

ACCOUNTING FOR THE HORIZON

By

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Abstract

Firms have long-term relationships with economic agents such as managers, auditors, and suppliers that can be characterized as sequences of shorter lived agents interacting with a longer lived firm. In this framework, the agent's tenure with the firm becomes the object of investigation. Thus, when the firm starts a multi-period relationship with an agent, what is the role played by beliefs about the duration of the relationship? If the relationship is governed by long-term contracts and if the length of the contract can be chosen ex-ante, and commitment to a certain tenure is possible, is there an optimal ex-ante tenure from the firm's owners' point of view?

This dissertation addresses several issues related to an agent's tenure in multi-period models, and is based on three essays. The first essay is on auditing and analyzes how beliefs about auditor tenure impact auditor independence and audit pricing. The second and third essays examine the question of optimal managerial tenure in a multi-period agency model, while at the same time investigating the value of commitment and how different assumptions about commitment impact the solution of the agency problem. One other common theme in the first and third essays is the lack of a "last period", so that, while the firm (the principal) has an infinite horizon and needs to consider a succession of auditors or managers, the agents (auditors or managers) have finite horizons. As a result, an agent's last period is not the last period of the model, the firm continues to exist and operate, and a new agent is hired.

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Chapter 1

Introduction

1.1 The horizon problem

Firms have long-term relationships with economic agents such as managers, auditors, and suppliers that can be characterized as sequences of shorter lived agents interacting with a longer lived firm. To stylize this to the extreme, the firm is infinitely lived, and the agents are finitely lived. In this framework, the agent's tenure with the firm becomes the object of investigation. For example, Hambrick and Fukutomi [15] propose a model of the dynamics of the CEO's tenure in office. Their analysis considers the CEO's performance in relation to the number of periods that have passed since the CEO started the current job. In this context, a natural question is to examine the role played by prior beliefs about the duration of tenure, where the duration of tenure is the total number of periods the agent will work for the firm. Thus, when the firm starts a multi-period relationship with an agent, what is the role played by beliefs about the duration of the relationship? If the relationship is governed by long-term contracts and if the length of the contract can be chosen ex-ante, and commitment to a certain tenure is possible, is there an optimal ex-ante tenure from the firm's owners' point of view? The same question arises when only short-term contracts are allowed. In this case, the commitment to a certain tenure may be implicit, as it appears to be in auditing.

For example, if a manager is hired with a long-term contract, what is the importance of the length of the contract (as determined at contracting time, and assuming commitment

issues away)? In auditing, a similar question arises from the fact that auditors stay with the same firm for long periods of time. For auditors, beliefs about tenure are important for multi-period pricing under perfect competition which leads to lowballing.

In principal-agent models, the agent's tenure is important because it impacts the amount and the characteristics of the information available for contracting. Specifically, if in each period the information system produces a piece of information that is used as a performance measure in contracting with the manager, the number of periods the manager is employed determines the informational environment for contracting.

In all the cases discussed above, the agent's behavior and the principal's welfare are influenced by the (explicit or implicit) contracting horizon. An auditor's pricing strategy depends on the horizon under consideration, resulting in more lowballing for longer horizons (expected tenure). The firm's auditing costs depend on the horizon, since keeping an auditor for a longer term spreads the switching costs over more periods.

In a dynamic agency model with correlated performance measures, the agent's effort depends both on the horizon (tenure) and on how far the agent is from the first or the last period of his tenure. It follows that the principal's welfare also depends on the agent's horizon.

This dissertation aims to address issues related to an agent's tenure in multi-period models, and is based on three essays. The first essay is on auditing and analyzes how beliefs about auditor tenure impact auditor independence and audit pricing. The second and third essays examine the question of optimal managerial tenure in a multi-period agency model, while at the same time investigating the value of commitment and how different assumptions about commitment impact the solution of the agency problem. One other common theme in the first and third essays is the lack of a "last period", so that, while the firm (the principal) has an infinite horizon and needs to consider a succession of auditors or managers, the agents (auditors or managers) have finite horizons. As a

result, an agent's last period is not the last period of the model, the firm continues to exist and operate, and a new agent is hired.

The main contributions to the literature are:

- A model of audit pricing with finite auditor tenure in which tenure beliefs determine auditor turnover in equilibrium.
- A testable hypothesis regarding the existence of ex-ante tenure beliefs.
- A better understanding of the multi-period LEN (Linear contracts, Exponential utility, Normal distributions) agency model with correlated periods.
- A new commitment concept that extends the idea of renegotiation to a series of short-term contracts.
- A new N-period model of ratcheting in a pure moral hazard context.
- Results regarding the impact of inter-period correlation of (accounting-based) performance measures on the ex-ante optimality of a manager's tenure.

The next three sections describe the main ideas and results of the three essays that are the main body of the thesis. Section 2 reviews Chapter 2 on audit pricing and auditor turnover in an infinite period horizon with finite auditor tenure. Section 3 presents Chapter 3 on commitment and ratcheting in a simple two-period world. Section 4 is devoted to Chapter 4, which deals mainly with the problem of optimal managerial tenure after generalizing some of the results from Chapter 3 to an N-period world.

1.2 Agency costs, audit pricing, and auditor turnover

The impact of auditor tenure – the length of time a client retains a particular auditor – on the quality of audit services has long been the subject of both speculation and controversy.

For example, Mautz and Sharaf [28] suggest that the personal relationship that develops between an auditor and the client over time can be expected to adversely influence the auditor's vigilance. The same view was expressed by the U.S. Senate's Metcalf Committee [32] which concluded that a lengthy association with a client may lead an accounting firm to identify closely with the interests of the client management, thereby impairing auditor independence. These arguments have led critics of the profession to propose mandatory auditor rotation policies for publicly held companies, but such regulations have never been implemented in North America. The principal reason that auditor rotation has not been mandated is the presumption that such a policy would harm audit efficiency and be cost increasing to clients.

Turning to research, to my knowledge, no formal analytical model of optimal auditor tenure has been developed and I do not develop such a model in Chapter 2. However, auditor tenure is an ingredient in multi-period pricing models where the traditional approach, as in DeAngelo [5], is to assume an infinite time horizon. In the presence of auditor learning and/or client switching costs, the result is initial 'lowballing' of audit fees followed by an infinite number of periods in which incumbent auditors earn client-specific quasi-rents. Moreover, there is no auditor turnover because incumbent auditors price engagements at the amount necessary to deter auditor change by clients.

However, infinite auditor tenure obviously does not exist in the real world. In the mid-1970's, average auditor tenure for audits of publicly held U.S. companies was about 18 years (Simunic [35]). More recently, average tenure appears to have decreased to about 10 years (O'Keefe, Simunic and Stein [31]). In order to introduce finite auditor tenure in analytical models, it is usual to assume that exogenous factors – such as the need for new financing, company growth, client financial distress, or auditor-client disagreement over a financial reporting issue (e.g. Dye [8]) – motivate voluntary auditor changes, whether initiated by the client or the auditor.

In Chapter 2, I develop a model of audit pricing and auditor change when investors, management, and the auditor all hold rational expectations that auditor tenure will be of finite length. I make no claim that this finite period is ex ante optimal in any sense, although – once specified – it is ex post optimal. Perfect competition among auditors bidding for an engagement is assumed. Lowballing and an implicit multi-period commitment are obtained, while the auditor is hired one period at a time. Consistent with intuition, the amount of lowballing is strictly increasing in the conjectured length of auditor tenure. Thus, the fee structure is a generalization of DeAngelo's infinite horizon model to a more realistic world with finite auditor tenure.

I also examine the effects of the fee structure on incentives to replace the auditor, and auditor independence. I show that the quasi-rents earned by the auditor do not impair independence because management dismissal threats are not credible when auditors are identical and do not disagree on auditing issues as in Magee and Tseng [26]. Fees are such that management cannot compensate the auditor (except through an explicit bribe) for any additional risk incurred by the auditor from compromising independence during the auditor's expected tenure.

The existence of the lowballing and the revision of beliefs about tenure by competing auditors ensure that management cannot threaten the auditor with replacement during the expected duration of the audit engagement. Thus, lowballing works to protect the auditor's independence through its impact on off the equilibrium path beliefs about tenure held by competing auditors. One can say that the other auditors punish the manager with higher fees if the lowball of the incumbent is not recovered.

Conversely, the auditor is replaced at the end of the conjectured number of periods because retention would allow the auditor to earn economic rents from the engagement. While these rents do not result in a compromise of the auditor's independence, the total audit cost to the firm (audit fee) would incorporate the expected litigation costs faced

by the manager as if the auditor's independence were impaired. This is true because, once retained past the expected tenure, the auditor could only be replaced by one who would systematically compromise his independence. Thus, having rational expectations of finite auditor tenure can be sufficient to induce auditor change within the same class of auditors (e.g. the Big Five). No external event is necessary to trigger the auditor change.

In deriving the pricing and auditor change model, I have assumed that tenure beliefs are exogenously given and common knowledge among the players. Maintaining the assumption that expected auditor tenure is an exogenous characteristic of the manager, it is sufficient to assume that the auditors' costs and fees are common knowledge. Indeed, once an auditor is hired, in the first period of his engagement, both competing auditors, and the investors can infer tenure beliefs from the observed amount of lowballing. In some sense, the manager can use lowballing to communicate his beliefs about tenure to the other players. Then, given a distribution of managers with different tenure beliefs, each manager can play an equilibrium based on his conjectured tenure N . Knowing that, auditors bidding for the engagement are more likely to be equally dispersed in their bids.

If lowballing cannot be inferred because fees are not disclosed, it is more likely that tenure beliefs N are formed by competing auditors and by the investors independently of the manager (for example, conjectured tenure could be the mean observed auditor tenure for the industry). Then, if a manager tries to play an equilibrium based on a personal conjecture of tenure $N' \neq N$, he will at some point make an off-equilibrium move (from the point of view of the outsiders) which results in an increase of auditing costs. Therefore, given a distribution of managers with different tenure beliefs, and an outsiders' conjectured N , managers are more likely to try to be closer to N . Auditors that bid for the engagement would also base their bids on N . Empirically, one may expect to see more variance around the mean tenure in a cross-section of audit engagements

when audit fees are disclosed than when they are not disclosed, since with fee disclosure, managers have more discretion over N .

Another interesting question related to the formation of the initial tenure beliefs is that of the existence of ex-ante optimal tenure. My auditor change model assumes a flat residual agency cost, independent of the conjectured tenure N and, as a result, the total audit cost is decreasing in N . That would imply optimal auditor tenure $N = \infty$. The empirical evidence of Johnson et al.[19] is consistent with the idea that very short and very long auditor tenure is suboptimal, leaving auditor tenure optimally undetermined in an interval of four to nine years in their paper. Their findings are consistent also with an U-shaped residual agency cost $C(N)$ that depends only on N and is constant for each period of the auditor's tenure. Presumably, such an agency cost structure would result from a combination of increasing audit quality in the first periods and a significant decrease of audit quality (or perceived auditor independence) as the auditor's tenure increases. Assuming that $C(N)$ increases sufficiently as N increases, it is possible to obtain an ex-ante optimal tenure that is neither 1 nor ∞ . However, in my model, once an equilibrium conjecture is agreed upon, the manager and the auditors are locked into it in all subsequent periods. Thus, there is no ex-ante optimal tenure, but common tenure beliefs, once reached, are ex-post optimal.

More generally, all auditor changes are anticipated since they are based on the conjectured N and convey no information to the market. This observation is consistent with the low explanatory power of empirical models of auditor change that relate auditor turnover to exogenous events. My model suggests that some auditor changes are endogenous, simply based on an expectation of auditor tenure, and are unrelated to exogenous events.

Anticipated exogenous events that are usually associated with auditor changes are outside the scope of model in Chapter 2. For example, if a company anticipates a major

business transaction at a given point in time that requires a new auditor, any auditor engaged prior to that event will expect to be replaced exactly at that time. Such tenure expectations are consistent with the finite horizon model of Magee and Tseng [26] but not with my model in which tenure is an endogenous horizon, rather than an exogenous event date. As a result, the auditor change is no news to the market when triggered by an anticipated event.

To conclude, the main empirical predictions of the model in Chapter 2 are that:

1. Lowballing is an increasing function of auditor tenure; this prediction provides a potential test between our endogenous horizon hypothesis and the exogenous event date hypothesis of Magee and Tseng. Recall that the model of Magee and Tseng predicts lowballing to be independent of tenure.
2. The variance of a cross-section of auditor tenure is higher in a market with fee disclosure than in a market without fee disclosure.

The model is also consistent with the low explanatory power of empirical models of auditor change, and explains why some auditor changes are no news to the market.

1.3 Correlated Noise, Commitment, and Ratcheting

The ratchet effect has been described in the economics literature in connection with centrally planned economies (see for example Litwack [25] and the references therein), and more generally in settings where the agent is privately informed. The book by Laffont and Tirole provides a detailed analysis and references [23]. Their description of the ratchet effect is as follows: “If [a regulated firm] produces at a low cost today, the regulator may infer that low costs are not hard to achieve and tomorrow offer a demanding incentive scheme. That is, the firm jeopardizes future rents by being efficient”. The essence of

the ratchet effect with a privately informed agent is that the agent can obtain a rent in future periods by hiding his type in the current period.

Weitzman [36] presents a multi-period model of the ratchet effect with moral hazard only, but in his model the ratcheting mechanism is exogenous. More recently, Milgrom and Roberts [29] and Indjejikian and Nanda [18] have shown that there is a ratchet effect in two period models with moral hazard but without adverse selection. In these models, the ratcheting is endogenous and driven by the lack of commitment by the principal regarding the use of available information. Milgrom and Roberts [29] define the ratchet effect as “the tendency for performance standards to increase after a period of good performance”. Given that the principal will use today’s outcome in writing tomorrow’s contract creates for the manager a link between today’s effort and tomorrow’s standard of performance. The ratchet effect is always inefficient in the models with adverse selection since good types will mimic bad types and earn rents. In the pure moral hazard model, the ratchet effect is also inefficient with respect to the full commitment solution as shown by Indjejikian and Nanda [18].

Ratcheting results from the principal’s ability to optimally adjust the agent’s second-period incentive for the lower ex-post variance of the second performance measure by using the first-period performance measure when the principal cannot commit fully to a long-term contract. The assumption that the two periods are correlated is thus crucial, and differentiates this model of ratcheting in a pure moral hazard setting from other models of sequential action choice. Repeated moral hazard with independent periods is analyzed by Lambert [24], Rogerson [33], Holmstrom and Milgrom [17], and Fudenberg, Holmstrom and Milgrom [12].

Matsumura [27] presents an analysis of sequential action choice with correlated outcomes in a single period in which the agent observes a first outcome before selecting the second action. However, in Matsumura’s model, there is no contracting after the

first outcome is observed, and this outcome is private agent information until the end of the period. As a consequence, second-period incentives are affected by the first-period performance, but there is no renegotiation.

In Chapter 3, I extend the analysis of the Indjejikian and Nanda model [18] to include commitment issues and the possibility of agent turnover. Indjejikian and Nanda present two types of commitment: full commitment to a long-period contract (which they refer to as “commitment”) and an intermediate form of commitment with a sequence of two short-term contracts (which they refer to as “lack of commitment”). I show that the two short-term contracts obtained by Indjejikian and Nanda correspond to a form of commitment which I call *commitment to fairness*. This form of commitment is an adaptation of the concept of fairness introduced by Baron and Besanko [1]. A contracting relationship is governed by *fairness* if the principal is restricted to fair wages and the agent must participate in all periods if he accepts the contract in the first period. Fair wages are paid when the agent gets his reservation wage as if he could leave in each period. That is, the agent’s certainty equivalent of future compensation, conditional on available information and on the principal’s conjecture of the agent’s first-period action is set to the reservation level at the start of each period.¹ Thus, in addition to the usual contract acceptance constraint at the start of the first period, there is a second constraint that the second-period contract is acceptable to the agent as if the agent had other employment opportunities and had not committed to stay for both periods. The agent trades off his ability to leave in the second period for the guarantee of fair compensation in the second period.

I also show that the two contracts under commitment to fairness and a long-term renegotiation-proof contract produce equivalent results. That is, the payoffs for the agent

¹Note that an equilibrium involves rational expectations regarding the agent’s first-period action. Thus, second-period fair wages are based on the principal’s conjecture of the agent’s first-period action, which is correct in equilibrium.

and the principal, and the induced actions coincide for the renegotiation-proof contract and the sequence of contracts under commitment to fairness.

The fairness constraint is not only sufficient for obtaining the solution of Indjejikian and Nanda, but also necessary. If the principal offers the agent the optimal sequence of contracts derived under commitment to fairness, and if the agent does not commit for both periods, the equilibrium breaks down. Restoring the agent's ability to leave in the second period allows the agent to act strategically in the first period, take an action other than that anticipated by the principal and then leave after the first-period compensation is paid. This situation parallels the "take-the-money-and-run" strategy that arises in ratcheting with adverse selection when the agent cannot commit to stay for both periods (see Laffont and Tirole [23]). Moreover, if the agent is able to leave in the second period, then there is no equilibrium with two short-term contracts in which the agent stays for both periods (see also Christensen and Feltham [2]). Thus, the fairness assumption helps to overcome the non-existence of an equilibrium problem.

Removing the commitment to fairness assumption leads to a solution in which the principal optimally employs a different agent in each period. The only assumption necessary to obtain the two-agent solution is that the principal can commit to replace the first agent in the second period. At the time the principal replaces the agent in the second period, he is indifferent between retaining and replacing the agent. Thus, the principal's commitment to replace the agent is not a strong assumption.

An analysis of the principal's welfare under the different commitment assumptions reveals that for negatively correlated performance measures, full commitment is preferred to all other forms of commitment, or lack thereof. The situation is somewhat reversed with positively correlated performance measures in that, although full commitment is better than renegotiation and commitment to fairness, no commitment (two agents) is better than full commitment. The driving force behind this result is that with negative

correlation, having the same agent in both periods reduces the total risk to which the agent is exposed by the optimal incentive scheme. When the correlation is positive, using a different agent in each period eliminates the risk premium due to the correlation between the optimal compensation schemes for each action.²

1.4 Multiperiod Ratchet Effect and Managerial Tenure

In Chapter 3, I extended the analysis of Indjejikian and Nanda [18] to include commitment issues and the possibility of agent turnover. The central theme of Chapter 3 is the analysis of different levels of commitment and a comparison of the principal's welfare under different commitment scenarios and different correlation of the performance measures. Regarding agent turnover, the main insight from Chapter 3 is that, in a two-period LEN model, the two agent solution (turnover or no commitment) is the least preferred when the performance measures are negatively correlated, with long-term full commitment and long-term commitment with renegotiation (or commitment to fairness) dominating. The result is the opposite for positive correlation of the performance measures, in that the two-agent solution (turnover or no commitment) dominates long-term full commitment.

These results regarding turnover indicate that, in a two-period world, the principal prefers to replace the agent after one period when the performance measures are positively correlated. On the other hand, the principal prefers even the weakest form of long-term commitment with a single agent when the performance measures are negatively correlated. These results can be extended to more than two periods. In a setting with more than two periods, one can consider the choice of alternative tenures in a given time horizon. In a realistic setting, tenure is a choice variable in a world with more than two

²The situation is as if there is a compensation scheme for the first-period action \tilde{c}_1 and a compensation scheme for the second-period action \tilde{c}_2 . With one agent, the risk for which the principal pays compensation is $\text{var}(\tilde{c}_1 + \tilde{c}_2) = \text{var}(\tilde{c}_1) + 2\text{cov}(\tilde{c}_1, \tilde{c}_2) + \text{var}(\tilde{c}_2)$. With two agents, the risk for which the principal pays compensation is $\text{var}(\tilde{c}_1) + \text{var}(\tilde{c}_2)$.

periods. In addition, when considering tenure (or replacement policies), the analysis can be framed in different ways. For example, regarding optimal tenure one may ask:

1. In a given finite number of periods, what is the optimal number of agents and their respective tenures? In other words, given a known finite life for the firm, how can one optimally partition that among several agents, and what are tenures of those agents?
2. In an infinite world, what is the optimal tenure for agents, if we assume a policy of replacing agents after the same number of periods (tenure)?

The main difference between the two frameworks, and between the way questions are answered is that in one world there is a “last period”, whereas in the other world, there is no “last period”. The infinite period world permits the comparison of different tenure lengths in a natural way. A similar comparison of these two frameworks is undertaken in Chapter 2 regarding auditor tenure. In Chapter 4, I extend the two-period model of Chapter 3 in order to answer the second question posed above: what is the optimal (stationary) agent turnover policy in an infinite-period world?

In Chapter 4 I develop an N-period model of the ratchet effect in a principal-agent problem with moral hazard but without adverse selection. Thus, while the agent’s action is unobservable by the principal, in equilibrium the principal has rational beliefs regarding the agent’s past actions and as a result, in equilibrium, information asymmetries do not develop over time between the principal and the agent. In addition, there is no learning of productivity or any agent characteristic that is unknown at the start. The only dynamic information effects are the adjustment of posterior beliefs about future performance measures, conditional on the sequential observation of past performance measures together with conjectures of the agent’s past actions. The model generalizes the two-period model of Chapter 3, and most results derived therein remain valid in the

N-period case. Primarily, the conclusions regarding the role of commitment in obtaining different solutions remain the same. However, the N-period model gives insights into the importance of the contracting horizon, or tenure. In addition, the N-period model offers insights into the agent's long-term performance that cannot be inferred from the two-period model.

While in Chapter 3 I have compared different commitment scenarios given a fixed two-period horizon, in Chapter 4 the emphasis is on tenure given a particular set of commitment assumptions. Since the very idea of tenure implies some form of implicit or explicit long-term commitment, the choice is between the three types of long-term commitment discussed in Chapter 3: full commitment to a long-term contract, commitment to a long-term contract with renegotiation, and commitment to fairness with short-term contracts. Full commitment to a long-term contract is too restrictive, in that, especially over longer horizons, renegotiation is more likely. Commitment to fairness is a mechanism that replicates the commitment to a long-term contract with renegotiation by using a sequence of short-term contracts. This makes the choice between the two forms of contracting almost a matter of taste.

In Chapter 4, I choose to restrict the analysis to contracting under commitment to fairness. Besides capturing the idea of renegotiation, it only requires short-term contracts that are settled at the end of each period, allowing for a breakdown of performance and surplus period-by-period. In addition, the principal and the agent only need to commit to "fair contracts" and to a tenure duration, no other long-term commitments or contracts are necessary.³ Finally, the commitment to fairness solution to the dynamic agency problem provides consistency and ease of comparison to previous literature such as Milgrom and Roberts [29] and Indjejikian and Nanda [18].

³The reader who finds the concept of "commitment to fairness" unpalatable can interpret all results in the context of commitment to a long-term contract with renegotiation.

The driving force behind the “ratchet effect” with pure moral hazard described in Chapter 4 is the principal’s inability to commit not to use past performance measures when setting a short-term contract with the agent in each period other than the agent’s first period of tenure. The principal has an incentive to set the agent’s compensation in a later period based on past performance measures in order to optimally adjust the risk in the agent’s contract for the lower posterior variances. In doing so, the principal makes the fixed pay in future periods depend on past performance measures. For the agent, it means that from an *ex ante* (previous periods) perspective, future fixed pay depend on earlier effort choices. In other words, “fixed compensation” in one period is fixed only in that period, given the past actions and performance measures, but is variable when anticipated from earlier periods. It follows that the agent’s incentives for effort in any one period are spread among the variable wage for that period and the fixed wages in all future periods.

The nature of the solution to the dynamic agency problem is such that, for longer horizons, the manager’s effort is close to some limit level for most periods. Thus, there are essentially two effort levels, the second best in the last period, and approximately a “third best” in most other periods. When correlation is positive, the “third best” is lower than the second best, generating inefficiencies relative to a repeated one-period problem (which is the multi-period problem when periods are independent). When the correlation is negative, the ratchet effect is efficient relative to the uncorrelated periods case since the “third best” effort level is closer to first best. To summarize, with positively correlated performance measures, incentives are stronger and the manager exerts less effort in most periods than in the last period. With negatively correlated performance measures, incentives are less strong and the manager exerts more effort in most periods than in the last period. The effort level in the last period serves as a benchmark because it coincides with the second-best solution, which is what one obtains with uncorrelated

performance measures.

A commitment problem could arise in the case of negatively correlated performance measures due to the decreasing performance of the agent towards the end of his tenure. The principal needs to be able to commit not to fire the manager before term, otherwise the efficiency gains from getting effort levels close to first-best are lost. This would not be a problem in the positive correlation case since there the principal gets better performance towards the last period, and has no incentive to fire the manager before term. In either case, commitment to fairness assumes that the principal commits to retain the agent for all N periods.

To answer the question of ex-ante choice of tenure by the principal, I examine optimal stationary replacement policies, whereby the agents are hired for N periods at a time within an infinite horizon for the firm, and the noise in the performance measures is firm-specific. The main result for positively correlated performance measures is that, in the presence of a switching cost, there exists a threshold switching cost such that optimal tenure is a single period whenever the switching cost is higher than the threshold value. On the other hand, optimal tenure is the maximum number of periods possible (the agent's maximum life) when the switching cost is lower than the threshold value. Thus, with positively correlated performance measures, the only optimal replacement (tenure) policies are the corner solutions of one period tenure or maximum possible tenure. The main result for negatively correlated performance measures is that the optimal replacement policy is always the maximum number of periods possible, irrespective of the switching cost.

In the case when the noise in the performance measures is agent-specific, there is an additional "learning" effect in the first few periods of a manager's tenure due to the (rapid) reduction of the posterior variances of the performance measures towards their limit value. This effect results in an increase of managerial effort, since it becomes less

costly over time for the principal to motivate managerial effort due to lower risk premia. With negatively correlated performance measures and agent-specific noise, the manager exerts the second-best effort in the last period, a higher effort level in all other periods, and that effort has an inverted U shape. This finding is consistent with evidence from the management literature that firm performance increases at first, reaches a maximum, and then declines during a manager's tenure. For a discussion of these findings, see the papers by Eitzen and Yetman [9], Katz [21], and Hambrick and Fukutomi [15].

Chapter 2

Agency Costs, Audit Pricing, and Auditor Turnover

2.1 Introduction

The impact of auditor tenure – the length of time a client retains a particular auditor – on the quality of audit services has long been the subject of both speculation and controversy. For example, Mautz and Sharaf [28] suggest that the personal relationship that develops between an auditor and the client over time can be expected to adversely influence the auditor's vigilance. The same view was expressed by the U.S. Senate's Metcalf Committee [32] which concluded that a lengthy association with a client may lead an accounting firm to identify closely with the interests of the client management, thereby impairing auditor independence. These arguments have led critics of the profession to propose mandatory auditor rotation policies for publicly held companies, but such regulations have never been implemented in North America. The principal reason that auditor rotation has not been mandated is the presumption that such a policy would harm audit efficiency and be cost increasing to clients.

Turning to research, to our knowledge, no formal analytical model of optimal auditor tenure has been developed and we do not develop such a model in this paper. However, auditor tenure is an ingredient in multi-period pricing models where the traditional approach, as in DeAngelo [5], is to assume an infinite time horizon. In the presence of auditor learning and/or client switching costs, the result is initial 'lowballing' of audit

fees followed by an infinite number of periods in which incumbent auditors earn client-specific quasi-rents. Moreover, there is no auditor turnover because incumbent auditors price engagements at the amount necessary to deter auditor change by clients.

However, infinite auditor tenure obviously does not exist in the real world. In the mid-1970's, average auditor tenure for audits of publicly held U.S. companies was about 18 years (Simunic [35]). More recently, average tenure appears to have decreased to about 10 years (O'Keefe, Simunic and Stein [31]). In order to introduce finite auditor tenure in analytical models, it is usual to assume that exogenous factors – such as the need for new financing, company growth, client financial distress, or auditor-client disagreement over a financial reporting issue (e.g. Dye [8]) – motivate voluntary auditor changes, whether initiated by the client or the auditor.

In this paper, we develop a model of audit pricing and auditor change when investors, management, and the auditor all hold rational expectations that auditor tenure will be of finite length. We make no claim that this finite period is *ex ante* optimal in any sense, although – once specified – it is *ex post* optimal. Perfect competition among auditors bidding for an engagement is assumed. Lowballing and an implicit multi-period commitment are obtained, while the auditor is hired one period at a time. Consistent with intuition, the amount of lowballing is strictly increasing in the conjectured length of auditor tenure. Thus, the fee structure is a generalization of DeAngelo's infinite horizon model to a more realistic world with finite auditor tenure. We also examine the effects of the fee structure on incentives to replace the auditor, and auditor independence. We show that the quasi-rents earned by the auditor do not impair independence because management dismissal threats are not credible when auditors are identical and do not disagree on auditing issues as in Magee and Tseng [26]. Fees are such that management cannot compensate the auditor (except through an explicit bribe) for any additional risk incurred by the auditor from compromising independence during the auditor's expected tenure. Conversely, the

auditor is replaced at the end of the conjectured number of periods because retention would allow the auditor to earn economic rents from the engagement. While these rents do not result in a compromise of the auditor's independence, the total audit cost to the firm (audit fee) would incorporate the expected litigation costs faced by the manager as if the auditor's independence were impaired. This is true because, once retained past the expected tenure, the auditor could only be replaced by one who would systematically compromise his independence. Thus, having rational expectations of finite auditor tenure can be sufficient to induce auditor change within the same class of auditors (e.g. the Big Five). No external event is necessary to trigger the auditor change.

The remainder of the paper is organized as follows. In section 2 we examine relevant literature dealing with auditor tenure and auditor change. Section 3 presents the pricing model given exogenous agency costs and exogenous auditor turnover. Section 4 presents the auditor change model. Section 5 contains a discussion of empirical implications and concludes the paper. The appendices contain some of the proofs.

2.2 Related literature

Following DeAngelo [5], the issues surrounding multi-period audit pricing – particularly the existence of ‘lowballing’ and its effects on auditor independence – have continued to interest researchers. Dye [8] introduced the notion of relative ‘bargaining power’ of the auditor vs. the client into the analysis and argued that DeAngelo’s pricing model implicitly assumes that the auditor possesses all (or most) of the bargaining power in setting audit fees. This allows the auditor to earn quasi-rents. Dye argues that if the client possessed all of the bargaining power, then an auditor would always be constrained to perform audits for a fee equal to the auditor’s costs, thereby eliminating quasi-rents and consequently lowballing. To resurrect the possibility of lowballing, Dye goes on to model

a situation where there may be information asymmetries between the auditor and the client about the client's financial statements being audited. In this setting, auditors earn quasi-rents (hence, lowballing occurs) only when quasi-rents are not disclosed to investors. This is so because the absence of disclosure prevents the (assumed) negative implications of quasi-rents on the auditor's independence from being impounded by investors in the firm's market value¹. Moreover, an incumbent auditor may be replaced if management's information reflected in proposed financial statements is more favorable than the auditor's information about the client, and the auditor is unwilling to attest to management's more favorable information. Note that in this more complex (relative to DeAngelo's) model, an auditor will still be used for an indefinite number of periods unless an information asymmetry concerning the client's financial statements arises.

Kanodia and Mukherji [20] develop a model of audit pricing, lowballing and auditor turnover under the assumption that the client has all of the bargaining power, but the auditor has better information about true audit costs. In their setting with information asymmetry about costs and a competitive market where clients make take-it-or-leave-it offers to purchase audit services, there will be some auditor turnover. However, in this model, auditor replacement also becomes less likely over time. More recently, Morgan and Stocken [30] take an intermediate view of bargaining power in which an incumbent auditor sets fees subject to competition from outside auditors. Their model provides for some probabilistic auditor replacement in an equilibrium with mixed strategies.

Finally, Magee and Tseng [26] analyze the complex and controversial issue of the effect of quasi-rents on auditor independence. That is, under what conditions will the existence of quasi-rents – independent of the causes that allow quasi-rents to arise – impair an auditor's independence? They show that in a rational market, these conditions are

¹One common feature of both the DeAngelo and Dye arguments is the implicit assumption that quasi rents impair auditor's independence because managers can make credible threats to replace the auditor.

quite limited, requiring – among other things – disagreement among auditors about the requirements of GAAP, a lack of auditor knowledge about his or her own position on a GAAP matter before it arises, and a lack of client knowledge about the incumbent auditor's position on a matter. In the absence of these conditions, the client simply employs the net benefit maximizing auditor and – as in DeAngelo – there is no motivation for auditor changes. It is also worth noting that while Magee and Tseng use a finite horizon pricing model, the horizon applies both to the life of the client firm as well as to the auditor's tenure. None of the existing models predict an auditor change simply because an auditor has been utilized a certain number of years by a client that will continue to exist in the future.

Empirical work, on the other hand, has identified a variety of circumstances under which auditor changes occur. Williams [37] finds that the most significant variables explaining auditor changes are the auditor's industry specialization, the length of auditor tenure (long tenure increases the probability of replacement), and the client's desire to signal favorable information following adverse publicity. Haskins and Williams [16] find a series of variables associated with auditor changes within the Big 8 group: client financial distress, client size and growth rate, and the industry dominance of audit firms. DeFond [6] finds that auditor changes occur in anticipation of, and in response to changes in client agency costs as measured by decreases in management ownership of shares and increases in leverage. These agency cost factors are significant after controlling for changes in client firm growth and the issuance of new securities. In general, while existing empirical work provides insights into factors motivating auditor changes, the studies are scattered (there is little replication) and the statistical models are characterized by low explanatory power. However, the evidence (weakly) suggests that an auditor change is more likely as the length of tenure increases.

Finally, there is limited empirical evidence linking auditor tenure with decreased audit

quality. Besides the negative effect of length of tenure on earnings response coefficients reported by Johnson et al [19], Krishnan and Krishnan [22] find that long auditor tenure is less likely to be associated with auditor resignation than with auditor dismissal by the client. Note that auditor dismissal could be motivated by weaker capital market reaction to positive unexpected earnings as auditor tenure increases. More directly, Deis and Giroux [7] and Copley and Doucet [4], using public sector data, find that the probability of receiving a substandard audit increases with the length of the auditor-client relationship.

The model of audit pricing, auditor tenure, and auditor change developed in this paper is consistent with the empirical findings of a negative relation between length of tenure and audit quality, and that auditor change is more likely as length of tenure increases. Moreover, the model is also consistent with the hypothesis that changes in agency costs can lead to auditor changes. However, the changes in agency costs arise endogenously, rather than being associated with exogenous changes in management ownership or other factors.

2.3 Audit pricing with exogenous costs and exogenous turnover

In this section we develop a model of audit pricing that extends DeAngelo [5] by allowing for finite tenure and a more general cost structure. We recover DeAngelo's results as a particular case (infinite tenure) and we show how lowballing depends on tenure when the auditor's tenure is finite. Our model differs from Magee and Tseng [26] in that we do not have a last period and the firm continues to exist after an auditor is replaced. Accordingly, the results on lowballing are different: in the Magee and Tseng model, with finite tenure, lowballing is independent of tenure, while in our model, lowballing is increasing with expected tenure.

Assumptions

The time variable is discrete and the unit of measure is one time period, typically one year. The firms exist for infinitely many periods and engage auditors one period at a time. Auditing is mandatory. A firm can switch to a new auditor each period. An incumbent auditor can choose not to continue providing audit services to the firm. The value of a firm is determined by outside investors through trading on the securities market. The quality of audit for any firm impacts the value of that firm through the impact of auditing on agency costs. For the rest of the paper, we analyze audit pricing and auditor switches for a single firm, referred to as *the firm*. The manager of this firm will be referred to as *the manager*.

The decision to hire an auditor is made by the manager. The manager has an exogenously specified compensation package and is risk neutral. The auditor is also risk neutral. The manager's compensation is increasing in firm value, and the manager incurs no personal cost in replacing the auditor. The manager's choice of auditor affects his wealth only through the effect of an audit on the value of the firm. Consequently, the manager will minimize the total cost of auditing when choosing an auditor. The firm hires an auditor at the start of period 1. The manager conjectures what the auditor's tenure with the firm is going to be. The auditor and the investors also have a conjecture of what the auditor's tenure will be. In a rational expectations equilibrium, the manager's conjecture, the auditor's conjecture, and the investors' conjecture of auditor tenure coincide. We denote this equilibrium conjecture of auditor tenure by N . In addition to finite tenure, we also allow for N to be infinite.

The impact of auditing on the value of the firm in any given period is represented by a residual agency cost function denoted by C . The cost C is defined as the difference between agency costs of the firm given the auditor it employs and agency costs in a world

with perfect auditing. The cost C can be equivalently characterized as the difference between the value of the firm in a world with perfect auditing and the value of the firm given the auditor it employs (and assuming the firm pays the same price for auditing in both worlds). The cost function C is an implied cost and characterizes the impact on the value of the firm of investors' perceptions of audit quality and of other agency cost variables such as leverage or management ownership of the firm. As a result, C will depend on determinants of audit quality such as brand name, reputation, size, and extent of consulting services. In addition to these variables, we assume that C also depends on two time variables:

- t , the number of periods the auditor has been employed by the firm;
- N , the conjectured auditor tenure.

In what follows we assume that all auditors are identical and that the firm characteristics that determine agency costs do not change in time. As a result, we can concentrate only on the dependence of C on N and t . Thus, C is a function of N and t only, and we denote it by $C(N, t)$. In general, the residual agency cost is endogenously determined. To simplify the analysis, we begin by assuming that the residual agency cost function is exogenously given, and we first solve the pricing problem. Once we know the structure of audit fees for any given cost function, we will endogenize the residual agency cost, given tenure beliefs and auditor independence considerations.

In auditing the firm's financial statements, an auditor incurs costs $A(t)$ in period t . The function $A(t)$ is nonincreasing and is the same for all auditors. The function $A(t)$ is also exogenously specified, and incorporates both production costs and expected litigation costs. Later, we will endogenize the expected litigation costs when we introduce auditor independence issues. As a result of perfect competition among auditors, the ex

ante net present value of the audit engagement to the auditor is zero.² The audit fees are such that in each period the net present value of future audit fees less audit costs is nonnegative. The firm incurs a cost S whenever it switches to a new auditor. This cost is incurred in the first period of the audit engagement. The manager and the investors estimate that the impact of audit costs on firm value is the net present value of all future audit costs as incurred by the firm. The audit costs incurred by the firm are the sum of audit fees and agency costs. The expectations for future audit costs are based on repeating identical auditor engagements to infinity.

The audit pricing model

The firm hires an auditor at the start of period 1. The firm pays the auditor a fee $F(N, t)$ at the end of period t . For uniformity, all other costs such as $A(t)$, S , and $C(N, t)$ are also considered as end of period costs. To abstract from all considerations other than audit pricing, we assume, in this section only, that the manager can commit to replace the auditor at the end of N periods. The switching cost S , the auditor's costs $A(t)$, and the residual agency cost function $C(N, t)$ are all exogenously specified. In the next section, we remove the assumption on commitment to turnover on the principal's part and we endogenize $A(t)$, $C(N, t)$ given auditor independence considerations.

The auditing market is assumed to be perfectly competitive in the following sense. The incumbent auditor sets fees at the start of each period, other than the first period of the engagement, such that the audit cost to the firm, conditional on retaining the incumbent are less than or equal to the audit cost conditional on replacing the incumbent. The audit cost to the firm is the present value of all future audit fees plus the residual

²The zero NPV condition applies only at the beginning of the first period of the audit engagement and is based on the conjectured N and the implied fee schedule derived from the conjectured N .

agency cost. Retaining the incumbent means retaining for the remainder of his conjectured tenure and continuing to replace auditors every N periods thereafter. Replacing the incumbent is also assumed to be followed by continuing to replace auditors every N periods. In the first period, the fee is set such that the net present value of the entire audit engagement (present value of future audit fees less audit costs) is zero.

For simplicity, we treat competitive pricing as an exogenous assumption in this section. Later, we show how competitive pricing arises in conjunction with the replacement of auditors and its impact on tenure beliefs. For now, setting aside the fact that an auditor switch may impact tenure beliefs, competitive pricing can be derived as follows. An incumbent auditor sets the highest fee such that he is retained. The firm will switch to a new auditor if the present value of all future auditing costs conditional on retaining the incumbent exceeds the present value of all future auditing costs conditional on hiring a new auditor. When these costs are equal, the firm retains the incumbent. An auditor switch does not change either the tenure beliefs under which a replacement is hired or the manager's commitment to switching auditors at the end of their conjectured tenure. In the first period, all auditors compete for the engagement, which drives the NPV to zero.

We shall refer to this set of assumptions as commitment to tenure and competitive pricing. The following proposition gives the equilibrium audit fee structure for any exogenously specified tenure N and costs $S, A(t), C(N, t)$ under the assumption of commitment to tenure and competitive pricing.

Proposition 2.3.1 *Let N be an exogenously given auditor tenure belief, and let $A(t)$, $C(N, t)$, S represent exogenously given auditing costs, residual agency costs, and switching cost, respectively. Assume further commitment to tenure N and competitive pricing. Then, the following audit fee schedule obtains in equilibrium for the N periods of the*

auditor's tenure:

$$F(N, t) = \begin{cases} Q(N)[\beta S + \sum_{t=1}^N \beta^t (A(t) + C(N, t))] - C(N, 1) - S & \text{if } t = 1 \\ Q(N)[\beta S + \sum_{t=1}^N \beta^t (A(t) + C(N, t))] - C(N, t) & \text{if } t \geq 2 \end{cases} \quad (2.1)$$

where $Q(N) = (\sum_{t=1}^N \beta^t)^{-1}$ is an annuity factor.

Proof. In each period the total auditing costs to the firm are denoted by $D(N, t)$,

$$D(N, 1) := F(N, 1) + C(N, 1) + S \quad (2.2)$$

$$D(N, t) := F(N, t) + C(N, t) \text{ for } 2 \leq t \leq N. \quad (2.3)$$

Let $K(N)$ be the present value of all costs of hiring and retaining an auditor for N periods,

$$K(N) := \sum_{t=1}^N \beta^t D(N, t), \quad (2.4)$$

where β represents the discount factor.³ Let $K_\infty(N)$ be the present value of all future auditing costs conditional on the firm hiring an auditor every N periods in perpetuity, and based on the assumption that the firm's agency costs determinants remain unchanged,⁴

$$K_\infty(N) := \sum_{i=0}^{\infty} \beta^{iN} K(N) = (1 - \beta^N)^{-1} K(N). \quad (2.5)$$

In particular, if $N = \infty$,

$$K_\infty(\infty) = K(\infty) = \sum_{t=1}^{\infty} \beta^t D(\infty, t).$$

The audit fees for the N periods are determined by perfect competition among auditors. Let $R(N, t)$ denote the present value of all future auditing costs to the firm

³As usual, $\beta = (1 + r)^{-1}$.

⁴That is, the cost function $C(N, t)$ remains the same in the future.

conditional on retaining the incumbent, discounted to time t ,

$$\begin{aligned} R(N, t) &:= \sum_{i=t}^N \beta^{i-t+1} D(N, i) + \beta^{N-t+1} K_{\infty}(N) \\ &= \beta[D(N, t) + R(N, t+1)] \text{ if } t < N. \end{aligned} \quad (2.6)$$

For the firm, the present value of all future auditing costs conditional on hiring a new auditor is $K_{\infty}(N)$. Thus, the incumbent sets the fees $F(N, t)$ such that $R(N, t) \leq K_{\infty}(N)$ for all $2 \leq t \leq N$. It follows that

$$R(N, t) = K_{\infty}(N) \text{ for } t = N, \dots, 2. \quad (2.7)$$

Equation (2.7) is equivalent to an entry-preventing condition, since it represents the highest fees for which the manager of the firm is indifferent between retaining and replacing the incumbent. It follows that the present value of future audit costs for the firm is constant through periods $2, \dots, N$ of the audit engagement. Furthermore, using equation (2.7) and backwards induction, $D(N, t) = rK_{\infty}(N)$ for $t = 2, \dots, N$. For $t = 1$, equations (2.4) and (2.5) determine $D(N, 1) = rK_{\infty}(N)$. For a proof, see Appendix A. Thus, $D(N, t)$ depends only on N and not on t . Let $D(N)$ denote the common value of $D(N, t)$ for $1 \leq t \leq N$.

The next task is to determine the equilibrium value of $F(N, t)$ as a function of N , and the variables S , $A(t)$, and $C(N, t)$. For the first period, perfect competition implies the zero NPV condition

$$\sum_{t=1}^N \beta^t (F(N, t) - A(t)) = 0. \quad (2.8)$$

Once $F(N, t)$ is known for $2 \leq t \leq N$, the fee $F(N, 1)$ is determined by solving equation (2.8).

For clarity of exposition, we will express the auditor's fees and the objective function of the firm in terms of $D(N)$. First, the equations that define $D(N, t)$, (2.2) and (2.3),

imply that

$$F(N, 1) = D(N) - C(N, 1) - S \quad (2.9)$$

$$F(N, t) = D(N) - C(N, t) \text{ for } 2 \leq t \leq N. \quad (2.10)$$

Substituting (2.9) and (2.10) into (2.8) gives an equation for $D(N)$,

$$\beta(D(N) - A(1) - S - C(N, 1)) + \cdots + \beta^N(D(N) - A(N) - C(N, t)) = 0. \quad (2.11)$$

Rearranging terms on both sides of (2.11) gives

$$D(N) \sum_{t=1}^N \beta^t = \beta S + \sum_{t=1}^N \beta^t A(t) + \sum_{t=1}^N \beta^t C(N, t),$$

which implies

$$D(N) = Q(N) \left(\beta S + \sum_{t=1}^N \beta^t A(t) + \sum_{t=1}^N \beta^t C(N, t) \right), \quad (2.12)$$

where $Q(N) = \left(\sum_{t=1}^N \beta^t \right)^{-1}$ is an annuity factor.⁵ Once the values of S and N , and the function $C(N, t)$ are known, $D(N)$ is uniquely determined by (2.12), and the audit fees in each period are uniquely determined by (2.9) and (2.10). \square

The fee structure is such that, in each period, the auditor's fee and the residual agency costs add up to the same amount, which is the sum of the annual equivalent values of the switching cost, the auditor's costs, and the residual agency costs over the auditor's tenure.⁶ The perfect competition among auditors ensures that, in present value terms, the auditor's fees add up to the same amount as the auditor's costs over the duration of

⁵ $Q(N)$ specifies the amount per year for N years that has a present value of one.

