AN INVESTIGATION OF HIGH SPEED, THIN STEEL ROTOR, ANNULAR, DOUBLE SIDED, LINEAR INDUCTION MOTORS

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Abstract

The objective of this dissertation is to analyse the performance of a linear induction motor suitable to drive a circular saw blade. A selection of analytical methods available from the field of electrical machine theory was used to investigate the particular type of motor. The theoretical analysis is supported by an extensive experimental investigation.

Although LIMs have been designed, analyzed and applied in other applications, significant differences exist between those LIMs and the one used for the new application. These include: the annular shaped motor, the smaller air gap, and the rotor which is thin and made of steel. Because of these differences, the methods used by previous investigators were not sufficient to design the LIM required.

The theoretical analysis used a selection of methods described in the literature to quantify the effect of the rotor material, the end effect and the edge effect. New methods are described to analyse the effect of the annular shape, the normal forces on the rotor and the coil connection. In addition, a new consideration in the optimisation of these type of motors is described.
An extensive experimental program was undertaken. Six different linear motors were constructed with output powers ranging from one to fifty kWatts. In addition, inverters, dynamometers, flux measurement apparatus, speed measurement, thrust measurement and friction measurement apparatus were designed and constructed. The effects on performance of slot harmonics, winding connections, the end effect and the edge effect were measured.

Several contributions to the field of electrical machine theory are presented. The first is a new annular disc motor resistivity correction factor. Second, is the analysis of the effects of poles in parallel versus in series in linear induction motors. Third, is the experimental comparison between odd and even pole designs. The fourth is a second optimum goodness consideration for LIMs, which had not previously been considered. The fifth is the analysis of the rotor/stator attractive force for magnetic rotor double sided motors and a description of the flux (crenelated flux) which causes the force. Finally, a criterion for when the re-entry effect may occur is presented.
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NOMENCLATURE

A = stator surface area
b = air gap flux
\(b_p = -\frac{iB_k k^2}{k^2 + is\omega}\)
\(b_1 = \frac{[\omega + k \tan \phi (1 - iv/k)/(r_1 - v - V\alpha)]b_p}{\mu_0 J/\mu_k}\)
\(c = \text{rotor thickness}\)
\(F_0 = \text{thrust produce by a motor without end effects}\)
\(F_{1n} = \text{thrust due to the end effect}\)
\(F_{r1} = \text{normal force on rotor due to stator 1}\)
\(F_{rs} = \text{normal force on rotor due to stator 2}\)
\(g = \text{air gap between the stator and rotor}\)
i = square root of -1
\(I = \text{current in the overhang regions of the rotor}\)
\(j = \text{rotor current current (A/m)}\)
\(J = \text{stator surface current density (A/m)}\)
k = \(2\pi/\eta\)
\(K = \frac{[1 - \exp((r_1 + ik)\eta)]}{[r_1 + ik]}\)
l = stator width
\(l_r = \text{width of rotor}\)
\(l_s = \text{stator width}\)
L = stator length
\(P = \text{number of stator pole pairs}\)
r = radial distance in cylindrical co-ordinates
\(r_1 = \frac{V\alpha/2[1 - 1 + 4(iw\alpha + v^2)/(\eta \omega^2)]}{[r_1 + ik]}\)
\(R_1 = \text{inner radius of the stator}\)
\(R_2 = \text{outer radius of the stator}\)
s = slip
\(S_{p1} = \text{distance on stator of one tooth pitch}\)
\(S_{p2} = \text{distance on stator to next adjacent stator tooth}\)
\(S_r = c(l - l)/2\)
t = stator tooth width
\(v = 4/\text{stator width/total rotor overhang}\)
\(V = \text{velocity}\)
w = stator frequency
\(x = \text{distance from entry end of stator}\)
y = distance from center line of stator
\(\alpha = \frac{\mu_0 c}{(p^* g)}\)
\(\beta = \frac{\mu_0}{\Omega g}\)
\(\gamma = c(l - l)/2\)
\(\delta = R_2/R_1\)
\[ \varepsilon = \frac{1}{1 + \frac{1}{\sqrt{1 + isG}} \tanh \beta a \tanh k(c - a)} \]

\( \theta \) = angular location on motor in cylindrical co-ordinates
\( \Theta \) = total angular section of motor in cylindrical co-ordinates
\( \eta \) = two pole pitches (one wavelength)
\( \mu_0 \) = permeability of free space
\( \pi \) = 3.1415
\( \rho \) = rotor resistivity
\( \sigma \) = magnetic reluctance
\( \tau \) = saturation flux density of rotor/peak flux in the stator
\( \phi \) = \( \tan^{-1}(sw\alpha/(k^2 + v^2)) \)
\( \Phi \) = magnetic flux
\( \Phi_c \) = crenelated flux (tooth-rotor-tooth leakage flux)
\( \omega \) = frequency in radians per second
\( \Omega \) = volume resistivity or ohm

Mathematical Operators

+ = addition
- = subtraction
* = multiplication (used if equation would be ambiguous)
/ = division
exp = the natural exponential
ln = the natural logarithm
db/dx = differential operator
d'b/d'x = partial differential operator
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1. INTRODUCTION

1.1 Objective of Thesis

A new type of circular saw is being developed which uses a linear induction motor (LIM) to drive the blade. Although LIMs have been designed, analyzed and applied in other applications, significant differences exist between those LIMs and the one used for the new application. These include: the annular shaped motor, the smaller air gap, and the rotor which is thin and made of steel. Because of these differences, the methods used by previous investigators were not sufficient to design the LIM required. An experimental investigation of this unusual type of LIM was undertaken aided by the use of various simulation and analysis methods. Specifically, in order to meet this objective, the following were undertaken:

1. Theoretical analysis was developed, one aspect of which involved applying conventional theories to a steady state electromagnetic model. The resulting equations were then set up for annular motors by using cylindrical coordinates and a resistivity correction factor. As a tool in developing this theoretical analysis, a spreadsheet program for simulating the performance of LIMs was developed. This spreadsheet method proved to have significant advantages over previous design methods, such as conventional Fortran simulation methods.

2. The normal forces between the rotor and the stator in this
type of machine were modelled and an experimental investigation undertaken in order to determine the factors affecting their magnitude.

3. Simulations and experiments were undertaken in order to analyze the motor parameters which affect efficiency, power-factor and power density since the maximizing of these factors is an important objective in the final design. The simulations and experiments consisted of using various rotor resistances, air gaps, frequencies and flux densities in machines with six different stators (These differed in terms of stator size, number of poles, type of connection and windings). These experiments required the construction of a variable frequency three-phase inverter, the various experimental machines and the measurement apparatus.

1.2 Application of Thesis Results

The Linear Induction Motor (LIM) has found applications in transportation and material handling. New applications now exist in sawmill and mining equipment for double sided annular induction motors with thin steel rotors.

One new application for LIMs which is being considered involves using a double-sided LIM to drive circular saw blades in wood cutting mills. The advantage of such a machine, the prototype of which is shown in Fig. 1.1, is that a thinner, straighter cut can
be made which in turn reduces wastage. Conventional large circular saws require a thick plate of 4-5 mm in order to support the thermal and mechanical stresses without the blade distorting. The proposed LIM driven saw has a design goal of a 2 mm blade thickness. Band saws, which use a comparable thickness of blade, are presently used in sawmills but they are very large and costly machines and expensive to maintain. An additional advantage of

Fig. 1.1 - Prototype LIM driven Saw
the proposed LIM driven saw is that it has a higher blade speed than present day band saws.

The objectives for the LIM are:
1) that it produce 50 kW at 140 m/s and an overload power of 70 kW at 110 m/s.
2) the power-factor/efficiency product should be above .25.
3) the motor should be no longer than 1 m and no wider than .3 m.

The primary application for the type of motor being investigated in this thesis is to drive circular saw blades in wood cutting mills. Other applications may exist in mining equipment to drive rock crushers or to drive sonic vibrators which enhance metal recovery in chemical reaction leaching tanks.

1.3 **Historical Background**

One of the first patents for a linear motor was filed by Page [1] in 1854 to use magnets to produce linear motion similar to that produced by a linear synchronous motor. The Electric Shuttle Company applied for a patent [2] in 1859 for a linear motor supplied by a mechanical inverter which energized sequential magnets. The first linear induction motor was most likely conceived during the time when Galilao Ferraris and Nicola Tesla first demonstrated working models of rotary induction motors.
during 1885. The first patent which closely describes the linear induction motor of today was given to Zehden [3] in 1902 for an electric traction system using a short primary and a long secondary, which is the configuration now used in most commercial applications.

Although patents were filed, no significant applications of LIMs were attempted until 1945 when Westinghouse Electric developed a catapult launcher for use on aircraft carriers [4]. The LIM developed a peak power of 7,000 kW and was successful at launching planes. However, the cost of the system made it impractical. Linear motors have also found applications in liquid metal pumps for atomic reactors [5] and some attempts have been made to use them in MHD generators [6].

The most active area for recent applications is in ground transportation (the automated light rapid transit system such as that used in Vancouver, Canada). When using a LIM the adhesion between the wheel and rail is not of concern at lower speeds and at higher speeds, above 300 km/h, where wheels cannot be used, then a LIM is the only electrical method of producing thrust (the Japanese HSST). Of all the possible LIM applications the Vancouver system is probably the greatest commercial success.

The idea of using a LIM in certain applications occurred almost simultaneously with the use of the rotary induction motor. An
obvious disadvantage of the LIM, when compared to the rotary induction motor, is its open configuration which is the opposite of what is required for a good electric motor. It is important that an electric motor has a "tight" structure so that the electric and magnetic circuits are tightly coupled and thus the magnetic flux does not leak. Flux leakage results in motors with poorer efficiency, power-factor and power density.

An apparent advantage of the Linear Induction Motor is that it does not require any gears, belts, pulleys or wheels to produce linear motion. However, these components are very durable and usually of negligible cost, so that their elimination alone does not justify the application of a Linear Induction Motor. This is the reason why so many attempts at using LIMs have failed to be a commercial success. The real advantage of a LIM is in applications which require high speed or where no mechanical slip in the drive system can be tolerated.

Although linear induction motors have been available since before 1905, they have not been extensively investigated until the last 40 years. It was only recently discovered [7] that LIMs behave differently from rotary motors at high speeds and require more complex analysis than conventional rotary motors. It was originally assumed that a LIM could be analyzed in the same way as a rotary motor. However, when high speed experiments were conducted, it was found that there was an
important difference - the LIM was less efficient and produced less power than

Fig. 1.2 - Entry and Exit Effect

expected. A LIM behaves differently from a rotary motor at high speed due to the entry and exit effects. The entry effect is caused by new rotor material entering the stator magnetic field
and disturbing it (see Fig. 1.2). As the magnetic field in the rotor cannot change instantaneously (because energy must be transferred) the rotor in the entry area does not produce significant thrust. At the exit area a disturbing field is created by the exiting rotor material which also decreases the thrust but this effect is generally insignificant.

The performance of the LIM was found to be strongly dependent on the resistance of the rotor. The optimum resistance for the rotor of a high speed LIM had not been investigated until recent work by Poloujadoff [8]. His results show that a higher resistance rotor may actually increase the efficiency of a high speed LIM. This is essentially the same conclusion previously reached by Nasar and Boldea [9]. They used the "goodness factor" as defined by Laithwaite [10] and determined an "optimum goodness factor" by finding an optimum resistance.

1.4 Summary

Various areas must be investigated during the design of such an unusual type of double sided LIM. The high blade speed (140 m/s) makes end effects important. Also, because the blade is made of steel, the attractive forces may be large and, since the rotor is only 2 or 3 mm thick, the blade must be well supported to withstand these forces. The possibility of a re-entry effect, which occurs when currents are still flowing in the rotor when it enters the stator, thereby affecting the performance, must also be
considered. In addition, the LIM must have good power density in order to keep the saw small. Moreover, a high power-factor efficiency product is essential since the motor is powered by an expensive inverter. Finally, the annular stator will not produce exactly the same performance as a conventional LIM of the same mean length. The amount of variation which will occur between the straight and annular LIM should be calculated if accurate motors designs are to be undertaken.

The above mentioned areas of analysis and design are set forth in the following chapters of the Thesis. Chapter 2 gives a description of LIMs, the basic theory, and a new analysis technique for annular LIMs. The analysis of the normal forces, which either attract or repel the rotor to the stator, is detailed in Chapter 3. The experimental apparatus is described in Chapter 4. The experimental results, which are a major component of this thesis, are presented in Chapter 5. Based on the experimental results from Chapter 5 and the analysis given in Chapter 2, optimization criteria and considerations are derived in Chapter 6. Chapter 7 reviews the results of the research and highlights the original work in the field of motor design presented in this thesis.
2. GENERAL FORMULATION

2.1 Description of LIMs

The linear induction motor can be considered as a rotary induction motor which has been cut and rolled out as shown in Fig. 2.1 [11,12]. The possible configurations are short stator or short rotor and single-sided or double-sided machines (The annular LIM to be analyzed is double-sided, short stator.). The stator is wound with sets of three-phase coils arranged so that a series of north and south magnetic poles sweep the stator and induce a current in the rotor. The thrust is produced by the interaction of the stator magnetic field and the rotor induced currents.

Fig. 2.1 - Motor rolled out to form a LIM
In addition to an electromagnetic thrust in the x-direction (see Fig. 1.2) there is an electromagnetic force in the y-direction, and if the rotor is made of magnetic material, a magnetic attractive force in the z-direction. If the LIM is double-sided and the rotor is centered in the air-gap the normal forces (z-direction) will be zero.

2.2 The Entry Effect

The performance of LIMs is influenced by the entry effect, which causes a parasitic drag on the rotor. This effect occurs due to new rotor material continuously entering the stator area. The new rotor material has no current flowing in it and as a result of the finite time required to establish the flux linkages of the rotor, the current will not instantaneously begin to flow as the rotor material enters the stator. This results in a reduction of the motor thrust at the entry area because the rotor current that begins to flow at the stator entry area is in neither the right phase, amplitude nor shape to produce large amounts of thrust. In fact the rotor currents which do initially form can produce negative rather than positive thrust [7]. These entry effect currents are shown in Fig. 2.2.

2.3 Review of Analysis Techniques

The development of a computer model for the annular LIM may be approached in a number of ways. Two methods are the simple
equivalent circuit model [13,14] and the mesh matrix equations model [15,16]. The most common method is the use of the field theory model [17,18,19,20]. Each of these is discussed below.

A simple equivalent circuit can be developed using the motor test parameters. Recently Duncan [14] obtained a reasonably accurate equivalent circuit model for performance modelling. The advantages of the equivalent circuit model are that it is mathematically
simple, requires no knowledge of the internal design of the LIM, and provides the motor terminal conditions (see Fig. 2.3). The disadvantage of this model is that it does not predict the internal conditions, such as the stator core flux density, of the motor and is thus less useful as a design tool.

The mesh matrix equations model the motor as sets of coils on the stator and on the rotor. The relationships between the coils are obtained by analyzing the structure of the machine, and the coil parameters are obtained by motor tests. This form of analysis is called "mesh matrix" as the mesh equations are manipulated in matrix form (see Fig. 2.4 and 2.5). This type of model is useful for transient studies as the input variable is either the current or voltage at each time step in the simulation, and so transient events are handled in the same manner as steady-state operation. The disadvantage of this model is that the entry effect found in LIMs is difficult to simulate.

The advantage of the field theory model is that it provides a good picture of the magnetic fields and current densities within the motor and thus provides the best model to use when designing new motors. The field theory model is also the only model which can be adapted to take into account the variation in current density and pole pitch which will occur over the face of the annular motor. There have been a number of theses written on field analysis [21-25], but to date none have modeled the annular LIM. The model proposed in this thesis will do this.
Fig. 2.3 - Equivalent Circuit Model

Fig. 2.4 - Mesh Matrix Derivation
**Fig. 25 - Mesh Matrix Equations**

<table>
<thead>
<tr>
<th>(v_a)</th>
<th>(v_b)</th>
<th>(v_c)</th>
<th>(i_a)</th>
<th>(i_b)</th>
<th>(i_c)</th>
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<td>(\text{PLSM})</td>
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<td>(\text{pMSRcos}(\Theta+\frac{2\pi}{3}))</td>
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<td>(\text{PLRM})</td>
<td>(\text{PLRM})</td>
<td>(\text{RR+pLRR})</td>
</tr>
</tbody>
</table>

**LSS** - per phase stator self inductance

**LSM** - per phase mutual inductance between stator windings

**LRR** - equivalent per phase rotor self inductance

**LRM** - equivalent per phase rotor mutual inductance

**MSR** - maximum value of inductance between rotor and stator winding

**RS** - per phase stator resistance

**RR** - equivalent per phase rotor resistance
2.4 One-Dimensional Field Theory Equations

Field theory models simulate the induction motor by modelling the stator as a block of material of negligible magnetic reluctance with an infinitely thin surface current, and the rotor as a thin plate with a specified volume resistivity.

Other assumptions made about the motor in most field theory models are:

- the slotted air gap is replaced by an unslotted one of greater length which is determined either by the Carter coefficient or by finite element analysis and does not include the thickness of the rotor if it is magnetic
- the stator windings are represented as surface currents
- the stator iron has negligible conductivity

Fig. 2.6 - Motor representation
The equations which describe the electromagnetic conditions of the LIM will now be solved to obtain the expression for the thrust of the motor.

The LIM is divided into areas as shown in Fig. 2.6. In one-dimensional analysis it is assumed that the EMF induced in the secondary by the primary exists only directly under the stator \((y \leq l_y/2)\) and is normal to the x-axis. Also, the currents directly under the stator flow only in the y-direction. In this model the current and flux densities are functions of the x component only. The following three equations describe these conditions. (all variables are steady-state values)

\[
- \frac{db}{dx} = \mu_0 \frac{(c * j + J) - \frac{g}{g}}{g} \quad (2.1)
\]

- the change in flux along the motor is equal to the current density flowing in the stator and rotor

\[
\frac{dI}{dx} = c*j \quad (2.2)
\]

- the change in current flowing along the return paths of the rotor is equal to the current density flowing in the rotor

\[
\frac{dj}{dx} - \frac{2*I}{l*S_r} = \frac{i*w*b + V*db}{p} \quad (2.3)
\]
the change in current density along the rotor minus the effect of the resistance is equal to the transformer and the speed induced voltage divided by the rotor resistivity.

When equations (2.1), (2.2) and (2.3) are combined, a third order differential equation (2.14) is obtained which can be solved from the boundary conditions and the given surface current density. (see Appendix 1)

\[
\frac{\mathrm{d}b}{\mathrm{d}x} = \frac{\mu_0 c^* j^* J}{g} \quad (2.1)
\]

\[
\frac{\mathrm{d}I}{\mathrm{d}x} = c^* j \quad (2.2)
\]

\[
\frac{\mathrm{d}j}{\mathrm{d}x} - \frac{2^* I}{l^* S_\tau} = \frac{i^* w^* b + V^* \frac{\mathrm{d}b}{\mathrm{d}x}}{p} \quad (2.3)
\]

\[
\frac{\mathrm{d}^3 b}{\mathrm{d}x^3} = \frac{V}{p^2 g} \left[ \frac{i^* w^* c}{p} + \frac{2}{l^* S_\tau} \right]^2 \frac{\mathrm{d}b}{\mathrm{d}x} \quad (2.14)
\]

\[
= - \frac{\mu_0}{g} \left[ \frac{\mathrm{d}^2 J}{\mathrm{d}x^2} \left( \frac{2}{l^* S_\tau} \right)^2 J \right]
\]

Equation (2.14) is then solved to obtain the magnetic field value [29].

