ESSAYS ON STRATEGIC DIVISIONALIZATION AND DECENTRALIZATION

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0.1 Abstract

The objective of the three essays of this doctoral dissertation is to investigate the strategic choices of organizational forms by competing firms in various environments.

The first essay, which is a joint work with Professor Guofu Tan, provides an alternative theory of divestitures that relies on product-line complementarities and product market competition. We consider a simple environment in which there are two firms, each supplying a group of complementary products and the products across groups are imperfect substitutes. We model the firms' choices of divesting and pricing as a two-stage game. The duopolists simultaneously choose their divestiture strategies in the first stage of the game and the independent divisions compete by setting prices in the second. It is shown that, when competing with each other, firms with complementary product-lines have incentives to split into multiple independent divisions supplying complementary products and services. Such divestitures increase prices and the parent firms' values but reduce aggregate social welfare. Moreover, the degree of divestiture, as we illustrate in the linear demand case, depends on the severity of competition and the nature of product-lines. Then, intensified competition due to deregulation, trade liberalization and entry may trigger divestitures. We further show that if two firms are able to coordinate their divestiture strategies, they can achieve the joint monopoly prices and profits in a non-cooperative price game.

The second essay analyzes the strategic incentive of oligopolists to create autonomous rival divisions when products are differentiated. We consider a two stage game where firms choose the number of autonomous divisions in the first stage and all the divisions engage in Cournot competition in the second. It is shown that product differentiation ensures the existence of an interior subgame perfect Nash equilibrium, and the equilibrium number of divisions increases with the degree of substitution among products and the number of firms. Further, if divisions are allowed to further divide, they always will, which leads to total rent dissipation. Thus, parent firms have incentives to unilaterally restrict their divisions from further dividing. In the free entry equilibrium, it is found that the possibility of setting up autonomous divisions is a natural barrier to entry. Incumbents may persistently earn abnormally high profits. In the cases where product differentiation is difficult, the only pure strategy free entry equilibrium is the monopoly outcome even if
the entry cost is relatively low.

The third essay develops a game theoretic model to analyze strategic leasing behaviors of landowners in a nonexclusively owned common oil pool. The oil field development is modeled as two more-or-less independent one-stage noncooperative game. The landowners choose leasing strategies in the first stage, and independent lease operators choose extraction strategies in the second. It is found that, in a nonexclusively owned oil field, it is individually rational for a landowner to unilaterally subdivide his landholding and delegate production rights to multiple independent firms, even though more dispersed production control leads to heavier common pool losses. Moreover, the degree of landownership concentration determines the degree of production concentration. The more fragmented the land ownership, the lower is the degree of production concentration in equilibrium. The analysis offers an explanation for the puzzling landowners' leasing behaviors in U.S. onshore oil fields.
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Chapter 1

Introduction

The modern economy is dominated by large corporations with sophisticated structures. Instead of being centralized to a single entity, production and marketing decisions in a firm are often made by various levels of management, subsidiaries, divisions, franchises, etc.. The institutional arrangement and decision structure in firms are critical factors for economic performance. The objective of this dissertation is to investigate how firms strategically set up autonomous units when facing competition in several simplified environments.

Coase is among the pioneers who first realized the importance of the study of the “institutional structure of production.” Stimulated by the great debate of the advantages of a free market system over a central planning system in the early thirties, Coase started to rationalize the very existence of the firm in the free market economy. According to economic theory, a competitive market economic system coordinated by prices would deliver the Pareto-efficient outcome. Then, if the pricing system provides all the coordination necessary, why is there a need for firms whose function is to coordinate in a planning fashion? In his famous 1937 article “The Nature of the Firm,” Coase acknowledged that there are transaction costs of using the price system. And, it is the avoidance of these costs that could explain the existence of the firm in which allocation of factors comes about as a result of administrative decisions. A firm could continue to exist if it performs its coordination function at a lower cost than would be occurred if it were achieved by means of market transactions and also at a lower cost than the same function could be performed by another firm.
Coase's theory convincingly justifies the emergence of the firm in the free market economy but stops short of explaining how and why a firm chooses a certain form of organization. Williamson (1975) investigates the increased use of multidivisional organization form in big corporations documented by Chandler (1962). He argues that big organizations inevitably encounter monitoring difficulties and moral hazard problems. Multidivisional form is an institutional innovation arising to reduce monitoring costs and to mitigate moral hazard problems by devolving operational decisions to smaller autonomous divisional units. Along a similar line of argument, Aron (1988) shows that stock market evaluation of separated units can reduce the costs of motivating managers. Meyer, Milgrom and Roberts (1992) argue that since influence costs, which arise from the possibility of manipulation of performance measures by poor performing units, reduce a firm's potential value, poor performing units should be divested. These theories based on asymmetric information or moral hazard problems do provide rationales for why firms set up multiple independently managed units. The explanation, however, is incomplete because the potential external effects of a firm's choice of organizational form on other firms in the market are neglected.

Recently, many authors have realized that there is a strategic aspect to the choice of the organizational form by a firm. Vickers (1984), Fershtman and Judd (1987), Fershtman, Judd and Kalai (1991), and Sklivas (1987) study the separation of ownership and control in management controlled firms in a duopolistic setting. The generic model is a two-period model. In the first period, each owner provides his manager with an incentive contract that is a linear combination of profit and sales revenue or output; in the second period, managers make production and sales decisions following their own interest in either a Cournot quantity game or a Bertrand price game. These articles show that the separation of ownership and management is the rational response of firms to oligopolistic competition. Moreover, the firms' goal-setting incentive contracts for managers are interdependent and are typically not the direct profit maximizing schemes. Under Cournot (Bertrand) competition, firms' profits are lower (higher) relative to the case where there is no separation of ownership and management.

These strategic delegation models have been helpful in explaining the existence of management control firms and the related compensation schemes for managers. However, by construction, these models can not explain the widespread existence of multiple au-
tonomously managed units (such as divisions, franchises, etc.) within a firm, for each owner is only allowed to hire a single manager.

More recently, there has been a growing interest in strategic divisionalization, which refers to firms' choices of multidivisional forms under oligopolistic competition. Schwartz and Thompson (1986) and Veendorp (1991) demonstrate that an incumbent monopoly can forestall entry through setting up multiple identical autonomous divisions. They argue that divisionalization can be used as a commitment device for the monopoly to produce an entry-deterring level of output. Corchon (1991), Polasky (1992), Corchon and Gonzalez-Meastre (1993), and Baye, Crocher and Ju (1996) consider a case of a symmetric duopoly with homogeneous products. The basic model is a two-stage game, where each duopolist chooses the number of autonomous divisions in the first stage, and all divisions engage in a Cournot competition in the second stage. The main result is that the duopolists have incentives to unilaterally set up multiple autonomous divisions. Such a strategy commits a parent firm to a higher level of output, mimicking a Stackelberg-type outcome in pursuing Cournot competition (Baye, Crocker and Ju, 1996). In equilibrium, the profits of competing firms are lower and social welfare is higher than in the no-divisionalization case.

This dissertation is an attempt to expand the investigation of how and why firms divide production among autonomous units into several different scenarios of competition. It consists of three essays. The first essay studies divisionalization of competing multiproduct firms when product-lines consist of complementary products or services. The second essay deals with divisionalization of firms with differentiated products. Finally, the third essay investigates strategic leasing in common pool oil production, which can be viewed as a special form of divisionalization. Without doubt, divisionalization in this thesis is used in the broadest sense possible. It refers to firms setting up independently managed units in whatever forms, such as independent leases, franchises, spinoffs, outright sales, etc..

In the first essay, a joint work with my thesis supervisor Professor Guofu Tan, we develop a theory of divisionalization (divestitures) that relies on product-line complementarities and product market competition. We consider a simple environment in which each of the competing firms supplies a group of complementary goods or services. Divisionalization of a multi-product firm is modeled as a partition of its product space. Hence,
divisions of the same firm produce products that are complementary to each other. The competition is modeled as a two-stage game. Each firm chooses its number of divisions in the first stage, and all divisions engage in a Bertrand price game in the second stage.

Under a general demand, it is shown that firms with complementary product-lines have incentives to set up multiple autonomous divisions unilaterally in competition. Prices and firms' profits are higher and social welfare is lower relative to the case where no divisionalization is permitted. More interestingly, if firms are able to coordinate their divisionalization decisions in the first stage, the subsequent noncooperative Bertrand competition in the second stage leads to monopoly prices and profits, as if all firms (or all divisions) are managed by a single agent. The number of divisions in a symmetric game is positively related to both the degree of competition, which is measured by the degree of the substitutability among the groups of products, and the number of groups. Therefore, this analysis builds a link between firms' business restructuring decisions or choices of organizational form and the competitive environment. Intensified competition that results from deregulation, trade liberalization or entries may trigger divisionalization. This may provide a partial explanation for the coincidence of the increased use of multidivisional forms (including divestitures) and the waves of deregulation and globalization in the developed economies since the 1980s.

The key to these results is the two opposite effects that the pricing of a product inflicts on complements and substitutes. It is known that an increase in the price of a product has a negative effect on the demand of its complements and a positive effect on the demand of its substitutes. In absence of divisionalization, the prices of products of two competing firms are too low relative to the monopoly prices, which maximize the aggregate profits of these firms. After divisionalization, independent divisions of the same firm ignore the negative externality of pricing on each other. Thus, the resulting prices and aggregate profits rise towards the monopoly level. The possibility that firms can tacitly collude in pricing through divisionalization makes antitrust regulation a more complex issue.

The second essay analyzes strategic divisionalization of oligopoly with product differentiation and the implication of free entry equilibrium. This analysis can be viewed as a generalization of the model of Corchon, Polasky and Baye et al., since it contains their model as a special case. Following convention, the competition is modeled as a two-stage game. Oligopolists choose the number of identical autonomous rival divisions in the first
stage, and all divisions engage in a Cournot competition in the second.

Like Corchon (1991), Polasky (1992) and Baye et al (1996), I show that oligopolists have incentives to set up unilaterally autonomous rival divisions, despite the fact that profits are lower for each of them in equilibrium relative to the no-divisionalization case. The equilibrium number of divisions of each firm increases with the substitutability of the differentiated products and with the number of competing firms.

Divisionalization also has significant implication on entry. An entry can considerably intensify divisionalization and, consequently, the competition. The credible threat of divisionalization in the case of an entry makes divisionalization a natural entry deterrent. As a result, an incumbent can enjoy an extremely high profit relative to the entry cost without worrying about an entry, when new products are difficult to differentiate from the existing ones. If an entry does occur, to soften competition, the entrant has much higher incentives to differentiate itself from the incumbent relative to the case where no divisionalization is allowed.

The third essay investigates strategic leasing of production rights in U.S. onshore oil fields. Oil production is modeled as a two-stage game. First, landowners choose leasing strategies, and then, all lease holders choose extraction strategies.

It is shown that landowners have incentives to subdivide unilaterally their landholdings and grant production rights to multiple independent operators, despite the fact that more dispersed production concentration leads to a more serious rent dissipation in a common pool oil field. Multiple leasing is a commitment mechanism for landowners to gain the Stackleberg-leader-advantage in the production stage.

This analysis provides an explanation to a long-standing puzzling phenomenon, i.e., the persistence of the widespread inefficient production organization in U.S. onshore oil fields. Moreover, it has significant implications on general common property problems. In the existing literature, the property owner is treated the same as the property operator. This article, however, shows that they are usually very different and that the operation structure can be much more dispersed than the ownership structure, because owners have incentives to grant operation rights to multiple independent agents. Therefore, distinguishing the ownership structure from the operation structure and modeling the relationship between them are essential to common property problems with privately controlled access (not free access).
The analysis in this dissertation is also closely related to, but distinct from the merger literature such as Salant et al. (1983) and Gaudet and Salant (1991), which examine the impact of mergers on firms' profits and on social welfare. Under plausible conditions, they show that merger of a subset of firms may result in profit loss for the merging firms, even though merger leads to a more concentrated oligopoly. This dissertation deals with the opposite issue: the incentive facing firms to divisionalize or to set up rival independent units. We show that, under several different and plausible scenarios, each firm has unilateral incentives to create rival competing units. Unlike most of the merger literature, our analysis permits one to analyze the equilibrium consequences of these incentives in a noncooperative setting that allows all firms to divisionalize freely. The merger literature, in contrast, implicitly views merger as a cooperative game among merging parties, for the set of firms that merge is usually exogenously selected.
Chapter 2

Complements, Substitutes and Strategic Divestiture

2.1 Introduction

Since the early eighties, corporate *divestitures* have been a major form of corporate re­structuring in the United States, accounting for 40 — 50% of merger and acquisition activities. Divestitures (or sell-offs) involve the transfer of a part of a firm’s business to a new owner, as opposed to the sale of the entire firm. The value of such sell-offs reached over $177 billion in the United States in 1996\(^1\), and similar markets exist in the other industrialized economies. Both the volume and the nature of these transactions raise questions for the economics of organization and for corporate strategy. What determines whether the complex bundle of business activities that comprises a corporation should be either a freestanding, independent enterprise or, instead, a unit of a large firm? What determines whether a particular business unit should be divested, and if so, how?

In this paper, we examine a possible answer to these questions. More specifically, we focus on firms’ strategic incentives of divestitures that rely on product-line complementarities and product market competition. We consider a simple environment in which there are two firms. Each firm supplies a group of complements, and the two groups of complements are imperfect substitutes. With this framework, we try to approximate competition among conglomerates, which are the main sources of divestitures. For ex-

\(^1\)See Mergerstat, 1997, p28-30.
ample, two competing airline alliances, each of which consists of several regional airline companies, have a conceptually similar structure. Another example is two competing shopping centers, each consisting of a number of stores. The existence of transportation costs makes stores within a mall, even when supplying substitutes in the normal sense, transactional complements ex ante (Ayres, 1985; Stahl, 1987; and Beggs, 1994). We address only the issue of strategic divestitures in such an environment, while the issue of mergers can be analyzed analogously in a setting where each complement is supplied by an independent firm. We model the firms' choices of divesting and pricing as a two-stage game. The duopolists simultaneously choose their divestiture strategies in the first stage of the game, and the independent divisions (the parent firms and the divested divisions) compete in prices in the second. In order to highlight purely strategic motives in our analysis, we strip away other factors that might affect the firms' decisions to divest, such as moral hazard problems and increasing return to scales. In particular, we assume that the technology of each firm exhibits constant returns to scale and scope.

In this framework we first show that, when facing competition, firms producing a set of complementary products or services may find divesting some of their complementary products a profitable strategy. By divesting, a firm credibly commits not to coordinate the pricing of complements and, therefore, softens competition to its rival firm. On one hand, this strategy makes the divesting firm more vulnerable to its rival firm's potential aggressive pricing. On the other hand, softened competition will increase the total profit in the market. If the gain in total profits outweighs the loss due to the vulnerability from divestiture, then divestiture is a profitable strategy for a firm. Once its rival has divested, a firm is more likely to follow the suit since the vulnerability resulted from divesting is reduced. In the symmetric equilibrium, both firms divest and neither suffers any loss in terms of relative strength in competition. Both firms then gain from the increase in total profits.

The results rely critically on product-line complementarity and competition between firms. In the absence of divestitures, two competing firms set prices lower than the monopoly prices, which maximize the aggregate profits, for each firm ignores the positive externality of its pricing on the rival firm's profit. After a firm divests into independent divisions producing complementary products, those divisions ignore the negative externalities of their pricing on each other's profits. When the degree of divestiture is small,
the own-group negative externality, which denotes the externality of pricing among divisions of the same firm, is smaller than the cross-group positive externality, which is the externality of pricing between firms. Therefore, a small degree of divestiture by one firm moves the prices closer to the monopoly prices, leading to higher profits. This reasoning works for both firms. Indeed, when we characterize the subgame perfect equilibrium of the two-stage game, we find that both firms have incentives to divest when competing. The degree of divestiture in equilibrium is determined by both the degree of substitution (or the severity of competition) and the degree of complementarity of each firm's product-line. The more severe the competition among firms, the higher the incentives to divest. Thus, changing the competition environment such as deregulation, trade liberalization and entry may trigger divestitures. By divesting, competing firms create own-group pricing externality to mitigate the opposite cross-group externality, resulting in higher prices and profits.

Our analysis has important implications for the social welfare consequences of divestitures. Conventional wisdom holds that mergers of firms supplying similar products reduce competition, increase the prices of the products, and decrease consumers' welfare and total surplus. Consequently, mergers have been the main concern of regulatory authorities. When divestitures are motivated by product-line complementarities, however, we find that divestitures increase prices, lower quantities, and decrease consumers' welfare and social surplus. Therefore, from a social welfare point of view, divestitures involving complementary goods or services should be discouraged as much as mergers involving substitutes. The fact that firms can achieve tacit collusion in pricing through divestiture when competing product-lines consist of complements makes regulation a more complex and difficult issue.

We also consider the situation in which firms are able to coordinate their divestiture decisions in the first stage. It is shown that there exists a pair of divestiture strategies such that the joint monopoly prices and profits are achieved in a non-cooperative price competition game in the second stage. In such cases, by choosing the appropriate degree of divestiture, competing firms create an own-group negative externality which exactly

\[\text{Our results in this context have a close parallel with those of Spence (1976) and Economides and Salop(1992), who argue that for complementary products, mergers reduce prices and increase consumers' welfare.}\]
offsets the cross-group externality. Such a coordinated number of divisions is greater than the number of divisions in the non-cooperative game. This is due to a positive externality between the firms' choices of divisions. After divesting, a firm commits not to coordinate the pricing of its divisions. This fat-cat strategy softens the second stage competition, raises prices, and thus benefits the rival firm. Coordination between the two firms internalizes the positive externality and thus results in more divisions than the non-cooperative equilibrium permits. With non-coordinated divestiture, the prices move up but remain below the monopoly prices. With coordinated divestiture, the equilibrium prices are exactly the same as monopoly prices, as if all divisions of both firms are operated by a single agent. Through divestitures, firms may achieve perfect collusion in pricing in a non-cooperative price game, which is a striking result.

A variety of other arguments have recently been put forward to explain divestitures. For instance, divesting is rationalized by some as an institutional innovation arising in response to the loss due to moral hazard problems in large corporations (Williamson, 1975; Aron, 1988; Hart and Moore, 1990; Holmstrom and Milgrom, 1991; Meyer, Milgrom and Roberts, 1992; etc.). These theories based on asymmetric information are certainly useful in explaining the decentralization of the control of assets or business activities. They have little power, however, in explaining that many divestitures are of units that had been previously acquired rather than started from scratch by divesting firms (Porter, 1987; Ravenscraft and Scherer, 1987; and Kaplan and Weisbach, 1990). Both Porter (1987) and Ravenscraft and Scherer (1987) interpret those sales as recognition of failure, which would account for the presumed performance-divestiture linkage. The evidence assembled by Kaplan and Weisbach (1990), however, suggests that less than one-third of acquisitions that were later divested could be considered failures ex post. Our theory differs from those arguments by relating divestitures to competition environments and the nature of product-lines. We predict that increased competition tends to intensify divestiture activities.

