THE EFFECTS OF PEER COLLABORATION ON STUDENT PERFORMANCE IN SOLVING PROCESS PROBLEMS IN MATHEMATICS

BY

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ABSTRACT

This study investigated the effects of group collaboration on children's performance in and their attitudes towards Math problem solving, compared to individual work. Twenty-eight grade six students in a single classroom were randomly assigned to either a group- or individual-work condition for eleven sessions of working with process problems in Math.

An analysis of covariance conducted on group means of problem-solving posttests with the pre-test and test administration order held as covariates, showed no significant difference between the group-work and individual-work conditions. An analysis of covariance on the means of posttests of the attitudes towards problem solving scale with pretest held as covariate, showed no significant difference between the two conditions in changes in attitude towards problem solving. An analysis of variance performed on the means of the summary scores of the answers to problems solved by students during the eleven sessions showed a significant difference between the two conditions, favouring the group-work condition.

Qualitative methods of analysis were used to assess the use of strategies and reveal group interaction activities that may have supported problem solving. Measures included transcriptions from individual interviews and six videotaped problem-solving sessions, answer sheets for problems solved, and recorded observations of students.

Analyses showed no difference between the two conditions in the type or number of strategies used. Interaction patterns were identified in group collaboration that may have allowed for better problem solving by groups of students during the intervention period: (a) suggesting strategies or ideas; (b) evaluation, checking and monitoring activities; (c) feedback in the form of explanations and clarifications that supported understanding of the problem by group members; (d) specific speech patterns that showed students mimicing or echoing each other's comments, speaking

simultaneously, or completing another's comment; (e) guiding discussion and facilitating the workload; and (f) group members persevering together in completing the assignment.

Although significant differences between the group-work and individual-work conditions on posttest problems were not observed when students solved problems alone, the significant difference favouring group-work when the students were working together suggests that peer interaction for elementary school students may have positive results.

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CHAPTER ONE: INTRODUCTION

Background Of The Problem

One of the difficulties many students have in Mathematics is the solving of problems. This study examined the impact of groups collaborating to solve problems, and individuals working alone to solve problems, on students' achievement in and attitudes to Math problem solving.

In their 1980 Agenda for Action, the National Council of Teachers of Mathematics (NCTM) placed first on their agenda their recommendation that problem solving be the focus of school mathematics. They suggested that students need to be able to solve real-world problems in unexpected, unplanned settings, and that the Mathematics curriculum should be organized around problem solving. In the province of British Columbia, a similar philosophy is apparent in the Response Draft to Problem Solving Across the Curriculum (1994) published by the Ministry of Education. In the Introduction, it is stated: "It is hoped that learning how to solve problems by applying knowledge and skills to real-life situations will assist students to grow intellectually and to make connections between various areas of study." (p.2)

Assessments of problem solving abilities of students in the province of British Columbia have shown that problem solving continues to be a critical area where more students need to participate fully in the activity. In 1990, Grades 4, 7 and 10 students were assessed across the province on their mathematics abilities and given questionnaires on their math experiences (Robitaille, 1991). Approximately 43 percent of students across the three grade levels achieved satisfactory or better levels in solving problems. One-quarter of students at each grade level did not attempt to solve the problems, showed no work, or identified only the data, in their problem solutions. Two extremes of ability were seen: students had little understanding of the problem and strategy implementation, or they had considerable understanding and strategy

implementation. A minority fell in the middle range (Szetela, 1991). Similar results were obtained in the 1977 and 1981 B.C. Mathematics Assessments (Sherrill, 1983) and in the United States (Duren & Cherrington, 1992; Silver & Thompson, 1984).

The Mathematics Curriculum/Assessment Framework Draft document (1992) published by the B. C. Ministry of Education defines problem solving:

> A problem may be a question requiring an answer, where that answer is not immediately evident and cannot be arrived at without inquiry. A problem may also be a situation requiring an explanation, where that explanation is not immediately evident and cannot be arrived at without inquiry. (p.24)

The National Council of Teachers of Mathematics (1987) suggests that to be good at solving problems, a student

> must be able to explore a situation, to understand fully what is involved, to know and apply strategies and heuristics to generate ideas, to translate the situation into different forms using mathematical models and ideas, to select the appropriate mathematical tools and procedures, to apply these tools and procedures to answer questions about the situation, to represent the situation in different ways, to reach valid conclusions, and to make reasonable predictions. (p. 153)

In addition to problem-solving ability, students' attitudes to problem solving affect their degrees of effort and participation. There is a growing perception that mathematics anxiety threatens both achievement and participation in mathematics, with anxiety increasing through junior high school, peaking at Grades 9 and 10, then leveling off (Hembree, 1990). The role of affect, which includes the students' interest in the task, willingness to take risks, motivation, perseverance, self-confidence, tolerance of ambiguity, and resistance to premature closure, is considered to be a key

component of successful problem solving (Charles, Lester & O'Daffer, 1987; Lester & Kroll, 1990).

The 1985 B. C. Math Assessment reported a strong correlation between students' achievement and their opinions of the importance, difficulty and enjoyment of Math. Two of the contributors to the report emphasized the importance of the relationship between the affective domain and achievement in the cognitive domain (Taylor & Robitaille, 1987). The 1990 B. C. Mathematics Assessment had shown that attitudes to problem solving declined throughout the grades, from two-thirds of grade 4 students, one-half of grade 7 students, to one-third of grade 10 students finding problem solving to be enjoyable or interesting (Robitaille, Schroeder & Nicol, 1990), paralleling the similar decrease in students' perceptions of their abilities in solving problems (Szetela, 1991).

If classroom practices can facilitate an increase in positive attitudes to problem solving, students' abilities in solving problems may improve. One practice that may be helpful for slowing the decline in students' positve attitudes to problem solving and in their perceptions of their abilities may be to have students work occasionally in collaborative groups. When students solve problems in groups, an environment may be created that minimizes anxiety and competition and where they feel safe to make mistakes. When more students have success in solving problems, their attitudes to problem solving may change (Weissglass, 1990).

In the last fifteen years, there has been a growing interest in using small groups for mathematics problem solving. The Commission on Standards for School Mathematics of the NCTM (1987) suggested that frequent opportunities to work together in small groups allow students to discuss strategies and solutions, ask questions, examine consequences and alternatives, and to verify their own thinking. Students working in groups can often handle more challenging problems than they can individually, where students working alone often experience severe and unnecessary frustration (Davidson, 1990).

The research literature on small-group learning generally supports claims for positive learning effects (Johnson & Johnson, 1990; Noddings, 1985; Webb, 1991). Collaborative contexts may allow students with different skills and points of view to work together (Johnson & Johnson, 1990; Weissglass, 1990) by assisting each other in the task and learning from each other (Davidson, 1990).

The claims for improved attitudes and abilities through work in small groups were a focus of this research. It appears that in most British Columbia classrooms, the use of small groups in Math is not often the norm. When B. C. students were assessed in 1990, they also reported on their time spent working in small groups. For grade 4 students, 31 percent reported they frequently worked in small groups, and 27 percent reported they seldom did so. (The remaining numbers fall in the "Sometimes" category.) For grade 7 students, the numbers were 31 percent and 43 percent, and for grade 10 students, 18 percent and 65 percent respectively (Robitaille, 1991).

Research on small-group learning in the 1960's and 1970's focused on student achievement, self-esteem, attitudes, motivation, and social and communication skills (Johnson & Johnson, 1990). The need for research on the cognitive effects of both individual and group problem-solving processes began to be expressed in the mid 1980's (Forman & Cazden, 1985; Noddings, 1985). In the 1980's to 1990's, researchers studying peer interaction pointed to the need for expanding the context of the research beyond the laboratory, for identifying which features of the interaction benefit cognitive development, and at what points in development social agents are more important for learning (Azmitia & Perlmutter, 1989; Davidson & Kroll, 1991; Slavin, 1992; Webb, 1989).

The research reviewed does not generally provide information on how students' interactions provide the support to more easily solve problems. For example, what do they say to each other that permits better representations of the problem presented? To the extent that students actually benefit from such collaboration, an in-depth analysis is needed. By studying students working together

in collaborative groups, we may become more aware of the diverse abilities of students that are not apparent when they work individually, how they think as they solve problems, the areas of problem definition that pose particular difficulties, the strategies and procedures they have in their inventory, the support level in groups, and their shared thinking in context.

Problem Statement And Questions

The purpose of this study was to investigate the effects of working in groups on students' achievement in and attitudes to Math problem solving. Using quantitative methods of analysis, group-work and individual-work students (the control group) in a single grade-six classroom of twenty-eight students were compared on pre- and posttest performance on solving Math problems, summative scores on answers to problems solved during the intervention period, and changes in attitude to Math problem solving. Information from problem-solving session transcripts provided qualitative data on interaction between peers. Strategies used by both group-work and individual-work students, problem answer sheets, attitude inventories, interviews with students selected for more intensive analysis, and research memoes, were also part of the information available for qualitative analyses.

It was proposed that there is a positive relationship between small-group work and Mathematics problem solving performance. Children should show increased ability to solve problems and an improved attitude to problem solving after working in groups. Their interaction in groups was expected to provide students with increased support in solving problems. Specific questions were raised to direct the inquiry.

Questions:

1. What is the effect of group problem-solving on students' abilities to solve Mathematics word problems?
2. What is the effect of group problem-solving on students' attitudes toward Math problem solving?
3. What types of problem-solving strategies do students use in collaborative problem solving? Are these different from the strategies used by students in individualized problem solving?
4. How does collaboration promote interactions that support and guide group members in the problem-solving task?

Assumptions, Justifications And Contributions Of The Study

While research has been conducted on paired or small-group students removed from the classroom setting (for example, Forman, 1989; Forman & Cazden, 1985; Glachan and Light, 1982; Phelps & Damon, 1989; Skon, Johnson & Johnson, 1981), there has been little research on students interacting in groups in a regular classroom setting on typical school problem solving, especially at the intermediate grade level (except see Duncan, 1985).

An in-depth examination of as many features of the process as possible, is needed in order to understand what is involved as peers collaborate to solve problems. When students solve problems together, it is possible to study the arguments, elaborations, justifications, and strategies they use, that would not be observable when one individual solves a problem alone. From a methodological point of view, collective problem solving can also provide for the study of the evolution of problem-solving strategies (Amigues, 1988).

It was expected that students working in groups would support each other to more successful conclusions than if they had to solve the problems on their own. By

working together, they would think of appropriate strategies more quickly, and the strategies raised would be more frequent in number and variety. By collaborating on tasks with their peers of diverse strengths and background knowledge, opportunities for more students to be involved would be possible.

It was also expected that student attitudes to problem-solving would be more positive after they experienced less stressful problem-solving in groups. By distributing the responsibility for solving a problem amongst group members, pressure to perform would be lessened. The more relaxed environment would allow students who normally would struggle over problems, to take a more active role in the task. Those students would also learn more effective problem-solving procedures from other members of the group, and learn appropriate strategies for problem-solving that they would have more difficulty understanding and applying otherwise. Students who normally would have little difficulty in solving problems would be able to develop more refined problem-solving procedures by having to explain accurately and coherently.

Hypotheses

It was hypothesized that:

1. students working in groups would be higher on math problem-solving posttests marked on an analytic rating scale than the group of students working individually;
2. group-work students' posttest attitudes towards problem solving would be higher than the group of students working individually;
3. the types of problem-solving strategies used by the group-work students would be more varied than those of students working individually; and
4. collaboration would:

   (a) specify interactions taking place during problem solving; and

   (b) those interactions would assist students in problem solving.

Summary Of Chapter One

The Introduction to the chapter outlined the importance of problem solving in the Mathematics curriculum. Results from assessments in problem solving of British Columbia's elementary school students in 1990 were reviewed. It was pointed out that the practice of having students involved in small-group arrangements may enhance their attitudes to and participation in the often-difficult task of solving Math problems. Reference was made to the voluminous amount of research conducted since the 1960s on small-group learning, and the point made that further research is needed on how peer interactions may provide students the support they need to more easily solve problems in a regular classroom setting.

The purpose of this investigation was to examine quantitatively the effects of working in groups or individually on students' achievement in and attitudes to Math problem solving. Qualitative analyses examined the types of strategies used by students in both conditions, and the types of peer interactions that promoted or did not promote problem-solving success.

CHAPTER TWO: REVIEW OF THE LITERATURE

Introduction

The research literature shows there are cognitive and affective benefits involved in the interaction between peers, but to quote Slavin (1990b, p. 54), the "research must broaden our understanding of *how* and *why* . . . (it) produces its various effects."

When students solve mathematical problems of a non-routine nature together, they may generate suitably effective procedures and strategies, and be stimulated to produce unique or unusual solutions. As students interact in groups, their discourse may allow researchers to observe and examine the features of their "talk" that produce particular effects on each other. Research will be reviewed on collaborative interaction within such a setting. The literature outlines both the positive effects and the cautions.

One disappointing finding is the sparsity of relevant research on upper elementary school students. The literature, though voluminous in the last twenty years on both mathematical problem solving and peer interaction, concentrates heavily on high school, college, or pre-school and early primary children.

Problem Solving Procedures And Strategies

The Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics (1989) states problem solving should be used as a method of inquiry and application so that students can:

- develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;
- verify and interpret results with respect to the original problem situation;
- generalize solutions and strategies to new problem situations; and
- acquire confidence in using mathematics meaningfully. (p. 75)

To analyze the processes involved in solving problems by individuals and groups, a composite of the key processes researchers suggest are involved in problem solving (Charles, Lester & O'Daffer, 1987; Krulik & Rudnick, 1993; Lester & Kroll, 1990; Polya, 1957) was used as a reference:

- Understanding the Problem
- Developing a Plan
- Carrying out the Plan
- Looking Back

To solve a problem, students first need to understand the problem, or be able to make sense of it. They need to understand the meaning of specific words, how the question relates to other statements in the problem, and be able to sift and select relevant data. Next, in the planning phase, subgoals or subproblems need to be identified, and appropriate solution strategies selected, followed by correct implementation of the solution strategy. Lastly, a determination of the reasonableness of the answer needs to be made. Throughout the procedure, students need to monitor their progress to avoid pursuing unfruitful leads, and verify and justify the choices they make. It may include checking the answer against the problem conditions, rereading the problem, or using estimation techniques (Lester & Kroll, 1990).

This study considered and included Szetela's (1991) suggestion that students' answer statements also provide indications about their understanding of the goals of the problem, and completing answer statements can clue the students to the reasonableness or completeness of an answer.

Researchers caution that elementary students do not routinely analyze problem information, monitor progress, or evaluate results (Garofalo & Lester, 1985; Hart, 1993). Students do not ask questions such as: "Do I understand this problem? What am I doing this for? This doesn't make sense. How will this help me?" (Hart, 1993, p. 169). Other factors that impede problem-solving performance include lack of an experiential framework, altering the goals of the problem statement, and unproductive

beliefs (Hart, 1993). Proudfit (1980) found after interviews with 5th grade children that most errors made were in understanding the problem.

Behaviours accompanying successful problem solving included: focusing on relevant information and on the question being asked, explaining the reason that a strategy seemed appropriate for a particular problem, explaining the implementation of the strategy, evaluating the strategy and the solution in light of the conditions of the problem, drawing a diagram or making a model, and making the problem more concrete (Proudfit, 1980).

This study also tracked the problem-solving strategies the students used. Mayer (1983, p. 374) defines a strategy as a "general problem-solving technique that helps guide in solving problems". After reviewing the lists researchers (Charles, Mason & Garner, 1985; Krulik & Rudnick, 1993; Moretti, Stephens, Goodnow & Hoogeboom, 1987; Szetela, 1991) suggest are the most commonly used strategies of upper elementary students in problem solving, the following composite lists those which the students in the study might have used:

Look for a pattern

Work backwards

Guess and test

Make it simpler

Use logical reasoning

Make a list or table

Act out or use objects

Draw a picture, diagram or model

Choose a numerical model

Write an equation

In groups, students may be exposed to various strategies others have in their repertoire. That students need to have exposure to various strategies is supported by the research. Less capable students do not acquire a variety of cognitive strategies unless they are given detailed and explicit instruction in their use, and the more complex the strategy in question, the more explicit the instruction needed, even for more capable students (Brown & Campione, 1990). However, if children are simply taught what strategy to apply, the effects will likely be short-lived. Borkowski (1992) refers to a criticism of strategy instruction that it does not place sufficient emphasis on the learner's active involvement in construction of knowledge.

When students are stimulated to explore new strategies through interactions with more competent problem solvers, strategies appear to evolve through a process of "guided discovery." (Borkowski, 1992, p. 255). Artzt and Armour-Thomas (1992) observed that throughout problem-solving sessions, groups of students went back and forth using different heuristics intermittently, a behaviour that seemed to play an important role in successful problem solving. Their students made frequent comments about the conditions of the problem, recognized by Polya (1957) as so important in the problem-solving process.

The research literature suggests a positive advantage in children working together to facilitate their mutual growth, with the recognition of the problems younger students have in regulating and monitoring their actions. It is also evident that elementary school students need to have a wide exposure to different strategies, and the exposure is best maintained when they have had experience in using the strategies on problems that are realistic and relevant to them. The teacher's task would normally be to guide her students in the use of self-monitoring functions and provide direct instruction in the use of strategies. However, the design of this study negated such instruction, as it would not provide information on strategies and procedures the students may have or ascertain on their own, and may have contaminated the results.

