{2(2+c-3p)} \in [0,p)$ (thus, $p \geq \frac{1+\sqrt{1+8c}}{4}$), $z^*(b,p) = \frac{p-c}{\gamma(2p-b^*(p))}$ and M's expected profit function in Stage 1 becomes:

$$E\Pi_M = \frac{p-c}{16\gamma} \cdot \frac{8(1-p)(p-c)}{2p-b^*(p)} = \frac{p-c}{16\gamma}H_2, \tag{3A.2}$$

where $H_2 = \frac{8(1-p)(p-c)}{2p-b^*(p)} = \frac{-6p^2+5p+4pc-3c+\sqrt{(2p^2-p-c)(-6p^2+7p+8pc-9c)}}{p}$. Using some algebra, one can show that for $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$, $E\Pi_M$ in (3A.2) is concave in p. Thus, the F.O.C. results with a unique equilibrium retail price p^* in Sequence 2. Accordingly, we can easily derive the equilibrium values of the other decision variables: $b^* = \frac{-3(2(p^*)^2-p^*-c)+\sqrt{(2(p^*)^2-p^*-c)(-6(p^*)^2+7p^*+8p^*c-9c)}}{2(2+c-3p^*)}$, $z^* = \frac{(p^*-c)}{\gamma(2p^*-b^*)}$, $Q^* = (1-p^*)z^*$ and $w^* = p^* - \frac{(p^*-b^*)(p^*-c)}{2p^*-b^*}$, and the equilibrium values of the expected profits are: $E\Pi^*_M = \frac{(p^*-c)H_2(p^*)}{8}$ and $E\Pi^*_R = \frac{(1-p^*)(p^*-b^*)(p^*-c)^2}{(2p^*-b^*)^2}$, where we recall that $\gamma = \frac{1}{2}$ for $t = 0$. □

Proof of Proposition 3.3.5. We study the four-stage problem in Sequence 4: M:w; R:Q; M:b; R:p, assuming a power distribution of $\xi \in [0,2]$ with $f(\epsilon) = \gamma(\epsilon)^t$ and that $D(p)$ is decreasing in p and satisfying $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$.

Stage 4: Given (w, Q, b), R chooses p to maximize his expected profit, given by (3.1), $E\Pi_R(p) = (p-w)Q-(p-b)E[Q-X]^+$, where $E[Q-X]^+ = QF(\frac{Q}{D(p)})-\int_0^{\frac{Q}{D(p)}} D(p)\epsilon f(\epsilon)d\epsilon = \frac{\gamma Q^{t+2}}{(t+1)(t+2)}D(p)^{-(t+1)}$, which is the expected unsold inventory. Since $D(p)$ is decreasing in p and satisfying $D(p)\frac{d^2D(p)}{dp^2} - (t+2)(\frac{dD(p)}{dp})^2 \leq 0$, it is not difficult to show that $\frac{dE[Q-X]^+}{dp} \geq 0$ and $\frac{d^2E[Q-X]^+}{dp^2} \geq 0$ (i.e., the expected lost sales increase in the retail price p at an increasing rate), and that $\frac{dE\Pi_R}{dp} = Q - E[Q-X]^+ - (p-b)\frac{dE[Q-X]^+}{dp}$ and $\frac{d^2E\Pi_R}{dp^2} = -2\frac{dE[Q-X]^+}{dp} - (p-b)\frac{d^2E[Q-X]^+}{dp^2} \leq 0$, which implies that $E\Pi_R$ is concave in p. Thus, the F.O.C. results with a unique stationary point, $p^0(Q,b)$, which satisfies: $b\frac{dE[Q-X]^+}{dp} = p\frac{dE[Q-X]^+}{dp} + E[Q-X]^+ - Q$. Note that $p^0(Q,b)$ is independent of w. Taking derivative of the F.O.C. equation under $p = p^0(Q,b)(> b)$ with respect to b and simplifying yields: $\frac{dE[Q-X]^+}{dp} = (2\frac{dE[Q-X]^+}{dp} + (p^0(Q,b)-b)\frac{d^2E[Q-X]^+}{dp^2})\frac{\partial p^0(Q,b)}{\partial b}$. Since $E[Q-X]^+$ is increasing and convex in p and $p^0(Q,b) > b$, we conclude that $\frac{\partial p^0(Q,b)}{\partial b} \geq 0$, and $p^0(Q,b)$ increases in b. Therefore, the optimal p^* for R in Stage 4 is either p^0 or it is attained at one of the extreme points[3A.1], w or $D^{-1}(\frac{Q}{2})$, which are independent of b.

Stage 3: Given (w, Q) and knowing p^* from Stage 4, which is either increasing or independent of b, we conclude that M's expected profit function, given by (3.1), is decreasing in b. Thus, $b^* = 0$, which implies that Sequence 4 coincides with the wholesale price-only contract sequence. □

Proof of Proposition 3.3.7. Consider Sequence 6: M:b; R:Q; M:w; R:p, assuming a general distribution of $\xi \in [0, U]$ and a general form of $D(p)$.

Stage 4: Given (b, Q, w), R sets p to maximize $E\Pi_R = (p-w)Q - (p-b)E[Q-X]^+$, given by (3.1). Assume that $p^0(b,Q)$ satisfies the F.O.C.: $Q - E[Q-X]^+ - (p-b)\frac{dE[Q-X]^+}{dp} = 0$, which is independent of w. Since $E\Pi_R$ is continuous in p, the optimal retail price p^* is either equal to $p^0(b,Q)$ or it is one of the two extreme points[3A.2], w and $D^{-1}(\frac{Q}{U})$, i.e.,

$$p^* = \begin{cases} \max(w, p^0(b,Q)) & \text{if } p^0(b,Q) \leq D^{-1}(\frac{Q}{U}), \\ D^{-1}(\frac{Q}{U}) & \text{if } p^0(b,Q) \geq D^{-1}(\frac{Q}{U}). \end{cases}$$

Stage 3: Given (b, Q) and knowing p^*, M chooses w to maximize $E\Pi_M = (w-c)Q - bE[Q - D(p^*)\xi]^+$, given by (3.1). Consider two scenarios: (A) When $p^0(b,Q) \geq D^{-1}(\frac{Q}{U})$, p^* is independent of w, and thus, $E\Pi_M$ is increasing in w. Therefore, $w^* = p^* = D^{-1}(\frac{Q}{U})$. (B) When $p^0(b,Q) \leq D^{-1}(\frac{Q}{U})$, M has two options regarding w: (B1) If $w \geq p^0(b,Q)$, then M sets $w(\geq p^0(b,Q))$ to maximize $E\Pi_M = (w-c)Q - bE[Q - D(w)\xi]^+$ and $w^* = p^*$. (B2) If $w \leq p^0(b,Q)$, then $w^* = p^* = p^0(b,Q)$. Since the optimal w^* in Option (B2) is on the edge of the feasible region, we conclude that M would choose option (B1), i.e., $w \in [p^0(b,Q), D^{-1}(\frac{Q}{U})]$ is chosen to maximize

[3A.1] Note that $Q \leq 2D(p)$ since $\xi \leq 2$. Thus, $p \leq D^{-1}(\frac{Q}{2})$.
[3A.2] Note that $Q \leq UD(p)$ since $\xi \leq U$. Thus, $p \leq D^{-1}(\frac{Q}{U})$.

$E\Pi_M = (w-c)Q - bE[Q - D(w)\xi]^+$, and at optimality, $w^* > \max(b, c)$. Thus, for any (b, Q), $p^* = w^*$.

Stage 2: Given b, R determines Q to maximize $E\Pi_R = -(w^* - b)E[Q - D(w^*)\xi]^+$. Let us consider two scenarios: (A) When $b < c$, we immediately have $w^* = p^* > b$ and $E\Pi_R < 0$ except for $Q = 0$. Thus, R would choose $Q^* = 0$ to avoid a negative expected profit. (B) When $b \geq c$, R has three choices: (B1) If $Q = UD(b)$ and $p^0(b, Q) \geq D^{-1}(\frac{Q}{U})(= b)$, then $w^* = p^* = b$ and $E\Pi_R = 0$. (B2) If $Q \neq UD(b)$ and $p^0(b, Q) \leq D^{-1}(\frac{Q}{U})(> b)$, then $E\Pi_R < 0$ except for $Q = 0$. Thus, $Q^* = 0$. (B3) If $p^0(b, Q) \leq D^{-1}(\frac{Q}{U})$, then $w^* > b$ and $E\Pi_R < 0$ except for $Q = 0$. Thus, $Q^* = 0$. R gets a zero profit in all three options in Scenario (B). Thus, for Scenario (B), the choice of either $Q^* = UD(b)$ or $Q^* = 0$ depends, by Assumption 3.3.1, on M's expected profit, which is $E\Pi_M = D(b)\{2(b-c) - E[2-\xi]^+\}$ (strictly less than the expected profit of the integrated channel for any value of b since $Q^* = 2D(b) \neq Q^I$) and $E\Pi_M = 0$, respectively.

Stage 1: M's decision on b in Stage 1 has two options: (A) If $b \leq c$, then $Q^* = 0$ and $E\Pi^*_M = 0$. (B) If $b \geq c$, then b is determined to maximize $E\Pi_M = \max(D(b)\{U(b-c) - E[U-\xi]^+\}, 0)$ and $E\Pi_R = 0$. □

Proof of Proposition 3.3.8. Assume a uniformly distributed ξ and $D(p) = 1 - p$.

$E\Pi^*_M(S2) > E\Pi^*_M(S1)$. From the analysis in the proof of Proposition 3.3.3, in Stage 4, in Sequence 2: $M{:}b$; $R{:}p$; $M{:}w$; $R{:}Q$, M decides upon $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$ to maximize: $E\Pi_M(S2) = \frac{p-c}{16\gamma p}(-6p^2 + 5p + 4pc - 3c + \sqrt{(2p^2 - p - c)(-6p^2 + 7p + 8pc - 9c)})$. Similarly, from the analysis in the proof of Proposition 3.3.2, in Stage 4, in Sequence 1: $M{:}w$; $R{:}p$; $M{:}b$; $R{:}Q$, M chooses $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$ to maximize: $E\Pi_M(S1) = \frac{(1-p)(p-c)}{8\gamma p}(\sqrt{2p^2 - p - c} + \sqrt{p-c})^2$. Next, we show that $E\Pi_M(S2) > E\Pi_M(S1)$ for any $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$, which is a sufficient condition for $E\Pi^*_M(S2) > E\Pi^*_M(S1)$. Note that $E\Pi_M(S2) > E\Pi_M(S1)$ is equivalent to $S21 \equiv -6p^2 + 5p + 4pc - 3c + \sqrt{(2p^2 - p - c)(-6p^2 + 7p + 8pc - 9c)} - 2(1-p)(\sqrt{2p^2 - p - c} + \sqrt{p-c})^2 > 0$, and it is not difficult to show that, indeed, for any $p \in [\frac{1+\sqrt{1+8c}}{4}, 1)$ and any $c \in [0, 1)$, $S21 > 0$.

$E\Pi^*_M(S1) > E\Pi^*_M(TS)$. Recall that the equilibrium value of the retail price and M's expected profit in the traditional sequence: $M{:}w, b$; $R{:}p, Q$ are: $p^* = \frac{5+\sqrt{1+8c}}{8}$ and $E\Pi^*_M(TS) = \frac{(3-\sqrt{1+8c})^3(1+\sqrt{1+8c})}{128}$, where $\gamma = \frac{1}{2}$. Evaluating $E\Pi_M(S1)$, given by (3A.1), at $p^* = \frac{5+\sqrt{1+8c}}{8}$ and simplifying yields: $E\Pi_M(S1)(p^*) = \frac{(-3+\sqrt{1+8c})(-5-\sqrt{1+8c}+8c)(\sqrt{3+3\sqrt{1+8c}-12c} + \sqrt{10+2\sqrt{1+8c}-16c})^2}{1024\gamma(5+\sqrt{1+8c})}$. Now, one can easily verify that $E\Pi_M(S1)(p^*) > E\Pi^*_M(TS)$ for any $c \in [0, 1)$, implying that $E\Pi^*_M(S1) > E\Pi^*_M(TS)$. □

Proof of Proposition 3.3.11. We analyze Sequence 7: $R{:}p$; $M{:}w$; $R{:}Q$; $M{:}b$, assuming that ξ

has a power distribution and $D(p) = 1 - p$.

Stage 4: M's problem in this stage is pretty straightforward, i.e., M would always choose $b^* = 0$.

Stage 3: Given (p, w) and $b^* = 0$, R determines $z = \frac{Q}{D(p)}$ (since Q is chosen after p) to maximize his expected profit function, given by (3.2), which can be rewritten as: $E\Pi_R = D(p)[(p-w)z - \frac{\gamma p z^{t+2}}{(t+1)(t+2)}]$, and is concave in z. Thus, z^* satisfies the F.O.C. $p - w - \frac{\gamma p z^{t+1}}{t+1} = 0$, and R's expected profit function becomes: $E\Pi_R = \frac{\gamma}{t+2} D(p) p z^{t+2}$.

Stage 2: Given p and knowing z^* and $b^* = 0$, M determines w to maximize $E\Pi_M = (w - c)D(p)z^*$. Again, we work with z instead of w for M's problem to maximize $E\Pi_M = D(p)[(p - c)z - \frac{\gamma}{t+1} p z^{t+2}]$, which is, again, concave in z. Thus, the F.O.C. $p - c - \frac{\gamma(t+2)}{t+1} p z^{t+1} = 0$ yields $z^*(p)$.

Stage 1: We work with z instead of p for R's problem in Stage 1. Consider two cases as follows.

(A) For $c = 0$, from Stage 2, we have $z^* = (\frac{t+1}{\gamma(t+2)})^{\frac{1}{t+1}}$, which is independent of p. Thus, R's expected profit function becomes: $E\Pi_R = \frac{\gamma}{t+2}(\frac{t+1}{\gamma(t+2)})^{\frac{t+2}{t+1}} D(p)p$, where $D(p) = 1 - p$. It is clear that $p^* = \frac{1}{2}$, and the equilibrium values of w^* and the expected profits can be computed accordingly.

(B) For $c > 0$, R chooses z to maximize $E\Pi_R = \frac{\gamma c(t+1)[(t+1)(1-c)-\gamma(t+2)z^{t+1}]z^{t+2}}{(t+2)[(t+1)-\gamma(t+2)z^{t+1}]^2}$. Similar to the analysis of Sequences 1 and 2, it is difficult to solve R's problem in Stage 1 for any value of t. Thus, to complete the proof of Proposition 3.3.11, we next consider a uniformly distributed ξ, i.e., $t = 0$.

When $t = 0$, $p(z) = \frac{c}{1-2\gamma z}$ (and since $p \in [c, 1)$, we must have $z \leq \frac{1-c}{2\gamma}$), and R's expected profit function reduces to: $E\Pi_R = \frac{\gamma c(1-c-2\gamma z)z^2}{2(1-2\gamma z)^2}$, which can be easily shown to be unimodal in z. Thus, the F.O.C. gives us the unique equilibrium $z^* = \frac{(3-\sqrt{1+8c})}{2}$. (Recall that ξ is distributed on $[0, 2]$ and $\gamma = \frac{1}{2}$.) Accordingly, we can compute the equilibrium values of the other decision variables and expected profits: $p^* = \frac{1+\sqrt{1+8c}}{4}$, $Q^* = \frac{(3-\sqrt{1+8c})^2}{8}$, $w^* = \frac{(1+\sqrt{1+8c})^2}{16}$, $b^* = 0$, and $E\Pi_M^* = 2E\Pi_R^* = \frac{(3-\sqrt{1+8c})^3(1+\sqrt{1+8c})}{128}$. □

Proof of Proposition 3.3.13. R's problem of determining p^* in Stage 3 coincides with R's problem in Stage 4 in Sequence 4: <u>M:w; R:Q; M:b; R:p</u>, which has been analyzed in the proof of Proposition 3.3.5. Therefrom, we conclude that p^* is either increasing in b or independent of b and $E[Q - D(p^*)\xi]^+$ increases in p, which implies that M's expected profit function in Stage 2, $E\Pi_M = (w-c)Q - bE[Q - D(p^*)\xi]^+$ is decreasing in b. Thus, $b^* = 0$, and M's expected profit is increasing in w. Thus, in Stage 2, $w^* = D^{-1}(\frac{Q}{2})$ and $b^* = 0$, which leads to $p^* = w^*$ and $E\Pi_R < 0$ except when $Q = 0$. To avoid a strictly negative expected profit, R would choose $Q^* = 0$ and, in equilibrium, $E\Pi_M^* = E\Pi_R^* = 0$. □

Proof of Proposition 3.3.14. Recall that the sequences <u>R:p; M:w; R:Q; M:b</u>, <u>R:Q; M:w, b; R:p</u>, <u>R:p; M:w, b; R:Q</u> and <u>R:Q; M:w; R:p; M:b</u> have been previously analyzed. Thus, below we cover the other three sequencing instances when R is the Stackelberg leader.

Sequence: $\underline{R{:}p;\ M{:}b;\ R{:}Q;\ M{:}w}$. The analysis in this four-stage sequence under a general distribution of ξ and a general form of $D(p)$ is pretty straightforward. In Stage 4, after p, b and Q have been determined, M would set her wholesale price as high as possible. Thus, $w^* = p$. Knowing $w^* = p$, R's expected profit would be strictly negative if he chooses a strictly positive order quantity and $b \neq p$. When $b = p$, R is actually indifferent regarding the value of Q, since his expected profit will always be zero. Thus, when $b \neq p$, R would choose $Q = 0$, and otherwise, consistent with Assumption 3.3.1, Q will be chosen to maximize M's expected profit, i.e., $F(\frac{Q^*}{D(p)}) = \frac{p-c}{p}$. Now, it leaves the choice on whether $b = p$ to M in Stage 2. If $b \neq p$, then $Q^* = 0$ and both M and R earn a zero profit. If $b = p$, then M would realize a positive profit. Since R gets a zero expected profit for any p, p is chosen to maximize M's expected profit function, which coincides with the integrated channel problem. Therefore, in this sequence, M would secure the entire expected profit of the integrated channel, while R gets nothing.

Sequence: $\underline{R{:}p, Q;\ M{:}w, b}$. The analysis in this sequence under a general ξ and a general $D(p)$ is, again, quite simple. After R's decisions on p and Q have been set, M will definitely set a high enough w and a low enough b. Thus, $w^* = p$ and $b = 0$. Taking M's response in Stage 2 into account, R would not order anything in order to avoid a negative profit. Therefore, there will be no contract between M and R and both of them will realize a zero profit.

Sequence: $\underline{R{:}Q;\ M{:}b{:}\ R{:}p;\ M{:}w}$, and a general ξ and a general $D(p)$. In Stage 4, knowing (Q, b, p), M will set w as high as possible. Thus, $w^* = p$. Given (Q, b) and knowing $w^* = p$, R's expected profit function in Stage 3 becomes: $E\Pi_R = -(p-b)E[Q-X]^+$, where X is the random demand and $E[Q-X]^+$ is the lost sales. Clearly, the lost sales increase in the retail price p. Thus, $E\Pi_R$ decreases in p. Therefore, in Stage 3, R will choose a retail price as small as possible, i.e., $p^* = \max(b, c)$. In Stage 2, given Q and knowing $w^* = p^* = \max(b, c)$, M has two options: (A) $b < c$ or (B) $b \geq c$. (A) If $b < c$, then $w^* = p^* = c$ and $E\Pi_M = -bE[Q-X]^+$, which decreases in b. Thus, $b^* = 0$, and accordingly, $E\Pi_M = 0$, and $E\Pi_R = -cE[Q - D(c)\xi]^+ < 0$ except for $Q = 0$. (B) If $b \geq c$, then $w^* = p^* = b$, which is a complete consignment contract, and accordingly, $E\Pi_M = (b-c)Q - bE[Q - D(b)\xi]^+$ and $E\Pi_R = 0$. M chooses $b(\geq c)$ to maximize $E\Pi_M = (b-c)Q - bE[Q - D(b)\xi]^+$. Thus, R in Stage 1 is indifferent between $Q = 0$ in Option (A) and a complete consignment contract in Option (B). Therefore, by Assumption 3.3.1, Q, together with b, is used to maximize M's expected profit function $E\Pi_M = (b-c)Q - bE[Q - D(b)\xi]^+$, which coincides with the expected profit of the integrated channel. Thus, in equilibrium, M attains the entire expected profit of the integrated channel and R gets a zero profit. □

Proof of Proposition 3.3.15. Let us first consider Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$, with a power distribution of ξ and a general form of $D(p)$. By the proof of Proposition 3.3.2 in the appendix, for any given w, in Stage 2, R's problem is to choose $p(\geq w)$ to maximize $E\Pi_R = \frac{t+1}{t+2}D(p)(p-w)z^*$, where $z^* = [\frac{t+1}{\gamma(t+2)} \cdot \frac{(t+1)(w-c)+p-c}{p}]^{\frac{1}{t+1}}$. Note that $p(z^*)^{t+1} - \frac{t+1}{\gamma}(p-w) = \frac{t+1}{\gamma(t+2)}[(t+1)(w-c)+p-c-(t+2)(p-w)] = b(z^*)^{t+1} > 0$. The last inequality is due to the fact that $b > 0$, which can be verified from the analysis in Stage 2 in the proof of Proposition 3.3.2, and $z^* > 0$.

In the wholesale price-only sequence: $\underline{M{:}w;\ R{:}p, Q}$ with a power distribution of ξ and any $D(p)$, for any given w, in Stage 2, R chooses p and Q to maximize R's expected profit function, $E\Pi_R = (p-w)Q - \frac{\gamma p Q^{t+2}}{D(p)^{t+1}(t+1)(t+2)}$, which is concave in Q for any p. Thus, $Q^* = D(p)(\frac{(t+1)(p-w)}{\gamma p})^{\frac{1}{t+1}}$ and R's expected profit function reduces to $E\Pi_R = \frac{t+1}{t+2}D(p)(p-w)z_w^*$, where $z_w^* = [\frac{(t+1)(p-w)}{\gamma p}]^{\frac{1}{t+1}}$. R is to choose $p(\geq w)$ to maximize his expected profit function in Stage 2.

For a given w, it is easy to show that $z^* > z_w^*$ for any value of p. Thus, for any values of w and p, R's expected profit in Sequence 1 is strictly larger than his expected profit in the wholesale price-only contract sequence. □

Proof of Proposition 3.4.1. Let us first consider Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$. Since the analysis of Stages 4 and 3 in Sequence 1, as carried out in the proof of Proposition 3.3.2, is valid for any form of $D(p)$, we continue with the analysis in Stage 2, assuming $D(p) = e^{-p}$ and a uniform ξ on $[0, 2]$ (i.e., $t = 0$).

Stage 2: From the analysis in Stages 4 and 3, we have: $b^*(w,p) = \frac{p(3w-p-2c)}{w+p-2c}$ and $z^*(w,p) = \frac{(w+p-2c)}{p}$, and M's and R's expected profit functions can be simplified to: $E\Pi_M = \frac{1}{4}\frac{e^{-p}(w+p-2c)^2}{p}$ and $E\Pi_R = \frac{1}{2}\frac{e^{-p}(p-w)(w+p-2c)}{p}$, respectively. In Stage 2, R chooses p to maximize $E\Pi_R$ for any given w. The F.O.C. yields $\frac{dE\Pi_R(p)}{dp} = A\frac{e^{-p}}{2p^2} = 0$, where $A = (-p^3 + 2p^2c + pw^2 - 2pwc + p^2 + w^2 - 2wc)$, which is a cubic function of p. Since $A(p = w) > 0$ if $w \neq c$ and $A(p \to \infty) < 0$, the optimal retail price $p^*(w) > w$ is an inner solution and satisfies $\frac{dE\Pi_R(p)}{dp} = 0$, i.e., $A(p) = 0$.

Stage 1: We work with p instead of w for M's problem in this stage. Note that A can be written as a quadratic function of w as follows: $A(w) = (1+p)w^2 - 2c(1+p)w + p^2(1+2c-p)$, and there is a unique $w^*(p) = \frac{c+pc+\sqrt{(1+p)(-c-pc+p^2-p)(p-c)}}{1+p}$, satisfying $A(w^*(p)) = 0$. Since $w^* \in [c, p)$, we must have $p \geq \frac{1+c+\sqrt{c^2+6c+1}}{2}$. Substituting $w^*(p)$ into M's expected profit function we have: $E\Pi_M = \frac{(p-c)(p^2-pc+\sqrt{(1+p)(-c-pc+p^2-p)(p-c)}-c)e^{-p}}{2p(1+p)}$.

Next, we consider Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$. Similar to Sequence 1, the analysis of Stages 4 and 3 in Sequence 2 in the proof of Proposition 3.3.3 is valid for any form of $D(p)$, and we can continue our analysis in Stage 2, assuming $D(p) = e^{-p}$ and a uniform ξ on $[0, 2]$ (i.e., $t = 0$).

Stage 2: From the analysis in Stages 4 and 3, we have: $w^*(b,p) = \frac{p+pc-bc}{2p-b}$ and $z^*(b,p) = \frac{2(p-c)}{2p-b}$, and M's and R's expected profit functions can be simplified to: $E\Pi_M = \frac{e^{-p}(p-c)^2}{2p-b}$ and $E\Pi_R = \frac{e^{-p}(p-b)(p-c)^2}{(2p-b)^2}$. R's problem in Stage 2 is to choose p to maximize $E\Pi_R$ for any given b. The F.O.C. yields $\frac{dE\Pi_R(p)}{dp} = A\frac{e^{-p}(p-c)}{(2p-b)^3}$, where $A = -2p^3+3p^2b+2p^2c-3pcb-b^2p+b^2c+2p^2-3bp+2pc-3bc+2b^2$. Since $A(p = \max(b,c)) > 0$ and $A(p \to \infty) < 0$, the optimal retail price $p^*(b)$ is an inner solution, $p^*(b) > \max(b,c)$, satisfying $\frac{dE\Pi_R(p)}{dp} = 0$, i.e., $A(p) = 0$. Note that $E\Pi_R = \frac{e^{-p}(p-b)(p-c)^2}{(2p-b)^2} > 0$ since $p^*(b) > \max(b,c)$.

Stage 1: We work with p instead of b for M's problem in this stage. Note that A can be written as a quadratic function of b as follows: $A(b) = (2+c-p)b^2+3(p^2-pc-p-c)b-2p(p^2-pc-p-c)$, and there is a unique $b^*(p) = \frac{-3(p^2-pc-p-c)+\sqrt{(p^2-pc-p-c)(p^2-pc+7p-9c)}}{2+c-p}$, satisfying $A(b^*(p)) = 0$. Since $b^* \in [0,p)$, we must have $p \geq \frac{1+c+\sqrt{c^2+6c+1}}{2}$, and M's expected profit function becomes: $E\Pi_M = \frac{e^{-p}(p-c)(-p^2+pc+5p-3c+\sqrt{(p^2-pc-p-c)(p^2-pc+7p-9c)})}{8p}$.

For any value of $p(\geq \frac{1+c+\sqrt{c^2+6c+1}}{2})$, let us compare M's expected profit in Sequences 1 and 2. Let $S21(p) \equiv E\Pi_M(S2) - E\Pi_M(S1)$, where $E\Pi_M(S1)$ and $E\Pi_M(S1)$ are M's expected profits in Sequences 1 and 2, respectively, for any value of p. Thus, $S21(p) = \frac{e^{-p}(p-c)}{8p(1+p)}\{\sqrt{p^2-pc-p-c}[(1+p)\sqrt{p^2-pc+7p-9c} - 4\sqrt{(1+p)(p-c)}] - p^3 + 2pc + p^2c + 5p + c\}$. One can verify that for any $p \geq \frac{1+c+\sqrt{c^2+6c+1}}{2}$ and any $c \geq 0$, $S21(p) > 0$, implying that $E\Pi_M(S2)(p) > E\Pi_M(S1)(p)$ and $E\Pi_M^*(S2) > E\Pi_M^*(S1)$.

Now, by (3.11) and (3.12), for $D(p) = e^{-p}$, $p^* = \frac{3+c+H}{2}$ and $E\Pi_M^*(TS) = \frac{(3+c-H)(1-c+H)}{4}e^{-\frac{3+c+H}{2}}$ in the traditional sequence. Evaluating M's expected profit function in Stage 1 in Sequence 1 at the equilibrium retail price p^* of the traditional sequence, and simplifying yields: $E\Pi_R(S1)(p^*) = \frac{(3-c+H)(5+c+3H+\sqrt{(5+c+H)(3-c+H))(1+H))}}{2(3+c+H)(5+c+H)}$, where $H = \sqrt{c^2+6c+1}$. By using some simple algebra, it is not difficult to verify that $E\Pi_M(S1)(p^*) > E\Pi_M^*(TS)$ for any value of c, which implies that $E\Pi_M^*(S1) > E\Pi_M^*(TS)$.□

Proof of Proposition 3.4.4. Consider Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$, assuming that ξ has a uniform distribution (i.e., $t = 0$ in the power distribution) and $D(p) = e^{-p}$. Since the analysis of Stages 4, 3 and 2 in Sequence 7 in the proof of Proposition 3.3.11 was done for an arbitrary $D(p)$, we only need to analyze R's problem in Stage 1 for $D(p) = e^{-p}$. Note that $w^* = p - \gamma pz$ and $E\Pi_M = (w-c)e^{-p}z$. In Stage 1, R sets p to maximize $E\Pi_R = \frac{\gamma}{2}e^{-p}pz^2$, where z satisfies:

$p - c - 2\gamma pz = 0$. R's expected profit function can be simplified to: $E\Pi_R = \frac{e^{-p}(p-c)^2}{4p}$, which can be easily verified to be unimodal in p, and the unique equilibrium retail price is $p^* = \frac{1+c+H}{2}$, where $H = \sqrt{c^2+6c+1}$. Accordingly, we can compute the equilibrium values of the other decision variables: $z^* = \frac{(3+c-H)}{2}$, $w^* = \frac{1+3c+H}{4}$ and $b^* = 0$, and the equilibrium values of the expected profits are: $E\Pi_M^* = 2E\Pi_R = \frac{(3+c-H)(1-c+H)}{8}e^{-\frac{1+c+H}{2}}$. By comparing these equilibrium values in Sequence 7 and those in the traditional sequence, as displayed by (3.11) and (3.12), we can derive: $E\Pi_M^*(S7) = 2E\Pi_R^*(S7) = \frac{e}{2}E\Pi_M^*(TS) = \frac{e}{2}E\Pi_R^*(TS)$, $Q^*(S7) = \frac{e}{2}Q^*(TS)$, $p^*(TS) = p^*(S7)+1$, $w^*(TS) = w^*(S7) + \frac{3+c-H}{4}$ and $b^*(TS) > b^*(S7) = 0$. $\square$

Proof of Proposition 3.4.5. Consider Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$. The analysis of Stages 4 and 3 in Sequence 1 was carried out in the proof of Proposition 3.3.2 for a power distribution of ξ and an arbitrary $D(p)$. We continue the analysis in Stage 2, assuming $D(p) = p^{-q}$ and a uniform $\xi \in [0,2]$.

Stage 2: From the analysis in Stages 4 and 3, we have: $b^*(w,p) = \frac{p(3w-p-2c)}{w+p-2c}$ and $z^*(w,p) = \frac{(w+p-2c)}{p}$, and M's and R's expected profit functions can be simplified to: $E\Pi_M = \frac{p^{-q}(w+p-2c)^2}{4p}$ and $E\Pi_R = \frac{p^{-q}(p-w)(w+p-2c)}{2p}$. In Stage 2, R sets p to maximize $E\Pi_R$, which is clearly unimodal in p and has a unique optimal p^* satisfying the F.O.C. $\frac{dE\Pi_R(p)}{dp} = 0$, which is equivalent to requiring that $A \equiv -qp^2 + 2qpc + qw^2 - 2qwc + p^2 + w^2 - 2wc = 0$.

Stage 1: We work with p instead of w for M's problem in Stage 1. Note that A can be written as a quadratic function of w as follows: $A(w) = (q+1)w^2 - 2(q+1)cw - qp^2 + 2qpc + p^2$, and there is a unique $w^*(p) = c + \sqrt{\frac{(p-c)(qp-p-qc-c)}{q+1}}$, satisfying $A(w^*(p)) = 0$. Since $w^* \in [c,p)$, we must have $p \geq \frac{(q+1)c}{q-1}$. M's expected profit function becomes: $E\Pi_M = \frac{p^{-(q+1)}(p-c)(\sqrt{\frac{qp-p-qc-c}{q+1}}+\sqrt{p-c})^2}{4}$.

Next, consider Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$. The analysis of Stages 4 and 3 in Sequence 2 was carried out in the proof of Proposition 3.3.3 for a power distribution of ξ and an arbitrary $D(p)$. We continue the analysis in Stage 2, assuming $D(p) = p^{-q}$ and a uniform $\xi \in [0,2]$.

Stage 2: From the analysis in Stages 4 and 3, we have: $w^*(b,p) = \frac{p+pc-bc}{2p-b}$ and $z^*(b,p) = \frac{2(p-c)}{2p-b}$, and M's and R's expected profit functions can be simplified to: $E\Pi_M = \frac{p^{-q}(p-c)^2}{(2p-b)}$ and $E\Pi_R = \frac{p^{-q}(p-b)(p-c)^2}{(2p-b)^2}$. In Stage 2, R determines p to maximize $E\Pi_R$ for any given b. The F.O.C. is $\frac{dE\Pi_R(p)}{dp} = A\frac{p^{-(q+1)}(p-c)}{(2p-b)^3}$, where $A = -2p^3q + 3p^2qb + 2p^2qc - 3pqcb - pqb^2 + qb^2c + 2p^3 - 3p^2b + 2p^2c - 3pcb + 2pb^2$. Since $A(p = \max(b,c)) > 0$ and $A(p \to \infty) < 0$, the optimal retail price $p^*(b)$ is an inner solution, $p^*(b) > \max(b,c)$, satisfying $\frac{dE\Pi_R(p)}{dp} = 0$, i.e., $A = 0$. Note that $E\Pi_R = \frac{p^{-q}(p-b)(p-c)^2}{(2p-b)^2} > 0$ since $p^*(b) > \max(b,c)$.

Stage 1: We work with p instead of b for M's problem in this stage. Note that A can be written as a quadratic function of b as follows: $A(b) = (2p - qp + qc)b^2 + 3p(qp - qc - p - c)b - 2p^2(qp - qc - p - c) = 0$, and there is a unique $b^*(p) = \frac{p((-3(qp-qc-c-p)+\sqrt{(pq-p-qc-c)(pq+7p-qc-9c)}))}{2(2p+qc-qp)}$, satisfying $A(b^*(p)) = 0$. Since $b^* \in [0, p)$, we must have $p \geq \frac{(q+1)c}{q-1}$. M's expected profit function becomes: $E\Pi_M = \frac{p^{-(q+1)}(p-c)(\sqrt{(qp-p-qc-c)(pq+7p-qc-9c)}-pq+qc+5p-3c)}{8}$.

Similar to the linear and exponential expected demand function cases, one can show, using simple algebra, that for any $p \geq \frac{(q+1)c}{q-1}$, $E\Pi_M(S2) > E\Pi_M(S1)$, implying that $E\Pi_M^*(S2) > E\Pi_M^*(S1)$.

Finally, by (3.13) and (3.14), in the traditional sequence under $D(p) = p^{-q}$, $p^* = \frac{q(q+1)c}{q-1}$ and $E\Pi_M^*(TS) = \frac{4(q-1)^{2q-1}}{c^{q-1}q^q(q+1)^{q+1}}$. Evaluating M's expected profit function in Stage 1 in Sequence 1 at the equilibrium retail price p^* in the traditional sequence, and simplifying yields: $E\Pi_R(S1)(p^*) = \frac{(q-1)^{q-1}(q^2+1)(q-1+\sqrt{q^2+1})^2}{4c^{q-1}(q(q+1))^{q+1}}$. It is not difficult to show that $E\Pi_R(S1)(p^*) > E\Pi_R^*(TS)$ for any value of q and c, implying that $E\Pi_R^*(S1) > E\Pi_R^*(TS)$. □

Proof of Proposition 3.4.7. Assume that ξ has a uniform distribution and $D(p) = p^{-q}$. The analysis of Stages 4, 3 and 2 in Sequence 7 in the proof of Proposition 3.3.11 is valid for an arbitrary $D(p)$, and we only need to analyze R's problem in Stage 1 for $D(p) = p^{-q}$. Note that $w^* = p - \gamma p z$ and $E\Pi_M = (w^* - c)D(p)z$. In Stage 1, R sets p to maximize $E\Pi_R = \frac{\gamma}{2}p^{-q+1}z^2$, where $z = \frac{p-c}{2\gamma p}$. It is easy to verify that $E\Pi_R$ is unimodal in p and uniquely maximized at $p^* = \frac{(q+1)c}{q-1}$. Accordingly, we can compute the equilibrium values of the other decision variables: $z^* = \frac{2}{q+1}$, $w^* = \frac{qc}{q-1}$ and $b^* = 0$, and the equilibrium values of the expected profits are: $E\Pi_M^* = 2E\Pi_R^* = \frac{(q-1)^{q-1}}{c^{q-1}(q+1)^{q+1}}$. By comparing these equilibrium values in Sequence 7 and those in the traditional sequence, as displayed by (3.13) and (3.14), we can easily derive: $E\Pi_M^*(S7) > E\Pi_M^*(TS)$ for $q > 2$, $E\Pi_M^*(S7) = E\Pi_M^*(TS)$ for $q = 2$, $E\Pi_M^*(S7) < E\Pi_M^*(TS)$ for $q < 2$, $E\Pi_R^*(TS) > E\Pi_R^*(S7)$, $Q^*(S7) = \frac{1}{2}(\frac{q}{q-1})^q Q^*(TS) > Q^*(TS)$, $p^*(S7) = \frac{q-1}{q}p^*(TS) < p^*(TS)$, $w^*(S7) = w^*(TS)$ and $b^*(S7) = b^*(TS) = 0$. □.