⁶The annual equivalent value of a stream of cash flows $c(1), \dots, c(N)$ with discount factor β is the amount $Q(N) \sum_{t=1}^N \beta^t c(t)$ that, if substituted for $c(t)$ in each period, would have the same NPV as the original cash flows. For a zero net discount rate ($\beta = 1$), the annual equivalent value is the average cash flow over N periods. See also Copeland and Weston [3], pp. 49–55.

the engagement. The firm's total cost is then the sum of the switching cost, the auditor's costs, and the residual agency costs.

Let $B(N)$ denote the sum of the annual equivalent value of the switching cost and the annual equivalent value of the auditing costs,

$$B(N) := Q(N) \left(\beta S + \sum_{t=1}^N \beta^t A(t) \right). \quad (2.13)$$

To simplify the analysis that follows, we assume that $C(N, t)$ does not depend on t for $t = 1, \dots, N$, that is the residual agency cost is the same in each period of the audit engagement. Let $C(N) = C(N, t)$ for $t = 1, \dots, N$ denote this common value. Then, the annual equivalent value of the residual agency costs is $C(N)$, and the firm's total cost of audit in each period becomes

$$D(N) = B(N) + C(N). \quad (2.14)$$

The first term, $B(N)$ is a decreasing function of N , since we assumed $A(t)$ to be a nonincreasing function of t . For a proof, see Appendix B. The equilibrium fee structure is then deduced from the equilibrium value of $D(N)$ according to (2.9) and (2.10),

$$F(N, t) = \begin{cases} D(N) - C(N) - S & \text{if } t = 1 \\ D(N) - C(N) & \text{if } t \geq 2. \end{cases} \quad (2.15)$$

From (2.14) it follows that

$$F(N, t) = \begin{cases} B(N) - S & \text{if } t = 1 \\ B(N) & \text{if } t \geq 2. \end{cases} \quad (2.16)$$

Lowballing occurs if $F(N, 1) < A(1)$. The net amount of lowballing is determined by (2.9),

$$LB = A(1) - F(N, 1) = A(1) + S - B(N) \quad (2.17)$$

and is increasing in N since $B(N)$ is decreasing in N . Furthermore, from equation (2.13) it follows that

$$\begin{aligned} LB &= Q(N) \left[\left(\sum_{t=1}^N \beta^t \right) (A(1) + S) - \left(\beta S + \sum_{t=1}^N \beta^t A(t) \right) \right] \\ &= Q(N) \sum_{t=2}^N \beta^t (A(1) - A(t) + S) . \end{aligned} \quad (2.18)$$

Thus, there is lowballing whenever $A(1) > A(t)$ for at least one period or $S > 0$. Lowballing is driven by perfect competition and either by the auditor's learning (if we interpret lower costs of auditing in later periods as a result of learning) or by the firm's switching costs (or by both).

In order to put the above results into perspective by comparing them to the pricing in the models of DeAngelo [5] and Magee and Tseng [26], we assume that all the auditor's learning costs are incurred in the first period:

$$A(t) = \begin{cases} A + L & \text{if } t = 1 \\ A & \text{if } t > 1 . \end{cases}$$

Given the simplifying assumptions for the audit costs, we can rewrite (2.13) as

$$B(N) := A + Q(N)\beta(S + L) . \quad (2.19)$$

The fee structure then becomes

$$F(N, t) = \begin{cases} A + \beta Q(N)(L + S) - S & \text{if } t = 1 \\ A + \beta Q(N)(L + S) & \text{if } t \geq 2 . \end{cases}$$

In particular, for $N = \infty$, $\beta Q(N) = 1 - \beta$ and the corresponding fee structure is the same as DeAngelo's,

$$F(N, t) = \begin{cases} A + (1 - \beta)L - \beta S & \text{if } t = 1 \\ A + (1 - \beta)(L + S) & \text{if } t \geq 2 . \end{cases}$$

Lowballing occurs if $F(N, 1) < A(1)$. The net amount of lowballing is determined by (2.9):

$$LB = A(1) - F(N, 1) = A + L + S - B(N) . \quad (2.20)$$

Using (2.19), we can rewrite (2.20) as

$$LB = (1 - Q(N)\beta) (S + L) . \quad (2.21)$$

It follows that there will be lowballing (that is positive LB) if either $L > 0$ or $S > 0$. Thus, lowballing is driven either by the auditor's learning costs or by the switching cost (or by both).

In particular, in the absence of switching costs ($S = 0$), the audit fee structure is flat ($F(N, t) = B(N)$) and there will be lowballing if the auditor has learning costs. In this case, there is lowballing without first period discounts. This shows that the implications of the model regarding lowballing generalize DeAngelo's [5] results to the case of finite tenure.

Since LB is increasing as a function of N , there will be a one-to-one correspondence between the amount of lowballing and the conjectured N . We also note for future reference that there is a one-to-one correspondence between the first period fee $F(N, 1)$ and N .

This result differs from the finite horizon pricing and lowballing of Magee and Tseng [26] in that lowballing depends on the time horizon. The difference in pricing between our model and theirs is due to the fact that we assume tenure beliefs of outside auditors to be independent of the number of periods remaining for the incumbent. Thus, our model is based on the assumption that tenure beliefs are a moving horizon, while Magee and Tseng's model is based on the assumption that the end of the auditor engagement is exogenously fixed and even if an incumbent auditor is replaced, the replacement auditor's

tenure does not extend beyond that fixed end date. With our notation, the fee structure of Magee and Tseng is

$$F(N, t) = \begin{cases} A + (1 - \beta)L - \beta S & \text{if } t = 1 \\ A + (1 - \beta)(L + S) & \text{if } 1 < t < N \\ A + L + S & \text{if } t = N \end{cases} \quad (2.22)$$

The outside auditors bid $F = A + (1 - \beta)L - \beta S$ in every period except when there is only one period left until an auditor change occurs, in which case they bid $F = A + L$. The lowball is then $LB = \beta(L + S)$ which corresponds to lowballing for infinite tenure in our model.

2.4 Auditor replacement in equilibrium

In the preceding section, we have derived the audit fee structure given the manager's commitment to replacing the auditor after N periods, competitive pricing, and exogenously specified auditing and residual agency costs. We remove now the assumption on the manager's ability to commit to replace the auditor after a given number of periods and we endogenize the residual agency costs and part of the auditor's costs. We maintain all the other assumptions we have made and introduce accounting issues over which the manager and the auditor may disagree, giving rise to the possibility of compromise and impaired auditor independence.

We also make detailed assumptions on how tenure beliefs are affected by the way in which the auditor is replaced/retained relative to the existing tenure beliefs. Tenure beliefs have to be consistent in equilibrium. However, we need to specify tenure beliefs off the equilibrium path, that is how tenure beliefs are updated by an observed off-equilibrium retention/replacement decision. The rules by which tenure beliefs are

updated are exogenously specified.⁷

We now proceed to describe the game between manager and auditor in which the manager decides whether to retain or to replace the auditor, and the auditor submits a fee bid and may have to decide on compromising/not compromising on an accounting issue in each period.

The manager and the incumbent auditor maximize utility, that is the manager minimizes the audit cost for the firm, net of any possible personal benefit that may result from the auditor compromising on an accounting issue. The auditor maximizes his audit fees less audit costs, where the audit costs include any increase in expected litigation costs that may arise from the auditor compromising on accounting issues. The competing auditors and the investors are not, properly speaking, players in the game since they do not maximize an objective function and their “strategies” are exogenously specified. However, the outside auditors submit competitive bids based on tenure beliefs, such that they earn a normal return on the audit engagement. Also, the investors are price protected, in that they increase the residual agency cost whenever they believe that the auditor’s independence is impaired.

2.4.1 The game

In each period, the game between manager and auditor unfolds as follows (for a graphical description, see Figure 1).

1. At the beginning of the period, all auditors submit bids to the manager. We assume bidding to be costless.
2. The manager chooses an auditor. Given the bids and the beliefs regarding tenure,

⁷Note that the use of the word “beliefs” is not in the usual sense. Tenure is not random, beliefs are not about a random variable, and there is no Bayesian updating involved.

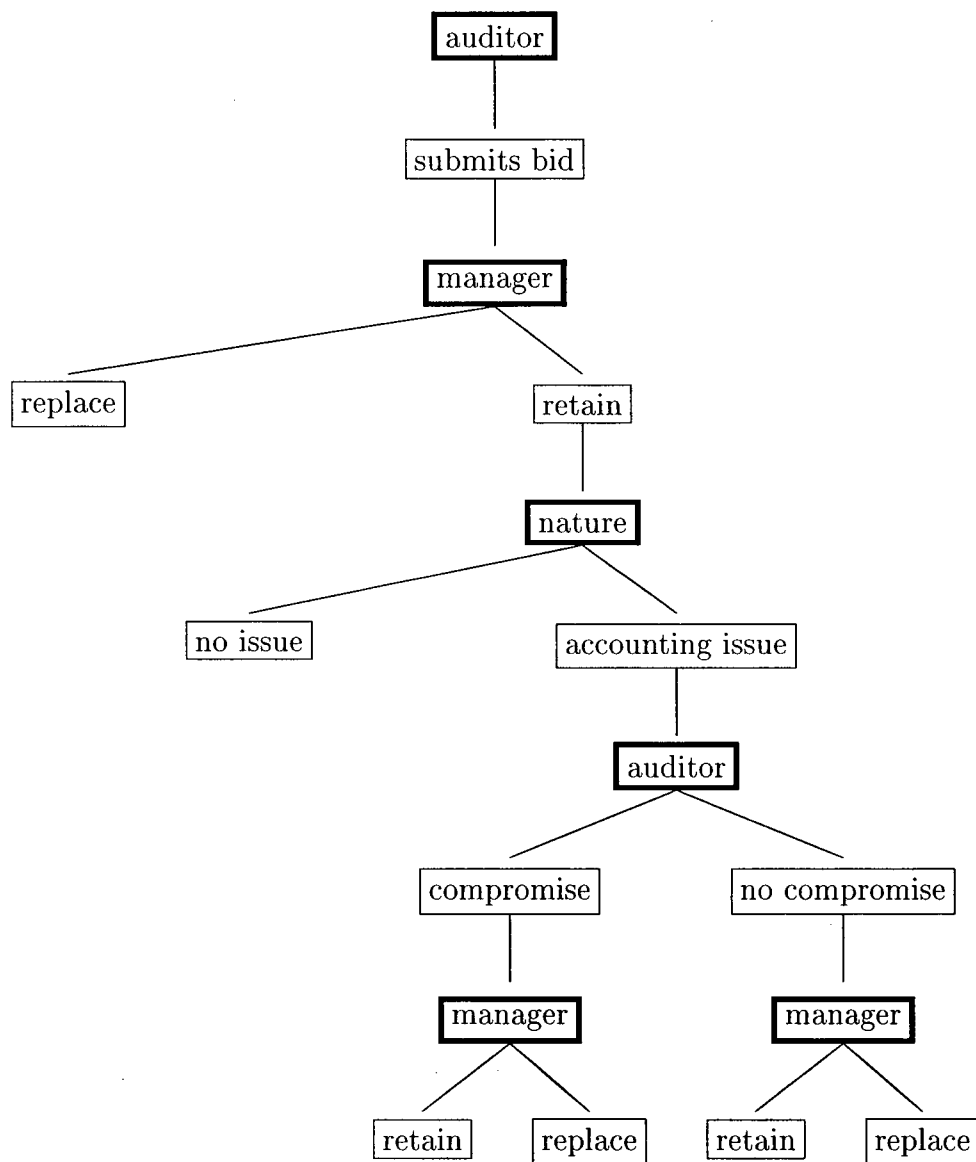


Figure 2.1: The game between manager and auditor

the manager retains the incumbent if indifferent between future audit costs conditional on retaining the incumbent and future audit costs conditional on hiring a new auditor. The manager anticipates that the agency costs may increase if the incumbent is retained. In the first period, there is no incumbent and an auditor whose fees are consistent with the tenure beliefs is chosen.⁸ The fee that was bid by the auditor who was ultimately engaged, or the fee bid by the incumbent in case an incumbent auditor was retained, is not subject to renegotiation for the remainder of the period. If the auditor is not replaced later in the period and provides the audit service, the fee bid at the start of the period is the final fee for that period.

3. The investors anticipate whether there is a positive probability of a compromise by the incumbent auditor (see 5 and 7 below). The investors cannot observe at any point in the game whether the auditor has compromised his independence or not. If they believe that there is a positive probability that the auditor's independence is impaired, agency costs increase by Δ as a result of a drop in share price. The amount Δ is assumed to be independent of the probability of impaired independence and the nature of the accounting issue.

We assume the investors believe that even the smallest compromise in auditor independence has very large consequences. When the investors believe that the auditor maintains his independence with probability one, they trust the auditor to be independent. When the investors believe that there is a positive probability of impaired independence, they do not trust the auditor anymore and lower the firm value by Δ , which is a penalty borne by the manager.⁹ As shown in 8 below, the

⁸We do not model how a common conjecture of auditor tenure is reached, or how the manager chooses one. In the first period we simply assume that the manager has chosen one auditor consistent with some tenure beliefs and that all other auditors and the investors have the same auditor tenure beliefs.

⁹The investors trust the auditor if, and only if, they believe that the auditor is independent with probability one. Equivalently, the investors trust the auditor if they believe the auditor's strategy in

penalty Δ represents the expected decrease in firm value, given that the investors know the distribution of accounting issues (see 5 and 7 below), but cannot observe the accounting issues themselves.

As it turns out, whether the investors trust the auditor to be independent or not depends on tenure beliefs at that time and on whether the auditor has been retained beyond the conjectured tenure.¹⁰

4. If the auditor is replaced at this stage, he is replaced either for pricing reasons (that is bidding too high) or because of an anticipated increase in agency costs in case he is retained. In either case, the competing auditors do not revise their beliefs regarding tenure.
5. Nature randomly picks an accounting issue over which the manager and the auditor may or may not disagree. This move by Nature is unobservable by investors and by the other auditors and is represented by the realization of two random variables $(\tilde{\varepsilon}, \tilde{\delta})$ (see 7, 8 below). The investors and the competing auditors know that each period there is a positive probability of a disagreement between the incumbent and the manager.

In the first period of the engagement, the auditor incurs the learning cost and the firm incurs the switching cost before a disagreement can arise. Neither cost can be recovered if the auditor is replaced. Thus, at the next stage, when the manager and the auditor choose their actions, these cost are sunk.

equilibrium is to reject any compromise.

¹⁰The assumptions above amount to the investors partitioning their beliefs about the auditor's compromise/no compromise strategy into two groups: the auditor does not compromise with probability one (auditor believed to be independent) and the auditor compromises with positive probability (auditor believed not to be independent). Thus, the question of investors having rational beliefs about the auditor's strategy, and their response to those beliefs, is greatly simplified. For a more detailed structure and its implications to the model, see Appendix D.

6. The manager chooses between asking for compromise and threatening dismissal versus doing nothing.¹¹
7. The incumbent accepts or rejects the proposed compromise. The auditor cannot report the accounting issue over which there is disagreement.¹² If the auditor rejects the proposed compromise, there is no change in the expected litigation costs for the auditor. If the auditor agrees to compromise, he believes that there is an increase in litigation risk resulting in an expected litigation loss amount $\delta > 0$. Thus, the auditor's cost in a period in which he compromises on an accounting issue is increased by δ , the expected litigation cost given the particular accounting issue over which he compromised. We assume that δ is the realization of a random variable $\tilde{\delta}$ with support in $\{0\} \cup [\delta_{min}, \delta_{max}]$ and with a mass point at zero. The mass point at zero corresponds to the cases when no disagreement arises. An example of such a distribution would be a uniform distribution on an interval $[\delta_{min}, \delta_{max}]$ with a mass point at zero.

The auditor incurs all the learning costs in the first period, and these are denoted by L . Otherwise, the auditor's cost of producing the audit is the same in each period and denoted by A . Let γ_t denote a period t variable that is equal to one whenever the auditor compromises, conditional on an accounting issue disagreement being present, and equal to zero otherwise. Thus, γ_t describes the auditor's compromise/no compromise strategy. Then, the auditor's cost in each period is,

¹¹The manager does nothing if there is no disagreement. The manager cannot commit to dismissal if the auditor does not compromise.

¹²The idea here is that, if the auditor accepts the proposed compromise, he issues an unqualified opinion. On the other hand, if the auditor rejects the proposed compromise, he does not issue a qualified opinion; instead, he makes a take-it-or-leave-it offer to the manager consisting of two options: replace the auditor or retain the auditor and agree to modify the financial statements according to the auditor's judgement. This mechanism also replaces the possibility of the auditor resigning instead of issuing a qualified report in case the manager refuses to modify the financial statements as required by the auditor.

conditional on the auditor being retained until the end of the period,

$$A(t, \gamma_t) = \begin{cases} A + L + \gamma_1 \delta & \text{if } t = 1 \\ A + \gamma_t \delta & \text{if } t \geq 2 . \end{cases}$$

8. The manager either replaces the incumbent or retains him. The manager benefits if the auditor is retained after having accepted the proposed compromise because the manager's compensation is higher under the accounting policy over which there is disagreement with the auditor. Specifically, let $\tilde{\zeta}$ represent the expected litigation cost against the manager in case the auditor compromises, and let $\tilde{\omega}$ represent the decrease in firm value if the auditor does not compromise and the manager changes the accounting policy accordingly. Note that, for consistency, we measure the litigation cost $\tilde{\zeta}$ by the equivalent decrease in firm value that would have the same effect on the manager's compensation. We assume that, for each accounting issue, the decrease in firm value due to the auditor not compromising the issue is always larger than the expected litigation cost to the manager in case the auditor compromises.¹³ Thus, the manager's incremental cost between the compromise/no compromise cases is defined as $\tilde{\varepsilon} = \tilde{\omega} - \tilde{\zeta} > 0$. With this notation, $\Delta = E[\tilde{\omega}]$ represents the expected decline in firm value when the investors believe that the auditor has compromised, but cannot observe the accounting issue over which the auditor compromised.

In terms of the residual agency cost borne by the manager,

$$C(N, t, \hat{\gamma}_t, \gamma_t) = C + \hat{\gamma}_t \Delta + \gamma_t \tilde{\zeta} + (1 - \gamma_t) \tilde{\omega} ,$$

where $\hat{\gamma}_t$ represents the investors' beliefs (see also point 3 above and Appendix D).

Note that, when the auditor compromises, the manager bears the cost of litigation

¹³Note that the auditor does not report the issue. No compromise by the auditor always results in a change of accounting policy by the manager, and thus in a modified financial report.

$\tilde{\zeta}$. On the other hand, if the auditor does not compromise, the manager bears the cost of changing the accounting policy $\tilde{\omega}$. In both cases, the penalty Δ is imposed by the investors if they believe the auditor's independence has been compromised. Given that the investors do not observe the disagreement, the compromise, or the auditor's refusal to compromise, the investor's beliefs are in general independent of the auditors compromise/no compromise decision. The differential cost to the manager between the compromise/no compromise cases is $-\gamma_t \tilde{\varepsilon}$ and can be thought as the manager's benefit from an auditor compromising.

Like the auditor's expected litigation cost δ , ε is the realization of a random variable $\tilde{\varepsilon}$ with the property that it has support in $\{0\} \cup [\varepsilon_{min}, \varepsilon_{max}]$. The distribution has a mass point at zero corresponding to the cases where there is no disagreement. An example of such a distribution would be a uniform distribution on $[\varepsilon_{min}, \varepsilon_{max}]$ with a mass point at zero. The manager does not replace the incumbent if the cost of audit increases as a result. If the manager is indifferent between replacing and retaining the incumbent, and he threatened to replace the auditor, he will carry through the dismissal threats.

Note also that the two variables $(\tilde{\varepsilon}, \tilde{\delta})$ completely characterize the accounting issue over which the manager and the auditor may disagree through its impact on the payoffs of both players. In addition, we assume that $S < E[\tilde{\varepsilon}] < \varepsilon_{min}$ and that $\delta_{max} < E[\tilde{\delta}] + L$.

9. If the incumbent auditor is replaced at this point, the competing auditors revise their tenure beliefs in a way that is detailed below. If the incumbent is retained, the beliefs about tenure remain unchanged.
10. If the incumbent is retained, the audit service is performed, costs are incurred, and

fees are paid at the end of the period.

Note that the game between manager and auditor is a game of symmetric information with observed actions. The realizations of the random variables associated with the accounting issue are known both by the manager and by the auditor. The investors and the outside auditors do not observe (ε, δ) , but they do know the distribution of $\tilde{\delta}$ and $\tilde{\varepsilon}$. The key issue in the information asymmetry between investors on one hand, and the manager and the auditor on the other hand, is that there is a positive probability of an accounting issue arising in each period. The investors do not know whether a disagreement over an accounting issue has occurred or not. The distributions of accounting issues in different periods are mutually independent.

Tenure beliefs

The competing auditors' tenure beliefs determine their bidding strategy at the bidding stage in each period and whenever an incumbent auditor is replaced. The beliefs about tenure are specified as a function of the history of the game and are consistent on the equilibrium path. Tenure beliefs are central to the model, since they determine the cost of audit in the event an incumbent auditor is replaced, and thus determine whether the manager's threats of replacing the auditor are credible or not. The next task is to describe how tenure beliefs are revised when an auditor is replaced or retained in a way that is not consistent with existing tenure beliefs at the time. There are two possibilities: either the incumbent auditor is replaced before the expected last period of his engagement, or the incumbent auditor is retained beyond the expected last period of the engagement.

The manager and an incumbent auditor have beliefs about the auditor's tenure that in order to be rational must be consistent on the equilibrium path. In addition, the manager and the auditor have beliefs about play in the event of a deviation from the

equilibrium path. Since all auditors are identical, a replacement auditor is assumed to follow the same equilibrium strategies given the tenure beliefs at that point in the game. Thus, the credibility of dismissal threats depends on current tenure beliefs and tenure beliefs conditional on the (off-equilibrium) replacement.

When tenure beliefs are finite, $N < \infty$, the off-equilibrium moves are a replacement before N periods or retaining an incumbent beyond N periods. When an incumbent auditor is replaced before the expected tenure N , the new tenure beliefs should be $N' < N$. For simplicity, we assume that $N' = 1$, that is replacing an auditor before term makes multi-period engagements not credible, and replacement auditors are only willing to believe single-period tenure possible. When an auditor is replaced before the end of his tenure, there is a loss of quasi-rents, which means that the auditor could not recover his lowball. Potential replacement auditors recognize the possibility that they will be faced with a similar loss, and so they cease to lowball, which is equivalent to bidding based on tenure $N = 1$.

When an incumbent auditor is retained beyond the conjectured tenure N , we assume that replacement auditors believe finite tenure is not credible anymore, and their tenure beliefs change to $N = \infty$. The key is that there are no equilibria consistent with a finite tenure $N' > N$ after an incumbent is retained beyond the expected tenure. The intuition is that if auditors believe it is possible to be retained longer, then an incumbent would be able to deter entry in perpetuity at the end of their tenure (as long as that initial tenure belief is $N' < \infty$).

When tenure beliefs are $N = \infty$, the only off-equilibrium move is replacing the incumbent auditor. There are two possibilities: contingent on an auditor replacement, tenure beliefs become finite, $N' < \infty$, or tenure beliefs remain unchanged, $N = \infty$. In the first case, we say that finite tenure is credible, and we assume that $N' = 1$ as in the case of replacement before term with finite tenure. The second case corresponds to a situation

in which finite tenure beliefs are no longer possible as a result of an incumbent being retained beyond term. In this case we assume that any subsequent auditor replacement will leave tenure beliefs unchanged at $N = \infty$ (since the alternative is finite tenure, which we assume no longer credible). This assumption is implicitly present in the models of DeAngelo [5] and Zhang [38], where tenure is assumed infinite, and that a replacement auditor also bids based on infinite tenure.

To summarize, when we say that tenure beliefs are $1 < N < \infty$, that means both that replacement is believed to take place at the end of N periods (finite tenure credible) and that replacement does not take place before the N periods (multi-period tenure credible). Beliefs of $N = 1$ require credible finite tenure, and replacement before the end of the period does not affect beliefs. Beliefs of $N = \infty$ can occur, however, both when finite tenure is credible and when it is not. In the former case, replacement of the incumbent changes tenure beliefs to $N = 1$, while in the latter case, tenure beliefs remain unchanged.

The above discussion of tenure beliefs is formalized by the following definitions and assumptions. Tenure beliefs are a pair $(N, A/B)$, where N is the number of periods an auditor is conjectured to be retained by the firm, and A/B are belief types as defined below. Tenure beliefs are said to be of type A if tenure is believed to be infinite and this belief is not affected by a replacement of the incumbent. Type A beliefs are also referred to as “finite tenure not credible”. Tenure beliefs are said to be of type B if tenure is believed to be finite, or if tenure is believed to be infinite but this belief is affected by a replacement of the incumbent. Type B tenure beliefs are also referred to as “finite tenure credible”.

Tenure beliefs (∞, A) remain unchanged whenever an incumbent is replaced. Tenure beliefs (N, B) change to $(1, B)$ whenever an incumbent is replaced before the end of N periods, and this holds for both finite and infinite N . Tenure beliefs (N, B) with finite N change to (∞, A) whenever an incumbent is retained beyond the end of period N . Tenure

beliefs (N, B) remain unchanged as long as retain/replace decisions are consistent with N (auditor never replaced if $N = \infty$, or auditor replaced every N periods otherwise).

Based on the assumptions we made regarding tenure beliefs and their revision, all possible histories in the game can be described as one of three types. A history here simply means all publicly available information at any given stage in the game. Thus a history consists of audit fees, and retain/replace decisions by the manager at all stages preceding the current one. Tenure beliefs can be inferred from past history and the above assumptions regarding their revision.

Type I. At some point in the history of the game, an incumbent auditor has been retained beyond N periods when beliefs were (N, B) with finite N . As a result, beliefs are (∞, A) , and either a new auditor is being hired, or there is an incumbent auditor such that beliefs were (∞, A) at the time he was hired.

Type II. There is an incumbent auditor that was hired when beliefs were (N, B) with finite N , and who was retained beyond period N . As a result tenure beliefs are (∞, A) .

Type III. No auditor hired under tenure beliefs (N, B) has been retained beyond term. As a result, tenure beliefs are (N, B) .

We assume that, at the start of the game, beliefs can only be of Type B. It follows that the game history starts out as Type III and can either remain Type III, or it can change to Type II and remain Type II, or it can go through Type II to Type I. Once the game history becomes Type I, it remains as such for ever. On the equilibrium path, only Type III histories can occur, Type II and Type I are results of off-equilibrium behavior. A strategy profile specifies actions at each possible history. We shall define separately strategy profiles for each history type. If the game were to be started at either type of history, we show that the given strategies are subgame perfect.

An equilibrium consists of a set of subgame-perfect strategies that are consistent with

tenure beliefs at each stage of the game. The manager and an incumbent auditor believe that a replacement auditor will play the same equilibrium strategy (given tenure beliefs in that event).¹⁴

Since off-equilibrium path moves can only change the history types from III→II→I, and these changes are irreversible, we characterize equilibrium strategies starting with Type I histories and then moving backwards to Types II and III. The following proposition characterizes equilibrium strategies when the game has reached a Type I history.

Proposition 2.4.1 *Assume that the players are at a Type I history in the game. Then the following strategy profile is subgame perfect.*

- *auditor bids*

$$F = \begin{cases} F_I - S & \text{in the first period} \\ F_I & \text{in subsequent periods} \end{cases}$$

$$F_I = A + E[\tilde{\delta}] + (1 - \beta)(L + S)$$

- *auditor compromises on all accounting issues that arise*
- *manager retains the auditor in the event of a disagreement if, and only if, the auditor compromises*
- *the total cost of audit to the manager is*

$$K_I = \frac{\beta}{1 - \beta}(A + E[\tilde{\delta}] + \Delta + E[\tilde{\zeta}]) + \beta(L + S) .$$

Proof. See Appendix C. \square

Note that the auditor's cost in each period is

$$A(t, \tilde{\delta} | \gamma_t = 1) = \begin{cases} A + \tilde{\delta} + L & \text{if } t = 1 \\ A + \tilde{\delta} & \text{if } t \geq 2 \end{cases}$$

¹⁴Some of the off-equilibrium actions are specified for a replacement auditor, and it is important that the manager and the incumbent auditor have common beliefs about the strategy of a new auditor in the event of auditor turnover.

because the auditor's strategy is to compromise on all accounting issues that the manager asks him to. Since the pricing stage precedes the disagreement stage, the realization of $\tilde{\delta}$ for the current period is not known to the auditor at the bidding stage. The fact that the auditor is risk neutral and $\tilde{\delta}$ is independent from period to period implies that the auditor perceives his costs when setting fees as

$$A(t|\gamma_t = 1) = \begin{cases} A + E[\tilde{\delta}] + L & \text{if } t = 1 \\ A + E[\tilde{\delta}] & \text{if } t \geq 2 . \end{cases}$$

The principal's residual agency cost is $C(N, t, \varepsilon|\gamma_t = 1) = C + \Delta + \zeta$ in each period, since the auditor compromises on each accounting issue that arises, and an accounting issue arises with positive probability in each period, resulting in the investors imposing the agency penalty Δ and the manager bearing the litigation cost ζ . This cost to the manager is from an ex-ante perspective $C(N, t|\gamma_t = 1) = C + \Delta + E[\tilde{\zeta}]$ (the ζ 's are also independent through time). Without loss of generality, we have normalized the residual agency cost such that $C = 0$, so $C(N, t) = \Delta + E[\tilde{\zeta}]$. Given $A(t)$ and $C(N, t)$ as described above, and tenure beliefs (∞, A) , the audit fee structure follows from Proposition 2.3.1.

The strategies presented in Proposition 2.4.1 are a subgame perfect equilibrium if the game were to reach a Type I history and will be referred to as Type I equilibrium strategies. We do not claim this to be a unique equilibrium in this case. However, there is no pure strategy equilibrium in which the auditor maintains his independence in each period. As a result, it is likely that if other equilibria exist, there will be a positive probability of the auditor compromising his independence in each period.¹⁵ In that case, the investors will impose the penalty Δ , and the expected litigation cost $E[\tilde{\zeta}]$

¹⁵The penalty Δ is imposed in every period. This would not occur if the equilibrium is such that the auditor never compromises, but such an equilibrium does not exist. Suppose, by contradiction, that it does exist. Then, the cost of a replacement auditor that does not compromise is $K_I = \beta A(1 - \beta) + \beta(L + S)$. In the first period, the dismissal threat is not credible since the switching cost is sunk when dismissal threats are made, and this increases the manager's end of period payoff by S if the incumbent is retained (in fact the incumbent's fee is $F - S$, while a replacement bids F to which the switching

will make such an equilibrium a very costly one. Our result is consistent with Zhang's modelling [38] of the fact that quasi-rents impair auditor independence under Type A tenure beliefs. The key in our model is that dismissal threats become credible once the auditor has lowballed and the competing auditors do not revise their tenure beliefs when they see an incumbent replaced (before term).

We now turn to a Type II history, that is we assume that there is an incumbent auditor who has been retained longer than expected (and was engaged under tenure beliefs (N, B) , and tenure beliefs are now (∞, A) . The following proposition characterizes subgame perfect strategies in this case.

Proposition 2.4.2 *Assume that the players are at a Type II history in the game and that the incumbent auditor's fee bid in the first period in which he was retained beyond his expected tenure is F' .¹⁶ Then the following strategy profile is subgame perfect.*

- *auditor bids $F = F_{II}$ in each period other than the first in which he is retained beyond his expected tenure where*

$$F_{II} = F_I + E[\tilde{\zeta}]$$

- *auditor makes no compromise on any accounting issues that arise*
- *manager retains the auditor in each period whether the auditor compromises or not on accounting issues*

cost must be added). However, once the switching cost is not subtracted from the incumbent's fee, the manager becomes indifferent between retaining the incumbent and replacing him after making the dismissal threats, in which case we have assumed that threats are credible. It follows that the penalty Δ is imposed in each period, since investors rationally believe that there is a positive probability of compromise in each period.

¹⁶There is no unique F' determined in equilibrium. Moreover, a Type II history is only reached after an incumbent submitted the bid F' and has been retained beyond the conjectured last period. Thus, for a Type II history, the fee F' is given as part of the information that characterizes the history of the game. On the equilibrium path, the auditor is never retained beyond his expected tenure. The strategies described in Proposition 2.4.2 characterize the off-equilibrium play for any possible bid F' off the equilibrium path.

The cost of audit to the manager is $K_{II} = K_I - 0$ in all periods other than the first period in which the incumbent is retained beyond his expected tenure, in which case the cost of audit is $\beta(F' + K_{II})$.

Proof. See Appendix C. \square

Note that the auditor's cost is $A(t) = A$ in each period and that the residual agency cost is $C(N, t) = C + \Delta$, since the auditor is an incumbent and does not compromise on any accounting issue. The pricing is derived along the lines of Proposition 2.3.1, although the proposition itself is not applicable here for determining audit fees due to the fact that perfect competition does not reduce the NPV of the engagement to zero for the incumbent in the first period.

If the manager has deviated from the equilibrium path by retaining the incumbent longer than expected, he will then optimally retain that incumbent in perpetuity. The incumbent in turn maintains his independence, but can extract very high rents because the replacement auditors would be playing a Type I equilibrium where they always compromise, the agency penalty Δ is always imposed, and the manager bears the litigation cost $E[\tilde{\zeta}]$. As a result, the equilibria that are played at Type I and Type II histories are both highly undesirable to the manager, since in both cases the total audit cost incorporates $E[\tilde{\zeta}]$, either as an expected litigation cost, or through the auditor's fees. In both cases the cost is as if the auditor's independence was impaired.

The following proposition characterizes the equilibrium strategies at game histories of Type III given tenure beliefs (N, B) .

Proposition 2.4.3 *Assume that the players are at a Type III history in the game, and that tenure beliefs are N . Then the following strategy profile is subgame perfect.*

- *auditor bids*

$$F = \begin{cases} F_{III} - S & \text{in the first period} \\ F_{III} & \text{in subsequent periods} \end{cases}$$

$$F_{III} = A + \beta Q(N)(L + S)$$

- *auditor makes no compromise on any accounting issues that arise*
- *manager retains the auditor in each period whether the auditor compromises or not on accounting issues*

The cost of audit to the manager is

$$K_{III} = K_{\infty}(N) = \frac{\beta}{1 - \beta} F_{III} = \frac{\beta}{1 - \beta} (A + \Delta + \beta Q(N)(L + S)) .$$

Proof. See Appendix C. \square

Note that the auditor's cost is $A + L$ in the first period of the engagement, and A thereafter, while the residual agency cost is constant $C(N, t) = C + \Delta$ in all periods. The equilibrium strategies presented in the proposition allow for infinite tenure if beliefs are (∞, B) . Since $Q(N)$ is increasing in N , the total audit cost K_{III} is decreasing in N , and the lowest value is attained when tenure beliefs are (∞, B) . The highest auditing costs are attained for tenure beliefs $(1, B)$.¹⁷

The three propositions together present a complete strategy profile that is subgame perfect in the game starting at any history. We are interested in the game starting at a Type III history with finite tenure beliefs and finite tenure credible. Then, the auditor is not replaced before term because that would increase the audit cost (tenure beliefs $(1, B)$ produce the costliest audit under (N, B) beliefs). The auditor is not retained beyond term because the game would switch to a Type II equilibrium and that has a higher cost

¹⁷This shows, in particular, that the tenure beliefs revision rule can be modified without altering the results significantly as follows: if an auditor hired under tenure beliefs (N, B) is replaced after $N' < N$ periods, the new tenure beliefs are N' . Such a rule would be consistent in equilibrium, since it would keep the manager's threats of dismissal from being credible, thus preserving auditor independence.

to the manager than the highest cost for a Type III equilibrium (beliefs $(1, B)$). As a result, on the equilibrium path, if an auditor is engaged under tenure beliefs (N, B) with finite N , the firm will maintain a stationary cycle of replacing auditors every N periods. The auditors maintain their independence and lowballing depends on the conjectured tenure N .

2.5 Empirical implications and conclusion

In this paper we develop a model of auditor change driven by agency costs and rational expectations of auditor tenure. When a new auditor is engaged, investors, management, and the auditors reach a rational expectations equilibrium belief of auditor tenure. Perfect competition among identical auditors ensures that the auditors earn only normal returns on the audit engagements (zero NPV over the entire expected duration of the audit engagement). Lowballing and a multi-period commitment are obtained while the auditor is hired one period at a time. The amount of lowballing is strictly increasing in the conjectured length of auditor tenure.

The existence of the lowballing and the revision of beliefs about tenure by competing auditors ensure that management cannot threaten the auditor with replacement during the expected duration of the audit engagement. Thus, lowballing works to protect the auditor's independence through its impact on off the equilibrium path beliefs about tenure held by competing auditors. We can say that the other auditors punish the manager with higher fees if the lowball of the incumbent is not recovered. The auditor is replaced at the end of the conjectured number of periods (tenure) because of increased agency costs and litigation costs to the manager. Retaining an incumbent auditor beyond the expected tenure would allow the auditor to earn economic rents from the engagement. While these rents do not result in a compromise of the auditor's independence, the total audit cost

to the firm (audit fee) would incorporate the litigation costs that would be faced by the manager if the auditor's independence were compromised. This is true because, once retained past the expected tenure, the auditor could only be replaced by one who would systematically compromise his independence.

In deriving the pricing and auditor change model, we have assumed that tenure beliefs are exogenously given and common knowledge among the players. Maintaining the assumption that expected auditor tenure is an exogenous characteristic of the manager, it is sufficient to assume that the auditors' costs and fees are common knowledge. Indeed, once an auditor is hired, in the first period of his engagement, both competing auditors, and the investors can infer tenure beliefs from the observed amount of lowballing. In some sense, the manager can use lowballing to communicate his beliefs about tenure to the other players. Then, given a distribution of managers with different tenure beliefs, each manager can play an equilibrium based on his conjectured N . Knowing that, auditors bidding for the engagement are more likely to be equally dispersed in their bids.

If lowballing cannot be inferred because fees are not disclosed, it is more likely that tenure beliefs N are formed by competing auditors and by the investors independently of the manager (for example, conjectured tenure could be the mean observed auditor tenure for the industry). Then, if a manager tries to play an equilibrium based on a personal conjecture of tenure $N' \neq N$, he will at some point make an off-equilibrium move (from the point of view of the outsiders) which results in an increase of auditing costs. Therefore, given a distribution of managers with different tenure beliefs, and an outsiders' conjectured N , managers are more likely to try to be closer to N . Auditors that bid for the engagement would also base their bids on N . Empirically, we may expect to see more variance around the mean tenure in a cross-section of audit engagements when audit fees are disclosed than when they are not disclosed, since with fee disclosure, managers have more discretion over N .

Another interesting question related to the formation of the initial tenure beliefs is that of the existence of ex-ante optimal tenure. Our auditor change model assumes a flat residual agency cost, independent of the conjectured tenure N and, as a result, the total audit cost is decreasing in N . That would imply optimal auditor tenure $N = \infty$. The empirical evidence of Johnson et al.[19] is consistent with the idea that very short and very long auditor tenure is suboptimal, leaving auditor tenure optimally undetermined in an interval of four to nine years in their paper. Their findings are consistent also with an U-shaped residual agency cost $C(N)$ that depends only on N and is constant for each period of the auditor's tenure. Presumably, such an agency cost structure would result from a combination of increasing audit quality in the first periods and a significant decrease of audit quality (or perceived auditor independence) as the auditor's tenure increases. Assuming that $C(N)$ increases sufficiently as N increases, it is possible to obtain an ex-ante optimal tenure that is neither 1 nor ∞ . However, in our model, once an equilibrium conjecture is agreed upon, the manager and the auditors are locked into it in all subsequent periods. Thus, we have no ex-ante optimal tenure, but common tenure beliefs, once reached, are ex-post optimal.

More generally, in our model, all auditor changes are anticipated since they are based on the conjectured N and convey no information to the market. The low explanatory power of empirical models of auditor change that relate auditor turnover to exogenous events leaves room for alternate theories of auditor turnover. Our model offers the alternative that some auditor changes are endogenous, simply based on an expectation of auditor tenure, and are unrelated to exogenous events.

Anticipated exogenous events that are usually associated with auditor changes are outside the scope of our model. For example, if a company anticipates a major business transaction at a given point in time that requires a new auditor, any auditor engaged prior to that event will expect to be replaced exactly at that time. Such tenure expectations

are consistent with the finite horizon model of Magee and Tseng [26] but not with our model in which tenure is an endogenous horizon, rather than an exogenous event date. As a result, the auditor change is no news to the market when triggered by an anticipated event.

To conclude, the main empirical predictions of our model are that:

1. Lowballing is an increasing function of auditor tenure; this prediction provides a potential test between our endogenous horizon hypothesis and the exogenous event date hypothesis of Magee and Tseng. Recall that the model of Magee and Tseng predicts lowballing to be independent of tenure.
2. The variance of a cross-section of auditor tenure is higher in a market with fee disclosure than in a market without fee disclosure.

The endogenous auditor changes described in our model might explain some of the numerous changes which empirical research reveals are unrelated to exogenous events. Furthermore, these endogenous changes might explain why, empirically, auditor changes generally convey little news to the market.

Appendix A

The entry-preventing condition (2.7) determines $D(N, t)$ as follows. For $t = N$,

$$\beta D(N, N) + \beta K_{\infty}(N) = K_{\infty}(N) \Rightarrow$$

$$\beta D(N, N) = K_{\infty}(N) (1 - \beta) \Rightarrow$$

$$D(N, N) = \frac{1 - \beta}{\beta} K_{\infty}(N) = r K_{\infty}(N).$$

For $t = N - 1$,

$$\begin{aligned}
 \beta D(N, N - 1) + \beta^2 D(N, N) + \beta^2 K_\infty(N) &= K_\infty(N) \Rightarrow \\
 \beta D(N, N - 1) &= \left(1 - \frac{1 - \beta}{\beta} \beta^2 - \beta^2\right) K_\infty(N) \\
 &= (1 - \beta + \beta^2 - \beta^2) K_\infty(N) = (1 - \beta) K_\infty(N) \Rightarrow \\
 D(N, N - 1) &= \frac{1 - \beta}{\beta} K_\infty(N) = r K_\infty(N).
 \end{aligned}$$

For $t = N - 2$,

$$\begin{aligned}
 \beta D(N, N - 2) + \beta^2 D(N, N - 1) + \beta^3 D(N, N) + \beta^3 K_\infty(N) &= K_\infty(N) \Rightarrow \\
 \beta D(N, N - 2) &= \left(1 - \frac{1 - \beta}{\beta} \beta^2 - \frac{1 - \beta}{\beta} \beta^3 - \beta^3\right) K_\infty(N) \\
 &= (1 - \beta + \beta^2 - \beta^2 + \beta^3 - \beta^3) K_\infty(N) \\
 &= (1 - \beta) K_\infty(N) \Rightarrow \\
 D(N, N - 2) &= \frac{1 - \beta}{\beta} K_\infty(N) = r K_\infty(N).
 \end{aligned}$$

Similar calculations for $t = N - 3, \dots, 2$ give

$$D(N, t) = K_\infty(N)r \text{ for } 2 \leq t \leq N.$$

Let $D(N)$ denote this common value,

$$D(N) := K_\infty(N)r. \quad (2.23)$$

From (2.4) it follows that

$$\begin{aligned}
 K(N) &= \beta D(N, 1) + D(N) \left(\sum_{t=2}^N \beta^t \right) \\
 &= \beta D(N, 1) + \frac{1 - \beta}{\beta} \beta^2 \frac{1 - \beta^{N-1}}{1 - \beta} K_\infty(N) \\
 &= \beta D(N, 1) + (\beta - \beta^N) K_\infty(N). \quad (2.24)
 \end{aligned}$$

Using (2.5) into (2.24) yields

$$(1 - \beta^N)K_\infty(N) = \beta D(N, 1) + (\beta - \beta^N)K_\infty(N). \quad (2.25)$$

Equation (2.25) can be now solved for $D(N, 1)$,

$$D(N, 1) = \frac{1 - \beta}{\beta} K_\infty(N) = r K_\infty(N) = D(N).$$

Appendix B

Given two continuously differentiable functions f, g , defined on $[0, \infty)$ let

$$T_g f(x) := \frac{\int_0^x f(t)g(t) dt}{\int_0^x g(t) dt}. \quad (2.26)$$

Given any discrete stream of cash flows $c(t)$, $t \geq 1$, there exist f, g as above such that $f(t) = c(t)$, $g(t) = \beta^t$ for all integers $t \geq 1$, and

$$T_g f(N) = \left(\sum_{t=1}^N \beta^t \right)^{-1} \left(\sum_{t=1}^N \beta^t c(t) \right). \quad (2.27)$$

The operator T_g corresponds to what is known in capital budgeting as *annual equivalent value* of a stream of cash flows discounted over a finite period of time. T_g is a linear positive operator if the function g is positive. In addition, if f is decreasing over the interval $[0, x_0]$, the function $T_g f$ is also decreasing over the same interval. Conditions on f that guarantee the convexity of $T_g f$ are trickier.

$$\begin{aligned} (T_g f)'(x) &= \frac{f(x)g(x) \int_0^x g(t) dt - g(x) \int_0^x f(t)g(t) dt}{\left(\int_0^x g(t) dt \right)^2} \\ &= \frac{g(x) \left(\int_0^x [f(x) - f(t)]g(t) dt \right)}{\left(\int_0^x g(t) dt \right)^2} \end{aligned} \quad (2.28)$$

Thus, given a positive function $g(x)$, $T_g f$ is an increasing function of x for all x such that

$$\int_0^x [f(x) - f(t)]g(t) dt > 0$$

and decreasing for all x such that

$$\int_0^x [f(x) - f(t)]g(t) dt < 0$$

Appendix C

The proofs for the three propositions are based on the one-stage-deviation principle. A strategy profile is subgame perfect if neither player has an incentive to deviate at the current stage and then return to playing the strategy (Fudenberg and Tirole [14], Theorem 4.1, Theorem 4.2).

Proof of Proposition 2.4.1

The proof shows that for the given strategies, no player has an incentive to deviate at the current stage and then return to playing his strategy. The table below indicates the continuation payoffs to the manager and the auditor in the first period of an audit engagement when at the start of the engagement tenure beliefs are (∞, A) , conditional on disagreement over an accounting issue (ε, δ) .¹⁸ The fee structure is given by Proposition 2.3.1 with

$$A(t) = \begin{cases} A + E[\tilde{\delta}] + L & \text{if } t = 1 \\ A + E[\tilde{\delta}] & \text{if } t \geq 2 \end{cases}$$

and $C(N, t) = \Delta + E[\tilde{\zeta}]$.