The intent therefore was to observe the different procedures and strategies the students already had in their repertoire, and whether there was a difference in the group-work and individual-work students' use of strategies. The next section deals with the critical features of peer interaction that promote effective problem solving.

The Role Of Small Groups

The importance of the role of small groups in mathematical communication is addressed in the *Curriculum and Evaluation Standards for School Mathematics* by the National Council of Teachers of Mathematics (1987):

> Small groups . . . provide an environment in which students can practice and refine their growing ability to communicate mathematical thought processes and strategies . . . for asking questions, discussing ideas, making mistakes, learning to listen to others' ideas, offering constructive criticism, and summarizing discoveries in writing. (p. 57)

The peer interaction in the proposed study was collaborative but used the individual accountability feature found in cooperative learning. Damon and Phelps (1989) define peer collaboration as relative novices of roughly the same level of competence working together to solve challenging learning tasks that neither could do on their own prior to the collaborative engagement. In the more relaxed environment of a discovery learning situation, mistakes and difficulties are not seen as personally threatening, and the context becomes mutually supportive, capable of provoking deeper conceptual insights and shifts in perspective (Damon & Phelps, 1989).

Comparative reviews by Davidson (1990), Johnson and Johnson (1992) and Slavin (1990a) show that small-group approaches significantly outscored individual performance measures. A combination of individual accountability and group goals produced consistently positive effects. More frequent discovery and use of higher

level reasoning strategies occurred within cooperative than competitive or individualistic settings. When exploring open-ended problems, making conjectures and testing them with data, students in groups can often handle challenging situations, while individuals make little progress and experience severe and unnecessary frustration attempting the same problems (Davidson, 1990).

Caution must be taken, however, against inferring from the research evidence supporting the use of small groups in problem solving, that problem-solving instruction can only be effective in small group work. Peer collaboration can be helpful in facilitating learning of deep, basic concepts, but not for fostering rote learning (Phelps & Damon, 1989). Thoughtful whole-class instruction can sometimes produce as good or even better results than small groups (Noddings, 1985).

It may be that forcing students to work "cooperatively" may have a debilitating effect on both short-term and long-term learning (Lester, 1988). Some members may do all or most of the work while others go along for the ride, sometimes called the "free-rider" effect (Slavin, 1990a). The "diffusion of responsibility" problem can occur when there is little incentive for peers to explain to others what they are doing, or help less advanced students, who can easily be ignored (Slavin, 1990a). Stacey (1992) found that grade 9 students working on test problems together did not perform better than individual-work students when they quickly accepted easy solutions and did not demand explanations from each other. Students with simpler approaches appeared confident; those with alternative strategies may have been less confident and needed more time to formulate and articulate their doubts.

The use of small groups in problem solving appears to be generally supported in the research literature, with cautions raised as to the composition of groups. When expectations are placed on group members, it seems that more productive participation results. What types of support are there in group work that produces effective results, if any? The next section outlines some of the factors that may be beneficial to students in the group-work context.

Researchers appear to be in agreement that the solving of problems is enhanced in a small-group format that encourages verbalizations that can be critically examined, and where models of higher quality reasoning are available (Dimant & Bearison, 1991; Phelps & Damon, 1989; Slavin, 1992). The role of questioning, argumentation and elaboration in bringing about both shifts in perspective and the development of task definitions and strategies, is supported in the literature as it provides a distinct means of promoting an active linguistic and cognitive response (Gallimore & Tharp, 1990; Resnick, 1988; Rogoff, 1990).

One feature of group collaboration found to be particularly important in enhancing problem-solving performance is the encountering of challenge or disbelief from peers that may lead individuals to examine their own knowledge, beliefs and strategies more closely and discard unworkable ones (Hart, 1993; Hatano, 1988; Noddings, 1985). When students are required to explain, elaborate, or defend their positions to others, it can help them evaluate, integrate, and elaborate knowledge in new ways (Brown and Campione, 1990; Duren & Cherrington, 1992; Webb, 1989).

Studies on interactions in which task-relevant disagreements, contradictions, or contrary solutions are expressed, generally conclude that disconfirming statements must be justified if they are to facilitate effective cognitive reorganization. In the Doise and Mackie (1981) experiments with five- to ten-year-old children, more advanced responses appeared in collective situations before individuals were capable of them, and more subjects advanced when they gave opposed solutions. The Dimant and Bearison (1991) study with college students solving reasoning problems during peer interaction showed similar results.

Studies examining the hypothesis that social interaction can be a source of cognitive development even if the partner is less advanced, have shown that more advanced children benefited from sharing the task with a less advanced partner, and

that the element of conflict in social interactions is the source of their impact (Hatano, 1988; Perret-Clermont, 1980; Weinstein & Bearison, 1985). Researchers with a Piagetian focus suggest the more competent partner needs to introduce reasoning at a level appropriate for the less competent partner's thinking, and the less competent partner needs to accept the higher level reasoning in the course of discussion (Tudge, 1992). Kuhn's (1972) study found that exposure to a model at the developmental stage immediately above the subject's is more effective than when the developmental gap is as large as two stages. The gap between the model and the subject should be small enough that the subject can understand the question at issue, yet large enough that the contradiction between the two viewpoints creates cognitive disequilibration.

Tutors are said to benefit more than tutees from interactive problem solving (Azmitia & Perlmutter, 1989). Verba and Winnykamen's (1992) study with 6-year-old partners found that high achieving experts performed facilitatory behaviours such as giving relevant information, explaining, giving instructions and helping the other perform.

However, if the interaction is dominated by one member, then it becomes a passive modelling situation for the non-dominant partner. In Glachan and Light's (1982) study, almost 90% of the moves made on a Tower of Hanoi task were made by one person in an unstructured interaction dyad. Tudge's (1989) study found that the more advanced member of 5-year-olds paired on a mathematical balance beam task regressed in their thinking while the less advanced member sometimes advanced.

Where the more-advanced child appears to play the role of tutor to the less-advanced child (Forman & Cazden, 1985; Gallimore & Tharp, 1990), researchers in the Vygotskian tradition liken the situation to apprenticeship, in which one child plays the role of the expert and the other child plays the role of the novice. When students need to acquire skills that do not extend beyond their conceptual reach, the novice is able to participate in skills beyond those that he or she is independently capable of handling. Both benefit by internalizing the shared intellectual processes experienced during the

interaction, appropriating what was carried out in collaboration to individual development (Brown, Collins & Duguid, 1989; Damon, 1984; Resnick, 1988; Rogoff, 1990, 1993).

The social setting seems to provide a kind of *scaffolding* for novices to participate in solving a problem that would be beyond their individual capacities. Scaffolding has been defined as a "process that enables a child or novice to solve a problem, carry out a task, or achieve a goal which would be beyond his unassisted efforts." (Wood, Bruner & Ross, 1976, p. 90).

Vygotsky (1978) postulated that the only "good learning" is that which is in advance of development. Through imitation or collective activity, children are capable of initiating a variety of actions well beyond the limits of their own capabilities. Speaking to the effects of collaboration on development, Forman and Cazden (1985), Rogoff (1993), and Tudge (1992) suggest that Vygotsky acknowledged a possible difference between individual and group problem solving with his hypothesis that children would be able to solve problems with assistance from an adult or more capable peer before they could solve them alone. Interactions should be within the less competent partner's *zone of proximal development*, which is defined by Vygotsky (1978):

> It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers. (Vygotsky, 1978, p. 86).

Forman and Cazden (1985) and Forman and McPhail (1993) report on studies of dyads of girls collaborating on problem-solving tasks that showed both girls becoming more involved in their partner's roles. The girls seemed able to create bi-directional zones of proximal development for each other, by assuming at different times the role of teacher or student. The dyads developed relatively sophisticated problem-solving

strategies compared to those working alone, but did not do better than singletons on all posttest measures.

The review of the literature on the merits of heterogeneous groupings does not show conclusively that both high-achieving and low-achieving students benefit from the interaction in groups. However, teachers concerned with group composition also keep in mind their students' affect, and attempt to ensure all students benefit in learning situations. The supportive structure of a group appears to provide for low-achieving students to participate to some extent, which they may not be able to do if they worked alone. Higher-achieving students may benefit from having to explain their reasoning to others, but may do just as well alone. In pondering group compositions for this study, those considerations were kept in mind. In this study, the students were the same age, which may mean they were closer together in developmental stages. The essence of heterogeneity in group composition was, however, maintained by structuring according to mathematics ability. Three-member groups were used to offset some of the expert-novice patterns of interaction that might have occurred.

The next section of the literature review looks at the role of language within the group setting that may assist students in their learning.

The Role of Language

One of the beneficial features of the group process is its heterogeneity. Students can learn by observing the behaviours of others, and learn by leading the group to a particular action or decision (Artzt & Armour-Thomas, 1992; Schoenfeld, 1985; Webb, 1989). Students with incomplete information may interact with others who have different perspectives and facts (Johnson & Johnson, 1992).

Peers may demonstrate effective ways of attacking problems, analyzing text, or constructing arguments. Orderly approaches to problem-solving situations communicated in a group, standard questions ("Does everyone agree?"), sticking to an agenda, participating in goal construction, turn taking (Noddings, 1985), and the planning of strategies in advance (Rogoff, 1993), may become internalized as powerful executive procedures in problem solving, and give students practice in the kinds of collaboration prevalent in real-world problem solving (Noddings, 1985; Schoenfeld, 1989). Students begin to reflect about their own strategies and understanding if they receive feedback that is discussed face-to-face (Johnson & Johnson, 1992; Noddings, 1985; Schoenfeld, 1989). "Thinking aloud" makes it possible for others to critique and shape a person's performance (Resnick, 1988). By explaining aloud how they reached their conclusions and how they know their conclusions are correct, students test and validate their answers. (Krulik and Rudnick, 1993).

Liedtke and Sales (1990) found that questions directed to cooperative group members by other students required them to rephrase or rethink their responses and explanations. They began to use questions such as "Why?" and "How do you know?" to challenge each other's answers. Those who had at one time been considered to have correct or perfect answers faced a situation where they had to explain themselves.

Schoenfeld (1989) relates competency in problem solving to the "internal dialogues" problem solvers undergo as a result of having internalized aspects of the cooperative problem-solving sessions in which they had been engaged. Lack of cooperative problem-solving experiences might therefore deprive the students access to models of behaviour and may hamper the development of individual control strategies. Individuals working by themselves may more frequently get lost in "wild-goose" chases and may have difficulty monitoring their own cognitive activity. In groups, members externalize their ideas and reasoning for critical examination and monitor the reasoning of others (Schoenfeld, 1989).

Some students may still learn from the group interaction when others describe or show their work explicitly or correctly, by internalizing problem-solving strategies that others use, or building on the work or ideas of others. It may be easier to understand explanations directed to others than explanations directed to oneself. Connections between prior knowledge and aspects of the problem situation can be made when heuristics such as interpreting, restating, clarifying, and adding concrete details are part of the interaction between peers (Noddings, 1985). Leal's (1993) literary peer group discussions saw occasions where 3rd and 5th grade children helped each other to consider alternate ways of understanding the text, and prior knowledge thus became shared knowledge.

Dees (1991) found that college students benefited from the cognitive rehearsal involved in explaining a concept to someone else. The following testimonial from one of the students reveals her thoughts on the experience of working together:

> When you work with someone else and you have to explain a process or definition to them, simply by saying the words aloud you not only help the other student, you tremendously help yourself. It seems that by saying the words aloud your ears hear what your brain knows and this very fact helps you immeasurably . . . Things you were unsure of or things you thought you understood but didn't, seem to come clearer when two or more work together. (Dees, 1991, p. 420).

It appears important that peers develop some form of cooperation in their speech as well, with some researchers suggesting the development of "intersubjectivity" between peers. The term, coined by Rommetveit (1979), describes the mutually-agreed-upon understanding members of a group attain when they discuss their differing viewpoints or their own subjective ways of making sense of a problem.

In one of the pilot studies conducted for this study, a comment written in the response journal of one of the students stated: " . . . it seemed (that) we could read each other's minds." When children are involved in a joint activity, language becomes the major vehicle for the development of intersubjectivity, the internalization of concepts, and the development of higher cognitive processes (Gallimore & Tharp, 1990). Tudge (1992) cautions, however, that the process of arriving at shared meaning or intersubjectivity can have either deleterious or beneficial effects. A critical element of the zone of proximal development is that the more competent thinking or performance in the collaborative process should be internalized or "appropriated". At times, the less competent partner might persuade the other of the correctness of his or her problem solution.

The participants in most of the studies reviewed were aged five to seven. Alternatively, other researchers have studied individuals of college age. Therefore little research has been conducted in the area of intermediate elementary school children. It also appears that the abilities of the young children in the reported studies to vocalize their actions and arguments was necessarily restricted by their level of language development. In this study, the participants have the advantage of being able to articulate to a greater degree, their proposed strategies and calculations.

Children speak directly to one another on a level that they can easily understand, without hedging words, and take the feedback of another child seriously. They need both to satisfy themselves, but others as well, about the best solution. When children explain their reasoning, they have to make sense to each other, and while trying to coordinate their points of view, the children often realize their own mistakes. This forces them to work out their own understanding so they are better able to express their views convincingly.

Another beneficial feature of peer interaction is the support system provided in the group setting when students are encouraged to try different approaches. Since they are not competing against one another to solve problems, more opportunities for success for all students are made available. Seeing other students struggle may alleviate some of their own insecurities as they realize they are not unique (Davidson, 1990; Duren & Cherrington, 1992; Schoenfeld, 1989). Schoenfeld (1985) cited the pathological behaviour observed in individual students working on a cells problem that was outside the range of their experience. The students read the problem and had no clear idea of how to approach it. They felt great pressure to produce something substantial for the experimenters who would be listening to the tapes of their work, and resorted to voluminous computations. Students working in pairs felt the same pressure, but alleviated it by saying things like, "I have no idea of what to do. Do you?" (Schoenfeld, 1985, p. 280).

A quote from Davidson (1990) may express the feelings many students have toward problem solving:

> The learning of mathematics is often viewed as an isolated, individualistic matter. One sits alone with paper, pencil, and perhaps calculator or computer and struggles to understand the material or solve the assigned problems. This process can often be lonely and frustrating. Perhaps it is not surprising that many students and adults are afraid of mathematics. In contemporary language, they are troubled by math avoidance or math anxiety. (p. 3)

There appears to be general agreement that affective factors, such as motivation, interest, self-confidence, anxiety, perseverance, willingness to take risks, and tolerance of ambiguity, play an important role in problem solving (Lester & Kroll,

1990; Schoenfeld, 1985; Silver and Thompson, 1984). However, the research literature is not conclusive on how these factors affect problem solving. The reported difficulty is in designing instruments that can reliably measure these factors (Lester, 1980; Lester & Kroll, 1990). Willingness, perseverance, and self-confidence were three important influences on problem-solving performance found in Lester's (1980) review of the research. In Hembree's (1990) meta-analysis of 151 studies examining mathematics anxiety, positive attitudes toward mathematics were found to consistently relate to lower mathematics anxiety, with strong inverse relations for enjoyment of mathematics and self-confidence in the subject.

Since this study also measured the impact of small-group work on the attitudes students hold towards problem solving, research on the effectiveness of such a practice on attitudes was reviewed. Classroom interventions such as small-group work did not seem effective in ameliorating mathematics anxiety in the studies reviewed by Hembree (1990), nor in changes in attitudes towards Math and Math self-concept in Taylor's (1994) study of socially-assisted problem solving.

Duren and Cherrington's (1992) study comparing cooperative-work and individual-work students found that students were more willing to spend a longer time working on a problem in groups compared to those working by themselves, who tended to give up quickly if an immediate solution was not available. Charles & Lester (1984) found an improvement in grade 5 and 7 students' willingness to engage in problem solving and in their confidence in their ability to succeed after training in a process-oriented problem solving program.

The assessment of children's interest in and attitudes to problem solving is important, as their willingness to persevere, their motivation, and self-confidence is construed as having a direct effect on their participation in groups or in their individual work. In this study, an attempt was made to determine whether working in a group led to a change in their attitudes. As an adjunct to the main study, the students' responses on their attitude scales and during individual interviews were

analyzed to provide further insight into the way they interacted with their groups or how they worked individually. Some of the attitudes assessed in the attitude scale included students' willingness to engage in problem-solving activities, their perseverance during the process, their self-confidence, and positive and negative feelings to problem-solving.

Summary Of Chapter Two

One of the purposes of this study was to attempt to discover how peers interacted in a group setting that resulted in benefits to all members. They were to solve puzzling problems that supposedly challenged them in creative ways, ones in which no member could have been classified as an "expert".

The positive reports found in the research literature for small-group work need to be examined in the context of regular classroom work. Do elementary school students analyze problem information, generate solution strategies, or evaluate their solutions, during group collaboration with resultant improved ability and attitude? What kinds of support systems, if any, do they build for each other in their collaborative efforts, keeping in mind the generally unpredictable nature of human interaction, and the unlikely probability of finding students consistently providing support for each other? Are the cautions raised by Slavin of "free-rider" effects, the dominant member reported by Webb, and the attitudinal and motivational levels of particular students, seen in elementary students' group problem solving?

