TOOL WEAR MONITORING IN FACE MILLING

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Abstract

The purpose of this thesis is to investigate tool wear monitoring using Fourier Series simulation of steady state cutting forces. These simulations show that mean and fundamental values are all that is required to accurately predict immersion and tool wear (high frequency terms are ignored). It is found that the ratio of the magnitude of the fundamental values of force over the quasi-mean resultant force are insensitive to wear, while the same ratio is found to change markedly with immersion. Due to the nature of wear and different cutting conditions, two different wear identification methods are proposed. The first type of wear is chipping of the primary edge; the ratio of quasi-mean resultant force over mean torque gives the necessary indication without being affected by normal wear. The second type of wear studied is the normal wear band, where the axial force, $F_z$, (which was modelled using equivalent chip thickness, $h_{eq}$, and equivalent approach angle, $\psi_e$) is found to be useful in the identification of this type of wear. The mean value of $F_z$ over the mean value of torque gives information about the state of normal wear while being insensitive to chipping.

Work on an insitu sensor is also reported. Preliminary investigation shows that a deposit comprising a hybrid resistor on the flank face of a throw-away insert has the potential to monitor wear due to the permanent increase in resistance of the deposit as cutting proceeds. A U.S. patent has been obtained for this idea.
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Nomenclature

$F_t$: tangential force
$F_r$: radial force
$F_X$: static cutting force in the $x$ direction
$F_Y$: static cutting force in the $y$ direction
$F_{wX}$: wear force in the $x$ direction
$F_{wy}$: wear force in the $y$ direction
$F_{XT}$: total force in the $x$ direction
$F_{YT}$: total force in the $y$ direction
$F_{qm}$: quasi-mean forces
$K_s$: specific cutting pressure
$a$: axial depth of cut
$s$: feed per tooth
$h$: instantaneous chip thickness
$h^*$: imaginary chip thickness in the flank wear zone
$h_{eq}$: equivalent chip thickness
$h_{eq}^*$: critical value of the equivalent chip thickness
$\phi$: instantaneous angle of immersion
$\phi_s$: swept angle of cut
$\psi_e$: effective approach angle
$r$: ratio of radial forces to tangential
$r_2$: ratio of radial forces to tangential on the flank zone
$N$: number of tooth
\( a_k, b_k: \) Fourier series (F.S.) coefficients for tangential force

\( a_{\alpha k}, b_{\alpha k}: \) F.S. coefficients for \( F_X \)

\( a_{\gamma k}, b_{\gamma k}: \) F.S. coefficients for \( F_Y \)

\( a_{\omega k}, b_{\omega k}: \) F.S. coefficients for tangential force in the flank zone

\( a_{\omega \alpha k}, b_{\omega \alpha k}: \) F.S. coefficients for \( F_{\omega X} \)

\( a_{\omega \gamma k}, b_{\omega \gamma k}: \) F.S. coefficients for \( F_{\omega Y} \)
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And he said to Giacomo:

Il poeta trovò la sua poesia.

Ora, andiamo da Momus.
Chapter 1

Introduction

Monitoring wear on cutting tools can give very valuable information about the useful life of a tool and its operating conditions. It is especially necessary in flexible manufacturing systems (FMS) and unmanned cutting operations to track the wear of the cutting tool, as the behaviour of the tool is critical for their functionality. The economic importance of monitoring in manufacturing processes has been illustrated by Tönshoff et al. [1]. In figure (1.1), it can be seen that monitoring a process not only prevents technical failures but it also increases the productivity.

If a tool is worn substantially, it can cause:

1. Dimensional errors on the workpiece;
2. Worsening surface-finish quality;
3. Fatal damage to the cutting tool;
4. Damage to the machine tool itself.

There are two ways to track the wear on the tool during the cutting process (also known as on-line wear detection). The first method, the direct method, involves taking dimensional measurements of the tool or the workpiece as cutting is progressing. Researchers find it difficult to implement such techniques because of the harsh cutting environment and those that can be implemented do not lend themselves readily to on-line detection. The alternative method is to look at various parameters that might be
affected by tool wear during cutting (this is known as the indirect method). This method has been implemented successfully and results look promising.

The rest of this thesis will be organized as follows: in chapter 2, a brief literature survey is presented. In chapter 3 the derivations necessary for modelling are shown. Chapter 4 is where mathematical expressions used in wear monitoring are derived. In chapter 5, experimental work to verify the derived methods is presented. Some conclusions and recommendations follow chapter 5.
Chapter 2

Literature Survey

2.1 Tool Wear

While it is beyond the scope of this work to look at wear mechanisms in detail, a brief introduction will be given for completeness. Because of the severe rubbing nature of the cutting process, tools wear on both the rake and clearance (flank) faces. Wear on the rake face is typically in the form of a crater and hence takes its name as crater wear (annotated by KT). The flank face is usually worn as a flat surface extending from the tip of the tool to the end of flank contact. This is usually called flank wear and is annotated by VB. Typical tools with flank and crater wear are shown below in figures (2.1) & (2.2).

2.2 The Wear Processes

There are four major wear mechanisms that have been identified and are described as follows:

2.2.1 Adhesion Wear Mechanism

The most straightforward explanation given is that welded junctions between the tool and workpiece take place and subsequently when they shear below the interface, wear particles are formed. Further research by Archard and Hirst, and Hirst and Lancaster [4] has revealed that two types of wear mechanism are at work depending on the cutting conditions. "Severe" wear occurs when the cutting loads and the associated interface
Figure 2.1: A tool with typical flank wear (after Tönnhoff et al.)

Figure 2.2: A tool with crater wear (after Sarin et al.)
stresses are high. A sudden step increase in wear rate is observed. Considerable welding and tearing of the softer rubbing surface and large wear particles are expected under these conditions. The major difference between this kind of wear and "mild" wear is the difference in wear rates. When the wear rate is low (in the order of $\approx 10^{-10} \text{cm}^3/\text{cm}$), mild wear is observed. As the wear rate increases proportional to load there is a sudden increase in wear rate corresponding to the severe wear mechanism (wear rates in the order of $\approx 10^{-7} \text{cm}^3/\text{cm}$). Mild wear conditions are characterized by the improvement of the sliding surface finish (due to the removal of uneven or high spots) and the formation of smaller wear particles. The mechanisms for mild and severe wear are explained further as follows. In both conditions the riders (cutting tools) are rubbing on a hard rotating surface (workpiece). In the case of severe wear, wear particles are transferred to the rotating surface, building up small contact regions. With time these small contact regions grow and become unstable so that they are removed by the cutting tool upon impact. Eventually an equilibrium wear rate is reached with some contact regions growing while others are removed as large free wear particles. In the case of mild wear, an oxide film forms on the contact regions. It is this layer that differentiates mild wear from severe wear. Mechanical "rubbing off" of the oxide film produces smaller wear particles. The wear rate in this case is largely controlled by the rate of the oxide formation. Under low loads metal to metal contact is prevented and the oxide film has time to form; however, if the loads are increased then the oxide layer is penetrated and conditions return to that of severe wear situation. The above considerations are for repeated rubbing of the tool over the same workpiece surface. Armarego & Brown in their book on metal cutting [4] mention their investigation to this end. They show that for a hard (HSS) tool on a softer (mild steel) workpiece, the tool wear rate is considerably lower for rubbing on an unworn surface than for repeatedly rubbed surface. The explanation for the wear is that a lump of material builds up on the tool in non-repeated rubbing and this lump turns out to be
the only contact with the rubbed surface. As this lump grows bigger it becomes unstable and breaks off eventually taking with it some of the tool material causing wear. This wear rate is higher for repeated rubbing.

2.2.2 Abrasive Wear Mechanism

The concept of abrasive wear is that of abrasion of high spots on one surface by the other surface. The process itself is a cutting process and therefore material properties such as hardness, elastic modulus and the geometry of the tool are important. In fact research has revealed that "the larger the amount of elastic deformation a surface can sustain, the greater will be its resistance to abrasion wear" [4, pp. 133]. The abrasion resistance was later found to be proportional to the elastic modulus of the workpiece. A good initial model for modelling the abrasive wear mechanism is that of a surface loaded by a sliding indentor (analogous to an asperity on a mating surface). The important parameter to consider in this mechanism is the criterion for transition from elastic to plastic deformation. While various researchers concluded differently with respect to the effect of the size of loads, it is accepted generally that plastic deformations for most surfaces take place under light loads. The first step is to consider a spherical indentor being pushed along a surface. Material will start piling up in front of the indentor. The forces on an element on the indentor will be along the sliding direction and normal to it. Resolving these forces into friction force and normal force (see figure (2.3)), the following equations can be written:

\[
\mu N = Q \cos \alpha - P \sin \alpha \quad (2.1)
\]

\[
N = Q \sin \alpha + P \cos \alpha \quad (2.2)
\]
where \( \mu \) is the coefficient of friction between the material and the indentor, \( \alpha \) is the angle the indentor makes at the point of contact with the vertical axis, \( Q \) and \( P \) are forces in the sliding direction and normal to it. The condition for material piling up in front of the indentor can be expressed mathematically as \( \mu N > Q \cos \alpha - P \sin \alpha \). At this critical condition, a simple manipulation of eqns. (2.1) & (2.2) gives,

\[
\tan \alpha = \frac{Q - \mu P}{\mu Q + P} \quad (2.3)
\]

If the depth of penetration can be written knowing \( \alpha \) and radius of the indentor,

\[
h = R(1 - \cos \alpha), \quad (2.4)
\]

it is now possible to write equation (2.3) to calculate \( h \). This equation will be a function of the shear stress of the material as well as work-hardening properties and the shape of the indentor:

\[
h \geq R \left[ 1 - \frac{\mu Q + P}{\sqrt{(P^2 + Q^2)(1 + \mu^2)}} \right]. \quad (2.5)
\]

If one assumes that the normal forces \( P \) and \( Q \) are equal then eqn. (2.5) will be reduced to:

\[
h \geq R \left[ 1 - \frac{\mu + 1}{\sqrt{2(\mu^2 + 1)}} \right]. \quad (2.6)
\]

This above expression is useful for estimating the approximate depth at which cutting starts, given the radius of indentor and the coefficient of friction.

### 2.2.3 Diffusion Wear Mechanism

Diffusion is a temperature dependent atomic transfer process. Initially diffusion was thought to occur at the contacting asperities (local high points on the surfaces of the
cutting tool and the chip). Recently however new work demonstrates that diffusion plays an important part in other wear mechanisms, particularly abrasion. Examples of this situation occur most readily when carbide tools are used to cut steels. Steel has a chemical affinity to attack the cobalt matrix of the carbide tool diffusing the $Co$ into the workpiece. This chemical attack leaves the tungsten carbide surface weakened and as a result usually an abrasive process wears it causing craters. The above situation is controlled by two methods: 1, by adding titanium and tantalum alloying elements; 2, by an aluminium oxide coating. The above situation is one scenario for diffusion wear. Researchers have found a number of other ways in which diffusion processes influence final tool life. They are listed as follows:

- Gross softening of the tool. Diffusion of carbon from the cutting tool into the workpiece can lead to softening of the tool and produce major plastic flows in the tool. Eventually this leads to a catastrophic tool failure.

\footnote{A comprehensive reference list can be found in The Machining of Metals by Armarego and Brown}
Chapter 2. Literature Survey

• Diffusion of major tool constituents into the workpiece. A major strengthening constituent of the tool matrix may be diffused into the workpiece or the chip surface as it passes the tool. This may be a problem particularly for carbide or ceramic tools.

• Diffusion of minor tool constituents into the workpiece. This is the situation that involves the cobalt binder in a carbide tool as initially discussed above.

• Diffusion of a work-material component into the tool. Unlike the previous cases here the material from the workpiece can diffuse into the tool and alter the physical strength of the tool. Diffusion of lead into the cutting tool may produce a thin brittle layer that can be removed by fracture.

• Diffusion of a tool constituent influencing the work material. A more interesting result of diffusion involves strengthening of the chip surface by the carbon present in the tool. This carbon enrichment of the chip surface strengthens it and causes a higher tool wear rate.

These above conditions can occur simultaneously or one mechanism can predominate depending on the cutting conditions; however, they are all manifestations of the diffusion process.

Initial work in modelling diffusion gave a relation of wear rate with that of temperature,

\[ W = AT^B \]  \hspace{1cm} (2.7)

where \(A, B\) are constants. Other independent research by Saibel and Ling [4] also revealed to a first approximation a relationship between the wear rate and temperature using the more general equation below:
where, \( N \) is the normal load, \( l \) is the sliding distance, \( \sigma_u \) is the yield stress for the weaker material in contact and finally \( k \) is the coefficient of wear. Using the chemical reaction theory, \( k \) could be written in terms of temperature as:

\[
k = Ke^{-\frac{E}{RT}}
\]

\( E \) being the activation energy in chemical reactions, \( R \) universal gas constant. Hence after the substitution of eqn. (2.9) into (2.8), it is shown that \( \ln(W/Nl) \propto 1/T \) and to a first approximation:

\[
\frac{W}{Nl} \propto \frac{1}{T}
\]

From this relationship in (2.10), it was deduced that "the shape of the crater in the rake face of the tool is directly related to the rake face temperature distribution" [4, pp. 146]. However, it should be pointed out that this would be good for an initial temperature distribution, since the shape of the crater with increasing wear will affect the temperature distribution around it which in its turn affects the wear rates.

### 2.2.4 Fatigue Wear Mechanism

It was demonstrated experimentally by Radchik and Radchik [4] that an asperity on a given tool surface will have stresses alternating between compression and tension as it is passing under the surface of the workpiece. At some distance ahead of the asperity, there will be compressive forces since the underlying surface will be compressed. Similarly, behind the asperity the surface will be elongated and therefore tensile forces will be built up. This change in the nature of stresses as asperities pass by the workpiece surface
can cause fatigue failure of the tool material. Cracks that build at the bottom of the asperities eventually propagate to the surface and result in the chipping of material from the cutting tool. In the more general case, it is believed that oxidation of the cutter surface takes place, and it is the successive removal and reformation of this oxide layer by fatigue that causes the wear process. The fatigue wear mechanism also explains wear in the presence of lubrication. The lubricant reduces the friction load on the tool, while the normal load acting on it remains the same. The material around the asperity on the tool is still subject to a fluctuating stress although there may be no metal to metal contact due to the lubricant film.

2.3 Tool Wear Monitoring

As is already mentioned, there are two basic approaches to monitor wear. In this chapter a brief survey of the previous work (both direct and indirect methods) in the field will be discussed.

2.3.1 Direct Methods

The idea behind the method is to track the wear by direct physical measurements of the tool and the workpiece. This surely gives the most reliable results but there are serious problems involved in implementation. The harsh cutting environment puts various constraints on what can be achieved under the circumstances. Nevertheless, there are proposals that are worth mentioning and for completeness are included here.

Tracking with stylus

Suzuki and Weinmann [5] have devised a simple tool wear tracking sensor. The device consists of two parts: 1) A displacement transducer, and 2) a stylus, see figure (2.4).
The stylus follows the cutter always in contact with the work surface. As wear occurs, the distance between the disk and proximity probe is reduced and the probe sends a signal directly proportional to that distance. The dynamic characteristics of the stylus are ignored totally and it is further assumed that the stylus itself will not wear. The major problem from this design comes from the fact that the stylus is always trailing the cutter and hence it is not a true on-line detection sensor. The disadvantage is the failure of the device to detect tool failure before it is too late. Further, because the sensor is designed for continuous cutting, in intermittent cutting such as milling this device cannot be used. The authors claim that their device can determine the wear on the tool with an accuracy of 90%.

**Pneumatic system**

Stöferle and Bellmann [6] propose a pneumatic device to measure wear on tool flank. The fluidic measuring system consists of a nozzle and a baffle plate (see figure (2.5) for a better understanding of the measuring principle).

A nozzle is placed on the tool away from the worn area. The gap is measured using
the differential pressure (i.e. the difference between the reference pressure and the nozzle outlet pressure) and the calibration graphs shown in figure (2.6).

Care must be taken to stay within the linear part of the graph in figure (2.6). The problems with this method arise from the working fluid (usually air). The high temperatures due to cutting can cause changes in viscosity of the fluid which is sufficient to affect the nozzle outlet pressure: hence the measurement becomes inaccurate. Another inherent problem with this system is the calibration that is necessary each time a new insert or a different workpiece is used, thus rendering the system impractical.
TV image method (optical techniques)

Giusti, Santochi, and Tantussi [9] built and tested a sensor that uses image analysis techniques. The principle behind the sensor is the computer analysis of the images obtained using proper lighting of the tool and proper angles of viewing. A schematic description of the system is represented in figure (2.7) with a diagram of the sensor.

A DEC VAX 11/730 host computer is used to analyze the stored digital data in a 512x512 pixel frame. Software used in conjunction with the above hardware gives the maximum wear, $V_{B_{\text{max}}}$, and mean wear $V_{B_{\text{mean}}}$ at several locations, the number of which depends on the time the tool is available for observation. The results of one experimental run along with the image of crater wear is shown in figure (2.8).
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The system is impressive in creating the image of wear but it has serious shortcomings in terms of practicality. First of all, it is an expensive set-up that requires expert knowledge in the field of image analysis. Furthermore, the system works only when the tool is completely disengaged from the workpiece and machining is stopped momentarily or when the tool is in the tool magazine. Therefore, from the point of view of in-process techniques, it renders itself impractical. As a last point, an extensive software must be written for the proper functioning of hardware.

Tool Tip Sensor

Spur & Leonards [10] suggested a tool tip sensor that would give a signal indicative of wear. Their sensor used the principle of an electric circuit that was closed when the cutting tool made contact with the workpiece. A group of resistors are deposited on the
flank surface of the insert perpendicular to the cutting edge. The length of each resistor is 50 \( \mu m \) shorter than the previous one. The principle behind this sensor can be explained with the help of figures (2.9) and (2.10).

A current source is fed to the resistive circuitry. As the tool is worn, more resistive elements are added to the circuit because a larger part of the flank surface makes contact with workpiece. The total resistance decreases because of this situation, the current rises and through the use of an ammeter this can be measured and related to wear. The benefit of this sensor is its low cost and simplicity of instrumentation. There are however two problems associated with it. First, the values of resistances depend strongly on temperature. Since the cutting zone generates a good deal of heat, this is an important setback. The second problem is the fact that the closing of the circuit depends on the contact between the workpiece and the insert. This is a problem since a chip or other metal particles could also complete the circuit giving false results.

