Financing Investment with External Funds

by

Nathalie Moyen

B.Sc. (Économie), Université de Moncton
M.A. (Economics), Queen's University at Kingston

A thesis submitted in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

in

the Faculty of Graduate Studies
the Faculty of Commerce and Business Administration

We accept this thesis as conforming
to the required standard

The University of British Columbia
June 1999

Faculty of Business Administration

The University of British Columbia
Vancouver, Canada

Date 28 June 1999

Abstract

This thesis presents various dynamic models of corporate decisions to address two main issues: investment distortions caused by debt financing and cash flow sensitivities.

In the first chapter, four measures of investment distortion are computed. First, the effect of financing frictions is examined. The tax benefit of debt induces firms to increase their debt capacity and to invest beyond the first-best level on average. The cost of this investment distortion outweighs the tax benefit of debt. Second, Myers's (1977) debt overhang problem is examined in a dynamic framework. Debt overhang obtains on average, but not in low technology states. Third, there is no debt overhang problem in all technology states when debt is optimally put in place prior to the investment decision. Finally, the cost of choosing investment after the debt policy is examined. Equity claimants lose value by choosing to invest after their debt is optimally put in place because they do not consider the interaction between their investment choice and the debt financing conditions.

The second chapter explores the impact of financial constraints on firms' cash flow sensitivities. In contrast to Fazzari, Hubbard, and Petersen (1988), cash flow sensitivities are found to be larger, rather than smaller, for unconstrained firms than for constrained firms. Then, why is investment sensitive to cash flow? In the two models examined in the second chapter, the underlying source of investment opportunities is highly correlated with cash flows. Investment may be sensitive to cash flow fluctuations simply because cash flows proxy for investment opportunities. This leaves two important questions. Can this chapter suggest a better measure of investment opportunities than Tobin's Q? Not a single measure for both the unconstrained and constrained firm models. Can this chapter suggest an easily observable measure of financial constraint? Yes: large and volatile dividend-to-income ratios.

Table of Contents

List of Tables

List of Figures

Acknowledgements
Wow! It's been a long road. I am most grateful to my travelling companion, Martin Boileau. After three years of meeting each other, Martin chose to follow me across the country on a SSHRC post-doctoral fellowship for the first two years of my Ph.D. program. He offered me his love and support, most of the closet space, and too many meaty dishes. How could I have done it without you?

Upon arriving in Vancouver, Khang Min Lee and Jim Storey immediately included me in their study group and their lives. They are great friends and challenging colleagues: Khàng Min is perfect (even her computational errors canceled out) and Jim understands the big picture. But without me, who would have reminded them of the various statistical formulas?

I am also lucky to have met Bryan Routledge, Murray Carlson, Lisa Kramer (great to see you alive and well), David Peterson, Patrick Savaria, Mary Kelly, Joshua Slive, Lorenzo Garlappi, and Yinghu Liu in the program.

Rob Heinkel and Burton Hollifield have been wonderful thesis supervisors, always encouraging and critical. I am inspired by your dedication and constructive outlook. I also wish to thank Glen Donaldson, Gerry Garvey, Ron Giammarino, Margaret Slade, Raman Uppal and Tan Wang for their support.

I am grateful to seminar participants at ASU, UBC, CMU, CU Boulder, Columbia, Maryland, McGill, Northwestern, Penn State, Queen's, Toronto, Utah, Waterloo, York, and at the 1998 WFA meetings. I am especially indebted to Rick Green, Charles Himmelberg, Michael Lemmon, Deborah Lucas, Vojislav Maksimovic, and Tony Smith for helpful discussions.

Financial support from the University of British Columbia Graduate Fellowship, the Social Sciences and Humanities Research Council of Canada, and the Fonds pour la Formation de Chercheurs et l'Aide à la Recherche is gratefully acknowledged.

Finally, I thank my family for accepting and encouraging my academic pursuit in a foreign land: Popsy, Momsy, Juju, et Pet, je vous aime beaucoup. For the same reason, I thank the Boileau family.

1 Introduction

The first chapter examines firms' investment decisions in a dynamic stochastic framework with an interest tax deduction benefit and a deadweight default cost of debt financing. It begins by investigating the model's implications regarding the cross-sectional and time-series properties of financial series, including investments, debt issues, revenues, dividends, equity returns, and interest rates. The series simulated from the model show that firms adjust their asset and debt levels to avoid default through time. A firm choosing a higher debt level today might not be able to repay debt claimants tomorrow, unless the firm also invests more today in order to generate higher revenues tomorrow. Investments are thus highly correlated with debt issues, consistent with the observed series.

Given that the model compares well with the data, investment distortions caused by the presence of debt in a firm's capital structure are measured. The firm's investment policy in the benchmark model is compared to policies derived in different economic environments. In turn, the benchmark investment policy is compared with the first-best policy derived from a Modigliani and Miller (1958) framework of no financing friction, with the policy derived from a Myers (1977) framework where the levered firm maximizes its equity value given the capital structure already in place, and with the policy derived from a modified-Myers framework where the debt in place is optimally chosen. Four conclusions obtain from this comparison of investment policies. First, financing frictions induce the firm to increase its debt capacity and to invest beyond the first-best level on average. The presence of financing frictions leads to an important reduction in equity value. Second, Myers's debt overhang problem obtains on average in the dynamic framework. That is, equity claimants maximize their own value by underinvesting in the presence of risky debt already in place. No debt overhang obtains in low technology states because equity claimants overinvest as a bid to avoid default tomorrow. Third, when debt is optimally put is place prior to the choice of investment, no debt overhang occurs: the investment level is first-best. Finally, equity claimants lose significant value by choosing to invest after their debt is in place rather than simultaneously choosing their investment and debt policies. With sequential decisions, equity claimants ignore the effect of investment on the debt financing conditions.

More generally, the first chapter contributes to the literature on investment and financing decisions which began with Modigliani and Miller (1958), who demonstrate that a firm's production decisions are independent of its financial decisions. Their irrelevance result is consistent with Tobin's Q theory in which a firm's investment decision is determined by maximizing its profits, regardless of its other sources and uses of funds. There also exist many studies of a firm's recapitalization decision that take the investment decision as given. Dynamic recapitalization studies include Fischer, Heinkel, and Zechner (1989), Goldstein, Ju, and Leland (1998), Ju (1998), Kane, Marcus, and McDonald (1984, 1985), and Wiggins (1990).

Modigliani and Miller's (1958) irrelevancy result is derived in an economic environment with no financing frictions. Since then, the literature has examined more realistic environments, including frictions such as recapitalization costs, asymmetric information, taxes, and a default cost of debt. With these frictions, the investment decision of firms depends on their financial decisions. Papers discussing the impact of these frictions on the investment decision include Bernanke and Gertler (1989, 1990), Brennan and Schwartz (1978), Calomiris and Hubbard (1990), Dammon and Senbet (1988), Décamps and Faure-Grimaud (1997), Dotan and Ravid (1985), Faig and Shum (1999), Froot, Scharfstein, and Stein (1993), Leland (1994), Leland and Toft (1996), Mayer (1986), Mella-Barral and Perraudin (1997), and Myers and Majluf (1984) among others. These papers are developed within a static framework, do not allow for changes in the debt level through time, or do not solve for the endogenous claims prices. In contrast, this chapter presents a model of investment in the presence of the traditional debt financing frictions – a tax benefit and a default cost – that is dynamic, allows for recapitalizations through time, and imposes consistent pricing.

Brennan and Schwartz (1984), Jensen and Meckling (1976), Leland (1998), Mauer and Triantis (1994), Mello and Parsons (1992), Myers (1977), and Parrino and Weisbach (1997) focus their attention on the distortionary effects of debt on firms' real decisions. The first chapter's main contributions to this literature are two-fold. First, the model extends previous dynamic studies by characterizing the investment scale decision. Leland (1998) concludes his presidential address by stating: "Dividend (payout) policies and investment scale are treated as exogenous. [...] Relaxing these assumptions remains a major challenge for future research." The model characterizes optimal dividend policies and optimal investment scale policies through time. More specifically, the model characterizes firms' investment decisions as they interact with debt financing decisions through the probability of default. The investment distortion costs thereby obtained can be compared to the operating distortion costs documented in the literature. Second, the investment distortion is quantified throughout various underlying technology states. For example, as discussed below, Myers's (1977) debt overhang problem obtains on average, but not in low technology states.

Myers (1977) illustrates the debt overhang problem according to which equity claimants invest less than the total firm value-maximizing level with risky debt in place. Equity claimants forgo positive net present value projects, because they maximize their own levered value rather than the total firm value. This chapter shows that debt overhang occurs on average, but not in low technology states and not when equity claimants optimally choose their debt in place.

Brennan and Schwartz (1984) were the first to examine the interaction of firms' investment and financing decisions in a dynamic framework. They develop a model of firm valuation in the presence of bond indenture provisions that disallow asset sales, debt levels greater than the asset base value, and debt levels violating a specified interest coverage test.

Mauer and Triantis (1994) and Mello and Parsons (1992) examine the operating distortion cost of debt in the presence of a tax benefit and a default cost. In both papers, the operating policy is a binary function that depends on an underlying price process related to the firm's cash flows. Mello and Parsons take the firm's capital structure as given. As such, they quantify Myers's (1977) debt overhang problem in a dynamic framework. The debt overhang is measured as the difference between the firm value with the first-best operating policy and the firm value with the operating policy that maximizes the levered equity value only. They find that this agency cost is significant. The results obtained in this chapter are consistent with a large debt overhang cost. Conversely, Mauer and Triantis allow for costly debt recapitalizations through time. The firm decides to produce or not and how much debt to carry at each point in time. They find that changes in the recapitalization cost impact the debt level, but has very little effect on the operating policy. That is, the investment distortion cost of debt frictions is not significant. Without a real option-pricing framework where there is value of waiting to invest, the investment distortion due to dèbt financing frictions is important.

Jensen and Meckling (1976) discuss the asset substitution problem according to which equity claimants invest in more risky projects when debt is already in place, thereby expropriating value from debt claimants. Obviously, the asset substitution problem and the debt overhang problem are closely related. The asset substitution problem refers to the variance distortion, while the debt overhang problem refers to the mean distortion. Both trigger agency costs because equity claimants choose an investment policy that maximizes the equity value only, once debt is already in place. Leland (1998) allows the firm to change both its risk strategy (low or high) and its debt structure through time. The distortion cost is measured by the difference between the firm value when both the risk strategy and debt structure is chosen simultaneously and the firm value when the risk strategy is chosen after debt is optimally put in place. Leland finds that the difference in firm values is very small.

Parrino and Weisbach (1997) conduct Monte Carlo experiments to quantify the magnitude of both agency problems: debt overhang and asset substitution. They quantify the wealth transfer from equity claimants to debt claimants arising from the adoption of low-risk positive-net-present-value projects, and the converse transfer arising from the adoption of high-risk negative-net-present-value projects. The Monte Carlo experiments suggest that these agency costs are unlikely to be important. In contrast to the previous papers, Parrino and Weisbach's investment and debt decisions are not obtained in a value-maximizing framework, but are described by given rules. Proxying agency costs with such rules may be misleading because the firm is not allowed to behave optimally.

The second chapter is motivated by the empirical findings of Fazzari, Hubbard, and Petersen (FHP, 1988). FHP present evidence on the investment behavior of U.S. manufacturing firms during the 1970-1984 period. They test the financing hierarchy hypothesis according to which equity and debt markets charge an information premium to certain firms with hard-to-evaluate investment opportunities. Firms facing such information problems prefer to finance their investments with retained earnings. Investments of these constrained firms (identified a priori as firms with low dividend-to-income ratios) should be explained by their cash flows, while investments of less constrained firms (identified a priori as firms with high dividend-to-income ratios) should be less sensitive to their cash flows. The empirical evidence is consistent with this hypothesis: investments of low-dividend firms are more sensitive to cash flow variations than investments of high-dividend firms.

FHP's results initiated an important and heated debate. On the empirical front, Kaplan and Zingales (KZ, 1997) took a different look at the subset of firms identified as most-financially-constrained by FHP. KZ consider various quantitative indicators of financial constraint and supplement this information with manager's statements about the firm's liquidity to build a new classification of financial constraint. KZ classify firms as constrained if they are constrained from investing more while FHP view firms as constrained if they are constrained from obtaining external funds to finance their investment. KZ find that, in 85 percent of firm-years, FHP's most-constrained firms were actually not constrained from investing more. This suggests that the dividend-to-income ratio may not proxy well for investment-constrained firms. Moreover, KZ show that, according to their classification of investment-constraints, most-constrained firms have lower cash flow sensitivities than least-constrained firms. This result contrasts with FHP's evidence that most-constrained firms exhibit higher sensitivities than least-constrained firms.

The second chapter examines two models to assess the impact of financial constraints on firms' cash flow sensitivities. Constrained firms are modeled as firms without access to external markets while unconstrained firms are modeled as firms that can choose their optimal amount of external financing in the presence of tax and default frictions. In contrast to FHP, cash flow sensitivities are larger, rather than smaller, for unconstrained firms than for constrained firms.

More importantly, the underlying source of investment opportunities in the two models is found to be highly correlated with cash flows. This suggests that investment may be sensitive to cash flow fluctuations simply because cash flows proxy for investment opportunities. Unfortunately, the model does not suggest a single measure of investment opportunities for both unconstrained and constrained firms because the marginal product of capital in these two models are too different.

This second chapter also suggests that FHP's identification of greater financial constraint with low dividend-to-income ratios may be misleading. Constrained firms are found to have higher dividend-to-income ratios than unconstrained firms. Indeed, larger and more volatile dividend-to-income ratios proxy for a greater degree of financial constraint. Firms with no financial flexibility cannot smooth dividends but promise larger dividends to compensate equity claimants for the default risk they face.

Hoshi, Kashyap, and Scharfstein (1991) provide empirical evidence in support of FHP. They divide Japanese firms into two groups using the natural identification of financial constraint provided by the *keiretsu* institution. A firm who belongs to a *keiretsu* has close ties to a main bank. This main bank is likely to be well informed about the firm and is likely to be the primary lender of funds to the firm. Firms are identified as less (more) constrained if they (do not) belong to a *keiretsu*. Hoshi, Kashyap, and Scharfstein find that constrained firms have investment policies that are more sensitive to cash flow fluctuations than unconstrained firms.

FHP's empirical findings have generated interest in conglomerates. Lamont (1997), Rajan, Servaes, and Zingales (RSZ, 1998), Scharfstein (1997), Shin and Park (1998), and Shin and Stulz (1998) examine the relation between internal funds transfers across divisions of diversified firms and their respective investment policies. All papers but RSZ use an empirical specification similar to FHP and find that a division's investment policy is sensitive to the cash flow fluctuations of another but not sensitive to its own investment opportunities.

A number of structural estimations have been performed to test the presence of a borrowing constraint. Bond and Meghir (1994), Hubbard and Kashyap (1992), Hubbard, Kashyap, and Whited (1995), and Whited (1992) describe the investment decision using a model of profit-maximizing firms under the null hypothesis of no borrowing constraint and under the alternative of an exogenous borrowing constraint. They find that the former is consistent with data for unconstrained firms while the latter fits the data for constrained firms.

Rather than taking as given the presence of a given borrowing limit, Gross (1995) and Pratap and Rendón (1998) model the investment decision under an endogenous financing constraint defined by the possibility of liquidation if the firm's cash flow falls to zero at any point in time. Both studies find that these cash flows are dynamically managed to avoid liquidation. In that sense, all firms behave as if they are constrained in their investment decisions, with constraint-binding firms being more sensitive to cash flow variations. Gross and Pratap and Rendón identify firms as constrained if the cash flow constraint is binding. This is similar to KZ's identification of investment-constrained firms when firms are constrained from investing more. However, the theoretical results of Gross and Pratap and Rendón contrast with KZ's empirical results that most-constrained firms are less sensitive to cash flow fluctuations. The chapter investigates whether Gross's and Pratap and Rendón's default definition is to blame. Their default definition implies that a highly valuable firm with a low cash flow in a particular year must default: it cannot sell assets, issue equity, or raise new debt. In this chapter, default is defined in reference to the firm value rather than the current cash flow. In spite of this value-based default point, the results obtained are similar to Gross and Pratap and Rendón, inconsistent with KZ.

Gilchrist and Himmelberg (1995) and Cummins, Hasset, and Oliner (1997) empirically investigate the possibility that Tobin's Q mismeasures investment opportunities such that cash flow predicts investment only because it contains valuable information about investment opportunities. Gilchrist and Himmelberg construct an alternative measure of Tobin's Q based on Abel and Blanchard (1986) and they find that the cash flow sensitivity survives this alternative measure of Tobin's Q. Hence, mismeasurement of investment opportunities by Tobin's Q does not seem to explain the cash flow sensitivity. On the other hand, Cummins, Hasset, and Oliner use analysts' earnings forecasts as a measure of a firm's opportunities. They find no cash flow sensitivity, suggesting, in contrast to Gilchrist and Himmelberg, that mismeasurement of investment opportunities by Tobin's Q explains the cash flow sensitivity. Gomes (1998) builds an investment model with exogenous financing costs where profit-maximizing firms also choose whether to exit at any point in time. Gomes shows that the real cash flow variable does not improve the fit of the investment regression when Tobin's Q is measured without error. The real cash flow variable only increases the fit of the investment regression when measurement error is introduced to Tobin's Q. Gomes lends support to Cummins, Hasset, and Oliner.