The following chapter outlines the design of the study to examine if and how peer collaboration affects student abilities and attitudes in problem solving.

CHAPTER THREE: DESIGN OF THE INQUIRY

Study Questions:

Question 1. What is the effect of group problem-solving on students' abilities to solve Mathematics word problems?

Question 2. What is the effect of group problem-solving on students' attitudes toward Math problem solving?

Question 3. What types of problem-solving strategies do students use in collaborative problem solving? Are these different from the strategies used by students in individualized problem solving?

Question 4. How does collaboration promote interactions that support and guide group members in the problem-solving task?

Introduction

This study was conducted in the natural environment of a class of grade six students, with their teacher as researcher. Information on the study participants forms the first part of this chapter, followed by the procedures used during the intervention period. Data collection measures and instruments are outlined, followed by methods of analyses and information on pilot studies.

Participants

Twenty-nine students in a grade six class (ages 11 and 12 years) were initially involved in the study, after parental permission was obtained. The students were members of the teacher-researcher's class. One student, in the group-work condition, moved away at the end of the intervention period and just before the final posttests were given, and her scores are not included in either pre- or posttest results, leaving 28

students for the comparisons. Approval had been received from the School Board to conduct the study in the classroom.

The classroom is in a school located in an inner-city area of Vancouver, and is one of the lowest socioeconomic areas of the city. Over 85 percent of the students have English as a second language, with the predominant ethnic background being Chinese. Therefore the study is further localized to a particular area of a particular city.

The class as a whole was average to above average in math computation, but those with less experience in elaborated English had major difficulties in problem solving, related to their level of language proficiency.

Using the technique of simple random sampling and a table of random numbers (Borg & Gall, 1989), the students were numbered alphabetically by last name from 1 to 29, then the first 14 numbers appearing on the table were used to select the students who would be working individually, as the control group.

The remainder, fifteen, were then assigned to group work (including the student who moved away), with three students in each group. The teacher-researcher assigned the students to groups to promote compatibility within each group. Additionally, each group was comprised of high-, medium-, and low-ability students as assessed through teacher judgement.

Two of the five groups were selected for more intensive study, as well as three of the individual-work students. (The two targeted groups are referred to as Group ABC and Group DEF, the students within the two groups referred to as Students A, B, C, D, E and F, and the targeted individual-work students referred to as Students G, H, and I.) The groups with the most articulate or vocal members were chosen, with the objective of more informative data being obtained. This also applied to the "individual-work" students, that is, selecting those more able to articulate their thoughts. This follows the recommendation of Stake (1994), whose choice would be to take "that case from which we feel we can learn the most", where the "potential for learning is a different and sometimes superior criterion to representativeness" (p. 243).

Although it is apparent that the small participant size (N=28) narrows the generalizability of any findings, and the one-grade study in one classroom restricts it further, the advantages of using the one class outweigh the disadvantages. They include the greater comfort level experienced by the students, the greater ease of administration of measures, lessened timetabling restrictions, ability to reschedule sessions to account for absenteeism, and greater control of extraneous variables such as noise and possible fatigue on the part of the students.

Procedures

The twenty-eight students (29 at pretest, but the student who moved away was dropped from the comparison study) were randomly given two forms of problem-solving tests (see Appendix A). Each test consisted of five problems to be solved within a one-hour time interval. A random half of the participants in the individual-work and the group-work sections were given Test A at the beginning of the test, then took Test B at the end of the study as a posttest. The other students completed Test B at pretest, then Test A at posttest. This counter-balancing was used to adjust for possible differences in difficulty levels of Tests A and B.

The *Attitudes Toward Problem Solving* scale (Szetela, 1991) (see Appendix B) was given prior to the "pre-" and "posttest" administrations of the problem solving tests. Individual (taped) interviews were conducted after the problem solving tests, following a scripted format (see Appendix C), with the nine students selected for more intensive study.

There was a total of 11 problem-solving sessions. Each session began with a whole-class introduction of the problem, clarification of the vocabulary, and restatement of the goal. This procedure was necessary in the classroom where English is a second language for most of the students, to ensure understanding of the problem's wording. There was no discussion of possible strategies, in order not to influence

student approach. The students were then dispersed to work in their assigned groups or alone, using two separate classrooms for the two conditions. Those groups or individuals who finished early were given a related problem extension to solve. It was estimated each problem would take about 20 to 40 minutes to solve. Each group and individual was asked to compose a summary statement explaining the method used, the reason it was chosen, and the problem solution, to encourage the students to clarify their ideas, and reflect on their thought processes (Krulik & Rudnick, 1994). During whole-class de-briefing following each work session, discussion of strategies used took place, but the teacher's role was as facilitator of the discussion only. She did not make any evaluative comments or suggest any "correct" approaches.

The researcher videotaped the two groups during sessions 1, 3, 5, 7, 9 and 11. In addition, both groups were audiotaped during the same sessions to provide a clearer audio track for transcription purposes. Students working individually, and other groups, were occasionally videotaped on alternate sessions, to reduce possible Hawthorne effects arising from targeted individuals. The Hawthorne Effect is the term used for the phenomenon in which participants, realizing they are part of an experiment or are receiving special attention, tend to improve their performance, interest or motivation (Borg & Gall, 1989).

Approximately two weeks after the last session, the *Attitudes to Problem Solving* scale and the alternate problem-solving test (A or B) were given to all students. The same individuals interviewed at pretest were interviewed after the attitude scale and problem-solving "post"test had been completed, using the same scripted format as at the beginning of the study.

Data Collection Measures And Instruments

Data collection measures and instruments were either developed as composites of other published work, or copied from studies with a similar focus to this study, with minor changes in wording or format. Attention was paid to the validity and reliability of such measures.

A. Pre- and Posttests

There was not available at the time of the study a suitable published problem-solving test that would be appropriate for this study. Therefore, pre- and posttests were constructed. Problems were chosen from a pool of 16 questions pilot-tested on grade six students from the same school not in the study (N=40), 4 questions given to each of four groups of 10 students. The problems were chosen from sources targeted for grade six students (Charles, R. I., Mason, R. P., & Garner, D., 1985; Malone, J. A., Douglas, G. A., Kissane, B. V., & Mortlock, R. S., 1980; Meyer, C. & Sallee, T., 1983; and Moretti, G., Stephens, M., Goodnow, J., and Hoogeboom, S., 1987). Problems which the students were able to satisfactorily complete were retained, scored using the *Analytic Scale for Problem Solving*, satisfactory completion set at a mark of 5 out of 10. Five problems were chosen for Test A, and five for Test B.

Both pre- and posttests were randomly sorted and marked at the same time, at the end of the study, to reduce effects of bias for one test period over the other. Tests were marked "blind" as to student name, type of condition (group-work or individual-work) and time of presentation (pre or post). The tests were also marked using the *Analytic Scale for Problem Solving* , by two researchers collaborating on marking. Each problem was scored with a possible maximum score of 10, with a total possible mark of 50 on each test. The cover pages of the test booklets (identifying the student and date of test) were numbered randomly by the researcher's collaborator, then the identifying

number entered on each test page. (The number was used later to re-sort the problems for each student.) Cover pages were then removed and filed away. Each test booklet was then separated into same-problem sets for marking.

To establish reliability, each problem set containing 28 answer sheets was first marked with the two collaborators marking five together to establish a common standard, then each marking four separately and comparing scores used. Inter-rater agreement was calculated at 82.5% on the four separately-marked answer sheets across ten problem sets. Discussion and negotiation then followed to establish a mutually-agreed-upon marking standard for each of the four separately-marked answer sheets. The remaining 19 answer sheets were then split with one person marking 9, the other marking 10. The same procedure was followed for all ten test problem sets.

The *Analytic Scale for Problem Solving* was used for scoring Tests A and B and problems solved during the sessions. A copy is attached as Appendix D. The scale evaluates:

Understanding of the Problem (rated from 0-No attempt, to 4-Complete understanding of the problem);

Solving the Problem (from 0-No attempt, to 4-A plan that could lead to a correct solution with no arithmetic errors), and

Answering the Problem (from 0-No answer or wrong answer based upon an inappropriate plan, to 2-Correct solution).

It is reported that the potential difficulties of using such rating scales are that although they help teachers focus on a single phase of the problem-solving process, it is difficult to separate scores for understanding and scores for the solution process, and there can be a wide range of scores and consistency (Szetela, 1988). Informal assessments of process problems by the researcher indicated the instrument as easy to

use and not time-consuming. However, some interpretation is necessary in assessing appropriate plans that do not follow the expected forms of solving.

B. Math Problems

The problems selected emphasized the process of problem solving rather than choosing which operation (addition, subtraction, multiplication or division) to use. Charles and Lester (1984) define a process problem as one that cannot be solved by simply selecting an algorithm. The problem solver has to make a guess about what to do, test it, play hunches, and use a variety of thinking processes. Process problems develop general strategies for understanding, planning, and solving problems, as well as evaluating attempts at solutions.

The problems selected for the eleven sessions were drawn from two sources, *Problem-Solving Experiences in Mathematics, Grade 6*, by Charles, Mason & Garner (1985) and *The Problem Solver 6*, by Moretti, Stephens, Goodnow & Hoogeboom (1987). The following is an example of the problems selected (see Appendix E for a complete description):

> Three grizzly bears, Chuck, Barry, and Amos, were very carefully weighed two at a time. Barry and Chuck weighed 248 kilograms together, Amos and Barry weighed 262 kilograms, and Amos and Chuck weighed 256 kilograms. How much does each bear weigh?
>
> *Problem Extension:*
>
> If Amos and Barry weigh 236 kilograms, Amos and Chuck weigh 242 kilograms, and Chuck and Barry weigh 220 kilograms, how much does each bear weigh? (Charles et al., 1985, p. 119)

The 19 answer sheets of the five groups and the 14 individual-work students were marked for all eleven sessions. Rather than use the full *Analytic Scale for Problem*

Solving , only the Answering the Problem section using the three-point rating scale was used to mark the answer portion of the problem-solving sheets:

0 - No answer or wrong answer based upon an inappropriate plan.

1 - Copying error; computational error;
partial answer for problem with multiple answers;
no answer statement; answer labeled incorrectly.

2 - Correct solution. (D. Wilson, 1991, cited in Szetela and Nicol, 1992)

The author marked the answer sheets for all 11 sessions, and a colleague marked the answer sheets for sessions 1, 3, 5, 7, 9 and 11 to establish reliability. The same procedure was followed in marking each set of commonly-marked answer sheets. Five sheets were first marked together to establish conformity in marking. Those answer sheets where ratings were different were then reviewed and agreement reached on each. Inter-rater agreement for the 6 answer sets was 94.6% before differences were resolved. A summary score for each student or group was derived by adding across all 11 sessions to capture problem solving performance during the intervention period.

C. *Attitude Inventories*

All students were given an *Attitudes Toward Problem Solving* scale to complete (see Appendix B), both directly before problem solving pretests and posttests, to provide information on changes in students' attitudes to problem solving from pre- to posttest. The attitude scale is the same as that completed by Grades 4, 7 and 10 students in the 1990 Math Assessment in British Columbia (Szetela, 1991). The eight items on the scale measure students' willingness to engage in problem-solving activities, perseverance during the problem-solving process, and self-confidence with respect to problem solving. Items are worded to reveal positive or negative feelings, for example: *I enjoy solving Math problems*, and *When my teacher gives us Math problems to*

solve, I get uncomfortable. A four-point rating scale was used (1-Strongly agree, 2-Agree, 3-Disagree, 4-Strongly Disagree). Items 2 and 4 were negatively worded, then the scores adjusted to be consistent with the other items. Both sets of tests were marked after the posttests were completed.

Attention was paid to the caution by the Commission on Standards for School Mathematics of the National Council of Teachers of Mathematics (1987) that attitude inventories can be too generalized to capture students' disposition to mathematics. Students may be thinking of the most recent situation that would not be representative of their range of responses. In this study, interviews of the nine selected students provided additional information about their dispositions. As well, informal observations of their willingness to explain their point of view, their tolerance of nontraditional procedures or solutions, their curiosity, and the kinds of questions they ask, were used as a reliability check on the attitude inventory.

D. Individual Interviews

Information from taped individual interviews with the nine students selected provided data on the students' problem-solving approaches both before and after the study. The students were encouraged to relate how they solved one of the test problems, with the researcher using a structured interview form (see Appendix C). The items are an amalgamation from two sources (Fortunato et al., 1991 and Montague, Bos, Doucette, 1991).

The focus was on eliciting from the students what they did as they began to solve the problem, as they worked the problem, after they finished working the problem, and any strategies they used. They were also encouraged to discuss their responses to the eight statements on the attitude inventory. The individual interviews allowed a comparative look at how students varied in their approaches, their strengths and weaknesses, and how they perceived their abilities in problem solving.

E. Transcripts from Interviews and Problem-Solving Sessions

The two selected groups were audio and videotaped during alternate sessions. A total of six sessions was therefore transcribed for analysis of peer interaction and problem-solving processes and strategies used by the groups. Additionally, transcripts of the individual interviews with the nine selected students both before and after the intervention period were available.

The benefits of using transcripts to study problem solving appear to be outweighed by their disadvantages. Researchers point to the ability to hear the thoughts of the students without interfering in the process (Artzt & Armour-Thomas, 1992), and the possibility that multi-person protocols may provide more verbalization than single-person protocols. An individual may say, "I think we should do X," and perhaps argue why, and responses may take the form of "Why should we?", resulting in rationales for the decisions being made overt (Schoenfeld, 1985)

However, researchers also point to the many difficulties in using verbal reports or protocols. In a small group, thoughts are not always verbalized and there can be a low incidence of the "interesting" events that are instrumental in producing learning during the problem solving situation (Artzt & Armour-Thomas, 1992; Brown, 1987; Resnick, 1988; Schoenfeld, 1985). They may result in serious omissions, for example, when a question from a partner may alter a person's intended solution path, and it is not voiced, and when a student does not assess the current status of a solution or the potential utility of a proposed approach (Schoenfeld, 1985).

F. Research Diary and Memoes

Emerging commonalities, insights, informal observations, observations by other teachers peripherally involved, comments by individual students, advice to oneself on research procedures and methods, the procedures used and relevant dates, and the

qualitative analysis measures taken to compile an audit trail, were some of the items entered in an on-going research diary. Memoes were completed throughout the study outlining observations and key procedures, and were referred to in the final analysis.

Methods Of Analysis

While the study used both quantitative and qualitative methods of analysis, each procedure was used to be able to make reference to multiple sources of evidence, and to add depth and quality to the other in the final analysis.

A. Quantitative Analyses

Question 1. What is the effect of group problem-solving on students' abilities to solve Mathematics word problems?

After scoring the pre- and posttests of problem solving, means and standard deviations were calculated for both conditions and testing times. An analysis of covariance was used for statistical analysis, to test the difference between group-work and individual-work students' posttests, with the pretest and order of test administration as covariates to adjust for initial differences in pretest means and possible mediating influence in the order of administration of the two forms of the test. The analysis of covariance was chosen as it allowed for attribution of mean change scores to the effect of the intervention rather than to differences in initial scores or test order (Borg & Gall, 1989).

The answer sheets for problems solved during the intervention period were scored for all groups and individual-work students. A summary score for each group or student was derived by adding across all 11 sessions to capture problem solving performance during the intervention period. Mean scores and standard deviations

were calculated on the summative scores across all 11 sessions for the groups (N=5) and the individual-work students (N=14).

An analysis of variance was used for statistical analysis of the difference between the means of group-work and individual-work students on the summative scores, with scores as the dependent variable.

Question 2. What is the effect of group problem-solving on students' attitudes toward Math problem solving?

The *Attitudes Towards Problem Solving* scales for both pre- and posttests were marked at the same time for both conditions. Means and standard deviations were then calculated.

To analyze the differences between pre- and posttest means of group- and individual-work students, the Analysis of Covariance, controlling for pretest, was used.

B. Qualitative Analyses

Qualitative methods were used to examine the relationships in the two target groups, the individuals within the groups, and the three targeted students working individually.

Categorical codes were developed after the transcripts from the first and second interviews of the nine students and six taped problem-solving sessions from each of the two groups had been reviewed twice. The codes used arose first from the general themes in the data, refined repeatedly, then given to a colleague to review for feedback. Trial runs of the codes on several of the protocols revealed areas where adjustments were needed. After about six revisions, a final categorical code list was prepared, attached as Appendix F. To provide a reliability check in the analysis, a colleague and

the researcher first coded one interview and one session transcript to establish a common scale of comparison, then separately coded another two interviews and two sessions, and finally reviewed any differences. The researcher coded the remaining protocols. Inter-rater agreements were obtained on the coding, of 82.8% on codes for the transcript for Group ABC in Session 5, and of 85.1% for codes on the transcript of Student I's post-interview.

Question 3. What types of problem-solving strategies do students use in collaborative problem solving? Are these different from the strategies used by students in individualized problem solving?

Strategies used by the two groups and the three individual-work students selected for more intensive study were tallied for each session. By recording and displaying visually the types and frequency of strategic decisions made by the two groups and the three individual-work students, comparisons were made and changes in use of strategies assessed over time. This was done to disclose the evolving nature of both group interaction and students' discovery of techniques that may have been helpful in their solutions.

