ACADEMIC INFORMATION AND FINANCIAL MARKETS: AN EMPIRICAL INVESTIGATION OF MARKET LEARNING FROM THE SIZE ANOMALY

by

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Abstract

This dissertation examines the impact of academic information on the capital markets. A test of market learning from academic information is performed by examining the impact of published research about the size anomaly on the underlying asset pricing process.

A theoretical framework to examine the effect of events that affect the equilibrium pricing process is first developed in a simple economy with one single risky asset. A learning model based on Bayesian updating is proposed and its empirical implications are derived. The model predicts a change in the asset prices in the case of market learning. The predictions about the learning path depend on the assumed information structure. The key hypotheses are motivated through an illustrative case in a multi-asset economy where there is more information available concerning large firms than about small firms.

The econometric model of switching regimes is used to analyze the hypothesized structural change in the mean returns associated with the size variable. We postulate two regimes, one prior to and another after the incorporation of research information on the size anomaly. We find evidence of a switch in regimes with estimated mean switch located in 1983. The estimated average size premium has declined from approximately 13.6% per annum in the first regime to about -2.8% per annum in the second regime. More importantly, the switch in 1983 is not explained by any of the hypothesized economic factors that explain a large part of the stochastic variation in the size effect in the periods prior to 1983. We also find evidence of a switch in regimes when the seasonal January size effect is excluded. The evidence also suggests an increase in the trading volume associated with the information arrival.

Our evidence strongly suggests that the market has undergone a change in its underlying equilibrium pricing process after the discovery of the size anomaly.

The evidence supports the hypothesis that academic research relating to the size anomaly has provided useful information to the investors and the market has learnt from this information.

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Chapter 1

Introduction

The question of whether the market learns and becomes more knowledgeable over time has been the focus of research in many recent empirical studies. Watts (1978) and Charest (1978) examine the market's ability to assimilate information in earnings announcements and stock split announcements respectively in different time periods and find that market inefficiencies exist but are confined to the early periods. Their findings imply that over time the capital market has become more efficient in assimilating such information in stock prices. Nicholas and Brown (1981) support these conclusions in general but find that with respect to unexpected changes in corporate earnings and for certain announcements of stock splits, the market does not appear to be any more efficient than it had been in the past. Halpern and Turnbull (1985) test the hypothesis that investors have become more knowledgeable over time about pricing of options by examining the probability and the magnitude of boundry condition violations in the Toronto Stock Exchange (TSE) options market over the period 1978-79. They find that both the frequency of violations and their magnitude have increased over time. However, they acknowledge that the sample period was a period of rapid growth in the TSE options market and the observed results cannot be generalized to the current period where its growth has levelled off.

Most of the above studies have examined the ability of market participants to learn from their experience. The tests of market learning are done by comparing the way the market reacts to similar announcements over time. An implicit assumption in these tests is that investors know the equilibrium asset pricing model and learning is only with respect to the interpretation and incorporation of information in stock prices. The purpose of this dissertation is to explore whether market participants also learn from academic research that debates the correct specification of equilibrium pricing models. This is an important issue in view of the recent discovery of many capital market anomalies that have characterized research. Examples include the size effect[1], the weekend effect[2] and the year-end effect[3]

The term anomaly is broad. This dissertation defines it as any empirical phenomenon that cannot be satisfactorily explained by accepted models of market equilibrium. In other words, anomalous evidence contradicts the conclusions of some widely accepted theory. Most of the anomalies have been discovered in joint tests of a theoretical valuation model[4] and market efficiency. The existence of these anomalies has been used to imply that either the market is inefficient or the current theory of capital markets is insufficient to fully comprehend the working of capital markets. A preponderance of evidence supports the proposition that capital markets are efficient. According to Jensen (1978) there is no other proposition in economics that has more solid empirical evidence supporting it than the efficient market hypothesis. Most researchers view the anomalies as misspecification of theoretical models rather than evidence of market inefficiency. Thus answers to these empirical puzzles have been sought either in alternative pricing models or in the mismeasurement of the data. However, none of the answers have satisfactorily explained the current extant body of anomalous evidence.

There is a third possible explanation for the existence of empirical anomalies. In a world where information is costly, academic research concerning the implications of equilibrium pricing models could provide valuable information to the market. Under this scenario, the market could react, or learn, even if it were originally efficient relative to the information held by the market participants. We investigate this issue by examining whether market participants learn from the discovery of empirical anomalies. An analysis of whether the information about an anomaly

[1]Banz (1981), Reinganum (1981)
[2]French (1980)
[3]Officer(1975), Rozeff and Kinney (1976)
[4]For example, the Capital asset pricing model or the Option pricing model.

causes it to go away or not provides a direct test of market learning. A market reaction to remove the anomaly will be consistent with the hypothesis that the market learns from the information about empirical anomalies, while no market reaction will be consistent with the hypothesis that there is really no anomaly. In the latter case, the apparent anomaly arises from some misspecification in the pricing model or in the empirical analysis.

This dissertation focuses on market learning from the research information relating to the size anomaly. The size effect refers to the empirical finding that small firm stocks have in the past earned, on average, higher risk adjusted returns than large firm stocks, where risk is measured by the standard capital asset pricing models. The estimates of the size premium vary from approximately 10% to about 20% per annum. Further, empirical support for the association between firm size and average stock returns is about as strong as the association between risk and average returns. Thus an alternative asset pricing model developed on the basis of size and expected return would seem to have as much empirical validity as a pricing model based on the assumption of risk averse expected utility maximizing participants in the market. Among the many anomalies discovered recently the size anomaly has received wide attention of both academicians and practitioners. The Journal of Financial Economics devoted a special issue of June 1983 to research on size and related anomalies. Since the publication of the first papers on the size anomaly in 1981, the size effect has been thoroughly examined for both statistical and economic explanations and there is a general consensus that it cannot be explained by current capital market theory.

The remainder of the dissertation is organized as follows. Chapter 2 provides a rationale for selecting the size anomaly and a summary of the research concerning the size anomaly. Chapter 3 presents a theoretical framework of market learning and the empirical implications that follow from the model. The key hypotheses are motivated through an illustrative case where research information on the size anomaly can provide useful information to investors. Chapter 4 contains the econometric model and chapter 5 the empirical analysis. Chapter 6 contains a summary of the findings, conclusions and a discussion of the direction for future research.

The main purpose of this dissertation is to develop a framework where the effect of the events that affect the equilibrium asset pricing process can be evaluated and to use this framework to examine the impact of the research concerning the size anomaly. Such a framework differs from that in a standard event study where the underlying equilibrium process is assumed to be unaffected by the information relating to an event. Also, the information effect may occur over a long period of time and may be compounded by many other effects of general nature that affect the pricing process during the information period.

Briefly, our findings strongly suggest that the market has undergone a change in its underlying pricing process after the discovery of the size anomaly. Using a model of switching regimes, we find evidence of a switch in the mean size effect in 1983. The estimated mean size effect has declined from approximately 13.6% to about −2.8% per annum. A similar pattern is found for non-January observations. More importantly, the switch in 1983 is not explained by any of the hypothesized economic factors that explain a large part of the stochastic variation in the size effect in the periods prior to 1983. The evidence indicates that the switch in 1983 is associated with the research documenting the size anomaly.

Chapter 2

Selection of an Anomaly for Empirical Analysis

A test of the market learning from the discovery of an anomaly requires assessing the impact of information relating to such anomalies on capital markets and thus in principle such a test parallels an event study. However, there are important differences between the two in the nature and the process of the information arrival in the market. In most of the event studies the occurrence of an event is signalled either by the release of some information or announcement regarding the event. The information received is normally routine. This facilitates the processing and interpretation of the information by the market with great speed resulting in an instantaneous reaction in an efficient market. In contrast, in the case of research information the process of the release of information is normally slow and prolonged. Also, the information is conceptual in nature. The research may raise more questions than provide answers and thus this information may be hard to process and interpret. Further, the research may go through various stages when information is released to different groups before it is finally published, for example its circulation as a working paper or its presentation at a conference. The publication of a research work, however, may only be the beginning and not the end of a debate about the conjectures and the findings of a research paper. The conclusions of the paper may eventually be accepted or rejected and thus research work may or may not signify an event. There may be long time periods, sometimes many years, before a consensus emerges about the findings of pathbreaking research. Additional time may elapse before it gets the attention of practitioners. In cases where the debate becomes too prolonged or the information about the various stages of the process is not known, it may be difficult to disentangle the effects of the academic research from the effects of other events in the market. Important criteria for the selection of a piece of research to test the impact in the capital markets include:

(i) There should be some consensus in the academic community about its conclusions and the consensus should emerge within a reasonable amount of time.

(ii) It should have received wide attention of both the academicians and practitioners.

(iii) A priori, the theory should predict some correction by the market, in fact, a strong reaction so that it is not swamped by the estimation errors.

Many empirical anomalies meet most of the above criteria. These anomalies are discoveries of systematic relationships between variables that cannot be explained by any current theory. While some anomalies have been resolved or explained over time others remain a puzzle.[1] Some of the unresolved anomalies in securities markets[2]that have received widespread attention include the following:

1. The size effect: discovered by Banz (1981) and Reinganum (1981) who reported a significant negative relation between abnormal returns and the market value of common equity for samples of NYSE and NYSE- AMEX firms, respectively.

2. The weekend effect: French (1980), and Gibbons and Hess (1981) have documented that the daily patterns of stock returns are not uniform across all trading days of the week. In contrast to Tuesday through Friday, Monday returns were on the average negative. Keim and Stambaugh (1984) point out that the negative Monday returns persisted even when the NewYork stock exchange was open during Saturdays.

[1]For example, Kleidon (1985) challenges Shiller (1980,1981)'s evidence based on variance bound tests that bond and stock prices are far more volatile by arguing that the assumptions required to conduct variance bound tests are violated empirically.

[2]The discovery of anomalies is not limited to the equity markets. Galai (1982) has reported the violations of the boundry conditions in the options market while Brennan and Schwartz (1982) have discovered the violations in the bond market.

3. The turn-of-the-year effect: Officer (1975) and Rozeff and Kinney (1976) detected a January seasonal effect in the stock returns series. Keim (1983) reported that the nature of the seasonal pattern is systematically related to market capitalization. About half of the annual size effect can be attributed to the month of January; Moreover, much of the January effect occurs during the first few trading days of the month.[3] Tinic and West (1984) find that January is the only month to show a consistently positive, statistically significant relationship between risk and expected return.

4. The earnings' yield effect: Basu (1977) has reported that portfolios of high (low) earnings' yield security trading on the NYSE earn higher (lower) absolute and risk adjusted rates of returns on average than portfolios consisting of randomly selected securities. While Reinganum (1981) concluded that the size effect subsumes the earnings' yield effect, Basu (1983) reexamined and confirmed his earlier results.

5. The period of listing effect: Barry and Brown (1984) have documented a period of listing anomaly associated with, but distinct from, the size anomaly. They find that the shorter is the period of listing of a security the larger is its risk adjusted rate of return.

6. The monthly effect: Ariel (1987) finds that the mean returns for stocks is positive only for days immediately before and during the first half of calender months and indistinguishable from zero for days during the last half of the month.

The selection of the size anomaly is appealing for various reasons. Firstly, among the many anomalies discovered recently the size anomaly has received the most attention. Since the first papers were published in 1981 on the size anomaly, it has intrigued many researchers. It has been thoroughly examined for both statistical and economic explanations, has been characterized in detail within a short period of time, and there is consensus that the size effect is strong. Moreover, the

[3]Also, this effect is not confined to the north American markets only but is prevalent even in the Australian market.

size effect has drawn the attention of practitioners since as early as 1980. For example, in the well-publicized Institutional Investor (1980, p.29) article ' Is Beta Dead ? ', Richard Michaud of Bache mentions using a market capitalization instead of a beta model. The American National Bank and Trust Company of Chicago even set up a ' passive management ... Market Expansion Fund ' of small firm stocks. The class of ' small firm growth stocks ' considered in the Wall Street Week (1980) program provides another illustration. Thus not only is the size anomaly well documented but it is well publicized both within the academic community and practitioners.

Secondly, the size anomaly has provided a very serious challenge to the capital asset pricing models that are the core of the modern financial theory. Schwert (1983) observes that the empirical support for a positive relationship between risk and return as predicted by the standard asset pricing models is weak and the association between firm size and average stock returns is about as strong as the association between risk and average returns. He compares the two statistics and reports that in Fama and MacBeth (1973) the t-statistic testing the hypothesis that the slope of the risk-return relation is zero is 2.57 for the 1935-1968 sample period, but it is only 1.92, 0.70, and 1.73 for the 1935-45, 1946-55, and 1956-68 subperiods respectively. The t-statistic testing the hypothesis that the size effect coefficients are zero is -2.54 for the 1936-75 period, and -1.88 and -1.91 for the 1936-55 and 1956-75 subperiods respectively. Thus an alternative asset pricing model developed on the basis of size and expected return has as much empirical validity as a pricing model based on the assumption of risk averse expected utility maximizing participants in the market. The size anomaly has not been explained within the framework of another theoretical model, the Arbitrage pricing theory. Furthermore, several studies have shown that anomalous return behaviour associated with other firm specific variables is largely subsumed by the size effect.

Finally, the size anomaly is also closely related to many other anomalies; for example the size effect depicts seasonality and about half of the annual size effect occurs in January. However, the January size premium and the average size effect exhibit different time series patterns. The January size effect has been strong and consistently increasing during the 1963-1979 period while the size effect has been unstable in various subperiods. A study of the impact of research concerning the size anomaly will also provide information about the relationship between different size related anomalies.

Appendix 1 provides a list of key research papers and appendix 2 contains empirical findings of these papers. Summarizing the empirical research we find that most of the researchers conclude that there is evidence of a strong average size effect. The estimates of the size effect vary between 10% and 20% per annum. The size effect is also more pronounced in daily data than in monthly data. The magnitude of the size effect has varied over different time periods and in the 1969-73 period it even reversed in sign. There is also a persistent and statistically significant seasonal size effect in January which has increased in magnitude from 1963 to 1979. So far the search for an explanation of the size anomaly has been unsuccessful. Many statistical and economic explanations including the mismeasurement of beta, excessive transaction costs for small firms, and measurement problems have been examined. The association between size and other variables such as dividend yield, the standard deviation of stock returns, and between firm size, dividend yield and co-skewness have also been examined. But none of these explanations provide a satisfactory answer. The general conclusion is that the small firm effect is a significant empirical anomaly.

Chapter 3

The Theoretical Framework

3.1 Main issues

Although the basic framework for the assessment of the impact of research information on capital markets is similar to that of a standard event study, there are important differences between the two which raise some special issues. Similar to the case of an event study, we need to specify an equilibrium asset pricing process as a benchmark to measure the impact of information. However, the information in a standard event study relates to firm specific economic events and the equilibrium pricing process is assumed to remain unaffected by such events. Further, the occurrence of an event is signalled by the release of some information or announcement regarding the event. This facilitates the processing and interpretation of the information by the market with great speed resulting in an instantaneous reaction in an efficient market. If the equilibrium model is correct, and if the market is efficient in incorporating all relevant information in prices, the residuals will capture the impact of firm specific events on stock prices in the information period.

In contrast, research information on the size anomaly pertains to a systematic relationship between risk and expected return since it focuses on whether size proxies a risk variable that may be priced by the market. Any market reaction to this information is likely to affect the equilibrium asset pricing process. Standard residual analysis cannot be used in this case because the observed residual in the information period will contain two effects; the effect of random firm specific shocks that hit the economy and the effect of information about the asset pricing model that is released in the information period. To disentangle the two effects, we need to specify, in addition to an equilibrium pricing process, an explicit model of learning by which agents incorporate research information into asset prices. In this chapter we formulate such a model of market learning and derive its empirical implications.

An additional difficulty arises because research information is conceptual in nature and does not provide definitive answers. A research paper may provide partial answers, raise some additional questions, and suggest some alternative explanations or suggestions for further research. Thus any reaction to the research information is likely to be slow and may span many periods. Many economic events of a general nature may occur during such a period which may confound the information effect. A careful analysis is needed to take into account these additional influences during the information period.

To deal with these issues, we first develop a theoretical framework in a simple economy of a riskless and a risky asset. The true risk of the risky asset is unknown to the investors. The investors receive noisy signals about the true risk of the asset. Information is assumed to be exogeneous to the economy. A rational equilibrium asset pricing process and a model of market learning through which information is incorporated in asset prices is developed in this economy. The equilibrium properties in such a model are described and the empirical implications in terms of ex-ante and ex-post returns that follow from the model are derived.

Although the simple model described above does not fully capture the impact of academic information, it illustrates in a clear way the distinction between the effects of information signals and the effects of random shocks on the equilibrium asset pricing process. The learning process developed in this simple model is useful to examine the relationship between ex-ante and ex-post returns which is important to derive testable implications in the case of market learning. This relationship is illustrated analytically and graphically in this simple economy. A general case where the information effect is confounded by other effects is also discussed and the empirical implications for the information effect in such a case are examined. We show that the qualitative results derived in the simple case remain unaffected in the general case.

If the market reacts to the release of academic information, then the mechanism by which academic information affects the market becomes important. We develop a simple model to provide an example in which academic research provides new information to rational investors. The model is based on differential information concerning small and large firms (section 3.3) in a world of costly information. Academic information is modelled as being exogeneous to the market. This could be justified in a broader model incorporating subsidies to academic research. Subsidies make it optimal for academic researchers to carry out analysis beyond the point that would be optimal for a typical market participant. Under this scenario the market is efficient relative to the information that is available.

Although learning by the market from academic information may be understood in the context of efficient markets, we recognize that an alternative scenario exists. It is possible that in the pre-information period market participants had full access to the information that would subsequently be published by researchers. If the market then reacted to the publication of research results, this would indicate that the market may not have been fully efficient in incorporating information in the pre-information period. Thus an evidence of market learning can be consistent with both scenarios of market efficiency and inefficiency. To enable us to discriminate between the two scenarios, we need a richer theoretical framework. Such analysis will require additional assumptions on the equilibrium pricing process in the pre-information period and on the information structure.

Our main purpose in this dissertation is to test whether the market learned from academic information concerning the size anomaly. There are several possible scenarios where academic information can influence the asset pricing process. The discrimination among the different scenarios, although an important issue, is beyond the scope of this dissertation and is left for future research.

3.2 A Model with a Single Risky Asset

3.2.1 The Economy and the Equilibrium Pricing Process

Assume that there are only two assets in the economy: a risk free asset and a risky asset. The return from the risky asset is received in the form of dividends. The expected dividend D is assumed to be the same in every period. Dividend is received at the end of each period.

The analysis is done in a multi-period framework and begins at time t=0. Trading takes place at time $t = 0, 1, 2, 3, \ldots$. The time interval between time t and t+1 is denoted as period t, $t = 0, 1, 2, \ldots$ To focus on the information effect we assume away all other effects during the period of interest. In other words, the only stochastic variation apart from the inherent risk of the security occurs due to new information arrival. In particular, no consumption occurs during this period and only portfolio decisions are made at time $t = 1, 2, 3, \ldots$. The risk free rate is assumed to be the same in each period. The actual dividend received in each period t is denoted by D_t,

$$D_t = D + e_t \tag{3.1}$$

where $e_t, t = 0, 1, 2, 3, \ldots$ are independent and identically distributed random variates with mean 0 and variance γ^*. The random shocks e_t are related to the true risk of the asset γ^*.

All agents are risk-averse expected utility maximizers and have homogeneous beliefs. The true risk γ^* of the asset is assumed to be unknown to the agents. Agents regard γ^* as a fixed but unknown parameter. At any time t, agents form beliefs about γ^* based on the available information at that time. Information is costly to obtain and all information is exogeneously provided.[1] All agents possess the same information and update their beliefs using Bayes's theorem. Any new information is instantaneously incorporated into asset prices.

The current value of the risky asset at time t is the discounted value of future

[1] In particular, agents do not learn from experience. This assumption is relaxed later.

expected cash flows:

$$P_t = \frac{D}{(1+r_t)} + \frac{D}{(1+r_t)^2} + \frac{D}{(1+r_t)^3} + \cdots = D/r_t \tag{3.2}$$

where P_t is the equilibrium price of the stock at time t, D is the per period expected dividend and r_t is the risk adjusted expected rate of return from the stock in period t, i.e. from time t to t+1. The discount rate r_t in periods $t+1, t+2, t+3, \ldots$ is the same as in period t since agents do not anticipate any further information. The discount rate r_t is an increasing function of the perceived risk of the security and can be thought of as derived by the agents in two steps: first based on the information available at any time t, agents form beliefs about γ^* and then based on their beliefs about γ^* they demand a rate of return r_t that is consistent with the perceived risk of the asset. The expected rate of return or the discount rate denoted by r_t is equal to this required rate of return. In general, the higher the perceived risk of the asset the larger will be the risk-adjusted discount rate.

3.2.2 Market Learning

At time t=0, agents' prior beliefs about γ^* are represented by a normal distribution with mean γ_0 and precision h_0 denoted by $N(\gamma_0, h_0)$.[2] Information about γ^* is received in the form of noisy signals y_t. Signals y_t, where subscript t denotes the time of the receipt of the signal, reveal γ^* but with a normally distributed noise η_t. $y_t = \gamma^* + \eta_t$, where η_t, $t = 0, 1, 2, \ldots$ is a sequence of independent but identically distributed normal variates with mean 0 and precision h_η. After receiving the signal y_t agents update their beliefs about γ^* using Bayes's theorem. The posterior beliefs of the agents about γ^* after receiving the signal y_t are represented by a normal distribution with mean γ_t and precision h_t denoted by $N(\gamma_t, h_t)$.

No new information is received prior to $t = t^*$ and the first signal is received at time t^*. All information is received in the period between $t = t^*$ and $t = t^{**}$, which is referred to as the information period. The periods prior to $t = t^*$ and after $t = t^{**}$ are referred to as the pre-information and post-information periods, respectively. All information signals are unanticipated. Signal y_t is received immediately prior to trading at any time t. Since all information is instantaneously incorporated into

[2]The precision of a normal distribution is the reciprocal of its variance.

asset prices,

$$E(r_t \mid \phi_t)_t = r_t = D/P_t \tag{3.3}$$

where ϕ_t denotes the information set available to the agents at time t and $E(r_t \mid \phi_t)_t$ is the expected rate of return for period t using all available information ϕ_t with expectations formed at time t. Since no new information is received in the pre-information period the initial beliefs remain unchanged till time $t^* - 1$. In other words, $N(\gamma_t, h_t) = N(\gamma_0, h_0)$ for $t < t^*$. Let r_0 be the risk-adjusted required rate of return consistent with these beliefs.

At time t^*, the first signal y_{t^*} is received. After observing the signal investors update their beliefs about γ^* using Bayes' theorem. Their new prior is normally distributed with mean and precision

$$E(\gamma^*)_{t^*} = \gamma_{t^*} = \frac{h_0\gamma_0 + h_\eta y_{t^*}}{h_0 + h_\eta}$$

$$h_{t^*} = h_0 + h_\eta$$

where subscript t denotes the expectations at time t after receiving the signal. Based on the updated beliefs about γ^* investors revise the risk-adjusted discount rate from r_0 to r_{t^*}, consistent with the revised beliefs about γ^*.

In general, if the posterior distribution of γ^* at time t is normal with mean γ_t and precision h_t, then the posterior distribution of γ^* at time t+1 is normal with mean γ_{t+1} and precision h_{t+1},

$$E(\gamma^*)_{t+1} = \gamma_{t+1} = \frac{h_t\gamma_t + h_\eta y_{t+1}}{h_t + h_\eta} = \frac{h_0}{(h_0 + n_{t+1}h_\eta)}\gamma_0 + \frac{n_{t+1}h_\eta}{(h_0 + n_{t+1}h_\eta)}\frac{\sum_{\tau=t^*}^{t+1} y_\tau}{n_{t+1}}$$

$$h_{t+1} = h_t + h_\eta = h_0 + n_{t+1}h_\eta$$

where n_t is the number of signals received till and including time t. The posterior beliefs are the weighted average of initial beliefs and the sample mean where weights are proportional to the precision of the signal. As the number of signals increases the weight on the initial beliefs goes to zero and the weight on the sample mean tends to 1. The equilibrium prices at time t+1 will be determined on the expected asset return r_{t+1} which is consistent with the revised beliefs about γ^*.

The change in expected mean of γ^* at time t+1 after receiving the signal y_{t+1} is given by

$$E(\gamma^*)_{t+1} - E(\gamma^*)_t = (\gamma_{t+1} - \gamma_t) = \frac{h_\eta}{(h_0 + n_{t+1}h_\eta)}(y_{t+1} - \gamma_t) \tag{3.4}$$

The increase in precision after receiving any signal is h_η.

3.2.3 Impact of Information on the Equilibrium pricing process

To assess the impact of information we examine the equilibrium prices in the pre-information, information and post-information periods.

Pre-Information period

In the pre-information period no new information is received. The beliefs about the risk of the asset as well as the risk adjusted expected return consistent with these beliefs are therefore unchanged during this period and are the same as at time t=0. The current asset price and all future expected prices will be based on the expected rate of return r_0 and we will observe the following schedule of prices:

$$P_t = D/r_0 = P_0$$

$$P_t = E(P_{t+1})_t = E(P_{t+2})_t = \ldots = P_0 \qquad \forall \quad t < t^*$$

where $E(P_t)_k$ denote the expected asset price at time t with expectations formed at time k,

Information Period

In the information period agents revise their beliefs about γ^* according to the Bayesian learning model specified above. At time t^* when the first signal is received the equilibrium prices and all expected future prices will be based on the expected rate of return r_{t^*} which is consistent with the updated beliefs $N(\gamma_{t^*}, h_{t^*})$ about γ^*. We will observe the following schedule of prices at time t^*:

$$P_{t^*} = D/r_{t^*}$$

$$P_{t^*} = E(P_{t^*+1})_{t^*} = E(P_{t^*+2})_{t^*} = \ldots =$$

In general, at any time t in the information period the equilibrium asset price and all future expected prices will be based on the expected rate of return r_t which is consistent with the beliefs $N(\gamma_t, h_t)$ about γ^* at that time. We will observe the following schedule of prices:

$$P_t = D/r_t \tag{3.5}$$

$$P_t = E(P_{t+1})_t = E(P_{t+2})_t = \ldots = \qquad \forall \quad t, t^* \leq t \leq t^{**} \tag{3.6}$$

After receiving the last signal at time t^{**} we will observe the following schedule of prices

$$P_{t^{**}} = D/r_{t^{**}}$$

$$P_{t^{**}} = E(P_{t^{**}+1})_{t^{**}} = E(P_{t^{**}+2})_{t^{**}} = \ldots =$$

Post-Information period

No additional signals are received in this period. Thus beliefs about γ^* and the expected return consistent with these beliefs remain unchanged from those at time t^{**}. We will observe the following sequence of prices for any $t > t^{**}$:

$$P_t = D/r_{t^{**}} = P_{t^{**}}$$

$$P_t = E(P_{t+1})_t = E(P_{t+2})_t = \ldots = P_{t^{**}} \qquad \forall \quad t > t^{**}$$

3.2.4 Empirical Implications: Ex-ante Returns

The learning model has empirical implications about market learning and the learning process from the arrival of new information.

Market Learning

The following null and alternative hypotheses about market learning can be formulated in terms of ex-ante returns:

H_0: No Market Learning: The signals y_t have no impact on the return generating process of the asset. In this case $r_{t^{**}}, r_{t^{**}-1}, \ldots, r_{t^*}, r_0$ are equal or $r_{t^{**}} = r_{t^{**}-1} = \ldots = r_{t^*} = r_0$.

H_1: Market Learning: The signals y_t have an impact on the return generating process of the asset. In this case $r_{t^{**}}, r_{t^{**}-1}, \ldots, r_{t^{**}}, r_0$ are not all equal.

Learning Paths

In the case of market learning we can also analyze the learning process. The actual learning path will depend on the impact of each signal. This impact can be measured in terms of the change in beliefs about γ^* after a signal is received. From equation 3.4, the change in the expected mean of γ^* after the incorporation of information at time t+1 is given by

$$\frac{h_\eta}{(h_t + h_\eta)}(y_{t+1} - \gamma_t)$$

which is positive, negative or zero depending on whether $(y_{t+1} - \gamma_t)$ is positive, negative or zero. The size of the change is a function of y_{t+1}, γ_t, h_0, h_η and the number of signals received till time t+1. Thus the actual adjustment pattern depends on the mean and precision of the signals received at each time t as well as on the initial beliefs. Since the expected return is an increasing function of the beliefs about true risk of the asset γ^*, any change in γ^* will result in a corresponding change in expected returns. In other words, the impact of any signal y_{t+1} can also be measured in terms of a change in expected returns $r_{t+1} - r_t$ after receiving a signal and this change is positive, negative or zero depending on whether $y_{t+1} - \gamma_t$ is positive negative or zero. Restating in terms of ex-ante returns, we conclude that the adjustment process will depend on the ex-ante returns in the pre-information period and the impact of each signal measured in terms of the change in ex-ante returns.

Many different patterns may be observed under this scenario. To predict a specific pattern will require further assumptions about the information structure. Examples of two such patterns corresponding to additional assumptions on the impact of signals are:

(i) $r_{t^{**}} \geq r_{t^{**}-1}, \geq \ldots, \geq r_0$: After receiving each signal y_t the change in expected returns $(r_t - r_{t-1})$ is > 0 which implies that the asset is perceived to be less risky.