Aoyama, Kishinami, and Saito [11] proposes an alternative system where the resistive
circuitry is independent of the contact between the workpiece and the insert. The idea behind their sensor is interesting. Again taking figure (2.11) as a reference, the working principle can be explained.

The resistive element is deposited on the flank surface parallel to the cutting edge. Unlike the Spur sensor it consists of only one resistor. As the flank face wears so will the resistor. This of course will cause the resistance to increase. It is this increase in resistance that the authors correlate to the magnitude of wear. This method does not isolate the increase in resistance due to cutting heat from that of area reduction. Hence, the temperature problem is not solved. Indeed, the graph below shows exactly this.

Further to the temperature problem, due to the condition of the contact between the titanium circuit and the workpiece, there is high frequency noise in the signal. The authors suggest because of the temperature problem and the noise present during cutting their sensor is valuable in the off-line monitoring of wear.

A third approach proposed by the author and Yellowley [12] takes the previous idea
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Figure 2.11: The Japanese sensor (after Aoyama et al.)

Figure 2.12: Correlation of resistance to wear (after Aoyama et al.)
one step further. As was seen the effect of temperature on resistance is important. This method looks precisely at the relationship of temperature on resistance and makes use of that relationship to detect wear. The resistive element is again deposited on the flank surface and parallel to the cutting edge. The idea is that as the flank is worn, the resistor is closer to the heat source and this permanently alters the resistance of the deposit. The reader should note that unlike the previous case no abrasion of the resistor needs to take place. With increasing wear, the cutting zone approaches the resistive deposit on the flank face. It is this approaching heat source that permanently damages the resistor and allows a correlation between wear and resistance. An experiment is devised to test how the temperature affects the resistance of the sensor. The induced permanent damage on the resistor is expected to increase the resistance. The set-up involves a gas burner and a stand to which the insert is attached. The burner is directly underneath the insert. The heat source is brought closer to the rake surface by lowering the insert holding arm (see figure (2.13) for the set-up). These readjustments in the distance to the heat source are done about every twenty minutes so that there is sufficient time for temperatures to reach steady state values on the rake face. To achieve temperature distributions comparable to the case while cutting a small hole was drilled in the insert holding apparatus allowing direct contact between the heat from the source and the rake face itself. The figures (2.14) and (2.15) show the surface of the resistive deposit that were and were not subjected to heat treatment respectively. The important point to note is the homogeneity of the particle size and the gaps between each particle for the untreated surface and the non-homogeneous particles in size and shape as well as the size of the gaps. It is believed that the elevated temperature of cutting causes permanent change in the crystal structure of the deposit such that the gaps between conducting particles are larger now causing it to have larger resistances.

The other important aspect this type of sensor differs from the earlier ones discussed
Figure 2.13: The setup for temperature experiments
Figure 2.14: The untreated resistive deposit
Figure 2.15: The treated resistive deposit
is due to the type of resistor material used. The previous designs uses thin film depositing techniques for the resistor material (used in I.C.'s extensively). The design proposed here uses thick films. Firstly, the deposition technique is simpler with thick films since the process involves basically an accurate silk screening process, as opposed to a virtually vacuum environment necessary for thin film depositions. This makes the thin film deposition a more expensive and specialised process.

Here it is a good place to discuss briefly some of the more common resistive materials used in thick film deposition techniques and their performance with elevated temperature comparable to that of the cutting zone temperatures. Thick film pastes, also referred to as inks, are used in forming conducting lines, resistors and dielectrics. The earlier resistive pastes consisted of combinations of silver, silver oxide, palladium, and palladium oxide. But because palladium oxide was readily reduced to palladium from the hydrogen generated from the chemical reaction of metals with moisture, they were highly unstable. More recently ruthenium is used in place of palladium. Ruthenium is not only stable against the chemical reduction of its oxide but thermodynamically more stable as well. In fact the resistance drift is expected to be ± 0.3 % under no load condition after 1000 hrs. at 150°C s. With 25 watts/in² at 85°C again after 1000 hrs. the drift is between 0.25 to 0.30 %. Thick film resistor pastes are available in sheet resistances of 1 \( \Omega \) per square to \( 10^9 \Omega \) per square millimeters.

A second advantage that thick films present is the control in the position of the conducting interface. This is important since it can be used to isolate changes in resistance due to material loss from changes due to temperature induced conditions. In real life situations, it will be difficult to prevent loss of resistive material due to the wear of the flank face. But this can be handled by having these sensors in the rake face as well as the flank face. Hence the sensor on the rake face would be affected by temperature only and can be used to calibrate the flank wear sensor so that the sensor on the flank face
Figure 2.16: Temperature experiments

would be affected by the change of the cross section area which is indirectly proportional to a change in resistance. The results of the temperature experiments is shown in figure (2.16)). The graph shows the resistance vs. temperature curve. The temperature is measured with a thermocouple. The graph indicates that the resistance is little affected by temperature until around 300°C, after this temperature the resistance seems to increase exponentially. When a second set of experiments under similar conditions is done with the same insert, the resistance of the deposit is not the same as at the beginning of the first test; a permanent change in resistance has been achieved. This permanent increase in resistance can be used to correlate wear. The justification as explained earlier is that as wear occurs the sensor on the wearland is approached by the heat source in the flank.

The sensor is still being developed, a patent application has been completed and the patent recently granted [12].
2.3.2 Indirect Methods

In the previous section, direct physical measurements of tool wear were discussed as a possible means of monitoring. In this section, parameters that are affected by tool wear are considered and methods to use these parameters for tracking wear on the tool will be proposed. These parameters are: 1, Static cutting forces; 2, Dynamic cutting forces; 3, Temperature; 4, Acoustic Emission.

Cutting Forces

A worn (dull) tool will generate larger cutting forces than a new tool. It is this fact that makes it possible for cutting forces to be used as an indication of wear for monitoring purposes. Various researchers have looked into this method. Only two such investigations will be discussed here.

Force Ratio Method

In their paper Colwell, Mazur, and DeVries [13] performed numerous orthogonal cutting tests while measuring cutting force, $F_C$ and feeding force, $F_N$. The results of these tests are shown in figure (2.17).

The figure shows that $F_C$ is higher than $F_N$ in steady state cutting. During a dwell cycle, which corresponds to zero depth of cut (the cutting edge is just rubbing the workpiece), both forces decrease to the same minimum value (this DC offset exists because of the worn flank that is rubbing the workpiece). The situation is different in the second figure, here $F_N$ is higher than $F_C$; furthermore, during the dwell, the behaviour is significantly different. The feeding force is twice the value of cutting force. The authors call the moment at which the $F_N$ and $F_C$ magnitude reversal occurs, the dog-leg (or "break in the wear vs. time curve" as seen in figure (2.18)). Furthermore, from the previous figures
Figure 2.17: Force signals for flank wear before and after tool-life (after Colwell et al.)
it is clear that $F_N$ is more sensitive to wear than $F_C$. After the above investigation, the authors propose a method for detecting wear (to be more precise wear rate).

It is apparent from the force history plots that any part of the graph (any window) can be used as the detection reference. Taking the dwell cycle as an example, it is possible to monitor the behaviour of wear at dwell after steady state cutting. If the dog-leg stage takes place then a signal can be sent to stop machining as the end of useful tool life nears. The problem is to be able to detect the steep change in slope of $R = F_N/F_C$. A least squares regression model is used to update $R_i$ at each time, $T_i$:

$$R_i = b_0^{(l)} + b_1^{(l)}[T_i - T_l - N - 1] + e_i \quad (2.11)$$
where $b_o^{(l)}$ & $b_1^{(l)}$ can be expressed in the matrix form.

$$
\begin{pmatrix}
  b_o^{(l)} \\
  b_1^{(l)} 
\end{pmatrix} = \begin{bmatrix}
  N & \sum_{i=l-N}^{l} (T_i - T_{i-N-1}) \\
  \sum_{i=l-N}^{l} (T_i - T_{i-N-1}) & \sum_{i=l-N}^{l} (T_i - T_{i-N-1})^2 
\end{bmatrix} \begin{pmatrix}
  \sum_{i=l-N}^{l} R_i \\
  \sum_{i=l-N}^{l} (T_i - T_{i-N-1}) 
\end{pmatrix}
$$

(2.12)

Moreover, a test can be devised to test if the slope of the $F_N/F_C$ vs. time graph has increased:

$$t = \frac{b_1^{(l)} - b_1^{(l-1)}}{\text{Var}[b_1^{(l-1)}]}$$

(2.13)

This test (hypothesis testing) has the advantage of showing if the new $b_1^{(l)}$ has changed significantly from the last point, taking errors into account. If the value of $t$ from equation (2.13) is greater than $t_\alpha(N - 2)$ then the slope has changed significantly and a signal should be sent to the system. The important two factors user should be aware of are: 1. The value of $N$ (how many past values to use); 2. $\alpha$, probability of false alarm. In table (2.1) the algorithm can be seen at work.

**Feed Force vs. Feed Method**

Uehara, Kiyosawa, Takeshita [14] suggest that, after various experiments, the relationship between the feed force and the feed per revolution is strongly influenced by tool wear. First, the effects of flank wear and crater wear on the feed force vs. feed curve are discussed. In figure (2.19) feed force vs. feed is plotted in the presence of flank wear. There is a visible peak in the feed force before it settles to a steady state increase. The authors explain this peak as the result of rubbing of the tool at low feeds. With large flank wear it is clear that the feed force is large too.

Similarly, feed force vs. feed in the presence of crater wear is examined. In figure (2.20), it is seen that large crater wear actually reduces the feed force at small feed rates.
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Table 2.1: Detection data using force ratio for steady state cutting of titanium (after Colwell et al.)

<table>
<thead>
<tr>
<th>$T_e$ (min)</th>
<th>$\beta_1$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.69</td>
<td>-0.014</td>
<td>0.73</td>
</tr>
<tr>
<td>3.26</td>
<td>-0.021</td>
<td>0.83</td>
</tr>
<tr>
<td>3.83</td>
<td>-0.024</td>
<td>0.63</td>
</tr>
<tr>
<td>4.60</td>
<td>-0.013</td>
<td>2.22</td>
</tr>
<tr>
<td>5.17</td>
<td>0.006</td>
<td>2.43</td>
</tr>
<tr>
<td>6.74</td>
<td>0.013</td>
<td>0.81</td>
</tr>
<tr>
<td>8.77</td>
<td>0.007</td>
<td>-0.93</td>
</tr>
<tr>
<td>9.84*</td>
<td>0.079</td>
<td>13.91</td>
</tr>
</tbody>
</table>

Series #2

<table>
<thead>
<tr>
<th>$T_e$ (min)</th>
<th>$\beta_1$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.57</td>
<td>-0.002</td>
<td>-0.472</td>
</tr>
<tr>
<td>8.07</td>
<td>-0.002</td>
<td>-0.045</td>
</tr>
<tr>
<td>9.50</td>
<td>-0.002</td>
<td>-0.041</td>
</tr>
<tr>
<td>10.30*</td>
<td>0.038</td>
<td>21.50</td>
</tr>
</tbody>
</table>

*Change Detected

Figure 2.19: Expected relationship between feed and feed force for a tool with flank wear only (after Uehara et al.)
This intuitively contradictory result can be explained as follows. With increased crater wear, the effective rake angle also increases. Because $F_f \propto \sin(\alpha)/\cos(\alpha)$ (where $\alpha =$ rake angle), it is easy to see that an increase in $\alpha$ translates in a decrease in $F_f$. If the feed is increased, cut chip thickness increases and the crater wear is not able to act as an effective rake angle increasing mechanism. Hence, after sufficiently large feedrates, the tool behaves like a new tool (as far as crater wear is concerned).

Experiments performed by Uehara et al. confirm these trends as can be seen in figure (2.21).

The objective of the experiments was to obtain a quantitative result for flank wear and crater wear. By setting limits to both types of wear (here $V_B_{max} = 0.15 \, mm$ & $K T_{max} = 20 \, \mu m$), an alarm signal is sent when maximum allowable wear is reached. The height of the peak in the $F_f$ vs. feed diagram gives the value of flank wear, while the length of the plateau gives the value of crater wear. Graphs for the above relationships must be obtained for each situation hence the system is suitable for high volume machining.
Figure 2.21: Feed force oscillograms (after Uehara et al.)
Cutting force ratio, $F_T/F_Z$

An additional study of the influence of flank wear on the force ratio was carried out by Lai [15]. The work was carried out for turning operations. The first important contribution of this author was his model of cutting forces in turning. He used the idea of equivalent chip thickness to take into consideration the geometry of cut (except the rake angle). The tangential force, $F_Z$ is written as:

$$F_Z = A(K_1 + \frac{K_2}{h_{eq}}),$$  \hspace{1cm} (2.14)

and the radial force, $F_T$, can be written as:

$$F_T = A(r_1K_1 + \frac{r_2K_2}{h_{eq}}).$$  \hspace{1cm} (2.15)

Normalizing the above equations with respect to area, $A$, it is possible to obtain the ratios $\frac{F_T}{A}$ and $\frac{F_Z}{A}$ which are also known as specific cutting pressures. The linearity of the above model is verified by Lai and as example one of the graphs that demonstrates this is shown in figure (2.22). This implies of course that it is possible to obtain the cutting constants $K_1, K_2, r_1, r_2$ from the slopes and intercepts of the graphs. After experimentally confirming the cutting force models, Lai looks at the ratio of $\frac{F_T}{F_Z}$ with increasing wear. Physically what happens is that the in-plane forces increase at a faster rate than the tangential force. This intuitively makes sense when one considers an increase in forces since the force in flank face is dependent on the area of wear (i.e. the pressure distribution on flank face integrated over area). The graph in figure (2.23) illustrates how the force ratio behaves for increasing VB. Lai observes a minimum of 20% difference in the force ratio between a sharp and worn tool. The effects of depths of cut and approach angle are shown to be not very significant within the practical range of cutting operations. The important point to remember however is the proper setting of force thresholds for correct wear detection of each tool-workpiece combinations.
Figure 2.22: Specific cutting pressure vs. $1/h_{eq}$ (after Lai)

Figure 2.23: Influence of flank wear on force ratio $F_T/F_Z$ (after Lai)
Dynamic Forces

In the previous section, the use of the cutting forces as a method of determining wear on the tool was discussed. In this section, the dynamic forces are examined as an indication method for tool wear. The frequency composition of the vibration signal and the changes in the signal can reveal the state of the wear. It is found that the vibration signal from cutting is very sensitive to tool wear [16].

A set-up for monitoring vibration of the tool can be seen in figure (2.24) (the low pass filter is to filter out fluctuations in the signal).

Two accelerometers, one set in the direction of the cutting force and the other in the direction of feed measure the vibration simultaneously. A recursive algorithm suggested by Jiang et al. monitors the average energy of the signal which is related to the vibration of the system:

$$\psi_n^{2BPF} = \frac{n-1}{n} \psi_{(n-1)BPF} + \frac{1}{n} \chi_{nBPF}^2$$  \hspace{1cm} (2.16)
where,

\[ \Psi_{x_{BPF}}^2 = \text{average energy} \]

\[ \chi = \text{signal from the filters} \]

\[ n = \text{data number}. \]

The system is sampled for a predetermined number of cycles, N, within a certain frequency range, \( f_l \) and \( f_h \), since the whole frequency range is not needed. In fact, more than one low pass filter can be used for as many frequency ranges so as to monitor the changes in those ranges. Examples of frequency ranges to monitor are:

1. First cutting resonance frequency range of the system.
2. Low frequency range (lower than the first resonance frequency).
3. Background noise which can be any frequency range provided not within any of the vibrating modes of the tool.

Various experiments were performed by Jiang and his colleagues to prove the practicality of the method in tracking wear. One such experiment consisted of orthogonal cutting with carbide tool of 45C hardened and tempered steel. The cutting speed was \( V_c = 174 \ m/min \), feed, \( f = 0.1 \ mm/rev \) and depth of cut, \( a = 1 \ mm \). The experiment is continued until the tool was broken.

The energy distribution of the vibration signal was plotted against frequency until the tool was broken producing an interesting history of energy distribution of vibration signal as seen in figure (2.25).

The results of this experiment reveal many details about tool wear:

1. The initial vibration energy is high; although, no peaks appear in the graph (this means that the background noise or the DC offset is high).
2. Low frequency vibration decreases in the initial wear stage.

3. During normal wear stage the low frequency energy level is stable while the higher frequencies (especially those of the first two modes) increases steadily.

4. The next stage, also identified as "micro-breakage" stage, is the time when an alarm signal is sent. Here the vibration energies at very low frequencies (<20 Hz) increases roughly ten fold. Although the tool does not break, the effects of this stage are small sparks and worsening of surface quality.

5. Tool breakage takes place. The background noise is high. No peaks seem to appear (including the resonant frequencies).

The method can determine the micro-breakage stage rather accurately, according to the authors. Furthermore, it is applicable to other machining processes as well. One final point is that this method can be used only if the transfer function of the system is known. Otherwise, it is possible to send false alarm signals.
Temperature

The role of temperature in metal cutting was first recognized earlier this century by Taylor who noted the influence of temperature on tool wear. It is now accepted that temperature controls the mode and the rate of wear. If a relationship between the temperature and tool wear is found then this can be used for implementation of monitoring wear by means of measuring temperatures. Lister & Barrow [17] survey several methods of measuring cutting temperatures and list them as follows:

1. Thermochemical reaction measurement,
2. Electromagnetic radiation,
3. Thermo-emf techniques.

Briefly, the feasibility of the above measurement techniques is discussed before presenting work on temperature-wear relationship.

Thermochemical Reaction Measurement

This type of measurement involves the use of thermosensitive paints which change colour with temperature. As an advantage the system has a desirable simplicity. Yet it has two major disadvantages: 1. The surface that is most affected by temperature is really very hard to get at; 2. the time it takes the colour to change is long. The last point renders this method for on-line tracking of wear impractical.

Electromagnetic Radiation

Every object emits electromagnetic radiation when heated. In fact the radiation emitted is related to the fourth power of temperature of the object. If a method is devised such that radiation of hot surface (in this case side of chip) can be measured, it is possible to obtain the cutting temperature. The sensitivity, quick response and
the independence of data obtained from the workpiece and tool material make this an attractive measuring system. However, once again the method requires access to the cutting zone. Hence the method is impractical since it is not easy to access this zone.