The thrust can then be calculated from the stator surface current
and the rotor magnetic field. The thrust developed by the motor is separated into two components. $F_0$ is the thrust described by standard induction motor theory and $F_1$ is the thrust produced by the parasitic end effects found in linear induction motors as given by Poloujadoff [29].

$$F_0 = \frac{1}{2} J_m^* B_0^* l^* p^* L^* \cos \phi^* \sin \phi$$  \hspace{1cm} (2.15)

$$F_1 = \frac{1}{2} J_m^* B_0^* l^* \text{Real}[(b_1/b_p^*)^*(b_p/B_0^*)^*K]$$  \hspace{1cm} (2.16)

\begin{align*}
b_1 &= [V^* \alpha + k^* \tan \phi (1-i^* v/k)/(r_1-v-V^* \alpha)]^* b_p \\
B_0 &= \mu_0^* J/(k^* g) \\
b_p &= -i^* B_0^* k^2/(k^2+i^* s^* w^* \alpha) \\
i &= \sqrt{-1} \\
r_1 &= V^* \alpha^2/[1-[1+4^*(i^* w^* \alpha^2 V^*/(V^* \alpha)^2]}
\end{align*}

$V$ = velocity  
$v$ = $4$/stator width/total rotor overhang  
$\alpha$ = $\mu_0^* c/p^* g$  
$p$ = rotor resistivity  
$K$ = $(1-\exp[(r_1+i^* k)\eta^*])/[r_1+i^* k]$  
$\phi$ = $\tan^{-1}(s^* w^* \alpha^2/k^2+v^2)$  
$L$ = length of stator  
$J_m^*$ = maximum primary current density  
$l$ = width of stator  
$\eta$ = wavelength  
$k$ = $2^* \pi^* \eta$
The total thrust is $F_0$ plus $F_1$. For rotary induction motors the value of $F_1$ is zero.

2.5 Effect of Annular Stator

Annular stators have been used to drive disc rotors for applications in linear induction motor test apparatus [26,27] and commercial machines [11]. These machines have curvature effects which have in the past been assumed to be negligible in simulation models. However, no analysis has yet been done to show when this is true or how the effects of curvature should be accounted for. One author [28] does analyze the effect of driving a disc with a straight LIM but he does not consider the curved LIM situation.

![Figure 2.7 - Curvature of the annular LIM stator.](image)
Curvature of an LIM (see Fig. 2.7) creates three factors which affect the performance. The first is the varying effective surface current density and the second is the varying pole pitch. The third is the increase in the effective resistance of the stator due to the wedge shaped current path. The first two of these three factors will now be analyzed.

To analyze the annular LIM, the equations described in section 2.4 were set up using cylindrical co-ordinates and an annular resistivity correction factor was developed. The notation for these equations is shown in Fig. 2.7.

First write out equations (2.1), (2.2) and (2.3) in cylindrical co-ordinates.

\[-\frac{db}{rd\theta} = \mu_0/g*(c*j+J) \quad (2.17)\]

\[\frac{dI}{rd\theta} = c*j \quad (2.18)\]

\[\frac{dj}{rd\theta}-(2/(8*S_r))*I \quad (2.19)\]

\[= -((i*\omega*b)+((r*d\theta/dt)*db/rd\theta))/p\]
Then write out the terms of equation (2.19)

\[
dj/rd\theta - 2/(\delta*S_r) \cdot I = -i*w*b/p - (r*d\theta/dt)*p*db/rd\theta
\]  

(2.20)

Taking the first derivative of (2.20) with respect to \( \theta \)

\[
d^2j/rd\theta^2 - 2/(\delta*S_r) \cdot dI/rd\theta = -i/p*w*db/rd\theta - (r*d\theta/dt)/p*d^2b/rd\theta^2
\]

(2.21)

Now taking the first derivative of equation (2.17)

\[
-d^2b/rd\theta^2 = \mu_o*c/g * dj/rd\theta + \mu_o/g*d\theta J/rd\theta
\]

(2.22)

Taking the first derivative with respect to \( \theta \)

\[
-d^3b/rd\theta^3 = \mu_o*c/g*d^2j/rd\theta^2 + \mu_o/g*d^2J/rd\theta^2
\]

(2.23)

From (2.23) obtain an expression for \( d^2j/rd\theta^2 \)

\[
d^2j/rd\theta^2 = -g/\mu_o/c*d^3b/rd\theta^3 - 1/c*d^2J/rd\theta^2
\]

(2.24)
and obtain an expression for \( j \) by rearranging (2.17)

\[
j = -\frac{g}{u_0}c db/rd \theta - \frac{J}{c}
\]

(2.25)

Substitute the expressions (2.18) and (2.24) into equation (2.21)

\[
d^2j/rd \theta^2 - \frac{2}{(\delta*S_r)} dI/rd \theta = -1/p*i*w*db/rd \theta - (r*d \theta/dt)/p*d^2b/rd \theta^2
\]

(2.26)

\[
\frac{-g}{\mu_o/c*J} - \frac{1}{c*d^2J/rd \theta^2} - \frac{2}{(\delta*S_r)} c \cdot j = -1/p*i*w*db/rd \theta - (r*d \theta/dt)/p*d^2b/rd \theta^2
\]

(2.27)

Substitute (2.25) into (2.27)

\[
\frac{-g/(\mu_o*c)*d^3b/rd \theta^3 - 1/c*d^2J/rd \theta^2 - 2*c/(\delta*S_r)*}{-g/(\mu_o*c)*db/rd \theta - 1/c*J} = -1/p*i*w*db/rd \theta - (r*d \theta/dt)/p*d^2b/rd \theta^2
\]

(2.28)
Gathering together terms

\[
\begin{align*}
-g/\mu_0/c^3 b/r d\theta^3 & - 1/c^2 J/r d\theta^2 + 2g/(S \mu_0) dB/r d\theta \\
+ 2J/S & = -1/p*i*w*db/r d\theta - (r*d\theta/dt)/p*d^2 b/r d\theta^2
\end{align*}
\]

Rearrange to get:

\[
\begin{align*}
-g/(\mu_0 * c)*d^3 b/r d\theta^3 & + (r*d\theta/dt)/p * d^2 b/r d\theta^2 + (2g/(S \mu_0) + i*w/p) = 1/c^2 J/r d\theta^2 - 2J/(S \mu_0)
\end{align*}
\]

Multiply through by \(-\mu_0 * c/g\) to get:

\[
\begin{align*}
d^3 b/r d\theta^3 & - (r*d\theta/dt)*\mu_0 * c/(g*p)*d^2 b/r d\theta^2 - (2c/(S \mu_0)) \\
+ \mu_0 * c*i*w/(g*p) dB/r d\theta = -\mu_0/g*d^2 J/r d\theta^2 + 2\mu_0 * c*J/S \mu_0 /g
\end{align*}
\]

Equation (2.30) is then solved (using the same general solution
method found in [29] but for cylindrical coordinates) to obtain the magnetic field value. The thrust can then be calculated from the stator surface current and the rotor magnetic field. The torque developed by the motor is separated into two components. $T_0$ is the torque described by standard rotary induction motor theory and $T_1$ is the torque produced by the parasitic end effects found in annular linear induction motors.

$$T_0 = r^2 O J_m B_0 (R_2 - R_1) \cos \phi \sin \phi$$  \hspace{1cm} (2.31)

$$T_1 = -r J_m B_0 (R_2 - R_1) \text{Real}[(b_1/b_p)(b_p/B_0)K]$$  \hspace{1cm} (2.32)

$$B_0 = \mu_0 J/(k*g)$$

$$b_p = -i B_0 k^2 / (k^2 + i s w \alpha)$$

$$b_1 = [(r^2 \theta/\alpha_0 + k \tan \phi (1 - i v/k)]$$

$$(r_1 - v - r^2 \theta/\alpha_0) b_p$$

$$K = [1 - \exp((r_1 + i k) P * \eta)] / (r_1 + i k)$$

$$k = 2 \pi / \eta$$

$$r_1 = (r^2 \theta/\alpha_0) / 2 [1 - (1 + 4 (i w \alpha + v^2) / (\alpha r^2 \theta / \alpha_0)^2)]$$

$$v = 4 / \text{stator width/total rotor overhang}$$

$$\alpha = \mu_0 c / (p g)$$

$$p = \text{rotor resistivity}$$

$$P = \text{number of pole pairs}$$

$$\phi = \tan^{-1} (s w \alpha / (k^2 + v^2))$$

$$\eta = \text{wavelength}$$

$$R_1 = \text{stator inner radius}$$

$$R_2 = \text{stator outer radius}$$

$$\delta = R_2 - R_1$$
The total torque is $T_0$ plus $T_1$. For continuous annular induction motors the value of $T_1$ is zero. The solution of the above equations will now enable the analysis of the first and second factors which affect the performance of the annular LIM.

Over the surface of an annular LIM the current density varies inversely with radius, and as the surface current density increases so does the thrust. The pole pitch varies in proportion to the radius, and similarly the synchronous speed of the machine will also increase. These two effects were analyzed using the above derived analysis for annular LIMs and applied to Experimental Motor #1 (acting about a .25 m radius). The net effect (at 50% slip) was found to be a 25% increase in power and a 12% increase in thrust at the outside of the stator (due to the greater stator length at the outside) compared to the center of the stator. This is shown in Figure 2.8.

The third factor, the increase in rotor resistivity, will now be analyzed. The effective increase in resistance can be calculated by integrating the resistance over the wedge shaped path of the rotor current (see Fig. 2.9). The correction factor to the rotor resistivity obtained is:

$$k_a = \frac{\ln\left(\frac{R_2}{R_1}\right)(R_2+R_1)}{2(R_2-R_1)}$$  \hspace{1cm} (2.33)
In order to illustrate the effect on performance due to the rotor resistivity correction factor for annular motors, three cases will be considered. The first case will be that of a hypothetical disc positioning motor, the second case will be for the Experimental Motor #1 and the third case will be for Experimental Motor.
#2. (Both the Experimental Motors, which have significantly larger mean radii, are described in Chapter 5.)

\[ \int_{R1}^{R2} \frac{p \, dr}{r} = \frac{p \ln \left( \frac{R2}{R1} \right)}{t \, \theta} \]

Resistance of Wedge = \[ \frac{p \ln \left( \frac{R2}{R1} \right)}{t \, \theta} \]

Resistance of Block = \[ \frac{p \, (R2 - R1)}{t \, \theta \, (R2 + R1)/2} \]

\[ K_a = \frac{\text{Resistance of Wedge}}{\text{Resistance of Block}} = \frac{\ln(R2/R1) \cdot (R2+R1)}{(R2-R1)^2} \]

Figure 2.9 - Effect of curvature on rotor resistivity

The hypothetical motor (see Table 2.1) has an outside radius of the stator (R₂) which is four times greater than the inside radius (R₁). The calculation for the annular resistivity correction factor is shown below.

\[ k_a = \frac{\ln(R2/R1) \cdot (R2+R1)}{[2*(R2-R1)]} \]

\[ k_a = \frac{\ln(0.0762/0.0190) \cdot (0.0762+0.0190)}{[2*(0.0762-0.0190)]} \]

\[ k_a = 1.15 \]
MOTOR PARAMETERS

Hypothetical Motor Number: 1

Type: Segmental

Winding Parameters

Number of Poles: 4
Number of Phases: 3
Pole Pitch: 6.19 cm
Pitch Factor: .966
Coil Pitch: 5.16 cm
Coil Span: 1:6
Distribution Factor: .966
Number of Coils: 24
Turns per Coil: 30
Turns in Series/Phase: 60 (4 poles in parallel)
Mean Length of Turn: .254 m
Equivalent Wire Gauge: 18
Winding Connection: Wye
Connection of Stators Together: parallel

Mechanical Parameters

Primary Width: 5.175 cm
Primary Thickness: 3.81 cm
Tooth Width: 5.16 mm
Slot Width: 5.16 mm
Slot Depth: 2.86 cm
Total Primary Length: .314 m
Active Primary Length: .298 m
Number of Stator Slots (single layer): 24
Stator Slots (total): 29
Stator Slots (active): 29
Stator Slots (half-filled): 19
Secondary Thickness: 2.5 mm
Primary-Secondary Gap per side: 1.4 mm
Secondary Overhang per Side: 2.54 cm
Magnetic Gap (total): 2.8 mm
Mean Radius: 4.76 cm

Electrical Parameters

Primary Resistance per Phase: .0569 Ω
Secondary Resistivity: 26 μΩcm
Frequency: 400 Hz
Voltage: 440 V
Current Rating (continuous): 40 A

Table 2-1
The annular rotor resistivity correction factors for Experimental Motor #1 and Experimental Motor #2 were calculated in the same manner and were found to be 1.0045 and 1.0003 respectively. These corrected values of rotor resistivity were used in the simulations shown in Figs. 2.10 to Fig. 2.12 for the three cases. The simulations show that only the Hypothetical Motor #1 shows a significant difference in performance when the annular resistivity correction factor is taken into account. It can be concluded that
the annular resistivity correction factor does not have to be used in order to obtain accurate simulations of the experimental motors that were constructed.

\[ \text{Fig. 2.11 - Annular Resistivity Correction Factor} \]

\[ \text{Applied (---) and Not Applied (—) To Experimental Motor #1.} \]

2.6 Re-Entry Effect

The re-entry effect could occur if the steel rotor still had currents flowing as it re-entered the stator area. In this case interference between the entry and exit effects—could affect the performance of the motor.
Fig. 2.12 - Annular Resistivity Correction Factor

Applied (- - -) and Not Applied (——) To Experimental Motor #2.

One way to determine whether re-entry will occur is to compare the length of the longitudinal entry wave to the distance that the currents would have to continue flowing before entering the stator again. If the entry wave is significantly shorter, then no interference will occur. The length of the longitudinal entry wave
is known and its value will have been calculated during the design and analysis of the LIM. For the Experimental Motor #2 these values were calculated using the analysis of Poloujadoff [29]. The value given for the entrance wave is:

\[
\text{entrance wave} = b_1 \exp(r_1 x)
\]  

(2.34)

where:
\[
b_1 = \frac{V\alpha + k\tan \phi (1 - i*v/k)/(r_1-v-V*\alpha)}{(r_1-v-V*\alpha)} * b_p
\]

\[
B_0 = \mu_0 J/(k*\gamma)
\]

\[
b_p = -i*B_0 * k^2/(k^2 + i*s*w*\alpha)
\]

\[
r_1 = V*\alpha/2*[1+4(i*w*\alpha+v^2/(V*\alpha)^2)]
\]

\[
V = \text{velocity}
\]

\[
v = 4/\text{stator width/total rotor overhang}
\]

\[
\alpha = \mu_0 c/p/\gamma
\]

\[p = \text{rotor resistivity}
\]

\[w = \text{stator electrical frequency}
\]

Using the above formula the distance before the end effect wave attenuates to .368 (1/e) of its peak value was calculated to be .63 m for the Experimental Motor #2 (see Table 5.2 for motor parameters). This is the worst case condition because the entry wave is supported by the steel of the stator which increases the time constant of a perturbation far longer on the rotor while under the stator than outside of the stator. For the Experimental Motor #2, re-entry is clearly not a problem as the distance before the blade will re-enter the stator is approximately 2 meters. If the above
analysis does indicate that the exit wave may enter the stator then a more exact calculation can be performed using the boundary conditions for the rotor outside of the stator.

2.7 Conclusions

The annular motor has been analyzed in this chapter using the electromagnetic field analysis equations in cylindrical co-ordinates. In addition an annular resistivity correction factor was developed and applied to one hypothetical and two experimental motors. It can be concluded that the annular stator will have a measurable effect on performance if the stator width is greater than half its mean radius. For the experimental motors constructed, the annular stator will have an almost negligible effect on performance.

The possibility of a re-entry effect was analyzed and found not to occur for the experimental machines. When one reviews the analysis, it is clear that the only time that a re-entry effect will ever occur (for physically realizable machines) is when two sets of annular stators are used to drive a very high speed rotor (greater than 140 m/s) and the exit point of one stator is very close (within a few centimeters) to the entrance of the other.
3. NORMAL FORCES

3.1 Introduction to Normal Forces

The analysis of the normal forces is important in the design of the steel rotor LIM because a large unbalanced normal force will create large friction losses which result in a low efficiency design. The large friction forces occur between the rotor and those pads which keep the rotor from touching the stator core.

There are two normal forces acting on the rotor. These are: the magnetic attractive forces due to areas in the machine where the reluctance of flux paths can be reduced by movement of the blade ("reluctance normal forces"), and the normal force due to interaction of the magnetic field of the stator and currents in the rotor ("electromagnetic normal forces"). These flux paths are shown in Figure 3.1. There are also others (see Alger p.200 [30]). In addition there is a large attractive force between the two stators which should not be confused with the forces on the rotor.

It is interesting first to compare the magnitude of the forces acting on the stators and rotor. The attractive force acting on the two stators due to the main flux is given by [31,32]:

\[ F_s = B^2 A / (2 \mu_0) \]  

(3.1)
where $A$ is the stator surface area and $B$ is the stator magnetic field. For the Experimental Machine #2 (parameters for this motor are found in Table 5.2), this was calculated to be 44.5 kN. The forces acting on the rotor will be shown to be approximately one to two orders of magnitude less.

The reluctance normal forces will be analyzed by first looking at the flux paths and saturation conditions. This analysis gives the peak attractive force (for the worst case of the rotor positioned against one stator), calculated from equations (3.2) and (3.3). Then the modulation of the peak attractive force, in space, along the

![Fig. 3.1 - Magnetic flux paths](image)
stator is analyzed and the result is given in equation (3.4). Once these values are determined, the relationship between the maximum value of attraction and the rotor position is analyzed and the resulting relationship for the experimental motor is given in equation (3.5). Finally the maximum normal force due to the rotor currents (electromagnetic normal force) is given in equation (3.6) and shown to be small in relation to the attractive force.

Following this, the analysis is first confirmed in the experimental results section, and then applied to obtain an expected value for drag during the full load operation of the motor.

3.2 Saturation Conditions

The force of attraction occurs when the rotor moves from the center of the air-gap, resulting in greater attractive force being exerted by one stator than the other. Once the rotor is far from the center position in the air-gap the attractive force can be determined by looking at the flux path through the rotor and determining the total maximum flux which can take this path before the rotor saturates.

If the rotor did not saturate (at approximately 2.0 T [33]) then the attractive force due to the pole-to-pole flux on the rotor would be as great as for the stator-to-stator force. However, the rotor does saturate, and is also relatively thin, so that the maximum flux that can pass through the rotor longitudinally is
only a small percentage of the main flux. The percentage of main flux carried by the blade is given by:

\[
\% \text{ main flux} = 4\pi c^2 \tau / \eta \times 100
\]  

(3.2)

where \( c \) is the rotor thickness, \( \eta \) is the length of two poles and \( \tau \) is ratio of saturation flux density of the rotor divided by the peak flux in the stator (see Fig. 3.2). For the Experimental

Machine #2 this value is 1.8 percent. When this value of flux is used to determine the rotor-to-stator attractive forces, a value of only 12 N is calculated. Experimental results, however, showed a much greater value. This greater attractive force can occur due to slot leakage flux which travels from tooth to tooth through the rotor blade. In a sheet rotor motor this flux will hereafter be

\[c\]
\[t\]

\[S_{pn}\]
\[S_{pn+1}\]

\[n\]

Fig. 3.2 - Saturated Rotor Flux Paths
called "crenelated flux" as compared to zigzag flux as described by Alger [30] which changes pattern depending on the position of the rotor bars in the squirrel cage motor. The attractive force can be determined, once again by looking at the flux path through the rotor and determining the total maximum flux which can take this path before the rotor saturates. This is given by:

\[
\% \text{main flux} = \frac{2c\sigma}{t} \times 100
\]  

(3.3)

where \( t \) is the width of one stator tooth. For the experimental machine (EM #2) this gives a maximum force of 2350 N for a 3 mm thick blade.

This force is modulated along the length of the stator due to the sinusoidal current distribution (which is producing the mmf) along the stator. The mmf which drives the crenelated flux along this path is the difference of mmf between adjacent stator teeth. By using magnetic circuit analysis and finding the reluctance for the crenelated flux path an equation can be obtained for the value of this flux. For a stator with nine teeth per pole (20 degree spacing) the maximum crenelated flux is given by:

\[
\Phi_c = \frac{[\sin(w*t) - \sin(w*t+20 \text{ degrees})] * \text{mmf}}{2\sigma}
\]  

(3.4)

and the force:

\[
F = \frac{\Phi_c^2 \times 10^7}{(A_t \times 8\pi)} \text{ neutons}
\]  

(3.5)
where $\sigma$ is the reluctance of the magnetic path from the stator tooth to the rotor and $A_t$ is the tooth area. The calculated stator-rotor attractive force along the stator length is shown in Figure 3.3. The per unit base is the maximum attractive force due to the pole to pole flux running through an infinitely thick rotor.

