ESSAYS ON BANKING

by

KIT PONG WONG

B. SSc., The Chinese University of Hong Kong, 1987
M. A., The University of Western Ontario, 1989

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES
THE FACULTY OF COMMERCE AND BUSINESS ADMINISTRATION

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

July 1993

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Department of Commerce

The University of British Columbia
Vancouver, Canada

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This dissertation contains three essays which look at the role of price competition in banking. The method of investigation is a theoretical one. The first two essays examine the relative efficiency of relationship banking and price banking. The third essay discusses the determination of bank interest margin.

Conventional wisdom suggests that increased interbank competition should improve social welfare and thus price banking should dominate relationship banking. Essay one shows that the opposite result may occur when the product market is imperfect and the lending instruments are loan commitments. Under relationship banking both banks and borrowers have bargaining power. The borrowers have substantial bargaining power when the costs of switching banks are small. In this case, it pays the banks to charge interest rates below the competitive rates in order to keep their customers. The interest losses are compensated for by higher commitment fees paid upfront by the borrowers. Since interest costs are lower under relationship banking than under price banking, borrowers produce more and output price declines. Social welfare thus unambiguously increases. Essay two goes on to examine the relative efficiency of relationship banking and price banking under the asset substitution problem. The bank-customer relationship is assumed to provide a credible commitment for a borrower to refrain from transacting with other banks. The outcome under relationship banking is second-best since underinvestment results in solving the asset substitution problem. The multilateral credit transactions permitted by price banking impose negative externalities to existing loans by inducing the borrower to substitute riskier project. More underinvestment is needed to resolve the dual incentive problem and equilibrium results in reduced welfare for borrowers. Essay three tackles the determination of bank interest margins using a simple production-based model of risk-neutral banks which face (i) loan default risk, (ii) interest rate risk, (iii) capital regulation, and (iv) deposit insurance. The optimal bank interest margin is shown to be increasing with the variability of the short-term money market rate, but decreasing with either a stiffer capital requirement or an increase in the flat-rate deposit insurance premium.
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ACKNOWLEDGEMENT

I wish to express my most sincere thanks to the members of my dissertation committee, Professor Josef Zechner and Professor Paul Fisher, for their assistance and, especially, to my dissertation supervisor, Professor Ron Giammarino, for his encouragement and helpful guidance. I would also like to thank Rob Heinkel, Burton Hollifield, Helena Mullins, Raman Uppal and seminar participants at the University of British Columbia for helpful comments. My special thanks also go to David Downie and Max Maksimovic who made personal effort to assist my work.

Financial support from the University of British Columbia and the Social Sciences and Humanities Council of Canada is gratefully acknowledged.
Chapter 1

Introduction and Overview

1.1 Introduction

This dissertation contains three essays which look at the role of price competition in banking. The importance of this issue is highlighted by the fact that financial market developments and financial deregulation have induced a move from less price competitive "relationship banking" to more price competition.\footnote{This transition is well documented by Crum and Meerschwan (1986) and Meerschwan (1989, 1991).} As Keeley (1990) points out, increases in competition seem to be related to decline in bank charter values, which in turn may have caused banks to increase default risk through increases in asset risk and reductions in capital.\footnote{See also Chan, Greenbaum and Thakor (1992).} The failure of the Continental Illinois Bank\footnote{See Swary (1986) and Saunders (1987).} in the United States (1984) and the collapse of the Canadian Commercial Bank and Northland Bank\footnote{See Estey (1986).} in Canada (1985) may be viewed as the most significant examples of the competition/financial stability tradeoff that accompanies the transition to the new price-driven system—all three failed banks had heavily purchased loans and deposits from others who had no ongoing relationship with the banks prior to default. The weakening of relationship links between banks and customers is suggested by Davis (1992) to be an important factor of the financial fragility and instability.
of recent years.

In spite of the significance of the above issues, very few studies—either empirical or theoretical—have been focused on them. Shockley and Thakor (1993) state: “[The] small but growing literature on the theoretical and empirical significance of relationship banking ... should shed further light on the role of banks in an era of intense debate about the relative importance of banks versus capital markets in the capital allocation process (p. 32).” In particular, little is known about what relationship banking means for welfare. The purpose of this dissertation is to add to the understanding of this welfare aspect.

Although relationship banking is discussed in the literature, it is not specifically defined. While related to a lack of competition, relationship banking is not explicitly linked to any particular market imperfection. In this study, the term “relationship banking” will refer to model specific imperfections. In chapter 2 the bank-customer relationship arises from the cost of educating more than one bank. Chapter 3 assumes that the bank-customer relationship takes the form of an exogenous mechanism to commit not to borrow further. In chapter 4, the bank is assumed to be a monopolist in the loan market.

The deregulation of banking and demise of local financial boundaries have squeezed bank interest margins (the difference between average lending rates and average borrowing rates) to an extent which “did not properly reflect the risks that were being taken on.” As pointed out by Blanden (1993): “The problem for the banks in every industrialized country is that the search for other sources of income has dominated their thinking (p. 20).” He concludes: “The banks have been diverted from their fundamental role in life [of borrowing and lending money]. They need to get back to basics, even if that means becoming even more unpopular with customers (p. 20).” Thus, this dissertation will also tackle the determination of bank interest margins in order to add to our understanding of the sources of change in bank interest margins.

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5This is a comment made by Robin Leigh-Pemberton, Governor of the Bank of England, in a speech to The Chartered Institute of Bankers in Scotland, as reported by Blanden (1993), p. 19.

6Words inside the brackets are added by the author.
The remaining part of this chapter is organized as follows. The next section presents a brief historical account of the changes in the banking systems within and between major developed countries. Section 1.3 provides a concise literature review on relationship banking. Section 1.4 offers an overview of this dissertation.

1.2 Historical Background\(^7\)

Until the mid-1970s, in the major developed countries, such as the United States, Japan, and the United Kingdom, the financial regulatory structure brought forth similar forms of banking systems. Crum and Meerschwam (1986) refer to these systems as “relationship banking.” Under these highly regulated banking regimes, the freedom to transact financially at mutually beneficial prices was seriously suppressed. For example, in the United States, a system of geographic and product market segmentation, imposed through branching restrictions (McFadden Act) and product restrictions (Glass-Steagall Act), was bolstered by various interest rate restrictions (e.g., Regulation Q). In the United Kingdom, interest rates were not allowed to reflect market conditions but were determined by a bank cartel composed of the dominant clearing banks. In Japan, a strict interest rate law (Temporary Interest Rate Adjustment Law, 1947) was imposed to link various rates throughout the economy. Therefore, non-price competition in the form of long-term relationships between banks and customers evolved as a primary means of competition. Through such relationships, lenders were able to accumulate information and build up confidence in borrowers; while borrowers, through continued good repayment, warranted the right to receive future credit. These ongoing relationships, fostered by regulation, permitted both parties to credibly commit to behaving responsibly and to forbear temporary problems.\(^8\) Relationship banking, therefore, “represented a stable, effective financial system, and one that developed as a common response to ... [different] national cultures and strategies (Crum and Meerschwam (1986) pp.

\(^7\)For a detailed documentation, see Meerschwam (1991).

\(^8\)Relationship banking is in fact a form of implicit contract upon which the nature of the agreement to provide credit (by the lender) and to remain a customer (by the borrower) cannot be formally contracted.
During the 1970s, in which the Bretton Woods system of fixed exchange rates collapsed and massive international capital flows began to emerge, interest rates became more volatile. This situation provided investors with the incentive to shop for the best financial product prices and banks with the incentive to be innovative—both for profits and market share. New, price-oriented financial instruments were rapidly developed to take advantage of these emerging market opportunities. This led to a move away from the well established bank-customer relationships and, as a result, the price-regulated domestic systems began to decay.

Many forces encouraged the observed changes from local relationships towards price-driven transactions. Below, some major types of deregulation are highlighted (see Davis (1992) p. 26):

- abolition of interest-rate controls, or cartels that fixed rates;
- abolition of exchange controls;
- removal of regulations restricting establishment of foreign institutions;
- development and improvement of money, bond, and equity markets;
- removal of regulations segmenting financial markets;
- deregulation of fees and commissions in financial services;
- tightening of prudential supervision.

Crum and Meerschwam (1986) refer to the new systems, which emphasized impersonal price-oriented financial transactions, as "price banking."
1.3 Literature Review

The importance of the bank-customer relationship to bank lending policy was first asserted by Hodgman (1961, 1963). Then, the bank-customer nexus was tied to the studies of credit rationing in a variety of contexts (see Kane and Malkiel (1965), Wood (1975), and Blackwell and Santomero (1982)). The basic ingredient of this literature is the presence of externalities to bank lending which yield additional benefits to the bank from either subsequent loans or cross-selling other services to the borrower. In a full information world, the loan pricing should reflect the externalities. Blackwell and Santomero (1982), therefore, suggest that “models based on the customer relation are inadequate to explain the special status of the prime customer in a competitive world (p. 129).” In fact, on the assumption that borrowers with low credit risk have a more elastic demand for credit, they show that these borrowers will be the least costly to ration, whatever the nature of the bank-customer relationships.

Greenbaum, Kanatas and Venezia (1989) point out that “the existing literature provides no formal explanation for ... the benefit of forming a bank-customer relationship in a competitive environment (p. 222).” Rather than taking the relationships as given, they provide a rationale by constructing a stylized search model. The incumbent bank is assumed to enjoy an advantage of information reusability (see Chan, Greenbaum and Thakor (1986)) over the competing banks about the borrower’s probability of repayment as a consequence of previous lending. In addition, the borrower is assumed to incur exogeneous search costs in shopping for more favourable loan terms. Thus, the incumbent bank obtains some

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9A broader survey of banking theory is provided by Bhattacharya and Thakor (1991).
10A preliminary outline of this idea is suggested by Greenbaum and Venezia (1985) in a context of pricing loan commitments. Easterwood and Morgan (1991) also establish a search model of a financial market (generalized to include costly contracting). However, their model is a certainty model and focuses on the borrower’s optimal choice between depository and brokerage financial intermediaries.
11Specifically, Greenbaum, Kanatas and Venezia assume symmetric information between the incumbent bank and the borrower, but asymmetric information between the incumbent bank and the competing banks.
12This captures Wood’s (1975) suggestion that there are fixed costs to borrowers of changing their banking connection.
monopoly power owing to its private information and to the borrower's search costs. They show that the longer the relationship between the borrower and the bank, the higher the loan rate that the bank will charge\(^{13}\) and the shorter the expected remaining duration of the relationship. They also show that the price charged by the incumbent bank will tend to be higher than the average price offered by competing banks, since the latter must price lower in order to lure the borrower, establish a relationship, and reap the future monopoly profits. They conclude: "From the bank's viewpoint, the client relationship is a wasting asset with a strictly finite duration (p. 232)."\(^{14}\)

Similar to Greenbaum, Kanatas and Venezia (1989), Sharpe (1990) casts the bank-customer relationships as a consequence of the asymmetric evolution of information. The incumbent bank is assumed to have superior interim information since it can observe the \textit{ex post} return on the borrower's investment.\(^{15}\) This creates \textit{ex post} monopoly power for the incumbent bank, who will then be tempted to opportunistically increase its loan interest rate to successful borrowers about whom it knows more than the competing banks. \textit{Ex ante} interbank competition results in banks bidding away these anticipated \textit{ex post} expected profits via lower initial loan interest rates. With diminishing marginal returns of investment in any given project, the final outcome is second-best. Because the size of the loan taken by borrowers depends on the interest rate, too much capital is loaned out when less is known about the quality of borrowers and too little capital is allocated to the successful borrowers. As a result, low quality borrowers employ a greater proportion of the capital loaned out, relative to the first-best symmetric information case. Bank-customer relationships arise endogenously in Sharpe's model not because the bank treats borrowers preferentially, but because high quality borrowers are, in a sense, "informationally captured"—the problem of adverse selection hinders the competing banks from drawing off the incumbent bank's good

\(^{13}\) This result seems to be consistent with Hester's (1979) empirical finding that new loans to borrowers who were profitable customers to banks in the past bear higher interest rates than loans to other borrowers.

\(^{14}\) For some empirical findings on borrowers' shopping behaviour, see Haines, Riding and Thomas (1991).

\(^{15}\) Specifically, Sharpe assumes that borrowers do not know their own types (either high quality or low quality). All agents (the borrower and all banks) are initially symmetrically informed.
customers without a concomitant attraction of the worse ones.

The papers mentioned so far have not addressed the important welfare question about the relative efficiency of relationship banking and price banking.\(^\text{16}\) This question has recently drawn a great deal of academic attention. Conventional wisdom suggests that increased interbank competition should improve social welfare. This assertion is formally verified by Besanko and Thakor (1992) in a context of spatial oligopolistic banking competition. They show that increased competition caused by a relaxation of entry barriers into banking improves the welfare of depositors and borrowers at the expense of banks' shareholders.

Besanko and Thakor (forthcoming) comment that their earlier model precludes the potential interesting interactions between the bank's portfolio choice and the banking structure. Due to the static nature of that model, relationship banking issues are also ignored. In fact, relationship banking adds value to the bank charter since relationship-specific informational advantages generate rents.\(^\text{17}\) The concern with protecting these rents mitigates the distortionary effects of the fixed-rate deposit insurance system\(^\text{18}\) and thus improves bank soundness.\(^\text{19}\) Besanko and Thakor are particularly interested in the manner in which deregulated entry into banking impinges on borrowers' welfare. They show that there are two conflicting effects that increased interbank competition has on a borrower's welfare. The direct effect is that his borrowing cost is lowered due to more fierce competition among banks, which benefits him. However, this increase in the borrower's surplus reduces the value of the bank-customer relationship to the bank, which in turn dampens the pivotal countervailing force to the bank's propensity to exploit the deposit insurance put option by appropriately increasing risk. The higher bank insolvency probability resulting from riskier loans increases the chance of disruption of the bank-customer relationship and the associated destruction of

\(^{16}\)Although Greenbaum, Kanatas and Venezia (1989) and Sharpe (1990) have shown that the outcome under relationship banking is second-best, they do not contrast it with the one under price banking.

\(^{17}\)See also Sharpe (1990) and Rajan (1992) for how informational rents can arise from bank-customer relationships.

\(^{18}\)See Merton (1977) for the moral hazard problem inherent in the risk-insensitive deposit insurance system.

\(^{19}\)This argument is similar to Keeley (1990).
valuable information. The cost of this for some borrowers may exceed the benefit of lower loan interest rates. Thus, the conventional wisdom that increased interbank competition improves borrowers' welfare is not quite right when the impact of market structure on banks' portfolio choices is accounted for.

Petersen and Rajan (1993) provide another argument for relationship banking. They show (both theoretically and empirically) that borrowers are more likely to get finance from banks in a concentrated market than in a competitive one. This is because the individual rationality constraints in concentrated markets do not bind period by period as they do for competitive markets so that surplus can be re-allocated between time periods. Mayer (1988) and Hellwig (1991) also make the observation that bank may rescue firms that are in financial difficulty if they anticipate being able to participate in the returns from such rescues.

Rajan (1992) points out that there are both advantages and disadvantages of relationship banking over price banking. Under relationship banking, banks have incentives to monitor borrowers and control their investment decisions. In the process of doing so, banks acquire the ability to appropriate borrowers' rents because of information monopoly. This adversely affects borrowers' incentives to exert effort. Thus, Rajan concludes that relationship banking and price banking represent two starkly different control-rent trade-offs such that any unidimensional comparisons are misleading.

Bhattacharya (1993) examines the relative efficiency of relationship banking and price banking for financing R&D-intensive investments by firms competing in production markets. The R&D investments may yield technological knowledge of potential benefit to society, as well as to competing firms. The generation of R&D knowledge requires unobservable effort with non-pecuniary costs on the part of borrowers. Such knowledge is not only private to borrowers, but also proprietary in the sense that disclosure of it to product market competitors would reduce its private value. Under relationship banking such knowledge is not
revealed to product market competitors; under price banking it may lead to such knowledge being shared across product market competitors. Bhattacharya is able to show that if the private costs are small, price banking will be weakly dominating. On the other hand, when these costs are sufficiently high, relationship banking induces greater incentives for borrowers to do R&D and this may or may not coincide with the level of R&D that is in the social interest.

The papers by Maksimovic (1990) and Bizer and DeMarzo (1992) also relate to this dissertation. Maksimovic (1990) shows that a bank loan commitment, by precommitting the firm to aggressive product market behaviour, increases the value of a firm in a Cournot oligopoly. While it is individually rational for each firm to acquire a loan commitment, however, when all firms in the industry enter into such arrangement, they are made worse off. The strategic value of a loan commitment is a cornerstone of the model explained in chapter 2. Bizer and DeMarzo (1992) study the negative externalities arising from additional borrowing in the presence of moral hazard. While each new bank does not pay for these externalities, prior banks recognize the potential victimization that they may suffer and thus react accordingly. This contrasts with a one-bank environment in which all effects on earlier loans are internalized by the sole lender. Bizer and DeMarzo show that borrowers are worse off than they would be if they could credibly commit to refraining from further borrowing.

