VISUALIZATION IN SYLLOGISTIC REASONING

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Abstract

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The purpose of this study was to compare the effects of three different methods for teaching syllogistic reasoning. Two of the instructional methods contained visual constructs and the other consisted only of non-visual (verbal) explanations. The literature suggested that visual aids enhance performance in problem solving tasks. Of primary concern, therefore, were the effects of the visual methods compared to the non-visual approach. All methods were developed for the micro-computer.

The visual approaches were modelled after two theories of syllogistic reasoning behaviour. Erickson (1974) provided the basis for the Venn diagram method and Johnson-Laird & Steedman (1978) developed the necessary details for the Sample Representation method.

The findings of this study were:

(1) The Venn diagram method resulted in significantly greater syllogistic reasoning than both the Sample Representation and the Non-visual methods.

(2) Surprisingly, the Non-visual method resulted in significantly greater syllogistic reasoning than the Sample Representation method.

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CHAPTER 1: STATEMENT OF THE PROBLEM

The subject under investigation for the current study is deductive reasoning. Reasoning is an important part of the thinking process. People use it often in everyday situations. Students use their reasoning capabilities in mathematics and other sciences. Reasoning is used by athletes in amateur or professional sport, by labourers in the workplace, and even by children at play. The ability to make a logical decision knowing what exists is common indeed, and essential.

It is important to acknowledge the two forms of logic most often encountered when investigating reasoning tasks. The first form of logical reasoning utilizes methods within a formal system to determine whether or not an argument is logically correct. Individuals trained in this form of logic are mathematicians, philosophers, and other 'abstract' scientists. The second form of logic is the type that is natural to every human. It is utilized by the layman in everyday experiences and consists of inferences governed by weakly defined rules and human biases.

Methods to determine solutions to reasoning tasks are very different within each of these logical domains. For example, suppose one were presented with the statement: "John came into the room". Most would agree that the probability that John opened the door and entered the room is 100%. However, within the realm of a formal logical system, this is not the case. There is an indeterminate probability that there was no door, or that John knocked the door down, or that he sawed a hole in the door and crawled through it, or that he had used some other means to enter the room. The point is that most people would not consider these as possibilities when presented with this statement, even though each is logically possible.

The question arises: Are humans who are not trained in formal logic logical? The popular answer to this question is "no". This question has been posed by several researchers, including Anderson (1980), Revlis (1975), and Erickson (1974), who stated: ". . . there exist a number of very simple situations in which

the typical subject does not behave the way a logician would say he ought to behave" (p. 305). Anderson (1980) stated that humans violate what he coins the "soundness criterion" because they "will accept an illogical conclusion . . . and firmly believe it to be true" (p. 300). Revlis (1975) claimed that "for some researchers, . . . findings suggest that untrained reasoners are not strictly logical in their inferences. . ." (p. 94), and Erickson (1974) stated that "the average college student doesn't seem to be very 'logical'" (p.305). He stated further that "they are quite willing to accept a variety of syllogistic arguments as valid when they are not" (p. 307). Henle (1962) maintained that human reasoning does not usually pattern itself upon structured logical form and that most people succumb to logical contradiction.

The fact that the "Aristotelian" syllogism generates a great deal of difficulty for most is, then, not surprising. Erickson (1974) affirmed that "when college-student subjects are presented with syllogistic arguments in the laboratory, they don't seem to do particularly well" (p. 307). For many years, researchers have investigated this field to determine the causes of erroneous inferencing, and then used this information in order to develop models of correct logical reasoning.

The subject of the majority of studies conducted in the field of reasoning is inferencing errors and models of accurate reasoning processes. It would be interesting to analyze other aspects of logical reasoning, especially those that pertain more directly to educational concerns, for instance, learning and instruction. The following questions should be addressed: How do reasoners learn to act more logically? Is there more than one way in which reasoners solve logic problems, and if so, which ones are most effective? How, then, should the process of reasoning be taught? It is difficult to answer these questions, but with more research devoted to reasoning within an educational context, a clearer picture should emerge.

Several psychologists such as Chase & Clark (1972), Peirce (1931, 1932), and Ceraso & Provitera (1971) have claimed that imagery and visualization lie behind the reasoning process. It is the purpose of the present investigation to

transfer a fundamentally psychological issue to the field of education by using visual constructs to teach correct syllogistic reasoning. Specifically, two visual approaches will be submitted as justifiable aids to the solution process and the results will be compared.

The research questions for the present investigation are:

1. Will there be a significant difference in syllogistic reasoning achievement between the two visual instructional methods (outlined in chapter 2)?
2. Will the two visual methods result in significantly greater syllogistic reasoning achievement than a non-visual method?

CHAPTER 2: REVIEW OF THE LITERATURE

The general aim of the literature review is to present a broad survey of the previous work that has been done in the areas of reasoning, imagery and problem solving. In the review process specific domains that have not been investigated as thoroughly as others were identified which will open up new areas for research in reasoning.

The first review section will discuss the various sources of error found to be common during reasoning processes. Numerous researchers such as Ceraso & Provitera (1971), Chapman & Chapman (1959), and Woodworth & Sells (1935) have presented explanations for erroneous inferencing, and others have supported the results.

Sources of syllogistic reasoning error can be categorized into seven major areas:

1. Misinterpretation of the Premises

It has been established that some reasoners do not proceed with faulty reasoning, but are misguided due to some obfuscation arising from premise interpretation. An example follows:

Some As are not Bs,
All Bs are Cs,
Therefore, Some As are Cs.

A misinterpretation of the first premise may lead a reasoner to accept the above invalid conclusion as valid. It is common to interpret Some As are not Bs as Some As are Bs, but it is also correct to interpret this premise as No As are Bs. This interpretation leads to the conclusion that there is no correct conclusion, i.e., one cannot tell if any As are Cs or not.

2. Probabilistic Inference

This source of error entails reaching a decision based on the *probability* of its occurrence, not on the accuracy of the reasoning involved. Kahneman & Tversky (1972, 1973) are the leading researchers in this field. They described the *judgement heuristics* that people use in order to come up with reasonable conclusions (but not necessarily correct ones). An example is:

Joe almost always drives to work.
Today, Joe arrived at work on time,
Therefore, Joe's car is in good working order.

There is a great probability that Joe's car *is* in good working condition, but the above premises do not necessarily guarantee that fact.

3. Personal Knowledge

Often, the reasoner's personal knowledge will enter into an inferencing process and alter a logically correct conclusion. This usually occurs without the person realizing it. The syllogism presented here illustrates this error:

Most Doberman Pinschers eat twice a day.
My pet is a Doberman Pinscher,
Therefore, My pet is dangerous.

The fact that this type of dog is generally perceived as dangerous is rarely disputed; however, this conclusion does not logically follow from the presented premises.

4. Personal Bias

The most common sort of error, however, is represented by the fourth area of errors. Several investigators have studied the phenomenon of basing a conclusion on individual convictions (Cantril, 1938; Feather, 1964; Henle & Michael, 1956; Janis & Frick, 1943; Kaufman & Goldstein, 1967; Lefford, 1946; Morgan & Morton, 1944; Thouless, 1959). Morgan & Morton (1944) have stated

that:

> A person is likely to accept a conclusion which expresses his convictions with little regard for the correctness or incorrectness of the inferences involved. Our evidence . . . indicate[s] that the only circumstance under which we can be relatively sure that the inferences of a person will be logical is when they lead to a conclusion which he has already accepted. (p. 39)

The following example shows how emotions and beliefs can lead a reasoner to a faulty conclusion:

All hippies are poets,
<u>Some poets are communists,</u>
Therefore, All hippies are communists.

5. <u>Atmosphere Effect</u>

This was developed by Woodworth & Sells in 1935 and is possibly the most famous of all sources of error in syllogistic reasoning. It entails the generation of responses due to a general psychological "aura" created by certain combinations of premises. The syllogism presented here shows a tendency to accept a negative conclusion to negative premises, even though it is invalid:

No As are Bs,
<u>No Bs are Cs,</u>
Therefore, No As are Cs.

6. <u>Language Ambiguity</u>

Researchers such as Revlis (1975), Frase (1966), and Chapman & Chapman (1959) have suggested that reasoners often have difficulty with the meanings of certain quantifiers, and as a result frequently have a more limited number of interpretations to deal with. A reasoner may have trouble accepting the following

logically correct argument because of a common misunderstanding of the meaning of the word "some":

All As are Bs,
<u>All Bs are Cs,</u>
Therefore, Some As are Cs.

7. <u>Henle's Error Theory</u>

Henle (1962) conceived of a theory explaining how all errors fall into one or more of the categories that she lists. Some of these sources of error have already been discussed or are a combination of what has already been dealt with, but she had offered a different perspective to the problem, which will become evident later in the chapter.

The second segment of this review will cover very important aspects to problem solving: imagery and visualization, and diagrammatic aids. After these are analyzed with respect to problem solving in general, the discussion will be focussed on subsets of this domain: reasoning, and finally syllogistic inference. Connections have been established between imagery and reasoning in general (Chase & Clark,1972; Beckman, 1977; Peirce, 1931, 1932). However, the only syllogistic inferencing that has been linked with visualization is the <u>ordered</u> reasoning task. Psychologists such as Handel, DeSoto, & London (1968) and Trabasso, Riley, & Wilson (1975) have investigated imagery and spatial constructions in ordered syllogisms and claimed that it plays a very large role in this type of reasoning. The Aristotelian syllogism has largely been ignored with respect to visualization. Ceraso & Provitera (1971) and Erickson (1974) have discussed Venn diagrams as premise interpretations, and Johnson-Laird & Steedman (1978) developed heuristics for the solution process which involves visual techniques.

It is the purpose of the present investigation to study the relationship between imagery and syllogistic reasoning and to use the results in the domain of education. The two visual processes cited above [Erickson (1974) and Johnson-Laird & Steedman (1978)] will be used to instruct subjects on the solving

of syllogisms. The overall outline of this literature review is made clear by the following diagram, which also includes the aim of the entire study.

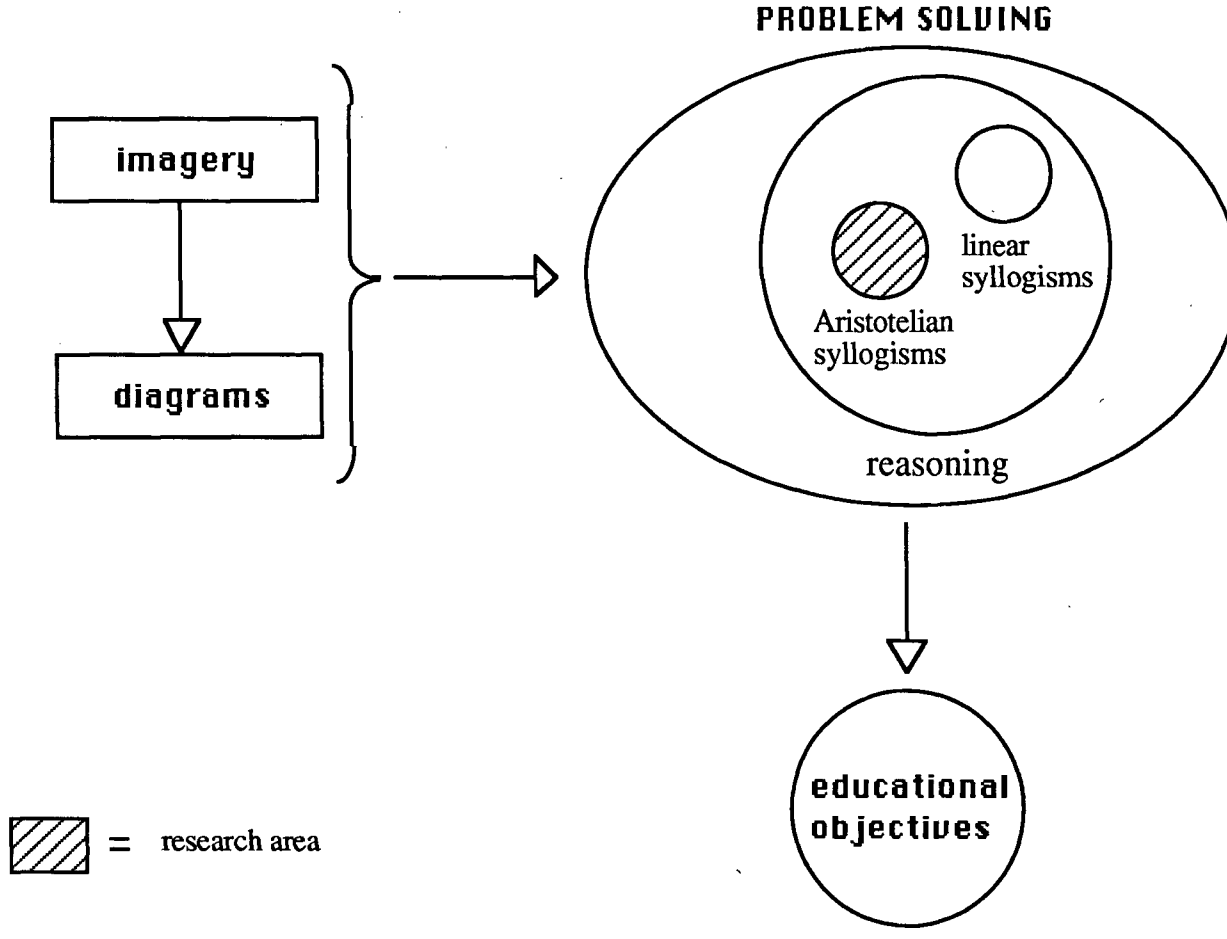


Figure 1. Outline of literature review.

DEFINITIONS

What is a Syllogism?

The definitions listed in this section are standard terminology used for Aristotelian syllogisms and are based on a long tradition.

A syllogism consists of two statements called *premises* (statements to be assumed true), and a final statement called the *conclusion*. The *validity* of the argument can be confirmed if and only if the truth of the premises guarantees the truth of the conclusion.

Three examples follow; one each of a valid conclusion, a contingent conclusion, and an invalid conclusion.

a) Valid conclusion: A conclusion which is true for all cases described by the premises is valid.

	All As are Bs,	(1)
	All Bs are Cs,	(2)
Therefore,	All As are Cs.	(3)

Statements (1) and (2) are the major and minor premises respectively. Line (3) is the valid conclusion.

b) Contingent conclusion: A conclusion which can be true in one particular case, but not for all cases, is a contingent conclusion.

	All As are Bs,
	Some Bs are Cs,
Therefore,	No As are Cs.

It is very important to note that in the literature, contingent conclusions are customarily considered invalid, as they are not valid under all circumstances.

c) Invalid conclusion: A conclusion which is not true for any case described by the premises is an invalid conclusion.

	All As are Bs,
	All Bs are Cs,
Therefore,	No As are Cs.

The quantifiers "all", "no", "some", and "not" are classed as "structural" characteristics which alone are enough to settle upon a valid conclusion. The other terms (As, Bs, and Cs, or meaningful words) are labelled the "syllogistic variables". They do not enter into the reasoning process, but can interfere with the

reasoner's objectivity (Frase, 1966).

There are four premise "types": the universal affirmative (UA), the universal negative (UN), the particular affirmative (PA), and the particular negative (PN). These types are described in Figure 2 in a chart based on that in Revlis' (1975, p.96) chapter.

Type	Features	Relation
UA	Universal Affirmative	All As are Bs
UN	Universal Negative	No As are Bs
PA	Particular Affirmative	Some As are Bs
PN	Particular Negative	Some As are not Bs

Figure 2. Proposition types

Syllogisms are customarily characterized by: the type of the first premise, the type of the second premise and the type of the conclusion (UA, UN, PA, or PN). For example, the syllogism in part a) on page 9 is of type UA-UA-UA; the syllogism in part b) on page 9 is of type UA-PA-UN, and the syllogism in part c) on page 9 is of UA-UA-UN type.

Let the form of the conclusion be denoted by S-P (subject-predicate). Then, the major premise expresses a relationship between the subject of the conclusion and an intermediate term M, and the minor premise expresses a relationship between the conclusion's predicate and the M term. Therefore, one form of the syllogism is as follows:

S - M,
M - P,
Therefore, S - P.

However, each term in each premise may appear in one of two orderings. As a result, the syllogism may take on any of four forms, as illustrated in the chart modified slightly from Revlis' (1975, p. 95), in Figure 3.

	Order 1	Order 2	Order 3	Order 4
major premise	S - M	M - S	S - M	M - S
minor premise	M - P	M - P	P - M	P - M
conclusion	S - P	S - P	S - P	S - P

Figure 3. The four "figures" of the syllogism.

Each of the orders (1, 2, 3, and 4) in Figure 3 is called a "figure". Since there exist four types of premises (UA, UN, PA, PN), then each figure branches out into 16 separate syllogisms, or "moods". Four figures will then allow for 64 different problems. (Johnson-Laird & Steedman (1978) suggest that the order of the premises makes a big difference in reasoning behaviour.) A syllogism such as A-B / B-C has a different response pattern from the syllogism that has the same premises but in reversed order. (This is termed the "figural effect", which will be discussed later.)

What is a Venn Diagram?

Euler (1707-1783), one of the most prolific mathematicians of the 18th century, was the first to depict mathematical sets and subsets and the relationships between them in a graphical way. A beneficial aid in the study of Boolean algebra, circles are used to represent different classes. Members belonging to the intersection of overlapping circles are considered elements of both sets. One circle placed completely within another signifies that all elements of the first set are also members of the other. Appropriately, they are designated Euler circles.

These representations are more commonly referred to as Venn diagrams. John Venn (1834-1923) was a British logician who taught logic and wrote Symbolic Logic (1881) and The Principles of Empirical Logic (1889) and in 1894, included rectangles denoting the universal set along with the set circles.

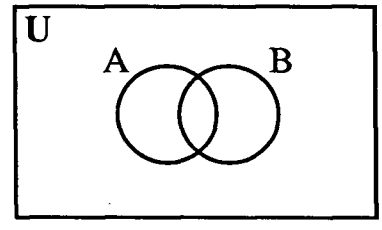


Figure 4. A Venn diagram.

For the sake of simplicity, the term 'Venn diagram' will be used to denote Euler circle representation.

SOURCES OF ERROR

Misinterpretation of the Premises

One popular claim is that subjects use proper reasoning methods but they are led to erroneous conclusions due to their misinterpretations of the premises. Revlis (1975) stated that "reasoners do not possess faulty inference mechanisms in any simple way: deductive errors are the result of the reasoner's applications of logical operations to a faulty data base" (p. 97). Ceraso & Provitera (1971) stated that "errors made on the traditional syllogisms can be accounted for by the

premise - misinterpretation hypothesis" (p. 409).

An example of this misinterpretation would be the faulty perception of the subject to equate the premises "All A are B" with "All B are A" (Chapman & Chapman, 1959). This is not surprising for subjects who were introduced to formal reasoning via introductory mathematics courses because in that domain, "are" is almost always interpreted as "are equal to" rather than "is included in", which is characteristic of set theory and syllogistic reasoning. Not only do subjects treat the converse of UA type statements to be true, but also of PN type statements.

This is why, in the study conducted by Ceraso & Provitera (1971), an effort was made to depict the premises so clearly that their subjects would have no difficulties whatsoever in interpreting them. These modified premises were characterized by Venn diagrams. It is important to note that Ceraso & Provitera (1971) also emphasized that their subjects did not perform in a non-logical way, but arrived at faulty conclusions due to their misinterpretation of the premises.

Other examples of misinterpretation were presented by Chapman & Chapman (1959). They discussed the confusion held between PA (Some As are Bs) and PN (Some As are not Bs) type propositions. They claimed that an

> . . . error pattern might be attributed to the fact that I [PA] and O [PN] propositions imply one another except when we are in a position to assert a contradictory universal. For example, the statement "Some A's are B's" implies that some A's are not B's unless we can assert that all A's are B's. (p. 226)

Another source of confusion is an inaccurate understanding of the conclusion "No As are Cs". In syllogisms with two particular premises, UN type conclusions are more popular than UA type conclusions. It has therefore been suggested by Chapman & Chapman (1959) that an UN conclusion tends to be interpreted as "nothing has been proven".

Probabilistic Inference

Errors in syllogistic reasoning have also been generated by probabilistic inference. It is important to note that syllogistic reasoning is a deductive process in logic i.e. "something that *must* be true if other things are true" (Glass & Holyoak, 1986). Probabilistic inference is based on inductive judgements: something that, according to previous events, has a probability of occurring. This type of inferencing is extremely useful in everyday contexts. For example,

Things that swim are often fish,
I see some things swimming in the lake,
Therefore, some things I see are fish.

The above syllogism is of type PA-PA-PA and is not valid; however, the conclusion has a high probability. This is the method by which individuals reason everyday. Without probabilistic inference, humans would have enormous difficulty understanding social scenarios, making decisions and acting upon them. The example presented in the opening paragraphs (John came into the room), is an excellent case in point.

This type of inferencing is "not allowed" in the structured domain of syllogistic reasoning. Chapman & Chapman (1959) described this type of error in terms of "S's regarding as proved something which is merely probable. If so, they . . . behav[e] as fairly reasonable but incautious people"(p. 225).

Personal Knowledge

Another cause of error described by Revlis (1975) is that some reasoners base their conclusions on personal knowledge. Revlis (1975) presented an example of this type of error:

All men are mortals
Socrates is a man
Therefore, Socrates is a mortal
*Socrates is a Greek (p. 93).

Revlis (1975) stated that "indeed, some reasoners may accept this inappropriate conlusion [*] because they believe it is empirically true, rather than because it follows from the information provided"(p. 93). "Socrates is a Greek" is a historically correct statement, and reasoners who are aware of this may let factual information mislead them.

Personal Bias

Attitudes held by the reasoner have also been shown to be sources of syllogistic error. Morgan & Morton (1944) claimed that 35% of reasoning errors are caused by personal convictions. Studies conducted by Feather (1964), Henle & Michael (1956), Janis & Frick (1943), Kaufman & Goldstein (1967), Lefford (1946), Morgan & Morton (1944), and Thouless (1959) have revealed that distortion in syllogistic reasoning is more prone to occur when the propositions contain subject matter that triggers a strong emotional component within the subject.

Feather (1964) collected data from 165 male students to assess the validity of 12 proreligious, 12 antireligious, and 16 interspersed neutral syllogisms. Feather (1964) concluded with the finding that "personality variables influence the tendency to evaluate arguments in a manner consistent with attitude"(p. 135).

Thouless (1959) studied the effect of prejudice on reasoning by designing syllogisms consisting of controversial subject matter such as socialism, life after death, and war. He tested the hypothesis that "individuals are inclined to judge the logic of an argument as sound if they agree with its conclusion and unsound if they do not agree with its conclusion" (p. 18). He tested two groups: one adult student group and one university graduate group. His results led him to accept his hypothesis for the former group, but not for the latter. Thouless suggested the

reason for this was the differing intelligence levels or educational backgrounds.

Lefford (1946) also studied the effects of emotional attitude and syllogistic reasoning and found a strong correlation between them. He analyzed this behaviour and suggested:

> Once this attitude. . . has become established, critical rational analysis of the structure of the syllogism is at a disadvantage. It acts to channelize the subject's thinking and prevent his accomplishing a reorganization of part processes which might solve the problem. . . . [For example,] if the subject has emotionally accepted the conclusion, then the judgement of valid would have been facilitated. . . . These emotional attitudes are thus the arch-enemies of objective and clear thinking. (p. 146)

Atmosphere Effect

Probably the most famous investigation concerning fallacious inferencing was that conducted by Woodworth & Sells (1935). They perceived reasoning errors to be caused by what they termed the "atmosphere effect": a "global feeling" or "impression" created by certain types and combinations of premises. They claimed that quantifiers (all, some, no) would induce conclusions with the same terms.