**Thermo-Emf Technique**

This method of measuring temperature was developed by Gottwein, Herbert and Shore. The principle of the thermometer (or the temperature measuring device) is that a "thermal potential" difference will generate an analogous electrical potential if two different types of metals are used as the two junctions (i.e. hot junction and cold junction). This idea can be illustrated in figure (2.26).

According to Barrow [18], there are two problems with this method:

1. Parasitic emf’s that can alter the reading of thermocouple.
2. Calibration of thermocouple output.

The mercury bath in figure (2.26) is included to prevent the first problem. By insulating the moving parts of machinery by means of a mercury bath slip rings as shown above, those parasitic emf’s can be eliminated.

Colwell [19] performed tests to correlate the cutting temperature and cutting forces with wear. He stressed the point that while the temperature signal alone might be
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Figure 2.27: Worn tool after six previous cuts (total time 6.04 min. Flank wear = 0.35 mm) (after Colwell)

Interesting, it takes on a greater importance when it is used in conjunction with cutting force. In his paper, Colwell shows that when a force cross-over occurs (i.e. $F_C \& F_N$ cross each other) after the dwell, the temperature curve takes a sharp dip, as can be seen in figure (2.27).

This is explained by the author as a result of a sharp decrease in the coefficient of friction. Further, with a more worn tool than that of the above test, an increase of temperature occurs in the second revolution of dwell. This is in contrast to that of sharper tools where an actual decrease in temperature takes place.

In general, the problems for this system arise from the increased complexity of tooling. Moreover, the response time of this approach to tool failure is slow so that it becomes unsuitable for the intended purpose. In fact, Lister and Barrow stress that temperature measurement method is useful in cases where temperatures are steady state. This lends itself to control of tool wear rates rather than the wear itself.
Acoustic Emission Methods

Acoustic emission (AE) methods have been used since the 1950's in the study of plastic deformations, fracture mechanics and fatigue. Practical applications of this method are in non-destructive testing of various structure types such as pressure vessels, pipelines, bridges. More recently, the AE method has found uses in diverse areas such as metal cutting. During the cutting process, acoustic pulses are transmitted through the work-piece when the strain energies stored in it are released as the material is undergoing deformation and fracture, or a combination of the two. These AE signals are very high frequency waves in the order of 0.1 MHz to 1.0 MHz. It is these high frequency signals that are high pass filtered and used in monitoring. Iwata and Moriwaki [7], in 1977, examined the application of this technique to metal cutting. Their experiments consisted of placing an AE sensor behind the tool holder away from the cutting process. The setup for their tests in schematic form is shown in figure (2.28). The authors suggest that there are two general types of AE signals; 1, Burst emission, characterized by high amplitudes
and short duration; 2, Continuous type, characterized by low amplitudes. The authors claim that in their tests, the AE signals were of burst type which is associated with crack extension. The first thing the authors examined was the frequency characteristics. The following series of graphs shows their findings. First, the noise of the whole equipment during idle running is shown to be insignificant. Secondly, the frequency spectrum as a whole shows an increase in magnitude, even at frequencies below the range of the high pass filters. This increase is particularly strong between 100 kHz to 250 kHz. The second important aspect of their work was to look at count rate and total count. By count rate it is meant count of AE bursts per cut (i.e. between each measurement) and by total count it is meant the addition (or integration) of AE bursts over the whole experiment for each cutting condition. Again their findings are shown below in figures (2.30) & (2.31).
Figure 2.30: Count of A.E. per cut plotted against flank wear (after Iwata et al.)
Figure 2.31: Total count of A.E. per cut plotted against flank wear (after Iwata et al.)
From this it is concluded that total count seems to be very indicative of the amount of wear and therefore is a good candidate for in-process monitoring. In concluding their work the authors are cautious in saying that setting of thresholds to characterize a burst is an important point that needs more work, and that disturbances caused by chips as they slide past the rake face and the friction on the newly cut surface by the flank face are significant contributors to the AE signals.

Iwata and Moriwaki's work was carried out using the turning process. More recently Ramalingam et al. [8] have examined the use of acoustic emission data in both turning and milling. They proceeded in their research with a complete redesign of the AE sensor such that it can be easily deposited on the throw away inserts. The advantages of such a system include close proximity to the AE signal source to avoid deterioration of the signal, costs less to build, and is simple to operate. The sensor is a piezo-electric transducer element constructed by a thin film deposition technique on one face of the throw-away insert. The AE signal is fed into an AC coupled amplifier with a flat gain of 60 dB from 50 Hz to 500 kHz. The 60 dB gain is low compared with 100 - 120 dB gain needed for commercially available systems. Hence it allows for more compact amplifiers that can be attached to the tool holder.

Ramalingam et al. first compare their system to other commercially available systems and arrive at the conclusion that their sensor gives results which are consistent with commercially available systems. While the authors claim that they can detect tool fracture in turning operations, no indication is given towards monitoring of wear. In fact tests done on various different types of metals prove that each metal will behave in a completely different way (see figure (2.32)), hence render any correlation very difficult. This difficulty is encountered in milling as well. Other serious practical difficulties in milling arise from entry and exit angles both of which give a burst of AE activity, making tool fracture and wear detection more difficult. Cross-talk between two or more sensors in cut
Figure 2.32: Frequency domain A.E. data for a, aluminium; b, brass; c, steel (after Ramalingam et al.)
also significantly alters the AE signal. While the authors suggest that this situation can be remedied by alternating between the instrumented insert and an uninstrumented one, so that only one sensor is in cut, the problem is that for face mills, with a large number of teeth in cut (e.g. an 8 tooth cutter in full immersion), this system can not be used.

2.4 Conclusions

In the previous sections, methods of tool wear tracking that are being used in fundamental manufacturing research today were discussed. Basically two different methods (i.e. direct methods and indirect methods) are available for the researcher in the area of tool wear monitoring. From the literature surveyed, it can be seen that direct methods are not very practical for research (or industry) applications. Of the several possibilities, tracking with a stylus seems to be the most feasible. This method however fails to detect tool breakage on time, but it can give valuable tool wear data with minimal cost. Two assumptions that the authors make, i.e. stylus itself does not wear and its dynamics are negligible, are reasonable ones according to them. As far as milling is concerned, the idea of stylus is not practical in intermittent cutting processes.

As far as the indirect methods are concerned, more understanding of the physics of the cutting mechanism is needed. Rather than measuring the wear itself, its effects are measured. Of the three methods surveyed, the temperature method is not reliable on its own for a tool wear tracking system. Whereas either the cutting forces or the dynamic forces lend themselves to accurate monitoring techniques. Furthermore, because many researchers use these two methods, many potential problems are already corrected.
Chapter 3

Theoretical Background

While various approaches involving cutting forces to monitor wear in milling have been proposed, it is shown by Yellowley [23] that the modelling of steady state cutting forces using Fourier Series could lead to a new more revealing insight to the problem of tool wear monitoring. In this chapter, the background necessary for developing the Fourier Series (F.S.) approximation of the instantaneous static cutting torques and forces will be developed.

3.1 F.S. Modelling of Static Cutting Forces for a Single Straight Tooth

The tangential and radial forces acting on a straight single tooth milling cutter are given by,

\[ F_T = K_s a S_t \sin \phi \]  \hspace{1cm} (3.1)

and,

\[ F_R = r F_T \]  \hspace{1cm} (3.2)

where

\[ K_s = \text{specific cutting pressure} \ (N/mm^2), \]
\[ a = \text{axial depth of cut} \ (mm), \]
\[ S_t = \text{feed per tooth} \ (mm/tooth.), \]
\[ \phi = \text{angle the cutting edge makes with the vertical axis} \ (rad), \]
\[ \phi_s = \text{swept angle of cut (rad)}, \]
\[ r = \text{ratio of radial force to tangential force}. \]

It must be mentioned here that throughout this analysis three assumptions are made. These are:

- \( K_s \) and \( r \) are constant. While this is not true, it has been shown by Fu et al. [24] that such an assumption is reasonable (It should be noted that later in the thesis the effective value of \( r \) and \( K_s \) will be seen as changing. This is because the influence of edge forces will be included).

- Only up-milling is considered. A similar analysis is possible for down-milling.

- The entry angle is 0° and the exit angle is \( \phi_s \). Again the model can easily accommodate other entry and exit angles.

Now it is possible to develop the Fourier Series of the cutting forces (which are functions of period \( 2\pi \)). Equation (3.3) represents tangential force component in terms of Fourier Series with coefficients \( a_k, b_k \) obtained by integrating eqns. (3.4) to (3.8).

\[
F_T = K_s a S \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos k\phi + b_k \sin k\phi \right) \right] \tag{3.3}
\]

\[
a_0 = \frac{1}{\pi} \int_{0}^{2\pi} \sin \phi d\phi \tag{3.4}
\]
\[
a_1 = \frac{1}{\pi} \int_{0}^{\Phi_s} \sin 2\phi d\phi \tag{3.5}
\]
\[
b_1 = \frac{1}{\pi} \int_{0}^{\Phi_s} \sin^2 \phi d\phi \tag{3.6}
\]
\[
a_k = \frac{1}{\pi} \int_{0}^{\Phi_s} \sin \phi \cos k\phi d\phi \tag{3.7}
\]
\[
b_k = \frac{1}{\pi} \int_{0}^{\Phi_s} \sin \phi \sin k\phi d\phi \tag{3.8}
\]
The results of the above integrations are shown below.

\[
\begin{align*}
a_0 &= \frac{1}{2\pi} (1 - \cos \phi_s) \\
a_1 &= \frac{1}{4\pi} (1 - \cos 2\phi_s) \\
b_1 &= \frac{1}{4\pi} (2\phi_s - \sin 2\phi_s) \\
a_k &= \frac{1}{2\pi} \left[ \frac{\cos((n-1)\phi_s)}{(n-1)} - \frac{\cos((n+1)\phi_s)}{(n+1)} - \frac{1}{(n-1)} + \frac{1}{(n+1)} \right] \\
b_k &= \frac{1}{2\pi} \left[ \frac{\sin((n-1)\phi_s)}{(n-1)} - \frac{\sin((n+1)\phi_s)}{(n+1)} \right]
\end{align*}
\] (3.9)

For up-milling, the forces in the \(x\) and \(y\) directions can be represented according to figure (3.1). In the above figure the forces acting on each tooth can be written in terms of \(F_T\) and \(F_R\). Hence,

\[
F_X = F_T \cos \phi + F_R \sin \phi
\] (3.10)
and

\[ F_Y = F_T \sin \phi - F_R \cos \phi \]  \hspace{1cm} (3.11)

Now, it is possible to write the forces in the \( x \) and \( y \) directions in terms of their Fourier Series representations. First, for a single tooth one can write \( F_X \) and \( F_Y \) as follows:

\[ F_X = F_T(\cos \phi + r \sin \phi) \]  \hspace{1cm} (3.12)

\[ F_Y = F_T(\sin \phi - r \cos \phi) \]  \hspace{1cm} (3.13)

Since the Fourier Series model of the tangential force \( F_T \) is known, it can be substituted into equations (3.12) & (3.13) to obtain

\[ F_X = C(\cos \phi + r \sin \phi) \left[ a_0 + \sum_{k=1}^{\infty} [a_k \cos k\phi + b_k \sin k\phi] \right] \]  \hspace{1cm} (3.14)

\[ F_Y = C(\sin \phi - r \cos \phi) \left[ a_0 + \sum_{k=1}^{\infty} [a_k \cos k\phi + b_k \sin k\phi] \right] \]  \hspace{1cm} (3.15)

where,

\[ C = K_s a S_t. \]

After some algebraic manipulation, these equations can be reduced to another Fourier Series with coefficients in terms of the Fourier Series coefficients for the tangential force as shown below:

\[ F_X = C \left[ a_{x0} + \sum_{k=1}^{\infty} [a_{xk} \cos k\phi + b_{xk} \sin k\phi] \right] \]  \hspace{1cm} (3.16)

where,

\[ a_{x0} = \frac{|a_1 + rb_1|}{2}, \]

\[ a_{x1} = a_0 + \frac{|a_2 + rb_2|}{2}, \]

\[ b_{x1} = ra_0 + \frac{|b_2 - ra_2|}{2}, \]

\[ a_{xk} = \frac{1}{2} [a_{(k-1)} - rb_{(k-1)} + a_{(k+1)} + rb_{(k+1)}], \]

\[ b_{xk} = \frac{1}{2} [b_{(k-1)} + ra_{(k-1)} + b_{(k+1)} - ra_{(k+1)}]. \]  \hspace{1cm} (3.17)
Similarly the force in the y direction can be shown to be given by,

\[ F_y = -C \left[ a_{y0} + \sum_{k=1}^{\infty} [a_{yk} \cos k\phi + b_{yk} \sin k\phi] \right] \tag{3.18} \]

where,

\[ a_{y0} = \frac{[ra_1 - b_1]}{2}, \]
\[ a_{y1} = ra_0 + \frac{[ra_2 - b_2]}{2}, \]
\[ b_{y1} = -a_0 + \frac{[rb_2 - a_2]}{2}, \]
\[ a_{yk} = \frac{1}{2} [ra_{(k-1)} + b_{(k-1)} + ra_{(k+1)} - b_{(k+1)}], \]
\[ b_{yk} = \frac{1}{2} [rb_{(k-1)} - a_{(k-1)} + a_{(k+1)} + rb_{(k+1)}]. \tag{3.19} \]

### 3.2 Fourier Series Modelling for a Multi Tooth Cutter

In the previous section the Fourier Series for a single straight tooth was obtained. Now a similar procedure will be used to obtain the Fourier Series for a multi tooth straight face milling cutter. First, consider the useful property of Fourier Series which allows the shifting of a curve by means of adding a phase to the sine and cosine terms of the series. The equation for an N teeth cutter can be written as an addition of N number of equations for N number of teeth shifted \( \frac{2\pi}{N} \) radians from the equation for the previous teeth. In mathematical form, these terms can be written as follows,

\[ F_{X,1st} = C \left[ a_{x0,1st} + \sum_{k=1}^{\infty} [a_{xk,1st} \cos k\phi + b_{xk,1st} \sin k\phi] \right] \tag{3.20} \]

for the second tooth,

\[ F_{X,2nd} = C \left[ a_{x0,2nd} + \sum_{k=1}^{\infty} [a_{xk,2nd} \cos \left( k\phi + \frac{2\pi}{N} \right) + b_{xk,2nd} \sin \left( k\phi + \frac{2\pi}{N} \right)] \right] \tag{3.21} \]

for the Nth tooth,

\[ F_{X,Nth} = C \left[ a_{x0,Nth} + \sum_{k=1}^{\infty} [a_{xk,Nth} \cos \left( k\phi + \frac{2\pi(r-1)}{N} \right) + b_{xk,Nth} \sin \left( k\phi + \frac{2\pi(r-1)}{N} \right)] \right] \tag{3.22} \]
Now a simple addition of these above terms will give the final expression for a multi tooth cutter. This expression after some manipulations is shown below.

\[ F_X = C_1 \left[ a_{x0} + \sum_{k=N,2N,3N,...}^{\infty} \left( a_{xk} \cos k\phi + b_{xk} \sin k\phi \right) \right] \]  

(3.23)

where \( N \) is the number of teeth and the coefficients \( a_{xk} \) \& \( b_{xk} \) are as calculated before. \( C_1 \), shown below is analogous to \( C \):

\[ C_1 = K_a S_t N. \]  

(3.24)

The above equation implies that for a two teeth cutter, every second coefficient of the one tooth case is non-zero (i.e. \( a_{x1}, b_{x1}, a_{x3}, b_{x3}, ... \) will be all zero.). Similarly for the three teeth case only every third term of the one tooth case will be non-zero. This is very convenient for simulations since the coefficients of the one tooth case can easily be calculated once and then be used for calculating the coefficients of 2,3,4, etc. tooth cutters by simply setting the appropriate coefficients to zero.

A similar argument can be made for \( F_Y \) where now the coefficients are \( a_{yk} \) and \( b_{yk} \).

### 3.3 Modelling of Wear

To this point, the modelling process has examined a multi tooth cutter with no flank wear or indeed edge forces. In real life situations, as cutting proceeds, wear can be expected to increase both \( F_T \) and \( F_R \). To accommodate this rise in forces, a modified cutting model is used. The new cutting forces are now expressed as an addition of two terms, the cutting forces and forces due to nose and flank contact which is proportional to the length of the cutting edge. In mathematical terms,

\[ F_T = K_a a h + xa \]  

(3.25)

where, \( x \) denotes the edge force constant. Now if one defines a critical chip thickness \( h^* \) that occurs when both the cutting force and the parasitic edge force have equal
magnitudes, then the following can be obtained:

\[ xa = K_s a h^* \]
\[ x = K_s h^* \]

Hence the above equation can be now rewritten as:

\[ F_T = K_s a h + K_s a h^* \]  \hspace{1cm} (3.26)

In reality, no cutting takes place to cause this but the concept is useful to model a worn tool. From the equation above it can be seen that wear causes a rectangular force pulse. It is also possible to express the influence of wear and edge forces on the radial force as follows:

\[ F_R = K_s a (r h + r_2 h^*) \]  \hspace{1cm} (3.27)

Here, \( r_2 \) is analogous to \( r \), an imaginary force ratio for wear forces. The Fourier series approximation can now be calculated just as before. Since the contribution of forces due to wear is rectangular, the Fourier series coefficients can be written as the integral of the following:

\[
aw_0 = \frac{1}{2\pi} \int_0^{\phi_*} \phi d\phi,
\]
\[
aw_1 = \frac{1}{\pi} \int_0^{\phi_*} \cos \phi d\phi,
\]
\[
bw_1 = \frac{1}{\pi} \int_0^{\phi_*} \sin \phi d\phi,
\]
\[
aw_2 = \frac{1}{\pi} \int_0^{\phi_*} \cos 2\phi d\phi,
\]
\[
bw_2 = \frac{1}{\pi} \int_0^{\phi_*} \sin 2\phi d\phi,
\]
\[
aw_k = \frac{1}{\pi} \int_0^{\phi_*} \cos k\phi d\phi,
\]
\[
bw_k = \frac{1}{\pi} \int_0^{\phi_*} \sin k\phi d\phi. \]  \hspace{1cm} (3.28)
Chapter 3. Theoretical Background

These simple integrals are evaluated and shown down below as:

\[
\begin{align*}
aw_0 &= \frac{\phi_s}{2\pi}, \\
aw_1 &= \frac{\sin \phi_s}{\pi}, \\
bw_1 &= \frac{1 - \cos \phi_s}{\pi}, \\
aw_2 &= \frac{\sin 2\phi_s}{2\pi}, \\
bw_2 &= \frac{1 - \cos 2\phi_s}{2\pi}, \\
aw_k &= \frac{\sin k\phi_s}{k\pi}, \\
bw_k &= \frac{1 - \cos k\phi_s}{k\pi}.
\end{align*}
\] (3.29)

The coefficients for the \(x\) and \(y\) directions can be calculated using the same equations for the steady state cutting only situation. Hence,

\[
\begin{align*}
aw_{x0} &= \frac{1}{2}(aw_1 + r_2bw_1), \\
aw_{x1} &= aw_0 + \frac{1}{2}(aw_2 + r_2bw_2), \\
bw_{x1} &= r_2aw_0 + \frac{1}{2}(bw_2 - r_2aw_2), \\
aw_{xk} &= \frac{1}{2}(aw_{k-1} - r_2bw_{k-1} + aw_{k+1} + r_2bw_{k+1}), \\
bw_{xk} &= \frac{1}{2}(bw_{k-1} + r_2aw_{k-1} + bw_{k+1} - r_2aw_{k+1}).
\end{align*}
\] (3.30)

Since the method of obtaining the coefficients above are the same for the \(y\) direction as well, only the results are given:

\[
\begin{align*}
aw_{y0} &= \frac{1}{2}(r_2aw_1 - bw_1), \\
aw_{y1} &= r_2aw_0 + \frac{1}{2}(r_2aw_2 - bw_2), \\
bw_{y1} &= -aw_0 + \frac{1}{2}(r_2bw_2 + aw_2), \\
aw_{yk} &= \frac{1}{2}(r_2aw_{k-1} + bw_{k-1} + r_2aw_{k+1} - bw_{k+1}),
\end{align*}
\]
Chapter 3. Theoretical Background

\[ bw_{yk} = \frac{1}{2}(r_2bw_{k-1} - aw_{k-1} + aw_{k+1} + r_2bw_{k+1}) \]  

(3.31)

After the calculation of the Fourier coefficients, the F.S. for the wear forces in the x and y directions can be written as:

\[ Fw_x = C_2 \left[ aw_{x0} + \sum_{k=N,2N,3N,...}^{\infty} [aw_{xk} \cos k\phi + bw_{xk} \sin k\phi] \right] . \]  

(3.32)

Here, the value of \( C_2 \) is:

\[ C_2 = K_sah^*N \]  

(3.33)

where \( N \) is the number of teeth. The force in the y direction can be written in the same way as the above equation with \( aw_{xk} \)s and \( bw_{xk} \)s being replaced by \( aw_{yk} \)s and \( bw_{yk} \)s.