1.4 Overview

Chapters 2 and 3 of this dissertation try to further explore the efficiency implications of market structure in banking. In chapter 2, I argue that relationship banking may improve social welfare which is measured as the sum of producers’ and consumers’ surplus when the product market is imperfect and the lending instruments are loan commitments. In my model, there are no asymmetric information or agency problems. The seemingly counter-

\[20\] This should be contrasted with Besanko and Thakor's (forthcoming) interest in borrowers' welfare only. They have not actually shown that relationship banking may benefit the whole society.
intuitive result is driven by the second-best nature of the economy where both the capital and product markets are imperfect. The adverse effects imposed by one imperfect market upon the economy may offset (at least partially) those caused by the other and, because of the bank-customer relationships, banks are able to exploit their borrowers. However, borrowers are free to switch to other banks and thus this provides a force to discipline the behaviour of the incumbent bank. Of course, the effectiveness of this force depends on how costly it is for borrowers to switch banks. Borrowers' search costs are modelled as costs of delayed production.\textsuperscript{21} I show that if borrowers are tough (i.e., the search costs are small), the incumbent bank has to charge an interest rate below the competitive one in order to keep its customers. The interest losses are compensated for by higher commitment fees paid upfront by the borrowers. Since interest costs are lower under relationship banking than under price banking, borrowers produce more and output price declines. Social welfare thus unambiguously increases.

Chapter 3 goes on to examine the relative efficiency of relationship banking and price banking under the situation where risk-neutral borrowers can choose among projects of varying degree of riskiness which is unobservable to banks (i.e., under the asset substitution problem). I assume that under relationship banking each borrower can only deal with one bank. I show that the outcome of this bilateral credit transaction is second-best since underinvestment results in solving the asset substitution problem. In the absence of such a bank-customer relationship (so that borrowers cannot credibly commit to refraining from future borrowing), I show that the outcome is third-best. Within the multilateral credit transactions, borrowers can borrow as many times as they want, each time going to a new bank. The crux of the problem is that additional loans impose negative externalities to existing loans by inducing borrowers to substitute riskier projects.\textsuperscript{22} Since new banks do not pay for these externalities, prior banks recognize the potential victimization that they

\textsuperscript{21}In explaining firm's dynamic financing and investment decisions, Berkovitch and Narayanan (1993) assume that switching banks involve dissipative costs which are proportional to the net present value of the firm's investment project. My model shares some of this view but endogenizes the switching costs.

\textsuperscript{22}In the existing literature, the negative externalities described are not formally modelled.
may suffer and thus react accordingly. More underinvestment is needed to resolve the dual incentive problem and equilibrium results in reduced welfare for borrowers.\footnote{This reinforces the finding by Bizer and DeMarzo (1992) who consider a similar situation in which risk-averse borrowers can choose the effort required to avoid bankruptcy.}

The final chapter of this dissertation tackles the determination of bank interest margins in response to the concerns raised by Blanden (1993) and others as described in section 1.1. I develop a simple production-based model of risk-neutral banks which face (i) loan default risk, (ii) interest rate risk, (iii) capital regulation, and (iv) deposit insurance. The model is richer in structure than the existing ones in explaining the behaviour of bank interest margins.\footnote{There are only a few theoretical models for this issue (Ho and Saunders (1981), McShane and Sharpe (1985), Allen (1988), Zarruk (1988) and Zarruk and Madura (1992)). In contrast to my model, all the existing ones focus on risk-averse banks and pay little attention to banking regulation.} I am able to decompose the optimal bank interest margin into two components: the spread due to default risk and the spread due to banking regulation. I show that the optimal bank interest margin increases with the variability of the short-term money market rate, but decreases with either a stiffer capital requirement or an increase in the flat-rate deposit insurance premium. The chapter concludes with empirical implications unique to this literature.
References


Chapter 2

Banking Competition and Efficiency under Product Market Imperfection

2.1 Introduction

The purpose of this chapter is to examine the relative efficiency of relationship banking and price banking. By the term relationship banking I refer to a system of repeated bilateral credit transactions between each borrower and a particular bank. Price banking, on the other hand, is characterized by many banks bidding competitively for each transaction undertaken by a borrower. As documented by Crum and Meerschwam (1986) and Meerschwam (1989, 1991), there is a transition from relationship banking to price banking in developed countries such as the United States, the United Kingdom and Japan. This gives rise to the question of the relative efficiency of these two systems. Conventional wisdom suggests that a competitive price mechanism should achieve an optimal allocation of all available funds. However, Meerschwam (1991) argues that imperfections such as asymmetric information and agency problems divert the achievement of the first best outcome, and a relationship between a borrower and a bank may be an efficient institutional attempt to deal with these information

1Besanko and Thakor (1992) verify this assertion using a spatial model of oligopolistic banking.
problems.  

In this chapter I demonstrate that, even in the absence of any information problems, relationship banking may yield a higher level of social welfare than price banking. The key factor that drives this result is product market imperfection. In a second-best world where both the capital market and the product market are imperfect, it is possible that the adverse effects imposed by one imperfect market upon the economy may offset (at least partially) those caused by the other.

To illustrate the above "two wrongs make a right" argument, I construct a two-period Cournot duopoly model with a stylized relationship-based banking system. Outputs are produced and sold to consumers in each period by firms that have no capital and therefore must borrow from banks. Each firm has a lock-in relationship with a bank at the beginning of the first period and prefers to start its production in that period so that it can capture a larger market share from its rivals. The firm's desire to produce earlier gives the incumbent bank a chance to exploit it. However, if the firm thinks that it is overly exploited, it can spend some time to educate other banks. After a new relationship is established, the firm can ask the two banks to bid for its loan. Assuming the banks cannot collude, the firm is able to obtain better terms for the loan. The firm's search cost is endogenized to the extent that the cost of switching banks comes from the loss of market share due to the delay of production. It does not pay for the firm to build up a relationship with other banks when the benefit from receiving better terms of the loan cannot fully cover the search cost. The ability to switch to other banks provides the firm with an outside option that defines its bargaining position (i.e., its reservation profit) when it bargains with the incumbent bank.

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2See Sharpe (1990) for a formal model that shows how such a relationship arises endogenously as a consequence of asymmetric information.

3Brander and Lewis (1986) and Maksimovic (1986) are the first seminal studies that emphasize how important the linkages between the product market and the capital market are on the equilibrium outcomes.

4Petersen and Rajan (1992) present empirical evidence that small firms tend to concentrate their borrowing from a single bank, although this tendency declines as firm size increases.

5A similar stylized imperfectly competitive financial market is used by Berkovitch and Narayanan (1993). They assume that switching banks involve dissipative costs which are proportional to the net present value of the borrower firm's investment project. My model goes further to endogenize these costs.
The argument, however, also requires that the bank's actions are able to affect the firm's product market behaviour. This is added to the model by letting banks offer terms for a loan commitment to firms. A loan commitment obligates a bank to grant loans to a firm up to a maximum amount called the commitment size at a predetermined interest rate. In addition to the interest paid when the commitment is used, the contract itself entails a cost called the commitment fee which the firm pays upfront to the bank for acquiring the standby credit option.\(^6\) Maksimovic (1990) argues that a loan commitment increases the value of a borrower firm from a Cournot oligopoly by enhancing the firm's strategic power.\(^7\) While it is individually rational for each firm to acquire a loan commitment, he shows that all firms in the industry taken together are made worse off by the existence of loan commitments.

The equilibrium for the above model is contrasted with the one for an otherwise equivalent model but with a transactions-based competitive banking system. The main findings are somewhat surprising: output price is lower (and thus consumers are better off) and social welfare (the sum of producers' surplus and consumers' surplus) is higher under relationship banking than under price banking. To understand these seemingly counterintuitive results, first note that banks are not passive investors under relationship banking. Suppose that a single bank is dealing with competing borrowers. The incumbent bank can exert substantial influence over the firm's production decision. This influence in turn indirectly affects the value of the firm's outside option. For example, suppose that an incumbent bank offers two competing firms low interest rates. If the firm rejects the offer while its rival accepts, its rival will flood the product market in the first period because of the low interest costs (i.e., low marginal production costs): the value of the firm's outside option is small in this case. Since the bank can extract all the surplus above what the firm can get from its outside option by

\(^6\)A typical loan commitment also contains restrictive covenants such as collateral requirements and limited dividends. These are designed either to monitor the behavior of the borrower or to enhance the safety of the contract. As documented by Duca and VanHoose (1990), bank loan commitments comprise about 80% of all commercial lending in the United States.

\(^7\)There are many theoretical papers in the banking literature showing the importance of loan commitments. See Campbell (1978), Thakor, Hong and Greenbaum (1981), Thakor (1982), Deshmukh, Greenbaum and Kanatas (1983), Kanatas (1987), Boot, Thakor and Udell (1987, 1991), and Berkovitch and Greenbaum (1991), to name just a few.
means of the commitment fee, it pays the bank to charge a low interest rate to keep the value of the firm’s outside option small. While doing this implies a smaller producers’ surplus, as long as the portion shared by the bank is larger, the bank is willing to pick this as the equilibrium outcome. As a result, relationship banking is more pro-competitive.

This chapter is related to the growing literature on relationship banking which is reviewed in chapter 1. My work is distinguished from this area of research on the basis of its special focus on the product market competition. The exceptions in the literature which also emphasize the product market side are the papers by Bhattacharya (1993) and Poitevin (1989). While Bhattacharya (1993) looks at the R&D race in the product market, I examine a more traditional Cournot model of competition. The rationale of the results is also very different. Bhattacharya shows that relationship banking may be efficient because it induces greater incentives for borrowers to do R&D. In contrast, the driving force in my model is the allocation of bargaining power between borrowers and banks.

My work is also related to the paper by Poitevin (1989). He shows that the extent of competition in downstream industries may depend on the choice of banks. It is now well-known that in a Cournot oligopoly, debt is pro-competitive, since it gives incentives to the borrowing firm to undertake an aggressive output strategy. Poitevin shows that members of the industry may be able to achieve a partial collusion in the output market by borrowing from the same bank. A common bank can better control the incentive effects induced by debt and thus limit the extent of competition in the output market. I extend Poitevin’s work by considering interbank competition and the important implications that arise from it.

The remaining part of this chapter is organized as follows. In the next section, I present a formal description of the model in the context of a linear Cournot duopoly and a stylized relationship-based banking system. Sections 2.3 and 2.4 characterize the equilibrium loan commitments and the product market outcome. In section 2.5, I contrast the above equilibria.

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8See Brander and Lewis (1986) for the arguments under standard debt contracts and Maksimovic (1990) under loan commitments.
rium with the one in an otherwise equivalent economy with a transactions-based competitive banking system and show that the former banking regime is in fact more pro-competitive. Section 2.6 concludes.

2.2 The Basic Model\textsuperscript{9}

Consider an industry comprised of two identical firms (indexed by $i = 1, 2$) producing a homogeneous good.\textsuperscript{10} Each firm has two production periods and is endowed with a point-input point-output technology that transforms one unit of capital into one unit of output. After production in each period, the output is sold immediately to consumers. The industry’s inverse demand function is given by $p = a - b(q_1 + q_2)$, where $p$ is the price of the homogeneous good, $q_i$ is firm $i$’s two-period cumulative output, and $a$ and $b$ are positive constants.

Each firm has zero capital and must therefore rely on a banking sector (consisting of at least two banks) to grant loans in the form of loan commitments. A loan commitment, $(r, f)$, is a pair that specifies a predetermined interest rate, $r$, at which the bank is obligated to guarantee future availability of credit to the firm, and a commitment fee, $f$, paid upfront by the firm to acquire this privilege of having standby credit.\textsuperscript{11} In other words, a loan commitment is viewed as a two-part tariff, which is one of the most basic and common pricing schemes in economics.\textsuperscript{12} For simplicity, each bank faces a perfectly elastic supply of deposits at the zero two-period interest rate. There is no discounting in the economy.

\textsuperscript{9}The model described below can be viewed as a two-period extension of Maksimovic’s (1990) model.

\textsuperscript{10}Since firms in the industry are asymmetric in the sense that they have different production costs, for the tractability reason I have to restrict my attention to the duopoly case.

\textsuperscript{11}Since it is not an important issue here, the question of where the firm obtains the funds for paying the commitment fee is suppressed. Little generality is lost by assuming that the firm’s initial wealth endowment accommodates the commitment fee but is insufficient to permit self-financing for production.

\textsuperscript{12}Another distinct feature of the loan commitment contract is the quantity setting clause. This commitment obligation places an upper limit (called commitment size) on the firm’s borrowing. Within this limit, the firm may set future loan size unilaterally at the agreed upon rate. From a survey by Ham and Melnik (1987), less than 20% of the firms in their sample ever reached the maximum limit of the firms’ commitment sizes. Hence, for simplicity, I assume away the possibility that the firm faces credit rationing, in the sense that it cannot borrow as much as it wants at the prespecified interest rate. This assumption is innocuous in my model because the commitment size is isomorphic to the production capacity of the firm. It is always optimal for the firm not to be capacity-constrained as this will reduce its competitiveness over its rival.
At the beginning of the first period, each firm has a lock-in relationship with a bank and the firm requests a loan commitment from this bank to fund its two-period production. The bank then makes a take-it-or-leave-it offer to the firm. If the firm accepts the offer, it receives capital to start production right away. If the firm is not satisfied by the bank’s offer, it is free to visit other banks and educate them. After a second relationship is built up, the firm asks both banks to bid for its loan. Of course, doing so is time-consuming and I assume that it inevitably leads to a delay of production for the entire first period. Hence, the firm’s search cost is to some extent endogenously determined: Getting the line of credit earlier provides the firm with a first-mover advantage on the one hand, but may involve less favourable terms of the loan on the other. The endogenous component of the search costs, not found in existing search models in the literature, allows me to consider welfare implications and leads directly to the surprising conclusions.

Formally, the set up is a two-period four-stage complete information game. At the beginning of the first period, each firm solicits a loan for production from the incumbent bank. Both firms may happen to have relationships with the same bank or different banks and I treat these two possibilities as two different full games, denoted as full game 1 for the one-bank case and full game 2 for the two-bank case. Without loss of generality, I assume that both firms have relationships with bank 1 in full game 1 and firm \( i \) has a relationship with bank \( i \) in full game 2.

Stage 1. In full game 1, bank 1 announces a pair of loan commitments \([(r_1, f_1), (r_2, f_2)]\), one for each firm. These are take-it-or-leave-it offers and firms can observe their rival’s offer.\(^\text{13}\) In full game 2, banks 1 and 2 simultaneously announce take-it-or-leave-it offers \((r_1, f_1)\) and \((r_2, f_2)\) to firms 1 and 2, respectively. Firms cannot observe their rival’s offer. For the rest of both full games, all actions are the same. Firms either accept or reject the

\(^{13}\)If firms cannot observe their rival’s offer, then they will fear “third-party” opportunism by the bank: Once a firm has paid its commitment fee the bank is no longer concerned with protecting that firm’s profit and is therefore tempted to reduce the interest rate to the other firm in exchange for a higher commitment fee. To rule out this incentive problem, I simply assume that offers by the bank are publicly observable. See, for example, Hart and Tirole (1990) and McAfee and Schwartz (1990).
offers. Accepting involves paying the commitment fee upfront to the bank. Each firm then learns its rival’s loan commitment and acceptance decision.

Stage 2. If both firms accepted the offers, they simultaneously choose their first-period outputs, \( q_{11} \) and \( q_{21} \). If firm \( i \) accepted the offer while firm \( j \) rejected, firm \( i \) chooses its first-period output \( q_{i1} \), knowing that \( q_{j1} = 0 \). If both firms rejected the offers, then \( q_{11} = q_{21} = 0 \). These outputs are then publicly known and sold to consumers at the end of the first period.

Stage 3. At the beginning of the second period, any firm \( i \) which rejected the offer in stage 1 now gathers banks to bid for its loan commitment, \( (\hat{r}_i, \hat{f}_i) \). Again, accepting involves firm \( i \) paying the commitment fee, \( \hat{f}_i \), upfront to the bank which wins the bid. The loan commitments are then publicly known.

Stage 4. Both firms choose simultaneously their second-period outputs, \( q_{12} \) and \( q_{22} \), which are then sold to consumers at the end of the second period.

The equilibrium concept employed is Selten’s (1975) subgame perfect Nash equilibrium (SPNE) and I restrict attention to pure strategy equilibria only. A set of pure strategies for a game is a SPNE if it is a Nash equilibrium for the entire game and its relevant action rules are a Nash equilibrium for every proper subgame. This is the appropriate equilibrium concept for the given complete information game. For the following analysis, a variable with an asterisk means a SPNE strategy for the subgame in question. I also adopt the tie-breaking rule that a firm will choose a loan commitment offered in the first period if the firm is indifferent between this loan commitment and another loan commitment offered in the second period.\(^{14}\)

2.3 Analysis of Production-Search-Production Subgames

In this section, I characterize the SPNE for subgames starting from stages 2 to 4 (hence-

\(^{14}\)This assumption is justified when there are transaction costs such as transportation costs and bidding arrangement costs, no matter how small they are.
forth called production-search-production subgames). Given a pair of loan commitments 
\[ (r_1, f_1), (r_2, f_2) \] offered to firms in stage 1, there are four possible histories associated with production-search-production subgames. These are denoted as follows: (accept, accept) denotes the history in which both firms accepted the offers; similarly, (reject, reject) denotes the history in which both firms rejected the offers; finally, (accept, reject) and (reject, accept) denote the histories in which one firm accepted its offer while the other rejected. I shall analyze each of these in turn, specifying how equilibrium play unfolds for each possible history. Since the two subgames (accept, reject) and (reject, accept) have parallel analyses, we shall address only the subgame (accept, reject) in detail. The extensive form of production-search-production subgames is depicted in figure 1.

