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Date July 14, 1993
ABSTRACT

This dissertation is a collection of three essays on strategic trade policy. The main purpose of this research is to make contributions to the strategic trade literature by dealing with some of its interesting and important issues.

The first essay extends the well-known Brander-Spencer (1985) model, by considering market uncertainty, endogenizing firms' choice of strategic variables, and introducing a quadratic export tax/subsidy scheme; and investigates the interrelationship between trade policies and market competition. It is shown that firms are not indifferent between setting prices and setting quantities in an uncertain environment. The quadratic export tax/subsidy scheme is proved superior to the linear export tax/subsidy scheme because the former has the unique ability to influence the domestic firm's choice of strategic variables and therefore to affect the market conduct in favour of the domestic country.

The second essay explores an incomplete information version of the Brander-Spencer (1985) model, in which the domestic firm's production cost is private information. This model has the distinguishing feature that it contains a mixture of screening and signalling problems. Because of this, policy makers are confronted by twin conflicting policy objectives: choosing between an information revealing policy menu and an information concealing uniform policy. We prove that the policy menu is preferred to the uniform policy in quantity competition while the opposite is true in price competition.
In essay three, we examine the implications of credit rationing for international trade. Development and production of new products often involves uncertainty (e.g. R&D uncertainty and demand uncertainty), which could lead to credit rationing. We find that the presence of credit rationing may give partial explanation for the indeterminacy of the pattern of trade between two similar countries. When credit is rationed, two-way trade is less likely to occur.
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OVERVIEW

It had been traditionally argued and widely accepted that under almost all situations free trade was the best trade policy a country could have. In fact, we have witnessed a gradual move of the world economy towards free trade since the introduction of the Bretton Wood system in 1944. This movement is mainly characterized by tremendous tariff reductions around the world, especially in developed countries. However, the world is still far from free trade. Use of non-tariff barriers has been significantly increased and other types of government intervention in international trade such as export subsidies are frequently found. It can hardly been denied that the rapid growth of the newly industrial countries (Korea, for example) can largely be attributed to these nations’ industrial and trade policies such as export promotion and import restriction. We can often hear complaints from American producers that they are facing unfair competition from their Japanese counterparts since the Japanese producers are helped directly by their government. If free trade is optimal, however, what is the rationale for government interventions in those economically successful countries? In the attempt to answer questions of this sort, we in the 1980s have witnessed a revolution in international trade theory: the emergence and development of the so-called new trade theory. Among those theories, the theory of strategic trade policies, pioneered by Jim Brander and Barbara Spencer, has received the greatest attention both in academic circle and in business. This theory, which is built

\[^1\text{They are now associated with U.B.C.. Naturally and fortunately, my work has benefitted a great deal from this local contact.}\]
upon the theory of industrial organization (especially oligopoly theory), was initially inspired by the observation that international market competition in many industries is indeed imperfect.\(^2\) The general idea underlying the strategic trade literature is that firms earn positive rents under imperfect competition and thus *appropriate* trade policies adopted by a government may affect the behaviour of those competing firms in such a way to shift some of the rents otherwise earned by the foreign firm to the domestic firm. Although it is elegant and powerful, the theory has been subject to many criticisms. As Harris (1989, p753) points out, “the basic problem as it turns out is that there are many models of imperfect competition, and answers to particular policy issues are quite sensitive to assumptions as to how markets work and the manner in which government intervenes in these markets.” To make the theory useful for policy prescriptions, further and rigorous studies are both desirable and valuable. In addressing and solving some of the existing problems, this thesis makes new contributions to this growing literature.

This thesis consists of three essays. Appropriate strategic policy design in various circumstances is the main concern of the first two essays (Chapters 1 and 2). The third essay (Chapter 3) turns to a different but also important issue. It examines the implications of credit market imperfection for the pattern of trade.

In Chapter 1, which is entitled “Quadratic Export Subsidy Scheme and Market Conduct”, we extend the well known Brander-Spencer (1985) model by introducing market uncertainty, endogenizing firm’s choice of strategic variables, and taking non-

\(^2\)In contrast, perfect competition is one of the basic assumptions in traditional trade theory.
linear export policies into consideration. In the strategic trade literature, all studies with only one exception (Laussel (1992)) have one thing in common: assuming that the type of market competition (or market conduct) is exogenously given. However, the equilibrium market conduct, which is jointly determined by both the domestic and the foreign firms' choice of strategic variables, could be affected by many factors including trade policies per se. Therefore, if market conduct is changeable, there is an obvious logical flaw in the derivation of optimal policies in many studies since they all assume fixed market conduct. Thus, policies so obtained are not necessarily optimal. The study contained in the first essay (Chapter 1) allows the type of market competition to change responding to different government policies. In particular, we show that when demand is uncertain, firms have strict preferences for setting prices or quantities, depending on their different cost structures. Thus, the equilibrium market conduct is determined by firms' cost structures which is in turn affected by the government's tax and subsidy policy. After defining a quadratic export subsidy scheme, we show that it is superior to the often-studied linear export subsidy scheme because the former has the ability to influence the domestic firm's choice of strategic variables and therefore affects the type of market competition in favour of the domestic country.

In Chapter 2, we analyze strategic trade policies taking into account the incentive compatibility constraints arising from information asymmetry. Optimal strategic trade policies depend upon competition in export market, production costs, and market demands, among others. However, policy makers generally lack such information
required for policy design. Because producers are in the frontier of an economy, they, in contrast, have this information or at least know more than their governments. This creates an information asymmetry between governments (regulating bodies, principals) and producers (regulated bodies, agents). Chapter 2 deals with one of these information problems, which is perhaps the most common one: asymmetric cost information.

Specifically, we consider an asymmetric information version of the Brander-Spencer model by assuming that neither the domestic government nor the foreign firm knows about the domestic firm’s production cost. This model is of particular interest since it is a mixture of screening and signalling problems, which has not been studied in the information economics literature or in the strategic trade literature. The screening problem is present when the domestic government attempts to overcome the incentive problem by offering a menu of policies to the domestic firm. The signalling problem is involved in the stage in which the domestic firm chooses a policy from the menu and then competes with the foreign firm. Although use of policy menus can accomplish the screening task, incentive compatible menus inevitably reveal the domestic firm’s cost information to the foreign firm. This information revelation might or might not be desirable from the point of view of the domestic country. If it is undesirable, the domestic government could instead adopt a uniform policy to conceal the information. Due to these conflicting informational consequences of different policies, therefore, the government is confronted by the following problem: choosing between a policy menu and a uniform policy. We find that policy menu is preferred to uniform policy in
Cournot competition while the opposite occurs in Bertrand competition.

Chapters 1 and 2 have respectively investigated issues in strategic trade policy in the presence of demand uncertainty and asymmetric cost information. In Chapter 3, we focus on another important situation when uncertainty and information asymmetry together affect international trade. In particular, we examine the pattern of trade between two similar countries in the presence of financial market credit rationing, which is caused by the coexistence of uncertain product market return and asymmetric information about borrowers' riskiness.

We consider the case in which two countries face the same opportunities to produce a series of new and risky products. Development of new products requires R&D and demand for these new products is generally uncertain. Because of this, investment in new product development is risky. If lenders and borrowers have asymmetric information about the riskiness of these investments, borrowers may be subject to credit rationing when these projects have to be (at least partly) financed externally. In these circumstances, whether one country would eventually produce a certain new product and export to the other partly depends on whether the producers in this country could borrow from a bank. This makes the pattern of trade indeterminate.
Chapter 1

QUADRATIC EXPORT SUBSIDY SCHEME
AND
MARKET CONDUCT
1.1 Introduction

The literature on strategic trade policy has been flourishing since early 1980s. There are at least two reasons for this. First, government intervention in international trade is commonly found in the world. Second, international markets in many industries are indeed imperfect due to entry barriers and/or product differentiation. The general idea underlying the studies in this literature is that firms earn positive rents under imperfect competition and appropriate trade policies adopted by the domestic government may affect the competing firms behaviour in the export market in such a way to shift some of the rents otherwise earned by the foreigners to the domestic producers. This is the familiar “rent-shifting” argument. Following this idea, various optimal trade policies are derived under different assumptions about the type of competition (market conduct), the nature of competing products, and the number of firms. All these studies have one thing in common: they take the type of market competition as exogenously given and confine themselves to linear policies. However, a firm’s choice of strategic variables (price or quantity) could be affected by many factors including trade policies per se. Therefore, the equilibrium type of market

\[ \text{For a survey of this literature, see Helpman and Krugman (1989) or Pomfret (1991).} \]

\[ \text{Laussel (1992) is the only exception. The discussion paper version of Laussel (1992) came to my attention after the first version of this chapter had been finished in April 1991. The reader should find that these two studies are very similar in their motivations, ideas, and results. However, I focus on price setting and quantity setting behaviour of the firms rather than using a more general approach — the supply-function equilibrium approach, which is used by Laussel. Moreover, the analysis of the present study emphasizes on the dependence of the optimal quadratic export subsidy scheme on the firms' cost structures, given a fixed degree of demand uncertainty. Laussel (1992), on the other hand, stresses the link between the optimal linear-quadratic schedule and the degree of uncertainty, under constant marginal cost.} \]
competition should be endogenously determined given a particular trade policy. We also realize that there are many other possible forms of policies which are as easily implemented as the linear policy, for example, a quadratic export subsidy scheme which will be defined later. The purpose of this study is to explore nonlinear policies and compare them with the linear policy when firms’ choice of strategic variables has been endogenized.

It has been well known in the strategic trade literature that the type of optimal strategic trade policies depends on the type of market competition. In their seminal paper, Brander and Spencer (1985) argue that certain policies precommitted by the home government could have strategic effects on the international market competition. In particular, they find that if firms compete in quantities, export subsidization will shift the market share in favour of the home firm and therefore increase the national welfare. However, as shown by Eaton and Grossman (1986), if firms compete in prices, the optimal policy involves export tax, which increases firms’ profits at the expense of consumers. Nevertheless, the central point is that government intervention puts the home firm in a Stackelberg leader’s position vis-à-vis the foreign firm.

Almost all studies in strategic trade policies focus on export tax-subsidy policies in a certainty environment with the exception of Cooper and Riezman (1989) and Arvan (1991). They consider export quota policies and compare them with export tax-subsidy policies in the presence of demand and cost uncertainties. They find that subsidization is preferred to quotas if demand uncertainty is high but a quota policy is better than subsidization if demand uncertainty is low. What is most im-
portant in their work is that they point out the importance of using different modes of intervention in different circumstances.

In a duopoly model with demand uncertainty but without government intervention, Klemperer and Meyer (1986) show that firms are not indifferent between choosing a quantity and a price as their strategic variables. In particular, they prove that the dominant strategy is to choose quantities (prices) if the cost curve is convex (concave). Therefore, depending on the firms’ cost structures, the equilibrium market conduct could be Cournot, Bertrand, or asymmetric competition (i.e., one sets a quantity and the other sets a price).

In this chapter, I extend the Brander-Spencer (1985) model by introducing market uncertainty, endogenizing firms’ choice of strategic variables, and considering nonlinear export subsidy policies. In particular, I propose a quadratic export subsidy scheme in which subsidy rates vary with export volumes and find that it is superior to the often-studied linear export subsidy scheme in which there is a uniform subsidy rate. Both the quadratic and linear subsidy schemes can shift the domestic firm to the Stackelberg leader’s position given a particular market conduct; but, in addition to that, the quadratic scheme has the ability to influence the domestic firm’s choice of strategic variable and therefore to affect the market conduct in favour of the domestic firm. Note that the practical two-step tax-subsidy policy is a special case of the quadratic subsidy scheme and implementing the quadratic scheme should not be more difficult than the linear scheme.

The remainder of this chapter is organized as follows. In Section 1.2, we develop
a model for analysis. In Section 1.3, the optimal linear scheme and the optimal quadratic scheme are derived and through a comparison we reach the conclusion that the quadratic scheme is superior to the linear scheme. The last section of this chapter, Section 1.4, summarizes the results and outlines future research. Most of the proofs are contained in Appendix I.

1.2 The Model

As in Brander and Spencer (1985), there are one domestic firm and one foreign firm and the domestic government assists its firm in export competition. Firms produce differentiated products, all of which are exported to a third country.\(^5\) There are demand uncertainties due to many unknown factors such as consumer preferences and income of the importing country. The domestic government is free to adopt a nonlinear policy or a linear policy.

The model can be viewed as a two-stage game. In the first stage, the domestic government designs its trade policy. We confine ourselves in this study to the following two types of policy schemes:

1. linear scheme, in which the domestic firm receives \(s_1q\) amount of subsidy from the government if it exports \(q\) units of goods (if \(s_1 < 0\), it pays an export tax);

and

2. quadratic scheme, in which the domestic firm receives \(s_2q^2\) amount of subsidy

\(^5\)This is to isolate producer's surplus and so to simplify welfare analysis.
from the government if it exports \( q \) units of goods (if \( s_2 < 0 \), it pays an export tax).

To determine the level of the chosen policy scheme, one needs to specify \( s_1 \) or \( s_2 \). The government chooses a policy by taking into account its impacts on the second stage game.

In the second stage, firms make their decisions simultaneously. A firm may set a price and then produce the amount demanded for its products or it may set an output level and then sells its products at the price which clears the market. In the first case, the firm chooses price as its strategic variable while in the last case, its strategic variable is quantity.\(^6\)

Let demand be characterized by the linear system proposed by Dixit (1979) with a slight modification. More specifically,

\[
p_i = a - bq_i - dq_j + \epsilon \quad i, j = 1, 2, \quad i \neq j, \quad a > 0, \quad b \geq d \geq 0
\]

(1)

where \( q_1 \) and \( q_2 \) denote the quantities of supply of (or demand for) the domestic firm’s and the foreign firm’s products, respectively; \( p_1 \) and \( p_2 \) are the prices of corresponding products; and \( \epsilon \) stands for a random variable with \( E(\epsilon) = 0 \) and \( E(\epsilon^2) = \sigma^2 > 0 \).

Assume that firms have the same cost functions:

\[
C(q) = c_1q + \frac{1}{2}c_2q^2
\]

where \( 0 < c_1 < a \) and \( c_2 \) can be positive or negative. The restriction on \( c_1 \) is obvious because \( c_1 \geq a \) implies that firms will not engage in production without government

\(^{6}\text{This is the same as in Klemperer and Meyer (1986). In contrast, Singh and Vives (1984) assume that firms first commit to setting a price or quantity, then each firm determines the level of its strategic variable after observing its rival’s choice of strategic variables.}\)
subsidization. Although our analysis is based on the linearity assumption about demand and the linear-quadratic specification about cost, the qualitative results hold in a more general case (see the discussion in Subsection 1.3.4).

Thus, given an export subsidy \( S \) (\( S = s_1 q_1 \) in the linear scheme and \( S = s_2 q_1^2 \) in the quadratic scheme), the domestic firm’s profit is

\[
\pi_1 = p_1 q_1 - C(q_1) + S
\]

and the foreign firm’s profit is

\[
\pi_2 = p_2 q_2 - C(q_2).
\]

The domestic government’s objective function is the domestic country’s expected welfare which is the expected domestic firm’s profit net subsidy:

\[
W = E(\pi_1 - S) = E[p_1 q_1 - C(q_1)].
\]

1.3 Analysis

Our setting is similar to Klemperer and Meyer (1986) except the presence of government intervention. In this section, we first derive a useful result similar to Lemma 1 in Klemperer and Meyer (1986). We then analyze the linear scheme (in Subsection 1.3.1) and the quadratic scheme (in Subsection 1.3.2). Finally, we compare the two schemes and thus obtain the optimal export policy for the domestic government.

Before we explore the linear and quadratic schemes individually, let us first examine a general export subsidy function which contains the linear and quadratic schemes as its special cases:
\[ S(q) = s_1 q + \frac{1}{2} s_2 q^2 \]  

(3)

where \( s_1 \) and \( s_2 \) can be positive, negative, or zero.

According to the Nash equilibrium concept, each firm maximizes its profit by choosing a strategic variable and a value for the chosen variable, having the conjecture that its rival’s action (i.e., the type of strategic variable and its value) is held constant. Thus, given any action chosen by the foreign firm, according to the demand system (1), the domestic firm faces a general residual demand:

\[ p_1 = A_1 - B_1 q_1 + \epsilon \]  

(4)

where the values of \( A_1 \) and \( B_1 \) depend upon the foreign firm’s choice and value of strategic variables.\(^7\) Similarly, the foreign firm faces the following residual demand:

\[ p_2 = A_2 - B_2 q_2 + \epsilon. \]

Lemma 1 is helpful in finding the equilibrium market conduct. The proof is contained in Appendix I.