(ii) $r_{t^{**}} \leq r_{t^{**}-1}, \leq \ldots, \leq r_0$: After receiving each signal y_t the change in expected returns $(r_t - r_{t-1})$ is < 0. The asset is perceived to be more risky.

3.2.5 Empirical Implications: Ex-Post Returns

The empirical implications of the preceding section have been derived in terms of expected returns. To facilitate the empirical analysis we need to formulate testable hypotheses in terms of ex-post returns in each period t, $t = 0, 1, 2, 3, \ldots$. Ex-post rate of return in period t denoted by $\tilde{r}_t$ is defined as the actual return from holding the asset from time t to t+1. Ex-post rate of return in period t will differ from the expected rate of return r_t for the same period with expectations formed at time t due to two possible effects (i) effect of information signal y_{t+1} and (ii) effect of random shock e_t related to the inherent risk of the asset.

In our simple model, the first effect causes the equilibrium asset price P_{t+1} at time t+1 to differ from the expected equilibrium price $E(P_{t+1})_t$ with expectations formed at time t. From equation 3.6 $E(P_{t+1})_t$ is equal to P_t since all signals are unanticipated. Thus the effect of information signal y_{t+1} is captured by the difference in the actual equilibrium asset price P_{t+1} at time t+1 and the expected equilibrium price P_t at time t+1 with expectations formed at time t. The second effect causes the actual dividend D_t in period t to differ from the expected dividend D in the same period. From equation 3.1, this difference is measured by e_t where e_t is a normally distributed random variable with mean 0 and variance γ^*. Thus ex-post rate of return $\tilde{r}_t$ in period t is:

$$\tilde{r}_t = \frac{D}{P_t} + \frac{(P_{t+1} - P_t)}{P_t} + \frac{e_t}{P_t} \tag{3.7}$$

The first term D/P_t on the right hand side expression in equation 3.7 is the expected rate of return r_t in period t, the second term $(P_{t+1} - P_t)/P_t$ measures the impact of information signal y_{t+1}, and the third term e_t/P_t measures the impact of random shock e_t related to the inherent risk of the asset. The information signal y_{t+1} reveals information about the true risk of the asset γ^*. This information affects the expected risk adjusted rate of returns demanded by the agents in all future periods and thus impacts the equilibrium asset pricing process. The second shock e_t, on the other hand, is a transitory effect and in our simple model does not affect equilibrium asset prices.

To focus on the information effect, we define a new term: expected ex-post rate of return. Expected ex-post rate of return for period t, denoted by $E(\tilde{r}_t \mid \phi_t, y_{t+1})_t$, is defined as the expected rate of return in period t conditional on the arrival of information signal y_{t+1} at time t+1 with expectations formed at time t. The information available at time t is incorporated in ϕ_t. Since the information effect in period t is measured by $(P_{t+1} - P_t)/P_t$,

$$E(\tilde{r}_t \mid \phi_t, y_{t+1})_t = \frac{D}{P_t} + \frac{(P_{t+1} - P_t)}{P_t} \tag{3.8}$$

Substituting $E(r_t \mid \phi_t)_t = D/P_t$ from equation 3.3 in equation 3.8,

$$E(\tilde{r}_t \mid \phi_t, y_{t+1})_t - E(r_t \mid \phi_t)_t = \frac{(P_{t+1} - P_t)}{P_t} \tag{3.9}$$

Thus the information effect $(P_{t+1} - P_t)/P_t$ present in the ex-post rate of return in period t is given by the difference between ex-ante and expected ex-post rate of returns in period t. The difference between the expected rate of return and expected ex-post rate of return in period t lies in the information set used by the agents to form expectations at time t, the former return is unconditional while the latter return is conditional on the arrival of information signal y_{t+1}.[3]

To assess the impact of information signals we need to examine the relationship between ex-ante and expected ex-post rates of return in each period. This relationship may be very complex with the precise relationship depending on the information structure. We illustrate this point analytically and graphically by comparing the impact on prices, ex-ante rate of return and expected ex-post rate of return in the information period under different information processes. All figures are drawn assuming sixty time periods; the first twenty periods comprise the pre-information period, the next twenty represent the information period and the last twenty represent the post-information period. In all cases the expected rate of return r_0 in the pre-information period and expected dividend D are assumed to be 0.1 and 1.0 respectively. To keep the analysis tractable we impose some additional assumptions on the stochastic process and deal with the case where expected returns follow a declining trend during the information period.[4] We first analyze

[3]This approach is based on the general framework suggested by Thompson(1985) for parameterizing event study problems. This approach can also be used to analyze the impact of partially anticipated events, See Malatesta and Thompson(1985).

[4]This is equivalent to the case in which the asset is considered to be less risky after the arrival of each signal.

the simple case of no market learning and then analyze the case of market learning under different information processes.

No Market Learning:

The case of no market learning is observationally equivalent to the case of no information arrival. We will observe the following schedule of prices:

$$P_0 = D/r_0 = P_t, \quad t = 0, 1, 2, \ldots$$

In this case ex-ante returns will be equal to expected ex-post returns in each period. This case is illustrated in figure 3.1.

Market Learning:

Ex-ante and expected ex-post returns will exhibit a different pattern in the case of market learning. The relationship between ex-ante and expected ex-post returns becomes more complex as the number of signals increases. Even in a simple case where the expected returns are declining many different time-series patterns for expected ex-post returns may emerge. However, there will be some common threads in various scenarios that can be used to derive empirical implications about the learning process. We illustrate this point in different scenarios where one, two and multiple signals are received. All signals are unanticipated. Expected returns are assumed to be declining in each case.

1. One information signal: Assume that only one information signal is received and it arrives at time t^*. Assume that after receiving the signal the expected rate of return declines from r_0 to $r_0 - \Delta_1$, where $\Delta_1 > 0$ measures the impact of the signal on the pricing process. The equilibrium asset price at time t^* and all future expected prices will be based on the revised expected rate of return $r_0 - \Delta_1$ and we will observe the following schedule of prices:

$$P_{t^*} = D/(r_0 - \Delta_1)$$

$$P_{t^*} = E(P_{t^*+1})_{t^*} = E(P_{t^*+2})_{t^*} = \ldots$$

The ex-post equilibrium price P_{t^*} that incorporates the information received at time t^* is higher than the ex-ante equilibrium price with expectations

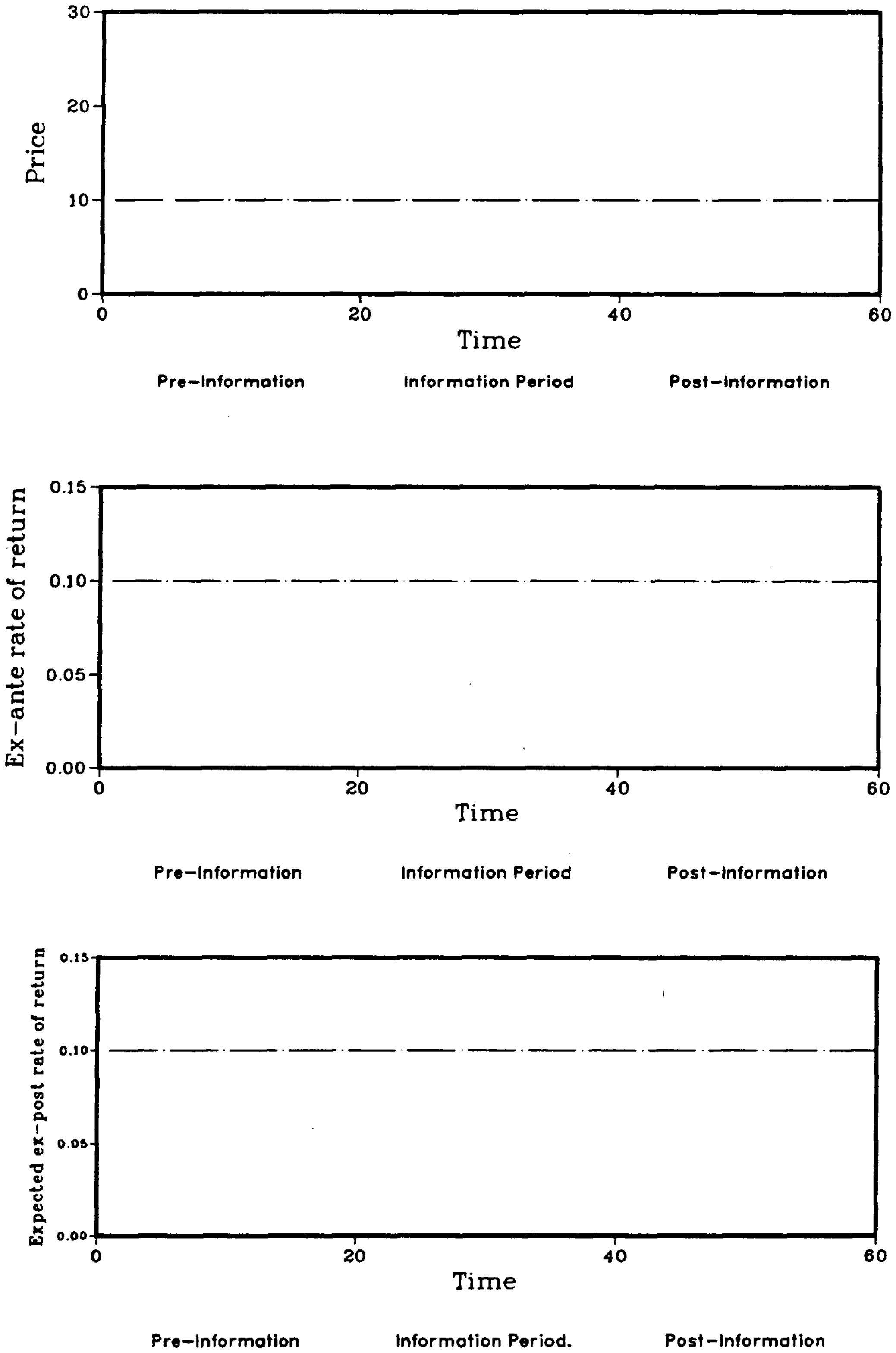

Figure 3.1: No Market Learning

formed at time $t^* - 1$. This follows because

$$P_{t^*} = D/(r_0 - \Delta_1) > D/r_0 = E(P_{t^*})_{t^*-1} = P_0$$

Higher ex-post prices at time t^* imply higher expected ex-post returns in period $t^* - 1$. From equation 3.9, the difference between expected ex-post and ex-ante rate of returns is

$$(P_{t^*} - P_0)/P_0 = \Delta_1/(r_0 - \Delta_1) \quad > 0$$

This difference is an increasing function of the impact of the signal measured by Δ_1 and a decreasing function of the expected rate of return r_0 in the case of no new information. Thus the larger the impact of the information, the higher will be the equilibrium price P_{t^*} and the larger will be the difference between expected ex-post and ex-ante returns. However, with the exception of the period $t^* - 1$, ex-ante returns are equal to expected ex-post returns in all periods. This case is illustrated in figure 3.2 assuming $\Delta_1 = 0.002$.

2. Two Information Signals: Assume that in addition to the signal received at time t^*, another signal is received at time time $t^* + 1$ and the second signal also confirms that the asset is less risky than initially perceived by investors. Assume that after receiving the second signal the expected rate of return on the stock declines from $r_0 - \Delta_1$ to $r_0 - \Delta_1 - \Delta_2$ where $\Delta_2 > 0$ measures the impact of the second signal on the pricing process. The equilibrium price of the asset at time $t^* + 1$ as well as all future expected prices will be based on the revised expected rate of return $r_0 - \Delta_1 - \Delta_2$. We will observe the following schedule of prices:

$$P_{t^*+1} = D/(r_0 - \Delta_1 - \Delta_2)$$

$$P_{t^*+1} = E(P_{t^*+2})_{t^*+1} = E(P_{t^*+3})_{t^*+1} = \ldots$$

Similar to the case of one signal, the expected ex-post price P_{t^*+1} is higher than ex-ante equilibrium price P_{t^*} with expectations formed at time $t^* + 1$ since

$$P_{t^*+1} = D/(r_0 - \Delta_1 - \Delta_2) > D/(r_0 - \Delta_1) = E(P_{t^*+1})_{t^*} = P_{t^*}$$

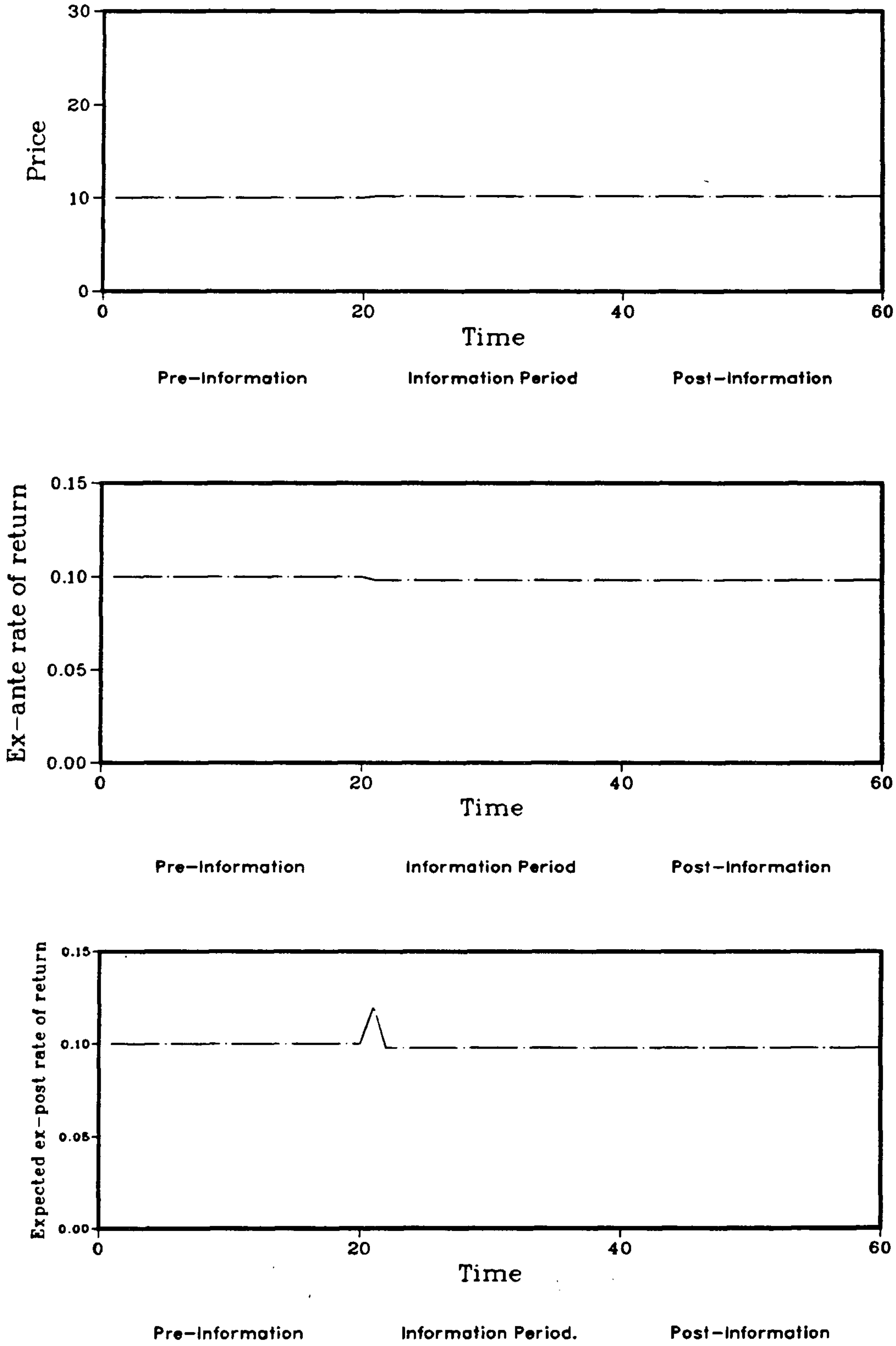

Figure 3.2: Market Learning: One Information Signal

Again higher ex-post prices at time $t^* + 1$ imply larger ex-post returns in period t^* compared to the ex-ante returns for the same period with expectations formed at time t^*. The difference between the ex-ante and ex-post rate of returns is

$$(P_{t^*+1} - P_{t^*})/P_{t^*} = \Delta_2/(r_0 - \Delta_1 - \Delta_2) \quad > 0$$

The larger the impact of the information measured by Δ_2, the higher will be the equilibrium price P_{t^*+1} and the larger will be the difference between the ex-ante and expected ex-post rate of returns. For all periods, with the exception of the periods $t^* - 1$ and t^* expected ex-post returns are equal to ex-ante rate of returns. The case of two information signals is illustrated in figure 3.3 assuming $\Delta_1 = 0.002$ and $\Delta_2 = 0.003$.

3. Multiple Signals: The case of multiple signals is an extension of the case of two signals. However, a larger variety of patterns are now possible under different assumptions of the information structure. We examine three different stochastic processess for the expected returns during the information period: $t^* \leq t \leq t^{**}$

$$\begin{aligned}
\text{Case(i) Linear Adjustment Process} : r_t &= r_0(1 - .02(t - (t^* - 1)) \\
\text{Case(ii) Nonlinear Adjustment Process} : r_t &= r_{t-1}(1 - .003(t - (t^* - 1)) \\
\text{Case(iii) Nonlinear Adjustment Process} : r_t &= r_{t-1}(1 - .001(t - (t^* - 1))^2
\end{aligned}$$

The three cases are illustrated graphically in figures 3.4, 3.5 and 3.6 respectively.

We observe that while ex-ante rate of returns follow the same pattern of decline, the expected ex-post rate of returns reveal different patterns in the information period in all cases. In the first case, expected ex-post rates of return rise for a few periods and then decline slowly, followed by a steep decline. In the second case, ex-post rates of return increase slowly in most of the information period and then decline sharply in the remaining period. The third case exhibits a slow increase at first, followed by a steep increase and a steep decline.

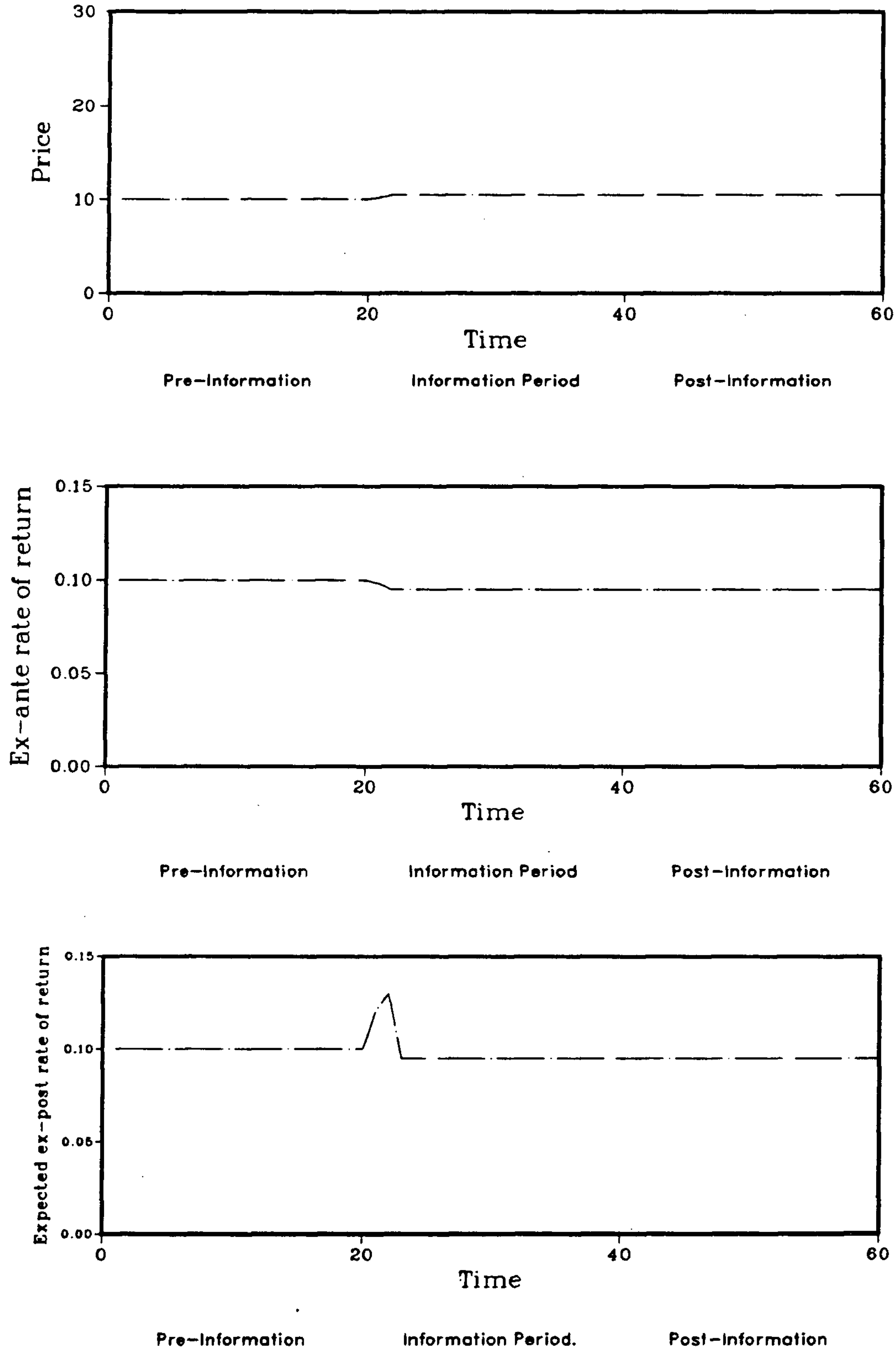

Figure 3.3: Market Learning: Two Information Signals

Case 1: Linear Trend

$$R_t = R_{20}\,[1.0 - 0.02[t-20]]$$

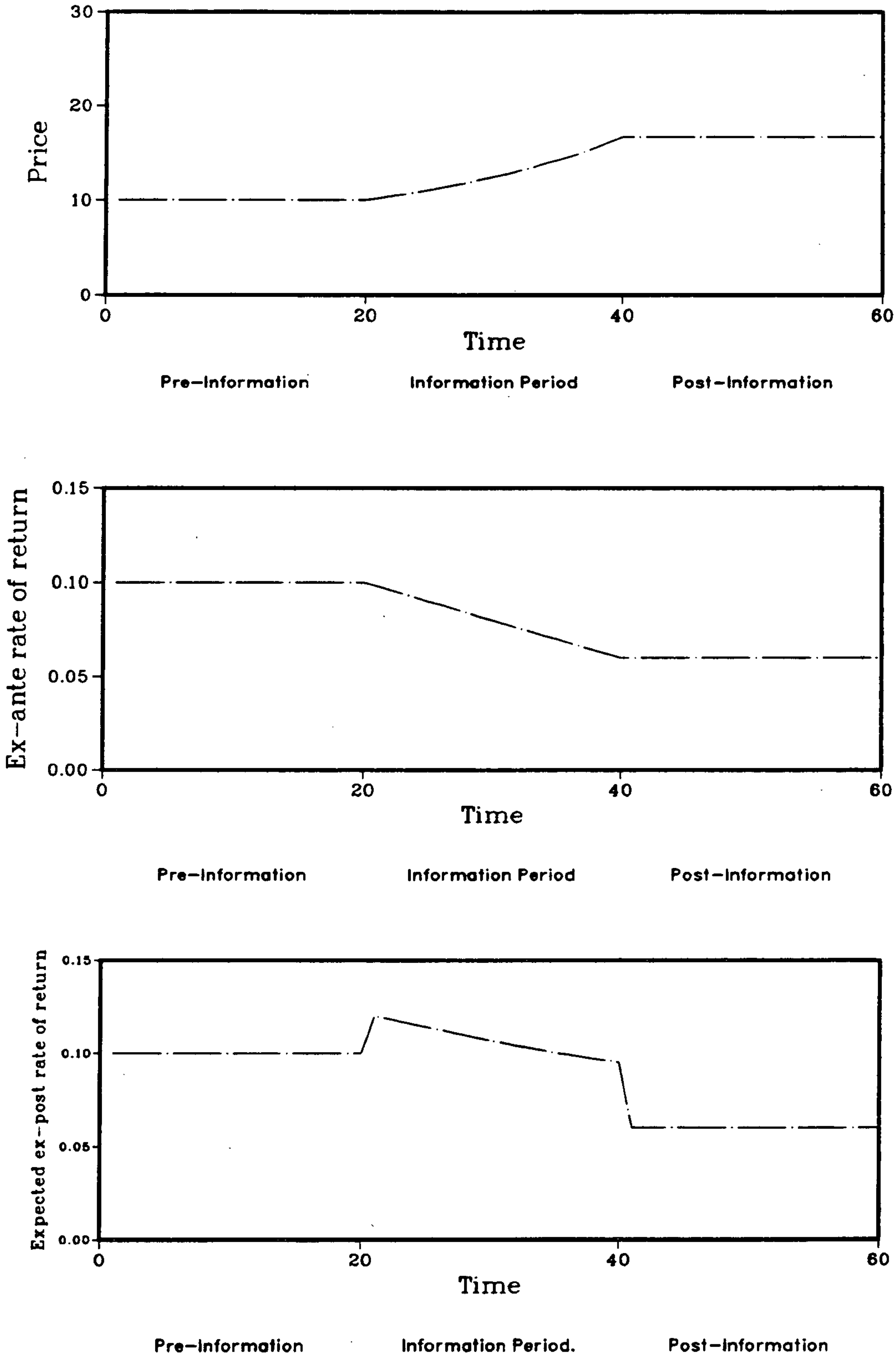

Figure 3.4: Market Learning: Multiple Signals

Case 2: Nonlinear Trend

$$R_t = R_{t-1}\,[1.0-0.003[t-20]]$$

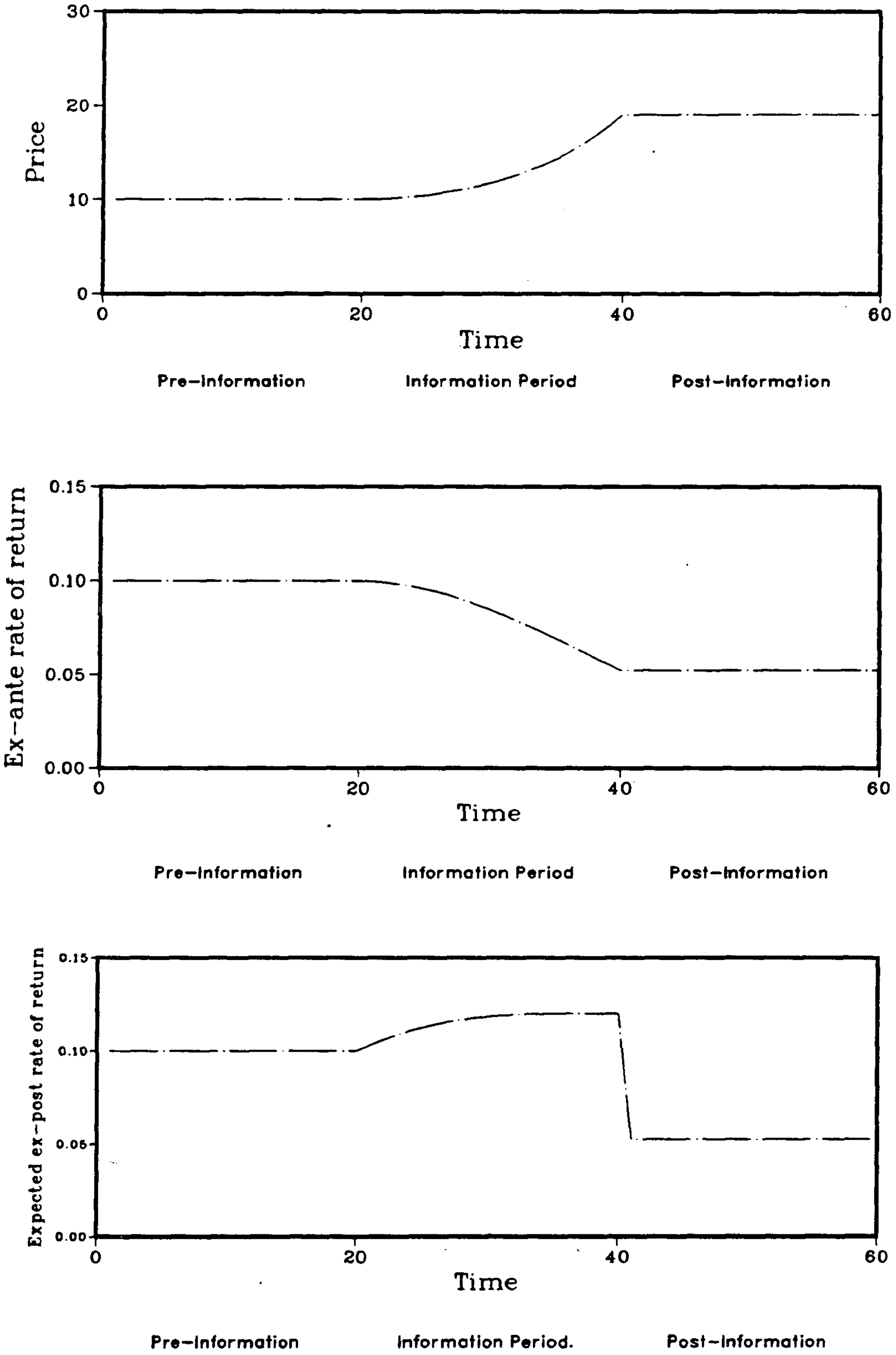

Figure 3.5: Market Learning: Multiple Signals

Although the expected ex-post returns reveal different patterns in all the three cases, there is one common thread in all scenarios. In all three cases expected ex-post returns first exhibit an increasing trend prior to following a declining trend. This provides a testable implication. In the case of a declining trend in ex-ante returns, we will observe an increasing trend in expected ex-post returns prior to a decline.[5] In such a case, expected ex-post returns in a subperiod of the information period will be even higher than those in the pre-information period.