¹⁸Recall that in the absence of a disagreement, i.e. $\varepsilon = \delta = 0$, the manager cannot replace the auditor after the pricing stage.

Type I, first period		Retain	Replace
Compromise	Manager	$\beta(\varepsilon - E[\tilde{\varepsilon}] + S) - K_I$	$-K_I$
	Auditor	$\beta(E[\tilde{\delta}] - \delta + L)$	0
No Compromise	Manager	$-\beta E[\tilde{\varepsilon}] + \beta S - K_I$	$-K_I$
	Auditor	$\beta E[\tilde{\delta}] + \beta L$	0

Table 2.1: Type I history, first-period payoffs

The continuation payoffs in the table are derived as follows. If the auditor is replaced, his continuation payoff is zero, given the perfect competition and zero NPV assumption on a new engagement. If the manager replaces the auditor, his cost is the total cost of all future engagements with an auditor that compromises in every period, that is residual agency cost $C(\infty, t) = E[\tilde{\zeta}] + \Delta$. This cost is given by K_I below based on Proposition 2.3.1.

If the auditor is retained, his continuation payoff in case he compromised consists of the present value of all future audit fees less audit costs. The audit fees are $A + E[\tilde{\delta}] + (1 - \beta)L - \beta S$ in the first period, and $A + E[\tilde{\delta}] + (1 - \beta)(L + S)$ in subsequent periods, while the costs are $A + \delta$ in the first period (since at this time the issue is known and the learning cost is sunk), and $A + E[\tilde{\delta}]$ in subsequent periods. If the auditor is retained and did not compromise, his payoff is greater by exactly $\beta\delta$, the avoided increase in expected litigation costs.

Similarly, for the manager, if the auditor is retained, the same argument works with S instead of L and ε instead of δ . The payoffs to the manager are the present value of all future audit costs. These are $\beta(\Delta + \zeta) + A + E[\tilde{\delta}] + (1 - \beta)L - \beta S$ in the first period, and $\beta(\Delta + E[\tilde{\zeta}] + A + E[\tilde{\delta}] + (1 - \beta)(L + S))$ in subsequent periods if the auditor compromises,

and lower by $\beta\varepsilon$ if the auditor does not compromise. In all these cases, compromise/no compromise refers only to the current period, in all subsequent periods the auditor is assumed to return to the equilibrium strategy, which is to compromise in each period.

Intuitively, the key to this payoffs is in the differences between the retain/replace columns. On the first line, by retaining the incumbent, the manager gains by avoiding an additional switching cost, the benefit from the current auditor compromising on a specific issue less the expected benefit of the auditor compromising which works like an opportunity cost. On the second line, by being retained, the auditor gains from the fee compensating for the learning cost (which is sunk and not counted at this stage), from passing the expected litigation cost to the manager, less the litigation cost associated to the particular accounting issue.

Assuming that $S < E[\tilde{\varepsilon}]$, the threat of replacement when the auditor does not compromise is credible.¹⁹ As a result, if $E[\tilde{\delta}] - \delta + L > 0$ for all δ , that is $\delta_{max} < E[\tilde{\delta}] + L$, the auditor always prefers to compromise. The manager retains the auditor that compromises since we assumed $\varepsilon_{min} > E[\tilde{\varepsilon}]$. The term K_I represents the cost of audit to the manager on the equilibrium path, and is given by

$$K_I = \frac{\beta}{1-\beta}(A + E[\tilde{\delta}] + E[\tilde{\zeta}] + \Delta) + \beta(L + S) .$$

Next we examine the continuation payoffs to the manager and the auditor in subsequent periods of an audit engagement when at the start of the engagement tenure beliefs are (∞, A) , conditional on disagreement over an accounting issue (ε, δ) . These are derived in a similar manner to those for the first period (see the discussion above).

The manager's replacement threat is still credible if the auditor does not compromise, the manager always prefers to retain the auditor if the auditor compromises, and the

¹⁹This assumption is not critical, since if $S < E[\tilde{\varepsilon}]$, the auditor's strategy will be no compromise if the disagreement arises in the first period of the engagement, and compromise on all disagreements in subsequent periods.

Type I, subsequent periods		Retain	Replace
Compromise	Manager	$\beta(\varepsilon - E[\tilde{\varepsilon}]) - K_I$	$-K_I$
	Auditor	$\beta(E[\tilde{\delta}] - \delta) + \beta(L + S)$	0
No Compromise	Manager	$-\beta E[\tilde{\varepsilon}] - K_I$	$-K_I$
	Auditor	$\beta E[\tilde{\delta}] + \beta(L + S)$	0

Table 2.2: Type I history, subsequent periods payoffs

auditor prefers to compromise and be retained instead of losing the engagement since $\delta_{max} < E[\tilde{\delta}] + L + S$. \square

Proof of Proposition 2.4.2

The proof shows that for the given strategies, no player has an incentive to deviate at the current stage and then return to playing his strategy. The table below indicates the continuation payoffs to the manager and the auditor in the first period in which the incumbent has been retained longer than expected, given that the incumbent's fee bid was F' . The tenure beliefs are (∞, A) as a result of the incumbent being retained longer than expected. The payoffs are derived as in the proof of Proposition 2.4.1.

Type II, first period		Retain	Replace
Compromise	Manager	$\beta\varepsilon - \beta(F' + K_{II})$	$-K_I$
	Auditor	$-\beta\delta + \beta(F' + K_{II}) - \beta A/(1 - \beta)$	0
No Compromise	Manager	$-\beta(F' + K_{II})$	$-K_I$
	Auditor	$\beta(F' + K_{II}) - \beta A/(1 - \beta)$	0

Table 2.3: Type II history, first period payoffs

The cost K_{II} represents the total audit cost on the equilibrium path in subsequent periods. If $\beta(F' + K_{II}) < K_I$, the manager's replacement threat is not credible in case

the auditor does not compromise. We note that there is a whole range of F' such that the auditor earns rents and this upper bound is satisfied. Otherwise, the fee F' is not uniquely determined. The payoffs are derived as in the proof of Proposition 2.4.1.

Type II, subsequent periods		Retain	Replace
Compromise ²⁰	Manager	$\beta\varepsilon - K_{II}$	$-K_I$
	Auditor	$-\beta\delta + K_{II} - \beta A/(1 - \beta)$	0
No Compromise	Manager	$-K_{II}$	$-K_I$
	Auditor	$K_{II} - \beta A/(1 - \beta)$	0

Table 2.4: Type II history, subsequent periods payoffs

The same discussion as in the first period applies here, the manager's threats are not credible as long as $K_{II} < K_I$. Thus, in equilibrium, the incumbent auditor can set audit fees such that the total audit cost is just under what it would be after a replacement. A replacement at this stage would move the game to a type I equilibrium, for which the cost is K_I . We write then $K_{II} = K_I - 0$. The incumbent tries to get the highest fee and earns rents for a range of audit fees. In addition, the total audit cost on the equilibrium path is either $\beta(F' + K_{II})$ or K_{II} , both of which are strictly less than the total audit cost on the Type I equilibrium path. If the manager has deviated from the equilibrium path by retaining the incumbent longer than expected (under a Type III equilibrium), he will then optimally retain that incumbent in perpetuity. The incumbent in turn maintains his independence, but can extract very high rents because the replacement auditors would be playing a Type I equilibrium where they always compromise, the agency penalty Δ is always imposed, and the manager faces the litigation cost $E[\tilde{\zeta}]$. \square

Proof of Proposition 2.4.3

The proof shows that for the given strategies, no player has an incentive to deviate at the current stage and then return to playing his strategy. The table below indicates the continuation payoffs to the manager and the auditor in each period under the assumption that tenure beliefs are $(1, B)$. The payoffs are derived as in the proof of Proposition 2.4.1.

Type III, $N = 1$		Retain	Replace
Compromise	Manager	$\beta(\varepsilon + S) - K_\infty(1)$	$-K_\infty(1)$
	Auditor	$-\beta\delta + \beta L$	0
No Compromise	Manager	$\beta S - K_\infty(1)$	$-K_\infty(1)$
	Auditor	βL	0

Table 2.5: Type III history, $N = 1$ payoffs

The cost $K_\infty(1)$ represents the total cost of replacing the auditor each period. The manager strictly prefers to retain the auditor regardless of whether the auditor compromises or not due to the presence of switching costs, and given that dismissal threats are not credible the auditor will never compromise. In case the manager retains an incumbent for a second period or longer, a Type II equilibrium is played with a total audit cost of at least βK_{II} (corresponding to a fee $F' = 0$). We assume $E[\tilde{\zeta}]$ is large enough so that $\beta K_{II} > K_\infty(1)$. In other words, the costs of retaining the auditor beyond the expected term are higher than the costs of replacing the auditor every period.

The table below indicates the payoffs to the manager and the auditor in the first period under the assumption that the tenure expectation is $N > 1$ and tenure beliefs are of type B. The payoffs are derived as in the proof of Proposition 2.4.1.

Type III, $N > 1$, first period		Retain	Replace
Compromise	Manager	$\beta(\varepsilon + S) - K_\infty(N)$	$-K_\infty(1)$
	Auditor	$-\beta\delta + \beta L$	0
No Compromise	Manager	$-K_\infty(N) + \beta S$	$-K_\infty(1)$
	Auditor	βL	0

Table 2.6: Type III history, $N > 1$, first period payoffs

The cost $K_\infty(1)$ represents the total cost of replacing the auditor every N periods. The manager strictly prefers to retain the auditor regardless of whether the auditor compromises or not, and given that dismissal threats are not credible the auditor will never compromise. If the manager replaces the incumbent at the disagreement stage, tenure beliefs change to $N = 1$. As a result, the manager's cost of replacement is $-\beta S - K_\infty(1)$. In subsequent periods, given tenure beliefs $N > 1$, the payoffs are given by the table below. The payoffs are derived as in the proof of Proposition 2.4.1.

Type III, $N > 1$, subsequent periods		Retain	Replace
Compromise	Manager	$\beta\varepsilon - K_\infty(N)$	$-K_\infty(1)$
	Auditor	$-\beta\delta + V_t$	0
No Compromise	Manager	$-K_\infty(N)$	$-K_\infty(1)$
	Auditor	V_t	0

Table 2.7: Type III history, $N > 1$, subsequent periods payoffs

The auditor's payoff in case he is retained depends on how many periods are left in his engagement and is denoted by V_t . The value of incumbency V_t represents the present

value of future expected quasi-rents and is determined by the fee structure (2.16), (2.19). As before, the manager cannot make credible threats, and the auditor never compromises. We note that the same arguments are valid for $N = \infty$, that is infinite tenure could be an equilibrium as long as finite tenure is credible and dismissing the auditor leads to a revision in beliefs about tenure. \square

Appendix D

In equilibrium, the investors have rational beliefs about the auditor's strategy on compromising. Let $\Gamma = (\gamma_t(\varepsilon, \delta))_t$ denote the auditor's strategy, where $\gamma_t = 1$ if the auditor compromises and $\gamma_t = 0$ otherwise. The auditor's strategy is a function of the game history and of the particular accounting issue over which there is disagreement. Let $\hat{\Gamma}$ denote the investors' beliefs about Γ . Then, in equilibrium $\hat{\Gamma} = \Gamma$.

For an accounting issue (ε, δ) , let V_1 denote the firm value given financial statements and the investors' belief that the auditor has not compromised and let V_2 denote the firm value given the same financial statements, the investors' belief that the auditor has compromised and assuming the investors could observe the accounting issue. Thus, if the investors know what the accounting issue is and they believe the auditor has compromised, the same financial statement information leads to different beliefs about the firm's future cash flows, and thus V_2 is in general less than V_1 (we assume that, whatever the accounting issue is, the manager tries to "fool" the investors into believing the firm value is higher than the auditor is willing to attest to). Now let $\omega = V_1 - V_2$ denote the difference in firm value under the two scenarios. Note that ω is associated with the accounting issue the same way as ε and δ , and therefore is the realization of a random variable $\tilde{\omega}$. If there is no accounting issue, $\omega = 0$.

Since investors cannot observe either the accounting issue or whether it arises or not,

their beliefs are summarized by knowledge of the distribution of $\tilde{\omega}$. Combined with the investors' beliefs about the auditor's strategy $\hat{\Gamma}$, the residual agency cost borne by the manager is

$$\begin{aligned} C(N, t) &= C + E[\hat{\Gamma}\tilde{\omega} + \hat{\Gamma}\tilde{\zeta} + (1 - \hat{\Gamma})\tilde{\omega}] \\ &= C + E[\hat{\gamma}_t(\tilde{\varepsilon}, \tilde{\delta})\tilde{\omega} + \hat{\gamma}_t(\tilde{\varepsilon}, \tilde{\delta})\tilde{\zeta} + (1 - \hat{\gamma}_t(\tilde{\varepsilon}, \tilde{\delta}))\tilde{\omega}]. \end{aligned} \quad (2.29)$$

In particular, if $\hat{\Gamma} = \text{"always compromise"}$, that is $\hat{\gamma}_t = 1$ at all histories, then $\Delta = E[\hat{\gamma}_t(\tilde{\varepsilon}, \tilde{\delta})\tilde{\omega}] = E[\tilde{\omega}]$, and it follows that $C(N, t) = C + \Delta + E[\tilde{\zeta}]$. If, on the other hand, $\hat{\Gamma} = \text{"never compromise"}$, that is $\hat{\gamma}_t = 0$ at all histories, then $C(N, t) = C + \Delta$.

The assumptions made in point 3) in the description of the game correspond to a restriction of the auditor's strategies to strategies that are independent of ε and δ . In that case, given the other assumptions, the auditor is never indifferent between compromising and not compromising, so mixed strategies are ruled out. It follows that the auditor is restricted to pure strategies independent of the accounting issue and that, in this case,

$$\begin{aligned} C(N, t) &= C + \hat{\gamma}_t(E[\tilde{\omega}] + E[\tilde{\zeta}]) + (1 - \hat{\gamma}_t)E[\tilde{\omega}] \\ &= C + \hat{\gamma}_t(\Delta + E[\tilde{\zeta}]) + (1 - \hat{\gamma}_t)\Delta = C + \Delta + \hat{\gamma}_tE[\tilde{\zeta}]. \end{aligned} \quad (2.30)$$

The above description of the agency cost describes exactly the situation in which the investors impose the agency cost Δ if, and only if, they anticipate the auditor to compromise, $\hat{\gamma}_t = 1$. On the equilibrium path, the investors' beliefs and the auditor's strategies coincide, $\hat{\Gamma} = \Gamma$. It follows that, in equilibrium, the differential cost to the manager between an auditor that compromises and an auditor that maintains his independence is, all else equal, the expected litigation cost $E[\tilde{\zeta}]$.

Under the assumptions made in this Appendix, none of the results presented in the paper change, since the equilibria presented involve pure strategies on the auditor's part of either "never compromise" or "always compromise". Of course, this still leaves the possibility that there may be strategies that depend on the accounting issue, and are

thus ex-ante, or from the point of view of the uninformed investors, random. We do not explore these possibilities, as they are intractable and do not add any new insights to the auditor replacement question.

At the time the manager decides to replace/retain the auditor after an accounting disagreement, the residual agency cost borne by the manager conditional on retaining the auditor is

$$C(N, t, \varepsilon, \hat{\gamma}_t, \gamma_t) = C + \hat{\gamma}_t \Delta - \gamma_t \varepsilon,$$

where $\hat{\gamma}_t$ represents the investors' beliefs and γ_t the auditor's decision on the compromise. The same cost borne by the manager, conditional on replacing the auditor, is given by

$$C(N, t, \varepsilon, \hat{\gamma}_t) = C + \hat{\gamma}_t \Delta - \hat{\gamma}_t E[\tilde{\varepsilon}],$$

where $\hat{\gamma}_t$ represents the manager's and the investors' common beliefs regarding the strategy of a replacement auditor.

Chapter 3

Correlated Noise, Commitment, and Ratcheting

3.1 Introduction

The ratchet effect has been described in the economics literature in connection with centrally planned economies (see for example Litwack [25] and the references therein), and more generally in settings where the agent is privately informed. The book by Laffont and Tirole provides a detailed analysis and references [23]. Their description of the ratchet effect is as follows: “If [a regulated firm] produces at a low cost today, the regulator may infer that low costs are not hard to achieve and tomorrow offer a demanding incentive scheme. That is, the firm jeopardizes future rents by being efficient”. The essence of the ratchet effect with a privately informed agent is that the agent can obtain a rent in future periods by hiding his type in the current period.

Weitzman [36] presents a multi-period model of the ratchet effect with moral hazard only, but in his model the ratcheting mechanism is exogenous. More recently, Milgrom and Roberts [29] and Indjejikian and Nanda [18] have shown that there is a ratchet effect in two period models with moral hazard but without adverse selection. In these models, the ratcheting is endogenous and driven by the lack of commitment by the principal regarding the use of available information. Milgrom and Roberts [29] define the ratchet effect as “the tendency for performance standards to increase after a period of good performance”. Given that the principal will use today’s outcome in writing tomorrow’s contract creates for the manager a link between today’s effort and tomorrow’s standard of

performance. The ratchet effect is always inefficient in the models with adverse selection since good types will mimic bad types and earn rents. In the pure moral hazard model, the ratchet effect is also inefficient with respect to the full commitment solution as shown by Indjejikian and Nanda [18].

Ratcheting results from the principal's ability to optimally adjust the agent's second-period incentive for the lower ex-post variance of the second performance measure by using the first-period performance measure when the principal cannot commit fully to a long-term contract. The assumption that the two periods are correlated is thus crucial, and differentiates this model of ratcheting in a pure moral hazard setting from other models of sequential action choice. Repeated moral hazard with independent periods is analyzed by Lambert [24], Rogerson [33], Holmstrom and Milgrom [17], and Fudenberg, Holmstrom and Milgrom [12].

Matsumura [27] presents an analysis of sequential action choice with correlated outcomes in a single period in which the agent observes a first outcome before selecting the second action. However, in Matsumura's model, there is no contracting after the first outcome is observed, and this outcome is private agent information until the end of the period. As a consequence, second-period incentives are not affected by the first-period performance.

In this paper, I extend the analysis of the Indjejikian and Nanda model [18] to include commitment issues and the possibility of agent turnover. Indjejikian and Nanda present two types of commitment: full commitment to a long-period contract (which they refer to as "commitment") and an intermediate form of commitment with a sequence of two short-term contracts (which they refer to as "lack of commitment"). I show that the two short-term contracts obtained by Indjejikian and Nanda correspond to a form of commitment which I call *commitment to fairness*. This form of commitment is an adaptation of the concept of fairness introduced by Baron and Besanko [1]. A contracting relationship

is governed by *fairness* if the principal is restricted to fair wages and the agent must participate in all periods if he accepts the contract in the first period. Fair wages are paid when the agent gets his reservation wage as if he could leave in each period. That is, the agent's certainty equivalent of future compensation, conditional on available information and on the principal's conjecture of the agent's first-period action is set to the reservation level at the start of each period.¹ Thus, in addition to the usual contract acceptance constraint at the start of the first period, there is a second constraint that the second-period contract is acceptable to the agent as if the agent had other employment opportunities and had not committed to stay for both periods. The agent trades off his ability to leave in the second period for the guarantee of fair compensation in the second period.

I also show that the two contracts under commitment to fairness are equivalent to the long-term renegotiation-proof contract: the payoffs for the agent and the principal, and the induced actions coincide for the renegotiation-proof contract and the sequence of contracts under commitment to fairness.

The fairness constraint is not only sufficient for obtaining the solution of Indjejikian and Nanda, but also necessary. Allowing the agent to leave in the second period gives the opportunity to the agent to take another action in the first period than that anticipated by the principal and then leave after the first-period compensation is paid. This situation parallels the "take-the-money-and-run" strategy that arises in ratcheting with adverse selection when the agent cannot commit to stay for both periods (see Laffont and Tirole [23]). Moreover, if the agent is able to leave in the second period, then there is no equilibrium with two short-term contracts in which the agent stays for both periods (see also Christensen and Feltham [2]). Thus, the fairness assumption helps to overcome

¹Note that an equilibrium involves rational expectations regarding the agent's first-period action. Thus, second-period fair wages are based on the principal's conjecture of the agent's first-period action, which is correct in equilibrium.

the non-existence of an equilibrium problem.

Removing the commitment to fairness assumption leads to a solution in which the principal optimally employs a different agent in each period. The only assumption necessary to obtain the two-agent solution is that the principal can commit to replace the first agent in the second period. At the time the principal replaces the agent in the second period, he is indifferent between retaining and replacing the agent. Thus, the principal's commitment to replace the agent is not a strong assumption.

An analysis of the principal's welfare under the different commitment assumptions reveals that for negatively correlated performance measures, full commitment is preferred to all other forms of commitment, or lack thereof. The situation is somewhat reversed with positively correlated performance measures in that, although full commitment is better than renegotiation and commitment to fairness, no commitment (two agents) is better than full commitment. The driving force behind this result is that with negative correlation, having the same agent in both periods reduces the total risk to which the agent is exposed by the optimal incentive scheme. When the correlation is positive, using a different agent in each period eliminates the risk premium due to the correlation between the optimal compensation schemes for each action.²

The remainder of the paper is organized as follows. Section 2 presents the basic assumptions of the model. Sections 3, 4, 5, and 6 derive the full commitment solution, the commitment and renegotiation solution, the commitment and fairness solution, and the no commitment solution, respectively. Section 7 concludes with an analysis of the principal's welfare under the different commitment assumptions. The proofs are collected in an Appendix.

²The situation is as if there is a compensation scheme for the first-period action \tilde{c}_1 and a compensation scheme for the second-period action \tilde{c}_2 . With one agent, the risk for which the principal pays compensation is $\text{var}(\tilde{c}_1 + \tilde{c}_2) = \text{var}(\tilde{c}_1) + 2\text{cov}(\tilde{c}_1, \tilde{c}_2) + \text{var}(\tilde{c}_2)$. With two agents, the risk for which the principal pays compensation is $\text{var}(\tilde{c}_1) + \text{var}(\tilde{c}_2)$.

3.2 The model

In this section I develop a simple two-period agency model that will serve as the basis for the analysis of commitment and ratcheting in a pure moral hazard context. A risk-neutral principal owns a production technology that requires productive effort from an agent in two periods $t = 1, 2$.

The agent is risk- and effort-averse with exponential utility and quadratic effort cost of the form $u(w, a) = -\exp[-r(w - \frac{1}{2}(a_1^2 + a_2^2))]$, where w is the agent's terminal wealth and $a = (a_1, a_2)$ is the agent's effort at the start of periods 1 and 2. The agent's certainty equivalent of terminal wealth \tilde{w} and effort a is, assuming \tilde{w} to be normally distributed,

$$\text{ACE}(\tilde{w}, a_1, a_2) = E[\tilde{w}] - \frac{1}{2}r \text{var}(\tilde{w}) - \frac{1}{2}(a_1^2 + a_2^2) . \quad (3.31)$$

The output from agent's effort $a_t \in \mathbb{R}$ is, for $t = 1, 2$,

$$\tilde{x}_t = b_t a_t + \tilde{\lambda}_t , \quad (3.32)$$

where λ_t is an arbitrary mean zero noise term which does not depend on a_t in any way. Both outcomes, x_1 and x_2 , are not observed until after the termination of the contract at the end of period 2. Hence, the output \tilde{x}_t only determines the principal's expected surplus, and since the principal is risk-neutral, no further distributional assumptions are needed regarding $\tilde{\lambda}_t$. The agent's actions are unobservable. Hence, neither the output nor the agent's actions are contractible.

A contractible performance measure y_t is observed at the end of each period. The agent's effort in period t affects only the mean of the performance measure in that period,

$$\tilde{y}_t = m_t a_t + \tilde{\varepsilon}_t , \quad (3.33)$$

where $\tilde{\varepsilon}_t$ are mean zero noise terms. The noise terms in the performance measures are

joint normally distributed with variance-covariance matrix

$$\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}. \quad (3.34)$$

The variances of the noise terms ε_t are normalized to one in order to focus on the inter-period correlation of the two performance measures. An alternative description of the performance measures can be given as follows. When the correlation is positive, the noise terms can be decomposed as

$$\tilde{\varepsilon}_1 = \tilde{\nu}_1 + \tilde{\delta} \quad (3.35)$$

$$\tilde{\varepsilon}_2 = \tilde{\nu}_2 + \tilde{\delta}, \quad (3.36)$$

where $\tilde{\nu}_1, \tilde{\nu}_2$ are period-specific, and $\tilde{\delta}$ is a common component in both periods. All three terms are independent of each other and $\text{var}(\tilde{\delta}) = \rho$. The common component $\tilde{\delta}$ can be thought of as a shock that persists from the first period to the second period. When the correlation of the performance measures is negative, the noise terms can be decomposed as

$$\tilde{\varepsilon}_1 = \tilde{\nu}_1 + \tilde{\delta} \quad (3.37)$$

$$\tilde{\varepsilon}_2 = \tilde{\nu}_2 - \tilde{\delta}, \quad (3.38)$$

where $\tilde{\nu}_1, \tilde{\nu}_2$ are period-specific, and $\tilde{\delta}$ is a common component in both periods. All three terms are independent of each other and $\text{var}(\tilde{\delta}) = |\rho|$. The common component $\tilde{\delta}$ can be thought of as first-period accruals that have to be reversed in the second period. Thus, negatively correlated noise in accounting-based performance measures reflects the nature of the accrual process. In this model, however, the accruals, as with all the other components of the noise in the performance measure, are outside the manager's control.

The principal owns the production technology for both periods and needs an agent to supply productive effort. There is more than one agent that the principal can employ

in each period. All agents are identical (the agents have the same ability and the same utility functions) and have alternative employment opportunities. Each agent's reservation certainty equivalent is normalized to zero in each period. Both the principal and the agents are assumed to have discount rates of zero. Utility functions, discount rates, reservation wages, the nature of the production technology, and the information structure are common knowledge.

Throughout the paper, I maintain the above assumptions and I analyze the principal-agent problem under different commitment assumptions. Specifically, I consider the following types of commitment:

1. full commitment to a long-term contract;
2. commitment with renegotiation (with a long-term contract);
3. commitment to fairness (with a sequence of two short-term contracts);
4. no commitment (with a sequence of two short-term contracts).

For each of these cases, there is a slightly different time line of events that will be detailed when that case is analyzed. In all four cases, the following basic six-date time line is present. At the start of the first period, the principal and the agent sign a contract. After contracting, in the first period, the agent provides effort a_1 , after which the first performance measure is reported. At the end of the first period, the agent may receive some compensation depending on the particular contracting setting. At the start of the second period, recontracting, or contracting for the second period may take place, depending on the particular setting. After contracting, the agent provides effort a_2 , and then the second performance measure y_2 is reported. At the end of the second period all remaining contracts are settled. After the end of the second period (i.e., after all contracts are settled), the outcomes x_1, x_2 are revealed to the principal. Thus, in each period, all or

some of the following events occur in this order: contracting (at the start of the period), productive effort supplied by agent, performance measure reported, contract(s) settled (at the end of the period).

The first two types of commitment (full commitment, commitment with renegotiation) assume that the agent is compensated by a long-term contract $c(\tilde{y}_1, \tilde{y}_2)$ to be settled at the end of the second period. The other two types of commitment (commitment to fairness, no commitment) assume that the agent is compensated by a series of short-term contracts $(c_1(\tilde{y}_1), c_2(y_1, \tilde{y}_2))$ to be settled at the end of each period.

Contracts are always assumed to be linear, and the only contractible information is given by the two performance measures. The resulting contracts are thus the *optimal linear contracts* in each case, although linear contracts are not optimal.³ The main issue here is the role of commitment in a two-period LEN⁴ framework and how lack of commitment gives rise to ratcheting.

Given linear contracts and normally distributed performance measures, the agent's wealth is normally distributed as well, which implies that the agent's certainty equivalent of wealth and effort is given by equation (3.31). More precisely, given a linear contract in both performance measures, the certainty equivalent of compensation at the start of the first period is given by

$$\text{ACE}(\tilde{w}, a_1, a_2) = E[\tilde{w}] - \frac{1}{2}r \text{var}(\tilde{w}) - \frac{1}{2}(a_1^2 + a_2^2) . \quad (3.39)$$

Similarly, for any contract that is linear in the second performance measure, the agent's

³The Holmström and Milgrom [17] framework is not applicable here primarily because the periods are not independent (the fact that the performance measures are correlated is a key assumption in my model, while independence is a key assumption in the Holmström and Milgrom model). Holmström and Milgrom [17] present sufficient conditions under which linear contracts are optimal in a multi-period agency. In their model, the agent's actions generate a sequence of independent binary signals (one in each period) and the agent's utility is exponential.

⁴Linear contracts, Exponential utility, Normal distributions.

certainty equivalent at the start of the second period is given by

$$\text{ACE}(\tilde{w}, a_2 | y_1, a_1) = E[\tilde{w} | y_1, a_1] - \frac{1}{2} r \text{var}(\tilde{w} | y_1, a_1) - \frac{1}{2} a_2^2. \quad (3.40)$$

Note that, while the conditional expectation $E[\tilde{w} | y_1, a_1]$ depends both on the observed value of y_1 and the agent's first-period action, the posterior variance $\text{var}(\tilde{w} | y_1, a_1)$ does not depend on either the realized value of y_1 , or on the agent's action a_1 .

The agent's wealth \tilde{w} represents the total compensation to be received by the agent, and the agent is indifferent as to the timing of consumption. Thus, the agent's utility ensures that there are no intertemporal consumption smoothing issues. In addition, the agent's exponential utility eliminates wealth effects, in that compensation paid (earned) does not impact the agent's risk preferences. The quadratic cost of effort, additive across tasks, means that the agent is not indifferent to the allocation of effort among tasks in the two periods.⁵

3.3 Full commitment, long-term contract

In this section I assume that the principal can commit at the start of the first period to a two-period contract. The terms of this contract are not subject to renegotiation. Furthermore, if the agent accepts the contract at the start of the first period, he commits for both periods, and cannot leave after the first period. These assumptions about the parties' ability to commit make the model equivalent (within the LEN framework) to a two task, two correlated performance measures, as analyzed by Feltham and Xie [11] and Feltham and Wu [10].

The time line of events is as follows.

- 1) At the start of the first period, the principal offers the agent a linear contract based

⁵The agent's exponential utility $u(w, a) = -\exp[-r(w - \frac{1}{2}(a_1^2 + a_2^2))]$ implies that, over two periods, his certainty equivalent is $\text{ACE}(\tilde{w}, a) = E[\tilde{w}] - \frac{1}{2} r \text{var}(\tilde{w}) - \frac{1}{2}(a_1^2 + a_2^2)$, which is not a function only of total effort $a_1 + a_2$.

on the two performance measures that are contractible information:

$$\tilde{c} = \alpha_0 + \beta_1 \tilde{y}_1 + \beta_2 \tilde{y}_2 . \quad (3.41)$$

The coefficient α_0 represents the fixed wage to be paid the agent, and the coefficients β_1 , β_2 represent the variable wages to be paid the agent for each unit of the performance measures.

2) If the agent accepts the contract, he provides productive effort a_1 in the first period and a_2 in the second period.

3) After the agent has provided effort a_t , which is not observed by the principal, the performance measure y_t is publicly reported.

4) At the end of the second period, the contract is settled. After the end of the second period, the outcomes x_1, x_2 are revealed to the principal.

Note that in this case, the timing of information and the timing of payments to the agent is irrelevant, the only constraint is that the performance measures are available before the contract is settled.

The assumption that the principal can commit to a long-term contract guarantees that after y_1 is observed, the principal will not be able to modify the contract. The LEN assumptions guarantee that the agent's choice of action in the second period will be independent of the observation of the first-period performance measure and of the agent's first-period action a_1 . The reason is that, at the start of the second period, the part of the agent's certainty equivalent of compensation conditional on y_1, a_1 that is variable in a_2 , and thus provides incentives for a_2 , does not depend on either y_1 or a_1 . Here, each of the assumptions of the LEN model is essential. The exponential utility guarantees that wealth does not affect the agent's risk preferences. The linearity of the contract and the fact that the performance measures are joint normally distributed with additive noise ensure that y_1 and a_1 impact on the agent's compensation separately from a_2 . In the LEN

model with full commitment, the timing of y_1 is not important, only its contractibility matters. However, in other settings, the timing of information is important, and if the agent observes the first-period performance measure y_1 before selecting the second-period action a_2 , then a_2 may depend on y_1 . See for example the model of sequential action choice with correlated periods of Matsumura [27].

With full commitment to a long-term contract, the only role played by the correlation between the two performance measures is through the impact on the total risk to which the agent is exposed for incentive purposes as measured by $\text{var}(\tilde{c}) = \beta_1^2 + 2\rho\beta_1\beta_2 + \beta_2^2$.

The principal's problem is to maximize, at the start of the first period, the expected total outcome net of the agent's compensation, subject to the agent's participation constraint and the agent's incentive compatibility constraint.

Proposition 3.3.1 *The optimal linear contract $\tilde{c} = \alpha_0 + \beta_1 \tilde{y}_1 + \beta_2 \tilde{y}_2$ with full commitment for both periods and the optimal actions are characterized by $\hat{a}_1 = \beta_1 m_1$, $\hat{a}_2 = \beta_2 m_2$,*

$$\alpha_0 = -\frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2) + \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1m_2} + \frac{\hat{a}_2^2}{m_2^2}\right), \quad (3.42)$$

$$\beta_1 = \frac{m_1(r + m_2^2)b_1 - r\rho m_2 b_2}{(r + m_1^2)(r + m_2^2) - r^2\rho^2}, \quad (3.43)$$

$$\beta_2 = \frac{m_2(r + m_1^2)b_2 - r\rho m_1 b_1}{(r + m_1^2)(r + m_2^2) - r^2\rho^2}. \quad (3.44)$$

The principal's surplus is

$$\pi = b_1\hat{a}_1 + b_2\hat{a}_2 - \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1m_2} + \frac{\hat{a}_2^2}{m_2^2}\right) - \frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2). \quad (3.45)$$

Proof. See Feltham and Xie [11] or the Appendix. \square

3.4 Commitment and renegotiation

In this section, I assume that the principal and the agent commit to a two-period contract subject to renegotiation in the second period. Specifically, the principal commits to

a two-period contract and the agent commits to stay for both periods if he finds the initial contract acceptable. However, the principal and the agent cannot commit not to renegotiate after the first performance measure is observed. The terms of the contract are subject to renegotiation in the usual sense: the existing contract can only be replaced by a new contract if both parties agree to it. The renegotiation takes the form of a take-it-or-leave-it offer by the principal.

The time line of events is as follows.

- 1) At the start of the first period, the principal offers the agent a linear contract based on the two contractible performance measures:

$$\tilde{c}^I = \alpha_0^I + \beta_1^I \tilde{y}_1 + \beta_2^I \tilde{y}_2 . \quad (3.46)$$

The coefficient α_0 represents the fixed wage to be paid the agent, and the coefficients β_1 , β_2 represent the variable wages to be paid the agent for each unit of the performance measures.

- 2) If the agent accepts the contract, he commits to the terms of the contract, unless both parties agree later to replace it by a new contract.
- 3) In the first period, after the agent has provided effort a_1 unobservable by the principal, the performance measure y_1 is publicly reported.
- 4) At the start of the second period, the principal can make the renegotiation offer. The principal cannot fire the agent without the agent agreeing to leave at the start of the second period, and the agent cannot leave in the second period without the principal agreeing to it. In addition, a new contract $\tilde{c}^R = \alpha_0^R(y_1) + \beta_2^R(y_1)\tilde{y}_2$ can replace the old one only if both the principal and the agent weakly prefer it. The renegotiation offer is exogenously assumed to be linear in keeping with the restriction to linear contracts in the model. The coefficients of the renegotiation offer may depend on the first performance

measure which is known at the time of renegotiation.⁶

5) After the renegotiation stage, the agent provides effort a_2 and then y_2 is reported.

6) After y_2 has been reported, the contract is settled at the end of the second period.

After the end of the second period, the outcomes x_1, x_2 are revealed to the principal.

As in the full-commitment case, the contract is settled at the end of the second period. The main difference is that the contract can be renegotiated after the agent has taken his first-period action and the first-period performance measure has been reported. In this context, a *renegotiation-proof contract* is a contract $\tilde{c} = \alpha_0 + \beta_1 \tilde{y}_1 + \beta_2 \tilde{y}_2$ such that, once agreed upon at the start of the first period, there does not exist a contract at the renegotiation stage which is weakly preferred by both parties and at least one party strictly prefers. The fact that a linear contract can be renegotiation-proof is due to the particulars of the LEN framework. It turns out that, for every initial contract that is linear in y_1, y_2 , the optimal renegotiation offer (which is restricted to be linear in y_2) has a second-period incentive independent of y_1 and a fixed wage that is linear in y_1 . In other words, the renegotiation offer is also linear in the two performance measures from an ex-ante (start of the first period) perspective.

An equilibrium in the principal-agent renegotiation game⁷ consists of a pair of contracts $(\tilde{c}^I, \tilde{c}^R)$, the agent's actions a_1, a_2 , and principal's beliefs about the agent's actions \hat{a}_1, \hat{a}_2 such that: (i) the agent accepts both the initial contract and the renegotiation offer, and rationally anticipates the renegotiation offer and its acceptance when selecting action a_1 ; (ii) the principal's beliefs are correct $\hat{a}_1 = a_1, \hat{a}_2 = a_2$; (iii) the pair $(\tilde{c}^I, \tilde{c}^R)$ is ex-ante (start of the first period) optimal and \tilde{c}^R is ex-post (start of the second period,

⁶At the time of renegotiation, the initial contract is linear in the second performance measure, which is still uncertain. I assume that a renegotiation offer is restricted to having the same linear form.

⁷Note that, in general, an equilibrium in the renegotiation game is not given by a renegotiation-proof contract.

conditional on y_1, a_1) optimal from the principal's point of view.

The following proposition shows that the analysis of the equilibrium can be restricted without loss of generality to renegotiation-proof contracts.

Proposition 3.4.1 *If $(\tilde{c}^I, \tilde{c}^R, a_1, a_2)$ is an equilibrium in the principal-agent renegotiation game and \tilde{c}^R is linear in \tilde{y}_2 at the time of renegotiation, then \tilde{c}^R is also ex ante (at the start of the first period) linear in \tilde{y}_1, \tilde{y}_2 and $(\tilde{c}^R, \tilde{c}^R, a_1, a_2)$ is an equivalent equilibrium with a single renegotiation-proof contract.*

Proof. First, I show that any renegotiation offer that gives the optimal second-period incentive and is acceptable to the agent is linear in the two performance measures from an ex-ante perspective. At renegotiation time, the principal is restricted to offer a linear contract $\tilde{c}^R = \alpha_0^R(y_1) + \beta_2^R(y_1)\tilde{y}_2$, whose coefficients may depend on the first-period performance measure. The second-period incentive β_2^R does not depend on y_1 since it is determined only by the conditional variance $\text{var}(\tilde{y}_2|y_1)$ which is independent of the actual value of y_1 . Furthermore, from the participation constraint at renegotiation time it follows that

$$\begin{aligned} \text{ACE}(\tilde{c}^I|y_1, a_1, a_2) &= \text{E}[\tilde{c}^I|y_1, a_1, a_2] - \frac{1}{2}r\text{var}(\tilde{c}^I|y_1) - \frac{1}{2}a_2^2 \leq \text{ACE}(\tilde{c}^R|y_1, a_1, a_2) \\ &= \alpha_0^R(y_1) + \beta_2^R\text{E}[\tilde{y}_2|y_1, a_1, a_2] - \frac{1}{2}r\text{var}(\tilde{c}^R|y_1) - \frac{1}{2}a_2^2. \end{aligned} \quad (3.47)$$

Since the initial contract is linear in y_1, y_2 and since the conditional expectation $\text{E}[\tilde{y}_2|y_1]$ is linear in y_1 (due to the joint normality of the distributions), it follows, solving the equation implied by assuming the participation constraint(3.47) to be binding for $\alpha_0^R(y_1)$, that α_0^R is linear in y_1 ,

$$\alpha_0^R = \text{E}[\tilde{c}^I|y_1, a_1, a_2] - \beta_2^R\text{E}[\tilde{y}_2|y_1, a_1, a_2] - \frac{1}{2}r[\text{var}(\tilde{c}^I|y_1) - \text{var}(\tilde{c}^R|y_1)]. \quad (3.48)$$

It then follows that, conditional on being accepted by the agent, the renegotiation offer is of the following form

$$\begin{aligned}\tilde{c}^R &= E[\tilde{c}^I | y_1, a_1, a_2] + \beta_2^R(\tilde{y}_2 - E[\tilde{y}_2 | y_1, a_1, a_2]) - \frac{1}{2}r[\text{var}(\tilde{c}^I | y_1) - \text{var}(\tilde{c}^R | y_1)] \\ &= \tilde{c}^I + (\beta_2^R - \beta_2^I)(\tilde{y}_2 - E[\tilde{y}_2 | y_1, a_1, a_2]) - \frac{1}{2}r[\text{var}(\tilde{c}^I | y_1) - \text{var}(\tilde{c}^R | y_1)] .\end{aligned}\tag{3.49}$$

From the above equation it follows that $\text{ACE}(\tilde{c}^R) = \text{ACE}(\tilde{c}^I)$ at the start of the first period, and that \tilde{c}^R is acceptable at the start of the first period if, and only if, \tilde{c}^I is acceptable. Thus, \tilde{c}^R is accepted by the agent if offered as a first-period contract. Since \tilde{c}^R is an optimal renegotiation offer given \tilde{c}^I , and since the efficient second-period incentive is independent of y_1, a_1 , the principal has no incentive to renegotiate \tilde{c}^R if it is offered as an initial contract. This proves that \tilde{c}^R is renegotiation-proof.

It remains to show that \tilde{c}^R induces the same actions as $(\tilde{c}^I, \tilde{c}^R)$, since the payments to the agents and the principal's surplus are determined by a_1, a_2 . The second-period action is determined by c^R in both cases since that is the contract in effect at the time the agent provides effort a_2 . Since $(\tilde{c}^I, \tilde{c}^R, a_1, a_2)$ is an equilibrium, the first period action is chosen in anticipation of the second-period renegotiation, and so is determined by the (linear) incentive contained in $\alpha_0^R(y_1)$. The reason is that it is suboptimal for the agent to select an action different from a_1 and then reject the renegotiation offer. Thus, the agent's first-period action is determined only by the incentives in \tilde{c}^R , and so is the same when \tilde{c}^R is offered as the first-period contract. \square

The main idea in the proposition is that if the contract is renegotiated in equilibrium, the agent's actions are completely determined by the renegotiated contract. The principal cannot gain by offering the agent a contract that will later be renegotiated as long as the agent anticipates the renegotiation.

The principal's problem is to maximize at the start of the first period the expected

total outcome net of the agent's compensation, subject to the agent's participation constraint, and the agent's incentive compatibility constraints. The contract must also be renegotiation-proof at the start of the second period. Since at the time the contract can be renegotiated, the first-period action has already been taken, and the first-period performance measure has been reported, the only part of the agent's compensation that remains uncertain is the second-period variable wage $\beta_2 \tilde{y}_2$. The requirement that the contract is renegotiation-proof means that at the start of the second period, the contract maximizes the principal's expected total outcome net of the agent's compensation conditional on first-period information and subject to the agent's participation constraint and the agent's second-period incentive compatibility constraint.

Proposition 3.4.2 *The optimal linear two-period renegotiation-proof contract $\tilde{c} = \alpha_0 + \beta_1 \tilde{y}_1 + \beta_2 \tilde{y}_2$ and the optimal actions are characterized by $\hat{a}_1 = m_1 \beta_1$, $\hat{a}_2 = m_2 \beta_2$*

$$\alpha_0 = -\frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2) + \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1m_2} + \frac{\hat{a}_2^2}{m_2^2}\right), \quad (3.50)$$

$$\beta_1 = \frac{[m_2^2 + r(1 - \rho^2)]b_1m_1 - r\rho b_2m_2}{(m_1^2 + r)(m_2^2 + r(1 - \rho^2))}, \quad (3.51)$$

$$\beta_2 = \frac{b_2m_2}{m_2^2 + r(1 - \rho^2)}. \quad (3.52)$$

The principal's surplus is given by

$$\pi = b_1\hat{a}_1 + b_2\hat{a}_2 - \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1m_2} + \frac{\hat{a}_2^2}{m_2^2}\right) - \frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2). \quad (3.53)$$

Proof. See the Appendix. \square

The renegotiation concept I use here is the same as that of Fudenberg and Tirole [13] in that both parties must agree to the renegotiated contract, but the timing is different. In my model, renegotiation takes place after the first performance measure is observed, while Fudenberg and Tirole have the renegotiation take place between the time the agent

takes the action and the time the performance measure is observed in a single period model. Having the renegotiation take place after y_1 is observed and before a_2 is taken avoids the insurance/adverse selection problem of Fudenberg and Tirole.⁸

The driving force behind renegotiation in my model is the principal's desire to optimally adjust the agent's induced effort in the second period to match the (reduced) ex-post variance of the second performance measure. In this context, the fact that y_1 is observed is important, and not the actual value of y_1 . As a result, the second-period action and the second-period incentive are independent of y_1 , the agent first-period action a_1^\dagger , and the principal's conjecture of the agent's first-period action \hat{a}_1 .

The inability of the principal to commit not to renegotiate is ex-ante inefficient relative to the full commitment case. The reason is that the renegotiation-proof contract imposes an additional binding constraint on the optimal contract. In both cases, the principal chooses the agent's actions to be induced by the contract in order to maximize his surplus as given by (3.53). In the full commitment case, the principal maximizes π unconstrained, while with renegotiation in the second period, the second-period action is constrained to $\hat{a}_2 = b_2 m_2^2 [m_2^2 + r(1 - \rho^2)]^{-1}$. Since the unconstrained optimum is at least as high as the constrained one, the full commitment contract is at least as efficient as the renegotiation-proof contract. If $\rho = 0$, that is if the performance measures in the two periods are independent, there is no difference between the two contracts, which take the form of repeatedly inducing the optimal action from the one period problem.

