SPATIAL COMPETITION AND NONLINEAR RESPONSES IN MARKETING

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES

(Commerce and Business Administration)

We accept this thesis as conforming
to the required standard.

THE UNIVERSITY OF BRITISH COLUMBIA

JULY 1993

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Abstract

Spatial competition, in the context of industry-wide changes in retailing formats and strategies, is addressed in this dissertation from a theoretical modelling perspective. Chapter 2 develops a normative individual choice model to explore how "power retailers" affect grocery shopping behaviour, and, consequently, market share. Power retailers are very large retail outlets that compete primarily on price, and are known variously as warehouse clubs, category killers, and superstores. The model shows that consideration of consumer stockpiling can lead to an "increasing returns" nonlinear response of market share to price reductions, and that the effect is not noticeable when competitors have small price differences. The model also differentiates between perishable and nonperishable goods, and shows that this may drive planned multistore shopping. Chapter 3 starts with the observation that competent management in many sectors of retailing, including grocery retailing, requires an ability to respond quickly and effectively to unexpected adversity. This dynamic is included in an oligopolistic spatial interaction model, and the system is shown to evolve to a novel and robust stochastic steady state known as self-organized criticality (SOC). One characteristic of the SOC state is that it allows small exogenous shocks to produce large responses at a rate greater than would be expected if the law of large numbers applied. This work represents the first known investigation of SOC in a marketing setting.
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Acknowledgements

It is with pleasure that I acknowledge the support and assistance of my committee members, Dr. Murray Frank and Dr. Ken Denike. It has been especially a privilege to work with my committee chairman and mentor, Dr. Charles Weinberg, whose encouragement, advice, and responsiveness were invaluable for the completion of this dissertation. Dr. Derek Atkins gave freely of his time to discuss inventory control theory, and an observation of his led to Lemma 7n.

I gratefully acknowledge the Social Sciences and Humanities Research Council, and the Faculty of Commerce and Business Administration for financial support.

Finally, to my wife Clair, who alone knows her contribution: thank you.

I dedicate this dissertation to my parents, Evert and Inga.
CHAPTER ONE

INTRODUCTION

Marketing academics and practitioners have an interest in spatial competition that dates back several decades. Of the two main branches of spatial competition research, namely positioning in product attribute space, and location of facilities, particularly retail outlets, in geographic space, this research is concerned with the latter. In this chapter, the structure of the dissertation is first briefly outlined, and then the two major sections discussed in more detail.

The context motivating the work is the grocery retailing business, a particularly dynamic, even volatile, sector of the retailing industry. It is characterized by intense competition and continually changing formats. This research explicitly recognizes this character and explores its consequences. Specifically, Chapter 2 develops a theoretical model to look at the impact of a retailing format—the "power retailer"—that has dramatically altered the nature of retail competition in many categories across North America over the last few years. In spite of power retailing's importance, only recently has it received research attention from marketing academics (e.g., Farris and Ailawadi, 1992).

In Chapter 2, a consumer decision model is developed that is particularly applicable in situations where consumers have a choice between power retailers (for example, "superstores") and more conventional outlets (e.g., supermarkets). One consequence of the model developed in Chapter 2 is the presence of increasing returns to scale to price reductions in the market studied. This nonlinearity suggests a number of interesting areas
Chapter One: Introduction
to pursue and Chapter 3 examines one of them. In particular, Chapter 3 tackles the nature of the long-run organization of an industry with the characteristics of the retail trade, and especially of food retailing. The rapid and major changes in formats, and the need to respond quickly and effectively to these changes because of the intense competition, are features that are captured in a simple model. The model, and the resulting stochastic steady state, are in the spirit of traditional theories like the "Wheel of Retailing" (McNair, 1958). The common thread that runs through the traditional work and the research reported here is the attempt to formalize the intuition that, in spite of the turbulent nature of retailing, there is some underlying order. Chapters 2 and 3 will now be discussed in more detail.

In Chapter 2, the impact of power retailers on the grocery trade is examined. "Power retailer" is a term referring to a retail format that has appeared in many retail categories over the last decade. It refers to very large retail outlets that compete primarily on price, and includes formats known as "superstores", "warehouse clubs", and "category killers". These stores become destinations in their own right, with large trade areas and dramatic impacts on suppliers, competitors, and customers. While channel relationships between suppliers and power retailers have been the topic of some research (e.g., Farris and Ailawadi, 1992), and the impact on competitors and customers is commonly discussed in the trade and popular press (e.g., Business Week, December 21, 1992; Financial Times of Canada, April 3, 1993), theoretical work on customers and competitors is very limited. In particular, there is a need to examine the fundamental issue of individual consumer choice behaviour in the presence of a power retailer, as it is consumer behaviour and the firm's ability to respond to it, which ultimately determines success or failure.
To address this gap, a model is developed and analyzed to determine timing and purchase quantities of optimizing consumers who trade off the costs of purchase price, travelling to and from the store, and holding inventory. Consumer stockpiling has long been an issue as a response to short-term promotions (e.g., Meyer and Assuncao, 1992), and has been modelled as a planned behaviour in the context of multipurpose shopping, and the resulting agglomeration of different types of retailers (that is, if consumers need to do some banking, drop off the dry cleaning, and pick up something for supper, it is more efficient for them to drive to one location where all three outlets are located close together, than to make several stops; see, for example, Ingene and Ghosh, 1990). The inclusion of consumer stockpiling as a factor affecting long-term choices between directly competing retailers is novel.

Food products have a characteristic which complicates the stockpiling problem: some goods are perishable and have much shorter storage lifetimes than others. This aspect of grocery retailing is very important to shopping behaviour and is explicitly taken into account in the model. While much research on perishable goods has been done in the operations research literature on inventory control, the inclusion in this context is unique.

In summary, Chapter 2 addresses the following questions:

• How does consumer grocery shopping behaviour change in the presence of a power retailer?
• How is market share affected when a power retailer enters a grocery market?
Chapter One: Introduction

One of the consequences of the model is a possible explanation of why incumbents in grocery retailing did not expect the new superstores to be as successful as they were: it is possible for the market share of a store to exhibit nonlinear increasing returns to price reductions, but only when the price differences between stores become great enough. These nonlinear returns were possibly not anticipated by traditional retailers (or by traditional models, as shown in Chapter 2). This leads to a more general observation, namely that retailing is such an intensely competitive and dynamic industry, that unexpected adverse consequences are the rule rather than the exception. To the extent that this is true, it follows that a component of successful management must be the ability to respond creatively and effectively to unexpected adversity. This leads to the main question addressed in Chapter 3:

• What is the nature of long-term industry structure when the dominant dynamic is unexpected adversity, followed by creative and effective response?

This response to competitive and environmental attacks is very nonlinear, and an unusual dynamic from the point of view of theoretical modelling; however, it will be argued in Chapter 3, that it is a commonly acknowledged characteristic of retailing (see, for example, Corstjens and Doyle, 1989).

This dynamic is formalized and embedded in a spatial interaction model. Spatial interaction models are well established probabilistic market share models used in empirical and decision-oriented work in retail outlet (and other facility) location problems (e.g., Jain
Chapter One: Introduction

and Mahajan, 1979).

Chapter 3 contributes not only by modelling an unusual and important dynamic, but also by showing that in the long run, this dynamic, operating in a spatial competition context, leads to a novel steady state. This state, known as self-organized criticality (SOC), has only once before been described in an economic context (Bak, Chen, Scheinkman, and Woodford, 1992). It has, however, received a very large amount of attention as a robust and general organizing principle in the physical and life sciences since it was first described in Bak, Tang, and Weisenfeld (1988).

The economic model of Bak et. al. (1992) is a highly stylized inventory and production model. While Bak et. al. (1992) is important in showing the possibility of SOC emerging in economics, it is mathematically isomorphic with the prototype model in statistical physics, with an economic interpretation imposed on it. The model developed in Chapter 3, in contrast, has its origins solidly in existing marketing models. As such, it is a first example of the possibility of self-organized criticality arising from a purely economic model.

The dissertation closes with Chapter 4. The results and contributions of the previous chapters are summarized. Limitations are noted, and resulting directions for future research suggested.

Research into spatial competition in marketing has a long and rich history. It is believed that the research reported here adds to our understanding of the field, particular in the complex and dynamic area of retailing.
Chapter 2

Grocery Shopping Behaviour in the Presence of a Power Retailer

2.1 Introduction

"Clout! More and More, Retail Giants Rule the MarketPlace", is the headline on a recent Business Week cover story. To quote, "In category after category, giant "power retailers" are using sophisticated inventory management, finely tuned selections, and, above all, competitive pricing to crowd out weaker players". Marketing academics, too, have recognized the phenomenon. The academic emphasis, however, has been primarily on the channel relationships. For example, Farris and Ailawadi (1992) discuss the apparent redistribution of channel power to the retailer. The nature of horizontal competition, and the impact on consumer shopping behaviour has yet to be investigated. The research in this chapter addresses those issues in the context of food retailing.

Specifically, a modelling approach is taken to answer two related questions: first, "How does consumer shopping behaviour change when a power retailer enters the grocery market?"; and, second, "What is the effect of the presence of a power retailer on share of market?". Assuming cost-minimizing shoppers, for whom three costs are important (trip costs, inventory costs, and price), and recognizing that some goods are perishable, the model provides several important insights. Two are that, first, stockpiling behaviour becomes dramatically more important as the price reduction of the power retailer becomes greater, leading to an "increasing returns" effect of price reductions; and, second, the difference between perishable and nonperishable goods can lead some consumers to have normative strategies of shopping at more than one store, even when there is no uncertainty. Combining
Chapter 2: Grocery Shopping Behaviour

these two results, it is shown that perishable goods are much less susceptible to the influence of a power retailer entering the market, than are nonperishables.

Food retailing is an appealing industry to initially investigate the power retailing phenomenon for two reasons. First, total demand for groceries is relatively inelastic compared to many other retail goods (Ghosh and Harche, 1992). This allows modelling to focus on the allocation of the food dollar to the competing retailers. Second, the food and drug category of retailing is relatively well defined. It will be helpful, for subsequent empirical work, to have well defined market boundaries.

One case which helps illustrate and motivate this research is the entry of the Real Canadian Superstore (RCS) to the western Canadian grocery retailing industry, during the last decade. In the major cities of western Canada (Winnipeg, Calgary, Edmonton, and Vancouver), the retail grocery business was dominated by supermarkets, led by Safeway, in the early eighties. The supermarkets were typically 20,000 to 40,000 square feet, and traded in a relatively localized area of a few kilometres in diameter. Price competition appeared to be healthy, with much promotion and advertising. RCS entered various cities in the mid-eighties, with an expansion westward, opening in Edmonton in 1984, and in Vancouver in 1989. RCS stores were over 100,000 square feet, of which about 60% to 70% was devoted to food and drugs. Labour and land costs of the RCS outlets were kept dramatically lower than Safeway's, by employing non-union staff, and locating in industrial areas. The resulting reduction in prices, as well as some increase in variety, appeared to make the outlets a destination, so that the unfavourable locations were not a disadvantage. RCS, like power retailers in other categories, drew tremendous market share with very few outlets. One
of the interesting aspects of the entry of RCS to the market, and one which a model should be able to explain, is Safeway’s early response. According to private communications with consultants and academics in Winnipeg, Edmonton, and Vancouver, Safeway was surprised by the impact of RCS on the market. The modelling approach in this chapter suggests a reason: the market’s highly nonlinear response to price differences that arises when consumers are allowed to stockpile. Consumers at unusually long distances from the Superstore can make large purchases, in order to make the long trip less frequently, and thus keep long-run average trip costs down. If the price advantage is great enough, the stockpiling costs will be offset, and the power grocery retailer can capture a huge trading area.
2.2 Literature Review

The model developed in this chapter draws on literature in three areas: consumer stockpiling in response to promotions, normative inventory control theory, and multipurpose shopping. First the relevance of these areas will be stated, and then the literature reviewed.

The literature on promotions in retailing establishes, first, the value of normative inventory models in consumer shopping (eg., Meyer and Assuncao, 1992). Second, it establishes that consumers make tradeoffs between the costs of the goods purchased ("price costs") and inventory holdings (eg., Blattberg, Eppen, and Lieberman, 1981). Third, it recognizes the "fill-in trip", a minor trip that occurs between major trips, which has a parallel with the shopping behaviour studied in this research (eg., Kahn and Schmittlein, 1992). Finally, this literature is also useful for structuring consumer's holding costs. Although perishability is recognized as a relevant factor in the promotion literature, it has not previously been explicitly treated as an inventory cost. There is, however, extensive work on perishable goods in the operations research inventory control literature (Nahmias, 1982). The form of the perishability costs are taken from this literature.

As an inventory control problem, the situation would fall into the class of joint replenishment problems (JRP). Joint replenishment refers to the problem of maintaining inventory of more than one good, when there is the possibility of reducing replenishment costs by ordering the goods simultaneously. The problem analyzed here is different from the solved JRP problems in the types of goods (perishable and nonperishable) analyzed, and in differences in cost structure associated with consumers, as opposed to firms. An example of a JRP that, like this problem, has periodic solutions, is Andres and Emmons, 1975.
Chapter 2: Grocery Shopping Behaviour

The notion that consumers trade off price against travel costs has a long history. In the last decade inventory costs have also been included in this tradeoff, in the multipurpose shopping literature (Ghosh and McLafferty, 1986; Ingene and Ghosh, 1990). Multipurpose shopping refers to the observation that consumers will make a single trip to a central location to purchase more than one good or service (Christaller, 1933). The model developed in this research is similar in spirit to the multipurpose shopping literature in that consumers are assumed to make a choice on the basis of cost minimization over transportation, inventory, and price costs. It differs in its objectives (choice between stores differentiated by price, rather than between "order", or level of agglomeration, of centre), the detailed cost structures, and types of goods (perishable vs nonperishable) considered.

Now, each of these three areas will be discussed more fully.

Promotions and Stockpiling

A large literature exists on consumer grocery shopping behaviour in the presence of promotions, and a substantial proportion of this deals with consumer stockpiling as a response to promotions. The usual concern is that if purchases are simply accelerated in time or quantity in response to promotions, they may not be beneficial to the manufacturer or retailer (Neslin, Henderson and Quelch, 1985; Gupta, 1988). This concern, and the context of short term responses to promotions, is quite different from the long term behaviour addressed in this chapter. (For an excellent current review of the promotions literature see Blattberg and Neslin, forthcoming). However, some of the results that are relevant to this study will be reviewed here.
Blattberg, Eppen, and Lieberman (1981) present a theoretical model where consumer stockpiling may be beneficial to the retailer. Under the assumptions of a single brand and a single optimizing retailer, and optimizing consumers (trading off price against holding costs) who have holding costs less than half the retailer's holding costs, the retailer may increase profits by reducing holding costs through dealing. This reason for dealing is contrasted with an alternate explanation based on manufacturer's trade deals offered to retailers, to force retail price reductions and encourage consumer trial. An empirical test finds the inventory explanation consistent with the data, and, more importantly for the model developed in this chapter, suggests that consumers do trade off price against holding costs.

Another important reason for dealing is to attract customers to the store, and then hope that they will buy non-deal items while they are there. Kumar and Leone (1988) investigate this store substitution effect in the disposable diapers product category. They find that price promotions in one store negatively affects the same-category sales in competing stores, indicating that promotions do attract customers to the store. However, the effect was dependent on geographic proximity of the competing stores, implying that there was a limit the customers would travel to take advantage of a promotion. Whether this is a planned tradeoff between price and travel costs, as assumed in the long-term context of the research in this chapter, or simply opportunistic shopping within a local set of stores, is not addressed. Walters (1991) found similar, although weaker, interstore effects in spaghetti and cake mix categories.

Customers may neutralize the promotional objective described above in a number of ways, such as "cherry-picking", (shopping at different stores and purchasing only the deal
items, e.g., Bucklin and Lattin 1992) or stockpiling deal items that would have been purchased in any case. Much of the research in this area involves testing for stockpiling or cherry-picking, based on panel data, and initially did not find consistent patterns. While Blattberg et al. (1981) found evidence of purchase quantity and timing acceleration, McAlister (1985) found minimal acceleration. This led to various attempts to sort out why some studies found stockpiling and some did not (eg., Helsen and Schmittlein, 1992). Litvack, Calantone, and Warshaw (1985) suggested that some goods were more susceptible to deals because they were "stock-up" goods. A "stock-up item" was defined as "any nonperishable good in a unit size that is consumed frequently by a purchaser's household". Similarly, a non-stock-up item was either perishable or infrequently consumed. Using expert judges to select 36 stock-up and 36 non-stock-up items, a four-store price experiment was run to determine elasticities. The hypothesis, that elasticity for stock-up products would be greater than for non-stock-up products, was strongly supported. This suggests not only that consumers stockpile in response to price reductions, but that the response varies according to "stockpilability", one component of which is perishability.

Raju (1992) examines the effect of promotional activity in a product category, as well as category characteristics, on sales variability. In particular, he found that "bulkiness" reduces sales variability, consistent with the notion that "stockpilability" is important to consumers when considering price reductions.

Another predictor of the effectiveness of promotions is whether the trip is "major" or "fill-in". The notion of a filler trip has a long history in the marketing literature (Kollat and Willet 1967; MacKay 1973) and is relevant because the model in this chapter predicts
a similar (although not identical) type of behaviour. Filler trips are defined in various ways in the literature, but always on the basis of expenditure. Major trips are large expenditure trips, and filler trips involve smaller expenditures. In applying Ehrenberg's negative binomial buying model to filler trips, Frisbie (1980) considers three definitions of a filler trip. One is based on an absolute dollar threshold as a cutoff; one is a threshold percentage of monthly food expenditure and varies by household; and one is a threshold based on a percentage of annual income, and also varies by household. Frisbie finds that the different definitions have little effect on the ability of the NBD to describe the shopping behaviour. Kahn and Schmittlein (1992) define the cutoff on a household basis by constructing a histogram of amounts spent on each trip for the household. If the histogram is unimodal, the mode is used as the cutoff. If it is bi-modal, the midpoint between modes is used as the cutoff. They then use the definition to correlate the type of trip with various types of promotions. In all of this research, the distinction between trip types is a defined quantity, based on expenditures on goods. In the research of this chapter, the different trip types are the result of optimizing behaviour; and not all customers will find it optimal to engage in fill-in shopping. Furthermore, the differences in trip types are not only size of expenditures. They are also related to expected long-term price differences, and the perishability of the goods purchased.

Fill-in trips also occur at shorter intervals than major trips. MacKay (1973) found that major trips tended to occur weekly, with fill-in trips more frequently. This pattern of regularly weekly shopping trips and irregular fill-in trips is commonly observed (eg., Kahn and Schmittlein, 1992). The general pattern is consistent with the results of this chapter;
however, the time scales are substantially increased. It will be argued that in the presence of a power grocery retailer, there are some consumers who add another level of trip—a "super trip" for which the previous "major trip" becomes a fill-in trip.

**Normative Inventory Theory**

One of the unique features of the model in this chapter, compared to existing consumer shopping models, is that perishable goods are treated explicitly and are a critical factor in governing shopping trip behaviour among competing stores. Perishable goods, however, have been treated extensively in the theoretical inventory management literature. Of the many kinds of perishability studied (e.g., deterministic and stochastic demand, deterministic and stochastic product lifetimes, and continual deterioration; see Nahmias, 1982 for a review) this chapter considers the case of fixed deterministic lifetimes and demand rates for the perishable good. This case is analyzed in the perishable inventory literature when the perishable good also has time-dependent holding costs, and under fairly general conditions it is shown that ordering will occur such that no items expire, that is, become non-usable. In contrast, the model here has one good with only holding costs, and a second good with only perishability costs. Results for this case are derived in section 2.4, and shown to be parallel to the case of one good with both holding and perishability costs.

Another inventory theory aspect of the model presented here concerns the costs associated with buying more than one good. In the model developed here, once customers have incurred one trip cost, they may purchase either or both goods with no additional trip, or "ordering cost". The inventory management literature refers to this as the joint replenishment problem (JRP). The consumer-based variation considered here could be
considered as an example of the infinite horizon JRP problem. There are two important
differences, however. First, the JRP literature assumes variable costs are constant and are
therefore ignored. In the consumer model here, we are concerned explicitly with the
competitive situation where there are price differences between stores, and prices of the
goods translate to the variable costs in inventory management. Second, the JRP literature
assumes there is a "major set-up cost" for an order, plus an additional "minor set-up cost"
for each type of good included. For the consumer problem, there is only a "trip cost",
which corresponds to the major set-up cost.

The consumer model may be considered as a member of the subclass of JRP
problems with continuous time, an infinite horizon, and deterministic constant parameters.
Quoting from Iyogun (1987),

The common denominator [of this subclass] is that they use equally spaced
replenishment epochs. These policies are not necessarily optimal... The
optimum referred to in all these papers where 'optimum' is specifically
mentioned should be qualified by the word 'periodic'. It is still an open
problem what problem parameters ensure the existence of periodic optimum
policies, except two cases. The first case is obvious and this is the case when
all items have equal parameter values... The second case... has item-dependent
set-up costs but with a fixed saving when all items are replenished together.
For this problem, Andres and Emmons (1975) showed that an optimum policy
has equally spaced replenishment epochs for each item. Even the problem of
finding optimal periodic policies is not a trivial task. Because of this,
researchers have focused mainly on developing approximate methods for
finding good periodic policies.

In this chapter, it is argued that the optimal policy for the consumer stockpiling model is
"almost always" (in the sense of the probability approaching unity) periodic, and that if there
is a non-periodic optimal policy, there is also a periodic optimal policy. This allows the
derivation of exact numerical solutions to the problem.
There are few examples of normative inventory theory applied to consumer stockpiling. Exceptions include Blattberg et. al. (1981), Jeuland and Narasimhan (1985), and Meyer and Assuncao (1992). The latter uses a set of results derived by Golabi (1985) in inventory control policy to describe optimal buying strategy. The application, again, is consumer buying strategy in the presence of promotions and the opportunity for stockpiling, which differs from the context of this chapter. However, the main thrust of the research is to empirically validate the normative policy in an experimental setting. As Meyer and Assuncao state, "It is a central tenet of this research that purchase quantity decisions are made through the use of heuristics which, while perhaps not optimal in structure, nevertheless serve to mimic the central properties of optimal inventory control policies." For the purposes of this chapter their main conclusion is that, while some systematic biases do exist, purchasing patterns were positively correlated with those predicted by the normative model.

Multipurpose Shopping

A final stream of literature that has much in common with this chapter is the multipurpose shopping literature. With its origins in Central Place theory (Christaller, 1933), this literature considers "high order" and "low order" centres, or shopping locations. The high order centre has more goods, some of which may be purchased less frequently, and are located further apart from each other than low order centres. In the last decade, several papers have appeared which model consumer shopping behaviour in order to show how these centres develop (McLafferty and Ghosh, 1986; McLafferty and Ghosh 1987; Ingene and Ghosh 1990). The consumers are trading off travel costs against storage costs (and
occasionally price of goods) in the presence of at least two goods, that have different consumption rates. As a result, the two different kinds of centres develop. While the context and objectives of these models differ from the model in this chapter, the form, as a cost minimization of travel, inventory, and price costs for more than one good at more than one location, is quite similar. The models also differ in the explicit form of the costs. In the papers referred to above, inventory costs are related to price. In the model here, it is argued that it is preferable to consider inventory costs as proportional to quantity, for non-perishable goods. Furthermore, the second good is differentiated by being perishable, so that the inventory cost for it is the loss associated with any expired goods. A second difference is that prices for both goods differ by store in this model. In the multipurpose shopping literature, price does not have a critical role, and may be left out. Finally, the solution method in the multipurpose shopping literature involves relaxing the constraint that the number of trips is a nonnegative integer, so that calculus may be applied. In this chapter, the integer constraint will be retained. In summary then, the multipurpose shopping literature is relevant primarily because the general structure of the consumer's cost minimization problem is the same as that used here.
2.3 Model Development

Assumptions, notation, and rationale for the model components follow.

**Goods:** Two goods, one perishable and one non-perishable, are available to consumers. Subscripts \( *_p \) and \( *_n \) denote quantities attached to the perishable and non-perishable goods respectively.

**Stores:** Each of the two goods is available at two stores, which, in keeping with the power retailer theme, are identified as the "large" and the "small" store, with subscripts \( *_l \) and \( *_s \). The two stores are differentiated by price, with the small store the most expensive on both goods, and location. Thus, there are four prices: \( P_{n,l}, P_{n,s}, P_{p,l}, \) and \( P_{p,s} \).

**Cost Minimization:** Consumers minimize long-run average costs over an infinite horizon. Total costs consist of trip costs \( C_t \), plus storage costs \( C_s \), plus the price of the goods \( C_p \).

**Trip Costs:** Trip costs consist of a fixed amount \( c_s \) or \( c_l \) incurred for each trip to the small or large store. Later, trip costs will be assumed proportional to the Euclidean distance between customer and store, but in the early sections of this chapter, they will remain general. It should be noted that trip costs correspond to order costs or set-up costs in the inventory management literature, but that, unlike the inventory management literature, the costs are *not* tied to the goods. In particular, once the customer is in the store, there is no further cost whether she purchases one or both of the goods.

**Non-perishable good inventory costs:** Consumers have quantity-dependent instantaneous storage costs \( s \cdot Q_n(t) \) for the non-perishable good, where \( s \) is the cost per unit quantity per unit time, and \( Q_n(t) \) is the quantity of the non-perishable good on hand at time.
This serves to limit the quantity purchased, which would be unlimited if only trip costs were considered. Note that since price appears explicitly elsewhere in the model, this formulation makes storage costs NOT dependent on price. This is in contrast to the inventory management literature, where it is implicitly assumed that the holding costs include the cost of capital; and in contrast to the multipurpose shopping models (e.g., Ghosh and McLafferty, 1987), where the holding cost is proportional to the monetary value of the stock. Quantity-dependent holding costs are used here for three reasons. First, given that the frequency of grocery shopping is on the order of weekly, it seems unlikely that the cost of capital would play a significant role in limiting the amount of consumers' purchases compared to storage and transportation capacity limitations (how big is the car and the cupboard?), and time limitations for a single trip (how many hours can the customer stand being in a grocery store?). Second, an objective of this research is to investigate the effects of price differences between stores, and that objective is facilitated by keeping price as a separate component. The third reason is empirical. Litvack, Calantine and Warshaw (1985) and Blattberg, Eppen and Lieberman (1981) both conclude that "bulkiness" lowers the incentive to stockpile in the presence of promotions. More recently, Raju (1992) investigated how the variability in category sales, where the variability is largely due to price discounting, depends on category characteristics. While the relationship between category expensiveness and variability was not statistically significant, bulky categories had significantly lower variability. To the extent that sales variability reflects a stockpiling response to discounts, this supports the modelling of storage costs as quantity-dependent and price-independent. Note that if there were more than one nonperishable good, and the goods
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differed in bulkiness, this argument would require that a scale factor on the Q variable be applied in the storage cost formulation to account for the difference. However, here there is only one nonperishable, so the relation between bulkiness and quantity may be ignored.

Storage costs over time are given by

\[ \int_{0}^{T} sQ(t)dt \]  \hspace{1cm} (1)

An amount Q purchased and immediately consumed at the constant rate D will have a storage cost (Figure 1):

\[ \int_{t_0}^{t_0+Qn/D_n} s(Q_n - D_nt)dt = \frac{Q_n^2}{2D_n} = \frac{Q_n\Delta t}{2} = \frac{D_n(\Delta t)^2}{2} \]  \hspace{1cm} (2)

![Diagram of stockpiling parameters](Figure 1: Stockpiling Parameters)

Perishable Good Inventory Cost: The perishable good has a lifetime \( \Delta t_e \) after purchase, and incurs an inventory cost equal to the price paid for any quantities that expire before they are consumed. If the expired quantities, which must be disposed of, are given
by $Q_{p, \text{expired}}$, the cost is $P_p Q_{p, \text{expired}}$. Inventory management models consider both sudden expiry and gradual deterioration. This model only considers the former, which, for groceries has face validity, in that many items are marked with an expiry date. It also seems reasonable that even for goods like vegetables that do not have marked expiry dates, the consumer has to decide whether to consume or dispose of the product. It is assumed that the quantity-dependent storage costs associated with the perishable good are negligible compared to the other costs in the model, but nonzero. The rationale for this negligibility is that the consumer always has enough room to transport and store (eg., in the refrigerator) more perishables than she can consume before they expire. In other words, the expiry time, rather than the quantity-dependent storage costs, impose the limitation on the amount of perishables purchased on a single trip, which simplifies the analysis. The nonzero assumption is also reasonable, since the perishable goods do take up some room, and allows the derivation of analytic results.

**Stockouts:** No stockouts of either good are allowed.

**Discounting:** The time value of money is assumed negligible compared to other costs, and not considered. This is consistent with the promotions literature, the multipurpose shopping literature, and much of the JRP literature. Although the model is infinite horizon, grocery shopping trips occur frequently enough and expenditures are large enough that it is difficult to imagine the time value of money entering into the consumer’s calculations.

**Demand:** Demands are determined by consumption rates, denoted $D_a$ and $D_p$, and are constant. Ingene and Ghosh (1990) discuss this assumption in the spatial shopping context:

A number of authors have worked with inelastic demand models (see, for example, Bacon 1984; Ghosh and McLafferty 1984; Hotelling 1929;
McLafferty and Ghosh 1986; and Thill 1985) One interpretation of the fixed quantities model is that it is representative of a rectangular demand curve, provided that delivered price does not exceed the maximal demand price (Ingene, 1975). The alternative approach of utility maximization has been explored in the multipurpose shopping context by Mulligan (1987), among others. However, its complexities have caused Thill and Thomas, citing the work of Bacon (1984), to write that "it seems that the neoclassical approach--has few prospects, unless the utility-maximizing behaviour is relaxed and replaced by a cost-minimizing one" (1987, p. 8). We recognize, of course, the importance of extending the analysis in the future to consider utility-maximizing behaviour.

Table 1: Notation

Subscripts:
- n: non-perishable
- p: perishable
- s: small (expensive) store
- l: large (cheap) store
- i: visit index to large store
- j: visit index to small store

$Q_{n,i}$, $Q_{n,s}$, $Q_{p,l}$, $Q_{p,s}$: Quantities of each good purchased at each store
(time subscript omitted)

$P_{n,i}$, $P_{n,s}$, $P_{p,l}$, $P_{p,s}$: Prices of each good at each store

$t_{i}$, $t_{ij}$: time of $i^{th}$ ($j^{th}$) visit to each store

$\Delta t_{i}$, $\Delta t_{ij}$: time intervals between purchases

$D_{n}$, $D_{p}$: consumption rate of each good

$\Delta t_{e}$: time between purchase and expiry of perishable good

$s$: instantaneous storage cost of nonperishable good

$c_{l}$, $c_{s}$: trip cost to each store

$C_{t}$: long-run average trip costs

$C_{s}$: long-run average storage costs

$C_{p}$: long-run average price costs

$C$: total long-run average costs

Decision Variables: The consumer's problem is to decide when to shop at which store, and how much to buy. This is formalized by identifying shopping patterns, or sequences of visits to the stores (for example the sequence "large, small, small, small,
large"), and optimizing over purchase quantities $Q_{n,l}$, $Q_{n,a}$, $Q_{p,l}$, $Q_{p,a}$, and associated purchase occasions $t_{i,j}$ and $t_{a,j}$. Through a series of lemmas and propositions, the huge number of possible shopping patterns are reduced to a manageably small set of possibly optimal patterns, for which analytic expressions for the cost function are derived. These cost functions can then be minimized over $Q_{c,j}$, and the smallest selected as the optimal pattern.