(Figure 1 about here)

The calculation of the SPNE for a production-search-production subgame proceeds by solving backward. Given complete information and zero discounting, rational consumers accurately anticipate all equilibrium actions taken by firms and the prices paid by them are the same in both periods and equal to the market clearing price: \( a - b(q_1 + q_2) \).

In the final stage, each firm knows its rival’s interest cost and first-period output. As the first-period profit from sales to consumers has already been realized, firm \( i \)'s second-period profit maximization problem given \( (q_{i1}, q_{j1}, q_{j2}, r_i, r_j) \) is

\[
\max_{q_{i2} \geq 0} [a - b(q_{i1} + q_{i2} + q_{j1} + q_{j2}) - (1 + r_i)q_{i2}], \quad i = 1, 2.
\]

It is straightforward to solve the second-period Cournot-Nash equilibrium outputs, \( q_{i2}^* \) and \( q_{j2}^* \), which are given by

\[
q_{i2}^* = q_{i2}(q_{i1}, q_{j1}, r_i, r_j) = \frac{1}{3b}[a - 1 - b(q_{i1} + q_{j1}) - 2r_i + r_j], \quad i = 1, 2. \quad (2.1)
\]

Now, go back one stage and suppose that firm \( j \) rejected the offer in stage 1. Then, \( q_{j1} = 0 \) and \( r_i \) and \( q_{i1} \) are known. In stage 3, banks compete for the loan commitment \( (\hat{r}_j, \hat{f}_j) \). For
any \((r_i, \hat{r}_j)\), they can correctly anticipate the second-period equilibrium outputs to be given by (2.1). Through Bertrand competition, they charge \(\hat{f}_j = -\hat{r}_j q_{j2}^*\) so that they break even. Again Bertrand competition drives them to set an interest rate that maximizes firm \(j\)'s ex-commitment fee profit, i.e., it solves the following profit maximization problem given \((q_{i1}, r_i)\):

\[
\max_{r_j} \left[ a - b(q_{i2}^* + q_{i1} + q_{j2}^*) - (1 + r_i)q_{j2}^* - \hat{f}_j \right] = \max_{r_j} \left[ a - b(q_{i2}^* + q_{i1} + q_{j2}^*) - 1\right]q_{j2}^* ,
\]

since \(\hat{f}_j = -\hat{r}_j q_{j2}^*\). Using (2.1), it is easy to show that the optimal interest rate is given by

\[
\hat{r}_j^* = \hat{r}_j(q_{i1}, r_i) = \frac{1}{4}(a - 1 - b q_{i1} + r_i).
\] (2.2)

In stage 2, looking ahead to the second-period equilibrium outputs, firm \(i\) is solving the following two-period profit maximization problem given \((q_{j1}, r_i, r_j)\):

\[
\max_{q_{i1} \geq 0} \left[ a - b(q_{i1} + q_{i2}^* + q_{j1} + q_{j2}^*) - (1 + r_i)(q_{i1} + q_{i2}^*), \quad i = 1, 2. \right. \] (2.3)

2.3.1 Subgame (reject, reject)

Since both firms rejected the offers in this subgame, \(q_{j1}^* = q_{21}^* = 0\). Solving (2.2) simultaneously for both firms, the equilibrium interest rates are \(\hat{r}_1^* = \hat{r}_2^* = -(a - 1)/5\), and the equilibrium ex-commitment fee profit for each firm is \(2(a - 1)^2/25b\).

2.3.2 Subgame (accept, reject)

Suppose that firm \(i\) accepted its offer \((r_i, f_i)\) but firm \(j\) rejected. Then \(q_{j1}^* = 0\) and firm \(j\) knows \(r_i\) and \(q_{i1}\) before it visits other banks. Since my goal is to characterize the SPNE for the full game, I have to consider the SPNE for this subgame following all possible pairs of loan commitments offered in stage 1.
The set of all possible loan commitment contracts offered in stage 1 can be partitioned into three mutually exclusive subsets according to the firms' optimal response in each subgame. The first subset is the one in which firm $i$ produces nothing in each period in equilibrium. Notice that firm $i$ can always earn $2(a - 1)^2/25b$ if it rejects the offer. Hence, this subgame cannot be on the equilibrium path. This implies that the consideration of this subset is irrelevant to the characterization of the full equilibrium.

The second subset is the one in which firm $i$ produces positive output but firm $j$ produces zero output in equilibrium. In the appendix, I show that no pair of loan commitments in this subset is subgame perfect. This result follows from the fact that the interest rate must be set low enough to induce firm $i$ to adopt a very aggressive output strategy so as to drive firm $j$ out of the industry. If the bank did so, it could not fully cover its interest loss from the commitment fee. As a result, I can restrict the attention to the third subset in which both firms produce positive outputs in equilibrium.

Since the equilibrium is interior, substituting (2.1) and (2.2) into (2.3) yields firm $i$'s two-period profit maximization problem given $r_i$. By simple calculation, the optimal first-period output of firm $i$ is given by

$$q_{i1}^* = q_{i1}(r_i) = \frac{1}{3b}(a - 1 - 3r_i).$$

Hence, firm $i$'s cum-commitment fee profit given $r_i$ is given by

$$\pi_i^f(r_i) = \frac{1}{12b}(a - 1 - 3r_i)^2,$$

and firm $j$'s ex-commitment fee profit given $r_i$ is given by

$$\pi_j^f(r_i) = \frac{1}{18b}(a - 1 + 3r_i)^2.$$

These subsets are mutually exclusive because the optimal response of each firm is unique following any pair of loan commitments offered in stage 1. To save notation, I am deliberately informal here.
2.3.3 Subgame (accept, accept)

From the analysis in the above subsection, it is no loss of generality to assume an interior equilibrium for this subgame. Substituting (2.1) into (2.3), the first-period Cournot-Nash equilibrium outputs are given by

\[ q_{i1}^* = q_{i1}(r_i, r_j) = \frac{1}{5b}(a - 1 - 3r_i + 2r_j), \quad i = 1, 2. \]  

(2.4)

Hence, the cum-commitment fee profit given \((r_i, r_j)\) for each firm in this subgame is

\[ \pi_i^*(r_i, r_j) = \frac{2}{25b}(a - 1 - 3r_i + 2r_j)^2, \quad i = 1, 2. \]

2.4 Equilibrium for the Full Game

Now, proceed to analyze the stage 1 decision. Since there are two different full games depending on whether firms have relationships with the same bank or not at the beginning of the first period, I shall analyze each of them in turn. In the next subsection, we consider full game 1 first and delay the analysis of full game 2.

2.4.1 Full Game 1: Both Firms Have Relationships with Bank 1 in Stage 1

Suppose that bank 1 offers the loan commitment \((r_1, f_1)\) to firm 1 and \((r_2, f_2)\) to firm 2 at the beginning of the first period. I summarize each firm’s ex-commitment fee profit calculated in section 2.3 by the normal form game depicted in figure 2.

(Figure 2 about here)

**Proposition 2.1.** In full game 1, any pairs of SPNE loan commitments solve: (P1)

\[ \max_{r_1, r_2, f_1 \geq 0, f_2 \geq 0} \sum_{i=1, i \neq j}^2 [r_i(q_{i1}^* + q_{i2}^*) + f_i] \]  

(2.5)
\[ \pi_i^*(r_i, r_j) - f_i - \pi_j^*(r_j) \geq 0, \quad (2.6) \]
\[ \pi_i^*(r_i) - f_i - \frac{2}{25b} (a - 1)^2 \geq 0, \quad (2.7) \]
\[ \pi_j^*(r_i, r_j) - f_j - \pi_j^*(r_i) \geq 0, \quad (2.8) \]

where \( q_{i1}^* \) and \( q_{i2}^* \) are defined by (2.4) and (2.1) respectively.

**Proof.** See the appendix. □

Conditions (2.6) and (2.7) are the necessary and sufficient conditions for accepting to be the dominant strategy of firm \( i \). Given that firm \( i \) accepts the offer, condition (2.8) is the necessary and sufficient condition for firm \( j \) to accept its offer. Hence, proposition 2.1 simply says that bank 1’s expected profit, (2.5), is maximized subject to the acceptance of the offers by both firms.

**Proposition 2.2.** There exists a unique SPNE for full game 1 which is asymmetric with firm \( i \)’s offer being worse than firm \( j \)’s offer. Firm \( i \) produces less than firm \( j \) and the industry output is 0.905187\((a - 1)/b\).

**Proof.** See the appendix. □

Given that firms \( i \) and \( j \) are identical, the fact that they are treated asymmetrically is surprising. The intuition supporting this result is as follows. Since the offers presented to the firms are publically observable, the bank can, through terms of the loan, credibly commit one of the firm to act more aggressively, thereby reducing the value of the other firm’s outside option. By giving firm \( j \) a lower rate, the bank is able to extract a larger fraction of the remaining surplus from firm \( i \) and increases total profits by doing so. The industry output is bigger than the traditional Cournot output \( 2(a - 1)/3b \) but smaller than the Pareto efficient output \((a - 1)/b\).\(^{16}\)

\(^{16}\)The Pareto efficient output is one in which the output price is equal to the marginal interest cost.
2.4.2 Full Game 2: Firm \( i \) Has a Relationship with Bank \( i \) in Stage 1

Consider full game 2 in which bank \( i \) offers the contract \((r_i, f_i)\) to firm \( i \) without observing bank \( j \)'s offer, \((r_j, f_j)\), to firm \( j \) (and vice versa) at the beginning of the first period. Bank \( i \) must offer a contract that firm \( i \) will definitely accept or otherwise it will earn zero profit. Thus, using similar argument as above, bank \( i \)'s profit maximization problem given \((r_j, f_j)\) is: (P2)

\[
\max_{r_i, f_i \geq 0} r_i(q_{i1}^* + q_{i2}^*) + f_i \quad \text{s.t.} \quad (2.6) \text{ and } (2.7),
\]

where \( q_{i1}^* \) and \( q_{i2}^* \) are defined by (2.4) and (2.1) respectively.

**Proposition 2.3.** There exists a unique SPNE for full game 2 which is symmetric. The industry output is \(0.9052(a - 1)/b\).

*Proof.* See the appendix. □

The following two propositions follow immediately from propositions 2.2 and 2.3.

**Proposition 2.4.** The equilibrium industry output in full game 1 is lower than that in full game 2.

**Proposition 2.5.** If firms are risk-neutral and there is an equal chance for them to be treated preferentially by the bank with which they both have relationships, they will strictly prefer to have relationships with the same bank.

These two results are similar to the findings of Poitevin (1989). Proposition 2.4 says that if both firms borrow from the same bank, the product market becomes more collusive. This outcome is achieved through a better coordination of the firms' production activities when the bank can partially internalize the external effect of one firm's marginal interest cost on its
rival's output. Proposition 2.5 says that the expected ex-commitment fee profits are higher when firms borrow from the same bank, despite the fact that the bank has stronger power to exploit them in this case. This is the case because the producers' surplus is larger due to a more collusive industry output and both the firms and the bank are able to benefit from sharing a bigger pie.

2.5 Banking Competition and Welfare

Consider an otherwise equivalent economy with a transactions-based banking system. Everything is the same as in the previous section except that banks compete directly in loan prices. Through competition, banks will charge $f_i = -r_i(q_i^* + q_{i2}^*)$ so that they break even, where $q_i^* + q_{i2}^*$ is the equilibrium total output of firm $i$ anticipated by banks given $(r_i, r_j)$. Again, competition drives them to set an interest rate that maximizes firm $i$’s ex-commitment fee profit, i.e., it solves the following profit maximization problem given $r_j$:

$$\max_{r_i} \pi_i^x(r_i, r_j) + r_i(q_i^* + q_{i2}^*),$$

where $q_i^*$ and $q_{i2}^*$ are defined by (2.4) and (2.1), respectively. It is straightforward to calculate the equilibrium interest rates to be $r_1^* = r_2^* = -(a - 1)/14$ and the equilibrium industry output to be $6(a - 1)/7b$. It follows immediately that the equilibrium industry output in this case is lower than that under the imperfectly competitive banking regime. Hence, the major results are summarized as follows:

**Proposition 2.6.** *The output price is lower (and thus consumers are better off) and social welfare is higher under relationship banking than under price-driven banking.*

The intuition of these seemingly inconsistent results is as follows. With perfectly competitive banks, firms have all the bargaining power and banks are passive. As a result, the main concern in the bargaining process is to maximize firms’ profits. However, under
relationship banking, both banks and firms have some bargaining power. The incumbent bank has bargaining power because it is a first mover and the firm has bargaining power because it possesses an outside option by switching to other banks. Also note that the bank can indirectly influence the value of firm’s outside option. Now suppose that the bank sets a low interest rate. If the firm rejects the offer while its rival accepts, its rival will flood the output market in the first period because its interest cost is low. Thus, setting a low interest rate reduces the firm’s bargaining strength through the reduction of the value of its outside option. Since the bank can use the commitment fee to extract the surplus above what the firm can get from its outside option, it pays for the bank to charge a low interest rate. Even though this implies a smaller producers’ surplus, as long as the portion shared by the bank is larger, the bank is willing to pick this as the equilibrium outcome. Thus, in equilibrium, interest rates are lower than those with perfectly competitive banks. This in turn causes firms to be more aggressive in production. Consequently, relationship banking is more pro-competitive.

2.6 Conclusion

In developed countries such as the United States, the United Kingdom and Japan, there is a transition from relationship banking to price-driven banking. This gives rise to some debate about the relative efficiency of these two banking systems. In this chapter, I developed a stylized model of relationship banking under product market imperfection. I find that even in the absence of any information problems, a relationship-based banking system may yield a higher social welfare than a transactions-based competitive system. Interactions between the imperfect product and capital markets may offset each market’s adverse effects on society and thus improve overall efficiency.
Appendix

Subgame (accept, reject). Consider the subgame (accept, reject) following a pair of loan commitments in the second subset in which firm $i$ becomes the monopoly producer of the industry in equilibrium. Notice first that firm $j$ chooses $q_{j2} = 0$ if and only if $q_{i1} \geq (a - 1 + r_i)/b$. The proof of this observation is as follows.

To prove the sufficiency part, suppose that firm $i$ anticipates that firm $j$ will cease production. Then, the monopoly second-period output of firm $i$ is

$$q^*_i = q_i(a, r_i) = \frac{1}{2b}(a - 1 - bq_{i1} - r_i). \quad (A2.1)$$

Using (A2.1), the output price, given that firm $j$ produces nothing is

$$a - b(q_{i1} + q^*_i) = \frac{1}{2}(a + 1 - bq_{i1} + r_i) \leq 1,$$

where the inequality follows from the restriction on $q_{i1}$. Since the output price is below firm $j$'s interest plus commitment fee (i.e., 1) even if it does not produce, firm $j$ will not produce and firm $i$'s conjecture is correct. To prove the necessity part, suppose that $q^*_i = 0$. Then, it must be the case that the output price is below firm $j$'s true interest cost. Using (A2.1), I get the restriction on $q_{i1}$.

Therefore, using (A2.1), firm $i$'s two-period profit maximization problem given $r_i$ and its monopoly position is

$$\max_{q_{i1} \geq \frac{1}{b}(a - 1 + r_i)} \frac{1}{4b}(a - 1 - bq_{i1} - r_i)(a - 1 + q_{i1} - r_i). \quad (A2.2)$$

The derivative of this objective function is $-bq_{i1}/2 < 0$ for all $q_{i1} > 0$. Hence, the optimal solution for this problem is

$$q^*_i = q_i(r_i) = \begin{cases} (a - 1 + r_i)/b & \text{if } r_i > 1 - a, \\ 0 & \text{if } r_i \leq 1 - a. \end{cases} \quad (A2.3)$$

Substituting (A2.3) into (A2.2), firm $i$’s sum-commitment fee monopoly profit given $r_i$ is

$$\pi^m_i(r_i) = \begin{cases} -r_i(a - 1)/b & \text{if } r_i > 1 - a, \\ (a - 1 - r_i)^2/4b & \text{if } r_i \leq 1 - a. \end{cases} \quad (A2.4)$$
Since firm $i$ can always earn the ex-commitment fee profit of $2(a - 1)^2/25b$ if it rejects the offer, bank 1 must allow it to retain at least this profit level. Thus, the maximum commitment fee given $r_i$ that bank 1 can charge is

$$f_i(r_i) = \pi_i^{ma}(r_i) - \frac{2}{25b}(a - 1)^2. \quad (A2.5)$$

Using (A2.1) to (A2.5), the bank’s profit as a function of $r_i$ given that firm $i$ is a monopoly is

$$\left\{ \begin{array}{ll}
-2(a - 1)^2/25b & \text{if } r_i > 1 - a, \\
-(a - 1 - r_i)(1 - a - r_i)/4b - 2(a - 1)^2/25b & \text{if } r_i \leq 1 - a,
\end{array} \right.$$

which is always negative. □

Proof of Proposition 2.1. To prove this proposition, I first prove the following lemma.

