A NUMERICAL MODEL FOR VORTEX SHEDDING
FROM SHARP WEDGES IN OSCILLATORY FLOW

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We accept this thesis as conforming
to the required standard

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April 1990

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Vancouver, Canada

Date 18th April 1990
Abstract

This thesis describes a numerical simulation and some flow visualization of vortex shedding from sharp edges in normal oscillatory flow.

The modelling of vortex shedding from sharp edges has been done using a discrete vortex method; the separated shear layer issuing from the separation point is represented by a system of discrete two dimensional vortices.

In Chapter Two, a finite wedge is modelled by considering the flow near the edge as the inner region of an oscillatory flow around an infinite wedge. This can be done if the Keulegan-Carpenter number is low, i.e. if the vortex shedding takes place mostly in the vicinity of the edge and is independent of shedding from any other edge(s). The mathematical formulation of this problem, although based on the combination of recent work of other researchers, represents a somewhat different approach when examined in detail. Each new vortex, called the nascent vortex, is introduced into the flow at a position not fixed in advance. Its position is dependent on the edge angle, the time step used in the numerical simulation and the influence of all the other vortices in the field. The expression describing the position of the nascent vortex can be derived as a natural development of the formulation. Therefore, it is not necessary to use empirical formulae to define the initial position of the nascent vortex and/or to fix this position for all time throughout the numerical simulation. Lamb vortices are used in the present study to delay the onset of instability in the numerical calculations. This results in very stable computations.
Numerical modelling results concerning vortex induced forces are presented in Chapter Three. These results are then compared to those obtained numerically and experimentally by other researchers.

Flow visualization experiments of vortex shedding from finite sharp wedges in an oscillatory flow are described in Chapter Four. The flow was produced using a sloshing tank, and visualized by hydrogen bubbles produced by the electrolysis of water. All results were recorded on video tape and photographs of flow visualizations have been produced through the use of a micro-computer based frame grabber. The kinematics of the numerical modelling are compared to those obtained from flow visualizations.

An application of the model to the roll decay of a simplified geometry of a single chine west coast trawler is presented in Chapter Five. No firm conclusions regarding the accuracy of the numerical prediction of roll decay can be drawn due to the gross simplification of the vessel section. However, the results do indicate that, with the absence of other forms of roll damping, vortex induced forces alone was able to cause roll extinction in the vessel. Therefore, it can be said that the prediction of roll extinction given by the present model is of an acceptable order of magnitude when compared to experimental roll decay results from previous work done in this department.
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Nomenclature

\[ A_n \]
Parameter used in numerical integration

\[ a_i \]
The \( i^{th} \) edge of a body with a number of edges

\[ \alpha_i \]
A defined angle at the \( i^{th} \) edge of a body with a number of edges

\[ \alpha_0 \]
Initial angle of heel for roll decay simulation

\[ B_n \]
Parameter used in numerical integration

\[ B_\phi \]
Total damping coefficient for a vessel in pure roll

\[ b \]
Waterline beam of a floating vessel

\[ \beta_n \]
Parameter used in numerical integration

\[ C \]
Contour of a vortex sheet

\[ C_{fu} \]
Vortex force coefficient for an infinite wedge

\[ C_{FV} \]
Vortex force coefficient for a finite wedge

\[ C_d \]
Drag coefficient for an infinite wedge

\[ C_m \]
Total inertia coefficient for an infinite wedge

\[ C_{m0} \]
Attached flow inertia coefficient for an infinite wedge

\[ C_{M0} \]
Attached flow inertia coefficient for a finite wedge

\[ C_D \]
Drag coefficient for a finite wedge

\[ C_M \]
Total inertia coefficient for a finite wedge

\[ C_0 \]
Factor for the initial placement of the nascent vortex

\[ C_n \]
Parameter used in numerical integration

\[ C_\phi \]
Hydrostatic restoration moment for a heeled floating vessel

\[ D_n \]
Parameter used in numerical integration
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Vortex induced drag coefficient for an infinite wedge</td>
</tr>
<tr>
<td>$d$</td>
<td>Characteristic length scale of the body</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Internal angle of a given wedge</td>
</tr>
<tr>
<td>$\delta_n$</td>
<td>Parameter used in numerical integration</td>
</tr>
<tr>
<td>$\Delta \tau$</td>
<td>Time step size for the numerical integration procedure</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Displacement of a floating vessel</td>
</tr>
<tr>
<td>$\Delta \phi$</td>
<td>Roll extinction for a vessel in free roll</td>
</tr>
<tr>
<td>$F(z)$</td>
<td>Complex potential in the $z$-plane</td>
</tr>
<tr>
<td>$F(\zeta)$</td>
<td>Complex potential in the $\zeta$-plane</td>
</tr>
<tr>
<td>$F_v$</td>
<td>Dimensional vortex induced forces for an infinite wedge</td>
</tr>
<tr>
<td>$GM$</td>
<td>Metacentric height of a floating vessel</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration in $m/s^2$</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Dimensional vortex strength</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Non-dimensional vortex strength</td>
</tr>
<tr>
<td>$I_\phi$</td>
<td>Virtual mass moment of inertia of a floating vessel</td>
</tr>
<tr>
<td>$i$</td>
<td>Complex number where $i = \sqrt{-1}$</td>
</tr>
<tr>
<td>$Im(...)$</td>
<td>Imagery part of a complex variable</td>
</tr>
<tr>
<td>$K$</td>
<td>Variable used in the determination of the nascent vortex position</td>
</tr>
<tr>
<td>$K_n$</td>
<td>Parameter used in numerical integration</td>
</tr>
<tr>
<td>$K_n^*$</td>
<td>Parameter used in numerical integration</td>
</tr>
<tr>
<td>$K_c$</td>
<td>Keulegan-Carpenter number of the flow where $K_c = \frac{V_0 T}{d}$</td>
</tr>
<tr>
<td>$K_d$</td>
<td>A parameter which determines the rate of vortex decay</td>
</tr>
<tr>
<td>$L_{equiv}$</td>
<td>An equivalent vessel length after simplification of its section</td>
</tr>
<tr>
<td>$L_z$</td>
<td>Flow length scale in the $z$-plane</td>
</tr>
<tr>
<td>$L_v$</td>
<td>Position of a vortex core at a given instant</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Non-dimensional parameter where $\lambda = 2 - \frac{\delta}{\pi}$</td>
</tr>
<tr>
<td>$M$</td>
<td>Vortex induced inertia coefficient for an infinite wedge</td>
</tr>
<tr>
<td>$m$</td>
<td>Variable used in the determination of the nascent vortex position</td>
</tr>
<tr>
<td>$N_m$</td>
<td>Number of time steps between the introduction of nascent vortices</td>
</tr>
<tr>
<td>$N_v$</td>
<td>Number of discrete vortices introduced per time cycle</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity of the fluid</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Roll angle of a floating vessel</td>
</tr>
<tr>
<td>$\phi_v$</td>
<td>Argument of a vortex core at a given instant</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Argument of a vortex core at a given instant from flow visualization</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>Argument of a vortex core at a given instant calculated by the model</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Position of a vortex core at a given instant from flow visualization</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>Position of a vortex core at a given instant calculated by the model</td>
</tr>
<tr>
<td>$Re(...) \quad$</td>
<td>Real part of a complex variable</td>
</tr>
<tr>
<td>$R_n$</td>
<td>Reynolds number of the flow where $R_n = \frac{\nu L}{\nu}$</td>
</tr>
<tr>
<td>$s$</td>
<td>Line element of a vortex sheet</td>
</tr>
<tr>
<td>$T$</td>
<td>Period of the oscillatory flow $V$</td>
</tr>
<tr>
<td>$t$</td>
<td>Dimensional time</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Non-dimensional time</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Initial argument of the nascent vortex</td>
</tr>
<tr>
<td>$U$</td>
<td>Horizontal velocity component pushing vortices away from the edge</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity vector of a fluid particle</td>
</tr>
<tr>
<td>$V$</td>
<td>Dimensional form of oscillatory velocity past a wedge</td>
</tr>
<tr>
<td>$v$</td>
<td>Non-dimensional form of oscillatory velocity past a wedge</td>
</tr>
<tr>
<td>$W$</td>
<td>Dimensional form of complex conjugate velocity</td>
</tr>
<tr>
<td>$w$</td>
<td>Non-dimensional form of complex conjugate velocity</td>
</tr>
</tbody>
</table>
$w_k$ Induced velocity at the $k^{th}$ vortex
$\omega$ Frequency of roll motion of a floating vessel
$Z$ Dimensional form of complex variable in the physical plane
$z$ Non-dimensional form of complex variable in the physical plane
$z_k$ Velocity of the $k^{th}$ vortex in the physical plane
$z_n$ Vortex position at the $n^{th}$ time step in the physical plane
$z_{n+1}$ Vortex position at the $(n + 1)^{th}$ time step in the physical plane
$z_0$ Initial position of the nascent vortex in the physical plane
$z_k$ Position of the $k^{th}$ vortex in the physical plane
$\hat{\xi}$ Dimensional form of complex variable in the transform plane
$\xi$ Non-dimensional form of complex variable in the transform plane
$\hat{\xi}_k$ Velocity of the $k^{th}$ vortex in the transform plane
$\xi_0$ Initial position of the nascent vortex in the transform plane
$\xi_k$ Position of the $k^{th}$ vortex in the transform plane
Acknowledgement

I wish to thank Professor Sander Calisal for his help and advice. As my research supervisor, he has been a constant source of ideas and guidance.

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To all those whose names are not mentioned here but have in one way or other contribute to making my research that much easier, I wish to say a sincere word of thanks.

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Chapter 1

Introduction

1.1 General

The study of flow past bluff bodies has many practical applications. This is amply demonstrated by its relevance to the effective design of offshore structures and marine vehicles. In particular, operational and safety considerations require reasonably accurate predictions of the motional behaviour of vessels in a seaway.

The theoretical prediction of ship motions in regular waves is very frequently based on strip theory, which was first proposed by Korvin-Kroukovsky and Jacobs [17]. Significant improvement to the original formulation has since been achieved by other researchers. A notable example is the work of Salvesen, Tuck and Faltinsen [26] in which account was taken of the ship’s heading and forward speed; factors not previously addressed. Strip theory based methods have been largely successful in the prediction of heavily damped motions; the same cannot be said of the prediction of lightly damped motions such as resonance roll of ships and barges. Such motions are normally overestimated by strip theory. The poor match between predicted and experimental roll response is thought to be due to the neglect of viscous damping in the theoretical model. This discrepancy of results is even larger for vessels with appendages and sharp-edged keels and fins. The reason for this probably lies in the fact that roll motion is very sensitive to the amount of roll damping, and flow separation from appendages promotes viscous damping.
Chapter 1. Introduction

Experiments have shown that flow separation does occur on vessels with sharp keels [8]. Moreover, the roll amplitudes of a barge with sharp edged bilges was found to be smaller than that of an identical vessel with rounded bilges [2]. Because strip theory solutions cannot give reasonably accurate predictions of vessel roll near resonance, the linear radiation damping term in the formulation is usually augmented with a non-linear, semi-empirical damping factor. Damping factors are based on experiments with typical sections of different ship types. As such, the scope of application is restricted to vessels with more or less similar sections. As a result, much attention has been focused on the construction of more generally applicable theoretical models for viscous damping since the mid-seventies. The work of Graham [13], Fink and Soh [10], and more recently Faltinsen et al. [1], Cozens [6], and Downie et al. [9] are relevant examples. In most cases, Schwarz-Christoffel transformations were used to map a sharp edge in the physical plane onto a transformed half-plane in which the flow problem was solved. The basis for the solution method is to approximate the vortex sheet rollup from sharp edges by discrete vortex representations. Viscous interactions can then be incorporated into the potential solution in strip theory using matching techniques. It was found in [9] that this treatment gives improved predictions of vessel roll response near resonance. The above method can also, in principle, be extended to other modes of ship motions although the relative worth of further improvement to the results for heavily damped motions has not been assessed.

The study of the phenomenon of vortex shedding from sharp edges is therefore of great practical interest. Application is, however, not limited to the prediction of ship response in a seaway. Problems with hydrofoils heaving in a seaway and flows at high angles of attack across rudder edges could probably be treated in a similar way. The latter example can be considered for the study of ship manoeuvring and directional stability in beam seas.
The work reported in this thesis deals primarily with the study of vortex shedding from infinite wedges and flat plates with a view to applying the results to hydrodynamic problems. It is generally divided into two main components: the numerical modelling of vortex shedding from sharp edges and experimental work to provide a basis for the judgement of the validity of the numerical results. Following that, the numerical method is then used in an application to the prediction of roll decay of a simple vessel.

1.2 Vortex Shedding from Sharp Edges

Flow separation occurs when the boundary layer on a body surface reaches a sharp edge, where the radius of curvature of the edge is very much smaller than the boundary layer thickness. In two dimensional flow, the pressure gradient set up results in the formation of an unstable shear layer which shed into the flow field as a free vortex sheet of infinitessimal thickness [12] and subsequently rolls up into a tight spiral.