Having formulated the edge forces, the next step is to combine them into the static cutting force equations already derived in the previous section. This is easily accomplished by adding the two components together. In algebraic terms this is shown below:

\[ F_{XT} = F_X + F_{wx}, \]

\[ F_{YT} = F_Y + F_{wy}. \]  

(3.34)
Chapter 4

Frequency Domain Analysis

In the previous chapter, models of the cutting forces due to cutting and parasitic phenomenon were obtained in the time domain using Fourier Series approximation. In this chapter, the usefulness of such an approach will be considered. One of the objectives of this work is to identify the width and the depth of cut independently of direction of cut (i.e. the magnitudes of cutting force and wear force will depend on the width of cut, the depth of cut, and the direction of cut when contouring). Once this is accomplished attention will be directed towards measures of cutting wear. Various approaches to achieve this objective are discussed and presented below.

4.1 Other Work Done with Fourier Series Modelling in Wear Monitoring

Previous work using Fourier Series representative of cutting force to detect and monitor wear have been carried out by Elbestawi et al [26,27]. The first method the authors examine involves looking at the frequency domain of the force and acceleration (i.e. vibration) signatures for 1/4 and full immersion. The authors obtain the ”total harmonic power” (THP) by summing the power spectrum at integral orders in both $x$ and $y$ direction:

$$THP = \sum_{m=1}^{N} G(m) \quad (4.1)$$

where,

$$G(m) = \text{power spectrum}$$
Chapter 4. Frequency Domain Analysis

\( N = \text{the largest integer that defines a frequency range of interest.} \)

The authors chose the first 5 harmonics of the force signature (corresponding to \( \approx 200 \) Hz). For the acceleration signal \( 0 - 2 \) kHz range was found adequate since "no consistent trend with tool wear was found in the high frequency contents of the signals (above 2 kHz)" [26, pp. 1011]. The expectation is that an increase in wear will cause the THP's in \( x \) and \( y \) directions for force and acceleration signature to increase as well. This is proven experimentally to varying degrees depending on immersion. Because of this dependence on immersion, the sensitivity of THP's to wear is significantly reduced in full immersion and authors combine the average THP for vibration and average THP for force to obtain a dimensionless wear feature \( WF \) that is sensitive for various immersion:

\[
WF(i) = \left( \frac{THPF_{av}(i)}{THPF_{av}(0)} \right) \left( \frac{THPA_{av}(i)}{THPA_{av}(0)} \right)
\]

(4.2)

where \( i = 1,2,3,... \) denotes the \( i \)th measurement, \( THPF_{av}(0) \) and \( THPA_{av}(0) \) refer to the tool when sharp. The following figures show this \( WF \) at \( 1/4 \) and full immersion for different feedrates. While it is understood that the progression of wear will cause increases in magnitudes of frequency components, it should be clear that in multi-tooth machining the useful bandwidth of the dynamometer will be considerably shortened (not to mention the mass of the workpiece attached to the dynamometer which of course will lower its natural frequency). Furthermore, from simulations it is clear that the high frequency components of the static cutting forces will give very little significant effect to wear monitoring. Meanwhile, the analysis of the total harmonic power for a large number of calculations is time consuming and not very practical from the point of view of in-process monitoring techniques. It is also interesting to note that the method does call upon the use of accelerometers attached to the sides of the dynamometer which complicates the experimental set-up and calculations necessary for the discriminant. In their second paper on the subject, the authors look at the low frequency components of
the static cutting signal. The authors studied various low frequency components under various cutting conditions. They discovered that certain frequency components were more influenced at different immersion than others. So depending what the immersion is then a most affected frequency term is divided by the least affected components and this is used as wear indicator. The authors do not make it clear which ratio is used for which immersion. A second problem with this method is that authors make the assumption that either the entry or the exit angle must be zero. There are situations where neither the exit nor the entry angles are zero (see figure (4.1)). This complicates the wear detection according to the above criteria. In such cases the tangential signal will be quite similar to the rectangular wear signal and it will be difficult to find a discriminating ratio of magnitudes for this situation. Other significant problems include the modelling of an insert with an infinitessimally small nose-radius rather than with a finite nose-radius. The effect of this nose-radius was studied by Colwell [28] who then introduced the idea of equivalent approach angle. The effect of this nose-radius will be particularly significant at

Figure 4.1: Cutting forces for non-zero entry and exit angles.
low depths of cut where the majority of the cutting forces will take place in the periphery of the nose-radius. For no wear situations there will still be a significant parasitic force due to this radius (as a secondary edge), rendering one major assumption in their work invalid.

### 4.2 The Frequency Content

Looking at the frequency content of the signal with wear some observations can be made. First, the signal with no wear exhibits a decrease of magnitude after the fundamental for one or more toothed cutters. With the inclusion of wear, at moderate immersion angles a beating phenomenon occurs. At the extreme case, when immersion is 180°, various harmonics simply disappear. The magnitude of the force signal with and without wear is shown in figure (4.2). The frequency content of the steady state cutting force signal depends on immersion, depth of cut, wear and lastly direction. If any one of the
cutting conditions change during cutting then it is impossible to relate this directly to wear and therefore cannot be directly used in monitoring wear (this is the situation in contouring operations). In straight cuts where immersion, depth of cut and direction do not change, the DC component (which is also the mean value of force) and the fundamental component can give a straight and effective wear monitoring system.

4.3 Previous Work on Identification of Radial Width and Axial Depth of Cut

An identification algorithm for peripheral milling was suggested by Altıntaş & Yellowley [29]. In their paper the authors developed a ratio using average cutting forces that is independent of the depth of cut. Once this ratio is shown to be dependent only on the immersion, a relationship is examined between actual immersion and this ratio. Below is a mathematical explanation of the method. First, looking at the ratio,

\[ x(t) = \frac{F_{x_{ave}}(t) - F_{y_{ave}}(t)}{F_{qm}(t)}, \]  

(4.3)

it is easy to see that the influence of axial depth of cut will be cancelled. It should be mentioned that in the presence of flank forces, the ratio above will also depend on \( S_t \), feed per tooth. \( S_t \) can be obtained experimentally leaving immersion the only parameter to be identified. The authors then correlate the ratio of \( x(t) \) to \( z(t) \), radial depth of cut through the use of a "bivariate" polynomial in the form of:

\[ z(t) = a_1 + a_2 x(t) + a_3 x^2(t) + a_4 x^3(t) + a_5 \alpha^2(t) + a_6 \alpha(t) \]  

(4.4)

where,

\[ \alpha(t) = \frac{h^*}{S_t}. \]

The axial depth of cut is obtained independently of the radial width of cut so as not to include any errors introduced in the immersion identification into the axial depth of cut.
Chapter 4. Frequency Domain Analysis

The authors looked at a normalised stress function \( y(t) \), which they defined as

\[
y(t) = \frac{F_{qm}(t)}{Ca(t)}
\]

where,

\[
C = \frac{K_aNS_t}{8\pi}
\]

\( y(t) \) is also a function of \( x(t) \) since it is now solely dependent on immersion (and feed per tooth if flank forces are present). Again a simple polynomial form is assumed for a relationship between \( y(t) \) and \( x(t) \). Hence,

\[
y(t) = b_1 + b_2x(t) + b_3x^2(t) + b_4x^3(t).
\]

The axial depth then is obtained using:

\[
a(t) = \frac{F_{qm}(t)}{Cy(t)}.
\]

The two graphs in figures (4.3),(4.4) show these algorithms at work. In the absence of entry and exit dynamics the algorithm predicts accurately both the immersion and the actual depth of cut.

Further work on identification was done by Altıntaş in his Ph.D. research [30]. The author looked at the immersion identification to obtain the necessary information in tuning the tool failure thresholds. Because most practical machining operations involve contours and sharp edges, the \( x \) and \( y \) components of the dynamometer would have to be resolved into its components to obtain a resultant force at each sampling interval. To circumvent this problem the "quasi-mean resultant force", \( F_{qm} \), is used so that force calculations would be necessary only at each tooth period as opposed to each sampling interval. It is evident that after investigating various methods that involve measurements of torque and steady state cutting forces "in order to be independent of direction any
Chapter 4. Frequency Domain Analysis

Figure 4.3: Predicted and actual values of immersion (after Altıntaş et al.)

Figure 4.4: Predicted and actual values of axial depth of cut (after Altıntaş et al.)
method must use either resultant force or quasi mean resultant force\textsuperscript{1} [30, pp. 140]. In fact the author looks at the ratio of mean value of resultant force to quasi-mean resultant force.

### 4.4 Relevant Work on Identification Using Quasi-Mean Resultant Force and Torque Values

The ratio of mean torque to quasi-mean resultant force to evaluate the swept angle of cut was used by Yellowley [23]. He encountered the problem of lack of sensitivity at low immersion. To circumvent this problem, he suggests the use of the ratio of the magnitude of the fundamental frequency of the torque signal over the mean value of torque. Such an approach has the advantage of being sensitive especially with more than one tooth cutters at low immersion. The author demonstrates this clearly in figure (4.5).

Once the immersion is identified there is enough information to identify depth from

\textsuperscript{1}It will be explained in the next section.
the mean torque alone. Other possible ratios that could be used in a similar way would be the axial force, $F_z$, and the quasi-mean resultant force. In all these cases the ratio of the magnitude of fundamental frequency to mean value will produce a ratio sensitive to immersion even at low swept angles of cut. Of course, once the immersion is identified, the mean value of the parameter ($F_z$ or $F_{qm}$ or $T$) can be used alone for depth identification.

The advantage this identification algorithm offers is the fact that very little computation is required since only the fundamental frequency and the mean values of forces and/or torques are required. The practical meaning of this ratio is a bit obscure at first but physically it means that from the locus of $F_X$ vs. $F_Y$ plot the area is dependent on immersion. In fact this is the method that is the subject of the next section and further explanation of the various parameters and physical interpretation will be given there.

### 4.5 Area over Quasi-mean Cutting Force Method

Plotting the $F_{XT}$ vs. $F_{YT}$ as shown in figure (4.6), it is possible to manipulate various graphical entities. One such possibility is to look at the area of $F_{XT}$ vs. $F_{YT}$ graph. First a formulation for the mathematical expression will be obtained then a physical explanation will be given. In equation (4.8), the dependence of area on radius can be seen easily. Here,

$$\text{Area} = \frac{1}{2} \int_{0}^{2\pi} R^2 d\theta$$

(4.8)

If the angle swept is constant and the radius, $R$, is assumed constant then a simplified relationship between area and $R$ can be obtained:

$$\text{Area} \propto R^2.$$  

(4.9)
Taking the mean value of $R$ makes it easier to calculate the area of the $F_{XT}$ vs. $F_{YT}$ curve since it is now possible to write $R$ as,

$$R^2 = f_x^2 + f_y^2 = (a_{x,1}^{\text{harm.}} + b_{x,1}^{\text{harm.}}) + (a_{y,1}^{\text{harm.}} + b_{y,1}^{\text{harm.}})$$  \hspace{1cm} (4.10)

(see next section for a detailed derivation of $R$). Here $a_{x,1}^{\text{harm.}}$ etc. denotes the first non-zero harmonic of the signal. Also note that the force signal is passed through a low-pass filter and only the fundamental and the first harmonics are allowed to pass.

The calculation of quasi-mean forces is straightforward since it is just the addition of squares of the mean $x$ and mean $y$ values of forces. In mathematical terms,

$$F_{qm}^2 = a_{x0}^2 + a_{y0}^2.$$  \hspace{1cm} (4.11)

Hence, the term $\frac{\text{Area}^2}{F_{qm}^2}$ can be written in its final form as,

$$\text{ratio} = \left( \frac{a_{x,1st}^2 + b_{x,1st}^2 + a_{y,1st}^2 + b_{y,1st}^2}{a_{x0}^2 + a_{y0}^2} \right).$$  \hspace{1cm} (4.12)
Chapter 4. Frequency Domain Analysis

It is important to note the physical significance of this method. The explanation for the quasi-mean forces are easy to understand since they are one form of averaging orthogonal forces. The difference of the quasi-mean from the real mean is that quasi-mean forces do not take into consideration the fluctuations in the orthogonal forces. Hence, if these fluctuations were negligibly small then indeed the quasi-mean would correspond to the real mean. The quasi-mean forces, because they are independent of direction, will give one of the unknown cutting parameters, namely the depth of cut.

The area represented in figure (4.6) envelopes a zone that represents the deviation of cutting forces from their average values. The shape of the curve signifies the deviation from the average deviation of forces. Therefore, when it is assumed above that the radius of the curve is constant then the deviation from the average deviation is assumed to be zero. Further, to clarify the physical significance of the area, if the fluctuations in forces in the x and y directions are large then the area will be large. However, if they are small the area will be small; in fact, in the limit the area would be reduced to a point (when slotting \( F_{XT} \) and \( F_{YT} \) will be constant so that the \( F_{XT} \) vs. \( F_{YT} \) is indeed a point).

In the table (4.1), values of the ratio for a 2 tooth cutter with various stages of wear and for various values of immersion are calculated. As it can be seen from the table the ratio seems fairly uninfluenced by the state of the wear (across the columns) yet influenced by immersion (along the columns). This of course means that it is now possible to obtain immersion independent of direction and the state of wear.

<table>
<thead>
<tr>
<th>( \phi_0 )</th>
<th>( \frac{h_r}{S_l} = 0.0 )</th>
<th>( \frac{h_r}{S_l} = 0.1 )</th>
<th>( \frac{h_r}{S_l} = 0.2 )</th>
<th>( \frac{h_r}{S_l} = 0.3 )</th>
<th>( \frac{h_r}{S_l} = 0.4 )</th>
<th>( \frac{h_r}{S_l} = 0.5 )</th>
<th>( \frac{h_r}{S_l} = 0.6 )</th>
<th>( \frac{h_r}{S_l} = 0.7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>3.76</td>
<td>3.72</td>
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Table 4.1: Effect of wear on \( \frac{\text{Area}^2}{\text{Area}_{\text{em}}} \) derived in section (4.5) for two tooth cutter.
4.6 The Magnitude of Deviation over Quasi-mean Method

In the previous section, the shape of the $F_{XT}$ vs. $F_{YT}$ curve was approximated to that of a circle. In this section, the deviation from the circular shape will be investigated to see if this information can be used to extract the state of wear. It is expected that the shape of this curve will be affected by the amount of wear. As usual, a mathematical derivation will be given first followed by the physical meaning of this method.

