CORPORATE VOLUNTARY DISCLOSURES OF
PRE-DECISION INFORMATION

by

MANDIRA ROY SANKAR

B.Sc., Indian Institute of Technology, 1977
M.Sc., Indian Institute of Technology, 1979
M.B.A., The University of British Columbia, 1984

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Department of **COMMERCE & BUSINESS ADMINISTRATION**

The University of British Columbia
Vancouver, Canada

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Abstract

This dissertation consists of two essays in the area of corporate voluntary disclosure of predecision information.

The first essay entitled, "Disclosure Choice in a Duopoly", focusses on the phenomenon of partial disclosure, where the manager of the firm discloses selected signals and withholds the rest. The manager may or may not receive private information which is related to both firm-specific and industry-wide common factors. The motivation for disclosure (non-disclosure) is derived from the proprietary nature of the manager's private information. The cost (benefit) of disclosure is modelled in an imperfectly competitive product market, where an uninformed opponent's reaction to a disclosure affects the manager's expected profit.

Our results indicate that the nature of the manager's optimal disclosure policy is crucially dependent on whether the signal is more informative about firm-specific or industry-wide common factors. Unfavourable news is disclosed and favourable news withheld if the signal is more informative about common factors. On the other hand, favourable news is disclosed and unfavourable news is withheld if the signal is more informative about firm-specific factors. Comparative statics show that the sensitivity of the optimal disclosure policy and the probability of disclosure to some key parameters are also dependent on this characteristic of a signal. The empirical implications of our results suggest that when testing hypotheses involving voluntary disclosures, failure to take the above characteristic into account may confound the results.

The second essay entitled, "Disclosure and Reputation in Credit Markets", deals with a different aspect of voluntary disclosures. A reputation game is modelled in the absence of credible disclosure. The manager's ability with respect to obtaining predecision information is of interest to the firm's creditors. The manager's future nominal interest charges depend on the creditors' belief about the manager's ability, i.e., on his reputation. Hence, the manager attempts to communicate this ability through sub-optimal production choice and creditors learn about the manager by observing the end of period revenue realization.

If credible disclosures are possible the manager may make direct disclosures to communicate his information gathering ability to the creditors. This alternative mechanism avoids the cost of reputation building incurred by selecting a suboptimal project. However, it is shown that if these two mechanisms for reputation acquisition are not "independent", then the possibility of disclosure increases the manager's incentive to select a sub-optimal action.
# Contents

Abstract

List of Figures

List of Tables

Acknowledgement

Dedication

1 Introduction

2 Literature Review

3 Disclosure Choice in a Duopoly

3.1 Introduction ........................................... 14

3.2 Duopoly Under Uncertainty ............................... 19

3.2.1 Symmetric Information ................................. 19

3.2.2 Asymmetric Information ............................... 20

3.3 Duopoly Under Asymmetric Information ................. 23

3.3.1 The Model ........................................... 23

3.3.2 Output Choice ....................................... 26

3.3.3 Disclosure Choice .................................... 28

3.4 Equilibrium Analysis .................................... 30

3.4.1 Belief Revision ....................................... 30

3.4.2 Disclosure Equilibrium ................................. 31

3.4.3 Disclosure Equilibrium under Multivariate Normality .... 33

3.4.4 Equilibrium Production and Profit .................... 42
3.4.5 Restricting Output to be Non-negative .............................................. 46
3.4.6 Multiple Uninformed Opponents ..................................................... 51
3.4.7 Discussion ....................................................................................... 52
3.5 Comparative Statics ........................................................................... 57
  3.5.1 Impact of the Probability of Information ......................................... 57
  3.5.2 Impact of Competition ..................................................................... 60
3.6 Empirical Implications ........................................................................ 64
3.7 Conclusion ........................................................................................... 67

4 Disclosure and Reputation in Credit Markets ........................................... 69
  4.1 Introduction ......................................................................................... 70
  4.2 The Model ........................................................................................... 74
    4.2.1 Revenues and Profits ..................................................................... 77
    4.2.2 Reputation and Nominal Return .................................................... 79
  4.3 The Non-Disclosure Regime ................................................................. 84
    4.3.1 Creditors’ Belief Revision in the Non-Disclosure Regime ............... 85
    4.3.2 Optimal Project Choice in the Non-Disclosure Regime ................. 86
    4.3.3 Equilibrium in the Non-Disclosure Regime .................................... 88
  4.4 The Disclosure Regime ........................................................................ 102
    4.4.1 Creditors’ Belief Revision in the Disclosure Regime ...................... 103
    4.4.2 Optimal Output and Disclosure Choice in the Disclosure Regime .... 105
    4.4.3 Equilibrium in the Disclosure Regime .......................................... 106
    4.4.4 Disclosure versus Non-Disclosure ................................................ 115
  4.5 Conclusion ........................................................................................... 118

Bibliography .............................................................................................. 120

A Appendices to Chapter 3 ......................................................................... 147
  A.1 Derivations .......................................................................................... 148
    A.1.1 Optimal Solutions to the Output Choice Game under Asymmetric Information ......................................................... 148
    A.1.2 Expressions for the equilibrium values of $s'$ and $s''$. .................. 151
    A.1.3 Output choice given the constraints $y_i \geq 0$ and $y_j \geq 0$. .......... 152
    A.1.4 Output choice with $M$ opponents .................................................. 156
A.2 Proofs ................................................................. 158
  A.2.1 Proofs of Lemmas .............................................. 158
  A.2.2 Proofs of Propositions ....................................... 173
  A.2.3 Proof of Corollary ........................................... 179

B Appendices to Chapter 4 ........................................... 180
  B.1 Derivations ...................................................... 181
    B.1.1 Density functions of $R_t$ given $\tau$, and $p_t$ .... 181
    B.1.2 Joint Densities of $R_t$ and $m_t$ given $\tau$, $p_t$ and $\beta_t$ 182
    B.1.3 Choice of Optimal Disclosure Policy .................. 183
  B.2 Proofs ........................................................... 186
    B.2.1 Proofs of Lemmas ........................................... 186
    B.2.2 Proofs of Propositions .................................... 189
List of Figures

3.1 Parabola Representations of Optimized Expected Profits ................. 125
3.2 Ranges for the Equilibrium Values of $s'$ and $s''$ ......................... 126
3.3 Optimal Output and Expected Profit in an Unconstrained Duopoly .......... 127
3.4 Optimal Output and Expected Profit in a Constrained Duopoly ............. 128
3.5 Optimal Output and Expected Profit in an Unconstrained Oligopoly .......... 129
3.6 Disclosure and Nondisclosure Sets in Different Studies ..................... 130
3.7 $s'$ and $s''$ as functions of $q$ .................................. 131
3.8 $P(D)$ as a function of $q$ ........................................ 132
3.9 Opponent’s Optimal Output and Expected Profit in a Constrained Duopoly . 133
3.10 Truncated Reaction Functions ..................................... 134
4.1 Decision Trees in the Non-Disclosure Regime ............................ 135
4.2 $\Delta V_2^\tau$ and $\Delta \pi_1^\tau$ for $\tau \in \{G_h, G_l\}$ in the Non-Disclosure Regime .... 136
4.3 $\Delta V_2^\tau$ and $\Delta \pi_1^\tau$ for $\tau \in \{G_u, B\}$ in the Non-Disclosure Regime .... 137
4.4 Example of the Existence of $A^1$, $A^2$, $A^3$ and $A^4$ ................. 138
4.5 $\Delta V_2^\tau$ and $\Delta \pi_1^\tau$ for $\tau \in \{G_u, B\}$ in the Disclosure Regime ... 139
4.6 Example of the Existence of $\hat{A}^1$, $\hat{A}^2$ and $\hat{A}^4$ ............... 140
List of Tables

3.1 Equilibrium Values if $\delta_i > \delta$ .................................................. 142
3.2 Equilibrium Value if $\delta_i < \delta$ .................................................. 143
4.1 Posterior Beliefs Given Different Pure Strategies in the Non-Disclosure Regime . 144
4.2 Existence of the Different Pure Strategy Equilibria in the Numerical Example . 145
4.3 Posterior Beliefs Given Different Pure Strategies in the Disclosure Regime . . . 146
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Dedicated to my parents

Prodyut Kumar Roy and Roma Roy
Chapter 1

Introduction
Corporate managers receive private information about various aspects of the operation of their firms, over the duration of their operating periods. Managers have frequently been observed to voluntarily disclose such information prior to the end of their operating periods. This phenomenon has been examined both empirically and theoretically in the accounting literature.

The 1980's have seen a spurt in the analytical research in the field of voluntary disclosure. The main focus of these papers has been on explaining the phenomenon of partial disclosure, where the manager discloses selected signals and withholds the rest.

This dissertation consists of two analytical essays in the area of corporate voluntary disclosure. Both essays consider situations where the manager may or may not receive private information prior to his operating decision. In the first essay we focus on the issue of partial disclosure, where the manager discloses selected signals and withholds the rest. The second essay focusses on the role of voluntary disclosure in the establishment of manager reputation.

The analytical research in voluntary disclosure is preceded by an extensive amount of empirical research on management earnings forecasts, much of it triggered by the SEC's concerns at different points in time. In the earlier part of the 1970's the SEC was concerned about ensuring uniform access by all investors to voluntarily released forecasts. Implicit in the SEC's concern was its belief that management forecasts influence investors beliefs and actions. Following this, empirical studies were interested in the issue of "information content" of management earnings forecasts. Patell (1976), Penman (1980) and Waymire (1984) examine common stock price behaviour that accompanies voluntary disclosures. A favorable forecast seems to be accompanied by an increase in stock price and an unfavorable forecast by a decrease. On the average, therefore, corporate earnings forecasts possess information relevant to the valuation of firms and hence to investors' decisions.

An important finding in many of these empirical studies is that the average stock price change, at the time of forecast release, is positive. A possible explanation for this observation, offered by Penman, is that firms with relatively poor earnings prospects do not reveal their relative position through an earnings forecasts. Since only firms that have released earnings forecasts are selected for empirical studies, it follows that the average stock price reaction should be positive. This explanation relies on a selection bias story which is also consistent with analytical models for discretionary disclosure proposed by Verrechhia (1983) and Dye (1985). These models predict that firms with favourable news make voluntary disclosures to differentiate themselves from firms with unfavourable news. However, firms with unfavourable news prefer

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1 Patell's data is from the period 1963-67, Penman and Waymire use data from the period 1968-73.
non-disclosure. However, the latter prediction is not supported by other empirical studies.\(^2\) In general, therefore, which signals are disclosed (withheld) depend on a manager's objective function. Hence, in general, it is not necessary that only managers with poor (bright) prospects withhold (disclose) their information.

An alternative explanation for the positive average stock price change was suggested by Patell. He conjectured that the act of voluntarily committing the firm to a prediction of earnings per share conveys information separate from the predicted numerical value. This can be interpreted as meaning that the "information" being used to revise stock prices is the very fact that the manager possesses a particular kind or quality of information. Implicit in this statement is the assumption that the kind (quality) of the manager's private information or his ability to generate the same is not common knowledge. Hence, the market learns about the manager's type with respect to his information quality through his disclosure.\(^3\) The market's learning process translates into the establishment of a reputation for a privately informed manager.

The first essay in this dissertation, "Disclosure Choice in a Duopoly", focusses on the partial disclosure issue. We explore the impact of firm-specific versus industry-wide common information on the optimal disclosure policies of managers. The manager is concerned about the production decisions made by himself and a competitor in his market. The manager's private signal is relevant in the production decisions of both members of the duopoly. Hence, the manager is concerned about the possible impact of any disclosures he may make on his competitor's output level. The results indicate that the firm's disclosure policy is crucially dependent on whether the manager's private information is related to firm-specific or industry-wide uncertainty. This implies that failure to consider the information's type may have a confounding effect on empirical tests of voluntary disclosure.

The second essay focusses on the role of disclosure in the establishment of manager reputation. The manager is concerned with the cost of raising capital for his production process. The manager's private information facilitates his choice among alternative risky investment projects or operating decisions. If the production decision is based on better quality information, the profits are higher and the probability of insolvency is lower. From the creditors' viewpoint, a better manager is one who can meet all his debt obligations.

The two essays differ with respect to their modelling of the manager's ability to acquire pre-decision information. In the first essay the manager's ability to acquire pre-decision infor-

\(^2\)LEV and PENMAN (1990).

\(^3\)The term "market" applies to any party in the firm's environment who has any effect on the firm's operations or profits.
mation is common knowledge. Hence, the disclosure choice problem can be restricted to a single period. In the second essay the manager’s ability to acquire private information is itself private information. The creditors merely begin with a prior distribution with respect to the manager’s ability. The asymmetric information about the manager’s ability to acquire pre-decision information results in a learning process on the part of the creditors. Hence, it is necessary to consider multiple periods. We show that in the absence of voluntary disclosures managers may use suboptimal decisions to signal their type. If direct disclosures are credible, then they may provide a “less costly” signal to enable the manager to build his reputation.

The rest of this dissertation is organized as follows. Chapter 2 provides of a review of the current literature on voluntary disclosure. Chapter 3 contains the first essay, “Disclosure Choice in a Duopoly”, and chapter 4 contains the second essay, “Disclosure and Reputation in credit markets”. Each of these two chapters begins with a detailed introduction and ends with a conclusion to the relevant essay. Detailed references for all citations are in the bibliography. This is followed by all tables and figures referred to in the text. Appendix A contains the appendices to chapter 3 and the appendices to chapter 4 are in appendix B.
Chapter 2

Literature Review
This chapter classifies and relates the recent analytical research in the area of voluntary disclosure. Some relevant empirical studies are also discussed and summarized in the light of the results obtained in the analytical literature.

For our purpose, the analytical research in the general area of communication of private management information can be partitioned into two broad areas. The first consists of models dealing with direct disclosure and the second consists of signalling models. The key difference between the models in these two areas of research arises due to the difference in their assumption about the credibility of a disclosure. Direct disclosure models assume that disclosures are credible. This is often justified by assuming that disclosures can be verified ex post and that false disclosures are discouraged by the threat of exogeneous penalties. On the other hand, signalling models assume that a direct disclosure of the manager’s private information is not credible. In this case, therefore, the manager uses a surrogate signal which credibly communicates his private information. Credibility of the signal is ensured by assuming an inverse relationship between the cost of signalling and the manager’s information.

In general, signalling models consider two agents one of whom possesses private information about a commodity or service and is interested in increasing its price and the other whose belief influences the price. Models differ in their precise assumptions about the commodity, information asymmetry and the bargaining positions of agents. The results usually obtain the existence of a fully separating equilibrium.

Akerlof (1970) analyzes the influence of informational asymmetry on markets where buyers are imperfectly informed about the quality of the product. If informational asymmetry persists in such markets, then all high quality sellers withdraw leading to a market failure. Since the worst sellers remain, this kind of market has been termed as a “market for lemons”. Spence (1973) analyzes the same issue with the variation that sellers can communicate their information to the buyers. If there is an inverse relationship between the cost of signalling and the signal itself, then signalling leads to a fully separating equilibrium. The information asymmetry between buyer and seller is eliminated, and hence, the market does not fail. This signalling model has been followed by others that apply a similar concept to different settings and problems.

In contrast to signalling models, direct disclosure models assume that a direct announcement of the manager’s private information is credible. The earliest papers to model communication by direct disclosure are Milgrom (1981) and Grossman (1981). These papers consider a seller and a buyer of a commodity where the seller possesses private information about the commodity
to be traded. Disclosures are assumed to be truthful and costless. Both papers show that under rational expectations the seller uses a strategy of full-disclosure, which results in a complete separation of seller types in equilibrium. The unravelling argument is the same as the one used by Akerlof (1970), and hence, the result is parallel to the fully separating equilibrium obtained in signalling models. However, even casual empiricism indicates that the behavior of corporate managers is not consistent with full-disclosure.

Following these initial models of direct disclosure, Jovanovic (1982) showed that a partial disclosure equilibrium is obtained by relaxing the assumption of costless disclosure. The model examines the disclosure of the quality of an item. The question investigated is whether or not the free market provides sellers enough incentives to disclose information about the quality of their product. The imposition of an exogenous cost of disclosure results in a partial disclosure equilibrium with a single threshold level of disclosure.

Intuitively, a disclosure equilibrium can be prevented from unravelling if non-disclosure cannot be interpreted unequivocally. This arises if there exists at least two types of informational asymmetries in the event of non-disclosure, which do not affect the informed firm's incentives in the same direction.

The voluntary disclosure research in the accounting area focusses on the issue of partial disclosure, which is achieved by relaxing one of two of the assumptions made by Grossman and Hart. Some researchers relax the assumption of costless disclosure as done by Jovanovic and others relax the assumption that the firm (seller) is always informed. In the presence of a positive cost of disclosure, both firms with extremely unfavorable news and those with high cost of disclosure withhold their information. Non-disclosing firms are hence valued at an average of the above two types rather than at an extremely unfavourable level. Similarly, if there is a possibility that the firm may be uninformed and cannot credibly disclose that fact, then again non-disclosure could be due to unfavourable news or the lack of information. This leads to a firm valuation which is higher than the most unfavourable level, and hence, to a partial disclosure equilibrium.

In addition to the above difference in modelling, the models of voluntary disclosure differ in their specification of the manager's objective function under disclosure and non-disclosure. Under asymmetric information, the expected end of period value and the market value of the firm may be affected differently by disclosure. Hence, these two different objective functions may yield different disclosure policies in equilibrium.

The earliest models of discretionary disclosure in the accounting literature are due to Ver-
recchia (1983) and Dye (1985a). Both assume that the privately informed firm maximizes current market value. A firm’s current market value is monotonically increasing in the relevant information variable. Therefore, the informed firm is motivated to disclose favourable information. In Dye’s model full disclosure is prevented by a non-zero probability that the firm may be uninformed. In equilibrium, the single threshold level of disclosure is below the mean of the information variable. Dye shows that as the probability that the manager receives information increases, the threshold level of disclosure falls. As the probability of information approaches one, the equilibrium disclosure policy unravels to one of full disclosure. Verrecchia’s model, on the other hand, introduces a constant proprietary cost if the informed firm discloses its information. He also obtains a single threshold level above which the increase in market value due to disclosure is higher than the proprietary cost, and below which the cost is too high for disclosure to be worthwhile. Verrehhia also shows that increasing the proprietary cost leads to an increase in the manager’s incentive to withhold information, and hence, to an increase in the threshold level of disclosure.

Verrechhia (1990) examines the impact of information quality on disclosure policy in the same setting as Verrechhia (1983). The quality of the manager’s information about the uncertain liquidating value of a risky asset is represented by the precision of the posterior distribution of the signal given the asset value. Under the assumption that the market prices the risky asset at its expected value, the paper shows that the threshold level (probability) of disclosure decreases (increases) as the quality of information increases. The paper also finds that as the precision of the prior beliefs increases, the threshold level (probability) of disclosure increases (decreases). If the probability that the firm is informed as used in Dye (1985) and Jung and Kwon (1988), is interpreted as the quality of information, then their results are consistent with Verrechhia’s.

Dye (1985b) focusses on the relation between mandatory and voluntary disclosure. The informed firm is engaged in an entry game where its future earnings are reduced by a proprietary cost if entry occurs. The firm’s optimal disclosure policy depends on how proprietary cost can be minimized. Hence, more discretion in the choice of disclosure policy makes the firm better off. It is also not necessary that imposing more detailed reporting requirements makes investors better informed about the firm’s future earnings prospects. Since mandatory and voluntary disclosures may be substitutes, a requirement of a more detailed mandatory report may decrease the firm’s voluntary disclosure thus leading to an overall decrease in the “amount”

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1Dye’s analysis had a technical error that was subsequently corrected by Jung and Kwon (1988). This correction did not change the basic form of the equilibrium disclosure policy. However, it yielded a unique partial disclosure equilibria where there was a potential multiplicity.
of information released. In addition, a more detailed but uniform reporting requirement may remove the possibility of signalling through accounting techniques. The paper also provides conditions under which a more detailed mandatory reporting requirement increases the amount of information disseminated.

Dye (1986) analyzes disclosure policy choice when the manager is endowed with both proprietary and non-proprietary information. By definition disclosure of the proprietary information imposes a cost on the firm. In addition it is assumed that disclosure of the non-proprietary information alone may reveal something about the proprietary signal, and hence, impose a cost on the firm. Given this structure of disclosure cost, the main results are as follows. Non-proprietary information may be withheld and good news is more likely to be disclosed or partially disclosed than bad news. If the proprietary signal is below some threshold value then non-disclosure of each signal of the non-proprietary information is preferred to full disclosure. The opposite is true for proprietary signals below the threshold value. Intuitively, these results derive from the fact that disclosure of non-proprietary information has two counteracting effects. It may increase the investors' expectations of future earnings of the firm, however, it also imposes a cost by divulging proprietary information.

Several papers have analyzed models of discretionary disclosure in which the proprietary cost of disclosure is endogenously determined. Bhattacharya and Ritter (1983) consider a setting where the informed firm is engaged in an R&D race with uninformed opponents. The informed firm can raise funds at better terms by disclosing good news about its innovation prospects. This, however, results in a proprietary cost since it decreases its chances of obtaining a patent, thus decreasing the conjectured intrinsic value of the invention.

Dontoh (1990) considers an N-firm oligopoly in which one firm is privately informed about the industry-wide common demand parameter. Note that due to the fact that only one firm is informed, its opponents can be thought of as a single representative firm. Hence, the setting is like a duopoly where the opponent is bigger. The informed firm's competitors respond to disclosures about industry-wide common cost by changing their output levels. The competitors' actions affect the informed firm's profit level, imposing a proprietary cost. There are two types of information asymmetries in Dontoh's model. In addition to information asymmetry about demand, the informed firm's objective function, which could be its either its current market value (type A) or its expected future value (type B), is also private information. If the firm is of type A, it is motivated to disclose favourable information. If, on the other hand, it is a type B firm, it is motivated to disclose unfavourable information in order to induce its competitors
to reduce their output levels. Therefore, in the event of non-disclosure it cannot be ascertained whether the informed firm is of type A and a recipient of unfavourable information or it is of type B and has favourable information. This uncertainty sustains a partial disclosure equilibrium in which extremely favourable as well as extremely unfavourable information is withheld.

Darrough and Stoughton (1990) (D/S), Wagenhofer (1990) and Feltham and Xie (1988) (F/X), analyse models of discretionary disclosure in which possible entry of an opponent provides the necessary noise in the interpretation of non-disclosure. The informed firm is a monopolist that has to share its product market if another firm enters the market. Thus the incumbent's post entry expected profit function is much lower than that prior to entry. The entrant's decision is dependent on the incumbent's disclosure policy. Wagenhofer assumes that the incumbent is a current value maximizer whereas the D/S and F/X assume that the incumbent maximizes expected future value and raises funds in the capital market. D/S use binary support for their information variable and, hence, instead of partial disclosure they obtain a mixed strategy equilibrium. F/X, on the other hand, use continuous support and hence obtain more general results. Both Wagenhofer and F/X obtain partial disclosure equilibria with one pair of disjoint disclosure sets and a pair of disjoint non-disclosure sets, under similar conditions. Basically, in the event of non-disclosure the investors (creditors) are unable to discern whether the firm is a monopolist and is withholding extremely unfavorable information or entry has occurred and the firm's information is not that unfavorable.

Trueman (1986) differs in its objective, from all of the papers outlined above. The question asked is "Why do managers issue earnings forecasts?" rather than "What do manager's disclose?". The model analyses the motivation for voluntary disclosure in a two period setting. The timings of the manager's information receipt in the two periods are positively related and earlier information receipt translates to a higher expected profit. Hence, when the manager receives information in the first period conveys it information about the expected profit in the second period. The results indicate that in equilibrium it is optimal for managers to reveal their signal as soon as they receive it, in the first period, irrespective of its value. The neutrality to signal value is achieved by omitting the first period expected profit from the manager's objective function.

We next summarize some empirical studies in the area of voluntary disclosure. The earlier empirical research in this area preceded the analytical research summarized above and was motivated by the concerns of the SEC in the early and mid-1970s. In February 1973 the SEC announced its intention to adopt rules that would require a firm, that issued a forecast to
any outsider, to make a public filing of that forecast with the SEC. The SECs main concern was that all investors did not have equal access to projections and forecasts which reflected management private information. While it was not clear that equal access to information is an appropriate objective, there seems to be an implicit assumption that earnings forecasts are useful in investors' belief revision. Several empirical studies address this "information content" issue. These studies find that forecast disclosures are accompanied by significant stock price adjustments, and therefore, conclude that forecast disclosures do have information content. In addition to the above, the average stock price reaction was found to be positive. Penman offers one possible explanation for this observation: only firms with relatively good news disclose their information, and hence, stock prices of disclosing firms increase, on the average. Patell hypothesizes that the act of disclosure may itself convey positive information separate from the actual signal, and thus lead to a positive stock price reaction, on the average.

Following these earlier studies, Ajinkya and Gift (1984) and McNichols (1988) conduct tests with more recent data and fail to find significant evidence of positive average forecast deviation. McNichols offers an explanation based on the selection bias story, for the apparent discrepancy between the results obtained with earlier and more recent data. She suggests that the management forecast practices in more recent years may differ from the prior periods studied. The lower proportion of unfavorable forecasts in the earlier data may be due to higher costs of disclosing negative forecasts. One might argue that the Safe Harbour Rule of the SEC, instituted in 1979, decreases the liability exposure of managers issuing unfavorable forecasts, making it less costly. This could result in the observed difference between the results obtained with earlier and more recent data.

The positive average stock price reaction observed by the above empirical studies is consistent with the analytical results obtained by Verrechhia (1983) and Dye (1985). The partial disclosure results obtained by Verrechhia and Dye imply that good news will be disclosed and bad news withheld. In Verrechhia’s model the threshold level of disclosure depends on the magnitude of the proprietary cost and in Dye’s model it depends on the probability that the

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2Refer to Gonedes, Dopuch and Penman (1976)
5Differential liability exposure for favorable versus unfavorable forecasts is not unlikely. SEC's Rule 10-5b stipulates that in order to sue a firm for misleading disclosure, shareholders must prove that they suffered a monetary loss due to the disclosure. This is probably easier to do in the case of unfavorable forecasts which decrease stock price.
manager is uninformed.

Lev and Penman (1990) test a modified "screening" hypothesis as suggested by Verrechhia (1983). They test the two following hypotheses. The first hypothesis is: managers with above "average" valuation make disclosures and experience upward revisions in stock price. The second hypothesis is: managers with below "average" valuation withhold their information and their stock prices are revised downwards. Their results support the first hypothesis. However, they find that nonforecasting firms in the same industries as forecasting firms cannot be characterized as having particularly poor earnings. In fact, the effect on the nonforecasting firm's stock tends to be in the same direction as the forecasting firm. In addition, despite the fact that average forecast news is good news, firms with "bad news" earnings generally revealed that information by publishing forecasts that decreased stock price. Since their results are not consistent with all the major predictions of the screening models, the authors suggest that other analytical models be examined.

Pownall and Waymire (1986) test the hypothesis implied by Trueman's model and suggested by Patell to explain the positive average stock price reaction. Their method controls for the deviations of forecasts from expectations and then tests for positive stock price returns associated with the act of disclosure. They find that the empirical evidence fails to support the hypothesis.

It has been suggested that while firms with good news disclose their information, firms with bad news delay the release of their information. Pastena and Ronen (1979) empirically test the implications of the existence of a disincentive to produce and disseminate negative information. They find support for the hypotheses that: (a) managers seem to delay the dissemination of negative information, relative to positive information; (b) they disclose primarily "soft" positive information as contrasted with "soft" negative information; and (c) they disclose negative information essentially only after such information becomes hard. The hypothesis that firms with good news disclose early and firms with bad news delay their disclosures is not supported by evidence found by Lev and Penman, discussed earlier.

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6Note that the original screening models of Grossman and Hart and Milgrom imply full disclosure.
Chapter 3

Disclosure Choice in a Duopoly
3.1 Introduction

Frequently firms make public disclosures of their private information prior to the end of an accounting period. Such disclosures include estimates of market demand for the firm's output, cost of production, and forecasts of future earnings. As a rule, these public disclosures are discretionary in the sense that firms do not usually reveal all their information, disclosing and withholding both favourable and unfavourable information. This is referred to as partial disclosure in the voluntary disclosure literature.

The key requirement for a partial disclosure equilibrium is the existence of at least two types of informational asymmetries in the event of non-disclosure. The dimensions in which the informational asymmetries exist must not affect the informed firm's incentives to disclose in the same direction. Under such conditions non-disclosure cannot be interpreted as implying that the firm has the worst possible information. This prevents the disclosure equilibrium from unravelling to one of full-disclosure. Two such conditions have been identified and modelled in the current literature. Partial disclosure can be sustained, in equilibrium, when either:

- the firm may be either informed or uninformed with positive probability; or
- the information is proprietary, making disclosure costly.

Under the first condition, if the firm cannot credibly communicate its lack of information, then non-disclosure may be either due to unfavourable information or due to lack of information. Therefore, non-disclosure cannot drive the firm's objective function to its lowest possible value. Hence, some types prefer non-disclosure. Under the second condition, disclosing firms incur a proprietary cost. In this case, non-disclosure may either be due to the fact that information is extremely unfavourable or just not favourable enough to justify the cost of disclosure. Hence, some types prefer to withhold their information. The proprietary cost is used to sustain a partial disclosure equilibrium and is not related to the credibility of the disclosure. Hence, unlike signalling costs, the only property required of the proprietary cost is that it be higher than the gains from disclosure for some signal value other than the lowest one.

The current paper assumes that the firm under consideration may be informed or uninformed and if uninformed cannot communicate this fact. In an earlier paper, Dye (1985a) analyses the question of partial disclosure under the same assumption. In his model the manager of a firm maximizes the firm's current market value. Prior to the end of the period the manager receives private information about the end of period cash flow. If the private information is favourable,
the manager is motivated to disclose it in order to increase the firm's market value. However, if
the information is unfavourable, the manager prefers to withhold it. Since there is a possibility
that the manager is uninformed and unable to convey that, a non-disclosing firm could either
be in possession of unfavourable information or be uninformed. Hence, investors value the non-
disclosing firm at an average of these two, thus sustaining a partial disclosure equilibrium. Dye
finds that the disclosure region expands if the probability of the disclosing firm being informed
is increased. A related paper by Jung and Kwon (1988) corrects a technical error in Dye's
analysis and presents further comparative statics results.

Verrechhia (1983) adopts the assumption that disclosure is costly, thus obtaining partial
disclosure in equilibrium. Several later papers explicitly model the the proprietary cost of
disclosure. Typically, this is achieved through an assumption of imperfect competition. The
disclosing firm is engaged in a production choice or an entry game and receives private informa-
tion about industry-wide common parameters such as industry demand and common marginal
cost of production. Hence, in the oligopoly and entry setting, information that has favourable
implications for the informed firm's cash flows is also favourable from the existing or potential
competitors' point of view. In the oligopoly setting, therefore, favourable information induces
the competitors to increase their output levels, driving down output price and the informed
firm's profits. In the entry game setting, favourable information increases the probability of
entry, thus increasing the probability that the informed firm has to share its profits. Clearly,
if we ignore the capital market in the two above settings, the informed firm is motivated to
withhold its favourable information and disclose only its unfavourable information.

Intuitively, if we consider information about firm-specific factors about the informed firm, we
would expect the incentives for disclosure to be reversed. Favourable information about factors
such as firm-specific marginal cost places the informed firm in a better competitive position,
with respect to its competitors. Hence, such information would induce the competitors to
reduce their output levels, raising output price and the informed firm's profits. Therefore, the
informed firm is motivated to disclose (withhold) its favourable (unfavourable) firm-specific
information.

Both types of information have been considered by authors in the industrial organization
literature. They deal with the issue of information sharing among rival firms, who maximize
their expected future value in an oligopolistic product market setting. In their approach,
however, firms commit to the sharing of information and hence cannot make their disclosure
decisions after observing their signals. Information sharing is modelled as a two stage game.
In the second stage, firms receive and share their information as well as choose their optimal output levels. It is in the first stage, however, that firms decide whether or not to share their information. Firms share all or none of their information. In some of the models in this area, partial sharing is achieved by adding a noise term to all signals prior to transmission.\(^1\) Note that this, in a sense, violates our assumption of truthful disclosure.

In general, the models in this area find that in a Cournot equilibrium, firms find it optimal to share all their information if it is about firm-specific factors.\(^2\) On the other hand, firms will agree not to share any of their information about industry-wide common factors.\(^3\) Since the decision about information sharing is made prior to its receipt, rival firms choose between full-disclosure and non-disclosure. The possibility of making a firm’s disclosure policy a function of its information signal is not considered. Hence, the results in these models cannot be compared to ours.

The present paper deals with discretionary disclosure in a duopoly where only one firm may be informed. We examine the informed firm’s disclosure decision process when the information variable is related to both its firm-specific marginal cost as well as the industry-wide common marginal cost. A firm-specific cost, for instance, arises due to idiosyncratic variations in the performance of a firm’s production machinery and personnel or in the input cost charged by its particular supplier. An industry-wide common cost, on the other hand, is governed by common factors in the production process, for instance, the price of common inputs. An estimate of the total cost is a signal that is related to both types of cost. We limit our attention to industry-wide marginal cost, rather than the more commonly used industry demand, to allow easier comparisons between the effects of industry-wide versus firm-specific factors.

We focus on the conflicting incentives for disclosure that arise due to the information signal’s "informativeness" about the two different types of cost, under imperfect competition. The main objective of this paper is to determine the equilibrium disclosure policy and to clearly delineate the impact of the signal’s informativeness with respect to each type of cost. We also examine the changes in the informed firm’s disclosure policy and probability of disclosure due to changes in the values of several key parameters.

It is important to distinguish between the signal’s informativeness with respect to firm-specific versus industry-wide common uncertainty. In an imperfectly competitive industry the disclosure policies in the two cases are expected to differ, thus giving rise to different empirical

\(^1\) Gal-Or (1985), Li (1985)
\(^2\) Li (1985), Gal-Or (1985), Shapiro (1986)
implications. Hence, this issue should be of concern in the design of empirical tests of hypothesis about discretionary and partial disclosure. The following major assumptions are made:

- We deal with a standard duopoly model under uncertainty, with a linear inverse demand function. Firms maximize their expected end of period payoffs.

- One firm may be informed with a certain probability and the other firm is uninformed.

- The information variable is jointly normal with both kinds of marginal costs. The joint distribution function is common knowledge.

- After receiving its information, the informed firm chooses its disclosure policy and implements it. This is followed by production choice by both firms.

The results depend on whether the information variable is more informative about firm-specific marginal cost (Case 1) or common marginal cost (Case 2). It is shown that in case 1, favourable information along with extremely unfavourable information is disclosed and "below average" information is withheld. Whereas in case 2, the informed firm withholds both extremely favourable and extremely unfavourable information and discloses "average" information. The basic result is robust with respect to the number of uninformed firms in the industry.

The equilibrium disclosure sets and the probability of disclosure are sensitive to the probability of the disclosing firm being informed. As this probability of being informed increases, the amount of disclosure increases in both cases 1 and 2. The probability of disclosure is also increasing in case 1, but may be either increasing or decreasing in case 2. We also find that if the firm is informed with certainty, then full disclosure results in case 1. However, in case 2, partial disclosure continues to be the equilibrium disclosure policy.

The degree of competition faced by the informed firm also affects its disclosure policy. Increasing competition always encourages (discourages) voluntary disclosure in case 1 (case 2). The probability of disclosure, on the other hand, may be either increasing or decreasing with increasing competition in case 1, but is decreasing in case 2.

The results in this paper suggest that in designing empirical tests a distinction should be made between firm-specific and industry-wide common information. If the signal is related to both types of uncertainty, the sample of disclosing firm's should be partitioned according to whether the signal is more informative about firm-specific (case 1) or industry-wide common (case 2) uncertainty.
In case 1 we would expect to find a predominance of “good” signals in disclosures and in case 2 a predominance of “bad” signals. In terms of stock price behavior at the time of disclosure, in case 1 the average abnormal stock return of disclosing firms is expected to be positive. However, in case 2 the average abnormal stock return of disclosing firms is expected to be negative. The non-disclosing firms are expected to have negative average abnormal returns in case 1 and positive average abnormal returns in case 2.

If the probability of the receipt of information is time dependent and the probability of receiving information increases over time, then in case 1 (case 2) better (worse) news is disclosed earlier with worse (better) news being disclosed later. The above empirical implications indicate that failure to partition a sample of disclosing or non-disclosing firms may confound the empirical evidence.

The paper is organized as follows. Section 2 contains a discussion of the incentives in a duopoly under uncertainty. Both homogeneous and heterogeneous beliefs are considered. Section 3 contains the main assumptions, and the optimal choices under asymmetric information. We present the equilibrium analysis in section 4 and comparative statics in section 5. Section 6 discusses some empirical implications of the analytical results and section 7 concludes the paper.
3.2 Duopoly Under Uncertainty

The basic model considers a duopoly setting with the two firms denoted $i$ and $j$. Each firm faces a constant marginal cost of production which is the sum of an uncertain firm-specific component $\tilde{c}_k, k = i, j$ and an uncertain industry-wide component $\tilde{c}$. A firm's profit function is given by

$$\pi_k = (P - \tilde{c} - \tilde{c}_k)y_k, \text{ for } k = i, j$$

where, $P = a - by$ is the inverse demand function for the industry. $P$ is the price per unit of output, $y_k$ is the quantity of output produced by firm $k$, $y = y_i + y_j$ is the total industry output, and $a$ and $b$ are constants.

The probability density function over $C_k$, the domain of $c_k$, is $h(c_k)$ and over $C$, the domain of $c$, it is $g(c)$. If the realizations of $\tilde{c}_k$ and $\tilde{c}$ are not known with certainty, the two firms use their respective expectations of their marginal costs to obtain their expected profits.

This section examines the equilibrium production choices that will be made by the two firms under two generic types of decision settings: symmetric and asymmetric information. The analysis here does not consider the specifics of the process by which these information structures arise. It merely provides a general characterization of the production choices in these two types of settings. In the case of asymmetric information we also demonstrate how incentives for communication information may emerge.

3.2.1 Symmetric Information

Here we assume that neither firm receives private information about the marginal costs $\tilde{c}_k$ and $\tilde{c}, k = i, j$. This is equivalent to the assumption that if any information is received then the signal is common knowledge. This leads to homogeneous beliefs and both firms have the same expectations of the marginal costs. We refer to these common expectations as prior expectations and denote them by $\tilde{c}_i, \tilde{c}_j$ and $\tilde{c}$.

The equilibrium output choices in this model is the same as in the standard duopoly model in a deterministic setting. The Nash Equilibrium outputs are found as follows.

Firm $i$ solves:

$$\max_{y_i} E(\pi_i) = [a - \tilde{c} - \tilde{c}_i - b(y_i + y_j^*)]y_i$$

Firm $j$ solves:

$$\max_{y_j} E(\pi_j) = [a - \tilde{c} - \tilde{c}_j - b(y_i^* + y_j)]y_j$$

19
After substituting $y_i^\ast$ and $y_j^\ast$, the following first order conditions are obtained:

\[ \text{foc (3.1)} : a - \bar{c} - \bar{c}_i - 2by_i^\ast - by_j^\ast = 0 \]
\[ \text{foc (3.2)} : a - \bar{c} - \bar{c}_j - 2by_i^\ast - by_j^\ast = 0 \]

It can be shown that the second order conditions are satisfied. The equilibrium output levels and expected profits are:

\[ y_k^\ast = \frac{a - \bar{c} - 2\bar{c}_k + \bar{c}_l}{3b} \]
\[ E(\pi_k^\ast) = \frac{(a - \bar{c} - 2\bar{c}_k + \bar{c}_l)^2}{9b} \]

where, \( k = i, j \) and \( l = i, j \) and \( l \neq k \).

The industry-wide component, \( \bar{c} \), and the firm-specific component, \( \bar{c}_i \), of firm \( i \)'s marginal cost both have a negative effect on the firm's optimal output level. The decrease in output due to an increase in the firm-specific component is twice as much as in the case of the industry-wide component. This is due to the fact that an increase in the industry-wide component also decreases the competitor's output level, such that firm \( i \) does not need to lower its output by as much.

The industry-wide component of marginal cost, \( \bar{c} \), and the firm-specific component of the competing firm, \( \bar{c}_j \), have opposite effects on optimal production level, \( y_i^\ast \). Although an increase in \( \bar{c}_j \) has no direct effect on firm \( i \)'s marginal cost, it results in a decrease in firm \( j \)'s production level, allowing firm \( i \) to increase its output.

### 3.2.2 Asymmetric Information

This section considers the above duopoly in a setting where firm \( i \) receives private information about its own firm specific marginal cost and/or the common marginal cost. We assume that the probability distributions of firm \( i \)'s posterior expectations are common knowledge as is the fact that firm \( i \) receives private information whereas firm \( j \) does not.

Due to the asymmetric information, firms form heterogeneous beliefs. This results in a different set of expected marginal costs for each firm. We refer to firm \( i \)'s expectations as \( E_i(c_i), E_i(c_j) \) and to firm \( j \)'s expectations as \( E_j(c_i), E_j(c_j) \) and \( E_j(c) \). Note that since neither firm receives any information about \( \bar{c}_j \), their expectation of \( \bar{c}_j \) is the same as the prior expectation, i.e., \( E_i(c_j) = E_j(c_j) = \bar{c}_j \). In addition, is this case we also refer to the
uninformed firm's expectations as prior expectations and to the informed firm's expectations as posterior expectations.

Firms compute their expected profit functions based on their respective expectations of marginal costs. The Nash equilibrium output levels are determined by the simultaneous maximization of each firm's expected profits with respect to its output level. Since firm i has private information whereas firm j does not, firm i knows firm j's maximization problem and hence knows \( y_i^* \). However, firm j does not know firm i's maximization problem and hence uses a conjecture, \( \hat{y}_i^* \), for firm i's optimal output level. This is determined by firm j maximizing its own expectation of firm i's profit. The maximization problems and their solutions are presented below:

Firm i solves:

\[
\max_{y_i} E_i(\pi_i) = \left[ a - E_i(c_i) - E_i(c_i) - b(y_i + y_j^*) \right] y_i
\] (3.3)

Firm j solves:

\[
\max_{y_j} E_j(\pi_j) = \left[ a - E_j(c_j) - \hat{c}_j - b(\hat{y}_i^* + y_j) \right] y_j \] (3.4)

\[
\max_{\hat{y}_i} E_j(\pi_i) = \left[ a - E_j(c_i) - E_j(c_i) - b(\hat{y}_i + y_j^*) \right] \hat{y}_i \] (3.5)

where \( \hat{y}_i^* \) is firm j's expectation of firm i's optimal output. \( \hat{y}_i^* \) will differ from firm i's actual optimal output, \( y_i^* \), which is based on firm i's private information. In order to determine its own optimal output, firm j uses its estimate of firm i's optimal output, which is found by maximization of j's expectation of i's profit, as in problem (3.5). Maximization of j's expectation of i's profit gives us j's expectation of i's optimal output because of the linearity of the first order conditions. Maximization problems (3.4) and (3.5) are solved to obtain \( y_j^* \) which is then used to solve problem (3.3), to obtain \( y_i^* \). The first order conditions are:

\[
foc (3.3) : a - E_i(c) - E_i(c_i) - 2by_i^* - by_j^* = 0
\]

\[
foc (3.4) : a - E_j(c) - \hat{c}_j - 2by_j^* - by_i^* = 0
\]

\[
foc (3.5) : a - E_j(c) - E_j(c_i) - 2by_i^* - by_j^* = 0
\]

These give the following optimal output levels and expected profits:

\[
y_i^* = \frac{a + \hat{c}_j - E_i(c) - 2E_i(c_i) + \frac{E_i(c) - E_i(c_i)}{2} - \frac{E_i(c_i) - E_i(c_i)}{2}}{3b}
\]

\[
E(\pi_i^*) = \frac{(a + \hat{c}_j - E_i(c) - 2E_i(c_i) + \frac{E_i(c) - E_i(c_i)}{2} - \frac{E_i(c_i) - E_i(c_i)}{2})^2}{9b}
\] (3.6)
From the above expressions it is evident that firm i has definite incentives to sometimes communicate its information to firm j, in order to change firm j's beliefs. This is clearly demonstrated in (3.6). If \([E_j(c) - E_i(c)]\) is increased and/or \([E_j(c_i) - E_i(c_i)]\) is decreased then firm i's optimal expected profit increases. By communicating its information, firm i can equalize \(E_j\) and \(E_i\). Hence if the prior belief about industry-wide common cost, \(E_j(c)\), is lower than the posterior belief, \(E_i(c)\), or if the prior belief about firm-specific cost, \(E_j(c_i)\), is higher than the posterior, \(E_i(c_i)\), then firm i has an incentive to communicate its information to induce firm j to revise its beliefs and reduce its output.
3.3 Duopoly Under Asymmetric Information

3.3.1 The Model

In this section we specifically model an information structure that gives rise to information asymmetry in the basic duopoly. Firm $i$ may or may not be informed ($I/U$) and the probability of firm $i$ being informed is $q$. If informed, firm $i$ receives a signal $s$. Firm $j$ is uninformed. For instant, firm $i$ may be a larger firm with more resources that it can apply to its research department which can sometimes obtain new information whereas firm $j$ does not employ any resources to obtain new information. In order to preserve informational asymmetry in the event of non-disclosure by the informed firm, we assume that production quantities are not observable.

As in the previous section, each firm faces a constant marginal cost of production which is the sum of an uncertain firm specific component $\bar{c}_k, k = i,j$ and an uncertain industry-wide component $\tilde{c}$. A firm’s profit function is given by

$$\pi_k = (P - \bar{c} - \tilde{c}_k)y_k, \text{ for } k = i,j$$

where, $P = a - by$ is the inverse demand function for the industry. $P$ is the price per unit of output, $y_k$ is the quantity of output produced by firm $k$, $y = y_i + y_j$ is the total industry output and $a$ and $b$ are constants.

Throughout most of our analysis we assume that output levels are unconstrained. Hence, the output levels, $y_i$ and $y_j$, could be negative. In order to conceptualize negative production, it is useful to think of the firm as a producer of at least two different products. The product in question may be sold to outside consumers as well as used internally as an input in the production of another product. The variable $y_k$, then, represents firm $k$’s net output. If the price offered by outside consumers is high then the optimal net output, $y_k$, is positive. On the other hand, if the output price is low, the optimal net output could become negative. Since we permit output levels to be negative, there is a possibility that the total industry output is negative. This possibility need not be ruled out to obtain the results in our model. However, a market equilibrium cannot be reached if there is net demand for the product from both the producers and the consumers of the product. Negative total output may be ruled out by assuming that it is a zero probability event (or we could view “consumers” as suppliers who satisfy the demand created by the producers). In a later section we examine how our model and results are affected by constraining output to be non-negative.
The random variables $s$, $c_i$, and $c$ are jointly distributed with density function $f(s, c_i, c)$ over $S \times C \times C$. It is assumed that the above joint density function is common knowledge. The following notation is used for the marginal density functions:

$$
\int_{c_i \in C} f(s, c_i, c) dc_i = g(s, c)
$$

$$
\int_{c \in C} f(s, c_i, c) dc = h(s, c_i)
$$

$$
\int_{s \in S} f(s, c_i, c) ds = f(c_i, c)
$$

$$
\int_{c_i \in C} h(s, c_i) dc_i = \int_{c \in C} g(s, c) dc = f(s)
$$

$$
\int_{s \in S} h(s, c_i) ds = h(c_i)
$$

$$
\int_{s \in S} g(s, c) ds = g(c)
$$

It is assumed that the random variables, $c_i$ and $c$ are independent, i.e., $f(c_i, c) = h(c_i).g(c)$. Firm $i$'s information signal $s$ can be thought of as a measure of its total cost. Thus $s = c_i + c$ or $s = c_i + c + \epsilon_i$, where $\epsilon_i$ is an error term. At this juncture it should be noted that we could have considered firm $i$'s demand parameter $a$ to be the sum of a firm-specific and an industry-wide part and let $s$ be a measure of $a$ instead of total marginal cost. This would not have made any qualitative difference to our results.

The connection between firm $i$'s information variable $s$ and its profit function is established through the joint density functions, $h(s, c_i)$ and $g(s, c)$, which are used to determine the expected marginal costs. Each firm maximizes its expected end of period profit, given its information. This objective function is justified by assuming that equity holders retain ownership of the firm until the end of the period. As demonstrated by Feltham and Christensen (1988), well-diversified investors in a large economy can achieve an efficient allocation of resources and consumption without knowing each manager's firm-specific information as long as the manager of each firm, in an investor's portfolio, acts so as to maximize the "true value" of investors' equity.

In such a setting the firm's market value is of relevance only if it has an impact on the true value of the investor's equity. For instance, if the firm needs to raise new capital during the period, its market valuation determines the cost of issuing new debt or equity, which impacts on the firm's cash flow. This is ruled out by assuming that the firm does not raise any new capital during the period under consideration.
Hence in a model with private information and possible non-disclosure, if investors are well-diversified, end of period value maximization is an appropriate objective. The fact that investors are well-diversified ensures that they behave in a risk neutral manner towards firm specific risk. We abstract from any agency issues by assuming that the managers are motivated to act optimally on the diversified investors' behalf and all insiders are precluded from trading for their individual benefit.

Given the above characteristics of the investors and no new capital, the current market value of the firm's equity has no impact on the firm's disclosure decision or its production choice. The sole purpose of a disclosure by an informed firm is to influence its competitor's production decision. Firms are assumed to be sequentially rational. The order of events are as follows:

I firm i learns whether it is informed (I) or uninformed (U). If informed, firm i receives its information signal s.

II firm i sends a message m, where

\[
m = \begin{cases} 
  s & \text{a truthful disclosure} \\
  \emptyset & \text{non-disclosure}
\end{cases}
\]

III Both firms make their output choices, given their information.

**Time Line:**

<table>
<thead>
<tr>
<th>(t_0)</th>
<th>(t_1)</th>
<th>(t_2)</th>
<th>(t_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>firm i receives</td>
<td>Stage 1</td>
<td>Stage 2</td>
<td>Cash flows</td>
</tr>
<tr>
<td>s or no signal</td>
<td>firm i sends</td>
<td>Each firm</td>
<td>are</td>
</tr>
<tr>
<td>chose (m = s)</td>
<td>message (m = s)</td>
<td>chooses its</td>
<td>realized</td>
</tr>
<tr>
<td>or no message</td>
<td>optimal output</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The informed firm first makes its disclosure decision and then its production decision. The uninformed firm revises its beliefs on the basis of the disclosure and then makes its production decision. A sequential Nash equilibrium is found by backward induction, where the production decision problems are solved first followed by disclosure choice.
In this model it is assumed that if firm $i$ is uninformed it is unable to credibly convey its uninformed status and hence has to resort to non-disclosure. Further, it is possible to detect false disclosures and to impose sufficiently large penalties on firms found making false disclosures. This ensures that disclosures are always truthful.\footnote{This is a standard assumption in the voluntary disclosure literature.} For example, if disclosures are accompanied by an announcement of the source of that information, then false disclosures can be detected and penalized \textit{ex post}. In this setting, any disclosure not accompanied by an announcement of source, lacks credibility. This prevents uninformed firms from making any credible disclosures. Empirical tests of the information content of voluntary disclosures indicate that positive (negative) disclosures are accompanied by positive (negative) stock price reactions. Therefore, empirical evidence indicates that investors revise their beliefs based on voluntary disclosures which implies that voluntary disclosures are credible.

### 3.3.2 Output Choice

The output choice game at $t_2$ is considered first. Firms maximize expected end of period profits given their information. Hence, the optimal output levels depend in part on whether firm $i$ receives information and if so, on the value of $s$ observed by firm $i$ at $t_0$. The output decisions also depend on how firm $j$ revises its expectations on observing a particular value of $m$, at $t_1$.

Hence, there are three different possible situations in which optimal outputs are chosen:

1. firm $i$ receives signal $s$ and discloses $m = s$.
2. firm $i$ receives signal $s$ but does not disclose it, $m = \emptyset$.
3. firm $i$ does not receive any information and cannot make a disclosure, $m = \emptyset$.

The output choice problem is solved and the optimal output, $y^*_k$, and expected profit, $E(\pi^*_k)$, are found for each firm, in each case.\footnote{The algebraic solutions are in Appendix A.1.1.} The notation described below is used in subsequent analysis.

\begin{align*}
E(c_i|s) & = \text{Expected firm-specific marginal cost, given signal } s. \\
E(c|m = \emptyset) & = \text{firm } j's \text{ expectation of firm } i's \text{ firm-specific marginal cost, given non-disclosure.} \\
E(c|m = \emptyset) & = \text{firm } j's \text{ expectation of industry-wide marginal cost, given non-disclosure.}
\end{align*}
non-disclosure.

\[ \bar{c}_j = \text{firm } j\text{'s expected firm-specific marginal-cost.} \]

\[ \bar{c}_i = \text{firm } i\text{'s expected firm-specific marginal-cost, with expectation based on prior beliefs.} \]

\[ \bar{c} = \text{Expected industry-wide marginal-cost, with expectation based on prior beliefs.} \]

1. **Firm i is informed (I), receives signal } s \text{ and discloses } m = s.

\[
y_i^s(m = s) = \frac{1}{3b} \{a - E(c|s) - 2\bar{c}_j + E(c_i|m = s)\}
\]

\[
y_i^s(s, m = s) = \frac{1}{3b} \{a - E(c|s) - 2E(c_i|m = s) + \bar{c}_j\}
\]

\[
E(\pi_i^s|m = s) = \frac{1}{9b} \{a - E(c|s) - 2\bar{c}_j + E(c_i|m = s)\}^2
\]

\[
E(\pi_i^s|s, m = s) = \frac{1}{9b} \{a - E(c|s) - 2E(c_i|m = s) + \bar{c}_j\}^2
\]

2. **Firm i is informed (I), receives signal } s \text{ but does not disclose its information, } m = \emptyset.

\[
y_i^s(m = \emptyset) = \frac{1}{3b} \{a - 2\bar{c}_j - E(c|m = \emptyset) + E(c_i|m = \emptyset)\}
\]

\[
y_i^s(s, m = \emptyset) = \frac{1}{3b} \{a - 2E(c_i|s) + \bar{c}_j - E(c|s) + \frac{1}{2}[E(c_i|s) - E(c_i|m = \emptyset)] - \frac{1}{2}[E(c|s) - E(c|m = \emptyset)]\}
\]

\[
E(\pi_i^s|m = \emptyset) = \frac{1}{9b} \{a - 2\bar{c}_j - E(c|m = \emptyset) + E(c_i|m = \emptyset)\}^2
\]

\[
E(\pi_i^s|s, m = \emptyset) = \frac{1}{9b} \{a - 2E(c_i|s) + \bar{c}_j - E(c|s) + \frac{1}{2}[E(c_i|s) - E(c_i|m = \emptyset)] - \frac{1}{2}[E(c|s) - E(c|m = \emptyset)]\}^2
\]

3. **Firm i is uninformed (U) and does not make a disclosure, } m = \emptyset.

\[
y_i^s(m = \emptyset) = \frac{1}{3b} \{a - 2\bar{c}_j - E(c|m = \emptyset) + E(c_i|m = \emptyset)\}
\]

\[
y_i^s(m = \emptyset) = \frac{1}{3b} \{a - 2\bar{c}_i + \bar{c}_j - \bar{c} + \frac{1}{2}[\bar{c}_i - E(c_i|m = \emptyset)] - \frac{1}{2}[\bar{c} - E(c|m = \emptyset)]\}
\]
\[ E(\pi^*_i|m = 0) = \frac{1}{9b} \{ a - 2\tilde{c}_j - E(c|m = \emptyset) + E(c_i|m = \emptyset) \}^2 \]

\[ E(\pi^*_i|m = 0) = \frac{1}{9b} \{ a - 2\tilde{c}_i + \tilde{c}_j - \tilde{c} + \frac{1}{2}[\tilde{c}_i - E(c_i|m = \emptyset)] \]

\[ -\frac{1}{2}[\tilde{c} - E(c|m = \emptyset)] \}^2 \]

### 3.3.3 Disclosure Choice

At \( t_1 \), firm \( i \) makes its disclosure choice. If firm \( i \) is uninformed it cannot release any signal. However, if firm \( i \) is informed, it may choose to either disclose or not disclose its information. Since firms are sequentially rational, the disclosure choice is made by firm \( i \) under the assumption that optimal output levels will be chosen given its disclosure choice. Hence, \( i \)'s disclosure choice is made by maximizing its expected profit, \( E(\pi^*_i|m, s) \).