The governing vorticity transport equation, which is obtained by taking the curl of the Navier-Stokes equation (with vorticity \( \omega = \nabla \times \vec{V} \)), is given by:

\[
\frac{D\omega}{Dt} = \frac{\partial \omega}{\partial t} + \vec{V} \cdot \nabla \omega = \omega \cdot \nabla \vec{V} + \nu \nabla^2 \omega
\]  

where \( \nu \) is the kinematic viscosity of the fluid and \( \vec{V} \) represents the velocity vector at any instant of time \( t \). Equation 1.1 implies that the rate of change of vorticity is due to the stretching of the vortex lines caused by both the motion of the fluid (term \( \omega \cdot \nabla \vec{V} \)) and by viscous diffusion (term \( \nu \nabla^2 \omega \)). For inviscid two dimensional flow, both these terms are equal to zero since \( \nu = 0 \) and \( \vec{\omega} \) is a vector normal to the x-y plane. Equation 1.1 becomes:

\[
\frac{D\omega}{Dt} = \frac{\partial \omega}{\partial t} + \vec{V} \cdot \nabla \omega = 0
\]  

(1.2)
This implies that the vorticity of a fluid element remains constant and convects with the flow. With the irrotational outer flow known, the velocity in the vicinity of the separation point can be found if the vorticity in the flow at a given time is calculated. The vortex sheet induced velocity is given by the *Biot-Savart* law:

\[ u - iv = \frac{i}{2\pi} \int_C \frac{\gamma(s)ds}{z - z(s)} \]  \hspace{1cm} (1.3)

where \( z(s) \) is the point on the sheet contour \( C \) with vortex strength \( \gamma(s) \).

Once the evolution of the vortex sheet is calculated, the velocity field and other quantities of interest, such as the time dependent forces acting on the body shedding the vorticity, can be determined. Unfortunately, the mathematical modelling of this problem is fraught with difficulties. For one thing, the induced velocity due to a vortex sheet spiral is given by a complicated integral. Generally, the solution can only be obtained numerically through the approximation of the vortex sheet by a number of discrete point vortices and/or vortex elements.

Discrete vortex methods are commonly applied to the calculation of two dimensional separated flows around bluff bodies at high Reynold's numbers. Except in the initial generation of vorticity in separation, the role of viscosity is neglected, see Equations 1.1 and 1.2. Point vortices are introduced into the flow very near to the separation point at a given time interval \( \delta t \) to satisfy the *Kutta-Joukowski condition* for smooth separation. The new point vortex thus created is referred to as the nascent vortex. With the passage of time, the total number of discrete vortices grows. Numerical time integration is used to simulate the growth of the shear layer into a larger vortex as well as to track the movement of the vortex clusters as they convect in the flow field. The hydrodynamic forces acting on the vorticity generating body can be calculated directly by the use of the Blasius force equation. Alternatively, such forces can be calculated
by the integration of pressures on the body surface found using the unsteady form of the Bernoulli equation.

In the present work, the vortex sheet will be approximated by multiple discrete vortices. Discretisation of the sheet requires that Equation 1.3 be modified. The induced velocity at the $k^{th}$ vortex can be written as:

$$ w(z_k) = \frac{i}{2\pi} \sum_{j=1,j\neq k}^{n} \frac{\Gamma_j}{z_k - z_j} $$

where $n$ is the total number of vortices in the flow field and $\Gamma_j$ and $z_j$ are the vortex strength and position of the $j^{th}$ vortex respectively. As an infinite Reynold's number approximation (i.e. ignoring viscous diffusion) the strength of each vortex can be taken to be constant and each vortex convects with the flow.

Discrete vortex methods are vulnerable to computational instabilities. To improve the stability of vortex shedding simulations, various researchers have devised ingenious schemes, some of which are more robust than others. A more detailed treatment of some of these schemes will be given when the problems involved in the numerical simulation of vortex shedding from flat plates and wedges in oscillatory flow is discussed.

1.3 Earlier Work On Discrete Vortex Methods

The use of discrete vortices to represent the gross features of separated shear layers shed from bluff bodies has been of interest to many for a long time. Rosenhead (1931) was among the first researchers to use this concept when he approximated a sinusoidally perturbed vortex sheet by 12 two-dimensional line vortices. He obtained a smooth roll-up of the shear layer into discrete vortex clusters spaced one wavelength apart.

Westwater (1935) applied the method to the shedding of the vortex sheet from an elliptically loaded wing. The results obtained were consistent with those predicted
analytically by Kaden in 1931.

Later, Hama and Burke (1960) repeated Rosenhead's calculations using more discrete vortices and a smaller time step. They found that, while the vortices do concentrate into clusters, the roll-up was not smooth. Also, it was found that the paths of some vortices cross: a physical impossibility. However, Hama and Burke extended the Rosenhead study by using unequal spacing of the vortices and were able to improve the stability of their calculations.

Moore (1974) reconsidered Westwater's work using very accurate time integration and a larger number of vortices. The study also revealed difficulties similar to those found by Hama and Burke: vortices occasionally cross paths and the sheet rolls up in a very irregular fashion. The reason for the latter difficulty was thought to be due to the tendency of vortices in near proximity to rotate around each other at high speeds. Moore concluded that this form of instability is caused by the method of discretising the sheet and was able to delay the onset of instability by merging the innermost vortices of the spiral into a central core.

The interaction between a pair of vortices result in uncharacteristically high induced velocities. Chorin and Bernard (1973) used vortices with a viscous core; that is, the velocity at the centre of each vortex is taken to be zero instead of being infinite for a pure potential vortex. In this way, the maximum induced velocity of a vortex is limited to a certain value after which it decreases as the centre of the vortex is approached. Smooth vortex sheet roll-up was achieved and path cross-overs were eliminated. Clements and Maull (1975) limited the induced velocities by amalgamating any pair of vortices that are too close together. They also found that the procedure increase the separation of vortices.

Fink and Soh (1974) pioneered a scheme of rediscretization in which the vortex sheet is rearranged into equidistant positions after each time step in the numerical
procedure. Although computation time is increased, this approach does result in more orderly roll-up of the vortex sheet.

Meanwhile, Moore revised his approach by replacing the innermost portion of each spiral by a single core and achieved smooth vortex sheet roll-up. Moore concluded that the problem with irregular vortex motion can be attributed to the representation of a tightly wound vortex spiral of infinite length by a finite number of discrete point vortices.

Graham (1977) applied the method to the calculation of vortex shedding from the sharp edge of an infinite wedge in oscillatory flow. Among other things, this study explored the benefits of representing the part nearest to the shedding edge by a vortex line element; based partly on the work of Giesing (1969). Roll-up of the vortex sheet appears to be smooth although the results were not compared to experimental data. Graham (1980) extended the method for use in calculating the vortex forces induced at a sharp edge in oscillatory motion at low Keulegan-Carpenter numbers ($K_e$). This was done by regarding vortex shedding from an infinite wedge as the inner region of flow past a large but finite body. The underlying assumption in this case is that the body length scale is large so that the vortex shed does not affect other parts of the body which are far away from the edge when compared to the flow length scale. The results obtained in this work were consistent with those obtained experimentally by Singh (1979). Downie, Bearman and Graham (1988) followed up on this and calculated the vortex damping forces on a rolling barge.

Cozens (1987) applied the discrete vortex method to the prediction of roll damping for vessels with sharp as well as rounded bilges with or without keel span attached. Here, rounded corners are approximated by a number of straight sections. Many features of this work concerning the discrete vortex method, for example: the use of a
vortex decay mechanism, the representation of the centre of the vortex spiral by a single core vortex, the use of the Kutta condition to determine the strength of the last vortex shed, and the fixing of the initial position of this nascent vortex, has been used by other researchers.

Dalton and Wang (1990) applied variable time steps, re-discretization and Lamb vortices to the problem of vortex shedding from an elliptically loaded wing. Although smooth roll-up was achieved and the calculations remain stable for much longer than that obtained by other researchers, the CPU time required for this formulation is comparatively high.

The above discussion on the historical development of the discrete vortex method applied to sharp edges has necessarily been brief. More comprehensive reviews on this subject can be found in Sarpkaya (1989), Clements and Maull (1975) and Fink and Soh (1974).
Chapter 2

Discrete Vortex Numerical Simulation

2.1 Summary

This chapter deals with the modelling of vortex shedding from infinite wedges in normal oscillatory flow. The discrete vortex method is well suited for application in this case since the separation point is not predicted naturally by the method. For bluff bodies without sharp edges, the separation point(s) would have to be fixed in advance empirically or an auxiliary boundary layer calculation is required.

The separated shear layer growing from the sharp edge is developed through the introduction of discrete vortices close to the edge at fixed time intervals and thereafter tracking their motion in the flow field. The body influence can either be represented by a distribution of singularities (sources and sinks or vortices) or by a transformation technique. The latter is used in the present work.

In the following sections, the problem of two dimensional vortex shedding from a sharp edge in normal oscillatory flow is modelled using complex variables. Conformal mapping is used to open out an infinite wedge in the physical plane. The velocity potential and hence velocity at each vortex in the flow is then derived. A flow length scale is constructed for normalising all variables in the problem. Then, a detailed description of how the nascent vortex position and strength are determined is given. Conditions for the amalgamation of core vortices are laid down. In the process, a force free condition is introduced in order to define the convection equation of a core vortex.
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The numerical procedure is given in Section (2.4). A system of discrete vortices are used to represent the developing shear layer. The total number of vortices in the flow field increases with time. At any given instant, the location of each of these vortices is found by numerically integrating the complex velocity of that vortex. After that various methods used for the avoidance of computational instability are given. In the present context, computational instability is defined as the sudden and rapid rise of calculated vortex velocities during the running of the computer program. Such large changes in computed values result in very high vortex induced forces and a very 'jagged' force characteristics. Moreover, computed values can get so large that the simulation breaks down. Both effects are not representative of the physical situation but are considered to be due to the discretisation of the vortex sheet as well as the use of potential vortices.

Following that the force coefficients associated to the above problem are derived.

2.2 Transformation of the Physical Plane

A numerical Schwarz-Christoffel transformation can be utilised to transform a body with a number of vertices $a_i$ into a half-plane as shown in Figure (2.1). The general form of the transform is given by:

$$
\frac{dZ}{d\zeta} = M \prod_{i=1}^{n} (\zeta - a_i)^{\frac{\alpha_i}{\pi}}
$$

(2.5)

where $M$ is a scaling constant between the physical $Z$ and the transformed $\zeta$ planes.

For a simple wedge with the sharp edge located at the origin in the physical plane, $a_1 = 0$. Taking the internal angle of the wedge as $\delta$, the angle $\alpha_1$ is given by $\alpha_1 = \pi - \delta$ and Equation 2.5 becomes:

$$
\frac{dZ}{d\zeta} = M \zeta^{1-\frac{\delta}{\pi}}
$$
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The transform equation for a plain isolated edge can be obtained by integrating Equation 2.6 to give:

\[
\frac{dZ}{d\zeta} = M \zeta^{\lambda-1}
\]  \hspace{1cm} (2.6)

Putting \( \lambda = 2 - \frac{4}{r} \) gives the following:

\[
\frac{dZ}{d\zeta} = M \zeta^{\lambda-1}
\]

The transform equation for a plain isolated edge can be obtained by integrating Equation 2.6 to give:

\[
Z = \frac{M}{\lambda} \zeta^{\lambda}
\]  \hspace{1cm} (2.7)

Figure 2.1: General Schwarz-Christoffel Transformation
2.3 The Theoretical Formulation

2.3.1 The Complex Potential and Velocity

The oscillatory flow velocity, \( V = V_0 \sin(\omega t) \) with the maximum velocity amplitude \( V_0 \) is normal to the edge bisector and the strength of the vortex at \( z \) and its corresponding point \( \zeta \) is given by \( \Gamma \).

The potential in the \( \zeta \)-plane, with a total of \( n \) vortices in the flow, is given by the following expression:

\[
F(\zeta) = iV\zeta + \frac{i}{2\pi} \sum_{k=0}^{n} \Gamma_k \ln \frac{\zeta - \zeta_k}{\zeta + \zeta_k}
\]  

Since the potential does not vary with transformation, for the \( j^{th} \) vortex:
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\[ F(\zeta_j) + \frac{i \Gamma_j}{2\pi} \ln(\zeta - \zeta_j) = F(Z_j) + \frac{i \Gamma_j}{2\pi} \ln(Z - Z_j) \quad (2.9) \]

where \( F(\zeta_j) \) and \( F(Z_j) \) are the potentials in the \( \zeta \) and \( Z \) planes respectively, not including the effect of the \( j^{th} \) vortex.