The precise pattern of expected ex-post returns is a function of the relative impact of different signals and the required rate of return in the case of no information. To estimate the learning curve or the learning process we need to impose further structure on the information process and need to be specific about the stochastic process associated with the information arrival.

Summarizing the empirical implications in terms of expected ex-post returns we conclude that:

- The empirical implications for market learning in the case of expected ex-post returns are the same as in the case of expected returns:

  1. In the case of no market learning, we would observe no change in expected ex-post returns.
  2. In the case of market learning, we would observe a change in expected ex-post returns.

- The empirical implications for the learning path depend on the assumed information structure. Different patterns in the expected ex-post returns series may be observed depending on the information processes. However, in the different scenarios examined the expected ex-post return series exhibit a pattern opposite to that of the ex-ante return series before following the same pattern.

[5] The case of an increase in ex-ante returns will be the opposite

Case 3: Nonlinear Trend

$$R_t = R_{t-1}\,[1.0-0.001[t-20]]^2$$

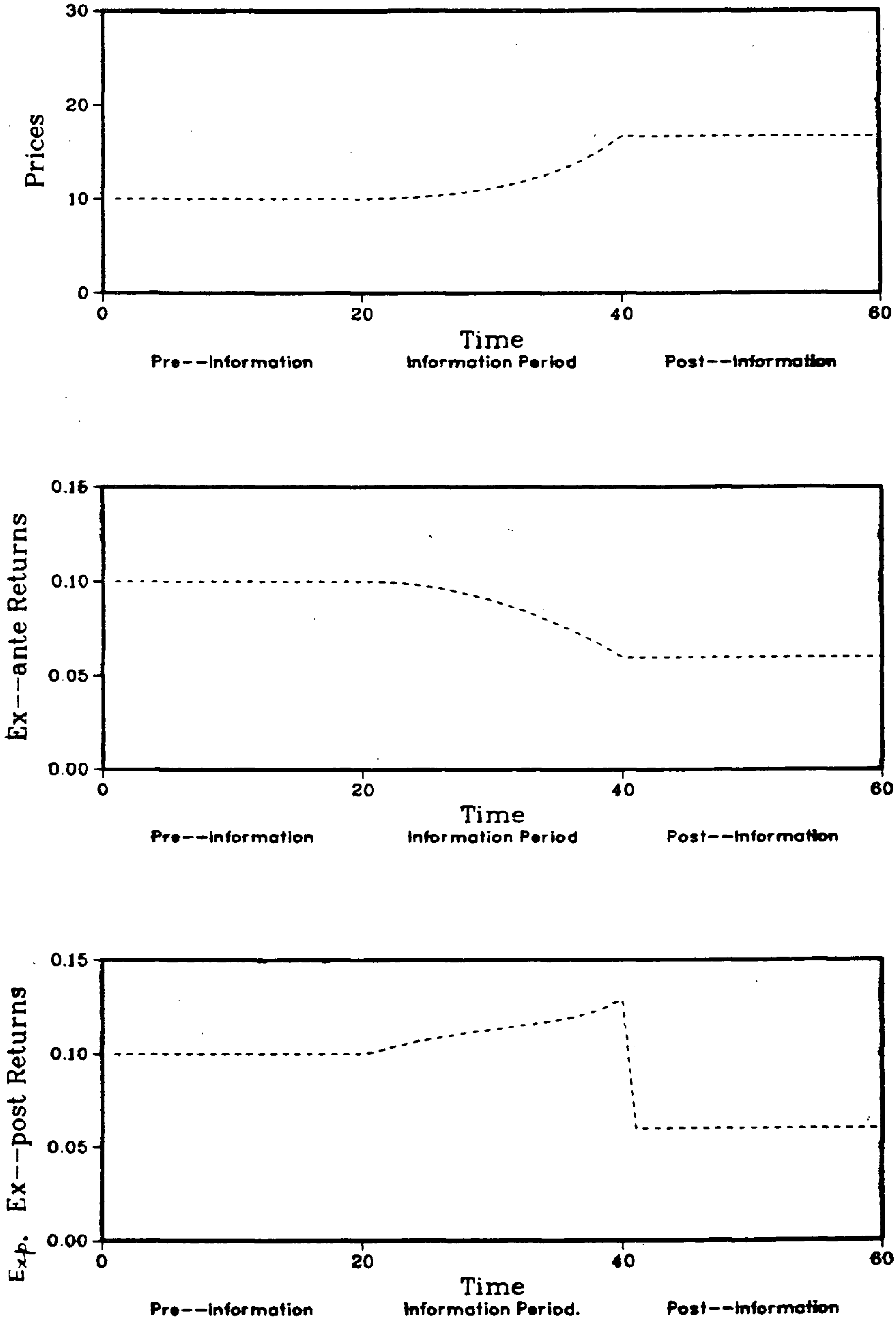

Figure 3.6: Market Learning: Multiple Signals

3.2.6 Relaxing Assumptions:

We have examined the impact of information in a simple economy with a single risky asset. We now discuss the effects of relaxing some of the assumptions of the simple model for empirical analysis.

- Economic effects: Asset risk premiums may be stochastic due to many economic factors affecting either the cash flows or the risk adjusted discount rate of different assets. Some factors may be seasonal in nature.[6] While the information effect in such scenario basically remains unchanged from that derived in the simple model, it is now confounded by other effects. To separate the information effect we need to specify the stochastic process generating the other factors. A test of market learning will examine the change in the return generating process after controlling for the stochastic variation due to the economic factors.

- Learning from Experience: An alternate or complementary effect can be present when agents learn from their experience. Many models of market learning that deal with different aspects of learning have been discussed in the literature.[7] Grossman, Khilstrom and Mirman (1974) develop a model based on Bayesian approach where agents learn by doing and by production of information. However, it may be difficult to discriminate between learning from experience and learning from the exogeneous information.

- Heterogeneous Investors: Many recent papers have examined the relationship between trading volume and information flows by relaxing the assumption of homogeneous investors.[8] Although a theory of trading volume is not fully developed, the models based on hetrogeneous investors are a more realistic description of capital markets where active trading is observed. Karpoff

[6] This issue has been discussed in many recent empirical studies which investigate some form of stochastic process or variability in prices, See Shiller(1981), Kleidon(1984), Keim and Staumbaugh (1986), Chen, Roll and Ross (1983).

[7] Some papers deal with situations in which agents are learning to form rational expectations. See Blume, Bray and Easley (1982), Bray (1982), Blume and Easley (1982), Bray and Kreps (1986) Taylor(1975), Townsend(1978, 1983), Frydman(1982). The focus in these papers is on the stability of and convergence to a rational expectations equilibrium.

[8] Karpoff(1986), Pfleiderer (1984), and Varian (1985)

(1986) develops a simple theoretical model where a normal trading volume occurs because of liquidity or speculative demands. The arrival of new information affects trading volume in two different ways; the first effect is through investor disagreement about the hypothesized effects and the second is through divergent prior expectations. The model predicts that information increases trading volume if it causes investors to revise their demand prices heterogeneously or if information is partially but not homogeneously anticipated. These predictions are consistent with empirical evidence. Since the information in our study pertains to an asset pricing model it is possible that such information may also affect the level of trading volume for some securities. A number of recent studies have used trading volume to address empirical issues concerning information effects.[9] An examination of a change in trading volume can provide additional evidence regarding the association of a change in asset prices with information arrival.

3.3 Academic Information in a Multi-asset Economy

3.3.1 The Size Anomaly: Differential Information case

In this section the key hypotheses are motivated through an illustrative case in a multi-asset economy. In particular, we present a scenario where research on the size anomaly may provide useful information to the rational investors. The example is developed in a differential information framework; a scenario where there is more information available concerning some securities than concerning others. We show that in such a framework the relative information risk is relevant for asset pricing and the arrival of new information for some assets can affect relative asset prices.[10]

[9] For example, see Pincus (1983), Asquith and Krasker (1984), Richardson, Sefcik, and Thompson (1986), Grundy (1985), and Lakoshinok and Vermaelen (1984)

[10] when the same amount of information is available for all securities the theoretical model is similar to the simple model with a single risky asset because the same information structure is assumed for all assets. The equilibrium pricing process in this model has has been examined by Kalymon (1971), Barry (1974), Brown (1979) and Bawa and Brown (1979).

3.3.2 Pre-Information Period

Consider an economy with only two types of securities; large and small firm securities. Assume that security returns are distributed according to a multivariate distribution with mean vector μ and covariance matrix Σ, both of which are unknown to the investors. Assume that there is more information available about large firms than about small firms.[11] Suppose there are N_L observed returns for large firms but only $N_S < N_L$ observed returns for small firms.[12] All agents have access to the same information. Assume that information is costly to obtain and that researchers' information is exogeneous. All information is instantaneously incorporated into asset prices.

The optimal portfolio choice problem in such an economy under the usual assumptions of homogeneous beliefs, single period expected utility maximization, and risk aversion among all participants in the market with no taxes or transaction costs and infinite divisibility of assets has been studied by Kalymon (1971), Barry (1974), Klein and Bawa (1977), Barry and Brown (1985), and Clarkson (1986). In this market setting, investors will take into account the estimation risk or the parameter uncertainty in optimal portfolio selection by using predictive distributions. The predictive distribution in the simple case of an unknown μ and a known Σ is $N(\mu^*, \Sigma^*)$ [13] with

$$\mu^* = \bar{r} \qquad \Sigma^* = \begin{pmatrix} h(N_L)\Sigma_{LL} & h(N_L)\Sigma_{SL} \\ h(N_L)\Sigma_{SL} & h(N_S)\Sigma_{SS} \end{pmatrix}$$

where $\bar{r}$ is the sample mean return vector for all securities Σ_{LL} denotes the submatrix of covariances among large firm securities, Σ_{SS} denotes the submatrix of covariances among small firm securities, Σ_{SL} denotes the submatrix of covariances between large and small firm securities, and $h(N) = 1 + 1/N$ represents an adjustment factor based on available sample information.[14] Average returns will be

[11] Such an assumption is consistent with the empirical evidence, See Barry and Brown (1984)

[12] There may be a wide variety of measures of the relative quantity of information. Raiffa and Schlaifer (1961) have pointed out that there is often an equivalent sample information interpretation for the posterior distribution. In other words, the posterior distribution may be formed on the basis of a wide variety of information sources, but it still may have a form as if it were the result of observing historical returns.

[13] See Barry and Brown (1985) p. 410

[14] In the case when both mean μ and Σ are unknown Σ^* will have the same general form as above

consistent with a capital asset pricing model that reflects investors' perception of differential information. Investors will adjust the measure of systematic risk that they employ for pricing securities to compensate for relative information risk. Barry and Brown (1985) demonstrate that even in the simple case of an unknown μ and a known Σ high information securities will have smaller betas under differential information than they would under a no differential information case. The opposite will be the case for low information securities. In a more realistic case when both μ and Σ are unknown the effect of relative information risk is exacerberated. The low information securities may have very high uncertainty and contribute high uncertainty to portfolios containing them, and such securities may require relatively large returns in comparison with high information securities. Klein and Bawa (1977) show that the effects of differential information produce relatively low estimation risk for the high information securities and arbitrarily large estimation risk for the low information securities.

Clarkson (1986) examines the issue of diversification of the estimation risk in a large economy where multiple low and high information securities exist. In his model, the adjusted beta for each security i is the form $\beta_i^* + b_i$ where β_i^* is the estimate of systematic risk based on equal information for small and large firms and b_i is an adjustment term to account for the relative information risk or the estimation risk. The adjustment factor b_S is positive for small firms[15] and is a decreasing function of two variables (i) the correlation ρ_{SL} between the high and low information securities' cash flows and (ii) the relative levels of precision associated with the securities as measured by $(1/N_L - 1/N_S)$.

Clarkson argues that in a large economy the estimation risk is diversifiable. His argument is that the reduction in the estimation risk comes from two sources. The first is through an inference based on what is known about related securities. A low information security that is highly correlated with a high information security effectively reflects the higher information. With many cross-correlated high

with two exceptions: the Σ assumed known is replaced by its sample counterpart $\hat{\Sigma}$ and $h(N)$ is modified to $h(N) = \frac{N-1}{N-S} \cdot \frac{N+1}{N}$ where S is the number of securities in the economy. The predictive distribution will be Student-t distribution with $N - S$ degrees of freedom. See Klein and Bawa (1977) and Barry and Brown (1986).

[15]For large firms the adjustment term b_L is negative.

information securities the market is able to infer the missing information for the low information securities from its counterpart for the high information securities. The second source of estimation risk reduction is through diversification across low information securities. If the source of uncertainty surrounding expected cash flows is largely uncorrelated across low information securities, portfolio formation will reduce the requirement for adjustment of beta. He argues that if the address or the identity of the missing data pieces is only partially correlated across low-information securities, the differential information risk can be reduced by holding a large number of small securities.

An implicit assumption in Clarkson's argument is that information about risk reduction from two sources is costlessly available to the investors. In the presence of costs of collecting this information, investors may not acquire it. To illustrate this point, we examine what information is required to enable an investor to diversify the estimation risk from the two sources.

The risk reduction from the first source depends on locating a large firm whose cash flows are correlated with that of a given small firm. If for a small firm a corresponding large firm is found the estimation risk of that small firm will be reduced. Such information is firm specific and has to be collected on a firm by firm basis. The risk reduction from the second source depends on whether the missing data is idiosyncratic or common across all firms. However, to draw an inference about whether the missing data is common or idiosyncratic across small firms requires the collection and comparison of missing data across small firms. Diversification of the estimation risk from this source depends on the extent to which small firms are similar. If small firms have little or no common risk characteristics, then estimation risk is idiosyncratic and can be reduced by holding a large portfolio of small firms.

Complete diversification of the estimation risk, even in a large economy, is possible only when either (i) for each small firm a matching large firm is found whose cash flows are perfectly correlated with the cash flows of the small firm, or (ii) by comparison of missing data of all small firms it is found that missing information is not correlated across small firms. This information may be costly and difficult to collect. The problem is further complicated by observing that the estimation risk in a single period model is a cross-sectional concept. The sample information about the possible risk reduction from the two sources can be obtained only through comparison across various economies or through a long time series of repeated realizations of one economy if the underlying process generating estimation risk is assumed to be constant.

In view of the implicit and explicit costs of collecting the information required to draw inferences about the diversification of estimation risk, we assume that such information is not available to the agents in the pre-information period. In the absence of such information, investors will price the securities as if the estimation risk were not diversifiable. Assuming that the risk free rate is zero, investors will price securities on the basis of expected returns derived by using an adjusted measure of systematic risk as follows:[16]

$$E(R_i) = \hat{\gamma}_1 \hat{\beta}_i \tag{3.10}$$

where

- $E(R_i)$= Expected return on asset i, i = L,S where L, S are large and small firm portfolios respectively.
- $\hat{\gamma}_1$ is an estimate of the expected return on the market portfolio.
- $\hat{\beta}_i$, i = L,S are the adjusted measures of the systematic risk of large and small firms respectively used by the agents.
- $\hat{\beta}_i = \beta_L^* - \hat{b}_L$, $\hat{\beta}_S = \beta_S^* + \hat{b}_S$, β_L^* and β_S^* are the estimates of the systematic risk of large and small firm portfolios respectively with equal information and are common knowledge.
- $\hat{b}_S$ is an estimate of the adjustment for estimation risk used by the agents for small firms with $\hat{b}_S > 0$. Since the beta of the market portfolio is 1, an upward adjustment in the systematic risk for small firms implies a downward adjustment in the systematic risk for large firms. Thus the adjustment for the large firms is $\hat{b}_L = q\hat{b}_S$, where $q = w_S/w_L \leq 1, w_S$ and w_L are the weights of small and large firms in the market portfolio respectively.

[16]We use Sharpe-Linter version of the CAPM and assume that risk free rate is known.

3.3.3 Information Period

Researchers estimate the following regression model:

$$E(R_i) = \gamma_1 \beta_i^* + \gamma_2 \theta_i \tag{3.11}$$

where

- $\theta_i = (\phi_m - \phi_i)/\phi_m$, i = L,S.
- ϕ_i = the size of the firm i, i = L,S.
- ϕ_m = the average size of a firm in the market
- γ_1= Expected return on the market portfolio.
- γ_2= Expected size premium measuring the contribution of ϕ_i to the returns of a security.

Researchers find a positive estimate of γ_2 which is called the size effect. This leads to further research concerning the relationship between size and various risk factors. It is possible that the size effect is fully consistent with the differential information equilibrium in the pre-information period. This follows from the relationship between γ_2 and $\hat{b}_S$ which is derived below.

Equating pricing equation 3.10 used by the agents with equation 3.11 used by the researchers

$$\gamma_1 \beta_L^* + \gamma_2 \theta_L = \hat{\gamma}_1 \hat{\beta}_L \tag{3.12}$$

$$\gamma_1 \beta_S^* + \gamma_2 \theta_S = \hat{\gamma}_1 \hat{\beta}_S \tag{3.13}$$

and solving the above equations we obtain the following relationships:

$$\gamma_1 = \hat{\gamma}_1 (1 - \hat{b}_S \frac{(q\theta_S + \theta_L)}{(\beta_L^* \theta_S - \beta_S^* \theta_L)}) \tag{3.14}$$

$$\gamma_2 = \hat{\gamma}_1 \hat{b}_S \frac{(q\beta_S^* + \beta_L^*)}{(\beta_L^* \theta_S - \beta_S^* \theta_L)} \tag{3.15}$$

Assuming that β_L^* and β_S^* are both positive implies the denominator $(\beta_L^* \theta_S - \beta_S^* \theta_L)$ in above equations is positive since θ_S is positive and θ_L is negative. The term $(q\theta_S + \theta_L)$ is positive because $\theta_S + \theta_L = 0$ and $q \leq 1$. Also, $(q\beta_S^* + \beta_L^*)$ is positive because q is positive and β_L^*, β_S^* are assumed to be positive. Thus from equation 3.14 $\gamma_1 \geq \hat{\gamma}_1$ and from equation 3.15 $\gamma_2 > 0$ since $\hat{b}_S > 0$. It follows that a positive estimate of γ_2 is a direct implication of a positive $\hat{b}_S$ used by the agents in the pre-information period. Since small firms are also low information firms, the size effect proxies the estimation risk associated with small firms. Thus, the discovery of a relationship between size and expected returns by itself may not be new information to the investors.

After the discovery of the size effect, researchers attempt to find explanations for the size effect parameters they have obtained. Apart from various statistical and economic explanations, the researchers also examine the relationship between size and various risk factors. Academic research on the systematic relationship between size and risk provides sample information about the common characteristics of small firms and the correlation ρ_{LS} between small and large firm portfolios. This sample information may be useful to the agents in drawing inferences about whether estimation risk is idiosyncratic or systematic across small firms. This follows from the possibility of diversification of estimation risk from two sources. The sample information about the correlation between small and large firms will be useful for diversification from the first source while the information relating to small firm and risk factors relates to the sample information about the second source of risk reduction. Based on this sample information, the investors may revise their prior beliefs about the diversification of estimation risk and thus revise their estimates of $\hat{b}_S$. The net effect of research information can be considered as increasing N_S, the amount of information available for small firm portfolios. In particular, as N_S approaches N_L the relative information risk approaches zero. This can affect relative asset prices.

3.3.4 Market Learning

In this framework, learning by agents is about whether the estimation risk associated with small firms is diversifiable. It corresponds to a revision in agents' prior beliefs concerning $\hat{b}_S$ of the estimation risk associated with the small firms after receiving the research information. Ex-ante, the research information may lead to downward, upward or no revision in the estimate $\hat{b}_S$ used by the agents in the pre-information period. Under the maintained hypothesis of market equilibrium based on differential information, there are two main scenarios:

Scenario 1: No market Learning

Academic research concerning the size effect provides no useful information to the market. The sample information provided by the researchers is consistent with the prior beliefs of the agents and there is no revision in the prior beliefs $\hat{b}_S$ of the agents.

Scenario 2: Market Learning

Academic research concerning the size effect provides useful information to the market. The sample information provided by the researchers is not consistent with the prior beliefs of the agents and there is a revision in the prior beliefs $\hat{b}_S$ of the agents.

3.3.5 Empirical Implications

Under scenario 2, if the new information concerning size and risk were known with certainty we would expect a once and for all instantaneous adjustment by the market. However, research of this nature does not normally give definitive results. Rather each research paper opens further possibilities and further research occurs. The simplest way to capture this idea is to suppose that the adjustment for the estimation risk, $\hat{b}_S$, is unchanged under scenario 1 and takes one other value under scenario 2. Academic research can then be modeled as affecting the probability as viewed by the agents that the market is in scenario 1 or scenario 2. Each research paper can be viewed as a noisy signal that provides information about the probabilities of the two values of $\hat{b}_S$. Upon receiving each signal, agents update their beliefs using Bayes' rule and revise the probabilities assigned to each branch.

From equation 3.15 since

$$\frac{d\gamma_2}{d\hat{b}_S} = \hat{\gamma}_1 \frac{(q\beta_S^* + \beta_L^*)}{(\beta_L^*\theta_S - \beta_S^*\theta_L)} \quad > 0 \tag{3.16}$$

any revision in $\hat{b}_S$ will affect the estimate of γ_2 obtained in the post information period. We will observe a decrease, increase or no effect on the estimates of γ_2

corresponding to a downward, upward or no revision in $\hat{b}_S$. Further, any revision in beliefs will have the opposite effect on large and small firms.

Chapter 4

The Econometric Model

4.1 Model Specification

The theoretical model of learning developed in chapter 3 predicts a change in a parameter of the equilibrium asset pricing model in the event of market learning. The parameter of interest in our analysis is the mean returns associated with the size variable and our aim is to test whether a change in this parameter is associated with the arrival of research information. Since the information arrival process spans a number of years, the hypothesized structural change is likely to be slow and may be realized over many time periods rather than in one single period. Econometric methods for estimating time-varying parameters are suitable for such analyses.

The economic literature on time-varying parameters includes many models based on different assumptions about the stochastic process generating parameter variation.[1] The most general specification allows for continuous parameter variation. Models of this type include Kalman filter[2] and random coefficient models. These models facilitate testing of a change in parameters as well as the estimation of the stochastic process of parameter variation. Alternatively, the number of possible parameter changes may be assumed to be finite where each possible state of the parameter vector may be called a regime.[3] Examples of this type include

[1]See Rosenberg (1973) and Sarris (1973) for excellent reviews on time-varying parameter techniques.

[2]Kalman (1960) and Kalman and Bucy (1961)

[3]These regimes may be associated with such things as the state of business cycle, other economic variables or other more fundamental structural changes. time varying parameters.

switching regressions.[4] The switching regimes model is useful for testing a shift in parameters as well as for estimating when a regime switch occurs.

For the market learning model posited in our study, a more general framework of continuous parameter variation is appropriate. However, this general framework is informationally more demanding since it requires information on the stochastic process as well as the time period when the effect occurs. This information is not available to us a priori because the learning model only predicts a change in the mean size effect associated with research information and does not specify the stochastic process generating the change. We observed in chapter 3 that many different forms of learning paths are possible under different assumptions about the information structure. In view of this limitation the empirical analysis is done in two stages. In the first stage we test for a change in mean returns associated with the size variable. This test is done by positing a simple stochastic structure of parameter variation by allowing only two possible parameter states and using a switching regimes model. If the evidence in the first stage supports the hypothesis of a switch in regimes then the second stage analysis is done in which the learning process generating the change is estimated from the data with some additional assumptions on the information structure.

The advantage of the two stage analysis is that the test of a switch in mean is done under the assumption of a simple stochastic process of change that is consistent with our a priori information. Further, as discussed in the theoretical model it is plausible that the information effect may be confounded by effects of other economic and seasonal factors. An appropriate test should assess the information effect after controlling for other effects. Such an analysis is proposed to be done in the first stage by specifying relevant economic factors and examining a switch in regimes after taking into account the variation attributable to these factors. Similarly, the seasonal variation in the month of January is proposed to be explored by testing a switch in non-January months. The switching regime method in addition to testing for a switch in regimes also provides us with estimates of a mean switch point, the switching period, and the parameters in different regimes. The estimates of the switching period can be used to estimate the learning process

[4] See Quandt (1958) and Goldfeld and Quandt (1973).

from the data under different a priori specifications and to discriminate among different models.[5] The test of a switch in the first stage is also consistent with the second stage analysis. If we reject the hypothesis of no switch using a restricted model of a single switch we will expect it to be rejected under the more general framework.

4.2 Switching Regimes Model: Formulation and Estimation

Let $R_{d1}, R_{d2}, R_{d3}, \ldots, R_{dn}$ be n observations of returns on a portfolio whose returns proxy the size effect. We postulate two regimes, one prior to and another after the incorporation of the research information on the size anomaly:

$$\text{Regime 1}: \quad R_{dt} = \alpha_1 + u_{1t} \tag{4.1}$$

$$\text{Regime 2}: \quad R_{dt} = \alpha_2 + u_{2t} \tag{4.2}$$

where:

- α_1 = mean of the time-series R_{dt} prior to the information about the size anomaly.
- α_2 = mean of the time-series R_{dt} after the incorporation of the information about the size anomaly.
- u_{it} = Normally and independently distributed error terms, $u_{it} \sim N(0, \sigma_i^2)$, $i = 1, 2$.

Using the generalized approach of Goldfeld and Quandt (1973), the switch in regimes is assumed to depend on a variable D_t and the model is written as:

$$R_{dt} = \alpha_1(1 - D_t) + \alpha_2 D_t + u_{1t}(1 - D_t) + u_{2t} D_t \tag{4.3}$$

We assume that D_t is a function of time and has a logistic functional form[6] Thus the model to be estimated is:

$$R_{dt} = \alpha_1 + \gamma D_t + \epsilon_t \tag{4.4}$$

[5] See Sarris (1973) for the method and Slade (1987) for an application of this technique.

[6] The particular functional form of the function D_t is of secondary importance to test whether a switch in regimes has occurred. This is because the general behaviour of the quantity ($\alpha_2 -$

where

- $\gamma = \alpha_2 - \alpha_1$
- $D_t = \dfrac{1}{1 + exp^{(\lambda + \mu t)}}$
- $\epsilon_t = u_{1t}(1 - D_t) + u_{2t} D_t$ and $\epsilon_t \sim N(0, \sigma_1^2 (1 - D_t)^2 + \sigma_2^2 D_t^2)$

The parameter γ measures the change in the mean of the distribution of R_{dt} from regime 1 to regime 2 and can be considered a weighting factor for means α_1 and α_2 during the transition from one regime to another. The central location parameter M and the standard deviation S of the variable D_t are given by $-\lambda/\mu$ and $-\pi/\sqrt{3}\mu$ respectively.[7] The parameter M provides the switch point and the standard deviation S provides an estimate of the transition period from regime 1 to regime 2. For example, $(M - 2S, M + 2S)$ provides an estimate of the time period during which approximately 95% of the change in mean occurred. A graphic illustration of the shape of D_t and of the relationship between D_t and R_{dt} for three different values of S is provided in figure 4.1 We observe that the change in regime is abrupt or smooth depending on whether the standard deviation is small or large.

The model of equation 4 is nonlinear and involves six parameters α_1, γ, σ_1^2, σ_2^2, λ and μ. We use maximum likelihood procedure to estimate the six parameters. The maximum likelihood estimates are consistent, asymptotically efficient and normally distributed. The logarithmic likelihood function is

$$\ln L = -\frac{n}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{n} \ln[{\sigma_1}^2(1 - D_t)^2 + {\sigma_2}^2(D_t)^2] - \frac{1}{2}\sum_{t=1}^{n} \frac{(R_{dt} - \alpha_1 - \gamma D_t)^2}{{\sigma_1}^2(1 - D_t)^2 + {\sigma_2}^2(D_t)^2} \tag{4.5}$$

Maximum likelihood estimates are obtained by maximizing equation 4.5 with respect to the six parameters $\alpha_1, \gamma, \sigma_1^2, \sigma_2^2, \lambda$ and μ. Let the maximum of the likeli-

$\alpha_1)D_t$ is only slightly affected by the particular form of the transition function. Further, the effect of variability in the data will generally mask any differences introduced by different transition functions. Thus the cumulative distribution function of any symmetric probability distribution function could be used as the transition function. See Bacon and Watts (1971) and Goldfeld and Quandt (1973).

[7] The distribution function of logistic curve with mean a and standard deviation $k\pi/\sqrt{3}$ is given by $\dfrac{1}{1 + exp^{-(x-a)/k}}$. Also note that, $\frac{\partial M}{\partial \mu} = \frac{\lambda}{\mu^2} > 0$ and $\frac{\partial M}{\partial \lambda} = \frac{-1}{\mu} > 0$ Thus mean M is an increasing function of both λ and μ. In general, $|\lambda/\mu^2| \geq |-1/\mu|$ which implies that location parameter M is more sensitive to small changes in μ than to small changes in λ.