Question 4. How does collaboration promote interactions that support and guide group members in the problem-solving task?

As the study had a key component of determining the effectiveness of group collaboration on cognitive change, it was important to determine what kinds of interaction supported growth, if any. If cognitive change took place in interaction with others before it became individualized, the specific interaction features that brought about such change needed to be identified.

The audio and videotape transcriptions were coded as noted above for the types of transactions that occurred within the group, who made them, and the change, if any, in types of transactions. Each group was expected to develop certain styles of interaction that may have inhibited or promoted group problem solving.

There was also an examination of how the students used the problem solving procedures of understanding the problem, developing and carrying out plans, and evaluating and justifying their solutions.

C. Analytic Induction Through the Grounded Theory Method of Data Analysis

This study used the grounded theory method developed by Glaser & Strauss (1967) as the major analytic approach. Theory is generated that is grounded in systematically gathered and analyzed data, where the theory evolves through a continuous interplay between analysis and data collection. The emphasis was on describing social interaction in its structural context, capturing the changing process, the why and how of action/interaction, and the consequences (Strauss & Corbin, 1994).

The general method has become known as the "constant comparative method" (Strauss & Corbin, 1994). Constant comparison focuses on identifying categories and on generating relationship statements, and allows the researcher to generate abstract categories and their properties (Glaser & Strauss, 1967; LeCompte & Preissle, 1993). A category is a conceptual element of the theory, and a property is a conceptual aspect or element of a category (Glaser & Strauss, 1967).

Initially, through a process of data reduction (Huberman & Miles, 1994), categories occurring most frequently were selected to determine how the units were alike and different. Concepts related to the same phenomenon were then grouped into categories of a higher, abstract level (Glaser & Strauss, 1967), highlighting similarities and differences. Properties differentially shared among units, relationships and

linkages, were established by comparison and contrast, and by identifying underlying associations, themes or patterns (LeCompte & Preissle, 1993).

Both an inductive process, in generating statements of relationships, and a deductive process, to test relationships, are involved in developing a theory or hypothesis that is grounded in the data. Successively formulating and testing hypotheses throughout the project refined the process of arriving at explanations for the behaviour exhibited (LeCompte & Preissle, 1993).

Coding was completed on all transcripts from interviews and problem-solving sessions using the categories generated. "Chunks of meaning" were used to determine where one code began and ended. Each chunk was therefore an event portraying a specific interaction, and could have comprised one or twenty lines of a transcript. Data displays or matrices were used to organize the data and to generate meaning more effectively and draw conclusions (Huberman & Miles, 1994).

Validity And Reliability

Measures were taken to ensure the study's procedures and measurement instruments recognized established criteria for both internal and external validity, and reliability.

A. Internal Validity

This study sought to provide an account of "what is really there" (Merriam, 1988, p. 166), that is, to ensure internal validity measures were taken. One threat to internal validity is whether history and maturation affects the nature of the data collected. The two-condition format within the same class ensured that both comparison groups were encountering the same influences.

Another measure taken to ensure internal validity in this study included triangulation of methods. Triangulation of methods provides multiple sources of evidence to confirm emerging findings (Merriam, 1988). Pre- and posttests on math problem solving by all students, evaluation of student and group participation from audio and videotapes, evaluation of problems solved during the intervention period, information from taped individual interviews with the nine students, and pre- and posttest attitude surveys from all students, provided a chain of evidence for the conclusions to be drawn. These multiple data sources allow others to authenticate the findings. Further, by tracing the process of data collection and analyses in depth, an audit trail was created that aided in establishing internal validity by describing in detail how data was collected, how categories were derived, and how decisions were made throughout the inquiry.

Triangulation of measures can reduce another threat to internal validity, the effect of the observer (LeCompte & Preissle, 1993). This will be especially notable in the interviews conducted by the teacher-researcher. The participants, as students of the teacher, may have told the researcher what they think the researcher should, or wanted, to hear. This threat can be reduced by corroboration through other measures, a sufficiently long enough study time, and the teacher's knowledge of the students.

B. External Validity

Other measures were taken to ensure that external validity, that the findings of this study could be applied to other situations (Merriam, 1988), were considered. One of the measures taken included the random assignment of students, using a random numbers table, to either the group-work or individual-work condition.

Certain other variables threatening the validity of this study could be raised (as suggested by Borg & Gall, 1989). Experimental treatment diffusion can occur when members of control and experimental groups are in close proximity to each other. The

students in the two conditions were separated, with one group using a classroom across the hall during the problem-solving sessions. The John Henry Effect can occur when control group members perform beyond their usual expected level to compete with the experimental group. Conversely, the Hawthorne Effect can operate when the experimental group feels stimulated and motivated by the attention given to them. Resentful demoralization of the control group (as raised by Borg & Gall, 1989), can occur when members perceive the experimental group as receiving favourable treatment, resulting in poorer performance on their part. Both the John Henry and Hawthorne Effects, and the resentful demoralization of the control group, may have been lessened in the classroom studied because of the numerous previous opportunities the students have had to work collaboratively, to the extent that it has become routine (26 of the 28 students were in my class the previous year in grade five). Additionally, the Hawthorne Effect may have been reduced as both the group-work and individual-work students encountered a stimulating new project at the same time.

Certain threats to external validity, or a study's comparability and translatability, were ameliorated in this study by careful identification and description of the phenomena to permit comparison with other phenomena of similar types (LeCompe & Preissle, 1993; Marshall & Rossman, 1989), and identifying and reporting disparities as attributes of the groups being examined (LeCompte & Preissle, 1993). By providing a "rich, thick description", generalization of the study's findings can be improved so that readers have a base of information from which they can appropriately ask how it applies to their own situation (Merriam, 1988).

C. Reliability of Observations

Although there are suggestions that qualitative studies by their nature cannot be replicated because the real world changes (Marshall & Rossman, 1989) measures taken to ensure reliability included: taking thorough notes and maintaining a diary

recording design decisions and rationales, allowing others to inspect or comment on procedures, protocols and decisions, and having organized, retrievable data available if the findings are challenged or other researchers want to reanalyze the data (Marshall & Rossman, 1989). In addition, general strategies used in analyzing the data were identified, and categories defined carefully, to reduce the danger of magnifying idiosyncracies or lack of comparability, and facilitating replication (LeCompte & Preissle, 1993).

Interobserver reliability (LeCompte & Preissle, 1993) or peer examination (Marshall & Rossman, 1989; Merriam, 1988) are terms used to describe agreement on the description or composition of events. This peer examination provides a check on biases the researcher may unwittingly impose on the assumptions and conclusions. A colleague, currently also working on a master's thesis, was asked to provide reliability checks on pre-post math test scoring, a review of coding categories, and coding of protocols for strategies and procedures used by students and interaction patterns within groups. Another colleague provided a reliability check on the marking of answer sheets for problems solved during the intervention period.

All pre- and post- math problem solving tests were marked blindly as to treatment or control condition, and time of test, by a colleague and the researcher, at the conclusion of the study. The tests were randomly sorted, with identification of student and date of test removed, to reduce effects of bias. Inter-rater agreement was calculated at 82.5% before discussion and negotiation to resolve differences. Additionally, an inter-rater agreement of 94.6% was obtained before discussion of differences on the marking of answer sheets for problems solved during the intervention period. Inter-rater reliabilities were also obtained on the coding of interview and session transcripts, of 82.8% on codes for the transcript for Group ABC in Session 5, and of 85.1% for codes on the transcript of Student I's post-interview.

Strategies used to reduce threats to internal reliability included using low-inference descriptors (concrete descriptions, verbatim accounts), searching for negative

instances in addition to data supporting conclusions, purposeful testing of possible rival hypotheses, integrating findings from other studies, and the use of recorded data to strengthen reliability of results (Marshall & Rossman, 1989; Merriam, 1988).

Pilot Studies

Two pilot studies were completed in the researcher's grade six class in an effort to determine the potential pitfalls and weaknesses in this study. The interaction that evolved in solving a process problem by a group of three students suggested several points for consideration in this study. The quick response (two minutes) and acceptance of one student's answer by other members of the group indicated that a more open-ended problem would have yielded more diverse strategies than those produced by the problem selected. The group failed to check whether they had answered the requirements of the problem (they had not), and pointed to the need to ensure students have full understanding of all parts of each problem presented. The students, in their review of their own processes, were able to articulate the strategies they had used, that is, "drawing a picture, figuring it out in our heads, and writing an equation." Therefore the potential for eliciting strategic feedback seemed to be available in this task. The pilot study also pointed to the need to have students' work-in-progress saved for the analysis, to determine the miscues, recursions, omissions, additions, they made.

A second group with a more open-ended problem had much difficulty arriving at a productive procedure for completing the problem. They groped for understanding of the intent of the question and were confused by the terminology. They tried a guess-and-test strategy but did not systematically eliminate wrong answers. This pilot study pointed to the importance of studying the potential pitfalls in the vocabulary of a problem when students lack fluency in English, or alternatively, when they may not have the strategies necessary to solve the problem.

Summary Of Chapter Three

When qualitative and quantitative approaches are combined, they can produce an effective and productive mixture. Miles and Huberman (1994) provide rationales for linking qualitative and quantitative data that apply to the present study. There is the enabling of confirmation or corroboration of each approach via triangulation of methods. Richer detail can be provided in the analysis. Quantitative data can supply background data and control early judgments, and can verify or cast new light on qualitative findings. Qualitative data can help by validating, interpreting, clarifying and illustrating quantitative findings.

The next section outlines the results of the quantitative and qualitative analyses of the data accumulated.

CHAPTER FOUR: RESULTS

The results of the quantitative analyses are shown for study questions 1 and 2, and qualitative analysis results are shown for questions 3 and 4.

Question 1: What is the effect of group problem solving on students' abilities to solve Mathematics word problems?

Analyses were conducted to see if there were any differences in the posttest results between the group- and individual-work conditions.

Analyses were first conducted to determine alternate form reliability of the problem-solving tests. How comparable were the two forms of the test? Pearson correlation coefficients were calculated. When the testing administration was of the order of Test A at pretest and Test B at posttest (N = 15), the Pearson correlation coefficient between the pre- and posttests was r= .77. With the test order of Test B at pretest and Test A at posttest (N = 13), the Pearson correlation coefficient between the pre- and posttests was .65. The overall test-retest order for all students resulted in a Pearson correlation coefficient (N = 28) of r = .49.

	Mean	Std Dev	Correlation Coefficients	N
Order 1 (A to B)	21.33	7.49	.77	15
Order 2 (B to A)	31.62	12.26	.65	13
All	26.11	9.98	.49	28

Since the overall correlation was lower than the two sub-group administrations, test order may have been a moderating influence or intervening variable for the total correlation. Therefore an Analysis of Variance was completed to determine whether the order of test administration, that is, Test A at pretest and Test B at posttest, or Test

B at pretest and Test A at posttest, accounted for any difference in the results. The dependent variable was posttest scores by levels of test order. The results of the Analysis of Variance showed a significant difference between the two test administrations, $F\ (1,26) = 7.39$, $p < .05$, N=28), favouring Test Order A to B. Test A was seen to be easier than Test B.

Analysis of Variance

Order of Test Administration

Source	Sum of Squares	df	Mean Square	F	Significance
Between Groups	736.27	1	736.27	7.39	.01
Within Groups	2590.41	26	99.63		

N = 28

The results from this analysis suggest that test order had a moderating influence (or acted as an intervening variable) on students' test scores. However, precautions had been taken in the study to ensure random distribution of the tests. The participants had been randomly assigned to either test order, by distributing Test A, then B, then A, and so on, around each table in which students were sitting, and in addition as previously mentioned, had been randomly assigned to either the group-work or individual-work condition.

To determine the effect of group problem solving on ability to solve math problems, the students' pre- and posttest scores were compiled, then mean scores calculated for each condition. Mean scores and standard deviations for pre- and posttest problem solving tests for both conditions (group-work and individual-work) are shown in Table 1. The posttest scores on the problem solving tests of the two conditions were then analyzed using an Analysis of Covariance, with the pretest and

Table 1. Problem Solving Mean Scores by Condition

		PreTests			PostTests		
	N	Mean	StdDev	Range	Mean	StdDev	Range
All Students	28	23.00	9.58	10-44	26.11	11.10	10-50
Group-Work Students	14	23.92	11.18	10-44	27.71	12.23	10-50
Order 1 (A to B)	6	27.50	12.61		23.67	10.61	
Order 2 (B to A)	8	21.25	9.97		30.75	13.14	
Individual-Work Students	14	22.07	7.99	13-40	24.50	10.04	13-46
Order 1 (A to B)	9	22.67	8.67		19.78	4.58	
Order 2 (B to A)	5	21.00	7.38		33.00	12.04	

Note: A refers to Test A, B to Test B

order of presentation of test forms (A-B and B-A) held as covariates. Since test order was a mediating variable in the posttest results, it was included as a covariate. The pretest was also included as a covariate to adjust for any effects from pretest scores. Testing of the hypothesis that there is a difference between students who worked in the group condition or worked individually showed the differences were not significant: F (1,24) = 0.08, $p > .05$, N = 28. Group-work did not effectively improve students' scores at posttest.

Analysis of Covariance

Math Problem Solving PostTests

Source of Variation	Sum of Squares	DF	Mean Square	F	Significance of F
PreTest	1148.65	1	1148.65	19.22	.00
Order of Test	1052.75	1	1052.75	17.62	.00
Condition (Group or Individual)	4.66	1	4.66	.08	.78
Error	1434.60	24	59.76		

N = 28

Table 2. Problem Solving Scores on Pre- and PostTests

Student	Pre Test	Test Form	Post Test	Test Form	Change
Group-Work Students					
A	41	B	37	A	- 4
B	19	B	31	A	+ 12
C	41	A	37	B	- 4
D	30	B	50	A	+ 20
E	27	A	22	B	- 5
F	23	A	23	B	0
Individual-Work Students					
G	34	B	44	A	+ 10
H	22	A	27	B	+ 5
I	24	A	17	B	- 7

Students targeted for the in-depth qualitative study included students coded as A, B and C, comprising the ABC group, students D, E and F, forming the DEF group, and individual-work students G, H and I. A student-by-student comparison of the problem-solving scores for group-work condition students as opposed to the individual-work students (see Table 2) shows that in Group ABC, both Students A and C showed no pre-post gain (both dropped by 4 points), whereas Student B gained by 12 points from pretest to posttest. In Group DEF, Student D gained 20 points, whereas Student E decreased by 5 points, and Student F remained the same. For the individual-work students, Student G gained by 10 points, Student H by 5 points, and Student I decreased by 7 points. The varied results appear to parallel the non-significant results from the quantitative analysis of posttest scores.

In sum, the analyses of posttests results indicated there was no significant difference in the effectiveness of group-work in comparison to individual-work in problem solving on posttest scores, that is, on tests completed individually after the problem-solving sessions were over. However, personal observation during the problem-solving sessions pointed to some differences between the two conditions on problem solving during group sessions. Also, spot checks of students' answer sheets had shown frequent wrong solutions by the individual-work students. Therefore further analysis of differences in problem solving performance that existed between the two conditions during the problem-solving sessions was undertaken.

The answer sheets of the 5 groups and the 14 individual-work students were marked for all eleven sessions. Only the answer portion of all students' problem-solving worksheets was used in the evaluation. The marks given each group or student over the eleven sessions are presented in Table 3. A summary score for each student was derived by adding across all 11 sessions to capture problem solving performance during the intervention period. Mean scores were then calculated for the groups (N=5) and the individual-work students (N=14). Condition 1 (Group-work students) had a mean of 9.8 and standard deviation of 3.11, while Condition 2 (Individual-work students) had a mean of 5.86 and a standard deviation of 3.39.

Table 3. Scores For Math Problems Solved During Sessions

Students	Sessions 1	2	3	4	5	6	7	8	9	10	11	Summary Total
Group-work												
* ABC	2	2	2	1	2	0	2	0	2	0	0	13
* DEF	2	2	1	0	2	2	2	2	0	0	0	13
JKL	1	1	1	1	2	0	1	0	0	1	0	8
MNO	1	1	1	1	0	0	0	0	1	1	0	6
PQR	2	1	1	0	0	0	2	0	2	1	0	9
											Total	49
												M = 9.8
Individual-Work Students												
* G	2	2	2	1	1	2	0	0	2	0	0	12
* H	1	2	1	0	0	0	0	0	0	2	0	6
* I	1	2	2	0	1	0	0	2	0	0	0	8
S	1	0	0	0	1	0	0	0	1	0	0	3
T	1	0	1	1	0	0	0	0	0	0	0	3
U	1	1	1	0	0	0	0	0	0	0	0	3
V	1	0	2	0	0	0	0	0	0	0	0	3
W	1	1	0	0	0	0	0	0	0	0	0	2
X	1	2	0	0	2	0	0	0	1	2	0	8
Y	1	2	0	1	0	0	2	0	1	0	0	7
Z	1	2	1	1	1	0	0	0	0	2	0	8
AA	1	1	2	0	0	0	0	0	0	0	0	4
BB	1	1	1	0	0	0	0	0	0	0	0	3
CC	2	2	2	0	0	2	1	0	1	2	0	12
											Total	82
												M = 5.86

* Students targeted for qualitative analysis

Scale:
0 - No answer or wrong answer based upon an inappropriate plan.
1 - Copying error; computational error; partial answer for problem with multiple answers; no answer statement; answer labeled incorrectly.
2 - Correct solution.