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 0$ (uniform)										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	5.000E-01	5.500E-01	6.000E-01	6.500E-01	7.000E-01	7.500E-01	8.000E-01	8.500E-01	9.000E-01	9.500E-01
b^*	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01
p^*	7.500E-01	7.927E-01	8.266E-01	8.555E-01	8.812E-01	9.045E-01	9.260E-01	9.461E-01	9.650E-01	9.829E-01
Q^*	2.500E-01	1.719E-01	1.203E-01	8.353E-02	5.648E-02	3.647E-02	2.188E-02	1.161E-02	4.890E-03	1.163E-03
$E\Pi_M^*$	6.250E-02	4.172E-02	2.726E-02	1.717E-02	1.023E-02	5.636E-03	2.758E-03	1.116E-03	3.180E-04	3.833E-05
$E\Pi_R^*$	3.125E-02	2.086E-02	1.363E-02	8.583E-03	5.116E-03	2.818E-03	1.379E-03	5.579E-04	1.590E-04	1.916E-05
Channel	9.375E-02	6.258E-02	4.089E-02	2.575E-02	1.535E-02	8.453E-03	4.137E-03	1.674E-03	4.770E-04	5.749E-05
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 1$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	5.000E-01	5.500E-01	6.000E-01	6.500E-01	7.000E-01	7.500E-01	8.000E-01	8.500E-01	9.000E-01	9.500E-01
b^*	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01
p^*	7.500E-01	7.845E-01	8.147E-01	8.423E-01	8.679E-01	8.922E-01	9.153E-01	9.375E-01	9.589E-01	9.797E-01
Q^*	2.500E-01	1.957E-01	1.530E-01	1.182E-01	8.922E-02	6.492E-02	4.463E-02	2.795E-02	1.471E-02	5.043E-03
$E\Pi_M^*$	8.333E-02	6.117E-02	4.381E-02	3.031E-02	1.998E-02	1.231E-02	6.861E-03	3.261E-03	1.156E-03	2.000E-04
$E\Pi_R^*$	4.167E-02	3.059E-02	2.191E-02	1.515E-02	9.989E-03	6.154E-03	3.430E-03	1.630E-03	5.782E-04	1.000E-04
Channel	1.250E-01	9.176E-02	6.572E-02	4.546E-02	2.997E-02	1.846E-02	1.029E-02	4.891E-03	1.735E-03	3.000E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 2$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	5.000E-01	5.500E-01	6.000E-01	6.500E-01	7.000E-01	7.500E-01	8.000E-01	8.500E-01	9.000E-01	9.500E-01
b^*	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01
p^*	7.500E-01	7.815E-01	8.102E-01	8.371E-01	8.626E-01	8.871E-01	9.108E-01	9.339E-01	9.564E-01	9.784E-01
Q^*	2.500E-01	2.047E-01	1.667E-01	1.339E-01	1.052E-01	7.985E-02	5.761E-02	3.823E-02	2.173E-02	8.430E-03
$E\Pi_M^*$	9.375E-02	7.109E-02	5.256E-02	3.757E-02	2.565E-02	1.643E-02	9.579E-03	4.810E-03	1.837E-03	3.590E-04
$E\Pi_R^*$	4.688E-02	3.554E-02	2.628E-02	1.879E-02	1.283E-02	8.214E-03	4.789E-03	2.405E-03	9.186E-04	1.795E-04
Channel	1.406E-01	1.066E-01	7.885E-02	5.636E-02	3.848E-02	2.464E-02	1.437E-02	7.216E-03	2.756E-03	5.384E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 4$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	5.000E-01	5.500E-01	6.000E-01	6.500E-01	7.000E-01	7.500E-01	8.000E-01	8.500E-01	9.000E-01	9.500E-01
b^*	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01	5.000E-01
p^*	7.500E-01	7.790E-01	8.063E-01	8.326E-01	8.579E-01	8.827E-01	9.069E-01	9.306E-01	9.540E-01	9.772E-01
Q^*	2.500E-01	2.125E-01	1.790E-01	1.485E-01	1.206E-01	9.493E-02	7.128E-02	4.961E-02	3.002E-02	1.288E-02
$E\Pi_M^*$	1.042E-01	8.108E-02	6.154E-02	4.519E-02	3.175E-02	2.099E-02	1.270E-02	6.668E-03	2.704E-03	5.828E-04
$E\Pi_R^*$	5.208E-02	4.054E-02	3.077E-02	2.259E-02	1.588E-02	1.050E-02	6.348E-03	3.334E-03	1.352E-03	2.914E-04
Channel	1.563E-01	1.216E-01	9.231E-02	6.778E-02	4.763E-02	3.149E-02	1.904E-02	1.000E-02	4.057E-03	8.742E-04

Table 3A.1: Equilibrium values in the traditional sequence: $\underline{M{:}w,b;\ R{:}p,Q}$ for $D(p) = 1 - p$

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 0$ (uniform)

c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.456E-01	5.230E-01	5.911E-01	6.531E-01	7.105E-01	7.644E-01	8.156E-01	8.644E-01	9.112E-01	9.564E-01
b^*	3.884E-01	4.326E-01	4.694E-01	5.016E-01	5.302E-01	5.563E-01	5.804E-01	6.028E-01	6.239E-01	6.438E-01
p^*	7.016E-01	7.501E-01	7.904E-01	8.255E-01	8.567E-01	8.851E-01	9.112E-01	9.355E-01	9.582E-01	9.797E-01
Q^*	2.439E-01	1.787E-01	1.301E-01	9.289E-02	6.418E-02	4.217E-02	2.567E-02	1.379E-02	5.873E-03	1.411E-03
$E\Pi^*_M$	6.996E-02	4.795E-02	3.193E-02	2.040E-02	1.231E-02	6.848E-03	3.381E-03	1.379E-03	3.956E-04	4.798E-05
$E\Pi^*_R$	3.123E-02	2.030E-02	1.297E-02	8.004E-03	4.691E-03	2.544E-03	1.227E-03	4.902E-04	1.380E-04	1.645E-05
Channel	1.012E-01	6.825E-02	4.490E-02	2.841E-02	1.700E-02	9.392E-03	4.608E-03	1.869E-03	5.336E-04	6.444E-05

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 1$

c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.604E-01	5.251E-01	5.858E-01	6.435E-01	6.989E-01	7.524E-01	8.043E-01	8.548E-01	9.042E-01	9.526E-01
b^*	3.806E-01	4.103E-01	4.372E-01	4.619E-01	4.851E-01	5.069E-01	5.276E-01	5.474E-01	5.663E-01	5.852E-01
p^*	7.159E-01	7.532E-01	7.872E-01	8.186E-01	8.482E-01	8.762E-01	9.029E-01	9.284E-01	9.531E-01	9.769E-01
Q^*	2.480E-01	2.013E-01	1.614E-01	1.271E-01	9.733E-02	7.168E-02	4.978E-02	3.145E-02	1.668E-02	5.753E-03
$E\Pi^*_M$	9.019E-02	6.725E-02	4.875E-02	3.405E-02	2.262E-02	1.403E-02	7.871E-03	3.761E-03	1.340E-03	2.328E-04
$E\Pi^*_R$	4.224E-02	3.062E-02	2.167E-02	1.484E-02	9.686E-03	5.915E-03	3.271E-03	1.544E-03	5.438E-04	9.334E-05
Channel	1.324E-01	9.787E-02	7.042E-02	4.888E-02	3.231E-02	1.995E-02	1.114E-02	5.304E-03	1.884E-03	3.261E-04

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 2$

c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.690E-01	5.290E-01	5.866E-01	6.424E-01	6.965E-01	7.494E-01	8.012E-01	8.520E-01	9.021E-01	9.514E-01
b^*	3.777E-01	4.019E-01	4.245E-01	4.459E-01	4.663E-01	4.858E-01	5.046E-01	5.228E-01	5.404E-01	5.577E-01
p^*	7.238E-01	7.571E-01	7.884E-01	8.181E-01	8.466E-01	8.740E-01	9.006E-01	9.264E-01	9.515E-01	9.760E-01
Q^*	2.494E-01	2.096E-01	1.739E-01	1.416E-01	1.125E-01	8.624E-02	6.271E-02	4.189E-02	2.395E-02	9.338E-03
$E\Pi^*_M$	9.963E-02	7.639E-02	5.699E-02	4.103E-02	2.819E-02	1.815E-02	1.063E-02	5.361E-03	2.055E-03	4.028E-04
$E\Pi^*_R$	4.765E-02	3.584E-02	2.630E-02	1.867E-02	1.266E-02	8.062E-03	4.675E-03	2.335E-03	8.877E-04	1.726E-04
Channel	1.473E-01	1.122E-01	8.329E-02	5.970E-02	4.085E-02	2.621E-02	1.531E-02	7.696E-03	2.943E-03	5.754E-04

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 4$

c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.785E-01	5.346E-01	5.894E-01	6.432E-01	6.960E-01	7.481E-01	7.995E-01	8.503E-01	9.007E-01	9.505E-01
b^*	3.756E-01	3.952E-01	4.140E-01	4.322E-01	4.499E-01	4.670E-01	4.837E-01	5.001E-01	5.162E-01	5.320E-01
p^*	7.322E-01	7.621E-01	7.911E-01	8.191E-01	8.465E-01	8.732E-01	8.994E-01	9.251E-01	9.504E-01	9.754E-01
Q^*	2.502E-01	2.162E-01	1.844E-01	1.545E-01	1.265E-01	1.002E-01	7.563E-02	5.289E-02	3.213E-02	1.383E-02
$E\Pi^*_M$	1.086E-01	8.513E-02	6.499E-02	4.796E-02	3.384E-02	2.245E-02	1.362E-02	7.175E-03	2.917E-03	6.302E-04
$E\Pi^*_R$	5.289E-02	4.099E-02	3.098E-02	2.265E-02	1.586E-02	1.044E-02	6.296E-03	3.296E-03	1.333E-03	2.865E-04
Channel	1.615E-01	1.261E-01	9.597E-02	7.061E-02	4.970E-02	3.290E-02	1.992E-02	1.047E-02	4.250E-03	9.166E-04

Table 3A.2: Equilibrium values in Sequence 1: $\underline{M{:}w;\ R{:}p;\ M{:}b;\ R{:}Q}$ for $D(p) = 1 - p$

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 0$ (uniform)										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.566E-01	5.419E-01	6.151E-01	6.803E-01	7.394E-01	7.935E-01	8.431E-01	8.886E-01	9.301E-01	9.673E-01
b^*	3.894E-01	4.552E-01	5.131E-01	5.668E-01	6.183E-01	6.690E-01	7.202E-01	7.731E-01	8.300E-01	8.951E-01
p^*	6.318E-01	6.940E-01	7.439E-01	7.866E-01	8.242E-01	8.582E-01	8.894E-01	9.185E-01	9.461E-01	9.727E-01
Q^*	2.661E-01	1.949E-01	1.429E-01	1.032E-01	7.239E-02	4.850E-02	3.024E-02	1.674E-02	7.417E-03	1.891E-03
$E\Pi_M^*$	8.407E-02	5.788E-02	3.886E-02	2.510E-02	1.535E-02	8.686E-03	4.375E-03	1.829E-03	5.417E-04	6.871E-05
$E\Pi_R^*$	2.331E-02	1.481E-02	9.202E-03	5.483E-03	3.069E-03	1.569E-03	6.994E-04	2.499E-04	5.921E-05	5.077E-06
Channel	1.074E-01	7.269E-02	4.806E-02	3.059E-02	1.842E-02	1.025E-02	5.074E-03	2.079E-03	6.009E-04	7.378E-05
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 1$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.813E-01	5.457E-01	6.066E-01	6.645E-01	7.197E-01	7.724E-01	8.229E-01	8.710E-01	9.167E-01	9.597E-01
b^*	3.532E-01	4.079E-01	4.621E-01	5.165E-01	5.718E-01	6.285E-01	6.876E-01	7.500E-01	8.177E-01	8.948E-01
p^*	6.277E-01	6.714E-01	7.123E-01	7.512E-01	7.886E-01	8.248E-01	8.601E-01	8.948E-01	9.291E-01	9.637E-01
Q^*	2.719E-01	2.269E-01	1.870E-01	1.513E-01	1.192E-01	9.048E-02	6.498E-02	4.266E-02	2.372E-02	8.737E-03
$E\Pi_M^*$	1.138E-01	8.644E-02	6.387E-02	4.550E-02	3.088E-02	1.959E-02	1.127E-02	5.541E-03	2.043E-03	3.708E-04
$E\Pi_R^*$	2.654E-02	1.900E-02	1.318E-02	8.749E-03	5.478E-03	3.159E-03	1.613E-03	6.773E-04	1.975E-04	2.317E-05
Channel	1.403E-01	1.054E-01	7.704E-02	5.425E-02	3.636E-02	2.275E-02	1.288E-02	6.218E-03	2.240E-03	3.940E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 2$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.938E-01	5.506E-01	6.060E-01	6.600E-01	7.129E-01	7.645E-01	8.147E-01	8.636E-01	9.109E-01	9.565E-01
b^*	3.107E-01	3.670E-01	4.245E-01	4.833E-01	5.438E-01	6.064E-01	6.717E-01	7.406E-01	8.147E-01	8.972E-01
p^*	6.169E-01	6.561E-01	6.948E-01	7.330E-01	7.709E-01	8.085E-01	8.461E-01	8.836E-01	9.214E-01	9.597E-01
Q^*	2.827E-01	2.458E-01	2.106E-01	1.772E-01	1.454E-01	1.152E-01	8.687E-02	6.042E-02	3.622E-02	1.504E-02
$E\Pi_M^*$	1.308E-01	1.025E-01	7.816E-02	5.754E-02	4.044E-02	2.667E-02	1.603E-02	8.320E-03	3.297E-03	6.736E-04
$E\Pi_R^*$	2.608E-02	1.945E-02	1.403E-02	9.693E-03	6.323E-03	3.810E-03	2.042E-03	9.064E-04	2.834E-04	3.672E-05
Channel	1.569E-01	1.220E-01	9.219E-02	6.723E-02	4.676E-02	3.048E-02	1.807E-02	9.226E-03	3.581E-03	7.103E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 4$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	5.023E-01	5.540E-01	6.054E-01	6.566E-01	7.074E-01	7.579E-01	8.079E-01	8.573E-01	9.060E-01	9.537E-01
b^*	2.306E-01	2.978E-01	3.661E-01	4.356E-01	5.065E-01	5.791E-01	6.538E-01	7.312E-01	8.125E-01	8.998E-01
p^*	5.913E-01	6.310E-01	6.708E-01	7.108E-01	7.509E-01	7.912E-01	8.318E-01	8.728E-01	9.142E-01	9.563E-01
Q^*	3.090E-01	2.753E-01	2.420E-01	2.090E-01	1.764E-01	1.442E-01	1.126E-01	8.171E-02	5.182E-02	2.356E-02
$E\Pi_M^*$	1.522E-01	1.218E-01	9.493E-02	7.153E-02	5.157E-02	3.500E-02	2.175E-02	1.177E-02	4.931E-03	1.105E-03
$E\Pi_R^*$	2.293E-02	1.767E-02	1.318E-02	9.433E-03	6.389E-03	4.009E-03	2.249E-03	1.054E-03	3.524E-04	5.038E-05
Channel	1.752E-01	1.395E-01	1.081E-01	8.096E-02	5.796E-02	3.901E-02	2.400E-02	1.282E-02	5.283E-03	1.155E-03

Table 3A.3: Equilibrium values in Sequence 2: $\underline{M{:}b;\ R{:}p;\ M{:}w;\ R{:}Q}$ for $D(p) = 1 - p$

Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 0$ (uniform)										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	2.500E-01	3.427E-01	4.265E-01	5.055E-01	5.812E-01	6.545E-01	7.260E-01	7.961E-01	8.651E-01	9.329E-01
b^*	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
p^*	5.000E-01	5.854E-01	6.531E-01	7.110E-01	7.624E-01	8.090E-01	8.521E-01	8.922E-01	9.301E-01	9.659E-01
Q^*	2.500E-01	1.719E-01	1.203E-01	8.353E-02	5.647E-02	3.647E-02	2.188E-02	1.161E-02	4.888E-03	1.164E-03
$E\Pi^*_M$	6.250E-02	4.172E-02	2.726E-02	1.717E-02	1.023E-02	5.636E-03	2.758E-03	1.116E-03	3.180E-04	3.833E-05
$E\Pi^*_R$	3.125E-02	2.086E-02	1.363E-02	8.583E-03	5.116E-03	2.818E-03	1.379E-03	5.579E-04	1.590E-04	1.916E-05
Channel	9.375E-02	6.258E-02	4.089E-02	2.575E-02	1.535E-02	8.453E-03	4.137E-03	1.674E-03	4.770E-04	5.749E-05
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 1$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	3.333E-01	4.126E-01	4.863E-01	5.564E-01	6.239E-01	6.896E-01	7.537E-01	8.166E-01	8.786E-01	9.397E-01
b^*	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
p^*	5.000E-01	5.690E-01	6.295E-01	6.846E-01	7.359E-01	7.844E-01	8.306E-01	8.750E-01	9.179E-01	9.595E-01
Q^*	2.887E-01	2.259E-01	1.767E-01	1.365E-01	1.030E-01	7.496E-02	5.153E-02	3.228E-02	1.699E-02	5.824E-03
$E\Pi^*_M$	9.623E-02	7.064E-02	5.059E-02	3.499E-02	2.307E-02	1.421E-02	7.922E-03	3.765E-03	1.335E-03	2.310E-04
$E\Pi^*_R$	3.208E-02	2.355E-02	1.686E-02	1.166E-02	7.690E-03	4.737E-03	2.641E-03	1.255E-03	4.451E-04	7.699E-05
Channel	1.283E-01	9.418E-02	6.746E-02	4.666E-02	3.076E-02	1.895E-02	1.056E-02	5.021E-03	1.780E-03	3.080E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 2$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	3.750E-01	4.473E-01	5.154E-01	5.807E-01	6.440E-01	7.057E-01	7.663E-01	8.258E-01	8.846E-01	9.426E-01
b^*	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
p^*	5.000E-01	5.630E-01	6.205E-01	6.742E-01	7.253E-01	7.743E-01	8.217E-01	8.677E-01	9.127E-01	9.568E-01
Q^*	3.150E-01	2.579E-01	2.100E-01	1.687E-01	1.325E-01	1.006E-01	7.257E-02	4.817E-02	2.738E-02	1.062E-02
$E\Pi^*_M$	1.181E-01	8.956E-02	6.623E-02	4.734E-02	3.232E-02	2.070E-02	1.207E-02	6.061E-03	2.315E-03	4.523E-04
$E\Pi^*_R$	2.953E-02	2.239E-02	1.656E-02	1.183E-02	8.079E-03	5.174E-03	3.017E-03	1.515E-03	5.787E-04	1.131E-04
Channel	1.476E-01	1.120E-01	8.278E-02	5.917E-02	4.040E-02	2.587E-02	1.509E-02	7.576E-03	2.893E-03	5.653E-04
Power distribution $f(\epsilon) = \gamma(\epsilon)^t$ with $t = 4$										
c	0.000E+00	1.000E-01	2.000E-01	3.000E-01	4.000E-01	5.000E-01	6.000E-01	7.000E-01	8.000E-01	9.000E-01
w^*	4.167E-01	4.817E-01	5.440E-01	6.043E-01	6.632E-01	7.211E-01	7.781E-01	8.344E-01	8.901E-01	9.453E-01
b^*	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
p^*	5.000E-01	5.581E-01	6.127E-01	6.652E-01	7.158E-01	7.653E-01	8.138E-01	8.613E-01	9.081E-01	9.543E-01
Q^*	3.494E-01	2.969E-01	2.501E-01	2.075E-01	1.686E-01	1.327E-01	9.962E-02	6.934E-02	4.195E-02	1.800E-02
$E\Pi^*_M$	1.456E-01	1.133E-01	8.601E-02	6.316E-02	4.438E-02	2.934E-02	1.774E-02	9.320E-03	3.780E-03	8.146E-04
$E\Pi^*_R$	2.426E-02	1.889E-02	1.433E-02	1.053E-02	7.396E-03	4.890E-03	2.957E-03	1.553E-03	6.300E-04	1.358E-04
Channel	1.699E-01	1.322E-01	1.003E-01	7.368E-02	5.177E-02	3.423E-02	2.070E-02	1.087E-02	4.410E-03	9.503E-04

Table 3A.4: Equilibrium values in Sequence 7: $\underline{R{:}p;\ M{:}w;\ R{:}Q;\ M{:}b}$ for $D(p) = 1 - p$

Chapter 4

Price and Order Postponement in a Decentralized Newsvendor Model with Price-Dependent Demand

4.1 Introduction

Over the past two decades, with global competition, faster product development, and increasingly flexible manufacturing systems, an unprecedented number and variety of products are competing in markets ranging from apparel and toys to power tools and computers. Despite the benefits to consumers, this phenomenon is making it more difficult for manufacturers and retailers to predict which of their goods will sell and to plan production, ordering and pricing decisions accordingly. To be able to make supply meet demand in an uncertain world, supply chain members have invested considerable resources to control demand variability and reduce risk due to its uncertainty. Postponing operational decisions has emerged as a strategic mechanism to manage some of the risks associated with uncertain demand.

Basically, postponement entails the delay of activities, such as pricing, production and ordering, until after some or all of the uncertain attributes of demand have been observed. Production postponement was inspired by its successful implementation in Benetton (e.g., Signorelli and Heskett (1984), Lee and Tang (1998)) and in Hewlett-Packard (HP) (e.g., Lee et al. (1993), Lee and Billington (1994), and Feitzinger and Lee (1997)). Indeed, postponement of one form or another has become a marketing, manufacturing and logistics business concept which is applied throughout the entire supply chain. Companies such as HP, Dell, and Honda are often cited in the operations literature as leading practitioners of postponement.

In this chapter, we study two kinds of postponement in a decentralized supply chain: order postponement and price postponement. In what follows, we motivate the postponement problem we are interested in by using instances from the Operations Management literature and practices.

Rapid advances in information technology allow supply chain members in many industries to routinely use effective postponement strategies in their decision process. For example, Amazon.com, which is the largest Internet retail company in the world, allows customers to place advance purchase orders for "future release" items. These pre-orders, in turn, help Amazon decide upon appropriate order quantities from its suppliers. Many other online stores, e.g., Cdnow.com, Chapters.indigo.ca, follow Amazon's practice regarding their future release items, which can be viewed as examples of order postponement. Indeed, Moe and Fader (2002) have provided the practices of Cdnow as an example wherein advance purchase orders are used to estimate demand for future release items.

Order postponement is apparently also used in the large appliances industries. Indeed, market research conducted by the washing machine company Whirlpool has revealed that in many cases customers do not expect, or need, immediate delivery of their orders. Rather, such orders are often required by customers only upon their occupancy of a new residence (Waller (2000)). This observation allows washing machine retailers to advantageously postpone their ordering decisions. Finally, we note that order postponement can also be employed by conference organizers (Xie and Shugan (2001)) by using information about abstract submissions, done online, to better estimate demand for conference space.

Van Miegham and Dada (1999) have suggested that the bargaining practices in a car dealership can be viewed as an example for price postponement. Specifically, price postponement is implemented if a car dealership allows for some bargaining and haggling about the final price, whereas price postponement is not implemented whenever the dealership does not allow bargaining and follow the "no negotiation pricing policy" (e.g., Saturn). In that respect, price postponement is implemented by any retailer, who does not insist that the posted price is the final price, and allows for some bargaining to determine the selling price.

Price postponement is used by GreatModels.com, which is an online retail store which provides buyers with scale models (e.g., scale helicopter models, scale car models, and scale artillery/cannons/missiles/guns), accessories, and decals, etc. Indeed, it is clearly stated on their website that "the price might not show if the item is a future release", which suggests that the price of such products would be determined after demand information from the pre-launch orders has been assessed.

The effect of various postponement strategies in a centralized setting was extensively analyzed in the operations literature, see, e.g., Lee (1996), and Lee and Tang (1997). Aviv and

Federgruen (2001) have investigated the benefits of postponement in more general settings, where parameters of the demand distributions are unknown or where demand in consecutive periods is correlated. Recently, Iyer et al. (2003) have analyzed the benefits of demand postponement as a strategy to handle potential demand surges. Anand and Mendelson (1998) have studied the relationship between a firm's information system and the value of delayed production, and have analyzed the benefits of production postponement, i.e., delayed product differentiation, in a multi-product supply chain setting. Yang and Burns (2001) have provided a conceptual framework of postponed manufacturing and its implementation as part of a global strategy. Van Mieghem and Dada (1999) have conducted a thorough analysis of postponement strategies in a two-stage decision model where a centralized firm makes the following three decisions: capacity investment, production (inventory) quantity, and retail pricing. They have shown, among other results, that postponement of either retail price or production would always (weakly) benefit the centralized firm. Or, equivalently, the expected value of perfect information (EVPI) of demand, either for pricing decision or ordering decision, is non-negative.

Now, while benefits of postponement for a single decision maker have been demonstrated, decentralized supply chains are comprised of individually rational players who may behave in a selfish way. Thus, a natural question that may arise is whether, for example, Amazon's postponement strategies as reflected via their practice regarding future release products, could hurt the supply chain? In general, we are interested in analyzing the following issues related to postponement in a decentralized system.

(I) Is the EVPI of demand information, either for pricing or ordering decision, non-negative, as is the case in a centralized system?

(II) What are the effects of price or order postponement on the equilibrium profits of the channel players? Would the party introducing these postponement strategies be able to keep most of the benefits, if any, of postponement?

(III) What would be the effect of these postponement strategies on the equilibrium values of the contract parameters?

(IV) Which postponement strategies are preferred by different players?

In order to answer the above questions, we consider the PD-newsvendor model described in the introduction chapter in §1.2, wherein a manufacturer (M) sells a product, possibly with a buyback rate, to an independent retailer (R) who sets an order quantity and a retail price that affects uncertain demand. Both M and R maximize their own profits. In the PD-newsvendor model without any postponement, all contract parameters (e.g., M's wholesale price and R's retail price and order quantity) are determined before the random component in demand is realized. In general, in this chapter, we attempt to investigate the effect of postponing R's decisions of order quantity and retail price on the equilibrium values of the contract parameters and expected profits.

We show that the PD-newsvendor model with either an additive or multiplicative demand function under order postponement coincides with the corresponding deterministic model wherein the demand function coincides with the expected demand function in the PD-newsvendor model. We further show a remarkable relationship between the multiplicative PD-newsvendor model with buybacks, with and without price postponement, and the corresponding deterministic model. Specifically, under some conditions, in the multiplicative PD-newsvendor model with buyback options, under price postponement, in equilibrium, the wholesale and buyback prices, profit allocation ratio between M and R and channel efficiency coincide with those values in the corresponding PD-newsvendor model with buyback options and no postponement. These equilibrium values, excluding the buyback rate but including the expected retail price, in turn, further coincide with their counterparts in the corresponding deterministic model.

We also show that in most cases, despite vertical competition, the effect of postponement in the multiplicative model is quite beneficial for M and R. In particular, when the channel profit is relatively small, the percentage improvement in the profits of M and R could be very substantial. However, we also demonstrate in this chapter that in some cases, e.g., when the manufacturing cost is relatively low, the effect of postponement in a decentralized system is qualitatively different from its effect in a centralized firm. Specifically, in the multiplicative PD-newsvendor model, postponement could make R, who presumably initiates such a postponement strategy, worse off. Moreover, it could make the channel worse off, and in fact, it could even make both M and R strictly worse off. Similar to the multiplicative model, in general, price or order postponement in the additive PD-newsvendor model is beneficial for both players, but there are cases when both of them can be worse off. These results are in stark contrast with those in a centralized firm. Our results also demonstrate that, in many cases, R is unable to retain the lion share of the benefits stemming from postponement, and that in the multiplicative model, the effect of postponement depends very significantly on the type of contract (wholesale price-only or buyback contract).

To our knowledge, Taylor (2002b) is the only paper that has analyzed the effect of postponement in a decentralized setting. Specifically, he has considered a decentralized model wherein M decides when to sell to R. It is assumed in his paper that the retail price is always set after demand uncertainty is resolved, and the objective is to investigate the effect of postponing the upstream pricing decision, i.e., M's wholesale price, on the channel and its members. The case wherein M sells early coincides with the PD-newsvendor model without buybacks and retail price postponement. There are other papers in the supply chain literature that also consider the issue of M's sales timing (e.g., Erhun et al. (2001), Anand et al. (2002), and Cachon (2004a)). Padmanabhan and Png (1997) have studied a PD-newsvendor problem with an additive demand model. The retail price is always assumed to be determined after observing demand, and the objective is to evaluate the implications of using wholesale price-only contracts versus full-credit returns contracts. This chapter, as explained above, focuses on the effect of postponing the downstream decisions. Namely, whether R should sell early or late to the end-customers, and whether he should order early or late from M.

The remainder of this chapter is organized as follows. Section 4.2 recalls the basic price-dependent newsvendor model and related notation, as introduced in §1.2 in Chapter 1. Sections 4.3, 4.4, 4.5 and 4.6 consider various postponement strategies in the PD-newsvendor model with a multiplicative demand function (i.e., the multiplicative PD-newsvendor model). More specifically, Section 4.3 introduces and analyzes order postponement. In Section 4.4 we study price postponement. The no postponement model is considered in Section 4.5, and the effects of price postponement and order postponement are discussed in Section 4.6. We extend the analysis to the PD-newsvendor model with an additive demand function (i.e., the additive PD-newsvendor model) in Section 4.7. Managerial insights and conclusions are presented in Section 4.8, and all proofs in this chapter are given in the appendix in Section 4.9.

4.2 Preliminaries and Notation

Consider the decentralized price-dependent newsvendor model described in Section 1.2, wherein a manufacturer sells a single product to an independent retailer who is facing stochastic demand from the end-customer market. Randomness in demand is price independent. Specifically, recall that demand is defined as $X = D(p) + \xi$ in the additive case and $X = D(p) \cdot \xi$ in the multiplicative case, where $D(p)$ is the deterministic part of X which decreases in the retail price p, and ξ captures the random factor of the demand function, which could have either a continuous or discrete distribution. For continuous ξ, $F(\cdot)$ and $f(\cdot)$ are the distribution and density functions, respectively.

See Section 1.2 for more detailed description of the demand models. In this chapter, in §4.3, §4.4, §4.5 and §4.6, we focus our attention on a multiplicative demand model, and in §4.7 we extend our analysis to an additive demand model.

Recall that the decision sequence in the PD-newsvendor model without postponement is as follows. M, who has unlimited production capacity and can produce the items at a fixed marginal cost c, is a Stackelberg leader. M initiates the process by offering a wholesale price w, possibly together with a buyback rate b. R then commits to an order quantity Q and a selling price p, and the retail price affects the expected demand function. Demand is realized thereafter. For simplicity, it is assumed that the salvage value of unsold inventory is zero for both M and R, unsatisfied demand is lost and there is no penalty cost for unmet demand. For feasibility, we assume: (i) $c \leq w \leq p$ and (ii) $0 \leq b \leq w$.

The objective of this chapter is to study the effects of various postponement strategies in the PD-newsvendor model. We will refer to the multiplicative (respectively, additive) PD-newsvendor model, wherein all decisions are made before demand uncertainty is resolved, as the multiplicative (respectively, additive) PD-newsvendor model under no postponement, or the multiplicative (respectively, additive) N-postponement model. The postponement strategies we study differ in the timing of the retailer's operational decisions (e.g., order quantity and retail price) relative to the realization of demand uncertainty. We will refer to the multiplicative (respectively, additive) PD-newsvendor model, wherein R only postpones his decision on the order quantity Q (respectively, retail price p) until after demand uncertainty is observed, as the multiplicative (respectively, additive) PD-newsvendor model under order postponement (respectively, price postponement), or the multiplicative (respectively, additive) Q-postponement (respectively, p-postponement) model.

Recall that, unless otherwise noted, we denote by $(\cdot)^I$, $(\cdot)^*$ and $\hat{(\cdot)}^*$ the equilibrium values in the integrated system, the system under a buyback contract and the system under a wholesale price-only contract, respectively, and $\hat{\epsilon}$ the observed value of demand uncertainty ξ.

4.3 Order Postponement in the Multiplicative Model

Under a multiplicative Q-postponement model, R postpones his ordering decision until after observing demand uncertainty. The decision sequence is as follows. M initiates the process by setting a wholesale price w (Stage 1). R then commits to a retail price p (Stage 2). Demand uncertainty, ξ, is resolved afterwards. Finally, after observing demand uncertainty, R sets his order quantity Q (Stage 3), and M then fulfills the order without delay. Since R, in Stage 3, places his order after observing demand uncertainty, he will order only as much as he can sell, i.e., the realized demand, if his marginal profit is nonnegative, i.e., $p \geq w$. Thus, $Q = X$, and the expected profit functions for M and R in Stage 2 are:

$$E\hat{\Pi}_M = E_X[(w-c)Q] = (w-c)E(X) \text{ and } E\hat{\Pi}_R = E_X[p\min(Q,X) - wQ] = (p-w)E(X). \quad (4.1)$$

Note that under Q-postponement, it is unnecessary to introduce buybacks since there is no unsold inventory.

Recall that $X = D(p)\xi$, $D(p)$ decreases in p, $\xi \in [L, U]$, and $L \geq 0$. Thus, $E(X) = \mu_\xi \cdot D(p)$, where $\mu_\xi = E(\xi)$. M's and R's expected profit functions, given by (4.1), become $E\hat{\Pi}_M = \mu_\xi(w - c)D(p)$ and $E\hat{\Pi}_R = \mu_\xi(p - w)D(p)$, which are independent of the distribution of ξ, and only dependent on μ_ξ. Recall that μ_ξ is normalized to be one (see §1.2 for details). However, for explanation purposes, we keep μ_ξ in all expressions in this section.

In Stage 2, before observing demand uncertainty, R must commit to a retail price p to maximize $E\hat{\Pi}_R = \mu_\xi(p - w)D(p)$, which is equivalent to choosing p to maximize $(p - w)D(p)$. Assume R's best p in Stage 2, denoted by $p(w)$, is unique, and observe that $p(w)$ is independent of ξ.

The manufacturer's expected profit function in Stage 1 becomes: $E\hat{\Pi}_M = \mu_\xi(w - c)D(p(w))$. It is evident that M's best w in Stage 1, which is assumed to be unique, is independent of ξ since $p(w)$ is independent of ξ. Thus, we can conclude that in the Q-postponement model, in equilibrium (assuming uniqueness, which will be discussed later),

(1) $\hat{w}^*$ and $\hat{p}^*$ are both independent of ξ,

(2) $\hat{Q}^* = D(\hat{p}^*)\hat{\epsilon}$, and

(3) $E\hat{\Pi}^*_M = \mu_\xi(\hat{w}^* - c)D(\hat{p}^*)$ and $E\hat{\Pi}^*_R = \mu_\xi(\hat{p}^* - \hat{w}^*)D(\hat{p}^*)$,

where $\hat{\epsilon}$ is the observed value of ξ. Based on the above analysis, we conclude that postponing R's ordering decision allows the retailer to match his order to realized demand, which reduces the model to a price setting problem with deterministic demand.

Observation 4.3.1 *The equilibrium values of the wholesale price, retail price, expected order quantity and the equilibrium values of the expected profits of the manufacturer and the retailer in the multiplicative Q-postponement model coincide with those values in the corresponding deterministic model, wherein the demand function coincides with the expected demand function in the multiplicative Q-postponement model.*

Note that Observation 4.3.1 holds as well for the corresponding centralized firm. That is, in a centralized firm under Q-postponement, demand uncertainty has no effect on the optimal retail price. This is in contrast to the effect of demand uncertainty in a centralized model without Q-postponement, wherein, e.g., the equilibrium value of the retail price in a model with multiplicative uncertain demand is higher than that in the corresponding model with deterministic demand, see also Petruzzi and Dada (1999).

Since the equilibrium values of decision variables and expected profits of channel members in the multiplicative PD-newsvendor model coincide with those in the deterministic model, for comparison purpose, we recall these equilibrium values in the deterministic model in Table 4.1 below, which are originally presented in Table 2.2 in Chapter 2. We will further refer to them in §4.5 and §4.6 in this Chapter.

$D(p)$	$\hat{\Pi}_M^*$	$\hat{\Pi}_R^*$	Profit dist.	$\hat{w}^*$	$\hat{p}^*$	$\hat{Q}^*$	Π^I	p^I	Q^I	Channel efficiency
$1-p$	$\frac{(1-c)^2}{8}$	$\frac{(1-c)^2}{16}$	$2:1$	$\frac{1+c}{2}$	$\frac{3+c}{4}$	$\frac{1-c}{4}$	$\frac{(1-c)^2}{4}$	$\frac{1+c}{2}$	$\frac{1-c}{2}$	75%
e^{-p}	e^{-c-2}	e^{-c-2}	$1:1$	$1+c$	$2+c$	e^{-2-c}	e^{-c-1}	$1+c$	e^{-1-c}	$\frac{2}{e} \approx 73.58\%$
p^{-q}	$\frac{(q-1)^{2q-1}}{q^{2q}c^{q-1}}$	$\frac{(q-1)^{2q-2}}{q^{2q-1}c^{q-1}}$	$q-1:q$	$\frac{qc}{q-1}$	$\frac{cq^2}{(q-1)^2}$	$(\frac{q}{q-1})^{-2q}c^{-q}$	$\frac{(q-1)^{q-1}}{q^q c^{q-1}}$	$\frac{qc}{q-1}$	$(\frac{q}{c(q-1)})^{-q}$	$\frac{(2q-1)(q-1)^{q-1}}{q^q}$

Table 4.1: Equilibrium values in the deterministic model

By comparing the above results we immediately conclude:

Corollary 4.3.2 *In the multiplicative Q-postponement model with $D(p) = 1-p$, in equilibrium,*

(i) $p^I = \hat{p}^* - \frac{1-c}{4}$, *where* $c < 1$,

(ii) $EQ^I = 2E\hat{Q}^*$, *and*

(iii) channel efficiency of the decentralized system is 75%.

Corollary 4.3.3 *In the multiplicative Q-postponement model with $D(p) = e^{-p}$, in equilibrium,*

(i) $p^I = \hat{p}^* - 1$,

(ii) $EQ^I = e \cdot E\hat{Q}^*$, *and*

(iii) channel efficiency of the decentralized system is $\frac{2}{e} \approx 73.58\%$.

Corollary 4.3.4 *In the multiplicative Q-postponement model with $D(p) = p^{-q}$, in equilibrium,* *(i)* $p^I = \frac{q-1}{q} \cdot \hat{p}^*$,

(ii) $EQ^I = (\frac{q}{q-1})^q \cdot E\hat{Q}^*$, *and*

(iii) channel efficiency of the decentralized system is $\frac{(2q-1)(q-1)^{q-1}}{q^q}$.

The double marginalization in the Q-postponement model for the three expected demand functions we have considered is reflected by the fact that the integrated channel offers the lowest retail price and highest production quantity.

4.4 Price Postponement in the Multiplicative Model

We analyze in this section the multiplicative p-postponement model. Under price postponement, the retailer postpones his pricing decision until after demand uncertainty is resolved. The sequence of events is as follows. The manufacturer initiates the process by offering a wholesale price w with or without a buyback rate b (Stage 1). The retailer then commits to buying an order quantity Q (Stage 2). Demand uncertainty, ξ, is realized afterwards, and finally, after observing demand uncertainty, the retailer sets the retail price p (Stage 3). If $b = 0$ in Stage 1, the model coincides with a wholesale price-only contract. We use backward induction to solve this three-stage Stackelberg game.

In Stage 3, given (w, b, Q) and observing demand uncertainty, $\hat{\epsilon}$, the retailer chooses p to maximize:

$$\Pi_R = (p - b)\min(Q; D(p) \cdot \hat{\epsilon}) - (w - b)Q = (p - b)D(p)\min(z, \hat{\epsilon}) - (w - b)Q, \tag{4.2}$$

where $z = \frac{Q}{D(p)}$ is the stocking factor defined in Petruzzi and Dada (1999), and $\hat{\epsilon}$ is the realized random component of demand.

In the next three subsections, we analyze the multiplicative p-postponement model for linear ($D(p) = 1 - p$), exponential ($D(p) = e^{-p}$) and negative polynomial ($D(p) = p^{-q}$) expected demand functions. As explained in §1.2 in Chapter 1, the analysis can be easily extended to general linear, exponential or negative polynomial expected demand functions. Recall, again, that $\mu_\xi = 1$.

4.4.1 Multiplicative p-postponement model with linear expected demand

Assume that $D(p) = 1 - p$. The retailer's optimal retail price is given by in the following lemma.

Lemma 4.4.1 *Given (w, b, Q) and observing demand uncertainty, $\hat{\epsilon}$, the retailer will never set a retail price which will induce excess demand. Thus, $\hat{\epsilon} \leq z(\equiv \frac{Q}{D(p)})$, and the optimal retail price for* *the retailer is:*

$$p^* = \begin{cases} \frac{1+b}{2} & \text{if } \hat{\epsilon} \le \frac{2Q}{1-b}, \\ 1 - \frac{Q}{\hat{\epsilon}} & \text{if } \hat{\epsilon} \ge \frac{2Q}{1-b}. \end{cases} \tag{4.3}$$

Accordingly, we are able to compute the expected value of the retail price:

$$Ep^* = 1 - \frac{1-b}{2}F(\frac{2Q}{1-b}) - Q\int_{\frac{2Q}{1-b}}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon. \tag{4.4}$$

In Stage 2, given (w, b) and knowing p^*, given by (4.3), the retailer determines his best order quantity, Q, before demand uncertainty is observed. By Lemma 4.4.1, $\hat{\epsilon} \le z$. Thus, the retailer chooses Q to maximize:

$$\begin{aligned} E\Pi_R &= E_\xi[(p^* - b)(1 - p^*)\xi] - (w - b)Q \\ &= \frac{(1-b)^2}{4}\int_L^{\frac{2Q}{1-b}} \epsilon f(\epsilon)d\epsilon + Q(1-b)(1 - F(\frac{2Q}{1-b})) \\ &\quad -Q^2\int_{\frac{2Q}{1-b}}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon - (w-b)Q. \end{aligned} \tag{4.5}$$

Lemma 4.4.2 *The retailer's expected profit function, given by (4.5), is strictly concave in Q. Thus, there exists a unique Q^* which is strictly decreasing in w and satisfies:*

$$w = 1 - (1-b)F(\frac{2Q}{1-b}) - 2Q\int_{\frac{2Q}{1-b}}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon. \tag{4.6}$$

Next, let us direct our attention to the manufacturer's problem in Stage 1. We consider, separately, the manufacturer's problem when she offers a wholesale price-only contract wherein $b = 0$, and a buyback contract wherein $0 < b \le w$. Let us first examine the integrated system under price postponement, wherein $w = c$ and $b = 0$.

The integrated system. Substituting $w = c$ and $b = 0$ into (4.6) and simplifying, implies that the optimal production quantity, Q^I, in the integrated channel satisfies:

$$c = 1 - F(2Q) - 2Q\int_{2Q}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon. \tag{4.7}$$

Thus, substituting $b = 0$ and the implicit expression for Q^I into the expected retail price $Ep*$, given by (4.4), and simplifying gives us: $Ep^I = \frac{1+c}{2}$, and substituting $w = c$, $b = 0$ and the implicit expression for Q^I into (4.5) results with the equilibrium profit of the integrated channel:

$$E\Pi^I = \frac{1}{4}\int_L^{2Q^I} \epsilon f(\epsilon)d\epsilon + \frac{Q^I}{2}[1 - c - F(2Q^I)]. \tag{4.8}$$

The system under buyback contracts. In Stage 1, the manufacturer determines w and b simultaneously to maximize her expected profit function. To simplify the analysis, we use an alternative expression for the manufacturer's problem in Stage 1. That is, instead of working with $Q^*(w, b)$, we work with $w^*(b, Q)$. According to Lemma 4.4.2, there is a one-to-one relationship between the equilibrium values of $Q^*(w, b)$ and $w^*(b, Q)$, which guarantees that the alternative approach is valid. Thus, the alternative approach for the manufacturer's problem is to choose b and Q that maximize:

$$E\Pi_M = (w^*(b, Q) - c)Q - b(Q - E_\xi[(1 - p^*)\xi]), \tag{4.9}$$

where $w^*(b, Q)$ is given by (4.6), and the second term in $E\Pi_M$ represents the expected value of payment from the manufacturer to the retailer due to the unsold inventory.