**Lemma 1**: Given export policy \( S(q) \) as defined in (3),

1. the domestic firm will choose to set a quantity (price) if \( c_2 - s_2 \geq (<)0 \);

2. the foreign firm will choose to set a quantity (price) if \( c_2 \geq (<)0 \); and

3. the expected welfare of the domestic country is

\[ W = \frac{[(A_1 - c_1)^2 - s_1^2](2B_1 + c_2 - s_2) - s_2(A_1 - c_1 + s_1)^2}{2(2B_1 + c_2 - s_2)^2} - \frac{s_2 c_2}{2B_1^2} \]  

(5)

\(^7\)This will become clear as the analysis goes on.
where $x$ is a dummy variable defined as $x = 0$ for $c_2 \geq s_2$ and $x = 1$ for $c_2 < s_2$.

A straight forward implication which is also the most important one is that a firm’s optimal choice of strategic variables depends on the slope of its marginal cost curve (and $s_2$ for the domestic firm) but not on its rival’s choice. The intuition behind this result, as given by Klemperer and Meyer (1986), is that for convex (concave) costs or concave (convex) profits, a fixed level of output is more (less) attractive than a random level with the same mean.

It is worth emphasizing here that with the nonlinear policy the domestic government can influence the domestic firm’s choice of strategic variables but not the foreign firm’s. However, if the government is confined to the linear scheme, i.e., $s_2 = 0$ in (3), it will no longer be able to manipulate the domestic firm’s choice of strategic variables.

1.3.1 Linear Scheme

The linear scheme is a special case of (3) with $s_2 = 0$. Although the domestic government can not control any firm’s choice of strategic variables via the linear scheme, it is able to anticipate the prevailing type of market conduct. We now derive the equilibria under all possible types of market competition in turn.

(i): $c_2 \geq 0$.

By Lemma 1, both firms set quantities. Thus, the market competition is represented by a Cournot game.

Suppose the domestic government adopts the linear export policy $S(q_1) = s_1 q_1$. 


In the Cournot game, the domestic firm chooses $q_1$ to maximize its expected profit $E\pi_1$, where $\pi_1$ is defined in (2), taking $q_2$ as constant. Given the demand system (1), we can easily obtain the firm’s reaction function $q_1 = (a - c_1 + s_1 - dq_2)/(2b + c_2)$. Similarly, the foreign firm’s reaction function is $q_2 = (q - c_1 - dq_1)/(2b + c_2)$. Thus, given $s_1$, the market equilibrium quantities are, letting $k \equiv 2b + c_2$,

$$q_1^* = \frac{(k - d)(a - c_1) + ks_1}{k^2 - d^2} \quad \text{and} \quad q_2^* = \frac{(k - d)(a - c_1) - ds_1}{k^2 - d^2}.$$  

(6)

Moreover, by comparing (1) and (4) and remembering that the foreign firm’s choice is quantity, we have $A_1 = a - dq_2$ and $B_1 = b$. Substituting these into (5) yields

$$W = \frac{(a - dq_2^* - c_1)^2 - s_1^2}{2k}$$  

(7)

where $q_2^*$ is given in (6).

The domestic government chooses $s_1$ to maximize the welfare $W$. This optimal subsidy rate is

$$s_1^c = \frac{d^2(k - d)(a - c_1)}{k(k^2 - 2d^2)} > 0$$  

(8)

where superscript $c$ indicates Cournot competition. The positive sign of $s_1^c$ implies that the optimal policy is export subsidization. In addition, the optimal export subsidy rate increases with the substitutability of the two competing products since

$$\frac{\partial s_1^c}{\partial d} = \frac{d(a - c_1)[(2k - 3d)(k^2 - 2d^2) + 4d^2(k - d)]}{k(k^2 - 2d^2)^2} > 0.$$  

(ii): $c_2 < 0$. 

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According to Lemma 1, firms will engage in Bertrand competition if $c_2 < 0$. From the demand system (1), we can derive the respective residual demands for the domestic and the foreign firms given that each firm’s rival’s choice is price. These residual demands are

$$p_i = \alpha + \gamma p_j - \beta q_i + (1 - \gamma)\epsilon, \quad i, j = 1, 2, \quad i \neq j$$

(9)

where $\alpha = a(b - d)/b, \beta = (b^2 - d^2)/b, \text{and } \gamma = d/b$.

Analogously (as in the Cournot game), we can derive the market equilibrium prices, given $s_1$:

$$p_1^* = \frac{\alpha(h - \beta) + \beta c_1}{h - (h - \beta)\gamma} - \frac{\beta h}{h^2 - (h - \beta)^2\gamma^2} s_1$$

$$p_2^* = \frac{\alpha(h - \beta) + \beta c_1}{h - (h - \beta)\gamma} - \frac{\beta \gamma(h - \beta)}{h^2 - (h - \beta)^2\gamma^2} s_1$$

(10)

where $h \equiv 2\beta + c_2$. Note the second order condition for profit maximization requires $h > 0$. Moreover, we assume $h - \beta > 0$ to preclude the possibility of having downward sloping reaction curves in Bertrand competition.\(^8\) Comparing (9) to (4), we have $A_1 = \alpha + \gamma p_2$ and $B_1 = \beta$. Thus, by Lemma 1, the government’s objective function becomes

$$W = \frac{(\alpha + \gamma p_2^* - c_1)^2 - s_1^2}{2h} - \frac{\sigma^2 c_2}{2h^2}$$

(11)

where $p_2^*$ is given in (10). Hence, the optimal export subsidy rate is

$$s_1^b = -\frac{\beta \gamma^2(a - c_1)(1 - \gamma)(h - \beta)[h + (h - \beta)\gamma]}{[h - (h - \beta)\gamma^2][h^2 - (h - \beta)^2\gamma^2 c_2]} < 0$$

(12)

\(^8\)One can derive the reaction functions and check $\partial p_i/\partial p_j = \gamma(h - \beta)/h < 0$ if $h - \beta < 0$ and $h > 0$. 

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where superscript $b$ stands for Bertrand competition and the negative sign implies an export tax.

### 1.3.2 Quadratic Scheme

A quadratic scheme can be obtained by setting $s_1 = 0$ in (2). Before we can make a comparison between the linear and the quadratic policies, we should first derive the optimal quadratic scheme by following the same procedure as in the preceding subsection.

(i): $c_2 \geq 0$.

According to Lemma 1, the foreign firm will set a quantity. However, the domestic firm’s choice depends on the policy level $s_2$ relative to $c_2$. Since the induced market conduct varies with the policy level, we should first divide all possible policy levels into two regions such that in each region, we have the same type of market conduct. We then study the firms’ behaviour in each region. Finally, we make a welfare comparison between these two regions. Through the comparison, we would be able to know which type of market conduct is better for the domestic country and so obtain the optimal policy which induces the preferred type of market conduct.

First, suppose the government is restricted to setting $s_2$ such that $s_2 \leq c_2$. This policy might be an export tax or subsidy, depending on the sign of $s_2$. By Lemma 1, the resulting market competition is Cournot.

A simple calculation (which is similar to the corresponding case in Subsection 1.3.1) produces the market equilibrium quantities, given $s_2$:  

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Again, using (5), we obtain

\[ W = \frac{(a - c_1 - dq_2^*)^2(k - 2s_2)}{2(k - s_2)^2} \]  

(14)

where \( q_2^* \) is given in (13). Then the government's objective is to maximize \( W \), which is defined above, subject to \( s_2 \leq c_2 \). By solving this constrained maximization problem, we obtain the sub-optimal export subsidy rate:

\[ s_2^* = \begin{cases} 
\frac{d^2}{k} & \text{if } d^2 \leq kc_2 \\
 c_2 & \text{otherwise} 
\end{cases} \]  

(15)

where the superscript \( c \) stands for Cournot competition.

Second, suppose the government's policy level is confined to \( s_2 > c_2 \). Then the market competition is characterized by an asymmetric game, with the domestic firm setting a price and the foreign firm setting a quantity. We refer to this as \((p,q)\) competition, where and hereafter the first element in a parenthesis stands for the domestic firm's strategy and the second for the foreign firm's strategy.

Similar to the derivation of Cournot-Nash and Bertrand-Nash equilibria, we obtain the market equilibrium in the \((p,q)\) competition, given \( s_2 \):\n
\[ p_1^* = \frac{ah - (\alpha - c_1)d(b + c_2 - s_2) + bhc_1}{(k - s_2)(k - d\gamma) - d^2} \]

\[ q_2^* = \frac{(a - c_1)(k - s_2 - d)}{(k - s_2)(k - d\gamma) - d^2}. \]  

(16)

Since the foreign firm sets a quantity, the domestic firm’s residual demand in (4) is specified by \( A_1 = a - dq_2 \) and \( B_1 = b \). Therefore, by Lemma 1, the domestic welfare, given \( s_2 \), is
\[ W = \frac{(a - c_1 - d q_2^*)^2(k - 2s_2) - \sigma^2 c_2}{2(k - s_2)^2} \]  

(17)

where \( q_2^* \) is given in (16). The domestic government’s problem is to maximize \( W \), which is defined above, subject to \( c_2 < s_2 < k \), where the upper bound for \( s_2 \) is required to satisfy the second order condition for the firms’ profit optimization. We here skip the long and tedious computation for the sub-optimal export subsidy level, which is obtained as

\[ s_2^{pq} = \begin{cases} 
\frac{d^2}{(k - d)k} & \text{if } \gamma^2 > \frac{b k c_2}{b + c_2} \\
\frac{c_2}{d} & \text{otherwise}
\end{cases} \]  

(18)

where the superscript \( pq \) denotes the \( (p,q) \) competition.

We now can start comparing the two sub-optimal policies in order to select the optimal one. To make the comparison relevant and interesting, we should focus on the interior solutions of the two sub-optimal policy levels. Substituting (15) into (14) yields the sub-optimal welfare in Cournot competition:

\[ W_c^c = \frac{(a - c_1)^2(k - d)^2}{2k(k^2 - 2d^2)}; \]  

(19)

Replacing \( s_2 \) in (17) by \( s_2^{pq} \) given in (18) gives the sub-optimal welfare in the \( (p,q) \) competition:

\[ W_{Q_{pq}}^{pq} = \frac{(a - c_1)^2(k - d - d\gamma)^2}{2k - d\gamma)[k(k - d\gamma) - 2d^2]} - \frac{\sigma^2 c_2}{2b^2}. \]  

(20)

By comparing \( W_c^c \) with \( W_{Q_{pq}}^{pq} \), we obtain the following important result:

**Lemma 2**: Suppose \( c_2 \geq 0 \). Then \( W_c^c > W_{Q_{pq}}^{pq} \) if \( d \leq \sqrt{k c_2} \). Thus, the optimal quadratic policy is \( s_2^* \) which is defined in (15) and therefore firms engage in Cournot competition.
Proof: See Appendix I.

We now explain this result. The key to understand this result is to know why it is better to induce the domestic firm to set a quantity rather than a price. As it is well known in the strategic trade literature, the optimal strategic trade policy places the domestic firm in a Stackelberg leader's position vis-à-vis the foreign firm.\(^9\) Thus, we could proceed with the following discussion as if the domestic firm was a Stackelberg leader, absent government intervention. In quantity competition, after the domestic firm has chosen a quantity, say \(q_1\), the foreign firm chooses its quantity facing the following residual demand: \(p_2 = a - dq_1 - bq_2\). However, in \((p,q)\) competition, the foreign firm sets its output level facing a different residual demand: \(p_2 = \alpha + \gamma \bar{p}_1 - \beta q_2\) if the domestic firm has chosen \(\bar{p}_1\). Note the foreign firm has a flatter residual demand curve in \((p,q)\) competition than in quantity competition since \(\beta = b - d\gamma < b\). Thus, the adverse effect of increasing output by the foreign firm on its price is smaller in the case of \((p,q)\) competition and therefore it will set a higher output level in the \((p,q)\) competition than in the quantity competition. Of course, the domestic firm is worse off if the foreign firm has a higher quantity. Hence, the domestic firm will be better off by setting a quantity.

Let us now briefly discuss the constraint \((d \leq \sqrt{kc_2})\) in Lemma 2. Like the linear scheme, the optimal export subsidy rate in the quadratic scheme \(s_2^e\) (given in (15)) is also increasing with the goods' substitutability since

\[
\frac{\partial s_2^e}{\partial d} = \frac{2d}{k} > 0.
\]

\(^9\)This was first pointed out by Brander and Spencer (1985).
Therefore, when the two products are close substitutes, i.e., when \( d \) is big, the desired subsidy rate is very high and it may become too high (i.e., \( s_2 > c_2 \)) to prevent the domestic firm from switching to setting a price. As a result, the restricted welfare for Cournot competition will not reach its optimum and thus the advantage of setting a quantity is undermined. Because of this, the welfare in Cournot competition \( Q^c_0 \) may be lower than the welfare in \((p,q)\) competition \( Q^{pq}_0 \).

(ii): \( c_2 < 0 \).

By Lemma 1, the foreign firm will set a price but the domestic firm’s choice depends on the government’s export policy. As in the previous case, we will examine the two possible choices by the domestic firm separately.

If \( s_2 > c_2 \), then the domestic firm will also set a price and so the market competition is characterized by a Bertrand game. Given \( s_2 \), the market equilibrium prices are

\[
p_1^* = \frac{1}{H} \left\{ \left[ \alpha(\beta + c_2 - s_2) + \beta c_1 \right] \left[ h + \gamma(\beta + c_2) \right] - \beta \gamma c_1 s_2 \right\}
\]

\[
p_2^* = \frac{1}{H} \left\{ \left[ \alpha(\beta + c_2) + \beta c_1 \right] \left[ h - s_2 + \gamma(\beta + c_2) \right] - \alpha \gamma (\beta + c_2) s_2 \right\}
\]

where \( H = h(h - s_2) - \gamma^2 (\beta + c_2)(\beta + c_2 - s_2) \). Since \( A_1 = \alpha + \gamma p_2 \) and \( B_1 = \beta \) which are obtained by comparing (4) with the residual demand function (9), the expected welfare is

\[
W = \frac{(\alpha - c_1 + \gamma p_2^*)^2(h - 2s_2)}{2(h - s_2)^2} - \frac{\sigma^2 c_2}{2}.
\]

where \( p_2^* \) is given in (21). The government maximizes \( W \) under the constraint \( c_2 < s_2 < 2\beta + c_2 \), where the second inequality is to satisfy the second order con-
dition. Routine calculation gives the sub-optimal export subsidy rate in Bertrand competition:

\[ s_2^b = \begin{cases} 
-d\gamma(k - b - d\gamma)/(k - d\gamma) & \text{if } \gamma^2(1 - \gamma^2) < -kc_2/b^2 \\
\frac{c_2}{2} & \text{otherwise}
\end{cases} \]  

(23)

where the superscript \( b \) stands for Bertrand competition. Since 

\[-d\gamma(k - b - d\gamma)/(k - d\gamma) = -\beta \gamma^2(h - \beta)/(h - \gamma^2(h - \beta)) > 0, \]

this policy is an export tax.

We now turn to consider the case in which \( s_2 \leq c_2 \). From Lemma 1, we know that there will be a \((q,p)\) competition, i.e., the domestic firm sets a quantity while the foreign firm sets a price. The market equilibrium, given \( s_2 \), is

\[ q_1^* = (a - c_1)(k - d)/M \]

\[ p_2^* = [(h - s_2)(ab + ac_2 + bc_1) - d(b + c_2)(\alpha - c_1)]/M \]

where \( M \equiv k(h - s_2) + d\gamma(b + c_2) \). Furthermore, the expected welfare is

\[ W = \frac{(\alpha - c_1 + \gamma p_2^*)^2(h - 2s_2)}{2(h - s_2)^2} \]  

(25)

where \( p_2^* \) is given in (24). Under the constraint \( s_2 \leq c_2 \), we obtain the sub-optimal export subsidy rate in the \((q,p)\) competition:

\[ s_2^{qp} = \begin{cases} 
-d^2(b + c_2)/bk & \text{if } d^2 \geq -kbc_2/(b + c_2) \\
\frac{c_2}{2} & \text{otherwise}
\end{cases} \]  

(26)

where the superscript \( qp \) denotes the \((q,p)\) competition.

Substituting (23) into (22) gives the sub-optimal welfare in the case of Bertrand competition, denoted by \( W^b \), and substituting (26) into (25) yields the sub-optimal welfare for \((q,p)\) competition, denoted by \( W^{qp} \). Let us leave the complicated welfare comparison to Appendix I and report the result here.
Lemma 3: Suppose $c_2 < 0$. Then $W_Q^p > W_Q^b$ if $d > \sqrt{-\frac{b c_2}{b+c_2}}$ and $\frac{\sigma^2(c_2)}{2\beta^2}$ is not too large. Thus, the optimal policy is an export tax $s^p$ as defined in (26), which induces the domestic firm to choose quantity as its strategic variable. The resulting market conduct is a (quantity, price) competition.

