OFDM/FM FRAME SYNCHRONIZATION
FOR MOBILE RADIO DATA COMMUNICATION

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Abstract

A synchronization scheme enabling the use of OFDM/FM in a pure ALOHA environment over a mobile radio channel is proposed, implemented, and tested. The synchronization scheme encodes synchronization information in parallel with data in the same manner in which data is encoded in the OFDM/FM frame. The encoded synchronization information is in the form of tones, centered in reserved frequency sub-channels of the OFDM signal. The receiver uses a correlation detector, implemented in the frequency domain, to accurately acquire synchronization on a packet by packet basis. Experimental results indicate that BER performance with synchronization is achieved to within 1.5 dB of the performance achievable with ideal synchronization.
# Table of Contents

Abstract .......................................................... ii

List of Tables ......................................................... vi

List of Figures ......................................................... vii

Acknowledgement ..................................................... ix

1 Introduction ......................................................... 1
   1.1 Mobile Data Communication ........................................ 1
   1.2 OFDM/FM for Mobile Radio Data Communication ..................... 3
   1.3 Scope of Thesis .................................................. 4
   1.4 Organization of Thesis .......................................... 4

2 OFDM Modulation, Implementation Configuration, FM Fading Channel 5
   2.1 OFDM Modulation ............................................... 5
      2.1.1 Historical Perspective and Motivation ......................... 5
      2.1.2 Basic Principles of Operation ................................ 6
   2.2 Implementation Configuration ..................................... 8
   2.3 FM Fading Channel ............................................. 10
      2.3.1 Rayleigh Fading Channel .................................... 10
      2.3.2 FM Transceiver Characteristics ............................. 10

3 Review of Current Synchronization Techniques .......................... 21
   3.1 Purpose and Hierarchy of Synchronization .......................... 21
   3.2 Serial Transmission ............................................ 22
# 6 Conclusions

6.1 Conclusions .......................................................... 75
6.2 Topics for Further Research ......................................... 76

## Bibliography

## A Models and Algorithms for Synchronization

A.1 Synchronization Model .............................................. 83
A.2 Frequency Domain Derivation of the Correlation Detector .......... 85
A.3 Fine Tune Algorithm .................................................. 86

## Appendices

## B DFT Algorithms

B.1 FFT Implementation .................................................. 88
B.2 DFT Implementation .................................................. 89
B.3 Sliding Window DFT ................................................. 89
B.4 Accumulated Round-Off Error for the Sliding Window DFT .......... 91

## C Digital Low Pass Filter

C.1 Filter Requirements ................................................. 93
C.2 IIR vs. FIR ............................................................ 93
C.3 FIR Filter Design ..................................................... 95

## D Modulating Signal Power Prediction

## E Estimation of FM Channel Attack Time
List of Tables

5.1 Experiment List Summary ........................................... 63
5.2 Synchronization Sub-channel selection for 1024 block size .......... 67
List of Figures

1.1 Typical Mobile Data Communication System ........................................ 1
1.2 A sample plot of the received signal level........................................... 2

2.1 Transient Response of the ICOM-2ATs connected back-to-back. ............. 12
2.2 Transmitter Attack Time of ICOM-2AT .............................................. 13
2.3 Estimated FM Channel Attack Time of ICOM-2AT ................................. 15
2.4 Receiver output power vs. IF SNR .................................................... 16
2.5 Noise Power Distribution for modulating signal with constant power across
  spectrum ......................................................................................... 17
2.6 Noise Power Distribution for modulating signal with 10 dB per decade de-
  emphasis ...................................................................................... 18
2.7 Scatter plot of received data without channel equalization ..................... 19
2.8 Scatter plot of received data with channel equalization ......................... 20
2.9 Channel Equalization Transform of transmitted OFDM signal for 1024 sam-
  ple block size with 10dB per decade de-emphasis ................................ 20

4.1 Synchronization Block Diagram ......................................................... 35
4.2 Synchronization - Phase I .................................................................. 36
4.3 Synchronization - Phase II .................................................................. 37
4.4 Synchronization - Phase III .................................................................. 38

5.1 FM Fading Channel ............................................................................ 45
5.2 Digital FM Channel ............................................................................ 47
5.3 Tx and Rx Gain Units .......................................................................... 50
5.4 Tx Switch ............................................................................................ 51
5.5 Mid-rise model of 8 level A/D quantization ........................................... 53
5.6 A/D SNR versus Gaussian Standard Deviation ................................. 54
5.7 OFDM Baseband Signal Spectrum .................................................. 57
5.8 OFDM RF Signal Spectrum ....................................................... 58
5.9 Discriminating among the four classifications of synchronization .. 62
5.10 Phase I performance in the absence of a transmitted signal ........... 64
5.11 Phase I performance in the presence of a transmitted signal. Doppler rate is: \( f_d = 10 \text{Hz} \) ......................................................... 64
5.12 Phase I performance in the presence of a transmitted signal. Doppler rates are: \( f_d = 20 \text{Hz} \) and \( f_d = 50 \text{Hz} \) ........................................ 65
5.13 Phase II performance .............................................................. 67
5.14 Phase III performance - Absolute Results .................................. 69
5.15 Phase III performance - Relative to Ideal. \( f_d = 10 \text{Hz} \) .............. 70
5.16 Integrated System Synchronization Performance .......................... 71
5.17 Integrated System BER Performance ............................................ 72
5.18 Performance Loss Due to Synchronization ....................................... 73

A.1 Sync signal phasor diagram ....................................................... 86

B.1 DFT calculations on sliding window ............................................. 90

C.1 Modified signal spectrum and LPF spectrum .................................. 94
C.2 Interpolation filter impulse response ........................................... 96

D.1 OFDM Baseband Signal Generation ............................................. 97

E.1 Hardware Configuration to Estimate the Attack Time of the ICOM-2AT FM Channel .......................................................... 100
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Chapter 1

Introduction

1.1 Mobile Data Communication

Figure 1.1: Typical Mobile Data Communication System

Reliable and efficient data communication between a base station and mobile users is of interest to many in the business and service industry. Examples of mobile user equipment currently in use are [1,2]:

- The 7100-11FK mobile data terminal and the KDT 840 portable data terminal from Mobile Data International Inc.
• The GL1110 data link radio modem and GL4160A fax compatible radiotelephone for wide area mobile phone service from Glenayre Electronics.

A system consisting of a base station interfaced to a central computer system and a multitude of mobile data terminals (MDT’s) allows access by a mobile user to a central database. This capability is used by law enforcement agencies to obtain motor vehicle information or criminal record information while on patrol. MDT’s are also beneficial to dispatch services such as taxi cabs and couriers. Figure 1.1 shows a typical mobile data communication system.

The communication link is achieved through radio transceivers (transmitter/receiver). The processing of the received radio signals to recover the transmitted data sent to and from mobile sources is a challenging problem [3]. In an urban environment, radio signals transmitted by a stationary source are reflected and scattered by buildings and other
objects. This results in a pattern of constructive and destructive interference forming a varying field strength. As the mobile moves through this interference pattern, the received signal level fluctuates. A similar effect occurs when a mobile transmits a signal to a stationary receiver. Figure 1.2, taken from Casas [4], was generated using the Rayleigh fading model. It shows the deep fades that are characteristic of the received signal.

1.2 OFDM/FM for Mobile Radio Data Communication

Orthogonal Frequency Division Multiplexing (OFDM) has been proposed and investigated as an approach to the problem of fading [5,4]. OFDM is a modulation technique that frequency division multiplexes a serial data stream and transmits the data in parallel. This allows the baud rate to be reduced without decreasing the bit rate. The serial data stream is frequency mapped into many quadrature amplitude modulated\(^1\) sub-carriers [6]. Each transmitted symbol can convey hundreds or thousands of bits of data. Thus, the symbol duration can be increased by hundreds or thousands without decreasing the data rate. The increased duration of each symbol makes it less likely that the entire symbol will be severely faded. Signal fading results in cross-talk among the sub-channels of the OFDM signal and degrades performance.

Cimini [5] investigated the use of OFDM Single Side Band (SSB) transceivers. Casas [4] also studied the use of OFDM. His work differed from Cimini's work in that narrow band Frequency Modulation (FM) transceivers were used. Both approaches had advantages: OFDM/SSB is more spectrally efficient and power efficient, but OFDM/FM does not suffer from effects of frequency offsets, and the system can be easily retrofitted to existing FM communications systems.

A simplifying assumption made in [4,5] was that perfect symbol synchronization could be achieved. It is the purpose of this thesis to propose and evaluate a technique to achieve

\(^1\)see section 2.1.2
Chapter 1. Introduction

this synchronization.

1.3 Scope of Thesis

The OFDM/FM configuration used in [4] is selected. The motivating reason is the availability of the actual equipment used by Casas [4] allowing direct comparison to his previously established performance results.

It is assumed that the only sources of signal degradation are additive white Gaussian noise and Rayleigh fading. The effects of log normal fading (shadow fading) are not considered.

The main contributions of this thesis are:

- proposal of an algorithm capable of providing good synchronization performance,
- refinement of the synchronization algorithm to reduce computational complexity,
- implementation of a prototype OFDM/FM system to measure the performance of the synchronization algorithm.

1.4 Organization of Thesis

Chapter 2 provides background information on OFDM modulation, characteristics of the FM fading channel, and outlines the configuration of the implemented test system.

Chapter 3 contains a review of some currently used synchronization techniques for serial and parallel data communication.

Chapter 4 presents the proposed synchronization technique in detail and its similarities and differences to current techniques.

Chapter 5 provides details of the test system, including parameter selection. Results of experiments conducted are presented and discussed.

Chapter 6 summarizes the contributions of the thesis and recommends topics for further research.
Chapter 2

OFDM Modulation, Implementation Configuration, FM Fading Channel

2.1 OFDM Modulation

2.1.1 Historical Perspective and Motivation

Orthogonal Frequency Division Multiplexing (OFDM) is a member of a larger class of modulation schemes referred to as Multi-Carrier Modulation (MCM). MCM has also been referred to as Multi-Frequency Modulation (MFM) [7]. MCM, first used over 30 years ago [8], has been of continuing interest since, but has not moved beyond the realm of peripheral interest until recently [9]. One of the reasons for the recent interest in MCM is that the long symbol time, characteristic of MCM, produces a much greater immunity to impulse noise and fast fades [9].

MCM's greater immunity to fast fades has prompted researchers [4,5] to propose and investigate the use of OFDM as an approach to the problem of fading in the mobile radio environment. In MCM, when efficient use of bandwidth is not required, the most effective parallel system uses conventional FDM, in which the frequency sub-channel spectra do not overlap, and passband filters [5]. In mobile radio communication, however, spectrum is a precious commodity and efficient use of bandwidth is important [5]. In contrast to conventional FDM, the individual spectra of the OFDM sub-carriers overlap each other providing a better utilization of available bandwidth. Despite this overlap, complete separation of the signals by the receiver is possible due to the designed orthogonality of the sub-carriers [10].

The greater immunity of OFDM to the fast fades of the mobile radio environment results from the effect of a fade being spread over all sub-carriers (assuming non-frequency
selective fading) [9]. Instead of a few bits of data being severely distorted, all bits of data in the block are only slightly or moderately distorted. The distribution, or averaging, of noise over all sub-carriers results in demodulated data values distorted by noise with a distribution similar to Gaussian. This is in comparison to a bursty noise distribution that would result if serial modulation techniques were used.

The process of modulation and demodulation can be implemented quite simply by using the Discrete Fourier Transform (DFT). Computational requirements can be reduced using the Fast Fourier Transform (FFT) implementation of the DFT. This technique was originally proposed by [10] and later used in [9,4,5,11,12]

2.1.2 Basic Principles of Operation

In OFDM, the sub-carrier frequencies are chosen to be spaced at the symbol rate, that is, if the OFDM symbol duration is $T$ seconds, the sub-carrier frequency spacing is $1/T$ Hz. With this frequency spacing, the sub-carriers are orthogonal when viewed over one symbol interval [6,10,13,5,4]. This allows modulation and demodulation using the DFT in its efficient FFT form [10].

During transmission of data, the sub-carriers are “keyed” by the stream of serial data. The incoming serial data is collected in blocks of $K$ bits. To each sub-carrier, a smaller grouping of bits (usually 2 to 5) is assigned. These bits are used to set the phase and magnitude of the sub-carrier. Typically, a $2^m$-QAM constellation is used to encode the data into the phase and magnitude of the sub-carrier, where $m$ is the number of bits assigned to the sub-carrier. From this perspective, OFDM can also be considered as Orthogonal Frequency Division Multiplexed Phase-Amplitude Shift Keying (OFDMPASK). For the purpose of this thesis, the more general term, OFDM, will be used.

Conversion of the serial data to the modulating signal is actually performed by an $N$-point Inverse FFT. The $m$-bit groupings of data are encoded as complex values, $a + jb$, defining points in the $2^m$-QAM constellation. The complex encoded data values become
the data for the Inverse FFT, their location in the Inverse FFT data array corresponding to their assigned sub-carrier channel position in frequency. If specific sub-carriers are not used, the corresponding data values in the Inverse FFT data array are set to zero. Only the lower half of the data array is used to encode sub-carrier information. The upper half of the data array is encoded as the conjugate symmetry of the lower half. The purpose of this is to produce an \(N\)-sample real modulating signal when the Inverse FFT is performed.

The real data samples generated by the Inverse FFT are converted to an analog waveform using the appropriate digital-to-analog (D/A) converters and reconstruction filters.

The analog modulating signal can be transmitted directly as a baseband signal, or as in the case of mobile radios, used to modulate an RF transmitter.

At the receiver, the reverse sequence of events occurs. The received baseband signal is sampled and converted to digital form. \(N\) consecutive samples are grouped to form an OFDM block. An \(N\)-point FFT is performed to determine the phase and magnitude of the sub-carriers. For each sub-carrier, the transmitted bit information is extracted by determining the location in the complex plane defined by the received sub-carrier and locating the nearest constellation point. Determination of the constellation point identifies the transmitted \(m\)-bit data sequence.

Assuming the following conditions, the transmitted data can be recovered without error.

- Ideal synchronization of OFDM frame.
- No distortion of the magnitude or phase of the sub-carriers due to system hardware.
- No degradation of the orthogonality of the sub-carriers due to fading and other sources of noise.

The above ideal conditions are not realistically achievable. In this and the following
chapters, these problems are more thoroughly examined.