**Lemma.** In the subgame (accept, reject), the maximum profit of bank 1 is $(a-1)^2/300b$.

**Proof.** Using an argument that is similar to the proof used above, the maximum commitment fee given $r_i$ that bank 1 can charge is

$$f_i(r_i) = \pi_i^a(r_i) - \frac{2}{25b}(a - 1)^2. \quad (A2.6)$$

Then, using (2.5) to (2.8) and (A2.6), bank 1’s profit maximization problem is

$$\max_{r_i} \frac{1}{2b} r_i(a - 1 - 3r_i) + \frac{1}{12b} (a - 1 - 3r_i)^2 - \frac{2}{25b}(a - 1)^2.$$ 

The first-order condition yields $r_i^* = 0$. Thus, in this case, the maximum profit of bank 1 is $(a - 1)^2/300b$. □

Now, I am ready to prove proposition 1. If both firms reject the offers, bank 1 earns zero profit. If bank 1 sets the pair of loan commitments such that only one firm accepts the offer, then by the above lemma bank 1 can at most earn $(a - 1)^2/300b$. Now, suppose that bank 1 sets $r_1 = r_2 = 0$ and $f_1 = f_2 = (a - 1)^2/300b$. It is easy to show that this pair of
symmetric loan commitments satisfies constraints (2.6) to (2.8) and gives bank 1 the profit of \((a - 1)^2/150b\). Hence, in any SPNE for the full game, the bank must induce both firms to accept the offers. □

**Proof of Proposition 2.2.** Denote \(Q(r_i, r_j, f_i, f_j)\) and \(A_k(r_i, r_j, f_i, f_j)\) for \(k = 1, 2\) and 3 be the objective function and constraints (2.6) to (2.8) of (P1) respectively. It is easy to check that \(Q\) is strictly concave while \(A_k\) is strictly quasi-concave for all \(k\). Then, by Theorems 1 and 2 in Arrow and Enthoven (1961) (see also Takayama (1985) pp. 114–115), the Kuhn-Tucker conditions are the necessary and sufficient conditions for a global maximum of the given problem. The Kuhn-Tucker conditions are

\[
\frac{\partial}{\partial r_i} Q(r_i^*, r_j^*, f_i^*, f_j^*) + \sum_{k=1}^{3} \lambda_k^* \frac{\partial}{\partial r_i} A_k(r_i^*, r_j^*, f_i^*, f_j^*) = 0, \quad (A2.7)
\]

\[
\frac{\partial}{\partial r_j} Q(r_i^*, r_j^*, f_i^*, f_j^*) + \sum_{k=1}^{3} \lambda_k^* \frac{\partial}{\partial r_j} A_k(r_i^*, r_j^*, f_i^*, f_j^*) = 0, \quad (A2.8)
\]

\[
\lambda_1^* + \lambda_2^* + \lambda_3^* = 1, \quad \lambda_3^* = 1, \quad (A2.9)
\]

\[
A_1(r_i^*, r_j^*, f_i^*, f_j^*) = 0, \quad (A2.10)
\]

\[
A_2(r_i^*, r_j^*, f_i^*, f_j^*) = 0, \quad (A2.11)
\]

\[
A_3(r_i^*, r_j^*, f_i^*, f_j^*) = 0, \quad (A2.12)
\]

\[
f_i^* > 0, \quad f_j^* > 0, \quad \lambda_k^* > 0 \text{ for all } k, \quad (A2.13)
\]

where \(\lambda_1, \lambda_2\) and \(\lambda_3\) are the Lagrangean multipliers and \((r_i^*, r_j^*, f_i^*, f_j^*, \lambda_1^*, \lambda_2^*, \lambda_3^*)\) is a solution to the Kuhn-Tucker conditions.

Subtracting (A2.11) from (A2.10), I get

\[
27r_i^{*2} - [18(a - 1) - 864r_j^*]r_i^* - [19(a - 1)^2 - 12(a - 1)r_j^* - 162r_j^{*2}] = 0. \quad (A2.14)
\]

Note that I can write

\[
r_j^* = r_i^* + k(a - 1) \quad (A2.15)
\]

for some constant \(k\). Since \(k\) is allowed to depend on \(a\), this representation of \(r_j^*\) is always possible. In fact, I shall show later that \(k\) is indeed a constant independent of \(a\). Substituting
(A2.15) into (A2.14), I can solve the quadratic equation (A2.14) to get the two real roots for $r_i^*, r_{ib}^*$ and $r_{is}^*$, as functions of $k$:

$$r_{ib}^* = -\frac{1188k - 6 - \sqrt{80064 - 64800k + 729000k^2}}{2106}(a - 1), \quad (A2.16)$$

$$r_{is}^* = -\frac{1188k - 6 + \sqrt{80064 - 64800k + 729000k^2}}{2106}(a - 1). \quad (A2.17)$$

Substituting (A2.9) into (A2.7), I can solve $\lambda_1^*$ as a function of $r_i^*$ and $r_j^*$. Similarly, substituting (A2.9) into (A2.8) yields $\lambda_i^*$ as another function of $r_i^*$ and $r_j^*$. These two functions must be the same and thus I get

$$2808r_i^*^2 + [243(a - 1) + 891r_j^*]r_i^* + [7(a - 1)^2 - 57(a - 1)r_j^* - 2592r_j^*^2] = 0. \quad (A2.18)$$

Substituting (A2.15) into (A2.18), I can solve the quadratic equation (A2.18) to get the two real roots for $r_i^*$ as functions of $k$:

$$r_{ib}^* = -\frac{186 - 4293k - \sqrt{3600 - 1344600k + 29907225k^2}}{2214}(a - 1), \quad (A2.19)$$

$$r_{is}^* = -\frac{186 - 4293k + \sqrt{3600 - 1344600k + 29907225k^2}}{2214}(a - 1). \quad (A2.20)$$

Since (A2.16) and (A2.19) must be the same, using numerical methods, I get $k = 0.0583$ which is independent of $a$. Substituting the corresponding value of $r_i^*$ into (A2.11) yields $f_i^* = -0.04(a - 1)^2/b$, which violates (A2.13). Hence, I know that the solution must be the one that solves (A2.17) and (A2.20). Using numerical methods, I get $k = -0.003$ which is independent of $a$. The uniqueness of the global maximum solution follows from the uniqueness of the solution for the Kuhn-Tucker conditions. The SPNE outcome for full game 1 is

$$r_i^* = -0.129983928(a - 1), \quad r_j^* = -0.132984454(a - 1),$$

$$f_i^* = 0.080997163(a - 1)^2/b, \quad f_j^* = 0.083107548(a - 1)^2/b, \quad q_{i1}^* = q_{i2}^* = 0.224796575(a - 1)/b, q_{j1}^* = q_{j2}^* = 0.227797101(a - 1)/b,$$

$$\pi_i^* = 0.020069836(a - 1)^2/b, \quad \pi_j^* = 0.02067549(a - 1)^2/b. \quad \Box$$
Proof of Proposition 2.3. By analogous arguments as used in the proof of proposition
2.2, the global maximum solution for (P2) given \((r_i^*, f_i^*)\) is one at which constraints (2.6) and
(2.7) are binding, i.e., it satisfies (A2.14). In equilibrium, (A2.14) must hold simultaneously
for \(i = 1\) and \(2\) at \((r_1^*, r_2^*)\). Subtracting (A2.14) for \(i = 2\) by (A2.14) for \(i = 1\) yields
\((r_1^* - r_2^*) [9(r_1^* + r_2^*) + 2(a - 1)] = 0. I have either \(r_1^* = r_2^*\) or
\[
 r_1^* + r_2^* = -\frac{2}{9}\,(a - 1). \hspace{1cm} (A2.21)
\]
Suppose that (A2.21) holds. Substituting (A2.21) into (A2.14) yields
\[
2025r_i^* + 450(a - 1)r_i^* + 41(a - 1)^2 = 0. \hspace{1cm} (A2.22)
\]
However, there are no real roots for (A2.22) because
\[
450^2(a - 1)^2 - 4(2025)(41)(a - 1)^2 = -[360(a - 1)]^2.
\]
Since the relevant domain of the objective function is compact, by the Weierstrass Theorem
(see Takayama (1985) p. 33), there must exist a real maximum solution. By the Arrow and
Enthoven Theorem, the maximum solution must satisfy the Kuhn-Tucker conditions. Hence,
(A2.21) cannot hold.

Since \(r_1^* = r_2^*\), I have \(k = 0\) from (A2.15). Thus, by (A2.16) and (A2.17), I get \(r_{ib}^* = 0.137205995(a - 1)\) and \(r_{is}^* = -0.13150799(a - 1)\). Using similar arguments as in the proof
of proposition 2.2, \(r_{ib}^*\) is not the solution. The uniqueness of the equilibrium follows from
the uniqueness of the solution for the Kuhn-Tucker conditions. The SPNE outcome for full
game 2 is
\[
 r_i^* = -0.13150799(a - 1), \quad f_i^* = 0.082058091(a - 1)^2/b,
\]
\[
 q_{i1}^* = q_{i2}^* = 0.226301598(a - 1)/b, \quad \pi_i^* = 0.020366734(a - 1)^2/b,
\]
for \(i = 1, 2. \) □
References


Figure 1. Extensive Form of Production-Search-Production Subgames
Figure 2. **The Normal Form of Production-Search-Production Subgames.**

The ex-commitment fee profits of firm 1 are in the upper left corners and those of firm 2 are in the lower right corners.

<table>
<thead>
<tr>
<th>firm 1</th>
<th>firm 2</th>
<th>accept</th>
<th>reject</th>
</tr>
</thead>
<tbody>
<tr>
<td>accept</td>
<td></td>
<td>$\pi_1^i(r_1, r_2) - f_1$</td>
<td>$\pi_1^i(r_1) - f_1$</td>
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<tr>
<td></td>
<td></td>
<td>$\pi_2^i(r_1, r_2) - f_2$</td>
<td>$\pi_2^i(r_1)$</td>
</tr>
<tr>
<td>reject</td>
<td>$\pi_1^i(r_2)$</td>
<td>2(a - 1)$^2$/25b</td>
<td>2(a - 1)$^2$/25b</td>
</tr>
<tr>
<td></td>
<td>$\pi_2^i(r_2) - f_2$</td>
<td>2(a - 1)$^2$/25b</td>
<td>2(a - 1)$^2$/25b</td>
</tr>
</tbody>
</table>
Chapter 3

Debt, Asset Substitution, and

Further Borrowing

3.1 Introduction

The previous chapter demonstrated that relationship banking may be more efficient than price banking when product markets are imperfect. The setting considered was one of symmetric information. This chapter goes on to examine this issue under moral hazard. Moral hazard arises when borrowers, after receiving their loans, take ex post unobservable actions that jeopardizes their loan repayments.1

When a borrower is allowed to engage in multilateral credit transactions with different banks, there exist negative externalities that are notably ignored by the contracting literature. These negative externalities stem from the public good nature of the borrower’s unobservable action: the effect of varying the loan amount provided by one bank affects the borrower’s action choice, and hence the profits of the other banks which transacted with him. Bizer and DeMarzo (1992) are the first to incorporate such externalities into the design of optimal loan contracts. They consider the effort-incentive problem under a two-period consumption smoothing model. The borrower borrows in the first period and repays his debt

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1See, for example, Jensen and Meckling (1976) and Myers (1977).
from his random second period income whose distribution is affected by his unobservable work effort. Unlike the traditional approach, the borrower is allowed to borrow sequentially from more than one bank and debt is fully prioritized so that prior loans retain seniority over new ones. However, future loan requests by the borrower are unobservable to those banks which have already transacted with the borrower. If the borrower is heavily leveraged so he risks bankruptcy in the bad state, increases in indebtedness will decrease his work effort which in turn lowers the probability of repayment of earlier loans. Since new banks do not pay for the negative externalities that they impose on prior lenders, their loan terms will not reflect the resulting devaluation of existing debt. This contrasts with a one-bank environment in which all effects on prior credit are internalized by the sole creditor. Bizer and DeMarzo show that the borrower is worse off than he would be if he could credibly commit to refraining from further borrowing.

While Bizer and DeMarzo's model is appropriate for the study of consumer loans, the effort-incentive problem may be less important for corporate loans than the risk-shifting incentive. Jensen and Meckling (1976) argue that the agency cost of corporate debt results mainly from the risk-incentive problem (hereafter referred as the asset substitution problem), which occurs when the managers of a firm can choose among projects of varying riskiness. As pointed out by Black and Scholes (1973), stockholders of the levered firm can be viewed as holders of a European call option on the firm with an exercise price equal to the face value of the debt. The value of this call option can be increased if stockholders can induce managers to shift into high risk projects in their investment policies. This type of incentive problem is the focus of this chapter.

My model is similar in spirit of Bizer and DeMarzo's in that I retain the assumptions of

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2 The possible impact of unobservable further borrowing under the asset substitution problem is also mentioned by Bizer and DeMarzo, although it is not possible to convert their model to address this issue. In contrast, my model can be easily modified to discuss the effort incentive problem. See section 3.5.

3 Other formal models discussing the asset substitution problem include Gavish and Kalay (1983), Green (1984), and Green and Talmor (1986).
full prioritization of debt and unobservability of future borrowing. The main issue of this chapter, however, is the asset substitution problem (vis-à-vis the effort incentive problem in Bizer and DeMarzo): the borrower can choose among projects of varying riskiness and his project choice is ex post unobservable to banks. I assume that under relationship banking the borrower can credibly commit not to borrow further and I show that underinvestment arises when the asset substitution problem is present. This generates a second-best outcome. I also assume that borrowers cannot credibly commit to refrain from future borrowing under price banking and I show that more underinvestment is needed to overcome the given dual incentive problem. This gives rise to a third-best outcome.

The incentive problem induced by further borrowing can in principle be eliminated by debt covenants restricting future debt issues. However, such covenants are not widely used in practice (see Asquith and Wizman (1990)). One reason for this may be the underinvestment problem identified by Myers (1977). If new investment of a firm can only be financed by new equity issues or by reduced dividends, then with risky debt outstanding part of the gains from the investment goes to debtholders, rather than shareholders. Covenants prohibiting future borrowing would induce perverse investment incentives so that not all positive net present value projects would be undertaken. Another way to overcome this incentive problem is through the use of contingent claims such as callable debts or warrants (see Green (1984)). Bizer and DeMarzo (1992) argue that, as a firm's investment opportunity set might evolve over time, pricing a fully state-contingent claim is extremely difficult.

The remainder of this chapter is organized as follows. Section 3.2 describes the basic model and the first-best equilibrium. Section 3.3 characterizes the equilibrium under relationship banking, along with a comparative statics analysis. Section 3.4 develops the equilibrium under price banking and also obtains some comparative statics results. Section 3.5 discusses the robustness of the analysis and suggests possible extensions. Section 3.6

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4See Bizer and DeMarzo for arguments and empirical evidence that justify these two assumptions.
5See also the discussions by Smith and Warner (1979) and Bizer and DeMarzo (1992).
concludes. All proofs are given in the appendix.

3.2 Model and the First-Best Solution

3.2.1 The Model

Consider a risk-neutral entrepreneur (hereafter called the borrower) who has monopoly access to two mutually exclusive, nontransferable, divisible investment projects, indexed by \( \theta \in \{L, H\} \). The borrower has no initial wealth and must solicit one or more loans from a risk-neutral competitive banking sector in order to fund either project. The banking sector consists of at least two banks. Deposit insurance is (de facto) complete and thus each bank’s deposit funding cost is the gross riskless interest rate \( i \geq 1 \).

In this subsection I adopt a one-shot perfect information game with two stages: the banking stage and the investment stage. In the banking stage, the borrower may request loans repeatedly, each time going to a new bank. In other words, there may be multiple rounds in this stage. At each round, the borrower visits a new bank and offers a loan contract. The bank either accepts or rejects the loan request based on the borrower’s current credit history.\(^6\) The borrower’s current credit history is a sequence of outcomes (loan requests and acceptance of banks) of each round of banking prior to the visit of the bank.\(^7\) The bank, however, cannot observe future borrowings by the borrower. The loan contract, \((r, I)\), is a pair that specifies a loan amount, \(I \geq 0\), provided by the bank if it accepts the contract, and a repayment amount, \(r \geq I\), that the borrower must repay to the bank after the project’s cash flow is realized. The project’s cash flow is ex post observable and verifiable. Debt is fully prioritized so that each loan is repaid only after all earlier commitments are satisfied. This assumption, together with the observability of the borrower’s previous borrowings, ensures

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\(^6\)The results are robust to alternative institutional arrangements, since there are no search or time costs for the borrower and banks are competitive.

\(^7\)If banks cannot observe the borrower’s current credit history, there will be a market failure similar to the one described in Akerlof (1970). Banks always infer that the borrower will only undertake the high-risk project.
that loan contract terms (based solely on the project’s realized cash flow) are enforceable. The borrower stays in the banking stage until he is satisfied with his loan portfolio, then he enters into the investment stage by choosing his optimal project given his loan portfolio. His project choice is \textit{ex post} unobservable to banks.