Non-Disclosure is preferred to Disclosure if :

\[ E(\pi^*_i|s, m = \emptyset) > E(\pi^*_i|s, m = s) \]

Disclosure is preferred if :

\[ E(\pi^*_i|s, m = \emptyset) < E(\pi^*_i|s, m = s) \]

The non-disclosure/disclosure sets are bounded by the solutions to :

\[ E(\pi^*_i|s, m = \emptyset) = E(\pi^*_i|s, m = s) \quad (3.12) \]

Using expressions (3.8) and (3.11), Equation (3.12) can be rearranged and expanded to yield :

\[ \frac{1}{9b} \{ a - 2E(c_i|s) + \tilde{c}_j - E(c|s) \]  

\[ + \frac{1}{2}[E(c_i|s) - E(c_i|m = \emptyset)] - \frac{1}{2}[E(c|s) - E(c|m = \emptyset)] \}^2 \]

\[ -\frac{1}{9b} \{ a - E(c|s) - 2E(c_i|s) + \tilde{c}_j \}^2 = 0 \]

\[ \Rightarrow \frac{1}{9b} \{ 2a - 4E(c_i|s) + 2\tilde{c}_j - 2E(c|s) \]  

\[ + \frac{1}{2}[E(c_i|s) - E(c_i|m = \emptyset)] - \frac{1}{2}[E(c|s) - E(c|m = \emptyset)] \} \]

\[ \times \{ \frac{1}{2}[E(c_i|s) - E(c_i|m = \emptyset)] - \frac{1}{2}[E(c|s) - E(c|m = \emptyset)] \} = 0 \]

\[ \Rightarrow \{ 12b\gamma_i(s, m = s) + [E(c_i|s) - E(c_i|m = \emptyset)] - [E(c|s) - E(c|m = \emptyset)] \]  

\[ \times \{ [E(c_i|s) - E(c_i|m = \emptyset)] - [E(c|s) - E(c|m = \emptyset)] \} = 0 \quad (3.13) \]
From the above, we can delineate the disclosure and non-disclosure sets as follows,

**Non-Disclosure is Optimal if:**

\[
(i) \left\{ \begin{array}{l}
12by_t^*(s, m = s) + [E(c_i|s) - E(c|s)] < [E(c_i|m = \emptyset) - E(c|m = \emptyset)] \\
\quad \text{and } [E(c_i|s) - E(c|s)] < [E(c_i|m = \emptyset) - E(c|m = \emptyset)] \\
\end{array} \right.
\]

**OR**

\[
(ii) \left\{ \begin{array}{l}
12by_t^*(s, m = s) + [E(c_i|s) - E(c|s)] < [E(c_i|m = \emptyset) - E(c|m = \emptyset)] \\
\quad \text{and } [E(c_i|s) - E(c|s)] < [E(c_i|m = \emptyset) - E(c|m = \emptyset)] \\
\end{array} \right.
\]

**Disclosure is Optimal if:**

\[
(iii) \left\{ \begin{array}{l}
12by_t^*(s, m = s) + [E(c_i|s) - E(c|s)] > [E(c_i|m = \emptyset) - E(c|m = \emptyset)] \\
\quad \text{and } [E(c_i|s) - E(c|s)] > [E(c_i|m = \emptyset) - E(c|m = \emptyset)] \\
\end{array} \right.
\]

**OR**

\[
(iv) \left\{ \begin{array}{l}
12by_t^*(s, m = s) + [E(c_i|s) - E(c|s)] > [E(c_i|m = \emptyset) - E(c|m = \emptyset)] \\
\quad \text{and } [E(c_i|s) - E(c|s)] > [E(c_i|m = \emptyset) - E(c|m = \emptyset)] \\
\end{array} \right.
\]

**Indifference occurs if:**

\[
12by_t^*(s, m = s) + [E(c_i|s) - E(c|s)] = [E(c_i|m = \emptyset) - E(c|m = \emptyset)] \\
\text{or } [E(c_i|s) - E(c|s)] = [E(c_i|m = \emptyset) - E(c|m = \emptyset)]
\]
3.4 Equilibrium Analysis

3.4.1 Belief Revision

The equilibrium concept employed here is that of a sequential equilibrium.\(^6\) In order to characterize a disclosure equilibrium, expressions for firm \(j\)'s conditional expectations of \(c_i\) and \(c\) under non-disclosure are required. In a sequential equilibrium, the posterior beliefs for these conditional expectations must be derived by Bayes' theorem.

In addition to being influenced by its prior beliefs, firm \(j\)'s posterior beliefs are also affected by firm \(j\)'s conjecture about firm \(i\)'s information disclosure policy. Firm \(j\) is assumed to have the following conjectures about firm \(i\)'s disclosure policy:

1. If firm \(i\) is informed (I) then there are values of \(s\) for which firm \(i\) does not disclose its information. The set of these non-disclosure values of \(s\) is denoted by \(N\).

2. If firm \(i\) is informed (I) and \(s \notin N\) firm \(i\) will disclose its information truthfully. This set of disclosure values of \(s\) is denoted \(D\).

3. If firm \(i\) is uninformed (U) then non-disclosure is the only option open to firm \(i\).

Using the above conjectures and the assumptions about the prior density function of \(s\), firm \(i\)'s posterior beliefs may be calculated as follows.

*Posteriors when \(m = s\):*

\[
\begin{align*}
P(s = x|m = z) &= 1 \\
P(c_i = x|m = z) &= \frac{h(c_i = x, s = z)}{f(s = z)} = h(c_i = x|s = z) \\
P(c = x|m = z) &= \frac{g(c = x, s = z)}{f(s = z)} = g(c = x|s = z)
\end{align*}
\]

*Posteriors when \(m = \emptyset\):*

From firm \(j\)'s conjectures,

\[
P(m = \emptyset|s \in N) = 1
\]

\(^6\)Due to Kreps and Wilson (1982).
\[ P(m = 0|s \in D) = 0 \]
\[ P(m = 0|U) = 1 \]

From firm j's Prior beliefs,
\[ P(I) = q \text{ and } P(U) = (1 - q) \]

Using the above and the density function for \( s \), defined before, we get the following joint probabilities,
\[ P(m = 0 \cap I) = q \int_{s \in N} f(s)ds \]
\[ = qF(s \in N) \]
\[ P(m = 0 \cap U) = (1 - q) \]

This gives the probability of non-disclosure as
\[ P(m = 0) = q \int_{s \in N} f(s)ds + (1 - q) \]

(3.14)

**Posterior Probabilities and Conditional Density Functions, when m = 0**:

Using Bayes' Rule,
\[ P(I|m = 0) = \frac{qF(s \in N)}{qF(s \in N) + (1 - q)} \]
\[ = p \] (3.15)
\[ P(U|m = 0) = \frac{(1 - q)}{qF(s \in N) + (1 - q)} \]
\[ = (1 - p) \] (3.16)

\[ P(c_i|m = 0) = \frac{P(c_i, m = 0)}{P(m = 0)} \]
\[ = \frac{q \int_{s \in N} h(s, c_i)ds + (1 - q)h(c_i)}{qF(s \in N) + (1 - q)} \]

\[ P(c|m = 0) = \frac{P(c, m = 0)}{P(m = 0)} \]
\[ = \frac{q \int_{s \in N} g(s, c)ds + (1 - q)g(c)}{qF(s \in N) + (1 - q)} \]

3.4.2 Disclosure Equilibrium

In a sequential equilibrium the posterior beliefs must satisfy the Bayes' consistency requirement. Hence, the conditional expectations of marginal costs, used by firm i to calculate expected profit.
must satisfy the following.

**Conditional Expectations given s**:  
\[
E(c_i|s) = \int_{c_i \in C_i} c_i h(c_i|s) dc_i \\
E(c|s) = \int_{c \in C} c g(c|s) dc
\]

**Conditional Expectations of firm j given non-disclosure by firm i**:  
\[
E(c_i|m = 0) = \int_{c_i} c_i P(c_i|m = 0) dc_i \\
= p\beta + (1 - p)\bar{c}_i \\
E(c|m = 0) = \int_{c} c P(c|m = 0) dc \\
= p\alpha + (1 - p)\bar{c}
\]

where,  
\[
\bar{c}_i = \int_{s \in S} \int_{c_i \in C_i} c_i h(s, c_i) dc_i ds \\
\bar{c} = \int_{s \in S} \int_{c \in C} c g(s, c) dc ds \\
\beta = \frac{\int_{s \in N} \int_{c_i \in C_i} c_i h(s, c_i) dc_i ds}{F(s \in N)} \\
= E(c_i|I, m = 0) \\
\alpha = \frac{\int_{s \in N} \int_{c \in C} c g(s, c) dc ds}{F(s \in N)} \\
= E(c|I, m = 0)
\]

In order to obtain firm i's equilibrium disclosure policy, we need to substitute the above expressions for conditional expectations into expression (3.12) from the last section, and solve it. With these substitutions, expression (3.12) becomes  
\[
E(\pi_i^*|s, m = 0) = E(\pi_i^*|s, m = s) \\
\Rightarrow \frac{1}{9b} \{a + \bar{c}_j - E(c|s) - 2E(c_i|s) + \frac{1}{2}[E(c_i|s) - E(c|s)] \\
- \frac{1}{2}[p\beta + (1 - p)\bar{c}_i - p\alpha - (1 - p)\bar{c}]^2 \\
= \frac{1}{9b} \{a + \bar{c}_j - E(c|s) - 2E(c_i|s)\}^2
\]  

(3.17)
To prove the existence of a disclosure equilibrium and to characterize this equilibrium we need to solve (3.17) above. In the absence of any further assumptions, \( E(c_i|s) \) and \( E(c|s) \) may have any functional form. Let the above conditional expectations be the following functions of \( s \),

\[
\psi_i(s) = E(c_i|s) \quad \text{and} \quad \psi(s) = E(c|s)
\]

Without any further assumptions on the functional form of \( \psi_i(s) \) and \( \psi(s) \) we can solve equation (3.17) in terms of \( \psi_i(s) \) and \( \psi(s) \).

**Lemma 3.1** The difference between the conditional expectations, i.e., \( \psi(s) - \psi_i(s) \) has two real values:

\[
\psi(s) - \psi_i(s) = 12by(s, m = s) + [p\alpha + (1 - p)\bar{c}] - [p\beta + (1 - p)\bar{c}_i]
\]

\[
\psi(s) - \psi_i(s) = [p\alpha + (1 - p)\bar{c}] - [p\beta + (1 - p)\bar{c}_i]
\]

This is shown by rearranging expression (3.17) to obtain a quadratic equation in the difference, \( \psi(s) - \psi_i(s) \), and solving it.\(^7\) From this we can conclude that at least two values of \( s \) exist at which the equation (3.17) holds. Without further assumptions we cannot be sure of the total number of such solutions or how many of them are real and how many are imaginary. If \( \psi(s) \) and \( \psi_i(s) \) have the appropriate shapes, expression (3.17) may have more than two real solutions. This would imply the existence of disclosure and non-disclosure sets, each composed of more than two disjoint regions.

To obtain more interesting results while keeping the algebra as simple as possible we assume that the conditional expectations are linear functions of \( s \). Linear conditional expectations may be obtained by assuming a joint normal distribution function over \( s, c \) and \( c_i \). In addition, several other different prior-posterior distribution pairs, e.g., gamma-Poisson and beta-binomial give us linear conditional expectations.\(^8\) However, we choose the joint normal distribution since it is simpler to handle and gives us results with more interesting and intuitive interpretations.

### 3.4.3 Disclosure Equilibrium under Multivariate Normality

From here on we assume that the signal and marginal costs are jointly normally distributed.\(^9\) This distributional assumption and its implications are formalized below.

---

\(^7\)Proofs of all lemmas are in Appendix A.2.1

\(^8\)Please refer to Li (1985)

\(^9\)The assumption of normality allows for the possibility of negative costs. Since negative cost is not an intuitively appealing concept, we assume that the parameter values are such that negative costs have only a
Distributional Assumptions:

We assume here that,

\[ f(s, c_i, c) \sim N(\mu, \Sigma) \]

where, \( \mu = \begin{bmatrix} \bar{s} \\ \bar{c_i} \\ \bar{c} \end{bmatrix} \) and \( \Sigma = \begin{bmatrix} \sigma^2_i & \sigma_{ci} & \sigma_{cs} \\ \sigma_{ci} & \sigma^2_{ci} & 0 \\ \sigma_{cs} & 0 & \sigma^2_c \end{bmatrix} \)

We get the following conditions on the marginal density functions,

\[ h(c_i) \sim N(\bar{c}_i, \sigma^2_{ci}), g(c) \sim N(\bar{c}, \sigma^2_c) \text{ and } f(s) \sim N(\bar{s}, \sigma^2_s) \]

\( h(c_i, s) \) is bivariate normal with covariance \( \sigma_{ci,s} \) and \( g(c, s) \) is bivariate normal with covariance \( \sigma_{cs} \). The distributional assumptions ensure that \( E(c_i|s) \) and \( E(c|s) \), the posterior expected marginal costs, given \( s \), are linear in \( s \), as follows,

\[ E(c_i|s) = \gamma_i + \delta_i s = \bar{c}_i + \delta_i(s - \bar{s}) \]
\[ E(c|s) = \gamma + \delta s = \bar{c} + \delta(s - \bar{s}) \]

where

\[ \gamma_i = \bar{c}_i - \delta_i \bar{s}, \quad \delta_i = \frac{\sigma_{ci} \rho_{ci,s}}{\sigma_c} \]
\[ \gamma = \bar{c} - \delta \bar{s}, \quad \delta = \frac{\sigma_c \rho_{cs}}{\sigma_c} \]
\[ \rho_{ci,s} = \frac{\sigma_{ci,s}}{\sigma_{ci} \sigma_s} \text{ and } \rho_{cs} = \frac{\sigma_{cs}}{\sigma_c \sigma_s} \]

\( \delta_i \) and \( \delta \) are the regression coefficients of \( c_i \) and \( c \) on \( s \), respectively, as shown in the initial parts of equations (3.18) and (3.19). From the right hand sides of equations (3.18) and (3.19), \( \delta_i \) and \( \delta \) can be interpreted in terms of adjustments made to the prior means of the marginal costs. For instance, \( \delta(s - \bar{s}) \) is the adjustment made to the prior expectation of industry-wide marginal cost, \( \bar{c} \), on observation of signal \( s \). If the observed signal \( s \) is equal to the mean of the possible signal values, this adjustment would be zero and the posterior mean \( E(c_i|s) \) would equal the prior mean \( \bar{c} \). The term \( \delta \) is, therefore, the rate of adjustment of the prior mean on receipt of information and can also be described as the "informativeness" of the information variable, \( s \), with respect to the industry-wide marginal cost.

very small probability of occuring. Under certain conditions, however, the concept of negative costs may be reasonable. Consider the situation where A and B are joint products with joint production cost \( c \). Let \( p_A \) and \( p_B \) be the selling prices of A and B, respectively. Then the profit from the process is \( p_A + p_B - c \). We can think of the quantity \( c - p_B \) as the production cost of A. Hence, if \( p_B > c \) then the production cost of product A is negative. If we assume that A has significant disposal cost, it is not produced unless it is going to be sold.

34
The rate of adjustment is increasing in the correlation between the industry-wide marginal cost and the information variable, $\rho_{cs}$. A higher $\rho_{cs}$ indicates that $s$ is a better predictor of $c_i$, resulting in a higher rate of adjustment, for the same value of $s$. In the extreme case, if say the correlation is zero, the rate of adjustment, $\delta$ is also zero. If $s$ and $c$ are uncorrelated, a signal $s$ does not give any new information about $c$ and hence does not warrant any adjustment of the prior expectation of $c$.

The rate of adjustment $\delta$ is increasing in the variance of the industry-wide marginal cost, $\sigma_c^2$, and decreasing in the variance of the information variable, $s$. The higher the variance of the distribution of $c$, the less precisely $c$ is known, and hence any information signal is taken more "seriously". On the other hand, a less precise signal is taken less "seriously", resulting in a lower rate of adjustment. The adjustment factor $\delta_i$, for firm-specific marginal cost, can be interpreted in the same fashion.

The assumptions of normality also gives us the following simplifications for expected costs, given non-disclosure.

**Lemma 3.2** If $h(c_i, s)$ and $g(c, s)$ are bivariate normal density functions, then

\[
\alpha = \gamma + \delta A \tag{3.23}
\]

\[
\beta = \gamma_i + \delta_i A \tag{3.24}
\]

\[
\bar{c} = \gamma + \delta \bar{s} \tag{3.25}
\]

\[
\bar{c}_i = \gamma_i + \delta_i \bar{s} \tag{3.26}
\]

where $A = \frac{\int_{s \in N} s f(s) ds}{F(s \in N)} = E[s | I, m = \emptyset]$

The term $A$ is the mean of $s$ where the distribution of $s$ has been truncated to exclude the values of $s$ in the disclosure set. It therefore represents the conditional expected value of $s$ given non-disclosure by firm $i$ and it is known to be informed. Given this interpretation of $A$, $\alpha$ and $\beta$ are the posterior means of industry-wide and firm-specific marginal costs, respectively, given informed non-disclosure by firm $i$. The posterior means, given informed non-disclosure by firm $i$, are $\bar{c}$ and $\bar{c}_i$.

\[^{10}\text{Refer to Appendix A.2.1 for the proof.}\]
Substitution of the above simplifications, into the expressions for expected costs, given non-disclosure, give the following.

\[ E(c|m = 0) = p\beta + (1 - p)c_i = \gamma_i + \delta_i[pA + (1 - p)s] \]  
\[ E(c|m = 0) = p\alpha + (1 - p)c = \gamma + \delta[pA + (1 - p)s] \]  
\( ^{(3.27)} \)

Note that from expressions (3.15) and (3.16), \( p \) and \( (1 - p) \) are the probabilities that firm \( i \) is informed and uninformed, respectively, given non-disclosure. Substituting for \( E(c_i|m) \), \( E(c|m) \), \( E(c_i|m = 0) \) and \( E(c|m = 0) \) into expressions (3.7) and (3.10) we obtain the following expressions for optimal output under multivariate normality.

\[ y_i^*(s, m = s) = \frac{1}{3b} \{ a + c_j - \gamma - 2\gamma_i - (\delta + 2\delta_i)s \} \]  
\[ y_i^*(s, m = 0) = \frac{1}{3b} \{ a + \bar{c}_j - \gamma - 2\gamma_i - \frac{3}{2}(\delta + \delta_i)s \} \]  
\[ -\frac{(\delta_i - \delta)}{2}[pA + (1 - p)s] \]  
\( ^{(3.29)} \)
\( ^{(3.30)} \)

The corresponding expressions for expected profits can be obtained by substitution for the marginal costs in expressions (3.8) and (3.11). This yields,

\[ E(\pi_i^*|s, m = s) = \frac{1}{9b} \{ a + c_j - \gamma - 2\gamma_i - (\delta + 2\delta_i)s \}^2 \]  
\[ E(\pi_i^*|s, m = 0) = \frac{1}{9b} \{ a + \bar{c}_j - \gamma - 2\gamma_i - \frac{3}{2}(\delta + \delta_i)s \} \]  
\[ -\frac{(\delta_i - \delta)}{2}[pA + (1 - p)s] \]  
\( ^{(3.31)} \)
\( ^{(3.32)} \)

Algebraically, the quadratic form of the maximized expected profits with respect to the information signals, arises due to the combined effects of two factors:

- the fact that the optimized profit is a quadratic function of expected marginal costs.\(^{11}\)

- and the assumption that the information variable and marginal cost are jointly normal.

This ensures that the conditional expectations of the marginal costs, \( E(c|m) \) and \( E(c_i|m) \) are linear functions of the signal, \( s \).

For any positive disclosed value of \( s \), the higher the values of \( \gamma, \gamma_i, \delta \) and \( \delta_i \) the lower is the expected profit, given that the optimal output level is non-negative. This is due to the fact that a higher \( s \) signals higher expected marginal costs and hence, a lower output level. If the

\(^{11}\)This is a standard result in a Cournot oligopoly. It is driven by the linearity of the inverse demand function and the assumption of constant marginal cost.
output level is positive, then this translates to a lower expected profit. However, if the output level is negative, then a decrease in that level results in a higher expected profit.

In the case of non-disclosure, however, in addition to the above effect, the relative magnitudes of $\delta$ and $\delta_i$, as well as the relative magnitudes of $s$ and $pA + (1 - p)\bar{s}$, are important in the determination of expected profit. Since $p$ is the probability that firm $i$ is informed, $pA + (1 - p)\bar{s}$ gives the expectation of $s$, given non-disclosure by firm $i$. Firm $i$’s optimal expected profit is increased or decreased by non-disclosure of its information, depending on the relative magnitudes of $\delta_i$ and $\delta$ and on whether its observed $s$ is greater or less than $pA + (1 - p)\bar{s}$ and whether its output level is positive or negative.

Determination of the optimal disclosure sets involves a comparison of the above expressions for expected profits under disclosure and non-disclosure, for each value of the signal $s$. If $E(\pi_i^* | s, m = 0)$ is greater than $E(\pi_i^* | s, m = s)$ then non-disclosure of $s$ is preferred. Whereas, if $E(\pi_i^* | s, m = 0)$ is less than $E(\pi_i^* | s, m = s)$ then disclosure of $s$ is preferred. If the two expected profits are equal then there is indifference between disclosure and non-disclosure at that value of $s$. Substitution for $E(\pi_i^* | s, m = s)$ and $E(\pi_i^* | s, m = 0)$ from expressions (3.31) and (3.32) into equation (3.17) yields:

$$E(\pi_i^* | s, m = 0) = E(\pi_i^* | s, m = s) \Rightarrow \{a + c_i - \gamma - 2\gamma_i - (\delta + 2\delta_i)s + \frac{(\delta_i - \delta)}{2}[s - pA - (1 - p)\bar{s}]\}^2$$

$$= \{a + c_i - \gamma - 2\gamma_i - (\delta + 2\delta_i)s\}^2 \quad (3.33)$$

The difference between the above optimized expected profit terms represents firm $i$’s “proprietary” cost or gain of disclosing signal $s$. If $E(\pi_i^* | s, m = 0) > E(\pi_i^* | s, m = s)$, then firm $i$ incurs a proprietary cost if $s$ is disclosed. If, on the other hand, $E(\pi_i^* | s, m = 0) < E(\pi_i^* | s, m = s)$, then disclosure of $s$ results in a proprietary gain for firm $i$. At values of $s$ where the two expected profits are equal, there is no proprietary cost or gain and the firm is indifferent between disclosure and non-disclosure.

The optimized expected terms are quadratic in $s$, hence, they can be represented geometrically by a pair of parabolas. Since any two parabolas can intersect at most twice, it follows that at most two threshold level for disclosure/non-disclosure are possible. Proposition 1 proves that in this specific context exactly two threshold levels, $s'$ and $s''$, exist.

**Proposition 3.1** Under the assumption that $h(c_i, s)$ and $g(c, s)$ are Bivariate Normal, there exists a unique equilibrium value for each of $s''$ and $s'$ determined by $\bar{y}_i^*$ and the parameters, $b$, $q$, $\delta_i$, $\delta$ and $\bar{s}$.
Proposition 3.1 is proved\footnote{The proof is in Appendix A.2.2.} by demonstrating that the function obtained by taking the difference between the two optimized expected profit functions has two real roots.

The intuition behind proposition 3.1 is best illustrated geometrically. As mentioned previously, the expected profit functions can be represented by parabolas of the form:

\begin{align}
V(s|D) &\equiv E(\pi^*_m | s, m = s) = \frac{1}{9b} (x_D - z_D s)^2 \\
V(s|N) &\equiv E(\pi^*_m | s, m = \emptyset) = \frac{1}{9b} (x_N - z_N s)^2
\end{align}

where

\begin{align}
x_D &= a + \bar{c}_j - \gamma - 2\gamma_i \\
z_D &= \delta + 2\delta_i \\
z_N &= x_D - \frac{\delta_i - \delta}{2}[pA + (1 - p)\bar{s}] \\
z_N &= x_D - \frac{\delta_i - \delta}{2}
\end{align}

Note that both these parabolas are convex with minimum values of zero. The function \(V(s|D)\) reaches its minimum at \(s = \frac{z_D}{x_D} \equiv \hat{s}_D\) and \(V(s|N)\) reaches its minimum at \(s = \frac{z_N}{x_N} \equiv \hat{s}_N\), with \(\hat{s}_D, \hat{s}_N > \bar{s}\). The curvatures of \(V(s|D)\) and \(V(s|N)\) are determined by the coefficients \(z_D\) and \(z_N\), respectively. These functions are illustrated in figure 3.1.

The key property is that two parabolas which can be represented as \(V(s|D)\) and \(V(s|N)\) will intersect exactly twice if they differ in both the value of \(s\) at which minimal profit occurs and the slope at any given distance from that minimal value of \(s\). Note that the latter quantity is related to the curvature of the parabola. Hence, \(V(s|D)\) and \(V(s|N)\) intersect exactly twice if, and only if,

\[ z_D \neq z_N \quad \text{and} \quad \frac{x_D}{z_D} \neq \frac{x_N}{z_N} \equiv \hat{s}_D \neq \hat{s}_N \]

It is clear from the algebraic expressions for \(x_D, x_N, z_D\) and \(z_N\), that the above necessary and sufficient condition is satisfied if \(\delta_i \neq \delta\). Hence, there are exactly two threshold levels of disclosure.

In Appendix A.1.2 we derive the following expressions, by rearranging equation (3.33) and solving the resulting quadratic equation in \(s\).

\[ s' = \bar{s} + p(A - \bar{s}) \]
\[ s'' = \frac{4[a + c_i - \gamma - 2\gamma_i] - (\delta_i - \delta)[\bar{s} + p(A - \bar{s})]}{7\delta_i + 5\delta} = \bar{s} + \frac{12b\bar{y}_i^* - (\delta_i - \delta)p(A - \bar{s})}{7\delta_i + 5\delta} \]  

(3.41)

where, \( \bar{y}_i^* = \frac{1}{3\delta_i} \{a + c_i - \gamma - 2\gamma_i - (\delta + 2\delta_i)\bar{s}\} \), is firm \( i \)'s optimal production level, based on prior beliefs, \( \bar{s} \). We assume that \( \bar{y}_i^* \) is always positive. Expressions (3.40) and (3.41) give us the parameters which determine the equilibrium values of \( s' \) and \( s'' \). Note that both \( A \) and \( p \) are endogenously determined since they are both functions of the equilibrium values of \( s' \) and \( s'' \).

Ordinarily we would solve equations (3.40) and (3.41), simultaneously, to obtain equilibrium values for \( s' \) and \( s'' \). However, it is not possible to solve analytically for the values of \( s' \) and \( s'' \), in this case, since \( A \) and \( p \) in expressions (3.40) and (3.41) contain integrals with limits equal to \( s' \) and \( s'' \). In the following lemma we find ranges for the equilibrium values of \( s' \) and \( s'' \), using expressions (3.40) and (3.41). The ranges are sufficient to give a characterization of the disclosure/non-disclosure sets. To obtain exact solutions for \( s' \) and \( s'' \) we need to solve numerically for a particular set of parameter values.

**Lemma 3.3**

In Case 1, with \( \delta_i > \delta \), \( s' \) and \( s'' \) lie in the intervals,

\[ s' \in [\bar{s}, \bar{s} + \frac{12b\bar{y}_i^*}{4\delta + 8\delta_i}] \quad \text{and} \quad s'' \in [\bar{s} + \frac{12b\bar{y}_i^*}{4\delta + 8\delta_i}, \bar{s} + \frac{12b\bar{y}_i^*}{5\delta + 7\delta_i}] \]

respectively.

In Case 2, with \( \delta_i < \delta \), \( s' \) and \( s'' \) lie in the intervals,

\[ s' \in [\bar{s} - \frac{12b\bar{y}_i^*}{\delta - \delta_i}, \bar{s}] \quad \text{and} \quad s'' \in [\bar{s}, \bar{s} + \frac{12b\bar{y}_i^*}{4\delta + 8\delta_i}] \]

respectively.

**Corollary 3.1** If \( \bar{y}_i^* \geq 0 \), then \( s'' > s' \) for both \( \delta_i > \delta \) and \( \delta > \delta_i \).

These ranges are shown in Figure 3.2.

To illustrate these results we present a numerical example. Tables 3.1 and 3.2 give equilibrium values for \( s' \) and \( s'' \), calculated for \( \bar{y}_i^* = 1 \), \( b = 0.1 \), \( \bar{s} = 0 \), \( \sigma_2^* = 1 \) and \( q \) ranging between

---

\(^{13}\)The derivations are in the proof of lemma 3.3 in Appendix A.2.1.

\(^{14}\)It is apparent from the ranges for \( s' \) and \( s'' \), given above, that the relative values of \( \bar{s}, s' \) and \( s'' \) depend on whether \( \bar{y}_i^* \) is greater or less than zero. In case 1, \( \bar{y}_i^* > 0 \) implies that \( \bar{s} < s' < s'' \) and \( \bar{y}_i^* < 0 \) implies that \( \bar{s} > s' > s'' \). In case 2, \( \bar{y}_i^* > 0 \) implies that \( s' < \bar{s} < s'' \) and \( \bar{y}_i^* < 0 \) implies that \( s' > \bar{s} > s'' \).
0.1 and 1.0. Table 3.1 presents case 1 in which $\delta_1 = 0.5$ and $\delta = 0.3$ while Table 3.2 presents case 2 in which $\delta_1 = 0.3$ and $\delta = 0.5$. These values are substituted into expressions (3.40) and (3.41), which are then solved simultaneously to obtain $s'$, $s''$, $p$ and $A$. Since a positive value is used for $\bar{g}_i$, $\bar{s} < s' < s''$ in case 1 and $s' < \bar{s} < s''$ in case 2, for all values of $q$.

Given the above parameter values, the ranges for $s'$ and $s''$, from lemma 3.3, have the following numerical values:

Case 1: $s' \in [0, 0.23077]$ and $s'' \in [0.23077, 0.24000]$.

Case 2: $s' \in [-6.0, 0]$ and $s'' \in [0, 0.27273]$.

The calculated values of $s'$ and $s''$ given in Tables 3.1 and 3.2 lie in their respective ranges.

**Disclosure/Non-disclosure sets**

Disclosure/non-disclosure sets are specific sets of values of the signal $s$ for which disclosure/non-disclosure is optimal. The threshold values described thus far separate the disclosure and non-disclosure sets. Since there are two threshold values, $s'$ and $s''$, there can be either two disjoint disclosure sets separated by a non-disclosure set or two disjoint non-disclosure sets separated by a disclosure set.

To determine firm $i$'s equilibrium disclosure/non-disclosure sets it is useful to re-examine the parabola representations of the optimized expected profit functions $V(s|D)$ and $V(s|N)$, illustrated in figure 1. Given that $z_D \neq z_N$ and $\delta_D \neq \delta_N$ there are exactly two threshold levels, $s'$ and $s''$. In addition, the geometry indicates that $V(s|D) > V(s|N)$ for $s \in (s', s'')$ if, and only if, $z_D < z_N$, given that $\delta_D \neq \delta_N$. Therefore, in case 1 $V(s|D) < V(s|N)$ for $s \in (s', s'')$ and $V(s|D) > V(s|N)$ for $s \notin (s', s'')$, since $\delta_1 > \delta$. In case 2, on the other hand, $V(s|D) > V(s|N)$ for $s \in (s', s'')$ and $V(s|D) < V(s|N)$ for $s \notin (s', s'')$, since $\delta_1 < \delta$.

Summarizing the above results,

**Case 1: $\delta_1 > \delta$** The equilibrium non-disclosure set is $[s', s'']$ and the disclosure set consists of $(-\infty, s']$ and $[s'', +\infty]$.

**Case 2: $\delta_1 < \delta$** The equilibrium disclosure set is $[s', s'']$ and the non-disclosure set consists of $(-\infty, s']$ and $[s'', +\infty]$.

Basically, if $\delta_1 > \delta$, we obtain two disjoint sections in the disclosure set separated by a single connected non-disclosure set. On the other hand, if $\delta_1 < \delta$, we obtain two disjoint sections in the non-disclosure set separated by a single connected disclosure set.

In order to intuitively interpret the above results we need to simultaneously consider the
above disclosure/non-disclosure sets and the optimal output levels at each $s$. However, prior to doing that, we compare the incentives for disclosure involved here with those suggested in section 2.

The threshold level $s'$ corresponds to the threshold level suggested by the analysis under heterogeneous beliefs, in section 2. Under heterogeneous beliefs, the optimal expected profit is given by expression (3.6), where $E_i$ denotes posterior and $E_j$ denotes prior expectations. An increase (decrease) in $\frac{[E_i(c) - E_j(c)]}{2}$ decreases (increases) expected profit. Hence if the prior is higher (lower) than the posterior, firm $i$ is motivated to communicate (withhold) its information. On the other hand, in the case of firm-specific marginal cost, an increase (decrease) in $\frac{[E_i(c_i) - E_j(c_i)]}{2}$ increases (decreases) expected profit. Hence, if the prior is higher (lower) than the posterior, firm $i$ is motivated to withhold (communicate) its information.

Under the assumption of private information and normality, the above terms become:\(^{15}\)

$$\frac{E_i(c) - E_j(c)}{2} = \frac{E[c|s] - E[c|m = 0]}{2} = \frac{\delta}{2} \{s - [pA + (1 - p)\bar{s}]\}$$

$$\frac{E_i(c_i) - E_j(c_i)}{2} = \frac{E[c_i|s] - E[c_i|m = 0]}{2} = \frac{\delta_i}{2} \{s - [pA + (1 - p)\bar{s}]\}$$

where

- $\frac{\delta}{2}$ is the adjustment factor for expected common cost.
- $\frac{\delta_i}{2}$ is the adjustment factor for expected firm-specific cost.
- $s$ is firm $i$'s information.
- $pA + (1 - p)\bar{s}$ is firm $j$'s estimate of firm $i$'s information, in the event of non-disclosure.

From expression (3.40), $s' = pA + (1 - p)\bar{s}$; therefore,

$$\frac{E_i(c) - E_j(c)}{2} = \frac{\delta}{2} [s - s']$$

$$\frac{E_i(c_i) - E_j(c_i)}{2} = \frac{\delta_i}{2} [s - s']$$

As described above, firm-specific and industry-wide costs have opposing effects on firm $i$'s expected profit and its incentives for disclosure. Their net effect on optimal output is proportional

\(^{15}\)Please refer to (3.19) and (3.28) for the first equation and to (3.18) and (3.27) for the second equation.
to
\[
\frac{1}{2} \left( [E_i(c_i) - E_j(c_i)] - [E_i(c) - E_j(c)] \right) = \frac{(\delta_i - \delta)}{2} [s - s']
\]

Depending on whether it is positive or negative, the above expression is proportional to the increase or decrease in firm i’s optimal output, due to the non-disclosure of its information. Hence, it is also related to the gain or loss made by firm i by not disclosing its information. Therefore, if \( \delta_i > \delta \) then \( s > s' \) motivates non-disclosure and \( s < s' \) motivates disclosure given a positive output level. On the other hand, if \( \delta_i < \delta \) then \( s > s' \) motivates disclosure and \( s < s' \) motivates non-disclosure given a positive output level.

The relative magnitudes of \( \delta_i \) and \( \delta \) and the disclosure strategy of firm i may be interpreted as the informativeness of \( s \) as follows,

\( \delta_i > \delta \Rightarrow s \) is more informative about \( c_i \) than \( c \) and the incentives due to \( c_i \) dominate firm i’s disclosure strategy.

\( \delta_i < \delta \Rightarrow s \) is more informative about \( c \) than \( c_i \) and the incentives due to \( c \) dominate firm i’s disclosure strategy.

The disclosure sets obtained under our specifications of private information and normality have two threshold levels and three disclosure/non-disclosure regions, in each case. The existence of \( s' \) is consistent with the discussion in the heterogeneous beliefs case, in section 2. The existence of \( s'' \) above which, in case 1, disclosure again becomes optimal and in case 2, non-disclosure becomes optimal, seems counterintuitive. To obtain the intuition behind both threshold levels and all three disclosure/non-disclosure regions, we need to examine the impact of the optimal output level on the optimal disclosure policy, and vice versa, under each signal. In the next section we first determine the equilibrium output as a function of the signal observed, as well as the optimal disclosure for that signal. The expected profit functions given optimal output and disclosure follow. A simultaneous examination of both the optimal disclosure policy and the optimal output choice given the optimal disclosure, provides insight into our results.

### 3.4.4 Equilibrium Production and Profit

In this section we first determine the equilibrium output function for different values of the signal, \( s \). This is followed by an intuitive discussion of the mutual impact of the output and disclosure choices on each other.

Using the shorthand notation given by (3.36), (3.37), (3.38) and (3.39) on the expressions
for $y^*_i(s, m = s)$ and $y^*_i(s, m = \emptyset)$, given by (3.29) and (3.30), we obtain

$$y(s|D) = \frac{1}{3b}(x_D - z_D s) \equiv y^*_i(s, m = s) \quad (3.42)$$

$$y(s|N) = \frac{1}{3b}(x_N - z_N s) \equiv y^*_i(s, m = \emptyset) \quad (3.43)$$

Hence, the optimal output can be written as the following function of the signal, $s$, and the optimal disclosure, $m^*(s)$.

$$y(s|m^*) = \begin{cases} y(s|D) & \text{if } s \in (s', s'') \\ y(s|N) & \text{if } s \notin (s', s'') \end{cases}$$

The optimal output function $y(s|m^*)$ equals $y(s|D)$ if $s$ lies in the disclosure set and $y(s|N)$ if $s$ lies in the non-disclosure set. At the threshold levels $s'$ and $s''$ firm $i$ switches from disclosure (non-disclosure) to non-disclosure (disclosure), depending on the relative values of $\delta_i$ and $\delta$. We compare the optimal output level given disclosure with the optimal level given non-disclosure at each of the threshold levels.

Lemma 3.4

$$y(s|m = s') - y(s|N) = 0$$

$$y(s|m = s'') + y(s|N) = 0$$

The above lemma is proved by substituting $s'$ and $s''$ in each of the expressions (3.29) and (3.30) and simplifying the resulting expressions. It states that the optimal output at $s = s'$ is the same whether or not $s$ is disclosed, whereas $s = s''$ the optimal output if $s$ is not disclosed is the negative of that if it is disclosed. This implies that although the slope of the output function $y^*_i[s, m^*(s)]$ changes at $s'$ and $s''$, it is continuous at $s'$ but discontinuous at $s''$. For a more concrete representation, cases 1 and 2 are considered separately below.

Case 1: $\delta_i > \delta$

The optimal output function $y(s|m^*)$ is written as follows:

$$y(s|m^*) = \begin{cases} y(s|D) & \text{if } -\infty < s < s' \text{ or } s' < s < \infty \\ y(s|N) & \text{if } s' < s < s'' \end{cases}$$

The slopes and intercepts of $y(s|D)$ and $y(s|N)$ are compared. Since, $x_D > x_N$ and $-z_D < -z_N$, the intercept of $y(s|D)$ is higher than that of $y(s|N)$ whereas, the slope of $y(s|D)$ is lower than that of $y(s|N)$. Also note that $V(s|D) = b[y(s|D)]^2$ and $V(s|N) = b[y(s|N)]^2$. Hence, $y(s|D) > y(s|N)$ and $V(s|D) > V(s|N)$, when $s < s'$. 

43
\[ y(s|D) < y(s|N) \quad \text{and} \quad V(s|D) < V(s|N), \quad \text{when} \quad s' < s < s'' \]
\[-y(s|D) > y(s|N) \quad \text{and} \quad V(s|D) > V(s|N), \quad \text{when} \quad s > s'' \]

Plots of firm i's optimal output level and optimized expected profit, against signal \( s \), are given in figures 3.3(a) and 3.3(c), respectively. For any given \( s \), firm i maximizes its expected profit. Therefore,

\[ y(s|m^*) = \begin{cases} 
  y(s|D) > 0 \quad \text{and} \quad m^* = s & \text{if} \quad -\infty < s < s' \\
  y(s|N) > 0 \quad \text{and} \quad m^* = 0 & \text{if} \quad s' < s < s'' \\
  y(s|D) < 0 \quad \text{and} \quad m^* = s & \text{if} \quad s'' < s < \infty 
\end{cases} \]

**Case 2: \( \delta_i < \delta \)**

The optimal output function \( y(s|m^*) \) is written as follows:

\[ y(s|m^*) = \begin{cases} 
  y(s|D) & \text{if} \quad s' < s < s'' \\
  y(s|N) & \text{if} \quad -\infty < s < s' \quad \text{or} \quad s'' < s < \infty 
\end{cases} \]

The slopes and intercepts of \( y(s|D) \) and \( y(s|N) \) are compared. Since, \( z_N > z_D \) and \( -z_N < -z_D \), the intercept of \( y(s|N) \) is higher than that of \( y(s|D) \) whereas, the slope of \( y(s|N) \) is lower than that of \( y(s|D) \). Since \( V(s|D) = b[y(s|D)]^2 \) and \( V(s|N) = b[y(s|N)]^2 \),

\[
\begin{align*}
y(s|D) & < y(s|N) \quad \text{and} \quad V(s|D) < V(s|N), \quad \text{when} \quad s < s' \\
y(s|D) & > y(s|N) \quad \text{and} \quad V(s|D) > V(s|N), \quad \text{when} \quad s' < s < s'' \\
y(s|D) & < -y(s|N) \quad \text{and} \quad V(s|D) < V(s|N), \quad \text{when} \quad s > s''
\end{align*}
\]

Plots of firm i's optimal output level and optimized expected profit, against signal \( s \), are given in figures 3.3(b) and 3.3(d), respectively. For any given \( s \), firm i maximizes its expected profit. Therefore,

\[ y(s|m^*) = \begin{cases} 
  y(s|N) > 0 \quad \text{and} \quad m^* = 0 & \text{if} \quad -\infty < s < s' \\
  y(s|D) > 0 \quad \text{and} \quad m^* = s & \text{if} \quad s' < s < s'' \\
  y(s|N) < 0 \quad \text{and} \quad m^* = 0 & \text{if} \quad s'' < s < \infty 
\end{cases} \]

44
Discussion:

As pointed out at the end of the last section, a key to the economic interpretation of the disclosure/non-disclosure sets is firm i’s optimized production function. This function is illustrated in figures 3.3(a) and 3.3(b), for cases 1 and 2, respectively. The general trend, in both cases, is to produce less as \( s \) increases since costs are expected to be higher. At values of \( s < s'' \), since \( y(s|m^*) > 0 \), firm i produces positive quantities of output. As \( s \) increases to higher than \( s'' \) it signals very high marginal costs. It is no longer optimal to produce positive quantities of output at such high levels of marginal cost and firm i’s optimal output level becomes negative. Later in this section, we discuss the case where output levels are constrained to be positive.

In case 1, the information variable \( s \) acts as a proxy for firm i’s firm-specific marginal cost. Optimal output is positive for \( s < s'' \). At less than average values of \( s \), firm i’s firm-specific marginal cost is less than its expected value and, hence, firm i produces a higher quantity. To induce firm j to decrease its output, and thereby prevent the output price from decreasing, firm i reveals its good news. Due to the possibility of firm i being uninformed and rational expectations by firm j, the threshold disclosure level unravels upwards to \( s' \).

A value of \( s \) between \( s' \) and \( s'' \) is bad news for firm i since it implies that its firm-specific marginal cost is high. Hence, its optimal output level, though positive, is lower. In order to prevent firm j from increasing its output and causing a drop in output price, firm i withholds its unfavorable information.

At values of \( s \) that are higher than \( s'' \), the firm-specific marginal cost, for firm i, is too high for firm i to find positive output optimal. Hence, at these high levels of \( s \), firm i becomes a net consumer. Firm i reveals \( s \) in order to inform firm j that i is a net consumer so that firm j will increase its output level, thus decreasing the product price.

In case 2, the information variable \( s \) acts as a proxy for \( c \), the industry-wide marginal cost. Firm i’s optimal output is positive for \( s < s'' \) and negative for \( s > s'' \). At less than average values of \( s \), firm i withholds its private information to prevent firm j from taking advantage of the low industry-wide marginal cost to increase its output, thus lowering the product price. Due to the positive probability of firm i being uninformed and rational expectations on the part of firm j, the threshold level of disclosure unravels from \( s \) down to \( s' \).

At values of \( s \) between \( s' \) and \( s'' \), firm i is still a net producer and it reveals these higher values of \( s \) in order to inform firm j that industry-wide marginal costs are high. At higher cost levels firm j produces less and keeps output price from falling.

When \( s \) is higher than \( s'' \), industry-wide marginal cost is too high for firm i to find it optimal.
to produce positive quantities. Since firm i is a net consumer, it withholds its information to prevent firm j from decreasing its output level and increasing the product price.

3.4.5 Restricting Output to be Non-negative

In this section we examine the effect of imposing the condition that output levels must be non-negative. Hence, neither firm i nor firm j can become a net consumer. This translates to two additional constraints on each output and disclosure choice problem.

Output Choice:

In the case of output choice given disclosure, the additional constraints are: \( y_i^*(s, m = s) \geq 0 \) and \( y_j^*(m = s) \geq 0 \). Under non-disclosure the corresponding additional constraints are: \( y_i^*(s, m = s) \geq 0 \) and \( y_j^*(m = s) \geq 0 \).\(^{16}\) Prior to presenting the solutions to the constrained problem, it is useful to briefly describe certain features of the unconstrained solution. Under disclosure in the unconstrained case firm i's output level becomes negative at values of \( s > \hat{s}_1 \) and firm j's output level becomes zero at \( s = \hat{s}_3 \), where

\[
\hat{s}_1 = \frac{a + \bar{c}_j - 2\bar{e}_i - \bar{c}}{2\delta_i + \delta},
\]

\[
\hat{s}_3 = \frac{a - 2\bar{e}_j + \bar{c}_i - \bar{c}}{\delta - \delta_i}.
\]

Hence, in the unconstrained case if \( \delta_i > \delta \) and \( \hat{s}_3 < 0 < \hat{s}_1 \), then firm j's output level becomes negative for \( s < \hat{s}_3 \). On the other hand, if \( \delta_i < \delta \) and \( \hat{s}_3 > \hat{s}_1 > 0 \), firm j's output level becomes negative for \( s > \hat{s}_3 \).

The imposition of the non-negativity constraints on the output choice problem yields the following optimal output choices, given disclosure.

Firm i is informed (I), receives signal \( s \) and discloses \( m = s \).

Case 1: \( \delta_i > \delta \)

(a) If \( s < \hat{s}_3 < \hat{s}_1 \), then

\[
y_j^*(m = s) = 0
\]

\[
y_i^*(s, m = s) = \frac{1}{2\delta} \left\{ a - (\gamma + \gamma_i) - (\delta + \delta_i)s \right\}
\]

\(^{16}\)The algebra is in Appendix A.1.3.
(b) If $s \geq \hat{s}_1 > \hat{s}_3$, then

\[
\begin{align*}
  y^*_j(m = s) &= \frac{1}{2b} \{a - \bar{c}_j - \gamma - \delta s\} \\
  y^*_i(s, m = s) &= 0
\end{align*}
\]

(c) If $\hat{s}_3 < s < \hat{s}_1$, then

\[
\begin{align*}
  y^*_j(m = s) &= \frac{1}{3b} \{a - 2\bar{c}_j + (\gamma_i - \gamma) + (\delta_i - \delta)s\} \\
  y^*_i(s, m = s) &= \frac{1}{3b} \{a + \bar{c}_j - (2\gamma_i + \gamma) - (2\delta_i + \delta)s\}
\end{align*}
\]

Note that the expected profit functions are:

\[
E(\tau^*_j|m = s) = b[y^*_j(m = s)]^2 \quad \text{and} \quad E(\tau^*_i|s, m = s) = b[y^*_i(s, m = s)]^2
\]

In (a), $s$ is very low implying that the expected firm-specific marginal cost for firm $i$ is very low. Hence, firm $j$ ceases production at $s \leq \hat{s}_3$ and firm $i$’s optimal output and expected profits reflect a monopoly status. In (b), these positions are reversed at $s \geq \hat{s}_1 > \hat{s}_3$. When the value of $s$ is as high as $\hat{s}_1$, it implies that firm $i$’s firm-specific marginal cost is expected to be very high and it is not optimal for firm $i$ to produce positive quantities. Hence, at $s \geq \hat{s}_1$ firm $i$ ceases production and firm $j$ becomes a monopolist. In (c) both firms produce positive quantities. The optimal output levels at these intermediate values of $s$ are the same as those in the unconstrained solution.

**Case 2 : $\delta_i < \delta$**

(a) If $s \geq \hat{s}_3 > \hat{s}_1$, then

\[
\begin{align*}
  y^*_j(m = s) &= 0 \\
  y^*_i(s, m = s) &= 0
\end{align*}
\]

(b) If $\hat{s}_3 \geq s \geq \hat{s}_1$, then

\[
\begin{align*}
  y^*_j(m = s) &= \frac{1}{2b} \{a - \bar{c}_j - \gamma - \delta s\} \\
  y^*_i(s, m = s) &= 0
\end{align*}
\]

(c) If $\hat{s}_3 \geq \hat{s}_1 \geq s$, then

\[
\begin{align*}
  y^*_j(m = s) &= \frac{1}{3b} \{a - 2\bar{c}_j + (\gamma_i - \gamma) + (\delta_i - \delta)s\} \\
  y^*_i(s, m = s) &= \frac{1}{3b} \{a + \bar{c}_j - (2\gamma_i + \gamma) - (2\delta_i + \delta)s\}
\end{align*}
\]
Note that the expected profit functions are:

\[ E(\pi^*_i|m = s) = b[y^*_i(m = s)]^2 \quad \text{and} \quad E(\pi^*_s|m = s) = b[y^*_s(s, m = s)]^2 \]

In (a), \( s \) is extremely high implying that the expected industry-wide common marginal cost is very high. At these high levels of \( s \) it is not optimal for either firm \( i \) or firm \( j \) to produce positive quantities. In (b), \( s \) is not high enough to prevent firm \( j \) from producing a positive quantity. However, \( s \) affects both the firm-specific and the industry-wide common components of firm \( i \)'s marginal cost. Hence, \( s_1 \leq s \leq s_3 \) is high enough to prevent firm \( i \) from producing positive quantities. In (c), \( s \) is low enough for both firms to produce positive quantities. The optimal output levels at these intermediate values of \( s \) are the same as those in the unconstrained solution.

Under non-disclosure firm \( i \)'s optimal output level becomes zero at \( s = \hat{s}_2 \), where

\[
\hat{s}_2 = \frac{2}{2(\delta_i + \delta)} \{(a + \bar{c}_j - (2\gamma_i + \gamma)} \\
- \frac{(\delta - \delta_i)}{2}[(1 - p)\hat{s} + pA] + \frac{\epsilon}{2}
\]

\[
A = E[s| I, s \in N] \\
A = E[s| I, s \in N, y^*_i(s, m = \emptyset) = 0] \\
\epsilon = \frac{p(1-\eta)}{2}[(2\gamma_i + \gamma) + (2\delta_i + \delta)A - (a + \bar{c}_j)] \\
\eta = \frac{F(s \in N, y^*_i(s, m = \emptyset) > 0)}{F(s \in N)}
\]

\( \eta \) is \( j \)'s belief that \( i \)'s output level is greater than zero given that \( s \in N \) and \( A \) is the expected signal given non-disclosure and \( y^*_i(s, m = \emptyset) = 0 \). Both \( \eta \) and \( A \) depend on the threshold levels of disclosure which are endogenously determined. In addition, both quantities are also dependent on the density function of \( s \) which is exogenously specified. Also note that, in general, the value of the expected signal given non-disclosure, \( A \), in the constrained solution is different from that in the unconstrained solution.