Their findings can be divided into two parts. The first states that subjects are more inclined to accept a positive conclusion (*all*) to positive premises, a negative conclusion (*no*) to negative premises, and a negative conclusion to mixed premises. Anderson (1980) mentioned, in regard to Woodworth & Sells' (1935) atmosphere effect, that subjects are prone to accept as valid the following contingent (invalid) conclusion:

No As are Bs,
<u>All Bs are Cs</u>,
Therefore, No As are Cs.

The second part of the theory considers universal and particular propositions. Given two universal premises, subjects are more likely to accept a

universal conclusion; given two particular premises, a particular conclusion; and given mixed premises, also a particular conclusion. Therefore, they will likely accept the following:

All As are Bs,
<u>Some Bs are Cs</u>,
Therefore, Some As are Cs

which is invalid.

Woodworth & Sells (1935) called this the "caution" or "wariness" effect, meaning that accepting a particular conclusion to a set of mixed premises seems the "safest" way out. In other words, agreeing to a strong, weighty conclusion supplies most subjects with a fear to err.

Language Ambiguity

Several researchers have commented on subjects' difficulties with the quantifier "some". In the everyday usage of the English language, "some" implies "few, but not all". Revlis (1975) called this its "normative" meaning. In syllogistic terminology, its "logical" meaning is "at least one, possibly all". This is why many subjects are trained to interpret this quantifier in the proper fashion before participating in reasoning experiments (Wason & Johnson-Laird, 1972).

It has been found that rates of error are significantly diminished when its logical meaning has been made clear (Revlis, 1975; Frase, 1966). Revlis (1975) has claimed that:

> . . .universally quantified premises engender fewer errors than ones with particular quantifiers. Similarly, fewer universally quantified conclusions are erroneously accepted than are particularly quantified ones. . . . the universal propositions have fewer possible alternative representations than do the particularly quantified propositions (p. 103).

Frase (1966) reviewed this situation and claimed that training subjects on the meaning of ambiguous logical terms is necessary in order to eliminate semantic confusion. He asserted: "the assumption that a verbal introduction to logical terms overcomes the effects of those terms should be substantiated" (p. 240).

Another example of language ambiguity arises in the conversion errors discussed by Chapman & Chapman (1959). As mentioned earlier, the conversion of UA type propositions may originate from the narrow and rather limited understanding of the word "are", meaning "equals" in everyday parlance and in introductory mathematics courses, whereas in logic and set theory, this term possesses the definition of "is included in" (Chapman & Chapman, 1959).

The precise definition of "validity" may be a source of confusion for some reasoners. As already mentioned, a "contingent" conclusion implies it is true for one or more of the possible configurations of the premises, but not all. Therefore, for a conclusion to be valid, it must work for every possible combination of representations of premises.

Henle's Error Theory

Henle (1962) has developed a quantitative theory of the sources of syllogistic error. She divided reasoning errors into four separate categories, outlined as follows:

1. Failure to Accept the Logical Task

Subjects confuse valid conclusions with statements that they agree with or that are empirically true. These errors have been previously discussed and studies have been conducted by Cantril (1938), Feather (1964), Henle & Michael (1956), Janis & Frick (1943), Kaufman & Goldstein (1967), Lefford (1946), Morgan & Morton (1944), Revlis (1975), and Thouless (1959).

Richter (1957) insisted, as do the above authors, that these errors occur not because subjects are not reasoning logically, but rather because they are swayed by the content of the conclusion. He made an additional statement that errors arise

from the subjects' "general failure to grasp the concept of logical validity" and their "specific inability to differentiate 'logical validity' from another attribute of syllogisms"(p. 341). Richter regarded syllogistic reasoning as a classifying process and this, he stated, serves to facilitate a more systematic understanding. Two errors in classification which he carried over to reasoning are: 1) careless mistakes; those that are not deliberately made, or those that are intuitive, random responses and 2) those that are made due to faulty classification operations.

2. Restatement of a Premise or Conclusion so that the Intended Meaning is Changed

It is suggested that as reasoners handle the premises, the meanings of particular quantifiers are sometimes altered to adopt more universal connotations. Henle claimed that everyday conversations often allow for these shifts of interpretation.

An argument such as the following is easily constructed:

Some cancer patients die due to their illness,
Most cancer patients die due to their illness
All cancer patients die due to their illness.

Because of the high incidence of cancer casualties, the transition from *most* to *all* is very often made in casual reflection and discussion.

Revlis (1975) described another example of premise restatement. It is noted that while processing or encoding the second of the premises, syntactic terms, chiefly negative structures, are sometimes lost from short term memory, and premises statements take on very different meanings. An example presented by Revlis is the following: the premise "Some As are not Bs" can be altered and therefore understood as "Some As are Bs" since the quantifier "not" has been omitted from memory.

3. Omission of a Premise

Often, reasoners fail to incorporate one of the two given premises when assessing the validity of a conclusion. Henle reported that "in all these cases in which a premise has been omitted, the subject correctly reports that the conclusion does not follow" (p. 372), which is not necessarily the acceptable response to the original syllogism.

4. Slipping in of Additional Premises

Henle claimed that this error occurs often in daily discussions, where facts that are taken for granted are added to the syllogistic information without being noticed. Again, correct reasoning occurs when subjects consider the syllogisms that *they* decode, but faulty conclusions are generated due to these errors.

IMAGERY, VISUALIZATION, AND DIAGRAMS IN RELATION TO PROBLEM SOLVING

It has already been established that the use of pictures and diagrams created by students aid in the learning of mathematical concepts and problem solving (Dirkes, 1980; Furby, 1971; Hortin, 1983; Melancon, 1985; Nelson, 1983; Sharma, 1985). However, the use of specific visualization approaches as aids to Aristotelian syllogistic reasoning has not been investigated in great depth. Various researchers have studied spatial strategies in linear syllogisms (example: Tom is taller than Sam, John is shorter than Sam. Who is the tallest?) and have revealed that

> . . . subjects. . .construct imaginary arrays to solve three-term series problems. . . The subjects' strategy for constructing these arrays is not simply an internalization of a single type of activity; it blends imaginary use of written symbols with imaginary manipulation of objects. (Huttenlocher, 1968, p.558, 560)

These results have been expressed by others such as Handel, DeSoto, & London (1968) and Trabasso, Riley, & Wilson (1975). The former researchers

also advanced the view that spatial representations underlie the ability to solve linear syllogisms. The latter authors shared this view with Barclay (1973), Potts (1972), and Scholz & Potts (1974), and claimed that ". . .adults use imaginal, linguistic, or spatial strategies to integrate and order precise information. . ." (p. 221).

It is important, therefore, to determine two things: firstly, how imagery plays a role in the various facets of the learning process (chiefly problem solving), and secondly, what role concrete visual implementation plays in this process so that these two phenomena can be combined to formulate a visual approach to problem solving, specifically Aristotelian reasoning tasks.

Imagery

Fleming (1977) defined "imagery" as an "internal representation" of external stimuli, and therefore it is a process involving memory more readily than perception. Glass & Holyoak (1986), on the other hand, claimed that imagery is a direct product of perception. Kosslyn (1987) defined imagery as seeing with the "mind's eye" what is not actually there. Whatever the differing definitions, it is essential to note the effects imagery can have on concept learning and understanding.

Imagery has been tested extensively for its role in free recall, recognition, concept learning, and problem solving, among other cognitive processes. Kosslyn (1987) argued that imagery can be used to think and learn in such contexts as scientific problem solving. He further asserted that it can be utilized for comprehension and planning. Shepard and Cooper (1982) discussed such phenomena as "imaged models" and that these models were found to be helpful in the problem solving process. Chase & Clark (1972) stated that "what *is* clear is that imagery is reported in abundance in almost every problem solving task" (p. 224). They continued to suppose that visual-imaginal methods are abundant in all types of problem solving exercises, especially those which demand spatio-mathematical techniques.

Problem Representation

There are two phases to the problem solving process: the first is the way in which external information about the problem is mentally represented (symbolic structuring) and the second is the way attempts are made to find a solution by operating on these representations in order to arrive at solution steps and answers (Trabasso, Riley, & Wilson, 1975). How these two phases are linked with imagery are discussed in the next section. It is known that some psychologists have disagreed with the point of view, but for this discussion, the representations and operations referred to will be imaginal processes pertaining to problem solving.

Mental Representation

There is much evidence that in attacking problems found in mathematics, physics, and engineering, imagery and pictorial thought processes are used. Even great scientists such as Einstein and Hadamard have rejected the idea that they "think in words". Chase & Clark (1972) stated that "composers and artists regularly report hearing and seeing the works they are about to compose or construct" (p. 230).

Lashley (1951) and Inhelder & Piaget (1964) have proposed that "spatial representations subserve even nonspatial orderings in thinking; even in abstract reasoning tasks people rely on internal spatial constructions as thought models"(Handel, DeSoto, & London, 1968, p. 351).

Larkin (1983) distinguished between problem representation for novices and expert problem solvers. She appropriately identified them as "naive" and "physical" representations; the difference between the two is that the latter involves more abstract entities, such as force and momentum, whereas the former entails only visible objects mentioned in the problem. Larkin contended that each of these representations ". . .seems to involve entities that might readily be imaged . . ." (p. 79). She continued to state that it is less evident that a physical representation is always imageable, "but it is worthy of comment that most physical representations seem to have this feature" (Larkin, 1983, p. 81). Larkin quoted a problem solver in the midst of constructing a naive representation as

saying: "Once I visualize it, I can probably get started" (p. 81).

The debate over the pictorial versus antipictorial (propositional) internal representations of stimuli has been raging for quite some time now. One camp consists of psychologists such as Kosslyn and Pomerantz (1977) and Paivio (1976), who have asserted that visual stimuli are encoded in terms of resources that are very spatial and modality specific: "A visual image should resemble seeing, and an auditory image should resemble hearing" (Glass & Holyoak, 1986, p. 132). This phenomenon is commonly referred to as the "picture in the mind" (Fleming, 1977). Kosslyn (1987) made reference to imagery as a "mental organ", implying a primary and independent function. Anderson (1978) referred to this view as the "imagery position". This position can be seen as analogous to the diagrammatic representation described by Larkin and Simon (1987). Even though they described external representations (such as diagrams marked on paper as opposed to images stored in the brain), they referred to components of a diagram depicting the problem. In imagery, this diagram corresponds to the mental image one forms when pictorially representing the relevant elements to the problem.

On the other hand, there are theorists like Pylyshyn (1973, 1976) and Reed (1974) who have affirmed that imagery "is encoded in an abstract propositional format and that this format is used to encode verbal information" (Anderson, 1978, p. 250). A salient point to be made is that this notion of imagery does not regard it as a well formed domain in its own right, but rather a special aspect of a more general processing system. This viewpoint can be made analogous to Larkin and Simon's (1987) sentential representation where the sentences in a natural-language description of the problem correspond to the propositional representation stored internally.

Mental Operations

Various researchers have shown interest in spatial abilities in relation to the solution phase of problem solving. Questions have been asked about imaginal operations such as "Can imagery be selectively and deliberately generated to serve

cognitive purposes?" and "Can imagery be voluntarily manipulated or transformed to solve problems?" (Fleming, 1977, p. 53). The answer seems to be affirmative. Fleming (1977) asserted that "people can voluntarily generate images from words and can even invent interactive relations between the images" (p. 53).

McGee (1979) claimed that spatial visualization draws in "the ability to mentally manipulate, rotate, twist, or invert . . . " (p. 893), and Fennema (1985) contended that these properties are important factors in the solving of mathematical problems. Fennema and Sherman (1977) and Krutetskii (1976) have proposed that these abilities are related positively to mathematical problem solving. Threadgill-Sowder, Sowder, Moyer, and Moyer (1985) have suggested two reasons for this positive relation: the first is that many of these types of problems "yield to a visual-spatial attack" (p. 56). The second reason for the positive relation between spatial ability and performance, according to Threadgill-Sowder et al. (1985), is that general intelligence yields spatial performance. This implies that good problem solvers use spatial talents to find solutions if it is assumed that intelligent people are good problem solvers.

Antonietti, Barolo, and Masini (1985) conducted a study investigating restructuring strategies in problem solving. They found that good geometric problem solvers were able to restructure mental images while looking for alternative solutions to the problems. They also found that those who best synthesized mental images were the best problem solvers.

Chase & Clark (1972) discussed the function of imagery in problem solving tasks. They noted that the solution steps for some of these tasks employ visual-perceptual processes rather than deductive ones. They stated that "the mind's eye is a working space. . .for generating new concepts and relations" (p. 226). The game of chess was used to illustrate the generation of visual images from which new, abstract structures are created to develop solution possibilites. Problem solvers extract necessary information from the visual images they construct just as perceivers would from a physical stimulus.

Shepard and his colleagues (Cooper and Shepard, 1973; Shepard and Metzler, 1971) have discussed imaginal manipulation in problem solving by

finding evidence supporting the existence of mental rotation. They conducted a study involving diagrams of three-dimensional objects and asked subjects to determine whether the members of a pair were the same. Glass and Holyoak (1986) summarized that "the greater the difference in orientation of the pair members, the longer it took the subjects to make a decision. . ." (p. 13). Shepard & Metzler (1971) concluded that a procedure applied to the mental images of the objects was responsible for spinning one image so that it matched its pair. This he termed "mental rotation", which, as has been demonstrated, can be very useful in problem solving.

Criticisms have been put forward, however, concerning the use of imagery processes and constructs as educational tools. Fleming (1977) was disapproving of three points. Firstly, much of the previous work in this area has been accomplished with oversimplified pictures and pairing tasks which, according to Fleming, is not typical of the learning that takes place in the school setting. He asserted that more information is needed about each of the processes involved before "jumping on the imagery bandwagon and proclaiming it the panacea for instructional ills" (p. 46).

Secondly, due to the battle currently being waged between the imagists and the propositionalists, it is unclear whether or not imagery can be considered a subordinate, secondary process, in which case it has no role in any instructional or explanatory context. It is questionable which side will be victorious in this psychological battle. However, due to convincing evidence, most would agree that imagery does present itself as an aid to learning. Exactly how this process is executed is currently ill-defined and unknown, but advances are being made.

This folds in with the third criticism Fleming has submitted. Imagery has been shown to have considerable effects on memory, recognition, and other cognitive processes, but it is still unclear whether pictorial apparati consistently have the advantage over verbal instruction because there are so many unknown factors involved.

All in all, however, Fleming has adhered to the established finding that imagery processes facilitate both concept learning as well as understanding of

verbal material. Reasons for this have been advanced by other psychologists. Levin, Ghatala, & Wilder (1974), for example, have proposed that images and pictures create such a strong impression in memory because they are confronted less often than words. As Fleming (1977) put it: "Any particular picture certainly occurs in public or in instructional media less often than the words used to describe it. Such pictures are more novel, more noteworthy, and perhaps for *that* reason, more memorable" (p.48).

Bower (1972) contended that instruction involving imagery and visualization is inherently more organized than verbal presentation. It has been established that material well thought out and systematically presented has a far superior effect on memory and learning than disorganized material. Bower conducted a study and found that words or phrases organized together are remembered more distinctly than words presented separately: i. e. "white horse" is remembered more readily than "white" and "horse". As well, organized mental images such as "square door" are recalled before a pair of abstract words are. "Thus the memory effect of imagery could be ascribed primarily to its organized character" (Fleming, 1977, p. 48).

As for concept learning, it is relevant to determine the actual characteristics of a "concept". The meanings extracted from concepts have been found to be collections of instances rather than lists of attributes. It has been implied that concepts are interpreted as images of representative examples and not abstract features.

Diagrammatic Aids to Problem Solving (External Representation)

Until now, the discussion has been directed toward internal representations rather than external ones. The effectiveness of external representations, as in recordings on paper or blackboard, has been tested extensively. Pure and applied scientists are incessantly using diagrams, sketches, and pictures as aids to problem solving. Larkin and Simon (1987), in their article *Why a Diagram is (Sometimes) Worth Ten Thousand Words*, have shown quite thoroughly that it is advantageous

to use diagrams while devising a solution strategy.

Chase & Clark (1972) have stated that resorting to pencil and paper (i.e. an external representation) is in all likelihood the same as manipulating visual images in the mind. Theories such as these give rise to new studies and more recognition in the area of picture research. Researchers such as Fleming (1967, 1977) have asserted that because of the current preoccupation with mental imagery as a cognitive function essential to the learning process, picture experimentation has taken off. Thus, an equivalence can be constructed from the two stages of the problem solving process established by Trabasso, Riley, & Wilson (1975) to a set of physically manipulative components essential to the picture and diagram theory. It is outlined in Figure 5.

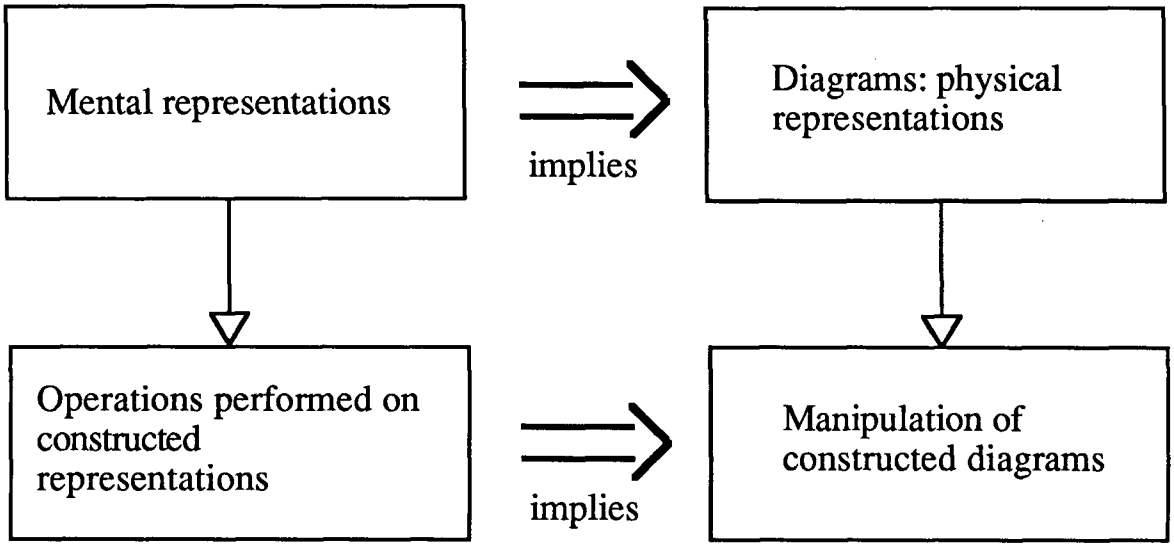


Figure 5. The problem solving process: Analogy between mental and physical phenomena.

Waller (1979) maintained that diagrams are extremely valuable vehicles for the facilitation of problem solving because even though it was found that most people have trouble drawing diagrams, few people have difficulties with their perception and interpretation. Threadgill-Sowder et al. (1985) have hypothesized that a diagrammatic representation of a problem is favourable to a verbal

representation because the former, while organizing the important elements visually, could result in a more succinct and explicit layout of the essential components of the problem.

Dirkes (1980) claimed that "pictures. . .serve to communicate mathematics in a way that words and symbols do not match. Some students claim that many concepts are understood only when they are illustrated through objects, diagrams, or pictures" (p. 10). Dirkes continued to state that "According to recent findings on the hemispheres of the brain, many persons function as if verbal information inhibits learning whereas spatial representations increase learning and communication" (p.10).

Nelson (1983) has also suggested that visual aids facilitate the problem solving process. He stated:

> Visualization is an effective technique for determining just what a problem is asking you to find. If you can picture in your mind's eye what facts are present and which are missing, it is easier to decide what steps to take to find the missing facts. (p. 54)

Waller (1979) analyzed the functions of graphic communication and stated that diagrams or visual representations are tools for enquiry and thought, aids to learning, and aids to problem solving; all three being of interest in this discussion. Diagrams, he suggested, are not merely descriptive constructs. Rather, they aid in the structuring and analysis of a problem. He considered graphic constructions as one type of classification process: "classification is central to science and is essentially a graphically organised task - the perception of pattern and structure among apparently chaotic data" (p. 220). Charles Sanders Peirce (1839-1914) advanced the creative aspect of diagrams: they do not simply list or describe known facts and elements of a problem.

Waller (1979) continued to assert that pictorial stimuli are essential for the comprehension and mastering of verbal problem solving tasks. He investigated the role of diagrams in this domain. He stated that "in order to make relationships and

functions clear, draughtsmen and illustrators use highly stylised representations of components" (p. 221).

MacDonald-Ross (1979) claimed that "the existence of the diagram makes possible certain kinds of mental operations which [are] almost inaccessible through language, or with other kinds of notations" (p. 227) and that the diagram also plays an important part in the generation of scientific hypotheses.

Using pictures and diagrams in an instructional setting seems to be the next logical step. If visualization (both internal and external) are of such great help in the area of problem solving, it follows then that teachers should, as much as they can, use illustrations to explain mathematical and scientific concepts. In addition, instructors should, wherever possible, encourage students to draw the problem elements before attempting a solution. Many textbooks stress the value of pictorially depicting the concepts involved in any given problem. For example, in Gilbert's (1976) Statistics, students are advised to "**Always** sketch the area needed" (p. 138) when calculating areas under the normal curve. Thus, in an instructional context, visual approaches are assets; in fact, they are vital to the realization of succinct communication. Waller (1979) treated graphic illustration as a communicative process and analyzes the encoding of information in graphic form. He stated that "graphic communication is richer than any one area of scholarship suggests" (p. 222). The notion of treating diagrams as a form of communication is not novel. C. S. Peirce (1931), in volume 1 of eight volumes of collected papers on the subject, has stated that:

> Every picture (however conventional its method) is essentially [an icon]. . . The only way of directly communicating an idea is by means of an icon; and every indirect method of communicating an idea must depend for its establishment upon the use of an icon. Hence, every assertion must contain an icon or set of icons, or else must contain signs whose meaning is only explicable by icons. (vol.1, p. 278).

Visual Techniques and Syllogistic Reasoning

A workable framework for visual techniques has already been established in the area of linear syllogisms (Handle, DeSoto, & London, 1968; Huttenlocher, 1968; Trabasso, Riley, & Wilson, 1975). However, a few researchers have indicated that imagery and visual techniques may play an important role in other, more general reasoning problems. Chase & Clark (1972) have asserted that "people probably use imagery in many kinds of tasks. It is quite possible, for example, that people scan mental images in deductive reasoning problems" (p. 229).

Beckman (1977) conducted an investigation to show that some children attack problems with spatial techniques and others with verbal techniques. She found that those who use non-verbal concept formation have an advantage when handling inferencing tasks.

C. S. Peirce (1939-1914), a leading philosopher in the area of imagery and diagrams in science, has done serious work in examining the relation between logic and imagery. He claimed that reasoning is diagrammatic, which is an opposing view to the traditional notion that "verbal syllogism is the foundation and symbols [are] used just to clarify and regularise the relationships found between the constituent proposition [sic] of the syllogism" (Macdonald-Ross, 1979, p. 227):

> . . .mathematical reasoning consists in constructing a diagram according to a general precept, in observing certain selections between parts of that diagram not explicitly required by the precept, showing that these relations will hold for all such diagrams, and in formulating this conclusion in general terms. All valid reasoning is in fact thus diagrammatic (vol. 1, 1931, p. 54).
> Deduction is that mode of reasoning which examines the state of things assessed in the premisses [sic], forms a diagram of the state of things, perceives in the parts of the diagram relations not explicitly mentioned in the premisses [sic], satisfies itself by mental experiments upon the diagram that these relations would always subsist. . . and concludes their necessary or probable

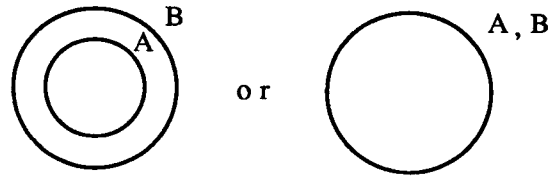
truth (vol. 1, 1931, p. 66; vol. 2, 1932, p. 216).