In general, there are additional conditions under which the full commitment contract is the same as the renegotiation-proof contract. The idea is that whenever the full commitment contract is renegotiation-proof, the two coincide. The precise conditions

⁸If renegotiation takes place before y_1 is observed, the agent is offered insurance by the principal for the risk in y_1 when the agent has private information about the action a_1 . As a result, the principal would like to perfectly insure the agent in order to avoid paying compensation for risk, and this destroys the agent's incentive to provide effort because the agent anticipates the renegotiation. Fudenberg and Tirole show that the only equilibria involve randomization by the agent in choosing his action.

under which the full commitment contract is renegotiation-proof are summarized in the following proposition (see Christensen and Feltham [2]).

Proposition 3.4.3 *The principal's surplus is the same under full commitment and under commitment and renegotiation if, and only if, at least one of the following conditions is satisfied:*

1. *the two periods are independent, $\rho = 0$;*
2. *the first-period performance measure is uninformative with respect to the first-period action, $m_1 = 0$;*
3. *$\rho m_1 \neq 0$ and $-\frac{b_1}{\rho m_1} = \frac{b_2 m_2}{m_2^2 + r(1 - \rho^2)}$.*

Proof. The principal's surplus is given both in the full commitment case and in the renegotiation case by the same function of the action induced $\pi(\hat{a}_1, \hat{a}_2)$ (see equations (3.53) and (3.45)). The principal's surplus is strictly concave in the two induced actions, which implies that the global maximum is unique. Let \hat{a}_1^C, \hat{a}_2^C represent the actions induced by the full commitment contract as given in (3.43), (3.44) and let \hat{a}_1^R, \hat{a}_2^R represent the actions induced by the commitment and renegotiation contract as given in (3.51), (3.52). Since \hat{a}_1^C maximizes $\pi(\hat{a}_1, \hat{a}_2)$ given \hat{a}_2^C and \hat{a}_1^R maximizes $\pi(\hat{a}_1, \hat{a}_2)$ given \hat{a}_2^R , it follows that $\hat{a}_1^C = \hat{a}_1^R$ if, and only if, $\hat{a}_2^C = \hat{a}_2^R$. Since the unique global maximum of $\pi(\hat{a}_1, \hat{a}_2)$ is attained only at $(\hat{a}_1^C, \hat{a}_2^C)$, it follows that the principal is strictly better off with full commitment relative to commitment and renegotiation if and only if $\hat{a}_2^C \neq \hat{a}_2^R$.

Note that the above argument also shows that $\hat{a}_1^C = \hat{a}_1^R$ if $\hat{a}_2^C = \hat{a}_2^R$. Thus, the two contracts coincide if and only if $\hat{a}_2^C = \hat{a}_2^R$. The remainder of the proof is straightforward algebra and shows that (1)–(3) are equivalent to $\hat{a}_2^C = \hat{a}_2^R$.⁹ \square

⁹See also Christensen and Feltham [2] for more details.

Another interesting special case is when the two periods are identical: $m_1 = m_2 = m$ and $b_1 = b_2 = b$. Then, if $\rho = -1$, the contracts again coincide, and in addition induce the first-best action in both periods $a_1^* = a_2^* = b^2$; the principal's surplus is the first-best $\pi^* = b^2$. When the two periods are identical, the full commitment contract induces the same action in both periods. When the noise terms in the two performance measures are perfectly negatively correlated, the total variance in the contract is zero since $\text{var}(\tilde{c}) = \beta_1^2 + 2\rho\beta_1\beta_2 + \beta_2^2$. Thus, the principal can motivate the agent to work without imposing any risk, and thus need not pay the agent any risk premium. The principal can then motivate the agent by paying only the cost of effort whenever $\beta_1 = \beta_2$, and thus achieve first-best. In the renegotiation case, the second-period action is the first-best one because the conditional variance of the second-period performance measure is zero. From the previous argument comparing the two contracts, it follows that the first-period actions are also the same, and the renegotiation contract also achieves first-best.

3.5 Commitment to fairness

In the previous two sections, I presented the two types of commitment that relate to two-period contracts: full commitment and renegotiation-proofness. In this section, I begin the analysis of short-term contracts in a two-period relationship. Short-term contracts are used whenever the parties cannot write long-term contracts. The inability to write a long-term contract does not necessarily imply that there is no long-term commitment (that is commitment beyond the first-period) in the principal-agent relationship, only that the principal cannot commit to a specific contract in a period other than the current one, or that the contracting environment is such that it prohibits the use of long-term contracts. The main idea in this section is an intermediate form of commitment, whereby the agent commits to stay for both periods, while the principal commits to offer contracts that

provide the agent with his reservation certainty equivalent for each period based on the public information at the contracting date and the principal's conjectures of the agent's actions.

This form of commitment is an adaptation of the concept of fairness introduced by Baron and Besanko [1]. A contracting relationship is governed by *fairness* if the principal is restricted to fair wages and the agent must participate in all periods if he accepts the contract in the first period. Fair wages are paid when the agent gets his reservation wage as if he could leave in each period. That is, the agent's certainty equivalent of future compensation, conditional on available information and conjectured actions is set to zero at the start of each period. Thus, in addition to the usual contract acceptance constraint at the start of the first period, there is a second constraint that the second-period contract is acceptable to the agent as if the agent had other employment opportunities, had not committed to stay for both periods, and had taken the conjectured first-period action. The key fact here is the agent's ability to commit to stay for both periods; removing the agent's ability to commit for both periods leads to a situation where there is no equilibrium in which the agent stays in the second period. The agent gives up his ability to leave in the second period for the guarantee of fair compensation in the second period.

The time line of events is as follows.

- 1) At the start of the first period, the principal offers the agent a linear contract based on the first-period performance measure y_1 :

$$\tilde{c}_1 = \alpha_1 + \beta_1 \tilde{y}_1 . \quad (3.54)$$

The coefficient α_1 represents the fixed wage to be paid the agent, and the coefficient β_1 represents the variable wages to be paid the agent for each unit of the performance measure.

- 2) If the agent accepts the contract, he commits to stay for both periods. The principal

commits to offer the reservation certainty equivalent of wages in each period, given all the information available at the time of contracting and given that the agent's actions correspond to the principal's conjectures.

3) In the first period, after the agent has provided effort a_1 unobservable by the principal, the performance measure y_1 is publicly reported.

4) After the performance measure y_1 is publicly reported, the first-period contract is settled at the end of the first period.

5) At the start of the second period, a new contract is offered by the principal

$$\tilde{c}_2 = \alpha_2 + \beta_2 \tilde{y}_2 . \quad (3.55)$$

This contract is subject to the agent reservation wage restriction and is accepted by the agent (since I assumed commitment for the second period on the agent's part). The terms of the second-period contract may depend on the performance measure y_1 and on the principal's conjecture about the agent's first period action, since they are informative about y_2 , and therefore can be used to reduce the noise of the second-period performance measure.

6) In the second period, the agent provides effort a_2 , then the performance measure y_2 is reported. The second-period contract is settled at the end of the second period. After the end of the second period, the outputs x_1, x_2 are revealed to the principal.

Short-term contracting with fair wages relies on the principal's conjectures regarding the (unobservable) agent actions. At the start of the second period, when \tilde{c}_2 is set, the terms of the contract depend on $E[\tilde{y}_2|y_1, \hat{a}_1]$, where \hat{a}_1 is the principal's conjecture. The concept of fair wages in the second period assumes that the principal's conjecture is correct, that is the agent actually has provided effort \hat{a}_1 in the first period. At the start of the first period, fair wages refer to the total future compensation paid to the agent

over the two periods $\tilde{c}_1 + \tilde{c}_2$, and involve both the principal's and the agent's conjectures of future actions \hat{a}_1, \hat{a}_2 . The principal and the agent are assumed to be in agreement over the conjectured future actions. For example, the principal states his conjecture \hat{a}_1 as part of the first-period contract \tilde{c}_1 . The agent then agrees to that conjecture when accepting the initial contract, and that will be the basis for setting the second-period contract.¹⁰

Thus, an optimal sequence of contracts under commitment to fairness is also characterized by a rational expectations equilibrium regarding the agent's actions. The agent's commitment to stay for both periods is essential in sustaining this rational expectations equilibrium, as will be shown later in this section.

The principal's problem is to maximize at the start of each period the remaining expected total outcome net of the agent's compensation, subject to the agent's reservation wage constraint and the agent's rationality constraint. Since the agent commits to stay for both periods at the start of the first period, the second-period contract \tilde{c}_2 is anticipated at the time the first-period contract is set.

Proposition 3.5.1 *The optimal sequence of short-term linear contracts ($\tilde{c}_1 = \alpha_1 + \beta_1 \tilde{y}_1, \tilde{c}_2 = \alpha_2(y_1) + \beta_2 \tilde{y}_2$) and the optimal actions induced under commitment and fairness are characterized by $\hat{a}_1 = m_1(\beta_1 - \rho\beta_2)$, $\hat{a}_2 = m_2\beta_2$,*

$$\alpha_1 = -\frac{1}{2}\hat{a}_1^2 - \rho\frac{m_1}{m_2}\hat{a}_1\hat{a}_2 + \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1m_2} + \rho^2\frac{\hat{a}_2^2}{m_2^2}\right), \quad (3.56)$$

$$\beta_1 = \frac{(m_2^2 + r(1 - \rho^2))b_1m_1 - r\rho b_2m_2}{(m_1^2 + r)(m_2^2 + r(1 - \rho^2))} + \rho\frac{b_2m_2}{m_2^2 + r(1 - \rho^2)}, \quad (3.57)$$

$$\alpha_2(y_1) = -\rho\frac{\hat{a}_2}{m_2}(y_1 - m_1\hat{a}_1) + \frac{1}{2}\hat{a}_2^2\left[\frac{r}{m_2^2}(1 - \rho^2) - 1\right], \quad (3.58)$$

$$\beta_2 = \frac{b_2m_2}{m_2^2 + r(1 - \rho^2)}. \quad (3.59)$$

¹⁰An alternative would be that both parties solve for the unique equilibrium conjecture \hat{a}_1 . In this case, the principal can use an expert witness (professor of accounting) to prove that the second-period wage is fair.

The principal's surplus is given by

$$\pi = b_1 \hat{a}_1 + b_2 \hat{a}_2 - \frac{1}{2} r \left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho \frac{\hat{a}_1 \hat{a}_2}{m_1 m_2} + \frac{\hat{a}_2^2}{m_2^2} \right) - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) . \quad (3.60)$$

Proof. See the Appendix. \square

Note also for future reference that

$$\beta_1 = \frac{\hat{a}_1}{m_1} + \rho \frac{\hat{a}_2}{m_2} , \quad (3.61)$$

$$\begin{aligned} \hat{a}_1 &= \frac{b_1 m_1^2}{m_1^2 + r} - r \rho \frac{m_1}{m_1^2 + r} \frac{\hat{a}_2}{m_2} \\ &= \frac{(m_2^2 + r(1 - \rho^2)) b_1 m_1^2 - r \rho b_2 m_1 m_2}{(m_1^2 + r)(m_2^2 + r(1 - \rho^2))} . \end{aligned} \quad (3.62)$$

The optimal actions induced by the sequence of contracts in Proposition 3.5.1 are the same as the actions induced by the renegotiation-proof contract . From (3.53) and (3.60), the principal's surplus is the same in both cases. Moreover, from an ex-ante (start of the first period) perspective, the contract $\bar{c} = \bar{c}_1 + \bar{c}_2$ is the same as the renegotiation-proof contract described in Proposition 3.4.2.

Under commitment to fairness, as with renegotiation, the second-period action and the second-period incentive do not depend on the first-period action, or on the specific value of first-period performance measure. The reason, as before, is that when contracting at the start of the second period, the role played by the available information is to reduce the variance in the second-period performance measure, and that variance does not depend on either the specific value of the first-period performance measure or the first-period action. It follows that the principal's optimal choice of risk for the agent when setting the second-period incentive does not depend on either the specific value of the first-period performance measure or the first-period action. Thus, the principal's problem in setting the incentive for the second period is the same in both cases, and as a result the induced second-period action is the same.

Once the second-period incentive rate is set, the fixed wage is given by the fairness restriction. Then, the second-period fixed wage depends linearly on y_1 because $E[\tilde{y}_2|y_1, \hat{a}_1]$ is linear in y_1 . From an ex-ante perspective, the second-period contract contains a (random) linear term in \tilde{y}_1 , which contributes in addition to the first-period incentives from the first-period contract to the agent's choice of action in the first period. At the start of the first period, the agent's cumulative future compensation $\tilde{c}_1 + \tilde{c}_2$ is linear in \tilde{y}_1 and \tilde{y}_2 , while the second-period incentive rate has the same value as in the renegotiation case. Both the principal and the agent anticipate the second-period contract \tilde{c}_2 and have a common conjecture regarding its terms. It follows that the principal's problem is the same as in the renegotiation case and that the *total* incentive for the first-period action is the same. The main difference is that, with commitment to fairness, the first-period incentives are split between the first-period variable wage and the second-period fixed wage. The agent's actions are the same and the principal's surplus is the same in both cases.

Thus, the two short-term contracts under commitment to fairness replicate the payoffs of the long-term renegotiation-proof contract. This particular sequence of contracts is the one derived by Indjejikian and Nanda [18] to illustrate how lack of commitment in a dynamic agency relationship leads to ratcheting. The principal's inability to commit to a long term-contract without renegotiation in the second period creates the same inefficiency with short-term contracts under commitment and fairness as the inefficiency of the renegotiation-proof contract. Here, as before, the inefficiency is relative to the full commitment long-term contract. The results in this section show that commitment to fairness is a sufficient assumption for obtaining the ratcheting with short-term contracts described by Indjejikian and Nanda [18]. However, their description of the commitment assumptions that are necessary for the solution they derive is incomplete and referred to as "the absence of commitment".

If “absence of commitment” means that the agent cannot commit to stay for both periods, then, as is shown in the following, there is no equilibrium with a sequence of short-term contracts in which the agent stays for both periods. As a result, the commitment to fairness assumption is not only sufficient, but also necessary to obtain the solution described in Proposition 3.5.1, which is to say Indjejikian and Nanda’s solution. The fairness assumption is necessary since, once the agent has committed to stay for both periods, there must be a restriction in the second period to prevent an “infinite” transfer from the agent to the principal through the second-period contract. The anticipation of such a contract renders any first-period contract in which the agent commits for both periods unacceptable to the agent. Thus, the simplest solution to this problem is a lower bound on the agent’s reservation certainty equivalent, which is precisely what the fairness constraint provides.

The following proposition shows that the sequence of optimal contracts derived under commitment to fairness always induces the agent to leave in the second period if the agent’s ability to leave is restored.

Proposition 3.5.2 *If $\rho m_1 \beta_2 \neq 0$, and the principal offers the sequence of contracts $(\tilde{c}_1, \tilde{c}_2)$ from Proposition 3.5.1, the agent works for the principal only in the first period, leaves in the second period, and earns a positive surplus.*

Proof. Suppose that the agent is unable to commit to the firm for both periods. Then, the agent’s ability to leave in the second period guarantees the same participation constraint for the second period as the assumption of fair wages. If the principal assumes that the agent will stay with the firm for both periods, he will offer the agent the contract \tilde{c}_1 in the first period, anticipating \tilde{c}_2 for a second-period contract (where \tilde{c}_1, \tilde{c}_2 are the same as those in Proposition 3.5.1).

At the time the agent chooses his first-period action, he can plan to stay for the

second period, in which case he will maximize

$$\text{ACE}(\tilde{c}_1 + \tilde{c}_2, a_1, \hat{a}_2 | \hat{a}_1) = E[\tilde{c}_1 + \tilde{c}_2 | a_1, \hat{a}_2] - \frac{1}{2}r \text{var}(\tilde{c}_1 + \tilde{c}_2) - \frac{1}{2}(a_1^2 + \hat{a}_2^2) ,$$

which is the agent's certainty equivalent of terminal wealth given that the agent stays in both periods and the principal's conjectures of the first- and second-period actions. The agent's certainty equivalent of compensation over the two periods depends on the agent's action choice in the first period a_1 , on the agent's conjectured second-period action \hat{a}_2 , and on the contract terms, which depend on the principal's conjectures regarding the agent's actions \hat{a}_1, \hat{a}_2 . In this case, the agent and the principal have a common conjecture for the agent's equilibrium action choice in the second period. The main point is that the agent's action choice in the second period is independent of the action chosen in the first period and of any other first-period information, which implies that the second-period action can be conjectured to be the same by both the principal and the agent at the start of the first period.

If the agent plans on leaving in the second period, he will maximize

$$\text{ACE}(\tilde{c}_1, a_1 | \hat{a}_1) = E[\tilde{c}_1 | a_1] - \frac{1}{2}r \text{var}(\tilde{c}_1) - \frac{1}{2}a_1^2 ,$$

which is the agent's certainty equivalent of terminal wealth given the agent collects the payment c_1 at the end of the first period and then leaves. The agent's certainty equivalent depends on his first-period action choice a_1 and on the contract terms which depend on the principal's conjecture of the agent's first-period action \hat{a}_1 . As in the previous case, the principal's conjecture \hat{a}_1 is based on the assumption that the agent stays for both periods. The principal's conjectures of the agent actions are the equilibrium actions described in Proposition 3.5.1, and the contracts offered to the agent are determined by these conjectures as in equations (3.56)-(3.59).

The key here is that the agent's certainty equivalent of compensation depends on the principal's conjectured actions (which are based on the assumption that the agent stays

in the second period) and on the agent's decision to leave/stay in the second period. The agent chooses his action strategically in conjunction with his decision to leave/stay. The contracts offered to the agent are based on the principal's conjectures \hat{a}_1, \hat{a}_2 , and are designed to be accepted by the agent conditional on the agent taking the conjectured actions. Also, the agent gets only his reservation certainty equivalent when he takes the conjectured actions. Thus, the agent may benefit by deviating from the actions conjectured by the principal.

If the agent plans to stay in the second period, the action he chooses is the same as the one the principal expects \hat{a}_1 , and in equilibrium his certainty equivalent of compensation is zero. On the other hand, if the agent plans to leave in the second period, $\tilde{c}_1 = \alpha_1 + \beta_1 \tilde{y}_1$, and the certainty equivalent of first-period compensation is

$$\text{ACE}(\tilde{c}_1|a_1, \hat{a}_1) = \alpha_1 + \beta_1 m_1 a_1 - \frac{1}{2} r \beta_1^2 - \frac{1}{2} a_1^2, \quad (3.63)$$

where α_1, β_1 are given by (3.56), (3.57). The first-order condition for the agent's action choice implies

$$a_1 = m_1 \beta_1 = \hat{a}_1 + \rho \frac{m_1}{m_2} \hat{a}_2. \quad (3.64)$$

Substituting in the agent's certainty equivalent of compensation gives, for all $\rho \neq 0$,

$$\begin{aligned} \text{ACE}(\tilde{c}_1|a_1, \hat{a}_1) &= \alpha_1 + a_1^2 - \frac{1}{2} r \beta_1^2 - \frac{1}{2} a_1^2 \\ &= -\frac{1}{2} \hat{a}_1^2 - \rho \frac{m_1}{m_2} \hat{a}_1 \hat{a}_2 + \frac{1}{2} r \left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho \frac{\hat{a}_1 \hat{a}_2}{m_1 m_2} + \rho^2 \frac{\hat{a}_2^2}{m_2^2} \right) \\ &\quad + \frac{1}{2} (m_1^2 - r) \left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho \frac{\hat{a}_1 \hat{a}_2}{m_1 m_2} + \rho^2 \frac{\hat{a}_2^2}{m_2^2} \right) \\ &= \frac{1}{2} \rho^2 m_1^2 \frac{\hat{a}_2^2}{m_2^2} > 0. \end{aligned} \quad (3.65)$$

The agent can thus earn a positive surplus if the principal offers a first-period contract in anticipation of a second-period contract, expecting the same agent to stay in the

second period, while the agent plans on leaving in the second period after the first-period compensation has been paid. The agent has a strict incentive to act strategically in the first period and then “take-the money-and-run” whenever he is offered the first contract of the sequence $(\tilde{c}_1, \tilde{c}_2)$ and the correlation between the performance measures is non-zero.

Now I will show that it is also optimal ex-post for the agent to leave in the second period, given that he acted strategically (i.e., he took the action (3.64)) in the first period. At the start of the second period, the principal offers the contract $\tilde{c}_2 = \alpha_2(y_1) + \beta_2\tilde{y}_2$ given by (3.58) and (3.59), and the agent’s certainty equivalent of second-period compensation is

$$\text{ACE}(\tilde{c}_2|y_1, a_1^\dagger, a_2) = \alpha_2 + \beta_2(m_2a_2 + \rho(y_1 - m_1a_1^\dagger)) - \frac{1}{2}r\beta_2^2(1 - \rho^2) - \frac{1}{2}a_2^2, \quad (3.66)$$

where $a_1^\dagger = \hat{a}_1 + \rho m_1 \hat{a}_2 / m_2$ is the action that the agent took in the first period. If the agent were to accept the second-period contract, his action choice in the second period would be the same as that anticipated by the principal \hat{a}_2 . The key here is that the second-period action \hat{a}_2 and the second-period incentive pay β_2 are independent of the actual action the agent took in the first period and of the specific value of the first-period performance measure. The second-period fixed wage is the one determined under commitment to fairness since I assumed that the principal designs the contracts based on having the same agent in both periods,

$$\alpha_2 = -\rho \frac{\hat{a}_2}{m_2} y_1 + \rho \frac{m_1}{m_2} \hat{a}_1 \hat{a}_2 + \frac{1}{2} r \frac{\hat{a}_2^2}{m_2^2} (1 - \rho^2) - \frac{1}{2} \hat{a}_2^2. \quad (3.67)$$

It follows that the agent’s certainty equivalent of compensation at the start of the second

period is

$$\begin{aligned}
 \text{ACE}(\tilde{c}_2|y_1, a_1^\dagger, \hat{a}_2) &= -\rho \frac{\hat{a}_2}{m_2} y_1 + \rho \frac{m_1}{m_2} \hat{a}_1 \hat{a}_2 + \frac{1}{2} r \frac{\hat{a}_2^2}{m_2^2} (1 - \rho^2) - \frac{1}{2} \hat{a}_2^2 \\
 &\quad + \frac{\hat{a}_2}{m_2} (m_2 \hat{a}_2 + \rho(y_1 - m_1 a_1^\dagger)) - \frac{1}{2} r \frac{\hat{a}_2^2}{m_2^2} (1 - \rho^2) - \frac{1}{2} \hat{a}_2^2 \quad (3.68) \\
 &= \rho \frac{m_1}{m_2} \hat{a}_2 (\hat{a}_1 - a_1^\dagger) = -\rho^2 \frac{m_1}{m_2} \hat{a}_2 < 0,
 \end{aligned}$$

which means that the agent will always reject the second-period contract, independent of y_1 . \square

The conclusion of the proposition holds for both positive and negative correlation between the performance measures in the two periods. However, the source of the agent's surplus is different in the two cases. If the performance measures are positively correlated, the agent works harder than the principal expects him to and the surplus comes from "too much" incentive pay. If the performance measures are negatively correlated, the agent works less than the principal expects him to, and the surplus comes from the fixed pay.

As shown by Christensen and Feltham [2], the fact that there is no equilibrium in which the agent stays for both periods when the agent is able to leave holds in greater generality, without having to specify that the principal offers the particular contract sequence from Proposition 3.5.1. The restriction to short-term contracts in addition to the agent's ability to leave is also essential for the equilibrium to break down. For example, it can be shown that if long-term contracting is possible, there are renegotiation equilibria such that the agent never leaves in the second period if he accepts the contract in the first period.

3.6 The agent cannot commit for two periods

Up to this point, I have assumed that the agent can commit to stay for both periods, either by signing a long-term contract (that may be subject to renegotiation) or by entering into a contractual arrangement based on commitment to fairness. In this section, I remove the agent's ability to commit to stay for more than one period and analyze two settings: one in which long-term contracts are possible, but are subject to renegotiation, and a second setting in which only short-term contracts are possible and the only commitment the principal can make is commitment to firing the agent hired in the first period.

3.6.1 Long-term contracts and renegotiation

In order to analyze the agent's ability to leave under commitment and renegotiation, a slight modification of the long-term contract is necessary. In the section on renegotiation, I assumed that the agent commits to stay for both periods and that the (renegotiated) contract is settled at the end of the second period. I now remove both assumptions as follows:

1. The long-term contract is settled over both periods as follows. Instead of a single payment at the end of the second period $\tilde{c} = \alpha_0 + \beta_{01}\tilde{y}_1 + \beta_{02}\tilde{y}_2$, the principal and the agent agree at the start of the first period on two payments to the agent, one at the end of each period, $\tilde{c}_1 = \alpha_1 + \beta_1\tilde{y}_1$, $\tilde{c}_2 = \alpha_2(y_1) + \beta_2\tilde{y}_2$. The total payments to the agent are set to be the same in both cases, $\tilde{c}_1 + \tilde{c}_2 = \tilde{c}$.
2. The agent can leave at the start of the second period. If the agent leaves, he takes payment \tilde{c}_1 and gives up \tilde{c}_2 . Thus, the agent cannot renege on c_1 after observing y_1 and then leave.

Note that this modification of the long-term contracting with renegotiation is different from the setting with short-term contracts. The key difference is that, with short-term contracts, the first-period contract \tilde{c}_1 has to be acceptable to the agent on its own at the start of the first period. With long-term contracts, the agent has to accept the total contract $\tilde{c} = \tilde{c}_1 + \tilde{c}_2$ at the start of the first period. The renegotiation mechanism is modified in a straightforward way: given an initial contract $\tilde{c}^I = (\tilde{c}_1^I, \tilde{c}_2^I)$, a renegotiation offer from the principal replaces \tilde{c}_2^I by \tilde{c}_2^R . At the start of the second period, the agent can respond to a renegotiation offer from the principal in three ways: accept the renegotiation offer and stay, reject the renegotiation offer and stay, and reject the renegotiation offer and leave.

An equilibrium in the renegotiation game in this modified form consists of an initial contract, a renegotiation offer, and the agent's actions $(\tilde{c}_1^I, \tilde{c}_2^I, \tilde{c}_2^R, a_1, a_2)$ with the same properties of an equilibrium as described in Section 4. Note again that an equilibrium assumes that the agent accepts the renegotiation offer and stays in the second period. This introduces an additional constraint on the renegotiation offer. When the agent is committed to stay, the renegotiation offer has to be weakly preferred to the initial contract, $\text{ACE}(\tilde{c}_2^I | y_1, a_1, a_2) \leq \text{ACE}(\tilde{c}_2^R | y_1, a_1, a_2)$. When the agent can leave, the additional constraint that staying is preferred to the reservation certainty equivalent, $\text{ACE}(\tilde{c}_2^R | y_1, a_1, a_2) \geq 0$, is needed to ensure the agent does not leave. As before, the analysis can be restricted without loss of generality to renegotiation-proof contracts (see Proposition 3.4.1).

The following proposition shows that a linear renegotiation-proof contract can be always rewritten so that the agent stays for both periods even when he can leave in the second period. Thus, having access to long-term linear contracts, makes the distinction whether the agent can leave or not unimportant.

Proposition 3.6.1 *For any renegotiation-proof equilibrium (\tilde{c}, a_1, a_2) , where the long-term contract \tilde{c} is linear in y_1, y_2 and the agent is committed to stay in the second period, there exist \tilde{c}_1, \tilde{c}_2 linear in y_1 and y_2 respectively such that $(\tilde{c}_1, \tilde{c}_2, a_1, a_2)$ is an equivalent equilibrium, $\tilde{c} = \tilde{c}_1 + \tilde{c}_2$, and the agent always stays in the second period without having committed to do so.*

Proof. The main idea of the proof is to rearrange the agent's payments across periods so that the acceptance decision in the second period, which in general may depend on the specific realization of y_1 when the agent is not committed to stay, does not depend on y_1 . To understand this idea, I start with a long-term renegotiation-proof contract for which the payments are all at the end of the second period. The restriction to a renegotiation-proof contract does not restrict the generality of the problem and eliminates the renegotiation constraint at the start of the second period, leaving only the acceptance constraint relative to the reservation certainty equivalent. Then, given the contract $\tilde{c} = \alpha_0 + \beta_1 \tilde{y}_1 + \beta_2 \tilde{y}_2$, the agent's participation constraint at the start of the second period is

$$\begin{aligned}
 \text{ACE}(\tilde{c} \mid y_1, a_1, a_2) &= \alpha_0 + \beta_1 y_1 + \beta_2 \text{E}[\tilde{y}_2 \mid y_1] - \frac{1}{2} r \text{var}(\tilde{c} \mid y_1) - \frac{1}{2} (a_1^2 + a_2^2) \\
 &= \alpha_0 + \beta_1 y_1 + \beta_2 (m_2 a_2 + \rho(y_1 - m_1 a_1)) - \frac{1}{2} r \beta_2^2 (1 - \rho^2) - \frac{1}{2} (a_1^2 + a_2^2) \\
 &= \alpha_0 + (\beta_1 + \rho \beta_2) y_1 + \beta_2 (m_2 a_2 - \rho m_1 a_1) - \frac{1}{2} r \beta_2^2 (1 - \rho^2) - \frac{1}{2} (a_1^2 + a_2^2) \geq 0 .
 \end{aligned} \tag{3.69}$$

It is easy to see that, starting with a long-term renegotiation proof contract and allowing the agent to leave at the start of the second period makes the agent's decision to stay depend on the particular realization of y_1 . If, instead of the whole payment, only one part is allocated to the second period $\tilde{c}_2 = \alpha_{02} + \beta_{21} \tilde{y}_1 + \beta_{22} \tilde{y}_2$, the agent's decision to

stay is given by

$$\begin{aligned}
 \text{ACE}(\tilde{c}_2 \mid y_1, a_1, a_2) &= \alpha_{02} + \beta_{21}y_1 + \beta_{22}\text{E}[\tilde{y}_2 \mid y_1] - \frac{1}{2}r \text{var}(\tilde{c}_2 \mid y_1) - \frac{1}{2}(a_1^2 + a_2^2) \\
 &= \alpha_{02} + (\beta_{21} + \rho\beta_{22})y_1 + \beta_{22}(m_2a_2 - \rho m_1a_1) - \frac{1}{2}r\beta_{22}^2(1 - \rho^2) - \frac{1}{2}(a_1^2 + a_2^2) \geq 0 .
 \end{aligned}
 \tag{3.70}$$

Then the agent will always stay if $\beta_{21} + \rho\beta_{22} = 0$, and α_{02} is set such that the participation constraint (3.6.1) is satisfied with a strict inequality. The second-period incentive rate is set such that $\beta_{22} = \beta_2$. The payment for the first period $\tilde{c}_1 = \alpha_{01} + \beta_{11}\tilde{y}_1 + \beta_{12}\tilde{y}_2$ is set such that $\tilde{c} = \tilde{c}_1 + \tilde{c}_2$. Thus $\beta_{12} = 0$, $\beta_{11} + \beta_{21} = \beta_1$, and $\alpha_{01} + \alpha_{02} = \alpha_0$. It follows that the sequence of payments $(\tilde{c}_1, \tilde{c}_2)$ is acceptable to the agent at the start of the first period since it is the same as the payment from the old contract \tilde{c} , while at the same being also ex-post (start of the second period) acceptable. \square

3.6.2 Two agents

I now remove all abilities to commit beyond a single period for both the principal and the agent. Thus, the principal cannot commit in the first period to anything that may happen in the second period. That means not only the terms of the second-period contract, but also terms such as fair wages as described in the previous section. The agent in turn, cannot commit to stay with the firm for more than one period and can leave at the end of the first period. In particular, long-term contracts are not possible in this setting.

Under the no commitment assumption, the sequence of contracts derived under the commitment to fairness assumption is no longer an equilibrium, as shown at the end of the previous section. Thus, the commitment to fairness assumption is also necessary for the contracts derived in that section to arise in equilibrium. The key problem is that if the principal offers a first-period contract based on the assumption that the same agent will be employed in the second period, the agent has an incentive to act strategically and

“take-the -money-and-run”.

On the other hand, the principal may design the contracts based on the assumption that he can commit to firing the first agent at the end of the first period, and will employ a different agent in each period. The time line of events is as follows.

1) At the start of the first period, the principal offers the first agent a linear contract based on the first-period performance measure y_1 :

$$\tilde{c}_1 = \alpha_1 + \beta_1 \tilde{y}_1 . \quad (3.71)$$

The coefficient α_1 represents the fixed wage to be paid the agent, and the coefficient β_1 represents the variable wages to be paid the agent for each unit of the performance measure. The principal commits to firing the agent at the end of the first period.

2) If the agent accepts the contract, he provides effort a_1 in the first period.

3) After the agent has provided effort a_1 unobservable by the principal, the performance measure y_1 is publicly reported. The contract is settled at the end of the period.

4) After the contract is settled, the principal fires the first agent.

5) At the start of the second period, a new contract is offered by the principal to a second agent

$$\tilde{c}_2 = \alpha_2 + \beta_2 \tilde{y}_2 . \quad (3.72)$$

This contract is subject to the agent participation constraint and is accepted by the agent. The terms of the second-period contract may depend on the performance measure y_1 since it is informative about y_2 , and affects the posterior beliefs (at the time of contracting) about the variance of the second-period performance measure.

6) In the second period, after the agent has provided effort a_2 , the second performance measure y_2 is reported. The second-period contract is settled at the end of the second period. After the end of the second period, the outputs x_1, x_2 are revealed to the principal.

The principal's problem is to maximize, at the start of each period, the remaining expected total outcome net of compensation for the agent(s), subject to the participation constraints and the agents' rationality constraints. In addition, the principal is committed to employing a different agent in each period.

Proposition 3.6.2 *The optimal sequence of short-term linear contracts ($\tilde{c}_1 = \alpha_1 + \beta_1 \tilde{y}_1$, $\tilde{c}_2 = \alpha_2 + \beta_2 \tilde{y}_2$) and the optimal actions induced with two different agents are characterized by $\hat{a}_1 = m_1 \beta_1$, $\hat{a}_2 = m_2 \beta_2$,*

$$\alpha_1 = -\frac{1}{2}\hat{a}_1^2 + \frac{1}{2}r \frac{\hat{a}_1^2}{m_1^2}, \quad (3.73)$$

$$\beta_1 = \frac{b_1 m_1}{m_1^2 + r}, \quad (3.74)$$

$$\alpha_2 = -\frac{1}{2}\hat{a}_2^2 + \frac{1}{2}r(1 - \rho^2) \frac{\hat{a}_2^2}{m_2^2} - \rho(y_1 - m_1 \hat{a}_1) \frac{\hat{a}_2}{m_2}, \quad (3.75)$$

$$\beta_2 = \frac{b_2 m_2}{m_2^2 + r(1 - \rho^2)}. \quad (3.76)$$

The principal's surplus is given by

$$\pi_1 = \frac{1}{2}b_1 \hat{a}_1 = \frac{1}{2} \frac{b_1^2 m_1^2}{m_1^2 + r}, \quad (3.77)$$

$$\pi_2 = \frac{1}{2}b_2 \hat{a}_2 = \frac{1}{2} \frac{b_2 m_2^2}{m_2^2 + r(1 - \rho^2)}. \quad (3.78)$$

Proof. See the Appendix. \square

Since I have proved in the previous section that the sequence of contracts based on a two-period tenure for the agent hired in the first period is not an equilibrium when the agent can leave in the second period, it follows that the two-agent solution is the only one possible in the absence of commitment over more than one period and under the assumption that the first agent's tenure is certain (either one period or two periods). That is, I assume that the principal and the agent are restricted to pure strategies regarding

firing/leaving decisions in the second period. Note, in addition, that the principal is assumed to be able to commit to firing the first agent before contracting in the second period takes place. This is a mild assumption on commitment since, in the absence of switching costs, the principal has no incentive to renege on the commitment to fire the first agent.

3.6.3 Random firing of the agent after one period

I now turn to the analysis of random firing of the agent at the end of the first period. The time line of events is the same as described at the start of the section on two agents. The only difference is that the principal commits to a random mechanism of firing for the first agent instead of committing to fire the first agent with certainty. To keep the analysis tractable and within the LEN framework, I assume that the principal can commit to firing the first agent at the end of the first period with probability $1 - p$, independent of \tilde{y}_1, \tilde{y}_2 . The firing decision is described by a random variable

$$\tilde{f} = \begin{cases} 1 & \text{if the agent is fired at the end of the period} \\ 0 & \text{if the agent is offered a contract in the second period ,} \end{cases} \quad (3.79)$$

which is independent of the performance measures. Thus, the agent's wealth from an ex-ante (start of the first period) perspective is $\tilde{w} = \tilde{c}_1 + (1 - \tilde{f})\tilde{c}_2$. The agent's expected utility of wealth is then given by

$$\begin{aligned} EU(\tilde{w}, \hat{a}_1, \hat{a}_2) &= pEU(\tilde{c}_1 + \tilde{c}_2, \hat{a}_1, \hat{a}_2) + (1 - p)EU(\tilde{c}_1, \hat{a}_1) \\ &= -p\exp(-r\text{ACE}(\tilde{c}_1 + \tilde{c}_2, \hat{a}_1, \hat{a}_2)) - (1 - p)\exp(-r\text{ACE}(\tilde{c}_1, \hat{a}_1)) . \end{aligned} \quad (3.80)$$

In the second period, the contract offered by the principal is the same, whether the first agent has been retained or not. The key is that the principal sets the second period fixed wage as a function of the observed value of the first-period performance measure

and his conjecture regarding the agent's first-period action. The principal's second-period surplus is then independent of his conjecture \hat{a}_1 . Conditional on the first agent not being fired, the agent's acceptance of the second-period contract depends on the agent's first-period action and on the principal's conjecture:

$$\text{ACE}(\tilde{c}_2|y_1, a_1, \hat{a}_1, \hat{a}_2) = \rho\beta_2 m_1(\hat{a}_1 - a_1) . \quad (3.81)$$

Since the agent's acceptance of the second-period contract is independent of y_1 , and the principal's firing decision is also independent of y_1 , it follows that the agent's beliefs about his continuing employment with the principal in the second period are not affected by the observation of y_1 . Thus, at the time the agent selects the first-period action, the only thing that matters is whether the agent believes that the principal's conjecture \hat{a}_1 will be correct, since that will determine whether the agent will accept the second-period contract conditional on not being fired. In a rational expectations equilibrium, the principal's conjecture is correct, $\hat{a}_1 = a_1$. It follows that, in a rational expectations equilibrium, the agent will always accept the second-period contract conditional on not being fired.

I now turn to proving that a rational expectations equilibrium is not possible when the agent can leave at the end of the first period. Assume, by contradiction, that there exists a rational expectations equilibrium in which $\hat{a}_1 = a_1$. Then, the principal offers a first-period contract based on the assumption that the agent accepts the second-period contract conditional on not being fired at the end of the first period. I now show that for the contract based on this assumption, the agent has an incentive to deviate from the equilibrium value of \hat{a}_1 and that he will, as a result, always turn down the second-period contract when not fired. Thus, an equilibrium in which $\hat{a}_1 = a_1$ is not possible when the agent is not fired with certainty.

As before, from the agent's participation constraint it follows that $\tilde{c}_2 = \alpha_2 + \beta_2 \tilde{y}_2$ with

$\hat{a}_2 = m_2\beta_2$ and the fixed wage given by $\alpha_2 = -\beta_2\rho(y_1 - \hat{a}_1) + \frac{1}{2}\beta_2^2[r(1-\rho^2) - m_2^2]$. Then, for any conjectured first-period action \hat{a}_1 , the two contracts are independent ex ante (start of the first period).¹¹ It follows that $\text{ACE}(\tilde{c}_1 + \tilde{c}_2, \hat{a}_1, \hat{a}_2) = \text{ACE}(\tilde{c}_1, \hat{a}_1) + \text{ACE}(\tilde{c}_2, \hat{a}_1, \hat{a}_2)$, and equation (3.80) becomes

$$\text{EU}(\tilde{w}, \hat{a}_1, \hat{a}_2) = \exp(-r\text{ACE}(\tilde{c}_1, \hat{a}_1))[-p \exp(-r\text{ACE}(\tilde{c}_2, \hat{a}_1, \hat{a}_2)) - (1-p)] . \quad (3.82)$$

At the time the agent selects his first-period action, let $A_1 = \text{ACE}(\tilde{c}_1, a_1, \hat{a}_1)$ and $A_2 = \text{ACE}(\tilde{c}_2, a_1, \hat{a}_1, \hat{a}_2)$. Then the first-order condition for the agent's action is

$$\frac{\partial}{\partial a_1} \text{EU}(\tilde{w}, \hat{a}_1, \hat{a}_2) = pr \left[\frac{\partial A_1}{\partial a_1} + \frac{\partial A_2}{\partial a_1} \right] \exp(-r(A_1 + A_2)) + (1-p)r \frac{\partial A_1}{\partial a_1} \exp(-rA_1) = 0 , \quad (3.83)$$

where

$$\frac{\partial A_1}{\partial a_1} = m_1\beta_1 - a_1 \quad (3.84)$$

$$\frac{\partial A_2}{\partial a_1} = -\rho m_1\beta_2 \quad (3.85)$$

Setting $a_1 = \hat{a}_1$ in the above equations simplifies the first-order condition (3.83) to

$$p(m_1\beta_1 - \hat{a}_1 - m_1\rho\beta_2) + (1-p)(m_1\beta_1 - \hat{a}_1) = m_1\beta_1 - \hat{a}_1 - pm_1\rho\beta_2 = 0 . \quad (3.86)$$

It follows that the equilibrium first-period action under random firing and rational expectations is

$$\hat{a}_1 = m_1(\beta_1 - p\rho\beta_2) . \quad (3.87)$$

Comparing the equilibrium action under rational expectations with random firing (3.87) to the equilibrium action under rational expectations and commitment to fairness as described in Proposition 3.5.1, it follows that the contracts offered with random firing

¹¹This is a straightforward calculation: $\text{cov}(\tilde{c}_1, \tilde{c}_2) = \text{cov}(\beta_1\tilde{y}_1, \beta_2\tilde{y}_2 - \beta_2\rho(\tilde{y}_1 - \hat{a}_1)) = 0$.

are equivalent to the contracts under commitment to fairness if the correlation ρ is replaced by $p\rho$. Then, following the same argument as in Proposition 3.5.2, if the agent is offered such a pair of contracts, he has the incentive to act strategically in the first period, deviate from the equilibrium conjecture \hat{a}_1 , and then leave in the second period taking a positive surplus.

3.7 Correlation and the value of commitment

In this section, I conclude by comparing the principal's surplus given the four different assumptions on commitment when the correlation between the two performance measures varies. Let π^C denote the principal's surplus under long-term contracting with full commitment (see Proposition 3.3.1), and let π^R denote the principal's surplus under long-term contracting with second-period renegotiation (see Proposition 3.4.2). Let π_t^F , $t = 1, 2$ denote the first- and second-period surplus under commitment to fairness (see Proposition 3.4.2), and let π_t^T , $t = 1, 2$ denote the first- and second-period surplus with two agents (see Proposition 3.6.2). Let $\pi^F = \pi_1^F + \pi_2^F$ and let $\pi^T = \pi_1^T + \pi_2^T$ denote the total surplus under short-term contracting.

Proposition 3.7.1 *If the two performance measures are uncorrelated, $\rho = 0$, the principal's surplus is the same in all cases and there is no difference between contracts:*

$$\pi^C = \pi^R = \pi^F = \pi^T . \quad (3.88)$$

Full commitment is weakly preferred to commitment with renegotiation or commitment to fairness:

$$\pi^C \geq \pi^R = \pi^F . \quad (3.89)$$

The inequality is strict whenever none of the conditions from Proposition 3.4.3 is satisfied.

Furthermore, assume that the correlation between the performance measures is positive, $\rho > 0$. Then, full commitment is preferred to renegotiation, but no commitment (two agents) dominates both

$$\pi^T > \pi^C \geq \pi^R = \pi^F . \quad (3.90)$$

Proof. The only thing that is left to prove is that for positive correlation, the two-agent solution dominates the full commitment solution. Suppose that the incentive rates from the full commitment contract are offered to two separate agents in the two periods. Then, the same actions are induced for the two agents as for the single agent. However, the risk premium paid to the two agents is lower since

$$\frac{1}{2}r[\beta_1^2 + 2\rho\beta_1\beta_2 + \beta_2^2] > \frac{1}{2}r[\beta_1^2 + \beta_2^2(1 - \rho^2)] . \quad (3.91)$$

Thus, two agents provide the same effort at a lower risk premium, which means that the principal's surplus is higher with two agents. The advantage of using two agents comes both from not having to pay compensation for the part of the total variance of the single-agent contract that is due to the performance measures being positively correlated (the term $2\rho\beta_1\beta_2$ in the above equation), and from the ability to use the lower ex-post variance with the second agent. \square

The above result can be strengthened under the additional assumption that the two periods are identical, in that for negatively correlated performance measures, the two agent solution is the least preferred.

Proposition 3.7.2 *Assume that the two periods are identical, $b_1 = b_2$ and $m_1 = m_2$. If the two performance measures are positively correlated, $\rho > 0$, full commitment is better than renegotiation, but no commitment (two agents) dominates both:*

$$\pi^T > \pi^C > \pi^R = \pi^F . \quad (3.92)$$

If the two performance measures are negatively correlated, $\rho < 0$, full commitment is better than renegotiation, and better than no commitment (two agents) :

$$\pi^C > \pi^R = \pi^F > \pi^T . \quad (3.93)$$

Finally, if $\rho = -1$,

$$\pi^* = \pi^C = \pi^R = \pi^F > \pi^T , \quad (3.94)$$

where $\pi^* = b^2$ is the first-best surplus.

Proof. See the Appendix. \square

With negatively correlated performance measures, the risk premium paid to a single agent through a long-term full commitment contract is reduced through the aggregation of the two performance measures. The key is that the incentive weights are identical on the two performance measures when the periods are identical. When the periods are not identical, the result regarding negatively correlated performance measures does not hold in general, but depends on the particular values of the parameters.