**Firm i is informed, receives signal \( s \) but withholds its information, \( m = \emptyset \).**

\[ y^*_i(m = \emptyset) = \frac{1}{3b} \{a - 2\bar{c}_j + (\gamma_i - \gamma) + (\delta_i - \delta)(1 - p)\hat{s} + pA\} - \epsilon \]

(a) If \( s < \hat{s}_2 \), then

\[ y^*_i(s, m = \emptyset) = \frac{1}{3b} \{a + \bar{c}_j - (2\gamma_i + \gamma) - \frac{3}{2}(\delta_i + \delta)s - \frac{(\delta_i - \delta)}{2}[pa + (1 - p)\hat{s}] + \frac{\epsilon}{2} \]
(b) If \( s \geq \delta_2 \), then \( y_i^*(s, m = \emptyset) = 0 \)

Again, the expected profit functions

\[
E(\pi_i^* | m = \emptyset) = b[y_i^*(m = \emptyset)]^2 \quad \text{and} \quad E(\pi_i^* | s, m = \emptyset) = b[y_i^*(s, m = \emptyset)]^2
\]

The term \( \frac{\delta_i}{2b} \) reflects the increase in the expectation of firm i’s output level given non-disclosure, when the non-negativity constraint is imposed. Since the disclosure set \( N \) contains some \( s \) that induce \( y_i^* = 0 \), firm i’s expected output given non-disclosure is higher in the constrained case. Since firm i’s expected output is higher, firm j’s optimal output level is lower, given non-disclosure in the constrained case. Similarly, since firm j’s output level is lower firm i’s optimal output level is higher.

Firm i’s optimal output functions given disclosure and non-disclosure for each of cases 1 and 2 are shown in figures 3.4(a) and 3.4(b), respectively. In case 1 \( \delta_1 > \delta \), therefore, firm i’s optimal output function given disclosure consists of three straight line segments. If \( s \geq \delta_1 \), then \( y_i^*(s, m = s) = 0 \), i.e., it coincides with the x-axis. If \( \delta_3 \geq s \geq \delta_1 \), then \( y_i^*(s, m = s) \) is the same as the corresponding output function in the unconstrained case in figure 3(a). We refer to this as the duopoly segment. If \( s \leq \delta_3 \), then firm j’s optimal output level is zero, and hence, firm i’s output function is the same as that of a monopolist and is referred to as the monopoly segment. Note that the slope of the monopoly segment is \( -\frac{\delta_1 + \delta}{2b} \) and the slope of the duopoly segment is \( -\frac{2\delta_1 + \delta}{2b} \). Hence, the monopoly segment of firm i’s optimal output function is flatter than the duopoly segment. In addition, the optimal output function is kinked but continuous at \( \delta_3 \).

Firm i’s optimal output function given non-disclosure consists of two segments. If \( s \geq \delta_2 \), then \( y_i^*(s, m = \emptyset) \) coincides with the x-axis. If \( s < \delta_2 \), then \( y_i^*(s, m = \emptyset) \) is parallel to but below the monopoly segment of \( y_i^*(s, m = s) \) in the constrained solution. Note that if firm j’s output level, \( y_j^*(m = 0) \), is zero then firm i’s optimal output function, \( y_i^*(s, m = \emptyset) \), is the same as the monopoly output function.

The constrained solution in case 2 is shown in figure 3.4(b). Firm i’s optimal output function given disclosure consists of two segments. If \( s \geq \delta_1 \), \( y_i^*(s, m = s) = 0 \) and if \( s < \delta_1 \), then \( y_i^*(s, m = s) \) is the same as the corresponding unconstrained solution. Note that in this case \( \delta_1 < \delta_3 \) and firm i is never a monopolist.

Firm i’s optimal output function, given non-disclosure, also consists of two segments. If \( s \geq \delta_2 \), \( y_i^*(s, m = \emptyset) \) coincides with the x-axis and if \( s < \delta_2 \), then the optimal output function \( y_i^*(s, m = \emptyset) \) has a lower slope but higher intercept than the corresponding \( y_i^*(s, m = s) \) function.
Disclosure Choice:

In the disclosure choice problem the relevant constraints are $y_i^*(s, m = s) \geq 0$ and $y_i^*(s, m = \emptyset) \geq 0$. In addition, we assume that firm $i$ withholds its information if its output level is zero. As before, the threshold level of disclosure is found by setting

$$E(\pi_i^*|s, m = \emptyset) = E(\pi_i^*|s, m = s) \equiv V(s|N) = V(s|D)$$

Under the additional constraints, $y_i^*(s, m = \emptyset) \geq 0$ and $y_i^*(s, m = s) \geq 0$ only one root survives, yielding,

$$y_i^*(s, m = \emptyset) - y_i^*(s, m = s) = 0 \equiv y(s|N) = y(s|D) = 0$$

Substitution for the optimal output levels and simplification yields one threshold level of disclosure,

$$s - pA - (1 - p)\bar{s} = 0 \Rightarrow \bar{s}' = \bar{s} + p(A - \bar{s})$$

Note, the threshold level $\bar{s}'$ in this constrained solution corresponds to $s'$ in the unconstrained solution but is not in general equal to $s'$.

**Lemma 3.5** If $\delta_i > \delta$, then $\bar{s}' > \bar{s}$ and if $\delta_i < \delta$, then $\bar{s}' < \bar{s}$.

The above lemma is proved by showing that if $\delta_i > \delta$ ($\delta_i < \delta$), then in the event of non-disclosure the opponent is uncertain as to whether firm $i$ has bad (good) news, i.e., $s$ is high (low), or no news, i.e., $s = \bar{s}$. Hence the threshold level of disclosure must lie above (below) $\bar{s}$. This yields the following disclosure (non-disclosure) sets:

If $\delta_i > \delta$, then the non-disclosure set is $(\bar{s}', +\infty)$ and the non-disclosure set $[-\infty, \bar{s}']$.

If $\delta_i < \delta$, then the non-disclosure set is $[-\infty, \bar{s}]$, $[\bar{s}_1, +\infty]$ and the disclosure set is $(\bar{s}', \bar{s}_1)$.

The disclosure (non-disclosure) sets found here are very similar to those found in the unconstrained solution. The threshold level $\bar{s}'$ in the constrained solution corresponds to the threshold level $s'$ in the unconstrained solution. In case 1 the upper threshold level goes to infinity in the constrained solution and in case 2 it goes up to $\bar{s}_1$. The relative values of $\bar{s}'$ and $s'$, however, are not independent of the density function of $s$.

It is useful to compare figures 3.3 and 3.4 at this point. The $y(s|D)$ functions in figures 3.3(a) and 3.3(b) are essentially the same as the duopoly segments of the $y(s|D)$ functions in figures 3.4(a) and 3.4(b), respectively. The $y(s|N)$ functions in figures 3.3(a) and 3.3(b)

\footnote{The proof is in Appendix A.2.1.}
have the same slopes but may have lower or higher intercepts than the corresponding \( y(s|N) \) functions in figures 3.4(a) and 3.4(b), respectively. The difference in the intercepts can be represented by the difference between 
\[
\frac{\delta_i - \delta}{2} - \left(1 - \frac{\delta_i - \delta}{2}\right) \left[1 - \frac{\delta_i - \delta}{2}\right] \left[1 - \frac{\delta_i - \delta}{2}\right] (1 - p) s + p A
\]
and 
\[
\frac{\delta_i - \delta}{2} - \left(1 - \frac{\delta_i - \delta}{2}\right) \left[1 - \frac{\delta_i - \delta}{2}\right] (1 - p) s + p A
\]
in the constrained solution. Note that the latter term is higher in the constrained than in the unconstrained solution for both case 1 and 2. This is due to the fact that \( A \) is higher (lower) in the constrained solution if \( \delta_i > \delta \) (\( \delta_i < \delta \)). In addition, the \( \frac{\delta}{2} \) term is positive. Therefore, the net difference could be either positive or negative.

The duopoly sections of the optimized expected profit functions \( V(s|D) \), depicted in figures 3.4(c) and 3.4(d) are essentially truncated versions of the \( V(s|D) \) functions in figures 3.3(c) and 3.3(d), respectively. The \( V(s|N) \) functions in figures 3.3(c) and 3.3(d), with their upper arms truncated have the same curvatures but different minima from the \( V(s|N) \) functions in figures 3.4(c) and 3.4(d).

### 3.4.6 Multiple Uninformed Opponents

In this section we examine the impact of increasing the number of firm i's opponents on its disclosure policy. As before firm i may or may not receive information. However, here firm i faces \( M \) uninformed opponents, indexed by \( j \), where \( j = 1, 2, \ldots, M \). The uninformed opponents are assumed to have homogeneous beliefs and their firm specific average costs are denoted by \( \bar{c}_j \). In such an oligopoly, firm j's optimal output functions under disclosure and non-disclosure are:

\[
y(s|D, M) = y^*_i(s, m = s, M) = \frac{1}{b(M + 2)} \left\{ a + \sum_{j=1}^{M} \bar{c}_j - \left[\gamma + (M + 1)\gamma_i\right] - \left[\delta + (M + 1)\delta_i\right] s \right\}
\]

\[
y(s|N, M) = y^*_i(s, m = \emptyset, M) = \frac{1}{b(M + 2)} \left\{ a + \sum_{j=1}^{M} \bar{c}_j - \left[\gamma + (M + 1)\gamma_i\right] - \frac{(M + 2)}{2} \left[\delta + \delta_i\right] s + \frac{M(\delta - \delta_i)}{2} [p A_M + (1 - p) \bar{s}] \right\}
\]

Note that similar to the duopoly case, the expected profit functions are:

\[V(s|D, M) = b[y(s|D, M)]^2 \quad \text{and} \quad V(s|N, M) = b[y(s|N, M)]^2\]

Assuming that all uninformed firms have the same expected firm-specific marginal cost, the above optimal output and profit functions can be depicted as in Figure 3.5. The \( y(s|D, M) \) functions intersect the duopoly \( y(s|D) \) functions at \( \hat{s}_3 \), where the opponents' total output is

\[18\text{Refer to Appendix A.1.4 for the algebraic derivation.}\]
zero. In case 1 if \( M > 1 \), the \( y(s|D,M) \) function which is downward sloping, is steeper, i.e.,
its slope is smaller than if \( M = 1 \). In case 2, however, the \( y(s|D,M) \) function is less steep,
i.e., its slope is larger, if \( M > 1 \) increases. The optimal output function given non-disclosure,
\( y(s|N,M) \), on the other hand, is merely shifted downward by a constant amount if \( M > 1 \).

It can be shown exactly as in the duopoly case, that in the oligopoly described above, two
threshold levels of disclosure \( s_{M}' \) and \( s_{M}'' \) exist. These are given by

\[
\begin{align*}
    s_{M}' &= \bar{s} + \frac{p(A_M - \bar{s})}{(M + 4)\delta + (3M + 4)\delta_i}, \\
    s_{M}'' &= \bar{s} + \frac{4b(M + 2)y(\bar{s}|D,M) + M(\delta - \delta_i)(s_{M}' - \bar{s})}{(M + 4)\delta + (3M + 4)\delta_i}
\end{align*}
\]

In case 1, \( s_{M}'' > s_{M}' > \bar{s} \) and in case 2, \( s_{M}' < \bar{s} < s_{M}'' \). The equilibrium non-disclosure and
disclosure sets are described as follows, exactly as in the duopoly case.

Case 1: \( \delta_i > \delta \) The equilibrium non-disclosure set is \([s_{M}', s_{M}'']\) and the disclosure set consists of
\([\bar{s}, +\infty]\) and \([s_{M}'', +\infty]\).

Case 2: \( \delta_i < \delta \) The equilibrium disclosure set is \([s_{M}', s_{M}''\]) and the non-disclosure set consists of
\([\bar{s}, +\infty]\) and \([s_{M}', +\infty]\)

The above expressions indicate that the basic form of the optimal output and expected profit
functions remain the same whether the number of uninformed opponents are greater than or
equal to one. It follows that the basic forms of the expressions for the threshold levels of
disclosure in an oligopoly are similar to those in a duopoly.

3.4.7 Discussion

This section compares the disclosure results obtained in this paper with those obtained in the
major classes of models in the voluntary disclosure literature. We first review the key features
of our model.

In our model, one of the firms in a duopoly setting may receive private information. Our
analysis focusses on the disclosure decision of the privately informed firm, labelled firm \( i \). In
our model.

- Firm \( i \) is an expected future value maximizer. The capital market and firm \( i \)'s possible
current market value are ignored in the entire analysis.

- Firm \( i \) may be informed, with probability \( q \). The information variable, \( s \), is correlated
with both the industry-wide and firm-specific marginal costs and have a multivariate
normal distribution function.
• The parameters of the joint distribution function are common knowledge.

• Firm i selects its output level and disclosure based on its information. Output levels may be positive or negative.

• Firm i’s opponent, firm j, behaves strategically, increasing or decreasing its output, based on firm i’s disclosure.

Given the above features, firm i’s private information may be termed “proprietary”. Disclosure/non-disclosure influences firm i’s expected profit, through the output price, which is a function of firm j’s reaction. However the proprietary nature of the private information does not play the same role in our model as in other models of voluntary disclosure of proprietary information.

Models used in the voluntary disclosure literature are classified below according to certain key attributes. The differences among these models, in their equilibrium disclosure policies, arise due to the differences in their attributes. A brief description of each class, along with the names of authors, are listed.
<table>
<thead>
<tr>
<th>Class</th>
<th>Attributes</th>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant proprietary cost of information</td>
<td>Verrecchia (1983)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dye (1986)</td>
</tr>
<tr>
<td>B</td>
<td>Current market value maximizer</td>
<td>Wagenhofer (1990)</td>
</tr>
<tr>
<td></td>
<td>Endogenous proprietary cost modelled in an entry game</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Current market value maximizer</td>
<td>Dye (1985)</td>
</tr>
<tr>
<td></td>
<td>Non-proprietary information</td>
<td>Jung and Kwon (1988)</td>
</tr>
<tr>
<td></td>
<td>Positive probability of being uninformed</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Expected future value maximizer in an entry game</td>
<td>Darrough and Stoughton (1990)</td>
</tr>
<tr>
<td></td>
<td>Endogenous proprietary cost affecting cost of capital</td>
<td>Feltham and Xie (1990)</td>
</tr>
</tbody>
</table>

In the papers classified in A, B and C, the informed firm is a current market value maximizer. The papers listed in A and B deal with information that is proprietary in nature. In A the proprietary cost, imposed by disclosure, is exogenously assumed to be constant. In B, on the other hand, the proprietary cost of disclosure is endogenously determined by the actions of a strategic opponent. The informed firm’s disclosure policy is determined by weighing its direct effect, on current market value due to the capital market’s reaction, against the indirect effect, through altered cash flow due to the opponent’s reaction. At the threshold levels of disclosure/non-disclosure these two effects are equal. The adverse selection argument,19 used to show how a disclosure equilibrium might unravel to one of full disclosure, does not hold in these papers. In the adverse selection argument, non-disclosure leads to the informed party being pooled with those possessing worse information, thus motivating all information to be disclosed. In A and B above, in the event of non-disclosure, the market is unable to ascertain whether it is due to the cost of disclosure being too high or due to firm \( i \) being uninformed. The “noise” introduced “noise” in the interpretation of non-disclosure, thus sustaining partial disclosure, in equilibrium.

In our paper, the informed firm \( i \) is an expected future value maximizer. The disclosure policy directly affects the objective function, due to the proprietary nature of the information. In the event of non-disclosure, the informed firm’s opponent is unable to ascertain whether it is due to the cost of disclosure being too high or due to firm \( i \) being uninformed. The “noise”

---

in the interpretation of non-disclosure, due to the possibility of firm \(i\) being uninformed, allows partial disclosure to exist in equilibrium. In this regard our model is similar to the models of voluntary disclosure of non-proprietary information listed in class C, above. In both Dye (1985) and Jung and Kwon (1988), the informed firm is a current market value maximizer. The capital market plays the same role as the product market does in ours. There is a positive probability that the informed firm is uninformed. Hence, full-disclosure is prevented, exactly as in our model.

Another class of models dealing with voluntary disclosure of proprietary information includes papers by Darrough and Stoughton (1990) and Feltham and Xie (1990), listed in D above. The informed firm is an expected future value maximizer with a strategic opponent imposing a proprietary cost of disclosure, as in our paper. In these papers, however, it is an entry decision by the opponent that imposes the proprietary cost. A third party considered in these models is the capital market, since the informed firm is assumed to need funds. The firm's disclosure determines the terms on which it can obtain funds. Hence the anticipated behavior of the capital market, in response to disclosure, influences the informed firm's disclosure decision. In these models, the "noise" in the interpretation of non-disclosure is provided by the capital market's reaction.

The equilibrium disclosure policy, in each model, is obtained by maximizing the objective function for each signal. Hence, the key determinants of the disclosure/non-disclosure sets are the shapes of the objective function under disclosure and non-disclosure. Therefore, the results are best illustrated by the use of diagrams, as in figure 3.6. In each diagram, the objective function under disclosure, \(V(s|D)\), and under non-disclosure, \(V(s|N)\),\(^{20}\) are plotted against \(s\), the information variable. Darrough and Stoughton (1990) is omitted since it uses binary supports for the information variable.

In both classes A and C, \(V(s|D)\) is monotonically increasing in \(s\) and \(V(s|N)\) is constant for all \(s\). Hence, in these classes, \(V(s|D)\) and \(V(s|N)\) intersect only once. This results in a single threshold level above which disclosure is optimal, and below which non-disclosure is optimal. In the models in classes B and D, \(V(s|D)\) has a sawtooth structure, intersected by a constant \(V(s|N)\) in class B and a positively sloped \(V(s|N)\) in class D. Hence, the equilibrium disclosure/non-disclosure sets, in both of these, consist of two disjoint disclosure regions, \(D_1\) and \(D_2\), and two disjoint non-disclosure regions, \(N_1\) and \(N_2\). In our model, both \(V(s|D)\) and

---

\(^{20}\)If production is involved in a particular model, then \(V(s|D)\) and \(V(s|N)\) are optimized with respect to output level.
$V(s|N)$ are convex quadratic functions of $s$, and hence, they intersect at two points.\textsuperscript{21} The relative curvatures of $V(s|D)$ and $V(s|N)$ determine the form of the equilibrium disclosure sets. When $\delta_i > \delta$, $V(s|D)$ is more curved than $V(s|N)$ and there are two disjoint disclosure regions and one non-disclosure region. On the other hand, when $\delta_i < \delta$, $V(s|N)$ is more curved than $V(s|D)$ and there are two disjoint non-disclosure regions and one disclosure region.

\textsuperscript{21}Please refer to figures 3.3c and 3.3d.
3.5 Comparative Statics

Firm $i$'s equilibrium disclosure set depends on the values of several different parameters. This section examines the effect of a change in the values of some of these parameters on the disclosure/non-disclosure sets in the duopoly setting.

3.5.1 Impact of the Probability of Information

As discussed above, in each class of model, “noise” is provided by different variables. In our model, this role is played by the probability $q$ of firm $i$ being informed. We next examine the relationship between $q$ and the equilibrium disclosure/non-disclosure sets, in our model.

Lemma 3.6

Case 1: $\frac{d\sigma'}{dq} > 0$, $\frac{d\sigma''}{dq} < 0$ and $\frac{d(\sigma''-\sigma')}{dq} < 0$

Case 2: $\frac{d\sigma'}{dq} < 0$, $\frac{d\sigma''}{dq} < 0$ and $\frac{d(\sigma''-\sigma')}{dq} > 0$

Lemma 3.6 is proved by signing the derivatives of $\sigma'$, $\sigma''$ and $(\sigma''-\sigma')$, with respect to $q$, in each case. In case 1, where $\delta_i > \delta$, the non-disclosure set is $[\sigma', \sigma'']$ and $\frac{d(\sigma''-\sigma')}{dq} < 0$, which implies that as $q$ increases, the non-disclosure (disclosure) set shrinks (expands). In case 2, where $\delta_i < \delta$, the disclosure set is $[\sigma', \sigma'']$ and $\frac{d(\sigma''-\sigma')}{dq} > 0$ which implies that as $q$ increases, the non-disclosure (disclosure) set shrinks (expands).

Increasing or decreasing $q$ affects the non-disclosure (disclosure) sets due to its effect on the opponent firm’s interpretation of non-disclosure. Non-disclosure affects the opponent’s estimate of expected marginal costs. As shown in expressions (3.27), (3.28) and (3.40),

$$E[c|m = \emptyset] = \gamma_i + \delta_i[pA + (1 - p)\tilde{s}] = \gamma_i + \delta_i\sigma'$$

$$E[c|m = \emptyset] = \gamma + \delta[pA + (1 - p)\tilde{s}] = \gamma + \delta\sigma'$$

where $[pA + (1 - p)\tilde{s}]$, the expected value of $s$ given non-disclosure, equals $\sigma'$, in equilibrium.

Inspection of the expression for the opponent’s optimal output, given by equation (3.9), shows that $E[c|m = \emptyset]$ affects the uninformed opponent’s output positively and $E[c|m = \emptyset]$ affects it negatively.

Case 1:

It is shown, in the proof of lemma 3.6 that when $\delta_i > \delta$, $\frac{d\sigma'}{dq} > 0$. This implies that $\frac{dE[c|m = \emptyset]}{dq} > 0$. Therefore, as $q$ increases, firm $j$’s optimal output increases. This causes the output price to fall, forcing the informed firm to lower its optimal output function, given non-disclosure. In
figure 3.3(a), the above reduction corresponds to a parallel, downward shift of the straight line, labelled, $y^*_i(s, m = \emptyset)$. This is also consistent with the fact that only the intercept of $y^*_i(s, m^* = \emptyset)$, in expression (3.30), is a function of $q$ through $p$ and $A$.

By lemma 3.4, $y^*_i(s', m^* = s') = y^*_i(s', m^* = \emptyset)$ and $y^*_i(s'', m^* = s'') = -y^*_i(s'', m^* = \emptyset)$. Inspection of figure 3.3(a) shows that, as $q$ increases, $y^*_i(s, m = \emptyset)$ shifts downwards. This has a positive impact on $s'$ since the point at which the two lines intersect moves to the right. The effect on $s''$ is negative since as the line $y^*_i(s, m = \emptyset)$ shifts downwards, the point at which $y^*_i(s, m = \emptyset) = -y^*_i(s, m = s)$ decreases. As a result of these, the non-disclosure set shrinks.

**Case 2:**

In the proof of lemma 3.6, it is shown that when $\delta_i < \delta$, $\frac{d\delta_i}{dq} < 0$. Since $E[c|m = \emptyset] = \gamma + \delta s'$, as $q$ increases, $E[c|m = \emptyset]$ decreases, increasing firm $j$'s optimal output. As in case 1, this causes the output price to fall and a downward shift in firm $i$'s optimal output function, given non-disclosure. In figure 3.3(b), this is a parallel, downward shift of the straight line $y^*_i(s, m^* = \emptyset)$. Again, due to lemma 3.4, $s'$ and $s''$ are determined such that, $y^*_i(s', m^* = s') = y^*_i(s', m^* = \emptyset)$ and $y^*_i(s'', m^* = s'') = -y^*_i(s'', m^* = \emptyset)$. Therefore, in figure 3.3(b), as $q$ increases and $y^*_i(s, m = \emptyset)$ shifts downward, $s'$ decreases. The effect on $s''$ is also negative since as the line $y^*_i(s, m = \emptyset)$ shifts downwards, the point at which $y^*_i(s, m = \emptyset) = -y^*_i(s, m = s)$ decreases. The net effect of these two reductions is a shrinkage of the disclosure set.

Tables 3.1 and 3.2 give values of $s'$ and $s''$ as $q$ changes from 0.1 to 1.0. These are plotted in figures 3.7(a) and 3.7(b), which show that $s'$ and $s''$ approach each other in case 1 and diverge in case 2, as $q$ increases.

Basically, in both cases, as the probability of being informed (uninformed) increases (decreases), non-disclosure by the informed firm is interpreted as being more likely due to high proprietary cost of disclosure than lack of information. This forces the informed firm to disclose more of its information. These results correspond to those in Dye (1985) and Jung and Kwon (1988). In these papers the non-disclosure set is the region below a threshold level. In both papers it is shown that the threshold level falls, i.e., the non-disclosure set shrinks, as the probability that the informed firm receives information increases.

The probability that firm $i$ is informed influences its probability of disclosure, in addition to the disclosure sets. The probability of disclosure is denoted by $P(D)$. From the probability of non-disclosure given by expression (3.14), $P(D)$ is defined as,

$$P(D) = q \left\{ 1 - \int_{s \in N} f(s) ds \right\} = qP(s \in D)$$

(3.45)
Proposition 3.2 In case 1, $\frac{dP(D)}{dq}$ is greater than zero, whereas, in case 2, $\frac{dP(D)}{dq}$ may be greater than or less than zero.

The probability of disclosure, $P(D)$, is the product of the probability of being informed, $q$, and the probability that an informed firm discloses, $P(s \in D)$. Increasing $q$ increases the first probability. The second probability may be increasing or decreasing in $q$. In case 1, the probability that an informed firm discloses is always increasing in $q$. As $q$ increases, the threshold levels $s'$ and $s''$ move toward each other, shrinking/expanding the non-disclosure/disclosure set. This results in an unambiguous increase in the areas in the tails of the density of $s$ which corresponds to an increase in $P(s \in D)$. Hence, in case 1 the probability of disclosure is increasing in $q$.

In case 2, however, the probability that an informed firm discloses may be either increasing or decreasing in $q$. As $q$ increases, both thresholds move in the same direction, $s'$ moving towards the lower tail of the density function of $s$ and $s''$ moving away from the upper tail. As shown in lemma 3.6, the magnitudes of the above movements are such that the disclosure set, $s'' - s'$, increases. This does not directly translate to an increase in $P(s \in D)$, which is a function of both the threshold levels and the density function of $s$. Since, in case 2, $s'$ and $s''$ are on opposite sides of the mean, $P(s \in D)$ may either increase or decrease depending on the relative values of the density function at $s'$ and $s''$. Hence, the probability of disclosure may either increase or decrease with $q$, in case 2.

Figure 3.8(a) and 3.8(b) plot the probability of disclosure against $q$, using the numerical values calculated in tables 3.1 and 3.2. The parameter values used give probability of disclosure increasing with probability of information, in both cases 1 and 2.

A natural question that arises at this point is, what are the forms of the disclosure (non-disclosure) sets when firm $i$ is informed with certainty, i.e., when $q = 1$. Dye (1985) and Kwon and Jung (1988) both find that as the disclosing firm's being informed becomes a certainty, the equilibrium disclosure policy unravels to one of full disclosure. This result is consistent with the adverse selection argument discussed earlier.

In our model, the limiting cases have the following characteristics.

Proposition 3.3 As the probability of firm $i$ being informed, $q$, becomes 1, the equilibrium disclosure policy becomes.

- **Full-disclosure**, in case 1, with $s' = A = s''$, and
- **Partial-disclosure**, in case 2, with $s' = A < s''$.  

59
The result in case 2 differs from that obtained by Dye (1985) and Jung and Kwon (1988). This is due to the quadratic nature of the objective function, which gives rise to incentives for non-disclosure over a set consisting of two disjoint extreme regions of $s$. At low and moderate values of industry-wide marginal cost, the output level is positive. Therefore, low $s$ are withheld and moderate $s$ are disclosed. However, at extremely high values of the industry-wide cost, the optimal output level becomes negative. Disclosure of such high $s$ would result in a reduction in the opponent's output and an increase in output price. Given that firm $i$ is a net consumer at these high values of $s$, the opponent's reaction to disclosure would impose a "proprietary cost" on the informed firm. Thus the firm would prefer to withhold its information for high values of $s$, in addition to low values.

In case 1 we obtain the same result as in Dye (1985) and Jung and Kwon (1988), when the probability of information becomes 1. This is due to the fact that firm $i$ is motivated to withhold its private information over a single continuous region of $s$. At low and moderate values of firm-specific cost, the optimal output level is positive. Therefore low $s$ are disclosed and moderate values of $s$ are withheld. Although the optimal output level is negative, at high levels of firm-specific cost, disclosure leads to an increase in the opponent's output, resulting in a decrease in the output price. This in effect amounts to a "proprietary gain" if the information is disclosed. Therefore high values of $s$ would be disclosed.

When $q = 1$, in case 1, firm $j$ interprets non-disclosure as bad news for firm $i$. Due to this unequivocal interpretation of non-disclosure and adverse selection, the equilibrium disclosure policy unravels to full-disclosure. In case 2, however, in the event of non-disclosure, firm $j$ is unable to ascertain whether firm $i$ has highly favourable news and is a net producer or it has unfavourable news and is a net consumer of the output. This uncertainty in the interpretation of non-disclosure provides the "noise" necessary to sustain a partial disclosure equilibrium.

These results are illustrated by the numerical example in tables 1 and 2 and the plots in figures 5a and 5b. The values of $A$, $s'$ and $s''$, corresponding to $q = 1$, show that at this limiting value $A = s' = s''$ in case 1 and $A = s' < s''$ in case 2.

### 3.5.2 Impact of Competition

The objective of this section is to examine the effect of the competition, faced by the informed firm, on its disclosure policy. A deterioration in the firm's competitive position, in the product market, may be achieved either by increasing the number of firms or by decreasing the existing opponent's marginal cost of production. Both result in an increase in the total output produced.
by the informed firm’s opponent/opponents, thus exerting a downward pressure on the informed firm’s output and profit. Conversely, an improvement in the informed firm’s competitive position may be achieved by decreasing the number of opponents or by increasing the existing opponent’s marginal cost of production. Here the informed firm’s competitive position is defined in terms of the opponent’s firm-specific marginal cost. Hence, a deterioration in the firm’s competitive position refers to a decrease and an improvement in the firm’s competitive position refers to an increase in the opponent’s firm-specific marginal cost. The effects of changes in the opponent’s firm-specific marginal cost on the lower and upper threshold levels of disclosure, and on the disclosure/non-disclosure sets, are given in the following lemma.

**Lemma 3.7**

*Case 1:* \( \frac{ds'}{d \epsilon_j} > 0 \) and \( \frac{ds''}{d \epsilon_j} > 0 \) and \( \frac{d(s''-s')}{d \epsilon_j} > 0 \)

*Case 2:* \( \frac{ds'}{d \epsilon_j} < 0 \), \( \frac{ds''}{d \epsilon_j} > 0 \) and \( \frac{d(s''-s')}{d \epsilon_j} > 0 \)

Lemma 3.7 may be reworded in terms of the informed firm’s competitive position, i.e., decreasing \( \bar{c}_j \), as follows. In case 1, as the competitive position of the informed firm deteriorates, i.e., as \( \bar{c}_j \) decreases, the threshold levels \( s' \) and \( s'' \) both decrease, and the non-disclosure region, \([s''-s']\), shrinks. Therefore, if \( \delta_i > \delta \), then a more competitive environment encourages voluntary disclosure. In case 2, as the competitive position of the informed firm improves, i.e., as \( \bar{c}_j \) increases, the threshold level \( s' \) increases and \( s'' \) decreases and, hence, the non-disclosure regions \([s'-\infty]\) and \([s'',+\infty]\) expand. Therefore, if \( \delta_i < \delta \), then voluntary disclosure is discouraged as the informed firm’s environment becomes more competitive.

In order to compare our results with those obtained by other authors, we ignore the possibility of negative optimal output. Then the non-disclosure region, in case 1, is \([s',+\infty]\) and in case 2, is \([-\infty,s']\). Since the relevant derivative, \( \frac{ds'}{d \epsilon_j} \), is greater than zero in case 1 and less than zero in case 2, the non-disclosure region shrinks as \( \bar{c}_j \) increases (i.e., competition decreases), in both cases. This result is consistent with the empirical interpretation provided for the corollary in Verrecchia (1983). The corollary states that the non-disclosure region expands as the proprietary cost increases. If we interpret the increase in the proprietary cost as resulting from a deterioration in the informed firm’s competitive position in the industry, then we can interpret the corollary as stating that disclosure of private information is discouraged by a more competitive environment.

Darrough and Stoughton (1990), however, can be interpreted as obtaining the opposite result, by modelling competition as the threat of entry. In their paper, the incumbent is an informed firm. An entrant will enter the product market depending on the informed firm’s
disclosure. The magnitude of the entrant's entry cost determines the degree of threat to the incumbent. The paper finds that full-disclosure exists, in equilibrium, when the prior is optimistic or the entry cost is relatively low, and non-disclosure exists, in equilibrium, when the prior is pessimistic relative to the cost of entry. Comparing the two results, Darrough and Stoughton state:

"An important implication of our model is that competition through threat of entry encourages voluntary disclosure. ... We conclude that since low entry costs lead to a higher entry probability, full disclosure ensues under competitive pressure."

There is a parallel between the variables used to define competition in Darrough and Stoughton's paper and in ours, in terms of their effects on the informed firm. Their "entry" cost, i.e., the cost incurred by the entrant if entry occurs, corresponds to the opponent's firm-specific marginal cost in our paper. Their probability of entry, coupled with the cost incurred by the incumbent if entry occurs, corresponds to the opponent's optimal output level in our paper. However, since they have binary support for their information variable they do not obtain any partial disclosure equilibria. Hence their results are non-comparable with ours.

Feltham and Xie (1990) and Wagenhofer (1990) also model competition as the threat of entry. Both these papers use continuous support for their information variable. The variables in their models parallel ours in a manner similar to Darrough and Stoughton's model. In these two models, however, entry cost refers to the cost incurred by the incumbent if entry occurs. Wagenhofer finds that as entry cost decreases (within the partial disclosure region) there is less expected disclosure. This result is consistent with ours. Hence, this model like ours, finds that competition or its threat discourages voluntary disclosure. Feltham and Xie does not deal explicitly with this issue. However, it appears that in their fixed entry cost case, an increase in the threat of entry may lead to a decrease in the disclosure set, as in Wagenhofer's model.

In case 1 of our model, if \( s'' \) is not ignored, we obtain the counter-intuitive result that disclosure is encouraged by a more competitive environment. This result is driven by the fact that \( \frac{df''}{dc} > \frac{df'}{dc} \).

We next examine the effect of the informed firm's competitive position on its probability of disclosure, \( P(D) \), which is given by expression (3.45).

**Proposition 3.4** In case 1, \( \frac{dP(D)}{dc} \) may be greater or less than zero, whereas, in case 2, \( \frac{dP(D)}{dc} \) is always greater than zero.
As pointed out before, the probability of disclosure is a product of the probability of being informed, $q$, and the probability that an informed firm discloses, $P(s \in D)$. The first probability, $q$, is independent of $c_j$. Hence, how the probability of disclosure varies with $c_j$ is determined by whether the second probability increases or decreases as $c_j$ increases.

In case 1, the increase in $s''$ due to an increase in $c_j$ is greater than the increase in $s'$. However since $s'' > s'$ and both are above the mean of $s$, $f(s'')$ is always less than $f(s')$. Therefore, as $c_j$ increases the probability between $s'$ and $s''$, i.e., the probability of non-disclosure, may be increasing or decreasing. Hence, in case 1, the probability of disclosure may increase or decrease as the opponent’s firm-specific marginal cost increases.

In case 2, on the other hand, as $c_j$ increases, $s'$ decreases and $s''$ increases leading to an expansion in the disclosure set. Hence there is an unambiguous increase in the probability of disclosure.
3.6 Empirical Implications

The results in this paper indicate that in an imperfectly competitive industry, a privately informed firm's voluntary disclosure policy depends on whether the information signal is more informative about firm-specific or industry-wide common factors. Specifically, the informed firm withholds information that imposes a proprietary cost on the firm and makes disclosures that yield benefits. In such a setting the informed firm can benefit by disclosing "good news" about firm-specific factors, thus forcing its opponents to reduce their output levels. The firm prevents its opponents from increasing their output levels and imposing a proprietary cost, by withholding "bad news" about firm-specific factors. In the case of industry-wide common factors the incentives are reversed. Hence, "bad news" is disclosed to reduce the opponents' output levels and "good news" is withheld to prevent the opponents from increasing their output levels. Note that negative output levels are ruled out in this discussion in order to maintain comparability with other empirical studies.

Empirically, a disclosing firm whose signal has a higher correlation with firm-specific uncertainty than with industry-wide uncertainty (case 1) will on the average disclose "good news" and withhold "bad news". Conversely, a firm whose signal has a higher correlation with industry-wide uncertainty than with firm-specific uncertainty (case 2) will on the average disclose "bad news" and withhold "good news". The first type of empirical prediction is consistent with that made by several other analytical models and has been tested empirically.

To judge whether a signal is good or bad news it has to be compared to a prior expectation about the signal by the market. Due to the difficulty of determining a prior, most empirical studies differentiate between good and bad news by its effect on the unexpected component of the return on the common stock of the disclosing firm. Using this methodology some empirical findings are consistent with disclosure of good news and non-disclosure of bad news. Several other studies have found that there is no significant bias in the unexpected returns of disclosing firms. Our analytical results offer a possible explanation for the above inconsistency.

Partitioning the sample of disclosing firms according to the type of uncertainty their signal is more informative about, we should expect to find the following.

- If the signal is more informative about firm-specific uncertainty, then the average abnormal

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return on the disclosing firms’ stock, associated with the disclosure should be positive.

- If the signal is more informative about industry-wide uncertainty, then the average abnormal return on the disclosing firms’ stock, associated with the disclosure could be either positive or negative.

The abnormal return for a disclosing firm, associated with the disclosure of a signal, is equivalent to \( V(s|D) - V(\bar{s}|D) \) in our model. Hence, the average abnormal return given disclosure is

\[
\int_{s \in D} [V(s|D) - V(\bar{s}|D)]f(s)ds
\]

Recall that \( V(s|D) \) is the informed firm’s expected profit function given disclosure. The above quantity is positive in case 1. However, in case 2 the convexity of the expected profit function prevents us from signing the above expression unequivocally. Figure 3.9 is useful in following the above argument.

As a counterpoint to the above predictions for the disclosing firms we present the following predictions for the non-disclosing firms.

- In case 1 the non-disclosing firms realize a negative (positive) abnormal return when the disclosing firm’s abnormal return is positive (negative).

- In case 2 the non-disclosing firms realize a positive (negative) abnormal return when the disclosing firm’s abnormal return is positive (negative).

These predictions are supported by the following comparisons of figure 3.5 and figure 3.9. We focus on the disclosure sets. In case 1 when \( s < \bar{s} \), then \( V_j(s|D) < V_j(\bar{s}|D) \) and \( V(s|D) > V(\bar{s}|D) \) and when \( \bar{s} < s < \bar{s}' \), then \( V_j(s|D) > V_j(\bar{s}|D) \) and \( V(s|D) < V(\bar{s}|D) \). On the other hand, in case 2 when \( \bar{s}' < s < \bar{s} \), then both \( V_j(s|D) > V_j(\bar{s}|D) \) and \( V(s|D) < V(\bar{s}|D) \) and when \( \bar{s} < s < \bar{s}_1 \), then \( V_j(s|D) < V_j(\bar{s}|D) \) and \( V(s|D) < V(\bar{s}|D) \). Note that in order to test this prediction it is necessary to distinguish between firms that are withholding information and its uninformed opponents.

In addition to the above predictions, we include an empirical prediction obtained from the comparative statics result with respect to probability of information. It has been shown that as the probability of being informed, \( q \), increases the non-disclosure set shrinks in both cases 1 and 2. However, the impact on the threshold level of disclosure differ in the two cases. In case 1 an increase in \( q \) results in an upward move in the threshold level of disclosure, \( s' \), which shrinks the non-disclosure set. Since the firm withholds bad news, an increase in \( q \) leads to disclosure.
of worse news which would be withheld at a lower $q$. If we assume that $q$ increases over time within a period, then the above result implies that "better" news is disclosed earlier in the period and "worse" news is revealed later in the period. This is consistent with the prediction made by Jung and Kwon (1988) and is supported by some empirical evidence on the timing of disclosure.$^{26}$

However, our results in case 2 indicate that as $q$ increases the threshold level of disclosure $s'$ decreases. Since the firm withholds good news, an increase in $q$ leads to disclosure of better news which would be withheld at a lower $q$. Hence as $q$ increases over a period, worse news is disclosed earlier and better news later in the period. Since no other analytical model makes a similar prediction, it would be interesting to test this hypothesis after partitioning the sample for cases 1 and 2.

$^{26}$Pastena and Ronen (1979).
3.7 Conclusion

In this paper we have examined the equilibrium discretionary disclosure policy of a privately informed firm in a duopoly. Since it is not possible to obtain even an existence result given a general distribution function, we have assumed that the information variable, the firm specific marginal cost, and the industry-wide marginal cost are jointly normal. This assumption gives us conditional expected cost functions which are linear in the information variable. We have also assumed that the parameters of the model are common knowledge.

We find that if the information variable is more informative about firm-specific cost, then below average signals are withheld with above average and extremely bad signals being disclosed. Whereas, if the informed firm is more informative about industry-wide common cost, then extreme values of the signal are withheld. The basic result is robust with respect to the number of uninformed opponents of the disclosing firm.

We also check the sensitivities of the disclosure sets and the probability of disclosure to the probability of being informed and to the degree of competition faced by the disclosing firm. These indicate that the range of disclosed values increases with increasing probability of being informed. The probability of disclosure may increase or decrease depending on the values of the other parameters in case 2, but is always increasing in case 1, as the probability of being informed increases. As the opponent’s competitive position improves, the range of the disclosed values increases in case 1, and contracts in case 2. Improvements in the opponent’s competitive position decreases the probability of disclosure in case 2, but in case 1 it may either increase or decrease the probability that the firm will disclose.

The results in this paper suggest that a distinction between firm-specific and industry-wide common information in the design of empirical tests may prove useful. Partitioning the sample along these lines, for example, would help eliminate any confounding effects of the relation of information to both firm-specific and industry-wide parameters, and lend more power to the empirical test.

In case 1 we would expect to find a predominance of “good” signals in disclosures and in case 2 a predominance of “bad” signals. In terms of stock price behavior at the time of disclosure, in case 1 the average abnormal stock return of disclosing firms is expected to be positive. However, in case 2 the disclosing firms average abnormal return is not independent of the other parameters of the model. In case 1 the non-disclosing firms’ price reactions are expected to be the reverse of those experienced by the disclosing firm and in case 2 both the
disclosing and non-disclosing firms are expected to experience similar price reactions.

If the probability of the receipt of information is time dependent then in case 1 the probability of disclosure increases over time in both cases. In case 1 (case 2) better (worse) news is disclosed earlier with worse (better) news being disclosed later. The above empirical implications indicate that failure to partition a sample of disclosing or non-disclosing firms as suggested, may have a confounding effect on the empirical evidence.
Chapter 4

Disclosure and Reputation in Credit Markets
4.1 Introduction

It has been suggested that the empirically observed positive average stock price change which accompanies the release of earnings forecasts is essentially a reaction to the act of disclosure per se.\footnote{Patell (1976).} This suggestion is consistent with the story that the possession of private information by a particular manager is positive news about that manager. An informed manager may hence be motivated to disclose to the market that he has private information.\footnote{The term market refers to any party with which the firm may interact.} Under certain conditions, this scenario may be the result of the manager’s attempt to establish a reputation.

The model in this paper is based on exactly such a motivation for disclosure. The manager in this model could be more or less talented at obtaining information which enables him to avoid outcomes that can lead to insolvency. The firm’s potential creditors are concerned about the possibility of insolvency. Voluntary disclosure helps a more talented manager to establish a reputation for himself, over time.

The idea that a reputation effect may be responsible for certain actions of firms has been discussed in a number of papers in the area of industrial organization and imperfect competition. However, the first formal modelling of reputation is found in Kreps and Wilson (1982) and in a more applied context in Milgrom and Roberts (1982). Prior to this, Selten had shown that although the idea of a reputation effect had intuitive appeal, in many contexts reputation building behaviour did not hold up in equilibrium. Selten examined the problem of entry deterrence in a multiperiod setting with perfect information. In any period a monopolist decides whether to incur costs by fighting back if entry occurs or to share the market quietly. If the monopolist wants to establish a reputation as a fighter to deter entry in the future, he would fight back. However, this kind of behaviour breaks down by the following logic. In the last period the monopolist is not motivated to fight since there are no future periods for which to build a reputation. Since this is common knowledge, entry occurs in the last period and the monopolist acquiesces. The monopolist knows that no matter what he does in the second last period, entry will occur in the last period; hence, he has no incentive to fight back in the second last period and entry occurs. Repeated application of this logic leads to an unravelling of any reputation building behaviour. This is referred to as the chain store paradox.

Kreps and Wilson (K/W) introduce asymmetric information about the monopolist’s type to Selten’s chain store problem. As a result, a potential entrant assigns a positive probability to the monopolist’s being a fighter even in a one shot game. If this probability is high enough
in any period it deters entry, thus preventing the reputation equilibrium from any further unravelling. Milgrom and Roberts use a somewhat different approach to the same problem. Their results indicate that a monopolist would use predation to discourage entry. To summarize the conditions under which reputation building arises we quote from the conclusion of Milgrom and Roberts' paper:

"....conditions will be necessary for reputation building to occur in general, and it would further seem that they are sufficient: in any situation where individuals are unsure about one another’s options or motivation and where they deal with each other repeatedly in related circumstances (or where past dealings with other people are observable), we would expect to see reputations develop.”

The concept of reputation has since been applied to several accounting and finance problems. Holmstrom (1982) introduces a reputational effect in a two-period agency setting. Kanodia, Bushman and Dickhaut (1988) apply the concept of reputation acquisition in a two period setting to explain the seemingly irrational phenomenon of escalation behavior or sunk cost effect.

Trueman (1986) presents a model in which voluntary disclosure conveys information about a manager’s ability to obtain information early in the period. Earlier information translates to a higher end of period expected profit and ultimately to a higher payoff to the manager. The earlier the manager receives information in the first period the earlier he is likely to receive information in the second period. Therefore when the manager receives information in the first period he also becomes privately informed about the timing of his information in the second period. The reputation effect comes into play and in equilibrium the manager discloses all his information as soon as he receives it. There is no possibility of non-disclosure in Trueman’s model. This is counter to casual empirical observation of voluntary disclosures.

Reputation acquisition in debt markets is discussed in Diamond (1988). The reputation game in Diamond’s analysis is played between a firm and its present and potential creditors in a multiperiod setting. The firm’s type is defined with respect to the kind of projects that are available to its manager in each period. The projects are either safe or risky where the risky project can lead to bankruptcy. The model assumes that end of period revenues are not publicly observable. Hence, current and potential creditors learn about the firm’s type when it goes bankrupt and cannot meet its debt obligations. The risky project is also assumed to yield a higher expected profit to the equityholders, thus giving rise to the classic conflict of interest between stock and bondholders. If this was a one shot game, equity value maximizing
managers would choose the risky project. However, due to the existence of multiple periods, the reputation effect comes into play. Motivated by the possibility of reducing future interest payments, the firm manager chooses the safe project to establish a reputation as a safe firm.

In general however, contrary to Diamond’s assumptions, a firm’s end of period revenue realizations are public information. Hence, as long as different decisions result in different probability distributions over the set of revenues, creditors learn about the firm’s decision by observing its revenue realizations. In addition, since the choice of a suboptimal project to signal type is costly, there is a strong incentive for the firm to use an alternative signalling mechanism or use the mechanism jointly with revenue to reduce cost.

This paper shows that a voluntary disclosure may be such an alternative mechanism. We analyze the reputational effect of the voluntary disclosure of a manager’s private information in a setting where the manager’s “opponents” in the reputation game are the potential creditor’s of the firm. The central analysis is carried out in a two period setting. Each period, the manager may or may not receive private information, and must decide whether to make a disclosure to inform potential creditors of his information gathering talent. If the manager receives pre-decision information in some periods, he is a “good” manager. On the other hand, if he never receives pre-decision information, he is a “bad” manager. Hence, the ability to obtain information prior to his operating decision defines a manager’s type.

The manager’s pre-decision information facilitates his output choice. If the manager is informed he can make a better output choice from the creditors’ viewpoint. The credit market establishes a direct link between reputation and future expected profit through future interest requirements. Since reputation is a function of disclosure, expected future profit becomes a function of disclosure. Hence, disclosure of private information can lead to higher reputation leading to lower future interest payments which result in higher expected profits in the future.

In the model presented here the manager obtains funds from the credit market to finance production. For modelling convenience we assume that the funds are obtained by repeated short term borrowing. This corresponds to a monopolist’s repeated encounter with new entrants, in Kreps and Wilson’s paper. This assumption, however, is not essential in our analysis since, unlike in K/W, the manager in our model can signal his type in a particular period even if he does not borrow in that period. In fact very similar results would be obtained if we replaced the assumption of repeated borrowing by an anticipation of borrowing in the last period. We would however have to ensure that the incremental effect of the action which increases reputation, decreases over time.
The reputation of the manager is affected by the end of period revenue and the disclosed signal. We first examine a benchmark case where the manager is restricted from making any disclosures. Following that we analyze a disclosure regime in which the manager can make truthful disclosures. The two different regimes enable us to isolate and better understand the impact of disclosure on reputation acquisition.

In the final period, under both regimes, the manager behaves in a myopic manner, irrespective of his type. In the first period, however, if the gain from reputation acquisition is high enough, it can motivate the manager to adopt a long run perspective. In the non-disclosure regime there are three possible reputation acquisition strategies and one myopic strategy. The choice among alternative projects involves trade-offs between the difference in the expectations of the current period gain versus the difference in future gain from these projects. If the revenue differential between the risky and the safe projects then the myopic strategy predominates in equilibrium. At intermediate values of the differential revenue, different strategies involving reputation building by the uninformed managers may become consistent with an equilibrium at intermediate values of the prior reputation. The myopic strategy continues to exist in equilibrium at extreme values of the prior reputation. At low values of the revenue differential, if the manager receives favourable private information and his prior reputation is not high, he may prefer to engage in reputation acquisition.

In the disclosure regime the creditors learn about the manager’s type from his disclosures as well as his end of period revenues. If the manager is informed he always discloses his information thus separating himself from the bad type. Hence, the informed manager does not need to make a sub-optimal project choice in order to acquire reputation. The uninformed firm who expects to receive information in the future has a “stronger” incentive to choose the sub-optimal project than in the non-disclosure regime. This is due to the fact that a sub-optimal project choice together with a truthful disclosure may result in a “perfect” reputation.

The rest of the paper is organized in the following manner. The basic description of the model and its notation are given in the first section. This is followed by a section containing a discussion of the single period setting and interest rate determination in a competitive credit market. The next section presents the non-disclosure regime in a two period setting. This is followed by a disclosure regime in the two period setting. The final section concludes the paper.
4.2 The Model

We consider a firm which has a two period time horizon. In period $t$, where $t = 1, 2$, the firm generates revenue $R_t$, which is a function of the project chosen in period $t$, $y_t$, and two state variables, $\theta_t$ and $\eta_t$. The firm can choose between two projects, $y_l$ and $y_h$. The state variable $\theta_t$ has binary support and may take on either a high or a low value in any given period, with the following probabilities:

$$\theta_t = \begin{cases} h & \text{with probability } n \\ l & \text{with probability } (1 - n). \end{cases}$$

$\theta_t$ is the state of nature at the time of project choice in period $t$. The state variable $\eta_t$, on the other hand, is not realized till the end of period $t$ and it is continuous with probability density function $g(\eta_t)$ over $E$. Both state variables are firm specific. The firm's revenue in period $t$ is $R_t = R(\theta_t, y_t, \eta_t)$. We use $g(R_t|\theta_t, y_t)$ to denote the conditional density function of $R_t$ given $\theta_t$ and $y_t$.

The firm's revenue is also dependent on its manager's "ability". The manager's ability may be either good ($G$) or bad ($B$). This is the manager's basic type that is unchanging over time. The difference in ability reflects the difference in the probability with which the manager acquires information prior to choosing his project. We assume that a type $B$ manager never receives information prior to his operating decision. However, the type $G$ manager's "ability" includes an time dependent component which changes randomly over time. In any period $t$ the type $G$ manager may either be informed with probability $q$ and observe either $\theta_t = h$ or $\theta_t = l$ or be uninformed. Hence, in period $t$ the type $G$ manager may actually be of type $G_t \in \{Gh, Gl, Gu\}$, where $Gh$ and $Gl$ indicate the manager's type if he observes $\theta_t = h$ and $\theta_t = l$, respectively, and $Gu$ indicates that the manager is uninformed. Hence, the manager's type in period $t$ is denoted by $\tau_t \in \{Gh, Gl, Gu, B\}$. However, the $t$ subscript is omitted from $\tau_t$ in the rest of this paper for ease of presentation.

In order to operate, the firm must invest one unit of capital. This is raised by issuing short term debt at the beginning of each period, with an obligation to pay back an amount $r$ at the end of the period.\(^3\) Hence, $r$ is the nominal return and the nominal interest rate is $r - 1$. In this model, it is not necessary that production be financed entirely by debt. Financing may consist of a mix of internal funds, equity and debt. This paper does not model the firm's financing choice. Since our focus is on the firm's production and disclosure choices and their impact on

\(^3\) $r = 1 +$ the nominal return rate.
learning by creditors we do not explicitly consider any other form of financing. It is, however, important that the firm be concerned about its reputation in the credit market or its credit rating. This feature can be incorporated in the model either through short term debt which is issued every period, through long term debt which is reissued whenever it matures, or a single borrowing in the last period. We choose the first option for algebraic convenience.

Creditors are risk neutral and credit markets are perfectly competitive, allowing creditors an expected return equal to that from a risk-free asset. We denote the risk-free return by \( r_f \), which is the nominal return specified in a risk-free debt contract. Without loss of generality we can set \( r_f = 1 \). In general the manager's type is not common knowledge. Hence, the nominal return required in period \( t \) is \( r_t \). This return is a function of \( \alpha_{t-1} \), the creditor assessed probability that the manager is type \( g \), at the beginning of period \( t \). The probability \( \alpha_{t-1} \) is a measure of the manager's reputation at the beginning of period \( t \). It can also be interpreted as the credit rating of any debt issued by the manager in period \( t \).

We examine a scenario in which direct disclosure of manager type is not verifiable and, therefore, is not credible. Hence \( \alpha_{t-1} \) is a function of other variables whose values are affected by manager type. Further, we ignore agency issues and do not differentiate between a firm and its manager; the two terms are used interchangeably.

At the beginning of each period the manager maximizes the expected net present value of future profits of the firm. Given our assumption that \( r_f = 1 \), the discount factor is unity. The following notation is used:

\[
\begin{align*}
    t & \quad \text{The end of period } t \\
    t_o & \quad \text{The beginning of period } t \\
    V_t & \quad \text{The expectation, at the beginning of period } t, \text{ of the net present value of all future profits} \\
    E_t[V_{t+1}] & \quad \text{The expectation of } V_{t+1}, \text{ at the beginning of period } t. \\
    \pi_t & \quad \text{Profit, net of the cost of the investment (including interest), in period } t \\
    E_t[\pi_t] & \quad \text{The expectation of } \pi_t, \text{ at the beginning of period } t
\end{align*}
\]

\(^4\)Myers and Majluf (1984) focus on the financing decision in a model where a firm has insufficient internal funds to finance a positive NPV project. The manager maximizes the value of the firm's existing equity and there is informational asymmetry between the manager and the investors. The model finds that if external financing is used then debt is preferred to equity.