Various researchers have presumed that subjects use Venn diagrams to represent the features present in each syllogism. For example, Ceraso & Provitera (1971) designed an experiment to observe the reasoning process by controlling for any possible misinterpretation of premises. They accomplished this by encoding each premise with Venn diagrams so as to avoid any confusion. This is demonstrated in Figure 6.

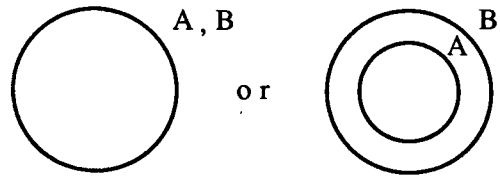
It was assumed in their study that subjects who understand premises very clearly represent them either mentally or externally as Venn diagrams. This assumption, however, was hastily made. It is not obvious that the representation of the premises presented in Figure 6 is the one constructed by correct reasoners. Nor is it apparent that the representation would necessarily facilitate the reasoning process at all. However, other investigators have assumed the same thing.

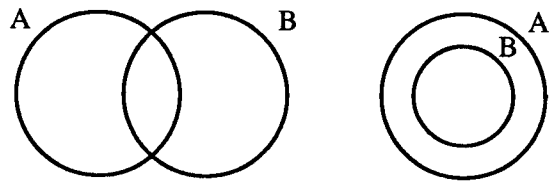
As an example, Revlis (1975) discussed the possible interpretations for each type of premise: UA, UN, PA, and PN (see page 10). He concluded that a hypothetical explanation for error rates is the fact that universal type propositions (UA, UN) induce fewer conceivable representations than the particular type (PA, PN). He used Venn diagrams to illustrate his point, making the assumption that reasoners use the same structural representation to arrive at solutions. Erickson (1974) called this hypothesis the "set-theoretic" interpretation of the premises. Neimark & Chapman (1975) studied the meaning subjects extract from the quantifiers "all" and "some" by portraying several propositions as combinations of Venn diagrams.

All As are Bs

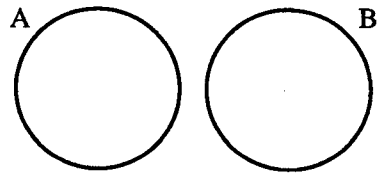


Some As are Bs

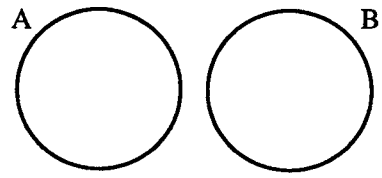




No As are Bs



Some As are not Bs



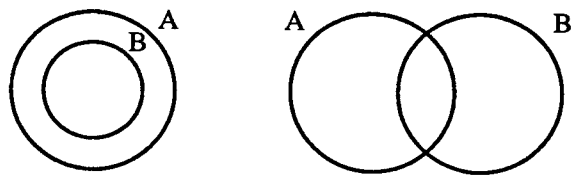


Figure 6. All possible Venn diagram interpretations of each premise.

Visual Models of the Inference Process: Set Versus Analogical Theory

It was Erickson (1974), however, who examined this sort of structured representation most meticulously. He asserted that the inference process involves three broad stages and that during the first two, Venn diagrams are used to interpret and combine the premises respectively. The three stages of Erickson's set analysis theory are shown in Figure 7 (pages 34 - 35) of his 1974 chapter "A set analysis theory of behaviour in formal syllogistic reasoning tasks" in Theories in cognitive psychology: The Loyola Symposium.

Erickson mentioned that during the first stage "the average subject does not consider all possible set relations"(p. 311), but instead chooses one or more of them (with an estimated probability) as the desired interpretation.

Stage 2 of Erickson's theory involves the reasoner's combining of the interpreted premises. There exist three likely ways that this is accomplished and they are labelled Models 1, 2, and 3.

Model 1 assumes that combinations are made from the premises that are interpreted and each is equally likely to be chosen. The subject makes a random selection from this set.

Model 2 describes the event that a reasoner studies all combinations made from his interpretation of the premises. He then chooses a conclusion that fits each of them.

Model 3 is similar to Model 1 in that not all the possible combinations are examined, but differs from it because the reasoner chooses one or more of them according to particular biases. There exists a certain probability that erroneous choices will be noticed by the reasoner.

Johnson-Laird & Steedman (1978) found three weaknesses to the above theory. The first shortcoming is, as Erickson (1974) himself mentioned, that in the first model possibly only one combination is available to select. In such a case, the subjects' alacrity to respond "no valid conclusion" is unexplained. Johnson-Laird & Steedman (1978) also added that "the fact that they also make this response to premises that allow a valid conclusion counts against the full-scale

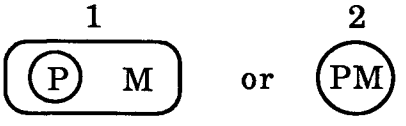
Task: Draw a conclusion from premises (or state that no conclusion is logical)

All P are M	Major Premise
All M are S	Minor Premise
? ? ?	Conclusion

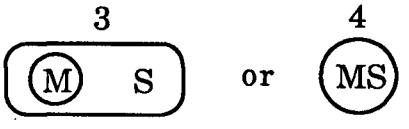
Stage I: Interpretation of the premises

Possible Interpretations

All P are M



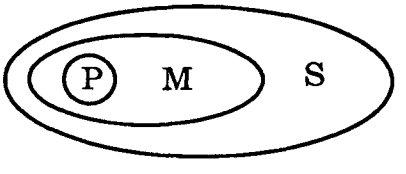
All M are S



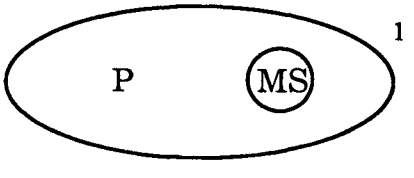
Stage II: Combination of interpreted premises

Possible Combinations

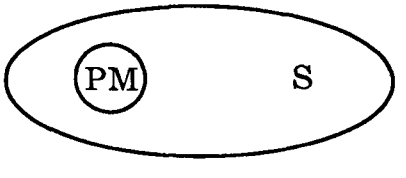
1 and 3



1 and 4



2 and 3



2 and 4

Stage III: Labeling of set relation of S to P[2]

Possible Labels

<u>1 and 3</u> Some S are P or Some S are not P	<u>1 and 4</u> Some S are P or Some S are not P[3]
<u>2 and 3</u> Some S are P or Some S are not P	<u>2 and 4</u> All S are P or Some S are P

<u>**Logical Conclusion**</u>: Some S are P (p. 309)

[1]Even though this appears in the original article, it is an error.

[2]Set relation of P to S could have been chosen without loss of generality.

[3]This label is correct according to the interpretations in Stage I, but incorrect according to the second combination in Stage II.

<u>Figure 7</u>. Erickson's (1974) Set Analysis Theory of behaviour in formal syllogistic reasoning tasks.

model in which all combinations are constructed"(p. 90).

The second weakness mentioned is that the set-theoretic representations of the premises are symmetrical (i.e. the representation of "No As are Bs" is identical to that of "No Bs are As", and so forth), and that nothing exists to account for the figural effect. (Johnson-Laird & Steedman (1978) suggested that it is careless of investigators of syllogistic reasoning not to account for the ordering of the premises, for instead of 64 separate syllogisms (16 moods X 4 figures) there exists twice that many, as each problem is different if its premises are switched. The "figural effect" is the phenomenon that there exists a bias toward A-C conclusions for the A-B / B-C premises and a bias toward C-A conclusions for the B-A / C-B premises. Johnson-Laird & Steedman (1978) have claimed that this effect is highly significant).

Erickson's (1974) set-theoretic interpretation of the premises cited earlier

supports the equality of the representations of statements such as "Some A are B" and "Some B are A". As well, the representations of "No A are B" and "No B are A" are indistinguishable. It was suggested by Johnson-Laird & Steedman (1978) that "the theory would need to be supplemented in some way in order to account for the figural effect" (p. 91).

Thirdly, it is noted that Erickson's (1974) set theory does not incorporate rules to predict subject performance: i.e. the cases where subjects invent their own conclusions, and do not choose from previously combined premises. Johnson-Laird & Steedman (1978) included "heuristics" in their theory which allows somewhat for the prediction of subject performance.

In response to these deficiencies, Johnson-Laird & Steedman put forward another approach to the representation of premises. They call it the "analogical theory of reasoning". The pivotal ingredient in this theory is: "syllogistic inference is based on an *analogical* representation of the premises that captures their logical properties within its structure"(p. 76). Their theory can be divided into the four stages outlined in Figure 8.

Stage 1 describes a rather new approach to the representation of the premises. A subject in an experiment conducted by Johnson-Laird & Steedman (1978) alluded to a visual-structural technique used to represent the following premise: "All artists are beekeepers". He mentioned: "I thought of all the little (sic) artists in the room and imagined that they all had beekeeper's hats on" (Johnson-Laird & Steedman, 1978, p. 77). Therefore, rather than a set representation presented by Erickson (1974), an analogical theory was introduced where "a class is represented simply by thinking of an arbitrary number of exemplars" (Johnson-Laird & Steedman, 1978, p. 77). Some reasoners use distinct, sharp images as sample items, others use more generic structures such as abstract or verbal components. For the sake of simplicity, <u>symbolic</u> sample units (such as A's and B's) and *not* pictorial sample units will be used to illustrate the propositions and subsequent stages of this theory.

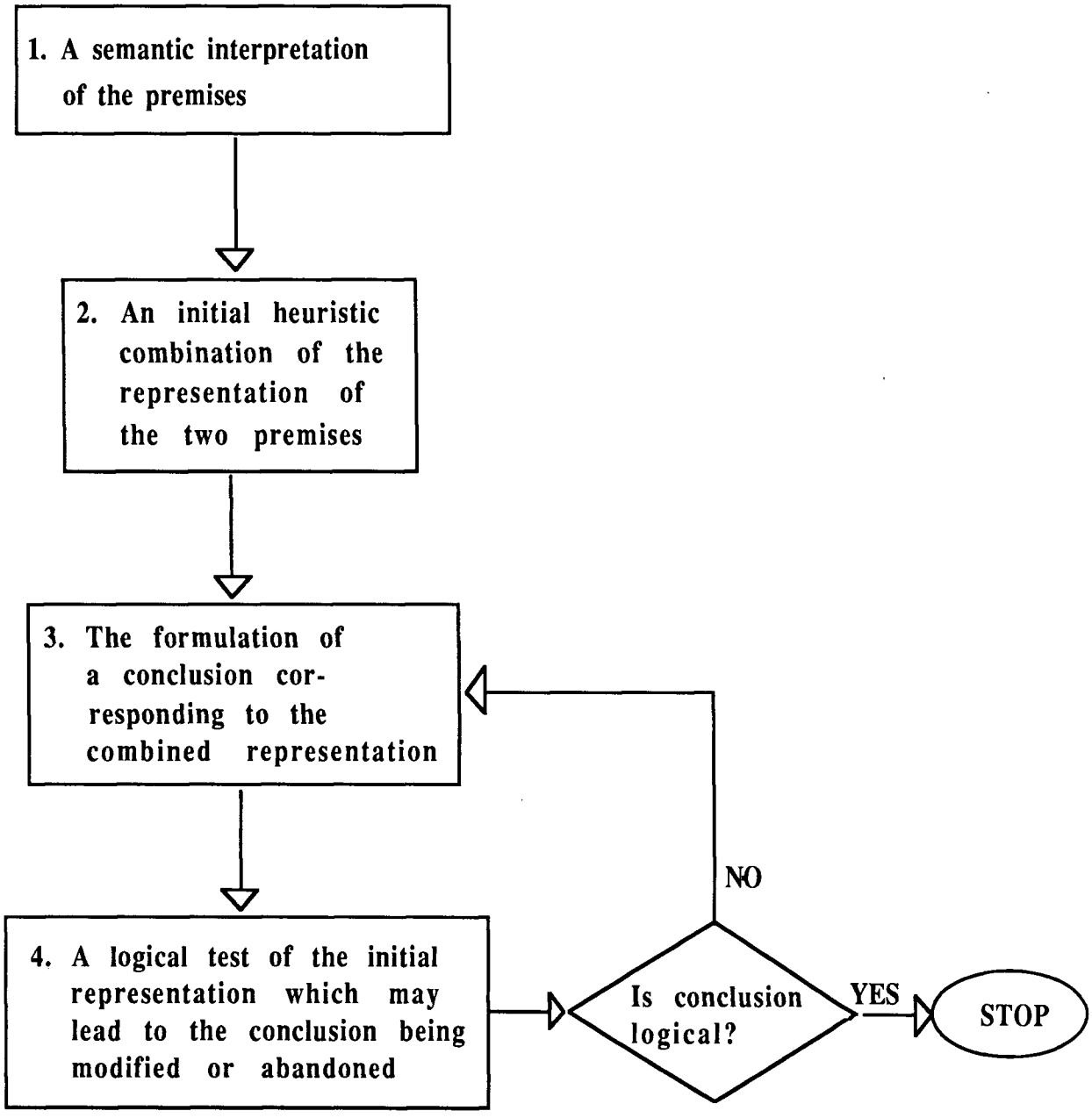


Figure 8. The four stages of the analogical theory of Johnson-Laird & Steedman (1978).

Of great importance are the relationships between particular items in each class. A vertical arrow (↓) indicates a "positive link" meaning that a member from one class is positively related to an item in another, and a "stopped" arrow (⊥) indicates a "negative link". A sample item in parentheses [i.e. (A)] means that there may or may not be other members in that class. Each premise type (UA,

UN, PA, PN) is illustrated below:

<u>UA type: All As are Bs</u>

$$\begin{array}{cc} \mathbf{A} & \\ \downarrow & \\ \mathbf{B} & \mathbf{(B)} \end{array}$$

The B in parentheses permits class B to have more members than class A, however, the arrows illustrate that each member of class A is "related" to one member of class B.

<u>PA type: Some As are Bs</u>

$$\begin{array}{cc} \mathbf{A} & \mathbf{(A)} \\ \downarrow & \\ \mathbf{B} & \mathbf{(B)} \end{array}$$

This representation allows for the interpretation that some As are not Bs, and also that some Bs are not As.

<u>UN type: No As are Bs</u>

$$\begin{array}{c} \mathbf{A} \\ \perp \\ \mathbf{B} \end{array}$$

The stopped arrow prevents any positive link to be established between the members of the first class and the members of the second.

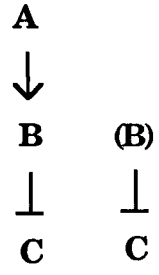
<u>PN type: Some As are not Bs</u>

$$\begin{array}{cc} \mathbf{A} & \mathbf{(A)} \\ \perp & \downarrow \\ \mathbf{B} & \mathbf{B} \end{array}$$

Here, it is possible that some As are Bs, but this representation also allows for the likelihood of no As being Bs.

Stage 2, the combination of premises, demonstrated using the following set of premises, consists of the following heuristic: link up items by way of middle terms:

All actuaries are boxers,
<u>No boxers are carpenters</u>
Conclusion



Johnson-Laird & Steedman (1978) developed another heuristic: if the path from a member of class A to a member of class C contains two positive links (two arrows), then the path is termed "positive" and if it contains at least one negative link, it is called "negative".

Stage 3 allows for the development of a conclusion based on the number of positive and negative paths in the combination of the premises. Johnson-Laird & Steedman (1978) described this heuristic most succinctly:

> Where there is at least one negative path, then the conclusion is of the form *Some X are not Y*, unless there are only negative paths in which case it is of the form *No X are Y*. Otherwise, where there is at least one positive path, the conclusion is of the form *Some X are Y* , unless there are only positive paths in which case it is of the form *All X are Y*. In any other case, no valid conclusion can be drawn, that is, where there are only indeterminate paths. (p. 79)

The conclusion derived from the above configuration is therefore "No actuaries are carpenters".

The depiction of premises for the analogical theory does not encounter the limitation of representations of certain premises and their converses being inherently equal as in the set theory since paths can only be formed in one

direction. Therefore, a strong figural effect is taken into account.

In stage 4, the subject manipulates the established combined representations in order to find an instance where the conclusion might fail. The idea is to construct as many correct possibilites in order to come up with the a conclusion that will hold under any combination of premises: i.e. to arrive at a valid result.

CONCLUSION

It is evident that a great deal of work has been conducted in the areas of visualization, visualization and problem solving, and the modelling of the inference process. It is important to understand the significance of these three cognitive processes in order to relate them to syllogistic reasoning.

Imagery, especially in the field of psychology, has been an issue recently and has been studied to provide new insights for a more complete understanding of memory, perception, and thinking. In the field of education, however, imagery and graphics have for the most part been overlooked. Waller (1979) has claimed that they are "essential ingredient[s] of many subjects, but [are] neglected in education" (p. 222). Only of late have educational psychologists attempted to link the effect pictorial stimuli have on thought processes with educational concerns such as learning and problem solving. Also, imagery has been extended only recently to other educational areas such as reading, cognitive development, special education, and vocational education (Fleming, 1977).

The link between imagery and the solving of reasoning tasks in particular is of special interest in this study. It is tempting to conduct an investigation to establish this link because, again, very little attention has been paid to this subject. Within the area of syllogistic reasoning, other phenomena apart from imagery have been extensively examined such as sources of inferencing errors, and models of accurate reasoning processes (Revlis, 1975). The only instance where imagery is directly related to the solution phase is within the analysis of linear syllogisms, where it has been confirmed that reasoners construct imaginary arrays to solve these types of reasoning tasks (Handel, DeSoto, & London, 1968; Huttenlocher,

1968; Trabasso, Riley, & Wilson, 1975).

Various psychologists have come close to regarding imagery as an important aspect of deductive reasoning. For instance, some have used visual structures to explain premise interpretation: in particular, Revlis (1975) utilized the Venn diagram to explain the ease with which subjects construct representations of universal quantifiers as opposed to particular quantifiers. Furthermore, while attempting to outline accurate inference models, only Johnson-Laird & Steedman (1978) and Erickson (1974) have used visualization techniques in the development of each stage of the reasoning process. In their work, they used structures involving imagery at the stages of premise interpretation, premise combination, and conclusion development.

In this literature review, it has been established that graphic instructional aid is an asset in many educational contexts due to the great impact illustrations and diagrams have on the learning of mathematical concepts. Thus, two of the purposes of the present investigation are to develop visual aids for the process of solving Aristotelian syllogisms, and to compare the effects on achievement resulting from two separate visual approaches. The advancement of visual techniques as *aids* to the solution process in the area of Aristotelian syllogistic reasoning has been for the most part ignored. The notion was, however, anticipated by Ceraso & Provitera (1971) who claimed that a Venn diagram depiction of the premises would help clarify any confusion that might have been generated in the reasoner's mind. They stated that:

> The strategy we followed . . . was to present Ss with syllogisms, modified so as to avoid misinterpretation of the premises. We tried to achieve this end by having each premise clearly refer to only one of the relations of Fig. 1 [Venn diagram representation]. (p. 402).

The first of the two visual approaches chosen for this experiment will be extracted from Erickson's (1974) set analysis theory of reasoning behaviour. Subjects will be introduced to formal syllogistic reasoning with a visual component; at each stage of the solution process, Venn diagrams will be used to

clarify any confusion possibly generated. Subjects will be able to manipulate Venn diagrams as necessary, so that each phase will be understood as thoroughly as possible.

The second technique will be patterned after the inferencing model designed by Johnson-Laird & Steedman (1978). For the purpose of the present experiment, the second visual approach will be termed the 'Sample Representation' method. The visual component arises from the links established between the representative elements of each class. Subjects will be instructed on the solution steps via the second visual approach (heuristics provided by Johnson-Laird & Steedman, 1978) with the aim to compare these performance results with the results obtained through the Venn diagram instructional approach. The two visual approaches will be compared to a group receiving non-diagrammed instructions on syllogistic reasoning.

The theory behind the present study stems from the hypothesis that imagery, visualization, and graphic instruction facilitates the reasoning process. Nevertheless, it is not apparent which instructional method will result in better performance, but it is the hope of the present investigation to provide a workable framework for future research in this area.

CHAPTER 3: RESEARCH DESIGN

RESEARCH HYPOTHESES

There emerge two hypotheses for this investigation. Since the research literature does not present convincing evidence that one visual method is superior to the other, and because this experiment is primarily a comparative study, the first hypothesis, concerning the relative effect of the two visual approaches, is non-directional. It is hypothesized, however, that a visual approach to the solving of syllogisms, regardless of which one, will yield better results on the posttest than an approach without a visual component; therefore, the second research hypothesis is directional. The two research hypotheses can be stated as follows:

H_1: There will be a significant difference in syllogistic reasoning achievement between the Venn diagram approach and the Sample Representation approach.

H_2: Both of the two visual approaches used for H_1 will result in significantly greater achievement in syllogistic reasoning than a non-visual approach.

METHODOLOGY

Population and Sample

The sample will be taken from the grade 8 population in a secondary school in a metropolitan school district. Grade 8 was selected since students at that level have not had the opportunity to formally study either visual approach to be used in the study nor have they studied syllogisms. Such prior knowledge by members of any one of the three sample groups could serve as a confounding variable which might contaminate the results. The large suburban school district was chosen to facilitate access to Macintosh computers.

For the purposes of the experiment, three sample groups (one for each treatment) need to be equated via random assignment of subjects to treatment. Operationally, however, 45 students in three intact groups will be provided for

the experiment. The choice of a random sample from this group would not supply the researcher with enough subjects. As an alternative, the researcher will randomly divide each of the three classes into three groups so that random assignment to the treatment groups is assured. It is important to note that three intact classes of honours students were provided for this study. The final configuration is shown in Figure 9.

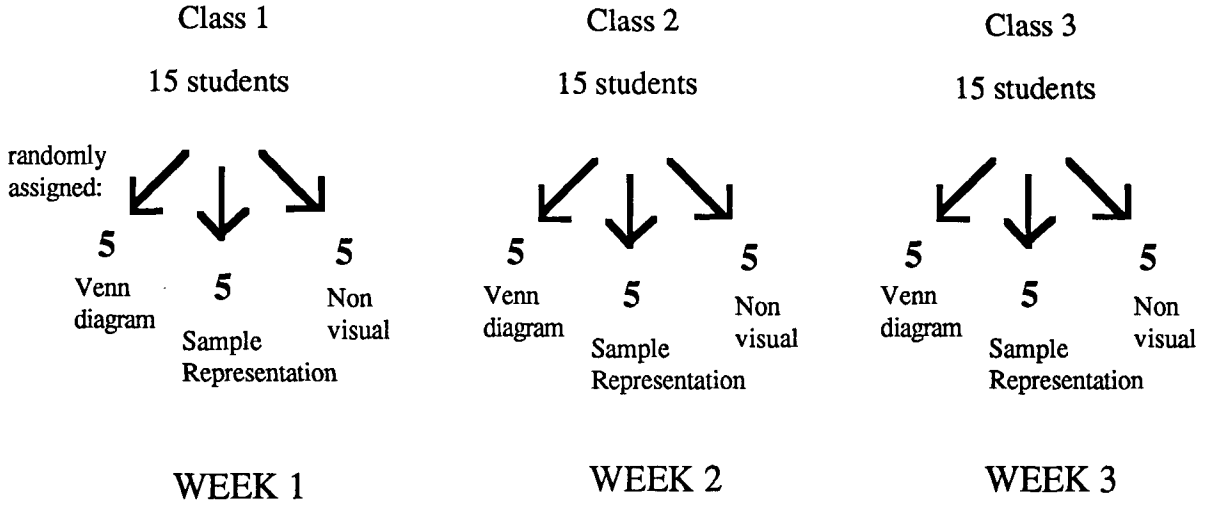


Figure 9. Configuration that allows for random assignment to each treatment group.

