Essays in Empirical Macroeconomics

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Abstract

This thesis consists of three essays that contribute empirical macroeconomics.

The first essay jointly tests several of the predictions of the intertemporal approach to the current account and one of its implications, the present value model of the current account (PVM). The intertemporal approach to the current account predicts that the current account of a small open economy is independent of world common disturbances. The PVM predicts that the response of the current account to a country-specific shock depends on the persistence of the shock. This essay combines these predictions to identify a structural vector autoregression (SVAR). The identification exploits the orthogonality of the world real interest rate and country-specific shocks as well as the lack of a long-run response of net output to transitory shocks. Estimates of the SVAR show that the Canadian and U.K. data support the intertemporal approach with two puzzling exceptions .

A recent study claims that habit formation in consumption improves the ability of the PVM of the current account to predict actual current account movements. The second essay shows that the habit-forming PVM of the current account is observationally equivalent to the canonical PVM augmented with a transitory consumption shock. To resolve the identification problem, this essay constructs a small open economy-real business cycle (SOE-RBC) models with habits and stochastic world real interest rates calibrated to Canadian postwar quarterly data. The results from Monte Carlo experiments reveal that to explain sample moments conditional on the habit-forming and standard PVMs, the SOE-RBC model with stochastic world real interest rates dominates the SOE-RBC

model with habit formation.

The third essay explores the ability of habit formation in consumption, in the context of the one-sector, closed economy-RBC model, to account for the U.S. growth rates of consumption and output. Existing studies show that habit formation helps successfully explain the negative response of labour input to a positive, permanent technology shock as well as the empirical puzzles of asset pricing behavior. This essay shows that the RBC model with habit formation fails to mimic not only the persistence of output growth over business cycle frequencies but that of consumption growth at zero frequency as well. Further, the model yields counterfactually low volatility of equity returns.

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To Kazuko, and Our Parents

Chapter 1

Overview and Summary

This thesis consists of three essays that contribute empirical macroeconomics.

Essay 1. A Structural VAR Approach to the Intertemporal Model of the Current Account

In the first essay, I jointly test several of the predictions of the intertemporal approach to the current account and one of its implications, the present value model of the current account (PVM). Given homogeneity across economies, the intertemporal approach predicts that the current account will not be affected by global disturbances; hence, country-specific shocks dominate current account fluctuations. The PVM predicts that the response of the current account to a country-specific output shock depends on the persistence of the shock.

This essay develops schemes to identify three shocks: global, country-specific permanent, and country-specific transitory shocks. The assumption of a small open economy requires that a country-specific shock be orthogonal to the world real interest rate.

This orthogonality condition, as well as the lack of a long-run response of output to a transitory shock, provides identification schemes for a structural vector autoregression (SVAR). The identified SVAR in turn makes it possible to test the predicted responses of the current account to the three shocks.

Using data of Canada and the U.K., I find support for the intertemporal approach to the current account in several dimensions. However, this essay reveals two puzzles that challenge the intertemporal approach. First, the impact response of the current account to a country-specific transitory shock is greater than that implied by the PVM. This is a puzzle because it implies that consumption responds negatively to a positive income shock. The second puzzle is that current account fluctuations are dominated by country-specific transitory shocks, which themselves explain very little of the fluctuations in income. This finding is inconsistent with the PVM.

Essay 2. Habit Formation, the World Real Interest Rate, and the Present Value Model of the Current Account

Habit formation in consumption is often employed to resolve puzzles between macro models and aggregate data. One example is the present value model of the current account (PVM) that includes habit formation. A recent study argues that habit formation improves the ability of the PVM to predict actual current account movements.

This essay shows that the habit-forming PVM is observationally equivalent to the canonical PVM augmented with a transitory consumption component that is serially correlated. This means that given the data, any test statistic constructed from the former PVM takes the same value as that from the latter PVM. Hence, by looking at the sample test statistics, a researcher cannot identify whether or not habit formation plays an important role in actual current account movements.

To resolve this identification problem, this essay constructs two small open economy-real business cycle (SOE-RBC) models: one with habit formation and the other with a stochastic world real interest rate, respectively. The two SOE-RBC models are calibrated to postwar Canadian quarterly data, and are used to generate artificial data to replicate the test statistics of the habit-forming PVM. The idea is: if the sample test statistics of the habit-forming PVM really reflect habit formation in consumption, the theoretical test statistics replicated by the SOE-RBC model with habit formation should be closer to the sample test statistics than those replicated by the SOE-RBC model with a stochastic world real interest rate.

Results from the Monte Carlo experiments reveal that to explain the sample test statistics of both the habit-forming and standard PVMs, the SOE-RBC model with the stochastic world real interest rate dominates the SOE-RBC model with habit formation; in other words, the former model does a better job of replicating the data. This suggests that future research in this literature should concentrate on the determinants of the world real interest rate rather than on alternative specifications of utility.

Essay 3. Habit Formation and Aggregate Dynamics in Real Business Cycle Models

The third essay explores the ability of habit formation in consumption, in the context of the one-sector, closed economy-real business cycle (RBC) model, to account for the U.S. growth rates of consumption and output. Existing studies have shown that habit formation helps successfully explain the negative response of labour input to a positive, permanent technology shock as well as the empirical puzzles of asset pricing behavior. This paper shows that this type of model (i) fails to mimic the persistence of output growth over business cycle frequencies, (ii) fails to mimic the hump-shaped impulse response of output growth to a transitory shock, (iii) overstates the persistence of consumption growth around zero frequency, and (iv) yields counterfactually low volatility of equity return. These failures of the one-sector, closed economy-RBC model cast doubt on habit formation as an important data generating mechanism to generate the dynamics of the U.S. aggregate data.

Chapter 2

A Structural VAR Approach to the Intertemporal Model of the Current Account

2.1 Introduction

The intertemporal current account approach provides an analytical framework to study current account movements of a small open economy because it emphasizes forward-looking behavior of economic agents[1]. The key message of the intertemporal approach is that domestic residents use the current account as a tool to smooth consumption against country-specific shocks by borrowing and lending in international capital markets. To the contrary, no global shock gives a small open economy an opportunity of consumption

[1]The small open optimal growth model of Hamada(1966) is an explicit precursor of the intertemporal approach to the current account. Obstfeld and Rogoff(1995) is an excellent review of this approach.

smoothing since all economies react symmetrically to a global shock. A global shock has no effect on the current account in a small open economy.

The present value model of the current account (PVM) expresses this consumption-smoothing motive in current account fluctuations as a linear closed-form solution of the intertemporal approach. With the assumption of the exogenous, constant world real interest rate, the PVM characterizes the current account to be negative of the discounted sum of expected future changes in net output[2]. This present value formula implies that when domestic residents expect future net output to increase temporarily by country-specific shocks, they lend out to the rest of the world to smooth consumption. Therefore, the current account moves into surplus. On the other hand, if an increase in future net output is expected to be permanent, the current account should not change because the permanent shocks to net output cannot be smoothed away[3].

[2]Sheffrin and Woo(1990), Otto(1992), Ghosh(1995) and Bergin and Sheffrin(2000) jointly test the cross-equation restrictions the PVM formula imposes on an unrestricted vector autoregressive, by applying the methodology originally developed by Campbell(1987) and Campbell and Shiller(1987) to test theories of consumption and stock price. Their tests statistically reject the basic PVM's cross-equation restrictions in the G-7 economies except for the U.S. This formal rejection of the PVM, however, does not necessarily imply that the PVM and the intertemporal approach are not useful to explain current account movements in a small open economy. For example, as Obstfeld and Rogoff(1995) discuss, the predictions of the PVM track historical current account movements fairly closely in some economies.

[3]More precisely, if net output follows a random walk, a country-specific shock permanently raises net output by the same amount. Sachs(1981,1982) shows that the current account does not respond to the shock since both consumption and current net output rise by the same amount in this case. Moreover when net output follows a more persistent process than a random walk, like an ARIMA process, the PVM predicts a negative response of the current account to a positive, country-specific shock.

Recent studies test the predictions of the intertemporal approach and the PVM in many different dimensions. Table 2.1 summarizes the main results of the past studies. First, by decomposing the Solow residuals into global and country-specific components, Glick and Rogoff(1995) and its successor İşcan(2000) observe in the post-1975 data of the Group of Seven (G-7) economies that the current account in fact responds little to a global technology shock. To the contrary, by exploiting a structural vector autoregression (SVAR) approach, Nason and Rogers(2002) show in the post-1975 Canadian data that the hypothesis of no response of the current account to a global shock is sensitive to identification[4]. Nason and Rogers(2002) also observe, as the second result in Table 2.1, that country-specific transitory shocks dominate current account fluctuations not only in the short run but also the long run.

Glick and Rogoff(1995) argue there is another puzzling observation in the joint dynamics of investment and the current account. The authors observe across the G-7 data that investment responds to the identified country-specific technology shock greater in the absolute value than the current account does. However, their intertemporal model predicts that when a country-specific technology shock is permanent, the current account should respond to the shock greater in the absolute value than investment because saving negatively responds to the permanent technology shock. They propose as a resolution a highly persistent but not permanent, country-specific technology shock. Similarly, the permanent-transitory decomposition of Hoffmann(2001) based on the vector error correction model (VECM), as well as the introduction of nontradable goods by İşcan(2000),

[4]In the appendix, they apply the same analysis to the other G-7 economies and obtain the almost same results as in the Canadian data.

provides a potential resolution for Glick and Rogoff's puzzle.

The purpose of this essay is to evaluate the predictions of the intertemporal approach and the PVM on responses of the current account to different shocks. This essay jointly tests the predictions of the intertemporal approach and the PVM on responses of the current account to three shocks to net output: global, country-specific permanent, and country-specific transitory shocks. This essay accomplishes this purpose by providing its own identification schemes. The three shocks are identified by a SVAR with two restrictions. The first restriction stems from the small open assumption maintained by the intertemporal approach. This assumption restricts the world real interest rate to be orthogonal to any country-specific shock at all forecast horizons. Together with the assumption of the small open economy, allowing the world real interest rate to vary stochastically makes it possible to identify global and country-specific shocks. The second identifying assumption this paper employs restricts transitory shocks to have no long-run effect on net output. This long-run restriction, based on Blanchard and Quah(1989), decomposes country-specific shocks into permanent and transitory components.

The assumption of the small open economy and the long-run restriction provide two identification schemes for the SVAR that contains the world real interest rate, the first difference of log of net output, and the current account-net output ratio as the endogenous variables. The identified SVAR in turn makes it possible to test jointly the predictions on the responses of the current account to the three shocks. The predictions are given as the cross-equation restrictions the intertemporal approach and the PVM impose on the SVAR[5].

[5]These cross-equation restrictions are conditional on the identification of the SVAR. Hence, this

This essay studies quarterly data of two proto-type small open economies, Canada and the U.K. The main results of this essay are summarized in Table 2.2. First, in Canada and the U.K., impulse responses of the current account to the identified shocks are consistent with the corresponding theoretical predictions. Second, tests of the cross-equation restrictions (CERs) show that the hypothesis that the current account does not respond to a global shock is sensitive to the identification, while the impact responses of the current account to country-specific shocks match the PVM's prediction. The test of the CERs also rejects the joint hypothesis related to the impact responses of the current account measure to all the three shocks. Third, given the identification, the data support the observation that the response of the current account-net output ratio to country-specific transitory shocks are *greater* than implied by the PVM. Fourth, the forecast error variance decompositions (FEVDs) of the current account reveal that country-specific transitory shocks dominate current account fluctuations not only in the short run but the long run as well, while the shocks explain almost none of the fluctuations in net output.

The first result supports the intertemporal approach and the PVM. This result adds to the literature that finds the intertemporal approach can explain many aspects of current account dynamics. The second result echoes Nason and Rogers(2002): the response of the current account to a global shock is sensitive to identification. The third result reveals a new aspect of Glick and Rogoff's(1995) puzzle: even when country-specific shocks are decomposed into permanent and transitory shocks, the impact response of the current

joint test is different from that of the cross-equation restrictions imposed on the reduced-form VAR as in Sheffrin and Woo(1990), Otto(1992), and Ghosh(1995).

account remains puzzling. Moreover, the third result implies that consumption negatively responds to a positive income shock. This implication is hard to be reconciled with the standard macroeconomics literature. The final result confirms the observation of Nason and Rogers(2002) with different identification. This result violates the PVM since the basic present value formula requires current account fluctuations need to be explained by the shocks that dominate net output fluctuations in the short run as well as the long run.

The following section introduces the model and develops the predictions of the intertemporal approach and the PVM as cross-equation restrictions on a structural VMA. Identification issues are discussed in section 3. Section4 reports the empirical results. Section 5 contains conclusions.

2.2 The Model and Its Predictions

This essay considers a world that consists of many small open economies. Following Glick and Rogoff(1995), assume that all the economies are homogeneous with respect to preferences, endowments and technologies. Furthermore, the international financial market is assumed to be incomplete in the sense that no household in a small open economy can buy or sell state-contingent claims to diversify away country-specific shocks. Only riskless bonds, which are denominated in terms of the single consumption good, are traded internationally[6].

[6]Incompleteness in the international financial market is one of the maintained assumptions in the intertemporal approach [see, for example, Obstfeld and Rogoff(1995) and Glick and Rogoff(1995)]

2.2.1 An Intertemporal, Small Open Economy Model

Consider an infinitely lived representative consumer in a representative small open economy. The assumption of the small open economy implies that this economy faces the world real interest rate r_t determined in the international financial market. The standard PVM of the current account, for example, Sheffrin and Woo(1990), Otto(1992) and Ghosh(1995), assumes the world real interest rate to be exogenous and constant. Instead, this essay allows the world real interest rate to vary stochastically, as in Bergin and Sheffrin(2000). The reason for this extension is that this essay exploits stochastic variations in the world real interest rate to identify global and country-specific shocks. In addition, this essay assumes the world real interest rate is covariance stationary.

Let C_t be consumption at period t, $u(C)$ be the period utility function of the consumer, and β be the subjective discount factor taking a value between 0 and 1, respectively. The consumer's expected lifetime utility function at period t is then given as

$$E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i}) \tag{2.1}$$

where E_t is the conditional expectation operator upon the information set at period t. Further defining B_t, Q_t, I_t and G_t to be the international bond holding, output, investment and government expenditure at period t, respectively, gives the consumer's

and the small open RBC models [see, for example, Mendoza(1991) and Cardia(1991)]. By contrast, the two-country RBC models [see, for example, Backus, Kehoe, and Kydland(1992) and Baxter and Crucini(1993)] assume the complete financial market. In this literature, agents in two countries can pool all idiosyncratic risks by trading any contingent claims.

budget constraint

$$B_{t+1} = (1 + r_t)B_t + Q_t - I_t - G_t - C_t. \tag{2.2}$$

The optimization problem of the representative consumer is then to maximize eq.(2.1) subject to eq.(2.2). The first order conditions of this problem comprise the budget constraint (2.2), the Euler equation

$$u'(C_t) = \beta E_t(1 + r_{t+1})u'(C_{t+1}), \tag{2.3}$$

and the transversality condition

$$\lim_{i\to\infty} E_t R_{t,i} B_{t+i} = 0 \tag{2.4}$$

where $R_{t,i}$ is the *ex post* market discount factor at period t for period $t+i$ consumption, which is defined as

$$R_{t,i} \equiv \begin{cases} 1/\left(\prod_{j=t+1}^{t+i}(1+r_j)\right) & \text{if } i \geq 1, \\ 1 & \text{if } i = 0. \end{cases} \tag{2.5}$$

For simplicity, let NO_t denote output net of investment and government expenditure at period t: $NO_t \equiv Q_t - I_t - G_t$. Taking the infinite sum of the consumer's budget constraint (2.2) toward the future and using the transversality condition (2.4) yield the *ex ante* intertemporal budget constraint of the consumer

$$\sum_{i=0}^{\infty} E_t R_{t,i} C_{t+i} = (1 + r_t)B_t + \sum_{i=0}^{\infty} E_t R_{t,i} NO_{t+i}. \tag{2.6}$$

To derive the present value representation of the current account measure, this essay takes a log-linear approximation of the Euler equation (2.3) and a linear approximation of the intertemporal budget constraint (2.6)[7]. The approximation begins by dividing the intertemporal budget constraint (2.6) by NO_t. After several steps of simple algebra, eq.(2.6) can be rewritten as

$$\begin{aligned}\frac{C_t}{NO_t}\left[1+\sum_{i=1}^{\infty}E_t\exp\left\{\sum_{j=t+1}^{t+i}(\Delta\ln C_j-\ln(1+r_j))\right\}\right]&\\ =\exp\{\ln(1+r_t)-\Delta\ln NO_t\}\frac{B_t}{NO_{t-1}}&\\ +\left[1+\sum_{i=1}^{\infty}E_t\exp\left\{\sum_{j=t+1}^{t+i}(\Delta\ln NO_j-\ln(1+r_j))\right\}\right].&\end{aligned}$$

Let c, b, γ^c, γ and μ denote the means of the consumption-net output ratio C_t/NO_t, the net foreign asset-net output ratio B_t/NO_{t-1}, the first difference of log of consumption $\Delta\ln C_t$, the first difference of log of net output $\Delta\ln NO_t$, and log of the gross world real interest rate $\ln(1+r_t)$, respectively. Eq.(2.6) is then linearly approximated by taking a first-order Taylor expansion around these means. Appendix A.1 shows the steps of the linear approximation of the intertemporal budget constraint in detail. For any variable X_t, let $\tilde{X}_t$ denote deviation from its mean value. The linear-approximated intertemporal

[7]Bergin and Sheffrin(2000) also conduct a linear approximation of the intertemporal current account model in order to involve stochastic variations of world real interest rates and terms of trade into the standard PVM. While they follow Huang and Lin's (1993) log-linear approximation, this essay develops an alternative linear approximation to derive a closed-form solution of the optimal current account-net output ratio.

budget constraint is given as

$$\widetilde{\frac{C_t}{NO_t}} \approx \frac{1-\alpha}{\kappa}\widetilde{\frac{B_t}{NO_{t-1}}} + \frac{1-\alpha}{\kappa} b\widetilde{\ln(1+r_t)} - \frac{1-\alpha}{\kappa} b\widetilde{\Delta \ln NO_t}$$
$$- c\sum_{i=1}^{\infty} \alpha^i E_t \left\{ \widetilde{\Delta \ln C_{t+i}} - \widetilde{\ln(1+r_{t+i})} \right\}$$
$$+ \frac{1-\alpha}{1-\kappa} \sum_{i=1}^{\infty} \kappa^i E_t \left\{ \widetilde{\Delta \ln NO_{t+i}} - \widetilde{\ln(1+r_{t+i})} \right\} \quad (2.7)$$

where $\alpha = \exp(\gamma^c - \mu) < 1$ and $\kappa = \exp(\gamma - \mu) < 1$ [8].

Notice that eq.(2.7) makes the consumption-net output ratio depend on the expected future path of consumption growth. To characterize the process of consumption growth, the Euler equation (2.3) is approximated log-linearly. Suppose that the period utility function is given as a power function

$$u(C) = \frac{C^{1-1/\sigma}}{1-1/\sigma}$$

where σ is the elasticity of intertemporal substitution. This specification of the utility function yields the Euler equation

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\sigma}} (1 + r_{t+1}) \right\}.$$

As shown in Campbell and Mankiw(1989) and Campbell(1993), when the world real interest rate and consumption are jointly conditionally homoscedastic and log-normally

[8] The conditions $\alpha < 1$ and $\kappa < 1$ are required to satisfy boundedness of the expected present discounted value terms of eq.(2.7). Through the following analysis, this essay assumes these conditions: the mean growth rates of consumption and net output are lower than the mean of world real interest rates, respectively. These conditions imply that on the balanced growth path the economy is *dynamically efficient*.

distributed, the above Euler equation can be rewritten as

$$
\begin{aligned}
E_t \Delta \ln C_{t+1} &= \delta + \sigma \ln \beta + \sigma E_t \ln(1 + r_{t+1}) \\
&= \delta + \sigma(\ln \beta + \mu) + \sigma E_t[\ln(1 + r_{t+1}) - \mu]
\end{aligned} \tag{2.8}
$$

where δ is a constant term including the variances of $\Delta \ln C_{t+1}$ and $\ln(1 + r_{t+1})$ and the covariance between the two terms[9].

Finally, to derive an approximated solution of the current account-net output ratio, recall the current account identity

$$
CA_t \equiv r_t B_t + NO_t - C_t. \tag{2.9}
$$

By assuming that the economy possesses a balanced growth path, $\alpha = \kappa$, and using the approximation $\ln(1 + r_t) \approx r_t$, Appendix A.2 shows that eqs.(2.7), (2.8) and (2.9) together give the present value representation of the current account-net output ratio:

$$
\widetilde{\frac{CA_t}{NO_t}} = b\widetilde{r}_t + [(\sigma - 1)c + 1] \sum_{i=1}^{\infty} \kappa^i \, E_t \, \widetilde{r}_{t+i} - \sum_{i=1}^{\infty} \kappa^i \, E_t \, \Delta \, \widetilde{\ln NO}_{t+i}. \tag{2.10}
$$

[9]It is important to note from the log-linearized Euler equation (2.8) that perfect consumption smoothing as in previous studies is not the case in this model. First, unless $\delta + \sigma(\ln \beta + \mu) = 0$, log of consumption has a deterministic trend, as shown by the first two constant terms in the RHS of (2.8). Second, the last term shows that the substitution effect of variations of world real interest rates on the consumption profile. A rise in the world real interest rate makes current consumption more expensive in terms of future consumption. Hence the representative consumer is induced to shift consumption toward the future with elasticity σ. These two effects together produce consumption profile that deviates from perfectly smoothed one.

Furthermore, a caveat of the log-linearized Euler equation (2.8) is that it only cares about first moments of logs of consumption and the world real interest rate. Higher moments of two series are assumed to be fixed.

Eq.(2.10) is the desired schedule of the current account-net output ratio, which is represented as a linear present value relation among the current account-net output ratio, the first difference of log of net output and the world real interest rate.

Eq.(2.10) says that the optimal current account-net output ratio is determined by three factors. The third term of the RHS of eq.(2.10) captures the consumption-smoothing motive. It implies that the representative consumer changes the current account-net output ratio to smooth consumption in response to expected changes in future path of net output growth. The second term represents a consumption-tilting factor due to expected variation of the world real interest rate. The coefficient $(\sigma - 1)c + 1$ on the second term implies the intertemporal substitution effect, the income effect and the wealth effect, respectively. If the world real interest rate is expected to change in future, the small open economy wants to deviate consumption from its smoothed, random walk path through the three effects. The first term of the RHS of eq.(2.10) is an additional consumption-tilting factor. When there is a change in the world real interest rate, net interest payment from abroad is changed given the net international asset position. For example, a rise in the world real interest rate increases net interest payment from (to) abroad if the country is a net creditor (debtor). This change in net interest payment prompts the consumer to alter the current account-net output ratio beyond its consumption-smoothing level.

2.2.2 Derivation of the Predicted Responses

This subsection derives the testable restrictions the present value formula (2.10) imposes on the responses of the current account measure to three orthogonal shocks to net output:

global, country-specific permanent and country-specific transitory shocks. Let ϵ_t^g, ϵ_t^{cp}, and ϵ_t^{cs} denote global, country-specific permanent, and country-specific transitory shocks, respectively, and be orthogonal each other. This essay assumes that the first difference of log of net output is linearly decomposed into three infinite-order MA components attributed to the three orthogonal shocks:

$$\widetilde{\Delta \ln NO_t} = \Gamma_g^{no}(L)\epsilon_t^g + \Gamma_{cp}^{no}(L)\epsilon_t^{cp} + \Gamma_{cs}^{no}(L)\epsilon_t^{cs} \tag{2.11}$$

where $\Gamma_i^{no}(L)$ for $i = \{g, cp, cs\}$ is an invertible, infinite-order polynomial with respect to the lag operator L, in which the impact coefficient $\Gamma_i^{no}(0)$ is not restricted to one[10]. Similarly, the process of the world real interest rate is linearly decomposed into three infinite-order MA components attributed to the three orthogonal shocks:

$$\widetilde{r}_t = \Gamma_g^r(L)\epsilon_t^g + \Gamma_{cp}^r(L)\epsilon_t^{cp} + \Gamma_{cs}^r(L)\epsilon_t^{cs}. \tag{2.12}$$

Given the processes of the first difference of log of net output and the world real interest rate, eqs.(2.11) and (2.12), the present value formula (2.10) yields the predictions on the impulse responses of the current account-net output ratio to the three shocks. The following structural moving average (SMA) representation of the current account-net output ratio represents the predictions (Appendix A.3 contains the details of this derivation.):

$$\frac{\widetilde{CA_t}}{NO_t} = \Gamma_g^{ca}(L)\epsilon_t^g + \Gamma_{cp}^{ca}(L)\epsilon_t^{cp} + \Gamma_{cs}^{ca}(L)\epsilon_t^{cs} \tag{2.13}$$

[10]Note that eq.(2.11) is a structural moving average (SMA) representation of the process $\Delta \ln NO_t$, rather than the Wold representation with the impact coefficient equal to one. Instead of being restricted to one, the impact coefficient is estimated.

where $\Gamma_i^{ca}(L)$ for an index $i \in \{g, cp, cs\}$ is an invertible, infinite-order polynomial with respect to the lag operator. The SMA (2.13) provides the testable hypotheses this paper studies.

The first hypothesis predicts that a global shock does not matter for the current account at any forecast horizons. Under the homogeneity assumption across economies, every economy has the same excess demand for international riskless bonds. In this case, as argued by Razin(1993) and Glick and Rogoff(1995), no economy can alter its net foreign asset position to a global shock because all the other economies react to the shock symmetrically. Therefore, a global shock has no effect on the current account at any forecast horizons. All that occurs is that the world real interest rate adjusts. Let $\mathcal{H}_g^i$ denote the impulse response of CA_t to ϵ_{t-i}^g. Then the first null hypothesis is given as

$$H_0: \quad \mathcal{H}_g^i = \frac{\partial CA_t}{\partial \epsilon_{t-i}^g} = 0 \quad \text{for any } i \geq 0. \qquad \text{(Hypothesis 1)}$$

To test this hypothesis, this essay recovers the impulse response functions (IRFs) of the *level* of the current account to a global shock from the IRFs of the current account-net output ratio and log of net output[11].

Next consider the impact responses of the current account-net output ratio to the two country-specific shocks ϵ_t^{cp} and ϵ_t^{cs}: $\Gamma_{cp}^{ca}(0)$ and $\Gamma_{cs}^{ca}(0)$ in eq.(2.13). To derive the second and third hypotheses, recall the small open economy assumption of the intertemporal approach. This assumption requires that a small open economy have no influence on

[11] To the contrary, the response of the current account-net output ratio to a global shock is ambiguous. For example, if a global shock has a positive impact on $\ln NO_t$ and the mean value of CA_t/NO_t is positive, then the current account-net output ratio should respond negatively to the shock.

the world real interest rate: a country-specific shock does not matter for the world real interest rate at any forecast horizons. In other words, this assumption implies that zero restrictions are imposed on the coefficients of the infinite-order polynomials related to the two country-specific shocks in the world real interest rate process (2.12): for any $i \geq 0$,

$$\Gamma^r_{cp,i} = \Gamma^r_{cs,i} = 0 \qquad \text{(Small Open Economy Assumption)} \tag{2.14}$$

where $\Gamma^r_{cp,i}$ and $\Gamma^r_{cs,i}$ are the i-th coefficients of the infinite-order polynomials $\Gamma^r_{cp}(L)$ and $\Gamma^r_{cs}(L)$ in eq.(2.12), respectively.

As shown in Appendix A.3 in detail, under the small open economy assumption (2.14), $\Gamma^{ca}_{cp}(0)$ and $\Gamma^{ca}_{cs}(0)$ should satisfy the following cross-equation restrictions, respectively:

$$\Gamma^{ca}_{cp}(0) = \Gamma^{no}_{cp}(0) - \Gamma^{no}_{cp}(\kappa) \tag{$\mathcal{R}_{cp}$}$$

and

$$\Gamma^{ca}_{cs}(0) = \Gamma^{no}_{cs}(0) - \Gamma^{no}_{cs}(\kappa) \tag{$\mathcal{R}_{cs}$}$$

where for an index $i \in \{cp, cs\}$, $\Gamma^{no}_{i}(\kappa)$ is the infinite polynomial $\Gamma^{no}_{i}(z)$ evaluated at $z = \kappa$.

The cross-equation restrictions $\mathcal{R}_{cp}$ and $\mathcal{R}_{cs}$ state that the impact response of the current account-net output ratio to a country-specific shock should be given as the difference between the impact and the discounted long-run responses of $\Delta \ln \widetilde{NO}_t$ to the shock. The current account identity (2.9) restricts the current account-net output ratio to be negatively related to the consumption-net output ratio. Therefore, if a country-specific shock raises net output above (below) consumption, the current account-net output ratio rises (falls). $\Gamma^{no}_{cp}(0)$ in $\mathcal{R}_{cp}$ captures the impact effect of the shock ϵ^{cp}_t on net output, while $\Gamma_{cp}^{no}(\kappa)$ shows the impact effect of the shock on consumption[12]. Hence the impact effect of the shock on the current account-net output ratio, $\Gamma_{cp}^{ca}(0)$, is given as the difference $\Gamma_{cp}^{no}(0) - \Gamma_{cp}^{no}(\kappa)$. The same explanation is applicable for $\mathcal{R}_{cs}$.

Define the statistics $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$ as $\mathcal{H}_{cp} \equiv \Gamma_{cp}^{ca}(0) - \Gamma_{cp}^{no}(0) + \Gamma_{cp}^{no}(\kappa)$ and $\mathcal{H}_{cs} \equiv \Gamma_{cs}^{ca}(0) - \Gamma_{cs}^{no}(0) + \Gamma_{cs}^{no}(\kappa)$, respectively. The cross-equation restrictions $\mathcal{R}_{cp}$ and $\mathcal{R}_{cs}$ then provide the following null hypotheses:

$$H_0 : \mathcal{H}_{cp} = 0 \qquad \text{(Hypothesis 2)}$$

and

$$H_0 : \mathcal{H}_{cs} = 0. \qquad \text{(Hypothesis 3)}$$

By construction, if $\mathcal{H}_i \neq 0$ for $i \in \{cp, cs\}$, the prediction of the PVM on the impact response of the current account-net output ratio to the shock ϵ_t^i is rejected because the observed response is considered to be greater or lesser than the prediction.

2.3 The SVMA and Identification Issues

Hypotheses 1-3 are constructed conditionally on identification of the three shocks. Testing the null hypotheses discussed in the last section requires the three shocks to be identified. To do so, this essay exploits the SVAR methodology, as in Nason and Rogers(2002). The most important difference in identification between this essay and the existing literature is that this paper allows the world real interest rate to vary stochastically and

[12]The underlying fact that consumption is determined by permanent net output makes the impact response of consumption be given as the discounted long-run response of the first difference of log net output. See, for example, Quah(1990).

combine the small open economy assumption with the stochastically varying world real interest rate to identify global and country-specific shocks. In this essay, as implied by the small open economy assumption, country-specific shocks are identified as shocks that are orthogonal to the world real interest rate in either the short-run or the long-run. Furthermore, country-specific shocks are decomposed into permanent and transitory components by Blanchard and Quah's(1989) long-run restriction.

To see this, consider a stationary column vector $X_t = [\widetilde{r}_t \; \widetilde{\Delta \ln NO_t} \; \widetilde{CA_t/NO_t}]'$. Let the probability distribution of the vector X_t be characterized by a p-th order unrestricted VAR. Since the vector X_t is stationary, it has a Wold-Vector Moving Average (VMA) representation, VMA(∞),

$$X_t = C(L)\upsilon_t \tag{2.15}$$

where $C(L)$ is an invertible, infinite-order matrix polynomial with respect to the lag operator L, and in particular the coefficient matrix of L^0 is the identity matrix. The reduced-form disturbance vector υ_t has a symmetric positive definite variance-covariance matrix Σ.

Stacking eqs.(2.11), (2.12) and (2.13) vertically implies that the vector X_t has the following structural VMA (SVMA) representation:

$$\begin{bmatrix} \widetilde{r}_t \\ \widetilde{\Delta \ln NO_t} \\ \widetilde{CA_t/NO_t} \end{bmatrix} = \begin{bmatrix} \Gamma^r_g(L) & \Gamma^r_{cp}(L) & \Gamma^r_{cs}(L) \\ \Gamma^{no}_g(L) & \Gamma^{no}_{cp}(L) & \Gamma^{no}_{cs}(L) \\ \Gamma^{ca}_g(L) & \Gamma^{ca}_{cp}(L) & \Gamma^{ca}_{cs}(L) \end{bmatrix} \begin{bmatrix} \epsilon^g_t \\ \epsilon^{cp}_t \\ \epsilon^{cs}_t \end{bmatrix},$$

or simply

$$X_t = \Gamma(L)\epsilon_t \tag{2.16}$$

where ϵ_t is the structural shock vector given as $\epsilon_t = [\epsilon_t^g\ \epsilon_t^{cp}\ \epsilon_t^{cs}]'$. In particular, following the standard exercise in the SVAR literature, this essay assumes that the variance-covariance matrix of the structural shock vector is given as the identity matrix: $E\epsilon_t\epsilon_t' = I$[13].

The small open economy assumption (2.14) implies $\Gamma_{cp}^r(L) = \Gamma_{cs}^r(L) = 0$ in the SVMA (2.16). This means that any country-specific shock has no influence on variations in the world real interest rate at any forecast horizons. Moreover, to decompose country-specific shocks into permanent and transitory components, this paper imposes on the SVMA (2.16) a restriction that the country-specific transitory shock ϵ_t^{cs} has no long-run effect on log of net output. This long-run restriction is given as

$$\Gamma_{cs}^{no}(1) = 0. \quad \text{(Long-Run Restriction)} \tag{2.17}$$

Imposing the small open economy assumption (2.14) and the long-run restriction (2.17) makes the impact and long-run matrices, $\Gamma(0)$ and $\Gamma(1)$, of the SVMA (2.16) be

$$\Gamma(0) = \begin{bmatrix} \Gamma_g^r(0) & 0 & 0 \\ \Gamma_g^{no}(0) & \Gamma_{cp}^{no}(0) & \Gamma_{cs}^{no}(0) \\ \Gamma_g^{ca}(0) & \Gamma_{cp}^{ca}(0) & \Gamma_{cs}^{ca}(0) \end{bmatrix}, \tag{2.18}$$

and

$$\Gamma(1) = \begin{bmatrix} \Gamma_g^r(1) & 0 & 0 \\ \Gamma_g^{no}(1) & \Gamma_{cp}^{no}(1) & 0 \\ \Gamma_g^{ca}(1) & \Gamma_{cp}^{ca}(1) & \Gamma_{cs}^{ca}(1) \end{bmatrix}. \tag{2.19}$$

[13]That is, the structural shocks are orthogonal at all leads and lags, and each shock has a unit variance. Therefore, in this essay, the impulse response function of a variable is interpreted as the response to a unit standard error shock.