The complex conjugate velocity \( W_j \) at the \( j^{th} \) vortex in the physical plane is given by the derivative of Equation 2.9 with respect to \( Z \).

\[
W_j = \left[ \frac{dF(Z_j)}{dZ} - \frac{i \Gamma_j}{2\pi} \frac{1}{Z - Z_j} \right]_{z_j} \\
= \left[ \frac{dF(Z_j)}{dZ} \right]_{z_j} \\
= \left[ \frac{dF(\zeta_j)}{d\zeta_j} \frac{d\zeta_j}{dZ} + \frac{i \Gamma_j}{2\pi} \frac{1 - \lambda}{2Z_j} \right]_{Z_j, \zeta_j} \\
\tag{2.10}
\]

The second term on the right hand side of the above equation is known as the Routh correction due to the transformation (Maull [21]). Assuming that the normal oscillatory velocity \( V \) dominates and there is a total of \( n \) vortices in the flow:

\[
W_j = \frac{\delta_j}{\lambda Z_j} \left[ iV + \sum_{k=0, k \neq j}^{n} \Gamma_k \left( \frac{1}{\zeta_j - \zeta_k} - \frac{1}{\zeta_j + \zeta_k} \right) - \frac{i \Gamma_j}{2\pi} \left( \frac{1}{\zeta_j + \bar{\zeta}_j} - \frac{1 - \lambda}{2\zeta_j} \right) \right] \\
\tag{2.11}
\]

In the above equation, the second term under the summation sign (in square brackets) is due to the influence of all vortices and their images except for the \( j^{th} \) vortex. The first part of the third term in square brackets represents the influence of the image of the \( j^{th} \) vortex while the second part is the Routh correction.

2.3.2 Normalising the Variables

The infinite wedge has no natural length scale for non-dimensionalising the equations. However, a length scale \( L \), in terms of an attached flow velocity scale in the physical
plane $V_a$ (after Cozens [6], 1988) can be deduced. The velocity at a given point $P_z$ in the physical plane in attached flow (that is without vortex shedding) is proportional to $V$. Taking the constant of proportionality as $\lambda$:

$$V_a = \lambda \left| \frac{dF}{dZ} \right|_{P_z}$$

where $F = iV\zeta$, the complex potential without vortex shedding. If $P_t$ is the point in the transformed plane corresponding to $P_z$ and taking the period of oscillation ($T$) as a time scale, then:

$$P_z = \lambda \frac{dF}{dZ} |_{P_z} T$$

$$= \lambda \frac{dF}{d\zeta} \frac{d\zeta}{dZ} |_{P_t} \frac{d\zeta}{dZ} |_{P_t} T$$

(2.12)

Where

$$\frac{dZ}{d\zeta} |_{P_t} = M(P_t)^{\lambda-1}$$

$$P_Z = \frac{M}{\lambda} (P_t)^{\lambda}$$

$$P_t = \left( \frac{M}{\lambda} \right)^{-\frac{1}{\lambda}} (P_Z)^{\frac{1}{\lambda}}$$

Let $L_z = |P_z|$ and $L_t = |P_t|$ be the length scales in the physical and transformed planes respectively. Then using Equation 2.12 and defining $V_0$ as the maximum attached flow velocity in the transformed plane, the length scale $L_z$ can be written thus:

$$L_z = \frac{\lambda V_0 T}{\frac{d\zeta}{dZ} |_{P_t}}$$

$$= \frac{\lambda V_0}{M(L_t)^{\lambda-1}}$$

$$= V_0 T \left( \frac{M}{\lambda} \right)^{-\frac{1}{\lambda}} (L_z)^{\frac{1}{\lambda}-1}$$

$$= \left( \frac{M}{\lambda} \right)^{-\frac{1}{\lambda}-1} (V_0 T)^{\frac{1}{\lambda}-1}$$

(2.13)
The length scale $L_t$ can then be found accordingly.

$$L_t = \left(\frac{M}{\lambda}\right)^{-\frac{3}{2\lambda-1}} (V_0T)^{\frac{1}{2\lambda-1}}$$  (2.14)

The main parameters in the problem can thus be non-dimensionalized with $V_0$ and $T$ in the following way.

$$Z = \left(\frac{M}{\lambda}\right)^{\frac{1}{2\lambda-1}} (V_0T)^{\frac{1}{2\lambda-1}} z$$
$$\zeta = \left(\frac{M}{\lambda}\right)^{\frac{2}{2\lambda-1}} (V_0T)^{\frac{1}{2\lambda-1}} \zeta$$
$$\Gamma = \left(\frac{M}{\lambda}\right)^{\frac{2}{2\lambda-1}} (V_0T)^{\frac{2\lambda-1}{2\lambda-1} - 1} \gamma$$
$$\tau = \frac{t}{T}$$
$$v = \sin(2\pi \tau)$$  (2.15)

where $\tau$ is the non-dimensional time. In the present analysis, all dimensional variables are written in upper case letters while the corresponding non-dimensional ones are in lower case (the only exception is $\zeta$ being the dimensional form and $\zeta$ is now non-dimensional). The scaling constant ($M$) need not be defined since it would be eliminated in the process of non-dimensionalization. The complex conjugate velocity, Equation 2.11, can thus be normalised with $W_j = \frac{L_t}{T} w_j$ to give:

$$w_j = \frac{i\zeta_j}{\lambda z_j} [v + \sum_{k=0, k \neq j}^n \frac{\gamma_k}{2\pi} \left(\frac{1}{\zeta_j - \zeta_k} - \frac{1}{\zeta_j + \bar{\zeta}_k}\right) - \frac{\gamma_j}{2\pi} \left(\frac{1}{\zeta_j + \bar{\zeta}_j} - \frac{1 - \lambda}{2\zeta_j}\right)]$$  (2.16)
2.3.3 Development of the Nascent Vortex

In order to fully define the nascent vortex one has to know both its strength and initial position. With one of these factors known, the other can be found by invoking the Kutta condition which ensures smooth separation at the edge. Sarpkaya [21] determined the strength of the nascent vortex using a relationship involving $Q_s$, defining $Q_s$ as the mean of the velocities of the newest four vortices in the shear layer, where:

$$\frac{d\Gamma}{dt} = \frac{1}{2} Q_s^2$$

(2.17)

Other similar expressions have been used by other researchers. With the strength of the nascent vortex known, the Kutta condition can be used to find its position and hence its velocity. The Kutta condition takes the form $\frac{dF(\xi)}{dt}_{\tau=0} = 0$ to ensure that the velocity at the sharp edge is finite. Hence from Equation 2.8, it follows that:

$$\frac{dF(\xi)}{d\xi}_{\tau=0} = t[V - \sum_{k=0}^{n} \frac{\Gamma_k}{2\pi} \left( \frac{1}{\xi_k} + \frac{1}{\xi_k} \right)]$$

$$= 0$$

(2.18)

$$\gamma_0 = \frac{v_n |\xi_0|^2}{Re(\xi_0)}$$

(2.19)

where

$$v_0 = \pi \sin(2\pi \tau)$$

$$v_n = \pi \sin(2\pi \tau) - \sum_{k=1}^{n} \gamma_k \frac{Re(\xi_k)}{|\xi_k|^2}$$

(2.20)
The above relationship can then be used to find the initial position, \( z_0 \), of the nascent vortex. Alternatively, as preferred by some researchers, the initial position of the nascent vortex is fixed at a small distance from the separation point depending on the length scale of the body and the numerical time step size [16]. The nascent vortex strength can then be found from Equation 2.19.

This author prefers to use a different approach since the methods mentioned above involve fixing one of two parameters in advance and thereby ignoring the influence of the other vortices in the field on the vortex shedding process. An expression can be derived for the initial position of the nascent vortex by considering the convection equation for a discrete vortex. It can be assumed that all vortices (except those whose strength changes with time, for example the core vortices at the centre of each cluster) convect with the flow field. In other words, the velocity of a vortex with constant strength is equal to the induced velocity at the centre of that vortex due to the influence of the free stream as well as all the other vortices in the field. Therefore, the convection equations in the physical and transformed planes for the \( k^{th} \) vortex are given by:

\[
\dot{z}_k = \omega_k \tag{2.21}
\]

\[
\dot{s}_k = \frac{s_k}{\lambda z_k} \dot{z}_k \tag{2.22}
\]

The Kutta condition is satisfied only at the beginning of each time step, at which time a new vortex enters the flow field with a given strength. Hence, Equation 2.21 also applies to the nascent vortex. For a time step \( \Delta t \), let the position of the nascent vortex at the end of its first time step:
\[ z_0 = K (\Delta \tau)^m e^{i\phi} \quad (2.23) \]

The following is a brief description of how the initial position of the nascent vortex is found. A full derivation for \( z_0 \) is given in Appendix A. The velocities \( z_0 \) and \( w_0 \) can be found using Equations 2.23 and 2.16 respectively. Then, from Equation 2.21 and assuming \( \gamma_0 = 0 \):

\[ \dot{z}_0 = mK(\Delta \tau)^{m-1}e^{i\phi} = -\frac{i\nu_n}{\lambda \pi} K^{\frac{1-\lambda}{\lambda}} (\Delta \tau)^{\frac{m(1-\lambda)}{\lambda}} e^{i\phi (1-\lambda)} (1 - \frac{1}{4\cos^2(\frac{\pi}{\lambda})} + \frac{(1 - \lambda)e^{\frac{i\phi}{\lambda}}}{4\cos(\frac{\pi}{\lambda})}) \quad (2.24) \]

Equating powers of \( \Delta \tau \), \( m = \frac{\lambda}{2\lambda - 1} \) and Equation 2.24 becomes:

\[ \frac{\lambda}{2\lambda - 1} K^{\frac{2\lambda - 1}{\lambda}} = -\frac{i\nu_n}{\lambda \pi} e^{-\frac{i\phi}{\lambda}} (1 - \frac{1}{4\cos^2(\frac{\pi}{\lambda})} + \frac{(1 - \lambda)e^{\frac{i\phi}{\lambda}}}{4\cos(\frac{\pi}{\lambda})}) \quad (2.25) \]

Equating the real and imaginary parts of the above equation yields the following results:

\[ \theta = -\lambda \cos^{-1} \left( \frac{\sqrt{\lambda}}{2} \right) \]

\[ K = \left( \frac{\nu_n \sqrt{4 - \lambda(\lambda - 1)(2\lambda - 1)}}{2\pi \lambda^3} \right)^{\frac{1}{2\lambda - 1}} \quad (2.26) \]

The nascent vortex is intuitively expected to leave the edge parallel to one face of
the wedge, depending on the direction of flow at that moment. However, the argument of $z_0$ given by Equation (2.26) is constant at all times for a given wedge internal angle. It appears that this is not always correct. Fortunately the nascent vortex have been known to adjust very rapidly to the correct path [12]. Hence, the nascent vortex will be introduced along the exterior edge bisector every time. That this does not make any difference has been shown by Cozens [6]. Therefore, with $K$ defined above, the initial position of the nascent vortex is given by:

$$z_0 = (K \Delta \tau)^{\frac{\lambda}{2\lambda - 1}}$$

(2.27)

It has, however, been found through numerical experimentation, see Cozens (1988) [6] that placing the nascent vortex at a position slightly away from that suggested by Equation 2.27 results in smoother vortex roll-up and improves the stability of the calculations. This is due to the fact that induced velocities are higher nearer to the edge. Therefore, the actual position at which the nascent vortex is introduced will be taken as $z_0 = C_0(K \Delta \tau)^{\frac{\lambda}{2\lambda - 1}}$. The value of $C_0$ varies from 1.10 to 1.25 for wedge angles from $0^\circ$ to $90^\circ$. The strength of the nascent vortex can then found using the Kutta condition given by Equation 2.19.

### 2.4 Amalgamation scheme for the core

The tendency of vortices in close proximity to orbit around each other is a major cause of the irregular roll-up of the vortex spiral. To avoid this problem, discrete vortices representing the shear layer are progressively wound into a central core over time. Merging of vortices nearest to the core would also give a more practical approximation
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of the infinite spiral with a finite number of discrete vortices. Amalgamation, by limiting the number of vortices in the calculations, also reduces computing time.

One cluster of vortices is produced in each half cycle. For each of these clusters, one vortex is amalgamated into the core for every 4 new vortices introduced. Cozens [6] had shown that using such an amalgamation rate does not result in any significant effect on the drag and added mass coefficients calculated. Therefore, if nascent vortices enter the flow at a rate of one per $2\Delta r$, then the amalgamation rate is $8\Delta r$.

The amalgamation scheme used by Cozens [6] is adopted in this study. This method yields an equivalent velocity field while maintaining representative vortex distributions around the edge. Two vortices at $z_1$ and $z_2$, with strengths $\gamma_1$ and $\gamma_2$ respectively, can be merged to give a new vortex at $z_3$ with strength $\gamma_3$ in the following way.

\[
\begin{align*}
z_3 &= \frac{z_1|\gamma_1| + z_2|\gamma_2|}{|\gamma_1| + |\gamma_2|} \\
\gamma_3 &= \gamma_1 + \gamma_2
\end{align*}
\]\ (2.28)

Since the core vorticity changes with time, Equation 2.21 does not hold for the convection velocity of the core. Instead, a force free condition is imposed on the core to reflect that fact that the fluid is unable to physically support any forces using momentum considerations. The forces acting on the vortex feeding sheet can be defined by $i\rho \Gamma_c(Z_c - Z_s)$ where $Z_c - Z_s$ represent the cut between the core and the rest of the vortex sheet. The lifting force due to the vortex sheet is given by $i\rho \Gamma_c(\dot{Z}_c - \dot{W}_c)$. Equating forces, the *force free condition* can be written in the form given below.

\[
\Gamma_c(\dot{Z}_c - \dot{W}_c) + (Z_c - Z_s)\dot{\Gamma}_c = 0
\]
\[ z_c = \omega_c - (z_c - z_e) \frac{\gamma_c}{\gamma_e} \quad (2.29) \]

2.5 The Numerical Procedure

A new vortex is introduced into the flow once in \( N_m \) number of time steps. The new position of each vortex after a time step can be found by numerically integrating its convection velocity. The initial position of the nascent vortex after a time step \( \Delta r \) is given by Equation 2.27. The strength of the nascent vortex can then be found using the Kutta condition given in Equation 2.18. The other vortices are then convected accordingly using Equations 2.16 and 2.21. The core vortices are convected using Equation 2.29.