Proof: See Appendix I.

The rationale for inducing the domestic firm to set a quantity has been discussed before (right after Lemma 2). We now discuss the constraints for the result to hold. If the first constraint in Lemma 3 is not met (i.e., $d < \sqrt{-\frac{b c_2}{b+c_2}}$), goods are close to independent and so the benefit from setting a quantity is very small. As we know from Klemperer and Meyer (1986), there is a benefit from setting a price when the marginal cost curve is downward sloping. When products are close to independent, the benefit from setting a price will dominate that from setting a quantity. The second restriction that $\frac{\sigma^2(c_2)}{2\beta^2}$ is not too large is also important. Let us concentrate on discussing the size of $\sigma^2$. It has been stressed by Klemperer and Meyer (1986) that uncertainty makes setting a quantity and setting a price nonequivalent. The higher the uncertainty is, (i.e., greater $\sigma^2$), the larger the difference will be. Thus, when $\sigma^2$ is very big and $c_2 < 0$, the domestic firm benefits more from setting a price than setting a quantity. In any one of the above two circumstances, it might not be worth altering the domestic firm’s choice by imposing a very high tax.

(iii): Summary

We conclude the analyses in (i) and (ii), especially Lemma 2 and Lemma 3, in the following proposition.
**Proposition 1** Under all restrictions in Lemma 2 and Lemma 3, the optimal quadratic scheme always induces the domestic firm to choose quantity as its strategic variable.

More specifically, the scheme is defined as \( S(q) = \frac{1}{2} s_2^* q^2 \) where

\[
s_2^* = \begin{cases} 
\frac{d^2}{k} > 0 & \text{if } c_2 \geq 0 \\
-\frac{d^2(b + c_2)}{bk} < 0 & \text{if } c_2 < 0.
\end{cases}
\] (27)

The equilibrium market conducts are (quantity, quantity) competition when \( c_2 \geq 0 \) and (quantity, price) competition when \( c_2 < 0 \). The domestic country's optimal expected welfare under this scheme is

\[
W_q^* = \frac{(a - c_1)^2(k - d)^2}{2k(k^2 - 2d^2)}.
\] (28)

Proof: See Appendix I.

Some remarks should be made on this proposition. First, the optimal quadratic scheme given in (27) is subject to some conditions, which limit the applications of the result. For example, it fails when the goods produced by two firms are close substitutes. Second, (28) should not be interpreted as that the optimal expected welfare are equal in the two cases, i.e., \( c_2 \geq 0 \) and \( c_2 < 0 \). Although the expressions are identical, \( k \) has different values in different cases, recalling that \( k = 2b + c_2 \). Finally and more importantly, while Lemma 1 shows that the government has the ability to manipulate the domestic firm's choice of strategic variables, Proposition 1 emphasizes the desirability of doing this.
1.3.3 Linear Scheme vs. Quadratic Scheme

(28) gives the maximum welfare under the quadratic scheme. In order to make a welfare comparison, we should also calculate the maximum welfare under the linear scheme.

By substituting $q_2^*$ defined in (6) and $s_1^c$ defined in (8) into (7), we obtain the maximum welfare under the linear scheme in the case of $c_2 \geq 0$:

$$W_{L}^c = \frac{(a - c_1)^2(k - d)^2}{2k(k^2 - 2d^2)}$$

which is identical to $W_Q^*$. Despite their differences, the two policies generate the same welfare for the domestic country.

When $c_2 < 0$, it is difficult to make a direct comparison between $W_Q^*$ and $W_L^b$, where $W_L^b$ stands for the maximum welfare under the linear scheme in the case of $c_2 < 0$ and can be derived by substituting $p_2^*$ defined in (10) and $s_1^b$ defined in (12) into (11). However, we can show (see Appendix I) that

$$W_L^b = W_Q^*$$

Note, when $c_2 < 0$, $W_Q^* = W_Q^{gr}$. By Lemma 3 and Proposition 1, we immediately have $W_Q^* > W_L^b$. That is, the quadratic scheme generates higher welfare for the domestic country than the linear scheme does. We now summarize the above discussion in Proposition 2. Let $W_L^*$ denote the maximum welfare under the linear scheme.

**Proposition 2**: Under all restrictions in Lemma 2 and Lemma 3, the quadratic scheme is superior to the linear scheme. In particular, $W_Q^* > (\geq)W_L^*$ when $c_2 < (\geq)0$. 

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It has been argued by Singh and Vives (1984) that from the point of view of strategic interaction between firms, a firm's dominant strategy is to set a quantity in order to discourage price cut from its rival. However, as shown in Klemperer and Meyer (1986), when demand is uncertain and its marginal cost is decreasing, the firm should choose a price to avoid price fluctuation. Therefore, in the case of uncertain demand and downward sloping marginal cost curve, there is a trade off between setting a quantity and setting a price. This trade off is resolved for the domestic country if the government uses the quadratic export subsidy scheme since the quadratic scheme can change the domestic firm's cost structure in such a way that the firm will face an upward sloping marginal cost curve. Consequently, the domestic firm will optimally set a quantity, which gives the domestic country higher welfare. The linear scheme, however, cannot alter the curvature of the firm's marginal costs and so cannot induce the firm to adopt a quantity strategy which is in the best interest of the domestic country. Therefore, the quadratic scheme is better than the linear scheme.

When \( c_2 \geq 0 \), the domestic firm's optimal choice of strategic variables is already quantity. Hence the quadratic scheme loses its strict superiority over the linear scheme since there is no need to change the curvature of the firm's marginal costs and so these two schemes are equivalent.

1.3.4 Robustness

The analysis in the preceding subsection is based on a model with many strong assumptions. To examine usefulness of these results, we discuss their robustness in
this subsection.

A. General Cost Structure

Firms have strict preferences on strategic variables whenever demand is uncertain and their marginal costs are not constant. As explained by Klemperer and Meyer (1986), when the slope of marginal cost curve is positive (negative), firms would set a quantity (price) to avoid output (price) fluctuation. Thus, in a more general setting (i.e., with a general cost function), the domestic firm might set a quantity or price, depending on the curvature of the cost function at the relevant (or potential equilibrium) output level. Note that given a cost function, theoretically, as long as the policy parameter $s_2$ is set at a sufficiently low level, the resulting cost function (i.e., production cost plus subsidy) would have a positive second order derivative at the relevant output level and the domestic firm would be induced to choose a quantity strategy. Thus, under a general cost structure, the quadratic scheme still has the ability to influence the domestic firm’s choice of strategic variables.

However, inducing the domestic firm to adopt a quantity strategy is not the sole purpose of any export policy. It is well known in the strategic trade literature that an optimal strategic trade policy should place the domestic firm in a Stackelberg leader’s position. Therefore, if the subsidy rate $s_2$ required to induce a quantity strategy is too distant from the optimal rate for the Stackelberg leader’s position, the government may find it better to let the domestic firm choose a price strategy. In this case, adopting a quadratic scheme will not give more benefits than using a linear scheme since altering the domestic firm’s strategic variable is no longer desirable.
Conceivably, so long as the marginal cost curve is not very steep, the quadratic scheme continuously dominates the linear scheme.

B. General Demand Structure

In their model with general demand and cost functions, Klemperer and Meyer (1986) show that the equilibrium market conduct is jointly determined by the curvature of demand and the slopes of marginal costs. They also point out that if the marginal cost is sufficiently steeper, with either positive or negative slope, the impact of cost structure dominates the demand. Since a quadratic scheme changes the slope of marginal cost, we can always get such a domination by setting the policy at a high level. Thus, quadratic scheme does not lose its power of affecting the domestic firm’s choice of strategic variables. The question then is whether altering the firm’s strategy is still desirable, as discussed above.

1.4 Conclusion

If demand uncertainty and nonconstant marginal costs are present in a model of international duopoly, firms have strict preferences on the choice of strategic variables, namely quantity and price. A quadratic export subsidy scheme can influence the domestic firm’s preference and therefore shift the equilibrium market conduct to the one which is in the best interest of the domestic country. The often-studied linear export subsidy scheme lacks such ability. As a result, the quadratic scheme dominates the linear scheme.

Ironically, the design of the optimal quadratic scheme is much simpler than the
optimal linear scheme in terms of the expression of the optimal subsidy rates. Moreover, the implementation of the quadratic scheme should not be more difficult than the linear scheme. In fact, the linear scheme is not the only policy form observed in reality. Two-step tax-subsidies are found in use, which could be viewed as a special case of the quadratic scheme.

The present study can be extended to include incomplete information problems. For example, if firms have private information about their costs, then the government's optimal strategic policies are subject to incentive compatibility constraints.

The study can also be extended in a different direction. Consider that the government of the product importing country is to protect its consumers by using import policies. Although there are a lot of studies on optimal tariffs, our approach could be very different. We are interested in finding an import policy which has the ability to influence the exporters' strategic variable selections. This is important because consumers in the importing country are affected differently by quantity competition and price competition between the two exporting firms.
Chapter 2

OPTIMAL STRATEGIC TRADE POLICY
UNDER
ASYMMETRIC INFORMATION
2.1 Introduction

As Stegemann (1989, p.89) has put in a survey article, "the evidence around us — increasing international economic interdependence, rekindled protectionism, and the increase in policy actions with real or perceived strategic intent — will continually force the economics profession to address the issues that authors of models of strategic trade policy have tried to address." One of the most influential models of strategic export policy is the well known Brander – Spencer model (1985), in which it is shown that the home government has a unilateral interest in adopting an export subsidy policy if the home firm competes with a foreign firm in quantities. The central motive for these types of strategic policies is to "shift profits" from foreign firms to the home country. Subsequently, Eaton and Grossman (1986) demonstrated that the optimal policy is an export tax when the home firm competes with a foreign firm in prices. One important implication of these two studies is that the type of optimal strategic export policy is sensitive to many industrial-specific factors such as market conduct. It then becomes clear that lacking the relevant information, the government could very likely adopt a wrong type of policy. Moreover, it is reasonable to be expect that policy makers have less information than firms concerning production and markets.

Information is also crucial in determining the appropriate policy. Wong (1990) has shown that if policymakers do not have complete information about the home firm's costs, some export policies could be welfare worsening. In particular, Wong (1990) demonstrated that in the Brander – Spencer model with asymmetric information (about cost), the optimal export subsidy scheme derived from the full information
case is no longer incentive compatible in general and may even be detrimental to the home country. The reason is simple. The home firm has an incentive to misreport its cost in order to maximize its subsidy-cum-profit. The production decision is thus distorted (compared to the equilibrium outcome without government intervention) and the distortion could be sufficiently large such that national welfare is reduced to a level lower than that under free trade. However, Wong (1990) did not go any further to explore incentive compatible policies.

The present study develops strategic trade policies taking into account the constraints arising from incentive compatibility. We assume that the home firm’s marginal cost is private information. Being uninformed, the home government faces two policy options. It can offer a menu of policies or simply a uniform policy. In the case of a policy menu, the home firm makes a selection from the menu, after which another uninformed party, the foreign firm, observes the policy selection. The two firms then compete in a third market for their exports.

The distinguishing feature of this model is that it is a mixture of screening and signalling problems. We can divide the game into two stages: the policy stage and the market stage. The former is a screening sub-model in which the uninformed party (government) offers a menu to the informed party (home firm). The latter is a signalling sub-model in which the informed party (home firm) takes an action (policy selection) before it meets another uninformed party (foreign firm).\(^\text{10}\) Moreover, the two tasks, screening and signalling, are accomplished in one action — when the home

\(^{10}\)See Kreps (1990, p.651) for the conceptual difference between screening and signalling models.
firm makes a selection from the given menu. The home government faces a dilemma because of this. On the one hand, it wants to discern the home firm’s type in order to provide proper export subsides or taxes for the purpose of profit shifting. On the other hand, it wishes to have the high cost firm sending out a low cost signal under Cournot competition in order to reduce the foreign firm’s production, or the low cost firm signalling a high cost under Bertrand competition to raise the foreign firm’s price. In this study we find that under Cournot competition it is optimal for the home government to offer a menu of policies which leads to a separating equilibrium, but under Bertrand competition it is optimal to adopt a uniform policy which results in a pooling equilibrium.

As is well known in the strategic trade literature, the role played by the government is to make the home firm’s output expansion credible. In the model with private cost information, we find that there is a second role for which the government can play. By offering a separation inducing menu, the government enables the home firm to credibly reveal its true type to its rival.

Recently Brainard and Martimort (1992) and Maggi (1992) also examine the impacts of asymmetric information in the Brander- Spencer model. However, the focus of their papers is somewhat different. Brainard and Martimort (1992) incorporate a cost of raising government funds and Maggi (1992) considers non-linear policies. Neither paper is concerned with the signalling aspect of the design of government policy.

11 These papers came to my attention after the initial submission of this paper to the Journal of International Economics.
The remainder of this chapter is organized as follows. In the next section, we present a model with screening and signalling characteristics when firms compete in a Cournot fashion. A complete analysis of the model is contained in Section 2.3. Section 2.4 discusses Bertrand competition and gives results contrary to those obtained under Cournot competition. Section 2.5 concludes this chapter.

2.2 The Model

Consider the following situation. Two firms, 1 and 2, who respectively locate in countries 1 and 2, produce homogenous goods for a third market. Both firms are assumed to produce strictly positive outputs and, unless stated otherwise, compete in a Cournot fashion. The inverse demand function takes the form: \( p = a - b(q_1 + q_2) \), where \( a > 0, b > 0 \), \( p \) is the price of the product, and \( q_i \) the output of firm \( i, i = 1, 2 \). Only the government in country 1 is involved in policy intervention.

The information structure is as follows. Firm 1’s marginal cost, \( c \), is constant and private but it is common knowledge that firm 1 is of either high cost, \( c_H \), or low cost, \( c_L \), with \( \text{Prob}(c=c_H) = \mu \). Firm 2’s marginal cost is assumed known to all parties and equal to zero for simplicity. The reason for this assumption is that in the present setting, even if there were some uncertainties associated with firm 2’s marginal cost, there is no way for firm 2 to signal it. Hence both government 1 and firm 1 would base their decisions on the expected value. Consequently, this will only complicate

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12 This supposition is common in the literature of trade policies and is made for simplicity. It can be justified, for instance, if government 2 unilaterally adopts a free trade policy.

13 Allowing government 1 and firm 2 to have different priors will not change our results as long as these priors are common knowledge.
the model without changing the qualitative aspect of the results. In contrast, firm 1 is able to signal its cost through its policy selection.

In the environment described above, we consider the following two-stage one-shot game. At the beginning of the first stage, called the policy stage, government 1 designs (and commits to) its policy.\textsuperscript{14} The government has two options: it can use a uniform policy or a menu of policies.\textsuperscript{15} Under the uniform policy regime, the government sets a specific (per unit) export subsidy rate for firm 1 regardless of its type. The alternative possibility, a menu of policies, gives firm 1 a choice as to the policies that will be applied. After the government has announced its menu of policies, firm 1 makes its policy choice. When information revealing is desirable, a policy menu plays a role in inducing this revelation. However, without such policy menus, a simple announcement by firm 1 about its cost is not credible.

The menu approach allows the policy to be conditional on the firm’s type. As is typical in asymmetric information models, at least two policy instruments are required for the underlying type of an agent to be revealed through self selection. In our setting, each choice on the menu consists of two policies: a specific export subsidy rate and a level of lump sum tax.\textsuperscript{16} If the subsidy rate were the only instrument available to

\textsuperscript{14}The government’s ability to commit is commonly assumed in the literature. If this is not assumed, precommitment effects should be carefully investigated as in Caillaud et al.(1990).

\textsuperscript{15}Policy menus have been explored extensively in the regulation literature. The studies of incentive compatible policies can also be found in the literature of trade policy. See, for example, Feenstra (1987) and Prusa (1990).

\textsuperscript{16}In a real world context, one could perhaps view the lump sum tax as a tax on profit. However, it is hard to find a case in which an industry-specific profit tax is tied to an export subsidy. As suggested by a referee, another possibility is that the fixed cost represents a cost of lobbying. To be viewed as a low cost firm, the firm must incur a lobbying cost that exceeds some threshold set by the government.
the government, firm 1 would have chosen the highest subsidy rate independently of its type and no information would be revealed.

We confine policy options to linear subsidies, i.e., the subsidy rate does not vary with other informative variables such as output and price. This has the advantage of simplicity. Some possible implications of non-linear policies are discussed in the concluding remarks.