Notice that the SVMA with the impact and long-run matrices (2.18) and (2.19) is overidentified. To see this, comparing the reduced-form VMA (2.15) with the SVMA (2.16) immediately provides the following relationships:

$$\Sigma = \Gamma(0)\Gamma(0)' \tag{2.20}$$

and

$$C(L)\Gamma(0) = \Gamma(L). \tag{2.21}$$

Moreover eq.(2.21) can rewrite eq.(2.20) as

$$\Sigma = C(1)^{-1}\Gamma(1)\Gamma(1)'C(1)'^{-1}. \tag{2.22}$$

Given estimates of Σ and $C(1)$, there are six linear independent equations and nine unknowns in eq.(2.22). Therefore, in general, three additional restrictions are needed for the SVMA (2.16) to be just-identified. On the other hand, the small open economy assumption (2.14) and the long-run restriction (2.17) impose an infinite number of restrictions on the coefficients in the SVMA (2.16): two impact restrictions, three long-run restrictions, and an infinite number of restrictions on IRFs. Since three restrictions are needed to just-identify the structural parameters, the SVMA (2.16) is an overidentified system. Following the identification strategy examined by King and Watson(1997) and Nason and Rogers(2002), this essay investigates two different identification schemes consisting of three restrictions from all the overidentifying restrictions in order to just-identify the system, and checks the robustness of the empirical results by comparing two identification schemes.

The first identification comes from the lower triangularity of the long-run matrix (2.19). The maintained assumptions in this paper provide three long-run restrictions.

The zero restrictions on the (1, 2)th and (1, 3)th elements of $\Gamma(1)$ reflect the small open assumption that requires country-specific permanent and transitory shocks to have no *long-run* effect on the world real interest rate, respectively. The zero restriction on the (2, 3)th element of $\Gamma(1)$ implies that a country-specific transitory shock has no *long-run* effect on log of net output, which is explicitly shown as the long-run restriction (2.17). Therefore, the lower triangular long-run matrix (2.19) is just-identified and the impact matrix can be recovered through eq.(2.21). Hereafter, this Blanchard and Quah's (1989) style identification is called *identification scheme I.*

Another identification scheme in this paper exploits together two impact restrictions in eq.(2.18) and the long-run restriction (2.17). The zero restrictions on the (1, 2)th and (1, 3)th elements of $\Gamma(0)$ reflect the small open assumption that requires country-specific permanent and transitory shocks to have no *instantaneous* effect on the world real interest rate. The zero restriction on the (2, 3)th element of $\Gamma(1)$ implies that a country-specific transitory shock has no long-run effect on log of net output[14].

Notice that the long-run restriction (2.17) can be rewritten as an impact restriction. To show this, let $A_{i,j}$ denote the (i,j)th element in any matrix A. The zero restriction on the $(2,3)$th element in $\Gamma(1)$ together with the zero restriction on the (1, 3)th element in $\Gamma(0)$ implies the restriction

$$C(1)_{2,2}\Gamma(0)_{2,3} + C(1)_{2,3}\Gamma(0)_{3,3} = 0. \tag{2.23}$$

Since $C(1)_{2,2}$ and $C(1)_{2,3}$ are estimated, eq.(2.23) can be considered as an impact re-

[14]The reason for choosing this long-run restriction from the others is that the restriction is essential for decomposing country-specific shocks into the permanent and transitory components.

striction. Together with the two impact restrictions shown in $\Gamma(0)$, eq.(2.23) makes it possible to just-identify $\Gamma(0)$ in eq.(2.18). Hence, the second identification scheme of this paper follows Galí's(1992) method that exploits the impact and long-run restrictions in concert. Hereafter, this identification is referred to as *identification scheme II.* Table 2.3 summarizes the two identification schemes of this essay.

2.4 Empirical Results

This section discusses the data, estimation methods, tests, and empirical results of this essay.

2.4.1 Data and Reduced-Form VAR Estimation

This essay studies two proto-type small open economies, Canada and the U.K. All data used in this essay are quarterly, span the period Q1:1960-Q4:1997, and are seasonally adjusted at annual rates. The estimation is based on the Q2:1963-Q4:1997 sample, with data prior to Q2:1963 used to construct lags. The world real interest rate is a weighted average of *ex ante* real interest rates across the G-7 economies. This follows the way in which Barro and Sala-i-Martin(1990) and Bergin and Sheffrin(2000) construct r_t. Net output and the current account are generated from the appropriate national accounting data. Appendix A.4 provides detailed information on the source and construction of the data.

The standard augmented Dickey-Fuller (ADF) tests provide evidence that the vector

X_t follows a stationary process[15]. Since the VMA (2.15) is invertible, it has an infinite-order VAR representation. The infinite-order VAR is approximated by truncating at a finite lag length. To select an optimal lag length, both the AIC and BIC criteria are calculated with a maximum lag length of fifteen. Both criteria select a lag length of one for each country. The first-order reduced-form VAR (RFVAR), $X_t = BX_{t-1} + v_t$, is estimated by OLS. Let $\hat{B}$, $\hat{\Sigma}$ and $C\hat{(1)}$ denote the estimates of the RFVAR coefficient matrix B, the variance-covariance matrix Σ and the implied infinite sum of the VMA coefficient matrices $C(1) \equiv [I_3 - B]^{-1}$ through the following analysis.

2.4.2 Joint Test of the PVM's Restrictions

Before estimating the SVMA (2.16), this essay conducts the traditional joint test of the cross-equation restrictions the PVM (2.10) imposes on the RFVAR, by following Sheffrin and Woo(1990), Otto(1992), Ghosh(1995) and Bergin and Sheffrin(2000). Let a 1×3 vector $\mathbf{e}_i$ be the ith row of the 3×3 identity matrix I_3. The PVM (2.10) then implies the following cross-equation restrictions on the RFVAR coefficient matrix $\hat{B}$ conditional on the parameters b, c, κ and σ:

$$\mathbf{e}_3 = \mathbf{e}_1 \left\{ b + [(\sigma - 1)c + 1]\kappa\hat{B}[I_3 - \kappa\hat{B}]^{-1} \right\} - \mathbf{e}_2\kappa\hat{B}[I_3 - \kappa\hat{B}]^{-1}. \tag{2.24}$$

[15]This essay constructs the demeaned series of the world real interest rate, the change in log of net output and the current account ratio, i.e. $\widetilde{r}_t$, $\widetilde{\Delta \ln NO_t}$ and $\widetilde{CA_t/NO_t}$; and perform unit root tests for them based on the ADF τ-test. Appendix A.5 summarizes the method and the results of the unit roots tests. The ADF tests reject the unit root null in all series at least at the 5 percent significance level. From this evidence, the series $\widetilde{r}_t$, $\widetilde{\Delta \ln NO_t}$ and $\widetilde{CA_t/NO_t}$ are considered to be stationary in the following analysis.

To test the cross-equation restrictions (2.24), define a statistic $\mathbf{k}(B)$ such that

$$\mathbf{k}(B) \equiv \mathbf{e}_1 \left\{b + [(\sigma-1)c+1]\kappa B[I_3 - \kappa B]^{-1}\right\} - \mathbf{e}_2 \kappa B[I_3 - \kappa B]^{-1} - \mathbf{e}_3.$$

Under the null of $\mathbf{k}(B_0) = 0$, the Wald statistic

$$\mathcal{W} = \mathbf{k}(\hat{B}) \left[\frac{\partial \mathbf{k}(\hat{B})}{\partial B} \hat{V} \frac{\partial \mathbf{k}(B)'}{\partial \hat{B}}\right]^{-1} \mathbf{k}(\hat{B})' \tag{2.25}$$

asymptotically follows the χ^2 distribution with the third degree of freedom.

Recall that the Wald statistic $\mathcal{W}$ is constructed conditional on the parameters κ, c, b, and σ. This paper calibrates κ, c, and b directly from the data; $\kappa = 0.993$, $c = 0.983$, $b = -0.712$ for Canada; $\kappa = 0.990$, $c = 0.988$, $b = 0.377$ for the U.K. The elasticity of intertemporal substitution σ is calibrated by matching the predictions of the PVM (2.10) on the current account-net output ratio with the actual series. The predictions $\mathcal{CA}/\mathcal{NO}_t^f$ are constructed as a function of σ by

$$\mathcal{CA}/\mathcal{NO}_t^f = \mathcal{F}(\sigma) X_t \tag{2.26}$$

where

$$\mathcal{F}(\sigma) = \mathbf{e}_1 \left\{b + [(\sigma-1)c+1]\kappa \hat{B}[I_3 - \kappa \hat{B}]^{-1}\right\} - \mathbf{e}_2 \kappa \hat{B}[I_3 - \kappa \hat{B}]^{-1}.$$

The elasticity of intertemporal substitution σ is then calibrated by minimizing the mean squared error of the prediction :

$$T^{-1} \sum_{t=1}^{T} \left(CA/NO_t - \mathcal{CA}/\mathcal{NO}_t^f\right)^2 = T^{-1} \sum_{t=1}^{T} [CA/NO_t - \mathcal{F}(\sigma) X_t]^2$$

The resulting σ is 0.001 for Canada, and 0.08 for the U.K. The small values of the elasticity of intertemporal substitution are close to the estimates of Bergin and Sheffrin(2000) in their two goods model. The first four rows of Table 2.4 summarize the calibrations in this paper.

The last two rows of Table 2.4 report the Wald statistics (2.25) for the joint test of the cross-equation restrictions (2.24), and the corresponding p-values based on the χ^2 distribution for Canada and the U.K. In the two economies, the Wald statistics are so large that the cross-equation restrictions are jointly rejected at any standard significance level. Figures 2.1(a) and (b) show the actual series of the current account-net output and the PVM's predictions $\mathcal{CA}/\mathcal{NO}_t^f$ for Canada and the U.K., respectively. Even though σ is chosen to minimize the mean squared error, the PVM's predictions are much smoother than the actual series in Canada. The result is much better in the U.K., but the PVM still cannot capture the huge deficits happened in the end of the 1980s.

In summary, the cross-equation restrictions the PVM imposes on the RFVAR is jointly rejected across the two economies. The predictions of the PVM closely tracks the U.K. series of the current account-net output ratio with the exceptional periods of the end of the 1980s, while those are still too smooth to match the Canadian series. This result suggests that especially in Canada, the source of the rejection of the PVM be attributed to something other than the fluctuations in net output as well as the world real interest rate.

2.4.3 SVAR Estimation and Test Statistics

The OLS estimates $\hat{\Sigma}$ and $C(\hat{1})$ make it possible to identify the impact matrix $\Gamma(0)$ with each of the identification schemes. This paper recovers the impact matrix $\Gamma(0)$ by the full information maximum likelihood (FIML) procedure[16].

Tests of Hypotheses 1, 2 and 3 are constructed as the Wald statistics. To do that, this essay exploits the fact that all restrictions provided by the hypotheses can be rewritten as linear restrictions on the impact matrix $\Gamma(0)$. Let $[A]_i^r$ and $[A]_i^c$ denote the i th row and column vectors of a matrix A, respectively. Furthermore, let R and R_i for an index $i \geq 0$ be 1×3 row vectors such that

$$R_i = \frac{CA}{CA/NO}[C_i]_3^r + CA \sum_{s=0}^{i} [C_s]_2^r$$

and

$$R = [C(\kappa)_{2,1} \quad C(\kappa)_{2,2} - 1 \quad C(\kappa)_{2,3} + 1]$$

where C_i, CA/NO, CA and $C(\kappa)_{i,j}$ denote the coefficient matrix of L^i in the VMA (2.15), the mean of the current account-net output ratio, the mean of the current account, and the (i, j)th element of the matrix $C(\kappa)$, respectively. It can be then easily shown that the statistics $\mathcal{H}_g^i$, $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$ are given as $\mathcal{H}_g^i = R_i[\Gamma(0)]_1^c$ for $i \geq 0$, $\mathcal{H}_{cp} = R[\Gamma(0)]_2^c$ and $\mathcal{H}_{cs} = R[\Gamma(0)]_3^c$. Appendix A.6 discusses derivation of the statistics in detail.

Let $\mathcal{W}_1$, $\mathcal{W}_2$ and $\mathcal{W}_3$ denote the Wald statistics for the null hypotheses $\mathcal{H}_g^0 = 0$, $\mathcal{H}_{cp} = 0$, and $\mathcal{H}_{cs} = 0$. In addition, let $\mathcal{W}_4$ and $\mathcal{W}_5$ be the Wald statistics for the joint null hypotheses $\mathcal{H}_g^0 = \mathcal{H}_{cp} = \mathcal{H}_{cs} = 0$ and $\mathcal{H}_g^0 = \mathcal{H}_g^1 = \mathcal{H}_g^2 = \mathcal{H}_g^3 = 0$. In particular, $\mathcal{W}_5$ is based on the null hypothesis that a global shock does not matter for the current

[16]Because of the lower triangular long-run matrix a numerical maximization procedure is not needed to recover the impact matrix in identification scheme I. In identification scheme II, the impact matrix is numerically recovered through the FIML procedure. See Amisano and Giannini(1997) and Hamilton(1994, chapter 11) for the FIML estimation of the SVAR models.

account up to a year after impact. For example, the Wald statistic $\mathcal{W}_1$ for Hypothesis 1 is constructed as

$$\mathcal{W}_1 = \hat{\mathcal{H}}_g^0 \left[\frac{\partial \mathcal{H}_g^0}{\partial \hat{B}} \hat{V} \frac{\partial \mathcal{H}_g^0}{\partial \hat{B}}' \right]^{-1} \hat{\mathcal{H}}_g^0$$

where $\hat{\mathcal{H}}_g^0$ is the point estimate of the statistic $\mathcal{H}_g^0$. The asymptotic theory states that $\mathcal{W}_1$ is distributed the χ^2 distribution with one degree of freedom[17].

To derive the Wald statistic $\mathcal{W}_4$ for the joint null hypothesis $\mathcal{H}_g^0 = \mathcal{H}_{cp} = \mathcal{H}_{cs} = 0$, construct a row vector $\lambda = [\hat{\mathcal{H}}_g^0 \quad \hat{\mathcal{H}}_{cp} \quad \hat{\mathcal{H}}_{cs}]$. Then the Wald statistic for the joint null is given as

$$\mathcal{W}_4 = \lambda \left[\frac{\partial \lambda}{\partial \hat{B}} \hat{V} \frac{\partial \lambda}{\partial \hat{B}}' \right]^{-1} \lambda'.$$

According to the asymptotic theory, $\mathcal{W}_4$ asymptotically follows $\chi^2(3)$. The same argument is applicable for the construction of the Wald statistic $\mathcal{W}_5$.

As in the standard exercise of the SVAR literature, the IRFs and the FEVDs of the endogenous variables to the identified shocks are estimated. The empirical standard errors of the IRFs and the FEVDs are calculated by generating 10,000 nonparametric bootstrapping replications based on the reduced-form disturbances. The 10,000 replications of the statistics $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$ generated by the bootstrapping exercise provide the empirical joint distribution of $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$.

[17]Notice that the statistics $\mathcal{H}_g^i$, $\mathcal{H}_{cp}$, and $\mathcal{H}_{cs}$ are constructed by the IRFs from the just-identified SVAR. Since the IRFs are nonlinear functions of the RFVAR parameters, as shown in Hamilton(1994, section 11.4), the asymptotic standard errors of the statistics $\mathcal{H}_g^i$, $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$ are obtained by using the asymptotic standard errors of the RFVAR parameters and the Delta method. Similarly, the asymptotic χ^2 statistics for the hypotheses can be constructed from knowledge of the asymptotic distribution of the RFVAR parameters. Of course, the asymptotic χ^2 test depends on identification, as the IRFs do.

2.4.4 Impulse Response Analysis

Recall from the introduction that the basic response predictions of the intertemporal approach and the PVM are (i) a global shock does not matter for the current account at all forecast horizons, (ii) a country-specific permanent shock to net output has no or a negative impact on the current account, and (iii) a country-specific transitory shock to net output has a positive impact on the current account. This subsection examines the IRFs of the current account to check whether or not these predictions are supported by the Canadian and the U.K. data.

Figure 2.2 shows the IRFs of the current account across the two economies under identification Scheme I. In each window, the dark line represents the point estimate and the dashed lines exhibit 95% confidence bands constructed by a nonparametric bootstrapping exercise. The results of the impulse response analysis are summarized as follows:

In Canada and the U.K.

- The IRFs of the current account to a global shock are not significant at any of the 40 periods after impact.[18]
- The IRFs of the current account to a country-specific permanent shock are positive but insignificant.
- The IRFs of the current account to a country-specific transitory shock are positive and significant. The positive responses remain significant for at least three years.

[18]A caveat is that the IRFs and the associated confidence bands are not a joint test statistic for hypothesis 1. They provide pointwise information about the response of the current account to a global shock.

As reported in Table 2.2, the results support the basic predictions of the intertemporal approach and the PVM: no response of the current account to a global shock, no response to a country-specific permanent shock, and a positive response to a country-specific transitory shock. Figure 2.3 shows the IRFs of log of net output in Canada and the U.K. under identification scheme I. Notice that the responses of log of net output to a country-specific permanent shock are almost flat after jumps at impact. This observation is consistent with the PVM's prediction that if a country-specific shock is random walk, the current account has no response to the shock.

The impulse response analysis, therefore, *qualitatively* supports the basic predictions of the intertemporal approach and the PVM: The predicted shapes of the impulse responses of the current account to the three shocks are consistent with the data. Although not reported, the same results are also observed even under identification scheme II[19]. Hence, this empirical result is robust for the two identification schemes.

2.4.5 Testing the Hypotheses

Notice that the *qualitative* validity of the predictions does not necessarily mean that the *quantitative* requirements of the intertemporal approach and the PVM - the cross-equation restrictions imposed on the SVMA - are supported at the same time. Testing Hypotheses 1-3 provides information about the validity of the cross-equation restrictions.

Tables 2.5(a) and (b) report the results of the asymptotic Wald tests under identification schemes I and II, respectively. Each table shows the Wald statistics and the

[19]The results under identification scheme II are available as Figures A.1, A.2, and A.3 and Table A.2.

corresponding p-values generated by asymptotic χ^2 distributions for the null hypotheses. The following results are observed:

- The single null $\mathcal{H}_g^0 = 0$ is not rejected in Canada and the U.K. in identification scheme I, but rejected in the two economies in identification scheme II.
- The single null $\mathcal{H}_{cp} = 0$ is not rejected in Canada and the U.K. across the two identification schemes.
- The single null $\mathcal{H}_{cs} = 0$ is not rejected in Canada and the U.K. across the two identification schemes.
- The joint null $\mathcal{H}_g^0 = \mathcal{H}_{cp} = \mathcal{H}_{cs} = 0$ is rejected in Canada and the U.K. across the two identification schemes.
- In Canada, the joint null $\mathcal{H}_g^0 = \mathcal{H}_g^1 = \mathcal{H}_g^2 = \mathcal{H}_g^3 = 0$ is rejected across the two identification schemes.

These results lead to the following inferences: (i) the validity of the hypothesis that the current account does not respond to a global shock is sensitive to the identification and the economy being studied, (ii) the PVM succeeds in making quantitative predictions on the impact responses of the current account to country-specific shocks, and (iii) the response predictions of the intertemporal approach and the PVM are jointly rejected.

Recall that the IRFs support the hypothesis that the current account do not respond to a global shock. From the two different tests, it is safe to say there is no robust evidence for this hypothesis. This confirms the inference drawn by Nason and Rogers(2002) that the hypothesis is sensitive to identification. On the other hand, the IRFs and the asymptotic Wald tests consistently support the predictions of the PVM on the responses of the current account to the country-specific shocks. Finally, the observation that the predictions of the PVM on the impact responses of the current account to the three shocks are jointly rejected reinforces the rejection of the cross-equation restrictions the PVM imposes on the RFVAR; see section 2.4.2.

A potential weakness of the Wald test is that it depends on the asymptotic χ^2 distribution, and with a small sample the Wald statistic does not necessarily follow the χ^2 distribution. Figure 2.4 shows the scatter plots of 10,000 pairs of the statistics $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$ replicated by nonparametric bootstrapping resamples under identification scheme I. In each window, the darkest square represents the point estimate and the joint null is given by the origin. Observe that in the two economies the scatter plots have strikingly similar shapes and almost all replicated pairs are concentrated on the upper regions of the windows. Therefore, the empirical distributions of the statistics $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$ provide information against the null hypothesis $\mathcal{H}_{cs} = 0$.

By construction, the observation that the empirical joint distribution of $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$ is concentrated in the upper region means that in Canada and the U.K.,

$$\Gamma^{ca}_{cs}(0) > \Gamma^{no}_{cs}(0) - \Gamma^{no}_{cs}(\kappa).$$

Under Hypothesis 3 the above equation must be satisfied with equality. Hence, this paper reveals that *in Canada and the U.K., the impact responses of the current account-net output ratio to a country-specific transitory shock are too large to support the PVM.* Again the same observation is obtained even in identification scheme II.

Since the calibrated values of κ in the two economies are very close to one (see Table

2.3), the long-run restriction (2.17) requires the term $\Gamma_{cs}^{no}(\kappa)$ to be almost zero. Hence the above inequality says that the impact response of the current account to a country-specific transitory shock is greater than that of net output. This observation is actually a puzzle. The current account identity requires that the impact response of the current account to a country-specific shock be the difference between the responses of net output and consumption. Thus, the greater response of the current account to a country-specific transitory shock than the response of net output implies that *consumption responds negatively to a positive country-specific shock to net output.* The basic intertemporal approach to the current account is not built on the prediction that consumption responds negatively to an positive income shock. This puzzle is a challenge to the current account literature.

2.4.6 Forecast Error Variance Decomposition Analysis

Another way to examine the effects of the three shocks on the current account is to look at the forecast error variance decompositions (FEVDs) of the current account. The FEVD provides information about the share of current account fluctuations that can be explained by an identified shock.

Table 2.6 provides the FEVDs of the current account attributed to the three shocks in Canada and the U.K. under identification scheme I. The table shows that at impact a country-specific transitory shock can explain almost 70 % of fluctuations in the current account across the two economies. Even at a year after impact, the shock can significantly explain 81 % and 71 % of fluctuations in the current account in Canada and the U.K., respectively. Therefore, the country-specific transitory shock can be considered as the dominant driving force of the current account in the short run.

A striking fact revealed by the FEVDs is that *even in the long-run the country-specific transitory shock dominates fluctuations in the current account in the two small open economies.* For example, at 40 quarters (10 years) after impact, about 80 % of fluctuations in the Canadian current account is attributed to the country-specific transitory shock. Similarly, at the same forecast horizon, the shock explains 72 % of fluctuations in the U.K. current account. This observation is also obtained under identification scheme II.

The result that country-specific transitory shocks dominate current account fluctuations not only in the short run but the long run as well echoes the finding of Nason and Rogers (2002). In their SVAR approach to study the joint dynamics of investment and the current account, they report the persistent dependence of the current account on country-specific transitory shocks across the G-7 economies. As they argue, at present there is no consensus intertemporal model that generates persistence in the current account to country-specific transitory shocks.

Table 2.7 shows the FEVDs of log of net output. Observe that in the two economies a country-specific transitory shock cannot significantly explain fluctuations in log of net output at any forecast horizons. The observation that a country-specific transitory shock having no significant effect on net output dominates fluctuations in the current account in the short run as well as the long run is the second puzzle of this essay. This observation violates the standard PVM as well as the augmented PVM with the stochastic world real interest rate because in these models current account fluctuations should be explained by a country-specific shock that dominates the fluctuations in net output. Combining with the joint rejection of the full cross-equation restrictions the PVM (2.10) imposes on the RFVAR, this puzzling observation suggests the importance of the consumption-tilting motive induced by country-specific shocks, rather than the consumption-smoothing behavior, to explain current account movements in the small open economies.

2.5 Conclusion

When the world real interest rate is allowed to vary stochastically, the intertemporal approach and its well-known closed-form solution, the PVM of the current account, jointly provide new identification for a SVAR. The small open assumption of the intertemporal approach gives the SVAR a restriction to identify global and country-specific shocks because the assumption requires any country-specific shocks to be orthogonal to the world real interest rate. By exploiting this orthogonality condition as well as the Blanchard and Quah's decomposition, this essay is able to develop two identifying schemes for the SVAR and recover its global, country-specific permanent and country-specific transitory shocks.

The identified SVAR based on the Canadian and the U.K. data then yields tests of the predictions the intertemporal approach and the PVM make on the responses of the current account to the three shocks. A part of the results of these tests reaffirms the result of the past studies. Even though the test jointly rejects the PVM's cross-equation restrictions on the RFVAR, the intertemporal approach and the PVM are still useful to explain some aspects of current account movements. In fact, the IRFs of this essay are consistent with the theoretical counterparts of the intertemporal approach and the PVM. Thus, this essay contributes to the current account literature by providing further evidence that small open economy models based on forward-looking economic agents are useful to understand current account dynamics.

This paper reveals two puzzles that challenge the intertemporal approach. First, the response of the current account-net output ratio to a country-specific transitory shock is too large to support the PVM. This observation in turn draws a puzzling inference that consumption negatively responds to a positive income shock. The second puzzling aspect this paper observe is that current account fluctuations are dominated by country-specific transitory shocks that explain almost none of the fluctuations in net output in the short run as well as the long run. This puzzle implies that the consumption-tilting motive induced by country-specific shocks, rather than the consumption-smoothing behavior that the past studies emphasize, is important to account for current account movements. These failures of the intertemporal approach to the current account suggest that more research about its theoretical structure is needed. For example, more general utility functions, non-tradable goods and endogenous risk premia may yield resolution of these puzzles. Seeking valid modifications of the basic intertemporal approach is a future task of the current account literature.

Chapter 3

Habit Formation, the World Real Interest Rate, and the Present Value Model of the Current Account

3.1 Introduction

A small open economy model endowed with rational, forward-looking agents serves as a benchmark for studying current account dynamics in the recent literature. This model, as known as the intertemporal approach to the current account, stresses the consumption-smoothing behavior of economic agents in the determination of the current account in a small open economy[1]. When they expect changes in future income, forward-looking agents smooth their consumption by borrowing or lending in international financial mar-

[1] Obstfeld and Rogoff(1995) provide a recent and most detailed survey of the intertemporal approach to the current account.

kets and hence by generating current account movements. This role of consumption-smoothing behavior in current account determination is clearly expressed by the present value model (PVM) of the current account, which is a closed-form solution of the intertemporal approach. For example, the PVM predicts that the current account moves into deficit when a country's income is expected to decline temporarily, while no change in the current account occurs if the decline in income is expected to be permanent[2].

Many empirical studies including Sheffrin and Woo(1990), Otto(1992), Ghosh(1995) and Bergin and Sheffrin(2000), however, fail to find empirical support for the standard PVM of the current account in postwar data of the G-7 economies. The cross-equation restrictions the standard PVM imposes on the unrestricted vector autoregression (VAR) are statistically rejected for all of the G-7 economies except the U.S. Moreover, the forecasts of the standard PVM are too smooth to track actual current account movements. The empirical failures of the standard PVM have led some researchers to explore the role of consumption-tilting motives in current account movements: the current account might be adjusted to factors that deviate consumption away from the random-walk, permanent income level, for example, stochastic variations in the world real interest rate[3].

[2]A crucial prediction of the PVM is that only country-specific shocks matter for the current account of a small open economy. A global shock does not give a small open economy an opportunity to borrow or lend in international financial markets because all economies have identical preferences, technologies and endowments and hence react to a global shock symmetrically. All that occurs is that the world real interest rate adjusts to the global shock.

[3]For example, by using a structural VAR approach to identify global and country-specific shocks, the second chapter of this thesis shows that almost all of Canadian current account movements are dominated by country-specific shocks unrelated to variations in the smoothed, permanent income. This

One way to introduce the consumption-tilting motive into the standard PVM is habit formation in consumption. Habit formation makes optimal consumption decisions depend not only on permanent income but also on past consumption. The household tends to maintain its past consumption level against unexpected shocks to permanent income; therefore, habit formation makes consumption smoother and more sluggish than in the basic permanent income hypothesis (PIH). The sluggishness of consumption in turn implies more volatile current account movements than the standard PVM predicts. Gruber(2000) uses habit formation in consumption to improve the ability of the PVM to track actual current account movements in the postwar quarterly data of the G-7 economies, of the Netherlands, and of Spain. He concludes that habit formation plays an important role in determining current account dynamics.

This essay shows that the habit-forming PVM is *observationally equivalent* to the canonical PVM augmented with a serially-correlated transitory consumption shock. In other words, given the information set studied by Gruber(2000), the two PVMs yield the same values of the sample test statistics. Because of this identification problem, Gruber's tests of the habit-forming PVM are not informative to detect the role of habit formation in current account movements.

In this essay, the source of the serially-correlated transitory consumption shock is specified with stochastic movements in the world real interest rate because of two reasons[4]. First, the stochastic real interest rate is a well-known way to introduce a

result empirically suggests the importance of consumption-tilting motives in Canadian current account movements.

[4]Another sources of the transitory consumption shocks are a transitory government expenditure

consumption-tilting motive into the PVM of the current account as well as the permanent income hypothesis of consumption[5]. Expected future changes in the world real interest rate tilt the consumption path away from the random-walk, permanent income level and, as a result, introduce the consumption-tilting component into the PVM of the current account. Second, recent studies on small open economy-real business cycle (SOE-RBC) model, Blankenau, Kose and Yi(2001) and Nason and Rogers(2003), provide evidence that the world real interest rate shocks play a crucial role in explaining net trade balance/current account movements in a small open economy.

To solve the identification problem, this essay conducts Monte Carlo experiments based on a small open-real business cycle model (SOE-RBC) that incorporates with either habit formation or the stochastic world real interest rate. To this end, the SOE-RBC model of Nason and Rogers(2003) is extended by introducing habit formation. The extended model is then used to generate artificial data that yield theoretical distributions of "moments" to be explained in this essay.

As in a standard calibration exercise, moments of the artificial data generated by SOE-RBC models are compared with their sample counterparts. However, as exam-

shock affecting the utility function and the stochastic terms of trade.

[5]See Campbell and Mankiw(1989) for tests of the permanent income hypothesis (PIH), and Bergin and Sheffrin(2000) and Kano(2003) for tests of the current account PVM. In particular, Bergin and Sheffrin(2000) extend the standard PVM by introducing stochastic variations in the world real interest rates as well as real exchange rates, which yield a serially-correlated transitory consumption component independent of permanent income. They observe that the extension improves the PVM prediction in Canada. The second chapter also shows the PVM of the current account in the presence of the stochastic world real interest rate using a different approach.

ined by Nason and Rogers(2003), the "moments" this essay studies are not standard unconditional variances and covariances of the sample. Instead, they are the sample statistics conditional on the habit-forming and standard PVMs of the current account: the sample estimate of the habit-formation parameter, the cross-equation restrictions implied by the habit-forming and standard PVMs, and the current account forecasts of the habit-forming and standard PVMs.

It is worth noting that by construction, the theoretical distributions have the null hypothesis of the underlying SOE-RBC model as the data-generating process (DGP) of the moments. This essay generates the theoretical distributions under two different null hypotheses. First, setting the structural parameters of the SOE-RBC model to rule out stochastic variations of the world real interest rate derives the theoretical distributions under the null of the SOE-RBC model with habit formation. Second, setting the habit parameter equal to zero provides the theoretical distributions under the null of the SOE-RBC model with the stochastic world real interest rate. The two different SOE-RBC models are evaluated from the viewpoint of classical statistics; that is to say, the sample statistics are used as critical values to derive empirical p-values. For example, if a sample statistic drops into the five percent tail of the theoretical distribution, the null is rejected at the five percent significance level.

The results from the Monte Carlo experiments support the SOE-RBC model with stochastic world real interest rates. Although the SOE-RBC model with habit formation can replicate a part of the empirical facts of the habit-forming PVM, the SOE-RBC model with the stochastic world real interest rate mimics all the relevant sample moments. The superiority of the SOE-RBC model with stochastic world real interest rates casts doubt on habit formation as the significant source of the consumption-tilting behavior needed to explain Canadian current account movements.

The structure of this essay is as follows. The next section introduces the habit-forming PVM and discusses the observational equivalence problem. The sample moments conditional on the habit-forming and standard PVMs are reported in section 3.3. Section 3.4 introduces the SOE-RBC models of this essay to mimic the sample moments. Section 3.5 reports the results of the Monte Carlo experiments. Concluding remarks are made in section 3.6.

3.2 The PVMs with Habit Formation and Transitory Consumption: Observational Equivalence

Gruber (2000) extends the standard PVM by introducing habit formation in consumption. Let C_t, B_t and NO_t denote consumption, international bond holding, and net output at period t, respectively. As in the standard literature, net output, which is defined as output minus domestic investment minus government expenditure, follows a nonstationary process having a country-specific, random-walk technology shock as the driving force[6]. The period utility function is specified as a quadratic form

$$u(C_{t+i} - h\bar{C}_{t+i-1}) = C_{t+i} - h\bar{C}_{t+i-1} - \frac{1}{2}(C_{t+i} - h\bar{C}_{t+i-1})^2, \quad 0 < h < 1$$

[6]The basic SOE-RBC model, which is well-known as the intertemporal approach to the current account, is a single-shock model containing a country-specific, unit-root technology shock. See Obstfeld and Rogoff(1995), Glick and Rogoff(1995), and Nason and Rogers(2003). Under this assumption, the intertemporal approach has the standard PVM as a closed-form solution.

where h represents the habit parameter. $\bar{C}_t$ represents aggregate consumption unaffected by any representative household decision. This specification of habit formation is related to *external* habit formation or the *catching up with the Joneses*, as in Abel(1990) and Campbell and Cochrane(1999)[7]. Note that $\bar{C}_t = C_t$ in equilibrium.