![Graph showing the instantaneous stator-rotor attractive force along the stator.]

Figure 3.3 - Instantaneous stator-rotor attractive force along the stator.

3.3 Normal Force Equations

The previous analysis determined the force exerted on the blade in the worst case position, i.e. when it is against one stator. Now the relationship between the force and position from the center
will be analyzed.

To determine the force on the blade dependent on the position in the gap, it should be remembered that, although the motor is supplied from a voltage source, the stator flux and the rotor currents set the stator current. The stator current is not affected by the position of the blade, so in effect the stator is supplied by a current source for this analysis. In other words, the current to the stator may increase due to greater load but the current will not increase due to the position (z-direction) of the rotor. This postulate is confirmed by the fact that the rotor is attracted to one stator or the other in the experimental motors.

The slot-to-rotor leakage flux of opposing stator teeth is of opposite polarity in the rotor and cancels out when the blade is in the center of the air-gap. However, when the blade moves from the center position, the flux increases linearly (from magnetic circuit theory) since the driving force is a constant current source, and so the force on the rotor varies with the square of distance from the center position up to the saturation point.

The reluctance normal force of one stator on the rotor is given by (see Fig. 3.1 for notation):

$$ F_{rs} = B_T^2 A_t / (2\mu_0) $$  \hspace{1cm} (3.6)
The net rotor to stator force for both stators is:

\[ F_{rsn} = F_{rs1} - F_{rs2} \]  \hspace{1cm} (3.7)

The force on the blade, for Experimental Motor #2, is then given by the equation:

\[ F_{rsn} = (.37/(a+g))^2 - (.37/(g-a))^2 \times 4450 \]  \hspace{1cm} (3.8)

This is plotted for various air gaps and rotor positions as shown in Figure 3.4. These "V" shaped curves are an important result of this analysis because they show that if low normal forces are to be obtained, either the rotor must be accurately maintained in the center of the air-gap or a large air-gap must be used.

The analysis for force on the rotor has so far ignored the effect of rotor current (electromagnetic normal force). This can cause a repulsive force on the rotor which will force it to the center of the air-gap. This effect and the complete analysis are
Figure 3.4 - Attractive force vs rotor position in the air-gap.

given by Poloujadoff [8]. The maximum value of this force when it is repulsive is given by:

\[ F = \mu_0 J_m^2 A/(g^4) \]  

(3.9)

Where \( J_m \) is the maximum stator surface current density. For the
experimental motor the formula predicts a peak centering force of 110 N, which is 5% of the previously analyzed reluctance normal force. This result is important because it shows that during normal operating conditions the total normal forces will always pull the rotor to one stator face or the other.

3.4 Experimental Results
In the previous analysis it was assumed that all the flux which causes the reluctance magnetic attraction must pass longitudinally through the blade. In order to verify this assumption and to estimate the normal and drag forces in Experimental Motor #2, three experiments were conducted. First, a simple measurement device was constructed to test the electromagnetic force that could be exerted on a sample of saw blade material. Second, the coefficient of drag for the normal force pads in the machine was measured for different speeds. Third, the force required to move the rotor backwards, and the forward thrust, while energized, were measured in order to calculate the friction and the electromagnetic thrust. (The term electromagnetic thrust refers to the thrust that would be measured if there was no friction in the machine).

The experimental test apparatus, shown in Fig. 3.5, was constructed and the specimen of saw steel is shown mounted in the jig in Fig. 3.6. The flux density in the saw steel was driven to saturation and the attractive force measured. The flux density in
experimental conditions. A diagram of the experimental set-up is

Fig. 3.5 - Attractive Force Experimental Apparatus
shown in Fig. 3.7. The measured force (356 N) under the test conditions compares favorably with the calculated value (360 N), given that fringing fields are not taken into account in the calculation method.

The last experiment was designed to find the thrust and friction while at 100 Hz energization (0.27% slip). These values can be
Fig. 3.7 - Test Set Up To Measure Attractive Force

Shown Diagrammatically

Fig. 3.8 - Coefficient of Friction vs Speed Test Set Up
found by measuring the forward thrust and the force required to turn the rotor backward against both the friction force and the electromagnetic thrust. The friction occurs between the rotor and the low friction plastic (trade name Delrin) pads which are mounted in the stator slots in the space above the windings.

The force required to turn the rotor backwards was 869 N and the forward thrust was 162 N. The calculation then gives an electromagnetic thrust of 507 N and a friction of 347 N.
If the 347 N of friction is accurate then a normal force of 2180 N (347/0.16 = 2180 N) is acting on the rotor of the four pole machine. The coefficient of friction for the stator pads was obtained with the disc brake apparatus shown in Fig. 3.8 to obtain the values shown in Fig. 3.9.

The results of these three experiments can now be used to calculate the expected drag in the machine during operation. This drag on the rotor, at full speed, would then be 190 N (2180*0.07). This value would be an extra 21 kW in losses. This calculation does not include the electromagnetic repulsive force that would be acting on the blade under these conditions.

3.5 Conclusions

The reluctance normal force has been analyzed by looking at the flux paths and saturation conditions. The analysis first identified the source of the large attractive force (the crenelated flux), then gave the peak attractive force and the modulation of the peak attractive force, in space, along the stator. Once these values were determined, the relationship between the maximum value of attraction and the rotor position was analyzed and the resulting relationship for the experimental motor was found. Finally the maximum normal force due to the rotor currents (electromagnetic normal force) was calculated and shown to be small in relation to the attractive force.
The analysis was then confirmed in the experimental results section and subsequently applied to obtain the expected value for drag during the full load operation of the motor.

The normal force, which will always occur in the double sided steel rotor LIM, has the potential to create large losses even with low friction guiding surfaces. With the proper design of the LIM, as shown in the analysis which describes the "V" shaped curves, it should be possible (by increasing the air-gap) to reduce the drag to a more acceptable value than that measured in Experimental Motor #2.
4. EXPERIMENTAL APPARATUS

4.1 Introduction

This chapter describes the experimental motors which were designed and constructed, the measurement apparatus and the possible error which may occur in the measurements. In Section 4.2 the parameters of the five motors are presented and a brief description of the construction techniques and materials is given. The equipment which was designed or obtained for use in the experiments is described in Section 4.3. The potential areas for errors in the experiments and the amount of expected error is discussed in Section 4.4. Section 4.5 concludes the chapter with a summary of the motors and the experimental apparatus.

4.2 Motor Descriptions

Six experimental motors were designed and constructed during the course of the investigation. The parameters for these machines are presented in Tables 4.1 - 4.6. The experimental motors were wound on three different stator cores called Stator Cores A, B and C. Experimental Motor #1 was wound on Core A (one-third scale model of the Rimsaw motor). Experimental Motors #2, #3 and #4 were wound on Stator Core B (full scale Rimsaw motor) and Experimental Motors #5 and #6 were wound on Stator Core C (small scale completely circular motor). The stator cores are shown in Fig. 4.1-4.3. The details of each stator core can be found in the mechanical parameter section of the motor parameter tables.
Experimental Motor #1 was used to test the effect of different rotor materials and to measure the flux variation in the radial direction. Experimental Motors #2-#4 were used to determine the effect of odd and even number of poles and the effect of series and parallel connected poles and to measure full size motor performance. Experimental Motors #5 and #6 were used to determine the magnitude of the end effect.

All the motors were constructed of M-19, 29 Gauge, non-oriented electrical steel with a standard lamination finish. Further information on the specifications for the steel can be found in Appendix 5. The laminations were inserted into grooves machined into solid blocks of aluminum. The slots for Experimental Motors #1, #5 and #6 were machined after the laminations were placed in the grooves. The slots for Experimental Motors #2-#4 (stator core B, the full sized machine) were punched into the laminations before insertion into the aluminum blocks. For the case of motors designed for experimental purposes it was found that machined slots were less expensive to construct. Surprisingly, the cores with the machined slots did not have significantly higher losses then the punched slotted cores. There are two reasons for this. First, the laminations have very low pressure forcing them together since they are slid into the machined grooves of the aluminum blocks and secondly, very little flux leaves the machined surfaces which would cause circulating currents to flow (see finite element analysis in Appendix 6). If the tops of the teeth are machined, however, the
steel laminations will heat prohibitively (this was found experimentally) even with the very low lamination pressure. The only disadvantage of the machined slots for an experimental machine is that there are no notches at the top of the slot so that spacers usually used to hold down the copper windings can not be used.

The active length of the machine is defined as that length of the stator steel encircled by active stator conductors.

Fig. 4.1 - Stator Core A
Fig. 4.2 - Stator Core B

Fig. 4.3 - Stator Core C
## MOTOR PARAMETERS

Experimental Motor Number: 1  
Type: Segmental  
Stator Core: A

### Winding Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Number of Poles</td>
<td>4</td>
</tr>
<tr>
<td>Number of Phases</td>
<td>3</td>
</tr>
<tr>
<td>Pole Pitch</td>
<td>6.19 cm</td>
</tr>
<tr>
<td>Pitch Factor</td>
<td>.966</td>
</tr>
<tr>
<td>Coil Pitch</td>
<td>5.16 cm</td>
</tr>
<tr>
<td>Coil Span</td>
<td>1:6</td>
</tr>
<tr>
<td>Distribution Factor</td>
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<tr>
<td>Number of Coils</td>
<td>24</td>
</tr>
<tr>
<td>Turns per Coil</td>
<td>30</td>
</tr>
<tr>
<td>Turns in Series/Phase</td>
<td>60 (4 poles in parallel)</td>
</tr>
<tr>
<td>Mean Length of Turn</td>
<td>.254 m</td>
</tr>
<tr>
<td>Equivalent Wire Gauge</td>
<td>18</td>
</tr>
<tr>
<td>Winding Connection</td>
<td>Wye</td>
</tr>
<tr>
<td>Connection of Stators Together</td>
<td>parallel</td>
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### Mechanical Parameters

<table>
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<th>Value</th>
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<td>Slot Width</td>
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<tr>
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<tr>
<td>Active Primary Length</td>
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</tr>
<tr>
<td>Stator Slots (active)</td>
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</tr>
<tr>
<td>Stator Slots (half-filled)</td>
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<tr>
<td>Secondary Thickness</td>
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<tr>
<td>Primary-Secondary Gap per side</td>
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</tr>
<tr>
<td>Secondary Overhang per Side</td>
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</tr>
<tr>
<td>Magnetic Gap (total)</td>
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### Electrical Parameters

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<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
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<td>Voltage</td>
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Table 4-1
MOTOR PARAMETERS

Experimental Motor Number: 2
Type: Segmental
Stator Core: B

Winding Parameters

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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Number of Poles</td>
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</tr>
<tr>
<td>Number of Phases</td>
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</tr>
<tr>
<td>Pole Pitch</td>
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</tr>
<tr>
<td>Pitch Factor</td>
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</tr>
<tr>
<td>Coil Pitch</td>
<td>15.2 cm</td>
</tr>
<tr>
<td>Coil Span</td>
<td>1.9</td>
</tr>
<tr>
<td>Distribution Factor</td>
<td>.9598</td>
</tr>
<tr>
<td>Number of Coils</td>
<td>45</td>
</tr>
<tr>
<td>Turns per Coil</td>
<td>5</td>
</tr>
<tr>
<td>Turns in Series/Phase</td>
<td>30 (5 poles in parallel)</td>
</tr>
<tr>
<td>Mean Length of Turn</td>
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<tr>
<td>Equivalent Wire Gauge</td>
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<tr>
<td>Connection of Stators Together</td>
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</table>

Mechanical Parameters

<table>
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<th>Value</th>
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</thead>
<tbody>
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<td>Primary Width</td>
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</tr>
<tr>
<td>Primary Thickness</td>
<td>11.4 cm</td>
</tr>
<tr>
<td>Tooth Width</td>
<td>9.52 mm</td>
</tr>
<tr>
<td>Slot Width</td>
<td>9.52 mm</td>
</tr>
<tr>
<td>Slot Depth</td>
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</tr>
<tr>
<td>Total Primary Length</td>
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<tr>
<td>Active Primary Length</td>
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<td>53</td>
</tr>
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<td>Stator Slots (half-filled)</td>
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<tr>
<td>Secondary Thickness</td>
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<tr>
<td>Primary-Secondary Gap per side</td>
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</tr>
<tr>
<td>Secondary Overhang per Side</td>
<td>2.54 cm</td>
</tr>
<tr>
<td>Air-Gap (total)</td>
<td>2.8 mm</td>
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Electrical Parameters

<table>
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<th>Parameter</th>
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<tbody>
<tr>
<td>Secondary Resistivity</td>
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<tr>
<td>Frequency</td>
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<td>Voltage</td>
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<td>Current Rating (continuous)</td>
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Table 4-2
MOTOR PARAMETERS

Experimental Motor Number: 3  
Type: Segmental  
Stator Core: B

Winding Parameters

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<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Number of Poles</td>
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</tr>
<tr>
<td>Number of Phases</td>
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</tr>
<tr>
<td>Pole Pitch</td>
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</tr>
<tr>
<td>Pitch Factor</td>
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</tr>
<tr>
<td>Coil Pitch</td>
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<tr>
<td>Coil Span</td>
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<tr>
<td>Number of Coils</td>
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</tr>
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<td>Turns per Coil</td>
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<tr>
<td>Turns in Series/Phase</td>
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<td>Mean Length of Turn</td>
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<td>Equivalent Wire Gauge</td>
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<td>Connection of Stators Together</td>
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Mechanical Parameters

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<td>Primary Width</td>
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<tr>
<td>Primary Thickness</td>
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</tr>
<tr>
<td>Tooth Width</td>
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</tr>
<tr>
<td>Slot Width</td>
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</tr>
<tr>
<td>Slot Depth</td>
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<tr>
<td>Active Primary Length</td>
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<td>53</td>
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<td>Stator Slots (half-filled)</td>
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<td>Secondary Thickness</td>
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<tr>
<td>Primary-Secondary Gap per side</td>
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<tr>
<td>Secondary Overhang per Side</td>
<td>2.54 cm</td>
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<tr>
<td>Air-Gap (total)</td>
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Electrical Parameters

<table>
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<th>Parameter</th>
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<td>Secondary Resistivity</td>
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<td>Frequency</td>
<td>360 Hz</td>
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<tr>
<td>Voltage</td>
<td>440 V</td>
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<td>Current Rating (continuous)</td>
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</table>

Table 4-3
MOTOR PARAMETERS

Experimental Motor Number: 4
Type: Segmental
Stator Core: B

Winding Parameters

Number of Poles 4
Number of Phases 3
Pole Pitch 17.1 cm
Pitch Factor .966
Coil Pitch 15.2 cm
Coil Span 1.9
Distribution Factor .9598
Number of Coils 45
Turns per Coil 2
Turns in Series/Phase 24 (4 poles in series)
Mean Length of Turn .670 m
Equivalent Wire Gauge #4
Winding Connection Wye
Connection of Stators Together parallel

Mechanical Parameters

Primary Width 6.19 cm
Primary Thickness 11.4 cm
Tooth Width 9.52 mm
Slot Width 9.52 mm
Slot Depth 6.35 cm
Total Primary Length 1.03 m
Active Primary Length 0.836 m
Number of Stator Slots (single layer) 36
Stator Slots (total) 53
Stator Slots (active) 44
Stator Slots (half-filled) 16
Secondary Thickness 3.0 mm
Primary-Secondary Gap per side 1.4 mm
Secondary Overhang per Side 2.54 cm
Air-Gap (total) 2.8 mm

Electrical Parameters

Secondary Resistivity 26 \(\mu\Omega\)·cm
Frequency 360 Hz
Voltage 440 V
Current Rating (continuous) 600 A

Table 4-4
### MOTOR PARAMETERS

Experimental Motor Number: 5  
Type: Annular  
Stator Core: C

#### Winding Parameters

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<tr>
<td>Number of Phases</td>
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<td>Coil Span</td>
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<td>Distribution Factor</td>
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<td>Turns per Coil</td>
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<td>Turns in Series/Phase</td>
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<td>Connection of Stators Together</td>
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#### Mechanical Parameters

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<td>Primary-Secondary Gap per side</td>
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#### Electrical Parameters

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<tr>
<td>Secondary Resistivity</td>
<td>26 μΩ·cm</td>
</tr>
<tr>
<td>Frequency</td>
<td>400 Hz</td>
</tr>
<tr>
<td>Voltage</td>
<td>440 V</td>
</tr>
<tr>
<td>Current Rating (continuous)</td>
<td>40 A</td>
</tr>
</tbody>
</table>

Table 4-5
### Motor Parameters

**Experimental Motor Number:** 6  
**Type:** Segmental  
**Stator Core:** C

#### Winding Parameters
- **Number of Poles:** 6  
- **Number of Phases:** 3  
- **Pole Pitch:** 8.76 cm  
- **Pitch Factor:** .966  
- **Coil Pitch:** 7.30 cm  
- **Coil Span:** 1.6  
- **Distribution Factor:** .966  
- **Number of Coils:** 60  
- **Turns per Coil:** 35  
- **Turns in Series/Phase:** 140 (3 pairs in parallel)  
- **Mean Length of Turn:** .254 m  
- **Equivalent Wire Gauge:** 18  
- **Winding Connection:** Wye  
- **Connection of Stators Together:** parallel

#### Mechanical Parameters
- **Primary Width:** 3.40 cm  
- **Primary Thickness:** 3.81 cm  
- **Tooth Width:** 7.3 mm  
- **Slot Width:** 7.3 mm  
- **Slot Depth:** 2.22 cm  
- **Total Primary Length:** .878 m  
- **Active Primary Length:** .527 m  
- **Number of Stator Slots (single layer):** 36  
- **Stator Slots (total):** 60  
- **Stator Slots (active):** 46  
- **Stator Slots (half-filled):** 10  
- **Secondary Thickness:** 3.8 mm  
- **Primary-Secondary Gap per side:** 1.4 mm  
- **Secondary Overhang per Side:** 2.54 cm  
- **Magnetic Gap (total):** 2.8 mm

#### Electrical Parameters
- **Secondary Resistivity:** 26 $\mu\Omega\cdot$cm  
- **Frequency:** 400 Hz  
- **Voltage:** 440 V  
- **Current Rating (continuous):** 40 A

Table 4-6
4.3 Description of Experimental Apparatus

In addition to the linear induction motors various other equipment was constructed or obtained for use in the experiments. This equipment is described below.

4.3.1 Power Supplies

Three different power supplies were used in the course of the experimental work. The 25 kVA supply was designed specifically for the experiments with Experimental Motors #1, #5 and #6. This power supply had programmable Volts per Hertz and digital readout of frequency to within one Hertz. Two larger power commercial units were used to run the larger motors #2, #3 and #5. The two commercial units did not have as advanced control circuitry however they did perform adequately.

1. Custom Made 8085 based, transistorized, 25 kVA
2. Yaskawa, Model # VS - 616 H 45B, 45 kVA
3. Yaskawa, Model # VS - 616 H 160B, 160 kVA

4.3.2 Speed Measurement

The speed measurement system was based on a Commodore Computer 64. The speed of the disc was measured by counting the number of revolutions of the disc over an interval of time. This number of was found by putting a hole in the rotating disc and directing the output from an infrared LED so that it could be measured whenever the hole passed by a receiving unit. The time
was measured and averaged over a number of revolutions so that a highly accurate and stable reading of the disc RPM could be made. The general set up of this equipment is shown in Fig. 4.4.

4.3.3 **Thrust Measurement**

The thrust was measured with an Omega DP-240 force transducer. The braking action was produced by an air activated friction disc brake unit. The disc pads were water cooled. This system proved to be very successful and allowed for very smooth operation and stable readings. The thrust measurement system is shown in Fig. 4.5.

4.3.4 **Current Measurement**

The instantaneous current was measured using a Tektronix Current Probe Model A6302 (20 Amp.) or A6303 (100 Amp.) connected to an AM 503 current probe amplifier. The average current was measured using a conventional current transformer placed around the motor leads and the reading was made from a panel meter.