The data collection for the three treatment groups will be completed after the third week. Even though collecting the data at different intervals for each group is not ideal, it is preferable to conducting the experiment without random assignment. There is no reason to expect that any non-treatment learning of syllogisms will occur between the first and third weeks as syllogisms are not in the mathematics curriculum and are not taught.

Variables

The independent variable is the treatment administered to each of the three experimental groups. Any member of the given sample will be instructed by one of three methods (Venn diagram, Sample Representation, and Non-visual) which comprise the three levels of this variable.

The treatment (instruction) will be designed for the Macintosh microcomputer. Three modules will be formulated: one for the Venn diagram solution method, one for the Sample Representation method, and one for the Non-visual method. The first two modules will be subdivided into two sections:

1) a 40 minute introductory session, outlining the diagrammatic notation needed to solve syllogisms (see Appendices A and B). Twelve short notation type problems will be presented for each subject to solve with accompanying worksheets (see Appendices F and G). Each subject will then write a 20 minute test on notation, which will not involve the computer (see Appendices I and J).

Even though $\begin{array}{c} A \\ \perp \\ B \end{array}$ is the correct representation for "No As are Bs", $\begin{array}{cc} A & A \\ \perp & \perp \\ B & B \end{array}$ will be used instead in the Sample Representation notation module to distinguish "No As are Bs" from "A is not B".

2) a 40 minute instruction session, illustrating the diagrammatic solution steps to syllogistic problems (see Appendices C and D). Each subject will attempt to solve six syllogisms with solutions explained after each exercise. Worksheets also accompany this section (see Appendix H).

The Non-visual unit will consist of only one lesson (see Appendix E), also accompanied by a worksheet (see Appendix H). Since it will entail only the *verbal* explanation of syllogistic problem solving, no notation session will be necessary. Because it is important to issue each step of the treatment for each group in parallel, the students in the Non-visual group will have a logic game to play on the computer during the first period of their treatment. This is illustrated in Figure 10. The computer game is entitled MasterGuess (Arning, 1985). This game can be played in two ways. The first way requires the student to formulate a pattern

code to match the one hidden by the computer. After each successive attempt, the student is given clues by the computer on how close they are to the secret code. This is done by awarding certain coloured pegs to denote correct pattern or position, or both. The game ends when the student arrives at the correct code within the allotted 12 attempts, or when the student fails to do so. The second way to play the game is by switching the role of student and computer. In this case, the student devises a secret code and the computer must reason out the pattern to match it. It is the student this time who helps the computer along the way by awarding the appropriate coloured pegs for pattern and position.

	GROUP 1 (Venn)	GROUP 2 (Sample Rep.)	GROUP 3 (Non-visual)
PERIOD 1	40 min. : instruction on notation 20 min. : test on notation	40 min. : instruction on notation 20 min. : test on notation	60 min. : play MasterGuess
PERIOD 2	40 min. : instruction on syllogisms	40 min. : instruction on syllogisms	40 min. : instruction on syllogisms
PERIOD 3	40 min. : posttest	40 min. : posttest	40 min. : posttest

Figure 10. Itinerary for data collection.

It is believed that MasterGuess is the computer game that most resembles the reasoning process under investigation for this study. The player is required, according to information provided, to arrive at a correct code. Similarly with syllogisms, using the premises provided, the students will be required to arrive at a valid conclusion.

Following the instruction period, the students will be tested with eight syllogisms. This will be a paper and pencil test (see Appendix K) and will consist of problems for which the students must come up with valid conclusions for two given premises. Each item will provide for the students a syllogism to be solved, a designated area to write their final conclusion, and a larger boxed area for their rough work to be shown. Every syllogism used in the instructional computer modules will have a valid conclusion; hence no item on the posttest will require 'no valid conclusion' as a response. The students will be taught to look for conclusions of the form A-C for every syllogism, and they will be instructed to answer each item on the posttest in this manner. The variable being observed, i.e. the outcome of the posttest (syllogistic reasoning achievement), is the dependent variable.

Scoring Scheme

The scoring scheme for posttest evaluation is outlined in Figure 11. It is divided into three sections: answer, method, and solution process. The latter section is subdivided further into two categories: premise notation and premise combination / reasoning.

Answer

The possible answers to any given syllogism must be analysed with respect to the correct, most general answer. If a student draws a conclusion that is not the most general solution, partial marks may or may not be awarded depending on how close or how "particular" that conclusion is. Figure 12 illustrates the categories of possible conclusions and their accuracy in relation to the most general solution.

In Figure 12, consider the conclusions "Some As are not Cs" and "All As are Cs". A statement that provides for each of these is "Some As are Cs". That is why the latter statement is called the most general conclusion. Each of the two former statements are called particular answers; one cannot imply the other, only together do they imply the most general solution. Particular answers are depicted as subsets of the set representing the correct conclusion.

		RESPONSE	POINTS
I.	Answer	Most general answer	4
		Particular answer: Special case	2
		Particular answer	1
		Wrong answer or no answer	0
II.	Method	Correct method	4
		Weak interpretatation of correct method	2
		Uses inappropriate method	0
III.	Solution Process		
	a. Premise notation	Both premises correct	3
		One premise correct	1.5
		Both premises incorrect	0
	b. Premise combination / reasoning		
		Correct reasoning	5
		Incorrect reasoning	0

Figure 11. Scoring scheme.

In this case, "No As are Cs" is considered an incorrect answer because under no circumstances can none of the As be Cs if the most general answer is "Some As are Cs". This response is depicted as an isolated, detached item relative to the other possible responses.

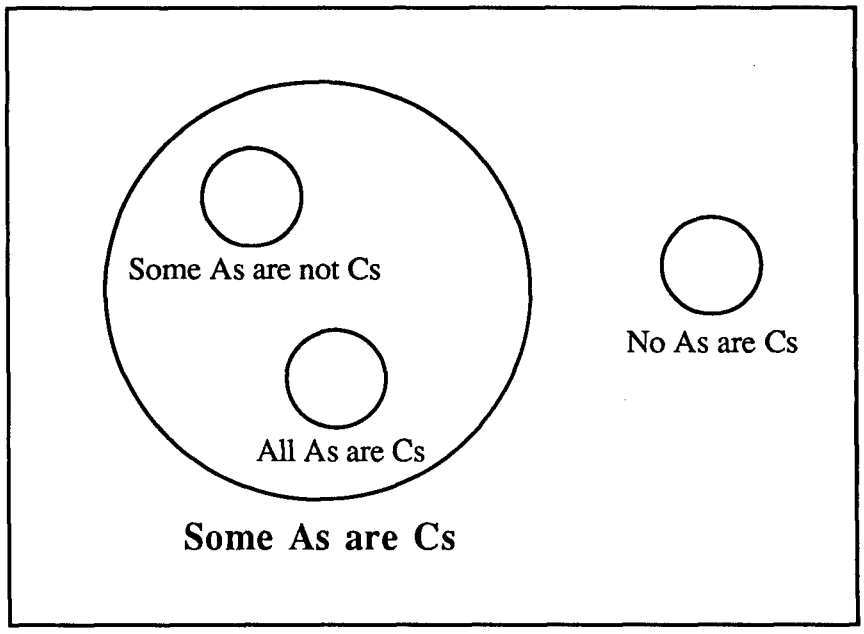


Figure 12. Case I: Most general answer - "Some As are Cs".

Among the two particular answers, "All As are Cs" is considered a special case of the most general conclusion. It is considered a more accurate conclusion than the other particular answer "Some As are not Cs", because the latter is definitely closer to "No As are Cs", which is a special case of "Some As are not Cs", and is also a completely incorrect answer.

Because students who arrive at the most general conclusion usually consider the two particular cases before reasoning out their final conclusion, this response is awarded 4 points, the maximum. Any particular answer is only awarded 1 point, as by and large, a particular answer can be derived from any correct combination of the premises involved and very little reasoning is necessary or used. An extra point is awarded if the particular answer is a special case of the correct conclusion. No points are given for an incorrect answer or for an item not attempted. The breakdown of points awarded for possible conclusions to a syllogism requiring "Some As are not Cs" as the most general answer is outlined in Figure 13. The justification for this distribution of points is similar to the argument presented above.

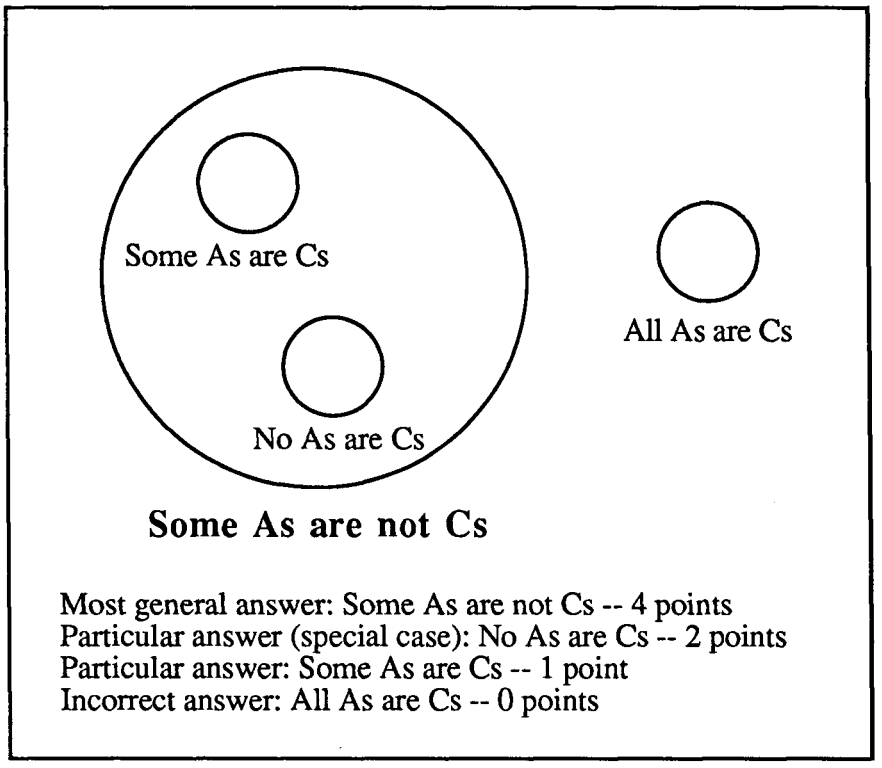
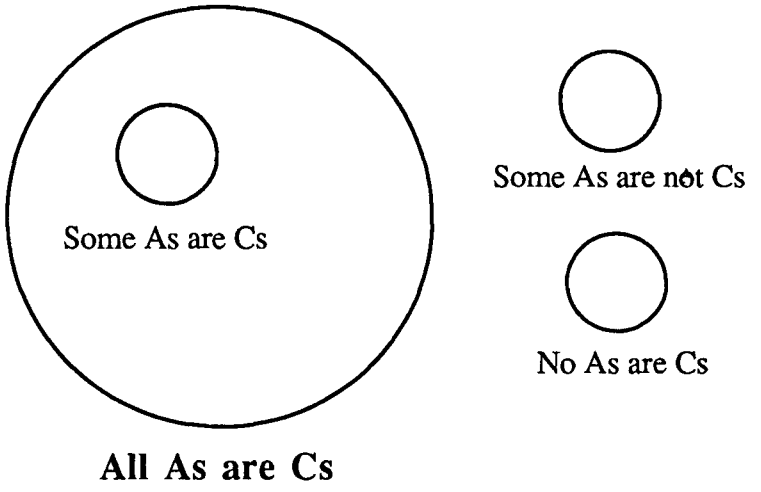


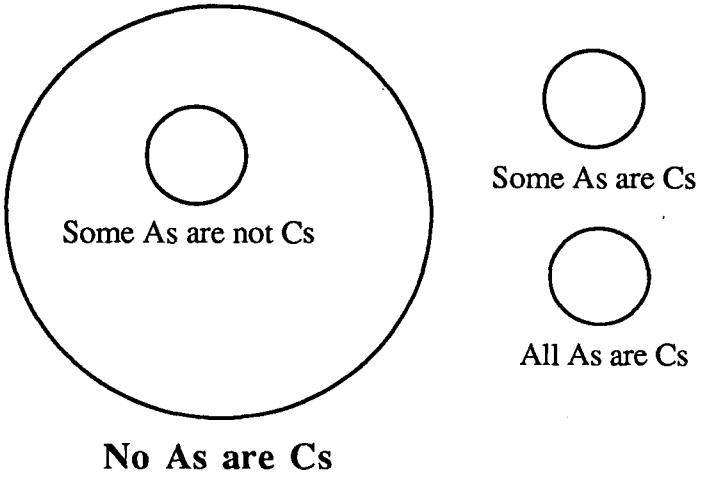
Figure 13. Case II: Most general answer - "Some As are not Cs".

Consider case III (Figure 14). It is difficult to call "All As are Cs" the most "general" conclusion in this case because to say that some of those As are Cs would also be correct. However, there are some syllogisms whose conclusions can be none other than "All As are Cs" or "No As are Cs" (case IV, Figure 15), so they will be called the most general answers for these types of syllogisms, Neither of them have true particular answers; however, in this case, "Some As are Cs" will be considered a particular answer worth 1 point. It is not worth more because it implies possible incorrect answers and is not a special case. The analogous particular answer for case IV is "Some As are not Cs", as is illustrated in Figure 15.



Most general answer: All As are Cs -- 4 points
Particular answer (special case): Does not exist
Particular answer: Some As are Cs -- 1 point
Incorrect answers: Some As are not Cs, No As are Cs -- 0 points

Figure 14. Case III: Most general answer - "All As are Cs".



Most general answer: No As are Cs -- 4 points
Particular answer (special case): Does not exist
Particular answer: Some As are not Cs -- 1 point
Incorrect answers: Some As are Cs, All As are Cs -- 0 points

Figure 15. Case IV: Most general answer - "No As are Cs".

Method

The purpose of this investigation is to compare the effects of three different methods of solving syllogisms. That is why points are awarded for the utilization of the correct method. It is important to note that methods are being compared and not the performances of individual students.

Four points for the employment of the correct method is given, the maximum awarded for this section. Students who interpret the method slightly differently interpret it incorrectly. Half of the maximum is allocated to these students. Subjects using a method not assigned to them (Venn, Sample Representation, or Non-visual) are given no points.

Solution Process

Premise notation.

Two premises must be represented correctly before the actual reasoning can take place. One and a half points are given for each correctly symbolized premise. No points are given if errors are found in both representations of the premises.

Premise combination / reasoning.

Independent of the final conclusion arrived at, 5 points are given to those who combine the two representations correctly and reason accurately from there. Those drawing incorrect conclusions will more than likely have reasoning errors, and are therefore awarded no points. Those arriving at either the most general answers, special cases, or even particular answers may very well carry through correct premise combination and accurate cognitive steps toward their conclusions. Posttest items will be studied closely for this occurrence, and points awarded accordingly.

Design

The emphasis in the experiment is not on how much each group will learn as a result of the exposure to the treatment in absolute terms, but how much each will learn in comparison to the other two groups. Owing to the comparative nature of the experiment, the Posttest-Only Control Group Design (presented diagramatically on page 178 in Gage, 1963) was modified and is diagrammed as follows:

RG_1	X_1	O
RG_2	X_2	O
RG_3	X_3	O

where G1 = Venn diagram group
G2 = Sample Representation group
G3 = Non-visual group

X1 = treatment for the Venn diagram group
X2 = treatment for the Sample Representation group
X3 = treatment for the Non-visual group and,

O = posttest on solving syllogisms.

Each of the three classes will be divided into three subgroups. These subgroups will be denoted as $g_{i,j}$, where i pertains to treatment group 1,2, or 3 and j pertains to the week in which the results will be collected, j =1, 2, 3. Therefore, $G_1 = g_{11} + g_{12} + g_{13}$, $G_2 = g_{21} + g_{22} + g_{23}$, and $G_3 = g_{31} + g_{32} + g_{33}$.

Specifically, the design can be elaborated according to the study at hand. Figure 16 illustrates the posttest-only control group design with each treatment group divided into three subgroups.

	Subgroup	Treatment		Posttest
WEEK 1	Rg_{11}	Venn	(X_1)	O
	Rg_{21}	Sample Rep.	(X_2)	O
	Rg_{31}	Non-visual	(X_3)	O
WEEK 2	Rg_{12}	Venn	(X_1)	O
	Rg_{22}	Sample Rep.	(X_2)	O
	Rg_{32}	Non-visual	(X_3)	O
WEEK 3	Rg_{13}	Venn	(X_1)	O
	Rg_{23}	Sample Rep.	(X_2)	O
	Rg_{33}	Non-visual	(X_3)	O

Figure 16. Research design.

There will be three days (or periods) in each week for data collection. The schedule for each week is outlined in the itinerary in Figure 10.

Verbal Instructions Given to Students

When assigned to their treatment groups and seats, the students will be told that they will be helping the researcher with a project being completed for the University of British Columbia. The nature of the investigation and the various stages of the study will be briefly explained to the students. They will always be given advance notice of when they will be tested and on what material. The students will be told that communication outside their own treatment groups is not permitted.

The students will be instructed on how the worksheets function in relation to the modules. Before the posttest is administered, the researcher will instruct the students to solve the items on the test using the method to which they are assigned,

as well as to show their work in the space provided. Furthermore, they will be told that they can score a higher grade by arriving at an invalid conclusion and showing their reasoning processes than by responding with a correct conclusion and no work shown. This is possible because of the way the scoring scheme was devised: the maximum amount of points awarded for the reasoning process is more than the maximum amount for the final answer. The different treatment groups will not be told they are all receiving the same posttest.

Steps Taken to Reduce Errors in the Study

It will be impossible to completely isolate the treatment groups within each week of data collection. This is owing to the way the computers are arranged in the laboratory. This arrangement forces some students from each group to be seated next to others using a different method. However, the maximum amount of treatment group segregation will be achieved: only 1 student from each of the Sample Representation and the Non-visual groups and two students from the Venn diagram group will be seated next to students using another method.

Many of the functions in Hypercard can be accessed through the keyboard. In order to avoid potential distractions caused by these functions, the computer modules will be designed so that the student does not need the use of the keyboard. Instead, they will be verbally instructed to use only the mouse, and on how to operate one.

For each test the students will receive, they will have worked through a worksheet identical in format. The worksheet accompanying the instructional modules will help the students to prepare for the tests to follow. It will reduce the possibility of misinterpretations of the problem tasks.

Analysis

The initial stage in testing both research hypotheses will be to test the following null hypothesis: $H_0 : \mu_1 = \mu_2 = \mu_3$. The method of analysis of variance (ANOVA) will be used to analyze the data obtained from the present

investigation. This analysis will determine whether or not there exists a significant difference among the three means. If a signficant F statistic is uncovered, then Tukey's (1949) method of multiple comparisons will be used to ascertain between which pair(s) of means the difference(s) were detected. The comparison of the means of the two visual approaches will determine whether or not the first hypothesis is supported. The comparisons of the means of each visual group to the mean of the Non-visual group will determine whether or not the second research hypothesis is supported.

Even though the second hypothesis is directional (each visual approach will result in greater achievement than the Non-visual approach), it was decided to use a two-tailed test. The one-tailed test provides more power, but, for all practical purposes, precludes identifying the possible situation where one or both of the visual approaches results in significantly lower achievement than the Non-visual approach. Although such a result would not reject the null hypothesis, it is worth identifying and discussing (Kirk, 1978).

EXPERIMENTAL VALIDITY

The internal validity of a study is a measure of the interpretability of its results. If the results of a certain study cannot be properly or completely explained due to the numerous extraneous uncontrolled factors which arise, then it lacks internal validity.

The extent to which the results of a study can be extrapolated to a general population (from which the sample was chosen, for example) is a measure of its external validity. Internal validity is a necessary condition for external validity. In other words, results of a study that cannot be interpreted with confidence cannot be extrapolated to a larger framework. But, on the other hand, if a study *is* internally consistent, it does not necessarily follow that it can be generalized.

Threats to Experimental Validity

Campbell & Stanley (1963) describe twelve threats to experimental validity; eight to internal and four to external. It should be noted that the posttest-only control group design is one of the most reliable experimental designs since internal and external validity are almost assured. This is why the design is called a "true experimental design".

Out of the aforementioned twelve threats, there are two potential sources of danger to the validity of the current experiment. Interaction of Selection and Treatment, in effect, does not cause any problems because the groups will not be created according to any particular factor, say intelligence, but rather will be randomly formed. With this procedure, the results will be generalizable. However, the Reactive Effects of Experimental Arrangements could play a role. The study may come up only with results which are representative of subjects who know that they are participating in an experiment and react differently due to the novelty of it (Hawthorne effect). Even though everyone in the study could be affected and there is no reason the reactive arrangements would differentially affect the treatment groups, it still may be true that the way humans reason logically in daily life may not be reflected in this study.

CHAPTER 4: RESEARCH FINDINGS

The statistical tests used for the current study were conducted to determine whether the method used for syllogistic problem solving had a significant effect on achievement and whether or not the two visual approaches significantly outperformed the Non-visual method. An analysis of variance (ANOVA) was used to determine whether any significant differences among the three means existed. The need for a multiple comparison test depended upon the significance of the ANOVA.

This chapter will describe the data collected, list the null hypotheses that were tested and discuss the results of each. In addition, the final section of the chapter will examine various observations made from the data that are worth special attention.

DESCRIPTION OF THE DATA

The dependent variable is syllogistic reasoning achievement. It was measured from the students' work on the posttest using the scoring scheme outlined in chapter 3. It was discovered after the data were collected that the first syllogism on the posttest required 'no valid conclusion' as the response. Since the students were not taught how to recognize these types of syllogisms, this item was omitted from the analysis.

The notation tests were evaluated not according to a scoring scheme, but according to how many final answers were correct out of eight test items. The raw data for the notation scores and the dependent variable are summarized in Table 1. The means for the Venn diagram, Sample Representation, and Non-visual methods will be designated by M_1, M_2, and M_3 respectively. One subject was absent from the Sample Representation group during the second week of the data collection and thus n_2 is 14 rather than 15. The Non-visual group was not tested for notation and hence no entries are in the table for this component.

Table 1

Data: Notation Test and Posttest

Method	n	Mean	S.D.[a]	Minimum score	Maximum score
Venn					
Notation test	15	87.29	14.41	55.13	100.00
Posttest	15	86.73	17.65	46.88	100.00
Sample Rep.					
Notation test	14	52.68	25.56	0.00	87.50
Posttest	14	47.99	13.68	23.21	76.34
Non-visual					
Notation test	---	---	---	---	---
Posttest	15	75.48	18.00	40.18	98.21

Note. Dashes indicate data not collected.

[a] S.D. = standard deviation

ANALYSIS

A comparison of the means for the three instructional methods can be made from Figure 17.