The problem the representative household faces is to maximize its expected discounted lifetime utility

$$E_t \sum_{i=0}^{\infty} \beta^i u(C_{t+i} - h\bar{C}_{t+i-1})$$

subject to the budget constraint

$$B_{t+1} = (1+r)B_t + NO_t - C_t$$

where r is the world real interest rate assumed to be constant and equal to the subjective discount rate. In this case, the first-order necessary conditions together with the transversality condition yield an optimal consumption decision rule. Letting ϵ_t denote a disturbance orthogonal to information at period t-1 and adding ϵ_t to the optimal consumption decision rule provide

$$C_t = \left(\frac{h}{1+r}\right) C_{t-1} + \left(1 - \frac{h}{1+r}\right)\left(\frac{r}{1+r}\right)\left[(1+r)B_t + \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t NO_{t+i}\right] + \epsilon_t \tag{3.1}$$

where the equilibrium condition $\bar{C}_t = C_t$ is imposed[8]. With habit formation, consumption is determined by a weighted average of permanent income and past consumption

[7]If habits are *internal*, as in Constantinides(1990), they depend on the household's own consumption and the household takes habits into account when choosing the amount of consumption.

[8]Campbell(1987) argues that a transitory consumption error uncorrelated with lagged information improves the ability of the PIH to fit the U.S. data.

with the weight $h/(1+r)$. This fact makes adjustments of consumption to permanent income shocks more sluggish than in the standard PIH.

Substituting the resulting consumption equation into the current account identity $CA_t \equiv rB_t + NO_t - C_t$ produces the PVM with habit formation

$$CA_t = hCA_{t-1} + \left(\frac{h}{1+r}\right)\Delta NO_t - \left(1 - \frac{h}{1+r}\right)\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_t\Delta NO_{t+i} - \epsilon_t. \quad (3.2)$$

Notice that the current account depends on its own past value. This makes the process of the current account more persistent than in the standard PVMs of Sheffrin and Woo(1990) and Otto(1992). Furthermore, the current account becomes sensitive to the current change in net output: the current account depends on not only the expected present value of future declines of net output but the current change of net output as well. This makes the current account more volatile than in the standard PVM.

An important point is that the present value formula (3.2) is observationally equivalent to the PVM derived from a multiple-shock model. Let C_t^T denote arbitrary transitory consumption that follows an exogenous AR(1) process

$$C_t^T = \rho_c C_{t-1}^T + \omega_t \quad |\rho_c| < 1 \quad (3.3)$$

where C_t^T may be observable or may not, and ω_t is a white noise shock. Assume that consumption C_t is linearly decomposed into the transitory consumption C_t^T and permanent income C_t^P:[9]

$$C_t = C_t^T + C_t^P \quad (3.4)$$

[9]Because the underlying SOE-RBC model has the unique stochastic trend, i.e. the country-specific, permanent, technology shock, it is possible to decompose consumption into a random-walk component C_t^P and a transitory component C_t^T: see King, Plosser and Rebelo(1988).

where permanent income C_t^P is determined by the standard PIH formula

$$C_t^P = \left(\frac{r}{1+r}\right)\left[(1+r)B_t + \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t NO_{t+i}\right]. \tag{3.5}$$

Appendix B.1 shows that the non-habit-forming, multiple-shock model specified by eqs.(3.3), (3.4), and (3.5) has the following present value representation of the current account

$$CA_t = \rho_c CA_{t-1} + \left(\frac{\rho_c}{1+r}\right)\Delta NO_t - \left(1 - \frac{\rho_c}{1+r}\right)\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_t \Delta NO_{t+i} - v_t \tag{3.6}$$

where v_t is a disturbance orthogonal to information at period t-1, which satisfies $E_{t-i}v_t = 0$ for $i \geq 1$.

Notice that the non-habit-forming PVM (3.6) is equivalent to the habit-forming PVM (3.2). Therefore, given the data of CA_t and ΔNO_t, any statistics based on eq.(3.2), for instance, an estimate of h, take the same values as those statistics from eq.(3.6). The habit-forming PVM is observationally equivalent to the non-habit PVM augmented with the AR(1) transitory consumption component. This implies that the statistics based on the habit-forming PVM (3.2) are not informative to identify whether or not habit formation plays an important role in explaining current account movements.

3.3 Sample Moments Conditional on the Habit-Forming and Standard PVMs

This section reports the sample moments conditional on the habit-forming and the standard PVMs. As mentioned in the introduction, this essay considers the sample test statistics of the two PVMs as the sample "moments" explained by SOE-RBC models. The next subsection discusses econometric issues related to estimation and test of the habit-forming PVM. The following subsection reports the sample moments.

3.3.1 Econometric Issues

Gruber(2000) exploits the generalized method of moments (GMM) procedure to estimate the habit parameter h in the habit-forming PVM (3.2). Define a variable $D_t \equiv CA_t - \Delta NO_t - (1+r)CA_{t-1}$ and rewrite the PVM (3.2) as

$$D_t = hD_{t-1} - \epsilon_t + (1+r)\epsilon_{t-1} + e_t \tag{3.7}$$

where ϵ_t, ϵ_{t-1} and e_t are disturbances orthogonal to the information set at period $t-2$, Ω_{t-2} [See Appendix B.2 for the detailed derivation of eq.(3.7).]. Let W_{t-2} denote a $k \times 1$ vector that contains k different variables in Ω_{t-2}. Eq.(3.7) then implies unconditional moment conditions

$$EW_{t-2}(D_t - hD_{t-1}) = 0 \tag{3.8}$$

where E is the unconditional expectation operator. Eq.(3.8) makes it possible to estimate h by the GMM/two step-two stage least square (2SLS) procedure by West(1988). Let $\hat{h}_{2SLS}$ be the 2SLS estimate of h. When $k > 1$, $\hat{h}_{2SLS}$ is overidentified. The J-statistic of Hansen(1982) tests the orthogonality conditions (3.8). Given $k(> 1)$ instruments, the J-statistic is asymptotically distributed χ^2 with $k-1$ degrees of freedom.

This essay proposes a more efficient estimate of the habit parameter than the 2SLS estimate $\hat{h}_{2SLS}$. In addition to the unconditional moment conditions (3.8), other theoretical restrictions the habit-forming PVM imposes on a p-th order bivariate vector autoregressive (VAR) of CA_t and ΔNO_t are used to estimate the habit parameter. Recall that a VAR(p) process has a corresponding first-order representation with a companion matrix $\mathcal{A}$:

$$\mathcal{Y}_t = \mathcal{A}\mathcal{Y}_{t-1} + \mathcal{U}_t \tag{3.9}$$

where $\mathcal{U}_t$ is a $2p \times 1$, zero mean, homoskedastic, serially uncorrelated error vector such that $\mathcal{U}_t \equiv [u_t^{\Delta NO} \quad 0 \quad \cdots \quad 0 \quad u_t^{CA} \quad 0 \quad \cdots \quad 0]'$, and $\mathcal{Y}_t$ is a $2p \times 1$ vector constructed as

$$\mathcal{Y}_t \equiv [\Delta NO_t \quad \Delta NO_{t-1} \quad \cdots \quad \Delta NO_{t-p+1} \quad CA_t \quad CA_{t-1} \quad \cdots \quad CA_{t-p+1}]'.$$

By assumption of the VAR, $\mathcal{Y}_{t-1}$ is orthogonal to the VAR disturbances $U_t = [u_t^{\Delta NO} \; u_t^{CA}]$. That is, the following unconditional moment conditions are satisfied:

$$E\mathcal{Y}_{t-1} \otimes U_t = 0 \tag{3.10}$$

where $\otimes$ is the operator of the Kronecker product.

Define a $1 \times 2p$ vector $\mathbf{e}_i$ that includes zeros except for the ith element equal to 1, i.e.

$$\mathbf{e}_i = [0 \cdots \underbrace{0}_{i-1st} \quad \underbrace{1}_{ith} \quad \underbrace{0}_{i+1st} \cdots 0].$$

The habit-forming PVM (3.2) then implies that under the null hypothesis, the following cross-equation restrictions should be the case:

$$\mathbf{e}_{p+1}\mathcal{A}\mathcal{Y}_t = \mathcal{K}^h\mathcal{A}\mathcal{Y}_t \tag{3.11}$$

where $\mathcal{K}^h$ is a $1 \times 2p$ vector such that

$$\mathcal{K}^h = h\mathbf{e}_{p+2} + \left(\frac{h}{1+r}\right)\mathbf{e}_1 - \left(1 - \frac{h}{1+r}\right)\left(\frac{1}{1+r}\right)\mathbf{e}_1\mathcal{A}\left[I - \left(\frac{1}{1+r}\right)\mathcal{A}\right]^{-1}.$$

Note that the cross-equation restriction (3.11) can be considered as an unconditional moment condition

$$E(\mathbf{e}_{p+1} - \mathcal{K}^h)\mathcal{A}\mathcal{Y}_t = 0. \tag{3.12}$$

Eq.(3.12) holds under the null hypothesis of the habit-forming PVM (3.2).

As a result, if the joint probability distribution of CA_t and ΔNO_t is specified by the unrestricted VAR (3.9), the habit-forming PVM (3.2) yields the unconditional moment conditions (3.10) and (3.12) in addition to (3.8)[10]. Construct a $(4p + k + 1) \times 1$ vector $g_t(\theta)$ such that

$$g_t(\theta) = \begin{bmatrix} W_{t-2}(D_t - hD_{t-1}) \\ \mathcal{Y}_{t-1} \otimes U_t \\ (\mathbf{e}_{p+1} - \mathcal{K}^h)\mathcal{A}\mathcal{Y}_t \end{bmatrix}$$

where θ is a vector constructed by stacking the habit parameter h and the elements of the companion matrix $\mathcal{A}$, i.e. $\theta \equiv [h \quad vec(\mathcal{A})']'$. The sample analogs of the theoretical moment conditions (3.8), (3.10), and (3.12) are given as

$$G(\theta) = T^{-1} \sum_{t=1}^{T} g_t(\theta) = 0$$

where T is the sample number. To obtain an efficient estimate of θ, this essay conducts the two-step GMM procedure of West(1988)[11]. Let $\hat{\theta}_{GMM}$ be the resulting two-step GMM estimate of θ with the asymptotic covariance matrix $\hat{V}_{\theta_{GMM}}$. In this case, the

[10]Gruber(2000) does not use the moment conditions (3.10) and (3.12) to estimate h. This fact makes Gruber's estimation and specification test based only on the overidentifying restrictions (3.8) inefficient since his procedure does not use all of information the model provides potentially.

[11]Appendix B.3 reviews the two-step GMM estimation in detail.

J-statistic $\mathcal{J}_T$ for the overidentifying restriction test, which satisfies

$$\mathcal{J}_T = TG(\hat{\theta}_{GMM})'M^*G(\hat{\theta}_{GMM})$$

under the optimal weighting matrix M^*, asymptotically follows the χ^2 distribution with degrees of freedom k.

Notice that the J-statistic jointly tests the overidentifying restrictions implied by the unconditional moment conditions (3.8), (3.10), and (3.12), but does not test the exact cross equation restrictions (3.11). To do so, define a $1 \times 2p$ vector $\mathcal{F}(\theta)$ as $\mathcal{F}(\theta) \equiv (\mathcal{K}^h - \mathbf{e}_{p+1})\mathcal{A} + \mathbf{e}_{p+1}$. Let θ_0 denote the true parameter vector under the null of the habit-forming PVM. Eq.(3.11) implies that $\mathcal{F}(\theta_0) = \mathbf{e}_{p+1}$ under the true parameter vector θ_0, i.e. the $p+1$st element of the vector $\mathcal{F}(\theta_0)$ should be one, while the others should be zero. The GMM estimate of the vector $\mathcal{F}(\theta)$, $\mathcal{F}(\hat{\theta}_{GMM})$, makes possible piecewise tests of the $2p$ cross-equation restrictions by the standard t-statistics, as well as joint test of those restrictions by the Wald statistic. The asymptotic standard error of the estimate $\mathcal{F}(\hat{\theta}_{GMM})$ is calculated from its covariance matrix numerically derived by the Delta method

$$\frac{\partial \mathcal{F}(\hat{\theta}_{GMM})}{\partial \theta'} \hat{V}_{\theta_{GMM}} \frac{\partial \mathcal{F}(\hat{\theta}_{GMM})}{\partial \theta'}'.$$

Let $\mathbf{k}(\theta) \equiv \mathbf{e}_{p+1} - \mathcal{F}(\theta)$. Then the estimates $\hat{\theta}_{GMM}$ and $\hat{V}_{\theta_{GMM}}$ yield the Wald statistic $\mathcal{W}_T$ satisfying

$$\mathcal{W}_T = \mathbf{k}(\hat{\theta}_{GMM}) \left[\frac{\partial \mathbf{k}(\hat{\theta}_{GMM})}{\partial \theta'} \hat{V}_{\theta_{GMM}} \frac{\partial \mathbf{k}(\hat{\theta}_{GMM})}{\partial \theta'}' \right]^{-1} \mathbf{k}(\hat{\theta}_{GMM})'.$$

Under the null hypothesis of $\mathbf{k}(\theta_0) = 0$, the Wald statistic $\mathcal{W}_T$ asymptotically follows the χ^2 with degrees of freedom $2p$.

Finally, the predictions of the habit-forming PVM on actual current account movements, denoted by $\mathcal{CA}_t^f$, are constructed as $\mathcal{CA}_t^f \equiv \mathcal{F}(\hat{\theta}_{GMM})\mathcal{Y}_t$. Under the null, it is the case that $\mathcal{CA}_t^f = CA_t$. Therefore, comparing the predictions with actual current account series provides another information to test the null hypothesis of the habit-forming PVM (3.2).

3.3.2 Empirical Results

This essay studies the quarterly, real, seasonally-adjusted Canadian data that spans the sample periods Q1:1963 and Q4:1997. The data construction follows Otto(1992) and Nason and Rogers(2003)[12]. The current account series and the first difference series of net output are demeaned to construct the sample vector $\mathcal{Y}_t$. The fourth lag $p = 4$ is chosen as the optimal lag by the general-to-specific likelihood ratio (LR) tests. To construct the series D_t, this essay uses the calibrated value of the constant world real interest rate $r = 0.0091$ [or equivalently 3.70 percent point on an annual basis: $r = (1.037)^{0.25} - 1$].

A crucial point for conducting the GMM/2SLS estimation is how to choose the instrument variables W_{t-2}. Theoretically, any variables in the information set Ω_{t-2} can be included in W_{t-2}. This essay lags the instruments more than one period and includes in W_{t-2} the fourth and fifth lagged values of CA_t and ΔNO_t to avoid potential correlation between $D_t - hD_{t-1}$ and any variable at period $t-2$ or $t-3$. In this case, W_{t-2} is a 4×1 vector satisfying

$$W_{t-2} = [\Delta NO_{t-4} \quad \Delta NO_{t-5} \quad CA_{t-4} \quad CA_{t-5}]'.$$

[12]All the data are distributed by Statistics Canada.

Therefore, $p = k = 4$ are chosen in the following analysis.

Table 3.1(a) summarizes the empirical results. First, the two estimates of the habit parameter, $\hat{h}_{2SLS}$ and $\hat{h}_{GMM}$, are reported in the first two columns. The 2SLS estimator based only on the unconditional moment conditions (3.8) yields $\hat{h}_{2SLS} = 0.931$ with the asymptotic standard error 0.192. This number is close to the estimate Gruber(2000) obtains ($\hat{h}_{2SLS} = 0.902$ and $s.e. = 0.257$, respectively). On the other hand, the GMM estimator based on the full moment conditions (3.8), (3.10), and (3.12) provides $\hat{h}_{GMM} = 1.002$ with the asymptotic standard error 0.152. Therefore, the GMM estimate based on the full moment conditions draws an inference of a larger habit parameter than the 2SLS estimate[13]. Although it is safe to claim that h is non-zero, either $\hat{h}_{2SLS}$ or $\hat{h}_{GMM}$ has a 95 % confidence interval including $h = 1$ [14]. This inference violates the constraint $h < 1$.

The statistic $\mathcal{J}_T$ is 0.455 with a p-value of 0.978, which means that the overidentifying restrictions out of the unconditional moment conditions (3.8), (3.10), and (3.12) cannot be jointly rejected even at 97.8 % significance level. However, the Wald statistic $\mathcal{W}_T$ for the cross-equation restrictions is 37.128 with a small p-value. This means that the cross-equation restrictions $\mathbf{k}(\theta_0) = 0$ are jointly rejected at any standard significance levels. Furthermore, the piecewise tests of the eight elements in the vector $\mathcal{F}(\theta)$ reflect this joint

[13]It is worth while mentioning that the standard error of the GMM estimate is smaller than that of the 2SLS. This means that the sampling uncertainty of the GMM estimate is smaller that that of the 2SLS estimate.

[14]If $h = 1$, the utility function implies that the household wants to smooth change in consumption, rather than level of consumption, across periods.

rejection of the cross equation restrictions. Recall that under the null, the fifth element $\mathcal{F}_5$ should be one, while all the other elements should be zero. The table reports that the GMM estimate $\hat{\mathcal{F}}_5$ is 1.276 with the asymptotic standard error 0.226. Hence, the estimate is not significantly different from one. The observation that two estimates $\hat{\mathcal{F}}_1 = -0.302$ and $\hat{\mathcal{F}}_6 = -0.400$ are statistically significant, however, violates the respective single null hypotheses. All the other estimates $\hat{\mathcal{F}}_i$ for $i \neq 1, 5, 6$ are statistically insignificant based on the two standard error rule.

Figure 3.1(a) plots the actual current account series, the predictions of the habit-forming PVM $\mathcal{CA}_t^f$, and the corresponding asymptotic two standard error band. Observe that the predictions of the habit-forming PVM track the actual current account fairly closely. The narrow standard error band reflects small sampling uncertainty attached to the predictions. The standard error band includes the actual current account in all the sample periods. These observations support the inference that the habit-forming PVM explains actual movements of the Canadian current account fairly well, as Gruber(2000) reports.

Comparing the empirical results of the habit-forming PVM (3.2) with those of the standard PVM demonstrates how introducing habit formation improves the ability of the PVM to track actual current account movements. Setting $h = 0$ and $\epsilon_t = 0$ in the habit-forming PVM (3.2) provides the following cross-equation restrictions imposed on the unrestricted VAR (3.9) under the null of the standard PVM

$$\mathbf{k}^*(\theta_0) \equiv \mathbf{e}_{p+1} - \mathcal{F}^*(\theta_0) = 0$$

where

$$\mathcal{F}^*(\theta) = -\mathbf{e}_1(1+r)^{-1}\mathcal{A}[I_8 - (1+r)^{-1}\mathcal{A}]^{-1}.$$

Note that θ includes only the VAR parameters. Hence, the unbiased estimate of θ is obtained by OLS. Let $\hat{\theta}_{OLS}$ denote the OLS estimate.

Table 3.1(b) reports the Wald statistic $\mathcal{W}_T^*$ to test the cross-equation restrictions $\mathbf{k}^*(\theta_0) = 0$ jointly, and the estimates of the eight elements of the vector $\mathcal{F}^*(\hat{\theta}_{OLS})$ to test the cross-equation restrictions piecewisely. First, the Wald statistic $\mathcal{W}_T^*$ is 20.589 with the asymptotic p-value 0.009. Therefore, the cross-equation restrictions are jointly rejected at any standard significance levels. The failure of the standard PVM is clearer in the piecewise tests of the null hypotheses. If the standard PVM holds, the fifth element of the vector $\mathcal{F}^*(\hat{\theta}_{OLS})$ should be one, while the other elements be zero. The estimate of the fifth element $\hat{\mathcal{F}}_5^*$ is -0.115 with the asymptotic standard error 0.408. Hence, the single null $\mathcal{F}_5^* = 1$ is strictly rejected by the standard t-statistic. All of the other estimates are statistically insignificant.

Figure 3.1(b) plots the actual Canadian current account series, the predictions of the standard PVM $\mathcal{CA}_t^{*f} = \mathcal{F}(\hat{\theta}_{OLS})\mathcal{Y}_t$, and the asymptotic two standard error band. The predictions are too smooth to track the actual series. The standard error band excludes the actual series at almost all periods. Hence, the standard PVM cannot predict the position of the Canadian current account. These observations clearly reveal the superiority of the habit-forming PVM to the standard PVM at least in the predicting ability.

The empirical results of this essay track those of Sheffrin and Woo(1990), Otto(1992), and Gruber(2000). Tables 3.2(a) and (b) summarize the empirical facts - the sample moments - of both the habit-forming and standard PVMs. In particular, this essay shares with Gruber(2000) the observation that taking habit formation into account greatly improves the PVM's prediction on the Canadian current account. The empirical results of both Gruber and this essay appear to support the claim that habit formation helps to explain Canadian current account movements.

However, the observational equivalence between the PVMs with habit formation and serially-correlated transitory consumption makes a researcher unable to identify whether the successful aspects of the habit-forming PVM are actually attributed to habit formation or other factor that generate consumption-tilting motives. A leading example for a small open economy is the stochastic world real interest rate. The next section discusses this essay's strategy to solve the identification problem.

3.4 Monte Carlo Investigation

Facing the identification problem, this essay conducts calibration-Monte Carlo exercises based on the SOE-RBC models with habit formation and the stochastic world real interest rate. The first task is to extend the SOE-RBC model of Nason and Rogers(2003) by introducing habit formation in consumption, as discussed in the next subsection.

3.4.1 The Small Open Economy Real Business Cycle Model

The lifetime utility function of the representative household is

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i u(C^*_{t+i}, L_{t+i}) \tag{3.13}$$

where $C^*_t = C_t - h\bar{C}_{t-1}$ and L_t is leisure at period t. Eq.(3.13) implies that the lifetime utility is non-separable not only across periods but also between consumption and leisure in each period. In particular, the period utility function $u(C^*, L)$ is parameterized as a constant relative risk aversion type

$$u(C^*, L) = \frac{(C^{*\phi} L^{1-\phi})^{1-\gamma} - 1}{1-\gamma}$$

for $\gamma \neq 1$. For $\gamma = 1$,

$$u(C^*, L) = \phi \ln C^* + (1-\phi) \ln L$$

and in either case $0 < \phi < 1$. Therefore, in the case of $\gamma = 1$ the preferences are separable between consumption and leisure.

Define Y_t, I_t, G_t and r_t to be output, investment, government consumption expenditure, and the real interest rate the representative household faces at period t. The household's budget constraint is

$$B_{t+1} = (1 + r_t)B_t + Y_t - I_t - G_t - C_t. \tag{3.14}$$

Output Y_t is produced by a Cobb-Douglas production function

$$Y_t = K_t^{\psi}[A_t N_t]^{1-\psi} \quad 0 < \psi < 1 \tag{3.15}$$

where K_t, A_t and N_t are capital stock, county-specific, labor-augmenting technology, and labor input at period t. Since the household is endowed with a unit hour to allocate between labour and leisure, the restriction $L_t + N_t = 1$ must be satisfied. The law of motion for capital is represented as

$$K_{t+1} = (1-\delta)K_t + \left(\frac{K_t}{I_t}\right)^{\varphi} I_t \quad 0 < \varphi < 1 \tag{3.16}$$

where $0 < \delta < 1$ is the depreciation rate. Eq.(3.16) includes adjustment costs of investment with the parameter φ. This specification of the adjustment costs follows Baxter and Crucini(1993).

As studied by Nason and Rogers(2003) and Schmitt-Grohé and Uribe(2003), the real interest rate r_t is decomposed into two components. The first component q_t is the exogenous and stochastic return that is common across the world. In this essay, q_t follows a covariance stationary process. The other component is the risk premium specific to this small open economy. The risk premium is given as a linear function of the economy's bond-output ratio. Following Nason and Rogers(2003), this essay specifies the stochastic real interest rate r_t to be

$$r_t = q_t - \eta\frac{B_t}{Y_t}, \quad 0 < \eta. \tag{3.17}$$

Eq.(3.17) implies that if the small open economy is a debtor (i.e. $B_t < 0$), the economy must pay a premium above q_t.[15]

The processes of the three exogenous variables G_t, A_t and q_t are specified as follows. Government consumption expenditure G_t is proportional to output Y_t with a constant

[15]The endogenous risk premium in eq.(3.17) excludes an explosive/unit root path of international bonds in the linearized solution of the equilibrium. Moreover it solves the famous problem in the SOE-RBC model that the deterministic steady state depends on the initial condition.

ratio g:[16]

$$G_t = gY_t. \tag{3.18}$$

The country-specific, labor-augmenting technology A_t is a random walk with drift

$$A_t = A_{t-1}\exp(\alpha + \epsilon_t^a), \quad \alpha > 0, \quad \epsilon_t^a \sim i.i.d.\mathbf{N}(0, \sigma_a^2). \tag{3.19}$$

Finally, the world real interest rate q_t follows an AR(1) process

$$1 + q_t = (1 + q^*)^{(1-\rho_q)}(1 + q_{t-1})^{\rho_q}\exp(\epsilon_t^q), \quad |\rho_q| < 1, \quad \epsilon_t^q \sim i.i.d.\mathbf{N}(0, \sigma_q^2) \tag{3.20}$$

where q^* is the deterministic steady state value of q_t. In the following analysis, ϵ_t^a and ϵ_t^q are assumed to be uncorrelated at all leads and lags.

3.4.2 The Optimality Conditions and Interpretations

The problem of the representative household is to maximize eq.(3.13) subject to eqs.(3.14)-(3.17), given the processes of the exogenous variables, eqs.(3.18)-(3.20), and the initial conditions $\bar{C}_{t-1} > 0, K_t > 0$, and $B_t \gtreqless 0$. The optimality conditions are

$$\Gamma_{t+1} = \beta\left(\frac{C_{t+1} - h\bar{C}_t}{C_t - h\bar{C}_{t-1}}\right)^{\phi(1-\gamma)-1}\left(\frac{1 - N_{t+1}}{1 - N_t}\right)^{(1-\gamma)(1-\phi)}, \tag{3.21}$$

$$1 = E_t\Gamma_{t+1}\left[1 + r_{t+1} - \eta\left(\frac{B_{t+1}}{Y_{t+1}}\right)\right], \tag{3.22}$$

[16]For example, consider the government budget that G_t is financed by lump-sum tax T_t satisfying $T_t = gY_t$. This assumption means that G_t and Y_t share not only a common trend but also a common cycle. Although this restriction is strict, it is reasonable for the Monte Carlo exercise in this essay because any shock to G_t can be considered as a shock to induce the consumption-smoothing motive, rather than the consumption-tilting motive.

$$\left(\frac{1-\phi}{\phi}\right)\left(\frac{C_t - h\bar{C}_{t-1}}{1-N_t}\right) = (1-\psi)\frac{Y_t}{N_t}\left[1+\eta\left(\frac{B_t}{Y_t}\right)^2\right], \tag{3.23}$$

and

$$\frac{1}{1-\varphi}\left(\frac{I_t}{K_t}\right)^{\varphi} =$$
$$E_t\Gamma_{t+1}\left\{\psi\frac{Y_{t+1}}{K_{t+1}}\left[1+\eta\left(\frac{B_{t+1}}{Y_{t+1}}\right)^2\right] + \left[\frac{1-\delta}{1-\varphi} + \frac{\varphi}{1-\varphi}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1-\varphi}\right]\left(\frac{I_{t+1}}{K_{t+1}}\right)^{\varphi}\right\}. \tag{3.24}$$

Recall that in equilibrium, the level of aggregate consumption must equal that of the representative household's consumption: $\bar{C}_t = C_t$. Any equilibrium path must satisfy the optimality conditions (3.21)-(3.24), the constraints (3.14)-(3.17), and the exogenous processes (3.18)-(3.20) with the transversality conditions

$$\lim_{i\to\infty} \beta^i E_t \lambda_{B,t+i} B_{t+i+1} = 0 \quad \text{and} \quad \lim_{i\to\infty} \beta^i E_t \lambda_{K,t+i} K_{t+i+1} = 0$$

where $\lambda_{B,t}$ and $\lambda_{K,t}$ are the shadow prices for the constraints (3.14) and (3.16), respectively.

Eq.(3.21) shows the stochastic discount factor, which turns out to be a familiar form $\beta(C_{t+1}/C_t)^{-1}$ when $h = 0$ and $\gamma = 1$. When $h \neq 0$ and $\gamma \neq 1$, the stochastic discount factor depends further on past consumption C_{t-1} and leisure at periods t and $t+1$, L_t and L_{t+1}. The higher C_{t-1} is, the lower Γ_{t+1} is because the marginal utility of consumption at period t rises due to habit formation and the marginal rate of the intertemporal substitution falls[17]. Similarly, the higher L_t is, the lower Γ_{t+1} is because the marginal utility of consumption at period t positively depends on leisure.

Eq.(3.22) is the optimality condition for holding the international bonds, i.e. the Euler equation. Notice that if $\eta = 0$, $h = 0$, $\gamma = 1$, and the world real interest is

[17]A rise in C_t increases the stochastic discount factor Γ_{t+1} as in the standard case.

constant, under the assumption of $\beta(1+r) = 1$, the Euler equation requires perfect smoothness of consumption across periods. Habit formation $h > 0$, the non-separable period utility over consumption and leisure $\gamma \neq 1$, and stochastic variations in the world real interest rate tilt consumption from the perfectly smoothed level through their effects on the stochastic discount factor[18]. The optimal consumption deviates away from the perfect smoothed level, i.e. permanent income. Hence, the deviation can be considered as the consumption-tilting motive or the transitory consumption component.

Eq.(3.23) is the optimality condition for the intratemporal substitution between consumption expenditure and leisure. It implies that the marginal rate of substitution between C_t and L_t should be equal to the marginal product of labour gross of the response of the endogenous risk premium to a change in labour. The Euler equation for capital, (3.24), has the interpretation that the expected loss of holding one more capital (represented by the LHS) should be equal to the expected benefit of the additional capital (represented by the RHS). The benefit consists of increased production gross of the risk premium, depreciation and smaller future adjustment costs of investment. On the other hand, the household needs to pay the cost that consists of the current utility loss due to investment in capital.

[18]Habit formation makes the household want to smooth not only consumption level but also consumption growth. The non-separable utility over consumption and leisure makes the household desire to smooth not only consumption but also leisure. Finally, if the real interest rate is expected to rise the future, the household wants to tilt consumption toward the future by lending out in international capital markets.

3.4.3 The Numerical Solution and Calibration

To derive the numerical solution of the equilibrium path, this essay takes linear approximation of the equilibrium conditions. First, all of the endogenous variables except for N_t and Γ_t are stochastically detrended by dividing them by the random walk technology shock A. Define the stochastically detrended variables $c_t \equiv C_t/A_t$, $i_t \equiv I_t/A_t$, $y_t \equiv Y_t/A_t$, $\varpi_t \equiv C_{t-1}/A_{t-1}$, $k_t \equiv K_t/A_{t-1}$ and $b_t \equiv B_t/A_{t-1}$. Next, a first-order Taylor expansion of each of the equilibrium conditions (3.14)-(3.17) and (3.21)-(3.24) is taken around the deterministic steady state. Let $\tilde{x}_t \equiv x_t - x$ and $\hat{x}_t \equiv x_t/x - 1$ for any variable x_t with the steady state x. Define vectors $\mathcal{P}_t$ and $\mathcal{S}_t$ by

$$\mathcal{P}_t = [\hat{c}_t \quad \hat{i}_t \quad \hat{y}_t \quad \hat{N}_t]' \quad \text{and} \quad \mathcal{S}_t = [\hat{\varpi}_t \quad \hat{k}_t \quad \hat{b}_t \quad \widetilde{\Delta \ln A_t} \quad \widetilde{\ln(1+q_t)}]'.$$

Then the solution method of Sims(2000) shows that there exists the unique equilibrium path and the vectors $\mathcal{P}_t$ and $\mathcal{S}_t$ follow the processes

$$\mathcal{P}_t = \mathcal{H}_1 \mathcal{S}_t \quad \text{and} \quad \mathcal{S}_t = \mathcal{H}_2 \mathcal{S}_{t-1} + \mathcal{H}_3 \epsilon_t \tag{3.25}$$

where $\epsilon_t = [\epsilon_t^a \quad \epsilon_t^q]$. Eq.(3.25) is the state space representation of the SOE-RBC model of this essay(see Appendix B.4 in detail).

Recall that there are fourteen structural parameters in the model. Table 3.3 gives the calibrated values of the structural parameters used in Monte Carlo experiments. This essay conducts two types of Monte Carlo experiments as discussed below. The baseline parameters β, γ, ϕ, ψ, φ, δ, η, g, α, σ_a and q^* are fixed across the experiments and set as the mean values of the prior distributions of Nason and Rogers(2003). In particular, across the experiments, the risk premium parameter η is chosen to be a very small number 0.000071 in order to cut the effect of the endogenous risk premium on the consumption-tilting motive/the transitory consumption component. In this case, the real interest rate r_t is almost equivalent to the world common real interest rate q_t[19].

The first Monte Carlo experiment is related to the SOE-RBC model with habit formation. This case sets the habit parameter depending on the estimated value. Although there are two candidates from two different estimations, the GMM estimate from the full moment conditions, $\hat{h}_{GMM}$, is suitable because it is more efficient than $\hat{h}_{2SLS}$. The problem is that $\hat{h}_{GMM}$ is greater than one, under which there exists no steady state in the SOE-RBC model. Therefore, in this experiment, the habit parameter is chosen to be 0.990, which is close to the estimate and included in the corresponding 90 % confidence interval. This experiment does not allow the world real interest rate to vary stochastically in order to maintain the assumptions of the habit-forming PVM: there is only a country-specific, unit-root technology shock. To this end, the persistence of the world real interest rate, ρ_q, and its standard deviation σ_q are set to be negligible: $\rho_q = \sigma_q = 1.00 \times 10^{-7}$. Therefore, the resulting theoretical distributions of the text statistics of the PVMs have the SOE-RBC model with habit formation as the null hypothesis.