As in the previous section, the signal is assumed to pass through a low-pass filter. Here,

\[ F_{XT} = a_{x0} + a_{x1} \cos \phi + b_{x1} \sin \phi \]
\[ F_{YT} = a_{y0} + a_{y1} \cos \phi + b_{y1} \sin \phi \] (4.13)

and

\[ \Delta F_{XT} = [F_{XT} - F_{XT-\text{ave}}] = a_{x1} \cos \phi + b_{x1} \sin \phi \]
\[ \Delta F_{YT} = [F_{YT} - F_{YT-\text{ave}}] = a_{y1} \cos \phi + b_{y1} \sin \phi \] (4.14)

Once this is obtained, eqns. (4.14) can be squared and summed to obtain the mean radius of circle. So after algebraic manipulations,

\[ \Delta F_{XT}^2 + \Delta F_{YT}^2 = \frac{1}{2} [a_{x1}^2 + b_{x1}^2 + a_{y1}^2 + b_{y1}^2] + \frac{1}{2} [a_{x1}^2 + a_{y1}^2 - b_{x1}^2 - b_{y1}^2] \cos 2\phi + [a_{x1} b_{x1} + a_{y1} b_{y1}] \sin 2\phi \] (4.15)

The deviatoric component is comprised of 2 parts as can be seen above. The first part is a constant term, the second part a variable one. The constant term is the radius of the circle so,

\[ R = \frac{1}{2} [a_{x1}^2 + b_{x1}^2 + a_{y1}^2 + b_{y1}^2] \] (4.16)

The variable terms are the deviations from the circle with radius $R$. The magnitude of this deviation is the term to be used in the calculations. Therefore, as a first step
rewriting these terms in the $x$ and $y$ directions results in:

$$\text{Dev}_{\Delta F_x}^2 = \frac{(a_{x1}^2 - b_{x1}^2)}{2} \cos 2\phi + a_{x1} b_{x1} \sin 2\phi$$

$$\text{Dev}_{\Delta F_y}^2 = \frac{(a_{y1}^2 - b_{y1}^2)}{2} \cos 2\phi + a_{y1} b_{y1} \sin 2\phi. \quad (4.17)$$

Hence the magnitude can now be calculated as:

$$\text{Mag}^2 = (\text{Dev}_{\Delta F_x}^2)^2 + (\text{Dev}_{\Delta F_y}^2)^2. \quad (4.18)$$

Or after simplifications,

$$\text{Mag}^2 = \frac{1}{4} (a_{x1}^2 + a_{y1}^2 - b_{x1}^2 - b_{y1}^2)^2 + (a_{x1} b_{y1} + a_{y1} b_{x1})^2. \quad (4.19)$$

Since the quasi-mean forces are calculated as before, the ratio, $\frac{\text{Mag}^2}{F_{qm}^2}$ is:

$$\frac{\text{Mag}^2}{F_{qm}^2} = C_1 \left[ \frac{1}{4} (a_{x1}^2 + a_{y1}^2 + b_{x1}^2 + b_{y1}^2)^2 - (a_{x1} b_{y1} - a_{y1} b_{x1})^2 \right] \frac{1}{a_{x0}^2 + a_{y0}^2}. \quad (4.20)$$

Note here $C_1$ is as defined before in the previous chapter. But in the calculations below $C_1$ is equal to $N$. The reason for this simplification is for ease of algebra only since the square of $K \alpha S_i$ is a large number and does not affect the outcome of the results shown in table (4.2).

As was discussed earlier, the magnitude of deviations are the deviations from that of a circular shape. It is believed that with wear this curve will look more circular. In table (4.2) a list of values for 2 teeth case are shown.

For increasing wear, the value of $\frac{\text{Mag}^2}{F_{qm}^2}$ gets bigger. Technically this allows $\frac{\text{Mag}^2}{F_{qm}^2}$ to be used as an indication for wear but practical problems still exist. The first problem is that the values of this ratio do not change very much for small wear (i.e. the amount of changes in value is not significant). Secondly, an error of a few degrees in immersion can be a significant source of errors.
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Table 4.2: Effect of wear on $\frac{Mag^2}{R_{qm}^2}$ derived in section (4.6) for two tooth cutter

4.7 Deviation over Radius Method

The idea of the previous section was to compare the deviation of the circle to the quasi-mean forces. These deviations can be compared to the radius of the circle itself. The idea is that the effects of wear on the circle will be amplified if compared to $R$. The derivation of the mathematical expression is straightforward since it is simply a manipulation of expressions derived earlier.

The magnitude of deviations can be written as:

$$Mag^2 = \frac{1}{4}(a_{z1}^2 + a_{y1}^2 + b_{z1}^2 + b_{y1}^2)^2 - (a_{z1}b_{y1} - a_{y1}b_{z1})^2. \quad (4.21)$$

Similarly, the radius can be written as:

$$R^2 = \left[\frac{1}{2}[a_{z1}^2 + b_{z1}^2 + a_{y1}^2 + b_{y1}^2]\right]^2. \quad (4.22)$$

So the ratio, $\frac{Mag^2}{R^2}$ is:

$$\frac{Mag^2}{R^2} = \frac{1}{2} - \frac{2(a_{z1}b_{y1} - a_{y1}b_{z1})^2}{(a_{z1}^2 + b_{z1}^2 + a_{y1}^2 + b_{y1}^2)^2}. \quad (4.23)$$

Table (4.3) shows the results of this method.

Again looking at the 2 teeth cutter, it becomes apparent that the sensitivity of the $\frac{Mag^2}{R^2}$ to wear is very low, especially so at low immersions. This trend can be observed for 3 and 4 teeth cutters as well (see end of chapter for complete simulation data). So it is obvious then that this method is not a likely candidate for use in wear identification.
Chapter 4. Frequency Domain Analysis

$\phi_s$ & $\frac{b_{x_s}}{a_{x_s}} = 0.0$ & $\frac{b_{x_s}}{a_{x_s}} = 0.1$ & $\frac{b_{x_s}}{a_{x_s}} = 0.2$ & $\frac{b_{x_s}}{a_{x_s}} = 0.3$ & $\frac{b_{x_s}}{a_{x_s}} = 0.4$ & $\frac{b_{x_s}}{a_{x_s}} = 0.5$ & $\frac{b_{x_s}}{a_{x_s}} = 0.6$ & $\frac{b_{x_s}}{a_{x_s}} = 0.7$

$30^\circ$ & 1.00 & 0.99 & 0.99 & 0.99 & 0.99 & 0.99 & 0.99 & 0.99

$60^\circ$ & 0.99 & 1.00 & 1.00 & 0.99 & 0.98 & 0.98 & 0.97 & 0.96

$90^\circ$ & 0.70 & 0.60 & 0.54 & 0.50 & 0.45 & 0.43 & 0.41 & 0.40

$180^\circ$ & 0.30 & 0.40 & 0.49 & 0.55 & 0.61 & 0.65 & 0.69 & 0.72

Table 4.3: Effect of wear on $\frac{M_{a_s}^2}{R^2}$ derived in section (4.7) for two tooth cutter

One reason for this insensitivity could be due to the number of harmonics after the fundamental taken from the low-pass filter.

4.8 The Phase Shift Method

In this section, the phase between the $x$ component and $y$ component of the cutting force will be examined for wear identification purposes. The algebra for the equation of the phase is presented below. Looking at the equation for cutting forces after the signal passed through the low-pass filter, $F_{XT}$ and $F_{YT}$ can be written as:

$$F_{XT} = a_{x0} + a_{x1} \cos \phi + b_{x1} \sin \phi$$

$$F_{YT} = a_{y0} + a_{y1} \cos \phi + b_{y1} \sin \phi$$

The phase of the deviatoric component of $x$ can be obtained by:

$$\tan \Phi_x = \frac{a_{x1}}{b_{x1}}.$$ \hspace{1cm} (4.24)$$

Similarly, for the $y$ component, it is:

$$\tan \Phi_y = \frac{a_{y1}}{b_{y1}}.$$ \hspace{1cm} (4.25)$$

Now using the trigonometric identity which gives the tangent of a difference of angles as,

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta},$$ \hspace{1cm} (4.26)$$
Chapter 4. Frequency Domain Analysis

\[ \tan(\Phi_x - \Phi_y) = \frac{a_{x1}b_{y1} - a_{y1}b_{x1}}{a_{x1}b_{y1} + a_{y1}b_{x1}}. \] (4.27)

Or after further simplifications,

\[ \tan(\Phi_x - \Phi_y) = \frac{a_{x1}b_{y1} - a_{y1}b_{x1}}{b_{x1}b_{y1} + a_{x1}a_{y1}}. \] (4.28)

The physical significance of the method can be explained as follows. The method looks at the phase of deviatory component of each cutting force. The force signal can be seen as made of two separate parts: 1, the mean; 2, the deviatory component. So the phase, \( \Phi_x \), would correspond to the phase difference between the deviatory component and the original signal. Subtracting \( \Phi_x \) from \( \Phi_y \) would give the relative phase between the two cutting force.

In the above table, it can be seen that the phase is sensitive to wear and is a promising method to identify wear. There are however problems on the data of the 3 and 4 tooth cutter (see end of chapter). In the case of \( 180^\circ \) immersion the data indicates a lack of usefulness from the point of view of identification purposes since no change in the phase is observed with increasing wear.

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Table 4.4: Phase difference between deviatoric force components for two tooth cutter

4.9 Forces in z-Direction

So far, the forces in the \( x \) and \( y \) directions have been used to obtain all the necessary information with respect to the cutting parameters. While the width of cut and the depth
of cut can be obtained from these in plane forces, wear monitoring is still not possible. This necessitates looking at the other force, the $z$-force, for monitoring purposes. First, a derivation and physical significance of the $z$-force will be given. Then simulations will be presented.

4.9.1 Derivation of Expression for $z$-Force

A good starting point for the understanding the derivation of $z$-forces is the turning operation with a non-straight cutting edge. First consider a tool with zero nose radius and an approach angle $\psi$, see figure (4.7). The thrust force, $F_{\text{thrust}}$, is made of two components: radial force, $F_R$, and axial force, $F_{\text{axial}}$. As it can be seen the thrust force will now have a radial component that is equivalent to the $z$-force, $F_Z$, in milling.
mathematical terms the $z$-force can be written as:

$$F_Z = F_{\text{thrust}} \sin \psi$$  \hspace{1cm} (4.29)$$

where,

$$F_{\text{thrust}} = K_s a S_{eq} \left( r_1 \sin \phi + r_2 \frac{h^*_{eq}}{h_{eq}} \right).$$  \hspace{1cm} (4.30)

$h^*_{eq}$ is the critical value of equivalent chip thickness $h_{eq}$ for which metal cutting and parasitic components are equal. In the above equation the thrust force is calculated using the idea of equivalent chip thickness, $h_{eq}$. It is a useful concept which really means the mean chip thickness. The practical reasoning comes from the geometry of the insert which acts as a chip thinning mechanism. The equation for $h_{eq}$ is formulated by dividing the undeformed area of cut by the length of active cutting edge:

$$h_{eq} = \frac{A}{l_a}. \hspace{1cm} (4.31)$$

Now consider a tool, with a 0° approach angle. Again figure (4.8) can be used for the tool geometry. The idea is to reduce this tool geometry to that of the earlier case where the insert had a finite approach angle. Then the suggestion is the cutting edge is now as shown in figure (4.8). The 'effective' angle of approach can now be calculated using the tool geometry.

$$\psi_e = \tan^{-1} \left( \frac{S_t}{a - S_t \tan \epsilon} \right) \hspace{1cm} (4.32)$$

The modelling of a tool with a tool tip radius is similar to the zero nose radius case except now the geometry is a little more complicated. Again the idea is to reduce the tool to the first geometry that was modelled. The effective approach angle is more involved to calculate since now the secondary cutting edge is not straight but curved. Figure (4.9) shows the details of the geometry. In particular, $\psi_e$ can be shown to be:

$$\tan \psi_e \approx \frac{a - r(1 - \sin \phi) \tan \phi + r \cos \phi + S_t/2}{a}. \hspace{1cm} (4.33)$$
Figure 4.8: Effective angle of approach for a straight edged tool
Figure 4.9: Effective angle of approach for a round tipped tool
4.9.2 Simulation Results

It is apparent that as the flank wear increases so will the z-force due to 'additional' secondary cutting edge forces. The idea is then to compare the z-force for a worn tool to that of sharp one by creating a wear index, $W_l$, where $W_l$ is:

$$W_l = \frac{F_z(\text{worn})}{F_z(\text{sharp})}.$$  \hfill (4.34)

For this simulation, $\psi_e$ will be kept constant. Then equations of forces in the z-direction for a worn and a sharp tool can be written as:

$$F_z(\text{worn}) = \tau_{\text{worn}} K_{s(\text{worn})} a S_t \sin \phi \sin \psi_e,$$  \hfill (4.35)

and,

$$F_z(\text{sharp}) = \tau_{\text{sharp}} K_{s(\text{sharp})} a S_t \sin \phi \sin \psi_e.$$  \hfill (4.36)

So the wear index is:

$$W_l = \frac{\tau_{\text{worn}} K_{s(\text{worn})}}{\tau_{\text{sharp}} K_{s(\text{sharp})}}.$$  \hfill (4.37)

where the cutting constants $K_{s(\text{worn})}$ and $\tau_{\text{worn}}$ are calculated from the $x$ and $y$ force equations for a given amount of wear at a given point in time. The assumption in making the calculations is that the estimated cutting coefficients are determined for the no wear case. The implication of this is of course that both $K_{s(\text{worn})}$ and $\tau_{\text{worn}}$ will be higher than the sharp case since they also carry information about wear. A set of simulations was done with $\tau_{\text{sharp}} = 0.3$ and $K_{s(\text{sharp})} = 1200 \text{ MPa}$. The results are in table (4.5). The results are promising and as expected. The z-force will indeed give the necessary information to detect wear.

At this point it is important to realise the nature of wear that will take place on the cutting edge. Two types of wear can be expected to arise from the cutting conditions in face milling. The first type of wear that should be considered is chipping of the primary
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Table 4.5: WI for 1 tooth cutter

cutting edge. This is a situation that occurs for example when the carbide or ceramic insert is subjected to stresses due to rapid changes of temperature. The second type of wear that takes place is the expected normal wear around the nose radius as well as along the primary cutting edge. Wear tracking parameters must be able to take these two different types of wear in addition to taking into consideration the two cutting edges (namely primary edge and radiused secondary edge). There are two logical ratios that are also physically meaningful. The first ratio is that of quasi-mean resultant force over mean torque. Since the quasi-mean resultant will be affected by the chipping of primary edge, it will be an indicator of its condition. Mathematically this ratio can be written as:

\[ \frac{F_{qm}^2}{T_{mean}^2} = \frac{1}{4} \cos^2 \psi_e (1 + r^2) \left( \frac{a_0^2 + b_0^2}{a_0^2} \right). \]  \hspace{0.5cm} (4.38)

However, because the wear indicator should be independent of depth of cut and width of cut, the ratio of \( F_T \) to \( F_R \) will be a better indicator. Therefore, a wear indicator can be defined from eqn. (4.38) as follows:

\[ WI_{primary} = (1 + r^2) = \frac{4}{\cos^2 \psi_e} \frac{T_{mean}^2}{F_{qm}^2} \frac{a_0^2}{a_1^2 + b_1^2}. \]  \hspace{0.5cm} (4.39)

Measuring the torque and calculating \( F_{qm} \) from the measured values of \( F_X \) and \( F_Y \), it is possible to calculate the value of \( WI_{primary} \). As can be seen it depends on the width of cut (coefficients of torque) as well as the depth of cut (\( \psi_e \)). In the experiments the values of \( T_{mean} \) were calculated from the values of torques obtained by the decoupling of \( F_X \) and \( F_Y \) equations. The second ratio that can be used is the ratio of mean axial force
over mean torque. This ratio will be dependent on the forces around the nose radius hence it can be used as a wear indicator around the secondary edge.

4.10 Wear Identification Strategy

In this chapter, the aim is to identify wear in a manner which is independent of width of cut, depth of cut, and direction of cut. From the simulations above, it is seen that various methods are successfully used to achieve this result. The identification strategy can be formally stated:

1. Obtain the cutting constants $K_r$, $r$, $r_2$ and $h^*$. These constants will be used in calculating the other unknowns. A detailed explanation of how the constants can be obtained is included in the next chapter;

2. Identify the immersion (width of cut) using $\frac{\text{Area}}{F_{qm}}$ ratio. Note that this ratio will also be independent of the depth of cut, $a$, since both Area and $F_{qm}$ are linearly related to the depth of cut.

3. Identify the depth of cut using the quasi-mean force, $F_{qm}$. The quasi-mean force depends both on the immersion and the depth of cut.

4. Identify wear from the $z$-force as well as the quasi-mean resultant force. The last step of the identification strategy involves the use of both the mean $z$-force and quasi-mean resultant force over mean torque to predict two different types of wear (that of the primary edge and secondary edge) knowing immersion and depth of cut.
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Table 4.6: Effect of wear on $\frac{\text{Area}^2}{F_{sm}^2}$ derived in section (4.5) for a one tooth cutter

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<th>$\frac{b^*}{S_1}$ = 0.2</th>
<th>$\frac{b^*}{S_1}$ = 0.3</th>
<th>$\frac{b^*}{S_1}$ = 0.4</th>
<th>$\frac{b^*}{S_1}$ = 0.5</th>
<th>$\frac{b^*}{S_1}$ = 0.6</th>
<th>$\frac{b^*}{S_1}$ = 0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>3.76</td>
<td>3.72</td>
<td>3.70</td>
<td>3.68</td>
<td>3.68</td>
<td>3.68</td>
<td>3.68</td>
<td>3.68</td>
</tr>
<tr>
<td>60°</td>
<td>3.2</td>
<td>3.14</td>
<td>3.10</td>
<td>3.06</td>
<td>3.04</td>
<td>3.02</td>
<td>3.00</td>
<td>3.00</td>
</tr>
<tr>
<td>90°</td>
<td>2.58</td>
<td>2.56</td>
<td>2.54</td>
<td>2.50</td>
<td>2.48</td>
<td>2.46</td>
<td>2.44</td>
<td>2.44</td>
</tr>
<tr>
<td>180°</td>
<td>2.00</td>
<td>2.00</td>
<td>2.02</td>
<td>2.02</td>
<td>2.04</td>
<td>2.04</td>
<td>2.06</td>
<td>2.06</td>
</tr>
</tbody>
</table>

Table 4.7: Effect of wear on ratio derived in section (4.5) for a two tooth cutter

<table>
<thead>
<tr>
<th>$\phi_s$</th>
<th>$\frac{b^*}{S_1}$ = 0.0</th>
<th>$\frac{b^*}{S_1}$ = 0.1</th>
<th>$\frac{b^*}{S_1}$ = 0.2</th>
<th>$\frac{b^*}{S_1}$ = 0.3</th>
<th>$\frac{b^*}{S_1}$ = 0.4</th>
<th>$\frac{b^*}{S_1}$ = 0.5</th>
<th>$\frac{b^*}{S_1}$ = 0.6</th>
<th>$\frac{b^*}{S_1}$ = 0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>3.48</td>
<td>3.38</td>
<td>3.34</td>
<td>3.32</td>
<td>3.30</td>
<td>3.30</td>
<td>3.30</td>
<td>3.28</td>
</tr>
<tr>
<td>60°</td>
<td>2.40</td>
<td>2.30</td>
<td>2.22</td>
<td>2.16</td>
<td>2.12</td>
<td>2.08</td>
<td>2.06</td>
<td>2.04</td>
</tr>
<tr>
<td>90°</td>
<td>1.46</td>
<td>1.42</td>
<td>1.38</td>
<td>1.36</td>
<td>1.32</td>
<td>1.30</td>
<td>1.28</td>
<td>1.26</td>
</tr>
<tr>
<td>180°</td>
<td>0.38</td>
<td>0.28</td>
<td>0.22</td>
<td>0.18</td>
<td>0.14</td>
<td>0.12</td>
<td>0.10</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 4.8: Effect of wear on ratio derived in section (4.5) for a three tooth cutter