Jensen's free-cash-flow hypothesis (1986) suggests that managers who are imperfect agents of stockholders will have a tendency to invest even in unprofitable businesses. The frequent asset sales following hostile takeovers can be interpreted as undoing excessive and unprofitable conglomerations. Increased discipline on managers from the strengthened market for corporate control in the 1980s reduced such investments and might also have
led managers to divest their previous bad investments to avoid having their companies become subject to hostile takeover. Jensen's hypothesis is consistent with the finding that the accounting performance of the divesting firm improves after the divestiture and that the announcement effects are positive (John and Ofek, 1995).

The puzzling side of this story is why the bad investments can be sold for more than they are worth to the current owner. One of the prominent explanations advanced is that divestitures are motivated by a desire to increase the focus of a firm's business and establish core competencies. The underlying hypothesis is that perhaps more focused firms are easier to manage and so create greater values or that different managers are skilled in managing different types of assets. Such an explanation is not very satisfactory, however, since it is hard to refute. In this paper, we provide an alternative explanation for a class of divestitures where product-line complementarities exist between the divesting units. Casual empiricism suggests that such complementarities often exist. Divestitures increase the total value of a firm because of the changed competition structure and the firms' strategic behaviors.

Our analysis is related to, but distinct from, the literature on mergers involving firms supplying complements. Salop (1990) and Economides and Salop (1992) show that, in a Cournot duopoly model with complements, mergers of two firms supplying complements reduce prices, because a merger allows coalition firms to absorb a positive externality.

For example, on September 20, 1995, AT&T announced that it would split into three independent firms, with the first offering long-distance telephone and credit card services, the second supplying telecommunication equipment, and the third dealing with computer businesses. These three businesses can be viewed as complementary. For details, see "AT&T's three-way split," The Economist, September 23rd, 1995. Many other telecom-equipment companies such as Sweden's Ericsson, Finland's Nokia, and the U.S.'s 3Com entered the computer market not long ago, but have now left the computer business.

Another example is shopping malls. We often observe that in-town shopping malls consist of many independently managed shops which usually sell ordinary complements. Different shopping malls compete by selling goods which are substitutes across malls. Furthermore, even if different stores in a mall offer similar products, consumers are not sure which product they would prefer prior to visiting stores. Their decisions are often based on expected prices. Therefore, the existence of shopping costs makes ordinary substitutes within a mall transaction complements (see Ayres, 1985; Stahl, 1987; and Beggs, 1994). Pashigian and Gould (1995) provide empirical evidence that positive agglomerate externalities exist when stores are located together in a mall. Our analysis explains why shops in shopping malls are not owned by a single firm.

The results are also similar to those in the vertical integration literature such as Greenhut and Ohta.
Our concern is with the opposite issue: competing firms' incentives to divest and the consequences. Further, unlike most of the merger literature, which usually models mergers in a cooperative game setting, our analysis of firms' choices of organization forms and pricing is conducted in a non-cooperative game setting.

This paper is also closely related to the recent strategic divisionalization literature, such as Corchon (1991), Corchon and Gonzales-Maestre (1993), Polasky (1992) and Baye, Crocker, and Ju (1996). These authors analyze the strategic incentives for firms to form independent divisions when competing in a homogeneous product market. They find that firms form multiple divisions in order to take a larger share of the market. Moreover, divisionalization leads to lower prices, lower profits and higher social welfare. This paper analyzes a different competitive environment and identifies a different incentive for firms to break up. In our framework, firms with a set of complementary products set up independent divisions to soften the competition from their rivals. As a result, divestiture of this kind increases prices and profits, and reduces the social welfare.

For simplicity, the main part of our analysis is conducted in a setting where there are two firms, each supplying a group of perfect complementary products or services. However, the intuition and qualitative results carry over to more complex settings in which there are more than two competing firms, and product-lines are imperfect complements.

The rest of the paper is organized as follows. The next section introduces a two-stage duopoly model with differentiated products. Each firm supplies a group of perfect complements, and across firms, the products are imperfect substitutes. Section 3 characterizes the equilibrium outcomes and provides the main results. Section 4 discusses possible extensions of the basic model. Section 5 concludes the paper.

### 2.2 The Model

Suppose that consumers demand a number of differentiated products, which are divided into two groups. Within each group the products are perfect complements and, across groups, they are imperfect substitutes. Let $N_k$ denote the set of products in group $k$, $k = 1, 2$. The demand functions are given by

(1979).
\[ q_{ki} = D(p_k, p_i), \quad i \in N_k, \]  
(2.1)

for \( k, l = 1, 2 \) and \( l \neq k \), where \( p_{ki} \) and \( q_{ki} \) denote the price and quantity of product \( i \) in group \( k \), respectively, and \( p_k = \sum_{i=1}^{n_k} p_{ki} \) is the aggregate price of products in group \( k \), and \( n_k \) is the number of products in group \( k \).

Notice that the demand system (2.1) is symmetric both within and between groups.\(^5\) We make the following assumptions regarding the function \( D(p_1, p_2) \).\(^6\) Let \( P = \{(p_1, p_2) \in R^2 \mid D(p_1, p_2) > 0, \ D(p_2, p_1) > 0 \} \).

(A1) \( P \) is convex and bounded, \( D(p_1, p_2) \) is twice continuously differentiable in \( P \), \( D_1(p_1, p_2) > 0, \) and \( D_1(p_1, p_2) + D_2(p_1, p_2) < 0 \) for \((p_1, p_2)\) in \( P \),

where \( D_k(p_1, p_2) \) denotes the first-order derivative of \( D(p_1, p_2) \) with respect to \( p_k, \) \( k = 1, 2 \). The assumption that \( D_2 > 0 \) represents demand substitutability between the two groups of the products. \( D_1 + D_2 < 0 \) states that the effect of the aggregate price within the group on the demand (own-group effect) dominates the effect of the aggregate price from the other group (cross-group effect). To illustrate our analysis, we frequently use the linear form of demand functions

\[ D(p_1, p_2) = \alpha - \beta p_1 + \gamma p_2, \]  
(2.2)

where \( \alpha > 0, \beta > \gamma > 0 \). The ratio, \( \gamma/\beta \), measures the degree of substitution between the two groups of products or the extent to which the own-group effect dominates the cross-group effect.\(^7\) When \( \gamma/\beta = 0 \), the two groups' products are independent; when \( \gamma/\beta = 1 \), the two groups are perfect substitutes.

There are two firms, 1 and 2. Firm 1 supplies all the goods in the first group and firm 2 offers all the complements in the second.\(^8\) To focus on the strategic incentives, we

\(^5\)The symmetry of demands across groups is not crucial for our discussions below.

\(^6\)These assumptions are standard in the literature on differentiated products. See Friedman (1977) and Deneckere and Davidson (1985).

\(^7\)Beggs (1994) uses the same linear demand function. There are only two complements within each group in his model.

\(^8\)An alternative specification of the model is to assume that the total number of firms is \( n_1 + n_2 \) and each firm supplies one product. The issue of strategic incentives to merge can be addressed in this setup.
assume that the production technology of each firm exhibits constant returns to scale and scope. That is, the total cost function for firm \( k \) is

\[
C_k(q_{k1}, \ldots, q_{kn_k}) = \sum_{i=1}^{n_k} c_{ki}q_{ki}, \quad k = 1, 2,
\]

where \( c_{ki} \) is the constant marginal cost of producing product \( i \) in group \( k \), and \( c_{ki} \geq 0 \).

We consider the subgame perfect equilibria of the following two-stage game with perfect information. In the first stage, the two firms simultaneously choose their restructuring strategies. In stage two, all independent firms compete by simultaneously setting prices. By restructuring, the parent firm keeps a subset of its product lines and sells the rest of its operations to independent entrepreneurs (not to its rival firm). It should be noted that, in our framework, the restructuring strategies of the firms can be any of divestiture, spin-off, breakup, or divisionalization, as long as the resulting firms independently choose their pricing strategies. In the following analysis, we simply refer to a restructuring choice as divisionalization, namely, a firm setting up autonomous rival divisions.

We model divisionalization of a firm as a partition of its product space. Let \( d_k \) denote a partition of \( N_k \), and each cell in the partition be a division of firm \( k \). Let \( m_k \) be the number of divisions in \( d_k \). Given a pair of strategies \((d_1, d_2)\), the profit function of division \( j \) in group \( k \) is

\[
\pi_{kj}(p_{kj}, p_k, p_l; d_k, d_l) = (p_{kj} - c_{kj})D(p_k, p_l), \quad l \neq k,
\]

where, for notational simplicity, \( p_{kj} \) and \( c_{kj} \) represent the aggregate price and the aggregate marginal cost of the products in division \( j \) of group \( k \), respectively, and \( p_k = \sum_{j=1}^{m_k} p_{kj} \) is the aggregate price of products in group \( k \). For \( k = 1, 2 \), let

\[
\Pi_k(p_k, p_l; d_k, d_l) = \sum_{j=1}^{m_k} \pi_{kj}(p_{kj}, p_k, p_l; d_k, d_l) = (p_k - c_k)D(p_k, p_l),
\]

where \( l \neq k \), and \( c_k = \sum_{j=1}^{m_k} c_{kj} \) is the aggregate marginal cost of the products in group \( k \).

A divisionalization (or divestiture) strategy can be viewed as a set of take-it-or-leave-it contracts signed between firm \( k \) (or the parent firm) and independent entrepreneurs. Each

\[\text{9The assumption of constant returns to scope implies that there is no operating synergy between different lines of businesses. Our model is motivated to address divestiture issues concerning conglomerates.} \]
contract specifies a reserve price at which the parent firm is willing to sell the operations of a subset of its products. We assume that the contracts are restrictive so that no division can further divide or subcontract prior to the price decisions in the second stage of the game.\footnote{If the divisions can further divide before they choose prices, the problem becomes very complicated. We are currently working on this issue.} It is then reasonable to assume that firm $k$ sets a reserve price equal to the profit that division $j$ can make in the second stage game, and that each entrepreneur is indifferent about accepting the contract or rejecting it. Therefore, the total profit of firm $k$ is given by (2.4).

The two-stage game can be solved via backward induction. In the second stage of the game, for any pair of strategies $(d_1, d_2)$, each division $j$ in group $k$ chooses its price $p_{kj}$ to maximize (2.3), given the price choices of other divisions within and across groups. In the first stage, firm $k$ chooses a divisionalization strategy, $d_k$, to maximize (2.4), taking into account the divisionalization choice of the other firm and the equilibrium prices of the second stage game.

To simplify our presentation and emphasize the effects of the nature of demands on divisionalization, we set all the marginal costs equal to zero. This simplification does not affect the qualitative results in the paper.

2.3 Analysis

In this section, we first introduce two benchmarks. One deals with competition between two firms without divestiture. The other is the joint profit maximization. We then characterize the equilibrium outcomes of the two-stage games, which can be solved by backward induction. For each set of divisions chosen by the two firms, the equilibrium of the second stage price game is determined, and comparative static properties of the equilibrium are discussed. Our main findings are then presented.

2.3.1 Two Benchmarks

In the first benchmark, two firms directly engage in Bertrand price competition without divestiture. That is, $d_1$ and $d_2$ are singletons and equal to $N_1$ and $N_2$, respectively. Given the simplification of the marginal costs, firm $k$ has the following profit function:
\[ \Pi_k(p_k, p_l; N_k, N_l) = p_k D(p_k, p_l), \]  

for \( k, l = 1, 2, l \neq k \), and \( p_k = \sum_{i=1}^{n_k} p_{ki} \). Notice that the profit functions depend only on the aggregate prices of the complements, \( p_k \) and \( p_l \). Each firm only needs to decide on its aggregate price, and individual prices are indeterminate. Furthermore, the profit function of firm \( k \) increases with the price of the other firm, since the products across groups are substitutes. In other words, there exists a positive externality between the two prices.

In equilibrium, firm \( k \) chooses \( p_k \) to maximize (2.5), given the aggregate price of the other group, \( p_l \). The first-order conditions are

\[ D(p_k, p_l) + p_k D_1(p_k, p_l) = 0, \]  

for \( k, l = 1, 2, \) and \( l \neq k \). The equilibrium price \( (p_1, p_2) \) is determined by (2.6). Let

\[ \epsilon_k(p_k, p_l) \equiv -p_k D_1(p_k, p_l)/D(p_k, p_l) \]

be the elasticity of demands in group \( k \) with respect to its own price, \( p_k \). Equations (2.6) then imply that, in equilibrium, the two firms set their prices such that their demand elasticities are equal to 1. Given the symmetry of demands across groups, in equilibrium \( p_1 = p_2 \), which we denote by \( p^0 \). Later, we will provide sufficient conditions for the existence and uniqueness of the equilibrium for this game.

In the case of linear demand function (2.2), the best-reply functions from (2.6) are easily computed as

\[ r_k(p_l) = \frac{\alpha + \gamma p_l}{2\beta}, \quad l \neq k, \]

which are linear and strictly increasing. Therefore, two prices are strategic complements, in the sense of Bulow, Geanakoplos, and Klemperer (1985). The equilibrium price is

\[ p^0 = \frac{\alpha}{2\beta - \gamma}, \]

and the equilibrium profit for each firm is
In the second benchmark, the two firms do not divide but collude by setting prices to maximize their joint profits:

\[
\Pi_1^0 = \Pi_2^0 = \frac{\alpha^2 \beta}{(2\beta - \gamma)^2}.
\]

Clearly, in this optimization problem only the aggregate prices matter. Assume for now that the global maximum is unique. Given the symmetry of \(\Pi(p_1, p_2)\), the optimal aggregate prices are identical and denoted by \(p_M\), which is determined by the first-order condition

\[
D(p_M, p_M) + p_M D_1(p_M, p_M) + p_M D_2(p_M, p_M) = 0.
\]  

(2.8)

Let \(\eta_{kl}(p_k, p_l) \equiv p_k D_2(p_k, p_l)/D(p_k, p_l)\) be the cross elasticity of demands between the two groups of products. Then (2.8) can be rewritten as

\[
\epsilon_k(p_M, p_M) = 1 + \eta_{kl}(p_M, p_M).
\]

Therefore, at the monopoly solution, the own price elasticity is set to be 1 plus the cross elasticity. Since the cross elasticity is positive, the own price elasticity at the monopoly solution is greater than 1.

Compared to the first benchmark, the joint profit maximization internalizes the externality between the prices of the two groups. As a result, the monopoly price, \(p_M\), is greater than the non-cooperative equilibrium price, \(p^0\). This point can be clearly illustrated for the linear demand function (2.2). In this case, \(\Pi(p_1, p_2)\) is strictly concave. The monopoly price and joint profits are computed as

\[
p_M = \frac{\alpha}{2(\beta - \gamma)}, \quad \Pi_M = \frac{\alpha^2}{2(\beta - \gamma)}.
\]

The monopoly price is greater than the duopoly price, and monopoly profits are higher.
2.3.2 Second-Stage Price Game

We next analyze the equilibrium outcome of the second-stage price game. For a given pair of divisionalization strategies \( (d_1, d_2) \), the profit function (2.3) of division \( j \) in group \( k \) can be written as

\[
\pi_{kj}(p_{kj}, p_k, p_l; d_k, d_l) = p_{kj}D(p_k, p_l),
\]

for \( k, l = 1, 2, \) and \( l \neq k \). Division \( j \) chooses its price, \( p_{kj} \), to maximize (2.9), given the price choices of the other divisions both within and across groups. The first-order condition for an interior solution is

\[
D(p_k, p_l) + p_{kj}D_1(p_k, p_l) = 0, \quad j = 1, \ldots, m_k,
\]

for \( k, l = 1, 2, \) and \( l \neq k \). Pure-strategy Nash equilibria are then determined by equations (2.10). In equilibrium, each division sets its own elasticity of demand, \( \epsilon_{kj}(p_{kj}, p_k, p_l) \), equal to 1, where

\[
\epsilon_{kj}(p_{kj}, p_k, p_l) \equiv -p_{kj}D_1(p_k, p_l)/D(p_k, p_l).
\]

Notice that, by (2.10), the equilibrium prices of the divisions within the group are identical. Thus, the equilibrium conditions (2.10) are equivalent to

\[
m_kD(p_k, p_l) + p_kD_1(p_k, p_l) = 0,
\]

for \( k, l = 1, 2, \) and \( l \neq k \). Equations (2.11) determine the best-reply function for group \( k \) and, hence, the equilibrium aggregate prices. There are interesting properties of the equilibrium. The first is that the own price elasticity of group \( k \) equals the number of divisions in that group, i.e., \( \epsilon_k(p_k, p_l) = m_k \). This implies that each firm can increase its own price elasticity by setting up autonomous rival divisions. The second property is that only the number of divisions matters, not the number of products in each division. Thus, a pair of divisionalization strategies \( (d_1, d_2) \) can be summarized by a pair of division numbers \( (m_1, m_2) \), where \( m_k \leq n_k \) for \( k = 1 \) and 2.
Further assumptions on the demand function are stated to guarantee the existence and uniqueness of equilibrium in the price-setting game (see Friedman 1977, p.72).

(A2) $D_{11}(p_1, p_2) \leq 0$ for $(p_1, p_2) \in P$.

(A3) $D_{11}(p_1, p_2) + D_{12}(p_1, p_2) \leq 0$ for $(p_1, p_2) \in P$.

(A4) $D_2(p_1, p_2) + p_1 D_{12}(p_1, p_2) \geq 0$ for $(p_1, p_2) \in P$.

Here $D_{kl}(p_k, p_l)$ denotes the second-order derivative of $D(p_k, p_l)$ with respect to $p_k$ and $p_l$, $k, l = 1, 2$. (A2) states that the demand function is concave with respect to the own-group price. This condition is sufficient for the concavity of $\pi_{kj}$ with respect to $p_{kj}$, which is the second-order condition for a maximum. (A3) states that the difference between own-group effect and cross-group effect falls as the own-group price goes up. (A2) guarantees the existence, and (A3) the uniqueness of equilibrium. (A4) implies that the best-reply functions determined by (2.11) are strictly increasing. Thus, the prices between the two groups are strategic complements. This is a common assumption in the literature on price competition with differentiated products. Some interesting comparative-static results can be obtained in this case. Let $\hat{p}_1$ and $\hat{p}_2$ denote the equilibrium aggregate group prices, respectively, and $\hat{q}_k = D(\hat{p}_k, \hat{p}_l)$, for $k, l = 1, 2$, and $l \neq k$ denote the equilibrium quantities.

Notice that the above assumptions are satisfied for the linear demand function (2.2). In this case, the best reply functions determined by (2.11) are

$$r_k(p_l) = \frac{m_k(\alpha + \gamma \beta)}{1 + m_k \beta}, \quad l \neq k,$$

which are linear and strictly increasing. Figure 2.1 illustrates the best-reply lines. As $m_k$ increases, the best-reply line for group $k$ shifts up, but the best-reply line for group $l$ does not change. Therefore, equilibrium prices for both groups increase with $m_k$. These prices can be easily computed as

$$\hat{p}_k = \frac{m_k \alpha [(1 + m_l) \beta + m_l \gamma]}{(1 + m_k)(1 + m_l) \beta^2 - m_k m_l \gamma^2}, \quad l \neq k, \quad k = 1, 2.$$

For the general form of demand functions, we have the following characterization and
Lemma 2.1: Suppose that (A1)-(A3) hold. Then, for each pair of \((m_1, m_2)\), there exists a unique equilibrium in the price-setting game. Furthermore,

(a) the equilibrium prices are identical within a group;
(b) the aggregate price in group \(k\), \(\hat{p}_k\), increases with \(m_k\) for \(k = 1, 2\);
(c) the aggregate price in group \(l\), \(\hat{p}_l\), increases with \(m_k\) for \(k, l = 1, 2\) and \(l \neq k\), if (A4) holds;
(d) the equilibrium quantity of each complement in group \(k\), \(\hat{q}_k\), decreases with \(m_k\) for \(k = 1, 2\); and
(e) if \(m_1 = m_2 = m\), then \(\hat{p}_1 = \hat{p}_2\) increases with \(m\), and \(\hat{q}_1 = \hat{q}_2\) decreases with \(m\).