The first chapter is organized as follows. The next section describes the model. Section 2.2 describes the financial series simulated from the model. Section 2.3 measures the amount and cost of investment distortions caused by debt. Section 2.4 concludes the chapter on investment distortions. The second chapter begins in Section 3.1 by describing the unconstrained firm model. Section 3.2 describes the constrained firm model. Section 3.3 presents the cash flow sensitivity results. Section 3.4 concludes the chapter on cash flow sensitivities.

2 Investment Distortions Caused by Debt Financing

This chapter examines firms' investment decisions in a dynamic stochastic framework with an interest tax deduction benefit and a deadweight default cost of debt financing.

2.1 The Model

Risk neutral claimants price the firm's equity according to

$$p = \beta E\left[(p' + D')1_{(V_u' \geq 0)}\right], \qquad (1)$$

where p is the ex-dividend equity price, β is the discount factor, D is the dividend, primed variables refer to tomorrow's beginning-of-the-period values, and $1_{(\cdot)}$ is the no-default state (to be defined below). Equation (1) shows that today's equity price equals tomorrow's expected discounted payoff. The equity payoff consists of the price and dividend if the firm does not default.

Substituting for the unlimited liability equity value, defined as

$$V_u = p + D, \tag{2}$$

equation (1) becomes

$$V_u = D + \beta E\left[V_u' 1_{(V_u' \geq 0)}\right],$$

where V_u is the unlimited liability equity value and the no-default indicator function is

$$1_{(V_u \geq 0)} = \begin{cases} 1 & \text{if } V_u \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

If no default occurs tomorrow, equity claims are valued at V_u'. Otherwise, equity claimants are protected from debt claimants by limited liability. Thus, default is defined to occur tomorrow when the equity value $V_u' 1_{(V_u' \geq 0)}$ is nil, i.e., when the equity value with unlimited liability V_u' is less than zero. Clearly, by maximizing the unlimited liability equity value V_u, the firm also maximizes the limited liability equity value $V_u 1_{(V_u \geq 0)}$.

The dividend is defined by the firm's sources and uses of funds equation

$$D = (1 - \tau_f) f(K; \theta) - I + \tau_f \delta K + B' - (1 + (1 - \tau_f)\iota)B, \tag{4}$$

where τ_f is the firm's tax rate, K is the asset base, θ is the technology state describing the underlying economic conditions, $(1 - \tau_f) f(K; \theta)$ is the after-tax operating income before depreciation, I is the investment, δ is the depreciation rate, $\tau_f \delta K$ is the capital cost allowance, B' is the new debt level, ι is the interest rate, and $(1 + (1 - \tau_f)\iota)B$ is the principal and tax-deductible interest payments.[1]

Although the debt B is modeled with a one-period maturity, the firm can decide at each time period to roll it over $\Delta B' = B' - B = 0$, to make a new issue $\Delta B' > 0$, or to retire a portion of its debt outstanding $\Delta B' < 0$. The one-period maturity debt can thus be viewed as an infinite maturity debt with a floating rate.

[1] For simplicity, the capital cost allowance rate is assumed equal to the true economic depreciation rate of the asset base.

The firm's operating income before depreciation is the difference between its revenues and expenses

$$f(K;\theta) = \theta K^{\alpha} - F, \tag{5}$$

where the Cobb-Douglas parameter $\alpha \in (0,1)$ specifies decreasing returns to scale and F is a fixed cost representing labor and other expenses.[2] $F = 9.5$ is chosen such that the mean of the debt-to-asset ratio B/K series generated from the model (0.4126) approximates the mean in the data (0.4031).

The asset base is subject to depreciation and takes time to build. It evolves according to the accumulation equation

$$K' = (1-\delta)K + I. \tag{6}$$

The technology state is represented by the following first-order autoregressive process:

$$\ln\theta' = \ln A + \rho \ln\theta + \sigma\epsilon', \tag{7}$$

where A is a constant and $\epsilon \sim iid\ N(0,1)$. The persistence ρ of the technology shock provides an exogenous source of dynamics.

The firm chooses how much dividend D to pay, how much to invest K', and how much debt to issue B' at which interest rate ι'. The firm makes these decisions after observing the beginning-of-the-period value for the technology state θ and last period's choices of asset base K, debt B, and interest rate ι. The following summarizes the timing of these decisions:

the firm observes θ	the firm observes θ'
given K, B, ι	given K', B', ι'
it chooses D, K', B', ι'	it chooses D', K'', B'', ι''

When making its dividend D, asset K', and debt financing (B', ι') decisions, the firm takes into account the pricing schedule at which the debt can be financed. Risk neutral debt claimants require an interest rate ι' such that the debt is fairly priced according to

$$\beta E\left[(1 + (1-\tau_{\iota})\iota')\, 1_{(V_u' \geq 0)} + \left(\frac{V_u'(K',0,0;\theta')}{B'} - X\right)(1 - 1_{(V_u' \geq 0)})\right] = 1, \tag{8}$$

[2] The firm's labor demand decision is not modeled.

where τ_ι is the debt claimants' interest income tax rate and X is the deadweight default cost as a proportion of the debt face value. Equation (8) shows that debt claimants require an interest rate such that one unit of debt lent to the firm today equals tomorrow's expected discounted payoff. The payoff on the debt claim consists of the face value and the after-tax interest payment if the firm does not default, or the net residual value if the firm defaults.

Default triggers an immediate reorganization process.[3] The residual accruing to debt claimants upon default is the reorganized value of the firm $V_u(K, 0, 0; \theta)$: the equity value with assets K, no debt, no interest, and a technology state θ. Debt claimants may then recapitalize the firm in an optimal manner. In fact, $V_u(K, 0, 0; \theta)$ takes into account the optimal recapitalization from that unlevered state.[4]

The firm does not choose whether to default or not. Although the firm positions itself to minimize the possibility of default tomorrow, default could nevertheless happen as a result of today's decisions K', B', and ι' when tomorrow's technology state θ' turns out to be much lower than expected.

Equations (4), (5), (6), and (8) are the only constraints facing the firm. The logarithmic technology process restricts revenues θK^α to be positive given that $A > 0$. The firm experiences operating losses before depreciation when expenses F exceed revenues θK^α. When net losses occur, the dividend is increased by a tax subsidy, $-\tau_f(f(K;\theta) - \delta K - \iota B) > 0$.[5] Dividends D are not restricted to be non-negative. Negative dividends are interpreted as rights offers. Equity claimants find it worthwhile to exercise these rights, otherwise default is triggered. In fact, the firm optimizes with respect to the dividend policy. The firm decides on the amount of dividends or rights issues that is optimal. In addition to dividends, investments I and debt

[3]This model does not distinguish between an informal reorganization process and a formal reorganization process through the bankruptcy court. It only specifies that reorganization is costly with a deadweight cost X and a one-period forgone tax benefit due to the reorganization $(\tau_f - \tau_\iota)\iota'$.

[4]By definition, the residual accruing to debt claimants upon default (when $V_u < 0$) is always less than the principal and after-tax interest income

$$\begin{aligned} V_u(K, B, \iota; \theta) = \quad & V_u(K, 0, 0; \theta) - \quad (1 + (1 - \tau_f)\iota)B < 0 \\ & \tfrac{V_u(K,0,0;\theta)}{B} \quad < (1 + (1 - \tau_f)\iota) < (1 + (1 - \tau_\iota)\iota) \end{aligned}$$

because the corporate tax rate τ_f is higher than the debt claimants' interest income tax rate τ_ι.

[5]Tax asymmetries such as limited carryback and carryforward provisions are not addressed.

issues $\Delta B'$ are not restricted to be non-negative. The firm is allowed to sell its assets and to retire its debt.

The Bellman equation describing the firm's intertemporal problem is

$$V_u(K, B, \iota; \theta) = \max_{\{D, K', B', \iota'\}} D + \beta E\left[V_u(K', B', \iota'; \theta')1_{(V_u' \geq 0)}\right]$$

subject to equations (4), (5), (6), and (8). The asset, debt, and coupon decisions of the firm are characterized by the following equations:

$$\beta E\left[\left\{(1-\tau_f)\theta'\alpha K'^{\alpha-1} + (1-(1-\tau_f)\delta)\right\}1_{(V_u' \geq 0)}\right] + \lambda v_{K'} = 1, \quad (9)$$

$$\beta E\left[(1+(1-\tau_f)\iota')1_{(V_u' \geq 0)}\right] = 1 - \lambda v_{B'}, \quad (10)$$

and

$$E\left[(1-\tau_f)B'1_{(V_u' \geq 0)}\right] = -\lambda v_{\iota'}, \quad (11)$$

where λ is the multiplier on the fair-bond-pricing equation (8), and v_K', v_B', and v_ι' represent marginal effects of the firm's decisions on the fair-bond-pricing equation (8) characterized in the appendix.

Equation (9) states that the firm invests up to the point where the cost of one unit of asset today equals tomorrow's expected discounted marginal contribution to dividends plus the benefits associated with better financing conditions. The marginal contribution to dividends consists of the asset resale price and the marginal after-tax income. The firm acts on behalf of current equity claimants by valuing tomorrow's contribution to dividends only in the no-default state. Equation (10) states that the firm issues debt up to the point where one unit of debt contributing to today's dividends net of the costs of deteriorated financing conditions equals the expected discounted face value and after-tax interest burden on tomorrow's dividends if the firm does not default. Equation (11) is used to determine the shadow value of claimants' debt holdings λ.

The tax and default frictions insure an interior solution for the debt level B' chosen by the firm. The tax benefit arises because the interest payments are deductible to the firm at a higher rate than the interest income is taxable to the debt claimant $\tau_f > \tau_\iota$. One unit of debt today is expected to generate $(\tau_f - \tau_\iota)\iota'$ funds if the firm does not default tomorrow. That unit of debt today is also expected to cost X funds if the firm defaults tomorrow.

2.2 Description of Simulated Series

The appendix details how the model is calibrated and solved. The resulting policy series K', B', p, and ι' are simulated from random outcomes of technology shocks ϵ. 1603 different series of 100 technology shocks are used to match the Compustat sample described in the appendix. Each series represents a simulated firm to match the Compustat sample size of 1603 firms. Only the last 20 shocks are kept to match the Compustat sample length of 20 years. From these policy series, investment I, new debt issues $\Delta B'$, revenues θK^{α}, dividend D_+ (where the $_+$ indicates that the dividend series does not include rights issues), and equity rate of return r are computed.

This section examines the ability of the model to describe the investment and debt choices observed in the data. Statistics describing the Compustat sample are compared to simulated statistics generated from the model. The Compustat data definitions for the investment, new debt issues, revenues, dividend, equity rate of return, and interest rate are provided in the appendix. Descriptive statistics on these Compustat series are presented in Table 2. First and second moments are computed for each of these 1603 firms and the resulting moments are averaged to represent the typical manufacturing firm. The promised interest rate ι' averages 0.1464, reflecting the riskiness of corporate claims. In fact, the 1603 firms in the sample survive for an average life of 13.9501 years. Equity rates of return r also reflect this riskiness with a mean rate of 0.2229.

The typical Compustat manufacturing firm invests nearly \$60 million per year and generates \$883 million in revenues each year. New debt issues represent less than \$8 million per year, but issues are very volatile with a standard deviation of \$45 million. Investments are positively correlated with both sources of funds, internal revenues θK^{α} and external new debt issues $\Delta B'$, with coefficients of 0.4320 and 0.2723 respectively. In contrast to new debt issues, dividends D are not very volatile with a standard deviation of \$8 million from a mean of \$18 million. Despite this evidence of smoothed dividends, dividends are highly positively correlated with revenues, with a coefficient of 0.4805. Because dividends and revenues move together through time, either of these variables may be used to proxy for economic conditions, i.e., the technology state.

First and second moments of the simulated series are computed for each of these 1603 simulated firms. These moments are averaged to represent the typical theoretical firm and reported in Table 3. The main difference between Tables 2 and 3 is that the theoretical firm never defaults. The promised coupon rate is equal to the riskfree rate $\iota' = \frac{1-\beta}{\beta(1-\tau_i)} = 0.0658$. Equity claimants are able to contract with debt claimants at the riskfree rate, thereby obtaining the lowest cost of debt financing and avoiding the default costs.[6] In turn, equity claims generate a lower mean rate of return (0.1688) than in the Compustat sample (0.2229).

The possibility of default plays an important role in the model. The threat of default defines the firm's optimal decisions. Decisions are made such that default is avoided in all states. Ex post, the possibility of default is always minimized. Note that default is avoided in all of the discretized states θ. If the domain of the state space was not discretized (this alternative is not numerically feasible), default would occur in the rare tail events and the interest rate would be slightly above the riskfree rate. Other qualitative results would remain unchanged.

The typical theoretical firm invests (4.0809) much more than it issues debt (0.0420), yet the standard deviation of debt issues 8.8701 is much greater than that of the investment 1.8952, as observed in the data. Investments are highly correlated with both sources of funds, internal revenues θK^α and external new debt issues $\Delta B'$, showing coefficients of 0.3245 and 0.9955. The typical theoretical firm generates 15.5025 in revenues each period and pays out 1.1353 in dividends. The two series are highly correlated, with a coefficient of 0.7929. Like in the data, dividends and revenues may be used to proxy for the technology state. Dividends are more volatile than in the data, with a standard deviation of 1.6249, because the risk neutral claimant does not care about smooth payouts.

Table 3 reveals that the operating income before depreciation is sometimes negative $f(K;\theta) < 0$. Revenues θK^α become lower than expenses F when revenues are more than 1.5 standard deviations away from their mean. Economic distress $(1-\tau_f)f(K;\theta) + \tau_f \delta K - I < 0$ occurs in the absence of any financial distress.

The highest correlation coefficient in Table 3 involves two variables chosen by the firm: investment and debt issues. A firm choosing a higher debt level today might not be able to repay debt claimants tomorrow, unless the firm also invests more today in order to generate higher revenues tomorrow. Investments covary with debt issues to avoid any possibility of default, leaving the interest rate required by debt claimants at a minimum. In reality,

[6] Mauer and Triantis (1994) also obtain riskless debt at the optimum. In this chapter, the firm does not default even without a deadweight default cost X, because the firm would otherwise lose the tax benefit for one period due to the reorganization.

investment and debt issuing decisions may not perfectly adjust to eliminate any possibility of default. Nevertheless, the observed correlation between investment and debt issues is positive, like the correlation obtained from the model.

Tables 2 and 3 also show that the correlation between the internal θK^{α} and external $\Delta B'$ sources of funds is positive both in the data and in the model. This indicates that firms seek out financing on the external debt market when their internal funds are larger. In other words, variations in external funds exacerbate rather than offset variations in internal funds.[7]

In sum, descriptive statistics of the simulated series suggest that firms fully adjust their asset and debt levels to eliminate any possibility of default. The resulting theoretical moments compare well with those from the Compustat sample.

Figure 1 graphs the policy functions K', B', and p. Because of the persistence ρ, firms experiencing low technology states θ today expect low states tomorrow and thus a low marginal productivity of their asset base. Firms invest only small amounts K' and carry very little debt B'. As the technology state increases, the marginal productivity of the asset base also improves. Firms invest greater amounts and this investment is financed by higher debt levels. Technology state improvements generate larger dividends, as valued into the equity price p.

The only source of dynamics in the model is through the technology state θ. With no technology persistence $\rho = 0$, $\log\theta \sim iid\ N(0, \sigma^2)$. The dynamic model reduces to a sequence of static decisions. In this case, the investment decision is constant through time and consists of replacing the depreciated asset base each period $I/K = \delta$. Debt levels B', equity prices p, and interest rates ι' are also constant. In contrast to the data, the model with no persistence does not generate any correlation between investment I and debt issuing $\Delta B'$ decisions.

Given policy functions K', B', and p, Figure 2 characterizes the minimum beginning-of-the-period funds $CF + K - B$ that the firm must have to avoid default today. No default occurs if

$$V_u = p + D \geq 0$$

$$p + CF + K - B - K' + B' \geq 0$$

$$CF + K - B \geq K' - B' - p,$$

[7]This fact was first noted by Fazzari, Hubbard, and Petersen (1988).

where cash flows $CF = (1-\tau_f)(f(K;\theta) - \delta K - \iota B)$. Firms with sufficiently high cash flows CF, high asset levels K, or low debt levels B do not default today. Figure 2 indicates that firms do not require as much beginning-of-the-period funds as the technology state improves.