In describing the basic principles of operation, it is assumed that the number of bits assigned to each sub-carrier is the same and that the total number of bits in the block is fixed. This is not a requirement of OFDM. As discussed in section 2.3.2 under *Noise Power Distribution*, the number of bits assigned to each sub-carrier can vary from one sub-carrier to the next.

The performance of OFDM is dependent on the size of the FFT block used. When a larger block size is used, the effect of a fade is spread over a greater number of sub-channels, reducing the distortion in each sub-channel. Casas [4] studied the performance of OFDM for FFT block sizes of 256, 512, 1024, 2048 and 4096. Results showed improving performance with increasing block size.

The performance of OFDM is also dependent on the rate of fading, a measure of the duration between fades and the duration of fades. The rate of fading is specified as the *doppler rate*, $f_d$, and is defined by

$$f_d = \frac{f_c v}{c}$$

(2.1)

where $f_c$ is the carrier frequency, $v$ is the vehicle speed, and $c$ is the velocity of propagation.

For OFDM/FM, Casas [4] examined the relation between block size, doppler rate and system performance. The product, $Tf_d$, of the block duration, $T$, and the doppler rate, $f_d$, was defined. Test conditions having equal $Tf_d$ values tended to produce similar performance.

### 2.2 Implementation Configuration

The implementation configuration chosen for the test system is based on a centralized packet radio broadcasting network. The characteristics of this type of network are [14]:

- packet transmission
• reception by many or all stations

• common transmission medium

• one central controller, multiple nodes

In the context of this thesis, the MDT, or mobile, is a node and the base station is the central controller. For the remainder of this thesis, the terms mobile and base station will be used.

The MDI NCP 3000 Network Control Processor [2] is an example of a mobile data transmission system based on a centralized packet radio broadcasting network.

Since all mobiles communicate with the central controller via a common medium, a Medium Access Control Protocol (MACP) is required to regulate the access to the channel. Packet radio networks typically use random access or contention techniques [14]. Contention techniques can be either slotted or un-slotted. In slotted systems, the receiver has a priori information as to when a data packet may arrive. In an un-slotted system, the receiver has no a priori information concerning the arrival times of data packets, which may arrive at random times.

Implementation of a MACP is beyond the scope of this thesis. The implemented system has one central controller and only one mobile. Transmission will occur in the direction of controller to mobile only. The mobile has no a priori information regarding arrival times of data packets.

In many data transmission systems, packet lengths are allowed to vary during transmission [14]. Information is contained within the packet identifying its length. In this thesis, only fixed packet lengths of 1024 samples are allowed. Implementation of variable length packets is left for future work.
Chapter 2. OFDM Modulation, Implementation Configuration, FM Fading Channel

2.3 FM Fading Channel

The OFDM baseband signal is used to modulate an FM transmitter, thereby broadcasting the signal to the mobile (from the base station) or to the base station (from the mobile).

Surrounding buildings and other structures create an interference pattern with areas of constructive interference and areas of destructive interference. Motion of the mobile through this interference pattern results in a received signal whose instantaneous power fluctuates greatly. This power fluctuation can be modelled by the Rayleigh fading model. A similar effect occurs for transmission from the mobile to the base station.

It is important to know the characteristics of the fading channel and the hardware used in the implementation, since limitations on system performance can be imposed by either. The following sections describe these characteristics and how they affect the design and performance of the OFDM/FM system.

2.3.1 Rayleigh Fading Channel

2.3.2 FM Transceiver Characteristics

Two ICOM-2AT transceivers [15] are used for the experimental work, one as the transmitter and one as the receiver. They were previously used by Casas in his initial study of OFDM/FM [4]. These are amateur radio hand-held transceivers. An operating frequency of 144.15 MHz was chosen.

Transient Response

When a modulating signal is input to the transmitter, a transition in modulating signal power from zero to maximum power occurs causing transients in the transmitter. The magnitude of the transients are dependent on the abruptness of the transition and result in a temporary distortion of the transmitted signal and thus affects the signal at the receiver output. Likewise, when the receiver experiences abrupt changes in the
modulation of the received carrier signal, transients are generated, further distorting the receiver output signal.

Transients in either the transmitter or the receiver result in a distorted received signal. This distortion degrades the orthogonality of the OFDM baseband signal. Therefore, it is important to allow transients to decay before decoding is performed. Casas ensured this by preceding the OFDM modulating signal with a periodic extension of the signal itself. The duration of this periodic extension was set excessively long to ensure that system transients were negligible when the received baseband signal was sampled for decoding. Note that the extension must be a periodic extension of itself, so that abrupt phase changes do not occur at the junction of the extension and the data block. In this thesis, the preceding periodic extension is referred to as the pre-extension.

The pre-extension of the OFDM block manifests itself as overhead, reducing the system throughput. Thus, it is desirable to minimize the duration of the pre-extension.

The transient response of the ICOM-2AT was measured for the combined effect of both the transmitter and receiver. A pulse was input to the transmitter and the output of the receiver was monitored. Monitoring of the signal was achieved using a Textronix 2232 digital storage oscilloscope triggered by the transmitter input pulse. Figure 2.1 gives the results of the transient response test. The lower trace shows the pulse input to the transmitter. The upper trace shows the resulting output from the receiver. Results are shown for 2 consecutive cycles with a repetition period of approximately 1.8 milliseconds. From the oscilloscope tracing, it appears that by the time the second pulse input occurs, the receiver output transients have faded significantly. To obtain a conservative estimate of the transient duration, the measured duration of 1.8 milliseconds is doubled and rounded up to the nearest integer. Therefore, the minimum duration of the pre-extension is set to 4 milliseconds. This corresponds to 32 samples at an 8 kHz sampling rate. For an OFDM block size of 1024 samples, this is approximately 3% overhead.
Chapter 2. OFDM Modulation, Implementation Configuration, FM Fading Channel

Radio Attack Time

Radio Attack Time is specified separately for transmitter and receiver [16,17,18].

- *Transmitter Attack Time*: time required to produce carrier power output after operation of the transmitter control switch.

- *Receiver Attack Time*: time required to produce audio power output after application of a modulated RF signal.

The minimum standard for Transmitter Attack Time is for the carrier level to increase to 50% of its maximum power in less than 100 milliseconds [16,18]. The minimum standard for Receiver Attack Time is for the audio power output to reach 90% of its rated power output in less than 150 milliseconds [17,18].
Radio attack time is a major source of transmission overhead. It imposes a lower limit on the time interval required between transmitted blocks. Larger time intervals are required for longer radio attack times.

![Figure 2.2: Transmitter Attack Time of ICOM-2AT](image)

Following the measurement procedures standardized by the Electronic Industries Association (EIA) [16,17,18], the transmitter attack time for the ICOM-2AT was measured. For the measurement, a spectrum analyzer was used to monitor the carrier power instead of connecting a linear peak carrier detector to the vertical plates of an oscilloscope as specified in the EIA standard. The result is shown in figure 2.2. The transmitter attack time was measured to be approximately 75 milliseconds. The equipment necessary to measure the receiver attack time was not available, hence the measurement was not performed.
Chapter 2. OFDM Modulation, Implementation Configuration, FM Fading Channel

A third parameter of interest, not defined by EIA standards, is the combined attack time for the Transmitter-Receiver system. The motivation for this measurement is that the response of the receiver to the transmitter during its power-on transition phase is not indicated by the EIA measurements. It may be possible that the receiver is sensitive enough to the presence of a carrier that it meets its specified criteria before the transmitter reaches its specified criteria. For purposes of this thesis, this measurement will be referred to as the FM Channel Attack Time. The FM Channel Attack Time is defined as the time required to produce audio power output from the receiver after application of a modulating signal to the transmitter and operation of the transmitter control switch. The measurement procedure is similar to that of the receiver attack time. Since the required equipment was not available, the measurement was not performed. A measurement procedure to estimate the true FM channel attack time is detailed in Appendix E.

The result of the FM channel attack time estimate is shown in Figure 2.3. The conclusion from the experiment is that the FM Channel Attack Time for the ICOM-2AT transmitter-receiver system is approximately 80 milliseconds.

Receiver Output Power

When the receiver squelch is disabled, the audio output power of the ICOM-2AT receiver is greater in the absence of a modulated or un-modulated carrier than in the presence of a carrier.

Figure 2.4 plots the receiver audio output power in the presence of OFDM modulated and un-modulated carriers at various IF SNR levels. The receiver output power approaches a maximum as the IF SNR level decreases. The receiver output power approaches a minimum as the IF SNR level increases.
Noise Power Distribution

If an OFDM modulating signal, whose power is distributed evenly over its spectrum, is transmitted over the FM channel, the baseband SNR of the received signal is not constant over its spectrum. Figure 2.5 shows this effect for the channel condition of IF SNR = 10dB and $f_d = 10$ Hz. Since the signal mean is constant over the band, the uneven distribution of noise power results in varying SNR over the sub-channels of the received signal. The trend shown in the figure 2.5 indicates that the SNR improves in the higher frequency sub-channels. This corresponds to the results of Casas [4], who noted that the lower sub-channels experienced higher BERs relative to the higher sub-channels, the effect becoming more pronounced at lower IF SNR levels. Casas determined that de-emphasizing the modulating signal by 10dB per decade tended to equalize the BERs of
Chapter 2. OFDM Modulation, Implementation Configuration, FM Fading Channel

the received signal sub-channels. The results of applying 10dB/decade de-emphasis to the modulating signal is shown in figure 2.6. The distribution of the noise power has flattened, giving a more constant SNR over the band. To maximize the attainable bit rate at a given error rate and SNR, Bingham [9] has indicated that the optimum power distribution should be calculated by a “water-pouring” procedure that is similar to that of Gallager [19].

Alternatively, BERs can be equalized over the sub-channels by employing a technique known as adaptive loading [9]. In this technique, more bits are assigned to the sub-channels with higher SNR and fewer bits are assigned to the sub-channels with lower SNR.

To achieve optimal BER performance, a combination of adaptive power distribution and adaptive loading should be used [19,20].

To maintain similarity to the system in [4], the method of 10dB per decade de-emphasis is employed. The implementation is simple and yields reasonably good results [4].
Chapter 2. OFDM Modulation, Implementation Configuration, FM Fading Channel

IFSNR = 10dB, $f_d = 10Hz$

Figure 2.5: Noise Power Distribution for modulating signal with constant power across spectrum

Channel Hardware Equalization

In Figure 5.2, a block diagram of the OFDM/FM data transmission system is presented. Several blocks have been grouped together and labelled as a Digital FM channel. This channel accepts a stream of serial digital data. The data is processed to generate digital data defining the OFDM baseband modulating signal. The digital OFDM signal is passed through a D/A converter and appropriate analog reconstruction filters and gain units to yield an analog OFDM signal suitable for modulating an FM transmitter. At the receiving end, the inverse functions occur in the reverse order.

The hardware comprising the digital FM channel, specifically the analog components, introduce phase and amplitude distortions. These phase and amplitude variations must be accounted for when decoding the OFDM signal (note: with 4-QAM encoding, amplitude equalization is not important since only the phase is used in the decoding process, however, when decoding larger constellations, such as 16-QAM, amplitude equalization becomes important.)

To measure the compensation required, a training procedure is performed in which
known blocks of data are transmitted and received. The received data blocks are compared to the transmitted data blocks to determine the mathematical transformation required to counteract the phase and magnitude distortions. To eliminate the necessity of synchronizing the transmitter and receiver, a loop-back configuration is employed in which only one modem, the base station, is used for both transmission and reception. Using a loop-back configuration will introduce error into the training sequence since the receiver characteristics of the mobile may differ from the receiver characteristics of the base station. Therefore, an end-to-end training sequence would be preferable. However, the loop-back configuration appears to give reasonable results and is simple to implement. It is, therefore, the chosen method.

The effect of channel equalization is shown in Figures 2.7 and 2.8. Both figures are scatter plots of QAM encoded data extracted from a received OFDM signal with a channel condition of: IF SNR = 15dB and no fading. In Figure 2.7, no channel equalization has been performed. Figure 2.8 is identical to Figure 2.7 except that channel equalization has been performed. It can be seen that with channel equalization, the received QAM encoded data is clustered around the four QAM constellation points. Without channel
equalization, the data is scattered and the signal points are not evident.

![Figure 2.7: Scatter plot of received data without channel equalization](image)

The training procedure was performed for the ICOM-2AT based digital FM channel. The results for 10 transmissions of OFDM blocks generated from random data are averaged and are shown in Figure 2.9. The training procedure was performed periodically over the course of experimentation with no significant visible changes in the equalization transform. However, with analog components, drift is inevitable. A practical system might employ an adaptive equalization vector to compensate for this drift.
Figure 2.8: Scatter plot of received data with channel equalization

Figure 2.9: Channel Equalization Transform of transmitted OFDM signal for 1024 sample block size with 10dB per decade de-emphasis.
Chapter 3

Review of Current Synchronization Techniques

In this chapter several synchronization techniques currently used in data communications are examined. Although OFDM is a parallel transmission scheme, the examination extends to both parallel and serial techniques.

3.1 Purpose and Hierarchy of Synchronization

The purpose of synchronization is to provide a frame of reference from which something or someone, a receiver, is able to correctly extract information from a signal that it has received from a sender.

In computer data communications, the process of synchronization occurs at one or more layers of abstraction. In many ways, the multi-layered process, or hierarchy of synchronization, is similar to the syntactic structure of the text of this thesis. Consider processing this text, from which all punctuation has been removed and the capitals beginning each sentence suppressed:

- individual characters can easily be identified but it would be very difficult if not impossible to accurately interpret the meaning of the text

Individual characters can easily be identified, but it would be very difficult, if not impossible, to accurately interpret the meaning of the text.

Reinserting the spaces allows the reader to easily identify each word. However, it is still likely that errors in interpretation will be made. Reinserting punctuation further simplifies the interpretation process and fewer errors are likely to occur.
In computer data communications, the implementation of synchronization may differ from the syntax of this text, but the concepts are similar.

3.2 Serial Transmission

Serial transmission of digital data can be achieved with either digital or analog signals [14]. For analog signalling, synchronization begins with carrier synchronization followed by bit synchronization. For digital signalling, synchronization begins with bit synchronization.

Two methods are common for digital signal synchronization, one for asynchronous transmission and the other for synchronous transmission [14].

3.2.1 Asynchronous Transmission

In asynchronous transmission, bits are sent one character at a time, with characters ranging in size from 5 to 8 bits. The receiver resynchronizes at the start of each character, and must maintain synchronization within each character. This technique is simple, but it requires a start and stop bit for each character. This results in 2 to 3 bits of overhead per character. Higher levels of synchronization are employed to group characters into messages.