3.8 Appendix: proofs

Proof of Proposition 3.3.1

The optimal contract is a solution of the following optimization problem:

$$\max_{\alpha_0, \beta_1, \beta_2} E[\tilde{x}_1 + \tilde{x}_2 - \tilde{c}] \quad (3.95)$$

subject to the participation constraint

$$ACE(\tilde{c}|\hat{a}_1, \hat{a}_2) \geq 0 \quad (3.96)$$

and the incentive compatibility constraint

$$\hat{a}_1, \hat{a}_2 \in \operatorname{argmax}_{a_1, a_2} ACE(\tilde{c}|a_1, a_2) , \quad (3.97)$$

where \hat{a}_1, \hat{a}_2 are the actions to be induced by the contract.

Given the basic structure of the model (4.274), (3.31), (3.33), and the linear contract (4.281), the principal's surplus is given by

$$\pi = E[\tilde{x}_1 + \tilde{x}_2 - \tilde{c}] = b_1\hat{a}_1 + b_2\hat{a}_2 - \alpha_0 - \beta_1 m_1 \hat{a}_1 - \beta_2 m_2 \hat{a}_2 . \quad (3.98)$$

The agent's certainty equivalent of compensation is given by

$$\begin{aligned} \text{ACE}(\tilde{c}|a_1, a_2) &= \alpha_0 + \beta_1 m_1 a_1 + \beta_2 m_2 a_2 - \frac{1}{2}r \text{var}(\beta_1 \tilde{y}_1 + \beta_2 \tilde{y}_2) - \frac{1}{2}(a_1^2 + a_2^2) \\ &= \alpha_0 + \beta_1 a_1 + \beta_2 a_2 - \frac{1}{2}r(\beta_1^2 + 2\rho\beta_1\beta_2 + \beta_2^2) - \frac{1}{2}(a_1^2 + a_2^2) \end{aligned} \quad (3.99)$$

Since the agent's participation constraint is binding at the optimum,

$$\alpha_0 + \beta_1 m_1 \hat{a}_1 + \beta_2 m_2 \hat{a}_2 = \frac{1}{2}r(\beta_1^2 + 2\rho\beta_1\beta_2 + \beta_2^2) + \frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2) , \quad (3.100)$$

which in turn gives for the principal's expected surplus

$$\pi = b_1\hat{a}_1 + b_2\hat{a}_2 - \frac{1}{2}r(\beta_1^2 + 2\rho\beta_1\beta_2 + \beta_2^2) - \frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2) . \quad (3.101)$$

The first-order conditions for the incentive compatibility constraint are

$$\frac{\partial}{\partial a_1} \text{ACE}(\tilde{c}|a_1, a_2) = m_1\beta_1 - a_1 = 0 \quad (3.102)$$

$$\frac{\partial}{\partial a_2} \text{ACE}(\tilde{c}|a_1, a_2) = m_2\beta_2 - a_2 = 0 , \quad (3.103)$$

which imply that the actions induced by the contract are

$$\hat{a}_1 = m_1\beta_1 , \hat{a}_2 = m_2\beta_2 . \quad (3.104)$$

It follows that the principal's problem can be rewritten as an unconstrained optimization problem (since the incentives β_1, β_2 uniquely determine the agent's actions, the principal chooses the optimal actions to be induced by the contract)

$$\max_{\hat{a}_1, \hat{a}_2} b_1\hat{a}_1 + b_2\hat{a}_2 - \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1m_2} + \frac{\hat{a}_2^2}{m_2^2}\right) - \frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2) . \quad (3.105)$$

The first-order conditions are

$$b_1 - \hat{a}_1 - r \frac{\hat{a}_1}{m_1^2} - r \rho \frac{\hat{a}_2}{m_1 m_2} = 0 \quad (3.106)$$

$$b_2 - \hat{a}_2 - r \frac{\hat{a}_2}{m_2^2} - r \rho \frac{\hat{a}_1}{m_1 m_2} = 0 . \quad (3.107)$$

Solving for \hat{a}_1, \hat{a}_2 gives

$$\hat{a}_1 = \frac{m_1^2(r + m_2^2)b_1 - r \rho m_1 m_2 b_2}{(r + m_1^2)(r + m_2^2) - r^2 \rho^2} \quad (3.108)$$

$$\hat{a}_2 = \frac{m_2^2(r + m_1^2)b_2 - r \rho m_1 m_2 b_1}{(r + m_1^2)(r + m_2^2) - r^2 \rho^2} . \quad (3.109)$$

Substituting \hat{a}_1, \hat{a}_2 in the agent's participation constraint (3.100) gives

$$\alpha_0 + \hat{a}_1^2 + \hat{a}_2^2 - \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1 m_2} + \frac{\hat{a}_2^2}{m_2^2}\right) - \frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2) = 0 , \quad (3.110)$$

which can be used to determine α_0 as

$$\alpha_0 = -\frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2) + \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1 m_2} + \frac{\hat{a}_2^2}{m_2^2}\right) . \quad (3.111)$$

The principal's surplus is

$$\pi = b_1 \hat{a}_1 + b_2 \hat{a}_2 - \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1 m_2} + \frac{\hat{a}_2^2}{m_2^2}\right) - \frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2) . \quad (3.112)$$

□

Proof of Proposition 3.4.2

At the start of the second period, the renegotiation-proof contract is a solution of the following optimization problem:

$$\max_{\alpha_0, \beta_1, \beta_2} E[x_1 + \tilde{x}_2 - \tilde{c} \mid y_1, \hat{a}_1, \hat{a}_2] \quad (3.113)$$

subject to the participation constraint

$$\text{ACE}(\tilde{c} \mid y_1, a_1^\dagger, \hat{a}_2) \geq \bar{w}_0 \quad (3.114)$$

and the incentive compatibility constraint

$$\hat{a}_2 \in \operatorname{argmax}_{a_2} \operatorname{ACE}(\tilde{c} \mid y_1, a_1^\dagger, a_2) , \quad (3.115)$$

where a_1^\dagger is the actual first-period action taken by the agent, y_1 is the reported value of the first-period performance measure, and \hat{a}_2 is the action to be induced by the contract in the second period. The participation constraint requires that the agent's certainty equivalent of compensation conditional on the first-period information is no less than the certainty equivalent of compensation under the existing contract \bar{w}_0 . In addition, \hat{a}_1 represents the principal's conjecture of what the agent's first-period action has been. In equilibrium, the principal's conjecture is correct, $\hat{a}_1 = a_1^\dagger$.¹² From now on, I will use only \hat{a}_1 for both the principal's conjecture and for the agent's actual first-period action. Moreover, before either action has been taken, the conjectured actions from the point of view of both the principal and the agent are also \hat{a}_1 and \hat{a}_2 . Both the principal and the agent have rational expectations regarding their own behavior and each other's behavior.

The information structure (3.33), (4.276) implies that

$$E[\tilde{y}_2 \mid y_1] = m_2 \hat{a}_2 + \rho(y_1 - m_1 \hat{a}_1) \quad (3.116)$$

$$\operatorname{var}(\tilde{y}_2 \mid y_1) = 1 - \rho^2 . \quad (3.117)$$

The principal's expected surplus at the start of the second period can be written as

$$\begin{aligned} \pi_2 &= E[\tilde{x}_1 + \tilde{x}_2 - \tilde{c} \mid y_1, \hat{a}_1, \hat{a}_2] = E[\tilde{x}_1 \mid y_1] + E[\tilde{x}_2 \mid y_1] - \alpha_0 - \beta_1 y_1 - \beta_2 E[\tilde{y}_2 \mid y_1] \\ &= b_1 \hat{a}_1 + b_2 \hat{a}_2 + E[\tilde{\lambda}_1 + \tilde{\lambda}_2 \mid y_1] - \alpha_0 - \beta_1 y_1 - \beta_2 (m_2 \hat{a}_2 + \rho(y_1 - \hat{a}_1)) . \end{aligned} \quad (3.118)$$

¹²In fact this is a subtle point: there is an equilibrium in which the principal's conjecture is correct. Thus, when renegotiation takes place, or at the start of the first period, *there is no uncertainty about the agent's past or future actions*; that is, although the agent's actions are unobservable by the principal, there is no contracting with private agent information.

The agent's certainty equivalent of compensation at the start of the second period is

$$\begin{aligned}
 \text{ACE}(\tilde{c} \mid y_1, a_1, a_2) &= \alpha_0 + \beta_1 y_1 + \beta_2 \text{E}[\tilde{y}_2 \mid y_1] - \frac{1}{2} r \text{var}(\beta_1 y_1 + \beta_2 \tilde{y}_2 \mid y_1) - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) \\
 &= \alpha_0 + \beta_1 y_1 + \beta_2 \text{E}[\tilde{y}_2 \mid y_1] - \frac{1}{2} r \beta_2^2 \text{var}(\tilde{y}_2 \mid y_1) - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) \\
 &= \alpha_0 + \beta_1 y_1 + \beta_2 (m_2 \hat{a}_2 + \rho(y_1 - m_1 \hat{a}_1)) - \frac{1}{2} r \beta_2^2 (1 - \rho^2) - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) .
 \end{aligned} \tag{3.119}$$

Since the agent's participation constraint is binding at the optimum,

$$\alpha_0 + \beta_1 y_1 + \beta_2 (m_2 \hat{a}_2 + \rho(y_1 - m_1 \hat{a}_1)) = \bar{w}_0 + \frac{1}{2} r \beta_2^2 (1 - \rho^2) + \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) , \tag{3.120}$$

which in turn gives for the principal's expected surplus

$$\pi_2 = b_1 \hat{a}_1 + b_2 \hat{a}_2 + \text{E}[\tilde{\lambda}_1 + \tilde{\lambda}_2 \mid y_1] - \bar{w}_0 - \frac{1}{2} r \beta_2^2 (1 - \rho^2) - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) . \tag{3.121}$$

The first-order condition for the incentive compatibility constraint is

$$\frac{\partial}{\partial a_2} \text{ACE}(\tilde{c} \mid y_1, \hat{a}_1, a_2) = m_2 \beta_2 - a_2 = 0 . \tag{3.122}$$

It follows that the action induced by the contract is

$$\hat{a}_2 = m_2 \beta_2 , \tag{3.123}$$

and the principal's problem can be rewritten as an unconstrained optimization problem

$$\max_{\hat{a}_2} b_1 \hat{a}_1 + b_2 \hat{a}_2 + \text{E}[\tilde{\lambda}_1 + \tilde{\lambda}_2 \mid y_1] - \frac{1}{2} r \frac{\hat{a}_2^2}{m_2^2} (1 - \rho^2) - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) - \bar{w}_0 . \tag{3.124}$$

The first-order condition is

$$b_2 - r(1 - \rho^2) \frac{\hat{a}_2}{m_2^2} - \hat{a}_2 = 0 . \tag{3.125}$$

Solving for \hat{a}_2 gives

$$\hat{a}_2 = \beta_2 m_2 = \frac{b_2 m_2^2}{m_2^2 + r(1 - \rho^2)} . \tag{3.126}$$

In summary, the restriction that the contract is renegotiation-proof at the start of the second period is equivalent to restricting the second-period incentive pay to the value above (4.300).

At the start of the first period, the optimal contract is a solution of the following optimization problem:

$$\max_{\alpha_0, \beta_1, \beta_2} E[\tilde{x}_1 + \tilde{x}_2 - \tilde{c}] \quad (3.127)$$

subject to the participation constraint

$$ACE(\tilde{c}|\hat{a}_1, \hat{a}_2) \geq 0, \quad (3.128)$$

the incentive compatibility constraint

$$\hat{a}_1, \hat{a}_2 \in \operatorname{argmax}_{a_1, a_2} ACE(\tilde{c}|a_1, a_2), \quad (3.129)$$

and the renegotiation-proofness constraint

$$\beta_2 = \frac{b_2 m_2}{m_2^2 + r(1 - \rho^2)}, \quad (3.130)$$

where \hat{a}_1, \hat{a}_2 are the actions to be induced by the contract.

It follows that the principal's problem is the same as the long-term contract problem (3.95), (3.96), (3.97) with the additional constraint (3.130). As a result, the optimization problem can be rewritten as (4.292) subject to (3.130), which gives the first-order condition (4.293). Solving for \hat{a}_1 gives

$$\begin{aligned} \hat{a}_1 = \beta_1 m_1 &= \frac{b_1 - r\rho \frac{\hat{a}_2}{m_1 m_2}}{1 + \frac{r}{m_1^2}} \\ &= \frac{[m_2^2 + r(1 - \rho^2)]b_1 m_1^2 - r\rho b_2 m_1 m_2}{(m_1^2 + r)(m_2^2 + r(1 - \rho^2))}. \end{aligned} \quad (3.131)$$

The fixed wage is determined in the same way as in (4.290), (4.291),

$$\alpha_0 = -\frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2) + \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1m_2} + \frac{\hat{a}_2^2}{m_2^2}\right). \quad (3.132)$$

Similarly, the principal's surplus is (see (4.292))

$$\pi = b_1\hat{a}_1 + b_2\hat{a}_2 - \frac{1}{2}r\left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho\frac{\hat{a}_1\hat{a}_2}{m_1m_2} + \frac{\hat{a}_2^2}{m_2^2}\right) - \frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2). \quad (3.133)$$

□

Proof of Proposition 3.5.1

The second-period contract specifies the second-period incentive for the action a_2 based on all information available at the time of contracting:

$$\max_{\alpha_2, \beta_2} E[\tilde{x}_1 + \tilde{x}_2 - \tilde{c}_2 \mid y_1, \hat{a}_1, \hat{a}_2] \quad (3.134)$$

subject to the participation constraint

$$\text{ACE}(\tilde{c}_2 \mid y_1, \hat{a}_1, \hat{a}_2) \geq 0 \quad (3.135)$$

and the incentive compatibility constraint

$$\hat{a}_2 \in \operatorname{argmax}_{a_2} \text{ACE}(\tilde{c}_2 \mid y_1, a_1^\dagger, a_2), \quad (3.136)$$

where y_1 is the reported value of the first-period performance measure, and \hat{a}_2 is the action to be induced by the contract in the second period. The first-period action a_1^\dagger taken by the agent is the same as the principal's conjecture \hat{a}_1 . The principal's expected surplus at the start of the second period can be written as

$$\begin{aligned} \pi_2 &= E[\tilde{x}_1 + \tilde{x}_2 - \tilde{c}_2 \mid y_1, \hat{a}_1, \hat{a}_2] = E[\tilde{x}_1 \mid y_1] + E[\tilde{x}_2 \mid y_1] - \alpha_2 - \beta_2 E[\tilde{y}_2 \mid y_1] \\ &= b_1\hat{a}_1 + b_2\hat{a}_2 + E[\tilde{\lambda}_1 + \tilde{\lambda}_2 \mid y_1] - \alpha_2 - \beta_2(m_2\hat{a}_2 + \rho(y_1 - m_1\hat{a}_1)). \end{aligned} \quad (3.137)$$

The agent's certainty equivalent of compensation at the start of the second period is

$$\begin{aligned}
 \text{ACE}(\tilde{c}_2 \mid y_1, a_1^\dagger, \hat{a}_2) &= \alpha_2 + \beta_2 \text{E}[\tilde{y}_2 \mid y_1] - \frac{1}{2} r \text{var}(\alpha_2 + \beta_2 \tilde{y}_2 \mid y_1) - \frac{1}{2} \hat{a}_2^2 \\
 &= \alpha_2 + \beta_2 \text{E}[\tilde{y}_2 \mid y_1] - \frac{1}{2} r \beta_2^2 \text{var}(\tilde{y}_2 \mid y_1) - \frac{1}{2} \hat{a}_2^2 \\
 &= \alpha_2 + \beta_2 (m_2 \hat{a}_2 + \rho(y_1 - m_1 a_1^\dagger)) - \frac{1}{2} r \beta_2^2 (1 - \rho^2) - \frac{1}{2} \hat{a}_2^2 .
 \end{aligned} \tag{3.138}$$

Since the agent's participation constraint is binding at the optimum, and in equilibrium $\hat{a}_1 = a_1^\dagger$,

$$\alpha_2 + \beta_2 (m_2 \hat{a}_2 + \rho(y_1 - m_1 \hat{a}_1)) = \frac{1}{2} r \beta_2^2 (1 - \rho^2) + \frac{1}{2} \hat{a}_2^2 , \tag{3.139}$$

which in turn gives for the principal's expected surplus

$$\pi_2 = b_1 \hat{a}_1 + b_2 \hat{a}_2 + \text{E}[\tilde{\lambda}_1 + \tilde{\lambda}_2 \mid y_1] - \frac{1}{2} r \beta_2^2 (1 - \rho^2) - \frac{1}{2} \hat{a}_2^2 . \tag{3.140}$$

The first-order condition for the incentive compatibility constraint is

$$\frac{\partial}{\partial a_2} \text{ACE}(\tilde{c} \mid y_1, a_1^\dagger, a_2) = m_2 \beta_2 - a_2 = 0 . \tag{3.141}$$

It follows that the action induced by the contract is

$$\hat{a}_2 = m_2 \beta_2 , \tag{3.142}$$

and the principal's problem can be rewritten as an unconstrained optimization problem

$$\max_{\hat{a}_2} b_1 \hat{a}_1 + b_2 \hat{a}_2 + \text{E}[\tilde{\lambda}_1 + \tilde{\lambda}_2 \mid y_1] - \frac{1}{2} r \frac{\hat{a}_2^2}{m_2^2} (1 - \rho^2) - \frac{1}{2} \hat{a}_2^2 . \tag{3.143}$$

The first-order condition is

$$b_2 - r(1 - \rho^2) \frac{\hat{a}_2}{m_2^2} - \hat{a}_2 = 0 . \tag{3.144}$$

Solving for \hat{a}_2 gives

$$\hat{a}_2 = m_2 \beta_2 = \frac{b_2 m_2^2}{m_2^2 + r(1 - \rho^2)} . \tag{3.145}$$

Thus, the second-period action induced by this contract is the same action induced by the renegotiation-proof contract. In addition, the second-period fixed wage is determined by (3.139),

$$\alpha_2 = -\rho \frac{\hat{a}_2}{m_2} (y_1 - m_1 \hat{a}_1) + \frac{1}{2} \hat{a}_2^2 \left[\frac{r}{m_2^2} (1 - \rho^2) - 1 \right] . \quad (3.146)$$

At the start of the first period, the principal and the agent anticipate the terms of the second-period contract \tilde{c}_2 as determined in (3.146) and (3.145). The optimal first-period contract is a solution of the following optimization problem:

$$\max_{\alpha_1, \beta_1} E [\tilde{x}_1 + \tilde{x}_2 - \tilde{c}_1 - \tilde{c}_2] \quad (3.147)$$

subject to the participation constraint

$$ACE(\tilde{c}_1 + \tilde{c}_2 | \hat{a}_1, \hat{a}_2) \geq 0 , \quad (3.148)$$

and the incentive compatibility constraint

$$\hat{a}_1 \in \operatorname{argmax}_{a_1} ACE(\tilde{c}_1 + \tilde{c}_2 | a_1, \hat{a}_2) , \quad (3.149)$$

where \hat{a}_1 is the action to be induced by the contract in the first period, and the second period contract is anticipated to be the one given by (3.146) and (3.145), with a second-period action $\hat{a}_2 = m_2 \beta_2$.

The principal's expected cumulative surplus is given by

$$\pi = E [\tilde{x}_1 + \tilde{x}_2 - \tilde{c}_1 - \tilde{c}_2] = b_1 \hat{a}_1 + b_2 \hat{a}_2 - \alpha_1 - \beta_1 m_1 \hat{a}_1 - E[\alpha_2] - \beta_2 m_2 \hat{a}_2 . \quad (3.150)$$

The agent's certainty equivalent of cumulative compensation is given by

$$\begin{aligned} ACE(\tilde{c}_1 + \tilde{c}_2 | \hat{a}_1, \hat{a}_2) &= \alpha_1 + \beta_1 m_1 \hat{a}_1 + E[\alpha_2] + \beta_2 m_2 \hat{a}_2 - \frac{1}{2} r \operatorname{var}(\tilde{c}_1 + \tilde{c}_2) - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) \\ &= \alpha_1 + \beta_1 m_1 \hat{a}_1 + E[\alpha_2] + \beta_2 m_2 \hat{a}_2 - \frac{1}{2} r (\operatorname{var}(\tilde{c}_1) + \operatorname{var}(\tilde{c}_2)) - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) \\ &= \alpha_1 + \beta_1 m_1 \hat{a}_1 + E[\alpha_2] + \beta_2 m_2 \hat{a}_2 - \frac{1}{2} r [\beta_1^2 + \beta_2^2 (1 - \rho^2)] - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) , \end{aligned} \quad (3.151)$$

where I have used that \tilde{c}_1 and \tilde{c}_2 are independent random variables, a fact which follows directly from the expression (3.146) for α_2 .

At the time the agent chooses his first-period action,

$$\begin{aligned} E[\alpha_2|a_1, \hat{a}_1] &= -\beta_2\rho(E[y_1] - m_1\hat{a}_1) + \frac{1}{2}\beta_2^2 [r(1 - \rho^2) - m_2^2] \\ &= -m_1\beta_2\rho(a_1 - \hat{a}_1) + \frac{1}{2}\beta_2^2 [r(1 - \rho^2) - m_2^2] , \end{aligned} \quad (3.152)$$

which implies that the agent's certainty equivalent is

$$\begin{aligned} \text{ACE}(\tilde{c}_1 + \tilde{c}_2|a_1, \hat{a}_1, \hat{a}_2) &= \alpha_1 + \beta_1 m_1 a_1 - \beta_2 m_1 \rho(a_1 - \hat{a}_1) + \frac{1}{2}\beta_2^2 [r(1 - \rho^2) - m_2^2] \\ &\quad + \beta_2 m_2 \hat{a}_2 - \frac{1}{2}r [\beta_1^2 + \beta_2^2(1 - \rho^2)] - \frac{1}{2}(\hat{a}_1^2 + \hat{a}_2^2) . \end{aligned} \quad (3.153)$$

The first-order condition for the agent's incentive compatibility constraint is

$$m_1\beta_1 - \rho m_1\beta_2 - a_1 = 0 . \quad (3.154)$$

Thus, the first-period action to be induced by the contract is given by

$$\hat{a}_1 = m_1(\beta_1 - \rho\beta_2) , \quad (3.155)$$

and the first-period incentive is

$$\beta_1 = \frac{\hat{a}_1}{m_1} + \rho \frac{\hat{a}_2}{m_2} . \quad (3.156)$$

The agent chooses his first-period action in anticipation of the second-period adjustment to his fixed wage based on the first-period performance, and this is the essence of the ratchet effect. The second-period fixed wage, to be determined at the start of the second period has a component that depends on y_1 , which makes it contribute to the incentives in the first period when y_1 is still uncertain.

In equilibrium, the agent's participation constraint is binding, and the principal's conjecture of the agent's action is correct. As a result, using (3.151) and $\text{ACE}(\tilde{c}_1 + \tilde{c}_2 | \hat{a}_1, \hat{a}_2) = 0$ in (3.150) gives for the principal's expected surplus

$$\pi = b_1 \hat{a}_1 + b_2 \hat{a}_2 - \frac{1}{2} r [\beta_1^2 + \beta_2^2 (1 - \rho^2)] - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2). \quad (3.157)$$

It follows that the principal's problem can be rewritten as

$$\max_{\hat{a}_1} b_1 \hat{a}_1 + b_2 \hat{a}_2 - \frac{1}{2} r \left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho \frac{\hat{a}_1 \hat{a}_2}{m_1 m_2} + \frac{\hat{a}_2^2}{m_2^2} \right) - \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2). \quad (3.158)$$

The first-order condition is

$$b_1 - r \frac{\hat{a}_1}{m_1^2} - r \rho \frac{\hat{a}_2}{m_1 m_2} - \hat{a}_1 = 0, \quad (3.159)$$

which gives the first-period action as

$$\hat{a}_1 = \frac{1}{1 + \frac{r}{m_1^2}} \left(b_1 - r \rho \frac{\hat{a}_2}{m_1 m_2} \right). \quad (3.160)$$

Substituting \hat{a}_2 from (3.145) gives

$$\hat{a}_1 = \frac{b_1 m_1^2 (m_2^2 + r(1 - \rho^2)) - r \rho b_2 m_1 m_2}{(m_1^2 + r)(m_2^2 + r(1 - \rho^2))}. \quad (3.161)$$

The first-period incentive is then

$$\begin{aligned} \beta_1 &= \frac{\hat{a}_1}{m_1} + \rho \frac{\hat{a}_2}{m_2} \\ &= \frac{b_1 m_1 (m_2^2 + r(1 - \rho^2)) - r \rho b_2 m_2}{(m_1^2 + r)(m_2^2 + r(1 - \rho^2))} + \rho \frac{b_2 m_2}{m_2^2 + r(1 - \rho^2)}. \end{aligned} \quad (3.162)$$

The first-period fixed wage is determined by the agent's participation constraint (see (3.151)),

$$\begin{aligned} \alpha_1 &= -\beta_1 m_1 \hat{a}_1 - \frac{1}{2} \beta_2^2 [r(1 - \rho^2) - m_2^2] - \beta_2 m_2 \hat{a}_2 + \frac{1}{2} r [\beta_1^2 + \beta_2^2 (1 - \rho^2)] + \frac{1}{2} (\hat{a}_1^2 + \hat{a}_2^2) \\ &= -\beta_1 m_1 \hat{a}_1 + \frac{1}{2} r \beta_1^2 + \frac{1}{2} \hat{a}_1^2 \\ &= -\frac{1}{2} \hat{a}_1^2 - \rho \frac{m_1}{m_2} \hat{a}_1 \hat{a}_2 + \frac{1}{2} r \left(\frac{\hat{a}_1^2}{m_1^2} + 2\rho \frac{\hat{a}_1 \hat{a}_2}{m_1 m_2} + \rho^2 \frac{\hat{a}_2^2}{m_2^2} \right). \end{aligned} \quad (3.163)$$

□

Proof of Proposition 3.6.2

In the first period, the principal offers a contract $\tilde{c}_1 = \alpha_1 + \beta_1 \tilde{y}_1$ anticipating that a new agent will be hired in the second period and offered a contract $\tilde{c}_2 = \alpha_2 + \beta_2 \tilde{y}_2$. The second-period contract can make use of all available information from the first period, so that the fixed and the variable wage in the second period may depend on y_1 .

The second-period contract is a solution of the following problem:

$$\max_{\alpha_2, \beta_2} E[\tilde{x}_1 + \tilde{x}_2 - \tilde{c}_2 \mid y_1, \hat{a}_1] \quad (3.164)$$

subject to the participation constraint

$$ACE(\tilde{c}_2 \mid y_1, \hat{a}_1) \geq 0 \quad (3.165)$$

and the incentive compatibility constraint

$$\hat{a}_2 \in \operatorname{argmax}_{a_2} ACE(\tilde{c}_2 \mid y_1, \hat{a}_1, a_2), \quad (3.166)$$

where y_1 is the reported value of the first-period performance measure, \hat{a}_2 is the action to be induced by the contract in the second period, and \hat{a}_1 is the second agent's and the principal's common conjecture of what the first agent's action has been.

Since $E[\tilde{x}_1 + \tilde{x}_2 \mid y_1, \hat{a}_1] = b_1 a_1 + b_2 a_2 + E[\tilde{\lambda}_1 + \tilde{\lambda}_2 \mid y_1, \hat{a}_1]$ and a_2 has no impact on $E[\tilde{\lambda}_1 + \tilde{\lambda}_2 \mid y_1, \hat{a}_1]$, I can substitute $\pi_2 = \tilde{x}_2 - \tilde{c}_2$ for the principal's objective function in (3.164).

Since the second agent's certainty equivalent of compensation is

$$\begin{aligned} ACE(\tilde{c}_2 \mid y_1, \hat{a}_1, a_2) &= \alpha_2 + \beta_2 E[\tilde{y}_2 \mid y_1, \hat{a}_1, a_2] - \frac{1}{2} r \beta_2^2 \operatorname{var}(\tilde{y}_2 \mid y_1) - \frac{1}{2} a_2^2 \\ &= \alpha_2 + \beta_2 (m_2 a_2 + \rho(y_1 - m_1 \hat{a}_1)) - \frac{1}{2} r \beta_2^2 (1 - \rho^2) - \frac{1}{2} a_2^2, \end{aligned} \quad (3.167)$$

the first-order condition for the agent's incentive compatibility constraint gives $\hat{a}_2 = m_2\beta_2$ as the action induced by the contract. The participation constraint is binding at the optimum and determines the fixed wage as

$$\begin{aligned}\alpha_2 &= -\beta_2(m_2\hat{a}_2 + \rho(y_1 - m_1\hat{a}_1)) + \frac{1}{2}r\beta_2^2(1 - \rho^2) + \frac{1}{2}\hat{a}_2^2 \\ &= -\frac{1}{2}\hat{a}_2^2 + \frac{1}{2}r(1 - \rho^2)\frac{\hat{a}_2^2}{m_2^2} - \frac{\hat{a}_2}{m_2}\rho(y_1 - m_1\hat{a}_1) .\end{aligned}\quad (3.168)$$

Substituting in the principal's second-period expected profit gives

$$\begin{aligned}\pi_2 &= b_2\hat{a}_2 + E[\tilde{\lambda}_2 | y_1, \hat{a}_1] - \alpha_2 - \beta_2 E[\tilde{y}_2 | y_1, \hat{a}_1] \\ &= b_2\hat{a}_2 + E[\tilde{\lambda}_2 | y_1, \hat{a}_1] - \frac{1}{2}r(1 - \rho^2)\frac{\hat{a}_2^2}{m_2^2} - \frac{1}{2}\hat{a}_2^2 .\end{aligned}\quad (3.169)$$

Solving for the action chosen by the principal from the unconstrained problem gives

$$\hat{a}_2 = \frac{b_2 m_2^2}{m_2^2 + r(1 - \rho^2)} , \quad (3.170)$$

and for the principal's second-period surplus

$$\begin{aligned}\pi_2 &= b_2\hat{a}_2 + E[\tilde{\lambda}_2 | y_1, \hat{a}_1] - \frac{1}{2}[m_2^2 + r(1 - \rho^2)]\frac{\hat{a}_2^2}{m_2^2} \\ &= \frac{1}{2}b_2\hat{a}_2 + E[\tilde{\lambda}_2 | y_1, \hat{a}_1] \\ &= \frac{1}{2} \frac{b_2 m_2^2}{m_2^2 + r(1 - \rho^2)} + E[\tilde{\lambda}_2 | y_1, \hat{a}_1] .\end{aligned}\quad (3.171)$$

From (3.171) it follows that $E[\pi_2] = \frac{1}{2}b_2\hat{a}_2$ which does not depend on a_1 or y_1 . As a result, in setting the first-period incentives, the principal can ignore π_2 . The principal's first-period problem is then

$$\max_{\alpha_1, \beta_1} E[\tilde{x}_1 - \tilde{c}_1] \quad (3.172)$$

subject to the participation constraint

$$ACE(\tilde{c}_1) \geq 0 \quad (3.173)$$

and the incentive compatibility constraint

$$\hat{a}_1 \in \operatorname{argmax}_{a_1} \operatorname{ACE}(\tilde{c}_1|a_1) . \quad (3.174)$$

Since the agent's certainty equivalent of compensation is

$$\begin{aligned} \operatorname{ACE}(\tilde{c}_1|a_1) &= \alpha_1 + \beta_1 m_1 a_1 - \frac{1}{2} r \beta_1^2 \operatorname{var}(\tilde{y}_1) - \frac{1}{2} a_1^2 \\ &= \alpha_1 + \beta_1 m_1 a_1 - \frac{1}{2} r \beta_1^2 - \frac{1}{2} a_1^2 , \end{aligned} \quad (3.175)$$

it follows that the agent's induced action is $\hat{a}_1 = m_1 \beta_1$. Since the participation constraint is binding at the optimum, the fixed wage is

$$\alpha_1 = -\frac{1}{2} \hat{a}_1^2 + \frac{1}{2} r \frac{\hat{a}_1^2}{m_1^2} , \quad (3.176)$$

and the principal's expected first-period profit is

$$\begin{aligned} \pi_1 &= b_1 \hat{a}_1 + \frac{1}{2} \hat{a}_1^2 \left(1 - \frac{r}{m_1^2}\right) - \hat{a}_1^2 \\ &= \hat{a}_1 - \frac{1}{2} \hat{a}_1^2 \left(1 + \frac{r}{m_1^2}\right) . \end{aligned} \quad (3.177)$$

Thus, the principal's optimal choice of action for the first agent is

$$\hat{a}_1 = \frac{b_1 m_1^2}{m_1^2 + r} , \quad (3.178)$$

and the expected first-period surplus is

$$\pi_1 = b_1 \hat{a}_1 - \frac{1}{2} b_1 \hat{a}_1 = \frac{1}{2} b_1 \hat{a}_1 = \frac{1}{2} \frac{b_1^2 m_1^2}{m_1^2 + r} . \quad (3.179)$$

The principal's total expected surplus for both periods is $\pi = \pi_1 + E[\pi_2]$. \square

Proof of Proposition 3.7.1

The proof is a straightforward computation. First, I calculate the principal's surplus separately for the two periods under commitment and fair wages. The expected second-period surplus is (see (3.143), (3.145))

$$\begin{aligned} \pi_2^F &= b \hat{a}_2 - \frac{1}{2} \frac{\hat{a}_2^2}{m^2} [m^2 + r(1 - \rho^2)] = \frac{1}{2} b \hat{a}_2 \\ &= \frac{1}{2} \frac{b^2 m^2}{m^2 + r(1 - \rho^2)} . \end{aligned} \quad (3.180)$$

For the first period, a similar calculation gives (see equations (3.163), (3.162), and (3.161))

$$\begin{aligned}
 \pi_1^F &= b\hat{a}_1 - \alpha_1 - m\beta_1\hat{a}_1 \\
 &= b\hat{a}_1 - \frac{1}{2} \frac{r}{m^2} (\hat{a}_1 + \rho\hat{a}_2)^2 - \frac{1}{2} \hat{a}_1^2 \\
 &= b\hat{a}_1 - \frac{1}{2} \frac{r}{m^2} (\hat{a}_1^2 + 2\rho\hat{a}_1\hat{a}_2 + \rho^2\hat{a}_2^2) - \frac{1}{2} \hat{a}_1^2 \\
 &= b\hat{a}_1 - \frac{1}{2} (m^2 + r) \frac{\hat{a}_1^2}{m^2} - r\rho \frac{\hat{a}_1\hat{a}_2}{m^2} - \frac{1}{2} r\rho^2 \frac{\hat{a}_2^2}{m^2} .
 \end{aligned} \tag{3.181}$$

Since

$$\hat{a}_1 = \frac{bm^2 - r\rho\hat{a}_2}{m^2 + r} , \tag{3.182}$$

it follows that

$$\begin{aligned}
 \pi_1^F &= \frac{b^2m^2 - r\rho b\hat{a}_2}{m^2 + r} - \frac{1}{2} \frac{(bm^2 - r\rho\hat{a}_2)^2}{m^2(m^2 + r)} - r\rho\hat{a}_2 \frac{bm^2 - r\rho\hat{a}_2}{m^2(m^2 + r)} - \frac{1}{2} r\rho^2 \frac{\hat{a}_2^2}{m^2} \\
 &= \frac{1}{2} \frac{1}{m^2(m^2 + r)} [2bm^2(bm^2 - r\rho\hat{a}_2) - (bm^2 - r\rho\hat{a}_2)^2 \\
 &\quad - 2r\rho\hat{a}_2(bm^2 - r\rho\hat{a}_2) - (m^2 + r)r\rho^2\hat{a}_2^2] \\
 &= \frac{1}{2} \frac{1}{m^2(m^2 + r)} [(bm^2 - r\rho\hat{a}_2)^2 - r\rho^2\hat{a}_2^2(m^2 + r)] \\
 &= \frac{1}{2} \frac{b^2m^2}{m^2 + r} - \frac{1}{2} \frac{1}{m^2 + r} r\rho\hat{a}_2(2b + \rho\hat{a}_2) .
 \end{aligned} \tag{3.183}$$

From Proposition 3.6.2, I have that

$$\pi^T = \pi_1^T + \pi_2^T = \frac{1}{2} \frac{b^2m^2}{m^2 + r} + \frac{1}{2} \frac{b^2m^2}{m^2 + r(1 - \rho^2)} , \tag{3.184}$$

which implies that

$$\pi^F - \pi^T = -\frac{1}{2} \frac{b^2m^2}{m^2 + r} r\rho\hat{a}_2(2b + \rho\hat{a}_2) . \tag{3.185}$$

The last calculation is for the difference in principal's surplus under full commitment and

no commitment

$$\begin{aligned}
\pi^C - \pi^T &= \frac{b^2 m^2}{m^2 + r + r\rho} - \frac{1}{2} \frac{b^2 m^2}{m^2 + r} - \frac{1}{2} \frac{b^2 m^2}{m^2 + r(1 - \rho^2)} \\
&= b^2 m^2 \frac{2(m^2 + r)(m^2 + r - r\rho^2) - (2(m^2 + r) - r\rho^2)(m^2 + r + r\rho)}{2(m^2 + r)(m^2 + r - r\rho^2)(m^2 + r + r\rho)} \\
&= \frac{2(m^2 + r)^2 - 2(m^2 + r)r\rho^2 - 2(m^2 + r)^2 - 2r\rho(m^2 + r) + (m^2 + r)r\rho^2 + r^2\rho^3}{2(m^2 + r)(m^2 + r - r\rho^2)(m^2 + r + r\rho)} \\
&= b^2 m^2 \frac{r\rho[r\rho^2 - 2(m^2 + r) - \rho(m^2 + r)]}{2(m^2 + r)(m^2 + r - r\rho^2)(m^2 + r + r\rho)} \\
&= b^2 m^2 \frac{-r\rho[m^2 + r(1 - \rho^2) + (m^2 + r)(1 + \rho)]}{2(m^2 + r)(m^2 + r - r\rho^2)(m^2 + r + r\rho)} .
\end{aligned}$$

(3.186)

□

Chapter 4

Multiperiod Ratchet Effect and Managerial Tenure

4.1 Introduction

Firms have long-term relationships with economic agents such as managers, auditors, and suppliers that can be characterized as sequences of shorter lived agents interacting with a longer lived firm. To stylize this to the extreme, the firm is infinitely lived, and the agents are finitely lived. In this framework, the manager's tenure with the firm becomes the object of investigation. For example, Hambrick and Fukutomi [15] propose a model of the dynamics of the CEO's tenure in office. Their analysis considers the CEO's performance in relation to the number of periods that have passed since the CEO started the current job. In this context, a natural question is to examine the role played by beliefs about the duration of tenure, where the duration of tenure is the total number of periods the agent will work for the firm. Thus, when the firm starts a multi-period relationship with an agent, what is the role played by beliefs about the duration of the relationship? If the length of the contract can be chosen ex-ante, and commitment to a certain tenure is possible, is there an optimal ex-ante tenure?

For example, if a manager is hired with a long-term contract, what is the importance of the length of the contract (as determined at contracting time, and assuming away commitment issues)? In auditing, a similar question arises from the fact that auditors stay with the same firm for long periods of time. For auditors, beliefs about tenure are important for multi-period pricing under perfect competition which leads to lowballing.

These issues are examined by Šabac and Simunic [34], who show that there is no ex-ante optimal tenure, but that tenure beliefs can lead to auditor replacements in equilibrium.

In principal-agent models, the agent's tenure is important because it impacts the amount and the characteristics of the information available for contracting. Specifically, if in each period the accounting system produces a piece of information that is used as a performance measure in contracting with the manager, the number of periods the manager is employed determines the informational environment for contracting.

As Feltham and Xie [11] show, multiple correlated performance measures are valuable because insurance effects allow the principal to reduce the risk imposed on the agent. Their setting corresponds to a multi-period world where the principal can commit to not renegotiate contract terms at later dates as subsequent information is revealed. If such commitment is not possible, and the principal chooses the sequentially optimal contract instead, then the agent's effort is influenced by the ratchet effect.

The ratchet effect has been described in the economics literature in connection with centrally planned economies (see for example Litwack [25] and the references therein), and more generally in settings where the agent is privately informed. The book by Laffont and Tirole provides a detailed analysis and references [23]. Their description of the ratchet effect is as follows: "If [a regulated firm] produces at a low cost today, the regulator may infer that low costs are not hard to achieve and tomorrow offer a demanding incentive scheme. That is, the firm jeopardizes future rents by being efficient". The essence of the ratchet effect with a privately informed agent is that the agent can obtain a rent in future periods by hiding his type in the current period.

Weitzman [36] presents a multi-period model of the ratchet effect with moral hazard only, but in his model the ratcheting mechanism is exogenous. More recently, Milgrom and Roberts [29] and Indjejikian and Nanda [18] have shown that there is a ratchet effect in two period models with moral hazard but without adverse selection. In these models,

the ratcheting is endogenous and driven by the lack of commitment by the principal regarding the use of available information. Milgrom and Roberts [29] define the ratchet effect as “the tendency for performance standards to increase after a period of good performance”. Given that the principal will use today’s outcome in writing tomorrow’s contract creates for the manager a link between today’s effort and tomorrow’s standard of performance. The ratchet effect is always inefficient in the models with adverse selection since good types will mimic bad types and earn rents. In the pure moral hazard model, the ratchet effect is also inefficient with respect to the full commitment solution as shown by Indjejikian and Nanda [18].

Ratcheting results from the principal’s ability to optimally adjust the agent’s second-period incentive for the lower ex-post variance of the second performance measure by using the first-period performance measure when the principal cannot commit fully to a long-term contract. The assumption that the two periods are correlated is thus crucial, and differentiates this model of ratcheting in a pure moral hazard setting from other models of sequential action choice. Repeated moral hazard with independent periods is analyzed by Lambert [24], Rogerson [33], Holmstrom and Milgrom [17], and Fudenberg, Holmstrom and Milgrom [12].

Matsumura [27] presents an analysis of sequential action choice with correlated outcomes in a single period in which the agent observes a first outcome before selecting the second action. However, in Matsumura’s model, there is no contracting after the first outcome is observed, and this outcome is private agent information until the end of the period. As a consequence, there is no renegotiation in the second period based on the first-period performance.

In Chapter 3, I extended the analysis of Indjejikian and Nanda [18] to include commitment issues and the possibility of agent turnover. The central theme of Chapter 3 is

the analysis of different levels of commitment and a comparison of the principal's welfare under different commitment scenarios and different correlation of the performance measures. Regarding agent turnover, the main insight from Chapter 3 is that, in a two-period LEN model, the two agent solution (turnover or no commitment) is the least preferred when the performance measures are negatively correlated, with long-term full commitment and long-term commitment with renegotiation (or commitment to fairness) dominating. The result is the opposite for positive correlation of the performance measures, in that the two-agent solution (turnover or no commitment) dominates long-term full commitment.

Both results are driven by the way in which the correlation of the performance measures impacts the risk premia that have to be paid to the agent(s). Due to the particular nature of the LEN framework, which essentially puts the agent in a mean-variance world, these risk premia are completely determined by the variances of the performance measures. The driving force behind this result is that, with negative correlation, having the same agent in both periods reduces the total risk to which the agent is exposed by the optimal incentive scheme. When the correlation is positive, using a different agent in each period eliminates the risk premium due to the correlation between the optimal compensation schemes for each action. The situation is as if there is a compensation scheme for the first-period action \tilde{c}_1 and a compensation scheme for the second-period action \tilde{c}_2 . With one agent, the risk for which the principal pays compensation is $\text{var}(\tilde{c}_1 + \tilde{c}_2) = \text{var}(\tilde{c}_1) + 2\text{cov}(\tilde{c}_1, \tilde{c}_2) + \text{var}(\tilde{c}_2)$. With two agents, the risk for which the principal pays compensation is $\text{var}(\tilde{c}_1) + \text{var}(\tilde{c}_2)$.

These results regarding turnover indicate that, in a two-period world, the principal prefers to replace the agent after one period when the performance measures are positively correlated. On the other hand, the principal prefers even the weakest form of long-term commitment with a single agent when the performance measures are negatively

correlated. These results can be extended to more than two periods. In a setting with more than two periods, one can consider the choice of alternative tenures in a given time horizon. In a realistic setting, tenure is a choice variable in a world with more than two periods. In addition, when considering tenure (or replacement policies), the analysis can be framed in different ways. For example, regarding optimal tenure one may ask:

1. In a given finite number of periods, what is the optimal number of agents and their respective tenures? In other words, given a known finite life for the firm, how can one optimally partition that among several agents, and what are tenures of those agents?
2. In an infinite world, what is the optimal tenure for agents, if we assume a policy of replacing agents after the same number of periods (tenure)?

The main difference between the two frameworks, and between the way questions are answered is that in one world there is a “last period”, whereas in the other world, there is no “last period”. The infinite period world permits the comparison of different tenure lengths in a natural way. A similar comparison of these two frameworks is undertaken in Chapter 3 regarding auditor tenure. In what follows, I extend the two-period model of Chapter 3 in order to answer the second question posed above: what is the optimal (stationary) agent turnover policy in an infinite-period world?

In this paper I develop an N-period model of the ratchet effect in a principal-agent problem with moral hazard but without adverse selection. Thus, while the agent’s action is unobservable by the principal, in equilibrium the principal has rational beliefs regarding the agent’s past actions and as a result, in equilibrium, information asymmetries do not develop over time between the principal and the agent. In addition, there is no learning of productivity or any agent characteristic that is unknown at the start. The only dynamic information effects are the adjustment of posterior beliefs about future

performance measures, conditional on the sequential observation of past performance measures together with conjectures of the agent's past actions. The model generalizes the two-period model of Chapter 3, and most results derived therein remain valid in the N-period case. Primarily, the conclusions regarding the role of commitment in obtaining different solutions remain the same. However, the N-period model gives insights into the importance of the contracting horizon, or tenure. In addition, the N-period model offers insights into the agent's long-term performance that cannot be inferred from the two-period model.

While in Chapter 3 I have compared different commitment scenarios given a fixed two-period horizon, in this paper the emphasis is on tenure given a certain choice of commitment assumptions. Since the very idea of tenure implies some form of implicit or explicit long-term commitment, the choice is between the three types of long-term commitment discussed in Chapter 3: full commitment to a long-term contract, commitment to a long-term contract with renegotiation, and commitment to fairness with short-term contracts. Full commitment to a long-term contract is too restrictive, in that, especially over longer horizons, renegotiation is more likely. Commitment to fairness is a mechanism that replicates the commitment to a long-term contract with renegotiation by using a sequence of short-term contracts. This makes the choice between the two forms of contracting almost a matter of taste.