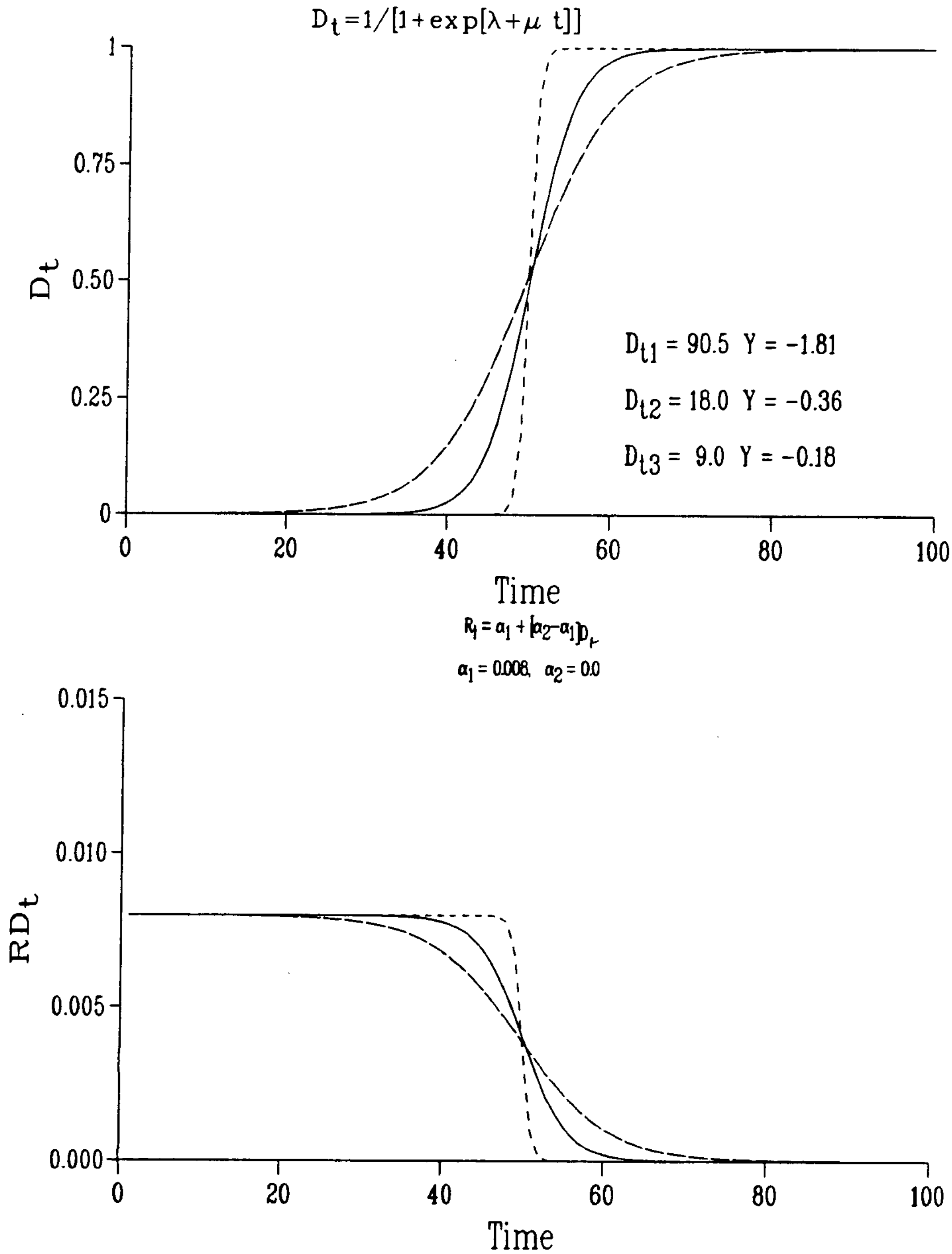

Figure 4.1: Illustration of the Econometric Model

hood function be denoted by $L(\hat{\alpha_1}, \hat{\gamma}, \hat{\sigma_1}, \hat{\sigma_2}, \hat{\lambda}, \hat{\mu})$ and the maximum under the null hypothesis by $L(\hat{\alpha}, \hat{\sigma})$. The likelihood ratio test statistic is

$$\theta = \frac{L(\hat{\alpha}, \hat{\sigma})}{L(\hat{\alpha_1}, \hat{\gamma}, \hat{\sigma_1}, \hat{\sigma_2}, \hat{\lambda}, \hat{\mu})}$$

and $-2\log\theta$ appears in finite samples to be well approximated by the χ^2 distribution with 4 degrees of freedom.[8]

The optimization problem is solved by minimizing $-\ln L$, the negative of the log likelihood function. The nonlinear function optimization package prepared by the university of British Columbia is used. We employ two nonlinear programming routines for the analysis. The first routine (GRG) employs the generalized reduced gradient method. The second routine (NLPQL) employs the quadratic approximation method.

The search for the optimum is done by providing an initial starting point. Some major problems in all nonlinear algorithms are convergence at a local optimum, false convergence or no convergence. For example, if an initial point is in the neighbourhood of a local maxima there is high probability that the algorithm may converge to the local maxima. On the other hand if the initial point is too far from the maximum the algorithm may not converge at all. The problem may be accentuated if multiple optima exist. To reduce execution time and the chances of encountering a local optimum, it is desirable that the initial point be close to the global optimum point. However, in our optimization problem there is minimum a priori information about the switch point. It is therefore important to search for the optimum from several initial points.

We select the initial points so that at least one initial switch point is provided in each year in the time period being examined. The value of log likelihood function is calculated for the convergent points obtained at each starting point and the maximum likelihood estimates are selected as that value of parameters which corresponds to the maximum maximorum.[9] The advantage of this approach is that it provides sensitivity analysis of the optimum point selected to various starting

[8]Goldfeld and Quandt (1973), page 479

[9]This approach is similar in spirit to that suggested by Quandt (1958), page 875, to estimate parameters of linear regression system obeying two separate regimes.

values. It also provides a comparison of the likelihood value at local optimum points with the likelihood value at the selected global optimum. Also, if a convergent point is an optimum we will expect that for all the initial points close to that convergent point convergence is achieved at that optimum.

The following estimation procedure is used with two algorithms. First, an initial point is chosen and the best solution is obtained using GRG with a tolerance value of 0.0001. The best solution obtained from GRG is then provided as initial point for routine NLPQL with tolerance value of 0.00001. This procedure is repeated with all starting values. The procedure of restarting from the best point found under one routine using another routine is useful in detecting the problem of false convergence.

Convergence in both routines is considered to have been achieved if the Kuhn-Tucker conditions are satisfied to within the specified accuracy, or if it appears that the objective function cannot be improved significantly when the constraints are satisfied to within a specified accuracy. The algorithm also provides the estimates of the t values and the standard errors at the convergent points. In both routines the standard errors are computed as the square root of the diagonal element in the estimated covariance matrix of the parameters and t values are computed as the parameter value divided by the standard error. Although we use the term t-statistics for t values at all convergent points, the t-statistics at local optimum points are t ratios.

In view of very small variable values the variables are scaled by ten and the econometric model used for estimation is:

$$R'_{dt} = \alpha'_1 + (\gamma')D_t + \epsilon'_t \tag{4.6}$$

where

$$R'_{dt} = 10R_{dt}, \ \alpha'_i = 10\alpha_i, \quad i = 1, 2.$$

$$\gamma' = 10\gamma, \quad \text{and } \epsilon'_t \sim N(0, 100(\sigma_1^2(1 - D_t)^2 + \sigma_2^2 D_t^2))$$

The null and the alternative hypotheses to be tested are:

H_0 : There is no switch in regimes ; $\gamma = 0.$[10]

[10] $\gamma' = 10\gamma = 0$ implies $\gamma = 0$. Note that when $\gamma = 0$, the model in equation 4.4 collapses to $R_{dt} = \alpha_1 + \epsilon_t$. This requires estimation of five parameters, $\alpha_1, \sigma_1, \sigma_2, \lambda$, and μ because $\epsilon_t \sim$

H_1 : There is a switch in regimes ; $\gamma \neq 0$.

$N(0, (\sigma_1^2(1 - D_t)^2 + \sigma_2^2 D_t^2))$.

Chapter 5

Empirical Analysis

5.1 Overview

This chapter is organized as follows. Section 5.2 describes the data and the sample period. In section 5.3 the stability of the size effect in the pre-information period is tested using our data set and the switching regimes method. The results are compared with those derived by Brown, Kleidon and Marsh (1983) who do similar analysis using Kalman filter and recursive residual techniques. Section 5.4 presents the results for the test of a switch in the mean size effect and section 5.5 examines some alternative explanations. Section 5.6 contains an analysis of the January seasonality in the size effect and section 5.7 contains an analysis of the transition period and of the learning process. Section 5.8 examines the relationship between the information effect and trading volume.

5.2 Data and Sample Period

We use the time-series of the difference between returns of equally-weighted and value-weighted indexes to proxy the returns associated with the size variable. A value-weighted index is more heavily invested in large firms than is an equally-weighted index and the difference between the two indexes is interpreted as a proxy for the size effect.[1]. The portfolio of equally weighted index less value weighted index is referred to as the difference portfolio and the time-series of returns on the difference portfolio is referred to as the difference series. Data are collected from

[1]See Roll(1981), Ariel (1987)

the CRSP daily index file (1986). These are available from July 1962 to December 1985 with a total of 5904 observations.

5.3 Instability of the size effect: Pre-information period

The purpose of this analysis is two-fold. Many researchers have analyzed the instablity of the size effect in our pre-information period, a period prior to the discovery of the size anomaly in 1981, using different techniques for estimating time-varying parameters and different proxy variables for the size effect. Our investigation using the switching regimes method and the difference series data serves to highlight the comparison between our method and the more general framework adopted by some researchers. This analysis also serves another purpose. Evidence of a switch in regimes in the pre-information period will suggest stochastic variation due to other factors. In that case we must control for the effects of these factors in order to draw valid inferences about a switch during the information period.

We focus on the period January 1967 to June 1979 which has also been examined by Brown, Kleidon and Marsh (1983). Brown, Kleidon and Marsh (BKM) form ten portfolios based on size and use the excess monthly mean returns series associated with each portfolio. They employ Kalman filter and recursive residuals[2] techniques for testing the hypotheses that mean excess returns associated with the size variable are constant for each of the ten portfolios from January 1967 to June 1979. BKM find that the assumption of a non-stochastic mean size effect is most seriously violated for the smallest and the largest portfolios. They also find that the difference between the returns on the portfolios of the smallest and largest firms is also not constant through time.[3] However, they find that while the excess returns are non-stationary over the entire period from January 1967 to June 1979, the excess returns are relatively stationary in two subperiods: January 1969 to December 1973 and January 1974 to June 1979. From January 1969 to December 1973 they find a relatively stable negative size effect and from January 1974 to

[2]Brown, Durbin and Evans (1975)

[3]BKM (1983) find that the excess returns of the six smaller firm portfolios are positively correlated as are those of the ninth and tenth (the largest portfolios).

June 1979 they find a positive size effect. The ordinary least squares estimates of the annualized difference in returns on the smallest and the largest portfolios in these two periods are -25% and 25% respectively.

We test for a switch in regimes in the period January 1967 to June 1979 as well as in the subperiods January 1969 to December 1973 and January 1974 to June 1979. To facilitate the comparison with the results of BKM (1983), we use monthly data. The monthly difference portfolio returns are compounded from the daily difference series returns.

Table 5.1 contains the likelihood ratio test-statistics and t-statistics for testing the null hypothesis of no switch against the alternative of a single switch. Tables 5.2, 5.3 and 5.4 contain the estimated parameters for periods January 1967 to June 1979, January 1969 to June 1973 and January 1974 to June 1979 respectively. The search for the optimum is done by providing eleven starting points. The starting points are selected so that at least one initial switch point is provided in each year. This is done by varying the initial values for λ. The standard deviation for all the initial points is 4.5 months. The same starting points are used in all subperiods.

Our analysis confirms the results obtained by BKM. Similar to the findings of BKM, we find a stable size effect in the subperiods January 1969 to December 1973 and January 1974 to June 1979. While the LR test statistics for the two subperiods are 5.5 and 19.8 respectively against the 5 percent level of 9.5, the t-statistics for $\gamma = 0$ for the two subperiods for the maximized log likelihood value are only -1.39 and -0.70 respectively against the 5 percent level of 1.98. The likelihood ratio test is a joint test on all parameters and the t-statistic is a test on the single parameter γ which measures the change in the size effect. Since the t-statistics for both subperiods are not significant at the 5% level, the hypothesis of no switch fails to be rejected in both subperiods. The estimates of the mean size effect in the two subperiods are -7.4% and 22% per annum respectively.

For the period January 1967 to June 1979 our results are also similar to those of BKM. The hypothesis of no switch in this period is rejected at the 1% level. The t-statistic for $\gamma = 0$ for the maximized likelihood estimates is -2.71 which is significant at the 1% level. The LR test statistic is 11.9 which is significant at the

TABLE 5.1 SWITCHING REGIMES MODEL MONTHLY DIFFERENCE SERIES : SUMMARY STATISTICS						
PERIOD	LnL_1	LnL_2	LR	t-statistic	M	S.D
1/69–12/73	-16.83	-13.99	5.5	-1.39	10/72	6.0
1/74–6/79	-35.06	-25.1	**19.8	-0.33	3/76	0.4
1/67–6/79	-66.49	-60.53	*11.92	**-2.7	11/68	4.0
1/67–12/73	-27.05	-17.69	**18.75	**-4.1	11/68	4.0
1/78–12/85	3.5	12.2	**17.2	**-2.9	12/83	2.2
8/62–12/85	-70.0	-54.9	**30.2	-0.9	5/79	2.4

LnL_1 is value of the Log Likelihood function assuming no switch.

LnL_2 is value of the Log Likelihood function assuming a single switch.

LR is the Likelihood ratio test statistic, $-2\ Ln(L_1 - L_2)$, for the test of no switch against the alternative of one switch and is distributed as χ^2 with 4 degrees of freedom.

t-statistic is for testing $\gamma' = (\alpha'_1 - \alpha'_2) = 0$ where α'_i is mean in regime i, $i = 1, 2$.

M is the maximum Likelihood estimate of the mean switch point.

S.D is the maximum Likelihood estimate of standard deviation (months).

* Significant at the 5 percent level.

** Significant at the 1 percent level.

Table 5.1: Switching Regime Model: Monthly Difference Series (Summary Statistics)

TABLE 5.2

SWITCHING REGIMES : MONTHLY DIFFERENCE SERIES (1/69-12/73)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	-0.10	0.20	0.10	0.12	3.6	-0.40	9/69	4.5	-20.8
P_f	-0.16	0.11	0.04	0.11	2.6	-0.46	6/69	4.0	-14.9
Stat.	-1.4	0.8	0.98	**4.7	0.71	-0.67			3.9
2. P_0	-0.10	0.20	0.10	0.12	8.4	-0.4	9/70	4.5	-22.8
P_f	-0.13	0.10	0.06	0.12	54.2	-2.8	7/70	0.6	-14.7
Stat.	*-2.2	1.3	**3.0	**4.5	1.7	-1.7			4.3
3. P_0	-0.10	0.20	0.10	0.12	13.6	-0.40	10/71	4.5	-20.7
P_f	-0.03	-0.05	0.12	0.09	20.2	-0.67	6/71	3.0	-15.3
Stat.	-0.5	-0.6	**3.6	**3.4	1.1	-1.0			3.1
4. P_0	-0.10	0.20	0.10	0.12	18.4	-0.40	10/72	4.5	-19.1
P_f	-0.03	-0.15	0.10	0.12	13.7	-0.3	10/72	6.0	-14.0
Stat.	-0.6	-1.3	**4.3	1.8	1.4	-1.4			5.5
5. P_0	-0.10	0.20	0.10	0.12	21.6	-0.40	6/73	4.5	-19.7
P_f	-0.03	-0.15	0.10	0.12	13.7	-0.3	10/72	6.0	-14.0
Stat.	-0.6	-1.3	**4.3	1.9	*2.4	*-2.3			5.5

P is parameter, P_0 is the initial point P_f is the convergent point.

α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.

$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).

LnL is the value of Log Likelihood function.

Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

* Significant at the 5 per cent level.

** Significant at the 1 per cent level.

Table 5.2: Switching Regime Model: Monthly Difference Series (1/69-12/73)

TABLE 5.3

SWITCHING REGIMES : MONTHLY DIFFERENCE SERIES (1/74-6/79)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.20	-0.10	0.10	0.12	2.4	-0.40	6/74	4.5	-42.9
P_f	1.0	-0.8	0.23	0.15	15.7	-9.4	2/74	0.2	-30.2
Stat.‡									*9.7
2. P_0	0.20	-0.10	0.10	0.12	7.2	-0.4	6/75	4.5	-45.9
P_f	0.22	-0.07	0.41	0.1	25.8	-1.6	4/75	1.1	-27.7
Stat.	1.4	-0.4	**2.6	**5.0	1.0	-1.0			**14.7
3. P_0	0.20	-0.10	0.10	0.12	12.0	-0.40	6/76	4.5	-43.2
P_f	0.22	-0.08	0.32	0.07	200.0	-7.4	3/76	0.24	-25.1
Stat.	*1.98	-0.7	**3.6	**4.4	**11.3	**-11.2			**19.8
4. P_0	0.20	-0.10	0.10	0.12	16.8	-0.40	6/77	4.5	-38.9
P_f	0.19	-0.04	0.34	0.08	5.4	-0.2	7/76	11.0	-26.2
Stat.	1.9	-0.3	**2.6	**3.7	1.7	-1.9			**17.7
5. P_0	0.20	-0.10	0.10	0.12	21.6	-0.40	6/78	4.5	-40.2
P_f	0.2	-0.04	0.34	0.07	5.4	-0.2	7/76	11.0	-26.2
Stat.	1.9	-0.3	**2.6	**3.7	1.7	-1.9			**17.7

P is parameter, P_0 is the initial point P_f is the convergent point.

α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.

$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).

LnL is the value of Log Likelihood function.

Stat. is test-statistic. (Col. 2-7) is the t-stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

*** Significant at the 5 per cent level.**

**** Significant at the 1 per cent level.**

‡ Not available.

Table 5.3: Switching Regime Model: Monthly Difference Series (1/74-6/79)

TABLE 5.4

SWITCHING REGIMES : MONTHLY DIFFERENCE SERIES (1/67-6/79)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	-0.10	0.20	0.10	0.12	6.5	-0.40	4/68	4.5	-77.4
P_f	0.25	-0.20	0.07	0.15	10.4	-0.45	11/68	4.0	-60.5
Stat.	**4.2	**-2.7	**3.1	**7.8	1.2	-1.2			*11.9
2. P_0	-0.10	0.20	0.10	0.12	13.2	-0.40	9/69	4.5	-76.3
P_f	0.25	-0.20	0.07	0.15	10.4	-0.45	11/68	4.0	-60.5
Stat.	**4.1	**-2.6	**2.6	**7.7	0.6	-0.6			*11.9
3. P_0	-0.10	0.20	0.10	0.12	18.0	-0.40	9/70	4.5	-78.4
P_f	0.15	-0.07	0.09	0.16	47.8	-1.4	9/69	1.3	-63.6
Stat.	**2.8	-1.1	**4.0	**7.6	**4.8	**-4.6			5.8
4. P_0	-0.10	0.20	0.10	0.12	23.2	-0.40	10/71	4.5	-76.3
P_f	0.04	0.1	0.1	0.18	500.0	-6.5	4/73	0.3	-62.2
Stat.	1.1	1.6	**6.2	**6.1	**25.4	**-24.9			8.6
5. P_0	-0.10	0.20	0.10	0.12	28.0	-0.40	10/72	4.5	-74.6
P_f	0.04	0.10	0.10	0.18	500.0	-6.5	4/73	0.3	-62.2
Stat.	1.1	1.6	**6.2	**6.1	**51.1	**-47.2			8.6
6. P_0	-0.10	0.20	0.10	0.12	31.2	-0.40	6/73	4.5	-76.3
P_f	0.04	0.09	0.10	0.18	280.0	-3.7	4/73	0.5	-62.4
Stat.‡									8.2
7. P_0	0.20	-0.10	0.10	0.12	36.0	-0.40	6/74	4.5	-83.3
P_f	0.04	0.13	0.13	0.15	108.4	-1.2	7/74	1.5	-63.8
Stat.	1.1	1.98	**6.7	**5.4	**14.3	**-13.6			5.4
8. P_0	0.20	-0.10	0.10	0.12	40.8	-0.40	6/75	4.5	-85.6
P_f	0.08	0.09	0.16	0.08	27.9	-0.23	3/77	8.0	-61.2
Stat.	*2.2	1.3	**7.5	**3.3	**9.7	**-9.4			*10.6
9. P_0	0.20	-0.10	0.10	0.12	45.6	-0.40	6/75	4.5	-82.3
P_f	0.08	0.09	0.16	0.08	27.9	-0.23	3/77	8.0	-61.2
Stat.	*2.2	1.3	**7.5	**3.3	**12.3	**-11.5			*10.6
10. P_0	0.20	-0.10	0.10	0.12	50.4	-0.40	6/77	4.5	-78.7
P_f	0.07	0.08	0.17	0.07	113.1	-1.0	6/76	1.8	-60.8
Stat.	1.8	1.3	**7.4	**4.2	**11.5	**-11.3			*11.5
11. P_0	0.20	-0.10	0.10	0.12	55.2	-0.40	6/78	4.5	-79.9
P_f	0.08	0.09	0.16	0.08	27.9	-0.23	3/77	8.0	-61.2
Stat.	*2.2	1.3	**7.5	**3.3	**11.7	**-11.0			*10.6

P is parameter, P_0 is the initial point P_f is the convergent point.
α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.
$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).
LnL is the value of Log Likelihood function.
Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.
*** Significant at the 5 per cent level.**
**** Significant at the 1 per cent level.**
‡ Not available.

Table 5.4: Switching Regime Model: Monthly Difference Series (1/67-6/79)

5% level. The estimated switch point is located at November 1968 with a standard deviation of about four months. Thus the switch in mean occurred between March 1968 and July 1969. The estimated mean size effects in the two regimes are 34.5% and -7.3% per year respectively.

We also examine the subperiod January 1967 to December 1973 to test the hypothesis of no switch. The summary test statistics and the estimated parameters for these subperiods are provided in table 5.1 and table 5.5 respectively. We obtain two convergent points for the three starting values. For three starting points convergence is obtained at a point with estimated mean switch point at November 1968 and a standard deviation of 4.5 months. The other three starting points converged to a point with estimated mean switch at July 1972 and a standard deviation of 6.7 months. The t-statistics for $\gamma = 0$ at the two convergent points are -4.1 and -2.7, respectively and are significant at the 1% level. The log likelihood values at both points are -17.7 and -21.9 respectively. The LR test statistics at the two points are 18.8 and 10.3 respectively providing the switch point at November 1968 as the maximum likelihood estimate; the same obtained for the period 1967-1979. The convergent point with switch point at July 1972 is therefore a local maximum.

The analysis in the 1967-73 subperiod indicates that during the period January 1967 to June 1979 there may be two or more switches in regimes. This hypothesis is tested by postulating a model of two switches during January 1967 to June 1979. The hypotheses of no switch and a single switch in regimes are rejected against the alternative of two switches in regimes at the 1% level. The LR test statistic of the hypothesis of two switches against the hypothesis of a single switch is distributed as χ^2 with four degrees of freedom. The value of LR test statistic is 26.0 and is significant at the 1% level. The estimated switch points are located at November 1968 and April 1973 respectively. The estimated mean size effect in the three regimes are 37.6%, -7.4%, and 18% with t-statistics 4.39, -1.28 and 2.78 respectively. The t-statistics for the hypotheses that the mean in the first regime is different from the mean in the second and that the mean in the second regime is different from the mean in the third are 4.6 and 2.98 respectively which are significant at the 1% level. The estimated variances are approximately equal in first two regimes but is almost twice as large in the third regime.

TABLE 5.5

SWITCHING REGIMES : MONTHLY DIFFERENCE SERIES (1/67-12/73)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	-0.10	0.20	0.10	0.12	6.5	-0.40	4/68	4.5	-38.8
P_f	0.27	-0.33	0.07	0.11	9.2	-0.40	11/68	4.5	-17.7
Stat.	**4.2	**-4.1	**2.9	**5.3	1.2	-1.2			**18.8
2. P_0	-0.10	0.20	0.10	0.12	13.2	-0.40	9/69	4.5	-37.6
P_f	0.27	-0.33	0.07	0.11	9.2	-0.40	11/68	4.5	-17.7
Stat.	**4.1	**-4.0	**2.9	**5.3	1.1	-1.1			**18.8
3. P_0	-0.10	0.20	0.10	0.12	18.0	-0.40	9/70	4.5	-39.7
P_f	0.27	-0.33	0.07	0.11	9.2	-0.40	11/68	4.5	-17.7
Stat.	**4.3	**-4.3	**3.1	**5.4	1.8	-1.8			**18.75
4. P_0	-0.10	0.20	0.10	0.12	23.2	-0.40	10/71	4.5	-37.6
P_f	0.08	-0.28	0.11	0.11	17.8	-0.27	7/72	6.7	-21.9
Stat.	1.9	**-2.7	**5.5	*2.2	*2.3	*-2.2			*10.3
5. P_0	-0.10	0.20	0.10	0.12	28.0	-0.40	10/72	4.5	-35.9
P_f	0.08	-0.28	0.11	0.11	17.8	-0.27	7/72	6.7	-21.9
Stat.	1.9	**-2.7	**5.5	*2.2	*2.3	*-2.2			*10.3
6. P_0	-0.10	0.20	0.10	0.12	31.2	-0.40	6/73	4.5	-36.6
P_f	0.08	-0.28	0.11	0.11	17.8	-0.27	7/72	6.7	-21.9
Stat.	1.9	**-2.7	**5.5	*2.2	*2.3	*-2.2			*10.3

P is parameter, P_0 is the initial point P_f is the convergent point.

α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.

$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).

LnL is the value of Log Liklihood function.

Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

* Significant at the 5 per cent level.

** Significant at the 1 per cent level.

Table 5.5: Switching Regime Model: Monthly Difference Series (1/67-12/73)

The estimated switch point located at November 1968 under the model of a single switch for the 1967-79 period is also a switch point under the model of two switches. However, the second switch point located at April 1973 under a two switch model is not a switch point under the one switch model. This is not surprising since the model of a single switch is more restrictive when the appropriate model is many switches in regimes. Thus rejecting the hypothesis of no switch in favour of one switch model also rejects it in favour of two or more switch model.

Based on the above analysis we conclude that, for testing the stability of the size effect in the pre-information period, the switching regimes method with its simplifying assumption of a finite number of regimes provides results that are similar to the more general Kalman Filter technique which allows for continuous changes in the parameters. The evidence also supports the hypothesis that there are at least several switches in regimes in the pre-information period. It is possible that these switches may be related to the stochastic variation in economic or other factors that may also affect the asset pricing process. We must take into account stochastic variations due to other factors in order to draw any valid inferences about the impact of research information on the size anomaly in the information period. This analysis is done in section 5.5 where we specify these factors and examine a switch in regime in the information period after controlling for variation due to these factors. Before proceeding with that analysis we first examine if there is a switch in regimes in the information period.

5.4 Empirical Analysis: Information Period

In this section we test the hypothesis of no switch in a period that is associated with the arrival of the research information about the size anomaly. We focus on a subperiod rather than the entire period of July 1962 to December 1985. The main reason for focusing on a subperiod is that the evidence in the previous section indicates that there may be three or more switches in regimes if the entire time period is considered. Thus an appropriate model for the entire series would posit three or more switches rather than a single switch in regimes. This will greatly increase the computational burden because the number of parameters to be estimated increases considerably with an increase in the number of switches in regimes. For example, in contrast to the estimation of only six parameters for a single switch model the two switch model requires the estimation of ten parameters and the three switch model requires the estimation of fourteen parameters. Moreover, since the estimation method is iterative the computational burden is further increased due to the large number of observations for the entire series.

For the switching regimes model, we need a time period that includes some portion of the pre-information period as well as the information period. The discovery of the size anomaly is generally attributed to the research papers by Banz and Reinganum that were published in March 1981. The earliest research information on the size anomaly can be considered to be the Ph.D dissertations by Banz and Reinganum which were completed in December 1978 and December 1979, respectively. Most of the research papers on the size anomaly, including a special issue on the size and related anomalies, were published between 1981 and 1983. (See appendix A for a list of the key information dates associated with the size anomaly.) We select the subperiod January 1978 to December 1985 to test a shift in the mean size effect associated with research information. This subperiod includes the entire information period, some portion of the pre-information as well as the post-information period. We also do sensitivity analysis using the entire time series from July 1962 to December 1985.

Table 5.6 provides the summary test statistics for the period January 1978 to December 1985. The null hypothesis of no switch is rejected at the 1% level. The LR test statistic for testing the null hypothesis of no switch against a single switch, which is distributed as χ^2 with four degrees of freedom, is 136 and is significant at any reasonable level. The t-statistics for $\gamma = 0$ for the maximized likelihood estimates is -3.97 which is significant at the 1% level.