<table>
<thead>
<tr>
<th>$\phi_s$</th>
<th>$\frac{b^*}{S_1}$ = 0.0</th>
<th>$\frac{b^*}{S_1}$ = 0.1</th>
<th>$\frac{b^*}{S_1}$ = 0.2</th>
<th>$\frac{b^*}{S_1}$ = 0.3</th>
<th>$\frac{b^*}{S_1}$ = 0.4</th>
<th>$\frac{b^*}{S_1}$ = 0.5</th>
<th>$\frac{b^*}{S_1}$ = 0.6</th>
<th>$\frac{b^*}{S_1}$ = 0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>3.14</td>
<td>2.98</td>
<td>2.90</td>
<td>2.86</td>
<td>2.84</td>
<td>2.84</td>
<td>2.82</td>
<td>2.82</td>
</tr>
<tr>
<td>60°</td>
<td>1.60</td>
<td>1.46</td>
<td>1.36</td>
<td>1.30</td>
<td>1.26</td>
<td>1.22</td>
<td>1.18</td>
<td>1.16</td>
</tr>
<tr>
<td>90°</td>
<td>0.64</td>
<td>0.60</td>
<td>0.56</td>
<td>0.52</td>
<td>0.50</td>
<td>0.48</td>
<td>0.46</td>
<td>0.46</td>
</tr>
<tr>
<td>180°</td>
<td>0.00</td>
<td>0.00</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>0.06</td>
<td>0.08</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 4.9: Effect of wear on ratio derived in section (4.5) for a four tooth cutter
Chapter 4. Frequency Domain Analysis

<table>
<thead>
<tr>
<th>$\phi_s$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.0$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.1$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.2$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.3$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.4$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.5$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.6$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>60°</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
<td>0.06</td>
<td>0.08</td>
<td>0.10</td>
<td>0.13</td>
<td>0.15</td>
</tr>
<tr>
<td>90°</td>
<td>0.07</td>
<td>0.09</td>
<td>0.12</td>
<td>0.14</td>
<td>0.18</td>
<td>0.21</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td>180°</td>
<td>0.08</td>
<td>0.08</td>
<td>0.09</td>
<td>0.09</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 4.10: Effect of wear on $\frac{M_{ag}}{F_{pm}}$ derived in section (4.6) for a one tooth cutter

<table>
<thead>
<tr>
<th>$\phi_s$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.0$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.1$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.2$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.3$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.4$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.5$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.6$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.05</td>
<td>0.07</td>
<td>0.09</td>
<td>0.12</td>
</tr>
<tr>
<td>60°</td>
<td>0.06</td>
<td>0.09</td>
<td>0.11</td>
<td>0.15</td>
<td>0.19</td>
<td>0.23</td>
<td>0.29</td>
<td>0.34</td>
</tr>
<tr>
<td>90°</td>
<td>0.11</td>
<td>0.13</td>
<td>0.15</td>
<td>0.17</td>
<td>0.19</td>
<td>0.22</td>
<td>0.24</td>
<td>0.28</td>
</tr>
<tr>
<td>180°</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
<td>0.11</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 4.11: Effect of wear on ratio derived in section (4.6) for a two tooth cutter

<table>
<thead>
<tr>
<th>$\phi_s$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.0$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.1$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.2$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.3$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.4$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.5$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.6$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.01</td>
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<td>0.04</td>
<td>0.07</td>
<td>0.09</td>
<td>0.13</td>
<td>0.17</td>
<td>0.21</td>
</tr>
<tr>
<td>60°</td>
<td>0.07</td>
<td>0.09</td>
<td>0.11</td>
<td>0.13</td>
<td>0.16</td>
<td>0.19</td>
<td>0.22</td>
<td>0.26</td>
</tr>
<tr>
<td>90°</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>180°</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 4.12: Effect of wear on ratio derived in section (4.6) for a three tooth cutter

<table>
<thead>
<tr>
<th>$\phi_s$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.0$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.1$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.2$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.3$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.4$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.5$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.6$</th>
<th>$\frac{h^*_{s_f}}{S_f} = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.02</td>
<td>0.03</td>
<td>0.06</td>
<td>0.08</td>
<td>0.12</td>
<td>0.16</td>
<td>0.21</td>
<td>0.26</td>
</tr>
<tr>
<td>60°</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>90°</td>
<td>0.01</td>
<td>0.02</td>
<td>0.02</td>
<td>0.03</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>180°</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 4.13: Effect of wear on ratio derived in section (4.6) for a four tooth cutter
Table 4.14: Effect of wear on $\frac{M_{ag^2}}{R^2}$ derived in section (4.7) for a one tooth cutter

<table>
<thead>
<tr>
<th>$\phi_*$</th>
<th>$\frac{h^*}{S_1} = 0.0$</th>
<th>$\frac{h^*}{S_1} = 0.1$</th>
<th>$\frac{h^*}{S_1} = 0.2$</th>
<th>$\frac{h^*}{S_1} = 0.3$</th>
<th>$\frac{h^*}{S_1} = 0.4$</th>
<th>$\frac{h^*}{S_1} = 0.5$</th>
<th>$\frac{h^*}{S_1} = 0.6$</th>
<th>$\frac{h^*}{S_1} = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>60°</td>
<td>0.98</td>
<td>0.97</td>
<td>0.97</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>90°</td>
<td>0.92</td>
<td>0.89</td>
<td>0.86</td>
<td>0.85</td>
<td>0.84</td>
<td>0.83</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>180°</td>
<td>0.55</td>
<td>0.59</td>
<td>0.62</td>
<td>0.66</td>
<td>0.69</td>
<td>0.72</td>
<td>0.74</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Table 4.15: Effect of wear on ratio derived in section (4.7) for a two tooth cutter

<table>
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<tr>
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<th>$\frac{h^*}{S_1} = 0.0$</th>
<th>$\frac{h^*}{S_1} = 0.1$</th>
<th>$\frac{h^*}{S_1} = 0.2$</th>
<th>$\frac{h^*}{S_1} = 0.3$</th>
<th>$\frac{h^*}{S_1} = 0.4$</th>
<th>$\frac{h^*}{S_1} = 0.5$</th>
<th>$\frac{h^*}{S_1} = 0.6$</th>
<th>$\frac{h^*}{S_1} = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
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<tr>
<td>60°</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>0.98</td>
<td>0.98</td>
<td>0.97</td>
<td>0.96</td>
</tr>
<tr>
<td>90°</td>
<td>0.70</td>
<td>0.60</td>
<td>0.54</td>
<td>0.50</td>
<td>0.45</td>
<td>0.43</td>
<td>0.41</td>
<td>0.40</td>
</tr>
<tr>
<td>180°</td>
<td>0.30</td>
<td>0.40</td>
<td>0.49</td>
<td>0.55</td>
<td>0.61</td>
<td>0.65</td>
<td>0.69</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Table 4.16: Effect of wear on ratio derived in section (4.7) for a three tooth cutter

<table>
<thead>
<tr>
<th>$\phi_*$</th>
<th>$\frac{h^*}{S_1} = 0.0$</th>
<th>$\frac{h^*}{S_1} = 0.1$</th>
<th>$\frac{h^*}{S_1} = 0.2$</th>
<th>$\frac{h^*}{S_1} = 0.3$</th>
<th>$\frac{h^*}{S_1} = 0.4$</th>
<th>$\frac{h^*}{S_1} = 0.5$</th>
<th>$\frac{h^*}{S_1} = 0.6$</th>
<th>$\frac{h^*}{S_1} = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
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<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>60°</td>
<td>0.86</td>
<td>0.76</td>
<td>0.70</td>
<td>0.66</td>
<td>0.64</td>
<td>0.62</td>
<td>0.61</td>
<td>0.60</td>
</tr>
<tr>
<td>90°</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>180°</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Table 4.17: Effect of wear on ratio derived in section (4.7) for a four tooth cutter

<table>
<thead>
<tr>
<th>$\phi_*$</th>
<th>$\frac{h^*}{S_1} = 0.0$</th>
<th>$\frac{h^*}{S_1} = 0.1$</th>
<th>$\frac{h^*}{S_1} = 0.2$</th>
<th>$\frac{h^*}{S_1} = 0.3$</th>
<th>$\frac{h^*}{S_1} = 0.4$</th>
<th>$\frac{h^*}{S_1} = 0.5$</th>
<th>$\frac{h^*}{S_1} = 0.6$</th>
<th>$\frac{h^*}{S_1} = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
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<td>0.95</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>60°</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>90°</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
<td>0.56</td>
<td>0.57</td>
<td>0.58</td>
<td>0.59</td>
</tr>
<tr>
<td>180°</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Chapter 4. Frequency Domain Analysis

<table>
<thead>
<tr>
<th>$\phi$, $S_1$</th>
<th>$h_1^*$ = 0.0</th>
<th>$h_1^*$ = 0.1</th>
<th>$h_1^*$ = 0.2</th>
<th>$h_1^*$ = 0.3</th>
<th>$h_1^*$ = 0.4</th>
<th>$h_1^*$ = 0.5</th>
<th>$h_1^*$ = 0.6</th>
<th>$h_1^*$ = 0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.27</td>
<td>-0.29</td>
<td>-0.17</td>
<td>-0.14</td>
<td>-0.12</td>
<td>-0.11</td>
<td>-0.10</td>
<td>-0.10</td>
</tr>
<tr>
<td>60°</td>
<td>0.18</td>
<td>0.30</td>
<td>0.44</td>
<td>0.61</td>
<td>0.83</td>
<td>1.12</td>
<td>1.53</td>
<td>2.19</td>
</tr>
<tr>
<td>90°</td>
<td>0.30</td>
<td>0.37</td>
<td>0.44</td>
<td>0.49</td>
<td>0.54</td>
<td>0.59</td>
<td>0.63</td>
<td>0.67</td>
</tr>
<tr>
<td>180°</td>
<td>2.42</td>
<td>2.72</td>
<td>3.03</td>
<td>3.36</td>
<td>3.69</td>
<td>4.03</td>
<td>4.37</td>
<td>4.71</td>
</tr>
</tbody>
</table>

Table 4.18: Phase difference between deviatoric force components for a one tooth cutter

<table>
<thead>
<tr>
<th>$\phi$, $S_1$</th>
<th>$h_1^*$ = 0.0</th>
<th>$h_1^*$ = 0.1</th>
<th>$h_1^*$ = 0.2</th>
<th>$h_1^*$ = 0.3</th>
<th>$h_1^*$ = 0.4</th>
<th>$h_1^*$ = 0.5</th>
<th>$h_1^*$ = 0.6</th>
<th>$h_1^*$ = 0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.53</td>
<td>-0.60</td>
<td>-0.36</td>
<td>-0.29</td>
<td>-0.25</td>
<td>-0.23</td>
<td>-0.21</td>
<td>-0.20</td>
</tr>
<tr>
<td>60°</td>
<td>0.35</td>
<td>0.59</td>
<td>0.85</td>
<td>1.15</td>
<td>1.52</td>
<td>2.00</td>
<td>2.64</td>
<td>3.55</td>
</tr>
<tr>
<td>90°</td>
<td>0.66</td>
<td>0.82</td>
<td>0.96</td>
<td>1.09</td>
<td>1.20</td>
<td>1.32</td>
<td>1.42</td>
<td>1.52</td>
</tr>
<tr>
<td>180°</td>
<td>$\infty$</td>
<td>-12.9</td>
<td>-7.08</td>
<td>-5.15</td>
<td>-4.19</td>
<td>-3.62</td>
<td>-3.24</td>
<td>-2.96</td>
</tr>
</tbody>
</table>

Table 4.19: Phase difference between deviatoric force components for a two tooth cutter

<table>
<thead>
<tr>
<th>$\phi$, $S_1$</th>
<th>$h_1^*$ = 0.0</th>
<th>$h_1^*$ = 0.1</th>
<th>$h_1^*$ = 0.2</th>
<th>$h_1^*$ = 0.3</th>
<th>$h_1^*$ = 0.4</th>
<th>$h_1^*$ = 0.5</th>
<th>$h_1^*$ = 0.6</th>
<th>$h_1^*$ = 0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.77</td>
<td>-0.98</td>
<td>-0.57</td>
<td>-0.45</td>
<td>-0.39</td>
<td>-0.36</td>
<td>-0.33</td>
<td>-0.31</td>
</tr>
<tr>
<td>60°</td>
<td>0.53</td>
<td>0.84</td>
<td>1.17</td>
<td>1.54</td>
<td>1.96</td>
<td>2.47</td>
<td>3.09</td>
<td>3.86</td>
</tr>
<tr>
<td>90°</td>
<td>1.22</td>
<td>1.48</td>
<td>1.74</td>
<td>1.99</td>
<td>2.24</td>
<td>2.49</td>
<td>2.74</td>
<td>2.98</td>
</tr>
</tbody>
</table>

Table 4.20: Phase difference between deviatoric force components for a three tooth cutter

<table>
<thead>
<tr>
<th>$\phi$, $S_1$</th>
<th>$h_1^*$ = 0.0</th>
<th>$h_1^*$ = 0.1</th>
<th>$h_1^*$ = 0.2</th>
<th>$h_1^*$ = 0.3</th>
<th>$h_1^*$ = 0.4</th>
<th>$h_1^*$ = 0.5</th>
<th>$h_1^*$ = 0.6</th>
<th>$h_1^*$ = 0.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>0.96</td>
<td>-1.49</td>
<td>-0.82</td>
<td>-0.64</td>
<td>-0.55</td>
<td>-0.50</td>
<td>-0.47</td>
<td>-0.44</td>
</tr>
<tr>
<td>60°</td>
<td>0.69</td>
<td>1.00</td>
<td>1.32</td>
<td>1.66</td>
<td>2.01</td>
<td>2.40</td>
<td>2.82</td>
<td>3.28</td>
</tr>
<tr>
<td>90°</td>
<td>2.43</td>
<td>2.66</td>
<td>2.94</td>
<td>3.27</td>
<td>3.67</td>
<td>4.14</td>
<td>4.70</td>
<td>5.37</td>
</tr>
<tr>
<td>180°</td>
<td>-0.55</td>
<td>-0.55</td>
<td>-0.55</td>
<td>-0.55</td>
<td>-0.55</td>
<td>-0.55</td>
<td>-0.55</td>
<td>-0.55</td>
</tr>
</tbody>
</table>

Table 4.21: Phase difference between deviatoric force components for a four tooth cutter
Chapter 5

Experimental Work

A set of experiments was conducted to test the ideas presented in the previous sections. A high alloy steel (4140 AISI) with a Brinell hardness of 350 was used as the workpiece material. The workpieces were of rectangular shape (see appendix C for dimensions). A dynamometer (either a Kistler model 9257A or an equivalent in-house built version) measured forces in three orthogonal directions, \( x, y, z \). A DT2801 data acquisition board and/or a digital scope was used in data acquisition. The tests were done on a knee type vertical CNC milling machine (refer to figure (5.1)). A five tooth three inch face milling cutter was fed into the workpiece at feedrates from 0.001 in/tooth to 0.004 in/tooth. Various widths of cut from 0.5 in. to 3 in. were tested. The tests were carried with both a sharp and a worn insert. The dynamometer output of forces for the sharp and worn tool in the \( x, y, z \) directions are shown at the end of the chapter. Further comments and discussions on these forces will also be included in the end of this chapter.

5.1 The Calculation of Cutting Constants

The first thing that must be discussed before analyzing the cutting force data is to formulate a method of obtaining the cutting constants \( K_s, r_1, r_2 \) and \( h^* \). The cutting forces and the wear forces were modelled in the previous chapters. Using those models it is possible to write the equations for average forces \( F_{XT} \) and \( F_{YT} \). The equation for
Figure 5.1: The set-up for cutting tests.
Chapter 5. Experimental Work

$F_{XT}$ can be written as:

$$F_{XT} = K_s a N S_t [\sin \phi \cos \phi + r_1 \sin^2 \phi + \frac{h^*}{S_t} (\cos \phi + r_2 \sin \phi)]. \quad (5.1)$$

The average force $F_{XT-ave}$ can now be obtained simply by integrating equation (5.1) from 0 to $2\pi$. Hence the equation for $F_{XT-ave}$ can be written as below:

$$F_{XT-ave} = \frac{K_s a N S_t}{2\pi} \left[ \int_0^{2\pi} (\sin \phi \cos \phi + r_1 \sin^2 \phi + \frac{h^*}{S_t} (\cos \phi + r_2 \sin \phi)) d\phi \right]. \quad (5.2)$$

The result of the integration is shown in equation (5.3) below,

$$F_{XT-ave} = \frac{K_s a N S_t}{8\pi} [(1 - \cos 2\phi_s) + r_1 (2\phi_s - \sin 2\phi_s) + \frac{4h^*}{S_t} (\sin \phi_s + r_2 (1 - \cos \phi_s))). \quad (5.3)$$

In a similar manner:

$$F_{YT-ave} = \frac{K_s a N S_t}{8\pi} [r_1 (1 - \cos 2\phi_s) - (2\phi_s - \sin 2\phi_s) + \frac{4h^*}{S_t} (r_2 \sin \phi_s - (1 - \cos \phi_s))]. \quad (5.4)$$

The above relations are derived for the up milling case. A similar exercise can be carried out for down milling. Since the procedures are identical, only the results will be given below:

$$F_{XT-ave} = \frac{K_s a N S_t}{8\pi} [r_1 (2\phi_s - \sin 2\phi_s) - (1 - \cos 2\phi_s) + \frac{4h^*}{S_t} (r_2 (1 - \cos \phi_s) - \sin \phi_s))] \quad (5.5)$$

and,

$$F_{YT-ave} = \frac{K_s a N S_t}{8\pi} [r_1 (1 - \cos 2\phi_s) + (2\phi_s - \sin 2\phi_s) + \frac{4h^*}{S_t} ((1 - \cos \phi_s) - r_2 \sin \phi_s)]. \quad (5.6)$$

The relationships shown in equations (5.3), (5.4), (5.5), (5.6) indicate that the average forces are linear functions of feedrate, $S_t$. Hence for a given range of feedrates and for a known immersion it is possible to find the cutting constants from the slope and intercept of equations (5.3) and (5.4). The force measurements are taken for various feedrates in
Table 5.1: Averages for full immersion and new insert

<table>
<thead>
<tr>
<th>( S_t ) (mm/tooth)</th>
<th>( F_{XT} ) (N)</th>
<th>( F_{YT} ) (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0254</td>
<td>125.5</td>
<td>-166.1</td>
</tr>
<tr>
<td>0.0508</td>
<td>213.5</td>
<td>-270.2</td>
</tr>
<tr>
<td>0.1016</td>
<td>328.3</td>
<td>-418.6</td>
</tr>
</tbody>
</table>

full immersion for the up-milling case. Substituting the immersion angle (\( \pi \) radians) into the average force equations, simpler relationships are obtained:

\[
F_{XT-ave} = \frac{K_s a}{8\pi} (S_t 2\pi r_1 + 8h^*r_2) \tag{5.7}
\]

and

\[
F_{YT-ave} = -\frac{K_s a}{8\pi} (S_t 2\pi + 8h^*). \tag{5.8}
\]

5.2 Experimental Results

In this section, the values of the cutting constants will be calculated using those equations and the experimental data.

