CAVITY PERTURBATION MEASUREMENTS
OF THE SHUNT IMPEDANCE OF A TUNED RESONATOR
WITH HIGHER-ORDER-MODE DAMPING

by

David Noah Joffe

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Department of PHYSICS

The University of British Columbia
Vancouver, Canada

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Abstract

The performance of a prototype higher-order-mode (HOM) damper for the proposed KAON Factory Booster Ring was determined by measuring the shunt impedance of a variable frequency tuned resonator with higher-order-mode damping at both the fundamental and higher resonant frequencies using the cavity perturbation technique. This technique consists of pulling a spherical dielectric bead across the cavity gap and recording the change in the frequency of the cavity resonance at each point along the path. Using the frequency shift values, the electrical properties of the bead, and the quality factor and frequency of the non-perturbed cavity resonance, one can find the relative electric field strengths along the path across the cavity gap, and these may be integrated to give the shunt impedance of the structure.

An automated system for performing cavity perturbation measurements was constructed with a single computer both controlling the pulling of the bead and recording the properties of the cavity resonance. This system was tested for accuracy on a cavity with no damper attached, and the effects of different cavity configurations were examined.

The impedances of two closely related designs of prototype higher-order-mode dampers were measured using the automated system. The first damper consisted of a five-element high-pass filter made up of three washer-like structures encircling the cavity axis, and the second damper consisted of a seven-element filter similar to the first but with the final two elements being made up of an additional washer and pipe structure (the horn) which were added in order to shield the structure from the beam. The two structures were found to have nearly identical properties in terms of their shunt impedances and abilities to damp higher-order-modes in the cavity.

Both damper designs were found to lower the shunt impedance of the cavity gap by less than 10% at the fundamental frequency, and by more than 99% for all higher-order-modes up to the 700 MHz range. Above 700 MHz the dampers were found to lower the shunt impedance of TEM modes by about 90%, and of the first TE mode by 50%. In all cases the shunt impedances of the system at frequencies above the fundamental were below 500 Ω, well below the 1 kΩ maximum allowable in order to maintain a stable beam [1, 2].
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Introduction

In the past 20 years, accelerator physics has seen a tremendous increase in the size and scale of new machines produced. With each new leap in energy or intensity, however, design problems which could previously be ignored or tolerated may become more and more detrimental to the proper functioning of the system. Eliminating potentially damaging instabilities has thus become an important design consideration.

Whether or not a beam is made unstable through interaction with the electromagnetic field set up purposely by each accelerator component, or through the interaction of the beam's own field with the physical structures around it, potential instabilities can be examined quantitatively by using the concept of impedance: the potential difference seen by the beam for a given beam current level.

In the case of the Booster cavities, which are designed to provide a strong electric field parallel to the beam path in order to give additional energy to the beam, some of the most likely destabilizing impedances are those caused by the higher-order modes of the cavity [3]. The strength of the effect of these modes, which exist at frequencies many times higher than the Booster's operating frequency, can be analyzed using the concept of shunt impedance; the impedance measured along the beam path across the cavity gap for any given frequency.

Many types of structures have been designed to lessen the effects of these higher frequency impedances. These higher-order-mode dampers, as they are called, all share the characteristic of being designed to absorb, or damp, the energy of the higher-order modes. Unfortunately, while they may do this quite successfully, many dampers also absorb significant amounts of energy from the Booster cavity at the fundamental mode as well, the mode in which the cavity transfers energy to the beam. Thus the overall rf efficiency of the cavity may be reduced even while the instabilities producing higher-order modes are damped. Furthermore, existing dampers tend to damp the higher-order modes only at specific fixed
frequencies, not the broad range required in cavities meant to operate over a sizable range of frequencies, such as the ferrite-tuned cavity designed for the proposed KAON Factory Booster Ring at TRIUMF [4].

In light of this problem, a new design for a higher-order-mode damper was developed at TRIUMF which would both conserve cavity rf efficiency, and operate at all frequencies above a certain cut-off point [5 - 8]. This new damper was meant to act as a multi-element high-pass filter, absorbing all energy from fields above the corner frequency of the filter, while leaving the fields at the fundamental frequency as undisturbed as possible. To test the effectiveness of this new type of damper it was necessary to measure the shunt impedance of the combined cavity/damper system at both the fundamental frequency and at the frequency of the higher-order modes.

This thesis describes the measurement of the shunt impedance of a test cavity with a model of the damper described above attached to it. The shunt impedance was measured using the cavity perturbation technique, in which a bead is pulled across the cavity gap, and the resulting change in the resonant frequency of the cavity is used to calculate impedance [9]. Chapter 1 deals with the fields in a resonating cavity, both at the fundamental and at the higher-order modes, as well as introduces the concept of higher-order-mode damping. Chapter 2 develops the theory of cavity perturbation measurements, and outlines how the perturbation technique may be used to measure shunt impedance. Chapter 3 describes the actual apparatus used to perform the cavity perturbation measurements and some of the difficulties encountered. Chapter 4 discusses the accuracy of perturbation measurements and strategies for improving the quality of the measurements. Chapter 5 describes how the apparatus was tested for accuracy and examined to check the consistency of results given a variety of different cavity conditions. Chapter 6 gives the results of the measurements of the shunt impedance of the model damper/cavity system performed using the cavity perturbation apparatus, followed by a discussion of their effectiveness and a comparison of the two
proposed damper designs. Finally, chapter 7 looks at other applications of, and improvements to, both the measurement system, and the prototype damper.
1 Shunt Impedance and Higher-Order-Mode Damping

1.1 Coaxial Cavities and Shunt Impedance

Coaxial cavities make up one of the most important classes of resonating structures used in particle accelerators. These cavities consist of a coaxial transmission line which can support both a principal mode of wave propagation along the transmission axis as well as a variety of higher-order modes which may or may not propagate along the cavity axis. For a transmission line which is terminated either by open or short circuits at both ends, the fundamental resonating frequency is determined by the overall length of the transmission line, and the type of terminations. The dominant mode of this structure will be that which has the fewest number of nodes/antinodes in the field pattern. For the doubly short-circuited transmission line, the dominant mode will have current maxima at either end, and for the transmission line which is open on both ends, the dominant mode will have voltage maxima at either end. In both cases, the longest resonant wavelength supported by the cavity will be twice the length of the transmission line, as one full wavelength is twice the distance between two adjacent current or voltage maxima. Since the length of the line is half that of the longest wavelength (\(\lambda\)), these types of cavities are known as \(\lambda/2\) structures.

An even more common type of coaxial structure is that in which the line is terminated by an open circuit at one end and a short circuit at the other. In this case the open-circuit end will have a voltage maximum and the short-circuit end a current maximum. Since maxima alternate between voltage and current, the distance between two adjacent voltage maxima will be twice the distance between a voltage maximum and the nearest current maximum. Thus the longest resonant wavelength that the cavity will support is four times the cavity length (twice the distance between two adjacent similar maxima). For this reason these types of cavities are known as \(\lambda/4\) or quarter-wavelength structures. It is this type of structure, the coaxial quarter-wavelength resonator, that will be dealt with in this thesis, although the concepts of shunt impedance, cavity perturbation measurements, and higher-order-mode
damping developed in Chapters 1 and 2 are applicable to a much broader range of resonating and accelerating structures.

Coaxial $\lambda/4$ cavities may be used to accelerate a beam of particles in a number of ways. In the first case the beam may be passed along the cavity axis, traveling in the field free zone inside the inner conductor, until it reaches the open-circuit end or voltage maximum. At that point the beam passes across the high-voltage gap in between the inner and outer conductors, with some gain in energy. The cavity in the proposed KAON Factory Booster Ring represents a modified example of this configuration, where ferrites have been added to the space between the two conductors near the short-circuited end in order to be able to modify the fundamental resonating frequency quickly, without changing the physical dimensions of the cavity.

Another possibility is to pass the beam perpendicularly to the cavity axis through a slice made along the side of the cavity. In this case the beam will cross two voltage gaps, first traveling from the outer to the inner conductor, passing through the field-free region inside the inner conductor, and then passing across the gap from the inner to the outer conductor. For this type of cavity the coaxial symmetry may have to be modified to ensure that the beam is in phase with the voltage across both the first and second gaps. Thus the cavity must be designed such that the time required for the beam to pass through the inside of the inner conductor is approximately one half of the total period of the resonance, enough time for the polarity of the field between the inner and outer conductors to reverse itself. The booster cavity inside the TRIUMF cyclotron is an example of this second type of cavity. Figure 1-1 shows the difference between the two types of structures, as well as an example of each. Although both configurations may be used as accelerating cavities, it is the first case, in which the beam travels along the cavity axis, which will be examined in this thesis.

The amount of energy transferred from the resonating cavity to the beam is determined by the voltage seen by the beam across the gap. Unfortunately, determining gap voltages at the frequencies at which many booster cavities operate (tens or hundreds of MHz)
can be a difficult undertaking. In order to compare the effects of different structures on the beam quantitatively without comparing input power levels, the concept of shunt impedance is used. The general definition of shunt impedance is the ratio of the voltage across a gap to the current. In general this shunt impedance will be a complex quantity dependent on frequency, however in the special case of a resonant frequency, the shunt impedance will always be real or purely resistive. In this case the shunt impedance across a given gap can be calculated as the square of the peak voltage seen by the beam across the gap divided by twice the input power level.

\[ R_S = \frac{V^2}{2P} \]  

(1)

As will be seen in the following derivation for the case of the coaxial \( \lambda/4 \) resonator, the shunt impedance can be completely determined by the physical structure of the cavity, and is not altered by a change in the input signal. Thus it represents the fundamental quantity which is used to measure the strength of the effect of any given structure on a beam of charged particles.

The concept of shunt impedance is useful for describing not only the basic potential difference seen by the beam across the gap, but also for effects caused by interactions between the beam's electromagnetic field and the cavity structure itself. The beam in a circular accelerator usually consists of a string of "bunches" at time intervals of one rf period. Any charged particle or group of charged particles, such as a bunch, passing through a conducting structure, will induce image currents on the surface of the conductor. The field set up by this current is known as the wakefield. For structures with axial symmetry, it is useful to separate this wakefield into its longitudinal and transverse components. The wakefields are transient, but will affect particles passing through the structure at a slightly later time, particularly the next bunch. Calculating the effects of such a wakefield for a large group of particles such as a beam, however, is a complicated task, as the fields themselves depend on the original beam current.
One would therefore like a method for describing the effect of the wakefield interaction on the beam without referring to the specific properties of the beam itself. To do this one takes the wakefield of a single particle in the ultra-relativistic limit, in which its charge distribution approaches that of a Dirac $\delta$-function. The wakefield of a group of particles is made up of a superposition of these $\delta$-function wakefields. Fourier transforms of the time-dependent longitudinal and transverse wakefields can then be taken; the results are the longitudinal and transverse shunt impedances $Z(\omega)$ which depend only on the frequency and the properties of the surrounding structure and not on those of the beam [10]. For structures such as the coaxial quarter-wavelength resonators described above, which support the strongest fields along the beam axis, it is the longitudinal shunt impedances which are of most concern.

Any mode which can exist in the cavity could potentially be set up by the wakefield, and thus one would like to know the value of these impedances for all the resonating modes of the cavity. The higher-order modes are of particular concern because their frequencies are commensurate with those of coupled-bunch instabilities, in which the wakefield of one bunch, passing through the cavity structure, has a coherent effect on the following bunch [11]. Unfortunately, the frequencies at which these instabilities can occur are not fixed relative to the operating frequency of the cavities, which increases by 33% as the protons are accelerated. Thus the whole spectrum of higher-order cavity modes will shift relative to the spectrum of beam instability frequencies, greatly increasing the chance of a coincidence, driving rf power into a beam instability. Moreover the effect is a mutual one between two oscillating systems; the beam mode excites the cavity mode and vice-versa, leading to exponential growth. It is thus important to eliminate any higher-order modes of the cavity which could be triggered by the beam and couple to an instability. Thus the shunt impedance, the measure of the interaction between the beam and the cavity, must be sharply reduced at the frequencies of the higher-order modes.
Figure 1-1(a): $\lambda/4$ resonator with beam parallel to cavity axis

Figure 1-1(b): Cavity for proposed KAON Factory Booster Ring

Figure 1-1(c): $\lambda/4$ resonator with beam perpendicular to cavity axis

Figure 1-1(d): TRIUMF cyclotron booster cavity

Figure 1-1 - Types of Coaxial Cavities
Figure 1-2(a) - $\lambda/4$ coaxial resonator

Figure 1-2(b) - $\lambda/4$ coaxial resonator with five-element filter

Figure 1-2(c) - $\lambda/4$ coaxial resonator with seven-element filter

Figure 1-2 - Equivalent circuit of a coaxial cavity
1.2 Field Patterns in Coaxial Cavities

In order to find the shunt impedance of the λ/4 resonator, let us take a coaxial line of length \( l \), shorted at one end, and determine the energy stored in the fields at the fundamental frequency. The equivalent circuit of this resonator, given in Figure 1-2, shows a transmission line, terminated in a shunt impedance \( R_s \), but with capacitance per unit length \( C' \) and inductance per unit length distributed along the line. The amount of energy stored in the field inside the cavity will be a combination of the energy stored in the distributed capacitance and the energy stored in the distributed inductance. The energy stored per unit length in the distributed capacitance is given by the following formula in analogy with the formula for the energy stored in a capacitor in an ac circuit.