Table 5.7 contains the estimated parameters and maximized likelihood value for nine different starting points. The starting points are selected by varying the switch points; with at least one switch point in each year between 1978 and 1985. The standard deviation for all starting points is 121 days. For one initial point convergence could not be achieved. For three starting points the same convergent

TABLE 5.6

SWITCHING REGIMES MODEL

DAILY DIFFERENCE SERIES : SUMMARY STATISTICS

PERIOD	LnL_1	LnL_2	LR	t-statistic	M	S.D
1/78–12/85	3786.89	3854.9	**136.0	**-3.97	6/83	13.0
1/78–12/82	2233.46	2253.4	**40.0	-0.08	6/82	25.0
1/84–1/85	1096.88	1108.95	**24.16	-0.53	4/85	37.0
7/62–12/85	11263.83	11307.8	**87.94	**-4.04	6/83	36.0
1/78–12/85 (NON–JAN.)	3464.8	3534.2	**139.0	**-3.7	6/83	13.0
1/78–12/85 (JANUARY)	333.13	336.5	6.8	-1.9	1/80	4.4
7/62–12/85 (JANUARY)	824.43	849.0	**49.0	**-3.8	1/77	13.0

NON–JAN. includes observations for non–January months only and JANUARY includes observations for the month of January only.

LnL_1 is value of the Log Likelihood function assuming no switch.

LnL_2 is value of the Log Likelihood function assuming a single switch.

LR is the likelihood ratio test statistic $-2Ln(L_1 - L_2)$ for the test of no switch against the alternative of one switch and is distributed as χ^2 with 4 degrees of freedom.

t-statistic is for testing $\gamma' = (\alpha'_1 - \alpha'_2) = 0$ where α'_i is mean in regime i, $i = 1, 2$.

M is the maximum Likelihood estimate of the mean switch point.

S.D is the maximum Likelihood estimate of standard deviation (days).

* Significant at the 5 percent level.

** Significant at the 1 percent level.

Table 5.6: Switching Regime Model: Daily Difference Series (Summary Statistics)

TABLE 5.7

SWITCHING REGIMES : DAILY DIFFERENCE SERIES (1/78-12/85)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.005	-0.006	0.0017	0.0007	3.0	-0.015	10/78	121	3421.8
P_f†	0.007	-0.005	0.0016	0.0014	10.9	-0.03	5/79	58	3806.9
Stat.	**3.4	*-2.1	**11.7	**28.4	**8.9	**-8.9			**40.0
2. P_0	0.005	-0.006	0.0017	0.0007	6.0	-0.015	8/79	121	3474.7
P_f	0.007	-0.005	0.0016	0.0014	10.9	-0.03	5/79	58	3806.9
Stat.	**3.4	*-2.1	**11.7	**28.4	**8.9	**-8.9			**40.0
3. P_0	0.005	-0.006	0.0017	0.0007	10.0	-0.015	8/80	121	3585.6
P_f	0.005	-0.004	0.0017	0.0011	112.6	-0.14	4/81	13	3810.9
Stat.	**3.8	*-2.2	**20.2	**24.3	**93.6	**-80.9			**48.0
4. P_0	0.005	-0.006	0.0017	0.0007	13.0	-0.015	6/81	121	3668.5
P_f	0.005	-0.004	0.0017	0.0011	113.6	-0.14	4/81	13	3810.9
Stat.	**3.8	*-2.2	**20.2	**24.3	**211	**-121			**48.0
5. P_0	0.005	-0.006	0.0017	0.0007	16.0	-0.015	3/82	121	3680.8
P_f	0.004	-0.003	0.0016	0.0011	76.0	-0.07	4/82	25	3809.0
Stat.	**3.6	-1.6	**22.9	**21.5	**163	**-84			**46.2
6. P_0	0.005	-0.006	0.0017	0.0007	19.0	-0.015	1/83	121	3767.9
P_f	0.005	-0.006	0.0017	0.0007	199.2	-0.14	6/83	13	3854.9
Stat.	**4.7	**-4.0	**26.1	**17.9	**323	**-208			**136
7. P_0	0.005	-0.006	0.0017	0.0007	22.0	-0.015	10/83	121	3839.7
P_f	0.005	-0.006	0.0017	0.0007	197.2	-0.14	6/83	13	3854.9
Stat.	**4.7	**-4.0	**26.1	**17.9	**318	**-207			**136
8. P_0	0.005	-0.006	0.0017	0.0007	24.0	-0.015	5/84	121	3827.6
P_f	0.005	-0.006	0.0017	0.0007	185.8	-0.14	6/83	13	3854.9
Stat.	**4.7	**-4.0	**26.1	**17.9	**454	**-234			**136
9. †P_0	0.005	-0.006	0.0017	0.0007	27.0	-0.015	2/85	121	3829.9

P is parameter, P_0 is the initial point P_f is the convergent point.
α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.
$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(days).
LnL is the value of Log Likelihood function.
Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.
* Significant at the 5 per cent level.
** Significant at the 1 per cent level.
† Convergent Problems.

Table 5.7: Switching Regime Model: Daily Difference Series (1/78-12/85)

point is obtained. The log likelihood value is also maximized at this point and is 3854.9 compared to 3810.9, 3809 and 3806.9 for the other three convergent points. The maximized likelihood parameter estimates for α_1, γ, σ_1^2, σ_2^2, γ, μ, are 0.00051, -0.00062, 0.0000167, 0.0000077, 197.18 and -0.143 respectively. The estimated daily means for the difference portfolio returns in the two regimes are 0.00051 and -0.00011 respectively which approximate to 13.7% and -2.8% per year respectively. The estimated standard deviations of the error terms in the two regimes are 0.00409 and 0.0028, respectively. The hypothesis of equal error variances against the alternative of different error variances in the two regimes is tested using the likelihood ratio test statistic. The maximized likelihood value under the assumption of equal variances in two regimes is 3839.9. The LR test statistic, which is distributed as χ^2 with one degree of freedom, is 30.0. The hypothesis of the equal variances against the alternative of different variances in the two regimes is rejected at the 1% level. The estimated switch point is located at June 15, 1983 and the estimated switching period is from May 1983 to July 1983.

Sensitivity analysis is done to check whether the estimates of the switch point and switching period are robust. We first test the hypothesis of no switch in regimes in the subperiods January 1978 to December 1982 and January 1984 to December 1985 excluding the 1983 period. A failure to reject the hypothesis of no switch in these subperiods will support the hypothesis that the switch occurred in the year 1983. The summary test statistics for both subperiods are reported in table 5.6. The hypothesis of no switch fails to be rejected in both subperiods. For the 1978-82 period, the estimated parameters are provided in table 5.8. All starting points converged to the same point. Although the LR test statistic at the maximum likelihood point is 40.0 which is significant at the 1% level, the t-statistic for $\gamma = 0$ is only -0.08 which is not significant at any reasonable confidence level. For the period 1984-85 also, The LR test statistic is 24.16 which is significant at 1% level but the t-statistic for $\gamma = 0$ is only -0.53 which is not significant at any reasonable level. Many convergence problems are encountered in this time period.

The sensitivity analysis of the switching period is done by estimating and comparing the maximized log likelihood value with switch point located at June 15,

TABLE 5.8

SWITCHING REGIMES : DAILY DIFFERENCE SERIES (1/78-12/82)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.005	-0.006	0.0017	0.0007	3.0	-0.015	10/78	121	1818.0
P_f	0.004	-0.004	0.0016	0.0003	44.9	-0.04	6/82	25	2253.4
Stat.	**3.5	-0.08	**23.3	**6.5	**124	**-76			**40.0
2. P_0	0.005	-0.006	0.0017	0.0007	6.0	-0.015	8/79	121	1871.1
P_f	0.004	-0.004	0.0016	0.0003	44.9	-0.04	6/82	25	2253.4
Stat.	**3.5	-0.08	**23.3	**6.5	**124	**-76			**40.0
3. P_0	0.005	-0.006	0.0017	0.0007	10.0	-0.015	8/80	121	1981.9
P_f	0.004	-0.004	0.0016	0.0003	44.9	-0.04	6/82	25	2253.4
Stat.	**3.5	-0.08	**23.3	**6.5	**124	**-76			**40.0
4. P_0	0.005	-0.006	0.0017	0.0007	13.0	-0.015	6/81	121	2064.9
P_f	0.004	-0.004	0.0016	0.0003	44.9	-0.04	6/82	25	2253.4
Stat.	**3.5	-0.08	**23.3	**6.5	**144	**-80			**40.0
5. P_0	0.005	-0.006	0.0017	0.0007	16.0	-0.015	3/82	121	2079.5
P_f	0.004	-0.004	0.0016	0.0003	44.9	-0.04	6/82	25	2253.4
Stat.	**3.5	-0.08	**23.3	**6.5	**117	**-74			**40.0

P is parameter, P_0 is the initial point P_f is the convergent point.

α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.

$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(days).

LnL is the value of Log Likelihood function.

Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

* Significant at the 5 per cent level.

** Significant at the 1 per cent level.

Table 5.8: Switching Regime Model: Daily Difference Series (1/78-12/82)

1983 but with different standard deviations. We estimate the log likelihood value with standard deviations of 30, 60, 90, 120, 180, and 250 days. We find that the log likelihood estimates decrease in a consistent manner as the standard deviation is increased. The likelihood estimates are 3853.9, 3851.9, 3847.7, 3841.5, 3828.97, and 3818.6 respectively. the same pattern is observed for the t-statistics for $\gamma = 0$ which decline from -3.95 for thirty days to -3.3 for 250 days. The sensitivity analysis supports the robustness of the switching period.

Sensitivity of the switch in regimes to a sample period is done by testing whether there is a switch at the optimal point derived from the 1978-1985 sample when the entire time series data from July 1962 to December 1985 is used. The analysis is first done by estimating four parameters α_1, α_2, σ_1^2 and σ_2^2 for the period July 1962 to December 1985 using maximium likelihood estimates of D_t obtained from the January 1978 to December 1985 period as the starting values. The results confirm that a switch in regimes occurred in 1983 even when the entire sample period is considered. The t-statistic for $\gamma = 0$ is -4.04 which is significant at the 1% level. The estimated daily mean size effect in the two regimes are 0.00038 and -0.00011 which on a yearly basis are 10% and -2.7% respectively.

We also test for a switch in mean for the series from July 1962 to December 1985 using the same starting points as in the analysis of 1978-85 period. The summary statistics are contained in table 5.6. The results are similar to those obtained using the 1978-85 subperiod although problems in convergence are encountered with six of the nine initial points. The hypothesis of no switch in regimes is rejected at the 1% level. The maximized log likelihood value using a one switch model is 11307.8 while the maximized log likelihood value under no switch is 11263.85. The LR test statistic is 88 which is significant at any reasonable level. The switch point corresponding to the maximum likelihood estimate is June 15, 1983 which is the same as obtained using the 1978-85 period. The t-statistic for $\gamma = 0$ is -4.01 which is also similar to those obtained in 1978-85 period and is significant at the 1% level. The estimated standard deviation at this point is 36 days which is larger than the estimate of 13 days obtained for the the 1978-85 period. The estimates of the mean returns in the two regimes are 0.00038 and 0.000114 respectively which correspond to about 10% and 2.8% per year respectively. Thus the sensitivity analysis for the entire series July 1962 to December 1985 supports the robustness of the convergent point obtained in the analysis of 1978-1985 period.

5.5 Switch in regimes: Alternative Explanations

The evidence in the previous section supports the hypothesis of a switch in regimes in the period January 1978 to December 1985. The estimated switch date is in 1983 and the annual estimated mean size effect declined from approximately 13.6% in regime 1 to about −2.8% in regime 2. This evidence is consistent with the impact of research information on the return generating process. However, the evidence in section 5.3 indicated that the the size effect was unstable in other periods. A plausible explanation for this variation may be that the magnitude of the size effect varies with the stochastic movement in some factors that affect the return generating process of different assets. A testable implication of this hypothesis is that the switch in 1983 and the earlier switches in the pre-information period are attributable to the stochastic movement in the same factors. The examination of this alternative hypothesis is the focus of our analysis in this section. The analysis is done by testing for a switch in regimes in the information period after controlling for the stochastic movement due to the factors. A switch in regimes after taking into account the effect of these factors will support the hypothesis of the information effect while no switch in regimes will support the hypothesis that the change in the mean size effect is due to the influence of other factors.

To formulate such a test we first need to specify the factors that affect the return generating process associated with the size variable and define the variables that proxy these factors. In recent years, many researchers have addressed the issue of stochastic movement in risk premiums of various assets and in the differences between risk premia of different types of assets.[4] We draw upon this literature and construct five variables with an aim to explore whether the current switch in the size effect can be attributed to any of these factors. The motivation for selecting these variables and the description of the proxy variables is discussed below.

[4] See Keim and Staumbaugh (1986), Chan, Chen and Hseigh (1985), Chen, Roll and Ross (1983) and Breen, Glosten and Jagannathan (1987)

1. The spread between the yield on low-grade corporate bonds and one-month treasury bills (YBLTB): Keim and Staumbaugh (1986) provide an intutive motivation for using this variable in a simple valuation model where an asset's price is equal to the present value of expected future cash flows. The discount rate used to calculate the present value of an asset is a function of expected future returns and is inversely related to the level of prices. The variable consisting of the spread between yields on low-grade corporate bonds and one-month Treasury bills is inversely related to the level of bond prices and thus should be positively associated with future returns if expected returns change, holding everything else constant.[5] For the proxy variable, we use the difference between yields on BBB-rated long-term corporate bonds and short-term (one-month) U.S. Treasury bills.[6]

2. The spread between the yield on low-grade corporate bonds and government bonds (YBLG): We use the difference between yields on BBB-rated long-term corporate bonds and long-term government bonds to proxy this variable. This is similar to the Bond market variable PREM, the difference between returns on low-grade (under Baa-rated) corporate bonds and government Bonds, used by Chan, Chen and Hseigh (1985) to proxy the change in the risk premium.[7] Chan, Chen and Hseigh argue that the risk premium is a function of the price of risk and risk, where price of risk is defined to be the marginal trade-off between consumption and risky investments. The risk premium is likely to be affected by economic conditions and the spread in returns on bonds of different perceived riskiness will measure the changing risk premium. The sensitivity of a stock's return to the changing risk premium can be measured by regressing the stock's returns on the bond-return

[5] Keim and Staumbaugh (1986) also use two other variables constructed from the stock market to reflect the level of prices: (i) minus the logarithm of the ratio of the the real Standard and Poor's index to its previous historic average and (ii) minus the logarithm of share price, averaged across NYSE firms in the quintile of the smallest market value. They note that all these variables are sufficiently collinear. We should expect similar results with the other variables.

[6] Keim and Staumbaugh (1986) use the yields on under BAA-rated corporate bonds provided by Ibbotson(1979) to proxy the yields on low-grade corporate bonds. This series ends in 11/1978. Therefore we use the yields on BBB-rated bonds the data on which is available for our period of study.

[7] We use the difference between yields rather than returns because the returns series on low-grade bonds available from Ibbotson (1979) ends in 11/77 and is not available for our period of study.

difference which they call PREM beta. Chan, Chen and Hseigh find that PREM has the most power in explaining the difference in returns between the smallest and the largest firm portfolios.[8]

3. The spread between yields on long-term Government bonds and one-month treasury bills (UTS): This variable is a measure of the change in the term structure of interest rates. The motivation for this is derived from a study by Chen, Roll and Ross (1983) designed to link stock returns to macro-economic variables. In a simple model of asset pricing where the current price of a financial asset equals the discounted expected cash flows in the future, the change in the term structure of interest rates is relevant since it can affect the discount rate used for pricing the assets. The difference between yields on long-term Government bonds and short-term (one-month) U.S. treasury bills is used as a proxy variable.

4. The change in the expected inflation rate (EI): The motivation for this variable is also derived from the study by Chen, Roll and Ross (1983). The difference between the yields on short-term (one-month) U.S. treasury bills at time t and at time t-1 is used as a proxy to measure unexpected inflation.

5. The value-weighted market index (VWNY): The motivation for this variable can be provided in two ways. First, in a mean-variance world of CAPM the systematic risk measure plays a significant role in explaining asset returns. The proxy portfolio, the difference between equally-weighted and value-weighted NYSE indexes, used in our study to measure the returns associated with the size variable may have some systematic risk. Including VWNY allows us to control for the effect of systematic risk during our sample period. A second reason is based on the motivation provided by Chen, Roll and Ross (1983) that the market index is a proxy for measuring the change in the expected long-run growth rate of real activity because any new information concerning future real activity should be reflected in the aggregate return on the market. We use the value-weighted NYSE index as the

[8]Other variables used by Chan, Chen and Hseigh (1985) are: a market index, the seasonally adjusted monthly growth rate of industrial production, change in expected inflation, unanticipated inflation and a measure of the change in the slope of the yield curve.

proxy variable.

The yield data on BBB-rated corporate bonds and long-term Government bonds is obtained from Standard and Poor's statistical service, security price index record, 1986. The per year bond yield is divided by twelve, and the yield spread is stated on the monthly basis. The yield data on short-term U.S. Treasury bills consists of the yield to maturity of the one-month treasury bills and are obtained from the term structure file constructed by Fama (1987). This file is available on the CRSP Government Bond files (1987). Data on the value-weighted market index are obtained from the CRSP monthly return series.

Since the data on the explanatory variables are available on a monthly basis, the investigation in this section is done using the monthly data. The monthly returns for the difference portfolio are obtained by compounding the daily difference series returns using the arithmetic mean (For details of the calculations see appendix C). This method corresponds to the daily rebalancing method and has been used by Reinganum (1981) and Keim (1983) for constructing monthly returns to study the size effect.

We first examine the hypothesis of no switch in monthly compounded difference returns series in 1978-85 subperiod. The summary test statistics and parameter estimates are contained in table 5.1 and table 5.9 respectively. Nine starting points are selected which correspond to the starting points used in the daily data.

The results using monthly data are similar to those obtained using daily data. The hypothesis of no switch is rejected at the 1% level. The LR test statistic and t-statistic for $\gamma = 0$ for the maximized likelihood estimates are 17.2 and -2.9 respectively and both are significant at the 1% level. Similar to the analysis in the daily data, the evidence supports a switch in regimes in 1983. The estimates of the switch point at December 1983 and the standard deviation of 2.2 months are slightly different from the estimates using daily data. The estimated mean size effects in the two regimes are 0.0094 and -0.0022, respectively which correspond to approximately 12.6% and -3.0% per year, respectively. The analysis of the subperiod from January 1978 to December 1982 supports the hypothesis that switch in 1983 is not a statistical artifact.

TABLE 5.9

SWITCHING REGIMES : MONTHLY DIFFERENCE SERIES (1/78-12/85)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.13	-0.16	0.10	0.12	3.0	-0.3	10/78	6.0	-13.4
P_f	0.17	-0.12	0.13	0.04	11.1	-0.7	3/79	2.5	10.6
Stat.	1.9	-1.3	*2.4	**6.3	1.4	-1.4			**14.2
2. P_0	0.13	-0.16	0.10	0.12	6.0	-0.3	8/79	6.0	-7.6
P_f	0.17	-0.12	0.13	0.04	11.1	-0.7	3/79	2.5	10.6
Stat.	1.9	-1.3	*2.4	**6.3	1.6	-1.5			**14.2
3. P_0	0.13	-0.16	0.10	0.12	9.6	-0.3	8/80	6.0	-7.8
P_f	0.11	-0.07	0.08	0.03	11.0	-0.24	9/81	7.4	10.7
Stat.	*2.4	-1.3	**3.94	**4.7	1.7	-1.8			**14.3
4. P_0	0.13	-0.16	0.10	0.12	12.6	-0.3	6/81	12.6	-5.6
P_f	0.11	-0.07	0.08	0.04	11.0	-0.24	9/81	7.4	10.7
Stat.	*2.4	-1.3	**4.2	**4.8	**3.5	**-3.6			**14.3
5. P_0	0.13	-0.16	0.10	0.12	15.3	-0.3	3/82	6.0	-3.8
P_f	0.11	-0.07	0.08	0.04	11.0	-0.24	9/81	7.4	10.7
Stat.	*2.3	-1.3	**3.7	**4.6	1.1	-1.2			**14.3
6. P_0	0.13	-0.16	0.10	0.12	18.3	-0.3	1/83	6.0	-4.2
P_f	0.1	-0.12	0.06	0.02	60.2	-0.84	12/83	2.2	12.1
Stat.	**3.3	**-2.9	**5.9	**3.4	**9.7	**-9.4			**17.2
7. P_0	0.13	-0.16	0.10	0.12	21.0	-0.3	10/83	6.0	-2.7
P_f	0.1	-0.12	0.06	0.02	60.1	-0.83	12/83	2.2	12.1
Stat.	**3.3	**-2.9	**5.9	**3.4	**2.8	**-2.7			**17.2
8. P_0	0.13	-0.16	0.10	0.12	23.1	-0.3	5/84	6.0	-1.7
P_f	0.1	-0.12	0.06	0.02	60.0	-0.83	12/83	2.2	12.1
Stat.	**3.3	**-2.9	**5.9	**3.4	**4.3	**-4.3			**17.2
9. P_0	0.13	-0.16	0.10	0.12	25.8	-0.3	2/85	6.0	-2.3
P_f†	0.08	-0.16	0.06	0.008	25.8	-0.28	8/85	6.4	9.1
Stat.‡									7.0

P is parameter, P_0 is the initial point P_f is the convergent point.
α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.
$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).
LnL is the value of Log Likelihood function.
Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.
* Significant at the 5 per cent level.
** Significant at the 1 per cent level.

Table 5.9: Switching Regime Model: Monthly Difference Series (1/78-12/85)

An alternative method of calculating monthly returns is to use the geometric mean which corresponds to the buy-and-hold strategy for the holding period of one month. Many researchers have provided evidence that there are substantial differences in the calculated size premium under the two methods.[9] In this section we are mainly interested in examining the co-movement of the hypothesized variables with the difference portfolio returns rather than the magnitude of the size effect per se. Thus we investigate whether the time series behaviour of the monthly returns series based on arithmetic and geometric mean differs significantly. The summary statistics for the time-series of difference portfolio returns based on the two methods for the period August 1962 to December 1985 are contained in table 5.10. We find that while means under the two methods are different, the time series behaviour under the two methods is very similar. The correlation coefficient between the two monthly difference series returns is .96. The correlation estimates for various sub-periods are found to be similar. Thus, the results in this section are expected to be similar under both methods. We use the daily rebalancing method because it provides estimates of the size effect which are closer to the estimates provided by daily data. For the value-weighted index the monthly series under both methods are virtually identical both in terms of mean and the time-series behaviour. The correlation between the two series is .998.[10] Table 5.10 also contains the correlation coefficients among variables. Most of the correlations are small except for UTS which is highly correlated with YLTB since both contain the treasury bill yields.

To control for stochastic variation due to the hypothesized factors we do the analysis in two stages. We first regress the monthly returns of the difference portfolio on the five variables:[11]

$$R_{dt} = a_0 + a_1(YBLTB)_{t-1} + a_2(YBLG)_t + a_3(UTS)_t + a_4(EI)_t + a_5(VWNY)_t + e_t \tag{5.1}$$

[9]Roll (1983) and Blume and Staumbaugh (1983) find that the monthly size premium is almost half under buy-and-hold strategy than under the daily rebalancing method.

[10]Our analysis supports the results of Roll (1983) and Blume and Staumbaugh (1983) that the differences in the method of compounding affect mainly the small firms returns.

[11]Lag variable for YBLTB is used on the basis of Keim and Staumbaugh (1986) study who use this variable as a pre-determined variable. We also examined the model with all variables except VWNY as pre-determined variables. The results were similar to those obtained using this model.

TABLE 5.10

SUMMARY STATISTICS : EXPLANATORY VARIABLES

MONTHLY DATA (AUGUST 1962 -DECEMBER 1985)

	DIFFA	LYBLTB	YBLG	UTS	EI	VWA	DIFFB	VWB
LYBLTB	0.033	1.0						
YBLG	-0.09	0.44	1.0					
UTS	0.075	0.93	0.06	1.0				
EI	-0.04	-0.33	-0.14	-0.31	1.0			
VWA	0.32	0.21	0.012	0.23	-0.12	1.0		
VWB	0.29	0.22	0.012	0.23	-0.12	0.99	1.0	
DIFFB	0.96	0.06	-0.069	0.098	-0.10	0.35	0.33	1.0

DIFFA is the monthly Difference Portfolio returns using daily rebalancing method; mean and variance are 0.0073 and 0.031 respectively.

DIFFB is the monthly Difference Portfolio returns using buy–and–hold method from CRSP tape; mean and variance are 0.0035 and 0.023 respectively.

VWA is the monthly value–weighted Portfolio returns using daily rebalancing method; mean and variance are 0.0097 and 0.043 respectively.

VWB is the monthly value–weighted Portfolio returns using buy–and–hold method from CRSP tape; mean and variance are 0.0091 and 0.043 respectively.

LYBLTB is the spread between yield on low–quality long term bonds and one–month U.S. treasury bills.

YBLG is the spread between yield on low–quality long term bonds and long–term Govt. Bonds.

UTS is the spread between yield on long– term Govt. bonds and one–month U.S. treasury bills.

EI is the difference between yield on one–month U.S. treasury bills at time t and t-1.

Table 5.10: Summary Statistics: Explanatory Variables Monthly Data (8/62-12/85)

The residuals (e_t) measure the unexplained portion of the returns. Next we use the switching regime model on the time-series of residuals e_t

$$e_t = \alpha_{e_1} + \gamma_e D_t + + \eta_t \tag{5.2}$$

where

- $\alpha_{e_1} =$ mean of residuals $\mathrm{e_t}$ in regime1
- $\alpha_{e_2} =$ mean of residuals $\mathrm{e_t}$ in regime2
- $\gamma_e = \alpha_{e_2} - \alpha_{e_1}$
- $D_t = \dfrac{1}{1 + exp^{(\gamma + \mu t)}}$
- $\eta_t = u_{e_1 t}(1 - D_t) + u_{e_2 t} D_t$ and $\eta_t \sim N(0, \sigma^2_{e_1}(1 - D_t)^2 + \sigma^2_{e_2} D^2_t)$

The analysis is first done for period January 1978 to December 1985. To compare the results with the previous analyses periods we also consider the pre-information period. Table 5.11 provides likelihood ratio test-statistics and t-statistics for testing a switch in regimes in the residuals obtained from regression 5.1. To facilitate comparison with the results obtained in the monthly difference series, the same starting values are used for the analysis.

For the 1978-85 period. the hypothesis of no switch for the residual series is rejected at the 1% level. The LR test statistic for the hypothesis of no switch against a switch is 14.84 which is significant at the 1% level. The t-statistic for $\gamma = 0$ is -2.64 which is significant at the 1% level. The parameter estimates for the residual series for 1978-85 period are provided in table 5.12. The estimates of the switch point using residuals are very similar to the estimates obtained using difference portfolio returns. The switch point is located at January 1984 with a standard deviation of 3.1 months. Thus assuming a stationary stochastic process for the size effect, we conclude that the decline in the size effect is not explained by any of the hypothesized variables.

We next examine whether any of the switches in the pre-information period are explained by the hypothesized factors. We analyze the subperiods 1967-73 and 1967-79 when a switch in regimes is evident as well as the periods 1969-73

TABLE 5.11

SWITCHING REGIMES MODEL

MONTHLY RESIDUAL SERIES : SUMMARY STATISTICS

PERIOD	LnL_1	LnL_2	$-2Ln(L_1 - L_2)$	t-statistic	M	S.D
1/69–12/73	-4.63	0.072	9.41	*-2.01	9/69	0.5
1/74–6/79	-33.17	-22.4	**21.2	-0.9	3/76	0.4
1/67–6/79	-52.27	-45.09	**14.34	0.98	4/73	0.4
1/67–12/73	-5.85	-4.10	3.5	-1.2	10/72	6.3
1/78–12/85	7.72	15.1	**14.8	**-2.6	1/84	3.1
8/62–12/85	-55.76	-43.7	**24.4	-**3.7	10/83	6.0

Residual Series is obtained from the regression of difference portfolio returns on on five variables LYBLTB, YBLG, UTS, EI and VW.

LnL_1 is value of the Log Likelihood function assuming no switch.

LnL_2 is value of the Log Likelihood function assuming a single switch.

LR is the Likelihood ratio test statistic for the $-2Ln(L_1 - L_2)$ test of no switch against the alternative of one switch and is distributed as χ^2 with 4 degrees of freedom.

t-statistic is for testing $\gamma' = (\alpha_1' - \alpha_2') = 0$ where α_i' is mean in regime i, $i = 1, 2$.

M is the maximum Likelihood estimate of the mean switch point.

S.D is the maximum Likelihood estimate of standard deviation (months).

* Significant at the 5 percent level.