3.2.2 Synchronous Transmission

A more common technique, used in demanding applications, is synchronous transmission. It provides more efficient communication than asynchronous transmission. To improve efficiency, blocks of characters or bits are transmitted without start or stop symbols. Synchronization within the block is accomplished by exploiting coding schemes that have embedded clocking information, such as biphase encoding, or using the carrier to maintain synchronization for analog signals. Synchronizing to the block requires the detection of the start and end of the block. To achieve start detection, each block begins with a
preamble bit pattern. This is a unique bit pattern that the receiver will recognize as the start of the block. To achieve end detection, either a postamble bit pattern is used at the end of the block, or control information is included within the block to indicate its length.

In a noisy environment, bit errors during transmission are likely to occur. If one or more of the bits in the preamble are in error, the receiver will not detect the start of the block and the data will be lost.

Barker [21] first examined the problem of detecting synchronization patterns in a noisy channel. Current techniques are based on his initial work. Barker's technique is based on the cross-correlation function, defined as:

$$ R_{xy}(n) = \sum_{k=0}^{M-1} x_k y_{n+k} \quad (3.1) $$

where $x_k$ and $y_k$ are the sampled values of two signals, and $M$ is the number of signal data samples used in estimating the cross-correlation. Generally speaking, the cross-correlation function measures the similarity between two sets of data. Synchronization patterns are prefixed to the data to be transmitted. During transmission, the synchronization pattern may be corrupted by errors. A receiver, implementing the cross-correlation function, measures the similarity between the expected synchronization pattern and the received signal pattern. If a high measure of similarity is detected, then it is likely that the synchronization pattern has been detected.

Although correlation detection has been shown to be sub-optimal [22], it provides good performance and is still used. The effectiveness of this method depends on the selection of the synchronization, or preamble, pattern. The ideal preamble pattern will have an auto-correlation defined as:

$$ R_{xx}(n) = \begin{cases} K, & n = 0 \\ 0, & \text{otherwise} \end{cases} \quad (3.2) $$

where $K$ is a finite value whose magnitude depends on the magnitude of the signal $x(n)$,
and a cross correlation (with random data) defined as:

\[ R_{xy}(n) = 0, \quad \text{all } n. \] (3.3)

Since the initial work by Barker, much work has been focused on finding synchronization patterns having the desired autocorrelation and cross correlation properties.

In practice, these two conditions cannot be achieved exactly; however, preamble patterns of various lengths have been discovered that provide excellent results [23].

The first set of synchronization patterns were discovered by Barker. These patterns, now known as Barker sequences [21], have the property that the maximum absolute value of the autocorrelation sidelobes is \( M = 1. \) Unfortunately, there are only a total of 8 known Barker sequences, ranging from 2 bits to 13 bits in length.

More recently, Linder performed a brute force computer search to find synchronization sequences of length up to 40 with good correlation properties. Barker sequences are a subset of Linder sequences [23].

It should be noted that a synchronization sequence that is optimal for one application is not necessarily optimal in general. For example, Maury and Styles found a set of optimal synchronization sequences for deep space telemetry, but, as Wu [23] pointed out, for digital satellite communications, sequences exist that exhibit better correlation properties than the Maury and Styles sequences.

Radar tracking systems [24] using pulse compression are similar. The transmitted signal is a carefully selected signal having an auto-correlation function with low sidelobes. The receiver picks up the reflected signal and uses a matched filter to detect the known pattern and determine the time duration from transmission of the signal until reception. The oldest, and best known, type of pulse compression code (PCC) is the chirp. Basically, the chirp is a sinusoid whose frequency is linearly increased throughout the duration of the signal. Binary phase coding is the second most common type of PCC. Of the family of binary phase codes, Barker codes are the most frequently used, primarily because they achieve sidelobes that are relatively small compared to other binary codes.
In binary phase codes, the transmitted signal is divided into a number of segments of equal duration. The phase of each segment is switched between two binary values in accordance with a predetermined code sequence. Frank codes [24], a family of polyphase codes closely related to the chirp and Barker codes, are also used. Frank codes are basically a digital version of the analog chirp [24]. These codes, and many others used in radar tracking systems, all seem to have one common element: the characteristics of the signal change during the duration of the signal (eg. the changing frequency in the chirp).

3.3 Parallel Transmission

The area of synchronization for parallel transmission has received much less attention compared to serial transmission. This is largely due to the greater use of serial transmission schemes in past years.

Most of the devised schemes are designed to maintain correct timing when slight deviations of timing are present. Also, many of the techniques accumulate information from consecutive baud intervals to improve timing estimates.

When Weinstein and Ebert [10] first proposed the use of the DFT in data transmission systems based on FDM, they recognized the need for synchronization, and indicated that "one or several channels of the transmitted signal can readily be utilized for this purpose". However, they did not investigate the details of implementing such a system, nor did they indicate expected levels of performance of such a technique.

Hirosaki, whose work followed Weinstein and Ebert's by ten years, took a different approach to synchronization [25,12,26]. Although his work revolved around a slightly different MCM system: Orthogonally Multiplexed QAM (OQAM), the conditions under which synchronization is applied are similar. Rather than using pilot tones to determine timing deviations, dual automatic equalizers were employed. The dual automatic equalizer is a slightly modified version of the complex automatic equalizer used in conventional
single channel QAM systems. Hirosaki showed that dual automatic equalizers, with appropriately spaced tapped delay lines, can equalize not only the transmission channel distortions, but the timing deviations and demodulating carrier phase deviations as well. One equalizer is employed for each sub-channel of the OQAM signal. During reception of an OQAM signal, an estimate of the timing deviation is determined by each sub-channel equalizer. The timing deviation estimates from each sub-channel are averaged together. The adjustment of the equalizer coefficients is continuous but response to deviations occurs over several baud intervals. Deviations in timing can be adjusted for only when the deviation in timing is slower than the loop tracking speed of the automatic equalizers.

Following his previous work, Hirosaki joined with others to design a 19.2 Kbps voice-band data modem based on his OQAM technique [13]. To achieve the desired data throughput, a 128 point QAM constellation had to be used. It was determined that with the large constellation, it would not be possible to track phase and gain hits using the data channels. Therefore, two pilot tones were used for synchronization. As in Hirosaki’s previous work, adaptive equalizers were used on each of the data and timing channels.

Paralleling the work of Hirosaki, Keasler studied the use of MFM techniques in voice band telephone modems [27,11]. His work was based on OFDM, similar to that of Weinstein and Ebert [10]. Correlation detection is employed at the receiver to extract, or demodulate, the information contained in the various sub-carriers. Synchronization timing information is provided by transmitting a pair of unmodulated sub-carriers in two reserved sub-channels. The two reserved sub-channels are identified as timing channels. The correlation process used in demodulation produces vectors indicating the phase of the sub-carriers in the timing channels. During demodulation of a received OFDM signal, the timing channel correlation vectors are monitored. When the monitored vectors are identical in phase, the proper correlation interval is established and synchronization is achieved. In addition, the relation between the timing channel phase in adjacent baud times is employed to correct for frequency offset.
Another orthogonal MFM based transmission scheme using pilot tones for synchronization was designed and marketed by the Telebit Corporation [28]. The product was a telephone modem, able to achieve up to 12000 bps over a dial up line with a simultaneous 300 bps reverse channel. To obtain the high level of data throughput, the modem used 64 sub-channels with 5 bits of data encoded in each sub-channel. To provide synchronization, one of the sub-channels was reserved. Unlike previous systems, however, timing information was provided by pulsing the channel with a carrier. That is, the timing channel cyclically transmitted at full intensity for one epoch (baud interval), off for two, on for one, off for two, etc. The amplitude of the received timing channel was used as an amplitude reference to counteract gain hits. The beginning and end of transmissions of the timing channel was used to establish the time boundaries for each epoch. Timing accuracy was maintained by fine tuning the time boundary estimates over consecutive frames.

A novel synchronization technique for differentially encoded MFM signals was recently developed by Moose [7]. The technique is similar in concept to synchronous serial transmission. Synchronization is achieved by inserting a known sync baud at the beginning of a packet of data bauds in a transmission. The receiver synchronizes to the sync baud using a matched filter and must maintain accurate timing for the remaining data bauds. Matched filter detection is performed on the received analog signal. Once synchronization has been established, the analog signal is sampled to allow data decoding in the digital domain.
Chapter 4

Proposed Synchronization Technique

This chapter presents the proposed synchronization technique for OFDM signals. The synchronization requirements are first defined. The basic concept is then presented, followed by a detailed examination of the synchronization process.

4.1 Synchronization Requirements

The synchronization requirements can be separated into two categories: (1) conditions under which synchronization must operate (2) the probability of synchronization error.

The designed implementation is a pure ALOHA based mobile radio communication network using FM transceivers. The requirements under this category are:

- The available spectrum being very restricted, the bandwidth overhead must be minimized.
- Due to the pure ALOHA implementation, each frame has to be synchronized independently.
- Due to the fading nature of the mobile radio channel, the synchronization procedure cannot use a simple energy detector to provide synchronization.
- The synchronization procedure must be relatively immune to phase and amplitude distortions induced by the channel noise.

The probability of synchronization error can be grouped into four sub-categories.

- **Probability of false alarm** is the probability that the synchronization algorithm detects the presence of a data block when none is present.
• **Probability of miss detection** is the probability that the synchronization algorithm does not detect the presence of a data block when one is present.

• **Probability of bad synchronization** is the probability that the synchronization algorithm detects the presence of a data block when one is present, but, does not correctly synchronize. This occurs when enough distortion of the signal occurs to prevent proper synchronization, but not enough distortion to cause a miss detection.

• **Probability of correct synchronization** is the probability that the synchronization algorithm detects the presence of a data block when one is present and correctly synchronizes to the data block.

Within the category of correct synchronization, the degree of accuracy can range from excellent to poor. During experimentation, the accuracy of the synchronization procedure will be determined indirectly through BER measurements. Channel noise and fading imposes a lower limit on the achievable BER. When synchronization is poor, the resulting error in the time frame of reference causes a rotation of the decoded QAM phasor. The magnitude of this rotation increases for the higher frequency sub-channels. Thus, ideal synchronization minimizes the BER for a given block. Comparing the BER achieved using the synchronization procedure with the ideal BER provides an indication of the accuracy of the synchronization procedure.

Prior to experimentation, desired synchronization performance levels are established. The primary motivation in establishing the performance goals is to define levels of synchronization performance such that the resulting inaccuracy of the synchronization procedure is not the major contributing factor limiting the achievable BER. That is, the achievable BER should be limited by the OFDM modulation technique, not the synchronization procedure.

The minimum requirements for the probabilities of false alarm, miss detection and
bad synchronization are set equal to the BER of the OFDM system given ideal synchronization. This should result in a measured BER that does not exceed two times the BER given ideal synchronization. The required probability of good synchronization is not specified; it is simply the probability that one of the three defined errors does not occur. With better synchronization performance, system performance would be improved only marginally, due to the limiting BER.

One last concern regarding the synchronization algorithm is that if it is too complex and too computationally expensive, it will not be practical.

4.2 Basic Concept

Synchronization techniques currently available for serial or parallel modulation schemes are inadequate for use in MFM based communication systems in a pure ALOHA mobile radio environment.

Serial techniques are inherently unsuitable because of channel fading. The fading causes burst errors that would make it difficult to detect the synchronization preamble. Also, the complexity of the transmitter and receiver would be increased in order to support both serial mode for synchronization and parallel mode for data transfer.

Current parallel techniques rely on at least one assumption that contradicts the synchronization requirements:

- Hirosaki’s [25,12,26] technique described in section 3.3 requires adaptive equalizers for each sub-channel. A typical OFDM signal may have up to one thousand or more sub-channels. The resulting complexity and amount of processing make the technique impractical. A modified version of Hirosaki’s technique [13] uses only two reserved sub-channels for timing control. Unfortunately, in this version and in Hirosaki’s original version, the technique can only handle slight deviations in timing, and the accuracy of timing is obtained over several baud periods through use of a feedback loop. That is, the technique assumes that synchronization has
already been established and that only fine tuning has to be performed and can be executed over consecutive baud intervals. Neither of these assumptions are valid in a pure ALOHA environment.

- Keasler’s system [27], using the difference in phase between the un-modulated carriers of two reserved sub-channels, has merit. However, the technique is sensitive to phase errors. As the channel noise increases, the phase errors introduced to the two reserved sub-channels (as well as to all data channels) also increase. Therefore, it is possible that when the data frame is ideally synchronized, the two reserved sub-channels will have a large phase error. When the synchronization procedure modifies the time reference to minimize the phase error in the two reserved sub-channels, the remaining data sub-channels will experience an increased BER. Therefore, the sensitivity of Keasler’s system to phase noise makes it unsuitable for the mobile radio environment.

- The method employed by the Telebit Corporation [28] is not useful for two reasons: 1) accurate detection of the start and stop of a baud interval (epoch) is required, but, due to the frequent fades of the received mobile radio signal, this is not possible; 2) inaccuracies in timing are averaged out over consecutive baud intervals; however, in a pure ALOHA environment each baud is independent and timing information from one baud interval is not useful in synchronizing to another baud.

- Moose’s [7] system is designed for use in mobile satellite communications and is relatively immune to the problem of fading. Moose’s system, however, uses a synchronization baud preceding a packet of data bauds. Synchronization is not achieved on individual bauds, and since the technique uses all available sub-channels, data cannot be transmitted in the synchronization baud. Therefore, Moose’s technique is not suitable.

The proposed solution utilizes two basic concepts:
1. A number of the sub-channels of the OFDM signal are reserved for synchronization tones.

2. At the receiver, a correlation detector is used to detect the synchronization signal imbedded in the OFDM signal and indicate synchronization timing.

Both concepts are used in current systems. The novel aspects of the proposed system are the way in which the concepts are combined and the method for implementation.

The transmitter encodes the reserved synchronization tone sub-channels with known phases and amplitudes. The remaining sub-channels are encoded with data as outlined in section 2.1.2. To the synchronization algorithm of the receiver, the received signal can be interpreted as a sync baud (as in [7]) imbedded in noise, where the noise is a combination of channel noise distorting the OFDM signal, and the signal components of the data sub-channels. Typically, to keep transmission overhead at a minimum, the number of reserved sub-channels is small compared to the number of available sub-channels. Resulting from this is a very poor $\text{SNR}^1$. For example, if 10% of the available sub-channels are reserved, the perceived SNR will be -9.54dB. For a correlation detector to work, the SNR would have to be much larger [29]. However, recall that the OFDM sub-channels are orthogonal; in a properly implemented correlation detector the noise components due to the data sub-channels would cancel, resulting in a much higher effective SNR. The only degradation to the effective SNR would result from degradations to the orthogonality of the OFDM sub-channels caused by fades and channel noise.