4.3.5 **Power Measurement**

The power to the inverter was measured with a Paladin #256 - TWMU three-phase 60 Hz power measurement system. The power to the motor was measured with a Load Controls Inc. model PH-3A three-phase variable frequency power measurement system (response time 15 milliseconds, frequency to 1000 Hz). The two power measurement systems were used so that the efficiency of the
Fig. 4.4 - General Set Up of Experimental Equipment

Fig. 4.5 - Friction Brake Load Set Up
inverter could be measured and so that accurate power input measurements could be made when the motor was running at low power-factor. The accuracy of the motor power measurement is poor during low power-factor operation because the power is low and is very sensitive to the phase angle between the voltage and current. This is not a problem at higher power-factor operation where the accuracy will be within 5%.

4.3.6 **Flux Measurement**

The flux was measured by flux sensing coils mounted on the stators. The voltage from the sensors was integrated to produce the actual flux value using a simply designed integrator and phase measurement system. The flux sensing coils specifications are:

- Experimental Motor #1: 100 turns of #38 AWG (see Fig. 4.6)
- Experimental Motor #2-#4: 20 turns of #28 AWG (see Fig. 4.7)
- Experimental Motor #5-#6: no sensors
Fig. 4.6 - Flux Sensors Mounted On Experimental Motor #1

Fig. 4.7 - Flux Sensors Mounted on Experimental Motor #2-#4
4.4 Experimental Accuracy

The measurement accuracy of the experiments is dependent on the accuracy of the instruments and the ability of the observer to read the values correctly. For the case of the frequency measurement of the inverters and the measurement of speed of the disc the accuracy will be greater than one percent as these values are measured referenced to crystal oscillators. The voltage and the current are measured with an accuracy of five percent from the output of the inverter (rms value measured). The motor input power is measured to an accuracy of five percent. The friction brake assembly, when properly calibrated and steady values are observed, will read to within five percent. All of the above are typical value for electrical machine experimental measurements.

The difficulty in obtaining an accurate value of motor output power cannot be overstated. In order to obtain an accurate value for output power all friction and windage losses must be added to the measured value of output power. In normal rotary induction motors friction and windage losses are a very small fraction of the total output power, but as was shown in Chapter 3, the power lost due to friction in the induction motors under investigation may be very high.

The difficulty in obtaining an accurate measurement for the friction can only be appreciated when one considers the constraints on the measuring device. The friction measuring
mechanism must fit between the face of the stator and the rotor which is less than 1 mm in height. The normal force that must be supported is approximately 2000 N and the mechanism cannot be made of magnetic or electrically conductive material. During the course of the investigation no such mechanism could be produced which worked properly.

The most elaborate mechanism consisted of a ladder network which supported two rails alongside each side of the stator on which the rotor would slide. The steps of the ladder were placed in the slots of the stator and were designed to flex in the longitudinal direction of the stator but not in the perpendicular direction. The amount of deflection for the whole ladder was to be measured with a position transducer. Unfortunately the amount of vibration in the motor during operation was far greater than the amount of deflection to be measured and no accurate results were obtained.

4.5 Conclusions
Various experimental motors and apparatus were constructed in order to investigate the annular disc LIM motors operating characteristics.

Experimental Motor #1 was used to test the effect of different rotor materials and to measure the flux variation in the radial direction. Experimental Motors #2-#4 were used to determine the effect of odd and even number of poles and the effect of series
and parallel connected poles and to measure full size motor performance. Experimental Motors #5 and #6 were used to determine the magnitude of the end effect.

The experimental apparatus which was used to load the machines and the instruments used to make the measurements was described in Section 4.3. The experimental accuracy, discussed in Section 4.4, of all measurements except electromagnetic thrust were accurately obtained.

The apparatus described in this chapter was used in the experiments described in the following chapter.
5. RESULTS AND DISCUSSION

5.1 Introduction

Experiments were conducted in which the motor parameters were varied in order to measure the effect on motor performance. In addition, those factors which reduce performance and may have been exacerbated by the unusual design of the thin steel rotor LIM were measured and analyzed. In order to clearly show the effect of different parameter variations, simulation results are also presented. In some cases the simulation results are used to highlight the effect of the parameter changes because in a simulation it is possible to make comparisons which cannot be made in the physical world. The results of these experiments and simulations, which were conducted on six experimental motors, are presented in this chapter.

In Section 5.2 various experimental results are compared to the one dimensional model to verify that the model will accurately predict the power produced by the experimental motors.

The end effect and the effect it has on the actual overall performance of a steel rotor machine is described in the experiments of Section 5.3.
Various rotor materials were considered and tested. The results of these experiments and simulations are presented in Section 5.4. Although none of the other rotor materials tested would produce a practical alternative to steel at the present time, the results of the experiments are very useful in that they show potential areas of improvements in the design of this type of machine.

The choice of whether to use an odd or even number of poles does not occur in standard rotary induction motors which must all have an even number of poles. However, in the LIM there is no such requirement since the entry pole does not have to be of the opposite polarity of the exit pole. It is thus possible, and sometimes done, to have an odd number of poles. The result of an experiment to compare the effect of odd or even number of poles is presented in Section 5.5.

An option in the design of any LIM is whether to connect the poles of the stator in series or parallel. The advantage of connecting the winding in series is that the current which flows is the same in all the coils whereas in parallel connected windings some of the coils carry more current than others. This is important if the current capability of the wire is near the limit that it can carry without overheating. However, more power may be produced if the windings are connected in parallel. An experiment was conducted to determine the actual effect of the
two alternative connection methods and these results are presented in Section 5.6.

The edge effect will cause the square pattern of rotor current shown in Fig. 1.1 to close in to form ovals. The result of the oval pattern (as compared to the more optimum square) is higher losses and lower output power. Three different experiments were conducted to determine the extent of this problem for a steel rotor LIM. These experimental results are described in Section 5.7.

Space and time harmonics cause higher losses and reduced output. The effect of slot harmonics (the space harmonics) is expected to be more pronounced in the type of motor under investigation than large air gap single sided LIMs due to the small effective air gap and the non-laminated rotor. Space harmonics also exist due to the winding distribution but these harmonics will not be any greater for this type of machine than for other LIMs. Slot harmonic data was obtained from air gap flux measurements. An experiment was devised to compare the performance of a solid steel blade with and without the effect of space harmonics, in order to determine the magnitude of their effect. The time harmonics were also measured and calculations undertaken to determine the magnitude of their effect. This work is presented in Section 5.8.
It was shown analytically in Chapter 2 that the effect of the annular stator would be very small on the performance of the annular LIMs under investigation (i.e. the increased rotor resistivity and the variation in power produced over the face of the motor). To obtain further confirmation of this conclusion, flux plotting was conducted and these results are presented in Section 5.9.

Finally, in Section 5.10 a summary of the experiments is presented and the important results are highlighted.

5.2 Verification of the Computer Model

The model used to simulate the performance of the experimental machines is a one-dimensional current source model. This model has been used by previous investigators [29]. The advantages of using the one-dimensional model are: 1) intermediate calculated values are more meaningful, 2) the equations can be solved on personal computers and 3) the one-dimensional model can be more accurate than very complex models when used with correction factors obtained from previous experimental results [29]. As there are numerous experimental results from which to obtain the correction factors the one-dimensional model is the best model for use in this investigation.

The five simulations (Figs. 5.1 - 5.5) presented in this section contain the correction factors for the air-gap due to the stator
teeth (the Carter coefficient [48]) and the effect on the rotor resistivity due to the finite width of the rotor [50,51]. The value for friction, which is included in these simulations, is obtained from coast down tests and friction measurements for the actual machines.

![Graph showing power vs. synchronous speed](image)

**Fig. 5.1 - Comparison of Experimental Points and Simulation Curve For Experimental Motor #1 With 3 mm Copper Rotor**
All of the simulation curves show reasonable accuracy in predicting the performance of the motors. Some discrepancies do occur and these are caused by changing friction in the machine. This will occur if the rotor goes through a mechanical resonance, the cooling water flow rate changes, or mechanical clearances change during the experiment due to thermal effects.

Fig. 5.2 - Comparison of Experimental Points and Simulation Curve For Experimental Motor #1 With 3 mm Stainless Steel Rotor
In Fig. 5.3 the simulation and the experimental result show the greatest discrepancy. The rotor of this motor was poorly balanced which caused severe vibration during the test. This would result in the friction factor used in the simulation not being as accurate as for the other simulations. The large amount of vibration did not permit high speed operation of the motor and this is why the experimental points do not include the no load operating point.

![Graph](image)

Fig. 5.3 - Comparison of Experimental Points and Simulation Curve For Experimental Motor #1 With 2 mm Stainless Steel/Copper Rotor
These results presented in this section will be referred to again in Section 5.4 which contains comparisons on the performance of different rotor materials.

Fig. 5.4 - Comparison of Experimental Points and Simulation Curve For Experimental Motor #1 With 2 mm Steel Rotor
Fig. 5.5 - Comparison of Experimental Points and Simulation Curve For Experimental Motor #3 With 3mm Steel Rotor
5.3 Effect of Segmented Rotor

In this section various measurements and the results of experiments will be presented to show experimentally the effect of the end-effect on the performance of the thin steel rotor annular LIM. The analysis for the end-effect was presented in Chapter 2.

The first experiment consisted of measuring the performance of an experimental motor (see Fig. 5.6) designed so that it could be simply converted from a continuous annular motor (thereby having no end-effects, Experimental Motor #5) to a segmental annular motor by disconnecting poles of the annular stator core (Experimental Motor #6). The two rotor materials chosen for this experiment were steel and aluminum. The steel rotor is expected to exhibit very little degradation in performance due to the end effect in this small prototype motor. The speed of the motor is slow so the rotor currents will have time to build up to the correct level. The aluminum rotor is five times more conductive than the steel rotor and should show the degradation in performance due to the end effect.

Fig. 5.7 shows the experimental results for Experimental Motor #5 and Experimental Motor #6 with the steel rotor. In this case neither the efficiency or the power factor of the motor are degraded by the end effect.
Fig. 5.6 - Experimental Motor #6 Derived By Disconnecting Two Fifths of Experimental Motor #5

Fig. 5.7 - Experimental Results of Motor #5 (-----) and Motor #6 (-----), Steel Rotor
Fig. 5.8 shows the simulated power curves for Experimental Motor #5 (three-fifths of the actual power to make the result comparable to Experimental Motor #6) and for Experimental Motor #6. Plotted on the figure are the actual experimental points (three-fifths of the actual power for Experimental Motor #5 to make the result comparable to Experimental Motor #6). The simulation curves for the two motors are so similar that it appears as one line. These simulations include the effect of friction.

![Simulation Results and Experimental Points](image)

Fig. 5.8 - Simulation Results and Experimental Points

For Experimental Motor #5 With (— —) and Without (——) End-Effects, Steel Rotor
Fig. 5.9 shows the experimental results for the aluminum rotor. It can be observed that the end effect is causing a significant reduction in efficiency for this motor. The peak efficiency drops from 0.48 to 0.27. The power factor, however, is almost identical for both motors at 0.31. This results in zeta (power factor times efficiency) being significantly worse for the aluminum rotor with end effects than without.

Fig. 5.10 shows the simulation curves and the experimental points for the power produced by these two rotors. The aluminum rotor with end effects shows the significant loss of power that the simulation predicts.
From these results it is clear that for the smaller annular motors (Experimental Motor #5) the end effect does not reduce the power or efficiency for the thin steel rotor machine significantly. However, the more conductive aluminum rotor does have a significant loss in efficiency due to the end effect and would not be a good rotor for a motor with the parameters of Experimental Motor #5.
The second experiment consisted of measuring the flux and phase of the steel rotor Experimental Motor #4 in order to determine the magnitude of the end-effect in the full size steel rotor machine. This motor is the full size prototype for the sawing application. The flux was measured with flux sensing coils wrapped around each tooth of the stator as shown in Fig. 5.11. The measured values for flux for three different conditions (no rotor, 15% slip and 20% slip) are shown in Fig. 5.12. It can be
seen that the flux near the entry area is reduced for the higher speed rotor but as a greater load is applied, and the rotor slows down, the flux moves more towards the no rotor shape. From the theory presented in Chapter 2 and from measuring the amount of reduction in flux at the entry of the motor, the ratio $b_1/b_p$ can be estimated ($b_p$ is the no load flux and $b_1$ is the maximum value of the end-effect flux which reduces the air gap flux during operation of the motor). From Fig. 5.12 it can be estimated that $b_1$ is approximately (measured at the 12th stator tooth) .35 times $b_p$ at 1956 rpm and approximately .32 $b_p$ at 1850 rpm when the machine is under greater load. This significant reduction shows the effect of the end effect and why the performance of the LIM is reduced at high speed.
Fig. 5.12 - Magnetic Flux Density Measured Along Experimental Motor #3
5.4 Effect of Rotor Variations

Various rotor materials were tested on Experimental Motor #1 to find the optimum rotor material in terms of maximum efficiency and power-factor and also to produce the maximum power for the size of stator (power density). It is especially important that the power-factor efficiency product is maximized because the motor must run from an inverter, which is the most costly single component of the sawing system. These experiments could not be conducted on the full size machine (Experimental Motors #2-#5) because only steel was rigid enough to run smoothly through the stators at high speed.

The experimental equipment consisted of a custom made 25 kVA, three-phase, voltage source inverter which could be accurately set to produce frequencies from 25 to 600 Hz. All of the experiments were conducted with Experimental Motor #1. The speed of the rotor was measured with a hand held digital tachometer and the torque was measured with a commercial strain gauge force transducer. A friction brake dynamometer was used as the load which means that only those operating points where the torque decreases with increasing speed could be measured. This is not an important disadvantage of the friction brake system since this the only operating region for most motors and is the operating region for the sawing application. The test results for four different rotor materials are presented in this section - they
are: copper, stainless steel, a stainless steel/copper layered blade, and steel.

The experimental results for the efficiency, power-factor, power-factor*efficiency product (zeta) and power output are shown in Figures 5.13 to 5.16. The curves drawn in on these figures are to show the trend of the data points and are not simulated values. The Figure 5.16 is Figures 5.1 to 5.4 combined onto one graph but without the simulation results.

The first rotor test result is for a 3 mm thick copper blade. This is a highly conductive rotor which results in higher losses due to the end-effect. In a normal rotary machine such a highly conductive rotor would reach maximum power very close to synchronous speed. In this case, however, maximum power is reached at 20% slip. Both the efficiency and power-factor are 50% at the maximum power point, resulting in a very high power-factor efficiency product (zeta) of 0.25. The maximum power measured was the second best produced of the four blades tested.

The next rotor material is a non-magnetic stainless steel blade (SS Type 3041). The stainless steel is a very resistive material (\( p = 68 \ \mu\Omega\text{-cm} \)) so it would be expected to have very little end-effect but would also require a very high slip frequency before much power is produced. This can be seen in the graph. The peak power was reached at 40% slip and this was also at a
high stator frequency of 600 Hz (to obtain good performance in the desired operating speed). The power-factor is not too unreasonable at 44% but the efficiency is very low, 24%, due to the high stator core losses which results from the high stator frequency. The output power was the lowest of the tested rotors at 1.07 kW.

The third rotor material is a stainless steel/copper sandwich rotor consisted of two 0.5 mm sheets of stainless steel covering each side of a 1 mm thick copper sheet. The stainless steel sections of the rotor would have very little effect on the performance because the stainless steel is both non-magnetic and approximately fifty times more resistive than the copper. Due to a mechanical imbalance in the rotor it was not possible to run the blade faster than 0.8 of synchronous speed so the no-load speed was not measured. The sandwich blade proved to be the optimum of the four tested. This rotor produced the maximum power (3.1 kW) and also had the highest power-factor. The efficiency of 48%, although not the highest compared to the 3mm copper blade, when combined with the high power-factor resulted in the highest power-factor efficiency product of all the rotors tested.

The last material tested was 2 mm, hardened steel with a resistivity of 26 $\mu\Omega$-cm. Since the rotor is made of a magnetic material the effective air-gap is approximately half of what it
would be with the other rotors. This results in half the magnetizing current and is expected to result in a higher power-factor which is what can be seen in the results. The high resistivity of the steel, approximately fifteen times that of copper, results in a blade that does not reach maximum power until 60% slip and thus has a low efficiency of 28%. The maximum power-factor efficiency product is only 0.16 for this rotor material.

Fig. 5.13 - Experimental Points and Approximating Curves

Showing Efficiency
Fig. 5.14 - Experimental Points and Approximating Curves

Showing Power-Factor
Fig. 5.15 - Experimental Points and Approximating Curves

Showing Power-Factor Efficiency Product (Zeta)
Fig. 5.16 - Experimental Points and Approximating Curves

Showing Power
From the test results and simulations presented in this section it can be concluded that a rotor material with the resistance of a 1 mm copper sheet would produce close to the optimum motor for a stator with the parameters of Experimental Motor #1. These experimental results may be extended to Experimental Motors #2-#4 if the proper scaling factors are used [49] to determine the optimum rotor material. This scaling to a larger size motor is presented in Chapter 6.

5.5 **Effect of Odd or Even Number of Poles**

Unlike its rotating induction motor counterpart, the linear induction motor can have either an odd or even number of poles. This is due to the way in which the LIM is constructed. Fig. 5.17 shows how a LIM can be imagined as a rotary induction motor which has been split and unrolled. Once the LIM has been formed it is possible to add on an extra pole if it is desirable to increase the length of the motor. This would not be possible in
the rotary induction motor because there is no place to add in a pole which would not be adjacent to a pole of the same polarity. It would have also been possible to increase the length of the LIM by increasing the pole pitch.

In theory, the odd number of poles may result in lower power-factor since when the magnetic circuit is analyzed, it is found that there is no path for the extra flux to return along, except around the sides and edge of the stator, as show in Fig. 5.18. In a real LIM, however, there are other paths available as described by Laithwaite [7] and as will be shown in the results of this section.

Fig. 5.18 - Analysis of the Flux For a Three Pole Stator
When the flux is integrated across the face of the stator in equation 5.1 there is a net flow which must return through air

\[ \text{Flux} = \frac{\mu_0}{g} \int_0^x j \, dx \]  \hspace{1cm} (5.1)

and this is what should result in a lower power-factor. In the analysis presented by Yoshido et al. [17] it is stated that this extending flux will result in lower efficiency, although quantitative comparisons are not made. In this section theoretical values for the expected increase in power and experimental values for the increase in power and the effect on efficiency and power-factor are obtained. In addition, an experiment was conducted to observe the effect of an odd number of poles on the power-factor and efficiency of the thin steel rotor, annular LIM. Designing an experiment which will accurately compare the effect of odd or even number of poles will always result in compromise. If the pole-pitch is increased to eliminate a pole, then the magnetizing current will decrease and the Goodness will increase (which will increase the power factor). On the other hand, the relative end effect force will increase which reduces the efficiency. If the other alternative - disconnecting a pole - is chosen, then the parasitic thrust of the end-effect will remain the same but the motor thrust is reduced because of the fewer poles
In this experiment, the second option was chosen. One pole was disconnected from Experimental Motor #3 (5 poles to four poles), which then became Experimental Motor #4 (this was the simplest method of obtaining the even number of poles), and then the expected changes in efficiency and power-factor were taken into account. Another disadvantage of this method of obtaining the even number of poles is that since all the poles are connected in series, the voltage applied per pole of the four pole motor is greater than for the five pole motor. To compensate for this factor the voltage gain of the inverter supply was turned up by 25% for the Experimental Motor #3 tests.

The experimental results for the odd and even comparison are shown in Fig. 5.19. From the results it can be seen that prior to increasing the voltage the four pole motor has a slightly greater power-factor but a lower efficiency. The resulting power-factor efficiency product is almost identical for the two motors over much of the operating range. When the output voltage of the inverter is now increased for the five pole motor (to produce the same voltage per pole as the four pole motor), then the efficiency and power-factor of both motors is about the same over the normal operating range.