Proposition 4.4.3 *In the multiplicative p-postponement model with a linear expected demand function and buybacks, the manufacturer's expected profit is globally maximized at* $(w^* = \frac{1}{2}(1 + c), b^* = \frac{1}{2})$*, and in equilibrium,* $Q^* \in (0, \frac{U}{4})$ *and satisfies*

$$1 - F(4Q) - 4Q \int_{4Q}^{U} \frac{1}{\epsilon} f(\epsilon) d\epsilon = c, \tag{4.10}$$

$Ep^* = \frac{3+c}{4}$*, and*

$$E\Pi_M^* = 2E\Pi_R^* = \frac{1}{8} \int_L^{4Q^*} \epsilon f(\epsilon) d\epsilon + \frac{Q^*}{2}[1 - c - F(4Q^*)]. \tag{4.11}$$

The system under wholesale price-only contracts. In this subsection, we analyze the wholesale price-only contract under price postponement, i.e., $b = 0$. Substituting $b = 0$ into (4.6), we conclude that the retailer's optimal order quantity $\hat{Q}^*$ under a wholesale price-only contract satisfies:

$$w = 1 - F(2Q) - 2Q \int_{2Q}^{U} \frac{1}{\epsilon} f(\epsilon) d\epsilon. \tag{4.12}$$

It is not difficult to verify that the retailer's profit function in Stage 1 and his expected profit function in Stage 2 under a wholesale price-only contract are well behaved. Thus, it is sufficient to substitute $b = 0$ into the optimal order quantity under buybacks, given by (4.6), to derive the optimal order quantity under no buybacks.

Again, we work with $\hat{w}^*(Q)$ instead of $\hat{Q}^*(w)$ to solve the manufacturer's problem in Stage 1, which is now reduced to:

$$E\hat{\Pi}_M = (\hat{w}^*(Q) - c)Q = [1 - F(2Q) - 2Q \int_{2Q}^{U} \frac{1}{\epsilon} f(\epsilon) d\epsilon - c]Q. \tag{4.13}$$

The equilibrium values of the decision variables and profits of channel members in the p-postponement model under wholesale price-only contracts are presented below.

Proposition 4.4.4 *In the multiplicative p-postponement model with a linear expected demand function and under a wholesale price-only contract, if the density function of ξ, $f(\epsilon)$, is continuous and differentiable, and $\epsilon f(\epsilon)$ is increasing in ϵ, the equilibrium order quantity $\hat{Q}^* \in (0, \frac{U}{2})$ is the unique solution which satisfies: $1 - c - F(2Q) - 4Q\int_{2Q}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon = 0$, and in equilibrium, $\hat{w}^* = \frac{1}{2}(1 + c - F(2\hat{Q}^*))$, $E\hat{p}^* = \frac{1}{4}(3 + c - F(2\hat{Q}^*))$, $E\hat{\Pi}_M^* = \frac{\hat{Q}^*}{2}(1 - c - F(2\hat{Q}^*))$ and $E\hat{\Pi}_R^* = \frac{1}{4}\int_L^{2\hat{Q}^*} \epsilon f(\epsilon)d\epsilon + \frac{\hat{Q}^*}{4}(1 - c - F(2\hat{Q}^*))$.*

The requirement that $\epsilon f(\epsilon)$ is increasing in ϵ is satisfied by the Beta distribution $f(\epsilon) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}(1-\epsilon)^{\beta-1}\epsilon^{\alpha-1}$, where $\Gamma(a) = \int_0^\infty y^{a-1}e^{-y}dy$, for values of the shape parameters (α, β) satisfying $\beta \leq 1$ and $\alpha \geq 1 - \beta$. When $(\beta = 1, \alpha \geq 1)$, the Beta distribution is reduced to the power distribution with a positive exponent. The requirement that $\epsilon f(\epsilon)$ is increasing in ϵ is also satisfied by many other common distributions, such as, Gamma and Weibull families, on a domain for which ϵ is relatively small. Note that this requirement is more restrictive than the increasing generalized failure rate (IGFR) property.

By comparing the integrated model and the decentralized p-postponement models with and without buybacks, we have:

Proposition 4.4.5 *In the multiplicative p-postponement model with $D(p) = 1 - p$, in equilibrium:*

(i) $Q^I = 2Q^*$.

(ii) Channel efficiency under buybacks is always 75%.

(iii) $Ep^I = Ep^* - \frac{1-c}{4} < Ep^*$.

(iv) The wholesale price, expected retail price, profit allocation ratio between M and R and channel efficiency coincide with those values in the corresponding deterministic model, wherein the demand function coincides with the expected demand function in the multiplicative p-postponement model.

If $\epsilon f(\epsilon)$ is increasing in ϵ, then we have:

(v) $Q^I = 2Q^* > \hat{Q}^*$.

(vi) $Ep^I < E\hat{p}^* < Ep^*$.

Example 4.4.6 Power demand distribution with linear expected demand.

From the analysis above, it follows that in the multiplicative p-postponement model with buybacks, in equilibrium, the wholesale and buyback prices, expected retail price, profit allocation ratio between M and R and channel efficiency are independent of the distribution of ξ, while the equilibrium values of the expected profits of M and R and the order quantity depend on the distribution of ξ. In the multiplicative p-postponement model under a wholesale price-only contract, all equilibrium values depend on the distribution of ξ. Thus, in order to evaluate the effect of price postponement in the multiplicative model, we will consider in this example a specific family of distribution functions of ξ. Specifically, we will consider the power distribution with a density function $f(\epsilon) = \gamma(\epsilon)^t$ for any $t \in [0, \infty)$. For simplicity, we assume $L = 0$, i.e., $\xi \in [0, U]$. To satisfy $\mu_\xi = 1$ and $F(U) = 1$, we let $\gamma = \frac{(t+1)^{t+2}}{(t+2)^{t+1}}$ and $U = \frac{t+2}{t+1}$. Note that the power distribution satisfies the condition that $\epsilon f(\epsilon)$ is increasing in ϵ, and for $t = 0$, $f(\epsilon)$ reduces to a uniform distribution. Under the power distribution, the equilibrium retail price, order quantity and profits of M and R are implicit functions of the exponent of the power distribution t (except for $t = 0$, where there are explicit expressions for the equilibrium values under a uniform distribution). For any given t, the computation of the equilibrium values in the integrated and decentralized channels are pretty straightforward. The corresponding results for $t = 0$, 1 and 4 are presented in Table 4A.1 in the appendix in §4.9. These results will be further discussed in §4.6.

4.4.2 Multiplicative p-postponement model with exponential expected demand

Assume that $D(p) = e^{-p}$. Again, we use backward induction to solve the three-stage Stackelberg game. In Stage 3, given (w, b, Q) and observing demand uncertainty, $\hat{\epsilon}$, the retailer chooses p to maximize his profit function given by (4.2). The retailer's optimal retail price is given by in the following lemma.

Lemma 4.4.7 *Given (w, b, Q) and observing demand uncertainty, $\hat{\epsilon}$, the retailer will never set a retail price which will induce excess demand. Thus, $\hat{\epsilon} \leq z$, and the optimal retail price for the retailer is:*

$$p^* = \begin{cases} 1 + b & \text{if } \hat{\epsilon} \leq e^{1+b}Q, \\ ln(\hat{\epsilon}) - ln(Q) & \text{if } \hat{\epsilon} \geq e^{1+b}Q. \end{cases} \tag{4.14}$$

Accordingly, we are able to derive the expected value of the retail price:

$$Ep^* = (ln(Q) + 1)F(e^{1+b}Q) + bF(e^{1+b}Q) - ln(Q) + \int_{e^{1+b}Q}^{U} ln(\epsilon)f(\epsilon)d\epsilon. \tag{4.15}$$

In Stage 2, given (w, b) and knowing p^*, given by (4.14), the retailer determines his best order quantity Q, before demand uncertainty is observed, to maximize his expected profit:

$$\begin{aligned} E\Pi_R &= E_\xi[(p^* - b)e^{-p^*}] - (w - b)Q \\ &= e^{-1-b}\int_L^{e^{1+b}Q} \epsilon f(\epsilon)d\epsilon - wQ + bQF(e^{1+b}Q) \\ &\quad +Q\int_{e^{1+b}Q}^U ln(\epsilon)f(\epsilon)d\epsilon - Qln(Q)(1 - F(e^{1+b}Q)). \end{aligned} \tag{4.16}$$

Lemma 4.4.8 *The retailer's expected profit function, given by (4.16), is strictly concave in Q. Thus, there exists a unique Q^* which is strictly decreasing in w and satisfies:*

$$w = bF(e^{1+b}Q) + \int_{e^{1+b}Q}^U ln(\epsilon)f(\epsilon)d\epsilon - (ln(Q) + 1) + (ln(Q) + 1)F(e^{1+b}Q). \tag{4.17}$$

Again, let us consider the manufacturer's problem in Stage 1. We consider, separately, the integrated system and the systems under buyback and wholesale price-only contracts.

The integrated system. Substituting $w = c$ and $b = 0$ into (4.17) and simplifying implies that the optimal production quantity, Q^I, in the integrated channel, satisfies:

$$c = \int_{eQ}^U ln(\epsilon)f(\epsilon)d\epsilon - (ln(Q) + 1)(1 - F(eQ)).$$

Thus, substituting $b = 0$ and the implicit expression for Q^I into the expected retail price Ep^*, given by (4.15), and simplifying gives us: $Ep^I = 1 + c$, and substituting $w = c$, $b = 0$ and the implicit expression for Q^I into (4.16) results with the equilibrium profit of the integrated channel:

$$E\Pi^I = e^{-1}\int_L^{eQ^I} \epsilon f(\epsilon)d\epsilon + Q^I(1 - F(eQ^I)).$$

The system under buyback contracts. In Stage 1, the manufacturer determines w and b simultaneously to maximize her expected profit function. Similar to the linear case, we work with $w^*(b, Q)$ instead of $Q^*(w, b)$. According to Lemma 4.4.8, there is a one-to-one relationship between the equilibrium values of $Q^*(w, b)$ and $w^*(b, Q)$, which guarantees that the alternative approach is valid. Thus, the alternative approach for the manufacturer's problem is to choose b and Q that maximize:

$$E\Pi_M = (w^*(b, Q) - c)Q - b(Q - E_\xi(e^{-p^*}\xi)), \tag{4.18}$$

where $w^*(b, Q)$ is given by (4.17). The manufacturer's optimal decisions are given in the following proposition.

Proposition 4.4.9 *In the multiplicative p-postponement model with $D(p) = e^{-p}$ and buybacks, the manufacturer's expected profit is globally maximized at $(w^* = 1 + c, b^* = 1)$, and in equilibrium, $Q^* \in (0, \frac{U}{e^2})$, which satisfies*

$$c = \int_{e^2Q}^{U} ln(\epsilon) f(\epsilon) d\epsilon - (ln(Q) + 2)(1 - F(e^2Q)),$$

$Ep^* = 2 + c$, *and*

$$E\Pi_M^* = E\Pi_R^* = Q^*(1 - F(e^2Q^*)) + e^{-2} \int_{L}^{e^2Q^*} \epsilon f(\epsilon) d\epsilon.$$

The system under wholesale price-only contracts. Let us consider the multiplicative p-postponement model under a wholesale price-only contract. Substituting $b = 0$ into (4.17), we obtain that the retailer's optimal order quantity $\hat{Q}^*$ under a wholesale price-only contract satisfies:

$$w = \int_{eQ}^{U} ln(\epsilon) f(\epsilon) d\epsilon - (ln(Q) + 1)(1 - F(eQ)). \tag{4.19}$$

As in the linear case under no buybacks, we are able to verify that the retailer's profit function in Stage 1 and his expected profit function under a wholesale price-only contract in Stage 2 are well behaved. Thus, it is sufficient to substitute $b = 0$ into the optimal order quantity under buybacks, given by (4.17), to derive the optimal order quantity in the model without buybacks.

Again, we work with $\hat{w}^*(Q)$ instead of $\hat{Q}^*(w)$ to solve the manufacturer's problem in Stage 1, which now becomes:

$$E\hat{\Pi}_M = (\hat{w}^*(Q) - c)Q = -Q(ln(Q) + 1)(1 - F(eQ)) - cQ + \int_{eQ}^{U} ln(\epsilon) f(\epsilon) d\epsilon. \tag{4.20}$$

The equilibrium values of the decision variables and the profits of channel members in the p-postponement model under a wholesale price-only contract are presented below.

Proposition 4.4.10 *In the multiplicative p-postponement model with $D(p) = e^{-p}$ and under wholesale price-only contracts, if the density function of ξ, $f(\epsilon)$, is continuous and differentiable, and $\epsilon f(\epsilon)$ is increasing in ϵ, the equilibrium order quantity $\hat{Q}^* \in (0, \frac{U}{e})$ is the unique solution satisfying $-(ln(eQ) + 1)(1 - F(eQ)) - c + \int_{eQ}^{U} ln(\epsilon) f(\epsilon) d\epsilon = 0$, and in equilibrium, $\hat{w}^* = 1 + c - F(e\hat{Q}^*)$, $E\hat{p}^* = 2 + c - F(e\hat{Q}^*)$, $E\hat{\Pi}_M^* = (1 - F(e\hat{Q}^*))\hat{Q}^*$ and $E\hat{\Pi}_R^* = e^{-1} \int_{L}^{e\hat{Q}^*} \epsilon f(\epsilon) d\epsilon + (1 - F(e\hat{Q}^*))\hat{Q}^*$.*

From the analysis in the integrated channel and Propositions 4.4.9 and 4.4.10, we have:

Proposition 4.4.11 *In the multiplicative p-postponement model with $D(p) = e^{-p}$, in equilibrium:*

(i) $Q^I = eQ^*$.

(ii) *Channel efficiency is always* $\frac{2}{e} = 73.58\%$.

(iii) $Ep^I = Ep^* - 1 < Ep^*$.

(iv) *The wholesale price, expected retail price, profit allocation ratio between M and R and the channel efficiency coincide with those in the corresponding deterministic model, wherein the demand function coincides with the expected demand function in the multiplicative p-postponement model.*

If $\epsilon f(\epsilon)$ *is increasing in* ϵ*, then we have:*

(v) $Q^I = eQ^* > \hat{Q}^*$.

(vi) $Ep^I < E\hat{p}^* < Ep^*$.

Example 4.4.12 Power demand distribution with exponential expected demand.

Similar to Example 4.4.6, we have derived the equilibrium values in the decentralized multiplicative models with and without buybacks for a power distribution of ξ with $t = 0$, $t = 1$ and $t = 4$ and an exponential expected demand function. These results are presented in Table 4A.2 in the appendix and will be further discussed in §4.6.

4.4.3 Multiplicative p-postponement model with negative polynomial expected demand

In this subsection, we let $D(p) = p^{-q}$, where $q > 1$. Again, we use backward induction to solve the three-stage Stackelberg game.

In Stage 3, given (w, b, Q) and observing demand uncertainty, $\hat{\epsilon}$, the retailer chooses p to maximize his profit function given by (4.2). The retailer's optimal selling price can be described as follows.

Lemma 4.4.13 *Given* (w, b, Q) *and observing demand uncertainty,* $\hat{\epsilon}$*, the retailer will never set a retail price which will induce excess demand. Thus,* $\hat{\epsilon} \leq z$*, and the optimal retail price for the retailer is:*

$$p^* = \begin{cases} \frac{qb}{q-1} & \text{if } \hat{\epsilon} \leq \delta, \\ (\frac{\hat{\epsilon}}{Q})^{\frac{1}{q}} & \text{if } \hat{\epsilon} \geq \delta, \end{cases} \tag{4.21}$$

where $\delta = Q(\frac{qb}{q-1})^q$.

Accordingly, we are able to calculate the expected value of the retail price:

$$Ep^* = \frac{qb}{q-1}F(\delta) + Q^{-\frac{1}{q}} \int_\delta^U (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon, \tag{4.22}$$

where, as we recall, $\delta = Q(\frac{qb}{q-1})^q$.

In Stage 2, given (w, b) and knowing p^*, given by (4.21), the retailer determines his best order quantity Q, before demand uncertainty is observed, to maximize his expected profit:

$$\begin{aligned} E\Pi_R &= E_\xi[(p^* - b)(p^*)^{-q}] - (w-b)Q \\ &= \frac{b}{q-1}(\frac{qb}{q-1})^{-q} \int_L^\delta \epsilon f(\epsilon)d\epsilon + Q^{1-\frac{1}{q}} \int_\delta^U (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon \\ &\quad -bQ(1-F(\delta)) - (w-b)Q. \end{aligned} \tag{4.23}$$

Lemma 4.4.14 *The retailer's expected profit function, given by (4.23), is strictly concave in Q. Thus, there exists a unique Q^* which is strictly decreasing in w and satisfies:*

$$w = (1-\frac{1}{q})Q^{-\frac{1}{q}} \int_\delta^U (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon + bF(\delta). \tag{4.24}$$

To analyze the manufacturer's problem we consider again the integrated system and the systems under a buyback contract and under a wholesale price-only contract.

The integrated system. Substituting $w = c$ and $b = 0$ into (4.24) and simplifying results with the optimal production quantity in the integrated channel:

$$Q^I = [\frac{(1-\frac{1}{q}) \int_L^U (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon}{c}]^q.$$

Thus, substituting $b = 0$ and the implicit expression for Q^I into the expected retail price Ep^*, given by (4.22), and simplifying results with $Ep^I = \frac{qc}{q-1}$, and substituting $w = c$, $b = 0$ and Q^I into (4.23) and simplifying results with the equilibrium profit of the integrated channel:

$$E\Pi^I = \frac{c}{q-1}Q^I.$$

The system under buyback contracts. In Stage 1, the manufacturer determines w and b simultaneously to maximize her expected profit function. Similar to the linear case, we work with $w^*(b, Q)$ instead of working with $Q^*(w, b)$. According to Lemma 4.4.14, there is a one-to-one relationship between the equilibrium values of $Q^*(w, b)$ and $w^*(b, Q)$, which guarantees that the alternative approach is valid. Thus, the alternative approach for the manufacturer's problem is to choose b and Q that maximize:

$$E\Pi_M = (w^*(b,Q) - c)Q - b(Q - E_\xi((p^*)^{-q} \cdot \xi)), \tag{4.25}$$

where $w^*(b, Q)$ is given by (4.24). The manufacturer's optimal decisions are given in the following proposition.

Proposition 4.4.15 *In the multiplicative p-postponement model with a negative polynomial expected demand function and buybacks, the manufacturer's expected profit is globally maximized at* ($w^* = \frac{qc}{q-1}, b^* = 0$)*, and in equilibrium,* $Q^* = [\frac{(1-\frac{1}{q})^2 \int_L^U (\epsilon)^{\frac{1}{q}} f(\epsilon) d\epsilon}{c}]^q$, $Ep^* = \frac{q^2 c}{(q-1)^2}$, *and*

$$E\Pi_M^* = \frac{q-1}{q} E\Pi_R^* = \frac{c}{q-1} Q^*.$$

Proposition 4.4.15 implies that under a negative polynomial expected demand function, buybacks are not implemented in equilibrium. Thus, the system under a buyback contract coincides with the system under a wholesale price-only contract.

From the analysis of the integrated channel and Proposition 4.4.15, we immediately have:

Proposition 4.4.16 *In the multiplicative p-postponement model with* $D(p) = p^{-q}$*, in equilibrium,*

(i) $Q^I = \frac{q^q}{(q-1)^q} Q^*$.

(ii) Channel efficiency is $\frac{(2q-1)(q-1)^{q-1}}{q^q}$.

(iii) $Ep^I = \frac{q}{q-1} Ep^*$.

(iv) The wholesale price, expected retail price, profit allocation ratio between M and R and channel efficiency coincide with those values in the corresponding deterministic model, wherein the demand function coincides with the expected demand function in the multiplicative p-postponement model.

Example 4.4.17 Power demand distribution with negative polynomial expected demand.

Similar to Examples 4.4.6 and 4.4.12, we have derived the equilibrium values in the decentralized multiplicative models for a power distribution of ξ with $t = 0$, $t = 1$ and $t = 4$ and a negative polynomial expected demand function with $q = 2$, i.e., $D(p) = p^{-2}$. The results are presented in Table 4A.3 in the appendix and will be further discussed in §4.6.

4.4.4 Summary on the multiplicative p-postponement model

Having derived the equilibrium values of the decision variables and profits in the multiplicative p-postponement model for three different expected demand functions, we are able to summarize the results in the multiplicative p-postponement model with and without buybacks.

By comparing the results in the multiplicative p-postponement model with buybacks and the results in the corresponding deterministic model for three different expected demand functions, which are displayed in Table 4.1, we have:

Theorem 4.4.18 *In the multiplicative PD-newsvendor model with buyback options, for a general distribution of ξ and linear, exponential or negative polynomial expected demand, under p-postponement, in equilibrium:*

(i) *The wholesale price, expected retail price, profit allocation ratio between M and R and channel efficiency coincide with those values in the corresponding deterministic model.*

(ii) *The buyback rate is independent of the distribution of ξ.*

In general, the results in the multiplicative p-postponement model under a wholesale price-only contract depend on the distribution of ξ. Based on the numerical results in the multiplicative PD-newsvendor model with linear and exponential expected demand functions and a power distribution of ξ, presented in Tables 4A.1 and 4A.2 in the appendix, we observe that under price postponement, in equilibrium, buyback contracts induce a higher order quantity than wholesale price-only contracts, i.e., $Q^* \geq \hat{Q}^*$.

As was the case in the multiplicative Q-postponement model, the double marginalization in the multiplicative p-postponement model for the expected demand functions we have considered is reflected by the fact that the integrated channel offers the lowest retail price and highest production quantity.

4.5 No Postponement in the Multiplicative Model

To be able to evaluate the effect of the various postponement strategies, we need to analyze the multiplicative PD-newsvendor model without any postponement, i.e., all decisions are made before demand is realized. Thus, in this case, the manufacturer initiates the process by offering a wholesale price w possibly with a buyback rate b (Stage 1), and then the retailer chooses a retail price p and an order quantity Q (Stage 2). Thereafter, demand is realized. Again, backward induction is used to solve this two-stage Stackelberg game. To make the manufacturer's expected profit function in Stage 1 well behaved, we assume that the distribution of ξ has the increasing failure rate (IFR) property, i.e., $\frac{f(\epsilon)}{1-F(\epsilon)}$ is increasing in ϵ. The expected profit functions of the manufacturer and the retailer are:

$$E\Pi_M = D(p)[(w-c)z - b\Lambda(z)] \quad \text{and} \quad E\Pi_R = D(p)\{(p-b)[z-\Lambda(z)] - (w-b)z\}, \tag{4.26}$$

where $\Lambda(z) = \int_L^z (z-\epsilon)f(\epsilon)d\epsilon$, which represents the unsold inventory due to demand uncertainty. Recall that $z = \frac{Q}{D(p)}$. Following Petruzzi and Dada (1999) and Wang et al. (2004), z is called the "stocking factor", and further, when Q is chosen after p is set, the problem of choosing a retail price p and an order quantity Q is equivalent to choosing a retail price p and a stocking factor z. Again, the deterministic demand function, $D(p)$, is assumed to take three different forms: linear, exponential and negative polynomial. Note that the multiplicative N-postponement model with and without buybacks under a uniform ξ has been analyzed in Chapter 2 in this thesis to study the effect of buybacks in the PD-newsvendor model.

4.5.1 Multiplicative N-postponement model with linear expected demand

In this subsection, we consider the multiplicative PD-newsvendor model under no postponement with a linear expected demand function, i.e., $D(p) = 1-p$. We discuss the models with and without buybacks separately.

The system under buyback contracts. In Chapter 2 we have derived explicit expressions for the equilibrium decisions and profits assuming the random component of demand, ξ, follows a uniform distribution, and Song et al. (2004) have extended our results to a distribution of ξ with the increasing failure rate (IFR) property. Let us recall their results:

$$w^* = \frac{1}{2}(1+c), b^* = \frac{1}{2}, \quad p^* = \frac{1}{2}[1 + \frac{z^* - \Lambda(z^*)}{z^* - \Lambda(z^*) + \int_L^{z^*} \epsilon f(\epsilon)d\epsilon}], \quad Q^* = (1-p^*)z^*, \tag{4.27}$$

$$E\Pi_M^* = 2E\Pi_R^* = \frac{1-p^*}{2}[z^* - \Lambda(z^*) - cz^*], \tag{4.28}$$

where z^* is the unique solution to $(1-F(z)) - c(1 + \frac{\int_L^z \epsilon f(\epsilon)d\epsilon}{z - \Lambda(z)}) = 0$, and, as we recall, $\Lambda(z) = zF(z) - \int_L^z \epsilon f(\epsilon)d\epsilon$.

To evaluate the effect of order postponement and price postponement, based on (4.27) and (4.28), we need to assume a specific distribution for ξ. Accordingly, we assume that ξ has a power distribution, $f(\epsilon) = \gamma(\epsilon)^t$, and we present in Table 4A.1 in the appendix the effect of p-postponement on equilibrium values in the multiplicative model under buybacks, for $t = 0$, $t = 1$ and $t = 4$. These values will be further discussed in the next section.

The system under wholesale price-only contracts. In the model without buybacks, we use backward induction to solve the two-stage Stackelberg game. In Stage 2, substituting $b = 0$ into the retailer's expected profit function, given by (4.26), and simplifying results with:

$$E\Pi_R = (1-p)(p[z - \Lambda(z)] - wz).$$

Following the analysis in Song et al. (2004), for any given w, the retailer's expected profit function is well-behaved in (p, z), and the first order conditions result with a unique solution, (p^*, z^*), to the retailer's problem of maximizing his expected profit for any distribution of ξ, which satisfies:

$$w = p(1 - F(z)) \quad \text{and} \quad p = \frac{z - \Lambda(z)}{z - \Lambda(z) + \int_L^z \epsilon f(\epsilon) d\epsilon}.$$

However, in general, the manufacturer's expected profit function in Stage 1, taking into account the retailer's reaction function of (p^*, z^*), may not be well behaved, see Song et al. (2004).

Chapter 2 has derived implicit expressions for the equilibrium values of the wholesale price, buyback rate, expected retail price, order quantity and the expected profits of M and R in the multiplicative N-postponement model under a wholesale price-only contract for a uniformly distributed ξ (i.e., $t = 0$). Fortunately, under a power distribution of ξ, for any given value of the exponent t, by using Maple, we can show that the manufacturer's expected profit function in Stage 1 is unimodal in $z \in [0, U]$ if we work with $w^*(z)$ instead of $z^*(w)$, i.e., the manufacturer chooses the stocking level to maximize her expected profit. Thus, implicit expressions for the equilibrium values as a function of t are available. In Table 4A.1 in the appendix, we present the percentage changes in the equilibrium values in the model under a wholesale price-only contract for a power distribution of ξ and for $t = 0$, $t = 1$ and $t = 4$.

4.5.2 Multiplicative N-postponement model with exponential expected demand

In this subsection, we study the multiplicative N-postponement model for $D(p) = e^{-p}$.

The system under buyback contracts. In Chapter 2 we have derived explicit expressions for the equilibrium decisions and profits under buybacks assuming that ξ follows a uniform distribution, and we have numerically extended the study to two families of distributions: power and triangular. We next extend the analysis to a more general distribution of $\xi \in [L, U]$.

Proposition 4.5.1 *In the multiplicative N-postponement model with buybacks and $D(p) = e^{-p}$, if the density function of ξ, $f(\epsilon)$, is continuous and differentiable, and $\epsilon f(\epsilon)$ is increasing in ϵ, then the manufacturer's expected profit function is globally maximized at $(w^* = 1 + c, b^* = 1)$, and in equilibrium, $E\Pi_M^* = E\Pi_R^*$.*

Song et al. (2004) have independently proven Proposition 4.5.1 for a distribution of ξ with the IFR property for an exponential expected demand function.

As explained in the paragraph following Proposition 4.4.4 in §4.4.1, the requirement that $\epsilon f(\epsilon)$ is increasing in ϵ is satisfied by the Beta distribution for values of the shape parameters (α, β)

satisfying $\beta \leq 1$ and $\alpha \geq 1-\beta$, and when $(\beta = 1, \alpha \geq 1)$, the beta distribution is reduced to the power distribution with a positive exponent.

Finally, in Table 4A.2 in the appendix, we present the percentage changes in equilibrium values, due to price postponement, in the multiplicative PD-newsvendor model with buybacks for a power distribution of ξ and $t = 0$, $t = 1$ and $t = 4$.

The system under wholesale price-only contracts. As it was in the linear expected demand function case, the manufacturer's profit function in Stage 1, taking the retailer's reaction into account, is not generally well-behaved. Thus, we need to assume some specific distributions of ξ. Implicit expressions for the equilibrium values for a uniformly distributed ξ have been derived in Chapter 2. Again, under a power distribution, for any given value of the exponent t, we are able to show that the manufacturer's expected profit function is indeed concave in $z \in [0, U]$, and implicit expressions of the equilibrium values in the model are available. In Table 4A.2 in the appendix, we present the percentage changes in equilibrium values, due to price postponement, in the multiplicative PD-newsvendor model under a wholesale price-only contract for a power distribution of ξ and $t = 0$, $t = 1$ and $t = 4$.

4.5.3 Multiplicative N-postponement model with negative polynomial expected demand

Similar to the linear case, for the negative polynomial expected demand function, $D(p) = p^{-q}$, we have derived in Chapter 2 explicit expressions for the equilibrium decisions and profits assuming that the random component of demand, ξ, follows a uniform distribution, and Song et al. (2004) have extended the results to a ξ which has the IFR (increasing failure rate) property. We, again, recall their results:

$$w^* = \frac{q}{q-1}c, \quad b^* = 0, \quad p^* = \frac{q}{q-1}\frac{c}{1-F(z^*)}, \quad Q^* = p^{-q} \cdot z^*,$$

$$E\Pi_M^* = \frac{q-1}{q} E\Pi_R^* = \frac{cQ^*}{q-1},$$

where z^* is the unique positive solution to $z[1-F(z)] = (q-1)\int_L^z \epsilon f(\epsilon)d\epsilon$.

Note that $b^* = 0$ for the negative polynomial expected demand function. Thus, a buyback contract reduces to a wholesale price-only contract. Similar to the linear and exponential expected demand cases, we present in Table 4A.3 in the appendix the percentage changes in equilibrium values, due to price postponement, in the multiplicative PD-newsvendor model for a power distribution of ξ with $t = 0$, $t = 1$ and $t = 4$ and $D(p) = p^{-2}$.

4.6 Effect of Postponement in the Multiplicative Model

Having derived the equilibrium values in the models with and without price and order postponement in the multiplicative PD-newsvendor model, we are now ready to evaluate the effects of these postponement strategies in the multiplicative model with and without buybacks.

Evidently, in view of the results derived, the effect of postponement in the multiplicative PD-newsvendor model depends crucially on whether a buyback option is offered. Specifically, our results reveal a remarkable relationship between the multiplicative Q-, p- and N-postponement PD-newsvendor models with buybacks and the corresponding deterministic model. Indeed, it was shown that for linear, exponential and negative polynomial expected demand functions: (1) for any distribution of ξ, in equilibrium, the wholesale price, expected retail price, profit allocation ratio between M and R, and channel efficiency coincide in the multiplicative p-postponement, Q-postponement and deterministic models, (2) under some conditions on the distribution of ξ, and with exception of the expected retail price, these equilibrium values coincide with their counterparts in the multiplicative N-postponement model, and (3) under some conditions on the distribution of ξ, the buyback rates in the multiplicative p-postponement and N-postponement models coincide.

We suggest that it is quite remarkable that the multiplicative Q-, p- and N-postponement models, as well as the corresponding deterministic model, have the same profit allocation ratio between M and R. By contrast, neither this invariant result, nor any of the others which hold for the multiplicative PD-newsvendor model with buybacks prevails under a wholesale price-only contract. In fact, as it is observed below, under a wholesale price-only contract, for example, M could strictly benefit from the introduction of p-postponement while R could be strictly worse off from such a postponement strategy.

Note that with the exception of the buyback rate, the effect of order postponement is qualitatively similar to the effect of price postponement in the multiplicative PD-newsvendor model with or without buybacks. Thus, in this section, we only present the effects of price postponement. In the following three subsections, based on the analysis in §4.4 and §4.5 and computational results presented in the three tables in the appendix, we discuss the effect of price postponement in the multiplicative PD-newsvendor model with a power distribution of ξ and linear, exponential and negative polynomial expected demand functions.

4.6.1 Linear expected demand

Table 4A.1 in the appendix presents the comparison between the multiplicative models with and without price postponement for a linear expected demand function. From Table 4A.1 we conclude that if c (or $\frac{c}{k}$ for a general expected demand function $D(p) = a(k - p)$) is not too small, price postponement is quite beneficial. Indeed, when the channel profit is relatively small, due to, e.g., a high manufacturing cost, price postponement can increase the manufacturer's (respectively, retailer's) expected profit by as much as 270% (respectively, 210%), as is the case for $c = 0.8$ under a wholesale price-only contract.

The results displayed in Table 4A.1 confirm that the effect of p-postponement in the multiplicative model depends strongly on the contract. Indeed, when returns are permitted, as expected, the equilibrium wholesale price, buyback rate and profit allocation ratio are unaffected by p-postponement. By contrast, when returns are not permitted, the equilibrium wholesale price can change quite significantly due to p-postponement, and for example, for $c = 0$, it increases by 32%. Moreover, while the retailer is always better off with p-postponement, if returns are permitted, for a very low manufacturing cost, R could end up being worse off due to p-postponement if returns are not permitted. For example, for $c = 0$, his profit would decrease by about 10%. In fact, for very low values of c, the entire channel could be worse off due to p-postponement, and for $c = 0$, it would decrease by about 4%. Thus, the expected value of perfect information about demand for the retail pricing decision could be negative in a decentralized firm. This is in stark contrast to p-postponement in a centralized firm, wherein the firm is always (weakly) better off due to p-postponement (Van Mieghem and Dada (1999)).

For $c = 0$, the multiplicative model with buybacks turns out to be a full-price buyback contract, i.e., $b = w$, and the introduction of p-postponement in this case has no effect on the equilibrium values. For $c > 0$, in both models with and without buybacks, due to p-postponement, production (i.e., Q^*) is decreasing when c is small and increasing when c is large, and in the model with buybacks, the expected retail price under p-postponement is somewhat smaller than the retail price without postponement.

Finally, we observe that R, who presumably initiates p-postponement, gains relatively less than M from such a strategy. Indeed, when returns are possible, for every dollar increase in R's expected profit, M's expected profit increases by two dollars, and from Table 4A.1 we conclude that when returns are not possible, the percentage increase in M's expected profit is always larger than that of R. This suggests that M may have an incentive to financially support a possible desire by R to implement p-postponement in the multiplicative model.

4.6.2 Exponential expected demand

Table 4A.2 in the appendix displays the comparison between the multiplicative models with and without price postponement under an exponential expected demand function.

Perhaps the most striking observation which follows from Table 4A.2 is that the introduction of p-postponement in the multiplicative PD-newsvendor model with a wholesale price-only contract and exponential expected demand will make both the manufacturer and the retailer worse off when the marginal manufacturing cost c is small. We also can conclude from Table 4A.2 that both for buyback contracts and wholesale price-only contracts, the effect of price postponement on the equilibrium values of the wholesale and retail prices, order quantity and expected profits is similar to that in the linear expected demand case, with the exception that the profit allocation ratio in the exponential case is one to one.

4.6.3 Negative polynomial expected demand

Note that with a negative polynomial expected demand function, buybacks are not implemented in both the multiplicative p- and N-postponement models, i.e., $b^* = 0$. Following Proposition 4.4.15 and the analysis of the multiplicative N-postponement model with a negative polynomial expected demand function in §4.5, we conclude that the percentage changes, due to price postponement, in the equilibrium values of the order quantity, expected retail price, and profits of M and R are independent of the manufacturing cost c. Additionally, from the analysis in §4.5 and Proposition 4.4.15, it follows that the equilibrium wholesale price is invariant to the introduction of p-postponement. All these results are confirmed in Table 4A.3.

Finally, based on the numerical results for a power distribution of ξ, we conclude from Table 4A.3 in the appendix that due to price postponement, both M and R are better off, and the retail price (respectively, order quantity) decreases (respectively, increases) in the multiplicative model.

4.7 Postponement in the Additive Model

In this section, we extend our analysis of postponement strategies to the PD-newsvendor model with an additive demand function, i.e., $X = D(p) + \xi$, where, as noted in Chapter 1, $\xi \in [L, U]$ and $D(p) + L \geq 0$ for some p. As it becomes apparent in this section, the analysis of the additive PD-newsvendor model could turn out to be significantly more difficult than that of the multiplicative model. For instance, the analysis of the additive model with a linear expected demand function without postponement is quite complex (see, e.g., Wang et al. (2004) and Song et al. (2004)), as is the analysis of this model with p-postponement. In fact, even for the case when ξ is uniformly distributed on $[0,2]$, it is difficult to derive a closed-form expression for, e.g., the equilibrium value of the wholesale price in the additive p-postponement model. Thus, for tractability reasons, we only consider in this section postponement under wholesale price-only contracts. For convenience, we will denote by $()^*$ instead of $\hat{()}^*$ the equilibrium values of decisions and profits in this section.

4.7.1 Additive Q-postponement model

In an additive demand model, $X = D(p) + \epsilon$. Thus, $E(X) = D(p) + E(\xi) = D(p) + \mu_\xi$, and M's and R's expected profit functions, given by (1.2), become $E\Pi_M = (w - c)(D(p) + \mu_\xi)$ and $E\Pi_R = (p - w)(D(p) + \mu_\xi)$, which are independent of the distribution of ξ, and only depend on μ_ξ.

In Stage 2, before observing demand uncertainty, R commits to a retail price p to maximize $E\Pi_R = (p - w)(D(p) + \mu_\xi)$. Assume that the optimal p, $p(w, \mu_\xi)$, for R in Stage 2 is unique and $D(p(w, \mu_\xi)) + L \geq 0$. Then, M's expected profit function becomes:

$$E\Pi_M = (w - c)(D(p(w, \mu_\xi)) + \mu_\xi). \tag{4.29}$$

Let w^*, p^*, Q^* denote the equilibrium values of w, p, Q, respectively. Then, we can conclude that in equilibrium,

(1) w^* and p^* are only functions of c and μ_ξ,

(2) $Q^* = D(p^*) + \hat{\epsilon}$, and

(3) $E\Pi_M^* = (w^* - c)(D(p^*) + \mu_\xi)$ and $E\Pi_R^* = (p^* - w^*)(D(p^*) + \mu_\xi)$,

where $\hat{\epsilon}$ is the observed value of ξ. As was the case in §4.3 for the multiplicative model, we can immediately make the following observation.

Observation 4.7.1 *The equilibrium values in the additive Q-postponement model coincide with those corresponding to the model with deterministic demand, wherein the demand function coincides with the expected demand function in the additive Q-postponement model.*

The effect of Q-postponement in the additive model is consistent with that in the multiplicative model described in Observation 4.3.1. The following example will be used subsequently.

Example 4.7.2 Additive PD-newsvendor model under Q-postponement.

Consider the additive PD-newsvendor model where $X = D(p) + \xi$, $D(p) = -p$, $\xi \in [L, U]$, and $D(p) + L \geq 0$ for some $p(\geq w)$. R's expected profit in Stage 2 of the additive Q-postponement model becomes $E\Pi_R = (p-w)(-p+\mu_\xi)$, which is concave in p. Thus, $p(w,\mu_\xi) = \frac{w+\mu_\xi}{2}(\geq w)$, and M's expected profit in Stage 1 becomes $E\Pi_M = (w-c)(-p(w,\mu_\xi)+\mu_\xi) = (w-c)(\frac{-w+\mu_\xi}{2})$, which, again, is concave in w. Thus, $w^* = \frac{c+\mu_\xi}{2}$. Accordingly, for the additive Q-postponement model, in equilibrium, $p^* = \frac{c+3\mu_\xi}{4}$, $EQ^* = E[\frac{-c-3\mu_\xi+4\xi}{4}] = \frac{-c+\mu_\xi}{4}$ and $E\Pi_M^* = 2E\Pi_R^* = \frac{(-c+\mu_\xi)^2}{8}$.

4.7.2 Additive p-postponement model

As previously mentioned, the analysis of the additive model is more complex. As a result we only analyze in this subsection the additive price postponement PD-newsvendor model under price postponement assuming that ξ is binary. More specifically, we adopt the demand model in Padmanabhan and Png (1997), wherein[4.1] $X = \xi - p$, and the random intercept, ξ, follows a binary distribution with two market states: *Low* (α_l) with a probability λ and *High* (α_h) with a probability $(1-\lambda)$. To avoid trivialities, $0 < \lambda < 1$. Let $\mu_\xi \equiv \lambda\alpha_l + (1-\lambda)\alpha_h$ denote the expected value of ξ. As mentioned in Padmanabhan and Png (1997), demand uncertainty is captured by two aspects of the demand model. Namely, the range of possible demand outcomes, denoted as $\theta \equiv \frac{\alpha_h}{\alpha_l}$, and the probabilities, λ and $(1-\lambda)$, of the two events. In the former case, uncertainty is increasing in α_h or θ, holding α_l constant. In the latter case, uncertainty is maximized when $\lambda = 0.5$. Note that $\theta > 1$. For further simplification and for tractability reasons, we assume that M's marginal manufacturing cost $c = 0$.

Backward induction is used to analyze the problem. In Stage 3, given (w, Q) and observing the random part of demand, $\hat{\epsilon}$, R chooses his best retail price p to maximize:

$$\Pi_R = p\ \min(Q, [\hat{\epsilon} - p]^+) - wQ. \tag{4.30}$$

Clearly, either $\hat{\epsilon} = \alpha_l$ or $\hat{\epsilon} = \alpha_h$. The best retail price for R is characterized as follows.