Similar to Moose's system, this technique does not rely on time-domain detection of the start of the data block and is therefore relatively insensitive to channel fading. Unlike Moose's system, however, synchronization is achieved separately for each block, so that the technique will work in a pure ALOHA environment.

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1In this interpretation of signal and noise, assuming negligible channel noise and equal distribution of power among the sub-channels, the SNR would be defined as: $\text{SNR} = 10 \log_{10} \frac{\text{no. of synchronization sub-channels}}{\text{no. of data sub-channels}}$. 

Error performance of the technique can be controlled by altering the number of synchronization sub-channels. For a given number of synchronization sub-channels, error performance can be optimized through judicious selection of sub-channels.

4.3 Detailed Examination

4.3.1 Synchronization Signal Model

In Appendix A, a model of the synchronization signal is presented. To reduce the complexity, simplifying assumptions are made. The derived model is periodic and has infinite duration. The auto-correlation of the synchronization signal model also has these two characteristics.

**Periodicity:** Each sub-carrier of an OFDM signal is sinusoidal and has an integer number of periods in the frame interval. Therefore, each sub-carrier is periodic over the frame interval. The synchronization signal is comprised of a number of phase and amplitude encoded sub-carriers. Therefore, the synchronization signal is also periodic over the frame interval. Depending on the specific sub-channels used for synchronization, it is possible for the synchronization signal to be periodic over a duration that is less than the frame period. The minimum periodic duration of the synchronization signal is the lowest common periodic duration of the synchronization sub-carriers. This will not be less than the period of the highest synchronization sub-channel (which has the smallest period), and will not exceed the frame duration.

**Infinite Duration:** In the model, the synchronization signal is assumed to be of infinite duration and is therefore band-limited.

**Auto-correlation:** In Appendix A, the synchronization model derivation shows that, assuming an infinite duration received synchronization signal, the modelled auto-correlation
functions of the received signal and the reference signal are also periodic and band-limited. The period of the auto-correlation is equal to the period of the synchronization signal. The maximum frequency component of the auto-correlation is equal to the maximum frequency component of the synchronization signal.

4.3.2 Three-phase Synchronization Acquisition

A three phase synchronization acquisition procedure is used:

1. Phase I detects when an OFDM signal is present by monitoring the power in the received signal. It does not attempt to acquire synchronization, but does determine a rough estimate of the location of the signal.

2. Phase II uses a correlation detector to extract the synchronization information from the received signal and to acquire synchronization to within \( \pm \frac{1}{2} \) of the sample period. This is considered as coarse synchronization.

3. Phase III provides fine tuning of Phase II synchronization by accurately locating the local peak of the correlation detector implemented in Phase II.

Figure 4.1 presents a block diagram of the synchronization data processing steps.

Phase I - Energy Detection

In Figure 2.4 it can be seen that the audio output power of the receiver is larger in the absence of a carrier and smaller in the presence of a carrier. In an ALOHA mobile radio environment, a transmitter only generates a carrier when transmitting data; therefore, if no transmitters are transmitting data, there will not be a carrier for a receiver to detect.

Referring to Figure 4.1, Phase I monitors the power level of the received signal. When the power level drops below a threshold, phase I concludes that an OFDM signal is present and phase II of synchronization is initiated.
Figure 4.2 provides greater detail on the operation of Phase I. The received data is stored in Data Buffer I prior to power estimation. The data buffer will keep a record of the most recent values. When a new data value is received from the A/D converter, it is loaded into the Data Buffer I and the oldest data value in the buffer is discarded. Each time a new data value is received, the power of the received signal is estimated and compared to the reference threshold.

The power of the signal is estimated by determining the power of the data in Data Buffer I. The equation used to calculate the power is:

\[ p = \sum_{i=0}^{N-1} (x_i)^2, \]  

(4.1)

where \( x_i \) are the data samples and \( N \) is the number of samples used in the estimate and the size of Data Buffer I. The accuracy of this estimate is dependent on the number of data samples used [30]. Given this, it might seem reasonable to use a very large number of data samples, as this would enable the receiver to determine the presence of a carrier more reliably. However, using a data set that is larger than the OFDM data frame would reduce its ability to determine the location of the OFDM data frame. This, in turn, would
make it necessary for phase II to work under more difficult conditions. Therefore, a compromise is made; the number of data samples used to estimate the signal power is set equal to the number of samples in the OFDM frame.

Determination of an appropriate threshold is achieved experimentally. If the threshold is set too high, the probability that the power estimate falls below the threshold when no carrier is present will be too high. This will result in a high probability of false alarm and a high probability of incorrect synchronization. If the threshold is set too low, the probability that the power estimate will not fall below the threshold when a carrier is present will be too high. This will result in a high probability of miss detection.

Phase II - Coarse Synchronization

Referring to Figure 4.1 and with more detail in Figure 4.3, following the detection of an OFDM signal and rough determination of its location, phase II is used to acquire synchronization alignment to within $\pm \frac{1}{2}$ sample period. This is considered coarse synchronization because it is unacceptable for decoding of the OFDM data. Consider the data sub-carrier...
corresponding to 2 kHz, the center of the OFDM baseband signal. At a sampling rate of 8 kHz, an alignment error of ±\(\frac{1}{8}\) of a sampling period will result in a phase error of ±45 degrees. This is unacceptable for QAM decoding. For the data sub-carriers corresponding to higher frequencies in the OFDM signal, the phase errors will be even greater.

Similar to Phase I, the received A/D sampled data values are temporarily stored in Data Buffer II. When Phase II begins, the last data value stored in Data Buffer I will also be the last data value stored in Data Buffer II. Initially, and each time a new data value is received, a correlation detector is used to determine the correlation of the data in the buffer and the reference synchronization signal. The reference synchronization signal is a copy of the transmitted synchronization signal. The minimum size required for Data Buffer II is equal to the number of data samples used by the correlation detector.

The output of the correlation detector is input into an interpolating filter to provide a higher rate of samples. A peak detector monitors the output of the interpolating filter and keeps track of the location of the peak value. Operation of the peak detector is controlled by a start/stop signal. The peak detector resets and begins operation when
phase II processing starts. A fixed time later, peak detection is halted. The peak location indicated by the detector corresponds to the point of synchronization. The Data Index Calculator converts the output of the Peak Locator to an index pointer that identifies the location, in the Main Data Buffer, of the synchronized data block. The control signal that halts operation of the peak detector also initiates phase III of synchronization.

The correlation detector and the interpolation filter are described in greater detail below.

**Correlation Detector** The correlation detector calculates the correlation between the data in Data Buffer II and the reference synchronization signal.

Often, correlation detectors are implemented in the time domain. However, in the OFDM system, time domain implementation may lead to difficulty in implementation. Recall, from *Channel Hardware Equalization* in section 2.3.2, that the received signal is phase and amplitude distorted due to analog filtering and the FM channel. This distortion must be compensated for before the results of the correlation detector can be used. One
technique is to pass the received data values through an equalization filter prior to storing them in Data Buffer II. Another technique is to use a phase and amplitude distorted version of the reference synchronization signal. The first technique has the advantage of being adaptable and the disadvantage of increasing the computational load. The second technique does not increase the computational load, but it is not easily adaptable, an important consideration if the transfer function of the hardware drifts. To provide adaption in the second technique would require an analysis of the change in transfer function and its effect on the transmitted signal. The phase and amplitude distorted version of the transmitted signal would then have to be regenerated.

A third technique is proposed that allows flexibility and adaptability without dramatically increasing the computational load: the correlation detector is implemented in the frequency domain. At first thought, it might seem that the transformation of the data to the frequency domain is computationally expensive. However, a special routine can be used that is order $N$, compared to order $N \log N$ for the FFT and order $N^2$ for the DFT. This routine, called the DFT update routine, is described in the following paragraphs and is derived in Appendix B.

Using the DFT update routine, the data in the phase II data buffer is processed to extract the phase and magnitude (in phasor form) of the sync tone sub-channels. To compensate for known phase and magnitude distortions, the received sync tone phasors are multiplied by equalization phasors, determined using a training sequence as described in section 2.3.2.

The equalized phasors and the synchronization reference phasors are used to calculate the correlation detector output. The equation defining the frequency domain implementation of the correlation detector, developed in Appendix A, is given as:

$$Y = \kappa \sum_{i=1}^{j} s_i \odot r_i$$

(4.2)

where $s_i$ is the equalized phasor of the $i^{th}$ sync tone of the received signal, $r_i$ is the phasor of the $i^{th}$ sync tone of the reference signal, $j$ is the number of synchronization tones used,
is the dot product, and $\kappa$ is a constant.

The DFT update routine achieves a reduction in computational complexity by updating the spectral estimates based on previous estimates, rather than by recalculating the spectral estimates, and by only updating the spectral estimates for the specific frequencies required. This is an ideal situation for the correlation detector which only requires the spectral information for the synchronization tones. From Appendix B, the DFT update equation is

$$X_{n+1}(k) = [X_n(k) - x(n) + x(n + N)] \exp^{j\frac{2\pi}{N}}$$

(4.3)

where $X_n(k)$ is the spectral information of the $k^{th}$ sub-channel with the trailing edge of the data window positioned at the $n^{th}$ data sample, $x(n)$ is the $n^{th}$ data sample and $N$ is the block size of the DFT/FFT. The equation is simple: from the previous spectral estimate, the newly received data value is added, the oldest data value is subtracted and the result is multiplied by a phase shift.

When phase II is initiated, a previous spectral estimate does not exist. Therefore, the sync tone spectral information of the received data is calculated using a DFT or FFT$^2$. Following this, the spectral estimates are updated using equation 4.3. A drawback of the DFT update routine is that round-off errors propagate and are accumulated at each invocation of the routine. Therefore, the DFT/FFT should be used periodically to recalculate the spectral estimates. Appendix B contains an analysis of the accumulated error and discusses how often the spectral estimates should be recalculated.

**Interpolation Filter** The correlation detector provides correlation results for the received OFDM sync signal and the reference synchronization signal. These results are snapshots, spaced at intervals equal to the sample period, of the continuous time correlation. If the width of the expected correlation detector output peak is less than the sample period, the peak may not appear in the output of the correlation detector. This

$^2$Depending on the number of sub-channels of interest, the DFT may be faster than the FFT. The DFT routine can be modified to calculate the spectral information for specific sub-channels.
possibility can be avoided by recalling that the auto-correlation of the synchronization signal is band-limited. Therefore, the correlation detector output will be band-limited, and can be passed through a digital interpolation filter to obtain more frequent snapshots of the signal and reduce the possibility of missing the correlation peak.

In Appendix C, a 63 tap finite impulse response low pass filter is derived that provides interpolated data values at four times the original data rate. This is sufficient to insure that the peak is detected to within 95% of its actual value.

Phase III - Fine Synchronization

Referring to Figures 4.1 and 4.4, fine synchronization is performed following the completion of coarse synchronization.

Due to the discrete nature of phase II, coarse synchronization can only be obtained to within $\pm \frac{1}{2}$ of a sample period. This degree of accuracy, however, results in large phase errors for the higher frequency sub-channels and, therefore, is inadequate.

Phase III uses a closed-form equation to provide fine synchronization by accurately determining the location of the local maximum of the correlation detector output from Phase II. In addition, phase III demodulates the OFDM signal and extracts the encoded data.

Since phase II achieves sync to within $\pm \frac{1}{2}$ of a sample period, it is guaranteed that the data frame will be aligned within the un-weighted pre-extension of the OFDM data frame. Therefore, an FFT can be performed on the received data block without loss of information. Following the FFT computations, all sub-channels of the OFDM signal are equalized to compensate for the known phase and magnitude distortions. Optimal synchronization is performed on this final set of data. The algorithm, derived in Appendix A, calculates the time shift that maximizes the correlation with the reference synchronization signal. The required phase shift for each sub-channel is then calculated and applied.
4.3.3 Synchronization Sub-Channel Selection

Selection of the sub-channels of the OFDM signal which should be reserved for synchronization is critical to the performance of the synchronization algorithm.

As with matched filter and correlation filter synchronization techniques for serial communications, different synchronization signals produce different side lobe patterns with local correlation peaks occurring away from the synchronization location. The variation of the side-lobe patterns results in differing synchronization performance. The ideal correlation function for serial techniques, equations 3.2 and 3.3, are also ideal for OFDM.

As previously indicated, the assumption of infinite duration signals allows the output of the correlation detector in phase II of synchronization to be modelled by the correlation of the infinite duration OFDM signal with one period of itself. The resulting correlation signal has a period equal to the period of the synchronization signal.

In phase II of synchronization, the output of the correlation detector is monitored for a fixed period of time. If this period of time is greater than the synchronization period, two occurrences of the synchronization peak may be detected which will cause synchronization errors. Therefore, the selection of synchronization tones must have a period greater than the duration of phase II. The duration of phase II, however, is dependent on the ability of phase I to detect the location of the data frame. For this reason, the selection of sync tones cannot be made until analysis of Phase I synchronization has been completed.

Given that the duration of the minimum synchronization period has been determined through analysis of Phase I results, selection of synchronization sub-channels can proceed. Ideally, the combination of $J$ sub-channels that has a period greater than the minimum required and the minimum side-lobe levels of all possible sub-channel combinations is to be determined. The value of $J$, the number of sub-channels used, must also be determined. In general, as $J$ increases, the minimum attainable side-lobe level will decrease and Phase II performance will improve.

Previous work on designing sinusoidal signals with desired correlation properties
could not be located, and a theoretical development was not readily apparent. Therefore, a computer search was used to determine the best combinations. This brute force technique has been used previously in designing synchronization patterns for serial communications [23]. An accurate theoretical prediction of the number of sub-channels required to meet synchronization performance requirements was unavailable. Experimentation was used: the number of sub-channels was increased until suitable performance was obtained.

Initially, a brute force search to select 4 from the 256 available sub-channels, was performed on a SUN SPARC Station 1. With 80% CPU utilization, the search took approximately 7 days to complete. As the number of desired sub-channels is increased, the number of possible combinations increases factorially. It is estimated that selecting 5 from the 256 available sub-channels will take 350 days to complete.