In the second stage, referred to as market stage, two firms compete in quantities on the basis of their respective information sets. Between the two stages, there is a transition period, in which firm 2 observes the policy choices made in the first stage and then updates its belief concerning firm 1's true marginal cost. This belief is correctly inferred by firm 1 and government 1. In particular, if government 1 designs a menu of policies inducing firm 1 to reveal its true type, then firm 2 observes this and the outputs of both firms are determined on the basis of full information. Otherwise both firm 2 and government 1 estimate the marginal cost of firm 1 at its expected value.

2.3 Analysis

We analyze the model by solving the game backwards. When we look at the second stage game, we know that its nature depends on the results of the first stage which are influenced by the types of policies used by government 1. Under all policies, however, there are only two different first stage results as far as the second stage information is concerned: the policy selection is either pooling or separating. The analysis proceeds
as follows. We first determine the optimal policy among the class of policies that lead to separating equilibria (in Subsections 2.3.1 and 2.3.2). We then consider pooling equilibria (in Subsection 2.3.3). Finally, we derive the optimal policy (in Subsection 2.3.4) by comparing the maximum social welfare achieved under these two types of policies. In considering pooling equilibria, we assume, without loss of generality, that the government sets a uniform policy. A uniform policy causes pooling (involuntarily) but a policy menu approach could also lead to pooling if both types of firms choose the same option.

For the purpose of comparison, we briefly illustrate the optimal policy in the case of full information. Obviously, neither a policy menu nor a lump sum tax is necessary. We denote variables in the full information case with a superscript $f$. Then, given any export subsidy rate, $s$, the second stage game yields

$$q'_1 = \frac{a - 2c + 2s}{3b} \quad \text{and} \quad q'_2 = \frac{a + c - s}{3b}.$$  

Government 1 chooses $s'$ to maximize its objective function, denoted $W'(s)$, which is simply the profit the home firm earns from exports less any subsidy payments. With linear demand, this can be expressed as:

$$W'(s) = [a - b(q'_1(s) + q'_2(s)) - c]q'_1(s).$$

Thus, we obtain

$$s' = \frac{a - 2c}{4}. \quad (30)$$

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2.3.1 Separation Inducing Menus

We now consider separating equilibria. Note that only policy menus could lead to separating equilibria. Let \( t = (t_L; t_H) \) denote a policy menu, where \( t_L \) is a policy intended for the low cost firm and \( t_H \) is a policy intended for the high cost firm. Each policy \( t_i = (s_i, \tau_i) \) for \( i = L, H \) consists of two elements, a specific export subsidy rate \( s_i \) and a lump sum tax \( \tau_i \).

Letting \( \pi^i(t_j) \) denote firm 1’s profit when it is of type \( i \) and chooses policy \( t_j \) for \( i, j = L, H \). A menu \( t \) is called a separation inducing menu if for \( i \neq j \)

\[
\begin{align*}
(i) & \quad \pi^i(t_i) \geq \pi^i(t_j), \\
(ii) & \quad \pi^i(t_i) \geq 0, \quad i, j = L, H.
\end{align*}
\]

Condition (i) is the separation constraint while condition (ii) is the participation constraint. As mentioned above, if a separation inducing menu is used by government 1, firm 2 will be able to discern firm 1’s type and it then follows that all parties have complete information at the second stage Cournot game.

Given a separation inducing menu \( t \), the market stage game can be described by the following maximization problems, \( i = L, H \),

\[
\begin{align*}
\pi^i(t_i) &= \max_{q_{si}} \{ [a - b(q_{si} + q_{2i}) - c_i + s_i]q_{si} - \tau_i \} \\
\max_{q_{2i}} [a - b(q_{si} + q_{2i})]q_{2i}
\end{align*}
\]

where \( q_{si} \) is firm 1’s output and \( q_{2i} \) firm 2’s output under the policy menu regime and when firm 1 is type \( i \). If \( t_L (t_H) \) is adopted, then \( i = L (H) \) in both (31) and (32), which constitute a Cournot game between firm 2 and a low (high) cost firm 1 with firm 2 knowing firm 1’s true type. Thus, the respective reaction functions are
\[ q_{si} = \frac{a - c_i + s_i - bq_{2i}}{2b} \quad i = L, H \tag{33} \]

\[ q_{2i} = \frac{a - bq_{si}}{2b} \quad i = L, H. \]

Given \( s \), the market stage equilibrium outputs are

\[ q_{si} = \frac{a - 2c_i + 2s_i}{3b} \quad \text{and} \quad q_{2i} = \frac{a + c_i - s_i}{3b} \quad i = L, H. \tag{34} \]

As mentioned in the introduction, the model incorporates both screening and signalling. Use of a menu of policies allows the government to screen for the type of firm so as to better design its export policy, but at the same time, firm 1's policy selection signals the same information to firm 2. To understand the implications of this, it is helpful to first consider a pure screening version of the model in which firm 2 knows firm 1's marginal cost. In this case, firm 2's output level will be affected by the policy imposed by the government, but the means by which the policy is chosen conveys no information and consequently has no effect on firm 2's output. In particular, if firm 1 chooses a policy \( t_j \) that is not consistent with its type \( i \), then firm 2 will produce \( q_{2i} = (a + c_i - s_j)/3b \), the Cournot output based on firm 1's true marginal cost \( c_i \) and subsidy \( s_j \). Therefore, we have

\[ \pi^i(t_j) = \frac{1}{9b}(a - 2c_i + 2s_j)^2 - \tau_j, \quad i, j = L, H. \tag{35} \]

If the firm selects policy \( t_i \) corresponding to its type \( i \), its profit \( \pi^i(t_i) \) is given by setting \( j = i \) in (35).

If menu \( t \) is separation inducing, condition (i) must be satisfied. Comparing \( \pi^i(t_i) \) with \( \pi^i(t_j) \) (\( j \neq i \)), we obtain Lemma 4.
Lemma 4: In the pure screening model, if \( t \) is a separation inducing menu, then \( s_L \geq s_H \) and \( \tau_L \geq \tau_H \).

Proof: See Appendix II.

Next, we provide some intuitions for Lemma 4. First and foremost, the two instruments in a separation inducing menu have to go hand in hand, with a high subsidy associated with a high tax and vice versa. Otherwise both types would pick the policy with higher subsidy and lower tax. Secondly, a separation inducing menu must have \( s_L \geq s_H \). To see this, suppose \( t \) is separation inducing with \( s_L < s_H \) (so \( \tau_L < \tau_H \)). Given that the high cost firm chooses \( t_H \) rather than \( t_L \), the firm’s benefit from the high subsidy rate \( s_H \) must be sufficient (relative to \( s_L \)) to more than offset the loss from the higher lump sum tax \( \tau_H \) (relative to \( \tau_L \)). The problem is that in this case the low cost firm will also prefer \( t_H \). Since the low cost firm produces more than the high cost firm, it benefits more than the high cost firm from any increase in the specific export subsidy rate. Thus by choosing \( t_H \), the low cost firm enjoys a greater increase in the total subsidy payment than would the high cost firm yet suffers the same penalty as would the high cost firm from the higher lump sum tax associated with \( t_H \). This contradiction implies that \( t \) cannot be separation inducing. It follows that in a pure screening model, \( s_L \geq s_H \) is a necessary condition for separation of types.

We now return to our original model incorporating both signalling and screening. Since firm 2 does not know firm 1’s marginal cost, its output level will depend on its updated belief which is influenced by firm 1’s policy selection. Suppose that
the government offers a separation inducing menu, firm 2 will believe firm 1’s type as signalled by the adopted policy. More precisely, if firm 1 chooses \( t_j \), then firm 2 believes that firm 1 is of type \( j \) and correspondingly sets \( q_{2j} = (a + c_j - s_j)/3b \). Taking this into account, firm 1 with type \( i \) will optimally set \( q_{si} = (2a - 3c_i - c_j + 4s_j)/6b \).

Consequently,

\[
\pi^i(t_j) = \frac{1}{36b}(2a - 3c_i - c_j + 4s_j)^2 - \tau_j \quad \text{and} \quad \pi^i(t_i) = \frac{1}{9b}(a - 2c_i + 2s_i)^2 - \tau_i. \tag{36}
\]

Lemma 5 gives a necessary condition for the separation of types (the proof is similar to that of Lemma 4).

**Lemma 5**: In the model with screening and signalling, if \( t \) is a separation inducing menu, then \( s_L - s_H \geq -(c_H - c_L)/4 \).

Lemma 5 differs from Lemma 4 in several aspects. First, in Lemma 5, there is no monotonicity property imposed on the two policy instruments. Monotonicity is a common requirement in the optimal contract and mechanism design literature because such models contain only a screening problem. Thus, the inclusion of signalling weakens the constraint required for the separation of types. Secondly and more specifically, when signalling is taken into account, the separation constraint does not rule out the possibility that \( s_L < s_H \). These results follow because there are now two basic forces influencing firm 1’s decision on policy selection. On the one hand, as in the case of the pure screening model, the low cost firm benefits more than the high cost firm from an increase in the export subsidy rate because the former produces more than the latter. On the other hand, the low cost firm loses more than the
high cost firm when firm 2’s belief about firm 1’s type changes from low to high. To see this, note that if firm 2 believes that firm 1 is a high cost firm, it will increase its output. Consequently, the price drops. The decrease in price hurts the low cost firm more than the high cost firm since the former has a greater output than the latter. Now suppose \( s_L < s_H \) (letting \( \tau_L = \tau_H \) for ease of analysis). We argue that it is quite possible that such a menu is separation inducing for particular levels of \( s_H \) and \( s_L \). The high cost firm might prefer \( t_H \) to \( t_L \) because the benefit from a higher subsidy rate at least covers the loss from being identified. Moreover, the low cost firm might choose \( t_L \). Although the low cost firm’s benefit from the higher subsidy rate associated with \( t_H \) is greater than the high cost firm’s, it might not be sufficiently great to offset the larger loss that arises when a low cost firm is identified as a high cost firm by firm 2.

### 2.3.2 Optimal Separation Inducing Menu

In designing its optimal profit shifting policy in stage 1, the objective of the government is to maximise the expected social welfare in country 1. This is simply the profit of each type of firm 1 less any net subsidy weighted by the probability of that type. When menus induce separation, the expected social welfare is given by

\[
W(t) \equiv \mu(\pi^L(t_L) - s_L q_{SL} + \tau_L) + (1 - \mu)(\pi^H(t_H) - s_H q_{SH} + \tau_H)
\]

where outputs \( q_{SL} \) and \( q_{SH} \) are given by (34) and \( \pi^L(t_L) \) and \( \pi^H(t_H) \) are given by (36).

To derive the optimal menu, we first maximize \( W(t) \) by ignoring the separation constraints. The first order condition yields the optimal subsidy rates \( s_L^* \) and \( s_H^* \):
\[ s_L^* = \frac{1}{4}(a - 2c_L) \quad \text{and} \quad s_H^* = \frac{1}{4}(a - 2c_H). \] (37)

from which and (34), we obtain the equilibrium outputs of each type of firm 1 and firm 2:

\[ q_{si} = \frac{a - 2c_i}{2b} \quad \text{and} \quad q_{2i} = \frac{a + 2c_i}{4b}, \quad i = L, H. \] (38)

As expected, the optimal subsidy rates (37) are just the full information subsidy rates corresponding to the firm's particular type. Since the optimal menu imposes no restrictions on the lump sum tax instrument, we are free to use lump sum taxes to ensure separation. For convenience, we first normalize the lump sum tax by setting \( \tau_{L}^* = 0 \). When \( s_L^* \) and \( s_H^* \) are set at their optimal levels as in (37), we demonstrate in Appendix II that the menu \( t^* \) induces separation (i.e., \( \pi^L(t_L^*) \geq \pi^L(t_H^*) \) and \( \pi^H(t_H^*) \geq \pi^H(t_L^*) \)) if

\[ \tau_L^* = \frac{1}{2b}(c_H - c_L)(a - c_H - c_L). \] (39)

These results are summarized in Proposition 3.

**Proposition 3** : \( t^* \) as defined in (37) and (39) is the optimal separation inducing menu.

The intuition behind Proposition 3 is clear. It is well known in the literature of strategic trade policy that when demand is linear, the optimal profit shifting policy requires that a higher export subsidy rate \( s_L^* \) (relative to \( s_H^* \)) be granted to the low cost firm.\(^{17}\) A lump sum tax \( \tau_L^* \) is levied on the low cost firm in order to prevent the

\(^{17}\)This can be seen from the formula for the optimal subsidy in Brander and Spencer (1985).
high cost firm from pooling so as to obtain the higher subsidy rate. The tax \( \tau^*_L \) is chosen so that the low cost firm still selects \( t^*_L \) but the high cost firm chooses \( t^*_H \).

### 2.3.3 Optimal Uniform Policy

We now consider uniform policies. Whenever a uniform policy is adopted by the government, firm 2 receives no information from the policy stage, so it remains uninformed. It follows that the market stage game is a quantity competition with asymmetric information (firm 2 has incomplete information about firm 1's cost). Letting \( s \) denote the common subsidy rates, \( q_{pi} \) denote firm 1's output under the uniform policy regime when it is type \( i (i = L, H) \), and \( \bar{q}_2 \) denote firm 2's output. Then the market stage game is characterized by the following maximization problems:

\[
\begin{align*}
\pi^i(s) &\equiv \max_{q_{pi}} [a - b(q_{pi} + \bar{q}_2) - c_i + s]q_{pi}, \quad i = L, H \\
\max_{q_2} [a - b(\bar{q}_1 + \bar{q}_2)]\bar{q}_2
\end{align*}
\]

where \( \bar{q}_1 \equiv \mu q_{pL} + (1 - \mu)q_{pH} \). Firm 1's reaction function is the same as (33) except \( q_{2i} = \bar{q}_2 \), but firm 2 now reacts to firm 1's expected output \( \bar{q}_1 \), giving rise to the reaction function,

\[
\bar{q}_2 = \frac{a - b\bar{q}_1}{2b}.
\] (40)

Hence, given \( s \), the market stage equilibrium outputs are

\[
q_{pi} = \frac{1}{6b}(2a - 3c_i - \bar{c} + 4s) \quad \text{and} \quad \bar{q}_2 = \frac{1}{3b}(a + \bar{c} - s), \quad i = L, H,
\] (41)

where \( \bar{c} \) stands for the expected value of firm 1's marginal cost, i.e., \( \bar{c} = \mu c_L + (1 - \mu)c_H \).

Thus, the resulting firm 1's profit given \( s \) is (using (41))
\[ \tilde{\pi}^i(s) = \frac{1}{36b}(2a - 3c_i + 4s - \hat{c})^2, \quad i = L, H. \]

Under the uniform policy regime, the expected social welfare in country 1 is the weighted sum of each type’s profit net of subsidy, which is given by

\[ \tilde{W}(s) \equiv \mu(\tilde{\pi}^L(s) - sq_{PL}) + (1 - \mu)(\tilde{\pi}^H(s) - sq_{PH}). \]

From the first order condition for the maximization of \( \tilde{W}(s) \), we can easily derive the optimal subsidy rate \( s^*_p \) and the equilibrium outputs:

\[ s^*_p = \frac{a - 2\hat{c}}{4}, \quad q_{pi} = \frac{a - c_i - \hat{c}}{2b} \quad \text{and} \quad \tilde{q}_i = \frac{a + 2\hat{c}}{4b}, \quad i = L, H. \] (42)

As expected, asymmetric information creates some distortions if the optimal uniform policy \( s^*_p \) is adopted. By comparing \( s^*_p \) in (42) with \( s^f \) in (30), we realize that \( s^*_p \) can never shift profit from firm 2 optimally since there exists no type of firm 1 with marginal cost equal to \( \hat{c} \). It oversubsidizes (undersubsidizes) firm 1 and causes overproduction (underproduction) when firm 1 is a high (low) cost firm.

### 2.3.4 Optimal Policy and Equilibrium

In this subsection, we derive the overall optimal policy by comparing the social welfare under the optimal separation inducing menu with the welfare under the optimal uniform policy.

We first illustrate the equilibria under the two different optimal policies using Figure 1. Note that the level of per unit subsidy rate affects the position of firm 1’s reaction curve. Since \( s^*_L > s^*_p \), the low cost firm’s reaction curve in the separating case \( R_{sL} \) is to the right of \( R_{PL} \), its reaction curve in the pooling case. Similarly, the
high cost firm’s reaction curve in the separating case $R_{SH}$ is to the left of $R_{PH}$, its reaction curve in the pooling case because $s^*_h < s^*_p$. However, firm 2’s reaction curve in the separating case, as shown by $R_2$, is unaffected by the subsidy level in country 1. Thus, in Figure 1, $S^{SL}$ and $S^{SH}$ are the equilibria in the separating case.