The proof of Lemma 2.1 is given in Appendix 2.A. The intuition for the monotonicity of the aggregate group price with respect to the number of divisions in the group is the following. Since the products within the group are complementary, there is a negative externality among the prices of these products. In other words, an increase in the price of one product reduces the demand for the other products within the same group and, hence, decreases the profits of those products. If the firm is divided into independent divisions, divisions will not take this externality into account. In equilibrium, the aggregate prices of the complements within the group increase when more divisions compete against each other.

From (2.11), an increase in \(m_k\) shifts up the best-reply curve of group \(k\), but does not change the best-reply curve of group \(l\). Given (A4), the best-reply curves for the two groups are upward-sloping. Therefore, both prices increase as \(m_k\) goes up (see Figure 2.1).

Lemma 2.1(d) states that the equilibrium quantity of each complement decreases with the number of divisions in that group. As the number of divisions in the other group goes up, however, the equilibrium quantity of the complement does not necessarily fall. It depends on the sizes of both \(m_1\) and \(m_2\). What we know is that when the firms divide symmetrically, i.e., \(m_1 = m_2 = m\), the equilibrium quantity of each complement decreases as \(m\) increases.
2.3.3 Strategic Incentives to Divest

We now examine whether a divestiture improves the firms' profits. Given the characterization of the equilibrium in the second-stage price game, we can write the reduced-form profit function of firm $k$ as

$$\bar{\Pi}_k(m_k, m_l) = \hat{p}_k D(\hat{p}_k, \hat{p}_l),$$

for $k, l = 1, 2$ and $l \neq k$, which depends only on the numbers of divisions $m_k$ and $m_l$.

In the rest of this section, we treat $m_1$ and $m_2$ as continuous variables. In Appendix 2.B, we illustrate that the qualitative results in this paper still hold when $m_1$ and $m_2$ are restricted to be integers. The profit of firm 1 depends on $m_1$ through the two prices. The derivative of firm 1's profit function with respect to $m_1$ can be computed as follows:

$$\frac{\partial \bar{\Pi}_1(m_1, m_2)}{\partial m_1} = [D(\hat{p}_1, \hat{p}_2) + \hat{p}_1 D_1(\hat{p}_1, \hat{p}_2)] \frac{\partial \hat{p}_1}{\partial m_1} + \hat{p}_1 D_2(\hat{p}_1, \hat{p}_2) \frac{\partial \hat{p}_2}{\partial m_1}. \quad (2.12)$$

Using the first-order conditions (2.11), we can write the own-group effect of the price on the profit as

$$D(\hat{p}_1, \hat{p}_2) + \hat{p}_1 D_1(\hat{p}_1, \hat{p}_2) = \hat{p}_1 D_1(\hat{p}_1, \hat{p}_2)(m_1 - 1)/m_1, \quad (2.13)$$

which is negative for $m_1 > 1$. Therefore, by Lemma 2.1(b) and (c), an increase in $m_1$ has two effects on firm 1's profit. The first is the own-group effect: it increases the aggregate price of products in group 1, which reduces firm 1's profit due to the negative externality of prices within the group. The second is the cross-group effect: it increases the aggregate price of products in group 2, which increases firm 1's profit due to the positive externality of prices across groups. When $m_1$ is close to 1, the own-group effect is close to zero, and the cross-group effect is positive. This means that a small degree of divestiture by firm 1 increases its total profit. If the degree of substitution between the two groups of products is high, then each firm has an incentive to divide unilaterally into multiple divisions.

Next, we determine the non-cooperative equilibrium of the first-stage division game, where the firms independently choose their numbers of divisions. Each firm chooses its number of divisions to maximize its profit, taking the other firm's number of divisions.
and the second stage pricing behavior as given. Using (2.12) and (2.13), we can write the first-order conditions for an interior solution as follows:

\[ D_1(\hat{p}_k, \hat{p}_l) \frac{m_k}{m_l} \frac{\partial \hat{p}_k}{\partial m_k} + D_2(\hat{p}_k, \hat{p}_l) \frac{\partial \hat{p}_l}{\partial m_k} = 0 \quad (2.14) \]

for \( k, l = 1, 2, \) and \( l \neq k \). The non-cooperative equilibrium is determined by (2.14). Since the reduced-form profit functions are symmetric in \( m_1 \) and \( m_2 \), which we denote by \( \hat{V}(m_1, m_2) \), i.e., \( \hat{V}_1(m_1, m_2) = \hat{V}(m_1, m_2) \), and \( \hat{V}_2(m_2, m_1) = \hat{V}(m_2, m_1) \), there exists a symmetric equilibrium. We denote by \( \hat{n} \) the symmetric equilibrium number of divisions.

To determine the size of the equilibrium number of divisions, \( \hat{n} \), we impose a further restriction on the demand function:

(A5) \( D_{12}(p_1, p_2) + D_{22}(p_1, p_2) \leq 0 \) for \( (p_1, p_2) \) in \( P \).

Similar to (A3), (A5) states that the difference between the own-group effect and the cross-group effect does not increase as the cross-group price goes up. Both assumptions impose a limit on how the net effect varies with prices. (A3) and (A5) together imply that the degree of substitution at the symmetric price, \( -D_2(p, p)/D_1(p, p) \), is non-increasing in price \( p \). In the case of the linear demand function, (A5) is obviously satisfied. The following lemma illustrates the important properties of the reduced-form profit function when the firms divest symmetrically.

**Lemma 2.2:** Suppose that (A1)-(A3) and (A5) hold. Then the reduced-form profit function \( \hat{V}(m, m) \) is a single-peaked function of \( m \) and reaches the maximum at \( m = m^* \), where

\[ m^* = \frac{D_1(p_{M1}, p_{M2})}{D_1(p_{M1}, p_{M2}) + D_2(p_{M1}, p_{M2})}. \quad (2.15) \]

The proof of Lemma 2.2 is presented in Appendix 2.A. Lemma 2.2 implies that a small degree of divestitures by both firms increases their profits, but too many divestitures can reduce their profits, provided that \( m^* \) is less than \( n_1 \) and \( n_2 \). We now state our main result.
Proposition 2.1: Suppose that (A1)-(A5) hold. Then the symmetric equilibrium number of divisions, \( \hat{m} \), satisfies the inequalities 1 < \( \hat{m} \) < \( m^* \).

The proof of Proposition 2.1 is presented in Appendix 2.A. The uniqueness of the equilibrium in the division game requires further restrictions on the demand function. We do not provide the exact restrictions here. Instead, we use the linear demand function (2.2) to illustrate Lemma 2.2 and Proposition 2.1. In this case, the equilibrium payoff from the price game can be computed as

\[
V(m_1, m_2) = \frac{m_1\alpha^2\beta[(1 + m_2)\beta + m_2\gamma]^2}{[(1 + m_1)(1 + m_2)\beta^2 - m_1m_2\gamma^2]^2}.
\] (2.16)

It can be easily verified that \( V(m_1, m_2) \) increases strictly with \( m_2 \). Thus, there exists a positive externality between the firms’ choices of divisions. The best-reply functions in the division game are \( m_1 = R(m_2) \) and \( m_2 = R(m_1) \), where

\[
R(m) = \frac{(1 + m)\beta^2}{(1 + m)\beta^2 - m\gamma^2},
\] (2.17)

which is strictly increasing and always greater than 1. Figure 2.2 illustrates the best-reply curves that cross at the point \((\hat{m}, \hat{m})\), where

\[
\hat{m} = \frac{\beta}{\sqrt{\beta^2 - \gamma^2}}.
\]

It can be verified that \( V(m_1, m_2) \) is a single-peaked function of \( m_1 \), which guarantees \((\hat{m}, \hat{m})\) to be a Nash equilibrium of the division game. Furthermore, the equilibrium is unique. It follows from Lemma 2.1 that the subgame perfect equilibrium of the two-stage game is unique and symmetric.

To illustrate Lemma 2.1 and Proposition 2.1, we can easily verify that \( V(m, m) \) is single-peaked and reaches the maximum at

\[
m^* = \frac{\beta}{\beta - \gamma}.
\]

Clearly, \( \hat{m} \) is greater than 1, but less than \( m^* \). For any fixed \( \beta \), both \( \hat{m} \) and \( m^* \) increase with \( \gamma \), which measures the degree of substitution between the groups of products. As \( \gamma \)
goes up, the competition between the two groups intensifies. Competing firms respond by divesting into more and finer independent divisions.

We now discuss the welfare implications of the equilibrium divestitures. Suppose that the demand system (2.1) is derived from an aggregate utility maximization subject to a budget constraint, where the aggregate utility function takes the following form:

\[ u = q_0 + U(q_{11}, \ldots, q_{1n_1}, q_{21}, \ldots, q_{2n_2}), \]

where the good \( q_0 \) is a numeraire, and \( U(\cdot) \) is a monotone increasing function. It follows that the consumers' surplus is

\[ CS = U(q_{11}, \ldots, q_{1n_1}, q_{21}, \ldots, q_{2n_2}) - \sum_{k=1}^{2} \sum_{i=1}^{n_k} p_{ki} q_{ki}, \]

and the social surplus is the consumer’s surplus plus the firms’ total profits, which is simply \( SC = U(q_{11}, \ldots, q_{1n_1}, q_{21}, \ldots, q_{2n_2}) \). Applying Lemmas 1 and 2 and Proposition 1, we obtain the following welfare implications of the equilibrium divestitures:

**Corollary 2.1:** Suppose that (A1)-(A5) hold. Then the equilibrium divestitures increase the firms’ profits and reduce the consumers’ welfare and social surplus.

The implications for the social welfare consequences of divestitures are noteworthy. Conventional wisdom tells us that mergers of firms supplying homogeneous products or imperfect substitutes (with quantity competition) reduce competition, increase the prices of the products, and decrease consumers’ welfare and social surplus. In our model, divestitures are motivated by product-line complementarities. Corollary 2.1 and Lemma 2.1 imply that such divestitures increase prices, lower quantities, and decrease consumers’ welfare and social surplus. Therefore, from a social welfare point of view, divestitures involving complementary goods or services should be discouraged as much as mergers involving substitutes. In practice, however, regulatory authorities are usually concerned only about mergers, but usually not about divestitures. The possibility that competing firms can achieve tacit collusion in pricing through divestitures, rather than through mergers, makes antitrust a more complex and difficult issue.

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2.3.4 Coordinated divestitures

In this subsection, we consider the situation in which the firms are able to coordinate their divestiture decisions in the first stage. We show that there exists a pair of division numbers \((m_1, m_2)\) such that the joint profit maximizing prices \((p_M, p_M)\) can be supported as a non-cooperative equilibrium outcome of the price-setting game. This pair of division numbers turns out to be \((m^*, m^*)\), defined in (2.15).

Indeed, comparing the equilibrium conditions (2.11) for the price game with the necessary condition (2.8) for the joint profit maximization problem, we find that the joint profit maximizing prices satisfy the Nash equilibrium conditions, if \(m_1 = m_2 = m^*\). Notice that \(m^* = \epsilon_k(p_M, p_M)\), i.e., the number of divisions is equal to the own price elasticity of each group at monopoly prices (or 1 plus the cross elasticity). A positive degree of substitution between the two groups of products implies that \(m^* > 1\). Since Lemma 2.1 provides sufficient conditions for the existence and uniqueness of the equilibrium, it follows that the joint profit maximizing prices can be supported as a unique equilibrium outcome of the price game if both firms break up into \(m^*\) number of independent divisions. In other words, when the firms are able to coordinate their choices of divisions, they can replicate the monopoly profit. This provides an alternative way for the firms to collude.

Proposition 2.2: Suppose that (A1)-(A3) hold and \(\min\{n_1, n_2\} \geq m^*\). Then, by setting \(m_1 = m_2 = m^*\), the second-stage price competition yields the joint profit maximizing prices and profits.

The logic behind Proposition 2.2 can be described as follows. Since the joint profit function (2.7) can be rewritten as the summation of the profit functions (2.9) over all independent divisions from both groups, the joint profit maximization problem can be equivalently solved by choosing prices \((p_{11}, \ldots, p_{1m_1})\) and \((p_{21}, \ldots, p_{2m_2})\). Notice that we can decompose the effect of an increase in price \(p_{1j}\) into the following three terms:

\[
\frac{\partial \Pi}{\partial p_{1j}} = [D(p_1, p_2) + p_{1j}D_1(p_1, p_2)] + \sum_{j' \neq j} p_{1j'}D_1(p_1, p_2) + p_2D_2(p_2, p_1). \tag{2.18}
\]

The first term in the square brackets represents the own-group effect of the price on the profit of division \(j\) in group 1, \(\pi_{1j}\), which is the same as the left-hand side of (2.10).
The second term is the aggregate intra-group effect of price on the profits of the other divisions in the same group, $\pi_{1j^{'}}$, for all $j^{'}$ different from $j$. It is negative since divisions within the same group supply complementary products. The third term is the aggregate inter-group effect of price on the profits of the divisions in the other group. Since products are imperfect substitutes across groups, the aggregate inter-group effect is positive.

Now, if there exists a number of divisions $m$ such that the negative intra-group effect exactly offsets the positive inter-group effect, then the necessary condition for joint profit maximization is equivalent to the first-order condition (2.10). In other words, given the number of divisions $m$, the necessary conditions for joint profit maximization are identical to the first-order conditions for a non-cooperative equilibrium in the price game. This can be accomplished by setting the second plus the third term on the left-hand side of (2.18) equal to zero. Imposing symmetry and substituting $p_M$ for $p_1$ and $p_2$, we obtain

$$(m_1 - 1)(p_M / m_1)D_1(p_M, p_M) + p_M D_2(p_M, p_M) = 0.$$  \hspace{1cm} (2.19)$$

The solution to (2.19) is $m_1 = m^*$. A similar argument determines $m_2 = m^*$. As a result, the monopoly price $p_M$ satisfies (2.8) and (2.11) and, hence, consists of the unique solution to the joint profit maximization problem and of the equilibrium of the price game.

The driving force behind this finding is the commitment power of divestiture combined with the extended product space, which includes substitutes as well as complements. Prior to divestiture, the prices of products in each group are set coordinately. After divestiture, the firm credibly commits not to set prices of the group coordinately, therefore inducing less competition from the rival group. As a result, the prices and profits increase. This implies that price coordination by a group of firms supplying complements does not necessarily benefit the firms and harm consumers. In our model, the lack of coordination among the prices of complements indeed leads to higher profits and lower consumer surplus. In other words, firms have incentives to tie their own hands in order to induce a better (more profitable) response from their rivals\(^{11}\).

The importance of incorporating both substitutability and complementarity within the same framework can be seen clearly in terms of externalities of each division’s pricing

\(^{11}\)A similar commitment effect works when firms use a most-favored-customer policy to raise the prices (see Cooper, 1986).
decision. There is a positive externality between the prices across groups due to substitutability. Divestiture in the first-stage creates a negative externality of prices among independent divisions within a group. This negative externality can offset the positive externality. When the degree of divestiture is small, the positive externality dominates. When the degree of divestiture is large, the negative externality outweighs the positive one. At $m = m^*$, all externalities are neutralized, and consequently, the monopoly outcome is achieved.

We now understand the second inequality in Proposition 2.1. It is driven by the positive externality between the firms' choices of divisions. Given (A5), an increase in $m_1$ will increase firm 2's profit, when both firms choose the same number of divisions. In the presence of such a positive externality, a lack of coordination between the firms results in a smaller equilibrium number of divisions than the coordinated number of divisions.

What determines the size of the optimally coordinated number of firms? Notice that the ratio, $-D_2(p_M, p_M)/D_1(p_M, p_M)$, represents the degree of substitution between the two groups of the products. The following corollary provides a comparative-static result:

**Corollary 2.2:** The optimally coordinated number of divisions, $m^*$, increases with the degree of substitution between the two groups of products.

In one extreme case, where there are no substitute goods, $m^*$ is equal to 1, and each firm should monopolize the supply of the complements and never divide. In the presence of competing substitutes, the firms have incentives to divide. As the relative degree of substitution between the two groups of complements increases, the positive externality increases and, hence, each firm should split into more divisions to increase the negative externality and mitigate the positive one. At the other extreme, if the number of complements in each group is small or the degree of substitution is large, further divestiture may not be possible. In this case, monopoly profit cannot be replicated, and the firms prefer complete divestiture in which each product is supplied by an independent firm.
2.4 Extensions

In the previous section, we have discussed the effect of product market competition (or product differentiation) on divisionalization and shown that a higher degree of substitution between the two groups of products leads to a greater number of divisions in both groups. There are other factors that determine the coordinated and non-cooperative divisionalization strategies and the scopes of the firms. They include marginal costs, asymmetric demands, imperfect complements, and several groups of complements. In this section, we briefly discuss only two of these factors, imperfect complements and the number of groups of complements. For simplicity, we use linear demand functions.

a) Imperfect Complements

In our basic model, we have considered only perfect complements within the group. Our analysis can be extended to the case of imperfect complements. To illustrate, we consider an example with the following linear demand functions:

\[ q_{ki} = \alpha - \beta' p_{ki} - \beta \sum_{i' \neq i} p_{i'i} + \gamma p_l, \]

for \( k, l = 1, 2, l \neq k, i = 1, 2, ..., n_k, \) where \( \alpha > 0, \beta' \geq \beta > \gamma > 0, \) and \( p_k = \sum_{i=1}^{n_k} p_{ki}. \) The assumption \( \beta' \geq \beta \) implies that product \( i \) and any other product within the group are imperfect complements and that the effect of the price \( p_{ki} \) on the demand for product \( i \) dominates the intra-group effect on the demand for any other product within the group. As before, \( \beta > \gamma \) means that intra-group effects outweigh inter-group effects. The demand system is symmetric both within the group and between groups.