All vortices, including the nascent vortex, are then convected over \( N_m - 1 \) time steps. At the end of the period \( N_m \Delta r \), the nascent vortex would become an 'old' vortex and a 'new' nascent vortex is introduced at time \((N_m + 1)\Delta r\). Thereafter, the process repeats itself.

If the number of vortices injected into the flow per time cycle is \( N_v \), then the time step size \( \Delta r = \frac{1}{N_v N_m} \). This means that the non-dimensional elapsed time \( \tau \) at any instant is expressed as number of time cycles into the simulation.

The Euler-Cauchy Method (a predictor-corrector method), see [18] was used for the numerical integration of the all vortex positions. The integration time step required would be \( \Delta r \). The integration procedure is given below where the velocity of a vortex located at \( z_n \) at the \( n^{th} \) time step can be integrated to find its new position \( z_{n+1} \) after one time step \( \Delta r \).

\[ \dot{z}_n = f(\dot{z}_n, \gamma_n) \]
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\[ A_n = f(z_n + \Delta t \dot{z}_n, \gamma_{n+1}) \]
\[ z_{n+1} = z_n + \frac{\Delta t}{2}(\dot{z}_n + A_n) \]  
(2.30)

2.6 Use of Lamb Vortices

Two vortices that are too close together induce very large velocities on each other. This is also the case for a vortex approaching the body too closely, due to the influence of its image. Large mutually induced velocities is one of the primary causes of instability of discrete vortex computations. Therefore, in order to increase the time span in which the calculations remain stable, Lamb vortices were used in this study \(^1\), in Equation 2.16.

The Lamb vortex has a viscous core such that as the distance from the core tends to zero, the vorticity will vanish exponentially. This can be defined mathematically as follows.

\[ \Gamma_k = \Gamma_{k,0}[1 - \exp(-\frac{D^2}{4\nu t})] \]  
(2.31)

where \( D \) is the distance between the \( k^{th} \) vortex and the vortex in question, at \( \zeta_j \), and \( t \) is the age of the \( k^{th} \) vortex. The kinematic viscosity of the fluid \( \nu \) can be expressed in terms of an 'equivalent' Reynolds number, \( R_n = V_0 L / \nu \). The normalised form of the above equation, with \( \gamma_{k,0} \) as the actual vortex strength, becomes:

\[ \gamma_k = \gamma_{k,0}[1 - \exp(-\frac{R_n L^2}{V_0 L_s T} \frac{\zeta_j - \zeta_k}{4r})] \]  
(2.32)

\(^1\)The Lamb vortex has successfully been used by Dalton and Wang \([7]\) to prolong the stability of their computations.
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The actual expression used in the vortex shedding program is:

\[
\gamma_k = \gamma_{k,0} [1 - \exp(-50|\xi - \xi_k|^2)]
\]  

(2.33)

The size of the core is not constant. Hence the limiting induced velocity depends on the age of the vortex. For example, for a vortex with an age of one time cycle, the induced velocity at a non-dimensional distance of 0.0142 from its centre would be only 1% of the actual.

Although the Reynolds number has been assumed to be infinite, the use of a finite \( R_n \) here does not affect the results obtained both because the viscous region is confined to a small area around each vortex and because the value of \( R_n \) used was large.

2.7 Decay mechanism for the discrete vortices

In order to achieve better computational stability, vortices are allowed to decay with time. Again one can argue that decay of vorticity is justified by the fact that discrete vortex methods typically overestimate the vortex induced forces [13] by about 30% and that the Reynolds number is large but finite. Hence vortices convected away are allowed to decay in strength in the following manner:

\[
\gamma_k = \gamma_{k,0}[1 - \exp\left(-\frac{K_d}{m\Delta r}\right)]
\]  

(2.34)

where \( m\Delta r \) specifies the age of the \( k^{th} \) vortex. The decay rate is determined by \( K_d \) which is taken as -0.3567 in the present numerical model. This effectively means that the strength of a fully developed spiral is reduced to about 30% of its original value over one time cycle.
The vortex decay rate used is not considered high in the context of the vortex pairing in the shedding process. This requires a little more explaining. The vortex spiral shed over one half cycle is swept back as the free stream reverses. It is carried towards the opposite side of the wedge. Meanwhile, another vortex spiral is issuing from the edge. These two vortex spirals will eventually pair up and be convected off the edge when they are of about equal but opposite strength. This process can be observed in flow visualization experiments and will be described in Chapter Four. Maull suggested in [21] that up to 50% of the total vorticity is lost in this way. In consideration of this, each vortex is allowed to decay to a small fraction of its original strength one and a half cycles after its introduction into the flow field.

In other words, a vortex is actually 'switched' off when it is far away from the edge. However, it should be mentioned that the mechanism of decay should be applied uniformly: decay comes into effect on every vortex as soon as it leaves the edge to ensure a smoother rollup. This does not contradict the condition that the vortex should be force free if the time step used, and hence the actual amount of vorticity change, is sufficiently small.

2.8 Pushing the vortices away from the edge

A velocity $U$, parallel to the real axis, is imposed at the start of the numerical simulation to push vortices well away from the edge and to start of the shedding process. The magnitude of $U$, which depends on the normal velocity $V$ as well as the wedge parameter $\lambda$, decreases exponentially with time in the form:

$$U = U_{init} e^{-t^2 / \lambda^2}$$

(2.35)

\(^2\)Kudo [20] removed any vortex that was above a pre-defined distance from the body.
The value of $U_{\text{init}}$ varies from $V$ to $2V$ and can be found, considering computational stability, by numerical experimentation. However, it can be noted that this velocity is reduced to only 1% of its initial value after three time cycles. Therefore, the imposition of this 'horizontal' velocity has no long term effect on the numerical solution.

2.9 Vortex Induced Forces

The force induced on a plain infinite wedge due to vortex shedding can be found using the unsteady form of the Blasius force equation:

$$ F_v = -i\rho \oint_c Fdz - \frac{i\rho}{2} \oint_c \frac{\partial F}{\partial z}(\frac{\partial F}{\partial z})^2 dz $$

where the integrals are taken around the perimeter of the body, $c$, in the physical plane $z = x + iy$ and $F$ is the complex potential.

Expanding $\frac{ds}{d\zeta}$ and $F$ for large $\zeta$ and applying the residue theorem gives the dimensional vortex force on a plain wedge as follows:

$$ F_v = i\rho \frac{\partial}{\partial t} \sum_{k=0}^{n} \frac{\Gamma_k}{2}(\zeta_k + \bar{\zeta}_k) \quad (2.36) $$

A vortex force coefficient can then be found by non-dimsionalizing Equation 2.36 and dividing by $\frac{1}{2} \rho L^2_s L_t T^{-2}$.

$$ C_{fv} = \frac{F_v}{\frac{1}{2} \rho L^2_s L_t T^{-2}} $$

$$ = 2i \frac{\partial}{\partial t} \sum_{k=0}^{n} \gamma_k \text{Re}(\zeta_k) \quad (2.37) $$
The above formulation is for an infinite wedge, that is, for the limit of zero Keulegan-Carpenter number since \( Kc = \frac{V_0T}{d} \) where \( d \) is the characteristic body length scale. For a finite wedge, the vortex force can be matched with the above following Graham [13] in the following way. Since \( (V_0T)^{\frac{2}{3}} \propto (Kc)^{\frac{2}{3}} \), the ratio of the flow length scale to the body length scale is proportional to \( (Kc)^{-\frac{2}{3}} \). The vortex force coefficient is obtained by non-dimensionalizing with \( \frac{1}{2} \rho V_0^2 d \).

\[
C_{fv} = \frac{F_v}{\frac{1}{2} \rho V_0^2 d} = (Kc)^{-\frac{2}{3}} C_{fv}
\]

(2.38)

### 2.10 Drag and Added Mass Coefficients

The vortex induced force \( F_v \) acts in a direction normal to the edge bisector since it is pure imaginary, see Equation 2.37. The dimensionless coefficient \( C_{fv} \) can be translated into drag and inertia coefficients by taking Fourier integrals over a time cycle.

The drag coefficient \( C_d \) is given by:

\[
C_d = D = \frac{3\pi}{4} \int_{r_n}^{r_{n+1}} C_{fv} \sin(2\pi \tau) d\tau
\]

(2.39)

and the inertia coefficient \( C_m \) by:

\[
C_m = C_{m_0} + M
\]

(2.40)

where \( M = \frac{2}{\pi^2} \int_{r_n}^{r_{n+1}} C_{fv} \cos(2\pi \tau) d\tau \) and \( C_{m_0} \) is the inertia coefficient due to the attached flow.
For small $K_e$, the above are given by (following Graham [13]):

$$C_D = (K_e)^{2\lambda-1} \frac{3\pi}{4} \int_{0}^{\frac{m+1}{2\lambda}} C_V \sin(2\pi \tau) d\tau$$

and

$$C_M = C_{M_0} + (K_e)^{2\lambda-1} \frac{2}{\pi^2} \int_{0}^{\frac{m+1}{2\lambda}} C_V \cos(2\pi \tau) d\tau.$$
Chapter 3

Results of the Numerical Simulation

3.1 General

Computer programs in Pascal have been written (using the Apollo system at the Mechanical Engineering Department) for the numerical simulation of vortex shedding from sharp wedges in normal oscillatory flow. Attention had been focused on two cases: the flat plate and the square edge, since numerical and experimental results are available in the literature for comparison purposes.

In this chapter, only the dynamics of the simulation will be assessed. The kinematic results of the simulation will be presented here but will be compared to the results of flow visualization experiments in the next chapter.

3.2 Vortex Development with Time

Figure (3.3) shows the development of the vortex sheet over one complete time cycle. Initially, the free stream accelerates uniformly towards a maximum at $t/T = 0.5$. The starting flow causes the shear layer to roll up in the expected sense as shown in Figure (3.3) until the flow begins to change in direction. This flow reversal occurs before the free stream reverses at $t/T = 0.5$ due to the induced velocities of the vortices already shed. This has been observed in flow visualization experiments and can also be seen in Figure (3.3).
Shear Layer Development $t/T = 0.25$

Shear Layer Development $t/T = 0.50$

- ○ ○ ○ ○ Discrete vortices
- ○ ○ ○ Represented shear layer
- --- 90 deg wedge

$N_v = 48$
Figure 3.3: Shear Layer Development for 90-degree Wedge
As the flow accelerates in the other direction, a second vortex sheet is forming at the edge in a way similar to that described above. Meanwhile, the 'old' vortex sheet has been convected over the real axis to the upper half plane. Figure (3.3) shows the vortex locations in the flow at $t/T = 0.75$.

The second vortex cluster continues to develop until its strength is more or less equal but opposite to the first cluster. When this occurs, the two vortex clusters will form a pair and convects downstream. Again, this does not necessarily happen at the end of the time cycle. A new vortex begins to form towards the end of the cycle and the process will repeat itself over subsequent cycles, see Figure (3.3).

### 3.3 Effect of Time Step Size

The time step size is determined by the number of vortices, $N_v$, introduced into the flow per cycle of oscillation. A larger integer $N_v$ means that the time step size is smaller. In general, better definition of the vortex sheet roll-up is obtained with larger $N_v$ and $N_m$. However, a smaller time step size increases the computation time. In fact, it was found that the increase in resolution of the sheet roll-up is not significant when $N_v$ is increased from 40 to 80. Figure (3.4) shows the difference in vortex sheet definitions for varying time step size. For a single point vortex representation of the 'centroid' of the sheet, that is $N_v = 2$, it can be seen that its position is further from the edge when compared to the core position for other values of $N_v$.