The second experiment is related to the SOE-RBC model with the stochastic world real interest rate. In this case, the world real interest rate is allowed to vary stochastically. Nason and Rogers(2003) also estimate the persistent parameter ρ_q and the standard deviation σ_q of the common component of the world real interest rate[20]. They give 0.903

[19]As Nason and Rogers(2003) study, the specific number 0.000071 implies that the risk premium in Canada is one basis point at an annual rate at the steady state.

[20]They calculate the world real interest rate by using Fisher's equation, the three-month Euro-dollar

and 0.004 as the means of the prior distributions of ρ_q and σ_q, respectively. This essay uses these values, and also set the habit parameter to zero to rule out the effect of the habit formation. The resulting theoretical distributions of the statistics of the PVMs have the null hypothesis of the multi-shock SOE-RBC model - the SOE-RBC model with the stochastic world real interest rate.

Each of the experiments generates 1000 sets of artificial data by which the theoretical distributions of the test statistics, $\hat{h}_{2SLS}$, $\hat{h}_{GMM}$, $\mathcal{W}_T$, $\mathcal{F}(\hat{\theta}_{GMM})$, $\mathcal{W}_T^*$, and $\mathcal{F}^*(\hat{\theta}_{OLS})$, are constructed. The GMM procedure is repeatedly applied to the sets of the artificial data, and the resulting 1000 replications of $\hat{\theta}_{GMM}$ are used to construct the theoretical distributions of the statistics. The matching of the theoretical moments with the sample moments is evaluated as in Christiano(1989) and Gregory and Smith(1991). That is, taking the sample statistics as critical values, this essay counts the proportion of times that the simulated number exceeds the corresponding sample point estimate. This proportion is considered as the empirical p-value of the corresponding sample point estimate under the null hypothesis that the data generating process - the underlying SOE-RBC model - is true. Extreme values below 5 % or above 95 %imply a poor fit in the dimension examined.

3.5 Results

This section reports the results of the Monte Carlo experiments. The first experiment is related to the SOE-RBC model with habit formation. Three successful aspects of

deposit rate, the Canadian dollar-U.S.dollar exchange rate, and the implicit GDP deflator of Canada.

the habit-forming SOE-RBC model should be mentioned. The third column of Table 3.4 summarizes the empirical p-values of the sample estimates. First, observe that the p-values of $\hat{h}_{2SLS}$ and $\hat{h}_{GMM}$ are 0.7245 and 0.3824, respectively. Figures 3.2(a) and (b) show the nonparametrically smoothed theoretical distributions of $\hat{h}_{2SLS}$ and $\hat{h}_{GMM}$[21]. Notice that the modes of the theoretical distributions are close to the sample estimates, especially in $\hat{h}_{GMM}$. Second, Table 3.4 reveals that there are no elements of the vector $\mathcal{F}(\hat{\theta}_{GMM})$ that take extreme p-values above 0.95 or below 0.05. The third successful aspect is observed in the predictions of the habit-forming PVM, $\mathcal{CA}_t^f$. Figure 3.4(a) plots the estimated predictions of the habit-forming PVM and the 90 % theoretical confidence band. Note that all the point estimates fall inside the confidence band. The probability that the sample predictions are inside the band through the whole periods is actually equal to 1. Hence at least from these observations, it is hard to reject an inference that the true distributions of $\hat{h}_{2SLS}$, $\hat{h}_{GMM}$, $\mathcal{F}(\hat{\theta}_{GMM})$ and $\mathcal{CA}_t^f$ are the theoretical distributions under the null of the SOE-RBC model with habit formation.

The habit-forming SOE-RBC model, however, fails to replicate the sample estimates $\mathcal{W}_T$, $\mathcal{W}_T^*$, $\mathcal{F}^*(\hat{\theta}_{OLS})$ and $\mathcal{CA}_t^{*f}$. The third column of Table 4 reports that the empirical p-values of the Wald statistics for both the habit-forming and standard PVMs, $\mathcal{W}_T$ and $\mathcal{W}_T^*$, are 0.0696 and 0.0141, respectively. The p-value of $\mathcal{W}_T^*$ implies that at the significance level of 5 %, the sample estimate rejects the habit-forming SOE-RBC model as the underlying DGP, while the p-value of $\mathcal{W}_T$ means rejection of the habit-forming SOE-RBC model on boundary and at least at 10 % significance level. The nonparametrically

[21]The smoothed distribution is obtained by the nonparametric kernel density estimation with the normal kernel.

smoothed theoretical distributions of $\mathcal{W}_T$ and $\mathcal{W}_T^*$ in Figures 3.2(c) and (d) visually show the failure of the habit-forming SOE-RBC model to replicate the test statistics of the habit-forming and standard PVMs, $\mathcal{W}_T$ and $\mathcal{W}_T^*$: the sample estimates are at the far right tails of the theoretical distributions. Moreover, all the p-values of the elements of the vector $\mathcal{F}^*(\hat{\theta}_{OLS})$ take extreme values above 0.95 or below 0.05, except for $\hat{\mathcal{F}}_7^*$ equal to 0.0605. Finally, Figure 3.4(b) plots the sample predictions of the standard PVM and the corresponding 90 % theoretical confidence band. Observe how frequently the sample predictions fall outside the confidence band. The probability that the sample predictions are inside the confidence band through the whole period equals to 0.3972.

The next Monte Carlo experiment is based on the SOE-RBC model with the stochastic world real interest rate. The surprising result of this experiment is that there is no clear evidence to reject the null hypothesis that the true DGP is the SOE-RBC model with the stochastic world real interest rate. The fourth column of Table 3.4 reports the empirical p-values of the sample estimates in this experiment. First, note that the empirical p-values of $\hat{h}_{2SLS}$ and $\hat{h}_{GMM}$ are 0.115 and 0.1070, which in turn imply that the underlying SOE-RBC model cannot be rejected even at 10 % significance level. Figures 3.3(a) and (b) draw the smoothed theoretical distributions of $\hat{h}_{2SLS}$ and $\hat{h}_{GMM}$. Although the dispersion of the theoretical distribution of $\hat{h}_{2SLS}$ is large, and the theoretical distribution of $\hat{h}_{GMM}$ is heavily skewed toward the left, their modal values are close to the sample estimates. Regarding the vector $\mathcal{F}(\hat{\theta}_{GMM})$, the empirical p-values of all the elements except for the first one support the SOE-RBC model with the stochastic world real interest rate as the true DGP. As shown in Figure 3.5 (a), even with a couple of exceptions, almost all of the sample predictions on the current account, $\mathcal{CA}_t^f$, fall inside the theoretical 90 % confidence band. The probability that the sample predictions are inside the band is equal to 0.9858.

The result of the Wald statistic $\mathcal{W}_T$ is the first clear difference between the two Monte Carlo experiments. In the SOE-RBC model with the stochastic world real interest rate, the empirical p-value of the Wald statistic $\mathcal{W}_T$ is 0.5499. This implies that the sample estimate is fairly close to the median of the theoretical distribution, and the underlying null cannot be rejected at any standard significance levels. Its smoothed theoretical distribution in Figure 3.3(c) visually repeats this inference. Furthermore, striking differences are observed regarding the sample statistics related to the standard PVM. The empirical p-value of the Wald statistics for the standard PVM, $\mathcal{W}_T^*$, is 0.3259, which in turn implies together with the smoothed theoretical distribution in Figure 3.3(d) that the null of the SOE-RBC model with the stochastic world real interest rate cannot be rejected in this dimension. Except for $\hat{\mathcal{F}}_4$, all the estimates of the elements of the vector $\mathcal{F}^*(\hat{\theta}_{OLS})$ have the p-values between 0.05 and 0.95. Moreover, Figure 3.5(b) shows that the sample predictions are inside the 90 % theoretical confidence band in greater number of periods than in the case of the habit-forming SOE-RBC model. Indeed, the probability that the sample predictions are inside the band through the whole periods is 0.8156. This observation echoes the main finding of Nason and Rogers(2003): stochastic variations in the world real interest rate can explain the rejections of the standard PVM observed in the literature.

The results of the two Monte Carlo experiments are summarized in Table 3.5. This essay therefore reveals the superiority of the SOE-RBC model with the stochastic world real interest rate to the habit-foming SOE-RBC model to explain the broad empirical facts of the habit-forming and standard PVMs. Better than habit formation in consumption, stochastic variations in the world real interest rate explain the transitory consumption component/the consumption-tilting behavior, which is a crucial factor of the DGP of the Canadian current account.

3.6 Conclusion

This essay issues a caution about interpreting the empirical results from the habit-forming PVM as evidence that habit formation in consumption plays a significant role in explaining current account movements. One reason is that the habit-forming PVM is observationally equivalent to the non-habit PVM associated with serially correlated transitory consumption. This makes identification of the habit-forming PVM of the current account problematic.

Monte Carlo simulations based on SOE-RBC models are one to avoid this identification problem. The simulation exercises study the ability of different SOE-RBC models to mimic the sample moments or the empirical facts conditional on the habit-forming and standard PVMs. Two SOE-RBC models are hypothesized as the true DGPs of the sample moments: the one with with habit formation and the other with the stochastic world real interest rate. The Monte Carlo simulations make it possible to construct the theoretical distributions of the sample moments from the two hypothesized DGPs.

The results of the matching exercise based on the post-war Canadian data support the SOE-RBC model with the stochastic world real interest rate. The model matches all the key sample moments of the habit-forming and standard PVMs. The SOE-RBC model with habit formation mimics only a part of the empirical facts of the habit-forming PVM. This model fails to mimic the cross-equation restrictions predicted by the habit-forming PVM and all the empirical facts related to the standard PVM. Thus, the SOE-RBC model with a world real interest rate shock dominates the habit forming SOE-RBC model. Recent studies of Lettau and Uhlig(2000) and Otrok, Ravikumar and Whiteman(2002) claim counterfactual predictions of habit formation on several aspects of macroeconomics, e.g. consumption volatility and the equity premium puzzle. This essay also casts doubts on habit formation as an important source for the Canadian current account movements.

Chapter 4

Habit Formation and Aggregate Dynamics in Real Business Cycle Models

4.1 Introduction

Habit formation in consumption is proposed as a way of resolving the empirical puzzles in behavior of asset prices. The habit-forming consumer takes care of past consumption in determining current consumption: having consumed a good deal in the past, she also tends to consume a good deal in the current period. Therefore, habit formation makes a consumption process smoother. The equity premium puzzle and the risk-free rate puzzle are solved by introducing habit formation simply because smoother consumption implies the higher marginal rate of intertemporal substitution, which in turn yields a lower risk-free rate even under moderate curvature of the utility function.

In spite of the success of habit formation in solving the two asset pricing puzzles, it is still controversial what implications habit formation has for aggregate economic dynamics in the context of the real business cycle (RBC) models. Francis and Ramey(2002) argue that the one-sector RBC model with habit formation and adjustment costs of investment can replicate the negative response of hours worked to a positive permanent technology shock, which Galí(1999) finds by applying his structural VAR (SVAR) identification to the U.S. data. To the contrary, in their one-sector RBC model with the habit-forming utility function of Campbell and Cochrane(1999), Lettau and Uhlig(2000) show that their model generates an extremely smoothed consumption path, which cannot match the sample volatility of the H-P filtered U.S. consumption. Furthermore, they find that habit formation tends to dampen volatilities of output and investment counterfactually. Finally, Boldrin, Christiano and Fisher(2001) develop the two-sector RBC model with habit formation and inflexible labour mobility across sectors, and show that their model is successful in explaining broad business cycle dimensions in the U.S. data[1]. One exception is that their model cannot replicate the negative response of labour input to a positive, permanent technology shock.

This essay evaluates Francis and Ramey's(2002) one-sector RBC model with habit formation and adjustment costs of investment by examining the model's ability to account for sample moments representing aggregate dynamics of the U.S. data. The main question this essay asks is whether or not the habit-forming RBC model resolving Galí's(1999)

[1]For example, their model is successful in explaining the sample first and second moments of asset prices, output, consumption and investment, the comovement of employment across sectors, the excess sensitivity of consumption to income, and the inverted leading indicator phenomenon.

observation can explain the dynamics of consumption and output in the U.S. data. The dynamics of consumption and output are characterized by three moments of the sample: (i) autocorrelation functions (ACFs) of the growth rates of consumption and output, (ii) spectral density functions (SDFs) of the growth rates of consumption and output, and (iii) impulse response functions (IRFs) of log of output to permanent and transitory shocks. As studied by Cogley and Nason(1995), the IRFs of log of output are identified by applying Blanchard and Quah's(1989) long-run restriction to a bivariate, second-order SVAR including the growth rate of output and hours worked. The equilibrium path of the RBC model is log-linearly approximated around the deterministic steady state. The resulting linear rational expectation model is solved to obtain the state space representation, which is used to conduct Monte Carlo experiments.

The results from the matching exercise are summarized in Table 4. 1. First, the habit model fails to mimic the significantly positive, first and second order ACFs of output growth in the sample. Second, the habit model cannot replicate the maximum power spectrum observed over business cycle frequencies in the sample. Third, the habit model fails to generate the hump-shaped IRFs of output to a transitory shock. Fourth, the habit model overstates the higher order ACFs of consumption growth. Fifth, the habit model overstates the power spectrum around zero frequency. The first three results confirm Cogley and Nason's(1995) conclusion: the propagation mechanisms embodied in standard RBC models do not generate the right kind of output dynamics. This essay reveals that this conclusion is also applicable to the habit-forming RBC model. The next two results echo the observation of Lettau and Uhlig(2000): habit formation in consumption makes the consumption path counterfactually smooth.

Moreover, this essay examines the implications of the habit model for the asset pricing puzzles. As many past studies show, habit formation in consumption can generate a high equity premium and a low risk-free rate on average; hence, the equity premium puzzle and the risk-free rate puzzle are solved by introducing strong habit formation. However, as the sixth result in Table 4.1, the habit model fails to yield the high volatility of the rate of returns on equity observed in the sample. This is because collaborating with adjustment costs of investment and elastic labour supply, habit formation in consumption dampens volatilities of output and investment. That is to say, as emphasized by Francis and Ramey(2002) in accounting for Galí's(1999) observation, the mechanism that generates the negative correlation between labour input and a permanent technology shock leads to a wrong implication for an aspect of asset pricing behavior. Therefore, it is hard for the one-sector RBC model with habit formation and adjustment costs of investment to survive as a restricted data generating process of the aggregate dynamics of the postwar U.S. economy.

The next section reviews the empirical facts of consumption and output dynamics. Section 4.3 introduces the habit RBC model of this essay. Sections 4.4 and 4.5 discuss the results summarized in Table 4.1 in detail. Finally, Section 4.6 makes conclusion.

4.2 Empirical Facts of Consumption and Output Dynamics

This section introduces the sample moments related to consumption and output dynamics. As in the standard RBC literature, this essay uses real, seasonally-adjusted GNP as output, and divides it by total population to obtain per capita output. The data of consumption are constructed by taking the sum of real, seasonally-adjusted personal expenditures on nondurable goods and services and dividing the result by total population. The data of hours worked are constructed from the average weekly hours of production workers[2]. The sample period spans between Q1:1954 and Q2:2002.

Figures 4.1(a) and (b) show the sample estimates of the ACFs and SDFs for the growth rate of output[3]. The figures repeat the well-known empirical fact regarding GNP growth: the GNP growth rate is positively and significantly autocorrelated over short horizons. At lags of 1 and 2 quarters, the sample ACFs are significantly positive. Furthermore, the SDF for output growth has its maximum power at roughly 14 quarters or 3.5 years per cycle. As discussed by Cogley and Nason(1995), this means that a relatively large portion of the variance of output growth occurs at business cycle frequencies.

[2]DRI Basic Economics distributes all the data. In particular, this essay uses the civilian noninstitutional population as total population. All the data series are seasonally adjusted at annual rates.

[3]The ACFs are estimated by the GMM procedure with the optimal weighting matrix calculated by the heteroskedasticity-autocorrelation consistent estimator of Newey and West(1987). Following Cogley and Nason(1995), this essay estimates the SDFs by smoothing the sample periodogram using a Bartlett window.

On the other hand, Figures 4.1(c) and (d), which respectively plot the ADFs and SDFs of the growth rate of consumption, show no clear evidence that the growth rate of consumption is persistent at business cycle frequencies. Although the ACFs of the first 6 quarter lags are positive and about 0.15 on average, all sample ACFs are insignificant except for the lag of 6. Furthermore, the maximum power of the estimated SDFs is at zero frequency.

Figures 4.2(a) and (b) plot the sample estimates and the corresponding 90 percent confidence band of the IRFs of log of output to both permanent and transitory shocks identified by Blanchard and Quah's(1989) long-run restriction[4]. Figure 4.2(b) repeats the most important observation of Blanchard and Quah(1989) and Cogley and Nason(1995): output has a significant, hump-shaped response to a transitory shock over the short-horizon. This observation implies that output appears to have an important trend-reverting component.

4.3 The Model

This section introduces a closed-economy, one-sector RBC model with adjustment costs of investment and habit formation in consumption, and asks whether or not the RBC model can mimic the empirical facts of the consumption and output dynamics found in the last section.

[4]The 90 percent confidence bands are calculated by 1000 non-parametric bootstrapping resamples.

4.3.1 A One-Sector RBC Model with Adjustment Costs of Investment and Habit Formation in Consumption

Let C_t and N_t denote consumption and labour supply at period t, respectively. As in Boldrin, Christiano and Fisher(2001) and Francis and Ramey(2002), the lifetime utility function of the representative household is

$$E_t \sum_{i=0}^{\infty} \beta^i [\log(C_{t+i} - hC_{t+1+i}) - N_{t+i}], \quad 0 < \beta < 1, \quad 0 \leq h < 1 \tag{4.1}$$

where E_t and β are the conditional expectation operator on the information set at period t and the subjective discount factor. The parameter h characterizes habit formation in consumption: if $0 < h < 1$, the representative household forms consumption habits, while if $h = 0$ the utility function turns out to be time-separable with unit relative risk aversion, as in the standard RBC model.

This essay adopts the "internal habit" specification. In this case, current utility depends on household's own past consumption, rather than aggregate past consumption as in the "external habit" or "catching-up-with-the-Joneses" specification studied by Abel(1990). As discussed in Constantinides(1990) and Boldrin, Christiano and Fisher(2001), the internal habit specification makes it possible to derive a high equity premium even under moderate levels of risk aversion. Eq.(4.1) also shows that the utility function is defined as the logarithm of the difference $C_t - hC_{t-1}$. This difference specification, as in Campbell and Cochrane(1999), yields time-varying risk aversion[5]. Habits depend on only 1 lag of consumption.

[5]On the other hand, the ratio specification studied by Abel(1990) and Fuhrer(2001) yields constant risk aversion.

The representative household owns capital and technology to produce consumption goods. Let Y_t, K_t, I_t, and A_t denote output, capital, investment, and the aggregate state of technology at period t. The production function is Cobb-Douglas:

$$Y_t = K_t^{\psi}(A_t N_t)^{1-\psi} \quad 0 < \psi < 1 \tag{4.2}$$

where ψ implies the capital share. The aggregate state of technology A_t follows an exogenous random walk with drift in log term:

$$A_t = A_{t-1} \exp(\alpha + \epsilon_t^a) \quad \epsilon_t^a \sim i.i.d.\mathbf{N}(0, \sigma_a^2). \tag{4.3}$$

The law of motion of capital is

$$K_{t+1} = (1-\delta)K_t + \left(\frac{K_t}{I_t}\right)^{\varphi} I_t \quad 0 < \delta < 1 \tag{4.4}$$

where δ is the depreciation rate of capital. The second term of the RHS of eq.(4.4) implies that the representative household faces adjustment costs of investment. Jermann(1998) and Boldrin, Chiristiano and Fisher(2001) find that when the utility function is habit-forming, adjustment costs of investment improve the ability of a one-sector RBC model to account for behavior of asset prices. Moreover, Francis and Ramey(2002) argue that the combination of habit formation and the adjustment costs of investment yields the negative response of N_t to a permanent technology shock, which Galí(1999) observes in his SVAR identification. This essay follows Baxter and Crucini(1993) in specifying the adjustment costs of investment.

The aggregate resource constraint is

$$Y_t = C_t + I_t + G_t \tag{4.5}$$

where G_t is government consumption spending that is assumed to be exogenous and stochastic[6]. This essay follows Nason and Rogers(2002) in specifying the stochastic process of G_t: G_t shares a common trend with Y_t, and the ratio of government spending to output $g_t = G_t/Y_t$ follows an exogenous stationary process in log term

$$g_t \equiv G_t/Y_t = (g^*)^{1-\rho_g} g_{t-1}^{\rho_g} \exp(\epsilon_t^g) \quad \epsilon_t^g \sim i.i.d.\mathrm{N}(0, \sigma_g^2). \tag{4.6}$$

Hence, as in Cogley and Nason(1995), the model is driven by two exogenous shocks-technology shocks and government spending shocks.

The equilibrium allocation is found by solving the household's optimization problem at period t: maximizing the lifetime utility (4.1) subject to the production function (4.2), the law of motion of capital (4.4), the budget constraint (4.5), two exogenous driving forces (4.3) and (4.6), and the initial conditions $K_t \geq 0$ and $C_{t-1} \geq 0$ given. Together with the transversality conditions for the state variables K_t and C_{t-1}, the first-order necessary conditions characterize the equilibrium path of the economy.

4.3.2 Numerical Solution, Calibration, and Evaluation

This essay log-linearly approximates the equilibrium path around the deterministic steady state. Solved by Sims's(2000) method, the resulting linear rational expectation model

[6]The reason stochastic variations in government consumption expenditure are allowed is that this essay repeats the SVAR exercise of Cogley and Nason(1995). The authors identify the IRFs of $\Delta \ln Y_t$ to both permanent and transitory shocks by applying the Blanchard and Quah's(1989) long-run restriction to a bivariate SVAR including $\Delta \ln Y_t$ and N_t as the endogenous variables. Without the government spending shock, the standard, single-shock RBC model with the permanent technology shock makes the bivariate SVAR stochastically singular.

derives the state space representation of the equilibrium path, which in turn is used to conduct Monte Carlo simulations to generate artificial data of aggregate variables.

The model is calibrated by the parameter values of Chiristiano and Eichenbaum(1992), Cogley and Nason(1995), Boldrin, Christiano and Fisher(2001), and Nason and Rogers (2002). Table 4.2 summarizes the parameter values this essay uses. In particular, g^* is calibrated to the U.S. data by taking the sample average of the government spending-GNP ratio. Given the other calibrated parameters, the habit parameter $h = 0.985$ is obtained by maximizing the ability of the model to account for the risk-free rate[7]. This essay conducts two Monte Carlo experiments: one with habit formation ($h = 0.985$) and the other without habit formation ($h = 0$). In other words, this essay considers the one-sector RBC model with adjustment costs of investment as the benchmark model, and compares the moment-matching performance of the benchmark model with that of the habit-forming RBC model.

The ability of the models to replicate the sample moments is evaluated from the viewpoint of classical statistics. The model is considered to be restricted data generating processes (DGP) for the sample moments. The synthetic data generated by Monte Carlo simulations yield the theoretical distributions of the sample moments under the null hypothesis that the RBC model is the restricted DGP. The sample moments are used as critical values to evaluate the null hypothesis: if a sample moment drops outside 5 percent of the corresponding theoretical distribution, the null hypothesis-the RBC

[7]The risk-free rate is calculated as the inverse of the expected stochastic discount factor(i.e. the marginal rate of intertemporal substitution of consumption) minus one. It is defined in detail in section 3.5.

model-is rejected by two side test at 10 percent significance level.

In addition, this essay constructs the generalized Q statistics for the ACFs for the growth rates of consumption and output, and for the IRFs of output to permanent and transitory shocks, as proposed by Cogley and Nason(1995). For example, the generalized Q statistic for the ACFs of the growth rate of output has the null hypothesis that all replicated ACFs of the first 8 quarter lags match their sample counterparts. The same is true for the generalized Q statistic for the ACFs of the growth rate of consumption. The generalized Q statistic for the IRFs of output has the null hypothesis that the replicated IRFs at the first 8 periods after impact match the sample counterparts. Under the null hypothesis, each Q statistic asymptotically follows the χ^2 distribution with 8 degrees of freedom. Hence, the matching performance of the model is also evaluated by the χ^2 test statistics[8].

4.4 Results

This section reports the results of the matching exercise.

4.4.1 ACFs and SDFs for the Output Growth Rate

First, the upper two windows of Figure 4.3 show the sample estimates of the ACFs and SDFs of the growth rates of output, and the corresponding 90 percent confidence bands constructed by 1000 artificial data generated under the null of the benchmark, non-habit model. Notice that the sample ACFs at lags of 1 and 2 are clearly outside the 90 percent

[8]For detailed derivation of the generalized Q statistics, see Cogley and Nason(1995).

confidence band: the benchmark model fails to mimic the sample first and second ACFs. Moreover, the maximum power spectrum observed around 14 quarters in the sample is also outside the confidence band: the benchmark model fails to replicate the empirical fact that a relatively large portion of the variance of output growth occurs at business cycle frequencies. Table 4.3 shows that for the ACFs of the growth rate of output, the benchmark model yields the generalized Q statistic of 40.994 with zero asymptotic p-value. Hence, the null hypothesis that the model-generated ACFs up to 8 lags match the sample ACFs is strictly rejected. The failure of the benchmark, non-habit model in mimicking output dynamics echoes the observations of Cogley and Nason(1995).

Can habit formation in consumption improve the matching performance of the RBC model with respect to output dynamics? The answer is no. Again, the lower two windows of Figure 4.3 show the sample estimates of the ACFs and SDFs of the growth rates of output, and the corresponding 90 percent confidence bands constructed by 1000 artificial data generated under the null of the habit model in this experiment. Notice that all of the above results of the benchmark model can be applied to the habit model: the habit model cannot replicate the sample ACFs with lags of 1 and 2 and the maximum power spectrum at the business cycle frequencies. The generalized Q statistics for the ACFs of the growth rate of output, which is given in Table 4.3, is slightly smaller than that of the benchmark model; however, the asymptotic p-value is still zero.

4.4.2 IRFs of Output

The next matching exercise involves the IRFs of output to permanent and transitory shocks. Figure 4.4 summarizes the results for the IRFs of log of output to permanent and transitory shocks. The upper window is related to the IRFs to a permanent shock, while the lower window is related to those to a transitory shock. In each window, the solid line shows the sample IRFs, the dashed line shows the mean of the IRFs generated by the benchmark, non-habit model, and the dotted line shows the mean of the IRFs generated by the habit model. It is important to mention the following three observations. First, on average, the habit model overstates the IRFs of output to a permanent shock, while the benchmark model understates them. Second, the two models fail to mimic the hump-shaped response of output to a transitory shock observed in the sample. Hence, habit formation is not a reliable propagation mechanism in explaining the humped-shaped response of output. This inference is further strengthened by the third observation: the habit model generates a transitory shock that falsely has a very persistent effect on output.

In addition to Figure 4.4, the generalized Q statistics related to the IRFs of output in Table 4.3 statistically indicate the poor matching performance of the habit and benchmark models in this dimension. For the IRFs of output to a permanent shock, the habit and benchmark models respectively yield the generalized Q statistics that are 112.525 and 40.152 with zero p-values, while for the IRFs of output to a transitory shock, the two models yield the generalized Q statistics of 236.037 and 24.189 with zero p-values. Although these chi-squared test statistics strictly reject both the habit and benchmark models as the restricted DGP of the IRFs of output, the habit model yields the generalized Q statistics about three times larger than those of the benchmark model. This is indirect evidence that habit formation deteriorates the ability of the RBC model to explain an important aspect of the output dynamics.

4.4.3 ACFs and SDFs for Consumption Growth

The clearest implication of habit formation in consumption is that it produces a smoother consumption path than that of the time-separable utility function. Consequently, checking the matching performance of the habit model for consumption dynamics is the most direct and helpful exercise in evaluating the habit model.

The upper two windows of Figure 4.5 illustrate the sample estimates of the ACFs and SDFs of the growth rates of consumption, and the corresponding 90 percent confidence bands constructed under the null of the benchmark model. Observe that in the two windows, the 90 percent confidence bands include almost all of the sample ACFs and SDFs with an exception of the sample SDFs around 28 quarters per cycle. Therefore, it is safe to say that the benchmark model can mimic the ACFs and SDFs of the growth rate of consumption fairly well. This successful aspect of the benchmark model is also confirmed by the generalized Q statistic in Table 4.3. This chi-squared test statistic is 7.917 with p-value 0.442. This means that at 10 percent significance level, the benchmark model cannot be rejected as the restricted DGP of the sample ACFs of the growth rate of consumption.

Notice in Table 4.3 that the generalized Q statistic is not able to reject the habit model as the DGP of the sample ACFs of consumption growth up to 8 lags: the corresponding generalized Q statistic is 6.94 with p-value 0.543. On the other hand, the lower two windows of Figure 4.5 show the sample estimates of the ACFs and SDFs of the growth rates of consumption as well as the corresponding 90 percent confidence bands constructed under the null of the habit model. Observe that the theoretical confidence band for the ACFs is shifted up relative to that of the benchmark model. As a result, the sample ACFs are almost on the lower(left) boundary of the confidence band. In particular, the sample ACFs of lags of 7, 8, 9, 13, 15, 16, and 17 are outside the the confidence band: these ACFs reject the habit model at least at 10 percent significance level. These facts lead to an inference that the habit model overstates the ACFs of consumption growth. Next, the sample SDFs around zero frequency are below the theoretical 90 percent confidence band. This means that the habit model overemphasizes volatilities of the growth rate of consumption around zero frequency.

The above inference that the habit model overstates the ACFs of consumption growth is shared with Lettau and Uhlig(2000) who argue that the habit-forming utility of Campbell and Cochrane(1999) yields a counterfactually smooth consumption path. Moreover, by using the concept of the spectral utility function, Otrok, Ravikumar and Whiteman(2002) show that the habit-forming utility (4.1) makes the household more averse to high-frequency fluctuations of consumption than to low frequency fluctuations. The model-generated SDFs of consumption growth are excessively concentrated around zero frequency, as shown in Figure 4.5, reflect that the habit-forming household prefers low frequency fluctuations of consumption growth to high-frequency fluctuations. However, because the sample SDFs around zero frequency are below the theoretical 90 percent confidence band of the habit model, this aspect of the habit model is simply rejected.

4.5 Implications for Asset Prices

The last section shows the difficulties involved when using the RBC model with habit formation to explain the consumption and output dynamics in the U.S. data. Despite these difficulties, Jermann(1998) and Boldrin, Chiristiano and Fisher(2001) claim that the one-sector RBC model with habit formation and adjustment costs of investment can solve two empirical puzzles of asset pricing behavior: the equity premium puzzle and the risk-free rate puzzle[9]. The habit model in this essay also solves the two asset pricing puzzles. However, this essay reveals another difficulty of the habit model: it fails to explain the high volatility of the rate of return on equity in the sample.

The risk-free rate and the rate of return on equity implied by the RBC model are calculated on the equilibrium path as in Boldrin, Christiano and Fisher(2001). First, the risk-free rate r_t^f is given as the inverse of the expected stochastic discount factor minus one:

$$r_t^f = \frac{1}{E_t \Gamma_{t+1}} - 1 \tag{4.7}$$

where Γ_{t+1} is the stochastic discount factor or the intertemporal marginal rate of sub-

[9]The equity premium puzzle is the empirical fact that returns on the stock market exceed returns on Treasury bills by an average of 6 percentage point. The standard consumption CAPM explains this phenomenon only by an extremely high risk aversion. Weil(1989) then points out the risk-free rate puzzle if, indeed, consumers are highly risk-averse. The return on Treasury bills is low on average, and consumption grows steadily. To reconcile these empirical facts with high risk-aversion requires that consumers be extremely patient with a low or even negative rate of time preference.

stitution of consumption. Second, the rate of return on equity r^e_{t+1} is

$$r^e_{t+1} = \frac{1}{q_t}\left[\psi\left(\frac{Y_{t+1}}{K_{t+1}}\right) + \left(\frac{\varphi}{1-\varphi}\right)\left(\frac{I_{t+1}}{K_{t+1}}\right)\right] + (1-\delta)\frac{q_{t+1}}{q_t} - 1 \tag{4.8}$$

where q_t is the relative price of capital to consumption, which is well-known as Tobin's q and calculated on the equilibrium path by

$$q_t = \left(\frac{1}{1-\varphi}\right)\left(\frac{I_t}{K_t}\right)^{\varphi}.$$

Intuitively, eq.(4.8) says that the rate of returns on equity equals the amount of goods the representative household can consume at period t+1 if investing a unit of consumption goods to capital at period t. To invest a unit of consumption goods to capital, the household has to pay q_t at period t. The invested capital, on the other hand, increases consumption at period t+1 by raising output and reducing adjustment costs [i.e. the first and second terms of the RHS of eq.(4.8)]. Moreover, if the relative price q_{t+1} rises, the household can consume capital gain net of depreciation [i.e. the third term].

The "Data" column of Table 4.4 shows estimates of the mean of the risk free rate Er^f_t, the mean of the equity premium $E(r^e_{t+1} - r^f_t)$, the standard deviation of the rate of return on equity σ_{r^e}, and the Sharpe ratio $E(r^e_{t+1} - r^f_t)/\sigma_{r^e}$, over the U.S. sample[10]. First, observe the "Benchmark" column in Table 4.4. This column implies that the benchmark, non-habit RBC model fails to solve the two asset pricing puzzles. That is to say, the replicated mean of the risk-free rate is too high to account for the sample mean, while the replicated mean of the equity premium is too low to match the sample mean. This model also fails to explain the high volatility of the equity return: the replicated

[10]These sample moments are provided by Ceccheti, Lam and Mark(1993). Boldrin, Chiristiano and Fisher(2001) also use these sample moments.

standard deviation of the rate of return on equity is 0.82, while its sample counterpart is 19.4. Finally, this model yields a lower Sharpe ratio than that in the sample.