\(^5\)Creditors will behave in a risk neutral manner if they hold well diversified portfolios since all risk of this firm is firm-specific and assumed to be diversifiable.
Then, \( \pi_t = \begin{cases} \frac{R_t - r_t}{R_t} & \text{if } R_t > r_t \\ 0 & \text{if } R_t \leq r_t \end{cases} \)

and \( V_t = E_t[\pi_t] + E_t[V_{t+1}] \)

Hence, the expectation at the beginning of period \( t \) of the net present value of future profits is a sum of the expected period \( t \) profit and the expected net present value of all future profits. The form of the profit term, \( \pi_t \), derives from the assumption of limited liability. \( E_t[\pi_t] \) depends on the manager's information at the beginning of period \( t \) and on \( y_t \) and \( r_t \), the production level and interest requirement, in period \( t \). It follows that the expected net present value of future profits, \( E_t[V_{t+1}] \), which is a sum of future expected profits, depends on future project choices and interest requirements. All future interest requirements are functions of the reputation acquired by the manager by the end of period \( t \).

If the firm faces insolvency in any period, the manager is able to costlessly reorganize or start a new firm. In other words, realization of a revenue which is insufficient to cover debt obligations, in a particular period, does not prevent the manager from borrowing and producing in future periods. It merely affects his reputation. Inclusion of a constant bankruptcy cost would not alter our results qualitatively.

The firm’s decision variables consist of \( y_t \), its project, and \( m_t \), any disclosure the firm might make. As stated above, \( y_t \) has a direct impact on \( \pi_t \) through the conditional distribution of \( R_t \). In addition, \( m_t \) and the realized value of \( R_t \) influence future expected profits through their impact on the manager’s reputation and future nominal interest rates. As a result, maximization of expected net present value involves a balance between short run profits versus long run interest savings. The sequence of events, described above are summarized in the following time line.

**Time Line:**

<table>
<thead>
<tr>
<th>( t-1 )</th>
<th>( t_o )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{t-1} ) publicly observed</td>
<td>( \theta ) observed by G</td>
<td>( R_t ) publicly observed</td>
</tr>
<tr>
<td>( r_t ) determined</td>
<td>( y_t ) and ( m_t ) are selected by maximizing ( E_{t_o}[V_t] )</td>
<td>( r_{t+1} ) determined</td>
</tr>
</tbody>
</table>
In the following we focus on the two key quantities which the manager trades off to maximize his expected net present value. First, the effects of the manager's project choice on the firm's short run profit is discussed. Since, project choice also affects the manager's reputation and thus his future nominal interest rates, he is compelled to maintain a long run perspective and consider the impact of his decision on future interest savings. Following that, we examine how the firm's reputation together with creditor's anticipation of the manager's project choice affect the nominal return. Disclosure choice and its impact on reputation are not explicitly considered in the following section. These are introduced into the analysis in a later section entitled "Disclosure Regime".

4.2.1 Revenues and Profits

In order to ensure that the two projects have a differential impact on current period revenues and expected profit we make further assumptions on the conditional distribution of revenue. The first assumption concerns the supports of the conditional distribution functions of revenue and ensures that the lower production level is preferred by creditors. The third assumption ensures that the higher production level is preferred by the manager if he is uninformed. The second assumption concerns the informed type $G$ manager's preferences.

**Assumption A1:**

The conditional density functions, $g(R_t | \theta_t = h, y_t = y_h)$, $g(R_t | \theta_t = h, y_t = y_l)$ and $g(R_t | \theta_t = l, y_t = y_l)$ have support $[L, U]$ where, $U > L > r_{\text{max}} > r_f$ and $g(R_t | \theta_t = l, y_t = y_h)$ has support $[L_0, U_0]$ where, $U_0 > r_{\text{max}} > r_f > L_0$. Where, $r_{\text{max}}$ represents the highest possible nominal return required.

This assumption ensures that only $y_t = y_h$ can lead to bankruptcy and therefore $y_h$ can be a "risky" project whereas $y_t = y_l$ is always a "safe" project from the perspective of the creditors. Given assumption A1, the expected profits, $E[\pi_t | \theta_t, y_t]$ and $E[\pi_t | y_t]$ are written as:

\[
E[\pi_t | \theta_t = h, y_t = y_h] = E[R_t | \theta_t = h, y_t = y_h] - r_t
\]
\[
E[\pi_t | \theta_t = h, y_t = y_l] = E[R_t | \theta_t = h, y_t = y_l] - r_t
\]
\[
E[\pi_t | \theta_t = l, y_t = y_l] = E[R_t | \theta_t = l, y_t = y_l] - r_t
\]
\[
E[\pi_t | \theta_t = l, y_t = y_h] = \int_{r_t}^{U_0} (R_t - r_t) g(R_t | l, y_h) dR_t
\]
\[
E[\pi_t | y_t = y_l] = E[R_t | y_t] - r_t
\]
\[
E[\pi_t | y_t = y_l] = n\{E[R_t | \theta_t = h, y_t = y_h] - r_t\} + (1 - n)E[\pi_t | \theta_t = l, y_t = y_h]
\]
where, \(E[R_t|\theta_t, y_t]\) is the expected revenue given \(\theta_t\) and \(y_t\) and \(E[R_t|y_t]\) is the expected revenue given \(y_t\). The second expectation is taken over \(\theta_t\), yielding

\[
E[R_t|y_t] = nE[R_t|\theta_t = h, y_t] + (1 - n)E[R_t|\theta_t = l, y_t].
\]

The following assumptions rank the above expected profit levels.

**Assumption A2:**

(a) \(E[\pi_t|h, y_h, r_t] - E[\pi_t|h, y_i, r_t] \equiv E[R_t|h, y_h] - E[R_t|h, y_i] > 0 \ \forall \ r_t\)

(b) \(E[\pi_t|l, y_i, r_{max}] - E[\pi_t|l, y_h, r_{max}] \equiv E[R_t|l, y_i] - E[R_t|l, y_h] \)

\[-\int_{\theta_{max}}^{r_{max}} (r_{max} - R_t)g(R_t|l, y_h)dR_t > 0\]

Assumptions A2(a) ensures that \(E[\pi_t|h, y_h] > E[\pi_t|h, y_i]\) for all possible values of \(r_t\). Similarly, assumption A2(b) ensures that \(E[\pi_t|l, y_i] > E[\pi_t|l, y_h]\) for all possible values of \(r_t\) less than \(r_{max}\). As a consequence of assumptions A2(a) and A2(b), a type \(G\) manager prefers \(y_t = y_h\) when he observes \(\theta_t = h\) and prefers \(y_t = y_i\) when he observes \(\theta_t = l\), respectively. These imply that an informed manager always prefers the “correct” project.

**Assumption A3:**

\(E[\pi_t|y_h, r_f] - E[\pi_t|y_i, r_f] \equiv E[R_t|y_h] - E[R_t|y_i] + \int_{L_0}^{r_f} (r_f - R_t)g(R_t|l, y_h)dR_t > 0.\)

This sufficient condition ensures that \(E[\pi_t|y_h, r_t] - E[\pi_t|y_i, r_t] > 0 \ \forall \ r_t > r_f\) This assumption ensures that in a one period setting, an uninformed manager prefers the risky project. Assumption A3 introduces a conflict of interest between the manager/equity-holders and the creditors of the firm.\(^6\) From the equity-holders’ perspective, the risky project is preferred since it yields a higher expected net profit. From the creditors’ perspective, however, the safe project is superior since the firm may default on its debt obligations if the risky project is chosen. As discussed by several authors, we may view the equity-holders’ claim as a call option on the firm with an exercise price equivalent to the nominal return specified in the debt contract.\(^7\) The myopic manager maximizes the value of this option by choosing the risky project.

**Myopic Project Choice:**

The manager’s incentives in the absence of a reputation effect may therefore be summarized as follows. Due to assumption A2, it is clear that an informed type \(G\) manager would rationally

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\(^6\)This conflict is commonly discussed in the finance literature, for example, see Fama and Miller(1972).

\(^7\)See Black and Scholes(1973), Jensen and Meckling(1976).
choose $y_t = y_h$ when he observes $\theta_t = h$ and $y_t = y_l$ when he observes $\theta_t = l$. Assumption A3 ensures that a type $B$ manager and an uninformed type $G$ manager will choose $y_t = y_h$.

For a more general representation of managerial choice and creditor anticipation we use $P_t \in \{P_{Ght}, P_{Gl}, P_{Gut}, P_{Bt}\}$ to denote the probabilities that the manager chooses $y_t = y_h$ if his type is $G_h$, $G_l$, $G_u$ and $B$, respectively. Creditor’s anticipation of $P_t$ is denoted by $p_t \in \{p_{Ght}, p_{Gl}, p_{Gut}, p_{Bt}\}$. Note that, in general, each of $P_{Ght}$, $P_{Gl}$, $P_{Gut}$ and $P_{Bt}$ can take on any value between and including zero and one. Corresponding to that, each of $p_{Ght}$, $p_{Gl}$, $p_{Gut}$ and $p_{Bt}$ are also in the closed interval $[0, 1]$. This allows for randomization between $y_t = y_h$ and $y_t = y_l$ by all types.

4.2.2 Reputation and Nominal Return

In this section we examine how the reputation acquired by a manager influences future nominal interest requirements, i.e., how $r_t$ is determined given a reputation/credit-rating of $\alpha_{t-1}$.

It is useful to first list the expected payments to creditors under each possible value of the state variable, $\theta_t$, and each project, $y_t$. These are,

\begin{align*}
D(r_t, \theta_t = h, y_t = y_h) &= r_t \\
D(r_t, \theta_t = h, y_t = y_l) &= r_t \\
D(r_t, \theta_t = l, y_t = y_l) &= r_t \\
D(r_t, \theta_t = l, y_t = y_h) &= r_t - [r_t - \mu(r_t)]G(r_t|l, y_h)
\end{align*}

where, $G(r_t|l, y_h) = \int_{L_0}^{R_t} g(R_t|l, y_h) dR_t$ and $\mu(r_t) = \frac{\int_{L_0}^{R_t} R g(R_t|l, y_h) dR_t}{G(r_t|l, y_h)}$.

The quantity $\mu(r_t)$ represents the expected payment to creditors if insolvency occurs and $G(r_t|l, y_h)$ represents the probability of insolvency, given that $(\theta_t, y_t) = (l, y_h)$. Since, insolvency occurs only if $R_t < r_t$, it must be true that $\mu(r_t) < r_t$. This implies that $D(r_t, l, y_h) < r_t$. Expressions (4.1) through (4.4) indicate that there is a possibility that the creditors are not paid in full only if $\theta_t = l$ occurs when the project chosen is $y_h$.

If the manager’s type is not common knowledge, then the creditors’ expected return is given by the following expression, which considers the manager’s reputation $\alpha_{t-1}$ and the creditors’ belief $p_t$ about the production strategy each type will employ:

\begin{align*}
D(\alpha_{t-1}, p_t, r_t) &= \alpha_{t-1}\left\{q_h\{p_{Ght}D(r_t, h, y_h) + (1 - p_{Ght})D(r_t, l, y_l)\} + q_l\{p_{Gl}D(r_t, h, y_l) + (1 - p_{Gl})D(r_t, l, y_l)\}
\right.
\end{align*}

\begin{align*}
&\left. + (1 - p_{Glt})D(r_t, l, y_l)\} + q_u\{p_{Gut}D(r_t, h, y_h) + (1 - p_{Gut})D(r_t, l, y_l)\} + (1 - \alpha_{t-1})\{p_{Bl}D(r_t, h, y_h) + (1 - p_{Bl})D(r_t, l, y_l)\}\right\}
\end{align*}

79
where $\bar{n} = (1 - n)$ and $\bar{q} = (1 - q)$.

Substituting for $D_t$ from expressions (4.1), (4.2), (4.3) and (4.4) into the above yields

$$D(\alpha_{t-1}, p_t, r_t) = r_t - \bar{n} \{ \alpha_{t-1} x_{lt} + (1 - \alpha_{t-1}) p_{B_t} \} \{ r_t - \mu(r_t) \} G(r_t | l, y_h)$$

(4.5)

where $x_{lt} = q \bar{p}_{Glt} + \bar{q} \bar{p}_{Blt}$. That is, the adjustment for possible bankruptcy depends on the probability that $y_t = y_h$ when $\theta_t = l$, i.e., on $\bar{n} \{ \alpha_{t-1} x_{lt} + (1 - \alpha_{t-1}) p_{B_t} \} G(r_t | l, y_h)$, and on the expected deficit incurred by the creditors, $r_t - \mu(r_t)$, if bankruptcy occurs.

We assume that the credit market is competitive; hence, creditors receive an expected payment equal to the risk free return. The nominal return required from the manager is determined by setting

$$D(\alpha_{t-1}, p_t, r_t) = r_f.$$ 

(4.6)

Equation (4.6) together with equation (4.5) indicates that the nominal return equals the risk free return if, and only if,

$$\bar{n} \{ \alpha_{t-1} x_{lt} + (1 - \alpha_{t-1}) p_{B_t} \} G(r_t | l, y_h) = 0,$$

since $r_t - \mu(r_t) > 0$, by definition. Note that $\bar{n} x_{lt} G(r_t | l, y_h)$ is the probability that a type $G$ manager will face insolvency and $\bar{n} p_{B_t} G(r_t | l, y_h)$ is the probability that a type $B$ manager will face insolvency in period $t$. Therefore, the $LHS$ of the above equation gives us the probability of insolvency given $\alpha_{t-1}$, $p_{Glt}$, $p_{Blt}$ and $p_{B_t}$. Observe that the manager can only borrow at the risk free return if creditors believe the probability of insolvency is zero, given his reputation. It is notable that $p_{Glt}$, the anticipated production strategy of a type $G$ manager who has observed $\theta_t = h$, does not appear in expression (4.5). This is due to the fact that a type $G_h$ manager never faces insolvency, no matter what his production strategy is.

Due to the presence of $r_t$ in the limits of the integrals contained in $G(r_t | l, y_h)$ and $\mu(r_t)$, it is not possible to solve for $r_t$ without further assumptions on $g(R_t | l, y_h)$. Therefore, at this point, we are limited to a discussion of some properties of $r_t$. Let

$$r_G = r_t \text{ required from a firm if its manager is known to be type } G$$

$$r_B = r_t \text{ required from a firm if its manager is known to be type } B$$

Then, $r_G$ and $r_B$ satisfy the following :

$$D(\alpha_{t-1} = 1, p_t, r_G) = r_G - \bar{n} x_{lt} \{ r_G - \mu(r_G) \} G(r_G | l, y_h) = r_f$$

(4.7)

$$D(\alpha_{t-1} = 0, p_t, r_B) = r_B - \bar{n} p_{Blt} \{ r_B - \mu(r_B) \} G(r_B | l, y_h) = r_f.$$ 

(4.8)
Recall that $\bar{G}(\alpha_1|\lambda, \gamma_h)$ is the probability of insolvency if the manager is known to be type $G$ and $\bar{B}(\alpha_1|\lambda, \gamma_h)$ is the probability of insolvency if the manager is known to be type $B$. The following lemma compares the values of $r_G$ and $r_B$.

**Lemma 4.1** $r_f < r_G < r_B < L$ if $p_{Bt} > x_{it} > 0$ and $r_f < r_B < r_G < L$ if $0 < p_{Bt} < x_{it}$.

In the above, $p_{Bt} > x_{it}$ implies that the type $B$ manager faces a probability of insolvency which is at least as high as that faced by a type $G$ manager. Hence, the nominal return required from a manager who is known to be type $B$ must be higher than that required from a manager who is known to be type $G$. If, however, $p_{Bt} < x_{it}$ then the type $G$ manager faces a probability of insolvency which is higher than that faced by a type $B$ manager. Hence, in this case the nominal return required from a manager who is known to be type $G$ is higher than that required from a manager who is known to be type $B$.

If the manager's type is not known with certainty then the assessed nominal return depends on the reputation $\alpha_{t-1}$ as well as the anticipated output strategies $p_{Gt}$, $p_{Bt}$, and $p_{Bt}$. The following lemma outlines some properties of the nominal return, $r_t(\alpha_{t-1}, p_t)$.

**Lemma 4.2** Assume $\alpha_{t-1} \in (0, 1)$.

1. If $p_{Gt} = p_{Bt} = p_{Bt} = 0$, then $r(\alpha_{t-1}, p_t = (p_{Gt}, 0, 0, 0)) = r_G = r_B = r_f$.

2. If $p_{Gt} > 0$ or $p_{Bt} > 0$ and $p_{Bt} > 0$, then $r(\alpha_{t-1}, p_t) \in (r_G, r_B)$, where $r_G$ and $r_B$ are both greater than $r_f$.

3. $\frac{dr_t}{d\alpha_{t-1}} = \frac{\bar{G}(\alpha_{t-1}|\lambda, \gamma_h)[r_{t-1} - m(r_t)]}{1 - \bar{G}(\alpha_{t-1}|\lambda, \gamma_h)(1 - m(r_t))}$.

Point (i) states that even if the manager's type is not known with certainty, the nominal return may still equal the risk free return if each of the types $G$, $G_u$ and $B$ managers are expected to choose the safe project level. Point (ii) states that if either of types $G$ or $G_u$ and type $B$ managers are expected to choose the risky project, then the nominal return must lie between $r_G$ and $r_B$, which are both greater than $r_f$.

The third point in lemma 4.2 concerns the effect of a change in the manager's reputation on the nominal return required from him. As in lemma 4.1, a key factor is the difference in the probability of insolvency of the type $G$ and type $B$ managers. If the type $B$ manager is more likely to take an action that might result in insolvency, i.e., if $p_{Bt} > x_{it}$, then the nominal return required from him is higher.

---

8Proofs of lemmas are in Appendix B.2.1.

9Refer to Appendix B.2.1 for the proof.
return decreases with increasing reputation. However, if the type \( G \) manager is more likely to face insolvency then the nominal interest is increasing in reputation. It follows that if both types face the same risk of insolvency, then the nominal return is independent of reputation.

It is instructive to examine the implications of some specific anticipated pure strategy sets for the nominal return and its change with reputation. Four possible anticipated pure production strategy sets, which are of particular relevance in a later section, are considered below.

**Case 1:** \( p_t = (1,0,0,0) \equiv p^1 \)

In this case \( p_{G\text{it}} = p_{G\text{ut}} = p_{Bt} = 0 \). This implies that \( \bar{n}(1 - G_t)(q_{G\text{it}} + \bar{q}_{G\text{ut}}) = 0 \) and \( \bar{n}(1 - G_t)p_{Bt} = 0 \) and hence, \( r(\alpha_{t-1} \in [0,1], p_t = p^1) = r_f \). Basically, if none of the manager types are expected to face insolvency, then the manager is charged the risk free rate, irrespective of his reputation. It follows that the manager’s nominal return requirement is independent of his reputation, i.e., \( \frac{dr_t}{d\alpha_{t-1}} = 0 \).

**Case 2:** \( p_t = (1,0,1,1) \equiv p^2 \)

Types \( G, \ G_u \) and \( B \) are expected to choose the risky project and the type \( G_i \) manager is expected to choose the safe project. This implies that \( p_{Bt} = 1 \) whereas \( q_{G\text{it}} + \bar{q}_{G\text{ut}} = \bar{q} \), i.e., the type \( B \) manager faces a higher probability of insolvency than the type \( G \) manager. Therefore, in this case, \( r_B > r(\alpha_{t-1} \in (0,1), p_t = p^2) > r_G > r_f \). It follows that the nominal return is decreasing in reputation i.e., \( \frac{dr_t}{d\alpha_{t-1}} < 0 \).

**Case 3:** \( p_t = (0,0,0,1) \equiv p^3 \)

In this case, the type \( G \) manager is never expected to choose the risky project whereas the type \( B \) manager is always expected to choose the risky project. This implies that the type \( G \) manager is not expected to face insolvency whereas the type \( B \) manager may. Hence, \( r_B > r(\alpha_{t-1} \in (0,1), p_t = p^3) > r_G = r_f \) and \( \frac{dr_t}{d\alpha_{t-1}} < 0 \).

**Case 4:** \( p_t = (1,0,0,1) \equiv p^4 \)

The type \( G \) manager who observes \( \theta_t = h \) and the type \( B \) manager are expected to choose the risky project. However only the type \( B \) manager may face insolvency. Hence the nominal return requirements and the effect of reputation are exactly the same as in case 3, above.

We next analyze managerial decision making in a two period setting, given that creditors determine the nominal return required from the manager by equating their expected payment to the risk free return, as given in equation (4.6). Possible sequential equilibria are characterized in which creditors have rational expectations, and hence, their anticipation of the manager’s
production strategies matches the manager's optimal project choice. As required in a sequential equilibrium, creditors are assumed to determine their posterior beliefs by Bayesian revision of their priors. The two period horizon enables us to examine the costs and benefits involved in the acquisition of reputation, in the simplest possible setting.

We first consider a non-disclosure regime in which disclosures are not permitted. Hence, creditors observe only end of period revenues, which they use to revise their belief about the manager. The problem is then analysed in a disclosure regime in which the manager can make truthful disclosures. In this regime creditors observe the end of period revenues as well as management disclosures, both of which are used in belief revision. The analysis of reputation acquisition in two different regimes helps us to isolate and better understand the role of disclosure on reputation acquisition.
4.3 The Non-Disclosure Regime

In this section we consider a situation in which the manager is not permitted to make any disclosures. Since the manager has a two period time-horizon, his expected net present value is the sum of the expected profits to be realized at the end of each period. Mere maximization of current period expected profit is not optimal in this setting. The impact of a particular action on the current period and the future period expected profits have to be traded off against each other to maximize the total expected net present value. As before, the manager's production strategy set is denoted by \( P_t \) and the creditors' anticipation of the manager's production strategy set is denoted by \( p_t \), where \( t = \{1, 2\} \).

At the beginning of the first period, creditors start with a prior belief \( \alpha_0 \). By the end of the period, as additional data become available, the prior belief is revised to \( \alpha_1 \). Being a probability, \( \alpha_t \) lies between 0 and 1. If \( \alpha_t = 1 \), the manager of the firm has acquired the reputation of being type \( G \) and, if \( \alpha_t = 0 \), then it is common knowledge that the firm has a type \( B \) manager. If \( \alpha_t \in (0, 1) \), then there is uncertainty about the manager's type. At any point in time, reputation is a function of the prior history of the values of all publicly observable variables related to it and, hence, represents the information contained in the entire history. In the non-disclosure regime the only publicly observable variable is the firm's end of period revenue. We assume that the actual project choice \( y_t \) and the realizations of the random variable \( \theta_t \) are never observed publicly. Hence, the prior belief in a particular period is revised based on that period's anticipated production strategies and realized revenue. The nominal return in the next period is a function of the anticipated production strategies in that period and the reputation established at the end of the previous period.

A sequential equilibrium concept is used to determine the optimal production plans of the firm, given its manager's type, and the nominal returns required by creditors from the firm, given its past history of realized revenues. Creditors set their required returns based on their beliefs about the manager's type and the anticipated production plans of each type. An equilibrium consists of an optimal project choice for each type of manager and a set of creditor belief revision rules, which are consistent with each other. Such a consistent set is required for each of the time periods in a two period equilibrium.

We first discuss the belief revision rules used by creditors and their nominal interest rate determination followed by a discussion of the manager's optimization problem. Possible equilibria of this reputation game are then characterized and examined.
4.3.1 Creditors' Belief Revision in the Non-Disclosure Regime

Every period potential creditors update their beliefs about the firm's manager. In a sequential equilibrium beliefs must be revised using Bayes' rule. In order to revise their beliefs the creditors use a set of conjectures about the behavior of the manager. It is then shown that, in equilibrium, the manager's optimal choices are consistent with these conjectures. Creditors' anticipation of the optimal production strategies in period $t$ are functions of the manager’s reputation at the beginning of period $t$. Thus the anticipated production strategy set is

$$p_t(\alpha_{t-1}) = \{p_{\text{cht}}(\alpha_{t-1}), p_{\text{ct}}(\alpha_{t-1}), p_{\text{cut}}(\alpha_{t-1}), p_{\text{st}}(\alpha_{t-1})\},$$

where $t \in \{1, 2\}$.

The posterior beliefs are functions of the priors, the anticipated optimal project choices, and the realized revenue. The prior belief, $\alpha_{t-1}$, at the beginning of period $t$ (or equivalently, at the end of period $t - 1$) is revised to $\alpha_t$, when realized revenues are announced at the end of period $t$, using Bayes theorem. If the manager has been identified as to his type he is no longer part of the reputation game. In other words, once $\alpha_t = 0$ or $1$ for the manager, it remains at that value for all subsequent periods. Therefore, we focus our attention on the manager as long as his type has not been revealed, i.e., as long as $0 < \alpha_{t-1} < 1$. The following are the creditors' prior and posterior beliefs about the manager.$^{10}$

**Prior Belief:** $\alpha_{t-1} = \text{Posterior belief at the end of period } t - 1.$

**Posterior Beliefs:**

$$\alpha_t(\alpha_{t-1}, R_t, p_t) = \frac{\text{Prob}(G|\alpha_{t-1}, R_t, p_t)}{g(R_t|G, p_{\text{ct}}) \cdot \alpha_{t-1} + g(R_t|B, p_{\text{ct}}) \cdot (1 - \alpha_{t-1})},$$

(4.9)

where

$$g(R_t|G, p_{\text{ct}}) = q \left\{ \begin{array}{l} [np_{\text{cht}}g(R_t|h, y_h) + np_{\text{ct}}g(R_t|l, y_h)] \\ +[n(1 - p_{\text{cht}})g(R_t|h, y_l) + n(1 - p_{\text{ct}})g(R_t|l, y_l)] \\ +\tilde{q} \left\{ p_{\text{cut}}[ng(R_t|h, y_h) + \tilde{n}g(R_t|l, y_h)] \\ +\tilde{q} \left\{ p_{\text{cut}}[ng(R_t|h, y_l) + \tilde{n}g(R_t|l, y_l)] \\ +(1 - p_{\text{cut}})[ng(R_t|h, y_l) + \tilde{n}g(R_t|l, y_l)] \end{array} \right\} \\ g(R_t|B, p_{\text{ct}}) = \left\{ p_{\text{ct}}[ng(R_t|h, y_h) + \tilde{n}g(R_t|l, y_h)] \\ +(1 - p_{\text{ct}})[ng(R_t|h, y_l) + \tilde{n}g(R_t|l, y_l)] \right\} \\ p_{\text{ct}} = (p_{\text{cht}}, p_{\text{ct}}, p_{\text{cut}}). \right.$$  

$^{10}$This follows directly by applying Bayes rule.
The firm’s reputation, in any period $t$, is a function of $\alpha_t$, the realized revenues and the anticipated production strategies, from the first to the $t$th period. Therefore, at any point in time, the reputation summarizes all the relevant information contained in the entire history of realized revenue and anticipated production strategies. This implies that at the end of period $t$, the firm’s reputation, $\alpha_t$, can be expressed as a function of $\alpha_{t-1}$, $R_t$ and $p_t$. The manager may have the same reputation irrespective of his actual type.

4.3.2 Optimal Project Choice in the Non-Disclosure Regime

At the beginning of each period, the manager selects his optimal production strategy. If the manager is an informed type $G$, he has observed the value of $\theta_t$ prior to selecting $y_t$. Whereas, if he is an uninformed type $G$ or a type $B$ manager, he has not. The selected production strategy affects period $t$ realized revenue, which not only affects the period $t$ expected profit, but also the manager’s reputation and his future interest rates. Figures 4.1(a) and 4.1(b) give tree diagrams for the manager’s decision problem if the manager is informed and uninformed, respectively. In any period $t$, the project is selected by maximizing the net present value of expected future cash flows, net of anticipated interest requirements. In calculating his expected net present value, it is assumed that the manager optimizes all future expected profits in the same manner. The following representation of the project choice problem is abbreviated with $\alpha_{t-1}$, $\alpha_t$, $y_t$ etc., not explicitly stated, for more convenient representation.

**Optimization Problem 1**:

$$V^*_t(\tau) = \max_{\mu} V_t(\tau) = E_t[\pi_t|\tau] + E_t[V^*_{t+1}|\tau]$$

subject to $r_t = r(\alpha_{t-1}, p_t)$

$$\alpha_t = \frac{g(R_t|G, p_{t1}) \cdot \alpha_{t-1}}{g(R_t|G, p_{t1}) \cdot \alpha_{t-1} + g(R_t|B, p_{t1}) \cdot (1 - \alpha_{t-1})}$$

where $\tau$ indicates the manager's type in period $t$.

The manager solves the optimization problem appropriate for his type, at $t = 1, 2$. Note that $E_2[V^*_3] = 0$ since no third period exists. The expected net present value in the first period contains the term $E_1[V^*_2]$ which is the optimized second period expected profit. Therefore, we need to maximize the second period expected profit prior to determining $E_1[V^*_2]$. We can then maximize the first period expected net present value after substituting for $E_1[V^*_2]$ into the expression for $V_1$. The constraints in the maximization problem ensure that at the time of project choice, in each period, the manager is aware of the belief revision rule used by creditors.
The manager’s maximization problem depends on his type since expectations of profits are taken differently for each type. If the manager is type $B$ he does not have prior knowledge of $\theta_t$. Hence the firm’s expected net present value, at the beginning of period $t$, is taken over both random variables $\theta_t$ and $\eta_t$. On the other hand, if the manager is type $G$ he may observe the realized value of its $\theta_t$, at the beginning of period $t$. Hence, the firm may have a different expected net present value for each value of $\theta_t$, taken over the random variable $\eta_t$. This implies that, in general, the optimal production strategies $P_{\theta t}^*$, $P_{\theta t}^*$, $P_{\theta t}^*$, and $P_{B t}^*$ differ from each other.

Since there are only two possible decisions, the manager’s optimization problem can be written in terms of trade-offs between short run and long run gains. In particular, the period $t$ optimal decision for type $\tau$ may be represented by the following set of conditions, where $P_t$ is the probability that the manager selects $y_h$.

$$P_{\tau t}^* = \begin{cases} 1 & \text{if } V_t(\tau, \alpha_{t+1}, y_t = y_h, p_t) - V_t(\tau, \alpha_{t+1}, y_t = y_l, p_t) > 0 \\ \Delta \pi_t^*(r_t(\alpha_{t+1}, p_t)) \equiv E[\pi_t|\tau, \alpha_{t+1}, y_t = y_h, p_t] - E[\pi_t|\tau, \alpha_{t+1}, y_t = y_l, p_t] \\ > \Delta V_{t+1}^*(\alpha_{t+1}, p_t) \equiv E[V_{t+1}|\tau, \alpha_{t+1}, y_t = y_l, p_t] - E[V_{t+1}|\tau, \alpha_{t+1}, y_t = y_h, p_t] \end{cases}$$

(4.11)

$$P_{\tau t}^* \in [0, 1] \text{ if } V_t(\tau, \alpha_{t+1}, y_t = y_h, p_t) - V_t(\tau, \alpha_{t+1}, y_t = y_l, p_t) = 0$$

$$\Rightarrow \Delta \pi_t^*(r_t(\alpha_{t+1}, p_t)) \equiv E[\pi_t|\tau, \alpha_{t+1}, y_t = y_h, p_t] - E[\pi_t|\tau, \alpha_{t+1}, y_t = y_l, p_t]$$

$$= \Delta V_{t+1}^*(\alpha_{t+1}, p_t) \equiv E[V_{t+1}|\tau, \alpha_{t+1}, y_t = y_l, p_t] - E[V_{t+1}|\tau, \alpha_{t+1}, y_t = y_h, p_t]$$

(4.12)

$$P_{\tau t}^* = 0 \text{ if } V_t(\tau, \alpha_{t+1}, y_t = y_h, p_t) - V_t(\tau, \alpha_{t+1}, y_t = y_l, p_t) < 0$$

$$\Rightarrow \Delta \pi_t^*(r_t(\alpha_{t+1}, p_t)) \equiv E[\pi_t|\tau, \alpha_{t+1}, y_t = y_h, p_t] - E[\pi_t|\tau, \alpha_{t+1}, y_t = y_l, p_t]$$

$$< \Delta V_{t+1}^*(\alpha_{t+1}, p_t) \equiv E[V_{t+1}|\tau, \alpha_{t+1}, y_t = y_l, p_t] - E[V_{t+1}|\tau, \alpha_{t+1}, y_t = y_h, p_t]$$

(4.13)

$V_t(\tau, \alpha_{t+1}, y_t = y_h, p_t)$ and $V_t(\tau, \alpha_{t+1}, y_t = y_l, p_t)$ depend on the manager’s type, $\tau$, and on $(p_t, p_{t+1}, \ldots)$ (i.e., the creditor perceptions) which can depend on $(\alpha_{t-1}, \alpha_t, \ldots)$.

In the above inequalities, depending on his type and information, the manager trades off the quantity represented by $\Delta \pi_t^*$ against $\Delta V_{t+1}^*$. $\Delta \pi_t^*$ represents the difference in the impact of the risky and the safe projects levels on the current period expected profit and is termed the profit differential. $\Delta V_{t+1}^*$, on the other hand, represents the differential impact of the safe and risky projects on future expected profits and is termed the reputation benefit. Basically, in choosing his production level, given his type and information, the manager makes a trade-off between a
short term gain ($\Delta \pi_t^f$) from the action that yields the highest expected current profit versus a long term benefit ($\Delta V_{t+1}^*$) due to decreased future nominal interest payments. Therefore, the manager chooses the risky project if his $\Delta \pi_t^f$ is greater than his $\Delta V_{t+1}^*$ and the safe project if the opposite is true.

In general, optimal project choice depends on the manager's type and on the creditors' anticipation of present and future production strategy sets and reputation. Note that the creditors' anticipation of the production strategies, $p_{rt}$, may differ from $P_{rt}^*$. However, in equilibrium the anticipated production strategy must be the same as the optimal production strategy.

4.3.3 Equilibrium in the Non-Disclosure Regime

In order to characterize an equilibrium we need to obtain solutions to optimization problem 1 which are also consistent with creditors' beliefs. Hence, it is necessary to compare the incremental benefit obtained in the current period with that obtained in the future from one action over the other. Assumptions A2 and A3 enable us to compare the $\Delta \pi_t^f$s (current period incremental benefit) terms. However, in order to compare the $\Delta V_{t+1}^*$ (future reputation benefit) terms we need to be more specific about the relative values of the future nominal returns given the two different projects in period $t$. A specification of the distributions of $R_t$ given $\theta_t$ and $y_t$ is required for this. The following distributional assumptions give us a simple structure without allowing an exact inference of the project choice from end of period revenue.

Assumption A4:

(a) $\theta_t = h, y_t = y_h : R_t \in \{R_{hh}\}$ and $\text{Prob}[R_{hh}|\theta_t = h, y_t = y_h] = 1$

(b) $\theta_t = l, y_t = y_l : R_t \in \{R_{ll}\}$ and $\text{Prob}[R_{ll}|\theta_t = l, y_t = y_l] = 1$

(c) $\theta_t = h, y_t = y_l : R_t \in \{R_{hl}, R_{lh}\}$ and $\text{Prob}[R_{hh}|\theta_t = h, y_t = y_l] = G_h$

(d) $\theta_t = l, y_t = y_h : R_t \in \{R_{lh}, R_{ll}\}$ and $\text{Prob}[R_{ll}|\theta_t = l, y_t = y_h] = G_l$

Assumption A4 states that the realized revenue can take on one of only four possible values, i.e., $R_t \in \{R_{hh}, R_{ll}, R_{hl}, R_{lh}\}$. It further restricts the the values $R_t$ can assume conditional on any given pair of values of $\theta_t$ and $y_t$. Given $(\theta_t, y_t) = (h, y_h)$ revenue must equal $R_{hh}$ and given $(\theta_t, y_t) = (l, y_l)$ revenue must equal $R_{ll}$. However, if the safe (risky) project is chosen when $\theta_t = h$ ($\theta_t = l$), then the conditional density of revenue has binary support. If $(\theta_t, y_t) = (h, y_l)$, then $R_t = R_{hh}$ with probability $G_h$ and $R_t = R_{hl}$ with probability $1 - G_h$. Similarly, if $(\theta_t, y_t) = (l, y_h)$, then $R_t = R_{ll}$ with probability $G_l$ and $R_t = R_{lh}$ with probability $1 - G_l$. 

88
Assumption A5: There exist bounds $U$, $L$, $U_0$ and $L_0$ where, $U \geq R_{hh} > R_{ll} \geq L$, $U \geq R_{hl} \geq L$ and $R_{lh} \in [L_0, U_0]$ where $U_0 \leq r_f < r_{\max} = r_B(p_B = 1) \leq L$.

Assumption A5 corresponds to assumption A1 but reflects the simplified setting specified by assumption A4. Basically, the assumption specifies different intervals within which particular sets of revenue realizations and the upper and lower bounds of nominal returns must be contained. The values $R_{hh}$, $R_{ll}$, and $R_{hl}$ are in the closed interval $[L, U]$ and $R_{lh}$ is in the closed interval $[L_0, U_0]$. The lowest possible nominal return, i.e., the riskfree return, and the highest possible nominal return, $r_{\max}$, are in the interval $[U_0, L]$. The highest possible nominal return is required from the type $B$ manager if he is expected to select the risky project, as shown in lemma 4.1. Two objectives are served by the bounds imposed by assumption A5. First, A5 ensures that if an informed manager chooses the "correct" project then he never faces insolvency and, second, project $y_i$ is "safe" whereas $y_h$ is "risky" from the viewpoint of the creditors.

Under assumptions A4 and A5 the expressions for expected revenues and profits can be written as follows:

$$E[R_t | \theta_t = h, y_t = y_h] = R_{hh}$$
$$E[R_t | \theta_t = h, y_t = y_l] = G_h R_{hh} + \bar{G}_h R_{hl} \equiv \bar{R}_{hl}$$
$$E[R_t | \theta_t = l, y_t = y_l] = R_{ll}$$
$$E[R_t | \theta_t = l, y_t = y_h] = G_l R_{ll} + \bar{G}_l R_{lh} \equiv \bar{R}_{lh}$$
$$E[R_t | y_t = y_h] = n R_{hh} + \bar{n} [G_l R_{ll} + \bar{G}_l R_{lh}] \equiv \bar{R}_h$$
$$E[R_t | y_t = y_l] = n [G_h R_{hh} + \bar{G}_h R_{hl}] + \bar{n} R_{ll} \equiv \bar{R}_l$$

$$E[\pi_t | \theta_t = h, y_t = y_h, r_t] = R_{hh} - r_t \quad (4.14)$$
$$E[\pi_t | \theta_t = h, y_t = y_l, r_t] = \bar{R}_{hl} - r_t \quad (4.15)$$
$$E[\pi_t | \theta_t = l, y_t = y_l, r_t] = R_{ll} - r_t \quad (4.16)$$
$$E[\pi_t | \theta_t = l, y_t = y_h, r_t] = \bar{R}_{lh} - G_l r_t - \bar{G}_l R_{lh} \quad (4.17)$$
$$E[\pi_t | y_t = y_l, r_t] = \bar{R}_l - r_t \quad (4.18)$$
$$E[\pi_t | y_t = y_h, r_t] = \bar{R}_h - [n + \bar{n} G_l] r_t - \bar{n} \bar{G}_l R_{lh}, \quad (4.19)$$

where $\bar{n} = 1 - n$, $\bar{G}_l = 1 - G_l$ and $\bar{G}_h = 1 - G_h$. Substitution of the more specific expressions for $g(R_t | \cdot)$ given by assumption A4 into that for the posterior beliefs given by expression (4.9)
\[ \alpha_t(\alpha_{t-1}, R_t = R_{hh}, p_t) = \frac{[x_{lt} + (1-x_{lt})G_t] \alpha_{t-1}}{[x_{lt} + (1-x_{lt})G_t] \alpha_{t-1} + [p_Bt + (1-p_Bt)G_t](1-\alpha_{t-1})} \] (4.20)

\[ \alpha_t(\alpha_{t-1}, R_t = R_{lt}, p_t) = \frac{[x_{lt}G_t + (1-x_{lt})] \alpha_{t-1}}{[x_{lt}G_t + (1-x_{lt})] \alpha_{t-1} + [p_BtG_t + (1-p_Bt)](1-\alpha_{t-1})} \] (4.21)

\[ \alpha_t(\alpha_{t-1}, R_t = R_{lh}, p_t) = \frac{x_{lt} \alpha_{t-1}}{x_{lt} \alpha_{t-1} + p_Bt(1-\alpha_{t-1})} \] (4.22)

\[ \alpha_t(\alpha_{t-1}, R_t = R_{lh}, p_t) = \frac{(1-x_{lt}) \alpha_{t-1}}{(1-x_{lt}) \alpha_{t-1} + (1-p_Bt)(1-\alpha_{t-1})} \] (4.23)

where \( x_{lt} \equiv p_{ght} + \bar{q}p_{gut} \), \( x_{lt} \equiv p_{glt} + \bar{q}p_{gut} \) and \( \bar{q} = (1-q) \).

Note, that the probabilities and supports specified for revenue, in assumption A4, permit the possibility that, given a set of anticipated production strategies, some values of revenue are not expected to occur. This is due to the fact that the conditional probabilities of \( R_t \) have moving support. The conditions under which this may happen are \((p_{ght}, p_{glt}, p_{gut}, p_Bt) = (1, p_{glt}, 1, 1) \) and \((p_{ght}, 0, 0, 0) \). If \( p_{ght} = p_{gut} = p_Bt = 1 \) then the realization \( R_t = R_{hl} \) is not expected to occur irrespective of the value of \( p_{glt} \). Similarly, if \( p_{glt} = p_{gut} = p_Bt = 0 \) then the realization \( R_t = R_{lh} \) is not expected to occur irrespective of the value of \( p_{ght} \). Under the above conditions, the expression for \( \alpha_t(\alpha_{t-1}, R_t = R_{hi}, p_t) \) and \( \alpha_t(\alpha_{t-1}, R_t = R_{lh}, p_t) \) obtained by Bayes' theorem are not defined. Creditors need to have a rule by which they revise their beliefs in the contingency that an off-equilibrium realization of revenue is observed. We specify the following off-equilibrium beliefs:

\[ \alpha_t(\alpha_{t-1}, R_t = R_{hi}, p_{ght} = 1, p_{glt}, p_{gut} = 1, p_Bt = 1) = 1 \]

\[ \alpha_t(\alpha_{t-1}, R_t = R_{lh}, p_{ght}, p_{glt} = 0, p_{gut} = 0, p_Bt = 0) = 0, \]

i.e., creditors believe that if there is any “error” in production choice then it is made by an uninformed type \( G \) manager if the safe project is chosen by mistake and by a type \( B \) manager if the risky project is chosen by mistake. Other off-equilibrium beliefs could be specified.

The nominal return \( r_t \), specified in a debt contract at the beginning of period \( t \), is a function of creditors’ posterior belief \( \alpha_{t-1} \), at the end of period \( t-1 \), as well as their expectations of the period \( t \) production choices of each type of manager.

\(^{11}\)Refer to Appendix B.1.1 for the expressions for \( g(R_t|\cdot) \).
Lemma 4.3 \(^{12}\) Under assumptions A4 and A5 the nominal return in period \(t\) is written as

\[
\begin{align*}
\tau_t &= R_{lh} + \frac{\tau_f - R_{lh}}{\phi_t(\alpha_{t-1}, p_t)},
\end{align*}
\]

where \(\phi_t(\alpha_{t-1}, p_t) = [1 - \bar{n}G_i x_{it}]\alpha_{t-1} + [1 - \bar{n}G_i p_{Bt}](1 - \alpha_{t-1}).\)

The above expression for \(\tau_t\) is derived by setting the creditors' expected return equal to the riskfree nominal return. The above expressions illustrate the fact that \(\tau_t\) decreases as either of \(p_{Bt}\), \(p_{Gt}\) or \(p_{Gt}\) increases, i.e., if creditors expect the probability of choosing the risky project to go up, their expectation of insolvency increases and they charge a higher nominal return. Note that \(p_{Gt}\) is absent from the expression for \(\tau_t\). This can be explained by the fact that if an informed good manager has observed \(\theta_t = h\), he is not expected to become insolvent irrespective of the value of \(p_{Gt}\).

In order to examine how \(\tau_t\) is affected by \(\alpha_{t-1}\) it is illustrative to restate the expression for \(\phi_t(\alpha_{t-1}, p_t)\) as

\[
\phi_t(\alpha_{t-1}, p_t) = 1 - \bar{n}G_i p_{Bt} + \bar{n}G_i(p_{Bt} - x_{it})\alpha_{t-1}.
\]

From this expression it is clear that \(\tau_t\) decreases with an increase in \(\alpha_{t-1}\) if \(p_{Bt} > x_{it}\), and increases with an increase in \(\alpha_{t-1}\) if \(p_{Bt} < x_{it}\). Intuitively, if creditors assign a higher probability to the event that the type \(B\) manager rather than the type \(G\) manager chooses the risky project, then an increase in the probability that the manager is type \(G\) decreases his risk of insolvency, thus decreasing \(\tau_t\). Similarly, if creditors assign a higher probability to the event that the type \(G\) manager rather than the type \(B\) manager chooses the risky project, then an increase in the probability that the manager is type \(G\) increases his risk of insolvency, thus increasing \(\tau_t\).

From the expression for \(\tau_t\) in lemma 4.3 we obtain the expression for \(\tau_G\) by substituting \(\alpha_{t-1} = 1\) and for \(\tau_B\) by substituting \(\alpha_{t-1} = 0\). Thus,

\[
\begin{align*}
\tau_G(p_t) &= R_{lh} + \frac{\tau_f - R_{lh}}{1 - \bar{n}G_i x_{it}}, \quad \text{(4.24)} \\
\tau_B(p_t) &= R_{lh} + \frac{\tau_f - R_{lh}}{1 - \bar{n}G_i p_{Bt}}. \quad \text{(4.25)}
\end{align*}
\]

The above expressions show that the nominal return assessed for the manager if his type is common knowledge depends on his anticipated production strategy. Basically, the required nominal return takes the manager's risk of insolvency into account.

Since the manager has a two period time horizon, the optimization problem, appropriate to the manager's type, has to be solved for each of the two periods. Note that the \(\Delta V_{t+1}\)

\(^{12}\) Proof of lemma 4.3 can be found in Appendix B.2.1.
terms consist of period $t + 1$ optimized expected profits. Hence, the optimal strategies for each type in the last period have to be determined first. These optimized expected profits can then be substituted into the expressions for the first period to determine the optimal production strategies in the first period. The optimal strategies thus determined are consistent with an equilibrium if they coincide with the anticipated production strategies, $p_1$, used in the expressions for $\Delta \pi_1^T$ and $\Delta V_2^T$, to find $P_{t1}^*$, Thus $p_1^* = P_{t1}^*$.

**Lemma 4.4** In the second period, the equilibrium strategies are $p_2^* = (1, 0, 1, 1)$.

Since there is no future beyond the second period, the $\Delta V_{t+1}^T$ terms corresponding to $t = 2$ are zero. By assumption A2, $\Delta \pi_2^{Gh}(r_2(\alpha_1, p_2)) > 0$ and $\Delta \pi_2^{Gl}(r_2(\alpha_1, p_2)) < 0$, and, due to assumption A3, $\Delta \pi_2^{Gh}(r_2(\alpha_1, p_2)) = \Delta \pi_2^{Gl}(r_2(\alpha_1, p_2)) > 0 \forall r_2$. Hence, in equilibrium the type $Gh$ manager chooses $y_h$, the type $Gl$ manager chooses $y_i$ and both the type $Gu$ and the type $B$ manager choose $y_h$. The resulting equilibrium production strategy set is $p_2^* = (1, 0, 1, 1)$.

Given the above production strategy set, the equilibrium nominal interest in the last period can be written as the following function of $\alpha_1$:

$$r_2^*(\alpha_1, p_2^* = (1, 0, 1, 1)) = R_{lh} + \frac{r_f - R_{lh}}{1 - \tilde{n}\tilde{G}_l[1 - q(1 - \alpha_1)]}$$

Note that $\tilde{\alpha}_G$ is the probability of insolvency faced by a type $G$ manager and $\tilde{\alpha}_B$ is the probability of insolvency faced by a type $B$ manager, in the second period. Since $p_0^{Gh} = 0$, $p_0^{Gu} = 1$ and $p_0^{Gl} = 1$, the probabilities of actions that may result in insolvency are $x_2^G = \tilde{q}$ if the manager is type $G$ and $p_{Bx} = 1$ if the manager is type $B$. Hence, as in point (iii) of lemma 4.2, the nominal return in the second period is decreasing in the manager’s reputation at the end of the first period.

The first period optimal project choice if the manager is type $\tau$ can be determined by comparing the $\Delta \pi_1^T$ with the $\Delta V_2^T$ term, after first substituting for $P_{t1}^* = p_2^* = (1, 0, 1, 1)$ into the expressions for $\Delta V_2^T$. In addition, we can obtain more specific expressions for $\Delta V_2^T$ and for the first period differential profits, $\Delta \pi_1^T$, by imposing assumption A4. These substitutions result in the following simplifications of the $\Delta V_2^T$ terms:

$$\Delta V_2^{Gh}(\alpha_0, p_1, p_2^*) = (1 - \tilde{n}\tilde{G}_l\tilde{q})\tilde{G}_h[r_2(\alpha_1(\alpha_0, R_1 = R_{h\ell}, p_1), p_2^*) - r_2(\alpha_1(\alpha_0, R_1 = R_{h\ell}, p_1), p_2^*])$$
$$\Delta V_2^{Gl}(\alpha_0, p_1, p_2^*) = (1 - \tilde{n}\tilde{G}_l\tilde{q})\tilde{G}_l[r_2(\alpha_1(\alpha_0, R_1 = R_{l\ell}, p_1), p_2^*) - r_2(\alpha_1(\alpha_0, R_1 = R_{l\ell}, p_1), p_2^*])$$
$$\Delta V_2^{Gu}(\alpha_0, p_1, p_2^*) = (1 - \tilde{n}\tilde{G}_l\tilde{q})\tilde{G}_u[r_2(\alpha_1(\alpha_0, R_1 = R_{u\ell}, p_1), p_2^*) - r_2(\alpha_1(\alpha_0, R_1 = R_{u\ell}, p_1), p_2^*)]$$

13In the last period managers make myopic production choices.
\[\Delta V_2^B(\alpha_o, p_1, p_f^2) = (1 - \hat{n} \hat{G}_I) \left\{ \hat{n} \hat{G}_I \left[ r_2(\alpha_1(\alpha_o, R_1 = R_{hh}, p_1), p_f^2) - r_2(\alpha_1(\alpha_o, R_1 = R_{hl}, p_1), p_f^2) \right] + n \hat{G}_h \left[ r_2(\alpha_1(\alpha_o, R_1 = R_{hh}, p_1), p_f^2) - r_2(\alpha_1(\alpha_o, R_1 = R_{hl}, p_1), p_f^2) \right] \right\}.\]

The \(\Delta V_2^T\) terms give us the expectations of the nominal return saved by the different types if the manager chooses \(y_1 = y_i\) instead of \(y_1 = y_h\). The corresponding expressions for differential expected profits in the first period are:

\[
\begin{align*}
\Delta \pi^G_i(r_1(\alpha_o, p_1)) &= R_{hh} - \hat{R}_{hl} \\
\Delta \pi^G_{ii}(r_1(\alpha_o, p_1)) &= \hat{R}_{lh} - \hat{R}_{il} + \hat{G}_I[r_1(p_1, \alpha_o) - \hat{R}_{il}] \\
\Delta \pi^G_{un}(r_1(\alpha_o, p_1)) &= \hat{R}_{h} - \hat{R}_{l} + \hat{n} \hat{G}_I[r_1(p_1, \alpha_o) - \hat{R}_{il}] \\
\Delta \pi^G_{i}(r_1(\alpha_o, p_1)) &= \hat{R}_{h} - \hat{R}_{l} + \hat{n} \hat{G}_I[r_1(p_1, \alpha_o) - \hat{R}_{il}].
\end{align*}
\]

The differential profits for types \(G_l, G_u\) and \(B\) consist of a differential revenue part and a part representing the benefit due to limited liability. Type \(G_l\)'s differential revenue is \((\hat{R}_{lh} - \hat{R}_{il})\) and limited liability benefit is \(\hat{G}_I(r_1 - R_{il})\). Types \(G_u\) and \(B\) have the same differential revenue, i.e., \((\hat{R}_h - \hat{R}_l)\), and the same limited liability benefit, \(\hat{n} \hat{G}_I(r_1 - R_{il})\). The limited liability benefits are increasing in the nominal return \(r_1\) reaching its lower bound at \(r_1 = r_f\) and its upper bound at \(r_1 = r_{max}\). Type \(G_h\)'s differential profit, on the other hand, consists of only a differential revenue portion, \((R_{hh} - \hat{R}_{hl})\), and hence, is independent of \(r_1\). This is due to the fact that the type \(G_h\) manager never faces insolvency.

Given the above expressions for the \(\Delta \pi^G_i\) and \(\Delta V_2^T\) terms, it can be shown that equilibrium production strategies exist for all values of \(\alpha_o\). In the following we denote the equilibrium value of \(p_1\) by \(p_1^e\). Note that in equilibrium the anticipated production strategy set is the same as the optimal production strategy set, i.e., \(p_1^e = P_1^e\). Hence, we can use \(p_1^e\) to denote the equilibrium production strategy sets.

**Proposition 4.1**

For every \(\alpha_o\), there exists at least one equilibrium strategy set \(p_1^e = \{p_{G_{hl1}}^e, p_{G_{il1}}^e, p_{G_{un1}}^e, p_{B_{1}}^e\}\) such that:

\[
\begin{align*}
\text{if } \Delta \pi^G_i(r_1(\alpha_o, p_1^e)) &= \Delta V_2^T(\alpha_o, p_1^e, p_2^2) \\
\Delta \pi^G_{i}(r_1(\alpha_o, p_1^e)) &= \Delta V_2^T(\alpha_o, p_1^e, p_2^2) \\
\Delta \pi^G_{ii}(r_1(\alpha_o, p_1^e)) &= \Delta V_2^T(\alpha_o, p_1^e, p_2^2) \\
\Delta \pi^G_{un}(r_1(\alpha_o, p_1^e)) &= \Delta V_2^T(\alpha_o, p_1^e, p_2^2)
\end{align*}
\]

\[\text{Refer to Appendix B.2.2 for the proof.}\]
Note, proofs of all propositions are in the appendix. In order to characterize equilibrium production strategies each type's $\Delta \pi_1$ term has to be compared to the same type's $\Delta V_2$ term. The following relationships between the $\Delta V_2$'s and the $\Delta \pi_1$'s, which hold for all values of $\alpha_o$, enable us to narrow down the feasible set of production strategies that are consistent with an equilibrium.