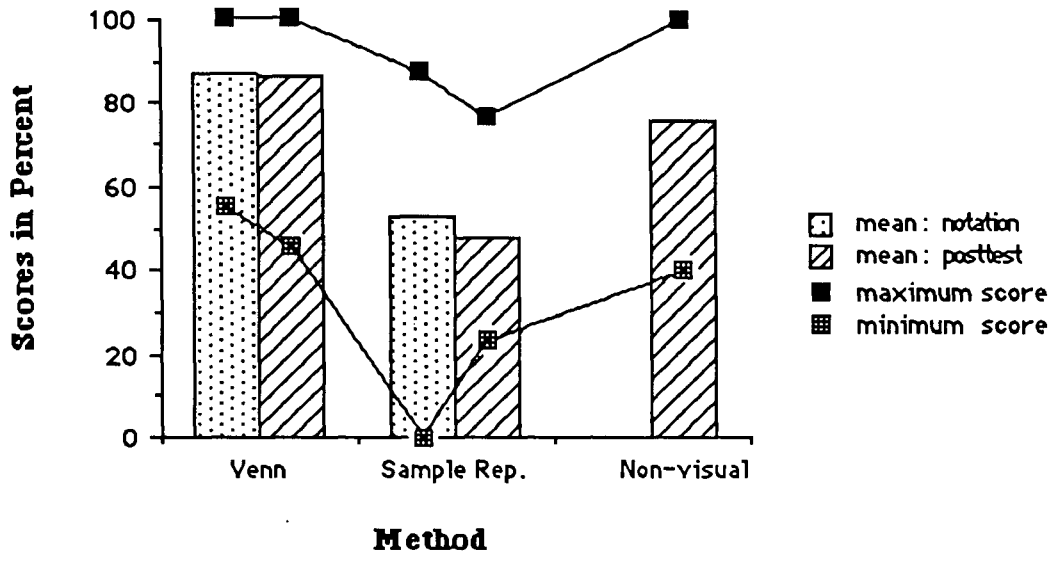


Figure 17. Means and extreme scores.

Without any analysis performed on the data, a few surprises arose. Correctly predicted was the substantial difference between the achievement resulting from the two visual approaches (the difference in their means being 38.74%); however, it was expected that *both* would substantially overshadow the effects of the Non-visual method. The posttest scores for the Venn diagram approach resulted in a mean of 11.25% higher than that for the Non-visual approach; this difference was statistically analyzed for significance and is discussed later in the chapter. However, the mean on the posttest for the Sample Representation method fell short of the mean for the Non-visual scores by 27.49%. Owing to this result, the second research hypothesis (both visual methods significantly outperforming the Non-visual one) is automatically rejected. The Tukey test comparing μ_2 and μ_3 was conducted because it was intriguing to determine an overall idea of the relative inefficacy of the Sample Representation method. The test comparing μ_1 and μ_3 was conducted to ascertain the effects of

the Venn diagram method on syllogistic reasoning.

Analysis of Variance and Tukey's Comparisons

All 44 scores were used for the analysis of variance. An outline of this analysis can be found in Table 2.

Table 2

Analysis of Variance for Data of Table 1

Source of variation	Sum of squares	Degrees of freedom	Variance estimate	F
Between	11 414.36	2	5 707.18	20.65 *
Within	11 330.01	41	276.34	---
Total	22 744.37	43	---	---

* $p \leq 0.01$.

The significant F statistic (20.65, $\alpha \leq 0.01$) allows one to conclude that the method did significantly affect syllogistic reasoning achievement, but Tukey's multiple comparisons are needed to find exactly where this significance lies. Tukey's multiple comparison test is used when a significant F statistic is computed from the analysis of variance. It determines between which pair or pairs of means the significance exists. The ensuing analyses are the tests for the two hypotheses discussed in chapter 3.

H_1: *There will be no significant difference in syllogistic reasoning achievement between the Venn diagram approach and the Sample Representation approach.*

$(\mu_1 - \mu_2 = 0)$.

Using Tukey's method of multiple comparisons, a significant F statistic of 39.32 was found ($\alpha \leq 0.01$). Based on this result, hypothesis H_1 was rejected.

Given that $M_1 > M_2$ and the F statistic was rejected, it was concluded that the Venn diagram instructional approach to syllogistic reasoning significantly outperformed the Sample Representation approach.

H_2: *There will be no significant difference in syllogistic reasoning achievement between either of the two visual approaches used for H_1 and the Non-visual approach.*

($\mu_1 - \mu_3 = 0$ and $\mu_2 - \mu_3 = 0$).

There must be no significant difference between the means of either the Venn diagram or the Sample Representation methods and the mean of the Non-visual method in order for H_2 to be supported.

Using Tukey's test, a significant F statistic (21.51, $\alpha \leq 0.01$) was found between the means of the Venn diagram method and the Non-visual method (comparison 2). It can be concluded that the Venn diagram approach resulted in significantly superior achievement in syllogistic reasoning than the Non-visual approach.

A significant F statistic (19.80, $\alpha \leq 0.01$) was found between the means of the Sample Representation approach and the Non-visual approach (comparison 3), again using Tukey's test. However, since $M_2 < M_3$, the conclusion is that the Sample Representation approach resulted in significantly lower reasoning achievement compared to the Non-visual approach.

Although comparisons 2 and 3 resulted in statistically significant differences, the direction of these differences led to the second research hypothesis being rejected (only the Venn diagram method outperformed the Non-visual method). The analysis of variance for all three Tukey orthogonal comparisons is outlined in Table 3.

Table 3

Analysis of Variance for Tukey's Orthogonal Comparisons

Source of variation	Sum of squares	Degrees of freedom	Variance estimate	F
Comparison 1	10 865.53	1	10 865.53	39.32*
Comparison 2	5 943.38	1	5 943.38	21.51*
Comparison 3	5 470.98	1	5 470.98	19.80*
Within	33 609.90	41	276.34	---
Total	55 889.79	44	---	---

Note . Dashes indicate non-essential or inapplicable data.

*$p \leq 0.01$.

OBSERVATIONS OF THE DATA

For each visual method, the mean posttest score was slightly lower than the mean score for the respective notation test by an average of almost 3% (see Figure 17). A reasonable guess as to why the mean posttest scores were lower than the respective notation test mean scores is that for the items on the posttest, the subjects had to use the notation learned earlier in order to understand each preliminary step of the solution process *and* to learn the reasoning needed to complete the problem. In other words, the material tested in the first instance was also included and marked in the second instance. Therefore, the scores based on combined techniques were not as high as scores on the previously learned individual technique.

The mean of the posttest scores for the Sample Representation group was lower than that for the Venn diagram group by almost 39%. It is therefore possible that the students using Venn diagrams obtained higher scores on the posttest because the required notation was easier to understand. This is further supported by the finding that the notation scores of the Venn diagram group exceeded those of the Sample Representation group by almost 35%.

Noteworthy is the large standard deviation for the notation scores under the Sample Representation instruction (see Table 1). This may imply that the scores originated either from a cluster at the low end of the scale or a cluster at the high end of the scale. It may be speculated that the nature of this notation did not allow for many scores to fall in the intermediate range. Therefore, it may be possible that the students either fully grasped the main ideas behind this notation, or they grasped few, if any, of the main ideas.

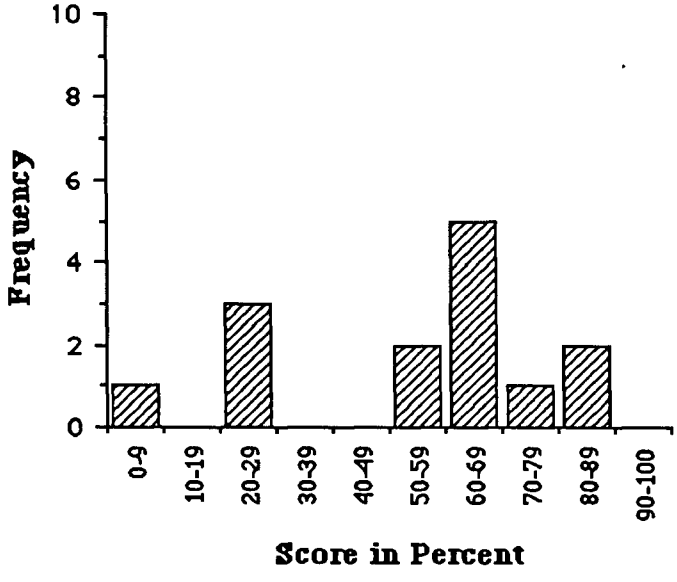


Figure 18. Frequency distribution for Sample Representation notation scores.

The frequency distribution for the Sample Representation notation scores can be studied from Figure 18. It is interesting to note that there were no scores between 30% and 50%. Just under 80% of the subjects achieved a score lower than 63%. Of these subjects scoring under 63%, almost 55% scored 50% or lower. At the high end of the scale, 14% received a score of 80% or higher. In comparison, 13% of the subjects under the Venn diagram instruction achieved a score of 63% or less on the notation section of the testing and 73% scored 80% or higher. This is illustrated in Figure 19.

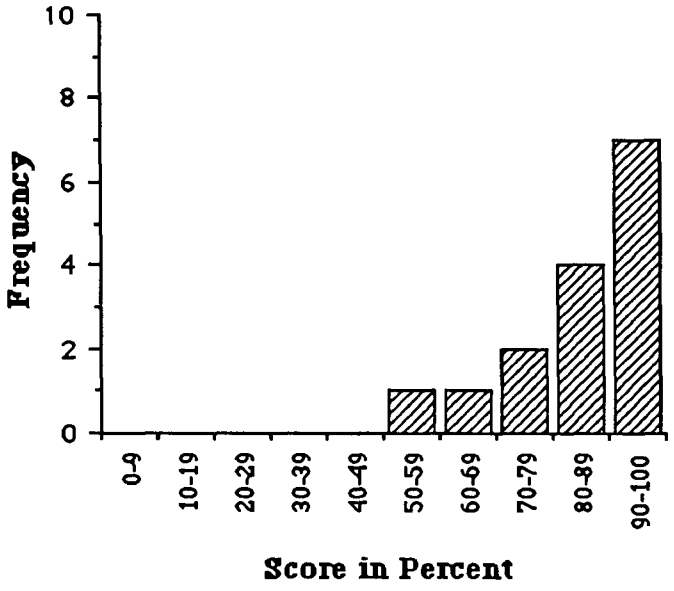


Figure 19. Frequency distribution for Venn diagram notation scores.

Another interesting observation emerges from the frequency distribution of the posttest scores under the Venn diagram instruction. This distribution can be found in Figure 20. Over 53% of the subjects received a score of 97% or higher and only 13% received a score of 54% or lower.

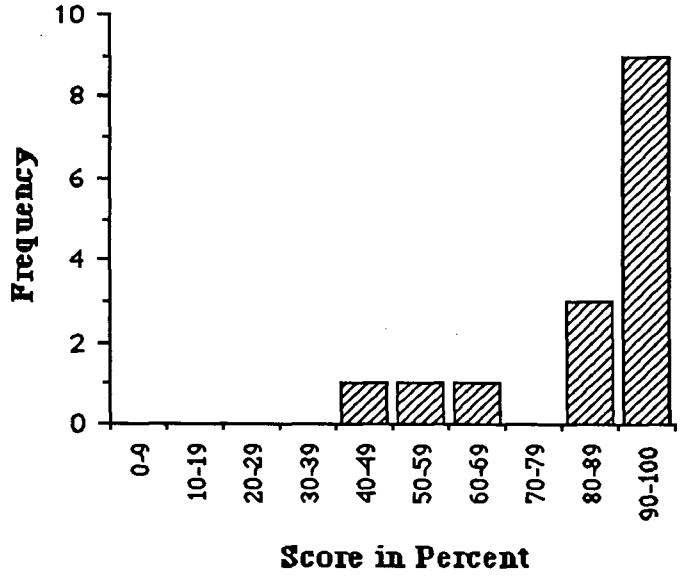


Figure 20. Frequency distribution for Venn diagram posttest scores.

Studying the data from the Sample Representation group, on the posttest, almost 79% achieved a grade less than or equal to 54%. Only 21% received a grade higher than 54%, and the maximum score was 76%. The frequency distribution for these scores can be found in Figure 21.

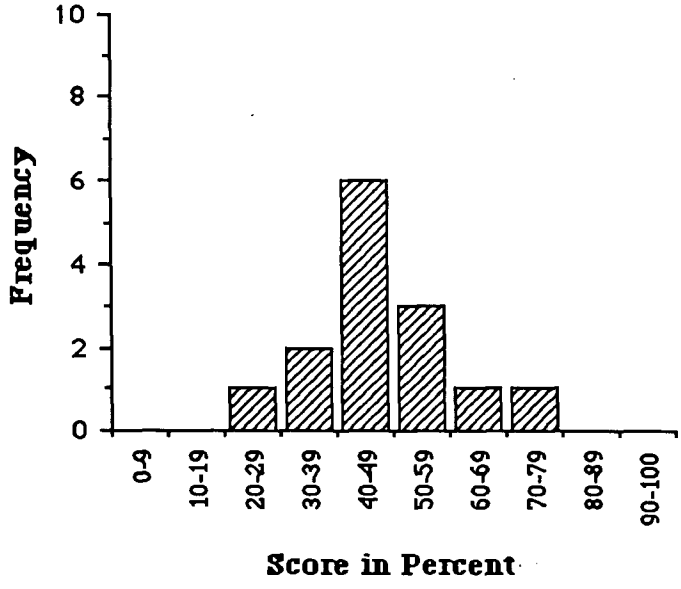


Figure 21. Frequency distribution for Sample Representation posttest scores.

Comparing the distributions presented in Figures 20 and 21, it is worth noting that while 80% of the Venn diagram group scored 80% or higher, no one in the Sample Representation group did so. At the other end of the scale, while only 7% of the Venn diagram group scored under 50%, 64% of the Sample Representation group did so.

The frequency distribution for the posttest scores for the Non-visual method is presented in Figure 22.

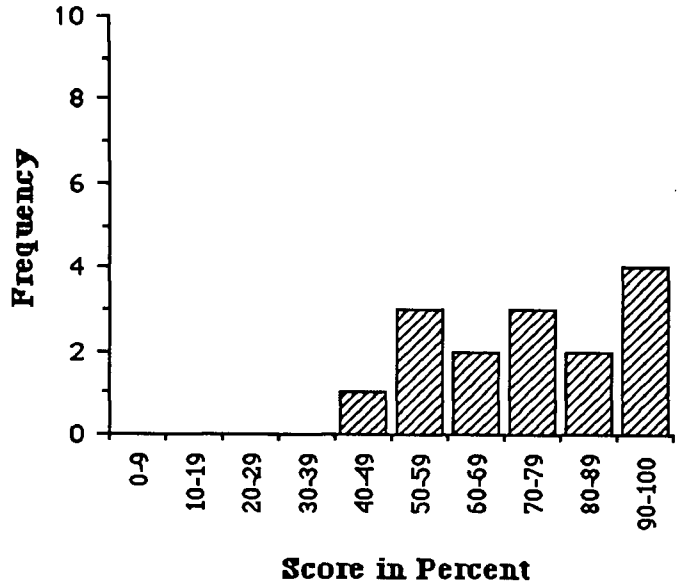


Figure 22. Frequency distribution for Non-visual posttest scores.

CHAPTER 5: SUMMARY AND DISCUSSION

SUMMARY

Restatement of the Problem

The phenomenon under study for the present investigation was syllogistic reasoning. To be specific, the syllogisms used for this study consisted of two premises, each expressing a relationship between two sets using quantifiers (all, some, no, some are not). Linear syllogisms were excluded for this experiment.

It was the purpose of this study to examine the effects of three different instructional methods on syllogistic reasoning achievement. Of particular interest were the two methods utilizing visual constructs: the Venn diagram method, discussed by Ceraso & Provitera (1971), Erickson (1974), Neimark & Chapman (1975), and Revlis (1975), and the Sample Representation method designed by Johnson-Laird & Steedman in 1978. The third instructional method was a purely semantic one. The lessons consisted of verbal explanations of the reasoning needed to solve Aristotelian syllogisms.

It was hypothesized that firstly, one of the two visual methods would significantly outperform the other, and secondly, that *both* visual methods would result in significantly greater syllogistic reasoning achievement than the Non-visual approach.

Worksheets accompanied each instructional module which were designed using Hypercard for the Macintosh microcomputer. The students worked with the modules independently and at their own pace within the allotted time.

Method

Forty-five grade 8 honours students from one school in a metropolitan school district served as the sample. Three intact classes of fifteen students were offered for the project, one for each week of the data collection period. The students in each class were randomly assigned to the treatment groups. One student assigned to the Sample Representation group was absent, and therefore did not participate in the study.

The materials needed to conduct this experiment were: a Macintosh microcomputer laboratory consisting of fifteen computers; approximately 20 floppy disks (3.5") on which to load the appropriate instructional modules; worksheets, for both stages of the treatment; and 44 posttests similar in format to the worksheets.

The design used for the study was the posttest-only control group design (with the Non-visual comparison group replacing the control group). The treatment for each of the visual methods was administered in two stages. The first stage introduced the student to the notation needed for the specific method; the second stage provided instructions on how to correctly draw valid conclusions for various Aristotelian syllogisms using the notation introduced in the first stage. After each notation lesson, the students were tested on their understanding of the material presented. After the second stage, the subjects were posttested with eight syllogisms. All students were tested simultaneously.

When they were tested on notation, all the students in both visual groups used the proper notation. In addition, all the students in the Venn diagram group used the appropriate notation on the posttest. However, many of the students in the Sample Representation group did not use the notation they were taught when solving the syllogisms on the posttest. Many chose to use modified versions of the representations presented to them during the first stage of the treatment. Perhaps the notation needed for the Venn diagram method was more easily remembered than the notation required for the Sample Representation method.

The students in the Non-visual comparison group were instructed with one module on reasoning, and were not instructed or tested on any notation. They, too, were given the same posttest at the same time as the students in the other two groups. During the notation sessions for the groups using visual approaches, the students in the Non-visual group were given a logic game to play on the computer. This game required students to formulate pattern codes, according to information provided them, to match a computer-generated code hidden from view. The game was entitled MasterGuess and it involved similar logical processes to syllogistic reasoning.

Analysis

An analysis of variance was conducted on the data collected in order to determine whether a significant difference existed among the three means. After a significant F statistic was computed ($F = 20.65$, $\alpha \leq 0.01$), Tukey's method of multiple comparisons was used to ascertain whether significant pairwise differences existed between any two groups. The research hypotheses will be discussed here.

Results

H_1: *There will be a significant difference in syllogistic reasoning achievement between the Venn diagram approach and the Sample Representation approach.*

The mean for the Venn diagram group was found to be significantly higher than the mean for the Sample Representation group ($F = 39.32$, $\alpha \leq 0.01$). The significant difference found between the effects of the Venn diagram and the Sample Representation methods leads one to conclude that Venn diagrams are a superior method to the teaching of syllogistic reasoning than the approach based on the theory proposed by Johnson-Laird & Steedman (1978). Various possible reasons for this result are discussed later.

H_2: *Both visual approaches used for H_1 will result in significantly greater syllogistic reasoning achievement than the Non-visual approach.*

To determine whether this hypothesis is rejected or not, two comparisons were necessary; both the Venn diagram and the Sample Representation approach had to be significantly superior to the Non-visual approach.

Upon analysis of the data, however, it was established that H_2 is rejected. This is owing to the mean of the Non-visual group being significantly higher ($F = 19.80$, $\alpha \leq 0.01$) than that of the Sample Representation group.

Even so, it was found that the mean for the Venn diagram group was found to be significantly higher than the mean for the Non-visual group ($F = 21.51$, $\alpha \leq$

0.01). This result cannot affect the support or rejection of H_2, but it aids in understanding the general theory behind the success of the Venn diagram method with respect to syllogistic reasoning. This also will be discussed in the next section.

INTERPRETATIONS

The results have shown that the Venn diagram approach has offered clear and straightforward representations of the premises as well as an easy way to use these representations to solve syllogisms accurately. Consequently, the Venn diagram method has shown to be the effective way to solve these types of reasoning tasks.

The students in the Sample Representation group performed very poorly on the posttest in comparison to the other groups. The required notation which is decidedly more demanding for these students may be largely responsible for this. The students in the Sample Representation group would probably have benefited from a longer notation instruction period in order to have been better prepared for the method.

The results show that there exist serious defects to the Sample Representation method itself. Because visual methods for problem solving are almost always encouraged over non-visual ones, it might be beneficial to locate these defects and work to remove as many as possible from the technique.

Because the results have revealed the success of the Venn diagram method over the Sample Representation method, the criticisms of the former method put forward by Johnson-Laird & Steedman (1978) must be re-examined. They will be reassessed later in the chapter.

The Non-visual method has emerged as a satisfactory method for syllogistic reasoning. However, the results could not reveal the effectiveness of the instruction itself as accurately as the other two methods. This is owing to the unstructured nature of the Non-visual method which allowed for the development of more individual techniques.

BEYOND THE STUDY

This section is included to go beyond the data of the current study. The current study has shown that the Venn diagram approach yields greater syllogistic achievement than the Sample Representation approach. Having established that point, several educational implications will be put forward. In addition, possible reasons why Venn diagrams proved so successful, such as intuitiveness, familiarity, and simplicity, are offered in this section. Two minor findings of this study and the reasons behind their occurrence will be discussed as well.

Implications

It has been established that the Venn diagram approach was the most effective of the three approaches for accurate syllogistic reasoning. This result has numerous educational implications. As was discovered in the literature review, individuals of every age have difficulty with formal logical problems (Anderson, 1980; Erickson, 1974; Henle, 1962; Revlis, 1975). It was therefore imperative to determine the most powerful method for teaching accurate syllogistic reasoning.

There are areas where syllogistic competence is required and many others where it is valued, if not required. In mathematics, the comprehension of the operations under such formal systems is an asset because of the deductive nature of mathematics. If researchers and educators are interested in expanding the secondary mathematics curriculum to include syllogistic reasoning, then it is proposed to slowly integrate syllogistic reasoning in the mathematics curriculum by first introducing it as enrichment material. The advantage to this is that syllogistic reasoning develops a formal logical approach to thinking and cultivates imaginative approaches to problem solving. This is not to say that syllogistic reasoning is always used in daily reasoning and decision making. [In fact, Schiller (1930) asserts that syllogistic reasoning "has nothing whatever to do with actual reasoning" (p. 282).] However, syllogisms will very likely aid in future mathematical and scientific endeavours that operate under formal logical systems.

This study has established the effectiveness of one of the three most commonly used approaches when compared to the other two. Based on the results

of the study, educators and curriculum designers should consider presenting syllogisms using Venn diagrams exclusively. In fact, Venn diagrams should be encouraged whenever syllogisms are encountered and should be made the standard method for solving problems of this nature.

If a similar instructional approach is to be used (i.e. computer modules), then improvements should be made to the instructional materials. In particular, the modules should be designed so that the student would be able to learn interactively with the computer. Students should be able to learn the solution process through their mistakes. In other words, the computer lessons should allow them the opportunity to develop their own heuristics rather than simply providing them with algorithms.

If the Venn diagram method were used in a secondary mathematics course (and not in an experimental context), the modules should then be modified to include non-abstract elements, as suggested by researchers Johnson-Laird & Steedman (1978), and Wason & Johnson-Laird (1972). The worksheets should include more diverse problems and be altered to match the remodelled lessons.

It is strongly advised that teachers personally review the characteristics of the syllogism and the Venn diagram method for arriving at valid conclusions if syllogisms are to be introduced in the curriculum. This is necessary because this particular form of reasoning is not a trivial task, nor is the utilization of Venn diagrams in a reasoning environment.

Using the results of this study, it is difficult to generalize about visual aids in relation to other problem solving tasks, academic or otherwise. Just because Venn diagrams work for syllogistic reasoning does not necessarily imply that all visual aids would work for any other problem solving task, although it would be tempting to declare the use of diagrams a universal must for any problem which requires creative thinking and reasoning. Because the other visual method in this study fared so poorly, and because the purely verbal method significantly outperformed the Sample Representation method, a general conclusion is difficult to make. Furthermore, mental diagrams are rarely employed in daily decision making thus narrowing the conclusions that can be extracted from this study. Still,

one can safely claim that using Venn diagrams provides an excellent approach for studying syllogistic reasoning.