Notice that the demand function for product \( i \) in the first group can be written as

\[ q_{1i} = -(\beta' - \beta)p_{1i} + \alpha - \beta p_1 + \gamma p_2, \]

which consists of two parts, the first depending only on the individual price \( p_{1i} \) and the second being the same as the demand function (2.2). Clearly, \( \beta' - \beta \) does not affect the demand externality arising from an increase in \( p_{1i}. \) Remember that the optimally coordinated number of divisions is determined by balancing the negative and positive externalities of the prices. Therefore, it is independent of \( \beta' - \beta \) and can be computed as
\[ m^* = \frac{\beta}{(\beta - \gamma)}. \] However, \( \beta' - \beta \) affects the Nash equilibrium number of divisions, \( \hat{m} \). It can be easily calculated that \( \hat{m} \) is determined by the following first-order condition

\[ \hat{m}^2(\beta^2 - \gamma^2) - \beta^2 + 2(\beta' - \beta)\hat{m}(\hat{m} - 1)/n = 0, \]

where \( n_1 \) and \( n_2 \) are assumed to be equal, for simplicity, and denoted by \( n \). Notice that \( \hat{m} \) decreases with \( \beta' - \beta \). When the demand elasticity with respect to own price increases (higher \( \beta' \)), the incentives for each firm to divide fall because the second-stage equilibrium prices decrease. As a result, the equilibrium number of divisions decreases as \( \beta' \) increases.

b) Multiple Groups of Complements

Our analysis can also be extended to the case of many groups of complements, where the products are imperfect substitutes across groups. Let \( K \) be the number of groups, \( K \geq 2 \). The demand function for a complement in group \( k \) is linear and presented by

\[ q_{ki} = \alpha - \beta p_k + \gamma \sum_{k' \neq k} p_{k'}, \]

for \( k = 1, ..., K \) and \( i = 1, ..., n_k \), where \( \alpha > 0, \beta > (K-1)\gamma > 0, n_k \) is the number of complements in group \( k \), and \( p_k = \sum_{i=1}^{n_k} p_{ki} \) is the total price of the complements in group \( k, k = 1, ..., K \). The demand for other products is symmetric with the group and across groups.

We can compute the optimally coordinated number of divisions for each group and the non-cooperative equilibrium number of divisions as follows:

\[ m^* = \frac{\beta}{\beta - (K-1)\gamma}, \]

\[ \hat{m} = \frac{\sqrt{(K-2)^2\gamma^2 + 4(\beta + \gamma)(\beta - (K-1)\gamma) - (K-2)\gamma}}{2(\beta + \gamma)(\beta - (K-1)\gamma)}. \]

As before, the optimally coordinated number of divisions and the equilibrium number of divisions increase when the degree of substitution between the groups of the products increases. An interesting comparative-static result is that both \( m^* \) and \( \hat{m} \) increase with \( K \). As the number of groups of the complements goes up, competition across groups
increases and, hence, the magnitude of the positive externalities among the prices across groups also increases. To mitigate the increased positive externalities, the firms need to create more negative externalities by dividing the firms into more independent divisions. Therefore, both the optimally coordinated number of divisions and the non-cooperative equilibrium number of divisions increase.

2.5 Concluding Remarks

In this paper, we have argued that a firm producing a group of complements can generate higher profits through divestitures when there is a competing firm supplying an imperfect substitute group of complements. By delegating pricing decisions to independent divisions, each firm credibly commits not to set the prices of the complements within the group coordinately, which softens the competition between the two firms. Our analysis suggests that industry structure and size of firms are closely related to the nature of heterogeneous products and the strategic considerations of the firms.

The welfare implications of the divisionalizations in our models are significant. It is shown that firms supplying complements have incentives to divest when facing competition, and the resulting divestitures raise prices and reduce consumers’ surplus. However, the profit gains are not large enough to compensate the consumers’ losses. As a result, the total surplus is reduced. This suggests that, from a social welfare point of view, divestitures involving complementary goods or services can be as harmful as mergers involving substitutes. Thus, antitrust authorities should be concerned about not only mergers, but also divestitures. Yet, in practice, mergers are usually the sole focus of antitrust policies.

Another implication of our analysis is that divestitures motivated by product-line complementarities can be viewed as a response towards entry. Suppose that, initially, there is a monopoly that supplies a group of complementary goods or services. Clearly, in our context the monopolist does not have any incentives to divest its operations. When potential rivals enter the market, the monopolist has two possible responses. One is to compete directly against the entrants. If the entrants do not make enough profits to cover their entry costs, the entries can then be deterred. If the products offered by the entrants are differentiated enough from the incumbent’s products, and if the entry costs are relatively small, the entries cannot be deterred. In this case, an optimal strategy for
the incumbent firm is to divest some of its operations to soften the competition.

It should be noted, finally, that our analysis is based upon a number of assumptions. One crucial assumption is that divisions cannot further divide before they choose their prices. This is reasonable in situations, where the parent firms still have major ownership control of the divisions, but do not make management decisions. Franchise contracts can be viewed as an example of such a divisionalization. In other situations, it can be difficult for firms or divisions to make this type of commitment. Divisions may then have incentives to divide further. This raises the issue of what determines a stable industry structure in the presence of both substitutes and complements. Further research along this line is needed. Our modeling of the demand structure also implies that products across groups are not compatible. When they are completely compatible, the demand function has to be adapted. Firms divesting decisions will be affected. Therefore, our analysis does not apply in the case of cross-group product compatibility.
Chapter 3

Product Differentiation, Strategic Divisionalization and Persistence of Monopoly

3.1 Introduction

It is generally accepted in economics that, in markets with similar products, competition reduces firms' profits. Yet, large firms often set up independently managed rival divisions\(^1\) supplying similar products and competing in the same market\(^2\). A similar puzzling phenomenon is that, to a certain degree, franchises of the same parent firm are often

\(^1\)For example, many automobile manufacturers have independently managed operating divisions. In the case of General Motors, Sloan (1963) notes “According to General Motors plan of Organization ... the activities of any specific operation are under absolute control of the General Manager of that Division, subject only to very broad contact with the general officers of the Corporation” (p.106). Moody’s Industrial Manual (1993) describes that each of GM operating divisions “is a self-contained administrative unit with a general manager responsible for all functional activities of his divisions.

\(^2\)Baye et. al. (1995) indicate that “For example, General Motors produce the LeSabre (Buick Motor Division) and the Olds 88 (Oldsmobile Division) which, while differing in styling, are built on the same chassis and, comparably equipped, sell for virtually identical prices. Similarly, the Ford Motor Co. produces the Sable (Lincoln-Mercury Division) and the Taurus (Ford Division), which are effectively the same car with different name plates, as are the Chrysler Corporation’s Plymouth Voyager and Dodge Caravan. Indeed, in just about every price range, all of the major domestic manufacturers have several divisions producing competing products.”
allowed to compete for the same customers. There are a few different explanations for such practices. First, production efficiency requires multiple production plants under decreasing returns to scale technology. Second, the need to satisfy heterogeneous consumers, in terms of either tastes or location, forces firms to set up multiple divisions producing different brands or multiple outlets operating at different locations. Third, it is argued by Williamson (1975) that setting up autonomous divisions alleviates incentive problems due to moral hazard within large organizations. Nevertheless, these explanations do not justify why a firm allows its divisions to compete rather than to cooperate with each other in production (or sales) strategies.

To search for a more satisfactory explanation, recent studies have focused on firms’ strategic incentives in divisionalization. Schwartz and Thompson (1986) and Veendorp (1991) show that the incumbent firm can forestall entry by setting up multiple rival divisions prior to entry. Corchon (1991) and Polasky (1992) analyze a two stage divisionalization game with a duopoly supplying homogeneous products. They show that each firm has an incentive to set up rival divisions, but there is no finite Nash equilibrium. Using a similar framework, Corchon and Gonzalez-Maestre (1993) and Baye, Crocker and Ju (1996) rectify the nonexistence problem by imposing either an exogenous bound of permissible number of divisions or a fixed division set-up cost.

The insight from these recent studies is the following: in the homogeneous product market, setting up a new independent division reduces the aggregate profit due to the increased competition, but increases a firm’s share in the aggregate output and profit. The second effect always dominates the first if products of different firms are perfect substitutes. Consequently, firms have a private incentive to divisionalize. Furthermore, there is no finite equilibrium because of the dominance of the second effect.

In this paper, we provide a model of differentiated products, which contains the previous framework considered by Corchon, Polasky and Baye et. al. as a special case. It is first shown that the problem of nonexistence of an interior subgame perfect Nash Equilibrium (SPNE) in this literature is due to the assumption of homogeneous products. Product differentiation alone ensures the existence of an interior SPNE. As shown in previous studies, divisionalization has two effects on a firm’s profit; setting up autonomous divisions increases a firm’s market share, on one hand, and creates competition for the firm’s existing divisions, on the other. We refer to the first effect as the business
stealing effect, which by itself enhances the firm's profitability, and the second as the competition effect, which reduces the firm's profitability. Product differentiation weakens the business stealing effect because of the reduced substitutability among products. The competition effect, however, increases with product differentiation, since the competition from additional divisions is borne more by existing divisions of the same firm when the substitutability among rival products decreases. Thus, a firm's incentive to divisionalize is reduced if products are differentiated, resulting in an interior SPNE.

We then show that, if divisions are allowed to divide further, they always will. The final outcome of the divisionalization game without restriction on further dividing is equivalent to the perfectly competitive equilibrium, where each firm earns zero profit. To prevent this disastrous outcome of total profit dissipation, parent firms have a unilateral incentive to restrict their divisions from further dividing. This finding provides a theoretical justification for a rather important assumption in the divisionalization literature that only parent firms are allowed to set up independent divisions, and that divisions themselves are not. Veendorp (1991) shows that parent firms delegate output decisions and the like to their divisions, but not investment decisions regarding divisions' capacity. We show why parent firms reserve the authority regarding the divisionalization decision. Our finding is also consistent with the general business practice in franchising. Franchisees are usually not allowed to sell franchises themselves. An alternative organizational set-up is that the parent firm functions as a holding company, allowing its subsidiaries to manage independently and, at the same time, taking away the authority of setting up subdivisions.

Finally, we study the effect of divisionalization on the free entry equilibrium. Because entry induces incumbent firms to set up more divisions, and the numbers of divisions of firms are strategic complements, entry can enormously intensify competition under divisionalization. An entrant faces competition not only from the existing divisions, but also from entry-induced additional divisions of incumbent firms. As a result, potential firms are more reluctant to enter a market if divisionalization is possible. In other words, divisionalization is a natural entry barrier, potentially generating high and persistent profits for the incumbent firms. In the cases where product differentiation is difficult, the only pure strategy free entry SPNE is the monopoly outcome. More interestingly, in such circumstances, it is the credible threat of divisionalization after entry occurs that ensures the monopoly outcome. The incumbent does not actually have to set up divisions.
prior to entry. This finding sharply contrasts to Eaton and Lipsey (1979, 1981), Gilbert and Newberry (1982), and Schwartz and Thompson (1986), who suggest the incumbent firm has either to build up capacities or to set up divisions prior to entry to prevent it effectively from happening.

The paper is closely related to, but distinct from the merger literature such as Salant et al. (1983) and Gaudet and Salant (1991), which examine the impact of mergers on firms’ profits and on social welfare. Under plausible conditions, they show that merger of a subset of firms may result in profit loss for the merging firms, even though merger leads to a more concentrated oligopoly. This paper deals with the opposite issue: the incentive facing firms to divisionalize or to set up rival independent units. We show that, in plausible scenarios, each firm has unilateral incentives to create rival competing units. Unlike most of the merger literature, our analysis permits one to analyze the equilibrium consequences of these incentives in a noncooperative setting that allows all firms to divisionalize freely. The merger literature, in contrast, implicitly views merger as a cooperative game among merging parties, for the set of firms that merge is usually exogenously selected.

The rest of the paper proceeds as follows. In the next section, we set up the basic model. Section 3 characterizes the equilibrium of the two stage divisionalization game. Section 4 considers the implication of the possibility of further divisionalization by divisions. Section 5 examines the free entry equilibrium. A final section concludes.

### 3.2 The Model

#### 3.2.1 Demand and Technology

We consider an environment where $n$ oligopolistic firms each produces a differentiated product. To simplify the analysis, demands for different brands are assumed to be represented by the following linear system:

$$p_k = a - bx_k - d \sum_{t \neq k} x_t,$$

for $k = 1, 2, \ldots, n$, where $d \in (0, b)$, and $x_k$ and $p_k$ are the consumption and price of the $k^{th}$ brand respectively. Notice that $d$ can be viewed as a parameter indicating the degree of
brand differentiation (or the degree of substitution between brands). As \( d \) approaches zero, brands become more differentiated, and in the limit \( (d = 0) \), demands are independent. As \( d \) approaches \( b \), brands become closer substitutes and, in the limit, become perfect substitutes.

In order to isolate the strategic motivation for firms to divisionalize, we assume that oligopolistic firms have constant-return-to-scale technologies with the same marginal cost. Then, without loss of generality, we set the marginal cost equal to zero. Further, divisions of the same parent firm inherit the same technology. Thus, divisions of the same firm supply perfect substitutes, and divisions of different firms supply imperfect substitutes. By construction, the intra-firm substitutability is greater than the inter-firm substitutability. This feature of our model will be shown to be critical to ensuring the existence of an interior SPNE. Moreover, divisionalization is assumed to be costless (i.e., the division setup cost is zero). However, to enter the market, a firm has to incur a fixed cost, \( F \), which can be interpreted as a R&D cost for developing a product.

### 3.2.2 The Divisionalization Game

Following the convention in the literature\(^3\), we refer to a firm setting up autonomous rival divisions as divisionalization hereafter. The divisionalization game we consider is a simultaneous-move two-stage game with perfect information. In the first stage, each oligopolistic firm simultaneously chooses its number of autonomous divisions. In the second stage, all divisions engage in a Cournot competition by setting levels of output simultaneously. Every market participant knows the demand structure and technologies. Divisions can be viewed as independent profit centers whose managers are compensated solely according to divisions' profits. The profit of a parent firm is the sum of the profits of its divisions. We solve the game by backward induction: solving first for the Cournot equilibrium outputs in stage two for a given set of numbers of divisions, then for the equilibrium number of divisions chosen by each firm in stage one.

Notice that, like other models in the divisionalization literature, we implicitly assume that autonomous divisions will not further divide into more independent subdivisions. The analysis of divisionalization depends critically on this assumption. We will provide

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\(^3\)For examples, see Schwartz and Thompson (1986) and Baye, Crocker and Ju (1995).
a theoretical justification for this assumption in Section 3.4.

3.3 Analysis

In this section, we first solve the two stage subgame perfect Nash equilibrium by backward induction. Then we discuss the comparative static results of the equilibrium and summarize our main findings.

3.3.1 The Second Stage Cournot Quantity Game

In the second stage, all independent divisions choose their levels of output simultaneously for a given set of numbers of divisions. Let \( m_k \) denote the number of divisions of firm \( k \), and \( x_{ki} \) be the output level of division \( i \) of firm \( k \), where \( i = 1, 2, \ldots, m_k \) and \( k = 1, 2, \ldots, n \). Subscripts \( k \) and \( i \) denote firm \( k \) and division \( i \) of firm \( k \), respectively. Notice that the total output of the \( k \)th firm \( x_k = \sum_{i=1}^{m_k} x_{ki} \). Then, the profit function of division \( i \) of firm \( k \) is

\[
\pi_{ki} = p_k x_{ki} = \left( a - b \sum_{i=1}^{m_k} x_{ki} - d \sum_{t \neq k} \sum_{j=1}^{n} x_{tj} \right) x_{ki}. \quad (3.2)
\]

Division \( i \) of firm \( k \) chooses \( x_{ki} \) to maximize its profit, taking the output decisions of the other divisions as given. The first order condition yields

\[
\frac{\partial \pi_{ki}}{\partial x_{ki}} = a - b \left( 2x_{ki} + \sum_{s \neq i} x_{ks} \right) - d \sum_{t \neq k} \sum_{j=1}^{n} x_{tj} = 0, \quad (3.3)
\]

for all \( k \) and \( i \). Pure strategy Nash equilibrium is then determined by the above equation system. Solving for \( x_{ki} \), we have

\[
x_{ki} = \frac{a}{b} - x_k - z \sum_{t \neq k} x_t, \quad (3.4)
\]

where \( z = \frac{d}{b} \), which measures the degree of substitution between products. When \( z = 1 \), products of different firms are perfect substitutes. When \( 0 < z < 1 \), products are imperfect substitutes and the degree of substitution increases with \( z \).

Notice that \( x_{ki} \) is the same for all \( i \). Summing the above equation over \( i \) yields

\[
x_k = \frac{a}{b} - x_k - z \frac{\sum_{t \neq k} x_t}{1 + \frac{1}{m_k}}, \quad (3.5)
\]
for all \( k = 1, 2, \ldots, n \). Equation (3.5) involves only the total output of each firm and, thus, can be viewed as the best response function at the firm level. Solving \( x_k \) in terms of the numbers of divisions, we have

\[
x_k = \frac{a \delta_k}{b(1 + z \Delta)},
\]

where \( \delta_t = \frac{1}{1 + \frac{1}{m_t - \epsilon}}, \forall t, \) and \( \Delta = \sum_{t=1}^{n} \delta_t \).

Lemma 3.1: Given the linear demand system, for each configuration of numbers of divisions of the oligopolistic firms \( m_1, m_2, \ldots, m_n \), there exists a unique Nash equilibrium in the Cournot quantity game. Moreover,

(a) a division's output decreases with the number of divisions of each firm;

(b) a parent firm's total output increases with its number of divisions and decreases with the number of divisions of every other firm; and

(c) outputs of every division and firm decrease with the number of firms.

(Proof: see Appendix 3.)

The driving force behind Lemma 3.1 is that divisions' outputs are strategic substitutes in the Cournot game.

Define the market share of firm \( k \) as the ratio of its output to the total output of the industry: \( s_k = \frac{x_k}{x} \), where \( x = \sum_{k=1}^{n} x_k \). An interesting corollary of Lemma 3.1(b) is in order:

**Corollary 3.1:** An increase in the number of divisions of one firm increases the market share of that firm and decreases that of every other firm.

Corollary 3.1 highlights a firm's private incentive to set up competing divisions. The dual effects of divisionalization on a firm's profit can also be easily shown from the following derivation.

Rearranging equation (3.3) and using the inverse demand function gets

\[
p_k = a - bx_k - d \sum_{t \neq k} x_t = bx_{ki}.
\]
Since \( x_{ki} \) is the same for all \( i \), the profit of firm \( k \) can be written as

\[
\pi_k = m_k \pi_{ki} = m_k p_k x_{ki} = b m_k x_{ki}^2.
\]

Then,

\[
\frac{d\pi_k}{dm_k} = \frac{\partial \pi_k}{\partial m_k} + \frac{\partial \pi_k}{\partial \pi_{ki}} \frac{\partial \pi_{ki}}{\partial m_k} = m_k \frac{\partial \pi_{ki}}{\partial m_k} + \pi_{ki},
\]

where \( \frac{\partial \pi_{ki}}{\partial m_k} = 2 b x_{ki} \frac{\partial \pi_{ki}}{\partial m_k} < 0 \). The first term of the above equation represents the effect of creating a new division on the profit of the existing divisions of the firm. We shall refer to this effect as the competition effect of divisionalization, since the new division increases the level of competition and reduces profits of the existing divisions. The second term represents the direct contribution of the new division towards the profit of the firm. We shall refer to it as the business stealing effect of divisionalization, since the output of the new division comes partly at the cost of other firms. The second effect highlights the motivation behind divisionalization.