\[
dE/dx = V_x^2 \cdot C'/2 \tag{2}
\]

Here \( dE/dx \) is the energy stored per unit length, and \( V_x \) is the voltage between the two conductors at the point \( x \). To determine the voltage along the transmission line for the fundamental (quarter-wavelength) mode we make use of the fact that for TEM standing waves along a transmission line the voltage varies sinusoidally between a maximum and zero. Thus the total energy \( E \) stored in the capacitance is given by the following integral:

\[
E = \frac{1}{2} \int_0^l \frac{V_{\text{max}}^2}{2} \sin^2(2\pi\frac{x}{\lambda}) \cdot C' \, dx
\]

\[
= \left( \frac{l}{\pi} \right) \int_0^{\pi/2} \frac{V_{\text{max}}^2}{2} \sin^2(\theta) \cdot C' \, d\theta
\]

\[
= l \, C' \, V_{\text{max}}^2 / 4
\]

Here \( V_{\text{max}} \) is the voltage at the open end of the transmission line and \( \theta = 2\pi \) times the ratio of the position \( x \) to the wavelength \( \lambda \). The length of the cavity \( l \), which is also given by \( \lambda/4 \), can also be given in terms of the frequency \( \omega \) as:

\[
l = \pi c / 2\omega \tag{6}
\]

Thus the reactive power (the product of stored capacitive energy and frequency) can be given as:

\[
\omega E = \pi c C' \, V_{\text{max}}^2 / 8 \tag{7}
\]

But the distributed capacitance \( C' \) for a coaxial transmission line can be expressed as:
Here $Z_0$ is the characteristic impedance of the transmission line as given by the formula:

$$Z_0 = (1/2\pi) (\mu_0/\epsilon_0)^{1/2} \ln(b/a)$$  \hspace{1cm} (9)

Here $b$ and $a$ are the radii of the outer and inner conductors respectively \[12, 13\]. Thus in terms of the characteristic impedance, the reactive power can be given as:

$$\omega E = \pi \beta_{\text{max}}^2 / 8 Z_0$$  \hspace{1cm} (10)

The quality factor $Q$ of a resonator is defined as the reactive power divided by the input power:

$$Q = \omega E / P$$  \hspace{1cm} (11)

Rewriting equation (10) in terms of $Q$ and rearranging terms gives:

$$\beta_{\text{max}}^2 / 2PQ = 4Z_0 / \pi$$  \hspace{1cm} (12)

Remembering that the maximum voltage occurs across the gap, and that the shunt impedance is defined as the voltage across the gap squared divided by twice the input power level (equation (1)), we may rewrite equation (12) in terms of the shunt impedance as:

$$R_s / Q = 4Z_0 / \pi$$  \hspace{1cm} (13)

Thus for the coaxial $\lambda/4$ cavity resonating at its fundamental frequency, the shunt impedance across the gap can be determined from the $Q$ factor of the cavity, and its characteristic impedance. This derivation, however, does not take into account the stored energy created by the capacitance of the end piece added to the outer conductor of the transmission line at the open end in order to give a gap voltage parallel to the beam axis (as in Figure 1-1). This capacitance is known as the tip capacitance, and its effect on the shunt impedance is to lower it by an amount which can be calculated by using the value of tip capacitance as compared to the total capacitance of the cavity. For a derivation of the formulas used to determine the magnitude of this tip loading effect, see Appendix A.
To find the field profile of the fundamental resonance of the cavity we make use of a few important qualities of propagating electromagnetic waves. Since we are dealing with a wave whose direction of propagation is along the transmission line axis (in order to make the cavity a $\lambda = 4l$ type resonator) we must have the electric and magnetic fields only in the radial and circumferential directions in order that they remain perpendicular to the direction of wave propagation. This type of wave on a transmission line is known as a TEM or transverse electromagnetic wave, and TEM waves make up the most important class of field solutions for transmission type cavities such as our coaxial quarter wavelength resonator. The field profile for a TEM wave on a coaxial transmission line can be found by solving Maxwell's equations using the boundary conditions $\phi = V_0$ at the surface of the outer conductor and $\phi = 0$ at the surface of the inner conductor to obtain the following field profile:

$$E = V_0 a_r e^{jkr} / r \ln(b/a)$$  \hspace{1cm} (14)

$$H = V_0 a_\phi e^{jkr} / r Z_0 \ln(b/a)$$  \hspace{1cm} (15)

Here $E$ is the electric field, $H$ is the magnetic field, $a_r$ is the unit vector in the radial direction, $a_\phi$ is the unit vector in the circumferential direction, $j$ is the square root of -1, $z$ is the distance along the cavity axis, $Z_0$ is the characteristic impedance of the transmission line, $r$ is the distance from the cavity axis, and $k$ is the wave number defined by the equation:

$$k = \omega \sqrt{(\mu_0 \varepsilon_0)}^{1/2}$$  \hspace{1cm} (16)

The most important higher-order modes of the cavity are TEM standing waves with this same field profile, propagating along the cavity axis, which are harmonics of the fundamental quarter-wavelength frequency. Since the transmission line has an open circuit at one end and a short circuit at the other, standing wave solutions with voltage maxima at the gap will exist only for the odd harmonics. Thus we may expect to see higher-order resonances at $3f_0$, $5f_0$, $7f_0$, etc. where $f_0$ is the fundamental cavity frequency. In practice, these higher-order modes may exist at frequencies which vary slightly from the integer multiples given above due to the effects of the tip capacitance and other edge effects.
Figure 1-3(a) - Field profile for TEM modes

Figure 1-3(b) - Cutoff wavelengths of higher modes [13]

Figure 1-3(c) - The electric field of some of the higher modes in a coaxial line [13]

Figure 1-3 - Field patterns in a coaxial cavity

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Apart from the TEM harmonics, there are two more important classes of waves on a coaxial transmission line. The first class is that in which only the electric field is perpendicular to the cavity axis; these are known as TE or transverse electric waves. The second class is that in which only the magnetic field is perpendicular to the axis; these are known as TM or transverse magnetic waves. Both of these classes contain an infinite number of solutions or field profiles each related to a different component of the multipole expansion of the electric or magnetic fields between two coaxial conductors. Figure 1-3 shows the electric and magnetic transverse field patterns for the TEM modes and the first few TE and TM modes, while the exact field solutions for TE and TM modes are found in Appendix B. The TE and TM modes have wavelengths which are in the range of the transmission line circumference or smaller; thus for cavities such as the ones discussed in this thesis, where the cavity length is significantly longer than the outer conductor radius, the TE and TM modes are seen only at frequencies higher than the first few odd TEM harmonics. In the case of the test cavity used in the bead pull experiment, the first of these circumferential higher order resonances seen was the TE_{1,1} mode, at a frequency slightly higher than 700 MHz, putting it between the 11th and 13th TEM harmonics. The next lowest circumferential resonance for the test cavity was that associated with the TE_{2,1} mode, but this was well above 1 GHz, and thus beyond the scope of the measurements, which were concerned primarily with the frequency range located around the fundamental frequency and the first few harmonics (up to 800 MHz). Other higher-order-modes may be excited in the cavity by smaller structures such as probes, supports, and damper components, but these generally exist with wavelengths of comparable size to these structures themselves, and thus the corresponding resonances are located at frequencies much higher than the range of interest.

1.3 Higher-Order-Mode Damping

The objective of higher-order-mode damping is to reduce the potentially destabilizing shunt impedances as much as possible for the higher modes while keeping the shunt
impedance at the fundamental mode unchanged. Given that one does not want to alter the basic transmission line properties of the cavity itself, damping the higher-order modes means adding a structure to the cavity which will absorb the potential across the gap, but only at frequencies higher than the fundamental. Thus the damper must be a structure which can act as an absorbing-type filter for the higher-order modes, passing undesirable frequencies to some load, but leaving the signal unchanged at the fundamental frequency. The equivalent circuit for two such resonator/filter systems are shown in Figures 1-2(b) and 1-2(c).

Two filter types are potential candidates for higher-order-mode damping. The first possibility is to use a series of bandpass filters which can eliminate specific higher-order-mode resonances. This is of no usefulness, however, in the case of a cavity with a variable fundamental frequency and thus variable higher-order-mode frequencies. For this scenario, one would like to absorb all frequencies above a certain cut-off, and thus the second possibility, a high-pass filter, is the more appropriate one.

Different designs of high-pass filters will give different frequency responses; what is desirable for higher-order-mode damping is that the corner frequency be slightly lower than the lowest possible higher-order-mode frequency, and that the filter response before the corner frequency be as steep as possible so as to induce the minimum possible amount of absorption at the fundamental. One would thus like a multi-element filter in order to give a steeper slope. For the five-element and seven-element filters examined in this thesis, the filter response is approximately 40 dB lower at the fundamental frequency than at the corner frequency. More elements in the filter would allow for an even sharper response at the low-frequency end, but might become more difficult to realize in a physical structure.

To achieve the capacitances and inductances shown in the circuit diagram for the cavity/damper structure in Figures 1-2(b) and 1-2(c), a combination of washers and connective rods were used, as shown in Figure 1-4. The washers were arranged at the outer end of the gap such that they circled the cavity axis. The washers had large surface areas and were placed close together so as to achieve a high enough capacitance to make the filter
work. In the case of the seven-element filter, the flat part of the horn made up the first washer/capacitance, with the pipe part of the horn making up the first inductance. The final washer was connected to the outer conductor through a series of four 50 Ω loads. In the case of the prototype damper for the cavity in the proposed KAON Factory Booster Ring the loads were water cooled so as to be able to absorb significant amounts of power if necessary, but the model damper for the test cavity used simple loads since the only power source in this case was the low-level signal (on the order of 10 mW) used for performing measurements by the network analyzer.

This combination of washer and rod structures formed an effective multi-element high-pass filter only for the coaxial TEM modes. For TE and TM modes the effective capacitances between the elements were far lower, and the damper was only partially effective in passing the gap voltage to the loads.

![Figure 1-4 - Higher-order-mode damper](image)
2 Cavity Perturbation Theory

2.1 Advantages of Perturbation Measurements

Measuring exact field strengths in an RF cavity can be a difficult process. Capacitive and inductive probes can give an idea of the relative field strengths for different points in the cavity, but getting absolute measurements requires having some point where the field strength is known to compare all of the other readings to. Furthermore, the field strengths depend directly on the power going into the cavity, which may fluctuate and can be difficult to measure exactly. Finally the probe itself may fundamentally alter the patterns of the field inside the cavity: the larger one makes the probe, and thus the larger the measurement signal, the more one is likely to fundamentally alter the fields themselves.

Some of these problems can be eliminated by measuring shunt impedance, which can be expressed as a ratio of absolute field strength squared to input power level. There are various methods by which one can use the specifics of the coaxial \( \lambda/4 \) structure to determine the shunt impedance without having to measure absolute power level or gap voltage [14, 15]. These methods are discussed more fully in the first section of Chapter 5.

These schemes allow one to determine shunt impedance without looking at absolute field strengths or power levels, but the one is required to run a conductor fully across the gap. Such an arrangement produces a significant effect on the fields in the cavity, drastically changing the position of the resonance even at low frequencies, a situation which only increases with higher frequency as the inductive effects of the conductor across the gap become stronger and stronger. Measurements using these methods done on both the test cavity and the proposed KAON Booster cavity show that meaningful results are possible only for frequencies up to the 500 MHz range (see Chapter 5); above this the fields are so distorted that it becomes quite difficult to distinguish between the individual resonances. Even below 500 MHz the frequencies are lowered by a significant amount, and there are no guaranties of the accuracy of the impedance measurements given the huge change in the field structure.
It would be desirable then to have a method for measuring fields or shunt impedances using a probe which would alter the fields only in some regularly defined way such that the change in the field patterns due to the probe could be taken into account. Cavity perturbation measurements start from this perspective; the most desirable type of probe is one which, when placed in a electric or magnetic field, will alter the field in as regular a way as possible. Simple objects such as metallic or dielectric spheres or ellipsoids suspended within the cavity, or cylinders protruding from the walls of the cavity, as long as they are small enough that the original field is more or less uniform over their volume, will perturb the field in a regular and predictable way. Furthermore, these objects will change the field inside the cavity in such a way that the overall frequency of a resonance will be pulled down by an amount directly related to the strength of the original field. By using the properties of the perturbing object, the frequency of the resonance (as well as its downward shift when the object is introduced), the $Q$ factor of the resonance, and the geometry of the cavity, the shunt impedance can be determined [16, 17].

2.2 Perturbation Theory as Applied to RF Cavities

Perturbation measurements of fields in rf cavities are based on a principle of thermodynamics known, among other things, as the Boltzmann-Ehrenfest adiabatic invariant theorem [9, 18]. This theorem states that "for a periodical and linear working lossless engine the product of energy (kinetic and potential) and period time is invariant for adiabatic deformations." Applying this principle to our cavity situation gives the relation:

$$ U T = \frac{2\pi U}{\omega} = \text{constant} \quad (17) $$

Here $U$ is the total stored energy in the cavity and $T$ is the period of the resonance. Taking logarithms and differentiating this expression gives:

$$ \frac{\delta U}{U} = \frac{\delta \omega}{\omega} \quad (18) $$
Here \( \delta U \) is the change in energy due to an "adiabatic deformation" of the system (our perturbation), and \( \delta \omega \) is the corresponding frequency shift.