(D. Wilson, 1991, cited in Szetela & Nicol, 1992)

An Analysis of Variance performed comparing the group to individual summary-answer means revealed that a significant difference existed between the group-work and individual-work conditions: $F(1, 17) = 5.17, p < .05$ (two-tailed). Therefore the group-work students were answering the problems more effectively during the intervention period than the individual-work students.

Analysis Of Variance

Answers to Problems Solved During Sessions

Source	Sum of Squares	DF	Mean-Square	F-Ratio	Probability
Condition	57.28	1	57.28	5.17	0.04
Error	188.51	17	11.09		

N=19

In sum, the results of the Analysis of Covariance on problem solving posttest scores showed no significant differences when the students were answering the tests individually. This suggests that the benefits of working as a group did not extend to independent problem solving in a test situation. However, analyses on problem solving during the intervention suggest that there was a significant advantage for individuals when working in groups. In subsequent sections, qualitative analyses of group interactions that might be associated with higher problem solving scores will be reported.

Question 2: What is the effect of group problem-solving on students' attitudes towards Math problem solving?

Students' attitudes towards problem solving were expected to change favourably with exposure to group interaction. The mean scores for both group- and individual-work students are presented in Table 4. Lower scores indicate more positive attitudes.

Table 4. Attitude Surveys Mean Scores by Condition and Time of Testing

		PreTests			PostTests		
	N	Mean	StdDev	Range	Mean	StdDev	Range
All Students	28	16.64	3.97	11-28	16.54	4.49	11-23
Group-Work Students	14	17.00	4.66	12-28	16.79	3.87	11-26
Individual-Work Students	14	16.29	3.29	11-22	16.29	5.18	13-23

An Analysis of Covariance comparing posttest scores between individual-work and group-work students, with pretest as covariate, revealed no significant differences between the fourteen group-work and fourteen individual-work students, $F(1,25) = 0.01$, $p > .05$, on their attitudes to problem solving after the intervention period.

In addition, the responses of the students targeted for specific study (Groups ABC and DEF and Students G, H, and I), revealed no differences between group-work and individual-work students in their responses to the *Attitudes Toward Problem Solving* scale (see Table 5), with the possible exception of Student I. Eight of the nine students indicated generally positive attitudes toward problem solving, with little change from pre- to posttest. Student I indicated on her pre- and posttests and in the interviews a lack of confidence in solving problems.

Analysis of Covariance

Attitude Surveys

Source of Variation	Sum of Squares	df	Mean Square	F	Significance of F
PreTest	124.95	1	124.95	7.47	.01
Condition (Group or Individual Work)	.087	1	.09	.01	.94
Error	418.27	25	16.73		

N=28

To summarize, there was no difference at post test between the individual-work and group-work students on attitudes to math problem solving. Working in a group did not result in a more favourable attitude toward problem solving.

Table 5. Student Responses: Attitudes Toward Problem Solving

			Students								
			Group-Work						Individual-Work		
			A	B	C	D	E	F	G	H	I
1.	I enjoy solving Math problems.	PreTest	SA	SA	A	SA	A	A	SA	SA	A
		PostTest	SA	SA	A	SA	A	A	SA	SA	D
2.	When my teacher gives us Math problems to solve, I get uncomfortable.	PreTest	D	D	D	SD	D	D	SD	SD	A
		PostTest	D	D	D	SD	D	D	SD	SD	A
3.	Once I start a Math problem, I don't give up until I solve it.	PreTest	A	A	A	A	A	SA	SA	S	D
		PostTest	A	A	A	A	A	A	D	SA	D
4.	I would rather solve only easy problems.	PreTest	SD	D	SD	SD	SD	SD	SD	D	D
		PostTest	D	SD	SD	SD	SD	D	D	A	D
5.	Problems that make you think are more fun than easy problems.	PreTest	SA	A	SA	SA	SA	SA	SA	SA	A[a]
		PostTest	SA	SA	A	SA	A	A	A	SA	A
6.	If I had the choice, I would rather solve Math problems than do Arithmetic drills or exercises.	PreTest	A	SA	D	A	D	D	D	A	A
		PostTest	D	SA	D	A	D	D	D	A	A
7.	Math would be more interesting if we had more problems.	PreTest	A	SA	A	D	A	A	A	A	D
		PostTest	A	SA	A	D	A	A	A	A	D
8.	I think I'm good at solving problems.	PreTest	A	A	A	SA	D	D	SA	SA	D
		PostTest	A	A	A	SA	D	D	A	A	D

Codes:

SA Strongly agree D Disagree

A Agree SD Strongly disagree

Note: [a] Changed during interview to an Agree response.

Question 3: What types of problem-solving procedures or strategies do students use in collaborative learning? Are these different from the procedures or strategies used by students in individualized learning?

In this section, I report analyses comparing problem solving strategies used by students working individually and those working in groups. This might inform or help explain the superior performance of the collaborative problem solving groups during the intervention period. The first point of analysis was to review the answer sheets of the two groups and three individual workers selected for intensive analysis to determine what strategies had been used. The strategies used by Groups ABC and DEF and Students G, H and I to solve the problems during all 11 sessions are summarized in Table 6 and itemized in Table 7. These data suggest that the kinds of strategies used by students in both conditions appear to be the same, that is, there was no difference in either the number or type of strategies used.

An examination of the number of strategies used by the groups and individuals shows both Group ABC and Group DEF used the same total number of strategies (22). The average number used by the three individual-work students (21.3) is similar to the number used by both groups.

Of the total 109 strategies used by Groups ABC, DEF and Students G, H and I, the most commonly used strategy was Logical Reasoning, representing 28% of the total strategies used. The next most common strategies used were Guess and Test for 22%, and Draw a Diagram or Picture at 20% (see Table 6). Other strategies used to a lesser extent included Choosing a Numerical Operation, Writing an Equation, Making a List or Table, Looking for a Pattern, and Working Backwards.

There was no appreciable difference between the two groups and the three individual-work students in the type or frequency of strategies used. In the process type of problems used in all sessions, different strategies could have been used for the

Table 6. Frequency of Use of Problem-Solving Strategies

Strategy	Groups ABC	Groups DEF	Individual-work Students G	H	I	Totals	Percent
Logical Reasoning	6	6	6	6	6	30	28
Guess and Test	5	5	7	3	4	24	22
Draw a Diagram or Picture	4	4	5	5	4	22	20
Choose a Numerical Operation	1	2	2	2	2	9	8
Work Backwards	2	2	2	2	1	9	8
Look for a Pattern	1	2	2	1	1	7	6
Make a List or Table	1	1	1	2	-	5	5
Write an Equation	3	-	-	-	-	3	3
Totals	22	22	25	21	18	109	

Note: 11 sessions.

same question, resulting in a correct answer. The data in Table 7 show that individuals and groups used similar strategies in solving these kinds of problems. For example, Session 3 shows that both groups and the three individual-work students were uniform in the use of three similar strategies: Draw a Diagram or Picture, Look for a Pattern, and Logical Reasoning (see Table 7). Group ABC included the strategy of Writing an Equation.

In general, the two groups were comparable in the strategies they used across problems. For example, Table 7 shows that Groups ABC and DEF were uniform in using the same type of strategies, differing by one or two strategies for 5 of the 11 problem-solving sessions (Sessions 1, 3, 4, 6, 10). The individual-work students also were comparable in using the same type of strategies, differing by one or two

strategies for 8 of the 11 sessions (Sessions 1, 4, 5, 6, 7, 9, 10, 11) when either Student G, H or I or a combination of two of the students, used one or two more strategies than each other.

In summary, the strategies used by group-work and individual-work students were not significantly different in type or number, and it is unlikely that differences in effectiveness for either group- or individual-work problem solving can be related to the different types of strategies used in the two conditions.

Table 7. Strategies Used by Students During Problem Solving Sessions

Session/ Group or Student		Choose a Numerical Operation	Write an Equation	Guess and Test	Draw a Diagram or Picture	Make a List or Table	Look for a Pattern	Use Logical Reasoning	Work Backwards	Totals
1	ABC					X				1
	DEF					X	X			2
	G				X	X	X			3
	H					X				1
	I				X					1
2	ABC							X		1
	DEF							X		1
	G							X		1
	H							X		1
	I							X		1
3	ABC		X		X		X	X		4
	DEF				X		X	X		3
	G				X		X	X		3
	H				X		X	X		3
	I				X		X	X		3
4	ABC		X	X					X	3
	DEF			X					X	2
	G			X					X	2
	H								X	1
	I			X					X	2
5	ABC				X			X		2
	DEF				X			X		2
	G			X	X			X		3
	H				X			X		2
	I							X		1
6	ABC			X						1
	DEF	X		X	X					3
	G	X		X						2
	H	X			X					2
	I	X		X						2

Continued on next page

Table 7 (Cont'd.) Strategies Used by Students During Problem Solving Sessions

Session/ Group or Student		Choose a Numerical Operation	Write an Equation	Guess and Test	Draw a Diagram or Picture	Make a List or Table	Look for a Pattern	Use Logical Reasoning	Work Backwards	Totals
7	ABC	X							X	2
	DEF	X							X	2
	G	X							X	2
	H	X							X	2
	I	X								1
8	ABC			X				X		2
	DEF			X				X		2
	G			X				X		2
	H			X				X		2
	I			X				X		2
9	ABC				X			X		2
	DEF				X			X		2
	G			X	X			X		3
	H				X			X		2
	I				X			X		2
10	ABC		X	X	X					3
	DEF			X						1
	G			X	X					2
	H			X	X					2
	I				X					1
11	ABC			X				X		2
	DEF			X				X		2
	G			X				X		2
	H			X		X		X		3
	I			X				X		2
Totals		9	3	24	22	5	7	30	9	109

Question 4: How does collaboration promote interactions that support and guide group members in the problem-solving task?

The previous analysis suggests that superior problem solving scores associated with working in groups cannot be accounted for by differences in strategies used. In this section, other characteristics of group dynamics that might be associated with effective group problem solving are examined. To examine group dynamics, results from two sets of analyses are reported. First, students' descriptions of the benefits of group problem solving from interviews are described, followed by analyses of interaction patterns of groups while problem solving.

To describe students' perceptions of the benefits of working in groups, the responses given by students during their individual interviews were arranged in a matrix or array form as recommended by Miles and Huberman (1994). During the first (at pretest time) and second (at posttest time) interviews, each of the nine students commented that they preferred to work in groups when the question was difficult, because they could then talk about the problem together. Some of their typical comments were:

> If I don't understand then I could talk with others to find out . . . My group helped me to understand more words . . . My group explained what is the question asking.
>
> Other people in my group can teach me how to get the answer.
>
> Other people have different ideas.
>
> When it's very hard, that one person won't get the answer that fast or something, it's better to share or talk with each other and solve it.
>
> One person's head can't be two. It's like two persons' minds together mix with their ideas is better than one. Two heads is better than one.

Students D and G both saw the advantage of working in groups on difficult problems, but added that they could learn more by themselves on simpler questions. (Both Students D and G also showed high self-confidence in their problem-solving abilities.)

The students' responses suggest that they identified a distinct advantage to working in groups. In the second analysis, transcripts of group problem solving sessions were examined in order to identify activities or interactions that might explain the advantage of and preference for group problem solving. To characterize group interactions and the difference between group and individual problem-solving dynamics, it was decided to analyze group interactions based on transcripts from sessions 1, 3, 5, 7, 9 and 11, and review the answer sheets for individuals and groups for all eleven sessions.

In analyzing the transcripts for the six problem-solving sessions, nine categories of interactions were identified. To arrive at the nine categories, a frequency count was first conducted of all events coded specifically for group interaction across the six taped sessions. Then, several categories were dropped from further analysis because the number of observations assigned to those codes was not sufficient for either or both groups for a satisfactory comparison (e.g. categories for dominance, off-task behaviour, and humour).

Table 8. Frequency of Occurrence of Group Interactions

Session/ Group	Suggest a strategy or idea	Evaluate, monitor check or summarize	Explain, clarify for others	Ask for clari-fication or Assis-tance	Mimic, echo, talk simulta-neously	Finish another's comment	Guide discussion, facilitate, divide labour	Urge group to complete assign-ment	"At least we tried; it's better than nothing"	Totals
1 ABC	21	17	8	7	1	8	9	5	1	77
DEF	23	16	7	6	14	8	5	-	-	79
3 ABC	32	16	9	5	7	1	8	2	-	80
DEF	28	7	4	3	7	4	2	-	-	55
5 ABC	13	8	4	7	-	5	3	3	1	44
DEF	8	17	2	2	12	12	6	-	-	59
7 ABC	2	13	4	3	1	4	2	-	3	32
DEF	3	3	5	7	18	5	-	-	-	41
9 ABC	4	13	1	3	15	3	4	4	-	47
DEF	5	8	1	1	6	4	4	1	-	30
11 ABC	8	11	4	3	9	3	1	4	3	46
DEF	6	11	5	4	22	4	2	1	1	56
Totals	153	140	54	51	112	61	46	20	9	646
Percent of Total	24	22	8	8	17	9	7	3	1	

A summary of the final set of categories coded for group interaction and the frequency of ocurrence of each type of interaction is shown in Table 8. These data show that in percentage terms, in descending order of frequency, 24% of the events related to students suggesting a strategy or idea, 22% related to evaluating, monitoring or checking, 17% related to students mimicing, echoing, or talking simultaneously, 16% related to students explaining or clarifying for each other, 9% saw students finish each other's comments, 7% involved students guiding the discussion and facilitating turn-taking, 3% related to students urging the group to complete the assignment or to think, and 1% related to students saying "At least we tried; it's better than nothing".

Results of analyses of each of the nine categories will be discussed further below.

A. Suggest a Strategy or Idea

The results of the analysis of strategy use (Study Question 3) by the nine group-work and all individual-work students (see Table 7) suggested there was no appreciable difference between either condition in terms of the type or number of strategies ultimately used. However, previous analyses had shown that eight of the nine students interviewed (the exception being Student G) stated that in groups, others may have different ideas or they could learn ways of solving from others.

In this section, analyses are reported of the extent to which group members in fact contributed ideas or suggestions during problem-solving sessions. To this end, analyses of session transcripts were conducted to determine the frequency with which students provided suggestions of strategies or ideas to one another during group problem solving.

The analyses show that, taken together, Groups ABC and DEF had 153 instances coded as suggesting a strategy or idea. The high incidence (24% of the

selected interactions) of occurrences of students suggesting a strategy or idea points to the volume of conversation that dealt specifically with idea-generation within the two groups during the six taped sessions. The following excerpts show the type of interaction the students engaged in when suggesting strategies or ideas:

C: Just figure out all the patterns. That's it.

A: Yeah. OK. We have to figure out how many ways we could make 50 cents exactly without using pennies.

C: Oh just wait. I've got an idea. Why don't we add all of this and then subtract it from this . . . and then get all the extras out. We'll add it to . . .

D: Let me find out the missing number here. We have to work backwards.

D: This will be the order. OK. I got it. This will be the order, right? Missing card. 3. Missing card. And then missing card, and a 4.

A: First we bring the dog, right, then we have to pour the seed out on to the floor, get bag, put the bird inside it, tie it so the bird go free, the dog won't eat the bird, the cat won't eat the seed.

These data suggest that multiple ideas for effective strategy use were available during group interactions. However, as the previous analysis showed (Question 3), there was no appreciable difference between the groups and individuals in type or number of strategies used. This suggests that the generation of ideas by group members (at least about strategies to use) alone cannot account for the superior performance of groups when problem solving.

B. Evaluate, Monitor, Check or Summarize

The second most frequent category of group interactions was labelled evaluating, monitoring, checking or summarizing (22% of the total interactions in Table 8). In this section, data are presented which address the questions: What kinds of checking or monitoring behaviour were taking place within the two groups? Were the individual workers also checking their solutions?

In order to address these questions, the first analysis to be conducted compared the amount of time spent solving problems by individual- and group-work problem solvers. Informal observations during sessions suggested that individual-work students had often been the first to be finished with the problems during problem-solving sessions, but that they often arrived at the wrong answers. At the same time, the groups had appeared to be spending a longer time on the problems. It is possible that longer times spent solving problems can be associated with more careful monitoring or evaluating of solution attempts.

To compare how quickly groups and individual students answered the problems, the numbers of problems and problem extensions completed by all students within the given time for each session were tabulated. (Problem extensions were available for students to complete after they had finished the main problem.) It was found that the 14 individual-work students had completed a total of 106 problem extensions over 11 sessions, while the group-work students (with each student in a group counted in the five groups) would have been pro-rated as completing 45 problem extensions. These data suggest that individual-work students worked more quickly through problems, as evidenced by their completing over twice as many problem extensions as the group-work students. (Note: Groups ABC and DEF, and students G, H and I, did not follow the general pattern evidenced by the groups and individuals as a whole. Groups ABC and DEF did not complete any problem extensions, and neither did students G, H or I, across the eleven sessions.)