### 3.3.2 The First Stage Game

Given the characterization of the equilibrium in the second stage, we now consider firms' divisionalization decisions in the first stage. The parent firm \( k \)'s total profit is the sum of profits of all its divisions. Then,

\[
\pi_k = \sum_{i=1}^{m_k} \pi_{ki} = p_k x_k = \frac{a^2 \delta_k^2}{b m_k (1 + z \Delta)^2}. \tag{3.7}
\]

Firm \( k \) chooses \( m_k \) to maximize its profit taking as given the number of other firms' divisions. We have the following first order condition:

\[
\frac{\partial \pi_k}{\partial m_k} = \frac{a^2 \delta_k^2}{b m_k^2 (1 + z \Delta)^3} \left[ \frac{1 + z \Delta_k}{m_k} - 1 - z \Delta_k (1 - z) \right] = 0, \tag{3.8}
\]

where \( k = 1, 2, \ldots, n \) and \( \Delta_k = \sum_{i \neq k}^{n} \delta_t \). Equation (3.8) defines the equilibria of the first stage divisionalization game\(^4\).

\(^4\)The second order derivative evaluating at the solution of the first order condition is

\[
\frac{\partial^2 \pi_k}{\partial m_k^2} = - \frac{a \delta_k^3}{m_k^3 (1 + z \Delta)^4} [1 + z \Delta_k (1 - z)] < 0;
\]
Lemma 3.2: The numbers of divisions of the oligopolistic firms are strategic complements.

Proof: Rearranging equation (3.8), we have the best response function of firm $k$

$$m_k = \frac{1}{1 - \frac{z^2}{\Delta_k + z}}$$ \hspace{1cm} (3.9)

It is easy to verify that $\frac{\partial m_k}{\partial \Delta_k} > 0$ and $\frac{\partial \Delta_k}{\partial m_t} > 0$ for $t \neq k$.

Then,

$$\frac{\partial^2 m_k}{\partial m_t} = \frac{\partial m_k \partial \Delta_k}{\partial \Delta_k \partial m_t} > 0.$$  

Q.E.D.

Lemma 3.2 implies that a firm responds to an increase in the number of divisions of another firm by increasing the number of its own divisions. In doing so, a firm attempts to mitigate its market share loss due to other firms' aggressiveness in divisionalization. Because the complementarity shown here implies a positive chain reaction among the numbers of divisions of firms, a change in a factor which affects a firm's number of divisions might have a profound effect on the number of total divisions and, in turn, on the level of competition. It will be shown later that this insight will have significant implications on the entry deterrence property of divisionalization.

Imposing a symmetry condition to the equation (3.8), we obtain the solution for the equilibrium number of divisions $m$:

$$m^2[(1 - z)((n - 1)z + 1)] - m(n - 2)z - 1 = 0$$

or,

$$m = \frac{(n - 2)z + \sqrt{(n - 2)^2z^2 + 4(1 - z)((n - 1)z + 1)}}{2(1 - z)((n - 1)z + 1)}.$$ \hspace{1cm} (3.10)

that is, the second order condition is satisfied.

It is shown in the Appendix that all possible solutions to the stage one game are symmetric; therefore, imposing the symmetric condition here does not constitute any real restriction on the solution.
Proposition 3.1: With linear demand and differentiated products, the two stage divisionalization game has a unique subgame perfect Nash equilibrium, in which each firm chooses a finite number of independent divisions. Furthermore, the equilibrium number of divisions of each firm increases with the degree of substitution between products and the number of firms (the number of differentiated products).

Proof: see Appendix 3.

Proposition 3.1 states that product differentiation ensures the existence of an interior SPNE for the divisionalization game. The effect of product differentiation on the determination of the equilibrium number of divisions is evident from the positive relationship between the equilibrium number of divisions and the degree of substitution between firms. As we have shown earlier, divisionalization enables a firm to steal business from its rivals, on one hand, and creates competition for the firm’s existing divisions, on the other. The business stealing effect decreases with the degree of differentiation among products, since lower substitutability among products naturally limits the firm’s ability to shift the demand from its rivals’ products to its own product. The competition effect, on the contrary, increases with the degree of product differentiation, because additional divisions have to compete mainly with the existing divisions of the same firm when product substitutability is low. Thus, firms’ incentive to divisionalize is reduced when products are differentiated, resulting in an interior SPNE.

The existence result of interior SPNE with product differentiation can be clearly illustrated with the help of equation (3.9), the best response function of firm $k$. With product differentiation, i.e., $z < 1$,

$$0 < \frac{\partial m_k}{\partial m_t} = \frac{z^2}{(\Delta k(1 - z) + 1)^2} \frac{1}{(m_t(1 - z) + 1)^2} < 1.$$ 

Then, if firm $t$ increases its number of divisions, $m_t$, firm $k$ has an incentive to increase its number of divisions but will not match the increase in $m_t$. Moreover, as $m_t \rightarrow \infty$, $\frac{\partial m_k}{\partial m_t} \rightarrow 0$ and $m_k \rightarrow \frac{1 + (n-2)z}{(1-z)(1+z(n-1))} \leq \frac{1}{1-z}$. Recall from Lemma 3.2 that $m_k$ increases with $m_t$. Thus, the best response function of firm $k$ is bound above. No matter how many divisions other firms set up, firm $k$ will not set up more than $\frac{1}{1-z}$ divisions. That guarantees the interior equilibrium.
Without product differentiation \( (z = 1) \), however, equation (3.9) becomes

\[
m_k = \sum_{i \neq k}^n m_i + 1,
\]

and

\[
\frac{\partial m_k}{\partial m_t} = 1.
\]

That is, firm \( k \) has an incentive to match any increase in the number of divisions of any other firms. There is no upper bound for the best response function \( m_k \). In fact, each firm has an incentive to set up more divisions than the number of divisions of all the other firms combined. That drives the equilibrium number of divisions to infinity.

In the case of duopoly, the two best response functions can be plotted in a diagram. When \( z = 1 \), they are two parallel lines (Figure 3.1 (a)). One always lies above, and the other lies under the 45 degree line. No interior equilibrium exists. When \( z < 1 \), however, the best response curves both bend towards the 45 degree line and are bound above at \( \frac{1}{1-z} \) (see Figure 3.1 (b)), resulting in an interior equilibrium with the number of divisions for each firm less than \( \frac{1}{1-z} \).

It is worth considering two extreme cases. If there is zero substitution among products, each firm faces totally independent markets. Divisionalization then creates the competition effect, but not the business stealing effect. Thus, the firm has no incentive to divisionalize. If products are perfect substitutes, however, the business stealing effect is at its maximum and the competition effect is at its minimum\(^6\). In fact, the former always dominates the latter. Each firm in equilibrium will set up infinite divisions. We summarize these two cases in the following corollaries:

**Corollary 3.2:** If there is zero substitution among products, firms do not divisionalize.

**Corollary 3.3:** If oligopolists produce perfect substitutes, the equilibrium number of divisions for each firm is infinity.

Corollary 3.3 is one of the main results of Corchon (1991), Corchon and Gonzalez-Maestre (1993), Polasky (1992), and Baye, Crocker and Ju (1995), who all consider a model of homogeneous products. Our results show that the nonexistence problem in the

---

\(^6\)The extra competition is shared equally by all existing divisions of all firms.
previous studies is mainly due to the assumption that firms supply homogeneous products. Product differentiation automatically guarantees an interior SPNE.

Proposition 3.1 also characterizes a positive relationship between the equilibrium number of divisions and the number of differentiated products. That is, firms get more aggressive in divisionalization when they face more competing firms. The intuition is that the larger the number of competing firms, the more sources a firm can steal business from and spread the additional competition to. Consequently, firms have a higher incentive to divisionalize and, as a result, the equilibrium number of divisions of each firm will be higher. This result also has a significant implication for entry deterrence. It implies that an entrant has to compete not only with the divisions existing prior to entry, but also with additional divisions induced by the entry. We will come back to this issue in section 3.5.

3.4 Possibility of Further Divisionalization

In the two stage divisionalization game, we implicitly assume that the original parent firms can set up divisions, but divisions themselves cannot. What happens if this assumption is relaxed? Will divisions then further divide? Will parent firms' profits decrease or increase? Do parent firms have incentives to restrict divisions unilaterally from further dividing?

To answer these questions, consider a case where divisions are allowed to divide further into autonomous subdivisions. Let a division's profit be the sum of profits of its subdivisions. We will first show by contradiction that if divisions are allowed to divide further, they always will.

Assume that there exists an equilibrium structure such that no divisions choose to divide further even though they are free to do so. Let \( m_k \) be the number of divisions of firm \( k \) in equilibrium, where \( k = 1, 2, ..., n \). By definition, it is not profitable for any division to divide further in equilibrium. Now, imagine a hypothetical further breakup of divisions. Let \( s_{ki} \) be the number of independent subdivisions in division \( i \) of firm \( k \). The total number of divisions of firm \( k \) will be \( N_k = \sum_{j=1}^{m_k} s_{kj} \). According to equation (3.7),
the total profit of firm \( k \) is

\[
\pi_k = \frac{a^2}{bN_k[(1 + \frac{1}{N_k} - z)(1 + z\Delta)]^2}.
\]

Then, the total profit of the \( s_{ki} \) subdivisions of division \( i \) of firm \( k \) is

\[
\pi_{ki} = \frac{s_{ki}}{N_k} \pi_k = \frac{a^2 s_{ki}}{b[N_k(1 + \frac{1}{N_k} - z)(1 + z\Delta)]^2},
\]

Division \( i \) of firm \( k \) chooses \( s_{ki} \) to maximize \( \pi_{ki} \) taking the number of subdivisions of other divisions as given. After simplifying the first order condition, we have

\[
s_{ki} = \sum_{j \neq i}^{j=m_k} s_{kj} + 1 + \frac{bz^2\Delta_k}{1+(1-z)z\Delta_k},
\]

Notice that \( \sum_{j \neq i}^{j=m_k} s_{kj} + 1 \) is the number of subdivisions division \( i \) of firm \( k \) would set up if there were no other firms besides \( k \), and \( \frac{bz^2\Delta_k}{1+(1-z)z\Delta_k} \) is the number of divisions induced by the existence of subdivisions of firms other than \( k \). For \( m_k \geq 2 \), the symmetric solution to the above equation is infinity, which is not surprising, since divisions of the same firm supply perfect substitutes, and, therefore, the business stealing effect dominates the competition effect in further divisionalization. This result obviously contradicts our assumption earlier that there exists an equilibrium structure in which no divisions have the incentive to divide further. Thus, if divisions are allowed to divide further, they always will.

Furthermore, the total profit of the firm which does not restrict its divisions from further dividing is

\[
\lim_{M \to \infty} \pi_k = \lim_{M \to \infty} \frac{a^2}{bM[(1 + \frac{1}{M} - z)(1 + z\Delta)]^2} = 0,
\]

and the price of \( k^{th} \) firm's product is

\[
\lim_{M \to \infty} p_k = \lim_{M \to \infty} \frac{a}{(1 + \frac{1}{M} - z)(1 + z\Delta)} = 0.
\]

That is, allowing divisions to divide further leads to zero markup and zero profit for the firm. The result holds regardless of whether other firms allow their divisions to divide.
further. In order to avoid the total dissipation of profit, each firm has an incentive to restrict its divisions unilaterally from further dividing. When a parent firm cannot effectively enforce this restriction, it is better not to set up divisions. The results are summarized as follows:

**Proposition 3.2:** If divisions of a firm are allowed to divide further, they always will, resulting in total profit dissipation for the parent firm. Thus, each firm has an incentive to restrict its divisions unilaterally from further dividing.

One implicit but critical assumption in the strategic divisionalization literature is that only the parent firms can set up divisions, and the divisions themselves cannot. Without this assumption, the corresponding analysis and results do not hold. Proposition 3.2 provides a theoretical justification for this assumption. This result complements Veendorp's (1991) finding that a firm in a multidivisional structure delegates to its divisions decisions regarding output and the like, but reserves for itself investment decisions regarding capacities. We show that a parent firm will reserve the divisionalization decision. This finding is also consistent with the general practice in franchising where franchisees are usually not allowed to set up franchises themselves.

### 3.5 Free entry equilibrium

In this section, we turn our attention to the free entry equilibrium. Formally, we consider the following simultaneous entry game with a large number of identical potential firms in the market:

- **Stage 0:** Firms make their entry decisions given the other firms' entry decisions.
- **Stage 1:** Firms which have decided to enter the market in the first stage choose their numbers of independent divisions.
- **Stage 2:** All the divisions engage in Cournot competition.

Notice that the last two stages of the free entry game are the same as the two stage divisionalization game we have solved in Section 3.3. Thus, we only need to solve the entry game in stage 0.
Free entry, by definition, entails zero profit for each firm in the market. That is, in the free entry equilibrium,

\[ \pi_k - F = 0, \]

or,

\[ \frac{a^2}{b m [1 + \frac{1}{m} + z(n - 1)]^2} = F, \]  

(3.11)

where \( m \) is the solution to equation (3.10), \( F \) is the fixed cost associated with entry, and \( \pi_k \) is the gross profit of firm \( k \). Equation (3.11) defines the number of firms in the free entry equilibrium.

**Proposition 3.3** For a given \( F \), there exists a pure strategy SPNE for the free entry game. Moreover, the equilibrium number of firms (or differentiated products) decreases with the degree of substitution between the products, as well as with the fixed entry cost, \( F \).

Similar to Salop (1979), we find that, in equilibrium, the number of firms is negatively related to the magnitude of the fixed entry cost. However, our results imply a much stronger case for the persistence of high profits (except the case where the zero profit condition produces an exact integer solution for the number of firms). Since the equilibrium number of divisions of a firm increases with the number of firms, the competition an entrant faces comes not only from incumbents' existing divisions, but also from additional divisions induced by entry. The number of additional divisions induced by entry can be rather large, due to the complementarity among the numbers of divisions of different firms\(^7\). Thus, entry might induce very severe competition and profit dissipation. As a result, with divisionalization, a potential firm is more reluctant to enter the market, and the incumbents may, in turn, enjoy abnormally high profits (this is clearly illustrated in the following example).

Like divisionalization itself, the entry deterrence effect of divisionalization increases with the degree of substitution among products. As \( z \) approaches 1, even a duopoly will generate so many divisions that the last stage Cournot game will generate an outcome close

\(^7\)See Lemma 2.
to the perfectly competitive equilibrium. Then, the only possible free entry equilibrium is the monopoly outcome, even if the entry cost is relatively low. Consequently, the incumbent can persistently earn the monopoly profit, which may be many times more than the entry cost, without worrying about entry. For example, in the case of a duopoly \((n = 2)\), \(m = \sqrt{\frac{1}{1-z^2}}\) (from equation (3.10)) and the profit of a duopolist \(\pi = \frac{a^2}{b} \frac{\sqrt{1-z^2}}{(1+z+\sqrt{1-z^2})^2}\) from (equation (11)). When \(z = .98\), \(m = 5\), a duopolist's profit is 8.4% of that of a monopoly, which is \(\frac{a^2}{2b}\). Hence, if \(F\), the fixed entry cost, is greater than or equal to 8.4% of the monopoly profit, the free entry equilibrium is a monopoly outcome, despite the fact that the monopoly may earn as much as 12 times of the entry cost.

It is worth noting that, in this case, it is mainly the threat of divisionalization (not the fixed entry cost) that causes the natural monopoly outcome. Interestingly, unlike what the previous literature has suggested, divisionalization does not need to occur to assure the monopoly result. The credible threat of divisionalization in case of entry is enough.

The above result is summarized in the following proposition:

**Proposition 3.4:** Under divisionalization, incumbents in free entry equilibrium may persistently earn abnormally high profits. In particular, for any given \(F\), there exists a \(z^* < 1\) such that, for any \(z > z^*\), a natural monopoly is the unique pure strategy free entry SPNE, where \(z^*\) satisfies \(\frac{\sqrt{1-z^2}}{(1+z+\sqrt{1-z^2})^2} = F \frac{b}{a^2}\).

**Remark:** There is a paradoxical implication of Proposition 3.4. Given the number of firms in the industry, divisionalization intensifies the level of competition and alleviates the distortion in pricing. It seems that, from a social point of view, divisionalization should be encouraged. Under free entry, however, the threat of divisionalization by incumbents reduces potential entrants' incentive to enter a market. The competition level in free entry equilibrium with divisionalization might be much lower than when no divisionalization is allowed. In summary, even though limiting the number of autonomous divisions of firms may weaken competition in the short run, it can strengthen competition in the long run.

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8For example, Schwartz and Thompson (1986) show how the incumbent forestalls entry by setting up independent divisions just prior to the date of a potential entry.

9In the example where \(z = .98\), if \(F\) is 8.4% of the monopoly profit, the free entry equilibrium with divisionalization is monopoly. If divisionalization is not allowed, it is easy to verify that there will be at least nine competing firms in equilibrium.
3.6 Concluding Remarks

In this paper, we consider an environment in which competing oligopolistic firms with
differentiated products can set up independent rival divisions. We analyze the strategic
incentives for a firm to divisionalize, characterize the equilibrium of a divisionalization
game and highlight the effect of product differentiation in ensuring an interior equilibrium.
By allowing for product differentiation, we demonstrate that the existence of an interior
equilibrium can be achieved without reliance on ad hoc assumptions, such as an exogenous
bound of permissible number of divisions or a costly divisionalization.

The existence of an interior equilibrium depends heavily on the assumption that
divisions of the same firm produce closer substitutes than divisions of different firms do.
However, the divisionalization result does not depend on this assumption. When all firms
produce homogeneous products (the model of Corchon, Polasky and Baye et al.), firms
still divisionalize. In fact, they set up an infinite number of divisions in equilibrium. We
can infer that, if divisions of the same firm are able to be differentiated such that divisions
of different firms produce closer substitutes than divisions of the same firm\textsuperscript{10}, the business
stealing effect will be strengthened, and the competition effect will be weakened. Thus,
firms would have stronger incentives to divisionalize. Assuming there is no fixed set-up
cost, each firm would set up as many such differentiated divisions as possible. However,
differentiation of divisions of the same firm often requires differentiated products, and dif­
ferentiated products often imply R & D costs. In such cases, according to Baye, Crocker
and Ju (1996), fixed set-up costs may limit the number of divisions of a firm.

To isolate the strategic aspect of divisionalization, we also assume a constant return to
scale technology. Yet, the nature of technologies is an important factor when firms choose
the number of divisions. Increasing return to scale technologies should reduce firms' in­
centives to divisionalize (dividing production), and decreasing return to scale technologies
should increase such incentives. For example, the increasing return of scale nature of ad­
tertising in the cereal industry may prevent cereal firms from setting up differentiated
divisions even though each cereal firm has a number of differentiated products.

We also consider the consequences of allowing divisions to divide further. It is found

\textsuperscript{10}For example, in the automobile industry, high-end market divisions compete more with other high-end
market divisions than with the low-end divisions from the same firm.
that, if divisions are allowed to divide further, they always will. Then, the only possible outcome is the one in which the firm that allows its divisions to divide further has an infinite number of divisions and zero profit. Hence, each firm has an incentive to restrict its divisions unilaterally from further breakup. Our finding provides a theoretical justification for the assumption in the strategic divisionalization literature that only parent firms can set up divisions, and divisions themselves cannot.