In order to use this perturbation formula to measure fields in the cavity we must be able to determine the change in total field energy \( \delta U \) caused by the insertion of a small regular object; the simplest case being that of a spherical object or bead. Figure 2-1 gives an illustration of the effects of inserting a metallic or dielectric sphere into a uniform electric field. For a metallic sphere the change in total energy when placed in a uniform field is given by the relation:

\[
\delta U = - (\varepsilon_0 E^2 - \frac{1}{2} \mu_0 H^2) \pi r^3
\]  

(19)

Here \( E \) is the electric field strength before perturbation, \( H \) is the magnetic field strength before perturbation, and \( r \) is the radius of the bead [9, 19]. To obtain a perturbation whose magnitude depends on the electric field only one must use a dielectric bead. In this case the change in total energy (derived in Appendix C) will be given by the formula:

\[
\delta U = -\varepsilon^* E^2 \pi r^3
\]  

(20)

Here \( \varepsilon^* \) is the modified dielectric constant which is related to the dielectric constant \( \varepsilon \) of the bead by the relation:

\[
\varepsilon^* = \frac{\varepsilon_0 (\varepsilon - \varepsilon_0)}{\varepsilon + 2\varepsilon_0}
\]  

(21)

Combining this expression for the change in total energy from a dielectric sphere with our original expression in equation (18) we obtain:

\[
\frac{- \varepsilon^* E^2 \pi r^3}{U} = \frac{\delta \omega}{\omega}
\]  

(22)

Remembering that the total energy \( U \) can be written as the product of the \( Q \) factor of the cavity and the input power divided by the angular frequency (see equation (11)), we may rewrite equation (22) as:
Solving this equation for the field strength $E$ gives:

$$|E| = \frac{\sqrt{\delta \omega QP}}{\omega \sqrt{\pi \varepsilon \rho^3}}$$  \hspace{1cm} (24)

Thus we have obtained a formula for measuring the absolute magnitude of the electric field strength at a given point in a resonating cavity. To measure a magnetic field strength one would need to use both dielectric and magnetic beads and compare the frequency shifts from each. Unfortunately, the difficulty of measuring the power going into the cavity means that equation (24) is of only limited value for finding absolute electric or magnetic field strengths. As will be shown in the next section, however, it can be used to obtain the shunt impedance without requiring either the absolute field strength or the absolute input power level.

### 2.3 Using Cavity Perturbation to Measure Shunt Impedance

The shunt impedance is defined as the ratio of the gap voltage squared to the input power. If we expand the voltage term as the integral of the electric field across the gap we obtain the expression:

$$R_s = \left( \int_{\rho} E \cdot dz \right)^2 / 2P$$  \hspace{1cm} (25)

Here the integral is taken along the path across the gap. In a situation where the gap electric field is known to be parallel to the path across the gap (such as along the axis of the coaxial $\lambda/4$ resonator) then we may substitute for $E$ in equation (25) the expression for the magnitude of the field strength as determined by the perturbation technique given in equation (24) (See Figure 5b). Thus we have:

$$R_s = \left( \int_{\rho} \frac{\sqrt{\delta \omega QP}}{4\omega P^2 \sqrt{\pi \varepsilon \rho^3}} \right)^2$$  \hspace{1cm} (26)
The $Q$ factor of the cavity, the power level, and the electric and physical properties of the bead are all constant as the bead is pulled along the path, and so $Q$, $P$, $r$, and $e^*$ may all be drawn out of the integral. The power term in the expression from the perturbation measurement then cancels with the power term in the shunt impedance definition to give the relation:

$$R_s = \frac{Q \left[ \int \sqrt{\omega} \, dz \right]^2}{2\pi e^* \omega^2 r^3} \quad (27)$$

Thus we have an expression for the shunt impedance which depends only on the physical and electrical properties of the bead, the frequency and frequency shift of the cavity, the $Q$ factor of the cavity, and the length of the path. To make the integral a measurable quantity one can replace it with a sum over a series of $n$ steps along the path with each step having length $\Delta z$. This gives the expression:

$$R_s = \frac{Q \left[ \sum \sqrt{\omega} \, \Delta z \right]^2}{2\pi e^* \omega^2 r^3} \quad (28)$$

If the field strength is changing rapidly along the path, then a larger number of steps $n$ will be needed, but for smoothly or slowly varying field strengths fewer steps will suffice. If the field is of almost uniform strength across the gap, one can make a quick approximation to the shunt impedance by taking the limit of equation (28) for small $n$. When $n = 1$ we have:

$$R_s \approx \frac{Q \, \delta \omega \, z^2}{2\pi e^* \omega^2 r^3} \quad (29)$$

For a cavity with a $Q$ factor of 2000, a bead with a radius of 4 mm and a dielectric constant of 10, a gap of 40 mm, a shunt impedance of 120 k$\Omega$, and a resonant frequency of 70 MHz, the above relation predicts a frequency shift of approximately 3000 Hz. Thus one of the keys to performing perturbation measurements is the ability to measure changes in the resonant frequency of a cavity of only a few kilohertz. The practical difficulties of this problem will be discussed further in Chapter 4.
The above relations for finding the shunt impedance of a cavity rest on a few assumptions which must be valid for the measurements to be accurate. In the first place the bead must be very small compared to the total volume of the cavity in order that its insertion really makes only a small adiabatic variation in the resonance. The unperturbed field must be more or less uniform over the volume of the sphere; for rapidly varying field patterns smaller beads will be necessary (see Figure 2-1). Also the bead must be at least one diameter away from any conducting cavity wall or edge in order to minimize additional effects caused by images of the bead in the conductor.

To use a metallic bead to measure shunt impedance requires the assumption that the magnetic field is zero over the bead's volume. Although theoretically this may be true exactly along the axis of the cavity, in practice this situation makes the measurements far too vulnerable to minute misalignments of the bead. A much better solution is to use a dielectric bead for which the presence of magnetic fields will not adversely effect the accuracy of the measurements. The most appropriate dielectric material will be one with a high enough dielectric constant (at least five) to give a sizable perturbation, and which can easily be machined into spherical beads.

The final assumption made in equations (26 - 29) is that the electric field is exactly parallel to the path along which the bead is being pulled (see Figure 2-2). This is true for special cases such as along the axis of a coaxial λ/4 structure, but even then one must still be very careful when aligning the bead. If the field is not parallel to the path, then the direction of the field strength must be determined as well as the magnitude. This requires using ellipsoids at each point and determining the different frequency shifts as the ellipsoid is oriented with its major axis along three perpendicular directions. While this method can give field direction as well as magnitude, it is much more difficult to perform than the spherical case, and it is desirable to use the symmetries of the system as much as possible in order to avoid having to use perturbative methods to measure field direction.
Figure 2-1 - Field around a dielectric sphere placed in a uniform electric field

Figure 2-2 - Bead being pulled through a gap
3 Cavity Perturbation Measurements - Apparatus

3.1 Apparatus - Introduction

The apparatus used for making cavity perturbation measurements consisted of several parts. In the first place was the test cavity being used, a coaxial $\lambda/4$ resonator (see Figure 3-1), and the damper model which was being evaluated. Several types of dielectric beads were pulled through the cavity along a nylon line by an electric micro-stepping motor. This motor was driven by an indexer/controller which was itself controlled by an IBM 486 computer. This computer also controlled the Hewlett Packard network analyzer used to measure the properties of the cavity resonance. To the computer was also attached a printer to print out data as well as storing it on disc as the measurements were made. The software used in the experiment contained some 40 pages of Quick Basic code developed during the course of the experiment to control all of the various devices. A diagram of the overall apparatus is given in Figure 3-2.

3.2 Test Cavity

The test cavity used in the experiment was a copper coaxial $\lambda/4$ resonator with a ratio of the inner and outer conductors such that the characteristic impedance of the coaxial transmission line was exactly 50 $\Omega$. To the open end of the inner conductor was attached a horn similar to that used in the second prototype damper design (as described in Section 1.3) which could be slid in and out along the inner conductor so as to change the frequency of the fundamental resonance. To the open end of the outer conductor was attached a copper barrel along which a wooden disc sheathed in copper foil could be slid so that the gap between it and the horn on the inner conductor remained constant as the latter was moved to tune the cavity. This tuning mechanism allowed one to lengthen or shorten the overall cavity by about 3 cm, although when the horn and disc were fully extended so as to give the longest possible resonator, both pieces were quite loose and it was very difficult to make sure that both sides
of the gap were aligned perpendicularly to the cavity axis. Thus the realistic change in cavity length was somewhat less than 2 cm, giving a possible frequency change of around 2 MHz for a fundamental frequency of 70 MHz.

Through the outer conductor of the cavity, near the open end, were attached two small loosely-coupled capacitive probes which could be connected to the two ports of the Hewlett Packard S-parameter Test Set. The probes could be placed at different positions around the axis as well as along the axis to determine which modes varied in strength along the length of the transmission line (TEM modes) and which modes varied in strength around the axis (TE and TM modes). The 50 Ω BNC cables which connected the probes to the network analyzer were as short as physically possible (two feet or less) in order to maintain the highest possible signal quality.

Through the center of both the disc on the outer side of the gap and the flat part of the horn on the inner side of the gap were cut circular apertures with diameters of 4 cm around the axis to allow the bead to pass through, and to simulate a beam aperture of approximately half the diameter of the inner conductor. Through the end of the inner conductor a smaller aperture, with a diameter of 1 cm, was cut to allow the line on which the bead was attached to be run through the center of the inner conductor. A plastic tube placed along the axis inside the inner conductor guided the line to the shorted end of the cavity.

The cavity had a \( Q \) factor close to 3000 immediately after being cleaned and having all the connections cleaned and tightened; however the value for \( Q \) quickly fell to slightly over 2000 after a few days exposure to air. The majority of the cavity perturbation measurements were made with the cavity having a \( Q \) factor of approximately 2000.

3.3 Bead and Line

The beads used for the perturbation experiments were dielectric spheres made of two separate materials; Macor and Stycast. Macor is a machinable aluminum based ceramic, and Stycast is a ceramic composed primarily of titanium dioxide which can be ordered with a
variety of high dielectric constants. The electrical properties of these two materials are summarized in Table 3-1. The particular Stycast rods used in the cavity perturbation apparatus had a dielectric constant of 27.3 at the 100 MHz range. The Macor and Stycast samples were milled into beads of various standard sizes from a bead diameter of 1/8" to a bead diameter of 1/2". Through the center of each bead was drilled a small hole, just big enough for a nylon fishing line of thickness 0.138 mm to be threaded through. The bead was attached permanently to the nylon line by a drop of very strong adhesive at the points at which the line entered and exited the bead.

The line was threaded through the inner conductor using the plastic tube as a guide and kept taut by tension from a plumb bob. Care was taken to make sure that the plumb bob was not too heavy to snap the nylon line, which had a tension limit of 5 lb. The other end of the line was wound around a spool which was attached to a motor driven shaft which could pull the bead towards the motor setup. The spool was exactly 200 mm in circumference so that moving the bead 1 mm meant exactly 25 microsteps of the motor, which was set to rotate at a rate of 5000 microsteps per revolution. The bead was always pulled through the cavity from the inner to the outer conductor so that the plumb bob was always being pulled upwards to avoid unnecessary vibrations of the bead. The apparatus was allowed to sit for several hours every time a new bead would be strung through to allow for the slight stretching of the line which would occur immediately after the plumb bob was attached.

<table>
<thead>
<tr>
<th>Material</th>
<th>Manufacturer</th>
<th>Dielectric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macor</td>
<td>Dow Corning</td>
<td>5.8 ± 0.1</td>
</tr>
<tr>
<td>Stycast</td>
<td>Emerson &amp; Cuming</td>
<td>27.3 ± 0.1</td>
</tr>
</tbody>
</table>

Table 3-1: Dielectric properties of bead materials.
3.4 Motor and Motor Control

The motor which pulled the bead was mounted on an aluminum frame and attached by a series of mechanical couplers to a 12" long 1/4" shaft, also mounted on the same frame. The shaft contained a threaded section, which would move a block of metal back and forth along the shaft axis as it turned. This metal block fit into a track running parallel to the shaft axis and limit switches could be placed at a number of locations along its length to be triggered by the guided block so that the motor could be shut off without relying on the computer if the shaft turned too far. The shaft ended at the spool around which the line attached to the bead was wound (see Figure 3-3).

The motor was controlled by an American Precision Industries P315X controller/indexer which was capable of turning the motor shaft in increments of 1/30000 of a complete revolution, although the setting used in the measurements was 5000 steps per revolution. The indexer/controller could also control the speed and acceleration and deceleration times of each step, although the fact that it took approximately 10 seconds to make a network analyzer measurement at each step meant that the speed-control capability of the motor and control system was rarely needed during the course of the measurements.

3.5 Network Analyzer and RF Measurements

All measurements of the properties of the cavity resonance were made with a Hewlett Packard 8753 (A or B) network analyzer attached to a Hewlett Packard S-Parameter Test Set. The network analyzer had a frequency range of 300 kHz to 3 GHz, more than sufficient for the frequencies used in the cavity perturbation measurements, which ranged from 60 MHz (the approximate fundamental frequency of the most fully tip-loaded configuration of the cavity) and 800 MHz (the approximate frequency of the first non-TEM mode).