Figure 3 shows the robustness of the benchmark results to different calibrations. Because the literature offers no guidance regarding the calibration of the revenues α and technology state A, ρ, σ parameters, the effect of different values than those estimated in this study is investigated. The impact of different values for financing frictions τ_f, τ_ι, X is also examined. As summarized in Figure 3, the qualitative results of the benchmark calibration are robust to the various calibrations.

A larger sensitivity of revenues to asset variations α increases the marginal productivity of the asset base, irrespective of the technology state. As the marginal productivity increases, the firm invests more and finances this greater investment with a larger debt capacity. Similarly, a larger technology state level A also increases the marginal productivity of the asset base, and thus the asset base and the debt level.

A larger technological persistence ρ means that the technology state facing the firm today is more likely to persist tomorrow. Hence, a firm facing a low technology state today expects a low marginal productivity of its asset base tomorrow. It invests less and decreases its debt level. Conversely, a firm facing a high technology state today expects a high marginal productivity tomorrow, invests more, and increases its debt level. Without technological persistence $\rho = 0$, the firm's policy functions are flat. As the persistence increases, the slopes of the investment and debt issuing policy functions become steeper. The technological volatility σ has the opposite effect. A larger volatility means that the technology state facing the firm today is less likely to predict tomorrow's technology state. A firm facing a low technology state today is less likely to face a low marginal productivity of its asset base tomorrow. It invests more and increases its debt level. Conversely, a firm facing a high technology state today is less likely to face a high marginal productivity of its asset base tomorrow. It invests and borrows less.

An increase in the interest income tax rate τ_ι decreases the marginal tax benefit of debt $(\tau_f - \tau_\iota)\iota'$. The firm chooses a lower debt level. This decrease in funds leads to a lower investment. An increase in the corporate tax rate τ_f has two conflicting effects. On one hand, it decreases the marginal productivity of the asset base, implying a lower asset (and debt) level. On the other hand, it increases the marginal tax benefit, implying a higher debt (and asset) level. The net effect is to increase the debt level, leaving the asset base virtually unchanged. Finally, an increase in the deadweight cost of defaulting X reduces the debt level and thus the asset base.

2.3 Investment Distortion

2.3.1 Distortion Caused by Financing Frictions

Modigliani and Miller (1958) show that a firm's investment decision is independent of its financing policy with frictionless markets. In other words, the presence of debt in a firm's capital structure does not distort the investment decision away from its first-best level. In reality, the U.S. tax code favors the use of debt financing by allowing firms to deduct their interest payments at a higher rate than the tax rate faced by debt claimants on the interest income they receive. There is also evidence of legal and other costs paid by distressed firms. The tax benefit and default cost not only define a firm's optimal debt policy, but they also distort its investment decision. Indeed, it is the presence of debt financing frictions, such as the tax benefit $(\tau_f - \tau_\iota)\iota B$ and the default cost XB, that links the investment decision to the debt financing decision. The impact of these financing frictions is examined by contrasting a firm's investment decisions in a world with and without financing frictions.

With no debt financing frictions, $\tau_f = \tau_\iota = \tau = 0.4$ and $X = 0$, the firm's intertemporal problem simplifies to

$$V_u(K, B, \iota; \theta) = \max_{\{D, K', B', \iota'\}} D + \beta E\left[V_u(K', B', \iota'; \theta')1_{(V_u' \geq 0)}\right]$$

subject to

$$D = (1-\tau)(\theta K^\alpha - F) + (1 - (1-\tau)\delta)K - K'$$

and

$$\beta E\left[(1 + (1-\tau)\iota')\, 1_{(V_u' \geq 0)} + \frac{V_u'(K', 0, 0; \theta')}{B'}(1 - 1_{(V_u' \geq 0)})\right] = 1.$$

As expected, the debt financing decision is indeterminate. The fair-bond-pricing equation above is obtained as a result of the firm's debt and coupon decisions. Thus, there is only one equation to identify two debt financing variables. Any combination of B' and ι' that satisfies the fair-bond-pricing equation represents a possible solution.

The investment chosen is now the first-best level

$$\beta E[(1-\tau)\theta'\alpha K'^{\alpha-1} + (1-(1-\tau)\delta)] = 1 \tag{12}$$

or

$$K' = \left\{ \frac{\beta(1-\tau)\alpha E[\theta']}{1-\beta(1-(1-\tau)\delta)} \right\}^{\frac{1}{1-\alpha}}.$$

Table 4 and Figure 4 document the amount $I - I_{mm}$ and value $V - V_{mm}$ of the investment distortion caused by the presence of a tax benefit and a default cost of debt. mm denotes the Modigliani and Miller (1958) framework without financing frictions described above, while variables without subscripts refer to the benchmark model with financing frictions as described in Section 2.1. Table 4 (up to Table 7) reports the investment distortion amount and value, computed as the average over the 1603 simulated firms and the 20 periods, while Figure 4 (up to Figure 7) displays the investment distortion amount and value, computed as the average over firms and periods around each discretized technology state.

Table 4 shows that the mean of $I - I_{mm}$ is equal to 0.7966, representing a large 13.61 percent of the first-best firm value V_{mm}. The tax benefit provides additional funds to the firm to overinvest on average. Because of the presence of a tax benefit, firms are induced to borrow more. This higher debt level today necessitates more investment today to avoid any possibility of default tomorrow. Figure 4 shows that, in low technology states, the firm actually underinvests to avoid the possibility of default. In such low states, the firm avoids defaulting tomorrow by carrying very little debt. As a result, the firm does not invest much. As the technology state improves, the firm levers up and increases its investment beyond the first-best level.

According to Table 4, the additional value provided by the tax benefit is outweighed by the cost of the suboptimal investment by an average of $V - V_{mm} = -8.4701$ or 14.88 percent of the first-best firm value V_{mm}. That is to say, equity claimants do not benefit from financing frictions. As displayed in Figure 4, the discrepancy of equity values worsens as the technology state improves. The firm takes on more debt, that finances more investment over and above the first-best level, resulting in a lower value accruing to equity claimants.

Table 4 and Figure 4 indicate that the amount and cost of the investment distortion caused by financing frictions are important. On average, the firm overinvests as a result of the tax benefit of debt financing thereby reducing their equity value. However, the firm actually underinvests in low technology states to avoid default despite the tax benefit of debt. Equity claimants lose value from the existence of financing frictions because the investment distortion cost outweighs the tax benefit of debt.

2.3.2 Debt Overhang

With debt, conflicts between equity and debt claimants may arise when a levered firm acts in the interest of equity claimants only. Myers (1977) discusses the debt overhang problem according to which a levered firm chooses an investment policy that maximizes the value of its equity claims rather than the total firm value. Myers shows that the firm underinvests due to the presence of debt in its capital structure. The impact of the debt overhang problem is examined by extending Myers static framework to include dynamic investment decisions when debt is already in place. The debt overhang problem is measured by contrasting the resulting investment decision I_m with the firm's first-best investment level I_{mm}.

The firm's problem is to choose its dividend D and investment K' policies to maximize the value of its equity, given an arbitrary and constant debt structure in place (B, ι). The Bellman equation describing the intertemporal investment problem is

$$V_u(K;\theta) = \max_{\{D,K'\}} D + \beta E[V_u(K';\theta')1_{(V_u' \geq 0)}]$$

subject to

$$D = (1-\tau_f)(\theta K^{\alpha} - F) + (1-(1-\tau_f)\delta)K - K' - (1-\tau_f)\iota B.$$

Given the arbitrary capital structure in place, the firm invests to maximize the equity value without considering the fair-bond-pricing equation. That is precisely the nature of the conflict between equity and debt claimants: equity claimants ignore the effect of their investment decision on the debt pricing equation. The investment decision is characterized by

$$\beta E[\{(1-\tau_f)\theta'\alpha K'^{\alpha-1} + (1-(1-\tau_f)\delta)\}1_{(V_u' \geq 0)}] = 1.$$

Myers (1977) takes the debt financing decisions as given. Similarly, interest payments are now considered a fixed cost, aggregating with the fixed cost of labor and other expenses F. Following the debt-to-asset calibration of the next model where the investment is chosen after the debt, $F + \iota B$ is set to 9.2, where $\iota = 0.0658$ and $B = 17.9083$ are the mean simulated interest rate and debt level from Section 2.1. A fixed labor cost of $F = 8.0216$ is applied to both the Modigliani and Miller (1958) framework and the Myers framework.

Table 5 and Figure 5 document the amount $I_m - I_{mm}$ and value $V_m - V_{mm}$ of the investment distortion caused by debt overhang. mm denotes the first-best Modigliani and Miller (1958) framework, while m denotes the Myers (1977) framework of no debt financing flexibility. Table 5 shows that the mean of $I_m - I_{mm}$ is equal to -2.8055, representing 8.23 percent of the first-best firm value V_{mm}. The debt overhang problem is important on average. Debt in place induces equity claimants to underinvest compared to the first-best level. Figure 5 shows that no overhang occurs in low technology states. In low technology states, equity claimants who do not manage the debt policy must invest more than the first-best level in order to generate higher revenues tomorrow and decrease the probability of defaulting tomorrow. Default happens in the Myers framework because the firm does not bear any cost of defaulting. In fact, the firm ignores the fair-bond-pricing equation.

As the technology state improves, equity claimants invest less than the first-best level because the marginal productivity of the asset base is mitigated by the possibility of default. Table 5 shows that this agency conflict is very costly to equity claimants, with an average of $V_m - V_{mm} = -34.9806$ representing a very large 92.23 percent of the first-best firm value V_{mm}. Figure 5 indicates that the agency cost increases with the technology state.

Table 5 and Figure 5 show that the amount and cost of the debt overhang problem is very important. With an arbitrary and constant debt policy of B and ι, the debt overhang problem occurs. The firm underinvests on average. However, in low technology states, the firm overinvests to decrease its probability of defaulting tomorrow.

2.3.3 Debt Overhang with Optimal Debt

Debt overhang presumes that there is debt already in place and that the debt policy does not anticipate future investment decisions of equity claimants. The firm's investment policy is now examined when the debt policy is optimally chosen each period before the investment decision is made. The investment level when debt is already, and optimally, put in place I_s is compared to the first-best level I_{mm}.

The firm's problem is now sequential: each period the firm chooses its debt policy B' and ι' in the first stage and it chooses its investment policy K' in the second stage. Solving backwards, the Bellman equation describing the firm's intertemporal investment problem is

$$V_u(K, B_f, \iota; \theta) = \max_{\{D,K'\}} D + \beta E[V_u(K', B', \iota'; \theta') 1_{(V_u' \geq 0)}]$$

subject to

$$D = (1-\tau_f)(\theta K^\alpha - F) + (1-(1-\tau_f)\delta)K - K' + B' - (1+(1-\tau_f)\iota)B_f.$$

The investment decision is not only a function of the state variables K, B_f, ι, θ but also a function of the first stage debt level B' and interest ι' chosen. The investment decision is characterized by

$$\beta E[\{(1-\tau_f)\theta' \alpha K'^{\alpha-1} + (1-(1-\tau_f)\delta)\} 1_{(V_u' \geq 0)}] = 1. \tag{13}$$

The benchmark investment equation (9) differs from the sequential investment policy by its effect on the fair-bond-pricing equation $\lambda v_{K'}$. With sequential decisions, equity claimants ignore the effect of their investment on debt financing conditions.

Working back to the first stage of the firm's problem, the intertemporal debt problem is represented by

$$V_u(K, B_f, \iota; \theta) = \max_{\{B',\iota'\}} D + \beta E[V_u(K', B', \iota'; \theta') 1_{(V_u' \geq 0)}]$$

subject to

$$D = (1-\tau_f)(\theta K^\alpha - F) + (1-(1-\tau_f)\delta)K - K' + B' - (1+(1-\tau_f)\iota)B_f$$

and the fair-bond-pricing equation

$$\beta E\left[\left(1+(1-\tau_\iota)\iota'\right) 1_{(V_u' \geq 0)} + \left(\frac{V_u'(K', 0, 0; \theta')}{B'} - X\right)\left(1 - 1_{(V' \geq 0)}\right)\right] = 1.$$

The debt level and coupon equations (10) and (11) remain unchanged

$$\beta E[(1+(1-\tau_f)\iota') 1_{(V_{um}' \geq 0)}] = 1 - \lambda v_{B'}$$

and

$$E[(1-\tau_f)B' 1_{(V_{um}' \geq 0)}] = -\lambda v_{\iota'}.$$

Table 6 and Figure 6 document the amount $I_s - I_{mm}$ and value $V_s - V_{mm}$ of debt overhang when the debt is optimally put in place. s refers to the sequential model where the asset level is chosen after the debt policy, while

mm refers to the first-best framework of Modigliani and Miller (1958). Both models are calibrated at $F = 9.2$ such that the sequential model replicates the mean debt-to-asset ratio observed in the data. Table 6 shows that $I_s = I_{mm}$. There is no overhang on investment caused by debt when the debt is optimally put in place. To understand this result, note that the only difference between investment equations (12) and (13) is the no-default indicator function $1_{(\cdot)}$. In the sequential model, the firm is always able to fully adjust its debt policy to avoid default. Hence, the resulting investment policy is first-best.

The fair-bond-pricing constraint reduces the equity value V_s when compared to the equity value V_{mm} without debt financing frictions. The cost of this additional constraint is $V_s - V_{mm} = -11.5729$, representing 47.54 percent of the first-best firm value V_{mm}.

Table 6 and Figure 6 show that there is no overhang on investment and the cost of optimally putting debt in place is much smaller than when debt is taken as given.

2.3.4 Distortion Caused by Sequential Decisions

The cost of choosing its investment policy after the firm optimally chooses its debt policy is now quantified. The firm's sequential investment decision I_s is compared to the investment I simultaneously chosen with the debt policy as described by the benchmark model of Section 2.1.

Table 7 and Figure 7 document the amount $I_s - I$ and value $V_s - V$ of the investment distortion caused by choosing investment after debt. Table 7 shows that the mean of $I_s - I$ is equal to -0.7787, representing a mere 1.68 percent of the firm value with simultaneous decisions V. This investment distortion $I_s - I$ is the opposite measure of the investment distortion caused by financing frictions $I - I_{mm}$, because the investment choice with debt optimally put in place I_s is equal to the first-best I_{mm}. The discrepancy between the means of $I - I_{mm} = 0.7966$ and $I_s - I = -0.7787$ results from a different calibration of fixed costs. $I - I_{mm}$ is obtained from $F = 9.5$ that replicates the debt-to-asset ratio observed in the data for the benchmark simultaneous model, while $I_s - I$ is obtained from $F = 9.2$ that replicates the observed debt-to-asset ratio for the sequential model. In this section, both the sequential investment I_s and the simultaneous investment I are obtained with $F = 9.2$.

When debt is chosen prior to the investment decision, the firm invests less on average. With sequential decisions, debt claimants are not willing to lend as much funds to the firm. Although there is no debt overhang, the firm suffers from a reduced borrowing capacity. The debt is still riskless $\iota' = 0.0658$ but the firm does not borrow as much on average $B_s = 13.9503 < 17.7360 = B$. Without these funds, the firm does not invest as much. As discussed above, it actually invests at the first-best level rather than overinvest due to the debt financing frictions. With sequential decisions, the investment decision is separated from the debt financing conditions and therefore it is not influenced by tax benefit of debt.

Figure 7 shows that, in low technology states, investment with debt already put in place is actually higher than investment chosen simultaneously with the debt level. The firm does not sell as much of its asset base because these asset proceeds do not change the interest rate required by debt claimants when default is more likely to occur. The investment policy is decided after the debt is put in place. Thus, selling more assets does not make debt financing less expensive. For the same reason, as the technology state improves, the firm maximizing the equity value with debt in place does not have any incentive to invest beyond the first-best level. It thus invests less than the firm whose investment decision is influenced by the tax benefit of debt. The value of the investment distortion caused by debt in place averages $V_s - V = -0.6039$ or 9.04 percent of the firm value with simultaneous decisions V. The cost of making the investment decision after the debt is in place increases as the technology state improves.

Table 7 and Figure 7 show that the cost of choosing the investment policy once debt is optimally put in place is non negligible, despite the small amount of investment distortion. The firm with sequential decisions invests at the first-best level, thereby investing more than a firm who makes simultaneous investment and debt decisions in low technology states and investing less than that firm in high technology states. The firm who makes sequential decisions does not take into account the tax benefit of debt when making its investment decision. It will therefore lose value compared to the firm who decides simultaneously on its investment and debt policies.