The "Habit" column of Table 4.4, on the other hand, reports evidence that habit formation in consumption helps solve the two puzzles of asset pricing. First, the replicated mean of the risk-free rate takes a value close to the sample mean: the former is 1.28, and the latter is 1.19. Second, the habit model increases the equity premium by 3.6 percent more than the benchmark model. Although the habit model still understates the sample mean of the equity premium by about 3 percent, as discussed by Boldrin, Christiano and Fisher(2001), this discrepancy is not important because the gap can be closed if a slightly higher curvature were introduced into the utility function.

Caveats, however, should be added to the above successful results of the habit model in the two asset pricing puzzles. First, this model fails to mimic the sample standard deviation of the rate of return on equity; indeed, habit formation makes the replicated standard deviation of the rate of return on equity lower than that of the benchmark model. Second, the high equity premium and the low standard deviation of the rate of return on equity automatically imply an implausibly high Sharpe ratio, which is observed in the last column of Table 4.4.

The failure of the habit model in explaining the high volatility of the rate of returns on equity stems from the following result of habit formation. Together with the adjustment costs of investment and elastic labour supply, habit formation in consumption conterfactually dampens the volatilities of output, investment, and capital, which jointly determine the volatility of the equity return through eq.(4.8). The habit-forming consumer desires an extremely smooth consumption path. To smooth consumption, he/she can adjust investment and labour supply in the closed-economy RBC setting. However, since the consumer faces adjustment costs of investment, he/she does not want to change investment a lot. The consumption-smoothing enforced by habit formation is implemented mainly by adjusting labour supply in a countercyclical way. In particular, as shown in Boldrin, Christiano and Fisher(2001) and Francis and Ramey(2002), this model implies that a positive, permanent technology shock reduces labour input over the short-horizons because the income effect overcomes the substitution effect. This negative correlation between a permanent technology shock and labour input makes output less volatile[11]. Hence, the volatilities of output and investment are counterfactually dampened by strong habit formation.

4.6 Conclusion

Francis and Ramey(2002) consider the one-sector RBC model with habit formation in consumption and adjustment costs of investment as a candidate for the restricted DGP for the negative response of labour input to a positive, permanent technology shock, which Galí(1999) find in his SVAR identification. This essay reexamines other dimensions of their model, and shows that this type of the RBC model fails to replicate the dynamics of consumption and output in the postwar U.S. data. As many past studies show, habit formation can help solve the two asset pricing puzzles. However, habit formation

[11] Jermann(1998) observes high volatility of the equity return in a one-sector RBC model with habit formation and adjustment costs of investment. This is because the model assumes constant labour supply. The representative household can adjust only investment to smooth consumption.

dampens volatilities of both output and investment and yield extremely low volatility of equity returns. Based on these results, this essay concludes that it is hard to find support for habit formation in consumption in the one-sector RBC model, even though the model is consistent with Galí's(1999) observation.

One way of future research is to abandon habit formation in consumption. There are several non-habit models that can generate a negative response of labour input to a positive, permanent technology shock: e.g. the sticky price model in Galí(1999), the Leontief model with labour-saving technology shocks in Francis and Ramey(2002), and the home production RBC model in Campbell and Ludvigson(2001) to give examples. However, it is still unclear what implications these models have for the asset pricing behavior. Another way is to keep habit formation as a resolution of the asset pricing puzzles but change other aspects of the RBC model. In this case, the two sector RBC model with habit formation and inflexible labour mobility of Boldrin, Chiristiano and Fisher(2001) is a guaranteed starting point for future research.

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Appendices

A: Appendices of Chapter 2

Appendix A.1: Derivation of the Linear-Approximated Intertemporal Budget Constraint (2.7)

Dividing the intertemporal budget constraint (2.6) by NO_t gives

$$\frac{C_t}{NO_t}\sum_{i=0}^{\infty} E_t\, R_{t,i}\frac{C_{t+i}}{C_t} = (1+r_t)\frac{B_t}{NO_t} + \sum_{i=0}^{\infty} E_t\, R_{t,i}\frac{NO_{t+i}}{NO_t}. \qquad (A.1.1)$$

Notice that for any variable X_t the relation $\frac{X_{t+i}}{X_t} = \frac{X_{t+1}}{X_t}\frac{X_{t+2}}{X_{t+1}}\cdots\frac{X_{t+i}}{X_{t+i-1}}$ holds. Therefore eq.$(A.1.1)$ can be rewritten as

$$\frac{C_t}{NO_t}\left[1+\sum_{i=1}^{\infty} E_t\, R_{t,i}\prod_{j=t+1}^{t+i}\left(\frac{C_j}{C_{j-1}}\right)\right] = (1+r_t)\frac{B_t}{NO_t} + \left[1+\sum_{i=1}^{\infty} E_t\, R_{t,i}\prod_{j=t+1}^{t+i}\left(\frac{NO_j}{NO_{j-1}}\right)\right]. \qquad (A.1.2)$$

Notice that for any variable X_t, the relation $\prod_{j=t+1}^{t+i} X_j = \exp\{\sum_{j=t+1}^{t+i} \ln(X_j)\}$ holds. From this relation and the definition of $R_{t,i}$, eq.$(A.1.2)$ can be further rearranged as

$$\frac{C_t}{NO_t}\left[1+\sum_{i=1}^{\infty} E_t \exp\left\{\sum_{j=t+1}^{t+i} (\Delta \ln C_j - \ln(1+r_j))\right\}\right]$$
$$= \exp\{\ln(1+r_t) - \Delta \ln NO_t\}\frac{B_t}{NO_{t-1}}$$
$$+\left[1+\sum_{i=1}^{\infty} E_t \exp\left\{\sum_{j=t+1}^{t+i} (\Delta \ln NO_j - \ln(1+r_j))\right\}\right]. \quad (A.1.3)$$

Taking a first-order Taylor expansion of the LHS of eq.$(A.1.3)$ around the mean values gives

$$\widetilde{\text{The LHS}} \approx \frac{1}{1-\alpha}\frac{\widetilde{C_t}}{NO_t} + \frac{c}{1-\alpha}\sum_{i=1}^{\infty} \alpha^i E_t\left\{\Delta \widetilde{\ln C_{t+i}} - \ln(\widetilde{1+r_{t+i}})\right\}. \quad (A.1.4)$$

where $\alpha = \exp(\gamma^c - \mu) < 1$. The RHS of eq.$(A.1.3)$ is also approximated as

$$\widetilde{\text{The RHS}} \approx \frac{1}{\kappa}\frac{\widetilde{B_t}}{NO_{t-1}} + \frac{b}{\kappa}\ln(\widetilde{1+r_t}) - \frac{b}{\kappa}\Delta \widetilde{\ln NO_t}$$
$$+\frac{1}{1-\kappa}\sum_{i=1}^{\infty} \kappa^i E_t\left\{\Delta \widetilde{\ln NO_{t+i}} - \ln(\widetilde{1+r_{t+i}})\right\}. \quad (A5)$$

where $\kappa = \exp(\gamma - \mu) < 1$. From the results of eqs.$(A.1.4)$ and $(A5)$, the linear-approximated intertemporal budget constraint (2.7) is finally given as

$$\frac{\widetilde{C_t}}{NO_t} \approx \frac{1-\alpha}{\kappa}\frac{\widetilde{B_t}}{NO_{t-1}} + \frac{1-\alpha}{\kappa} b\ln(\widetilde{1+r_t}) - \frac{1-\alpha}{\kappa} b\Delta \widetilde{\ln NO_t}$$
$$- c\sum_{i=1}^{\infty} \alpha^i E_t\left\{\Delta \widetilde{\ln C_{t+i}} - \ln(\widetilde{1+r_{t+i}})\right\}$$
$$+\frac{1-\alpha}{1-\kappa}\sum_{i=1}^{\infty} \kappa^i E_t\left\{\Delta \widetilde{\ln NO_{t+i}} - \ln(\widetilde{1+r_{t+i}})\right\}.$$

Appendix A.2: Derivation of the Approximated Solution of the Optimal Current Account Ratio(2.10)

Substitute the log-linearized Euler equation (2.8) into the linear-approximated intertemporal budget constraint (2.7). For simplicity, assuming that the economy is around the balanced growth path; $\alpha = \kappa$ and using the approximation $\ln(1 + r_t) \approx r_t$, I can obtain the consumption-net output ratio equation as

$$\widetilde{\frac{C_t}{NO_t}} = \frac{1-\kappa}{\kappa}\widetilde{\frac{B_t}{NO_{t-1}}} + \frac{1-\kappa}{\kappa} b\widetilde{r}_t - \frac{1-\kappa}{\kappa} b\Delta \widetilde{\ln NO_t}$$
$$- (\sigma - 1)c \sum_{i=1}^{\infty} \kappa^i \, E_t \, \widetilde{r}_{t+i} - \sum_{i=1}^{\infty} \kappa^i \, E_t \, \widetilde{r}_{t+i}$$
$$+ \sum_{i=1}^{\infty} \kappa^i \, E_t \, \Delta \widetilde{\ln NO}_{t+i}. \quad \text{(A.2.1)}$$

To derive the optimal current account-net output ratio equation, consider the current account identity (2.9). Dividing eq.(2.9) by NO_t rewrites eq.(2.9) as

$$\frac{CA_t}{NO_t} = 1 + \frac{\exp[\ln(1 + r_t)] - 1}{\exp(\Delta \ln NO_t)} \frac{B_t}{NO_{t-1}} - \frac{C_t}{NO_t}.$$

Taking a first-order Taylor expansion of the above equation gives

$$\widetilde{\frac{CA_t}{NO_t}} \approx \left[\frac{1}{\kappa} - \frac{1}{\exp(\gamma)}\right] \widetilde{\frac{B_t}{NO_{t-1}}} + \frac{b}{\kappa}\widetilde{r}_t - \left[\frac{1}{\kappa} - \frac{1}{\exp(\gamma)}\right] b\Delta \widetilde{\ln NO_t} - \widetilde{\frac{C_t}{NO_t}}. \quad \text{(A.2.2)}$$

Substituting the consumption equation (A.2.1) into (A.2.2), I can obtain the equation of the optimal current account-net output ratio:

$$\widetilde{\frac{CA_t}{NO_t}} = \left[1 - \frac{1}{\exp(\gamma)}\right] \widetilde{\frac{B_t}{NO_{t-1}}} + b\widetilde{r}_t - \left[1 - \frac{1}{\exp(\gamma)}\right] b\Delta \widetilde{\ln NO_t}$$
$$+ [(\sigma - 1)c + 1] \sum_{i=1}^{\infty} \kappa^i \, E_t \, \widetilde{r}_{t+i} - \sum_{i=1}^{\infty} \kappa^i \, E_t \, \Delta \widetilde{\ln NO}_{t+i}.$$

Since $\exp(\gamma)$ takes a close value to one, it might be a reasonable approximation to set the coefficient $[1 - 1/\exp(\gamma)]$ to zero. Then the optimal current account-net output ratio equation (2.10) is constructed as

$$\frac{\widetilde{CA_t}}{NO_t} = b\widetilde{r}_t + [(\sigma - 1)c + 1] \sum_{i=1}^{\infty} \kappa^i \, E_t \, \widetilde{r}_{t+i} - \sum_{i=1}^{\infty} \kappa^i \, E_t \, \Delta \, \widetilde{\ln NO}_{t+i}.$$

Appendix A.3: Derivation of Cross-Equation Restrictions $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$

To derive the cross-equation restrictions $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$, I exploit the *Wiener-Kolmogorov formula*, which is well-known as Hansen and Sargent's(1980) *distributed predicted leads formula*. For exposition, I give this formula as the following lemma without proof;

Lemma (Hansen and Sargent(1980)). *For a covariance-stationary process X_t with a Wold MA representation $X_t = A(L)\nu_t$ and $\beta \in (0, 1)$, it is the case that*

$$\sum_{i=1}^{\infty} \beta^i E_t X_{t+i} = \beta \left[\frac{A(L) - A(\beta)}{L - \beta} \right] \nu_t.$$

By using the present value relation (2.10), the maintained DGPs of the first difference of log of net output and the world real interest rate, (2.11) and (2.12), and the above lemma, I can derive a structural MA representation of the current account-net output ratio

$$\frac{\widetilde{CA_t}}{NO_t} = \Gamma_g^{ca}(L)\epsilon_t^g + \Gamma_{cp}^{ca}(L)\epsilon_t^{cp} + \Gamma_{cs}^{ca}(L)\epsilon_t^{cs} \tag{A.3.1}$$

where $\Gamma_g^{ca}(L)$, $\Gamma_{cp}^{ca}(L)$ and $\Gamma_{cs}^{ca}(L)$ are infinite-order polynomials, respectively, which satisfy

$$\Gamma_g^{ca}(L) = b\Gamma_g^r(L) + [c(\sigma - 1) + 1]\kappa \left[\frac{\Gamma_g^r(L) - \Gamma_g^r(\kappa)}{L - \kappa}\right] - \kappa \left[\frac{\Gamma_g^{no}(L) - \Gamma_g^{no}(\kappa)}{L - \kappa}\right], \quad \text{(A.3.2)}$$

$$\Gamma_{cp}^{ca}(L) = -\kappa \left[\frac{\Gamma_{cp}^{no}(L) - \Gamma_{cp}^{no}(\kappa)}{L - \kappa}\right], \quad \text{(A.3.3)}$$

and

$$\Gamma_{cs}^{ca}(L) = -\kappa \left[\frac{\Gamma_{cs}^{no}(L) - \Gamma_{cs}^{no}(\kappa)}{L - \kappa}\right] \quad \text{(A.3.4)}$$

under the assumption of the small open economy (2.14). Since the impact responses of the current account ratio to ϵ_t^{cp} and ϵ_t^{cs} are given as $\Gamma_{cp}^{ca}(0)$ and $\Gamma_{cs}^{ca}(0)$, respectively, $\mathcal{H}_{cp}$ and $\mathcal{H}_{cs}$ are obvious from (A.3.3) and (A.3.4).

Appendix A.4: Data Description and Construction

This essay uses quarterly data of four G-7 economies, Canada, Japan, the U.K. and the U.S., which span the sample period Q1:1960-Q4:1997. All data are seasonally adjusted at annual rates and provided by Datastream and IFS.

To construct a measure of the world real interest rate, r_t, I follow the method of Barro and Sala-i-Martin(1990) and Bergin and Sheffrin(2000). I collect short-term nominal interest rates, three-month Treasury bill rates or money market rates, on the G-7 economies from IFS. The inflation rate in each country is calculated by using that country's CPI and the expected inflation rate is constructed by regressing the inflation rate on its own eight lags. The nominal interest rate is then subtracted by the expected inflation rate to compute an *ex-ante* real interest rate. The world real interest rate is computed by taking a weighted average of *ex ante* real interest rates across the G-7 economies, with time-varying weights for each country based on its share of real GDP in the G-7 total.

To construct the net output and current account series, I use each country's national accounting data distributed by Datastream. All nominal series are converted to real series by using the GDP price deflators. The resulting real series are divided by population. Following definition, I construct the net output series, NO_t, by subtracting gross fixed capital formation, change in stocks and government consumption expenditure from GDP. Taking a log of the net output series and a first difference of the resulting logarithmic series provides the first difference of log net output $\Delta \ln NO_t$. The current account series, CA_t, is constructed by subtracting gross fixed capital formation, change in stocks, government consumption expenditure and private consumption expenditure from GNP. Dividing CA_t by NO_t provides the series of the current account-net output ratio, CA_t/NO_t.

Finally the three series, r_t, $\Delta \ln NO_t$ and CA_t/NO_t, are demeaned to construct the series, $\widetilde{r}_t$, $\widetilde{\Delta \ln NO_t}$ and $\widetilde{CA_t/NO_t}$.

Appendix A.5: Unit Root Tests

To check whether $\widetilde{r}_t$, $\widetilde{\Delta \ln NO_t}$ and $\widetilde{CA_t/NO_t}$ are stationary, I apply the augmented Dickey-Fuller test (the ADF test) for the three series. The ADF τ-statistic for time series y_t is given as a t-statistic of the coefficient λ in the following OLS regression

$$\Delta y_t = \lambda y_{t-1} + \sum_{i=1}^{n} \Delta y_{t-i} + \eta_t \tag{A.5.1}$$

where the lag length n is chosen to render η_t white noise. Since the demeaned series $\widetilde{r}_t$, $\widetilde{\Delta \ln NO_t}$ and $\widetilde{CA_t/NO_t}$ fluctuate around zero and have no clear time trend, I do not include either constant or a time trend in the ADF regression (A.5.1). Davidson and MacKinnon(1993) provide asymptotic 10 %, 5 % and 1 % critical values for the Dickey-Fuller τ-statistics equal to -1.62, -1.94 and -2.56, respectively. I perform this test for three choices of the lag length, one, three and five.

Table A.1 summarizes the results of the unit root tests. Except for $\widetilde{CA_t/NO_t}$ of the U.S., the ADF tests reject the unit root null in all series at least at the 5 % significance level for all cases of the lag length. In the case of $\widetilde{CA_t/NO_t}$ of the U.S., the ADF tests reject the unit root null at the 10 % significance level for three and five lags, while the unit root null cannot be rejected even at 10 % significance level for the case of one lag.

Appendix A.6: Predicted Linear Restrictions on the Impact Matrix

In this appendix, I show that all the hypotheses can be rewritten as linear restrictions on the impact matrix $\Gamma(0)$. For exposition, let $[A]_i^r$ and $[A]_i^c$ denote the i th row and column vectors of a matrix A, respectively. First of all, recall that two of three restrictions in identification scheme II are zero restrictions on the impact matrix, which means that they are linear restrictions on the impact matrix in nature. More precisely, let e_i denote a 1×3 row vector which has zeros as the $j \neq i$ th elements and one as the i th element. Then two exclusion restrictions are rewritten as $\Gamma(0)_{1,2} = e_1[\Gamma(0)]_2^c = 0$ and $\Gamma(0)_{1,3} = e_1[\Gamma(0)]_3^c = 0$, respectively. Second, it is the case from eq.(2.22) that $C(1)\Gamma(0) = \Gamma(1)$. This relation implies that $\Gamma(1)_{i,j} = [C(1)]_i^r[\Gamma(0)]_j^c$ for any $i, j = 1, 2, 3$. Therefore a long-run restriction $\Gamma(1)_{i,j} = 0$ should be equal to an orthogonality condition between the ith row vector of $C(1)$ and the j th column vector of $\Gamma(0)$ and can be rewritten as a linear restriction on the impact matrix.

To find the impulse response functions (the IRFs) of CA_t to a global shock ϵ_{t-i}^g for any $i \geq 0$, I take a derivative of the identity $CA_t \equiv (CA_t/NO_t)NO_t$ and obtain the following relation

$$\frac{\partial CA_t}{\partial \epsilon_{t-i}^j} = \frac{CA}{CA/NO}\frac{\partial CA_t/NO_t}{\partial \epsilon_{t-i}^g} + CA\frac{\partial \ln NO_t}{\partial \epsilon_{t-i}^g}$$

where CA/NO and CA are means of CA_t/NO_t and the CA_t, respectively. In particular, the last term in the RHS of the above relation can be given by the accumulated impulse response of $\widetilde{\Delta \ln NO_t}$ to ϵ_{t-i}^g. Hence the IRF of CA_t to ϵ_{t-i}^g is given by

$$\frac{\partial CA_t}{\partial \epsilon_{t-i}^g} = \frac{CA}{CA/NO}\Gamma_{g,i}^{ca} + CA\sum_{s=0}^{i}\Gamma_{g,s}^{no} \tag{A.6.1}$$

where $\Gamma_{g,i}^{no}$ and $\Gamma_{g,i}^{ca}$ are the impulse responses of $\widetilde{\Delta \ln NO_t}$ and $\widetilde{CA_t/NO_t}$ to ϵ_{t-i}^g, respectively.

Let C_i denote the coefficient matrix of L^i in the VMA (2.16). Since $\Gamma_i = C_i\Gamma(0)$ for any $i \geq 0$, the IRFs, $\Gamma_{g,i}^{no}$ and $\Gamma_{g,i}^{ca}$, can be written as follows:

$$\Gamma_{g,i}^{no} = (C_i\Gamma(0))_{2,1} = [C_i]_2^r[\Gamma(0)]_1^c$$

$$\Gamma_{g,i}^{ca} = (C_i\Gamma(0))_{3,1} = [C_i]_3^r[\Gamma(0)]_1^c$$

These equations and eq.(A.6.1) rewrite hypothesis 1 as

$$\left\{[C_i]_3^r + \frac{CA}{NO}\sum_{s=0}^{i}[C_s]_2^r\right\}[\Gamma(0)]_1^c = R_i[\Gamma(0)]_1^c = 0 \quad \forall i \geq 0 \tag{A.6.2}$$

where R_i is a 1×3 row vector such that $R_i = \left\{[C_i]_3^r + (CA/NO)\sum_{s=0}^{i}[C_s]_2^r\right\}$. Therefore hypothesis 1 is also rewritten as a linear restriction on the impact matrix.

Next notice from eq.(2.21) that $\Gamma(\kappa) = C(\kappa)\Gamma(0)$ and thus $\Gamma_{i,j}(\kappa) = [C(\kappa)]_i^r[\Gamma(0)]_j^c$ for any $i, j = 1, 2, 3$. By using this fact, I can rewrite hypothesis i for *i=2,3* as

$$\Gamma(0)_{3,i} = \Gamma(0)_{2,i} - [C(\kappa)]_2^r[\Gamma(0)]_i^c$$

or more compactly, with a 1×3 row vector $R = [C_{2,1}(\kappa) \quad C_{2,2}(\kappa) - 1 \quad C_{2,3}(\kappa) + 1]$,

$$R[\Gamma(0)]_i^c = 0. \tag{A.6.3}$$

Eq.(A.6.3) shows that hypothesis 2 and 3 are also given as the linear restrictions on the impact matrix.

A striking fact is that *under the joint null hypotheses the impact matrix* $\Gamma(0)$ *should be singular*. To show this, first consider identification Scheme I. Notice that there are three linear restrictions on $[\Gamma(0)]_3^c$ under the null: $[C(1)]_1^r[\Gamma(0)]_3^c = 0$, $[C(1)]_2^r[\Gamma(0)]_3^c = 0$ and $R[\Gamma(0)]_3^c = 0$. Since these restrictions are linearly independent and $[\Gamma(0)]_3^c$ is a 3×1 vector, a unique solution for $[\Gamma(0)]_3^c$ exists and should be equal to zero. This implies then that the impact matrix $[\Gamma(0)]$ should be singular under the null. The same result is obtained even with identification Scheme II. In this case, three linearly independent restrictions on $[\Gamma(0)]_3^c$ under the null are given as $e_1[\Gamma(0)]_3^c = 0$, $[C(1)]_2^r[\Gamma(0)]_3^c = 0$ and $R[\Gamma(0)]_3^c = 0$. Therefore a unique solution for $[\Gamma(0)]_3^c$ exists and equals to zero. The impact matrix should be singular under the null.

The singularity of the impact matrix makes it impossible to examine the LR and LM tests for the null since these asymptotic tests depend on the restricted ML estimates of the test statistics. On the other hand, the asymptotic Wald test, which exploits only the unrestricted ML estimates, is applicable for this situation.

B: Appendices for Chapter 3

Appendix B.1: Derivation of Eq.(3.6)

Let CA_t^P denote the standard PVM under $h = 0$ and $C_t^T = 0$:

$$CA_t^P \equiv rB_t + NO_t - C_t^P = -\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_t \Delta NO_{t+i}. \tag{B.1.1}$$

Substituting the decomposition $C_t = C_t^P + C_t^T$ into the current account identity and using eq.(B.1.1) yield

$$\begin{aligned} CA_t &\equiv rB_t + NO_t - C_t \\ &= rB_t + NO_t - C_t^P - C_t^T \\ &= CA_t^P - C_t^T. \end{aligned} \tag{B.1.2}$$

Applying the AR(1) process of C_t^T to eq.(B.1.2) gives

$$\begin{aligned} CA_t &= CA_t^P - C_t^T \\ &= CA_t^P - \rho_c C_{t-1}^T - \omega_t \\ &= \rho_c CA_{t-1} + CA_t^P - \rho_c CA_{t-1}^P - \omega_t \end{aligned} \tag{B.1.3}$$

Several steps of algebra easily show that the term $CA_t^P - \rho_c CA_{t-1}^P$ has the following representation

$$CA_t^P - \rho_c CA_{t-1}^P = \left(\frac{\rho_c}{1+r}\right)\Delta NO_t - \left(1 - \frac{\rho_c}{1+r}\right)\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_t \Delta NO_{t+i}$$
$$- \left(\frac{\rho_c}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i (E_t - E_{t-1})\Delta NO_{t+i}.$$

Note that the last term of the RHS represents revision of expectation for future changes in net output between periods t and $t-1$. Let this term be, say, ξ_t, and notice that expectation of ξ_{t+s} conditional on the information set at period t is zero for any $s \geq 1$ by law of the iterated expectation. Substituting back the term $CA_t - \rho_c CA_{t-1}$ into eq.(B.1.3) and setting $v_t = \xi_t + \omega_t$ provide eq.(3.6).

Appendix B.2: Derivation of Eq.(3.7)

Substituting the PVM (3.2) into the definition of D_t yields

$$\begin{aligned} D_t &\equiv CA_t - \Delta NO_t - (1+r)CA_{t-1} \\ &= -(1+r-h)CA_{t-1} - \left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t\Delta NO_{t+i} - \epsilon_t \\ &= hD_{t-1} - hD_{t-1} - (1+r-h)CA_{t-1} - \left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t\Delta NO_{t+i} - \epsilon_t. \end{aligned} \tag{B.2.1}$$

Substituting the definition $D_{t-1} \equiv CA_{t-1} - \Delta NO_{t-1} - (1+r)CA_{t-2}$ into the second term in the RHS of eq.(B.2.1) and using the PVM (3.2) to eliminate the resulting term

CA_{t-1} further rewrite eq.(B.2.1) as

$$D_t = hD_{t-1} - \epsilon_t + (1+r)\epsilon_{t-1}$$
$$+ (1+r-h)\sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_{t-1}\Delta NO_{t+i-1} - \left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t\Delta NO_{t+i}. \tag{B.2.2}$$

Note that the fourth term in the RHS of eq.(B.2.2) equals

$$\left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_{t-1}\Delta NO_{t+i}.$$

Therefore eq.(3.7) is the case:

$$D_t = hD_{t-1} - \epsilon_t + (1+r)\epsilon_{t-1} - \left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i (E_t - E_{t-1})\Delta NO_{t+i}$$
$$= hD_{t-1} - \epsilon_t + (1+r)\epsilon_{t-1} + e_t.$$

Note that expectation of e_{t+s} conditional on the information set at period t is zero for any $s \geq 1$ because

$$E_t e_{t+s} = -\left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i E_t(E_{t+s} - E_{t+s-1})\Delta NO_{t+i+s}$$
$$= -\left(1 - \frac{h}{1+r}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i (E_t - E_t)\Delta NO_{t+i+s}$$
$$= 0$$

by the law of the iterated expectation.

Appendix B.3: The Two-Step GMM Estimation

In the first step, the criterion function $J(\theta) = G(\theta)'MG(\theta)$ is minimized with respect to θ under the restriction that the weighting matrix M is the identity matrix I. The resulting estimate of θ, say θ^*, is used to construct the optimal weighting matrix M^* such that

$$M^* = \left[T^{-1} \sum_{t-1}^{T} g_t(\theta^*) g_t(\theta^*)' \right]^{-1}$$

when $g_t(\theta^*)$ follows an i.i.d. process. Because there is a possibility of serial correlation of $g_t(\theta^*)$ in the first step, this essay exploits the heteroskedasticity-autocorrelation consistent estimator of Newey and West(1987) to calculate the optimal weighting matrix M^*. In the second step, minimizing the criterion function $J(\theta)$ under the optimal weighting matrix M^* yields the second step estimate $\hat{\theta}_{GMM}$ with the asymptotic variance-covariance matrix

$$\hat{V}_{\theta_{GMM}} = T^{-1} \left[\frac{\partial G(\hat{\theta}_{GMM})}{\partial \theta'} M^* \frac{\partial G(\hat{\theta}_{GMM})}{\partial \theta'}' \right]^{-1}.$$

Appendix B.4: The State Space Representation of the Equilibrium Path

The purpose of this appendix is to explain in detail the derivation of the state space representation from the system of stochastic difference equations, which contains eqs.(3.14)-(3.24). The first step is to convert the system to the stationary one. To do that, it is convenient to introduce a new variable ϖ_t satisfying

$$\varpi_t = C_{t-1}/A_{t-1}. \tag{B.4.1}$$

That is, ϖ_t is stochastically detrended consumption at period $t-1$.

B.4.1: Deriving the Stationary System

Using the stochastically detrended variables and eqs.(3.17), (3.18) and (B.4.1) rewrites the system of equations (3.14)-(3.16) and (3.21)-(3.24) as the following stationary system:

The Stationary System

$$b_{t+1} = \left[1 + q_t - \eta\left(\frac{b_t}{y_t}\right)\exp(-\Delta\ln A_t)\right]\exp(-\Delta\ln A_t)b_t + (1-g)y_t - i_t - c_t \quad (3.14')$$

$$y_t = k_t^{\psi}N_t^{1-\psi}\exp(-\psi\Delta\ln A_t) \quad (3.15')$$

$$k_{t+1} = (1-\delta)\exp(-\Delta\ln A_t)k_t + \left(\frac{k_t}{i_t}\right)^{\varphi} i_t\exp(-\varphi\Delta\ln A_t) \quad (3.16')$$

$$\beta\exp\{[\phi(1-\gamma)-1]\Delta\ln A_{t+1}\}\left[\frac{c_{t+1} - h\exp(-\Delta\ln A_{t+1})\varpi_{t+1}}{c_t - h\exp(-\Delta\ln A_t)\varpi_t}\right]^{\phi(1-\gamma)-1}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\phi)(1-\gamma)} = \Gamma_{t+1} \quad (3.21')$$

$$1 = E_t\Gamma_{t+1}\left[1 + q_{t+1} - 2\eta\exp(-\Delta\ln A_{t+1})\left(\frac{b_{t+1}}{y_{t+1}}\right)\right] \quad (3.22')$$

$$\frac{1-\phi}{\phi}\left[\frac{c_t - h\exp(-\Delta\ln A_t)\varpi_t}{1-N_t}\right] = (1-\psi)\frac{y_t}{N_t}\left[1 + \eta\exp(-2\Delta\ln A_t)\left(\frac{b_t}{y_t}\right)^2\right] \quad (3.23')$$

$$\begin{aligned}\frac{1}{1-\varphi}\left(\frac{i_t}{k_t}\right)^{\varphi}\exp(\varphi\Delta\ln A_t) = {}& \\ E_t\Gamma_{t+1}\left\{\frac{1-\delta}{1-\varphi} + \frac{\varphi}{1-\varphi}\exp[(1-\varphi)\Delta\ln A_{t+1}]\left(\frac{i_{t+1}}{k_{t+1}}\right)^{1-\varphi}\right\}&\exp(\varphi\Delta\ln A_{t+1})\left(\frac{i_{t+1}}{k_{t+1}}\right)^{\varphi} \\ + E_t\Gamma_{t+1}\psi\exp(\Delta\ln A_{t+1})\frac{y_{t+1}}{k_{t+1}}&\left[1 + \eta\exp(-2\Delta\ln A_{t+1})\left(\frac{b_{t+1}}{y_{t+1}}\right)^2\right] \quad (3.24')\end{aligned}$$

and eq.(B.4.1). The stationary system contains the eight equations, the eight endogenous variables and the two exogenous variables following the processes (3.19) and (3.20).

B.4.2: The Deterministic Steady State

Let c, y, i, N, k, b, Γ and ϖ denote the deterministic steady state values of the corresponding variables. From the stationary system, the deterministic steady state is characterized as follows. First, from eq.(3.21'), the steady state value of the stochastic discount factor, Γ, is given as

$$\Gamma = \beta \exp\{[\phi(\gamma - 1) - 1]\alpha\}$$

where α is the unconditional mean of $\Delta \ln A_t$. Eq.(B.4.1) shows that the steady state value ϖ is equal to c

$$\varpi = c.$$

From eqs.(3.16') and (3.22'), the steady state ratios i/k and b/y are determined by

$$\frac{i}{k} = [1 - (1 - \delta) \exp(-\alpha)]^{\frac{1}{1-\varphi}} \exp(\varphi\alpha)^{\frac{1}{1-\varphi}}$$

and

$$\frac{b}{y} = \left[\frac{1 + q^* - \frac{1}{\Gamma}}{2\eta \exp(-\alpha)}\right].$$

Given i/k and b/y, the steady state ratio y/k is determined as a solution of the equation

$$\begin{aligned}
\frac{1}{1-\varphi}\left(\frac{i}{k}\right)^{\varphi} \exp(\varphi\alpha) = \\
\Gamma\left\{\frac{1-\delta}{1-\varphi} + \frac{\varphi}{1-\varphi}\exp[(1-\varphi)\alpha]\left(\frac{i}{k}\right)^{1-\varphi}\right\} \exp(\varphi\alpha)\left(\frac{i}{k}\right)^{\varphi} \\
+ \Gamma\psi \exp(\alpha)\frac{y}{k}\left[1 + \eta \exp(-2\alpha)\left(\frac{b}{y}\right)^2\right].
\end{aligned}$$

Because i/k and y/k have been already derived, the steady state ratio i/y can be constructed by dividing i/k by y/k. Eqs.(3.15') and (3.23') then yield the steady state ratios

k/N and c/y as

$$\frac{k}{N}=\left[\frac{y}{k}\exp(\psi\alpha)\right]^{\frac{1}{\psi-1}}$$

and

$$\frac{c}{y}=\left[1+q^{*}-\eta\left(\frac{b}{y}\right)\exp(-\alpha)\right]\exp(-\alpha)\left(\frac{b}{y}\right)+(1-g)-\left(\frac{i}{y}\right)-\left(\frac{b}{y}\right).$$

Finally, eq.(3.23') determines the steady state level of N as a solution of the equation

$$\frac{1-\phi}{\phi}[1-h\exp(-\alpha)]\frac{c}{y}=(1-\psi)\frac{1-N}{N}\left[1+\eta\exp(-2\alpha)\left(\frac{b}{y}\right)^{2}\right].$$

Given N, the steady state level k is obtained by multiplying the ratio k/N by N. The steady state level y is obtained by multiplying y/k by k. Similarly, the other steady state levels c and i are constructed by multiplying c/y and i/y by y, respectively[12].