The network analyzer was used for measuring the $Q$ factor and frequency of the resonance using the $S_{21}$ scattering parameter (transmission) of the signal between the two ports, which were attached by short 50 Ω cables to two of the capacitive probes along the
cavity wall, and looking at the logarithmic magnitude of the signal. The frequency shift as the bead was pulled through the cavity was measured by looking at the phase of the $S_{21}$ signal, finding the phase at the exact center of the resonance, and setting a marker to show the frequency of that phase as the resonance was pulled down in frequency by the perturbation caused by the insertion of the bead. In this way the frequency shift $\delta\omega$ in the perturbation formula (Chapter 2, Equation 28) could be measured directly by the network analyzer.

The network analyzer was set to a signal power level of 20 dBm (0.1 W) during the course of the measurements, but this signal was attenuated by approximately 13 dBm by the $S$-Parameter test set, so that the power level of the signal going into the cavity was in the order of 7 dBm (5 mW). The number of points taken by the analyzer was set at the maximum (1601) for measurements performed once (such as finding the Q factor), but was lowered to 201 or 101 when measuring the frequency shift as the bead moved along the path in order to reduce the time required for each step. The time taken by the analyzer for each measurement was set to a minimum: approximately 1 ms for each point taken during a sweep. Triggering and averaging was done externally from the controlling computer. No smoothing or data-altering algorithm options were used during the measurements and all frequency sweeps were of the linear type. Because only relative signal measurements such as the Q factor or the difference between two phases (frequency shift) were ever done, calibration could be kept to a minimum while running the program and was generally done only at the beginning of each measurement session as a check of the proper functioning of the analyzer.

The printer was not used with the network analyzer during the measurements due to a conflict of addresses on the GPIB line, although the printer could be attached when the computer control was disengaged and the analyzer was being operated in local mode.

3.6 Computer Control of Devices

The IBM 486 computer used in the cavity perturbation apparatus had to act as the controller of several devices during the course of each measurement, the first of these was the
printer used to print out the data. The second device controlled by the computer was the motor indexer/driver, which had an RS-232 attachment to one of the serial ports on the IBM 486. The serial port was opened as a file in the source code and instructions were sent to the indexer/driver by printing into the file (and thus sending through the communications port) a series of commands written in ICL, the language used to program American Precision Instruments motor controllers.

The third and most difficult device to control from the computer was the Hewlett Packard network analyzer. A GPIB (General Purpose Interface Bus) cable attached the network analyzer to a special National Instruments GPIB card which was installed on one of the card slots on the controlling computer. In order to allow the non-Hewlett Packard device (the controlling computer) to send coherent signals to the analyzer, a Universal Language Interface (ULI) program had to be installed on the computer to run simultaneously with the main program. The GPIB could then be opened as two files (one for input and one for output) in the source code and using the ULI Hewlett Packard commands could be issued to devices along the GPIB line (in this case the network analyzer at the GPIB address number 16). Commands could also be issued to the GPIB board itself, which was GPIB address number 0, in order to clear the board and change its configurations during the running of the main program. One of the main configuration changes was to disable the time-outs on the board so that the program would not be affected by the extreme slowness of the network analyzer, which could take up to two seconds to perform one measurement operation.

There was one serious communications problem which had to be overcome in order to be able to send coherent signals to and from the network analyzer; this was to find an acceptable end-of-statement convention. Hewlett Packard 8753 network analyzers of model B and later are designed to accept only semicolons as a legitimate end of statement character. Anything other than a semicolon would result in a syntax error in the running of the computer inside the network analyzer. Thus not only did semicolons have to be appended to all messages sent into the GPIB from inside the program, but the GPIB had to be reconfigured to
set no End of Statement or End or Identify character such as a line feed or a carriage return.
While this meant that a coherent set of signals could be sent from the computer to the network analyzer, data sent back to the computer through the GPIB had only a semicolon appending each data statement, and thus the input buffer would remain uncleared after each piece of data was entered. In order to clear the input buffer to receive multiple pieces of data, the input file had to be closed and reopened after each statement sent to the computer by the network analyzer. This technique is applicable to all Hewlett Packard instruments which can be controlled through a GPIB interface.

3.7 Software

The software for the cavity perturbation measurement system was written in Quick Basic to provide maximum capability for annotation and an easily understood source code. Since the network analyzer performed functions at such a slow rate, the speed of the program was not a concern, and Quick Basic was an acceptable language to use, if somewhat cumbersome for the programmer.

The program was designed to be menu driven, with the structure of the subroutines in the program more or less mirroring the structure of the menus. The source code was heavily annotated, with a tree diagram of the subroutine structure appearing at the beginning of the code, as well as a complete explanation appearing each time a command in another language (such as ICL or Hewlett Packard commands) appears. This heavy annotation accounts for about half of the total length of the program, which is approximately 40 pages of code. A diagram of the subroutine structure of the program is given in Appendix D.

When the program was run, first the GPIB board was cleared and configured, then a connection to the network analyzer was established and confirmed, and the user was presented with the main menu. The options on this menu include calibration of the network analyzer (for transmission, one-port, or full two-port calibrations), turning on or off the dual channel capability of the analyzer, saving or recalling a series of analyzer settings, testing the
motor system, changing the default settings for the perturbation measurement calculations, changing the network analyzer settings, making a measurement or exiting the program. Of these options, only changing the settings and making a measurement led to a secondary menu, the others simply went through their tasks and returned to the main menu.

The settings menu presented the user with a simultaneous display of all of the network analyzer and allowed the user to reset them. Thus all of the basic functions of the network analyzer normally done from the network analyzer control panel could be done completely through the controlling computer using the program.

The measurement menu presented the user with two main options. The first was to measure the $Q$ factor of a resonance. This option allowed the user simply to give the general range of frequencies where the resonance might be, and the program would automatically find the peak, optimize the scale, and find the $Q$ factor averaged over a number of sweeps determined by the user. The program also gave the user the option of modifying the $Q$ measurement by taking a VSWR measurement and automatically making the appropriate corrections to the $Q$ factor. All the options for the $Q$ measurement had default settings which could be changed from the main menu.

The second option on the measurement menu was the perturbation measurement option. In this case the program would ask the user for all the properties of the bead (the defaults to which could be set from the main menu), remind the user about the nature of perturbation measurements and remove the bead from the cavity, find the unperturbed $Q$ of the cavity (as described above), find the frequency of the resonance peak, find the phase at that frequency, and set a marker to follow that phase and a second marker to remain fixed at the original frequency. The difference in frequency between the two markers was then the frequency shift of the cavity.

Once this was done the user was presented with two options. In the first option the system simply set the bead to the center of the gap and used the gap length to calculate an approximate shunt impedance according to formula (29) given in Chapter 2. The second
option allowed the user to have the computer pull the bead step by step across the gap and measure the frequency shift at each step, calculating the shunt impedance at the end according to formula (28) in Chapter 2. In both cases the program would attempt to correct for inaccuracies in the measurement caused by the instability of the unperturbed resonating frequency of the cavity. A discussion of the accuracy of the measurements and techniques for improving accuracy is given in the next chapter.

The data from the measurements could be sent according to the choice of the user to a data file or the printer or both. Default settings for the data collection options could be changed from within the main menu. The raw data contained the maximum amount of information about each measurement: the Q factor and center frequency of the unperturbed resonance for each trial, as well as the average of these, their standard deviation, and any corrections made from a VSWR measurement, the frequency shift and corrected frequency shift of the resonance for each step along the path, and the total shunt impedance along the path. Since 20 - 100 steps were normally taken along the path, and each measurement might be repeated several times, not all the measurements were printed out, but all were recorded on files on the hard drive.

![Diagram of Test Cavity]

Figure 3-1 Test cavity
Figure 3-2 - Perturbation apparatus overall schematic
Figure 3-3 - Motor mount for cavity perturbation apparatus
4 Cavity Perturbation Measurements - Accuracy

4.1 Sources of Error in Perturbation Measurements

The cavity perturbation technique for measuring shunt impedance is based on equation (28) derived in Section 2.2 which states that, given a certain set of conditions, by perturbing a resonating cavity with a dielectric spherical bead pulled along a path through the accelerating gap, the shunt impedance can be calculated from the values of the unperturbed resonating frequency, the frequency shift caused by the bead at each point along the path, the length of each step along the path, the $Q$ factor of the resonance, the radius of the bead and its dielectric constant. The accuracy of the value of the shunt impedance calculated using this technique depends then on both satisfying the conditions which make equation (28) valid, as well as being able to measure the six quantities used in the calculation accurately. This section discusses the relative weights of these factors in making accurate shunt impedance measurements.

Several conditions need to be met for equation (28) for finding shunt impedance to be valid. In the first place, the volume of the perturbing object must be very small compared to the total volume of the cavity. Although this may be a problem for measuring shunt impedances in smaller microwave cavities, for the volumes used in rf cavities beads the ball bearings with diameters of a few millimeters are more than small enough. In the case of the 6" $\lambda/4$ test cavity, the cavity volume was 15,500 cm$^3$, while the volume of the largest bead used was slightly larger than 1 cm$^3$, so the assumption of a small perturbing object was essentially correct.

Another condition which needs to be true for the perturbation formula to hold is that the bead is spherical. By using machinable high dielectric ceramics such as Macor and Stycast this condition was easily met; and measurements of bead diameter for a large number of different orientations showed that the maximum deviation from the average diameter was
0.3%. These measurements were performed on Stycast beads of various sizes using electronic calipers.

The perturbation technique also assumes that the line along which the bead is pulled is essentially non-perturbing. This assumption can be realized by using a thin line with as low a dielectric constant as possible; the best choice is nylon, for which lines are readily available which have thicknesses of the order of 100 μm. For the nylon line used in the test cavity, the total volume of the line in the cavity was of the order of 10^{-4} \text{ cm}^3, which is less than 0.5% of the volume of the smallest bead. This small volume means that the fact that nylon is quite lossy can be effectively ignored. Even if the line were thicker, as long as its dielectric properties are uniform and it is not a conductor, it should not fundamentally affect the measurements, since the small perturbation caused by the line affects the frequency of the cavity with and without the bead equally, as the line remains in the cavity even for the unperturbed measurements.

A much more serious assumption is that the region around the bead (approximately one bead diameter around the bead itself) is one of uniform electric field. This means that the bead can never be allowed to come within one bead diameter of any conductor, such as a cavity wall, since the image of the bead in the wall will significantly perturb the electric field. Although a correction factor exists to take into account the effects of the bead's image in a flat conducting surface [16], the best solution is simply to keep the bead away from all cavity walls, which could easily be done for the 6" λ/4 test cavity by running the bead only along the cavity axis and making sure that the apertures in the gap were large enough for the bead to pass through without image problems.

Even if there is no conducting surface nearby to worry about, care still has to be taken that the field is not varying strongly in the region of the bead. Fields across a simple gap generally have maximum strength at the center of the gap, with the strength dropping off on either side. The bead must be small enough that the field strength does not change significantly along its length. If there is more than a few percent variation of the field
strength at points on opposite sides of the bead, then a smaller bead must be used. For the smoothly varying fields across the gap of a coaxial quarter-wavelength cavity, however, any bead small enough never to come within one diameter of any conducting surface should also be small enough for the assumption of field uniformity to hold.

The most serious assumption made in equation (28) is that the bead is being pulled along a path which is exactly parallel to the direction of the electric field. If the nylon line is not parallel to the field direction, but is tilted from it by an angle $\theta$, then equation (28) must be corrected by a factor of $\cos \theta$. For a one-degree deviation from the field direction, this gives only a 0.02% variation, but for a ten-degree deviation the variation is closer to 2%, large enough to affect the impedance values. Thus the nylon line must be kept within only a few degrees of the field direction. This can be a serious concern, since it means that the cavity perturbation technique using a bead pulled along a single line can only be used in the case where the field direction remains constant along the path for which the shunt impedance is being calculated. In the case of the 6" $\lambda/4$ test cavity the only straight line path across the gap which is parallel to the field direction for its entire length is the path directly along the cavity axis. Off axis the field may also have transverse (specifically axial) components, and if the field direction deviates significantly from the direction of the cavity axis then even if the beadline is exactly parallel to the cavity axis, the shunt impedance measurement could be off by several percent. Thus the beadline must be placed directly along the bead axis, not simply parallel to it. In practice, however, the axial components of the fields were quite weak compared to the longitudinal components, particularly near the center of the gap, and the angular alignment of the nylon line was more important than its distance from the axis itself.

Even if all of the conditions which make equation (28) valid are met, the accuracy of the shunt impedance value obtained is dependent on the accuracy to which the six quantities from which it is calculated are known. Four of these quantities can be measured so accurately that they add very little uncertainty to the final measurement. The Hewlett Packard network analyzer can measure the unperturbed frequency of the resonance to better than one part in
ten million at the 50-100 MHz range, so the accuracy of the frequency measurement will be limited not by the instrument but by the stability of the cavity itself, which will be discussed in the next section. The network analyzer is also capable of measuring the Q factor of the cavity to better than one percent; again, the accuracy of the measuring device exceeds the stability of the cavity for the purpose of shunt impedance measurements. The radius of the bead, assuming that it has been machined properly, can also be measured extremely accurately with a set of electronic calipers. Even though the radius is raised to the power of three in equation (28), the fact that it could be determined to better than one part in a thousand for the beads used in the measurements meant that it did not seriously affect the accuracy of the final impedance value. The length of each step could also be determined to better than one part in a thousand using the micro-stepping controller/indexer system.