In the pooling case, firm 2’s output $q_2$ is a function of firm 1’s expected output $\bar{q}_1$ (see (40)). If firm 1 is actually a low cost firm, then the equilibrium is at $S^{PL}$ where the horizontal line at $\bar{q}_2$ intersects firm 1’s reaction function $R_{PL}$. Similarly, the equilibrium is at $S^{PH}$ if firm 1 is a high cost firm. The expected value of these two equilibria is shown by $S^P$, the intersection of $\bar{q}_2$ and firm 1’s expected reaction curve $\bar{R}_1$ (which is derived as if firm 1 had marginal cost $\bar{c}$).

It is well-known in the strategic trade literature that the optimal subsidies place the home firm (firm 1) in a Stackelberg leader position vis-à-vis the foreign firm (firm 2). Thus, $S^{SL}$ and $S^{SH}$ are the respective Stackelberg quantity equilibria in the separating case when firm 1 is respectively the low cost firm and the high cost firm. In the pooling case, $S^P$ could be viewed as a modified Stackelberg equilibrium in which a Stackelberg leader reveals only its average output $\bar{q}_1$ (and firm 2 responds by committing to $\bar{q}_2$) but the leader’s actual output subsequently varies with its type. Due to the linear reaction function ($R_2$), it is not difficult to see that firm 1’s average outputs are the same under both cases and that firm 2’s output in the pooling case is equal to its average output in the separating case. Our previous results in Subsections 2.3.2 and 2.3.3 also prove this. By (38) and (42).
\[
\tilde{q}_1 = \mu q_{PL} + (1 - \mu)q_{PH} = \frac{a - 2\tilde{c}}{2b} = \mu q_{SL} + (1 - \mu)q_{SH}
\]

\[
\tilde{q}_2 = \frac{a - 2\tilde{c}}{4b} = \mu q_{2L} + (1 - \mu)q_{2H}.
\]

Moreover, the total outputs in the market are unchanged in the two cases since using (38) and (42)

\[
q_{si} + q_{2i} = \frac{3b - 2c_i}{4b} = q_{pi} + \tilde{q}_2 \quad i = L, H.
\]

Thus, the optimal separation inducing menu and the optimal uniform policy give rise to the same equilibrium price for the product, i.e.,

\[
p_{SL} = p_{PL} \quad \text{and} \quad p_{SH} = p_{PH}.
\]

Although firms 1 and 2’s expected outputs and the equilibrium price do not vary in the separating and pooling cases, Lemma 6 below shows that the optimal separation inducing menu dominates the optimal uniform policy. If we denote the expected social welfare under these two optimal policies as \(W^*\) and \(\tilde{W}^*\), respectively, where

\[
W^* = \mu(p_{SL} - c_L)q_{SL} + (1 - \mu)(p_{SH} - c_H)q_{SH}
\]

\[
\tilde{W}^* = \mu(p_{PL} - c_L)q_{PL} + (1 - \mu)(p_{PH} - c_H)q_{PH},
\]

then we have Lemma 6.

**Lemma 6 :** The optimal separation inducing menu gives country 1 higher welfare than the optimal uniform policy: \(W^* > \tilde{W}^*\).

Proof: Using (43)-(44) in (45)-(46), the welfare difference can be expressed by
\[ W^* - \tilde{W}^* = \mu(p_{SL} - c_L)(q_{SL} - q_{PL}) + (1 - \mu)(p_{SH} - c_H)(q_{SH} - q_{PH}) \]  

(47)

\[ = \mu(q_{SL} - q_{PL})[(p_{SL} - c_L) - (p_{SH} - c_H)]. \]  

(48)

Because the low cost firm produces more in the separating case than in the pooling case, i.e., \( q_{SL} > q_{PL} \), and the low cost firm’s price-cost margin is higher than the high cost firm’s since

\[ p_{SL} - c_L = \frac{a - 2c_L}{4} > \frac{a - 2c_H}{4} = p_{SH} - c_H, \]  

(49)

(48) shows \( W^* - \tilde{W}^* > 0 \). Q.E.D.

Note that the first term in (47) is positive (since \( q_{SL} > q_{PL} \)) but the second term is negative (since \( p_{SH} < p_{PH} \)). This implies that use of the optimal separation inducing menu instead of the optimal uniform policy results in welfare gain if firm 1 turns out to be a low cost firm but a welfare loss if it is in fact a high cost firm. Thus, there is an ex ante trade off using the optimal separation inducing menu.

To understand how the trade off can be resolved, let us make use of Figure 1. Since points \( S^{SH} \) and \( S^{PH} \) entail the same total output and the same price, all points on the segment (which is not depicted in the figure) between these two points also entail the same total output and the same price. Similarly, all points on the segment between points \( S^{SL} \) and \( S^{PL} \) (including these two points) entail the same total output and the same price. It follows that firm 1 has equal price-cost margin \( (p_{SH} - c_H) \) at all points on the segment between \( S^{SH} \) and \( S^{PH} \) and equal price-cost margin \( (p_{SL} - c_L) \) at all points on the segment between \( S^{SL} \) and \( S^{PL} \). Note that at both the pooling equilibrium (shown by \( (S^{PH}, S^{PL}) \)) and the separating equilibrium (shown by
(S^{SH}, S^{SL}), firm 1 has the same expected output (\bar{q}_1) but a higher variance in its output at the separating equilibrium. Thus, the issue is whether country 1’s welfare is increasing or decreasing in the variance of firm 1’s output. To answer this, let us consider a continuum of “outcomes” between the pooling and separating equilibria. An “outcome” here is represented by a pair of points with one on the segment between \( S^{PL} \) and \( S^{SL} \), the other on the segment between \( S^{PH} \) and \( S^{SH} \), and the expectation at \( S^p \). Then an increase in the variance in firm 1’s output is achieved by an increase in the low cost firm’s output (a point on the segment between \( S^{PL} \) and \( S^{SL} \)) and a decrease in the high cost firm’s output (a point on the segment between \( S^{PH} \) and \( S^{SH} \)) keeping expected output unchanged at \( \bar{q}_1 \). Since the low cost firm’s price-cost margin is higher than the high cost firm’s (see (49)), welfare in country 1 must rise as the variance in firm 1’s output increases. Hence, the optimal separation inducing menu is preferred to the optimal uniform policy.

With Lemma 6 we are now ready to analyze the entire game described in Section 2.2. To do this, we must consider all policy options, both menu and uniform, at the same time.

Since government 1 is a Stackelberg leader vis-à-vis the firms, it will choose a policy which results in the highest social welfare in country 1. This, together with Lemma 6, rules out the uniform policies. Moreover Lemma 6 and Proposition 3 indicate that \( t^* \) is the optimal one among all policies. Once the government adopts \( t^* \), we have all results obtained in Subsection 2.3.2. We now conclude the above analysis in Proposition 4.
Proposition 4: In the two-stage sequential game with asymmetric information and Cournot competition, there exists a separating equilibrium with \( t^* \) as the optimal separation inducing menu. The full information equilibrium allocation is achieved. Pooling is never an equilibrium.

The result that the full information allocation is attained is not surprising. On the one hand, the export subsidy rate which firm 1 receives in the optimal separation inducing menu is identical to that in the full information case corresponding to the firm’s type.\(^{18}\) On the other hand, the additional policy instrument used in our model has no impact on the export market, i.e., the introduction of a lump sum tax per se does not cause the firms’ outputs and price to differ from those in the full information model.\(^{19}\)

Lemma 6 and Proposition 4 seem to suggest that it is always preferable for the government to set a policy so as to induce firm 1 to reveal its information. This result, however, is sensitive to the nature of market competition. We will see this in the next section.

\(^{18}\)Because of this, the government’s commitment is not a problem in the Cournot model. Although the government knows the firm’s type after the firm has chosen a policy from this optimal menu, there is no need to revise the level of that policy because it is already equal to the optimal level in the full information case.

\(^{19}\)This result of course depends upon a common assumption that monetary transfer either from the government to firm 1 (i.e., subsidy) or from firm 1 to the government (i.e., tax) is costless. The government is never reluctant to introduce an additional instrument when it is necessary, for example, a lump sum tax in this model, as it can be implemented at no expense. For the possible effects of relaxing this assumption, see Caillaud et al.(1988) and Brainard and Martimort (1992).
2.4 Bertrand Competition

The government always wants to learn firm 1’s true cost in order to design a precise policy. However, if firm 1 tries to inform the government or if the government uses some rules to induce firm 1 to reveal its cost, in our setting, the information is also released to firm 1’s rival (firm 2), which may not be desirable. Earlier results indicate that information revelation is desirable in a Cournot game. The question is whether revelation is always optimal. To answer this, we now consider another model which differs from the earlier one in two respects: the firms produce imperfect substitutes and compete in prices.

Suppose the demand system is given by

\[ q_i = \alpha - \beta p_i + \gamma p_j, \quad \text{where } i, j = 1, 2, \quad i \neq j, \quad \alpha > 0, \quad \text{and} \quad \beta > \gamma > 0. \]  

(50)

We adopt the same notations defined before but interpret them in the context of a Bertrand game. For brevity, we only present results related to Lemma 6 and Proposition 4. In doing so, we first derive the optimal uniform policy. We then characterize the optimal separation inducing menu. Finally, we compare the social welfare under these two optimal policies. Results are summarized in Proposition 6.

Given \( s \) under the uniform policy regime, the market stage game is characterized by the following two maximization problems:

\[
\max_{p_{pi}} (p_{pi} - c_i + s)(\alpha - \beta p_{pi} + \gamma \check{p}_2), \quad i = L, H, \quad \text{and} \quad \max_{\check{p}_2} (\alpha - \beta \check{p}_2 + \gamma \check{p}_1)
\]

where \( p_{pi} \) is firm 1’s price when it is type \( i \) in the pooling case, \( \check{p}_1 \equiv \mu p_{pL} + (1 - \mu) p_{pH} \) is the expected price charged by firm 1, and \( \check{p}_2 \) is firm 2’s price in the pooling case.
By solving these problems simultaneously, we obtain the market stage equilibrium prices \((i = L, H)\):

\[
\bar{p}_2 = \frac{1}{4\beta^2 - \gamma^2} \left[ \alpha(2\beta + \gamma) + \beta \gamma (\bar{c} - s) \right]
\]

\[
p_{pi} = \frac{1}{2(4\beta^2 - \gamma^2)} \left[ 2\alpha(2\beta + \gamma) + \gamma^2 (\bar{c} - c_i) + 4 \beta^2 (c_i - s) \right].
\]

Maximizing the expected social welfare, which is

\[
\tilde{W}(s) \equiv \mu(p_{pL} - c_L)(\alpha - \beta p_{pL} + \gamma \bar{p}_2) + (1 - \mu)(p_{pH} - c_H)(\alpha - \beta p_{pH} + \gamma \bar{p}_2)
\]

gives the optimal uniform policy \(s^*_p\):

\[
s^*_p = -\frac{\gamma^2}{4\beta^2(2\beta^2 - \gamma^2)} \left[ \alpha(2\beta + \gamma) - (2\beta^2 - \gamma^2)\bar{c} \right].
\]

Therefore, the equilibrium prices are

\[
p_{pi} = \frac{1}{2} \left[ c_i + \frac{\alpha(2\beta + \gamma)}{2\beta^2 - \gamma^2} \right] \quad i = L, H \\
\bar{p}_2 = \frac{1}{4\beta} \left[ \alpha + \gamma \bar{c} + \frac{2\alpha \beta (\beta + \gamma)}{2\beta^2 - \gamma^2} \right]
\]

and analogously to (46), country 1’s expected welfare at the pooling equilibrium is

\[
\tilde{W}^* = \mu(p_{pL} - c_L)q_{pL} + (1 - \mu)(p_{pH} - c_H)q_{pH},
\]

where

\[
q_{pL} = \alpha - \beta p_{pL} + \gamma \bar{p}_2 \quad \text{and} \quad q_{pH} = \alpha - \beta p_{pH} + \gamma \bar{p}_2.
\]

To avoid repetition, we omit the derivation of the optimal separation inducing menu and the resulting market equilibrium. The optimal separation inducing menu has the per unit subsidy rates\(^\text{20}\)

\(\text{20}The levels of lump sum tax are chosen to make the policy satisfying the separation constraints.
\[ s_i^* = -\frac{\gamma^2}{4\beta^2(2\beta^2 - \gamma^2)}[\alpha(2\beta + \gamma) - (2\beta^2 - \gamma^2)c_i] \quad i = L, H \]

and the market equilibrium prices are

\[ p_{si} = \frac{1}{2}[c_i + \frac{\alpha(2\beta + \gamma)}{2\beta^2 - \gamma^2}] \quad \text{and} \quad p_{2i} = \frac{1}{4b}[\alpha + \gamma c_i + \frac{2\alpha\beta(\beta + \gamma)}{2\beta^2 - \gamma^2}] \quad i = L, H. (53) \]

Moreover, analogously to (45), the separating equilibrium welfare of country 1 is

\[ W^* = \mu(p_{SL} - c_L)q_{SL} + (1 - \mu)(p_{SH} - c_H)q_{SH} \quad (54) \]

where

\[ q_{SL} = \alpha - \beta p_{SL} + \gamma p_{2L} \quad \text{and} \quad q_{SH} = \alpha - \beta p_{SH} + \gamma p_{2H}. \]

We illustrate the above equilibria in Figure 2. Note that the position of firm 1’s reaction curve (shown as \( R_{SL}, R_{SH}, R_{PL}, \) and \( R_{PH} \) in Figure 2) depends on its type and the tax rate. When changing from the optimal uniform policy to the menu, the low cost firm’s reaction curve is shifted downward (because \( s_L^* < s_p^* \), i.e., the firm faces a higher tax); and the high cost’s reaction curve is shifted upward (since \( s_H^* > s_p^* \), i.e., the firm faces a lower tax). Also in Figure 2, \( S^{SL} \) and \( S^{SH} \) represent the Stackelberg price equilibria in the separating case. \( S^P \) would be the Stackelberg price equilibrium if firm had marginal cost \( \bar{c} \). \( S^{PL} \) and \( S^{PH} \) are the actual equilibria in the pooling case.

Similar to the Cournot case, linearity of the reaction function implies that the price \( \bar{p}_2 \) charged by firm 2 in the pooling case is equal to the average price in the separating case: i.e., by (51) and (53)
\[ \mu p_{2L} + (1 - \mu)p_{2H} = \frac{1}{4b}[\alpha + \gamma \bar{c} + \frac{2\alpha \beta (\beta + \gamma)}{2\beta^2 - \gamma^2}] = \bar{p}_2. \]  

Also by (51) and (53), we have

\[ p_{SL} = p_{PL} \quad \text{and} \quad p_{SH} = p_{PH} \]  

meaning that each type of firm 1 sets the same price under these two cases, but

\[ p_{2L} < \bar{p}_2 \quad \text{and} \quad p_{2H} > \bar{p}_2. \]  

Due to the strategic complementarity of Bertrand competition, firm 2 sets a lower price if it knows that its rival is a low cost firm than if it does not know. Similarly, firm 2 sets a higher price when it knows that it is competing with a high cost firm.

We now make welfare comparison between the pooling and separating equilibria. From (52) and (54), the welfare difference can be expressed in the same form as (47) for the Cournot case:

\[ W^* - \tilde{W}^* = \mu(p_{SL} - c_L)(q_{SL} - q_{PL}) + (1 - \mu)(p_{SH} - c_H)(q_{SH} - q_{PH}) \]  

Note from the demand system (50) and (57)

\[ q_{SL} - q_{PL} = \gamma(p_{2L} - \bar{p}_2) < 0 \quad \text{and} \quad q_{SH} - q_{PH} = \gamma(p_{2H} - \bar{p}_2) > 0. \]  

Thus, the first term of (58) is negative, implying a loss from using the separation inducing menu when firm 1 is a low cost firm. However, the second term is positive, which captures the gain from adopting the menu when firm 1 is a high cost firm. Moreover, the gain and loss are proportional to firm 1’s price-cost margin. It follows that to show that welfare is lower at the separating equilibrium than the pooling...
equilibrium, we need only to show that firm 1 earns a higher price-cost margin when it is low cost than when it is high cost. To do this, we use (55) and (59) to rewrite the welfare difference (58) as

\[ W^* - \hat{W}^* = \frac{1}{\gamma} \mu(q_{SL} - q_{PL})(p_{SL} - c_L) - (p_{SH} - c_H). \]  

(60)

By (53),

\[ p_{SL} - c_L = \frac{1}{2} \left[ \frac{\alpha(2\beta + \gamma)}{2\beta^2 - \gamma^2} - c_L \right] > \frac{1}{2} \left[ \frac{\alpha(2\beta + \gamma)}{2\beta^2 - \gamma^2} - c_H \right] = p_{SH} - c_H. \]  

(61)

Using (59) and (61) in (60), we immediately obtain \( W^* < \hat{W}^* \). This establishes Proposition 5.