2.4 Concluding Comments on Investment Distortions

In this chapter, the interaction between investment and debt issuing decisions of a firm in the presence of the traditional tax benefit and default cost frictions is examined. The model generates investment and new debt issuing decisions that are positively correlated to avoid default through time, as observed in the data. Given that the model performs well compared to the data, the chapter proceeds to measure various investment distortions caused by debt financing. First, the distortion caused by debt financing frictions is measured. The tax benefit of debt induces the firm to increase its debt capacity and invest beyond the first-best level on average. The cost of the overinvestment outweighs the tax benefit of debt thereby reducing the equity value below the first-best level. In low technology states, the firm actually underinvests to avoid default despite the tax benefit of debt. Second, Myers's (1977) debt overhang problem is measured. The debt overhang problem obtains on average and becomes more important at higher technology states. Third, the debt overhang problem with optimal debt is measured. When debt is optimally put is place, there is no debt overhang: the resulting investment level is first-best. Finally, the cost of choosing investment after the debt policy is measured. Equity claimants lose value by choosing to invest after their debt is optimally put in place because they do not consider the effect on their investment decision on the debt financing conditions.

Unlike previous papers, the model characterizes the optimal investment scales chosen by the firm at each point in time. In line with Mello and Parsons (1992), this chapter finds that the debt overhang cost $V_m - V_{mm}$ is very important. Mauer and Triantis's (1994) conclusion that the operating policy is not affected by the capital structure is due to their real option-pricing framework where there is value of waiting to invest. Without such a feature, this chapter shows that the investment distortion due to debt financing frictions $I - I_{mm}$ is important.

3 Why is Investment Sensitive to Cash Flow?

In order to understand how financing constraints influence the sensitivity of investment to cash flow fluctuations, the cash flow sensitivity derived from a model of a firm without any constraint is compared to the cash flow sensitivity derived from a model of a firm without access to equity and debt markets. In the spirit of FHP, firms who cannot raise any funds from external markets are called constrained firms, while firms who face no financing constraint are called unconstrained firms. The unconstrained firm model is identical to the benchmark model of Section 2.1, but repeated here for comparability with the constrained firm model.

3.1 Unconstrained Firms

The Consumer
The risk neutral consumer "c" maximizes its expected lifetime utility

$$U_0 = E_0\left[\sum_{t=0}^{\infty} \beta^t C_t\right].$$

The Bellman equation describing its intertemporal problem is

$$U(S_c, B_c) = \max_{\{C;S_c',B_c'\}} C + \beta E\left[U(S_c', B_c')\right]$$

subject to the budget constraint

$$\begin{aligned} C + pS_c' + B_c' = \;& \{(p+d)S_c + (1+(1-\tau_\iota)\iota)B_c\}\, 1_{(V_u \geq 0)} \\ & + (g - X)\, B_c(1 - 1_{(V_u \geq 0)}). \end{aligned}$$

The consumer maximizes its utility by choosing how many goods C to consume, how many equity claims S_c' to buy, and how much debt B_c' to hold, taking as given the ex-dividend share price p, the dividend-per-share ratio d, the interest rate ι, the firm residual g that accrues to debt claimants upon default as a proportion of the debt face value, and the no-default state $1_{(\cdot)}$ (to be defined below), where β is the discount factor, τ_ι is the interest income tax rate, X is the deadweight default cost as a proportion of the debt face value, and primed variables refer to tomorrow's beginning-of-the-period values.

The risk neutral consumer prices equity and debt claims according to the following two equations:

$$p = \beta E\left[(p' + d')1_{(V_u' \geq 0)}\right] \tag{14}$$

and

$$1 = \beta E\left[\left(1 + (1-\tau_\iota)\iota'\right) 1_{(V_u' \geq 0)} + (g' - X)(1 - 1_{(V_u' \geq 0)})\right]. \tag{15}$$

Equation (14) shows that the consumer prices the equity claim such that today's price equals tomorrow's expected discounted payoff. The equity payoff consists of the price and dividend if the firm does not default. Similarly, equation (15) shows that the consumer requires an interest rate such that one unit of debt lent to the firm today equals tomorrow's expected discounted payoff. The payoff on the debt claim consists of the face value and the after-tax interest payment if the firm does not default, or the net residual value if the firm defaults.

The Firm

The firm "f" maximizes the value to its equity claimants. The model assumes that there is no dilution. The firm cannot change its number of shares outstanding: $S'_f = S_f = 1$. Then, the ex-dividend share price p becomes the ex-dividend equity value. The unlimited liability equity value is defined as

$$V_u = p + d \tag{16}$$

and the dividend payment as

$$D = d. \tag{17}$$

The equity value equation (14) is rewritten as

$$V_u = D + \beta E\left[V'_u 1_{(V'_u \geq 0)}\right],$$

where the indicator function is defined by

$$1_{(V_u \geq 0)} = \begin{cases} 1 & \text{if } V_u \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

If no default occurs tomorrow, equity claims are valued at V'_u. Otherwise, equity claimants are protected from debt claimants by limited liability. Thus, default is defined to occur tomorrow when the equity value $V'_u 1_{(V'_u \geq 0)}$ is nil, i.e., when the equity value with unlimited liability V'_u is less than zero. Clearly, by maximizing the unlimited liability equity value V_u, the firm also maximizes the limited liability equity value $V_u 1_{(V_u \geq 0)}$.

The firm chooses how much dividend D to pay, how much to invest I, and how much debt to issue $\Delta B'_f = B'_f - B_f$ at which interest rate ι', given its after-tax operating income before depreciation $(1-\tau_f)f(K;\theta)$, its capital cost allowance $\tau_f \delta K$, and its debt face value and tax-deductible interest payments $(1+(1-\tau_f)\iota)B_f$, where τ_f is the firm's tax rate and δ is the capital cost allowance rate.[8] The firm makes its decisions after observing the beginning-of-the-period value for the technology state θ and last period's choices of asset base K, debt B, and interest rate ι. The following summarizes the timing of these decisions:

[8]For simplicity, the capital cost allowance rate is assumed equal to the true economic depreciation rate of the asset base.

the firm observes θ	the firm observes θ'
given K, B, ι	given K', B', ι'
it chooses D, K', B', ι'	it chooses D', K'', B'', ι''

Although the debt B_f is modeled with a one-period maturity, the firm can decide at each time period to roll it over $\Delta B'_f = 0$, to make a new issue $\Delta B'_f > 0$, or to retire a portion of its debt outstanding $\Delta B'_f < 0$. The one-period maturity debt can thus be viewed as an infinite maturity debt with a floating rate.

The dividend is defined by the firm's sources and uses of funds equation

$$D = (1 - \tau_f) f(K; \theta) - I + \tau_f \delta K + B'_f - (1 + (1 - \tau_f)\iota) B_f. \tag{19}$$

The firm's operating income before depreciation is the difference between its revenues and expenses

$$f(K; \theta) = \theta K^{\alpha} - F, \tag{20}$$

where the Cobb-Douglas parameter $\alpha \in (0, 1)$ specifies decreasing returns to scale and F is a fixed cost representing labor and other expenses.[9]

The asset base is subject to depreciation and takes time to build. It evolves according to the accumulation equation

$$K' = (1 - \delta) K + I. \tag{21}$$

The technology state is represented by the following first-order autoregressive process:

$$\ln \theta' = \ln A + \rho \ln \theta + \sigma \epsilon', \tag{22}$$

where A is a constant and $\epsilon \sim iid\ N(0, 1)$. The persistence ρ of the technology shock provides an exogenous source of dynamics.

When making its dividend D, asset K', and debt financing (B'_f, ι') decisions, the firm must also take into account the pricing schedule at which the debt can be financed. Debt claimants require an interest rate ι' such that the debt is fairly priced according to equation (15), restated here for convenience,

$$\beta E \left[(1 + (1 - \tau_\iota)\iota')\, 1_{(V'_u \geq 0)} + (g' - X)(1 - 1_{(V'_u \geq 0)}) \right] = 1.$$

[9]The firm's labor demand decision and the consumer's labor supply decision are not modeled.

The firm knows that the residual gB_f accruing to debt claimants upon default is the reorganized value of the firm

$$gB_f = V_u(K, 0, 0; \theta),$$

the equity value with assets K, no debt, no interest, and a technology state θ.[10] Debt claimants may then recapitalize the firm in an optimal manner. In fact, $V_u(K, 0, 0; \theta)$ takes into account the optimal recapitalization from that unlevered state. The consumer's debt pricing equation (15) becomes

$$\beta E\left[(1 + (1 - \tau_\iota)\iota')\, 1_{(V_u' \geq 0)} + \left(\frac{V_u'(K', 0, 0; \theta')}{B_f'} - X\right)(1 - 1_{(V_u' \geq 0)})\right] = 1. \tag{23}$$

The firm does not choose whether to default or not. Although the firm positions itself to minimize the possibility of default tomorrow, default could nevertheless happen as a result of today's decisions D, K', B_f', and ι' when tomorrow's technology state θ' turns out to be much lower than expected. Default triggers an immediate reorganization process.

Equations (19), (20), (21), and (23) are the only constraints facing the firm. The logarithmic technology process restricts revenues θK^α to be positive given that $A > 0$. The firm experiences operating losses before depreciation when expenses F exceed revenues θK^α. When net losses occur, the dividend is increased by a tax subsidy, $-\tau_f(f(K;\theta) - \delta K - \iota B_f) > 0$.[11] Dividends D are not restricted to be non-negative. Negative dividends are interpreted as rights offers. Equity claimants find it worthwhile to exercise these rights, otherwise default is triggered. In fact, the firm optimizes with respect to the dividend policy. The firm decides on the amount of dividends or rights issues that is optimal. In addition to dividends, investments I and debt issues $\Delta B'$ are not restricted to be non-negative. The firm is allowed to sell its assets and to retire its debt.

[10]By definition, the residual g accruing to debt claimants upon default (when $V_u < 0$) is always less than the principal and after-tax interest income $(1 + (1 - \tau_\iota)\iota)$

$$\begin{aligned} V_u(K, B, \iota; \theta) &= V_u(K, 0, 0; \theta) - (1 + (1 - \tau_f)\iota)B_f < 0 \\ g &= \tfrac{V_u(K,0,0;\theta)}{B_f} < (1 + (1 - \tau_f)\iota) < (1 + (1 - \tau_\iota)\iota) \end{aligned}$$

because the corporate tax rate τ_f is higher than the income income tax rate τ_ι.

[11]Tax asymmetries such as limited carryback and carryforward provisions are not addressed.

The Bellman equation describing the firm's intertemporal problem is

$$V_u(K, B_f, \iota; \theta) = \max_{\{D, K', B'_f, \iota'\}} D + \beta E\left[V_u(K', B'_f, \iota'; \theta')1_{(V'_u \geq 0)}\right]$$

subject to equations (19), (20), (21), and (23). The asset, debt, and coupon decisions of the firm are characterized by the following equations:

$$\beta E\left[\left\{(1-\tau_f)\theta'\alpha K'^{\alpha-1} + (1-(1-\tau_f)\delta)\right\} 1_{(V'_u \geq 0)}\right] + \lambda v_{K'} = 1, \tag{24}$$

$$\beta E\left[(1+(1-\tau_f)\iota')1_{(V'_u \geq 0)}\right] = 1 - \lambda v_{B'}, \tag{25}$$

and

$$E\left[(1-\tau_f)B'_f 1_{(V'_u \geq 0)}\right] = -\lambda v_{\iota'}, \tag{26}$$

where λ is the multiplier on the consumer's fair-bond-pricing equation (23), and v'_K, v'_B, and v'_ι represent marginal effects of the firm's decisions on the fair-bond-pricing equation (23) characterized in the appendix.

Equation (24) states that the firm invests up to the point where the cost of one unit of asset today equals tomorrow's expected discounted marginal contribution to dividends plus the benefits associated with better financing conditions. The marginal contribution to dividends consists of the asset resale price and the marginal after-tax income. The firm acts on behalf of current equity claimants by valuing tomorrow's contribution to dividends only in the no-default state. Equation (25) states that the firm issues debt up to the point where one unit of debt contributing to today's dividends net of the cost of deteriorated financing conditions equals the expected discounted face value and after-tax interest burden on tomorrow's dividends if the firm does not default. Equation (26) is used to determine the shadow value of the consumer's debt holdings λ.

The tax and default frictions insure an interior solution for the debt level B'_f. The tax benefit arises because the interest payments are deductible to the firm at a higher rate than the interest income is taxable to the consumer $\tau_f > \tau_\iota$. One unit of debt today is expected to generate $(\tau_f - \tau_\iota)\iota'$ funds if the firm does not default tomorrow. That unit of debt today is also expected to cost X funds if the firm defaults tomorrow.

Equilibrium

Finally, the equilibrium requires that all markets clear. There are two financial markets and one goods market. Clearing in the equity market requires that the number of shares purchased by the consumer be equal to the number of shares outstanding $S'_c = S'_f = 1$. Similarly, clearing in the debt market requires $B'_c = B'_f = B'$. Given that the budget constraint of the consumer and the sources and uses of funds equation of the firm are satisfied, the goods market also clears by Walras's law.

3.2 Constrained Firms

Without access to external markets, the model is somewhat simplified. The consumer's equity pricing equation (14) remains unchanged

$$p = \beta E\left[(p' + d')1_{(V'_u \geq 0)}\right].$$

The firm's problem is to choose its dividend D and investment K' policies to maximize the value of equity claims. The firm is constrained from financing itself with a debt issue $B' = B = 0$ or with a rights issue $D \geq 0$. The Bellman equation describing the intertemporal investment problem is

$$V_u(K;\theta) = \max_{\{D,K'\}} D + \beta E[V_u(K';\theta')1_{(V'_u \geq 0)}]$$

subject to

$$D = (1 - \tau_f)(\theta K^\alpha - F) + (1 - (1 - \tau_f)\delta)K - K' \geq 0.$$

The investment decision is characterized by

$$\beta E[\{(1 - \tau_f)\theta'\alpha K'^{\alpha-1} + (1 - (1 - \tau_f)\delta)\}(1 + \eta')1_{(V'_u \geq 0)}] = 1 + \eta,$$

where η is the Kuhn-Tucker multiplier disallowing rights issues. Clearing in the equity market is insured by $S'_c = S'_f = 1$. Because the budget constraint of the consumer and the sources and uses of funds equation of the firm are satisfied, the goods market clears by Walras's law.

3.3 Results

The appendix details how the two models are calibrated, solved, and simulated. Figure 8 graphs the policy functions K', B', and p of the unconstrained firm. Because of the persistence ρ, firms experiencing low technology states θ today expect low states tomorrow and thus a low marginal productivity of their asset base. Firms invest only small amounts K' and carry very little debt B'. As the technology state increases, the marginal productivity of the asset base also improves. Firms invest greater amounts and this investment is financed by higher debt levels. The firm is able to fully adjust the asset and debt levels to avoid the possibility of default, as reflected by the constant interest rate $\iota' = 0.0658$. Technology state improvements generate larger future dividends, as valued into the equity price p.

Figure 9 shows that the policy functions K' and p of the constrained firm behave similarly to those of the unconstrained firm. Constrained firms have no access to debt or equity markets. Hence, firms with a low revenues-generating asset base lack funds to invest as much as desired. In those states, the Kuhn-Tucker multiplier restricting rights issues is binding $\eta > 0$.

The only source of dynamics in the model is through the technology state θ. With no technology persistence $\rho = 0$, $\log\theta \sim iid\ N(0, \sigma^2)$. The dynamic model reduces to a sequence of static decisions. In this case, the investment decision is constant through time and there is no cash flow sensitivity.

Information contained in Table 8 is taken from FHP. FHP classify Value Line firms during the 1970 to 1984 period into three classes, from most-financially-constrained to least-financially-constrained. Class 1 firms represent the most-constrained firms as identified by dividend-to-income ratios lower than 0.1, Class 2 firms have ratios between 0.1 and 0.2, and Class 3 firms represent the least-constrained firms as identified by ratios greater than 0.2. Table 8 summarizes FHP's descriptive statistics on the investment $\frac{I}{K}$ and cash flow $\frac{CF}{K}$ variables. Most-constrained Class 1 firms invest more than Class 2 firms, who in turn invest more than least-constrained Class 3 firms. Tables 9 and 10 indicate that unconstrained firms simulated from the model invest slightly more than theoretical constrained firms, with medians of 0.1000 and 0.0862 respectively. Table 8 also shows that most-constrained Class 1 firms have more cash flows than Class 2 firms, who in turn have more cash flows than least-constrained Class 3 firms. This is similar to simulated statistics of Tables 9 and 10. Theoretical unconstrained firms have less cash flows than theoretical constrained firms, with medians of 0.0027 and 0.0139 respectively.

Table 8 also summarizes FHP's cash flow sensitivity results. Cash flow $\frac{CF}{K}$ sensitivities decrease monotonically from the most-constrained class to the least-constrained class. Tables 9 and 10 report the regression results of the simulated investment $\frac{I}{K}$ series on the simulated Tobin's Q and cash flow $\frac{CF}{K}$ series. Table 9 indicates that the theoretical unconstrained firm has a cash flow sensitivity of 4.5348, while Table 10 indicates that the theoretical constrained firm has a cash flow sensitivity of 0.5715. The cash flow sensitivity results obtained from the models are not consistent with FHP.