One quantity which could not be easily measured with any great accuracy was the dielectric constant of the material used in making the beads. This was particularly true for the Stycast, for which the dielectric constant was given by the manufacturer as being 25 (± 10%). Several attempts were made to verify this independently. A first attempt, based on measuring the effect of the dielectric on the capacitance of two parallel copper plates, proved insensitive and unsatisfactory.

A more effective method which was tried used a specially constructed coaxial λ/4 cavity in which a sample of the material to be measured could be placed so as to fill the gap between the two conductors at the open end of the cavity. The change in frequency of the cavity resonance could then be measured as different materials were placed across the gap, and these frequencies could be compared with SUPERFISH calculations done for the cavity with the gap filled with materials of various dielectric constants. This method gave a dielectric constant for Teflon of 2, as expected, and a dielectric constant for Stycast of between 18 and 19. Unfortunately, the accuracy of this measurement was difficult to estimate since the SUPERFISH calculations were unable to take into account the loss factor of the Stycast, which while small, was still large enough to lower the Q factor of the cavity by more
than 50%. As well, the field was highly concentrated inside the dielectric, and thus the SUPERFISH calculated frequencies were also dependent on the size of mesh used.

A third method, suggested by the manufacturer, was to measure the density of the sample. Since the various types of Stycast are made of titanium dioxide grains held together by a filler, the only difference between them being the ratio of titanium dioxide to filler, and since both the densities and dielectric constants of titanium dioxide and the filler were known fairly accurately, finding the overall density of a sample allowed the determination of its composition, and thus of its dielectric constant. Comparing the density measured at TRIUMF with the data supplied by the manufacturer, the dielectric constant was found to have a value of 25 ± 3.

Finally, a small sample of the Stycast batch received at TRIUMF was sent back to the manufacturer, who had a special transmission line structure for measuring the dielectric constants of materials already set up. In this set up the transmission and reflection coefficients were measured of a signal sent through a coaxial transmission line with one section of the line having the space between the two conductors filled with the Stycast sample. The result was a value for the dielectric constant of 27.3 ± 0.1.

Fortunately, the difficulty in determining the dielectric constant of the Stycast had very little effect on the shunt impedance values obtained, for several reasons. In the first place, the dielectric constant of the beads remained unchanged throughout the entire experiment, so that all measurements made with the beads would be off by a constant factor if an inaccurate value of the dielectric constant were used in the calculation. Since measurements made using the value of 27.3 for the dielectric constant gave results for shunt impedance similar to results from measurements made with the Macor beads, as well as those made with the non-perturbative techniques, this constant factor was limited to a few percent. Furthermore, for bead materials with dielectric constants in the range of 25, a change of ten percent in the dielectric constant results in a change of less than one percent in the final value for shunt impedance, since the effective dielectric constant ε* given in equation (28) is

39-
related to the real dielectric constant of the material by the relation given in equation (21) (See Section 2.2). From equation (21) we can see that $\varepsilon^*$ approaches $\varepsilon_0$ for large values of the dielectric constant:

$$
\varepsilon^* = \frac{\varepsilon_0 (\varepsilon - \varepsilon_0)}{\varepsilon + 2\varepsilon_0}
$$

(21)

The fractional deviation from $\varepsilon_0$ is given by $3 / (\varepsilon / \varepsilon_0 + 2)$ so that $\varepsilon^*$ only varies appreciably for materials with dielectric constants of less than 10. Thus not only did the Stycast have a dielectric constant which was more than high enough to guarantee the largest possible perturbation for a given size bead, but it was also high enough to make inaccuracies in the measurement of the dielectric constant tangential to the accuracy of the cavity perturbation shunt impedance measurement.

The final quantities used in the shunt impedance calculations were the frequency shifts of the cavity produced by the bead at different points along its path. The network analyzer is capable of measuring shifts in frequency as small a few cycles per second, much smaller than the shifts of several kHz caused by the bead's perturbation. In spite of the network analyzer's ability to measure small frequency shifts, this quantity was the least certain of all the values used to calculate the shunt impedance. The accuracy of this measurement was, as with the case of the unperturbed frequency measurements, limited not so much by the measurement device as by the stability of the cavity.

4.2 Frequency Instability and Strategies for Minimizing its Effect

No real rf cavity will resonate at a completely fixed frequency. Mechanical vibrations, movement of conductors around cavity openings, and thermal variations can all affect the resonance. Although these effects may be small in magnitude, accurate measurements made using the cavity perturbation technique require being able to measure resonant frequency to within a few hundred cycles per second. Thus even small instabilities in the cavity can prove
to be detrimental and strategies are needed to minimize them, and if possible to compensate for them.

Most rf cavities are not completely enclosed, but contain some openings through the outer conducting walls. This is always true of accelerating cavities which must contain apertures to allow for the passage of the beam. These openings allow the resonating field to extend outside the cavity, where it may interact with other conductors. In general, the larger the openings in the cavity, the larger a proportion of the total field will exist outside the cavity walls, and the more the resonant frequency will be affected by changes in the positions of objects placed around the cavity. Many rf cavities are also designed to as to be able to be tuned in some way. In some cases, such as the booster cavity for the TRIUMF cyclotron (see Figure 1-1(c)), the tuning element consists of a piece of the conducting surface which can be moved back and forth. The cavity for the proposed KAON Factory Booster Ring was designed to be tuned not by the physical movements of a conductor, but by the use of ferrites. In either case, mechanical or electrical instabilities in the tuning system result in instabilities in the overall resonance. This was particularly noticeable in the case of the ferrite-tuned cavity, where fluctuations in the bias current governing the ferrites resulted in frequency swings of the cavity resonance on the order of tens of kHz.

Thus for both the TRIUMF cyclotron booster cavity and the cavity for the KAON Factory Booster Ring, performing any perturbation measurement of shunt impedance meant that the perturbing object had to be big enough to give a perturbation larger than the random fluctuations of the cavity frequency due to either movement of surrounding objects or change in bias current. In both cases this meant using beads at least several inches in diameter. Particularly in the case of the ferrite-tuned cavity, these beads were large enough to guarantee being close enough to a conducting surface to cause imaging problems, as well as making the unperturbed field non-uniform over the volume of the bead. For this reason, perturbation measurements made on these cavities could at best provide rough estimates of the shunt impedance; accurate measurements were all but impossible.
The 6" $\lambda/4$ test cavity used in the perturbation measurements avoided these two most severe frequency-instability problems. The main opening in the cavity, the aperture in the outer conductor side of the gap, was made as small as possible while still allowing the largest bead to pass through it without being less than one bead diameter from any conducting surface. Furthermore, the sliding disc and horn used to tune the cavity were not moved back and forth along their full range, but were kept in the region where they were mechanically most stable and rigid. This meant that the overall frequency of the cavity could only be varied by a few MHz, but instabilities from fluctuations in the tuning device were kept to a minimum.

A more serious source of fluctuations in the test cavity resonance was mechanical vibration. The current on the cavity surface was at a maximum at the shorted end of the cavity, with the short consisting of a washer-shaped conductor ringed with finger contacts around both its outer and inner radii, which was placed in between the outer and inner conductors. The effectiveness of this short depended on its making good rf contact with both the outer and inner conductor all the way around the cavity axis. This contact could be changed by vibrations or movements of the cavity, which in turn would affect the current passing through the short, and the overall frequency and $Q$ factor of the resonance. Even use of the machine tools near the cavity could often be seen to affect the cavity resonance; for this reason measurements were always performed after 5:00 p.m. or before 7:00 a.m. when the machine shop was not in use.

The most serious source of frequency instability in the test cavity was thermal fluctuation. Changes in the temperature of the cavity affect both the dielectric properties of the air in between the two conductors, and the resistivity of the cavity walls. This results in changes of both the overall frequency of the resonance and its $Q$ factor. If there were an abrupt change in temperature in the room, such as that caused by opening the outside door on a hot or a cold day, then the frequency of the resonance could often be seen to jump by tens of kHz, and all measurements would have to be repeated. Even when the measurements were
performed at night, when the door was not being opened or closed and the temperature in the room remained fairly stable, fluctuations in the resonant frequency of the cavity of up to one kHz were observed.

Several things were done to minimize the effect of these fluctuations. In the first place, since the temperature changed slowly, over the course of minutes rather than seconds, measurements were performed as quickly as possible. Unfortunately, the speed of the measurement system was limited by the amount of time required to take a network analyzer sweep. Even with each network analyzer sweep reduced from 1600 points to 100 points (and thus ~ 100 ms), each step of the measurements required several sweeps in order to optimize the measurements, and thus each step could take on the order of one second. Keeping the total measurement time for the whole path to within a couple of minutes thus meant taking no more than 100 steps along the path for any given measurement. Fortunately this was sufficient to observe the profile of the field in detail. Unfortunately, even for measurements which were kept to below one minute, the cavity frequency could undergo fluctuations of its fundamental frequency of several hundred cycles per second, significant given that the bead perturbation itself was on the order of a couple of kHz.

The controlling program contained an algorithm to attempt to compensate for these thermal fluctuations. Since in general the temperature in the room might rise or fall continuously for several minutes, one could naively attempt to approximate the change in temperature (and its effect on resonant frequency) as being linear over the course of a one or two minute measurement. The unperturbed frequency was thus measured both before and after the bead was pulled through the path, and the frequency shift measured at each point was adjusted accordingly to account for this linear unperturbed frequency shift.

Although this simple compensation system did make the measurements somewhat more stable, thermal fluctuations and their effects on the cavity resonance were rarely strictly linear, and in practice, any set of measurements taken for more than 50 steps along the path (enough to see the field pattern in detail) could be expected to give a set of shunt impedance
values for which the standard deviation was between 5 and 10 percent of the average, depending on conditions in the room. For this reason, shunt impedance measurements recorded in this experiment were always taken as the average of 10 trials. Even so, the standard deviation of these averaged measurements would themselves be one or two percent of the overall average, due in part to the thermal fluctuations, and in part to the effect of realigning and repositioning the nylon line. Thus even with averaging and measurements performed under optimum conditions, shunt impedances could only be found to an accuracy of one percent. This is the lowest value of the uncertainty in the shunt impedance measurements which is used in the discussion on the testing of the system in the next chapter.
5 Cavity Perturbation Measurements - Testing

5.1 Comparison with Calculated Shunt Impedance Values and Values Measured by Other Methods

The accuracy of the cavity perturbation measurement system was tested by performing shunt impedance measurements on the simple test cavity (with no higher-order-mode damper attached) and comparing them with the shunt impedance values calculated for the cavity from the equations given in Chapter 1 and Appendix A. From Equation (13) in Section 1.1 we would expect a value for \( R_s / Q \) of 200/\( \pi \) \( \Omega \), given that the characteristic impedance of the coaxial structure is 50 \( \Omega \). After the tip-loading effect is taken into account (see Appendix A), the calculated value for \( R_s / Q \) becomes 63.5 \( \Omega \), as compared to the value of 63.4 ± 0.6 \( \Omega \) measured by the perturbation technique. This value was measured using a Stycast bead of 0.25" diameter, with 60 steps taken along the path and the horn completely non-extended.

For TEM harmonics of the fundamental \( \lambda/4 \) mode the voltage and current distributions will be sinusoidal along the length of the transmission line, as in the case of the fundamental mode, but with periodicities which are greater by odd integer multiples. Integrating the current or voltage along the length of the structure (as in Chapter 1) gives the reactive power of the system, from which the shunt impedance may be calculated. The result is that (neglecting the tip-loading effect) higher-order TEM harmonics have \( R_s / Q \) values which are related to the \( R_s / Q \) value at the fundamental mode by the following relation:

\[
(R_s / Q)_{\text{fundamental}} = n \times (R_s / Q)_{\text{harmonic}}
\] (30)

Here \( n \) is the number of the harmonic [13]. At higher orders the signal level is lower and cavity perturbation measurements of the value of \( R_s / Q \) have an uncertainty of approximately 1 \( \Omega \). A comparison of the calculated values for \( R_s / Q \) with the values measured by the cavity perturbation measurements for odd harmonics up to the ninth showed that they agree to within 10% in all cases (See Table 5-1).
A further check performed on the $R_s/Q$ values measured using the cavity perturbation method was to compare them to $R_s/Q$ values measured using a non-perturbative technique. This technique consisted of connecting a single probe across the gap of the test cavity, and measuring the change in phase with frequency of the reflecting signal ($S_{11}$ parameter) from the probe using the network analyzer. The value of $R_s/Q$ is then given from the change in phase with frequency by the following relation [14,15]:

$$ (\partial \psi / \partial \omega) \times R_s/Q = -4 Z_o / \omega \quad (31) $$

Here $\partial \psi / \partial \omega$ is the change in phase with change in frequency, $Z_o$ is the characteristic impedance of the coaxial transmission line, and $\omega$ is the angular frequency of the relevant mode. Measurements of $R_s/Q$ using this technique are most accurate at lower frequencies; at higher frequencies the slope of the phase becomes masked by the effects of the inductance of the conducting probe connected across the cavity gap. Beyond the 700 MHz range, the resonances become so tip-loaded by the connecting conductor that it becomes difficult even to distinguish between them, let alone determine their $R_s/Q$ values. Values for $R_s/Q$ measured using this technique in comparison with the calculated values and those measured using the cavity perturbation technique are given in Table 2 for odd harmonics up to the ninth.