**Domain:** All parameters are positive real. Quantities are non-negative real.

Notation is summarized in Table I.
2.4 Results for Some Simple Cases

Before addressing the full problem of two goods at two stores, results for some restricted cases are presented. This helps to clarify the issues involved, and some of the results will be used later.

2.4.1 Non-perishable goods and one store only

For a single good available at a single store, we may ignore price, and the well-known Economic Order Quantity (EOQ) result from inventory theory applies (e.g., Clark and Howe, 1962). Equal quantities are ordered at equal time intervals, with the inventory just at zero at each repurchase occasion ("zero inventory rule"). Omitting the price the customer pays for the goods ("price costs") for now, because they only have an impact when shopping from two sources with different prices, the optimal long-run average quantities, times, and costs are:

\[
Q_n^* = \sqrt{\frac{2cD_n}{s}}
\]

\[
\Delta t_i^* = \sqrt{\frac{2c}{sD_n}}
\]

\[
C^* = \sqrt{2scD_n}
\]

Now suppose there are two non-perishable goods, differing only on storage costs, \(s_1\) and \(s_2\), and purchased at a single store. Note that even if the goods have different prices,
price will not affect the solution, because goods are available at only one store and the consumer must always have some on hand (and we are not considering the time value of money). It is necessary to consider the possibilities that only one or the other good is purchased on a trip, or that both goods are purchased on a trip. Consider first a fixed horizon, during which time \( f_1 \) purchases of good 1, and \( f_2 \) purchases of good 2 are made. Let the number of trips to purchase only good 1 be \( m_1 \); the number of trips to purchase only good 2 be \( m_2 \); and the number of trips to purchase both goods be \( m_{12} \). We note that \( m_1 + m_{12} = f_1, m_2 + m_{12} = f_2, m_1 + m_2 + m_{12} = f_1 + f_2 \), and that the zero inventory rule still holds for each good (if any quantity of the good were on hand at repurchase time, storage costs could always be reduced by purchasing less there time before), so that the integrated storage costs may be expressed as the summation of terms like (2). The consumer's problem is then

\[
\text{MIN } C_T \quad \text{subject to } \begin{align*}
m_1, m_2, m_{12}, f_1, f_2, Q_1, Q_2
\end{align*}
\]

where

\[
C_T = c(m_1 + m_2 + m_{12}) + \sum_{i_1=0}^{f_1-1} s_1 \frac{Q_{i_1}}{2} \Delta t_{i_1} + \sum_{i_2=0}^{f_2-1} s_2 \frac{Q_{i_2}}{2} \Delta t_{i_2} \quad (4)
\]

and, as in the usual single good case, the no-stockout and zero-inventory rules imply constraints on the total quantity purchased over the horizon \( T \):
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With only one store, and two non-perishable goods with different storage costs, the following proposition states that the customer purchases both goods each time he shops.

**Proposition 1:** For $s_1, s_2 > 0$, (4) above is minimized only when $m_1 = m_2 = 0$.

Proof: Suppose not. Suppose that for an optimal $C_T^*$, $m_1^* \neq 0$. Consider $C_T'$, where $m_1' = m_1^* - 1$, and $m_2' = m_2^* + 1$, that is we replace one of the "good-one-only" trips with a "both-goods" trip. The first and second terms of equation (4), namely the trip costs and the summed storage costs for good 1, are unaffected. The third term, the summed storage costs for the second good, has one of its summands (say, the $j^{th}$) replaced with two summands. It suffices to examine this summand, as all others are unaffected.

For the second good (dropping the goods subscript), we have that the $j^{th}$ time interval, and the $j^{th}$ quantity purchased are now divided: $\Delta t_j^* = \Delta t_j' + \Delta t_{j+1}'$, and $Q_j^* = Q_j' + Q_{j+1}'$. Let the affected term be $K_j^* = sQ_j^*\Delta t_j^*$, and the new terms $K_j'$ and $K_{j+1}'$ be similarly defined. Then

$$K_j' = \frac{s}{2}(Q_j' + Q_{j+1}')\Delta t_j' + \Delta t_{j+1}'$$

$$= K_j' + K_{j+1}' + \frac{s}{2}(Q_j'\Delta t_{j+1}' + Q_{j+1}'\Delta t_j')$$

$$= \sum_{i_1=0}^{f_1-1} Q_{i_1} = Q_1 = D_1 T$$

$$\sum_{i_2=0}^{f_2-1} Q_{i_2} = Q_2 = D_2 T$$
implying $K_j^* > K_j' + K_{j+1}'$, and hence $C_T^* > C_T'$. Thus $C_T^*$ cannot be optimal, and $m_1$ (and similarly $m_2$) must be zero $\Box$.

Proposition 1 gives $f_1 = f_2 = f$, and identical interpurchase time intervals for both goods, $\Delta t_{1i}^* = \Delta t_{2i}^*$ (although not yet necessarily equal for all $i$). Since each pair of interpurchase intervals are the same, we also have $Q_{1i} = (D_1/D_2)Q_{2i}$, so we may write the cost function as

$$C_T = cf + \sum_{i=0}^{f-1} (s_1 + D_2 s_2) \frac{Q_{1i}}{D_1} \Delta t_i$$

which is identical to the single good cost function, with the storage cost replaced by a weighted sum of the different storage costs. Hence the usual EOQ solutions given in Eqn. (3) apply, with $s$ replaced by the weighted sum of $s_1$ and $s_2$, and therefore interpurchase times are equal for all $i$. Times, quantities, and long-run average costs for the case of two non-perishable goods available at one store are thus:
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2.4.2 Perishable goods at one store only

The optimal shopping policy for one perishable good at one store only is obviously to shop at intervals equal to the expiry time of the good, and purchase just enough to last to expiry. This simple case will be used as an opportunity to demonstrate the roles of the various model assumptions by deriving the result formally.

The no-stockout assumption has two slightly different expressions depending on the relation between the amount purchased on a trip, the consumption rate, and the expiry time:

\[ \frac{Q_{p,i}}{D_p} > \Delta t_e \Rightarrow \Delta t_i < \Delta t_e \]  \hspace{1cm} (9)
Equations (9) and (10) simply state that if more perishable goods are purchased than can be consumed before expiry, the repurchase interval must not exceed the expiry time; and if less are purchased than can be consumed before expiry, the repurchase interval must not exceed the purchased quantity divided by the consumption rate.

To minimize inventory costs, which for perishable goods are assumed to be the price of any expired unconsumed goods, the consumer must avoid purchasing more than she can consume in the expiry time:

$$\frac{Q_{p,i}}{D_p} \leq \Delta t_e \implies \Delta t_i \leq \frac{Q_{p,i}}{D_p}$$

(10)

Hence, only (10) above is allowed. Finally, minimizing the long-run average travel costs implies that, since each trip cost is fixed, the interpurchase interval $\Delta t_i$ must be maximized. In the right hand side of (10), then, equality must obtain. Maximizing the interpurchase interval implies maximizing the quantity purchased, so that equality also obtains in (11). The optimal policy is (ignoring price costs as in the non-perishable case):
\[
\Delta t_i^* = \Delta t_e
\]

\[
Q_{p,i}^* = D_p \Delta t_i^* = D_p \Delta t_e
\]

\[
C_T^* = \frac{c}{\Delta t_e}
\]

Minimum long run average costs are just the averaged trip costs, which occur at intervals equal to the lifetime of the good.

2.4.3 Perishable and non-perishable goods

A well-established result in the perishable inventory management literature is that if a good has both holding costs and perishability costs, the optimal order quantity is (Nahmias, 1982):

\[
Q^* = \min\left( \sqrt{\frac{2cD}{s}}, D\Delta t_e \right)
\]

In other words, the optimum is the amount considering only the storage costs, unless that is more than can be consumed before expiry, in which case the consumer purchases only enough to last to expiry.

The case here is similar, in that two goods, each of which has only one type of cost, are purchased at a single store. The results parallel Equation (13). To clarify the model structure and assumptions, first we will consider the total costs that would be incurred for two goods if each had both quantity-dependent storage costs and perishability costs, for the
finite horizon. The formulation will then be explicitly restricted so that there is one good of each type, and, using previous results, solved for the number of trips of each type over the finite horizon. Finally, the optimal timing of trips for the infinite horizon will be found.

The general formulation for trip and inventory costs over a finite horizon $T$, for two perishable goods, (identified by subscripts 1 and 2) with storage costs and expiry times is

$$C_T = c (m_1 + m_2 + m_{12}) + \sum_{i_1=0}^{f_1-1} s_1 \frac{Q_{i_1}}{2} \Delta t_{i_1} + \sum_{i_2=0}^{f_2-1} s_2 \frac{Q_{i_2}}{2} \Delta t_{i_2} \quad (14)$$

such that the sum of the quantities purchased over the horizon $T$ are fixed by the demand rate,

$$\sum_{i_1=0}^{f_1-1} Q_{i_1} = D_1 T \quad (15)$$

$$\sum_{i_2=0}^{f_2-1} Q_{i_2} = D_2 T$$

and repurchase must occur at or before expiry:
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where \( m_1, m_2, m_{12}, f_1, \) and \( f_2 \) are defined as in the previous section. Note the expression of the integrated storage costs in (14) as summations result, again, from the "zero inventory rule": it is never optimal to be holding inventory when a shopping trip is made.

The above formulation is now specialized as follows. Assume good 1 has negligible storage costs \( (s_1 \to 0) \), so that the summation in (14) for good 1 is much less than the remaining terms. Good 1 is the perishable good, and as argued before, its quantity-dependent storage costs are ignored because of the assumption that perishability limits the quantity purchased much more severely than any other consideration; for example, it is easy to buy and store more tomatoes than can be eaten before they spoil. Now assume good 2 is not perishable \( (\Delta t_2 \to \infty) \); spaghetti will keep in the cupboard for a long, long time, so the related constraint in (16) may be ignored.

Since \( s_1 \) and \( s_2 \) are both strictly positive, Proposition 1 applies: it is always less expensive to buy both goods on each trip: \( m_1 = m_2 = 0, f_1 = f_2 = f, \) and \( \Delta t_{i1} = \Delta t_{i2} = \Delta t_i = Q_{i1}/D_1 = Q_{i2}/D_2. \) This means the timing problem is isomorphic with the problem of purchasing one good that has both costs. If the EOQ solution for the non-perishable good by itself gives an interpurchase time \textit{less} than the expiry time of the perishable good, then \textit{that} interpurchase time controls the purchase of both goods. The perishability constraint (16) is not binding. On the other hand, if the EOQ interpurchase time exceeds the expiry time,
then the perishability constraint is binding, and the expiry time controls the purchase of both goods.

Formally, the cost function with the above restrictions is now

\[
C_T = \sum_{i=0}^{f-1} \left( c + s_n \frac{Q_{i_p} \Delta t_i}{2} \right)
\]  

such that

\[
\sum_{i=0}^{f-1} Q_{i_p} = D_p T
\]  

\[
\sum_{i=0}^{f-1} Q_{i_n} = D_n T
\]

\[
\Delta t_i \leq \Delta t_{e_p}
\]

where the goods subscripts have been relabeled \( n \) and \( p \). Note that the perishable good does not enter the cost function directly; it only operates through constraint (20). Thus, if (20) is not binding, the problem is the same as the single good case with storage costs only.

Perishability Non-Binding Case:

Assuming the perishability constraint is not binding, and following the standard procedure for determining the EOQ in inventory theory, we note that the purchase quantities are equal, and equally spaced, for a convex cost function (Clark and Howe, 1962). Alternatively, explicitly minimizing (17) over \( Q_m \) subject to (18) gives the finite horizon
solutions. With the occasion subscript, "i", dropped on the purchase quantities because they are all identical, but retained on the interpurchase time \( \Delta t_i \) to distinguish it from the expiry time \( \Delta t_e \), the optimal quantities and times, and minimum cost, are easily shown to be

\[
Q_n^* = D_n \Delta t_i^* \\
\Delta t_i^* = \frac{T}{f} \\
C^*_T = fc + \frac{sD_n T^2}{2f}
\]

(21)

To extend to the infinite horizon case, consider the long-run average cost

\[
\bar{C}_T^* = \frac{C_T^*}{T} = \frac{c}{\Delta t_i^*} + \frac{sD_n \Delta t_i^*}{2}
\]

(22)

and minimize over the interpurchase interval \( \Delta t_i^* \). This gives the usual EOQ solutions (where we now take the " * " to mean optimal over quantity and time, over the long run) as in equation (3):

\[
Q_n^* = D_n \Delta t_i^* \\
\Delta t_i^* = \sqrt{\frac{2c}{s_n D_n}} \\
\bar{C}_T^* = \sqrt{2s_n D_n c}
\]

(23)
and invoking Proposition 1,

\[ Q^*_p = D_p \Delta t^*_i \]  

(24)

Once again, we note that since there is only one store, it is not necessary to consider price, and hence the perishable good has no costs uniquely associated with it.

**Perishability Binding Case:**

Assume now that the perishability constraint (20) is binding. Then (dropping the subscript "p" on the perishability constraint),

\[ Q^*_n = D_n \Delta t_e \]

\[ Q^*_p = D_p \Delta t_e \]  

(25)

\[ C^*_T = \frac{c}{\Delta t_e} + \frac{s_n D_n \Delta t_e}{2} \]

From (20) and (23), the condition for the perishability constraint to be binding is

\[ \sqrt{\frac{2c}{s_n D_n}} > \Delta t_e \]  

(26)

Finally, to establish that this is the complete set of solutions, we note that the Kuhn-Tucker first-order conditions are sufficient for the minimization problem, because the cost function is convex and the inequality constraint, (16), linear. We may now write, in analogy with (13),
\[ Q^*_n = \min\left( \sqrt{\frac{2cD_n}{s_n}}, D_n\Delta t_e \right) \]

\[ Q^*_p = \min\left( D_p\sqrt{\frac{2c}{s_nD_n}}, D_p\Delta t_e \right) \]

\[ \Delta t^*_i = \min\left( \sqrt{\frac{2c}{s_nD_n}}, \Delta t_e \right) . \]
2.5 Two Goods at Two Stores: Feasible Normative Behaviour

With the introduction of two stores, each with its own prices, the problem is complicated by the huge variety of qualitatively different shopping behaviours. Given a sequence of store visits (e.g., small, large, small, small...) and an associated sequence specifying which goods are purchased, one could in principle minimize the total cost over the quantities purchased, and purchase timing. However, with an infinite horizon, there are an infinity of these sequences of shopping patterns. The same problem is encountered in the JRP literature, where the analysis is simplified by restricting the problem to periodic policies, which allows the analysis of only one period. That a periodic policy is in fact generally optimal has only been demonstrated for a few situations (Andres and Emmons, 1975; for a review, see Iyogun, 1987), all of which are simpler than that considered here.

Figure 2: A complete and mutually exclusive classification of shopping behaviour.

First, consider the three-way classification of shopping only at the small store, only at the large store, and at both (Figure 2). The minimal costs (over quantities) for each of these cases must be compared to see which one obtains. For the single store cases, the results have been found in the previous sections. For the mixed case, there remains an
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infinity of possibilities. In this section, a series of results restricting the optimal patterns of shopping, in the case where both stores are visited, will be proven.

In the previous sections the "zero inventory rule" and the associated requirement that both goods be purchased on each trip greatly facilitated the solution by defining the only possible shopping pattern. When there are two stores, it may well be optimal to visit one store, i.e., the more expensive (small) store, while still holding goods purchased at the cheap (large) store. The zero inventory rule thus requires careful consideration.

The next result states that the "zero inventory rule" still applies to the large store.

**Lemma 1:** Any trip to the large store that is part of an optimal policy will only occur when both goods have just been depleted.

**Proof:** First note that it is never optimal to make a trip without purchasing at least one of the goods, say good $a$. Let the amount purchased at the large store on trip $i$ be $Q_{a,i}^*$. The no-stockout assumption implies that inventory on hand any time before the trip is made must be non-zero. This means that the inventory on hand immediately after the last trip where good $a$ was purchased must be:

$$Q_{a,i-1}^* \geq D_a(t_i - t_{i-1})$$  \hspace{1cm} (28)

Suppose the above inequality is strict, and the amount on hand at time $t_i$ is

$$\delta Q_{a,i}^* = Q_{a,i-1}^* - D_a(t_i - t_{i-1}) > 0$$  \hspace{1cm} (29)

The total amount on hand immediately after the trip at time $t_i$ is then $Q_{a,i}^* + \delta Q_{a,i}^*$. 

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The total costs consist of inventory costs, trips costs, and price. If \( Q_{a,i}^* \) (and hence \( \delta Q_{a,i}^* \)) is reduced, and \( Q_{a,i}^* \) increased by the same amount, and no other changes made to the policy, then

1. Total storage costs are decreased.
2. Trip costs remain the same.
3. Price costs either decrease (if some of the inventory \( \delta Q_{a,i}^* \) was purchased at the small expensive store) or remains the same (if all the inventory \( \delta Q_{a,i}^* \) was purchased at the large store).
4. All goods are stored for the same, or shorter, time, so that perishability costs do not increase.

Therefore total costs decrease, and the policy cannot be optimal. Equation (28) must be an equality, and hence from (29), \( \delta Q_{a,i}^* = 0 \).

Now consider the second good, say good \( b \). If it is optimal to purchase any amount of good \( b \), the same argument as for the first good applies. If not then some amount \( \delta Q_{b,i}^* \) must be on hand from the previous purchase. But if an amount \( \delta Q_{b,i}^* \) is purchased at the large store at time \( i \), and the purchase of good \( b \) at time \( t_{i-1} \) is decreased by the same amount, then:

1. Since the customer is already at the store to purchase one good, no extra trip costs are incurred by purchasing the second good.
2. Since the store is the cheapest, no extra price costs are incurred.
3. Storage costs are decreased, as above.
4. Perishability costs are not increased, as above.
Therefore it cannot be optimal to have any of the second good on hand at time $t_i$.

Lemma 1 establishes the "zero-inventory rule" for visits to the large store. It also shows explicitly why such a lemma doesn't hold for the small store, in that condition (3) regarding the price costs doesn't hold. Note also that lemma 1, like Proposition 1, relies on the perishable good having some nonzero storage cost. Lemma 1 also leads immediately to the following result.

**Corollary 1-1:** Both goods are purchased on each trip to the large store in any optimal policy.

**Proof:** Suppose not. From Lemma 1, both goods must be just depleted when the trip at time $t_i$ occurs. The no-stockout rule therefore requires that both goods must be purchased somewhere at time $t_i$. If either good (or both) is not purchased on the trip to the large store at $t_i$, another trip must immediately be made, incurring extra trip costs, and possibly (if the trip is to the small store) extra price costs, with no savings. Therefore the policy cannot be optimal.

**Lemma 2:** If the optimal policy has three successive occasions where both goods are at zero inventory, then the time-average cost between the first and second occasion is equal to the time average cost between the second and third.

**Proof:** Suppose not. Then replacing the higher cost interval with a copy of the lower cost interval will produce an overall lower long-run cost policy, and the original policy cannot be optimal.

The equality of costs required by lemma 2 can always be achieved, for any arbitrary set of parameters, by having identical shopping patterns, or policies, in the two intervals.
It is possible for some special sets of parameters to allow different policies with the same minimal costs in the two intervals. Such parameters could be found by first finding the cost functions $C_1^*(\cdot)$ and $C_2^*(\cdot)$ defined over the n-dimensional parameter space, where the subscripts correspond to two different shopping patterns, and the optimization is over the quantities purchased. Imposing the constraint $C_1^* = C_2^*$ defines a lower-dimensional subspace of the parameter space where the two cost functions are equal. (This, in fact, is the procedure that will be used later on to find the transition, or crossover points, in trip cost parameter space, between different optimal policies). Except for such special cases, lemma 2 requires that the detailed policies in the two intervals be identical. Since an arbitrarily selected set of parameters will lie in a lower-dimensional subspace of parameter space with vanishingly small probability, we will say that the detailed policies in the two intervals are "almost always" identical. On the basis of this heuristic argument, we state, without proof:

**Corollary 2-1:** If the optimal policy has three successive occasions where both goods are at zero inventory, then the policy between the first and second occasion is (almost always) identical to the policy between the second and third.

**Proposition 2:** The optimal policy for the two good, two store scenario is periodic (almost always), with period defined by the interval between trips to the large store.

**Proof:** Follows immediately from lemma 1, corollary 1-1, and successive application of corollary 2-1.

Proposition 2 is a major restriction on the variety of shopping patterns that need to be examined. It restricts the possibilities not only to periodic policies, but to a specific subset of periodic policies, and allows application of finite horizon results (although not fixed
horizon, so some care must still be taken). From here on, it is only necessary to consider the interval between two trips to the large store, with the inventory level of both goods at zero at the beginning and end of the interval, and both goods purchased at the large store at the beginning of the interval.

Note also that, while a nonperiodic policy may (rarely) be optimal, it occurs when the long-run average costs of two or more different shopping patterns are identical, so that the long run pattern can be any sequence of the individual cycles. Replacing each cycle with just one of the possible cycles will give exactly the same costs, and be periodic. Therefore, even if there is a nonperiodic optimal policy, there will also be a periodic optimal policy.

Since we are examining the mixed shopping case, we also know there is at least one trip to the small store. Note, however, that nothing has been said about the purchase patterns at the small store. One cycle, with notation, for m visits to the small store, is depicted below:

\[
\begin{align*}
\text{large} & \quad \text{small} & \quad \text{small} & \quad \ldots & \quad \text{small} & \quad \text{large} \\
\Delta t_0 & \quad \Delta t_{s,1} & \quad \Delta t_{s,2} & \quad \ldots & \quad \Delta t_{s,m} & \quad \Delta t_{m+1} \\
t_0 & \quad t_{s,1} & \quad t_{s,2} & \quad \ldots & \quad t_{s,m} & \quad t_{m+1}
\end{align*}
\]

One of the two goods purchased at the large store, either the perishable or the non-perishable, must last the longest. The following results are designated with an "n" or a "p" depending on which good lasts longest.
Case n:

Refer to Figure 3 for the following lemma and corollary.

**Figure 3**: Timing of first purchase of non-perishable at small store. Diagram refers only to lemma 3n. Subsequent results will further restrict this general picture.

**Lemma 3n**: Consider the optimal policies where the non-perishable good purchased at the large store lasts at least as long as the perishable good, i.e.,

$$\frac{Q_{n,l}}{D_n} \geq \frac{Q_{p,l}}{D_p}$$  \hspace{1cm} (30)

The first purchase of the non-perishable at the small store is on the trip immediately before, or at, the depletion of the non-perishable purchased at the large store, i.e., the trip defined by
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\{ i \mid \max t_{s,i}^* \leq \frac{Q_{n,l}^*}{D_n} \} \quad (31)

Proof: The no-stockout assumption and lemma 1 (zero inventory at \( t_o \) and \( t_{m+1} \)) give the total amount of the non-perishable purchased at the small store as

\[ Q_{n,s}^* = D_n \Delta t_l^* - Q_{n,l}^* \quad (32) \]

The no stockout constraint implies that \( t_{s,i}^* \) is the latest the first purchase can occur. Purchasing earlier increases storage costs without affecting any other costs, so that any earlier purchase than \( t_{s,i}^* \) cannot be optimal \( \Box \).

Corollary 3n-1: Any trips to the small store before occasion \( t_{s,i}^* \) defined in (31) must be for the perishable good only.

Proof: Follows immediately from lemma 3n and the fact that something must be purchased on each trip \( \Box \).

The same results, with parallel proofs, apply to the case where the depletion time for the perishable good purchased at the large store is longest. Since the results are essentially identical, they will not be restated, but will be referred to as Lemma 3p and Corollary 3p-1.

Lemma 4n: Again consider the case given by (30). The only time the non-perishable will be purchased at the small store, if at all, is the last trip in the period, \( t_{s,m}^* \).

Proof: Suppose not. Then between time \( Q_{n,l}^*/D_n \) and \( t_{s,m+1}^* \) (interval a in Figure 4) the customer must optimize his purchases with respect to only one store (he has no inventory on hand from the large store, and doesn’t visit the large store). Thus he will buy both
perishables and non-perishables simultaneously, according to the single store policy; and at
time \( t_{s,i+1} \), his inventory is zero. But by corollary 2-1, this can only be optimal in the knife-edge
case that the average costs over the intervals before and after \( t_{s,i+1} \) are equal.
Generally, either one or the other interval will have lower costs, and the policy cannot be
optimal.

If trips to both stores are optimal, then the first interval must have lower costs and
be the repeated interval. Hence \( t_{s,i+1} = t_{s,m+1} \), or \( i = m \).

Figure 4: Visits to the small store to purchase both goods more than once per period
cannot be optimal.

Again, the same results apply to the case where the perishable good from the large
store lasts longer, and will be referred to as Lemma 4p, but not explicitly stated. It is, of
course, possible, and more economical, to combine the two cases for arbitrary goods for
Lemmas 3 and 4, as the argument is the same to this point. Henceforth, however, the
situation at the end of the interval is slightly different between the two cases. Also, later on,
it will be necessary to distinguish the two cases. Therefore, economy has been sacrificed in the hope that clarity is gained by distinguishing the cases early on.

Note that lemmas 3n and 3p specify the earliest (the trip just before both goods purchased at the large store have run out) that a trip to the small store to purchase both goods can occur, and 4n and 4p specify the latest (when the inventory of both goods purchased at large store has just run out) such a trip can occur. Together this may be restated as:

**Proposition 3:** If both goods are purchased at the small store in an optimal policy that includes shopping at both stores, then the second good will only be purchased once per period, and on the last trip to the small store in the period.

**Proof:** Follows immediately from lemmas 3n, 3p, 4n, and 4p □.

![Shopping Pattern Possibilities Diagram](image-url)
Figure 5 indicates the possibilities so far. Next, we will examine the non-perishable case (referred to as the "4n case" in Figure 5, or simply the "n" case later on) in more detail, and restrict the possible policies further.

**Proposition 4n:** The amount of perishable good purchased at the large store, in the "n" mixed trip shopping case, is just enough for the lifetime $\Delta t_e$ of the good, which must be less than the consumption time $Q_{n,l}^*/D_n$ of the non-perishable good purchased at the large store:

$$\frac{Q_{p,l}^*}{D_p} = \Delta t_e < \frac{Q_{n,l}^*}{D_n}$$

**Proof:** Consider the three possibilities relating the lifetime of the perishable good to the consumption time of the non-perishable purchased at the large store.

(a) **Lifetime less than consumption time:** $\Delta t_e < Q_{n,l}^*/D_n$. (See Figure 6). Any value of $Q_{p,l}^*$ less than $D_p \Delta t_e$ would increase the price cost by requiring the purchase of more at the small store, and have either no effect on, or an increase in, trip costs.

(b) **Lifetime equal to consumption time:** $\Delta t_e = Q_{n,l}^*/D_n$. As in (a), $Q_{p,l}^* = D_p \Delta t_e$. But that implies a zero inventory point for both goods at the time $\Delta t_e$. By Corollary 2-1, that implies that, almost always, shopping would occur only at the large store, or only at the small store, which violates the assumption of mixed trip shopping.

(c) **Lifetime greater than consumption time:** $\Delta t_e > Q_{n,l}^*/D_n$. By Corollary 2-1, as in (b), and the assumption of restriction to the "n" case, the consumption time for the perishable must be less than the nonperishable. But costs can
always be reduced by increasing the quantity of the perishable purchased, so this cannot be optimal □.

**Proposition 5n:** The amount of perishable goods purchased on each trip to the small store, and hence the interpurchase intervals, is identical, except possibly for the last trip and interval.

**Proof:** For any number of trips to the small store, only one good is being purchased on all but possibly the last trip. Since the perishable good has a small but nonzero storage cost associated with it, the standard result from perishable inventory theory applies (eg., Nahmias, 1982): equally spaced trips, with equal purchase quantities on each trip, minimizes costs □.

**Corollary 5n:** If the non-perishable is never purchased at the small store, proposition 5n applies to all trips in the interval.
Proof: As for Proposition 5n □.

**Lemma 5n:** If some non-perishable is purchased at the small store, then the depletion of the nonperishable from the large store cannot coincide with depletion of the perishable purchased at the small store.

**Proof:** When there is zero inventory of both goods, Corollary 2-1 applies, and since mixed-store shopping is optimal, the cycle would be repeated without ever purchasing non-perishable goods at the small store □.

At the end of the period, the following proposition shows that there are only two possibilities to consider:

**Lemma 6n:** If there is a purchase of the non-perishable good at the small store, at the end of the period, either it is purchased by itself when the nonperishable purchased at the large store runs out, i.e., the last trip to the small store in the period, at time \( t_{s,m}^* \); or it is purchased with the perishable good on the last trip to the small store before the non-perishable runs out.

**Proof:** First note that lemma 5n eliminates the possibility of the two cases coinciding.

The purchase of the extra non-perishable must occur between the time \( t_{s,m}^* \) of the last trip to the small store and the depletion of the existing stock (in the interval \( \Delta t_s \) in Figure 7). Purchase at time \( t_{s,m}^* \) incurs extra storage costs \( A \) through the interval \( \Delta t_s \). If it is optimal to delay the purchase, and save some of the storage costs \( A \) at the expense of an extra trip, then the total costs can be minimized by delaying the trip until the goods on hand from the large store are depleted, thereby reducing \( A \) to zero □.
Next, these two possibilities are examined in detail, and it is found that one can be eliminated.

**Lemma 7n:** An optimal policy in the "n" case will almost never include the purchase of any of the non-perishable good at the small store at the time of perishable good purchase.

**Proof:** Proceeding by contradiction, assume some $Q_{n,s}^* \ (0 \leq Q_{n,s}^* \leq D_n(\Delta t_a + \Delta t_b))$ is purchased at time $t_{n,s}^*$. Let the total amount of non-perishable purchased in the period be $Q_a^* = Q_{n,l}^* + Q_{n,s}^*$. For any given shopping pattern, then, the only costs which vary with $Q_{n,s}$ are the storage costs and price costs associated with different allocations of the non-perishable purchases between the large and small store ($Q_a^*$ held fixed). Storage costs for the non-perishable are
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\[ C_s = \frac{s Q_{n,l}}{2 \Delta t_l} \Delta t_{n,l} + \frac{s Q_{n,s}}{\Delta t_l} \Delta t_a + \frac{s Q_{n,s}}{2 \Delta t_l} \Delta t_b \]  

(34)

Let \( T \) be the time from the beginning of the period to \( t_{a,m} \), the last trip to the small store, to buy perishables. Noting that

\[ Q_{n,l} = Q_n - Q_{n,s} \]; \hspace{1cm} \Delta t_{n,l} = (Q_n - Q_{n,s})/D_n;

\[ \Delta t_a = (Q_n - Q_{n,s})/D_n - T \]; \hspace{1cm} \Delta t_b = Q_{n,s}/D_n

equation (34) becomes

\[ C_s = \frac{s}{\Delta t_l} \left[ \frac{(Q_n - Q_{n,s})^2}{2D_n} + Q_{n,s} \left( \frac{Q_n - Q_{n,s}}{D_n} - T \right) + \frac{Q_{n,s}^2}{2D_n} \right] \]

\[ = \frac{s}{\Delta t_l} \left[ \frac{Q_n^2 - 2Q_n Q_{n,s} + Q_{n,s}^2}{2D_n} + \frac{Q_{n,s} Q_n - Q_{n,s}^2}{D_n} - Q_{n,s} T + \frac{Q_{n,s}^2}{2D_n} \right] \]

\[ = \frac{s}{\Delta t_l} \left[ \frac{Q_n^2}{2D_n} - Q_{n,s} T \right] \]

(36)

which implies that as \( Q_{n,s} \) increases (and \( Q_{n,l} \) decreases) storage costs decrease in proportion.