** Significant at the 1 percent level.

Table 5.11: Switching Regime Model: Monthly Residual Series (Summary Statistics)

TABLE 5.12

SWITCHING REGIMES : MONTHLY RESIDUAL SERIES (1/78-12/85)

P	α_1'	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.13	-0.16	0.10	0.12	3.0	-0.3	10/78	6.0	-7.3
P_f	0.10	-0.11	0.11	0.04	9.8	-0.65	3/79	2.8	14.0
Stat.	1.1	-1.2	*2.3	**6.3	1.2	-1.1			*12.6
2. P_0	0.13	-0.16	0.10	0.12	6.0	-0.3	8/79	6.0	-3.3
P_f	0.10	-0.11	0.11	0.04	9.8	-0.65	3/79	2.8	14.0
Stat.	1.1	-1.2	*2.3	**6.3	**2.7	*-2.4			*12.6
3. P_0	0.13	-0.16	0.10	0.12	9.6	-0.3	8/80	6.0	-5.5
P_f	0.10	-0.11	0.11	0.03	9.8	-0.65	3/79	2.8	14.0
Stat.	1.1	-1.2	*2.3	**6.3	1.4	-1.3			*12.6
4. P_0	0.13	-0.16	0.10	0.12	12.6	-0.3	6/81	12.6	-4.6
P_f	0.03	-0.06	0.07	0.03	13.4	-0.30	9/81	6.0	12.9
Stat.	0.8	-1.2	**3.8	**4.7	**3.9	**-2.8			*10.4
5. P_0	0.13	-0.16	0.10	0.12	15.3	-0.3	3/82	6.0	-3.6
P_f	0.04	-0.07	0.07	0.03	84.7	-2.1	5/81	0.9	13.6
Stat.	1.0	-1.5	**4.4	**5.3	**16.8	**-15.5			*11.8
6. P_0	0.13	-0.16	0.10	0.12	18.3	-0.3	1/83	6.0	-4.4
P_f	0.04	-0.07	0.07	0.03	84.8	-2.1	5/81	0.9	13.6
Stat.	1.0	-1.5	**4.4	**5.3	**15.3	**-14.2			*11.8
7. P_0	0.13	-0.16	0.10	0.12	21.0	-0.3	10/83	6.0	-4.3
P_f	0.03	-0.11	0.06	0.02	42.0	-0.58	1/84	3.1	15.1
Stat.	0.1	**-2.6	**5.9	**3.3	**6.0	**-6.0			**14.8
8. P_0	0.13	-0.16	0.10	0.12	23.1	-0.3	5/84	6.0	-4.0
P_f	0.03	-0.11	0.06	0.02	42.0	-0.58	1/84	3.1	15.1
Stat.	0.1	**-2.6	**5.9	**3.3	**4.4	**-4.4			14.8
9. P_0	0.13	-0.16	0.10	0.12	25.8	-0.3	2/85	6.0	-5.5
P_f	0.03	-0.11	0.06	0.02	42.0	-0.58	1/84	3.1	15.1
Stat.	0.1	**-2.6	**5.9	**3.3	**6.0	**-6.0			**14.8

Residual Series is obtained from the regression of difference portfolio returns on on five variables LYBLTB, YBLG, UTS, EI and VW.
P is parameter, P_0 is the initial point P_f is the convergent point.
α_i' is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha_1' - \alpha_2'$.
$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).
LnL is the value of Log Likelihood function.
Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

Table 5.12: Switching Regime Model: Monthly Difference Series (Summary Statistics).

and 1974-79 when there is no switch in regimes found using the difference series. The summary statistics using the residual series are contained in table 5.11 and parameter estimates are provided in tables 5.13, 5.14, 5.15 and 5.16, respectively. The same starting values are used to facilitate comparison.

The results are strikingly different for the residual series in the pre-information subperiods January 1967-June 1969 and January 1967-December 1973 when a switch in regimes was evident using the difference series. For the January 1967-June 1979 period when there were two switches in regimes for the difference series there are no switches using the residual series. The switch corresponding to 1968 in the difference series is now located at October 1968 but the LR test statistic is only 7.3 at this point compared to the 5% level of 9.5. Also the t-statistic for $\gamma = 0$ is only 1.5 compared to -2.7 for the difference series. Thus the switch in 1968 appears to be explained by the stochastic variation in the economic variables. The maximized likelihood value for the residual series for this period is obtained at a convergent point with estimated switch point at 4/73. This was the second switch point for the difference series. However, although the LR test statistic is significant at the 1% level at this convergent point, the t-statistic for $\gamma = 0$ is only 0.97 and is not significant at the 5% level. Thus there is no evidence of a switch in regimes for the residual series in 1967-79 period.

The results for the period 1967-1973 when there was a significant decline for the difference portfolio returns are even more striking. Using the difference series, there is a switch in regimes at two convergent points and the t-statistics for $\gamma = 0$ are significant at the 1% level at both points. In contrast, there is no switch in regimes using the residual series. Also the LR test statistic is less than 3.5 for all convergent points relative to the 5% level of 9.5. The convergent points are located at approximately the same location. The t-statistics for $\gamma = 0$ at these points vary between 0.2 to -0.9 compared to the t-statistics of -4.05 and -2.7 for the unconditional mean size effect. This evidence strongly suggests that a large part of the variation in the size effect in 1967-73 period can be attributed to economy wide factors. The regression coefficients on all variables are significant at the 5% level and R^2 for the regression equation 5.1 is 0.4 which reveals that a significant portion of the stochastic variation in the size effect during this period is explained

TABLE 5.13

SWITCHING REGIMES : MONTHLY RESIDUAL SERIES (1/69-12/73)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	-0.10	0.20	0.10	0.12	3.6	-0.40	9/69	4.5	-8.06
P_f	-0.10	0.11	0.01	0.08	31.7	-3.7	9/69	0.5	0.07
Stat.	*-2.5	*2.1	*2.0	**5.1	**3.1	**-3.1			9.41
2. P_0	-0.10	0.20	0.10	0.12	9.6	-0.40	12/70	4.5	-11.8
P_f	0.04	-0.06	0.08	0.06	17.1	-0.60	6/71	3.2	-2.8
Stat.	0.6	-0.8	**3.2	**3.4	0.6	-0.6			3.7
3. P_0	-0.10	0.20	0.10	0.12	13.6	-0.40	10/71	4.5	-10.2
P_f	0.04	-0.06	0.08	0.06	17.1	-0.60	6/71	3.2	-2.8
Stat.	0.6	-0.8	**3.2	**3.4	0.6	-0.6			3.7
4. P_0	-0.10	0.20	0.10	0.12	18.4	-0.40	10/72	4.5	-8.9
P_f	0.02	-0.10	0.07	0.08	13.4	-0.30	9/72	6.0	-3.0
Stat.	-0.3	-1.1	**4.3	1.9	1.4	-1.4			3.3
5. P_0	-0.10	0.20	0.10	0.12	21.6	-0.40	6/73	4.5	-8.8
P_f	0.02	-0.10	0.07	0.08	13.4	-0.30	9/72	6.0	-3.0
Stat.	-0.3	-1.1	**4.3	1.9	1.4	-1.4			3.3

Residual Series is obtained from the regression of difference portfolio returns on on five variables LYBLTB, YBLG, UTS, EI and VW.

P is parameter, P_0 is the initial point P_f is the convergent point.

α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.

$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).

LnL is the value of Log Likelihood function.

Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

*** Significant at the 5 per cent level.**

**** Significant at the 1 per cent level.**

Table 5.13: Switching Regime Model: Monthly Residual Series (1/69-12/73)

TABLE 5.14

SWITCHING REGIMES : MONTHLY RESIDUAL SERIES (1/74-6/79)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.20	-0.10	0.10	0.12	2.4	-0.40	6/74	4.5	-43.0
P_f	1.0	-1.0	0.13	0.13	16.9	-10.0	2/74	0.2	-27.3
Stat.‡									*11.7
2. P_0	0.20	-0.10	0.10	0.12	7.2	-0.40	6/75	4.5	-46.7
P_f	0.08	-0.10	0.40	0.09	25.4	-1.6	4/75	1.2	-25.6
Stat.	0.5	-0.7	**2.6	**5.0	1.0	-1.0			**15.2
3. P_0	0.20	-0.10	0.10	0.12	12.0	-0.40	6/76	4.5	-48.2
P_f	0.06	-0.10	0.30	0.06	130.7	-4.8	3/76	0.4	-22.4
Stat.	0.6	-0.9	**3.6	**4.4	**7.4	**-7.1			**21.2
4. P_0	0.20	-0.10	0.10	0.12	16.8	-0.40	6/77	4.5	-45.4
P_f	0.03	-0.06	0.34	0.07	5.1	-0.2	8/76	11.0	-23.5
Stat.	0.3	-0.5	*2.4	**3.6	1.5	-1.8			**19.3
5. P_0	0.20	-0.10	0.10	0.12	21.6	-0.40	6/78	4.5	-48.4
P_f	0.06	-0.10	0.30	0.06	130.7	-4.8	3/76	0.4	-22.4
Stat.	0.6	-0.9	**3.6	**4.4	**7.4	**-7.1			**21.2

Residual Series is obtained from the regression of difference portfolio returns on on five variables LYBLTB, YBLG, UTS, EI and VW.

P is parameter, P_0 is the initial point P_f is the convergent point.

α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.

$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).

LnL is the value of Log Likelihood function.

Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

* Significant at the 5 per cent level.

** Significant at the 1 per cent level.

‡ Not available, This is not a feasible point.

Table 5.14: Switching Regime Model: Monthly Residual Series (1/74-6/79)

TABLE 5.15

SWITCHING REGIMES : MONTHLY RESIDUAL SERIES (1/67-6/79)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	-0.10	0.20	0.10	0.12	6.5	-0.40	4/68	4.5	-60.0
P_f	0.08	-0.10	0.06	0.13	12.5	-0.60	10/68	3.2	-48.3
Stat.	1.5	1.5	**3.1	**7.8	1.2	-1.2			7.3
2. P_0	-0.10	0.20	0.10	0.12	13.2	-0.40	9/69	4.5	-57.3
P_f	0.008	-0.01	0.06	0.13	37.7	-1.1	10/69	1.6	-48.6
Stat.	0.2	-0.2	**4.0	**7.6	**4.4	**-4.4			7.3
3. P_0	-0.10	0.20	0.10	0.12	18.0	-0.40	9/70	4.5	-57.9
P_f	-0.02	0.02	0.07	0.14	58.6	-1.4	7/70	1.3	-48.8
Stat.	-0.4	0.4	**4.6	**7.3	**3.7	**-3.7			6.8
4. P_0	-0.10	0.20	0.10	0.12	23.2	-0.40	10/71	4.5	-55.4
P_f	-0.03	0.06	0.07	0.16	500.0	-6.6	4/73	0.3	-45.1
Stat.	-0.9	1.0	**6.2	**6.1	**42.4	**-41.1			**14.3
5. P_0	-0.10	0.20	0.10	0.12	28.0	-0.40	10/72	4.5	-53.7
P_f	-0.03	0.06	0.07	0.16	500.0	-6.6	4/73	0.3	-45.1
Stat.	-0.9	1.0	**6.2	**6.1	**42.4	**-41.1			**14.3
6. P_0	-0.10	0.20	0.10	0.12	31.2	-0.40	6/73	4.5	-53.9
P_f	-0.03	0.06	0.07	0.16	500.0	-6.6	4/73	0.3	-45.1
Stat.	-0.9	1.0	**6.2	**6.1	**60.1	**-56.3			**14.3
7. P_0	0.20	-0.10	0.10	0.12	36.0	-0.40	6/74	4.5	-83.3
P_f	-0.03	0.06	0.07	0.16	500.0	-6.6	4/73	0.3	-45.1
Stat.	-0.9	1.0	**6.2	**6.1	**50.1	**-48.0			**14.3
8. P_0	0.20	-0.10	0.10	0.12	40.8	-0.40	6/75	4.5	-79.1
P_f	0.01	-0.05	0.13	0.06	38.9	-0.30	4/77	6.0	-47.6
Stat.	0.3	-0.8	**7.6	**3.4	**12.0	**-11.6			9.34
9. P_0	0.20	-0.10	0.10	0.12	45.6	-0.40	6/75	4.5	-80.3
P_f	0.01	-0.05	0.13	0.06	38.9	-0.30	4/77	6.0	-47.6
Stat.	0.3	-0.8	**7.6	**3.4	**4.2	**-4.1			9.34
10. P_0	0.20	-0.10	0.10	0.12	50.4	-0.40	6/77	4.5	-78.1
P_f	0.02	-0.06	0.14	0.06	500.0	-4.5	4/76	0.40	-47.1
Stat.	0.4	-1.2	**7.4	**4.3	**74.2	**-68.4			*10.4
11. P_0	0.20	-0.10	0.10	0.12	55.2	-0.40	6/78	4.5	-81.2
P_f	0.02	-0.06	0.14	0.06	500.0	-4.5	4/76	0.4	-47.1
Stat.	0.4	-1.2	**7.4	**4.3	**88.0	**-79.0			*10.4

Residual Series is obtained from the regression of difference portfolio returns on on five variables LYBLTB, YBLG, UTS, EI and VW.

P is parameter, P_0 is the initial point P_f is the convergent point.

α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.

$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).

LnL is the value of Log Likelihood function.

Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

* Significant at the 5 per cent level.

** Significant at the 1 per cent level.

Table 5.15: Switching Regime Model: Monthly Residual Series (1/69-6/79)

TABLE 5.16

SWITCHING REGIMES : MONTHLY RESIDUAL SERIES (1/67-12/73)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	-0.10	0.20	0.10	0.12	6.5	-0.40	4/68	4.5	-15.3
P_f	0.05	-0.06	0.06	0.07	15.8	-0.8	9/68	2.4	-4.3
Stat.	0.8	-1.0	**3.0	**5.5	1.9	-1.8			3.2
2. P_0	-0.10	0.20	0.10	0.12	13.2	-0.40	9/69	4.5	-12.3
P_f	-0.009	0.01	0.06	0.08	30.9	-0.95	8/69	1.9	-4.8
Stat.	-0.2	0.2	**3.8	**5.0	0.8	-0.8			2.1
3. P_0	-0.10	0.20	0.10	0.12	18.0	-0.40	9/70	4.5	-15.2
P_f	-0.009	0.01	0.06	0.08	30.9	-0.95	8/69	1.9	-4.8
Stat.	-0.2	0.2	**3.9	**5.0	**3.4	**-3.4			2.1
4. P_0	-0.10	0.20	0.10	0.12	23.2	-0.40	10/71	4.5	-13.9
P_f	0.02	-0.06	0.07	0.07	29.9	-0.27	7/72	3.3	-4.2
Stat.	0.7	-0.9	**5.0	**3.5	**3.6	**-3.6			3.5
5. P_0	-0.10	0.20	0.10	0.12	28.0	-0.40	10/72	4.5	-12.5
P_f	0.01	-0.10	0.07	0.10	20.1	-0.29	10/72	6.3	-4.1
Stat.	0.3	-1.2	**5.6	1.8	**2.8	**-2.8			3.4
6. P_0	-0.10	0.20	0.10	0.12	31.2	-0.40	6/73	4.5	-12.3
P_f	0.01	-0.10	0.07	0.10	20.1	-0.29	10/72	6.3	-4.1
Stat.	0.3	-1.2	**5.6	1.8	1.2	-1.2			3.4

Residual Series is obtained from the regression of difference portfolio returns on on five variables LYBLTB, YBLG, UTS, EI and VW.

P is parameter, P_0 is the initial point P_f is the convergent point.

α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.

$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).

LnL is the value of Log Likelihood function.

Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

* Significant at the 5 per cent level.

** Significant at the 1 per cent level.

Table 5.16: Switching Regime Model: Monthly Residual Series (1/67-12/73)

by economic variables.

For the subperiods 1969-73 and 1974-79 when there is no switch in the difference series, the results are very similar using the residual series. For the 1969-73 period The LR test statistic is not significant at the 5% level for any convergent point. For the 1974-79 period, although The LR test statistic is significant at the 5% for all convergent points the t-statistics for $\gamma = 0$ is not significant for any of these points.

We also compare the results for the difference series and the residual series for the entire time series from August 1962 to December 1985.[12] 1962 to December 1985. We postulate a model of single switch and the analysis is done using all 19 starting points used in various subperiods from 1967-1985. Although a model of one switch is restrictive in view of many switches evident in the difference series, it will serve the limited aim of our analysis to explore (i) whether there is a switch in regimes near October 1983 when the entire data is used and (ii) whether this switch can be explained by the hypothesized variables.

The summary test statistics for the difference series are provided in table 5.1 and the estimated parameters in table 5.17. The first part of Table 5.17 contains the analysis using the starting points between 1978-1985 and the other part using the starting points in the 1967-79 period. We find that for six starting values the convergent point is obtained at the switch point of October 1983 with a standard deviation of 4.6 months similar to the estimates obtained in the 1978-85 period. The LR test statistic and t-statistic for $\gamma = 0$ at this point are 26.4 and -3.0 respectively and are both significant at the 1% level. However, this convergent point is not the maximized likelihood estimate. The value of the log likelihood is maximized at a convergent point that provides the estimated switch point at 5/79. The LR test statistic at this point is 30.2 which is significant at the 1% level. However, the t-statistic for $\gamma = 0$ at this point is only -0.9 and is not significant at the 5% level. Thus the evidence fails to reject the hypothesis of no switch at this convergent point. This result is not surprising because the model of a single switch is too restrictive for the entire series. The evidence, however, does support the existence of a switch in regimes at October 1983 even when all the data are considered. We

[12] We use August 1962 instead of July 1962 because of lag variable LYBLTB

TABLE 5.17

SWITCHING REGIMES : MONTHLY DIFFERENCE SERIES (8/62-12/85)

Starting Points : (Between January 1978 and December 1985)

P	α_1'	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.13	-0.16	0.10	0.12	58.5	-0.3	10/78	6.0	-82.3
P_f	0.08	0.001	0.12	0.05	74.9	-0.43	3/79	4.0	-57.5
Stat.	**2.9	0.03	**9.3	**7.1	**40.7	**-36.5			**25.0
2. P_0	0.13	-0.16	0.10	0.12	61.5	-0.3	8/79	6.0	-76.6
P_f	0.08	-0.05	0.11	0.04	45.0	-0.2	9/81	9.0	-55.1
Stat.	**3.7	-1.3	**10.4	**5.0	**23.1	**-22.0			**29.9
3. P_0	0.13	-0.16	0.10	0.12	65.1	-0.3	8/80	6.0	-76.9
P_f	0.08	-0.05	0.11	0.04	45.0	-0.2	9/81	9.0	-55.1
Stat.	**3.7	-1.3	**10.4	**5.0	**23.1	**-22.0			**29.9
4. P_0	0.13	-0.16	0.10	0.12	68.1	-0.3	6/81	12.6	-74.6
P_f	0.08	-0.11	0.1	0.02	101.6	-0.40	10/83	4.6	-56.9
Stat.	**4.1	**-3.0	**11.2	**3.4	**15.8	**-15.7			**26.4
5. P_0	0.13	-0.16	0.10	0.12	70.8	-0.3	3/82	6.0	-72.8
P_f	0.08	-0.11	0.1	0.02	101.6	-0.40	10/83	4.6	-56.9
Stat.	**4.1	**-3.0	**11.2	**3.4	**44.6	**-42.2			**26.4
6. P_0	0.13	-0.16	0.10	0.12	73.8	-0.3	1/83	6.0	-73.3
P_f	0.08	-0.11	0.1	0.02	101.6	-0.40	10/83	4.6	-56.9
Stat.	**4.1	**-3.0	**11.2	**3.4	**17.3	**-17.2			**26.4
7. P_0	0.13	-0.16	0.10	0.12	76.5	-0.3	10/83	6.0	-71.7
P_f	0.08	-0.11	0.1	0.02	101.6	-0.40	10/83	4.6	-56.9
Stat.	**4.1	**-3.0	**11.2	**3.4	**16.7	**-16.6			**26.4
8. P_0	0.13	-0.16	0.10	0.12	78.6	-0.3	5/84	6.0	-70.8
P_f	0.08	-0.11	0.1	0.02	101.6	-0.40	10/83	4.6	-56.9
Stat.	**4.1	**-3.0	**11.2	**3.4	**53.5	**-49.5			**26.4
9. P_0	0.13	-0.16	0.10	0.12	81.3	-0.3	2/85	6.0	-71.3
P_f	0.08	-0.11	0.1	0.02	101.6	-0.40	10/83	4.6	-56.9
Stat.	**4.1	**-3.0	**11.2	**3.4	**47.4	**-47.4			**26.4

P is parameter, P_0 is the initial point P_f is the convergent point.
α_i' is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha_1' - \alpha_2'$.
$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).
LnL is the value of Log Likelihood function.
Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

Table 5.17: Switching Regime Model: Monthly Difference Series (8/62-12/85)

TABLE 5.17 (Continuation)

SWITCHING REGIMES : MONTHLY DIFFERENCE SERIES (8/62-12/85)

Starting Points: (Between January 1967 to June 1979)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	-0.1	0.2	0.1	0.12	27.6	-0.4	4/68	4.5	-86.9
P_f	0.07	0.004	0.04	0.11	104.6	-2.2	7/66	0.8	-62.4
Stat.‡									**15.2
2. P_0	-0.1	0.2	0.1	0.12	34.4	-0.4	9/69	4.5	-85.8
P_f	0.11	-0.05	0.06	0.11	48.5	-0.64	11/68	2.8	-64.2
Stat.	**4.0	-1.46	**6.1	**10.0	**5.2	**-5.0			*11.6
3. P_0	-0.1	0.2	0.1	0.12	38.8	-0.4	9/70	4.5	-87.0
P_f	0.11	-0.05	0.06	0.11	48.5	-0.64	11/68	2.8	-64.2
Stat.	**4.0	-1.46	**6.1	**10.0	**5.2	**-5.0			*11.6
4. P_0	-0.1	0.2	0.1	0.12	44.4	-0.4	10/71	4.5	-85.8
P_f	0.07	0.04	0.08	0.11	91.5	-0.8	8/71	2.0	-67.7
Stat.	**2.6	0.12	**7.3	**9.2	**17.7	**-17.2			4.6
5. P_0	-0.1	0.2	0.1	0.12	49.2	-0.4	10/72	4.5	-84.2
P_f	0.06	0.03	0.08	0.11	500.0	-4.0	11/72	0.5	-67.6
Stat.‡									4.6
6. P_0	-0.1	0.2	0.1	0.12	52.4	-0.4	6/73	4.5	-85.8
P_f	0.07	0.04	0.08	0.11	91.5	-0.8	8/71	2.0	-67.7
Stat.	**2.6	0.12	**7.3	**9.2	**17.7	**-17.2			4.6
7. P_0	0.2	-0.1	0.1	0.12	57.2	-0.4	6/74	4.5	-68.6
P_f	0.046	0.06	0.1	0.09	500.0	-3.5	6/74	0.5	-68.6
Stat.	1.7	1.5	**8.4	**8.3	**20.9	**-20.9			2.8
8. P_0	0.2	-0.1	0.1	0.12	62.0	-0.4	6/75	4.5	-54.9
P_f	0.08	-0.03	0.12	0.04	153.3	-0.76	5/79	2.4	-54.9
Stat.	**3.4	-0.9	**9.9	**6.3	**58.1	-51.8**			**30.2
9. P_0	0.2	-0.1	0.1	0.12	66.8	-0.4	6/76	4.5	-91.6
P_f	0.08	-0.03	0.12	0.04	153.3	-0.76	5/79	2.4	-54.9
Stat.	**3.4	-0.9	**9.9	**6.3	**58.1	-51.8**			**30.2
10. P_0	0.2	-0.1	0.1	0.12	71.6	-0.4	6/77	4.5	-87.3
P_f	0.08	0.001	0.12	0.05	71.6	-0.4	6/77	4.5	-57.5
Stat.	**2.9	0.03	**8.3	**7.2	**18.0	**-17.5			**25.0

P is parameter, P_0 is the initial point P_f is the convergent point.
α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.
$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).
LnL is the value of Log Likelihood function.
Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.
* Significant at the 5 per cent level.
** Significant at the 1 per cent level.
‡ Not available.

next explore whether this switch is explained by any of the hypothesized factors.

The summary statistics for the residual series are given in table 5.11 and the parameter estimates are provided in table 5.18. The evidence for the residual series supports the hypothesis of a switch in regimes at October 1983. More importantly, the convergent point with estimated switch at October 1983 is now also the point where the value of the log likelihood function is maximized. The LR test statistic at this point is 24.4 which is significant at the 1% level. The t-statistic for $\gamma = 0$ is -3.7 which is even higher than the corresponding t-statistic of -3.0 obtained for the difference series. The evidence strongly suggests that the switch in 1983 cannot be explained by any of the economic factors.

The evidence in this section supports the hypothesis that the switch in 1983 and the earlier switches in the pre-information period are driven by different factors. The switches in the pre-information period are associated with the movement in some economic factors, the switch in 1983 is not explained by any of those factors. Additional corroborating evidence is obtained from the estimates of the switch point and the standard deviation for the switch in 1983. These estimates are similar irrespective of whether the residual or the difference series are used or whether the entire series from July 1962 to December 1985 or only the subperiod January 1978 to December 1985 is used.

5.6 Seasonality in the size effect: January size effect

Many researchers have reported a strong January seasonal in stock returns. Rozeff and Kinney (1976) find a positive January seasonal in stock returns. The high January returns vary cross-sectionally for assets in different size categories with much larger returns for small firms. Keim (1983) and Roll (1983) find that over the period 1963-1979 nearly fifty percent of the average magnitude of the risk-adjusted premium of small firms relative to large firms is due to anomalous January returns, with more than twenty-six percent of the size premium attributable to large abnormal returns during the first trading week in the year and almost eleven percent to the first trading day. Many researchers argue that there is no size

TABLE 5.18

SWITCHING REGIMES : MONTHLY RESIDUAL SERIES (8/62-12/85)

Starting Points: (Between January 1978 and December 1985)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.13	-0.16	0.10	0.12	58.5	-0.3	10/78	6.0	-76.4
P_f	0.009	-0.03	0.11	0.04	86.5	-0.4	4/79	4.2	-44.3
Stat.	0.4	-1.0	**9.9	**6.3	**32.4	**-30.1			**22.8
2. P_0	0.13	-0.16	0.10	0.12	61.5	-0.3	8/79	6.0	-73.0
P_f	0.009	-0.03	0.11	0.04	86.5	-0.4	4/79	4.2	-44.3
Stat.	0.4	-1.0	**9.9	**6.3	**32.4	**-30.1			**22.8
3. P_0	0.13	-0.16	0.10	0.12	65.1	-0.3	8/80	6.0	-75.5
P_f	0.004	-0.008	0.11	0.05	95.0	-0.54	5/77	3.4	-45.0
Stat.	0.2	-0.2	**9.3	**7.1	**43.3	**-38.6			**21.6
4. P_0	0.13	-0.16	0.10	0.12	68.1	-0.3	6/81	12.6	-74.5
P_f	0.01	-0.14	0.1	0.02	68.1	-0.26	10/83	6.0	-43.7
Stat.	0.7	**-3.7	**11.2	**3.3	**15.0	**-14.8			**24.4
5. P_0	0.13	-0.16	0.10	0.12	70.8	-0.3	3/82	6.0	-73.0
P_f	0.01	-0.14	0.1	0.02	70.1	-0.28	10/83	6.0	-43.7
Stat.	0.7	**-3.7	**11.2	**3.3	**15.0	**-14.8			**24.4
6. P_0	0.13	-0.16	0.10	0.12	73.8	-0.3	1/83	6.0	-73.6
P_f	0.004	-0.008	0.11	0.05	95.0	-0.54	5/77	3.4	-45.0
Stat.	0.2	-0.2	**9.3	**7.1	**43.3	**-38.6			**21.6
7. P_0	0.13	-0.16	0.10	0.12	76.5	-0.3	10/83	6.0	-73.9
P_f	0.01	-0.14	0.1	0.02	76.5	-0.30	10/83	6.0	-43.7
Stat.	0.7	**-3.7	**11.2	**3.3	**15.0	**-14.8			**24.4
8. P_0	0.13	-0.16	0.10	0.12	78.6	-0.3	5/84	6.0	-73.9
P_f	0.01	-0.14	0.1	0.02	78.6	-0.31	10/83	6.0	-43.7
Stat.	0.7	**-3.7	**11.2	**3.3	**15.0	**-14.8			**24.4
9. P_0	0.13	-0.16	0.10	0.12	81.3	-0.3	2/85	6.0	-76.2
P_f	0.01	-0.05	0.1	0.04	170.8	-0.75	7/81	2.4	-46.0
Stat.	0.5	-1.5	**10.6	**5.2	**35.3	**-33.7			**19.5

Residual Series is obtained from the regression of difference portfolio returns on on five variables LYBLTB, YBLG, UTS, EI and VW.
P is parameter, P_0 is the initial point P_f is the convergent point.
α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.
$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).
LnL is the value of Log Likelihood function.