**Proposition 5**: In the two-stage sequential game with asymmetric information and Bertrand competition, \( \hat{W}^* > W^* \), i.e., the optimal uniform policy achieves higher social welfare than a separation inducing menu. The equilibrium is pooling.

The welfare difference (58) for the Bertrand case is identical in form to (47) for the Cournot case. Moreover, since firm 1 has the same expected output \( \bar{q}_1 \) at the separating and pooling equilibriums,\(^{21}\) it is again true that the difference between the separating and pooling equilibriums could be explained on the basis of the difference in the variance of output. However, in the Bertrand case, it is more convenient to use Figure 2 to express the explanation in terms of the difference in the variance of firm 2's price. As can be seen from (59), when firm 2's price is high, then firm 1's output is high (and vice versa) and an increase in the variance of firm 2's price causes a proportional increase in the variance of firm 1's output.

\(^{21}\)Following (55) and (56), one can easily check this by calculating the two expected outputs.
In Figure 2 and from (56), firm 1 has the same price and price-cost margin at all points on the segment between $S^{PL}$ and $S^{SL}$ and likewise on the segment between $S^{PH}$ and $S^{SH}$. However, firm 2’s price varies as an “outcome” moves from the separating equilibrium (shown by $(S^{SL}, S^{SH})$) to the pooling equilibrium (shown by $(S^{PL}, S^{PH})$). The variance in firm 2’s price is higher at the separating equilibrium than at the pooling equilibrium. In fact, a reduction in firm 2’s price variance is achieved by a rise in firm 2’s price when firm 1 is a low cost firm (a point moving upwards from $S^{SL}$ towards $S^{PL}$) and a drop in firm 2’s price when firm 1 is a high cost firm (a point moving downwards from $S^{SH}$ towards $S^{PH}$) keeping the expected price unchanged at $\bar{p}_2$. Thus, to select between these two equilibria, we must know whether country 1’s welfare is increasing or decreasing in firm 2’s price variance.

Since the welfare difference (58) for the Bertrand case is identical in form to (47) for the Cournot case and that the low cost firm has a higher price-cost margin than the high cost firm in both cases (see (49) and (61)), as discussed before (for Lemma 6), we prefer a higher output from the low cost firm. Note that a higher output for the low cost firm is achieved by a lower variance in firm 2’s price (and thus a low variance in firm 1’s output). Consequently, the optimal uniform policy which results in a smaller variance in firm 2’s price is preferred to the menu in Bertrand competition.

The sharp difference between Lemma 6 and Proposition 5 arises because the pooling and separating equilibria have significantly different consequences for outputs in the Bertrand case than in the Cournot case. It is known from (59) that, in the
Bertrand case, the low cost firm produces less while the high cost firm produces more in the separating equilibrium than in the pooling equilibrium. However, in the Cournot case, the low cost firm produces more but the high cost firm produces less in the separating equilibrium than in the pooling equilibrium, i.e., the expressions in (59) have opposite signs in the Cournot case.\textsuperscript{22}

When government 1 adopts an optimal uniform policy, the equilibrium necessarily differs from the full information equilibrium. Interestingly enough, Proposition 5 indicates that country 1 achieves higher social welfare in a world with incomplete information than in a world with full information. This is because firm 1's cost information has positive net effects on the social welfare if it is being kept private. One important implication of this finding is that in a "rivalrous agency" model,\textsuperscript{23} in some cases it is better not to induce the agent to reveal its information because the revelation has a signalling effect.

\subsection*{2.5 Conclusion}

When information asymmetry problems are present in the Brander–Spencer model, the home government can design a menu of policies, by introducing an additional policy instrument, to induce information revelation and achieve the full information social welfare.

\textsuperscript{22}It can be easily verified that this is also true if we had differentiated products instead of homogenous products in the Cournot case.

\textsuperscript{23}Our model differs from Ferstman and Judd's (1987) owner-manager model. In a "rivalrous agency" environment we consider a hidden information problem while they deal with issues of hidden action.
However, if the home firm’s cost is private information, the model is characterized by a mix of screening and signalling. In models of this kind, it is not always optimal for the uninformed government to design a mechanism which induces the informed home firm to reveal its information because this firm’s rival (the foreign firm) also becomes informed. The information nature of the optimal policy is very sensitive to the type of competition between home and foreign firms. Under Cournot competition, the government offers a menu of policies which induces the home firm to reveal its type. Under Bertrand competition, however, the government chooses a uniform policy which helps the home firm to conceal its information.

As suggested by a referee, it would be of interest to investigate non-linear policies that do not reveal the home firm’s cost information to the foreign firm and compare them with linear policies. Under the non-linear policy regime, the government first offers a subsidy scheme in which subsidy rates are contingent on output levels, then firms produce, and finally the home firm receives subsidy payments at the rate corresponding to its output. This has the advantage that different types of the home firm receive different subsidy rates which are better designed for them. Also the types are separated only after the foreign firm has produced and so the home firm’s cost information does not influence the foreign firm’s output decision. Such non-linear policies might dominate linear policies, especially when information concealing is desirable, for the reason that the non-linear policies share the advantages which are

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24In a model in which the foreign firm knows the home firm’s marginal cost (no signalling), Maggi (1992) shows that non-linear policies enhance the government’s degree of freedom to implement the Stackelberg outcome.
respectively attained by separation inducing menus and uniform polices.
Chapter 3

NEW PRODUCTS, CREDIT RATIONING, AND

INTERNATIONAL TRADE
3.1 Introduction

According to product cycle theory, new products are first introduced and produced in developed countries (the North), then after a certain time period, these products, which become old, are imitated and produced in less developed countries (the South), while newer products emerge in the North. This can explain the pattern of North-South trade as a function of differences between the two regions in their R&D capabilities, capital availabilities, labour qualities, and market structures. However, it is less clear what shapes the North-North and South-South trade when countries are very similar in every respect. The purpose of this study is to explore the pattern of North-North trade in those new and risky products.

Development of new products involves the following two stages: scientific research and product commercialization. In developing new products, especially hi-tech products, there is a risk in the first stage, which is referred to as R&D (research and development) uncertainty. On the other hand, it is always difficult to predict demands for new products. This is the risk in the second stage, which will be referred to as market uncertainty. In this chapter, we study trade in new products in association with these two types of uncertainties. Since these uncertainties severely affect

\[25\] International trade in product cycle was first studied by Vernon (1966). Krugman (1979) was the first to make formal modelling. More recent works can be found in the references of Grossman and Helpman (1991).

\[26\] Increasing returns and product differentiation are two of the major explanations of intra-industry trade in which each country simultaneously produces, exports, and imports products of the same industry (see Krugman (1980)).

\[27\] These two important aspects of the development of new products have also been emphasized by Ben-Zion (1981) who stated that a project is often subject to major uncertainty about both the probability of its scientific success and commercialization.
the profitability of potential products, risk neutral lenders may be reluctant to invest in these risky projects. As a result, the demand for loans may exceed the supply of funds and credit could be rationed.\footnote{There are generally three types of credit rationing: (1) a limit on the number of loans, (2) a limit on the size of each loan, and (3) different interest rates on different loans. As in Stiglitz and Weiss (1981), we consider the first type of credit rationing. For discussions of the other two types, see Jaffee and Russell (1976) and Gale and Hellwig (1985). In the credit rationing literature, it has been argued that contractual mechanism, such as loan commitments and collateral, may mitigate the rationing problem. For a brief review of the theoretical debate on whether credit rationing may persist in equilibrium and the empirical tests on whether credit rationing might be a significant macroeconomic phenomenon, see Berger and Udell (1992).}

To examine the pattern of trade in risky products, we develop a model in which there are two identical countries facing the possibility of innovating and producing a set of products. Potential producers must borrow to finance their R&D and production. Because these projects are risky, as in Stiglitz and Weiss (1981), credit is rationed, i.e., banks will only lend to some of the loan applicants. Those who receive loans can then undertake R&D and if successful they start to produce. Thus, whether a new product can eventually be produced in one country and exported to the other depends on whether the potential producer of the product is able to borrow and whether the R&D is successful. Although the pattern of trade may be somewhat indeterminate,\footnote{In a different model, Grossman and Helpman (1990) found that even if two countries differ in their initial stock of knowledge capital but which is globally accessible, the pattern of trade would be completely indeterminate.} we can calculate the probabilities of the following three possible outcomes for a product: (i) it is produced in both countries and sold in both markets (two-way trade); (ii) it is only produced in one country and exported to the other (one-way trade); and (iii) it is produced by none (no trade).
The Stiglitz and Weiss (1981) framework has been used previously in the study of international trade. For example, Copeland (1990) and Flam and Staiger (1991) have developed models in which they reconsider the infant industry argument for protection. In these models, capital market imperfections lead to insufficient entry of the domestic infant industry, but a tariff improves welfare in Flam and Staiger (1991) while it may or may not do so in Copeland (1990) due to their different assumptions about direct foreign investment. In contrast to these studies, the present study focuses on trade between two developed economies, without any one having any advantages, and examines the pattern of trade.

Kletzer and Bardhan (1987) have also investigated the impact of credit rationing on the pattern of trade. However, they consider North-South trade under a different type of credit rationing in which borrowers face different interest rates. Unlike their model, we study North-North trade in new and risky products under credit rationing in which the number of loans is constrained.

The rest of this chapter is organized as follows. Section 3.2 presents a model in which information asymmetry between lenders and borrowers about the riskiness of new products leads to credit rationing. The pattern of trade is analyzed in Section 3.3. In Section 3.4, we argue that the home and foreign banks could mutually benefit from making their decisions sequentially rather than simultaneously and we briefly discuss its implication for the pattern of trade. In Section 3.5, we conclude and outline future research.
3.2 The Model

There are two countries: home and foreign. They are identical in all aspects and so we shall just illustrate the situation at home.

There are N potential and (for simplicity) independent products. Each product is to be developed and, if successful, produced by a single firm, the innovating firm.\(^{30}\) Firms face two types of uncertainty. First, whether a new product emerges depends upon whether the R&D is successful or not. For simplicity, we assume that all products have equal probabilities of success in R&D, which is denoted by \(\alpha \in (0, 1)\).\(^{31}\) Second, even if R&D is successful, the demand for the new product is uncertain.

We now specify market uncertainty. Let \(\epsilon_i\) stand for an additive random variable in the inverse demand function of product \(i\), i.e.,

\[
P_i(q) = \bar{P}(q) + \epsilon_i, \quad i = 1, \ldots, N
\]  

(62)

where \(q\) is the demand of product \(i\) and \(\bar{P}(q)\) is the inverse demand function when \(\epsilon_i = 0\) (i.e., without uncertainty).\(^{32}\) \(\bar{P}'(q) < 0.\) \(\epsilon_i \in [\underline{\epsilon}, \bar{\epsilon}]\) with

\[
E(\epsilon_i) = 0 \quad \text{for all } i.
\]  

(63)

As in Stiglitz and Weiss (1981), we adopt the definition of mean preserving spreads

\(^{30}\)This is the case if each producer has a national patent for her invention. Our analysis and results are still valid if we allow a number of producers to compete in each product (oligopoly).

\(^{31}\)This assumption could be replaced by a less restrictive one, for example, assuming that the probabilities of succeeding in R&D are different but they are private information to the firms. Alternatively, to characterize these products' risk differentials, we could emphasize on R&D uncertainties by assuming different probabilities of successful R&D but identical market uncertainties for these products.

\(^{32}\)The assumption that all products have identical non-random inverse demand function \(\bar{P}(q)\) can be relaxed. See Footnote 33 for a discussion.
to measure riskiness. In particular, we order the products increasingly by riskiness, i.e. if $1 \leq i < j \leq N$ and $x \leq \tilde{\epsilon}$, then

$$\int_x^\infty \tilde{F}_j(\epsilon) d\epsilon \geq \int_x^\infty \tilde{F}_i(\epsilon) d\epsilon$$

(64)

where $\tilde{F}_i(\cdot)$ and $\tilde{F}_j(\cdot)$ are cumulative distribution functions (c.d.f.) of $\epsilon_i$ and $\epsilon_j$, respectively.

All firms have identical cost functions. Each requires the same amount of funds, $B$, for R&D and the expense must be financed by borrowing from banks. There are many banks which compete among themselves by choice of interest rate to maximize their profits. In this competitive banking system, we hereafter refer to a representative bank as “the bank”. There is an information asymmetry between the bank and the borrowers. Firm $i$ knows the distribution of $\epsilon_i$ ex ante and realizes the value of $\epsilon_i$ ex post. The bank is aware that there is a firm with demand uncertainty $\epsilon_i$ but it cannot tell which one. Given an interest rate, the demand for loans is the number of firms which want to borrow and the supply of loans is the number of loans which the bank is willing to provide.

The sequence of moves is as follows. At the very beginning, the bank announces an interest rate $r$. Then, each firm decides whether to submit an application for a loan at the given interest rate. Upon receiving all applications, the bank makes a loan offer. In the case when supply exceeds or equals demand, all applicants receive

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33The assumptions of identical demands and identical costs are not essential. Our results still hold so long as all firms have equal expected profits or more realistically the lenders lack information about demands and costs and so they can not discern the firms' differences in their expected profits.
loans from the bank; otherwise (i.e., when demand exceeds supply) the bank only provides loans to some of the applicants. Credit rationing occurs in the latter case. When credit is rationed, the bank makes a random offer. Those applicants who are able to receive loans (the eventual borrowers) can then undertake their R&D activities and if successful, they start producing and selling to both the home and foreign markets. Market uncertainties are realized after firms have made their output decisions. Finally, the bank receives payments (principal plus interest) from the borrowers. There is limited liability and the minimum the bank can claim back from each loan is the prerequisite collateral \(K > 0\).\(^{34}\) This happens when the loan receiver fails in R&D. The bank may receive the maximum (gross) return \((1 + r)B\) from a loan, providing that the loan receiver earns a sufficient profit from the product market.\(^{35}\) Depending on the firm’s profit from the markets, any partial return between \(K\) and \((1 + r)B\) to the bank is possible.

3.3 Trade

3.3.1 Firm’s Expected Profit

In this and the next two subsections, the analysis is focused on the home country. Analogous arguments apply to the foreign country.

We define market return as sales revenue minus production cost. A firm’s profit then is its market return net of R&D expenditure and payment to the bank. We con-

\(^{34}\)As shown by Stiglitz and Weiss (1981), credit rationing may still occur even if the bank uses collateral combined with interest rate in a contract. Here, for simplicity, we focus on the case where only interest rate is used as a screening device.

\(^{35}\)Obviously, \((1 + r)B > K\), otherwise the bank bears no risk.
sider the following hierarchy in each firm: the owner makes the borrowing decision 
while the production and sales manager makes the output decision. More specifically, 
the manager chooses an output level to maximize the expected market return. Al-
though it is true at least for many firms and in many cases that the production and 
sales manager has a goal different from the owner's, this separation assumption is 
made only to simplify the following analysis. Our results (which are qualitative) hold 
without this assumption.

A firm's market return varies with the market structure which is determined by 
the R&D results of the firm and its foreign counterpart. If firm $i$ succeeds in R&D 
but its foreign counterpart fails, firm $i$ becomes a monopolist in both the home and 
foreign markets of product $i$. We denote the maximum expected market return as 
$\pi^m$ and the optimal output as $q^m$ for this case. If, however, the foreign counterpart 
also succeeds in R&D, then both the home and foreign markets are characterized by 
duopolistic competition. In this case, we use $\pi^d$ to denote firm $i$'s maximum expected 
market return and $q^d$ the optimal output level. Thus, the actual returns to firm $i$ 
in these two cases are

$$R_i^k = \pi^k + \epsilon_i q^k \quad k = m, d,$$

36 Under the specification of market uncertainty (62), property (63), and identical cost function, these output levels and expected returns are the same for all industries. Hence, a subscript $i$ is unnecessary.

37 The production and sales director faces market uncertainty only, but the owner faces both R&D and market uncertainties. In the case of monopoly, the production and sales director chooses $q$ to maximize $E[2(\bar{P}(q/2) + \epsilon_i)(q/2) - C(q)]$ where $C(\cdot)$ is the cost function. At optimal output $q^m$, the maximum expected return $\pi^m = \bar{P}(q^m/2)q^m - C(q^m)$ and the actual return $R_i^m = 2(\bar{P}(q^m/2) + \epsilon_i)(q^m/2) - C(q^m) = \pi^m + \epsilon_i q^m$. The derivation of $R_i^d$ is similar.
and therefore the expected market returns over these two cases are

\[ R_i = (1 - \alpha)R_i^m + \alpha R_i^e = \pi^e + \epsilon_i q^e \]  

(65)

where \( \pi^e \equiv (1 - \alpha)\pi^m + \alpha \pi^d \) and \( q^e \equiv (1 - \alpha)q^m + \alpha q^d \).