Now consider price costs of the non-perishable.

\[ C_p = \frac{Q_{n,s}P_{n,l}}{\Delta t_l} + \frac{Q_{n,s}P_{n,s}}{\Delta t_l} \]

\[ = \frac{Q_n}{\Delta t_l} + \frac{Q_{n,s}(P_{n,s} - P_{n,l})}{\Delta t_l} \]  

(37)
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Price costs also vary linearly with $Q_{n,s}$. Hence, we have the following boundary solutions for $Q_{n,s}$: if price costs dominate, i.e.,

$$\frac{P_{n,S} - P_{n,l}}{\Delta t_l} > \frac{sT}{\Delta t_l} \quad (38)$$

then $Q_{n,s} = 0$. If storage costs dominate, then

$$Q_{n,S} = D_n(\Delta t_a + \Delta t_b) = D_n\left[\frac{Q_{p,n,m}}{D_p}\right] \quad (39)$$

In the case (which almost never occurs) that (38) is an equality, any value of $Q_{n,s}$ between 0 and the value given by (39) will be optimal.

But in the case that storage costs dominate, (39) implies that the non-perishable purchased at the large store runs out at time $t_{a,m}$. Hence both goods are at zero inventory and by corollary 2-1, that is almost never optimal. Therefore $Q_{n,s} = 0$ almost always $\square$.

Combining the above lemmas into a summary proposition:

**Proposition 6n:** For an optimal policy in the "n" case, if any non-perishable is purchased at the small store, it is purchased only once, and by itself, on the last trip to the small store in the period, at the the time when the non-perishable purchased at the large store is just depleted (almost always).

**Proof:** Follows immediately from lemmas 5n, 6n, and 7n $\square$.

The "n" case, then, has only two shopping pattern possibilities, that vary by whether or not any non-perishable is ever purchased at the small store. Even if some is purchased,
the shopping pattern differs from the never-purchase case only by an extra "fill in" trip to the small store to top up only nonperishables, at the end of the period.

**P-case:**

Now consider the "p" case: shopping still occurs at both stores, but the depletion time of the perishable purchased at the large store is longer than the time for the non-perishable good purchased at the large store. This case has questionable face validity--the implication is that a trip is made to the large, inexpensive store for tomatoes and spaghetti, but the spaghetti runs out before the tomatoes, and is replenished, by itself, at the small store. Given that the tomatoes expire (rather quickly), spaghetti trips must be quite frequent. The parameters necessary for such behaviour to be optimal (e.g., a large price cost differential on tomatoes, a small differential on spaghetti, large storage costs, and/or small trip costs to the small store) are possible within the model structure, but somewhat pathological. While acknowledging that this is slightly bizarre behaviour, the "p" case will still be examined here for completeness. Refer to Figure 8.

**Lemma 5p:** If any perishable good is purchased at the small store, then the depletion of the perishable good purchased at the large store cannot coincide with depletion of the non-perishable purchased at the small store.

**Proof:** As for Lemma 5n □.

**Lemma 6p:** If any perishable good is purchased at the small store, it must be purchased simultaneously with the last trip to purchase non-perishables.
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Figure 8: Notation for the "p" case.

Proof: By Proposition 3, the good must be purchased on the last trip in the period to the small store, so it cannot be purchased any earlier than the last trip to purchase non-perishables. Delaying the purchase would incur an extra trip, with no savings (storage costs for the perishable good are negligible compared to other costs), so the last non-perishable trip is also the last perishable trip □.

Lemma 7p: If any perishable is purchased at the small store then just enough perishable must be purchased at the large store to last to expiry, and the period must be longer than the expiry time:

\[
\frac{Q_{p,l}^*}{D_n} = \Delta t_{p,l}^* = \Delta t_e < \Delta t_i^* \tag{40}
\]
**Proof:** First recall that the stock of perishable good must be depleted at the beginning and end of the period (Proposition 2). Increasing the amount $Q_{p,i}$ decreases price costs without increasing any other costs, up to the limit imposed by either the expiry time or the end of the period. If the end of the period imposes the limit by being less than or equal to the expiry time, then no perishable good will be purchased at the small store. Therefore the expiry time must be the limit $\square$.

**Proposition 7p:** If any perishable good is purchased at the small store in the "p" case, then it is purchased simultaneously with the non-perishable good on the last trip to the small store in the period, before the perishable good at the large store expires, which in turn is before the end of the period.

**Proof:** Follows immediately from Lemmas 5p, 6p, and 7p $\square$.

**Proposition 8p:** In the "p" case, the non-perishable purchased is purchased at the small store in equal quantities, at equal intervals.

**Proof:** Any other distribution of purchases increases storage costs, as in standard EOQ results $\square$. 
Figure 9: The complete set of shopping possibilities.
2.6 Optimal Shopping Behaviour: Solution to a Special Case

The previous section reduced the possible shopping behaviours to six patterns, each of which may be optimized over the quantities purchased, which then immediately induces the optimal timing. In the most general case, for a given set of parameters, the six optimizations could be done, and the minimal of the six would determine the unrestricted optimal shopping pattern. Then, in principle, the parameters could be varied to determine where the shopping patterns change. With 6 possible patterns, and a 10-dimensional parameter space, the results would be difficult to interpret. In this section, a subset consisting of three important cases is examined and solved.

Two of the cases to be considered are when the customer shops at one store or the other exclusively. For the mixed-store shopping, only the "n" case will be considered, because the "p" case (as noted previously) has little face validity. Of the two "n" case possibilities, only the case where the non-perishables are purchased exclusively at the large store will be considered. The modified version of the "n" case, where some non-perishable may be purchased on the last trip to the small store, will be left for future investigation.

2.6.1 Shopping at the small store only

This problem was solved in section 2.4.3, without price costs. If the perishability constraint (26) is binding, the long-run average cost with prices, from (25), is

\[
C^*_T,s,b = \frac{c_s}{\Delta t_e} + \frac{sD_n \Delta t_e}{\Delta t_e} + D_p P_{p,s} + D_n P_{n,s}
\]  

(41)

When the perishability constraint is not binding, the cost from (23) is
\[
C_{T,s, nb}^* = \sqrt{2c_s D_n s} + D_p P_{p,s} + D_n P_{n,s}
\]  

(42)

where the cost has been subscripted to indicate "binding" and "not binding", and "small".

### 2.6.2 Shopping at the large store only

Costs for the large-store only case are as above with the appropriate subscripts:

\[
C_{T,l, b}^* = \frac{c_l}{\Delta t_e} + \frac{sD_n \Delta t_e}{2} + D_p P_{p,l} + D_n P_{n,l}
\]  

(43)

\[
C_{T,l, nb}^* = \sqrt{2c_l D_n s} + D_p P_{p,l} + D_n P_{n,l}
\]  

(44)

From here on, upper case "C" will mean total long-run average costs, and the bar and the subscript "T" will be dropped from the cost symbol.

### 2.6.3 Shopping at Both Stores

The shopping pattern being investigated is the periodic policy with period defined by the time between trips to the large store, and where non-perishables are purchased only at the large store (Figure 10).

The problem is

\[
\text{MIN} \quad C_m
\]

\[Q_{n, l}, Q_{p, l}, Q_{p, s}, m\]

(where the "m" subscript on the cost refers to the mixed case described above and in Figure 10, and minimization is over quantities, and m, the number of trips per period to the small...
store) subject to the perishability constraints,

\[ \Delta t_{p,s} \leq \Delta t_e \]  \hspace{1cm} (45)\]

\[ \Delta t_{p,l} \leq \Delta t_e \]  \hspace{1cm} (46)\]

Note (Figure 10) that Proposition \(4n\) requires the quantity of the perishable good purchased at the large store to be exactly enough to last to expiry, so that the perishability constraint (46) is always binding on \(Q_{p,l}\), so it is unnecessary to minimize over that quantity. Corollary \(5n\) also fixes the total amount purchased at the small store during the period as the number of trips, \(m\), to the small store, times \(Q_{p,s}\), which can be expressed in terms of \(Q_{n,l}\):
\[ mQ_{p,s} = (\Delta t_{n,l} - \Delta t_e)D_p \]

\[ Q_{p,s} = \left( \frac{Q_{n,l}}{D_n} - \Delta t_e \right) \frac{D_p}{m} \]

Consequently, the minimization need only be done over \( Q_{n,l} \) and \( m \).

The cost function is

\[ C = \frac{D_n}{Q_{n,l}} \left( c_l + mc_s + \frac{sQ_{n,l}^2}{2D_n} + Q_{n,l}P_{n,l} + Q_{p,s}P_{p,l} + mQ_{p,s}P_{p,s} \right)^{(48)} \]

Substituting for \( Q_{p,l} \) and \( Q_{p,s} \) from above, and letting \( P_{p,l} - P_{p,s} = \Delta P_p \),

\[ C = \frac{D_n}{Q_{n,l}} \left( c_l + mc_s + \frac{sQ_{n,l}^2}{2D_n} + Q_{n,l}P_{n,l} - D_p \Delta t_e \Delta P_p + \frac{D_p P_{p,s}Q_{n,l}}{D_n} \right)^{(49)} \]

The effect of the binding perishability constraint on \( Q_{p,l} \) has already been incorporated into equation (49), but the constraint is not necessarily binding on \( Q_{p,s} \). Since \( Q_{p,s} \) has been eliminated from the cost function, the constraint is rewritten as a minimum on the number of trips that must be taken to the small store in the period:

\[ m \geq \frac{\Delta t_{n,l} - \Delta t_e}{\Delta t_e} \]

or, in terms of \( Q_{n,l} \):
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\[ m + 1 - \frac{Q_{n,t}}{\Delta t_e D_n} \geq 0 \quad (51) \]

Figure 11: Long-run average trip costs to the small store as a function of length of the periodic interval.

A brief discussion of how the perishability constraint (51) affects optimal shopping patterns at the small store, and how that effects the length of the period, may be helpful. Since the only costs associated with changing the trip pattern to the small store are trip costs, an optimal policy will minimize the number of small store trips for any given period length. However, the perishability constraint and the no-stockout rule puts a lower bound on the number of trips for any given period length. Now consider what happens if the parameters of the model are varied in such a way that the length of the period \( \Delta t_{n,l} \) increases; refer to Figure 11, which shows just the (time-average) optimal (small store) trip costs as the period increases. As the period increases from once to twice the expiry time, one trip must be made to the small store, and the fixed cost of that trip translates to a decreasing average trip cost. If the period exceeds twice the expiry time, and extra trip must be made, which increases the trip costs discontinuously. The pattern repeats itself, with a fixed upper bound
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c/\Delta t_e on average trip costs. At each discontinuity, the lower bound is a factor \((m - 1)/m\) times the upper bound. Since the remaining costs (storage and price) are continuous in the model parameters, what must happen as the parameters change is that the optimal period length increases to an integer multiple of the expiry time \(\Delta t_e\), and then locks there: adding a continuous function to the discontinuous function of Figure 11 cannot produce a minimum total cost at the upper bound on the trip cost. As the parameters continue to change in such a way as to force the period length to increase, the period will eventually jump discontinuously to a length near, or at the next integer multiple of the expiry time.

When the period is locked at an integer multiple of \(\Delta t_e\), the perishability constraint, (45) or (51), is binding. The subsequent jump in period length, as parameters change, may be to a value less than the next integer multiple, at which point the perishability constraint is not binding. As the parameters change to increase optimal period length continuously from this point, the period will eventually hit the next integer multiple, at which point the constraint is again binding. It is also possible that the period will jump directly from the binding \(m^{th}\) to the binding \(m^{th} + 1\) case. However, the nonbinding case must always become binding before an extra trip is added. These results will be demonstrated formally later.

In the following, the subscripts \(n, l\) will be dropped from \(Q_{n, l}\), as that is the only quantity being dealt with in the optimization.

The cost function (49) is convex in \(Q\) and the constraint (51) is linear. Therefore the Kuhn-Tucker first-order conditions are sufficient for minimization with respect to \(Q\). The Lagrangian is
\[
L = C - \mu \left( m + 1 - \frac{Q_{n,l}}{\Delta t_e D_n} \right)
\]
\[
= \frac{D_n}{Q_{n,l}} \left(c_i + mc_s + \frac{sQ_{n,l}^2}{2D_n} + Q_{n,l}P_{n,l} - D_p \Delta t_e \Delta P_p + \frac{D_p P_{p,s} Q_{n,l}}{D_n} \right)
\]
\[
- \mu \left( m + 1 - \frac{Q_{n,l}}{\Delta t_e D_n} \right)
\]  
(52)

where \( \mu \) is the Kuhn-Tucker multiplier. The first order condition with respect to \( Q \) is

\[
\frac{\partial L}{\partial Q} = \frac{\partial C}{\partial Q} + \frac{\mu}{\Delta t_e D_n} = 0
\]  
(53)

The other FOC is the perishability constraint (51).

If the constraint is binding, (51) gives

\[
Q_{b,m}^* = D_n \Delta t_e (m + 1)
\]  
(54)

where the subscripts on \( Q \) indicate "binding" and dependence on the trip parameter "m".

The binding case implies \( \mu > 0 \); differentiating the cost and substituting in (53),

\[
sQ_{b,m}^2 + 2D_n (\Delta P_p \Delta t_e D_p - c_i - c_s m) < 0
\]  
(55)

substituting (51) into (55) and rearranging gives the parameter range where the constraint binds:
Finally, the minimal (binding) cost is

\[ C_{b,m}^* = \frac{c_l + mc_s}{\Delta t_e(1 + m)} - \frac{\Delta P_p D_p}{1 + m} + D_n P_{n,l} + D_p P_{p,s} + \frac{s\Delta t_e D_n(1 + m)}{2} \]  

(57)

When the constraint is not binding, \( \mu = 0 \), and

\[ Q < (m + 1)D_n\Delta t_e \]  

(58)

The first-order condition (53) becomes, after differentiation and simplifying,

\[ Q_{nb,m}^* = \sqrt{\frac{2D_n}{S}(c_l + mc_s - \Delta P_p \Delta t_e D_p)} \]  

(59)

Substituting into the constraint gives

\[ \frac{2D_n}{S}(c_l + mc_s - \Delta P_p \Delta t_e D_p) < (m + 1)^2 D_n^2 \Delta t_e^2 \]  

(60)

or

\[ c_l + mc_s < \frac{(m + 1)^2 D_n \Delta t_e^2 S}{2} + \Delta P_p \Delta t_e D_p \]  

(61)

which is the complement of the set defined in (56). The (not binding) cost is
Let us consider how the minimum cost varies with the travel cost per period, \( c_i + mc_i \).

From equations (57) and (62), the minimum cost is linear with travel cost in the binding case, and varies with the square root of travel cost in the nonbinding case. At the point where the constraint just binds, i.e., equality in (56), the two cost functions (substituting (56) at equality) and their first derivatives are equal:

\[
C^*_{\text{b or nb,m}} = D_n P_{n,l} + D_p P_{p,s} + \Delta t_e D_n s(1 + m)
\]

where the notation is intended to indicate that the derivatives are evaluated at the point where equality obtains in (56).

So, ceterus paribus, the perishability constraint binds when the trip costs are large, as indicated in Figure 12. As can be readily seen in Figure 12, the binding of the \( m \)th perishability constraint leads to a higher cost (\( C^*_b > C^*_{\text{nb}} \)).

Also note that if trip costs are some positive monotonically increasing function of distance, and the stores are separated, so that there is a minimum total trip cost, that it is quite possible for the constraint to be binding for all customers that are homogeneous on other parameters. This would be the case where the period \( \Delta t_{n,1} \) only takes values equal to integer multiples of the expiry time \( \Delta t_e \).
Figure 12 is for a fixed value of m. How does it change with different values of m? Note that in (56) or (61) that the RHS increases as $m^2$, while the LHS as m, implying that for larger values of m, there is greater chance of the constraint being nonbinding. The point of tangency in Figure 12, defined by equality in (56), moves to the right. So, as trip costs increase for fixed m, the optimal cost function will switch from nonbinding to binding (if nonbinding is ever feasible); but for larger values of m, the switch will come at higher trip costs. One useful application is the following: suppose one were working with a particular set of parameter values, and distance related trip costs, and found that, because of fixed store separation, the constraint was always binding for the $m = 3$ case. Then one would know that the constraint was always binding for $m = 1$ and $m = 2$.

The final step is to determine the optimal number of trips to the small store, m, where m is restricted to integers greater than or equal to one. One approach is to assume m is continuous, and solve the first order conditions. Then for any particular numerical values of the parameters, take the value of m rounded up or down as appropriate.
If one is interested in a particular parameter (or parameters), an alternate approach is to take the cost functions (57) and (62), and determine at what parameter value the cost with m trips equals the cost with m+1 trips. For a sequence of values for m, this will give the points where the shopping behaviour changes by addition of an extra trip to the small store, as a function of the parameter. Because the next section addresses the change in shopping behaviour as a function of household location (which translates to trip costs), the latter approach will be taken here. The method can easily be adapted for other parameters.

We want to find when \( C_m^* = C_{m+1}^* \). Because there are different cost functions for the binding and non-binding cases, there are four possibilities for the transition region:

1) \( C_{b,m}^* = C_{b,m+1}^* \)
2) \( C_{b,m}^* = C_{nb,m+1}^* \)
3) \( C_{nb,m}^* = C_{nb,m+1}^* \)
4) \( C_{nb,m}^* = C_{b,m+1}^* \)

A brief geometric discussion of these transitions in the context of Figure 11, which shows trip costs, follows. As noted before, the remaining costs are convex functions of \( Q_{m,t} \). Adding a convex function to the trip costs, and varying model parameters so that the minimum in the total cost function moves to the right (increasing period length) in Figure 11, can produce transitions from binding to binding, as in 1) above, and from binding to nonbinding, as in 2) above. These possibilities are displayed graphically in Figures 13 (a) and (b) respectively, which shows in a stylized fashion the total cost as a function of period length. These costs are the sum of the trip costs of Figure 11 and the convex remaining costs. The minimum total cost occurs at a period length which increases as the parameters change; and the increase in optimal period length must change discontinuously. However,
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Figure 13: Possible transitions from m trips to m+1 trips as period $\Delta \tau_{n,1}^*$ increases: (a) perishability constraint binding for both m and m+1 (b) constraint binding for m and not binding for m+1.

it is also apparent from Figure 13 that transitions given by 3) and 4) above cannot occur.

As the minimum cost moves to the right, the non-binding, m trip case must first switch to the binding case, which then jumps discontinuously to either the binding or nonbinding, m+1 trip case.

**Proposition 9:** For optimal mixed-store shopping, as parameters vary, direct transition from the non-binding case, to a case with one extra trip to the small store, will not occur.
Proof: Recall $c_s$ and $c_i$ are assumed positive real. The proposition states that 3) and 4) above will not occur. First consider 3), substituting in the cost functions, and eliminating common terms:

$$\sqrt{2D_n(c_i + mc_s) - 2D_n\Delta t_e \Delta P_p D_p}$$

which implies $c_s = 0$, a contradiction.

Next consider 4) above, substituting the cost functions and eliminating common terms:

$$\sqrt{2D_n(c_i + c_s m - \Delta P_p \Delta t_e D_p)}$$

Squaring equation (66) gives a quadratic in $c_i$, which may be solved for $c_i$:

$$c_i = \Delta P_p D_p \Delta t_e + \Delta t_e^2 D_n s (m + 2)^2 - c_s (m + 1)$$

which also has no real solution for positive $c_s$.

Referring to Figure 13, we can see that the $c_s = 0$ condition which arises in the above proof corresponds the disappearance of the discontinuity in the cost function; this is consistent with the intuitive argument that it is the discontinuity in trip cost that forces the
discontinuity in the optimal period length, \( \Delta t_{n,1} \) (from the binding case, \( m \) trips, to the \( m+1 \) trips case).

Proposition 9 eliminates two of the four possibilities for transitions. For the numerical optimization to be carried out below, it now remains to find the relation between the parameters that defines the region of parameter space where the remaining two allowed transitions occur. Because the examples later in this chapter focus on the effects of trip costs, the relation between the parameters are solved for \( c_l \), the trip cost to the large store.

The first case, binding \( m \) trips, to binding \( m+1 \) trips is

\[
\frac{c_l + mc_s}{\Delta t_e(1 + m)} = \frac{P_P}{1 + m} + \frac{sD_n \Delta t_e (1 + m)}{2}
\]

which simplifies to

\[
c_l = c_s + \Delta t_e \left[ P_P + \frac{sD_n \Delta t_e (1 + m)}{2} \right] (68)
\]

The last case, binding \( m \) trips to non-binding \( m+1 \) trips, is

\[
\sqrt{2D_n s (c_l + c_s (m + 1)) - \Delta P_p \Delta t_e D_p} = \frac{c_l + c_s m}{\Delta t_e (m + 1)} - \frac{P_P}{(m + 1)} + \frac{s \Delta t_e D_n (m + 1)}{2}
\]

Squaring and solving for \( c_l \) simplifies (70) to
\[ c_i = \Delta P_p D_p \Delta t_e - c_s m + \frac{s}{2} \Delta t_e^2 D_n (m + 1)^2 \]

\[ \pm \frac{\Delta t_e (1 + m)}{2} \sqrt{8c_s D_n s} \]

The above transition equations (69) and (71) are bounds of the areas where various values of \( m \) are optimal in the mixed shopping cases. With the constraint equations (56) and (61), which give the regions in parameter space where, for each value of \( m \), the constraint is binding or not binding, the transition equations are helpful in the task of comparing the optimal cost surfaces for the mixed and single store shopping behaviours.
2.7 A Numerical Application: Two-Dimensional Trading Areas

The model assumes that customers shop deterministically, with the shopping behaviour that minimizes long-run average costs. It is natural to then ask what this sort of shopping behaviour implies for the market areas of the grocery stores. This, of course, will depend on the specific parameters describing each customer in the competitive market space. The two parameters which are immediately related to customer location, and hence market area, are the trip costs to the large and small store. In the absence of information on other customer-related parameter values, trip costs based on distance can provide a first approximation of market areas, assuming customers are homogeneous on other parameters.

In this section, the shopping behaviour of spatially heterogeneous customers, and the resulting market areas of the two stores, will be derived using numerical and graphical methods.

In particular, assume trips costs are directly proportional to the Euclidean distance between the customer's household and the store. This is consistent with many theoretical models (Hotelling 1929; Ingene and Ghosh 1990) as well as being a special case of the many empirical Huff-type spatial interaction models (Huff, 1962). For the small store:

\[ c_s = \tau \sqrt{(x - x_s)^2 + (y - y_s)^2} \]  

(72)

where \( \tau \) is the sensitivity to travel in dollars per kilometre, the customer is located at \((x,y)\), and the small store is located at \((x_s,y_s)\). The expression for the large store trip cost is similar. Table 2 shows the arbitrary, but reasonable, parameter values used in the numerical and graphical calculations.
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Table 2: Parameter values used in numerical calculations.

<table>
<thead>
<tr>
<th>Locations:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Large store:</td>
<td>$(x,y) = (-5,0)$</td>
</tr>
<tr>
<td>Small store:</td>
<td>$(x,y) = (5,0)$</td>
</tr>
<tr>
<td>(store separation = 10 km.)</td>
<td></td>
</tr>
<tr>
<td>Prices:</td>
<td></td>
</tr>
<tr>
<td>Perishable, large store:</td>
<td>$P_{p,l} = 40.00/\text{unit}$</td>
</tr>
<tr>
<td>Perishable, small store:</td>
<td>$P_{p,s} = 50.00$</td>
</tr>
<tr>
<td>Perishable difference:</td>
<td>$\Delta P_p = 10.00$</td>
</tr>
<tr>
<td>Non-perishable, large store:</td>
<td>$P_{n,l} = 40.00$</td>
</tr>
<tr>
<td>Non-perishable, small store:</td>
<td>$P_{n,s} = 50.00$</td>
</tr>
<tr>
<td>Demand rates:</td>
<td></td>
</tr>
<tr>
<td>Perishable:</td>
<td>$D_p = 1 \text{ unit/week}$</td>
</tr>
<tr>
<td>Non-perishable:</td>
<td>$D_n = 1$</td>
</tr>
<tr>
<td>Travel cost:</td>
<td>$\tau = 4.00/\text{km}$</td>
</tr>
<tr>
<td>Storage cost:</td>
<td>$s = 2.00/\text{unit/week}$</td>
</tr>
<tr>
<td>Expiry time:</td>
<td>$\Delta t_e = 1 \text{ week}$</td>
</tr>
</tbody>
</table>

The households represented by these parameters would spend $100.00 per week if they shopped exclusively at the small store, and $80.00 per week if they shopped exclusively at the large store. Travel costs can be compared to this savings: a difference in 1 km. to the stores means a savings of $4.00. Storage costs of $2 per unit per week appear relatively small by comparison, but since they are quadratic in quantity purchased, if (for example) the customer stocks up on 4 weeks supply, the storage cost will be $16.00 for that purchase. A lifetime of 1 week seems reasonable for many vegetables and dairy products. The consumption rates are fixed at one unit per week, and are essentially scaling parameters. Alternatively, one could think of the above parameters as representing some average of all perishables and all nonperishables purchased by the household. The point is to illustrate the nature of shopping behaviour and trading areas that emerge from the model.
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The parameter values are used to evaluate the optimal (over quantity) cost functions derived in the last section. The smallest of these functions at each point in the plane determines the shopping pattern at that point. First, for the single-store cases, we determine the region in the plane where the constraints are binding or nonbinding, using equation (26 part 1).

Figures 14 and 15 show these regions: note that the horizontal axis is shifted in the two diagrams, as the stores are located at different points in the plane. Also note that the binding case dominates except for a small region of about one-quarter kilometer radius around each store. Outside this region, the perishability constraint binds. For the majority of the plane, then, we need to consider only the binding cost functions, (41) and (43). (While the size of the region that is nonbinding depends on the parameters, in the typical case there will be both binding and nonbinding areas in a similar geometric relation).
Figure 16 shows the cost surfaces with perishability constraint binding for the single-store only shopping behaviours. If multi-store shopping were not available to the customer, the intersection of these surfaces would determine the trading areas of the two stores.

(a): Large store
(b): Small store
(c): (a) and (b) superimposed.

Figure 16: Single-store-only, binding constraint, optimal cost surfaces.

Note that the large store’s shopping area would be larger than the small store’s, and that the lowest cost at the large store is less than the lowest cost at the small store.
The interesting aspect of the model examined in this research, however, is that it allows shopping at both stores. This means we must superimpose the mixed-trip cost surfaces as well. Moreover, we need to consider both binding and non-binding optimal costs for each value of \( m \), the number of trips to the small store per period. As with the single store cases, first we determine where the mixed trip cases switch from binding to non-binding. For the parameters here, it turns out that for \( m \) less than or equal to 4, the perishability constraint is binding for all locations in the plane. This occurs because the stores are separated by a fixed distance, so that there is a non-zero minimum on the travel cost, \( c_i + mc_s \), and this minimum is greater than the transition value (binding to nonbinding... the point of tangency in Figure 12) as long as \( m \) is less than 5. To explicitly show this, consider the following. If \( m = 1 \), (trips to the stores alternate between large and small), the minimum travel cost in the plane lies on any point on the line joining the two stores. (Since trips alternate between the stores, total travel cost is the same for any household on this line). If, on the other hand, \( m > 1 \), the minimum travel cost is the for the customer who lives in (on?) the small store, i.e., \( c_s = 0 \). Therefore, for any fixed value of \( m \), no customer engaging in mixed trip shopping will have lower costs than the customer at this point. Therefore, (again referring to Figure 12), if this customer is in the binding region, all customers in the plane will be in the binding region. At this point, \( c_i = 10 \text{ km} \). Substituting
these values and the numerical parameter values above into the condition (61) for the constraint to be nonbinding gives

$$40 < (m + 1)^2 + 10 \quad (73)$$

$$4.47 < m$$

i.e., \(m\) must be 5 or more for the perishability constraint to be nonbinding anywhere in the plane.

For \(m = 5, 6,\) and 7, there is a progressively larger region around the small store where the constraint is not binding (Figure 17). Referring to Figure 12, the change in the nonbinding area as \(m\) changes can be seen as the change in the relation between the minimum cost and the point of tangency. For each different \(m\), the actual cost curves differ, and so does the point where the minimum cost occurs. For \(m\) less than 5, the point of tangency is to the left of the minimum cost. As \(m\) increases, the point of tangency moves to the right relative to the minimum cost.

In summary, then, we need only consider the nonbinding mixed-trip cost as long as \(m\) is less than five. At this stage, however, we haven’t eliminated the possibility that higher values of \(m\) may be optimal, and if \(m\) exceeds 4, we will have to consider both binding and nonbinding. However, from Figure 17, we need only examine limited regions around the small store when comparing the nonbinding, mixed-trip cost surfaces, with the other cost surfaces.

Figure 18 shows the mixed-trip, constraint binding, cost surfaces for \(m = 1\) and \(m = 3\). Note that for \(m = 1\), the optimal cost is constant and minimal between the two stores,
and for $m = 3$, it is linear between the stores and minimal at the small store, as required by the preceding argument. Also, the smallest cost for $m = 3$ is less than for $m = 1$.

(a): $m = 1$.
(b): $m = 3$.
(c): (a) and (b) superimposed.

Figure 18: Optimal cost surfaces for mixed-store, binding constraint, shopping behaviour.

The next step is to determine where the intersections between the surfaces for different values of $m$ are. These will indicate where the transition occurs from one type of discrete shopping behaviour ($m$ trips per period) to the next ($m + 1$ trips per period). Using the transition formula (69) for binding, $m$, to binding, $m+1$, the regions of the plane where each shopping pattern is minimal can be found. Figure 19 shows the regions where $m = 0$ (which is equivalent to the large-store-only shopping pattern), $m = 1$, $m = 2$, $m = 3$, and $m = 4$. 
There is no intersection between $m = 4$ and $m = 5$, indicating that 4 trips to the small store, between trips to the large store, is the maximum number that can be optimal (for the binding case) in the market. In other words, we have now eliminated nonbinding mixed-trip costs for $m$ less than 5, and binding, mixed-trip costs for $m$ greater than 4, from further consideration.

A note regarding the resolution of the graphical approach is in order here. Consider the line $y = 0$ for $x > 5$, i.e., to the right of the small store. On this half-line, the distance to the large store is 10 km. more than the distance (say, $d_s$) to the small store, so that $c_l + mc_s = 40 + 4(m + 1)d_s$. Substituting this into (57) gives the the mixed, binding optimal cost along the half-line for any value of $m$:

$$C_{b,m}^* = \frac{40 + 4(m + 1)d_s}{1 + m} - \frac{10}{1 + m} + 90 + 1 + m$$

(74)

When either $m = 4$, or $m = 5$, this reduces to

$$C_{b,m}^* = 101 + 4d_s$$

(75)

That is, along this half-line, $C_{b,m=4}^* = C_{b,m=5}^*$: the two cost surfaces are osculating. This means there should be a degenerate contour in Figure 19 extending from (5,0) in the positive
direction along the x axis, indicating this equality. The graphics software, however, cannot pick up this degenerate contour, and it appears that there is no possibility of the $m = 5$ case. All of the intersection contours represent subspaces where either of the equal-cost shopping patterns can occur. They represent the vanishingly small regions of the market, referred to in Corollary 2-1, where customers "almost never" live. In the case of the degenerate contour, we can say, further, that the $m = 5$ behaviour almost never occurs.

**Figure 20:** Contours of intersection of the small-store only cost surface with the mixed trip (from left to right: $m = 0, 1, 2,$ and $3$) surfaces.

**Figure 21:** Superposition of Figure 20 on Figure 19. The Figure 20 intersections occur to the right of the corresponding Figure 19 regions.