In this paper, I choose to restrict the analysis to contracting under commitment to fairness. Besides capturing the idea of renegotiation, it only requires short-term contracts that are settled at the end of each period, allowing for a breakdown of performance and surplus period-by-period. In addition, the principal and the agent only need to commit to "fair contracts" and to a tenure duration, no other long-term commitments or contracts are necessary.¹ Finally, the commitment to fairness solution to the dynamic

¹The reader who finds the concept of "commitment to fairness" unpalatable can interpret all results

agency problem provides consistency and ease of comparison to previous literature such as Milgrom and Roberts [29] and Indjejikian and Nanda [18].

The driving force behind the “ratchet effect” with pure moral hazard described in this paper is the principal’s inability to commit not to use past performance measures when setting a short-term contract with the agent in each period other than the agent’s first period of tenure. The principal has an incentive to set the agent’s compensation in a later period based on past performance measures in order to optimally adjust the risk in the agent’s contract for the lower posterior variances. In doing so, the principal makes the fixed pay in future periods depend on past performance measures. For the agent, it means that from an *ex ante* (previous periods) perspective, future fixed pay depend on earlier effort choices. In other words, “fixed compensation” in one period is fixed only in that period, given the past actions and performance measures, but is variable when anticipated from earlier periods. It follows that the agent’s incentives for effort in any one period are spread among the variable wage for that period and the fixed wages in all future periods.

The nature of the solution to the dynamic agency problem is such that, for longer horizons, the manager’s effort is close to some limit level for most periods. Thus, there are essentially two effort levels, the second best in the last period, and approximately a “third best” in most other periods. When correlation is positive, the “third best” is lower than the second best, generating inefficiencies relative to a repeated one-period problem (which is the multi-period problem when periods are independent). When the correlation is negative, the ratchet effect is efficient relative to the uncorrelated periods case since the “third best” effort level is closer to first best. To summarize, with positively correlated performance measures, incentives are stronger and the manager exerts less effort in most periods than in the last period. With negatively correlated performance

in the context of commitment to a long-term contract with renegotiation.

measures, incentives are less strong and the manager exerts more effort in most periods than in the last period. The effort level in the last period serves as a benchmark because it coincides with the second-best solution, which is what one obtains with uncorrelated performance measures.

A commitment problem could arise in the case of negatively correlated performance measures due to the decreasing performance of the agent towards the end of his tenure. The principal needs to be able to commit not to fire the manager before term, otherwise the efficiency gains from getting effort levels close to first-best are lost. This would not be a problem in the positive correlation case since there the principal gets better performance towards the last period, and has no incentive to fire the manager before term. In either case, commitment to fairness assumes that the principal commits to retain the agent for all N periods.

To answer the question of ex-ante choice of tenure by the principal, I examine optimal stationary replacement policies, whereby the agents are hired for N periods at a time within an infinite horizon for the firm, and the noise in the performance measures is firm-specific. The main result for positively correlated performance measures is that, in the presence of a switching cost, there exists a threshold switching cost such that optimal tenure is a single period whenever the switching cost is higher than the threshold value. On the other hand, optimal tenure is the maximum number of periods possible (the agent's maximum life) when the switching cost is lower than the threshold value. Thus, with positively correlated performance measures, the only optimal replacement (tenure) policies are the corner solutions of one period tenure or maximum possible tenure. The main result for negatively correlated performance measures is that the optimal replacement policy is always the maximum number of periods possible, irrespective of the switching cost.

In the case when the noise in the performance measures is agent-specific, there is

an additional “learning” effect in the first few periods of a manager’s tenure due to the (rapid) reduction of the posterior variances of the performance measures towards their limit value. This effect results in an increase of managerial effort, since it becomes less costly over time for the principal to motivate managerial effort due to lower risk premia. With negatively correlated performance measures and agent-specific noise, the manager exerts the second-best effort in the last period, a higher effort level in all other periods, and that effort has an inverted U shape. This finding is consistent with evidence from the management literature that firm performance increases at first, reaches a maximum, and then declines during a manager’s tenure. For a discussion of these findings, see the papers by Eitzen and Yetman [9], Katz [21], and Hambrick and Fukutomi [15].

The remainder of the paper is organized as follows. In Section 2, I present the principal-agent model and the informational environment for contracting. In Section 3, I present the full-commitment long-term contract solution to the agency problem. In section 4, I discuss long-term contracts with renegotiation, and the optimal renegotiation-proof contract. In section 5, I turn to short-term contracts under commitment to fairness and I derive the first key result, the optimal sequence of “fair” short-term contracts and induced actions. I also show that the commitment to fairness mechanism replicates the payoffs of the renegotiation-proof contract. Section 6 is devoted to the discussion of the dynamics of effort and incentives given the “fair contracts” solution derived in Section 5. In Section 7 I examine the question of agent replacement policies with a fixed switching cost and derive the second key result of the paper, the solution to the optimal tenure problem. Section 8 concludes the paper and the Appendix contains some of the more technical proofs.

4.2 The principal-agent model

In this section I present an N -period generalization of the principal-agent model from Chapter 3, in which the agent's performance measures are correlated. The model is a pure moral hazard model similar to the one found in Indjejikian and Nanda [18]; the agent does not have any private information other than his own effort level.

A risk-neutral principal owns a production technology that requires productive effort from an agent in N periods $t = 1, \dots, N$. The agent is risk- and effort-averse with exponential utility and quadratic effort cost of the form $u(w, a) = -\exp[-r(w - \frac{1}{2}(a_1^2 + \dots + a_N^2))]$, where w is the agent's terminal wealth and $a = (a_1, \dots, a_N)$ is the agent's effort at the start of periods 1 through N . The agent's certainty equivalent of terminal wealth \tilde{w} and effort a is, assuming \tilde{w} to be normally distributed,

$$\text{ACE}(\tilde{w}, a_1, \dots, a_N) = E[\tilde{w}] - \frac{1}{2}r \text{var}(\tilde{w}) - \frac{1}{2}(a_1^2 + \dots + a_N^2) . \quad (4.187)$$

The output from agent's effort $a_t \in \mathbb{R}$ is, for $t = 1, \dots, N$,

$$\tilde{z}_t = b_t a_t + \tilde{\lambda}_t , \quad (4.188)$$

where λ_t is an arbitrary mean zero noise term which does not depend on a_t in any way. The outcomes z_t are not observed until after the termination of the contract at the end of period N . Hence, the output \tilde{z}_t only determines the principal's expected surplus, and since the principal is risk-neutral, no further distributional assumptions are needed regarding $\tilde{\lambda}_t$. The agent's actions are unobservable. Hence, neither the output nor the agent's actions are contractible.

A contractible performance measure x_t is observed at the end of each period. The agent's effort in period t affects only the mean of the performance measure in that period,

$$\tilde{x}_t = m_t a_t + \tilde{\varepsilon}_t , \quad (4.189)$$

where $\tilde{\varepsilon}_t$ are mean zero noise terms. The noise terms in the performance measures are joint normally distributed with variance-covariance matrix Σ_N . In addition, for each t , denote by Σ_t the variance-covariance matrix of $\tilde{x}_1, \dots, \tilde{x}_t$. The variance-covariance matrix Σ_N is defined recursively as follows.

$$\Sigma_1 = \begin{bmatrix} \sigma \end{bmatrix} \quad (4.190)$$

$$\Sigma_t = \begin{bmatrix} \sigma & \rho e'_1(t-1) \\ \rho e_1(t-1) & \Sigma_{t-1} \end{bmatrix} \quad (4.191)$$

where $e'_1(t) = (1, 0, \dots, 0)$ is a row vector of length t .

For each $1 \leq t \leq N$, let $\underline{a}_t = (a_1, \dots, a_t)$, and $\underline{x}_t = (x_1, \dots, x_t)$ denote the histories of means and outcomes from the first t variables in the sequence.

Important notation convention. Throughout the paper the variance-covariance matrices are written such that the lower right corner corresponds to the lowest value of the time index and the upper left corner corresponds to the highest value of the time index. For example, the matrix Σ_t is written as follows:

$$\Sigma_t = \begin{bmatrix} \text{cov}(\tilde{x}_t, \tilde{x}_t) & \text{cov}(\tilde{x}_t, \tilde{x}_{t-1}) & \dots & \text{cov}(\tilde{x}_t, \tilde{x}_1) \\ \text{cov}(\tilde{x}_{t-1}, \tilde{x}_t) & \text{cov}(\tilde{x}_{t-1}, \tilde{x}_{t-1}) & \dots & \text{cov}(\tilde{x}_{t-1}, \tilde{x}_1) \\ \dots & \dots & \dots & \dots \\ \text{cov}(\tilde{x}_1, \tilde{x}_t) & \text{cov}(\tilde{x}_1, \tilde{x}_{t-1}) & \dots & \text{cov}(\tilde{x}_1, \tilde{x}_1) \end{bmatrix}$$

In terms of the interpretation of the signals \tilde{x}_t as managerial action (effort) plus noise, the noise terms are joint normally distributed and $\tilde{\varepsilon}_t \sim N(0, \sigma)$, $\text{cov}(\tilde{\varepsilon}_t, \tilde{\varepsilon}_{t \pm 1}) = \rho$ and $\text{cov}(\tilde{\varepsilon}_t, \tilde{\varepsilon}_{t \pm k}) = 0$ for all $k \geq 2$.

The particular form of Σ_N gives the simplest variance-covariance matrix in which there is correlation among periods. The structure is such that, in the sequential model, the entire history is relevant in conditioning the current variable, although variables more

than one period in the past are independent of the current one. When $\rho = 0$, $\Sigma_N = \sigma I_N$, where I_N is the $N \times N$ identity matrix and in this case the noise terms $\tilde{\varepsilon}_t$ are i.i.d.

The variances and covariances of the noise terms need to be restricted for the matrix Σ_N to be positive definite. The key requirement is that the absolute value of the covariance between two adjacent periods' noise does not exceed half the variance of each noise term. The following Lemma formally proves this result.

Lemma 4.2.1 *Let $D_t = \det(\Sigma_t)$ for all $t \geq 1$ and set $D_0 = 1$. Then, for all $t \geq 1$,*

$$D_{t+1} = \sigma D_t - \rho^2 D_{t-1} . \quad (4.192)$$

In addition, $\sigma > 0$ and $0 \leq |\rho| \leq \sigma/2$ are sufficient conditions for Σ_t to be positive definite for all $t \leq N$.

Proof. See the Appendix. \square

An alternative description of the performance measures can be given as follows. When the correlation is positive, the noise terms can be decomposed as

$$\tilde{\varepsilon}_{t-1} = \tilde{\nu}_{t-1} + \tilde{\delta}_{t-2} + \tilde{\delta}_{t-1} \quad (4.193)$$

$$\tilde{\varepsilon}_t = \tilde{\nu}_t + \tilde{\delta}_{t-1} + \tilde{\delta}_t \quad (4.194)$$

$$\tilde{\varepsilon}_{t+1} = \tilde{\nu}_{t+1} + \tilde{\delta}_t + \tilde{\delta}_{t+1} , \quad (4.195)$$

where $\tilde{\nu}_t$, are period-specific, and $\tilde{\delta}_t$ is a common component in adjacent periods. All terms are mutually independent and $\text{var}(\tilde{\delta}_t) = \rho$. The common (between adjacent periods) component $\tilde{\delta}_t$ can be thought of as a shock that persists from one period to the next period. The noise component δ_t first appears in period t , persists for one period and then disappears. In period $t + 1$, a new noise term δ_{t+1} appears, and so on.

When the correlation of the performance measures is negative, the noise terms can be decomposed as

$$\tilde{\varepsilon}_{t-1} = \tilde{\nu}_{t-1} - \tilde{\delta}_{t-2} + \tilde{\delta}_{t-1} \quad (4.196)$$

$$\tilde{\varepsilon}_t = \tilde{\nu}_t - \tilde{\delta}_{t-1} + \tilde{\delta}_t \quad (4.197)$$

$$\tilde{\varepsilon}_{t+1} = \tilde{\nu}_{t+1} - \tilde{\delta}_t + \tilde{\delta}_{t+1} , \quad (4.198)$$

where $\tilde{\nu}_t$, are period-specific, and $\tilde{\delta}_t$ is a common component in adjacent periods. All terms are mutually independent and $\text{var}(\tilde{\delta}_t) = \rho$. The common (between adjacent periods) component $\tilde{\delta}_t$ can be thought of as period t accruals that have to be reversed in the next period, $t + 1$. Thus, negatively correlated noise in accounting-based performance measures reflects the nature of the accrual process. In this model, however, the accruals, as with all the other components of the noise in the performance measure, are outside the manager's control.

In view of the above discussion, the result of Lemma 4.2.1 can be easily interpreted. First note that, given the mutual independence of the terms in the decompositions of noise ε_t ,

$$\sigma = \text{var}(\tilde{\varepsilon}_t) = \text{var}(\tilde{\nu}_t) + \text{var}(\tilde{\delta}_{t-1}) + \text{var}(\tilde{\delta}_t) = \text{var}(\tilde{\nu}_t) + 2\rho . \quad (4.199)$$

It then follows, that unless there is no period-specific noise $\tilde{\nu}_t$, the covariance ρ is always less than half the variance σ . The limit case is when there is no period-specific noise, and the noise terms consist only of the components that carry over from period to period. This restriction is specific to the N -period model, since there are two terms that carry over from period to period in each of the "middle" noise terms $\tilde{\varepsilon}_t$ with $t \neq 1, N$. By contrast, in the two-period case, there is only one term that carries over from period to period, and the correlation is unrestricted between -1 and 1.

The principal owns the production technology for N periods and needs an agent to supply productive effort. There is more than one agent that the principal can employ in each period. All agents are identical (the agents have the same ability and the same utility functions) and have alternative employment opportunities. Each agent's reservation certainty equivalent is normalized to zero in each period. Both the principal and the agents are assumed to have discount rates of zero. Utility functions, discount rates, reservation wages, the nature of the production technology, and the information structure are common knowledge.

Throughout the paper, the following basic time line is present. At the start of the first period, the principal and the agent sign a contract. After contracting, in the first period, the agent provides effort a_1 , after which the first performance measure is reported. At the end of the first period, the agent may receive some compensation depending on the particular contracting setting. At the start of period $t > 1$, recontracting, or contracting for period t may take place, depending on the particular setting. After contracting, the agent provides effort a_t , and then the performance measure x_t is reported. At the end of period N , all remaining contracts are settled. After the end of period N (i.e., after all contracts are settled), the outcomes z_1, \dots, z_N are revealed to the principal. Thus, in each period, all or some of the following events occur in this order: contracting (at the start of the period), productive effort supplied by agent, performance measure reported, contract(s) settled (at the end of the period).

As in the two-period case, four types of commitment can be considered: full commitment to a long-term contract, commitment with renegotiation, commitment to fairness, and no commitment. The first two types of commitment (full commitment, commitment with renegotiation) assume that the agent is compensated by a long-term contract $c(\tilde{x}_1, \dots, \tilde{x}_N)$ to be settled at the end of period N . The other two types of commitment (commitment to fairness, no commitment) assume that the agent is compensated by a

series of short-term contracts $(c_1(\tilde{x}_1), \dots, c_N(x_1, \dots, \tilde{x}_N))$ to be settled at the end of each period.

Contracts are always assumed to be linear, and the only contractible information is given by the N performance measures. The resulting contracts are thus the *optimal linear contracts* in each case, although linear contracts are not optimal.² The main issue here is agent tenure in an N -period LEN³ framework and how lack of commitment gives rise to ratcheting.

The above description of the information environment gives the ex-ante (time zero) structure. In developing the principal-agent model in which contracting and action choice take place sequentially, I need to describe the information structure at each point in time given a realization of history up to that point. Since $\tilde{x}_1, \dots, \tilde{x}_N$ are joint normally distributed, it follows (see for example Greene(1997), Theorem 3.6) that the conditional distribution of $\tilde{x}_{t+1}, \dots, \tilde{x}_N$ given $\tilde{x}_1, \dots, \tilde{x}_t$ is also normal.

Lemma 4.2.2 *The conditional distribution of future information given the history from the first t periods, specified past actions \underline{a}_t , and specified future actions a_{t+1}, \dots, a_N is normal and given by*

$$(\tilde{x}_{t+1}, \dots, \tilde{x}_N) | (\underline{x}_t, \underline{a}_t, \hat{a}_{t+1}, \dots, \hat{a}_N) \sim N((E_t[\tilde{x}_{t+1}], \dots, E_t[\tilde{x}_N]), \Sigma_{t+1,N})$$

where the conditional means are determined by

$$E_t[\tilde{x}_{t+1}] = m_{t+1}a_{t+1} + \rho e'_1(t) \Sigma_t^{-1} (\underline{x}_t - \underline{m}_t \underline{a}_t)' \quad (4.200)$$

²The Holmström and Milgrom [17] framework is not applicable here primarily because the periods are not independent (the fact that the performance measures are correlated is a key assumption in my model, while independence is a key assumption in the Holmström and Milgrom model). Holmström and Milgrom [17] present sufficient conditions under which linear contracts are optimal in a multi-period agency. In their model, the agent's actions generate a sequence of independent binary signals (one in each period) and the agent's utility is exponential.

³Linear contracts, Exponential utility, Normal distributions.

$$E_t[\tilde{x}_{t+2}] = m_{t+2}a_{t+2}, \dots, E_t[\tilde{x}_N] = m_N a_N \quad (4.201)$$

and the conditional variances are determined by

$$\Sigma_{t+1,t+1} = \begin{bmatrix} \sigma_{t+1} \end{bmatrix} \text{ where } \sigma_{t+1} = \sigma - \rho e_1'(t) \Sigma_t^{-1} \rho e_1(t) \quad (4.202)$$

$$\Sigma_{t+1,k} = \begin{bmatrix} \sigma & \rho e_1'(t-1) \\ \rho e_1(t-1) & \Sigma_{t+1,k-1} \end{bmatrix} \text{ for } k \geq t+2 \quad (4.203)$$

In other words, the information structure is the same at time t as at time zero except that the conditional expectation and the conditional variance of \tilde{x}_{t+1} are affected. I will use the notation $E_t[\cdot]$ for conditional expectation given history \underline{x}_t and $\text{cov}_t(\cdot, \cdot)$ for conditional covariance given history \underline{x}_t . Throughout the paper, the notation $\underline{m}_t \underline{a}_t$ refers to multiplication component by component, that is $\underline{m}_t \underline{a}_t = (m_1 a_1, \dots, m_t a_t)$.

Since the means of the performance measures given past outcomes are influenced by managerial effort, conditional expectations will depend on either observed or conjectured past managerial effort. Specifically, the manager knows his past actions \underline{a}_{t-1} , so from his perspective, $E_t[\cdot] = E_t[\cdot | \underline{x}_t, \underline{a}_t]$. The principal, on the other hand, does not observe the manager's actions, but has conjectures about the agent's past actions $\hat{\underline{a}}_t = (\hat{a}_1, \dots, \hat{a}_t)$. Thus, from the principal's perspective, $E_t[\cdot] = E_t[\cdot | \underline{x}_t, \hat{\underline{a}}_t]$. In addition, the conditional expectations at time t for any future performance measures $E_t[x_{t+k}]$ implicitly assume conjectured future actions \hat{a}_{t+k} both from the principal's and the agent's perspective.

Note that $\sigma_t = \text{var}(\tilde{x}_t | \underline{x}_{t-1})$ is the conditional variance of \tilde{x}_t given history \underline{x}_{t-1} . The conditional variances do not depend on the agent's actions and represent the common posterior beliefs of the principal and the agent about the variance of future performance measures given past observations of the performance measures. In addition, note that while the conditional expectations depend on past observed values of the performance

measures, conditional variances do not depend on these values, but only on time and the observability of these performance measures.

I now turn to the derivation of explicit formulas for σ_t and $E_t[\tilde{x}_{t+1}]$.

Lemma 4.2.3 $E_t[\tilde{x}_{t+1}] = m_{t+1}a_{t+1} + R_t \cdot (\underline{x}_t - \underline{m}_t \underline{a}_t)'$, where

$$\begin{aligned} R_t &= \frac{\rho}{D_t} (D_{t-1}, -\rho D_{t-2}, \dots, (-\rho)^{t-1}) \\ &= \rho \left(\frac{1}{\sigma_t}, \frac{-\rho}{\sigma_t \sigma_{t-1}}, \frac{(-\rho)^2}{\sigma_t \sigma_{t-1} \sigma_{t-2}}, \dots, \frac{(-\rho)^{t-1}}{\sigma_t \dots \sigma_1} \right) \end{aligned} \quad (4.204)$$

$$\text{and } \sigma_{t+1} = \text{var}_t(\tilde{x}_{t+1}) = \frac{D_{t+1}}{D_t}.$$

In addition, $(\sigma_t)_{t \geq 1}$ is a decreasing sequence and $\sigma_t > \mu_1$, where $\mu_1 = \frac{\sigma + \sqrt{\sigma^2 - 4\rho^2}}{2}$.

Proof. See the Appendix. \square

Lemma 4.2.3 shows how past history influences conditional expectations for the current period's variable. Thus, to obtain the current conditional expectation, the unconditional mean is corrected by a sum of weighted deviations of past results from their means. The signs of the weights of these corrections are alternating if the correlation between periods is positive; otherwise, all weights are negative. In absolute value, the weights attached to events from the more distant past are smaller relative to more recent events. This follows from the fact that $\rho \leq \sigma/2 \leq \mu_1 < \sigma_t$ and the fact that each weight is a product of the form $\pm(\rho/\sigma_{k-1}) \dots (\rho/\sigma_{k-p})$. The intuition is that the influence of distant events is weaker than that of closer ones because the correlation is not direct but through a chain of pairwise correlated variables.

Also note that R_t can be rewritten as

$$R_t = \frac{\rho}{\sigma_t} (1, R_{t-1}), \quad (4.205)$$

which implies

$$E_t[\tilde{x}_{t+1}] = m_{t+1}a_{t+1} + \frac{\rho}{\sigma_t}(x_t - E_{t-1}[\tilde{x}_t]) . \quad (4.206)$$

When the correlation between periods is positive, the current conditional expectation is adjusted upwards from the mean by a term proportional to the amount that the last outcome exceeded *previous expectations*. Thus, an increase in past expectations $E_{t-1}[\tilde{x}_t]$ will decrease the current expectation $E_t[\tilde{x}_{t+1}]$. When the correlation between periods is negative, the opposite is true; an increase in the last period's outcome will decrease the current expectation, while an increase in last period's expectation will increase the current expectation. The proportionality factor in equation (4.206), ρ/σ_t is essentially a conditional correlation since $\rho/\sigma_t = \text{cov}_{t-1}(\tilde{x}_t, \tilde{x}_{t+1})/\text{var}_t(\tilde{x}_t)$.

Given linear contracts and normally distributed performance measures, the agent's wealth is normally distributed as well, which implies that the agent's certainty equivalent of wealth and effort is given by equation (4.187). More precisely, given a linear contract in all N performance measures, the certainty equivalent of compensation at the start of the first period is given by

$$\text{ACE}(\tilde{w}, a_1, \dots, a_N) = E[\tilde{w}] - \frac{1}{2}r \text{var}(\tilde{w}) - \frac{1}{2}(a_1^2 + \dots + a_N^2) , \quad (4.207)$$

where a_1, \dots, a_N are the actions that the agent expects to take in the N periods. Similarly, for any contract that is linear in performance measures x_t, \dots, x_N , the agent's certainty equivalent at the start of period t is given by

$$\begin{aligned} \text{ACE}(\tilde{w}, \hat{a}_t, \dots, \hat{a}_N | \underline{x}_{t-1}, \underline{a}_{t-1}) \\ = E_t[\tilde{w} | \underline{x}_{t-1}, \underline{a}_{t-1}] - \frac{1}{2}r \text{var}_t(\tilde{w} | \underline{x}_{t-1}, \underline{a}_{t-1}) - \frac{1}{2}(a_t^2 + \dots + a_N^2) , \end{aligned} \quad (4.208)$$

where \underline{a}_{t-1} is the history of actions already taken, and $\hat{a}_t, \dots, \hat{a}_N$ are the actions that the agent expects to take in periods t, \dots, N . Note that, while the conditional expectation

$E_t[\tilde{w}|\underline{x}_{t-1}, \underline{a}_{t-1}]$ depends both on the observed values of x_1, \dots, x_{t-1} and the agent's previous actions, the posterior (conditional) variance $\text{var}_t(\tilde{w}|\underline{x}_{t-1}, \underline{a}_{t-1})$ does not depend on either the realized value of x_1, \dots, x_{t-1} , or on the agent's actions a_1, \dots, a_{t-1} . Throughout the paper I consider the actions \underline{a}_{t-1} a sunk cost at the start of period t , and therefore not directly included in the agent's certainty equivalent (4.208). Past actions impact the agent's welfare at the start of period t only through their impact on the means of past performance measures in the conditional expectation $E_t[\tilde{w}|\underline{x}_{t-1}, \underline{a}_{t-1}]$.

The agent's wealth \tilde{w} represents the total compensation to be received by the agent, and the agent is indifferent as to the timing of consumption. Thus, the agent's utility ensures that there are no intertemporal consumption smoothing issues. In addition, the agent's exponential utility eliminates wealth effects, in that compensation paid (earned) does not impact the agent's risk preferences. The quadratic cost of effort, additive across tasks, means that the agent is not indifferent to the allocation of effort among tasks in the N periods.⁴

4.3 Full commitment, long-term contract

In this section, I assume that the principal can commit at the start of the first period to a N -period contract. The terms of this contract are not subject to renegotiation. Furthermore, if the agent accepts the contract at the start of the first period, he commits for N periods, and cannot leave in a later period. These assumptions about the parties' ability to commit make the model equivalent (within the LEN framework) to a N task, N correlated performance measures, as analyzed by Feltham and Xie [11] and Feltham and Wu [10].

⁴The agent's exponential utility $u(w, a_1, \dots, a_N) = -\exp[-r(w - \frac{1}{2}(a_1^2 + \dots + a_N^2))]$ implies that, over N periods, his certainty equivalent is $\text{ACE}(\tilde{w}, a_1, \dots, a_N) = E[\tilde{w}] - \frac{1}{2}r \text{var}(\tilde{w}) - \frac{1}{2}(a_1^2 + \dots + a_N^2)$, which is not a function only of total effort $a_1 + \dots + a_N$.

The time line of events is as follows.

- 1) At the start of the first period, the principal offers the agent a linear contract based on the N performance measures that are contractible information:

$$\tilde{c} = \alpha_0 + \beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N . \quad (4.209)$$

The coefficient α_0 represents the fixed wage to be paid the agent, and the coefficients β_1, \dots, β_N represent the variable wages to be paid the agent for each unit of the performance measures.

- 2) If the agent accepts the contract, he provides productive effort a_1, \dots, a_N in each of the N periods.
- 3) After the agent has provided effort a_t , which is not observed by the principal, the performance measure x_t is publicly reported.
- 4) At the end of period N , the contract is settled. After the end of the period N , the outcomes z_1, \dots, z_N are revealed to the principal.

Note that in this case, the timing of information and the timing of payments to the agent is irrelevant, the only constraint is that the performance measures are available before the contract is settled.

The assumption that the principal can commit to a long-term contract guarantees that after x_1, \dots, x_t are observed, the principal will not be able to modify the contract. The LEN assumptions guarantee that the agent's choice of action in period t will be independent of the observation of performance measures \underline{x}_{t-1} and of the agent's actions \underline{a}_{t-1} . The reason is that, at the start of period t , the part of the agent's certainty equivalent of compensation conditional on $\underline{x}_{t-1}, \underline{a}_{t-1}$ that is variable in a_t , and thus provides incentives for a_t , does not depend on either \underline{x}_{t-1} or \underline{a}_{t-1} . Here, each of the assumptions of the LEN model is essential. The exponential utility guarantees that wealth does not affect the agent's risk preferences. The linearity of the contract and the fact that the

performance measures are joint normally distributed with additive noise ensure that \underline{x}_{t-1} and \underline{a}_{t-1} impact on the agent's compensation separately from a_t . In the LEN model with full commitment, the timing of x_1, \dots, x_{t-1} is not important, only their contractibility matters. However, in other settings, the timing of information is important, and if the agent observes the first-period performance measure y_1 before selecting the second-period action a_2 , then a_2 may depend on y_1 . See for example the two-period model of sequential action choice with correlated periods of Matsumura [27].

With full commitment to a long-term contract, the only role played by the correlation between the two performance measures is through the impact on the total risk to which the agent is exposed for incentive purposes as measured by $\text{var}(\tilde{c}) = \text{var}(\beta_1 \tilde{x}_1 + \dots + \beta_N \tilde{x}_N)$.

The principal's problem is to maximize, at the start of the first period, the expected total outcome net of the agent's compensation, subject to the agent's participation constraint and the agent's incentive compatibility constraint. Before presenting the solution to the agency problem under full commitment, I need some notation. Let M be a diagonal $N \times N$ matrix with m_1, \dots, m_N on the main diagonal. Then, the performance measures can be written in vector form as $\tilde{x} = Ma' + \tilde{\varepsilon}$, where a' is a column vector with components a_1, \dots, a_N . Let $b = (b_1, \dots, b_N)'$ be the column vector with components b_1, \dots, b_N such that the principal's gross payoff is $\tilde{z} = b'a + \tilde{\lambda}_1 + \dots + \tilde{\lambda}_N$. With this notation, the N -period full commitment contract is characterized by the following Proposition.

Proposition 4.3.1 *The optimal linear contract $\tilde{c} = \alpha_0 + \beta_1 \tilde{x}_1 + \dots + \beta_N \tilde{x}_N$ with full commitment for N periods and the optimal actions are characterized by $\hat{a}_1 = \beta_1 m_1, \dots, \hat{a}_N =$*

$\beta_N m_N$,

$$\beta = QMb, \quad (4.210)$$

$$a = M\beta = MQMb, \quad (4.211)$$

where $Q = (M^2 + r\Sigma_N)^{-1}$. The principal's expected surplus is

$$\pi = b \cdot a - \frac{1}{2}r \text{var}(\beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N) - \frac{1}{2}(\hat{a}_1^2 + \cdots + \hat{a}_N^2). \quad (4.212)$$

Proof. See Feltham and Xie [11]. \square

4.4 Commitment and renegotiation

In this section, I assume that the principal and the agent commit at the start of the first period to a N -period contract subject to renegotiation in subsequent periods. Specifically, the principal commits to a N -period contract and the agent commits to stay for N periods if he finds the initial contract acceptable. However, the principal and the agent cannot commit not to renegotiate after the performance measures are observed. The terms of the contract are subject to renegotiation in the usual sense: the existing contract can only be replaced by a new contract if both parties agree to it. The renegotiation takes the form of a take-it-or-leave-it offer by the principal.

The time line of events is as follows.

- 1) At the start of the first period, the principal offers the agent a linear contract based on the N contractible performance measures:

$$\tilde{c}^{I1} = \alpha_0^{I1} + \beta_1^{I1} \tilde{x}_1 + \cdots + \beta_N^{I1} \tilde{x}_N. \quad (4.213)$$

The coefficient α_0 represents the fixed wage to be paid the agent, and the coefficients β_1, \dots, β_N represent the variable wages to be paid the agent for each unit of the performance measures.

- 2) If the agent accepts the contract, he commits to the terms of the contract, unless both parties agree later to replace it by a new contract.
- 3) In the first period, after the agent has provided effort a_1 unobservable by the principal, the performance measure x_1 is publicly reported.
- 4) At the start of the second period, the principal can make a renegotiation offer. The principal cannot fire the agent without the agent agreeing to leave at the start of the second period, and the agent cannot leave in the second period without the principal agreeing to it. In addition, a new contract $\tilde{c}^{R1} = \alpha_0^{R1}(x_1) + \beta_2^{R1}(x_1)\tilde{x}_2 + \cdots + \beta_N^{R1}(x_1)\tilde{x}_N$ can replace the old one only if both the principal and the agent weakly prefer it. The renegotiation offer is exogenously assumed to be linear in keeping with the restriction to linear contracts in the model. The coefficients of the renegotiation offer may depend on the first performance measure which is known at the time of renegotiation.⁵
- 5) After the renegotiation stage, the agent provides effort a_2 and then x_2 is reported. At the end of the second period, the contract in effect after renegotiation is denoted by C^{I3} since this is also the contract in effect at the start of the next period.
- 6) At the start of period t , the principal can make a renegotiation offer $\tilde{c}^{Rt} = \alpha_0^{Rt}(\underline{x}_{t-1}) + \beta_t^{Rt}(\underline{x}_{t-1})\tilde{x}_t + \cdots + \beta_N^{Rt}(\underline{x}_{t-1})\tilde{x}_N$. At this time, the contract under renegotiation is \tilde{c}^{It} , which is the outcome of period $t - 1$ renegotiation. The contract that is renegotiated consists of a fixed payment determined by the history of past performance measures and variable payments for the remaining performance measures that are still uncertain at the time of renegotiation. The renegotiation offer is then restricted to a contract of the same form, that is the principal may offer a different fixed payment and different variable payments only for the performance measures that are reported in the future. All payments, fixed and variable, may in general depend on past history.

⁵At the time of renegotiation, the initial contract is linear in the performance measures $\tilde{x}_2, \dots, \tilde{x}_N$, which are still uncertain. I assume that a renegotiation offer is restricted to having the same linear form.

7) After x_N has been reported, the contract is settled at the end of period N . After the end of period N , the outcomes z_1, \dots, z_N are revealed to the principal.

As in the full-commitment case, the contract is settled at the end of period N . The main difference is that the contract can be renegotiated each time after the agent has taken period t action and the period t performance measure has been reported. In this context, a *renegotiation-proof contract* is a contract $\tilde{c} = \alpha_0 + \beta_1 \tilde{x}_1 + \dots + \beta_N \tilde{x}_N$ such that, once agreed upon at the start of the first period, there does not exist a contract at any later renegotiation stage which is weakly preferred by both parties and at least one party strictly prefers. The fact that a linear contract can be renegotiation-proof is due to the particulars of the LEN framework. It turns out that, for every initial contract that is linear in x_1, \dots, x_N , the optimal renegotiation offers (which are restricted to be linear in x_t, \dots, x_N) have period t, \dots, N incentives independent of \underline{x}_{t-1} and a fixed wage that is linear in \underline{x}_{t-1} . In other words, the renegotiation offers are also linear in the N performance measures from an ex-ante (start of the first period) perspective.

An equilibrium in the principal-agent renegotiation game⁶ consists of a sequence of contracts $(\tilde{c}^{I1}, \tilde{c}^{R1}, \dots, \tilde{c}^{IN}, \tilde{c}^{RN})$, the agent's actions a_1, \dots, a_N , and principal's beliefs about the agent's actions $\hat{a}_1, \dots, \hat{a}_N$ such that: (i) the agent accepts both the initial contract and all the renegotiation offers, and rationally anticipates all renegotiation offers and their acceptance when selecting his actions; (ii) the principal's beliefs are correct $\hat{a}_t = a_t$ for all t ; (iii) the sequence $(\tilde{c}^{I1}, \tilde{c}^{R1}, \dots, \tilde{c}^{IN}, \tilde{c}^{RN})$ is ex-ante (start of the first period) optimal and $(\tilde{c}^{It}, \tilde{c}^{Rt}, \dots, \tilde{c}^{IN}, \tilde{c}^{RN})$, is ex-post (start of period t , for all t , conditional on $\underline{x}_{t-1}, \hat{a}_{t-1}$) optimal from the principal's point of view.

The following proposition shows that the analysis of the equilibrium can be restricted

⁶Note that, in general, an equilibrium in the renegotiation game is not given by a renegotiation-proof contract.

without loss of generality to renegotiation-proof contracts.

Proposition 4.4.1 *If the sequence $(\tilde{c}^{I1}, \tilde{c}^{R1}, \dots, \tilde{c}^{IN}, \tilde{c}^{RN}, a_1, \dots, a_N)$ of contracts and actions is an equilibrium in the principal-agent renegotiation game and \tilde{c}^{Rt} is linear in $\tilde{x}_t, \dots, \tilde{x}_N$ at the time of renegotiation, then \tilde{c}^{Rt} is also ex ante (at the start of period $1, \dots, t-1$) linear in $\tilde{x}_1, \dots, \tilde{x}_N$ and offering the contract \tilde{c}^{RN} in every period $(\tilde{c}^{RN}, \dots, \tilde{c}^{RN}, a_1, \dots, a_N)$ is an equivalent equilibrium with a single renegotiation-proof contract.*

Proof. First, I show that any renegotiation offer that gives the optimal last period incentive and is acceptable to the agent is linear in the performance measures from an ex-ante perspective. At renegotiation time, the principal is restricted to offer a linear contract $\tilde{c}^{RN} = \alpha_0^{RN}(\underline{x}_{N-1}) + \beta_N^{RN}(\underline{x}_{N-1})\tilde{x}_N$, whose coefficients may depend on the first $N-1$ performance measures. The period N incentive β_N^{RN} does not depend on \underline{x}_{N-1} since it is determined only by the conditional variance $\text{var}(\tilde{x}_N|\underline{x}_{N-1})$ which is independent of the actual values of \underline{x}_{N-1} . Furthermore, from the participation constraint at renegotiation time it follows that

$$\begin{aligned} \text{ACE}(\tilde{c}^{IN}, a_N|\underline{x}_{N-1}, \underline{a}_{N-1}) &= \text{E}[\tilde{c}^{IN}|\underline{x}_{N-1}, \underline{a}_{N-1}, a_N] - \frac{1}{2}r \text{var}(\tilde{c}^I|\underline{x}_{N-1}) - \frac{1}{2}a_N^2 \\ &\leq \text{ACE}(\tilde{c}^{RN}, a_N|\underline{x}_{N-1}, \underline{a}_{N-1}) = \alpha_0^{RN}(\underline{x}_{N-1}, \underline{a}_{N-1}) + \beta_N^{RN} \text{E}[\tilde{x}_N|\underline{x}_{N-1}, \underline{a}_{N-1}, a_N] \\ &\quad - \frac{1}{2}r \text{var}(\tilde{c}^{RN}|\underline{x}_{N-1}) - \frac{1}{2}a_N^2 . \end{aligned} \tag{4.214}$$

Since the initial contract is linear in x_1, \dots, x_N and since the conditional mean of the last period performance measure $\text{E}[\tilde{x}_N|\underline{x}_{N-1}]$ is linear in x_1, \dots, x_{N-1} (due to the joint normality of the distributions), it follows, solving the equation implied by assuming the

participation constraint(4.214) to be binding for $\alpha_0^{RN}(\underline{x}_{N-1})$, that α_0^{RN} is linear in \underline{x}_{N-1} ,

$$\begin{aligned} \alpha_0^{RN} = & E[\tilde{c}^{IN} | \underline{x}_{N-1}, \underline{a}_{N-1}, a_N] - \beta_N^{RN} E[\tilde{x}_N | \underline{x}_{N-1}, \underline{a}_{N-1}, a_N] \\ & - \frac{1}{2} r [\text{var}(\tilde{c}^{IN} | \underline{x}_{N-1}) - \text{var}(\tilde{c}^{RN} | \underline{x}_{N-1})] . \end{aligned} \quad (4.215)$$

It then follows that, conditional on being accepted by the agent, the renegotiation offer is of the following form

$$\begin{aligned} \tilde{c}^{RN} = & E[\tilde{c}^{IN} | \underline{x}_{N-1}, \underline{a}_{N-1}, a_N] + \beta_N^{RN} (\tilde{x}_N - E[\tilde{x}_N | \underline{x}_{N-1}, \underline{a}_{N-1}, a_N]) \\ & - \frac{1}{2} r [\text{var}(\tilde{c}^{IN} | \underline{x}_{N-1}) - \text{var}(\tilde{c}^{RN} | \underline{x}_{N-1})] \\ = & \tilde{c}^{IN} + (\beta_N^{RN} - \beta_N^{IN}) (\tilde{x}_N - E[\tilde{x}_N | \underline{x}_{N-1}, \underline{a}_{N-1}, a_N]) \\ & - \frac{1}{2} r [\text{var}(\tilde{c}^{IN} | \underline{x}_{N-1}) - \text{var}(\tilde{c}^{RN} | \underline{x}_{N-1})] . \end{aligned} \quad (4.216)$$

From the above equation it follows that $\text{ACE}(\tilde{c}^{RN}) = \text{ACE}(\tilde{c}^{IN})$ at the start of period $N - 1$, and that \tilde{c}^{RN} is acceptable at the start of period $N - 1$ if, and only if, \tilde{c}^{IN} is acceptable. Thus, \tilde{c}^{RN} is accepted by the agent if offered as a period $N - 1$ renegotiation offer. (Note that, in equilibrium, c^{IN} is the outcome of renegotiation from period $N - 1$.) Since \tilde{c}^{RN} is an optimal renegotiation offer given \tilde{c}^{IN} , and since the efficient period N incentive is independent of $\underline{x}_{N-1}, \underline{a}_{N-1}$, the principal has no incentive to renegotiate \tilde{c}^{RN} if it is the initial contract in period $N - 1$. This proves that \tilde{c}^R is renegotiation-proof in period N and it can replace \tilde{c}^{RN-1} in period $N - 1$.

It remains to show that \tilde{c}^{RN} induces the same actions as $(\tilde{c}^{IN}, \tilde{c}^{RN})$, since the payments to the agent and the principal's surplus are uniquely determined by the agent's actions. The period N action is determined by c^{RN} in both cases, since that is the contract in effect at the time the agent provides effort a_N . Since $(\tilde{c}^{IN-1}, \tilde{c}^{RN}, a_{N-1}, a_N)$ is an equilibrium in the last two periods, the period $N - 1$ action is chosen in anticipation of the period N renegotiation, and so is determined by the (linear) incentive contained in $\alpha_0^{RN}(x_{N-1})$. The reason is that it is suboptimal for the agent to select an action different

from a_{N-1} and then reject the renegotiation offer. Thus, the agent's period $N - 1$ action is determined only by the incentives in \tilde{c}^{RN} , and so is the same when \tilde{c}^{RN} is the period $N - 1$ renegotiation offer.

In the same way, it is shown by backwards induction that \tilde{c}^{RN} can be offered as a renegotiation offer in periods $N - 2, \dots, 2$, and as an initial contract \tilde{c}^{I1} at the start of the first period. \square

The main idea in the proposition is that if the contract is renegotiated in equilibrium, the agent's actions are completely determined by the renegotiated contract. The principal cannot gain by offering the agent a contract that will later be renegotiated as long as the agent anticipates the renegotiation.

The principal's problem is to maximize at the start of the first period the expected total outcome net of the agent's compensation, subject to the agent's participation constraint, and the agent's incentive compatibility constraints. The contract must also be renegotiation-proof at the start of subsequent periods. Since at the time the contract can be renegotiated, the actions a_1, \dots, a_{t-1} have already been taken, and the performance measures x_1, \dots, x_{t-1} have been reported, the only part of the agent's compensation that remains uncertain is the period t, \dots, N variable wage $\beta_t \tilde{x}_t + \dots + \beta_N \tilde{x}_N$. The requirement that the contract is renegotiation-proof means that at the start of periods $t = 2, \dots, N$, the contract maximizes the principal's expected total outcome net of the agent's compensation conditional on information from periods $1, \dots, t - 1$ and subject to the agent's participation constraint and the agent's period t, \dots, N incentive compatibility constraints.

Proposition 4.4.2 *Given the optimal linear N -period renegotiation-proof contract $\tilde{c} = \alpha_0 + \beta_1 \tilde{x}_1 + \dots + \beta_N \tilde{x}_N$, the optimal actions are characterized by $\hat{a}_t = m_t \beta_t$, for all t and*

$$\alpha_0 = -\frac{1}{2}(\hat{a}_1^2 + \dots + \hat{a}_N^2) + \frac{1}{2}r \text{var}(\beta_1 \tilde{x}_1 + \dots + \beta_N \tilde{x}_N) . \quad (4.217)$$

The principal's surplus is given by

$$\pi = b_1 \hat{a}_1 + \cdots + b_N \hat{a}_N - \frac{1}{2} r \text{var}(\beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N) - \frac{1}{2} (\hat{a}_1^2 + \cdots + \hat{a}_N^2) . \quad (4.218)$$

Proof. At the time the agent selects action a_t at the start of period t , his certainty equivalent of compensation given conjectured future actions $\hat{a}_{t+1}, \dots, \hat{a}_N$ is

$$\begin{aligned} & \text{ACE}(\tilde{c}, \hat{a}_{t+1}, \dots, \hat{a}_N \mid \underline{x}_{t-1}, \underline{a}_{t-1}, a_t) \\ &= \alpha_0 + \beta_1 x_1 + \cdots + \beta_{t-1} x_{t-1} + \beta_t \text{E}[\tilde{x}_t \mid \underline{x}_{t-1}, \underline{a}_{t-1}, a_t] \\ &+ \beta_{t+1} \text{E}[\tilde{x}_{t+1} \mid \underline{x}_{t-1}, \underline{a}_{t-1}, a_t, \hat{a}_{t+1}] + \cdots + \beta_N \text{E}[\tilde{x}_N \mid \underline{x}_{t-1}, \underline{a}_{t-1}, a_t, \hat{a}_N] \\ &- \frac{1}{2} r \text{var}(\tilde{c} \mid \underline{x}_{t-1}) - \frac{1}{2} (a_t^2 + \hat{a}_{t+1}^2 + \cdots + \hat{a}_N^2) . \end{aligned} \quad (4.219)$$

Note that, at the time the agent chooses action a_t , effort in the previous periods is sunk, and does not enter the agent's calculations. Furthermore, the fixed portion of the agent's compensation $\alpha_0 + \beta_1 x_1 + \cdots + \beta_{t-1} x_{t-1}$, the variable payments in future periods $\beta_{t+1} \text{E}[\tilde{x}_{t+1} \mid \underline{x}_{t-1}, \underline{a}_{t-1}, a_t, \hat{a}_{t+1}] + \cdots + \beta_N \text{E}[\tilde{x}_N \mid \underline{x}_{t-1}, \underline{a}_{t-1}, a_t, \hat{a}_N]$, the risk premium for the variance of the contract, and the cost of future actions do not depend on the choice of period t action. It then follows that the only part of the agent's certainty equivalent (4.219) that depends on a_t is

$$\begin{aligned} & \beta_t \text{E}[\tilde{x}_t \mid \underline{x}_{t-1}, \underline{a}_{t-1}, a_t] - \frac{1}{2} a_t^2 \\ &= \beta_t (m_t a_t + \rho e'_1 (t-1) \Sigma_{t-1}^{-1} (\underline{x}_{t-1} - \underline{m}_{t-1} \underline{a}_{t-1}))' - \frac{1}{2} a_t^2 . \end{aligned} \quad (4.220)$$

The first-order condition for the incentive compatibility constraint is

$$\frac{\partial}{\partial a_t} \text{ACE}(\tilde{c}, \hat{a}_{t+1}, \dots, \hat{a}_N \mid \underline{x}_{t-1}, \underline{a}_{t-1}, a_t) = m_t \beta_t - a_t = 0 . \quad (4.221)$$

It follows that the action induced by the contract is

$$\hat{a}_t = m_t \beta_t . \quad (4.222)$$

As in the full commitment case, the principal's expected surplus, π is the gross benefit $b_1 a_1 + \dots + b_N a_N$ less compensation for the agent's effort $\frac{1}{2}(\hat{a}_1^2 + \dots + \hat{a}_N^2)$ and a risk premium for the variance the performance measures $\frac{1}{2}r \text{var}(\beta_1 \tilde{x}_1 + \dots + \beta_N \tilde{x}_N)$. \square

The renegotiation concept I use here is the same as that of Fudenberg and Tirole [13] in that both parties must agree to the renegotiated contract, but the timing is different. In my model, renegotiation takes place after the performance measure x_t is observed, while Fudenberg and Tirole have the renegotiation take place between the time the agent takes the action and the time the performance measure is observed in a single period model. Having the renegotiation take place after x_t is observed and before a_{t+1} is taken avoids the insurance/adverse selection problem of Fudenberg and Tirole.⁷

The driving force behind renegotiation in my model is the principal's desire to optimally adjust the agent's induced effort in subsequent periods to match the (reduced) ex-post variance of the performance measures. In this context, the fact that x_t is observed is important, and not the actual value of x_t . As a result, period t, \dots, N actions and period t, \dots, N incentives are independent of \underline{x}_{t-1} , the agent's actions \underline{a}_{t-1} , and the principal's conjectures of the agent's actions $\hat{\underline{a}}_{t-1}$.