Speculations

The conclusion based on the confirmation of the first research hypothesis was that Venn diagrams resulted in significantly greater syllogistic reasoning achievement than Sample Representations. The conclusion leads to the following speculation: the Venn diagram method is more "intuitive" than the Sample Representation method.

Upon casual inspection, many of the qualities of intuition and insight that Bastick (1982) lists (see Table 4) are also properties of the Venn diagram method of reasoning. Very few of them characterize the Sample Representation method, however, and using this list as a measure of intuitiveness, it can be said that Venn diagrams therefore present a method that utilizes more immediate and instinctive reasoning techniques. Several relevant properties will be selected to illustrate this point. This selection is somewhat arbitrary, but the characteristics chosen seem to apply more readily to the discussion.

Property No.1: Quick, immediate, sudden appearance.

According to what was observed during the data collection, the students in the Venn diagram group completed their assigned tasks much earlier than the students in the Sample Representation group. Their worksheets were completed with seemingly less effort, and the majority of them handed in their test papers before any of the Sample Representation subjects did. Although no data were collected to support this, the researcher has come to these opinions through what was seen and heard during the administration of the treatment.

The procedure of drawing a valid conclusion after premise combination using the Venn diagram method requires a visual inspection of the final configuration. This step in the reasoning process is rather immediate and based on sudden appearance. The Sample Representation method, on the other hand, requires many more analytic, computational-type steps in order to arrive at the correct conclusion to the given syllogism. This behavioural approach can be

labelled less instinctive than the gestalt-style reasoning that the Venn diagrams offer.

Table 4

Numbered Properties of Intuition and Insight

No.	Properties
1	Quick, immediate, sudden appearance
2	Emotional involvement
3	Preconscious process
4	Contrast with abstract reasoning, logic, or analytic thought
5	Influenced by experience
6	Understanding by feeling - emotive not tactile
7	Associations with creativity
8	Associations with egocentricity
9	Intuition need not be correct
10	Subjective certainty of correctness
11	Recentring
12	Empathy, kinaesthetic [sic] or other
13	Innate, instinctive knowledge or ability
14	Preverbal concept
15	Global knowledge
16	Incomplete knowledge
17	Hypnagogic reverie
18	Sense of relations
19	Dependence on environment
20	Transfer and transposition

(Bastick, 1982, p. 25)

Property No. 5: Influenced by experience.

The concept of grouping objects within closed sets inherent in the Venn diagram set theory is suggestive of a practical, experiential influence. In daily experience, one is constantly exposed to discrete objects enclosed in containers of various sizes. Examples of this are easily found as it is common and directly applicable to everyday problem solving and decision making. Features of the Sample Representation theory, on the other hand, involve either positive or negative links between two objects, and these are less concrete and less commonplace in daily experience than the features of the Venn diagram method. In particular, negative links are not practically obvious, and negative links between set members are even less so.

Property No. 7: Associations with creativity.

The Sample Representation method is completely closed to any type of reorganization, reformulation or remodelling of any kind. It does not lend itself to creative manipulation. By contrast, the Venn diagram method is very open to creative operations. Each premise, except for one, is presented with at least two possible representations. In the premise combination stage of the solution process, many configurations are indeed possible. In addition, each set can be moved around as long as the intersections remain consistent.

The Sample Representation method allows for no rearrangement of components at all. As a matter of fact, each premise itself can be represented only one way and once two are combined, the entire reasoning procedure remains rigid and inflexible. This serves as a disadvantage because the student has only one chance at representing each premise correctly. When using Venn diagrams, on the other hand, the student has several options for premise representation. Therefore, the solution process for the solving of a given syllogism is more likely to be correct when Venn diagrams are used.

Property No. 13: Innate, instinctive knowledge or ability.

The Venn diagram method is more intuitive than the Sample Representation method because the former relies on ingrained, instinctive reactions. Sample Representations rely more on memorization than understanding and therefore

they stray from instrinsic abilities.

Property No. 14: Preverbal concept.

Venn diagrams involve *familiar* imagery-based ideas; ideas that become recognizable at an early age. In fact, Paivio (1971) asserts that verbal language acquisition depends on an earlier stage of imagery development. Woodson (1984) summarizes his theory: "The infant, through exposure to objects and events, develops a storehouse of images. Language builds from this storehouse and remains interlocked with it" (p. 4). These images include items in closed containers, rearrangement of objects, and the determination of common items. Many children's games involve just these notions.

It may be hypothesized that the Sample Representation method involves both pre- and post-verbal concepts. The idea of representing a set with typical semantic-based members constitutes the post-verbal concept and the determination of positive or negative links between two set members contributes to the pre-verbal nature of the method, even though, as mentioned above, links, especially negative ones, are not a commonplace notion.

Property No. 15: Global knowledge.

Venn diagrams tend to be highly intuitive due to the fact that they are, after all, an internationally accepted way to diagrammatically represent sets and their members. They have become, since their conception in the eighteenth century, global knowledge. The method developed by Johnson-Laird & Steedman (1978) has not. Their Sample Representation approach was created as an alternate method to Erickson's (1974) set theory (Venn diagram approach) processes for syllogistic reasoning. It has not become popularly known, and is in no secondary mathematics curriculum.

Property No. 19: Dependence on environment.

This property overlaps with property no. 5 because experiences usually occur within and due to the environment. The environment offers many practical examples of sets found in sociological contexts. For example, from an early age, children learn that all automobiles are divided up into separate categories or sets (i.e cars, trucks, vans and so on). The Sample Representation approach does not

relay the set theme as clearly as do Venn diagrams. It is therefore difficult for those using the Sample Representation approach to rely on environmental familiarity to help them with the notion of set representation and manipulation.

The conclusion based on the rejection of the first hypothesis leads to a second speculation: the Venn diagram method is more familiar to grade 8 students than the Sample Representation method.

In grade 8 and in previous years, students are not introduced to set theory in relation to logical processes. However, as early as grade 1, students are introduced to the notion of grouping objects together in order to perform certain operations. The British Columbia Mathematics Curriculum Guide, grades 1-8 (1987), lists diagrammed exercises for grades 1 through 4 that use grouped objects for the introduction of basic concepts such as addition, subtraction, repetition of sets, and comparison of objects from set to set. In grades 7 and 8, the students learn to determine one-to-one correspondences between two sets, and to specify, draw, or state the union of two or more sets and the intersection of two or more sets.

It is apparent that the students in the Venn diagram group were more familiar with the components they were manipulating than the students in the Sample Representation group were with theirs. It should be noted, however, that the students' familiarity with illustrated set concepts did not necessarily result in an advantage over the reasoning process itself. In addition, the exercise of determining correspondences between two sets, an intended learning outcome listed in The British Columbia Mathematics Curriculum Guide, grades 1-8 (1987), may have familiarized the students in the Sample Representation group with the notion of 'linking' two set members together, a concept highly necessary for this method.

Thirdly, it may be speculated that the Sample Representation method is more complex than the Venn diagram method. Each premise representation in Johnson-Laird & Steedman's (1978) theory allows for all possibilities and special cases in one configuration. For example, the representation for "Some As are Bs" allows for all cases in one diagram: All As are Bs (the case where sets A and B are the same size *and* the case where set A is completely contained within set B), and

strictly some As are Bs (the case where sets A and B of the same size overlap, *and* the case where set B is completely contained within set A). This appears to have presented the learner with too many ideas to process at once. The Venn diagram approach handles every special case in a separate representation. The learner was required to master a given diagram before moving on to the representation of another case.

The Sample Representation scheme requires a thorough understanding of what exists and what *may* exist. This looks as if it may have obstructed the clarity of the method. Venn diagrams appeared to have been easier to grasp because they deal only with the actual elements existing in the depicted sets. This method is less ambiguous and offers a more concrete illustration of premise representation.

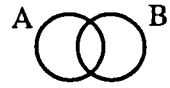
The Sample Representation method tends to be too 'discrete'. By this it is meant that it contains too many small, disconnected components such as set members (letters), arrows, negative links, brackets, plus and minus signs, and so forth. Furthermore, the process linked with this method requires many more steps than the Venn diagram method to arrive at a valid conclusion. A comparative investigation of the steps involved for each of these two methods is outlined in Figure 23. Three basic steps are needed when utilizing each method, however, upon closer inspection, the Sample Representation method actually requires at least two more steps than the Venn diagram approach. In addition, in step 2, some restructuring while combining the premises might be necessary. Furthermore, it is impossible after premise combination to arrive at a valid conclusion soley on the basis of inspection. Some calculations are needed, such as determining possible paths between sets A and C, constructing *existing* links between these two sets, and determining *possible* links between these two sets. Only then can a valid solution be determined.

It is therefore speculated that one of the reasons for the Venn diagram method's success is its lack of complexity. Its simplicity is exhibited by the similarity of the first solution step to the final one. In other words, the process required in drawing a valid, final conclusion resembles the process of determining the representation of the individual premises. Both involve the same

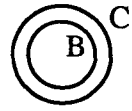
Venn Diagram Solution Process

1. separate representation of the two premises

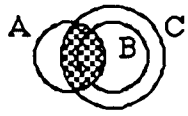
a. Some As are Bs



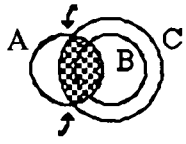
b. All Bs are Cs



2. combination of the premises



3. drawing of a valid conclusion
   This step entails:
   i. inspection of the final configuration
   ii. determination of a relationship between sets A and C



⇒ Some As are Cs

Sample Representation Solution Process

1. separate representation of the two premises

a. Some As are Bs

A (A)
↓
B (B)

b. All Bs are Cs

B
↓
C (C)

2. combination of the premises (may require some restructuring)

A (A)

B (B)
↓ ↓
C C (C)

3. drawing of a valid conclusion
   This step entails:
   i. determination of existing paths between members of sets A and C
   ii. where two separate links already exist, construction of the link between members of sets A and C (positive or negative)

A (A)
↓ ?
(B)
↓
C C (C)

   iii. where only one link exists, construction of possible links between members of sets A and C
   iv. according to what links exist and what links may exist, the determination of a valid conclusion.

A (A)
↓ or ⊥
(B)
↓
C C (C)

⇒ Some As are Cs

Figure 23: A comparitive analysis of solution steps.

principle: the utilization of the intersection of two sets to represent a relation between them. The only difference between these two steps is that the separate representations of the premises requires a construction of the suitable intersections (if any), and the final step requires the extraction of the meaning of the intersection formed after premise combination.

The Sample Representation method cannot be analyzed so neatly. The final computational steps involved in reasoning out a conclusion differs from premise representation because the diagram for each premise remains constant whereas the solution steps differ from syllogism to syllogism, depending on which two premises are supplied.

Another finding of the current study was that Venn diagrams resulted in significantly greater syllogistic reasoning achievement than non-visual representations. The explanation of this expected result stems directly from the review of the literature. The literature suggests that diagrammatic aids enable the learner to constructively organize the various components of a problem in such a way as to facilitate the solution process. This result confirms studies conducted by Dirkes (1980), Larkin & Simon (1987), MacDonald-Ross (1979), Nelson (1983), Threadgill-Sowder et al. (1985), and Waller (1979).

A third finding of the study was that non-visual representations resulted in significantly greater syllogistic reasoning achievement than Sample Representations. It is proposed that the Non-visual method produced significantly higher reasoning achievement than the Sample Representation method because of the fact that the instruction consisted only of words. The instruction gave the students no structured visual patterns to rely on, in contrast to the other two methods. Sadoski (1983) conducted a study to determine the mental images third grade students reported after reading a story. The story utilized contained three illustrations, but the climax was not pictured. It was found that two groups of about equal size surfaced; one group did not visualize the unillustrated climax, the other did. In a replication of the study conducted by Sadoski in 1985, a story was used that had no illustrations whatsoever and it was found that the occurrence of imagery during the reading of the story was reported twice as much. Sadoski

(1985) suggests that "perhaps unillustrated stories invite more reader imagery" (p. 664). Thus, regarding the present investigation, it is proposed that the students given no visual stimuli were forced to create their own. Evidence for this proposition will be discussed later in the chapter. In comparison to the Sample Representation method, the students assigned to the Non-visual group used a method which was not as restrictive, and they were free to mentally represent and combine sets as they wished.

Other researchers have studied mental imagery generated by verbal text. Woodson (1984), for example, studied the relationship between semantic language and mental imagery. She claims that mental images are "produced directly through descriptive language - color words, texture words, size words and so on . . . " (p. 4). Bruner (1964) studied language acquisition and determined three distinct stages, the second being the 'ikonic' stage, in which an individual's surroundings are represented by mental images. The stage following this is the verbal or 'symbolic' stage, implying that image formation is necessary for semantic acquisition.

Paivio (1971) claims that even the most abstract written (or verbal) passages require the formation of mental images for understanding and interpretation to take place. Woodson (1984) states: "Certain conceptual pegs in language, both descriptive and abstract, trigger non-verbal images which help the reader to retain not the syntactic properties of a language but the semantic interpretations" (p. 4). She continues to claim that the reader uses the semantic symbols in written text to form the necessary images, and then stores those images in order to be able to interpret the text. Langer (1942) proposes that the two processes, verbal understanding and image generation occur at the same time. At the paragraph level, Woodson (1984) asserts that the images are generated sequentially sentence by sentence and then are put in storage for later development of a general, ordered understanding, or form, of the paragraph.

Other proponents of this theory are Juola, Schadler, Chabot, & Mccaughey (1978) and Lesgold & Curtis (1981). Further researchers who have pointed out the substantial importance of mental imagery in the recognition and

understanding of either words, phrases, sentences, or paragraphs are Bower (1970), Bugelski (1968), Johnson (1970), Paivio (1969), Anderson & Hidde (1971), Begg (1971), Begg & Paivio (1969), Levin, Davidson ,Wolff, & Citron (1973), Montague & Carter (1973), Pompi & Lachman (1967), Steingart & Glock (1979), and Yuille & Paivio (1969).

Thus, due to the research supporting the theory that written text contains image-producing features, it can be hypothesized that the students in the Non-visual group developed their own, by and large internal, visual techniques superior to the ones presented to the students in the Sample Representation group.

Why was the Non-visual approach significantly superior to the Sample Representation method if the latter *already* offered a visual strategy for the reasoning process? It is suggested that the type of mental imagery created by the Non-visual group greatly resembled the Venn diagram approach and evidence of this will be presented in a later section of the chapter. It is proposed that these students used a collective, closed, and rather compact representation for sets A, B, and C. It is guessed that they chose a method less 'discrete' than the Sample Representation method; a method using visual aids which confined the members of the sets. The reasons for this choice are largely due to environmental influence, as described in the interpretation of finding #1.

Interestingly, despite the fact that all the students were told to solve each syllogism using the method they were assigned by showing their work in the space provided on their sheets, two students in the Non-visual group actually used circles to represent sets. One student combined the circles correctly; however it was devoid of shading and the important intersections were not identified. Regardless of these imperfections, the student answered every syllogism with the valid, most general conclusion except one, for which he gave the special case of the conclusion.

The second student also used circles to represent the sets but the separate premise representations were incorrect, as well as their combination. This instance is particularly interesting because even though the Venn diagrams were not properly used, the student wrote out in sentences her reasoning processes

underneath the diagrams. It is as if she needed to use circles for the manipulation of the three sets, even though incorrect, to write out her logical thought patterns.

Therefore, because the Non-visual method probably triggered more such responses, it is suggested that this method enabled the students to either mentally or physically use *effective* visual aids during the reasoning processes on the posttest, more effective in fact, than the ones provided by the Sample Representation method.

Concluding Remarks

An interesting question to be addressed is: Why did the Venn diagram method significantly outperform the Non-visual method if the research on the relationship between imagery and written text suggests that the students in the Non-visual group were free to use their own visual patterns? As reported earlier, there are some clues to suggest that most students in the group assigned to the Non-visual approach used effective Venn diagram type processes; however, these processes were freely generated by the students and were not a result of formal instruction. In fact, one student who demonstrated a visual technique using Venn diagrams while reasoning out the syllogisms used the method incorrectly.

If, in fact, the students assigned to the Non-visual group used their own *visual* aids for the reasoning process, the method then automatically takes on two stages. In the first stage, the student must mentally gather all the written information provided for them on the screen. In the second stage, he or she must transfer this information to form adequate representations in order to fully understand it. Only then are logical steps ready to be included in the solution process. This two stage procedure immediately makes the Non-visual method more complex than the Venn diagram method. The latter approach does not require as much of a transition from words to pictures as the diagrams are placed immediately following each statement. It was also previously suggested that complexity was a factor in the lack of success of the Sample Representation approach.

Therefore, the students in the Venn diagram group have substantial

advantages over the students in the other two groups. Firstly, due to reasons such as intuitiveness, simplicity, and familiarity, this approach significantly outperformed the Sample Representation method. Furthermore, because the Venn diagram instruction modules formally trained the students in the use of Venn diagrams, they fared significantly better than those in the Non-visual group, who probably used a similar method, but with ad hoc procedures. The Non-visual method emerges as the unexpected success. This is due not only to the imagery generated from the instructional text on the screen, but also to the failure of the Sample Representation method. The latter, due to its complex and unspontaneous nature, was expected to fare reasonably well, but instead resulted in unsatisfactory syllogistic reasoning.

DISCUSSION OF THE CRITICISMS OF ERICKSON'S (1974) SET ANALYSIS THEORY BY JOHNSON-LAIRD & STEEDMAN (1978)

It is interesting to review the criticisms of the Venn diagram approach put forward by Johnson-Laird & Steedman (1978). Firstly, they point out that subjects may represent each premise only one way, allowing for only one possible configuration for their combination. In this case, the response, 'no valid conclusion' is therefore highly unlikely. Johnson-Laird & Steedman (1978) claim that this response is popular, therefore contradicting the aforementioned possibility. For the present investigation, however, syllogisms with no valid conclusion were not included in the instructions or on the posttest. Therefore, this criticism is not relevant.

The second criticism indicates that the Venn diagram representations for "No As are Bs" and "No Bs are As" are the same, and similarly for "Some As are Bs" and "Some Bs are As", and so forth. This can prove to be responsible for the figural effect, according to Johnson-Laird & Steedman (1978), and they claim that the Sample Representation approach accounts for this significant effect. However, for the present experiment, the separate meanings of "No As are Bs" and "No Bs are As", for example, are specifically explained in the notation section

of the Venn diagram treatment. In addition, on the notation tests, this type of confusion was not encountered. Furthermore, the figural effect was not relevant to this study, as the modules instructed the students to always look for conclusions of the form A-C, regardless of the form of the syllogism.

The third questionable point raised about the Venn diagram approach is the fact that there does not exist a suitable heuristic accompanying the approach to predict student performance. This again, is not relevant to the study because the logical tests developed by Johnson-Laird & Steedman (1978) for the verification of conclusions were not included in the treatment for the Sample Representation method.

METHODOLOGICAL LIMITATIONS OF THE STUDY

1. Guessing as opposed to solving the problems

The flaws of the study were found mostly in the data collection process. The raw data obtained were tricky to evaluate. The following is an example of this. Although substantial weight was given to the method shown by each student on the posttest, a maximum of 4 points out of 16 was awarded for the correct answer to the syllogism. For those subjects who did not show their work in the space provided for them, (even though they were expressly instructed to do so), or for those who incorrectly interpreted the method they were assigned to, it is very difficult to say how they decided upon their answers. In other words, for any particular item, students who did not show their work and who yet arrived at the correct conclusion, got 4 out of 16 points for that item, even though they may have guessed the solution. Indeed, there is a 1 in 4 chance of guessing rather than reasoning out the correct answer.

2. Students from previous weeks exposing material and/or methods to students undergoing the same treatment in later weeks

Due to the manipulation of the subjects in order to obtain randomization, each treatment group was divided into three subgroups, each undergoing treatment at three separate intervals. Students from earlier weeks may have

revealed items on the posttest to other students writing the same posttest in later weeks. This occurrence, however, is unlikely for two reasons. Firstly, it would have been difficult for the students to have remembered verbatim any one or more syllogisms as they are all so similar in form. The posttest items could not have been recalled exactly as they may easily have been confused with the statements, representations, and syllogisms studied during the instruction periods. Secondly, the students were unaware that the posttests given to all three treatment groups within each week were identical, and also that the posttests given to students undergoing the same treatment in later weeks were also identical. Because within each week they were divided into three groups, they were under the impression that three different tests were distributed for each group and similarly, that different posttests were distributed every week.

3. Students may have used methods other than the ones in which they had been instructed

Because of the way the computers were physically arranged in the laboratory, some students were seated beside others using a different method. That is to say, some students may have been exposed to a method not their own. Furthermore, some students may have "taught" their peers in different treatment groups how to solve syllogisms their way. This is possible because the treatment groups were not isolated. However, all subjects were instructed to use the method they were taught to solve the problems on the posttest (by showing their work) and they were aware that they would be penalized if they did not. They also knew that they would be penalized even more severely if they utilized an inappropriate method such as one used in any of the other treatment groups.

Using the data obtained, it is nearly impossible to determine the methods of the students in the Non-visual group. Therefore, especially if these students mentally devised their own "visual" methods to arrive at valid conclusions, the effects of the methods cannot be accurately determined.

Indeed, two students in the Non-visual group tried to solve their syllogisms using Venn diagrams. One can only speculate as to why this happened, but it could be attributed to the exposure to other methods via seat arrangement or via inter

group communication or possibly because Venn diagrams are the most intuitive and easiest way to solve syllogisms. This point, however, was discussed earlier in the chapter.

4. Method proves immaterial

Even though most students used the method they were instructed with and demonstrated this fact by using the 'show your work' space provided for them on their test papers, even then it is difficult to determine if it was indeed this process that enabled them to solve given syllogistic reasoning tasks. It is possible that some students used strictly mental means (visual or otherwise) to arrive at valid conclusions and that the method assigned to them was inconsequential. In this case, again, the scores on the posttests could not adequately measure method effectiveness.

5. Known sources of syllogistic error

Three known errors in particular may have interfered with a student's ability to correctly reason any given posttest item. The errors most likely to have blocked understanding of a particular solution method are, as discussed in chapter 3, misinterpretation of the premises, the Atmosphere effect, and language ambiguity. These effects could have lead the student to arrive at faulty conclusions regardless of the method. Due to the way the posttests were graded, these errors were most likely attributed to the method and not to possible, known errors in syllogistic reasoning.

6. Generalizability is limited

One weakness can be found regarding sampling procedures. The only available students for this experiment were at the grade 8 honours level. Due to the effects of randomness, a regular grade 8 sample would most probably not have changed the present results. The means would be proportional but probably lower. Because of the sample used for the present investigation, it is impossible to extrapolate the results to the entire population of grade 8 students. Only the extrapolation to the population of all honours grade 8 pupils is possible.

Types of Weaknesses

The methodological limitations discussed above can either be attributed to the research design or to the execution of the study. This section examines the characteristics of most of these limitations.

There is one flaw that is inherent in the research design, and that is the second limitation discussed above (students from previous weeks revealing their methods to students undergoing the same treatment in later weeks). The design called for random assignment to the treatment groups. Because only three intact classes were available for the study, it was imperative to randomly divide each class into three subgroups, one for each treatment. Therefore, students in the same treatment group were exposed to the material at different stages of the data collection period and could have prepared their peers for the treatment awaiting them in later weeks.

The seating arrangement is one flaw inherent in the execution of the study. As much as possible was done to arrange the treatment groups in such a way as to hinder their exposure to other treatments.

The other flaws discussed earlier such as guessing, unknown reasoning methods, and known sources of error, are confounding variables that could not be eliminated by altering either the research design or the execution of the study. These variables, internal processes within each subject, could only have been controlled through interviewing techniques and qualitative analysis.