All calculations can be done with $N_v = 40$ and $N_m = 4$, or $\delta t = \frac{1}{180}$. This would give good representations of the shear layer while keeping the CPU time requirement for the simulation at a practical level. For simulations over long periods of time, it was found (through numerical experimentation) that a value of $N_m = 2$ can be used without appreciable loss of accuracy.
Chapter 3. Results of the Numerical Simulation

Comparison for Different Nv at t/T = 0.5

Wedge angle 90 deg
Nv = 2

Comparison for Different Nv at t/T = 0.5

Wedge angle 90 deg
Nv = 20
Comparison for Different \( N_v \) at \( t/T = 0.5 \)

Wedge angle 98 deg

\( N_v = 48 \)

Figure 3.4: Vortex Sheet For Varying \( N_v \)
Chapter 3. Results of the Numerical Simulation

3.4 Effect of Constant Nascent Vortex Argument

In Chapter Two Section (2.3.3), it was mentioned that the argument of the nascent vortex would be taken as $\theta = 0$ regardless of the value of the edge angle whereas the value of $\theta$ was given by the formulation as:

$$\theta = -\lambda \cos^{-1}(\frac{\sqrt{\lambda}}{2})$$

Numerical results show that taking the value of $\theta = 0$ has no effect on the kinematic development of the vortex sheet. This is illustrated in Figure (3.5) for the flat plate at $t/T = 0.25$ from which it can be seen that at the same vortex decay rate, the vortex spiral is identical whether $\theta = 0$ or $\theta = -1.571$ radians as given by the formulation.

Figure (3.5) also shows that the vortex decay rate has no significant effect on the shape of the vortex spiral.
Chapter 3. Results of the Numerical Simulation

3.5 Effect of Vortex Decay Rate

The vortex decay rate was introduced into the simulation mainly because of stability considerations and that the vorticity in the field is known to be overestimated by the discrete vortex method. It is therefore necessary to determine the effects of the vortex decay rate used.

The decay rate is a function of the constant $K_d$. A smaller value of $K_d$ gives a higher rate of vortex decay. Although changes in $K_d$ does not appear to have any effect on the phase lag of the coefficient $C_{fV}$ in the first cycle, the magnitude of the $C_{fV}$ decreases with an increase in the vortex decay rate as shown in Figure (3.6).

The maximum amplitudes of $C_{fV}$ in the second half-cycle without decay and with $K_d = 0.3567$ (occurring around $t/T = 0.70$) are $-12.75$ and $-9.24$ respectively. The present model uses $K_d = 0.3567$, that is the value of $C_{fV}$ found is 27.5% lower due to the use of vortex decay. Since it is well known that discrete vortex methods typically
Chapter 3. Results of the Numerical Simulation

3.6 Vortex Induced Forces for Different Internal Angles

In the following discussion, only the induced forces due to the shedding of vorticity is considered. Figures (3.8) and (3.9) shows typical force traces from numerical calculations using the present model for a square diamond and a flat plate respectively. Similar force characteristics have been obtained for other wedge angles.

The stability of the computations have been greatly improved by the schemes given in Chapter 2. For the flat plate, the simulation had been run for more than 46 time
Chapter 3. Results of the Numerical Simulation

Figure 3.8: Vortex Induced Forces Square Diamond

Figure 3.9: Vortex Induced Forces Flat Plate
Chapter 3. Results of the Numerical Simulation

Vortex Induced Force: Flat Plate

Figure 3.10: Stability of Vortex Force Calculations

cycles (68 hours in real time) without any sign of computational instability, see Figure (3.10). It can be observed from Figure (3.10) that the coefficient $C_{fv}$ appears to return a reasonable value at the 47th positive peak. However, the simulation was stopped as the stability of the calculations was deemed to be well illustrated by then. The vortex induced force coefficient $C_{fv}$ are virtually identical when time cycle 5 is compared to time cycle 35. This comparison is shown in Figure (3.11).

The maximum induced force, that is the average of peak force magnitudes, can then be found for different angles $\delta$ and plotted in Figure (3.12). Such forces are obviously dependent on wedge internal angles ($\delta$) and the Keulegan-Carpenter number ($K_c$).

The drag and inertia coefficients, D and M, can be derived by taking Fourier integrals of the vortex induced force over one time cycle, see Equations 2.39 and 2.40 in the previous chapter. The variations of D and M with wedge angle are then plotted and shown in Figures (3.13) and (3.14).

Values of coefficient D obtained in the present work are generally lower than those
Chapter 3. Results of the Numerical Simulation

Figure 3.11: Comparison of Late Cycle to an Early Cycle

Figure 3.12: Peak Vortex Force For Different Wedge Angles
of Graham [13]. This is possibly due to the fact that high rates of vortex decay are used in the present model.

### 3.7 Comparison with the Results of Others

Results obtained from the present model are compared to those of other researchers and presented in Table (3.1).

The inertia coefficient, \( M \), is sensitive to the amount of vortex decay used, the velocity \( U \) and the factor \( C_0 \) used to increase the distance of the nascent vortex from the edge. It is not possible to conclude which of the numerical results obtained is accurate. Inspection of Table (3.1) suggests that the magnitude of \( M \) is correct. Fortunately, \( M \) is so much smaller than \( D \) as to render the former unimportant in the present analysis.
Figure 3.14: Variation of Coefficient M with Wedge Angle

Table 3.1: Comparison of Coefficients D and M
The drag coefficient, \( D \), obtained for the flat plate (Figure 3.14) in the present work is nearer to those found in experiments by Singh (1979)\(^1\) and Bearman et al (1985)\(^2\) than the numerical results of Graham [13] and Cozens [6]. On the other hand, calculated \( D \) for the 90° edge is 30% lower than Singh's experimental values while that of Graham is 12% higher. This is reflected in Figures (3.15) and (3.16) where the coefficients \( C_D \) and \( C_M \) for a range of Keulegan-Carpenter numbers are compared. However, it is not unreasonable to say that there is always some uncertainty attached to the results obtained from experiments of this difficult nature.

The experimental results of other researchers presented in Figures (3.15) and (3.16), for the flat plate and the square diamond respectively, were digitized from graphs in Reference [13].

\(^1\)Taken from Graham [13].
\(^2\)Taken from Cozens [6].
Chapter 3. Results of the Numerical Simulation

3.8 Comments on the Limits of Validity of the Model

During the development of the numerical model in the previous chapter, some assumptions have been made. These assumptions place constraints on the range of validity of the method.

The matching of isolated edge flows to finite bodies with sharp edges require that vortex shedding from each edge be independent of that at the other edges. As the Keulegan-Carpenter ($K_c$) number becomes higher, that is with an increase of the parameter $V_0 T$, the size of the vortex spiral becomes larger. Interactive vortex shedding will eventually occur. Graham [13] suggested that non-interactive vortex shedding is limited to a $K_c$ of 5 or less.

According to Graham [13], the experimental values of $D$ and $M$ for the circular cylinder are 0.10 and $-0.005$ respectively. The analysis discussed here should not be applied to the circular cylinder (that is $\delta = 180^\circ$) since the separation point is no longer
fixed and it is difficult to satisfy the Kutta condition in a meaningful way. Furthermore, it has not been shown that the influence of vortices far away from the body is negligible. This means that the equations for $C_D$ and $C_M$ for a finite body at non-zero $K_e$ derived in Chapter Two is probably not valid for the circular cylinder. Fortunately, wedge angles are unlikely to be near $180^\circ$ for applications to ship sections. So the value of $\delta$ in the present analysis has been limited to a maximum of $135^\circ$. Moreover, Figure (3.12) shows that vortex forces for angles larger than $135^\circ$ are small.

The magnitude of the inertia coefficient $M$ in the limit of $K_e = 0$ is much smaller than that of the drag coefficient $D$. Computed values for $M$, with the possible exception of the flat plate results, do not agree well with experimental results. The computed values of $M$ should therefore be viewed with some scepticism.
Chapter 4

Report on Experimental Work Done

4.1 Experimental Objectives

The main purpose of conducting the experiments reported in this chapter is to provide a basis for comparison and subsequently an assessment regarding the accuracy and usefulness of the numerical methods described in Chapter Two. It is also hoped that enough is learned to enable some preliminary conclusions concerning the general applicability of the theoretical model to wedges with different internal angles.

Having said that, it is noted that experimental limitations have made difficult the exact simulation of the conditions specified in the theoretical formulation. One obvious example is the provision of an oscillatory flow past a wedge. It is not possible to conveniently recreate an oscillatory flow in a controlled environment in order to allow proper flow visualization to take place. Alternatively, the wedge model can be set into oscillatory motion in water. This is kinematically the same as having oscillatory flow past the model but dynamically different by the Froude-Krylov force which is in phase with the acceleration. Unfortunately, the equipment needed is not available. Even if they are, there would remain the problem of wedge submergence (to avoid free surface effects) as well as the satisfactory visualization of the flow around the moving wedge model. Other difficulties were also encountered, but these will be dealt with in the following sections of this chapter.
The experiments conducted were designed for the gathering of information to adequately expose two principal parameters in the problem of vortex shedding from sharp edges. These are the vortex roll up visualization (kinematic) and determination of the in-line forces on the test body due to the flow (dynamic). However, it was not possible to meet the latter objective due to limitations of the sloshing tank system. This will be explained in Section (4.6).

4.2 Test Bodies Used

The wedge models used were geometrically simple wedges with the sides piercing the free surface and edges deep enough in the water so as to minimise the effect of the free surface on shed vortices, See Figure (4.17). One thin flat plate was also used for the experiments. All models were fabricated from plexiglass components which were then assembled with screws prior to usage.
4.3 Flow Visualization Experiments

The set up for flow visualization experiments is shown schematically in Figure (4.18).

Oscillating motion was produced by a stepping motor and actuator arrangement and controlled by an IBM PC, See Figure (4.18). A detailed description of the sloshing tank and its associated control system is given in Appendix B. The period and amplitude of oscillation are defined by the user. The output control signal from the computer goes to the control module and is in turn passed to the translator which provides the appropriate power to set the sloshing tank into roll motion.

The above method of moving the water in a sloshing tank obviously does not simulate pure oscillatory flow. One major problem is that of shallow water effects on the flow, since the water depth has necessarily been shallow (500 mm or less) due to the limited height of the tank. Another related problem is that the free surface might exert an influence on the oscillatory flow and vortex formation at the edge. Also, the width of the tank is only 900 mm so that wave reflection from the sides will affect the flow.

However, since the flow visualization exercise is mainly focused on the first one or two periods of oscillation but especially on the starting flow, the above sources of error can be considered secondary. The author is not claiming that these errors can be ignored but rather proposing that they be considered when analysing the experimental results.

There are a number of ways to visualize the flow in the vicinity of the sharp edge. The more common ones can roughly be categorized into the following:

- direct injection methods,
- suspended particle methods, and
- electrolytic methods.
Figure 4.18: Schematic Set-up for Flow Visualization.
The use of dye injected into the non-circulating water in the sloshing tank is not a suitable choice because the water will require frequent changing. Moreover, the injection rate for the visualization medium is difficult to control. Suspended particles in the sloshing tank could have been a good alternative but for the fact that it does not yield well defined visualizations of the vortex sheet rollup. It is probably possible to use some form of chemical reaction method for flow visualization, for example it is known that the release of chlorine gas into a solution of hydrogen peroxide will produce luminous bubbles that can be an excellent way of obtaining photographic documentation of the flow around the sharp edge. However, the electrolytic method of introducing hydrogen bubbles into the flow was chosen for the present work. This choice was made partly because the equipment for this technique was already available, and partly because the size of bubbles produced and the rate of production can easily be controlled by electronic means.

Hydrogen bubbles can be generated by passing a current through two electrodes in an electrolyte [22]. Appendix C contains a description of the PC II Hydrogen Bubble Generator used in the present series of flow visualizations. This electro-chemical process can be controlled by varying the current supplied to the electrodes. Two chromium alloy steel wires of diameter 0.2mm were used as electrodes. One wire was attached to the test body while the other was positioned very near to the sharp edge. Alternating current was supplied to the hydrogen bubble generator module for rectification into direct current before being supplied to the electrodes, See Figure (4.18). Current supply to the electrodes can be switched on and off by a double pole double throw relay within the hydrogen bubble generator module via a control signal from a function generator. In this way, current (DC) to the electrodes and hence bubbles leaving the electrode at the edge can be pulsed at a specific rate. The velocity at which the hydrogen bubbles rise to the surface can be limited to a low value by making the size of the bubbles as
small as possible. This ensures that bubble buoyancy does not have a significant effect on the vortex location visualized. Such a system is very suitable for experimental work in small tanks without water circulation arrangements.

Although precipitation does occur (due to electrolysis), the residues form small lumps which do not seriously affect the clarity of the water even when they are floating around in the tank. The precipitates are periodically siphoned out without much effort.

Two slide projectors with 200 kW bulbs were used to focus two slits of light at the edge. The light slits on the two sides of the wedge were aimed at a central position so as to avoid any end effects on the visualization. With all other lights in the laboratory switched off, these light slits effectively provide a two dimensional illuminated 'cut' of the flow around the sharp edge. A blue food dye was used to colour the electrolyte to give better colour contrast. The overall effect was most satisfactory and the bubbles emanating from the electrode at the edge appear to glow in the water.

The flat plate and wedges of internal angles 30°, 45° and 90° were used for the flow visualization, see Figure (4.17). The draught for each test body was between 85 mm to 110 mm to minimise free surface effects on the one hand and to ensure enough depth of water below the sharp edge so as not to affect the vortex shedding on the other. The oscillation of the sloshing tank was varied with period of 8 to 12 seconds and amplitudes of 5° to 6° most of the time as these settings were found to produce the least sloshing of the water in the tank. Current supply to the hydrogen bubble generating electrode at the sharp edge was fixed at 30 volts AC at the variable transformer used as a power supply.