Finally, we discuss the free entry issue. We find that divisionalization has a natural entry deterrence property, for it can significantly magnify the severities of competition in the face of entry. As a result, incumbent firms may persistently earn abnormally high profits in free entry equilibrium, relative to the no divisionalization case. In fact, when firms have difficulty differentiating from each other because of either the concentration of consumers' tastes or technological reasons, the only pure strategy subgame perfect free entry equilibrium is the monopoly outcome, even if the entry cost is relatively low. By limiting the number of independent divisions or franchises, regulators can actually help to increase competition. In addition, in contrast to the previous literature which suggests that the incumbent actually has to set up divisions to deter entry, we show that the threat to divisionalize may be enough to ensure the monopoly outcome. Considering that the equilibrium we analyze is a free entry SPNE, this result is rather surprising.
Chapter 4

Divide and Conquer: Strategic Leasing in Common Pool Oil Fields

"The overdrilling that has taken place ... represents a tremendous economic waste, not only in the expenditure of capital, but in the dissipation of natural resources." "The overdevelopment cannot be said to be entirely the fault of the operator, for the landowner is equally to blame." [Bulletin of the American Association of Petroleum Geologists, Vol. VIII, July-August, 1924]

4.1 Introduction

Since petroleum was first found in the U. S. in the middle of the 19th century, oil production has been plagued by serious common pool wastes\(^1\). They are usually attributed to excessive drilling, unnecessary surface storage, overextraction, and reduced ultimate oil recovery. Under the common law rule of capture, the property rights to oil are only assigned upon extraction. When multiple firms compete for migratory oil in a common pool reservoir, each has an incentive to drill competitively and drain oil from its neighbors. Common pool losses arise as capital costs are driven up by the drilling of excessive numbers of wells (more than geologic and fluid conditions warrant) and the construction

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\(^1\)For example, in 1914, the U.S. Bureau of Mines estimated annual losses from competitive extraction at $50 million, approximately one-quarter of the total value of U.S. production; the Federal Oil Conservation Board (1926, p.30; 1929, p.10) estimated recovery rates of only 20-25 percent with competitive extraction, while 85-90 percent was possible with controlled withdrawal.
of surface storage. Rapid production also prematurely depletes the subsurface pressure and in turn reduces the total recovery.

Given the extraordinary costs of competitive production in common pool fields, one would expect that landowners have incentives to limit the number of independent operators in order to reduce the common pool loss associated with competitive extraction. However, a widespread phenomenon in onshore oil production is that landowners in the same oil fields often divide their landholding into smaller pieces and grant production rights to multiple different operators\(^2\)(a phenomenon I shall refer to as *multiple leasing* hereafter). Landowners' multiple leasing practice can seriously aggravate the common pool problem. Given the competitive behavior of independent operators, why does a landowner grant more than one lease? This paper attempts a game theoretic explanation to this puzzling phenomenon.

Table 4.1 (see Appendix 4.2) summarizes the landholding and leasing information of the 42 U.S. onshore oil fields, which I collected from Zingery and Southwest oil production maps in various issues of *Oil Weekly* from February 1938 to April 1940. Columns 2-4 are the number of landowners, the number of independent operators, and the number of leases in each field respectively\(^3\). The fifth column is the ratio of the number of operators to the number of landowners. The last column is the average number of leases per landowner. In terms of leases, Table I shows that, in all oil fields sampled, leases are more numerous than landowners. In 36 out of the 42 fields (or 86%), there are at least twice as many leases as landowners. Averaging across fields, each landowner has 3.8 leases. In terms of independent operators, similar results persist: 35 out of the 42 (or 83%) oil fields sampled have more operators than landowners, and the average operator-landowner ratio across fields is 2.6. The field level data shows that multiple leasing is a rather widespread practice in the early stage of U.S. onshore oil fields development.

Formally, we view that the oil field development consists of two stages. In the first stage, the landowners simultaneously choose leasing strategies. In the second stage, independent lease operators produce oil competitively by choosing extraction strategies simultaneously. Not surprisingly, it is found that competitive extraction by multiple in-

\(^2\)In the sample Libcap and Wiggins (1984) assembled, for example, Yates field has 16 independent operators but only 2 initial landowners and Hendrick field has 18 operators but only 3 landowners.

\(^3\)In some cases, one operator owns more than one lease in one field so that the number of leases may differ from that of operators. That is why data on both of them are presented.
dependent operators leads to overdrilling, overextraction, and reduced recovery. More importantly, it is shown that, in a non-exclusively owned oil field, it is individually rational for a landowner to subdivide his landholding unilaterally and delegate production rights to multiple independent lease operators. Consequently, the production lease ownership is usually more dispersed than the landownershi. Landowners are as responsible as the operators for the serious common pool wastes.

Given that lower concentration of production leads to more serious rent dissipation in a common pool, it seems irrational for a landowner to grant leases to multiple operators. The key to understanding this puzzle is that operators of a multi-lease landowner as a whole are more aggressive in oil production than the landowner. This is because they ignore the externality of their extraction on each other, in addition to that on operators of other landowners. Essentially, the multiple leasing strategy enables a landowner to behave as a Stackleberg leader and credibly commit to a higher production level and, in turn, captures a bigger share of the aggregate output and of the fieldwide economic rent. Multiple leasing decreases the fieldwide rent, but increases the landowner’s share. The effect of the increasing share can dominate that of the decreasing total rent. The tradeoff of these two effects determines how many leases a landowner will grant.

Unfortunately for the landowners, if they all follow the same strategy, everyone will be worse off in equilibrium, for multiple leasing increases the extraction rate and, therefore, the common pool losses, but landowners’ shares of output (or rent) remain unchanged in equilibrium. Nonetheless, in the non-cooperative game, given other landowners’ strategies, a landowner has to pursue the multiple leasing strategy. Otherwise, the landowner will do even worse, for it will capture a smaller share of the shrunk total output and rent.

This paper is closely related to, but distinct from, the common property literature. Much of the common property literature (Gorden, 1954; Hardin, 1968; and Dasgupta and Heal, 1979, for examples) focuses attention on the economic wastes under competitive production, but not on the determination of the organization of production itself. In this literature, resource owners are usually implicitly assumed to be the same as resource developers, and thus the property ownership structure is the same as the resource operation structure. The traditional approach leaves the choices of inefficient organization of production unexplained. The main focus of our analysis is the strategic choices of the organization of production by property owners (landowners). More precisely, we show
that landowners have incentives to delegate production rights to multiple independent operators. As a result, the landownership structure is, in general, different from the production operation structure. Therefore, the traditional theory tends to under-estimate the tragedy of the commons, if resource ownership structure is used as an approximate of the operation structure.

This analysis is also related to the literature on strategic divisionalization in Cournot competition setting. Many authors, Corchon (1991), Polasky (1992), and Baye, Crocker and Ju (1996), for example, analyze a two stage divisionalization game. In the game, parent firms choose the number of independently managed divisions in the first stage, and divisions compete in a Cournot fashion in the second stage. It is shown that each firm has an incentive to set up rival divisions. This shows that a parallel argument can be constructed to explain a long and puzzling phenomenon (multiple leasing) in oil field development and organization of production.

The chapter proceeds as follows. Section 4.2 provides a brief review of oil production technology. Section 4.3 presents a model of oil field development. Section 4.4 provides an analysis of landowners’ strategic leasing behaviors in a common pool. Section 4.5 concludes.

4.2 Oil Extraction Technology in a Common Pool

In this section, we discuss oil production in a common pool based on models of oil extraction reviewed and developed by Ben-Zvi (1985).

To simplify the analysis, we assume that an oil field consists of a perfectly-connected common pool without the stratification and separating faults. Thus, the oil can potentially flow to any corner of the field. Without a doubt, this is an oversimplification of the reality but it captures the essence of the common pool problem.

4.2.1 The Extraction Rate

Oil reservoirs are usually compressed between a layer of natural gas and a layer of water. The underground pressure drives the oil to the surface when the surrounding formation is punctured by wells. The instantaneous extraction rate depends on the geological and fluid parameters of the reservoir formation and the number of wells in the field. Given the
geological and fluid characteristics of an oil reservoir, the extraction rate for the pool is proportional to the difference between the wellhead pressure and the underground pressure and is a concave function in the number of wells. That is,

\[ q(t) = \eta \Phi(N)(p(t) - p_w(t)), \]  

(4.1)

where \( q(t) \) is the instantaneous extraction rate; \( \eta \) is a parameter which measures the effect of geological and fluid characteristics of the reservoir; \( N \) is the number of wells in the field and \( \Phi(N) \) is a concave function in \( N \); \( p(t) \) is the underground pressure; and \( p_w(t) \geq 0 \) is the wellhead pressure.

Theoretically, operators can control the extraction rate by choosing \( p_w(t) \). Since we are mainly interested in competitive production, we assume that wells produce at full capacity, i.e., \( p_w(t) = 0 \). To simplify the analysis, we also assume that \( \Phi(N) = N^d \), where \( d > 1 \) approximates the degree of concavity of the instantaneous production function with respect to the number of wells. The extraction function then becomes

\[ q(t) = \eta N^d p(t). \]  

(4.2)

4.2.2 The Pressure Depletion Dynamics

Unlike many other exhaustible natural resources, the ultimate recovery of oil depends on the time path of output. With a high extraction rate, the ratio of natural gas and water to oil produced increases, leading to premature loss in subsurface pressure. Due to the loss of pressure, the natural gas dissolved in the oil leaves the solution, reducing the oil's mobility and leaving significant reserves permanently trapped. It is very costly to extract oil once the pressure is exhausted. Hence, characterizing the behavior of \( p(t) \) is an important issue in oil production. The rate of change in \( p(t) \) is generally believed to be a function of both the extraction rate, \( q(t) \), and the pressure, \( p(t) \). Following Ben-Zvi (1985), we adopt the following functional form for the rate of change in \( p(t) \):

\[ \frac{dp(t)}{dt} = -\alpha q^b(t)p^{-a}(t), \]  

(4.3)

where \( \frac{dp(t)}{dt} \) is the rate of pressure depletion; \( \alpha \) is a constant; and \( b (a + 2 > b > 1) \).
measures the degree of the convexity of the rate of pressure depletion in terms of the extraction rate, where \( a > 0 \). Notice that this functional form implies that there is no pressure depletion if no oil is extracted; otherwise, the pressure declines. Moreover, the marginal pressure loss increases with the extraction rate.

### 4.2.3 The Ultimate Recovery

The ultimate recovery is defined as

\[
Q = \int_0^\infty q(t) dt,
\]

where \( Q \) is the ultimate recovery. Solving for \( dt \) in equation (4.3) yields

\[
dt = -\frac{1}{a} q^{-b}(t) p^a(t) dp.
\]

Substituting equations (4.2) and (4.5) into (4.4), we have

\[
Q = UN^{\frac{1-b}{d}} = UN^{-s},
\]

where \( U = \frac{n p(0)^{2+a-b}}{a(2+a-b)} \), and \( s = \frac{b-1}{d} \).

Equation (4.6) shows that the ultimate recovery declines with the total number of producing wells in a common pool. When there is only one well \( (N = 1) \), the ultimate recovery reaches its maximum, where \( Q = U \). Thus, \( U \) is the maximum recoverable oil reserve. The decline rate of the ultimate recovery is measured by \( s \), which we shall refer to as the gross rent dissipation rate (GRDR). GRDR characterizes how fast the gross rent (ultimate recovery) decreases with the number of wells. Notice that, when \( s = 0 \) (or \( b = 1 \)), the ultimate recovery is independent of the number of wells. The Federal Oil Conservation Board (1926, p.30; 1929, p.10) estimated that the ultimate recovery at competitive extraction is about 30% of that under controlled withdrawal. Since the number of wells under competitive extraction is usually many times greater than that under controlled withdrawal, it can be inferred that \( s \) is a number between zero and one, and perhaps relatively close to zero. We therefore assume hereafter that \( 0 \leq s < 1 \).
4.3 Analysis of Competitive Oil Extraction

In this section, we analyze the equilibrium of competitive extraction of multiple independent operators in a common pool oil field.

In order to simplify the analysis, the following assumptions are made:

A1: The drilling cost function is assumed to be $D(N) = NC$, where $C$ denotes the marginal cost of drilling a well, and $D$ is the total drilling cost.

A2: The crude oil market is assumed to be perfectly competitive. Without the loss of generality, we further assume the crude oil price to be constant over time and normalize it to be one.\(^4\)

A3: The discount rate is assumed to be zero.

Usually, drilling cost is assumed to be proportional to the drilling depth. In assumption A1, I implicitly assume that all wells in the same oil field have the same depth. My justification for it is that, before drilling occurs, the relevant drilling cost for potential lessees is the expected value, which is more or less the same in the same field.

Assumption A2 is not unreasonable. First, in addition to numerous world oil producers, there are in the U.S. alone thousands of oil fields and many times more independent operating firms. Thus, oil producers in a relatively small oil pool face an approximately competitive market, unless they mainly supply for a relatively independent local market. Second, in making leasing decisions, the relevant oil price is the expected future price. It is not uncommon to assume the expected oil price to be constant in the future.

Assumption A3 is obviously an oversimplification of reality. However, it helps to make a tedious derivation tractable. Under a zero discount rate, the present value of oil production becomes the ultimate recovery. A nonzero discount rate does not affect the main results of the paper but only complicates the derivation.

Consider a simple setting, where $L$ independent operators extract oil competitively in a common pool oil field. More specifically, we consider a game where $L$ operators

\(^4\)The constant-price assumption seems contradictory to Hotelling pricing rule, but it is based on historical facts. One of the reason that hotelling rule fails, is that oil reserved estimated has been increasing over the last century.
choose their extraction strategy simultaneously. Since drilling is costly, each well drilled will produce to its full capacity. Since we assume that an oil field consists of a perfectly-connected homogeneous common pool without the stratification and separating faults, the oil can freely flow to any corner of the field, and the geological conditions are same for each well. Therefore, each well will produce at the same capacity. Thus, an operator’s extraction strategies are simply choosing the number of wells.

Let \( N_l \) be the number of wells that operator \( l \) chooses to drill, where \( l = 1, 2, ..., L \). The total number of wells in the common pool is \( N = \sum_{l=1}^{L} N_l \). Since each well produces at the same rate, operator \( l \’\)s output share and instantaneous extraction rate are then \( \frac{N_l}{N} \) and \( \frac{N_l}{N} q(t) \), respectively, where \( q(t) \) is the total instantaneous extraction rate in the common pool. Operator \( l \’\)s net return is the present value of its revenue flow, minus the drilling cost and the fixed lease fee, \( \Lambda \), that is

\[
\pi_l = \int_0^\infty \frac{N_l}{N} q(t) dt - N_l C - \Lambda
\]

\[
= \frac{N_l}{N} Q(N) - N_l C - \Lambda. \tag{4.7}
\]

Taking as given other operators’ number of wells, operator \( l \) chooses \( N_l \) to maximize his net return. The first order condition yields

\[
\frac{\partial}{\partial N_l} Q(N) + \frac{N_l dQ(N)}{N} dN - C = 0. \tag{4.8}
\]

On the left-hand-side of the above equation, each of the three terms represents a different effect of drilling an extra well on operator \( l \’\)s profit: the first term measures the gain due to the subsequent increase in operator \( l \’\)s output share; the second term measures the loss due to the decrease in ultimate recovery; and the third term measures the loss due to the increased drilling cost. The first order condition describes the state where these three effects are balanced.

Rewriting equation (3.8), we have

\[
\frac{N - N_l}{N^2} Q(N) + \frac{N_l dQ(N)}{N} dN - C
\]

\[
= U((N - N_l)N^{-(2+s)} - sN_l N^{-(2+s)}) - C = 0. \tag{4.9}
\]
Notice that the $N_l$ that solves equation (4.9) is independent of the subscript $l$. That is, $N_l$ is the same for $l = 1, 2, \ldots, L$. Denote the equilibrium number of wells per operator by $n$. Then, the number of wells in the field is $N = nL$.

Multiplying equation (4.9) by $L$ and solving for $N$, we have

$$N = \left[ \frac{1 - \frac{s+1}{L}}{c} \right]^{\frac{1}{s+1}},$$

for $L > s + 1$, where $c = \frac{G}{U}$ is the relative marginal drilling cost of a well. Then,

$$N_l = n = \frac{1}{L} \left[ \frac{1 - \frac{s+1}{L}}{c} \right]^{\frac{1}{s+1}},$$

and

$$Q(N) = UN^{-s} = U \left( \frac{1 - \frac{s+1}{L}}{c} \right)^{-\frac{s}{s+1}}.$$

Call the return before subtracting the fixed lease fee the gross return. Then, the field-wide gross return, denoted by $\Pi$, is

$$\Pi = (Q(N) - CN)$$

$$= UL^{-\frac{1}{s+1}} \left( \frac{c}{L - (s + 1)} \right)^{\frac{s}{s+1}}.$$

Similarly, operator $l$'s gross return, denoted by $\Pi_l$, is

$$\Pi_l = \frac{1}{L} \left[ UL^{-\frac{1}{s+1}} \left( \frac{c}{L - (s + 1)} \right)^{\frac{s}{s+1}} \right].$$

**Proposition 4.1:** Under A1-A3, the following results hold:

(a) both the ultimate recovery of oil and the aggregate gross return of the common pool decrease with the number of independent operators;

(b) the value of a lease granted to an operator decreases with the number of independent leases in the common pool field.
The proof of Proposition 4.1 can be easily obtained by differentiating equation (4.12) to (4.14).

Proposition 4.1 (a) states two separate claims regarding the ultimate recovery and the aggregate rent respectively. The ultimate recovery of oil declining with the number of operators can be said to be an oil extraction phenomenon, resulting from the underground pressure depletion dynamics of oil production. That a large number of independent operators causes low economic rent, however, is the usual outcome associated with common property problems. Here, the number of operators measures the degree of production fragmentation. More operators represent a lower degree of production concentration, which results in higher rent dissipation.

Proposition 4.1 (b) can be viewed as a corollary of Proposition 4.1 (a). More operators share a shrinking fieldwide rent. The value per lease of course declines with the number of leases in the field.

4.4 A Game Theoretic Model of Strategic Leasing in a Common Pool

In this section, we analyze landowners’ leasing behaviors. Consider a simple environment, where \( M \) landowners each owns a fraction of the land surface that covers a homogeneous common oil pool. In anticipation of the competitive behavior of lease operators, each landowner has to decide how to grant production rights to operators which specialize in oil production. If a landowner decides to produce oil himself, we can always view him as both the landowner and an operator, as if he signs a lease contract to himself. Our main focus is how the landowners take advantage of the common pool by strategically choosing the production organization (multiple leasing). More specifically, we model landowners’ decisions as a game in which each of them chooses simultaneously the number of leases he or she will grant to independent operators. Following convention, we refer to landowners and operators as lessors and lessees respectively.

In order to simplify the analysis further, the following assumption about lease contracts is made:

A4: The lease contract takes the following form:
1. Having paid a fixed fee to a lessor, a lessee gains the full production right on a prespecified oil tract and retains the right to the oil produced.

2. Lessors have all the bargaining power; that is, the leasing market is perfectly competitive. Therefore, a lessor extracts all the economic rent of oil production through a fixed fee, and lessees make zero profit.

3. Leasing is costly. The marginal transaction cost of signing a lease is \( \Omega \).

Assumption A4 attempts to capture the main characteristics of a typical oil lease contract. In order to make lessees aggressive in production, a lease contract usually requires a lessee to pay a large fee up front and grants the lessee up to 90 percent of the oil produced. The assumption of a fixed fee lease contract is an approximation of contracts of such a type. We can think of the contracting process of leasing as follows. A landowner first proposes contracts in a take-it-or-leave-it fashion to many potential independent operators with similar opportunity costs. The operators then decide whether they will accept the contracts. The competition among those operators will leave them with their opportunity costs and landowners with the entire rents. Finally, the leasing transaction cost is assumed to capture the cost and frictions associated with lease contracting.