Following KZ's classification of constrained firms when firms are restricted from investing more, constrained firms are further classified into two sub-groups: investment-constrained if the Kuhn-Tucker multiplier restricting rights issues is binding and investment-unconstrained otherwise. In accordance with Gross and Pratap and Rendón, investment-constrained firms have investment policies that are more sensitive to cash flow fluctuations than investment-unconstrained firms, with sensitivities of 1.3514 and 0.5444 respectively. These results are not consistent with KZ.

The cash flow variable is highly correlated with the technology state θ, with coefficients of 0.9867 for unconstrained firms and 0.3840 for constrained firms. This suggests that investment may be sensitive to cash flow fluctuations only because cash flows proxy well for investment opportunities.

In this chapter, both Tobin's Q and cash flow $\frac{CF}{K}$ are endogenously constructed from realizations of the technology state θ. Hence both variables are allowed to contain information about investment opportunities. Moreover, the technology state θ represents the only source of uncertainty. With only one source of uncertainty, there is a close link between cash flow and investment. For example, if some noise were to be added to cash flow $\frac{CF}{K}$, the sensitivity of investment to cash flow fluctuations may be reduced.

The unconstrained firm model yields the highest investment correlation with the dividend-to-income ratio $\frac{D_+}{Inc}$, while the constrained firm model yields the highest investment correlation with the technology state θ. There is no single measure of investment opportunities that fits for all firms irrespective of their degree of financial constraint. The marginal productivity of the asset base is different across different degrees of financial constraint and cannot be captured by a single measure.

The median dividend-to-income ratio $\frac{D_+}{Inc}$ is equal to 0 for unconstrained firms and to 0.9324 for constrained firms. Because unconstrained firms never default, they are able to contract with debt claimants at the riskfree rate, thereby obtaining the lowest cost of debt financing and avoiding the default cost. In turn, unconstrained firms pay out a lower risk compensation, i.e., a lower dividend, to its equity claimants than constrained firms. Constrained firms must promise larger dividends to compensate equity claimants for the default risk they face. Constrained firms cannot raise debt or issue equity to better manage their solvency through various technology shocks. As a result, firms with no financial flexibility show more volatile dividends. The model of constrained firms, compared to the model of unconstrained firms, suggests that large and volatile dividend-to-income ratios proxy for greater financial constraints. Low dividend-to-income ratios are associated with unconstrained (as opposed to FHP's most-constrained) firms and the high dividend-to-income ratios are associated with constrained (as opposed to FHP's least-constrained) firms.

Note that, dismissing the model identification of financial constraint to follow FHP's a priori dividend-to-income identification, FHP's results obtain. Low-dividend firms, a priori identified by FHP as constrained firms but modeled here as unconstrained firms, have larger cash flow sensitivities than high-dividend firms, a priori identified as unconstrained firms but modeled here as constrained firms.

3.4 Concluding Comments on Cash Flow Sensitivities

Many questions have been addressed. Can this chapter determine why investment is sensitive to cash flow? Yes: because cash flows proxy for investment opportunities θ. This chapter cannot replicate FHP's empirical result that cash flow sensitivities are larger for more constrained firms. This chapter also cannot replicate KZ's empirical result that cash flow sensitivities are lower for more investment-constrained firms. Table 11 summarizes the main results of the cash flow sensitivity literature. A star * indicates that this chapter provides evidence in support of the result.

Can this chapter suggest a better measure of investment opportunities than Tobin's Q? Not a single measure for both the unconstrained and constrained firm models.

Can this chapter suggest an easily observable measure of financial constraint? Yes: large and volatile dividend-to-income ratios.

References

[1] Abel, A.B. and O.J. Blanchard (1986) The Present Value of Profits and Cyclical Movements in Investment, *Econometrica* **54**, 249-273.

[2] Altman, E.I. (1984) A Further Empirical Investigation of the Bankruptcy Cost Question, *Journal of Finance* **39**, 1067-1089.

[3] Andrade, G. and S.N. Kaplan (1998) How Costly is Financial (Not Economic) Distress? Evidence from Highly Leveraged Transactions that Became Distressed, *Journal of Finance* **53**, 1443-1493.

[4] Bernanke, B. and M. Gertler (1990) Financial Fragility and Economic Performance, *Quarterly Journal of Economics* **105**, 87-114.

[5] Bernanke B. and M. Gertler (1989) Agency Costs, Net Worth, and Business Fluctuations, *American Economic Review* **79**, 14-31.

[6] Bond, S. and C. Meghir (1994) Dynamic Investment Models and the Firm's Financial Policy, *Review of Economic Studies* **61**, 197-222.

[7] Brennan, M.J. and E.S. Schwartz (1984) Optimal Financial Policy and Firm Valuation, *Journal of Finance* **39**, 593-609.

[8] Brennan, M.J. and E.S. Schwartz (1978) Corporate Income Taxes, Valuation, and the Problem of Optimal Capital Structure, *Journal of Business* **51**, 103-114.

[9] Calomiris, C.W. and R.G. Hubbard (1990) Firm Heterogeneity, Internal Finance, and 'Credit Rationing,' *Economic Journal* **100**, 90-104.

[10] Cochrane, D. and G.H. Orcutt (1949) Application of Least Squares Regressions to Relationships Containing Autocorrelated Error Terms, *Journal of the American Statistical Association* **44**, 32-61.

[11] Coleman, W.J. II (1990) Solving the Stochastic Growth Model by Policy-Function Iteration, *Journal of Business and Economic Statistics* **8**, 27-29.

[12] Cummins, J.G., K.A. Hasset, and S.D. Oliner (1997) Investment Behavior, Observable Expectations, and Internal Funds, NYU mimeo.

[13] Dammon, R.M. and L.W. Senbet (1988) The Effect of Taxes and Depreciation on Corporate Investment and Financial Leverage, *Journal of Finance* **43**, 357-373.

[14] Décamps, J.-P. and A. Faure-Grimaud (1997) Pricing the Gamble for Resurrection and the Consequences of the Effects of Renegotiation and Debt Design, GREMAQ mimeo.

[15] Dotan, A. and S.A. Ravid (1985) On the Interaction of Real and Financial Decisions of the Firm under Uncertainty, *Journal of Finance* **40**, 501-517.

[16] Faig, M. and P.M. Shum (1999) Irreversible Investment and Endogenous Financing: An Evaluation of Corporate Tax Effects, *Journal of Monetary Economics* **43**, 143-171.

[17] Fazzari, S.M., R.G. Hubbard, and B.C. Petersen (1988) Financing Constraints and Corporate Investment, *Brookings Paper on Economic Activity* **1**, 141-195.

[18] Fischer, E.O., R. Heinkel, and J. Zechner (1989) Dynamic Capital Structure Choice: Theory and Tests, *Journal of Finance* **44**, 19-40.

[19] Froot, K.A., D.S. Scharfstein, and J.C. Stein (1993) Risk Management: Coordinating Corporate Investment and Financing Policies, *Journal of Finance* **48**, 1629-1658.

[20] Gilchrist S. and C.P. Himmelberg (1995) Evidence on the Role of Cash Flow in Reduced-Form Investment Equations, *Journal of Monetary Economics* **36**, 541-572.

[21] Goldstein, R., N. Ju, and H. Leland (1998) Endogenous Bankruptcy, Endogenous Restructuring and Dynamic Capital Structure, mimeo.

[22] Gomes, J.F. (1998) Financing Investment, Wharton mimeo.

[23] Gross, D.B. (1995) The Investment and Financing Decisions of Liquidity Constrained Firms, MIT Ph.D. Thesis.

[24] Hoshi, T., A.K. Kashyap, and D. Scharfstein (1991) Corporate Structure, Liquidity, and Investment: Evidence from Japanese Panel Data, *Quarterly Journal of Economics* **106**, 33-60.

[25] Hubbard, R.G. and A.K. Kashyap (1992) Internal Net Worth and the Investment Process: An Application to U.S. Agriculture, *Journal of Political Economy* **100**, 506-534.

[26] Hubbard, R.G., A.K. Kashyap, and T.M. Whited (1995) Internal Finance and Firm Investment, *Journal of Money, Credit, and Banking* **27**, 683-701.

[27] Jensen, M.C. and W.H. Meckling (1976) Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, *Journal of Financial Economics* **3**, 305-360.

[28] Ju, N. (1998) Dynamic Optimal Capital Structure and Maturity Structure, mimeo.

[29] Kane, A., A.J. Marcus, and R.L. McDonald (1985) Debt Policy and the Rate of Return Premium to Leverage, *Journal of Financial and Quantitative Analysis* **20**, 479-499.

[30] Kane, A., A.J. Marcus, and R.L. McDonald (1984) How Big is the Tax Advantage to Debt? *Journal of Finance* **39**, 841-855.

[31] Kaplan, S.N. and L. Zingales (1997) Do Investment-Cash Flow Sensitivities Provide Useful Measures of Financing Constraints? *Quarterly Journal of Economics* **112**, 169-215.

[32] Kydland, F. and E.C. Prescott (1982) Time to Build and Aggregate Fluctuations, *Econometrica* **50**, 1345-1370.

[33] Lamont, O. (1997) Cash Flow and Investment: Evidence from Internal Capital Markets, *Journal of Finance* **52**, 83-110.

[34] Leland, H.E. (1998) Agency Costs, Risk Management, and Capital Structure, *Journal of Finance* **53**, 1213-1243.

[35] Leland, H.E. (1994) Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance* **49**, 1213-1252.

[36] Leland, H.E. and K.B. Toft (1996) Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, *Journal of Finance* **51**, 987-1019.

[37] Mauer, D.C. and A.J. Triantis (1994) Interactions of Corporate Financing and Investment Decisions: A Dynamic Framework, *Journal of Finance* **49**, 1253-1277.

[38] Mayer, C. (1986) Corporation Tax, Finance and the Cost of Capital, *Review of Economic Studies* **53**, 93-112.

[39] Mella-Barral, P. and W. Perraudin (1997) Strategic Debt Service, *Journal of Finance* **52**, 531-556.

[40] Mello, A.S. and J.E. Parsons (1992) Measuring the Agency Cost of Debt, *Journal of Finance* **47**, 1887-1904.

[41] Modigliani, F. and M.H. Miller (1958) The Cost of Capital, Corporation Finance and the Theory of Investment, *American Economic Review* **48**, 261-297.

[42] Myers, S.C. (1977) Determinants of Corporate Borrowing, *Journal of Financial Economics* **5**, 147-175.

[43] Myers, S.C. and N.S. Majluf (1984) Corporate Financing and Investment Decisions when Firms Have Information that Investors do not Have, *Journal of Financial Economics* **13**, 187-221.

[44] Parrino, R. and M.S. Weisbach (1997) On the Magnitude of Stockholder-Bondholder Conflicts, University of Texas mimeo.

[45] Pratap, S. and S. Rendón (1998) Firm Investment in Imperfect Capital Markets: A Structural Estimation, NYU mimeo.

[46] Rajan, R., H. Servaes, and L. Zingales (1998) The Cost of Diversity, the Diversification Discount and Inefficient Investment, NBER Working Paper No. 6368.

[47] Scharfstein, D.S. (1997) The Dark Side of Internal Capital Markets II: Evidence from Diversified Conglomerates, NBER Working Paper No. 6352.

[48] Shin, H. and Y.S. Park (1998) Financing Constraints and Internal Capital Market: Evidence from Korean Chaebols, California Polytechnic State University mimeo.

[49] Shin, H. and R.M. Stulz (1998) Are Internal Capital Markets Efficient? *Quarterly Journal of Economics* **113**, 531-552.

[50] Warner, J. (1977) Bankruptcy Costs: Some Evidence, *Journal of Finance* **32**, 337-348.

[51] Whited, T.M. (1992) Debt, Liquidity Constraints, and Corporate Investment: Evidence from Panel Data, *Journal of Finance* **47**, 1425-1460.

[52] Wiggins, J.B. (1990) The Relation between Risk and Optimal Debt Maturity and the Value of Leverage, *Journal of Financial and Quantitative Analysis* **25**, 377-386.

4 Appendix

4.1 Effects of the Firm's Decisions on the Fair-Bond-Pricing Equation

The marginal effects of the firm's decisions on the fair-bond-pricing equation (8) are

$$\begin{aligned} v_{K'} = & -\beta E\left[\left(\tfrac{(1-\tau_f)\theta'\alpha K'^{\alpha-1}+(1-(1-\tau_f)\delta)}{B'}\right)(1-1_{(V_u'\geq 0)})\right] \\ & +\beta\left\{(\tau_f-\tau_\iota)\iota'+X\right\}\tfrac{\partial\Phi(\bar\theta)}{\partial K'}, \end{aligned} \tag{27}$$

$$v_{B'} = -\beta E\left[\frac{V_u(K',0,0;\theta')}{B'^2}(1-1_{(V_u'\geq 0)})\right] - \beta\left\{(\tau_f-\tau_\iota)\iota'+X\right\}\frac{\partial\Phi(\bar\theta)}{\partial B'}, \tag{28}$$

and

$$v_{\iota'} = E\left[(1-\tau_\iota)1_{(V_u'\geq 0)}\right] - \left\{(\tau_f-\tau_\iota)\iota'+X\right\}\frac{\partial\Phi(\bar\theta)}{\partial\iota'}, \tag{29}$$

where Φ is the standard normal cumulative density function and $\bar\theta$ is the technology state at the default point. More specifically, $\bar\theta$ is defined by

$$V_u(K',B',\iota';\bar\theta)=0.$$

Substituting for equations (2), (4), (5), and (6), the default point is expressed as

$$\bar\theta = \frac{(1-\tau_f)F - p' - (1-(1-\tau_f)\delta)K' + K'' - B'' + (1+(1-\tau_f)\iota')B'}{(1-\tau_f)K'^{\alpha}}.$$

The marginal effects of the firm's decisions on the probability of default are

$$\frac{\partial\Phi(\bar\theta)}{\partial K'} = -\phi(\bar\theta)\left(\frac{(1-(1-\tau_f)\delta)}{(1-\tau_f)K'^{\alpha}} + \frac{\alpha\bar\theta}{K'}\right) < 0, \tag{30}$$

$$\frac{\partial\Phi(\bar\theta)}{\partial B'} = \phi(\bar\theta)\frac{(1+(1-\tau_f)\iota')}{(1-\tau_f)K'^{\alpha}} > 0, \tag{31}$$

and

$$\frac{\partial\Phi(\bar\theta)}{\partial\iota'} = \phi(\bar\theta)\frac{B'}{K'^{\alpha}} > 0, \tag{32}$$

where ϕ is the standard normal probability density function. Equations (30), (31), and (32) indicate that more investment decreases the probability of default, while more debt or a higher coupon rate increases it. Equations (27) to (29) show how the firm's decisions affect the pricing schedule of debt claimants. Equation (27) shows that one unit of asset affects the expected discounted residual claim obtained by debt claimants upon default and the costs at the default point (both the deadweight cost X and the forgone tax benefit due to the reorganization $(\tau_f - \tau_\iota)\iota'$). Equation (28) shows that one unit of debt today affects tomorrow's expected discounted residual obtained by debt claimants upon default and the costs at the default point. Equation (29) shows that the interest rate affects the payoff of debt claimants when no default occurs and the costs at the default point.

4.2 Data and Calibration

In order to obtain a solution, parameter values for β, δ, τ_f, τ_ι, X, α, A, ρ, σ, and F are required. The discount factor β is set to 0.95 and the depreciation rate δ is set to 0.1, in accordance with most dynamic investment studies since Kydland and Prescott (1982). According to the U.S. tax code, it is reasonable to assume that a representative firm faces a 0.35 federal flat rate and a 0.05 state flat rate. Hence the corporate tax rate τ_f is set to 0.4. Using individual income tax return data from the U.S. Internal Revenue Service, the personal interest income tax rate is proxied by the ratio of federal, state, and local income taxes to adjusted gross income $\tau_\iota = 0.2$.

Warner (1977) estimates direct bankruptcy costs using data from eleven bankrupt U.S. railroad firms. These costs include the legal, accounting, and administrative costs directly related to the bankruptcy process. He shows that direct costs amount to one percent of a railroad's market value seven years prior to the petition date, and 5.3 percent at the petition date. Altman (1984) includes the indirect costs of lost profits. He estimates the total bankruptcy costs with a sample of eighteen industrial firms who went bankrupt during the 1970-1978 period. On average, total bankruptcy costs represent 12.4 percent of the firm value three years prior to the petition date, and 16.7 percent at the petition date. Andrade and Kaplan (1998) obtain results that are consistent with Altman's results. They estimate both direct and indirect financial distress costs and find that these represent between ten and twenty percent of firm value.[12] I follow previous dynamic

[12] Andrade and Kaplan further show that the subsample of financially but not economically distressed firms have little financial costs. However, in this thesis, default is triggered by low technology shocks. Hence firms who are financially distressed are also economically distressed.

recapitalization models in representing the default cost as a proportion of the debt face value, rather than as a proportion of the firm value as estimated by the empirical literature. In their calibration, Fischer, Heinkel, and Zechner (1989) set their bankruptcy cost to five percent of the debt face value. Kane, Marcus, and McDonald's (1984) calibration assumes a higher value, fifteen percent of the debt face value. As a compromise, I set the deadweight default cost X at ten percent of the face value.