Some Relationships between $\Delta \pi_1$'s and $\Delta V_2$'s:

1. $\Delta V_2^G > \Delta V_2^B$ since $1 - \bar{n}G_iG_j > 1 - \bar{n}G_i$.
2. $\Delta \pi_1^G = \Delta \pi_1^B > 0$ due to assumption A3.
3. $\Delta \pi_1^G < 0$ by assumption A2(b), $\Delta V_2^G \geq 0$ if $p_1 \geq x_{11}$, and $\Delta V_2^G < 0$ if $p_1 < x_{11}$.
4. $\Delta \pi_1^G > 0$ by assumption A2(a), $\Delta V_2^G \leq 0$ if $x_{11} \geq p_1$, and $\Delta V_2^G > 0$ if $x_{11} < p_1$.
5. $\Delta \pi_1^u = n\Delta \pi_1^h + \bar{n}\Delta \pi_1^G$, and $\Delta V_2^G = n\Delta V_2^h + \bar{n}\Delta V_2^G$.

These properties impose constraints on the production strategies possible in equilibrium. As a result only the following four pure strategy sets remain feasible.

Feasible Pure Strategy Sets $p_1$:

$p_1 = (1, 0, 0, 0), p_2 = (1, 0, 1, 1), p_3 = (0, 0, 0, 1), p_4 = (1, 0, 0, 1)$.

The pure strategy set $p_1$ is one in which the uninformed firms engage in reputation building. On the other hand, all types choose their optimal production in a myopic manner in $p_2$. In $p_3$, the type $G$ manager engages in reputation building and in $p_4$ only the uninformed type $G$ manager does so. Note that these strategy sets were examined briefly in section 4.2.2.

We next identify the equilibrium production strategy sets given different values of the prior $\alpha_o$, from the above feasible set. The expressions for $\Delta V_2^r$ and $\Delta \pi_1^r$ are basically functions of $\alpha_o$ and $p_1$. Hence, our objective can be achieved by focussing on how $\Delta \pi_1^r$, the short run "expected profit differential", and $\Delta V_2^r$, the long run "reputation benefit", vary with $\alpha_o$. 

94
The Profit Differential $\Delta \pi_1^T$:

For any given $p_l$, the $\Delta \pi_1^T$ terms are affected by $\alpha_o$ through its effect on $r_1$, the nominal return in period 1. In addition, $\alpha_o$ affects $\alpha_1$ which in turn affects $r_2$ and thus $\Delta V_2^T$. To understand how $\Delta \pi_1^T$ and $\Delta V_2^T$ vary with $\alpha_o$, it is useful to first examine how the generic interest $r_l(\alpha_{t-1}, p_l)$ varies with $\alpha_{t-1}$ for alternative $p_l$. Differentiating the functional form given in lemma 4.3 with respect to $\alpha_{t-1}$ we obtain,

$$\frac{dr_l}{d\alpha_{t-1}} = \frac{-\bar{G}_I(r_f - R_{lh})(p_{gt} - x_{lt})}{\phi_l^2}$$

where $x_{lt} = gp_d + qpg_{ut}$. This indicates that if $p_{gt} > x_{lt}$ then the nominal return is decreasing in reputation and if $p_{gt} < x_{lt}$ then the nominal return is increasing in reputation. If, however, $p_{gt} = x_{lt}$ then reputation does not effect the nominal return. This implies that:

- if $p_l = p^2$, $p^3$ or $p_4$ then $\frac{dr_l}{d\alpha_{t-1}} < 0$ and
- if $p_l = p^1$ then $\frac{dr_l}{d\alpha_{t-1}} = 0$.

The $\Delta \pi_1^T$ terms are increasing in $r_1$ for all types except $Gh$, because of the larger limited liability benefit associated with the risky action $y_h$. The $\Delta \pi_1^{Gh}$ term, on the other hand, is constant since it is independent of $r_1$. The resultant effect of $\alpha_o$ on $r_1$ and of $r_1$ on $\Delta \pi_1^T$ is that $\Delta \pi_1^{G1}$, $\Delta \pi_1^{G1}$ and $\Delta \pi_1^B$ are decreasing and $\Delta \pi_1^{Gh}$ remains constant, as $\alpha_o$ increases. However, if $p_l = p^1$, then insolvency cannot occur and, hence, the nominal return is set equal to the riskfree return, irrespective of the prior belief. This implies that if $p_l = p^1$ then none of the $\Delta \pi_1^T$ terms are affected by $\alpha_o$. The following summarizes the relationship between the different $\Delta \pi_1^T$.

- $\Delta \pi_1^{Gh} = \overline{G}_h (R_{hh} - R_{hl})$ is constant over $p_l$ and $\alpha_o$.
- $0 > \Delta \pi_1^{G1}(\alpha_o, p^2) > \Delta \pi_1^{G1}(\alpha_o, p^3) = \Delta \pi_1^{G1}(\alpha_o, p^4) > \Delta \pi_1^{G1}(\alpha_o, p^1)$
  and $\Delta \pi_1^{G1}(\alpha_o = 0, p^2) = \Delta \pi_1^{G1}(\alpha_o = 0, p^3) = \Delta \pi_1^{G1}(\alpha_o = 0, p^4)$
- $\Delta \pi_1^{G2}(\alpha_o, p^2) > \Delta \pi_1^{G2}(\alpha_o, p^3) = \Delta \pi_1^{G2}(\alpha_o, p^4) > \Delta \pi_1^{G2}(\alpha_o, p^1) > 0$
  and $\Delta \pi_1^{G2}(\alpha_o = 0, p^2) = \Delta \pi_1^{G2}(\alpha_o = 0, p^3) = \Delta \pi_1^{G2}(\alpha_o = 0, p^4)$
- $\Delta \pi_1^{G3}(\alpha_o, p_l) = \Delta \pi_1^{G3}(\alpha_o, p_1)$ for all $\alpha_o$ and $p_l$.

Figures 4.2(a)-(d) depict $\Delta \pi_1^{Gh}$ and $\Delta \pi_1^{G1}$ as functions of $\alpha_o$, for the different $p_l$ and figures 4.3(a)-(d) depict $\Delta \pi_1^{G2}$ and $\Delta \pi_1^B$ as functions of $\alpha_o$. 

95
The Reputation Benefit $\Delta V^*_2$:

The impact of $\alpha_o$ on $\Delta V^*_2$ depends on how $\alpha_o$ influences the difference in $r_2$ for pairs of $R_1$ values. In particular, $\Delta V^G_{2h}$ is proportional to the difference in the $r_2$ that is assessed if $R_1 = R_{hh}$ versus $R_1 = R_{hl}$. Similarly, $\Delta V^G_{2l}$ is proportional to the difference in the $r_2$ that is assessed if $R_1 = R_{lh}$ versus $R_1 = R_{ll}$. The $\Delta V^G_{2u}$ term is a weighted sum of $\Delta V^G_{2l}$ and $\Delta V^G_{2h}$ and the $\Delta V^G_{2l}$ term is proportional to $\Delta V^G_{2u}$.

Since the equilibrium production strategy in period 2 is $p^*_2 = (1, 0, 1, 1)$, the nominal return $r_2$ is a decreasing function of reputation $\alpha_1$, for all values of $R_1$. Hence, the discussion focuses on the impact of alternative revenue realizations on $\alpha_1$, for different production strategy sets. Table 4.1 summarizes the expressions for $\alpha_1$, for the four different revenue realizations, for each of the alternative pure strategies. We use these expressions to determine how $\Delta V^G_{2h}$ and $\Delta V^G_{2l}$ vary across $\alpha_o$ for alternative output strategy sets. These are given in figures 4.2(a)-(d). Each case is discussed separately below.

Case 1: $p_1 = (1, 0, 0, 0) \equiv p^1$. In this case $R_{lh}$ is an off-equilibrium outcome. A comparison of the appropriate expressions for $\alpha_1$ in table 1 indicates that $\Delta V^G_{2h}$ is negative because $R_{hh}$ induces a higher reputation than $\alpha_o$, while $R_{hl}$ induces a lower reputation, and the former is assured with $y_h$. $\Delta V^G_{2h}$ approaches zero as $\alpha_o$ goes to either zero or one because $R_{hh}$ and $R_{hl}$ induce very little change in $\alpha_1$ at these extreme values. The largest impact on $\alpha_1$ occurs when $\alpha_o$ is close to 1, i.e., when there is the greatest prior uncertainty about the manager's type.

$\Delta V^G_{2l}$ is positive because $R_{hl}$ maintains the manager's reputation, while $R_{hh}$ destroys it, and the former is assured with $y_l$. The maintenance of the manager's reputation increases in value as $\alpha_o$ increases.

Case 2: $p_1 = (1, 0, 1, 1) \equiv p^2$. In this case $R_{hl}$ is an off-equilibrium outcome. $\Delta V^G_{2h}$ is positive because a "perfect" reputation is achieved by selecting $y_l$ if it results in $R_{hl}$, while $R_{hh}$ merely maintains the existing reputation. This benefit goes to zero as $\alpha_o$ goes to one, since the reputation is close to one even if $R_{hh}$ occurs. If $\alpha_o$ is close to zero, then there is a strictly positive benefit from obtaining $R_{hl}$. As $\alpha_o$ increases the reputation from $R_{hl}$ remains the same, while the reputation from $R_{hh}$ increases. This implies that the incremental reputation benefit of $R_{hl}$ over $R_{hh}$ decreases as the existing reputation increases.

$\Delta V^G_{2l}$ is positive because $R_{ll}$ induces a higher reputation than $\alpha_o$, while $R_{lh}$ induces a lower reputation. The spread reaches a maximum near $\alpha_o = \frac{1}{2}$, i.e., when there is the greatest uncertainty about the manager's type. For $\alpha_o$ close to zero, both $R_{ll}$ and $R_{lh}$ result in $\alpha_1$ close
to zero, and for $\alpha_o$ close to one both $R_{II}$ and $R_{III}$ result in $\alpha_1$ close to one. Hence, the differences in these two cases approach zero.

**Case 3**: $p_1 = (0, 0, 0, 1) \equiv p^3$. In this case, $R_{III}$ reduces the manager's reputation, while $R_{II}$ reveals that the manager is type $G$. Although $R_{III}$ is not an off-equilibrium outcome it produces the same effect as in case 1, i.e., creditors know that the manager is type $B$. On the other hand, $R_{II}$ improves the manager's reputation instead of maintaining it. Hence, the reputation benefit to type $G_1$ is strictly greater than in case 1 and is increasing in $\alpha_0$.

**Case 4**: $p_1 = (1, 0, 0, 1) \equiv p^4$. The impact of $R_{II}, R_{III}$ and $R_{II}$ on $\alpha_1$ are the same as in case 3 and the impact of $R_{III}$ is the same as case 1 (which is similar to case 3). Hence, the $\Delta V^G_2$ term is exactly the same as in case 3, whereas the $\Delta V^S_2$ term behaves the same way as $\alpha_0$ changes but is strictly smaller.

The above relationships between $\Delta V^G_2$ and $\Delta V^G_1$ are summarized as follows:

- $\Delta V^G_2(\alpha_o, p^1) < 0 < \Delta V^G_2(\alpha_o, p^2) < \Delta V^G_2(\alpha_o, p^4) < \Delta V^G_2(\alpha_o, p^3)$
  and $\Delta V^G_2(\alpha_0 = 0, p^2) = \Delta V^G_2(\alpha_0 = 0, p^4) = \Delta V^G_2(\alpha_0 = 0, p^3)$

- $0 < \Delta V^G_1(\alpha_o, p^2) < \Delta V^G_1(\alpha_o, p^3) < \Delta V^G_1(\alpha_o, p^4) < \Delta V^G_1(\alpha_o, p^1)$
  and $\Delta V^G_1(\alpha_0 = 1, p^1) = \Delta V^G_1(\alpha_0 = 1, p^3) = \Delta V^G_1(\alpha_0 = 1, p^4)$

Given the above properties of the $\Delta V^G_2$ and $\Delta V^G_1$ terms, we next determine how $\Delta V^G_u$ and $\Delta V^S$ vary with $\alpha_o$. As noted before, $\Delta V^G_u$ is a convex combination of $\Delta V^G_2$ and $\Delta V^G_1$ and $\Delta V^S$ is a fraction of $\Delta V^G_u$. More specifically,

$$\Delta V^G_u = \eta \Delta V^G_2 + \bar{\eta} \Delta V^G_1$$
$$\Delta V^S = \frac{1 - \bar{\eta} G_1}{1 - \eta G_2} \Delta V^G_u$$

Figures 4.3(a)-(d) illustrate how $\Delta V^G_u$ and $\Delta V^S$ vary with $\alpha_o$, obtained by combining $\Delta V^G_2$ and $\Delta V^G_1$. In case 1, $p_1 = p^1$, both $\Delta V^G_u$ and $\Delta V^S$ are increasing in $\alpha_o$. In case 2, $p_1 = p^2$, both $\Delta V^G_u$ and $\Delta V^S$ are decreasing in $\alpha_o$, after an initial section where both functions may be increasing in $\alpha_o$. In case 3, $p_1 = p^3$, both functions could be either increasing or decreasing in $\alpha_o$, depending on the values of $n, G_l$ and $G_h$. Neither function, however, goes to zero at either $\alpha_o = 0$ or 1. Case 4, $p_1 = p^4$, is similar to case 3. The relationships between the $\Delta V^G_u$s and $\Delta V^S$s for different values of $p_1$ and $\alpha_o$ can be summarized as follows:

- $\Delta V^G_u(\alpha_o, p^3) > \Delta V^G_u(\alpha_o, p^4), \Delta V^G_u(\alpha_o, p^4) > \Delta V^G_u(\alpha_o, p^2)$
  and $\Delta V^G_u(\alpha_o, p^4) > \Delta V^G_u(\alpha_o, p^1)$
\[ \Delta V_2^u(\alpha_0 = 1, p^3) = \Delta V_2^u(\alpha_0 = 1, p^4) = \Delta V_2^u(\alpha_0 = 1, p^1) > \Delta V_2^u(\alpha_0 = 1, p^2) = 0 \]

and \[ \Delta V_2^u(\alpha_0 = 0, p^3) = \Delta V_2^u(\alpha_0 = 0, p^4) = \Delta V_2^u(\alpha_0 = 0, p^2) > \Delta V_2^u(\alpha_0 = 0, p^1) = 0 \]

- The various \( \Delta V_2^u \)s satisfy relationships which are identical to the above.

We next determine the conditions under which each of the above feasible pure strategy sets is consistent with an equilibrium. This is done by comparing each \( \Delta \pi_1^f \) with the corresponding \( \Delta V_2^u \) function, under each of \( p^1, p^2, p^3 \) and \( p^4 \). It is important to note that the magnitude of \( \Delta \pi_1^f \) can be increased arbitrarily by increasing the differential revenue portion of that term. If the differential revenue part is large enough it can dominate the relationship between \( \Delta \pi_1^f \) and \( \Delta V_2^u \). Similarly, the magnitude of \( \Delta V_2^u \) can be increased by either increasing the amount of the debt in the future period or by increasing the number of periods in the future. In the following proposition the specific conditions under which each of the above feasible pure strategy sets is consistent with an equilibrium are identified. The values of \( \alpha_0 \) referred to in the following proposition are depicted in figures 4.2(a)-(d) and 4.3(a)-(d).

Recall that by assumption A3, \( \tilde{R}_h - \tilde{R}_l > -\bar{n}f(r_f - R_{ih}) \). We denote \( \bar{n}f(r_f - R_{ih}) \) by \( N \).

Note that \( N \) represents the uninformed firms' limited liability benefit when the nominal return is at its lowest value of \( r_f \). Let \( A^1, A^2, A^3 \) and \( A^4 \) denote the set of \( \alpha_0 \) for which \( p^1, p^2, p^3 \) and \( p^4 \) are equilibrium strategies.

**Proposition 4.2** In the non-disclosure regime

**I** If \( \tilde{R}_h - \tilde{R}_l > -X.N \) where \( X = \frac{[1-\bar{n}f((1+q) - q_{il})]_{\bar{G}_l}}{1-\bar{n}f} \), then

(a) \( A^3 = \emptyset \), (b) \( A^4 = \emptyset \) (c) \( A^1 = \emptyset \) and (d) \( A^2 = [0,1] \).

**II** If \(-X.N > \tilde{R}_h - \tilde{R}_l > -Y.N \) where \( Y = \frac{[1-\bar{n}f(1+q) - q_{il}]_{\bar{G}_l}}{1-\bar{n}f} \), then

(a) \( A^3 = \emptyset \), (b) \( A^4 \neq \emptyset \) if max_{\alpha_0} \{ \Delta V_2^u(\alpha_0, p^4) - \Delta \pi_1^f(\alpha_0, p^4) \} > 0 

(c) \( A^1 \neq \emptyset \) if, and only if, \( \tilde{R}_h - \tilde{R}_l < -W.N \) where \( W = \frac{1-\bar{n}f}{1-\bar{n}f} \) and

(d) \( A^2 = \left\{ [0, \alpha^3], [\alpha^4, 1]\right\} \).

**III** If \(-Y.N > \tilde{R}_h - \tilde{R}_l > -N. \) then

(a) \( A^3 \neq \emptyset \) if \( R_{hh} - R_{hl} < \frac{2}{(1-\bar{n}f)} \cdot N \) and \( \tilde{R}_h - \tilde{R}_l > -Z.N \) where

\[ Z = \left\{ \frac{1-\bar{n}f}{1-\bar{n}f} + \frac{q_{il} \bar{G}_l}{(1-\bar{n}f)/(1-\bar{n}f)} \right\} < R_{hh} - R_{hl} < \frac{2}{(1-\bar{n}f)} \cdot N, \]

(b) \( A^4 \neq \emptyset \) if \( \tilde{R}_h - \tilde{R}_l > -W.N \).
(c) $A^1 \neq \emptyset$ if, and only if, the condition in II(c) hold and

(d) $A^2 = [\alpha^4, 1]$.

III is feasible if and only if $qnG_h > nG_l$.

The proof of the above proposition makes use of the relationships between $\Delta V_f^I$'s and $\Delta \pi_f^I$'s at extreme values of $\alpha$. Comparisons of the values of $\Delta V_f^I$'s and $\Delta \pi_f^I$'s at $\alpha = 0$ and $\alpha = 1$ for each of the feasible $p_1$ helps to identify the above conditions.

The conditions outlined in the above proposition place bounds on the differential revenue terms of types $G_h$, $G_u$ and $B$, over and above those placed by assumptions A2(a), A2(b) and A3. Given assumption A2(b) the type $G_l$'s profit differential is always negative, and hence, this type's reputation benefit is always positive. Hence, type $G_l$ always prefers the safe project. The type $G_h$ manager's preference between the risky and the safe projects depends on the relative values of his revenue differential $R_{Ah} - R_{Al}$ and reputation benefit which is a function of $N$. The uninformed managers' preference over alternative projects depends on the relative values the revenue differential $R_h - R_l$ and different functions of $N$ which represent differences between the reputation and limited liability benefits.

Under condition I the revenue differentials of the type $G_h$ and uninformed managers are high enough to rule out the choice of the safe project by any of them. Hence, only the myopic strategy exists in equilibrium at each value of $\alpha$. Under condition II, on the other hand, both the uninformed types may select the safe project in equilibrium but the revenue differential is not high enough for the type $G_h$ to choose the safe project. Note that the myopic strategy exists in equilibrium for extreme values of the prior belief. Under condition III the revenue differentials are low and, hence, the type $G_h$ manager may prefer the safe project. Also, under this condition the myopic strategy exists in equilibrium at the upper extreme values of the prior. Condition III is feasible, if and only if, $qnG_h > nG_l$. If this condition is violated, the values of $\bar{R}_h - \bar{R}_l$ indicated in III are ruled out by assumption A3.

The proposition and its proof contains partial characterizations of the sets $A^1$, $A^2$, $A^3$ and $A^4$, if they exist in the different ranges of $R_h - R_l$ specified in the proposition. These characterizations are summarized as follows.$^{15}$

Under I : $A^1$, $A^3$ and $A^4$ are empty, and $A^2 = [0, 1]$.

Under II : $A^3 = \emptyset$, $A^4 \subset [0, \alpha^{11}]^c$ since $\alpha^{11} > 0$, $A^2 = ([0, \alpha^2], [\alpha^4, 1])$ and $A^4 = [\alpha^4, 1]$.

Under III : $A^2 = [0, \alpha^5]$, $A^4 = [\alpha^4, 1]$, $A^1 = [\alpha^1, 1]$ and $A^4 \subset [0, \alpha^{10}]^c$

$^{15}$Refer to the proof of proposition 4.2 in Appendix B.2.1.
When $\hat{R}_h - \hat{R}_l$ is high as under condition I, the myopic strategy set is the only one among the pure strategies which is consistent with an equilibrium at every $\alpha_o$.

Under II, $\hat{R}_h - \hat{R}_l$ is lower than in I. In this range $p^3$ is not consistent with an equilibrium for any $\alpha_o$. Under II, the type $Gh$ manager’s revenue differential is not high enough to motivate him to choose the the safe project for any prior reputation. Given that the type $Gh$ manager is expected to choose the risky project the uninformed types may both choose the safe or the risky project or may choose different projects. Intuitively, if creditors expect both $Gu$ and $B$ to choose the risky project, then the current interest requirement is set high resulting in a high limited liability benefit. This leads to both the uninformed types preferring the risky project at extreme values of the prior of reputation, $\alpha_o$. At high $\alpha_o$ the reputation is already high and the uninformed manager makes use of his existing reputation instead of attempting to increase it. On the other hand, at low $\alpha_o$ the reputation is too low to engage in reputation acquisition and, hence, types $Gu$ and $B$ prefer the risky project. Therefore, $p^2$ is consistent with an equilibrium at extreme values of $\alpha_o$.

If creditors expect the uninformed types to choose the safe project, the interest requirement in the current period is set low, resulting in a small benefit from limited liability in the current period. Hence, the safe project is chosen by both types at high $\alpha_o$. If the prior reputation is high, however, the reputation benefit is high enough to exceed the revenue differential. Therefore, $p^1$ is consistent with an equilibrium at high values of $\alpha_o$. On the other hand, if creditors expect $Gu$ to choose the safe project and $B$ to choose the risky project then the reputation benefit is high for intermediate values of the prior reputation and may exceed the profit differential at these $\alpha_o$s. Therefore, $p^4$ may be consistent with an equilibrium at intermediate value of $\alpha_o$.

Under condition III, $\hat{R}_h - \hat{R}_l$ is very low. The type $Gh$’s revenue differential, $R_{hh} - R_{hl}$ is also low enough to motivate the type $Gh$ to choose the safe project, if the prior reputation is very low. The $Gh$ type’s reputation benefit is highest at low $\alpha_o$, falling to zero at $\alpha_o = 1$ and, hence, at higher $\alpha_o$ does not exceed his profit differential. Given that the type $Gh$ prefers the safe project at low $\alpha_o$, the reputation benefits of the uninformed types are also high at low $\alpha_o$. Therefore $p^3$ is consistent with an equilibrium at low $\alpha_o$, provided the $B$ type’s reputation does not also exceed his profit differential. These results are illustrated through a numerical example.

A Numerical Example

The results in the proposition are illustrated by the following numerical example. The
parameter values used are: \( n = 0.7, G_I = 0.5, G_h = 0.5, q = 0.9, r_f = 1.0 \) and \( R_{hl} = 0 \). These yield the following values for the intervals: \( X.N = -0.0706, Y.N = -0.1209 \) and \( N = -0.15 \). We find the values of \( \Delta V_2(a_o, p_1) \) and \( \Delta \pi_1^f(a_o, p_1) \) for different values of \( a_o \in [0, 1] \) for each of \( p^1, p^2, p^3 \) and \( p^4 \). This enables us to determine the sets \( A^1, A^2, A^3 \) and \( A^4 \). These intervals are given in table 4.2(a) and illustrated in figure 4.4.

The reputation benefit is relatively small in the above model. It can be increased by increasing the impact of the future, which can be achieved in either of two ways. One is to increase the capital that the firm borrows in the future period. The other is to increase the number of periods over which the firm borrow in the future. If the reputation benefit is increased sufficiently, the myopic strategy will cease to exist in equilibrium, for almost all prior beliefs. The exceptions are prior beliefs which are very close to one. This is due to the fact that under the anticipation that the myopic strategy will be played, the reputation benefit is almost zero when the prior reputation is close to one. Hence, the current period profit differential exceeds the reputation benefit for types \( G_h, G_u \) and \( B \).
4.4 The Disclosure Regime

In this section we consider a regime in which a manager can make disclosures to convey information about his type. As stated before, we assume that a direct disclosure of manager type is not verifiable and hence is not credible. In addition, neither the realized value of \( \theta_t \) nor the selected project \( y_t \) ever publicly observed or verifiable. Hence, an announcement of either \( \theta_t \) or \( y_t \) individually is not credible. The manager can communicate information about his type by issuing earnings forecasts or announcing both his information and his project choice. It is assumed that the manager discloses his information at the time of output choice in each period, i.e., the disclosure occurs after the borrowing for the current period. Hence, the disclosure in a particular period does not affect the required nominal return in that period. This sequence of events enables us to avoid any short run effects of disclosure and focus on its reputation effect.

A message released by the manager in period \( t \) is denoted by \( m_t \). The manager can make one of the following disclosures:

\[
m_t \in \{E[R|\theta_t, y_t], E[R|I, y_t], E[R|I, y_t], E[R|\theta_t, y_t], E[R|y_t], E[R|y_t], \emptyset\}
\]

where the expectations given above are conditioned on either \( \theta_t \) and \( y_t \) or \( y_t \) alone. The above indicates that a manager may signal any expected revenue or he may opt for non-disclosure.\(^{16}\)

Note that a disclosure of expected revenue is equivalent to a simultaneous disclosure of \( \theta_t \) and \( y_t \). Since the realized revenue \( R_t \), is publicly observed at the end of period \( t \), a comparison of a firm’s disclosure and its end of period revenue can facilitate identification of false disclosures. A manager who is discovered falsifying disclosures may be penalized. We assume that the magnitudes of such penalties are sufficiently high to discourage any false disclosures that can be detected by observation of realized revenues, at the end of the period.

Only an informed manager who selects his signal “correctly” realizes a revenue consistent with his signal. An uninformed manager, on the other hand, cannot issue any of the signals \( E[R|\theta_t, y_t], E[R|I, y_t], E[R|I, y_t], E[R|\theta_t, y_t] \) without the risk of an inconsistency between his signal and his realized revenue. However, the uninformed can truthfully disclose \( m_t = E[R|y_t] \) when his production level is \( y_t = y_h \) and \( m_t = E[R|y_t] \) when his production level is \( y_t = y_l \). In addition to these, both types of managers may choose non-disclosure.

We next discuss the creditors’ belief revision and the manager’s optimal production and disclosure choice followed by the characterization of two period sequentially rational equilibria.

\(^{16}\)Expected revenue and expected profit can be used interchangeably.
which consist of mutually consistent sets of creditor beliefs and project and disclosure strategies in each period.

4.4.1 Creditors' Belief Revision in the Disclosure Regime

In the disclosure regime, potential creditors update their beliefs about the firm twice each period. One revision occurs when earnings forecasts are released and the other occurs when realized revenues are announced at the end of a period. As in the non-disclosure case, we first assume that creditors have a certain set of conjectures about the production and disclosure choice behavior of the firm. It is then shown that in equilibrium the firm's optimal choices are consistent with creditors’ conjectures and vice versa. Creditors' anticipation of the optimal production strategies in period \( t \) are functions of the manager’s reputation at the beginning of period \( t \). Thus the anticipated production strategy set is

\[
\mathcal{P}_t(\alpha_{t-1}) = \{p_{gh_t}(\alpha_{t-1}), p_{gt_t}(\alpha_{t-1}), p_{gu_t}(\alpha_{t-1}), p_{B}(\alpha_{t-1})\},
\]

where \( t \in \{1, 2\} \). Creditor's anticipation of optimal disclosure policies given the manager’s type and production level are denoted by

\[
\beta_t(m_t, \tau, y_t) = \text{Prob}(m_t|\mathcal{G}h, y_t)
\]

where, \( y_t \in \{y_h, y_l\} \) and \( \tau \in \{\mathcal{G}h, \mathcal{G}l, \mathcal{G}u, B\} \). Note that we allow for the possibility that the disclosure and project choices may not be consistent with each other, i.e., \( 0 \leq \beta_t(m_t, \tau, y_t) \leq 1 \) for any pair of \( m_t \) and \( y_t \) values. This also allows for non-disclosure by the firm when it is informed and disclosure when it is uninformed.

The time of disclosure in period \( t \) is denoted by \( t_0 \) and the time of announcement of realized revenue, i.e., the end of the period, by \( t \). Note that the nominal return for period \( t \) is assessed at the end of period \( t - 1 \) which is prior to \( t_0 \). The factors affecting the posterior beliefs at time \( t_0 \) are: the priors, the anticipated production and disclosure strategies, and the disclosure itself. Similarly, the factors affecting the posterior beliefs at time \( t \) are: the prior \( \alpha_{t-1} \), the anticipated production and disclosure strategies, the realized revenue \( R_t \), and the disclosure made at time \( t_0 \). The following are the creditors’ posterior beliefs about the firm at \( t_0 \) and \( t \).\(^{17}\) The prior belief at the beginning of period \( t \) is \( \alpha_{t-1} \).

Posterior Beliefs at \( t_0 \):

\[
\alpha_{t_0}(\alpha_{t-1}, R_t, p_t, m_t, \beta_t) = \frac{\text{Prob}(m_t|\mathcal{G}, p_{gt}, \beta_{gt}) \cdot \alpha_{t-1}}{\text{Prob}(m_t|\mathcal{B}, \beta_{gt}, p_{gt}) \cdot \alpha_{t-1} + \text{Prob}(m_t|\mathcal{B}, \beta_{gt}, p_{gt}) \cdot (1 - \alpha_{t-1})}
\]

\(^{17}\) These follow using Bayes rule.
where

\[
\text{Prob}(m_t | G, p_G, \beta_G) = q \left\{ [np_Ght \beta(m_t | G_h, y_h) + \bar{\beta} p_Gu \beta(m_t | G, y_h)] \\
+ [n(1 - p_Ght) \beta(m_t | G_h, y_l) + \bar{\beta} (1 - p_Gu) \beta(m_t | G, y_l)] \right\} \\
+ (1 - q) \left\{ p_Gu \beta(m_t | G_u, y_h) + (1 - p_Gu) \beta(m_t | G_u, y_l) \right\}
\]

\[
\text{Prob}(m_t | B, p_B) = p_B \beta(m_t | B, y_h) + (1 - p_B) \beta(m_t | B, y_l)
\]

**Posterior Beliefs at t:**

\[
\alpha_t (\alpha_{t-1}, R_t, p_t, m_t, \beta_t) \equiv \text{Prob}(G | \alpha_{t-1}, R_t, p_t, m_t, \beta_t) = \frac{g(R_t, m_t | G, p_G, \beta_G) \cdot \alpha_{t-1}}{g(R_t, m_t | G, p_G, \beta_G) \cdot \alpha_{t-1} + g(R_t, m_t | B, p_B, \beta_B) \cdot \alpha_{t-1}}
\]

where

\[
g(R_t, m_t | G, p_G, \beta_G) = q \left\{ [np_Ght g(R_t | h, y_h) \cdot \beta(m_t | G_h, y_h)] \\
+ \bar{\beta} p_Gu g(R_t | l, y_h) \cdot \beta(m_t | G, y_h)] \\
+ [n(1 - p_Ght) g(R_t | h, y_l) \cdot \beta(m_t | G_h, y_l)] \\
+ \bar{\beta} (1 - p_Gu) g(R_t | l, y_l) \cdot \beta(m_t | G, y_l)] \right\} \\
+ (1 - q) \left\{ p_Gu [ng(R_t | h, y_h) + \bar{\beta} g(R_t | l, y_h)] \cdot \beta(m_t | G_u, y_h)] \\
+ (1 - p_Gu) [ng(R_t | h, y_l) + \bar{\beta} g(R_t | l, y_l)] \cdot \beta(m_t | G_u, y_l)] \right\}
\]

\[
g(R_t, m_t | B, p_B, \beta_B) = \left\{ p_B [ng(R_t | h, y_h) + \bar{\beta} g(R_t | l, y_h)] \cdot \beta(m_t | B, y_h)] \\
+ (1 - p_B) [ng(R_t | h, y_l) + \bar{\beta} g(R_t | l, y_l)] \cdot \beta(m_t | B, y_l)] \right\}
\]

\[
p_t = (p_Ght, p_Gu, p_Gu, p_B)
\]

\[
p_Gt = (p_Ght, p_Gu, p_Gu)
\]

\[
\beta_Gt = (\beta_t(m_t | G_h, y_t), \beta_t(m_t | G_l, y_t), \beta_t(m_t | G_u, y_t))
\]

\[
\beta_t = (\beta_Gt, \beta_Bt)
\]

The manager's reputation, at the end of period \( t \), is a function his prior reputation, the end of period revenue realization, the disclosed signal, and creditors' anticipation of the production
and disclosure policies. Although the posterior belief \( \alpha_{t_0} \) gives advance information about the manager's type and his firm's future revenue, it has no effect on the firm's nominal return in either period \( t \) or \( t+1 \). This is due to the fact that the nominal return for period \( t \) is determined prior to \( t_0 \) and the nominal return for period \( t+1 \) is determined at time \( t \) and is thus based on the reputation at time \( t+1 \). However, the value of \( \alpha_{t_0} \) does affect the firm's market value at the time of forecast issue. Therefore the expression for \( \alpha_{t_0} \) is included for completeness.

4.4.2 Optimal Output and Disclosure Choice in the Disclosure Regime

As in the non-disclosure regime, the manager selects his optimal project at the beginning of each period. An informed manager observes the value of \( \theta_t \) prior to selecting \( y_t \), whereas an uninformed manager does not. Motivated by the incentive to distinguish himself from type \( B \) managers, an informed manager also discloses his information at the beginning of each period subsequent to receipt of information. An uninformed type \( G \) manager may or may not make a disclosure depending on what the type \( B \) manager is expected to do and on whether or not any penalties exist for false disclosures. A type \( B \) manager wishes to remain indistinguishable from type \( G \) managers and hence is motivated to mimic either the informed or the uninformed type \( G \) manager's behavior.

In addition to affecting the firm's period \( t \) realized revenue, \( y_t \) together with \( m_t \) also influences the firm's reputation and, hence, its future interest rates. Therefore, in any period \( t \), the project and disclosure are selected by maximizing the net present value of expected future cash flows, net of anticipated capital repayment. In calculating a firm's future cash flows it is assumed that all future production levels will be optimized in the same manner. The firm's production choice problem is written as follows. Note that as in the non-disclosure regime, the following is an abbreviated representation of the problem in which \( \alpha_{t-1}, \alpha_t, y_t \) etc., are not explicitly stated.

**Optimization Problem 2**:

\[
V_i^*(\tau) = \max_{y_t, m_t} V_i(\tau) = E_t[p_t | \tau] + E_t[V_{i+1}^* | \tau] \\
(4.28)
\]

subject to

\[
r_t = r(\alpha_{t-1}, p_t) \\
\alpha_t = \frac{g(R_t, m_t[G, p_{G_t}, \beta_{G_t}] \cdot \alpha_{t-1}}{g(R_t, m_t[G, p_{G_t}, \beta_{G_t}] \cdot \alpha_{t-1} + g(R_t, m_t[B, p_{B_t}, \beta_{B_t}] \cdot (1 - \alpha_{t-1})}
\]

The manager determines his optimal production and disclosure policies by solving the above optimization problem for \( t = 1, 2 \). Note that \( E[V^*_3] = 0 \) since no third period exists. The
expected net present value in the first period contains the term \( E_1[V^*_2] \) which is the optimized second period expected profit. Therefore, we need to maximize the second period expected profit first to determine \( E_1[V^*_2] \). We can then maximize the first period expected net present value after substituting for \( E_1[V^*_2] \) into the expression for \( V_1 \). The constraints in the maximization problem ensure that the creditors' belief revision rules are common knowledge. As in the non-disclosure regime, the manager's objective function is dependent on his type, since the expectations of the profit terms depend on whether or not the manager has prior knowledge of \( \theta_t \) and on the probability with which he expects to be informed in the future.

### 4.4.3 Equilibrium in the Disclosure Regime

An equilibrium consists of solutions to optimization problem 2, which are consistent with creditors' beliefs. The disclosure equilibria are characterized under assumption A4 and A5. Under assumption A4 the general disclosure set becomes,

\[
m_t \in \{R_{hh}, R_{ll}, R_{hl}, R_{lh}, R_l, R_h, \emptyset\}
\]

Given assumptions A4 and A5, we can derive expressions for \( g(R_t, m_t, \tau, p_t, \beta_t) \), the joint densities of \( R_t \) and \( m_t \) conditional on manager type, creditors' beliefs about project choices and \( \beta_t \). Substitution of these expressions for \( g(R_t, m_t, \tau, p_t, \beta_t) \), into expression (4.27) yield expressions for the posterior beliefs.

Given creditors' posteriors and expectations of project choices, nominal returns are determined by using the expression in lemma 4.3. Hence, the period \( t \) return is a function of \( \alpha_{t-1} \) and \( p_t \). Since project choice is affected by a manager's disclosure choice, in equilibrium \( p_t \) is a function of \( m_t \). Therefore, the period \( t \) nominal return is affected by the period \( t \) disclosure choice, through the anticipated production strategy, \( p_t \). Past disclosures and project choices also affect \( r_t \) through the posterior belief \( \alpha_{t-1} \).

According to optimization problem 2, the manager chooses his project and disclosure levels simultaneously. However, this can be represented as a two stage problem. Since the project set has binary support whereas there are several possible disclosure levels, it is easiest to first determine an optimal disclosure policy, for each project level, and then determine the optimal project given the optimal disclosure function.

\[\text{18 Refer to Appendix B.1.2 for the expressions for } g(R_t, m_t, \tau, p_t, \beta_t).\]
Equilibrium Disclosure Policy

Under assumptions A4 and A5, each type of manager's disclosure choice problem in period \( t \) can be written, for different project choices, as

\[
W_{t+1}(\tau, \alpha_{t-1}, y_t) = \max_{\tau_t, m_t} \mathbb{E}_{R_t}[V_{t+1}^* (\tau, \alpha_{t-1}, R_t, m_t, p_t, \beta_t)] \tau, y_t
\]

where, \( \tau \in \{G_h, G_l, G_u, B\} \), \( y_t \in \{y_h, y_l\} \) and the expectation \( \mathbb{E}_{R_t} \) is taken over the different revenue outcomes, given \( y_t \) and \( \tau \). \( W_{t+1}(\tau, \alpha_{t-1}, y_t) \) represents the expected net present value of future period profits which have been optimized with respect to the future project and disclosure choices.

In the two period setting \( V_{t+1}^* \) refers to the optimized final period expected profit. To determine this quantity we first solve the disclosure and production choice problems for the last period. Since there are no future periods, reputation acquisition and disclosure are irrelevant in the final period. Hence, the equilibrium production strategy set is exactly the same as in the non-disclosure regime. Taking this into account, the first period disclosure choice problem can be written as follows. Note that although \( \alpha_1 \) also depends on the anticipated disclosure and production strategy sets, \( \beta_t = \beta_1 \) and \( p_t = p_1 \), respectively, the latter are suppressed in the following expressions. Also note that \( q [nR_{hh} + \bar{n}R_{hl}] + \bar{q}[\bar{R}_h - \bar{n}\bar{G}_lR_{lh}] \) is denoted by \( \bar{R}_G \), for notational convenience and \( \bar{R}_G \) represents the last period expected revenue of the type \( G \) manager.

If the manager is an informed type \( G \) and has observed \( \theta_1 = h \), then his optimal disclosure \( m_1^*(G_h, \alpha_o, y_l) \) for each project is determined by:

\[
W_2(G_h, \alpha_o, y_1 = y_h) = \max_{\alpha_1} \bar{R}_G - (1 - \bar{n}\bar{G}_l\bar{q})r_2(\alpha_1(\alpha_o, R_1 = R_{hh}, m_1))
\]

\[
W_2(G_h, \alpha_o, y_1 = y_l) = \max_{\alpha_1} \bar{R}_G - (1 - \bar{n}\bar{G}_l\bar{q})\{\bar{G}_h r_2(\alpha_1(\alpha_o, R_1 = R_{hh}, m_1)) + \bar{G}_h r_2(\alpha_1(\alpha_o, R_1 = R_{hl}, m_1))\}
\]

If the manager is an informed type \( G \) and has observed \( \theta_1 = l \), then his optimal disclosure \( m_1^*(G_l, \alpha_o, y_1) \) for each project is determined by:

\[
W_2(G_l, \alpha_o, y_1 = y_l) = \max_{\alpha_1} \bar{R}_G - (1 - \bar{n}\bar{G}_l\bar{q})r_2(\alpha_1(\alpha_o, R_1 = R_{ll}, m_1))
\]

\[
W_2(G_l, \alpha_o, y_1 = y_h) = \max_{\alpha_1} \bar{R}_G - (1 - \bar{n}\bar{G}_l\bar{q})\{\bar{G}_l r_2(\alpha_1(\alpha_o, R_1 = R_{ll}, m_1))
\]
If the manager is an uninformed type $G$ then his optimal disclosure $m_1^*(G, \alpha, y_1)$ for each project is determined by,

$$W_2(G, \alpha, y_1 = y_h) = \max_{m_1} \bar{R}_G - (1 - \tilde{n}G_i)\{nr_2(\alpha_1(\alpha_0, R_1 = R_{hh}, m_1)) + \tilde{n}G_i r_2(\alpha_1(\alpha_0, R_1 = R_{hh}, m_1)) + \tilde{n}G_i r_2(\alpha_1(\alpha_0, R_1 = R_{lh}, m_1))\}$$

$$W_2(G, \alpha, y_1 = y_l) = \max_{m_1} \bar{R}_G - (1 - \tilde{n}G_i)\{nr_2(\alpha_1(\alpha_0, R_1 = R_{ll}, m_1)) + \tilde{n}G_i r_2(\alpha_1(\alpha_0, R_1 = R_{ll}, m_1)) + \tilde{n}G_i r_2(\alpha_1(\alpha_0, R_1 = R_{lh}, m_1))\}$$

If the manager is a type $B$ then his optimal disclosure $m_1^*(B, \alpha, y_1)$ for each project is determined by,

$$W_2(B, \alpha, y_1 = y_h) = \max_{m_1} \bar{R}_h - (1 - \tilde{n}G_i)\{nr_2(\alpha_1(\alpha_0, R_1 = R_{hh}, m_1)) + \tilde{n}G_i r_2(\alpha_1(\alpha_0, R_1 = R_{hh}, m_1)) + \tilde{n}G_i r_2(\alpha_1(\alpha_0, R_1 = R_{hl}, m_1))\}$$

$$W_2(B, \alpha, y_1 = y_l) = \max_{m_1} \bar{R}_h - (1 - \tilde{n}G_i)\{nr_2(\alpha_1(\alpha_0, R_1 = R_{ll}, m_1)) + \tilde{n}G_i r_2(\alpha_1(\alpha_0, R_1 = R_{ll}, m_1)) + \tilde{n}G_i r_2(\alpha_1(\alpha_0, R_1 = R_{hl}, m_1))\}$$

Note that the last period expected revenue of the type $B$ manager is $\bar{R}_h$. The quantities being maximized represent the net present values of expected profits given the manager’s type and his reputation at the end of period 1. The assumptions of this model enable us to narrow down the feasible disclosure sets and then identify the disclosure policies which result in perfect reputation.

**Feasible Disclosure Sets:**

Since disclosures that are identifiable as false, ex post, are ruled out by assumption, the message sets by type of manager are constrained in the following manner.

- A type $Gh$ manager’s feasible disclosure set is :
  
  \{ $R_{hh}$, $\bar{R}_{hh}$, $\bar{R}_h$, $R_l$, $\emptyset$ \} if $P_{Gh} = 1$
  
  \{ $\bar{R}_{hl}$, $\bar{R}_l$, $\emptyset$ \} if $P_{Gh} = 0$. 

108
- A type $\mathcal{G}l$ manager's feasible disclosure set is:
  \begin{align*}
  \{R_{lh}, \tilde{R}_{lh}, \tilde{R}_{h}, \tilde{R}_{l}, \emptyset\} & \text{ if } P_{Glt} = 0 \\
  \{\tilde{R}_{lh}, \tilde{R}_{h}, \emptyset\} & \text{ if } P_{Glt} = 1.
  \end{align*}

- A type $\mathcal{G}u$ manager's feasible disclosure set is:
  \begin{align*}
  \{\tilde{R}_{h}, \emptyset\} & \text{ if } P_{Gut} = 1 \\
  \{\tilde{R}_{l}, \emptyset\} & \text{ if } P_{Gut} = 0.
  \end{align*}

- A type $\mathcal{B}$ manager's feasible disclosure set is:
  \begin{align*}
  \{\tilde{R}_{h}, \emptyset\} & \text{ if } P_{Bt} = 1 \\
  \{\tilde{R}_{l}, \emptyset\} & \text{ if } P_{Bt} = 0.
  \end{align*}

Note that $\tilde{R}_{h}$ and $\tilde{R}_{l}$ are included in the informed type $\mathcal{G}$ manager's message sets. Although these are not truthful for an informed manager, they are included in his feasible disclosure set since he can make these disclosures without being detected. If the manager is uninformed, however, he cannot issue forecasts $R_{hh}$, $R_{ll}$, $\tilde{R}_{lh}$ or $\tilde{R}_{hl}$ without a positive probability of being detected and penalized, ex post. Hence, these are not included in the disclosure sets of uninformed managers.

**Optimal Disclosure Policies:**

Given the above feasible disclosure sets, the manager's optimal disclosure policy is a function of his type, project choice and the anticipated production strategy set. It is shown later that the only pure strategy production sets that can exist in equilibrium for $\alpha_o \in [0,1]$ are:

\begin{align*}
  p^1 &= (1,0,0,0), \quad p^2 = (1,0,1,1) \quad \text{and} \quad p^4 = (1,0,0,1)
\end{align*}

Under these production strategy sets, the optimal disclosure policies are given below.

(i) A type $\mathcal{G}h$ manager's optimal disclosure policy is:
\[ m^*_h(\mathcal{G}h, \alpha_o, y_h) \in \{R_{hh}, \tilde{R}_{hl}\} \text{ and } m^*_h(\mathcal{G}h, \alpha_o, y_l) = \tilde{R}_{hl} \forall \alpha_o. \]

(ii) A type $\mathcal{Gl}$ manager's optimal disclosure policy is:
\[ m^*_l(\mathcal{Gl}, \alpha_o, y_l) \in \{R_{ll}, \tilde{R}_{lh}\} \text{ and } m^*_l(\mathcal{Gl}, \alpha_o, y_h) = \tilde{R}_{lh} \forall \alpha_o. \]

(iii) The type $\mathcal{G}u$ and $\mathcal{B}$ managers' optimal disclosure policies are:
\begin{align*}
  &- m^*_l(\tau, \alpha_o, y_h) = \tilde{R}_{h} \text{ and } m^*_l(\tau, \alpha_o, y_l) \in \{\tilde{R}_{l}, \emptyset\} \text{ if } p_1 = p^1, \\
  &- m^*_l(\tau, \alpha_o, y_l) \in \{\tilde{R}_{h}, \emptyset\} \text{ and } m^*_l(\tau, \alpha_o, y_l) = \tilde{R}_{l} \text{ if } p_1 = p^2, \\
  &- m^*_l(\tau, \alpha_o, y_h) \in \{\tilde{R}_{h}, \emptyset\} \text{ and } m^*_l(\tau, \alpha_o, y_l) = \tilde{R}_{l} \text{ if } p_1 = p^4,
\end{align*}
where $\tau \in \{G_u, B\}$

The optimality of these disclosure functions is shown in Appendix B.1.3. The manager prefers a disclosure policy and project choice pair that yields a higher reputation and is indifferent among disclosure policies that yield the same reputation for a given type and project. Basically, a dominance ordering is established amongst the feasible disclosures for each type and the best disclosure or disclosure set is then selected for each type.

Types $Gh$ and $Gl$ can achieve a perfect reputation, $\alpha_1 = 1$, from their optimal disclosures. The type $G_u$ manager, on the other hand, cannot always distinguish himself from the type $B$ manager. If their project choices are different, then the type $G_u$ manager can achieve a perfect reputation by truthful disclosure. However, if they choose the same project, then the type $G_u$ manager cannot separate himself from the type $B$ manager, and hence, is indifferent between disclosure and non-disclosure. The type $B$ manager’s incentives are the opposite of the type $G_u$ manager, since he is better off mimicking the type $G_u$ manager.

Observe that in several of the cases the manager is indifferent over a set of different disclosures. These sets can be thought of as interval estimates of the signal. The cases where there are single values can be thought of as point estimates. Given the above optimal disclosures as functions of production strategy we next determine the first period equilibrium production strategy sets.

**Equilibrium Project Choice in the First Period**

As in the non-disclosure regime the manager chooses the production strategy that yields the highest expected net present value. Hence, conditions (4.11), (4.12) and (4.13) can be applied.

The expressions for $\Delta \pi_T^1$ are the same as in the non-disclosure regime. Although $m_1$ does not appear in these expressions the incremental expected profit is affected by the creditors’ belief $\beta_1$ through its influence on $p_1$. The $\Delta V_2^{G_h}$ terms are, however, different from the corresponding terms in the non-disclosure regime. This is due to the fact that the reputation at the end of the first period now depends on both the disclosure and the revenue outcome in the first period.

Given their respective optimal disclosures $m^*_1(Gh, \alpha_0, y_1)$ and $m^*_1(Gl, \alpha_0, y_1)$, the type $Gh$ and $Gl$ managers achieve perfect reputation at the end of the first period, irrespective of the output level chosen. Hence, $\Delta V_2^{G_h}$ and $\Delta V_2^{G_l}$ are zero. Since $\Delta \pi_1^{G_h}$ is positive and $\Delta \pi_1^{G_l}$ is negative, this implies that $P_{Gh1}^* = 1$ and $P_{Gl1}^* = 0$. Hence, the only production strategy sets that are potential candidates for pure strategy equilibria are: $p_1 = (1,0,0,0)$, $p_2 = (1,0,1,1)$, $p_4 = (1,0,0,1)$ and $(1,0,1,0)$. 

110
The reputation benefit functions for the type $G_u$ and $B$ managers are as follows.

$$
\Delta V^G_u(\alpha_o, p_1, p^*_2, \beta^*_1) = (1 - nG_I)\left\{ [nr^2(\alpha_1(\alpha_o, R_1 = R_{hh}, p_1), p^*_2, m^*_1(G_u, \alpha_o, y_h), \beta^*_1) + nG_r^2(\alpha_1(\alpha_o, R_1 = R_{ll}, p_1), p^*_2, m^*_1(G_u, \alpha_o, y_h), \beta^*_1) + nG_r^2(\alpha_1(\alpha_o, R_1 = R_{ll}, p_1), p^*_2, m^*_1(G_u, \alpha_o, y_h), \beta^*_1) - [nr^2(\alpha_1(\alpha_o, R_1 = R_{hh}, p_1), p^*_2, m^*_1(G_u, \alpha_o, y_l), \beta^*_1) + nG_r^2(\alpha_1(\alpha_o, R_1 = R_{ll}, p_1), p^*_2, m^*_1(G_u, \alpha_o, y_l), \beta^*_1) + nG_r^2(\alpha_1(\alpha_o, R_1 = R_{ll}, p_1), p^*_2, m^*_1(G_u, \alpha_o, y_l), \beta^*_1) \right\}
$$

$$
\Delta V^B_2(\alpha_o, p_1, p^*_2, \beta^*_1) = (1 - nG_I)\left\{ [nr^2(\alpha_1(\alpha_o, R_1 = R_{hh}, p_1), p^*_2, m^*_1(B, \alpha_o, y_h), \beta^*_1) + nG_r^2(\alpha_1(\alpha_o, R_1 = R_{ll}, p_1), p^*_2, m^*_1(B, \alpha_o, y_h), \beta^*_1) + nG_r^2(\alpha_1(\alpha_o, R_1 = R_{ll}, p_1), p^*_2, m^*_1(B, \alpha_o, y_h), \beta^*_1) - [nr^2(\alpha_1(\alpha_o, R_1 = R_{hh}, p_1), p^*_2, m^*_1(B, \alpha_o, y_l), \beta^*_1) + nG_r^2(\alpha_1(\alpha_o, R_1 = R_{ll}, p_1), p^*_2, m^*_1(B, \alpha_o, y_l), \beta^*_1) + nG_r^2(\alpha_1(\alpha_o, R_1 = R_{ll}, p_1), p^*_2, m^*_1(B, \alpha_o, y_l), \beta^*_1) \right\}
$$

Since from the optimal disclosure policy table, $m^*_1(G_u, \alpha_o, y_h) = m^*_1(B, \alpha_o, y_h)$, if both types choose the same project, and $(1 - nG_I) > (1 - n\tilde{G}_I)$, the reputation benefit terms

$$
\Delta V^G_u(\alpha_o, p_1, \beta^*_1) > \Delta V^B_2(\alpha_o, p_1, \beta^*_1)
$$

As in the non-disclosure regime,

$$
\Delta \pi^G_u(r_1(\alpha_o, p_1)) = \Delta \pi^B(r_1(\alpha_o, p_1))
$$

This implies that $p_{B1} \geq p_{G1}$, thus eliminating $p_1 = (1, 0, 1, 0)$ as a feasible production strategy set. Hence, given the above constraints we obtain the following feasible production strategy sets.

**Feasible Pure Strategy Sets**: $p^1 = (1, 0, 0, 0), p^2 = (1, 0, 1, 1)$ and $p^4 = (1, 0, 0, 1)$

Note that these strategy sets are identical to the feasible pure strategy sets, $p^1, p^2$ and $p^4$, in the non-disclosure regime. As described before, $p^1$ and $p^4$ are reputation building strategy sets. In the former both uninformed types choose the safe project, whereas, in the latter only the uninformed type $G$ chooses the safe project. In the pure strategy set $p^2$, on the other hand, all types choose their projects myopically. As in the non-disclosure regime, the existence of
any of these pure strategy sets in equilibrium depends on the relative magnitudes of the profit
differential and the reputation benefit under rational expectations by creditors. Since, the
type $Gh$ and the type $Gi$ always choose the risky and safe projects, respectively, it is sufficient
to focus on the project choices of the type $Gu$ and the type $B$ managers.