B.4.3: Derivation of the State Space Representation

The next step is to take a first-order Taylor expansion of the system (B.4.1), (3.14')-(3.16') and (3.21')-(3.24') around the deterministic steady state. Let $\tilde{x}_t \equiv x_t - x$ and $\hat{x}_t \equiv x_t/x - 1$ for any variable x_t with the deterministic steady state x. Note that the linear approximations of eqs.(3.15') and (3.23') are static equations. By using this fact, $\hat{y}_t$ and $\hat{N}_t$ can be solved as linear functions of $\hat{c}_t$, $\hat{k}_t$, $\hat{b}_t$ and $\widetilde{\Delta \ln A_t}$, respectively, which in turn are used to solve out $\hat{y}_t$ and $\hat{N}_t$ in the other linear approximations of eqs.(3.14'), (3.21'), (3.22'), (3.24'). Furthermore, eq.(3.21') characterizes the process of the stochastic discount factor. Using the linear approximation of this equation can solve out the

[12]It is important to note that the above derivation of the steady state does not require solving a nonlinear simultaneous equation system. This fact makes the following numerical exercise simple.

stochastic discount factor in the other equations. As a result, the linear approximations of (3.14'), (3.16'), (3.22'), (3.24') and (B.4.1) are given as linear stochastic difference equations with respect to the five endogenous variables $\hat{c}_t$, $\hat{\imath}_t$, $\hat{\varpi}_t$, $\hat{k}_t$ and $\hat{b}_t$, and the two exogenous variables $\widetilde{\Delta \ln A_t}$ and $\widetilde{\ln(1+q_t)}$ that follows the stochastic processes eqs.(3.19) and (3.20), respectively.

Let $\mathcal{X}_t = [\hat{c}_t \quad \hat{\imath}_t \quad \hat{\varpi}_t \quad \hat{k}_t \quad \hat{b}_t \quad \widetilde{\Delta \ln A_t} \quad \widetilde{\ln(1+q_t)}]$. Then it is shown that the system of the linear stochastic difference equation has the matrix representation:

$$\Theta_0 \mathcal{X}_t = \Theta_1 \mathcal{X}_{t-1} + \Psi \epsilon_t + \Pi \nu_t \tag{B.4.2}$$

where Θ_0 and Θ_1 are 7×7, Ψ and Π are 7×2, $\epsilon_t = [\epsilon_t^a \quad \epsilon_t^q]'$ and ν_t is the vector of expectational errors satisfying

$$\nu_t = \begin{bmatrix} \hat{c}_t - E_{t-1}\hat{c}_t \\ \hat{\imath}_t - E_{t-1}\hat{\imath}_t \end{bmatrix}.$$

The leading matrix Θ_0 is non-singular and invertible.

This essay solves the linear rational expectation model (B.4.2) by following Sims(2000). Sims argues that the disturbance vector $\Psi\epsilon_t + \Pi\nu_t$ is not exogenous as ϵ_t itself is, because ν_t depends on the endogenous variables $\hat{c}_t$ and $\hat{\imath}_t$ and their expectations. Hence solving the linear rational expectation model (B.4.2) needs to determine ν_t from ϵ_t.

Since the leading matrix is invertible, premultiplying eq.(B.4.2) by Θ_0^{-1} yields

$$\mathcal{X}_t = \tilde{\Theta}_1 \mathcal{X}_{t-1} + \tilde{\Psi}\epsilon_t + \tilde{\Pi}\nu_t \tag{B.4.3}$$

where $\tilde{\Theta}_1 = \Theta_0^{-1}\Theta_1$, $\tilde{\Psi} = \Theta_0^{-1}\Psi$ and $\tilde{\Pi} = \Theta_0^{-1}\Pi$. The matrix $\tilde{\Theta}_1$ has the eigenvalue decomposition such that

$$\tilde{\Theta}_1 = V \Lambda V^{-1}$$

where Λ is the diagonal matrix that contains the eigenvalues of $\tilde{\Theta}_1$ in the descending order in absolute value, and V is the matrix constructed by the corresponding eigenvectors. Premultiplying eq.(B.4.3) by V^{-1} and defining a new vector $\mathcal{Z}_t = V^{-1}\mathcal{X}_t$ yield

$$\mathcal{Z}_t = \Lambda \mathcal{Z}_{t-1} + V^{-1}[\tilde{\Psi}\epsilon_t + \tilde{\Pi}\nu_t]. \tag{B.4.4}$$

Let Λ_1 be the diagonal matrix that contains the eigenvalues greater than or equal to one in absolute value. Also let $\mathcal{Z}_t^1$ is the vector containing the elements of $\mathcal{Z}_t$ corresponding to the explosive eigenvalues. Then eq.(B.4.4) implies that $\mathcal{Z}_t^1$ follows the process

$$\mathcal{Z}_t^1 = \Lambda_1 \mathcal{Z}_{t-1}^1 + B_1[\tilde{\Psi}\epsilon_t + \tilde{\Pi}\nu_t]. \tag{B.4.5}$$

where B_1 comes from the partition $V^{-1} = [B_1' \quad B_2']'$. Since all the diagonal elements of Λ_1 are explosive eigenvalues, eq.(B.4.5) has a forward solution such that

$$\mathcal{Z}_t^1 = -\sum_{i=0}^{\infty} \Lambda_1^{-i-1} B_1[\tilde{\Psi}\epsilon_{t+i+1} + \tilde{\Pi}\nu_{t+i+1}]. \tag{B.4.6}$$

Notice that $E_t\mathcal{Z}_t^1 = \mathcal{Z}_t^1$ because all the elements of $\mathcal{X}_t$ are included in the information set at period t. Since $E_t\epsilon_{t+i} = E_t\nu_{t+i} = 0$ for any $i \geq 1$,

$$\mathcal{Z}_t^1 = E_t\mathcal{Z}_t^1 = -\sum_{i=0}^{\infty} \Lambda_1^{-i-1} B_1[\tilde{\Psi}E_t\epsilon_{t+i+1} + \tilde{\Pi}\nu_{t+i+1}] = 0. \tag{B.4.7}$$

Comparing eq.(B.4.6) and (B.4.7) shows that the following equality must be satisfied

$$\sum_{i=0}^{\infty} \Lambda_1^{-i-1} B_1[\tilde{\Psi}\epsilon_{t+i+1} + \tilde{\Pi}\nu_{t+i+1}] = 0. \tag{B.4.8}$$

For eq.(B.4.8) to be satisfied it is the case that

$$B_1\tilde{\Pi}\nu_t = -B_1\tilde{\Psi}\epsilon_t \tag{B.4.9}$$

for all t. Therefore, as Sims argues, the necessary and sufficient condition for the existence of a solution satisfying eq.(B.4.8) is that the column space of $B_1\tilde{\Psi}$ be contained in that of $B_1\tilde{\Pi}$. That is, for any realization of ϵ_t, there must exist some ν_t satisfying eq.(B.4.9) for the existence of a solution.

Next consider the stable part of eq.(B.4.4):

$$\mathcal{Z}_t^2 = \Lambda_2 \mathcal{Z}_{t-1}^2 + B_2[\tilde{\Psi}\epsilon_t + \tilde{\Pi}\nu_t]. \tag{B.4.10}$$

where Λ_2 is the diagonal matrix that contains the eigenvalues of $\tilde{\Theta}_2$, which are less than one in absolute value. The problem here is the uniqueness of the solution. Since the existence requires eq.(B.4.9), $B_1\tilde{\Pi}\nu_t$ can be determined from a known stochastic process for ϵ_t. However, eq.(B.4.10) requires that $B_2\tilde{\Pi}\nu_t$ should be known at the same time. It is possible that knowing $B_1\tilde{\Pi}\nu_t$ is not enough to show $B_2\tilde{\Pi}\nu_t$ when the solution is not unique. Sims gives as the necessary and sufficient condition for the uniqueness that the row space of $B_2\tilde{\Pi}$ be contained in that of $B_1\tilde{\Pi}$. In other words, it should be the case that there exists some matrix Φ satisfying

$$B_2\tilde{\Pi} = \Phi B_1\tilde{\Pi}. \tag{B.4.11}$$

Suppose then that the necessary and sufficient conditions for the existence and uniqueness of the solution is satisfied, i.e. eq.(B.4.9) is satisfied and a matrix Φ satisfying eq.(B.4.11) exists. Then eq.(B.4.10) can be rewritten as

$$\mathcal{Z}_t^2 = \Lambda_2 \mathcal{Z}_{t-1}^2 + (B_2 - \Phi B_1)\tilde{\Psi}\epsilon_t. \tag{B.4.12}$$

To derive the state space representation, it is convenient to partition the vector $\mathcal{X}_t$

and the matrices V and V^{-1} as

$$\mathcal{X}_t = [\mathcal{X}_t^{1'} \quad \mathcal{X}_t^{2'}]', \quad \mathcal{X}_t^1 = [\hat{c}_t \quad \hat{i}_t]', \quad \mathcal{X}_t^2 = [\hat{\varpi}_t \quad \hat{k}_t \quad \hat{b}_t \quad \widetilde{\Delta \ln A_t} \quad \widetilde{\ln(1+q_t)}]',$$

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \quad V^{-1} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}.$$

Moreover assume that from eq.(B.4.11) the matrix Φ is obtained as

$$\Phi = B_2\tilde{\Pi}(B_1\tilde{\Pi})^{-1}.$$

For the inverse to exist, the matrix $B_1\tilde{\Pi}$ must be square. This then implies that the row number of B_1 must be 2 because the column number of $\tilde{\Pi}$ is 2. Since by construction B_{11} is square, B_{11} and B_{12} should be 2×2 and 2×5. Recall that eq.(B.4.7) requires $\mathcal{Z}_t^1 = B_1\mathcal{X}_t = 0$. Hence it is the case that

$$\mathcal{X}_t^1 = -B_{11}^{-1}B_{12}\mathcal{X}_t^2 \tag{B.4.13}$$

Eq.(B.4.13) shows the cross-equation restrictions characterizing the saddle path. Using these cross-equation restrictions (B.4.13) can rewrite eq.(B.4.12) as the process of $\mathcal{X}_t^2$:

$$\mathcal{X}_t^2 = \mathcal{G}^{-1}\Lambda_2\mathcal{G}\mathcal{X}_{t-1}^2 + \mathcal{G}^{-1}(B_2 - \Phi B_1)\tilde{\Psi}\epsilon_t. \tag{B.4.14}$$

where $\mathcal{G}$ is a 5×5 matrix satisfying $\mathcal{G} = B_{22} - B_{21}B_{11}^{-1}B_{12}$. Let $\mathcal{X}_t^2 = \mathcal{S}_t$. Eq.(B.4.14) is then the transition equation of the state variables $\mathcal{S}_t$ in eq.(3.25). Recall that $\hat{y}_t$ and $\hat{N}_t$ are given as linear functions of $\mathcal{X}_t$. This fact and eq.(B.4.13) yields the observation equation with respect to $\mathcal{P}_t$ of the state space representation eq.(3.25).

Table 2.1: Three Empirical Results of the Intertemporal Approach and the PVM of the Current Account:

How Does the Current Account Respond to the Shocks?

1. **Does a Global Shock Have No Impact on the Current Account?**
   - **Yes**: Glick and Rogoff(1995), İşcan(2000)
   - **Sensitive to Identification**: Nason and Rogers(2002)

2. **Country-Specific Transitory Shocks Dominate the Current Account Fluctuations in the Short-Run as Well as the Long-Run**: Nason and Rogers (2002)

3. **Glick and Rogoff's (1995) Puzzle**
   - **Persistent Country-Specific Shock**: Glick and Rogoff(1995)
   - **Permanent and Transitory Decomposition by the VECM**: Hoffmann(2001)
   - **Nontradable Goods**: İşcan(2000)

Table 2.2: Findings of This Essay

1. Impulse Responses of the Current Account to the Identified Shocks are Consistent with the Corresponding Theoretical Predictions

2. Tests for the Cross-Equation Restrictions on the SVAR Show

   - The Hypothesis the Current Account Does Not Respond to a Global Shock is Sensitive to the Identification.
   - The Impact Responses of the Current Account to Country-Specific Shocks Match the PVM's Predictions.
   - The Joint Hypothesis Related to the Impact Responses of the Current Account to All the Three Shocks is Rejected.

3. The Data Support the Observation that the Current Account Responds to a Country-Specific Transitory Shock Greater than Net Output.

4. The FEVDs Show that Country-Specific Transitory Shocks Dominate Current Account Fluctuations Not Only in the Short Run But the Long Run As Well, While the Shocks Explain Almost None of the Fluctuations in Net Output.

Table 2.3: Identification Schemes

(a) Identification Scheme I

Economic Meaning	*Restriction*
A Country-Specific Permanent Shock Has No *Long-Run* Effect on the World Real Interest Rate	$\Gamma(1)_{1,2} = 0$
A Country-Specific Transitory Shock Has No *Long-Run* Effect on the World Real Interest Rate	$\Gamma(1)_{1,3} = 0$
A Country-Specific Transitory Shock Has No *Long-Run* Effect on Log of Net Output	$\Gamma(1)_{2,3} = 0$

(b) Identification Scheme II

Economic Meaning	*Restriction*
A Country-Specific Permanent Shock Has No *Instantaneous* Effect on the World Real Interest Rate	$\Gamma(0)_{1,2} = 0$
A Country-Specific Transitory Shock Has No *Instantaneous* Effect on the World Real Interest Rate	$\Gamma(0)_{1,3} = 0$
A Country-Specific Transitory Shock Has No *Long-Run* Effect on Log of Net Output	$\Gamma(1)_{2,3} = 0$

Note 1: In addition to three restrictions, each identification scheme requires the structural shocks to be orthogonal and have unit variances.

Note 2: $\Gamma(0)$ and $\Gamma(1)$ are the impact and the long-run matrices of the SVMA, respectively. For a matrix A, $A_{i,j}$ shows the (i, j)th element of the matrix A.

Table 2.4: Calibrated Parameters and Joint Test of the Present Value Restrictions

	Canada	the U.K.
κ	0.993	0.990
c	0.983	0.988
b	-0.712	0.377
σ	0.001	0.080
$\mathcal{W}$	18.193	23.224
p-value	0.000	0.000

Note 1: To calibrate b requires the data of international bond holdings B_t. This essay uses as B_t the international net investment position (IIP) in the balance of payment statistics. Statistics Canada (http://www.statcan.ca) distributes the annual IIP for Canada from 1926 to 2001. This essay converts the annual series to quarterly series, divides the resulting series by nominal net output and takes the sample average from Q1:1963-Q4:1997 to construct b. On the other hand, National Statistics (http://www.statistics.gov.uk) provides the annual IIP series of the U.K. only from 1966. Nevertheless, the value of b for the U.K. is calibrated by applying the same method as in the Canadian case for the whole sample period 1966-1997.

Note 2: The elasticity of intertemporal substitution σ is calibrated by minimizing the mean squared error of the PVM prediction on the current account-net output ratio.

Note 3: The Wald statistic $\mathcal{W}$ is calculated by eq.(2.25) conditional on the calibrated parameters κ, c, b, and σ. The corresponding p-value is based on the chi-squared distribution with the third degree of freedom.

Table 2.5 Asymptotic Wald Tests of the Cross-Equation Restrictions

(a) Identification Scheme I

	Canada	U.K.
W1	0.190	0.758
p-value	*0.663*	*0.384*
W2	0.069	0.001
p-value	*0.793*	*0.983*
W3	1.562	1.589
p-value	*0.211*	*0.208*
W4	379.392	320.599
p-value	*0.000*	*0.000*
W5	20.010	0.823
p-value	*0.001*	*0.935*

(b) Identification Scheme II

	Canada	U.K.
W1	10.416	24.100
p-value	*0.001*	*0.000*
W2	0.782	1.297
p-value	*0.376*	*0.255*
W3	1.827	3.212
p-value	*0.176*	*0.073*
W4	14.603	34.944
p-value	*0.002*	*0.000*
W5	14.809	24.809
p-value	*0.005*	*0.000*

Note 1: The null of W1 is that Hypothesis 1 is satisfied.
Note 2: The null of W2 is that Hypothesis 2 is satisfied.
Note 3: The null of W3 is that Hypothesis 3 is satisfied.
Note 4: The null of W4 is that Hypotheses 1, 2, 3 are jointly satisfied.
Note 5: The null of W5 is that Hypothesis 1 is satisfied up to a year.
Note 6: P-values are constructed by asymptotic chi-square distributions.

Table 2.6 FEVDs of the CA under Identification Scheme I

Canada

Periods	g	cp	cs
0	*0.0555*	*0.1744*	*0.7700*
	(0.1526)	(0.1538)	(0.1970)
1	*0.0544*	*0.1535*	*0.7921*
	(0.1521)	(0.1440)	(0.1893)
2	*0.0499*	*0.1485*	*0.8016*
	(0.1492)	(0.1424)	(0.1863)
3	*0.0457*	*0.1457*	*0.8086*
	(0.1466)	(0.1413)	(0.1837)
4	*0.0422*	*0.1440*	*0.8139*
	(0.1443)	(0.1405)	(0.1816)
12	*0.0344*	*0.1381*	*0.8275*
	(0.1378)	(0.1362)	(0.1725)
20	*0.0396*	*0.1355*	*0.8249*
	(0.1391)	(0.1336)	(0.1708)
40	*0.0440*	*0.1354*	*0.8206*
	(0.1408)	(0.1318)	(0.1706)

U.K.

Periods	g	cp	cs
0	*0.2419*	*0.0598*	*0.6983*
	(0.2143)	(0.1130)	(0.2180)
1	*0.2396*	*0.0485*	*0.7118*
	(0.2125)	(0.1056)	(0.2123)
2	*0.2383*	*0.0469*	*0.7149*
	(0.2111)	(0.1054)	(0.2106)
3	*0.2371*	*0.0458*	*0.7171*
	(0.2100)	(0.1050)	(0.2091)
4	*0.2361*	*0.0454*	*0.7186*
	(0.2091)	(0.1049)	(0.2080)
12	*0.2319*	*0.0459*	*0.7223*
	(0.2061)	(0.1047)	(0.2038)
20	*0.2311*	*0.0468*	*0.7221*
	(0.2063)	(0.1044)	(0.2035)
40	*0.2311*	*0.0479*	*0.7210*
	(0.2079)	(0.1042)	(0.2044)

Note 1: g, cp and cs represent global, country-specific permanent and transitory shocks, respectively.
Note 2: The numbers in parentheses denote the standard errors based on 10000 nonparametric bootstrapping resamples.

Table 2.7 FEVDs of ln NO under Identification Scheme I

Canada

Periods	g	cp	cs
0	*0.0104*	*0.8521*	*0.1375*
	(0.1077)	(0.3269)	(0.3210)
1	*0.0105*	*0.8643*	*0.1251*
	(0.1010)	(0.3204)	(0.3152)
2	*0.0105*	*0.8756*	*0.1139*
	(0.0948)	(0.3114)	(0.3068)
3	*0.0103*	*0.8857*	*0.1041*
	(0.0896)	(0.2999)	(0.2949)
4	*0.0100*	*0.8947*	*0.0953*
	(0.0854)	(0.2867)	(0.2805)
12	*0.0064*	*0.9399*	*0.0537*
	(0.0976)	(0.1926)	(0.1584)
20	*0.0047*	*0.9588*	*0.0366*
	(0.1310)	(0.1664)	(0.0959)
40	*0.0067*	*0.9729*	*0.0204*
	(0.1871)	(0.1899)	(0.0438)

U.K.

Periods	g	cp	cs
0	*0.1034*	*0.8506*	*0.0460*
	(0.0900)	(0.2213)	(0.2204)
1	*0.1105*	*0.8510*	*0.0385*
	(0.0893)	(0.2080)	(0.2094)
2	*0.1178*	*0.8500*	*0.0322*
	(0.0903)	(0.1953)	(0.1981)
3	*0.1252*	*0.8477*	*0.0272*
	(0.0925)	(0.1838)	(0.1868)
4	*0.1325*	*0.8444*	*0.0232*
	(0.0958)	(0.1740)	(0.1758)
12	*0.1853*	*0.8050*	*0.0097*
	(0.1335)	(0.1508)	(0.1106)
20	*0.2253*	*0.7686*	*0.0061*
	(0.1659)	(0.1689)	(0.0748)
40	*0.2800*	*0.7169*	*0.0031*
	(0.2103)	(0.2081)	(0.0347)

Note 1: g, cp and cs represent global, country-specific permanent and transitory shocks, respectively.
Note 2: The numbers in parentheses denote the standard errors based on 10000 nonparametric bootstrapping resamples.

Table 3.1: The Sample Statistics of the PVMs

(a) The Habit-Forming PVM

$\hat{h}_{2SLS}$	$\hat{h}_{GMM}$	$\mathcal{J}_T$	$\mathcal{W}_T$
0.931	1.002	0.455	37.128
(0.192)	(0.152)	[0.978]	[0.000]

$\hat{\mathcal{F}}_1$	$\hat{\mathcal{F}}_2$	$\hat{\mathcal{F}}_3$	$\hat{\mathcal{F}}_4$	$\hat{\mathcal{F}}_5$	$\hat{\mathcal{F}}_6$	$\hat{\mathcal{F}}_7$	$\tilde{\mathcal{F}}_8$
-0.302	-0.068	0.017	0.006	1.276	-0.400	0.062	0.138
(0.130)	(0.059)	(0.073)	(0.049)	(0.226)	(0.157)	(0.073)	(0.079)

(b)The Standard PVM

$\mathcal{W}_T^*$	$\hat{\mathcal{F}}_1^*$	$\hat{\mathcal{F}}_2^*$	$\hat{\mathcal{F}}_3^*$	$\hat{\mathcal{F}}_4^*$	$\hat{\mathcal{F}}_5^*$	$\hat{\mathcal{F}}_6^*$	$\hat{\mathcal{F}}_7^*$	$\hat{\mathcal{F}}_8^*$
20.589	0.229	0.066	0.010	0.106	-0.115	0.046	-0.019	-0.095
[0.009]	(0.171)	(0.179)	(0.126)	(0.088)	(0.408)	(0.106)	(0.113)	(0.106)

Note: Table 3.1(a) reports the sample statistics of the PVM with habits. $\hat{h}_{2SLS}$ is the 2SLS estimate of the habit parameter based on the single unconditional moment conditions (3.8) while $\hat{h}_{GMM}$ is the GMM estimate of the habit parameter based on the full unconditional moment conditions (3.8), (3.10) and (3.12). $\mathcal{J}_T$ is the χ^2 statistic with the fourth degree of freedom for the overidentifying restriction test. $\mathcal{W}_T$ is the χ^2 statistic with the eighth degree of freedom for the cross-equation restrictions (3.11). The brackets below $\mathcal{J}_T$ and $\mathcal{W}_T$ show the corresponding asymptotic p-values. $\hat{\mathcal{F}}_i$ represents the estimate of the ith element in the vector $\mathcal{F}(\hat{\theta})$. The numbers in parentheses give the asymptotic standard errors for the corresponding estimates.

Table 3.1(b) shows the sample statistics for the standard PVM. $\mathcal{W}_T^*$ is the χ^2 statistic with the eighth degree of freedom for the cross-equation restrictions of the standard PVM. $\hat{\mathcal{F}}_i^*$ represents the estimate of the ith element in the cross-equation restrictions of the standard PVM.

Table 3.2: Empirical Facts of the Present Value Models

(a) **The PVM with Habit Formation**

1. The Habit Parameter is Close to One.
2. The Cross-Equation Restrictions are Jointly Rejected.
3. The Fifth Element of $\mathcal{F}(\hat{\theta}_{GMM})$ is Close to One.
4. The Predictions Tracks the Actual Series Closely.

(b) **The Standard PVM**

1. The Cross-Equation Restrictions are Jointly Rejected.
2. The Fifth Element of $\mathcal{F}^*(\hat{\theta}_{OLS})$ is Close to Zero.
3. The Predictions are Too Smooth.

Table 3.3: Calibrated Parameters of SOE-RBC Models

Baseline Parameters

β	ϕ	γ	ψ	φ	δ
0.994	0.371	2.000	0.350	0.050	0.020

η	g	α	σ_a
0.071×10^{-4}	0.230	0.0024	0.012

Monte Carlo Experiments with Habit Formation

h	ρ_q	σ_q
0.990	1.000×10^{-7}	1.000×10^{-7}

Monte Carlo Experiments with the World Real Interest Rate

h	ρ_q	σ_q
0.000	0.903	0.004

Table 3.4: Sample Estimates and Empirical P-values under the Nulls of SOE-RBC Models

		Empirical P-values	
	SampleEstimates	Habit Formation	World Real Interest Rates
$\hat{h}_{2SLS}$	0.931	0.7245	0.1150
$\hat{h}_{GMM}$	1.002	0.3824	0.1070
$\mathcal{W}_T$	37.128	0.0696	0.5499
$\hat{\mathcal{F}}_1$	-0.302	0.7326	0.9536
$\hat{\mathcal{F}}_2$	-0.068	0.6297	0.5217
$\hat{\mathcal{F}}_3$	0.017	0.4733	0.4571
$\hat{\mathcal{F}}_4$	0.006	0.4904	0.6670
$\hat{\mathcal{F}}_5$	1.276	0.2593	0.1493
$\hat{\mathcal{F}}_6$	-0.400	0.7841	0.7215
$\hat{\mathcal{F}}_7$	0.062	0.4198	0.3885
$\hat{\mathcal{F}}_8$	0.138	0.3481	0.1766
$\mathcal{W}_T^*$	20.589	0.0141	0.3259
$\hat{\mathcal{F}}_1^*$	0.229	0.0000	0.7164
$\hat{\mathcal{F}}_2^*$	0.066	0.0000	0.8073
$\hat{\mathcal{F}}_3^*$	0.010	0.0071	0.7952
$\hat{\mathcal{F}}_4^*$	0.106	0.0131	0.0363
$\hat{\mathcal{F}}_5^*$	-0.115	0.9980	0.4773
$\hat{\mathcal{F}}_6^*$	0.046	0.0151	0.7548
$\hat{\mathcal{F}}_7^*$	-0.019	0.0605	0.8295
$\hat{\mathcal{F}}_8^*$	-0.095	0.0111	0.9072

Note: Empirical p-values are constructed as the frequency of times that the simulated number exceeds the corresponding sample point estimate.

Table 3.5: The Monte Carlo Experiments: Which SOE-RBC Model Mimics the Empirical Facts?

1. The SOE-RBC model with habit formation mimics the first, third and fourth facts of the habit-forming PVM.

2. The SOE-RBC model with habit formation fails to mimic the second fact of the habit-forming PVM: the Wald statistics for the cross-equation restrictions.

3. The SOE-RBC model with habit formation fails to mimic all the facts of the standard PVM.

4. The SOE-RBC model with stochastic world real interest rates mimics all the facts of the habit-forming PVM.

5. The SOE-RBC model with stochastic world real interest rates mimics all the facts of the standard PVM. In particular, the model does a better job in replicating the third fact of the standard PVM than the SOE-RBC model with habit formation does.

Table 4.1: Failures of the One-Sector RBC Model with Habit Formation and Adjustment Costs of Investment

The Habit Model

1. Fails to Mimic the Significantly Positive, First and Second Order ACFs of Output Growth.

2. Fails to Mimic the Maximum Power Spectrum of Output Growth Over Business Cycle Frequencies.

3. Fails to Mimic the Hump-Shaped IRFs of Output to a Transitory Shock.

4. Overstates the Higher-Order ACFs of Consumption Growth.

5. Overstates the Power Spectrum of Consumption Growth around Zero Frequency.

6. Fails to Yield the High Volatility of Equity Return.

Table 4.2: Calibrated Parameters of the Model

Parameter	Calibrated Value	Source
β	0.992	CE, CN
α	0.004	CE, CN, BCF
ψ	0.360	BCF
δ	0.021	CE, CN, BCF
g^*	0.228	U.S. data
φ	0.050	NR
ρ_g	0.960	CE, CN
σ_a	0.018	BCF
σ_g	0.021	CE
h	0.985	

Notes: CE, CN, BCF, and NR denote Chiristiano and Eichenbaum(1992), Cogley and Nason(1995), Boldrin, Christiano, and Fisher(2001), and Nason and Rogers(2002b), respectively. In particular, g^* is calibrated to the U.S. data. Given the other parameters, h is calibrated to maximize the ability of the model to account for the risk free rate.

Table 4.3: Generalized Q Statistics

	ACFs		IRFs	
Model	$\Delta \ln Y_t$	$\Delta \ln C_t$	Permanent	Transitory
Benchmark	40.994	7.917	40.152	24.189
	(0.000)	(0.442)	(0.000)	(0.002)
Habit	39.411	6.94	112.525	236.037
	(0.000)	(0.543)	(0.000)	(0.000)

Note: In the table, each number denotes the generalized Q statistic, and the number in the parenthesis shows the corresponding p-value. For derivation of the generalized Q statistic, see Cogley and Nason(1995). All the generalized Q statistics in the table approximately follow the chi-squared distribution with 8 degrees of freedom.

Table 4.4: Asset Price Statistics

Statistic	Data	Benchmark($h = 0$)	Habit ($h = 0.985$)
Er_t^f	1.19	4.94	1.28
	(0.81)		
$E(r_{t+1}^e - r_t^f)$	6.63	0.05	3.67
	(1.78)		
σ_{r^e}	19.4	0.82	0.40
	(1.56)		
$E(r_{t+1}^e - r_t^f)/\sigma_{r^e}$	0.34	0.06	9.16
	(0.09)		

Notes: (i) The "Data" column reports estimates of the mean of the risk free rate, the mean of the equity premium, the standard deviations of the rate of return of equity, and the Sharpe ratio, with standard errors in parentheses, over the period 1892-1987 for U.S. data. These numbers are taken from Cecchetti, Lam and Mark(1993) and Boldrin, Chiristiano and Fisher(2001). (ii) All statistics are annualized and in percent terms. (iii) The statistic from the models are based on 1000 Monte Carlo experiments.

Table A.1 Unit Root Tests

no of lags	1	3	5
r	-2.572 ***	-2.217 **	-2.012 **
		Canada	
$\Delta\ln NO$	-9.520 ***	-6.010 ***	-5.140 ***
CA/NO	-2.864 ***	-2.167 **	-2.393 **
		Japan	
$\Delta\ln NO$	-10.863 ***	-6.083 ***	-4.427 ***
CA/NO	-3.018 ***	-2.502 **	-2.825 ***
		U.K.	
$\Delta\ln NO$	-9.938 ***	-6.220 ***	-5.430 ***
CA/NO	-2.325 **	-2.448 **	-2.672 ***
		U.S.	
$\Delta\ln NO$	-7.090 ***	-5.060 ***	-4.756 ***
CA/NO	-1.605	-1.816 *	-1.911 *

Note 1: The unit root tests are based on the ADF t-test. Since each variable is demeaned, the ADF regression does not include both constant and trend.

Note 2: ***, ** and * denote that the unit root null is rejected at 1%, 5% and 10% significance levels, respectively.

Note 3: Asymptotic 1%, 5% and 10% critical values are provided by Davidson and MacKinnon(1993) and equal to -2.56, -1.94 and -1.62, respectively.

Table A.2 FEVDs of the CA under Identification Scheme II

Canada

Periods	g	cp	cs
0	*0.0005*	*0.2002*	*0.7993*
	(0.0137)	(0.1891)	(0.1893)
1	*0.0008*	*0.1778*	*0.8214*
	(0.0147)	(0.1792)	(0.1791)
2	*0.0020*	*0.1722*	*0.8258*
	(0.0181)	(0.1773)	(0.1766)
3	*0.0037*	*0.1689*	*0.8274*
	(0.0232)	(0.1758)	(0.1746)
4	*0.0058*	*0.1668*	*0.8273*
	(0.0295)	(0.1747)	(0.1731)
12	*0.0267*	*0.1590*	*0.8142*
	(0.0733)	(0.1683)	(0.1708)
20	*0.0395*	*0.1558*	*0.8046*
	(0.0898)	(0.1651)	(0.1725)
40	*0.0452*	*0.1556*	*0.7992*
	(0.0957)	(0.1631)	(0.1732)

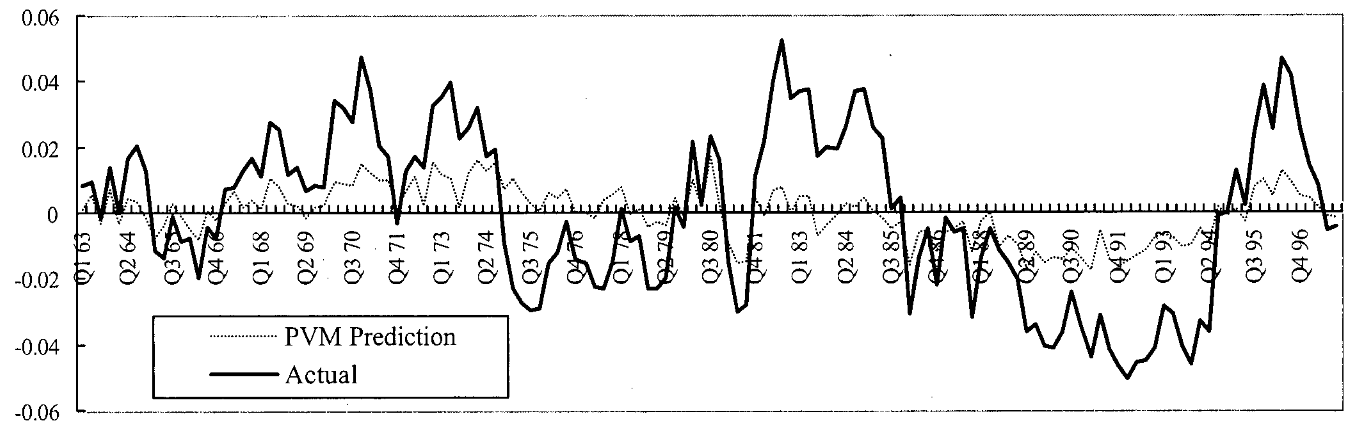
U.K.