To be of any practical use, the search time must be greatly reduced. Two simplifications were found and implemented.

1. Results from the initial experiments were examined and a pattern was found. In all cases, each of the 4 sub-channels selected had an integer (or close to an integer) number of periods in the minimum required periodic duration. Therefore, the search space was reduced to include only those sub-channels that had an integer number of periods in the minimum required periodic duration.

2. The search was divided into several consecutive steps, each step building on the results of the previous step. In each step, the selected sub-channels from the previous step are kept and an additional 3 sub-channels are selected that provide the smallest sidelobes.

Because of the above simplifications, the resulting set of sub-channels may not be optimal. However, a larger set of synchronization tones can be used to compensate for this sub-optimality.
Chapter 5

Experiments

This chapter describes the experimental setup used to measure the performance of the proposed synchronization system. The results of the experiments are also presented.

5.1 Experimental Setup

5.1.1 Hardware

Analog FM fading channel

A block diagram depicting the implementation of an FM fading channel is given in in Figure 5.1. The implementation is that used by Casas [4]. A baseband signal is used to modulate an ICOM-2AT narrow-band FM transmitter. Transmission occurs over a co-axial cable link to provide isolation from external electromagnetic interference and to allow control of signal fading and SNR levels. The RF output of the transmitter is passed through a Rayleigh fading channel simulator. The simulator [31], provides for Doppler rate selection of 2 Hz to 126 Hz in increments of 2 Hz. The output from the Rayleigh fading channel simulator is passed through a step-wise variable attenuator (Kay model no. 437A) and combined with the output of an RF noise source using a power splitter/combiner (Mini-Circuits model no. ZSC-2-1). The Kay attenuator allows levels of 0 dB to 102.5 dB attenuation, in increments of 0.5 dB, to be selected. A power amplifier (Mini-Circuits model no. ZHL-2-8) is used as the RF noise source. To prevent amplification of any stray signals present in the environment, the input to the amplifier was terminated with a $50 \Omega$ shielded load. The output of the power combiner is connected to
the antenna input of the ICOM-2AT receiver. The recovered baseband signal is monitored via the audio output jack of the receiver.

The SNR of the RF signal seen by the receiving ICOM-2AT is difficult to measure. However, as in [4], the RF SNR can be approximated by measuring the IF SNR within the ICOM-2AT receiver. The IF SNR level is controlled by varying the attenuation setting of the Kay step attenuator. Increasing the attenuation level reduces the signal power and thus decreases the IF SNR. Conversely, a decrease in the attenuation level results in an increased IF SNR. For the setup of Figure 5.1, Casas calibrated the IF SNR level resulting in an equation relating the attenuation level and the IF SNR:

\[
\text{IF SNR (dB)} = 75(\text{dB}) - \text{attenuation (dB)}
\] (5.1)

The IF SNR measurement procedure, as detailed in [4,32,33,34], was reproduced and equation (5.1) shown to be accurate.
Digital FM channel

The digital FM channel, shown in Figure 5.2, is comprised of the analog FM fading channel shown in Figure 5.1, and additional hardware and software which allows the input of data to the transmitter to be in digital form, and for the output of data from the receiver to be in digital form. The OFDM modulating technique is imbedded within the hardware and software of the digital FM channel.

The generation of the OFDM signal is shared between the host computer and special Digital Signal Processing (DSP) hardware. The host computer is an IBM PC/AT compatible. Residing on the bus internal to the host is a DSP56001 Processor Board from Spectrum Signal Processing of Burnaby, Canada [35]. This is a specialized signal processing board based on Motorola's DSP56001 digital signal processor. Connected to this board, and also residing on the host's internal bus, is a 4-channel I/O board which provides Digital-to-Analog (D/A) and Analog-to-Digital (A/D) conversion. The I/O board is also from Spectrum Signal Processing [36].

Data Input  At the transmitting end serial digital data, in binary form, is accepted. The data is QAM encoded, grouped into blocks and converted to parallel.

Software De-emphasis  The blocks of data received from the data input serial to parallel converter are interpreted as frequency domain information from which the OFDM baseband signal is constructed. The QAM encoding of data results in equal energy in each of the frequency components of the OFDM signal. De-emphasis of the encoded data by 10 dB per decade is provided in software to improve the performance of the system (see Noise Power Distribution in section 2.3.2).

OFDM Signal Generation  The sampling frequency of the system is set at 8 kHz, the Nyquist\(^1\) frequency corresponding to a maximum signal frequency of 4 kHz. In the

\(^1\)The Nyquist rate is defined as twice the maximum frequency of the signal [37].
Chapter 5. Experiments

Figure 5.2: Digital FM Channel
experiments, 256 frequency sub-channels between 1 kHz and 3 kHz are used to carry data and synchronization information. An inverse FFT is used to construct the OFDM signal with these characteristics from the QAM encoded data. To implement this, an inverse FFT block size of 1024 is used. Within this block, data elements from 0 to 127 and from 384 to 511 are set to zero. The QAM encoded data is transferred to data elements 128 to 383. Data elements 512 to 1023 are defined to be the complex conjugates of elements 0 to 511. Performing an inverse FFT on this data results in a signal with no complex component. Data element 128 in the constructed data block corresponds to the 1 kHz frequency sub-channel of the OFDM signal. Data element 384 corresponds to the 3 kHz frequency sub-channel.

Following the application of the inverse FFT, the resulting block of data is converted back to serial form. This serial digital data represents the OFDM modulating signal in digital form.

D/A Loading Factor  Prior to D/A conversion, the amplitude of the signal is modified to optimize the loading factor (LF) [38], defined as:

\[
LF = \frac{\text{Peak amplitude}}{\text{RMS amplitude}}.
\]

Correct choice of the loading factor maximizes the SNR following D/A conversion. Also affected by the loading factor is FM transmitter performance. Frequency deviation in a FM transmitter is controlled by the amplitude of the modulating signal. To prevent frequency deviations from exceeding the allowed limit, the FM transmitter clips the amplitude of the modulating signal. According to Jakes [3], the optimal loading factor for voice signals over FM channels is: \( LF = 3.16 \). This level was used in [4]. However, the characteristics of the OFDM baseband signal differ from voice signals. The OFDM baseband signal has a Gaussian distribution, while human speech is often modelled with a Laplace, or more accurately with a Gamma distribution [39]. Jayant [38] provides an

\[^2\text{Jakes specifies the Peak to RMS power ratio to be 10 dB, which corresponds to LF = 3.16}\]
analysis of optimal loading factor for A/D conversion. The process of D/A conversion followed by reconstruction filtering is different from A/D conversion, with the distortions of each process affecting the signal characteristics differently. Therefore, Jayant's analysis of optimal loading factor may not be precise for D/A conversion. However, the analysis provides a reasonably good estimate of the optimal D/A loading factor. The analysis of the optimal A/D loading factor\(^3\) identified LF \(\approx 4.0\) to be the optimal loading factor for A/D conversion. A loading factor of 4.0 is selected for D/A conversion. It is achieved by applying a software gain to the OFDM signal. In the fixed point implementation of the DSP56001, the magnitude of the data is bounded by \(\pm 1.0\)^4. When the magnitude of a number exceeds \(\pm 1.0\), it is limited to \(\pm 1.0\). Thus, multiplying a set of data by a scalar value greater than 1.0 will increase the RMS value of the data but will not increase the peak magnitude beyond \(\pm 1.0\). Multiplication of data by a factor greater than 1.0 can be done by realizing that shifting the data to the left by one bit is equivalent to multiplying by two.

The RMS value of the OFDM signal data generated by the inverse FFT is approximately \(1.1 \times 10^{-2}\) (refer to Appendix D for derivation)^5. To achieve a loading factor of 4.0, assuming a peak of 1.0, the signal must have an RMS value of 0.25. This can be obtained by multiplying the signal data by 22.7. For ease of implementation, a factor of 32 is chosen. This can be implemented by shifting all data values 5 bits to the left. The DSP56001 properly saturates to \(\pm 1.0\) when this value is exceeded during bit shifting [40]. The effect of choosing a multiplying factor that is larger than optimal will decrease the loading factor resulting in an increase in overload distortion. However, the effect on overall performance appears to be negligible\(^6\).

---

\(^3\)see A/D Loading Factor in this section.

\(^4\)Actually, \(+1.0\) cannot be represented in the fixed point architecture of the DSP56001. The largest positive value is 0.9999998.

\(^5\)This is a representation internal to the DSP. No units are associated with this number.

\(^6\)Phase III testing with LF = 22.7 resulted in BER performance similar to testing with LF = 32
Gain and Filtering  The output signal of the D/A converter has a measured RMS level of 620 millivolts. The signal contains high frequency components which are replicas of the desired frequency spectrum at multiples of the sampling frequency. This characteristic of D/A conversion is compensated for using a reconstruction filter. This is a low pass filter with a cutoff frequency usually set equal to half of the sampling rate.

In the experimental setup, an 8th-order Butterworth low pass filter with a cutoff frequency of 4 kHz is used. As a precaution, an 8th order high pass filter with a cutoff frequency of 100 Hz is used to remove any unexpected DC component from the signal. Krohn-Hite filters (Model no. 3342) are used.

The desired level for the modulating signal is 6 millivolts (RMS). The attenuation provided by the Tx Gain unit, the filters and the Tx switch reduce the signal level from
620 millivolts (RMS) to the desired level. The circuit for the Tx Gain unit, shown in Figure 5.3, is designed to additionally provide a high input impedance and a low output impedance.

**Tx Switch** The Tx switch, shown in Figure 5.4, serves a dual purpose. It provides the last stage of attenuation required to reduce the level of the modulating signal to the desired level. It also provides a means to turn the FM transmitter on and off.

Within the ICOM-2AT transceiver, a load detector is attached to the microphone input. When a load is detected, the transmitter's **Push To Talk (PTT)** is enabled. When the load is removed, PTT is disabled.

The Tx switch is an electronically controlled switch that when closed provides a 25 kΩ load on the line connected to the transmitter microphone input, and when open presents an open circuit to the microphone input. When the switch is closed, the attenuated OFDM signal is gated to the switch output.

The electronic control of the switch allows the FM transceiver to be turned on and off by the DSP hardware at the times appropriate for data transmission. For the ICOM-2ATs, a trigger advance of 80 milliseconds is used. That is, the PTT is enabled 80 milliseconds
prior to the start of data transmission by the DSP. This allows adequate time for the transmitter to power-up (see Radio Attack Time in Section 2.3.2).

**Receiver Section** The components of the receiver section perform the inverse functions to the transmitter section in the reverse order.

**Anti-Aliasing Filters** A 16th-order Butterworth low pass filter with a cutoff frequency of 4 kHz is used to remove all frequencies above 4 kHz. At an A/D sampling rate of 8 kHz, any frequency components above 4 kHz will be aliased down into the 0 to 4 kHz range. Therefore, these frequencies are removed from the signal.

**Rx Gain** An Rx gain of 2.4 is used to achieve a signal level of 620 millivolts (RMS). For A/D sampling with a signal input range of ±2.5 volts, this corresponds to a loading factor of 4.0.

**A/D Loading Factor** Following the analysis in Jayant [38], the optimal A/D loading factor for zero mean, gaussian distributed signals, when considering the SNR following conversion, can be calculated. The optimal loading factor minimizes the sum of the granular error variance, $\sigma^2_{\text{granular}}$, and the overload error variance, $\sigma^2_{\text{overload}}$, thus maximizing the SNR of the A/D conversion. Granular error arises because the A/D converter has a finite number of sample values and thus cannot exactly represent the signal level. Overload error arises due to the magnitude limitation of the A/D converter. The equation for total error variance is given as:

$$\sigma_q^2 = \sigma^2_{\text{granular}} + \sigma^2_{\text{overload}}$$

(5.3)

where

$$\sigma^2_{\text{granular}} = \sum_{k=2}^{L-1} \int_{x_k}^{x_{k+1}} (x - y_k)^2 P_x(x) dx + 2 \int_{x_L}^{x_{\text{overload}}} (x - y_L)^2 P_x(x) dx$$

(5.4)
Figure 5.5: Mid-rise model of 8 level A/D quantization

\[ \sigma_{\text{overload}}^2 = 2 \int_{x_{\text{overload}}}^{\infty} (x - y_L)^2 P_x(x) dx \]  

In equations 5.4 and 5.5, \( P_x(x) \) is the Probability Density Function (PDF) of the signal input to the quantizer, and \( L \) is the number of discrete levels of the A/D converter. The \( x \) variables and \( y \) variables, corresponding to the continuous input signal and the quantized output signal, are identified in Figure 5.5. The variable \textit{Peak Amplitude} in equation 5.2 corresponds the variable \( x_{\text{overload}} \) in equations 5.4 and 5.5. The variable \textit{RMS amplitude} corresponds to the variable \( \sigma_x \), the standard deviation of the PDF \( P_x(x) \).

Figure 5.6 shows the A/D conversion SNR as a function of the standard deviation of a Gaussian signal and the bit resolution of the A/D converter. For all curves, \( x_{\text{overload}} = 1.0 \), and the loading factor is equal to the reciprocal of the standard deviation. For 8-bit A/D conversions, an SNR level of 40dB is obtained when the loading factor is 4.0. For each curve, there is a point where the SNR level begins to drop off dramatically as the loading factor is increased. This is the loading factor at which the overload distortion becomes
Figure 5.6: A/D SNR versus Gaussian Standard Deviation
dominant [38]. A loading factor of 4.0 is chosen.

**Rx Software Gain**  In the transmitter, a software gain of 32 was applied to the signal to modify the loading factor. In the receiver, a software gain of 1/32 is applied to counteract this. Without this gain, processing of the data by the DSP will result in saturation and, therefore, distortion of the received data.

**Data Recovery**  The serial data from the Rx Software Gain section is grouped into blocks and converted to parallel. An FFT is then performed on this data to recover the transmitted QAM encoded data.

**Channel Equalization**  The hardware comprising the digital FM channel distorts the amplitude and phase of the signal by a fixed amount. Equalization is performed to compensate for this distortion. Determination of the proper equalization is made through a training sequence procedure (see *Channel Hardware Equalization* in Section 2.3.2).

**Data Output**  The parallel data, following equalization, is converted back to serial form and QAM decoded to recover the transmitted bit sequence.