Since \( \pi^e \) and \( q^e \) are positive constants, (65) is an affine transformation from \( \epsilon_i \) to \( R_i \). This allows us to define riskiness directly on the random market return instead of on the random market demand because any affine transformation preserves the property of mean preserving spread (64). That is, for all \( 1 \leq i < j \leq N \) and \( R \leq x \leq \bar{R} \), where \( \bar{R} = \pi^e + \epsilon q^e \) and \( \bar{R} = \pi^e + \bar{\epsilon} q^e \),

\[ \int_R^x F_j(R)dR \geq \int_R^x F_i(R)dR \]  

(66)

where \( F_i(\cdot) \) and \( F_j(\cdot) \) are the c.d.f. of \( R_i \) and \( R_j \). For simplicity and without loss of generality, let \( \epsilon = -\pi^e/q^e \) to normalize \( \bar{R} = 0 \).

Firm \( i \)'s profit \( \pi(R_i, r) \) is a function of its market return and the interest rate. Note when \( R_i + K < (1 + r)B \), the firm's net loss is \( K \), since, in addition to the collateral (\( K \)), the earning from the market (\( R_i \)) is totally used up to pay the bank; when \( (1 + r)B - K < R_i < (1 + r)B \), part of the collateral will be claimed by the bank because the market return is not sufficient to pay the bank; when and only when \( R_i > (1 + r)B \), the firm earns a positive profit \( (R_i - (1 + r)B) \). In sum,

\[ \pi(R_i, r) = \max\{R_i - (1 + r)B, -K\} \]

38 Although the word "expected" is used here, \( R_i \) is still a random return because the expectation is taken over all possible results of the foreign firm's R&D for a given market uncertainty.

39 An affine transformation from \( x \) to \( y \) is defined as \( y = a + bx \), where \( a \) and \( b \) are any constants and \( b > 0 \).

40 This result can be easily proved and so the proof is omitted here.
Therefore, the expected profit to firm $i$, given interest rate $r$, is

$$
\pi_i(r) = (1 - \alpha)(-K) + \alpha \int_0^R \pi(R, r) dF_i(R)
$$

$$
= -(1 - \alpha)K + \alpha \int_0^{(1+r)B-K} (-K) dF_i(R) + \alpha \int_{(1+r)B-K}^R [R - (1 + r)B]dF_i(R)
$$

$$
= -(1 - \alpha)K + \alpha A_i(r) + \alpha [\pi^e - (1 + r)B], \quad (67)
$$

where

$$
A_i(r) \equiv \int_0^{(1+r)B-K} [(1 + r)B - K - R]dF_i(R) > 0. \quad (68)
$$

Since $\pi(R, r)$ is convex in $R$, $\pi_i(r)$ must be increasing with the degree of riskiness (see Rothschild and Stiglitz (1970)). Thus, from (67), we know that $A_i(r)$ increases in $i$.

In the last expression of (67), the first term is the firm’s loss when R&D fails; the last term is the profit when R&D succeeds under unlimited liability; the second term is the benefit from limited liability.

Firm $i$ borrows if and only if its expected profit is nonnegative or equivalently

$$
A_i(r) \geq \frac{(1 - \alpha)K}{\alpha} + (1 + r)B - \pi^e. \quad (69)
$$

Since $A_i(r)$ increases in $i$ and the right hand side of (69) is constant with respect to $i$ but increases in $r$, we immediately obtain our first result.

**Result 1** (Theorems 1 and 2 in Stiglitz and Weiss (1981)): For a given interest rate $r$, there exists $i^*$ such that firm $i$ borrows from the bank if and only if $i > i^*$. As the interest rate increases, the critical value $i^*$ increases.
An important implication of this result is that if the bank raises the interest rate, the demand for loans drops and the average degree of risk from the set of the remaining applicants rises and hence the applicant pool becomes worse.

### 3.3.2 The Bank's Expected Return

We now consider the (gross) return to the bank from a loan to firm $i$. If firm $i$'s R&D fails, the bank claims the collateral $K$. If the R&D succeeds, the return to the bank denoted as $\rho(R_i, r)$ is a function of firm $i$'s market return and interest rate. The bank gets the full return $(1 + r)B$ only in the case that the firm has the ability to pay, i.e. $R_i + K \geq (1 + r)B$; otherwise the bank receives partial payment $R_i + K$. Thus,

$$\rho(R_i, r) = \min\{R_i + K, (1 + r)B\}.$$

Hence, the expected return to the bank is

$$\rho_i(r) = (1 - \alpha)K + \alpha \int_0^R \rho(R, r) dF_i(R)$$

$$= (1 - \alpha)K + \alpha \int_{(1 + r)B - K}^{(1 + r)B - K} (R + K) dF_i(R) + \alpha \int_{(1 + r)B - K}^R (1 + r)B dF_i(R)$$

$$= (1 - \alpha)K - \alpha A_i(r) + \alpha (1 + r)B. \quad (70)$$

In the last expression of (70), the first term is the return to the bank from a loan when R&D fails; the last term is the return to the bank when R&D succeeds but under unlimited liability; the second term adjusts the return to the bank due to limited liability.

Recall that $A_i(r)$ increases in the riskiness of the products, (70) leads to Result 2.
Result 2 (Theorem 3 in Stiglitz and Weiss (1981)): The expected return to the bank from a loan decreases as the riskiness of the loan increases.

3.3.3 Credit Rationing

Let $\bar{\rho}(r)$ denote the average return to the bank at interest rate $r$ if the bank lends to all applicants. From Results 1 and 2 it should be clear that there exists an interest rate $r^*$ that maximizes $\bar{\rho}(r)$.\footnote{To see this, note, as pointed out by Stiglitz and Weiss (1981), that an increase in interest rate has two conflicting effects: the positive effect which gives the bank a higher return due to the interest payment; and the adverse-selection effect — the average riskiness of the loans goes up (by Result 1) and so the return to the bank decreases (by Result 2). Initially, the positive effect dominates the adverse-selection effect but the domination is altered after $r^*$.} The bank’s supply of loans can then be obtained from the supply function at interest rate $r^*$, regardless of the demand. Similar to Theorem 5 in Stiglitz and Weiss (1981), we know that there exist supply functions which give rise to credit rationing.

Result 3: At the equilibrium interest rate $r^*$, there is credit rationing, i.e., the demand for loans exceeds the supply of funds.

For ease of later discussion, let $i^*$ be the critical value such that all firms with $i > i^*$ apply for loans at interest rate $r^*$; $I^* = \{\text{firms with } i > i^*\}$, which is the set of all applicants at interest rate $r^*$; $I^{**}$ be the set of applicants whose applications are not denied, i.e., the eventual borrowers. From Result 3, $I^{**} \subset I^*$. We define the degree of credit rationing $x^*$ as the fraction of the number of applications denied over the total number of applications, i.e., $x^* = \frac{\text{number of firms in } I^* - \text{number of firms in } I^{**}}{\text{number of firms in } I^*}$.
3.3.4 Pattern of Trade

Since the two countries are identical, the preceding analysis and results apply to the foreign country as well. In particular, these countries have the same equilibrium interest rate $r^*$, the same applicant pool $I^*$, and the same degree of credit rationing $x^*$. However, since the lucky applicants are determined randomly in each country, the set of the eventual borrowers at home $I^*_h$ may differ from that in the foreign country $I^*_f$. There are three possibilities: (i) $I^*_h = I^*_f$; (ii) $I^*_h \cap I^*_f = \emptyset$, where $\emptyset$ denotes the empty set; and (iii) neither (i) nor (ii) holds.

In case (i), two countries support the same set of products. Although the final entry to each product market in this set depends upon the success of R&D, we are likely to see more duopoly markets in this case than in cases (ii) and (iii) because every product in this set is financed at home and abroad. Thus the competition is high. However, the product variety is small since the number of different products being financed in the world is small.

Case (ii) becomes possible only if $x^* > .5$, i.e., more than half of the applicants are not able to borrow from the bank. This could happen when a large number of products are highly risky so the demand for loans is high (by Result 1). For similar reasons given above, we are likely to observe low market competition and large product variety in this case. In fact, all markets which come to exist are supplied by monopolists.

Observing case (iii) is most plausible. The resulting market competition and product variety will be between those in cases (i) and (ii).

When both firm $i$ at home and its foreign counterpart receive loans and succeed in
R&D, two-way trade occurs and both the home and foreign markets are characterized by duopolistic competition. If, however, the foreign firm does not get a loan or it obtains a loan but fails in R&D, the home firm will be the sole producer of product \( i \) and becomes a monopolist in both the home and foreign markets. In the last case, we say that the home country is a pure exporter and the foreign country is a pure importer of product \( i \).

We now establish Proposition 6.

**Proposition 6:**

(i) Products with \( i \leq i^* \) will not be produced in either country; all products with \( i > i^* \) have equal chances \((\alpha(1 - x^*))\) to be produced in both countries.

(ii) The home country may become a pure importer or a pure exporter of product \( i \) \((i > i^*)\), each with probability \( \alpha(1 - x^*)[1 - \alpha(1 - x^*)]\); the probability of having two-way trade in product \( i \) is equal to \( \alpha^2(1 - x^*)^2 \).

By observing that \( \alpha^2(1 - x^*)^2 > 2\alpha(1 - x^*)[1 - \alpha(1 - x^*)] \) if and only if \( \alpha(1 - x^*) > 2/3 \), we immediately obtain a corollary from Proposition 6.

**Corollary 1:** Two-way trade is more likely than one-way trade if the R&D successful rate \( \alpha \) is high and credit is not severely rationed (i.e., \( x^* \) is small).

### 3.4 The Timing of Moves

In the above analysis, the home and foreign banks are assumed to make their decisions simultaneously. We now relax this restriction by allowing the banks to choose the time

\[42\text{We distinguish this from the case of two-way trade, in which the home country (and the foreign country as well) produces, imports, and exports product } i \text{ at the same time.}\]
of moves, i.e., when to make offering decisions, and then briefly discuss its impact on the pattern of trade. In particular, we ask whether a bank, say the home bank, has an incentive to delay its loan offer. In other words, we want to know if the home bank can benefit from making its offering decision after it has been aware of the foreign bank’s offering decision.

If the home and foreign banks move simultaneously, there are two possibilities: (i) \( I_h^* \cap I_f^* \neq \emptyset \) and (ii) \( I_h^* \cap I_f^* = \emptyset \). In case (i), some firms at home and their foreign counterparts receive loans from their respective banks, while in case (ii), there exists no such a firm who and its foreign counterpart both receive loans. For ease of analysis, we first investigate a special case of (i) by supposing \( I_h^* \cap I_f^* = \{x\} \), i.e., only product \( x \) is financed in both countries. We then discuss the general case of (i) and finally case (ii).

Suppose the home and foreign banks make their loan offer simultaneously and \( I_h^* \cap I_f^* = \{x\} \). We are going to show that the home bank will be better off if it redistributes loans by cancelling the one which was designated for \( x \) and giving it to another firm, say \( y \), which is not in the previously determined set of loan receivers, i.e., \( y \notin I_h^* \).

Although the home bank cannot discern the riskiness of products \( x \) and \( y \), it is aware that firm \( x \)'s counterpart abroad gets a loan from the foreign bank but firm \( y \)'s counterpart does not. If it lends to firm \( y \), firm \( y \) will be a monopolist. However, if it lends to firm \( x \), firm \( x \) may become a duopolist. It is this difference that makes the loan for \( y \) more attractive than the loan for \( x \). We now provide proof of this assertion.
Recall that the bank does not even know the probability distributions of market uncertainty associated with products $x$ and $y$. The bank then will take the two distributions as the same when it calculates market returns to these firms. We use $\epsilon$ to denote the representative distribution. Thus firm $y$’s market return is $\hat{R}(\epsilon) = \pi^m + \epsilon q^m$ because the firm faces no competition. However, firm $x$’s market return is $R(\epsilon) = \pi^e + \epsilon q^e$ as given by (65) in Section 3.3. Let $m(\epsilon)$ denote the difference between these two market returns:

$$m(\epsilon) \equiv \hat{R}(\epsilon) - R(\epsilon) = \alpha([\pi^m - \pi^d] + \epsilon(q^m - q^d]).$$

(71)

Then, we find Lemma 7 useful in deriving the main result of this section.

**Lemma 7**: Assume $\pi^m q^d - \pi^d q^m \geq 0$. Then $m(\epsilon) > 0$ and so $\hat{R}(\epsilon) > R(\epsilon)$ for every $\epsilon$.

Proof:

$$m(\epsilon) = \frac{\alpha(\pi^m q^d - \pi^d q^m)}{(1 - \alpha)q^m + \alpha q^d} > 0 \quad \text{and} \quad m'(\epsilon) = \alpha(q^m - q^d) > 0.$$

Therefore, $m(\epsilon) > 0$ for all $\epsilon$. Q.E.D.

If $q^d \geq q^m$, obviously the assumption in Lemma 7 holds since $\pi^m > \pi^d$. Even if $q^d \leq q^m$, the assumption is not stringent. Let us rewrite the assumption as

$$\frac{\pi^m}{q^m} \geq \frac{\pi^d}{q^d} \quad \text{or} \quad \hat{P}(\frac{1}{2}q^m) - \frac{C(q^m)}{q^m} > \hat{P}(q^d) - \frac{C(q^d)}{q^d}.$$

By noting $2q^d > q^m$, we can see that this assumption holds if the average cost does not increase in output very rapidly. We only preclude the rare case that a monopolist
produces a lot more than a duopolist but the monopolist’s profit is only slightly higher than the duopolist’s.

By definition, the expected returns to the bank from lending to firms $y$ and $x$ are derived by taking expectation of the same return function $\rho(R, r)$ over different distributions $F_y(R)$ and $F_x(R)$ (see (70)) or equivalently, they can be calculated by taking expectation of different return functions over the same distribution $F_x(R)$. For purpose of comparison, we adopt the latter approach. Let $\hat{R}_0$ and $\hat{R}_1$ denote firm $y$’s minimum and maximum market returns and define $\epsilon_0$ as a critical point such that firm $y$ is at even, i.e., $\hat{R}(\epsilon_0) = (1 + r)B - K$. Then, $R(\epsilon_0) + m(\epsilon_0) = (1 + r)B - K$ and by (71) and Lemma 7, we obtain

$$
\rho_y(r) = (1 - \alpha)K + \alpha \int_{R_0}^{R_1} \rho(R, r) dF_y(R)
$$

$$
= (1 - \alpha)K + \alpha \int_{0}^{(1 + r)B - K - m(\epsilon_0)} [R + m(\epsilon) + K - (1 + r)B] dF_x(R) + \alpha(1 + r)B
$$

$$
> (1 - \alpha)K + \alpha \int_{0}^{(1 + r)B - K - m(\epsilon_0)} [R + K - (1 + r)B] dF_x(R) + \alpha(1 + r)B
$$

$$
= \rho_x(r).
$$

Thus, inequality (72) indicates the benefit to the bank from loan redistribution, switching the loan from firm $x$ to firm $y$.

We now consider the general case of (i), i.e., when there are more than one industries receiving loans in both countries. Following the same arguments above, we know that the home bank will be better off by cancelling a loan previously designated to a
firm whose foreign counterpart also receives a loan and offering the loan to another applicant whose foreign counterpart does not receive a loan. The bank will stop doing so when all loans, which are previously designated to those firms whose foreign counterparts also receive loans, have been cancelled or all applicants, whose foreign counterparts do not receive loans, have received loans. This is because no further loan relocation will be able to create a new potential monopolist by sacrificing a potential duopolist and no gain will accrue to the bank. In case (ii), since all loan receivers are already potential monopolists in their respective product markets, clearly there exists no benefit from any loan redistribution.

The above discussion indicates that if the foreign bank does not change its time of loan offer, the home bank would delay its loan offer. By so doing, the home bank could avoid loan overlapping like case (i), which would often occur if it has made its decision before it knows the foreign bank’s offer, and therefore achieve a better outcome. Moreover, the above discussion also implies that the foreign bank equally benefits from the home bank’s delay of offer. To see why, recall from the special case of (i) that the home bank gains from replacing a potential duopolist \( x \) with a potential monopolist \( y \). Consequently, the foreign potential duopolist \( x \) becomes a potential monopolist, which gives the foreign bank a higher expected return. Therefore, both the home bank and the foreign bank are (equally) better off if they announce their loan offer sequentially instead of simultaneously.