Table 5.18: Switching Regime Model: Monthly Residual Series (8/62-12/85)

TABLE 5.18 (Continuation)

SWITCHING REGIMES : MONTHLY RESIDUAL SERIES (8/62-12/85)

Starting Points: (Between January 1967 to June 1979)

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	-0.1	0.2	0.1	0.12	27.6	-0.4	4/68	4.5	-70.7
P_f	0.003	-0.003	0.06	0.1	52.6	-0.7	10/68	2.5	-51.2
Stat.	0.1	-0.08	**6.0	**10.1	**8.2	**-8.5			8.9
2. P_0	-0.1	0.2	0.1	0.12	34.4	-0.4	9/69	4.5	-68.4
P_f	-0.01	0.02	0.06	0.1	115.1	-1.3	7/69	1.4	-50.6
Stat.	-0.4	0.47	**6.5	**9.8	**30.7	**-28.0			*10.3
3. P_0	-0.1	0.2	0.1	0.12	38.8	-0.4	9/70	4.5	-68.7
P_f	-0.01	0.02	0.06	0.1	115.1	-1.3	7/69	1.4	-50.6
Stat.	-0.4	0.47	**6.5	**9.8	**30.7	**-28.0			*10.3
4. P_0	-0.1	0.2	0.1	0.12	44.4	-0.4	10/71	4.5	-66.3
P_f	-0.02	0.03	0.07	0.1	66.8	-0.61	9/71	3.0	-51.9
Stat.	-0.7	0.7	**7.3	**9.2	**11.3	**-11.2			7.8
5. P_0	-0.1	0.2	0.1	0.12	49.2	-0.4	10/72	4.5	-64.5
P_f	-0.03	0.06	0.07	0.1	500.0	-3.9	4/73	0.5	-50.8
Stat.	-1.4	1.6	**8.0	**8.7	**206	**-139			*9.9
6. P_0	-0.1	0.2	0.1	0.12	52.4	-0.4	6/73	4.5	-64.4
P_f	-0.03	0.06	0.07	0.1	500.0	-3.9	4/73	0.5	-50.8
Stat.	-1.4	1.6	**8.0	**8.7	**85.7	**-78.3			*9.9
7. P_0	0.2	-0.1	0.1	0.12	57.2	-0.4	6/74	4.5	-102.1
P_f	-0.03	0.06	0.07	0.1	500.0	-3.9	4/73	0.5	-50.8
Stat.	-1.4	1.6	**8.0	**8.7	**81.9	**-75.4			*9.9
8. P_0	0.2	-0.1	0.1	0.12	62.0	-0.4	6/75	4.5	-103.7
P_f	0.005	-0.008	0.11	0.05	94.7	-0.5	5/77	3.4	-45.0
Stat.	0.2	-0.2	**9.3	**7.1	**34.1	**-31.6			**21.6
9. P_0	0.2	-0.1	0.1	0.12	66.8	-0.4	6/75	4.5	-102.8
P_f	0.005	-0.008	0.11	0.05	94.7	-0.5	5/77	3.4	-45.0
Stat.	0.2	-0.2	**9.3	**7.1	**34.1	**-31.6			**21.6
10. P_0	0.2	-0.1	0.1	0.12	71.6	-0.4	6/77	4.5	-99.4
P_f	0.005	-0.008	0.11	0.05	94.7	-0.5	5/77	3.4	-45.0
Stat.	0.2	-0.2	**9.3	**7.1	**34.1	**-31.6			**21.6

Residual Series is obtained from the regression of difference portfolio returns on on five variables LYBLTB, YBLG, UTS, EI and VW.

P is parameter, P_0 is the initial point P_f is the convergent point.

α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.

$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(months).

LnL is the value of Log Likelihood function.

Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

* Significant at the 5 per cent level.

** Significant at the 1 per cent level.

premium in non-January months and that the size anomaly pertains only to the month of January.

Other researchers have noted the differences in the time-series behaviour of the January size premium and the average size premium. Keim (1983) observes that in the period 1963-1979, the January size premium is persistent, consistently increasing and statistically significant although the average size premium has been unstable. The January size effect is strong even in the years when on average large firms earn larger risk adjusted returns than small firms. For example, even during the period 1969-1973 when a reversal of the size effect has been documented the size premium has been significantly positive in January. Keim concludes that we can separate the size effect into two distinct components; a large premium every January and a much smaller and, on average, positive differential between risk adjusted returns of small and large firms in every other month. In his view, a complete explanation of the size effect requires two separate explanations for these very different phenomena.

Various explanations have been advanced for the January size effect, with tax-loss selling pressure as the most common reason. The empirical evidence has been mixed in this regard. Roll (1983) concludes that the seasonal in stock prices is induced by tax-loss-selling and that transaction costs prevent arbitrageurs from eliminating the seasonal. Reinganum (1983) and Chan (1985) find that tax-loss-selling cannot explain the entire excess returns in January. There is also evidence of the January size effect in overseas markets with non-January tax year starting dates.[13]

January seasonal is also not unique to the stock market. There is evidence of a positive January seasonal in bond returns.[14] Similar to the case in the stock market there is evidence of cross-sectional differences in returns of bonds in different categories. Keim and Staumbaugh (1986) report a January seasonal in the difference in returns between low and high quality bonds. They find strong January seasonality in the bond and stock market price variables they use to predict the risk premiums. Keim and Staumbaugh observe that while the price variables have low explanatory

[13]Brown, Keim, Kleidon and Marsh (1983), Gultekin and Gultekin (1982)

[14]Schneeweiss and Woolridge (1979), Keim and Smirlock (1983)

power for the non-January size premium they do explain a substantial proportion of the January premium for both the difference between large and small firm stocks and the difference between low and high grade bonds.

The evidence on the January size effect points to three possible explanations of January seasonality in the size premium:

(i) The January size premium is driven by the same factors as the long-run average size premium. The long-term premium on small firms accrues to an investor during two distinct calender periods: January and the rest of the year.

(ii) The January seasonal is due to factors that are not related to the long-run average size-premium. For example, tax-loss selling or other economy wide factors that have a strong seasonal may drive the January size premium.

(iii) All the size premium occurs in the month of January and there is no average size-premium in non-January months. The observed size premium in non-January months is only a statistical artifact.

In this section we examine these hypotheses using the switching regimes framework to collect additional evidence about whether the switch in mean size effect in 1983 is related to research documenting the size anomaly. This research information relates to the long-term average risk premium since it explores the relationship between average risk and return characteristics of small firms. If this information is useful to investors it is likely to affect the size premium in all months, January and non-January. To examine this hypothesis, we divide the sample data into January and non-January observations and test for a switch in regimes in both subsamples. Three possible scenarios are:

- a change in regimes in both the January and non-January subsamples which is consistent with academic information effect.

- No change in regimes in the January subsample, but a change in regimes in the non-January subsample which is consistent with academic information effect.

- a change in regimes in the January subsample, but no change in regimes in the non-January subsample which is inconsistent with academic information effect. This is because the January size premium may contain the effect of other seasonal factors.

Table 5.6 reports the test-statistics for the January and non-January subsamples for the period 1978-1985. The hypothesis of no switch in regimes for non-January observations is rejected at the 1% level. The LR test statistic for the maximized likelihood value is 139.0 which is significant at any reasonable level. The t-statistics for $\gamma = 0$ at the maximized log likelihood value is -3.68 which is also significant at the 1% level. For the January observations, the hypothesis of no switch in the 1978-1985 period fails to be rejected. -2 Log likelihood ratio for the hypothesis of a switch against the hypothesis of no switch is only 6.8 against the 5% critical value of 9.5. The t-statistic for $\gamma = 0$ is 1.9 and is also not significant at the 5% level.

Table 5.19 contains the parameter estimates for the non-January observations. The same starting points as in the case of all observations are used to facilitate comparison. The similarity of the parameter estimates for each convergent point to those obtained in the case of all observations is striking. The convergent point with maximized likelihood value provides the same estimates of the switch point and standard deviation as obtained for all observations; the switch point is located at June 1983 with a standard deviation of 13 days.[15] The t-statistic for $\gamma = 0$ at this point is -3.68 compared to -3.9 in the case of all observations. The estimates of α_1 and α_2 for the maximized value in two regimes are 0.0003969 and -0.000189, respectively which on a yearly basis are 10.4% and -4.8%, respectively. Corresponding estimates for the case of all observations are 0.00051 and -0.00011, respectively which on a yearly basis are 12% and -2.8%, respectively. Other convergent points also provide parameter estimates that are very close to the case of all observations. However, the t-statistics for $\gamma = 0$ are lower in almost every case. For example, in the case of all observations the t-statistics for $\gamma = 0$ for convergent points with switch points located at 5/79 and 4/81 are -2.1 and -2.2 respectively

[15] For NLPQL routine convergence problems were encountered at the three starting points no.6,7,8 (See table 19.). Three additional starting points with switch point located close to these points were tried. In two points convergence was achieved in both routines at the same convergent point as obtained in starting points no.6,7,8.

TABLE 5.19

SWITCHING REGIMES : DAILY DIFFERENCE SERIES (1/78-12/85)

NON-JANUARY OBSERVATIONS

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.005	-0.006	0.0017	0.0007	2.7	-0.015	10/78	121	3133.4
P_f	0.005	-0.004	0.0016	0.0014	10.7	-0.035	4/79	51	3482.8
Stat.	*2.5	-1.5	**11.0	**27.1	**9.6	**-9.4			**36.0
2. P_0	0.005	-0.006	0.0017	0.0007	5.4	-0.015	8/79	121	3169.6
P_f†	0.005	-0.004	0.0016	0.0014	10.7	-0.035	4/79	51	3482.8
Stat.	*2.5	-1.5	**11.0	**27.1	**9.6	**-9.4			**36.0
3. P_0	0.005	-0.006	0.0017	0.0007	9.0	-0.015	8/80	121	3289.5
P_f	0.004	-0.003	0.0018	0.0011	106.5	-0.14	4/81	13	3489.3
Stat.	**2.6	-1.6	**19.2	**23.5	**224.5	**-128.2			**49
4. P_0	0.005	-0.006	0.0017	0.0007	11.7	-0.015	6/81	121	3359.1
P_f	0.004	-0.003	0.0018	0.0011	102.6	-0.14	4/81	13	3489.3
Stat.	**2.6	-1.6	**19.2	**23.5	**224.5	**-128.2			**49
5. P_0	0.005	-0.006	0.0017	0.0007	14.4	-0.015	3/82	121	3368.9
P_f	0.003	-0.002	0.0017	0.0011	64.5	-0.07	4/82	28	3487.3
Stat.	**2.4	-1.3	**21.7	**20.7	**144	**-79			**45.0
6. P_0	0.005	-0.006	0.0017	0.0007	17.1	-0.015	1/83	121	3434.1
P_f†	0.004	-0.006	0.0017	0.0007	191.0	-0.15	6/83	13	3534.2
Stat.	**3.4	**-3.7	**24.8	**17.3	**183	**-148			**139
7. P_0	0.005	-0.006	0.0017	0.0007	20.1	-0.015	10/83	121	3517.3
P_f†	0.004	-0.006	0.0017	0.0007	193.6	-0.15	6/83	13	3534.2
Stat.	**3.4	**-3.7	**24.8	**17.3	**413	**-219			**139
8. P_0	0.005	-0.006	0.0017	0.0007	21.8	-0.015	5/84	121	3509.7
P_f†	0.004	-0.006	0.0017	0.0007	197.8	-0.16	6/83	13	3534.2
Stat.	**3.4	**-3.7	**24.8	**17.3	**502	**-229			**139
9. P_0	0.005	-0.006	0.0017	0.0007	24.5	-0.015	2/85	121	3510.5
P_f	0.003	-0.008	0.0017	0.0009	8.6	-0.005	12/84	51	3526.8
Stat.	**3.0	**-2.9	**23.5	**7.4	**15.8	**-14.3			**124

P is parameter, P_0 is the initial point P_f is the convergent point.
α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.
$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(days).
LnL is the value of Log Likelihood function.
Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.

Table 5.19: Switching Regime Model: Daily Difference Series (1/78-12/85) Non-January Observations

which are significant at the 5% level. The t-statistics for the same convergent points for the non-January observations are only -1.5 and -1.6 respectively which are not significant at the 5% level.

Table 5.20 contains the parameter estimates and convergent points for the January observations in the 1978-1985 period. Five different starting values corresponding to switch points located in 1979, 1981, 1982, 1983 and 1984 are used. Three different convergent points with switch point at January 1980, January 1981 and January 1984 are obtained. However, in all cases the LR test statistic for the test of a switch against the hypothesis of no switch as well as the t-statistics for $\gamma = 0$ are not significant at conventional levels.

We also test for a switch in regimes for the January observations from the entire 1963-1985 period to examine whether there is any change in the January size effect prior to the arrival of the information about the size anomaly. The evidence supports the hypothesis of a switch in the January effect for the overall period 1963-85. Summary test statistics for this period are reported in table 5 and parameter estimates are provided in table 5.21. For some starting points, problems in convergence are encountered. The convergent point corresponding to the maximized log likelihood value provides the estimated switch point located in January 1977 with a standard deviation of 13 days. The LR test statistic at this point is 49.0 which is significant at any reasonable significance level. The t-statistics for $\gamma = 0$ is -3.75 which is significant at the 1% level. This point is obtained for three starting values. Other convergent points provide estimated switch points at January 1968, January 1972, January 1981 and January 1983. The first two points correspond to an increase and the other two points to a decline in the January size premium at the switch points. Although the LR ratio at these points is significant at the 1% level, the t-statistics for $\gamma = 0$ is not significant for the first two points. For the other two points the likelihood ratio is approximately 21 which is much smaller than its value of 49 observed at the January 1977 switch point. The evidence suggests that there is a switch in the overall January size premium and that this switch took place between January 1976 and January 1978 prior to the arrival of research information on the size anomaly. The estimates of the mean in two regimes are 0.0029 and 0.0013 daily which approximate to 6.3% and 2.7%

TABLE 5.20

SWITCHING REGIMES : DAILY DIFFERENCE SERIES (1/78-12/85)

JANUARY OBSERVATIONS

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.02	-0.01	0.0012	0.001	3.5	-0.1	1/79	18	330.1
P_f	0.024	-0.012	0.0008	0.0013	13.8	-0.40	1/80	4.4	336.5
Stat.	**4.7	-1.9	**3.95	**8.05	**5.5	**-4.9			6.8
2. P_0	0.02	-0.01	0.0012	0.001	8.0	-0.1	1/81	18	325.7
P_f	0.019	-0.0076	0.0011	0.0012	113.1	-1.4	1/81	1.3	334.9
Stat.	**5.2	-1.45	**6.3	**6.7	**48.4	**-37.4			3.5
3. P_0	0.02	-0.01	0.0012	0.001	10.0	-0.1	1/82	18	322.8
P_f	0.019	-0.0076	0.0011	0.0012	112.7	-1.4	1/81	1.3	334.9
Stat.	**5.2	-1.45	**6.3	**6.7	**38.4	**-32.4			3.5
4. P_0	0.02	-0.01	0.0012	0.001	12.0	-0.1	1/83	18	330.0
P_f	0.017	-0.009	0.0012	0.0009	41.5	-0.3	1/84	6.0	336.3
Stat.	**5.7	-1.45	**8.0	**3.9	**2.3	**-2.3			6.4
5. P_0	0.02	-0.01	0.0012	0.001	14.0	-0.1	1/84	18	224.5
P_f †	0.017	-0.009	0.0012	0.0009	41.5	-0.3	1/84	6.0	336.3
Stat.	**5.7	-1.45	**8.0	**3.9	**2.3	**-2.3			6.4

P is parameter, P_0 is the initial point P_f is the convergent point.
α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.
$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is standard deviation(days).
LnL is the value of Log Likelihood function.
Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for a test of no switch against a switch.
* Significant at the 5 per cent level.
** Significant at the 1 per cent level.
† Convergent Problems with NLPQL routine.

Table 5.20: Switching Regime Model: Daily Difference Series (1/78-12/85) January Observations

TABLE 5.21

SWITCHING REGIMES : DAILY DIFFERENCE SERIES (1/63-12/85)

JANUARY OBSERVATIONS

P	α'_1	γ'	σ_1^2	σ_2^2	λ	μ	M	S.D	LnL
1. P_0	0.02	-0.01	0.0012	0.001	9.0	-0.1	1/67	18.0	696.0
P_f	0.028	-0.03	0.002	0.002	9.7	-0.02	1/83	83.0	837.8
Stat.	**10.9	**-2.7	**12.2	**3.1	**5.7	**-5.0			**27.0
2. P_0	0.02	-0.01	0.0012	0.001	14.0	-0.1	1/69	18.0	688.6
P_f†	0.021	0.005	0.0009	0.0024	10.8	-0.1	1/68	19.0	845.3
Stat.	**7.0	1.2	**7.0	**13.1	*2.3	*-2.4			**42.0
3. P_0	0.02	-0.01	0.0012	0.001	20.0	-0.1	1/72	18.0	703.1
P_f	0.022	0.003	0.0012	0.003	54.2	-0.26	1/72	7.0	844.9
Stat.	**9.3	0.9	**10.1	**11.7	**7.5	**-7.4			**41.0
4. P_0	0.02	-0.01	0.0012	0.001	24.0	-0.1	1/74	18.0	616.2
P_f†	0.022	0.003	0.0012	0.003	54.2	-0.26	1/72	7.0	844.9
Stat.	**9.3	0.9	**10.1	**11.7	**15.3	**-15.0			**41.0
5. P_0	0.02	-0.01	0.0012	0.001	31.0	-0.1	1/79	18.0	775.4
P_f	0.03	-0.015	0.0025	0.0012	42.2	-0.14	1/77	13.0	849.0
Stat.	**10.6	**-3.8	**12.1	**9.4	**4.5	**-4.4			**49.0
6. P_0	0.02	-0.01	0.0012	0.001	35.4	-0.1	1/79	18.0	780.5
P_f	0.03	-0.015	0.0025	0.0012	42.2	-0.14	1/77	13.0	849.0
Stat.	**10.6	**-3.8	**12.1	**9.4	**4.5	**-4.4			**49.0
7. P_0	0.02	-0.01	0.0012	0.001	39.9	-0.1	1/81	18.0	808.2
P_f	0.03	-0.015	0.0025	0.0012	42.2	-0.14	1/77	13.0	849.0
Stat.	**10.6	**-3.8	**12.1	**9.4	**27.3	**-25.5			**49.0
8. P_0	0.02	-0.01	0.0012	0.001	41.9	-0.1	1/82	18.0	773.1
P_f†	0.028	-0.017	0.002	0.0013	41.9	-0.11	1/81	17.0	834.5
Stat.‡									**20.0
9. P_0	0.02	-0.01	0.0012	0.001	43.9	-0.1	1/83	18.0	780.3
P_f	0.026	-0.019	0.002	0.0011	45.9	-0.1	1/83	18.0	834.7
Stat.	**11.7	**-3.13	**14.5	**4.1	**24.1	**-22.9			**21.0
10. P_0	0.02	-0.01	0.0012	0.001	45.9	-0.1	1/84	18.0	782.2
P_f†	0.026	-0.019	0.002	0.0011	45.9	-0.1	1/83	18.0	834.7
Stat.	**11.7	**-3.13	**14.5	**4.1	**24.1	**-22.9			**21.0

P is parameter, P_0 is the initial point P_f is the convergent point.
α'_i is mean and σ_i^2 is variance in regime i, i=1,2; $\gamma' = \alpha'_1 - \alpha'_2$.
$M = -\lambda/\mu$ is the mean switch point, $S.D = -\pi/\sqrt{3}\mu$ is the standard deviation.
LnL is the value of Log Likelihood function.
Stat. is test-statistic. (Col. 2–7) is the t–stat. for parameter=0. (Col. 10) is the likelihood ratio test stat. for test of no switch against a switch.
* Significant at the 5 per cent level.
** Significant at the 1 per cent level.
† Convergent Problems.
‡ Not available.

Table 5.21: Switching Regime Model: Daily Difference Series (1/63-12/85) January Observations

per year respectively. Thus while the January size effect declined in magnitude after 1977, it is still positive and different from zero.

The evidence of a switch in non-January observations and no switch in January observations in the period 1978-85 supports the explanation that the January and non-January size premiums are driven by different factors. We find additional evidence in support of the distinction between the time-series patterns of the January and average size premium noted by many other researchers. The different timings of the switch points for January and non-January observations along with the evidence that while the average size effect after 1983 is close to zero, the January size effect is still positive supports the explanation that the January size effect and average size premium are different phenomena. More importantly, the evidence in the 1978-85 sample period also rejects the hypothesis that the size premium in non-January months is merely a statistical artifact and all the size premium occurs in the month of January. We find that during this period the magnitude of the size premium in non-January months is significantly different from zero at the 1% level prior to the switch in 1983. Whether the January premium is driven by economy wide factors or other reasons needs to be investigated further. However, the evidence of a switch in regimes in non-January months supports the hypothesis that the switch in 1983 is associated with the research information on the size effect.

5.7 Transition Period: Estimation of the learning process

The transition period is when the information on the size anomaly is incorporated in the asset pricing process. More specifically, in our learning model it is the period associated with declining expected returns on small firm stocks. However, as discussed in chapter 3, the time-series behaviour of ex-post returns during the learning period may be different from that of the ex-ante returns series. The exact nature of the adjustment path will depend on the expected returns prior to any information arrival and the signals received in each period. Even in the simple case of continuously declining returns many different patterns for the expected ex-post return series were observed under different specifications of stochastic processes

(See figures. 4-6). However, in all cases examined expected ex-post returns first revealed an upward trend prior to the decline. Thus in the case of market learning we would expect the ex-post returns to exhibit an upward trend prior to its decline. Testing this prediction of the theoretical model is the focus of our analysis in this section.

To estimate and test the hypothesis of an increasing trend prior to a declining trend, we have to specify a stochastic process for the increasing trend during the information period. We assume there is a continuously increasing trend in ex-post returns prior to the decline.[16] We first test the hypothesis of an increasing trend prior to the switch in 1983 and then examine whether a switching regime model with an increasing trend prior to a decline is a more appropriate model than the model specified in equation 4.3.

We estimate the following models in the period January 1978 to May 1983 which includes the period immediately prior to the decline in observed returns:

$$\text{Trend} \quad R_{dt} = \alpha_1 + \beta Ln(t - t^*) + \epsilon_t \tag{5.3}$$

$$\text{No Trend} \quad R_{dt} = \alpha_1 + \epsilon_t \tag{5.4}$$

where α_1 is the mean for $t < t^*$, $\beta Ln(t - t^*)$ is the increase over α_1 for $t \geq t^*$. $Ln(t - t^*)$ is assumed to be zero for $t < t^*$.

The model with no trend is nested in the model with a trend and we can use a likelihood ratio test to select the appropriate model. The maximized log likelihood values under the trend and no trend models are 2437.19 and 2433.63 respectively. The value of -2 log liklihood ratio, which is distributed as χ^2 with two degrees of freedom, is 7.12 and is significant at the 5% level. We reject the hypothesis of no increasing trend prior to decline in the mean at the 5% level. The maximized likelihood estimates of α_1 and β are 0.00039 and 0.000189 respectively. The t-statistic for $\beta = 0$ is 2.67 which is significant at the 1% level. The estimate of t^* is 1172.0 which corresponds to August 1982. Thus the evidence supports an upward trend in ex-post returns beginning in August 1982.

[16]Such an assumption is equivalent to a process where the impact of each subsequent signal is relatively larger than that of a prior signal in the transition period

Next, we estimate a switching regimes model with an upward trend in the first regime:

$$\text{Regime 1}: \quad R_{dt} = \alpha_1 + \beta Ln(t - t^*) + u_{1t} \tag{5.5}$$

$$\text{Regime 2}: \quad R_{dt} = \alpha_2 + u_{2t} \tag{5.6}$$

where:

- α_1 = mean of the time-series R_{dt} for $t < t^*$ where t^* is the start of the trend.
- $\beta Ln(t - t^*)$ is the increase over α_1 for $t \geq t^*$ in regime 1.
- α_2 is mean of the time-series R_{dt} after the incorporation of the information about the size anomaly.
- u_{it} = Normally and independently distributed error terms, $u_{it} \sim N(0, \sigma_i^2)$, $i = 1, 2$.

The switch is assumed to depend on a variable $D(t)$ and the model to be estimated is written as:

$$R_{dt} = \alpha_1 + (\alpha_2 - \alpha_1)D(t) + \beta(Ln(t - t^*))(1 - D(t)) + u_{1t}(1 - D(t)) + u_{2t}D(t) \tag{5.7}$$

The model in equation 5.7 is illustrated in figure 5.1 with three different specifications of $D(t)$ for 100 observations. The data are generated by a combination of two different stochastic processes and appear to be a reasonable approximation for the model. The trend in first regime starts at t=10 and continues till t=50 with $\beta = 0.0012$ and $\alpha_1 = 0.006$.[17] The switching regimes model estimated in equation 4.3 is nested in this model since $\beta = 0$ provides the switching regime model with a continuous decline. We can also select between the two competing models using the likelihood ratio test statistic. Two additional parameters β and t^* are to be estimated in this model compared to the specification in equation 4.3.

We estimate the model for the period January 1978-December 1985. Six starting points are selected by varying the switch points. We encountered many convergence problems particularly with NLPQL routine. The maximized log likelihood is

[17] An alternate and perhaps a better way will be to specify a function that facilitates an upward trend prior to decline in place of logistic function. Such a function will be advantageous when different a priori functional forms are to be estimated and discriminated. The advantage of the model specified in equation 7 is that the earlier model is nested in this.

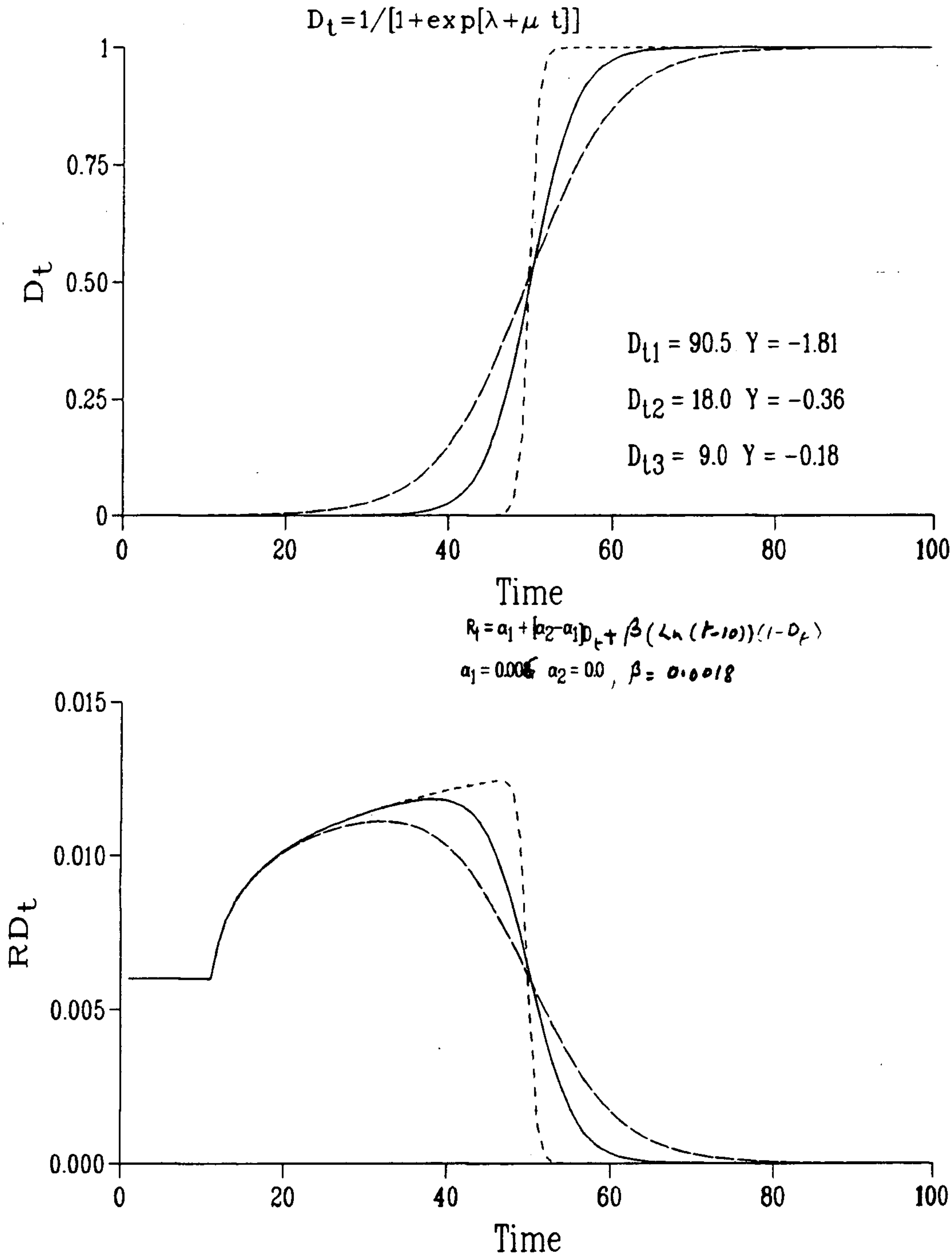

Figure 5.1: Illustration of the Econometric Model (Trend in Regime 1)

3858.5. The LR test statistic for the hypothesis of no switch against a switch under this model is 143.2 which is significant at the 1% level. The t-statistics for $\gamma = 0$ and $\beta = 0$ at the maximized log likelihood value are -3.14 and 2.6 respectively which are both significant at the 1% level. Thus the hypothesis of no switch is rejected using this model at the 1% level. The estimates of the switch point at this convergent point are June 1983 with a standard deviation of 13 days similar to the earlier estimates.

To select between models of switching regimes with no trend in the first regime specified in equation 4.3 and with a trend in the first regime specified in equation 5.7, we use the LR test statistic which is distributed as χ^2 with two degrees of freedom. The value of the LR test statistic is 7.2 which is significant at the 5% level. The evidence favours the model in equation 5.7 over the model in equation 4.3.