From these experimental results it can be concluded that whether a thin steel rotor, high speed LIM has an odd or even number of
Fig. 5.19 - Experimental Results For A Comparison Between Four (——) And Five (— —) Number of Poles

poles does not effect the overall performance of this type of motor operating under the conditions described in the table of Motor Parameters.
5.6 Effect of Series and Parallel Connection

Very little has been written on the subject of connecting poles of a LIM in series or parallel. Laithwaite [11] discusses the flux pattern of parallel connected LIMs but does not discuss the effect of the parallel connection on the performance of the LIM. Yamamura [56] describes parallel connected compensation windings in his book but Dukowicz [57] shows in his paper that these type of windings do not improve the overall performance of a LIM.

The reason that the series or parallel connection question is seldom considered is that most LIM stators have windings which are current limited and the entry coil would not be able to withstand the greater current resulting from a parallel connection. The entry coil carries more current because in the parallel connection the full line voltage is applied across the coil and so the flux will be constant. At the same time the rotor material entering the stator has no flux associated with it and the steady-state result is higher current in the entry coils and higher flux density at the entry end of the stator than for the series connection. For double sided LIMs with small air gaps, the stator windings can be designed to carry the extra current. If power density of the motor is important then the parallel connection may be advantageous.

In order to investigate the effect of the parallel connection on
the performance of the LIM, the following tests were carried on a parallel machine: a) simulations of the power produced by the entry pole for both the parallel and series connected motor were made, b) the flux at the entry area of the motor was measured and c) an experiment was conducted to compare the power, power-factor and efficiency for the two different connections.

First an experiment was conducted to measure the current of the entry coils of a parallel connected motor (Experimental Motor #2) to determine the increase in current density during different operating conditions. These values are given in Table 5.1.

<table>
<thead>
<tr>
<th>% of synchronous speed</th>
<th>Motor Current (A)</th>
<th>Coil Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>285</td>
<td>120</td>
</tr>
<tr>
<td>84</td>
<td>300</td>
<td>124</td>
</tr>
<tr>
<td>80</td>
<td>320</td>
<td>106</td>
</tr>
<tr>
<td>75</td>
<td>345</td>
<td>106</td>
</tr>
</tbody>
</table>

Table 5.1 - Entry Coil Current

The above values can then be used to simulate the power produced by the entry pole of the stator for the parallel connected machine. The simulation for the entry pole of the series connected machine is obtained from the standard series connected simulation technique described in Chapter 2. The results of these simulations (Fig. 5.20) show that the power produced by the first pole of the stator is greater for the parallel connected stator at all
operating points and is 15 kW (150 %) greater at 75 % of synchronous speed.

The next step of the experiment was to measure the flux at the entry end of both the parallel and the series connected stator. The results of these measurements are shown in Fig. 5.21. One can observe in the figure that the flux of the parallel connected machine is not reduced as much as for the series connected
machine. This confirms that the parallel connected machine should produce more power for the same length of machine.

Finally, direct measurements were made for the power produced, the efficiency and the power-factor for two motors which were wound on the same stator core, but differed in that one was parallel connected (Experimental Motor #2) and one was series connected (E.M. #3) and a Series Connected Motor (E.M. #3).

![FLUX DENSITY AT ENTRY OF THE MOTOR](image)

**Fig. 5.21 - Measured Flux at the Entry End of a Parallel Connected (E.M. #2) and a Series Connected Motor (E.M. #3)**
connected (Experimental Motor #3). These experimental results, in Fig. 5.22, confirm the simulation results shown in Fig. 5.20. The measured increase in power was 17.5 kW (70% increase).

The power-factor remains the same for both but the parallel connected machine has greater efficiency and thus has a greater efficiency power-factor product.

Fig. 5.22 - Experimental Results for a Comparison of the Performance for a Parallel Connected (——) and a Series Connected (— —) Motor

These results are of importance for the design of the double-sid-
ed, thin steel rotor LIM because one of the greatest difficulties of using a thin steel rotor is obtaining good power densities. The parallel connected LIM has been shown to have a significantly greater power density with no decrease in the power-factor efficiency product. This increase in power for the parallel connected LIM also demonstrates the deterioration due to the end effect.

5.7 **Edge Effect**

The current which flows in the rotor of a sheet rotor motor does not flow in a perfect rectangular pattern as in the rotor bars of a squirrel cage induction motor. (The sheet rotor motor in Fig. 1.1 shows a rectangular pattern only because it is the idealized case.) Rather, the pattern is more like that shown in Fig. 5.23. The curved current paths are caused by the edges of the rotor not being of zero resistance. These curved paths result in poorer than predicted performance of sheet rotor motors and this effect is called the edge effect.

![Pattern of Rotor Currents](image)

**Fig. 5.23 - Pattern of Rotor Currents**
The curved lines of current cause a disruption of the flux pattern, with higher flux density at the edge of the stator and lower flux density near the center as shown in Fig. 5.24 [50].

![Diagram of Flux Density]

Fig. 5.24 - Diagram of Flux Density

This effect was studied by Hugh Bolton [50] among others [51]. Although Bolton does not analyze the specific case of the solid iron rotor, he states at the conclusion of his paper: "Finally, pronounced transverse flux redistribution is known to take place in rotary induction motors with solid-iron rotors". As the motor types under investigation use solid-iron rotors this effect was
analyzed and experiments conducted in order to determine its magnitude.

Bolton's paper describes how the various motor parameters can be used to decide whether or not severe edge effects can be expected. The analysis of the edge effect and how it is affected by the motor parameters will now be presented.

The analysis of the edge-effect requires the two-dimensional analysis of the stator and rotor in the x-y plane. The definitions of the variables can be seen in Fig. 5.25 [50].

![Reference Frame For the Edge-Effect Analysis](image)
The air gap equation for the rotor and the stator is given by equation 5.2. (\(\frac{d'b}{dx}\) refers to partial derivatives)

\[
\frac{d^2 b^2}{dx^2} + \frac{d^2 b^2}{dy^2} + \frac{\mu_0 d'b}{\Omega g d't} = -\frac{\mu_0 d'j}{g d'x}
\]

(5.2)

Assuming sinusoidal functions, equation (5.2) can be written as:

\[
\frac{d^2 B}{dy^2} - k^2 + \frac{j s w \mu_0}{\Omega g} B = \frac{j \mu_0 k J}{g}
\]

(5.3)

The solution of this equation is given by:

\[
B = \frac{j \mu_0 J}{g k} Z^2 \left( 1 + \frac{(1 - Z)}{Z^2} \right) e^{Cosh \beta y}
\]

(5.4)

where:

\[
\varepsilon = \frac{1}{1 + \frac{1}{\sqrt{1 + i s G}}} \frac{Tanh \beta a Tanh k(c - a)}{Tanh \beta a Tanh k(c - a)}
\]

\[
\beta = k^2 + \frac{i s w \mu_0}{\Omega g}
\]

\[
= k^2 (1 + i s G)
\]

\[
Z = \frac{1}{1 + i s G}
\]

To find the ratio, \(U\), of the flux density at the edge to the mean flux density, first calculate the mean flux density:

\[
B_{mean} = \frac{1}{2a} \int_{-a}^{a} B \ dy
\]
\[ U = \frac{B_{\text{side}}}{B_{\text{mean}}} \frac{1 + i s G e}{1 + i s G F \tanh \beta a} \] (5.5)

From the above equations Bolton was able to develop a family of curves based on the stator width, rotor width, the pole pitch and the Goodness Factor for a sheet rotor induction motor. These curves are shown in Fig. 5.26 and give a good insight into the parameters of the motor which can be varied to reduce the edge-effect. The parameters for Experimental Motor #2 are plotted on the curves and it can be seen that the edge effect is expected to have minimal effect on the performance of the motor.

Fig. 5.26 - Flux Density Variation Factor [50]
For the test motors the parameters were checked against these curves, which predicted that severe edge effects were not expected. However, because the analysis is based on a non-magnetic rotor the effect of the steel rotor on the edge effect is included in the above analysis only partially (by a reduction in the effective air gap), so that the actual effect of the steel rotor is not certain. For this reason experiments were conducted to determine the magnitude of the edge effect, first by measuring the variation in flux density, and second, by decreasing the effect of the edge effect on the performance of the thin steel rotor annular LIM. In the first experiment, flux search coils were laid across the stator and the flux measured for Experimental Motor #1. In the second, slots were cut into the rotor of Experimental Motor #4 to eliminate the possibility of the longitudinal currents flowing over the stator area and causing the edge effect to reduce the performance of sheet rotor motors.

In the first experiment, flux sensing coils were laid across the face of Experimental Motor #1 to measure the variation in flux density due to the edge effect. The positioning of these sensors is shown in Fig. 5.27. It was not possible to measure the lateral variation in the flux of the full size prototype motor because the flux sensors are approximately 1.5 mm thick, which could not be made to fit in the air gap of Experimental Motor #4, so Experimental Motor #1 was used with a larger air gap (to accommodate
the flux sensors) but the rotor was coated with a copper layer so that the Goodness of the LIM would be the same as that for Experimental Motor #3. The Goodness partially determines the extent of the edge effect.

Figure 5.23 shows that the flux profile does not reduce significantly in the center (less than five to one difference; see Bolton [50]), indicating that the edge effect is not significant. The reason for the sloping lateral flux density curve is due to the
The second experiment consisted of cutting slots into the steel rotor of Experimental Motor #4. This was done to eliminate the possibility of longitudinally flowing currents over the stator area reducing the performance of the LIM (which is the edge effect). Approximately three slots per pole (3.14 slots per pole length) were cut with a laser through the steel plate rotor. A laser was used so that the slots could be cut thin enough that the amount of rotor material removed would have a negligible

![Figure 5.28](image-url)

**Fig. 5.28 - Measured and Expected Value of Flux Across Stator Face For experimental Motor #1**
effect on rotor resistivity. These slots are shown in Fig. 5.29.

Experimental Motor #4 was tested with the rotor described above and the results of this experiment are given in Table 5.2. It can be seen from the table that the performance of the machine improved slightly after the slots were cut into the rotor. If the edge effect had been significant than the power input would have dropped significantly for the case where the rotor had slots cut into it.

Fig. 5.29 - Slots Cut Into the Rotor of Experimental Motor #4
### Table 5.2 - Measurements For Experimental Motor #4

<table>
<thead>
<tr>
<th></th>
<th>Power Input</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without Slots</td>
<td>82 kW</td>
<td>2024</td>
</tr>
<tr>
<td>With Slots</td>
<td>80 kW</td>
<td>2001</td>
</tr>
</tbody>
</table>

With and Without Slots at 380 Hz Supply

The results of these two experiments confirmed the analysis that the edge effect did not have a significant effect on the performance of the type of motors being analyzed.

#### 5.8 Effect of Harmonics

Space and time harmonics in the air gap flux can have detrimental effects on the performance of induction motors. Space harmonics occur in motors due to the slotted nature of the stators. Time harmonics are produced by the switching of the transistors which supply the variable frequency voltage source. These harmonic effects can be expected to be greater than for a conventional induction motor due to the un laminated rotor of the LIM.

##### 5.8.1 Space Harmonics

If the magnitudes of the space harmonics is large in relation to the fundamental, then a significant decrease in motor efficiency and power may be expected [53].

The slot harmonics will appear to the rotor at a frequency of at
least 36 times that of the fundamental flux (see Fig. 5.29). This is because the slot harmonic is due to the stator teeth and the number of stator teeth is usually at least twelve per fundamental and most motors run at no less than 75% of synchronous speed (.75*12/[1-.75] = 36). Due to skin effect this high frequency will usually be attenuated and the drag due to the slot harmonic will be quite low.

Three experiments were conducted on Experimental Motor #4 to determine whether slot harmonics would decrease the efficiency.

In order to determine the magnitude of the slot harmonic, the air gap flux was measured (along the center line of the stator) with the use of a Hall Effect flux probe. The measurement was done at low flux density so that the abnormal saturation effects of using DC would not occur. Saturation should not occur during normal operation because as is shown in Fig. 5.11 (the flux profile plot) the flux density is not into the saturation region for the steel (see Appendix 4 for the steel specifications). The air gap flux measurement for two poles of the stator is shown in Fig. 5.30.
From the figure it can be estimated that the first slot harmonic is one-fifth the amplitude of the fundamental flux. For a rotor running at 75% of synchronous speed fed with a 360 Hz supply the effective frequency that the rotor would see is given by:

$$\text{Freq.} = 0.75 \times 360 \text{ Hz} \times \frac{18 \text{ slots}}{\text{two pole pitches}}$$

$$= 4860 \text{ Hz}$$

Using the value for the effective stator to rotor harmonic frequency, the magnitude of the flux slot harmonic, and the simulation technique described in Chapter 2, it is possible to obtain a relation between torque and rotor resistivity as shown in Fig. 5.31.
The rotor resistivity will be affected by skin depth [52] which is difficult to determine due to the non-linear effect of saturation in the steel rotor [27]. For this reason a possible range of force is shown in the graph. The figure shows that if the effective rotor thickness becomes 50 μm then the slot harmonics would produce the maximum drag of 90 N which is 10% of the
full load power of the motor. From the above analysis it was shown that the slot rotor harmonics could contribute to significant losses in the LIM depending on the skin depth of the induced slot harmonic currents. In order to obtain an exact value for the slot harmonic loss the experiments described in the following were undertaken.

The first experiment consisted of misaligning the teeth of the stator as shown in Fig. 5.32, in order to reduce the air gap flux harmonic.

Two parameters will actually change in this experiment. The effective air gap will actually increase in addition to the reduction in space harmonics. Discussions regarding the effect of misaligning the teeth have been made with some experimenters claiming improved performance [29, p. 220]. However, in the experiment undertaken no significant improvement was measured as is shown in Table 5.3. From the geometry of the motor shown in the above diagram one can conclude that very few motors would expect to have greater space harmonic to fundamental ratio than is found in Experimental Motor #2. This is because Experimental Motor #2 has a very small air gap and is a double sided motor.
The one problem with this experiment and the ones conducted by other researchers is that the slot harmonics may not be reduced significantly right at the surface of the rotor which is where the induced rotor slot harmonic currents will flow due to the skin effect. To eliminate the possibility that skin effect was causing the misaligned teeth experiment to produce incorrect conclusions a second experiment was conducted.
A second experiment was conducted which involved cutting slots into the blade so that no harmonic currents could flow (see Fig. 5.33). By skewing the slots and having their pitch the same as the tooth pitch, no current path exists directly in the shape of the slot harmonics. The resistance of the rotor current path is changed only slightly; by less than 1% when the end paths are included. This means that the slot harmonics can be totally eliminated with almost no effect on the fundamental. When this experiment was conducted a small decrease in power draw was observed (Table 5.4) which did indicate that the slot harmonics were decreasing the efficiency by approximately 5%.

<table>
<thead>
<tr>
<th></th>
<th>Power</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aligned Teeth</td>
<td>93 kW</td>
<td>1850 RPM</td>
</tr>
<tr>
<td>Misaligned Teeth</td>
<td>93 kW</td>
<td>1850 RPM</td>
</tr>
</tbody>
</table>

Table 5.3 - The effect On the Performance Of
Experimental Motor #2 With and Without Misaligned Teeth
Fig. 5.33 - Slots Cut In Rotor To Eliminate Rotor Slot Harmonics

<table>
<thead>
<tr>
<th></th>
<th>Power</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unslotted Rotor</td>
<td>82</td>
<td>2024</td>
</tr>
<tr>
<td>Slotted Rotor</td>
<td>78</td>
<td>2001</td>
</tr>
</tbody>
</table>

Table 5.4 - The effect On the Performance Of Experimental Motor #4 With and Without Slotted Rotor
5.8.2 Time Harmonics

The time harmonics of the current were measured with a spectrum analyzer to determine their magnitude and these are presented in table 5.5. The third order harmonics, which cancel out in a balanced supply, are present in the inverter supply due to timing inaccuracies in the control circuits. The fifth and seventh harmonics, which are the strongest ones in this particular case, will produce a pulsating force. These amplitudes of harmonics have been shown by George John [55] not to decrease the efficiency or thrust of the LIM.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Order</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>350</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1050</td>
<td>3</td>
<td>0.044</td>
</tr>
<tr>
<td>1750</td>
<td>5</td>
<td>0.11</td>
</tr>
<tr>
<td>2100</td>
<td>6</td>
<td>0.068</td>
</tr>
<tr>
<td>2450</td>
<td>7</td>
<td>0.177</td>
</tr>
</tbody>
</table>

Table 5.5 - Harmonic Components of Current Wave Form to LIM

5.9 The Effect of the Annular Motor

In the analysis presented in Chapter 2 the effect of the annular stator was simulated with a constant current source of varying
surface current density. The flux that was generated by the surface current sheet was assumed not to travel in a radial direction. This resulted in a higher flux density at the inner radius of the stator. However, due to the physical construction of the stator there is actually less stator area to carry the flux than was assumed in Chapter 2. This is because the conductors which carry the stator current are of constant diameter so that the sides of the stator slots must be parallel and the result is that the stator teeth are a wedged shape as shown in Fig. 5.34.

![Fig. 5.34 - The Stator Tooth And Slot Profile](image)

For the Annular Stator

The analysis presented in Chapter 2 predicted that more torque and power would be generated at the outer edge of the stator.
The measured flux plot of the stator in Fig. 5.35 however, shows a greater air gap flux density at the outer edge of the stator so in actual fact the variation in power shown would actually be greater.

Fig. 5.35 - Lateral Variation in Flux For Experimental Motor #1

A simulation of the variation in flux and the power produced by the motor is shown in Fig. 5.36. The figure shows the extra increase power and torque due to the increased flux which is now accounted for.
5.10 Discussion

Various experiments were conducted to determine the effect of different parameter variations and to experimentally determine the effect of the steel rotor on the performance of a high speed annular LIM. The results of these experiments, which were conducted on six experimental motors, were presented in this chapter.
The end effect and the effect it has on the actual overall performance of a steel rotor machine is described in the experiments of Section 5.3.

Various rotor materials were considered and tested. The results of these experiments and simulations are presented in Section 5.4. Although none of the other rotor materials tested would produce a practical alternative to steel at the present time, the results of the experiments are very useful in that they show potential areas of improvements in the design of this type of machine and the results of these experiments can used with scaling factors to find a more optimum design. This is presented in Chapter 6. This section also allowed for the direct comparison between simulated and actual results. The simulation when corrected for friction and rotor resistivity agreed well with the actual experimental results.

The choice of whether to use an odd or even number of poles does not occur in standard rotary induction motors which must all have an even number of poles. However, in the LIM there is no such requirement since the entry pole does not have to be of the opposite polarity of the exit pole. It is thus possible, and sometimes done, to have an odd number of poles. The result of an experiment to compare the effect of odd or even number of poles was presented in Section 5.5 which showed that the an odd number of poles did not reduce the performance of the LIM.
An option in the design of any LIM is whether to connect the poles of the stator in series or parallel. The advantage of connecting the winding in series is that the current which flows is the same in all the coils whereas in parallel connected windings some of the coils carry more current than others. This is important if the current capability of the wire is near the limit that it can carry without overheating. However, more power may be produced if the windings are connected in parallel. An experiment was conducted to determine the actual effect of the two alternative connection methods and these results showed that a parallel connected LIM has greater power density with no decrease in the power-factor/efficiency product.

The edge effect will cause the square pattern of rotor current shown in Fig. 1.1 to close in to form ovals. The result of the oval pattern (as compared to the more optimum square) is higher losses and lower output power. Three different experiments were conducted to determine the extent of this problem for a steel rotor LIM. These experimental results showed that the edge effect did not decrease the efficiency or power output of the experimental motors.

Space and time harmonics cause higher losses and reduced output. The effect of slot harmonics (the space harmonics) was expected to be more pronounced in the type of motor under investigation
due to the small effective air gap and the non-laminated rotor. Slot harmonic data was obtained from flux measurements. An experiment was devised to compare the performance of a solid steel blade with and without the effect of space harmonics, in order to determine the magnitude of their effect. The space harmonics were found to have some effect on the efficiency of the double sided steel rotor LIM but not a great amount.

The time harmonics were also measured and calculations undertaken to determine the magnitude of their effect. The time harmonics were found to have insufficient amplitude to have an effect on the performance of the LIM.