Before solving for the equilibrium of the leasing game, we show first, by the use of an example, the potential gains of the multiple leasing strategy to a landowner.

4.4.1 The Potential Gain of a Multiple Leasing Strategy: An Example

Consider a common oil pool, where \( M = 5 \), \( U = \$1,000,000 \), \( c = .001 \), and \( s = .5 \). Among the five landowners, we assume that four are strategically innocent, and one is strategically sophisticated. An innocent landowner either produces oil herself or grants only one production lease, whereas the sophisticated landowner may grant production leases to multiple independent operators. Denote by \( l \) the number of independent leases the sophisticated landowner grants; thus the total number of leases in the common pool is \( l + 4 \). Let \( \pi^s \) and \( \pi^i \) be the resulting profits of a typical sophisticated and innocent landowner, respectively. Using the equilibrium solution of the competitive extraction game, we obtain
\[ \pi^s = \frac{l}{l+4} Q(N) - nlC \]
\[ = \frac{U_{cl} \left( \frac{l+3-s}{(l+4)c} \right)^{1/(s+1)}(\frac{1+s}{l+3-s})}{l+4}, \]

and

\[ \pi^i = \frac{1}{l+4} Q(N) - nC \]
\[ = \frac{U_{cl} \left( \frac{l+3-s}{(l+4)c} \right)^{1/(s+1)}(\frac{1+s}{l+3-s})}{l+4}, \]

and

\[ \Pi = \pi^s + 4\pi^i, \quad (4.15) \]

where \( \Pi \) is the total rent in the common pool.

A simple calculation generates the following table:

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^s )</td>
<td>6757</td>
<td>9172</td>
<td>9952</td>
<td>10047</td>
<td>9839</td>
<td>7947</td>
<td></td>
</tr>
<tr>
<td>( \pi^i )</td>
<td>6757</td>
<td>4586</td>
<td>3317</td>
<td>2512</td>
<td>1938</td>
<td>1324</td>
<td></td>
</tr>
<tr>
<td>( \Pi )</td>
<td>33785</td>
<td>28516</td>
<td>23220</td>
<td>20095</td>
<td>17591</td>
<td>13243</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2

Table 4.2 shows that, as the number of independent leases granted by the sophisticated landowner grows, the total rent in the common pool steadily decreases. The sophisticated landowner's profit, however, increases initially and then decreases with the number of independent leases she grants. When she has four leases, her profit reaches $10,047, which is $3290 more than the profit she makes if she grants only one lease strategy. Thus,
this example shows that, given that other landowners grant one lease each, the remaining
landowner has incentive to grant unilaterally multiple leases. The question, then, is will
landowners choose to grant multiple leases if each of them is free to do so?

4.4.2 Analysis of the Leasing Game

Equipped with the characterization of the equilibrium in the competitive extraction game,
we now turn to analyzing the equilibrium of the landowners' leasing game. Under the
assumptions about leasing outlined earlier, lessors have all the bargaining power. Thus,
lessors extract all the economic rent from leases through a fixed lease fee, leaving lessees
to earn zero profits.

Let \( M \) denote the number of lessors in a common pool, and \( L_m \) the number of leases
the \( m \)th lessor chooses to grant, where \( m = 1, 2, \ldots, M \). Denote the profit of lessor \( m \)
by \( \pi_m \), which is her leasing revenue, netting the transaction cost of signing these leases.
Hence, we have

\[
\pi_m = \Lambda L_m - \Omega L_m \\
= \frac{L_m}{L} \left[ UL^{-\frac{\Lambda}{s+1}} \left( \frac{c}{L - (s + 1)} \right)^{\frac{\Lambda}{s+1}} \right] - \Omega L_m. \tag{4.16}
\]

The example in section 4.4.1 shows that a landowner has an incentive to grant multiple
independent leases, if others do not. We now turn to the symmetric Nash equilibrium of
the leasing game.

In the Nash leasing game, a typical lessor \( m \) maximizes \( \pi_m \) by choosing \( L_m \), taking as
given other lessors' number of leases. The first order condition yields

\[
(s + 1)UC^{\frac{s}{s+1}} \frac{-L_m^2 + L_m + \Delta_m(\Delta_m - s - 1)}{L^{(2s+3)/(s+1)}(L - s - 1)(2s+1)/(s+1)} - \Omega = 0, \tag{4.17}
\]

for \( m = 1, 2, \ldots, M \). Notice that equation (4.16) is symmetric for all \( m \)s, which implies
the equilibrium solution of \( L_m \) is the same for all \( m \). Denote the equilibrium number of
leases of a typical lessor by \( l \). Then, we obtain
for $M > 2$, where $\omega = \frac{\theta}{\rho}$ is the relative marginal transaction cost of leasing. Replacing $L$ by $\frac{L}{M}$ in the above equation yields

$$(M - 2)Ml - (s + 1)(M - 1) + 1 - \frac{\omega}{s + 1} c^{-\frac{\omega}{s + 1}} M^{-\frac{\omega}{s + 1}} [Ml - (s + 1)]^{\omega + 1} = 0, \quad (4.18)$$

Equation (4.17) determines the equilibrium number of leases per landowner. Equation (4.18) characterizes the relationship between the degree of production concentration and the degree of landownership concentration.

**Proposition 4.2**: If the marginal transaction cost of leasing is sufficiently low and $M > 2$, the subgame perfect equilibrium has the following properties: (a) each landowner grants multiple leases; (b) the number of independent leases increases with the number of landowners in the field; and, (c) the number of leases per landowner decreases with the number of landowners in the field.

(See Appendix 4.1 for the proof)

Proposition 4.2 (a) predicts landowners’ multiple leasing behavior in a common pool with more than two landowners. It also implies that the degree of production concentration is lower than the degree of landownership concentration. Proposition 4.2 (b) states that the degree of production concentration is determined by and positively related to the degree of landownership concentration. Hence, more fragmented landownership leads to more dispersed production control. Proposition 4.2 (c) indicates that there is a limit to landowners’ strategic leasing. This is because the marginal transaction cost of leasing is a constant, but the total economic rent has an upper bound and declines as the number of leases increases.

In order to understand the forces behind the landowners’ leasing consideration, we rewrite the first order condition of the leasing game as follows:
\[
\frac{dL}{dL_m} Q(N) + \frac{L_m}{L} \frac{dQ(N)}{dN} \frac{\partial N}{\partial L_m} \frac{\partial N_m}{\partial L_m} - C - \Omega = 0.
\] (4.20)

The four terms on the LHS of the equation measure four different effects of granting an extra lease on the profit of lessor \( m \): the first term measures the gain in lessor \( m \)'s share of ultimate recovery, which we shall refer to as the share-enhancing effect; the second term measures the loss of profit due to reduced total recovery, which we shall refer to as the size-of-the-pie effect; the third term measures the profit loss due to increased drilling cost, which we shall refer to as the overinvestment effect; and the fourth term measures the loss due to the marginal transaction cost of leasing, which we shall refer to as the transaction effect. The last three effects are all negative. Obviously, the share-enhancing effect of multilateral leasing is what induces a lessor to create multiple leases. If a lessor's share in total output and rent were fixed under some rules, she would not grant multiple leases.

**Remark:** Multiple leasing strategy benefits a lessor, not because lessees have superior technologies, but because lessees face different externalities. Lessees of a multi-lease lessor ignore the externality that extraction inflicts not only on other lessors, but also on other lessees of the lessor. This makes lessees more aggressive in extraction than the lessor. By granting production rights to multiple independent lessees, a lessor credibly commits to a higher level of extraction and, consequently, increases her share of the total output and economic rent. However, if all lessors pursue the same strategy, in equilibrium, everyone is worse off. Nevertheless, from an individual landowner's point of view, multiple leasing always dominates granting a single lease. The outcome of the leasing game is similar to that of the prisoner's dilemma. Without interventions from outsiders (the government, for example), individual rationality will defeat the common interest.

### 4.5 Conclusion

This paper examines leasing behaviors of landowners in a common oil pool. We show that, despite the rent dissipation associated with nonconcentrated oil extraction, it is profitable for a landowner to grant production rights to multiple independent firms. The key to this puzzle is the power of commitment in a multi-stage noncooperative game.
Through multiple leasing, a landowner credibly commits to a higher extraction rate and, consequently, captures a higher share of output and rent. When the gain in share dominates the loss due to the shrinking of fieldwide rent, it is rational for a landowner to grant an additional lease to an independent operator. This analysis provides an explanation for the puzzling leasing behaviors of landowners in U.S. onshore oil fields. Our results also indicate that insights from this phenomenon may have significant implications for the formation of effective regulation policies.

Moreover, the structural characteristics that lead to landowners’ multiple leasing strategies are also present in many other common property problems. A conceptually similar problem is that of fishery in international waters. In this case, each country would have incentives to license multiple fishing firms to increase its share in output. In principle, the framework of this paper can be applied to any common property problem with private access rights.
Chapter 5

Conclusion

Three essays comprise this thesis. They study how firms strategically set up autonomous units when facing competition. Our results illustrate the importance of the institutional arrangement and decisions structure in a firm, not only to its performance, but also to its rivals' profits and consumers’ welfare.

In the first essay, we show that a firm that produces a group of complements can generate higher profits through divestitures, when there is a competing firm supplying an imperfect substitute group of complements. By delegating pricing decisions to independent divisions, each firm credibly commits not to set the prices of the complements within the group collusively, which softens the competition. In other words, by dividing, the firms create negative externalities among the prices within the group that offset the positive externality between the prices across groups. The same insight also applies to the case when firms supplying imperfectly complementary goods or services compete by setting quantities. Our analysis suggests that industry structure and size of firms are determined by the nature of products and the strategic considerations of the firms.

The welfare implications of the divisionalization in our model are significant. Divestitures by the firms supplying complements raise prices and reduce consumers' surplus. However, the profit gains are not large enough to compensate the consumers’ losses. As a result, the total social surplus is reduced.

Another implication of our analysis is that divestiture motivated by product-line complementarity can be viewed as a response towards entry. Suppose that, initially, there is a monopoly that supplies a group of complementary goods or services. Clearly, in our
context, the monopolist does not have any incentives to divest its operations. When potential entrants enter the market and supply differentiated products, the monopolist has two possible responses. One is to compete against the entrants. If the entrants do not make enough profits to cover their entry costs, the entries are deterred. If the products offered by the entrants are differentiated enough from the incumbent’s products, and if the entry costs are relatively small, the entry cannot be deterred. In this case, an optimal strategy for the incumbent firm is to soften competition by divesting some of its operations. Our analysis suggests that the optimal number of divisions for the incumbent increases as more entrants enter the market.

It should be noted finally that our analysis is based upon a number of assumptions. One crucial assumption is that divisions cannot further divide before they choose their prices. This is reasonable in certain situations where the parent firms still have major ownership control of the divisions, but do not make management decisions. Franchise contracts can be viewed as an example of such a divisionalization. In other situations, it can be difficult for firms or divisions to make this type of commitment. Divisions may then have incentives to divide further. This raises the issue of what determines a stable industry structure in the presence of both substitutes and complements. Further research along this line is needed.

In the second essay, we consider an environment in which competing oligopolistic firms with differentiated products can set up independent rival divisions. We analyze the strategic incentives for a firm to divisionalize, characterize the equilibrium of a divisionalization game, and highlight the effect of product differentiation in ensuring an interior equilibrium. By allowing for product differentiation, we demonstrate that the existence of an interior equilibrium can be achieved without reliance on ad hoc assumptions such as an exogenous bound of permissible number of divisions or a costly divisionalization.

We also consider the consequences of allowing divisions to further divide. It is found that, if divisions are allowed to further divide, they always will. Then, the only possible outcome is the one in which the firm that allows its divisions to further divide has infinite number divisions and zero profit. Hence, each firm has an incentive to unilaterally restrict its divisions from further breakup. Our finding provides a theoretical justification for the assumption in the strategic divisionalization literature that only parent firms can set up divisions, and divisions cannot.
Finally, we discuss the free entry issue. We find that divisionalization has a natural entry deterrence property, for it can significantly magnify the severities of competition in the face of entry. As a result, incumbent firms may persistently earn abnormally high profits in free entry equilibrium relative to the no divisionalization case. In fact, when firms have difficulty to differentiate from each other because of either the concentration of consumers' tastes or technological reasons, the only pure strategy subgame perfect free entry equilibrium is the monopoly outcome, even if the entry cost is relatively low. By limiting the number of independent divisions or franchises, regulators can actually help to increase competition. In addition, in contrast to the previous literature, which suggests that the incumbent actually has to set up divisions to deter entry, we show that the threat to divisionalize is enough to ensure the monopoly outcome.

The third essay examines leasing behaviors of landowners in a common oil pool. We show that, despite the rent dissipation associated with nonconcentrated oil extraction, it is profitable for a landowner to grant production rights to multiple independent firms. The key to this puzzle is the power of commitment in a multi-stage noncooperative game. Through multiple leasing, a landowner credibly commits to a higher extraction rate and, consequently, captures a higher share of output and rent. When the gain in share dominates the loss due to the shrinking of fieldwide rent, it is rational for a landowner to grant an additional lease to an independent operator. This analysis provides an explanation for the puzzling leasing behaviors of landowners in the U.S. onshore oil fields. Our results also indicate that insights from this phenomenon may have significant implications for the formation of effective regulation policies.

Moreover, the structural characteristics that lead to landowners' multiple leasing strategies are also present in many other common property problems. A conceptually similar problem is that of fishery in international waters. In this case, each country would have incentives to license multiple fishing firms to increase its share in output. In principle, the framework of this paper can be applied to any common property problem with private access rights.
Bibliography


[52] *Oil Weekly*, various issues.


5.1 Appendix 2.A

Proof of Lemma 2.1: The existence and uniqueness of the equilibrium follow from Friedman (1977). The symmetry of the equilibrium prices within the group follows from the necessary conditions (2.9). In what follows, we show that the equilibrium aggregate prices, \( \hat{p}_1 \) and \( \hat{p}_2 \), increase with \( m_1 \). Denoting \( x_1 = (\hat{p}_1, \hat{p}_2) \) and \( x_2 = (\hat{p}_2, \hat{p}_1) \) and applying standard comparative-static techniques to (11), we obtain

\[
\begin{align*}
\frac{\partial \hat{p}_1}{\partial m_1} &= -D(x_1)\left[(1 + m_2)D_1(x_2) + \hat{p}_2D_{11}(x_2)\right]/\Delta, \\
\frac{\partial \hat{p}_2}{\partial m_1} &= D(x_1)\left[m_2D_2(x_2) + \hat{p}_2D_{21}(x_2)\right]/\Delta,
\end{align*}
\]

where

\[
\Delta = [(1 + m_1)D_1(x_1) + \hat{p}_1D_{11}(x_1)][(1 + m_2)D_1(x_2) + \hat{p}_2D_{11}(x_2)] - [m_1D_2(x_1) + \hat{p}_1D_{12}(x_1)][m_2D_2(x_2) + \hat{p}_2D_{21}(x_2)].
\]

Assumptions (A1)-(A3) imply that \( \Delta > 0 \) and \( \partial \hat{p}_1/\partial m_1 > 0 \). And \( \partial \hat{p}_2/\partial m_1 > 0 \) follows from (A4). The proofs of statements (d) and (e) are analogous. Q.E.D.

Proof of Lemma 2.2: First notice that, by Lemma 1, when \( m_1 = m_2 = m \) the equilibrium aggregate price for each group of the complements is symmetric, which we denote by \( \hat{p}(m) \). Let \( x = (\hat{p}(m), \hat{p}(m)) \). Then,

\[
\frac{dV(m, m)}{dm} = \frac{d\hat{p}(m)}{dm} (D(x) + \hat{p}(m)D_1(x) + \hat{p}(m)D_2(x)).
\]

From the first-order conditions (2.11), \( \hat{p}(m) = -mD(x)/D_1(x) \). It follows that

\[
\frac{dV(m, m)}{dm} = mD(x) \frac{d\hat{p}(m)}{dm} \left( \frac{1 - m}{m} - \frac{D_2(x)}{D_1(x)} \right).
\]

(A5) implies that \(-D_2(p, p)/D_1(p, p)\) is non-increasing with \( p \), Lemma 1(e) implies that \( \hat{p}(m) \) increases strictly with \( m \), and \((1 - m)/m \) strictly increases with \( m \). It follows that \((1 - m)/m - D_2(x)/D_1(x)\) strictly decreases with \( m \) and is equal to zero at \( m = m^* \).
Thus, \( dV(m, m)/dm \) is positive for \( m < m^* \) and negative for \( m > m^* \). The claim follows. Q.E.D.

Proof of Proposition 2.1: Let \( \hat{m} \) be the number of divisions in the symmetric equilibrium. We first show that \( \hat{m} > 1 \). Using (2.12) and (2.13), we obtain, for any \( m_2 \),

\[
\frac{\partial \Pi_1(m_1, m_2)}{\partial m_1} = \hat{p}_1 D_1(\hat{p}_1, \hat{p}_2) \frac{m_1 - 1}{m_1} + \hat{p}_1 D_2(\hat{p}_1, \hat{p}_2) \frac{\partial \hat{p}_2}{\partial m_1}.
\]

Lemma 2.1(c) implies that \( \partial \hat{p}_2/\partial m_1 > 0 \). The claim follows from the following inequality

\[
\frac{\partial \Pi_1(m_1, m_2)}{\partial m_1} \bigg|_{m_1=1} > 0.
\]

Next, we show that \( \hat{m} < m^* \). Suppose to the contrary that \( \hat{m} \geq m^* \). By Lemma 2.1, when \( m_1 = m_2 = m \) the equilibrium aggregate price for each group of the complements is symmetric, which we denote by \( \hat{p}(m) \). Let \( x = (\hat{p}(\hat{m}), \hat{p}(\hat{m})) \). It follows from (2.12) and (2.13) that, at \( m_1 = m_2 = \hat{m} \),

\[
\frac{\partial \Pi_1(m_1, m_2)}{\partial m_1} = \hat{p}(\hat{m}) \left( \frac{\hat{m} - 1}{\hat{m}} D_1(x) \frac{\partial \hat{p}_1}{\partial m_1} + D_2(x) \frac{\partial \hat{p}_2}{\partial m_1} \right),
\]

where \( \partial \hat{p}_k/\partial m_1 \) is the derivative of the equilibrium group price \( \hat{p}_k \) with respect to \( m_1 \) evaluated at \( m_1 = m_2 = \hat{m} \), \( k = 1, 2 \).