If the restriction that banks make their loan offer simultaneously is dropped off, banks might move sequentially and so the chance to see two-way trade will be greatly
reduced. In this case, production specialization occurs not because of any comparative advantage but because banks dislike competition.

3.5 Concluding Remarks and Future Research

Since developing new products is risky (it involves R&D uncertainty and demand uncertainty), credit is rationed in the presence of asymmetric information between lenders and borrowers. When credit rationing occurs, a country may not produce some products which it has the ability to produce and has no comparative disadvantage in producing. A new product will be produced at home only if the innovating firm can borrow from a bank and also succeed in R&D. This may give partial explanation for the indeterminacy of the pattern of international trade between two similar countries.

For future research, this model can be used to examine trade policy implications in the presence of credit rationing. In this regard, we now outline two possible directions and discuss some potential difficult issues that certainly deserve special attention.

Let us first consider the case in which the home government imposes a tariff on a particular product. The following discussion is based on our original model in which the home and foreign banks make decisions simultaneously. Suppose the home government announces a tariff rate on a product before banks set their interest rates. To derive a policy implication, we should pay special attention to the banks' determination on the interest rate of the loan intended to the protected firm and the consequent market equilibrium of the corresponding product. However, since banks make their decisions on the interest rate for the protected firm basing on their
expectation about the return from such a lending, we shall first analyze the tariff’s impacts on the protected firm’s profitability at a given interest rate and then return to discuss the possible influences on the banks’ decisions.

As it is well known in the strategic trade literature, in a competition between a home firm and a foreign firm, tariffs imposed by the home government will shift some market shares from the foreign firm to the home firm. Note also from the analysis in Section 3.2 that a firm’s output decision is independent of the interest rate at which it borrows. These together will imply that at any given interest rate the tariff will increase the protected firm’s expected profit.

One should realize that a higher expected profit to a firm might not necessarily imply a higher expected return to the bank from a loan to this firm or at least this relationship is not obvious since the bank’s return is affected by not only the firm’s expected market return but also the riskiness of the loan and a tariff may change the loan’s riskiness through its impact on the firm’s output decision. However, it may be possible to show that under some not unrealistic assumptions the expected return to the bank from the loan intended to the protected firm will also be increased at any given interest rate.

It is then expected that all banks in the home country will compete among themselves (by using interest rates) for the loan to the protected firm since it is more profitable than the loan to an average firm at the same interest rate. Presumably, elaboration on the model is needed for the existence of an equilibrium interest rate. Only after determining the equilibrium interest rate can we start discussing the tariff
implication. One of the most interesting questions is whether a tariff may help the targeted industry get out of the credit rationing situation.

Policy analysis can also be carried out for R&D subsidization. Presumably the same qualitative results shall also be obtained as those from the case of tariffs. A welfare comparison, however, might invite some difficulties.

Another possibility for future work (not just for policy analysis) is to concentrate on R&D uncertainty in our original model by assuming no demand uncertainty. This will be more interesting and will substantially simplify our analysis. However, we must carefully model the R&D uncertainty in order to give rise to credit rationing.
References


A. Proof of Lemma 1:

(i). Given any strategy of the foreign firm, the domestic firm faces the residual demand (4).

If the domestic firm maximizes its expected profit \( E\pi_1 \) by choice of quantities, then the optimal quantity is

\[
\hat{q}_1 = \frac{A_1 + s_1 - c_1}{2B_1 + c_2 - s_2}
\]  
(73)

and the maximum expected profit given \( s_1 \) and \( s_2 \) is

\[
E\pi_{1q} = \frac{(A_1 + s_1 - c_1)^2}{2(2B_1 + c_2 - s_2)}.
\]  
(74)

However, if the domestic firm chooses a price to maximize its expected profit, the optimal price is

\[
\hat{p}_1 = \frac{(A_1 - s_1 + c_1)B_1 + (c_2 - s_2)A_1}{2B_1 + c_2 - s_2}
\]  
(75)

and the maximum expected profit given \( s_1 \) and \( s_2 \) is

\[
E\pi_{1p} = E\pi_{1q} - \frac{\sigma^2(c_2 - s_2)}{2B_1^2}.
\]  
(76)

The first result of the lemma follows.

(ii). The proof for the foreign firm is similar. We have, if the foreign firm sets a quantity,

\[
\hat{q}_2 = \frac{A_2 - c_1}{2B_2 + c_2} \quad \text{and} \quad E\pi_{2q} = \frac{(A_2 - c_1)^2}{2(2B_2 + c_2)}.
\]
If the foreign firm sets a price, then

$$\hat{p}_2 = \frac{(A_2 + c_1)B_2 + c_2 A_2}{2B_2 + c_2} \quad \text{and} \quad E_{\pi_{2p}} = E_{\pi_{2q}} - \frac{2^2 c_2}{2B_2^2}.$$

Therefore, the second result of the lemma follows.

(iii). (5) is obtained by using (73) and (74) in the welfare function $W$ if $c_2 > s_2$, but using (75) and (76) if $c_2 < s_2$. Q.E.D.

B. Proof of Lemma 2:

First, we show $W_Q^c > W_Q^{pq}$ when $d_1 \leq d \leq d_2$, where $d_1 = \sqrt{kc_2/(b + c_2)}$ and $d_2 = \sqrt{kc_2}$. Define a function $g(y) = \frac{(k-d-y)^2}{(k-y)(k(k-y)-2d^2)}$ for $y \leq d$. Then by (18) and (20), $W_Q^c - W_Q^{pq} > \frac{1}{2}(a - c_1)^2[g(0) - g(d\gamma)]$. But $\partial g(y)/\partial y = -2d(k - d - y)[(k - d)(k - y) - d^2]/(k - y)^2[k(k - y) - 2d^2]^2 < 0$ since $k - d - y > 2(b - d) + c_2 > 0$ and $(k - d)(k - y) - d^2 > (k - d)^2 - d^2 > (b + c_2)^2 - d^2 > 0$. Thus $g(0) > g(d\gamma)$. So the desired inequality holds within the given range.

Second, we extend the result to the case $d < d_1$ without comparing them directly. Suppose that the domestic firm still sets a price when $d < d_1$. Then the optimal expected net-subsidy profit when the domestic firm sets a price is

$$\bar{W}_Q^{pq} = \frac{(a - c_1)^2(k - d\gamma - d)^2}{2(k - d\gamma)[k(k - d\gamma) - 2d^2]} - \frac{\sigma^2 c_2}{2b^2} \quad \text{for} \quad d \leq d_2$$

which can be obtained by going through the two-stage game without having the constraint $s_2 > c_2$. From above, we can infer $W_Q^c > \bar{W}_Q^{pq}$ for $d \leq d_2$. However, both $W_Q^{pq}$ and $\bar{W}_Q^{pq}$ are derived by maximizing the same objective functions but the former has one more constraint $s_2 \geq c_2$. Thus $W_Q^{pq} \leq \bar{W}_Q^{pq}$ for $d \leq d_1$ since the maximized
profit with more constraints can not be greater than the one with fewer constraints.

The result follows. Q.E.D.

C. Proof of Lemma 3:

First, we show

\[ W^p_Q = \frac{(a - c_i)^2(k - d)^2}{2k^2(h - 2s^p_k)} \] (77)

\[ W^b_Q = \frac{(a - c_i)^2[\gamma(\beta + c_2) - \beta\gamma]^2}{2k^2(h - 2s^b_k)} + \frac{\sigma^2(-c_2)}{2\beta^2}. \] (78)

According to (5) in Lemma 1, in a \((q,p)\) competition, the government's problem can be written as

\[
\max_{s_2}\left[\frac{(A_1 - c_1)^2(2B_1 + c_2 - 2s_2)}{2(2B_1 + c_2 - s_2)^2}\right] \quad \text{s.t.} \quad A_1 = \alpha + \gamma p^*_2, s_2 < c_2.
\]

Since \(d > \sqrt{-kbc/(b + c_2)}\), the optimal tax is an interior solution. Then from the F.O.C. we may obtain

\[ A_1 - c_1 = \frac{1}{s_2}\gamma(h - s_2)(h - 2s_2)(\frac{\partial p^*_2}{\partial s_2}). \]

Thus,

\[ W^p_Q = \frac{\gamma^2(h - 2s_2)^3}{2s_2} \left( \frac{\partial p^*_2}{\partial s_2} \right)^2. \] (79)

Differentiating (24) w.r.t. \(s_2\) yields

\[ \frac{\partial p^*_2}{\partial s_2} = -\frac{d(b + c_2)(a - c_1)(k - d)}{k^2(h - 2s^2)^2} = s_2 \frac{(a - c_1)(k - d)}{\gamma k(h - 2s^2)^2}. \]

Substituting the above into (79) gives (77). We can also obtain (78) in the same way (but omit).

Second, we show

\[ W^p_Q = \frac{(a - c_1)^2(k - d)^2}{2k(k^2 - 2d^2)} \] (80)
\[ W_{Q}^{b} = \frac{(a - c_{1})^{2}(k - d - d')^{2}}{2(k - d\gamma)[k(k - d\gamma) - 2d']^{2}} + \frac{\sigma^{2}(-c_{2})}{2\beta^{2}}. \] (81)

Note that \( h = k - 2d\gamma \). If we substitute this and (26) into (77), we have \( W_{Q}^{b} = \frac{1}{2k^{2}}(a - c_{1})^{2}(k - d)^{2}/[k - 2d\gamma + 2d\gamma(b + c_{2})/k]. \) Note also that \( 2d\gamma(b + c_{2}) = 2d\gamma(k - b) = 2kd\gamma - 2bd\gamma = 2kd\gamma - 2d^{2}. \) Then (80) follows. To show (81), we have

\[ h - \gamma^{2}(\beta + c_{2}) = h - \gamma^{2}(h - \beta) = (1 - \gamma^{2})(k - 2d\gamma) + \beta\gamma^{2} = (1 - \gamma^{2})(k - d\gamma) \]

since \( \beta = (1 - \gamma^{2})b \) and \( \gamma^{2}b = d\gamma \). Note that \( \beta\gamma = b(1 - \gamma^{2})\gamma = (1 - \gamma^{2})d \), so

\[ h - \gamma^{2}(\beta + c_{2}) - \beta\gamma = (1 - \gamma^{2})(k - d\gamma - d). \]

Note also that

\[ h - 2s_{2}^{b} = h + \frac{2\beta\gamma^{2}(\beta + c_{2})}{h - \gamma^{2}(\beta + c_{2})} = \frac{h(1 - \gamma^{2})(k - d\gamma) + 2\beta\gamma^{2}(\beta + c_{2})}{h - \gamma^{2}(\beta + c_{2})} \]

but the numerator \( = (1 - \gamma^{2})[h(k - d\gamma) + 2b\gamma^{2}(h - \beta)] = (1 - \gamma^{2})[(k - 2d\gamma)(k - d\gamma) + 2d\gamma(k - 2d\gamma - \beta)] = (1 - \gamma^{2})[k(k - d\gamma) - 2d^{2}] \) because \( d\gamma + \beta = b \) and \( b\gamma = d \). After using all these results and substituting them into (78), (81) follows.

The rest of the proof is the same as that of Lemma 2. Note that the condition that \( \sigma^{2}(-c_{2})/2\beta^{2} \) is small is necessary because it is a positive component in \( W_{Q}^{b} \). Q.E.D.

D. Proof of Proposition 1:

(27) is an immediate result of Lemma 2, Lemma 3, (15), and (26). (28) follows because of (14) and (80). Q.E.D.

E. Proof of (29):

First, \( h - \gamma^{2}(h - \beta) - \beta\gamma = h(1 - \gamma^{2}) - \beta\gamma(1 - \gamma) = (1 - \gamma)[h + \gamma(h - \beta)]. \)

Substituting this into (12) yields

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\[ s_1^b = -\frac{\beta \gamma^2(a - c_1)(h - \beta)(h - \gamma^2(h - \beta) - \beta \gamma)}{[h - (h - \beta)\gamma^2][h^2 - (h - \beta)\gamma^2c_2]} \]

\[ = s_2^b (a - c_1)[h - \gamma^2(h - \beta) - \beta \gamma] \frac{1}{h^2 - (h - \beta)\gamma^2c_2}. \]

Second, from the F.O.C. of maximizing \( W^b_L \) we obtain \( \alpha + \gamma p_2^* - c_1 = \frac{s_1^b}{(\gamma \frac{\partial p_2^*}{\partial s_1})} \), but from (10) \( \frac{\partial p_2^*}{\partial s_1} = -\beta \gamma(h - \beta)/[h^2 - \gamma^2(h - \beta)^2] \). Using these results, the maximum expected welfare becomes

\[ W^b_L = \frac{1}{2h} \left[ \frac{(s_1^b)^2}{\gamma^2(\frac{\partial p_2^*}{\partial s_1})^2} - (s_1^b)^2 \right] + \frac{\sigma^2(-c_2)}{2\beta^2} \]

\[ = \frac{(a - c_1)^2[h - \gamma^2(\beta + c_2) - \beta \gamma]^2}{2[h - \gamma^2(\beta + c_2)][h^2 - \gamma^2(\beta + c_2)c_2]} + \frac{\sigma^2(-c_2)}{2\beta^2}. \]

Finally, we show \( [h - \gamma^2(\beta + c_2)](h - 2s_2^b) = h^2 - \gamma^2(\beta + c_2)c_2. \) Utilizing (23) we have \( \text{LHS} = [h - \gamma^2(\beta + c_2)][h + \frac{2\beta \gamma^2(\beta + c_2)}{h - \gamma^2(\beta + c_2)}] = h^2 - h\gamma^2(\beta + c_2) + 2\beta \gamma^2(\beta + c_2) = h^2 - \gamma^2(\beta + c_2)(h - 2\beta) = \text{RHS}. \) Using this result and comparing (83) with (78), (29) immediately follows. Q.E.D.
APPENDIX II

A. Proof of Lemma 4:

By (35), $\pi^L(t_H) - \pi^H(t_H) = 4(c_H - c_L)(a - c_L - c_H + 2s_H)/9b$. Since $\pi^L(t_L) \geq \pi^L(t_H)$ from the separation constraint, we obtain

$$\pi^L(t_L) - \pi^H(t_H) \geq \frac{4}{9b}(c_H - c_L)(a - c_L - c_H + 2s_H).$$

(84)

Similarly,

$$\pi^H(t_H) - \pi^L(t_L) \geq -\frac{4}{9b}(c_H - c_L)(a - c_L - c_H + 2s_L).$$

(85)

Combining (84) and (85) yields

$$\frac{4}{9b}(c_H - c_L)(a - c_L - c_H + 2s_H) \leq \frac{4}{9b}(c_H - c_L)(a - c_L - c_H + 2s_L).$$

So $s_L \geq s_H$. Using $\pi^H(t_H) \geq \pi^H(t_L)$ once again, we have

$$\frac{(a - 2c_H + 2s_H)^2}{9b} - \tau_H \geq \frac{(a - 2c_H + 2s_L)^2}{9b} - \tau_L$$

which confirms $\tau_H \leq \tau_L$. Q.E.D.

B. Proof of Proposition 3:

We shall show that $t^*$ induces separation. By using optimal subsidy levels (37) and the normalization $\tau^*_H = 0$, we obtain

$$\pi^L(t^*_L) = \frac{(a - 2c_L)^2}{4b} - \tau^*_L, \quad \pi^H(t^*_H) = \frac{(a - 2c_H)^2}{4b} - \tau^*_H.$$
Therefore, $\pi^L(t^*_L) \geq \pi^L(t^*_H)$ iff

$$\tau^*_L \leq \frac{1}{4b}[(a - 2c_L)^2 - (a - c_L - c_H)^2] = \frac{1}{4b}(c_H - c_L)^2 + \frac{1}{2b}(c_H - c_L)(a - c_L - c_H)$$

and $\pi^H(t^*_H) \geq \pi^H(t^*_L)$ iff

$$\tau^*_L \geq \frac{1}{4b}[(a - c_L - c_H)^2 - (a - 2c_H)^2] = \frac{1}{2b}(c_H - c_L)(a - c_L - c_H) - \frac{1}{4b}(c_H - c_L)^2.$$

Hence the separation constraints are satisfied iff

$$-\frac{1}{4b}(c_H - c_L)^2 \leq \tau^*_L - \frac{1}{2b}(c_H - c_L)(a - c_L - c_H) \leq \frac{1}{4b}(c_H - c_L)^2.$$ 

Obviously, $\tau^*_L$ defined as in (39) satisfies the above condition. Thus $t^*$ induces separation. Q.E.D.
Figure 1

Figure 2

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