Unlike the parameters just discussed, the literature does not offer guidance on calibrating α, A, ρ, σ, and F. The Cobb-Douglas parameter α, the level A of the technology state, its persistence ρ, and its volatility σ are set such that the firm's income equation (5) and its technology process equation (7) represent U.S. manufacturing firms. Equation (5) is log-linearized

$$\ln(f(K_{it};\theta_{it}) + F) = \alpha_i \ln K_{it} + \ln \theta_{it}, \tag{33}$$

where $f(K_{it};\theta_{it}) + F$ represents revenues, i denotes the firm and t the year. Equations (33) and (7) are then simultaneously estimated for each manufacturing firm using the Cochrane and Orcutt (1949) procedure.

Annual Compustat data from the 1977-1996 period are used, where manufacturing firms are defined as those with SIC codes from 2000 to 3999. Estimating equations (33) and (7) requires data on firms' revenues $f(K_{it};\theta_{it})+F$ and assets K_{it}. Revenues are captured by Compustat's Net Sales variable (data item number 12). The asset base is constructed from the Gross Property, Plant, and Equipment variable (data item number 7), the Capital Expenditures variable (data item number 128), and the Sale of Property, Plant, and Equipment variable (data item number 107). Asset book values, represented by the Gross Property, Plant, and Equipment variable, are converted into market values. First, the market value is set equal to the book value for the first year a firm appears in the sample. Then, the subsequent market values are generated with the restated accumulation equation (6)

$$K_{it+1} = (1-\delta)K_{it} + I_{it},$$

where the investment is measured as the Capital Expenditures net of the Sale of Property, Plant, and Equipment. Book values of the asset base also serve to filter out firms with large discontinuities. These discontinuities are assumed to result from mergers, acquisitions, or divestitures. Firms are included in the sample if they satisfy the M&A filter that variations in book values net of investment do not exceed fifty percent. Finally, the annual data, expressed in millions of U.S. dollars, are deflated at the firms' fiscal year-ends using the *U.S. Bureau of Labor Statistics*' monthly producer price index for all commodities.

The Cobb-Douglas parameter α_i and the autoregressive parameters A_i, ρ_i, and σ_i are estimated for each firm. More than four years of data is needed to estimate these four parameters. Out of the population of 7196 manufacturing firms, 1603 firms have at least ten years of data (on all *Compustat* series used in this thesis) and survive the M&A filter, while 2218 firms show a minimum of eight years of data and survive the filter. The parameter values used for the benchmark calibration are the means of the ten-year sample estimates. Table 1 documents the parameter values and the dispersion of the estimates.

Firms that have been present for at least ten years during the 1977-1996 window are characterized by a sensitivity of their revenues to asset base variations of $\alpha = 0.4365$, a technology state level of $A = 2.9679$, a persistence of $\rho = 0.5866$, and a volatility of $\sigma = 0.1836$. Firms that have been present for a minimum of eight years show a similar sensitivity to asset variations of $\alpha = 0.4295$, a similar technology state level $A = 2.9164$, and a similar volatility of $\sigma = 0.1885$, but differ by a lower persistence of $\rho = 0.5048$.

Although the labor demand and labor supply decisions are not modeled, the presence of expenses is acknowledged. Labor and other expenses F are represented by a fixed cost. The calibrated value varies across the different models presented in this thesis such that the mean of the debt-to-asset ratio B/K series generated from the model approximates the mean in the data (0.4031). The debt-to-asset ratio is used for the calibration of F because the interaction between investment and debt issuing decisions is the main focus of the thesis. As such, the calibration of the thesis should be based on the observed mean ratio of these two variables.

In addition to the asset base K, investment I, and revenues θK^{α} series, other series are constructed from *Compustat*. The debt level B is measured by the Long Term Debt variable (data item number 9). The price p is represented by the Close Price at the Fiscal Year-End (data item number 199) multiplied by the number of Common Shares Outstanding (data item number 25) because the number of shares is standardized to one in the model. As for the interest rate ι, the Interest Expense on Long Term Debt (data item number 101) is not available for most firms in Compustat. Instead, the interest rate ι is proxied by today's Interest Expense (data item number 15) divided by the sum of yesterday's Long Term Debt and yesterday's Debt in Current Liabilities (data item number 34). Finally, dividends D are measured as Common Dividends (data item number 21). These series are deflated by *U.S. Bureau of Labor Statistics*' producer price index.

4.3 Numerical Method

The model's equilibrium cannot be solved analytically, but can be approximated using numerical methods. Because the default indicator defined by equation (3) introduces so much curvature in the policy functions, the solution is approximated with finite element methods following Coleman's (1990) algorithm. Accordingly, the policy functions K', B', p, and ι' are approximated by piecewise linear interpolants of the state variables K, B, ι, and θ. Because the consumer is risk neutral, the endogenous state variables K, B, and ι do not appear in the pricing and decision equations. Thus, the four-dimensional interpolant effectively simplifies to a unidimensional one.

The state variable θ is discretized using a uniform grid. This grid consists of ten uniformly-spaced points between the unconditionally lowest outcome of the technology state $\theta_l = [A\exp(-\sigma)]^{\frac{1}{(1-\rho)}}$ and its unconditionally highest outcome $\theta_h = [A\exp(\sigma)]^{\frac{1}{(1-\rho)}}$.

The approximation coefficients of the piecewise linear interpolants are chosen by collocation, i.e., to satisfy the Euler equations at all grid points. The approximated policy interpolants are substituted in the Euler equations (1), (8), (9), (10) and the coefficients are chosen such that the Euler residuals are set to zero at all grid points. The time-stepping algorithm is used to find these root coefficients. Given initial coefficient values for all grid points, the time-stepping algorithm finds the optimal coefficients that minimize the Euler residuals at one grid point, taking coefficients at other grid points as given. In turn, optimal coefficients for all grid points are determined. The iteration over coefficients stops when the maximum deviation of optimal coefficients from their previous values is lower than a specified tolerance level, e.g., 0.0001.

The numerical integration involved in computing the Euler residuals is approximated with a Gauss-Hermite quadrature rule. Only two quadrature nodes are used, reducing the stochastic process to a binary process in which an up move of σ occurs with probability 1/2 and a down move of $-\sigma$ occurs with probability 1/2.

Following the homotopy principle, according to which policy functions of a well-behaved problem are approximated, the indicator function (3) is transformed to

$$1_{(V_u \geq 0)} \approx \frac{1}{1+\exp\left(-sV_u\right)}.$$

The policy approximations are first solved using a small slope, e.g., $s = 1$. Starting values of the policy approximation coefficients are set to the deterministic steady state. Then, the slope is iteratively increased to a large value $s = 1000$, using coefficients from the previous iteration as starting values. Increasing the slope beyond $s = 1000$ does not affect the solution.

4.4 Simulation

The policy series K', B', p, and ι' are simulated from random outcomes of technology shocks ϵ. Policy series are generated for 1603 firms of 100 periods, keeping the last 20 periods to replicate the Compustat sample length of 20 years. From these policy series, investment I, cash flows $CF = (1 - \tau_f)(f(K;\theta) - \delta K - \iota B)$, Tobin's $Q = \frac{p+B}{(1-\tau_f)K}$, dividend D_+ (where the $_+$ indicates that the dividend series does not include rights issues), and income $Inc = f(K;\theta) - \delta K$ are computed.

Table 1: Calibration of the Revenues Function

	10-year sample	8-year sample
α	0.4365 (0.7081)	0.4295 (0.9320)
A	2.9679 (3.2524)	2.9164 (3.7617)
ρ	0.5866 (0.3322)	0.5048 (0.3875)
σ	0.1836 (0.1611)	0.1885 (0.1639)

Note: α is the sensitivity of revenues to asset base variations. A, ρ, and σ are the level, persistence, and volatility parameters of the technology process. Standard deviations appear in parenthesis.

Table 2: Descriptive Statistics from the Compustat Sample

	I	$\Delta B'$	θK^{α}	D	r	ι'
mean	59.9474	7.7573	883.1633	17.9536	0.2229	0.1464
standard deviation	22.9612	45.0239	219.2099	7.9950	0.6300	0.1732
correlation						
I	1.0000					
$\Delta B'$	0.2723	1.0000				
θK^{α}	0.4320	0.0609	1.0000			
D	0.3666	0.1056	0.4805	1.0000		
r	-0.0710	-0.0266	0.0355	-0.1066	1.0000	
ι'	-0.0371	-0.0879	-0.1392	-0.0713	0.0290	1.0000

Note: I is investment, $\Delta B'$ is the new debt issue, θK^{α} is revenues, D is the dividend paid to equity claimants, r is the equity rate of return, and ι' is the promised interest rate. All level variables are reported in millions of dollars.

Table 3: Descriptive Statistics Simulated from the Model

	I	$\Delta B'$	θK^α	D_+	r	ι'
mean	4.0809	0.0420	15.5025	1.1353	0.1688	0.0658
standard deviation	1.8952	8.8701	3.9525	1.6249	0.4182	0.0000
correlation						
I	1.0000					
$\Delta B'$	0.9955	1.0000				
θK^α	0.3245	0.2420	1.0000			
D_+	0.6292	0.5811	0.7929	1.0000		
r	0.9514	0.9304	0.4507	0.7049	1.0000	
ι'	NaN	NaN	NaN	NaN	NaN	NaN

Note: r is the equity rate of return, ι' is the promised interest rate, I is investment, $\Delta B'$ is the new debt issue, θK^α is revenues, D_+ is the dividend paid to equity claimants and does not include rights issues, r is the equity rate of return, and ι' is the promised interest rate. The above statistics are based on the benchmark calibration, where $\beta = 0.95$, $\delta = 0.1$, $\tau_f = 0.4$, $\tau_\iota = 0.2$, $X = 0.1$, $\alpha = 0.4365$, $A = 2.9679$, $\rho = 0.5866$, $\sigma = 0.1836$, and $F = 9.5$. NaN means Not a Number.

Table 4: Investment Distortion Caused by Financing Frictions

	$I - I_{mm}$	$V - V_{mm}$
mean	0.7966	-8.4701
% of V_{mm}	(13.61)	(14.88)

Note: $I - I_{mm}$ is the amount and $V - V_{mm}$ is the value of the investment distortion caused by the presence of a tax benefit and a default cost of debt. mm denotes the Modigliani and Miller framework of no financing friction described in Section 2.3.1, while variables without subscripts refer to the benchmark model with financing frictions of Section 2.1. F is calibrated at 9.5 such that the benchmark model replicates the mean debt-to-asset ratio observed in the data.

Table 5: Debt Overhang

	$I_m - I_{mm}$	$V_m - V_{mm}$
mean	-2.8055	-34.9806
% of V_{mm}	(8.23)	(92.23)

Note: $I_m - I_{mm}$ is the amount and $V_m - V_{mm}$ is the value of debt overhang. m denotes the Myers framework of investment decisions with arbitrary debt-in-place described in Section 2.3.2, while mm refers to the first-best Modigliani and Miller framework described in Section 2.3.1. F is calibrated at 8.0216.

Table 6: Debt Overhang with Optimal Debt

	$I_s - I_{mm}$	$V_s - V_{mm}$
mean	0	-11.5729
% of V_{mm}	(0)	(47.54)

Note: $I_s - I_{mm}$ is the amount and $V_s - V_{mm}$ is the value of debt overhang with debt optimally put in place. s denotes the sequential framework of investment decisions following optimal debt financing decisions described in Section 2.3.3, while mm refers to first-best Modigliani and Miller framework described in Section 2.3.1. F is calibrated at 9.2 such that the sequential model replicates the mean debt-to-asset ratio observed in the data.

Table 7: Investment Distortion Caused by Sequential Decisions

	$I_s - I$	$V_s - V$
mean	-0.7787	-0.6039
% of V	(1.68)	(9.04)

Note: $I_s - I$ is the amount and $V_s - V$ is the value of distortion caused by debt in place. s denotes the sequential framework of investment decisions following optimal debt financing decisions described in Section 2.3.3, while variables without subscript refer to the benchmark model of simultaneous investment and debt decisions of Section 2.1. F is calibrated at 9.2 such that the sequential model replicates the mean debt-to-asset ratio observed in the data.

Table 8: Cash Flow Sensitivity Results of Fazzari, Hubbard, and Petersen (1988)

	Class 1 Most Constrained	Class 2	Class 3 Least Constrained
$\frac{I}{K}$: mean	0.26	0.18	0.12
standard deviation	0.17	0.09	0.06
$\frac{CF}{K}$: mean	0.30	0.26	0.21
standard deviation	0.20	0.09	0.06
regress $\frac{I}{K}$ on:			
$\frac{CF}{K}$	0.461	0.363	0.230
	(0.027)	(0.039)	(0.010)
Q	0.0008	0.0046	0.0020
	(0.0004)	(0.0009)	(0.0003)

Note: K denotes the capital stock, I the investment, CF the cash flow, and Q is Tobin's average q. Standard errors are in parenthesis.

Table 9: Cash Flow Sensitivities of Unconstrained Firms

	$\frac{I}{K}$	$\frac{CF}{K}$	Q	θ	$\frac{D_+}{Inc}$	Intercept
median	0.1000	0.0027	1.0351	1.0029	0.0000	
mean	0.1235	0.0049	1.0030	1.0300	0.2055	
standard deviation	0.2140	0.0511	0.2569	0.2336	0.2446	
correlation	1.0000					
	0.4726	1.0000				
	-0.4576	0.5546	1.0000			
	0.4427	0.9867	0.5505	1.0000		
	0.6770	0.8388	0.2168	0.8173	1.0000	
regress $\frac{I}{K}$ on:		4.5348	-0.8754			0.9843
		(0.0027)	(0.0005)			(0.0006)

Note: K denotes the capital stock, I the investment, $CF = (1-\tau_f)(f(K;\theta)-\delta K - \iota B)$ the cash flow, $Q = \frac{p+B}{(1-\tau_f)K}$ Tobin's average q, θ the technology state, D_+ the dividend paid to the equity claimants, and $Inc = f(K;\theta)-\delta K$ the income. Standard errors are in parenthesis.

Table 10: Cash Flow Sensitivities of Constrained Firms

	$\frac{I}{K}$	$\frac{CF}{K}$	Q	θ	$\frac{D_+}{Inc}$	Intercept
median	0.0862	0.0139	0.7607	1.0268	0.9324	
mean	0.0914	0.0121	0.7481	1.0540	2.4248	
standard deviation	0.0804	0.0787	0.2737	0.2348	160.0979	
correlation	1.0000					
	0.4535	1.0000				
	0.0967	0.5259	1.0000			
	0.7539	0.3840	0.2979	1.0000		
	0.0030	0.0004	-0.0011	0.0003	1.0000	
regress $\frac{I}{K}$ on:		0.5715	-0.0881			0.1500
		(0.0119)	(0.0031)			(0.0022)
investment-unconstrained:		0.5444	-0.0860			0.1491
		(0.0117)	(0.0030)			(0.0021)
investment-constrained:		1.3514	-0.4257			0.1491
		(0.0504)	(0.0350)			(0.0021)

Note: K denotes the capital stock, I the investment, $CF = (1-\tau_f)(f(K;\theta) - \delta K)$ the cash flow, $Q = \frac{p}{(1-\tau_f)K}$ Tobin's average q, θ the technology state, D_+ the dividend paid to the equity claimants, and $Inc = f(K;\theta) - \delta K$ the income. Standard errors are in parenthesis.

Table 11: Main Results of the Cash Flow Sensitivity Literature

Literature	Result
Empirical	
FHP	Lower $\frac{D_+}{Inc}$ identifies more constrained firms. More constrained firms have larger $\frac{CF}{K}$ sensitivities. Lower $\frac{D_+}{Inc}$ firms have larger $\frac{CF}{K}$ sensitivities.*
KZ	Lower $\frac{D_+}{Inc}$ does not identify more I-constrained firms. More I-constrained firms have lower $\frac{CF}{K}$ sensitivities.
HKS	More constrained firms have larger $\frac{CF}{K}$ sensitivities.
GH	$\frac{CF}{K}$ sensitivities not because Tobin's Q mismeasures θ.
CHO	$\frac{CF}{K}$ sensitivities because Tobin's Q mismeasures θ.*
Theoretical	
Gr, PR	More I-constrained firms have larger $\frac{CF}{K}$ sensitivities.*
Go	$\frac{CF}{K}$ sensitivities because Tobin's Q mismeasures θ.*

Note: FHP refers to Fazzari, Hubbard, and Petersen (1988), KZ to Kaplan and Zingales (1997), HKS to Hoshi, Kashyap, and Scharfstein (1991), GH to Gilchrist and Himmelberg (1995), CHO to Cummins, Hasset, and Oliner (1997), Gr to Gross (1995), PR to Pratap and Rendón (1998), and Go to Gomes (1998). I denotes investment, $\frac{CF}{K}$ the cash flow-to-asset ratio, Q Tobin's average q, θ the underlying investment opportunities, and $\frac{D_+}{Inc}$ the dividend-to-income ratio. A star * indicates that the second chapter provides evidence in support of the result.