None of the five flaws listed above, however, can be used to explain the failure to confirm the second hypothesis. The reasons for the rejection of H_2 were speculated in the previous section of this chapter.

IDEAS FOR FUTURE RESEARCH

This study opens many doors for future investigation in this area. Because the Venn diagram method performed so well compared to the Sample Representation method, it is suggested that the latter should not be seriously considered in future research. Perhaps a deeper analysis of why it did not result in satisfactory reasoning achievement would be relevant to proponents of this

theory; however, in this discussion, it will not be present.

Most future work should concentrate on the effectiveness of the Venn diagram method. In particular, features of the method that contributed to its success must be inspected closely. A few of those features were put forward earlier as reasons for the method's success, but they were not based on previous research. It is therefore necessary to initiate new research studies along these lines.

The predominant claim was that Venn diagrams provided an intuitive means to solving syllogisms. The numerous properties of intuition can be considered individually as interesting, new research topics. For example, a study comparing the amount of time needed and the amount of effort expended toward solving syllogisms using two or more methods can be conducted. This is proposed because it was noticed how the students in the Venn diagram group solved their reasoning problems with considerably less effort and in less time than students using the other two methods, particularly the Sample Representation method. Perhaps this is due to factors such as simplicity and familiarity with geometric shapes, (possibly an academic or preconscious familiarity, or both).

Another relevant study would be to investigate the amount of environmental and experiential elements existing in the Venn diagram approach to reasoning. Although it was suggested that there was much in the way of these influences present, and that they induced positive reactions to the method, no evidence in the literature exists to support this claim. More detailed conclusions regarding the popularity and the applicability of Venn diagrams are needed as well as their environmental affiliations.

The following issue should be addressed as well: Several researchers believe that teaching creative thinking is indeed an important part of education. It would be interesting to investigate whether or not the Venn diagram method to syllogistic reasoning produced in the students *creative* thinking skills and reasoning processes. Furthermore, the type of thinking skills generated by the Non-visual method could be included in a comparative study.

An interesting problem would be to study the apparent simplicity of Venn

diagrams in relation to their applicability to set theory, logical thinking, and more specifically, syllogistic reasoning. The following questions need to be answered: Is simplicity a contributing factor in the performance of the Venn diagram method? If so, how can Venn diagrams be used in other problem solving areas? What are the characteristics of Venn diagrams that make them 'simple' to use and understand? Are there ways to make the method even simpler?

Perhaps the most limiting aspect of this study involved the unknown techniques used by the students in the Non-visual group to solve syllogisms. The possibility that the majority of them used visual constructs to complete the posttest must be observed. A qualitative study investigating exact reasoning methods, visual or not, would make for a compelling endeavour. If, indeed, most students in the Non-visual group solved the reasoning tasks using some type of visual aid, it would be necessary to determine the similarities of these various aids to the Venn diagram method. This would explain whether the Non-visual method resulted in significantly greater syllogistic reasoning achievement than the Sample Representation method because of its similarity to the Venn diagram method, or because an entirely semantic approach to reasoning is superior to the complex, ambiguous nature of Sample Representations. This appears to be the most pressing issue and one which should be investigated in further detail.

By examining more closely the contents of the 'show your work' section of the posttest for the students in the Non-visual and Sample Representation groups, it was found that one intriguing yet individually fabricated method was used by several students. Each premise was transformed into 'equation' form and a conclusion was derived from these representations. An example of one student's markings can be studied in Figure 24. The two premises not represented in Figure 24 are "No As are Bs" and "Some As are not Bs". Throughout the posttest in question, these two premises were represented as "$A \neq B$" and "Some $A \neq B$" respectively.

TEST ITEM #4

Syllogism: All Bs are As
Some Cs are Bs

Conclusion: Some As are Cs

Show your work:

B = A
Some C = B

Some A = C

Figure 24. Example of the work shown on a Non-visual posttest.

Could the students have stumbled across the beginnings of a new, potentially effective method for syllogistic reasoning? Can these invented methods be fully developed into a valuable, practical means for solving syllogisms? This presents future researchers in this area with new approaches to investigate.

Several researchers have studied the effects of syllogistic reasoning using sensible, everyday content rather than abstract, symbolic materials. It is their conclusion that future research should be conducted using everyday elements because

> a psychologist who studies reasoning with abstract materials is not so much studying a pure deduction, unsullied by his subjects' knowledge or attitudes, as a very special sort of reasoning designed to compensate for the absence of everyday content. (Johnson-Laird & Steedman, 1978, p. 66)

Abstract content was employed for this investigation for the sake of simplicity. The three instructional methods were designed for a computer with which the students had very little or no familiarity. The intent was to make the instructional modules as simple as possible to allow the students to concentrate on the material presented to them.

To modify the existing lessons to include everyday content would prove to be a challenging venture. A syllogism such as the following might present an idea of what illustrations would then be necessary:

	All animals are living creatures
	Some animals are reptiles,
Therefore,	Some living creatures are reptiles.

In this case, using actual objects as set members would invite the students in the Non-visual group to imagine the individual elements of each set (animals, reptiles etc.) and the intention was to discourage the use of mental imagery for the students in this group. Admittedly, the picturing of set *elements* would not necessarily lead to the development of a visual *method* to solve the syllogism, but it is proposed that the tendency to do so would be greater in this case. This provides another stimulating area for future research.

In conclusion, this study provides evidence for accepting Venn diagrams as useful tools for syllogistic reasoning. The Venn diagram approach provides many opportunities for future research in the areas of cognitive, developmental, and educational psychology.

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Appendix A

Venn Diagram Notation Module

For this research project, you will learn how to solve syllogisms.

NEXT CARD

What is a syllogism?

A syllogism is a word problem that has:
a) two clues; and
b) a solution.
Click on NEXT CARD for an example.

LAST CARD NEXT CARD

Here is an example of a syllogism:

Clue #1: All As are Bs
Clue #2: No Bs are Cs
Solution: No As are Cs.

LAST CARD NEXT CARD

We will refer to the "clues" as "statements" and the correct solution as the "conclusion". Therefore, this syllogism takes the form of:

Statement #1: All As are Bs
Statement #2: No Bs are Cs
Conclusion: No As are Cs.

LAST CARD NEXT CARD

Statement #1 always involves As and Bs: All As are Bs

Statement #2 always involves Bs and Cs: No Bs are Cs

The Conclusion always expresses a relationship between As and Cs: No As are Cs.

LAST CARD NEXT CARD

Here is a problem to solve:

Clue #1: All As are Bs
Clue #2: No Bs are Cs
Solution: ?

What is the relationship between the As and the Cs? Click on NEXT CARD to find out.

LAST CARD NEXT CARD

The correct solution for this syllogism is:
"No As are Cs".

You will learn how to use the two given clues to figure out the correct solution.

LAST CARD NEXT CARD

Let's learn how to solve various syllogisms! But first, let's start with a few basics.

LAST CARD NEXT CARD

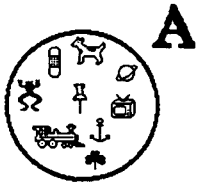
DEFINITION

A group of objects is called a set. The objects making up a set belong to the set. These items which belong to the set are called members.

LAST CARD NEXT CARD

Example . . .

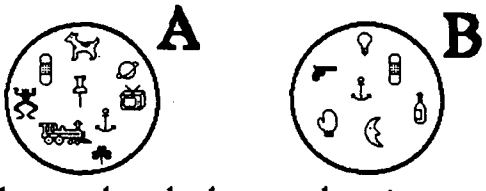
We will represent a set with a circle. The members of the set are in the circle:



LAST CARD NEXT CARD

Another Example. . .

Consider 2 sets: set A and set B.

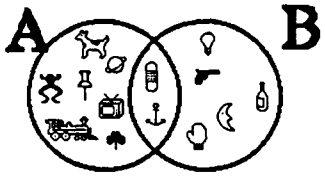


What members do these sets have in common?

Click here to find out:

LAST CARD NEXT CARD

Therefore, we can represent sets A and B as follows:



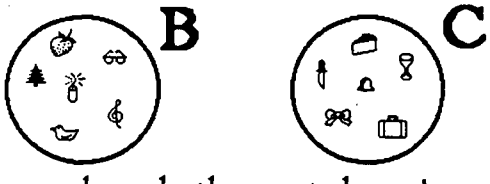
NEXT CARD

Set A and set B have overlapping members.

LAST CARD

Another Example. . .

Consider the two sets B and C:



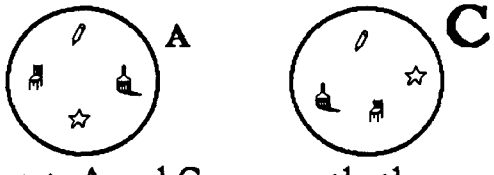
What members do these sets have in common

Click here to find out:

LAST CARD

Another Example. . .

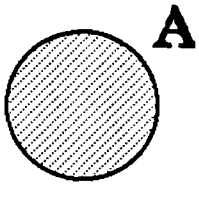
Consider the two sets A and C:



Here, sets A and C are exactly the same because they have exactly the same members

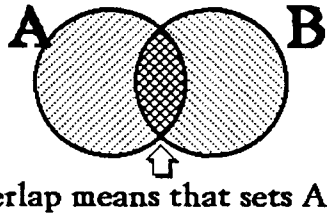
LAST CARD NEXT CARD

Instead of drawing each object in a set as in the previous examples, we will draw a typical set A as follows:



LAST CARD NEXT CARD

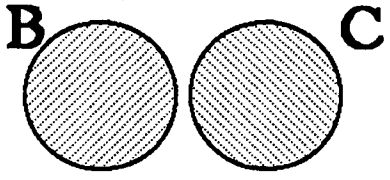
Also, we will represent overlapping sets A and B as follows:



This overlap means that sets A and B have one or more members in common.

LAST CARD NEXT CARD

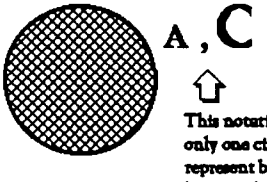
Sets that have no members in common must remain separated, and will be represented as follows:



Here, there are no overlapping members.

LAST CARD NEXT CARD

If sets A and C have all their members in common, we represent them as follows:



This notation means that only one circle is needed to represent both sets A and C because they have exactly the same members.

LAST CARD NEXT CARD

NOTE. . .

From now on, we will be talking about sets A, B, and C. The members of set A are called As, the members of set B are Bs, and the members of set C are Cs.

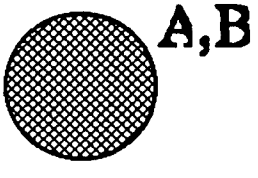
LAST CARD NEXT CARD

Let's look at some statements and see how they can be represented.
Click on NEXT CARD to continue.

LAST CARD NEXT CARD

Statement: All As are Bs

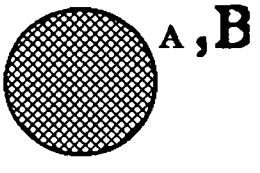
Representation:



LAST CARD NEXT CARD

Explanation:

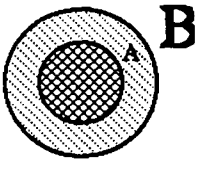
What would the picture look like if set B were larger than set A? Click here to find out: ☐



LAST CARD

Explanation:

What would the picture look like if set B were larger than set A?



☞ The criss-cross area represents all the As contained within set B.

LAST CARD NEXT CARD

The statement we have studied so far is:

All As are Bs

What if we reverse the letters?

LAST CARD NEXT CARD

The statement we have studied so far is:

All Bs are As

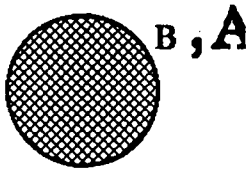
It becomes a totally new and different statement and we must learn its representations also!

LAST CARD NEXT CARD

Therefore, we can say:

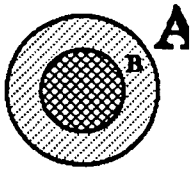
Statement: All Bs are As

Representation:



LAST CARD NEXT CARD

And, if set A were larger than set B, we can represent it as follows:

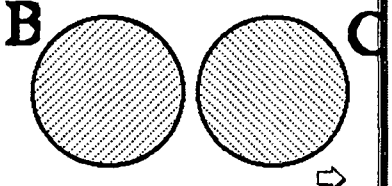


All Bs are As

LAST CARD NEXT CARD

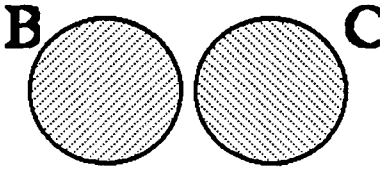
Statement: No Bs are Cs

Representation:



LAST CARD NEXT CARD

Explanation:



There is no overlap between sets B and C

LAST CARD NEXT CARD

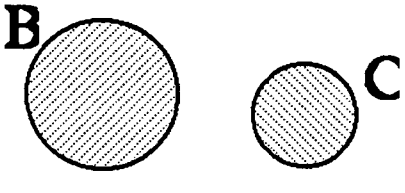
Even if sets B and C were different in size, there still would be no overlap because they don't have any elements in common.

Example 1: Click here ☐

Example 2: Click here ☐

LAST CARD

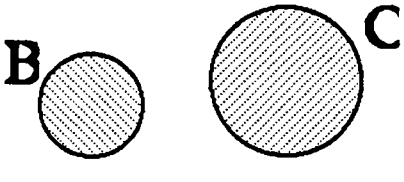
Even if sets B and C were different in size, there still would be no overlap because they don't have any elements in common.



Example 2: Click here ☐

LAST CARD

Even if sets B and C were different in size, there still would be no overlap because they don't have any elements in common.

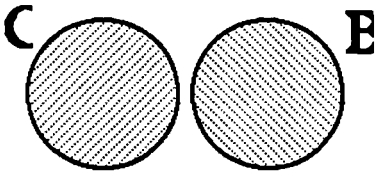


LAST CARD NEXT CARD

In the same way, we can say:

Statement: No Cs are Bs

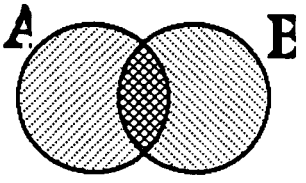
Representation:



LAST CARD NEXT CARD

Statement: Some As are Bs

Representation:

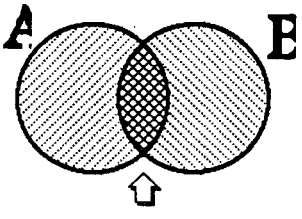


For the explanation, click here: ☐

LAST CARD

Statement: Some As are Bs

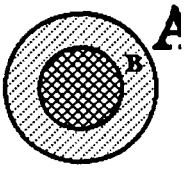
Representation:



These are the As that are Bs

LAST CARD NEXT CARD

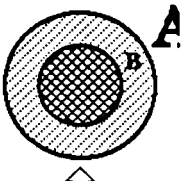
Another Representation:



For the explanation, click here: ☐

LAST CARD

Another Representation:



The criss cross area represents the As that are also Bs.

LAST CARD NEXT CARD

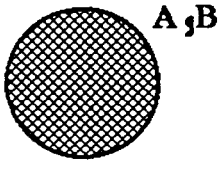
Another Representation:

What if set A and set B were exactly the same sets? Can we still say that some As are Bs in this case? Click here to find out: ☐

LAST CARD

Another Representation:

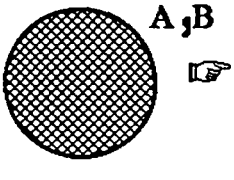
What if set A and set B were exactly the same sets? Can we still say that some As are Bs in this case?



LAST CARD

Another Representation:

What if set A and set B were exactly the same sets? Can we still say that some As are Bs in this case?

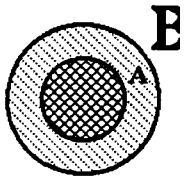


☞Yes! Even though All As are Bs, we can still say some of those As are Bs.

Click again: ☐

LAST CARD

Another Representation:



☞ Again, though All As are Bs, we can still say that some of those As are Bs.

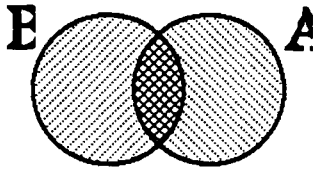
LAST CARD NEXT CARD

In the same way we can say:

MORE Click here for more

Statement: Some Bs are As

Representation:



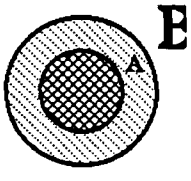
LAST CARD

Or. . .

MORE Click here for more

Statement: Some Bs are As

Another Representation:



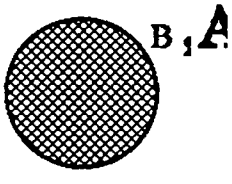
LAST CARD

Or. . .

MORE Click here for more

Statement: Some Bs are As

Another Representation:



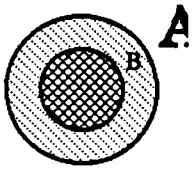
(All Bs are As)

LAST CARD

Or. . .

Statement: Some Bs are As

Another Representation:

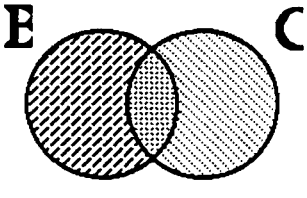


(All Bs are As)

LAST CARD NEXT CARD

Statement: Some Bs are not Cs

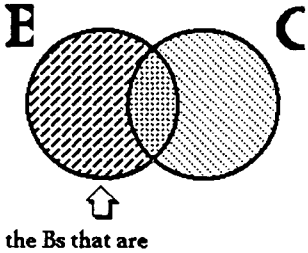
Representation:



For the explanation, click here: ☐

LAST CARD

Some Bs are not Cs



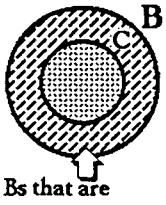
Explanation:

These are the Bs that are not Cs Click again: ☐

LAST CARD

Statement: Some Bs are not Cs

Another Representation:



These are the Bs that are not Cs Click again: ☐

LAST CARD

Another Representation:

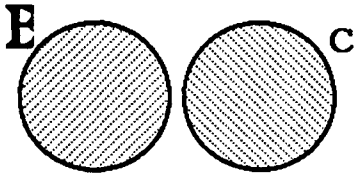
What if sets B and C were completely different sets (i.e. they have no members in common)? Can we still say that Some Bs are not Cs in this case?

Click here to find out: ☐

LAST CARD

Another Representation:

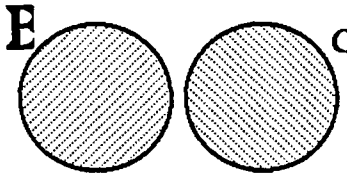
What if sets B and C were completely different sets (i.e. they have no members in common)? Can we still say that Some Bs are not Cs in this case?



LAST CARD

Another Representation:

What if sets B and C were completely different sets (i.e. they have no members in common)? Can we still say that Some Bs are not Cs in this case?



Yes! Even though No Bs are Cs, we can still say that Some Bs are not Cs

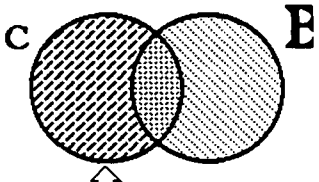
LAST CARD

NEXT CARD

In the same way, we can say:

Statement: Some Cs are not Bs

Representation:



These are the Cs that are not Bs

Click again: ☐

LAST CARD

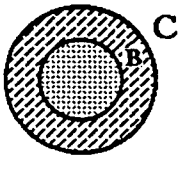
Or. . .

Statement: Some Cs are not Bs

MORE

Click here for more

Another Representation:

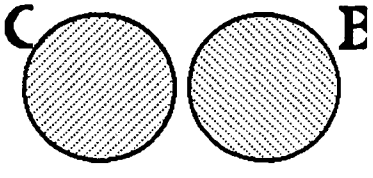


LAST CARD

Or. . .

Statement: Some Cs are not Bs

Another Representation:



(No Cs are Bs)

LAST CARD

NEXT CARD

In the following examples, write your answers in the spaces provided on your worksheets. After completing each exercise, study the correct answer and compare it to your answer. Do the two answers match?

LAST CARD

NEXT CARD

Example 1.

Give 1 representation for the following statement:

Statement: All Bs are As

Representations: ?

LAST CARD

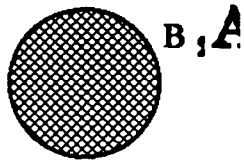
NEXT CARD

On your worksheet #1, write down your answer NOW!

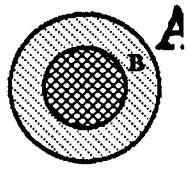
Click here for the answer:

LAST CARD

Was your answer one of the following?:



and



Write on your worksheet any representations that you did not get.

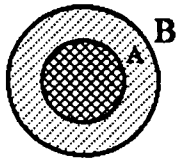
LAST CARD NEXT CARD

Example 2:

Give 1 statement for the following representation:

Statement: ?

Representation:



LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

Was your answer among the following three?:

1) All As are Bs
   and therefore,
2) Some As are Bs.
   also,
3) Some Bs are not As.

Write on your worksheet any statements that you did not get.

LAST CARD NEXT CARD

Example 3.

Give 1 representation for the following statement:

Statement: All Bs are Cs

Representation: ?

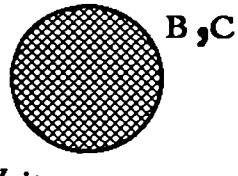
LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

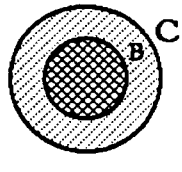
Click here for the answer:

LAST CARD

Was your answer one of the following?:



or



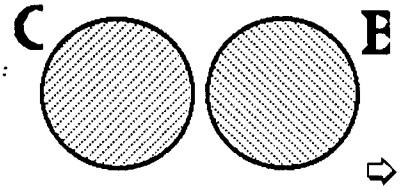
Write on your worksheet any representations that you did not get.

LAST CARD NEXT CARD

Example 4.
Give 1 statement for the following representation:

Statement: ?

Representation:



LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

Was your answer one of the following?:

No Cs are Bs.

It is also possible to say:

No Bs are Cs.

Write on your worksheet any statements that you did not think of.

LAST CARD NEXT CARD

Example 5.
Give 1 representation for the following statement:

Statement: Some Bs are As

Representations: ?

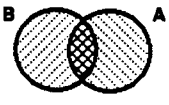
LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

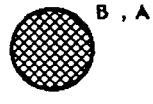
Click here for the answer:

LAST CARD

Was your answer one of the following four? Write on your worksheet any representations that you did not get.

1) 

2) 

3) 

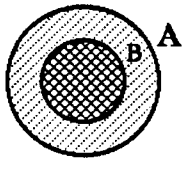
4) 

LAST CARD NEXT CARD

Example 6:
Give 1 statement for the following representation:

Statement: ?

Representation:



LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

Was your answer among the following four?:

1) Some As are Bs
2) All Bs are As
   and therefore,
3) Some Bs are As.
   also
4) Some As are not Bs.

Write down on your worksheet any statements you did not get.

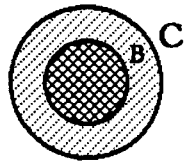
LAST CARD NEXT CARD

Example 7:

Give 1 statement for the following representation:

Statement: ?

Representation:



LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

Was your answer one of the following?:

1) All Bs are Cs and
2) Some Bs are Cs.

Other correct answers would be:

3) Some Cs are not Bs,
4) Some Cs are Bs.

Write on your worksheet any statements that you did not think of.

LAST CARD NEXT CARD

Example 8.

Give 1 representation for the following statement:

Statement: Some Cs are Bs

Representations: ?