### 5.1.2 Pseudo-Random Bit Sequence Generator

A *Pseudo-Random Bit Sequence (PRBS)* generator is used to generate the binary bit stream injected into the digital FM channel. The PRBS generator is implemented as a shift register with feedback. The generator polynomial, taken from [41], generates a maximal length sequence of period $2^{23} - 1$. The PRBS generator was implemented in the host computer.
5.1.3 Raised Cosine Weighting Function

Transmission of the OFDM signal block is preceded by a 32 sample periodic extension, called the pre-extension, of the OFDM signal to allow time for transients to dissipate (see Transient Response in Section 2.3.2) and is followed by a 32 sample periodic extension, called the post-extension. An improvement is made to further reduce the transient power at the start of the OFDM data. The first 16 samples of the pre-extension and the last 16 samples of the post-extension are weighted by a raised cosine function. This provides a smooth transition in signal power from zero to full power during transmission of the pre-extension and from full to zero power during transmission of the post-extension. Since transient magnitudes are dependent on the abruptness of the signal transition, the weighting function, which provides a smoother transition, should reduce the magnitude of the transients.

The purpose of weighting the post-extension is to reduce the effect of transients on an immediately following OFDM signal block.

5.1.4 OFDM Baseband Signal Spectrum

Figure 5.7 shows the spectral plot of the OFDM baseband signal following D/A conversion and reconstruction filtering. Due to the resolution bandwidth limitation of the Tektronix 497P spectrum analyzer, it is not possible to distinguish individual frequency sub-channels. However, it can be seen that only sub-channels in the frequency range from 1 kHz to 3 kHz are used.

The power distribution among the sub-channels is not constant; lower frequency sub-channels have more power than higher frequency sub-channels. This is due to the software de-emphasis of 10 dB per decade.

The spectral plot also indicates the presence of signal content above 5 kHz. Recall that D/A conversion results in replicas of the desired spectrum at multiples of the sampling frequency. Ideally, the reconstruction filter removes these higher frequency components.
Figure 5.7: OFDM Baseband Signal Spectrum
In practice, it is impossible to remove them completely. The signal above 5 kHz is the portion of the replicas that the reconstruction filter was unable to remove.

5.1.5 OFDM RF Signal Spectrum

The Department of Communications (DOC) in Canada and the Federal Communications Commission (FCC) in the U.S.A. strictly regulate RF emissions. Included in these regulations are specifications on the out-of-band emission attenuation for FM transmitters [42,43]. Figure 5.8 shows the spectral image of the ICOM-2AT transmitter RF output when modulated by a typical OFDM signal. Superimposed on the spectral plot is the spectral hat that indicates the maximum signal power output of the transmitter. Examination of the spectral plot indicates that the ICOM-2AT transmitters are well within the
Chapter 5. Experiments

guidelines.

5.1.6 Loop-Back Configuration

During the training sequence and in several phases of experimentation, perfect synchronization is assumed. This is achieved with a loop-back configuration. In this configuration, the host and its internal DSP processor board act as both transmitter and receiver. The clock that is used to trigger D/A conversion for the transmitter also triggers A/D conversion for the receiver. The design of the implemented DSP software allows simultaneous operation of transmit and receive functions.

5.1.7 End-to-End Configuration

In the final test of the synchronization technique, the receiver must be unaware of transmitter timing. This is provided by the end-to-end configuration. In this configuration, A/D sampling and data recovery by the receiver is performed using a second host, with a DSP processor board and I/O board, operating independently of the transmitter. While it is possible to install both sets of DSP boards in the same host and have the host perform the necessary functions for both transmitter and receiver, the DSP boards are installed in separate hosts. This is to reassure outside observers that the transmitter and receiver are functioning independently!

5.1.8 Received Data Analyzer

In the experiments, five performance indicators are measured to indicate synchronization performance:

- Bit Error Rate (BER)
- Probability of false alarm
- Probability of miss detection
• Probability of bad synchronization

• Probability of correct synchronization

In the loop-back configuration only the BER needs to be determined. Since ideal synchronization is obtained in the loop-back configuration, the remaining performance measures are meaningless. The BER is determined by comparison of the transmitted bit sequence with the received bit sequence.

In the end-to-end configuration, all performance indicators are required. To provide BER testing, the receiver's host is designed to generate a PRBS bit stream identical to that generated by the transmitter's host. Comparison by the receiving host of the generated bit stream and the received bit stream yields the BER.

Determination of the other performance indicators is more difficult. When a block of bits are received, they are compared to the expected data and a BER for that block, referred to as the block BER, is determined. If the block BER is small, then it is unlikely that an error in synchronization has occurred since the expected block BER with synchronization in error is 0.5. BER threshold levels are used to classify whether a block is the result of correct synchronization. The design of Phase III experiments provides information used to determine the BER thresholds. During Phase III testing, the host monitors the block BER for ideal synchronization and keeps track of the maximum block BER experienced during each experiment. This measured BER provides a lower bound on the setting of the BER threshold. If the threshold is set below this bound, it is likely that during end-to-end testing, a block that is correctly synchronized will be classified as bad. In practice, the BER threshold levels are set between the measured maximum block BERs and 0.5. This provides a safety margin to ensure that mis-classification of synchronization errors is unlikely.

If a data block is determined to be in synchronization error, there are three possibilities:

1. The block is the result of a false alarm.
2. A transmitted block was missed and the received block is a subsequently transmitted block.

3. The received block corresponds to the transmitted block, but due to inaccuracies in synchronization, the number of bit errors is large.

To differentiate among the possibilities, the receiver keeps a buffer of the expected data blocks. The received data block is compared to each block in the data buffer until a match is found or the limits of the buffer are exceeded. When a match is found, the position of the match within the buffer is examined. Knowledge of the location of the previous match, the number of blocks received since that are in error, and the location of the current match allow for determination of which error has occurred and if the error has occurred consecutively. It should be noted that it is not possible to differentiate between the case of a false alarm followed by a miss and the case of a bad sync. In this instance, the error is classified as a bad sync.

Figure 5.9 illustrates the occurrence and detection of each synchronization classification. In each illustrated situation, the expected blocks are blocks n, n+1, n+2, etc., and the received blocks are blocks k, k+1, k+2, etc.

- In the case of a false alarm, block k is matched with block n, block k+1 is matched with block n+1, and block k+3 is matched with block n+2. Note that no match is found for received block k+2, but matches are found for each of the expected blocks. Received block k+2 is the result of a false alarm.

- In the case of a miss sync, each of the received blocks are matched with expected blocks. However, no match is found for expected block n+2. The block that should have been matched with block n+2 was not detected, thus a miss sync has occurred.

- In the case of a bad sync, block k is matched with block n, block k+1 is matched with block n+1 and block k+4 is matched with block n+4. Received blocks k+2 and
Figure 5.9: Discriminating among the four classifications of synchronization
5.2 Experimental Results

5.2.1 Categorization of Results

Experiments were conducted to measure, separately, the performance of each phase of synchronization. Experiments were also conducted for the integrated system.

Table 5.1 summarizes the list of experiments conducted. The interpretation of the table is as follows: for Phase I with IF SNR = 10dB, experiments were made for doppler rates of 10Hz, 20Hz, and 50Hz; for Phase I with IF SNR = 15dB, experiments were only performed for a doppler rate of 10Hz. Additional to this list is the test for probability of False Alarm, which is independent of the IF SNR and doppler rate.

Results from the experiments are presented in graphical form. The estimated mean and 95% confidence intervals [44] are identified.

5.2.2 Phase I Results

Phase I experimentation measures two performance indicators:
Chapter 5. Experiments

Figure 5.10: Phase I performance in the absence of a transmitted signal.

Figure 5.11: Phase I performance in the presence of a transmitted signal. Doppler rate is: $f_d = 10\text{Hz}$
Figure 5.12: Phase I performance in the presence of a transmitted signal. Doppler rates are: $f_d=20\text{Hz}$ and $f_d=50\text{Hz}$

- probability of False Alarm

- ability of Phase I to determine location of OFDM data frame

Figure 5.10 shows the performance of Phase I in the absence of a transmitted signal. From this, the probability of false alarm for a range of threshold levels can be predicted. During testing, 50000 trials were performed for each indicated threshold level. One trial represents the transmission and reception of one OFDM data block. During each trial, no RF carrier is present. The power of the demodulated baseband signal received from the FM receiver is estimated using 1024 samples of the received signal. Power estimation continues each time a new sample is received and concludes when a total of 1024 additional samples have been received. A false alarm is detected if any of the 1025 power estimations has a value less than the specified threshold. The trend is an increase in the probability of False Alarm with increasing threshold level. For thresholds of 0.375 and lower, no false alarm errors were detected. Later, in this section, it is shown that
threshold levels of 0.15, 0.18, 0.21 and 0.25 are selected for IF SNR levels of 25db, 20db, 15db and 10db respectively. These thresholds are below the level for which false alarms were detected.

Figures 5.11 and 5.12 show the performance of Phase I in the presence of a transmitted OFDM signal. The results are used to interpret the ability of Phase I to determine the location of the OFDM data frame. Two sets of curves appear. The curves rising to the right give the probabilities that the estimated signal power drops below the threshold prior to that location. The horizontal axis indicates the location of detection prior to the start of the OFDM frame. The second set of curves (rising to the left) is generated by subtracting the first set from 1.0. They represent the probabilities that the estimated signal power does not drop below the threshold prior to that location. The thresholds used for each IF SNR level is determined experimentally. The goal is to reduce the probability of detection occurring after the point of synchronization to satisfy the specified probability of miss sync. At the same time, the probability of detection at a point well before synchronization is to be reduced, since this will improve the conditions under which Phase II must operate.

The results of these experiments are used to determine the required duration of Phase II. For a given IF SNR, the location at which the curve rising to the right equals the required synchronization performance is determined. The distance from this location to the point of synchronization indicates the necessary duration of Phase II. This process is repeated for each IF SNR level. For the experimental results of Figure 5.11, a duration of 700 sample periods is sufficient to meet the required synchronization performance for all IF SNR levels of interest.

Figure 5.12 shows the effect of increasing the doppler rate. Increasing the doppler rate reduces the necessary duration of Phase II.

5.2.3 Phase II Results
Figure 5.13: Phase II performance

<table>
<thead>
<tr>
<th>Number of Sync Tones</th>
<th>Selected Sub-channels (cumulative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>162 246 312</td>
</tr>
<tr>
<td>6</td>
<td>170 310 358</td>
</tr>
<tr>
<td>9</td>
<td>176 214 256</td>
</tr>
<tr>
<td>12</td>
<td>206 230 370</td>
</tr>
<tr>
<td>15</td>
<td>132 190 224</td>
</tr>
<tr>
<td>18</td>
<td>140 234 372</td>
</tr>
<tr>
<td>21</td>
<td>184 276 344</td>
</tr>
</tbody>
</table>

Table 5.2: Synchronization sub-channel selection. OFDM block size is 1024, therefore available sub-channels are [0,512]. Of these, only [128,384] are used.
Phase II experimentation measures the ability of Phase II to correctly detect the correlation detector output peak corresponding to correct synchronization and to ignore the sidelobes of the correlation detector output.

Figure 5.13 displays the performance of Phase II as a function of the IF SNR level for a number of synchronization tones and doppler rates. Three curves appear for the doppler setting of 10Hz, corresponding to the use of 15, 18, and 21 synchronization tones. The sub-channels used for synchronization are listed in Table 5.2. The selection of sub-channels was determined using the computer search algorithm presented in Section 4.3.3. The list of selected sub-channels in Table 5.2 is cumulative. For example, the sub-channels selected for 6 sync tones are \{162, 246, 312, 170, 310, 358\}. Comparing these curves with the reference curve indicating the required performance (also shown in the figure), it is apparent that the specified requirements can be met using 15 sync tones.

Also appearing in Figure 5.13 is a performance plot for 21 synchronization tones and a doppler rate of 20Hz. The effect of increasing the doppler rate from 10Hz to 20Hz results in a 2dB improvement for the case of 21 sync tones.
Chapter 5. Experiments

5.2.4 Phase III Results

Phase III experimentation measures the ability of the fine tune synchronization algorithm to accurately locate the position of synchronization. This ability is measured indirectly by determining the BER resulting from fine tune synchronization and comparing with the BER that could be obtained if ideal synchronization is achieved.

Figures 5.14 and 5.15 present the experimental results. In Figure 5.14, the dotted curves represent the best achievable BER for a doppler rate of 10Hz, obtained when ideal synchronization is achieved. Figure 5.15 displays the same data as 5.14, but as a ratio of actual BER compared to the BER for ideal synchronization. Except for the case of IF SNR=25dB, the relative results are quite similar. Also shown is the BER performance requirement curve. In general, fine tune synchronization improves as the number of synchronization tones is increased. The rate of improvement is greatest when the number of tones is small, and decreases as the number of tones increases. When
6 or more synchronization tones are used, performance of the fine tune synchronization algorithm exceeds the requirements.

The effect of increasing the doppler rate is shown in Figure 5.14 by the dashed lines. Minimal improvement results for IF SNR = 10dB. The degree of improvement increases with increasing IF SNR. For IF SNR = 25dB, increasing the doppler rate from 10Hz to 20Hz reduces the BER by a factor of 3 when 7 or more synchronization tones are used.

Integrated testing, presented in the following section, uses 21 synchronization tones for phase II, and uses all of them again for phase III. The performance of phase III using these synchronization tones is identified by the solid triangles in Figures 5.14 and 5.15.
5.2.5 Integrated Results

The results of the integrated system tests are given in Figures 5.16, 5.17, and 5.18. For each test case, 100,000 trials were conducted.

Figure 5.16 shows the performance of the synchronization algorithm with respect to the probability of bad synchronization, the probability of missed synchronization and the probability of false alarm. For IF SNR levels of 15dB or more, no false alarms were detected. For 25dB IF SNR and 20Hz doppler rate, no synchronization errors at all were detected. The results show that the achieved synchronization performance is better than the specified requirements. For 10dB IF SNR and 10Hz doppler, this margin is approximately 4dB. An increase in the doppler rate to 20Hz improves performance by an additional 4dB.

Figure 5.17 shows the resulting BER performance. Two sets of curves appear. One set indicates BER when only correctly synchronized OFDM blocks are considered. The
second set of curves has been adjusted to indicate BER performance when incorrectly synchronized OFDM blocks are included. In adjusting the BER it is assumed that incorrectly synchronized blocks have a block BER of $\frac{1}{2}$.

The specified BER requirements are met by both the adjusted and unadjusted BER measurements. For a 10Hz doppler rate, BER performance more than 1.5dB better than the specified requirements. An additional 1.5dB performance gain is obtained when the doppler rate is increased to 20Hz.