Figure 1: Policy Functions

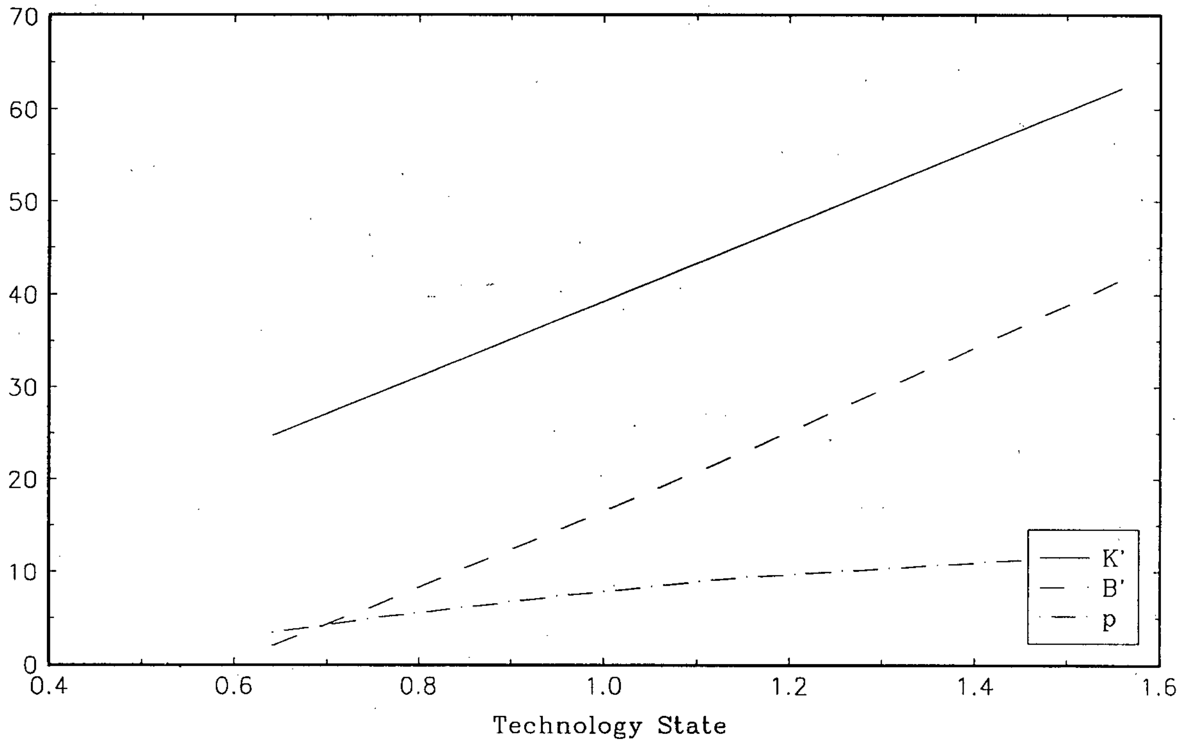

Figure 2: Minimum Beginning-of-the-Period Funds
CF+K-B

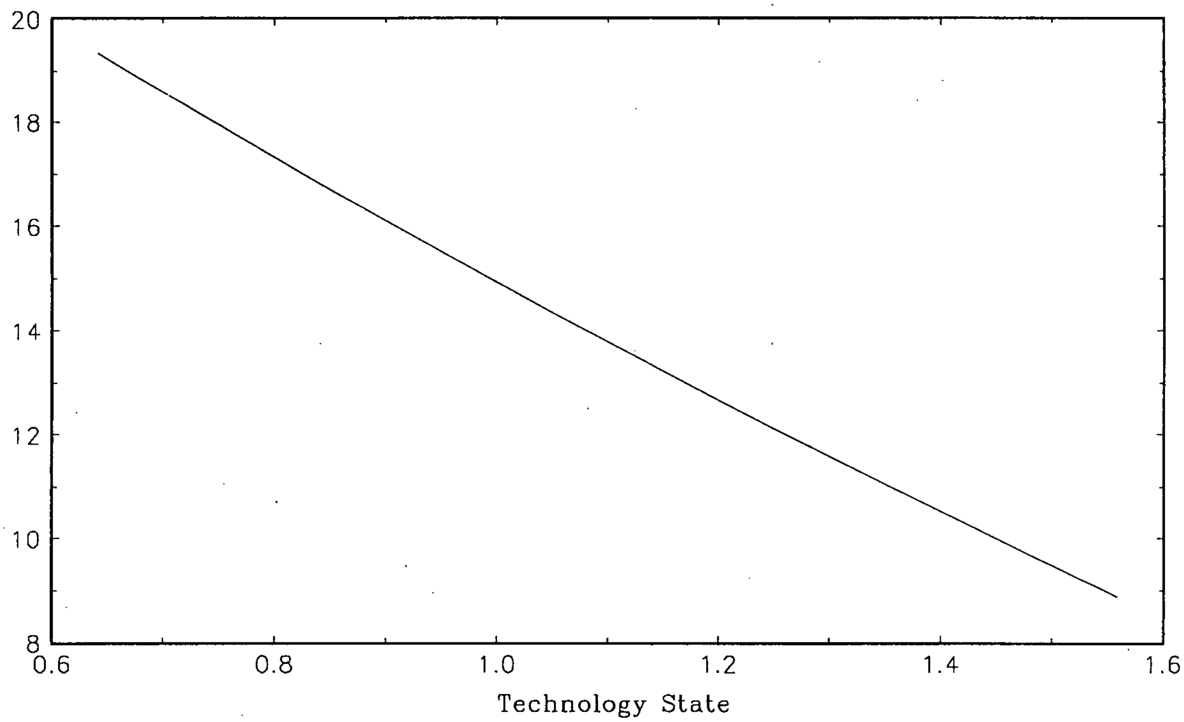

Figure 3a: Sensitivity Analysis α = 0.435

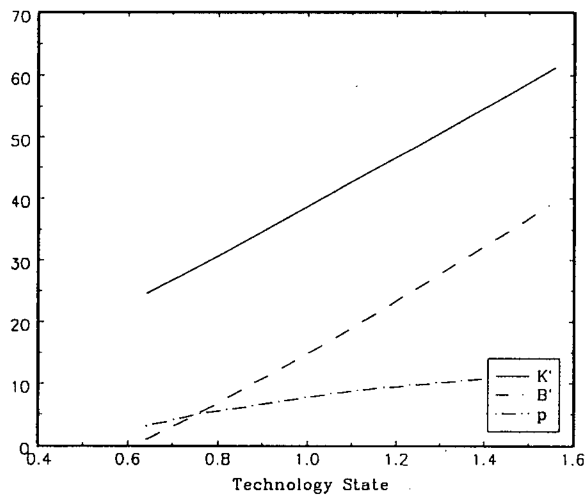

Figure 3b: Sensitivity Analysis α = 0.440

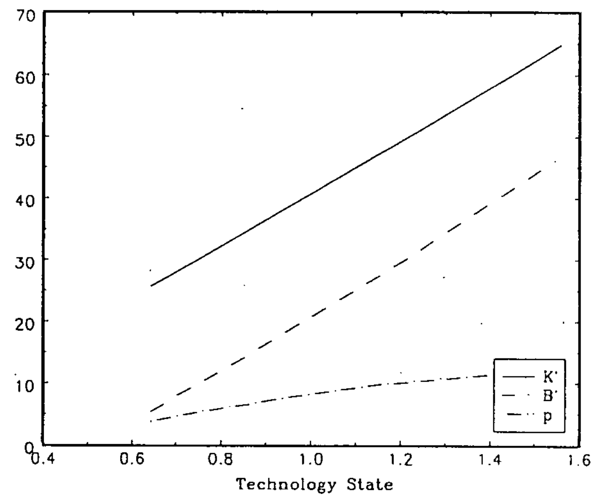

Figure 3c: Sensitivity Analysis A = 2.95

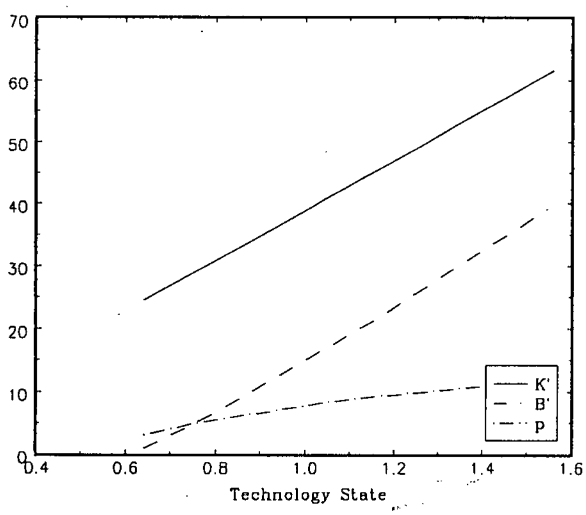

Figure 3d: Sensitivity Analysis A = 3.00

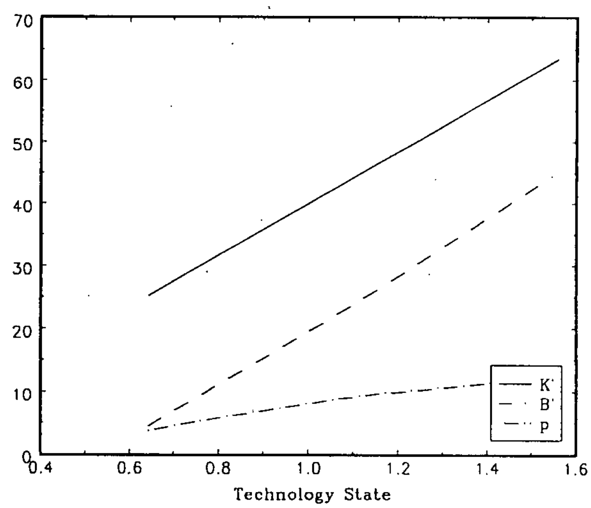

Figure 3e: Sensitivity Analysis ρ = 0.55

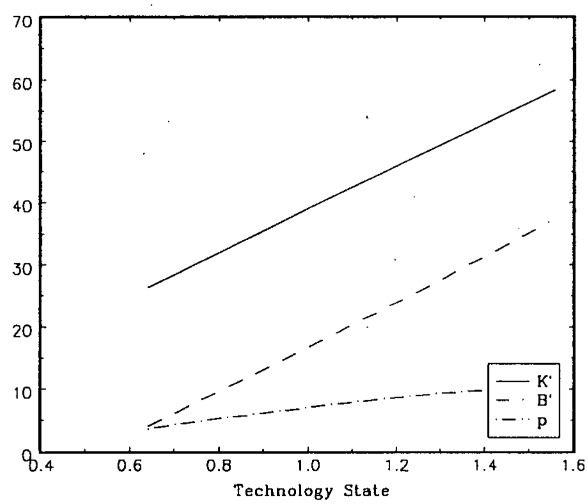

Figure 3f: Sensitivity Analysis ρ = 0.60

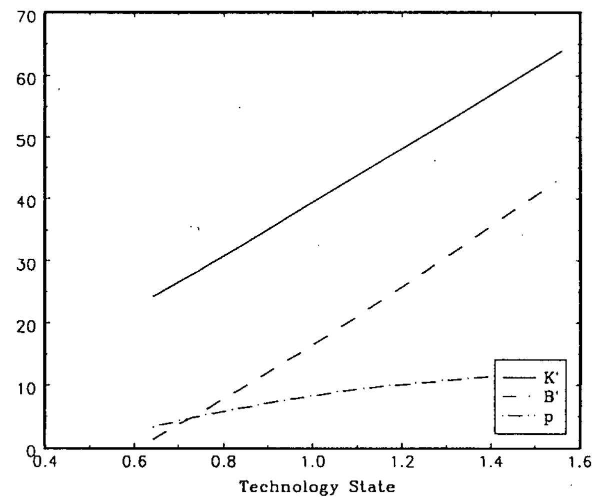

Figure 3g: Sensitivity Analysis σ = 0.15

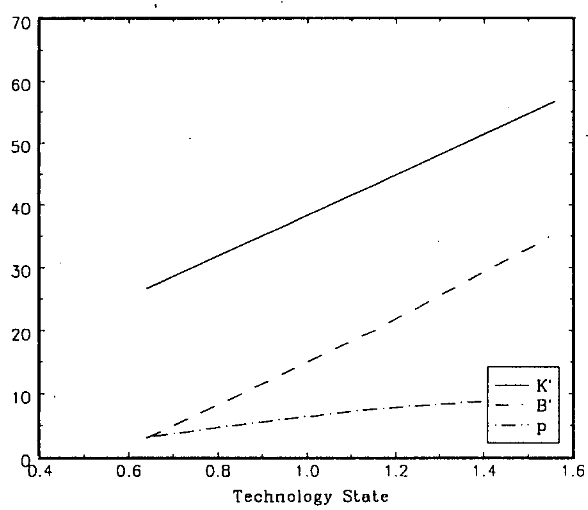

Figure 3h: Sensitivity Analysis σ = 0.20

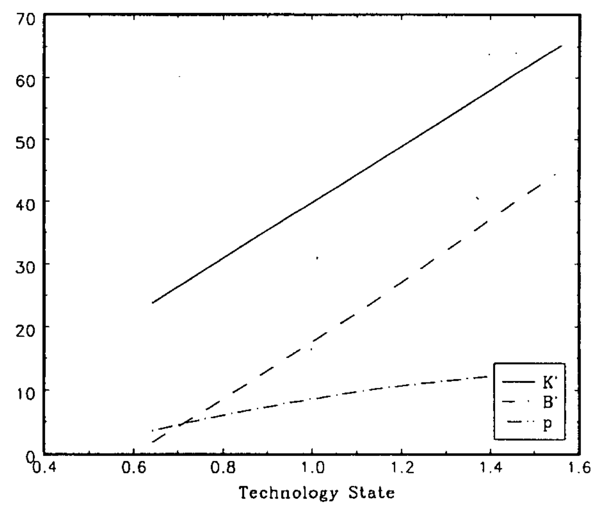

Figure 3i: Sensitivity Analysis τ_i = 0.15

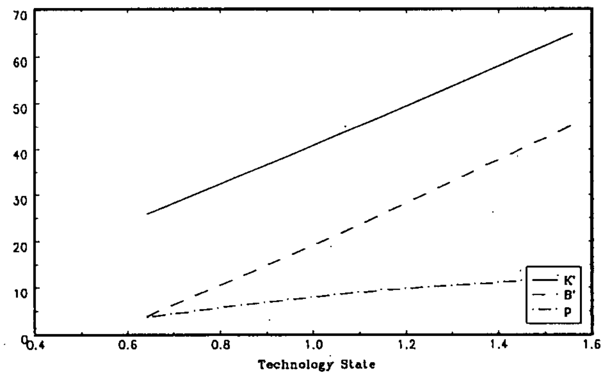