Figure 20 shows where the small-store only, constraint binding cost surface intersects each of the cost surfaces in Figure 19. (The small store surface is less than the mixed-store surfaces on the right of each of the contours). By superimposing Figure 19 on Figure 20, (Figure 21) it can be seen that each of the contours in Figure 20 is to the right of the corresponding region in Figure 19. This means that none of the Figure 20 intersections occur in the region of Figure 19 where that mixed-trip surface is smallest. In other words, moving from the large store towards the small store (left to right), the customer would
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switch from the \( m \)th to \( m+1 \) surface before encountering the small-store-only surface. So for these parameter values, the optimal binding, small-store-only shopping pattern is always more costly than the lowest binding, mixed-trip pattern.

Next consider the small regions around each store where the single-store cost function is not binding (Figures 14 and 15). Since that region for the large store is already in the large-store only shopping region, the pattern won’t change...just the form of the optimal solution. For the small store, however, we need to check to see if the non-binding, small-store only cost is less than the mixed trip costs in Figure 19, inside the region where the small-store cost is the non-binding cost. From Figures 14 and 19, this specifically means comparison of the small store non-binding cost with mixed trip binding cost for \( m = 4 \).

First we can look at the location of the small store, \((x, y) = (5,0)\), or \((c_l, c_s) = (40,0)\). At this point the optimal costs are, from equations (42) and (57),

\[
C_{s,nb}^{*} \bigg|_{c_s=0} = 100 \\
C_{b,m=4} \bigg|_{c_s=0} = 101
\]

There is, then, some region right around the store where the nonbinding, small-store only cost is less than the binding, mixed, \( m=4 \) cost. The intersection of these two cost surfaces is shown in Figure 22. However, the small store non-binding surface here has not been truncated by the curve that indicates where the constraint must be binding (Figure 14).

Superimposing Figure 14 on Figure 22 gives the region where the nonbinding, small only, cost function occurs, and the region where it is less than the binding, \( m=4 \), mixed
function. The result, shown in Figure 23 as the smallest intersection region inside the circle, is where shoppers prefer patronizing only the small store, to the \( m = 4 \), mixed trip behaviour.

The remaining consideration is the non-binding mixed trip cases, which are feasible for some values of \( m \) greater than 4 (Figure 17). First we note from equation (62) that the optimal cost for this case, regardless of the value of \( c_s \) and \( c_l \) (and hence of customer position \((x,y)\) in this numerical example), is increasing in \( m \). Therefore it suffices to consider the case \( m = 5 \), which is the smallest cost that is feasible for this case. First note that at the location of the small store, \((x,y) = (5,0)\), the cost for the mixed, \( m = 5 \), nonbinding case is, from (62),

\[
C_{nb,m=5}^{*}\bigg|_{c_s = 0} = 90 + \sqrt{120} = 101
\]  

(78)

Comparison with (76) indicates that at this point, the single store behaviour is optimal. Next, where, if at all, do these surfaces intersect?

---

Figure 22: Intersection of mixed, \( m = 4 \), binding optimal cost surface with small-only, nonbinding optimal cost surface.

Figure 23: Perishability constraint for small-only cost function is nonbinding inside the circle.
Figure 24: Intersection of nonbinding cost surfaces for small-store only, and mixed, \( m = 5 \).

Figure 25: Perishability constraint is nonbinding, for small-only shopping, inside the small circle.

Figure 26: Perishability constraint is nonbinding on the mixed, \( m = 5 \), case inside the small circle.

The intersection contour, shown in Figure 24, encloses the region where the single store cost is greater than the mixed, \( m = 5 \), cost (both nonbinding). As before, it is necessary to restrict the surfaces to the region where the constraints, for both surfaces, are actually nonbinding. Superimposing the constraint contours for the small-only case (Figure 14), and the mixed \( m = 5 \) case (the inner contour in Figure 17) on the intersection (Figure 24) gives the results shown in Figures 25 and 26. The central portion of the intersection contour lies inside the non-binding region of both cost functions, so that portion indicates a transition
in minimum costs from the small store only to the mixed, \(m = 5\) case, as the customer's position moves away from the store.

It was asserted that it was only necessary to inspect the case when \(m = 5\) because the optimal costs, for the nonbinding mixed cases, increase with \(m\) regardless of the customer's location. As a check on this assertion, the region in Figure 24, where the \(m = 5\), nonbinding case has lower cost than the small-only, nonbinding case, should be larger than the corresponding region with \(m = 6\). The latter region is shown in Figure 27, and is in fact much smaller.

The graphical approach to determining the regions where different shopping behaviours occur will be summarized shortly, but first a note regarding the resolution of the preceding few figures is in order. Two different contours are nearly identical in the preceding: the intersection of the small only nonbinding cost with the mixed, binding, \(m = 4\) contour (Figure 22); and the intersection of the small only nonbinding cost with mixed, nonbinding, \(m = 5\) contour (Figure 24). This indicates that the latter cost surfaces are as nearly identical as the graphical approach can discern here. This is consistent with the previous observation that the binding costs for \(m = 4\) and \(m = 5\) are identical to the right of the small store. Furthermore, the transition from binding to nonbinding for both the small, and the mixed \(m = 5\) costs (the circles in Figures 25 and 26) occur very close to the

![Figure 27: The intersection of the nonbinding, mixed, \(m = 6\) contour with the nonbinding, small only, contour.](image-url)
small store. Hence, in the region around the small store, all six cost functions are nearly identical, so that in the immediate vicinity of the small store, and particularly to the right, the precise shopping behaviour cannot be determined from the preceding diagrams.

Since this is an artifact of the particular parameters arbitrarily chosen here, it will not be explored further. We can nonetheless describe the shopping behaviour in the plane in some detail (see Figure 28). There is a tiny region immediately around the small store that
is its own market area exclusively. At about one-quarter of a kilometre away, shopping behaviour switches to a mixed, $m = 4$, or $m = 5$ type of behaviour. Moving strictly to the right along the x axis, this behaviour doesn't change. In any other direction, however, it changes to the mixed, $m = 4$ behaviour. In directions towards the large store, the behaviour goes through progressively fewer fill-in trips to the small store, until eventually shopping is done only at the large store.

From these market areas, market shares can be calculated from a given population density distribution. For purposes of illustration, assume that the city is the twenty by twenty kilometer square area that has been plotted, and that the population is uniformly distributed up to the boundaries of the city, and zero outside the boundaries. Everyone except for those living inside the small store's exclusive region purchases all of their nonperishables at the large store. Assuming this area is a circle of radius 0.2 km, the small store's share of nonperishable good is 0.03%, and the large store's share is 99.97%.

Share of the perishables varies with the different regions. The detailed calculations may be found in Appendix A. For the entire region, the large store captures 71.86% of the perishables, and the small store has a 28.14% share.

**Smaller Price Differences:** The model has two parameters under the control of management: price and store location. Price, of course, is a much shorter term strategic variable, and it is interesting to see what happens to the market areas if the price difference is less than the 20% postulated in the previous numerical example. Let us reduce the price difference to 10% by increasing the price of both goods at the large store from $40 to $45. Leaving all other parameters the same gives the trading areas shown in Figure 29. (The procedure followed is the same as in the preceding example, but not duplicated here). For comparison,
Figure 29: Market areas with a 10\% price difference between stores. From left to right, the regions are exclusively the large store's; mixed with $m = 1$ and $m = 2$; and exclusively the small store's.

Chapter 2: Grocery Shopping Behaviour

refer to Figures 19 or 28. Unlike the case with 20\% price difference, the small store now has a large market area exclusively its own. The mixed behaviour is confined to a narrow region between the two exclusive areas. The market shares that this shopping pattern corresponds to (using the same approach as for the 10\% price difference) are 36.6\% of the nonperishable for the small store and 63.4\% for the large store; and 40.5\% of the perishables for the small store and 59.5\% of perishables for the large store. Changing the price difference between stores from 20\% to 10\% affects the share of non-perishables much more than perishables. As price differences increase, it becomes worthwhile to travel further, but less frequently, for goods that can be stored. Where trip frequency is governed by perishability, the effect of price difference is not so dramatic.

Increased Consumption Rate: One would expect that larger households would be more sensitive to price differences, if only from income effects. However, even if all sensitivity parameters are kept the same, the increased consumption rate will make it worthwhile for a larger household to stockpile. Figure 30 shows the trading areas for the same prices (10\% difference between stores), and all other parameters the same, except for doubling the consumption rates: $D_a = D_p = 2$. The regions of mixed shopping are increased again, although not to the same extent as in Figure 19. Market shares for the nonperishable are
Figure 30: A greater consumption rate increases the tendency to stockpile. From left to right market areas are exclusively the large store’s; mixed with \( m = 1 \) and \( m = 2 \); and exclusively the small store’s.

18.5% and 81.5% for the small and large store respectively; and, for the perishable, 29.1% and 70.9%. The small store still has a substantial region of exclusivity. The market areas are, not surprisingly, sensitive to the absolute differences in consumer expenditure rates at the two stores. Another way of saying this is that people with large families will go further for a deal, even if they have the same price, travel, and stockpiling sensitivities as a single person.

The small store’s market shares for the three preceding sets of parameters are summarized in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>20% price difference</th>
<th>10% price difference</th>
<th>10% price difference; 2x consumption rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>nonperishable shares</td>
<td>0.03%</td>
<td>36.6%</td>
<td>18.5%</td>
</tr>
<tr>
<td>perishable shares</td>
<td>28.14%</td>
<td>40.5%</td>
<td>29.1%</td>
</tr>
</tbody>
</table>
2.8 Discussion

There are several interesting results from the preceding model. Perhaps the two most interesting are that, first, consideration of perishable and nonperishable goods allows multi-store shopping to be the outcome of a purely deterministic rational consumer choice process; and second, consideration of consumer stockpiling allows a grocer's share response to price to be dramatically nonlinear.

Multi-store shopping parallels the multi-purpose shopping results (Ingene and Ghosh, 1990). It differs in that the stores in the model here each offer both goods. The multipurpose models, on the other hand, seek to explain how stores offering frequently purchased (low-order) goods will agglomerate with stores offering infrequently purchased (high order) goods, due to savings associated with multipurpose shopping, thereby creating a high order centre. It seems less surprising that consumers will shop at several stores when they offer different goods, than it does when they offer the same goods. Similarly, while the retailing and promotions literature recognize that multi-store shopping strategies are the norm, and that loyalty to one store is low (Uncles and Ehrenberg, 1990), the possibility of such a strategy resulting from the combination of a power retailer (who has a large price advantage, but few locations) with the fact that some groceries are perishable and others nonperishable, has not been considered.

Nonlinear response, the second interesting result, is the way the market area of the price leader can expand dramatically as the price difference between stores increases. In the numerical example, dropping the price from $45 to $40, against the small store's $50, produced almost complete domination of the nonperishables market by the large store. This "increasing returns" effect is due to the consumers ability to reduce long-run average trip
costs by stockpiling and shopping less frequently at the large store, and thus being able to travel longer distances to take advantage of low prices.

Figure 31: Share response of the large store to its own price reductions, using the numerical example of the last section. At small price differences, no mixed shopping occurs, and the two goods have the same share.

Figure 31 shows this effect, using the shares calculated in the preceding section, as well as intermediate points calculated at 5% and 15% price differences. Note that the perishable goods are much less sensitive to price differences than nonperishables; and that the shares of the two goods are the same when the optimal shopping pattern for everyone in the region is exclusively one store or the other, which occurs when price differences are small.
One of the questions posed in the introduction was "Why was Safeway taken off-guard by the entry of the Real Canadian Superstore?" One possibility lies in the highly nonlinear response to the price differences that arises when stockpiling is considered.

The retailing literature has many examples of location and trading area models that do not consider stockpiling. These usually trade off store attractiveness (which may be a function of several different variables such as price or image) against distance, and can be traced from Reilly's (1931) deterministic gravity model, through Huff's (1962) probabilistic model, and subsequent spatial interaction models. (For a review, see Ghosh and McLafferty (1987). These models are discussed in more detail in Chapter 3). When the "attraction" of these models is due to price, then there is some tradeoff between price and distance. Typically these models are empirically tested, and are calibrated to determine some model parameters; these parameters can then be used to evaluate the sales potential of a proposed outlet, considering the existing market and the competition. The models have been quite successful, and also have a strong intuitive appeal. In fact, even without a formal model, a manager can readily look at trading areas and create a reasonable model of the price-distance tradeoff.

For purposes of comparison with the trading areas implied by the stockpiling shopping model, let us take a simple gravity-type model. Assume that consumers trade off prices and distances in such a way that the boundary of the trading area between stores is given by the line where the ratio of distances to the stores is equal to the inverse of the ratio of prices:

where $d_s$, $d_l$ are the distances from the consumer to the small and large store, and $p_s$ and $p_l$ are the expected prices at the small and large store. In an empirical application, $\lambda$ would
be estimated. Here, for purposes of illustration, it is simply assumed to be unity.

\[
\frac{d_s}{d_l} = \left( \frac{p_l}{p_s} \right)^\lambda
\]  

(79)

Figure 32: Trading area defined by the gravity model, (42), with a 10% price difference; cf. Figure 32.

Figure 33: Trade areas with 10% price difference using the stockpiling model (from Figure 29).

For the two stores located at (-5,0) and (5,0), and the 10% price difference ($45 and $50 at the large and small store respectively), the trading area boundary is shown in Figure 32. For comparison, the stockpiling model, with the same parameters, is shown again in Figure 33. If Figure 33 represented actual data, the much simpler gravity model would be a good approximation of the trading areas. An analyst, or manager operating intuitively, would be justified in applying Occam’s razor, and accepting equation (79) as a fair model of shopping behaviour.
If the competition was always within a 10% price difference, the analyst, or manager, would have a reasonable tool for predicting trading areas. But what if there is a new market entrant with 20% price reduction? Applying the model (79) to predict trading areas would give the map in Figure 34. The incumbent would see the threat of some erosion of market share, but perhaps not enough to elicit a dramatic response. This would be even more true if the incumbent was a chain with many stores, located every few kilometres in the plane, and the entrant was known to have a strategy of widely dispersed stores. Unfortunately, for the established retailer, consumers can stockpile, and the resulting trade areas turn out to be those shown in Figure 35. The market area, and hence share of the established store, has declined dramatically, from 41% to 28% for perishables, and from 37% to near zero for nonperishables. "Safeway" has been taken completely off-guard.

If this argument is reasonable, then we should expect to see substantially greater price differences between RCS and the supermarkets, than between supermarkets. A western
Canadian supermarket that is similar to Safeway is IGA. Both stores predate the entry of RCS. A weekly survey of grocery prices in Edmonton, Alberta (Satanove, unpublished) shows that RCS is indeed dramatically less expensive than either Safeway or IGA. Each week, a "shopping basket" (different each week), of approximately $100 value was compared at the stores. In 1989, IGA was 16% more expensive than RCS on average, and Safeway was 19% more expensive. In 1992, IGA was 18% higher, and Safeway was, again, 19% higher. Figure 36 shows the relative value of the IGA and Safeway baskets compared to the RCS basket for each week in 1992. Although data is not available on prices before the entry
of RCS, if the relative prices between IGA and Safeway were similar before the entry of RCS, we have a scenario very similar to the one described above.
Chapter 2: Grocery Shopping Behaviour

2.9 Contributions and Research Opportunities

This research addresses the problem of consumer response to power retailers in the context of the grocery industry. While power retailing has received much attention in the popular media, it has as yet received little attention from marketing academics, presumably because of the recency of the phenomenon. This research contributes to the retailing literature by showing how dramatic and unexpected changes may occur in consumer shopping behaviour when a power retailer enters the market.

To capture the unique characteristics of the retail food industry, three diverse research streams are drawn from: the promotions literature, to assist in understanding consumer stockpiling behaviour; the inventory management literature, to understand the structure of joint replenishment problems; and the multipurpose shopping behaviour, to structure the cost minimization problem. The model developed here contributes indirectly to each of these literatures. It shows how multi-store shopping, so well documented in the promotions literature, may arise from planned behaviour of rational consumers making a purely deterministic choice. It provides another example of a joint-replenishment problem that has periodic optimal policy. The model also extends the multipurpose shopping models by including goods differentiated by perishability and stores differentiated by prices. It shows that with these characteristics, multi-store shopping doesn’t require that the stores sell different goods, or have temporary price reductions.

There are at least four general directions that future research could take. First and foremost, there is the need for empirical validation of the model. While actual calibration would be ideal, it would also be a major project. The model, however, makes predictions that could be much more easily tested. On the consumer side, this includes the relation
between trip patterns to the different stores, distances travelled, and types of goods purchased. From the stores' perspectives, it suggests the power retailer should have a larger share of nonperishables than perishables.

A second direction is strategic. What are the theoretical implications for equilibrium prices and locations? From the large literature on spatial competition (Hotelling, 1929; de Palma et. al., 1985; Vandenbosch and Weinberg, 1992), it is known that increased separation on spatial dimensions (either geographic space, or product attribute space) decreases price competition. In this model, consumers travel further for nonperishables than perishables, which suggests that price competition should be fiercer for nonperishables. However, the fact that each store carries both goods may create interactions that affect this conclusion. Other strategic issues include how perishables and nonperishables might be treated differently, and how advertising might augment this treatment.

A third direction is model extensions. Additional variables, such as advertising, service, or product quality may be considered. The dynamic effects of short term promotions could be incorporated. This could lead to consumer uncertainty about prices, and require a stochastic approach. Consumers could also be uncertain about their own sensitivities to the various costs in the model; or, perhaps more interestingly from a managerial point of view, these sensitivities might be susceptible to manipulation; in fact, one of Safeway's responses to RCS has been to emphasize convenience of location in their advertising, presumably to increase the consumer's sensitivity to travel costs. For any model extension, an overriding issue is the robustness of the increasing returns effect.

An interesting issue is the structure of the trip costs. In the numerical section, trip costs were assumed to be only travel costs. However, it has long been acknowledged that
there is a cost associated with being in a store. Baumol and Ide (1957) suggest that the larger a store becomes, the more time consuming, and hence costly, shopping becomes. Ingene and Ghosh (1990) explicitly state that their trip costs include a fixed store-specific cost. This is intuitively reasonable, and of interest to management because, once location is set, the only possible short term control over trip costs is the in-store costs. The RCS, for example, is not only large, but often has long check-out lines. Is it worth their while to increase the number of checkouts? One step they have taken is to introduce express checkouts, but that only assists the small-purchase consumer. The effect of making the trip cost to the large store greater by a fixed amount for each customer would be interesting to know. It would certainly reduce the large store’s exclusive area, but would it have a greater effect on the mixed-trip region, or the small store’s exclusive area?

The final issue relates to the dynamic and unpredictable nature of retailing. In this chapter, a specific example is given of how consumers following a relatively simple optimizing model can dramatically alter their shopping behaviour in response to a relatively small change in the price structure of the market, and that while this change may seem perfectly sensible in hindsight, it would be asking much of managers to predict it before the fact. There are many rapid changes in competitive environments, store formats, consumer needs, and regulatory environments that are even less predictable. While this is true of any industry in a market economy, retailing is perhaps more dynamic and uncertain than most. Nonetheless, even if management is unable to exercise much foresight in this environment, it can be very clever in its response. Retailers typically engage in experimentation and imitation of successful strategies, and respond quickly to adverse situations. The question is, then, if static equilibria are unlikely to occur, are there other patterns that may arise?
Many authors have long felt that there are patterns in retailing, and have sought to describe them; however, they tend to not be empirically verifiable. The issue of dynamic patterns arising from many spatially competing agents with limited foresight, but the ability to react effectively to adversity, is addressed in the next chapter.
CHAPTER THREE

SPATIAL COMPETITION

AND SELF ORGANIZED CRITICALITY

3.1 Introduction

The preceding chapter demonstrated how retail market share can exhibit increasing returns to price reductions when the price differential between competitors becomes large enough. This effect is offered as an explanation for incumbents being surprised by the success of a new retailing format in the particular case of food retailing in western Canada. In this chapter, I consider the implications of unexpected adversity more generally for long run industry structure.

The model of competitive dynamics is again inspired by the Canadian food retailing industry: while the attack of the superstores may have been unexpectedly successful, incumbents have generally not been exterminated. Rather, they are fighting back, and with some success. To quote a recent (April, 1993) Financial Times of Canada article,

Safeway, too, has been struggling with declining market share--although it still holds the top spot in the West with nearly 24% of the region’s sales in 1992. Sales throughout the company were down an unhealthy 2.7% last year --and 3.7% in the fourth quarter. Safeway, in fact, used these numbers to convince its Alberta workers to accept wage rollbacks in March as part of its effort to try to match the warehousers’ costs of doing business. But the company is not scrimping on store remodelling. It’s halfway through a five-year, $3.2-billion (U.S.) capital investment plan-- double that spent during the previous cycle.

Supermarkets have responded to the entry of power retailers in a wide variety of ways: cost cutting, dramatically increasing service, adding high-margin delicatessens, and introducing their own superstores. Safeway’s television advertising now promotes the
locational convenience of their outlets (the message is that you can walk to Safeway, in contrast to the Superstores), in a clear attempt to increase customer's sensitivity to travel costs. As the Financial Times article concludes,

Little wonder, then, that Price Co.'s [a "superstore" company] earnings dipped last year for the first time. Or that investors have sent Price Co. and Costco share prices tumbling in the past year. Even so, Loblaws, Loeb and the rest have no room for complacency. Loblaws' Gilles Potvin [a store manager], for instance, survived the first wave of the warehouse invasion by scrambling astutely [emphasis added] to put his store on a sound footing. He'll survive the next wave because he's discovered the warehousers can't be all things to all people.

The need to "scramble astutely" is not confined to supermarket managers faced with new competitive forms. Corstjens and Doyle introduce a recent (1989) Marketing Science article as follows:

A central facet of modern retailing management is repositioning--adapting the business to a changing retail environment. A retailer's existing positioning base is continually being eroded by maturing markets and aggressive competitors seeking opportunities for profit and growth. Often the repositioning required is small and gradual...Sometimes, however, the repositioning has to be more radical--a switch into new types of stores, a change into major new merchandise areas or a total re-presentation of the stores.

This notion of continual erosion and repositioning is consistent with historical views of retailing as a very dynamic, continually changing industry. McNair's(1958) "wheel of retailing", and associated notions like "the accordion of retailing" (Mason and Mayer, 1981), suggest that retailers cycle through various formats.

The objectives of this chapter are:

1) to develop a model of competitive retail markets that recognizes the interdependency among the outlets, and captures the micro-level behaviour of
smart management in the face of unexpected adversity ("scrambling astutely");

and

2) to show that such markets reach a steady state, known as self-organized criticality (SOC), and to thereby introduce a novel equilibrium concept to the marketing literature.

The model described in the first objective has the following main features. Many firms located in a two dimensional plane compete for market share through a probabilistic share attraction, or spatial interaction, model. In each period, customers allocate their expenditures to firms according to distance and intrinsic attractiveness of the firms. Each period, a randomly selected firm experiences an exogenous shock in the form of a decrement to its market share. Firms monitor their own revenues, and react when they fall below a threshold level. The reaction is "astute" in the sense that it increases revenues, at the expense of the competition. The model is implemented numerically, and a series of computer experiments conducted to investigate dynamic behaviour.

The self-organized critical state of the second objective is discussed in detail in the next section. The remainder of the chapter is organized as follows. The relevant competition literature is reviewed and related to the model components in section three. Section four develops the model, and section five describes the results of numerical experiments. Results are summarized in section six, and the final section identifies contributions and research opportunities.
Chapter Three: Spatial Competition and Self Organized Criticality

3.2 Self-Organized Criticality

The self-organized critical state has been introduced to the economics literature by Bak, Chen, Scheinkman, and Woodford (BCSW) (1992). It has its origins in several independent developments in such diverse fields as theoretical biology (Kauffman and Johnson, 1992), solid state physics (Bak, Tang, and Weisenfeld, 1988) and computer simulations of artificial life (Langton, 1989, 1992), although the term originated with Bak et. al. (1988).

BCSW (1992) consider a model of production and inventory dynamics for an artificial economy with a large number of firms. The highly stylized model consists of a two dimensional network of producers on a cylinder, each of whom buy supplies from two of their neighbours at a higher level and sell goods to two other neighbours at a lower level. At one end of the cylinder, final goods are demanded randomly, from the last row of producers, by consumers. This demand creates a flow of goods from progressively higher levels in the supply network. The system converges to a state known as self-organized criticality (SOC). SOC will be discussed in more detail shortly, but first consider BCSW’s main point: the law of large numbers does not apply in this situation'. The central limit theorem states that the sum of independent random variables with finite mean and variance converges to a normal distribution, and that the variance of the distribution of the average of the independent random variables approaches zero as the number of random variables increases; roughly speaking, in the large number limit, aggregated independent shocks should tend to cancel out. In BCSW’s SOC state, however, in the limit of a large number of firms, the (appropriately scaled) aggregate response (production) to the independent exogenous

\[\footnote{See Judge, Griffiths, Hill, Lutkepohl, and Lee, (1985), page 156 for a discussion of various central limit theorems and assumptions.} \]
shocks (consumer demand) does not converge to a distribution with zero variance, but to a
Pareto-Levy distribution with nonzero variance. In other words, for large but finite
economies, the probability of large shocks decreases according to a power law distribution;
that is, much more slowly than the exponential decrease that would be expected if the central
limit theorem applied.\footnote{For a discussion of Pareto-Levy, or "scaling", or "stable" distributions, and an empirical
should be noted that the evidence for stable infinite-variance distributions in many financial
been challenged. Blattberg and Gonedes (1974) suggest that Fama's 1965 data could be fit by
a finite-variance t-distribution. Others (e.g., Westerfield 1977) have shown that if time is
redefined so that the analysis is based on prices per transaction, rather than per calendar time,
the distribution is normal. The generalized central limit theorem and the resulting stable distributions are discussed in Levy, (1925,1954).} To quote the introduction to the BCSW article,

Explaining the observed instability of economic aggregates is a long-standing puzzle for economic theory. A number of possible reasons for variation in the pace of production are easily given, such as stochastic variation in the timing of households' desired consumption of produced goods, or stochastic variation in the costs of production. But it is hard to see why there should be large variations in those factors that are synchronized across the entire economy—why most households should want to consume less at exactly the same time, or why most firms should find it an especially opportune moment to produce at the same time. Instead it seems more likely to suppose that variations in demand or in production costs in different parts of the economy should be largely independent. Thus, one might ask, should one not expect these local variations to cancel out, for the most part, in their effects on the aggregate economy, due to the law of large numbers? Fluctuations in activity of macroeconomic significance, it might be thought, should occur only when many independent shocks happen by coincidence to have the same sign, and this should be an extremely unlikely event (with the probability of occurrence decreasing exponentially with the square of the size of the event, by the central limit theorem).
There are a number of ways in which the law of large numbers can be made to fail\(^3\), and more ways still that aggregate instability can arise\(^4\). BCSW offer SOC as one way in which the law of large numbers may fail.

In fields other than economics, SOC has been shown to arise in an amazing variety of very different models and circumstances. While it would be possible to describe SOC in the context of BCSW's model, I will use the prototype "sandpile" model introduced by Bak, Tang, and Weisenfeld (1988) in statistical physics, which is mathematically isomorphic to the subsequent economic model of BCSW. This prototype is also the simplest model that displays SOC, and has a strong and concrete intuition associated with it that gives it substantial face validity, and also makes it easier to describe and understand. For more detail in relatively painless prose, the reader is referred to Bak and Chen's (1991) *Scientific American* article.

Consider a flat tabletop on which grains of sand are individually dropped. Eventually, the sand will pile up and start falling off the edge. At some point, the pile will reach a maximum height, with constant slope in all directions to the edge of the table. At this point in time, the addition of a grain of sand may have no effect, or it may trigger a small avalanche of sand, or, occasionally, it may trigger a large avalanche, on the order of the size of the whole sandpile. The distribution of avalanches follows a power law: the probability of an avalanche involving \( N \) grains of sand occurring is proportional to \( N^{-\alpha} \),

\(^3\) These include Shleifer's (1986) innovations, to be discussed in the next section; see Jovanic (1987) for more discussion and examples.

\(^4\)For example, periodicities or deterministic chaos arising from low dimensional nonlinear equations involving relations between macro variables; see Frank and Stengos (1988); Boldrin and Woodford (1990). A little known marketing example is the Bass diffusion equation, which is theoretically capable of producing chaotic behaviour.
where $\alpha$ is a constant. This state is arrived at whether the grains of sand are dropped at random locations, or at a single location. It can also be arrived at from the other direction--by putting barriers around the edge of the table, filling the resulting box with sand, and then removing the barriers, allowing the pile to relax to its natural slope. The resulting state of the sandpile will be "critical".

The notion of criticality comes from condensed matter physics, which has described "critical states" in a variety of circumstances, usually associated with phase transitions. In the subcritical state, local disturbances have only a weak effect on neighbouring parts of the system, and die out in a finite distance. Correlations between different parts of the system approach zero exponentially as the distance between the parts increases. By varying a "tuning parameter" such as temperature, however, the fluctuations can be made to propagate further. At a certain critical value of the tuning parameter, a state is reached where disturbances can just barely propagate to infinity. At this point, correlations no longer fall off exponentially, but with a power law. As the tuning parameter is changed further, a "phase transition" typically occurs, a new structure forms, and correlations again fall off exponentially. Disturbances, or shocks, can propagate to infinity only at the critical point.

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5For example, from a solid to liquid phase, or from a magnetized to an unmagnetized phase. The "phases" in the sandpile are a stable phase when the slope is below the critical value, and a turbulent high energy dissipation phase, as the sand everywhere flows downwards, when the slope is above the critical value.

6The classic example is spontaneous magnetization of a ferromagnetic material in the presence of a magnetic field as the temperature drops below the Curie point. On either side of the Curie point, the external field has no macroscopic effect. However, at the Curie point, macroscopic fluctuations of the aggregate magnetization are possible, and the weak field can determine the final aggregate magnetization. This effect was key in establishing plate tectonics, by "freezing" the orientation of the earth's magnetic field, in cooling minerals, at various times in geologic history.
Chapter Three: Spatial Competition and Self Organized Criticality

The critical state is a region where spontaneous macroscopic instability may occur. BCSW state,

The problem with this as a model of spontaneous macroeconomic instability is that, traditionally, critical states were thought to be associated with certain "critical" parameter values (such as temperature), that would almost certainly not occur in any existing system unless they were "tuned" to be at the critical value in a laboratory experiment. (Jovanovic's (1987) examples of economic models in which independent sectoral shocks produce aggregate fluctuations no matter how large the number of sectors...are special in exactly this sense). But [the sandpile model provides a mechanism whereby] large interactive dynamical systems can "self-organize" into a critical state. That is, the critical state can actually be an attractor for the dynamical system, toward which the system naturally evolves, and to which it returns after being perturbed by some large external shock.

The power law distribution of the sizes of disturbances, or "avalanches", is mixed news for a competitor living in a SOC state. The bad news is that a competitor may be affected by a shock happening anywhere in the system. The good news is that the probability of being affected by the shock decreases as the distance from the shock increases. This "mixed news" has substantial intuitive appeal.

There is another interesting way in which the state is "critical": in terms of how sensitive the evolution of the system is to small changes in initial conditions (that is, "small" relative to the range of the system variables normally observed in the system over a long period of time). If, at a fixed point in time, we take two realizations of any dynamic system that are separated by a "small" distance in state space, and watch their evolution over time, three general kinds of behaviour are possible. The two systems may converge or diverge in state space, or they may remain at a constant separation, either absolutely or on the average. The steady state achieved in the model developed here is in the constant-separation category. A constant separation in a nonlinear system seems surprising--it appears to be a
knife-edge effect. In fact, since divergent systems are commonly referred to as chaotic\(^7\), the SOC state is often said to be "on the edge of chaos"\(^8\).