In the investment stage, the borrower may invest any $I$ dollars in either project but not both.\footnote{Since the borrower is an expected terminal payoffs maximizer, he has no incentive to invest less than what he has borrowed in the banking stage.} In return, project $H$ yields a random terminal cash flow of $R_H(I)$ with probability $p_H$, or zero with probability $1 - p_H$.\footnote{The assumption of zero cash flow in the bad state is for the sake of simplicity. Relaxing this assumption will not affect the results qualitatively.} Project $H$ is referred to as the high-risk project. Project $L$ yields a random terminal cash flow of $R_L(I) = kR_H(I)$ with probability $p_L$, or zero with probability $1 - p_L$, where $0 < p_H < p_L \leq 1$ and $p_H/p_L < k$.\footnote{The assumption of $R_L(I)$ being proportional to $R_H(I)$ gives nice graphical illustrations of the results. In fact, the following less restrictive assumptions are necessary: (i) $p_L R_L(I) > p_H R_H(I)$, (ii) $R_L(I) < R_H(I)$, (iii) $R'_L(I) < R'_H(I)$, for all $I$ larger than a sufficiently small number.} Project $L$ is referred to as the low-risk project. The restriction on the scaling factor, $k$, is a sufficient condition for the low-risk project to be first-best, since it ensures that the low-risk project always generates a higher expected value than the high-risk project does.\footnote{See proposition 3.1.} The return function $R_H : \mathcal{R}_+ \rightarrow \mathcal{R}_+$ is strictly increasing and concave with $R_H(0) = 0$, $\lim_{I \to \infty} R_H(I) < \infty$ and $\lim_{I \to 0^+} R'_H(I) = \infty$.

Suppose that the borrower obtains a loan portfolio, $\{(r_t, I_t)\}_{t=1}^n$, where $t$ denotes the priority of the loan contract, and undertakes project $\theta$. Then his expected terminal payoff is

$$p_\theta \left[ R_\theta \left( \sum_{t=1}^n I_t \right) - \sum_{t=1}^n r_t \right].$$

For the loan contract $(r_\tau, I_\tau)$, the borrower’s total debt commitment prior to it is $\sum_{t=1}^{\tau-1} r_t$. Since debt is fully prioritized, a bank with this contract receives

$$p_\theta \min \left[ r_\tau, R_\theta \left( \sum_{t=1}^n I_t \right) - \sum_{t=1}^{\tau-1} r_t \right] - i I_\tau.$$

The equilibrium concept employed is Selten's (1975) subgame perfect Nash equilibrium.
A strategy for the borrower is a combination of the loan portfolio choice and the project choice. A strategy for a bank is a set of loan proposals that the bank is willing to accept from the borrower. A subgame perfect Nash equilibrium (hereafter called equilibrium) requires that the equilibrium strategy of each player maximizes his expected terminal payoff given the strategies of the other players, contingent upon all possible current credit histories of the borrower. An allocation, \([(r, I), \theta]\), is a pair consisting of an initial loan contract, \((r, I)\), and a project, \(\theta\), chosen by the borrower. Any equilibrium can then be specified by an allocation.

3.2.2 The First-Best Solution

In the first-best case, the borrower’s choice of project is \textit{ex post} observable to banks and is contractible. The borrower chooses the project that yields the highest expected value net of the investment compounded at the riskless rate \(i\). That is, he solves

\[
\max_{\theta \in \{L, H\}, I \geq 0} p_\theta R_\theta(I) - iI.
\]

Denote \(I_L\) and \(I_H\) be the respective solutions of the following first-order conditions:

\[
p_L k R_H'(I_L) = i, \tag{3.1}
\]

and

\[
p_H R_H'(I_H) = i. \tag{3.2}
\]

By the assumptions on \(R_H\), \(I_L\) and \(I_H\) are the unique interior maximum solutions. Clearly, the low-risk project is optimal when

\[
p_L k R_H(I_L) - iI_L > p_H R_H(I_H) - iI_H. \tag{3.3}
\]

It is easy to show that (3.3) holds if \(k > p_H/p_L\). Thus, the following proposition is immediate (all proofs are given in the appendix).

\textbf{Proposition 3.1.} \textit{The first-best equilibrium is the allocation \([(i^{FB}/p_L, I^{FB}), L]\), where \(I^{FB}\) solves (3.1).}
3.3 Asset Substitution under Relationship Banking:

The Second-Best Equilibrium

Under relationship banking, the borrower’s choice of project is ex post unobservable to banks, but the borrower can credibly commit to borrow from one bank only. The credibility of the commitment may arise from the fact that there are high costs to prevent the borrower from switching banks. The asset substitution problem is said to be present if the borrower chooses the high-risk project when he receives the first-best loan contract \((iF^B/p_L, I^F)\).

The condition for this to be true is

\[
p_H \left[ R_H(I^F) - \frac{iF^B}{p_L} \right] > p_L \left[ kR_H(I^F) - \frac{iF^B}{p_L} \right],
\]

which is equivalent to

\[
k < \frac{p_H}{p_L} + \left( 1 - \frac{p_H}{p_L} \right) \frac{iF^B}{p_L R_H(I^F)}.
\]

Since \(p_L kR_H(I^F) > iF^B\), it follows immediately that (3.4) implies \(k < 1\). Throughout the paper, condition (3.4) is assumed.

The second-best equilibrium is obtained by solving the following program: (P1)

\[
\max_{I \geq 0, \ r \geq I} p_\theta^* [R_\theta(I) - r]
\]

s.t. \(p_\theta^* r - iI \geq 0\),

\[
\theta^* \in \arg \max_{\theta \in \{L, H\}} p_\theta [R_\theta(I) - r].
\]

In words, (P1) says that the borrower’s expected terminal payoff, (3.5), is maximized subject to the bank at least breaking even, (3.6), and the borrower choosing an incentive-compatible project \(\theta^*\), (3.7).

Proposition 3.2. The second-best equilibrium (under asset substitution and relationship banking) is the allocation \([(iI^{SB}/p_L, I^{SB}), L]\), where \(I^{SB}\) solves

\[
(p_L k - p_H) R_H(I^{SB}) = \left( 1 - \frac{p_H}{p_L} \right) iI^{SB},
\]

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if the following condition holds:

\[ p_L k R_H(I^{SB}) - i^{SB} \geq p_H R_H(I_H) - i_H, \]

(3.9)

where \( I_H \) solves (3.2). Otherwise, the second-best equilibrium is the allocation \([(i_I^H/p_H, I^H), H]\).

Condition (3.9) states that the borrower’s expected payoff when he receives the loan contract \((i_I^{SB}/p_L, I^{SB})\) (and he chooses the low-risk project) is no less than that when he receives the loan contract \((i_I^H/p_H, I^H)\) (and he chooses the high-risk project). It can be interpreted as requiring the scaling factor \( k \) to be large enough.\(^{12}\) Since it is not an interesting issue if the low-risk project is not the second-best project, hereafter condition (3.9) is assumed.

(Figure 1 about here)

The second-best equilibrium is illustrated in figure 1. The borrower’s expected payoff increases to the northwest, and the bank’s expected profit increases to the southeast. Denote \( U_\theta \) and \( \Pi_\theta \) respectively as the borrower’s indifference curve (iso-expected-payoff curve) and the bank’s break-even line given that the borrower undertakes project \( \theta \in \{L, H\} \). It is easy to show that the borrower’s indifference curves are strictly increasing and convex. Following from the fact that \( k < 1 \), \( U_L \) (having slope \( 1/kR'_H(I) \)) is steeper than \( U_H \) (having slope \( 1/R'_H(I) \)). The I.C. curve depicts the locus of the loan contracts at which the incentive compatibility constraint, (3.7), binds, that is,

\[ p_L[kR_H(I) - r] = p_H[R_H(I) - r]. \]

It is easy to see that the I.C. curve is strictly increasing and convex with slope

\[ \frac{dI}{dr} \bigg|_{\text{I.C. curve}} = \frac{p_L - p_H}{(p_L k - p_H)R'_H(I)} > \frac{1}{kR'_H(I)}, \]

(3.10)

\(^{12}\)It is easy to show that condition (3.9) implies \( k > p_H/p_L \).
which is steeper than the borrower’s indifference curves. Loan contracts on or above the I.C. curve induce the borrower to choose the low-risk project. However, for these contracts to be admissible, the bank must be willing to participate. Hence, they must be on or below \( \Pi_L \). Thus, the shaded region is the set of feasible loan contracts which induce the borrower to choose the low-risk project.\(^ {13} \) Contract \( F = (iI^{FB}/p_L, I^{FB}) \) is the first-best contract which, by condition (3.4) (the presence of the asset substitution problem), lies below the I.C. curve. Thus, this contract cannot be implemented. Contract \( E = (iI^{SB}/p_L, I^{SB}) \) maximizes the borrower’s expected payoff in the shaded region. On the other hand, the borrower’s maximum expected payoff given that he chooses the high-risk project and the bank breaks even is corresponded by \( U_H \) passing through contract \( G = (iI_H/p_H, I_H) \). Since this indifference curve lies below contract \( E \) (this follows from condition (3.9) which guarantees the optimality of the low-risk project), inducing the borrower to undertake the low-risk project by contract \( E \) is indeed optimal.

**Proposition 3.3.** The second-best loan amount and repayment amount are less than their respective first-best levels.

In the second-best equilibrium, the borrower can be motivated to choose his first-best project by undercutting the loan amount. The reason why this can resolve the asset substitution problem is as follows. The differential between the cash flows of the high-risk project and the low-risk project in the successful state is \((1-k)R_H(I)\), which is strictly increasing in the investment amount, \( I \). By reducing the loan amount, the high-risk project becomes less superior in the successful state and, at the same time, the probability of success is lower if the borrower undertakes this project. Hence, provided that the loan amount is small enough, the low-risk project becomes the optimal project for the borrower.

\(^{13}\)Note that since the I.C. curve has a slope of zero as \( I \) approaches \( 0^+ \), the shaded region is always non-empty.
Proposition 3.4. The second-best loan amount and repayment amount are increasing with (i) an increase in the scaling factor, k, (ii) a decrease in the gross riskless interest rate, \(i\), (iii) an increase in the probability of success of the low-risk project, \(p_L\), or (iv) a decrease in the probability of success of the high-risk project, \(p_H\).

This proposition provides some comparative statics properties of the second-best equilibrium. The intuition underlying (i) is that an increase in the scaling factor increases the borrower’s net payoff from the low-risk project in the successful state for a given repayment amount. Thus, the bank can raise this amount at least a little bit without causing the borrower to switch to the high-risk project. By the same token, the loan size can be increased. To see why (ii) obtains, note that a decrease in the riskless rate lowers the bank’s cost of funds. The bank can now demand a smaller repayment amount while still breaking even. This in turn reduces the borrower’s incentive to substitute projects. As the borrower is suffering from underinvestment, it is efficient for the bank to increase the loan size at the same time. The intuition behind (iii) is similar to that of (i). An increase in the probability of success of the low-risk project raises the net expected payoff from the low-risk project. Hence, both the loan amount and the repayment amount can be increased. Finally, (iv) obtains because a decrease in the probability of success of the high-risk project dampens the asset substitution problem and hence less underinvestment is needed. Larger loan size implies a larger repayment obligation.

3.4 Asset Substitution under Price Banking: The Third-Best Equilibrium

I assume that price banking allows the borrower to obtain additional loans sequentially, each time from a new bank, after an initial loan contract has been signed.

3.4.1 Incentives for Further Borrowing
Suppose that the borrower has received the second-best loan contract as his initial loan. Figure 2 illustrates why the borrower may prefer to find a new bank from whom to seek an additional loan. Given the additional loan, the borrower switches to the high-risk project. Nonetheless, the borrower can offer an additional loan contract that gives the bank a nonnegative profit even if the bank assumes that the borrower will undertake the high-risk project and the borrower finds this additional loan desirable.

(Figure 2 about here)

Figure 2 is drawn such that the second-best loan size, $I^{SB}$, is smaller than the optimal investment amount, $I_H$, given that the borrower undertakes the high-risk project.\footnote{If this condition does not hold, there will be no incentive for the borrower to borrow further. See proposition 3.6.} If the borrower does not borrow further, his expected payoff is corresponded by his indifference curves passing through the second-best loan contract, $E$, regardless of whether he chooses the low-risk or high-risk project. Suppose now that the borrower borrows further by offering an additional loan contract (say contract $H$) in the lower shaded region. Obviously, this additional loan will be acceptable by any new banks irrespective of their beliefs about the borrower's project choice. The borrower's loan portfolio (the combined loan contract, say contract $E + H$) will end up in the upper shaded region.\footnote{These two shaded regions are simply one-to-one transformations of each other by shifting point $E$ to the origin. It is easy to show that $U_H'$, which passes through contract $E$, has a slope less than $p_H/i$ at point $E$, which is the slope of $\Pi_H$ (as well as $\Pi_H'$). This follows from the fact that $I^{SB} < I_H$ and from (3.2). Hence, the shaded regions are non-empty.} The borrower will be better off to offer this additional contract and switch to the high-risk project. Given that the borrower chooses the high-risk project, the initial bank which supplies contract $E$ will then sustain a loss. Thus, the second-best outcome is not attainable in this case if the borrower can engage in unobservable future borrowing.

The intuition behind the above observation is that the additional loan imposes a negative externality to the initial loan. Given that $I^{SB} < I_H$, the borrower will have an incentive to
switch to the high-risk project as long as he receives more funds. The asset substitution then reduces the probability of repayment of the initial loan and causes a loss to the initial bank which supplies it. It is this externality that hinders the implementability of the second-best outcome.

4.2 The Third-Best Solution

Banks, however, are not naive. They will take the borrower’s vulnerable behaviour into considerations. As a result, only those initial loan requests under which the borrower can credibly promise to refrain from future borrowing are acceptable to banks. Thus, the following proposition follows.

**Proposition 3.5.** When the borrower can engage in unobservable future borrowing, the equilibrium is characterized by solving the following program: (P2)

\[
\max_{I \geq 0, \, r \geq I} \ p_{\theta^*} [R_{\theta^*}(I) - r] \tag{3.11}
\]

s.t. \[ p_{\theta^*} r - i I \geq 0, \tag{3.12} \]

\[ \theta^* \in \arg \max_{\theta \in \{L, H\}} p_{\theta} [R_{\theta}(I) - r], \tag{3.13} \]

\[ [(0, 0), \, \theta^*] \in \arg \max_{(\Delta r, \, \Delta I) \in T, \, \theta \in \{L, H\}} p_{\theta} [R_{\theta}(I + \Delta I) - (r + \Delta r)], \tag{3.14} \]

(\text{where } T = \{(\Delta r, \, \Delta I) \in \mathbb{R}_+^2 : p_{H} \Delta r - i \Delta I \geq 0\}), \text{ there is no further borrowing in equilibrium.} \]

(P2) looks similar to (P1), the program solving the second-best outcome, except the imposition of constraint (3.14). Loosely speaking, (3.14) is the no further borrowing constraint which guarantees that the borrower will not offer any additional loans. In words, it says that there is no additional loan request from the borrower that is desirable to him and, at the same time, acceptable to new banks no matter what the borrower’s project choice. Clearly,
(3.14) provides a necessary condition for no further borrowing. However, it may not be clear that (3.14) is also sufficient to rule out all self-enforcing additional loans since it only considers those additional contracts which make nonnegative profits for new banks irrespective of the borrower's project choice. Given the structure of the problem, (3.14) happens to be sufficient as well. The reason is as follows: Note that the second-best project is the low-risk project. If the borrower finds it desirable to make an additional loan, it must be the case that he wants to switch to the high-risk project given his new loan portfolio. New banks anticipate the borrower's motive and price the additional loan request accordingly as if the borrower chooses the high-risk project. Thus, the solution of (P2) has the property that the borrower has no incentive to make any self-enforcing future borrowing.

**Proposition 3.6.** The second-best loan contract \((iI_{SB}^{SB}/p_L, I_{SB})\) is the optimal initial loan contract when the borrower can engage in unobservable future borrowing if, and only if, \(I_{SB} \geq I_H\).

This proposition says that the second-best outcome is implementable when \(I_{SB} \geq I_H\), even though the borrower is free to make unobservable additional loans. The underlying intuition is that for any investment level greater than \(I_H\), the borrower will be worse off if he increases the investment amount and switches to the high-risk project. Hence, given \(I_{SB} \geq I_H\), receiving \(I_{SB}\) as the initial loan ensures that the borrower will have no incentive to borrow further.

**Proposition 3.7.** \(I_{SB} < I_H\) if, and only if,

\[
k < \frac{p_H}{p_L} + \left(1 - \frac{p_H}{p_L}\right) \frac{p_H \lambda}{p_L(1 + \lambda)},
\]

where \(\lambda\) is the Lagrange multiplier defined in the appendix as (A3.1).

This proposition says that the second-best outcome is not attainable when the scaling
factor, $k$, is sufficiently small so that a substantial underinvestment is needed to resolve the incentive problem in the second-best case. In order to study the third-best outcome when the second-best outcome is not feasible, condition (3.15) is assumed from now on.

**Proposition 3.8.** If there exists an $I \in [0, I_H)$ satisfying

\[
p_L k R_H(I) - \left(2 - \frac{p_H}{p_L}\right)iI \geq p_H R_H(I_H) - iI_H, \tag{3.16}
\]

then the third-best equilibrium (under asset substitution and further borrowing) is the allocation $[(iI_T/p_L, I_T), L]$, where $I_T$ solves (3.16) with equality. Otherwise, the third-best equilibrium is the allocation $[(iI_H/p_H, I_H), H]$.