An Olympus 8mm video camera on a tripod was used to record the experiments.
4.4 Results of the Flow Visualization Experiments

Wedges of three internal angles and one flat plate (as described earlier) was used in the flow visualization.

Visualizations of the flow past a 30° wedge and a square wedge for one complete time cycle are presented in Figures (4.19) and (4.20). These frames have been extracted from video tape recordings of the flow visualization. Although the core appears to be a little 'fuzzy' in the visualization tape, resolution is deemed sufficient for comparison with the kinematic development of the vortex sheet in the present numerical model. See Appendix D for a description of the method used to process flow visualization records.

Some low frequency pulsing of the rolled up vortex sheet after two time cycles has been observed during an analysis of the video tape recordings of the flow visualization experiments. As the pulsing rate of the hydrogen bubbles was much higher, the sloshing effect of the water in the tank was the most likely cause of the instability of the vortex sheet images. This instability restricted the use of flow visualization results to the first two time cycles as the error due to water sloshing has not been assessed. Consequently, the assessment of the numerically simulated shear layer development is only possible for the first two cycles. In fact, the kinematic results from the numerical model will be compared to experimental visualizations at $t/T = 0.25$ in Section (4.5).
Flow visualization $t/T = 0.25$.

Flow visualization $t/T = 0.50$. 

Chapter 4. Report on Experimental Work Done
Figure 4.19: Flow Visualization over One Time Cycle, $\delta = 30^\circ$. 

Flow visualization $t/T = 0.75.$ 

Flow visualization $t/T = 1.00.$ 

Figure 4.19: Flow Visualization over One Time Cycle, $\delta = 30^\circ$. 
Flow visualization $t/T = 0.25$.

Flow visualization $t/T = 0.50$. 
Chapter 4. Report on Experimental Work Done

Flow visualization $t/T = 0.75$.

Flow visualization $t/T = 1.00$.

Figure 4.20: Flow Visualization over One Time Cycle, $\delta = 90^\circ$. 
4.5 Assessment of the Calculated Vortex Sheet

The basic characteristics of a vortex spiral issuing from a particular wedge is similar regardless of the Keulegan-Carpenter number. In other words, vortex shedding from a sharp edge with a given internal angle is self-similar. Therefore, it is only necessary to check the numerically calculated development of vortex shedding from such edges at one $K_c$.

In order to determine the acceptability of the calculated shear layer development, two criteria will be used in the present work: the size and the position of the vortex spiral. An appropriate measure of these parameters can be deduced by the definition of the distance of the core vortex and its angular position in the physical plane at a given time in the flow cycle, see Figure (4.21).
Table 4.2: Comparison of Spiral Size and Position at $t/T = 0.25$

<table>
<thead>
<tr>
<th>Wedge Angle (deg)</th>
<th>Numerical $L_e$</th>
<th>Numerical $\phi_e$ (deg)</th>
<th>Experimental $L_e$</th>
<th>Experimental $\phi_e$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.11</td>
<td>7</td>
<td>0.26</td>
<td>13</td>
</tr>
<tr>
<td>30</td>
<td>0.11</td>
<td>23</td>
<td>0.19</td>
<td>15</td>
</tr>
<tr>
<td>90</td>
<td>0.07</td>
<td>0</td>
<td>0.06</td>
<td>4</td>
</tr>
</tbody>
</table>

The measured distance of the core from the edge can be non-dimensionalized by the flow length scale factor $(V_0T)^{\frac{1}{2}}$. The angular position of the core for both the experimental and numerically simulated spiral can be measured directly. These are presented in Table (4.2). The match between experimental and calculated 'size' for $\delta = 90^\circ$ is remarkable. However, the experimental 'size' of the vortex spiral $(L_e)$ for the flat plate and the $30^\circ$ wedge are about twice that of the calculated values $(L_e)$.

On the other hand, the 'positions'$(\phi_e)$ for all cases appear to be well predicted, within limits of experimental and measurement errors.

4.6 Failure of Force Measurement Experiments

Normal forces, acting perpendicular to the edge bisector, were not measured. Attempts have been made to make such measurements with a load cell mounted on the wedge model. However, the magnitude of the forces to be measured were much smaller than the control signal noise level $^1$. The smallest load cell available to the author has a range of 0-100 lbs. So, even with a gain of 1000 (using a signal amplifier with a maximum gain of 1000), the load cell output signal is still not discernible from the noise. Also the magnitude of the interference is not constant but tends to drift. Hence, it was not possible to obtain force measurements for the wedge models with reasonable accuracy.

$^1$Appendix B gives more details on this noise problem.
Apart from the noise problem, another factor which influenced the accuracy of the force measurements is the effect of the free surface. The free surface effect cannot be reduced to a negligible level due to the limited depth of the sloshing tank. Moreover, the effect of finite depth effect on the vortex induced force is not clear.

Due to the above reasons, the author decided to use experimental force data obtained by other researchers in the assessment of the present numerical model for vortex shedding.
Chapter 5

Simulating Vortex Roll Damping of a Floating Vessel

5.1 General

A ship in motion in a seaway has six degrees of freedom: three are translational and the others rotational. The two translations in the horizontal plane are surge (which is in the direction of the ship) and sway (which is sideways) while heave is the vertical translation. Rotations about the axes of these directions of motions are roll, pitch and yaw respectively.

The theoretical prediction of ship motions in regular waves is very frequently based on strip theory, which was first proposed by Korvin-Kroukovsky and Jacobs (1957). In this method, the three dimensional problem is reduced to a series of two dimensional ones by dividing the ship into a number of transverse strips. The response to each section is calculated and these are summed over the length of the hull using a suitable numerical integration procedure, for example Simpson’s rule, to give the predicted ship response. It should be noted that surge prediction is not possible using strip theory. However, the hydrodynamic forces associated with surge is not as important as the other modes of motion. Hence the response of the ship is found by the solution of five linear coupled equations of motion given in Equation 5.41 below.

\[
\sum_{k=2}^{6} \left( -\omega^2 (M_{jk} + A_{jk}) + i \omega B_{jk} + C_{jk} \right) \dot{\eta}_k = F_j
\]

(5.41)

where A,B,C are the added mass, damping and hydrostatic restoration coefficients for
motion in the kth direction due to excitation F from the jth direction. The frequency and amplitude of response are \( \omega \) and \( \eta_k \) respectively. An improvement to the method was made by Salvesen, Tuck and Faltinsen (1970) in which account was taken of ship heading and forward speed.

The above problem is based on linear potential theory and solved in the frequency domain. This is possible only as a consequence of a number of assumptions. Regular, long crested harmonic waves are used as the source of excitation so that solutions in the frequency domain can be obtained. Linearisation of the problem carries with it the requirement that both ship motion amplitude as well as wave slope and wave amplitude are small. It also implies that the principle of superposition is valid. This means that the solution of the problem of a ship in regular waves can be considered to be the sum of the solutions of a rigid \(^1\) body in forced oscillation (in otherwise still water) and that of incoming harmonic waves on a fixed body. Apart from this, the behaviour of the vessel in regular waves may, through spectral analysis, be applied to the inference of ship motions in random seas. The fluid is assumed to be ideal and irrotational.

Potential theory is unable to include the effects of non-linear viscous damping in the prediction of ship motions. As a consequence, the response of a vessel in roll motion near resonance with only linear wave damping is usually overestimated.

In this chapter, the viscous damping due to vortex shedding from a simple vessel in free roll without excitation is calculated to simulate roll decay with time.

\(^1\)The ship hull is considered rigid in most practical applications although attempts have been made to account for the non-linear effects of a flexing hull.
5.2 The Problem of Roll Decay

5.2.1 The Equation of Motion

For a ship in pure roll, the single degree of freedom results in a simplification of Equation 5.41 as follows:

\[ I\ddot{\phi} + B\dot{\phi} + C\phi = M\phi(\omega t) \]  \hspace{1cm} (5.42)

where \( I \) is the virtual mass moment of inertia through the centre of gravity, \( B \) the damping coefficient, \( C \) the hydrostatic restoration, \( \phi \) the roll amplitude and \( M \) the roll excitation moment.

5.2.2 Simplifying Assumptions

The added mass moment of inertia in free roll is considered unimportant by assuming low rates of change of velocity. Therefore, the term \( I \) can be approximated by \( \frac{mb^2}{10} \) where \( m \) is the mass and \( b \) the beam of the section in question. The hydrostatic restoration coefficient is given by \( \Delta GM \). Hence, in the absence of an exciting moment Equation 5.42 becomes:

\[ \left(\frac{\Delta b^2}{16g}\right)\ddot{\phi} + B\dot{\phi} + \Delta GM\phi = 0 \]  \hspace{1cm} (5.43)

Assuming low frequency and small amplitude roll motion, the linear contribution due to wave radiation in the damping coefficient is ignored. The non-linear damping term is taken to be entirely due to vortex shedding. Hence if the flow around the shedding edge is taken to be the inner flow region of the flow around the hull, then
where \( n \) is the total number of vortex shedding edges considered and \( r \) is the moment arm from the vertical centre of gravity of the hull to the shedding edge and \( F_v \) is the in-line force due to vortex shedding acting against the direction of motion.

In order to fix the position of the centre of flotation at the centreline of the section, and hence maintain a constant value of \( R \) for each edge, the underwater part of the section must be symmetrical. This requires that the amplitude of roll be small. It should also be noted that the linear summation of the vortex damping force is only possible provided that vortex shedding from each edge is independent of that at all the other edges. Also the model does not account for the interference of the free surface. As a result of the above assumptions, Equation 5.43 can be rewritten thus:

\[
S_1 \ddot{\phi} + S_2 \phi = \sum_{i=1}^{n} F_{v,i} \cdot r_i
\]

(5.44)

where

\[
S_1 = \frac{\Delta b^2}{16g}
\]

\[
S_2 = \Delta GM.
\]

Since \( F_v \) is dependent on the roll velocity and thereby on roll angle, Equation 5.44 has to be solved numerically.

5.2.3 Finding the Roll Extinction Coefficient

The non-linear damping coefficient \( B_\phi \) can be estimated using free roll tests, see [15]. The vessel is heeled to a given angle and released. The motion of the vessel is then recorded as a function of time. Such a record is shown in Figure (5.22), taken from
experimental data of roll decay tests by Rohling (1986) [25], on the single chine seiner of this department. If $\phi_n$ is the roll angle at the $n_{th}$ peak, then the roll extinction, $\Delta \phi$ (in degrees) is defined as a third degree polynomial as follows:

$$\Delta \phi = a\phi_m + b\phi_m^2 + c\phi_m^3 \quad (5.45)$$

where

$$\Delta \phi = |\phi_{n-1}| - |\phi_n|$$
$$\phi_m = \frac{1}{2}(|\phi_{n-1}| + |\phi_n|) \quad (5.46)$$

In Equation 5.45, a, b and c are known as the extinction coefficients. These are related to an equivalent linear damping factor $B_\phi$ where $B_\phi = B_\phi \phi$ following Himeno [15]. Therefore, once the extinction characteristics are known, the viscous damping coefficient can be found.

The roll extinction characteristics of a UBC single chine seiner can be predicted
using an adaptation of the numerical model described in Chapter Two. However, since vortex shedding from rounded edges were not considered in the present work, gross simplifications of the actual hull are necessary.

5.2.4 Simplified Geometry of Single Chine Seiner

A body plan of a model of the single chine seiner is shown in Figure (5.23). The relevant particulars of this model are given Table (5.3).

A simplified form of the single chine vessel is derived, see Figure (5.24), having a total of three sharp edges of equal internal angles of 120°. The beam and draught are maintained equal to that of the actual model and the metacentric height $GM$ is taken to be 50mm $^2$. However, in order to maintain the same vessel displacement, the length

\footnote{One of the $GM$ values used in the experiments by Rohling [25].}
Chapter 5. Simulating Vortex Roll Damping of a Floating Vessel

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length Overall (LOA)</td>
<td>1.805 m</td>
</tr>
<tr>
<td>Length bet Perpendiculars (LBP)</td>
<td>1.640 m</td>
</tr>
<tr>
<td>Beam (b)</td>
<td>0.539 m</td>
</tr>
<tr>
<td>Depth (D)</td>
<td>0.352 m</td>
</tr>
<tr>
<td>Draught (d)</td>
<td>0.246 m</td>
</tr>
<tr>
<td>Displacement (Δ)</td>
<td>115.2 kg</td>
</tr>
<tr>
<td>Block Coefficient C)b</td>
<td>0.531</td>
</tr>
<tr>
<td>Midship Section Coefficient (Cm)</td>
<td>0.775</td>
</tr>
<tr>
<td>KM</td>
<td>0.301 m</td>
</tr>
<tr>
<td>GM</td>
<td>0.050 m</td>
</tr>
</tbody>
</table>

Table 5.3: Some Details of the Single Chine Seiner

of the simplified model, \( L_{\text{equiv}} \) is shorter than that of the actual model \( L_{\text{model}} \).