5.2.1 Calculation for a New Insert

A new chamfered insert (see figure (5.2)) is used for cutting at 360 rpm with feeds of 0.001, 0.002, and 0.004 inch/tooth. The forces are then averaged over the full period of the tooth. The results are shown in table (5.1).

A line is fitted through the points as shown in figure (5.3). From the graph the slope and intercept of the curve are obtained and used for the necessary calculations. For the new insert, the cutting constants are:

\[
K_s = 2565 \, N/mm^2, \]
\[
r_1 = 0.8, \]
Chapter 5. Experimental Work

Figure 5.2: A chamfered tool

Figure 5.3: Average forces vs. feed for full immersion, sharp insert
Table 5.2: Averages for full immersion and worn insert

<table>
<thead>
<tr>
<th>$S_t$ (mm/tooth)</th>
<th>$F_{XT}$ (N)</th>
<th>$F_{YT}$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0254</td>
<td>250.4</td>
<td>-234.9</td>
</tr>
<tr>
<td>0.0508</td>
<td>334.2</td>
<td>-321.9</td>
</tr>
<tr>
<td>0.1016</td>
<td>448.3</td>
<td>-476.0</td>
</tr>
</tbody>
</table>

$\tau_2 = 0.74,$

$h^* = 0.022 \text{ mm.}$

(5.9)

5.2.2 Calculation for a Worn Insert

A chamfered insert with 0.008 inch flank wear is used as the worn insert. The cutting test information is included in table (5.2). The results of the cutting constants are shown below:

$K_s = 2477 \text{ N/mm}^2,$

$r_1 = 0.81,$

$r_2 = 1.22,$

$h^* = 0.0394 \text{ mm.}$

(5.10)

Comparing the values for a new insert to that of a worn one, one can see that $K_s$ and $r_1$ are very similar. In the case of $r_2$ and $h^*$ these values are considerably different. This difference is attributed to the flank zone cutting forces that occur as the wear-land increases.
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5.3 Calculation of Experimental F.S. Coefficients

The next step after the calculation of cutting variables is the estimation of F.S. coefficients experimentally. The method of experimentally obtaining Fourier Series coefficients involves dividing the force vs. time plot at 12 equidistant times per spindle revolution (equivalent to 30° angular displacement of the spindle). The forces at those 12 instances can then be used to obtain the mean and fundamental coefficients as well as the first 2 frequency component coefficients after the fundamental. More detail concerning this method is included in the appendix B. The following equations give the necessary coefficients:

\[
\begin{align*}
a_{xt0} &= \frac{1}{12} (y_0 + y_1 + \ldots + y_{11}), \\
a_{xt3} &= \frac{1}{6} (y_0 - y_2 + y_4 - y_6 + y_8 - y_{10}), \\
b_{xt3} &= \frac{1}{6} (y_1 - y_3 + y_5 - y_7 + y_9 - y_{11}), \\
a_{xt1} &= \frac{1}{2} (y_0 - y_6) - a_{xt3}, \\
b_{xt1} &= \frac{1}{2} (y_3 - y_9) + b_{xt3}, \\
a_{xt2} &= \frac{1}{4} (y_0 - y_3 + y_6 - y_9), \\
b_{xt2} &= \frac{1}{4} (y_1 - y_2 + y_3 - y_4).
\end{align*}
\]  

(5.11)

Here, \(y_0, y_1, \ldots, y_{11}\) are the forces obtained experimentally at 0, 30°, 60°, ..., 330° of spindle revolution, and \(\gamma_1, \gamma_2, \gamma_3, \gamma_4\) corresponds to 45°, 135°, 225°, 315° of spindle revolution.

The experimental coefficients (especially their magnitudes) are now compared with the exact method of calculating the Fourier Series coefficients already shown in earlier sections (first for a new tool then for a worn tool).

A second set of experiments were done with round inserts (i.e. inserts with radiused cutting edge) after the problems experienced with chamfered inserts. An insert with
Chapter 5. Experimental Work

<table>
<thead>
<tr>
<th>F.S. coeff.</th>
<th>theoretical</th>
<th>experimental</th>
<th>$\text{magnitude}_\text{theor.}$</th>
<th>$\text{magnitude}_\text{exper.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{xt0}(N)$</td>
<td>332.4</td>
<td>295.8</td>
<td>332.4</td>
<td>295.8</td>
</tr>
<tr>
<td>$a_{xt1}(N)$</td>
<td>424.5</td>
<td>269.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{xt1}(N)$</td>
<td>555.7</td>
<td>457.0</td>
<td>699.3</td>
<td>530.6</td>
</tr>
<tr>
<td>$a_{xt2}(N)$</td>
<td>-309.9</td>
<td>-209.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{xt2}(N)$</td>
<td>452.8</td>
<td>544.5</td>
<td>548.7</td>
<td>583.4</td>
</tr>
<tr>
<td>$a_{xt3}(N)$</td>
<td>-168.5</td>
<td>-269.7</td>
<td>191.0</td>
<td>272.7</td>
</tr>
<tr>
<td>$b_{xt3}(N)$</td>
<td>-90.0</td>
<td>40.0</td>
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<td></td>
</tr>
</tbody>
</table>

Table 5.3: F.S. coeff. comparison for a new tool

<table>
<thead>
<tr>
<th>F.S. coeff.</th>
<th>theoretical</th>
<th>experimental</th>
<th>$\text{magnitude}_\text{theor.}$</th>
<th>$\text{magnitude}_\text{exper.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{xt0}$</td>
<td>451.0</td>
<td>427.1</td>
<td>451.0</td>
<td>427.1</td>
</tr>
<tr>
<td>$a_{xt1}$</td>
<td>519.2</td>
<td>329.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{xt1}$</td>
<td>741.9</td>
<td>686.7</td>
<td>905.5</td>
<td>761.5</td>
</tr>
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<td>$a_{xt2}$</td>
<td>-387.3</td>
<td>-302.5</td>
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<td>709.1</td>
</tr>
<tr>
<td>$b_{xt2}$</td>
<td>530.0</td>
<td>641.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{xt3}$</td>
<td>-162.7</td>
<td>-309.2</td>
<td>184.9</td>
<td>314.3</td>
</tr>
<tr>
<td>$b_{xt3}$</td>
<td>-87.9</td>
<td>56.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.4: F.S. coeff. comparison for a worn tool
chamfer is used for improved surface finish quality. The clearance angle on the chamfered edge is nearly zero. This poses the difficulty of determining the effects of secondary edge cutting. It would be difficult to estimate the equivalent chip thickness for this situation, since cutting would take place on this nearly flat edge. Consequently, the estimation of the depth of cut would also be difficult. The second set of tests were done with aluminium as well as the steel from the same stock as before. Before the test results are shown, a second method of cutting constant calculations are presented below.

5.4 Cutting Constant Calculations Using $F_T$ and $F_R$

The tangential force and the radial force are linearly dependent on the uncut chip thickness. Exploiting this relationship, one could plot $F_T$ and $F_R$ vs. $h$ and from this obtain directly the four constants. The only consideration is the realization that $h$ is dependent on the immersion angle, $\phi$, through the relationship $h = S_i \sin \phi$. For a given $\phi$ (rad), the $F_X$ and $F_Y$ forces will be known from the dynamometer data. These forces can be resolved into their components namely $F_T$ and $F_R$. It is now possible to plot $F_T$ and $F_R$ vs. $h$, since both can be calculated knowing $F_X, F_Y$ and $\phi$. If this is done for a range of $\phi$ a linear graph can be obtained. From the graph of $F_T$ vs. $h$, the slope and intercept will give $K_s$ and $h^*$ This is shown mathematically below:

$$F_T = K_s a h + K_s a h^* = m_1 h + b_1$$

(5.12)

where,

$$m_1 = K_s a$$

$$b_1 = K_s a h^*.$$  

Similarly, the slope and intercept of the $F_R$ curve will give $r_1$ and $r_2$:

$$F_R = r_1 K_s a h + r_2 K_s a h^* = m_2 h + b_2$$

(5.13)
where,

\[ m_2 = r_1 K_s a \]
\[ b_2 = r_2 K_s a h^* \]

The experimental results for aluminium are shown below. The aluminium used in these tests were classified into four different types depending on their metallurgical compositions as: 1, primary unmodified (PU); 2, secondary unmodified (SU); 3, primary modified (PM); 4, secondary modified (SM).

For PM(14),

\[ K_s \approx 1500 \text{ [N/mm}^2\text{]} \]
\[ r_1 = 0.42 \]
\[ r_2 = 4.75 \]
\[ h^* = 0.0129 \text{ [mm]} \]

For SM(6),

\[ K_s \approx 1200 \text{ [N/mm}^2\text{]} \]
\[ r_1 = 0.49 \]
\[ r_2 = 2.56 \]
\[ h^* = 0.0200 \text{ [mm]} \]

For SU(6),

\[ K_s \approx 1000 \text{ [N/mm}^2\text{]} \]
\[ r_1 = 0.99 \]
\[ r_2 = 7.5 \]
\[ h^* = 0.00303 \text{ [mm]} \]
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For PU(8),

\[
K_* \approx 1100 \text{ [N/mm}^2]\text{]
\]

\[
r_1 = 0.83
\]

\[
r_2 = 3.33
\]

\[
h^* = 0.00426 \text{ [mm].}
\]

The interesting observation from this data is the high value of \(r_1\) and lower value of \(K_*\) for unmodified samples in comparison with the modified. The plot of \(F_T\) and \(F_R\) vs. \(h\) is shown in figure (5.4). In this figure, it is important to note that for immersions over 90° the points deviate significantly from the expected linear relationship. Pandy and Shan [31] try to explain this phenomenon as follows. When the tool is cutting at \(\phi \leq 90°\), the increase in uncut chip thickness causes the surface slope to decrease which causes the shear angle to increase by an amount \(\delta\), which itself causes the normalized forces (with respect to uncut chip thickness) to decrease (see figure (5.5) for the physical meaning of \(\delta\)).

Similarly, once the tool passes the 90° mark, the surface slope increases and the shear angle becomes smaller by an amount \(\delta\). This change in the shear angle is the cause of the higher forces.

Having verified the method of obtaining the cutting constants using aluminium, experiments on wearing a carbide insert were carried out with a steel workpiece using the same procedure. For various widths of cut ranging from 21.8 mm to full immersion (75 mm), various passes were completed with a sharp round insert. These results are shown below.

For 21.8 mm immersion:

\[
K_* = 3450 \text{ [N/mm}^2]\text{]}
\]
Figure 5.4: A graph of tangential and radial forces vs. $h$
Figure 5.5: Shear angle considerations for full immersion cutting (after Pandy et al.)

\[ r_1 = 0.33 \]
\[ r_2 = 3.57 \]
\[ h^* = 0.0199 \text{ [mm]} \].

For 35 mm immersion:

\[ K_s = 3500 \text{ [N/mm}^2\text{]} \]
\[ r_1 = 0.37 \]
\[ r_2 = 3.15 \]
\[ h^* = 0.01078 \text{ [mm]} \].

For 50 mm immersion:

\[ K_s = 3180 \text{ [N/mm}^2\text{]} \]
\[ r_1 = 0.39 \]
\[ r_2 = 3.4 \]
\[ h^* = 0.01280 \text{ [mm]} \]

For 75 mm immersion (full immersion):
\[ K_s = 3450 \text{ [N/mm}^2\text{]} \]
\[ r_1 = 0.23 \]
\[ r_2 = 2.45 \]
\[ h^* = 0.03082 \text{ [mm]} \]

The idea is to use these tests as the basis of experimental tests for the coming sections, concentrating on the full and half immersion data for the conclusions.

### 5.5 Wear Detection

In the last chapter, the z-force, \( F_z \) was used to detect wear. In this section experimental justification can now be given for using \( F_z \). It is important to show the behaviour of the fundamental component of the steady state forces since they are used in immersion identification. As was discussed earlier in this chapter, the first three frequency components can be obtained experimentally once the force data is divided into 12 equidistant parts. These experimental values are shown in table (5.5).

As discussed earlier, two different types of wear are expected in the normal wearing process: 1, chipping; 2, normal wear land. The next step in the process of identification having calculated the Fourier Series coefficients is to calculate the immersion and wear ratios already discussed.

The experimental investigation of these two parameters along with the immersion identification parameters are discussed next. The experimental results for full and half immersion data are used. First, the immersion identification parameter is shown (figure
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<table>
<thead>
<tr>
<th>$V Bx(0.001&quot;)$</th>
<th>2.5</th>
<th>4</th>
<th>4.5</th>
<th>5.5</th>
<th>7.5</th>
<th>8.5</th>
<th>10.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{xt0}(N)$</td>
<td>108</td>
<td>97.1</td>
<td>81.3</td>
<td>103.3</td>
<td>98.1</td>
<td>115.4</td>
<td>225.2</td>
</tr>
<tr>
<td>$a_{xt1}(N)$</td>
<td>269.2</td>
<td>281.3</td>
<td>255.4</td>
<td>277.5</td>
<td>318.3</td>
<td>326.7</td>
<td>405.4</td>
</tr>
<tr>
<td>$b_{xt1}(N)$</td>
<td>110</td>
<td>100</td>
<td>64.2</td>
<td>97.5</td>
<td>81.7</td>
<td>105</td>
<td>261.7</td>
</tr>
<tr>
<td>$a_{xt2}(N)$</td>
<td>92.5</td>
<td>130.6</td>
<td>101.9</td>
<td>125</td>
<td>126.9</td>
<td>123.8</td>
<td>85</td>
</tr>
<tr>
<td>$a_{xt3}(N)$</td>
<td>173.8</td>
<td>187.5</td>
<td>165.6</td>
<td>186.3</td>
<td>200</td>
<td>190.6</td>
<td>198.1</td>
</tr>
<tr>
<td>$b_{xt3}(N)$</td>
<td>-34.2</td>
<td>-16.3</td>
<td>-25.4</td>
<td>-17.5</td>
<td>-23.3</td>
<td>-26.7</td>
<td>-47.9</td>
</tr>
<tr>
<td>$a_{yt0}(N)$</td>
<td>-208</td>
<td>-196.5</td>
<td>-208.1</td>
<td>-193.3</td>
<td>-217.5</td>
<td>-230</td>
<td>-222</td>
</tr>
<tr>
<td>$a_{yt1}(N)$</td>
<td>27.5</td>
<td>-7.5</td>
<td>21.3</td>
<td>-4.2</td>
<td>30.8</td>
<td>70</td>
<td>168.3</td>
</tr>
<tr>
<td>$b_{yt1}(N)$</td>
<td>-357.5</td>
<td>-317.1</td>
<td>-351.7</td>
<td>-319.2</td>
<td>-401.7</td>
<td>-399.6</td>
<td>-410</td>
</tr>
<tr>
<td>$a_{yt2}(N)$</td>
<td>236.3</td>
<td>208.8</td>
<td>215</td>
<td>212.5</td>
<td>263.8</td>
<td>237.5</td>
<td>371.3</td>
</tr>
<tr>
<td>$b_{yt2}(N)$</td>
<td>177.5</td>
<td>138.8</td>
<td>156.3</td>
<td>143.8</td>
<td>195</td>
<td>210.6</td>
<td>316.3</td>
</tr>
<tr>
<td>$a_{yt3}(N)$</td>
<td>32.5</td>
<td>40</td>
<td>33.8</td>
<td>39.2</td>
<td>61.7</td>
<td>37.5</td>
<td>19.2</td>
</tr>
<tr>
<td>$b_{yt3}(N)$</td>
<td>55</td>
<td>67.9</td>
<td>28.3</td>
<td>65.8</td>
<td>38.3</td>
<td>57.9</td>
<td>135</td>
</tr>
</tbody>
</table>

Table 5.5: F.S. coeff. using the experimental method for full immersion (5.6)). The theoretical lines are shown as dashed lines and the superimposed experimental values show good agreement with the theoretical values. Next the wear tracking parameters for half immersion and full immersion are shown. In the case of full immersion, as the parameters suggest as well in figure (5.7), the inserts sustained normal wear around the nose radius as well as the primary edge (see figure (5.8) for detailed tool condition).

The situation is different for the half immersion case. In figure (5.9), it can be seen that the $WI_{primary}$ indicator shoots up considerably while the normal wear component gradually changes. This indicates that chipping on the primary cutting edge occurred. A close-up view of the insert proves this point (figure (5.10)).

5.6 Comments on Experimental and Simulation Comparisons

The following pages show the experimental as well as simulation reconstruction of cutting forces in the $x$ and $y$ directions using Fourier Series. The first observation that can be
Figure 5.6: Immersion identification for full and half immersion compared to theoretical values.
Figure 5.7: Wear tracking in full immersion
Figure 5.8: Insert used in full immersion
Figure 5.9: Wear tracking in half immersion
Figure 5.10: Insert used in half immersion
made is that the number of terms taken in Fourier Series will undoubtedly influence the shape of the force curve and this can be seen readily from the simulation results. The second observation can be seen in the Fourier Series modelling especially with a 4 tooth cutter and 90° immersion cutting. The values of forces at 90° is actually non-zero. This is because of the disappearance of the sine term from the Fourier Series model. Taking more terms would be one solution for this situation but since this thesis is concerned with the mean and fundamental values those effects are ignored. Comparing the simulation results with experimental ones overlayed, the most important difference can be seen by the effect of the analog filter on the experimental results as smoothing the force curves. Further with respect to the modelling of wear, it is observed that when the worn tool enters the workpiece in up-milling from 0°, the $F_x$ and $F_y$ forces are zero and then build up at a rapid rate that also corresponds to the influence of the additional wear force. The model assumes a rectangular wave form for wear and because of this there is a DC offset. But as cutting progresses the simulation results compare more favourably to the experimental results.