The inability of the principal to commit not to renegotiate is ex-ante inefficient relative to the full commitment case. The reason is that the renegotiation-proof contract imposes additional binding constraints on the optimal contract. In both cases, the principal chooses the agent's actions to be induced by the contract in order to maximize his surplus as given by (4.218). In the full commitment case, the principal maximizes π unconstrained, while with renegotiation in periods $2, \dots, N$, the contracts are constrained. Since the unconstrained optimum is at least as high as the constrained one,

⁷If renegotiation takes place before x_t is observed, the agent is offered insurance by the principal for the risk in x_t when the agent has private information about the action a_t . As a result, the principal would like to perfectly insure the agent in order to avoid paying compensation for risk, and this destroys the agent's incentive to provide effort because the agent anticipates the renegotiation. Fudenberg and Tirole show that the only equilibria involve randomization by the agent in choosing his action.

the full commitment contract is at least as efficient as the renegotiation-proof contract. If $\rho = 0$, that is if the performance measures in the two periods are independent, there is no difference between the two contracts, which take the form of repeatedly inducing the optimal action from the one period problem.

Explicit formulas for the optimal actions will be derived in the next section, where a mechanism for replicating the renegotiation-proof solution by short-term contracts is presented.

4.5 Short-term contracts under commitment to fairness

In the previous two sections, I presented the two types of commitment that relate to N -period contracts: full commitment and renegotiation-proofness. In this section, I turn to the analysis of short-term contracts in an N -period relationship. Short-term contracts are used whenever the parties cannot write long-term contracts. The inability to write a long-term contract does not necessarily imply that there is no long-term commitment (that is commitment beyond the first-period) in the principal-agent relationship, only that the principal cannot commit to a specific contract in a period other than the current one, or that the contracting environment is such that it prohibits the use of long-term contracts. The main idea in this section is an intermediate form of commitment, whereby the agent commits to stay for N periods, while the principal commits to offer contracts that provide the agent with his reservation certainty equivalent for each period based on the public information at the contracting date and the principal's conjectures of the agent's actions.

This form of commitment is an adaptation of the concept of fairness introduced by Baron and Besanko [1]. A contracting relationship is governed by *fairness* if the principal is restricted to fair wages and the agent must participate in all periods if he accepts the

contract in the first period. Fair wages are paid when the agent gets his reservation wage as if he could leave in each period. That is, the agent's certainty equivalent of future compensation, conditional on available information and conjectured actions is set to zero at the start of each period. Thus, in addition to the usual contract acceptance constraint at the start of the first period, there are additional constraints that the period t contract is acceptable to the agent as if the agent had other employment opportunities, had not committed to stay for all periods, and had taken the conjectured actions in periods $1, \dots, t - 1$. The key fact here is the agent's ability to commit to stay for N periods; removing the agent's ability to commit for all N periods leads to a situation where there is no equilibrium in which the agent stays in all N periods (see also the detailed analysis in Chapter 3 of the agent's commitment and its impact on the equilibrium). The agent gives up his ability to leave in subsequent periods for the guarantee of fair compensation in periods $2, \dots, N$.

The time line of events is as follows.

- 1) At the start of the first period, the principal offers the agent a linear contract based on the first-period performance measure x_1 :

$$\tilde{c}_1 = \alpha_1 + \beta_1 \tilde{x}_1 . \quad (4.223)$$

The coefficient α_1 represents the fixed wage to be paid the agent, and the coefficient β_1 represents the variable wages to be paid the agent for each unit of the performance measure.

- 2) If the agent accepts the contract, he commits to stay for all N periods. The principal commits to offer the reservation certainty equivalent of wages in each period, given all the information available at the time of contracting and given that the agent's actions correspond to the principal's conjectures.
- 3) In the first period, after the agent has provided effort a_1 unobservable by the principal,

the performance measure x_1 is publicly reported.

4) After the performance measure x_1 is publicly reported, the first-period contract is settled at the end of the first period.

5) At the start of the second period $t, t \geq 2$, a new contract is offered by the principal

$$\tilde{c}_t = \alpha_t(\underline{x}_{t-1}, \hat{a}_{t-1}) + \beta_t(\underline{x}_{t-1}, \hat{a}_{t-1})\tilde{x}_t. \quad (4.224)$$

This contract is subject to the agent reservation wage restriction and is accepted by the agent (since I assumed commitment for all N periods on the agent's part). The terms of the period t contract may depend on the past observed values of the performance measures and on the principal's conjectures about the agent's past actions $(\underline{x}_{t-1}, \hat{a}_{t-1})$, since they are informative about x_t , and therefore can be used to reduce the noise of the period t performance measure.

6) In period t , the agent provides effort a_t , then the performance measure x_t is reported. The period t contract is settled at the end of period t .

7) After the end of period N , the outputs z_1, \dots, z_N are revealed to the principal.

Short-term contracting with fair wages relies on the principal's conjectures regarding the (unobservable) agent actions. At the start of period t , when \tilde{c}_t is set, the terms of the contract depend on $E[\tilde{x}_t | \underline{x}_{t-1}, \hat{a}_{t-1}]$, where \hat{a}_{t-1} are the principal's conjectures of the agent's actions in periods $1, \dots, t-1$. The concept of fair wages in period t assumes that the principal's conjectures are correct, that is the agent actually has provided effort \hat{a}_{t-1} in the first $t-1$ periods. At the start of the first period, and any subsequent period t , fair wages refer to the total future compensation paid to the agent over the remaining $N-t+1$ periods $\tilde{c}_t + \dots + \tilde{c}_N$, and involve both the principal's and the agent's conjectures of future actions $\hat{a}_t, \dots, \hat{a}_N$. The principal and the agent are assumed to be in agreement over the conjectured future actions. For example, the principal states his conjecture \hat{a}_1 as part of

the first-period contract \tilde{c}_1 . The agent then agrees to that conjecture when accepting the initial contract, and that will be the basis for setting the second-period contract.⁸ While the agreement on conjectured actions restricts the principal's contract choices (given fairness and past performance measures), the agent's actions are unconstrained, even though, in equilibrium, the agent will choose to implement the conjectured actions.

Thus, an optimal sequence of contracts under commitment to fairness is also characterized by a rational expectations equilibrium regarding the agent's actions. The agent's commitment to stay for all N periods is essential in sustaining this rational expectations equilibrium, as will be shown later in this section.

The principal's problem at the start of each period is to maximize the remaining expected total outcome net of the agent's compensation, subject to the agent's reservation wage constraint and the agent's rationality constraint. Since the agent commits to stay for all N periods at the start of the first period, subsequent contracts $\tilde{c}_{t+1}, \dots, \tilde{c}_N$ are anticipated at the time the period t contract is set.

Let \tilde{w}_t denote the agent's cumulative compensation net of personal effort cost from period t to period N

$$\tilde{w}_t = \tilde{c}_t - \frac{1}{2}a_t^2 + \dots + \tilde{c}_N - \frac{1}{2}a_N^2. \quad (4.225)$$

At the start of period t (at contracting time), the manager's objective is the certainty equivalent of \tilde{w}_t conditional on all available information \underline{x}_{t-1} , \underline{a}_{t-1} , and on conjectures about future actions $\hat{a}_t, \dots, \hat{a}_N$, denoted $\text{ACE}_{t-1}(\tilde{w}_t | \underline{x}_{t-1}, \underline{a}_{t-1}, \hat{a}_t, \dots, \hat{a}_N)$.

$$\begin{aligned} \text{ACE}_{t-1}(\tilde{w}_t | \underline{x}_{t-1}, \underline{a}_{t-1}) &= E_{t-1}[\tilde{c}_t | \underline{x}_{t-1}, \underline{a}_{t-1}] - \frac{1}{2}\hat{a}_t^2 \\ &+ \dots + E_{t-1}[\tilde{c}_N | \underline{x}_{t-1}, \underline{a}_{t-1}] - \frac{1}{2}\hat{a}_N^2 - \frac{1}{2}r \text{var}_{t-1}(\tilde{c}_t + \dots + \tilde{c}_N) \end{aligned} \quad (4.226)$$

⁸An alternative would be that both parties solve for the unique equilibrium conjecture \hat{a}_1 . In this case, the principal can use an expert witness (professor of accounting) to prove that the second-period wage is fair.

The values $\hat{a}_t, \dots, \hat{a}_N$ represent at this time the agent's conjectures of his future actions. The agent correctly anticipates that the terms of future contracts will depend on all history available at contracting time.

At the start of period t , the principal chooses α_t and β_t , taking into account all available information \underline{x}_{t-1} and his conjectures regarding the manager's actions, past and future, $\hat{a}_1, \dots, \hat{a}_N$. In a rational expectations equilibrium, at each point in time, the principal's conjectures about past managerial actions are correct, and the principal and the manager have the same conjectures regarding the manager's future actions. In other words, $a_t = \hat{a}_t$ for all values of t .

The following is a key technical result needed for the solution to the agency problem. The Lemma proves that, from the perspective of the start of period $t + 1$, future performance measures in periods $k \geq t + 2$, adjusted for conditional expectations given start of period k information, are mutually independent.

Lemma 4.5.1 *Given information history \underline{x}_t , the following relations hold:*

$$\text{cov}_t(\tilde{x}_k - E_{k-1}[\tilde{x}_k], \tilde{x}_l - E_{l-1}[\tilde{x}_l]) = 0 \text{ for } k \neq l \geq t + 2 \quad (4.227)$$

$$\text{cov}_t(\tilde{x}_{t+1}, \tilde{x}_k - E_{k-1}[\tilde{x}_k]) = 0 \text{ for } k \geq t + 2. \quad (4.228)$$

Proof. See the Appendix. \square

The intuition for this result is that adjusting for expectations based on available information removes the common components in the decompositions (4.193)-(4.195) or (4.196)-(4.198), so that (4.193)-(4.195) become

$$\tilde{\varepsilon}_{t-1} - E_{t-2}[\tilde{\varepsilon}_{t-1}] = \tilde{\nu}_{t-1} + \tilde{\delta}_{t-1} \quad (4.229)$$

$$\tilde{\varepsilon}_t - E_{t-1}[\tilde{\varepsilon}_t] = \tilde{\nu}_t + \tilde{\delta}_t \quad (4.230)$$

$$\tilde{\varepsilon}_{t+1} - E_t[\tilde{\varepsilon}_{t+1}] = \tilde{\nu}_{t+1} + \tilde{\delta}_{t+1}, \quad (4.231)$$

The Lemma implies that, if the agent's certainty equivalent of period k compensation, conditional on information available at the start of period k , is zero for $k \geq t + 1$, then the contracts $\tilde{c}_t, \dots, \tilde{c}_N$ are mutually independent as seen from the start of period t . The key is that $ACE_{k-1}(\tilde{c}_k, \hat{a}_k | \underline{x}_{k-1}, \hat{a}_{k-1}) = 0$ implies $E_{k-1}[\tilde{c}_k | \hat{a}_k, \underline{x}_{k-1}, \hat{a}_{k-1}] - \frac{1}{2}a_k^2 - \frac{1}{2}r \text{var}_{k-1}(\tilde{c}_k) = 0$ and, as a result the fixed wage in period k is determined by $\alpha_k = -\beta_k E_{k-1}[\tilde{x}_k | \hat{a}_k, \underline{x}_{k-1}, \hat{a}_{k-1}] + \frac{1}{2}a_k^2 + \frac{1}{2}r \text{var}_{k-1}(\tilde{c}_k)$. It follows that the period k contract can be rewritten as

$$\tilde{c}_k = \beta_k(\tilde{x}_k - E_{k-1}[\tilde{x}_k]) + \frac{1}{2}a_k^2 + \frac{1}{2}r \text{var}_{k-1}(\tilde{c}_k) , \quad (4.232)$$

and that the variable compensation for the agent is based on the performance measure adjusted for expectation. These adjusted performance measures are mutually independent as shown in Lemma 4.5.1, and from an ex-ante perspective (relative to period k), the agent's compensation in period k consists of the risk premium, the cost of effort, and a zero-mean variable compensation term. Thus, although the performance measures are correlated, it turns out that the fair contracts mechanism decouples the contracts by adjusting the fixed wage based on the history available at contracting time. Note also that the fixed wage in period k provides incentives for the agent's actions in periods $1, \dots, k-1$, since the period k fixed wage depends on the realized values of past performance measures \underline{x}_{k-1} . As a result, the fixed wage α_k depends on the random variables $\tilde{x}_l, \dots, \tilde{x}_{k-1}$ when anticipated at the start of period $l < k$.

The fact that future contracts are independent of each other implies that the variance of the agent's future compensation in the agent's certainty equivalent (4.226) can be easily calculated as

$$\text{var}_{t-1}(\tilde{c}_t + \dots + \tilde{c}_N) = \text{var}_{t-1}(\tilde{c}_t) + \dots + \text{var}_{t-1}(\tilde{c}_N) .$$

The following lemma shows that, in a rational expectations equilibrium, the fairness constraints determine the fixed wage for the agent in the period t contract as a function

only of the incentives in \tilde{c}_t and conditional expectations of period t variables given history \underline{x}_{t-1} and conjectures \hat{a}_{t-1} (note that the fixed wage and all other terms of the contract are set by the principal based on his conjectures of the agent's actions). Moreover, as in a single-period problem, the fixed wage compensates the agent for his cost of effort plus a risk premium for the posterior variance of the period t compensation contract (performance measure).

Lemma 4.5.2 *If the fairness constraint is satisfied at the start of each period, that is if $ACE_{t-1}(\tilde{w}_t, \hat{a}_t, \dots, \hat{a}_N | \underline{x}_{t-1}, \hat{a}_{t-1}) = 0$ for all $1 \leq t \leq N$, then the agent's certainty equivalent of future compensation at the start of period t is the same as the certainty equivalent of period t compensation*

$$ACE_{t-1}(\tilde{w}_t, \hat{a}_t, \dots, \hat{a}_N | \underline{x}_{t-1}, \hat{a}_{t-1}) = E_{t-1}[\tilde{c}_t | \underline{x}_{t-1}, \hat{a}_{t-1}, \hat{a}_t] - \frac{1}{2}\hat{a}_t^2 - \frac{1}{2}r \text{var}_{t-1}(\tilde{c}_t), \quad (4.233)$$

and the fixed wage set by the principal is

$$\alpha_t = -\beta_t E_{t-1}[\tilde{x}_t | \underline{x}_{t-1}, \hat{a}_{t-1}, \hat{a}_t] + \frac{1}{2}\hat{a}_t^2 + \frac{1}{2}r \text{var}_{t-1}(\tilde{c}_t). \quad (4.234)$$

In addition, at the time the agent selects the period t action,

$$\begin{aligned} ACE_{t-1}(\tilde{w}_t, a_t, \hat{a}_{t+1}, \dots, \hat{a}_N | \underline{x}_{t-1}, \underline{a}_{t-1}, \hat{a}_{t-1}, \hat{a}_t) \\ = E_{t-1}[\tilde{c}_t | \underline{x}_{t-1}, \underline{a}_{t-1}, a_t] - \frac{1}{2}a_t^2 - \frac{1}{2}r \text{var}_{t-1}(\tilde{c}_t) \\ + \beta_{t+1} E_{t-1} [\tilde{x}_{t+1} - E_t[\tilde{x}_{t+1} | \hat{a}_t] | \underline{x}_{t-1}, \underline{a}_{t-1}, a_t] \\ + \dots + \beta_N E_{t-1} [\tilde{x}_N - E_{N-1}[\tilde{x}_N | \hat{a}_N] | \underline{x}_{t-1}, \underline{a}_{t-1}, a_t] \end{aligned} \quad (4.235)$$

Proof. See the Appendix. \square

At contracting time, the agent and the principal have common conjectures regarding the agent's future actions $\hat{a}_t, \dots, \hat{a}_N$. Based on these common conjectures, and assuming

that the agent's past actions correspond to the principal's conjectures, $\underline{a}_{t-1} = \hat{a}_{t-1}$, the agent's certainty equivalent at the start of period t is zero $\text{ACE}_{t-1}(w_t) = 0$. Furthermore, each line in equation (4.235) is equal to zero. However, when the agent's action in period t becomes a choice variable, each of the lines in equation (4.235) captures the impact of action a_t on compensation in periods t, \dots, N . The compensation in future periods is affected by a_t since the fixed part of the agent's compensation in future periods is adjusted to extract all surplus from the agent based on past information.

After the agent has accepted the contract \tilde{c}_t , at the time of choosing the action a_t , let \hat{a}_t denote the principal's conjecture of what action the agent will choose. Then, equation (4.294) in the proof of Lemma 4.5.2 implies

$$\begin{aligned} \text{E}_{t-1} [\tilde{x}_k - \text{E}_{k-1}[\tilde{x}_k|\hat{a}_t]|a_t] &= \text{E}_{t-1} [-R_{k-1} \cdot (\underline{x}_{k-1} - \underline{m}_{k-1}\underline{a}_{k-1})|a_t, \hat{a}_t] \\ &= \frac{(-\rho)^{k-t}}{\sigma_{k-1} \dots \sigma_t} (\text{E}_{t-1} [\tilde{x}_t - m_t \hat{a}_t|a_t] - R_{t-1} \cdot (\underline{x}_{t-1} - \underline{m}_{t-1}\underline{a}_{t-1})) \\ &= \frac{(-\rho)^{k-t}}{\sigma_{k-1} \dots \sigma_t} m_t (a_t - \hat{a}_t) . \end{aligned} \quad (4.236)$$

Substituting in the agent's certainty equivalent (4.235) gives the agent's objective function at the time of selecting action a_t ,

$$\begin{aligned} \text{ACE}_{t-1}(\tilde{w}_t, a_t|\hat{a}_t) &= \text{E}_{t-1}[\tilde{c}_t|a_t, \hat{a}_t] - \frac{1}{2}a_t^2 - \frac{1}{2}r \text{var}_{t-1}(\tilde{c}_t) \\ &\quad + m_t \left[\beta_{t+1} \frac{-\rho}{\sigma_t} (a_t - \hat{a}_t) + \dots + \beta_N \frac{(-\rho)^{N-t}}{\sigma_{N-1} \dots \sigma_t} (a_t - \hat{a}_t) \right] \end{aligned} \quad (4.237)$$

From equation (4.234), given the principal's conjecture \hat{a}_t , I have that

$$\alpha_t = -\beta_t \text{E}_{t-1}[\tilde{x}_t|\hat{a}_t] + \frac{1}{2}\hat{a}_t^2 + \frac{1}{2}r\sigma_t\beta_t^2 \text{ and} \quad (4.238)$$

$$\text{E}_{t-1}[\tilde{c}_t|a_t, \hat{a}_t] = m_t\beta_t(a_t - \hat{a}_t) + \frac{1}{2}r\sigma_t\beta_t^2 . \quad (4.239)$$

As a consequence, the agent's objective function (4.237) becomes

$$\begin{aligned} \text{ACE}_{t-1}(\tilde{w}_t, a_t | \hat{a}_t) &= \frac{1}{2} \hat{a}_t^2 - \frac{1}{2} a_t^2 + m_t \beta_t (a_t - \hat{a}_t) \\ &+ m_t \beta_{t+1} \frac{-\rho}{\sigma_t} (a_t - \hat{a}_t) + \cdots + m_t \beta_N \frac{(-\rho)^{N-t}}{\sigma_{N-1} \cdots \sigma_t} (a_t - \hat{a}_t) \end{aligned} \quad (4.240)$$

Equation (4.240) captures the essence of the ratchet effect, in that it shows the agent's expectation of the impact that the period t action will have on setting performance standards in future periods. Note that an increased effort in period t has a mixed impact in future periods if ρ is positive; expected compensation will be reduced in some periods and increased in others. The magnitude of the effect decreases as the distance in time increases. In addition, future incentives $\beta_{t+1}, \dots, \beta_N$ directly influence these future impacts of a_t . If the correlation between periods is negative, increased effort in the current period increases compensation in all future periods. In addition, note that in each period, the agent's action choice is independent of past history and only depends on the impact of that action on future compensation as described by equation (4.240).

The principal's problem is to maximize at the start of each period the remaining expected total outcome net of the agent's compensation, subject to the agent's reservation wage constraint and the agent's rationality constraint.

$$\max_{\alpha_t, \beta_t} \mathbb{E}_{t-1} [(\tilde{z}_t - \tilde{c}_t) + \cdots + (\tilde{z}_N - \tilde{c}_N) | \underline{x}_{t-1}, \underline{a}_{t-1}, \hat{a}_t, \dots, \hat{a}_N] \quad (4.241)$$

subject to the reservation wage constraint

$$\text{ACE}_{t-1}(\tilde{w}_t) = 0 \quad (4.242)$$

and the agent's rationality constraint

$$\hat{a}_t \in \operatorname{argmax}_{a_t} \text{ACE}_{t-1}(w_t | a_t, \hat{a}_t) \quad (4.243)$$

Theorem 4.5.1 *The actions induced by the sequence of optimal contracts and the sequence of optimal incentive intensities are given by*

$$\hat{a}_N = m_N \beta_N = \frac{m_N^2 b_N}{m_N^2 + r \sigma_N} \quad (4.244)$$

$$\beta_t = \frac{m_t b_t + \frac{\rho}{\sigma_t} \frac{m_t^2}{m_{t+1}^2} m_{t+1} b_{t+1}}{m_t^2 + r \sigma_t} - \rho r \frac{\sigma_{t+1}}{m_t^2 + r \sigma_t} \frac{m_t^2}{m_{t+1}^2} \beta_{t+1} \quad (4.245)$$

$$\hat{a}_t = m_t \beta_t - \frac{\rho}{\sigma_t} \frac{m_t}{m_{t+1}} \hat{a}_{t+1} . \quad (4.246)$$

The fixed part of the agent's compensation is

$$\alpha_t = -\beta_t E_{t-1}[\tilde{x}_t | \underline{x}_{t-1}, \underline{a}_{t-1}] + \frac{1}{2} \hat{a}_t^2 + \frac{1}{2} r \sigma_t \beta_t^2 . \quad (4.247)$$

Proof. See the Appendix. \square

The next task is to show that the short-term contracts with commitment to fairness replicate the payoffs of the renegotiation-proof contract as in the two-period case. This is done by showing that the optimal actions induced by the sequence of contracts in Theorem 4.5.1 are the same as the actions induced by the renegotiation-proof contract described in Proposition 4.4.2. From (4.218) and (4.247), the principal's surplus is the same in both cases, in that the agent is only compensated for his effort and a risk premium for the posterior variance of each performance measure. Since the payoffs are uniquely determined by the induced actions it follows that, from an ex-ante (start of the first period) perspective, the contract $\tilde{c} = \tilde{c}_1 + \dots + \tilde{c}_N$ is the same as the renegotiation-proof contract described in Proposition 4.4.2.

The proof that both types of contracts induce the same actions is by backwards induction, starting with the last period. Under commitment to fairness, as with renegotiation, the period N action and the period N incentive do not depend on the previous actions, or on the specific value of previous performance measures. The reason, as before, is that

when contracting at the start of period N , the role played by the available information is to reduce the variance in the period N performance measure, and that variance does not depend on either the specific values of the previous performance measures or the previous actions. It follows that the principal's optimal choice of risk for the agent when setting the period N incentive does not depend on either the specific value of the previous performance measures or the previous actions. Thus, the principal's problem in setting the incentive for period N is the same in both cases, and as a result the induced period N action is the same.

Once the period t incentive rate is set, the fixed wage is given by the fairness restriction. Then, the period t fixed wage depends linearly on \underline{x}_{t-1} because $E[\tilde{x}_t | \underline{x}_{t-1}, \hat{a}_{t-1}]$ is linear in \underline{x}_{t-1} . From an ex-ante perspective, the period t contract contains a (random) linear term in \tilde{x}_{t-1} , which contributes in addition to the incentives from the period $t-1$ contract to the agent's choice of action in period $t-1$. At the start of period $t-1$, the agent's cumulative future compensation $\tilde{c}_{t-1} + \dots + \tilde{c}_N$ is linear in $\tilde{x}_{t-1}, \dots, \tilde{x}_N$, while the induced actions a_t, \dots, a_N are the same as in the renegotiation case. Both the principal and the agent anticipate future contracts $\tilde{c}_t, \dots, \tilde{c}_N$ and have a common conjecture regarding their terms. It follows that the principal's problem is the same as in the renegotiation case and that the *total incentive* for the period $t-1$ action is the same. The main difference is that, with commitment to fairness, the period $t-1$ incentives are split between the period t variable wage and the period t, \dots, N fixed wages. The agent's actions are the same and the principal's surplus is the same in both cases.

Thus, the sequence of short-term contracts under commitment to fairness replicates the payoffs of the long-term renegotiation-proof contract. This particular sequence of contracts generalizes the one derived by Indjejikian and Nanda [18] for the case of two periods to illustrate how lack of commitment in a dynamic agency relationship leads to

ratcheting. The principal's inability to commit to a long term-contract without renegotiation in the second period creates the same inefficiency with short-term contracts under commitment and fairness as the inefficiency of the renegotiation-proof contract. Here, as before, the inefficiency is relative to the full commitment long-term contract. The results in this section show that commitment to fairness is a sufficient assumption for obtaining the ratcheting with short-term contracts described by Indjejikian and Nanda [18]. However, their description of the commitment assumptions that are necessary for the solution they derive is incomplete and referred to as "the absence of commitment".

If "absence of commitment" means that the agent cannot commit to stay for all N periods, then, as is shown in the following, there is no equilibrium with a sequence of short-term contracts in which the agent stays for all N periods. As a result, the commitment to fairness assumption is not only sufficient, but also necessary to obtain the solution described in Theorem 4.5.1. The fairness assumption is necessary since, once the agent has committed to stay for all N periods, there must be a restriction in subsequent periods to prevent an "infinite" transfer from the agent to the principal through the period $t \geq 2$ contracts. The anticipation of such a contract renders any first-period contract in which the agent commits for all N periods unacceptable to the agent. Thus, the simplest solution to this problem is a lower bound on the agent's reservation certainty equivalent, which is precisely what the fairness constraint provides.

The following proposition shows that the sequence of optimal contracts derived under commitment to fairness always induces the agent to leave in a later period ($t \geq 2$) if the agent's ability to leave is restored.

Proposition 4.5.1 *If $\rho m_1 \beta_1 \dots m_N \beta_N \neq 0$, and the principal offers the sequence of contracts $(\tilde{c}_1, \dots, \tilde{c}_N)$ from Theorem 4.5.1, the agent can always earn a positive surplus by leaving before the end of the last period.*

Proof. The proof is by contradiction. Assume that the agent does not leave until the end of period N . Then the agent is faced with the pair of contracts $(\tilde{c}_{N-1}, \tilde{c}_N)$ at the start of period $N - 1$. If the agent has not left until the start of period $N - 1$, it is assumed that his actions are the ones conjectured by the principal up to this point, $\underline{a}_{N-2} = \hat{a}_{N-2}$. In that case, a direct application of Proposition 5.2 from Chapter 2 shows that the agent acts strategically in period $N - 1$, chooses an action $a_{N-1}^\dagger \neq \hat{a}_{N-1}$ different from the principal's conjecture, and leaves in period N . This contradicts the assumption that the agent stays for all N periods and concludes the proof. \square

4.6 Dynamic incentives and ratcheting

In this section, I examine the impact of noise correlation on the dynamics of incentives and induced managerial effort based on the commitment to fairness solution developed in the previous section. As opposed to the two-period case, to make different periods readily comparable, I assume that all periods are identical, in that the manager's effort level impacts the performance measures and the outcome in the same way in each period. That means $m_1 = \dots = m_N = m$ and $b_1 = \dots = b_N = b$. With this simplification, equations (4.244), (4.245), and (4.246) become

$$a_N = m\beta_N = \frac{m^2b}{m^2 + r\sigma_N} \quad (4.248)$$

$$\beta_t = mb \frac{1 + \frac{\rho}{\sigma_t}}{m^2 + r\sigma_t} - \rho r \frac{\frac{\sigma_{t+1}}{\sigma_t}}{m^2 + r\sigma_t} \beta_{t+1} \quad (4.249)$$

$$a_t = m\beta_t - \frac{\rho}{\sigma_t} a_{t+1} . \quad (4.250)$$

I now turn to the dynamic behavior of a_t and β_t . First, I use equation (4.299) in the expression for a_t (4.250) to derive

$$a_t = \frac{m^2b}{m^2 + r\sigma_t} - \frac{\rho r}{m^2 + r\sigma_t} a_{t+1} . \quad (4.251)$$

Using equations (4.248), (4.249), and (4.251), it can be shown that, for positively correlated performance measures $\rho > 0$, $a_{N-1} < a_N$ and $\beta_{N-1} > \beta_N$. These results are the same as those from the two-period model (see Chapter 2 or Indjejikian and Nanda [18]). The next task is to see what happens in periods $N - 2$, $N - 3$, etc.

First, I note that the posterior variances σ_t converge rapidly to a limit value, denoted by σ_∞ . I will then use the limit variance as an approximation for posterior variances.

Lemma 4.6.1 *Let $\sigma_\infty = \mu_1 = \frac{\sigma + \sqrt{\sigma^2 - 4\rho^2}}{2}$ (see also Lemma 4.2.1), and let $e_t^\sigma = \sigma_t - \sigma_\infty$. Then,*

$$\lim_{t \rightarrow \infty} \sigma_t = \sigma_\infty \quad (4.252)$$

$$e_{t+1}^\sigma = \frac{\rho^2}{\sigma_t \sigma_\infty} e_t^\sigma. \quad (4.253)$$

Proof. From Lemma 4.2.3 it follows that the sequence σ_t is decreasing and bounded from below by μ_1 , so it converges and $\lim_{t \rightarrow \infty} \sigma_t \geq \mu_1$. On the other hand, $\lim_{t \rightarrow \infty} \sigma_t$ is a solution to the equation $\sigma - \frac{\rho^2}{x} = x$ (recall that $\sigma_{t+1} = \sigma - \frac{\rho^2}{\sigma_t}$), and since μ_1 is the largest of the two solutions, $\lim_{t \rightarrow \infty} \sigma_t = \mu_1$.

For the second part of the lemma, $e_{t+1}^\sigma = \sigma_{t+1} - \sigma_\infty$ and $\sigma_{t+1} = \sigma - \frac{\rho^2}{\sigma_t}$, $\sigma_\infty = \sigma - \frac{\rho^2}{\sigma_\infty}$ imply that

$$e_{t+1}^\sigma = -\frac{\rho^2}{\sigma_t} + \frac{\rho^2}{\sigma_\infty} = \frac{\rho^2}{\sigma_t \sigma_\infty} (\sigma_t - \sigma_\infty) = \frac{\rho^2}{\sigma_t \sigma_\infty} e_t^\sigma.$$

□

In particular, if $|\rho| < \sigma/2$,

$$e_{t+1}^\sigma < \frac{\rho^2}{\sigma_\infty^2} e_t^\sigma < \left(\frac{\rho^2}{\sigma_\infty^2} \right)^t e_1^\sigma, \quad (4.254)$$

and $|\rho| < \sigma_\infty$, which imply that σ_t converges exponentially fast to σ_∞ . Note that when $|\rho|$ is increasing from 0 to $\sigma/2$, the convergence factor ρ^2/σ_∞^2 increases from 0 to 1. The smaller the convergence factor, the faster the convergence, which implies that the smaller

the correlation across periods, the faster the convergence of σ_t to σ_∞ . In the limit case when $|\rho| = \sigma/2$, $\sigma_\infty = \sigma/2$, and $e_{t+1}^\sigma = \frac{\rho}{\sigma_t} e_t^\sigma$, which implies

$$e_{t+1}^\sigma = \frac{\sigma_\infty}{e_t^\sigma + \sigma_\infty} e_t^\sigma, \quad (4.255)$$

and the convergence is slower.

The above result means that for a long enough history, the posterior variances are approximately constant for the later periods of the manager's tenure. In particular, if we assume that at the time the manager is hired, all previous history is common knowledge, and if we assume that this previous history is long enough, the assumption that posterior variances are approximately constant holds for all periods of the manager's tenure. It is also implicitly assumed that the noise terms in the performance measures are firm-specific and not agent-specific. Thus, when a new agent is hired, past history is relevant to determining conditional expectations and conditional variances of the performance measures.

The other possibility is that noise is agent-specific, and history that precedes the current agent's tenure is irrelevant. In this case, the assumption that posterior variances are constant throughout the agent's tenure is not reasonable. The problem is particularly significant during the first few periods of the agent's tenure, when most of the change in posterior beliefs about variance takes place.

In what follows, I will assume that noise is firm-specific, and that beliefs about the distribution of the performance measures are conditioned on a long history that precedes the hiring of the current agent.⁹ Thus $\sigma_t = \sigma_\infty$ is a constant for $1 \leq t \leq N$. Given this

⁹As a limiting case, if the firm has been in existence for infinitely many periods, the posterior variances are actually constant and equal to the limit value.

assumption, the recursive relations (4.249) and (4.251) become

$$\beta_t = mb \frac{1 + \frac{\rho}{\sigma_\infty}}{m^2 + r\sigma_\infty} - \frac{\rho r}{m^2 + r\sigma_\infty} \beta_{t+1} \quad (4.256)$$

$$a_t = \frac{m^2 b}{m^2 + r\sigma_\infty} - \frac{\rho r}{m^2 + r\sigma_\infty} a_{t+1} . \quad (4.257)$$

Then,

$$a_t - a_{t-1} = -\frac{\rho r}{m^2 + r\sigma_\infty} (a_{t+1} - a_t) \quad (4.258)$$

$$\beta_t - \beta_{t-1} = -\frac{\rho r}{m^2 + r\sigma_\infty} (\beta_{t+1} - \beta_t) , \quad (4.259)$$

which implies that if ρ is positive the two sequences are alternating, while if ρ is negative, the two sequences are monotonic. Specifically, if ρ is positive, then $a_N > a_{N-1}$, $a_{N-1} < a_{N-2}$, $a_{N-2} > a_{N-3}$, etc, and $\beta_N < \beta_{N-1}$, $\beta_{N-1} > \beta_{N-2}$, $\beta_{N-2} < \beta_{N-3}$, etc. In this case, the ratchet effect looks like a “sawtooth effect”. If ρ is negative, actions are increasing $a_N < a_{N-1} < a_{N-2} < \dots$, and incentives are decreasing $\beta_N > \beta_{N-1} > \beta_{N-2} > \dots$. In both cases a_t and β_t converge to limit values¹⁰ which I denote \bar{a} and $\bar{\beta}$. Taking limits in both sides of equations (4.256) and (4.257) gives

$$\bar{\beta} = mb \frac{1 + \frac{\rho}{\sigma_\infty}}{m^2 + r\sigma_\infty} - \frac{\rho r}{m^2 + r\sigma_\infty} \bar{\beta} \quad (4.260)$$

$$\bar{a} = \frac{m^2 b}{m^2 + r\sigma_\infty} - \frac{\rho r}{m^2 + r\sigma_\infty} \bar{a} . \quad (4.261)$$

solving for the limit values $\bar{\beta}$ and \bar{a} gives

$$\bar{a} = \lim_{t \rightarrow \infty} a_{N-t} = \frac{m^2 b}{m^2 + r(\sigma_\infty + \rho)} , \quad (4.262)$$

$$\bar{\beta} = \lim_{t \rightarrow \infty} \beta_{N-t} = mb \frac{1 + \frac{\rho}{\sigma_\infty}}{m^2 + r(\sigma_\infty + \rho)} . \quad (4.263)$$

¹⁰All the analysis is backward in time, so these limits are reached towards the first periods of the agent's tenure.

The convergence is very fast for both sequences since

$$|a_t - \bar{a}| = \frac{|\rho|r}{m^2 + r\sigma_\infty} |a_{t+1} - \bar{a}| ,$$

$$|\beta_t - \bar{\beta}| = \frac{|\rho|r}{m^2 + r\sigma_\infty} |\beta_{t+1} - \bar{\beta}| ,$$

and $|\rho|r/(m^2 + r\sigma_\infty) < 1$ for all admissible values of ρ including the extremes $|\rho| = \sigma_\infty = \sigma/2$.¹¹

To gain some insight into the nature of the limit effort level \bar{a} , let $\rho = k\sigma$ so that k represents the ex-ante correlation between adjacent period performance measures (outcomes). Then, $\sigma_\infty = \frac{1}{2}\sigma(1 + \sqrt{1 - 4k^2})$, and $\sigma_\infty + \rho = \frac{1}{2}\sigma(1 + 2k + \sqrt{1 - 4k^2})$. A separate analysis shows that for $k \in [-1/2, 0]$, as k increases from $-1/2$ to 0 , $\sigma_\infty + \rho$ increases from 0 to σ . On the other hand, for $k \in [0, 1/2]$, as k increases from 0 to $1/2$, $\sigma_\infty + \rho$ has a maximum of $(1 + \sqrt{2})\sigma/2$ at $k = 1/2\sqrt{2}$ and has its minimum equal to σ at both endpoints.

For \bar{a} the implications are that for negative correlation k , \bar{a} decreases from b to $bm^2/(m^2 + r\sigma)$ when k increases from $-1/2$ to 0 ; for positive correlation k , \bar{a} decreases from $bm^2/(m^2 + r\sigma)$ to a minimum value of $bm^2/(m^2 + r\sigma(1 + \sqrt{2})/2)$, and then increases back to $bm^2/(m^2 + r\sigma)$ when k increases from 0 to $1/2$. To put these results into perspective, note that the first-best effort level in the static one-period problem is $a^* = b$, and that in the dynamic problem, if effort is contractible, the first-best solution is also $a_t^* = b$ in every period with a fixed wage that compensates the manager for his effort. Thus, when the performance measures are negatively correlated, the manager's effort is moved towards first-best. In the extreme case in which $k = -1/2$, the first-best effort level is approximated arbitrarily close (limited only by the total number of periods available). On the other hand, when the performance measures are positively correlated,

¹¹Since, in general $|\rho| \leq \sigma/2 < \sigma_\infty$, it follows that $|\rho|r < m^2 + r\sigma_\infty$.

the effort level is always less than or equal to the second-best from the one-period problem $a^\dagger = bm^2/(m^2 + r\sigma)$. In the next section I will analyze the impact of managerial effort on the principal's surplus which will shed some light on the inefficiencies or the efficiencies (relative to the single-period static solution) associated with the multiperiod ratchet effect.

4.7 Managerial tenure with fixed switching cost

In this section, I analyze the principal's ex-ante preferences for the manager's tenure assuming that the principal can commit to retain the manager a given number of periods. The principal and the agent sign one-period contracts subject to commitment to fairness. Alternately, the results in this section can be interpreted as relevant to the optimal tenure under long-term contracting with renegotiation.

Although a similar question can be asked regarding the full commitment long-term contract, it is a question of optimal inter-period insurance effects for the agent. In that case, the principal's problem is to determine the optimal length of contracts so that a particular correlation structure for the performance measures gives the optimal amount of risk for the agent's compensation. Here I do not address the question of optimal tenure with full commitment to long-term contracts and focus instead on the relation between the impact of renegotiation on managerial effort ("ratcheting") and managerial tenure.

As in the two-period case, the principal's welfare is lower with either a renegotiation-proof contract or its equivalent, a sequence of contracts under commitment to fairness, than with the full commitment long-term contract for the same duration. The other results derived in Chapter 3 regarding the principal's welfare with or without agent turnover can be generalized only in a limited way to the N -period case. For example, the natural generalization of the fact that, with positive correlation, turnover dominates

full commitment is as follows.

Proposition 4.7.1 *Given a fixed finite horizon of N periods, and any $1 \leq T < N$, assume that the inter-period covariance is positive $\rho > 0$ and that all periods are identical. Then, the principal is always better off employing two agents with full commitment long-term contracts of length T and $N - T$, respectively than employing a single agent with a full commitment long-term contract for all N periods.*

Proof. The proof is a natural extension of the proof of Proposition 7.1 in Chapter 2. Suppose that the incentive rates from the full commitment contract are offered to two separate agents in the two periods. Then, the same actions are induced for the two agents as for the single agent. However, the risk premium paid to the two agents is lower since

$$\begin{aligned} & \frac{1}{2}r \text{var}(\beta_1 \tilde{x}_1 + \cdots + \beta_N \tilde{x}_N) \\ & > \frac{1}{2}r \text{var}(\beta_1 \tilde{x}_1 + \cdots + \beta_T \tilde{x}_T) + \frac{1}{2}r \text{var}(\beta_{T+1} \tilde{x}_{T+1} + \cdots + \beta_N \tilde{x}_N) \\ & > \frac{1}{2}r \text{var}(\beta_1 \tilde{x}_1 + \cdots + \beta_T \tilde{x}_T) + \frac{1}{2}r \text{var}(\beta_{T+1} \tilde{x}_{T+1} + \cdots + \beta_N \tilde{x}_N | \underline{x}_T) \end{aligned} \quad (4.264)$$

Thus, two agents provide the same effort at a lower risk premium, which means that the principal's surplus is higher with two agents. The advantage of using two agents comes both from not having to pay compensation for the part of the total variance of the single-agent contract that is due to the performance measures being positively correlated and from the ability to use the lower ex-post variance with the second agent. \square

The above proposition shows that, within a given N -period horizon, and without switching costs, using more than one agent is always preferred. The optimum in this case is to use a different agent in each period. However, this type of result is not relevant in the case of infinite horizons with stationary replacement policies and non-zero switching costs.

For the remainder of the paper, I consider only the case of short-term contracts under commitment to fairness. From equation (4.297) it follows that

$$\tilde{c}_t = \beta_t(\tilde{x}_t - E_{t-1}[\tilde{x}_t]) + \frac{1}{2}a_t^2 + \frac{1}{2}r\sigma_t\beta_t^2, \quad (4.265)$$

which implies

$$E_0[\tilde{z}_t - \tilde{c}_t] = b_t a_t - \frac{1}{2}a_t^2 - \frac{1}{2}r\sigma_t\beta_t^2, \quad (4.266)$$

since $E_0[\tilde{x}_t - E_{t-1}[\tilde{x}_t]] = 0$ for all $1 \leq t \leq N$. Using $a_t = b_t - r\sigma_t m_t^{-1}\beta_t$ to substitute for a_t in equation (4.266) gives the principal's ex-ante expected surplus for period t

$$\begin{aligned} E_0[\pi_t] &= E_0[\tilde{z}_t - \tilde{c}_t] \\ &= b_t^2 - r \frac{b_t \sigma_t}{m_t} \beta_t - \frac{1}{2} \left(b_t^2 - 2r \frac{b_t \sigma_t}{m_t} \beta_t + r^2 \frac{\sigma_t^2}{m_t^2} \beta_t^2 \right) - \frac{1}{2} r \sigma_t \beta_t^2 \\ &= \frac{1}{2} b_t^2 - \frac{1}{2} r \sigma_t \left(1 + \frac{r \sigma_t}{m_t^2} \right) \beta_t^2. \end{aligned} \quad (4.267)$$

The principal's total ex-ante expected surplus is then

$$E_0[\pi_1] + \cdots + E_0[\pi_N] = \frac{1}{2} \sum_{t=1}^N b_t^2 - \frac{1}{2} r \sum_{t=1}^N \sigma_t \left(1 + \frac{r \sigma_t}{m_t^2} \right) \beta_t^2. \quad (4.268)$$

A similar calculation gives $E[\pi_t]$ as a function of a_t only,

$$\begin{aligned} E_0[\pi_t] &= b_t a_t - \frac{1}{2} a_t^2 - \frac{1}{2} r \sigma_t \frac{m_t^2}{r^2 \sigma_t^2} (b_t - a_t)^2 \\ &= b_t a_t - \frac{1}{2} a_t^2 - \frac{1}{2} \frac{m_t^2}{r \sigma_t} (b_t^2 - 2b_t a_t + a_t^2) \\ &= \frac{1}{2r\sigma_t} [-(m_t^2 + r\sigma_t)a_t^2 + 2(m_t^2 + r\sigma_t)b_t a_t - m_t^2 b_t^2]. \end{aligned} \quad (4.269)$$

Since

$$\frac{\partial E[\pi_t]}{\partial a_t} = \left(1 + \frac{m_t^2}{r\sigma_t} \right) (b_t - a_t) \geq 0 \quad (4.270)$$

for all $a_t \leq b_t$, it follows that the expected period t profit is strictly increasing in managerial effort and attains a maximum at $a_t = b_t$.