We also do sensitivity analysis by specifying different $D1(t)$ and $D2(t)$ functions. Table 5.22 reports the maximized log likelihood values for four different specifications for $D1(t)$ and $D2(t)$ and compares these with the log likelihood value obtained at the optimal switch point under the assumption of no trend in the first regime. We observe that in each case the log likelihood value is higher under the assumption of a trend in the first regime. The highest log likelihood value under the assumption of a trend is 3857.3 while log likelihood value under the assumption of no trend is 3854.9. The LR test statistic is 4.48 and is not significant at the 5% level. Also the LR test statistic is not very different when different slope coefficients or different t^* are specified. Thus we cannot determine which trend process fits better. A possible reason for this may be the high unexplained variance in the data compared to the variance in the mean. Thus while the data can discriminate between a switch and no switch in regimes it cannot discriminate between the models of different stochastic processes.

The evidence in this section supports the hypothesis of an increase in ex-post returns prior to its decline as predicted by the theoretical model. We also conclude that the switching period estimate of May to July 1983 obtained under switching regime method in section 5.4 is a part of the transition period but not the entire transition period. The estimation of the latter will require further additional assumptions.

TABLE 5.22 TRANSITION PERIOD : ESTIMATION DAILY DIFFERENCE SERIES (JANUARY 1978–DECEMBER 1985)			
$D1(t)$	LnL_2	LnL_1	$-2Ln(L_1 - L_2)$
No trend in mean :1/78–3/82 Slow increase : 4/82–2/83 Fast increase : 3/83–4/83	3857.30**	3854.9	4.8
No trend in mean :1/78–1/82 Slow increase : 2/82–2/83 Fast increase : 3/83–4/83	3857.27**	3854.9	4.8
No trend in mean : 1/78–3/82 Slow increase : 8/81–6/82 Fast increase : 7/82–5/83	3857.04*	3854.9	4.28
No trend in mean : 1/78–1/82 Slow increase : 2/82–2/83 Fast increase : 3/83–4/83	3857.05*	3854.9	4.28

MODEL : $R_{dt} = \alpha_1 D1(t)(1 - D2(t)) + \alpha_2(D2(t) + \epsilon_1(1 - D2(t)) + \epsilon_2(D_2(t))$

LnL_1 is log Likelihood under no trend in regime 1.

LnL_2 is log Likelihood under hypothesized trend in regime 1.

* $D_2(t) = 1/(1 + exp^(197.16 - 0.143t))$

** $D_2(t) = 1/(1 + exp^(204.07 - 0.1479t))$

Table 5.22: Switching Regime Model: Daily Difference Series (1/78-12/85)Sensitivity Analyais

5.8 Information effect and Trading Volume

The relationship between information arrival and trading volume has been examined in many recent empirical studies. Most of these studies find an increase in trading volume associated with information arrival. The increase in volume is generally attributed to heterogeneous information or beliefs. We examine the impact on trading volume of small size firms during the information period to collect additional evidence about the association of the change in size effect in 1983 with the research information arrival. In the theoretical model developed in our study, the research information relates to the risk return characteristics of small firm portfolios relative to the large firm portfolios. The impact of this information will be through the change in perceived risk associated with small firms. It is likely that in the post-information period more investors are willing to hold small firm portfolios relative to the pre-information period.[18] If this is the case then in addition to the change in trading volume due to heterogeneous investors during the information period we will also observe a change in the normal trading volume of small firm portfolios in the post-information period. Further since the information impact is hypothesized to be slow and spans many periods we should observe a similar trend in the change in normal trading volume. To test this hypothesis is our main objective in this section.

In the case of portfolio returns autocorrelation is a convenient proxy variable for normal trading volume. It is well documented that portfolio returns are more autocorrelated than individual security returns and many researchers have confirmed that non-synchronous trading is a primary reason for higher autocorrelation in portfolio returns. Roll (1981) discusses the relationship between trading frequency, autocorrelation and riskiness and concludes that the longer is the average time between trades the greater is the induced autocorrelation in portfolios of such firms. Many empirical and theoretical studies also find a negative relationship between

[18]There is theoretical and empirical evidence supporting the conjecture that the firms which are perceived highly risky may be held by fewer investors, See Klein and Bawa (1977). Regulatory reasons may also restrict investment of some investors in some type of firms, See Reinganum and Smith (1983). Reily (1975) documents that during the 1970s, large institutions concentrated their attention on a universe of less than 700 stocks of large firms, while the number of public companies that did not qualify for investment by institutions likely exceeded 8000.

volume and bid-ask spread. Since small-firm portfolios have lower trading volumes, less synchronous trading and larger bid-ask spreads, small firm portfolios have relatively higher autocorrelation of returns than large-firm portfolios. By similar reasoning, the equally-weighted index daily returns are more autocorrelated than the value-weighted index returns. This has testable implications for the hypothesized change in normal trading volume of small firm portfolios:

H_0 There is no change in the autocorrelation in the difference series in 1978-85 period.

To test this hypothesis we estimate an ordinary least square regression of daily difference portfolio returns on twelve laged values for the entire July 1962-December 1985 period and for July 1962-December 1977 and July 1978-December 1985 subperiods. We use Chow F-statistic to draw inferences. The results are contained in table 5.23. The value of Chow F-statistic has 13 and 5866 degrees of freedom is 10.57 and is significant at any reasonable confidence level. The null hypothesis of no change in autocorrelation in the difference series in the 1962-85 period is rejected. We find that the difference series in 1962-77 period displays significant autocorrelation. The first three and fifth autocorrelation coefficients are 0.305, 0.08, 0.063 and 0.038 respectively. In contrast, in the period 1978-85 these coefficients are -0.03, 0.04, -0.002, and 0.04 respectively and none of these are significant at the 5% level.

We also do sensitivity analysis by specifying different years between 1978 to 1985 as the time period for the change in autocorrelation and examining the Chow F-statistic. The results are provided in table 5.23. We find that the Chow F-statistics are approximately the same when 1978, 1979 or 1980 are specified as the switch years. The Chow F-statistic follows a consistently declining pattern as the switch year is varied from 1981 to 1984. The evidence indicates that the change in volume initiated around 1980 or 1981.

We also test the hypothesis of no change in autocorrelation for the subperiod 1978-85. The results are provided in table 5.24. A similar pattern to that in the July 1962-1985 time period is obtained. We reject the hypothesis of no change in autocorrelation at the 1% level. The Chow F-statistic is significant at any

TABLE 5.23

IMPACT ON AUTOCORRELATION IN THE INFORMATION PERIOD

DAILY DIFFERENCE SERIES : JULY 1962–DECEMBER 1985

ST.	PERIOD	SUBPERIOD 1	SUBPERIOD 2	F
1.	7/62–12/85	7/62–12/77	1/78–12/85	**10.6
Auto.	0.2 0.1 0.05	0.3 0.1 0.08	0.04 0.05 -0.013	
t	**(15.3) **(7.4) **(3.8)	**(18.4) **(5.6) **(4.8)	(1.6) *(2.2) (-0.6)	
2.	7/62–12/85	7/62–12/78	1/79–12/85	**13.9
Auto.	0.2 0.1 0.05	0.3 0.08 0.06	-0.03 0.04 -0.02	
t	**(15.3) **(7.4) **(3.8)	**(19.6) **(4.9) **(3.9)	(-1.2) (1.7) (-0.07)	
3.	7/62–12/85	7/62–12/79	1/80–12/85	**12.4
Auto.	0.2 0.1 0.05	0.3 0.07 0.07	-0.04 0.06 0.006	
t	**(15.3) **(7.4) **(3.8)	**(19.8) **(4.8) **(4.2)	(-1.5) *(2.3) (0.02)	
4.	7/62–12/85	7/62–12/80	1/81–12/85	**7.2
Auto.	0.2 0.1 0.05	0.26 0.09 0.06	-0.003 0.05 -0.016	
t	**(15.3) **(7.4) **(3.8)	**(17.2) **(6.2) **(3.7)	(-0.4) (1.4) (0.2)	
5.	7/62–12/85	7/62–12/81	1/82–12/85	**5.5
Auto.	0.2 0.1 0.05	0.24 0.09 0.06	-0.003 0.05 -0.016	
t	**(15.3) **(7.4) **(3.8)	**(16.6) **(6.3) **(4.2)	(-0.08) (1.6) (-0.5)	
6.	7/62–12/85	7/62–12/82	1/83–12/85	**2.4
Auto.	0.2 0.1 0.05	0.22 0.1 0.06	-0.013 0.07 -0.025	
t	**(15.3) **(7.4) **(3.8)	**(15.6) **(6.7) **(4.0)	(0.36) (1.9) (-0.7)	
7.	7/62–12/85	7/62–12/83	1/84–12/85	*1.8
Auto.	0.2 0.1 0.05	0.2 0.1 0.6	0.02 0.03 -0.06	
t	**(15.3) **(7.4) **(3.8)	**(15.3) **(7.1) **(3.9)	(0.5) (0.7) (-1.3)	
8.	7/62–12/85	7/62–12/84	1/85–12/85	*0.42
Auto.	0.2 0.1 0.05	0.2 0.1 0.5	0.05 0.03 0.008	
t	**(15.3) **(7.4) **(3.8)	**(15.3) **(7.1) **(3.9)	(0.5) (0.4) (0.12)	

ST. is statistics.

Auto. is regression coefficients of difference series on its tweleve lag values, first three regression coefficients.

t is t-statistic for the parameters equal to zero.

F is Chow F statistic for testing no change in regression coefficients against a change in regression coefficients in the entire period.

* Significant at the 5 percent level.

** Significant at the 1 percent level.

Table 5.23: Impact on autocorrelation in the information period Daily difference Series (7/62-12/85)

TABLE 5.24

IMPACT ON AUTOCORRELATION IN THE INFORMATION PERIOD

DAILY DIFFERENCE SERIES : JAN.1978–DECEMBER 1985

ST.	PERIOD			SUBPERIOD 1			SUBPERIOD 2			Chow-F
1.	1/78–12/85			1/78–12/78			1/79–12/85			**4.1
Auto.	0.04	0.05	-0.01	0.4	-0.05	0.07	-0.02	0.05	0.003	
t	(1.6)	*(2.2)	(-0.6)	**(6.0)	(-0.8)	(-1.1)	-(0.9)	*(1.98)	(0.14)	
2.	1/78–12/85			1/78–12/79			1/80–12/85			**3.7
Auto.	0.04	0.05	-0.01	0.3	-0.07	-0.02	-0.04	0.06	0.006	
t	(1.6)	*(2.2)	(-0.6)	**(6.5)	(-1.4)	(-0.3)	-(1.5)	*(2.3)	(0.02)	
3.	1/78–12/85			1/78–12/80			1/81–12/85			*1.8
Auto.	0.04	0.05	-0.01	0.1	0.04	-0.03	-0.01	0.04	0.005	
t	(1.6)	*(2.2)	(-0.6)	**(2.6)	(1.2)	(-0.8)	-(0.4)	(1.4)	(0.18)	
4.	1/78–12/85			1/78–12/81			1/82–12/85			1.5
Auto.	0.04	0.05	-0.01	0.06	0.04	-0.004	-0.003	0.05	-0.016	
t	(1.6)	*(2.2)	(-0.6)	*(2.0)	(1.1)	(-0.1)	-(0.08)	(1.6)	(-0.5)	
5.	1/78–12/85			1/78–12/82			1/83–12/85			0.7
Auto.	0.04	0.05	-0.01	0.04	0.04	-0.01	-0.01	0.07	-0.03	
t	(1.6)	*(2.2)	(-0.6)	(1.5)	(1.4)	(-0.4)	(0.4)	(1.9)	(-0.7)	
6.	1/78–12/85			1/78–12/83			1/84–12/85			0.8
Auto.	0.04	0.05	-0.01	0.04	0.05	-0.01	-0.02	0.04	-0.06	
t	(1.6)	*(2.2)	(-0.6)	(1.4)	(1.9)	(-0.3)	(0.5)	(0.8)	(-1.3)	
7.	1/78–12/85			1/78–12/84			1/85–12/85			0.2
Auto.	0.04	0.05	-0.01	0.03	0.05	-0.014	-0.05	0.03	0.008	
t	(1.6)	*(2.2)	(-0.6)	(1.4)	*(2.1)	(-0.6)	(0.8)	(0.07)	(0.12)	

Auto. is regressio coefficients on lagged difference series returns at lag 1, 2 and 3.
t is t-statistic for the parameters equal to zero.
Chow–F is Chow F statistic for testing no change in regression coefficients against a change in regression coefficients in entire period.
* Significant at the 5 percent level.
** Significant at the 1 percent level.

Table 5.24: Impact on autocorrelation in the information period Daily difference Series (1/78-12/85)

reasonable significance level for 1979 and 1980 as the years for change in volume. For the 1981 year, the Chow F-statistic is significant at the 5% level and shows a continuous decline as different years after 1981 are specified as switch years. The first three autocorrelation coefficients also reveal a similar trend of decline in significance. The evidence indicates that the change in trading volume was initiated in or after 1980.[19]

We also examine the autocorrelation in difference series returns for many pre-information subperiods for comparison. Table 5.25 provides the results. The Portmanteau Q statistic for 24 lags which is approximately distributed as χ^2 with 24 degrees of freedom is used to test the null hypothesis that there is no autocorrelation and is also reported in table 5.25 for each period. We observe that the difference series displays significant autocorrelation in all subperiods prior to 1980. In contrast, the daily auto-correlation coefficients for the 1980-85 period are much smaller than those in any other subperiod at every lag for at least 9 lags. For example, the first auto-correlation coefficient for 1980-85 is only -0.03 compared to the coefficients of 0.31, 0.39 , 0.44, 0.15, 0.35 for the subperiods 1974-79, 1967-75, 1969-73, 1962-65 and 1962-79 respectively. Portmanteau Q values for the 1974-79, 1967-75, 1969-73 and 1962-73 subperiods are 326, 804, 595 and $(1000)^3$ respectively while the 5% and 1% levels of Q are 36.4 and 43.98 respectively. The Q statistic for the period 1980-85 is only 41 and is less than the 1% level. More importantly, significant autocorrelation in the difference series is a common feature of all the pre-information periods irrespective of whether the size effect is strong, weak or even reversed in that time period. Thus the magnitude of the autocorrelation seems to be a distinguishing feature of the time period prior to the discovery of the size effect and a change in the autocorrelation after the discovery supports the hypothesis of a market reaction associated with this discovery.

[19]We also estimate a switching regimes model under the assumption that both a change in information effect was evidenced in volume as well as on the mean of the size effect. We only use one lag variable for comutational convenience and use three starting points. The results of a switch in regime are similar to the results in section 4.

TABLE 5.25

COMPARISON OF AUTOCORRELATION IN SUBPERIODS

DAILY DIFFERENCE SERIES : JULY 1962-DECEMBER 1985

PER.	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	Q(24)
62-85	0.15	0.21	0.14	0.11	0.12	0.09	0.09	0.03	0.07	0.03	0.05	-0.01	**141
	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.04)	(.04)	(.04)	(.04)
62-79	0.35	0.23	0.18	0.14	0.13	0.11	0.10	0.09	0.09	0.06	0.08	0.05	**1,000
	(.02)	(.02)	(.02)	(.02)	(.02)	(.02)	(.02)	(.02)	(.02)	(.02)	(.02)	(.02)	(.02)
80-85	-0.03	0.06	0.0	0.0	0.04	0.06	0.03	-0.01	0.04	0.04	0.03	0.0	*41
	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)
74-79	0.31	0.18	0.16	0.12	0.10	0.09	0.09	0.05	0.06	0.02	0.06	0.04	**326
	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)
67-75	0.39	0.25	0.21	0.15	0.12	0.10	0.10	0.09	0.08	0.06	0.07	0.05	**804
	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)	(.03)
69-73	0.44	0.25	0.18	0.15	0.16	0.10	0.07	0.12	0.13	0.15	0.08	0.07	595**
	(.03)	(.03)	(.03)	(.04)	(.04)	(.04)	(.04)	(.04)	(.04)	(.04)	(.04)	(.04)	(.04)

Ai, is autocorrelation coefficient at lag i, i=1,2,...,12.
Per. is time period.
Standard error is in parentheses.
Q is Portmanteau Q, with 24 coefficients with 5 percent level is 36.4 and 1 percent level is 42.98.
*** Significant at the 5 per cent level.**
**** Significant at the 1 per cent level.**

Table 5.25: Impact on autocorrelation in different periods: Daily difference Series (7/62-12/85)

Chapter 6

Summary, Conclusions and the direction of Future Research

This dissertation has investigated the impact of academic information on the size anomaly in the capital markets. The main hypothesis examined in this study is that research by academics on the size anomaly provides useful information to investors. A testable implication of this hypothesis is that we should observe a change in the size effect after the incorporation of the research information relating to the size effect. Most researchers conclude that the premium associated with size is not consistent with any theory of asset pricing and is an anomaly. We should expect a decline in the size premium after the discovery of the size anomaly if investors were not fully aware of the nature of the anomaly before the research was conducted.

A general framework for addressing this question was developed in this study. A learning model based on Bayesian updating to incorporate information pertaining to the asset pricing process was presented. The stochastic process of change is a function of the size premium and the information structure.

The econometric model of switching regimes that allows for a finite number of switches is used to test the hypothesis of a change in the size premium in the 1978-1985 period. We find a significant decline in the size effect during this period. The estimated average size premium declined from approximately 13.6% to about −2.8% per year during this period. For the entire time period July 1962-December 1985, the size premium declined from about 10% to about −2.8% per annum. The evidence indicates that the decline was initiated in 1982 but the major impact occurred in 1983.

There have been many switches in regimes, including a reversal in the size effect in 1969-1973, in the period prior to any information on the size anomaly. A plausible explanation for this phenomenon can be that the size effect varies with stochastic movement in economic factors. This also provides an alternative explanation for the observed change in the size effect in 1983. We test this hypothesis by specifying five economic factors that might explain the stochastic movement in the size effect. We find that these factors explain the decline in the size effect in the 1967-1979 period but do not explain the decline in 1983. This evidence strongly suggests that the 1983 decline in the size premium is not related to the economic factors that likely generated the prior switches.

We also examine January seasonality in the size effect by dividing the sample observations in the 1978-1985 period into January and non-January observations. The results for the non-January observations are very similar to the results obtained for all observations. We find a decline in the size effect in 1983. In contrast, there is no change in the January size effect in the 1978-1985 period and the January size effect is still positive. In the overall period 1963-1985, we find a decline in the January size effect in 1977 prior to the research information period. Our evidence suggests that the January size premium may be related to different factors than the average size premium. The evidence of the 1978-85 period also rejects the hypothesis that the size premium is confined to only January months.

We also examine the implications of the theoretical model about the transition period. The evidence supports the existence of an upward trend prior to its decline as predicted by the learning model. The upward trend is estimated under different a priori assumptions but the data fails to discriminate between different models. The estimation of the transition period and the stochastic process generating the change will require additional assumptions about the impact of the research information. This may be explored in future research. The evidence does support that the learning period spans many periods.

We also observe a significant change in the autocorrelation of the difference series during the 1978-85 period. Significant autocorrelation in the difference series is a common feature of all subperiods in the pre-information period irrespective of whether the size effect was strong, weak or even reversed in that subperiod.

In contrast, we find a significant decrease in autocorrelation after the arrival of research information. Since trading volume and autocorrelation are negatively correlated, the evidence of a change in autocorrelation provides additional evidence on the association of the current switch in the size effect with research information. However, the exact implications of such evidence need to be explored more carefully in a theoretical framework. This is proposed for future research.

Many anomalies have recently been discovered. Examination of the impact of research information on other anomalies will provide additional information about whether changes in asset pricing can be attributed to research information. The impact depends on the extent to which such research may be useful to investors, which may be different for each anomaly. Anomalies may arise for different reasons. We have presented and examined one plausible reason for the existence of anomalies that can be used as a basis to distinguish between different anomalies. The theoretical and empirical framework provided in this dissertation will be useful in carrying out further academic investigations in the future into this problem.

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Appendix A

The Size Anomaly: Key Information dates

Banz	Limited Diversification and Market equilibrium:	Ph.D. Dissertation 1978
	an empirical analysis	Univ. of Chicago.
Reinganum	Misspecification of capital asset pricing: Empirical	Ph.D. Dissertation Aug. 1979
	anomalies based on earning yields and market values	Univ.of Chicago
Banz	Relationship between return and market value	JFE, March 1981
	of common stocks	Vers. June 79, Sept.80
Reinganum	Misspecification of CAPM: Empirical anomalies	JFE March 1981
	based on yields and market values	Vers. Dec.79,June 80
Reinganum	Abnormal returns in small firm portfolios	March 1981
Roll	A possible explanation of the small firm effect	JF Sept. 1981, W.P. Oct 80.
Christie and	Capital asset pricing anomalies: Size and other	W.P. 1981
Hertzel	correlations	
Reinganum	The arbitrage pricing theory: some empirical	JF May 1981
	results	Vers.W.P Aug.80
Reinganum	Abnormal returns in small firm portfolio	FAJ March 1981.
Lakonishok	Partial diversification as an explanation of the	March 1981.
	Manu. 1982	
and Shapiro	small firm effect: an empirical analysis	
Cook and	Size,Dividend yield and co-skewness effects on stock	Manu. 1982
Rozeff	returns: some empirical tests	
Reinganum	A direct test of Roll's conjecture on the firm	JF March 1982

	size effect	
Arbel and Strebel	The neglected and small firm effects	FR 1982
Keim	Size related anomalies and stock return seasonality: further empirical evidence	JFE June 83. Vers. June 81, June 82
BKM	New evidence on the nature of size related anomalies in stock prices	JFE June 83 Vers.WP 80, June 81,July 82
Roll	On computing mean returns and the small firm premium	JFE Nov. 83 Nov. 1983. Vers. WP 82, Jan.83, June 83
Roll	Vast ist das ? The turn of the year effect and the return premium of small firms	JPM 1983
Stoll and Whaley	Transaction costs and the small firm effect	JFE June 1983 Vers. Oct. 81, March 82.
Schultz	Transaction costs and the small firm effect : a comment	JFE June 83 Vers. July 82, Sept.82
Chan,Chen and Hseigh	An exploratory investigation of the firm size effect	JFE 1985 WP 83
James and Edmister	The relationship between common stock returns trading activity and market value	JF Sept. 83 WP 81
Reinganum	Portfolio strategies based on market capitalization	JPM 83.
Reinganum and Smith	Investor preference for large firms: new evidence on economies of scale	JIE Dec. 83 Vers. July 82
Blume and Stambaugh	Biases in computed returns: an application to the size effect	JFE 83 Vers. Feb.83, Aug.83.
Berges, McConnell, Schlarbaum	An investigation of the turn-of-the-year effect, the small firm effect, and the tax-loss-selling pressure hypothesis in Canadian stock returns	JF 84 JF 84.
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Barry and	Differential information and Security:	JFQA 85

Appendix B

The Size anomaly

B.1 Discovery of the size anomaly

The size anomaly was first reported by Banz (1981) and Reinganum (1981). Banz examines monthly data from 1926 to 1975 for samples of NYSE firms. The sample includes all common stocks quoted on the NYSE for at least five years between 1926 and 1975. Banz finds a negative association between abnormal returns and market value of stocks after controlling for risk. The small firms on average earned excess risk adjusted returns of about 12% per annum. The effect is prevalent in various subperiods also although the magnitude of the effect varies.

Reinganum (1981) analyzes NewYork as well as American Stock exchange companies using daily data from June 1963 to December 1977 and reports dramatic size effect. Reinganum forms ten portfolios based on the market values. The excess return is defined as the daily portfolio return less the equally weighted NYSE-AMEX index return. Reinganum confirms Banz's findings that small firms earned excess risk adjusted returns on the average but reports much larger size effect as the small firms earn excess return of about 0.05 percent per day. Reinganum also examines the P/E (Price to Earnings ratio) anomaly and concludes that the P/E anomaly and value anomaly seem to be related to the same set of missing factors. However, the size effect largely subsumes the E/P effect. He finds that after controlling for any E/P effect , a strong firm size effect still emerged. But, after controlling returns for any market value effect, a separate E/P effect was not found.

B.2 Statistical and Economic explanations

Roll (1981) suggests that the small firm effect may be attributed to the improper estimation of security betas. He contends that since the stocks of the small firms are traded less frequently than the stocks of the larger firms, the estimates of the systematic risk from security returns will be biased downwards. Roll conjectures that the autocorrelation in a series causes a downward bias in the variance of returns. Since small firms have more autocorrelated daily return series they will have downward biased betas.[1] Reinganum (1982) tests Roll's conjecture by using market capitalization data and Dimson's (1979) aggregated coefficients method to account for the non-synchronous trading. The results reveal that while non-trading is a much more serious concern for small firms than for the large firms, the failure to account for this understatement of beta is not sufficient to explain the size effect. The average returns of the small firms exceed those of the large firms by about 36 percent on an annual basis, while the difference between the estimated betas of the small firm and large firm portfolio is about 0.7 (Reinganum p.29). Thus, while the direction of the bias in beta estimation is consistent with Roll's conjecture the magnitude of the bias appears to be too small to explain the firm size effect. [2] Christie and Hertzel (1981) argue that the non-stationarity in the risk measures may be one plausible reason for the size effect. However, they find that adjustment for this bias does not eliminate the size effect. Barry and Brown (1984) examine the association of potential misspecification of the market model with size and find evidence that size anomaly in excess returns is associated with misspecifications in the market model used to estimate systematic risk. However, they find that like the size premium itself the bias in measured beta is not particularly stable across subintervals of the data and is yet to be fully explained.

The main economic explanation has been the existence of transaction costs. Stoll and Whaley (1983) and Schultz (1983) examine the magnitude of transaction costs for stocks of firms in different categories. Stoll and Whaley using monthly

[1] Autocorrelation is more severe in daily data compared to monthly or yearly data due to non-synchronous trading.

[2] Reinganum points out that in order for this difference in estimated betas to account for a 36% return differential, the expected market return must exceed the risk free return by more than 50%.

data conclude that the transactions costs are sufficient to eliminate the size effect, but Schultz concludes that for the holding period of one year the small firm portfolio earns average risk adjusted returns of about 31% per year net of transaction costs. Other possible explanations including the association between dividend yield and firm size , between firm size and the standard deviation of stock returns, and between firm size , dividend yield and co-skewness have also been examined by researchers but none of these provide a satisfactory answer and the general conclusion is that the small firm effect is a significant economic and empirical anomaly.

B.3 Characteristics of the size anomaly

Many researchers have examined in detail the nature and the magnitude of the size anomaly. Brown, Kleidon and Marsh (1983) find that size effect is non-stationary and analyze the size effect in subperiods in which it is stationary. They find that from Jan. 69-Dec.73 small firms had ex-ante negative excess returns of about 25% per annum, while from January 1974 to June 1979 they had ex-ante positive excess returns of about 25% per annum. Reinganum (1983) examines portfolio strategies based on market capitalization using daily return series from July 1962 to December 1980. He finds that for the smallest firms the average annual return equals 32.77%. On the other hand, the largest firms earn only on average about 9.47% per year. Only in four out of the eighteen years from 1963 to 1980, the large firm portfolio experience greater returns than the small firm portfolio. The years in which the size effect is most strikingly reversed are 1969 and 1973. [3]

Keim (1983) examines month-to-month stability of the size anomaly in the period 1963-1979. He finds that nearly fifty percent of the average magnitude of the risk adjusted premium of small firms relative to large firms over this period is due to anomalous January abnormal returns. Furthermore, more than twenty six percent of the size premium is attributable to large abnormal returns during the first week of trading in the year and almost 11% is attributable to the first trading day. Roll (1983) confirms Keim's findings about a striking annual pattern in stock returns for the small firms. He compares the average annual return differential between

[3]This confirms BKM (1983)'s finding that the size effect was reversed in the period 1969-73.

equally-weighted and value-weighted indexes of NYSE and AMEX stocks and finds it to be 9.31% for years 1963-1980. About 37% of the entire year differential occurs in just five trading days and 67% of the annual differential occurs during the first twenty days of January plus the last day of December.

Table 1 provides empirical findings of key papers on the size anomaly. Summarizing the empirical research on the size anomaly we find that

(i) There is preponderance of evidence that the size effect is a significant economic and empirical anomaly. Although the effect is not uniform either in magnitude or in sign from month to month or year to year, most studies find a strong and stable negative size effect in the period 1974-79. There is also persistent and statistically significant seasonal size effect in January. Moreover, while the size effect has been unstable overtime the January size effect has not only been stable but increasing in magnitude from 1963 onward.

(ii) So far the search for an explanation for the size anomaly has been unsuccessful. Neither the economic nor the statistical reasons have been able to fully

Appendix C

Calculation of Monthly Returns

The monthly difference portfolio returns R_{mk} for the month of k are calculated by using arithmetic average as follows:

$$R_{mk} = (\sum_{t=1}^{n} 1 + R_{dt}/n)^n - 1$$

where

- R_{di} = the return on the difference portfolio on day i, i=1,2,...,5904.
- n = the number of trading days in month k, k=1,2,...,282.

An alternative method to calculate returns is based on geometric average as follows:

$$R_{mk} = (\prod_{t=1}^{n}(1 + R_{dt})) - 1$$

The arithmetic average corresponds to daily rebalancing of the difference portfolio by buying and holding the difference portfolio each day. The geometric average method corresponds to buy-and-hold method of investing in the difference portfolio in the beginning of the month and holding the.portfolio till the end of the month. and holding the portfolio till the end of the month.