5.7 Further Comments on the Experimental Values of the Cutting Forces

The following graphs show the axial force $F_z$ as obtained from the dynamometer for a tool with increasing wear. The graphs are for full immersion, 1.57 mm depth of cut and 0.1067 mm/tooth feedrate. As expected, the in-plane forces $F_x$ and $F_y$ are both affected by the wear in terms of their magnitudes. Something more interesting takes place in $F_z$ forces. With a practically sharp insert, the $F_z$ force goes up sharply as the insert makes contact with workpiece and starts cutting, as can be seen by the relatively smooth sinusoidal signal (see figure (5.14)). As the insert is worn, an interesting trend is forming (see figure (5.15)). There is a local maximum in the forces as the insert enters
Figure 5.11: F.S. simulation for 1,2,4 tooth 90° immersion with 25 terms
Figure 5.12: F.S. simulation for 1,2,4 tooth 90° immersion with 50 terms
Chapter 5. Experimental Work

Figure 5.13: Comparison of experimental vs. simulation results
workpiece. This is believed to be due to ploughing. When the insert makes contact with workpiece the insert is pushed against it. This forces the insert to plough through the material until there comes a point such that the material begins to shear and cutting starts, (at this point a sudden decrease in the z-force is observed). Such a trend is of course not expected in the exit of the tooth form the workpiece because of the nature of down milling. Forces are large enough to force shearing hence cutting right to the exit of the tooth from the workpiece. This trend continues as wear increases gradually until a similar condition now begins forming in the exit (see figure (5.16)). This is shorter in duration than the entry condition. The reason for this is that it takes a longer time to accumulate forces large enough to start cutting in up-milling than in down-milling. After sufficient wear another phenomenon appears. A second shorter local maximum before the first one develops slowly (see figure (5.17)). This is believed to be due to rubbing of flank side of the tool with the newly cut surface. Again this first local maximum seems to appear in the entry first but soon a second one in the exit is also formed (see figure (5.18)).
Figure 5.15: The force in z-direction for VB = 0.004 in.

Figure 5.16: The force in z-direction for VB = 0.0075 in.
Figure 5.17: The force in z-direction for VB = 0.010 in.

Figure 5.18: The force in z-direction for VB ≈ 0.011 in.
Chapter 6

Conclusions and Future Work

Fourier Series modelling of steady state cutting forces has been shown to have promise in the identification of immersion and tool wear. Preliminary experiments support this claim but further experimental work must be done before a real time system may be achieved. The benefits of using this method are that only the fundamental frequency and mean values are necessary for identification purposes. The dynamometer signal can be used directly even for multi-tooth cutting. The axial force component $F_x$, is found to be an important part of the algorithm since it is independent of the direction of cut. Two different types of wear were observed in the experiments: chipped flank edge (primary edge), and normal wear band on the flank. The ratio $F_{zm}/T_{mean}$, was indicative of the chipped edge while the ratio $F_{z-mean}/T_{mean}$, was not affected very much by it. This enabled the prediction of the two types of wear independent of each other.

Work on a tool wear sensor to directly measure wear, has also been carried out. Preliminary results showed that a resistive deposit on the flank face could be used to correlate to wear. For a practical tool further testing must be planned. A U.S. patent has been obtained for this sensor.
Appendix A

Listing of F.S. Modelling Program for Steady State Face Milling

/* Artist: Yetvart Hosepyan */
/* Updated: February 13, 1990 */
/* Version: Turbo C */
/* Address: Dept. of Mech. Engr. */
/* Phone: 22B 2817 */

/* This program simulates the torques and the milling forces on a */
/* straight cutting edge milling cutter. Two important assumptions */
/* are: 1, the specific cutting pressure is constant; 2, the ratio */
/* of radial to tangential components of cutting force is constant. */
/* The program also simulates the edge forces that occur due to */
/* wear while cutting. These edge forces have components in the */
/* x and y directions naturally. Further, the magnitudes of the */
/* coefficients are calculated. This helps the user to find out */
/* which frequencies are worthwhile to watch to detect wear. */

#include <stdio.h>
#include <math.h>
define arr 50

int dphi, j, z, t;
double magnitude_x, magnitude_y, magnitude_wx, magnitude_wy, SUMW, SUMWX, SUMXY;
double a[50], b[50], sumT, SUMX, SUMY, Torque, Fx, Fy, Fwx, Fwy, phi_s;
double k, d, d1, d2, Cl, C2, x, y, n, phi, phi, phi, phi, phi, phi, phi, phi, phi, phi, phi,

double aw[50], bx[50], by[50], bT[50], b2[50];
double aw[50], bx[50], by[50], bT[50], b2[50];
double dimax, dimy, dimb, dimby, temp1, temp2;
FILE *fl, *f2, *f3, *f4;
PI = 3.14159265... /

main()
{
  fl = fopen("a:force.prn","w");
  f2 = fopen("a:torque.prn","w");
  f3 = fopen("a:coefI").prn", "w") ;
  f4 = fopen("a:results.prn", "w");
  PI = 4.0*atanU.0 J;
  /* input machining variables */
  printf("What is the number of teeth? ");
  scanf("id",iz);
  printf("What is the immersion angle? ");
  scanf("lf",idegi;
  printf("What is the specific cutting pressure? ");
  scanf("lf",ik);
  printf("what is the axial depth of cut? ");
  scanf("lf",&d);
  printf("What is the feed per tooth? ");
  scanf("lf",&St);
  printf("What is the radius of the tool? ");
  scanf("lf",&R);
  printf("What is the cutting ratio? ");
  scanf("lf",&r);
  printf("What is the edge force cutting ratio? ");
  scanf("lf",&r2);
  printf("Please specify h star: ");
  scanf("lf",&H);
  printf(" ");
  /* initialize the variables. */
  /* d=5.0; k=1200.0; St=0.2; R=15.0; */
  r=0.3; r2=0.8; 

  t = 0;
  phi s = deg*PI/180.0;
  aT[0] = 0.0;
  aT[1] = 0.0; bT[1] = 0.0;
  aT[2] = 0.0; bT[2] = 0.0;
  aw[1] = 0.0; bw[1] = 0.0;
  aw[2] = 0.0; bw[2] = 0.0;
  sumT = 0.0; SUMX = 0.0; SUMY = 0.0; SUMW = 0.0;
Listing of F.S. Modelling Program for Steady State Face Milling

\[ \begin{align*}
C_1 &= k*d*St+z; \\
C_2 &= k*d*r; \\
a(0) &= (1 - \cos(\phi_s))/(2*\pi); \\
a(1) &= (1 - \cos(2*\phi_s))/(4*\pi); \\
b(1) &= (2*\phi_s - \sin(2*\phi_s))/(4*\pi); \\
a(2) &= (\cos(\phi_s) - \cos(3*\phi_s))/3 - 2.0/3.0/(2.0*\pi); \\
b(2) &= (\sin(\phi_s) - \sin(3*\phi_s))/3.0/(2.0*\pi); \\
aw(0) &= \phi_s/(2*\pi); \\
aw(1) &= \sin(\phi_s)/\pi; \\
bw(1) &= (1 - \cos(\phi_s))/\pi; \\
aw(2) &= \sin(2*\phi_s)/(2*\pi) + 1; \\
bw(2) &= (1 - \cos(2*\phi_s))/(2*\pi);
\end{align*} \]

if (z==1)

\[ \begin{align*}
t(1) &= a(1); \\
t(2) &= a(2); \\
tw(1) &= aw(1); \\
tw(2) &= aw(2); \\
bw(1) &= bw(1); \\
bw(2) &= bw(2); 
\end{align*} \]

else

\[ \begin{align*}
t(1) &= \phi; \\
tw(1) &= tw(1); \\
bw(1) &= bw(1); \\
\end{align*} \]

if (z==2)

\[ \begin{align*}
t(2) &= a(2); \\
tw(2) &= aw(2); \\
bw(2) &= bw(2); 
\end{align*} \]

else

\[ \begin{align*}
t(1) &= a(1) + r*b(1)/2.0; \\
tw(1) &= r*a(1) - b(1)/2.0; \\
bw(1) &= (r*a(1) + b(1))/2.0; \\
aw(0) &= a(0) + (aw(2) + r*bw(2))/2.0; \\
aw(1) &= a(0) + (aw(2) + r*bw(2))/2.0; \\
bw(1) &= r*bw(1) + bw(2)/2.0; \\
awx(1) &= a(0) + (awx(2) + r*bwx(2))/2.0; \\
bwy(1) &= (awx(1) + a(0))/2.0; \\
magnitude_x &= C2*sqrt(ax(1)*ax(1) + bx(1)*bx(1)); \\
magnitude_y &= C2*sqrt(awx(1)*awx(1) + bwx(1)*bwx(1)); \\
magnitude wx &= C2*sqrt(ax(1)*ax(1) + bx(1)*bx(1)); \\
magnitude wy &= C2*sqrt(awx(1)*awx(1) + bwx(1)*bwx(1)); \\
fprintf(f3, "X %lf Y %lf %lf %lf %d
", magnitude_x, magnitude_y, magnitude wx, magnitude wy, 1); \\
else

\[ \begin{align*}
a(1) &= 0.0; \\
aw(0) &= 0.0; \\
awx(1) &= 0.0; \\
bwy(1) &= 0.0; \\
\end{align*} \]

for (dphi=0; dphi<=360; dphi++)

\[ \begin{align*}
phi &= (\text{double})dphi*\pi/180.0; \\
sumT &= a(0) + aw(1)*\cos(phi) + bw(1)*\sin(phi); \\
SUMX &= ax(0) + ax(1)*\cos(phi) + bx(1)*\sin(phi); \\
SUMY &= ay(0) + ay(1)*\cos(phi) + by(1)*\sin(phi); \\
SUMW &= aw(0) + aw(1)*\cos(phi) + bw(1)*\sin(phi); \\
SUMWX &= ax(0) + awx(1)*\cos(phi) + bx(1)*\sin(phi); \\
SUMWY &= awx(0) + awx(1)*\cos(phi) + by(1)*\sin(phi); \\
\text{temp1} &= a(1)*aw(1) + bw(1)*bw(1); \\
\text{temp2} &= aw(1)*aw(1) + bw(1)*bw(1); \\
\end{align*} \]

for (j=2; j<arr-1; j++)

\[ \begin{align*}
n &= (\text{double}); \\
\end{align*} \]
Appendix A. Listing of F.S. Modelling Program for Steady State Face Milling

\[ Cx_1 = (n-1)*\phi_s; \]
\[ Cx_2 = (n-1)*\phi_s; \]
while (\( Cx_1 \geq 2*\pi \)) \( Cx_1 = Cx_1 - 2*\pi; \)
while (\( Cx_2 \geq 2*\pi \)) \( Cx_2 = Cx_2 - 2*\pi; \)
\[ d[j] = ((\cos(Cx_1))/(n-1) - (\cos(Cx_2))/(n+1)) - (1/(n-1) + (1/(n+1)))/(2*\pi); \]
\[ b[j] = ((\sin(Cx_1))/(n-1) - (\sin(Cx_2))/(n+1))/(2*\pi); \]
\[ PHI_S = n*\phi_s; \]
while (\( PHI_S \geq 2*\pi \)) \( PHI_S = PHI_S - 2*\pi; \)
\[ aw[j] = \sin(PHI_S)/(PI*n); \]
\[ bw[j] = (1 - \cos(PHI_S))/(PI*n); \]
if (\( j \geq z \))
  \[ a[j] = a[j]; \]
  \[ b[j] = b[j]; \]
  \[ aw[j] = aw[j]; \]
  \[ bw[j] = bw[j]; \]
else
  \[ a[j] = 0.0; \]
  \[ b[j] = 0.0; \]
  \[ aw[j] = 0.0; \]
  \[ bw[j] = 0.0; \]
\[ temp1 = a[j]*a[j] + b[j]*b[j]; \]
\[ temp2 = a[j]*a[j] + b[j]*b[j]; \]
\[ PHI = n*\phi_s; \]
\[ SUMX = (ax[j]*\cos(PHI) + (bx[j]*\sin(PHI)) + SUMX; \]
\[ SUMY = (ay[j]*\cos(PHI) + (by[j]*\sin(PHI)) + SUMY; \]
\[ SUMWX = (awx[j]*\cos(PHI) + (bwx[j]*\sin(PHI)) + SUMWX; \]
\[ SUMWY = (awy[j]*\cos(PHI) + (bwy[j]*\sin(PHI)) + SUMWY; \]
else
  \[ ax[j] = 0.0; \]
  \[ bx[j] = 0.0; \]
  \[ ay[j] = 0.0; \]
  \[ by[j] = 0.0; \]
  \[ awx[j] = 0.0; \]
  \[ bwx[j] = 0.0; \]
  \[ awy[j] = 0.0; \]
  \[ bwy[j] = 0.0; \]
\[ PHI = n*\phi_s; \]
while (\( PHI \geq 2*\pi \)) \( PHI = PHI - 2*\pi; \)
\[ SUMX = (ax[j]*\cos(PHI)) + (bx[j]*\sin(PHI)) + SUMX; \]
\[ SUMY = (ay[j]*\cos(PHI)) + (by[j]*\sin(PHI)) + SUMY; \]
\[ SUMWX = (awx[j]*\cos(PHI)) + (bwx[j]*\sin(PHI)) + SUMWX; \]
\[ SUMWY = (awy[j]*\cos(PHI)) + (bwy[j]*\sin(PHI)) + SUMWY; \]
*/if (dphi <= 10)
  \[ dimax = C1*ax[1] + C2*awx[1]; \]
  \[ dimay = C1*ay[1] + C2*awy[1]; \]
  \[ dimbx = C1*bx[1] + C2*bwx[1]; \]
Appendix A. Listing of F.S. Modelling Program for Steady State Face Milling

dimy = C1*by[1]+C2*bwy[1];
fund = pow(pow(dimax,2) + pow(dimay,2) + pow(dimbx,2) + pow(dimby,2),2);
ratio = (fund/(pow(C1*ax[0]+C2*awy[0],2)+pow(C1*ay[0]+C2*awy[0],2)));
dev = (dimax-dimay-dimbx)/sqrt(fund);
printf("%lf %lf %lf %lf
",fund,ratio,dev);
fprintf(f4,"%lf %lf %lf %lf\n",fund,ratio,dev);
}
/*
t = 1;
wear_torque = C2*R*SUMW;
Torque = C1*R*sumT;
F_x = C1*SUMX;
F_y = -C1*SUMY;
F_wx = C2*SUMW;
F_wy = -C2*SUMY;
fprintf(f2,"%lf %lf %lf\n",Torque,wear_torque);
fprintf(f1,"%lf %lf %lf %lf %lf %lf\n",F_x,F_y,F_wx,F_wy);
*/
fclose(f1);
close(f2);
close(f3);
close(f4);
Appendix B

Approximate Method of Finding the First Three F.S. Coefficients

Given the function, \( f(x) \), it is possible to find its Fourier Series expansion coefficients, \( a_0, a_n, b_n \) after integrating \( \int f(x)dx, \int f(x) \cos nx dx, \int f(x) \sin nx dx \) respectively. If, however only a limited number of values for \( f(x) \) is known the above method cannot be used.

The method explained below is suitable for experimental situations where only the first three values of the Fourier Series coefficients are needed. The insight to the method becomes clear when it is assumed that the Fourier Series expansion of \( f(x) \) contains no terms beyond the third harmonics. So the Fourier Series expansion of \( f(x) \) can be written as:

\[
f(x) = a_0 + a_1 \cos(x) + b_1 \sin(x) + a_2 \cos(2x) + b_2 \sin(2x) + a_3 \cos(3x) + b_3 \sin(3x) \tag{B.1}
\]

Next, it is assumed that 12 values of \( f(x) \), \( y_0, y_1, y_2, \ldots, y_{11} \) are available experimentally at intervals of 30° (for one period of the function). Now taking the mean values for the 12 given values of \( f(x) \), it is possible to obtain \( a_0 \) as:

\[
a_0 = \frac{1}{12}(y_0 + y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8 + y_9 + y_{10} + y_{11}) \tag{B.2}
\]

Similarly, \( a_3 \) and \( b_3 \) can be obtained by adding the influence of \( a_3 \) and \( b_3 \) terms at each 30° interval. Hence,

\[
a_3 = \frac{1}{6} \sum y \cos 3x = \frac{1}{6}(y_0 - y_2 + y_4 - y_6 + y_8 - y_{10}) \tag{B.3}
\]

\[
b_3 = \frac{1}{6} \sum y \sin 3x = \frac{1}{6}(y_1 - y_3 + y_5 - y_7 + y_9 - y_{11}) \tag{B.4}
\]
Appendix B. Approximate Method of Finding the First Three F.S. Coefficients

Now putting $x = 0, 180, 90, 270$ into equation (B.1), it is possible to obtain $a_1, b_1, a_2$:

\[
\begin{align*}
    a_1 &= \frac{1}{2}(y_0 - y_6) - a_3 \quad \text{(B.5)} \\
    b_1 &= \frac{1}{2}(y_3 - y_9) + b_3 \quad \text{(B.6)} \\
    a_2 &= \frac{1}{4}(y_0 - y_3 + y_6 - y_9) \quad \text{(B.7)}
\end{align*}
\]

The value of $b_2$ remains to be found. The method to obtain $b_2$ involves obtaining the values of $y$ corresponding to $x = 45, 135, 225, 315$ and call these values of $y$'s $\gamma_1, \gamma_2, \gamma_3, \gamma_4$. Choosing the eight equidistant ordinates corresponding to $x = 0, 45, 90, ..., 315$ it is possible to write $b_2$ as:

\[
b_2 = \frac{2}{8} \sum y \sin 2x = \frac{1}{4}(\gamma_1 - \gamma_2 + \gamma_3 - \gamma_4) \quad \text{(B.8)}
\]
Appendix C

Workpiece Dimensions

All Dimensions in Inches.

$\frac{5}{8}$ DIA, $\frac{7}{16}$ DIA BORE

4.90

5.90

1.19

4.50

1.66
Appendix C. Workpiece Dimensions

\[
\begin{align*}
\frac{3}{4} \text{ DIA, } \frac{1}{2} \text{ DIA BORE} \\
3.10 & \quad 0.45 \\
1.29 & \\
3.86 & \\
1.35 &
\end{align*}
\]

All Dimensions in Inches.
Bibliography


16th NAMRC, May 1988, pp. 245–255.