Another feature of this complex extended system is that the degrees of freedom remain high, even after the self-organized state is reached. As BCSW point out, "This is in contrast to a macroscopic description of economics in terms of a few global variables, where micro economic fluctuations are assumed to average out in the final analysis".

SOC models have proliferated at an amazing rate since introduced by Bak, Tang and Weisenfeld in 1988. Perhaps the most successful empirical application has been as an explanation of the Gutenberg-Richter power law distribution of the magnitude of earthquakes. (Sornette and Sornette, 1989) Another class of models in theoretical biology, which have an entirely different structure, also show SOC (Kauffman and Johnson, 1992). The biological models will not be discussed here, as the model developed in this research is more akin to the models of Bak et. al. It should be noted, however, that the biological models have substantial promise for economic applications because of their explicitly optimizing approaches.

\(^7\)This is the "butterfly effect" due to Lorenz (1969), and the rational behind empirical methods to detect chaos. See Grassberger and Procaccia(1983); Sugihara and May (1990). The butterfly effect states that the difference between a butterfly fluttering its wings or not in Beijing can make the difference between whether or not a thunderstorm occurs in New York a month later. Erickson (1993) provides a theoretical example of the possibility of chaotic behaviour in a marketing context: closed-loop Nash equilibrium strategies of duopolistic competitors in a Lanchester advertising model may be chaotic. While this is a micro-level effect, Erickson's concluding statement is relevant: "Chaos theory can be useful in marketing, because market responses to marketing activities are dynamic and nonlinear. It is also the case that rarely are markets observed to be in steady state, or in a state of repeating cycles, so that exclusively empirical approaches are not likely to be sufficient in the study of dynamical behavior in marketing settings."

nature. Routledge(1993), in a finance context, has shown behaviour similar to Kauffman and Johnson's in a model of many agents playing a repeated prisoner's dilemma game.

In summary, the appeal of SOC is, first that it provides a mechanism for generating aggregate fluctuations from independent random shocks that do not have vanishing variance in the large limit; in particular, these fluctuations follow power-law (or Pareto-Levy) distributions, which have been shown to occur in economic time series (Mandelbrot, 1982). Second, the self-organized state is generally robust to model details. This means that it is not necessary to have very special conditions. Third, it has been shown to arise in very different contexts. It remains unclear what criteria are necessary for SOC to arise, but one common feature seems to be that systems have many degrees of freedom, with dynamically interacting elements: not unlike the many agents in an economy.

In conclusion, the research presented in this chapter can be considered in the context of Moorthy's (1993) article on the role of theoretical modelling in marketing research. Moorthy persuasively suggests that this type of research can be considered "logical experimentation"; progress occurs by a series of "treatments", consisting of substantively different model assumptions, "very likely [made] by different researchers", which eventually build a picture of which assumptions are responsible for which outcomes. The research in this chapter may be considered, first, as a second "treatment" of economic SOC. The advance here is that, unlike BCSW's model, the model is not just an economic reinterpretation of an existing model from physics. Rather, it is drawn directly from marketing models of spatial competition, with a dynamic inspired by the apparent micro-dynamics in retailing. On a second level, this research may be considered as a novel "treatment" of micro-level decision dynamics in the context of spatial competition in
retailing. The resulting macro-level behaviour is unusual, but contains much of the spirit of verbal descriptions of the volatile and cyclic nature of retailing, as in the wheel of retailing.
3.3 Literature Review

Eliashberg and Chatterjee (1985) provide a classification framework and review, from a marketing perspective, of analytical models of competition. They describe their view of a marketing perspective as follows:

For marketing scholars and managers, the principal focus is on the conduct of the competing firms in the market, recognizing that the activities of one firm affect the performance of other firms in the market. Thus, marketers are primarily interested in competitive models based on an oligopolistic market structure, where the interdependence among the competing firms is recognized explicitly.

This section relates the model in this chapter to the relevant literature on theoretical models of competition and to Eliashberg and Chatterjee's classification. Table IV gives their scheme, and positions the model of this chapter according to those categories.

The first classification is the objective of the model: basic understanding versus decision-oriented. "Models directed at basic understanding study the industry as a whole rather than one specific company. The industry analysis involves such questions as, 'Under various scenarios, what is the nature of the dynamic evolution of the industry...?'. The model developed in this research is solidly in this category. In the concluding section of the same paper, Eliashberg and Chatterjee address the issue of "informational, motivational, and behavioral assumptions". They state, "From the perspective of basic understanding of competitive markets (i.e., from a "detached" viewpoint), we need a descriptive model based on how competitors believe the others act and how the competitors actually act (more precisely, the modeller's "best judgement of how they act")." While I don't propose that the behaviour as modelled here captures the entire dynamics of real decision processes in spatial competition, the introduction to this chapter suggests that "scrambling astutely" is an
### Table 4: Model Characteristics in Chatterjee and Eliashberg’s (1985) Classification.

<table>
<thead>
<tr>
<th>I. Problem</th>
<th>Effect of limited information and foresight in adverse environments</th>
</tr>
</thead>
<tbody>
<tr>
<td>II. Objective of model: basic understanding vs decision-oriented</td>
<td>Basic understanding</td>
</tr>
<tr>
<td>III. Basic Assumptions</td>
<td></td>
</tr>
<tr>
<td>III.1 Demand Characteristics</td>
<td></td>
</tr>
<tr>
<td>Number of segments and the nature of their interrelationship</td>
<td>1</td>
</tr>
<tr>
<td>Factors affecting primary demand</td>
<td>Intrinsic attractiveness</td>
</tr>
<tr>
<td>Factors affecting market share</td>
<td>Intrinsic attractiveness</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>Random exogenous shocks</td>
</tr>
<tr>
<td>III.2 Supply Characteristics</td>
<td></td>
</tr>
<tr>
<td>Number of products</td>
<td>N/A (store choice)</td>
</tr>
<tr>
<td>Product differentiation</td>
<td>Explicit</td>
</tr>
<tr>
<td>Barriers to entry</td>
<td>Not considered</td>
</tr>
<tr>
<td>Cost structure</td>
<td>linear f(attraction)</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>Not considered</td>
</tr>
<tr>
<td>III.3 Competitive Activity and Decision Making Process</td>
<td></td>
</tr>
<tr>
<td>Number of competitors</td>
<td>Many (64 - 100)</td>
</tr>
<tr>
<td>Decision variables</td>
<td>Intrinsic attractiveness</td>
</tr>
<tr>
<td>Competitive behavioral mode</td>
<td>Noncooperative</td>
</tr>
<tr>
<td>Decision makers' objectives</td>
<td>Satisfice: maintain profits</td>
</tr>
<tr>
<td>Decision makers' attitudes to risk</td>
<td>Not considered</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>Not considered</td>
</tr>
<tr>
<td>IV. Mode of Analysis</td>
<td></td>
</tr>
<tr>
<td>Level of aggregation</td>
<td>Individual</td>
</tr>
<tr>
<td>Static vs dynamic</td>
<td>Dynamic</td>
</tr>
<tr>
<td>Equilibrium conditions</td>
<td>Derived SOC</td>
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<tr>
<td>Analytic vs simulation</td>
<td>Simulation</td>
</tr>
<tr>
<td>V. Basic Results</td>
<td>System evolves to SOC</td>
</tr>
</tbody>
</table>

important component. Just as game theoretic models with full information and perfect
foresight can provide substantial insight into the nature of spatial competition, in spite of being an idealization, so should this model provide insight by examining a neglected component of dynamic competitive processes.

The remainder of this section is divided into two parts. The first discusses the market context, namely probabilistic customer choice in spatial competition. The second deals with the decision environment and competitive dynamics.

Spatial Competition: Structure of Supply and Demand

The model assumptions regarding supply and demand are in the context of spatial competition. Hotelling (1929) is generally credited with the original work on spatial competition, in his analysis of two firms competing for market share in a linear market. This work has been expanded in many directions. Much of it assumes demand arises from a deterministic customer choice process, and is not directly relevant to the research in this chapter, which uses probabilistic choice. A notable exception is dePalma, Ginsburgh, Papageorgiou, and Thisse (1985), where a logit model is used to account for unknown heterogeneity in consumer tastes. The probabilistic model removes discontinuities that plague Hotelling first-choice models, and has equilibrium configurations where firms minimally differentiate, as originally proposed by Hotelling. Probabilistic customer choice is intuitively appealing; however, it is analytically more difficult. Choi, DeSarbo, and Harker (1990;1992) also use a logit choice model in a product-attribute space, and use a numerical solution method to find simultaneous product-price Nash equilibria for many firms. The model is demonstrated with residential telecommunications equipment. These models are generally concerned with strategic issues of location, pricing, and other marketing
mix variables. In contrast, the model developed here is concerned with the dynamic evolution of the industry.

A second stream of spatial competition literature, which is more relevant to this research, is the location literature in marketing, geography, and regional science. This is generally a much more empirically oriented literature. A major branch is concerned with optimally locating retail outlets. An early version of this problem was addressed by Reilly (1931), who used a "gravitational" model to determine intermetropolitan trading area boundaries. Customers are assumed to make a deterministic choice based on a tradeoff between the size of the centre and their distance from it. Christaller's (1933) Central Places theory places primary importance on distance to the centre.

Unlike the theoretical spatial competition literature, the spatial location literature adopted probabilistic choice early. Huff (1962) is generally credited with the original formulation, although the probabilistic content is Luce's choice axiom (Luce, 1959). A rich empirical modelling literature, known as location-allocation modelling, has followed the general Huff formulation. Customers trade off travel related costs against intrinsic store attractiveness, which may be a function of any number of marketing mix variables. The tradeoff function gives a utility or attractiveness for each store. However, rather than allocating all of their expenditures deterministically to the winner of the tradeoff, it is allocated probabilistically according to share of attraction. The allocation is also often interpreted in the aggregate, rather than strictly probabilistically, so that each customer actually allocates shares of expenditure according to share of attraction. The relation between this work and the model here is examined in more detail in the model development section. See Ghosh and McLafferty (1987) for a review of location-allocation models.
Chapter Three: Spatial Competition and Self Organized Criticality

The likely reason that probabilistic customer choice is usually assumed in these models is because of their empirical and decision-focused nature, which must capture uncertain heterogeneity in customers. A primary justification for using first choice in theoretical work, even though empirical work uses probabilistic choice, is tractability. If the theoretical analysis is numerical, rather than analytic, tractability is less of a concern. Consequently, in the model developed in this chapter, the more realistic probabilistic choice model is used.

It is most commonly assumed in models of spatial competition that the industry-wide primary demand is fixed, and that firms compete for market share. This is also the more common assumption in the representative set of models reviewed by Eliashberg and Chatterjee (1985), except when the marketing mix variable of concern is advertising. In that case, it is unreasonable to assume inelastic total demand (e.g., Erickson, 1985).

In the probabilistic spatial competition models, one way to make primary demand elastic is to include an option for the customer that is outside the market. Choi, DeSarbo, and Harker (1990, 1992) use this device, and refer to it as either a generic choice, or a "no purchase" option. Customers still have a fixed total expenditure to allocate each period, but some of it may not go to any of the competitors. In this way, the total expenditures within the industry varies with the total industry attractiveness, relative to the total external attractiveness. The model in this chapter also uses this device, and thus has elasticity of total demand. However, the main reason here for the external option is to capture dynamic adversity, which will be discussed shortly.

Demand is most often expressed as a function of some quite specific decision variables, such as price, advertising, or location. In the model developed here, demand is
affected (through the attraction function) by store location relative to the customers, and the firm's intrinsic attractiveness. Because we want to capture the ability of firms to make innovative changes—that is, discontinuous and qualitatively different changes—the intrinsic attractiveness itself is the variable under control of management. It is not specified as a function of more specific marketing mix elements.

The industry structure consists of a fixed number of firms (typically 64 to 100) at fixed locations. Eliashberg and Chatterjee (1985), in their review of competition models, consider three categories:

1. Deterministic models addressing competition among incumbent firms.
2. Deterministic models addressing competitive entry issues.
3. Models addressing competitive decision making under uncertainty.

In this scheme, the model is partially in category 1—deterministic models addressing competition among incumbent firms. The model might be considered in category three—models addressing competitive decision making under uncertainty—because of the exogenous random shocks. However, the decision process itself does not take uncertainty into consideration, because the decision makers do not have explicit expectations. The dynamics and competitive activities are discussed in the next section.

A feature which occasionally appears in both the theoretical economic modelling literature (Carpenter, 1989) and the applied location-allocation modelling literature (Ghosh and Craig, 1991) is "reservation distance". Analogous to reservation price, this represents a distance beyond which customers will not travel to patronize the firm. In many situations,

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9As the number of firms is increased, the statistics improve but the strain on computer resources also increases. At 64 firms, the main results could be demonstrated; At 100 firms, resource limitations began to be severe.
this has substantial appeal. For example, it would not seem reasonable for a customer to allocate some portion of his dry cleaning, however small, to every dry cleaner in a city. Aside from that appeal to intuition, a major reason for introducing a distance cutoff is for tractability in analytic models, and to reduce the pressure on computing resources in numerical models. The latter is the dominant reason for the cutoff in my model.

Competitive Dynamics

As briefly described in the introduction, the approach in this model is to assume a rule-based decision dynamic, and allow the system to evolve, rather than assume a particular equilibrium concept. This is not a common approach to competitive dynamics, but neither is it highly unusual. The dynamics may be divided into four main features. First, the decision maker operates in a low information environment; second, the heuristic involves satisficing behaviour; third, the challenged firm has the ability to react with a successful innovation; and fourth, the entire system is driven by exogenous shocks in the form of indirect outside competition hitting individual firms randomly. First, rule-based heuristics generally will be reviewed, and then these four specific features.

The game-theoretic equilibrium approach has well-known advantages and shortcomings, which have been addressed eloquently by many, including Gould and Sen (1984) and Kreps (1990). Regarding the Nash equilibrium, Gould and Sen state that "the definition is essentially ad hoc in the sense that it is not endogenously motivated by the model itself. Among other things, this means that in a comparative static model the mechanism which assures equilibrium is unspecified". Gould and Sen conclude that
It is not yet clear whether the ambiguity will be resolved empirically or theoretically (most likely a combination of both), but it is clear that a resolution of these issues is important to marketing and economics. Indeed one hopes that some of the questions marketers are working on will be the means by which more is learned about this topic.

Kreps (1990, p405) introduces the Nash equilibrium as

...an [author's emphasis] answer to the question: If there is an obvious way to play the game, what properties must that "solution" possess? ... But this is a very weak question, and it is clear that having the answer "Nash equilibrium" is pretty thin gruel if what we are after is a way to solve games. All we have is a test of solutions derived by some other means."

The dynamic modelling approach which uses only recursive decision rules, in the form of sensible heuristics, directly deals with this problem of "how do we get where we're going". Examples are Baumol and Quandt (1964); Day (1967); Day and Tinney (1968); and Cohen and Axelrod (1984). These models, however, usually involve some explicit form of adaptation or learning behaviour. An example where learning is via Bayesian updating is Eliashberg (1981), who distinguishes circumstances where the competition may evolve cyclically, or it may converge to the game-theoretic static equilibrium. In contrast, my concern is not with the explicit modelling of the adaptation process; rather, the model simply assumes that firms can adapt in response to adversity.

Much of the work on decision rules is designed to show how heuristics can arrive at states that are close, or identical to, the game-theoretic equilibria implied by optimal strategies of fully informed managers with perfect foresight. A recent marketing example is Jeck (1991). He examines three heuristic price setting strategies, in a competitive channel context involving two manufacturers and two retailers. Jeck's work is also quite relevant to my model because of the "low information environment" assumed. The information is
low in the sense that each decision maker observes only his own cost, his own decision, and his own quantity demanded. Two of Jeck's decision rules lead to states that closely approximate a Bertrand-Nash equilibrium. The third, which involves more "memory" than the other two, leads (approximately) to a collusive equilibrium.

Turning now to low information environments, Jeck contends that, in spite of being under-researched, low information environments are important:

There is considerable evidence that managers have great difficulty deducing the form and parameters of the consumer demand curve even under full information about the elements of the marketing mix of all competitors. It is true that the firm can employ very sophisticated statistical analysis to develop estimates for the firm's demand function, but only a few such analyses have been performed. Perhaps more importantly, perfect information about competitors' actions even when it is available often is not taken into account when decisions are made. For example The Marketing Workbench Laboratory at Duke University has found that store by store reports of prices, which can be obtained by the decision makers, have not been used by many firms even though it is felt that many consumer purchase decisions are based on available stimuli at the point of purchase (Russo, 1977; Aaker and Ford, 1983). The decisions arrived at by many managers arise in decision making environments that do not match those used to derive high information closed-form equilibrium descriptions of markets. Given managers' apparent predilection to make decisions in environments that do not match those used to derive Bertrand-Nash equilibrium, it would appear beneficial to study the "equilibrium" conditions for markets when the decision makers are ill-informed about their competitors and their reactions.

My model has this low information condition, in that decision makers observe, and respond to, only their own revenue levels.

In the quote in the previous section, Corstjens and Doyle characterize as a "central facet" of retailing the need to respond to "continual erosion" of the retailer's position. The Financial Times of Canada article, in reference to the retail grocery industry, speaks of "scrambling". To capture this phenomenon, the model assumes that the players in the
market are under continual external pressure, from both exogenous shocks and endogenous competition. Furthermore, decision makers react when their revenues fall below a fixed threshold. This is a "satisficing" (Simon, 1965) type of behaviour, rather than optimizing. An example of recursive decision rules which involve satisficing can be found in Day (1967). In Day's model, however, the decision maker keeps attempting to improve his lot until the incremental improvement (in profit) falls below some satisfactory level. As this "satisficing level" is set smaller, the outcome approaches optimal "marginal costs equals marginal revenues" solution. In my model, the satisficing level is a fixed revenue level, rather than a differential level; in the turbulent environment, the retailer simply scrambles to keep his head above the threshold. This dynamic is novel, and perhaps unpalatable, in that it precludes any possibility of producing outcomes that could be described as optimal. It must be borne in mind, however, that description, rather than prescription, is the objective here.

The third feature of the model is that the decision maker can make a good decision, once prodded, in that the decision results in improved revenues. In the context of the spatial competition model, this means that the store attractiveness can be increased, drawing in more revenue. Thus we imagine, rather than a specific setting of some marketing mix variable, a qualitative change in the nature of an innovation. For a food retailer, this might mean a change in advertising strategy (perhaps to increase consumer's sensitivity to travel costs), or an introduction of new high-margin departments, or the extraction of wage concessions from

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10The implication, of course, is that the revenue more than offsets the cost of implementing the decision, so that profits increase. The model can be implemented with profits, rather than revenues, and an example is provided in the results section. The issue of costs, however, is outside the main focus of this work.
unions followed by price reductions. It might also be an imitation of a successful strategy—in some Western Canadian and U.S. cities, Safeway introduced its own low-cost superstore chain, Food-For-Less. A key assumption here is that there is always some revenue-increasing "innovation" available to the firm. It simply requires pressure on revenues for the innovation to be implemented. So the "innovations" could be referred to not only as "imitations" but as "implementations" (Shleifer, 1986). In the interest of verbal economy, and in keeping with common marketing terminology, however, the decision will be referred to as an innovation. The possibility that innovations are available to firms, but not implemented, has been addressed by Judd (1985) and Shleifer (1986). In both articles, "innovation cycles" (implementation cycles, in Shleifer's case) occur. However, the cycles are driven by almost opposite mechanisms. In Judd's model, the introduction of too many new products within a short time causes competition for consumer resources, and reduces profits for each one. Subsequent imitation puts even more price pressure on the products. Thus, after a period of innovation, entrepreneurs hold off on introducing new products until introduction is again profitable. Shleifer's model, on the other hand, is driven by expectations of large profits. Entrepreneurs would like to release their inventions when they can get the most profits, namely when the economy is booming. But a boom is driven by investment in the release of innovations. Hence, if firms share beliefs about the timing of a boom, they can make the boom a reality. Foresight leads to cyclical equilibria.

As in the above articles, a central issue of my research is to explore how innovative reactions sweep across a market. Both the driving mechanisms and the nature of the response, however, are entirely different. My driving mechanism, to be contrasted with the above mechanisms, is innovative response by firms to unexpected adversity in an industry
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that competes through spatial interaction. My response is waves, or avalanches, that occur on all scales, and with no periodicity.

My model shares one important characteristic with Shleifer's. To quote, "The possibility of a cyclical equilibrium sheds doubt on a frequently articulated view that a market economy smooths exogenous shocks. Inventions here can be interpreted as shocks hitting the economy, which are essentially identical each period. But these shocks can be "saved"...The economy follows a cyclical path when a smoother path is available (p. 1165)."

In my model, the exogenous shocks come in the form of a decrease in revenues for an individual firm, modelled as the appearance of competition outside the industry. The response of a firm, by innovating, can then become an endogenous shock for its nearest competitors. The resulting SOC state is characterized by responses that have a power law distribution; that is, the distributions' tails fall off as a power law.

This leads to the fourth feature: the nature of the shocks. In a share attraction model, the total market demand may be constant, allowing the modeller to focus on the distribution of share among firms. Alternately, the customers may be given a choice that is outside the industry, in the form of an additional term in the denominator of the share expression. This device is used by Choi, DeSarbo, and Harker (1990) to introduce price elasticity into their logit model of spatial competition. Choi et. al. refer to this as a "no purchase option" and use it to preclude the possibility of an equilibrium with infinite prices and infinite profits. In my model, the additional term is used to model the effect of "continual erosion", as described by Corstjens and Doyle, by incrementing the term each period at a randomly

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11See Bell, Keeney, and Little (1975) for an axiomatic development of the share attraction model and a discussion of its features and limitations.
chosen store. Although this "erosive shock" enters the model as another option for the customer, and hence may be easily thought of as either "no purchase", or as indirect competition, its main purpose is to create revenue erosion.
3.4 Model Development

In this section, the components of the model and numerical analysis are introduced. Two important points regarding the research strategy guiding model development will first be briefly summarized.

The first point is that the focus is the long-run behaviour of a model industry characterized by features of the retailing industry that have been often commented on: it is dynamic, intensely competitive, and innovative. The model assumes that successful, innovative responses to adversity will be made. These may arise from some evolutionary or learning process, as for example Eliashberg's (1981) Bayesian learning; the details of the origin of the successful response, however, is not the issue being addressed here. Nor is the precise nature of the innovation. It is simply assumed that an effective response can be made. Modelling at this level is necessary to capture the long-run behaviour, because the retail industry changes dramatically and qualitatively in the long run. It is similar in spirit to Shleifer (1986), who also examines innovations.

The second point relates to the outcome state, self-organized criticality. Marketing has a long tradition of borrowing from other fields, and this research is no exception. In borrowing concepts, however, there is always an issue of appropriateness. Unlike BCSW (1992), the approach here was not to find marketing labels to place on a model from statistical physics. Rather, the issue was whether the SOC state could arise in an established marketing context. In particular, the model for the interactions between the elements is the spatial interaction model which has been used in many empirical settings, and which involves substantially more complex computations that the sandpile and related models.
3.4.1 Customer Choice

The geography and retail location literature use a probabilistic customer choice mechanism (e.g., Huff 1962), where the market allocates its expenditures to stores according to relative attractiveness. The economic literature in the Hotelling tradition, in contrast, almost exclusively uses a first choice model, where the entire expenditure is allocated to the most attractive alternative. One exception is dePalma, Ginsburgh, Papageorgiou, and Thisse, (1985), where a logit model is used to account for unknown heterogeneity in consumer tastes. The probabilistic model removes discontinuities that plague Hotelling first-choice models, and is intuitively appealing; however, it is analytically more difficult. Because the majority of this work is simulation, and to maintain contact with the retail location literature, this model uses probabilistic customer choice.

In general, the utility of store \( j \) to customer \( i \) is assumed to take the form

\[
U_{ij} = \prod_{k=1}^{K} [f_{k}(A_{jk})]^{\alpha_k} \prod_{l=1}^{L} [g_{l}(D_{ijkl})]^{\beta_l}
\]

The utility--often referred to as the attraction in share attraction models (Cooper and Nakanishi, 1988)--is a function of \( K \) characteristics \( A_{jk} \) intrinsic to store \( j \); the \( \alpha_k \) are usually assumed positive, so that utility increases with increasing \( A_{jk} \). Ghosh and McLafferty (1987) state, "The [intrinsic] attractiveness of a store results from a number of factors, including its size (which is often a surrogate for breadth and assortment of goods carried), its relative prices, and consumer perceptions of quality of merchandise and service" (p.63). Utility is also a function of \( L \) travel cost components, \( D_{ijkl} \), usually measured as distance or
travel time between the ith customer and the jth store; the $\beta_i$ are assumed negative to capture disutility for travel. If the time-honoured assumption is made that either aggregate market shares or individual choice probabilities (Luce’s choice axiom) are proportional to the share of utility (or attraction). The jth store’s share of ith customer’s purchases is given by

$$M_{ij} = \frac{U_{ij}}{\sum_h U_{ih}}$$  \hspace{1cm} (81)

In this relation, total demand is inelastic. Elasticity is introduced by including an extra term, $K_j$, in the denominator, representing a generic choice, or a no-purchase option. (See, for example, Choi, DeSarbo, and Harker,1990). This operates like indirect competition, in the sense of being outside the market. Further, this term is store specific, so that each store may have a different level of "indirect competition" to deal with. It is through this term that adversity in the environment is modelled. The customer's allocation then takes the form

$$M_{ij} = \frac{U_{ij}}{\left( \sum_h U_{ih} \right) + K_j}$$  \hspace{1cm} (82)

In the marketing retail location literature, the functions $f_k$ and $g_i$ are almost always the identity function, giving the multiplicative competitive interaction (MCI) model. Early versions were limited to $K = L = 1$, with size as the surrogate for intrinsic attractiveness, and Euclidean distance for $D_{ij}$. If the absolute value of beta is much greater than alpha, the model approaches "nearest centre" models, as in Christaller’s (1933) Central Places
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formulation. In Reilly's 1931 "Law of Retail Gravitation", which focuses on intermetropolitan trading area boundaries, alpha is one and beta is -2. Huff (1962) assumes alpha is one, and estimates beta to be 2.1 to 3.7 for various types of outlets in Los Angeles. Later authors consider more attributes, such as sales area, number of checkout counters, and whether credit cards are taken for the $A_k$ (Jain and Mahajan, 1979); and auto travel time, transit travel time, and travel cost per unit income for the $D_{ij}$ (Weisbrod, Parcells, and Kern, 1984). Typical parameter values (for $\alpha$ and $\beta$) are between one and two, with extremes of 0.1 to 3.7. Fixing $\beta = 0$ gives a more usual MCI market share model, as, for example, in Hansen and Weinberg's (1979) analysis of retail banking outlets.

Exponential $f_k$ and $g_i$ result in the multinomial logit (MNL) share model. An example in the economics theoretical-equilibrium literature is dePalma, Ginsburgh, Papageorgiou, and Thisse (1985), who consider the intrinsic attractiveness to have two components, product valuation and price; and $D_{ij}$ to be the linear distance on a Hotelling "beach". The associated exponents are $\alpha_1 = 1/\mu$, $\alpha_2 = -1/\mu$, and $\beta = -c/\mu$, where $\mu$ is a population heterogeneity parameter and $c$ is travel costs.

The above discussion suggests that the minimal requirement for capturing spatial competition is one "intrinsic attractiveness" parameter and one distance parameter. For expositional intuition, the attractiveness parameter will be referred to as "size", as suggested by Ghosh and McLafferty (1987), although it may be equally well thought of as any number of other quantities (such as product valuation minus price). Because the focus here is on geographic space, distance will be taken as the usual Euclidean distance on geographic coordinates.
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There is relatively little reason to prefer either the identity or the exponential forms for the functions \( f \) and \( g \). The greater analytic tractability of the exponential form in optimization and equilibrium analysis is not relevant for this research, which uses simulation. The marketing retail location literature commonly uses the identity, and the dynamic geographic literature uses the identity for \( f \) and the exponential for \( g \), the distance function. For this research, \( f \) and \( g \) are taken to be the identity function, to be consistent with the marketing retail location literature.

As modelled so far, each customer considers all possible retail outlets, which is not only unrealistic and ignores the notion of consideration sets, but is computationally cumbersome. In a product space context, Carpenter (1989) introduces a "reservation distance" to limit the customer's consideration set. Similarly, Ghosh and Craig (1991) use a reservation distance in a location model for franchises. Applied to the retail location problem, from the firm's point of view, the reservation distance determines the outlet's trading area.

In the basic spatial interaction model, then, customer \( i \) allocates a proportion \( M_{ij} \) of his expenditures to store \( j \) in each period:

\[
M_{ij} = \begin{cases} 
\frac{S_j^\alpha D_{ij}^\beta}{\left( \sum_k S_k^\alpha D_{kj}^\beta \right) + K_j} , & D_{ij} \leq R \\
0 , & D_{ij} > R 
\end{cases}
\]  

(83)

where \( R \) is the reservation distance, and
\[ \alpha \geq 0, \]
\[ \beta \leq 0. \]

### 3.4.2 Firm Revenues

In each period, each firm's revenues are the sum of attracted customer's shares of expenditures:

\[ R_j = \sum_i M_{ij} \]  \hspace{1cm} (84)

### 3.4.3 Market Configuration

Customers are uniformly distributed on a bounded plane. For the simulation the plane is rectangular, with customer-origin points in a regular grid. It is slightly more intuitive to think of the origin points as city blocks, rather than individual customers, in this setup. Stores are located in a coarser grid in the market--for example, every third block in one direction and every fourth block in the other. In each period, each customer-origin spends one unit (e.g., dollar), allocating the unit to all the stores within the customer-origin's reservation distance according the share of attraction.

### 3.4.4 Dynamics

At time \( t = 0 \), the system is initialized by assigning store sizes and unique advantage randomly to all the stores in the plane. This ensures that the results do not depend on a
uniform distribution of store sizes. Revenues are then calculated for each store. A revenue threshold is initialized at some fraction of this initial revenue. The actual value of the fraction doesn’t matter—it is only set different from unity in order to investigate the transient behaviour of the system. Because of the random initial store sizes, each store starts with different revenues, and will have a different threshold. Again, this guards against results arising from uniformity in the model.

Stores are shocked by the addition of an increment $\delta k$, which remains constant over stores and time, to $K_j$. Stores innovate, when their revenues drop below their individual
Chapter Three: Self-Organized Criticality

revenue thresholds, by the addition of an increment $\delta S$, fixed over stores and time, to $S_j$.

After initialization, the following algorithm is implemented.

ALGORITHM: SCRAMBLING ASTUTELY

1. Shock a store chosen at random.
2. Calculate revenues of all stores.
3. If all stores have revenues above their threshold, increment time and return to 1.
4. All stores whose revenues have dropped below their threshold innovate.
5. Increment time and return to 2.

3.4.5 Software

The details of the software used to carry out the numerical experiments described in the next section is now briefly discussed.