The $\Delta V^*_2$ and $\Delta \pi^*_1$ terms are compared assuming that the optimal disclosure policy, given
a project choice, is adopted. Substituting for $m^*(\tau, \alpha_0, y_1)$, we obtain the following expressions
for $\Delta V^*_2$ and $\Delta \pi^*_1$, for $\tau \in \{Gu, B\}$ given each feasible production strategy set.

Note that

$$\Delta \pi^*_1(\alpha_0, p_1) = \Delta \pi^*_1(\alpha_0, p_1)$$

and that

$$\Delta V^*_2(\alpha_0, p_1) = \frac{1 - \bar{n}G_1}{1 - \bar{n}G_1} \Delta V^*_2(\alpha_0, p_1)$$

The $\Delta \pi^*_1$ and $\Delta V^*_2$ terms are

$$\Delta \pi^*_1(\alpha_0, p_1) = (1 - \bar{q}nG_1)((1 - \bar{n} - \bar{G}_1)r_G(p_2) + \bar{n}G_1r_B(p_2) - \bar{r}(p_2))$$

$$\Delta \pi^*_1(\alpha_0, p_2) = (1 - \bar{q}nG_1)\{\bar{r}(p_2) - r_G(p_2)\}$$

$$\Delta \pi^*_1(\alpha_0, p_4) = (1 - \bar{q}nG_1)\{r_B(p_2) - r_G(p_2)\}$$

where $r_G$ and $r_B$ are given by (4.24) and (4.25), respectively.

The uninformed types’ reputation benefit is a function of the future interest differentials
which are in turn functions of $\bar{n}G_1(r_f - R_{ih})$, and the posterior reputation. Intuitively, the
nominal return required by creditors’ is a function of the potential loss they expect to suffer
due to bankruptcy.

The two key components of the uninformed types’ profit differential terms are the revenue
differential $\bar{R}_h - \bar{R}_l$ and the limited liability benefit, $\bar{n}G_1[r_1(\alpha_0, p_1) - R_{ih}]$ which is a function of
$\bar{n}G_1(r_f - R_{ih})$. Hence, a tradeoff between the reputation benefit, $\Delta V^*_2$, and the profit differential, $\Delta \pi^*_1$, basically involves a comparison of $\bar{R}_h - \bar{R}_l$ and particular functions of $\bar{n}G_1(r_f - R_{ih})$.

The following proposition imposes sufficient conditions on the revenue differential term
which governs the existence of the different strategy sets in equilibrium. The $\Delta V^*_2(\alpha_0, p_1)$ and
$\Delta \pi^*_1(\alpha_0, p_1)$ are depicted as functions of $\alpha_0$ for $p^1$, $p^2$ and $p^4$, in figures 4.5(a), (b) and (c),
respectively.
As in the non-disclosure regime, we denote the subsets of \( \alpha \) for which \( p^1, p^2 \) and \( p^4 \) are equilibrium strategies, as \( \hat{A}^1, \hat{A}^2 \) and \( \hat{A}^4 \), respectively. \( N \) is used to denote \( n\tilde{G}_i(r_f - R_{ih}) \).

**Proposition 4.3** In the disclosure regime

I If \( \bar{R}_h - \bar{R}_l > -E.N \) where \( E = \frac{1 - \eta G}{1 - \eta G} \), then

(a) \( \hat{A}^4 = \emptyset \), (b) \( \hat{A}^1 = \emptyset \) and (c) \( \hat{A}^2 = [0,1] \).

II If \( -E.N > \bar{R}_h - \bar{R}_l > -F.N \) where \( F = \frac{1 - \eta G}{1 - \eta G} \), then

(a) \( \hat{A}^4 \neq \emptyset \),
(b) \( \hat{A}^1 = \emptyset \)
(c) \( \hat{A}^2 \neq \emptyset \)

III If \( -F.N > \bar{R}_h - \bar{R}_l > -N \), then

(a) The reputation building strategy set

\[
\hat{A}^4 = \begin{cases} 
\emptyset & \text{if } \bar{R}_h - \bar{R}_l < -\frac{\alpha(1-\eta G) + \eta \tilde{G}_1}{(1-\eta G)(1-\eta G) + \eta \tilde{G}_1} \cdot N \\
[0, \hat{A}^4] & \text{otherwise}
\end{cases}
\]

(b) \( \hat{A}^1 = [\hat{A}^1, 1] \)
(c) \( \hat{A}^2 = [\hat{A}^2, 1] \)

The above proposition is proved by comparisons of \( \Delta V^*(\mu, p_1) \) and \( \Delta \pi^*(\alpha, p_1) \) at the extreme values of \( \alpha \) for the different \( p_1 \).

Since the type \( \mathcal{G}h \) and \( \mathcal{G}l \) managers can distinguish themselves from the type \( \mathcal{B} \) by truthful disclosures, they choose the risky and safe project, respectively, for all prior beliefs. Proposition 4.3 gives ranges of the type \( \mathcal{G}u \) and type \( \mathcal{B} \) managers’ differential revenue term, \( \bar{R}_h - \bar{R}_l \), over which the three different pure strategy equilibria exist, given truthful disclosure. At very low levels of \( \bar{R}_h - \bar{R}_l \), as under condition III, \( p^4 \) may not exist in equilibrium but \( p^1 \) and \( p^2 \) do for high values of prior reputation. If, on the other hand, \( \bar{R}_h - \bar{R}_l \) is high, as under condition I, then neither of the reputation building strategies, \( p^1 \) and \( p^4 \), are consistent with equilibrium and both the uninformed types choose myopically. At intermediate values of \( \bar{R}_h - \bar{R}_l \) as under condition II, \( p^1 \) is inconsistent with an equilibrium but \( p^2 \) and \( p^4 \) exist in equilibrium.

Under condition III the revenue differential is very low resulting in a low profit differential. If creditors expect the type \( \mathcal{G}u \) manager to choose the safe project and the type \( \mathcal{B} \) to choose the risky project, the reputation benefit of the safe project is high. Hence, the type \( \mathcal{G}u \) prefer
the safe project to the risky project. The type $B$ manager, on the other hand, may prefer the safe project or the risky project if his reputation benefit is not high enough. Therefore, $p_4$ may or may not exist in equilibrium stated in III(a).

If, however, the creditors expect both types to choose the risky project, then the current period interest requirement is high, yielding a high limited liability benefit. In addition, under $p_1 = p_2$ a perfect reputation is associated with the project $y_i$ and the reputation benefit is decreasing in $\alpha_o$. Therefore, the limited liability benefit exceeds the reputation benefit if the prior reputation is high, as indicated in III(c).

If both types are expected to choose the safe project, then the current period interest requirement is high, yielding a high limited liability benefit. In addition, under $p_1 = p_2$ a perfect reputation is associated with the project $y_i$ and the reputation benefit is decreasing in $\alpha_o$. Therefore, the limited liability benefit exceeds the reputation benefit if the prior reputation is high, as indicated in III(c).

If both types are expected to choose the safe project, then the current period interest requirement is high, yielding a high limited liability benefit. In addition, under $p_1 = p_2$ a perfect reputation is associated with the project $y_i$ and the reputation benefit is decreasing in $\alpha_o$. Therefore, the limited liability benefit exceeds the reputation benefit if the prior reputation is high, as indicated in III(c).

Under condition II the revenue differential is not too high or low, and hence, all three strategy sets exist in equilibrium. If the creditors expect both types to choose the risky project, then the profit differential becomes high enough to lead both types to prefer the risky project as indicated in II(c).

If, however, the type $G_u$ (type $B$) manager is expected to choose the safe (risky) project, the reputation benefit of the safe project is at its highest level. However, the type $B$ manager's reputation benefit is not high enough to exceed the profit differential. If the prior reputation is already high, the type $G_u$ manager's reputation benefit exceeds his profit differential as stated in II(a).

If both types are expected to choose the safe project, then the reputation benefit of the safe project for both types is very low. As a result, it is exceeded by the profit differential for all prior reputations, as indicated by II(b).

Under condition I the revenue differential is very high and as a result the profit differential exceeds both types' reputation benefits, irrespective of $\alpha_o$. Hence, both types prefer the risky project for all values of $\alpha_o$.

These results are illustrated by the following numerical example. The parameter values used are the same as those in the non-disclosure regime. These yield $-E.N = 0.0088$, $-F.N = -0.1218$ and $-N = -0.15$. The sets $\hat{A}_1$, $\hat{A}_2$ and $\hat{A}_4$ obtained in this example are given in table 4.2(b) and illustrated in figure 4.6.
4.4.4 Disclosure versus Non-Disclosure

In this section, we compare the results obtained in the disclosure regime with those obtained in the non-disclosure regime. The discussion is led by a review of the major features of our model.

This model examines the decision problem of a manager who borrows funds to engage in production.

- The manager's "ability" determines whether he sometimes receives predecision information or never receives predecision information. In each period the manager's "type" is determined by his ability and "information" state, i.e., whether he (i) has received good news, (ii) has received bad news, (iii) is currently uninformed but may be informed in future, or (iv) is currently uninformed and will never receive information. Hence, although the manager's ability is fixed, his "type" is period specific. The manager's "type" influences his production decision in each period, which together influence the probability of insolvency.

- In each period creditors set required returns based on their beliefs about the manager's "ability" and production choice. The belief about his ability constitutes the manager's reputation in the credit market.

- In the non-disclosure regime, managers are prohibited from making any disclosures. Hence, creditor's learn about the manager's ability from his end of period revenues, which are at best noisy signals of ability. The probability density function of revenue is determined by the manager's "information" state and project choice in that period. Due to his expected future encounter with the credit market, the manager may attempt to improve his reputation through appropriate revenue realizations. Since the manager's project choice affects his revenue realizations, he may make a sub-optimal production choice in the presence of the reputation effect, relative to a one shot game.

- In the disclosure regime, the manager is permitted to make direct disclosures. Credibility is ensured by the assumption that revealed false disclosures can be penalized ex-post. Under the distributional assumptions of the model attempts by an uninformed manager to claim that he is informed have a positive probability of being detected (and penalized). Sufficiently high penalties are available to completely discourage false disclosures. Hence, an informed manager can signal his type perfectly and thus separate himself from the type $B$ manager. In addition, a high ability manager who is uninformed, i.e., type $Cu$, can
distinguish himself completely from the low ability manager by choosing the sub-optimal project and making a truthful disclosure, when the low ability manager is expected to choose the optimal project.

From the above, it is clear that the high ability manager, labelled \( G \), faces two opposing incentives, depending on whether he is informed or uninformed in that period.

1) If the \( G \) manager is informed, truthful disclosure can fully separate him from the low ability or \( B \) manager, thus he avoids any sub-optimal production choice.

2) If the \( G \) manager is uninformed, however, truthful disclosure may not enable him to distinguish himself from the \( B \) manager. Hence, not only does his incentive for sub-optimal project choice remain in place, it is in fact enhanced. This increased incentive, relative to the non-disclosure regime, is due to the fact that the uninformed \( G \) manager can distinguish himself completely from the type \( B \) manager by making a sub-optimal production choice and a truthful disclosure. In contrast, in the non-disclosure regime complete separation from the \( B \) manager is not guaranteed by any sub-optimal production decision since realized revenue is a noisy signal of the manager’s ability. Hence, this produces a lower reputation benefit than in the disclosure regime.

A comparison of the \( \Delta V_{2}^{\mathcal{G}u} \) terms in the disclosure and non-disclosure shows that \( \Delta V_{2}^{\mathcal{G}u} \), in the disclosure regime, is always higher than in the non-disclosure regime. This results in a stronger incentive for the uninformed \( G \) manager to engage in reputation acquisition in the disclosure regime. A similar comparison for the informed manager indicates that although the reputation benefit may be positive (negative) for the manager with good (bad) news in the non-disclosure regime, it is zero in the disclosure regime. Hence, the manager with the good (bad) news has no incentive to choose the safe (risky) project in the disclosure regime.

The two opposing incentives faced by the informed \( G \) versus the uninformed \( G \) are key to the interpretation of our results. A key result is that the informed type \( G \) manager with good news does not engage in reputation acquisition (through suboptimal production choice) in the disclosure regime. This is evidenced by the non-existence of \( p^0 \), in equilibrium, for any value of the revenue differential, in the disclosure regime. If disclosure is prohibited this type engages in reputation acquisition under condition III in proposition 4.2. However, in the disclosure regime this type of manager substitutes reputation acquisition through choice of the safe project, i.e., a sub-optimal decision, with reputation acquisition through truthful disclosure.
Another key result is that the uninformed type $G$ manager has a stronger incentive to acquire reputation by choosing the safe (sub-optimal) project and making a truthful disclosure. An examination of $I$ in both proposition 4.2 and 4.3 indicates that

\[-X.N < -E.N \quad \text{since,} \quad n\tilde{G}_t + n\tilde{G}_h < 1.\]

The threshold level of $R_h - \tilde{R}_t$ above which the myopic strategy set, $p^2$, is the only pure strategy set consistent with an equilibrium, is higher in the disclosure regime than in the non-disclosure regime. This result is driven by the increased benefit from reputation acquisition for the uninformed $G$ manager in the disclosure regime.

Below the threshold level $-X.N$ and $-E.N$ reputation acquisition strategies may be consistent with an equilibrium. The strategy set where only the type $G_u$ manager engages in reputation acquisition is $p^4$. Note that in the intervals denoted by $II$ in propositions 4.2 and 4.3 $p^4$ exists in equilibrium for some values of prior reputation in disclosure regime, whereas it may not exist for any prior reputations in the non-disclosure regime. This is again due to the uninformed $G$ manager’s higher incentive to build reputation in the disclosure regime. In contrast, it can be seen that the conditions for the existence of the reputation building strategy $p^4$ are identical in the disclosure and non-disclosure regimes. Intuitively, the uninformed $G$ manager makes a disclosure to separate himself from the type $B$ manager. However, if the type $B$ manager also chooses the safe project then he can exactly mimic the uninformed $G$ manager’s disclosure, and hence, the uninformed $G$ manager does not gain anything from disclosure.
4.5 Conclusion

In this chapter we have examined a model of reputation acquisition by managers who may possess private pre-decision information. In the non-disclosure regime the manager is prohibited from making any disclosures. Creditors learn about the manager from end of period revenue realization. Hence, the manager’s tries to signal his type by engineering his revenue realization through the choice of a project which is sub-optimal from the investor’s viewpoint. The manager can choose between a safe and a risky project, where the safe project is suboptimal since it provides a lower current period profit than the risky project. In the disclosure regime, however, the manager may make truthful disclosures to reveal his type.

The manager trades off the current period differential profit between alternative projects with the future reputation benefit. A key determinant of the magnitude of the former is the difference in expected revenue between the two projects. The latter, on the other hand, depends on various other factors, the most important ones being: the creditors’ prior belief about the manager’s ability to obtain information and their expectations about which strategy will be played. Hence, the results are stated in terms of these factors. Note that the creditors’ prior belief is also referred to as the manager’s prior reputation.

In the non-disclosure regime the myopic strategy set predominates if the current period revenue differential between the risky and the safe projects is high. At intermediate levels of differential revenue, however, strategy sets where the uninformed firms engage in reputation acquisition are consistent with an equilibrium at intermediate values of the prior reputation. If the revenue differential and the prior reputation are low, the manager who has received favourable information also engages in reputation acquisition by choosing the safe-project.

In the disclosure regime the myopic strategy predominates if the revenue differential is high. At intermediate levels of the revenue differential the uninformed type G manager engages in reputation building. The value of the differential revenue at which the uninformed manager prefers the safe project is higher in the disclosure than in the non-disclosure regime. At low values of the revenue differential both the uninformed types may engage in reputation acquisition if the prior reputation is high.

Signalling through sub-optimal project choice is costly mechanism for reputation acquisition. Hence, disclosures may provide a more economical method of communication. We find that the informed type G and the uninformed type G managers are affected differently by the possibility of disclosure. The type G manager who has received good news, indeed replaces suboptimal...
project choice with direct disclosure to signal his ability. However, the uninformed type $G$ manager cannot make an analogous substitution, since the disclosure which distinguishes him from the type $B$ manager has to be accompanied by choice of the safe project. This enables the uninformed type $G$ manager to achieve the maximum reputation, and hence, offers a higher reputation benefit relative to the non-disclosure regime. Hence, if credible disclosures are possible, the uninformed type $G$ manager's incentive for sub-optimal production choice increases.
Bibliography


Figures
Figure 3.1: Parabola Representations of Optimized Expected Profits

Case 1: $\delta_i > \delta$

Case 2: $\delta_i < \delta$
Figure 3.2: Ranges for the Equilibrium Values of $s'$ and $s''$

Case 1: $\delta_i > \delta$

Case 2: $\delta_i < \delta$
Figure 3.3: Optimal Output and Expected Profit in an Unconstrained Duopoly

Optimal Output

(a) Case 1 : \( \delta_i > \delta \)

\[ y(s|D) \]
\[ y(s|N) \]
\[ y(s|N) \]
\[ y(s|D) \]

(b) Case 2 : \( \delta_i < \delta \)

\[ y(s|D) \]
\[ y(s|N) \]
\[ y(s|N) \]
\[ y(s|D) \]

Optimal Expected Profit

(c) Case 1 : \( \delta_i > \delta \)

\[ V(s|D) \]
\[ V(s|N) \]
\[ V(s|N) \]
\[ V(s|D) \]

(d) Case 2 : \( \delta_i < \delta \)

\[ V(s|D) \]
\[ V(s|N) \]
\[ V(s|N) \]
\[ V(s|D) \]
Figure 3.4: Optimal Output and Expected Profit in a Constrained Duopoly

Optimal Output

(a) Case 1: $\delta_i > \delta$  

(b) Case 2: $\delta_i < \delta$

Optimal Expected Profit

(c) Case 1: $\delta_i > \delta$  

(d) Case 2: $\delta_i < \delta$
Figure 3.5: Optimal Output and Expected Profit in an Unconstrained Oligopoly

Optimal Output

(a) Case 1: $\delta_i > \delta$

(b) Case 2: $\delta_i < \delta$

Optimal Expected Profit

(c) Case 1: $\delta_i > \delta$

(d) Case 2: $\delta_i < \delta$

129
Figure 3.6: Disclosure and Nondisclosure Sets in Different Studies

Optimal Output

(A) \[ V(s|I) \]

(B) \[ V(s|D) \] \[ V(s|D) - C \] \[ V(s|N) \]

Optimal Expected Profit

(C) \[ V(s|I) \] \[ V(s|D) \] \[ V(s|N) \]

(D) \[ V(s|I) \] \[ V(s|D) \] \[ V(s|N) \]
Figure 3.7: $s'$ and $s''$ as functions of $q$

(a) Case 1: $\delta_i > \delta$

(b) Case 2: $\delta_i < \delta$
Figure 3.8: $P(D)$ as a function of $q$

(a) Case 1: $\delta_i > \delta$

(b) Case 2: $\delta_i < \delta$
Figure 3.9: Opponent's Optimal Output and Expected Profit in a Constrained Duopoly

Optimal Output

(a) Case 1: $\delta_i > \delta$

(b) Case 2: $\delta_i < \delta$

Optimal Expected Profit

(c) Case 1: $\delta_i > \delta$

(d) Case 2: $\delta_i < \delta$
Figure 3.10: Truncated Reaction Functions
Figure 4.1: Decision Trees in the Non-Disclosure Regime

Type B manager

Type Gu manager
Figure 4.2: $\Delta V_2^\tau$ and $\Delta \pi_1^\tau$ for $\tau \in \{Gh, Gl\}$ in the Non-Disclosure Regime

(a) $p_1 = (1, 0, 0, 0) \equiv p^1$

(b) $p_1 = (1, 0, 1, 1) \equiv p^2$

(c) $p_1 = (0, 0, 0, 1) \equiv p^3$

(d) $p_1 = (1, 0, 0, 1) \equiv p^4$
Figure 4.3: $\Delta V_2^\tau$ and $\Delta \pi_1^\tau$ for $\tau \in \{G, B\}$ in the Non-Disclosure Regime

(a) $p_1 = (1, 0, 0, 0) \equiv p^1$

(b) $p_1 = (1, 0, 1, 1) \equiv p^2$

(c) $p_1 = (0, 0, 0, 1) \equiv p^3$

(d) $p_1 = (1, 0, 0, 1) \equiv p^4$
Figure 4.4: Example of the Existence of $A^1$, $A^2$, $A^3$ and $A^4$
Figure 4.5: $\Delta V_2^\tau$ and $\Delta \pi_1^\tau$ for $\tau \in \{G, B\}$ in the Disclosure Regime

(a) $p_1 = (1, 0, 0, 0) \equiv p^1$

(b) $p_1 = (1, 0, 1, 1) \equiv p^2$

(c) $p_1 = (1, 0, 0, 1) \equiv p^4$
Figure 4.6: Example of the Existence of $\hat{A}^1$, $\hat{A}^2$ and $\hat{A}^4$
Tables
Table 3.1: Equilibrium Values if $\delta_i > \delta$

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</tr>
<tr>
<td>0.75</td>
<td>0.3896</td>
<td>0.05538</td>
<td>0.23778</td>
<td>0.1421</td>
<td>0.6961</td>
</tr>
<tr>
<td>0.80</td>
<td>0.4437</td>
<td>0.06534</td>
<td>0.23739</td>
<td>0.1473</td>
<td>0.7458</td>
</tr>
<tr>
<td>0.85</td>
<td>0.5081</td>
<td>0.07820</td>
<td>0.23687</td>
<td>0.1539</td>
<td>0.7969</td>
</tr>
<tr>
<td>0.90</td>
<td>0.5885</td>
<td>0.09598</td>
<td>0.23616</td>
<td>0.1631</td>
<td>0.8504</td>
</tr>
<tr>
<td>0.95</td>
<td>0.6997</td>
<td>0.12429</td>
<td>0.23503</td>
<td>0.1776</td>
<td>0.9087</td>
</tr>
<tr>
<td>1.00</td>
<td>0.9999</td>
<td>0.23070</td>
<td>0.23077</td>
<td>0.2307</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Parameter values:

$\ddot{s} = 0.000$

$\sigma_i^2 = 0.300$

$b = 0.100$

$y = 1.000$

$\delta_i = 0.500$

$\delta = 0.300$
Table 3.2: Equilibrium Value if $\delta_i < \delta$

<table>
<thead>
<tr>
<th>$q$</th>
<th>$p$</th>
<th>$s'$</th>
<th>$s''$</th>
<th>$A$</th>
<th>$P(D)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.0000</td>
<td>-0.0000</td>
<td>0.26087</td>
<td>-0.0544</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.05</td>
<td>0.0350</td>
<td>-0.0019</td>
<td>0.26079</td>
<td>-0.0546</td>
<td>0.0052</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0709</td>
<td>-0.0038</td>
<td>0.26070</td>
<td>-0.0548</td>
<td>0.0104</td>
</tr>
<tr>
<td>0.15</td>
<td>0.1078</td>
<td>-0.0059</td>
<td>0.26061</td>
<td>-0.0549</td>
<td>0.0158</td>
</tr>
<tr>
<td>0.20</td>
<td>0.1457</td>
<td>-0.0080</td>
<td>0.26052</td>
<td>-0.0551</td>
<td>0.0212</td>
</tr>
<tr>
<td>0.25</td>
<td>0.1846</td>
<td>-0.0101</td>
<td>0.26043</td>
<td>-0.0552</td>
<td>0.0267</td>
</tr>
<tr>
<td>0.30</td>
<td>0.2247</td>
<td>-0.0124</td>
<td>0.26033</td>
<td>-0.0554</td>
<td>0.0323</td>
</tr>
<tr>
<td>0.35</td>
<td>0.2661</td>
<td>-0.0147</td>
<td>0.26023</td>
<td>-0.0555</td>
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</tr>
<tr>
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<td>-0.0171</td>
<td>0.26012</td>
<td>-0.0557</td>
<td>0.0438</td>
</tr>
<tr>
<td>0.45</td>
<td>0.3530</td>
<td>-0.0197</td>
<td>0.26001</td>
<td>-0.0558</td>
<td>0.0497</td>
</tr>
<tr>
<td>0.50</td>
<td>0.3989</td>
<td>-0.0223</td>
<td>0.25990</td>
<td>-0.0559</td>
<td>0.0557</td>
</tr>
<tr>
<td>0.55</td>
<td>0.4465</td>
<td>-0.0250</td>
<td>0.25978</td>
<td>-0.0561</td>
<td>0.0619</td>
</tr>
<tr>
<td>0.60</td>
<td>0.4961</td>
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<td>0.25966</td>
<td>-0.0562</td>
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<td>0.0745</td>
</tr>
<tr>
<td>0.70</td>
<td>0.6021</td>
<td>-0.0339</td>
<td>0.25939</td>
<td>-0.0564</td>
<td>0.0811</td>
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<tr>
<td>0.75</td>
<td>0.6591</td>
<td>-0.0372</td>
<td>0.25925</td>
<td>-0.0565</td>
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</tr>
<tr>
<td>0.80</td>
<td>0.7191</td>
<td>-0.0406</td>
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<td>0.0947</td>
</tr>
<tr>
<td>0.85</td>
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<td>-0.0442</td>
<td>0.25895</td>
<td>-0.0565</td>
<td>0.1018</td>
</tr>
<tr>
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<td>0.8502</td>
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<td>-0.0565</td>
<td>0.1091</td>
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<td>-0.0521</td>
<td>0.25860</td>
<td>-0.0565</td>
<td>0.1167</td>
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<td>1.0000</td>
<td>-0.0564</td>
<td>0.25842</td>
<td>-0.0565</td>
<td>0.1245</td>
</tr>
</tbody>
</table>

Parameter values:

\[
\begin{align*}
\bar{s} & = 0.000 \\
\sigma^2_s & = 0.300 \\
b & = 0.100 \\
y & = 1.000 \\
\delta_i & = 0.300 \\
\delta & = 0.500
\end{align*}
\]
Table 4.1: Posterior Beliefs Given Different Pure Strategies in the Non-Disclosure Regime

<table>
<thead>
<tr>
<th>$p_1$</th>
<th>$\alpha_1(\alpha_o, R_{hh}, p_1)$</th>
<th>$\alpha_1(\alpha_o, R_{hi}, p_1)$</th>
<th>$\alpha_1(\alpha_o, R_{hi}, p_1)$</th>
<th>$\alpha_1(\alpha_o, R_{li}, p_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1 \equiv (1, 0, 0, 0)$</td>
<td>$\frac{(q + q G_h) \alpha_o}{(q + q G_h) \alpha_o + G_h (1 - \alpha_o)}$</td>
<td>$\frac{G_h \alpha_o}{G_h \alpha_o + (1 - \alpha_o)}$</td>
<td>0</td>
<td>$\alpha_o$</td>
</tr>
<tr>
<td>$p_2 \equiv (1, 0, 1, 1)$</td>
<td>$\alpha_o$</td>
<td>1</td>
<td>$\frac{G_h \alpha_o}{G_h \alpha_o + (1 - \alpha_o)}$</td>
<td>$\frac{(q + G_l) \alpha_o}{(q + G_l) \alpha_o + G_l (1 - \alpha_o)}$</td>
</tr>
<tr>
<td>$p_3 \equiv (0, 0, 0, 1)$</td>
<td>$\frac{G_h \alpha_o}{G_h \alpha_o + (1 - \alpha_o)}$</td>
<td>1</td>
<td>0</td>
<td>$\frac{\alpha_o}{\alpha_o + G_l (1 - \alpha_o)}$</td>
</tr>
<tr>
<td>$p_4 \equiv (1, 0, 1)$</td>
<td>$\frac{(q + q G_h) \alpha_o}{(q + q G_h) \alpha_o + (1 - \alpha_o)}$</td>
<td>1</td>
<td>0</td>
<td>$\frac{\alpha_o}{\alpha_o + G_l (1 - \alpha_o)}$</td>
</tr>
</tbody>
</table>
Table 4.2: Existence of the Different Pure Strategy Equilibria in the Numerical Example

(a) Non-Disclosure Regime

<table>
<thead>
<tr>
<th>$\tilde{R}_h - \tilde{R}_l$</th>
<th>$A^1$</th>
<th>$A^2$</th>
<th>$A^3$</th>
<th>$A^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.050</td>
<td>$\emptyset$</td>
<td>[0, 1]</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>-0.100</td>
<td>$\emptyset$</td>
<td>[0, 1]</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>-0.124</td>
<td>[0.9, 1]</td>
<td>[0.5, 1]</td>
<td>[0.0, 1]</td>
<td>[0.0, 1]</td>
</tr>
<tr>
<td>-0.140</td>
<td>[0.9, 1]</td>
<td>[0.9, 1]</td>
<td>[0.0, 4]</td>
<td>[0.1, 0.3]</td>
</tr>
</tbody>
</table>

$-X \cdot N = -0.0706, -Y \cdot N = -0.1209, -N = -0.15$

(b) Disclosure Regime

<table>
<thead>
<tr>
<th>$\tilde{R}_h - \tilde{R}_l$</th>
<th>$\hat{A}^1$</th>
<th>$\hat{A}^2$</th>
<th>$\hat{A}^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+0.05</td>
<td>$\emptyset$</td>
<td>[0, 1]</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>-0.05</td>
<td>$\emptyset$</td>
<td>[0, 1]</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>-0.10</td>
<td>$\emptyset$</td>
<td>[0.5, 1]</td>
<td>[0.3, 1]</td>
</tr>
<tr>
<td>-0.13</td>
<td>[0.9, 1]</td>
<td>[0.9, 1]</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

$-E \cdot N = -0.0088, -F \cdot N = -0.1218, -N = -0.15$
Table 4.3: Posterior Beliefs Given Different Pure Strategies in the Disclosure Regime

<table>
<thead>
<tr>
<th>p_1 = (1, 0, 0, 0) \equiv p^1</th>
<th>\alpha_1(a_o, R_{hh}, m_1(y_h) = \bar{R}<em>h) = 1 \quad \alpha_1(a_o, R</em>{hh}, m_1(y_l) \in {\bar{R}_l, \emptyset}) = \frac{\gamma a_o}{\gamma a_o + (1-a_o)}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\alpha_1(a_o, R_{ll}, m_1(y_h) = \bar{R}<em>h) = 1 \quad \alpha_1(a_o, R</em>{ll}, m_1(y_l) \in {\bar{R}_l, \emptyset}) = \frac{\gamma a_o}{\gamma a_o + (1-a_o)}</td>
</tr>
<tr>
<td></td>
<td>\alpha_1(a_o, R_{lh}, m_1(y_h) = \bar{R}<em>h) = 1 \quad \alpha_1(a_o, R</em>{lh}, m_1(y_l) \in {\bar{R}_l, \emptyset}) = \frac{\gamma a_o}{\gamma a_o + (1-a_o)}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p_1 = (1, 0, 1, 1) \equiv p^2</th>
<th>\alpha_1(a_o, R_{hh}, m_1(y_h) \in {\bar{R}<em>h, \emptyset}) = \frac{\gamma a_o}{\gamma a_o + (1-a_o)} \quad \alpha_1(a_o, R</em>{hh}, m_1(y_l) = \bar{R}_l) = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\alpha_1(a_o, R_{ll}, m_1(y_h) \in {\bar{R}<em>h, \emptyset}) = \frac{\gamma a_o}{\gamma a_o + (1-a_o)} \quad \alpha_1(a_o, R</em>{ll}, m_1(y_l) = \bar{R}_l) = 1</td>
</tr>
<tr>
<td></td>
<td>\alpha_1(a_o, R_{lh}, m_1(y_h) \in {\bar{R}<em>h, \emptyset}) = \frac{\gamma a_o}{\gamma a_o + (1-a_o)} \quad \alpha_1(a_o, R</em>{lh}, m_1(y_l) = \bar{R}_l) = 1 \quad (\text{Off-Eqibm.})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>p_1 = (1, 0, 0, 1) \equiv p^4</th>
<th>\alpha_1(a_o, R_{hh}, m_1(y_h) \in {\bar{R}<em>h, \emptyset}) = 0 \quad \alpha_1(a_o, R</em>{hh}, m_1(y_l) = \bar{R}_l) = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\alpha_1(a_o, R_{ll}, m_1(y_h) \in {\bar{R}<em>h, \emptyset}) = 0 \quad \alpha_1(a_o, R</em>{ll}, m_1(y_l) = \bar{R}_l) = 1</td>
</tr>
<tr>
<td></td>
<td>\alpha_1(a_o, R_{lh}, m_1(y_h) \in {\bar{R}<em>h, \emptyset}) = 0 \quad \alpha_1(a_o, R</em>{lh}, m_1(y_l) = \bar{R}_l) = 1</td>
</tr>
</tbody>
</table>

| p_1 = (1, 0, 1, 0) \equiv p^3 |
Appendix A

Appendices to Chapter 3
A.1 Derivations

A.1.1 Optimal Solutions to the Output Choice Game under Asymmetric Information

1. Firm $i$ is informed (I), receives signal $s$ and discloses $m = s$.

Firm $i$ solves
\[
\max_{y_i} E[\pi_i|s, m = s] = \{a - E[c_i|s] - E(c|s) - b(y_i + y_j)\}y_i
\]
Firm $j$ solves
\[
\max_{y_j} E[\pi_j|s, m = s] = \{a - \bar{c}_j - E(c|s) - b(y_i + y_j)\}y_j
\]

First Order conditions
\[
\frac{d\{E[\pi_i|s, m = s]\}}{dy_i} = a - E[c_i|s] - E[c|s] - 2by_i - by_j = 0
\]
\[
\frac{d\{E[\pi_j|m = s]\}}{dy_j} = a - \bar{c}_j - E[c|s] - 2by_j - by_i = 0
\]

Solving the first order conditions simultaneously, gives
\[
y_i^*(s, m = s) = \frac{1}{3b} \{a + \bar{c}_j - 2E[c_i|s] - E[c|s]\}
\]
\[
y_j^*(m = s) = \frac{1}{3b} \{a + E[c_i|s] - 2\bar{c}_j - E[c|s]\}
\]

Substituting for $y_i^*$ and $y_j$ in $E[\pi_i|\cdot]$ and $E[\pi_j|\cdot]$,
\[
E[\pi_i^*|s, m = s] = \frac{1}{9b} \{a + \bar{c}_j - 2E[c_i|s] - E[c|s]\}^2
\]
\[
E[\pi_j^*|m = s] = \frac{1}{9b} \{a + E[c_i|s] - 2\bar{c}_j - E[c|s]\}^2
\]

2. Firm $i$ is informed (I), receives signal $s$ but discloses $m = \emptyset$.

Firm $i$ solves
\[
\max_{y_i} E[\pi_i|s, m = \emptyset] = \{a - E[c_i|s] - E(c|s) - b(y_i + y_j)\}y_i
\]
Firm $j$ solves
\[
\max_{y_j} E[\pi_j|s, m = \emptyset] = \{a - \bar{c}_j - E(c|m = \emptyset) - b(\hat{y}_i + y_j)\}y_j
\]
where $\hat{y}_i$ is firm $i$'s expectation of firm $i$'s output given non-disclosure. $\hat{y}_i$ must solve
\[
\max_{\hat{y}_i} E[\pi_i|m = \emptyset] = \{a - E[c_i|m = \emptyset] - E(c|m = \emptyset) - b(\hat{y}_i + y_j)\}\hat{y}_i
\]
First Order conditions

\[
\begin{align*}
\frac{d\{E[\pi_i|s, m = 0]\}}{dy_i} &= a - E[c_i|s] - E[c|s] - 2by_i - by_j = 0 \\
\frac{d\{E[\pi_j|m = 0]\}}{dy_j} &= a - \bar{c}_j - E[c|m = 0] - 2by_j - \bar{b}y_i = 0 \\
\frac{d\{E_j[\pi_i|m = 0]\}}{d\bar{y}_i} &= a - E[c_i|m = 0] - E[c|m = 0] - 2b\bar{y}_j - by_j = 0
\end{align*}
\]

Solving the first order conditions simultaneously, gives

\[
\begin{align*}
y^*_i(s, m = 0) &= \frac{1}{3b} \{a + \bar{c}_j - 2E[c_i|s] - E[c|s]\} \\
&\quad + \frac{1}{2} (E[c_i|s] - E[c_i|m = 0]) \\
&\quad - \frac{1}{2} (E[c|s] - E[c|m = 0]) \\
y^*_j(m = 0) &= \frac{1}{3b} \{a + E[c_i|m = 0] - 2\bar{c}_j - E[c|m = 0]\} \\
E[\pi^*_i|s, m = 0] &= \frac{1}{9b} \{a + \bar{c}_j - 2E[c_i|s] - E[c|s]\} \\
&\quad + \frac{1}{2} (E[c_i|s] - E[c_i|m = 0]) \\
&\quad - \frac{1}{2} (E[c|s] - E[c|m = 0])^2 \\
E[\pi^*_j|m = 0] &= \frac{1}{9b} \{a + E[c_i|m = 0] - 2\bar{c}_j - E[c|m = 0]\}^2
\end{align*}
\]

3. Firm \(i\) is informed \((U)\), and does not make a disclosure, \(m = 0\).

Firm \(i\) solves

\[
\max_{y_i} E[\pi_i|m = 0] = \{a - \bar{c}_i - \bar{c} - b(y_i + y_j)\}y_i
\]

Firm \(j\)'s problem is the same as in (2) above.

First Order conditions

\[
\begin{align*}
\frac{d\{E[\pi_i|m = 0]\}}{dy_i} &= a - \bar{c}_i - \bar{c} - 2by_i - by_j = 0 \\
\frac{d\{E[\pi_j|m = 0]\}}{dy_j} &= a - \bar{c}_j - E[c|m = 0] - 2by_j - \bar{b}y_i = 0 \\
\frac{d\{E_j[\pi_i|m = 0]\}}{d\bar{y}_i} &= a - E[c_i|m = 0] - E[c|m = 0] - 2b\bar{y}_j - by_j = 0
\end{align*}
\]

Solving these three first order conditions simultaneously, yields

\[
y^*_i(m = 0) = \frac{1}{3b} \{a + \bar{c}_j - 2\bar{c}_i - c + \frac{1}{2} (\bar{c}_i - E[c_i|m = 0]) - \frac{1}{2} (\bar{c} - E[c|m = 0])\}
\]
\[ y_j^*(m = 0) = \frac{1}{3b} \{a + E[c_i|m = 0] - 2\bar{c}_j - E[c|m = 0]\} \]

\[ E[\pi^*_i|m = 0] = \frac{1}{9b} \{a + \bar{c}_j - 2\bar{c}_i - c + \frac{1}{2}(\bar{c}_i - E[c_i|m = 0]) - \frac{1}{2}(\bar{c} - E[c|m = 0])\}^2 \]

\[ E[\pi^*_j|m = 0] = \frac{1}{9b} \{a + E[c_i|m = 0] - 2\bar{c}_j - E[c|m = 0]\}^2 \]
A.1.2 Expressions for the equilibrium values of $s'$ and $s''$.

Note that $\psi(s) = E(c|s)$ and $\psi_i(s) = E(c_i|s)$, in (A.L.3.1-1) in the proof of lemma 3.1. Substituting for $E(c_i|s)$ and $E(c|s)$ from (3.18) and (3.19) and for $\alpha, \beta, \bar{c}$ and $\bar{c}_i$ from (3.23), (3.24), (3.25) and (3.26) in expression (A.L.3.1-1) we get

$$ (\gamma - \gamma_i) + (\delta - \delta_i)s'' = 12b\psi_i^*(s'') + (\gamma - \gamma_i) + p(\delta - \delta_i)A + (1 - p)(\delta - \delta_i)s $$  \hspace{1cm} (A.1.2-1)

and

$$ (\gamma - \gamma_i) + (\delta - \delta_i)s' = (\gamma - \gamma_i) + p(\delta - \delta_i)A + (1 - p)(\delta - \delta_i)s $$  \hspace{1cm} (A.1.2-2)

Note that

$$ 12b\psi_i^*(s) = 4\{a\bar{c}_j - (\gamma + 2\gamma_i) - (\delta + 2\delta_i)s\} $$

$$ = 4\{a\bar{c}_j - (\gamma + 2\gamma_i) - (\delta + 2\delta_i)s\} - (8\delta_i + 4\delta)(s - \bar{s}) $$

$$ = 12b\bar{y}_i^* - (8\delta_i + 4\delta)(s - \bar{s}) $$

Substituting for $\psi_i^*(s)$ in (A.1.2-1) and rearranging both (A.1.2-1) and (A.1.2-2) yields

$$ s' = \bar{s} + p(A - \bar{s}) $$  \hspace{1cm} (A.1.2-3)

$$ s'' = \bar{s} + \frac{12b\bar{y}_i^* - (\delta_i - \delta)p(A - \bar{s})}{7\delta_i + 5\delta} $$  \hspace{1cm} (A.1.2-4)
A.1.3 Output choice given the constraints $y_i \geq 0$ and $y_j \geq 0$.

1. Firm $i$ is informed discloses $m = s$.

Firm $i$ solves:

$$\max_{y_i} \{a - E(c_i|s) - E(c|s) - b(y_i + y_j)\} y_i$$

Firm $i$ solves:

$$\max_{y_j} \{a - \bar{c}_j - E(c|s) - b(y_i + y_j)\} y_j$$

subject to $y_i, y_j \geq 0$. Differentiating the above objective functions, we get the following first order conditions

$$\text{FOC } i: \quad a - E(c_i|s) - E(c|s) - 2by_i - by_j = 0$$

$$\text{FOC } j: \quad a - \bar{c}_j - E(c|s) - 2by_j - by_i = 0$$

FOC $i$ and FOC $j$ are firm $i$ and firm $j$'s reaction functions for a given $s$.

Under the constraints $y_i, y_j \geq 0$, these reaction functions are truncated at $y_i = 0$ and $y_j = 0$ respectively. This is illustrated in Figure 3.10.

The equilibrium values of $y_i$ and $y_j$, for each $s$ is given by the point of intersection of FOC $i$ and FOC $j$ given $s$. If FOC $i$ and FOC $j$ do not intersect in the positive quadrant, then we get corner solutions.

**Corner Solutions**:

a) If $2[a - \bar{c}_j - E(c|s)] < a - E(c_i) - E(c|s)$, then

$$y_i^* = \frac{a - E(c_i|s) - E(c|s)}{2b} \quad \text{and} \quad y_j^* = 0$$

b) If $2[a - E(c_i|s) - E(c|s)] > a - \bar{c}_j - E(c|s)$, then

$$y_j^* = \frac{a - \bar{c}_j - E(c|s)}{2b} \quad \text{and} \quad y_i^* = 0$$

**Interior Solution**:

b) If $2[a - E(c_i|s) - E(c|s)] > a - \bar{c}_j - E(c|s)$ and $2[a - E(c_i|s) - E(c|s)] < a - \bar{c}_j - E(c|s)$, then

$$y_i^* = \frac{1}{3b} \left\{ a + \bar{c}_j - 2E(c_i|s) - E(c|s) \right\}$$

$$y_j^* = \frac{1}{3b} \left\{ a + E(c_i|s) - 2\bar{c}_j - E(c|s) \right\}$$
Under the assumption of multivariate normality, the above solutions translate to the following:

(a) If $(\delta - \delta_i)(s - \bar{s}) > (a - 2\bar{c}_j - \bar{c} + c_i)$, then
\[
y^*_i(s, m = s) = \frac{1}{2b} \{a - (\gamma + \gamma_i) - (\delta + \delta_i)s\} \quad \text{and} \quad y^*_j(m = s) = 0
\]

(b) If $(2\delta_i + \delta)(s - \bar{s}) > (a + \bar{c}_j - 2\bar{c}_i - \bar{c})$, then
\[
y^*_i(s, m = s) = \frac{1}{2b} \{a - \bar{c}_j - \gamma - \delta s\} \quad \text{and} \quad y^*_j(m = s) = 0
\]

(c) If $(\delta - \delta_i)(s - \bar{s}) < (a - 2\bar{c}_j - \bar{c} + c_i)$ and $(2\delta_i + \delta)(s - \bar{s}) < (a + \bar{c}_j - 2\bar{c}_i - \bar{c})$, then
\[
y^*_i(s, m = s) = \frac{1}{3b} \{a + \bar{c}_j - (2\gamma_i + \gamma) - (2\delta_i + \delta)s\}
y^*_j(s, m = s) = \frac{1}{3b} \{a - 2\bar{c}_j - (\gamma - \gamma_i) - (\delta - \delta_i)s\}
\]

If
\[
\bar{s}_3 = \frac{a - 2\bar{c}_j + \bar{c}_i - \bar{c}}{\delta - \delta_i}
\]
and
\[
\bar{s}_1 = \frac{a + \bar{c}_j - 2\bar{c}_i - \bar{c}}{2\delta_i + \delta}
\]
then the above conditions become:

Case 1 : $\delta_i > \delta$

(a) If $s < s_3 < s_1$, then $y^*_j(m = s) = 0$ and
\[
y^*_i(s, m = s) = \frac{1}{2b} \{a - (\gamma + \gamma_i) - (\delta + \delta_i)s\}
\]

(b) If $s > s_1 > s_3$, then $y^*_i(s, m = s) = 0$ and
\[
y^*_j(m = s) = \frac{1}{2b} \{a - \bar{c}_j - \gamma - \delta s\}
\]

(c) If $s_3 < s < s_1$, then
\[
y^*_j(m = s) = \frac{1}{3b} \{a - 2\bar{c}_j - (\gamma - \gamma_i) - (\delta - \delta_i)s\}
y^*_i(s, m = s) = \frac{1}{3b} \{a + \bar{c}_j - (2\gamma_i + \gamma) - (2\delta_i + \delta)s\}
\]

Case 2 : $\delta_i < \delta$

(a) If $s \geq s_3 > s_1$, then $y^*_j(m = s) = y_i(s, m = s) = 0$

(b) If $s_3 \geq s \geq s_1$, then $y^*_i(s, m = s) = 0$ and
\[
y^*_j(m = s) = \frac{1}{2b} \{a - \bar{c}_j - \gamma - \delta s\}
\]
(c) If $\hat{s}_3 \geq \hat{s}_1 \geq s$, then
\[
y^*_i(m = s) = \frac{1}{3b} \{a - 2\hat{\epsilon}_j - (\gamma - \gamma_i) - (\delta - \delta_i)s\}
\]
\[
y^*_i(s, m = s) = \frac{1}{3b} \{a + \hat{\epsilon}_j - (2\gamma_i + \gamma) - (2\delta_i + \delta)s\}
\]

2. Firm $i$ is informed but withholds the information $m = \emptyset$.

Firm $i$ solves:
\[
\max_{y_i} \{a - E(c_i|s) - E(c|s) - b(y_i + y_j)\}y_i
\]

Firm $j$ solves:
\[
\max_{y_j} \{a - \hat{\epsilon}_j - E(c|\emptyset) - b(\hat{y}_i + y_j)\}y_j
\]
subject to $y_i, y_j \geq 0$. Differentiating the above objective functions, we get the following first order conditions.

FOC $i$ : 
\[
a - E(c_i|s) - E(c|s) - 2by_i - by_j = 0
\]

FOC $j$ : 
\[
a - \hat{\epsilon}_j - E(c|\emptyset) - 2by_j - \hat{b}y_i = 0
\]

and
\[
\hat{y}_i = E_j[y^*_i(s, m = \emptyset)] = \frac{1}{2b} \{a - E(c|s \in \emptyset) - E(c_i|s \in \emptyset) - by_j\}
- \frac{p(1 - \eta)}{2b} \{a - E(c|s \in N^o) - E(c_i|s \in N^o) - by_j\}
\]

where $Prob(U) = (1 - p)$, $Prob(s \in N^+, I) = p\eta$ and $Prob(s \in N^o, I) = p(1 - \eta)$, and where

$N$ = Non-disclosure set

$N^+$ = Subset of $N$ where firm $i$'s output is positive

$N^o$ = Subset of $N$ where firm $i$'s output is zero

Substituting for $\hat{y}_i$ into FOC $j$ we get
\[
2by_j = a - E(c|m = \emptyset) - \hat{\epsilon}_j - \frac{1}{2} \{a - E(c|m = \emptyset) - E(c_i|m = \emptyset) - by_j\}
+ \frac{p(1 - \eta)}{2} \{a - E(c|s \in N^o) - E(c_i|s \in N^o) - by_j\}
\]

This can be rearranged to
\[
y^*_j(m = \emptyset) = \frac{1}{3b} \{a - E(c|m = \emptyset) - 2\hat{\epsilon}_j + E(c_i|m = \emptyset)\}
+ \frac{2p(1 - \eta)}{b[3 + p(1 - \eta)]} \{a + \hat{\epsilon}_j - E(c_i|N^o) - 2E(c_i|N^o)
- \frac{E(c|N^o) - E(c|m = \emptyset)}{2} + \frac{E(c_i|N^o) - E(c_i|m = \emptyset)}{2}\}
\]
Substituting for $y_j^*(m = 0)$ into FOC $i$ and rearranging, we get

$$y_i^*(s, m = 0) = \frac{1}{3b} \{a + \tilde{c}_j - E(c|s) - 2E(c_i|s)\} - \frac{E(c|s) - E(c|m = 0)}{2}$$

$$+ \frac{E(c_i|s) - E(c_i|m = 0)}{2} - \frac{p(1 - \eta)}{b[3 + p(1 - \eta)]} \{a + \tilde{c}_j - E(c|N^o) - 2E(c_i|N^o)\}$$

$$- \frac{E(c_i|N^o) - E(c|m = 0)}{2} + \frac{E(c_i|N^o) - E(c_i|m = 0)}{2}$$

Under multivariate normality

$$y_i^*(s, m = 0) = \frac{1}{3b} \{a + \tilde{c}_j - (2\gamma_i + \gamma) - \frac{3}{2}(\delta_i + \delta)s - \frac{\delta_i - \delta}{2}[pA + (1 - p)s]\} + \frac{\epsilon}{2}$$

$$y_j^*(m = 0) = \frac{1}{3b} \{a - 2\tilde{c}_j + (\gamma_i - \gamma) + (\delta_i - \delta)[(1 - p)s + pA]\} - \epsilon$$

where

$$A = E[s|N^o]$$

$$\epsilon = \frac{2p(1 - \eta)}{b[3 + p(1 - \eta)]} \{2\gamma_i + \gamma + \frac{3}{2}(\delta_i + \delta)A + \frac{\delta_i - \delta}{2}[pA + (1 - p)s] - (a + \tilde{c}_j)\}$$

$\epsilon$ is positive since it represents the change in firm $i$'s expected output over $s \in N^o$ due to the constraint $y_i \geq 0$. 