It was shown analytically in Chapter 2 that the effect of the annular stator would be very small on the performance of the annular LIMs under investigation (i.e. the increased rotor resistivity and the variation in power produced over the face of the motor). To obtain further confirmation of this conclusion, flux plotting was conducted and these values were then used to conduct a simulation based on the measured value of flux. The result of this simulation showed that the effect of the annular motor was more pronounced than the initial analysis indicated but it still did not significantly affect the performance of the LIMs under investigation.
6. FACTORS AFFECTING OPTIMIZATION

6.1 Introduction to Optimization

Optimization of electric motors generally means the maximizing of efficiency. However, in this application the size of the inverter supply (the most expensive component of the machine) is determined by the power-factor/efficiency product. If this value can be maximized while a high power density is obtained then the motor can be said to be optimized for this application. The following sections discuss various factors which can be modified to improve power density or the power-factor/efficiency product.

6.2 Goodness Factor

One criterion which is often used in the design of induction motors is that of goodness factor. The goodness factor is a measure of how well the magnetic and electrical circuits are utilized. The goodness factor is defined as:

\[ G = \frac{\mu_0 c V^2}{p g w} \]  

where \( w \) is the stator frequency. The goodness factor increases with the velocity squared and the thickness of the rotor. It will decrease for greater rotor resistivity, larger air gap and higher frequency. The calculated value of goodness for a machine must also include correction terms to take into account the actual paths of the current and flux [42,51]. Generally speaking the higher the
goodness factor the better the power density and the power-factor/efficiency product.

However, due to the end effects in LIMs, the highest goodness factor achievable is often not optimal. This fact led to the "optimum goodness factor" as defined by Nasar and Boldea [9].

6.3 **Optimum Goodness Factor**

Nasar and Boldea defined the optimum goodness factor as that value of goodness for a given LIM such that it will have zero thrust at zero slip. In this way the performance of a LIM and of a conventional rotary motor are the same. However, in order to make this one characteristic the same it is usually required that the rotor resistance must be increased, the number of poles increased or the air gap increased; in other words to do those things which are known to make for an inferior rotary motor.

The reason why an inferior motor should be produced is to reduce the negative thrust of the longitudinal end effect. It has been shown by Nasar and Boldea and others, that if a LIM is produced with very good performance in the conventional sense then the low slip performance will be quite poor. For this reason a poorer machine will be more efficient and have better power-factor. However, it will also produce less power than it could. The optimum goodness factor for an eight pole motor, about the greatest number of poles usually found in a LIM, is twenty. This
is much lower than for conventional motors which have goodness factors greater than 100. A LIM using the optimum goodness design rule (zero thrust at zero slip) will have some improvement in efficiency but certainly will not require less material and the power density will be less.

**Relative End Effect**

![Graph showing relative end effect force.](image)

**Fig. 6.1 - Relative end effect force.**
6.4 Analytical Results

It is important to observe the thrust due to the longitudinal end effect at higher slips. In Fig. 6.1 [9] it can be observed that, if the motor operates at higher than typical slips, say ten percent, then the negative thrust due to the longitudinal end-effect is actually less for a machine with a goodness factor of forty-two than for one with the optimal factor of twenty. If the analysis included a motor with goodness factor of an even higher value, then the end effect parasitic thrust will go to zero. From the above example it is clear that the "optimum goodness-factor criterion" is not optimum for all LIMs as implied in the analysis of Nasar and Boldea. The original hypothesis, however, is still true that if the motor is to run very close to synchronous speed then the motor should be designed according to the optimum-factor goodness criterion.

From a design point of view, the double sided LIM will be flux limited; the stator steel will be driven to its magnetic saturation point. The other case is the single sided LIM which will be current limited due to the smaller area to place the copper windings on the stator (only one stator). If a constant current design is used then a higher resistivity rotor, for example, will produce greater power at lower slip [8]. The constant current limitation may be important from the design point of view for the large air gap single sided machine, which is admittedly a common and important machine. For that type of machine the stator
resistive losses and cooling problems can be significant. However, for the constant voltage double sided machine the saturation of the stator steel, and not the stator resistive losses, is the limitation on the power density. The double sided machine with small air gap can have very good power to active surface area ratio and has low leakage inductance and stator winding losses. For the thin steel rotor machine a double sided LIM is the only practical design due to the attractive forces and the requirement of a return path for the magnetic flux.

Figure 6.2 shows the speed vs power curves for an optimally designed EM#1 (by goodness factor) and for the same LIM with a lower value of rotor resistivity. The optimized machine has such a high rotor resistivity that it produces very little power at low slip (although the end effect thrust is also low) and produces a peak power of only 50 kw at 50% slip and 20 kw at 20% slip. The goodness factor is at the optimum value of ten. However when a far more conductive rotor material is used (resistivity= 27 \(\mu\)ohm-cm; alloyed steel) then the power peaks at 100 kw and is 40 kw at 20% slip. For an actual machine the core loss is 4 kW so that the optimized machine is actually less efficient than the motor with the high goodness factor.
Fig. 6.2 - Power for various rotor resistances.

6.5 Optimization By Scaling Up

In Section 5.3 various rotor materials were tested to find the optimum rotor material with respect to maximum power-factor/efficiency product and maximum power density. Using scaling factors [54] it is possible to scale up the results of the smaller
motor experiments (Experimental Motor #1) and use them to predict some of the performance characteristics of the full size machines (Experimental Motors #2, #3 and #4).

The most fundamental of these factors is the Goodness Factor described in Section 6.2. The equation for the Goodness Factor is shown again in (6.2).

\[
G = \frac{\mu_0 c V^2}{pgw}
\]  

(6.2)

An example for a scaling up between the optimum conditions found for Experimental Motor #1 to the full size machine, Experimental Motor #4, will now be described. The Experimental Motors' parameters can be found in Tables 4.1 and Table 4.4. The parameters found in (6.2) are shown in Table 6.1. The objective

<table>
<thead>
<tr>
<th>E M #1</th>
<th>E M #4</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>1 mm</td>
</tr>
<tr>
<td>V</td>
<td>50 m/s</td>
</tr>
<tr>
<td>p</td>
<td>1.7 (\mu) cm</td>
</tr>
<tr>
<td>g</td>
<td>4 mm</td>
</tr>
<tr>
<td>freq</td>
<td>400 Hz</td>
</tr>
<tr>
<td>G</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 6.1 - Comparison of Motor Parameters
will be to obtain the same Goodness factor for the full size machine as for the smaller prototype. Using the above parameters the Goodness Factor was found to be approximately 2.4 times greater for the full size machine. To reduce the Goodness Factor, the most reasonable parameter to adjust is the rotor thickness since the final objective is to produce the thinnest rotor LIM. Experimental Motor #4 would produce more power and have a higher power-factor/efficiency product if it had a thinner rotor (of approximately 1.22 mm) in order to reduce the Goodness Factor to the more optimum value found in the scale model experiments. (Unfortunately the mechanical design of the full size machine did not allow for such a thin rotor to be tested.)

The actual value of the efficiency and the power-factor for the Optimum Goodness LIM can be estimated from the experimental results conducted on the smaller scale model motor. Both of these values will increase slightly for the larger machine (as is the case for all larger electrical machines) but the exact value cannot be determined using the techniques presented in this thesis.

6.6 **Series and Parallel Connection Optimization**

As was mentioned in the introduction of this chapter, optimization for the sawing application means obtaining a high power-factor/efficiency product while also maintaining a high power density. The power density is of critical importance in the
sawing application because a longer motor means a smaller depth of cut and a less powerful motor means that the saw would have to either cut slower or again decrease the depth of wood which it cuts. The connection of the stator poles in parallel has been shown in Chapter 5 to produce more power and thus increase the power density at the same or greater power-factor/efficiency product as the series connection.

Two approaches are possible for the design of the parallel connected stator. The first uses heavier copper windings and wider slots for the entry pole of the LIM and the second uses twice as heavy wire (and twice as deep slots) as would be required for the series connected machine. The first alternative would require the use of three different slot punches and two different coil wire diameters. The second alternative is to use an extra deep core and punch in deeper slots which would than be wound with a gauge of wire throughout the core which was heavy enough to carry the current which occurs in the entry coils. These are shown in Fig. 6.3 along with the equivalent series connected stator. Of the possible designs the extra deep slots would be the less costly to manufacture.
6.7 General Comments On Optimization

The effects of different parameter variations have been analyzed and experimental results have been obtained, some of which have been shown to be important in the design of the annular, high speed, thin steel rotor, double sided LIM.
The most important of the factors are the magnetic attractive forces analyzed in Chapter 3, the parallel/series connection of the stators and the resistivity of the rotor. A less important factor is the air gap flux harmonic caused by the slots. On the other hand, some factors which were originally thought to affect the performance and therefore the design of this type of machine but did not in fact have a significant effect. These are: the edge effect, supply harmonics, re-entry and the annular stator.

Scale model testing of the annular LIM was shown to be a valuable tool in determining the expected values for efficiency, power-factor and output power including frictional losses. The scale model LIM also allowed for testing of some rotor materials which was not possible on the full size machine. In addition, the optimum value of goodness found for the small scale machine can be applied to the design of the full size machine.

The answer to the question as to what constitutes the optimum design of a LIM is still very difficult and that answer must be decided by looking at all the parameter restrictions which are being placed on the machine by the particular application before general optimization criteria are applied to the design.
7. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

7.1 Summary
A type of induction motor with unusual parameters has been analyzed and the results presented in this thesis. The induction motor has a thin steel rotor, a double sided stator, an annular shape and is operated at high speed. This type of motor has been analyzed for the purposes of developing new types of machines for use in sawmill and mining applications. There may also be other applications for this type of motor in material handling or metal processing industries.

Previous theories and formulas developed by other authors are outlined in chapter two and have been used in the analysis of these machines. A one-dimensional linear induction motor model was developed which was implemented in both a Fortran program and a spreadsheet version. This analysis was supplemented with extensive experimental results including flux plotting both longitudinally and transversely for the stators.

In addition, new and original work has been presented;
- a new annular disc motor resistivity correction factor (p. 24)
- analysis of the effects of poles in parallel or series in linear induction motors. (Chapt. 5)
- experimental comparisons between odd and even pole designs
- a second optimum goodness consideration for LIMs which has
not previously been considered (p.123)
- rotor/stator attractive force analysis for magnetic rotor double sided motors and description of the flux (crenelated flux) (p. 34)
- a criterion for when the re-entry effect may occur (p. 29)
- the introduction of the use of spreadsheets for machine design and analysis (Appendix 4)

7.2 Conclusions
The annular motor was analyzed using the electromagnetic field analysis equations in cylindrical co-ordinates. In addition an annular resistivity correction factor was developed and applied to one hypothetical and two experimental motors. It can be concluded that the annular stator has a measurable effect on performance if the stator width is greater than half its mean radius. For the experimental motors constructed, the annular stator will have an almost negligible effect on performance.

The possibility of a re-entry effect was analyzed and found not to occur for the experimental machines. When one reviews the analysis, it is clear that the only time that a re-entry effect will ever occur (for physically realizable machines) is when two sets of annular stators are used to drive a very high speed rotor (greater than 140 m/s) and the exit point of one stator is very close (within a few centimeters) to the entrance of the other.
The reluctance normal force was analyzed by looking at the flux paths and saturation conditions. The analysis first identified the source of the large attractive force (the crenelated flux), then gave the peak attractive force and the modulation of the peak attractive force, in space, along the stator. Once these values were determined, the relationship between the maximum value of attraction and the rotor position was analyzed and the resulting relationship for the experimental motor was found. Finally the maximum normal force due to the rotor currents (electromagnetic normal force) was calculated and shown to be small in relation to the attractive force.

The analysis was then confirmed experimentally and applied to obtain the expected value for drag during the full load operation of the motor.

The normal force, which will always occur in the double sided steel rotor LIM, has the potential to create large losses even with low friction guiding surfaces. With the proper design of the LIM, as shown in the analysis which describes the "V" shaped curves, it should be possible (by increasing the air-gap) to reduce the drag to a more acceptable value than that measured in the experiments.

Various experimental motors and apparatus were constructed in
order to investigate the annular disc LIM motors operating characteristics. One motor was used to test the effect of different rotor materials and to measure the flux variation in the radial direction. Three other motors, based on the same stator core, were used to determine the effect of odd and even number of poles and the effect of series and parallel connected poles and to measure full size motor performance. A completely annular type motor was used to determine the magnitude of the end effect.

The experimental apparatus which was used to load the machines and the instruments used to make the measurements was described. The experimental accuracy of all measurements except electromagnetic thrust were accurately obtained.

Various experiments were conducted to determine the effect of different parameter variations and to experimentally determine the effect of the steel rotor on the performance of a high speed annular LIM.

One parameter that was investigated was the rotor material. Although none of the other rotor materials tested would produce a practical alternative to steel at the present time, the results of the experiments are very useful in that they show potential areas of improvements in the design of this type of machine and the results of these experiments can used with scaling factors to find a
more optimum design. This section also allowed for the direct comparison between simulated and actual results. The simulation when corrected for friction and rotor resistivity agreed well with the actual experimental results.

The choice of whether to use an odd or even number of poles does not occur in standard rotary induction motors which must all have an even number of poles. However, in the LIM there is no such requirement since the entry pole does not have to be of the opposite polarity of the exit pole. It is thus possible, and sometimes done, to have an odd number of poles. The result of an experiment to compare the effect of odd or even number of poles was presented which showed that the an odd number of poles did not reduce the performance of the LIM.

An option in the design of any LIM is whether to connect the poles of the stator in series or parallel. The advantage of connecting the winding in series is that the current which flows is the same in all the coils whereas in parallel connected windings some of the coils carry more current than others. This is important if the current capability of the wire is near the limit that it can carry without overheating. However, more power may be produced if the windings are connected in parallel. An experiment was conducted to determine the actual effect of the two alternative connection methods and these results showed that a parallel connected LIM has greater power density with no de-
crease in the power-factor/efficiency product.

The edge effect will cause the square pattern of rotor current to close in to form ovals. The result of the oval pattern (as compared to the more optimum square) is higher losses and lower output power. Three different experiments were conducted to determine the extent of this problem for a steel rotor LIM. These experimental results showed that the edge effect did not decrease the efficiency or power output of the experimental motors.

Space and time harmonics cause higher losses and reduced output. The effect of slot harmonics (the space harmonics) was expected to be more pronounced in the type of motor under investigation due to the small effective air gap and the non-laminated rotor. Slot harmonic data was obtained from flux measurements. An experiment was devised to compare the performance of a solid steel blade with and without the effect of space harmonics, in order to determine the magnitude of their effect. The space harmonics were found to have some effect on the efficiency of the double sided steel rotor LIM but not a great amount.

The time harmonics were also measured and calculations undertaken to determine the magnitude of their effect. The time harmonics were found to have insufficient amplitude to have an effect on the performance of the LIM.
It was shown analytically that the effect of the annular stator would be very small on the performance of the annular LIMs under investigation (i.e. the increased rotor resistivity and the variation in power produced over the face of the motor). To obtain further confirmation of this conclusion, flux plotting was conducted and these values were then used to conduct a simulation based on the measured value of flux. The result of this simulation showed that the effect of the annular motor was more pronounced than the initial analysis indicated but it still did not significantly affect the performance of the LIMs under investigation.

The effects of different parameter variations was analyzed and experimental results obtained, some of which were shown to be important in the design of the annular, high speed, thin steel rotor, double sided LIM.

The most important of the factors are the magnetic attractive forces, the parallel/series connection of the stators and the resistivity of the rotor. A less important factor is the air gap flux harmonic caused by the slots. On the other hand, some factors which were originally thought to affect the performance and therefore the design of this type of machine but did not in fact have a significant effect. These are: the edge effect, supply harmonics, re-entry and the annular stator.
Scale model testing of the annular LIM was shown to be a valuable tool in determining the expected values for efficiency, power-factor and output power including frictional losses. The scale model LIM also allowed for testing of some rotor materials which was not possible on the full size machine. In addition, the optimum value of goodness found for the small scale machine can be applied to the design of the full size machine.

7.3 Recommendations For Further Research

The machines which were analyzed in this thesis may have many commercial applications. The analysis presented is the first to deal with these particular machines with these unusual parameters and so not all areas could be covered in sufficient depth. In particular the possibility that a second goodness value may exist would in itself be a complete PhD thesis topic and one which would be very critical to the design of future machines of the type described. In addition, the effect of slot harmonics on the performance of this type of machine and the analysis of tooth shape and stator cross gap alignment would be another suitable topic. This last topic has been debated by other authors but has never been analyzed in detail. A further topic area is in the metallurgical field were the properties of suitable high conductivity and high strength materials for use in LIM driven saws should be studied.
REFERENCES


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Appendix 1
Derivation of Field Theory Equations [29]

The LIM is divided into areas as shown in Fig. 2.3. In one-di­mensional analysis it is assumed that the EMF induced in the secondary by the primary exists only directly under the stator \((y \leq l_g/2)\) and is normal to the x-axis. Also, the currents directly under the stator flow only in the y-direction. In this model the current and flux densities are functions of the x component only. The following three equations describe these conditions.

\[-\frac{db}{dx} = \mu_0/g^*(c^*j+J) \quad (2.1)\]

- the change in flux along the motor is equal to the current density flowing in the stator and rotor

\[\frac{dI}{dx} = c^*j \quad (2.2)\]

- the change in current flowing along the return paths of the rotor is equal to the current density flowing in the rotor

\[\frac{dj}{dx} - 2I/(l_g*S_r) = -(i*w*b + Vdb/dx)/p \quad (2.3)\]

- the change in current density along the rotor minus the effect of the resistance is equal to the transformer and the speed
induced voltage divided by the secondary resistivity

When equations (2.1), (2.2) and (2.3) are combined, a third order differential equation (2.4) is obtained which can be solved from the boundary conditions and the given surface current density.