Further, during sessions 3, 5, 6, 7 and 8, the length of time taken by both group-work and individual-work students to complete the problems was recorded for all students. With the exception of session 7, when students in both conditions finished the problem in 10 minutes, individual workers generally took less time to solve problems than did the students working in groups. In session 3, 2 of the 5 groups and 11 of the 14 individual-work students had started on the problem extension while the remainder were still working on the main problem. Several individual-work students started on the problem extension after spending less than 10 minutes on the main problem. In session 5, 11 of the 14 individual workers had finished the main problem and then started on the problem extension before any of the groups had finished the main problem. In session 6, 7 of the 14 individual workers had finished both the problem and extension before any of the groups had finished the problem. In session 8, 8 of the 14 individual workers finished both the problem and extension after 12 minutes, while none of the five groups had finished the problem.

The earlier analysis on the summary scores of answers to problems using the Analysis of Variance had shown that the groups performed better than the individual-work students on the problems during the intervention period. More detailed analyses (see Table 9) suggest that the scores of the groups and the individual workers for the five timed sessions showed that their scores were similar for sessions 3, 6 and 8, while for sessions 5 and 7, the groups' mean scores on answers to the problems was triple that of the individual-workers' mean scores. Taken together with the data on problems completed, these analyses suggest that while the individual workers accomplished more during the intervention period in the quantity of problems and extensions completed, their accuracy in solving the problems suffered.

To evaluate the possibility that increased time, and improved problem-solving ability, could be related to differences between individuals and groups in checking and evaluating, an analysis was conducted of the answer sheets from each session for the two groups and three individuals chosen for intensive analysis.

Table 9. Scores For Problems Solved by Groups ABC, DEF, and Students G, H, and I

	Sessions											
	1	2	3	4	5	6	7	8	9	10	11	Totals
Group-work												
ABC	2	2	2	1	2	0	2	0	2	0	0	13
DEF	2	2	1	0	2	2	2	2	0	0	0	13
Individual-Work Students												
G	2	2	2	1	1	2	0	0	2	0	0	12
H	1	2	1	0	0	0	0	0	0	2	0	6
I	1	2	2	0	1	0	0	2	0	0	0	8

(from Table 3)

Given the difference in successful problem-solving performance between individuals and groups for sessions 5 and 7, these sessions were selected for more in-depth examination. Analysis of students' answer sheets shows that in session 5, individual workers G, H and I either did not completely understand the problem or were able to get only part way through their solutions, while both groups ABC and DEF were successful. The problem (see Appendix E for complete problem set) required students to determine how Renaldo could carry his cat, bird, and a bag of seed across a river one at a time so that he could not leave the cat alone with the bird or the bird alone with the seed. Student G did not complete the problem, but got part way: "He can take the bird because the cat won't do anything to the seeds." Student I got part way also: "He could carry the bird, so the cat can't eat the bird and so the bird can't eat the bird seeds." Student H did not fully understand the problem: "He first leave [sic] the cat and the seed together, then he carries the bird. After that he carries the seed and the cat together, or the bird eat the seed first, then carries the cat and the bird later."

In contrast, during Session 5, Group DEF got an answer quickly, then as they were trying to write the explanation, realized they had not solved the problem. The

following excerpts show the group members checking the logic of their answers to the problem.

D: OK. Takes the bird first. And take the seed.

F: Bird first, second seed. No. I think we should bring the seed and then the cat and then the bird.

D: OK, if the cat goes first, right, then the bird will eat the seed.

E: No, look. He cannot leave the cat alone with the bird. The cat and bird was alone first.

All: (pondering).

(The group starts writing the explanation.)

D: The bird goes first.

F: (writing) The bird goes first, then the seed. And then bring back the bird.

D: What? You forgot the cat.

E: Oh yeah. Wait. First the bird, right?

D: First the bird, then second the cat, then

E: Then, third the bird goes back, after the seed?

F: Yeah. 4 things.

D: Oh yeah, so this will be first step , and then later the seed will be second step. Take the bird back will be third step. And taking cat.

E: The cat goes fourth.

Session 7 showed similar findings. During Session 7, Groups ABC and DEF solved the problem correctly, while Students G, H and I did not. (Ms. Mazzeroni found that 20 of her fish had died, and bought the same amount as were still living. She then divided the fish equally and sold them to 6 customers who each bought 15. Students were required to determine how many fish Ms. Mazzeroni had to start with.) Each of the individual-work students correctly got to 15 times 6 equals 90. Student G

stopped there. Student H took no chances and put down two answers. For the first answer, she added 20 to 90 to get 110, and for the second answer, doubled the 110 to get 220. Student I doubled the 90 to get 180.

On reading the Session 7 transcript for Group ABC, the interaction within the group appeared to contribute to a correct answer. The following excerpt shows the back-and-forth discussion that allowed them to focus on the task more clearly and check the accuracy of their answers:

A: 6 times 15.

B: Why?

A: Because there are 6 customers, and each customer got 15.

C: But then 20 of them died. So then you add 20. Equals 110.

B: So you minus 20.

C: No, no. To start with. Then you have to divide by this, too, right?

B: It's 110, right?

C: It's not 110 (realized her error). Because she bought twice. Double it.

Group DEF (particularly Student D) realized they had to divide 90 by 2 to get 45, then add 20. They worked backwards to check their answer.

D: If she raised 65, right, 20 of them died. Which gets to 45, right? She bought the same amount.

E: She bought the same amount as were still alive.

D: OK. 65. 20 of them had died. 45 times 2. 90, right? 90 could be divided by 15, equals 6 customers. Perfect.

The above illustrations suggest that the group interaction activity of evaluating, monitoring or checking the logic or accuracy of solutions may have aided the groups in their problem-solving task. It may have paid to have someone else check the accuracy of one's statements. Thus, the evidence reported here suggests that

individuals did less checking and spent less time on many problems, which may in part account for superior group problem solving. What other patterns of group interaction were taking place during the problem-solving sessions that might have assisted the groups in solving the problems? The next section deals with the interaction pattern of explaining or asking for clarification of the problem.

C. Explaining, Clarifying for Others, or Asking for Clarification or Assistance

To review, interview data reported earlier showed that the more difficult the problem, the more a group-work situation was seen by students to be valuable. Again eight of the nine interviewed students (the exception was Student D) commented that others in the group may have different ideas or ways of solving, or may provide help in understanding the problem. Individual-work Students H and I also said they learned ways of solving problems from the class discussions at the end of each session. During the intervention period, individual-work students had often asked questions of the teacher during the sessions about the wording of the problems. In groups, they appeared to work out their understanding together.

To determine in what manner groups explained or clarified information for each other during problem solving, the frequency of interactions wherein students asked for clarification or explained to others was tabulated. Table 8 shows that a total of 105 such events were coded. While no comparative information is available regarding help seeking by individual-work students, the frequency count of interaction events in Table 8 suggests that this was a frequent occurrence in group interactions. Specifically, in Group ABC, 30 instances of explaining or clarifying the problem for others took place, along with 28 instances of students asking for clarification or assistance, while Group DEF had 24 instances of explaining or clarifying the problem for others, and 23 instances of students asking for clarification or assistance. These accounted for 16% of the total interactions in Table 8.

A review of problem-solving session transcripts provides numerous examples of ways in which students explained or clarified for one another. For example, there were frequent references such as group members asking each other, "What does that mean? What is the question asking?"

Further, sessions 3 and 7 provide examples of how the two groups interacted so that when one student did not understand, the others explained the procedure to successfully solve the problem:

B: I want to understand this. Why do we use 4 times 3?

A: There's 4 passageways down here.Then each one times 3, right? . . . And then 12, you go 12 and then . . . 12 times 2, because all these will equal to 12, right? And there's 2 more passageways so . . .

B: Where did you get the 65? How did you know 65?

A: I just explain to check if it works, OK? She subtracted 20. That means that's 45. Later she bought the same amount. That means she bought another 45. Equals to 90. Then later she divided the fish equally. Each person got 15.

B: OK. Hold on. What is this?

C: Let me explain it to you. There were 6 customers, right? Each of them bought 15, right? So we multiply this, equals 90. 90 divided by 2. At the beginning, right? She didn't have 90, she had 45. It says from the start. 45 plus.

B: Why divide by 2?

C: Divide by 2 from the start. 45 plus 20 equals 65. But then in the very very beginning, her 20 fish were still living. So we plus 20.

A: OK? Do you get it? (to B).

B: Oh. Beginning. Oh! I see. OK.

E: This is the amount of fishes, right? That means do we subtract 20?

D: What we do is, she had 20, right? That would be 90, right?

F: Is that the answer or is it, it's part of it?

D: Part.

F: Oh. So now I know. Now. What do we need to find out?

D: How many she had to start with.

E: Wait. Each got 15, so then there's 6 customers, so do we multiply 15 to 6?

D: That's what we do.

F: Do we plus 90 plus 20? No, right?

D: Wait. No, no, no. That had died, that's why.

Note that Student B asked for clarification several times. Students A and C both took turns explaining and clarifying for Student B's understanding. Both Students E and F were asking for clarification or giving explanations. Student D often provided explanations or clarifications, and seldom asked for assistance. In both groups, then, explaining and clarifying took place to ensure all members understood both the problem vocabulary and the procedure for solving.

In sum, although no comparative data was available for the individual-work students, 16% of the group interactions shown in Table 8 were coded as students explaining and clarifying for each other, suggesting such actions may be related to effective group problem solving. Further analyses suggest several other interaction patterns were used by the two groups.

D. Use of specific speech patterns: *(a) Mimicing, echoing, talking simultaneously;*
(b) Finishing another's comment

Coding of group interactions showed that students in groups often mimiced or echoed an immediately-previous comment made by another member of the group, or spoke in unison with other members. In both groups, 17% of the interactions shown in Table 8 were of students mimicing or echoing another's comment, and speaking simultaneously. Additionally, 9% of the interactions on Table 8 saw the students using a form of telegraphic speech where they finished another person's comment. These data suggest that members of both groups often "piggybacked" on each other's comments. Combined, these types of interactions comprised a total of 26% of the patterns noted in Table 8.

The following examples illustrate the type of talk that took place during the group interaction:

B: OK. We got it by

A: Guessing, right?

B: Got it by adding

A: By adding coins. By adding quarters and nickels and

B: Dimes

A: Dimes, to make up to 50 cents.

F: Seed. Because if you bring the dog, that means

D: That means

E: The dog will

D: Wait. The dog will be. If we take the seed over, the dog will be

F: Alone with the cat.

D: OK. 4 times 25.

F: 4 times

All: Equals 100. 100 divided by 2 equals 50.

F: 50?

D&E: Yup.

F: And then

All: 20 plus 50 equals 70.

These examples show that the students were often able to spontaneously follow the sequence of another person's thought or had the same idea at the same time. This description of the specific speech patterns of group members provides interesting qualitative information about the character of group interactions during collaborative problem solving. Nonetheless, in the absence of comparative data from individual-work students, the relation between these interactions and effective problem solving remains unclear.

E. Guiding discussion, facilitating, or dividing labour

An examination of the transcripts showed that 7% of the coded interactions in Table 8 involved students guiding the discussion, facilitating, or turn-taking and dividing the labour amongst themselves. The students working in groups appeared to establish a system for sharing the workload. The following excerpts illustrate how the two groups established patterns of work:

Anyone got any ideas?

Your turn . . . You write it . . . You do that . . . I'll do this . . .

No way . . . Wait . . . I'll draw it.

C, you say first . . . OK, let's split it up.

OK, let's first get this whole simplest way first, then . . .

It'll be easier if one person just writes and the other person just talks.

Isn't it my turn? . . . What shall I do?

Just don't fool around . . . People. People. Let's get working here.

You have to take a part of this, too.

These data suggest that the students showed cooperative turn-taking behaviour and made comments to facilitate their work. This pattern was evident across all sessions except for Group DEF in Session 7 (see Table 8).

F. Group Perseverence

Within the groups, members appeared to be urging each other on towards successful completion of the task. This interpretation is supported by data in two of the coded categories: Urging the group to complete the assignment or to think, and At least we tried; it's better than nothing. These two patterns comprised a combined total of 4% of the interactions (see Table 8). In Group ABC, 26 comments were in the two codes related to dealing with the pressures of completion, or consoling each other that at least they tried, and in Group DEF, 29 entries were noted. Interactions coded in these categories included such statements as:

A: Hurry, hurry . . . Come on, come on . . . We could use all our mind power . . . B, don't waste time this time.

C: There's no more hope then. It's better than nothing.

B. No. I want to get this diagram. . . There's no way.

C: What shall I do?

B: Do it on the side. Your own. Help us. If you got any ideas.

C: I'm thinking. I'm thinking.

B: Yesterday we were so lucky. At least we still get the answer. A lot of people didn't. You know, I was trying to get every answer in every problem, because I don't want to miss one. I want to finish.

D: We don't have much time here.

E: I'll try something. Think also.

D: She's trying another one. I'm trying this one. So it's faster.

D: Too bad. We nearly had it. . . . This is our only hope.

The above excerpts show students persevering and urging each other to complete the problem. In the following examples, they console each other that at least they tried.

A: Hope we get it. At least we tried it, right? If we don't

C: Yeah. It's better than

A: Don't you think so?

B: Good. Good. Good so far.

C: If we do something, A, we tried, right? It's better than not doing anything.

D: So we can't. There's no possible way.

F: Yeah. Just put the answer No.

F: No way. Maybe there is a way. We can't figure it out.

A: I'm tired.

C: We'll never get it.

A: Wait. We'll get it. Think. Think. OK. Let's start all over, OK?

In the above situation, the group were also not averse to starting all over again. The following examples show students expressing feelings of tiredness or frustration, after which another person in the group often encouraged them on.

A: Oh. Lost it. Again. Lost it. Darn. Darn. (head cradled in hands) We lost it. We didn't even get an answer.

B: We have to check our adding, you know. I'm scared we got a wrong . . . And then we waste a lot of time. If I'm wrong I'm going to kill myself.

C: 24 ways?

A: I don't care. I'm going to try any old thing.

C: (sigh)

A: Huh? It won't work. Darn. (groan) Who cares? (then starts singing)

A&C: (sigh)

The above statements show the students releasing their tension within the group. The students openly expressed reactions to their own imposed pressure to complete an assignment.

Although no comparative information was available for the individual-work students, the presence of these interaction patterns during group discussion shows the students were able to verbalize their frustrations and to provide support to others in the group. The encouragement or presence of others may have allowed the students to continue to persevere with the task, and may be related to the longer time groups spent in working on the problems. However, insufficient data was available from this study to formally test this possibility.

Summary of Question 4

To summarize, the analysis of session transcripts showed that several identifiable group interactions took place. The presence of these interaction patterns points to their possible benefits for group collaboration. The interaction patterns summarized in this section were: (a) Suggest a strategy or idea; (b) Evaluate, monitor, check or summarize; (c) Explain or clarify or ask for clarification or assistance; (d) Use of specific speech patterns of mimicing, echoing or talking simultaneously, and finishing another's comment; (e) Guide discussion, facilitate or divide labour; and (f) Group perseverence.

First, each of the nine students had stated during the interviews their preference for working in groups when the question was difficult, and they pointed to the opportunities available in groups to talk about the problem together. Consistent with this suggestion, data reported here indicated that there was a high volume of conversation that dealt specifically with idea-generation within the two groups. This suggests that group members did offer lots of ideas to one another. At the same time, these ideas did not translate to differences in type or number of strategies used, as compared to students working independently. Thus, the relationship between strategy and idea generation and effective problem solving remains unclear.

Second, the interaction activity of evaluating, monitoring or checking the logic or accuracy of solutions may have aided the groups in their problem-solving task. When compared to individual problem solvers, groups spent more time and more frequently evaluated their work.

Third, data suggested that groups often interacted so that when one student did not understand, the others in the group explained the procedure for solving or clarified the problem vocabulary. Although speculative, it may be that these group activities are also associated with effective problem solving.

Other interaction patterns were observed that describe group interactions during collaborative problem solving and that may be associated with effective problem solving in groups. Specifically, students in groups often appeared to be mimicing or echoing an immediately-previous comment made by another member of the group, spoke in unison with another or both members, or used a form of telegraphic speech in finishing another person's comment. Students established a system for sharing the workload, guiding the discussion, or turn-taking to facilitate their work. The two groups shared responsibility for completion of their assignment, where members appeared to be urging each other on for successful completion of the task, or consoling each other that at least they had tried. Their statements showed the students releasing their tension within the group, by openly expressing feelings of tiredness or frustration, whereby another person in the group would encourage them on.

Summary of Chapter Four

To summarize , quantitative analyses of posttest performance on problem solving revealed no difference between the group-work and individual-work conditions when the students answered the posttests alone. However, there was a difference during the intervention period when the groups and individual workers solved problems, favouring the group workers. There was no difference in posttest results on their attitudes to problem solving. There was also no difference between the two conditions on the type or number of strategies used to solve problems. Interaction patterns were observed that describe group problem solving, and that may have contributed to successful group problem solving. The next section discusses the conclusions that can be drawn from these results.