Each experiment consists of a run of the simulation routine with a particular set of parameters. The parameters for each run are set in a parameter file, which is read by the simulation routine. These parameters describe the market configuration, the size and distance exponents $\alpha$ and $\beta$, the initial store sizes, the reservation distance, the number of iterations, and the size of the exogenous shocks and innovation responses. Parameters which vary across stores are store location, external environment $K_j$ and sizes $S_j$. The latter two, of course, also vary with time. All other parameters are held constant for each run, but can be changed from one run to the next.
Once the parameters are set, the simulation is run. The routine reads in the parameters and initializes the simulated market. If one is not interested in the transient behaviour, there is a method for getting the system to the steady state more quickly, which is described in the next section. The system is allowed to run until a fixed number of shocks have been delivered, determined in the input parameter file. Typically, 64 stores are used, and the simulation is run for 1000 to 2000 periods. The size of each innovation response, or avalanche, is measured as the number of stores which innovate after each shock. This number is recorded, and a histogram of response sizes is output at the end of the run. As well, matrices of the revenue level and innovation level are recorded for each innovation cycle. This data can then be input to an animation routine, which is used to observe the dynamics of the system as it evolves.

All software is written in C. The code for the simulation may be found in Appendix B. Simulations were run on both a 486 PC, and an IBM RS/6000 560 mainframe. Animations were run on 386 and 486 PC's with VGA.
3.5 Results of Numerical Experiments

In this section, the characteristics of the model are examined. First, the transient behaviour is examined in terms of total system revenues to demonstrate convergence to a fixed value.

Table 5: Parameters Investigated in Sensitivity Tests

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Range</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>size exponent</td>
<td>0.5 - 3.0</td>
<td>Determines customer sensitivity to &quot;size&quot; or &quot;intrinsic attractiveness&quot; of store. Empirical estimates of $\alpha$ are in this range.</td>
</tr>
<tr>
<td>$\beta$</td>
<td>distance exponent</td>
<td>0.5 - 3.0</td>
<td>Determines customer sensitivity to distance. A larger value means store attractiveness drops more rapidly with distance, implying weaker competitive interactions between stores. Range is representative of published estimates.</td>
</tr>
<tr>
<td>$R$</td>
<td>reservation distance</td>
<td>2.9 - 6.5</td>
<td>Maximum distance customers will travel to a store. Units are related to &quot;customer origin points&quot;, which are separated by 1 unit of distance, and store locations, which are separated by 3 units. $R$ also is the radius of a store's trading area. Like $\beta$, $R$ affects the strength of competitive interaction. At the low value (2.9), no store is within any other's trading area, and each has a monopoly in a small area (of radius 0.1). If $R$ dropped to 1.5, the system would be entirely decoupled, and each store would be a monopolist. When $R$ is 6.5, the trade area is 5 stores in diameter, which approaches the size of the 8 x 8 system.</td>
</tr>
</tbody>
</table>
Then the size distribution of avalanches in the steady state is examined, and found to follow a power law. The robustness of the power law distribution to model changes is then investigated. Table 5 gives the parameters and ranges that were investigated. Finally, the sensitivity of the system to initial conditions is examined by introducing a small perturbation at a single point in time, and then tracking the subsequent evolution in state space.

In most of this chapter, the concern is with the behaviour of the system as time approaches infinity, that is, the steady state of the system. Since the system is being drive at all times by shocks delivered to stores chosen at random, the steady state is stochastic. This means that some care must be taken to ensure that any initial transients have decayed before starting to count responses, as it is not apparent simply by examining the responses of the system in terms of the innovation avalanches. The issue of transient behaviour is examined in the next subsection.

3.5.1 Transient Behaviour

The nature of the environment and the decision process ensure that each firm’s revenues will eventually be close to its threshold level, in particular within a distance determined by the size of the exogenous shocks and the size of the innovative response. To demonstrate that the system will actually converge on this general region, the threshold level is set away from the initialization level, and the total revenues in the system monitored over time. In the following, the threshold is set at 80% of the initial revenues. The parameters used in this run are shown in Table 6.
Chapter Three: Spatial Competition and Self-Organized Criticality

Table 6: Parameter values for transient test.

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 3</td>
<td>STORE SEPARATION IN x &amp; y. These are the number of customer origin points between each store in the x and y directions.</td>
</tr>
<tr>
<td>23, 23</td>
<td>MARKET GRID SIZE. The number of customer origin points in the market in x and y directions. This gives a store grid of 8 x 8, or 64 stores.</td>
</tr>
<tr>
<td>1, 1</td>
<td>EXPONENTS of DIST &amp; SIZE. Alpha and beta in the choice model.</td>
</tr>
<tr>
<td>30</td>
<td>SEED. Initializes the random number generator.</td>
</tr>
<tr>
<td>3.0, .0001</td>
<td>SIZE INITIALIZATION. Size = a + b * ran; a, b are the parameters and ran a random number between 1 and 32,767.</td>
</tr>
<tr>
<td>4.5</td>
<td>RESERVATION DISTANCE. In units of customer origin points.</td>
</tr>
<tr>
<td>1000</td>
<td># OF ITERATIONS. Total number of exogenous shocks delivered.</td>
</tr>
<tr>
<td>0.8 0.2</td>
<td>MINREVENUE and INNOVATION. The threshold level and the size of $a_i$.</td>
</tr>
<tr>
<td>0.05 1.</td>
<td>ADVERSITY. Initial value of $K_j$ and size of shock.</td>
</tr>
</tbody>
</table>

Figure 37 shows the sum of the revenues of all 64 stores over 1000 iterations. Note that it does indeed converge, and to 80% of the starting value, as expected by the setting of the threshold.

The break in the slope around period 150 indicates the point where enough firms have been driven below their revenue threshold, and consequently innovate, to noticeably slow the decline of total industry revenues. Up to that point, only the external shocks have any impact on revenues, and so they are continually being driven down. As more and more firms become involved in innovation, the total revenue levels out at its steady state value.

Figure 38 repeats the environment of Figure 37, but now with much larger shocks. The size of the innovation increment is increased from 0.2 to 10.0, and the size of the
Figure 37: Transient behaviour: convergence of total profits in 64 store system. Parameters as in Table 5.

external shock from 1 to 50. This fifty-fold increase in the magnitude of the adjustment dynamics causes a faster initial approach to the steady state, and has slightly larger fluctuations, particularly noticeable in the transient region. One might expect a more dramatic change in the behaviour of the revenues, when shocks to market share and resulting responses are fifty time larger. Recall, however, that once stores’ revenues have crossed the threshold once, the numerator in the share allocation function will thereafter increase in response to the increase in the denominator, regardless of the size of the shock and innovation increments. The share allocations and the revenues will remain in the same neighbourhood as time progresses. Furthermore, the initial store sizes have numerical values
Figure 38: Convergence of system profits with shock and innovation magnitude 50 times those in Figure 37.

in the range of 3 to 6. With innovation increments to store size of 0.2, the initial values have an effect for some time; with increments of 10, they very quickly become washed out. In both cases, once a few hundred shocks have been delivered, the magnitudes of the shocks and innovations relative to the attraction functions is the same. To check this, the increments were increase by a factor of 500 over the initial values. The results, shown in Figure 39, are essentially identical to Figure 38, confirming the explanation.
Figure 39: Transient behaviour with increments 500 times those in Figure 37 is the same as with increments 50 times those in Figure 37, indicating that large shocks and innovations quickly remove the effect of the initial conditions.

3.5.2 Size Distribution of Avalanches of Innovation in the Steady State

A shock to an individual firm's market share through the attraction function reduces that firm's revenues. This reduction may or may not drive the firm to react, depending on whether or not its minimum revenue threshold is crossed. If it doesn't react, another shock is delivered to another random site. If it does react, however, it increases its intrinsic attractiveness, and captures share (and hence revenues) from not only the extrinsic sources ($K_j$), but from any competitors with whom it shares customers--that is, any stores that have overlapping trading areas with its own trading area. This causes a reduction in revenues of
those stores, some of which may be also be driven to respond. In this way, it is possible for innovative responses to cascade across the market through overlapping trade areas.

Once the system is in steady state, we would like examine its behaviour. To do this, we keep track of the size of the avalanche produced by each shock, and plot the frequency distribution of avalanche sizes.

Two technical details to briefly note have to do with initialization, and the relative sizes of the shocks and innovative responses. To get the system quickly to the steady state, all stores are initialized above their threshold; then shocks are delivered to the entire system simultaneously, rather than to individual stores, so as to drive down the revenues quickly. Once the first store is forced to innovate, the shocks are stopped, and all stores allowed to innovate until all are just above their own threshold, which they would be in the steady state. The random shock algorithm is then implemented, and run for 100 shocks, before starting to record avalanche sizes.

The second issue has to do with the relative sizes of the shocks. Since each store is only allowed to innovate once for each shock, the shocks must not be too large relative to the innovation. More precisely, the effect on revenue reduction of the shock must be less than the effect of revenue increase, on average, of innovations; if not the revenues in the system will eventually all go to the external source; revenues in the system will approach zero. If the shocks are relatively small, on the other hand, it will take many shocks to drive revenues down after an innovation. Since we are only interested in counting innovation avalanches, small shocks with no response place extra demands on computer resources. In practice, the relative sizes of the shocks are set so that about half of them produce at least
one innovation. This is a compromise between keeping computer runs from being expensive, and allowing the system to recover between shocks.

Figure 40: Size Distribution of avalanches in the steady state, for the base case, with approximately 60% of shocks producing avalanches

Size Distribution--Initial Results

Figure 40 is a log-log plot of the size distribution of the avalanches for the case given in Table 5, except that the innovation size is 1.0; this gives avalanches for 776 of the 1300 shocks. The striking feature is the apparent power law behaviour. The probability of large responses does not fall off exponentially, but rather with an exponent of about -1.6 (the

\[ \text{Logarithms are to base } e, \text{ that is, natural.} \]
slope of the regression line, superimposed on the plot\textsuperscript{11}). This is the footprint of self-organized criticality.

Since the relative sizes of the innovation and shocks are set to conserve computer resources in the remaining tests, we should ensure that the power-law distribution is not highly sensitive to this adjustment. Figures 41 and 42 show the distribution with the innovation increment at 0.8 and 3.0 respectively. The change in the size of the innovation increment relative to the shock increment changes the proportion of shocks that cause some (at least one innovation) effect, from 80\% (for 0.8) down to 20\% (for 3.0). There is little effect on the shape of the distribution, at least in this range. In all that follows, the increments are set so that about half the shocks produce a response\textsuperscript{12}.

The size of the system, of course, places an absolute limit on the maximum size of the avalanches. There is a further effect on an avalanche of any size, however. Whenever a propagating cascade of innovations encounters a boundary, it must stop. Had the boundary not been there, the size of that avalanche may have been larger. For example an avalanche that started near a boundary and involves only 10 stores may well have involved 20 or 30 had it started in the centre. The result is that the distribution of avalanches tends to shifted toward the small size of the distribution. This is a well-known "finite-size effect" in simulations of critical phenomena (Bak, Tang, and Weisenfeld, 1988). The problem is a generic one of trying to infer system behaviour in the large limit, using only a finite system.

\begin{footnotesize}
\textsuperscript{11}The regression line and reported slope on this, and subsequent plots, are intended only as reference points to help with interpretation of the data.

\textsuperscript{12}Many of the following plots have a code, such as "4L62", on them. This is an identifier for cross-referencing to data files and may be ignored.
\end{footnotesize}
Distribution of Avalanche Sizes

Figure 41: Avalanche size distribution when shocks are relatively large.

Figure 42: Size distribution when shocks are relatively small.

The plots here show finite size effects to varying degrees. It appears as a reduction
in the frequency of avalanches at the large end. The effect can be seen most dramatically in the plots of tests of sensitivity to the distance exponents, and in that section it will be discussed further.

3.5.3 Robustness to model parameters

One of the characteristics of the self-organized critical state is that the power law distribution is relatively insensitive to model details. In this section, the size distribution of the avalanches will be investigated for various values of the size and distance exponents, and the reservation distance.

Empirical estimates of the exponents of the spatial interaction model have been made in a variety of contexts. In Huff's (1962) original work, the size exponent $\alpha$ was assumed to be unity, and the distance exponent estimated. In suburban Los Angeles, a beta value of 2.6 to 3.7 was found for clothing stores, and 2.1 to 3.2 for furniture stores. Haines, Simon, and Alexis (1972) estimated beta, again assuming alpha fixed, for grocery stores in various suburban and inner city neighbourhoods in a U.S. city. They found values between 0.5 and 1.8. Jain and Mahajan (1979) estimated a multiattribute model for supermarkets in a "large northeastern metropolitan area", and found alpha values between .02 and .56 for their four intrinsic attractiveness attributes (sales area, number of checkout counters, credit cards accepted, and intersection location), and a beta value of 0.3, for the distance exponent.

The range of exponents estimated empirically is roughly from 0 to 3. Since the case where both are equal to one has already been described, the values of 0.5, 2.0 and 3.0 will examined in the following, first for $\alpha$ (size), and then for $\beta$. 
Figures 43, 44, and 45 show the avalanche size distribution for values of alpha equal to 0.5, 2.0 and 3.0. For the high and low values, we see more deviation from a straight line than before. In both cases, the falloff appears faster than a power law.

Consider the dynamics of incrementing the intrinsic attraction and the external attraction (S_j and K_j respectively) in Equation (4). When alpha is greater than one, the effect of constant increments of innovation on S becomes progressively greater as S grows. However, the effect of the increments to K remain linear. As the system evolves, there is a systematic increase in the effects of the innovation on revenues, relative to the effects of the exogenous shocks on revenues. A similar argument (in the opposite direction) applies when alpha is less than one. This systematic effect in the dynamics may well keep the
system away from the steady state. To eliminate this effect, we would like to keep the
absolute magnitude of the intrinsic attractiveness, \( S \), relatively constant as the system evolves. To accomplish this, the magnitude of \( S \) can be adjusted downwards across the entire system after each avalanche. It is important, however, that this adjustment doesn’t affect any store’s revenues; therefore, the value of \( K \) is adjusted as well. The following adjustment is used. Equation (4) is repeated below for convenience.

\[
M_{ij} = \begin{cases} 
\frac{S_j^\alpha D_{ij}^\beta}{(\sum_k S_k^\alpha D_{ij}^\beta) + K_j} & , \quad D_{ij} \leq R \\
0 & , \quad D_{ij} > R 
\end{cases} 
\]

(6)

Once the initialization of the system is complete, calculate the average initial value of \( S \):

\[
\overline{S}_0 = \frac{1}{N} \sum_{k=1}^{N} S_{k,0} 
\]

(7)

After each avalanche is complete (step 5 in the "scrambling astutely" algorithm) calculate the new average value of \( S \):

\[
\overline{S}_t = \frac{1}{N} \sum_{k=1}^{N} S_{k,t} 
\]

(8)

Reset each value of \( S \) by reducing it by the ratio of the initial to the new means:

\[
S_{j,t}' = S_{j,t} \left( \frac{\overline{S}_0}{\overline{S}_t} \right) 
\]

(9)

Finally, reduce each \( K \) value by the following amount:
\[ K_{j,t}^* = K_{j,t} \left( \frac{S_0}{S_t} \right) \]  

(10)

With these new values, the next random shock may be delivered, and the process repeated.

This adjustment keeps the average value of \( S_{j,t} \) over the system at its initial value, for each shock. Furthermore, the customers' expenditure allocations, given by (6), are unaffected by the adjustment given in (9) and (10); hence the revenues of each store remain unaffected, as required. The only effect on revenues is the exogenous shocks and the innovative response, as before.

Figures 46 and 47 show the avalanche size distributions using this adjustment algorithm, using the same parameters as in Figures 43 and 45 (the size exponents of 0.5 and 3.0). The avalanche size distribution is now much closer to a power law, particularly for \( \alpha = 0.5 \) (Figure 46), indicating that the deviation was due to the systematic change over time of the dynamics, as described above, rather than the sensitivity of the steady state to the magnitude of the exponent. There remains some systematic deviation in the case of \( \alpha = 3.0 \). It is unclear how much of this is due to finite size effects, and how much is a due to the value of the exponent. One might well ask if the curvature in Figure 47 could be captured by an exponential decay of the distribution. Figure 48 shows a log-linear plot of the same data, which, if the decay were exponential, would fall on a straight line. It appears that, at the smaller avalanche sizes, the distribution is not exponential; however, the increased variance at the larger sizes, (due to fewer samples) plus finite size effects, makes it difficult to say much about the shape of the distribution at the large limit tail.
Figure 46: The adjustment algorithm restores the power law (cf Fig. 43).

Figure 47: Distribution with adjustment algorithm for alpha = 3.0 (cf Fig 45).

In summary, the SOC state appears quite robust to changes in the value of the size exponent.
Figure 48: Log-linear plot of the data in Figure 47. Note the curvature at small sizes.

over the range of values that have been reported in the empirical literature. At larger values, around $\alpha = 3$, the conclusion is not as strong; the SOC state may be breaking down. This issue will be discussed again in the section on research opportunities.

**Distance Exponent**

The distance exponent, $\beta$, determines how rapidly the attraction falls off with distance. It reflects the relative sensitivity of customers to travel costs. In the context of the large system, it reflects how strongly the stores are coupled together. As $\beta$ increases, the coupling strength decreases.

Figures 49, 50, and 51 show results for $\beta = 0.5$, 2.0 and 3.0 respectively, with $\alpha$ held at one. As in the case of $\alpha$, this is representative of the range of empirically determined values for the distance exponent. In these diagrams the break from a straight line is particularly pronounced in Figures 49 and 51, the cases where $\beta$ is 0.5 and 3.0,
Figure 49: Size distribution, $\beta = 0.5$.

Figure 50: Size distribution when $\beta = 2.0$.

respectively. To emphasize this break, the reference line has been fit only to the first 12
data points in the first case, and the first 9 in the second case.

At smaller avalanche sizes, where there are many samples of each size and the variance is low, the distribution follows an extremely precise power law. Since the break at larger sizes, which is noticeable in all the plots, is pronounced here, the opportunity will be taken to briefly explore the finite size effect. Figures 52 to 55 repeat Figure 49, with smaller and larger markets. Figure 52 is the results of the run on 36 stores in a 6 x 6 array; Figure 53 is for 64 stores in an 8 x 8 array; Figure 54 is for 121 stores in an 11 x 11 array; and Figure 55 is 324 stores in an 18 x 18 array. The change in the location of the break can be seen between Figures 52 and 53. With the smaller array, the deviation from a power law occurs at smaller sizes, consistent with a finite size effect. A similar increase between 64 and 121 stores (8 x 8 to 11 x 11) is not noticeable, but with 324 stores in an 18 x 18
Figure 52: A small array: The distribution departs from a power law at around 8 firms.

Figure 53: With 64 stores, distribution breaks from a power law at around 12 firms.

array, (Figure 55) the increase is again apparent.
DISTRIBUTION OF AVALANCHE SIZES
Finite Size Effect

Figure 54: The break in the power law for the large array is also around 12 stores.

DISTRIBUTION OF AVALANCHE SIZES
Finite Size Effect

Figure 55: For an 18 x 18 array, the break is around 16 stores.

While the conclusion that the break is a finite size effect is tentative, it remains a
likely candidate explanation. A complete investigation of finite size effects is beyond the scope of this work. It would involve investigating a wide range of array sizes, including those much larger than investigated here. Bak, Tang, and Weisenfeld (1988) demonstrate the effect for their sandpile model, which is a cellular automaton which involves relatively simple and fast code. The model here is much more complex in its interactions and hence calculations, and the load placed on computer resources correspondingly greater, particularly as array sizes become large. The resources necessary to examine large arrays and many iterations were prohibitive for this model\textsuperscript{13}. Within these limitations, however, the effect appears to be identical to that described by Bak et. al.

\textsuperscript{13}For example, Figure 55 required 10 hours on the IBM RS/6000 560.
Reservation Distance

The reservation distance is a limit on how far the customer is willing to travel. It determines the number of stores to which the customer allocates his expenditures. From the stores' point of view, it represents the radius of the trading area. The larger the reservation distance, the more stores compete directly with each other. Conversely, the smaller the reservation distance, the more monopolistic each store can become. As the reservation distance decreases, the whole system will eventually decouple.

Recall that the store separation is 3 (measured in terms of customer origin units) and the runs so far have used a reservation distance of 4.5. (This puts eight stores in the trading area of every store not on the boundary of the system). Figures 56 and 57 show results with reservation distances of 6.5 (with 20 competitor stores in a trade area), and 2.9. The latter is small enough that, even though stores share customers, no store is actually in any other's trade area.

The distribution remains essentially the same for both the larger and smaller reservation distances. For the small reservation distance, the slope of the reference line is steeper; however, as before, one could fit a shallower line to the smaller avalanches.

At some point, the reservation distance will become small enough that the system will become entirely decoupled. An interesting technical issue (beyond the scope of this research) is the system behaviour as the stores become independent monopolists. Does the SOC state hold as long as there is at least one customer with divided loyalties? It would undoubtedly require very long computer runs to answer this question; and in any case, the rational for monopolists "scrambling astutely" is unclear.
Figure 56: Distribution of avalanche sizes with a larger reservation distance.

Figure 57: Distribution of sizes with a smaller reservation distance.
The results of the tests of sensitivity to parameter values are summarized in Table 7.

Table 7: Summary of parameter sensitivity tests.

<table>
<thead>
<tr>
<th></th>
<th>α</th>
<th>β</th>
<th>R.D.</th>
<th>RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>1</td>
<td>1</td>
<td>4.5</td>
<td>power law size distribution</td>
</tr>
<tr>
<td>Alpha tests</td>
<td>0.5</td>
<td>1</td>
<td>4.5</td>
<td>similar to base case</td>
</tr>
<tr>
<td></td>
<td>2.0</td>
<td>1</td>
<td>4.5</td>
<td>similar to base case</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>1</td>
<td>4.5</td>
<td>slight deviation at large sizes</td>
</tr>
<tr>
<td>Beta tests</td>
<td>1</td>
<td>0.5</td>
<td>4.5</td>
<td>pronounced finite size effect</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2.0</td>
<td>4.5</td>
<td>similar to base case</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3.0</td>
<td>4.5</td>
<td>deviation from power law at large sizes</td>
</tr>
<tr>
<td>R.D. tests</td>
<td>1</td>
<td>1</td>
<td>2.9</td>
<td>steeper slope, deviation from power law</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>6.5</td>
<td>similar to base case</td>
</tr>
</tbody>
</table>

3.5.4 Robustness to Dynamic Structure

How dependent is the steady state on the particular dynamics used in the preceding? To investigate this question, a variation on the innovation dynamics was implemented. Rather than incrementing the size, or intrinsic attraction only, the increment was added to the complete attraction function. This represents an innovation that uniformly increases the store’s attractiveness to all its customers. The customer now allocates expenditures according to
The model structure was changed to introduce a cost linear with $S_i$, and the decision to innovate based on profits rather than revenues. (Profits are not dependent on $a_i$). This is a relatively minor change. Since the store sizes are all initialized at different values, the costs, profits, and profit thresholds will all be different across the system. The costs and profit thresholds remain constant with time; revenues and profits themselves change with time. The shocking dynamic remains the same, as does the response dynamic, except of course for being triggered by the profit threshold and being added to $a_j$.

Figure 58 is the usual size distribution for the base case parameters ($\alpha, \beta = 1$; reservation distance $= 4.5$). The model was also run with a variety of other parameters. The distribution only departs substantially from the power law when the reservation distance becomes small, that is when the stores are only weakly coupled through competition in the market. In all other cases, the distribution of avalanche sizes followed a reasonable power law. These results parallel the original model. The steady state is robust to these changes in system structure and dynamics.
3.5.5 Sensitivity to Initial Conditions

We would like to know what happens in this market when the butterfly flaps its wings. When two realizations of the system start out very close to each other in state space, do they converge or diverge? To test this, the system was initialized and run for 200 iterations (shocks). At this point, three copies were made of the system. The first was perturbed by reducing the external attraction of one store ($K_j$) by an amount equivalent one quarter of one shock (the small change). The second had reduction in $K_j$ equivalent to one shock (the medium change) and the third had a reduction equivalent to one shock applied to three stores (the large change). The four systems, including the base case, were then allowed to run for another 500 iterations. The state space examined was the 64 dimensional space of store...
sizes. The Euclidean distance between each of the three perturbed systems and the original system was calculated each period. The distance is given by

\[
D = \sqrt{\sum_{k=1}^{64} (S_k - S'_k)^2}
\]  

(12)

where the prime indicates the perturbed system.

The distances are plotted in Figures 59, 60, and 61. The average separation over the 500 shocks is 2.3, 7.8, and 21.2 respectively. Note that the average separation increases with the initial separation across the three cases.

**Figure 59:** Separation in store size space after one store is perturbed.

For the small initial separation (Figure 59), the two systems repeatedly return to the same point in state space, that is, zero separation in Figure 59. They don’t diverge to separations that are seen in Figures 60 and 61. In order to achieve those larger separations,
the initial separation must be greater. This means that, at least by this measure, the aggregate system is not chaotic in the steady state. Another way of saying this is that at an aggregate level, the evolution of the system is predictable within the accuracy of initial measurements, and the variance induced by the random shocks.

Figure 60: As in Figure 59, with initial perturbation 4 times greater.

Neither do the systems show evidence of permanent convergence within the time frame examined here. Once apart, even if they return to the same point in state space, they will separate again. If one considers the constant exogenous shock rate and the adjustment
mechamism, it is perhaps not surprising that there is no point attractor within the store size state space for the system.¹⁴

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¹⁴One could also examine sensitivity to initial conditions in the store revenues state space. Since an attractor does exist in this state space, the maximum separation would be bounded, and the issue of divergence/convergence less obvious. This question is left for future research.
3.6 Summary of Numerical Experiments

The retail interaction model investigated here converges to a stochastic steady state, characterized by power law avalanche distributions, and is quite robust to changes in model parameters and structure. Changing the customers' sensitivities to travel costs and intrinsic store attractivenesses over the ranges where these values have been empirically determined doesn't affect the power law distributions until the sensitivities become high, i.e. the attraction varies with the cube of the size or distance attributes.

The steady state is sensitive to systematic changes in the relative sizes of the effects of the shocks and the innovation responses. When the shocks cause the external attraction to grow linearly, but the size, or internal attractiveness, to grow nonlinearly, the power law distribution breaks down. A normalizing adjustment that keeps both external and internal attractions from growing, without affecting revenues for any store, restores the power law distribution, for the values of the size exponent tested.

The steady state appears insensitive to changes in the reservation distance until it becomes small enough that firms become monopolistic, and the system decouples.

In most of the tests, there appears to be a distinct break in the power law curve in the large-avalanche limit. It is quite likely that the deviation is due to the finite-size of the market simulated. A full exploration of this effect has been precluded by the limitation on computer resources, however, it appears identical to that described in the literature.

The steady state is also robust to a different specification of the dynamic, with innovation being triggered by profit levels rather than revenues, and the innovative response added to the entire attraction, rather than the intrinsic attraction.
Finally, a comparison of two systems with initially small separations in the store size (intrinsic attraction) state space indicates that they evolve along approximately parallel paths, neither converging nor diverging.

These characteristics are those of "self-organized criticality", a phenomena described, to the best of my knowledge, only once previously in an economic model, but with many applications in a variety of other fields, including theoretical biology and statistical physics.
3.7 Contributions, Limitations, and Research Opportunities

Contributions

This chapter's two main contributions are, first, the modelling of a facet of retail dynamics often discussed, but not modelled; and second the demonstration of steady state model behaviour known as self-organized criticality in a marketing context.

The conventional wisdom that much of competitive retailing involves repositioning in response to continual erosion, or scrambling astutely, is captured by a shock-and-innovation dynamic embedded in a spatial competition model. The resulting stochastic steady state is characterized by a continually innovating industry, with firms innovating in response to the shocks, in waves, or avalanches, of various sizes. These responses are interesting generally because of the similarity in spirit to long-standing notions that there is some underlying order to the apparent turbulence in retailing, as exemplified by traditional theories of wheels and accordions in retailing. More specifically they are interesting because the dynamic allows a small shock to have a large response. The response size falls of as a power law, with exponent typically around -1.5, rather than exponentially, as would be expected from a simple linear aggregate of independent shocks.

The state the system evolves to has the characteristics of self-organized criticality (SOC). This state has been described once previously in an economic context by Bak, Chen, Scheinkman, and Woodford (1992). This chapter is the first application in a marketing context, and is an advance over BCSW in that it uses established marketing models as the context. BCSW, on the other hand, impose an economic interpretation on the prototype sandpile cellular automaton introduced by Bak, Tang, and Weisenfeld (1988).
One reason why SOC is potentially important to marketing is because of its robustness at several levels. At the lowest level, SOC is the end state over a range of parameter values investigated in this model. At the next level, model structural details can be changed without affecting the final state. At the highest level, SOC arises in entirely different fields. This robustness increases the probability that the phenomena may occur in a marketing context. Furthermore, although the necessary requirements for SOC have yet to be spelled out, the models in all the varied fields have in common many dynamically interacting degrees of freedom, a situation that describes many marketing contexts.

Limitations to this research, and implications for future work, will be discussed in three areas: model structure, method of analysis, and empirical evidence.

**Model Structure**

The model structure involves shocks of adversity and effective response; the possibility of positive shocks, and ineffective response, have been abstracted away. Closely related to ineffective response is the possibility of exit, and, of course entry. These are certainly features of retailing not captured in this model, and represent possible model variations or extensions. Incorporating such features in a simulation is relatively easy; however, the extra layers of complexity only increase the difficulty of exploring the model and generalizing results. Furthermore, the gain is likely only incremental.

A more important extension is to investigate optimizing behaviour explicitly. Although the model assumes the ability of management to make good decisions, it doesn’t address the specifics of the decision. This is an important next step for this line of research to lead to normative results. The models of theoretical biology, which incorporate short-term optimizing, will be helpful. The likely approach is to consider agents who operate on
Chapter Three: Spatial Competition and Self-Organized Criticality

the basis short-term profit maximizing heuristics, such as those described by Jeck (1991), and to compare the various steady states that can arise (static Nash equilibrium, cyclic, SOC, etc.) on the basis of system-wide profits. The hypothesis one would infer from the biological literature is that the SOC state, with its very dynamic nature, provides the highest system profits.

Method of Analysis

The numerical method of analysis, which allows the investigation of complex models, lacks the generality of analytic methods. Analytic approaches to SOC is an active area of research in statistical physics, and BCSW borrow from some of this work to demonstrate the Pareto-Levy distribution of avalanche sizes in their producer-supplier network model. The analytic work to date is in the context of relatively simple discrete cellular-automaton models. It is not at all clear that it will be applicable to the more complex model described here, and it is therefore likely that numerical approaches will dominate for the foreseeable future.

An important limitation of this research is the inability to completely investigate the finite size effect, which itself is a limitation of the numerical method of analysis. There are two possible approaches here. One is simply brute force--let the model run on large arrays for a long time. The other is to simplify the model, attempting to retain the basic features, but making the computations much less intensive. The former is probably the more desirable, as the latter still leaves open the question of whether or not the deviations from power law behaviour in this particular model are actually finite size effects.

Empirical Issues

The final issue is empirical. A crucial problem in detecting the kind of behaviour implied by this research, whether in the context of retailing or elsewhere, is that of counting
 avalanches. In this, and other SOC models, each shock triggers a single avalanche, which comes completely to rest, before another avalanche is triggered. This is unlikely to occur in any real setting. If shocks are occurring more rapidly, avalanches may run into each other. Even if they don’t physically interfere with each other, there is the likelihood that several avalanches in any large system are occurring simultaneously and econometric data sets will only record the aggregate response. The general question of aggregation of responses is also an active area of research in statistical physics. It is well known that when events, or pulses, of various shapes and sizes in time, are superimposed at random starting times, to produce a time series, the resulting series is highly correlated. The correlation is most often expressed in the frequency domain in terms of the power spectra, which can be shown to be a power law (A. van der Ziel, 1950). The power law behaviour of power spectra arising from aggregates of the pulses produced by systems in the SOC state has been an important part of SOC research from the beginning. Exponents between one and two were suggested originally by Bak, Tang, and Weisenfeld (1988), depending on the dimensionality of the system. Chau and Cheng (1992) suggest that whether the exponent is one or two depends on whether the model is continuous or discrete.¹⁵

Relating power law power spectra in economic time series to SOC models seems to be the best hope for empirical support for the model. The difficulties inherent in sorting out the values of the exponents are compounded by the possibilities of other explanations.