155
A.1.4 Output choice with $M$ opponents

1. Firm $i$ is informed and discloses $m = s$.

Firm $i$ and each $j$ solves:

$$\max_{y_i} \{a - E(c_i|s) - E(c|s) - b\sum_{j=1}^{M} (y_j + y_i)\}$$

$$\max_{y_j} \{a - \bar{c}_j - E(c|s) - b\left(\sum_{k=1}^{M} y_k + y_i\right)\} \quad \forall j = 1, \ldots, M$$

The first order conditions are then obtained as

$$\text{FOC } i : \quad a - E(c_i|s) - E(c|s) - 2by_i^* - b\sum_{j=1}^{M} y_j = 0$$

$$\text{FOC } j : \quad a - \bar{c}_j - E(c|s) - 2by_j^* - b\left(\sum_{k \neq j}^{M} y_k + y_i\right) = 0$$

Summing FOC $j$ over $j = 1, \ldots, M$ yields

$$M[a - E(c|s)] - \sum_{j=a}^{M} \bar{c}_j - b(M - 1)\sum_{j=1}^{M} y_j - by_i - 2b\sum_{j=1}^{M} y_j = 0$$

Substituting into FOC $i$ and rearranging yields

$$y(s|D, M) = \frac{1}{b(M + 2)} \left\{ a + \sum_{j=1}^{M} \bar{c}_j - E(c|s) - (M + 1)E(c_i|s) \right\}$$

Under joint normality

$$y(s|D, M) = \frac{1}{b(M + 2)} \left\{ a - \sum_{j=1}^{M} \bar{c}_j - [\gamma + (M + 1)\gamma_i] - [\delta + (M + 1)\delta_i|s] \right\}$$

2. Firm $i$ is informed but withholding its information, i.e. $m = \emptyset$.

Firm $i$ and each $j$ solves

$$\max_{y_i} \{a - E(c_i|\emptyset) - E(c|\emptyset) - b\sum_{j=1}^{M} (y_j + y_i)\}$$

$$\max_{y_j} \{a - \bar{c}_j - E(c|\emptyset) - b\left(\sum_{k=1}^{M} y_k + \tilde{y}_i\right)\} \quad \forall j = 1, \ldots, M$$

where $\tilde{y}_i$ is firm $j$'s expectation of firm $i$'s output choice given non-disclosure. The first order conditions are then obtained as

$$\text{FOC } i : \quad a - E(c_i|\emptyset) - E(c|\emptyset) - 2by_i^* - b\sum_{j=1}^{M} y_j = 0$$

$$\text{FOC } j : \quad a - \bar{c}_j - E(c|\emptyset) - 2by_j^* - b\left(\sum_{k \neq j}^{M} y_k + \tilde{y}_i\right) = 0$$

156
where
\[
\hat{y}_i = \frac{p}{2b} \left\{ a - E(c|s \in N) - E(c_i|s \in N) - b \sum_{j=1}^{M} y_j \right\} + \frac{1-p}{2b} \left\{ a - \bar{c} - \bar{c}_i - b \sum_{j=1}^{M} y_j \right\}
\]

Hence,
\[
\hat{y}_i = \frac{1}{2b} \left\{ a - E(c|\emptyset) - E(c_i|\emptyset) - b \sum_{j=1}^{M} y_j \right\}
\]

After substituting for \(\hat{y}_i\) into FOC \(j\), we sum FOC \(j\) over \(j\) and then solve the resulting equation simultaneously. This yields the optimal output as
\[
y(s|N, M) = \frac{1}{b(M + 2)} \left\{ a + \sum_{j=1}^{M} \bar{c}_j - E(c|s) - (M - 1)E(c_i|s) \right. \\
- \frac{M}{2} [E(c|s) - E(c|\emptyset)] + \frac{M}{2} [E(c_i|s) - E(c_i|\emptyset)] \left. \right\}
\]

Under the assumption of joint normality this becomes
\[
y(s|N, M) = \frac{1}{b(M + 2)} \left\{ a + \sum_{j=1}^{M} \bar{c}_j - [\gamma + (M + 1)\gamma_i] - \frac{M + 2}{2} (\delta + \delta_i)s \right. \\
+ \frac{M}{2} (\delta - \delta_i)[pA_M + (1-p)s] \left. \right\}
\]

where \(A_M = E[s|N] \equiv E[s|I, m = \emptyset]\).
A.2 Proofs

A.2.1 Proofs of Lemmas

Proof of Lemma 3.1 The threshold values of disclosure must satisfy equation (3.17). From the text,

\[ E(c_i | s) = \psi_i(s) \quad \text{and} \quad E(c | s) = \psi(s) \quad \text{(A.L.3.1-1)} \]

Let firm i’s optimal output given s and m = s be

\[ y_i^*(s) = \frac{1}{3b} \{a + \beta - E(c_i | s) - 2E(c_i | s)\} \]

Then expression (3.17) becomes \( E(\pi_i^* | s, m = \emptyset) - E(\pi_i^* | s, m = s) \)

\[ \Rightarrow \frac{1}{9b} (3by_i^*(s) + \frac{1}{2} [\psi_i(s) - p\beta - (1 - p)\beta_i]) \]

\[ - \frac{1}{2} [\psi_i(s) - p\alpha - (1 - p)\beta_i]^2 - \frac{1}{9b} [3by_i^*(s)]^2 = 0 \]

\[ \Rightarrow \frac{1}{9b} (3by_i^*(s) [\psi_i(s) - p\beta - (1 - p)\beta_i] - [\psi_i(s) - p\alpha - (1 - p)\beta_i]) \]

\[ + \frac{1}{4} ([\psi_i(s) - p\beta - (1 - p)\beta_i]^2 - [\psi(s) - p\alpha - (1 - p)\beta_i])^2 = 0 \]

Expanding the squared term and letting

\[ C_i = p\beta + (1 - p)\beta_i \quad \text{and} \quad C = p\alpha + (1 - p)\beta \]

the above expression becomes,

\[ \frac{1}{36b} \{12by_i^*(s) [\psi_i(s) - \psi(s)] - 12by_i^*(C_i - C) + [\psi_i(s) - \psi(s)]^2 \}

\[ - 2[\psi_i(s) - \psi(s)][C_i - C] + (C_i - C)^2 \} = 0 \]

\[ \Rightarrow \frac{1}{36b} \{[\psi(s) - \psi_i(s)]^2 + 2[\psi(s) - \psi_i(s)][C_i - C - 6by_i^*(s)] \}

\[ + 2[C_i - C][C_i - C - 12by_i^*(s)] \} = 0 \]

Note that if \( a = 1, b = 2[C_i - C - 6by_i^*(s)] \) and \( c = (C_i - C)[c_i - C - 12by_i^*(s)] \), the solution to the above quadratic equation is

\[ \psi(s) - \psi_i(s) = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(A.L.3.1-2)} \]

\[ b^2 - 4ac = 4 \{C_i^2 + C^2 + 36b^2[y_i^*(s)]^2 - 2C_i C - 12C_i by_i^*(s) + 12C by_i^*(s) \}

\[ - 4 \{C_i^2 - 2C_i C - 12C_i by_i^*(s) + C^2 + 12C by_i^*(s) \}

\[ = 4 \{36b^2[y_i^*(s)]^2 \} = \{12by_i^*(s)\}^2 \quad \text{(A.L.3.1-3)} \]
Since $b^2 - 4ac$ is a square it is always positive. Hence, the roots of $\psi(s) - \psi_i(s)$ are real. These roots are

$$\psi(s) - \psi_i(s) = \frac{-2[C_i - C - 6b\gamma_i^*(s)] \pm 12b\gamma_i^*(s)}{2}$$

Therefore,

$$\psi(s) - \psi_i(s) = \begin{cases} 12b\gamma_i^*(s) - C_i + C & \text{or} \\ C - C_i & \end{cases}$$

Equivalently,

$$\psi(s) - \psi_i(s) = \begin{cases} 12b\gamma_i^*(s) - [p\beta + (1 - p)\tilde{c}_i] + [p\alpha + (1 - p)\tilde{c}] & \text{or} \\ [p\alpha + (1 - p)\tilde{c}] - [p\beta + (1 - p)\tilde{c}_i] & \end{cases}$$

Proof of Lemma 3.2 To show that $\alpha = \gamma + \delta A$, we first write the expanded form of $\alpha$

$$\alpha = \int_{s \in N} \int_{-\infty}^{\infty} c g(c, s) dc ds \left/ F(s \in N) \right.$$  

$$= \int_{s \in N} \left[ \int_{-\infty}^{\infty} c g(c|s) dc \right] f(s) ds \left/ F(s \in N) \right.$$  

$$= \int_{s \in N} E(c|s) f(s) ds \left/ F(s \in N) \right.$$  

From (19), in the text, we can substitute for $E(c|s)$ to get,

$$\int_{s \in N} [\gamma + \delta s] f(s) ds \left/ F(s \in N) \right. = \gamma + \delta A$$

By similar algebraic substitutions we can obtain,

$$\beta = \gamma_i + \delta_i A$$

$$\tilde{c}_i = \gamma_i + \delta_{i\bar{s}} \quad \text{and}$$

$$\tilde{c} = \gamma + \delta \bar{s}$$
Proof of Lemma 3.3 To find the equilibrium values for \( s' \) and \( s'' \), we need to solve equations (3.40) and (3.41) simultaneously. Since (3.40) and (3.41) cannot be solved analytically, we find ranges for the equilibrium values of \( s' \) and \( s'' \). The values of \( A \) and \( p \) in both (3.40) and (3.41), given any set of values for \( s' \) and \( s'' \), depend on whether the equilibrium disclosure set, \( N \), is convex or non-convex. Since both are possible, as seen in figure 1, each case is considered separately below.

"Case 1" is used to refer to the case where the non-disclosure set, \( N \), is convex, i.e.,

\[
N = \begin{cases} 
[s', s''] & \text{if } s'' > s' \\
[s'', s'] & \text{otherwise}
\end{cases}
\]

"Case 2" is used to refer to the case where the non-disclosure set, \( N \), is non-convex, i.e.

\[
N = \begin{cases} 
(-\infty, s'] \cup [s'', \infty) & \text{if } s'' > s' \\
(-\infty, s'] \cup [s', \infty) & \text{otherwise}
\end{cases}
\]

The following expression, obtained by substitution for \( s' \) from (3.40) into (3.41), will be required:

\[
s'' = \bar{s} + \frac{12by_1^* - (\delta_i - \delta)(s' - \bar{s})}{7\delta_i + 5\delta}
\]

(A.L.3.3-1)

**Case 1 : \( \delta_i > \delta \)**

If \( s' > s'' \), then \( A \) and \( p \) are as follows

\[
A = \frac{\int_{s'}^{s''} s f(s) ds}{\int_{s''}^{s'} f(s) ds} = \frac{s' F(s') - s'' F(s'') - \int_{s'}^{s''} F(s) ds}{F(s') - F(s'')}
\]

\[
p = \frac{q \int_{s'}^{s''} f(s) ds}{q \int_{s''}^{s'} f(s) ds + (1 - q)} = \frac{q[F(s') - F(s'')]}{q[F(s') - F(s'')] + (1 - q)}
\]

Substitution of \( A \) and \( p \) into (3.40) gives

\[
(s' - \bar{s}) = \frac{q[F(s') - F(s'')]}{q[F(s') - F(s'')] + (1 - q)} \left\{ \frac{s' F(s') - s'' F(s'') - \int_{s'}^{s''} F(s) ds}{F(s') - F(s'')} - \bar{s} \right\}
\]

This then yields

\[
(1 - q)(s' - \bar{s}) = q \{(s' - s'')F(s'') - \int_{s''}^{s'} F(s) ds\}
\]

(A.L.3.3-2)

If \( s' < s'' \), \( A \) and \( p \) are as follows

\[
A = \frac{\int_{s'}^{s''} s f(s) ds}{\int_{s'}^{s''} f(s) ds} = \frac{s'' F(s'') - s' F(s') - \int_{s'}^{s''} F(s) ds}{F(s'') - F(s')}
\]

\[
p = \frac{q \int_{s'}^{s''} f(s) ds}{q \int_{s'}^{s''} f(s) ds + (1 - q)} = \frac{q[F(s'') - F(s')]}{q[F(s'') - F(s')] + (1 - q)}
\]

(A.L.3.3-3)
Substitution of $A$ and $p$ into (3.40) gives

$$s' - \bar{s} = \frac{q[F(s'') - F(s')]}{q[F(s'') - F(s')] + (1 - q)} \left\{ \frac{s''F(s'') - s'F(s') - \int_{s'}^{s''} F(s)ds}{F(s'') - F(s')} - \bar{s} \right\}$$

This then yields

$$\begin{align*}
(1 - q)(s' - \bar{s}) &= q\{(s'' - s')F(s'') - \int_{s'}^{s''} F(s)ds\} \\
(1 - q)(s' - \bar{s}) &= q\int_{s'}^{s''} [F(s'') - F(s)]ds \\
\end{align*}$$

(A.L.3.3-5)

Combining the expressions (A.L.3.3-2) and A.L.3.3-5), we get

$$\begin{cases} 
q \int_{s'}^{s''} [F(s'') - F(s)]ds & \text{if } s'' < s' \\
q \int_{s'}^{s''} [F(s'') - F(s)]ds & \text{otherwise} 
\end{cases}$$

(A.L.3.3-6)

The intervals within which the equilibrium values of $s'$ and $s''$ must lie are first identified. We then show that both the left hand side (LHS) and right hand side (RHS) of expression (A.L.3-6) are monotonic and, therefore, the equilibrium values of $s'$ and $s''$ are unique.

**Identification of Intervals**

We compare the values of the LHS of (A.L.3.3-6) with its RHS, at different values of $s'$ relative to $s''$. We show that at one extreme of this interval the LHS > RHS and at the other extreme the LHS < RHS. Since (A.L.3.3-6) is a continuous function, the LHS must be equal to the RHS at least once within this interval.

Note, since $F(s)$ is the cumulative distribution function over $s$,

$$\begin{align*}
F(s'') > F(s) & \text{ if } s'' > s > s' \\
F(s'') < F(s) & \text{ if } s' > s > s''
\end{align*}$$

Therefore, the RHS of (A.L.3.3-6) can be signed as

$$\begin{align*}
\text{RHS} > 0 & \text{ if } s'' > s' \\
\text{RHS} < 0 & \text{ if } s'' < s'
\end{align*}$$

If $s' = s''$, then

$$s' = \bar{s} + \frac{12b\check{y}_i^*}{8\delta_i + 4\bar{s}}$$

from (A.L.3.3-1). Therefore,

$$\begin{align*}
\text{LHS} &= (1 - q) \cdot \frac{12b\check{y}_i^*}{8\delta_i + 4\bar{s}} > 0 & \text{since } \check{y}_i^* > 0 \\
\text{RHS} &= q \int_{s'}^{s''} [F(s') - F(s)]ds = 0
\end{align*}$$

161
If \( s' > s'' \), then

\[
s' > \bar{s} + \frac{12b\bar{y}_i^*}{8\delta_i + 4\delta}
\]

Therefore,

\[
\text{LHS} > (1 - q) \frac{12b\bar{y}_i^*}{8\delta_i + 4\delta}
\]
\[
\text{RHS} < 0 \quad \text{shown above}
\]

If \( s' > \bar{s} \), then

\[
s'' = \bar{s} + \frac{12b\bar{y}_i^*}{7\delta_i + 5\delta} > s' \quad \text{since} \quad \bar{y}_i^* > 0 \text{ in (A.L.3-3-1)}
\]

Therefore,

\[
\text{LHS} > (1 - q)0 = 0
\]
\[
\text{RHS} > 0 \quad \text{since} \quad s'' > s'
\]

Summarizing the above,

At \( s' = s'' = \bar{s} + \frac{12b\bar{y}_i^*}{8\delta_i + 4\delta}, \) LHS > 0, and RHS = 0.
At \( s' = \bar{s} < s'' = \bar{s} + \frac{12b\bar{y}_i^*}{7\delta_i + 5\delta}, \) LHS = 0, and RHS > 0.

Therefore, LHS = RHS at least once in the interval characterized by

\[
\begin{align*}
\text{at} & \quad s' < \bar{s} < s'' \\
& \text{and}
\end{align*}
\]

\[
\text{LHS} > (1 - q)0 = 0
\]
\[
\text{RHS} > 0 \quad \text{since} \quad s'' > s'
\]

\[
\text{Monotonicity:}
\]

We find the derivatives of the LHS and RHS with respect to \( s' \) and examine their signs in the interval \( \bar{s} < s' < s''. \)

\[
\frac{\partial \text{LHS}}{\partial s'} = (1 - q) > 0
\]

\[
\frac{\partial \text{RHS}}{\partial s'} =
\begin{cases}
- q \left\{ \frac{(\bar{s} - s')}{7\delta_i + 5\delta} (s' - s'') f(s') + \int_{s'}^{s''} f(s') ds \right\} & \text{if} \quad s' > s'' \\
- q \left\{ \frac{(\bar{s} - s')}{7\delta_i + 5\delta} (s'' - s') f(s') + \int_{s'}^{s''} f(s') ds \right\} & \text{if} \quad s' < s'' \\
0 & \text{if} \quad s' = s''
\end{cases}
\]

Now, when \( s' > s'' \),

\[
\left\{ \left( \frac{\delta_i - \delta}{7\delta_i + 5\delta} \right) (s' - s'') f(s'') + \int_{s''}^{s'} f(s) ds \right\} > 0
\]
and when \( s' < s'' \),
\[
\left\{ \left( \frac{\delta_i - \delta}{7\delta_i + 5\delta} \right) (s'' - s') f(s'') + \int_{s'}^{s''} f(s) \, ds \right\} > 0
\]
Therefore, \( \frac{\partial \text{RHS}}{\partial s'} \) is non-positive for all values of \( s' \). Hence, LHS=RHS can be equal only once.
Therefore, the equilibrium values of \( s' \) and \( s'' \) are unique and are in the intervals
\[
s' \in \left[ \bar{s}, \bar{s} + \frac{12b\bar{y}'_i}{7\delta_i + 5\delta} \right]
\]
and
\[
s'' \in \left[ \bar{s} + \frac{12b\bar{y}'_i}{8\delta_i + 4\delta}, \bar{s} + \frac{12b\bar{y}'_i}{7\delta_i + 5\delta} \right]
\]
**Case 2 : \( \delta_i < \delta \)**

If \( s' > s'' \), then \( A \) and \( p \) are as follows
\[
A = \frac{\int_{-\infty}^{s''} s f(s) \, ds + \int_{s'}^{s''} f(s) \, ds}{\int_{-\infty}^{s'} f(s) \, ds + \int_{s'}^{s'} f(s) \, ds} = \frac{s'' - s' f(s') + s'' f(s''') + \int_{s'}^{s''} f(s) \, ds}{1 - F(s') + F(s'')}
\]
\[
p = \frac{q \left\{ \int_{-\infty}^{s'} f(s) \, ds + \int_{s'}^{s''} f(s) \, ds \right\}}{q \left\{ \int_{-\infty}^{s'} f(s) \, ds + \int_{s'}^{s''} f(s) \, ds \right\} + (1 - q)} = \frac{q[1 - F(s') + F(s'')]}{q[1 - F(s') + F(s'')] + (1 - q)}
\]
Substitution of \( A \) and \( p \) into (3.40) gives
\[
(s' - s) = q \left\{ \int_{s'}^{s''} [F(s) - F(s'')] \, ds \right\} = q \int_{s'}^{s''} [F(s) - F(s'')] \, ds \quad (A.L.3.3-7)
\]
If \( s' < s'' \), \( A \) and \( p \) are as follows
\[
A = \frac{\int_{-\infty}^{s' s'} f(s) \, ds + \int_{s'}^{s''} f(s) \, ds}{\int_{-\infty}^{s'} f(s) \, ds + \int_{s'}^{s''} f(s) \, ds} = \frac{s'' - s' f(s') + s'' f(s'') + \int_{s'}^{s''} f(s) \, ds}{1 - F(s'') + F(s')} \quad (A.L.3.3-8)
\]
\[
p = \frac{q \left\{ \int_{-\infty}^{s'} f(s) \, ds + \int_{s'}^{s''} f(s) \, ds \right\}}{q \left\{ \int_{-\infty}^{s'} f(s) \, ds + \int_{s'}^{s''} f(s) \, ds \right\} + (1 - q)} = \frac{q[1 - F(s''') + F(s')]}{q[1 - F(s'') + F(s')] + (1 - q)} \quad (A.L.3.3-9)
\]
Substitution of the above \( A \) and \( p \) into (3.40) gives
\[
(s' - s) = q \left\{ \int_{s'}^{s''} [F(s) - F(s'')] \, ds \right\} = q \int_{s'}^{s''} [F(s) - F(s'')] \, ds \quad (A.L.3.3-11)
\]

163
Combining the expressions (A.L.3.3-7) and (A.L.3.3-11), we get

\[
(s' - \bar{s}) = \begin{cases} 
q \int_{s'}^s [F(s) - F(s')] ds & \text{if } s' > s'' \\
q \int_{s'}^{s''} [F(s) - F(s'')] ds & \text{otherwise}
\end{cases}
\text{(A.L.3.3-12)}
\]

As in case 1, the intervals within which the equilibrium values of \(s'\) and \(s''\) must lie are identified.

We then show that both the left hand side (LHS) and right hand side (RHS) of expression (A.L.3.3-12) are monotonic and therefore the equilibrium values of \(s'\) and \(s''\) are unique.

**Identification of Intervals**

As in case 1, we compare the values of the LHS of (A.L.3.3-12) with its RHS, at different values of \(s'\) relative to \(s''\). Note, since \(F(s)\) in expression (A.L.3.3-12) is the cumulative distribution function over \(s\),

\[
F(s'') < F(s) \quad \text{if} \quad s'' < s < s'
\]

\[
F(s'') > F(s) \quad \text{if} \quad s'' > s > s'
\]

Therefore, the RHS of (A.L.3.3-12) can be signed as

\[
\text{RHS} > 0 \quad \text{if} \quad s' > s''
\]

\[
\text{RHS} < 0 \quad \text{if} \quad s' < s''
\]

If \(s' = s'' = \bar{s} + \frac{12b\bar{\gamma}_i}{8\delta_i + 4\delta} (s' - \bar{s})\), then \(s' = \bar{s} + \frac{12b\bar{\gamma}_i}{8\delta_i + 4\delta}\). Therefore,

\[
\text{LHS} = \frac{12b\bar{\gamma}_i}{8\delta_i + 4\delta} > 0 \quad \text{since } \bar{\gamma}_i > 0
\]

\[
\text{RHS} = q \int_{s'}^{s''} [F(s) - F(s')] ds = 0
\]

If \(s' > s''\), then \(s' > \bar{s} + \frac{12b\bar{\gamma}_i}{8\delta_i + 4\delta}\). Therefore,

\[
\text{LHS} > \frac{12b\bar{\gamma}_i}{8\delta_i + 4\delta} > 0
\]

\[
\text{RHS} > 0 \quad \text{since } s' > s''
\]

If \(s' = \bar{s}\), then \(s'' = \bar{s} + \frac{12b\bar{\gamma}_i}{7\delta_i + 5\delta} > s'\), since \(\bar{\gamma}_i > 0\) in (A.L.3.3-7). Therefore,

\[
\text{LHS} = (1 - q).0 = 0
\]

\[
\text{RHS} < 0 \quad \text{since } s' < s''
\]

If \(s' = \bar{s} - \frac{12b\bar{\gamma}_i}{\delta - \delta_i}\), and \(s'' = \bar{s} > s'\) then

\[
\text{LHS} = -\frac{12b\bar{\gamma}_i}{\delta - \delta_i} < 0
\]

\[
\text{RHS} = q \int_{s'}^{s''} [F(s) - F(s'')] ds = q \int_{s'}^{s''} F(s) ds - q \frac{12b\bar{\gamma}_i}{\delta - \delta_i} F(\bar{s})
\]

164
Since \( q F(\tilde{s}) < 1 \), this leads to

\[
q \left\{ \frac{12b\tilde{y}_i^*}{\delta - \delta_i} \right\} F(\tilde{s}) < \frac{12b\tilde{y}_i^*}{\delta - \delta_i}
\]

\[
\Rightarrow -q \left\{ \frac{12b\tilde{y}_i^*}{\delta - \delta_i} \right\} F(\tilde{s}) > -\frac{12b\tilde{y}_i^*}{\delta - \delta_i}
\]

Therefore,

\[
q \int_{\frac{\tilde{s}}{\delta - \delta_i}}^{\tilde{s}} F(s) ds - q \left\{ \frac{12b\tilde{y}_i^*}{\delta - \delta_i} \right\} F(\tilde{s}) > -\frac{12b\tilde{y}_i^*}{\delta - \delta_i}
\]

This implies that \( \text{RHS} > \text{LHS} \). Summarizing the above,

At \( s' = \bar{s}, \ s'' = \bar{s} + \frac{12b\tilde{y}_i^*}{7\delta_i + 5\delta} > s' \), \( \text{LHS} = 0 > \text{RHS} \).

At \( s' = \bar{s} - \frac{12b\tilde{y}_i^*}{7\delta_i + 5\delta}, \ s'' = \bar{s} > s' \) and \( \text{LHS} < \text{RHS} \).

Therefore, \( \text{LHS} = \text{RHS} \) at least once in the interval characterized by

\[
s' \in \left[ \bar{s} - \frac{12b\tilde{y}_i^*}{\delta + \delta_i}, \bar{s} \right] \text{ and } s'' \in \left[ \bar{s}, \bar{s} + \frac{12b\tilde{y}_i^*}{7\delta_i + 5\delta} \right]
\]

From this, \( s' < \bar{s} < s'' \).

**Monotonicity**:

We find the derivatives of the LHS and RHS with respect to \( s' \) and examine their signs in the interval \( s' < \bar{s} < s'' \).

\[
\frac{\partial \text{LHS}}{\partial s'} = 1
\]

\[
\frac{\partial \text{RHS}}{\partial s'} = \begin{cases} 
- \frac{12b\tilde{y}_i^*}{\delta - \delta_i} f(\tilde{s}) & \text{if } s' > s'' \\
-q \int_{s'}^{s''} f(s) ds - \frac{\delta - \delta_i}{7\delta_i + 5\delta} (s'' - s') f(s''') & \text{if } s' < s'' \\
0 & \text{if } s' = s''
\end{cases}
\]

Note that since \( f(s) \) is a Normal density function, and since both \( s' \) and \( s'' \) are finite,

\[
1 > \int_{s'}^{s''} f(s) ds > 0 \text{ and } (s'' - s') f(\bar{s}'') > 0
\]

Therefore, no matter what the relative values of \( \int_{s'}^{s''} f(s) ds \) and \( (s'' - s') f(\bar{s}'') \) are,

\[
\int_{s'}^{s''} f(s) ds - \frac{\delta - \delta_i}{7\delta_i + 5\delta} (s'' - s') f(s''') < \int_{s'}^{s''} f(s) ds < 1
\]

Therefore, \( \frac{\partial \text{RHS}}{\partial s'} \) is always less than \( \frac{\partial \text{LHS}}{\partial s'} \). This implies that R.H.S and L.H.S can be equal only once.
Proof of Lemma 3.4 Setting \( s = s' \) in expressions (3.29) and (3.30) gives,

\[
y_i^*(s', m = s') = \frac{1}{3b}\{a + \bar{c}_j - (2\gamma_i + \gamma) - (2\delta_i + \delta)s'\}
\]

\[
y_i^*(s', m = \emptyset) = \frac{1}{3b}\{a + \bar{c}_j - (2\gamma_i + \gamma) - \frac{3}{2}(\delta_i + \delta)s' - \frac{\delta_i - \delta}{2}[pA + (1 - p)\bar{s}]\}
\]

From expression (3.40) \( s' = pA + (1 - p)\bar{s} \). Hence, it follows that

\[
y_i^*(s', m = s') = y_i^*(s', m = \emptyset)
\]

Setting \( s = s'' \) in expressions (3.29) and (3.30) gives

\[
y_i^*(s'', m = s'') = \frac{1}{3b}\{a + \bar{c}_j - (2\gamma_i + \gamma) - (2\delta_i + \delta)s''\}
\]

\[
y_i^*(s'', m = \emptyset) = \frac{1}{3b}\{a + \bar{c}_j - (2\gamma_i + \gamma) - \frac{3}{2}(\delta_i + \delta)s'' - \frac{\delta_i - \delta}{2}[pA + (1 - p)\bar{s}]\}
\]

(3.41) implies that

\[(\delta_i - \delta)\bar{s} + p(A - \bar{s}) = 4[a + \bar{c}_j - \gamma - 2\gamma_i] - [7\delta_i + 5\delta]s''\]

Substituting into \( y_i^*(s'', m = \emptyset) \) and simplifying leads to

\[
y_i^*(s'', m = \emptyset) = -y_i^*(s'', m = s'')
\]

Proof of Lemma 3.5 In case 1, \( \delta_i > \delta \). The slope of \( y(s|N) \) exceeds the slope of \( y(s|D) \), since

\[-\frac{3}{2}(\delta_i + \delta) > -(2\delta_i + \delta)\]

Therefore, \( A = \int_{s'}^{\infty} sf(s)ds \) is always greater than \( \bar{s} \).

Since, \( \bar{s}' = (1 - p)\bar{s} + pA \), where \( p \in [0, 1] \), it follows that \( \bar{s} \leq \bar{s}' \leq A \). Therefore, \( \bar{s}' \geq \bar{s} \).

In case 2, \( \delta_i < \delta \). The slope of \( y(s|D) \) exceeds the slope of \( y(s|N) \). Therefore,

\[A = \int_{-\infty}^{\bar{s}'} sf(s)ds + \int_{\bar{s}'}^{\infty} sf(s)ds\]

Now suppose \( \bar{s}' \geq \bar{s} \). Then the non-disclosure region

\[
{[-\infty, \bar{s}'] \cup [\bar{s}, \infty]}\]
is such that $A < \tilde{s}$. But if $A < \tilde{s}$, then $\tilde{s}' < \tilde{s}$ since $\tilde{s}' = (1 - p)\tilde{s} + pA$. This leads to a contradiction. Hence

$$\tilde{s}' < \tilde{s}$$

Since, $\tilde{s}' = (1 - p)\tilde{s} + pA$, where $p \in [0, 1]$, it follows that $\tilde{s} \leq \tilde{s}' \leq A$. Therefore, $\tilde{s}' > \tilde{s}$.

**Proof of Lemma 3.6** We differentiate expressions (3.40) and (3.41), with respect to $q$, to obtain $\frac{dA}{dq}$, $\frac{dp}{dq}$, and $\frac{d(s'' - s''')}{dq}$. These derivatives are then signed in each case. Further, since the expressions for $A$ and $p$ are different in each case, the cases have to be dealt with separately.

**Case 1 : $\delta_i > \delta$**

By corollary 1, $s'' > s'$ in case 1. Therefore, $A$ and $p$ are as follows.

$$A = \frac{\int_{s''}^{s'} sf(s)ds}{F(s'') - F(s')}$$

$$p = \frac{q[F(s'') - F(s')]}{q[F(s'') - F(s')] + (1 - q)}$$

Further, since

$$\frac{d}{dq} \left[ F(s'') - F(s') \right] = f(s'') \frac{ds''}{dq} - f(s') \frac{ds'}{dq}$$

and

$$\frac{d}{dq} \left[ F(s'') - F(s') \right] = f(s'') \frac{ds''}{dq} - f(s') \frac{ds'}{dq},$$

we have

$$\frac{dA}{dq} = \frac{f(s'') \frac{ds''}{dq} \left[ \int_{s''}^{s'} sf(s)ds - s'F(s') \right] - f(s'') \frac{ds''}{dq} \left[ \int_{s''}^{s'} sf(s)ds - s''F(s'') \right]}{\left[ F(s'') - F(s') \right]^2}$$

and

$$\frac{dp}{dq} = \frac{F(s'') - F(s') + q(1 - q) \left[ f(s'') \frac{ds''}{dq} - f(s') \frac{ds'}{dq} \right]}{B_1^2}$$

where $B_1 = q[F(s'') - F(s')] + (1 - q)$.
Differentiating (3.40) with respect to \( q \),

\[
\frac{ds'}{dq} = \frac{dp}{dq}(A - \bar{s}) + p \frac{dA}{dq}
\]

\[
\Rightarrow \frac{ds'}{dq} = \frac{[F(s'') - F(s')][A - \bar{s}] + qf(s'') \frac{dA}{dq} [(1 - q)(A - \bar{s}) - B_1(A - s'')]}{B_1^2 + qf(s')[(1 - q)(A - \bar{s}) - B_1(A - s')]} \tag{A.L.3.6-1}
\]

Differentiating (3.41) with respect to \( q \),

\[
-\frac{7\delta_i + 5\delta}{\delta_i - \delta} \frac{ds''}{dq} + \frac{dp}{dq}(A - \bar{s}) + p \frac{dA}{dq} = \frac{ds'}{dq} \tag{A.L.3.6-2}
\]

From (3.40) and (3.41) it can be shown that

\[
A - s' = \frac{1 - q}{B_1} (A - \bar{s}) \tag{A.L.3.6-3}
\]

\[
A - s' = \frac{1 - q}{B_1} (A - \bar{s}) + \frac{12b \beta_i^* - (8\delta_i + 4\delta)(s'' - \bar{s})}{\delta_i - \delta} \tag{A.L.3.6-4}
\]

Substituting for \((A - s')\), \((A - s'')\) and \(\frac{ds''}{dq}\) into (A.L.3.6-1) gives

\[
\frac{ds'}{dq} = \frac{[F(s'') - F(s')][(A - \bar{s})(-7\delta_i - 5\delta)]}{B_1qf(s'')[12b \beta_i^* - (8\delta_i + 4\delta)(s'' - \bar{s})] - (7\delta_i + 5\delta)B_1^2} \tag{A.L.3.6-5}
\]

Substituting for \((A - s')\), \((A - s'')\) and \(\frac{ds'}{dq}\) into (A.L.3.6-2) gives

\[
\frac{ds''}{dq} = \frac{[F(s'') - F(s')](\delta_i - \delta)}{B_1qf(s'')[12b \beta_i^* - (8\delta_i + 4\delta)(s'' - \bar{s})] - (7\delta_i + 5\delta)B_1^2} \tag{A.L.3.6-6}
\]

Therefore,

\[
\frac{d(s'' - s')}{dq} = \frac{[F(s'') - F(s')][A - \bar{s})(8\delta_i + 4\delta)]}{B_1qf(s'')[12b \beta_i^* - (8\delta_i + 4\delta)(s'' - \bar{s})] - (7\delta_i + 5\delta)B_1^2}
\]

From the ranges found for \(s''\), in lemma 3.3, if \(\delta_i > \delta\),

\[
s'' > \bar{s} + \frac{12b \beta_i^*}{8\delta_i + 4\delta} \Rightarrow 12b \beta_i^* - (8\delta_i + 4\delta)(s'' - \bar{s}) < 0
\]

Further, since \(s'' > s' \geq \bar{s}, A > \bar{s} \) and \(F(s'') > F(s')\),

\[
\frac{ds'}{dq} > 0, \quad \frac{ds''}{dq} < 0 \quad \text{and} \quad \frac{d(s'' - s')}{dq} < 0
\]

**Case 2 : \(\delta_i < \delta\)**

By lemma 4, \(s'' > s'\) in case 2. Therefore, \(A\) and \(p\) are as follows.

\[
A = \frac{\bar{s} - \int_{s'}^{s''} sf(s)ds}{1 - F(s'') + F(s')}\]

\[
p = \frac{q[1 - F(s'') + F(s')]}{q[1 - F(s'') + F(s')] + (1 - q)}
\]

168
Further, since

\[ \frac{d[\bar{s} - f(s')sf(s)ds]}{dq} = s'f(s')\frac{ds'}{dq} - s''f(s'')\frac{ds''}{dq} \]

and

\[ \frac{d[1 - F(s'') + F(s')]}{dq} = f(s')\frac{ds'}{dq} - f(s'')\frac{ds''}{dq} \]

we have

\[ \frac{dA}{dq} = \frac{[1 - F(s'') + F(s')]\left(s'f(s')\frac{ds'}{dq} - s''f(s'')\frac{ds''}{dq}\right) - [\bar{s} - \int_{s'}^{s''} sf(s)ds]\left(f(s')\frac{ds'}{dq} - f(s'')\frac{ds''}{dq}\right)}{[1 - F(s'') + F(s')]^2} \]

and

\[ \frac{dp}{dq} = \frac{1 - F(s'') + F(s') + q(1 - q)\left[f(s')\frac{ds'}{dq} - f(s'')\frac{ds''}{dq}\right]}{B_2^2} \]

where \( B_2 = q[1 - F(s'') + F(s')] + (1 - q) \).

Therefore,

\[ \frac{ds'}{dq} = \frac{[1 - F(s'') + F(s')]\left[A - \bar{s} + qf(s'')\frac{ds''}{dq}\left(B_2(A - s'') - (1 - q)(A - \bar{s})\right)\right]}{B_2^2 + qf(s')\left[B_2(A - s') - (1 - q)(A - \bar{s})\right]} \]  

(A.L.3.6-7)

and as before

\[ \frac{ds''}{dq} = -\frac{\delta_i - \delta}{7\delta_i + 5\delta} \frac{ds'}{dq} \]  

(A.L.3.6-8)

Substituting for \((A - s'), (A - s'')\) and \(\frac{ds''}{dq}\) into (A.L.3.6-7) gives (Note that (A.L.3.6-3) holds in case 2 as well)

\[ \frac{ds'}{dq} = \frac{[1 - F(s'') + F(s')]\left[A - \bar{s} + qf(s'')\frac{ds''}{dq}\left(\left[8\delta_i + 4\delta\right](s'' - s') - 12b\tilde{y}_i^*\right)\right]}{B_2^2 + qf(s')\left[\left(8\delta_i + 4\delta\right)(s'' - s') - 12b\tilde{y}_i^*\right] - (7\delta_i + 5\delta)B_2^2} \]

Substituting for \((A - s'), (A - s'')\) and \(\frac{ds''}{dq}\) into A.L.3.6-2) gives (Note that (A.L.3.6-4) holds in case 2 as well)

\[ \frac{ds''}{dq} = \frac{[1 - F(s'') + F(s')]\left[A - \bar{s} + \delta\right]}{B_2^2 + qf(s'')\left[\left[8\delta_i + 4\delta\right](s'' - s') - 12b\tilde{y}_i^*\right] - (7\delta_i + 5\delta)B_2^2} \]

Therefore,

\[ \frac{d(s'' - s')}{dq} = \frac{[1 - F(s'') + F(s')]\left[A - \bar{s} + \delta\right]}{B_2^2 + qf(s'')\left[\left[8\delta_i + 4\delta\right](s'' - s') - 12b\tilde{y}_i^*\right] - (7\delta_i + 5\delta)B_2^2} \]

From the ranges found for \(s''\), in lemma 3.3, if \(\delta_i < \delta\),

\[ s'' < \bar{s} + \frac{12b\tilde{y}_i^*}{8\delta_i + 4\delta} \Leftrightarrow (8\delta_i + 4\delta)(s'' - s') - 12b\tilde{y}_i^* < 0 \]
Further, since $s' < s$, and by (3.40), $s' - s = p(A - s)$ and $A < s$

$$\frac{ds'}{dq} < 0, \quad \frac{ds''}{dq} < 0 \text{ and } \frac{d(s'' - s')}{dq} > 0.$$ 

**Proof of Lemma 3.7** We find the derivatives $\frac{ds'}{d\bar{c}_j}$ and $\frac{ds''}{d\bar{c}_j}$ by differentiating (3.40) and (3.41) with respect to $\bar{c}_j$.

**Case 1 : $\delta_i > \delta$**

$A$ and $p$ are given by (A.L.3.3-3) and (A.L.3.3-4) Since,

$$\frac{d}{d\bar{c}_j} \left[ \int s f(s) ds \right] = s'' f(s'') \frac{ds''}{d\bar{c}_j} - s' f(s') \frac{ds'}{d\bar{c}_j}$$

and

$$\frac{d}{d\bar{c}_j} [F(s'') - F(s')] = f(s'') \frac{ds''}{d\bar{c}_j} - f(s') \frac{ds'}{d\bar{c}_j},$$

we have

$$\frac{dA}{d\bar{c}_j} = \frac{(s'' - A) f(s'') \frac{ds''}{d\bar{c}_j} - (s' - A) f(s') \frac{ds'}{d\bar{c}_j}}{F(s'') - F(s')}$$

and

$$\frac{dp}{d\bar{c}_j} = \frac{q(1 - q)}{B_1^2} \left\{ f(s'') \frac{ds''}{d\bar{c}_j} - f(s') \frac{ds'}{d\bar{c}_j} \right\}$$

where $B_1 = q[F(s'') - F(s')'] + (1 - q)$.

Differentiating (3.40) with respect to $\bar{c}_j$ and rearranging,

$$\frac{ds'}{d\bar{c}_j} = \frac{q f(s'') \left\{ \frac{1 - q}{B_1} (A - s) - (A - s'') \right\} \frac{ds''}{d\bar{c}_j}}{1 + \frac{q f(s')}{B_1} \left\{ \frac{1 - q}{B_1} (A - s) - (A - s') \right\}}$$

(A.L.3.7-1)

By (A.L.3.7-1),

$$\frac{1 - q}{B_1} (A - s) - (A - s') = 0$$

Substituting for $(A - s'')$ from (A.L.3.6-4) into (A.L.3.7-1) gives

$$\frac{ds'}{d\bar{c}_j} = \frac{q f(s'')}}{B_1(\delta_i - \delta)} ((8\delta_i + 4\delta)(s'' - s) - 12b \delta_i) \frac{ds''}{d\bar{c}_j}$$

(A.L.3.7-2)
Differentiating (A.L.3.3-1) with respect to $\varepsilon_j$

$$\frac{ds''}{d\varepsilon_j} = \frac{4 - (\delta_i - \delta) \frac{ds'}{d\varepsilon_j}}{7\delta_i + 5\delta} \quad (A.L.3.7-3)$$

Solving (A.L.3.7-2) and (A.L.3.7-3) simultaneously,

$$\frac{ds'}{d\varepsilon_j} = \frac{4B_1}{qf(s'')[8\delta_i + 4\delta](s'' - \bar{s}) - 12b\bar{y}_i^*] + (7\delta_i + 5\delta)B_1}$$

$$\frac{ds''}{d\varepsilon_j} = \frac{4qf(s'')[8\delta_i + 4\delta](s'' - \bar{s}) - 12b\bar{y}_i^*]}{(\delta_i - \delta)\{qf(s'')[8\delta_i + 4\delta](s'' - \bar{s}) - 12b\bar{y}_i^*] + (7\delta_i + 5\delta)B_1}\}$$

Since in case 1, $(8\delta_i + 4\delta)(s'' - \bar{s}) > 12b\bar{y}_i^*$, the denominator of $\frac{ds''}{d\varepsilon_j}$ and $\frac{ds'}{d\varepsilon_j}$, and the numerator of $\frac{ds'}{d\varepsilon_j}$ are positive.

Since $B_1 > 0$, the numerator of $\frac{ds''}{d\varepsilon_j}$ is also positive. Therefore, $\frac{ds''}{d\varepsilon_j}$ and $\frac{ds'}{d\varepsilon_j}$ are both positive. Further,

$$\frac{d(s'' - s')}{d\varepsilon_j} = \frac{ds''}{d\varepsilon_j} - \frac{ds'}{d\varepsilon_j} > 0$$

$$\Rightarrow (\delta_i - \delta)B_1 > qf(s'')[8\delta_i + 4\delta](s'' - \bar{s}) - 12b\bar{y}_i^*]$$

$$= qf(s'')[(\delta_i - \delta)(s'' - \bar{s}) + (7\delta_i + 5\delta)(s'' - \bar{s}) - 12b\bar{y}_i^*]$$

From the expression for the equilibrium value of $s'$ (i.e. expression (A.L.3.3-1)),

$$(7\delta_i + 5\delta)(s'' - \bar{s}) = 12b\bar{y}_i^* - (\delta_i - \delta)(s'' - \bar{s})$$

Substituting for $(7\delta_i + 5\delta)(s'' - \bar{s})$, the above inequality may be written as

$$(\delta_i - \delta)B_1 > qf(s'')(\delta_i - \delta)(s'' - s')$$

Substituting back $B_1 = q[F(s'') - F(s')] + (1 - q)$, we get

$$q[F(s'') - F(s')] + (1 - q) > qF(s'')(s'' - s')$$

$$\Rightarrow F(s'') - F(s') + \frac{1 - q}{q} > f(s'')(s'' - s')$$

From $s'' > s' > \bar{s}$ and the assumption of normality of $s$, it follows that $f(s'') < f(s')$. Hence

$$F(s'') - F(s') > f(s'')(s'' - s')$$

Therefore,

$$F(s'') - F(s') + \frac{1 - q}{q} > f(s'')(s'' - s')$$
This then implies that
\[ \frac{d(s'' - s')}{d\bar{c}_j} > 0 \]

**Case 2 :** $\delta_i < \delta$

$A$ and $p$ are given by (A.L.3.3-8) and (A.L.3.3-10). Since,
\[
\frac{d \left[ \bar{s} - f(s') s f(s) ds \right]}{d\bar{c}_j} = s' f(s') \frac{ds'}{d\bar{c}_j} - s'' f(s'') \frac{ds''}{d\bar{c}_j}
\]
and
\[
\frac{d [1 - F(s'') + F(s')]}{d\bar{c}_j} = f(s') \frac{ds'}{d\bar{c}_j} - f(s'') \frac{ds''}{d\bar{c}_j},
\]
we have
\[
\frac{dA}{d\bar{c}_j} = \frac{(A - s'') f(s'') \frac{ds''}{d\bar{c}_j} - (A - s') f(s') \frac{ds'}{d\bar{c}_j}}{1 - F(s'') + F(s')}
\]
and
\[
\frac{dp}{d\bar{c}_j} = q(1 - q) \left\{ f(s') \frac{ds'}{d\bar{c}_j} - f(s'') \frac{ds''}{d\bar{c}_j} \right\}
\]
where $B_2 = q[1 - F(s'') + F(s')] + (1 - q)$.

Differentiating (3.40) with respect to $\bar{c}_j$ and substituting for $\frac{dp}{d\bar{c}_j}$ and $\frac{dA}{d\bar{c}_j}$ leads to
\[
\frac{ds'}{d\bar{c}_j} = \frac{qf(s'')}{1 + \frac{qd(s' - s'')}{B_2}} \left\{ \left( A - s'' \right) - \frac{B_2}{1 - q} \left( A - s' \right) \right\} \frac{ds''}{d\bar{c}_j}
\]
(A.L.3.7-4)

By (A.L.3.6-3),
\[ \frac{1 - q}{B_2} (A - \bar{s}) - (A - s') = 0 \]
Substituting for $(A - s'')$ from (A.L.3.6-4) into (A.L.3.7-4) yields
\[
\frac{ds'}{d\bar{c}_j} = \frac{qf(s'')}{B_2(\delta_i - \delta)} \left( 12b^*y_i - (8\delta_i + 4\delta)(s'' - \bar{s}) \right) \frac{ds''}{d\bar{c}_j}
\]
(A.L.3.7-5)

Solving (A.L.3.7-5) and (A.L.3.7-3) simultaneously,
\[
\frac{ds''}{d\bar{c}_j} = \frac{4B_2}{qf(s'')[12b^*y_i - (8\delta_i + 4\delta)(s'' - \bar{s})] + (7\delta_i + 5\delta)B_2}
\]
\[
\frac{ds'}{d\bar{c}_j} = \frac{4qf(s'')[12b^*y_i - (8\delta_i + 4\delta)(s'' - \bar{s})]}{(\delta_i - \delta)[qf(s'')[12b^*y_i - (8\delta_i + 4\delta)(s'' - \bar{s})] + (7\delta_i + 5\delta)B_2]}
\]

Since in case 2, $(8\delta_i + 4\delta)(s'' - \bar{s}) < 12b^*y_i$, the denominator of $\frac{ds''}{d\bar{c}_j}$ and the numerator of $\frac{ds'}{d\bar{c}_j}$ are positive.
Since in addition, $\delta_i > \delta$, the denominator of $\frac{ds'}{d\tilde{c}_j}$ is negative. Therefore

$$\frac{ds'}{d\tilde{c}_j} < 0$$

Since $B_2 > 0$, the numerator of $\frac{ds''}{d\tilde{c}_j}$ is also positive. Therefore,

$$\frac{ds''}{d\tilde{c}_j} > 0$$

This then implies that

$$\frac{d(s'' - s')}{d\tilde{c}_j} > 0$$
A.2.2 Proofs of Propositions

Proof of Proposition 3.1 This proof follows from the proof of lemma 3.1. Note that in lemma 3.1, \( \psi_i(s) = E(c_i|s) \) and \( \psi(s) = E(c|s) \). Substituting for \( E(c_i|s) \) and \( E(c|s) \) from expression (3.18) and (3.19) into expression (A.L.3.1-2), in the proof of lemma 3.1, we get

\[
(\gamma - \gamma_i) + (\delta - \delta_i)s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[\Rightarrow s = \frac{1}{\delta - \delta_i} \left\{ -\frac{b \pm \sqrt{b^2 - 4ac}}{2a} - (\gamma - \gamma_i) \right\} \]

\( s \) has two real values if \( b^2 - 4ac > 0 \). From (A.L.3.1-3),

\[ b^2 - 4ac = (12b\psi_i(s))^2 \]

which is always positive. Hence, \( s \) has two real values in equilibrium.

Proof of Proposition 3.2

Probability of disclosure = \( P(D) = q[1 - \int_{s \in S} f(s)ds] \)

Case 1 : \( N = s \in [s', s''] \)

\[
P(D) = q[1 - \int_{s'}^{s''} f(s)ds]
\]

\[
dP(D) = dP(D) = [1 - F(s'') + F(s')] - q[f(s'')\frac{ds''}{dq} - f(s')\frac{ds'}{dq}]\]

From lemma 3.6. \( \frac{ds''}{dq} < 0 \) and \( \frac{ds'}{dq} > 0 \) when \( \delta_i > \delta \). Therefore,

\[ f(s')\frac{ds'}{dq} - f(s'')\frac{ds''}{dq} > 0 \Rightarrow \frac{dP(D)}{dq} > 0 \]

Case 2 : \( N = s \in [-\infty, s'] \cup [s'', \infty] \)

\[
P(D) = q \int_{s'} f(s)ds
\]

\[
dP(D) = dP(D) = [F(s'') - F(s')] + q[f(s'')\frac{ds''}{dq} - f(s')\frac{ds'}{dq}]
\]

From lemma 3.6. \( \frac{ds''}{dq} < 0 \) and \( \frac{ds'}{dq} < 0 \) and \( \frac{ds'''}{dq} > \frac{ds'}{dq} \).

\[ \frac{dP(D)}{dq} \geq 0 \Rightarrow F(s'') - F(s') \geq q[f(s')\frac{ds'}{dq} - f(s'')\frac{ds''}{dq}] \]
Proof of Proposition 3.3 Case 1: The proof, in this case, is by contradiction. By (3.15), \( p = 1 \) if \( q = 1 \). Then by (3.40), \( s' = A \) if \( q = 1 \). Suppose \( s'' \neq s' \). From the definition of \( A \) in lemma 3.3

\[
A[F(s'') - F(s')] = \int_{s'}^{s''} sf(s)ds
\]

Integrating by parts

\[
\int_{s'}^{s''} sf(s)ds = s''F(s'') - s'F(s') - \int_{s'}^{s''} F(s)ds
\]

Hence

\[
A[F(s'') - F(s')] = s''F(s'') - s'F(s') - \int_{s'}^{s''} F(s)ds
\]

Substituting \( s' = A \), we get

\[
s''F(s') - s'F(s') = s''F(s'') - s'F(s') - \int_{s'}^{s''} F(s)ds
\]

This then leads to the following result

\[
(s'' - s')F(s'') = \int_{s'}^{s''} F(s)ds
\]

However, since \( F(s) \) is monotonically increasing in \( s \), and since \( s'' > s' \),

\[
(s'' - s')F(s'') > \int_{s'}^{s''} F(s)ds
\]

This contradicts the previous expression. Therefore, \( s'' \neq s' \).

Similarly, we can show that \( s'' \neq s' \). The two results together imply that \( s'' = s' \).

Case 2: We first show that full-disclosure is not an equilibrium when \( q = 1 \), in case 2. Note, that when \( q = 1 \), by (3.15), \( p = 1 \). Assume that full-disclosure is an equilibrium when \( q = 1 \), i.e. \( s' = -\infty \) and \( s'' = \infty \), in equilibrium.

Let the out of equilibrium non-disclosure belief be represented by \( \hat{s} \).

If \( j \) holds posterior belief \( \hat{s} \), given non-disclosure, then we obtain the expected profit from non-disclosure by replacing \( pA + (1 - p)\hat{s} \) with \( \hat{s} \) in expression (3.32).

\[
E[\pi_1^*|s, m = \emptyset] = \frac{1}{9\delta}(3by^*_1(s, m = s) - \frac{\delta_i - \delta_j}{2}(s - \hat{s}))^2 \tag{A.P.3.3-1}
\]

Also from (3.31),

\[
E[\pi_1^*|s, m = s] = \frac{1}{9\delta}(3by^*_1(s, m = s))^2 \tag{A.P.3.3-2}
\]

Let \( s_1 \) and \( s_2 \) be any two numbers such that

\[
\frac{\hat{s} - \int_{s_1}^{s_2} sf(s)ds}{1 - F(s_2) + F(s_1)} \quad \text{for} \quad -\infty < s_1 < s_2 < \infty \tag{A.P.3.3-3}
\]
and where

\[ y_i^*(s, m = s) > 0 \quad \forall \quad s < s_1 \]
\[ y_i^*(s, m = s) < 0 \quad \forall \quad s > s_2 \]

Since \( s_1 < s < s_2 \), it then follows that for all \( s \leq s_1 \) and \( s \geq s_2 \),

\[ E[\pi^*_i|s, m = \emptyset] > E[\pi^*_i|s, m = s] \]

Therefore, \( m^* = \emptyset \) for all \( s \in (-\infty, s_1) \cup (s_2, \infty) \).

Note: For any belief \( \tilde{s} \in (-\infty, \infty) \), there exist \( s_1 \) and \( s_2 \) that satisfy the above condition, due to the fact that \( y^*_i(s, m = s) \) is a downward sloping straight line function of \( s \).

Therefore, \( m^*(s) = \emptyset \) for all \( s \leq s_1 \) and \( s \geq s_2 \). Note that, this optimal message choice is consistent with the out of equilibrium belief, given by (A.P.3.3-3). Therefore, full disclosure is not an equilibrium.

We next show that neither is full non-disclosure an equilibrium.

Under full disclosure, (A.P.3.3-3) becomes \( A = a \). Note that by assumption, \( y^*_i(s, m = s) \) is decreasing in \( s \). Hence, there exists a signal \( s_0 > \tilde{s} \) such that \( y^*_i(s_0, m = s_0) = 0 \). Further,

\[ y^*_i(s, m = s) < \forall \quad s \leq s_0 \]

Now consider \( s \) where \( \tilde{s} < s < s_0 \). Since \( A = \tilde{s} \),

\[ E[\pi^*_i|s, m = \emptyset] = \frac{1}{9n} (3by^*_i(s, m = s) - \left[ \frac{\delta - \delta_i}{2} \right] (s - \tilde{s}))^2 \]
\[ E[\pi^*_i|s, m = s] = \frac{1}{9n} (3by^*_i(s, m = s))^2 \]

Comparison of the above two expected profit terms indicates that

\[ E[\pi^*_i|s, m = \emptyset] < E[\pi^*_i|s, m = s] \]

Therefore, non-disclosure is not optimal at \( s \). Hence, full disclosure is not an equilibrium.

The existence result in Proposition 1 is independent of the value of \( q \). Therefore, we know that an equilibrium exists. Since full disclosure and full non-disclosure are ruled out, a partial disclosure equilibrium exists. We next characterize this partial disclosure equilibrium.

Let \( P(D) \) be the probability of disclosure, if firm \( i \) is informed. Then,

\[ [1 - P(D)]A + P(D)\bar{C} = \hat{s} \]
where

\[ A = \frac{\int_{s \in N} s f(s) ds}{\int_{s \in N} f(s) ds} \]
\[ C = \frac{\int_{s \in D} s f(s) ds}{\int_{s \in D} f(s) ds} \]

Here, \( A \) is the mean of the non-disclosure set \( N \) and \( C \) is the mean of the disclosure set \( D \).

Since \( 0 < P(D) < 1 \), \( \bar{s} \) is a convex combination of \( A \) and \( C \). Therefore either \( A < \bar{s} < C \) or \( A > \bar{s} > C \).

Now let

\[ \bar{C} = \frac{\int_{a}^{b} s f(s) ds}{\int_{a}^{b} f(s) ds} = \int_{a}^{b} g(s) ds \]

where \( a \) and \( b \) are any two values of \( s \) and

\[ \int_{a}^{b} g(s) ds = 1 \]

Then,

\[ b > C > a \]  \hspace{1cm} (A.P.3.3-4)

From (3.40), if \( p = 1 \), then \( s' = A \). We need to show that \( s' \prec \bar{s} \prec s'' \). Substituting \( p = 1 \) and \( s' = A \) into (3.41), we get

\[ 12b\bar{y}_{i} - (7\delta_{i} + 5\delta)(s'' - \bar{s}) = (\delta_{i} - \delta)(s' - \bar{s}) \]  \hspace{1cm} (A.P.3.3-5)

(i) If \( A > \bar{s} > C \), then since \( A = s' \), we can write \( s' > \bar{s} > C \). Due to (A.P.3.3-4), \( s' > \bar{s} > C > s'' \).