CHAPTER FIVE: CONCLUSIONS AND DISCUSSION

Conclusions

The research reported was undertaken to determine the impact of group collaboration on elementary-school students working on process problems in Mathematics. The major conclusions arrived at after the quantitative and qualitative analyses are as follows:

1. There is evidence that:
   - (a) Group collaboration allowed for superior solving of math problems during the intervention period;
   - (b) Better problem solving in groups did not translate into improved student ability to solve problems when working alone.
2. Group interaction did not result in an increase in positive attitudes towards problem solving.
3. Group-work and individual-work students did not differ in the type or number of strategies used to solve problems.
4. Specific activities took place during group interaction that showed students collaborating, supporting and guiding each other in the problem-solving task. Those activities identified included:
   - (a) Suggesting strategies and ideas.
   - (b) Evaluating and checking solutions and problem-solving procedures.
   - (c) Providing feedback in the form of explanations and clarifications.
   - (d) Engaging in specific speech patterns of mimicing, talking simultaneously and finishing another's comment.
   - (e) Guiding others' actions and facilitating the work.
   - (f) Persevering as a group.

In the following sections, each of the above conclusions will be discussed and related to previous research, if applicable, followed by comments or other insights obtained from the study directed specifically to teachers. Recommendations will then be made for revision of this study, if it were to be repeated in the future, and for future research directions.

1. (a) *Group collaboration allowed for superior solving of math problems during the intervention period*

There were no differences in the pre- and posttest problem solving abilities of the students in either the group-work or individual-work conditions, but there was a difference in their answers to problems solved during the intervention period, favouring the group-work students.

The fact that there was a significant difference between the group-work and individual-work students on problems solved during the intervention indicates that group interactions facilitated problem-solving performance. In the determination of what were the causes of the superior problem solving in groups, some support can be given to the view that the groups provided certain advantages that the individual-work students did not have in completing the problems, such as having others available to check on the accuracy of their answers, to provide help in understanding the question, to provide ideas, to facilitate their work, and for sharing the responsibility for completing the task.

However, it also appears that good ability to solve problems may not require group collaboration for success. For example, individual-work Student G was able to successfully solve problems and in the interviews stated he did not feel the need for the help of a group, except in very difficult problems. He also was capable on both pre- and posttests.

1. (b) Better problem solving in groups did not translate into improved student ability to solve problems when working alone.

The greater effectiveness of group-work students on problems during the intervention was not maintained in posttest scores when the students were required to complete the tests individually. This difference between the impact of working collaboratively in groups as opposed to working individually needs further investigation in a longer-term study. It had been expected that some improvement in problem solving ability may have resulted if students learned about problem solving from each other.

Previous studies had determined that social interaction can be a source of cognitive development if students experienced forms of cognitive conflict (Hatano, 1988; Perret-Clermont, 1980; Weinstein & Bearison, 1985), or if students internalized the shared intellectual processes experienced during the interaction (Brown, Collins & Duguid, 1989; Damon, 1984; Resnick, 1988; Rogoff, 1990, 1993). In groups, students externalize their ideas and reasoning for critical examination and monitor the reasoning of others. The "internal dialogues" problem solvers undergo as a result of having internalized aspects of the cooperative problem-solving sessions in which they had been engaged, might, as suggested by Schoenfeld (1989), have resulted in increased competency in individual problem solving. Or, as Dees (1991) had found, students might have benefited from the cognitive rehearsal involved in explaining a concept to someone else. Such improvements were not found in this study on posttest.

The short term of the intervention (eleven sessions over four weeks) may have contributed to the lack of any significant posttest differences between the group-work and individual-work conditions. It is possible that if the study had taken place over a term of at least six months, the effects might have been different. With increased exposure to group interaction, more opportunities for learning together may have

been obtained by the group-work students. However, a caution against lengthening this study to six months in the same class is suggested. Negative effects such as the Hawthorne Effect, may result, where one group is seen to be favoured over the other.

In addition, the compilation of problem solving test booklets in Test Form A and Test Form B resulted in a difference between the level of difficulty of problems, with Test A being easier than Test B. This difference unfortunately accounted for some of the variance in the problem-solving scores for all students. A future study should control for problem solving difficulty on pre- and posttest sets.

2. *Group interaction did not result in an increase in positive attitudes towards problem solving.*

Analysis of the differences between the two conditions on the *Attitudes Toward Problem Solving* scale proved to be insignificant. The results of this study parallel the findings of other researchers (Hembree, 1990; Taylor, 1994) that classroom interventions such as implementation of small-group work did not seem effective in ameliorating mathematics anxiety or effecting changes in attitudes towards Math self-concept.

The results of this study also underline the difficulty that others have found (Lester & Kroll, 1990) in designing instruments that can reliably measure the role affective factors such as motivation, interest, self-confidence, and anxiety, play in problem solving. It may be that the attitude scale used could have been amended to include questions on group-work versus individual-work on problem solving.

The lack of changes in attitude scores may also have been influenced by the shortness of the length of the study.

3. Group-work and individual-work students did not differ in the types of strategies used to solve problems.

All students used strategies spontaneously during the intervention period, as determined by examining the answer sheets. They were not pre-instructed to use any particular strategy for a particular type of problem, to avoid restricting students to a set of strategies they felt they were expected to use, and to determine what strategies the students would use on their own. Further, given the process types of problems used in this study, different strategies could have been successfully used for the same question. It is interesting, then, that both group-work and individual-work students chose to employ similar strategies, suggesting that they had similar strategies available for use in their repertoires.

A future study could include discussions with individual-work students while they were solving problems during the intervention period, to reveal how they chose the strategies they used. Additionally, reducing the range of problem types may permit more detailed analysis of the specific strategies generated by groups or individuals by problem type.

Data from this study suggested a number of interaction patterns that portray group collaboration during problem solving. These patterns may account for more effective problem solving in groups. Students were seen generating ideas and strategies, monitoring and checking their solutions, explaining and clarifying for each other, using specific speech patterns of mimicing, echoing, talking simultaneously, or finishing another's comments, guiding discussion and facilitating the task, or sharing responsibility for completing the problem. The next section discusses the specific patterns noted during the analyses.

4. (a) Suggesting strategies and ideas.

In this study, the two groups also showed back-and-forth idea generation. There was a high volume of events of students suggesting ideas or strategies. They were never at a loss for words, even though they occasionally stumbled or were stumped. Ideas for effective strategy use were therefore available in group interaction.

Since both group-work and individual-work students used similar strategies, it is unclear whether the strategies raised by the group members was related to more effective problem solving. In previous research, Artzt and Armour-Thomas (1992) had observed that groups of students went back and forth using different heuristics intermittently, a behaviour that seemed to have played an important role in their groups' successful problem solving.

4. (b) Evaluating and checking solutions and problem-solving procedures.

The group collaboration during the taped sessions showed students "checking" and challenging each other. Individuals do not have anyone to challenge their answer. Therefore do the individual-work students really learn whether their strategy or solution is correct? The individual-work students frequently finished ahead of the group-work students, and checking of their answers showed frequent errors. The significant differences between the two conditions during the intervention period on answers to the problems supports the view that the individual workers often did not check their answers. In this study, the checking and monitoring behaviours of the groups appeared to be important for accuracy of solutions.

The analysis showed group-work students checking and monitoring their activities extensively. When such feedback is not available, as when students are working individually, it may be that students do not know whether their answers are correct. For example, during the pilot test of the pre- and posttest math problems,

students were asked to rate how difficult they felt each question was. It was noted that those students rated questions as easy or medium easy even when they did not answer the question correctly. It therefore appeared that they got an answer, but did not realize it was wrong. The pilot test students may have failed to analyze appropriately the information in the problem or evaluate their results, similar to that found by Garofalo and Lester (1985) showing elementary students failing to monitor their progress and solutions.

A future study could investigate whether elementary school students can learn to develop effective procedures or monitoring behaviours for challenging their own or others' answers systematically.

4. (c) Feedback in the form of explanations and clarifications.

All nine of the students articulated the need for others on difficult problems: to help each other, share ideas, and learn strategies. Analysis of interaction patterns in this study revealed group members engaging in these types of behaviours. For example, one pattern observed was that the two groups were able to pinpoint the critical points through back-and-forth discussion, and when one person did not understand, others would provide the help.

This study's results support Leal's (1993) findings where 3rd and 5th grade children helped each other to consider alternate ways of understanding the text, and prior knowledge became shared knowledge. In this study, members of the two groups took the time to ensure other members understood the procedure for solving a problem, or asked each other for clarification. This study also parallel's Liedtke and Sales' (1990) study on cooperative learning in mathematics where students began to challenge each other's answers with questions such as "Why?" and "How do you know?". The same questions were asked in this study by both groups. Students challenged each other's comments or ideas.

The results provide some support for the concept of a kind of scaffolding process being used by the students, as was evidenced by their attempts to explain each step of a procedure so another member was able to follow the procedure. The literature review had not shown conclusively whether both high-achieving and low-achieving students benefited from the interaction in groups. However, this study's results show that the students in the two study groups were able to participate in the process of problem solving, which they might not all have been able to do if they had worked alone. Forman and Cazden (1985) and Forman and McPhail (1993) reported on dyads creating bi-directional zones of proximal development for each other, by assuming at different times the role of teacher or student. Their results were suggested at but not fully able to be drawn out in this study. It was seen in this study when the students were explaining the processes to each other.

4. (d) Specific speech patterns of mimicing, talking simultaneously and finishing another's comment.

The high frequency of use of spontaneous speech patterns where students mimiced or echoed another's comment, or talked simultaneously, was noted during the group collaborations. Such speech patterns may have been related to a scaffolding process taking place, but further research is needed to interpret or decode these interactions. It is not clear how such speech patterns benefited group problem solving.

Also, the high number of findings of spontaneous finishing of another person's comments made it appear as though the students were "reading each other's minds". Perhaps the students were developing some form of shared thinking or "intersubjectivity" (Rommetveit, 1979) in their speech through their intense involvement in the activities.

The role of language as the major vehicle for the development of intersubjectivity, the internalization of concepts and the development of higher

cognitive processes (Gallimore & Tharp, 1990), was noted in the review of the literature. Further research is needed on the forms of discourse that appeared during group collaboration. What is going on as students "copy" each other's speech? Does the mimicing have an effect on their learning? Does their use of telegraphic speech show development of some form of shared thinking?

4. (e) Guiding actions and discussion.

This study saw students guiding discussion and turn-taking, appearing to establish a framework or a system for sharing the workload. We saw them saying, "It's your turn," or "It's my turn," or "You say next." Their turn-taking behaviours may also account for some of the longer time spent by groups on problems. A future study focusing specifically on guidance and turn-taking may reveal further evidence of cooperative behaviours that enhance problem solving, and may also demonstrate whether participation by more members of a group takes place with such behaviours.

The two groups studied also appeared to show collaborative forms of interaction and there appeared to be respect for each other's role in the group. Each member attempted to explain to others what they were doing and all students in both groups were intensely involved in their activities. More dynamically opposed group structures may show different facets of group interaction.

4. (f) Group Perseverence

Duren and Cherrington (1992) found that groups spend a longer time working on a problem, and individuals give up. During the problem-solving sessions of this study, the individual workers did not give up, but often finished in half the time it took the groups to complete a problem. It was noted earlier that the groups in this

study spent a large part of their time checking and monitoring their procedures and solutions.

It is conceivable that students feel a certain degree of pressure to complete a problem. The groups still pushed to achieve, or get the answer, or told each other that at least they tried. Group members acknowledged each other's frustration and appeared to help each other push on and persevere. In addition to checking and monitoring, then, more time spent by the groups may have been related to keeping going in the face of uncertainty.

Damon and Phelps (1989) suggested that the more relaxed environment of a discovery learning situation promotes mutual support and provokes deeper conceptual insights and shifts in perspective. Although the design of the study did not permit assessment of shifts in perspective, personal threats from mistakes and difficulties appeared to be alleviated. Schoenfeld's (1985) finding of pairs alleviating pressures by saying things like, "I have no idea of what to do. Do you?" were replicated in this study. Members of the two groups appeared to show they shared the responsibility for completing the task.

Slavin's (1990a) cautions of "diffusion of responsibility" and "free-rider" effects occurring in some groups were also not seen in either Group ABC or DEF. In neither group was there anyone who did not participate fully to the extent of their abilities. However, there was some imbalance in Group DEF, with Student D initiating most of the ideas. That may have resulted in fewer participation opportunities for Students E and F. In the post interview, Student D showed amazing insight when he said he should have let others do more thinking, rather than "hogging all the glory".

Findings of Interest to Educators

As a byproduct, this study revealed certain areas that may be of interest, particularly to teachers, that are not directly related to the results or conclusions. One of the benefits of this study is that it allowed the teacher-researcher to hear what and how students thought. One of the interesting comments was made by Student C, who showed awareness of the applicability of problem solving: "If there were (problems), then you can think about it (Math) more". This suggested that if we ask the students, it is amazing what they could teach US. The following comments originate in reflections while reading the transcripts of the interviews and sessions, observations of and the comments by students not in the group condition, and observations by other teachers peripherally involved.

First, student affect must be an important consideration for teachers when considering whether or not to use group-work and for what purpose. The repeated comments by students wishing to be in the group condition spoke to the need they felt for group support. Additionally, the nine students interviewed had articulated the need for group assistance when problems were difficult.

Second, seven of the nine students said they preferred challenging problems. Their comments illustrate why they prefer more challenging Math problems:

> If they make you think, and later when you finally get it, you feel very good about it . . . then you will be more happy than easy ones cause the easy ones you just know it right away . . . and you're not using your brain . . .
>
> When you try to solve it more, it gets more funner and funner . . .

Rather than looking for easy problems, the students actually wanted the harder, challenging problems, both for the challenge itself, to "tease the brain", and for more practical reasons, getting ready for high school studies and to get "smarter".

Third, the students in the study mainly spoke other languages before entering the school system, and their language proficiency would be considered behind

students in other areas of the city. Their understanding of both the problems used in the study and other problems found in their regular school work was sometimes problematic. Using visual aids such as pictures of the setting of the problem, objects used in the problem, or the use of concrete objects for manipulation, would assist students in understanding the often-times difficult vocabulary and would help them in getting started on a problem.

Fourth, students spontaneously used or developed a "context" for some of the problems, supposedly to make it easier for them to relate to the problem or to solve it. Does that provide relevance to the students or increase their interest in solving the problem? One of the students commented, "It's like a story to read. It makes us think of that place or doing the same thing. It makes us imagine maybe one time we've been to that place and done that kind of thing and then we will know how to solve it." Another said she was imagining she was delivering the sandwiches in one of the problems.

Perhaps we need to look at the context of the problems in Math textbooks, and reword or re-work them to place them in the students' contexts. Recent research by Hart (1996) has shown that personalizing word problems by including persons known to the students and using contexts familiar to the students, affected the students' attitudes and abilities in solving problems.

Lastly, the use of extensions for each problem during the problem-solving sessions may have helped to consolidate learning by providing an immediate review of the strategies used. Additionally, their use may have provoked the students to evaluate their procedures and solutions for the main problems.

Suggested Revisions to and Limitations Of The Study

If this study were to be replicated, changes to some aspects of the design of the study need to be made. First, an extended intervention period, of at least six months, may have increased effect sizes. However, an extended period should be accompanied with changes to selection of participants. They should not all be selected from the same class in the same school, to prevent the occurrence of the Hawthorne Effect, where improvements are associated with the increased effort by a group seen as being favoured. Second, review of the assessment instruments is needed. The problem-solving test booklets were not equal in difficulty, and a future study would control for problem difficulty on pre- and posttest sets. The attitude scale used could have been amended to include questions on group-work versus individual-work on problem solving. Third, systematic observations and discussion with the individual-work students could be conducted while they were solving problems. Perhaps more differences might become apparent. Systematic comparisons were also needed to detect group vs. individual differences in perceived pressures to complete a problem. Fourth, further investigation is needed to uncover more of the variables in the situation where groups of students perform better than the same individuals do when working alone on tests.

The fact the researcher was the teacher involved in the classroom can lead to suggestions of lack of objectivity, role conflict, and personal biases contaminating the conclusions drawn or reducing their validity. The small participant size (N=28) and use of one class in one school further limited generalizability of the findings. There was, however, advantage in the in-depth knowledge the teacher-researcher had of the participants in the study. Such knowledge provided for a more detailed understanding of the students and their relationships with each other, and provided a different focus of explanation that other researchers who do not know the children, would have been unable to provide. Interviews, observations, audio- and videotaped

evidence, student productions, all provided for triangulation of measures that served as a check on any biases the researcher may have had, while enhancing the scope of the study.

Concluding Comments

This study attempted to determine the impact of group-work on individual problem solving. The intervention of group-work did not translate into improved individual posttest performance on problem solving tests or improvement on attitude scales. However, it did show there were significant benefits to group-work when the students were solving problems together, benefits that were not visible when students were solving the problems alone. Interaction patterns were identified in group collaboration that may have allowed for better problem solving by groups of students.

One of the benefits of this study is that it studied peer interaction in a regular classroom activity. It examined what occurs as students work together to solve challenging problems in their natural environment. In that manner then, this study adds to the findings of other researchers. It also was conducted by the students' teacher. Reference has been made to possible suggestions of researcher bias. However, there was the added benefit of the students' more relaxed or natural responses in the "experiment" conducted in the natural course of classroom interaction by someone they knew well.

Another of the benefits of this study is the combination of quantitative and qualitative measures and methods. It allowed for the use of multiple sources of evidence, the reinforcement of one form to the other, and added a "thick description" of peer collaboration.