This simple observation, together with the discussion of the impact of the correlation k (see the discussion at the end of the previous section) on managerial effort imply the following. When the performance measures are positively correlated, all else equal, the principal is best off with the extreme values of the correlation, 0 or $1/2$, and better off the farther that correlation is from $1/2\sqrt{2}$. In all cases, the principal is worse off than in the one-period setting with the second-best solution.

In the case of negatively correlated performance measures, the worst performance is in the last period, and the best is at most \bar{a} . The higher the absolute value of the correlation, the higher \bar{a} , and the better off is the principal. In the extreme case $k = 1/2$, the first-best solution (first-best effort and first-best surplus) can be approximated arbitrarily close given sufficiently long tenure. A commitment problem could arise in this case due to the decreasing performance of the agent towards the end of his tenure. The principal needs to be able to commit to not fire the manager before term, otherwise the efficiency gains from getting effort levels close to \bar{a} are lost. This would not be a problem in the positive correlation case since there the principal gets better performance towards the last period, and has no incentive to fire the manager before term. In either case, commitment to fairness assumes that the principal commits to retain the agent for all N periods.

I turn now to addressing the issue of ex-ante optimal tenure. The efficiencies or inefficiencies generated by the ratchet effect depend on knowing when the last period is. Since there is managerial turnover, I will assume that there is a long-lived principal and a shorter-lived agent, such that the firm continues to operate after replacing a manager. First, I will assume away any problems that the principal may have in committing to a certain managerial tenure. The principal can offer a contract that guarantees employment for a number of periods, with compensation determined at the start of each period. The

manager can commit not to leave as long as the contract terms are acceptable at the start of each period. The setting is commitment to fairness for N periods at a time, for an infinite sequence of identical agents.

In order to simplify the analysis, I will assume again that all periods are identical, $m_t = m$ and $b_t = b$ for all t . In addition, I will assume that the entire history of the firm prior to hiring the current agent is common knowledge, and that the history is long enough so the approximation of posterior variances by a constant holds: $\sigma_t \approx \sigma_\infty$. The history $(\underline{x}_0, \underline{a}_0)$ represents all history available at the time a manager is hired, and I will use the notation $E_0[\cdot]$ to denote expectations conditional on available history at the start of the manager's tenure, $E_0[\cdot | \underline{x}_0, \underline{a}_0]$. Since I need to compare different lengths of managerial tenure from the point of view of a long-lived firm, I assume that the firm is infinitely lived and decides on a stationary tenure policy, that is managers are replaced every N periods in perpetuity. Then, the available history at the start of a manager's tenure is always the same $(\underline{x}_0, \underline{a}_0)$. If past history is at all relevant to the tenure decision, it only matters at the start of a manager's tenure, since that is the time at which the principal decides on tenure, and both the principal and the agent commit to it. Note in addition that, in this context, given the infinite horizon, although the output \tilde{z}_t may be observed by the principal, it is never publicly reported and is not contractible.¹²

I now turn to the principal's objective function when choosing the agent's tenure. Given an infinite horizon and identical histories $(\underline{x}_0, \underline{a}_0)$ each time a new manager is hired, the principal's expected surplus for each future period is

$$E_0[\pi_t] = E_0[\lambda_t] + \frac{1}{2}b^2 - \frac{1}{2}r\sigma_\infty \left(1 + \frac{r\sigma_\infty}{m^2}\right) \beta_t^2. \quad (4.271)$$

The sequence $E_0[\lambda_t]$ which reflects the principal's posterior beliefs about noise in the output does not depend on the agent's actions, and therefore does not depend on the

¹²The results that follow do not change by assuming instead that either the output is never revealed to the principal, or that the output is uncorrelated to any of the performance measures.

tenure variable N . It follows that the principal will ignore $E_0[\lambda_t]$ when choosing tenure for the agents.¹³ Since the principal only chooses among stationary tenure policies, and the discount rate is assumed to be zero, the principal's objective function is the average surplus per period for the entire tenure of one agent, and ignoring the noise terms $E_0[\lambda_t]$ in equation (4.271),

$$\frac{1}{N}E_0[\Pi(N)] = \frac{1}{N} \sum_{t=1}^N \left[\frac{1}{2}b^2 - \frac{1}{2}r\sigma_\infty \left(1 + \frac{r\sigma_\infty}{m^2} \right) \beta_t^2 \right], \quad (4.272)$$

where

$$\Pi(N) = \sum_{t=1}^N (\tilde{z}_t - \tilde{\lambda}_t - \tilde{c}_t).$$

In other words, $\Pi(N)$ is the total surplus for the N periods of a manager's tenure ignoring the noise terms in the output.

The average surplus per period for the entire tenure of one manager is the natural measure with infinite periods and no discounting, since it corresponds to the annual equivalent value of the infinite stream of surpluses (see also the discussion of the total auditing cost in Chapter 2). In addition, I assume that there is an upper limit on the agents' tenure (mandatory retirement) N_{max} . Assuming a fixed switching cost C that is incurred only when a new manager is hired and that the principal's objective function is expected surplus per period net of switching costs, I have the following theorem.

Theorem 4.7.1 *Assume that the correlation between adjacent periods is positive, $\rho > 0$. There exists C_0 such that if $C < C_0$, ex-ante optimal tenure is one period, and if $C > C_0$, and there is an upper bound on tenure N_{max} , then the optimal tenure is N_{max} .*

Proof. See the Appendix. \square

¹³This term is completely eliminated if we assume that the noise in the performance measures is manager-specific and thus $E_0[\cdot] = E[\cdot]$. The same is true if we assume that the performance measures are uninformative about the output, in other words if the noise terms $\tilde{\lambda}_t$ and $\tilde{\epsilon}_t$ are mutually independent.

This theorem is a mainly negative result, in that it shows that there is no ex-ante interior optimal tenure when the variance-reducing “learning” at the start of the manager’s tenure is absent. The manager’s effort is higher, but if there is no benefit from retaining the manager for the first periods, either the switching cost is low enough to allow turnover in each period, or the switching cost is so high that, once the manager is retained for more than one period, he will be retained forever.

The intuition behind this result is that, with positive correlation switching agents dominates any longer tenure. It is only when the switching cost is high enough that longer tenure is preferred. Specifically, $E_0[\Pi(N)]$ grows (almost) linearly in N , and as a result, the average benefit per period is essentially of the form $\gamma_1 + (\Pi_1 - C)/N$, which is either strictly increasing or strictly decreasing over the entire domain depending on whether $\Pi_1 > C$ or $\Pi_1 < C$. The first case leads to maximum tenure, while the second case leads to turnover every period.

For negative correlation, the optimum is always N_{max} regardless of the magnitude of the switching cost as the following result shows.

Theorem 4.7.2 *Assume that the correlation between adjacent periods is negative, $\rho < 0$. Then, for any value of the switching cost C , if there is an upper bound on tenure N_{max} , the optimal tenure is N_{max} .*

Proof. The main idea is to show that $\Pi(N)/N$ is increasing in N . This is proved in a straightforward way by induction. Let $a_1^{N+1}, \dots, a_{N+1}^{N+1}$ and a_1^N, \dots, a_N^N denote the agent’s actions in each period for the case of tenure $N + 1$ and N , respectively. Then, the following inequalities hold:

$$a_{N+1}^{N+1} = a_N^N, a_N^{N+1} > a_{N-1}^N, \dots, a_2^{N+1} > a_1^N$$

and

$$a_1^{N+1} > a_2^{N+1} \geq a_t^N \text{ for all } t.$$

It then follows that

$$\pi_{N+1}^{N+1} = \pi_N^N, \pi_N^{N+1} > \pi_{N-1}^N, \dots, \pi_2^{N+1} > \pi_1^N$$

and

$$\pi_1^{N+1} > \pi_2^{N+1} \geq \pi_t^N \text{ for all } t.$$

The above imply that

$$\pi_2^{N+1} + \dots + \pi_{N+1}^{N+1} > \pi_1^N + \dots + \pi_N^N$$

and

$$\pi_1^{N+1} > \frac{1}{N}(\pi_1^N + \dots + \pi_N^N).$$

This last equation implies that

$$\frac{1}{N+1}(\pi_1^{N+1} + \pi_2^{N+1} + \dots + \pi_{N+1}^{N+1}) > \frac{1}{N}(\pi_1^N + \dots + \pi_N^N)$$

which concludes the proof by showing that $(\pi_1^N + \dots + \pi_N^N)/N$ is increasing in N . So, regardless of the switching cost, the principal's gross benefit per period increases with tenure; the presence of the switching cost only adds to this conclusion since the average switching cost per period decreases with tenure. \square

4.8 Conclusions

In this paper I develop an N-period model of the ratchet effect in a principal-agent problem with moral hazard but without adverse selection. Thus, while the agent's action is unobservable by the principal, in equilibrium the principal has rational beliefs regarding the agent's past actions and as a result, in equilibrium, information asymmetries do not develop over time between the principal and the agent. In addition, there is no learning of productivity or any agent characteristic that is unknown at the start. The

only dynamic information effects are the adjustment of posterior beliefs about future performance measures, conditional on the sequential observation of past performance measures together with conjectures of the agent's past actions. The model generalizes the two-period model of Chapter 3, and most results derived therein remain valid in the N-period case. Primarily, the conclusions regarding the role of commitment in obtaining different solutions remain the same. However, the N-period model gives insights into the importance of the contracting horizon, or tenure. In addition, the N-period model offers insights into the agent's long-term performance that cannot be inferred from the two-period model.

While in Chapter 3 I have compared different commitment scenarios given a fixed two-period horizon, in this paper the emphasis is on tenure given a certain choice of commitment assumptions. Since the very idea of tenure implies some form of implicit or explicit long-term commitment, the choice is between the three types of long-term commitment discussed in Chapter 3: full commitment to a long-term contract, commitment to a long-term contract with renegotiation, and commitment to fairness with short-term contracts. Full commitment to a long-term contract is too restrictive, in that, especially over longer horizons, renegotiation is more likely. Commitment to fairness is a mechanism that replicates the commitment to a long-term contract with renegotiation by using a sequence of short-term contracts. This makes the choice between the two forms of contracting almost a matter of taste.

In this paper, I choose to restrict the analysis to contracting under commitment to fairness. Besides capturing the idea of renegotiation, it only requires short-term contracts that are settled at the end of each period, allowing for a breakdown of performance and surplus period-by-period. In addition, the principal and the agent only need to commit to "fair contracts" and to a tenure duration, no other long-term commitments or

contracts are necessary.¹⁴ Finally, the commitment to fairness solution to the dynamic agency problem provides consistency and ease of comparison to previous literature such as Milgrom and Roberts [29] and Indjejikian and Nanda [18].

The driving force behind the “ratchet effect” with pure moral hazard described in this paper is the principal’s inability to commit not to use past performance measures when setting a short-term contract with the agent in each period other than the agent’s first period of tenure. The principal has an incentive to set the agent’s compensation in a later period based on past performance measures in order to optimally adjust the risk in the agent’s contract for the lower posterior variances. In doing so, the principal makes the fixed pay in future periods depend on past performance measures. For the agent, it means that from an *ex ante* (previous periods) perspective, future fixed pay depend on earlier effort choices. In other words, “fixed compensation” in one period is fixed only in that period, given the past actions and performance measures, but is variable when anticipated from earlier periods. It follows that the agent’s incentives for effort in any one period are spread among the variable wage for that period and the fixed wages in all future periods.

The nature of the solution to the dynamic agency problem is such that, for longer horizons, the manager’s effort is close to some limit level for most periods. Thus, there are essentially two effort levels, the second best in the last period, and approximately a “third best” in most other periods. When correlation is positive, the “third best” is lower than the second best, generating inefficiencies relative to a repeated one-period problem (which is the multi-period problem when periods are independent). When the correlation is negative, the ratchet effect is efficient relative to the uncorrelated periods case since the “third best” effort level is closer to first best. To summarize, with positively

¹⁴The reader who finds the concept of “commitment to fairness” unpalatable can interpret all results in the context of commitment to a long-term contract with renegotiation.

correlated performance measures, incentives are stronger and the manager exerts less effort in most periods than in the last period. With negatively correlated performance measures, incentives are less strong and the manager exerts more effort in most periods than in the last period. The effort level in the last period serves as a benchmark because it coincides with the second-best solution, which is what one obtains with uncorrelated performance measures.

A commitment problem could arise in the case of negatively correlated performance measures due to the decreasing performance of the agent towards the end of his tenure. The principal needs to be able to commit not to fire the manager before term, otherwise the efficiency gains from getting effort levels close to first-best are lost. This would not be a problem in the positive correlation case since there the principal gets better performance towards the last period, and has no incentive to fire the manager before term. In either case, commitment to fairness assumes that the principal commits to retain the agent for all N periods.

To answer the question of ex-ante choice of tenure by the principal, I examine optimal stationary replacement policies, whereby the agents are hired for N periods at a time within an infinite horizon for the firm, and the noise in the performance measures is firm-specific. The main result for positively correlated performance measures is that, in the presence of a switching cost, there exists a threshold switching cost such that optimal tenure is a single period whenever the switching cost is higher than the threshold value. On the other hand, optimal tenure is the maximum number of periods possible (the agent's maximum life) when the switching cost is lower than the threshold value. Thus, with positively correlated performance measures, the only optimal replacement (tenure) policies are the corner solutions of one period tenure or maximum possible tenure. The main result for negatively correlated performance measures is that the optimal replacement policy is always the maximum number of periods possible, irrespective

of the switching cost.

In the case when the noise in the performance measures is agent-specific, there is an additional “learning” effect in the first few periods of a manager’s tenure due to the (rapid) reduction of the posterior variances of the performance measures towards their limit value. This effect results in an increase of managerial effort, since it becomes less costly over time for the principal to motivate managerial effort due to lower risk premia. With negatively correlated performance measures and agent-specific noise, the manager exerts the second-best effort in the last period, a higher effort level in all other periods, and that effort has an inverted U shape. This finding is consistent with evidence from the management literature that firm performance increases at first, reaches a maximum, and then declines during a manager’s tenure. For a discussion of these findings, see the papers by Eitzen and Yetman [9], Katz [21], and Hambrick and Fukutomi [15].

4.9 Appendix: proofs

Proof of Lemma 4.2.1. For the first part, simply write

$$\Sigma_{t+1} = \begin{bmatrix} \sigma & \rho & 0 & \dots & 0 \\ \rho & \sigma & 0 & \dots & 0 \\ 0 & \rho & & & \\ \dots & & \Sigma_{t-2} & & \\ 0 & 0 & & & \end{bmatrix}.$$

For the second part of the proof, let μ_1 be the largest solution¹⁵ of the equation $\sigma - \frac{\rho^2}{x} = x$, or equivalently, $x^2 - \sigma x + \rho^2 = 0$. Since

$$D_t = \frac{D_t}{D_{t-1}} \frac{D_{t-1}}{D_{t-2}} \dots \frac{D_1}{D_0},$$

¹⁵This is where I need $\sigma^2 \geq 4\rho^2$.

it is sufficient to show that $\frac{D_t}{D_{t-1}} > 0$ for all t . I prove by induction the stronger inequality

$$\frac{D_t}{D_{t-1}} > \mu_1 .$$

First, since $\mu_1 = \frac{\sigma + \sqrt{\sigma^2 - 4\rho^2}}{2}$, it follows that $\frac{D_1}{D_0} = \sigma > \mu_1$. Then, I assume that $\frac{D_t}{D_{t-1}} > \mu_1$, and I show that $\frac{D_{t+1}}{D_t} > \mu_1$. From $D_{t+1} = \sigma D_t - \rho^2 D_{t-1}$, I have that $\frac{D_{t+1}}{D_t} = \sigma - \rho^2 \frac{D_{t-1}}{D_t}$. By assumption, $\frac{D_t}{D_{t-1}} > \mu_1$, which implies $-\rho^2 \frac{D_{t-1}}{D_t} > \frac{\rho^2}{\mu_1}$. Adding σ on both sides gives the desired result since

$$\frac{D_{t+1}}{D_t} = \sigma - \rho^2 \frac{D_{t-1}}{D_t} > \sigma - \frac{\rho^2}{\mu_1} = \mu_1 .$$

□

Proof of Lemma 4.2.3. For the first part of the proof, I need to calculate the first row of the matrix Σ_t^{-1} .

$$\Sigma_t = \begin{bmatrix} \sigma & \rho & 0 & 0 & 0 & 0 & \dots & 0 \\ \rho & \sigma & \rho & 0 & 0 & 0 & \dots & 0 \\ 0 & \rho & \sigma & \rho & 0 & 0 & \dots & 0 \\ 0 & 0 & \rho & \sigma & \rho & 0 & \dots & 0 \\ 0 & 0 & 0 & \rho & \sigma & \rho & \dots & 0 \\ 0 & 0 & 0 & 0 & \rho & & & \\ \dots & & & & & & \Sigma_{t-5} & \\ 0 & 0 & 0 & 0 & 0 & & & \end{bmatrix}$$

A straightforward calculation then gives the first row of Σ_t^{-1} as

$$e'_1(t)\Sigma_t^{-1} = \frac{1}{D_t}(D_{t-1}, -\rho D_{t-2}, (-\rho)^2 D_{t-3}, \dots, (-\rho)^{t-1} D_0) . \quad (4.273)$$

As for the variance, $\sigma_{t+1} = \sigma - \rho e'_1(t) \Sigma_t^{-1} \rho e_1(t)$ implies

$$\begin{aligned}\sigma_{t+1} &= \sigma - \rho^2 \frac{D_{t-1}}{D_t} \\ &= \frac{\sigma D_t - \rho^2 D_{t-1}}{D_t} = \frac{D_{t+1}}{D_t}.\end{aligned}\quad (4.274)$$

To conclude the proof of equation (4.204), I use equation (4.274) to rewrite (4.273). For the second part of the proof, I note that

$$\sigma_{t+1} = \sigma - \frac{\rho^2}{\sigma_t}.\quad (4.275)$$

Thus, $\sigma_1 = \sigma > \sigma - \frac{\rho^2}{\sigma} = \sigma_2$. Assuming $\sigma_t < \sigma_{t-1}$, it follows that

$$\frac{1}{\sigma_{t-1}} < \frac{1}{\sigma_t} \Rightarrow -\frac{\rho^2}{\sigma_{t-1}} < -\frac{\rho^2}{\sigma_t} \Rightarrow \sigma - \frac{\rho^2}{\sigma_{t-1}} < \sigma - \frac{\rho^2}{\sigma_t} \Rightarrow \sigma_{t+1} < \sigma_t$$

which proves the inequality by induction. \square

Proof of Lemma 4.5.1. From Lemma 4.2.3 it follows that

$$\tilde{x}_k - E_{k-1}[\tilde{x}_k] = \tilde{x}_k - (m_k a_k + R_{k-1} \cdot (\underline{x}_{k-1} - \underline{m}_{k-1} \underline{a}_{k-1}))\quad (4.276)$$

$$\begin{aligned}R_{k-1} &= \frac{\rho}{D_{k-1}} (D_{k-2}, -\rho D_{k-3}, (-\rho)^2 D_{k-4}, \dots, (-\rho)^{k-3} D_1, (-\rho)^{k-2} D_0) \\ &= \rho \left(\frac{1}{\sigma_{k-1}}, \frac{-\rho}{\sigma_{k-1} \sigma_{k-2}}, \frac{(-\rho)^2}{\sigma_{k-1} \sigma_{k-2} \sigma_{k-3}}, \dots, \frac{(-\rho)^{k-2}}{\sigma_{k-1} \dots \sigma_1} \right)\end{aligned}\quad (4.277)$$

Denote by $RV(\tilde{\zeta} | \underline{x}_t)$ the random part of $\tilde{\zeta}$ given \underline{x}_t , and denote $\tilde{\zeta}_k = RV(\tilde{x}_k - E_{k-1}[\tilde{x}_k] | \underline{x}_t)$.

With this notation I have

$$\tilde{\zeta}_k = \tilde{x}_k + \frac{-\rho}{\sigma_{k-1}} \tilde{x}_{k-1} + \frac{(-\rho)^2}{\sigma_{k-1} \sigma_{k-2}} \tilde{x}_{k-2} + \dots + \frac{(-\rho)^{k-t+1}}{\sigma_{k-1} \dots \sigma_{t+1}} \tilde{x}_{t+1}\quad (4.278)$$

Similarly,

$$\tilde{\zeta}_l = \tilde{x}_l + \frac{-\rho}{\sigma_{l-1}} \tilde{x}_{l-1} + \frac{(-\rho)^2}{\sigma_{l-1} \sigma_{l-2}} \tilde{x}_{l-2} + \dots + \frac{(-\rho)^{l-t+1}}{\sigma_{l-1} \dots \sigma_{t+1}} \tilde{x}_{t+1}\quad (4.279)$$

It follows that

$$\text{cov}_t(\tilde{x}_k - E_{k-1}[\tilde{x}_k], \tilde{x}_l - E_{l-1}[\tilde{x}_l]) = \text{cov}_t(\tilde{\zeta}_k, \tilde{\zeta}_l) \quad (4.280)$$

To calculate the above covariance, I analyze separately the cases $k = l + 1$, $k = l + 2$, etc. For $k = l + 1$,

$$\tilde{\zeta}_k = \frac{-\rho}{\sigma_l} \tilde{\zeta}_l + \tilde{x}_{l+1} . \quad (4.281)$$

It follows that

$$\text{cov}_t(\tilde{\zeta}_k, \tilde{\zeta}_l) = \text{cov}_t\left(\frac{-\rho}{\sigma_l} \tilde{\zeta}_l + \tilde{x}_{l+1}, \tilde{\zeta}_l\right) = \text{cov}_t(\tilde{\zeta}_l, \tilde{x}_{l+1}) - \frac{\rho}{\sigma_l} \text{var}_t(\tilde{\zeta}_l) . \quad (4.282)$$

Since $\text{cov}_t(\tilde{\zeta}_l, \tilde{x}_{l+1}) = \rho$, it remains to show that $\text{var}_t(\tilde{\zeta}_l) = \sigma_l$. If $l = t + 2$, $\tilde{\zeta}_l = \tilde{x}_l + \frac{-\rho}{\sigma_{l-1}} \tilde{x}_{l-1}$ and

$$\begin{aligned} \text{var}_t(\tilde{\zeta}_l) &= \text{var}_t(\tilde{x}_l) + 2 \frac{-\rho}{\sigma_{l-1}} \text{cov}_t(\tilde{x}_l, \tilde{x}_{l-1}) + \frac{(-\rho)^2}{\sigma_{l-1}^2} \text{var}_t(\tilde{x}_{l-1}) \\ &= \sigma - 2 \frac{\rho^2}{\sigma_{l-1}} + \frac{\rho^2}{\sigma_{l-1}^2} \sigma_{l-1} \\ &= \sigma - \frac{\rho^2}{\sigma_{l-1}} = \sigma_l \end{aligned} \quad (4.283)$$

Now I prove by induction that $\text{var}_t(\tilde{\zeta}_l) = \sigma_l$. It is sufficient to show that $\text{var}_t(\tilde{\zeta}_l) = \sigma_l \Rightarrow \text{var}_t(\tilde{\zeta}_{l+1}) = \sigma_{l+1}$. From $\tilde{\zeta}_{l+1} = \frac{-\rho}{\sigma_l} \tilde{\zeta}_l + \tilde{x}_{l+1}$, and assuming that $\text{var}_t(\tilde{\zeta}_l) = \sigma_l$, I have

$$\begin{aligned} \text{var}_t(\tilde{\zeta}_{l+1}) &= \text{var}_t(\tilde{x}_{l+1}) + 2 \frac{-\rho}{\sigma_l} \text{cov}_t(\tilde{x}_{l+1}, \tilde{\zeta}_l) + \frac{(-\rho)^2}{\sigma_l^2} \text{var}_t(\tilde{\zeta}_l) \\ &= \sigma - 2 \frac{\rho^2}{\sigma_l} + \frac{\rho^2}{\sigma_l^2} \sigma_l \\ &= \sigma - \frac{\rho^2}{\sigma_l} = \sigma_{l+1} \end{aligned} \quad (4.284)$$

Thus, I proved that $\text{var}_t(\tilde{\zeta}_l) = \sigma_l$ for all l and that $\text{cov}_t(\tilde{\zeta}_l, \tilde{\zeta}_{l+1}) = 0$ for all l . Next, I show that $\text{cov}_t(\tilde{\zeta}_l, \tilde{\zeta}_k) = 0$ implies $\text{cov}_t(\tilde{\zeta}_l, \tilde{\zeta}_{k+1}) = 0$ for all $k \geq l + 2$. Since $\tilde{\zeta}_{k+1} = \frac{-\rho}{\sigma_k} \tilde{\zeta}_k + \tilde{x}_{k+1}$,

$$\text{cov}_t(\tilde{\zeta}_l, \tilde{\zeta}_{k+1}) = -\frac{\rho}{\sigma_k} \text{cov}_t(\tilde{\zeta}_l, \tilde{\zeta}_k) + \text{cov}_t(\tilde{\zeta}_l, \tilde{x}_{k+1}) = 0 . \quad (4.285)$$

This concludes the proof of the first part of the lemma by induction.

For the second part of the lemma, I need to show that $\text{cov}_t(\tilde{x}_{t+1}, \tilde{\zeta}_k) = 0$ for all $k \geq t + 2$. This is straightforward since

$$\begin{aligned} \text{cov}_t(\tilde{x}_{t+1}, \tilde{\zeta}_k) &= \frac{(-\rho)^{k-t}}{\sigma_{k-1} \dots \sigma_{t+2}} \text{cov}_t(\tilde{x}_{t+1}, \tilde{x}_{t+2}) + \frac{(-\rho)^{k-t+1}}{\sigma_{k-1} \dots \sigma_{t+1}} \text{cov}_t(\tilde{x}_{t+1}, \tilde{x}_{t+1}) \\ &= \frac{(-\rho)^{k-t}}{\sigma_{k-1} \dots \sigma_{t+2}} \rho + \frac{(-\rho)^{k-t+1}}{\sigma_{k-1} \dots \sigma_{t+1}} \sigma_{t+1} = 0 . \end{aligned} \quad (4.286)$$

□

Proof. of Lemma 4.5.2 The proof is by backwards induction. For $t = N$, there is nothing to prove since by definition $\text{ACE}_{N-1}(w_t) = E_{N-1}[\tilde{c}_N] - \frac{1}{2}a_N^2 - \frac{1}{2}r \text{var}_{N-1}(\tilde{c}_N)$, and α_N is set by the reservation wage constraint.

Now assume that $\text{ACE}_{k-1}(w_k) = E_{k-1}[\tilde{c}_k] - \frac{1}{2}a_k^2 - \frac{1}{2}r \text{var}_{k-1}(\tilde{c}_k) = 0$ for all $t+1 \leq k \leq N$, and that $\alpha_k = -\beta_k E_{k-1}[\tilde{x}_k] + \frac{1}{2}a_k^2 + \frac{1}{2}r \text{var}_{k-1}(\tilde{c}_k)$. I need to show that $\text{ACE}_{t-1}(w_t) = E_{t-1}[\tilde{c}_t] - \frac{1}{2}a_t^2 - \frac{1}{2}r \text{var}_{t-1}(\tilde{c}_t) = 0$. By definition,

$$\begin{aligned} \text{ACE}_{t-1}(w_t) &= E_{t-1}[\tilde{c}_t] - \frac{1}{2}a_t^2 + \dots + E_{t-1}[\tilde{c}_N] - \frac{1}{2}a_N^2 \\ &\quad - \frac{1}{2}r \text{var}_{t-1}(\tilde{c}_t + \dots + \tilde{c}_N) . \end{aligned} \quad (4.287)$$

Substituting α_k in equation (4.287) for $t+1 \leq k \leq N$, I obtain

$$\begin{aligned} \text{ACE}_{t-1}(w_t) &= E_{t-1}[\tilde{c}_t] - \frac{1}{2}a_t^2 \\ &\quad + E_{t-1} \left[-\beta_{t+1} E_t[\tilde{x}_{t+1}] + \frac{1}{2}a_{t+1}^2 + \frac{1}{2}r \text{var}_t(\tilde{c}_{t+1}) + \beta_{t+1} \tilde{x}_{t+1} \right] - \frac{1}{2}a_{t+1}^2 \\ &\quad + \dots \\ &\quad + E_{t-1} \left[-\beta_N E_{N-1}[\tilde{x}_N] + \frac{1}{2}a_N^2 + \frac{1}{2}r \text{var}_{N-1}(\tilde{c}_N) + \beta_N \tilde{x}_N \right] - \frac{1}{2}a_N^2 \\ &\quad - \frac{1}{2}r \text{var}_{t-1}(\tilde{c}_t + \dots + \tilde{c}_N) . \end{aligned} \quad (4.288)$$

To simplify equation (4.288) further, I write

$$\tilde{c}_k = -\beta_k E_{k-1}[\tilde{x}_k] + \frac{1}{2}a_k^2 + \frac{1}{2}r \text{var}_{k-1}(\tilde{c}_k) + \beta_k \tilde{x}_k , \quad (4.289)$$

and I note that the random part of \tilde{c}_k conditional on $\underline{x}_{t-1}, \underline{a}_{t-1}$ is simply $\beta_k(\tilde{x}_k - E_{k-1}[\tilde{x}_k])$. It follows from Lemma 4.5.1 that $\tilde{c}_t, \dots, \tilde{c}_N$ are independent at the start of period t , and as a consequence

$$\text{var}_{t-1}(\tilde{c}_t + \dots + \tilde{c}_N) = \text{var}_{t-1}(\tilde{c}_t) + \dots + \text{var}_{t-1}(\tilde{c}_N) . \quad (4.290)$$

Substituting back in equation (4.288) gives the following

$$\begin{aligned} \text{ACE}_{t-1}(w_t) &= E_{t-1}[\tilde{c}_t] - \frac{1}{2}a_t^2 - \frac{1}{2}r \text{var}_{t-1}(\tilde{c}_t) \\ &\quad + \beta_{t+1}E_{t-1}[\tilde{x}_{t+1} - E_t[\tilde{x}_{t+1}]] \\ &\quad + \dots \\ &\quad + \beta_N E_{t-1}[\tilde{x}_N - E_{N-1}[\tilde{x}_N]] \end{aligned} \quad (4.291)$$

To conclude the proof of the lemma, I show that $E_{t-1}[\tilde{x}_k - E_{k-1}[\tilde{x}_k]] = 0$ for all $t+1 \leq k \leq N$. Since $E_{t-1}[\tilde{x}_k] = m_k a_k$ and $E_{k-1}[\tilde{x}_k] = m_k a_k + R_{k-1} \cdot (\underline{x}_{k-1} - \underline{m}_{k-1} \underline{a}_{k-1})$ it suffices to show that $E_{t-1}[R_{k-1} \cdot (\underline{x}_{k-1} - \underline{m}_{k-1} \underline{a}_{k-1})] = 0$. Expanding the term $R_{k-1} \cdot (\underline{x}_{k-1} - \underline{m}_{k-1} \underline{a}_{k-1})$ gives

$$\begin{aligned} -R_{k-1} \cdot (\underline{x}_{k-1} - \underline{m}_{k-1} \underline{a}_{k-1}) &= \frac{-\rho}{\sigma_{k-1}}(\tilde{x}_{k-1} - m_{k-1} a_{k-1}) \\ &\quad + \frac{(-\rho)^2}{\sigma_{k-1} \sigma_{k-2}}(\tilde{x}_{k-2} - m_{k-2} a_{k-2}) + \dots + \frac{(-\rho)^{k-t-1}}{\sigma_{k-1} \dots \sigma_{t+1}}(\tilde{x}_{t+1} - m_{t+1} a_{t+1}) \\ &\quad + \frac{(-\rho)^{k-t}}{\sigma_{k-1} \dots \sigma_t}(\tilde{x}_t - m_t a_t) \\ &\quad + \frac{(-\rho)^{k-t+1}}{\sigma_{k-1} \dots \sigma_{t-1}}(\tilde{x}_{t-1} - m_{t-1} a_{t-1}) + \dots + \frac{(-\rho)^{k-1}}{\sigma_{k-1} \dots \sigma_1}(\tilde{x}_1 - m_1 a_1) . \end{aligned} \quad (4.292)$$

Taking expectations of both sides in equation (4.292), it follows that

$$\begin{aligned}
E_{t-1} [-R_{k-1} \cdot (\underline{x}_{k-1} - \underline{m}_{k-1} \underline{a}_{k-1})] &= E_{t-1} \left[\frac{(-\rho)^{k-t}}{\sigma_{k-1} \dots \sigma_t} (\tilde{x}_t - m_t a_t) \right] \\
&+ \frac{(-\rho)^{k-t+1}}{\sigma_{k-1} \dots \sigma_{t-1}} (x_{t-1} - m_{t-1} a_{t-1}) + \dots + \frac{(-\rho)^{k-1}}{\sigma_{k-1} \dots \sigma_1} (x_1 - m_1 a_1) \\
&= \frac{(-\rho)^{k-t}}{\sigma_{k-1} \dots \sigma_t} E_{t-1} [\tilde{x}_t - m_t a_t] \\
&+ \frac{(-\rho)^{k-t}}{\sigma_{k-1} \dots \sigma_t} \left(\frac{-\rho}{\sigma_{t-1}} (x_{t-1} - m_{t-1} a_{t-1}) + \dots + \frac{(-\rho)^{t-1}}{\sigma_{t-1} \dots \sigma_1} (x_1 - m_1 a_1) \right)
\end{aligned} \tag{4.293}$$

Finally, I have

$$\frac{-\rho}{\sigma_{t-1}} (x_{t-1} - m_{t-1} a_{t-1}) + \dots + \frac{(-\rho)^{t-1}}{\sigma_{t-1} \dots \sigma_1} (x_1 - m_1 a_1) = -R_{t-1} \cdot (\underline{x}_{t-1} - \underline{m}_{t-1} \underline{a}_{t-1}) ,$$

which gives

$$\begin{aligned}
E_{t-1} [-R_{k-1} \cdot (\underline{x}_{k-1} - \underline{m}_{k-1} \underline{a}_{k-1})] \\
= \frac{(-\rho)^{k-t}}{\sigma_{k-1} \dots \sigma_t} (E_{t-1} [\tilde{x}_t - m_t a_t] - R_{t-1} \cdot (\underline{x}_{t-1} - \underline{m}_{t-1} \underline{a}_{t-1})) = 0 .
\end{aligned} \tag{4.294}$$

□

Proof. of Theorem 4.5.1 The first-order condition for the agent's rationality constraint determines a_t given α_t and β_t (see (4.240)).

$$\begin{aligned}
\frac{\partial}{\partial a_t} \text{ACE}_{t-1}(w_t | a_t, \hat{a}_t) \\
= m_t \beta_t - a_t + m_t \beta_{t+1} \frac{-\rho}{\sigma_t} + \dots + m_t \beta_N \frac{(-\rho)^{N-t}}{\sigma_{N-1} \dots \sigma_t} = 0
\end{aligned} \tag{4.295}$$

It follows that

$$\begin{aligned}
a_t &= m_t \left(\beta_t + \beta_{t+1} \frac{-\rho}{\sigma_t} + \dots + \beta_N \frac{(-\rho)^{N-t}}{\sigma_{N-1} \dots \sigma_t} \right) \\
&= m_t \beta_t - m_t \frac{\rho}{\sigma_t} \left(\beta_{t+1} + \beta_{t+2} \frac{-\rho}{\sigma_{t+1}} + \dots + \beta_N \frac{(-\rho)^{N-t-1}}{\sigma_{N-1} \dots \sigma_{t+1}} \right) \\
&= m_t \beta_t - \frac{\rho}{\sigma_t} \frac{m_{t+1}}{m_t} a_{t+1} .
\end{aligned} \tag{4.296}$$

In particular, for $t = N$, $a_N = m_N \beta_N$. The agent's reservation wage constraint is used to determine α_t . From Lemma 4.5.2, I have

$$\begin{aligned}\alpha_t &= -\beta_t E_{t-1}[\tilde{x}_t] + \frac{1}{2}a_t^2 + \frac{1}{2}r \text{var}_{t-1}(\tilde{c}_t) \\ &= -\beta_t E_{t-1}[\tilde{x}_t] + \frac{1}{2}a_t^2 + \frac{1}{2}r \beta_t^2 \text{var}_{t-1}(\tilde{x}_t)\end{aligned}\quad (4.297)$$

I can use now equations (4.296) and (4.297) to substitute in the principal's problem for the action induced by β_t . The first-order condition is

$$\begin{aligned}&\frac{\partial}{\partial \beta_t} E_{t-1}[(\tilde{z}_t - \tilde{c}_t) + \cdots + (\tilde{z}_N - \tilde{c}_N)] \\ &= \frac{\partial}{\partial \beta_t} E_{t-1}[\tilde{z}_t - \tilde{c}_t] \\ &= \frac{\partial}{\partial \beta_t} E_{t-1}\left[\tilde{z}_t + \beta_t E_{t-1}[\tilde{x}_t] - \frac{1}{2}a_t^2 - \frac{1}{2}r \beta_t^2 \text{var}_{t-1}(\tilde{x}_t) - \beta_t \tilde{x}_t\right] \\ &= \frac{\partial}{\partial \beta_t} E_{t-1}\left[\tilde{z}_t - \frac{1}{2}a_t^2 - \frac{1}{2}r \beta_t^2 \sigma_t\right] \\ &= b_t \frac{\partial a_t}{\partial \beta_t} - a_t \frac{\partial a_t}{\partial \beta_t} - r \beta_t \sigma_t = 0.\end{aligned}\quad (4.298)$$

Since $\frac{\partial a_t}{\partial \beta_t} = m_t$, it follows that

$$r \beta_t \sigma_t = m_t(b_t - a_t). \quad (4.299)$$

Using equation (4.299) to substitute for a_t and a_{t+1} in equation (4.296), I obtain

$$a_t = b_t - \frac{r \beta_t \sigma_t}{m_t}$$

and

$$b_t - \frac{r \beta_t \sigma_t}{m_t} = m_t \beta_t - \frac{\rho}{\sigma_t} \left(b_{t+1} - \frac{r \beta_{t+1} \sigma_{t+1}}{m_{t+1}}\right) \frac{m_t}{m_{t+1}}. \quad (4.300)$$

Solving for β_t gives

$$\left(m_t + r \frac{\sigma_t}{m_t}\right) \beta_t = b_t + \frac{\rho}{\sigma_t} \frac{m_t}{m_{t+1}} b_{t+1} - r \rho \frac{\sigma_{t+1}}{\sigma_t} \frac{m_t}{m_{t+1}^2} \beta_{t+1},$$

$$\beta_t = (m_t^2 + r\sigma_t)^{-1} \left[m_t \beta_t + \frac{\rho}{\sigma_t} \frac{m_t^2}{m_{t+1}} b_{t+1} - r\rho \frac{\sigma_{t+1}}{\sigma_t} \frac{m_t^2}{m_{t+1}^2} \beta_{t+1} \right] ,$$

which simplifies to

$$\beta_t = \frac{m_t b_t + \frac{\rho}{\sigma_t} \frac{m_t^2}{m_{t+1}^2} m_{t+1} b_{t+1}}{m_t^2 + r\sigma_t} - \rho r \frac{\sigma_{t+1}}{\sigma_t} \frac{m_t^2}{m_{t+1}^2 + r\sigma_t} \beta_{t+1} . \quad (4.301)$$

□

Proof. of Theorem 4.7.1 The proof is based on describing the asymptotic behavior of $\Pi(N)/N$. The main idea is that there exist coefficients $\gamma_1 > 0$, $\gamma_2 > 0$, $\gamma_3 < 0$, and $\delta < 0$, with $|\delta| < 1$ such that

$$\Pi(N) = \gamma_1 N + \gamma_2(1 - \delta^N) + \gamma_3(1 - \delta^{2N}) . \quad (4.302)$$

Then, I will show that taking $C_0 = \gamma_2(1 - \delta) + \gamma_3(1 - \delta^2) > 0$ gives the desired result.

I now turn to the details of the proof. Let $e_t^\beta = \beta_t - \beta$. To simplify notation, let

$$\delta = \frac{-\rho r}{m^2 + r\sigma_\infty} .$$

Note that $|\delta| < 1$ determines how fast the actions and the incentive rates converge to their respective limits. It follows that $e_t^\beta = \delta e_{t+1}^\beta$ and that

$$e_{N-k}^\beta = \delta^k e_N^\beta = \delta^{k+1} \frac{m^2 \bar{\beta}}{r(\rho + \sigma_\infty)} .$$

Since $e_{N-k}^\beta = \beta_{N-k} - \bar{\beta}$, it follows that

$$\beta_{N-k} = \bar{\beta} + e_{N-k}^\beta = \bar{\beta} \left(1 + \frac{m^2}{r(\rho + \sigma_\infty)} \delta^{k+1} \right) .$$

Then (4.272) gives

$$\begin{aligned}
E_0[\Pi(N)] &= \frac{1}{2}Nb^2 - \frac{1}{2}r\sigma_\infty(1 + r\sigma_\infty m^{-2}) \sum_{k=0}^{N-1} \beta_{N-k}^2 \\
&= \frac{1}{2}Nb^2 - \frac{1}{2}r\sigma_\infty(1 + r\sigma_\infty m^{-2}) \bar{\beta}^2 \sum_{k=0}^{N-1} \left(1 + 2 \frac{m^2}{r(\rho + \sigma_\infty)} \delta^{k+1} + \frac{m^4}{r^2(\rho + \sigma_\infty)^2} \delta^{2k+2} \right) \\
&= \frac{1}{2}Nb^2 - \frac{1}{2}r\sigma_\infty(1 + r\sigma_\infty m^{-2}) \bar{\beta}^2 \left(N + \frac{2\delta m^2}{r(\rho + \sigma_\infty)} \frac{1 - \delta^N}{1 - \delta} + \frac{\delta^2 m^4}{r^2(\rho + \sigma_\infty)^2} \frac{1 - \delta^{2N}}{1 - \delta^2} \right) \\
&= \gamma_1 N + \gamma_2(1 - \delta^N) + \gamma_3(1 - \delta^{2N}) .
\end{aligned} \tag{4.303}$$

The three coefficients on the last line of equation (4.303), γ_1 , γ_2 , and γ_3 are given by

$$\gamma_1 = \frac{1}{2}b^2 m^2 \frac{m^2 \sigma_\infty + r(\sigma_\infty^2 - \rho^2)}{\sigma_\infty(m^2 + r(\rho + \sigma_\infty))^2} > 0 \tag{4.304}$$

$$\gamma_2 = m^2 b^2 \frac{r\rho(\rho + \sigma_\infty)(m^2 + r\sigma_\infty)}{\sigma_\infty(m^2 + r(\rho + \sigma_\infty))^3} > 0 \tag{4.305}$$

$$\gamma_3 = -\frac{1}{2}m^4 b^2 \frac{r\rho^2(m^2 + r\sigma_\infty)}{\sigma_\infty(m^2 + r(\rho + \sigma_\infty))^3(m^2 + r(\sigma_\infty - \rho))} < 0 \tag{4.306}$$

Let $C_0 = \gamma_2(1 - \delta) + \gamma_3(1 - \delta^2)$. Also note that $\gamma_2(1 - \delta) + \gamma_3(1 - \delta^2) > 0$. Given the switching cost C , the principal's problem is the maximization of $(E_0[\Pi(N)] - C)/N$. The principal's objective function can be rewritten as follows:

$$\frac{E_0[\Pi(N)] - C}{N} = \gamma_1 + \frac{\gamma_2(1 - \delta^N) + \gamma_3(1 - \delta^{2N}) - C}{N} .$$

Now rewrite $C = C_0 + \varepsilon$, where $\varepsilon > 0$ if $C > C_0$, and $\varepsilon < 0$ if $C < C_0$. Then

$$\frac{E_0[\Pi(N)] - C}{N} = \gamma_1 + \frac{\gamma_2(1 - \delta^N) + \gamma_3(1 - \delta^{2N}) - C_0 - \varepsilon}{N} \tag{4.307}$$

$$\frac{E_0[\Pi(1)] - C}{1} = \gamma_1 + \gamma_2(1 - \delta) + \gamma_3(1 - \delta^2) - C_0 - \varepsilon \tag{4.308}$$

$$= \gamma_1 - \varepsilon . \tag{4.309}$$

If $C > C_0$, and thus $\varepsilon > 0$, for $N \geq 2$,

$$\frac{E_0[\Pi(N)] - C}{N} = \gamma_1 + \frac{\gamma_2(\delta - \delta^N) + \gamma_3(\delta^2 - \delta^{2N}) - \varepsilon}{N} \quad (4.310)$$

$$< \gamma_1 - \frac{\varepsilon}{N}, \quad (4.311)$$

since $\gamma_2(\delta - \delta^N) + \gamma_3(\delta^2 - \delta^{2N}) < 0$. It follows that

$$\frac{E_0[\Pi(N)] - C}{N} < \gamma_1 - \frac{\varepsilon}{N}$$

for all N . On the other hand,

$$\lim_{N \rightarrow \infty} \frac{E_0[\Pi(N)] - C}{N} = \gamma_1.$$

Thus, for any given N , there exists $N' > N$ such that

$$\frac{E_0[\Pi(N)] - C}{N} < \frac{E_0[\Pi(N')] - C}{N'}.$$

In addition, there exists N_0 such that the function $(E_0[\Pi(N)] - C)/N$ becomes increasing for $N \geq N_0$. Then, for all $N < N_0$, $(E_0[\Pi(N)] - C)/N < \gamma_1 - \varepsilon/N_0$. The maximum cannot be reached between $N = 1$ and $N = N_0$, because for large enough N' , $(E_0[\Pi(N')] - C)/N' > \gamma_1 - \varepsilon/N_0$. The maximum is attained at N_{max} since the function $(E_0[\Pi(N)] - C)/N$ is increasing for $N \geq N_0$ and we assumed N_{max} to be large enough.

If $C_0 > C$, and thus $\varepsilon < 0$,

$$\frac{E_0[\Pi(N)] - C}{N} = \gamma_1 + \frac{\gamma_2(\delta - \delta^N) + \gamma_3(\delta^2 - \delta^{2N}) - \varepsilon}{N} \quad (4.312)$$

$$< \gamma_1 - \frac{\varepsilon}{N} < \gamma_1 - \varepsilon, \quad (4.313)$$

for all $N \geq 2$. However, $\gamma_1 - \varepsilon = (E_0[\Pi(1)] - C)/1$ which proves that

$$\frac{E_0[\Pi(1)] - C}{1} > \frac{E_0[\Pi(N)] - C}{N}$$

for all N .

□

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