Figure 3j: Sensitivity Analysis τ_i = 0.25

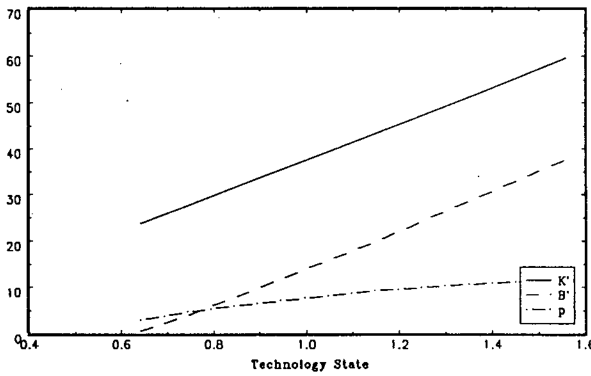

Figure 3k: Sensitivity Analysis τ_f = 0.35

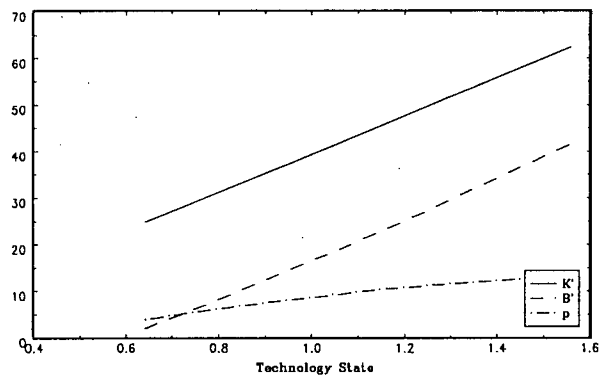

Figure 3l: Sensitivity Analysis τ_f = 0.45

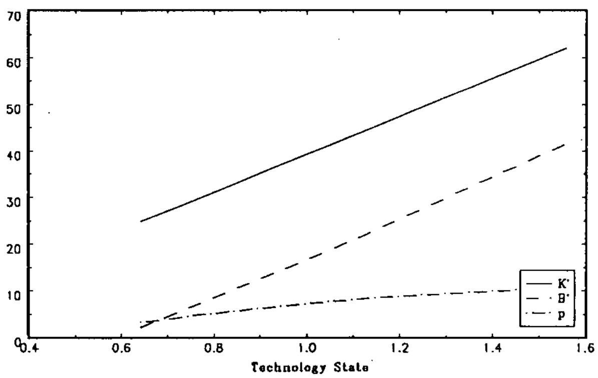

Figure 3m: Sensitivity Analysis X = 0.05

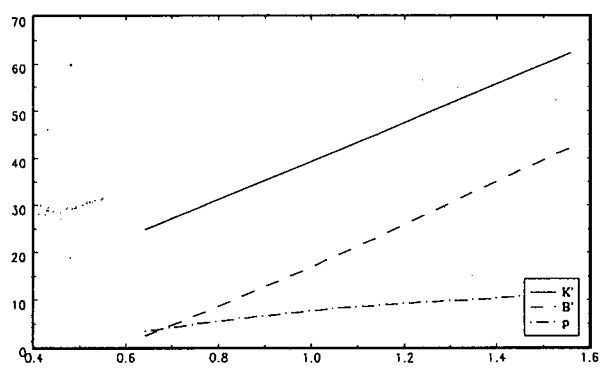

Figure 3n: Sensitivity Analysis X = 0.15

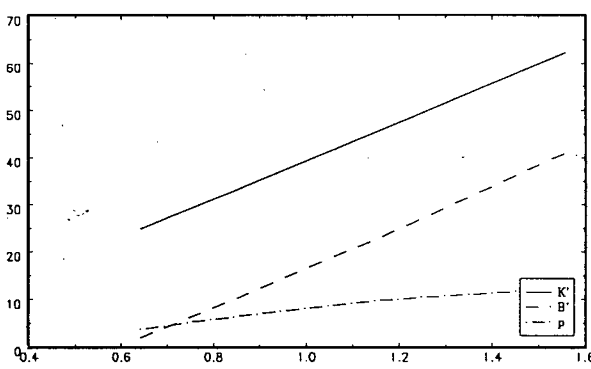

Figure 4: Investment Distortion Caused by Financing Frictions

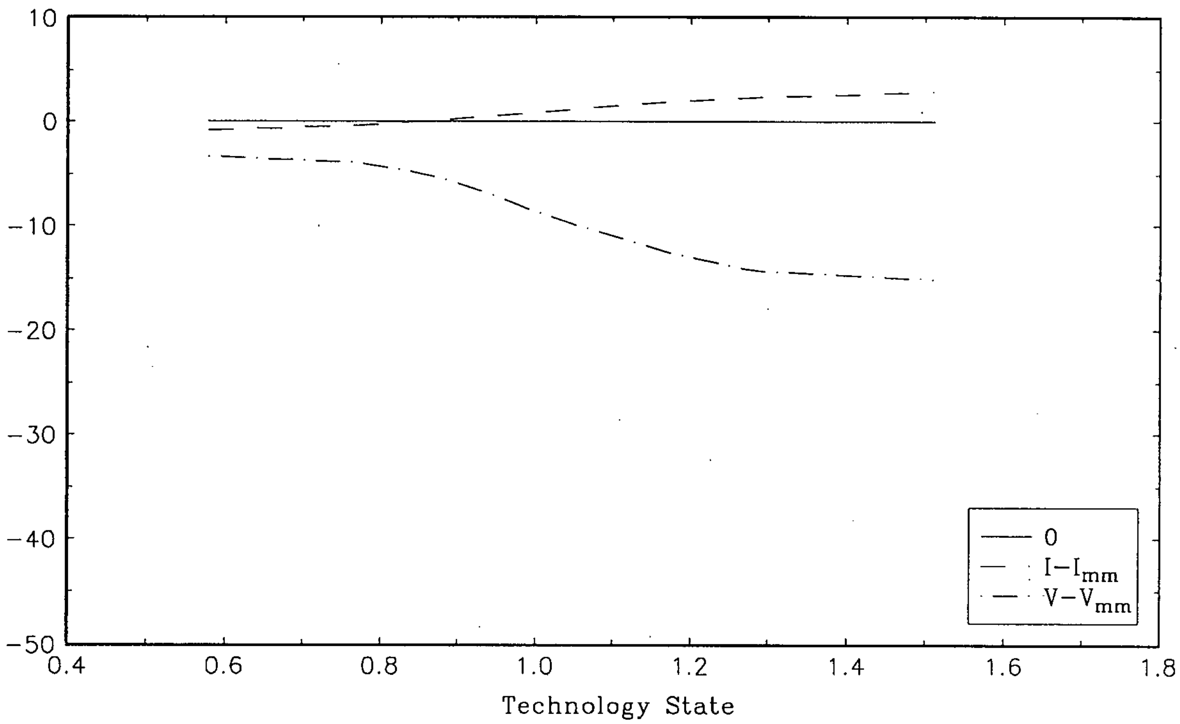

Figure 5: Debt Overhang

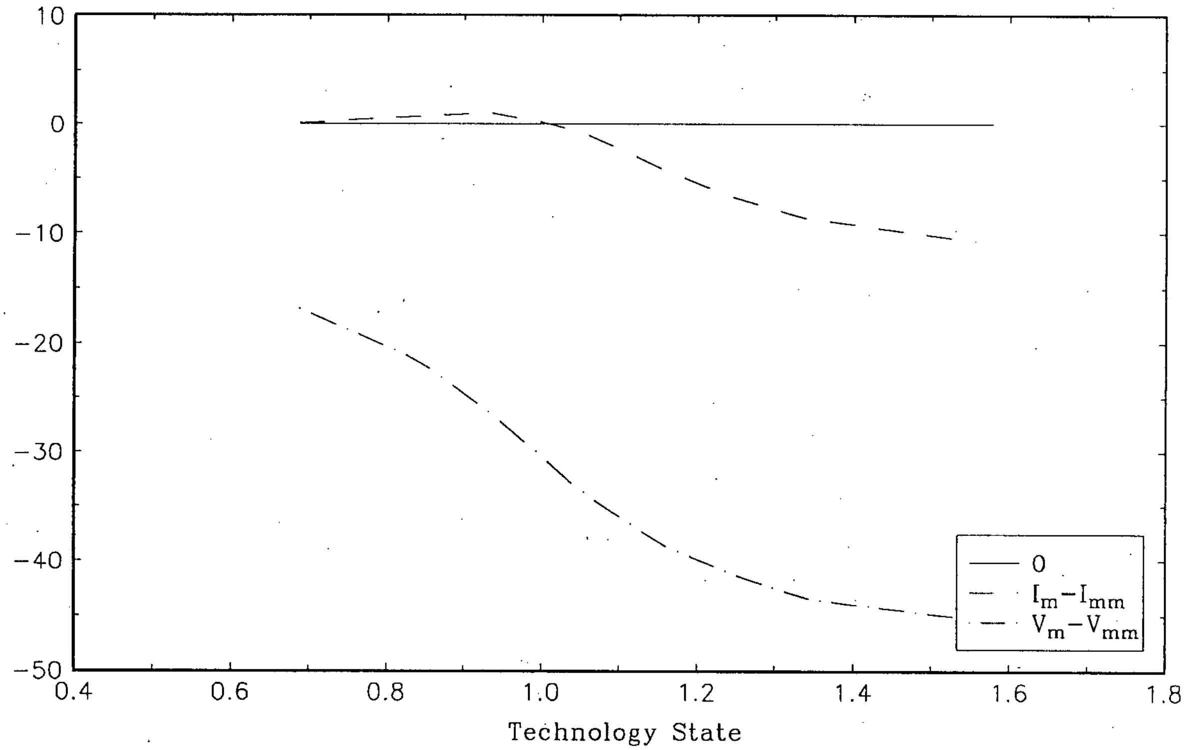

Figure 6: Debt Overhang with Optimal Debt

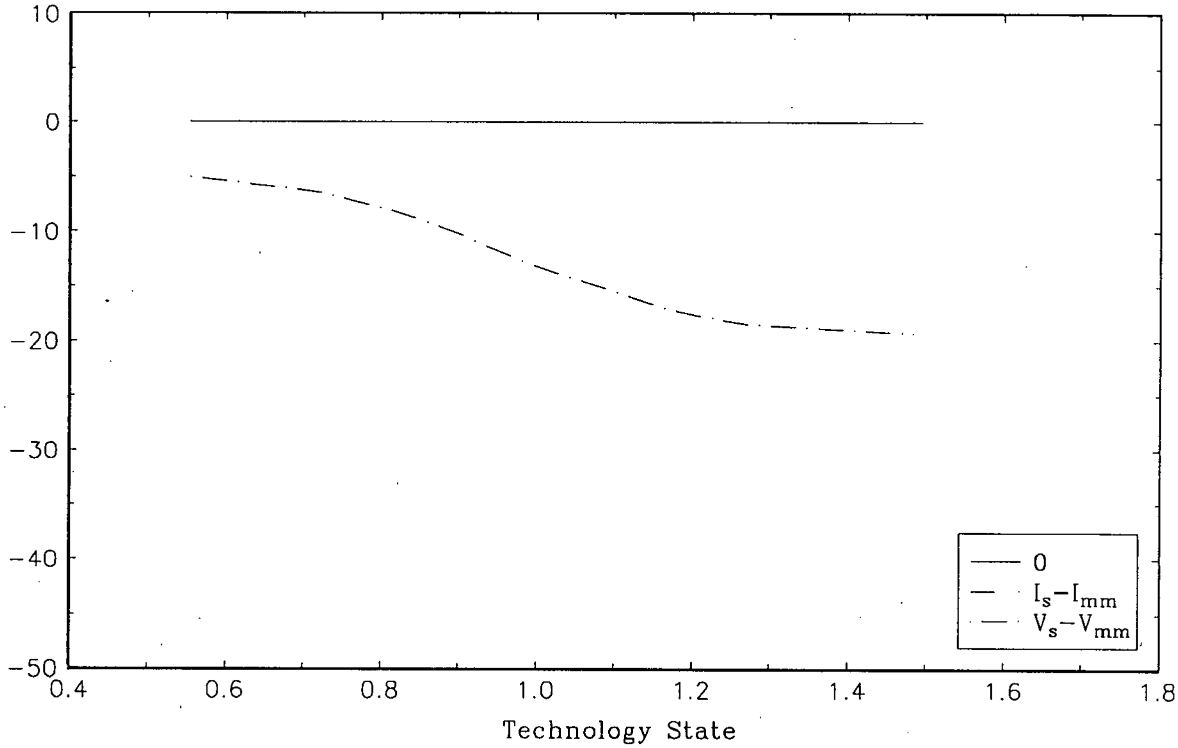

Figure 7: Investment Distortion Caused by Sequential Decisions

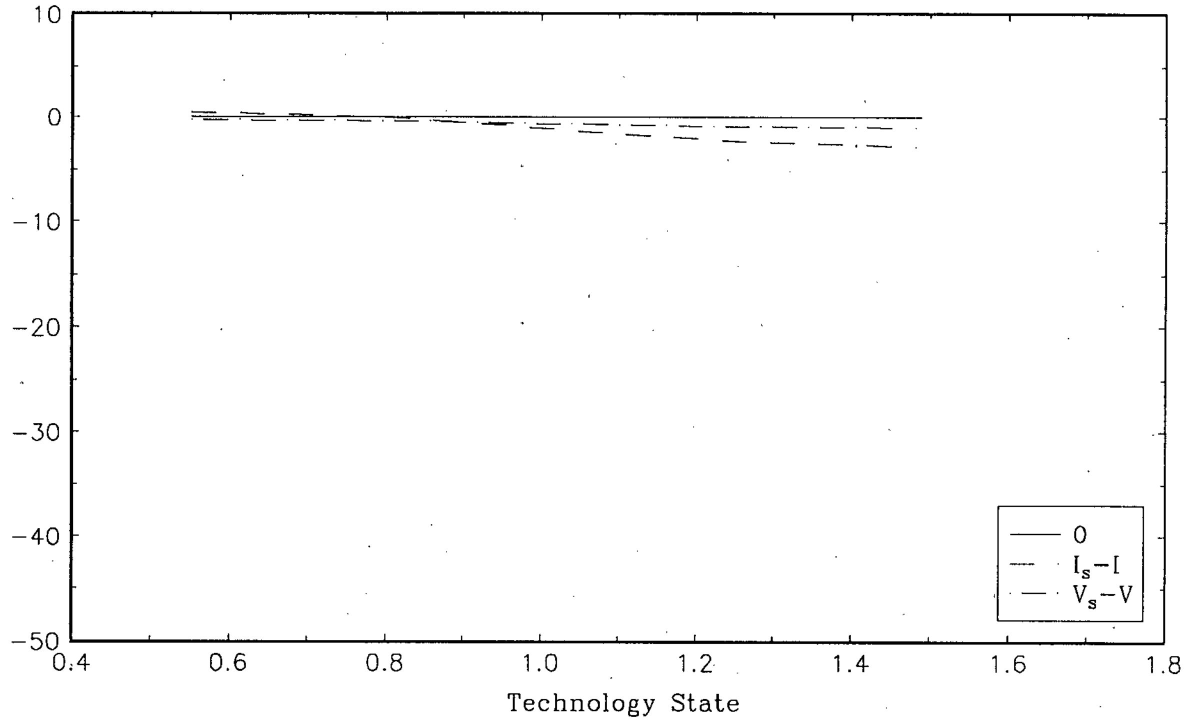

Figure 8: Policy Functions of Unconstrained Firms

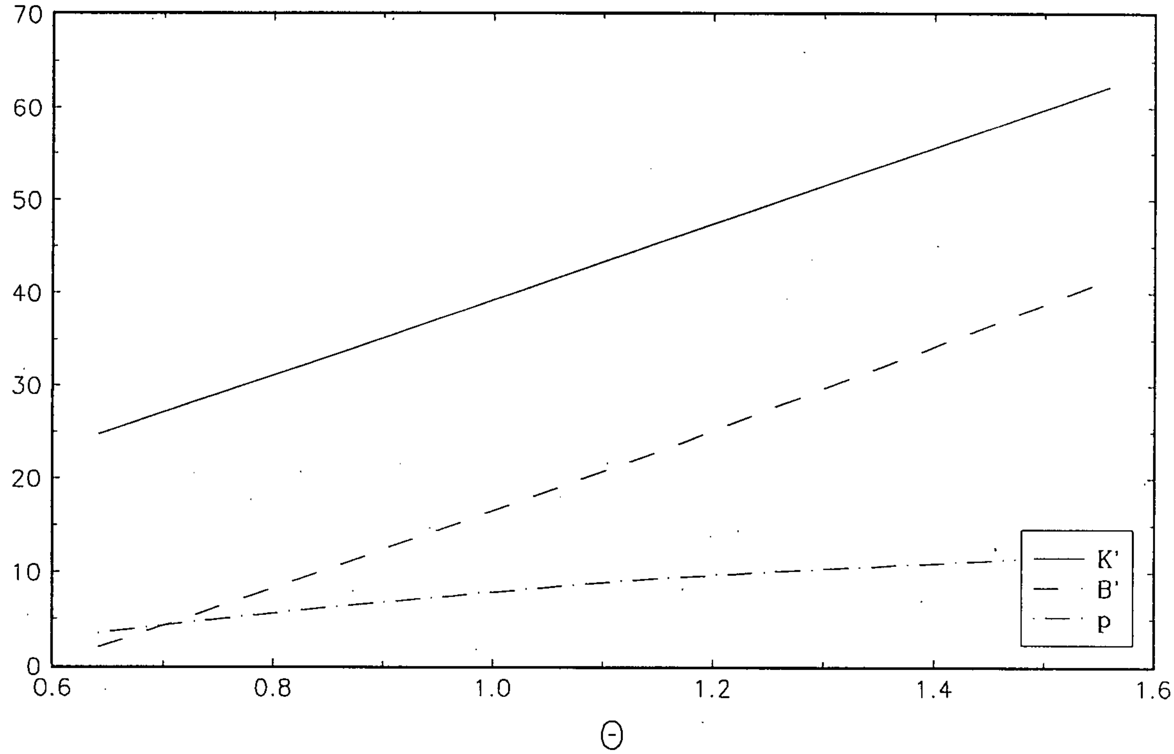

Figure 9: Policy Functions of Constrained Firms