Proposition 4.7.3 *Given (w, Q) and the resolved demand uncertainty, α_s, R's optimal retail price is:*

$$p_s^* = \begin{cases} \alpha_s - Q & \text{if } Q \leq \frac{\alpha_s}{2}, \\ \frac{\alpha_s}{2} & \text{if } Q \geq \frac{\alpha_s}{2}, \end{cases} \tag{4.31}$$

where $s \in \{l, h\}$.

Proposition 4.7.3 implies that a market clearing price is not always optimal for R. Indeed, when the quantity, ordered in Stage 2, is found to be relatively larger than the realized market state, i.e., $Q \geq \frac{\alpha_s}{2}$, it is optimal for R to charge a higher retail price than the market clearing price, which will induce some leftover inventories at the end of the selling period.

[4.1] The same demand model is used elsewhere in the literature, see, e.g., Anand and Mendelson (1997).

In Stage 2, before demand uncertainty is realized, for any given w and knowing p_l^* and p_h^*, given by (4.31), R chooses Q to maximize his expected profit:

$$E\Pi_R = \lambda p_l^* \,(\alpha_l - p_l^*) + (1-\lambda)p_h^* \,(\alpha_h - p_h^*) - wQ. \tag{4.32}$$

Note that the expected sales are $\lambda(\alpha_l - p_l^*) + (1-\lambda)(\alpha_h - p_h^*)$. R's best order quantity is characterized in the following proposition.

Proposition 4.7.4 *Given w and knowing p_l^* and p_h^*, given by (4.31), R's optimal order quantity and his corresponding optimal retail prices are:*

$$(Q^*, p_l^*, p_h^*) = \begin{cases} \left(\frac{(1-\lambda)\alpha_h - w}{2(1-\lambda)}, \frac{\alpha_l}{2}, \frac{(1-\lambda)\alpha_h + w}{2(1-\lambda)}\right) & \text{if } w \leq (1-\lambda)(\alpha_h - \alpha_l), \\ \left(\frac{\mu_\xi - w}{2}, \frac{2\alpha_l - \mu_\xi + w}{2}, \frac{2\alpha_h - \mu_\xi + w}{2}\right) & \text{if } (1-\lambda)(\alpha_h - \alpha_l) \leq w \leq \mu_\xi, \\ (0, -, -) & \text{if } w \geq \mu_\xi, \end{cases} \tag{4.33}$$

where $\mu_\xi = \lambda\alpha_l + (1-\lambda)\alpha_h$.

It follows from Proposition 4.7.4 that if the wholesale price is too high, i.e., $w \geq \mu_\xi$, then R would not accept the contract, i.e., $Q^* = 0$. On the other hand, whenever R accepts the contract, he would order, in Stage 2, a stock, Q, for which the market clearing price in Stage 3 is optimal for him when the market state is high. The market clearing price is still optimal for R when the market state is low if the wholesale price is high enough, i.e., $w \geq (1-\lambda)(\alpha_h - \alpha_l)$. However, if the wholesale price is relatively low, i.e., $w \leq (1-\lambda)(\alpha_h - \alpha_l)$, then the market clearing price is not optimal for him. Rather, in this case, his best price would induce some leftover inventories at the end of the period.

We are now ready to solve M's problem in Stage 1. In Stage 1, taking into account of R's reaction functions p_s^* and Q^*, given by (4.33), M chooses w to maximize her own expected profit function: $E\Pi_M = wQ^*$. (Recall that $c = 0$.) M's best choice of w and the corresponding equilibrium values of the other decision variables and the profits of the channel and its members are presented in the following proposition.

Proposition 4.7.5 *In the additive p-postponement model under a wholesale price-only contract, for $c = 0$, the equilibrium values are presented in Table 4.2 in the following page:*

	w^*	p_l^*	p_h^*	Q^*	$E\Pi_M^*$	$E\Pi_R^*$	$E\Pi_{M+R}^*$
$\theta \geq 1 + \frac{1}{\sqrt{(1-\lambda)}}$	$\frac{(1-\lambda)\alpha_h}{2}$	$\frac{\alpha_l}{2}$	$\frac{3\alpha_h}{4}$	$\frac{\alpha_h}{4}$	$\frac{(1-\lambda)\alpha_h^2}{8}$	$\frac{\lambda\alpha_l^2}{4} + \frac{(1-\lambda)\alpha_h^2}{16}$	$\frac{\lambda\alpha_l^2}{4} + \frac{3(1-\lambda)\alpha_h^2}{16}$
$\theta \leq 1 + \frac{1}{\sqrt{(1-\lambda)}}$	$\frac{\mu_\xi}{2}$	$\frac{4\alpha_l-\mu_\xi}{4}$	$\frac{4\alpha_h-\mu_\xi}{4}$	$\frac{\mu_\xi}{4}$	$\frac{\mu_\xi^2}{8}$	$\frac{\mu_\xi^2}{16}$	$\frac{3\mu_\xi^2}{16}$

Table 4.2: Equilibrium values in the additive p-postponement model

Note that the model analyzed in this subsection has been explicitly studied by Padmanabhan and Png (1997). However, their analysis is somewhat incomplete. Specifically, they have only considered the case where $\theta \geq 1 + \frac{1}{\sqrt{1-\lambda}}$, and they did not evaluate $E\Pi_R^*$ (and $E\Pi_{M+R}^*$).

4.7.3 Additive N-postponement model

In this subsection, we study the additive PD-newsvendor model under a wholesale price-only contract without any postponement. Again, for simplicity, we assume $c = 0$ and $X = \xi - p$, where, as we recall, ξ follows a binary distribution with two market states: α_l with a probability λ and α_h with a probability $(1-\lambda)$, and $\alpha_l < \alpha_h$. The timeline is as follows. M, as the Stackelberg leader, initiates the process by offering a wholesale price w (Stage 1), and then, R commits to both a retail price p and an order quantity Q (Stage 2). Demand is realized thereafter.

We first consider R's problem. Given w, R chooses p and Q to:

$$\begin{aligned}\text{Maximize} \quad E\Pi_R(p,Q) &= p\,Sale - wQ \\ &= p(\lambda \min(Q, [\alpha_l - p]^+) + (1-\lambda)\min(Q, [\alpha_h - p]^+)) - wQ, \quad (4.34)\\ &\text{s.t.}\ \ 0 \leq Q \leq \alpha_h - p \ \text{ and } \ w \leq p \leq \alpha_h.\end{aligned}$$

Note that the highest market state is α_h. Thus, the highest demand for any given p is $\alpha_h - p$. Therefore, we assume that $Q \leq \alpha_h - p$. We divide the feasible region, $\{(p,Q)|0 \leq Q \leq \alpha_h - p\}$, into three subregions in the (p,Q) plane (Figure 4.1 in the following page), and analyze R's problem in each subregion in order to find the optimal point (p^*, Q^*) for R.

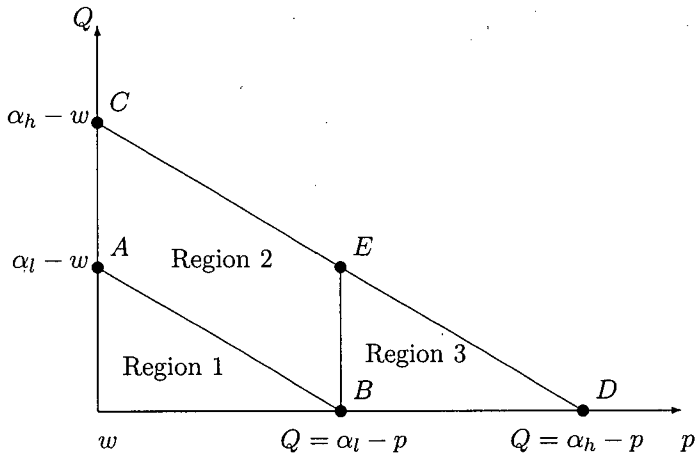

Figure 4.1: Three feasible subregions for R in Stage 2

Proposition 4.7.6 *For any given w, R's optimal p^* and Q^* lie either on Segment AB or on Segment CD. Thus, either $Q^* = \alpha_h - p^*$ or $Q^* = \alpha_l - p^*$.*

Following Proposition 4.7.6 and its proof in the appendix, R's optimal profit, obtained from (4.34), is equivalent to:

$$E\Pi_R^* = \max(E\Pi_R^{AB}, E\Pi_R^{CE}, E\Pi_R^{ED}),$$

where the superscripts AB, CE, and ED stand for the corresponding segments in Figure 4.1, and

$$E\Pi_R^{AB} = \max\{E\Pi_R(p) = (p - w)(\alpha_l - p), \ \ s.t. \ \ w \le p \le \alpha_l\}, \tag{4.35}$$

$$E\Pi_R^{CE} = \max\{E\Pi_R(p) = p(\mu_\xi - p) - w(\alpha_h - p), \ \ s.t. \ \ w \le p \le \alpha_l\}, \tag{4.36}$$

$$E\Pi_R^{ED} = \max\{E\Pi_R(p) = (p(1-\lambda) - w)(\alpha_h - p), \ \ s.t. \ \ \alpha_l \le p \le \alpha_h\}. \tag{4.37}$$

Since in all three problems, (4.35), (4.36) and (4.37), R's expected profit function is strictly concave in p, one can easily verify that R's optimal decisions in these problems are as displayed below.

For (4.35), $p^* = \frac{\alpha_l + w}{2}$, $Q^* = \frac{\alpha_l - w}{2}$, and $E\Pi_R^{AB} = \frac{(\alpha_l - w)^2}{4}$.
For (4.36), $p^* = \min(\alpha_l, \frac{\mu_\xi + w}{2})$, or

$$p^* = \begin{cases} \frac{\mu_\xi + w}{2} & \text{if } w \le (2-\lambda)\alpha_l - (1-\lambda)\alpha_h, \\ \alpha_l & \text{if } w \ge (2-\lambda)\alpha_l - (1-\lambda)\alpha_h, \end{cases} \tag{4.38}$$

and $Q^* = \alpha_h - p^*$. $E\Pi_R^{CE}$ can be calculated accordingly. (Recall that $\mu_\xi = \lambda\alpha_l + (1-\lambda)\alpha_h$.)

For (4.37),

$$p^* = \begin{cases} \alpha_l & \text{if } w \le (1-\lambda)(2\alpha_l - \alpha_h), \\ \frac{(1-\lambda)\alpha_h + w}{2(1-\lambda)} & \text{if } (1-\lambda)(2\alpha_l - \alpha_h) \le w \le (1-\lambda)\alpha_h, \\ \alpha_h & \text{if } w \ge (1-\lambda)\alpha_h, \end{cases} \tag{4.39}$$

and $Q^* = \alpha_h - p^*$. Similarly, $E\Pi_R^{ED}$ can be calculated accordingly.

Note that when $w \ge \alpha_l$, $p \ge \alpha_l$. Thus, (p^*, Q^*) must lie on Segment ED, and $E\Pi_R^* = E\Pi_R^{ED}$. When $w \le \alpha_l$, we need to compare $E\Pi_R^{AB}$, $E\Pi_R^{CE}$ and $E\Pi_R^{ED}$ in order to find (p^*, Q^*). Unfortunately, it is quite tedious to derive closed form expressions for p^* and Q^* for any given $w \in [0, \alpha_l]$ and any relationship between the parameters θ and λ (see also Footnote 9 in Padmanabhan and Png (1997)). Thus, for simplicity, we next consider just two special scenarios regarding the relationships between θ and λ: Scenario I: $\theta \ge 1 + \frac{1}{1-\lambda}$, and Scenario II: $\frac{1}{1-\lambda} \le \theta \le 2$.

Scenario I: $\theta \ge 1 + \frac{1}{1-\lambda}$. Under Scenario I, R's optimal reaction function (p^*, Q^*) can be characterized as follows:

Proposition 4.7.7 *When $\theta \ge 1 + \frac{1}{1-\lambda}$, i.e., $(2-\lambda)\alpha_l - (1-\lambda)\alpha_h \le 0$, for any given w, the optimal solution (p^*, Q^*) for R is on Segment ED (i.e., $Q^* = \alpha_h - p^*$) and*

$$p^* = \begin{cases} \frac{(1-\lambda)\alpha_h + w}{2(1-\lambda)} & \text{if } 0 \le w \le (1-\lambda)\alpha_h, \\ \alpha_h & \text{if } (1-\lambda)\alpha_h \le w \le \alpha_h. \end{cases}$$

Knowing p^* and Q^*, M's problem in Stage 1 is to choose w to maximize $E\Pi_M = wQ^*$. We next consider two choices for w. (1) $w \le (1-\lambda)\alpha_h$. Then $p^* = \frac{(1-\lambda)\alpha_h + w}{2(1-\lambda)}$, $Q^* = \frac{(1-\lambda)\alpha_h - w}{2(1-\lambda)}$, and $E\Pi_M = \frac{((1-\lambda)\alpha_h - w)w}{2(1-\lambda)}$, which is strictly concave in w. Thus, $w^* = \frac{(1-\lambda)\alpha_h}{2}$ and $E\Pi_M^* = \frac{(1-\lambda)\alpha_h^2}{8}$. (2) $w \ge (1-\lambda)\alpha_h$. Then $p^* = \alpha_h$, $Q^* = 0$, and $E\Pi_M^* = 0$. Evidently, M prefers (1), and Table 4.3 below provides the equilibrium solution and profits under Scenario I.

	w^*	p^*	Q^*	$E\Pi_M^*$	$E\Pi_R^*$	$E\Pi_{M+R}^*$
$\theta \ge 1 + \frac{1}{(1-\lambda)}$	$\frac{(1-\lambda)\alpha_h}{2}$	$\frac{3}{4}\alpha_h$	$\frac{\alpha_h}{4}$	$\frac{(1-\lambda)\alpha_h^2}{8}$	$\frac{(1-\lambda)\alpha_h^2}{16}$	$\frac{3(1-\lambda)\alpha_h^2}{16}$

Table 4.3: Equilibrium values in the additive N-postponement model for $c = 0$ under Scenario I

Scenario II: $\frac{1}{1-\lambda} \le \theta \le 2$. Under Scenario II, R's optimal (p^*, Q^*) can be characterized as follows:

Proposition 4.7.8 *When $\frac{1}{1-\lambda} \le \theta \le 2$, i.e., $(1-\lambda)(2\alpha_l - \alpha_h) \ge 0$ and $(1-\lambda)\alpha_h \ge \alpha_l$, for any given w, the optimal solution (p^*, Q^*) for R satisfies $Q^* = \alpha_h - p^*$, where*

$$p^* = \begin{cases} \frac{\mu_\xi + w}{2} & \text{if } 0 \le w \le (1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l), \\ \frac{(1-\lambda)\alpha_h + w}{2(1-\lambda)} & \text{if } (1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l) \le w \le (1-\lambda)\alpha_h, \\ \alpha_h & \text{if } (1-\lambda)\alpha_h \le w \le \alpha_h. \end{cases}$$

Observe that according to Proposition 4.7.8 and its proof in the appendix, in Scenario II, the optimal (p^*, Q^*) is attained either on Segment CE (when $w \le (1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l)$), or on Segment ED (when $w \ge (1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l)$). Now, knowing p^* and Q^*, given by Proposition 4.7.8, M chooses w in Stage 1 to maximize $E\Pi_M = wQ^*$. According to Proposition 4.7.8, it is easy to verify that

$$E\Pi_M(w \in [(1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l), (1-\lambda)\alpha_h]) \ge 0 = E\Pi_M(w \in [(1-\lambda)\alpha_h, \alpha_h]).$$

Thus, a choice of w for which $(1-\lambda)\alpha_h \le w \le \alpha_h$ is never optimal for M. Therefore, we only need to compare M's profit in Subproblem (1): $w \le (1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l)$, wherein Segment CE is optimal for R, and Subproblem (2): $w \ge (1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l)$, wherein Segment ED is optimal for R. Unfortunately, it is still too tedious to derive a closed-form expression for w^* due to the many possible relationships between the parameters λ and θ. Therefore, for simplicity, we need to further restrict the values of λ and θ. Now, note that in Scenario II, $\lambda \le 0.5$. Then, we set $\lambda = 1/8$, and we scale demand by normalizing the low demand state value, α_l, to 1, and thus, since $\theta \equiv \frac{\alpha_h}{\alpha_l}$, we have that $\alpha_h = \theta$. Under these simplifications, the equilibrium values of the contract parameters and profits for M and R are characterized in Proposition 4.7.9 below.

Proposition 4.7.9 *In the additive N-postponement model under a wholesale price-only contract with $c = 0$, $\lambda = \frac{1}{8}$ and $\frac{1}{1-\lambda} \le \theta \le 2$, the equilibrium wholesale price and the corresponding expected profit for M are:*

$$(w^*, E\Pi_M^*) = \begin{cases} \left(\frac{9\theta-1}{16}, \frac{(9\theta-1)^2}{512}\right) & \text{if } \frac{8}{7} \le \theta \le \frac{15+4\sqrt{14}}{23}, \\ \left(\frac{2\sqrt{14}-7\theta+7}{8}, \frac{(2\sqrt{14}-7\theta+7)(8\theta-4-\sqrt{14})}{64}\right) & \text{if } \frac{15+4\sqrt{14}}{23} \le \theta \le \bar{\theta}, \\ \left(\frac{7\theta}{16}, \frac{7\theta^2}{64}\right) & \text{if } \bar{\theta} < \theta \le 2, \end{cases}$$

and the corresponding equilibrium retail price, order quantity and expected profit for R are:

$$(p^*, Q^*, E\Pi_R^*) = \begin{cases} \left(\frac{23\theta+1}{32}, \frac{9\theta-1}{32}, \frac{-47\theta^2+110\theta+1}{1024}\right) & \text{if } \frac{8}{7} \le \theta \le \frac{15+4\sqrt{14}}{23}, \\ \left(\frac{4+\sqrt{14}}{8}, \frac{8\theta-4-\sqrt{14}}{8}, \frac{(14\theta-2\sqrt{14}-7)^2}{224}\right) & \text{if } \frac{15+4\sqrt{14}}{23} \le \theta \le \bar{\theta}, \\ \left(\frac{3\theta}{4}, \frac{\theta}{4}, \frac{7\theta^2}{128}\right) & \text{if } \bar{\theta} < \theta \le 2, \end{cases}$$

where $\bar{\theta} \equiv \frac{23\sqrt{14}+84+6\sqrt{7}+7\sqrt{2}}{126}$.

4.7.4 Effect of postponement in the additive model

Table 4.4 below summarizes the effects of p-postponement and Q-postponement in the additive model, i.e., $X = D(p) + \xi$, under a wholesale price-only contract with a linear expected demand, where ξ has two market states: high (α_h) and low (α_l). Recall that $\theta \equiv \frac{\alpha_h}{\alpha_l}$, and that for simplicity, it is assumed that $c = 0$.

	Scenario I: $\theta(= \frac{\alpha_h}{\alpha_l}) \geq 1 + \frac{1}{1-\lambda}$		Scenario II: $\lambda = \frac{1}{8}$ and $\frac{1}{1-\lambda} \leq \theta \leq 2$			
	Column 1	Column 2	Column 3			
	Effect of p-postponement	Effect of Q-postponement	Effect of p-postponement			
w^*	Unaffected	Increases	Decreases for a small θ, and then increases when θ becomes larger			
$p^*(Ep^*)$	Decreases	Increases	Decreases for a small θ, and then increases for an intermediate θ, and again decreases when θ is large enough			
$Q^*(EQ^*)$	Unaffected	Decreases	Decreases			
Profit dist.	Affected	Unaffected $(2:1)$	Affected			
			$1.14 < \theta < 1.39$	$1.39 < \theta < 1.52$	$1.52 < \theta < 1.55$	$1.55 < \theta < 2$
$E\Pi_M^*$	Unaffected	Decreases	Decreases	Decreases	Increases	Increases
$E\Pi_R^*$	Increases	Decreases	Increases	Decreases	Decreases	Increases
$E\Pi_{M+R}^*$	Increases	Decreases	Increases for $\theta < 1.33$, and then decreases	Decreases	Decreases	Increases

Table 4.4: Summary of the effect of postponement in the additive model with a binary distribution

Columns 1 & 3: p-postponement. Under Scenario I, since $(1-\lambda) \leq (1-\lambda)^{1/2}$, we have $\theta \geq 1+\frac{1}{1-\lambda} \geq 1+(1-\lambda)^{-1/2}$. Thus, following Tables 4.2 (p-postponement) and 4.3 (N-postponement) we derive Column 1 in Table 4.4. Accordingly, when the high market state is at least twice as large as the low market state (i.e., $\theta \geq 1 + \frac{1}{1-\lambda}$), R is able to extract the entire increase in channel profit, due to p-postponement, which solely stems from a reduction in the expected retail price. That is, the equilibrium order quantity and wholesale price, and thus, M's expected profit, are not affected by p-postponement.

Under Scenario II, we have $\theta \leq 2 < 1 + \frac{1}{1-\lambda}$. Thus, from Table 4.2 ($p$-postponement) and the analysis of Scenario II in §4.7.3 under N-postponement, we can derive the following proposition, which is summarized in Column 3 in Table 4.4.

Proposition 4.7.10 *In the additive PD-newsvendor model under a wholesale price-only contract with* $c = 0$, $\lambda = \frac{1}{8}$, *and* $\frac{1}{1-\lambda} \leq \theta \leq 2$, *in equilibrium,*

(i) $E\Pi_M^p < E\Pi_M^N$ *and* $E\Pi_R^p \geq E\Pi_R^N$, *if* $\frac{8}{7} \leq \theta \leq \frac{81}{121} + \frac{164\sqrt{14}}{847}$,

(ii) $E\Pi_M^p \leq E\Pi_M^N$ *and* $E\Pi_R^p \leq E\Pi_R^N$, *if* $\frac{81}{121} + \frac{164\sqrt{14}}{847} \leq \theta \leq \bar{\theta}$,

(iii) $E\Pi_M^p \geq E\Pi_M^N$ *and* $E\Pi_R^p < E\Pi_R^N$, *if* $\bar{\theta} \leq \theta \leq \frac{23\sqrt{14}+84+6\sqrt{7}+7\sqrt{2}}{126}$,

(iv) $E\Pi_M^p > E\Pi_M^N$ *and* $E\Pi_R^p > E\Pi_R^N$, *if* $\frac{23\sqrt{14}+84+6\sqrt{7}+7\sqrt{2}}{126} < \theta \leq 2$,

(v) w^p *is smaller (respectively, larger) than* w^N *when* θ *is small (respectively, becomes larger),*

(vi) Ep^p *is smaller than* p^N *when* θ *is small, it is larger for intermediate values of* θ, *and when* θ *is large enough,* Ep^p *is again smaller than* p^N, *and*

(vii) $Q^p < Q^N$,

where $\bar{\theta} = \frac{92\sqrt{14}+329+8\sqrt{56+14\sqrt{14}}}{497} \approx 1.52$, *and the supscripts "p" and "N" stand for p-postponement and N-postponement, respectively.*

Columns 1 and 3 in Table 4.4 and Proposition 4.7.10 clearly reveal that the effect of p-postponement depends strongly on the demand distribution. Indeed, when the value of the high market state is sufficiently larger than the value of the low market state, as is the case in Column 1 in Table 4.4, the channel's profit strictly increases, neither party is worse off, R is strictly better off, and among the equilibrium values of the decision variables, only the expected retail price is affected. However, when the value of the high market state is not sufficiently larger than the value of the low market state, p-postponement affects the equilibrium values of all decision variables. Moreover, under Scenario II (Column 3 in Table 4.4), p-postponement can decrease the channel profit. In fact, when $\theta \in (\frac{81}{121} + \frac{164\sqrt{14}}{847}, \frac{92\sqrt{14}+329+8\sqrt{56+14\sqrt{14}}}{497}) \approx (1.39, 1.52)$, both M and R are strictly worse off due to p-postponement. These results, again, are in stark contrast to the effect of p-postponement in a monopolistic setting.

__Column 2: Q-postponement.__ The equilibrium values under Q-postponement can be easily derived from Example 4.7.2 after substituting therein $c = 0$, and are as follows:

$$w^* = \frac{\mu_\xi}{2}, \quad p^* = \frac{3\mu_\xi}{4}, \quad EQ^* = \frac{\mu_\xi}{4} \text{ and } E\Pi_M^* = 2E\Pi_R^* = \frac{(\mu_\xi)^2}{8}, \tag{4.40}$$

where $\mu_\xi = \lambda\alpha_l + (1-\lambda)\alpha_h$. The effect of Q-postponement under Scenario I can be easily derived by comparing Table 4.3 for N-postponement with display (4.40) for Q-postponement, and is summarized in Column 2 in Table 4.4.

It follows from Column 2 in Table 4.4 that both M and R could be strictly worse off due to Q-postponement. This is, again, in stark contrast to Q-postponement in a monopolistic setting, wherein the centralized system is always (weakly) better off due to Q-postponement (Van Mieghem and Dada (1999)).

4.8 Managerial Insights and Conclusions

In general, in the multiplicative PD-newsvendor model, despite vertical competition and aside for some cases that are further discussed below, the effect of either p-postponement or Q-postponement are quite beneficial for the channel and its members. As such, postponement could be viewed as a viable strategy to increase channel efficiency. In particular, we note that the effect of either p-postponement or Q-postponement is quite substantial, percentage-wise, when the total channel profit of the supply chain is relatively small. For example, when this profit is relatively small, due, e.g., to a high manufacturing cost, p-postponement in the multiplicative model with a uniform ξ (i.e., $t = 0$ for a power distribution) can increase M's and R's expected profits by as much as 270% for M and 210% for R, as is the case for $c = 0.8$ and linear expected demand in the wholesale price-only contract in Table 4A.1. This suggests that when the total channel profit is relatively small, there is a significant advantage in obtaining more reliable information about market demand. Similarly, in general, postponement is beneficial for both players in the additive model.

Notwithstanding the benefits of postponement, it is clearly demonstrated in this chapter that for some parameter values, e.g., when the manufacturing cost is relatively low in the multiplicative model, the effects of postponement in a decentralized system are qualitatively different than their effect in a centralized system. Indeed, both in the multiplicative and additive models, p-postponement and Q-postponement can make the channel worse off, and in some instances, they could even make both M and R strictly worse off. In that regard, as far as we know, we are the first to provide examples wherein the expected value of perfect information in a competitive environment, modeled as a Stackelberg game, is negative.

Our results also demonstrate that in a decentralized setting, the party, i.e., R, who initiates postponement, does not necessarily end up gaining the lion share of the increase in the expected profit. For example, in the multiplicative case, when returns are possible and the expected demand function is linear, for every dollar increase in R's expected profit, M's expected profit increases by two dollars. Moreover, R's expected profit due to postponement could decrease, even though M's expected profit in this case increases. Thus, a strategic retailer should not implement p- or Q-postponement before, e.g., reaching a favorable agreement with M as to how any additional benefit due to postponement should be shared between them.

Our results in the multiplicative model also quite clearly demonstrate that the effect of postponement depends on the type of contract. Specifically, with buyback options, either p or Q-postponement does not affect the equilibrium wholesale price, profit allocation ratio and channel efficiency. However, without buybacks, such postponement strategies can significantly change the equilibrium values. In particular, as explained above, such strategies can make both M and R strictly worse off, which does not happen when a buyback option is offered.

4.9 Appendix

Proof of Lemma 4.4.1. Substituting $D(p) = 1 - p$ into the retailer's profit function, given by (4.2), results with $\Pi_R = (p-b)(1-p)\min(z,\hat{\epsilon}) - (w-b)Q$. Consider two options for p: (Option I) If $z \leq \hat{\epsilon}$, i.e., $p \leq 1 - \frac{Q}{\hat{\epsilon}}$, then the retailer's profit function reduces to $\Pi_R = (p-w)Q$, which is increasing in p. Thus, $p = 1 - \frac{Q}{\hat{\epsilon}}$. (Option II). If $z \geq \hat{\epsilon}$, i.e., $p \geq 1 - \frac{Q}{\hat{\epsilon}}$, then the retailer's profit function becomes $\Pi_R = (p-b)(1-p)\hat{\epsilon} - (w-b)Q$, which is strictly concave in p. Thus, $p^* = \max(\frac{1+b}{2}, 1 - \frac{Q}{\hat{\epsilon}})$. By comparing these two options we conclude that Option II gives us the optimal retail price. □

Proof of Lemma 4.4.2. Taking derivatives of $E\Pi_R$, given by (4.5), with respect to Q gives us $\frac{\partial E\Pi_R}{\partial Q} = 1 - (1-b)F(\frac{2Q}{1-b}) - 2Q\int_{\frac{2Q}{1-b}}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon - w$ and $\frac{\partial^2 E\Pi_R}{\partial Q^2} = -2\int_{\frac{2Q}{1-b}}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon < 0$, which implies that the retailer's expected profit function in Stage 2 is strictly concave. Thus, the first order condition (F.O.C.) gives us the unique maximizer of the retailer's expected profit function, i.e., the optimal order quantity, Q^*, satisfies $1 - (1-b)F(\frac{2Q}{1-b}) - 2Q\int_{\frac{2Q}{1-b}}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon - w = 0$. □

Proof of Proposition 4.4.3. Substituting $w^*(b,Q)$, given by (4.6), into the manufacturer's expected profit function, given by (4.9), and simplifying gives us:

$$E\Pi_M = (1-c)Q - QF(\frac{2Q}{1-b}) - 2Q^2\int_{\frac{2Q}{1-b}}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon + \frac{1}{2}b(1-b)\int_{L}^{\frac{2Q}{1-b}} \epsilon f(\epsilon)d\epsilon. \tag{4A.1}$$

Taking the first order derivative of $E\Pi_M$ with respect to b results with $\frac{\partial E\Pi_M}{\partial b} = \frac{1}{2}(1-2b)\int_L^{\frac{2Q}{1-b}} \epsilon f(\epsilon)d\epsilon$. Thus, there is a unique value of b for stationary point(s) of $E\Pi_M$, if there are any, which is $b^* = \frac{1}{2}$. Taking first order derivative of $E\Pi_M$ with respect to Q and evaluating it at $b^* = \frac{1}{2}$ results with $\frac{\partial E\Pi_M}{\partial Q}(b^* = \frac{1}{2}) = G(Q)$, where $G(Q) \equiv 1 - F(4Q) - 4Q\int_{4Q}^{U}\frac{1}{\epsilon}f(\epsilon)d\epsilon - c$. It is not difficult to show that $\frac{dG(Q)}{dQ} = -4\int_{4Q}^{U}\frac{1}{\epsilon}f(\epsilon)d\epsilon < 0$, $G(Q=0) = 1-c > 0$ and $G(Q = \frac{U}{4}) = -c < 0$. Thus, there is a unique $Q^* \in (0, \frac{U}{4})$ such that $\frac{\partial E\Pi_M}{\partial Q}(b^* = \frac{1}{2}) = 0$. Therefore, we conclude that there is a unique stationary point $(b^* = \frac{1}{2}, Q^*)$. Taking second order derivatives of $E\Pi_M$ with respect to b and Q and evaluating them at the unique stationary point gives us the Hessian matrix:

$$\begin{vmatrix} \frac{\partial^2 E\Pi_M}{\partial Q^2} & \frac{\partial^2 E\Pi_M}{\partial b \partial Q} \\ \frac{\partial^2 E\Pi_M}{\partial Q \partial b} & \frac{\partial^2 E\Pi_M}{\partial b^2} \end{vmatrix} = \begin{vmatrix} -4\int_{4Q^*}^{U}\frac{1}{\epsilon}f(\epsilon)d\epsilon & 0 \\ 0 & -\int_L^{4Q^*}\epsilon f(\epsilon)d\epsilon \end{vmatrix},$$

which is negative definite. Thus, the unique stationary point $(b^* = \frac{1}{2}, Q^*)$ is the global maximizer of the manufacturer's problem.

Substituting (b^*, Q^*) into $w^*(b, Q)$, given by (4.6), and simplifying results with $w^* = \frac{1+c}{2}$. Similarly, substituting (b^*, Q^*) into the expected retail price Ep^*, given by (4.4), and simplifying gives us: $Ep^* = \frac{3+c}{4}$, and substituting (b^*, Q^*) into the expected profit functions of the retailer and the manufacturer, given by (4.5) and (4A.1), respectively, and simplifying results with $E\Pi_M^* = 2E\Pi_R^* = \frac{1}{8}\int_L^{4Q^*}\epsilon f(\epsilon)d\epsilon + \frac{Q^*}{2}[1 - c - F(4Q^*)]$. □

Proof of Proposition 4.4.4. By (4.13), $\frac{dE\hat{\Pi}_M}{dQ} = 1 - c - F(2Q) - 4Q\int_{2Q}^{U}\frac{1}{\epsilon}f(\epsilon)d\epsilon$, $\frac{d^2E\hat{\Pi}_M}{dQ^2} = 2f(2Q) - 4\int_{2Q}^{U}\frac{1}{\epsilon}f(\epsilon)d\epsilon$ and $\frac{d^3E\hat{\Pi}_M}{dQ^3} = 4[f'(2Q) + \frac{f(2Q)}{Q}]$. Since $\epsilon f(\epsilon)$ is increasing in ϵ, we have $\epsilon f'(\epsilon) + f(\epsilon) \geq 0$. Therefore, $f'(2Q) + \frac{f(2Q)}{Q} > f'(2Q) + \frac{f(2Q)}{2Q} \geq 0$, which implies that $\frac{d^2E\hat{\Pi}_M}{dQ^2}$ strictly increases in Q. Since $\frac{dE\hat{\Pi}_M}{dQ}(Q=0) = 1-c > 0$, $\frac{dE\hat{\Pi}_M}{dQ}(Q = \frac{U}{2}) = -c < 0$ and $\frac{d^2E\hat{\Pi}_M}{dQ^2}(Q = \frac{U}{2}) = 2f(U) \geq 0$, we have $\frac{d^2E\hat{\Pi}_M}{dQ^2}(Q=0) < 0$. Thus, we conclude that $\frac{dE\hat{\Pi}_M}{dQ}$ first decreases in Q and then increases, and crosses the horizontal line only once at the unique solution $\hat{Q}^* \in (0, \frac{U}{2})$ satisfying:

$$1 - c - F(2Q) - 4Q\int_{2Q}^{U}\frac{1}{\epsilon}f(\epsilon)d\epsilon = 0. \tag{4A.2}$$

Substituting the implicit expression for $\hat{Q}^*$ into (4.12) and simplifying results with $\hat{w}^* = \frac{1}{2}(1 + c - F(2\hat{Q}^*))$. Substituting the resulting $\hat{w}^*$, $b = 0$ and $\hat{Q}^*$ into (4.4), the equilibrium expected value of the retail price is $E\hat{p}^* = \frac{1}{4}(3 + c - F(2\hat{Q}^*))$. Similarly, substituting $(\hat{w}^*, b = 0, \hat{Q}^*)$ into the retailer's expected profit function, given by (4.5), and $(\hat{w}^*, \hat{Q}^*)$ into the manufacturer's expected profit function, given by (4.13), and simplifying results with: $E\hat{\Pi}_M^* = \frac{\hat{Q}^*}{2}(1 - c - F(2\hat{Q}^*))$ and $E\hat{\Pi}_R^* = \frac{1}{4}\int_L^{2\hat{Q}^*}\epsilon f(\epsilon)d\epsilon + \frac{\hat{Q}^*}{4}(1 - c - F(2\hat{Q}^*))$, which completes the proof. □

Proof of Proposition 4.4.5. (i) is implied by (4.7) and (4.10), and (ii) is implied by (i), (4.8) and (4.11).

For (iii), following the analysis of the integrated channel, $p^I = \frac{1+c}{2}$. By Proposition 4.4.3, $Ep^* = \frac{3+c}{4}$. Thus, (iii) follows. By comparing the results in Proposition 4.4.3 and those in Table 4.1, (iv) follows.

For (v), following (4.7) and (4A.2), $1 - F(2Q^I) - 2Q^I \int_{2Q^I}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon = c = 1 - F(2\hat{Q}^*) - 2\hat{Q}^* \int_{2\hat{Q}^*}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon - 2\hat{Q}^* \int_{2\hat{Q}^*}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon$. It is easy to verify that $1 - F(2Q) - 2Q \int_{2Q}^{U} \frac{1}{\epsilon} f(\epsilon)d\epsilon$ is strictly decreasing in $Q(> 0)$. Thus, (v) follows. Finally, (vi) is implied by the analysis of the integrated channel and Propositions 4.4.3 and 4.4.4. □

Proof of Lemma 4.4.7. Substituting $D(p) = e^{-p}$ into the retailer's profit function, given by (4.2), results with $\Pi_R = (p - b)e^{-p} \min(z, \hat{\epsilon}) - (w - b)Q$. Following the same analysis process as in the linear case, we need to consider two options for p, and it is not difficult to verify that the optimal retail price, p^*, is given by (4.14). We do not repeat the analysis here. □

Proof of Lemma 4.4.8. Taking derivatives of $E\Pi_R$, given by (4.5), with respect to Q and simplifying gives us $\frac{\partial E\Pi_R}{\partial Q} = bF(e^{1+b}Q) + \int_{e^{1+b}Q}^{U} ln(\epsilon)f(\epsilon)d\epsilon - (ln(Q)+1) + (ln(Q)+1)F(e^{1+b}Q) - w$ and $\frac{\partial^2 E\Pi_R}{\partial Q^2} = -\frac{1}{Q}(1 - F(e^{1+b}Q)) < 0$, which implies that the retailer's expected profit function in Stage 2 is strictly concave. Thus, the F.O.C. gives us the unique maximizer of the retailer's problem, and the optimal order quantity, Q^*, satisfies $bF(e^{1+b}Q) + \int_{e^{1+b}Q}^{U} ln(\epsilon)f(\epsilon)d\epsilon - (ln(Q) + 1) + (ln(Q) + 1)F(e^{1+b}Q) - w = 0$. □

Proof of Proposition 4.4.9. Substituting $w^*(b, Q)$, given by (4.17), into the manufacturer's expected profit function, given by (4.18), and simplifying gives us:

$$E\Pi_M = -Q(ln(Q)+1) + Q(ln(Q)+1)F(e^{1+b}Q) - cQ + Q\int_{e^{1+b}Q}^{U} ln(\epsilon)f(\epsilon)d\epsilon + be^{-1-b}\int_{L}^{e^{1+b}Q} \epsilon f(\epsilon)d\epsilon. \tag{4A.3}$$

Using the same approach we have used in the linear case, we are able to verify that there is a unique stationary point $(b^* = 1, Q^*)$, where Q^* satisfies $-(ln(Q) + 2) + (ln(Q) + 2)F(e^2Q) + \int_{e^2Q}^{U} ln(\epsilon)f(\epsilon)d\epsilon - c = 0$, and the Hessian matrix at this stationary point is:

$$\begin{vmatrix} \frac{\partial^2 E\Pi_M}{\partial Q^2} & \frac{\partial^2 E\Pi_M}{\partial b \partial Q} \\ \frac{\partial^2 E\Pi_M}{\partial Q \partial b} & \frac{\partial^2 E\Pi_M}{\partial b^2} \end{vmatrix} = \begin{vmatrix} -\frac{1}{Q^*}(1 - F(e^2Q^*)) & 0 \\ 0 & -e^{-2}\int_{L}^{e^2Q^*} \epsilon f(\epsilon) f\epsilon \end{vmatrix},$$

which is negative definite. Thus, the unique stationary point $(b^* = 1, Q^*)$ is the global maximizer of the manufacturer's problem in Stage 1.

Substituting (b^*, Q^*) into $w^*(b, Q)$, given by (4.17), and simplifying results with $w^* = 1 + c$. Similarly, substituting (b^*, Q^*) into (4.15) and simplifying gives us the equilibrium expected retail price: $Ep^* = 2 + c$, and substituting (b^*, Q^*) into the expected profit functions of the retailer and the manufacturer, given by (4.16) and (4A.3), respectively, and simplifying results with $E\Pi_M^* = E\Pi_R^* = Q^*(1 - F(e^2Q^*)) + e^{-2}\int_L^{e^2Q^*} \epsilon f(\epsilon)d\epsilon$. □

Proof of Proposition 4.4.10. By (4.20), the derivatives of $E\hat{\Pi}_M$ with respect to Q are: $\frac{dE\hat{\Pi}_M}{dQ} = -(ln(Q)+2)(1-F(eQ))-c+\int_{eQ}^U ln(\epsilon)f(\epsilon)d\epsilon$, $\frac{d^2E\hat{\Pi}_M}{dQ^2} = e(f(eQ)-\frac{1-F(eQ)}{eQ})$ and $\frac{d^3E\hat{\Pi}_M}{dQ^3} = e^2(f'(eQ)+\frac{f(eQ)}{eQ} + \frac{1-F(eQ)}{(eQ)^2})$. Since $\epsilon f(\epsilon)$ is increasing in ϵ, we have $\epsilon f'(\epsilon) + f(\epsilon) \geq 0$. Therefore, $f'(eQ) + \frac{f(eQ)}{eQ} \geq 0$, $\frac{d^3E\hat{\Pi}_M}{dQ^3} \geq 0$, which implies that $\frac{d^2E\hat{\Pi}_M}{dQ^2}$ increases in Q. Since $\frac{d^2E\hat{\Pi}_M}{dQ^2}(Q = 0) < 0$ and $\frac{d^2E\hat{\Pi}_M}{dQ^2}(Q = \frac{U}{e}) > 0$, evidently, $\frac{dE\hat{\Pi}_M}{dQ}$ first decreases and then increases in Q. Note that $\frac{dE\hat{\Pi}_M}{dQ}(Q = 0) > 0$ and $\frac{dE\hat{\Pi}_M}{dQ}(Q = \frac{U}{e}) < 0$. Thus, there is a unique solution $\hat{Q}^* \in (0, \frac{U}{e})$, which satisfies:

$$c = \int_{eQ}^{U} ln(\epsilon)f(\epsilon)d\epsilon - (ln(Q) + 2)(1 - F(eQ)).$$

Substituting $\hat{Q}^*$ into (4.19) and simplifying results with $\hat{w}^* = 1 + c - F(e\hat{Q}^*)$. Substituting the resulting $\hat{w}^*$, $b = 0$ and $\hat{Q}^*$ into (4.15), reveals that $E\hat{p}^* = 2 + c - F(e\hat{Q}^*)$. Similarly, substituting $(\hat{w}^*, b = 0, \hat{Q}^*)$ into the retailer's expected profit function, given by (4.16), and $(\hat{w}^*, \hat{Q}^*)$ into the manufacturer's expected profit function, given by (4.20), and simplifying results with: $E\hat{\Pi}_M^* = (1 - F(e\hat{Q}^*))\hat{Q}^*$ and $E\hat{\Pi}_R^* = e^{-1}\int_L^{e\hat{Q}^*} \epsilon f(\epsilon)d\epsilon + (1 - F(e\hat{Q}^*))\hat{Q}^*$, which completes the proof. □

Proof of Proposition 4.4.11. The proof is analog to the proof of Proposition 4.4.5 in the linear expected demand function case. We do not repeat it here. □

Proof of Lemma 4.4.13. Substituting $D(p) = p^{-q}$ into the retailer's profit function, given by (4.2), results with $\Pi_R = (p - b)p^{-q}\min(z, \hat{\epsilon}) - (w - b)Q$. Following the same analysis approach as in the linear case, we need to consider two options for p. Again, it is easy to verify that the optimal retail price is given by (4.21). We do not repeat the analysis here. □.