¹⁵Some initial work on aggregate traces produced by superposition of the pulses generated by the model in this chapter gives a power spectra of $1/f$, that is an exponent of 1. By comparison, a time series of monthly Canadian bankruptcy data for retail businesses for 12 years has a power spectra at low frequencies that goes as $1/f^2$. The hypothesized connection is that if some fixed proportion of the firms involved in each avalanche are unable to effectively innovate, and exit the system, they will show up in bankruptcy data.
Random walks, for example have power spectra that obey power laws, with the simple Gaussian random walk having an exponent of two (Mandelbrot, 1982). This implies that the SOC explanation for power law behaviour will, at least, have to be contrasted with existing explanations of the origin of random walks.
CHAPTER FOUR

CONCLUSIONS AND CONTRIBUTIONS,
LIMITATIONS, AND FUTURE RESEARCH

This dissertation is concerned with spatial competition in the currently prevailing dynamic retail environment in North America. Chapter 2 shows the possibility of nonlinear increasing returns to scale to price reductions in grocery retailing, and suggests that the effect would have been difficult to detect, let alone predict, before the appearance of superstores. Chapter 3 builds on this result, and the conventional wisdom that struggling with unexpected adversity is the norm in retailing, to analyze the long-run behaviour of a model of the industry that captures the elements of successful reaction to unexpected adversity. Though the chapters are linked, they differ in terms of specifics, and so no overall conclusions are offered. Instead, the purpose of this chapter is to summarize the results of the previous two chapters in turn. First, the conclusions and contributions of the research are highlighted. This is followed by a discussion of limitations and of relevant future research.
4.1 Grocery Shopping Behaviour in the Presence of a Power Retailer

4.1.1 Conclusions and contributions

In Chapter 2, a normative model of individual consumer choice in grocery shopping is developed. The model assumes long-run cost minimizing consumers, who trade off inventory costs, travel costs, and price of goods purchased. Customers are heterogeneous in household location, and make a deterministic choice between two stores. The stores are differentiated by location and price; in addition, each store sells two goods, one perishable and one nonperishable. Except for their price, the goods are identical between stores. The broad objective of this model structure is to investigate shopping behaviour in the presence of a power retailer, an area not previously addressed in the literature.

Multistore shopping strategies

The first interesting finding is a new explanation for multi-store shopping by consumers among direct competitors in the grocery trade. While it is well documented that loyalty to any one store is low (Uncles and Ehrenberg, 1990; Bucklin and Lattin, 1992), the multi-store shopping strategy is either treated as an exogenous fact, or explained in terms of responses to promotions or search for deals. In contrast, the multi-store strategy in Chapter 2 is an endogenous result of rational optimizing in the face of long-term expectations about average store prices. Applied to the context of the entry of extremely price-competitive retailers, i.e., superstores, to the market, this may be more than a new explanation--it may be a reason for multi-store shopping that did not exist before the superstores. This possibility is supported by the numerical analysis that showed very small "mixed-store" shopping regions until price differences were on the order of 20%.
Another established reason for mixed store shopping arises from uncertainty in the choice process. The spatial interaction models originating with Huff (1962), as well as some theoretical positioning models (dePalma et. al., 1985; Choi et. al. 1990) consider a stochastic choice process that allocates expenditures probabilistically to more than one firm. In contrast, the model developed here involves a purely deterministic choice process; and, again, the multistore shopping arises endogenously.

The structure and results of the model have a strong parallel with the multipurpose shopping literature (Ghosh and McLafferty, 1987; Ingene and Ghosh, 1990). In both cases, deterministic choice and rational long-term planning in the presence of known parameters leads to the resulting behaviour. However, the emphasis in the multipurpose shopping literature is on different categories of retailers, rather than on direct competitors, and how these categories may agglomerate as the result of efficiencies achieved by multipurpose shopping. The research reported in Chapter 2 extends this literature by considering stores differentiated by price, rather than category, and by introducing two goods--perishables and nonperishables--with very different dynamic characteristics.

The fact that consumers in the model treat these two goods very differently has important implications, and raises some interesting strategic questions. These questions will be addressed in the "future research" section.

Increasing returns

It was shown that consideration of the time dimension in spatial competition models through consumer stockpiling leads to the possibility of high sensitivity of trade areas to price differences between competitors. Furthermore, this sensitivity may not be apparent when price differences are small. In other words, market share may exhibit an "increasing
returns" effect to price reductions. This effect is offered as a candidate explanation for why, according to several independent industry sources, Safeway was surprised by the success of the Real Canadian Superstore in western Canada.

**Periodic policies and the joint replenishment problem (JRP)**

From the theoretical standpoint of the JRP literature in inventory management, this research contributes another example to the limited set of JRP’s where periodic replenishment policies are optimal. A periodic replenishment policy is one where a particular fixed pattern of replenishment in a finite time interval is repeated indefinitely. It reduces the infinite horizon case to a finite horizon problem.

From this result, a series of propositions further constrains the possible shopping patterns to six. Three of these are argued to be either unlikely to occur or minor in impact. Analytic solutions are obtained for the optimal shopping policy for the remaining three patterns, and a method for determining which of the three will give minimal costs as a function of household location is developed.

The setting is unusual in that it involves consumer stockpiling, rather than firm inventory management. From the practical standpoint of inventory management, however, it is not clear that the properties of this setting translate to any firm-based inventory management problem. The insights it provides for consumer shopping behaviour, however, are interesting, as purchase timing issues are increasing in importance.

In summary, the model and results of Chapter 2 should be of interest to marketing academics and practitioners, in that it provides a theoretical basis, rich with implications, for examining an important current phenomenon in retailing.
4.1.2 Limitations

The store choice model developed here combines and extends existing models in several ways. However, the results need to be viewed in the light of a number of limitations, discussed below.

1. The numerical portion of the analysis has the usual limitation of lack of generality. The results must be considered as special numerical cases, if quite plausible ones. In particular, even though the analytic choice results are on an individual basis, for the aggregation analysis to determine market shares, customers are assumed to be homogeneous except for their spatial origin, or household location. This is a common assumption in theoretical work in spatial competition, usually made for reasons of tractability. The assumption, however, is obviously not tenable in any real setting. Consumers will vary on parameters such as price and distance sensitivity. The aggregate results should therefore be viewed as the behaviour of a particular segment of customers, who have homogeneous parameters. It would be possible, although cumbersome, to solve the numerical problem for different homogeneous segments, and combine the results.

2. The model has only two marketing mix variables—the long-run variable of location, and the short-run variable of price. While these two variables are the most obvious determinants of grocery store choice, other mix elements are also important. For incumbent firms in particular, who may be unable to match the power retailers’ prices, strategies based on attributes such as quality and service may be viable options.
3. The analysis is limited to three of the six possible optimal shopping patterns identified. One pattern not examined, namely shopping for nonperishables more frequently than perishables, lacks face validity. The other pattern not investigated, however, is more plausible. The possibility of special trips to the small store for nonperishables at the end of each periodic interval, and its impact on market share, remains to be investigated. It is expected, however, that this is likely to be a minor effect.

4. In any urban setting there will be many directly competing retail outlets, whereas this model only considers two independent outlets. Increasing the number of outlets is well within the framework of the model, although it would be numerically cumbersome. A more important issue is that much of retailing is dominated by chains, which qualitatively changes the nature of the strategic implications. For example, incumbent chain stores with many locations have the option of varying prices across locations to respond more effectively to the entry of a single power retailer.

5. The analysis is confined to existing firms. One of the important issues with superstore entry is that they can drive some stores out of business. This interacts with the issue of chains, and the possible strategic response of incumbents to superstore entry of closing some outlets and opening new ones.

4.1.3 Future Research

Four directions for future research are identified, some of which arise from the limitations of the previous section.
Chapter Five: Contributions, Limitations and Future Research

1. **Empirical:** Several model predictions could be tested fairly easily. Examples are a) consumers at large distances from superstores make less frequent trips and purchase larger quantities per trip than closer consumers b) customers at large distances from superstores make more fill in trips to supermarkets than closer consumers c) average purchase quantities at supermarkets are smaller than at superstores d) average trip frequency to supermarkets is greater than to superstores e) superstores share of nonperishables is larger than their share of perishables.

2. **Strategic:** Theoretical research in spatial competition suggests that equilibrium configurations involve the tradeoff between market share maximizing forces, which lead to minimal differentiation and maximum price competition, and a strategic force to reduce price competition by maximal differentiation. Since this model results in consumers willing to travel further for perishables than nonperishables, one would hypothesize that price competition should be more intense on nonperishables. The fact that both stores carry both goods would make this an interesting, if difficult, setting for an equilibrium analysis. Other strategic issues include how perishables and nonperishables might be treated differently by the different stores, and how advertising might augment this treatment.

3. **Extensions:** Several of the results here were derived by graphical and numerical methods, because the model is already complex enough to make analytic solutions impractical. Relaxing the requirement of discrete trips in favour of continuous trips would allow taking analytics further, and that step
may be necessary to incorporate extensions without getting bogged down in highly specialized numerical analyses.

Possible extensions include consideration of additional variables, such as advertising, service, and product quality. Short term dynamics in the form of promotions are another possibility. This could lead to consumer uncertainty about prices and require a stochastic approach. A stochastic approach could also be used to address the issue of consumer heterogeneity in sensitivities to various costs, as in dePalma et al. (1985). The possibility of customer sensitivities being susceptible to marketing activities is also a very interesting avenue of research.

Different cost structures are also possible. Perhaps the most obvious change is to include a fixed in-store cost that is different across stores, in the trip cost. This captures the effect of the very large stores being more time-consuming to shop in.

In all model extensions, an issue to investigate is whether or not the increasing returns effect holds.

4. Another research direction is that taken in Chapter 3, namely the long term effect on industry structure of a competitive dynamic involving effective response to unexpected adversity.
Chapter Five: Contributions, Limitations and Future Research

4.2 Spatial Competition and Self-Organized Criticality

4.2.1 Conclusions and Contributions

The academic, trade, and popular literature have often referred to the dynamic nature of retailing. Phrases such as "scrambling astutely" (Financial Times of Canada, April 3, 1993) and "repositioning in response to continual erosion" (Corstjens and Doyle, 1989) refer to the fact that much of retail management is a matter of making major creative changes in response to adversity. Chapter 3 formalizes this notion as a shock-and-innovation dynamic set in an oligopolistic spatial competition model. To the best of my knowledge, this is the first attempt to formally capture this type of dynamic behaviour.

The model’s steady state has some interesting characteristics. The exogenous shocks driving the system are delivered at random to firms in the model industry, so it is natural to look at distributions of responses. The probability of a large response, where the size is measured as the number of firms responding to a single shock, declines according to a power law. The model therefore contributes by describing a mechanism where the law of large numbers does not apply, as one would expect an exponential decline in that case. In other words, the likelihood of a system-wide wave of innovation in response to a small shock is relatively high.

This parallels the observation that retailing as whole tends to undergo dramatic shifts, such as described in the December 21, 1992 Business Week article on power retailers. The fact that a micro-level dynamic motivated by the conventional wisdom that retailers must "scramble astutely," leads to a stochastic steady state involving sweeping changes at the macro-level, which also parallels conventional wisdom, is intuitively appealing.
Chapter Five: Contributions, Limitations and Future Research

The steady state has the characteristics of self-organized criticality, and the research therefore contributes by providing, to the best of my knowledge, the first example of SOC in marketing, and the second in economics. Furthermore, SOC arises here from a purely marketing model, whereas the previous economic example (Bak et. al., 1992) is mathematically isomorphic to the prototype sandpile model of statistical physics, with an economic interpretation imposed. The research thus contributes to the SOC literature by providing an example in marketing, and a purely economic model.

SOC is of interest not only because of its rather unusual characteristics, but because it is quite robust to parameter values and model details. Furthermore, it appears to be a general organizing principle that has been applied to models in many different areas of the physical and life sciences, as well as economics. While the models vary widely, one common feature is the dynamic interaction of entities with many degrees of freedom, a characteristic of many marketing situations. This, plus the robustness and generality of SOC, increase the probability of it occurring in marketing situations.

4.2.2 Limitations

A number of relevant limitations of Chapter 3 will now be identified.

1. Like the majority of work on SOC, this research involves simulation, which, while it allows the investigation of complex systems, limits generality in comparison with analytic approaches. While analytic work on systems displaying SOC is an active area of research, it is based on simple cellular automaton models. Whether the methods can be applied to the more complex model here is an open question.
2. The simulation is based on a finite number of stores, and so the maximum size an avalanche can attain is limited. Although the strong power law behaviour at smaller avalanche sizes and general nature of the deviation at larger sizes is entirely consistent with the "finite size effect," it remains to be conclusively demonstrated that the effect is the reason for the deviation. This, again, is a limitation of the numerical approach, combined with the complexity of the interactions in this particular model.

3. The negative exogenous shock and effective response captures a subset of possible behaviours. The question of positive shocks, and/or the possibility of failure to respond effectively, is not addressed. The latter in particular is important, as exit is a common feature in retailing. Whether models that incorporate these features will still exhibit SOC is not dealt with here.

4. The deliberate focus on low information environments, surprise and satisficing ignores the more usual approach to modelling competitive activity, where some form of optimizing and foresight is generally considered. To the extent that retailers do more than "scramble astutely," this model is limited.

4.2.3 Future Research

Several research possibilities present themselves. Two of the more important directions will be discussed in this section.

1. Incorporating positive shocks, entry, and exit adds realism, but increases complexity. While it would be interesting to further test the robustness of SOC on these dimensions, it is believed that it would be more productive to
construct a new model that involves at least short-term optimizing behaviour.

This is a potentially interesting direction because of results in theoretical biology (Kauffman and Johnson, 1992) that relate SOC to Nash equilibria with short-term optimizing entities. They show that a SOC state occurs when Nash equilibria are just barely stable, and tenuously extend across a system. The competing species make single period adjustments which increase their competitive advantage, or "fitness." Furthermore, average fitness of the system is maximal in the SOC state. It would be very interesting to investigate circumstances under which industry-wide, or even economy-wide profits were maximal in an SOC state; and to investigate further the relation between Nash equilibria and self-organized criticality.

2. The footprint of SOC is power law distributions, and power laws are common in economics. For example, Pareto-Levy distributions (with power-law tails) which Bak, Chen, Scheinkman and Woodford (1992) have shown to be a consequence of SOC, have also been identified in various economic series (see Mandelbrot, 1982). SOC also gives rise to power spectra that have power law dependency on frequency, as do any time series that can be modelled as a random walk. Empirical and theoretical research should be undertaken to investigate the possibility of SOC being at the root of these observed power law distributions. While interesting and important, it is expected to be a very challenging undertaking.
Chapter Five: Contributions, Limitations and Future Research

In Conclusion

The research reported in this dissertation addresses some unique aspects of spatial competition inspired by the retailing industry. The results contribute in a number of ways to our understanding of this dynamic, even turbulent, industry.
Bibliography


Bibliography

Appendix A

Calculation of Market Shares

This appendix presents the calculations for market shares of the perishable good for the case shown in Figure 28, section 2.7.

Share of the perishables varies with the different regions. The large store captures all of the perishables in its exclusive region. In the mixed regions, since the constraint is binding, on each trip exactly enough perishable to last to expiry time is purchased. Thus in the $m = 1$ region, trips alternate between stores, and the large store has half the share. In the $m = 2$ region, the large store has on third the share; and so on. (This may also be easily shown formally by substituting the solution for the optimal quantities of the non-perishable purchased at the large store in the binding, mixed case, equation (54), into the expression for the perishable quantity purchased at the small store, equation (47), and comparing with constraint (46) when binding).

To determine the areas in Figure 28, first express the boundary contours, from equation (69) and the parameter values given in Table 2, Section 2.7, as $x = f(y,m)$:

$$x(y,m) = \pm \frac{1}{8} \left[ \frac{144 + 72m + 33m^2 + 6m^3 + m^4 + 64y^2}{102400y^2} \right]^{1/2}$$

To find the area to the right of each of the contours, transform the $x$ coordinate by shifting the origin to the right edge of the city and reflecting: $x' = 10 - x(y,m)$. Now the integral
Appendix A: Calculation of Market Shares

w.r.t \( y \) will give the area between the contour and the right edge of the city. For the \( m = 0 \) to \( m = 1 \) transition, the area to the right is

\[
A_l = \int_{-10}^{10} [10 - x(y,0)] dy
\]

\[
= \left[ 10y - \frac{3y\sqrt{91} + 4y^2}{4\sqrt{91}} - \frac{3}{8}\sqrt{91} \text{Arcsinh}\left(\frac{2y}{\sqrt{91}}\right) \right]_{-10}^{10}
\]

\[
= 154.525
\]

Similar integrations give the areas to the right of the remaining three contours. From these areas, and the total area of the city of 400 km\(^2\), the areas of each of the regions can be found, and the areal shares calculated. The large store's exclusive region is 61.37%. The mixed shopping areas, for \( m = 1, 2, 3, \) and 4 respectively are 4.12%, 6.97%, 12.18%, and 15.33%. The last region has the .03% market share of the small store's exclusive region removed. Therefore, the large store's share of the perishables is

\[
61.37 + \frac{4.12}{2} + \frac{6.97}{3} + \frac{12.18}{4} + \frac{15.33}{5}
\]

\[
= 71.86\%
\]

The small store's perishables share is 28.14%.
Appendix B

C Code

C1. Simulation Routine

/* SIM4LB.C descendent of sim4fb.c
   Size incremented directly with inoadv
   Size and ghat normalized after each avalanche
   
   fixed mkt radii
   descendant of sim4c.c; threshold = minprof*refprof
   any firm can innovate--no restriction on availability of
   innovation
   Elastic total demand through ghostatt, which increases
   uniformly by ghostinc across market
   Store advantage added rather than multiplied
   Nash only found once; after, Size remains fixed during
   innovations
   Then ghost attraction increases uniformly until avalanche
   occurs
   Then ghost attraction (ghat[][0]) incremented at random sites
   (oloop to ninov now counts number of these)
   Avalanche size distribution recorded in hist
   simultaneous moves
   simulation program */

#include <stdio.h>
#include <string.h>
#include <stdlib.h>
#include <math.h>

int igridx, igridy, inszf, inadvf, intrup, iseed, ninov;
double storsepx, storsepy, alpha, beta, gc, gama, ghostatt, ghostinc;
double arn,brn,crn,drn, attmin,dsize,damp,nashtol, minprof, inoadv;

char c[13][50];
char *chat[] = { &c[0][0], &c[1][0], &c[2][0], &c[3][0], &c[4][0],
                &c[5][0], &c[6][0], &c[7][0], &c[8][0], &c[9][0],
                &c[10][0], &c[11][0], &c[12][0], &c[13][0] };
Appendix B: C Code

int parfilin( char *parfnm );
void sizfilin( char *sizfnm, double storsiz[32][20], int maxnumx, int
maxnumy );
void advfilin( char *advfnm, double storadv[32][20], int maxnumx, int
maxnumy );
void sizout( int maxnumx, int maxnumy, double siz[32][20] );
void revenue( int maxnumx, int maxnumy, double storlocx[32], double
storlocy[20],
    double storadv[32][20], double storsiz[32][20], double
rev[32][20],
    double distmax[32][20], double ghat[32][20] );
void profit( int maxnumx, int maxnumy, double storsiz[32][20], double
rev[32][20],
    double prof[32][20] );
/*void tograph(int maxnumx, int maxnumy, double storsiz[32][20],
    double prof[32][20], int storino[32][20], FILE *bf );*/
int innovate(double prof[32][20], double refprof[32][20], double
storsiz[32][20],
    int storino[32][20], int maxnumx, int maxnumy );
void adjust( double storsiz[32][20], double ghat[32][20],
    int maxnumx, int maxnumy, double initmsize );

main ( int argc, char *argv[] )
{
    FILE *bf, *lfp;
    double dummy[32][20], storadv[32][20], prof[32][20], xstart,
ystart, distmax[32][20];
    double reprof[32][20], ghat[32][20], initmsize;
    double storsiz[32][20], storlocx[32], storlocy[20], rev[32][20];
    int retin, ntol, storino[32][20], oloop, immflag, immmflag;
    int maxnumx, maxnumy, i, j, k, l, iloop, avsize, hist[640];
char temp;
    /* printf( "\n PARFILE: %s
 SIZEFILE: %s
 ADVANTAGE FILE: %s\n", 
argv[1], argv[2], argv[3] );
    printf( "\nGRAPH OUT FILE: %s\n", argv[4] );
    printf( "\n If this is OK, Hit Any key to continue \n" );
    printf( " Otherwise, Hit 0 to break out\n" );
    temp = getchar();
    if( temp == '0' )
Appendix B: C Code

goto Break;/
retin = parfilin( argv[1] );
if (retin == 1 )
{
/* ******CALCULATE SOME STORE PARAMETERS****** */

    maxnumx = ((igridx + storsepx/2)/storsepx) - 1;
    maxnumy = ((igridy + storsepy/2)/storsepy) - 1;
    /* printf ("\nmaxnumx = %i maxnumy = %i\n", maxnumx, maxnumy ); */

    xstart = ( igridx - ((maxnumx) * storsepx)) / 2;
    ystart = ( igridy - ((maxnumy) * storsepy)) / 2;
    for(i = 0; i <= maxnumx; i++)
        storlocx[i] = i*storsepx + xstart;
    for(j = 0; j <= maxnumy; j++)
        storlocy[j] = j*storsepy + ystart;
    /* for(i = 0; i <= maxnumx; i++)
        printf( "\nstorlocx[%i] = %f", i, storlocx[i] );
        printf( "\n" );
        for(j = 0; j <= maxnumy; j++)
            printf( "\nstorlocy[%i] = %f", j, storlocy[j] );
        printf( "\nAny key to continue \n" );
        printf( "0 to break out\n" );
        temp=getchar();
        if( temp == '0' )
            goto Break;

    */

    /* ************************************************************* */

    /* ****ASSIGN INITIAL STORE SIZE AND DIFFERENTIAL ADVANTAGE**** */

    if( inszf == 0 )
    {
        srand( (unsigned)iseed );
        for( i = 0; i <= maxnumx; i++)
        {
            for( j = 0; j <= maxnumy; j++)
            {
                storsiz[i][j] = arn + brn *(double)rand();
            }
        }
    }
else
sizfilin( argv[2], storsiz, maxnumx, maxnumy );

printf( "\n Initial Store Sizes\n\n" );
sizout( maxnumx, maxnumy, storsiz );

printf( "\n Any key to continue \n" );
printf( "0 means break out\n" );
temp = getchar();
if( temp == '0' )
goto Break;
if( inadvf == 0 ) {
    srand( (unsigned)(iseed+1) );
    for( i = 0; i <= maxnumx; i++ )
    {
        for( j = 0; j <= maxnumy; j++ )
        {
            storadv[i][j] = crn + drn*rand();
        }
    }
}
else
advfilin( argv[3], storadv, maxnumx, maxnumy );

/* printf( "\n Initial Store Advantages \n\n" );
sizout( maxnumx, maxnumy, storadv ); */
/*
 printf( "\n Any key to continue \n" );
 printf( "0 means Break out\n" );
temp = getchar();
if( temp == '0' )
goto Break;
*/

/****INITIALIZE INNOVATION & GHOST ATTRACTION ARRAY***************/

for( i = 0; i <= maxnumx; i++ )
{
    for( j = 0; j <= maxnumy; j++ )
    {
        storino[i][j] = 0;
        ghat[i][j] = ghostatt;
    }
}
Appendix B: C Code

/* OPEN LOGFILE */
if ( (lfp = fopen(argv[5], "at") ) == NULL )
    printf("Unable to open Logfile\n");
    goto Break;
}

/* Opening binary graphics file */
* **************** OPEN AND START BINARY GRAPHICS FILE *** *
if( argv[4] == NULL )
    { if( (bf = fopen( "simtograph", "w" ) ) == NULL )
        printf("Unable to open graphics file\n");
            goto Break;
    }
else
    { if( (bf = fopen( argv[4], "w" ) ) == NULL )
        printf("Unable to open graphics file\n");
            goto Break;
    }
printf("writing graphics header\n");

fprintf( bf, "%lf %lf %i %i %i \n", storsepx, storsepy, maxnumx, maxnumy, igridx, igridy );
for( i = 0; i <= maxnumx; i++ )
    { fprintf( bf, "%lf \n", storlocx[i] );
    fprintf( bf, "%lf \n", storlocy[i] );
    fprintf( bf, "%lf %lf %lf %lf %i %i \n", alpha, beta, gc, gama, inszf, inadvf );
    fprintf( bf, "%i %lf %lf %lf %lf \n", iseed, am, brn, crn, drn
    fprintf( bf, "%lf %lf %lf %i %i \n", attmin, dsize, damp, ninov, intrup );
    fprintf( bf, "%lf %lf %lf %i %i \n", nashtol, minprof,inoadv,ghostatt,ghostinc );
*/
/***/ INITIALIZE REFERENCE PROFIT /***/

revenue( maxnumx, maxnumy, storlocx, storlocy, storadv, storsiz,
rev, distmax, ghat );
printf("\n\n INITIAL REVENUE MATRIX ");
sizout( maxnumx, maxnumy, rev );
profit( maxnumx, maxnumy, storsiz, rev, prof );
printf("\n\n PROFIT MATRIX ");
sizout( maxnumx, maxnumy, prof );

for( i = 0; i <= maxnumx; i++ )
{ for( j = 0; j <= maxnumy; j++ )
  { refprof[i][j] = prof[i][j];
  }
}
temp=getchar();
if( temp == '0' )
goto Break;

printf("\n Initial Store Sizes\n\n");
sizout( maxnumx, maxnumy, storsiz );
printf("\n Any key to continue \n");
printf( "0 means break out\n" );
temp = getchar();
if( temp == '0' )
goto Break;

/******** SOC SECTION, IHOPE *******************/

immflag = 0;

/**  *****Push down all the profits*****/

while(immflag == 0)
{
  for( i = 0; i <= maxnumx; i++ )
  { for( j = 0; j <= maxnumy; j++ )
     ghat[i][j] = ghat[i][j] + ghostinc;
  }

  printf("\n\n pushing down profits ");
}
Appendix B: C Code

revenue(maxnumx, maxnumy, storlocx, storlocy, storadv, storsiz,
    rev, distmax, ghat);
/* printf("\n\n    REVENUE MATRIX ");
sizout(maxnumx, maxnumy, rev);
*/    profit(maxnumx, maxnumy, storsiz, rev, prof);
/* printf("\n\n    PROFIT MATRIX ");
sizout(maxnumx, maxnumy, prof);

tograph(maxnumx, maxnumy, storsiz, prof, storino, bf);
*/
    /**** Check store's profit threshold */
    /**** If below, they innovate */
immflag = innovate(prof, refprof, storsiz, storino, maxnumx,
    maxnumy);
}
printf("\n\n    Profit threshold exceeded \n    Avalanche commences\n");

/*  ****Let the avalanche go*****/

    while( immflag != 0 )
    {
        revenue(maxnumx, maxnumy, storlocx, storlocy, storadv, storsiz,
            rev, distmax, ghat);
/* printf("\n\n    REVENUE MATRIX ");
sizout(maxnumx, maxnumy, rev);
*/    profit(maxnumx, maxnumy, storsiz, rev, prof);
printf("\n\n    PROFIT MATRIX ");
sizout(maxnumx, maxnumy, prof);
printf(" \n\n    Any key to continue \n\n    0 means break out\n");
temp = getchar();
if( temp == '0' )
goto Break;
/*
tograph(maxnumx, maxnumy, storsiz, prof, storino, bf);
*/
    /**** Check store's profit threshold */
    /**** If below, they innovate */
immflag = innovate(prof, refprof, storsiz, storino, maxnumx,
    maxnumy);
}
printf("\n\n    Avalanche Complete\n    Commence Random incrementing\n");
/**
 **** Increment ghosts at random sites ****/

/* *** First init'z hist array and mean size */

for( i = 0; i <= (maxnumx+1)*(maxnumy+1)-1; i++ )
    hist[i] = 0;
initmsize = 0;
for( i = 0; i <= maxnumx; i++ )
    { for( j = 0; j <= maxnumy; j++ )
        initmsize = initmsize + storsiz[i][j];
    }
initmsize = initmsize/((maxnumx+1)*(maxnumy+1));
printf("\n initial average storsize %f\n", initmsize);

/*
*********** Loop ***********
*/

for( oloop = 0; oloop <= ninov; oloop++ )
    { 
        i = (int)( (float)rand()*(float)(maxnumx+1)/(float)32768 );
        j = (int)( (float)rand()*(float)(maxnumy+1)/(float)32768 );
        ghat[i][j] = ghat[i][j] + ghostinc;
        immmflag = 1;
        avsize = 0;
        while(immmflag != 0)
            { 
                revenue( maxnumx, maxnumy, storlocx, storlocy, storadv,
                        storsiz,
                        rev, distmax, ghat);
                /*
                printf("\n\n                     REVENUE MATRIX ");
                sizout( maxnumx, maxnumy, rev );
                */
                profit( maxnumx, maxnumy, storsiz, rev, prof );
                /*
                printf("\n\n                     PROFIT MATRIX ");
                sizout( maxnumx, maxnumy, prof );
                */
                tograph( maxnumx, maxnumy, storsiz, prof, storino, bf );
                /*
                 /**** Check store's profit threshold */
                 /**** If below, they innovate */
                immmflag = innovate( prof, refprof, storsiz, storino,
                        maxnumx, maxnumy );
                /*
                printf("\n immmflag on return from innovate: %i", immmflag);
                */
    }
Appendix B: C Code

```c
/*
   avsize = avsize + immmflag;
   printf("\n avsize on incrementing by immflag: %i", avsize);
*/
}
hist[avsize] = hist[avsize] + 1;
printf("\n aval. size = %i",avsize);
printf("\n PRE ADJUSTED REVENUES \n");
sizout( maxnumx, maxnumy, rev );
    printf( "0 means break out\n" );
    temp = getchar();
adjust( storisz,ghat,maxnumx,maxnumy,initmsize );
printf("\n POST ADJUSTED REVENUES \n");
sizout( maxnumx, maxnumy, rev );
    printf( "0 means break out\n" );
    temp = getchar();
if(temp == '0' )
goto Break;

for( l = 0; l <= maxnumy; l++ )
{
    fprintf(lfp, "\n %i ", l*(maxnumx+1));
    for( k = 0; k <= maxnumx; k++ )
    {
        fprintf(lfp,"[%i] %i ", l*(maxnumx+1) +k,
        hist[l*(maxnumx+1) +k]);
    }
}
printf("\n\n Probe at location %i %i complete", i, j );
printf("\n oloop = %i, ninov = %i\n", oloop, ninov );
/*
temp = getchar();
if(temp == '0' )
goto Break;
*/
}
fprintf( lfp, "\n\n avalanche size histogram" );
for( j = 0; j <= maxnumy; j++ )
{
    fprintf(lfp, "\n %i ", j*(maxnumx+1));
    for( i = 0; i <= maxnumx; i++ )
    {
        fprintf(lfp,"[%i] %i ", j*(maxnumx+1) +i, hist[j*(maxnumx+1) +i]);
```
Appendix B: C Code

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break:
    fclose( bf );
    fclose( lfp );
    return( 0 );
}