By the definition of \( m^* \), \( \hat{p}(m^*) = p_M \) and the following equation

\[
\frac{m - 1}{m} = -\frac{D_2(\hat{p}(m), \hat{p}(m))}{D_1(\hat{p}(m), \hat{p}(m))},
\]

holds at \( m = m^* \). The left-hand side of the above equation strictly increases with \( m \), (A5) implies that \( -D_2(p, p)/D_1(p, p) \) is non-increasing in \( p \), and Lemma 2.1(e) implies that \( \hat{p}(m) \) increases with \( m \). It follows that \( -D_2(\hat{p}(m), \hat{p}(m))/D_1(\hat{p}(m), \hat{p}(m)) \) is non-increasing in \( m \). Thus,

\[
\frac{\hat{m} - 1}{\hat{m}} \geq -\frac{D_2(\hat{p}(\hat{m}), \hat{p}(\hat{m}))}{D_1(\hat{p}(\hat{m}), \hat{p}(\hat{m}))}.
\]

Therefore, at \( m_1 = m_2 = \hat{m} \),

78
\[
\frac{\partial \Pi_1(m_1, m_2)}{\partial m_1} \leq \hat{p}(\hat{m})D_2(x) \left( -\frac{\partial \hat{p}_1}{\partial m_1} + \frac{\partial \hat{p}_2}{\partial m_1} \right) \\
= \frac{\hat{p}(\hat{m})D_2[D_1 + \hat{m}(D_1 + D_2) + \hat{p}(\hat{m})(D_{11} + D_{12})]}{(1 + \hat{m})D_1 + \hat{p}(\hat{m})D_{11}]^2 - [\hat{m}D_2 + \hat{p}(\hat{m})D_{12}]^2} \\
< 0
\]

where the last inequality follows from (A1) and (A3). This contradicts with the necessary condition for \( \hat{m} \) to be the symmetric Nash equilibrium number of divisions. The claim follows. Q.E.D.
5.2 Appendix 2.B: An Integer Divisionalization Game

In this appendix we analyze an integer game in which the numbers of divisions, \( m_1 \) and \( m_2 \), chosen by the firms are restricted to be integers. The questions are whether it is still possible for the firms to achieve the monopoly profits by coordinating their divisionalization strategies and whether there exists a pure strategy Nash equilibrium in the non-cooperative division game.

For simplicity, we consider only the linear demand function (2.2). The reduced-form payoff functions are represented in (2.16). Let \( m^* \) and \( \hat{m} \) be the optimally coordinated number of divisions and Nash equilibrium number of divisions in the continuous division game discussed in Section 2.3, respectively. We argue that for large \( n_1 \) and \( n_2 \) the firms may not in general be able to replicate the maximum joint profit by coordinating their division strategies, but can almost achieve the maximum joint profit. The optimally coordinated number of divisions in the integer game is close to \( m^* \). Moreover, there always exists a pure strategy Nash equilibrium in the integer game and the equilibrium number is close to \( \hat{m} \).

Let \( [m] \) be the integer part of a real number \( m \), i.e., the integer such that \( m - 1 < [m] \leq m \). Consider first the non-cooperative division game. Denote \( [\hat{m}] \) by \( I \). Since the payoff for firm 1, \( V(m_1, m_2) \), is a single-peaked function of \( m_1 \), the best-reply of firm 1 to an integer \( m_2 \) is either \( [R(m_2)] \) or \( [R(m_2)] + 1 \). Moreover, since \( R(m) \) is monotonically increasing and slopes of \( R(m) \) are always less than 1, the best-reply to \( I \) is either \( [R(\hat{m})] \) or \( [R(\hat{m})] + 1 \). Similarly, the best-reply to \( I + 1 \) is either \( [R(\hat{m} + 1)] \) or \( [R(\hat{m} + 1)] + 1 \). It follows that \((I, I)\) is an equilibrium if

\[
V(I, I) \geq V(I + 1, I),
\]

and \((I + 1, I + 1)\) is an equilibrium if

\[
V(I + 1, I + 1) \geq V(I, I + 1).
\]

If neither inequality holds, then both \((I, I + 1)\) and \((I + 1, I)\) are Nash equilibria. Therefore, there exists at least one pure strategy Nash equilibrium that is close to the equilibrium.
### Table 5.1: The Payoff Matrices in the Division Game

<table>
<thead>
<tr>
<th>Firm 1 \ Firm 2</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(9.38, 9.38)</td>
<td>(11.98, 9.07)</td>
<td>(13.5, 8)</td>
<td>(14.49, 7.01)</td>
</tr>
<tr>
<td>2</td>
<td>(9.07, 11.98)</td>
<td>(12, 12)</td>
<td>(13.78, 10.78)</td>
<td>(14.96, 9.56)</td>
</tr>
<tr>
<td>3</td>
<td>(8, 13.5)</td>
<td>(10.78, 13.78)</td>
<td>(12.5, 12.5)</td>
<td>(13.66, 11.16)</td>
</tr>
<tr>
<td>4</td>
<td>(7.01, 14.49)</td>
<td>(9.56, 14.96)</td>
<td>(11.16, 13.66)</td>
<td>(12.24, 12.24)</td>
</tr>
</tbody>
</table>

in the continuous division game.

The coordinated divisionalization works in a similar way. The joint profit is $V(m_1, m_2) + V(m_2, m_1)$, denoted by $\bar{V}(m_1, m_2)$, which is symmetric in $m_1$ and $m_2$. It can be easily shown that, for any $m_2$, $\bar{V}(m_1, m_2)$ is single-peaked in $m_1$. Suppose that the peak of $\bar{V}(m_1, m_2)$ is reached at $m_1 = R(m_2)$ (i.e., the best reply function). Clearly, $m^* = R(m^*)$.

It can be verified that the slopes of $R(m_2)$ is between 0 and $-1$. The single-peakedness of $\bar{V}(m_1, m_2)$ implies that for any $m_2$ the maximum is reached either at $[R(m_2)]$ or at $[R(m_2)] + 1$. Since the slope of $R(m_2)$ is between 0 and $-1$, the symmetry then implies that the maximum of $\bar{V}(m_1, m_2)$ is reached at one of the following pairs: $([m^*], [m^*]), ([m^*], [m^*] + 1), ([m^*] + 1, [m^*])$, or $([m^*] + 1, [m^*] + 1)$.

In the following, we provide an example in which $\alpha = 10$, $\beta = 6$, $\gamma = 4$, and $n_1 = n_2 = 4$. Notice that $m^* = 3$ and $\hat{m} = 1.34$. The payoff matrices in the integer division game are represented in Table 1. The joint profits are maximized at $m_1 = m_2 = 3$. There are two pure strategy equilibria, $(m_1, m_2) = (1, 1)$ and $(m_1, m_2) = (2, 2)$.

---

1It should be noted that, when $\gamma/\beta$ is greater than $8/9$, the joint profit is strictly increasing in $m_1$ for small $m_2$. In this case, $R(m_2)$ is equal to $N_1$ for small $m_2$. 

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Figure 2.1: The Best-Reply Lines in the Price Game (the Linear Demand Functions)
Figure 2.2: The Best-Reply Curves in the Division Game (the Linear Demand Functions)
5.3 Appendix 3

**Proof of Lemma 3.1:** The existence is trivially shown from equation (3.6).

(a) Divide equation (3.6) by $m_k$ and rearrange it to obtain the output of a division in firm $k$:

$$x_{ki} = \frac{a}{b(m_k + 1) + \Delta_k((1 - z)m_k + 1)}.$$

Then,

$$\frac{\partial x_{ki}}{\partial m_k} = -\frac{a}{[b(m_k + 1) + \Delta_k((1 - z)m_k + 1)]^2} (b + \Delta_k(1 - z)) < 0,$$

and

$$\frac{\partial x_{ki}}{\partial m_t} = -\frac{a}{[b(m_k + 1) + \Delta_k((1 - z)m_k + 1)]^2} ((1 - z)m_k + 1) \frac{\partial \Delta_k}{\partial m_t} < 0,$$

where $t \neq k$.

(b) Differentiate equation (3.6) with respect to $m_k$ and $m_t$ to get

$$\frac{\partial x_k}{\partial m_k} = \frac{a(b + \Delta_k)}{[b(m_k + 1) + \Delta_k((1 - z)m_k + 1)]^2} > 0,$$

and

$$\frac{\partial x_k}{\partial m_t} = -\frac{a}{[b(\frac{1}{m_k} + 1) + \Delta_k((1 - z) + \frac{1}{m_k})]2} ((1 - z) + \frac{1}{m_k}) \frac{\partial \Delta_k}{\partial m_t} < 0.$$

**Proof of Proposition 3.1:** Lemma 3.2 shows that numbers of divisions are strategic complements. Thus, the first stage solution must be symmetric. If not, without loss of generality, denote $m_1 \leq m_2 \leq ... \leq m_n$ the solution, where at least one of the inequalities holds strictly. Then, $m_1 < m_n$. By the symmetry of the first order condition, any ordering of $m_t$ is a solution. In particular, consider the ordering $m_n, m_2, m_3, ..., m_{n-1}, m_1$. In the new ordering, the number of divisions of firm 1 increases from $m_1$ to $m_n$ and that of firm 2 to firm $n - 1$ stays the same. By Lemma 3.2, the number of divisions of firm $n$ should increase. However, it decreases from $m_n$ to $m_1$. It is a contradiction. Therefore, the solution for the first stage game is symmetric. The uniqueness is directly implied by equation (3.10).
To show the comparative static results, define $G(m, z, n) = m^2(1 - z)((n - 1)z + 1) - m(n - 2)z - 1 = 0$. Then,

$$
\frac{\partial G}{\partial m} = 2m(1 - z)((n - 1)z + 1) - (n - 2)z = m(1 - z)((n - 1)z + 1) + 1 > 0;
$$

$$
\frac{\partial G}{\partial n} = mz(1 - z)(m - \frac{1}{1 - z});
$$

and

$$
\frac{\partial G}{\partial z} = m[(n - 2)((1 - z)(m - \frac{1}{1 - z}) - mzn].
$$

Recall from section 3.2 that $m < \frac{1}{1 - z}$. Thus, \( \frac{\partial G}{\partial n} < 0 \) and \( \frac{\partial G}{\partial z} < 0 \) for $n \geq 2$. Then,

$$
\frac{\frac{\partial G}{\partial n}}{\frac{\partial G}{\partial m}} = -\frac{\frac{\partial G}{\partial n}}{\frac{\partial G}{\partial m}} > 0,
$$

and

$$
\frac{\frac{\partial G}{\partial z}}{\frac{\partial G}{\partial m}} = -\frac{\frac{\partial G}{\partial z}}{\frac{\partial G}{\partial m}} > 0.
$$

Q.E.D.

**Proof of Proposition 3.3:** Rearrange equation (3.11) as the following:

$$
a^2[b^2 \sqrt{m} + (1 + z(n - 1))\sqrt{m}]^2 = F.
$$

First, totally differentiate the above equation w.r.t. $n$ and $z$ to get

$$
[(n-1)\sqrt{m} + \frac{z(n-1)}{2\sqrt{m}} + \frac{1}{2} m^{-\frac{1}{2}} (m-1)] \frac{\partial G}{\partial z} \Delta z + [z \sqrt{m} + \frac{z(n-1)}{2\sqrt{m}} + \frac{1}{2} m^{-\frac{1}{2}} (m-1)] \frac{\partial G}{\partial n} \Delta n = 0.
$$

Then, \( \frac{\partial G}{\partial z} < 0 \). Similarly, totally differentiate equation (3.11) w.r.t. $n$ and $F$ to get

$$
[\frac{1}{\sqrt{m}} + (1 + z(n - 1))\sqrt{m}] \Delta F + 2F [z \sqrt{m} + \frac{z(n-1)}{2\sqrt{m}} + \frac{1}{2} m^{-\frac{1}{2}} (m-1)] \frac{\partial G}{\partial z} \Delta n = 0.
$$
Then, $\frac{\partial n}{\partial F} < 0$.

Q.E.D.

*Proof of Proposition 3.4:* At $n = 2$, $m = \sqrt{\frac{1}{1-z^2}}$ (from equation (3.10)) and the profit of a duopolist $\pi = \frac{a^2}{b} \frac{\sqrt{1-z^2}}{(1+z+\sqrt{1-z^2})^2}$. From equation (3.11), we have $\frac{\sqrt{1-z^2}}{(1+z^*+\sqrt{1-z^*})^2} = F \frac{b}{a^2}$. Then, the duopolist's profit is no greater than $F$ if $z \leq z^*$. Q.E.D.
Figure 3.1

Figure 1a

Figure 1b
5.4 Appendix 4.1

S.O.C. of the extraction game:

\[
\frac{\partial^2 \pi_l}{\partial N_l^2} = -sN^{-(2+s)} - (2 + s)(N - (1 + s)N_l)N^{-(3+s)} < 0
\]

S.O.C. of the leasing game:

\[
\frac{\partial^2 \pi_m}{\partial L_m^2} = -\frac{(s + 1)Uc^{\frac{1}{s+1}}(2L_m - 1) - \Omega \frac{\partial B}{\partial L_m}}{B} < 0
\]

where \(B = L^{(2s+3)/(s+1)}(L - s - 1)^{(2s+1)/(s+1)} > 0\).

Proof of Proposition 4.2.

Define

\[
F(l, M, \omega, c, s) = (M-2)Ml-(s+1)(M-1)+1-\frac{\omega}{s+1}c^{-\frac{s+1}{s+1}}M^{\frac{s+3}{s+1}}l^{\frac{s+3}{s+1}}[Ml-(s+1)]^{\frac{2s+1}{s+1}} = 0
\]

Differentiate \(F\) w. r. t. its arguments:

\[
F_l = M(M - 2) - \frac{s + 2}{s + 1} + \frac{2s + 1}{s + 1} \frac{M}{Ml - s - 1} = M(M - 2)l - (s + 1)(M - 1) + 1
\]

\[
< M(M - 2)l - (s + 1)(M - 1) + 1
\]

\[
= \frac{2M(M - 2)l - (s + 1)(M - 1) + 1}{l}
\]

\[
< \frac{2M(M - 2)l - (s + 1)(M - 1)}{l}
\]

\[
< \frac{M(2M - 2)l - (s + 1)}{l}
\]

\[
< 0
\]

for \(M > 2\).

\[
F_\omega = -\frac{1}{s+1}c^{-\frac{s+1}{s+1}}M^{\frac{s+3}{s+1}}l^{\frac{s+3}{s+1}}[Ml - (s + 1)]^{\frac{2s+1}{s+1}} < 0
\]

\[
F_c = \omega sc^{-\frac{2s+1}{s+1}}M^{\frac{2s+3}{s+1}}l^{\frac{2s+3}{s+1}}[Ml - (s + 1)]^{\frac{2s+1}{s+1}} > 0
\]

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\[ F_M = \frac{2(M - 1)l - (s + 1) - \frac{2s + 1}{s + 1}}{M} + \frac{2s + 1}{s + 1} \frac{l}{l - (s + 1)}(M(M - 2)l - (s + 1)(M - 1) + 1) \]

\[ < 2(M - 1)l - (s + 1) - \frac{4}{M} (M(M - 2)l - (s + 1)(M - 1) + 1) \]

\[ = -\frac{1}{M} (2M^2l - M(6l + 3(s + 1)) + 4(s + 2)) \]

\[ < -\frac{2l}{M} (\frac{M - 6l + 3(s + 1)}{4l} + 2(s + 2) \frac{1}{l} - \frac{6l + 3(s + 1)}{4l}) \]

\[ < 0 \]

Then,

\[ \frac{\partial l}{\partial \omega} = -\frac{F_M}{F_l} < 0, \text{ for } M > 2. \]

\[ \frac{\partial l}{\partial M} = -\frac{F_M}{F_l} < 0, \text{ for } M > 2. \]

Define

\[ D(L, M, \omega) = (M - 2)L - (s + 1)(M - 1) + 1 - \frac{\omega}{s + 1} e^{-l} M L^{t+2} [L - (s + 1)]^{2t+1} = 0 \]

Differentiating \( D(L, M, \omega) \) w.r.t. \( L, M, \omega \), yields

\[ D_L = M - 2 - \frac{s + 2}{s + 1} L + \frac{2s + 1}{s + 1} \frac{1}{L - (s + 1)}((M - 2)L - (s + 1)(M - 1) + 1) \]

\[ < M - 2 - \frac{3}{L} ((M - 2)L - (s + 1)(M - 1) + 1) \]

\[ = -\frac{1}{L} ((M - 2)(2L - (s + 1)) - s) \]

\[ < -\frac{1}{L} (2L - 2s - 1) \]

\[ < 0 \]

for \( M > 2. \)

\[ D_M = L - (s + 1) - \frac{1}{M} ((M - 2)L - (s + 1)(M - 1) + 1) = \frac{2L + s}{M} > 0 \]

\[ D_{\omega} = -\frac{1}{s + 1} e^{-l} M L^{t+2} [L - (s + 1)]^{2t+1} < 0 \]

Then,

\( a \) \( \frac{\partial l}{\partial \omega} = -\frac{D_{\omega}}{D_L} < 0, \text{ for } M > 2; \)
(b) \( \frac{\partial I}{\partial \omega} = -\frac{F_p}{P_l} < 0 \), for \( M > 2 \);
(c) \( \frac{\partial L}{\partial M} = -\frac{D_M}{D_l} > 0 \), for \( M > 2 \);
(d) \( \frac{\partial L}{\partial M} = -\frac{F_M}{P_l} < 0 \), for \( M > 2 \).

(c) and (d) validate (a) and (c) of Proposition 4.2 respectively.

The equilibrium number of leases \( l \) is one, when \( \omega \geq \omega^* = \frac{(s+1)((M-2)M-(s+1)(M-1)+1)c^{2+1}}{M^{2+1}[M-(s+1)]^{2+1}} \). From (b) above, \( l > 1 \) if \( \omega < \omega^* \). Thus, we have Proposition 4.2 (b). Q.E.D.
5.5 Appendix 4.2

Table 4.1: Leases and Operators Distribution by Fields

<table>
<thead>
<tr>
<th>Oil Field</th>
<th># of Landowners</th>
<th># of Operators</th>
<th># of Leases</th>
<th>Leases/Landowners</th>
<th>Operators/Landowners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Livermore-Wright</td>
<td>2</td>
<td>7</td>
<td>8</td>
<td>4.0</td>
<td>3.5</td>
</tr>
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<td>Lovington</td>
<td>2</td>
<td>19</td>
<td>21</td>
<td>10.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Slaughter</td>
<td>3</td>
<td>18</td>
<td>22</td>
<td>7.3</td>
<td>6.0</td>
</tr>
<tr>
<td>Keystone</td>
<td>3</td>
<td>6</td>
<td>8</td>
<td>2.7</td>
<td>2.0</td>
</tr>
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<td>3</td>
<td>11</td>
<td>13</td>
<td>4.3</td>
<td>3.7</td>
</tr>
<tr>
<td>North Cowden</td>
<td>3</td>
<td>9</td>
<td>11</td>
<td>3.7</td>
<td>3.0</td>
</tr>
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<td>16</td>
<td>17</td>
<td>5.7</td>
<td>5.3</td>
</tr>
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<td>19</td>
<td>6.3</td>
<td>4.7</td>
</tr>
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<td>4</td>
<td>17</td>
<td>27</td>
<td>6.8</td>
<td>4.3</td>
</tr>
<tr>
<td>Ace</td>
<td>4</td>
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<td>10</td>
<td>2.5</td>
<td>1.5</td>
</tr>
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<td>1.8</td>
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<td>5.3</td>
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<td>25</td>
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</table>

Average: 11.5  21.6  35.2  3.8  2.6