Proof of Lemma 4.4.14. Taking derivatives of $E\Pi_R$, given by (4.23), with respect to Q: $\frac{\partial E\Pi_R}{\partial Q} = (1 - \frac{1}{q})Q^{-\frac{1}{q}}\int_\delta^U (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon + bF(\delta) - w$, and $\frac{\partial^2 E\Pi_R}{\partial Q^2} = -\frac{1}{q}(1 - \frac{1}{q})\int_\delta^U (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon < 0$, which implies that the retailer's expected profit function in Stage 2 is strictly concave. Thus, F.O.C. gives us the unique maximizer of the retailer's problem, and the optimal order quantity, Q^*, satisfies $(1 - \frac{1}{q})Q^{-\frac{1}{q}}\int_\delta^U (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon + bF(\delta) - w = 0$. □

Proof of Proposition 4.4.15. Substituting $w^*(b, Q)$, given by (4.24), into the manufacturer's expected profit function, given by (4.25), and simplifying gives us:

$$E\Pi_M = (1-\frac{1}{q})Q^{1-\frac{1}{q}} \int_{\delta}^{U} (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon - cQ + b(\frac{qb}{q-1})^{-q} \int_{L}^{\delta} \epsilon f(\epsilon)d\epsilon. \tag{4A.4}$$

Taking first order derivative of $E\Pi_M$ with respect to b and simplifying results with $\frac{\partial E\Pi_M}{\partial b} = (1-q)(\frac{qb}{q-1})^{-q}\int_L^\delta \epsilon f(\epsilon)d\epsilon < 0$. Thus, for any Q, the manufacturer's expected profit is strictly decreasing in b. Thus, $b^* = 0$ and $\frac{dE\Pi_M}{dQ}(b^*=0) = (1-\frac{1}{q})^2 Q^{-\frac{1}{q}} \int_L^U (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon - c$ and $\frac{d^2 E\Pi_M}{dQ^2}(b^* = 0) = -\frac{1}{q}(1-\frac{1}{q})^2 Q^{-1-\frac{1}{q}} \int_L^U (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon < 0$. We conclude that $E\Pi_M(b^*=0)$ is strictly concave in Q and the F.O.C. gives us the unique maximizer $Q^* = [\frac{(1-\frac{1}{q})^2 \int_L^U (\epsilon)^{\frac{1}{q}} f(\epsilon)d\epsilon}{c}]^q$. Therefore, $(b^* = 1, Q^*)$ is the global maximizer of the manufacturer's problem in Stage 1.

Substituting (b^*, Q^*) into $w^*(b, Q)$, given by (4.24), and simplifying results with $w^* = \frac{qc}{q-1}$. Similarly, substituting (b^*, Q^*) into (4.22) and simplifying gives us the equilibrium expected retail price: $Ep^* = \frac{q^2 c}{(q-1)^2}$, and substituting (b^*, Q^*) into the expected profit functions of the retailer and the manufacturer, given by (4.23) and (4A.4), respectively, and simplifying, results with $E\Pi_M^* = \frac{q-1}{q}E\Pi_R^* = \frac{c}{q-1}Q^*$. □

Proof of Proposition 4.4.16. The proof is similar to the proof of Proposition 4.4.5 in the linear expected demand function case, and will not be repeated here. □

Proof of Proposition 4.5.1. Backward induction is used to solve the two-stage Stackelberg game. In Stage 2, taking the partial derivative of $E\Pi_R$, given by (4.26), with respect to p and simplifying results with: $\frac{\partial E\Pi_R}{\partial p} = e^{-p}[(z-\Lambda(z))(1+b-p)+(w-b)z]$. Evidently, since $z > \Lambda(z)$ and $w \geq b$, there exists a unique $p^*(z)$ such that $\frac{\partial E\Pi_R}{\partial p} > 0$ for $p < p^*(z)$, $\frac{\partial E\Pi_R}{\partial p} = 0$ for $p = p^*(z)$ and $\frac{\partial E\Pi_R}{\partial p} < 0$ for $p > p^*(z)$, which implies that $E\Pi_R$ is unimodal in p for any given z, and the unique optimal p for the retailer is:

$$p^*(z) = 1 + b + \frac{w-b}{z-\Lambda(z)}z.$$

We can now express the retailer's expected profit function as a function of z only:

$$E\Pi_R(p^*(z), z) = e^{-p^*(z)}\{(p^*(z)-b)[z-\Lambda(z)] - (w-b)z\}.$$

By using the chain rule, $\frac{dE\Pi_R(p^*(z),z)}{dz} = \frac{\partial E\Pi_R(p^*(z),z)}{\partial p} \cdot \frac{\partial p^*(z)}{\partial z} + \frac{\partial E\Pi_R(p^*(z),z)}{\partial z} = \frac{\partial E\Pi_R(p^*(z),z)}{\partial z}$. The last equality is due to the fact that $\frac{\partial E\Pi_R(p^*(z),z)}{\partial p} = 0$. Thus, $\frac{dE\Pi_R(p^*(z),z)}{dz} = e^{-p^*(z)}\{(p^*(z)-b)[1-F(z)] - (w-b)\} = \frac{e^{-p^*(z)}}{z-\Lambda(z)}G(z)$, where

$$G(z) = [z - \Lambda(z) + (w-b)z][1-F(z)] - (w-b)[z-\Lambda(z)].$$

Taking the derivative of $G(z)$ with respect to z and simplifying gives us:

$$\frac{dG(z)}{dz} = [1-F(z)]^2 - [z-\Lambda(z)+(w-b)zf(z)] = [1-F(z)]\{1-F(z)-\frac{z-\Lambda(z)}{1-F(z)}-(w-b)zh(z)\},$$

where $h(z) = \frac{f(z)}{1-F(z)}$ is the failure rate. Note that $F(z)$ and $\frac{z-\Lambda(z)}{1-F(z)}$ increase in z, and so does $zh(z)$, since $zf(z)$ increases in z. Thus, $\frac{dG(z)}{dz}$ is decreasing in z, and $G(z)$ is concave in z. Since we have that $G(z=L) = L \geq 0$ and $G(z=U) = -(w-b)\mu_\xi \leq 0$, there is a unique $z^* \in (L,U)$ such that $G(z^*) = 0$ and $\frac{dG(z^*)}{dz} < 0$. Therefore, z^* is the unique solution to $\frac{dE\Pi_R(p^*(z),z)}{dz} = 0$ and one can easily verify that $\frac{d^2E\Pi_R(p^*(z),z)}{dz^2} < 0$. Thus, z^* is the unique maximizer of the retailer's expected profit function, which is now proved to be well-behaved in (p,z). Therefore, the first order conditions of $E\Pi_R$, given by (4.26), give us the expressions of the unique maximizer of the retailer's problem in Stage 2:

$$(1+b-p)[z-\Lambda(z)]+(w-b)z = 0 \quad \text{and} \quad (p-b)[1-F(z)]-(w-b) = 0. \tag{4A.5}$$

Note that $z = L$ does not satisfy the above two equations. Thus, in equilibrium, $z > L$.

Now, let us look at the manufacturer's problem in Stage 1: Choose (w,b) to maximize her expected profit. Following Song et al. (2004), we work with (p,Q) instead of (w,b) to solve for the equilibrium values in the model. From the two equations in (4A.5), we are able to derive:

$$w(p,z) = p-1-\frac{\Lambda(z)[1-F(z)]}{zF(z)-\Lambda(z)} \quad \text{and} \quad b(p,z) = p-1-\frac{z[1-F(z)]}{zF(z)-\Lambda(z)}, \tag{4A.6}$$

where, we recall $\Lambda(z) = \int_L^z (z-\epsilon)f(\epsilon)d\epsilon$ and $zF(z)-\Lambda(z) = \int_L^z \epsilon f(\epsilon)d\epsilon$. Thus, the manufacturer's expected profit function in Stage 1 becomes:

$$E\Pi_M(p,z) = e^{-p}\{[w(p,z)-c]z - b(p,z)\Lambda(z)\}.$$

Taking the partial derivatives of $E\Pi_M$ with respect to p and z, we obtain:

$$\begin{aligned}\frac{\partial E\Pi_M}{\partial p} &= -e^{-p}[(w-c-\frac{\partial w(p,z)}{\partial p})z+(\frac{\partial b(p,z)}{\partial p}-b)\Lambda(z)], \quad \text{and}\\ \frac{\partial E\Pi_M}{\partial z} &= e^{-p}[w-c+\frac{\partial w(p,z)}{\partial z}z-bF(z)-\Lambda(z)\frac{\partial b(p,z)}{\partial z}].\end{aligned}$$

Now, using (4A.6), we can derive that $\frac{\partial w(p,z)}{\partial p} = 1$, $\frac{\partial b(p,z)}{\partial p} = 1$, $\frac{\partial w(p,z)}{\partial z} = -\frac{F(z)[1-F(z)]}{\int_L^z \epsilon f(\epsilon)d\epsilon} + \frac{\Lambda(z)f(z)[z-\Lambda(z)]}{(\int_L^z \epsilon f(\epsilon)d\epsilon)^2}$ and $\frac{\partial b(p,z)}{\partial z} = -\frac{1-F(z)}{\int_L^z \epsilon f(\epsilon)d\epsilon} + \frac{zf(z)[z-\Lambda(z)]}{(\int_L^z \epsilon f(\epsilon)d\epsilon)^2}$. Thus, substituting these partial derivatives into $\frac{\partial E\Pi_M}{\partial p}$ and $\frac{\partial E\Pi_M}{\partial z}$, given above, and simplifying results with:

$$\frac{\partial E\Pi_M}{\partial p} = -e^{-p}\{[w-(1+c)+(1-b)F(z)]z+(1-b)\int_L^z \epsilon f(\epsilon)d\epsilon\} \quad \text{and} \tag{4A.7}$$

$$\frac{\partial E\Pi_M}{\partial z} = e^{-p}[w-(1+c)+(1-b)F(z)]. \tag{4A.8}$$

Evidently, there exists a unique solution, i.e., $(w^* = 1+c, b^* = 1)$, to the first order conditions, given by (4A.7) and (4A.8), since $z > L \geq 0$. Further, substituting this stationary point into (4A.6) reveals that the value of (p, z) at this stationary point satisfies:

$$c\int_L^z \epsilon f(\epsilon)d\epsilon = [1-F(z)][z-\Lambda(z)] \quad \text{and} \quad p = 2 + \frac{cz}{z-\Lambda(z)}. \tag{4A.9}$$

Next, we show that under a certain sufficient condition on the distribution of ξ, i.e., $\epsilon f(\epsilon)$ increases in ϵ, the unique stationary point is indeed the global maximizer of the manufacturer's problem in Stage 1, by proving that the Hessian matrix of $E\Pi_M$ at this unique stationary point is negative definite. Following (4A.7) and (4A.8), we can derive the second order partial derivatives of $E\Pi_M$ with respect to p and z at the unique stationary point: $\frac{\partial^2 E\Pi_M}{\partial p^2} = -e^{-p}[z-\Lambda(z)](<0)$, $\frac{\partial^2 E\Pi_M}{\partial z^2} = -e^{-p}\frac{f(z)[z-\Lambda(z)]}{\int_L^z \epsilon f(\epsilon)d\epsilon}(<0)$ and $\frac{\partial^2 E\Pi_M}{\partial p \partial z} = \frac{\partial^2 E\Pi_M}{\partial z \partial p} = e^{-p}[1-F(z)]$. Thus, the determinant of the Hessian matrix equals: $\frac{\partial^2 E\Pi_M}{\partial p^2}\cdot\frac{\partial^2 E\Pi_M}{\partial z^2} - (\frac{\partial^2 E\Pi_M}{\partial p \partial z})^2 = \frac{e^{-2p}}{\int_L^z \epsilon f(\epsilon)d\epsilon}K(z)$, where $K(z) = f(z)[z-\Lambda(z)]^2 - [1-F(z)]^2\int_L^z \epsilon f(\epsilon)d\epsilon = [1-F(z)]^2[z^2 f(z) - \int_L^z \epsilon f(\epsilon)d\epsilon] + 2zf(z)[1-F(z)]\int_L^z \epsilon f(\epsilon)d\epsilon + f(z)[\int_L^z \epsilon f(\epsilon)d\epsilon]^2$. Denote $\bar{K}(z) = z^2 f(z) - \int_L^z \epsilon f(\epsilon)d\epsilon$. Since $\epsilon f(\epsilon)$ is increasing in ϵ, evidently, $\bar{K}(z)$ is increasing in z. Note that at the unique stationary point, $z > L$. Therefore, $\bar{K}(z) \geq \bar{K}(L) = L^2 f(L) \geq 0$, which implies that at the unique stationary point, $K(z) > 0$ and the Hessian matrix is negative definite. Thus, $(w^* = 1+c, b^* = 1)$ is indeed the global maximizer of the manufacturer's problem in Stage 1.

Accordingly, the equilibrium values of z and p are given by (4A.9), and the equilibrium values of the expected profits of the manufacturer and the retailer are: $E\Pi_M^* = E\Pi_R^* = e^{-p^*}[z^* - \Lambda(z^*)]$. □

Proof of Proposition 4.7.3. Given (w, Q) and knowing α_s, R chooses p to maximize his profit function, given by (4.30). When $\alpha_s = \alpha_l$, $\Pi_R = p_l \min(Q, [\alpha_l - p_l]^+) - wQ$, where p_l represents the retail price if $\alpha_s = \alpha_l$. It is obvious that R will never choose a retail price which is too high so as to induce negative demand or too low so as to generate excess demand. Thus, $0 \leq \alpha_l - p_l \leq Q$, and R's profit function reduces to: $\Pi_R = p_l(\alpha_l - p_l) - wQ$, which is strictly concave in p_l. Thus, $p_l^* = \max(\alpha_l - Q, \frac{\alpha_l}{2})$, which is evidently less than α_l. Similarly, when $\alpha_s = \alpha_h$, we have $p_h^* = \max(\alpha_h - Q, \frac{\alpha_h}{2})$. This completes the proof of Proposition 4.7.3. □

Proof of Proposition 4.7.4. Given w and knowing p_l^* and p_h^*, R chooses Q to maximize his expected profit function, given by (4.32). Let us consider three possible choices of Q for R.

(1) $0 \leq Q \leq \frac{\alpha_l}{2}$. Then $p_l^* = \alpha_l - Q$ and $p_h^* = \alpha_h - Q$, which implies that R chooses a quantity Q for which the market clearing retail price is optimal regardless whether the actual market state is high or low. Thus, $E\Pi_R = (\mu_\xi - Q)Q - wQ$, which is strictly concave in Q, and $Q_1^* = [\min(\frac{\alpha_l}{2}, \frac{\mu_\xi - w}{2})]^+$. (Recall that $\mu_\xi = \lambda\alpha_l + (1-\lambda)\alpha_h$.)

(2) $\frac{\alpha_l}{2} \leq Q \leq \frac{\alpha_h}{2}$. Then $p_l^* = \frac{\alpha_l}{2}$ and $p_h^* = \alpha_h - Q$. In this case, R chooses a Q for which he will stock out when the actual market state is high and he will hold some inventories when the actual market state is low. Thus, $E\Pi_R = (1-\lambda)(\alpha_h - Q)Q - wQ + \frac{\lambda\alpha_l^2}{4}$, which is, again, concave in Q, and $Q_2^* = \max(\frac{\alpha_l}{2}, \frac{(1-\lambda)\alpha_h - w}{2(1-\lambda)}) \leq \frac{\alpha_h}{2}$. The last inequality follows since $\alpha_l \leq \alpha_h$ and $w \geq 0$.

(3) $Q \geq \frac{\alpha_h}{2}$. Then $p_l^* = \frac{\alpha_l}{2}$ and $p_h^* = \frac{\alpha_h}{2}$. In this case, R chooses a very high Q which would result with some leftover inventories regardless of the actual market state. Thus, $E\Pi_R = \frac{\lambda\alpha_l^2}{4} + \frac{(1-\lambda)\alpha_h^2}{4} - wQ$, which decreases in Q, and $Q_3^* = \frac{\alpha_h}{2}$.

Compare R's optimal expected profit in the three cases. Since $E\Pi_R$ is continuous in Q and Q_3^* is on the edge of the feasible interval for (2), (2) dominates (3), and we only need to consider (1) and (2). We need to consider three different ranges for w.

(a) $w \geq \mu_\xi$. Then, $Q_1^* = 0$ and since $w > (1-\lambda)(\alpha_h - \alpha_l)$, $Q_2^* = \frac{\alpha_l}{2}$. Accordingly, one can easily verify that $E\Pi_R^1 = 0$ and $E\Pi_R^2 = \frac{-\alpha_l^2 + 2\alpha_l(\mu_\xi - w)}{4} \leq 0$, where the last inequality follows since $w \geq \mu_\xi$, and $E\Pi_R^i$ designates R's expected profit function in Choice (1). Thus, Choice (1) is optimal for R, $Q^* = 0$ $p_l^* = \alpha_l$ and $p_h^* = \alpha_h$.

(b) $(1-\lambda)(\alpha_h - \alpha_l) \leq w \leq \mu_\xi$. Then, $Q_1^* = \frac{\mu_\xi - w}{2}$ and $Q_2^* = \frac{\alpha_l}{2}$. Note that $E\Pi_R$ is continuous in Q and Q_2^* is on the edge of the feasible interval for Choice (1). Thus, Choice (1) is optimal for R, and $Q^* = \frac{\mu_\xi - w}{2}$, $p_l^* = \alpha_l - Q^*$ and $p_h^* = \alpha_h - Q^*$.

(c) $w \leq (1-\lambda)(\alpha_h - \alpha_l)$. Then, $Q_1^* = \frac{\alpha_l}{2}$ and $Q_2^* = \frac{(1-\lambda)\alpha_h - w}{2(1-\lambda)}$. Now, Q_1^* is on the edge of the feasible interval for Choice (2). Thus, Choice (2) is optimal for R, and $Q^* = \frac{(1-\lambda)\alpha_h - w}{2(1-\lambda)}$, $p_l^* = \frac{\alpha_l}{2}$ and $p_h^* = \alpha_h - Q^*$.

Combining the results in the three cases yields the display of Q^* in Proposition 4.7.4.□

Proof of Proposition 4.7.5. Taking Q^* and p_s^* into account, we next consider three possible choices for M to find her optimal wholesale price.

(1) $w \geq \mu_\xi$. Then, according to Proposition 4.7.4, $Q^* = 0$. Therefore, $E\Pi_M^1 = 0$.

(2) $\mu_\xi - \alpha_l \leq w \leq \mu_\xi$. Then, according to Proposition 4.7.4, $Q^* = \frac{\mu_\xi - w}{2}$, and $E\Pi_M = wQ^* = \frac{1}{2}w(\mu_\xi - w)$, which is strictly concave in w. Thus, $w_2^* = \max(\mu_\xi - \alpha_l, \frac{\mu_\xi}{2}) \leq \mu_\xi$.

(3) $w \leq \mu_\xi - \alpha_l$. Then, $Q^* = \frac{(1-\lambda)\alpha_h - w}{2(1-\lambda)}$, and $E\Pi_M = wQ^* = \frac{1}{2(1-\lambda)}w((1-\lambda)\alpha_h - w)$, which is strictly concave in w. Thus, $w_3^* = \min(\mu_\xi - \alpha_l, \frac{(1-\lambda)\alpha_h}{2})$.

Next, we evaluate M's expected profit in the above three choices. Note that $E\Pi_M \geq 0$ in both (2) and (3). Thus, (1) is dominated by the other two choices, and we only need to compare M's expected profit in (2) and (3). We consider three different cases as follows.

(a) $(1-\lambda)\alpha_h - 2(1-\lambda)\alpha_l \leq 0$, i.e., $\theta \leq 2$. Then, $\mu_\xi - 2\alpha_l < 0$. Thus, $w_2^* = \frac{\mu_\xi}{2}$ and $w_3^* = \mu_\xi - \alpha_l$. Since $E\Pi_M$ is continuous on $w \in [0, \alpha_h]$ and w_3^* is on the edge of the feasible interval for Choice (2), Choice (2) dominates Choice (3). Thus, $w^* = \frac{\mu_\xi}{2}$, and $E\Pi_M^* = \frac{\mu_\xi^2}{8}$.

(b) $\mu_\xi - 2\alpha_l \leq 0 \leq (1-\lambda)\alpha_h - 2(1-\lambda)\alpha_l$, i.e., $2 \leq \theta \leq 1 + \frac{1}{1-\lambda}$. Thus, $w_2^* = \frac{\mu_\xi}{2}$ and $w_3^* = \frac{(1-\lambda)\alpha_h}{2}$. Accordingly, $E\Pi_M^2 = \frac{\mu_\xi^2}{8}$ and $E\Pi_M^3 = \frac{(1-\lambda)\alpha_h^2}{8}$, and $E\Pi_M^2 - E\Pi_M^3 = \frac{(\mu_\xi - \sqrt{1-\lambda}\alpha_h)(\mu_\xi + \sqrt{1-\lambda}\alpha_h)}{8}$. Thus, $E\Pi_M^2 > E\Pi_M^3$ when $\mu_\xi > \sqrt{1-\lambda}\alpha_h$, i.e., $\theta \leq 1 + \frac{1}{\sqrt{1-\lambda}}$, $E\Pi_M^2 = E\Pi_M^3$ when $\theta = 1 + \frac{1}{\sqrt{1-\lambda}}$, and $E\Pi_M^2 < E\Pi_M^3$ when $\theta \geq 1 + \frac{1}{\sqrt{1-\lambda}}$.

(c) $\mu_\xi - 2\alpha_l \geq 0$, i.e., $\theta \geq 1 + \frac{1}{1-\lambda}$. Then, $(1-\lambda)\alpha_h - 2(1-\lambda)\alpha_l > 0$. Thus, $w_2^* = \mu_\xi - \alpha_l$ and $w_3^* = \frac{(1-\lambda)\alpha_h}{2}$. Since $E\Pi_M$ is continuous on $w \in [0, \alpha_h]$ and w_2^* is on the edge of the feasible interval for Choice (3), Choice (3) dominates Choice (2) and is optimal for M. Therefore, $E\Pi_M^* = \frac{(1-\lambda)\alpha_h^2}{8}$.

Combining the results in the above three Cases (a), (b) and (c), we conclude as follows.

[1] When $\theta \leq 1 + \frac{1}{\sqrt{1-\lambda}}$, Choice (2) is optimal for M. Thus, $w^* = \frac{\mu_\xi}{2}$, $E\Pi_M^* = \frac{\mu_\xi^2}{8}$. Substituting w^* into Q^* and p_s^*, given by (4.33), and simplifying results with $Q^* = \frac{\mu_\xi}{4}$, $p_l^* = \frac{4\alpha_l - \mu_\xi}{4}$ and $p_h^* = \frac{4\alpha_h - \mu_\xi}{4}$. Substituting in R's profit function, given by (4.32), and simplifying results with $E\Pi_R^* = \frac{\mu_\xi^2}{16}$.

[2] When $\theta \geq 1 + \frac{1}{\sqrt{1-\lambda}}$, Choice (3) is optimal for M. Then, it can be easily verified that $w^* = \frac{(1-\lambda)\alpha_h}{2}$, $E\Pi_M^* = \frac{(1-\lambda)\alpha_h^2}{8}$, $Q^* = \frac{(1-\lambda)\alpha_h}{4(1-\lambda)}$, $p_l^* = \frac{\alpha_l}{2}$, $p_h^* = \frac{3(1-\lambda)\alpha_h}{4(1-\lambda)}$, and $E\Pi_R^* = \frac{\lambda\alpha_l^2}{4} + \frac{(1-\lambda)\alpha_h^2}{16}$. □

Proof of Proposition 4.7.6. We analyze R's problem in Regions 1 − 3.

Region 1: $\{(p,Q) | w \leq p \leq \alpha_l, 0 \leq Q \leq \alpha_l - p\}$. $E\Pi_R$, given by (4.34), reduces to: $E\Pi_R = (p-w)Q$, which increases in both p and Q. Thus, $Q^* = \alpha_l - p^*$. Therefore, R attains the optimal profit on Segment AB in Region 1.

Region 2: $\{(p,Q) | w \leq p \leq \alpha_l, \alpha_l - p \leq Q \leq \alpha_h - p\}$. $E\Pi_R$, given by (4.34), can be simplified to: $E\Pi_R = p\lambda(\alpha_l - p) + (p(1-\lambda) - w)Q$, which is linear in Q and concave in p. Evidently, $Q^* = \alpha_l - p^*$ or $Q^* = \alpha_h - p$, depending on the sign of $(p(1-\lambda) - w)$. Therefore, R attains his optimal profit on either Segment AB or Segment CE.

Region 3: $\{(p,Q) | \alpha_l \leq p \leq \alpha_h, 0 \leq Q \leq \alpha_h - p\}$. $E\Pi_R$ reduces to $\Pi_R = (p(1-\lambda) - w)Q$, which is linearly increasing in p and linear in Q. Thus, $p^* = \alpha_h - Q^*$, i.e., $Q^* = \alpha_h - p^*$. Therefore, R's optimal (p^*, Q^*) lies on Segment ED.

Note that R's profit function is well behaved in all three subregions. $\square$

Proof of Proposition 4.7.7. Since $\theta \geq 1 + \frac{1}{1-\lambda}$, it can be easily shown that $(1-\lambda)(2\alpha_l - \alpha_h) \leq (2-\lambda)\alpha_l - (1-\lambda)\alpha_h \leq 0 \leq \alpha_l \leq (1-\lambda)\alpha_h$. Consider $w \in [0, \alpha_l]$. According to (4.38), $p^* = \alpha_l$ and $E\Pi_R^{CE} = (\alpha_h - \alpha_l)((1-\lambda)\alpha_l - w)$. According to (4.39), $p^* = \frac{(1-\lambda)\alpha_h + w}{2(1-\lambda)}$ and $E\Pi_R^{ED} = \frac{((1-\lambda)\alpha_h - w)^2}{4(1-\lambda)}$. We have $E\Pi_R^{AB} = \frac{(\alpha_l - w)^2}{4}$. Thus, $E\Pi_R^{ED} \geq E\Pi_R^{AB}$ since $(1-\lambda)\alpha_h \geq \alpha_l \geq w$, and $E\Pi_R^{ED} - E\Pi_R^{CE} = \frac{((1-\lambda)\alpha_h + w - 2(1-\lambda)\alpha_l)^2}{4(1-\lambda)} \geq 0$. Thus, Segment ED is optimal for R when $w \in [0, \alpha_l]$. It is further known from Proposition 4.7.6 that Segment ED is optimal when $w \in [\alpha_l, \alpha_h]$. Therefore, from (4.39) and since $(1-\lambda)(2\alpha_l - \alpha_h) \leq 0$ when $\theta \geq 1 + \frac{1}{1-\lambda}$,

$$p^* = \begin{cases} \frac{(1-\lambda)\alpha_h + w}{2(1-\lambda)} & \text{if } 0 \leq w \leq (1-\lambda)\alpha_h, \\ \alpha_h & \text{if } (1-\lambda)\alpha_h \leq w \leq \alpha_h, \end{cases}$$

which completes the proof of Proposition 4.7.7. $\square$

Proof of Proposition 4.7.8. Since $\frac{1}{1-\lambda} \leq \theta \leq 2$, it can be easily verified that $0 \leq (1-\lambda)(2\alpha_l - \alpha_h) \leq (2-\lambda)\alpha_l - (1-\lambda)\alpha_h \leq \alpha_l \leq (1-\lambda)\alpha_h$. Recall that $E\Pi_R^{AB} = \frac{(\alpha_l - w)^2}{4}$ and that Segment ED is optimal for R when $\alpha_l \leq w \leq \alpha_h$. For $0 \leq w \leq \alpha_l$, we need to consider the following three cases.

(1) $w \in [0, (1-\lambda)(2\alpha_l - \alpha_h)]$. According to (4.38), $p^* = \frac{\mu_\xi + w}{2}$ and $E\Pi_R^{CE} = \frac{(\mu_\xi)^2 - w^2}{4} - w(\alpha_h - \frac{\mu_\xi + w}{2})$. According to (4.39), $p^* = \alpha_l$ and $E\Pi_R^{ED} = (\alpha_h - \alpha_l)((1-\lambda)\alpha_l - w)$. Thus, $E\Pi_R^{CE} - E\Pi_R^{ED} = \frac{(\mu_\xi + w - 2\alpha_l)^2}{4} \geq 0$. Therefore, $E\Pi_R^{CE} \geq E\Pi_R^{ED}$. Similarly, $E\Pi_R^{CE} - E\Pi_R^{AB} = \frac{\alpha_h - \alpha_l}{4}[(1-\lambda^2)\alpha_l + (1-\lambda)^2\alpha_h - 2(1+\lambda)w] > 0$, where the last inequality follows since $[(1-\lambda^2)\alpha_l + (1-\lambda)^2\alpha_h - 2(1+\lambda)w] > 0$ for any $w \in [0, (1-\lambda)(2\alpha_l - \alpha_h)]$. Thus, $E\Pi_R^{CE} > E\Pi_R^{AB}$ for any $w \leq (1-\lambda)(2\alpha_l - \alpha_h)$. Therefore, Segment CE is optimal for R.

(2) $w \in [(1-\lambda)(2\alpha_l - \alpha_h), (2-\lambda)\alpha_l - (1-\lambda)\alpha_h]$. According to (4.38), $p^* = \frac{\mu_\xi + w}{2}$ and $E\Pi_R^{CE} = \frac{(\mu_\xi)^2 - w^2}{4} - w(\alpha_h - \frac{\mu_\xi + w}{2})$. According to (4.39), $p^* = \frac{(1-\lambda)\alpha_h + w}{2(1-\lambda)}$ and $E\Pi_R^{ED} = \frac{((1-\lambda)\alpha_h - w)^2}{4(1-\lambda)}$. Thus, $E\Pi_R^{ED} \geq E\Pi_R^{AB}$ since $(1-\lambda)\alpha_h \geq \alpha_l \geq w$. Now, we need to compare $E\Pi_R^{CE}$ and $E\Pi_R^{ED}$. Equating $E\Pi_R^{CE}$ and $E\Pi_R^{ED}$ and solving it, one can verify that $E\Pi_R^{CE} \geq E\Pi_R^{ED}$ when $w \leq (1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l)$, and $E\Pi_R^{CE} \leq E\Pi_R^{ED}$ when $w \geq (1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l)$.

(3) $w \in [(2-\lambda)\alpha_l - (1-\lambda)\alpha_h, \alpha_l]$. According to (4.38), $p^* = \alpha_l$ and $E\Pi_R^{CE} = (\alpha_h - \alpha_l)((1-\lambda)\alpha_l - w)$. According to (4.39), $p^* = \frac{(1-\lambda)\alpha_h + w}{2(1-\lambda)}$ and $E\Pi_R^{ED} = \frac{((1-\lambda)\alpha_h - w)^2}{4(1-\lambda)}$. Following the same argument as in Case (2), $E\Pi_R^{ED} \geq E\Pi_R^{AB}$. Similarly, $E\Pi_R^{ED} - E\Pi_R^{CE} = \frac{((1-\lambda)\alpha_h + w - 2(1-\lambda)\alpha_l)^2}{4(1-\lambda)} \geq 0$. Thus, Segment ED is optimal for R.

Combining the analysis in Cases (1), (2), (3), and the case when $\alpha_l \leq w \leq \alpha_h$, for which Segment ED is optimal, we have verified the display for p^* in Proposition 4.7.8. Note that since Segment AB was found never to be optimal, we have also shown that $Q^* = \alpha_h - p^*$ for $0 \leq w \leq \alpha_h$. □

Proof of Proposition 4.7.9. Since $\lambda = 1/8$, $\alpha_l = 1$ and $\alpha_h = \theta$, the feasible regions in M's two Subproblems (1): $w \leq (1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l)$ and (2): $(1-\lambda)^{1/2}\alpha_l - (1-\lambda)(\alpha_h - \alpha_l) \leq w \leq (1-\lambda)\alpha_h$ can be simplified to: (1): $0 \leq w \leq \frac{2\sqrt{14}-7\theta+7}{8}$, and (2): $\frac{2\sqrt{14}-7\theta+7}{8} \leq w \leq \frac{7\theta}{8}$. We analyze separately these two subproblems. Note that in Subproblem (1), Segment CE is optimal for R, and in Subproblem (2), Segment ED is optimal for R.

Subproblem (1): $0 \leq w \leq \frac{2\sqrt{14}-7\theta+7}{8}$. Then since Segment CE is optimal for R, $p^* = \frac{\mu_\xi + w}{2}$, $Q^* = \theta - \frac{\mu_\xi + w}{2}$, and $E\Pi_M = w(\theta - \frac{1}{2}\mu_\xi - \frac{1}{2}w)$, which is concave in w. Thus, since $\mu_\xi = \frac{7\theta+1}{8}$, we have

$$(w^*, E\Pi_M^{CE}) = \begin{cases} (\frac{9\theta-1}{16}, \frac{(9\theta-1)^2}{512}) & \text{if } \frac{8}{7} \leq \theta \leq \frac{15+4\sqrt{14}}{23}, \\ (\frac{2\sqrt{14}-7\theta+7}{8}, \frac{(2\sqrt{14}-7\theta+7)(8\theta-4-\sqrt{14})}{64}) & \text{if } \frac{15+4\sqrt{14}}{23} \leq \theta \leq 2. \end{cases} \tag{4A.10}$$

Subproblem (2): $\frac{2\sqrt{14}-7\theta+7}{8} \leq w \leq \frac{7\theta}{8}$. Then, since Segment ED is optimal for R, $p^* = \frac{7\theta+8w}{14}$, $Q^* = \frac{7\theta-8w}{14}$, and $E\Pi_M = \frac{(7\theta-8w)w}{14}$, which is, again, concave in w. Thus, we obtain:

$$(w^*, E\Pi_M^{ED}) = \begin{cases} (\frac{2\sqrt{14}-7\theta+7}{8}, \frac{(2\sqrt{14}-7\theta+7)(14\theta-2\sqrt{14}-7)}{112}) & \text{if } \frac{8}{7} \leq \theta \leq \frac{4\sqrt{14}}{21}(1+\frac{\sqrt{14}}{4}), \\ (\frac{7\theta}{16}, \frac{7\theta^2}{64}) & \text{if } \frac{4\sqrt{14}}{21}(1+\frac{\sqrt{14}}{4}) \leq \theta \leq 2. \end{cases} \tag{4A.11}$$

Now, let us evaluate M's equilibrium profit as a function of θ.

(1) Consider $\frac{8}{7} \leq \theta \leq \frac{15+4\sqrt{14}}{23}$. Comparing $E\Pi_M^{CE} = \frac{(9\theta-1)^2}{512}$, given in (4A.10), and $E\Pi_M^{ED} = \frac{(2\sqrt{14}-7\theta+7)(14\theta-2\sqrt{14}-7)}{112}$, given in (4A.11), we have $E\Pi_M^{CE} > \Pi_M^{ED}$. Thus, $w^* = \frac{9\theta-1}{16}$ and $E\Pi_M^* = \frac{(9\theta-1)^2}{512}$. Correspondingly, $p^* = \frac{23\theta+1}{32}$, $Q^* = \frac{9\theta-1}{32}$ and $E\Pi_R^* = \frac{-47\theta^2+110\theta+1}{1024}$.

(2) Consider $\frac{15+4\sqrt{14}}{23} \leq \theta \leq \frac{4\sqrt{14}}{21}(1+\frac{\sqrt{14}}{4})$. Comparing $E\Pi_M^{CE} = \frac{(2\sqrt{14}-7\theta+7)(8\theta-4-\sqrt{14})}{64}$, given in (4A.10), and $E\Pi_M^{ED} = \frac{(2\sqrt{14}-7\theta+7)(14\theta-2\sqrt{14}-7)}{112}$, given in (4A.11), results with $E\Pi_M^{CE} > \Pi_M^{ED}$. Thus, $w^* = \frac{2\sqrt{14}-7\theta+7}{8}$ and $E\Pi_M^* = \frac{(2\sqrt{14}-7\theta+7)(8\theta-4-\sqrt{14})}{64}$. Correspondingly, $p^* = \frac{4+\sqrt{14}}{8}$, $Q^* = \frac{8\theta-4-\sqrt{14}}{8}$ and $E\Pi_R^* = \frac{(14\theta-2\sqrt{14}-7)^2}{224}$.

(3) Consider $\frac{4\sqrt{14}}{21}(1+\frac{\sqrt{14}}{4}) \leq \theta \leq 2$. Since M's profit function on Segments CE and ED is quadratic in the parameter θ, one can easily verify that $E\Pi_M^{CE} > E\Pi_M^{ED}$ if $\theta < \bar{\theta}$, $E\Pi_M^{CE} = E\Pi_M^{ED}$ if $\theta = \bar{\theta}$ and $E\Pi_M^{CE} < E\Pi_M^{ED}$ if $\theta > \bar{\theta}$, where $\bar{\theta} \equiv \frac{23\sqrt{14}+84+6\sqrt{7}+7\sqrt{2}}{126}$. In order to compute the optimal values of p, Q and $E\Pi_R$, let us consider separately these three cases. When $\theta < \bar{\theta}$, Subproblem (1) is optimal for M, and thus, Segment CE is optimal for R.

Therefore, $w^* = \frac{2\sqrt{14}-7\theta+7}{8}$ and $E\Pi_M^* = \frac{(2\sqrt{14}-7\theta+7)(8\theta-4-\sqrt{14})}{64}$. Accordingly, $p^* = \frac{4+\sqrt{14}}{8}$, $Q^* = \frac{8\theta-4-\sqrt{14}}{8}$ and $E\Pi_R^* = \frac{(14\theta-2\sqrt{14}-7)^2}{224}$. When $\bar{\theta} < \theta \leq 2$, Subproblem (2) is optimal for M, and thus, Segment ED is optimal for R. Therefore, $w^* = \frac{7\theta}{16}$ and $E\Pi_M^* = \frac{7\theta^2}{64}$. Accordingly, $p^* = \frac{3\theta}{4}$, $Q^* = \frac{\theta}{4}$ and $E\Pi_R^* = \frac{7\theta^2}{128}$. When $\theta = \bar{\theta}$, M is indifferent between Subproblem (1) and Subproblem (2), while R strictly prefers Segment CE. Thus, for the benefit of R, M would choose Subproblem (1) when $\theta = \bar{\theta}$.

By summarizing Cases (1), (2) and (3), we have verified the displays of w^*, $E\Pi_M^*$, p^*, Q^* and $E\Pi_R^*$ in Proposition 4.7.9. □

Proof of Proposition 4.7.10. Note that $\lambda = 1/8$, $\frac{1}{1-\lambda} \leq \theta \leq 2$, and the low demand state, α_l, is set to 1 and the high demand state becomes $\alpha_h = \theta$. According to Table 4.2, under a wholesale price-only contract with p-postponement, $w^p = \frac{7\theta+1}{16}$, $p_l^p = \frac{31-7\theta}{32}$, $p_h^p = \frac{25\theta-1}{32}$, $Q^p = \frac{7\theta+1}{32}$, $E\Pi_M^p = \frac{(7\theta+1)^2}{512}$ and $E\Pi_R^p = \frac{(7\theta+1)^2}{1024}$. Thus, $Ep^p = \lambda p_l^* + (1-\lambda)p_h^* = \frac{21\theta+3}{32}$. The optimal values of the decision variables and profits under a wholesale price-only contract without postponement is given by Proposition 4.7.9. Next, let us evaluate the effect of p-postponement on optimal profits and equilibrium values of decision variables. We consider the following three cases with respect to the value of θ.