The values of $R_s/Q$ as measured by the cavity perturbation technique compare well with the calculated values, as well as those measured using the $\partial \psi / \partial \omega$ technique at low frequencies. Thus the ability of the technique to give valid measurements of the $R_s/Q$ value was confirmed.
<table>
<thead>
<tr>
<th>Mode</th>
<th>$f$ (MHz)</th>
<th>$Q$ Factor</th>
<th>$R_s/Q$ (Ω)</th>
<th>$R_s/Q$ (Ω)</th>
<th>$R_s/Q$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Calculated</td>
<td>$\partial q/\partial \omega$</td>
<td>Bead Pull</td>
</tr>
<tr>
<td>Fund.</td>
<td>71.4</td>
<td>2800</td>
<td>63.5</td>
<td>64 ± 1</td>
<td>63.4 ± 0.6</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>215.2</td>
<td>4812</td>
<td>21.2</td>
<td>21 ± 2</td>
<td>21 ± 1</td>
</tr>
<tr>
<td>$n = 5$</td>
<td>358.9</td>
<td>5366</td>
<td>12.7</td>
<td>14 ± 2</td>
<td>13 ± 1</td>
</tr>
<tr>
<td>$n = 7$</td>
<td>501.6</td>
<td>4422</td>
<td>9.1</td>
<td>11 ± 2</td>
<td>10 ± 1</td>
</tr>
<tr>
<td>$n = 9$</td>
<td>645.2</td>
<td>1770</td>
<td>7.1</td>
<td>7 ± 2</td>
<td>7 ± 1</td>
</tr>
</tbody>
</table>

Table 5-1: $R_s/Q$ of the test cavity as determined by a variety of methods.

5.2 Consistency of Perturbation Methods for Various Cavity Configurations

Once the validity of the cavity perturbation measurement of the shunt impedance had been shown, various aspects of the measurement set up were altered to determine their effect on the measured value of $R_s/Q$. The horn and disc were moved so as to change the fundamental cavity frequency (and the frequency of the higher-order modes). The path length and number of steps were varied, as was the size and dielectric constant of the bead. Finally, the nylon line was misaligned to determine the effect of alignment on the measurement. The frequency shift in the resonator as a function of bead position in the gap is shown in Figure 5-1.

5.2.1 Effect of Different Fundamental Cavity Frequencies

The horn and disc were extended 2 cm from their original positions (see Figure 3-2) so as to determine the effect of a change in the fundamental frequency on the measured value of $R_s/Q$. The $R_s/Q$ value of $63.4 \pm 0.6 \, \Omega$ measured at the highest fundamental cavity frequency (71.4 MHz) was found not to change as the horn and disc were extended. The frequency of the cavity was lowered to 69 MHz, while keeping the gap between the horn and the disc constant. Thus the tip-loading effect changed very little from its original value, and the frequency of the cavity was lowered without any appreciable effect on the measurement.
of $R_s/Q$. The horn could not be extended any further without becoming misaligned, so 69 MHz represented the lower limit of fundamental frequencies of the test cavity with no damper attached.

The $R_s/Q$ value of the third harmonic was also measured as the horn and disc were being extended. The measured $R_s/Q$ value of $21 \pm 1$ $\Omega$ did not change as the fundamental frequency of the cavity was lowered from 71.4 to 69 MHz.

5.2.2 Effects of Different Path Lengths and Number of Steps

The $R_s/Q$ value of the system was measured using different numbers of steps, from one to fifty, along the path. To reduce the time required, each measurement in the series was taken only once, not averaged over ten measurements. The results are shown graphically in Figure 5-1. The measured $R_s/Q$ value was highest for the measurement taken with only one step, since the frequency shift was measured only once, at its peak value halfway across the path, and this maximum value was taken for the whole path. This one-step measurement gave an $R_s/Q$ value of 215.8 $\Omega$, over three times the value obtained from the sixty-step measurement. As more frequency shift measurements were taken, the $R_s/Q$ value slowly decreased, and after twenty steps the random fluctuation in the measured $R_s/Q$ value (from thermal instabilities and bead misalignments) overshadowed any further gain in accuracy. These random fluctuations remained fairly constant in size from $n = 20$ to $n = 50$. One might suspects that extending the investigation to measurements taken with very large numbers of steps (>200) would show the fluctuations increasing as the amount of time needed to measure the $R_s/Q$ value increased, but such an increase is not evident for $n \leq 60$.

The frequency shifts caused by the bead at different points along a path which extends several centimeters outside the cavity are shown in Figure 5-2; the field strength is proportional to the square of the shifts.

The effects of different path lengths on the measured value of $R_s/Q$ were also examined. When the path was extended to include a long "field free" portion outside the
cavity (as in Figure 5-2), the \( R_s/Q \) value increased slightly, as the random frequency shifts seen by the bead outside the cavity were added into the sum in Equation (28). Conversely, when the path was reduced to cover only a portion of the cavity gap, the measured \( R_s/Q \) value decreased dramatically, as only part of the field was being taken into account. Thus it was preferable to slightly overestimate the length of the path needed to cover the entire gap, and the bead was allowed to travel a centimeter or two outside of the aperture on either side of the gap in order to make sure that the shunt impedance of the entire field was being measured.

5.2.3 Effects of Different Sizes and Dielectric Constants of the Bead

The value of \( R_s/Q \) was measured using both Macor and Stycast beads with diameters of 1/2", 1/4" and 1/8". The 1/8" diameter Stycast bead and the 1/4" and 1/8" diameter Macor beads all had dielectric volumes smaller than the original 1/4" diameter Stycast bead, since Stycast has a much larger dielectric constant, and thus gave larger frequency shifts. Thus, although the overall \( R_s/Q \) value measured remained constant for the smaller beads, the accuracy decreased, with the standard deviation of a group of ten \( R_s/Q \) measurements becoming larger for the smaller beads.

The 1/2" diameter beads gave values of \( R_s/Q \) similar to those measured using the 1/4" Stycast beads; although using bigger beads gave larger frequency shifts relative to the random thermal fluctuations, they were also more sensitive to misalignment of the nylon line and more likely to come within one bead diameter of a conducting surface, so that the overall accuracy gain was negligible. The larger beads were, however, much more time consuming to set up, because more care had to be taken to prevent the bead from coming within one diameter from the edge of either aperture. For this reason, the 1/4" Stycast bead was used throughout the rest of the experiment.
Figure 5-1 - $R_s/Q (\Omega)$ measured vs. number of steps taken along the path
Figure 5-2 - Frequency shift vs. distance across cavity gap
(path length = 100 mm - center of gap is at step number 50)
5.2.4 Effects of Misalignment of the Nylon Line

The nylon line was tilted at angles of several degrees from its original position parallel to the cavity axis. This effect was small for misalignments of a couple of degrees, but grew rapidly as the misalignment approached ten degrees. The effect of the nylon line misalignment on the values of the $R_S/Q$ measurement is summarized in Table 5-2.

<table>
<thead>
<tr>
<th>Approximate misalignment</th>
<th>$R_S/Q$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line parallel to cavity axis</td>
<td>63.4 ± 0.6</td>
</tr>
<tr>
<td>2° misalignment</td>
<td>64 ± 1</td>
</tr>
<tr>
<td>5° misalignment</td>
<td>65 ± 3</td>
</tr>
<tr>
<td>10° misalignment</td>
<td>71 ± 4</td>
</tr>
</tbody>
</table>

Table 5-2: Effects of bead line misalignments on $R_S/Q$ measurements.

The effect on the $R_S/Q$ measurement of the displacement of the line parallel to the cavity axis could not be systematically determined as the inner end of the line was forced to be on the axis in order to fit through the small aperture in the surface of the inner conductor.
6.1 Damper Structure with and without Horn

A high-pass filter higher-order-mode damper structure, as described in Chapter 1 (see Figure 1.4), was attached to the open end of the quarter-wavelength coaxial test cavity, replacing the barrel and disc structure (see Figure 3.2). The damper was first attached with the horn, and its effect on the shunt impedance of the overall system was examined, then the horn was removed and the process was repeated.

The combined cavity/damper structure had a fundamental resonant frequency of 65.5 MHz (with the horn), a much lower frequency than the value of ~ 70 MHz for the cavity without the damper structure. This 4 MHz reduction in the overall fundamental frequency is due primarily to the heavy capacitive tip loading of the damper structure. For the damper without the horn, the tip loading was not quite as large and the fundamental frequency was found at 65.9 MHz. The harmonics of the fundamental TEM mode were also observed at frequencies which were slightly reduced due to tip loading. In contrast to the reduced frequencies of the TEM modes, the frequency of the circumferential mode (TE_{1,1}) at 714 MHz remained basically unchanged when the damper structure was added to the cavity.

The addition of the damper structure to the system not only altered the existing modes of the cavity, it also added new modes. The first type of new mode added to the system was that of a quarter-wavelength coaxial resonator made up of the damper itself, with the horn acting as the inner conductor, and the outer wall acting as the outer conductor. There were two similar modes with slightly higher frequencies for which the inner conductor terminated not at the horn but at either of the two washers making up the high-pass filter elements. All of these modes occurred at frequencies much higher than the fundamental, with the lowest being found at around 600 MHz, because the inner conductor of the damper structure was much shorter than the inner conductor of the test cavity itself. The second mode type introduced was that of a half-wavelength coaxial resonator made up of the combined test cavity and
damper structure. In this case the inner conductors of the damper and cavity acted as a single inner conductor broken by a capacitance across the gap. These half-wavelength resonances were stronger for the case of the damper with the horn, for which the capacitance across the gap was larger. They were also stronger for higher frequency harmonics, since at higher frequencies the capacitance appeared more like a short circuit across the gap in the inner conductor, and the system became a better half-wavelength resonator. These resonances were observed starting at around 255 MHz for the case of the damper with the horn attached, and starting at around 500 MHz when no horn was attached to the damper.

In the case of both of these new types of modes observed in the combined damper/cavity structure, the shunt impedances were much lower than that of the fundamental resonance. The largest shunt impedance of the new modes was that of the lowest observable half-wavelength resonance for the damper with the horn attached, for which the magnitude of the shunt impedance when the damper was unloaded was in the range of 3 kΩ, less than 2% of the impedance at the fundamental. All of the new modes were damped strongly once the damper was fully loaded, and in no case did the new modes have impedances which exceeded 100 Ω for the fully loaded case. This is significant since it means that the addition of the damper structure did not add any modes for whom the shunt impedance across the gap would be greater than 1 kΩ for the fully damped system, and thus could potentially result in beam instabilities. It is also worth noting that the addition of the damper structure did not add any new circumferential modes in the relevant frequency range (up to 800 MHz).

The high-pass filter which made up the higher-order-mode damper structure was terminated in a set of four removable 50 Ω loads placed around the outer wall of the damper. The frequency, $Q$ factor, and shunt impedance of each mode was measured using the bead-pull apparatus for the combined cavity/damper structure, both with no loads attached (unloaded) and with all four loads attached (fully loaded). When all four loads were attached the frequencies of the resonances were often lowered slightly, and the $Q$ factor and shunt impedances were reduced dramatically for all modes other than the fundamental. The
effectiveness of the damper was determined by comparing the unloaded (undamped) and loaded (damped) $Q$ factors and shunt impedances for all of the higher-order-modes.

### 6.2 Shunt Impedances of Cavity/Damper Structure with Horn

The frequency, $Q$ factor, and shunt impedance for the cavity/damper structure with the horn attached were measured using the cavity perturbation apparatus for both the loaded and unloaded cases for resonances up to 800 MHz. The change in the $Q$ factor of each mode with damping is recorded in Table 6.1. The change in the shunt impedance of each mode with damping is recorded in Table 6.2.