(Figure 3 about here)

Graphically, the third-best outcome is illustrated in figure 3. The NFB curve depicts the locus of the loan contracts at which the no further borrowing constraint (3.14) binds. Suppose that $\theta^* = L$. Take $\theta = H$ for the right hand side of (3.14). Then, the optimal additional loan $(\Delta r, \Delta I) \in T$ that maximizes it given an initial loan $(r, I)$ can be computed. It is easy to show that the optimal additional loan is $(0, 0)$ when $I \geq I_H$ and is $(i(I_H - I)/p_H, I_H - I)$ when $I < I_H$. Hence, for $I \geq I_H$, the NFB curve coincides with the I.C. curve, and for $I < I_H$, the NFB curve is given by

\[
p_L [k R_H(I) - r] = p_H [R_H(I_H) - r] - i(I_H - I). \tag{3.17}
\]

In words, (3.17) says that at any point on this portion of the NFB curve (say at contract $E'$), the borrower’s expected payoff given that he undertakes the low-risk project with this contract (say corresponded by $U_L^*$) is the same as the one corresponded by the indifference curve assuming high-risk project (say $U_H^*$) that is tangent at $I_H$ to the half-line (say $\Pi_H^*$) extended from the point parallel to $\Pi_H$ (i.e. $U_L^*$ and $U_H^*$ cross at the I.C. curve). It is easy to see that this portion of the NFB curve is strictly increasing and convex with slope

\[
\frac{dI}{dr}_{\text{NFB curve}} = \frac{p_L - p_H}{p_L k R_H'(I) - i}, \tag{3.18}
\]

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which is flatter than the I.C. curve, since \( R_H'(I) > i/p_H \) for \( I < I_H \). From (3.17), this portion of the NFB curve lies above the I.C. curve. Hence, any loan contracts on or above the NFB curve must not give the borrower the incentive to borrow further and switch to the high-risk project. For these contracts to be admissible they must be on or below \( \Pi_L \) so the bank is willing to participate. Condition (3.16) simply says that the NFB curve intersects \( \Pi_L \) at some point with \( I < I_H \). This implies that the shaded region is non-empty and gives the set of feasible initial loans. Contract \( E' = (i'I^T_B/p_L, I^T_B) \) maximizes the borrower’s expected payoff in the shaded region. As \( I^T_B \) solves (3.16) with equality, the borrower obtains an expected payoff which is bigger than \( p_H R_H(I_H) - iI_H \), the borrower’s maximum expected payoff given that he chooses the high-risk project and banks price the initial loan contract correctly. Hence, the low-risk project is indeed optimal given condition (3.16). However, if condition (3.16) does not hold, the shaded region is empty and thus there is no feasible initial loan that can induce the borrower to choose the low-risk project. In this case, undertaking the high-risk project is the only credible outcome.

**Proposition 3.9.** The third-best loan amount and repayment amount are less than their respective second-best levels.

In the third-best equilibrium, the borrower is induced to undertake the low-risk project by a further cut in the initial loan size. The intuition underlying this result is that the freedom of seeking additional loans by the borrower makes the high-risk project even more attractive. As a result, to resolve the given dual incentive problem, a larger underinvestment is needed to ensure the low-risk project to be optimal.
Proposition 3.10. The third-best loan amount and repayment amount are increasing with (i) an increase in the scaling factor, $k$, (ii) a decrease in the gross riskless interest rate, $i$, (iii) an increase in the probability of success of the low-risk project, $p_L$, or (iv) a decrease in the probability of success of the high-risk project, $p_H$.

The comparative static properties of the third-best equilibrium are the same as those of the second-best equilibrium. The intuition described before applies here.

3.5 Discussion and Extensions

3.5.1 Renegotiation

In the third-best equilibrium, there is an additional efficiency loss. An immediate question arises as whether the reported equilibrium is robust to renegotiation between the borrower and the initial bank. Suppose that the borrower is allowed to seek new loans from the initial bank after he started his project but before the project's cash flow is realized (the project once started is irreversible). The bank can either accept or reject the new loan. My claim is that the third-best loan contract is also an optimal initial loan under renegotiation. That is, given the third-best contract, the borrower optimally chooses the low-risk project. In the renegotiation stage, the borrower requests the additional loan that yields a combined loan contract equal to the second-best one. The reason why renegotiation can improve efficiency is as follows. If the borrower offers a new loan that is below the I.C. curve, the bank will price the loan as if the borrower has chosen the high-risk project. From figure 3, the maximum expected payoff that the borrower can get is corresponded by $U_H$ if the high-risk project is chosen. This is definitely less than his second-best expected payoff. The borrower anticipates this outcome and thus optimally undertakes the low-risk project given the third-best contract as his initial loan.\textsuperscript{16}

\textsuperscript{16}See Wong (1992) for a similar observation when debt contracts include collateral, and Matthews (1991)
3.5.2 Effort-Incentive Problem

A trivial extension of the model is to consider the effort incentive problem. Suppose that the borrower has monopoly access to a nontransferable, divisible project that requires both capital and the borrower’s work effort to operate. The borrower can choose a work effort \( e \in \{\underline{e}, \bar{e}\} \), where \( 0 < \underline{e} < \bar{e} \leq 1 \). For an investment amount \( I \) and a work effort \( e \), the project yields a random terminal cash flow of \( R_e(I) \) with probability \( e \), or zero with probability \( 1 - e \).\(^\text{17}\) The borrower’s disutility of effort in monetary terms is \( V(e) \), with \( \infty > V(\bar{e}) > V(\underline{e}) \geq 0 \). The return function \( R_e : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) is strictly increasing and concave, with \( \lim_{I \to \infty} R_e(I) < \infty \) and \( \lim_{I \to 0^+} R'_e(I) = \infty \), for all \( e \). Moreover, the return functions satisfy (i) \( R_{\bar{e}}(I) > R_{\underline{e}}(I) \), (ii) \( \bar{e}R_{\bar{e}}(0) - V(\bar{e}) > \underline{e}R_{\underline{e}}(0) - V(\underline{e}) \geq 0 \), and (iii) \( \bar{e}R'_{\bar{e}}(I) < \underline{e}R'_{\underline{e}}(I) \), for all \( I > 0 \). Condition (i) says that work effort is productive. Conditions (i) and (ii) guarantee that high work effort always generates a higher expected payoff than low work effort does. Condition (iii) says that capital and work effort are substitutes. This gives rise an incentive for the borrower to increase his borrowing and shirk. All the qualitative results reported before remain valid in this modified problem. The borrower will borrow less in the third-best case. This contrasts with the findings by Bizer and DeMarzo (1992) who show that the converse may be true. The key reason for this difference is that the borrower seeks loans in order to smooth his intertemporal consumption in Bizer and DeMarzo’s model. Thus, the only way to correct the dual incentive problem is by charging a high interest rate to internalize the potential impact of unobservable further borrowing. However, resolving this problem by interest rates alone is inefficient, as the borrower may borrow more as well to ease part of the inefficiency.

\(^\text{17}\)Since the probability of success increases with effort, normalizing the effort level to be equal to this probability is without loss of generality.
3.5.3 Managerial Compensation

Another possible extension is to use the given framework to study the conflict of interests among managers, stockholders and bondholders. If managers’ objective is different from stockholders’, stockholders should design an optimal compensation scheme for managers before the banking stage begins. Obviously it is inefficient for stockholders to write a compensation contract that can perfectly align managers’ interests with theirs, as this will simply produce the third-best outcome discussed above. Thus, the optimal compensation scheme will be departed from those characterized in the principal-agent literature in order to alleviate the efficiency loss. In my framework, there exists a trivial managerial compensation contract that can achieve the first-best outcome: Stockholders offer managers a fixed salary when the firm is solvent and zero otherwise. Then, it is always in the managers’ best interest to choose the low-risk project, as it has a higher probability of success. As long as side-payments are not allowed, this contract credibly commits the firm to the choice of the low-risk project and thus removes the agency cost of debt. Of course, with a more complicated situation, such an efficient managerial compensation contract may not exist. Nevertheless, the agency problem can still be mitigated by an appropriate choice of managerial compensation. For further discussion, see Brander and Poitevin (1992).

3.6 Conclusion

In this chapter, I study loan contract design problems under both relationship banking and price banking. The opportunity to bank sequentially under price banking is shown to affect the equilibrium outcome by introducing a time-consistency requirement. Borrowers would be better off if they could credibly commit to refrain from additional loan requests but their vulnerable behaviour cracks the commitment. Borrowers end up receiving fewer funds for investment and this generates an efficiency loss. The crux of the problem is that new loans impose negative externalities to existing loans by inducing borrowers to substitute
riskier projects. Since new banks do not pay for these externalities, prior banks recognize the potential victimization that they may suffer and thus react accordingly. Equilibrium results in reduced welfare for borrowers.

In an empirical study of firms involved in leveraged buyouts, Asquith and Wizman (1990) report that corporate bonds containing clauses restricting total debt tend to maintain or even gain value in leveraged buyouts. In contrast, corporate bonds with weaker covenants enforcing priority alone suffered a decline in value from the substantial increase in junior debt associated with leveraged buyouts. This empirical evidence seems to be consistent with the finding that future borrowing imposes negative externalities on existing debt.

The negative externalities addressed here are not unique to banking markets. Kahn and Mookherjee (1991) suggest that “the externalities from side- trades are likely to pose difficulties whenever hidden information or hidden actions are present.” A deeper understanding on this issue is no doubt needed in order to improve the efficiency of optimal contract design.

\footnote{See Kahn and Mookherjee (1991) for other possible applications.}
Appendix

Proof of Proposition 3.1. Since $I_L$ is the maximum solution of $p_LkR_H(I) - iI$, it must be true that $p_LkR_H(I_L) - iI_L \geq p_LkR_H(I_H) - iI_H$, which in turn is greater than $p_HR_H(I_H) - iI_H$ by $k > p_H/p_L$. □

Proof of Proposition 3.2. Suppose that the low-risk project is optimal (i.e. $\theta^* = L$). Then, the Lagrangian for (P1) is

$$p_L[kR_H(I) - r] + \mu(p_Lr - iI) + \lambda[(p_Lk - p_H)R_H(I) - (p_L - p_H)r],$$

where $\mu$ is the Lagrange multiplier for (3.6) and $\lambda$ is the multiplier for (3.7). By the assumptions on $R_H$, the first-order conditions are necessary and sufficient for a global maximum. The first-order condition with respect to $r$ yields

$$\mu = 1 + \lambda \left( 1 - \frac{p_H}{p_L} \right) > 0,$$

since $\lambda \geq 0$. Therefore, (3.6) is binding. The first-order condition with respect to $I$ yields

$$\lambda = \frac{p_LkR'_H(I_{SB}) - i}{(p_L - p_H)i/p_L - (p_Lk - p_H)R'_H(I_{SB})},$$

using (3.6) and substituting the value of $\mu$. The denominator of (A3.1) is positive if, and only if,

$$\frac{p_L - p_H}{(p_Lk - p_H)R'_H(I_{SB})} > \frac{p_L}{i}.$$  \hfill (A3.2)

By (3.10), the left hand side of (A3.2) is equal to the slope of the I.C. curve. In words, (A3.2) means that the I.C. curve cuts $\Pi_L$ from below, which is always true. By (3.4), the first-best loan contract lies below the I.C. curve. Hence, using (3.4), (A3.2) and a simple geometric argument, I have $I_{SB} < I_{FB}$. But then by (3.1), the numerator of (A3.1) is positive since it can be rewritten as $p_Lk[R'_H(I_{SB}) - R'_H(I_{FB})]$. Thus, $\lambda > 0$ and (3.7) is binding. Solving (3.6) and (3.7) yields (3.8).
Now, suppose that the high-risk project is optimal (i.e. $\theta^* = H$), then there is no incentive problem. Hence, the optimal loan contract is $(iI_H/p_H, I_H)$, the one characterized in section 3.2.2. Which project is optimal then hinges on condition (3.9). □

Proof of Proposition 3.3. By (3.4) and (3.8), I have

$$\frac{R_H(I_{SB})}{I_{SB}} > \frac{R_H(I_{FB})}{I_{FB}}. \quad (A3.3)$$

Since $R_H$ is strictly concave, (A3.3) implies $I_{SB} < I_{FB}$. □

Proof of Proposition 3.4. Denote $D = (1 - p_H/p_L)\alpha - (p_Lk - p_H)R'_H(I_{SB})$ and $r_{SB} = iI_{SB}/p_L$. By (A3.2), $D > 0$.

Differentiating (3.8) with respect to $k$ and rearranging terms, I get

$$\frac{\partial I_{SB}}{\partial k} = \frac{p_LR_H(I_{SB})}{D} > 0.$$

Differentiating $r_{SB}$ with respect to $k$ yields

$$\frac{\partial r_{SB}}{\partial k} = \frac{i}{p_L} \frac{\partial I_{SB}}{\partial k} > 0.$$

Differentiating (3.8) with respect to $i$ yields

$$\frac{\partial I_{SB}}{\partial i} = -\frac{(1 - p_H/p_L)I_{SB}}{D} < 0.$$

Differentiating $r_{SB}$ with respect to $i$ yields

$$\frac{\partial r_{SB}}{\partial i} = \frac{I_{SB}}{p_L} + \frac{i}{p_L} \frac{\partial I_{SB}}{\partial i}
= \frac{(p_Lk - p_H)R'_H(I_{SB})}{p_LD} < 0.$$

Differentiating (3.8) with respect to $p_L$ yields

$$\frac{\partial I_{SB}}{\partial p_L} = \frac{p_LkR_H(I_{SB}) - (p_H/p_L)iI_{SB}}{p_LD}
\geq \frac{p_HR_H(I_H) - iI_H + (1 - p_H/p_L)iI_{SB}}{p_LD} > 0,$$

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where the first inequality follows from (3.9). Differentiating \( r^{SB} \) with respect to \( p_L \) yields

\[
\frac{\partial r^{SB}}{\partial p_L} = -\frac{iI^{SB}}{p_L^2} + \frac{i}{p_L} \frac{\partial I^{SB}}{\partial p_L} = \frac{i[p_L k R_H(I^{SB}) - iI^{SB} + (p_L k - p_H) R_H'(I^{SB}) I^{SB}]}{p_L^2 D} > 0.
\]

Differentiating (3.8) with respect to \( p_H \) yields

\[
\frac{\partial I^{SB}}{\partial p_H} = \frac{iI^{SB}/p_L - R_H(I^{SB})}{D} = \frac{-p_L(1-k)R_H(I^{SB})}{(p_L - p_H)D} < 0,
\]

where the first inequality follows from (3.8). Differentiating \( r^{SB} \) with respect to \( p_H \) yields

\[
\frac{\partial r^{SB}}{\partial p_H} = \frac{i}{p_L} \frac{\partial I^{SB}}{\partial p_H} < 0. \quad \Box
\]

**Proof of Proposition 3.5.** First, I show that constraint (3.14) gives a necessary condition for no further borrowing. Suppose that (3.14) does not hold. Then, there exists an additional loan contract \((\Delta r, \Delta I) \in T\) such that

\[
p_\theta[R_\theta(I + \Delta I) - (r + \Delta r)] > p_{\theta^*}[R_{\theta^*}(I) - r],
\]

for some \( \theta \in \{L, H\} \). That is, the additional loan contract makes the borrower better off if he offers it and switches to project \( \theta \). At the same time, new banks are willing to enter into any loan contracts in \( T \) irrespective of the borrower's project choice. This implies that the allocation \([(r, I), \theta^*]\) is not immune to this additional loan.

Next, an obvious sufficient condition for no further borrowing is: (SC) There does not exist an additional loan contract \((\Delta r, \Delta I) \) such that

\[
\hat{\theta} \in \arg \max_{\theta \in \{L, H\}} p_\theta[R_\theta(I + \Delta I) - (r + \Delta r)], \quad (A3.4)
\]

\[
p_{\hat{\theta}}[R_{\hat{\theta}}(I + \Delta I) - (r + \Delta r)] > p_{\theta^*}[R_{\theta^*}(I) - r], \quad (A3.5)
\]

and

\[
p_{\hat{\theta}} \Delta r - i \Delta I \geq 0. \quad (A3.6)
\]

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In words, there is no additional loan request from the borrower that the borrower finds it desirable, (A3.5), and new banks will accept, (A3.6), knowing that the additional loan will give them nonnegative profits as long as the borrower chooses his project optimally given his new loan portfolio, (A3.4). If I can show that maximizing (3.11) subject to (3.12), (3.13) and (SC) is equivalent to (P2), we are done.

For any additional contract \((\Delta r, \Delta I)\) that induces the borrower to choose the high-risk project and is acceptable to new banks, by (3.14), the borrower will not be better off by offering it. For any additional contract \((\Delta r, \Delta I)\) that induces the borrower to choose the low-risk project and is acceptable to new banks, maximizing (3.11) subject to (3.12) and (3.13) guarantees that (A3.5) cannot hold at the optimal initial loan contract. Hence, the solution of (P2) turns out to have the property that the borrower has no incentive to make any self-enforcing future borrowing. □

Proof of Proposition 3.6. By proposition 3.2, the second-best contract \((iI^{SB}/p_L, I^{SB})\) solves (P2) without imposing constraint (3.14). If I can show that this contract satisfies (3.14) if, and only if, \(I^{SB} \geq I_H\), I am done.