(1) $\frac{8}{7} \leq \theta \leq \frac{15+4\sqrt{14}}{23}$.

$E\Pi_M^N - E\Pi_M^p = \frac{(9\theta-1)^2}{512} - \frac{(7\theta+1)^2}{512} = \frac{\theta(\theta-1)}{16} > 0$ since $\theta > 1$.

$E\Pi_R^N - E\Pi_R^p = \frac{-47\theta^2+110\theta+1}{1024} - \frac{(7\theta+1)^2}{1024} = -\frac{3\theta(\theta-1)}{32} < 0$.

Therefore, in this case, $E\Pi_M^N > E\Pi_M^p$ and $E\Pi_R^N < E\Pi_R^p$.

$w^N - w^p = \frac{\theta-1}{8} > 0$, $p^N - Ep^p = \frac{\theta-1}{16} > 0$, and $Q^N - Q^p = \frac{\theta-1}{16} > 0$.

Thus, in this case, $w^N > w^p$, $p^N > Ep^p$ and $Q^N > Q^p$.

(2) $\frac{15+4\sqrt{14}}{23} \leq \theta \leq \frac{23\sqrt{14}+84+6\sqrt{7}+7\sqrt{2}}{126}$.

$E\Pi_M^N - E\Pi_M^p = \frac{(2\sqrt{14}-7\theta+7)(8\theta-4-\sqrt{14})}{64} - \frac{(7\theta+1)^2}{512}$, which is a strictly concave function of θ. Thus, one can easily show that $E\Pi_M^N > E\Pi_M^p$ for $\theta < \theta_0$, $E\Pi_M^N = E\Pi_M^p$ for $\theta = \theta_0$, and $E\Pi_M^N < E\Pi_M^p$ for $\theta > \theta_0$, where $\theta_0 = \frac{92\sqrt{14}+329+8\sqrt{56+14\sqrt{14}}}{497}$.

$E\Pi_R^N - E\Pi_R^p = \frac{(14\theta-2\sqrt{14}-7)^2}{224} - \frac{(7\theta+1)^2}{1024}$, which is a strictly convex function of θ. Thus, it can be easily shown that $E\Pi_R^N < E\Pi_R^p$ when $\theta < \hat{\theta}$, $E\Pi_R^N = E\Pi_R^p$ when $\theta = \hat{\theta}$, and $E\Pi_R^N > E\Pi_R^p$ when $\theta > \hat{\theta}$, where $\hat{\theta} \equiv \frac{81}{121} + \frac{164\sqrt{14}}{847}$.

$w^N - w^p = \frac{4\sqrt{14}+13-21\theta}{16}$, which is first positive when θ is small, and then becomes negative when θ becomes large, similarly for $p^N - Ep^p = \frac{4\sqrt{14}+13-21\theta}{32}$. $Q^N - Q^p = \frac{25\theta-17-4\sqrt{14}}{32} > 0$. Thus, $Q^N > Q^p$.

(3) $\frac{23\sqrt{14}+84+6\sqrt{7}+7\sqrt{2}}{126} < \theta \leq 2$.

$E\Pi_M^N - E\Pi_M^p = 2(E\Pi_R^N - E\Pi_R^p) = \frac{7\theta^2}{64} - \frac{(7\theta+1)^2}{512} = \frac{7\theta^2-14\theta-1}{512}$, which is strictly convex in θ. Thus, it is easy to verify that $E\Pi_M^N < E\Pi_M^p$ and $E\Pi_R^N < E\Pi_R^p$.

$w^N - w^p = -\frac{1}{16} < 0$, $p^N - Ep^p = \frac{3(\theta-1)}{32} > 0$ and $Q^N - Q^p = \frac{\theta-1}{32} > 0$. Thus, $w^N < w^p$, $p^N > Ep^p$ and $Q^N > Q^p$.

Combining the results in the above three cases completes the proof of Proposition 4.7.10. □

		$t=0$						$t=1$						$t=4$					
	c	0.00	0.05	0.10	0.20	0.50	0.80	0.00	0.05	0.10	0.20	0.50	0.80	0.00	0.05	0.10	0.20	0.50	0.80
Buybacks	w^*	0.50	0.53	0.55	0.60	0.75	0.90	0.50	0.53	0.55	0.60	0.75	0.90	0.50	0.53	0.55	0.60	0.75	0.90
	b^*	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
	Ep^*	0.75	0.76	0.78	0.80	0.88	0.95	0.75	0.76	0.78	0.80	0.88	0.95	0.75	0.76	0.78	0.80	0.88	0.95
	Q^*	0.50	0.35	0.29	0.22	0.09	0.03	0.38	0.29	0.26	0.21	0.11	0.04	0.30	0.25	0.23	0.20	0.12	0.05
	$E\Pi^*_M$	0.13	0.11	0.09	0.06	0.02	2E-03	0.13	0.11	0.10	0.07	0.03	4E-03	0.13	0.11	0.10	0.08	0.03	5E-03
	$E\Pi^*_R$	0.06	0.05	0.04	0.03	9E-03	1E-03	0.06	0.05	0.05	0.04	0.01	2E-03	0.06	0.06	0.05	0.04	0.02	2E-03
Percentage improvement relative to N-postponement	w^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	b^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Ep^*	0.00	-1.35	-2.23	-3.21	-3.26	-1.56	0.00	-0.71	-1.21	-1.81	-1.93	-0.93	0.00	-0.29	-0.51	-0.78	-0.87	-0.42
	Q^*	0.00	-15.06	-14.54	-8.90	27.95	156.00	0.00	-12.17	-12.64	-9.70	12.79	79.38	0.00	-8.16	-8.64	-6.93	5.89	33.25
	$E\Pi^*_M$	0.00	2.71	6.73	16.77	68.38	244.35	0.00	1.57	3.97	10.10	40.24	124.00	0.00	0.82	2.10	5.31	19.31	47.91
	$E\Pi^*_R$	0.00	2.71	6.73	16.77	68.38	244.35	0.00	1.57	3.97	10.10	40.24	124.00	0.00	0.82	2.10	5.31	19.31	47.91
Wholesale	$\hat{w}^*$	0.36	0.40	0.44	0.51	0.71	0.89	0.44	0.48	0.51	0.57	0.74	0.90	0.49	0.52	0.55	0.60	0.75	0.90
	$E\hat{p}^*$	0.68	0.70	0.72	0.75	0.85	0.94	0.72	0.74	0.75	0.78	0.87	0.95	0.75	0.76	0.77	0.80	0.87	0.95
	$\hat{Q}^*$	0.28	0.25	0.23	0.18	0.08	0.02	0.25	0.23	0.21	0.18	0.10	0.04	0.24	0.23	0.22	0.19	0.12	0.05
	$E\hat{\Pi}^*_M$	0.10	0.09	0.08	0.06	0.02	2E-03	0.11	0.10	0.09	0.07	0.03	4E-03	0.12	0.11	0.10	0.08	0.03	5E-03
	$E\hat{\Pi}^*_R$	0.07	0.06	0.05	0.04	0.01	1E-03	0.06	0.06	0.05	0.04	0.01	2E-03	0.06	0.06	0.05	0.04	0.02	2E-03
Percentage improvement relative to N-postponement	w^*	31.79	26.85	23.01	17.38	7.89	2.66	25.41	22.21	19.51	15.19	7.02	2.25	14.71	12.90	11.33	8.73	3.75	1.09
	$E\hat{p}^*$	-2.35	-2.70	-2.91	-3.08	-2.40	-1.00	1.31	1.02	0.77	0.39	-0.14	-0.15	2.17	1.89	1.63	1.19	0.34	0.02
	$\hat{Q}^*$	-23.40	-19.95	-16.19	-7.56	34.35	173.80	-18.14	-15.76	-13.17	-7.29	19.52	92.18	-9.56	-8.00	-6.35	-2.76	11.29	38.74
	$E\hat{\Pi}^*_M$	0.95	5.61	10.66	22.23	78.86	270.33	2.66	5.70	8.97	16.35	49.80	140.46	3.74	5.32	6.98	10.58	24.83	53.70
	$E\hat{\Pi}^*_R$	-9.63	-6.02	-2.03	7.27	53.54	209.69	-11.20	-8.95	-6.46	-0.70	26.34	101.22	-8.99	-7.47	-5.85	-2.29	11.91	39.75

Table 4A.1: Multiplicative p-postponement with power demand distributions and linear expected demand

		$t=0$						$t=1$						$t=4$					
	c	0.00	0.10	0.50	1.00	2.00	5.00	0.00	0.10	0.50	1.00	2.00	5.00	0.00	0.10	0.50	1.00	2.00	5.00
Buybacks	w^*	1.00	1.10	1.50	2.00	3.00	6.00	1.00	1.10	1.50	2.00	3.00	6.00	1.00	1.10	1.50	2.00	3.00	6.00
	b^*	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Ep^*	2.00	2.10	2.50	3.00	4.00	7.00	2.00	2.10	2.50	3.00	4.00	7.00	2.00	2.10	2.50	3.00	4.00	7.00
	Q^*	0.27	0.17	0.08	0.04	0.01	7E-04	0.20	0.14	0.08	0.05	0.02	8E-04	0.16	0.13	0.08	0.05	0.02	9E-04
	$E\Pi_M^*$	0.14	0.12	0.07	0.04	0.01	7E-04	0.14	0.12	0.08	0.05	0.02	8E-04	0.14	0.12	0.08	0.05	0.02	9E-04
	$E\Pi_R^*$	0.14	0.12	0.07	0.04	0.01	7E-04	0.14	0.12	0.08	0.05	0.02	8E-04	0.14	0.12	0.08	0.05	0.02	9E-04
Percentage improvement relative to N-postponement	w^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	b^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Ep^*	0.00	-3.87	-10.10	-12.13	-12.31	-9.58	0.00	-2.04	-5.71	-6.97	-7.04	-5.29	0.00	-0.84	-2.49	-3.08	-3.10	-2.27
	Q^*	0.00	-18.99	-8.42	11.36	55.04	199.70	0.00	-15.12	-8.51	5.07	31.17	92.24	0.00	-9.79	-5.14	2.60	14.10	34.43
	$E\Pi_M^*$	0.00	3.33	21.44	45.00	93.37	243.73	0.00	1.93	12.88	26.43	50.68	107.28	0.00	1.00	6.43	12.35	21.39	38.80
	$E\Pi_R^*$	0.00	3.33	21.44	45.00	93.37	243.73	0.00	1.93	12.88	26.43	50.68	107.28	0.00	1.00	6.43	12.35	21.39	38.80
Wholesale	$\hat{w}^*$	0.80	0.93	1.40	1.94	2.98	6.00	0.94	1.05	1.48	1.99	3.00	6.00	1.00	1.10	1.50	2.00	3.00	6.00
	$E\hat{p}^*$	1.80	1.93	2.40	2.94	3.98	7.00	1.94	2.05	2.48	2.99	4.00	7.00	2.00	2.10	2.50	3.00	4.00	7.00
	$\hat{Q}^*$	0.15	0.13	0.07	0.04	0.01	7E-04	0.13	0.12	0.08	0.05	0.02	8E-04	0.13	0.12	0.08	0.05	0.02	9E-04
	$E\hat{\Pi}_M^*$	0.12	0.11	0.07	0.04	0.01	7E-04	0.13	0.11	0.08	0.05	0.02	8E-04	0.13	0.12	0.08	0.05	0.02	9E-04
	$E\hat{\Pi}_R^*$	0.13	0.12	0.07	0.04	0.01	7E-04	0.13	0.12	0.08	0.05	0.02	8E-04	0.13	0.12	0.08	0.05	0.02	9E-04
Percentage improvement relative to N-postponement	$\hat{w}^*$	42.31	34.80	20.38	13.06	6.97	2.16	30.40	25.53	14.84	9.01	4.37	1.21	14.93	12.52	7.06	4.14	1.94	0.52
	$E\hat{p}^*$	-3.44	-4.93	-8.01	-9.38	-9.88	-8.40	1.59	0.44	-2.45	-4.11	-5.11	-4.54	2.09	1.35	-0.50	-1.53	-2.13	-1.91
	$\hat{Q}^*$	-31.26	-26.84	-9.58	12.08	57.15	202.22	-23.17	-19.86	-7.17	7.50	33.44	93.32	-11.58	-9.52	-2.39	4.67	15.21	34.79
	$E\hat{\Pi}_M^*$	-2.18	2.95	22.74	47.03	95.97	246.16	0.18	3.37	15.33	28.87	52.61	108.21	1.62	3.10	8.32	13.71	22.18	39.08
	$E\hat{\Pi}_R^*$	-5.06	-0.21	18.81	42.49	90.89	241.62	26.69	30.92	47.15	65.91	98.97	175.34	1.47	3.32	9.77	16.19	25.89	44.34

Table 4A.2: Multiplicative p-postponement with power demand distributions and exponential expected demand

		$t=0$						$t=1$						$t=4$					
	c	0.01	0.10	0.50	1.00	2.00	5.00	0.01	0.10	0.50	1.00	2.00	5.00	0.01	0.10	0.50	1.00	2.00	5.00
Buybacks (Wholesale)	w^*	0.02	0.20	1.00	2.00	4.00	10.00	0.02	0.20	1.00	2.00	4.00	10.00	0.02	0.20	1.00	2.00	4.00	10.00
	b^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Ep^*	0.04	0.40	2.00	4.00	8.00	20.00	0.04	0.40	2.00	4.00	8.00	20.00	0.04	0.40	2.00	4.00	8.00	20.00
	Q^*	555.56	5.56	0.22	0.06	0.01	2E-03	600.00	6.00	0.24	0.06	0.02	2E-03	619.83	6.20	0.25	0.06	0.02	2E-03
	$E\Pi_M^*$	5.56	0.56	0.11	0.06	0.03	0.01	6.00	0.60	0.12	0.06	0.03	0.01	6.20	0.62	0.12	0.06	0.03	0.01
	$E\Pi_R^*$	11.11	1.11	0.22	0.11	0.06	0.02	12.00	1.20	0.24	0.12	0.06	0.02	12.40	1.24	0.25	0.12	0.06	0.02
Percentage improvement relative to N-postponement	w^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	b^*	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	Ep^*	-33.33	-33.33	-33.33	-33.33	-33.33	-33.33	-20.00	-20.00	-20.00	-20.00	-20.00	-20.00	-9.09	-9.09	-9.09	-9.09	-9.09	-9.09
	Q^*	50.00	50.00	50.00	50.00	50.00	50.00	29.10	29.10	29.10	29.10	29.10	29.10	12.89	12.89	12.89	12.89	12.89	12.89
	$E\Pi_M^*$	50.00	50.00	50.00	50.00	50.00	50.00	29.10	29.10	29.10	29.10	29.10	29.10	12.89	12.89	12.89	12.89	12.89	12.89
	$E\Pi_R^*$	50.00	50.00	50.00	50.00	50.00	50.00	29.10	29.10	29.10	29.10	29.10	29.10	12.89	12.89	12.89	12.89	12.89	12.89

Table 4A.3: Multiplicative p-postponement with power demand distributions and negative polynomial expected demand

Chapter 5

Competition and Cooperation in a Multi-Supplier and Single-Assembler Supply Chain

5.1 Introduction

We consider a supply chain wherein a assembler buys n completely complementary components (or products) from n suppliers. The assembler assembles the n components into a final product, and sells it over a single selling season to consumers at a given (fixed exogenously) retail price. In this chapter, we consider two contracting systems between the assembler and the suppliers: push and pull. In the push system, the suppliers initiate the process by offering their wholesale prices to the assembler, and the assembler then orders from the suppliers well in advance of the selling season. Stochastic demand is realized thereafter. In this case, the assembler bears all of the supply chain's inventory risk due to demand uncertainty. In contrast to push, in the pull system, the assembler first sets the wholesale prices for the different suppliers, and then the suppliers decide how much to produce. Their products are shipped to the assembler on consignment, i.e., the assembler only pays for the quantity sold. Thus, the suppliers bear all of the supply chain inventory risk.[5.1] In both systems, suppliers can form coalitions among themselves or act independently.

Examples which are similar to our push and pull systems are numerous. Indeed, in the last fifty years, many industries have been moving from producing internally all of their final products' components and services to buying (outsourcing) them from external suppliers. Such businesses

[5.1] The pull system studied in this chapter was first introduced by Cachon (2004a), and it is different from the pull promotion strategy in the Marketing literature. In the pull system described above, all decisions, e.g., pricing and production, are made before observing customer orders. It is called a "pull system" because it is the downstream assembler who initiates the process by offering a wholesale price to the upstream suppliers and pulls inventory from suppliers when real orders occur. By contrast, in the pull promotion strategy analyzed in the Marketing literature, customers' orders trigger the production process.

include the personal computer industry, e.g., Hewlett-Packard (HP) subcontracts their personal computer components to suppliers in Taiwan, the automobile industry (e.g., Toyota and Ford), the software industry (e.g., the rise of contract software designers in the "three I's" – India, Ireland and Israel), the shoe industry (e.g., Nike), and even the management consulting industry, wherein a consulting company may divide a large project into small parts and subcontract some parts to other smaller consulting companies. See Wang (2004) for other examples.

In both the push and pull systems, we consider two levels of problems: The first level is concerned with the interaction among the suppliers who are considering whether to form alliances or act independently when they deal with the assembler. The second level is concerned with the interaction between the assembler and the suppliers, which is modeled as a two-stage Stackelberg game. The objective of this chapter is to understand how the players in our decentralized supply chain would behave competitively and cooperatively in equilibrium, and how the channel profit would be allocated among players. More specifically, we are interested in the following research questions.

(1) Which coalitions of suppliers would be formed in the process of their interaction with the assembler?

(2) What would be the equilibrium outcomes in the Stackelberg game between the suppliers and the assembler?

(3) What are the preferences of the suppliers and the assembler between the push and pull systems?

We use two different approaches to analyze the first level problem, i.e., alliance formation among the suppliers. The first approach is based on Nash equilibrium, wherein a player can defect from its current coalition, in a given coalition structure, either to be independent or to join another coalition if such a coalition becomes strictly better off. Clearly, in such an approach players are myopic in the sense that they fail to anticipate future deviations by other players brought about by their own deviations. The second approach that we use to identify stable coalition structures is based on farsighted stability concepts, introduced by Chwe (1994) and Mauleon and Vannetelbosch (2004), and embodied by their notions of the largest consistent set (LCS) and the largest cautious consistent set (LCCS), respectively. The LCS concept has been previously applied in the supply chain literature, e.g., Granot and Sošić (2005), Nagarajan and Bassok (2002), and Nagarajan and Sošić (2004).

Our main results are described as follows.

(I) For a given coalition structure among the suppliers, in equilibrium:

(i) In the push system, coalitions share the total profit of all suppliers equally.

(ii) In the pull system, each coalition's share of the total profit of all suppliers is proportional to its manufacturing cost.

(II) In the push system with two suppliers, the grand coalition is the unique element in the Nash-stable set, the LCS and the LCCS.

(III) In the push system with $n \geq 3$ and a general power distribution of demand:

(i) The independent structure is the unique element in the Nash-stable set.

(ii) The grand coalition is the unique element in the LCCS.

(iii) The grand coalition is the unique element in the LCS for $n \leq 4$, and it is also the unique element in the LCS for $n \geq 5$ when the coefficient of variation of demand is small enough.

(IV) The assembler always prefers the pull system, and suppliers with relatively lower (respectively, higher) manufacturing cost prefer push (respectively, pull) to pull (respectively, push).

There are relatively few papers that are closely related to this chapter. In the context of a single-supplier single-retailer newsvendor model, Cachon (2004a) has investigated three different types of wholesale price contracts: push, pull and advanced-purchase discounts. He has provided motivating examples for the pull system, and has studied the impact of inventory risk allocation on supply chain efficiency and its members' performance. Nagarajan and Bassok (2002) have considered a supply chain with a single assembler who buys complementary products from n suppliers. They have designed a bargaining framework, based on the Nash bargaining problem (Nash, 1950), through which prices that the suppliers charge the assembler and the quantity of components that the assembler purchases from each supplier are determined. Wang (2004) has independently analyzed a model which is related to our push model. In his model, suppliers producing a set of complementary products need to choose, independently, a selling price together with a production quantity for their individual products before the selling season, wherein stochastic demand depends on the sum of all n selling prices. He has assumed that all suppliers act independently, and has explored and compared two settings with respect to the sequence of decisions taken by the independent suppliers. When suppliers act simultaneously in his model, Wang (2004) has independently obtained a result which is similar to Result I(i) described above. A similar result to Result I(ii) has been independently derived in Wang and Gerchak (2003) and Gerchak and Wang (2004). Wang and Gerchak (2003) have considered capacity games in assembly systems wherein firms need to construct their production capacity before demand uncertainty is resolved. They have examined two game settings with respect to how contract terms are determined, and they have shown that when the assembler sets a unit price to each supplier and orders after demand uncertainty is resolved, in equilibrium, each supplier share of the total expected profit of all suppliers is proportional to its capacity cost. Gerchak and Wang (2004) have analyzed and compared revenue-sharing contracts and wholesale price-only contracts in assembly systems. In their revenue-sharing contracts, the assembler pays each supplier a share of the retail price for each unit sold. They have shown that if the share to each supplier is set so as to maximize the assembler's expected profit, then the share of each supplier of the total suppliers' share of the profit is proportional to its manufacturing cost.

The second level problem in our model, when there is only a single supplier, i.e., $n = 1$, has been studied extensively in the operations literature. See, e.g., Lariviere and Porteus (2001), who have examined the channel performance and profit allocation between channel members under a wholesale price contract, and Larviere (1999) and Cachon (2004b) for excellent reviews.

The remainder of this chapter is organized as follows. §5.2 introduces the model setup and some notation, and in §5.3 we analyze the integrated channel. §5.4 examines the push system and §5.5 investigates the pull system. A comparison between the push and pull systems is presented in §5.6, and conclusions are discussed in §5.7. Again, all proofs in this chapter are presented in the appendix in §5.8.

5.2 Model Setup and Notation

Consider a system in which a risk-neutral assembler buys n different products (components) from n risk-neutral suppliers. The assembler assembles the n components into a final product and sells it over a single selling season to end consumers at a given retail price, i.e., the price-independent newsvendor model. The suppliers can act independently or form coalitions among themselves. Demand, X, for the final product during the selling season is stochastic. Let $F(x)$ and $f(x)$ be the distribution and density functions of demand, and let $\bar{F}(x) = 1 - F(x)$. Assume that F is strictly increasing, differentiable and $F(0) = 0$.

We investigate two types of contracts between the suppliers and the assembler: push and pull. Figure 5.1 in the following page presents the two systems corresponding to the two contracts.

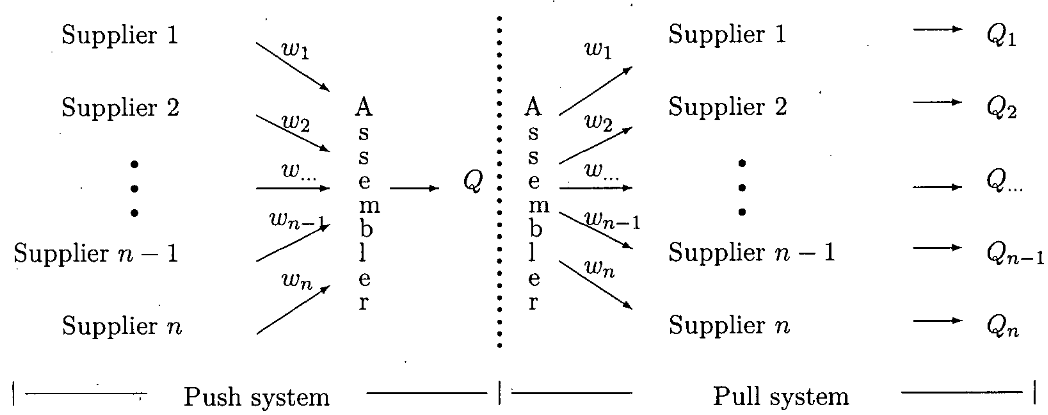

Figure 5.1: Push and pull systems

With a push contract each coalition of suppliers chooses its wholesale price, and then the assembler orders from each coalition well in advance of the selling season, and all suppliers produce what the assembler orders. The assembler bears all of the supply chain's inventory risk and the suppliers bear none. In contrast to push, with a pull contract the assembler sets the wholesale price for each coalition of suppliers, and then all coalitions choose how much to produce and their products are shipped to the assembler on consignment (i.e., the assembler only pays for the quantity sold), or the suppliers hold the inventory and replenish the assembler frequently and in small batches during the season. In a pull contract the suppliers bear all of the supply chain's risk. See Cachon (2004a). In both cases, it is assumed that there is no salvage value for unsold inventory, unmet demand is lost without any penalty cost and the assembly cost for the assembler is negligible. Note that all qualitative results in this chapter hold if the assembler incurs an assembly cost. Finally, we assume that the information is symmetric among players.

Under both push and pull contracts, we consider two levels of problems: The first level is concerned with the interaction among the suppliers who are considering whether to form alliances or act independently when they deal with the assembler. The second level is concerned with the interaction between the assembler and the suppliers, which is modeled as a two-stage Stackelberg game.

For convenience, we summarize some of the notation used in the sequel as follows.

$N = \{1, ..., n\}$: Set of suppliers.

M_i: Supplier $i \in N$.

w_{M_i}: Wholesale price of M_i.

c_{M_i}: Marginal manufacturing cost of M_i.

p: Fixed retail price for the final product.

$\mathcal{B} = \{B_1, ..., B_m\}$: Coalition structure. Thus, $\bigcup_{j=1}^{m} B_j = N$, and $B_h \bigcap B_k = \emptyset$, $h \neq k$.

$m(\leq n)$: Number of coalitions in a specific coalition structure $\mathcal{B}$.

B_j: Any coalition in a coalition structure $\mathcal{B}$, $j \in \{1, ..., m\}$.

$|B_j|$: The cardinality of Coalition B_j.

$\mathcal{B} = \{B_1, ..., B_m\}$ is symmetric if and only if $|B_h| = |B_k|$ for $h, k \in \{1, ..., m\}$.

$\tau_{\mathcal{B}} =< |B_1|, ..., |B_m| >$ is the profile of coalition structure $\mathcal{B} = \{B_1, ..., B_m\}$.

W_{B_j}: Total wholesale price of Coalition $B_j \in B$. Thus, $W_{B_j} = \Sigma(w_{M_i} : M_i \in B_j)$.

$W = \Sigma_{i=1}^{n}(w_{M_i}) = \Sigma_{j=1}^{m}(W_{B_j})$: Total wholesale price of all suppliers or all coalitions.

C_{B_j}: Total manufacturing cost of Coalition B_j. Thus $C_{B_j} = \Sigma(c_{M_i} : M_i \in B_j)$.

$C = \Sigma_{i=1}^{n}(c_{M_i}) = \Sigma_{j=1}^{m}(C_{B_j})$: Total manufacturing cost of all suppliers or all coalitions.

$\bar{c} = \frac{C}{n}$: Average manufacturing cost.

Q_{B_j}: Production quantity of Coalition B_j.

$\vec{Q} = (Q_{B_1}, ..., Q_{B_m})$: Vector of production quantities.

Q_{-B_j}: Vector of production quantities of all coalitions but B_j.

$E\Pi_{B_j}$: Expected profit of Coalition B_j.

$E\Pi_M^{Total}$: Expected profit of all suppliers.

$E\Pi_R$: Expected profit of the assembler.

$E\Pi_{Channel}$: Expected profit of the channel.

Note that $m = n$ implies the *independent coalition* structure, and $m = 1$ implies the *grand coalition* structure.

5.3 Integrated Channel

As a benchmark, we first consider the integrated supply chain which maximizes the sum of the assembler and suppliers profits. The expected channel profit is:

$$E\Pi = pS(Q) - CQ = p(Q - \int_0^Q F(x)dx) - CQ,$$

where the expected sales $S(Q) = \int_0^Q xf(x)dx + \int_Q^\infty Qf(x)dx$. The channel has to determine a production level Q to maximize its expected profit. This problem is the traditional newsvendor model with a fixed retail price, which has been studied extensively in the literature. For ease of future comparison, we recall the results in the integrated system as follows. The optimal production quantity, Q^I, satisfies the critical fractile:

$$F(Q^I) = \frac{p - C}{p}, \tag{5.1}$$

where, as we recall, the superscript "I" stands for "integrated". Denote the optimal expected value of the integrated channel profit as $E\Pi^I$. The efficiency of the channel is defined to be: $\frac{E\Pi(Q)}{E\Pi^I}$, where $E\Pi(Q)$ is the channel profit in any decentralized supply chain with production quantity Q.

5.4 Push System

In a push system, under a coalition structure $\mathcal{B} = \{B_1, ..., B_m\}$, each coalition B_j, in Stage 1, sets its wholesale price, W_{B_j}, $j \in \{1, ..., m\}$. Then, being provided with $\{W_{B_1}, ..., W_{B_m}\}$, the assembler orders inventory from each coalition in Stage 2, resulting with a vector of order quantities that is denoted as $\vec{Q} = \{Q_{B_1}, ..., Q_{B_m}\}$. Thereafter, each member in the coalition produces exactly what the assembler orders. In this system, the suppliers sell to a newsvendor who bears all of the supply chain's inventory risk.

In §5.4.1, we analyze the push model under a given coalition structure of the suppliers, and in §5.4.2 we evaluate the cost efficiency of the suppliers. In §5.4.3, we analyze the stability of coalition structures by using cooperative and non-cooperative game theory methodologies.

5.4.1 The push model under a given coalition structure

For a given coalition structure, $\mathcal{B} = \{B_1, ..., B_m\}$, backward induction is used to solve the two-stage Stackelberg game between the suppliers and the assembler. For any given vector of the wholesale prices, $\{W_{B_1}, ..., W_{B_m}\}$, the assembler's problem in Stage 2 is to choose $\vec{Q} = \{Q_{B_1}, ..., Q_{B_m}\}$ to maximize his expected profit function:

$$E\Pi_R = pS(\vec{Q}) - \Sigma_{j=1}^m (W_{B_j} Q_{B_j}) = pE_X(\min(\min(\vec{Q}), X)) - \Sigma_{j=1}^m (W_{B_j} Q_{B_j}), \tag{5.2}$$

where Q_{B_j} is the order quantity for Coalition B_j, and $\min(\vec{Q}) = \min\{Q_{B_j} : j = 1, ..., m\}$. The assembler's optimal inventory vector is characterized by the following lemma.

Lemma 5.4.1 *For a coalition structure $\mathcal{B} = \{B_1, ..., B_m\}$ and a corresponding wholesale price vector $\{W_{B_1}, ..., W_{B_m}\}$, the assembler's optimal inventory vector with a push contract satisfies:*

(i) $Q_{B_1} = ... = Q_{B_m} = Q$, *and*

(ii) $F(Q) = \frac{p-W}{p}$, *where* $W = \Sigma_{j=1}^m (W_{B_j})$.

The conclusion that the assembler always orders the same inventory from each coalition is quite intuitive. Indeed, if he orders unequal quantities from different coalitions, then by reducing a bit the higher orders the assembler would achieve the same sales level of the final product while reducing his cost, which strictly benefits him. Thus, the assembler will never order unequal inventories of different components. See also Wang (2004). Knowing that the assembler's order quantities are equal, his problem to choose the optimal Q does not depend on the individual wholesale price set by each coalition. Rather, it depends on the total wholesale price for all components. Thus, the assembler's problem is similar to the case when all suppliers form the grand coalition, which coincides with the single-supplier single-assembler newsvendor model, that is well studied in the literature. Note that substituting $W = p(1 - F(Q))$, given in Lemma 5.4.1, into $E\Pi_R$, given by (5.2), and simplifying results with

$$E\Pi_R = p\int_0^Q xf(x)dx, \tag{5.3}$$

which is strictly increasing in Q for $Q > 0$. See also Cachon (2004a).

Under a coalition structure $\mathcal{B} = \{B_1, ..., B_m\}$, knowing the assembler's reaction function of Q, the suppliers choose their corresponding optimal wholesale prices. The following theorem characterizes the equilibrium values of the wholesale prices and the corresponding production quantity in a push contract. We assume that the demand distribution function has an increasing general failure rate (IGFR), i.e., $g(Q) \equiv QH(Q) = \frac{Qf(Q)}{1-F(Q)}$ is weakly increasing in Q, where $H(Q) = \frac{f(Q)}{1-F(Q)}$ is the failure rate function. See Lariviere and Porteus (2001) for a detailed discussion of this property in the context of the classical newsvendor model.

Theorem 5.4.2 *In the push contract, under a coalition structure $\mathcal{B}$, the equilibrium values of the wholesale price for each coalition and production quantity satisfy:*

(i) $W^*_{B_1} - C_{B_1} = W^*_{B_2} - C_{B_2} = ... = W^*_{B_m} - C_{B_m} = \frac{W^*-C}{m} = pQ^*f(Q^*)$, *and*

(ii) $p(\bar{F}(Q^*) - mQ^*f(Q^*)) = C$.

Theorem 5.4.2 (i) implies that given any coalition structure, in equilibrium, each coalition will have the same marginal profit, and thus the same profit. Upon reflection, this result is quite intuitive. Indeed, any possible fixed cost invested by any coalition to produce its items is not incorporated in the model. Additionally, the products of all coalitions are needed in order to assemble the final product. Thus, in a sense, all products are equally important. Finally, none of the coalitions bears any inventory risk due to demand uncertainty. So differences in the values of coalitions' manufacturing costs are irrelevant. Therefore, it is intuitive that in equilibrium, all coalitions should have the same profit margin.

Wang (2004) has independently obtained a similar result in a related push model, wherein n suppliers producing a set of complementary products need to choose, independently, a selling price together with a production quantity for their individual products. He has shown that if all suppliers simultaneously make their pricing and production decisions, then suppliers have the same profit.

When $m = 1$, all suppliers form the grand coalition, and our problem coincides with the single-supplier single-assembler push system studied, e.g., by Lariviere and Porteus (2001) and Cachon (2004a). Indeed, Theorem 5.4.2 is consistent with the corresponding results derived in their papers. Theorem 5.4.2 also implies that the equilibrium values of the total wholesale price and the production quantity depend on m, the number of coalitions, but are independent of how these coalitions are formed. Note further that Theorem 5.4.2 (ii) implies that the equilibrium production quantity Q^* in the push system depends on the ratio $\frac{C}{p}$ only, instead of on an individual supplier's marginal cost and the selling price p.

Using Theorem 5.4.2, we are able to compute the equilibrium profits for each coalition, the assembler and the channel: $E\Pi^*_{B_j} = (W^*_{B_j} - C_{B_j})Q^* = \frac{(p\bar{F}(Q^*)-C)Q^*}{m} = p(Q^*)^2 f(Q^*)$ for any j, $E\Pi^*_R = p \cdot S(Q^*) - W^*Q^* = p(S(Q^*) - \bar{F}(Q^*)Q^*) = p\int_0^{Q^*} xf(x)dx$ and $E\Pi^*_{Channel} = mE\Pi^*_{B_j} + E\Pi^*_R = pS(Q^*) - CQ^*$, where Q^* satisfies $p(\bar{F}(Q^*) - mQ^*f(Q^*)) = C$. Thus, we can conclude that the equilibrium profits of each coalition in the coalition structure, and the assembler and the channel equilibrium expected profits depend only on m, rather than the actual composition of the coalition structure. For example, the equilibrium profits of each coalition, the assembler and the channel in the two structures $\mathcal{B}_1 = \{B_1 = (1,2), B_2 = (3,4)\}$ and $\mathcal{B}_2 = \{B_1 = (1), B_2 = (2,3,4)\}$ coincide.

Corollary 5.4.3 *In a push contract, in equilibrium:*

(i) $\frac{\partial E\Pi^*_{B_j}}{\partial m} \leq 0$ *for any* $j \in \{1, ..., m\}$ *and equality holds iff* $m = 1$.

(ii) $\frac{\partial E\Pi^{Total}_M}{\partial m} \leq 0$ *and equality holds iff* $m = 1$.

(iii) $\frac{\partial E\Pi^*_R}{\partial m} < 0$.

(iv) $\frac{\partial Q^*}{\partial m} < 0$, *and*

(v) $\frac{\partial W^*}{\partial m} > 0$.

Corollary 5.4.3 implies that the equilibrium quantity and the equilibrium profits of each supplier, and of the assembler and thus, of the entire channel, decrease in the number of coalitions, m. Thus, consumers surplus and all these equilibrium profits are maximized when the suppliers form the grand coalition.

5.4.2 Cost efficiency of the suppliers

According to Theorem 5.4.2 (i), under a push contract, all coalitions have the same profit margin in equilibrium. Thus, more cost efficient coalitions have no profit advantage in equilibrium. However, we wonder whether coalitions have an incentive to improve their cost efficiency. We next investigate the effect of changes in the marginal manufacturing cost on the channel profit and its members' profits, and on the equilibrium values of decision variables.

Proposition 5.4.4 *In the push contract, under a coalition structure $\mathcal{B}$, in equilibrium, for any $j \in \{1, ..., m\}$:*

(i) $\frac{\partial(Q^*)}{\partial(C_{B_j})} < 0$.

(ii) $\frac{\partial(W^*)}{\partial(C_{B_j})} > 0$ *and* $\frac{\partial(W^*_{B_j})}{\partial(C_{B_j})} > 0$.

(iii) $\frac{\partial(E\Pi^*_{B_j})}{\partial(C_{B_j})} < 0$ *and* $\frac{\partial(E\Pi^*_{B_i})}{\partial(C_{B_j})} < 0$ *for* $i \neq j$, *and*

(iv) $\frac{\partial(E\Pi^*_R)}{\partial(C_{B_j})} < 0$.

We conclude from Proposition 5.4.4 that each supplier has an incentive to improve her cost efficiency, i.e., reducing her marginal manufacturing cost, and that such a reduction in cost has a beneficial effect for the other suppliers, for the assembler, and for the end-consumers. We note, though, that the effect of changes in one coalition's marginal manufacturing cost on other coalitions' optimal wholesale prices depends on the demand distribution function. Specifically, the sign of $\frac{\partial(W^*_{B_i})}{\partial(C_{B_j})}$ depends on whether $Qf(Q)$ decreases or increases in Q. If $Qf(Q)$ decreases (or increases) in Q, then $\frac{\partial(W^*_{B_i})}{\partial(C_{B_j})} > 0$ (respectively, $\frac{\partial(W^*_{B_i})}{\partial(C_{B_j})} < 0$), and IGFR does not guarantee that $Qf(Q)$ increases in Q.

5.4.3 Stability of coalition structures

The equilibrium analysis of the Stackelberg game between the suppliers and the assembler in §5.4.1 is conducted for a given coalition structure. In this subsection, we investigate the stability of coalition structures.

We use two different approaches to investigate stability. The first approach is based on Nash equilibrium, wherein a player can defect from its current coalition in a given coalition structure if s/he is strictly better off from such a deviation. Clearly, in such an approach players are myopic in the sense that they fail to anticipate future deviations by other players brought about by their own deviations. The second approach to identify stable coalition structures is based on farsighted stability concepts introduced by Chwe (1994) and Mauleon and Vannetelbosch (M&V) (2004).

We will assume in this subsection that members of a coalition in any coalition structure share the coalition's profit equally. This is a reasonable assumption, since, by Theorem 5.4.2 (i), in equilibrium, all coalitions in any coalition structure gain equal profits.

Nash-stable coalition structures

We assume that a *feasible deviation* by a supplier, M_i, in any given coalition structure is either from her coalition to become independent, or to join another coalition, provided that the receiving coalition becomes strictly more profitable from having M_i joining it.

Definition 5.4.5 *The Nash-stable set of coalition structures consists of all coalition structures in which no supplier has a strictly profitable feasible deviation.*

Recall that $|B_j|$ is the cardinality of coalition B_j.

Proposition 5.4.6 *Let $\mathcal{B} = \{B_1, ..., B_m\}$ be a given coalition structure, and assume, without loss of generality, that $|B_1| \leq ... \leq |B_m|$. $\mathcal{B}$ is Nash-stable if and only if the following conditions hold:*
(i) If $1 \leq m < n$ and $|B_1| = 1$, then: $\frac{(p\bar{F}(Q^*_m)-C)Q^*_m}{m} \geq \max(\frac{|B_m|(p\bar{F}(Q^*_{m+1})-C)Q^*_{m+1}}{m+1}, \frac{(p\bar{F}(Q^*_{m-1})-C)Q^*_{m-1}}{(|B_2|+1)(m-1)})$.
(ii) If $1 \leq m < n$ and $|B_1| \geq 2$, then: $\frac{(p\bar{F}(Q^*_m)-C)Q^*_m}{m} \geq \frac{|B_m|(p\bar{F}(Q^*_{m+1})-C)Q^*_{m+1}}{m+1}$.
(iii) If $m = n$, then: $\frac{(p\bar{F}(Q^*_n)-C)Q^*_n}{n} \geq \frac{(p\bar{F}(Q^*_{n-1})-C)Q^*_{n-1}}{2(n-1)}$,
where Q^*_s, the equilibrium production quantity under a coalition structure with s coalitions, satisfies $p(\bar{F}(Q) - sQf(Q)) = C$.

To illustrate the conditions identified in Proposition 5.4.6 on stability, let us consider the following two examples.

Example 5.4.7 2-supplier system.