Periods	g	cp	cs
0	*0.0012*	*0.3005*	*0.6983*
	(0.0145)	(0.2341)	(0.2344)
1	*0.0014*	*0.2759*	*0.7227*
	(0.0157)	(0.2247)	(0.2245)
2	*0.0015*	*0.2724*	*0.7261*
	(0.0180)	(0.2240)	(0.2231)
3	*0.0016*	*0.2701*	*0.7283*
	(0.0214)	(0.2231)	(0.2215)
4	*0.0018*	*0.2691*	*0.7291*
	(0.0255)	(0.2225)	(0.2204)
12	*0.0023*	*0.2697*	*0.7280*
	(0.0531)	(0.2196)	(0.2158)
20	*0.0024*	*0.2714*	*0.7262*
	(0.0634)	(0.2182)	(0.2153)
40	*0.0024*	*0.2730*	*0.7246*
	(0.0680)	(0.2172)	(0.2157)

Note 1: g, cp and cs represent global, country-specific permanent and country-specific transitory shocks, respectively.
Note 2: The numbers in parentheses denote the standard errors based on 10000 nonparametric bootstrapping resamples.

Figure 2.1: PVM Predictions on the Actural Current Account-Net Output Ratio

(a) Canada



(b) The U.K.

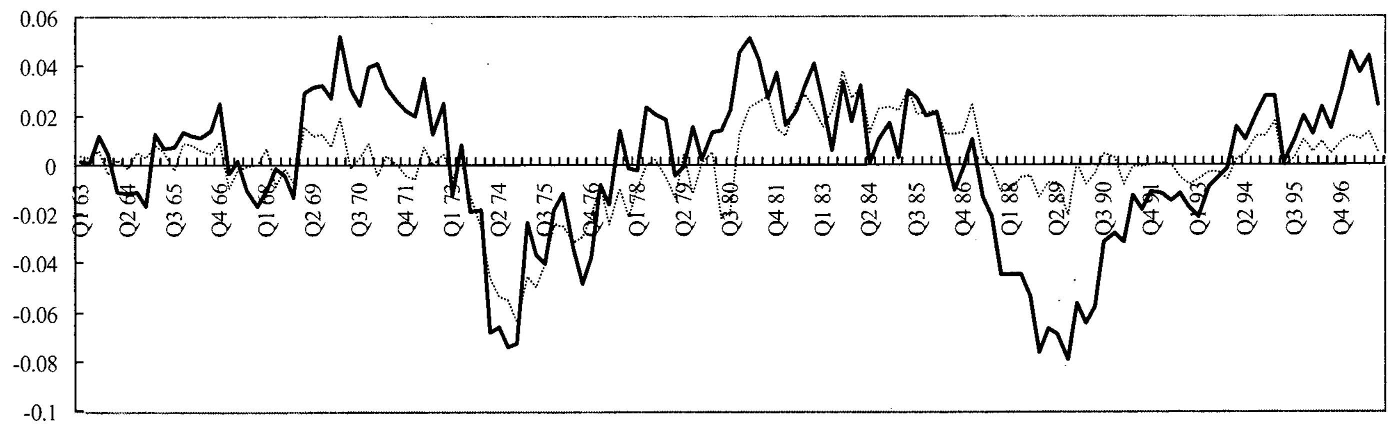
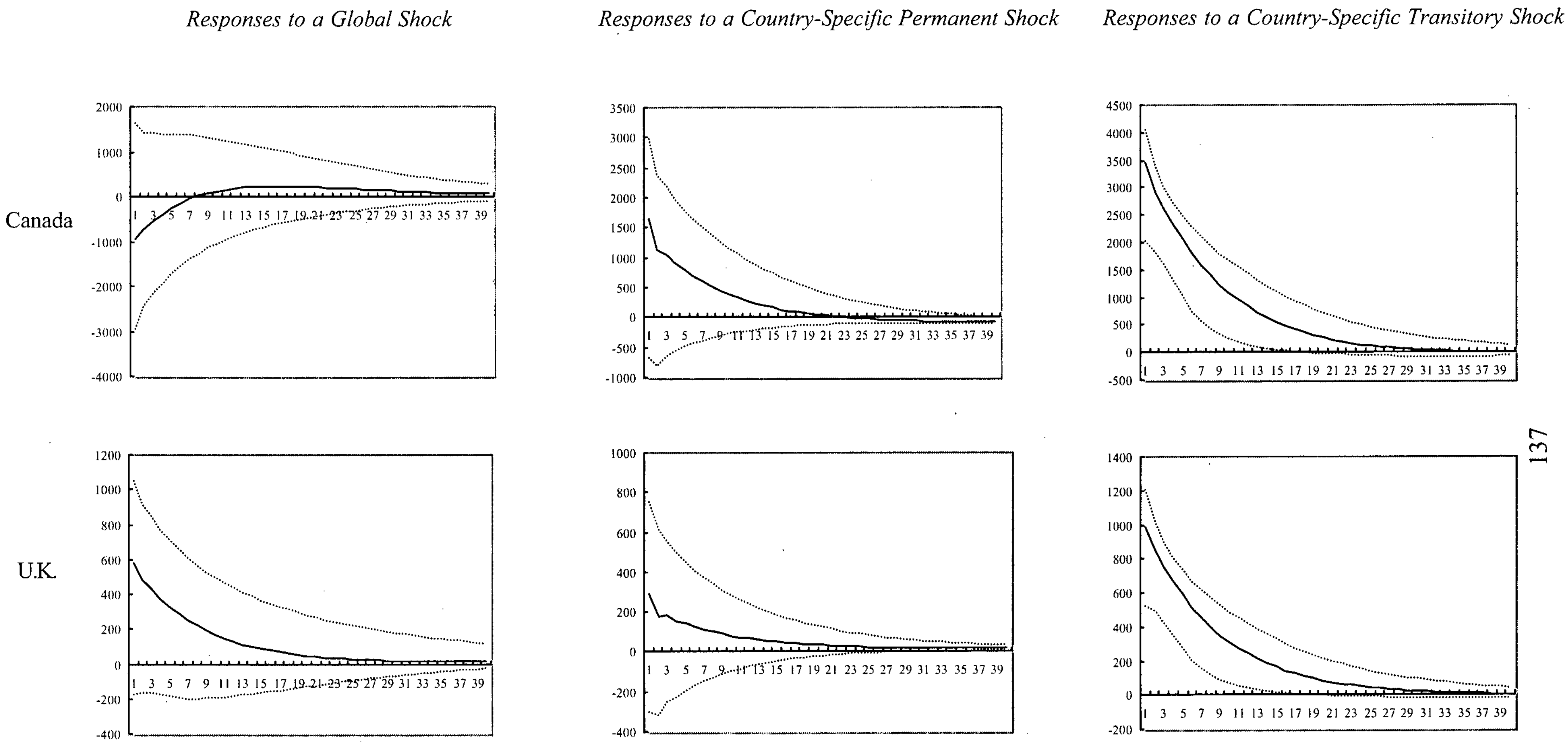
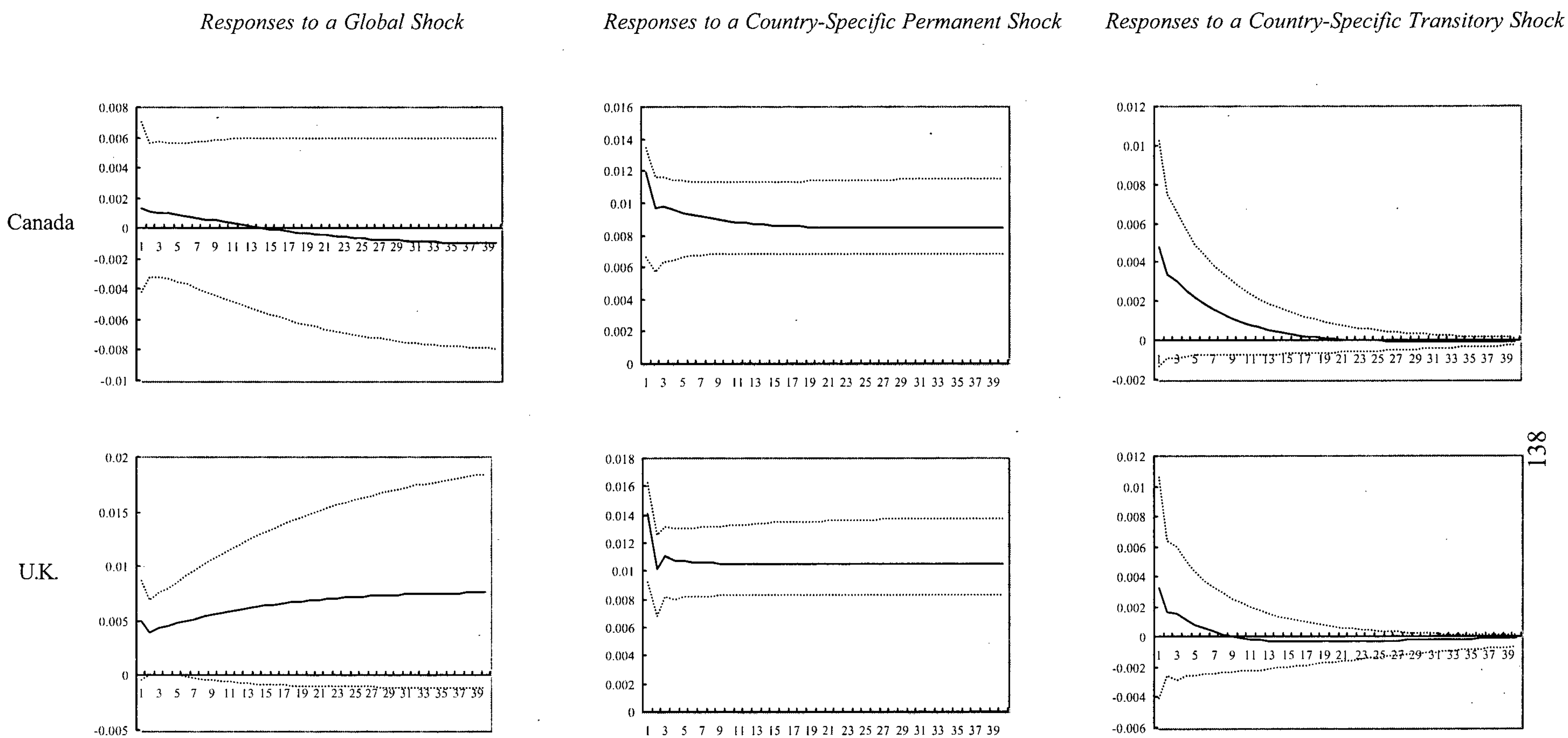


Figure 2.2: Impulse Responses of the CA under Identification Scheme I



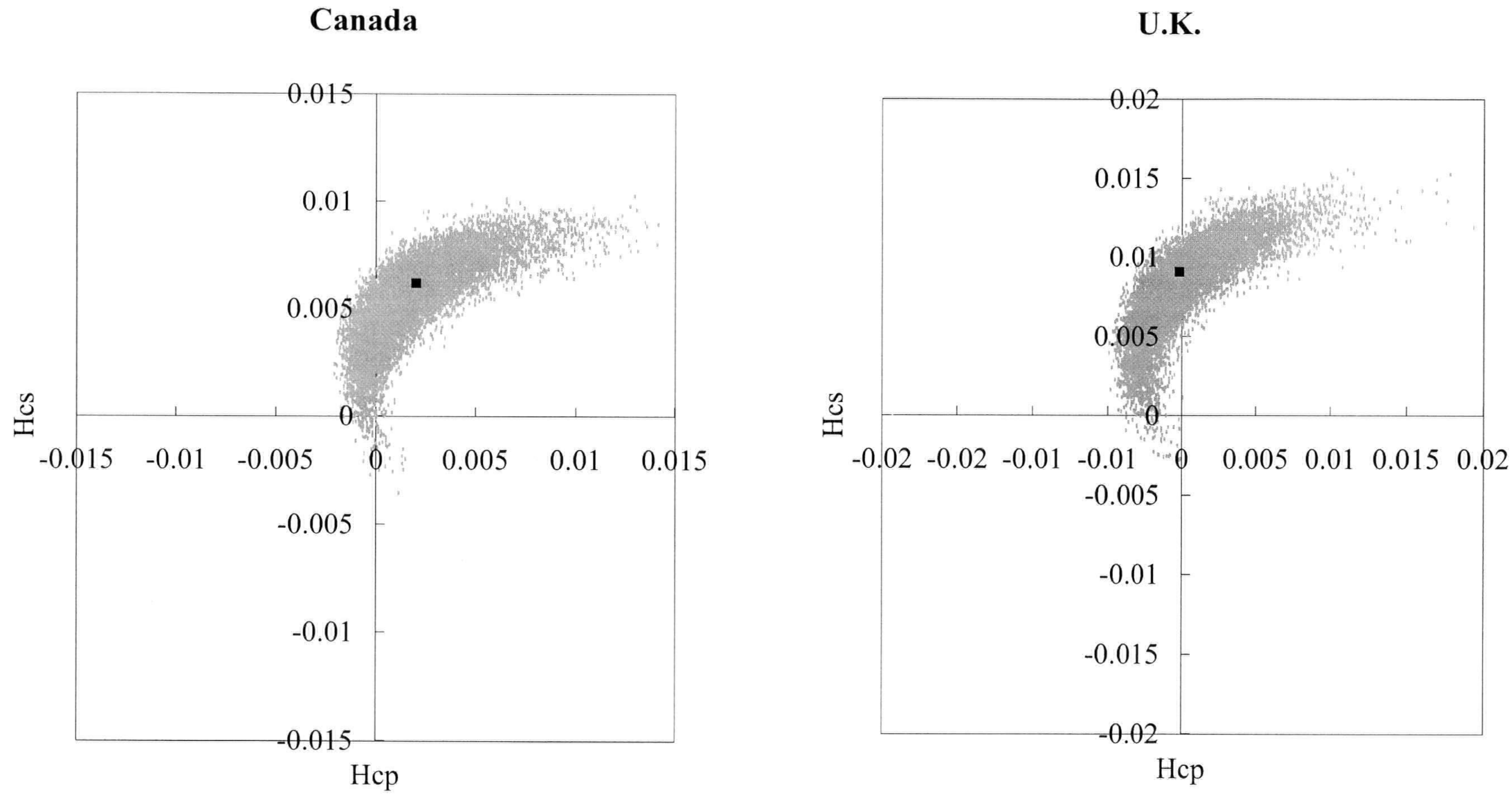
Note: The dark line shows the point estimates. The dashed lines represent 95% confidence intervals based on 10000 nonparametric bootstrapping resamples.

Figure 2.3: Impulse Responses of lnNO under Identification Scheme I



Note: The dark line shows the point estimates. The dashed lines represent 95% confidence intervals based on 10000 nonparametric bootstrapping resamples.

Figure 2.4 Empirical Joint Distributions of the Statistics Hcp and Hcs under Identification Scheme I



Note 1: The scatter plots are based on 10000 nonparametric bootstrapping resamples of the RFVAR residuals.
Note 2: The darkest squares are the ML point estimates.

Figure 3.1 PVM Predictions on the Canadian Current Account

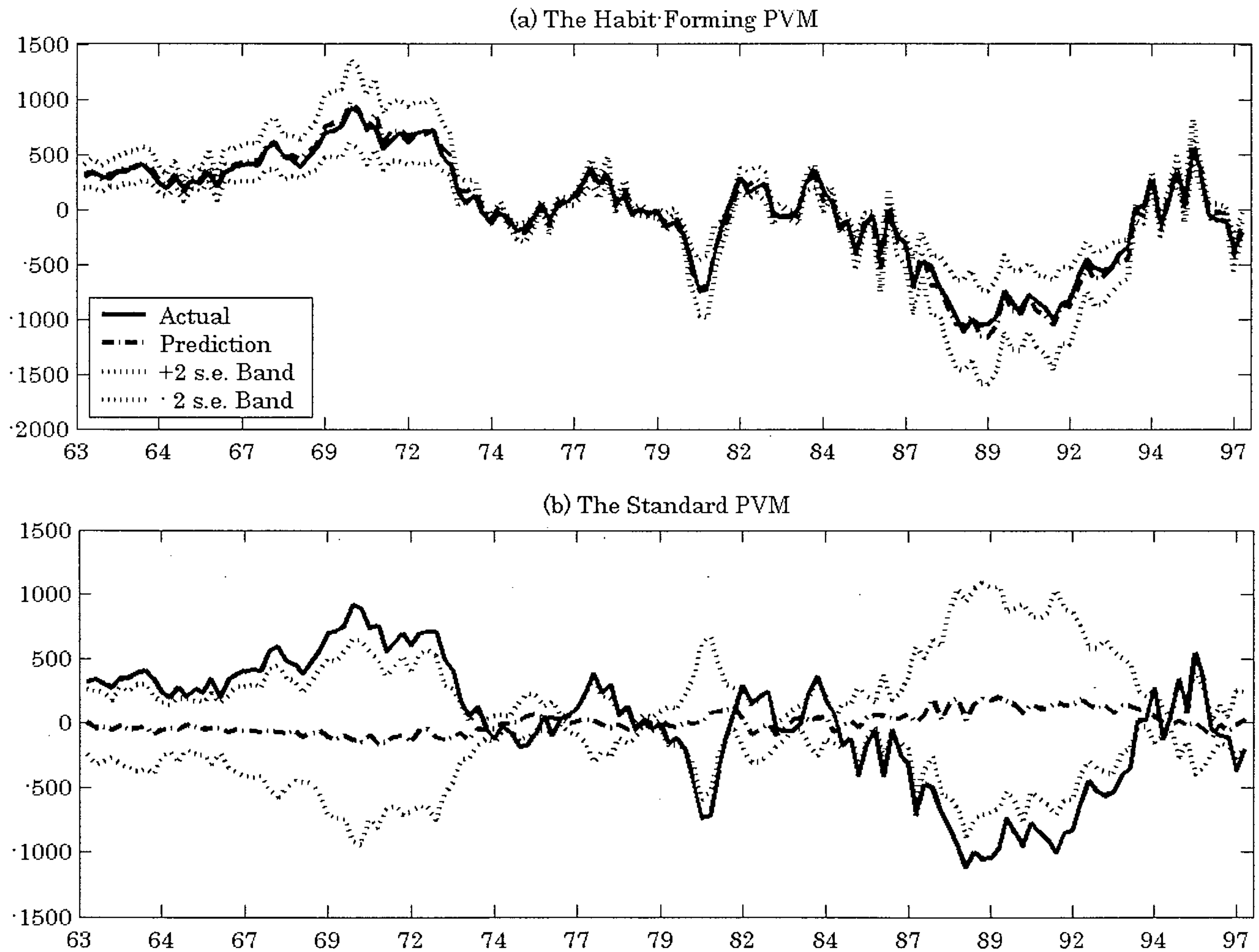


Figure 3.2 Theoretical Distributions of Test Statistics
The SOERBC with Habit Formation

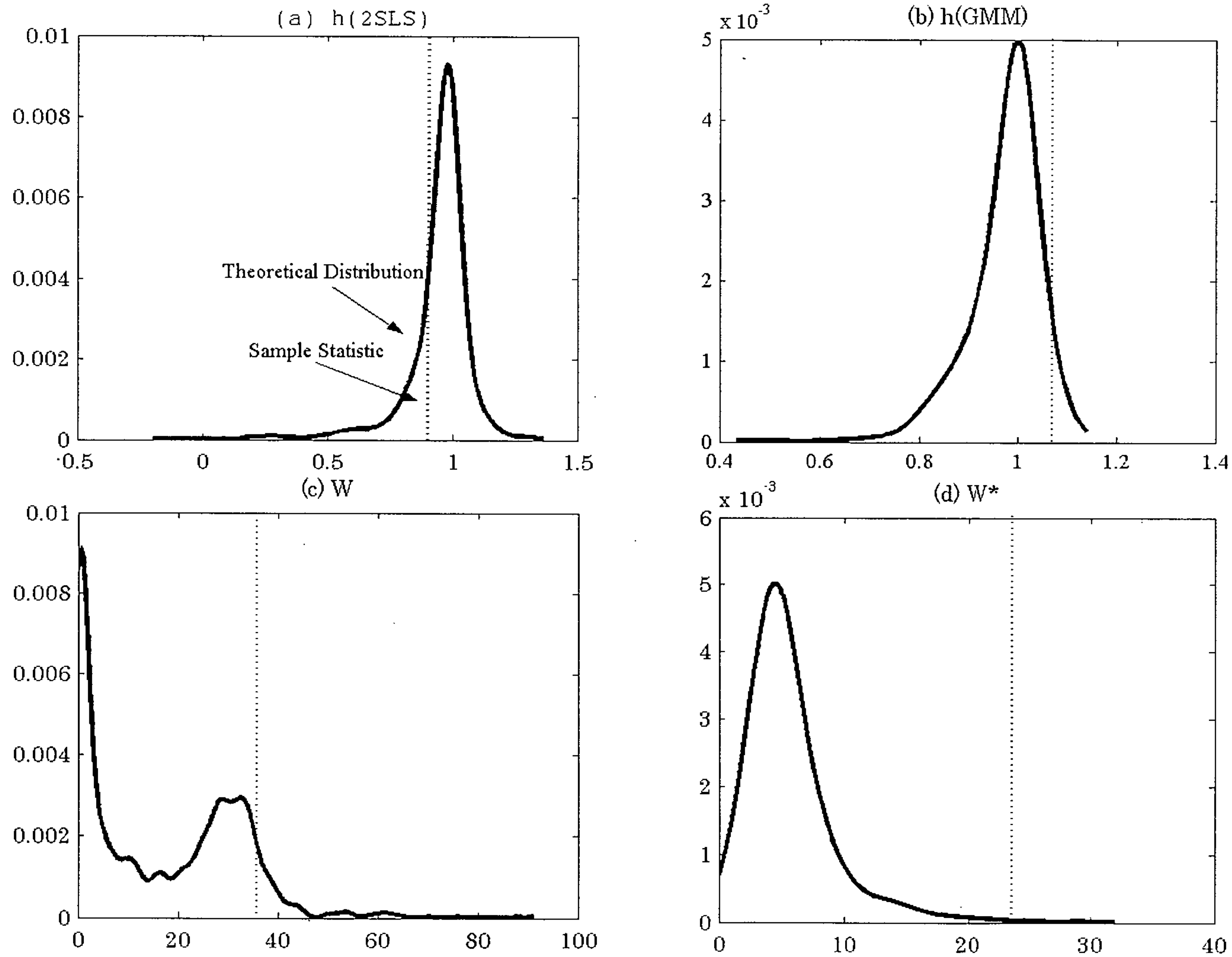


Figure 3.3 Theoretical Distributions of Test Statistics
The SOERBC with Stochastic World Real Interest Rates

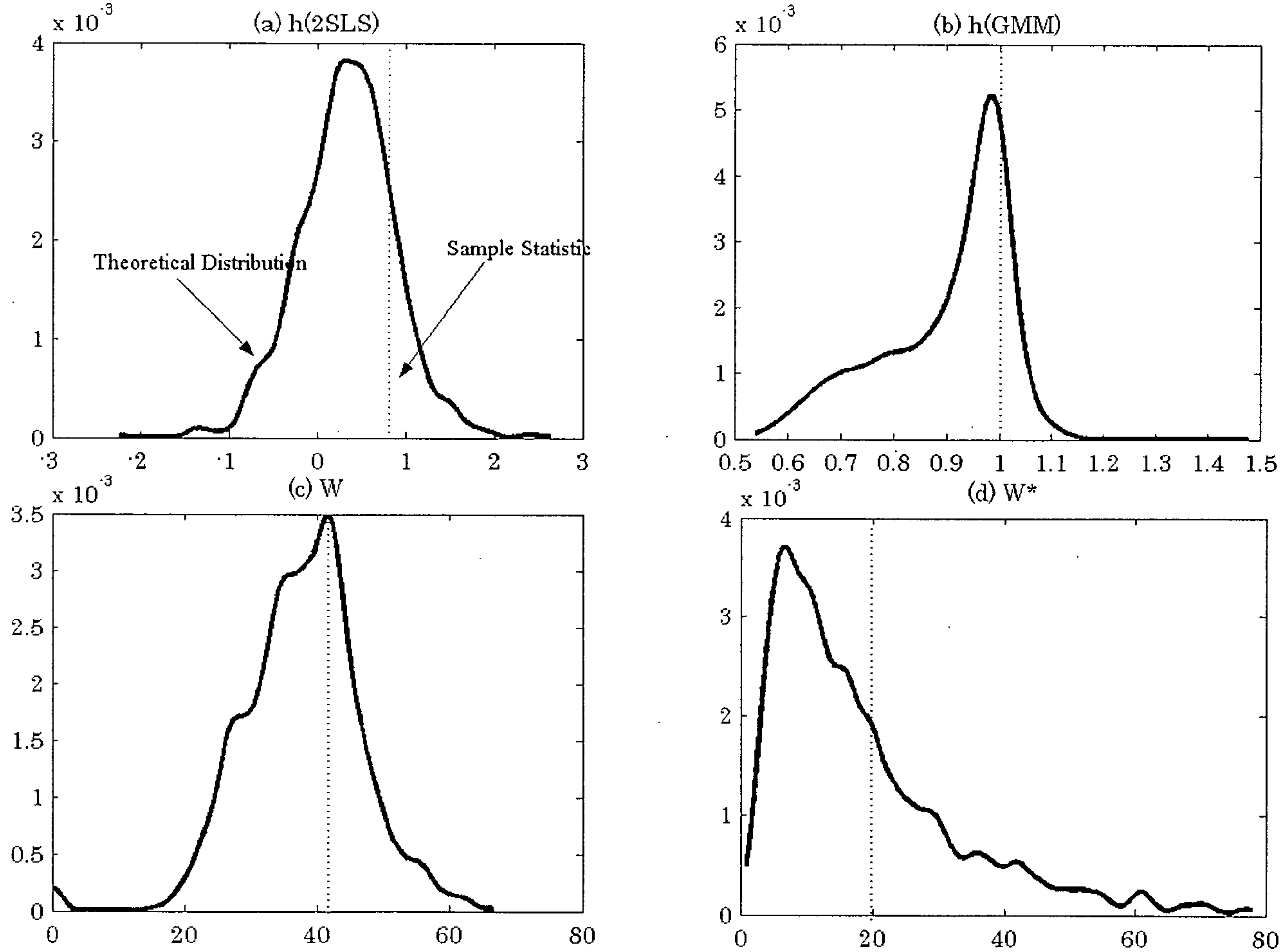


Figure 3.4 Sample Predictions and Theoretical Distributions:
The SOE-RBC Model with Habit Formation

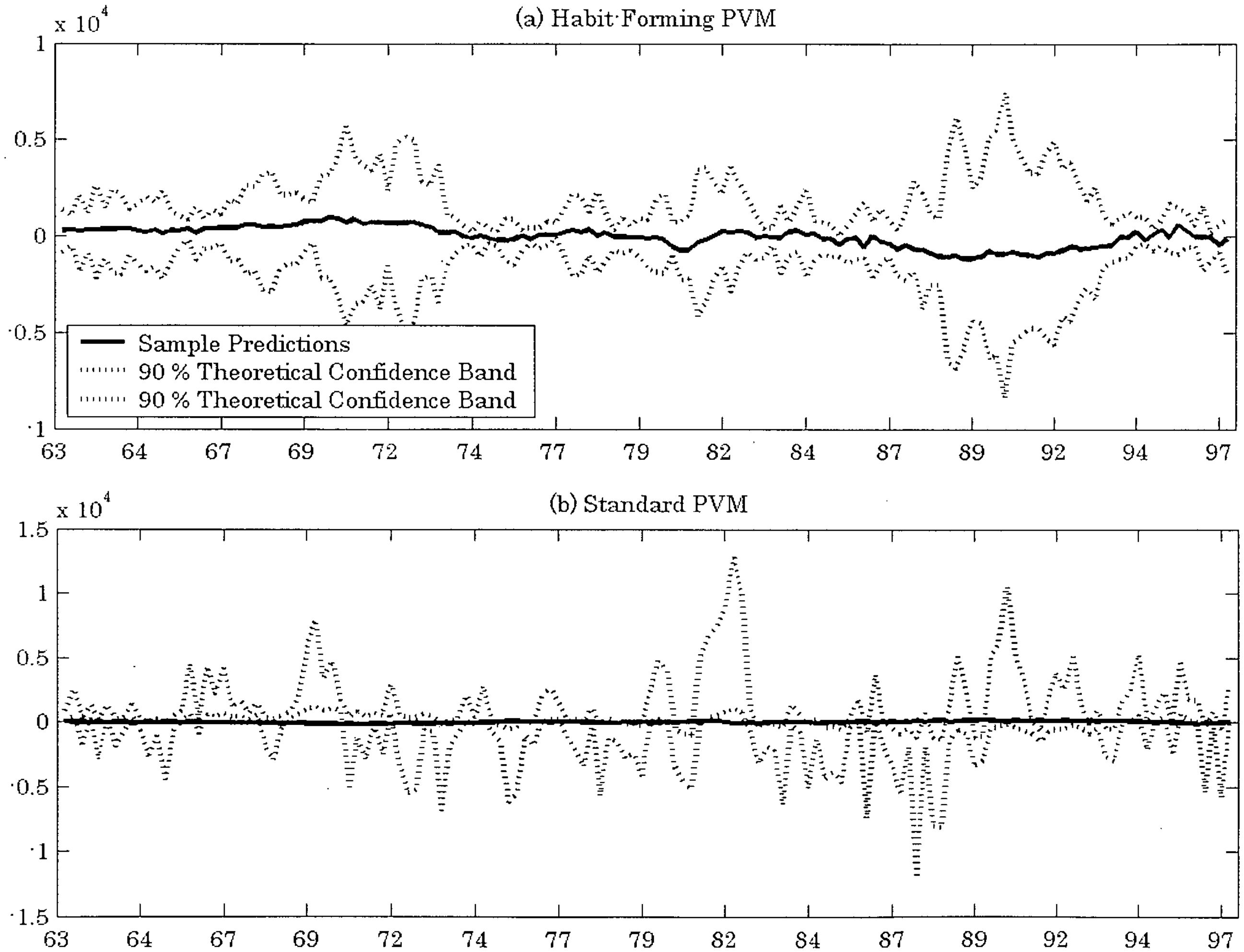


Figure 3.5 Sample Predictions and Theoretical Distributions:
The SOE-RBC Model with Stochastic World Real Interest Rates

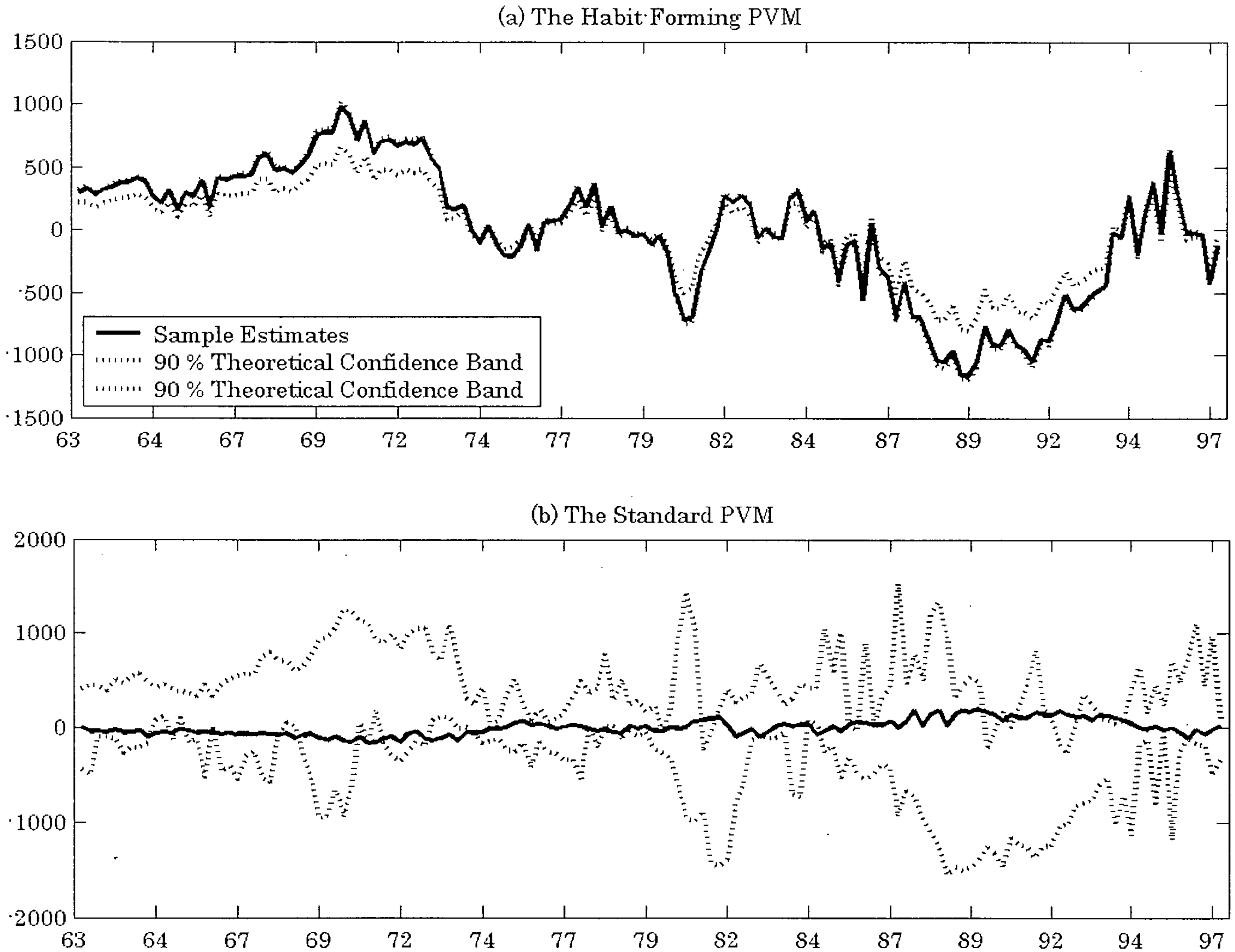


Figure 4.1: The Sample Estimates of the ACFs and SDFs of the Growth Rates of Consumption and Output