Suppose that the low-risk project is optimal (i.e. \(\theta^* = L\)). For any \(I \in [0, I_H)\), (3.14) can be written as

\[
p_L[kR_H(I) - r] \geq p_H[R_H(I_H) - r] - i(I_H - I).
\]

(A3.7)

For any \(I \in [I_H, \infty)\), (3.14) becomes

\[
p_L[kR_H(I) - r] \geq p_H[R_H(I) - r].
\]

Hence, the second-best contract \((iI^{SB}/p_L, I^{SB})\) satisfies (3.14) if, and only if, \(I^{SB} \geq I_H\). The low-risk project is indeed optimal by proposition 3.2. □

Proof of Proposition 3.7. From (A3.1), I have

\[
R_H'(I^{SB}) = \frac{i[p_L(1 + \lambda) - p_H \lambda]}{p_L[p_L k(1 + \lambda) - p_H \lambda]}.
\]

(A3.8)
Since $R_H$ is strictly concave, $I_s^{SB} < I_H$ if, and only if, $R'_H(I_s^{SB}) > R'_H(I_H)$, which in turn is equivalent to
\[
\frac{i[p_L(1 + \lambda) - p_H \lambda]}{p_L[p_L k(1 + \lambda) - p_H \lambda]} > \frac{i}{p_H},
\]
by (A3.8) and (3.2). Subtracting the left hand side of (A3.9) from its right hand side yields
\[
\frac{i[p_H \lambda(p_L - p_H) - p_L(p_L k - p_H)(1 + \lambda)]}{p_L p_H[p_L k(1 + \lambda) - p_H \lambda]},
\]
which is positive if, and only if, (3.15) holds. □

Proof of Proposition 3.8. Given that condition (3.16) holds. Suppose that the low-risk project is optimal (i.e. $\theta^* = L$). I shall verify this conjecture later.

By propositions 3.6 and 3.7, the third-best loan size must be less than $I_H$, or otherwise it will be equal to $I_s^{SB}$, which violates condition (3.15). Hence, I can restrict $I \in [0, I_H)$ from which (3.14) can be written as (A3.7). Since $I < I_H$, (A3.7) implies (3.13). The Lagrangian for (P2) is
\[
p_L[kR_H(I) - r] + \xi(p_L r - i) + \zeta[p_L kR_H(I) - p_H R_H(I_H) - (p_L - p_H)r + i(I_H - I)],
\]
where $\xi$ and $\zeta$ are the Lagrange multipliers for (3.12) and (A3.7) respectively. By the assumptions on $R_H$, the first-order conditions are necessary and sufficient for a global maximum. The first-order condition with respect to $r$ yields
\[
\xi = 1 + \zeta \left(1 - \frac{p_H}{p_L}\right) > 0,
\]
since $\zeta \geq 0$. Therefore, (3.12) is binding. The first-order condition with respect to $I$ yields
\[
\zeta = \frac{p_L kR_H'(I^{TB}) - i}{(2 - p_H/p_L)i - p_L kR_H'(I^{TB})},
\]
using (3.12) and substituting the value of $\xi$. By (3.2), $p_L kR_H'(I^{TB}) - i = p_L kR_H'(I^{TB}) - p_H R'(I_H) > 0$ since $k > p_H/p_L$ and $I^{TB} < I_H$. It follows that $\zeta > 0$ if, and only if,
\[
\frac{p_L - p_H}{p_L kR_H'(I^{TB}) - i} > \frac{p_L}{i}.
\]

(A3.10)
By (3.18), the left hand side of (A3.10) is equal to the slope of the NFB curve. In words, (A3.10) means that the NFB curve cuts $\Pi_L$ from below, which is always true when the NFB curve and $\Pi_L$ have non-empty intersection. Hence, given condition (3.16), (A3.7) is binding. Solving (3.12) and (A3.8) yields $I^{TB}$.

Finally, it remains to verify the initial conjecture. Since $I^{TB}$ solves (3.16) with equality, I know $p_L k R_H(I^{TB}) - i I^{TB} > p_H R_H(I_H) - i I_H$. The former is the expected payoff for the borrower if he undertakes the low-risk project while the latter is that for the high-risk project. Hence, I can conclude that the low-risk project is indeed optimal. $\square$

**Proof of Proposition 3.9.** Since $I_H$ maximizes $p_H R_H(I) - i I$, it must be true that

$$p_H R_H(I_H) - i I_H > p_H R_H(I^{TB}) - i I^{TB}. \tag{A3.11}$$

Using (A3.11) and (3.16), I have

$$(p_L k - p_H) R_H(I^{TB}) > \left(1 - \frac{p_H}{p_L}\right) i I^{TB}. \tag{A3.12}$$

By (3.9) and (A3.12), we know

$$\frac{R_H(I^{TB})}{I^{TB}} > \frac{R_H(I^{SB})}{I^{SB}}. \tag{A3.13}$$

Since $R_H$ is strictly concave, (A3.13) implies $I^{TB} < I^{SB}$. $\square$

**Proof of Proposition 3.10.** Denote $D' = (2 - p_H/p_L) i - p_L k R_H(I^{TB})$ and $r^{TB} = i I^{TB}/p_L$.

By (A3.10), $D' > 0$.

Differentiating (3.16) with respect to $k$, I get

$$\frac{\partial I^{TB}}{\partial k} = \frac{p_L R_H(I^{TB})}{D'} > 0.$$

Differentiating $r^{TB}$ with respect to $k$ yields

$$\frac{\partial r^{TB}}{\partial k} = \frac{i \partial I^{TB}}{p_L \partial k} > 0.$$
Differentiating (3.16) with respect to \( i \) yields

\[
\frac{\partial I^{TB}}{\partial i} = -\frac{(2 - p_H/p_L)I^{TB} + I_H}{D'} < 0,
\]

where I use (3.2) to simplify the numerator. Differentiating \( r^{TB} \) with respect to \( i \) yields

\[
\frac{\partial r^{TB}}{\partial i} = \frac{I^{TB}}{p_L} + \frac{i}{p_L} \frac{\partial I^{TB}}{\partial i} = \frac{-p_LkR_H'(I^{TB})I^{TB} + iI_H}{p_LD'} < 0.
\]

Differentiating (3.16) with respect to \( p_L \) yields

\[
\frac{\partial I^{TB}}{\partial p_L} = \frac{p_LkR_H(I^{TB}) - (p_H/p_L)iI^{TB}}{p_LD'} = \frac{p_HR_H(I_H) - iI_H + 2(1 - p_H/p_L)iI^{TB}}{p_LD'} > 0,
\]

where the second equality follows from (3.16). Differentiating \( r^{TB} \) with respect to \( p_L \) yields

\[
\frac{\partial r^{TB}}{\partial p_L} = -\frac{iI^{TB}}{p_L^2} + \frac{i}{p_L} \frac{\partial I^{TB}}{\partial p_L} = \frac{i[p_HR_H(I_H) - iI_H + [p_LkR_H'(I^{TB}) - (p_H/p_L)i]I^{TB}]}{p_L^2D'} > 0.
\]

Differentiating (3.16) with respect to \( p_H \) yields

\[
\frac{\partial I^{TB}}{\partial p_H} = -\frac{R_H(I_H) - iI^{TB}/p_L}{D'} = \frac{-p_LkR_H(I^{TB}) - iI^{TB} + i(I_H - I^{TB})}{p_HD'} < 0,
\]

where the numerator of the first equality is simplified by using (3.2) and the second equality follows from (3.16). Differentiating \( r^{TB} \) with respect to \( p_H \) yields

\[
\frac{\partial r^{TB}}{\partial p_H} = \frac{i}{p_L} \frac{\partial I^{TB}}{\partial p_H} < 0. \quad \square
\]
References


Figure 1. The Second-Best Equilibrium. $\bar{U}_\theta$ and $\Pi_\theta$ denote respectively the borrower’s indifference curve and the bank’s break-even line given that the borrower undertakes project $\theta \in \{L, H\}$. The shaded region is the set of incentive compatible and individually rational contracts which induce the borrower to choose the low-risk project. Contract $E$ maximizes the borrower’s expected payoff in the shaded region and lies above $\bar{U}_H$. Hence, in the second-best equilibrium, the borrower chooses the low-risk project with contract $E$. 
Figure 2. Incentives for Further Borrowing Given $I_{SB} < I_H$. $U'_H$ ($U''_H$) and $\Pi_H$ denote respectively the borrower's indifference curve and the bank's break-even line given that the borrower undertakes the high-risk project. $U'_H$ corresponds to the borrower's second-best expected payoff. $\Pi'_H$ is a line parallel to $\Pi_H$. Given that contract $E$ (the second-best loan contract) is the initial loan, any additional loan (say contract $H$) in the lower shaded region will give the borrower a new loan portfolio (say contract $E + H$) in the upper shaded region. This additional loan is acceptable by the bank irrespective of the borrower's project choice, and the borrower is better off to offer it and switch to the high-risk project. Hence, the second-best outcome is not implementable in this case.
Figure 3. The Third-Best Equilibrium. $U_\theta$ and $\Pi_\theta$ denote respectively the borrower's indifference curve and the bank's break-even line given that the borrower undertakes project $\theta \in \{L, H\}$. The shaded region is the set of incentive compatible and individually rational contracts which give the borrower no incentive to borrow further and choose the high-risk project. Contract $E'$ maximizes the borrower's expected payoff in the shaded region. The borrower's third-best expected payoff can be corresponded by either $U_L^*$ or $U_H^*$. Since $U_H^*$ lies above $\Pi_H$, in the third-best equilibrium the borrower chooses the low-risk project with contract $E'$ as the optimal initial loan.
Chapter 4

On the Determinants of Bank Interest Margin under Capital Regulation and Deposit Insurance

4.1 Introduction

Unlike the previous two chapters which examine the relative efficiency of relationship banking and price banking, this chapter is devoted to the study of the determinants of bank interest margin (the difference between loan and deposit rates).

The move away from relationship banking towards price banking is believed to create a frenzy of competition among banks to attract funds and to pass them onto borrowers—both domestically and internationally. It is argued that bank interest margins are squeezed to an extent which ignores the possibility of the world economic downturn such that banks would need to prepare against the threat of loan losses. Robin Leigh-Pemberton, Governor of the Bank of England, has recently commented on this problem:¹

It can be argued, particularly with hindsight, that banks expanded their balance

¹See the report by Blanden (1993).
sheets too rapidly, notably by lending to risky businesses (including property companies) at margins that did not properly reflect the risks that were being taken on. ... It is certainly true that there was a very pronounced reduction in bank margins during the 1980s. Moreover, although there have been many claims to the contrary, margins have in fact not risen very much in the early 1990s. And, given their new freedom and a growing economy, it was not surprising that banks sought to increase the volume of their lending. ... There are several questions banks must ask themselves. Do they really pay attention to the lessons of history—for example the property crisis of the early 1970s? Did they really monitor the credit criteria which had served them well in the past? And were the incentives given to loan officers really appropriate, or did they encourage new business at the expense of sound business? ... There are lessons for bankers. Close attention to the control and pricing of risk is a theme that I have brought to your attention before now, but I make no apology for doing so again. It is at the heart of the banker's professional life, and no amount of competition or marketing strategy should ever divert us from it.

What went wrong? Blanden (1993) argues that banks forgot the fundamental role of borrowing and lending money. Furthermore, he feels that banks spent too much effort searching for other sources of income and that they need to get back to basics. But it is not clear whether changes in margins are due to increased competition, lax regulatory supervision, or greater economic risk. It is, therefore, the purpose of this chapter to study the determinants of bank interest margins.

To date, few theoretical models have been developed to analyze the determinants of optimal bank interest margins. Ho and Saunders (1981) utilize a framework similar to the bid-ask spread model of Stoll (1978). In their model, a bank is viewed as a risk-averse dealer who pays for deposits at a bid price and lends funds at an ask price. This modeling strategy is further extended by McShane and Sharpe (1985) and Allen (1988). Lerner (1981),
the discussant of Ho and Saunders' paper, argues that by "considering banking to be only a
trading activity, the insights that arise from reconizing that a production function exists may
be lost. (p. 601)" In addition, it is hard to address regulatory issues in this setting. Since
banks are among the most heavily regulated firms in the economy, an important component
of the banking environment is eliminated.

Recently, Zarruk (1989) and Zarruk and Madura (1992) go some distance in filling this
gap by adopting a production-based model of risk-averse banks under uncertainty as an
alternative approach. Their model is similar to traditional banking models. However, they
suffer from two shortcomings: First, bank risk aversion seems to be an awkward assumption.
As argued by Santomero (1984) and Flannery (1989), given perfect capital markets the
appropriate objective for a bank is to maximize its market value. Second, in their models
banks are not subject to limited liability. This excludes the moral hazard problem arising
from the mispriced deposit insurance system from their analysis. Thus, the deposit insurance
system is nothing but a supplementary administrative cost to deposits in their framework.

The purpose of this chapter is to present a production-based model of risk-neutral banks
that are subject to prevailing capital regulation and deposit insurance. Banks are protected
by limited liability and deposit insurance is interpreted as a put option (see Merton (1977)).
Uncertainty in the model arises from random loan defaults and a stochastic short-term
money market rates. The model is used to characterize interest margins and comparative
statics provide alternative explanations of a number of empirical observations concerning the
behavior of a bank's interest margin. The results indicate that a bank's interest margin in-
creases with the short-term money market rate variability (in the sense of a mean-preserving
spread). Empirical evidence consistent with this is documented by Ho and Saunders (1981)
and McShane and Sharpe (1985). The results also show that a bank's interest margin is a
decreasing function of its capital-to-deposits ratio and the flat-rate deposit insurance pre-
mium. Indirect empirical support of the former result is provided by Furlong (1992), who
shows that bank loan growth rates and capital-to-assets ratio are positively related.
The remaining part of this chapter is organized as follows. Section 4.2 presents the basic structure of the model. The solution of the model is characterized in section 4.3, and is contrasted with the one in which deposit insurance is fairly priced. Section 4.4 develops the comparative static properties of the model. The final section summarizes and concludes.

4.2 The Model

Consider an insured risk-neutral bank which operates for one period. At the beginning of the period, the bank issues two types of liabilities: deposits, \( D \), and equity capital, \( K \). Regulators provide full deposit insurance and charge the bank a flat-rate premium, \( p > 0 \), per dollar of deposits. The deposit supply is perfectly elastic at the one-plus deposit rate \( R_D \). The amount of equity capital, \( K \), is assumed to be fixed and is not part of the bank’s decision, instead it is tied by regulation to satisfy the following capital requirement constraint:

\[
K \geq \kappa D, \tag{4.1}
\]

where \( \kappa > 0 \) is the required minimum capital-to-deposits ratio.\(^3\)

Using the proceeds from its liabilities, the bank can acquire two kinds of assets: risky loans, \( L \), and short-term money market assets, \( C \). It has monopoly access to a segment of the risky loan market and is a loan rate setter.\(^4\) Loan demand is given by a downward-sloping function, \( L(R_L) \), where \( R_L \) is the one-plus loan rate and \( L'(R_L) < 0 \). The initial balance sheet identity of the bank is given by\(^5\)

\[
L(R_L) + C = D(1 - p) + K. \tag{4.2}
\]

Uncertainty in the short-term money market rate is modeled through the use of an

\(^2\)The qualitative results are robust to an upward-sloping deposit supply function.

\(^3\)The constant minimum capital-to-deposits ratio is adopted to simplify the exposition. None of the qualitative results will change if the ratio is an increasing function of the amount of risky loans held by the bank (i.e. risk-based), as modeled by Zarruk and Madura (1992).

\(^4\)Loan rate-setting behavior by banks is well documented by Slovin and Sushka (1983) and Hancock (1986).

\(^5\)The inclusion of legal reserve requirements does not alter the qualitative results.
an additive random variable, \( \epsilon \), having a known cumulative distribution function, \( F(\epsilon) \), over \([\xi, \xi^2]\) with mean zero and variance \( \sigma^2 \). Succinctly, the one-plus short-term money market rate is given by \( R + \gamma \epsilon \), where \( R \geq R_D \) is the expected one-plus rate and \( \gamma > 0 \) is a shift parameter affecting interest rate uncertainty. Since investment in short-term money market assets should be relatively safe, throughout the paper I assume the following condition:

\[
R(1 - p + \kappa) \geq R_D.
\]

Condition (4.3) simply implies that should the bank invest all its equity capital, \( K \), and deposits (with the capital requirement constraint binding), \( K/k \), less the deposit insurance premium, \( pK/k \), into short-term money market assets, it is expected to remain solvent.\(^6\)

At the end of the period, loans are repaid and the bank receives an uncertain gross loan revenue, \( \hat{\theta}G(R_L) = \hat{\theta}R_LL(R_L) \), where \( \hat{\theta} \) is a random variable distributed over \([0, 1]\) and is independent of \( \epsilon \).\(^7\) Thus, for a given level of risky loans, \( L(R_L) \), the bank will receive at most its total contractual loan repayments, \( G(R_L) \), and it may receive less, depending on the uncertain state of nature, \( \hat{\theta} \). The bank also receives an uncertain payment from its investment in short-term money market assets, \( (R + \gamma \epsilon)C \). For the sake of tractability, I assume that \( \hat{\theta} \) is uniformly distributed with unit density. It should be noted that most of the qualitative results also hold under more general distribution functions.

Equity is a residual claim against the bank’s assets. If the bank’s end-of-period revenues exceed its deposit obligations, the remainder is distributed as liquidating dividends to shareholders. Otherwise, the bank defaults, and the regulators take control of the remaining assets and pay all deposit obligations. The expected value of the bank’s equity is given by\(^8\)

\[
V = \int_{\xi}^{\xi^2} \int_0^1 \max \{ \theta G(R_L) + (R + \gamma \epsilon)C - R_D D, 0 \} \, d\theta \, dF(\epsilon) \\
= \frac{1}{2} G(R_L) + R[D + K - L(R_L)] - pRD - R_D D
\]

\(^6\)In practice, \( \kappa \geq p \) and thus this assumption is satisfied.