The natural period of roll was given as 1.74 seconds from experiments carried out by Rohling [25]. With the roll frequency \( \omega \) given by \( \omega = \frac{2\pi}{T} \) where \( T \) is the natural period of roll, the moment of inertia in roll \( (I_\phi) \) can be found using:

\[
\omega = \sqrt{\frac{\Delta GM}{I_\phi}}
\]  

(5.47)

5.3 Adaptation of the Discrete Vortex Method

Since all edges are assumed to have the same internal angle, the full discrete vortex calculation need to be carried out for one wedge angle only. The instantaneous vortex induced force found from these calculations can then be multiplied by three to give the total damping force due to vortex shedding. The formulation of the problem is essentially similar to that given in Chapter Two except for a few modifications.

Since the roll period is not known, equations of motion for each vortex are not normalised. This does not affect the results in any way because the vessel dimensions
are used directly without non-dimensionalising. The roll angle is given in radians but converted to degrees when the results are presented.

The free stream velocity, \( V \), is now a function of the roll velocity such that \( V = -\dot{\phi}R \). The velocity in the x-direction, that is \( U \) in Chapter Two, is taken to be \( U = (1 + \delta)Ve^{-0.5t^2} \), where \( t \) is the elapsed time (in seconds). Each cluster produced per time cycle is progressively merged into a single core according to the amalgamation procedure described in Chapter Two. The vortex decay rate remains unchanged and Lamb vortices are also used.

At the beginning of each time step, the roll acceleration, \( \ddot{\phi} \), is numerically integrated to find the roll velocity, \( \dot{\phi} \), which is in turn integrated to find the roll angle \( \phi \). This can be done using the Runge-Kutta-Nystrom method [18], for second order differential equations as shown below for time step \( \Delta t \).

\[
A_n = \frac{\Delta t}{2} f(\tau_n, \phi_n, \dot{\phi}_n)
\]
\[
B_n = \frac{\Delta t}{2} f(\tau_n + \frac{\Delta t}{2}, \phi_n + \beta_n, \dot{\phi}_n + A_n)
\]
Chapter 5. Simulating Vortex Roll Damping of a Floating Vessel

\[
C_n = \frac{\Delta \tau}{2} f(\tau_n + \frac{\Delta \tau}{2}, \phi_n + \beta_n, \dot{\phi}_n + B_n)
\]

\[
D_n = \frac{\Delta \tau}{2} f(\tau_n + \Delta \tau, \phi_n + \delta_n, \dot{\phi}_n + 2C_n)
\]

Where

\[
\beta_n = \frac{\Delta \tau}{2} (\phi_n + \frac{1}{2} A_n)
\]

\[
\delta_n = \Delta \tau (\phi_n + C_n)
\]

The values of \( \phi_{n+1} \) and \( \dot{\phi}_{n+1} \) for the next time step is given by:

\[
\phi_{n+1} = \phi_n + \Delta \tau (\dot{\phi}_n + K_n)
\]

\[
\dot{\phi}_{n+1} = \dot{\phi}_n + K_n^*
\]

where

\[
K_n = \frac{1}{3} (A_n + B_n + C_n)
\]

\[
K_n^* = \frac{1}{3} (A_n + 2B_n + 2C_n + D_n).
\]

The above integration procedure require some initial conditions to start off with. If the model is heeled to an angle \( \alpha_0 \) and released, then the initial conditions from Equation 5.44 are:

\[
\bar{\phi}_0 = -\frac{S_2 \alpha_0}{S_1}
\]

\[
\dot{\phi}_0 = 0
\]

\[
\phi_0 = \alpha_0
\]

\[
F_{v,0} = 0
\]

(5.48)
5.4 Assessment of the Simulation Results

Programs in Pascal has been written for the roll decay simulation. For a test case, the simplified model described in Section (5.2.4) was used. The moment arms between the centre of flotation and each edge were assumed to be equal and an average value of 0.284 m was used. The equivalent length $L_{equiv}$ was 1.268 m so that the vessel displacement was the same as that of the single chine seiner model. The vessel displacement was taken to be 115.2 kg. There was a total of three vortex shedding edges with internal angle 120°. In the simulation, $N_v = 40$, that is 40 vortices were introduced per second. The model was heeled to 25° and released at the beginning of the simulation. The resulting roll angle variation over time is plotted in Figure (5.25).

The form of this curve is typical of a damped second order oscillation and is similar to the experimentally obtained curve given in Figure (5.22). Roll extinction, $\Delta \phi$, can then be obtained using Equation 5.46 and plotted against the mean roll amplitude $\phi_m$,
Roll extinction is underestimated at low mean roll angles. There are a number of possible reasons for this discrepancy, the major ones being the neglect of linear wave damping and hull added mass. It should also be noted that the discrete vortex formulation used tends to underestimate the vortex induced forces for edge angles of 90° or more.

The maximum velocity near each edge is less than 0.5 m/s so that the maximum
Keulegan-Carpenter number in the above case is around 2.4 (for the first two cycles) and hence well below the limit of $K_e = 5$ suggested in Section (3.8).

5.5 Time Domain Prediction of Roll Motion in Waves

The roll damping coefficient, that is the non-linear component of the damping coefficient, can be calculated through a simulation in the time domain.

For a time simulation of a 2-D ship section rolling in waves, the governing potential equation for the flow field around the hull is the two dimensional Laplace equation. Numerical methods, such as the Boundary Integral Method, can be used to solve for the velocities and pressure distribution around the section. The virtual mass moment of inertia and the linear wave damping coefficient can then be found. At the same time, a subroutine simulates vortex shedding from those edges considered as likely to provide significant viscous damping. The total viscous damping moment for each section can, subject to the assumptions given in Section (5.2.2), be taken as the linear sum of the calculated viscous damping moment for all edges. The free stream velocity in this case is a function of the roll velocity, see Section (5.3).

The added mass, damping and hydrostatic restoration coefficients are obtained by numerically integrating the pressure. Then, with the excitation moment $M(\omega t)$ known, Equation 5.42 for pure roll can be solved as a second order differential equation for the unknown $\phi$.

The time marching process repeats itself thereafter.
Chapter 6

Discussion and Conclusions

6.1 Summary of the Work Done

A brief introduction to the use of discrete vortex methods for the modelling of vortex shedding from sharp edges has been given in Chapter One. The discrete vortex method was then used in a numerical model for vortex shedding from an infinite wedge in normal oscillatory flow in Chapter Two. The results obtained using this model were presented in Chapter Three. These results were compared to those obtain by other researchers.

Flow visualization experiments were described and the results presented in Chapter Four. The kinematics of vortex shedding from a sharp wedge as predicted by the numerical model were then assessed using flow visualization experiments.

The numerical model was then applied to the prediction of roll decay in a simple geometry with three vortex shedding edges in Chapter Five.

6.2 Discussion

The simulation of vortex shedding using the discrete vortex method does not constitute an exact solution of the unsteady two dimensional Navier-Stokes equations since viscosity is ignored in the vorticity transport equation. A numerical technique has to be utilised to emulate vortex diffusion, both near the separation point as well as in the wake. The vortex decay rates used are somewhat arbitrary and not firmly based on
theory. This is due to the fact that the complex nature of the mechanism of viscous diffusion is not fully understood at the present time. However, the various schemes (and they should be regarded as such) used to remove vorticity from the flow field have been more or less successful in coping with this difficult problem in that the results obtained from discrete vortex models have been approximately consistent with experimentally obtained values. Nevertheless, the removal of vorticity from the field is not consistent with the initial assumption of inviscid fluid flow. These two apparently contradictory assumptions need to be reconciled in some way.

The discrete vortex method is unable to predict the position of the separation point, which means that as the edge angle increases, the method becomes less reliable. From the results of the present model, it appears that this problem begins to surface well before the edge angle reaches 180°. For large edge angles, the separation point would either have to be fixed empirically in advance or, alternatively, boundary layer calculations would be required to determine the position of the separation point before the introduction of each nascent vortex.


6.3 Conclusions

The numerical model predicts vortex induced forces on sharp wedges with acceptable accuracy. However, vortex forces are overestimated by 11% for the flat plate and underestimated by 30% for the square edge when compared to the experimental data of other researchers. It appears that the amount of underestimation of vortex induced forces progressively increases with the opening out of the wedge angle. Caution should therefore be exercised when applying the present model to wedge angles above 120°. The numerical results obtained for the flat plate case are superior to the numerical results of Graham [13] and Cozens [6] when compared with experimentally determined forces.

Although drag coefficients are predicted reasonably well by the present model, the same does not apply to added mass coefficients. However, the relatively small magnitude of the added mass coefficients render them unimportant in the problem considered.

The numerical model is also able to represent the kinematics of vortex shedding from sharp wedges in normal oscillatory flow well.

The sloshing tank can be used to provide oscillatory flow past an immersed body for a short duration only. It is not ideal for the measurement of steady state force characteristics due to the numerous sources of error detailed in Chapter Four. Hydrogen bubbles provide an excellent way of visualizing flow in non-circulating media such as in a sloshing tank.

Lamb vortices have been successfully used to prolong the computational stability of the present discrete vortex model. The simulation can be run for at least 46 time cycles (for the flat plate case) without any hint of instability. Unfortunately, to the author's knowledge, none of the other researchers actually report data concerning the stability of their methods. Hence it is not possible to say whether the present work
Amalgamation of the innermost portion of the spiral into a central core proved to be a useful technique in delaying the randomization of the vortex paths and reducing the computation time.

Subject to the conditions given in Chapter Five, the application of the present model to the prediction of viscous roll damping appears feasible. Although the model has been used for the prediction of free roll decay in the present work, it could possibly be extended for use in the calculation of ship's roll in waves as explained at the end of Chapter Five.
6.4 Suggestions for Future Work

1. The theoretical formulation for the numerical model assumed that the normal oscillatory velocity $V$ dominates the free stream. The symmetric velocity component $U$ was introduced only as a numerical tool to ensure that vortices were convected well away from the edge during the initial stages of the simulation. $U$ was decayed to a negligible value after three time cycles. However, the author has done boundary integral computations for a 90° wedge in oscillatory flow and found that the velocity along the sides is of roughly the same magnitude as $V$. Thus, there is reason to believe that the symmetric velocity component $U$ need not decay with time. Further investigation into this problem would prove beneficial to the improvement of the present model.

2. Adaptation of the present model for bodies with more than one sharp edge would be useful considering the fact that ship's bilges are often rounded and sometimes have a plate keel attached. The rounded keel could be approximated by a number of straight sections while the bilge keel can be represented directly by the numerical transformation given in Chapter Two.

3. A better vortex decay mechanism would serve to consolidate the theoretical base of the present model.

4. The method can be used in association with a boundary integral solution (to the two dimensional Laplace equation) to give a more accurate prediction of ship roll in a seaway.

5. Only normal oscillatory flow has been considered in this work. The method can, in principle, be extended to the modelling of sharp edges in unsteady flow at
given angles of attack.

6. Apart from the prediction of roll, another envisaged application of the discrete vortex method would be in ship manoeuvring where cross flow past sharp rudder edges might be modelled.

7. The problem of pitching hydrofoils/airfoils can probably be treated using the discrete vortex method.
Bibliography


Appendix A: Initial Position of the Nascent Vortex

The full derivation of the position at which the nascent vortex is introduced, given by Equations (2.23) and (2.24), is given here.

Let $z_0 = K(\Delta r)^m e^{i\phi}$.