**Table 6.1: $Q$ factor damping of the modes of the cavity/damper structure with horn**

<table>
<thead>
<tr>
<th>Unloaded Frequency (MHz)</th>
<th>Unloaded $Q$ Factor</th>
<th>Loaded Frequency (MHz)</th>
<th>Loaded $Q$ Factor</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>65.4940 ± 0.0001</td>
<td>2423 ± 2</td>
<td>65.4934 ± 0.0002</td>
<td>2341.4 ± 0.4</td>
<td>3%</td>
</tr>
<tr>
<td>195.692 ± 0.002</td>
<td>2000 ± 10</td>
<td>194.521 ± 0.001</td>
<td>63.31 ± 0.02</td>
<td>97%</td>
</tr>
<tr>
<td>255.547 ± 0.001</td>
<td>1260 ± 5</td>
<td>255.04 ± 0.01</td>
<td>9.135 ± 0.005</td>
<td>99.3%</td>
</tr>
<tr>
<td>339.629 ± 0.003</td>
<td>1674 ± 4</td>
<td>No 4</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>416.650 ± 0.001</td>
<td>1259.2 ± 0.4</td>
<td>393.7 ± 0.2</td>
<td>9.6 ± 0.2</td>
<td>99.2%</td>
</tr>
<tr>
<td>520.528 ± 0.0006</td>
<td>3168 ± 5</td>
<td>506.33 ± 0.01</td>
<td>45.4 ± 0.1</td>
<td>99%</td>
</tr>
<tr>
<td>644.19 ± 0.02</td>
<td>407 ± 3</td>
<td>No 3</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>656.849 ± 0.009</td>
<td>657 ± 1</td>
<td>No 1</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>661.123 ± 0.004</td>
<td>1281.7 ± 0.3</td>
<td>650.8 ± 0.1</td>
<td>40.2 ± 0.1</td>
<td>97%</td>
</tr>
<tr>
<td>714.295 ± 0.007</td>
<td>724 ± 3</td>
<td>714.073 ± 0.001</td>
<td>303.9 ± 0.1</td>
<td>60%</td>
</tr>
</tbody>
</table>
Table 6.2: Shunt impedance damping of the modes of the cavity/damper with horn

<table>
<thead>
<tr>
<th>Unloaded Frequency (MHz)</th>
<th>Unloaded $R_S$ (kΩ)</th>
<th>Loaded Frequency (MHz)</th>
<th>Loaded $R_S$ (kΩ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65.4940 ± 0.0001</td>
<td>157 ± 1</td>
<td>65.4934 ± 0.0002</td>
<td>148 ± 1</td>
</tr>
<tr>
<td>195.692 ± 0.002</td>
<td>33 ± 3</td>
<td>194.521 ± 0.001</td>
<td>&lt;1</td>
</tr>
<tr>
<td>255.547 ± 0.001</td>
<td>3.0 ± 0.3</td>
<td>255.04 ± 0.01</td>
<td>&lt;0.02</td>
</tr>
<tr>
<td>339.629 ± 0.003</td>
<td>6.2 ± 0.6</td>
<td>No Resonance</td>
<td></td>
</tr>
<tr>
<td>416.650 ± 0.001</td>
<td>1.9 ± 0.2</td>
<td>393.7 ± 0.2</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>520.5281 ± 0.0006</td>
<td>0.22 ± 0.02</td>
<td>506.33 ± 0.01</td>
<td>&lt;0.003</td>
</tr>
<tr>
<td>644.19 ± 0.02</td>
<td>&lt;0.1</td>
<td>No Resonance</td>
<td></td>
</tr>
<tr>
<td>656.849 ± 0.009</td>
<td>&lt;0.05</td>
<td>No Resonance</td>
<td></td>
</tr>
<tr>
<td>661.123 ± 0.004</td>
<td>&lt;2</td>
<td>650.8 ± 0.1</td>
<td>&lt;0.06</td>
</tr>
<tr>
<td>714.295 ± 0.007</td>
<td>&lt;0.7</td>
<td>714.073 ± 0.001</td>
<td>&lt;0.3</td>
</tr>
</tbody>
</table>

6.3 Shunt Impedances of Cavity/Damper Structure without Horn

The frequency, $Q$ factor, and shunt impedance for the cavity/damper structure with no horn attached were measured using the cavity perturbation apparatus for both the loaded and
unloaded cases for a resonances up to 800 MHz. The change in the Q factor of each mode with damping is recorded in Table 6.3. The change in the shunt impedance of each mode with damping is recorded in Table 6.4.

Table 6.3: Q factor damping of the modes of the cavity/damper structure with no horn

<table>
<thead>
<tr>
<th>Unloaded Frequency (MHz)</th>
<th>Unloaded Q Factor</th>
<th>Loaded Frequency (MHz)</th>
<th>Loaded Q Factor</th>
<th>Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>65.8665 ± 0.0004</td>
<td>2041 ± 3</td>
<td>65.8657 ± 0.0001</td>
<td>2036 ± 3</td>
<td>0.3%</td>
</tr>
<tr>
<td>204.984 ± 0.001</td>
<td>1759 ± 6</td>
<td>198.97 ± 0.01</td>
<td>38 ± 1</td>
<td>98%</td>
</tr>
<tr>
<td>327.999 ± 0.003</td>
<td>1182 ± 1</td>
<td>297.21 ± 0.02</td>
<td>6.8 ± 0.1</td>
<td>99.4%</td>
</tr>
<tr>
<td>413.638 ± 0.006</td>
<td>1150 ± 20</td>
<td>396.47 ± 0.02</td>
<td>39.0 ± 0.2</td>
<td>97%</td>
</tr>
<tr>
<td>515.328 ± 0.002</td>
<td>2210 ± 1</td>
<td>No Resonance</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>571.63 ± 0.05</td>
<td>1400 ± 100</td>
<td>No Resonance</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>640.60 ± 0.04</td>
<td>228 ± 7</td>
<td>No Resonance</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>653.965 ± 0.004</td>
<td>864.8 ± 0.8</td>
<td>642.94 ± 0.01</td>
<td>38.45 ± 0.03</td>
<td>96%</td>
</tr>
<tr>
<td>713.94 ± 0.04</td>
<td>619 ± 3</td>
<td>713.73 ± 0.03</td>
<td>300.4 ± 0.1</td>
<td>50%</td>
</tr>
<tr>
<td>797.239 ± 0.004</td>
<td>559.3 ± 0.6</td>
<td>790.105 ± 0.002</td>
<td>35.8 ± 0.3</td>
<td>94%</td>
</tr>
</tbody>
</table>
6.4 Conclusions

The damper was effective in damping the higher-order-mode resonances without strongly affecting the fundamental resonance both with and without the horn attached. The purpose of the horn is to make the cavity/damper system a single gap structure rather than to
improve the filter performace, however the additional two filter elements introduced when
the horn is added do have a small effect on the damping. At the fundamental frequency, the
reduction in $Q$ factor as a result of damping (which is a measure of the power lost) was
measured at 3% for the damper with the horn attached, and 0.3% for the damper with no horn
attached. Although this would seem to indicate that the filter without the horn gives a better
performance at the fundamental frequency, the power lost by the filter with the horn could
probably be reduced by optimizing the capacitances between the horn and the inner
conductor and the horn and the first washer. The shunt impedances also remained relatively
unchanged at the fundamental frequency, with the damper reducing the fundamental
impedance by 6% in the case of the damper with the horn attached, and by 2% in the case of
the damper with no horn attached.

At the higher-order-mode frequencies, both damper designs were effectively able to
damp both the $Q$ factors and shunt impedances of the TEM modes up to 800 MHz. Damping
of the $Q$ factors of these modes was between 95% and 100% both with and without the horn,
and in all cases the shunt impedance of the higher-order modes with damping were measured
to be less than 1 k$\Omega$. There was very little difference in the ability of the two damper designs
to damp the higher-order TEM modes, but the horn may be required to shield the beam from
interacting with the filter structure. The higher-order-mode damper had a much less dramatic
effect on the non-TEM modes for which the high-pass filter was less able to couple to the
modes. For the $TE_{1,1}$ mode both damper designs reduced the $Q$-factor of the resonance by
approximately 50% (see Tables 6.1 and 6.3), bringing the shunt impedance to below 1 k$\Omega$ in
both cases.

Not only did the addition of the horn to the cavity not fundamentally change the
overall performance of the damper, it also left the pattern of the field across the gap relatively
unchanged. Figure 6.1 shows the frequency shift (which is proportional to the square root of
the electric field strength) as the bead was pulled across the gap for the structure both with
the horn attached and without. Although the gap between the inner conductor and the first
damper element was set the same for both configurations, the effective total gap of the cavity/damper structure with no horn attached was larger, so that the washers were now inside the overall gap rather than beyond it. The change in the field pattern caused by removing the horn was to distribute the field along the entire length of this new extended gap, while keeping the shunt impedance more or less constant.

6.5 Improvements

Several improvements could be made to the cavity perturbation measurement apparatus which would allow measurements to be made on a wider variety of cavities. One change would be to use a series of three ellipsoids as beads rather than one spherical bead. This much more complicated system would allow one to measure not only field strength but also direction. Thus one could determine the field patterns of structures which have fields which are not necessarily parallel to the direction along which the bead is being pulled.

The system could also be modified to use a combination of metallic and dielectric beads. This would allow one to measure the magnetic field strength as well as the electric. The use of three metallic and three dielectric ellipsoidal beads would allow one to determine the magnitude and direction of both the magnetic and electric fields. This would, however, dramatically increase the complexity of the measurement system.

Finally, the system could be adapted to attempt to compensate for bead imaging in the case of beads passing near a flat conducting surface. Formulas have been derived to estimate the perturbative effects of such images, which could be incorporated into the program. This would allow for measurements on a wider variety of structures than those allowed by the current system.

Several improvements could also be made to the high-pass-filter-type higher-order-mode damper in order to enhance its performance. In the first place, one could attempt to accurately measure the capacitances between the washer elements and the inductances of the connecting rods. One could then adjust the lengths of the rods and the distances between the
washers in order to optimize the high-pass-filter structure for specific corner frequencies. Axial slots could also be cut in the washers in order to cut the currents of potential circumferential modes. This is an effective method of blocking such modes without fundamentally affecting the capacitive function of the washers in the high-pass filter. Blocking such circumferential modes becomes more important as the circumference of the damper elements becomes greater relative to the overall length of the resonating cavity, in which case the circumferential modes will come much closer to the fundamental frequency. One could also attempt to reduce the inductances of the connections between the final filter element and the absorbing loads since these inductances adversely effect the response of the filter at high frequencies. By making these changes, the damper's performance could be optimized for application to specific cavity structures.

Figure 6-1 - Frequency shift (Hz) vs. distance (mm) with and without horn

Distance from inner conductor (mm)
References


Appendix A

Tip Loading of the Shunt Impedance of a \(\lambda/4\) Coaxial Resonator

A \(\lambda/4\) coaxial resonator is made up of a coaxial transmission line which has a short placed between the two conductors at one end (the short circuit end) and no connection between the two conductors at the other end (the open circuit end). In order to make this a completely closed structure, the outer conductor is extended a small distance beyond the length of the inner conductor, and a flat conducting surface is added to the end of the outer conductor. Due to the capacitance of the additional portion of the outer conductor, the shunt impedance of this structure, as measured across the gap between the two conductors at the open circuit end, will no longer be given by equation (13) in Chapter 1 which states:

\[
R_s/Q = 4Z_o/\pi \tag{A-1}
\]

Equation (A-1) must be modified to account for the additional structure. This can be done, as in Chapter 1, by calculating the shunt impedance from the reactive power in the modified cavity, but in this case using the inductive reactive power (which remains unchanged at the tip by the modification) rather than the capacitive reactive power.

Let the distance along the cavity from the short circuited end to the open circuit end be given by \(x\), the length of the inner conductor be given by \(x_i\) and the length of the outer conductor by \(x_o\). Define the angle \(\theta\) by the relation:

\[
\theta = \frac{\pi x}{2x_o} \tag{A-2}
\]

Define the angle \(\varphi\) by the relation:

\[
\varphi = \frac{\pi x_i}{2x_o} \tag{A-3}
\]

Note that \(\varphi = \theta\) when \(x = x_i\). The transmission line voltage and current expressed as a function of distance from the short circuited end \((V_x\) and \(I_x)\) are given by the relations:
Here $V_{\text{max}}$ and $I_{\text{max}}$ are maximum current and voltage values, which are found at the values $\theta = \pi/2$ and $\theta = 0$ respectively. The distributed inductance $L'$ of the transmission line is given by the relation:

$$L' = Z_0/c \quad (A-6)$$

Here $Z_0$ is the characteristic impedance of the transmission line and $c$ is the speed of light. The energy per unit length stored in the distributed inductance is given by the relation:

$$dE = \frac{1}{2} (I_x)^2 L' dx = \frac{1}{2} (I_x)^2 (2x_0 / \pi) d\theta \quad (A-7)$$

This can be rewritten as:

$$dE = (I_{\text{max}} \cos \theta)^2 (Z_0 x_0 / \pi c) d\theta \quad (A-8)$$

The frequency of the fundamental cavity resonance is given by the relation:

$$\omega = 2\pi c / \lambda = \pi c / 2x_0 \quad (A-9)$$

Multiplying both sides of equation (A-9) by the frequency and canceling terms gives:

$$\omega dE = \frac{1}{2} (I_{\text{max}} \cos \theta)^2 Z_0 d\theta \quad (A-10)$$

Integrating the equation (A-10) along the length of the inner conductor gives the reactive power in the distributed inductance:

$$\omega E = \int_0^\varphi \frac{1}{2} (I_{\text{max}} \cos \theta)^2 Z_0 d\theta = \frac{1}{2} (I_{\text{max}})^2 Z_0 \int_0^\varphi \cos^2 \theta d\theta \quad (A-11)$$

$$\omega E = \frac{1}{4} (I_{\text{max}})^2 Z_0 \left( \varphi + \frac{1}{2} \sin 2\varphi \right) \quad (A-12)$$

For a transmission line, the relationship between peak voltage and peak current is given by:

$$Z_0 = V_{\text{max}} / I_{\text{max}} \quad (A-13)$$

Thus, equation (A-12) can be rewritten in terms of the peak voltage as:
\[ \omega E = \frac{1}{4} \left( \frac{(V_{\text{max}})^2}{Z_0} \right) (\varphi + \frac{1}{2} \sin 2\varphi) \] (A-14)

The voltage at the tip of the inner conductor (\(\theta = \varphi\)) is given by the relation:

\[ V_{\text{tip}} = V_{\text{max}} \sin \varphi \] (A-15)