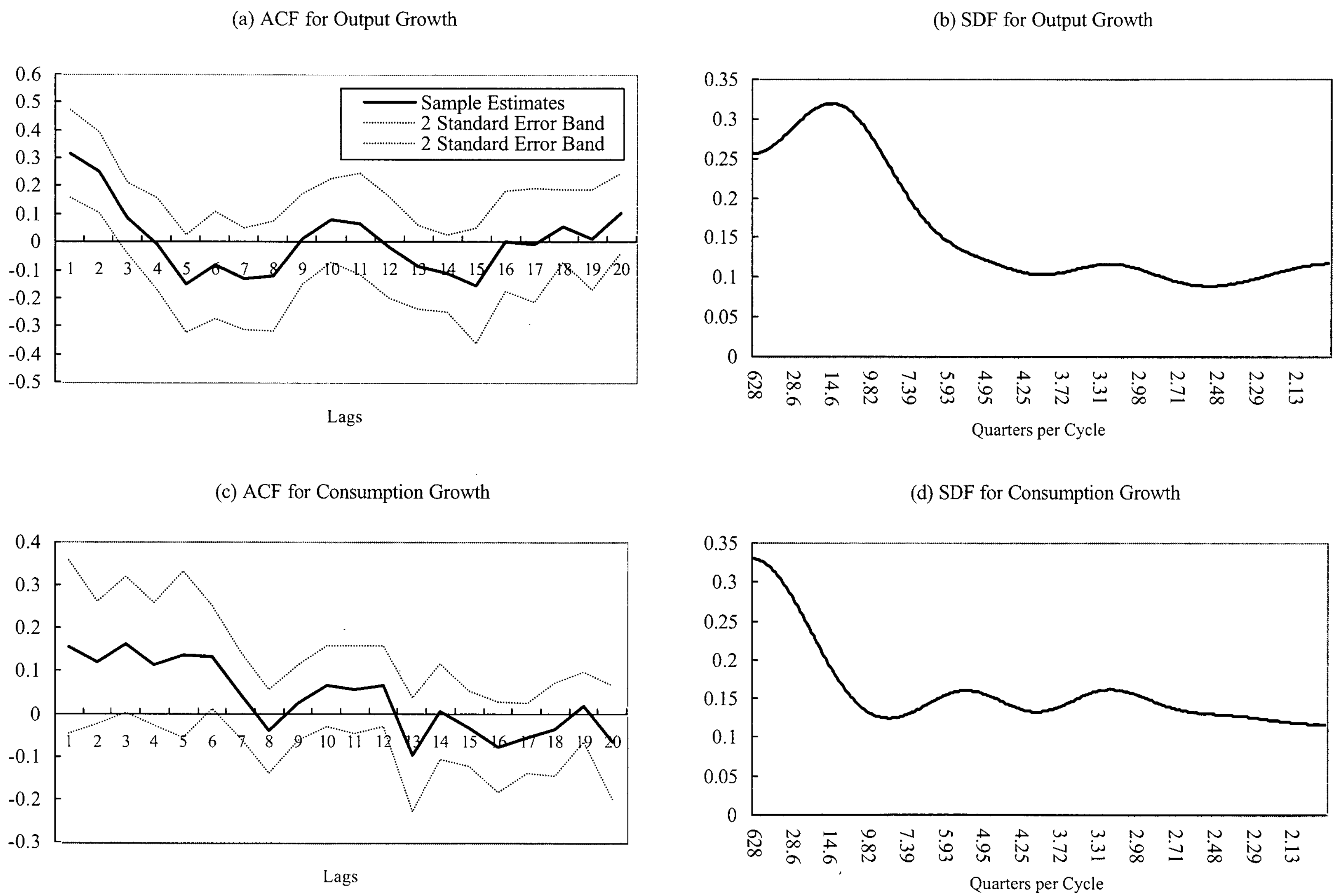
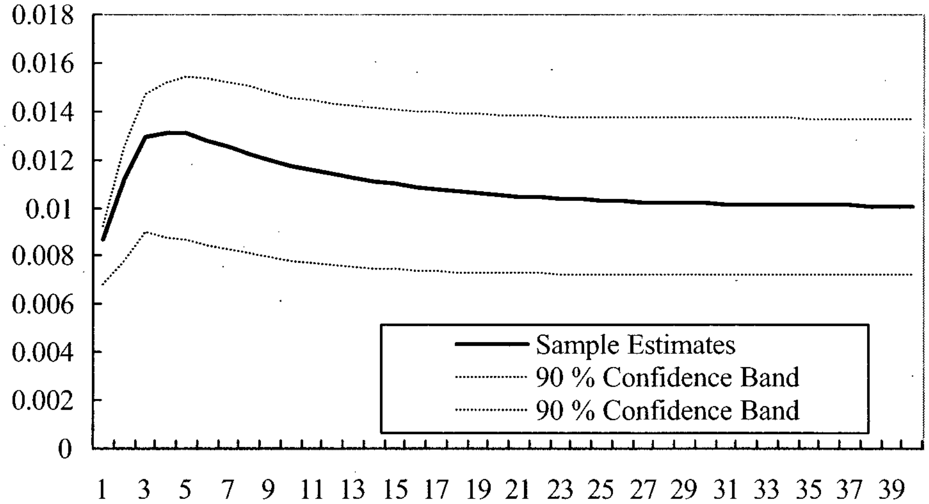


Figure 4.2: The IRFs of Log of Output to Permanent and Transitory Shocks

(a) To a Permanent Shock



(b) To a Transitory Shock

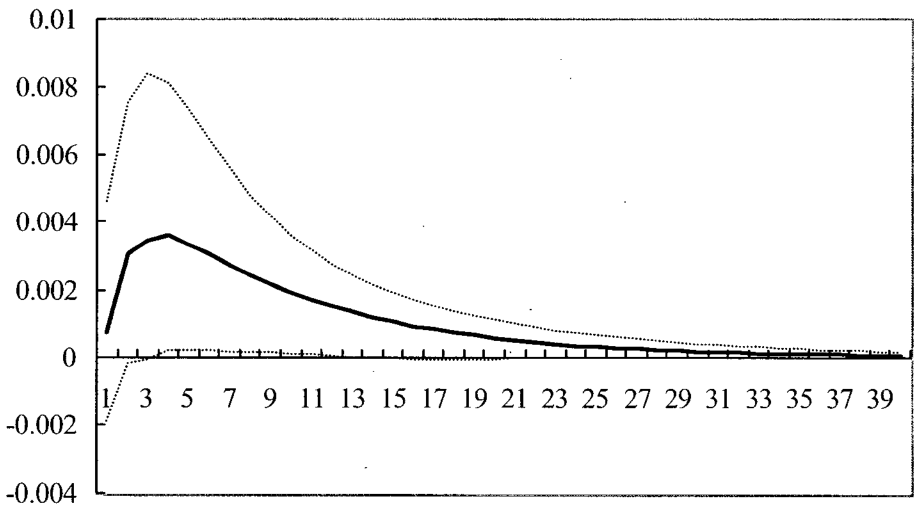
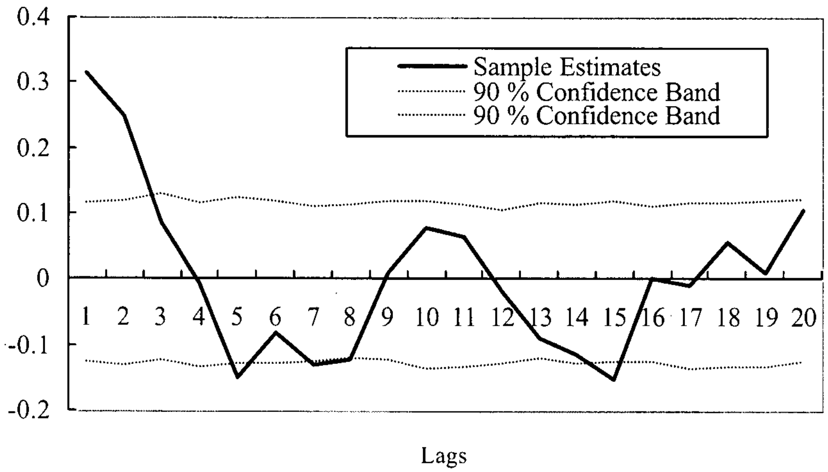


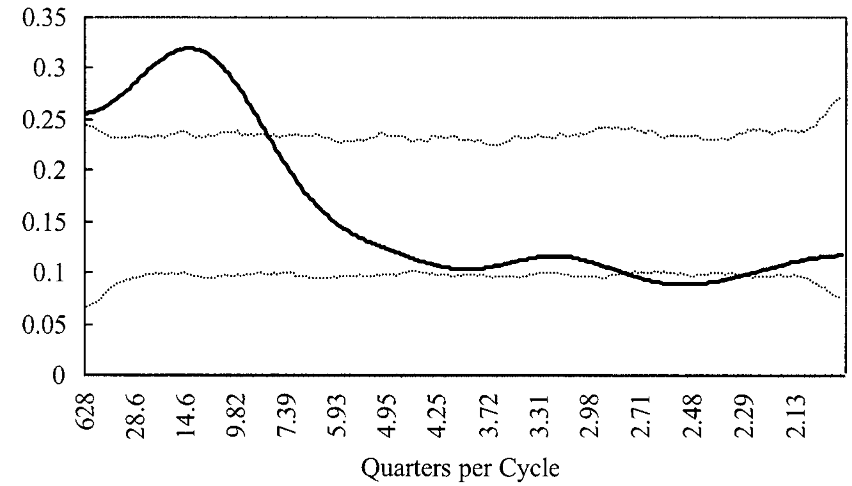
Figure 4.3: The Sample Estimates and Theoretical Distributions of the ACFs and SDFs of the Output Growth Rate

The Benchmark Model

ACFs

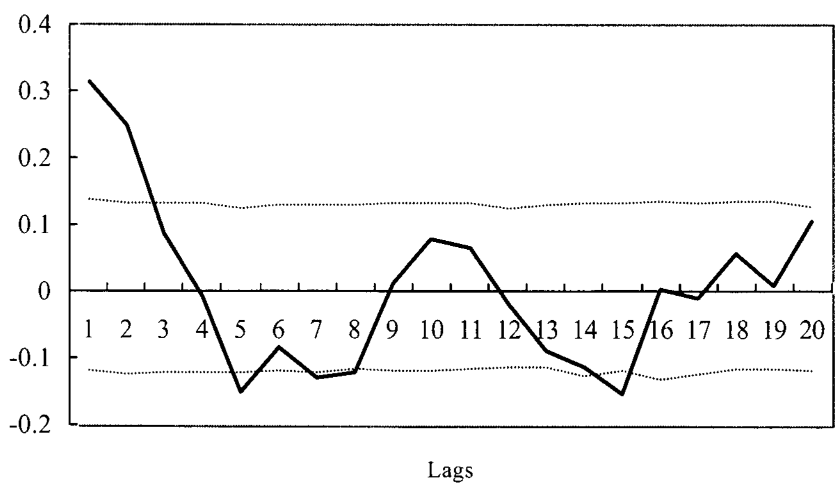


SDFs



The Habit Model

ACFs



SDFs

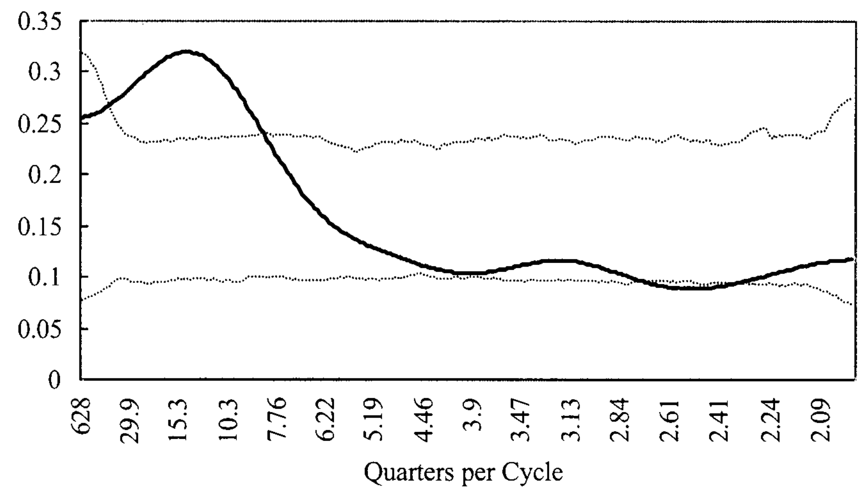
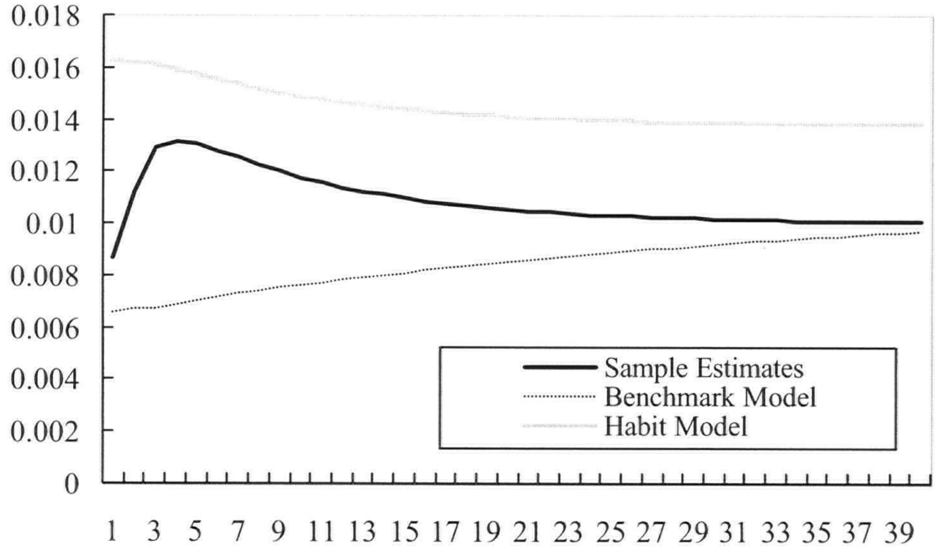


Figure 4.4: The IRFs of Log of Output to Permanent and Transitory Shocks: The Sample Estimates and Theoretical Mean Responses

(a) To a Permanent Shock



(b) To a Transitory Shock

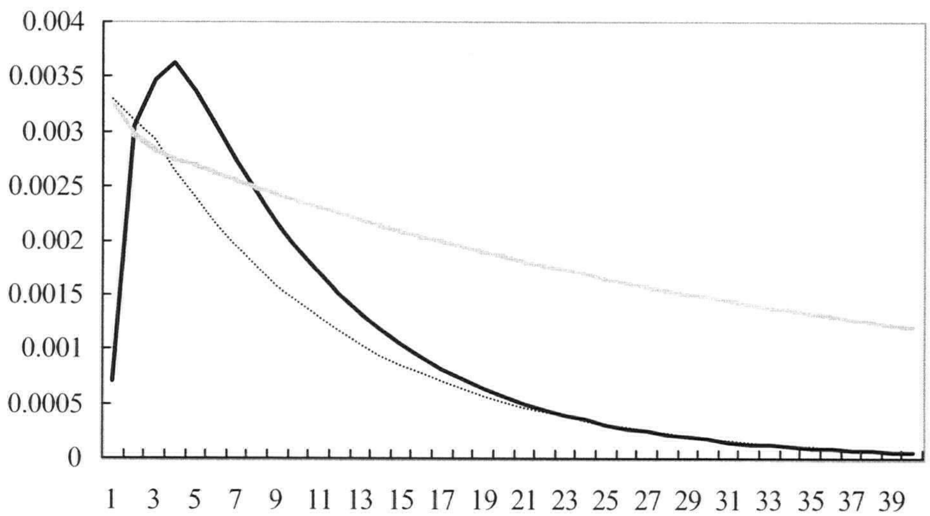
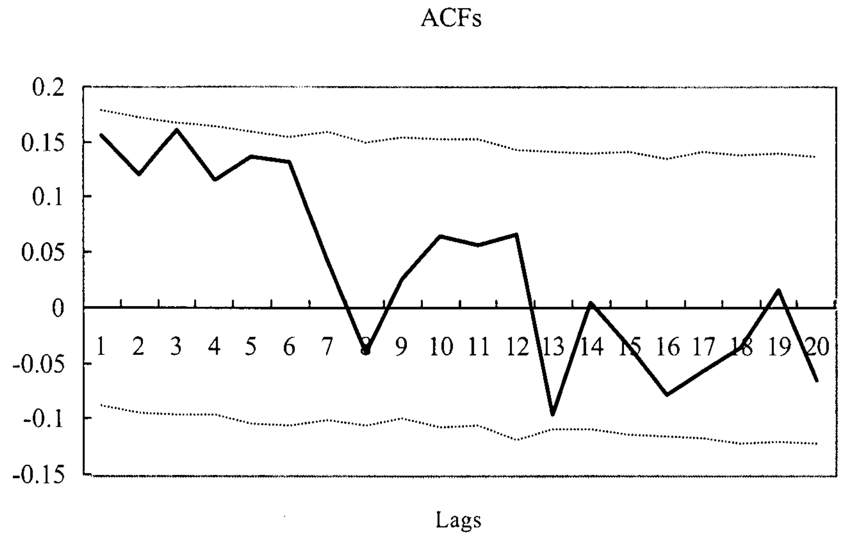
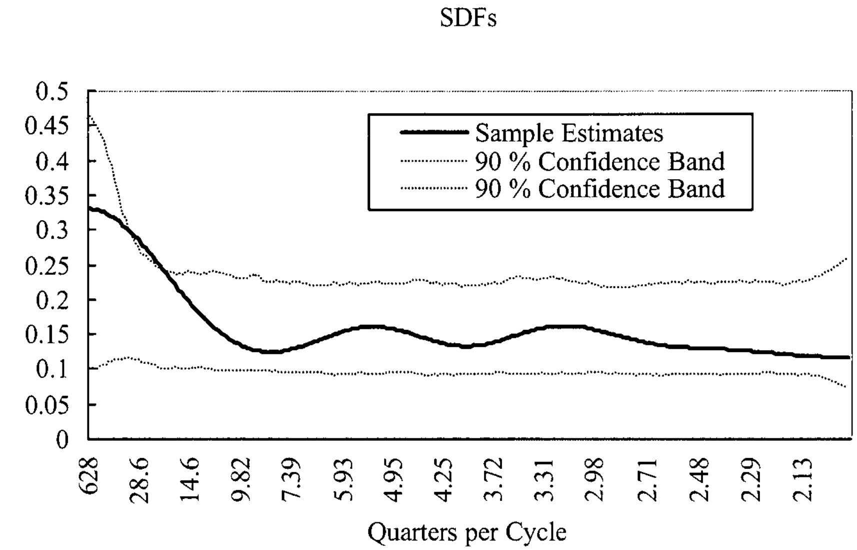


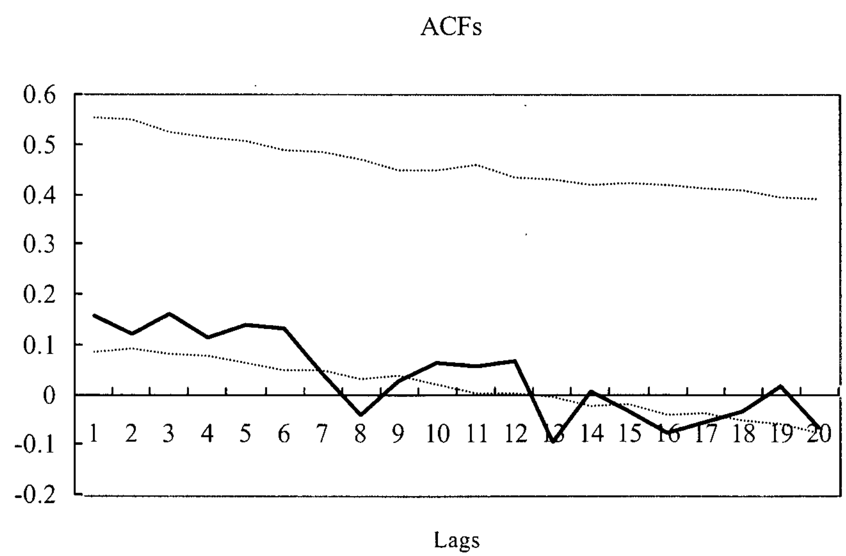
Figure 4.5: The Sample Estimates and Theoretical Distributions of the ACFs and SDFs for Consumption Growth

The Benchmark Model





The Habit Model



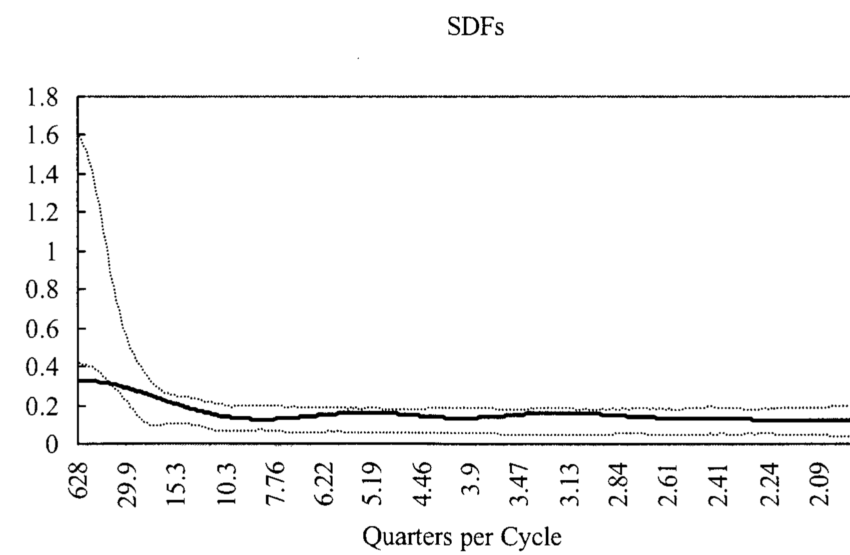
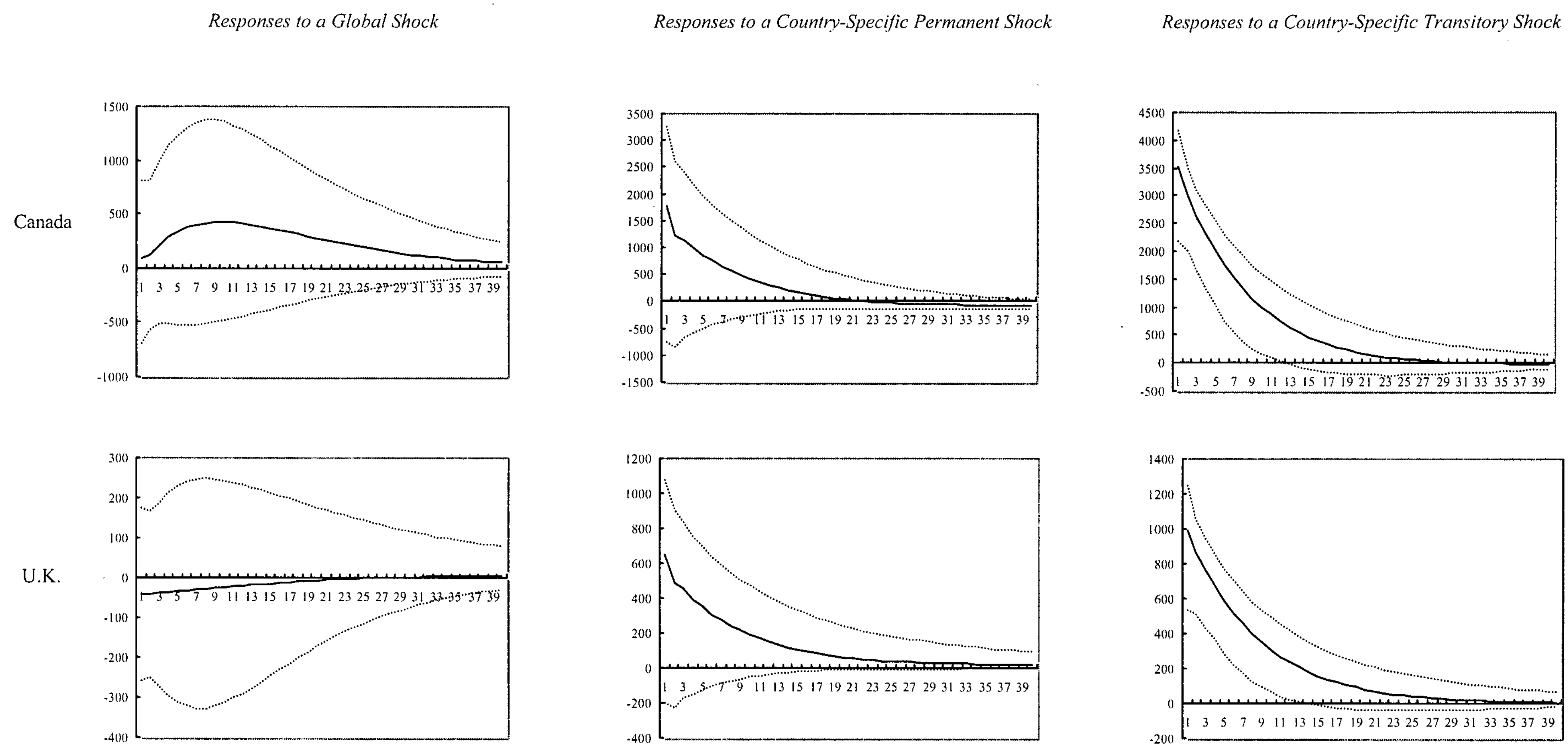


Figure A.1: Impulse Responses of the CA under Identification Scheme II



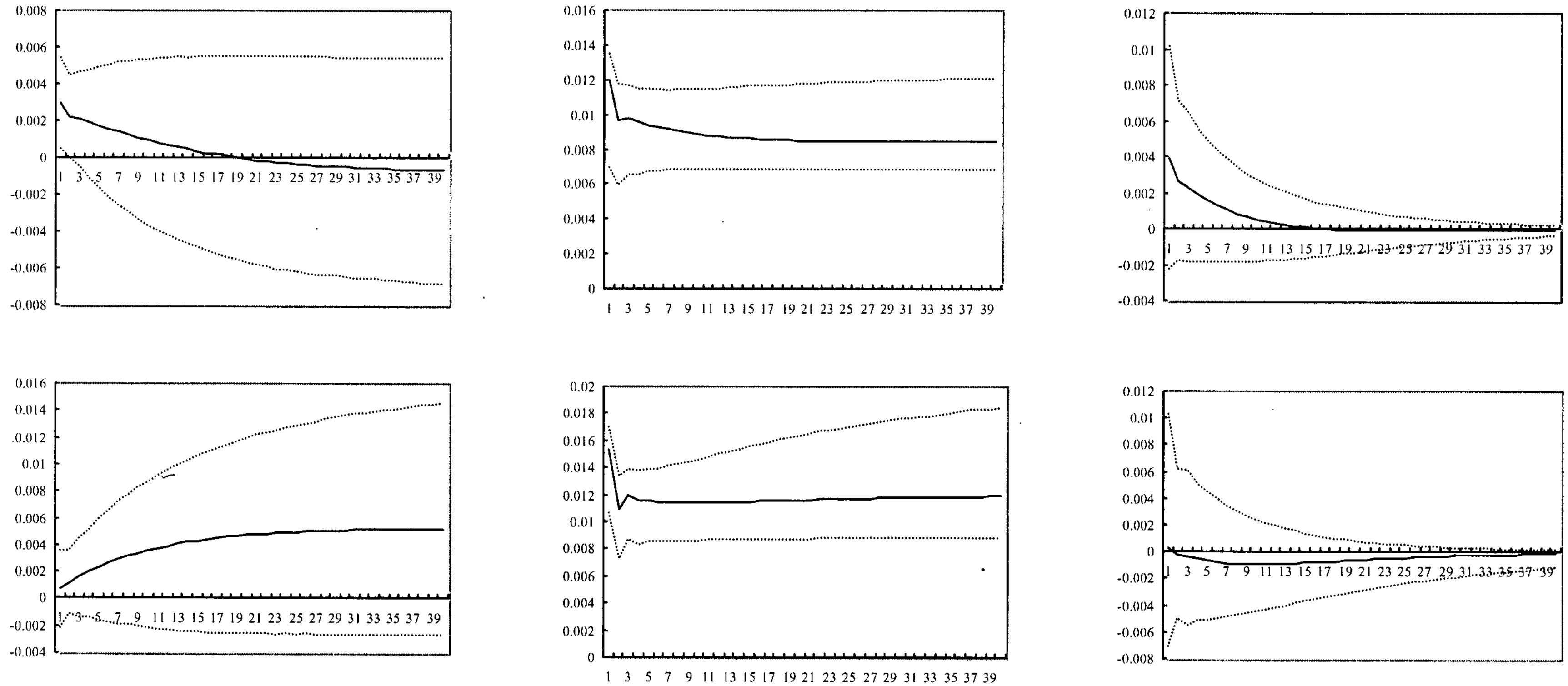
Note: The dark line shows the point estimates. The dashed lines represent 95% confidence intervals based on 10000 nonparametric bootstrapping resamples.

Figure A.2: Impulse Responses of lnNO under Identification Scheme II

Responses to a Global Shock | *Responses to a Country-Specific Permanent Shock* | *Responses to a Country-Specific Transitory Shock*

Canada

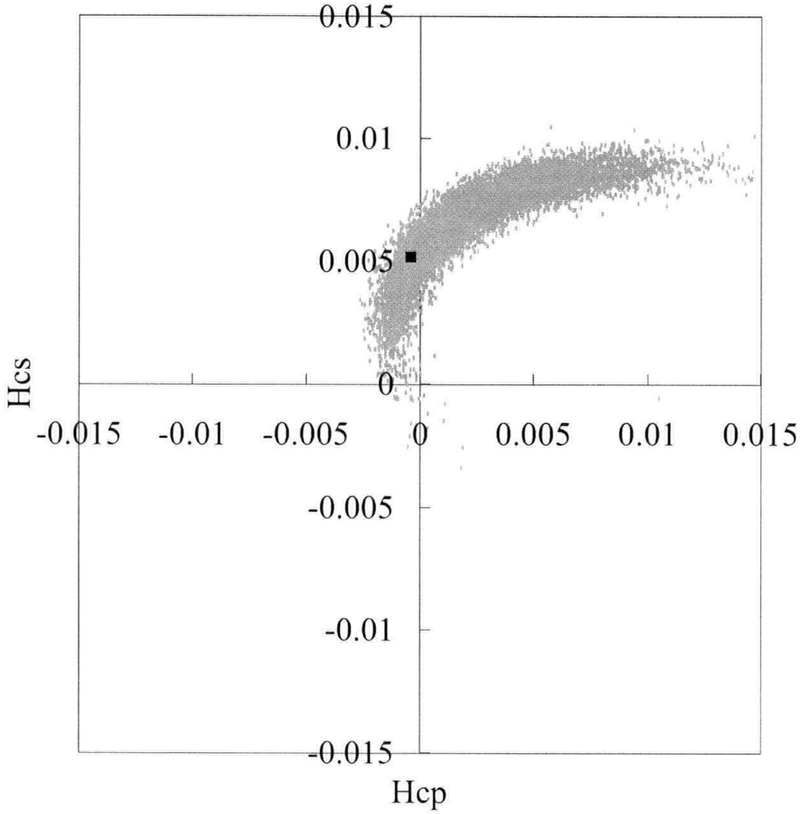
U.K.



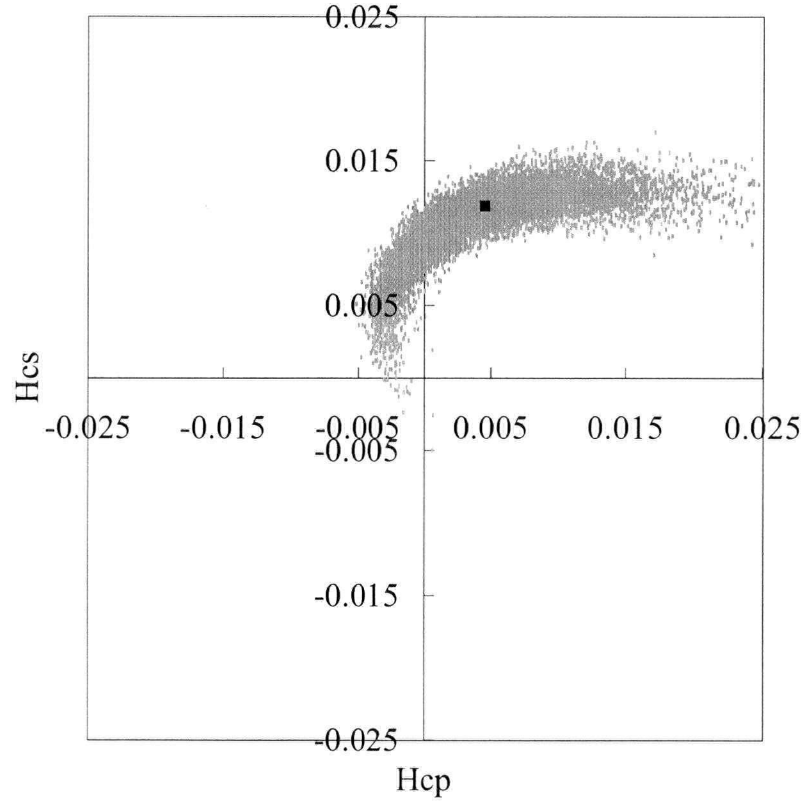
Note: The dark line shows the point estimates. The dashed lines represent 95% confidence intervals based on 10000 nonparametric bootstrapping resamples.

Figure A.3 Empirical Joint Distributions of the Statistics Hcp and Hcs under Identification Scheme II

Canada



U.K.



Note 1: The scatter plots are based on 10000 nonparametric bootstrapping resamples of the RFVAR residuals.
Note 2: The darkest squares are the ML point estimates.