First write out equations (2.1), (2.2) and (2.3)

\[-db/dx = \mu_0/g^*[(c^*j)+J] \quad (2.1)\]

\[dI/dx = c^*j \quad (2.2)\]

\[dj/dx - (2/(l_s S_r))*I = -(i*w*b)+(V*db/dx)/p \quad (2.3)\]

Then write out the terms of equation (2.3)

\[dj/dx - 2/(l_s S_r)*I = -i*w*b/p - V/p*db/dx \quad (2.4)\]

Taking the first derivative of (2.4)

\[d^2j/dx^2 - 2/(l_s S_r)*dI/dx = -i/p*w*db/dx - V/p*d^2b/dx^2 \quad (2.5)\]
Taking the second derivative of equation (2.1)

\[-d^2 b/dx^2 = \mu_o * c/g * \frac{dj}{dx} + \mu_o/g * dJ/dx\]  \hspace{1cm} (2.6)

Now taking the second derivative

\[-d^3 b/dx^3 = \mu_o * c/g * d^2 j/dx^2 + \mu_o/g * d^2 J/dx^2\]  \hspace{1cm} (2.7)

Obtain an expression from (2.7) for \(d^2 j/dx^2\)

\[d^2 j/dx^2 = -g/\mu_o/c * d^3 b/dx^3 - 1/c * d^2 J/dx^2\]  \hspace{1cm} (2.8)

Obtain an expression for \(j\) by rearranging (2.1)

\[j = -g/\mu_o/c * db/dx - J/c\]  \hspace{1cm} (2.9)

Substitute expressions (2.2) and (2.8) into equation (2.5)

\[d^2 j/dx^2 - 2/(l_s S_r) * dI/dx = -1/p * i * w * db/dx - V/p * d^2 b/dx^2\]  \hspace{1cm} (2.10)
\[-g/(\mu_0 c) d^3 b/dx^3 - 1/c d^2 J/dx^2 - 2/(l_s S_r) c^* j = \]
\[-1/p i^* w^* db/dx - V/p d^2 b/dx^2 \]

(2.11)

Substitute (2.9) into (2.11)

\[-g/(\mu_0 c) d^3 b/dx^3 - 1/c d^2 J/dx^2 - 2/c/(l_s S_r) \]
\*[\(-g/\mu_0 c^* db/dx - 1/c^* J\)] = \[-1/p i^* w^* db/dx - V/p d^2 b/dx^2 \]

(2.12)

Expand to get:

\[-g/\mu_0 c d^3 b/dx^3 - 1/c d^2 J/dx^2 + 2 g/(l_s S_r \mu_0) db/dx \]
\[+ 2 J/l S_r = 1/p i^* w^* db/dx - V/p d^2 b/dx^2 \]

Gathering together terms:

\[-g/\mu_0 c d^3 b/dx^3 + V/p d^2 b/dx^2 + (2 g/l S_r \mu_0 + i^* w/p) db/dx = \]
\[1/c d^2 J/dx^2 - 2 J/l S_r \]

Multiply through by \((-\mu_0 c/g)\) to get:

\[d^3 b/dx^3 - V^* \mu_0 c/g p d^2 b/dx^2 - (2 c/(l_s S_r) + \mu_0 c^* i^* w/g/p) db/dx = \]
\[-\mu_0 g^* d^2 J/dx^2 + 2 \mu_0 c^* J/(l_s S_r) g \]

(2.13)
$$\frac{d^3b}{dx^3} - V*\mu_0*c/(p*g)d^2b/dx^2 - i*w*\mu_0*c/(p*g) + (2*c/(l_s*S_r))db/dx = -\mu_0/g(d^2J/dx^2 - (2c/l_s*S_r)*J)$$  \hfill (2.14)

Equation (2.14) is then solved to obtain the magnetic field value. The general solution of this differential equation is of the form:

$$b_0 + b_1 \exp(r_1)$$

and the particular form is:

$$b_p = -i\mu_0 J/(kg) \cos\phi \exp(i\phi)$$

$$= -i B_0 \cos\phi \exp(i\phi)$$

The thrust can then be calculated from the stator surface current and the rotor magnetic field as given by the equation:

$$F = \int_{-\frac{1}{2}l_s}^{\frac{1}{2}l_s} \operatorname{Re}(J*b) \, dx$$

The thrust developed by the motor is separated into two components. $F_0$ is the thrust described by standard rotary induction motor theory and $F_1$ is the thrust produced by the parasitic end
effects found in linear induction motors.

\[ F_0 = \frac{1}{2} J_m B_0 l_s p L \cos \phi \sin \phi \]  
\( (2.15) \)

\[ F_1 = -i J_m B_0 l_s \text{Reall}\left(\frac{b_1}{b_p}\right)\left(\frac{b_p}{B_0}\right)k_1 \]  
\( (2.16) \)

\[ b_1 = \left[ V \alpha + k \tan \phi (1 - i*v/k)/(r_1 - v - V*\alpha) \right] b \]

\[ B_0 = \mu_0 J/(k*g) \]

\[ b_p = -iB_0 k^2/(k^2 + i*s*w*\alpha) \]

\[ r_1 = V*\alpha/2[1 - 1 + 4* (i*w*\alpha + v^2)/(V*\alpha)^2] \]

\[ V = \text{velocity} \]

\[ v = 4/\text{stator width/total rotor overhang} \]

\[ \alpha = \mu_0 c/p/g \]

\[ p = \text{rotor resistivity} \]

\[ K = [1 - \exp(r_1 + ik)\eta]/(r_1 + ik) \]

\[ \phi = \tan^{-1}(s*w*\alpha/k^2 + v^2) \]

\[ L = \text{length of stator} \]

\[ J_m = \text{maximum primary current density} \]

\[ l = \text{width of stator} \]
This is a linear induction motor (LIM) simulation program based on the steady-state analysis which is summarized in Dr. Poloujadoff's book.

For this simulation the magnetic flux begins at the entry of the motor and trails from the exit end.

This section will define all the variables used in the program.

- \( v_{not} \):
- \( v_{rotsq} \):
- \( v_{el} \):
- \( v_{elsyn} \):
- \( f_{req} \):
- \( f_{frad} \):
- \( b_{prime} \):
- \( b_{one} \):
- \( b_{two} \):
- \( \alpha \):
- \( \rho_{one} \):
- \( \rho_{two} \):
- \( j_{st} \):
- \( \kappa \):
- \( \mu_{not} \):
- \( \sigma_{o} \):
- \( \sigma_{row} \):
- \( \sigma_{ee} \):
- \( \sigma_{row} \):
- \( \sigma_{ gaps} \):
- \( \sigma_{el} \):
- \( \sigma_{br} \):
- \( \sigma_{bnot} \):
Define the types of variables for the program.

Real*8 vnot, vnotsq, vel, velsyn, freq, frad, alpha, jm, kay, mewnot,
+cee, row, gap, el, elpr, slip, lambda, psi, length, x, force,
+force1, force2, dreal, force0, force1, force2, forces, bnot,
+rcalc1, rcalc2, rcalc3, rcalc4, rcalc5, rcalc6, rcalc7, flbs,
+polen, forct0, forct1, forct2
complex*16 bprime, bone, btwo, rone, rtwo, btotal, cdsort,
+cdexp, negeye, poseye, calc1, calc2, calc3, calc4, calc5, test1, test3
character*64 result, greslt

*****Open a file for results*****
open(10, file='result')
open(11, file='greslt')

*****Set parameter values for temporary testing. *****
These must all be specified.
freq=396.
jm=8.102d4
cee=.00200
row=35.0d-8
gap=.00300
el=.0570
elpr=0.108
lambda is the wavelength
lambda=.1460
polen is the number of poles
polen=4.0
Set physical constants
mewnot=12.56628d-7
Calculate other motor parameters not specified
D Line# 1 7
60 alpha=mewnot*cee/row/gap
61 length=polen/2.0*lambda
62 velsyn=freq*lambda
63 frad=6.28318*freq
64 c Calculate all other variables.
65 vnotso=4.0/el*(elpr-el)
66 vnot=dsqrt(vriot*SQ)
67 kay=6.28314/lambda
68 c Define math constants.
69 nepeye=(0.0,-1.0)
70 poseye=(0.0,1.0)
71 c Set initial values to 0.0
72 force=0.0
73 forcet=0.0
74 c Set up output table
75 write(10,004)
76 004 format(1x,'THESE ARE THE RESULTS OF A 1-DIMENSIONAL LINEAR',
77 +' INDUCTION MOTOR SIMULATION')
78 write(10,005)
79 005 format(1x,'Frea', 'Amps/M', 'Rotor Thk', 'Resist', 'Gap',
80 ' S.Width', ' R.Width', ' W.Len.', '#poles')
81 write(10,006) freq, jm, cee, row, gap, el, elpr, lambda, polen
82 006 format(1x,f5.0,f7.0,1x,f7.4,1x,e9.2,1x,f6.3,1x,f6.3,1x,
83 +f6.3,3x,f6.3,4x,f4.1)
84 write(*,050)
85 write(10,050)
86 050 format(1x,'Slip', ' Horsepower', ' F0', ' F1', ' F2',
87 '+ Thrust(N)', ' Thrust(lbs)')
88 c ***** start main program routine *****
90    do 700 j=10,100,10
1 91    forct=0.0
1 92    forct0=0.0
1 93    forct1=0.0
1 94    forct2=0.0
1 95    vel=real(j)/100.*velsyn
1 96    slip=1.0-kay*vel/frad
1 97    psi=datan(slip*frad*alpha/(kay**2+vnotsq))
1 98    bnot=mewnot*jm/kay/gap
1 99    bprime=(negeye*mewnot*jm/gap/kay)*((kay**2+vnotsq)/
1 100    + (kay**2+vnotsq+ (poseye*slip*frad*alpha))
1 101    rone=vel*alpha/2.0*(1.0-cdsqrt(1.0+4.0*((poseye*frad*alpha
1 102    +vnotsq)/(vel*alpha)**2))))
1 103    rtwo=vel*alpha/2.0*(1.0+cdsqrt(1.0+4.0*((poseye*frad*alpha
1 104    +vnotsq)/(vel*alpha)**2))))
1 105    bone=bprime*(((vel*alpha)+(kay*dtan(psi))*(1.+negeye*vnot
1 106    +/kay))/(rone-vnot-vel*alpha))
1 107    calc1=negeye*psi
1 108    calc2=poseye*psi
1 109    btwo=bprime*(rone+negeye*dsin(psi))*(rone-negeye*Cexp
1 110    +(calc1))/((rtwo-rone)/dcos(psi)*cexp(calc2))
1 111    WRITE(*,205)BPRIME,BONE,BTW0
1 112    C 205 FORMAT(1X,'bprime, bone, btwo',2E14.7)
1 113    C write(*,206)rone,rtwo
1 114    C 206 format(1x,'rone=',2e14.7, ' rtwo=',2e14.7)
1 115    do 500 i=1,99
2 116    C write(*,201)i
2 117    C 201 format(1x,i3)
2 118    x=real(i)/100.*length
D Line#  1    7
  119   calc1=negeye*kay*x
  120   calc2=one*x
  121   calc3=rtwo*(x-length)
  122 c   write(*,225)calc1,calc2,calc3
  123 c 225 format(1x,'calc1=',2e14.7,1x,'calc2=',2e14.7,'calc3=',2e14.7)
  124   btotal=bone*cexp(calc2)+
  125   +btwo*cexp(calc3)
  126   calc1=negeye*kay*x
  127   test1=cexp(calc1)
  128   test3=jm*test1*btotal
  129 c   write(*,203)test1,test3
  130 c 203 format(1x,'This is test1 and test3',4e14.7)
  131   test2=dimag(test3*poseye)
  132 c   write(*,202)test1,test2
  133 c 202 format(1x,3e14.7)
  134 c   write(*,210)el,jm,calc1,btotal,length
  135 c 210 format(1x,7e14.7)
  136 c ******** calculate the force produced ********
  137 c ******** calculate force with no end effects ***
  138 c   force0=bnott+jm*dcos(psi)*dsin(psi)*.005*el*length
  139 c   force1=dimag(poseye*bone*cexp(rone*x))*jm*cexp(negeye*kay*x))
  140 c   **.005*el*length
  141 c   force2=dimag(poseye*btwo*cexp(rtwo*(x-length))*jm*cexp
  142 c   +(negeye*kay*x))*.005*el*length
  143 c ******** try a different way March 16 ********
  144 c ******** calculate entrance effect force *****
  145 c   rcalc1=dimag(poseye*rone*x)
  146 c   rcalc2=dimag(rone*x)
  147 c   rcalc6=dimag(poseye*negeye*kay*x)
  148 c   rcalc7=dimag(negeye*kay*x)
  149 c   rcalc3=dimag(poseye*bone)
rcalc4 = dimag(bone)
rcalc5 = datan2(rcalc3, rcalc4)
rcalc3 = datan2(rcalc1, rcalc2) - datan2(rcalc6, rcalc7) + rcalc5
force1 = .005 * e1 * length * cdabs(bone) * dexp(rcalc1)
rcalc5 = datan2(rcalc3, rcalc4)
rcalc3 = datan2(rcalc1, rcalc2) - datan2(rcalc6, rcalc7) + rcalc5
force2 = .005 * e1 * length * cdabs(btwoc) * dexp(rcalc1)

forces = force0 - force1 - force2
forcet = forct + forces
forct0 = forct0 + force0
forct1 = forct1 + force1
forct2 = forct2 + force2
force3 = .005 * e1 * test3 * length
forcet = forct + force
write (*, 600), forces, force0, force1, force2,

force3 = test3 * length
forcet = forct + force
write (*, 600), forces, force0, force1, force2,
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178  continue
179     flbs=forcet/4.448
180     forct0=0.5*el*length*bnot*dcos(psi)*dsin(psi)*jm
181     write(*,610)forcet, flbs
182     c 610 format(1x,'Forcet=',e13.6,' Neut. ',e13.6,' lbs.')
183     power=vel*3.281*flbs/550.0
184     write(*,615)slip, power, forct0, forct1, forct2, forct, flbs
185     write(10,615)slip, power, forct0, forct1, forct2, forct, flbs
186     615 format(1x,f4.2,1x,f10.3,5x,f6.1,1x,f6.1,1x,f6.1,1x,f11.2,f12.2)
187     write(11,650)slip, vel, power, flbs, forct
188     650 format(1x,5f10.2)
189     write(*,617)el, elpr, freq, lambda, alpha, vel
190     617 format(1x,6f10.3)
191     write(*,620)forct0, forct1, forct2
192     620 format(1x,' Thrust0=',f7.3,' Entrance effect=',f7.3)
193     +' Exit effect=',f7.3)
194     700 continue
195     close(10)
196     close(11)
197     stop
198     end
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### Appendix 3

#### Spreadsheet Simulation Program

**ANALYSIS OF LIN PERFORMANCE - MOTOR TYPE & MODEL NO.:** Experimental Motor #3

**File:** VOLTAGE 03-Mar-87

Note: To obtain a printout of this file enter "F7* then "Print".

#### SPECIFIED PARAMETERS

Ref: Effect of Annular Stator 1.0045

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<tr>
<td>Supply Frequency: (f_0) (Hz)</td>
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#### CALCULATED PARAMETERS

- **Electrical Parameters**

  RMS Voltage/Phase: \(E\) (V) = 254.0

  Peak Stator Surface Current Density: \(J_m\) (A/m²) = 5.23E+04

  RMS Magnetizing Current/Phase: \(I_m\) = 41.3

  Inductance/Phase: \(L_m\) (H) = 2.93

  Fringing Factor: \(fr\) = 2.33

  Effective Stator-Stator Airgap: \(G_e\) (\(m\)) = 2.85E-03

  Goodness Factor: \(G\) = 7

- **Geometrical Parameters**

  Stator Length: \(D_s\) (\(m\)) = 0.876

  Wavelength: \(\lambda\) (\(m\)) = 1.75E-03

  Pole Pitch: \(p\) (\(m\)) = 8.76E-02

  No. of Slots/Phase Belt: \(n\) = 2.0

  Synchronous Velocity: \(v_s\) (m/s) = 63

  Synchronous Frequency: \(f_s\) (Hz) = 72.0

  Synchronous RPM: \(SRPM\) = 4320

- **Peak Flux and Flux Densities**

  Peak Air Gap Magnetic Field: \(B_0\) (Tesla) = 0.642

  Total Flux/Pole: \(\phi\) (Wb) = 1.22E-03

  Average Flux Density: \(B_{av}\) (Wb/m²) = 0.409

  Average Tooth Flux Density: \(B_t\) (Wb/m²) = 0.817

  Average Core Flux Density: \(B_c\) (Wb/m²) = 1.183

- **Coil Parameters**

  Pitch Factor: \(K_p\) = 0.9659

  Distribution Factor: \(K_d\) = 0.9659

  Winding Factor: \(K_w\) = 0.9330

  Fill Factor: \(F_f\) = 0.3423

- **Copper Losses in Windings**

  Length of Coil/Turn: \(l_c\) (\(m\)) = 2.87E+01

  Resistance/Winding of \(N_s\) Turns: \(R_{w}\) (ohms) = 0.819

  Current/Winding: \(I_w\) (A) = 4.1

  Total Copper Losses: \(W_{cu}\) (KW) = 0.452

- **Other Constants**

  \(\alpha\) = 3.6

  \(v_0\) = 2315.9

  \(k\) = 35.9
### THRUST AND POWER CALCULATIONS

- **Main Thrust** = \( F_0 \)
  \[ F_0 = 14\pi \times \omega \times \sqrt{2 \times \cos(\psi) \times \sin(\psi)} \times k_1 \times \cos(\psi) \times \sin(\psi) \]
  \[ k_1 = 499,481 \]
  \[ \tan(\psi) = \frac{2\pi \times \# \times \alpha}{k_2 + v_2} \]

- **Entrance Thrust** = \( F_1 \)
  \[ F_1 = \frac{\omega}{4} \times \pi \times k_1 \times (1 - \exp((-\psi) \times k_1)) \]
  \( \psi = \text{not calculated, but is usually small} \)

- **Optimum Rotor Resistivity for Maximum Thrust as a Function of**
  \( s(x) = \frac{4.15E-08 \times 8.29E+07 \times 1.24E-07 \times 1.66E-07 \times 2.07E-07 \times 2.49E-07 \times 2.90E-07 \times 3.32E-07 \times 3.73E-07}{s(x)} \)

### CONSTANT FLUX MODEL

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<th>( \psi )</th>
<th>( J_0 ) (A/m)</th>
<th>( J_1 ) (A/m)</th>
<th>( F_0 ) (N)</th>
<th>( F_1 ) (N)</th>
<th>Ft (lb)</th>
<th>Output HP</th>
<th>Output kW</th>
<th>Be = el ( \times f_1 )</th>
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### CONSTANT CURRENT MODEL

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<th>( J_0 ) (A/m)</th>
<th>( J_1 ) (A/m)</th>
<th>( F_0 ) (N)</th>
<th>( F_1 ) (N)</th>
<th>Ft (lb/in²)</th>
<th>Output HP</th>
<th>Output kW</th>
<th>Be = el ( \times f_1 )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>0.0</td>
<td>0.000 -0.642</td>
</tr>
</tbody>
</table>
A4.1 Introduction to Computer Modelling

This Chapter describes two methods of implementing the equations required to simulate the performance of the linear induction motor. A traditional method, using a Fortran program, is described in Section A4.2. The new method, using a spreadsheet program, is described in Section A4.3. This second method is also described in [39] and [40].

Computers are an important tool in the complex task of designing and analyzing the behavior of motors. Major manufacturers of motors have computer programs to aid in the design of motors and books have been written specifically on the topic [41] or have sections that discuss areas in which computers can be used [42]. Computers are also used to calculate the steady-state and transient performance of motors under unusual conditions [43,44]. These applications of computers for the design and analysis of motors have traditionally relied on the large mainframe computer and usually the Fortran programming language. Recently, with the introduction of more powerful personal computers, it has been possible to do some of the complex tasks of motor design on an inexpensive machine, especially since the Fortran programming language is now available on personal computers [45,46].
Another new tool to emerge for the motor designer is the spreadsheet program. The advantages of using a spreadsheet program include the clear and logical organization of the input data, the ease with which the spreadsheet can be built up to include greater detail of analysis as required, the built in graphics capability, the low cost of the program and the high degree of portability since the program will run on any IBM-PC compatible computer. In addition, many design engineers will already be familiar with the use of a spreadsheet program and, if not, the programs are designed to be easy to learn, with many good manuals also available on the use of particular programs.

For the analysis of induction motors, two versions of a worksheet have been developed; one does a design based on minimum input data, and the second does a simulation based on a complete set of motor parameters. The normal design procedure is to first use the design worksheet to obtain a starting point for choosing the motor parameters and to then use the analysis worksheet to determine if the motor performance can be improved.

A4.2 Fortran Analysis Method

A program to simulate the performance of Linear Induction Motors was written in the Fortran programming language. This program is found in Appendix 2 of the thesis.
The program uses the one-dimensional field theory equations presented in Chapt. 2. There are two versions of the program; one for constant current conditions and one for constant flux. The required input variables are the pole pitch, stator width, number of poles, surface current density, rotor width, rotor thickness, rotor resistivity frequency of the supply and the effective air-gap. The effective air-gap must be precalculated and may be found either by the Carter co-efficient [48] or finite element analysis.

The output data is produced in two forms: tabular form to be read out, and in another form suited for the graphics routines used in the Lotus spreadsheet program. These graphics routines provide the basic X-Y plots very quickly and with a minimum amount of learning time required.

The advantage of programming in Fortran is that it is the most widely used simulation program and has many powerful subroutines available. The disadvantages are that it is very difficult to read data and to output data, to write correct code and to debug.

Some of these disadvantages were not found when a spreadsheet version of the simulation program was written as described below.

A4.3 Spreadsheet Analysis Method

The first spreadsheet program was introduced in 1978 under the
name of VisiCalc (the name is derived from "visible calculator"). The original application for the program was to do financial analysis. In 1983, the Lotus Development Corporation introduced Lotus 1-2-3 which had many more features than previous spreadsheets. The most important of these features, for the motor designer, are the graphics and trigonometric functions. The Symphony program is basically the 1-2-3 program with word processing, data base management and data communications.

A spreadsheet program lays out the computer screen as an array of cells which are labeled by rows (1,2,3..) and columns (A,B,C..)-so that a particular cell will have an address, for example, of A1. Each cell can have a number, a statement or a formula entered into it. In this way information can be entered into some cells, documentation describing what different cells represent can be written into others, and the results of computations performed by the formulas appear in others. The spreadsheets can become very large. The Symphony spreadsheet can accommodate up to two million cells but in actual fact the computer's memory would limit the available number of cells to a much smaller number.