Thus, equation (A-13) can be rewritten in terms of the tip voltage as:

\[ \omega E = \frac{(V_{\text{tip}})^2 (\varphi + \frac{1}{2} \sin 2\varphi)}{4 Z_0 \sin^2 \varphi} \] (A-16)

Equation (A-16) is an expression for the reactive power in the resonator as a function of the angle \(\varphi\). The \(Q\) factor of the resonator is related to the reactive power by the relation:

\[ Q = \omega E / P \] (A-17)

Here \(P\) is the input power. Equation (A-16) can thus be rewritten in terms of the \(Q\) factor as:

\[ QP = \frac{(V_{\text{tip}})^2 (\varphi + \frac{1}{2} \sin 2\varphi)}{4 Z_0 \sin^2 \varphi} \] (A-18)

Rearranging the terms in equation (A-18) one obtains:

\[ \frac{(V_{\text{tip}})^2}{2P} = \frac{2Q Z_0 \sin^2 \varphi}{\varphi + \frac{1}{2} \sin 2\varphi} \] (A-19)

The left-hand side of this equation, however, is simply the shunt impedance at the tip of the inner conductor, as defined in equation (1) of Chapter 1. Thus equation (A-19) may be rewritten in terms of the shunt impedance \(R_s\) as:

\[ \frac{R_s}{Q} = \frac{4Z_0 \sin^2 \varphi}{2\varphi + \sin 2\varphi} \] (A-20)

This is the expression (A-1) for the shunt impedance of a coaxial \(\lambda/4\) resonator modified to take into account the additional length of the outer conductor.
Appendix B

Solutions of Maxwell's Equations for a Coaxial Transmission Line

Maxwell's equations allow for three distinct types of modes of propagation of fields on a transmission line. The first type of mode is the TEM or transverse electromagnetic, for which in a coaxial transmission line the electric and magnetic fields are pointed in the radial and circumferential directions respectively. The exact field profile is given by:

\[
E = \frac{V_0 a_r e^{jkr}}{r \ln(b/a)} \quad \text{(B-1)}
\]

\[
H = \frac{V_0 a_\varphi e^{jkr}}{r Z_0 \ln(b/a)} \quad \text{(B-2)}
\]

Here \(E\) is the electric field, \(H\) is the magnetic field, \(V_0\) is the peak voltage between the two conductors, \(a_r\) is the unit vector in the radial direction, \(a_\varphi\) is the unit vector in the circumferential direction, \(j\) is the square root of -1, \(z\) is the distance along the cavity axis, \(Z_0\) is the characteristic impedance of the transmission line, \(r\) is the distance from the cavity axis, \(b\) is the radius of the outer cavity, \(a\) is the radius of the inner cavity, and \(k\) is the wave number defined by the equation:

\[
k = \omega (\mu_e \varepsilon_0)^{1/2} \quad \text{(B-3)}
\]

For TE (transverse electric) modes in a coaxial transmission line, only the electric fields are perpendicular to the transmission line axis. The set of all such modes forms a doubly infinite series, denoted by two subscripted indices which run from 0 to \(\infty\) (TE\(_{1,1}\), TE\(_{0,2}\), etc.). The exact field profile of the mode TE\(_{n,m}\) is given by the following relations:

\[
H_z = \left( k^2/j\omega \mu \right) e^{j\omega t - kr} \left[ AJ_n(kr) + BN_n(kr) \right] \cos n\theta \quad \text{(B-4)}
\]

\[
H_\theta = - (\gamma / r k^2) \partial H_z / \partial \theta \quad \text{(B-5)}
\]

\[
H_r = - (\gamma / k^2) \partial H_z / \partial r \quad \text{(B-6)}
\]

\[
E_z = 0 \quad \text{(B-7)}
\]

\[
E_\theta = - Z_0 H_r \quad \text{(B-8)}
\]
\[ E_r = Z_w H_\theta \] (B-9)

Here \( E_r, H_r, E_\theta, H_\theta, E_0, \) and \( H_0 \) are the electric and magnetic field strengths in the longitudinal, circumferential and radial directions respectively, the directions \( z, \theta, \) and \( r \) are as defined above for the TEM modes, \( t \) is time, \( n \) is the first subscripted index of the mode, \( J_n(kr) \) and \( N_n(kr) \) are Bessel functions of the first and second kind respectively of order \( n \), and \( A \) and \( B \) are constants given by the boundary condition:

\[
\frac{-B/A}{N_n'(ka)} = \frac{J_n'(ka)}{J_n'(kb)} \quad \frac{-B/A}{N_n'(ka)} = \frac{J_n'(kb)}{N_n'(kb)} \tag{B-10}
\]

Here \( a \) and \( b \) are the inner and outer conductor radii as defined for TEM waves, the primes denote differentiation with respect to \( k \), and \( k \) is the \( m \)th root of equation (B-10) where \( m \) is the second subscripted index of the mode. This modified wave number \( k \), is also given by the relation:

\[ k^2 = k_0^2 + \gamma^2 \tag{B-11} \]

Here \( k_0 \) is the wave number as defined in equation (B-3), and \( \gamma \) is the attenuation constant which is related to the wave impedance \( Z_w \) by the relation:

\[ Z_w = j\omega\mu / \gamma \tag{B-12} \]

Here \( j \) and \( \omega \) are as defined above for TEM modes and \( \mu \) is the magnetic permeability of the material between the two conductors.

For TM (transverse magnetic) modes in a coaxial transmission line, only the magnetic fields are perpendicular to the transmission line axis and the set of all such modes forms a doubly infinite series as in the case of the TE modes. The exact field profile of the mode \( \text{TM}_{n,m} \) is given by the following relations:

\[
E_z = (k^2 / j\omega\epsilon) e^{j\omega t - kr} \left[ AJ_n(kr) + BN_n(kr) \right] \cos n\theta \tag{B-13}
\]

\[
E_\theta = - (\gamma / rk^2) \partial E_z / \partial \theta \tag{B-14}
\]

\[
E_r = - (\gamma / kr^2) \partial E_z / \partial r \tag{B-15}
\]

\[
H_z = 0 \tag{B-16}
\]

\[
H_\theta = E_r / Z_w \tag{B-17}
\]
\[ H_r = -\frac{E_\theta}{Z_\omega} \]  \hspace{1cm} (B-18)

Here everything is defined as for the TE modes except that \( \varepsilon \) represents the electric permitivity of the material between the dielectrics, and the constants \( A \) and \( B \) are given by the boundary condition:

\[
\frac{-B}{A} = \frac{J_n(ka)}{N_n(ka)} = \frac{J_n(kb)}{N_n(kb)} \quad (B-19)
\]

Note that there is no differentiation of the Bessel functions with respect to \( k \) for the TM case, although \( k \) remains the \( m \)th root of the above equation where \( m \) is the second subscripted index of the mode as described above.
Appendix C

Change in Field Energy Caused by Placing a Dielectric Sphere in a Uniform Electric Field

The energy $U$ stored in a uniform electric field of volume $V_{\text{total}}$ is given by the relation:

$$U = \int_{V_{\text{total}}} \frac{1}{2} \varepsilon \varepsilon_0 E_u^2 \, dv$$  \hspace{1cm} (C-1)

Here the integral is taken over the total volume, $\varepsilon_0$ is the electric permittivity of free space, and $E_u$ is the strength of the uniform electric field. When a dielectric sphere is placed inside the field, the field inside the sphere will be homogeneous, in the direction of the original field, and with a field strength $E_{\text{sphere}}$ given by the relation:

$$E_{\text{sphere}} = \frac{3E_u}{\varepsilon + 2}$$  \hspace{1cm} (C-2)

Here $\varepsilon$ is the dielectric constant of the sphere, and the factor 3 stems from the depolarization factor $N$ of a sphere which is 1/3 [9].

Outside of the sphere, the electric field will be a linear combination of the original uniform field and a dipole field centered around the sphere, with the dipole pointed in the direction of the original uniform field. This dipole field has radial and longitudinal components $E_r$ and $E_\theta$ given by the relations:

$$E_r = \frac{pcos\theta}{2\varepsilon_0 r^3}$$  \hspace{1cm} (C-3)

$$E_\theta = \frac{psin\theta}{4\varepsilon_0 r^3}$$  \hspace{1cm} (C-4)

Here $r$ is the distance from the center of the sphere, $\theta$ is the longitudinal angle, and $p$ is the strength of the dipole moment of a sphere given by the relation:

$$p = 3V_{\text{sphere}} E_u \frac{\varepsilon_0 (\varepsilon - 1)}{\varepsilon + 2}$$  \hspace{1cm} (C-5)
Here $E_u$ is the original electric field vector [9].

The change in energy $\Delta U$ as the sphere is placed in the uniform electric field is given by the difference of the total energy as calculated using equation (C-1) and the sum of the energies of the parts of the perturbed field.

\[
\Delta U = U_{\text{total}} - (U_{\text{sphere}} + U_{\text{dipole}})
\]  

(C-6)

For the case of beads which are much smaller than both the total volume of the original field, and the wavelength of the field, the above equation can be approximated by the following relation:

\[
\Delta U = U_{\text{total}} - U_{\text{dipole}}
\]  

(C-7)

\[
\Delta U = \int \varepsilon_0 E_u^2 - \varepsilon_0 (E_u + E_{\text{dipole}})^2 dV
\]  

(C-8)

Here $E_{\text{dipole}}$ is the dipole electric field vector which is the sum of the two components given in equations (C-3) and (C-4) and the integral is taken in the area outside the bead. One can make further approximations to equation (C-8) by dropping the term which contains the square of the dipole fields to give:

\[
\Delta U = -\int \varepsilon_0 E_u \cdot E_{\text{dipole}} dV
\]  

(C-9)

Here the integral contains the dot product of the two field vectors. Performing this integral over all angles $\theta$ and $\phi$ (where $\theta$ is the latitudinal angle for which the product of the two fields remains constant) and over all radii from the radius of the sphere to infinity, one obtains the following solution:

\[
\Delta U = -\frac{1}{2} \pi \rho E_u = -3 V_{\text{sphere}} E_u \frac{\varepsilon_0 (\varepsilon_\gamma - 1)}{2(\varepsilon_\gamma + 2)}
\]  

(C-10)

Here all terms in the equation are defined as for the dipole moment above [9]. Taking the time average of the electric field and using the radius rather than the volume of the sphere one obtains the relation:

\[
\Delta U = -\varepsilon^* E_u^2 \pi r^3
\]  

(C-11)

Here $\varepsilon^*$ is the modified dielectric constant which is related to the dielectric constant $\varepsilon$ of the bead by the relation:
\[ \varepsilon^* = \frac{\varepsilon_0 (\varepsilon - \varepsilon_0)}{\varepsilon + 2\varepsilon_0} \]  

(C-12)

This is the result used in the derivation of the perturbation formula in Chapter 2.
Appendix D

Structure of the Network Analyzer Control Program

The subroutines (and thus the operating menus and functions) of the program ANALYZE used during the cavity perturbation menus had the following tree structure:

```
ANALYZE
  |--- PROBLEM
  |     |--- PORTSET
  |     |     |--- WHICHCHANNEL
  |     |     |--- SAVESETTINGS
  |     |     |--- RECALLSETTING
  |     |--- CALIBRATION
  |     |--- MOTOR
  |     |--- DEFAULT
  |     |--- SETTINGS
  |     |     |--- PROBLEM
  |     |     |     |--- CONFIGURATIONCHECK PROBLEM
  |     |     |--- CHANNELCHANGE
  |     |     |--- CONFIGCHANGE
  |     |     |--- FORMATCHANGE
  |     |     |--- SCALECHANGE
  |     |     |--- REFPCHANGE
  |     |     |--- REFVCHANGE
  |     |     |--- STARTCHANGE
  |     |     |--- STOPCHANGE
  |     |     |--- CENTERCHANGE
  |     |     |--- SPANCHANGE
  |     |     |--- POINTCHANGE
  |     |     |--- POWERCHANGE
  |     |     |--- TIMECHANGE
  |     |     |--- IFBANDCHANGE
  |     |     |--- AVERCHANGE
  |     |     |--- SMOOTHCHANGE
  |     |     |--- SAPERCHANGE
  |     |     |--- SWEEPCHANGE
  |     |     |--- CALCHANGE
  |--- MEASUREMENT
  |     |--- QMEASUREMENT
  |     |     |--- PROBLEM
  |     |     |     |--- CONFIGURATIONCHECK
  |     |     |     |--- CONFIGCHANGE
```

to next page
PERTURBATION
  "DIRADIUSFIND"
  "QMEASUREMENT"
    "PROBLEM"
      "CONFIGURATIONCHECK"
      "CONFIGCHANGE"
  "FIELDMEASUREMENT"
    "SPANSET"
      "TRIALNUMSET"
      "SPANFIND - MARKER2FIND"
      "SHIFTFIND - MARKER2FIND"
      "MARKER2FIND"
      "ZEROERRORFIND"
      "STRENGTHPRINT"
      "SHUNTCALC"
  "IMPEDANCEMEASUREMENT"
    "PORTSET"
      "SPANSET"
      "TRIALNUMSET"
      "SPANFIND MARKER2FIND"
      "MARKER2FIND"
      "ZEROERRORFIND"
      "STRENGTHPRINT"