In this example, $n = 2$, and there are only two possible scenarios. Either $m = 1$ (grand coalition structure) or $m = 2$ (independent structure). When $m = 1$, $|B_1| = 2$, and according to Corollary 5.4.3, the total equilibrium profit of all suppliers decreases in m. Thus, $(p\bar{F}(Q^*_1) - C)Q^*_1 \geq$

$(p\bar{F}(Q_2^*) - C)Q_2^*$, which satisfies Condition (ii) in Proposition 5.4.6. Therefore, the grand coalition structure is Nash-stable. When $m = 2(= n)$, since $(p\bar{F}(Q_1^*) - C)Q_1^* \geq (p\bar{F}(Q_2^*) - C)Q_2^*$, we have $\frac{(p\bar{F}(Q_1^*)-C)Q_1^*}{2} \geq \frac{(p\bar{F}(Q_2^*)-C)Q_2^*}{2}$, which does not satisfy Condition (iii) in Proposition 5.4.6. Thus, the independent structure is not Nash-stable.

We immediately have the following conclusion.

Corollary 5.4.8 *In a supply chain with two suppliers, i.e., $n = 2$, in equilibrium, the grand coalition is the unique Nash-stable coalition structure.*

Example 5.4.9 Power distribution function of demand.

In this example we assume that demand X follows a power function distribution on $[0, U]$. Therefore, $f(x) = x^q$ and $F(x) = \frac{x^{q+1}}{q+1}$, where $q \geq 0$, and $U = (q+1)^{\frac{1}{q+1}}$ to guarantee $F(U) = 1$. (Note that for $q = 0$, $f(x)$ is uniformly distributed.) For a given coalition structure, $\mathcal{B} = \{B_1, ..., B_m\}$, assume, without loss of generality, that $|B_1| \leq ... \leq |B_m|$. By Theorem 5.4.2 (ii), $p(\bar{F}(Q_m^*) - mQ_m^* f(Q_m^*)) = C$. Substituting $F(x)$ and $f(x)$ into this equation and simplifying, results with $Q_m^* = (\frac{1-\frac{C}{p}}{\frac{1}{q+1}+m})^{\frac{1}{q+1}}$.

If $m < n$ and $|B_1| = 1$, then $m \geq 2$ and $n \geq 3$. By Condition (i) in Proposition 5.4.6, the necessary and sufficient conditions for $\mathcal{B}$ to be Nash-stable are:

$$|B_m| \leq (1 + \frac{1}{\frac{1}{q+1} + m})^{1+\frac{1}{q+1}} \quad \text{and} \quad |B_2| \geq (1 + \frac{1}{\frac{1}{q+1} + m - 1})^{1+\frac{1}{q+1}}. \tag{5.4}$$

Note that $|B_m| \geq 2$, since $m < n$, and $|B_2| \geq 1$. Thus, to satisfy (5.4), $m \leq \frac{1}{2^{\frac{q+1}{q+2}}-1} - \frac{1}{q+1}$. The right hand side of the last inequality is strictly decreasing in $q(\geq 0)$, and is bounded between $\sqrt{2}$ (when $q = 0$) and 1 (when $q \to \infty$). Therefore, $m = 1$ is the unique value which satisfies (5.4). We conclude that a coalition structure with $m < n$ and $|B_1| = 1$ is never Nash-stable.

If $m < n$ and $|B_1| \geq 2$, then by Condition (ii) in Proposition 5.4.6, the necessary and sufficient condition for Nash-stability is $|B_m| \leq (1 + \frac{1}{\frac{1}{q+1}+m})^{1+\frac{1}{q+1}}$, which is identical to the first inequality in (5.4). Thus, by following the same analysis as in the case with $m < n$ and $|B_1| = 1$, we have that $m = 1$ is the unique value which satisfies the first part in (5.4). When $m = 1$, $|B_m| \leq (1 + \frac{q+1}{q+2})^{\frac{q+2}{q+1}}$. The right hand side of the last inequality is strictly decreasing in q, and is bounded between 2.25 (when $q = 0$) and 2 (when $q \to \infty$). Thus, $|B_m| = 2$. Therefore, if $m < n$ and $|B_1| \geq 2$, then the unique Nash-stable coalition structure is the grand coalition consisting of the two suppliers, i.e., $m = 1$ and $n = 2$.

If $m = n$, then by Condition (iii) in Proposition 5.4.6, the necessary and sufficient condition for the independent structure to be Nash-stable is:

$$(1 + \frac{1}{\frac{1}{q+1} + n - 1})^{1+\frac{1}{q+1}} \leq 2,$$

which can be simplified to $n \geq \frac{1}{2^{\frac{q+1}{q+2}}-1} + \frac{q}{q+1}$. The right hand side of the last inequality is strictly decreasing in $q(\geq 0)$, and is bounded between $\sqrt{2}+1$ (when $q = 0$) and 2 (when $q \to \infty$). Thus, if $m = n \geq 3$, the independent structure is always Nash-stable.

We note that the analysis in Example 5.4.9 can be easily extended to a more general power distribution function $f(x) = \gamma x^q$ for any $\gamma > 0$. We immediately have the following result.

Theorem 5.4.10 *In equilibrium:*

(i) If $n = 2$, then the grand coalition is the unique Nash-stable coalition structure.

(ii) If $n \geq 3$, and the demand distribution is a power function, i.e., $f(x) = \gamma x^q$ for $q \geq 0$, then the independent structure is the unique Nash-stable coalition structure.

The result above that for $n \geq 3$, the independent structure is the unique Nash-stable coalition structure for a power demand distribution is somewhat disappointing. Indeed, by Corollary 5.4.3, the assembler, the suppliers and the end-consumers would all be better off if the grand coalition of all suppliers is formed, which suggests that a legal objection to a possible collusion among the suppliers for the purpose of alliance formation is unlikely. Moreover, in reality, we often observe alliances of suppliers rather than the independent structure, see, e.g., alliances among the outsourcing vendors in the IT industry (Gallivan and Oh 1999). This result, of the unique stability of the independent structure, is likely due to the fact that stability based on Nash equilibrium is "myopic" and does not incorporate farsightedness, i.e., only one-step deviations are considered. In the next subsection we attribute "farsightedness" to coalitions of suppliers, as modeled by Chwe (1994) and M&V (2004) and embodied by their notions of the largest consistent set (LCS) and the largest cautious consistent set (LCCS), respectively, to overcome the shortcoming of the Nash equilibrium approach to identify stable coalition structures.

Finally, note that if a feasible deviation, by a supplier, in Definition 5.4.5 of the Nash-stable concept, is restricted to be only to become independent, then feasible deviations can only be made by the Type I suppliers, i.e., suppliers in a non-singleton coalition. Thus, the independent structure (i.e., $m = n$) is always Nash-stable, since there are no feasible deviations from it, and by Proposition 5.4.6, any coalition structure $\mathcal{B} = \{B_1, ..., B_m\}$ is Nash-stable if and only if $\frac{(p\bar{F}(Q_m^*)-C)Q_m^*}{m} \geq \frac{|B_m|(p\bar{F}(Q_{m+1})-C)Q_{m+1}^*}{m+1}$ for $m < n$. Thus, we conclude that in Example 5.4.7 with two suppliers, both the grand coalition and the independent structure are Nash-stable, and in Example 5.4.9 with a power distribution of demand, the independent structure is the unique Nash-stable coalition structure for $n \geq 3$.

Farsighted stable coalition structures

Farsightedness, according to Chwe, allows a coalition to consider the possibility that, once it acts, another coalition might react, a third coalition might in turn react, and so on without limit.

Before presenting the LCS concept, we need to introduce some more definitions and notation. Let P be the finite set of coalition structures. Denote by $\{\rightharpoonup_S\}_{S\subseteq N, S\neq\emptyset}$ the *effectiveness relation* on P, where $\mathcal{B}_1 \rightharpoonup_S \mathcal{B}_2$ if coalition structure $\mathcal{B}_2$ is derived when S deviates from the coalition structure $\mathcal{B}_1$. A coalition formation game in effectiveness form G is $(N, P, E\Pi, \{\rightharpoonup_S\}_{S\subseteq N, S\neq\emptyset})$.

Definition 5.4.11 *A coalition structure $\mathcal{B}_1$ is indirectly strictly dominated by $\mathcal{B}_m$, or $\mathcal{B}_1 \ll \mathcal{B}_m$, if there exist sequences $\{\mathcal{B}_1, \mathcal{B}_2, ..., \mathcal{B}_m\}$ and $\{S_1, S_2, ..., S_{m-1}\}$, such that $\mathcal{B}_j \rightharpoonup_{S_j} \mathcal{B}_{j+1}$ and $E\Pi_{M_i}(\mathcal{B}_m) > E\Pi_{M_i}(\mathcal{B}_j)$ for all $M_i \in S_j$, for all $j = 1, ..., m-1$.*

Direct strict dominance is obtained by setting $m = 2$ in Definition 5.4.11. A coalition structure $\mathcal{B}_1$ is *directly strictly dominated* by $\mathcal{B}_2$, or $\mathcal{B}_1 < \mathcal{B}_2$, if these exists a coalition S such that $\mathcal{B}_1 \rightharpoonup_S \mathcal{B}_2$ and $E\Pi_{M_i}(\mathcal{B}_2) > E\Pi_{M_i}(\mathcal{B}_1)$ for all $M_i \in S$. Clearly, if $\mathcal{B}_1 < \mathcal{B}_2$, then $\mathcal{B}_1 \ll \mathcal{B}_2$.

Definition 5.4.12 *A coalition structure $\mathcal{B}_1$ is indirectly weakly dominated by $\mathcal{B}_m$, or $\mathcal{B}_1 \underline{\ll} \mathcal{B}_m$, if there exist sequences $\{\mathcal{B}_1, \mathcal{B}_2, ..., \mathcal{B}_m\}$ and $\{S_1, S_2, ..., S_{m-1}\}$, such that $\mathcal{B}_j \rightharpoonup_{S_j} \mathcal{B}_{j+1}$ and $E\Pi_{M_i}(\mathcal{B}_m) \geq E\Pi_{M_i}(\mathcal{B}_j)$ for all $M_i \in S_j$ and $E\Pi_{M_i}(\mathcal{B}_m) > E\Pi_{M_i}(\mathcal{B}_j)$ for some $M_i \in S_j$, for $j = 1, ..., m-1$.*

Direct weak dominance is obtained by setting $m = 2$ in Definition 5.4.12. A coalition structure $\mathcal{B}_1$ is *directly weakly dominated* by $\mathcal{B}_2$, or $\mathcal{B}_1 \leq \mathcal{B}_2$, if these exists a coalition S such that $\mathcal{B}_1 \rightharpoonup_S \mathcal{B}_2$ and $E\Pi_{M_i}(\mathcal{B}_2) \geq E\Pi_{M_i}(\mathcal{B}_1)$ for all $M_i \in S$ and $E\Pi_{M_i}(\mathcal{B}_2) > E\Pi_{M_i}(\mathcal{B}_1)$ for some $M_i \in S$. Obviously, if $\mathcal{B}_1 \leq \mathcal{B}_2$, then $\mathcal{B}_1 \underline{\ll} \mathcal{B}_2$.

Definition 5.4.13 (The largest consistent set based on indirect strict dominance: $LCS(G, \ll)$, Chwe (1994).) *A set Y is called consistent if $\mathcal{B} \in Y$ if and only if for all $\mathcal{V}$ and S, such that $\mathcal{B} \rightharpoonup_S \mathcal{V}$, there is an $\mathcal{B}' \in Y$, where $\mathcal{V} = \mathcal{B}'$ or $\mathcal{V} \ll \mathcal{B}'$, such that we do not have $E\Pi_{M_i}(\mathcal{B}) < E\Pi_{M_i}(\mathcal{B}')$ for all $M_i \in S$. The largest consistent set is the union of all consistent sets.*

Definition 5.4.14 (The largest consistent set based on indirect weak dominance: $LCS(G, \underline{\ll})$, M&V (2004).) *A set Y is called consistent if $\mathcal{B} \in Y$ if and only if for all $\mathcal{V}$ and S, such that $\mathcal{B} \rightharpoonup_S \mathcal{V}$, there is an $\mathcal{B}' \in Y$, where $\mathcal{V} = \mathcal{B}'$ or $\mathcal{V} \underline{\ll} \mathcal{B}'$, such that we do not have $E\Pi_{M_i}(\mathcal{B}) \leq E\Pi_{M_i}(\mathcal{B}')$ for all $M_i \in S$ and $E\Pi_{M_i}(\mathcal{B}) < E\Pi_{M_i}(\mathcal{B}')$ for some $M_i \in S$. The largest consistent set is the union of all consistent sets.*

Since every coalition considers the possibility that, once it acts, another coalition might react, a third coalition might in turn react, and so on, a consistent set incorporates *farsighted* coalitional stability. If Y is consistent and $\mathcal{B} \in Y$, it does not imply that $\mathcal{B}$ is necessary stable, but that it is *possible* for $\mathcal{B}$ to be stable. However, if $\mathcal{B} \notin Y$, $\mathcal{B}$ cannot be stable. The largest consistent set (LCS) is unique, and has the merit of "ruling out with confidence". For a more detailed discussion and analysis of farsighted coalitional stability, see Chwe (1994).

To illustrate the notion of the largest consistent set, let us consider the following example, presented in Figure 5.2, which is a modification of an example in Chwe (1994). Figure 5.2 presents a three-player coalition formation game where the feasible coalition structures are: $\{1,2,3\}$, $\{12,3\}$, $\{1,23\}$ and $\{13,2\}$. The payoff vectors associated with these partitions and the possible moves from each partition are given in Figure 5.2. The effectiveness relations are represented by labeled directed arcs. Here Players $\{1,3\}$ will surely move from $\{1,2,3\}$ to $\{13,2\}$ because they are both better off in the coalition structure $\{13,2\}$. Similarly, none of the other coalition structure is Nash-stable, and it follows that the Nash-stable set is empty. The largest consistent set, however, is $LCS(G, \ll) = LCS(G, \underline{\ll}) = \{\{1,23\},\{12,3\},\{13,2\}\}$. Thus, both sets exclude the independent structure.

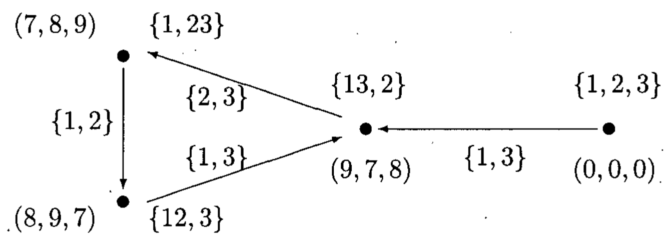

Figure 5.2: The Nash-stable set cannot make a "clear" prediction

The largest consistent set does not tell what *will* happen but what can *possibly* happen. In the example above, any coalition structure in $\{\{1,23\},\{12,3\},\{13,2\}\}$ can possibly be stable, but the structure $\{1,2,3\}$ cannot possibly be stable. The concept of the largest consistent set has been previously applied in the supply chain literature, e.g., Granot and Sošić (2005), Nagarajan and

Bassok (2002), and Nagarajan and Sošić (2004).

The *largest cautious consistent set* (LCCS), introduced by M&V (2004), is a refinement of the LCS. As pointed out by M&V, the largest consistent set may include coalition structures wherein coalitions could deviate without ending up being worse off in subsequent deviation, and possibly ending up being better off. Namely, a coalition structure may be in the LCS because a deviation from it is deterred by a *likely* sequence of subsequent deviations leading to an outcome where the initial deviators are equal off, in spite of the fact that any other *likely* subsequent deviations would not make the initial deviators worse off, and at least one of them would make the initial deviators better off. Then, according to M&V's LCCS concept, a coalition of *cautious* players, who assigns a positive weight to all likely subsequent deviations, will deviate for sure from the original coalition structure.

Similar to Chwe's LCS, M&V's LCCS is a farsighted concept. Once a coalition S deviates from a coalition structure $\mathcal{B}$ to another $\mathcal{B}'$, this coalition S should consider the possibility to end up with a positive probability at any coalition structure $\mathcal{B}''$ not ruled out and such that $\mathcal{B}' = \mathcal{B}''$ or $\mathcal{B}' \ll \mathcal{B}''$.

Formally, we have:

Definition 5.4.15 (The largest cautious consistent set based on indirect strict dominance: $LCCS(G, \ll)$, M&V (2004).) *A set Z is called cautious consistent if $\mathcal{B} \in Z$ if and only for all $\mathcal{V}$ and S, such that $\mathcal{B} \rightharpoonup_S \mathcal{V}$, there is a vector $\vec{\alpha} = (\alpha(\mathcal{B}^1), ..., \alpha(\mathcal{B}^m))$ satisfying $\Sigma_{j=1}^{m} \alpha(\mathcal{B}^j) = 1$, $\alpha(\mathcal{B}^j) \in (0,1)$, that gives only positive weight to each $\mathcal{B}^j \in Z$, where $\mathcal{V} = \mathcal{B}^j$ or $\mathcal{V} \ll \mathcal{B}^j$, such that we do not have*

$$E\Pi_{M_i}(\mathcal{B}) < \sum_{\mathcal{B}^j \in Z,\ \mathcal{V}=\mathcal{B}^j \text{ or } \mathcal{V} \ll \mathcal{B}^j} \alpha(\mathcal{B}^j) \cdot E\Pi_{M_i}(\mathcal{B}^j) \quad \text{for all } M_i \in S.$$

The largest cautious consistent set is the union of all cautious consistent sets.

Intuitively, a coalition structure $\mathcal{B}$ is not a member of the LCCS if a coalition S can make a deviation from $\mathcal{B}$ to $\mathcal{V}$ and by doing so there is no *risk* that some coalition members will end up being worse off.

Definition 5.4.16 (The largest cautious consistent set based on indirect weak dominance: $LCCS(G, \underline{\ll})$, M&V (2004).) *A set Z is called cautious consistent if $\mathcal{B} \in Z$ if and only for all $\mathcal{V}$ and S, such that $\mathcal{B} \rightharpoonup_S \mathcal{V}$, there is a vector $\vec{\alpha} = (\alpha(\mathcal{B}^1), ..., \alpha(\mathcal{B}^m))$ satisfying*

$\Sigma_{j=1}^{m}\alpha(\mathcal{B}^j) = 1$, $\alpha(\mathcal{B}^j) \in (0,1)$, *that gives only positive weight to each* $\mathcal{B}^j \in Z$, *where* $\mathcal{V} = \mathcal{B}^j$ *or* $\mathcal{V} \underline{\ll} \mathcal{B}^j$, *such that we do not have*

$$E\Pi_{M_i}(\mathcal{B}) \leq \sum_{\mathcal{B}^j \in Z,\ \mathcal{V}=\mathcal{B}^j \text{ or } \mathcal{V}\ll\mathcal{B}^j} \alpha(\mathcal{B}^j)\cdot E\Pi_{M_i}(\mathcal{B}^j) \quad \text{for all } M_i \in S \quad \text{and}$$

$$E\Pi_{M_i}(\mathcal{B}) < \sum_{\mathcal{B}^j \in Z,\ \mathcal{V}=\mathcal{B}^j \text{ or } \mathcal{V}\ll\mathcal{B}^j} \alpha(\mathcal{B}^j)\cdot E\Pi_{M_i}(\mathcal{B}^j) \quad \text{for some } M_i \in S.$$

The largest cautious consistent set is the union of all cautious consistent sets.

The following example, depicted in Figure 5.3 below and taken from M&V (2004), illustrates the difference between the LCS and the LCCS. Figure 5.3 presents a three-player coalition formation game where the feasible coalition structures are: $\{123\}$, $\{1,23\}$, $\{13,2\}$ and $\{1,2,3\}$. The payoff vectors associated with those partitions and the possible moves from each partition are given in Figure 5.3. The effectiveness relations are represented by labeled directed arcs. We have $\{123\} < \{1,23\}$, $\{1,23\} < \{1,2,3\}$, $\{1,23\} < \{13,2\}$ and $\{123\} \ll \{13,2\}$. It follows that $LCS(G,\ll) = LCS(G,\underline{\ll}) = \{\{123\}, \{1,2,3\}, \{13,2\}\}$. The independent structure belongs to both the LCS and the LCCS since no deviation from it is possible. The coalition structure $\{123\}$ belongs to $LCS(G,\ll)$ and $LCS(G,\underline{\ll})$ because the deviation to $\{1,23\}$ is deterred by the subsequent deviation by player 2 to $\{1,2,3\}$, wherein the original deviator, player 1, is equal off. But player 1 cannot end up being worse off by deviating from $\{123\}$, compared to what he gets in $\{123\}$. So, if player 1 thinks that he could end up with certain positive probability in any of the two coalition structures, $\{1,2,3\}$ or $\{13,2\}$, that indirectly dominate $\{123\}$, then he will for sure deviate from $\{123\}$ to $\{1,23\}$. Thus, by contrast with the LCS, $\{123\}$ is not contained in the LCCS.

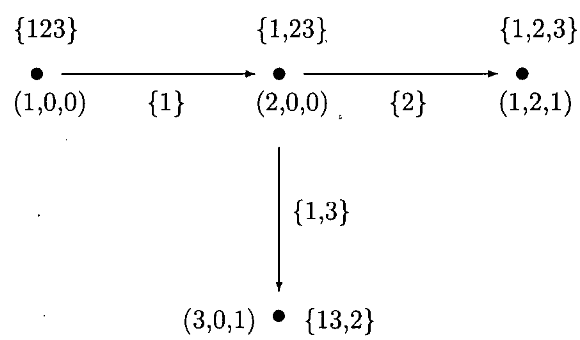

Figure 5.3: The largest consistent set is too conservative

We next characterize the LCS and the LCCS in the suppliers coalition formation game when demand has a power distribution. By suppliers coalition formation game we mean the game wherein the suppliers have the option of freely forming coalitions among themselves in advance of determining their decision variables.

Theorem 5.4.17 *In the suppliers coalition formation game, under a power demand distribution with* $f(x) = x^q$*, where* $x \in [0, (q+1)^{\frac{1}{q+1}}]$*,* $LCCS(G, \ll) = LCCS(G, \lesssim) = \{\mathcal{P}^*\}$.

Theorem 5.4.18 *In the suppliers coalition formation game, under a power demand distribution,* $f(x) = x^q$*, where* $x \in [0, (q+1)^{\frac{1}{q+1}}]$*:*

(i) $\mathcal{P}^* \in LCS(G, \ll)$ *and* $\mathcal{P}^* \in LCS(G, \lesssim)$.

(ii) For $n \leq 4$, $LCS(G, \ll) = LCS(G, \lesssim) = \{\mathcal{P}^*\}$.

(iii) For $n \geq 5$*, there exists some threshold value* q_n *such that* $LCS(G, \ll) = LCS(G, \lesssim) = \{\mathcal{P}^*\}$ *for any* $q \in (q_n, \infty)$.

From Theorems 5.4.17 and 5.4.18 we conclude that, by contrast with Nash-stability, which is based on myopic considerations, when coalitions are farsighted and able to account for future deviations, the grand coalition of all suppliers emerges as the most likely outcome in the suppliers game. Indeed, it is the unique element in the LCCS, and the unique element in the LCS for $n \leq 4$ and for $n \geq 5$ when the coefficient of variation is small enough, or, equivalently, when the exponent is large enough, for a general power demand distribution. This result is somewhat pleasing since, in view of Corollary 5.4.3, the channel, the assembler, all suppliers and the end-consumers are better off when the grand coalition is formed.

5.5 Pull System

In a pull system, R initiates the process by offering W_{B_j} to Coalition B_j (Stage 1), and then the coalitions choose the production quantities simultaneously, and bear the inventory risk (Stage 2). Cachon (2004a) has provided some examples for the pull system, which include the cases when suppliers' products are shipped to the assembler on consignment, or suppliers hold the inventory and replenish the assembler frequently and in small batches during the selling season (e.g., Vendor Managed Inventory with consignment). Saturn is a good example of the pull model. Indeed, the different metal sheet suppliers supply Saturn with complementary metal sheets and manage the inventories of their own products. Saturn pays, upon withdrawal, only for the quantities taken to their production line.

In §5.5.1 and §5.5.2, we analyze the pull model for a given coalition structure, and in §5.5.3, we evaluate the stability of coalition structures.

5.5.1 The pull model under a given coalition structure

Backward induction is again used to solve the two-stage Stackelberg game in the pull system between the suppliers and the assembler. We first consider the coalitions' production quantity problems in Stage 2 for any given set of wholesale prices $\{W_{B_1}, ..., W_{B_m}\}$. We assume that every coalition has a positive marginal manufacturing cost, i.e., $C_{B_j} > 0$ for all $j \in \{1, ..., m\}$.

Let us consider Coalition B_j's problem. Given W_{B_j} and vector Q_{-B_j}, B_j chooses Q_{B_j} to maximize:

$$E\Pi_{B_j} = W_{B_j} E_X(\min(\min(\vec{Q}), X)) - C_{B_j} Q_{B_j}, \tag{5.5}$$

where demand X is random. Consider the following two choices for Q_{B_j}:

Choice I: $Q_{B_j} \geq \min(Q_{-B_j})$. Thus, B_j's profit function, given by (5.5), becomes: $E\Pi_{B_j} = W_{B_j} E_X(\min(\min(Q_{-B_j}), X)) - C_{B_j} Q_{B_j}$, which is strictly decreasing in Q_{B_j}. Thus, $Q^*_{B_j} = \min(Q_{-B_j})$.

Choice II: $Q_{B_j} \leq \min(Q_{-B_j})$. Thus, B_j's profit function, given by (5.5), reduces to: $E\Pi_{B_j} = W_{B_j} E_X(\min(Q_{B_j}, X)) - C_{B_j} Q_{B_j}$, which coincides with the supplier's problem in a single-supplier single-assembler system. The unconstrained optimal production quantity in this case satisfies $F(Q_{B_j}) = 1 - \frac{C_{B_j}}{W_{B_j}}$. Therefore, the constrained optimal production quantity is:

$$Q^*_{B_j} = \min(\min(Q_{-B_j}), F^{-1}(1 - \frac{C_{B_j}}{W_{B_j}})). \tag{5.6}$$

By comparing these two choices, we conclude that Choice II is optimal for B_j since the optimal Q for Choice I is on edge of the feasible region of Choice II. Thus, B_j's optimal production quantity is given by (5.6).

Define $\frac{C_{B_e}}{W_{B_e}} \equiv \max_{j \in \{1,...,m\}}(\frac{C_{B_j}}{W_{B_j}})$. The equilibrium production quantities in the pull system are presented in the following lemma.

Lemma 5.5.1 *In the pull system, for any given coalition structure $\mathcal{B} = \{B_1, ..., B_m\}$, the equilibrium production quantities are:*

(i) $Q = F^{-1}(1 - \frac{C}{W})$ *if* $m = 1$, *and*

(ii) $Q^*_{B_1} = ... = Q^*_{B_m} \leq F^{-1}(1 - \frac{C_{B_e}}{W_{B_e}})$, *if* $m \geq 2$.

Lemma 5.5.1 implies that for any given wholesale price set, $\{W_{B_1}, ..., W_{B_m}\}$, each coalition will produce the same amount of each component. However, note also that the suppliers' production quantities have multiple Nash equilibria. We next use a refinement of Nash equilibrium, the strong Nash equilibrium, to reduce the number of the equilibrium production quantities.

Definition 5.5.2 (The Strong Nash Equilibrium, Aumann (1959).) *A strategy profile is a strong Nash equilibrium if there is no coalition of players (including the grand coalition) that can profitably deviate from the prescribed profile.*

By definition, a strong Nash equilibrium is Pareto efficient, but in many cases, the strong Nash equilibrium set is empty. Fortunately, in our game for the suppliers' production quantities, there is a unique strong Nash equilibrium.

Lemma 5.5.3 *In the pull system, under coalition structure $\mathcal{B} = \{B_1, ..., B_m\}$, for any given wholesale price set, $\{W_{B_1}, ..., W_{B_m}\}$, $0 \leq Q^*_{B_1} = ... = Q^*_{B_m} = F^{-1}(1 - \frac{C_{B_e}}{W_{B_e}})$ is the unique strong Nash equilibrium.*

In view of Lemma 5.5.3, we assume in the sequel that the coalitions in $\mathcal{B}$ provide the assembler with the strong Nash equilibrium production quantities, i.e., $Q^*_{B_1} = ... = Q^*_{B_m} = F^{-1}(1 - \frac{C_{B_e}}{W_{B_e}})$.

Knowing the equilibrium production quantities, the assembler chooses the wholesale price set to maximize his expected profit. His problem is the following constrained optimization problem:

$$\text{Maximize } E\Pi_R = (p - W)E_X(\min(Q, X)) \tag{5.7}$$

$$\text{s.t. } Q = F^{-1}(1 - \frac{C_{B_e}}{W_{B_e}}) \tag{5.8}$$

$$\frac{C_{B_e}}{W_{B_e}} = \max_{j\in\{1,2,\ldots,m\}} (\frac{C_{B_j}}{W_{B_j}}), \tag{5.9}$$

where Q is the production quantity of every coalition.

The equilibrium wholesale price for each coalition is presented in the following result.

Theorem 5.5.4 *In the pull system, for a given coalition structure $\mathcal{B} = \{B_1, B_2, ..., B_m\}$, the equilibrium values of the wholesale prices and production quantities satisfy:*

(i) $\frac{C_{B_1}}{W^*_{B_1}} = \frac{C_{B_2}}{W^*_{B_2}} = ... = \frac{C_{B_m}}{W^*_{B_m}} = 1 - F(Q^*)$, *and*

(ii) $Q^*_{B_1} = ... = Q^*_{B_m} = Q^*$, *where* $\frac{1-F(Q^*)}{1+J(Q^*)H(Q^*)} = \frac{C}{p}$, $J(Q) = \frac{S(Q)}{1-F(Q)}$ *and* $H(Q) = \frac{f(Q)}{1-F(Q)}$.

Theorem 5.5.4 (ii) implies that the equilibrium production quantity, Q^*, in the pull system depends on the ratio $\frac{C}{p}$ only, instead of on the individual suppliers' marginal costs and the selling price p.

By Lemma 1 in Cachon (2004a), $J(Q)H(Q)$ is increasing in Q for $Q > 0$. Accordingly, we have:

Corollary 5.5.5 *In equilibrium,*

(i) $W^*_{B_j} = \frac{C_{B_j}}{C}\Sigma_{i=1}^{m}(W^*_{B_i})$, $\Sigma_{i=1}^{m}(W^*_{B_i}) = W^* = \frac{C}{1-F(Q^*)}$, *and*

(ii) $E\Pi^*_{B_j} = \frac{C_{B_j}}{C}E\Pi_M^{Total*}(m = 1)$,

where $E\Pi_M^{Total*}(m = 1)$ *is the total equilibrium expected profit of all suppliers when the grand coalition was formed.*

Theorem 5.5.4 and Corollary 5.5.5 imply that in the pull system with multiple suppliers, if all suppliers provide the assembler with the strong Nash equilibrium production quantity, $Q^* = F^{-1}(1 - \frac{C_{B_e}}{W_{B_e}})$, then in equilibrium, the total wholesale price and the production quantity coincide with those in the case when all suppliers form the grand coalition. Thus, the total profit for all suppliers, $E\Pi_M^{Total}$, and the assembler's profit are independent of the coalition structure. A coalition's expected profit depends on the total manufacturing cost, which is also independent of the coalition structure. Therefore, the equilibrium values of the contract parameters and expected profits of each supplier and the assembler are all independent of the coalition structure under a pull system. By contrast, recall that in the push model, the total profit for all suppliers, the assembler's profit and any coalition's profit depend on the number of coalitions in the coalition structure.

Corollary 5.5.5 (ii) reveals that the share of each coalition of suppliers in the total suppliers profit is proportional to their coalitional cost, and thus, by contrast with the push model, coalitions do not realize equal profits in equilibrium. The intuition for this qualitative difference is as follows. Under pull, coalition B_j's expected profit is $E\Pi_{B_j} = (W_{B_j} - C_{B_j})S(Q) - C_{B_j}(Q - S(Q))$, and accordingly, B_j bears a demand uncertainty risk which is proportional to its manufacturing cost C_{B_j}. Since the production quantities are identical for all coalitions, coalitions share the total expected profit proportionally to their manufacturing costs.

As previously mentioned in Section 5.5.1, Corollary 5.5.5 has been independently derived in Wang and Gerchak (2003) and Gerchak and Wang (2004).

By Corollary 5.5.5, $W^*_{B_j} = \frac{C_{B_j}}{1-F(Q^*)}$. Substituting $W^*_{B_j}$ into B_j's profit function, given by (5.5), and simplifying results with

$$E\Pi^*_{B_j} = C_{B_j}(\frac{S(Q^*)}{1 - F(Q^*)} - Q^*). \tag{5.10}$$

5.5.2 Cost efficiency of the suppliers

We consider in this subsection the effect of the suppliers' marginal costs on the equilibrium profits of channel members and the equilibrium values of the decision variables.

Proposition 5.5.6 *In the pull system, for any coalition structure $\mathcal{B} = \{B_1, ..., B_m\}$, in equilibrium, coalitions with higher manufacturing costs earn higher profits.*

Proposition 5.5.7 *In the pull system, for any coalition structure $\mathcal{B} = \{B_1, ..., B_m\}$, in equilibrium, for any $j \in \{1, ..., m\}$:*

(i) $\frac{\partial Q^*}{\partial C_{B_j}} < 0$.

(ii) $\frac{\partial W^*}{\partial C_{B_j}} > 0$, $\frac{\partial W^*_{B_j}}{\partial C_{B_j}} > 0$ *and* $\frac{\partial W^*_{B_i}}{\partial C_{B_j}} < 0$ *for any* $i \neq j$.

(iii) $\frac{\partial E\Pi^*_{B_i}}{\partial C_{B_j}} < 0$ *for any* $i \neq j$, *and*

(iv) $\frac{\partial E\Pi^*_R}{\partial C_{B_j}} < 0$.

While some of the results in Proposition 5.5.7 and Proposition 5.4.4 in the push model are similar, the main difference between them, however, is that it is not clear how $E\Pi_M^{Total}$ or $E\Pi^*_{B_j}$ would change with respect to C_{B_j}. Thus, it is difficult to predict whether any supplier has an incentive to reduce its manufacturing cost.

Let us next consider an extreme case about cost efficiency. Suppose there is some coalition, B_k, which is extremely cost efficient, i.e., $C_{B_k} = 0$. What would happen to the channel? Evidently, for any given partition, the suppliers' (i.e., the coalitions') problems are not affected. Thus, the results in Lemma 5.5.1 are still valid, similarly for Lemma 5.5.3. However, the analysis in the assembler's problem has to be modified. Indeed, the equilibrium condition that all coalitions have the same ratio of cost to wholesale price cannot be satisfied since $C_{B_k} = 0$. However, according to Constraints (5.8) and (5.9) in the assembler's problem, it is evident that since $C_{B_k} = 0$, Q in the objective function, given by (5.7), is independent of W_{B_k}. Thus, the assembler would definitely choose $W^*_{B_k} = 0$ to maximize his profit. For those coalitions which have a positive manufacturing cost, the assembler will follow the same analysis process as in Subsection §5.5.1. We conclude that for coalitions with positive manufacturing cost the results presented in Theorem 5.5.4 hold, and that zero-manufacturing cost coalitions would attain a zero profit.

5.5.3 Stability of coalition structures

Note that by Corollary 5.5.5, under any coalition structure, in equilibrium, coalitions share the total expected profit of all suppliers proportionally to their coalition manufacturing cost. We suggest in this subsection that members in the same coalition should split the coalition profit proportionally to their own individual manufacturing costs. For example, the profit of M_i in Coalition B_j should be $E\Pi^*_{M_i} = \frac{c_{M_i}}{C_{B_j}} E\Pi^*_{B_j}$, given that $C_{B_j} > 0$, which is equivalent to having members in the same coalition sharing the total wholesale price of their coalition proportionally to their manufacturing cost, i.e., $w^*_{M_i} = \frac{c_{M_i}}{C_{B_j}} W^*_{B_j}$. This suggestion is consistent with the fact that coalitions share the total profit proportionally to their manufacturing cost, and zero-manufacturing cost coalitions realize zero profit in equilibrium. In fact, we have the following result.

Proposition 5.5.8 *In the pull system under the coalitional proportional splitting rule for all coalitions, if the same proportional allocation rule is used to allocate the profit among members of the same coalition, i.e.,* $E\Pi^*_{M_i} = \frac{c_{M_i}}{C_{B_j}} E\Pi^*_{B_j}$, *or equivalently,* $w^*_{M_i} = \frac{c_{M_i}}{C_{B_j}} W^*_{B_j}$, *for any* $M_i \in B_j$ *and any* B_j, *then any coalition structure is stable.*

Finally, observe that for any coalition structure $\mathcal{B}$, if the coalitional proportional splitting rule is not used for members in the same coalition, then $\mathcal{B}$ is never Nash-stable, neither is it contained in the LCS or the LCCS.

5.6 Push and Pull Systems

Having analyzed the push and pull systems, we would like to find the preferences of the assembler and the suppliers between the two systems. Now, we recall that the results in the pull system are independent of the coalition structure. Further, in the push system and a power distribution of demand, x^q, the grand coalition is the unique structure in the LCCS, and for q large enough, it is also the unique structure in the LCS. Thus, for simplicity, we will compare the two systems under the assumption that the grand coalition was formed. We note that under this assumption, and when all suppliers are viewed as a single supplier, our model coincides with that studied in Cachon (2004a), and the reader is referred to his paper for an excellent analysis and comparison of the two models in this case.

Denote the equilibrium values in the push system with a subscript *"push"* and those in the pull system with *"pull"*. Recall that in the push system, under the grand coalition assumption, the expected profit of the assembler, in terms of Q, given by (5.3), is: $E\Pi_{R,push} = p\int_0^Q xf(x)dx$, which is strictly increasing in Q for $Q > 0$, and the total expected profit of all suppliers, in terms of Q, is: $E\Pi^{Total}_{M,push} = (p(1-F(Q))-C)Q$, which is unimodal in Q and has a unique maximizer. Similarly, in the pull system, the expected profit of the assembler, in terms of Q, is: $E\Pi_{R,pull} = (p-\frac{C}{1-F(Q)})S(Q)$, which is strictly concave in Q, and the expected profit of all suppliers, in terms of Q, given by (5.10), is: $E\Pi^{Total}_{M,pull} = C(\frac{S(Q)}{1-F(Q)} - Q)$, which is strictly increasing in Q.

Let us first recall two important results from Cachon (2004a).

Theorem 5.6.1 (Cachon (2004a), Theorem 3) *In equilibrium,*

(i) $E\Pi^{*}_{R,pull}(Q^{*}_{pull}) > E\Pi^{Total*}_{M,push}(Q^{*}_{push})$, *and*

(ii) $Q^{*}_{pull} > Q^{*}_{push}$.

Lemma 5.6.2 (Cachon (2004a), Lemma 4) *The following hold:*

(i) There exists a unique Q' *such that* $E\Pi_{R,push}(Q') = E\Pi_{R,pull}(Q')$.

(ii) There exists a unique Q'' *such that* $E\Pi^{Total}_{M,push}(Q'') = E\Pi^{Total}_{M,pull}(Q'')$.

(iii) Q^P *is the unique maximizer of* $E\Pi_{R,pull}(Q) - E\Pi^{Total}_{M,push}(Q)$.

(iv) $Q^P = Q' = Q''$, *and*

(v) $Q^P > Q^{*}_{pull}$.

Theorem 5.6.1 and Lemma 5.6.2 (v) imply that $Q^{*}_{push} < Q^{*}_{pull} < Q^P$. Based on these results, we are able to derive the following Proposition, which is used for subsequent results.

Proposition 5.6.3 *In equilibrium,*

(i) $E\Pi_{R,pull}(Q^{*}_{pull}) > E\Pi_{R,push}(Q^{*}_{push})$, *and*

(ii) $E\Pi^{Total}_{M,push}(Q^{*}_{push}) > E\Pi^{Total}_{M,pull}(Q^{*}_{pull})$.

It follows from Proposition 5.6.3 that the assembler prefers the pull system to the push system and that the total profit of all suppliers with push is greater than their total profit with pull. The preference of any individual supplier between the two systems is revealed in the next proposition.

Proposition 5.6.4 *An individual supplier* M_i *with marginal cost* c_{M_i}, *for* $i \in \{1, ..., n\}$,

(i) prefers push to pull if $c_{M_i} < \alpha \cdot \bar{c}$,

(ii) is indifferent between push and pull if $c_{M_i} = \alpha \cdot \bar{c}$, *and*

(iii) prefers pull to push if $c_{M_i} > \alpha \cdot \bar{c}$,

where $\alpha \equiv \frac{E\Pi^{Total}_{M,push}(Q^*_{push})}{E\Pi^{Total}_{M,pull}(Q^*_{pull})} (> 1)$ *and* $\bar{c} = \frac{C}{n}$.

Basically, Proposition 5.6.4 asserts that suppliers with low manufacturing cost prefer push to pull since under pull, they are apparently not compensated enough for the risk they need to cover, which stems from the uncertain demand. On the other hand, suppliers with high manufacturing cost prefer pull to push since they are compensated proportionally to their cost.

Let us consider an extreme case when all suppliers have the same marginal cost, i.e., $c_{M_i} = c$ for all $i \in N$. Thus, $\bar{c} = c$, and by Proposition 5.6.4, since $\alpha > 1$, $c_{M_i} = c < \alpha \cdot \bar{c}$. Therefore, in this case all suppliers prefer push to pull.

Let us next consider an example wherein demand, again, follows a power distribution.