The specification of synchronization performance in section 4.1 is somewhat arbitrary, so that comparison of actual performance with the specified requirements is a bit artificial. To provide a better indication of the achieved synchronization performance, the BER versus IF SNR curves are converted to BER versus $E_b/N_o$. The conversion from IF SNR to $E_b/N_o$ is performed using the relationship

$$\frac{E_b}{N_o} = \text{SNR} \frac{B}{R}$$ (5.6)
where $B$ is the IF noise bandwidth, and $R$ is the bit rate.

The IF noise bandwidth of the ICOM-2AT receiver was measured to be $B = 14.9 \text{kHz}$ [4]. In the current implementation, a maximum of 256 data sub-channels are used, each transmitting 2 data bits per block. Since 21 sub-channels are reserved for synchronization, the actual number of data sub-channels is 235. Therefore, the data rate is 470 bits per block. Accounting for the overhead of the pre-extension and post-extension, the block rate is 7.35 blocks per second. Thus, the overall bit rate is 3456 bits per second. Expressed in dB units,

$$\frac{E_b}{N_o} (\text{dB}) = \text{SNR} (\text{dB}) + 10 \log \frac{14900}{3456} (\text{dB})$$

which can be simplified to

$$\frac{E_b}{N_o} (\text{dB}) = \text{SNR} (\text{dB}) + 6.35 (\text{dB})$$

Figure 5.18 shows the adjusted BER results versus the calculated values of $E_b/N_o$. 

![Figure 5.18: Performance Loss Due to Synchronization](image)
The ideal performance curves are also shown to provide a reference from which the performance loss due to synchronization is measured. The ideal curves are determined based on the assumptions of ideal synchronization and no overhead required for synchronization information or for periodic extensions. Thus, the ideal bit rate is 4000 bits per second. The ideal BER values are obtained from Phase III experimental results. The results obtained by Casas [4] for a doppler rate of 20Hz and assumption of ideal synchronization are shown. These results allow comparison with the measured results of this thesis.

The curves in Figure 5.18 show that for both the 10Hz and 20Hz doppler rate, the use of synchronization results in a BER performance that is approximately 1.5dB worse than the BER performance with ideal synchronization. Increasing the doppler rate from 10Hz to 20Hz results in a 2dB improvement in BER performance. The conditions of 20Hz doppler rate and ideal synchronization are comparable to the conditions for the displayed results from [4] and so are the performance curves.
Chapter 6

Conclusions

6.1 Conclusions

OFDM/FM is a multi-carrier modulation technique for use with FM transceivers. It has previously been proposed for data communication over mobile radio channels. A new synchronization technique has been proposed enabling the use of OFDM/FM in a pure ALOHA environment. The synchronization technique utilizes the OFDM modulation principles, thus reducing its sensitivity to signal fades which occur on the mobile radio channel.

A simple model for synchronization was developed and used in selecting the operating parameters of the synchronization algorithm. Determination of the optimal parameters is a difficult and unsolved problem. A simple algorithm was proposed to select a good set of parameters.

The proposed synchronization algorithm relies on frequency domain analysis of the received signal and extensively uses the FFT implementation of the DFT. A modified version of the DFT was used that performs significantly faster than the FFT in its intended application. An analysis is provided defining the limitations of the modified DFT.

An experimental OFDM/FM system was implemented using unmodified commercial VHF FM radio equipment, a fading channel simulator, and commercially available DSP processors. A three phase synchronization procedure, based on the proposed synchronization model, was implemented. Test procedures were devised to separately measure the performance of each phase and to measure the performance of the integrated synchronization algorithm. Performance results from each phase were used to adjust the
operating parameters of the other phases. Final experimental results indicated that the BER performance of the system implementing the proposed synchronization algorithm is only 1.5 dB worse than the BER performance achievable given ideal synchronization.

6.2 Topics for Further Research

The current technique for determining which sub-channels should be reserved for synchronization is sub-optimal. No analysis has been done to determine how sub-optimal the current selection is, nor has any analysis been done to determine the effect of this sub-optimality on the performance of the synchronization algorithm. If it is determined that a significant improvement in performance can be obtained through better selection of synchronization sub-channels, work should proceed in formulating an improved selection process.

Differential encoding of data has been previously proposed as a way to circumvent the need to perform channel equalization. Synchronization, however, will still be required. Modifications of the proposed technique may yield a synchronization algorithm capable of operating with differentially encoded data.

Phase I of the synchronization algorithm does not exploit OFDM's insensitivity to signal fades. Replacing Phase I with a variation of Phase II, where the output of the Phase II correlator is threshold detected, may provide improved performance.

The modulating signal levels into the FM transmitter are based on values suitable for the human voice. The characteristics of an OFDM signal are different from those of human voice. An improvement in the IF SNR may result with different modulating signal levels.
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Appendix A

Models and Algorithms for Synchronization

A.1 Synchronization Model

The OFDM signal contains a synchronization signal that is composed of a number of tones located in reserved sub-channels of the OFDM signal. The OFDM signal has a finite duration and is preceded and followed by a weighted periodic extension of itself. Therefore, the synchronization signal also has finite duration and is preceded and followed by a weighted periodic extension of itself. From this, it follows that the synchronization signal is deterministic and can be represented as

\[ s(t) = W_s(t) \sum_{i=1}^{N_r} \sin(\omega_i t) \]  

(A.1)

where,

- \( N_s \) = number of synchronization tones
- \( \omega_i \) = angular frequency of the \( i^{th} \) synchronization tone
- \( W_s(t) \) = weighting function

A correlation detector, implementing the correlation equation

\[ R_{ss}(t) = \int_{-\infty}^{+\infty} s_r(t + \tau) r(\tau) d\tau \]  

(A.2)

can be used in a receiver to measure the similarity between the received signal, \( s_r(t) \), and a known reference signal, \( r(\tau) \). The reference signal is similar to the synchronization signal (equation A.1), except that it does not have the weighted periodic extensions. The
reference signal is defined as

$$r(\tau) = W_r(\tau) \sum_{i=1}^{N_s} \sin(\omega_i \tau)$$  \hspace{1cm} (A.3)

where \(i, N_s, \text{and} \ \omega_i\) are as above and \(W_r(\tau)\) is a window function equal to unity over a width equal to the duration of the OFDM signal, not including the periodic extensions, and zero elsewhere.

In the absence of channel distortions, the received signal will be identical to the transmitted signal and, therefore, can be modelled by equation (A.1). However, channel degradations cause phase and magnitude distortions in the received signal. Considering phase distortions only, the model of the received signal can be written as

$$s_r(t) = W_s(t) \sum_{i=1}^{N_s} \sin(\omega_i t + \psi_i)$$  \hspace{1cm} (A.4)

where \(\psi_i\) is the phase noise term.

From equations (A.2), (A.3), and (A.4), the output of the correlation detector is given by

$$R_{rs}(t) = \int_{-\infty}^{+\infty} \left[ W_r(\tau) W_s(t+\tau) \sum_{i=1}^{N_s} \sin(\omega_i \tau) \sum_{j=1}^{N_s} \sin(\omega_j(t+\tau) + \psi_j) \right] d\tau$$  \hspace{1cm} (A.5)

Defining \(T\) to be the symbol duration of the OFDM signal, \(W_r(\tau) = 1\) for \(\tau = [0,T]\), and zero elsewhere, (A.5) becomes

$$R_{rs}(t) = \int_{0}^{T} \left[ W_s(t+\tau) \sum_{i=1}^{N_s} \sin(\omega_i \tau) \sum_{j=1}^{N_s} \sin(\omega_j(t+\tau) + \psi_j) \right] d\tau$$  \hspace{1cm} (A.6)

The solution to (A.6) is not readily apparent. If it is assumed that the weighting function, \(W_s(t)\), is unity everywhere, (A.6) can be further simplified, yielding

$$R_{rs}(t) = \int_{0}^{T} \left[ \sum_{i=1}^{N_s} \sin(\omega_i \tau) \sum_{j=1}^{N_s} \sin(\omega_j(t+\tau) + \psi_j) \right] d\tau.$$

As defined in section 2.1.2, each synchronization tone has an integral number of periods in the interval \([0,T]\). Therefore,

$$\int_{0}^{T} \sin(\omega_i \tau) \sin(\omega_j(t+\tau) + \psi_i) d\tau = \begin{cases} \frac{T}{2} \cos(\omega_i t + \psi_i), & i = j \\ 0, & i \neq j \end{cases}$$  \hspace{1cm} (A.8)
Substituting (A.8) into (A.7) gives

\[ R_{r_s}(t) = \frac{T}{2} \sum_{i=1}^{N_s} \cos(\omega_i t + \psi_i). \]  

(A.9)

From \( W_s(t) \) as defined in section 5.1.3, the use of the simplifying assumption is valid when the correlation interval is within 16 sample periods of the true synchronization position, since then \( W_s(t) \) is unity over the range of integration. Therefore, (A.9) provides reasonably accurate results in the region of interest.

In regions farther away from the true synchronization position, \( W_s(t) \) is not unity over the range of integration. Hence, (A.9) is less accurate further away from the true synchronization position.

### A.2 Frequency Domain Derivation of the Correlation Detector

Given the spectral information of the sinusoidal components of a received periodic signal, it is possible to implement the correlation detector in the frequency domain.

Consider two phasors, \( P_r \) and \( P_s \), in the complex plane shown in Figure A.1, representing a single sinusoidal component of the received signal and a single sinusoidal component of the same frequency in the reference signal. The phasors can be defined as

\[ P_r = k_1 e^{j\phi_r} \]  
\[ P_s = k_2 e^{j\phi_s} \]  

(A.10) \hspace{1cm} (A.11)

Let \( \phi_r = \omega t_r \) and \( \phi_s = \omega t_s \), where \( \omega \) is the angular frequency of the two sinusoids and \( t \) is time. The angle between the phasors is \( \theta = \omega(t_s - t_r) \). The dot product of \( P_r \) and \( P_s \) is

\[ P_r \cdot P_s = |P_r| |P_s| \cos \theta = k_1 k_2 \cos(\omega(t_s - t_r)) = k \cos(\omega(t_r - t_s)) \]  

(A.12)

where \( k, k_1, k_2 \) are scalar constants. Letting \( \tau = t_r - t_s \), (A.12) can be re-written as

\[ P_r \cdot P_s = k \cos(\omega \tau). \]  

(A.13)
For synchronization signals with multiple sinusoidal components, the dot products of each signal/reference phasor pair are summed. The result is an equation that, with $\kappa = \frac{T}{2}$, is identical to the synchronization model (equation A.9). Therefore, a time domain correlation detector can be realized in the frequency domain as the sum of the dot products of the corresponding received synchronization signal and reference signal phasors.

**A.3 Fine Tune Algorithm**

It is assumed that synchronization is achieved when the output of the synchronization correlation detector is maximum. Using the synchronization model, equation (A.9), $\mathcal{R}_{rs_r}(t)$ is maximized when the condition $\omega_i t + \psi_i = 0$ is satisfied for all $N_s$ synchronization tones. Due to the presence of the phase noise term, $\psi_i$, it is unlikely that a value of $t$ can be found to satisfy this condition. However, the value of $t$ that maximizes the correlation can be determined. Equation (A.9) is modified to include the variable $\Delta t$, a time adjustment factor, yielding

$$\mathcal{R}_{rs_r}(t) = \frac{T}{2} \sum_{i=1}^{N_s} \cos(\omega_i(t + \Delta t) + \psi_i) \quad (A.14)$$
In equation (A.9), the term \((\omega t + \psi_i)\) represents the phase difference between the received signal synchronization components and the corresponding reference synchronization components. Therefore, equation (A.14) can be re-written as

\[
R_{rs}(t) = \frac{T}{2} \sum_{i=1}^{N_s} \cos(\omega_i \Delta t + \theta_i) \tag{A.15}
\]

where \(\theta_i \Delta (\omega_i t + \psi_i)\) is the phase difference. The time axis shift, \(\Delta t\), that maximizes the correlation, can be determined by setting the derivative of \(R_{rs}(t)\) equal to zero and solving for \(\Delta t\), yielding

\[
0 = \frac{dR_{rs}(t)}{dt} = - \sum_{i=1}^{N_s} \omega_i \sin(\omega_i \Delta t + \theta_i) \tag{A.16}
\]

or,

\[
0 = \sum_{i=1}^{N_s} \omega_i \sin(\omega_i \Delta t + \theta_i). \tag{A.17}
\]

The objective of the algorithm is to make the phase difference between the received sync signal and the reference as small as possible. That is, \((\omega_i \Delta t + \theta_i) \to 0\) for all synchronization tones. Assuming that the phase differences are small, we can use the small angle approximation, \(\sin \theta \approx \theta\), in (A.17), to obtain

\[
0 \approx \sum_{i=1}^{N_s} \omega_i(\omega_i \Delta t + \theta_i) \tag{A.18}
\]

or

\[
\Delta t \approx - \frac{\sum_{i=1}^{N_s} \omega_i \theta_i}{\sum_{i=1}^{N_s} \omega_i^2}. \tag{A.19}
\]

For a fixed set of synchronization tones, let constant \(\beta\) be defined as

\[
\beta = - \frac{1}{\sum_{i=1}^{N_s} \omega_i^2} \tag{A.20}
\]

Then

\[
\Delta t \approx \beta \sum_{i=1}^{N_s} \omega_i \theta_i. \tag{A.21}
\]
Appendix B

DFT Algorithms

This appendix presents three DFT algorithms used in this thesis for generating the OFDM baseband signal, decoding the OFDM baseband signal and acquiring frame synchronization during reception of an OFDM transmission. Also included is an analysis of the accumulated roundoff error for the third DFT algorithm.

B.1 FFT Implementation

Generation of the OFDM baseband signal is performed by an inverse DFT. Decoding of the received OFDM signal is performed by a DFT. To reduce computational requirements, the DFT block size is chosen to be \(2^n\). This allows use of the very efficient Fast Fourier Transform (FFT) algorithm [45,46].

The implemented FFT is based on the program \texttt{fftr2a.asm}, obtained through Dr. BuB, Motorola’s DSP electronic bulletin board. The basic algorithm is the decimation-in-time (DIT), radix 2 FFT algorithm using 24 bit fixed-point arithmetic with a sine-cosine lookup table for the FFT coefficients.

The algorithm is very restricted in its operation. It provides only the basic FFT for a fixed block size determined at compile time. Modifications were made to improve its flexibility. The following added features are run-time selectable through software control.