The study has been personally worthwhile to the teacher-researcher. It has allowed for a step back to observe and listen to the students, and to learn many things from them not readily available in daily classroom activities. It has been a revealing look at what is "going on in their heads" and provided for discovery of students' individual strengths. It seems fitting to finish with a quote from one of the students: "One person's head can't be two. It's like two persons' minds together mix with their ideas is better than one. Two heads is better than one. "

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Appendix A

Pre- and Posttests of Math Problem Solving

Test Booklet A

1. My little sister decided that she just had to have a new softball. So she went to her piggy bank in order to get the 90¢ needed for the ball. She had 45 coins. How many pennies, nickels, dimes, or quarters did she have with her?

2. A fireman stood on the middle rung of a ladder, directing water into a burning building. As the smoke lessened, he stepped up three rungs. A sudden flare-up forced him to go down five rungs. Later he climbed up seven rungs and worked there until the fire was out. Then he climbed the remaining six rungs to the top of the ladder and entered the building. How many rungs did the whole ladder have?

3. Betty, Hilda, Adam, and Renee have decided to try out for parts for the school play. Hilda wants a part that has twice as many lines as the part that Betty wants. Adam has chosen a part twice as long as Hilda's plus three lines. Renee is going to try out for a part with four more lines than Betty's part. If they get the parts they want, altogether they will have a total of 47 speaking lines. How many lines would each actor have in the play?

4. Mindy, Ned, Opal, and Paul were skipping rocks in a lake. Paul's rock skipped 8 more times than Mindy's. Mindy's skipped 3 more times than Ned's. Ned's rock skipped 1/2 as many times as Opal's. Opal's rock skipped 8 times. How many times did Paul's rock skip?

5. Program cards are being handed out in 5th period P.E. As he passes out the cards to the members of his class, Arthur notices that 8 students are on the basketball team as well as on the student council; 7 students are on the track team as well as on the student council; and 5 students are on the track team as well as on the basketball team. How many more students are on the student council than are on the track team?

Test Booklet B

1. Daphne stares wide-eyed at the display of stickers before her in the store. She has $1.50 in babysitting money and plans to spend all of it on stickers. The unicorn sticker costs 80 cents, the rainbow sticker costs 70 cents, the teddy bear sticker is 40 cents, the whale sticker is 25 cents, and the ice-cream soda sticker is 15 cents. Find how many different combinations of stickers Daphne can buy with her $1.50.

Appendix A. Pre- and Posttests of Math Problem Solving

2. Darryl has a summer job with real ups and downs: he delivers packages in the Pacific Centre Tower. On his first day, he is given a box of sandwiches to deliver during lunchtime. He takes the turkey sandwich to an office 3 floors above the basement. He delivers the ham and cheese sandwich 13 floors above the turkey and 7 floors below the egg salad sandwich. He delivers the pastrami sandwich 9 floors below the ham and cheese sandwich and 8 floors below the submarine sandwich. He takes the tuna to an office 12 floors below the submarine sandwich.
What is the number of the floor to which Darryl delivered each sandwich?

3. How many minutes did Heidi, Saul, and Joy each travel to get to the skating rink on Saturday? Joy came by skateboard, Heidi came by bike, and Saul came on the bus. It took Heidi twice as long as Joy to get there. It took Saul 10 minutes more than it took both the girls together. All three skaters together took 64 minutes to get to the rink.

4. Lisa, Mary, Debbie and Jane were discussing diets. Lisa lost three more pounds than Debbie. Mary lost four more pounds than Jane did and twice as much as Debbie. Jane thought that dropping from 107 to 99 pounds was fine. Lisa only weighs 93 pounds now, but before her diet she used to weigh .... What?

5. Delia, Tracy, and Bella are going cross-country skiing with a school group. They are carrying packs and each pack can weigh up to and including 10 kilograms. The packs are weighed two at a time. Delia and Tracy weigh their packs together and the total is 24 kilograms. When Delia and Bella weigh their packs, the total is 20 kilograms. Tracy's and Bella's packs together weigh 18 kilograms. Which skiers have packs that are too heavy, and by how much?

Appendix A

Sources for Test Problems

Test Question	
A-1	Make it simpler: A practical guide to problem solving in mathematics, p. 135
A-2	Problem solving in school mathematics: 1980 Yearbook, p. 212
A-3	The Problem Solver 6, p. T-32
A-4	Problem-Solving Experiences in Mathematics, Grade 6, p. 19
A-5	The Problem Solver 6, p. 26
B-1	The Problem Solver 6, p. P-8
B-2	The Problem Solver 6, p. T-28
B-3	The Problem Solver 6, p. T-30
B-4	The Problem Solver 6, p. P-26
B-5	The Problem Solver 6, p. T-77

Charles, R. I., Mason, R. P., & Garner, D. (1985). *Problem-solving experiences in mathematics, Grade 6: Teacher sourcebook..* Don Mills, Ont: Addison-Wesley.

Malone, J. A., Douglas, G. A., Kissane, B. V., & Mortlock, R. S. (1980). Measuring problem-solving ability. In S. Krulik & R. E. Reys, (Eds.). *Problem solving in school mathematics: 1980 Yearbook,* (pp. 204-215). Reston, VA: National Council of Teachers of Mathematics.

Meyer, C. & Sallee, T. (1983). *Make it simpler: A practical guide to problem solving in mathematics.* Menlo Park, CA: Addison-Wesley.

Moretti, G., Stephens, M., Goodnow, J., and Hoogeboom, S. (1987). *The problem solver (Vol.) 6 - Activities for learning problem-solving strategies.* Palo Alta, CA: Creative Publications.

Appendix B

Attitudes Toward Problem Solving

Name: _____ Date: _____

CIRCLE THE ANSWERS THAT TELL BEST HOW YOU FEEL ABOUT MATH PROBLEMS.

1. I enjoy solving math problems.

   Strongly agree Agree Disagree Strongly disagree

2. When my teacher gives us math problems to solve, I get uncomfortable.

   Strongly agree Agree Disagree Strongly disagree

3. Once I start a math problem, I don't give up until I solve it.

   Strongly agree Agree Disagree Strongly disagree

4. I would rather solve only easy problems.

   Strongly agree Agree Disagree Strongly disagree

5. Problems that make you think are more fun than easy problems.

6. If I had the choice, I would rather solve math problems than do arithmetic drills or exercises.

   Strongly agree Agree Disagree Strongly disagree

7. Math would be more interesting if we had more problems.

   Strongly agree Agree Disagree Strongly disagree

8. I think I'm good at solving problems.

   Strongly agree Agree Disagree Strongly disagree

From: Szetela, W. (1991, p. 191). Performance on open-ended problems. In D. F. Robitaille, Ed. *The 1990 British Columbia Mathematics Assessment: Technical Report* (pp. 181-209). Victoria, B. C.: Ministry of Education.

Appendix C
Interview Questions

1. Before you began to solve the problem, what did you do?
2. Is there anything you don't understand about the problem?
3. Try to put the problem into your own words now.
   What question is asked in the problem?
4. As you worked the problem, what were you saying to yourself?
5. Did you use any special problem-solving strategies? Which ones?
6. Are you sure this is the correct answer to the problem? Why?
7. After you finished working the problem, what did you do?
   Did you check your answer in any way?
8. Do you think this problem could be solved in another way? How?
9. Did you try a method that didn't work and have to stop and try another way?
   How did you feel about that?
10. How would you explain to another person how to do this problem?
11. How did you feel while you were solving this problem?
12. Did you ever feel confused or frustrated or could not decide what to do? When?
13. Did you ever feel that you wanted to give up? When?
14. Did you enjoy solving this problem? Why or why not?
15. Would you rather have worked by yourself or with others? Why?
16. Tell me about your answers on the Attitudes Toward Problem Solving sheet.
    1. Do you enjoy solving Math problems?
    (N) 2. When my teacher gives us math problems to solve, I get uncomfortable.
    3. Once I start a math problem, I don't give up until I solve it.
    (N) 4. I would rather solve only easy problems.
    5. Problems that make you think are more fun than easy problems.
    6. If I had the choice, I would rather solve math prolems than do arithmetic drills or exercises.
    7. Math would be more interesting if we had more problems.
    8. I think I'm good at solving problems.
17. Describe your Math skills:
    very poor poor average good very good
    Why did you rate yourself that way?
18. What do you think of the experience (of problem solving)? (last interview only)

Adaptation of:

Fortunato, I., Hecht, D., Tittle, C. K., and Alvarez, L. (1991, Dec.). Metacognition and problem solving. *Arithmetic Teacher, 39*(4), 38-40.

Montague, M., Bos, C., & Doucette, M. (1991). Affective, cognitive, and metacognitive attributes of eighth-grade mathematical problem solvers. *Learning Disabilities Research & Practice, 6*, 145-151.

Appendix D

Analytic Scale for Problem Solving

UNDERSTANDING THE PROBLEM

0 No attempt
1 Completely misinterprets the problem
2 Misinterprets major part of the problem
3 Misinterprets minor part of the problem
4 Complete understanding of the problem

SOLVING THE PROBLEM

0 No attempt
1 Totally inappropriate plan
2 Partially correct procedure but with major fault
3 Substantially correct procedure with minor omission or procedural error
4 A plan that could lead to a correct solution with no arithmetic errors

ANSWERING THE PROBLEM

0 No answer or wrong answer based upon an inappropriate plan
1 Copying error; computational error; partial answer for problem with multiple answers; no answer statement; answer labeled incorrectly.
2 Correct solution.

TOTAL MARK POSSIBLE: 10.

From: D. Wilson, 1991, cited in Szetela, W., and Nicol, C. (1992). Evaluating problem solving in mathematics. Educational Leadership, *49*(8), 42-45.

Appendix E

Problems Solved During Sessions

Session

1 The sixth grade math teacher did an experiment with her students. One at a time, students were to give her change for a 50-cent piece without using pennies. No student could use the same set of coins as someone else. How many students will be able to give her change?
Problem Extension:
How many ways could they give her change for a quarter using dimes, nickels, and pennies?

2 On the planet Blipp, the monetary system is much like ours. A yak is worth more than an ugh. A bah is not the smallest unit. A yak is not the largest unit. Only 1 coin is worth less than an ipp. A gluk is worth more than a bah. There is more than 1 coin larger than a bah. Match their coins to our pennies, nickles, dimes, quarters, and half-dollars.
Problem Extension:
Suppose only 1 coin is greater than a bah. Can you match them to ours now?

3 A 3-storey birdhouse has 4 entrances in the bottom floor, 3 passageways between the first and second floors, and 2 passageways between the second and third floors. A mother bird builds her nest in the top floor. How many different ways can she get to the top floor from the outside?
Problem Extension:
Suppose there were 3 rather than 2 passageways from the second to the third floor. How many ways could she get to the top floor?

4 The Canadian Space Agency was taking applications for places in the experimental space station city being built near the moon. Half of the applications came from Canada, 1/4 came from the United States, 1/8 came from Russia, 1/16 came from England, 1/32 came from India, and 18 applications came from Mexico. How many applications in total have been received for the space city, and how many came from each country?
Problem Extension:
Make up a similar problem and solve it.

5 Renaldo needs to carry his cat, his bird, and a bag of seed across the river. Unfortunately, he can only swim across safely carrying one item at a time. He cannot leave the cat alone with the bird or the bird alone with the seed. How can he get them all safely across the river?
Problem Extension:
Could he also get a dog across if the dog and cat could not be left alone and the dog and bird could not be left alone?

Appendix E. Problems Solved During Sessions

Session

6 Chip and Dale placed peanuts in two piles--one for doubles (2 peanuts in a shell) and one for triples. When they shelled their peanuts, they counted 78 peanuts and 35 pairs of shells. How many of their peanuts were doubles?
Problem Extension:
Suppose they count 30 pairs of shells and 76 peanuts. How many doubles will they have?

7 Ms. Mazzeroni raises guppies. One day she found that 20 of her fish had died. She bought the same amount as were still living. She divided the fish equally and sold them to 6 customers. Each customer bought 15. How many fish did she have to start with?
Problem Extension:
Suppose she sold 25 fish to each of 4 customers, instead of 15 to each of 6 customers. Now how many fish would she have started with?

8 Three grizzly bears, Chuck, Barry, and Amos, were very carefully weighed two at a time. Barry and Chuck weighed 248 kilograms together, Amos and Barry weighed 262 kilograms, and Amos and Chuck weighed 256 kilograms. How much does each bear weigh?
Problem Extension:
If Amos and Barry weigh 236 kilograms, Amos and Chuck weigh 242 kilograms, and Chuck and Barry weigh 220 kilograms, how much does each bear weigh?

9 Armando has a card trick for Andrew:
"I have 10 cards, numbered from 1 to 10. I have arranged the cards in a stack in a special way. The first card facing up is a 1, and I'm putting it on the table. The second card I'm putting at the bottom of the stack in my hand. The third card, which is a 2, I'm putting on the table next to 1. Then the fourth card goes to the bottom of the stack. I'll continue putting one card on the table and the next card to the bottom of the stack until I put card 10 on the table. How did I first arrange the cards in the stack?"
Problem Extension:
Try the same trick, but use 16 cards, numbered 1 to 16.

10 A caravan is stranded in the desert with a 6-day walk back to the nearest town. Each person in the caravan can carry a 4-day supply of food and water. A single person cannot go alone for help because one person cannot carry enough food and water and would die. How many people must start out in order for 1 person to get to help and for the others to get back to the caravan safely?
Problem Extension:
If 4 people start out, how many days could 1 person travel to get help?

Appendix E. Problems Solved During Sessions

Session

11 Albert is squirming excitedly in his bleacher seat, awaiting the results of the men's 50-metre freestyle. His older brother, David, was one of the swimmers, along with David's friends Mick and Rick. Mick's and Rick's combined times totaled :93:00 seconds. Rick's and David's times totaled :93:22, and David's and Mick's times totaled :92.74. Who had the best time, and by how much?
Problem Extension:
Make up a similar problem and solve it.

Sources For Problems Solved During Sessions

Session	Source
1	Problem-Solving Experiences in Mathematics, Grade 6, p. 67
2	Problem-Solving Experiences in Mathematics, Grade 6, p. 75
3	Problem-Solving Experiences in Mathematics, Grade 6, p. 111
4	The Problem Solver 6, p. T-87
5	Problem-Solving Experiences in Mathematics, Grade 6, p. 88
6	Problem-Solving Experiences in Mathematics, Grade 6, p. 108
7	Problem-Solving Experiences in Mathematics, Grade 6, p. 48
8	Problem-Solving Experiences in Mathematics, Grade 6, p. 119
9	The Problem Solver 6, p. T-73
10	Problem-Solving Experiences in Mathematics, Grade 6, p. 119
11	The Problem Solver 6, p. T-79

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Charles, Randall I., Mason, Robert P., and Garner, Dianne. (1985). Problem-solving experiences in mathematics. Don Mills, Ont.: Addison-Wesley Publishing Company.

Moretti, G., Stephens, M., Goodnow, J., and Hoogeboom, S. (1987). The Problem solver (Vol.) 6 - Activities for learning problem-solving strategies. Palo Alto, CA: Creative Publications

Appendix F. Coding Categories

Question 1. What is the effect of problem-solving on students' abilities to solve Mathematics word problems?

A-1 Ability to solve problems

Question 2. Does group problem solving affect students' attitudes toward Math problem solving?

A-1 Perception of abilities in problem solving and of overall Math ability
A-2 Motivation/Persistence/Resistance to Frustration/Personal pressure to achieve or complete task
A-3 Challenging problems are fun/thinking feels good/learn more
A-4 Relevance/usefulness of problem solving/Degree of interest in, enjoyment of
Transfer of learning to other situations/settings/later life
A-5 Attitudes to/merits of working individually or in a group

Question 3. What types of problem-solving strategies do students use in collaborative learning? Are these different from the strategies used by students in individualized learning?

P-1 Interpreting and Understanding the Problem Statement
Clarify, restate in own words, problem statement or vocabulary.
Identify conditions or variables, data needed to solve
Provide context for the problem/personalize the problem
P-2 Confusion with/misunderstanding of the problem statement or vocabulary;
Ask for clarification of problem statement from others.
S-1 Strategies: suggest, select; develop a plan of attack
S-2 Implement the plan/strategy
Perform calculations, solve subproblems
Use representations, translations, aids in solving (e.g. charts, diagrams).
S-3 Flexibility/Adjust/shift to alternate ways/strategies of solving the problem
S-4 Monitoring/Evaluation
Summarize/Review/Monitor progress, procedures used
Check reasonableness of answer or calculations
Reflect, evaluate, verify, justify to self or others, solutions/choices made

Question 4. How do students collaborate, support, and guide each other in the problem-solving task?

G-1 Conflict/Tension/Difficulties in cooperating with others
Reject/Ignore others' suggestions or ideas/Dominate/Control
G-2 Peacemaker/Provide support, encouragement
G-3 Division of Labour/Move discussion along/Bring others back to task
G-4 Off task/Tension Reduction/Humour/Joking
G-5 Give explanations, directions, elaborations, answers or suggestions
Share prior knowledge, give background information
G-6 Agree with other's ideas, suggestions for solving/Passive modelling/mimicry
Talk simultaneously - Finish other's comments
G-7 Ask for help or confirmation from others in solving problem/Ask questions
G-8 Achievement
Responsibility for completing task distributed amongst group
Pressure for group to complete task; Urging others in group to think