\(^7\)This formulation of uncertain loan defaults is taken from Taggart and Greenbaum (1977).

\(^8\)The inclusion of administrative costs of loans and deposits does not affect the qualitative results.
+ \int_{\xi}^{\xi} \int_0^1 \max [RD - \theta G(R_L) - (R + \gamma \epsilon)[(D(1 - p) + K - L(R_L)]] 0 \, d\theta \, dF(\epsilon),

where I have used the fact that \(\max [X - Y, 0] = X - Y + \max [Y - X, 0]\) and the balance sheet identity, (4.2). The sum of the first two terms of (4.4) is the total expected value of the bank. The third term is the deposit insurance premium paid to the regulators. The fourth term is the payoff to depositors. The fifth term is equal to the expected payout to depositors by the regulators when the bank’s assets fall short of the claims of depositors. This term corresponds to Merton’s (1977) put option value of deposit insurance.

The objective and decisions of the bank are as follows. At the beginning of the period, the bank chooses the one-plus loan rate, \(R_L\), and the amount of deposits, \(D\), (before any uncertainty is resolved) to maximize its equity claim, (4.4), subject to the capital requirement constraint, (4.1).

4.3 The Solution

Let \(L\) represent the Lagrange function, and \(\lambda \geq 0\) denote the Lagrange multiplier associated with the capital requirement constraint, (4.1). The Kuhn-Tucker conditions are

\[
\frac{\partial L}{\partial R_L} = \frac{1}{2} G'(R^*_L) - RL'(R^*_L) - \int_{\xi}^{\xi} \int_0^{\theta^*_1(\epsilon)} [\theta G'(R^*_L) - (R + \gamma \epsilon)L'(R^*_L)] \, d\theta \, dF(\epsilon) = 0,
\]

\[
\frac{\partial L}{\partial D} = \int_{\xi}^{\xi} \int_0^{\theta^*_1(\epsilon)} [(R + \gamma \epsilon)(1 - p) - RD] \, d\theta \, dF(\epsilon) - \lambda^* \kappa = 0,
\]

\[
\frac{\partial L}{\partial \lambda} = K - \kappa D^* \geq 0,
\]

\[
\lambda^* \frac{\partial L}{\partial \lambda} = 0,
\]

where \((R^*_L, D^*, \lambda^*)\) is the optimal interior solution, and \(\theta^*_1(\epsilon)\) determines the maximum loan revenue given \(\epsilon\) below which the bank is insolvent, i.e.,

\[
\theta^*_1(\epsilon) = \frac{1}{G'(R^*_L)} \left\{ RD^* - (R + \gamma \epsilon)[D^*(1 - p) + K - L(R^*_L)] \right\}.
\]

In order to study the impact of the capital regulation, throughout the paper the capital requirement constraint is assumed to be binding. Thus, we have \(\lambda^* > 0\) and \(D^* = K/\kappa\). The
first-order conditions then reduce to

\[ V_{R_L} = \frac{1}{2} G'(R_L^*) - RL'(R_L^*) = \int_{\theta'}^{\theta''} \left[ \theta G'(R_L^*) - (R + \gamma \epsilon) L'(R_L^*) \right] d\theta dF(\epsilon) = 0, \quad (4.5) \]

and

\[ \int_{\theta'}^{\theta''} \left[ (R + \gamma \epsilon)(1 - p) - RD \right] d\theta dF(\epsilon) > 0, \quad (4.6) \]

where

\[ \theta''(\epsilon) = \frac{1}{G(R_L^*)} \left\{ RD \frac{K}{K} - (R + \gamma \epsilon) \left[ \frac{K}{K} (1 - p) + K - L(R_L^*) \right] \right\}. \quad (4.7) \]

The first-order condition (4.5) implies that the bank sets the loan rate up to the point where the expected marginal appreciation of the bank's value exactly offsets the expected marginal depreciation of the deposit insurance put option value.

To gain more insight into the above solution, it is contrasted with the one in which deposit insurance is actuarially fair. In this case, the deposit insurance premium (the third term of (4.4)) is always set equal to the value of the deposit insurance subsidy (the fifth term of (4.4)). Thus, the expected value of the bank's equity is simply equal to the total expected bank value net of the payoff to depositors:

\[ V = \frac{1}{2} G(R_L) + R[D + K - L(R_L)] - RD. \quad (4.8) \]

The bank's objective is to maximize its equity claim, (4.8), subject to the capital requirement constraint, (4.1). The first-order condition with respect to \( R_L \) is given by

\[ \frac{1}{2} G'(R_L^*) - RL'(R_L^*) = 0, \quad (4.9) \]

where \( R_L^* \) is the optimal loan rate with fairly priced deposit insurance. Comparing (4.5) and (4.9) yields

\[ \frac{1}{2} G'(R_L^*) - RL'(R_L^*) > \frac{1}{2} G'(R_L^{**}) - RL'(R_L^{**}) = 0. \]

By the second-order condition of the latter optimization problem, it follows immediately that \( R_L^* < R_L^{**} \). This implies that the bank will grant more risky loans when deposit insurance is mispriced. The difference between \( R_L^{**} \) and \( R_L^* \), \( \Delta R_L = R_L^{**} - R_L^* \), represents a subsidy from
deposit insurance. It is noteworthy mentioning that given actuarially fair deposit insurance the bank’s optimal loan rate depends only on the loan demand function and the expected short-term money market rate.

The insolvency risk of the bank is given by

$$P(R_L) = \int_{\underline{\theta}}^{\overline{\theta}} \int_{0}^{\theta_{1}(\epsilon)} d\theta \ dF(\epsilon).$$

Differentiating $P(R_L)$ with respect to $R_L$ yields

$$P'(R_L) = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial \theta_{1}(\epsilon)}{\partial R_L} dF(\epsilon)
= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{K R_L L'(R_L)}{\kappa G(R_L)^2} [(R + \gamma \epsilon)(1 - p + \kappa) - R_D] - \frac{\theta_{1}(\epsilon)}{R_L} \right\} dF(\epsilon)
= \frac{K R_L L'(R_L)}{\kappa G(R_L)^2} [R(1 - p + \kappa) - R_D] - \frac{1}{R_L} \int_{\underline{\theta}}^{\overline{\theta}} \theta_{1}(\epsilon) dF(\epsilon)
< 0.$$ 

The insolvency risk is a decreasing function of the loan rate. As a result, the presence of the mispriced deposit insurance induces the bank to increase its portfolio risk.

Now, I proceed to decompose the optimal bank interest margin. Suppose that all loans are safe (i.e., $\theta = 1$ with probability one) and deposit insurance is fairly priced. In this case, the expected value of the bank’s equity value is

$$V = G(R_L) + R[D + K - L(R_L)] - R_D D. \quad (4.10)$$

The bank’s objective is to maximize its equity claim, (4.10), subject to the capital requirement constraint, (4.1). The first-order condition yields

$$G'(R_L^{**}) - RL'(R_L^{**}) = 0,$$

where $R_L^{**}$ is the optimal loan rate with fairly priced deposit insurance and without loan default risk. Since $G'(R_L^{**}) < 0$, it must be the case that

$$\frac{1}{2} G'(R_L^{**}) - RL'(R_L^{**}) > G'(R_L^{**}) - RL'(R_L^{**}) = \frac{1}{2} G'(R_L^{**}) - RL'(R_L^{**}) = 0.$$
Thus, it follows from the second-order condition that $R_{L}^{***} < R_{L}^{**}$. The difference between $R_{L}^{**}$ and $R_{L}^{***}$, $\Delta R_{L}^2 = R_{L}^{**} - R_{L}^{***}$, represents a premium for loan default risk. As a result, the optimal bank interest margin can be decomposed as

$$R_{L}^{*} - R_D = R_{L}^{***} - R_D - \Delta R_{L}^1 + \Delta R_{L}^2.$$ 

In words, the optimal bank interest margin is equal to the interest spread under fairly priced deposit insurance and no loan default risk minus a component due to mispriced deposit insurance and plus a component due to loan default.

### 4.4 Comparative Statics

Having examined the solution to the bank's optimization problem, I am ready to analyse the comparative statics of the bank's optimal interest margin. The following observations will prove to be useful in the comparative statics analysis.

Rewritting (4.5) as

$$V_{R_L} = \int_{\theta(\epsilon)}^{1} \int \left[ \theta G'(R_{L}^{*}) - (R + \gamma \epsilon) L'(R_{L}^{*}) \right] \, d\theta \, dF(\epsilon) = 0,$$

implies that

$$G'(R_{L}^{*}) < 0.$$ 

Denote $M(\epsilon) = \theta_1(\epsilon)G'(R_{L}^{*}) - (R + \gamma \epsilon)L'(R_{L}^{*})$. Then, substituting (4.7) and rearranging terms yields

$$M(\epsilon) = \frac{L(R_{L}^{*})}{R_{L}^{*}}(R + \gamma \epsilon) - \frac{K G'(R_{L}^{*})}{\kappa G(R_{L}^{*})}[(R + \gamma \epsilon)(1 - p + \kappa) - R_D]. \quad (4.11)$$

Note that the effect of a change in any parameter, $x$, of the model on the bank's optimal loan rate (and thus the bank's interest margin) is given by

$$\frac{dR_{L}^{*}}{dx} = -\frac{V_{R_L}}{V_{R_L,R_L}}.$$
By the second-order condition, $V_{R_L R_L} < 0$ and the sign of $dR^*_L/dx$ is the same as that of $V_{R_L x}$. The comparative static analysis can thus be carried out by considering the sign of $V_{R_L x}$.

First consider the effect of a change in the capital-to-deposits ratio, $\kappa$. Partially differentiating (4.7) with respect to $\kappa$ yields

$$\frac{\partial \theta_1^*(\epsilon)}{\partial \kappa} = \frac{K}{\kappa^2 G(R^*_L)} [(R + \gamma\epsilon)(1 - p) - R_D].$$

(4.12)

From (4.5), (4.11) and (4.12), we have

$$V_{R_L \kappa} = -\int_{\underline{\epsilon}}^{\bar{\epsilon}} M(\epsilon) \frac{\partial \theta_1^*(\epsilon)}{\partial \kappa} dF(\epsilon)$$

$$= \frac{K^2 G'(R^*_L)}{\kappa^2 G(R^*_L)^2} \left\{ [R(1 - p) - R_D][R(1 - p + \kappa) - R_D] + \gamma^2 \sigma^2 (1 - p) (1 - p + \kappa) \right\}$$

$$= \frac{-K}{\kappa^2 R^*_L^2} \left\{ R[R(1 - p) - R_D] + \gamma^2 \sigma^2 (1 - p) \right\}.$$

If $R(1 - p) - R_D \geq 0$, it is clear that $V_{R_L \kappa} < 0$ and thus $dR^*_L/d\kappa < 0$. On the other hand, if $R(1 - p) - R_D < 0$, it is not difficult to show that $V_{R_L \kappa} < 0$ when

$$\gamma^2 \sigma^2 \geq \frac{R(1 - p) - R_D}{1 - p} R.$$  

(4.13)

Substituting (4.7) into the left hand side of (4.6) and rearranging terms yields

$$[R(1 - p) - R_D]\left[1 - \int_{\underline{\epsilon}}^{\bar{\epsilon}} \theta_1^*(\epsilon) dF(\epsilon) \right] + \frac{C^*}{G(R^*_L)} \gamma^2 \sigma^2 (1 - p)$$

$$= \frac{1}{G(R^*_L)} [R(1 - p) - R_D]\left\{ R^*_L L(R^*_L) - R_D \frac{K}{\kappa} + RC^* \right\} + \frac{C^*}{G(R^*_L)} \gamma^2 \sigma^2 (1 - p).$$

(4.14)

If $R(1 - p) - R_D < 0$, then for (4.14) to be positive it must be the case that

$$\gamma^2 \sigma^2 > \frac{R(1 - p) - R_D}{1 - p} \left[ \frac{R^*_L L(R^*_L) - R_D K/\kappa}{C^*} + R \right],$$

which implies (4.13). Thus, we can state

**Proposition 4.1.** An increase in the capital-to-deposits ratio decreases the bank’s optimal interest margin.
To understand the intuition behind this result, note that, ceteris paribus, an increase in $\kappa$ lowers the critical bankruptcy state, $\theta^*_1(\epsilon)$, only for sufficiently bad realizations of $\epsilon$, and raises the critical bankruptcy state otherwise. This implies that the deposit insurance subsidy is more valuable on the margin to the bank facing a stiffer capital requirement. Thus, the distortion inducing the bank to take on inordinate risky loans is fortified. Given that the loan market is imperfect, the bank reduces the size of its interest margin to attract more loans.\(^9\)

Beginning in 1990, the stiffening of capital regulation is argued to have curtailed bank lending, and, thereby, contributed to a credit crunch. Furlong (1992) does find empirical evidence that suggests the stiffening of capital regulation in the 1990s, but contrary to the aforementioned view, his analysis shows a positive relationship between bank loan growth rates and capital-to-assets ratio. Thus, proposition 4.1 provides a theoretical rationale for his observation and suggests that the so-called credit crunch in the 1990s is not due to changes in capital regulation.

Now consider the effect of changes in the flat-rate deposit insurance premium, $p$. Partially differentiating (4.7) with respect to $p$ yields

$$\frac{\partial \theta^*_1(\epsilon)}{\partial p} = \frac{K}{\kappa G(R_L^*)} (R + \gamma \epsilon). \quad (4.15)$$

From (4.5), (4.11) and (4.15), I have

$$V_{R_{LP}} = -\int_{\epsilon}^{\epsilon} M(\epsilon) \frac{\partial \theta^*_1(\epsilon)}{\partial p} \, dF(\epsilon)$$

$$= \frac{K^2 G'(R_L^*)}{\kappa^2 G(R_L^*)^2} \left\{ R [-R(1 - p + \kappa) - R_D] + \gamma^2 \sigma^2 (1 - p + \kappa) \right\} - \frac{K}{\kappa R_L^2} (R^2 + \gamma^2 \sigma^2)$$

$$< 0.$$

\(^9\)Zarruk and Madura (1992) obtain the same result in their model. However, their derivation is not correct because they ignore the term $L(R_L)$ on the upper limit of their integral when they take the first-order condition. By Leibniz’s rule, this term will affect the first derivative. Mullins and Pyle (forthcoming) derive similar result using simulation.
Proposition 4.2. An increase in the flat-rate deposit insurance premium decreases the bank's optimal interest margin.

The intuition underlying this result follows a similar argument as in the case of a change in $\kappa$. Increases in the cost of deposit insurance increase the critical bankruptcy state, $\theta_1^*(\epsilon)$, for all realizations of $\epsilon$, other things being equal. This encourages the bank to shift investments to risky loans, and thus the optimal loan rate drops.

Next, I examine the impact of mean-preserving spread changes in the short-term money market rate uncertainty, $\gamma$. Partially differentiating (4.7) with respect to $\gamma$ yields

$$
\frac{\partial \theta_1^*(\epsilon)}{\partial \gamma} = -\frac{\epsilon C^*}{G(R^*_L)}.
$$

(4.16)

From (4.5), (4.11) and (4.16), I have

$$
V_{R_L\gamma} = \int_{\hat{\gamma}}^{c} \int_{\theta_1^*(\epsilon)}^{1} -\epsilon L'(R^*_L) \, d\theta \, dF(\epsilon) - \int_{\hat{\gamma}}^{c} M(\epsilon) \frac{\partial \theta_1^*(\epsilon)}{\partial \gamma} \, dF(\epsilon)
$$

$$
= \frac{\gamma \sigma^2 C^*}{G(R^*_L)} \left[ \frac{L(R^*_L)}{R^*_L} - \frac{KG'(R^*_L)}{\kappa G(R^*_L)} (1 - p + \kappa) - L'(R^*_L) \right]
$$

$$
> 0.
$$

Proposition 4.3. A mean-preserving spread increase in the short-term money market rate uncertainty increases the bank's optimal interest margin.

Intuitively, increased uncertainty about the short-term money market rate induces the bank to shift investments to short-term money market assets. Doing so allows the bank to further exploit the mispriced deposit insurance. Thus, the optimal loan rate goes up to reduce loan demand. Ho and Saunders (1981) and McShane and Sharpe (1985) present strong empirical evidence that there is a positive relationship between bank interest margins and interest rate risk. Thus, proposition 4.3 provides a theoretical rationale for this empirical observation.
4.5 Conclusions

In this chapter I combine the option view of bank value maximization with the determinants of optimal bank interest margins. The results offer alternative explanations of the observed behavior of bank interest margins, based on the fact that deposit insurance is not truly risk-adjusted (i.e. actuarially fair). Specifically, the results rationalize the observations that the bank interest margin increases with short-term money market rate variability. Further, the results have implications for bank regulation: Propositions 4.1 and 4.2 show that either a stiffer capital requirement or a higher deposit insurance premium reduces the bank interest margin. This directly weakens the ability of the bank to sustain loan losses and thus the soundness of the bank. In the restructuring of the deposit insurance system and capital regulation, these effects must be taken into consideration when making a policy prescription.
References