/* ***function INPUT PARAMETER FILE *** */

int parfilin( char *parfnm )
{
    FILE *pfp;
    int result;
    char temp;
    if( pfp = fopen( parfnm, "r" ) )
    {
        fscanf( pfp, "%lf %lf", &storsepx, &storsepy );
        fgets( chat[0], 50, pfp );
        fscanf( pfp, "%i %i", &igridx, &igridy );
        fgets( chat[1], 50, pfp );
        fscanf( pfp, "%lf %lf", &alpha, &beta );
        fgets( chat[2], 50, pfp );
        fscanf( pfp, "%lf %lf", &gc, &gama );
        fgets( chat[3], 50, pfp );
        fscanf( pfp, "%i %i", &inszf, &inadvf );
        fgets( chat[4], 50, pfp );
        fscanf( pfp, "%i", &iseed );
        fgets( chat[5], 50, pfp );
        fscanf( pfp, "%lf %lf", &arn, &brn );
        fgets( chat[6], 50, pfp );
        fscanf( pfp, "%lf %lf", &crn, &drn );
        fgets( chat[7], 50, pfp );
        fscanf( pfp, "%lf", &attmin );
        fgets( chat[8], 50, pfp );
        fscanf( pfp, "%lf %lf", &dsize, &damp );
        fgets( chat[9], 50, pfp );
        fscanf( pfp, "%i %i", &ninov, &intrup );
        fgets( chat[10], 50, pfp );
        fscanf( pfp, "%lf %lf %lf", &nashtol, &minprof, &inoadv );
Appendix B: C Code

```c
fgets( chat[11], 50, pfp );
fscanf( pfp, "%lf %lf", &ghostatt, &ghostinc );
fgets( chat[12], 50, pfp );
close( pfp );
result = 1;
return result;
}
else
{
    perror("Couldnt open parameter file");
    result = 0;
    return result;
}
}

void sizout( int maxnumx, int maxnumy, double siz[32][20] )
{
    int i, j, temp;
    for( j = 0; j <= maxnumy; j++ )
    {
        printf( "\n%4i: ", j );
        for( i = 0; i <= maxnumx; i++ )
        {
            printf( "%2.4f ", siz[i][j] );
        }
    }
}

/* *****function**REVENUE CALCULATION FOR ALL STORES ON GRID************* */
/* Attractions to stores at a distance greater than attmin are set = 0 */
/* For each customer grid point, one unit of revenue is apportioned according to attractions */

void revenue( int maxnumx, int maxnumy, double storlocx[32], double storlocy[20],
              double storadv[32][20], double storsiz[32][20], double rev[32][20],
              double distmax[32][20], double ghat[32][20] )
{  

```
double bufatt[32][20];
double dist, bufsav, hold;
int i, j, k, l, kpref, lpref, temp;

/* printf("Calculating Attractions for each point on %i by %i
grid\n",igridx,igridy );
printf("to each of %i stores,\n",(maxnumx+1)*(maxnumy+1) );
printf("and assigning 1 unit of revenue proportionately.\n");
printf( "\nThis may take a while so RELAX...\n");
temp = getcharO;
*/

for( k = 0; k <= maxnumx; k++ )
{  for( 1 = 0; 1 <= maxnumy; 1++ )
{  
rev[k][1] = 0;
/* hold = storadvv[k][1] * pow( storsiz[k][1], beta ) / attmin;
distmax[k][1] = pow( hold, 1/alpha );  */
distmax[k][1] = attmin;
}
}

for( i = 0; i <= igridx; i++ )
{  for( j = 0; j <= igridy; j++ )
{  
bufsav = 0;
for( k = 0; k <= maxnumx; k++ )
{  for( 1 = 0; 1 <= maxnumy; 1++ )
{  
dist = sqrt( pow((i - storlocx[k]), 2) + pow((j - storlocy[l]), 2) );
if( dist <= attmin )
{  
bufatt[k][1] = pow(storsiz[k][1],beta )
  * pow( dist + 1, -alpha );
/* printf("\n\ni,j = %i, %i",i,j);
printf("\n stlocx[%i] stlocy[%i] = %f %f",k,l,storlocx[k],storlocy[l] );
printf("\nnnSBeta: %f tothe %f = %f", storsiz[k][l],beta, 
pow(storsiz[k][l],beta ) );
printf("\nD+lAlpha: %f tothe %f = %f", dist + 1, -alpha, 
pow(dist + 1, -alpha ) );
printf("\n attraction = %f", bufatt[k][1] );
temp = getchO;
*/
bufsav = bufsav + bufatt[k][1];
Appendix B: C Code

```c
} 
else 
    bufatt[k][l] = 0;
}
}
if( bufsav != 0 )
{ 
    for( k = 0; k <= maxnumx; k++ )
        for( 1 = 0; 1 <= maxnumy; 1++ )
            rev[k][l] = rev[k][l] + bufatt[k][l] / (bufsav +
ghat[k][l]);
    }
}
}

/* ***************PROFIT FUNCTION********************************** */

void profit( int maxnumx, int maxnumy, double storsiz[32][20], double 
rev[32][20],
        double prof[32][20] )
{
    int i, j;
    for( i = 0; i <= maxnumx; i++ )
        for(j = 0; j <= maxnumy; j++ )
            prof[i][j] = rev[i][j] - gc*pow( storsiz[i][j], gama );
}

/* **********INPUT INITIAL SIZE FILE ********************* */

void sizfilin( char *sizfnm, double storsiz[32][20], int maxnumx, int 
maxnumy )
{
    FILE *sfp;
    int i, j;
    if( sfp = fopen( sizfnm, "r" ) )
    {
        for( i = 0; i <= maxnumx; i++ )
            for(j = 0; j <= maxnumy; j++ )
                fscanf( sfp, "%lf", &storsiz[i][j] );
    }
```
Appendix B: C Code

```c
void advfilin( char *advfnm, double storadv[32][20], int maxnumx, int maxnumy )
{
    FILE *afp;
    int i, j;
    if( afp = fopen( advfnm, "r" ) )
    {
        for( i = 0; i <= maxnumx; i++ )
        {
            for(j = 0; j <= maxnumy; j++ )
            {
                fscanf( afp, "%lf", &storadv[i][j] );
            }
        }
    }
    fclose( afp );
    else
        perror("Could not open initial advantage file ");
}

/* ******** TO BINARY FILE FOR GRAPHICS ********* */

/*void tograph(int maxnumx, int maxnumy, double storsiz[32][20],
    double prof[32][20], int storino[32][20], FILE *bf )
{
    int i,j;
    for( i = 0; i <= maxnumx; i++ )
    {
        for( j = 0; j <= maxnumy; j++ )
        {
            fprintf( bf, "%lf %lf %i \n", storsiz[i][j], prof[i][j], storino[i][j] );
        }
    }
} */

/* **** INNOVATION ************************************* */
```
Appendix B: C Code

```c
int innovate(double prof[32][20], double refprof[32][20], double
storsiz[32][20],
    int storino[32][20], int maxnumx, int maxnumy )
{
    int immflag = 0, i, j;
    for( i = 0; i <= maxnumx; i++)
    {
        for( j = 0; j <= maxnumy; j++)
        {
            if(prof[i][j] < (minprof*refprof[i][j]) )
            {
                storsiz[i][j] = storsiz[i][j] + inoadv;
                storino[i][j] = storino[i][j] + 1;
                immflag = immflag +1;
                /*
                printf("\nStore at %i, %i innovates",i,j);
                */
            }
        }
    }
    return immflag;
}

/* **************** ADJUST  Readjustment of dynamic parameters **************/

void adjust( double storsiz[32][20], double ghat[32][20],
    int maxnumx, int maxnumy, double initmsize )
{
    int i,j;
    char temp;
    double msize = 0;

    for( i = 0; i <= maxnumx; i++)
    {
        for( j = 0; j <= maxnumy; j++)
        {
            msize = msize + storsiz[i][j];
        }
    }
    msize = msize/(( maxnumx + 1 )*( maxnumy + 1 ));
    for( i = 0; i <= maxnumx; i++)
    {
        for( j = 0; j <= maxnumy; j++)
        {
            storsiz[i][j] = storsiz[i][j]*(initmsize/msize);
            ghat[i][j] = ghat[i][j]*(pow( (initmsize/msize), beta ) );
        }
    }
}
```
Appendix B: C Code

printf("\n current mean storsize = \f", msize);
printf("\n ADJUSTED SIZES \n");
sizout( maxnumx, maxnumy, storsiz );
 printf(" 0 means break out\n");
 temp = getchar();

printf("\n ADJUSTED GH ATT\n");
sizout( maxnumx, maxnumy, ghat );
 printf(" 0 means break out\n");
 temp = getchar();
}

C2. Parameter File Set Routine

/* SET4D.C INITIALIZATON program */

#include <stdio.h>
#include <string.h>
#include <stdlib.h>

int igridx, igridy, inszf, inadvf, intrup, ninov, iseed;
double storsepx, storsepy, alpha, beta, gc, gamma, arn, brn, attmin;
double dsize, damp, crn, drn, nashtol, minprof, inoadv, ghostatt, ghostinc;

char c[13][50];
char *chat[] = { &c[0][0], &c[1][0], &c[2][0], &c[3][0], &c[4][0],
 &c[5][0], &c[6][0], &c[7][0], &c[8][0], &c[9][0],
 &c[10][0], &c[11][0], &c[12][0], &c[13][0] };

int parfilin( char *parfnm );
void parfilch( char *parfnm );

main ( int argc, char *argv[] )
{
 FILE *bf;
 int retin;
 char temp;

 printf("\n PARFILE: %s\n SIZEFILE: %s\n ADVANTAGE FILE: %s\n", 
 argv[1], argv[2], argv[3] );
 printf("\n If this is OK, Hit Any key to continue \n");
 printf(" Otherwise, Hit 0 to break out\n");
Appendix B: C Code

```c
retin = parfilin( argv[1] );
if (retin == 1 )
{
    printf(" \n Hit any key to change any of these parameters. \n" );
    printf(" Hit ENTER for none. \n" );
    if( ( temp = getch() ) != '\r')
    {
        parfilch( argv[1] );
    }
    temp = getch();
}
else
    perror( "couldn't open parameter file " );
}
/* ***function INPUT PARAMETER FILE *** */
int parfilin( char *parfnm )
{
    FILE *pfp;
    int result;
    if( pfp = fopen( parfnm, "r" ) )
    {
        fscanf( pfp, "%lf %lf", &storsepx, &storsepy );
        fgets( chat[0], 50, pfp );
        fscanf( pfp, "%i %i", &igridx, &igridy );
        fgets( chat[1], 50, pfp );
        fscanf( pfp, "%lf %lf", &alpha, &beta );
        fgets( chat[2], 50, pfp );
        fscanf( pfp, "%lf %lf", &gc, &gamma );
        fgets( chat[3], 50, pfp );
        fscanf( pfp, "%i %i", &inszf, &inadvf );
        fgets( chat[4], 50, pfp );
        fscanf( pfp, "%i", &iseed );
        fgets( chat[5], 50, pfp );
        fscanf( pfp, "%lf %lf", &arn, &brn );
        fgets( chat[6], 50, pfp );
        fscanf( pfp, "%lf %lf", &crn, &drn );
        fgets( chat[7], 50, pfp );
        fscanf( pfp, "%lf", &attmin );
        fgets( chat[8], 50, pfp );
        fscanf( pfp, "%lf %lf", &dsize, &damp );
        fgets( chat[9], 50, pfp );
        fscanf( pfp, "%i %i", &ninov, &intrup );
```
fgets( chat[10], 50, pfp );
fscanf( pfp, "%lf %lf %lf", &nashtol, &minprof, &inoadv );
fgets( chat[11], 50, pfp );
fscanf( pfp, "%lf %lf", &ghostatt, &ghostinc );
fgets( chat[12], 50, pfp );

printf( "INPUT FILE PARAMETERS

COMMENTS 

" );
printf( "storsepx: %5.1f 	 storsepy: %5.1f 
", storsepx, storsepy );
printf( chat[0] );
printf( "igridx: %0.3i 	 igridy: %0.3i 
", igridx, igridy );
printf( chat[1] );
printf( "alpha: %5.3f 	 beta: %5.3f 
", alpha, beta );
printf( chat[2] );
printf( "gc: %5.3f 	 gamma: %5.3f 
", gc, gamma );
printf( chat[3] );
printf( "inszf: %0.1i 	 inadvf: %0.1i 
", inszf, inadvf );
printf( chat[4] );
printf( "iseed: %i 
", iseed );
printf( chat[5] );
printf( "arn: %5.3f 	 brn: %5.5f 
", arn, brn );
printf( chat[6] );
printf( "crn: %5.3f 	 drn: %5.5f 
", crn, drn );
printf( chat[7] );
printf( "attmin: %5.3f 
", attmin );
printf( chat[8] );
printf( "dsize: %5.3f 	 damp: %5.3f 
", dsize, damp );
printf( chat[9] );
printf( "ninov: %0.1i 	 intrup: %0.1i 
", ninov, intrup );
printf( chat[10] );
printf( "nashtol: %5.3f 	 minprof: %5.3f 	 inoadv: %5.3f 
", nashtol, minprof, inoadv );
printf( chat[11] );
printf( "ghostatt: %5.3f 	 ghostinc: %5.3f 
", ghostatt, ghostinc );
printf( chat[12] );

fclose( pfp );
result = 1;
return result;
}
else
{
 perror("Couldnt open parameter file");
Appendix B: C Code

```c
result = 0;
return result;
}
}

/* *****function CHANGE PARAMETER FILE ***** */

void parfilch( char *parfnm )
{
    FILE *pfp;
    int num;
    do
    {
        printf( "\n\nType the number of the parameter from the list below,\n" );
        printf( "followed by a space and the parameter’s new value.\n\n" );
        printf( "0 MEANS NO MORE CHANGES\n\n" );

        printf( "1 storsepx: %5.1f\n2 storsepy: %5.1f \", storsepx, storsepy );
        printf( "3 igridx: %0.3i\n4 igridy: %0.3i \", igridx, igridy );
        printf( "5 alpha: %5.3f\n6 beta: %5.3f \", alpha, beta );
        printf( "7 gc: %5.3f\n8 gamma: %5.3f \", gc, gamma );
        printf( "9 inszf: %O.li\n10 inadvf: %0.li \", inszf, inadvf );
        printf( "11 iseed: %i \n\", iseed );
        printf( "12 arn: %5.3f\n13 brn: %5.7f \", arn, brn );
        printf( "14 crn: %5.3f\n15 drn: %5.7f \", crn, drn );
        printf( "16 attmin: %5.3f\n17 dsize: %5.3f\n18 damp: %5.3f \", ds
        printf( "19 ninov: %O.li\n20 intrup: %O.li \", ninov, intrup );
        printf( "21 nashtol: %5.4f\n22 minprof: %6.3f\n23 inoadv: %5.3f \n\", nashtol, minprof, inoadv );
        printf( "24 ghostatt: %5.3f\n25 ghostinc: %5.3f \", ghostatt, ghostinc );

        scanf( "%i\", &num );

        switch( num )
        {
            case 1:
                scanf( "%lf\", &storsepx );
                break;
            case 2:
                scanf( "%lf\", &storsepy );
                break;
            case 3:
```
```
Appendix B: C Code

```c
scanf( "%i", &igridx );
break;
case 4:
    scanf( "%i", &igridy );
    break;
case 5:
    scanf( "%lf", &alpha );
    break;
case 6:
    scanf( "%lf", &beta );
    break;
case 7:
    scanf( "%lf", &gc );
    break;
case 8:
    scanf( "%lf", &gamma );
    break;
case 9:
    scanf( "%i", &inszf );
    break;
case 10:
    scanf( "%i", &inadvf );
    break;
case 11:
    scanf( "%i", &iseed );
    break;
case 12:
    scanf( "%lf", &arn );
    break;
case 13:
    scanf( "%lf", &brn );
    break;
case 14:
    scanf( "%lf", &crn );
    break;
case 15:
    scanf( "%lf", &drn );
    break;
case 16:
    scanf( "%lf", &attmin );
    break;
case 17:
    scanf( "%lf", &dsize);
```
break;
case 18:
    scanf( "%lf", &damp );
    break;
case 19:
    scanf( "%i", &ninov );
    break;
case 20:
    scanf( "%i", &intrup );
    break;
case 21:
    scanf( "%lf", &nashtol );
    break;
case 22:
    scanf( "%lf", &minprof );
    break;
case 23:
    scanf( "%lf", &inoadv );
    break;
case 24:
    scanf( "%lf", &ghostatt );
    break;
case 25:
    scanf( "%lf", &ghostinc );
    break;
default:
    printf( "\nNo such parameter---execution continues\n" );
}
} while( num != 0 );
if( pfp = fopen( parfnm, "w" ) )
{
    fprintf( pfp, "%f %f%s\n", storexp, storsey, chat[0] );
    fprintf( pfp, "%i %i%s\n", igridx, igridy, chat[1] );
    fprintf( pfp, "%f %f%s\n", alpha, beta, chat[2] );
    fprintf( pfp, "%f %f%s\n", gc, gamma, chat[3] );
    fprintf( pfp, "%i %i%s\n", inszf, inadvf, chat[4] );
    fprintf( pfp, "%i%s\n", iseed, chat[5] );
    fprintf( pfp, "%f %5.7f%s\n", am, brn, chat[6] );
    fprintf( pfp, "%f %5.7f%s\n", crn, drn, chat[7] );
    fprintf( pfp, "%f%s\n", attmin, chat[8] );
    fprintf( pfp, "%f %f%s\n", arn, brn, chat[6] );
    fprintf( pfp, "%f %5.7f%s\n", crn, drn, chat[7] );
    fprintf( pfp, "%f%s\n", attmin, chat[8] );
    fprintf( pfp, "%f %f%s\n", dsize, damp, chat[9] );
    fprintf( pfp, "%i %i%s\n", ninov, intrup, chat[10] );
    fprintf( pfp, "%5.4f %6.3f %5.3f%s\n", nashtol, minprof, inoadv, chat[11] );
Appendix B: C Code

```c
fprintf( pfp, "%f %f%s\n", ghostatt, ghostinc, chat[12] );

printf( "\nNew parameters written to file\n" );

fclose( pfp );
}
else
  perror("\nCouldn’t open parameter file for write" );
```

C3. Example Parameter File

3.000000 3.000000 STORE SEPARATION IN x & y (float)
23 23 MARKET GRID SIZE (max 320, 200) (int)
0.000000 1.000000 EXPONENTS of DIST & SIZE (float)
0.000000 0.000000 CONST. AND EXP. OF COST FUNC. (float)
0 0 1 == > INITIALIZE FROM FILES (int)
30 IF inszf=0, USE THIS SEED AND
3.000000 0.0001000 TO INIT. SIZE: size= arn + brn*ran
0.001000 0.0000100 TO INIT. ADV: adv = crn + drn*ran
4.500000 MINIMUM ATTR. FOR PURCHASE (float)
0.100000 0.010000 GROWTH DSIZE & DAMPING
100 2 # OF INNOVATIONS; MAX # ITERATIONS
0.2000 0.950 0.600 TOLERANCE; MINPROFIT; INNOVATION
1.000000 1.000000 GHOST ATTRACTION & INCREMENT

C4. Animation Routine

#include <graph.h>
Appendix B: C Code

#include <stdlib.h>
#include <conio.h>
#include <stdio.h>
#include <math.h>
#include <errno.h>

main( int argc, char *argv[])
{
    FILE *lfp;
    char temp;
    double prof[32][20], storlocx[32], storlocy[20],
           sizmax, arn, brn, crn, drn, maxdistmax, dsize, damp;
    /* double histdens[20], histsz[20], histmaxsz, histminsiz,
       histmaxdens, histmindens;
       */
    double nashtol, minprof, inoadv, ghostatt, ghostinc;
    int ihist = 12, ninov, ihit, jhit, isavhit, jsavhit, first, last;
    double storsepx, storsepy, scrsepx, scrsepy, scale, alpha, beta,
           gc, gamma, pbc, rho, attmin;
    int igridx, igridy, maxnumx, maxnumy, iseed, intrup, hitflag[32][20], aggflag;
    int i, j, k, l, loop, iattminx, iattminy, radx[32][20],
       rady[32][20], inszf, inadvf, storino[32][20], savino[32][20];
    unsigned char diagmask[8] =
    { 0x93, 0xC9, 0x64, 0xB2, 0x59, 0x2C, 0x96, 0x4B }; 
    unsigned char solid[8] =
    { 0xFF, 0xFF, 0xFF, 0xFF, 0xFF, 0xFF, 0xFF, 0xFF }; 
    int istorlocx[32], istorlocy[20];
    short oldvpage, oldapage, vpage, apage;

    oldapage = _getactivepage();
    oldvpage = _getvisualpage();

    printf( "In Current visual page: %d, active page: %d", oldvpage, oldapage);

    /* ****************** GET DATA ****************** */

    if( (lfp = fopen( argv[1], "r" )) == NULL )
    {
        printf("\nUnable to open file\n" );
        goto Break;
    }
    else

    /* ****************** GET DATA ****************** */

    if( (lfp = fopen( argv[1], "r" )) == NULL )
    {
        printf("\nUnable to open file\n" );
        goto Break;
    }
    else
Appendix B: C Code

```
printf("\n File Opened ");

fscanf( lfp, "%lf %lf %i %i %i %i ",
  &storsepx, &storsepy, &maxnumx, &maxnumy, &igridx, &igridy );
for( i = 0; i <= maxnumx; i++ )
{
  fscanf( lfp, "%lf ",&storlocx[i] );
  fscanf( lfp, "%lf \n ",&storlocy[i] );
}

fscanf( lfp, "%lf %lf %lf %i %i \n ",
  &alpha, &beta, &gc, &gamma, &inszf, &inadvf );

fscanf( lfp, "%lf %lf %lf %lf \n ", &iseed, &arn, &brn, &crn, &drn );

fscanf( lfp, "%lf %lf %lf %i %i \n 
 
", &attmin, &dsize, &damp, &ninov, &intrup );
fscanf( lfp, "%lf %lf %lf %lf %i %i \n 

", &nashtol,&minprof,&inoadv,&ghostatt,&ghostinc);

printf("\n SIMTOGRAPH FILE PARAMETERS\n
");
printf( "storsep x and y: %f %f\n", storsepx, storsepy );
printf("maxnum x and y: %i %i\n", maxnumx, maxnumy);
printf("igrid x and y: %i %i\n", igridx, igridy );

printf( "STORLOCATIONS X:\n" );
for( i = 0; i <= maxnumx; i++ )
  printf( " %f\n", storlocx[i] );

printf( "STORLOCATIONS Y:\n" );
for( j = 0; j <= maxnumy; j++ )
  printf( " %f\n", storlocy[j] );

printf( "\n\nattraction exps alpha & beta: %f %f\n", alpha, beta );
printf( "cost parameters gc & gamma: %f %f\n", gc, gamma );
printf("input switches inszf & inadvf: %i %i\n", inszf, inadvf );
printf( "random params iseed, arn & brn: %i %f %f\n", iseed, arn, brn );
printf("attraction distance limit attmin: %f\n", attmin );
printf("Nash convergence parameters dsize & damp: %f %f\n", dsize, damp );
printf("iterations & innovation counts: %i %i\n", intrup, ninov );

printf("nashtol,minprof,inoadv,ghostatt,ghostinc: %f %f %f %f\n",
  nashtol,minprof,inoadv,ghostatt,ghostinc );

printf("\n\nENTER' to continue,\n'n' to continue without histogram,\n'0' to break out\n ");

temp = getch();
switch( temp )
```
{  
case '0':
    goto Break;
    break;
case 'n':
    ihist = 0;
    break;
case 'r':
    ihist = 12;
    break;
case '1':
    ihist = 1;
    break;
case '2':
    ihist = 2;
    break;
case '3':
    ihist = 3;
    break;
case '4':
    ihist = 4;
    break;
case '5':
    ihist = 5;
    break;
case '6':
    ihist = 6;
    break;
case '7':
    ihist = 7;
    break;
case '8':
    ihist = 8;
    break;
case '9':
    ihist = 9;
    break;
case 'q':
    ihist = 10;
    break;
case 'w':
    ihist = 11;
    break;  
}
case 'e':
    ihist = 13;
    break;
case 'r':
    ihist = 14;
    break;
case 't':
    ihist = 15;
    break;
}

/*** ***** GET FIRST SIZE MATRIX ************* */

fscanf(lfp, "%i %i ", &ihit, &jhit);
isavhit = ihit;
jsavhit = jhit;
for( j = 0; j <= maxnumy; j++ )
{
    for( i = 0; i <= maxnumx; i++ )
    {
        fscanf(lfp, "%lf ", &prof[i][j]);
        fscanf(lfp, "%i ", &storino[i][j]);
        savino[i][j] = storino[i][j];
    }
}

printf( "\n\n PROFIT MATRIX\n\n" );
sizmax = 0;
for( j = 0; j <= maxnumy; j++ )
{
    printf( "\n%i: ", j );
    for( i = 0; i <= maxnumx; i++ )
    {
        if( prof[i][j] > sizmax )
            sizmax = prof[i][j];
        printf( " %f", prof[i][j] );
    }
}
printf( "\n\nsizmax = %f", sizmax );
printf( "\nEnter Profit Scale Factor--1 is OK\n" );
scanf( "%lf", &scale );
Appendix B: C Code

printf( "Enter first and last iterations to plot\n");  
scanf( "%i %i",&first,&last );  
if( last > ninov )  
{  
    printf( "Error: Only %i iterations in file", ninov );  
goto Break;  
}  

/* *** ******************** GRAPHICS *********************** */  

if(_setvideomode(_ERESCOLOR ) == 0 )  
{  
    printf("VIDEOMODE NOT AVAILABLE\n" );  
    getch();  
    exit( 0 );  
}  

vpage = 0;  
apage = 1;  
_setactivepage(0);  
_setvisualpage(1);  

scrsepx = storsepx*(640/igridx);  
scrsepy = storsepy*(350/igridy);  

for(i = 0; i <= maxnumx; i+ + )  
    istorlocx[i] = (int)(storlocx[i] *640/igridx);  
for(j = 0; j <= maxnumy; j+ + )  
    istorlocy[j] = (int)(storlocy[j] *350/igridy);  

iattminx = (int)(attmin*640/igridx);  
ialttmny = (int)(attmin*350/igridy);  

/********* ****** ****** LOOP ****** ********** ***********/  

for( loop = 1; loop <= last; loop++ )  
{  
    if( loop >= first )  
    {  
        temp = getch();  
        apage = _getvisualpage();  
        _setvisualpage( _getactivepage() );  
        _setactivepage( apage );  
}
printf( "%i", loop - 1 );

/* set flag for hit */

for(i = 0; i <= maxnumx; i++)
{ for(j = 0; j <= maxnumy; j++)
    hitflag[i][j] = 0;
}

aggflag = 0;
if( (isavhit != ihit) || (jsavhit != jhit) )
{
    hitflag[ihit][jhit] = 1;
    aggflag = 1;
}
isavhit = ihit;
jsavhit = jhit;

for(i = 0; i <= maxnumx; i++)
{ for(j = 0; j <= maxnumy; j++)
{
    radx[i][j] = (int)(scale * (prof[i][j]/sizmax)*scrsepx);
    rady[i][j] = (int)(scale * (prof[i][j]/sizmax)*scrsepy);
}
}

_setviewport(0,0,640,350);
_clearscreen(_GCLEARSCREEN);
_setbkcolor(_BLUE);
_setfillmask(solid);
_setcolor(11);
for(i = 0; i <= maxnumx; i++)
{ for(j = 0; j <= maxnumy; j++)
{
    if(savino[i][j] != storino[i][j] )
    {
        _rectangle(_GFILLINTERIOR, istorlocx[i] - radx[i][j] + 10,
                     istorlocy[j] - rady[i][j] + 10,
                     istorlocx[i] + radx[i][j] + 10,
                     istorlocy[j] + rady[i][j] + 10 );
    }
Appendix B: C Code

    savino[i][j] = storino[i][j];

    
    
    _setcolor(14);

    for(i = 0; i <= maxnumx; i++)
    {
        for(j = 0; j <= maxnumy; j++)
        {
            
            _rectangle(_GFILLINTERIOR, istorlocx[i] - radx[i][j],
                        istorlocy[j] - rady[i][j],
                        istorlocx[i] + radx[i][j],
                        istorlocy[j] + rady[i][j]);

            }  // inner for loop
        }  // outer for loop
    }  // if

    if(aggflag != 0 )
    {
        for(i = 0; i <= maxnumx; i++)
        {
            for(j = 0; j <= maxnumy; j++)
            {
                if( hitflag[i][j] == 1 )
                {
                    
                    _setcolor(12);

                    _rectangle(_GFILLINTERIOR, istorlocx[i] - radx[i][j]-6,
                                istorlocy[j] - rady[i][j]-6,
                                istorlocx[i] + radx[i][j]-6,
                                istorlocy[j] + rady[i][j]-6 );

                }  // if hitflag[i][j] == 1
            }  // inner for loop
        }  // outer for loop
    }  // if

    _setcolor(13 );
    for( k = 0; k <= 1; k++)
    {
        for( j = 2; j <= 3; j++)
        {
            _ellipse(_GBORDER, istorlocx[2] - iattminx - k,
                      istorlocy[j] - iattminy - k,
                      istorlocx[j] + iattminx + k,
                      istorlocx[2] + iattminx + k,
                      ...};
istorlocy[j] + iattminy + k);

          istorlocy[j] - (int)(rady[2][j]/3),
          istorlocx[2] + (int)(radx[2][j]/3),
          istorlocy[j] + (int)(rady[2][j]/3) );
}

/******************** Calculate and Draw histogram ******************

Count number in each logarithmic interval
if(ihist != 0)
{
    for( k = 0; k <= 19; k++ )
        histdens[k] = 0;

    for(i = 0; i <= maxnumx; i++ )
    {
        for(j = 0; j <= maxnumy; j++ )
        {
            k = 0;
            while( (k <= 19) & ( log(storsiz[i][j]) >= histsiz[k] ) )
                k++;
            if( k <= 19 )
                histdens[k]++;
        }
    }

    Turn frequency "histdens[k]" (which is now just the number
    of stores in each interval) into logarithm of density
    of stores in each interval

    if( histdens[0] != 0 )
        histdens[0] = log( histdens[0] ) - histsiz[0];
    else
        histdens[0] = histmindens;
    for( k = 1; k <= 19; k++ )
    {
        if( histdens[k] != 0 )
            histdens[k] = log( histdens[k] ) - log( exp( histsiz[k] )
            - exp( histsiz[k-1] ) );
        else
            histdens[k] = histmindens;
    }
Appendix B: C Code

```c
_setviewport(320,20,610,150);
_setwindow( 1, histminsiz, histmindens, histmaxsiz, histmaxdens );
_setcolor( ihist );
_rectangle_w(_GBORDER,histminsiz,histdens[0],histsiz[0],histmindens );
for( k = 1; k <= 19; k++ )
    _rectangle_w(_GBORDER,histsiz[k-1],histdens[k],histsiz[k],histmindens );

*/

/****************Get next frame of data**********************/
_fscanf( lfp, "%i %i ", &ihit, &jhit );
for( j = 0; j <= maxnumy; j++ )
{
    for( i = 0; i <= maxnumx; i++ )
    {
        _fscanf(lfp," %lf ", &prof[i][j]);
        _fscanf(lfp," %i ", &storino[i][j]);
    }
}

/**** ***** ***** END LOOP ***** ***** ***** *************/

apage = _getvisualpage();
_setvisualpage( _getactivepage() );
_setactivepage( apage );
getch();
_setvideomode(_DEFAULTMODE);
_setactivepage( oldapage );
_setvisualpage( oldvpage );
Break:
_fclose( lfp );
```