Example 5.6.5 Power demand distribution.

Recall that under a power demand distribution, $f(x) = x^q$ and $F(x) = \frac{x^{q+1}}{q+1}$ for $q \geq 0$. By Theorem 5.4.2 (ii), we have that $Q^*_{push} = (\frac{1-\frac{C}{p}}{\frac{1}{q+1}+1})^{\frac{1}{q+1}}$, and

$$E\Pi^{Total}_{M,push}(Q^*_{push}) = p(Q^*_{push})^2 f(Q^*_{push}) = p(Q^*_{push})^{q+2}.$$

From Theorem 5.5.4, Q^*_{pull} satisfies $\frac{1-F(Q^*_{pull})}{1+J(Q^*_{pull})H(Q^*_{pull})} = \frac{C}{p}$, where $J(Q) = \frac{S(Q)}{1-F(Q)}$ and $H(Q) = \frac{f(Q)}{1-F(Q)}$. Furthermore,

$$E\Pi^{Total}_{M,pull}(Q^*_{pull}) = C(\frac{S(Q^*_{pull})}{1-F(Q^*_{pull})} - Q^*_{pull}).$$

Note that the value of Q^*_{pull} depends on the ratio of C to p, instead of the individual suppliers' marginal costs and the selling price. Since it is difficult to derive an explicit expression of Q^*_{pull} as a function of $\frac{C}{p}$ and a general q, we present in Table 5.1 in the following page the values of Q^*_{pull} and α as a function of $\frac{C}{p}$ for $q = 0, 1, 2, 4$.

	$q=0$			$q=1$			$q=2$			$q=4$		
$\frac{C}{p}$	Q^*_{push}	Q^*_{pull}	α	Q^*_{push}	Q^*_{pull}	α	Q^*_{push}	Q^*_{pull}	α	Q^*_{push}	Q^*_{pull}	α
0.01	0.4950	0.8273	12.36	0.8124	1.2419	19.22	0.9055	1.3013	24.89	0.9623	1.2791	34.29
0.05	0.4750	0.6690	5.56	0.7958	1.1105	8.46	0.8932	1.1924	10.95	0.9544	1.2000	15.25
0.10	0.4500	0.6142	4.14	0.7746	1.0219	6.24	0.8772	1.1184	8.08	0.9441	1.1460	11.34
0.20	0.4000	0.5000	3.20	0.7303	0.8997	4.78	0.8434	1.0154	6.20	0.9221	1.0705	8.81
0.50	0.2500	0.2747	2.40	0.5774	0.6367	3.57	0.7211	0.7888	4.67	0.8394	0.9030	6.81
0.90	0.0500	0.0507	2.05	0.2582	0.2622	3.07	0.4217	0.4279	4.08	0.6084	0.6157	6.09
0.99	0.0050	0.0050	2.01	0.0816	0.0818	3.01	0.1957	0.1960	4.01	0.3839	0.3843	6.01

Table 5.1: Q^*_{push}, Q^*_{pull} and the preference factor, α, under a power demand distribution

Based on Table 5.1, we conclude with the following observation.

Observation 5.6.6 *(i) The equilibrium values of the production quantity under push and pull are decreasing in C, which is consistent with Propositions 5.4.4 and 5.5.7, and they are increasing in the exponent q of the distribution function.*
(ii) $Q^*_{push} < Q^*_{pull}$, which is consistent with Theorem 5.6.1 (i.e., Theorem 3 in Cachon (2004a)).
(iii) The value of α is decreasing in $\frac{C}{p}$ and increasing in q.

Recall that some suppliers prefer the pull system, since they are well compensated for the risk they bear in this system due to demand uncertainty. Now, an increase in q can be easily shown to result with a decrease in the coefficient of variation in a power distribution. Apparently, as reflected by Observation 5.6.6 (iii), a decrease in the coefficient of variation results with a decrease in the level of compensation to suppliers in the pull system to an extent that more of them prefer the push to the pull system.

Similarly, recall that the share of the profit of suppliers in the pull system is larger if their manufacturing cost is higher. As suggested by Observation 5.6.6 (iii), it appears that a decrease in the total manufacturing cost decreases the profit of suppliers in the pull system so that more of them prefer the push system wherein their profit is independent of the manufacturing costs.

5.7 Conclusions and Further Research

We have studied in this chapter a supply chain model consisting of a single assembler who buys complementary components or products from n suppliers under two contracting systems: push and pull. In both systems, we have investigated the stability of coalition structures in the suppliers coalition formation game, and the Stackelberg game between the assembler and the suppliers. We have demonstrated that push and pull contracts, which allocate differently inventory risk among players, induce, in equilibrium, qualitatively different outcomes.

For example, for a given coalition structure, push and pull induce different profit allocation among coalitions. In a push system, coalitions realize equal profits in equilibrium. By contrast, in a pull system, coalitions share the total suppliers profit proportionally to their own manufacturing costs. In a push system, the assembler, the suppliers and the end-consumers would all be better off if the grand coalition of all suppliers is formed, while in a pull system, the coalition structure has no impact on the equilibrium profits of players. Finally, in a push contract, each supplier has an incentive to improve her cost efficiency, while in a pull contract, suppliers may not benefit from a reduction in their cost.

We have also derived in this chapter some sharp predictions about alliance formation among suppliers in a push system. For example, it is shown that in a system with two suppliers, the grand coalition is the unique element in the Nash-stable set, the largest consistent set (LCS) and the largest cautious consistent set (LCCS). Further, in a system with more than two suppliers, wherein demand follows a power distribution, the grand coalition is the unique element in the LCCS. It is also the unique element in the LCS for $n \leq 4$, and for $n \geq 5$ when the exponent is large enough, i.e., the coefficient of variation of demand is small enough. By contrast, for a power demand distribution, the Nash-stable set, in which, of course, players are known to be myopic, is shown to consist of the independent structure. In reality, we often observe alliances of suppliers, e.g., alliances among the outsourcing vendors in the IT industry (Gallivan and Oh 1999), and it is pleasing that farsighted concepts, such as the LCS and LCCS, predict alliance formation.

We have found out that the assembler always prefers the pull system to the push system. However, the suppliers' preferences between these two systems depend on their own manufacturing costs. More specifically, suppliers with relatively lower manufacturing costs prefer push to pull since under pull, they are apparently not compensated enough for the risk they bear due to uncertain demand. On the other hand, suppliers with relatively higher manufacturing costs prefer pull to push since they are compensated proportionally to their cost. It is interesting to note that if all suppliers have the same manufacturing cost, then all suppliers prefer push to pull.

Finally, regarding possible extensions, we recall that it was assumed in this chapter that all products are completely complementary. However, as pointed out by Wang (2004), the degree of complementarity in some business models may not be 100%. Additionally, components may have their own individual demand, in addition to demand for the final product. Thus, it would be important to extend our analysis to these more general situations.

5.8 Appendix

Proof of Lemma 5.4.1. Suppose the assembler chooses an inventory vector $\vec{Q}$. Let $Q_{B_k} = \min(\vec{Q})$. Then, the assembler's profit function becomes: $E\Pi_R = pE_X(\min(Q_{B_k}, X)) - \Sigma_{j=1}^m (W_{B_j} Q_{B_j})$, which is strictly decreasing in each element of Q_{-B_k}, which is the vector whose components are the production quantities of all coalitions in $\mathcal{B}$ but B_k. Thus, at optimality, the assembler will order the same inventory from all coalitions. Therefore, $Q_{B_1} = ... = Q_{B_m} = Q$, and the assembler's profit function reduces to: $E\Pi_R = pE_X(\min(Q, X)) - Q\Sigma_{j=1}^m (W_{B_j}) = p(Q\bar{F}(Q) + \int_0^Q xf(x)dx) - QW$, which has derivatives $\frac{dE\Pi_R}{dQ} = p\bar{F}(Q) - W$ and $\frac{d^2E\Pi_R}{d^2Q^2} = -pf(Q)(< 0)$. Thus, $E\Pi_R$ is strictly concave in Q and the global maximizer satisfies: $\frac{dE\Pi_R}{dQ} = 0$, i.e., $p\bar{F}(Q) = W$. □

Proof of Theorem 5.4.2. (i) Let us consider the problem facing coalition B_j. The profit function for B_j is: $E\Pi_{B_j} = (W_{B_j} - C_{B_j})Q^*$, where by Lemma 5.4.1 (ii), $p\bar{F}(Q^*) = W$, and Q^* is the assembler's optimal reaction function of Q. Accordingly, $\frac{\partial E\Pi_{B_j}}{\partial W_{B_j}} = Q^* + (W_{B_j} - C_{B_j})\frac{\partial Q^*}{\partial W_{B_j}}$. Taking derivative with respect to W_{B_j} on both sides of $p\bar{F}(Q^*) = W$ results with $\frac{\partial Q^*}{\partial W_{B_j}} = -\frac{1}{pf(Q^*)}$. Thus, $\frac{\partial E\Pi_{B_j}}{\partial W_{B_j}} = Q^* - \frac{W_{B_j} - C_{B_j}}{pf(Q^*)}$. First order condition for optimality results with: $W_{B_j}^* - C_{B_j} = pQ^*f(Q^*)$ for any $j \in \{1, 2, ..., m\}$. Now, similar to Lariviere and Porteus (2001) analysis in the context of the single-supplier single-assembler newsvendor model, in our model, the profit function for Coalition B_j,

$$E\Pi_{B_j} = (p\bar{F}(Q) - (C_{B_j} + W_{-B_j}))Q, \tag{5A.1}$$

is unimodal in Q and has a unique maximizer, and, in view of the one-to-one relationship between Q and W_{B_j}, it is also unimodal in W_{B_j}. Thus, in the maximization problem of $E\Pi_{B_j} = (W_{B_j} - C_{B_j})Q^*$, where $p(1 - F(Q^*)) = W = W_{B_j} + \Sigma_{k \neq j} W_{B_k}$, with respect to W_{B_j}, the first order condition, $\frac{\partial E\Pi_{B_j}}{\partial W_{B_j}} = 0$, is both necessary and sufficient to find the unique maximizer $W_{B_j}^*$. Therefore, we have $W_{B_j}^* - C_{B_j} \equiv pQ^*f(Q^*), j \in \{1, ..., m\}$ and (i) follows. Thus, at optimality, $p\bar{F}(Q^*) = W^* = mpQ^*f(Q^*) + C$, and (ii) follows. □

Proof of Corollary 5.4.3. Let us first prove (iv). By Theorem 5.4.2, $p(\bar{F}(Q^*) - mQ^*f(Q^*)) = C$. Thus, $1 - m\frac{Q^*f(Q^*)}{1-F(Q^*)} = \frac{C}{p}(\bar{F}(Q^*))^{-1}$, and the equilibrium value of Q^*, where $Q^* > 0$ (since $p > C$), is an implicit function of the number of coalitions m. Taking the derivative of Q^* with respect to m on both sides of $1 - m\frac{Q^*f(Q^*)}{1-F(Q^*)} = \frac{C}{p}(\bar{F}(Q^*))^{-1}$, and simplifying results with $-\frac{Q^*f(Q^*)}{1-F(Q^*)} = [m\frac{\partial(\frac{Q^*f(Q^*)}{1-F(Q^*)})}{\partial Q^*} + \frac{C}{p}(\bar{F}(Q^*))^{-2}f(Q^*)]\frac{\partial Q^*}{\partial m}$. Since the demand function is assumed to have an increasing general failure rate, $\frac{\partial(\frac{Q^*f(Q^*)}{1-F(Q^*)})}{\partial Q^*} \geq 0$. Thus, since $Q^* > 0$, it follows that $\frac{\partial Q^*}{\partial m} < 0$.

(v) Since $W^* = p\bar{F}(Q^*)$, and Q^* strictly decreases in m, it is easy to verify that W^* strictly increases in m.

(ii) By Theorem 5.4.2, $E\Pi_M^{Total} = (p\bar{F}(Q^*)-C)Q^*$. Taking derivatives on both sides with respect to m and simplifying results with $\frac{\partial E\Pi_M^{Total}}{\partial m} = (p\bar{F}(Q^*)-C-pf(Q^*)Q^*)\frac{\partial Q^*}{\partial m} = (m-1)pQ^*f(Q^*)\frac{\partial Q^*}{\partial m}$, where the last equality holds since $p\bar{F}(Q^*) - C = mpQ^*f(Q^*)$. Since $m \geq 1$ and $\frac{\partial Q^*}{\partial m} < 0$, $\frac{\partial E\Pi_M^{Total}}{\partial m} \leq 0$, and equality holds iff $m = 1$.

(i) Since $E\Pi_M^{Total}$ decreases in m, we immediately have that, for any $j \in \{1, ..., m\}$, $E\Pi_{B_j}^* = \frac{E\Pi_M^{Total}}{m}$ decreases in m.

(iii) Note that $E\Pi_R^* = p\int_0^{Q^*} xf(x)dx$. Then, $\frac{\partial E\Pi_R^*}{\partial m} = Q^*f(Q^*)\frac{\partial Q^*}{\partial m}$. Since $Q^*f(Q^*) > 0$ and $\frac{\partial Q^*}{\partial m} < 0$, we have that $\frac{\partial E\Pi_R^*}{\partial m} < 0$, which completes the proof of Corollary 5.4.3. □

Proof of Proposition 5.4.4. (i) By Theorem 5.4.2 (ii), $p(\bar{F}(Q^*) - mQ^*f(Q^*)) = C$. Thus, $\bar{F}(Q^*)(1 - m \cdot g(Q^*)) = \frac{C}{p}$, where, as we recall $g(Q) = \frac{Qf(Q)}{1-F(Q)}$. Due to the IGFR assumption, and since $F(x)$ is increasing in x, $\bar{F}(Q^*)(1 - mg(Q^*))$ decreases in Q^*. Thus, since C increases in C_{B_j}, $\frac{\partial(Q^*)}{\partial(C_B)_j)} < 0$.

(ii) From Lemma 5.4.1 (ii), $W^* = p\bar{F}(Q^*)$. Thus, since Q^* decreases in C_{B_j} and $\bar{F}(Q^*)$ is a decreasing function of Q^*, W^* increases in C_{B_j}. By Theorem 5.4.2 (i), $W_{B_j}^* = C_{B_j} + \frac{p\bar{F}(Q^*)-C}{m}$. Thus, $\frac{\partial(W_{B_j}^*)}{\partial(C_{B_j})} = \frac{m-1}{m} - \frac{1}{m}pf(Q^*)\frac{\partial Q^*}{\partial(C_{B_j})} > 0$, where the last inequality follows since $m \geq 1$ and $\frac{\partial Q^*}{\partial(C_{B_j})} < 0$.

(iii) $E\Pi_{B_j}^* = (W_{B_j}^* - C_{B_j})Q^* = \frac{p\bar{F}(Q^*)-C}{m}Q^*$. Taking derivatives with respect to C_{B_j} on both sides of $E\Pi_{B_j}^*$ results with $\frac{\partial(E\Pi_{B_j}^*)}{\partial(C_{B_j})} = \frac{1}{m}\frac{\partial(Q^*)}{\partial(C_{B_j})}[p\bar{F}(Q^*) - C - pQ^*f(Q^*)] < 0$, where the last inequality follows since $\frac{\partial(Q^*)}{\partial(C_{B_j})} < 0$ and $p\bar{F}(Q^*) - C - pQ^*f(Q^*) \geq p\bar{F}(Q^*) - C - mpQ^*f(Q^*) = 0$. Since $E\Pi_{B_j}^* = (W_{B_j}^* - C_{B_j})Q^* = \frac{p\bar{F}(Q^*)-C}{m}Q^*, j \in \{1, ..., m\}$, $\frac{\partial(E\Pi_{B_i}^*)}{\partial(C_{B_j})} = \frac{\partial(E\Pi_{B_j}^*)}{\partial(C_{B_j})} < 0$ for any $i \neq j$.

(iv) $E\Pi_R^* = p(S(Q^*) - \bar{F}(Q^*)Q^*)$. Since $S(Q) - \bar{F}(Q)Q$ increases in Q and Q^* decreases in C_{B_j}, $E\Pi_R^*$ decreases in C_{B_j}, which proves (iv) and completes the proof of Proposition 5.4.4. □

Proof of Proposition 5.4.6. (i) When $m < n$ and $|B_1| = 1$, we consider two types of suppliers: suppliers in a coalition which has more than one member (Type I) and independent suppliers (Type II).

Now, recall our assumption that members of a coalition share the coalition's profit equally. Then, by Theorem 5.4.2 (i), a deviation by a Type I supplier to join another coalition, B_k, is not feasible, since members of B_k will be strictly worse off from such a deviation. Thus, the only feasible deviation for a Type I supplier is to be independent. Consider any member, M_i, in B_m, which, by Theorem 5.4.2 (i) and the above assumption of equal profits for coalition members, earns the least profit. Before a deviation by M_i to be independent, M_i's equilibrium profit is: $E\Pi^*_{M_i}(before) = \frac{(p\bar{F}(Q^*_m)-C)Q^*_m}{|B_m|m}$, where Q^*_m, by Theorem 5.4.2 (ii), satisfies: $p(\bar{F}(Q) - mQf(Q)) = C$. After M_i deviates to be independent, there are $m+1$ coalitions instead of m. The equilibrium profit for M_i now becomes: $E\Pi^*_{M_i}(after) = \frac{(p\bar{F}(Q^*_{m+1})-C)Q^*_{m+1}}{m+1}$, where Q^*_{m+1} satisfies: $p(\bar{F}(Q) - (m+1)Qf(Q)) = C$. Thus, M_i cannot be strictly better off by deviating to be independent if and only if $E\Pi^*_{M_i}(before) \geq E\Pi^*_{M_i}(after)$, which results with $\frac{(p\bar{F}(Q^*_m)-C)Q^*_m}{m} \geq \frac{|B_m|(p\bar{F}(Q^*_{m+1})-C)Q^*_{m+1}}{m+1}$.

For Type II suppliers, since $|B_1| = 1$, it is sufficient to find conditions such that either the only member in B_1, M_i, has no incentive to join B_2, or B_2 will not accept M_i. Before deviation, $E\Pi^*_{M_i}(before) = E\Pi^*_{B_2}(before) = \frac{(p\bar{F}(Q^*_m)-C)Q^*_m}{m}$. After deviation, $E\Pi^*_{M_i}(after) = \frac{(p\bar{F}(Q^*_{m-1})-C)Q^*_{m-1}}{(|B_2|+1)(m-1)}$, and the equilibrium profit of Coalition B_2 in the new coalition structure is: $E\Pi^*_{B_2}(after) = \frac{|B_2|(p\bar{F}(Q^*_{m-1})-C)Q^*_{m-1}}{(|B_2|+1)(m-1)}$. Accordingly, $M_i(\in B_1)$ has no incentive to deviate or B_2 will not accept M_i if and only if $\frac{(p\bar{F}(Q^*_m)-C)Q^*_m}{m} \geq \frac{(p\bar{F}(Q^*_{m-1})-C)Q^*_{m-1}}{(|B_2|+1)(m-1)}$. Combining the conditions for the Type I and Type II suppliers yields (i).

(ii) When $m < n$ and $|B_1| \geq 2$, there are only Type I suppliers, and the same proof approach as that used in (i) can be employed to verify (ii).

(iii) When $m = n$, i.e., the independent coalition structure was formed, we only have Type II suppliers and $|B_1| = ... = |B_m| = 1$. Using the same analysis as in (i) would verify (iii). □

Proof of Theorem 5.4.17. For $n = 2$, by Theorem 5.4.10 (i), neither of the two suppliers has an incentive to deviate from the grand coalition. Thus, $LCCS(G, \ll) = LCCS(G, \lll) = \{\mathcal{P}^*\}$. For $n \geq 3$, Proposition 5 in M&V (2004) asserts that under four conditions, the largest cautious consistent set, based on both indirect strict and weak dominance, singles out the grand coalition. We next demonstrate that these conditions hold in our case.

(P.1) *Positive spillovers.* Positive spillovers require that the formation of a coalition by other players increases the payoff (i.e., the expected profit) of a player. Now, from Corollary 5.4.3 (i), $\frac{\partial E\Pi_{B_j}}{m} < 0$ for all $j \in \{1, ..., m\}$ for $m \geq 2$. Thus, clearly, the suppliers coalition formation game has positive spillovers.

(P.2) *Negative association.* Negative association requires that small coalitions have higher per-member expected profit than big coalitions. Theorem 5.4.2 (i) reveals that each coalition would have the same profit margin, thus the same expected profit. Since we assume that members in the same coalition share their profit equally, it is obvious that smaller coalitions have higher per-member payoff than larger coalitions. Therefore, Condition (P.2) is satisfied.

(P.3) *Individual free-riding.* Individual free-riding requires that if a player leaves any coalition to be alone, then s/he is better off. Theorem 5.4.10 (ii) implies Condition (P.3).

(P.4) *Efficiency.* Efficiency implies that the grand coalition is the only efficient coalition structure with respect to the expected profit. Corollary 5.4.3 implies Condition (P.4).

Thus, we have verified that all four required conditions of M&V are satisfied, and the proof of Theorem 5.4.17 follows. □

Proof of Theorem 5.4.18. Recall that a coalition structure $\mathcal{B}$ is symmetric if and only if $|B_h| = |B_k|$ for all $B_h, B_k \in \mathcal{B}$. We denote by $\mathcal{P}^* = \{N\}$ the grand coalition and by $\bar{\mathcal{P}}$ the stand-alone (i.e., independent) coalition structure: $\bar{\mathcal{P}} = \{B_1, ..., B_n\}$ with $|B_j| = 1$ for all $B_j \in \bar{\mathcal{P}}$ ($\mathcal{P}^*$ and $\bar{\mathcal{P}}$ are symmetric coalition structures).

(i) By Lemma 3 in M&V (2004), under Conditions (P.1) – (P.4), $\mathcal{P}^* \in LCS(G, \ll)$ and $\mathcal{P}^* \in LCS(G, \lll)$. Since (P.1) – (P.4) were shown to be satisfied in the proof of Theorem 5.4.17 under a power demand distribution, (i) follows.

(ii) By Proposition 4 in M&V (2004), a sufficient condition for the LCS to consist only of the grand coalition is that Conditions (P.1)-(P.4), as well as Condition (A) hold, where Condition (A) requires: *Each non-symmetric coalition structure contains at least one coalition whose members receive less than in the stand-alone coalition structure.* Since Conditions (P.1)-(P.4) are satisfied for a power demand distribution, we only need to verify that Condition (A) is valid.

Let us first consider the suppliers' expected profits in the stand-alone structure where $m = n$. By Theorem 5.4.2 (ii), $Q_n^* = (\frac{\frac{p-C}{p}}{\frac{1}{q+1}+n})^{\frac{1}{q+1}}$, and each supplier's expected profit is: $E\Pi_M^*(\bar{\mathcal{P}}) = p(Q_n^*)^2 f(Q_n^*) = p(Q_n^*)^{q+2} = p(\frac{\frac{p-C}{p}}{\frac{1}{q+1}+n})^{\frac{q+2}{q+1}}$.

For $n = 2$, Theorem 5.4.10 (i) implies that neither of the two suppliers would have an incentive to deviate from the grand coalition. Thus, in this case, $LCS(G, \ll) = LCS(G, \lll) = \{\mathcal{P}^*\}$.

For $n = 3$, there is a unique *coalition profile*, $< 2, 1 >$, corresponding to non-symmetric coalition structures. Such a profile represents coalition structures consisting of one coalition of two members and another coalition of a single member. Consider any supplier, M_i, in the coalition of two players. Then, by Theorem 5.4.2, $E\Pi_{M_i}^* = \frac{p}{2}(\frac{\frac{p-C}{p}}{\frac{1}{q+1}+2})^{\frac{q+2}{q+1}}$. Thus, $\frac{E\Pi_M^*(\bar{\mathcal{P}})}{E\Pi_{M_i}^*} = 2(\frac{\frac{1}{q+1}+2}{\frac{1}{q+1}+3})^{\frac{q+2}{q+1}}$, which is strictly increasing in q, and is bounded between $\frac{9}{8}$, for $q = 0$, and $\frac{4}{3}$, for $q \to \infty$. Therefore, such M_i receives in the coalition profile $< 2, 1 >$ less than in the stand-alone coalition structure, and Condition (A) is satisfied.

For $n = 4$, we use the same approach as above to verify Condition (A). There are two coalition profiles, $\mathcal{A}_1 =< 3, 1 >$ and $\mathcal{A}_2 =< 2, 1, 1 >$, corresponding to non-symmetric coalition structures. Let us first analyze $\mathcal{A}_1$. Consider any supplier, M_i, which is in the coalition of three players. Then, by Theorem 5.4.2, $E\Pi^*_{M_i} = \frac{p}{3}(\frac{\frac{p-C}{p}}{\frac{1}{q+1}+2})^{\frac{q+2}{q+1}}$. Therefore, $\frac{E\Pi^*_M(\bar{\mathcal{P}})}{E\Pi^*_{M_i}} = 3(\frac{\frac{1}{q+1}+2}{\frac{1}{q+1}+4})^{\frac{q+2}{q+1}}$, which is strictly increasing in q, and is bounded between $\frac{27}{25}$, for $q = 0$, and $\frac{3}{2}$, for $q \to \infty$. Let us now study $\mathcal{A}_2$. Consider any supplier M_j which is in the coalition of two players. Then, by Theorem 5.4.2, $E\Pi^*_{M_j} = \frac{p}{2}(\frac{\frac{p-C}{p}}{\frac{1}{q+1}+3})^{\frac{q+2}{q+1}}$. Therefore, $\frac{E\Pi^*_M(\bar{\mathcal{P}})}{E\Pi^*_{M_j}} = 2(\frac{\frac{1}{q+1}+3}{\frac{1}{q+1}+4})^{\frac{q+2}{q+1}}$, which is strictly increasing in q, and is bounded between $\frac{32}{25}$, for $q = 0$, and $\frac{3}{2}$, for $q \to \infty$. Thus, we conclude that for $n = 4$, each non-symmetric coalition structure contains at least one coalition whose members receive less than in the stand-alone coalition structure.

(iii) For $n \geq 5$, denote by $\mathcal{X}_m$ the distinct collection of coalition structure profiles corresponding to non-symmetric coalition structures having m coalitions. For $\tau_m =< \gamma_1, ..., \gamma_m >\in \mathcal{X}_m$, let $\gamma(\tau_m) \equiv \max\{\gamma_i : i = 1, ..., m\}$, and let $\gamma(\mathcal{X}_m) \equiv \min\{\gamma(\tau_m) : \tau_m \in \mathcal{X}_m\}$. The following two observations can be easily verified:

Observation A.1: For any values of n and m, in order to verify whether each non-symmetric coalition structure with m coalitions contains at least one coalition whose members receive less than in the stand-alone coalition structure, it is sufficient to consider an arbitrary coalition B such that $|B| = \gamma(\mathcal{X}_m)$.

Observation A.2: Given any values of n and $m \in [2, n-1]$, $\gamma(\mathcal{X}_m) = \lfloor \frac{n}{m} \rfloor + 1$, where $\lfloor x \rfloor$ is the integer component of x.

Based on Observations A.1 and A.2, we merely need to consider a coalition whose cardinality is $\lfloor \frac{n}{m} \rfloor + 1$. By Theorem 5.4.2, each member in this coalition earns an expected profit $E\Pi^*_M = \frac{1}{\lfloor \frac{n}{m} \rfloor+1}(\frac{\frac{p-C}{p}}{\frac{1}{q+1}+m})^{\frac{q+2}{q+1}}$. Thus, $\frac{E\Pi^*_M(\bar{\mathcal{P}})}{E\Pi^*_M} = (\lfloor \frac{n}{m} \rfloor+1)(\frac{\frac{1}{q+1}+m}{\frac{1}{q+1}+n})^{\frac{q+2}{q+1}}$, which is strictly increasing in $q \in [0, \infty)$ for any given n and m, and is bounded from above by $(\lfloor \frac{n}{m} \rfloor+1)(\frac{m}{n})$, which is strictly greater than 1. Thus, there exists a $q^m \in [0, \infty)$ such that $\frac{E\Pi^*_M(\bar{\mathcal{P}})}{E\Pi^*_M}(q > q^m) > 1$ for any $2 \leq m \leq n-1$ and $n \geq 5$. Let $q_n \equiv \max(q^2, q^3, ..., q^m)$. Therefore, for $q > q_n$, each non-symmetric coalition structure contains at least one coalition whose members receive less than in the stand-alone coalition structure, which completes the proof of Theorem 5.4.18. □

Proof of Lemma 5.5.1. When $m = 1$, the grand coalition was formed. Thus, according to (5.6), $Q^* = F^{-1}(1 - \frac{C}{W})$. When the grand coalition is formed, our model is equivalent to the single-supplier single-assembler system, which has been studied by Cachon (2004a). Our result is consistent with that derived in Cachon's paper.

When $m \geq 2$, since $\frac{C_{B_e}}{W_{B_e}} \equiv \max_{j\in\{1,...,m\}}(\frac{C_{B_j}}{W_{B_j}})$ and $F(x)$ is increasing, $F^{-1}(1 - \frac{C_{B_e}}{W_{B_e}}) \equiv$

$\min_j(F^{-1}(1-\frac{C_{B_j}}{W_{B_j}}))$. By (5.6), in equilibrium, $\min(\vec{Q}^*) \le F^{-1}(1-\frac{C_{B_e}}{W_{B_e}})$. Assume that there exists an equilibrium such that $Q_{B_k} \neq Q_{B_h}$ and $\min(Q_{B_k}, Q_{B_h}) \le F^{-1}(1-\frac{C_{B_e}}{W_{B_e}})$, where $k, h \in \{1, ..., m\}$, and $k \neq h$. Without loss of generality, assume $Q_{B_k} > Q_{B_h}$. Thus, according to (5.6), B_k has an incentive to reduce its quantity from Q_{B_k} to Q_{B_h}. Contradiction. Thus, $Q^*_{B_1} = ... = Q^*_{B_m}$. By (5.6), it is easy to verify that none of the suppliers has an incentive to deviate when (ii) holds. □

Proof of Lemma 5.5.3. According to Lemma 5.5.1, $0 \le Q^*_{B_1} = ... = Q^*_{B_m} = Q^* \le F^{-1}(1-\frac{C_{B_e}}{W_{B_e}})$ is a Nash equilibrium. No proper collection of coalitions $\mathcal{B}'$ in $\mathcal{B}$, $\mathcal{B}' \subset \mathcal{B}$, taking the actions of all other coalitions in $\mathcal{B}$ as given, can profitably deviate from $Q^*_{B_1} = ... = Q^*_{B_m} = Q^*$. Indeed, the expected profit of any coalition in $\mathcal{B}'$ is increasing in Q, for $Q \in [0, Q^*]$, and it is decreasing in Q, for $Q > Q^*$. However, a profitable deviation from $Q^*_{B_1} = ... = Q^*_{B_m} < F^{-1}(1-\frac{C_{B_e}}{W_{B_e}})$ exists, if the entire collection of coalitions in $\mathcal{B}$ deviates to $Q^*_{B_1} = ... = Q^*_{B_m} = F^{-1}(1-\frac{C_{B_e}}{W_{B_e}})$. From $Q^*_{B_1} = ... = Q^*_{B_m} = F^{-1}(1-\frac{C_{B_e}}{W_{B_e}})$, there is no profitable deviation by any collection, $\mathcal{B}'$, of coalitions, including the case when $\mathcal{B}' = \mathcal{B}$. □

Proof of Theorem 5.5.4. We first show that the assembler chooses a wholesale price set which induces an identical ratio of manufacturing cost to wholesale price for each coalition, i.e., $\frac{C_{B_1}}{W_{B_1}} = ... = \frac{C_{B_m}}{W_{B_m}}$.

Suppose there exist at least two coalitions, B_k and B_h, such that $\frac{C_{B_k}}{W_{B_k}} \neq \frac{C_{B_h}}{W_{B_h}}$, where $k \neq h$ and $k, h \in \{1, 2, ..., m\}$. Recall that $\frac{C_{B_e}}{W_{B_e}} = \max_{j \in \{1,2,...,m\}}(\frac{C_{B_j}}{W_{B_j}})$, and observe that $E_X(\min(Q, X))$ in the assembler's profit function, given by (5.7), and the constraint, given by (5.8), depend on $\frac{C_{B_e}}{W_{B_e}}$ only. Thus, the assembler's profit function, given by (5.7), is strictly decreasing in W_{B_j} for any $j \neq e$, as long as $\frac{C_{B_j}}{W_{B_j}} < \frac{C_{B_e}}{W_{B_e}}$. Therefore, by Constraint (5.9), at optimality, we have that $\frac{C_{B_j}}{W_{B_j}} = \frac{C_{B_e}}{W_{B_e}}$ for any $j \neq e$. Thus, $\frac{C_{B_1}}{W_{B_1}} = \frac{C_{B_2}}{W_{B_2}} = ... = \frac{C_{B_m}}{W_{B_m}} \equiv a$, and the assembler's profit function, given by (5.7), results with: $E\Pi_R = (p - \frac{1}{a}C)E_X(\min(Q, X))$, which is strictly decreasing in $\frac{1}{a}$ and strictly increasing in Q. Constraint (5.8) becomes: $\frac{1}{a} = \frac{1}{1-F(Q)}$. Thus, the assembler's problem coincides with his problem when all suppliers form the grand coalition, i.e., $m = 1$. By Lemma 5.5.1, when $m = 1$, $Q^* = F^{-1}(1-\frac{C}{W})$, i.e., $W = \frac{C}{1-F(Q)}$, and the assembler's profit function becomes:

$$E\Pi_R = (p - W)S(Q) = (p - \frac{C}{1-F(Q)})S(Q). \tag{5A.2}$$

It is shown by Cachon (2004a) that $E\Pi_R$ is strictly concave in Q, and that the unique optimal production quantity satisfies:

$$\frac{1-F(Q^*)}{1+J(Q^*)H(Q^*)} = \frac{C}{p}, \tag{5A.3}$$

where $J(Q) = \frac{S(Q)}{1-F(Q)}$ and $H(Q) = \frac{f(Q)}{1-F(Q)}$. Thus, $a^* = \frac{C_{B_j}}{W^*_{B_j}} = 1 - F(Q^*)$ for any j. □

Proof of Corollary 5.5.5. (i) follows from Theorem 5.5.4 (i). Further, by (i), $W^*_{B_j} = \frac{C_{B_j}}{1-F(Q^*)}$. Substituting $W^*_{B_j}$ into B_j's profit function, given by (5.5), and simplifying results with $E\Pi^*_{B_j} = \frac{C_{B_j}}{C}(\frac{CS(Q^*)}{1-F(Q^*)} - CQ^*) = \frac{C_{B_j}}{C}(W^*S(Q^*) - CQ^*) = \frac{C_{B_j}}{C}E\Pi^{Total*}_M(m=1)$. □

Proof of Proposition 5.5.6. Suppose coalitions B_j and B_k are in $\mathcal{B}$. Without loss of generality, assume that $C_{B_j} > C_{B_k}$. In equilibrium, $\frac{C_{B_j}}{W^*_{B_j}} = \frac{C_{B_k}}{W^*_{B_k}}$. Thus, $W^*_{B_j} = \frac{C_{B_j}}{C_{B_k}}W^*_{B_k}$. Coalition B_j's equilibrium profit is: $E\Pi^*_{B_j} = W^*_{B_j}S(Q^*) - C_{B_j}Q^* = \frac{C_{B_j}}{C_{B_k}}W^*_{B_k}S(Q^*) - C_{B_j}Q^* = \frac{C_{B_j}}{C_{B_k}}(W^*_{B_k}S(Q^*) - C_{B_k}Q^*) > (W^*_{B_k}S(Q^*) - C_{B_k}Q^*) = E\Pi^*_{B_k}$, where the last inequality follows since $C_{B_j} > C_{B_k}$. □

Proof of Proposition 5.5.7. (i) According to (5A.3), in equilibrium, $C \cdot G(Q^*) = p(1 - F(Q^*))$, where $G(Q) = 1 + J(Q)H(Q)$. Recall that $J(Q)H(Q)$ increases in Q. Thus, $G(Q)$ increases in Q. Taking derivatives with respect to C_{B_j} on both sides of the equation $C\cdot G(Q^*) = p(1-F(Q^*))$ results with $G(Q^*) + C \cdot G'(Q^*)\frac{\partial(Q^*)}{\partial(C_{B_j})} = -pf(Q^*)\frac{\partial(Q^*)}{\partial(C_{B_j})}$. Thus, $\frac{\partial(Q^*)}{\partial(C_{B_j})}(C \cdot G'(Q^*) + pf(Q^*)) = -G(Q^*)$. Therefore, $\frac{\partial(Q^*)}{\partial(C_{B_j})} < 0$ since $C \cdot G'(Q^*) + pf(Q^*) > 0$ and $G(Q^*) > 0$.

(ii) $W^* = \frac{p}{1+J(Q^*)H(Q^*)}$ decreases in Q^* since $J(Q)H(Q)$ increases in Q. Since Q^* decreases in C_{B_j}, W^* increases in C_{B_j}, i.e., $\frac{\partial(W^*)}{\partial(C_{B_j})} > 0$. Similarly, $W^*_{B_i} = \frac{C_{B_i}}{1-F(Q^*)}$ increases in Q^*, which decreases in C_{B_j} for $i \neq j$. Thus, $W^*_{B_i}$ decreases in C_{B_j}, i.e., $\frac{\partial(W^*_{B_i})}{\partial(C_j)} < 0$. Therefore, $\frac{\partial W^*_{B_j}}{\partial C_{B_j}} > 0$ since $W^* = \Sigma^m_{i=1}(W^*_{B_i})$.

(iii) For any $i \neq j$, $E\Pi^*_{B_i} = W^*_{B_i}S(Q^*) - C_{B_i}Q^*$, and taking derivatives, $\frac{\partial(E\Pi^*_{B_i})}{\partial(C_{B_j})} = \frac{\partial(W^*_{B_i})}{\partial(C_{B_j})}S(Q^*) - (W^*_{B_i}(1 - F(Q^*)) - C_{B_i})\frac{\partial(Q^*)}{\partial(C_{B_j})} = \frac{\partial(W^*_{B_i})}{\partial(C_{B_j})}S(Q^*) < 0$, where the last equality follows since $W^*_{B_i}(1 - F(Q^*)) - C_{B_i} = 0$ and the last inequality follows since $\frac{\partial(W^*_{B_i})}{\partial(C_{B_j})} < 0$.

(iv) $E\Pi^*_R = (p - W^*)S(Q^*)$. Since W^* increases in C_{B_j} and Q^* decreases in C_{B_j}, $E\Pi^*_R$ decreases in C_{B_j}. □

Proof of Proposition 5.5.8. Under the proportional splitting rule, both for coalitions and individual suppliers of the same coalition, the equilibrium profit of each supplier is independent of the coalition structure, and is proportional to its own marginal cost. Thus, all suppliers are indifferent among all coalition structures. □

Proof of Proposition 5.6.3. (i) Since $E\Pi_{R,pull}(Q)$ is unimodal in Q and uniquely maximized at Q^*_{pull}, and $Q^*_{pull} < Q^P$, $E\Pi_{R,pull}(Q^*_{pull}) > E\Pi_{R,pull}(Q^P)$. Further, $E\Pi_{R,push}(Q^*_{push}) < E\Pi_{R,push}(Q^P)$ since $E\Pi_{R,push}(Q)$ is strictly increasing in Q for $Q > 0$ and $Q^*_{push} < Q^P$. By Lemma 5.6.2, $E\Pi_{R,pull}(Q^P) = E\Pi_{R,push}(Q^P)$. Thus, $E\Pi_{R,pull}(Q^*_{pull}) > E\Pi_{R,push}(Q^*_{push})$ and (i) is proved.

(ii) Similar to the proof of (i), $E\Pi^{Total}_{M,push}(Q^*_{push}) > E\Pi^{Total}_{M,push}(Q^P)$, since $E\Pi^{Total}_{M,push}(Q)$ is unimodal in Q and uniquely maximized at Q^*_{push}, and $Q^*_{push} < Q^P$. Further, $E\Pi^{Total}_{M,pull}(Q^*_{pull}) < E\Pi^{Total}_{M,pull}(Q^P)$ since $E\Pi^{Total}_{M,pull}(Q)$ is strictly increasing in Q and $Q^*_{pull} < Q^P$. By Lemma 5.6.2, $E\Pi^{Total}_{M,push}(Q^P) = E\Pi^{Total}_{M,pull}(Q^P)$. Thus, $E\Pi^{Total}_{M,push}(Q^*_{push}) > E\Pi^{Total}_{M,pull}(Q^*_{pull})$. □

Proof of Proposition 5.6.4. Given any supplier M_i with marginal cost c_{M_i}, under push, her equilibrium expected profit is $E\Pi^*_{M_i,push} = \frac{E\Pi^{Total}_{M,push}(Q^*_{push})}{n}$, and her corresponding equilibrium expected profit under pull is $E\Pi^*_{M_i,pull} = \frac{c_{M_i}}{C} E\Pi^{Total}_{M,pull}(Q^*_{pull})$. Thus, $E\Pi^*_{M_i,push} > E\Pi^*_{M_i,pull}$ is equivalent to $c_{M_i} < \frac{E\Pi^{Total}_{M,push}(Q^*_{push})}{E\Pi^{Total}_{M,pull}(Q^*_{pull})} \cdot \frac{C}{n}$, which completes the proof of (i). Accordingly, (ii) and (iii) follow immediately. □

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