- forward/inverse FFT
- scale/noscale by \(N\)
- variable block size, \(2^n\) where \(n=1,2,3,...,15\).
This is a general purpose complex valued FFT algorithm. It was chosen because of its simplicity and flexibility. Although it is much faster than a DFT, it is not as fast as many optimized FFT algorithms. Improvements in speed can be obtained by using higher radix algorithms and other techniques discussed in [45]

### B.2 DFT Implementation

If spectral information is required for only a few subchannels, it is more efficient to perform a DFT for those selected frequencies than to perform an FFT.

Two DFT algorithms were considered, the direct calculation algorithm and the 2nd order Goertzel algorithm [46]. The Goertzel algorithm uses fewer mathematical operations than direct calculation and is faster on a general purpose computer. However, due to the parallel bus structure of the Motorola DSP56001, the direct calculation algorithm is both simpler to implement and is faster. The direct calculation algorithm was used in this work.

### B.3 Sliding Window DFT

During the establishment of frame sync, the data window on which the DFT is performed is shifted by increments of one data sample. At each window position, the DFT is used to calculate the spectral information for the sync tones.

When the window is shifted, most of the samples remain the same, except for their position within the window. This can be used to formulate an algorithm which updates the spectral estimates based on estimates from the previous window position. The result is a much faster algorithm.

Figure B.1 illustrates the sliding window concept. $X_0(k)$ and $X_1(k)$ represent the spectral information for two consecutive window positions.
Using the definition of the DFT equation [45], \( X_0(k) \) can be expressed as

\[
X_0(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi nk}{N}}. \tag{B.1}
\]

Similarly,

\[
X_1(k) = \sum_{n=1}^{N} x(n)e^{-\frac{j2\pi (n-1)k}{N}} \tag{B.2}
\]

which can be re-written as

\[
X_1(k) = e^{\frac{j2\pi k}{N}} \sum_{n=1}^{N} x(n)e^{-\frac{j2\pi nk}{N}} \tag{B.3}
\]

The summation term in B.3 is the same as that in B.1, except for the range of the variable of summation. Expanding (B.3) yields

\[
X_1(k) = e^{\frac{j2\pi k}{N}} \left[ X_0(k) - x(0)e^0 + x(N)e^{-j2\pi k} \right] \tag{B.4}
\]

Equation B.4 can be further simplified and written in general terms as

\[
X_{n+1}(k) = e^{\frac{j2\pi k}{N}} \left[ X_n(k) - x(n) + x(n + N) \right]. \tag{B.5}
\]
B.4 Accumulated Round-Off Error for the Sliding Window DFT

Since the sliding window DFT algorithm updates spectral estimates from one window position to the next, round-off error accumulates and progressively degrades the accuracy of the estimate. To ensure that the degradation will not be significant, an analysis of accumulated round-off error is performed.

The sliding window DFT (equation B.5) can be rewritten symbolically as the product of two complex terms,

\[ X_{n+1}(k) = (a + jb)(c + jd) = (ac - bd) + j(bc + ad) \]  \hspace{1cm} (B.6)

The Motorola DSP56001 is capable of performing the multiplications and additions in (B.6) within its arithmetic logic unit without incurring round-off errors. Round-off occurs only when intermediate or final results are transferred to data memory. From [47], the round-off errors can be modelled as additive noise with variance \( \sigma_i^2, \sigma_r^2 \), where

\[ \sigma_i^2 = \sigma_r^2 = \frac{2^{-2p}}{3} \]  \hspace{1cm} (B.7)

and \( p \) is the word size, in bits, of the data memory. The round-off error variance for each frequency component for a given update is

\[ \sigma^2 = \sigma_i^2 + \sigma_r^2 = \frac{2^{1-2p}}{3} \]  \hspace{1cm} (B.8)

Assuming that the round-off noise does not add coherently from one update to the next, the variance of the round-off noise for \( K \) consecutive updates is

\[ \sigma_K^2 = \frac{K2^{1-2p}}{3} \]  \hspace{1cm} (B.9)

The FFT implemented on the Motorola DSP56001 results in a round-off error given by

\[ \sigma_{\text{FFT}}^2 = \frac{(N - 1)2^{1-2p}}{3} \]  \hspace{1cm} (B.10)

where \( N \) is the FFT block size. If the round-off error of an FFT is used as an upper allowable limit, the maximum number of updates, \( K \), can be calculated as

\[ \frac{K2^{1-2p}}{3} = \frac{(N - 1)2^{1-2p}}{3} \]  \hspace{1cm} (B.11)
Appendix B. DFT Algorithms

Solving for $K$,

$$K = N - 1.$$  \hspace{1cm} (B.12)

For a DFT block size of $N = 1024$ samples, the maximum number of updates that could be performed before round-off error becomes a factor is 1023.
Appendix C

Digital Low Pass Filter

As discussed in section 4.3.2 a digital Low Pass Filter (LPF) is used to oversample (or interpolate) a band-limited input signal.

C.1 Filter Requirements

For this implementation, the input signal is specified as a signal with a maximum frequency of 3 kHz which is sampled at a rate of 8 kHz. An oversampling factor of 4 is required.

The standard method requires insertion of three zero's in the data stream for every data sample of the signal to be interpolated. The sample rate then becomes $4 \times 8 \text{ kHz} = 32 \text{ kHz}$. The addition of zeros in the data stream does not affect the shape of the spectrum below 4 kHz. The spectrum of the modified signal is periodic with period 8 kHz, not 32 kHz as with signals sampled normally at 32 kHz. The aliased signal components between 4 kHz and 28 kHz must be filtered out. To accomplish this without filtering out any of the frequencies below 3 kHz, the transition band of the designed filter must be from 3 kHz to 5 kHz. This results in a transition bandwidth of 2 kHz. Normalizing 32 kHz to $2\pi$ radians, the transition bandwidth is specified as $\pi/8$. Figure C.1 shows the spectrum of the signal modified with zero insertion and the spectrum of the required LPF.

C.2 IIR vs. FIR

The filter can be designed as either an Infinite Impulse Response (IIR) or Finite Impulse Response (FIR) filter. IIR filters have the advantage of superior amplitude response at the
expense of non-linear phase [48]. FIR filters, however, have exactly linear phase and are often preferred for interpolation. A common example of FIR filters used for interpolation are Compact Disk (CD) audio systems [49]. For this implementation, an FIR design was chosen.

Two basic techniques to design FIR filters are an iterative design method and the window method [48]. The iterative design method tends to give better results at the cost of increased design complexity and is normally implemented on a computer. The window method is straightforward in its application and can provide quite reasonable results. The window technique was used in this project.
C.3 FIR Filter Design

The first step in designing an FIR filter using the window technique is to approximate the ideal infinite impulse response with a finite impulse response. For LPF's, the ideal response is the sinc(x) = \( \frac{\sin x}{x} \) function. The finite impulse response approximation to this is simply a truncated (windowed) version of the sinc(x) function. The process of truncation results in a smearing of the filter's frequency response [48]. The extent of smearing can be reduced by increasing the size of the truncation window and/or modifying the window with a weighting function. Extensive analysis has been done in the design of windows [50]. In the present design, a Hanning window was chosen.

The transition width of the Hanning window is equal to \( \frac{8\pi}{N_w} \), where \( N_w \) is the number of samples in the window. As discussed in section C.1 the maximum allowable transition band is \( \frac{\pi}{8} \). Therefore, \( N_w = 64 \). If \( N_w \) is even, the filter will impose a phase shift equal to a non-integral number of sample periods resulting in a filter that will not preserve the samples of the original sequence [51]. However, if \( N \) is odd the filter will have a phase shift equal to an integer number of sample periods and, therefore, will preserve the original samples. In this case, an immediate processing reduction of 25% can be achieved, since only 3 interpolated values between the original samples have to be calculated. Choosing \( N = 63 \), results in always having 16 samples of the original data sequence and 47 added zeros in the window [51]. Further reduction in processing can be realized by noting that the zero samples of the signal do not contribute to the output of the filter. As a result, only the contributions of the 16 samples from the original data sequence have to be determined.

For the Hanning window, the window weights are given by

\[
w(n) = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi n}{N-1} \right) \right].
\]  

(C.1)

The unweighted filter coefficients of the approximated ideal LPF are given by the sampled
(\sin x)/x function centered at \( n = 32 \):

\[
h_d(n) = \frac{\sin[\omega_c(n - \alpha)]}{\omega_c(n - \alpha)}
\]  

(C.2)

where \( \omega_c = \pi/4 \) is the filter cutoff frequency, and \( \alpha = (N - 1)/2 \) is the phase delay.

The unit impulse response of the FIR linear-phase causal filter of length \( N \) is

\[
h(n) = h_d(n)w(n).
\]  

(C.3)

Expanding we can obtain

\[
h(n) = \frac{\sin[\omega_c(n - \alpha)]}{2\omega_c(n - \alpha)} \left[ 1 - \cos\left(\frac{2\pi n}{N-1}\right) \right]
\]  

(C.4)

Figure C.2 illustrates the truncated ideal LPF impulse response, the weights of the Hanning window, and the impulse response of the implemented filter.
Appendix D

Modulating Signal Power Prediction

Figure D.1: OFDM Baseband Signal Generation

Figure D.1 shows the basic steps in the generation of an OFDM baseband signal. A pseudo random bit sequence is QAM encoded, interpreted as frequency domain information and a conjugate symmetric spectrum is constructed. The spectrum data values are denoted by $X_k$. The OFDM baseband signal is generated by performing an inverse FFT on the encoded data, $X_k$. The resulting baseband signal data values are denoted by $x_i$. If the QAM constellation points are defined as $\pm 1 \pm j$, the mean of $X_k$ will be $\mu_X = 0$ and the variance will be $\sigma_X^2 = 2$.

Due to the properties of the IDFT, $x_i$ can be approximated by a Gaussian random variable. The mean of $x_i$ is then given by

$$\mu_x = E[x_i] = E\left[\frac{1}{N} \sum_{k=0}^{N-1} X_k W^{-ik}\right]$$

which can be reduced to

$$\mu_x = \frac{1}{N} \sum_{k=0}^{N-1} E[X_k] W^{-ik}.$$
However, $E[X_k] = \mu_X = 0$. Substituting into D.2 and simplifying, we get

$$\mu_x = 0.$$  \hfill (D.3)

The variance of $x_i$ is given by

$$\sigma_x^2 = E[x_i x_i^*] - |E[x_i]|^2$$

$$= E[x_i x_i^*]$$

$$= E \left[ \frac{1}{N} \sum_{k=0}^{N-1} X_k W^{-k} \cdot \frac{1}{N} \sum_{l=0}^{N-1} X_l W^{l} \right]$$

$$= \frac{1}{N^2} E \left[ \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} X_k X_l^* W^{-j(k-l)} \right]$$

$$= \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} E[X_k X_l^*] W^{-j(k-l)}. \hfill (D.4)$$

However,

$$E[X_k X_l^*] = \begin{cases} 
\sigma_x^2 & \text{for } k = l, k < \frac{N}{2} \\
0 & \text{for } k = N - l, k < \frac{N}{2} \\
0 & \text{otherwise} \end{cases} \hfill (D.5)$$

Therefore,

$$\sigma_x^2 = \frac{1}{N^2} \sum_{m=0}^{N-1} E[X_m X_m^*] \hfill (D.6)$$

Let $M$ be the number of subchannels used. For the subchannels not used, $X_i = 0$ and $E[X_m X_m^* | \text{subchannel } m \text{ not used}] = 0$. Therefore, (D.6) can be simplified and written as

$$\sigma_x^2 = \frac{2M}{N^2} \sigma_X^2 \hfill (D.7)$$

In summary, the statistics of the digital OFDM baseband signal can be expressed as

$$\mu_x = 0$$ \hfill (D.8)

$$\sigma_x^2 = \frac{2M}{N^2} \sigma_X^2$$ \hfill (D.9)
where $N$ is the size of the IDFT, $M$ is the number of sub-channels used, and $\sigma_X^2$ is the variance of the QAM encoded data.

In the implemented system, QAM encoding is $X_k = \pm 0.35 \pm 0.35j$. If the effect of the 10dB per decade de-emphasis is ignored, the mean is $\mu_X = 0$ and the variance is $\sigma_X^2 = 0.245$. The number of sub-channels containing data is $M = 256$, and the size of the IDFT is $N = 1024$. Therefore, the variance of the data following processing by the IDFT is

$$\sigma_x^2 = \frac{2 \times 256}{1024^2} \sigma_X^2 = 1.2 \times 10^{-4}. \quad (D.10)$$

and the standard deviation is

$$\sigma_x = 1.1 \times 10^{-2}. \quad (D.11)$$
Appendix E

Estimation of FM Channel Attack Time

Figure E.1: Hardware Configuration to Estimate the Attack Time of the ICOM-2AT FM Channel.

Figure E.1 shows the hardware configuration used to estimate the FM channel attack time for the ICOM-2AT transmitter/receiver system.

The baseband input to the FM channel is connected to a 50 kΩ termination via a voltage controlled switch. When a control voltage of zero volts is applied to the switch, the FM channel input leads are open circuited resulting in no RF carrier transmission by the FM transmitter. When the switch is closed by applying a positive +5 volt level at the control input, a 50 kΩ load is applied to the FM channel input leads causing the FM
transmitter to emit an RF carrier.

In the absence of an RF carrier, and with squelch turned off, the baseband output of the FM receiver is noise. In the presence of an RF carrier, the baseband output of the FM receiver is the recovered transmitted signal. Since the transmitted signal is zero volts DC, the recovered signal should be zero volts DC plus noise incurred due to the FM channel.

The output of the FM channel is captured with a digital storage oscilloscope. Triggering of the oscilloscope is provided by the control signal used to trigger FM transmission.

The following details the procedure for estimating the FM channel attack time.

1. Connect the FM channel, voltage controlled switch, and the digital storage oscilloscope as in Figure E.1.

2. Trigger the oscilloscope on the rising edge of input B.

3. Adjust the oscilloscope to the appropriate voltage and time base settings. This will be approximately 1 to 5 Volts per division vertically and 20 milliseconds per division horizontally.

4. Set trigger signal to zero volts.

5. Set oscilloscope for single trace and arm the trigger mechanism.

6. Set trigger signal to +5 volts DC.

7. Examine oscilloscope tracing and measure the duration from the rising edge of the trigger signal to the location where the noise output of the FM channel has diminished to its final level. This measurement provides an estimate of the FM channel attack time.