A4.3.1 Problem Formulation
As stated earlier, the advantage of the field theory model is that it provides a good picture of the magnetic fields and current
densities, and is the simplest method of representing the longitudinal end effect which occurs in high speed linear induction motors. The induction motor is approximated as a stator of negligible magnetic reluctance with an infinitely thin surface current and the rotor as a thin plate with a specified volume resistivity. The field theory equations described in Chapter 2 were implemented in the spreadsheet program as described in Section A4.3.2.

A4.3.2 Organization and Documentation of the Calculations

The design and analysis of electrical motors was systemized and documented in detail using the Symphony program. Figure A4.1 shows the design spreadsheet and Figure A4.2 shows the analysis spreadsheet. The section which calculates the parameters of the motor is common to both the analysis and the design spreadsheet and is shown in Figures A4.3 and A4.4. Figure A4.5 shows the table of output values, which each program generates. Both of the spreadsheets are organized into three sections as described below (see Appendix 3 to see the spreadsheet as it normally appears).
### ANALYSIS OF LIM PERFORMANCE - MOTOR TYPE & MODEL NO.: MOTOR #621

File: LIM_DES

21-Jun-86

Note: To obtain a printout of this file:

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**SPECIFIED PARAMETERS**

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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref. DESIGN A</td>
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</tr>
</tbody>
</table>

#### Power Supply

- Rated RMS Line Voltage: $E_l (V) = 220$
- Supply Frequency: $f_0 (hz) = 60$
- Type of Connection: 0 for Delta, 1 for Y = 1
- No. of Phases: $q = 3$

#### Electrical and Magnetic Specifications of Motor

- Rotor: 1 for non-magnetic, 0 for magnetic = 1
- Peak Air Gap Magnetic Field ($s=0$): $B_0 (Wb/m^2) = 0.800$
- Permittivity of Free Space: $\mu_0 (H/m) = 1.26E-06$
- Rotor Electrical Resistivity: $\rho_{ohm-m} = 1.72E-08$
- Number of Poles: $P = 6$

---

#### Motor Geometry

- Rotor Thickness: $c (m) = 4.76E-03$
- Lamination Width: $l (m) = 7.62E-02$
- Rotor Width: $ll (m) = 1.27E-01$
- No. of Stators: $S_n = 2$
- Tooth Depth: $td (m) = 5.56E-02$
- Each Side of Rotor ($m$) = $1.81E-03$
- Stator Back Iron: $dl (m) = 5.00E-02$
- Mean Diameter: $D (m) = 0.470$
- SEC (deg) = 360.0

---

Fig. A4.1 - Design Spreadsheet
ANALYSIS OF LIM PERFORMANCE - MOTOR TYPE & MODEL NO.: MOTOR #621

File: CUR_ANAL

21-Jun-86

Note: To obtain a printout of this file

SPECIFIED PARAMETERS

Ref: DESIGN B

o Power Supply

Rated RMS Supply Current: Io (A) = 300.0
RMS Magnetizing Current: I (A) = 250.0
Supply Frequency: f0 (hz) = 60
No. of Phases: q = 3
Type of Connection: 0 for Delta, 1 for Y = 1

o Motor Geometry

Rotor Thickness: c (m) = 4.76E-03
Each Side of Rotor (m) = 1.81E-03
Lamination Width: l (m) = 7.62E-02
Stator Back Iron: dl (m) = 5.08E-02
Rotor Width: li (m) = 1.27E-01
Mean Diameter: D (m) = 0.470
by Motor: SEC (deg) = 360.0
Span/Pole Pitch): wp = 0.889
Mean Tooth Width: t (m) = 1.37E-02
Mean Slot Width: aw (m) = 1.37E-02
Tooth Depth (m) = 2.54E-02
No. of Stators: Sn = 2

o Electrical and Magnetic Specifications of Motor

Rotor: 1 for non-magnetic, 0 for magnetic = 1
No. of Parallel Coil Groups/Stator/Phase: Np = 6
No. of Turns/Slot: N = 36
No. of Turns/Coil: Nc = 18
No. of Turns in Series: Na = 54
Number of Poles: P = 6

Rotor Electrical Resistivity: rho (ohm-m) = 1.72E-08
Diameter of Copper Wire: dw (m) = 4.12E-03

Permittivity of Free Space: muO (H/m) = 1.26E-06

Fig. A4.2 - Analysis Spreadsheet
CALCULATED PARAMETERS

o Electrical Parameters

Rated RMS Voltage/Phase: \( E \) (V) = 127 = \( E_1 \) for Delta & \( E_1/\sqrt{3} \) for \( Y \)

Peak Stator Surface Current Density: \( J_m \) (A/m) = 7.84E+04 = \( k\cdot Ge\cdot B_0/m_0 \) (At \( s = 0 \))

RMS Magnetizing Current/Phase: \( I_m \) = 267.3 = 4.42E5\( \cdot P^2\cdot Ge\cdot \phi_i/(q\cdot Kw\cdot D\cdot l\cdot Na) \)

Inductance/Phase: \( L_m \) (H) = 1.26E-03 = \( E/(2\pi f_0 I_m) \)

Fringing Factor: \( f_r \) = 1.20 (p. 184 Alger)

Effective Stator-Stator Airgap: \( G_e \) (m) = 9.64E-03 = \( g\cdot(t+sw)/(1+f_r\cdot g) \)

Goodness Factor: \( G \) = 83 = \( \mu_0\cdot c\cdot vs\cdot \pi^2/(\rho\cdot Ge\cdot \omega) \)

o Geometrical Parameters

Stator Length: \( D_s \) (m) = 1.476 = \( D_s/360 \cdot \pi \cdot SEC \)

Wavelength: \( \lambda \) (m) = 4.92E-01 = \( 2\cdot D_s/P \)

Pole Pitch: \( p \) (m) = 2.46E-01 = \( D_s/P \)

No. of Slots/Phase Belt: \( n \) = 3 = \( p/(t+sw)/q \)

Mean Tooth Width: \( t \) (m) = 1.37E-02 = \( p/(2\cdot n\cdot q) \)

Mean Slot Width: \( sw \) (m) = 1.37E-02 = \( sw = t \)

Synchronous Velocity: \( v_s \) (m/s) = 30 = \( \lambda f_0 \)

Synchronous Frequency: \( f_a \) = 20.0 = \( v_s/\pi D \)

Synchronous RPM: \( SRPM \) = 1200 = \( f_a\cdot 60 \)

o Peak Flux and Flux Densities

Total Flux/Pole: \( \phi_i \) (Wb) = 9.55E-03 = \( (2/\pi)\cdot B_0\cdot l\cdot p \)

Average Flux Density: \( B_{av} \) (Wb/m^2) = 0.509 = \( (2/\pi)\cdot B_0 \)

Average Tooth Flux Density: \( B_{ta} \) (Wb/m^2) = 1.019 = \( \phi_i/(n\cdot l\cdot q\cdot t) \)

Peak Tooth Flux Density: \( B_{tp} \) (Wb/m^2) = 1.600 = \( \pi/2\cdot B_{ta} \)

Peak Core Flux Density: \( B_c \) (Wb/m^2) = 1.253 = \( \phi_i/1/dl \) (Divided by 2 for a 360

(Maximum acceptable = 1.5 Wb/m^2)

Fig. A4.3 - Calculated Parameters (First Half)
### Coil Parameters

- **Winding Pitch**: $W_p = 0.8889 = \text{Coil Span/Pole Pitch}
- **Pitch Factor**: $K_p = 0.9848 = \sin(W_p\pi/2)
- **Distribution Factor**: $K_d = 0.9598 = \sin(\pi/(2q))/(n\sin(\pi/2n/q))
- **Winding Factor**: $K_w = 0.9452 = K_pK_d$
- **No. of Turns in Series**: $N_a = 54 = E/(4.44f_0\phi)
- **No. of Turns/Coil**: $N_c = 18 = N_a/N$
- **No. of Turns/Slot**: $N = 36 = 2N_c$
- **No. of Parallel Coil Groups/Stator/Phase**: $N_p = 6$

### Wire Gauge & Copper Losses in Windings

- **Slot Cross-Sectional Area**: $S_a (m^2) = 7.60E-04 = tdaw$
- **Useable Area/Conductor**: $A_w (m^2) = 1.06E-05 = 0.5S_a/N$ (50% Packing Factor)
- **Diameter of Copper Wire**: $d_w (m) = 4.12E-03$ (from Look-Up Table)
- **Length of Coil/Turn**: $l_c (m) = 8.09E-01 = 2\pi d_w p W_p p$
- **Resistance/Winding of $N_a$ Turns**: $R_w (ohm) = 5.66E-02 = N_a l_c \rho_{ohm} 4/\pi d_w^2$
- **Current/Winding**: $I_w (A) = 22.27 = I_m/(S_n N_p)$
- **Total Copper Losses due to $I_m$**: $W_{cu} (KW) = 1.01E+00 = I_w^2R_w N_p S_n q/1000$ (3 phases)
- **Current Density in Wire**: $J_w (A/cm^2) = 167 = 4I_w/(\pi d_w^2)$ (Maximum Acceptable

---

Fig. A4.A4 - Calculated Parameters (Second Half)
THRUST AND POWER CALCULATIONS

- Main Thrust = FO
  \[ FO = l \times Jm \times B0 \times D/2 \times \cos(\psi) \times \sin(\psi) \times k1 \times \cos(\psi) \times \sin(\psi) \]
  \[ k1 = \frac{3075.007}{\tan(\psi)} \]
  \[ \alpha = \frac{2 \times \pi \times s 	imes F0 \times \alpha}{(k^2 + \nu_0^2)} \]

- Entrance Thrust = F1
  \[ F1 = Jm \times l/2 \times \text{Re}(b1 \times (1 - \exp((r1 + i \times k) \times D)) / (r1 + i \times k)) \]

- Exit Thrust is not calculated, but is usually small.

Optimum Rotor Resistivity for Maximum Thrust as a Function of Slip

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<th>(a) (°)</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
</tr>
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<tr>
<td>(\rho) (ohm-m)</td>
<td>9.77E-09</td>
<td>1.95E-08</td>
<td>2.93E-08</td>
<td>3.91E-08</td>
<td>4.89E-08</td>
<td>5.86E-08</td>
<td>6.84E-08</td>
<td>7.82E-08</td>
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CONSTANT FLUX MODEL

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<th>Slip</th>
<th>(v) (m/s)</th>
<th>(\psi)</th>
<th>(Jm) (A/m)</th>
<th>(Im) (A)</th>
<th>(FO) (N)</th>
<th>(F1) (N)</th>
<th>(Ft) (N)</th>
<th>(Ft) (lb)</th>
<th>(Ft) (lb/in^2)</th>
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<td>0.000</td>
<td>73179</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.00</td>
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<tr>
<td>2%</td>
<td>28.9</td>
<td>0.223</td>
<td>75045</td>
<td>256</td>
<td>699</td>
<td>0</td>
<td>699</td>
<td>157</td>
<td>0.90</td>
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<tr>
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<td>0</td>
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<td>314</td>
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<td>6%</td>
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<td>337</td>
<td>2795</td>
<td>0</td>
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<td>628</td>
<td>3.60</td>
</tr>
</tbody>
</table>

Fig. 3 - Analysis Worksheet
A4.3.2.1 Specified Parameters

In the first section, there are three sub-sections for entering the parameters on which the analysis is based.

Supply Power: The rated supply current and/or voltage, the frequency, the number of phases, and the type of connection (Y or Delta) are entered in this section.

Electrical and Magnetic Specifications: The parameters entered in this section depend on whether the analysis is of an existing motor or a new motor. In the latter case, only the rotor material, the peak air gap magnetic field, the rotor resistivity and the number of poles are specified, while in the first case the various coil parameters are also entered, as illustrated in Figures A4.1 and A4.2, which are printouts of parts of the Design Worksheet and the Analysis Worksheet.

Motor Geometry: The rotor and stator dimensions and geometry are specified in this area. Again, fewer parameters are needed if a new motor is being designed, as illustrated in Figures A4.1 and A4.2.

Immediately to the left of the cell in which the data is entered, a description of the parameter is entered, along with its abbreviation and the units used. A global cell protection feature can be used to restrict users to entering data only in the desired
cells, thus minimizing the chance of an inexperienced operator accidentally modifying the formulae or the documentation.

A4.3.2.2 Calculated Parameters
In this section of the spreadsheet, the remaining electrical and geometrical parameters and the coil parameters are calculated, along with magnetic fluxes and copper losses in the windings. In the case of a motor design, a wire gauge for the coils is recommended.

In the cell immediately to the left of the parameter value, a description of the parameter, along with its abbreviation and the units used, is placed. Immediately to the right of the parameter value, the actual formula used to calculate the value is recorded, thus providing complete documentation in the spreadsheet and on the printout.

A4.3.2.3 Thrust and Power Calculations
In the final section of the spreadsheet, a complete table of thrust and power, under varying operating conditions, is generated. The calculations required are involved since they contain complex numbers. Symphony does not have the capability of handling complex numbers directly; however, they can be handled as normal calculations by keeping track of the real and imaginary parts of the calculation in separate columns.
Once one set of calculations is formulated correctly, using relative and absolute cell addresses as required, the powerful copy features of Symphony are used to generate a complete table of parameters for printing or for generating graphs. (The terms "relative" and "absolute" refer to the manner in which cell addresses are treated in formulae. For example, if a set of formulae on one line is copied to the line below, all relative cell addresses remain unchanged.)

An example of how the table generation is handled is shown in Table A4.1, where the formulae for two lines of the Thrust and Power Table are printed. Once the first line of the table has been formulated, the remaining lines are immediately generated by the Copy command. Note that when line 88 was copied to line 89, the cells modified with the $ signs are absolute references and are not incremented. On the other hand, cells without the $ are relative addresses and are incremented by one row. This is seen by referring to the formulae in cells F94 and F95.

Note that more columns than expected are used to perform the calculations, because of the complex numbers. The intermediate calculations are used to keep track of the real and imaginary parts of the calculations. The intermediate calculation columns are kept to the right side of the worksheet where they do
not appear as part of the normal viewing area or printout. The results are available in tabular or graphical form. One form of the graphical results is shown in Fig. A4.6. The graphical results can also be displayed simultaneously with the tabular results which is very helpful to students learning about the effect of motor parameter variation on motor performance.

A4.3.3 Input/Output

The spreadsheet programs have good graphics capabilities which can be used to obtain graphs from data produced by other computer programs, for example Fortran programs. The procedure is to write the output results to a separate file using the F10 format code. The resulting file is then imported into the spreadsheet program where the data can be used to produce graphs or be integrated into word processing documents.
A4.4 Comparison With Experimental Result

To verify that the computer model would simulate the performance of LIMs a comparison was made between the experimental results of motor tests and the simulation. One of these comparisons is shown in Fig. A4.7 where the test and simulation results for Experimental motor #3 (this motor is completely described in Chapter 5) are presented. Although the model does not appear to be as accurate as one would expect, the windage loss due to large amounts of water applied to the rotor results in a large unknown drag on the rotor. The model can be refined to obtain increased accuracy, if required, but this was not considered to be important as the model is used as a tool to analyse the effect of parameter
variations in this thesis.

Fig. A4.7 - Comparison Between Simulation (— — —) and Experimental Result (———) For Experimental Motor #3

A4.5 Conclusion

The spreadsheet analysis method has been shown to be a valuable new tool in the analysis of induction machines. The original
simulation program was developed using the Fortran programming language and took approximately two man months to write, debug, obtain graphical output and document. The spreadsheet analysis method required only five man days to obtain the same results. In addition, the spreadsheet program was better documented and could be used by other engineers with very little instruction. The computational time of both programs was the same at approximately 23 seconds. The results from these two simulations are discussed in the next Chapter.

The use of spreadsheet programs and personal computers has resulted in an analysis tool with many new applications. These may include the use of spreadsheets for finite element analysis and transient analysis of electrical machines. When newer versions of the spreadsheet programs with more mathematical functions become available, than even greater numbers of applications will be possible.
ARMCO DI-MAX M-19 CRFP
29 GAGE (.35 mm) THICK

TYPICAL CORE LOSS
& EXCITING POWER
AT 60 AND 400 HERTZ

TEST: AS; 50/50, A347 AND A348
CURVE NO.: 8354

CORE LOSS OR EXCITING POWER - W/LB OR VA/LB (x 2.205 = W/KG OR VA/KG)
Appendix 6

Finite Element Analysis of Experimental Motor #2
(Courtesy of G.E. Dawson, Queens University)
Zoom on (5, 40) - (25, 60)
This Program Does The Analysis of Relative End Effect

Real*8 RELEN,SLIP,GOOD,PAIR,PI,BONE,KN,KP,TEST2
COMPLEX*16 ALPHA,O,ALPHATB,NEGEYE,CALFAOB,CALFATB,TEST1,TEST3,
+TEST4,TEST5,TEST6,CRELEN,TEST7,TEST8
NEGEYE=(0.0,-1.0)
PI=3.1415
GOOD=42.0
PAIR=4.0
DO 100 J=20.42.22
GOOD=J
DO 100 I=0,99,1
SLIP=I/100.0
SLIPSQ=(1.0-SLIP)**2
BONE=SQRT(1.0+(4.0/(GOOD-SLIPSQ))**2)
KP=SQRT(0.5*(BONE+1.0))
KN=SQRT(0.5*(BONE-1.0))
TEST2=0.5*GOOD*(1.0-SLIP)
TEST3=CMPLX(KP,KN)
TEST4=TEST3-1.0
ALPHAOB=0.5*GOOD*(1.0-SLIP)*(TEST3+1.0)
ALPHATB=-0.5*GOOD*(1.0-SLIP)*(TEST4)
CALFAOB=DCONJG(ALPHAOB)
CALFATB=DCONJG(ALPHATB)
TEST5=2.0*PI*PAIR*(CALFATB+NEGEYE)
2 29  TEST6=CDEXP(TEST5)
2 30  TEST1=2.0*PI*PAIR*(CRLFATB+NEGEYE)
2 31  C  WRITE(*,50)PI,PAIR,BONE,KP,KN,TEST2,ALPHATB,CALFATB,NEGEYE,TEST1.
2 32  C  *TEST3,TEST4,TEST5,TEST6
2 33  C  50  FORMAT(1X,'REAL',6F10.3,'COMPLEX',8F14.7,8F14.7)
2 34  C  CRELEN=(NEGEYE*(CALFAOB+SLIP*GOOD)*(TEST6-1.0))
2 35  C  */((1.0+NEGEYE*SLIP*GOOD)*(CALFATB-CALFAOB)*(CALFATB+NEGEYE))
2 36  C  TEST7=(NEGEYE*(CALFAOB+SLIP*GOOD)*(TEST6-1.0))
2 37  C  TEST8=(1.0+NEGEYE*SLIP*GOOD)*(CALFATB-CALFAOB)*(CALFATB+NEGEYE)
2 38  C  RELEND=REAL(CRELEN)
2 39  C  WRITE(*,110)slip.relend
2 40  110  FORMAT(1X,'SLIP=',F10.3,'RELATIVE END EFFECT=',F10.3)
2 41  C  WRITE(*,115)TEST7,TEST8
2 42  115  FORMAT(1X,'TEST7=',2F10.3,'TEST8=',2F10.3)
2 43  100  CONTINUE
2 44  C  STOP
2 45  END

Name   Type           Offset   P Class
ALPHAO  COMPLEX*16   178      INTRINSIC
ALPHAT  COMPLEX*16   210      INTRINSIC
BONE    REAL*8        114      INTRINSIC
CALFAO  COMPLEX*16   242      INTRINSIC
CRLFAT  COMPLEX*16   258      INTRINSIC
CDEXP   CMPLX         193      INTRINSIC
CMPLX   COMPLEX*16   255      INTRINSIC
CRELEN  COMPLEX*16   370      INTRINSIC
DCONJG  REAL*8        132      INTRINSIC
GOOD    REAL*8        246      INTRINSIC
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