Since \( s' > \bar{s} \) and \( \delta_{i} < \delta \) (in case 2), the right hand side of (A.P.3.3-5) is less than zero. This implies that

\[ (s'' - \bar{s}) > \frac{12b\bar{y}_{i}}{7\delta_{i} + 5\delta} \]  \hspace{1cm} (A.P.3.3-6)

Since it has been assumed in lemma 4 that \( \bar{y}_{i} > 0 \), (A.P.3.3-6) implies that \( s'' > \bar{s} \). Hence,

\[ s' > \bar{s} \Rightarrow s'' > \bar{s} \]

which is a contradiction of (A.P.3.3-4). Therefore,

\[ s' \not\prec \bar{s} \]

(ii) If \( A < \bar{s} < C \), then since \( A = s' \), we can write \( s' < \bar{s} < C \). Due to (A.P.3.3-4),

\[ s' < \bar{s} < C < s'' \]

177
Since $s' < \bar{s}$ and $\delta_i < \delta$, the right hand side of (A.P.3.3-5) is positive. This implies that

$$(s'' - \bar{s}) < \frac{12b\bar{y}_i^*}{l\delta_i + 5\delta} \tag{A.P.3.3-7}$$

(A.P.3.3-7) does not contradict

$s' < \bar{s} < c < s''$

Therefore, $A = s' < \bar{s} < s''$.

**Proof of Proposition 3.4**

$$P(D) = q \int_{s \in D} f(s)ds$$

Case 1

$$\int_{s \in D} f(s)ds = \int_{-\infty}^{s'} f(s)ds + \int_{s'}^{\infty} f(s)ds = 1 - \int_{s'}^{s''} f(s)ds$$

From this it follows that

$$P(D) = q\left\{1 - \int_{s'}^{s''} f(s)ds\right\}$$

$$\frac{dP(D)}{dc_j} = -q\left\{f(s'')\frac{ds''}{dc_j} - f(s')\frac{ds'}{dc_j}\right\}$$

Therefore,

$$\frac{dP(D)}{dc_j} \geq 0 \text{ if } f(s')\frac{ds'}{dc_j} \geq f(s'')\frac{ds''}{dc_j}$$

From lemma 3.7. $\frac{ds''}{dc_j} > \frac{ds'}{dc_j}$ and both $\frac{ds''}{dc_j}$ and $\frac{ds'}{dc_j}$ are positive. Further, since $s'' > s' > \bar{s}$ and since $s$ is normal, $f(s'') < f(s')$.

Therefore, $f(s')\frac{ds'}{dc_j}$ could be greater or smaller than $f(s'')\frac{ds''}{dc_j}$. Further, $\frac{dP(D)}{dc_j}$ could be greater or smaller than zero.

Case 2

$$P(D) = q \int_{s'}^{s''} f(s)ds$$

$$\frac{dP(D)}{dc_j} = q\left\{f(s'')\frac{ds''}{dc_j} - f(s')\frac{ds'}{dc_j}\right\}$$
From lemma 3.7, \( \frac{dx}{dc_j} < 0 \) and \( \frac{dy}{dc_j} > 0 \) Therefore,

\[
\frac{dP(D)}{dc_j} > 0
\]
A.2.3 Proof of Corollary

Proof of Corollary 3.1 This proof follows directly from lemma 3.3. If $\delta_i > \delta$ and $\bar{y}_i^* > 0$, then $8\delta_i + 4\delta > 7\delta_i + 5\delta$. Therefore,

$$\bar{s} + \frac{12b\bar{y}_i^*}{8\delta_i + 4\delta} < \bar{s} + \frac{12b\bar{y}_i^*}{7\delta_i + 5\delta}$$

Therefore every point in the interval,

$$[\bar{s} + \frac{12b\bar{y}_i^*}{8\delta_i + 4\delta}, \bar{s} + \frac{12b\bar{y}_i^*}{7\delta_i + 5\delta}]$$

is greater than every point in the interval

$$[\bar{s}, \bar{s} + \frac{12b\bar{y}_i^*}{8\delta_i + 4\delta}]$$

Therefore, $s'' > s'$. 

If $\delta > \delta_i$, every point in the interval $[\bar{s}, \bar{s} + \frac{12b\bar{y}_i^*}{8\delta_i + 4\delta}]$ is greater than any point in the interval $[\bar{s} - \frac{12b\bar{y}_i^*}{7\delta_i - 5\delta}, \bar{s}]$. Therefore, $s'' > s'$.
Appendix B

Appendices to Chapter 4
B.1 Derivations

B.1.1 Density functions of $R_t$ given $\tau$, and $p_t$

Imposing assumption A4 on the expressions for $g(R_t|G, p_{G_t})$ and $g(R_t|B, p_{B_t})$ from the text, we obtain the following.

\[
g(R_t = R_{hh}|G, p_{G_t}) = n\left\{ [qg_{ht} + (1-q)p_{g_{ut}}] + [q(1-p_{g_{ht}}) + (1-q)(1-p_{g_{ut}})]G_h \right\}
\]

\[
g(R_t = R_{ll}|G, p_{G_t}) = (1-n)\left\{ [qg_{lt} + (1-q)p_{g_{ut}}]G_l + [q(1-p_{g_{lt}}) + (1-q)(1-p_{g_{ut}})] \right\}
\]

\[
g(R_t = R_{hl}|G, p_{G_t}) = n(1-G_h)[q(1-p_{g_{lt}}) + (1-q)(1-p_{g_{ut}})]
\]

\[
g(R_t = R_{lh}|G, p_{G_t}) = (1-n)(1-G_l)[qg_{lt} + (1-q)p_{g_{ut}}]
\]

\[
g(R_t = R_{hh}|B, p_{B_t}) = n[p_{B_t} + (1-p_{B_t})G_h]
\]

\[
g(R_t = R_{ll}|B, p_{B_t}) = (1-n)[p_{B_t}G_l + (1-p_{B_t})]
\]

\[
g(R_t = R_{hl}|B, p_{B_t}) = n(1-G_h)(1-p_{B_t})
\]

\[
g(R_t = R_{lh}|B, p_{B_t}) = (1-n)(1-G_l)p_{B_t}
\]
B.1.2 Joint Densities of $R_t$ and $m_t$ given $\tau$, $p_t$ and $\beta_t$

Imposing assumption A4 on the expressions for $g(R_t, m_t|G, p_G, \beta_G)$ and $g(R_t, m_t|B, p_B, \beta_B)$ from the text, we obtain the following.

\begin{align*}
g(R_t = R_{hh}, m_t|G, p_G, \beta_G) &= n \left\{ q[p_{gh}\beta(m_t|Gh, y_h) + (1-p_{gh})G_h\beta(m_t|Gh, y)] \\
&+ (1-q)[p_{gu}\beta(m_t|Gu, y_h) + (1-p_{gu})G_h\beta(m_t|Gu, y)] \right\} \\
g(R_t = R_{hl}, m_t|G, p_G, \beta_G) &= (1-n) \left\{ q[p_{gl}\beta(m_t|Gl, y_h) + (1-p_{gl})\beta(m_t|Gl, y)] \\
&+ (1-q)[p_{gu}\beta(m_t|Gu, y_h) + (1-p_{gu})\beta(m_t|Gu, y)] \right\} \\
g(R_t = R_{lh}, m_t|G, p_G, \beta_G) &= n(1-G_h)[q(1-p_{gh})\beta(m_t|Gh, y)] \\
&+ (1-q)(1-p_{gu})\beta(m_t|Gu, y)] \\
g(R_t = R_{lh}, m_t|G, p_G, \beta_G) &= (1-n)(1-G_l)[q[p_{gl}\beta(m_t|Gl, y_h) \\
&+ (1-q)p_{gu}\beta(m_t|Gu, y_h)] \\
g(R_t = R_{hh}, m_t|B, p_B, \beta_B) &= n[p_{bh}\beta(m_t|B, y_h) + (1-p_B)G_h\beta(m_t|B, y)] \\
g(R_t = R_{hl}, m_t|B, p_B, \beta_B) &= (1-n)[p_{bl}\beta(m_t|B, y_h) + (1-p_B)\beta(m_t|B, y)] \\
g(R_t = R_{lh}, m_t|B, p_B, \beta_B) &= n(1-p_B)(1-G_h)\beta(m_t|B, y) \\
g(R_t = R_{hh}, m_t|B, p_B, \beta_B) &= (1-n)p_B(1-G_l)\beta(m_t|B, y) \\
\end{align*}
B.1.3 Choice of Optimal Disclosure Policy

As stated in the text, the optimal disclosure policy will depend on the manager’s type, project choice and the creditors’ anticipation of the equilibrium production strategy set, at that particular value of \( \alpha_0 \).

In the following we use the expressions for \( E_R [V_2^* (\tau, \alpha_1 (\alpha_0, R_1, m_1, p_1, \beta_1)) | \tau, y_1] \) given in the text for different \( \tau \). Recall that this quantity is the expectation of the net profits in period 2, taken at the time of disclosure choice in period 1.

1. Type \( G_h \) manager’s optimal disclosure choice:

   (i) Consider the \( G_h \) type’s feasible disclosure set when \( P_{Gh1} = 1 \). Since \( R_{hh} \) and \( \bar{R}_{hl} \) are not included in the type \( B \) manager’s feasible disclosure set,

   \[
   E_{R_1} [V_2^* (G_h, \alpha_1 (\alpha_0, R_{hh}, m_1 = \bar{R}_{hh}, p_1, \beta_1)) | G_h, y_h] = R_G - (1 - \bar{n} \bar{q}) r_G = E_{\text{max}}^G [V_2^*]
   \]

   which denotes the maximum possible period 2 expected profit. Note that \( r_G \) is the lowest possible interest requirement.

   However, since \( \bar{R}_{hh}, \bar{R}_{hl} \) and \( \emptyset \) are also included in the type \( B \) manager’s feasible disclosure set,

   \[
   E_{R_1} [V_2^* (G_h, \alpha_1 (\alpha_0, R_{hh}, m_1 = \bar{R}_{hh}, p_1, \beta_1)) | G_h, y_h] < E_{\text{max}}^G [V_2^*] \quad \text{if} \quad p_{B1} > 0 \]

   \[
   E_{R_1} [V_2^* (G_h, \alpha_1 (\alpha_0, R_{hh}, m_1 = \bar{R}_{hl}, p_1, \beta_1)) | G_h, y_h] < E_{\text{max}}^G [V_2^*] \quad \text{if} \quad p_{B1} > 0 \quad \text{and}
   \]

   \[
   E_{R_1} [V_2^* (G_h, \alpha_1 (\alpha_0, R_{hh}, m_1 = \emptyset, p_1, \beta_1)) | G_h, y_h] < E_{\text{max}}^G [V_2^*] \quad \forall \quad p_{B1}
   \]

   Therefore, to ensure the highest future payoff at every \( \alpha_0 \), irrespective of type \( B \)’s anticipated action, the type \( G_h \) manager’s optimal action is,

   \[
   m_1^* (G_h, \alpha_0, y_h) \in \{ R_{hh}, \bar{R}_{hl} \} \quad \forall \quad \alpha_0.
   \]

(ii) Consider the \( G_h \) type’s feasible disclosure set when \( P_{Gh1} = 0 \). Since \( \bar{R}_{hl} \) are not included in the type \( B \) manager’s feasible disclosure set,

   \[
   E_{R_1} [V_2^* (G_h, \alpha_1 (\alpha_0, \{ R_{hh}, R_{hl} \}, m_1 = \bar{R}_{hl}, p_1, \beta_1)) | G_h, y_h] = E_{\text{max}}^G [V_2^*]
   \]

   Since \( \bar{R}_{hl} \) and \( \emptyset \) are also included in the type \( B \) manager’s feasible disclosure set,

   \[
   E_{\text{max}}^G [V_2^*] > E_{R_1} [V_2^* (G_h, \alpha_1 (\alpha_0, \{ R_{hh}, R_{hl} \}, m_1 = \bar{R}_{hl}, p_1, \beta_1)) | G_h, y_h] \quad \text{if} \quad p_{B1} > 1 \quad \text{and}
   \]

   \[
   E_{\text{max}}^G [V_2^*] > E_{R_1} [V_2^* (G_h, \alpha_1 (\alpha_0, \{ R_{hh}, R_{hl} \}, m_1 = \emptyset, p_1, \beta_1)) | G_h, y_h] \quad \forall p_{B1}
   \]

184
Therefore, to ensure the highest future payoff at every $\alpha_o$, irrespective of type $B$’s anticipated action, the type $Gh$ manager’s optimal action is,

$$m^*_1(Gh, \alpha_o, y_l) = \bar{R}_{hl} \quad \forall \; \alpha_o.$$  

2. Type $Gl$ manager’s optimal disclosure choice:

Using arguments similar to those used for the type $Gh$ manager, it can be shown that

$$m^*_1(Gl, \alpha_o, y_l) \in \{R_{hl}, \bar{R}_{lh}\} \quad \text{and} \quad m^*_1(Gl, \alpha_o, y_h) = \bar{R}_{lh} \quad \forall \; \alpha_o.$$  

3. Type $Gu$ manager’s optimal disclosure choice:

Note that type $Gu$ and $B$ managers’ feasible disclosure sets are the same.

(i) Consider the type $Gu$ manager’s feasible disclosure set when $P_{gu1} = 1$.

$$E_{R^1_1}[V^*_2(Gu, \alpha_1(\alpha_o, \{R_{hh}, R_{ll}, R_{hl}\}, m_1 = \bar{R}_h, p_1, \beta_1))|Gu, y_h] = E^G_{max}[V^*_2] \quad \text{if} \; p_{B1} = 0,$$

whereas,

$$E_{R^1_1}[V^*_2(Gu, \alpha_1(\alpha_o, \{R_{hh}, R_{ll}, R_{hl}\}, m_1 = \emptyset, p_1, \beta_1))|Gu, y_h] < E^G_{max}[V^*_2] \quad \forall \; p_{B1}$$

Therefore, $m^*_1(Gu, \alpha_o, y_h) = \bar{R}_h$ if $p_{B1} = 0$.

In addition,

$$E_{R^1_1}[V^*_2(Gu, \alpha_1(\alpha_o, \{R_{hh}, R_{ll}, R_{hl}\}, m_1 = \bar{R}_l, p_1, \beta_1))|Gu, y_l] = E^G_{max}[V^*_2] \quad \text{if} \; p_{B1} = 1,$$

$$= E_{R^1_1}[V^*_2(Gu, \alpha_1(\alpha_o, \{R_{hh}, R_{ll}, R_{hl}\}, m_1 = \emptyset, p_1, \beta_1))|Gu, y_l] \quad \text{if} \; p_{B1} = 1$$

Therefore, $m^*_1(Gu, \alpha_o, y_h) \in \{\bar{R}_h, \emptyset\}$ if $p_{B1} = 1$

(ii) Consider the type $Gu$ manager’s feasible disclosure set when $P_{gu1} = 0$.

$$E_{R^1_1}[V^*_2(Gu, \alpha_1(\alpha_o, \{R_{hh}, R_{ll}, R_{hl}\}, m_1 = \bar{R}_l, p_1, \beta_1))|Gu, y_l] = E^G_{max}[V^*_2] \quad \text{if} \; p_{B1} = 1,$$

whereas,

$$E_{R^1_1}[V^*_2(Gu, \alpha_1(\alpha_o, \{R_{hh}, R_{ll}, R_{hl}\}, m_1 = \emptyset, p_1, \beta_1))|Gu, y_l] < E^G_{max}[V^*_2] \quad \forall p_{B1}$$

Therefore, $m^*_1(Gu, \alpha_o, y_l) = \bar{R}_l$ if $p_{B1} = 1$.

In addition,

$$E_{R^1_1}[V^*_2(Gu, \alpha_1(\alpha_o, \{R_{hh}, R_{ll}, R_{hl}\}, m_1 = \bar{R}_l, p_1, \beta_1))|Gu, y_l]$$

$$= E_{R^1_1}[V^*_2(Gu, \alpha_1(\alpha_o, \{R_{hh}, R_{ll}, R_{hl}\}, m_1 = \emptyset, p_1, \beta_1))|Gu, y_l] \quad \text{if} \; p_{B1} = 0$$

Therefore, $m^*_1(Gu, \alpha_o, y_h) \in \{\bar{R}_l, \emptyset\}$ if $p_{B1} = 0$.
4. type $B$ manager's optimal disclosure choice:

Consider the type $B$ manager's feasible disclosure sets and the appropriate expression for $E_{R_1[V_2^*(B, \alpha_1(\alpha_0, R_1, m_1, p_1, \beta_1))]|B, y_1]}$. Note that the type $B$ manager's maximum possible period 2 expected profit is:

$$E_{\text{max}}^B[V_2^*] = \tilde{R}_h - (1 - \tilde{n}G_{i})\tau_{G}$$

Applying the same arguments as the ones used for the type $Gu$ manager after replacing $E_{\text{max}}^G[V_2^*]$ with $E_{\text{max}}^G[V_2^*]$, we obtain,

$$m_1^*(B, \alpha_0, y_t) = \tilde{R}_h \quad \text{if} \quad p_{S_1} = 0$$
$$m_1^*(B, \alpha_0, y_t) \in \{\tilde{R}_h, \emptyset\} \quad \text{if} \quad p_{S_1} = 1$$
$$m_1^*(B, \alpha_0, y_t) = \tilde{R}_l \quad \text{if} \quad p_{S_1} = 1$$
$$m_1^*(B, \alpha_0, y_t) \in \{\tilde{R}_l, \emptyset\} \quad \text{if} \quad p_{S_1} = 0$$

Since in equilibrium, $p_{r_t} = P_{r_t}$ for $\forall \tau$, the equilibrium disclosure strategies shown in the text follow from the above.
B.2 Proofs

B.2.1 Proofs of Lemmas

Proof of Lemma 4.1  From assumption A1, \( r_f < r_{max} < L \). Therefore, both \( r_g \) and \( r_B \) must
be less than \( L \).

From expression (4.7), since \( r_g - \mu(r_g) > 0 \), \( r_g > r_f \). Similarly, from expression (4.8), since
\( r_B - \mu(r_B) > 0 \), \( r_B > r_f \). Hence,
\[
    r_f < r_g < L \quad \text{and} \quad r_f < r_B < L
\]

We next compare \( r_g \) and \( r_B \). Consider a general expression for the RHS's of (4.7) and (4.8).

\[
    \text{RHS} = r_t - x(r_t|y_h)[r_t - \mu(r_t)]
\]

Note that
\[
    \frac{d \text{RHS}}{dr_t} = 1 - xG(r_t|y_h) > 0
\]

In other words, RHS increases with \( r_t \) for a given \( x \).

(i) Consider \( p_{Bt} > q_{Pitt} + q_{P_{ut}} \)

Let \( r_t \leq r_B \), then since \( r_B \) must satisfy (4.8), it must be true that
\[
    r_t - \bar{n}[q_{P_{ut}} + \bar{q}_{P_{ut}}]G(r_t|y_h)[r_t - \mu(r_t)]
    \geq r_B - \bar{n}p_{Bt}G(r_B|y_h)[r_B - \mu(r_B)]
    = r_f
    \Rightarrow r_t - \bar{n}[q_{P_{it}} + \bar{q}_{P_{it}}]G(r_t|y_h)[r_t - \mu(r_t)] > r_f
\]

and since \( r_g \) must satisfy (4.7), \( r_g \neq r_t \) for any \( r_t \geq r_B \). Therefore, it follows that
\[
    r_f < r_g < r_B < L \quad \text{if} \quad p_{Bt} > q_{P_{it}} + \bar{q}_{P_{it}}
\]

(ii) Consider \( p_{Bt} < q_{P_{it}} + q_{P_{ut}} \)

Let \( r_t \leq r_B \), then since \( r_B \) satisfies (4.8) and
\[
    \frac{d \text{RHS}}{dr_t} > 0.
\]
it must be true that

\[ r_t - \bar{n}[q_{cut} + \bar{q}_{cut}]G(r_t|l_y_h)[r_t - \mu(r_t)] \]
\[ < r_t - \bar{n}p_{Bt}G(r_t|l_y_h)[r_t - \mu(r_t)] \]
\[ \leq r_B - \bar{n}p_{Bt}G(r_B|l_y_h)[r_B - \mu(r_B)] \]
\[ = r_f \]
\[ \Rightarrow r_t - \bar{n}[q_{cut} + \bar{q}_{cut}]G(r_t|l_y_h)[r_t - \mu(r_t)] < r_f \]

and since \( r_G \) must satisfy (4.7), \( r_G \neq r_t \) for any \( r_t \leq r_B \). Therefore, it follows that

\[ r_f < r_B < r_G < L \quad \text{if} \quad p_{Bt} < q_{cut} + \bar{q}_{cut} \]

(iii) Consider \( p_{Bt} = q_{cut} + \bar{q}_{cut} \)

In this case, if we substitute for \( p_{Bt} \) in (4.8), we get \( r_B \) satisfying (4.7). Therefore, \( r_B = r_G \). Hence,

\[ r_f < r_G = r_B < L \quad \text{if} \quad p_{Bt} = q_{cut} + \bar{q}_{cut} \]

Proof of Lemma 4.2 (i) By substituting \( q_{cut} = p_{cut} = p_{Bt} = 0 \) into (4.5), we get

\[ D_t(\alpha_{t-1}, r_t, p_t = (p_{gh_t}, 0, 0, 0)) = r_t - 0 \]

Equating this to \( r_f \) to obtain the required \( r_t \), we get

\[ r_t(\alpha_{t-1}, r_t, p_t = (p_{gh_t}, 0, 0, 0)) = r_f \]

(ii) Note that \( r_t \) is set by solving (4.6), i.e.

\[ r_f = r_t - \bar{n}\{\alpha_{t-1}[q_{cut} + \bar{q}_{cut}] + (1 - \alpha_{t-1})p_{Bt}\}G(r_t|l_y_h)[r_t - \mu(r_t)] \]

Note that by lemma 4.1.

\[ r_G \leq r_B \quad \text{if} \quad p_{Bt} \geq q_{cut} + \bar{q}_{cut} \quad \text{and} \quad r_G = r_B \quad \text{if} \quad p_{Bt} = q_{cut} + \bar{q}_{cut} \]
Since, $\alpha_{t-1} \in [0,1]$,

$$r_g \leq r_B \Rightarrow p_{Bt} \geq \alpha_{t-1}(q\gamma + \gamma_{cut}) + (1 - \alpha_{t-1})p_{Bt}$$
$$\geq q\gamma + \gamma_{cut} \quad \text{and}$$

$$r_g = r_B \Rightarrow p_{Bt} = \alpha_{t-1}(q\gamma + \gamma_{cut}) + (1 - \alpha_{t-1})p_{Bt}$$
$$= q\gamma + \gamma_{cut}$$

Since $r_t$ must satisfy (4.6), $r_g$ must satisfy (4.7) and $r_B$ must satisfy (4.8), we can show that

$$r_g \leq r_t \leq r_B \quad \text{or} \quad r_g = r_t = r_B$$

in the same manner as the proof of lemma 1.

(iii) Fully differentiating (4.6) with respect to $\alpha_{t-1}$

$$\frac{dr(t)}{d\alpha_{t-1}} = \frac{-\bar{n}(\alpha_{t-1}(q\gamma + \gamma_{cut}) + (1 - \alpha_{t-1})p_{Bt})G(r_t|l,y_h)[r_t - \mu(r_t)]}{1 - \bar{n}(\alpha_{t-1}(q\gamma + \gamma_{cut}) + (1 - \alpha_{t-1})p_{Bt})G(r_t|l,y_h)}$$

If $p_{Bt} > q\gamma + \gamma_{cut}$, then $\frac{dr(t)}{d\alpha_{t-1}} < 0$.

Whereas, if $p_{Bt} = q\gamma + \gamma_{cut}$, then $\frac{dr(t)}{d\alpha_{t-1}} = 0$.

**Proof of Lemma 4.3** The expression for $r_t$ is obtained by setting expression (4.5) to zero.

Given assumption A4 we obtain the following simplifications for the terms $\mu(r_t)$ and $G(r_t|l,y_h)$ in (4.5).

$$\mu(r_t) = R_{lh} \quad \text{and} \quad G(r_t|l,y_h) = (1 - G_l)$$

Substitution for these into (4.5) yields

$$D(\alpha_{t-1}, p_t, r_t) = r_t - \bar{n}\{\alpha_{t-1}x_{lt} + (1 - \alpha_{t-1})p_{Bt}\}[r_t - R_{lh}](1 - G_l)$$

Setting this to $r_f$ and solving yields

$$r_t = R_{lh} + \frac{r_f - R_{lh}}{\phi_t(\alpha_{t-1}, p_t)}$$

where $\phi_t(\alpha_{t-1}, p_t) = [1 - \bar{n}\tilde{G}_t x_{lt}]\alpha_{t-1} + [1 - \bar{n}\tilde{G}_t p_{Bt}](1 - \alpha_{t-1})$.
B.2.2 Proofs of Propositions

Proof of Proposition 4.2 The conditions for the existence of each of the feasible pure strategy sets in equilibrium are:

\[ p_1 = p^1 : \Delta V_2^{Gh}(\alpha_0, p^1) - \Delta \pi_1^{Gh}(\alpha_0, p^1) < 0 \quad \text{and} \]
\[ \Delta V_2^T(\alpha_0, p^1) - \Delta \pi_1^T(\alpha_0, p^1) > 0 \quad \text{for } \tau \in \{G_l, G_u, B\} \]
\[ p_1 = p^2 : \Delta V_2^{Gl}(\alpha_0, p^1) - \Delta \pi_1^{Gl}(\alpha_0, p^1) > 0 \quad \text{and} \]
\[ \Delta V_2^T(\alpha_0, p^2) - \Delta \pi_1^T(\alpha_0, p^2) < 0 \quad \text{for } \tau \in \{G_h, G_u, B\} \]
\[ p_1 = p^3 : \Delta V_2^G(\alpha_0, p^3) - \Delta \pi_1^G(\alpha_0, p^3) < 0 \quad \text{and} \]
\[ \Delta V_2^T(\alpha_0, p^3) - \Delta \pi_1^T(\alpha_0, p^3) > 0 \quad \text{for } \tau \in \{G_l, G_u\} \]
\[ p_1 = p^4 : \Delta V_2^T(\alpha_0, p^4) - \Delta \pi_1^T(\alpha_0, p^4) < 0 \quad \text{for } \tau \in \{G_h, B\} \quad \text{and} \]
\[ \Delta V_2^T(\alpha_0, p^4) - \Delta \pi_1^T(\alpha_0, p^4) > 0 \quad \text{for } \tau \in \{G_l, G_u\} \]

Given the above necessary conditions, this proof identifies conditions under which the above relations hold for some \( \alpha_0 \in [0, 1] \), i.e., the sets \( A^1, A^2, A^3 \) and \( A^4 \) are non-empty.

Prior to a partial characterization of \( A^1, A^2, A^3 \) and \( A^4 \) we rewrite certain key relationships between the \( \Delta V_2^T \) and \( \Delta \pi_1^T \) terms from the text. Given \( p^2, p^3 \) and \( p^4 \), the following hold for all \( \alpha_0 \) and all parameter values.

\[ \Delta V_2^{Gl}(\alpha_0, p_1) > 0 > \Delta \pi_1^{Gl}(\alpha_0, p_1) \Rightarrow \Delta V_2^{Gl}(\alpha_0, p_1) - \Delta \pi_1^{Gl}(\alpha_0, p_1) > 0 \quad \text{(B.P.4.2-1)} \]

Since \( \Delta V_2^{Gh} = n\Delta V_2^{Gh} + \bar{n}\Delta V_2^{Gl} \) and \( \Delta \pi_1^{Gu} = n\Delta \pi_1^{Gh} + \bar{n}\Delta \pi_1^{Gl} \), (B.P.4.2-1) implies that

\[ \Delta V_2^{Gh}(\alpha_0, p_1) - \Delta \pi_1^{Gh}(\alpha_0, p_1) > 0 \Rightarrow \Delta V_2^{Gu}(\alpha_0, p_1) - \Delta \pi_1^{Gu}(\alpha_0, p_1) > 0 \quad \text{(B.P.4.2-2)} \]
\[ \Delta V_2^{Gu}(\alpha_0, p_1) - \Delta \pi_1^{Gu}(\alpha_0, p_1) < 0 \Rightarrow \Delta V_2^{Gh}(\alpha_0, p_1) - \Delta \pi_1^{Gh}(\alpha_0, p_1) < 0 \quad \text{(B.P.4.2-3)} \]

Note that (B.P.4.2-2) and (B.P.4.2-3) imply that the following is always true.

\[ \alpha^2 < \alpha^4, \quad \alpha^5 < \alpha^9 \quad \text{and} \quad \alpha^{10} < \alpha^{14} \quad \text{(B.P.4.2-4)} \]

[Refer to figures 4.2(b)-(d) and 4.3(b)-(d).]

Since \( \Delta V_2^G(\alpha_0, p_1) = \frac{1 - \bar{n}\pi_1^{Gh}}{1 - \bar{n}\pi_1^{Gl}} \cdot \Delta V_2^{Gu}(\alpha_0, p_1) \) and \( \Delta \pi_1^G(\alpha_0, p_1) = \Delta \pi_1^{Gu}(\alpha_0, p_1) \),

\[ \Delta V_2^{Gu}(\alpha_0, p_1) - \Delta \pi_1^{Gu}(\alpha_0, p_1) < 0 \Rightarrow \Delta V_2^G(\alpha_0, p_1) - \Delta \pi_1^G(\alpha_0, p_1) < 0 \quad \text{(B.P.4.2-5)} \]
\[ \Delta V_2^G(\alpha_0, p_1) - \Delta \pi_1^G(\alpha_0, p_1) > 0 \Rightarrow \Delta V_2^{Gu}(\alpha_0, p_1) - \Delta \pi_1^{Gu}(\alpha_0, p_1) > 0 \quad \text{(B.P.4.2-6)} \]
From the shapes of $\Delta \pi_1^*$ and $\Delta V_2^*$ functions in figures 4.2 and 4.3, the general forms of the $A^1$, $A^2$, $A^3$ and $A^4$ sets, if they exist, are:

$$A^1 = [\alpha^1, 1]$$

$$A^2 = \begin{cases} 
([\alpha^2, \alpha^3], [\alpha^4, 1]) & \text{if } \alpha^2 < \alpha^3 \\
[\alpha^4, 1] & \text{if } \alpha^3 > \alpha^2 
\end{cases}$$

$$A^3 = \begin{cases} 
[\alpha^5, \min\{\alpha^5, \alpha^7\}] & \text{if } \alpha^5 < \alpha^8 \\
[\alpha^8, \alpha^5] & \text{if } \alpha^5 > \alpha^8 
\end{cases}$$

$$A^4 = \begin{cases} 
[\min\{\alpha^{10}, \alpha^{11}\}, \alpha^{12}[\alpha^{13}, \alpha^{14}]] & \text{if } \alpha^{10} < \alpha^{12} \\
[\max\{\alpha^{10}, \alpha^{13}\}, \alpha^{14}] & \text{if } \alpha^{10} > \alpha^{12} 
\end{cases}$$

Sufficient conditions that ensure that the above sets are non-empty are listed below.

(a) $A^3 \neq \emptyset$ if $\alpha^5 > \alpha^6$ and $\alpha^7 > 0$.

- $\alpha^5 > \alpha^6$ if and only if, (B.P.4.2-2) holds at $\alpha_o$, i.e.,
  $$R_{hh} - R_{hl} < \frac{q}{1 - nG_l} \cdot N \Rightarrow \bar{R}_h - \bar{R}_l < -\frac{1 - qn\bar{G}_h}{1 - nG_l} \cdot N \quad \text{(B.P.4.2-7)}$$

  Note that this also implies that $\alpha^6 = 0$

- $\alpha^7 > 0$ if and only if, (B.P.4.2-6) does not hold at $\alpha_o = 0$, i.e.,
  $$\bar{R}_h - \bar{R}_l > \left\{ \frac{1 - qn\bar{G}_h}{1 - nG_l} + \frac{qnG_l}{(1 - nG_l)(1 - nG_lq)} \right\} \quad \text{(B.P.4.2-8)}$$

  Hence, (B.P.4.2-7) and (B.P.4.2-8) ensure that $A^3 \neq \emptyset$.

(b) $A^4 \neq \emptyset$ if $\alpha^{10} < \alpha^{14}$ and

- $\alpha^{10} < \alpha^{14}$ is always true by (B.P.4.2-4).

- $\alpha^{13} < 1$ if and only if, (B.P.4.2-6) does not hold at $\alpha_o = 1$, i.e.,
  $$\bar{R}_h - \bar{R}_l > -\frac{1 - n\bar{G}_l}{1 - nG_lq} \cdot N \quad \text{(B.P.4.2-9)}$$

- $\alpha^{14} > \alpha^{11}$ if and only if,
  $$\max_{\alpha_o} \left\{ \Delta V_2^u(\alpha_o, p^4) - \Delta \pi_1^u(\alpha_o, p^4) \right\} > 0 \quad \text{(B.P.4.2-10)}$$

(c) $A^1 \neq \emptyset$ if, and only if, $\alpha^1 < 1$ which holds if, and only if, (B.P.4.2-6) holds at $\alpha_o = 1$, i.e.,
  $$\bar{R}_h - \bar{R}_l < -\frac{1 - n\bar{G}_l}{1 - nG_lq} \cdot N \quad \text{(B.P.4.2-11)}$$
(d) $A^2 \neq \emptyset$ if $\alpha^3 > \alpha^2$ or $\alpha^2, \alpha^4 < 1$

- $\alpha^4 < 1$ is always true due to the fact that $\Delta V_2^{q_u}(\alpha_o = 1, p^2) = 0 < \Delta r_1^{q_u}(\alpha_o = 1, p^2)$
- $\alpha^2 < \alpha^4$ is always true by (B.P.4.2-4)
- $\alpha^3 > \alpha^2$ if, and only if, (B.P.4.2-2) does not hold at $\alpha_o = 1$, i.e.,

$$R_{hh} - R_{hl} > \frac{q}{1 - \bar{n}G_{l}} \cdot N$$

(B.P.4.2-12)

Note that if (B.P.4.2-12) holds then $\alpha^2 = 0$ and $\alpha^3 > 0$.

From the above conditions for existence we can determine which sets among $A^1, A^2, A^3$ and $A^4$ are non-empty in each of the ranges for $\bar{R}_h - \bar{R}_l$ given in proposition 4.2. The ranges considered are:

$$0 > \frac{1 - \bar{n}G_{l}(1 + q) - qnG_{h}}{1 - \bar{n}G_{l}} \cdot N > \frac{1 - qnG_{h}}{1 - \bar{n}G_{l}} \cdot N > -N$$

Note that $\frac{1 - qnG_{h}}{1 - \bar{n}G_{l}} \cdot N > -N$ if, and only if, $qnG_{h} > \bar{n}G_{l}$.

If $qnG_{h} < \bar{n}G_{l}$, then the relevant ranges include I and part of II. The interval implied by III is ruled out by assumption A3.

I $\bar{R}_h - \bar{R}_l > -X \cdot N$ where $X = \frac{1 - \bar{n}G_{l}(1 + q) - qnG_{h}}{1 - \bar{n}G_{l}}$

(a) Since $X < \frac{1 - qnG_{h}}{1 - \bar{n}G_{l}}$, condition B.P.4.2-7 is violated and, hence, $\alpha^5 < \alpha^6$.

Therefore, $A^3 = \emptyset$.

(b) Note that I implies that

$$\bar{R}_h - \bar{R}_l > \left\{1 - \frac{q(nG_{l} + nG_{h})}{1 - \bar{n}G_{l}}\right\} \cdot N$$

$$\Rightarrow \bar{R}_h - \bar{R}_l + N > \frac{q(nG_{l} + nG_{h})}{1 - \bar{n}G_{l}} \cdot N$$

The RHS of the above inequality can be written as

$$(1 - \bar{n}G_{l}q)(nG_{l} + nG_{h})[r_{G}(p_{5}) - r_{G}(p_{2})]$$

which is an upper bound for all $\Delta V_2^{q_u}$ terms. Note that $r_{G}(p_{5})$ and $r_{G}(p_{2})$ can be determined by substituting $p_{t} = p_{5} = (1, 0, 1, 1)$ into expressions (4.25) and (4.24), respectively. The LHS, on the other hand provides a lower bound for all $\Delta \pi_1^{q_u}$ terms. Therefore I implies that

$$\Delta V_2^{q_u}(\alpha_o, p_{1}) < \Delta \pi_1^{q_u}(\alpha_o, p_{1}) \ \forall \ \alpha_o, p_{1}$$

(B.P.4.2-13)

This implies that (B.P.4.2-10) is violated and $\alpha^{14} = \alpha^{13}$.

Therefore, $A^4 = \emptyset$.
(c) Since, \( X < \frac{1-nG_i}{1-nG_{i+1}} \), it implies that (B.P.4.2-11) is violated. Therefore, \( A^i = \emptyset \).

(d) If \( I \) holds (B.P.4.2-12) is violated, implying that (B.P.4.2-2) holds. Hence, \( \alpha^3 > \alpha^2 \Rightarrow \alpha^2 = 0 \) and \( \alpha^3 > 0 \). In addition, due to (B.P.4.2-13) \( \alpha^3 = \alpha^4 \).

Therefore, \( A^2 = [0,1] \).

\[ II \ -X.N > \bar{R}_h - \bar{R}_l = -Y.N \text{ where } Y = \frac{1-nG_i}{1-nG_{i+1}} \]

(a) Since (B.P.4.2-7) is violated if \( II \) holds, \( \alpha^5 < \alpha^6 \).

Therefore, \( A^3 = \emptyset \).

(b) Under \( II \) \( \Delta n_2^G(\alpha_o, p^4) - \Delta V_2^G(\alpha_o = 0, p^4) > 0 \) implying that \( \alpha^{11} > 0 \).

Therefore, \( A^4 = \emptyset \) if (B.P.4.2-10) holds.

(c) Condition (B.P.4.2-11) is not ruled out by \( II \).

Therefore, \( A^1 \neq \emptyset \) if (B.P.4.2-11) holds.

(d) Condition \( II \) implies that (B.P.4.2-12) holds [implied by (B.P.4.2-3)] and, hence, \( \alpha^3 > \alpha^2 \). In addition, \( II \) also implies that (B.P.4.2-13) does not hold and, hence, \( \alpha^3 < \alpha^1 \).

Therefore, \( A^2 = \{[0, \alpha^3], [\alpha^4, 1]\} \).

\[ III \ -Y.N > \bar{R}_h - \bar{R}_l = -N. \]

(a) Condition (B.P.4.2-7) is implied by \( III \) and, hence, \( \alpha^5 > \alpha^6 \). (B.P.4.2-8) holds over the entire interval given by \( III \) if

\[
-\left\{ \frac{1-qn\tilde{G}_h}{1-n\tilde{G}_i} + \frac{qn\tilde{G}_l}{(1-n\tilde{G}_l)(1-n\tilde{G}_i/q)} \right\} \cdot N > -N
\]

(B.P.4.2-8) holds over the higher part of the interval given by \( III \) if

\[
-\frac{1-qn\tilde{G}_h}{1-n\tilde{G}_i} \cdot N > -\left\{ \frac{1-qn\tilde{G}_h}{1-n\tilde{G}_i/q} + \frac{qn\tilde{G}_l}{(1-n\tilde{G}_l)(1-n\tilde{G}_i/q)} \right\} \cdot N
\Rightarrow (1-qn\tilde{G}_h) < 1
\]

Since this is always true, \( \alpha^7 > 0 \) in this interval.

Therefore, \( A^3 \neq \emptyset \) if

\[
-\frac{1-qn\tilde{G}_h}{1-n\tilde{G}_i} \cdot N > \bar{R}_h - \bar{R}_l > -\left\{ \frac{1-qn\tilde{G}_h}{1-n\tilde{G}_i/q} + \frac{qn\tilde{G}_l}{(1-n\tilde{G}_l)(1-n\tilde{G}_i/q)} \right\} \cdot N
\]
(b) Since condition III implies that \( \alpha^{11} = 0 \), a sufficient condition for \( A^4 \) to be non-empty is \( \alpha^{13} < 1 \), i.e., that (B.P.4.2-9) holds. However, (B.P.4.2-9) does not hold over the entire interval specified by III since, \(-\frac{1 - n\tilde{G}_l}{1 - n\tilde{G}_l} \cdot N > -N\). However, (B.P.4.2-9) may hold over the higher part of the interval.

Therefore, \( A^4 \neq \emptyset \) if \( \frac{1 - n\tilde{G}_l}{1 - n\tilde{G}_l} > \frac{1 - n\tilde{G}_l}{1 - n\tilde{G}_l} \).

(c) Condition (B.P.4.2-11) is the reverse of (B.P.4.2-9), and hence, (B.P.4.2-11) holds over the lower part of the interval specified by III.

Therefore, \( A^1 \neq \emptyset \) if \( -N < \bar{R}_h - \bar{R}_l < -\frac{1 - n\tilde{G}_l}{1 - n\tilde{G}_l} \).

(d) Condition (B.P.4.2-12) is violated by III, implying that \( \alpha^3 \leq \alpha^2 \).

Therefore, \( A^2 = [\alpha^4, 1] \).

**Proof of Proposition 4.3** We compare the value of \( \Delta V_2^{p^u} \) with \( \Delta \pi_1^{p^u} \) and \( \Delta V_2^p \) with \( \Delta \pi_1^p \) for each different strategy set. If \( \Delta V_2^{p^u} - \Delta \pi_1^{p^u} > 0 \) for some value of \( \alpha_0 \), then type \( G \) prefers the safe project at \( \alpha_0 \). Similarly, if \( \Delta V_2^p - \Delta \pi_1^p > 0 \) for some value of \( \alpha_0 \), then type \( B \) prefers the safe project at \( \alpha_0 \).

**Case 1:** \( p_1 = (1, 0, 0, 0) \equiv p^1 \)

It can be seen from figure 4.6(a) that \( p^1 \) is consistent with an equilibrium if \( \alpha_0 \in [\tilde{\alpha}^1, 1] \), where \( \tilde{\alpha}^1 < 1 \) if, and only if,

\[
\Delta V_2^B(\alpha_0 = 1, p^1, p^2_2) > \Delta \pi_1^B(\alpha_0 = 1, p^1).
\]

Note that

\[
\Delta V_2^B(\alpha_0, p^1, p^2_2) = (1 - \tilde{n}\tilde{G}_l)(1 - \tilde{n}\tilde{G}_l)r_B(p^2_2) + \tilde{n}\tilde{G}_l r_B(p^2_2) - \bar{r} \quad \text{(B.P.4.3-1)}
\]

\[
\Delta \pi_1^B(\alpha_0, p^1) = \bar{R}_h - \bar{R}_l + \tilde{n}\tilde{G}_l(r_f - R_{lh}) \quad \text{(B.P.4.3-2)}
\]

where \( \bar{r}(p^2_2) = \bar{R}_{lh} + \frac{r_f - R_{lh}}{(1 - n\tilde{G}_l)} (1 - \alpha_1) \) and \( \alpha_1 = \frac{q}{\theta_{\alpha_0} + (1 - \alpha_0)} \). Substituting for \( r_C \) and \( r_B \) from (4.24) and (4.25), respectively, the necessary and sufficient condition, above, becomes:

\[
\tilde{\alpha}^1 < 1 \quad \text{if, and only if,} \quad \bar{R}_h - \bar{R}_l < \frac{(1 - \tilde{n}\tilde{G}_l)}{1 - n\tilde{G}_l} \cdot \tilde{n}\tilde{G}_l(r_f - R_{lh}) \quad \text{(B.P.4.3-3)}
\]

**Case 2:** \( p_1 = (1, 0, 1, 1) \equiv p^2 \)
Figure 4.6(b) indicates that $p^2$ is consistent with an equilibrium if $\alpha_0 \in [\hat{\alpha}^2, 1]$, where $\hat{\alpha}^2 > 0$ if, and only if,

$$\Delta V^G_2(\alpha_0 = 0, p^2, p^2_2) > \Delta \pi^G_1(\alpha_0 = 0, p^2).$$

Note that

$$\Delta V^G_2(\alpha_0, p^2, p^2_2) = (1 - \bar{n}G_l \bar{q}) \left\{ \hat{r}(p^2_2) - r_g(p^2_2) \right\}$$

$$\Delta \pi^G_1(\alpha_0, p^2) = \bar{R}_h - \bar{R}_l + \frac{n \bar{G}_l (r_f - R_{1h})}{(1 - \bar{n}G_l)\alpha_0 + (1 - \bar{n}G_l)(1 - \alpha_o)}$$

where $\hat{r}(p^2_2)$ is the same as in case 1. Substituting for $r_g$ and $r_S$ from (4.24) and (4.25), respectively, and for $\hat{r}(p^2_2)$ the above necessary and sufficient condition becomes:

$$\hat{\alpha}^2 > 0 \text{ if, and only if, } \bar{R}_h - \bar{R}_l < -\frac{\bar{q}}{1 - \bar{n}G_l} \cdot \bar{n}G_l (r_f - R_{1h})$$

Note that $\hat{\alpha}^2 < 1$ is always true.

Case 4: $p_1 = (1, 0.0, 1) \equiv p^4$

From figure 4.6(c), $p^4$ is consistent with an equilibrium if $\alpha_0 \in [\hat{\alpha}^3, \hat{\alpha}^4]$, where $\hat{\alpha}^4 > 0$ if, and only if,

$$\Delta V^G_2(\alpha_0 = 0, p^4, p^2_2) < \Delta \pi^G_1(\alpha_0 = 0, p^4),$$

$\hat{\alpha}^3 < 1$ if, and only if,

$$\Delta V^G_2(\alpha_0 = 1, p^4, p^2_2) > \Delta \pi^G_1(\alpha_0 = 1, p^4),$$

$\hat{\alpha}^3 > 0$ if, and only if,

$$\Delta V^G_2(\alpha_0 = 0, p^4, p^2_2) < \Delta \pi^G_1(\alpha_0 = 0, p^4),$$

and $\hat{\alpha}^4 < 1$ if, and only if,

$$\Delta V^G_2(\alpha_0 = 1, p^4, p^2_2) > \Delta \pi^G_1(\alpha_0 = 1, p^4).$$

Note that

$$\Delta V^G_2(\alpha_0, p^4, p^2_2) = (1 - \bar{n}G_l \bar{q}) \left\{ r_S(p^2_2) - r_g(p^2_2) \right\}$$

$$\Delta V^B_2(\alpha_0, p^4, p^2_2) = (1 - \bar{n}G_l) \left\{ r_S(p^2_2) - r_g(p^2_2) \right\}$$

$$\Delta \pi^G_1(\alpha_0, p^4) = \bar{R}_h - \bar{R}_l + \frac{n \bar{G}_l (r_f - R_{1h})}{\alpha_0 + (1 - \bar{n}G_l)(1 - \alpha_o)}$$

$$\Delta \pi^B_1(\alpha_0, p^4) = \Delta \pi^G_1(\alpha_0, p^4)$$
Substituting for \( r_g \) and \( r_s \) from (4.24) and (4.25), respectively, the above necessary and sufficient conditions become:

\[
\hat{\alpha}^4 > 0 \quad \text{if, and only if,} \quad \tilde{R}_h - \tilde{R}_l > -\left( \frac{\bar{q}}{1 - \bar{n}G_i} + \frac{q\bar{n}G_i}{(1 - \bar{n}G_i)(1 - \bar{n}G_i\bar{q})} \right) \quad \text{(B.P.4.3-11)}
\]

\[
\hat{\alpha}^3 < 1 \quad \text{if, and only if,} \quad \tilde{R}_h - \tilde{R}_l < -\frac{1 - \bar{n}G_i - q}{1 - \bar{n}G_i} \cdot \bar{n}G_i(r_f - R_{lh}) \quad \text{(B.P.4.3-12)}
\]

\[
\hat{\alpha}^3 > 0 \quad \text{if, and only if,} \quad \tilde{R}_h - \tilde{R}_l > -\frac{\bar{q}}{1 - \bar{n}G_i} \cdot \bar{n}G_i(r_f - R_{lh}) \quad \text{(B.P.4.3-13)}
\]

\[
\hat{\alpha}^4 < 1 \quad \text{if, and only if,} \quad \tilde{R}_h - \tilde{R}_l < -\frac{q(1 - \bar{n}G_i)}{1 - \bar{n}G_i\bar{q}} \cdot \bar{n}G_i(r_f - R_{lh}) \quad \text{(B.P.4.3-14)}
\]

\[
\hat{\alpha}^4 < 1 \quad \text{if, and only if,} \quad \tilde{R}_h - \tilde{R}_l < -\frac{q(1 - \bar{n}G_i)}{1 - \bar{n}G_i\bar{q}} \cdot \bar{n}G_i(r_f - R_{lh}) \quad \text{(B.P.4.3-15)}
\]

Note that \( \hat{\alpha}^3 < \hat{\alpha}^4 \) always holds and, hence, sufficient conditions for the existence of \( p^4 \) in equilibrium are \( \hat{\alpha}^3 < 1 \) and \( \hat{\alpha}^4 > 0 \) which require (B.P.4.3-11) and (B.P.4.3-12) to hold, respectively.

We next determine which among the sets \( \hat{A}^1, \hat{A}^2 \) and \( \hat{A}^4 \) are non-empty within each of the ranges for \( \tilde{R}_h - \tilde{R}_l \) given in proposition 4.3. Recall that \( N = \bar{n}G_i(r_f - R_{lh}) \)

\[
\tilde{R}_h - \tilde{R}_l > \frac{1 - \bar{n}G_i - q}{1 - \bar{n}G_i} \cdot N
\]

(a) (B.P.4.3-3) is violated by I implying that \( \hat{\alpha}^1 = 1 \).

Therefore, \( \hat{A}^1 = \emptyset \).

(b) (B.P.4.3-6) is violated by I implying that \( \hat{\alpha}^2 = 0 \).

Therefore, \( \hat{A}^2 = [0, 1] \).

(c) (B.P.4.3-12) is violated by I implying that \( \hat{\alpha}^3 = 1 \).

Therefore, \( \hat{A}^4 = \emptyset \).

\[
(II) \quad \frac{1 - \bar{n}G_i - q}{1 - \bar{n}G_i} \cdot N > \tilde{R}_h - \tilde{R}_l > \frac{1 - \bar{n}G_i(1 + q)}{1 - \bar{n}G_i} \cdot N > -\frac{1 - \bar{n}G_i}{1 - \bar{n}G_i\bar{q}} \cdot N
\]

(a) (B.P.4.3-3) is violated over the entire interval given by I. This implies that \( \hat{\alpha}^1 = 1 \).

Therefore, \( \hat{A}^1 = \emptyset \).

(b) Since \( \frac{1 - \bar{n}G_i - q}{1 - \bar{n}G_i} \cdot N > -\frac{q}{1 - \bar{n}G_i} \), (B.P.4.3-6) is violated in this interval resulting in \( \hat{\alpha}^2 = 0 \). However, if \( -\frac{1 - \bar{n}G_i(1 + q)}{1 - \bar{n}G_i} < \frac{q}{1 - \bar{n}G_i} \), then (B.P.4.3-6) is not violated by II in that interval implying that \( \hat{\alpha}^2 > 0 \).

Therefore.

\[
\hat{A}^2 = \begin{cases} 
[0, 1] & \text{if } -\frac{1 - \bar{n}G_i - q}{1 - \bar{n}G_i} \cdot N > \tilde{R}_h - \tilde{R}_l > -\frac{q}{1 - \bar{n}G_i} \\
[\hat{\alpha}^2, 1] & \text{if } -\frac{q}{1 - \bar{n}G_i} > \tilde{R}_h - \tilde{R}_l > -\frac{1 - \bar{n}G_i(1 + q)}{1 - \bar{n}G_i} \cdot N
\end{cases}
\]
(c) (B.P.4.3-12) is the same as the upper inequality of II. This implies that $\hat{\alpha}^3 < 1$. In addition, (B.P.4.3-11) is not ruled out by II implying that $\hat{\alpha}^4 \geq 0$.

Therefore, $\hat{A}^4 = [\hat{\alpha}^3, \hat{\alpha}^4]$.

(III) $-\frac{1-\hat{n}\hat{G}_t}{1-\hat{n}\hat{G}_i} \cdot N > \hat{R}_h - \hat{R}_l > -N$

(a) (B.P.4.3-1) is violated by III if $-\left\{\frac{-\hat{q}}{1-\hat{n}\hat{G}_i} + \frac{\hat{n}\hat{G}_i}{(1-\hat{n}\hat{G}_i)(1-\hat{n}\hat{G}_i)}\right\} \cdot N > -\frac{1-\hat{n}\hat{G}_i}{1-\hat{n}\hat{G}_i} \cdot N$ implying that $\hat{\alpha}^4 = 0$ in that interval, i.e., $\hat{A}^4 = \emptyset$. If this inequality holds in reverse then $\hat{A}^4 = \emptyset$ in the upper part and $\hat{A}^4 \neq \emptyset$ in the lower part of the interval.

Since $\frac{1-\hat{n}\hat{G}_i}{1-\hat{n}\hat{G}_i} > -\frac{1-\hat{n}\hat{G}_i}{1-\hat{n}\hat{G}_i} \cdot N$, $\hat{\alpha}^3 < 1$.

Therefore,

$$\hat{A}^4 = \begin{cases} \emptyset & \text{if } -\left\{\frac{-\hat{q}}{1-\hat{n}\hat{G}_i} + \frac{\hat{n}\hat{G}_i}{(1-\hat{n}\hat{G}_i)(1-\hat{n}\hat{G}_i)}\right\} \cdot N < -\frac{1-\hat{n}\hat{G}_i}{1-\hat{n}\hat{G}_i} \cdot N \\ [\hat{\alpha}^3, \hat{\alpha}^4] & \text{otherwise.} \end{cases}$$

(b) (B.P.4.3-6) holds if $-\frac{-\hat{q}}{1-\hat{n}\hat{G}_i} \cdot N > -\frac{1-\hat{n}\hat{G}_i}{1-\hat{n}\hat{G}_i} \cdot N$ implying $\hat{\alpha}^2 > 0$. Therefore,

$$\hat{A}^2 = \begin{cases} [\hat{\alpha}^2, 1] & \text{if } -\frac{-\hat{q}}{1-\hat{n}\hat{G}_i} \cdot N > -\frac{1-\hat{n}\hat{G}_i}{1-\hat{n}\hat{G}_i} \cdot N \\ [0, 1] & \text{if } -\frac{1-\hat{n}\hat{G}_i}{1-\hat{n}\hat{G}_i} \cdot N > \hat{R}_h - \hat{R}_l > -\frac{-\hat{q}}{1-\hat{n}\hat{G}_i} \cdot N \end{cases}$$

(c) (B.P.4.3-3) is the consistent with III implying that $\hat{\alpha}^1 < 1$ over the entire interval.

Therefore, $\hat{A}^1 = [\hat{\alpha}^1, 1]$. 

197