From Equation (2.13), the complex conjugate velocity is given by:

$$w_0 = \frac{is_0}{\lambda z_0} [\sin(2\pi \tau) + \sum_{k=1}^{n} \frac{\gamma_k}{2\pi} \left( \frac{1}{s_0 - s_k - \frac{1}{\lambda}} - \frac{1}{s_0 + s_k} \right) - \frac{\gamma_0}{2\pi} \left( \frac{1}{s_0 - s_0} - \frac{1}{2s_0} \right)]$$

Now assuming that $s_k \gg s_0$:

$$\sum_{k=1}^{n} \frac{\gamma_k}{2\pi} \left( \frac{1}{s_0 - s_k - \frac{1}{\lambda}} - \frac{1}{s_0 + s_k} \right) = -\sum_{k=1}^{n} \frac{\gamma_k Re(s_k)}{\pi |s_k|^2}$$

From Equation (2.16) the strength of the nascent vortex is given by:

$$\gamma_0 = v_n \frac{|s_0|^2}{Re(s_0)}$$

where

$$v_n = \pi \sin(2\pi \tau) - \sum_{k=1}^{n} \frac{Re(s_k)}{|s_k|^2}$$

Therefore substituting into the complex conjugate velocity expression:
Appendix A: Initial Position of the Nascent Vortex

\[ w_0 = \frac{\imath v_n z_0^{(\frac{1}{\lambda} - 1)}}{\lambda \pi} \left[ 1 - \frac{|s_0|^2}{2 \text{Re}(s_0)} \left( \frac{1}{2 \text{Re}(s_0)} - \frac{(1 - \lambda)e^{-\imath \theta/\lambda}}{2s_0} \right) \right] \]

\[ = \frac{\imath v_n}{\lambda \pi} K^{(\frac{1}{\lambda} - 1)}(\Delta \tau)^{m(\frac{1}{\lambda} - 1)}e^{\imath \theta(\frac{1}{\lambda} - 1)} \left[ 1 - \frac{1}{4 \cos^2 \frac{\theta}{\lambda}} + \frac{(1 - \lambda)e^{-\imath \theta/\lambda}}{4 \cos \frac{\theta}{\lambda}} \right] \]

Equation (2.18) gives \( w_0 = z_0 \). Hence with \( z_0 = mK(\Delta \tau)^{m-1}e^{\imath \theta} \) Equation (2.18) becomes:

\[ mK(\Delta \tau)^{m-1}e^{\imath \theta} = \frac{-\imath v_n}{\lambda \pi} K^{(\frac{1}{\lambda} - 1)}(\Delta \tau)^{m(\frac{1}{\lambda} - 1)}e^{-\imath \theta(\frac{1}{\lambda} - 1)} \left[ 1 - \frac{1}{4 \cos^2 \frac{\theta}{\lambda}} + \frac{(1 - \lambda)e^{\imath \theta/\lambda}}{4 \cos \frac{\theta}{\lambda}} \right] \]

\[ mK^{(2-\frac{1}{\lambda})}(\Delta \tau)^{(m-1)} = \frac{-\imath v_n}{\lambda \pi} (\Delta \tau)^{m(\frac{1}{\lambda} - 1)}e^{-\imath \theta/\lambda} \left[ 1 - \frac{1}{4 \cos^2 \frac{\theta}{\lambda}} + \frac{(1 - \lambda)e^{\imath \theta/\lambda}}{4 \cos \frac{\theta}{\lambda}} \right] \]

Comparing terms in \( \Delta \tau \) an expression can be found for the variable \( m \):

\[ (m - 1) = m \left( \frac{1}{\lambda} - 1 \right) \]

\[ m = \frac{\lambda}{2\lambda - 1} \]

and the above equation becomes:

\[ \frac{\lambda}{2\lambda - 1} K^{2(\frac{1}{\lambda} - 1)} = \frac{-v_n}{\lambda \pi} \left[ \sin \frac{\theta}{\lambda} \left( 1 - \frac{1}{4 \cos^2 \frac{\theta}{\lambda}} \right) + \imath \left( \frac{1 - \lambda}{4 \cos \frac{\theta}{\lambda}} + \cos \theta \left( 1 - \frac{1}{4 \cos^2 \frac{\theta}{\lambda}} \right) \right) \right] \]

\[ = \frac{-v_n}{\lambda \pi} \left[ \sin \frac{\theta}{\lambda} \left( 1 - \frac{1}{4 \cos^2 \frac{\theta}{\lambda}} \right) + \imath \left( \cos \frac{\theta}{\lambda} - \frac{\lambda}{4 \cos \frac{\theta}{\lambda}} \right) \right] \]

Equating the imaginary part:
Appendix A: Initial Position of the Nascent Vortex

\[ 4 \cos^2 \frac{\theta}{\lambda} \lambda - \lambda = 0 \]

\[ \theta = -\lambda \cos^{-1} \frac{\sqrt{\lambda}}{2} \]

This also gives the expression \( \sin \frac{\theta}{\lambda} = -\sqrt{1 - \frac{\lambda}{4}} \).

Finally, equating the real part yields an expression for \( K \) as follows:

\[ K^{2\lambda - 1} = \frac{v_n(2\lambda - 1)\sqrt{1 - \frac{1}{\lambda}(1 - \frac{1}{\lambda})}}{\pi \lambda^2} \]

\[ K = \left\{ \frac{v_n(2\lambda - 1)(\lambda - 1)\sqrt{4 - \lambda}}{2\pi \lambda^3} \right\}^{\lambda - 1} \]

The position of the nascent vortex at \( \Delta \tau \), a small time after it is supposed to have been shed at the edge is given by:

\[ z_0 = K(\Delta \tau)^m e^{i\theta} \]

where

\[ K = \left\{ \frac{v_n(2\lambda - 1)(\lambda - 1)\sqrt{4 - \lambda}}{2\pi \lambda^3} \right\}^{\lambda - 1} \]

\[ m = \frac{\lambda}{2\lambda - 1} \]
Appendix B: The Sloshing Tank and its Control System

A schematic diagram of the sloshing tank and its associated control system is given in Figure (4.15) in Chapter Four. The sloshing tank is supported by bearings at the centre of rotation on an aluminium structure. Reversing flow past the test body can be simulated by oscillating the sloshing tank. A data acquisition board (Data Translation DT2801), with 16 analog to digital and 2 digital to analog outputs, is mounted on an IBM PC and connected to the control module via a 50-pin ribbon. The control module has a priming circuit which sends TTL signals to the translator. High voltage output signals from the translator then passes to the stepping motor, causing the actuator, and hence the sloshing tank, to move vertically upwards/downwards as desired. The motion of the sloshing tank is thus controlled by input signals from the microcomputer.

A control program (in BASICA), using PCLAB callable subroutines, reads an ASCII type data file (S1875C2.DAT) defining a sinusoidal velocity profile. Although only one data file is required in the present application, it should be noted that different velocity profiles can be used for other applications. The data read from file S1875C2.DAT is scaled according to the maximum amplitude of roll and period of oscillation to produce the specified roll motion of the sloshing tank. The amplitude and period of roll is interactively selected by the user. This is done by choosing option (1) in the control program, see Figure (B.1) below.

The experimental work described in this thesis was the first application of the sloshing tank constructed in 1988. As such, a number of teething problems have been revealed. Noisy signals in the control circuitry occasionally cause erratic motion of the sloshing tank. This problem seems to disappear when the stepping motor was operated
Figure B.1: Sloshing Tank Control Program Menu.

Figure B.2: Error in Actual and Desired Roll Period.
Appendix B: The Sloshing Tank and its Control System

at reduced current. The most likely offending source of the high noise levels is the translator. This, in fact, prevented the author from conducting force measurements since the noise levels are higher than the signal level produced by the load cells used for force measurements.

Although the amplitude of motion of the sloshing tank appears to be as specified, the same cannot be said for the period of oscillation. There is a slight discrepancy between the period specified (desired) and that measured for the sloshing tank roll. The error is shown, for the range of periods used by the author, in Figure (B.2). This error is probably due to non-linearities in the control. Some form of feedback from the actual moving parts might prove useful in correcting the situation.

Full details concerning the hardware and their specifications can be found Reference (1) of this Appendix.

Reference:

Appendix C: The Hydrogen Bubble Generator

The hydrogen bubble generator is based on the principle of electrolysis. Current passing through two immersed electrodes causes electrolysis of water into hydrogen and oxygen bubbles at the cathode and the anode respectively. In the process, hydrogen bubbles are produced. A schematic diagram of the hydrogen bubble generating system used for flow visualization experiments in the present work is given in Figure (4.15).

Alternating current supplied from a variable transformer is converted to constant Direct current by a rectifier bridge and capacitor arrangement in the Hydrogen Bubble Generator PC II unit. The DC, when sent directly to the electrodes will cause electrolysis in the electrolyte, which in this case is water doped with 1% sodium chloride. Hydrogen bubbles and oxygen bubbles discharge at the cathode and anode respectively. Reversing the current will change the polarity of the electrodes. Thus, hydrogen bubbles can be produced at either electrode.

The size of these bubbles can be controlled by varying the current supply at the variac. The rate of bubble production can be controlled by on/off signals to the hydrogen bubble generator unit. The control signal passes through a MOSFET, which switches the supply current on and off at regular intervals. Thus a pulsing of the bubbles can be produced at the electrodes and the period of these pulses can be changed by variation of the control (on/off) signal.

The system was designed for computer control using a data acquisition card (Data Translation DT2801) and an IBM PC through a BASICA program named Bubbles. The control signal, in the form of a rectangular wave, can be specified by the user for the duration of ‘on’ and ‘off’ signal level. In other words, the duration of a bubble pulse...
BUBBLES

Enter duration of pulse (in sec, min=.015 sec) : 0.25

Enter time interval between pulses (in sec, min=.025 sec) : 0.25

This program uses digital port #0 for digital output. Bit #0 of port #0 controls the voltage pulse while bit #1 automatically switches the polarity of the electrodes. Please connect the wires to the screw on terminal now.

Please press <RETURN> to begin program execution.

Figure C.1: Hydrogen Bubble Generator Control Program.
and the time interval between the pulse is to be given by the user, see Figure (C.1).

However, since only one data acquisition card was available, the author used a square wave generator as a source of the control signal.

Chromium alloy bare steel wires of diameter 0.2 mm were used as electrodes throughout the whole series of experiments. Although platinum wires of 0.1 mm gauge or less would be ideal as electrodes, they were not necessary in this case since the bubbles produced using the steel wires mentioned above gave bubbles that are sufficiently small for the flow visualization.

The positioning of the electrodes on the model is crucial for the proper visualization of flow around the sharp edge of the wedge. There should be enough exposure of the bare electrodes to the electrolyte to ensure that enough bubbles are produced. One electrode should be laid out slightly away from the edge to avoid concentrated bursts of bubbles rising along the sides of the wedge to the surface which tend to give intermittent instead of a more or less continuous visualization.

Reference:

Appendix D: Processing of Flow Visualization Recordings

The PC Vision Plus frame grabber by Imaging Technology was used for the conversion of video images into a bitmap. The intensity level of each pixel in the 512 by 512 frame ranges from 0 (black) to 255 (white). A photograph of such a frame is given in Figure (D.1) below.

![Frame Grabber Image](image.jpg)

Figure D.1: Photograph of Frame Grabber Image.

A maximum of 30 frames can be acquired per second. However, this was not necessary since the video tape has a time record (definition to 0.1 seconds) function. A frame at any particular instant in the cycle can be frozen on an analog monitor with the frame grabber on 'continuous grab'. The image can then be saved onto 360 Kb diskettes as a structured 8-bit binary file \(^1\). This file can then be transferred to the

\(^1\)Each image takes up about 256 Kb of memory.
Appendix D:  

Processing of Flow Visualization Recordings

VAX 11/750 using the telephone line transfer software KERMIT.

A graphics package (PLOTDATA), supplied to the author through the kind generosity of TRIUMF Research of Vancouver, Canada, can be used to produce density plots of the transferred video images. Unfortunately, the bitmap of the image created by PLOTDATA requires an hardcopy output device (such as the HP Laserjet 150) which was not available to the author. Nevertheless an image was plotted on the Digital LN03 printer dedicated to the departmental VAX system but the hardcopy thus produced was not clear enough to be of much use. However, it should be mentioned that hardcopies of captured images can, in principle, be produced using the PLOTDATA graphics software.

An alternative approach was used in this work. The required images from flow visualization experiments were frozen on the screen using the frame grabber as described earlier. A single lens reflex (SLR) 35 mm camera was then used to obtain black and white photographs of the images. Some of these photographs were presented in Chapter Four, Section (4.4).

The aperture and exposure speed settings on the camera is important for this application. These were set with the camera aimed at a 'white' screen. The required image is then retrieved and the room lights switched off. The film can then be exposed using a flexible extension cord connected to the shutter control. The camera was mounted on a firm tripod in all cases.

The shutter speed should be lower than $\frac{1}{30}$ second since the image is refreshed at a rate of 30 hertz. Otherwise, the camera may capture dark horizontal streaks (not visible to the naked eye when viewing the monitor). An illustration of a photograph taken with shutter speed at $\frac{1}{60}$ second is shown in Figure (D.1).
Appendix E: Description of the Development of a Discrete Vortex

The following contains a detail description of the development of a single discrete vortex from the moment it is released into the flow field to its amalgamation into a core.

The nascent vortex is introduced into the flow with position and strength as given in Section (2.3.3) at the end of the time step ($\Delta \tau$). All the other vortices already shed are moved according to Equation (2.16). The nascent vortex is then convected a further $N_m - 1$ time steps with a velocity given by Equation (2.16). During this time, all the other vortices are also convected accordingly.

Another vortex is now ready to be introduced while the nascent vortex becomes an 'old' vortex and will be referred to as DV1 from now on. The vortex DV1 will henceforth be amongst the 'other' vortices in the flow. The process will then repeat itself.

Vortex DV1 begins to decay in strength using Equation (2.34). Its convection velocity is a function of the positions and strengths of all other vortices in the flow, the free stream velocity and the parameter $\lambda$ as given in Equation (2.16). However, DV1 is regarded as a Lamb vortex. This means that its strength is limited by Equation (2.33) when calculating its convection velocity.

As more and more vortices are introduced, the vortex DV1 will be convected in the way described above; its strength, position and velocity calculated at every time step. DV1 makes a contribution to the total vortex induced force on the body through Equation (2.37) and will eventually be amalgamated into the core of the cluster to which it belongs. The cluster will eventually be merged into one core which moves away from the sharp edge.