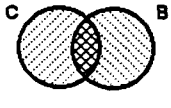
LAST CARD NEXT CARD

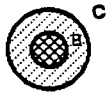
On your worksheet #1, write down your answer NOW!

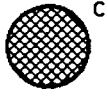
Click here for the answer:

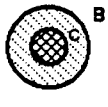
LAST CARD

Was your answer one of the following four? Write on your worksheet any representations that you did not get.

1) 

2) 

3) 

4) 

LAST CARD NEXT CARD

Example 9.

Give 1 representation for the following statement:

Statement: Some As are not Cs

Representations: ?

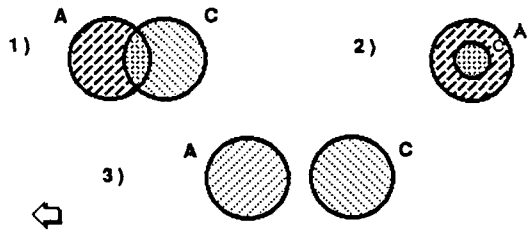
LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

Was your answer one of the following three? Write on your worksheet any representations that you did not get.



LAST CARD

NEXT CARD

<u>Example 10.</u>

Give 1 representation for the following statement:

Statement: Some As are not Bs

Representations:

LAST CARD

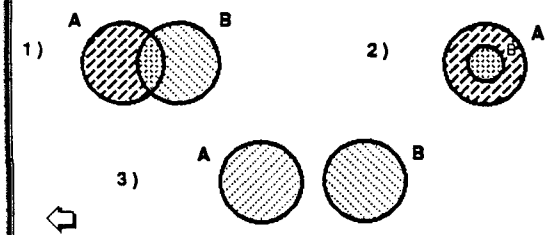
NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

Was your answer one of the following three? Write on your worksheet any representations that you did not get.



LAST CARD

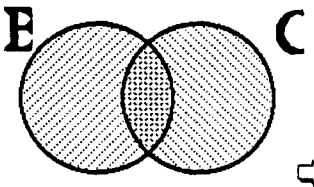
NEXT CARD

<u>Example 11:</u>

Give 1 statement for the following representation:

Statement:

Representation:



LAST CARD

NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

Was your answer one of the following four? Write on your worksheet any statements that you did not get.

1) Some Cs are not Bs
2) Some Cs <u>are</u> Bs
3) Some Bs are not Cs
4) Some Bs <u>are</u> Cs.

LAST CARD

NEXT CARD

Example 12.

Give 1 representation for the following statement:

Statement: Some Bs are Cs

Representations:

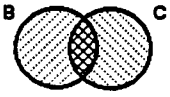
LAST CARD NEXT CARD

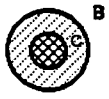
On your worksheet #1, write down your answer NOW!

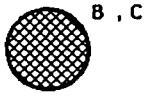
Click here for the answer:

LAST CARD

Was your answer one of the following four? Write on your worksheet any representations that you did not get.

1) 

2) 

3) 

4) 

LAST CARD NEXT CARD

Appendix B

Sample Representation Notation Module

For this research project, you will learn how to solve syllogisms.
Click on NEXT CARD to continue.

NEXT CARD

What is a syllogism?

A syllogism is a word problem that has:
a) two clues; and
b) a solution.
Click on NEXT CARD for an example.

LAST CARD NEXT CARD

Here is an example of a syllogism:

Clue #1: All As are Bs
Clue #2: No Bs are Cs
Solution: No As are Cs.

LAST CARD NEXT CARD

We will refer to the "clues" as "statements" and the correct solution as the "conclusion". Therefore, this syllogism takes the form of:

Statement #1: All As are Bs
Statement #2: No Bs are Cs
Conclusion: No As are Cs.

LAST CARD NEXT CARD

Statement #1 always involves As and Bs: All As are Bs

Statement #2 always involves Bs and Cs: No Bs are Cs

The Conclusion always expresses a relationship between As and Cs: No As are Cs.

LAST CARD NEXT CARD

Here is a problem to solve:

Clue #1: All As are Bs
Clue #2: No Bs are Cs
Solution: ?

What is the relationship between the As and the Cs?
Click on NEXT CARD to find out.

LAST CARD NEXT CARD

The correct solution for this syllogism is:
"No As are Cs".

You will learn how to use the two given clues to figure out the correct solution.

LAST CARD NEXT CARD

Let's learn how to solve various syllogisms! But first, let's start with a few basics.

LAST CARD NEXT CARD

↓ = "is"

This symbol is called a "positive link".

LAST CARD NEXT CARD

For example,

A is B

A
↓
B

LAST CARD NEXT CARD

Or, we can say:

B is A

A
↑
B

LAST CARD NEXT CARD

The same rule applies when there are many As and Bs:

Many As are Bs

A A A
↓ ↓ ↓
B B B

LAST CARD NEXT CARD

Or, we can say:

Many Bs are As

A A A
↑ ↑ ↑
B B B

LAST CARD NEXT CARD

⊥ = "is not"

This symbol is called a "negative link".

LAST CARD NEXT CARD

For example,

A is not B

A
⊥
B

LAST CARD NEXT CARD

Or, we can say:

B is not A

A
T
B

LAST CARD NEXT CARD

The same rule applies when there are many As and Bs:

Many As are not Bs

A A A
⊥ ⊥ ⊥
B B B

LAST CARD NEXT CARD

Or, we can say:

Many Bs are not As

A A A
T T T
B B B

LAST CARD NEXT CARD

DEFINITION

A group of objects is called a set. The objects making up a set belong to the set. These items which belong to the set are called members.

LAST CARD NEXT CARD

DEFINITION (con'd)

From now on, we will be talking about sets A, B, and C. The members of set A are As, the members of set B are Bs, and the members of set C are Cs.

LAST CARD NEXT CARD

MORE DEFINITIONS . . .

When you see A, this means that set A must have at least one member.

EXAMPLE:

A

This is set A and it has at least one member.

LAST CARD NEXT CARD

When you see:

A (A)

This means that set A has one member and that it may have more members.

LAST CARD NEXT CARD

EXAMPLE:

A (A)

Set A has one member and these brackets mean that set A may have more than one member.

LAST CARD NEXT CARD

Let's look at some statements and see how they can be represented.
Click on NEXT CARD to continue.

LAST CARD NEXT CARD

Statement: All As are Bs

Representation:

A
↓
B (B)

LAST CARD NEXT CARD

All As are Bs

Explanation:

A
↓
B (B)

What would the representation look like if sets A and B were the same size? Click here to find out: ☐

What would the representation look like if set B were larger than set A? Click here to find out: ☐

☞ **Every A in set A must be positively linked with a member in set B.**

LAST CARD NEXT CARD

The statement we have studied so far is:

All As are Bs

What if we reverse the letters?

LAST CARD NEXT CARD

The statement we have studied so far is:

All Bs are As

It becomes a totally new and different statement and we must learn its representation also!

LAST CARD NEXT CARD

Therefore, we can say:

All Bs are As

A (A)
↑
B

LAST CARD NEXT CARD

Statement: No Bs are Cs

Representation:

B B
⊥ ⊥
C C

LAST CARD NEXT CARD

No Bs are Cs

Explanation:

B B
⊥ ⊥
C C

What would the representation look like if set B were smaller than set C? Click here to find out: ☐

What would the representation look like if sets B and C were the same size? Click here to find out: ☐

☞ **Every member in set B must be negatively linked with a member in set C.**

LAST CARD NEXT CARD

In the same way, we can say:

No Cs are Bs

B B
T T
C C

LAST CARD NEXT CARD

Statement: Some As are Bs

Representation:

A (A)
↓
B (B)

LAST CARD NEXT CARD

Some As are Bs

Explanation:

A (A)
↓
B (B)

What if set A were smaller than set B? Click here: ☐

What if set A were larger than set B? Click here: ☐

What if sets A and B were the same size? (There are 2 choices):

☐ Representation 1 ☐ Representation 2

☞ At least one A is B.

LAST CARD NEXT CARD

In the same way, we can say:

Some Bs are As

A (A)
↑
B (B)

LAST CARD NEXT CARD

Statement: Some Bs are not Cs

Representation:

B (B)
⊥ ↓
C C

LAST CARD NEXT CARD

Some Bs are not Cs

Explanation:

B (B)
⊥ ↓
C C

What if set B were smaller than set C? Click here: ☐

What if set B and set C were the same size? Click here: ☐

☞ At least one B is not C, and some Bs may be Cs.

LAST CARD NEXT CARD

In the same way, we can say:

Some Cs are not Bs

B B
T ↑
C (C)

LAST CARD NEXT CARD

If we have two statements, such as: A is B and B is C

we can represent it as:

A
↓
B
↓
C

LAST CARD NEXT CARD

If we want to determine the relation between A and C in this example, there are two steps to follow:

① Find out whether the first link (the link between A and B) is positive or negative.

② Find out whether the second link (the link between B and C) is positive or negative.

LAST CARD NEXT CARD

If the two links are positive,

Click here to see this explanation again:

A
↓
B
↓
C

LAST CARD

If we have two statements such as A is B and B is not C we can represent it as:

A
↓
B
⊥
C

LAST CARD NEXT CARD

If one link is positive and the other is negative,

Click here to see this explanation again:

A
↓
B
⊥
C

LAST CARD

If we have two statements such as: A is not B and B is not C we can represent it as:

A
⊥
B
⊥
C

LAST CARD NEXT CARD

If both links are negative,

Click here to see this explanation again:

A
⊥
B
⊥
C

LAST CARD

Here Are Some More Examples. . .

A
↓
B
T
C

LAST CARD NEXT CARD

A is B and C is not B

Click here to see this explanation again:

A
↓
B
T
C

LAST CARD

Another Example. . .

A
T
B
↓
C

LAST CARD NEXT CARD

B is not A and B is C

A
T
B
↓
C

Click here to see this explanation again:

LAST CARD

In the following examples, write your answers in the spaces provided on your worksheets. After completing each exercise, study the correct answer and compare it to your answer. Do the two answers match?

LAST CARD NEXT CARD

Example 1.

Statement: A is B

Representation:

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

The answer is:

A
↓
B

LAST CARD NEXT CARD

Example 2.

Statement:

Representation:

A
↑
B

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

The answer is:

B is A

LAST CARD NEXT CARD

Example 3:

Statement: C is not B

Representation:

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

The answer is:

B

T

C

LAST CARD NEXT CARD

Example 4:

Statement:

Representation:

A

↓

B (B)

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

The answer is:

All As are Bs

LAST CARD NEXT CARD

Example 5:

Statement: All Bs are Cs

Representation:

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

The answer is:

B
↓
C (C)

LAST CARD NEXT CARD

Example 6:

Statement: ?

Representation:

B B
T T
C C

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

The answer is:

No Cs are Bs

LAST CARD NEXT CARD

Example 7:

Statement: ?

Representation:

A (A)
↓
B (B)

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

The answer is:

Some As are Bs

LAST CARD NEXT CARD

Example 8:

Statement: Some Cs are Bs

Representation: ?

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

The answer is:

B (B)
↑
C (C)

LAST CARD NEXT CARD

Example 9:

Statement: Some As are not Bs

Representation:

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

The answer is:

A (A)
⊥ ↓
B B

LAST CARD NEXT CARD

Example 10:

Statement:

Representation:

B B
⊤ ↑
C (C)

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

The answer is:

Some Cs are not Bs

LAST CARD NEXT CARD

Example 11: Determine the path between A and C:

A
↑
B
⊥
C

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

B is A and B is not C

Answer:

A
↑
B
⊥
C

Click here to see this answer again:

LAST CARD

Example 12: Determine the path between A and C:

A
↑
B
↑
C

LAST CARD NEXT CARD

On your worksheet #1, write down your answer NOW!

Click here for the answer:

LAST CARD

B is A and C is B

Answer:

A
↑
B
↑
C

Click here to see this answer again:

LAST CARD

Appendix C

Venn Diagram Syllogism Instruction Module

HOW TO SOLVE SYLLOGISMS

Here is an easy way to solve syllogisms!
Let's review the basics:
Click on NEXT CARD to continue.

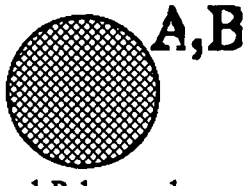
NEXT CARD

Let's review some statements and their representations.
Click on NEXT CARD to continue.

LAST CARD NEXT CARD

Statement: **All As are Bs**

Representation:



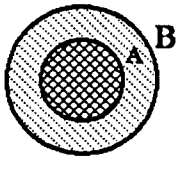
Explanation: **Sets A and B have the same members, so All As are Bs.**

LAST CARD NEXT CARD

What would the picture look like if All As are Bs and B is a larger set than A?
Click here to find out ☞ □

LAST CARD

New representation:



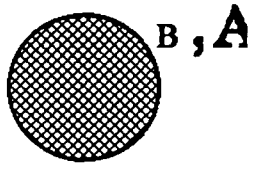
Explanation: ☞ **The criss-cross area represents all the As that are Bs.**

LAST CARD NEXT CARD

In the same way, we can say:

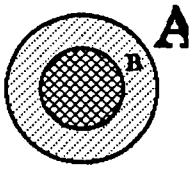
Statement: **All Bs are As**

Representation:



LAST CARD NEXT CARD

And, if set A were larger than set B, we can represent it as follows:

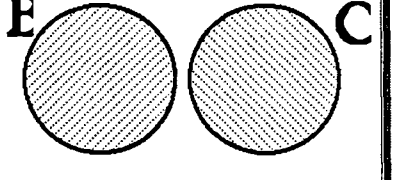


All Bs are As

LAST CARD NEXT CARD

Statement: **No Bs are Cs / No Cs are Bs**

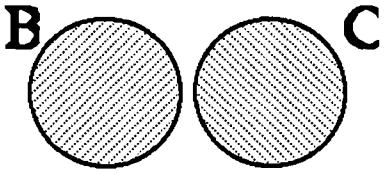
Representation:



For the explanation, click here: ☞ □

LAST CARD

Explanation:

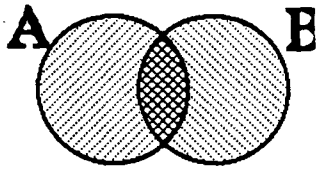


There is no overlap between sets B and C

LAST CARD NEXT CARD

Statement: Some As are Bs

Representation:

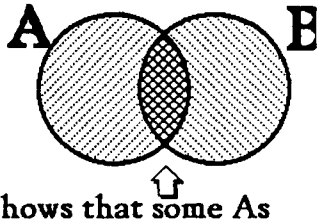


For an explanation, click ☞ □

LAST CARD

Statement: Some As are Bs

Representation:



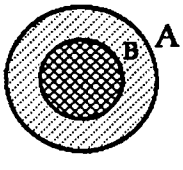
This overlap shows that some As are also Bs.

LAST CARD NEXT CARD

For another representation of Some As are Bs *, click here:* ☞ □

LAST CARD

Some As are Bs



Explanation: The criss-cross area shows the As that are also Bs.

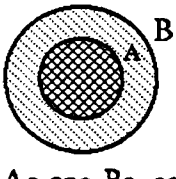
LAST CARD NEXT CARD

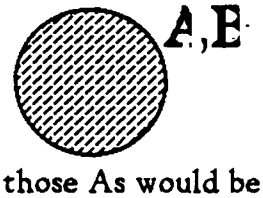
What if All As were Bs? Would it still be correct to say that Some As are Bs? Click here to find out: ☞ □

LAST CARD

Statement: Some As are Bs

Representations:

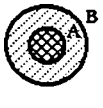




If All As are Bs, some of those As would be Bs, that's why we can say Some As are Bs.

LAST CARD NEXT CARD

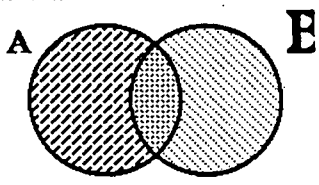
The representations for Some Bs are As *are:*



LAST CARD NEXT CARD

Statement: **Some As are not Bs**

Representation:

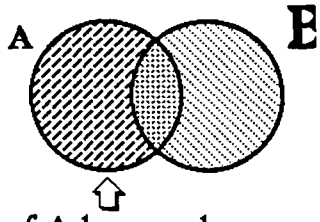


For an explanation, click here: ☞ □

LAST CARD

Statement: **Some As are not Bs**

Representation:



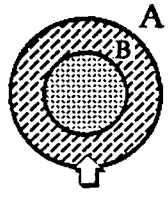
This part of A has no elements in B.

LAST CARD NEXT CARD

For another representation of **Some As are not Bs**, click here: ☞ □

LAST CARD

Some As are not Bs



Explanation:

This part of set A has no elements in set B.

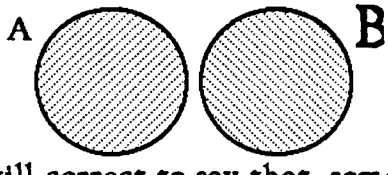
LAST CARD NEXT CARD

What if No As were Bs? Would it still be correct to say that Some As are not Bs? Click here to find out: ☞ □

LAST CARD

Statement: **Some As are not Bs**

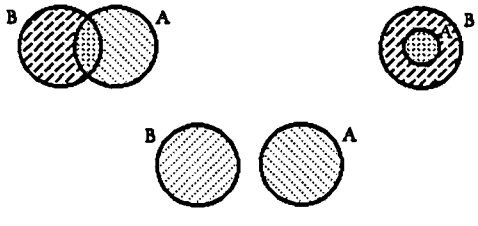
Representation:



It is still correct to say that some As are not Bs even though no As are Bs.

LAST CARD NEXT CARD

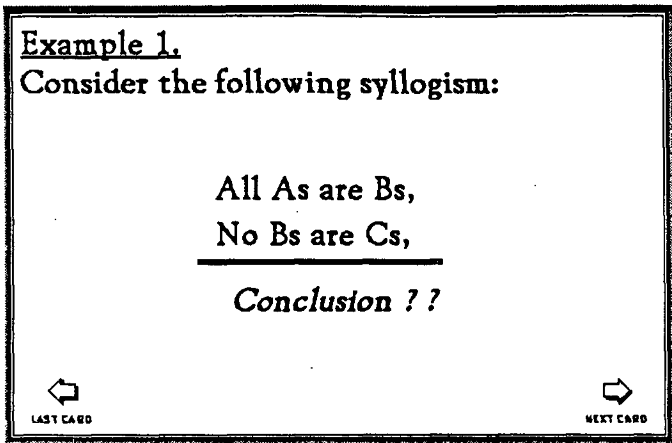
The representations for **Some Bs are not As** are:



LAST CARD NEXT CARD

Now, let us study the 5 steps needed to solve a syllogism: Click on NEXT CARD to continue.

LAST CARD NEXT CARD



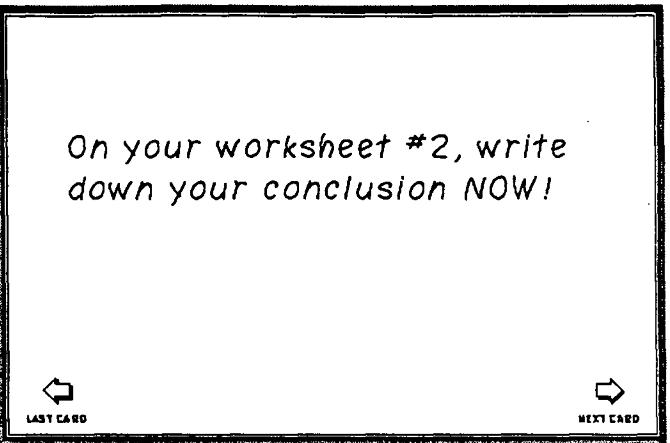
Example 1.

Consider the following syllogism:

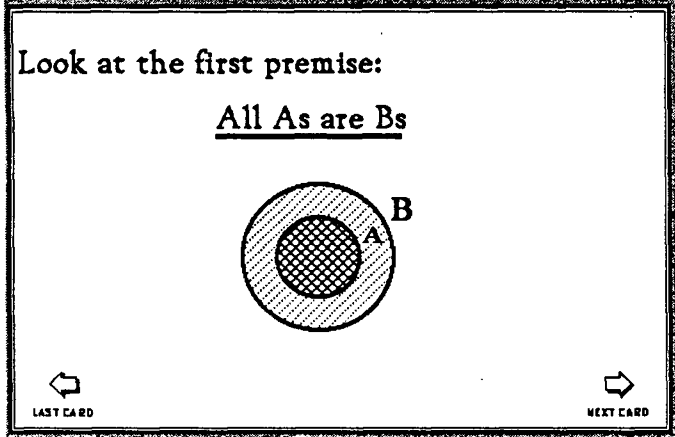
All As are Bs,

No Bs are Cs,

Conclusion ? ?

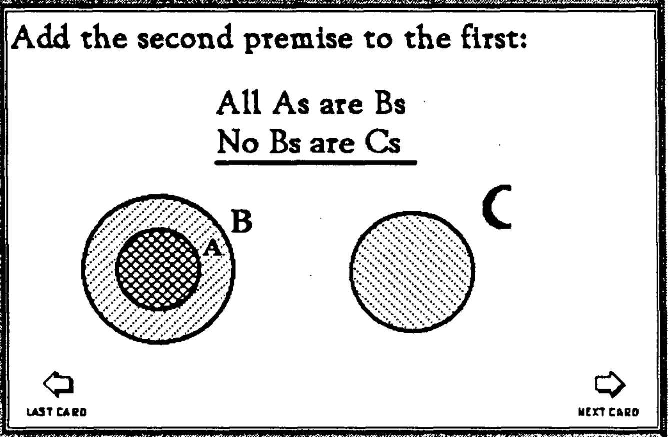


On your worksheet #2, write down your conclusion NOW!



Look at the first premise:

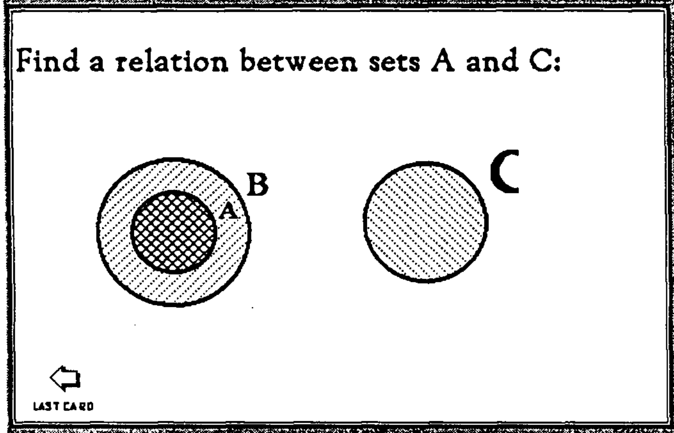
All As are Bs



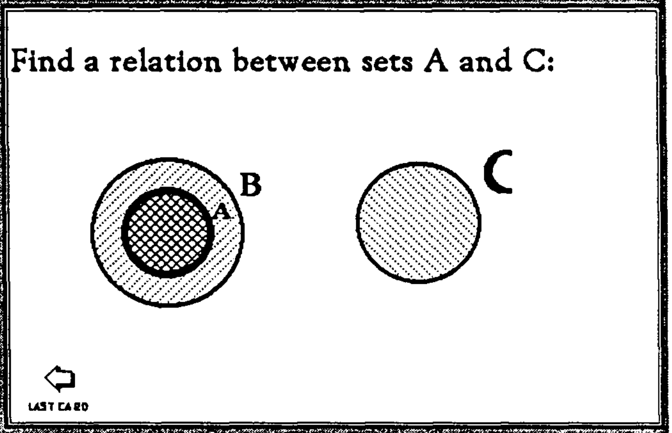
Add the second premise to the first:

All As are Bs

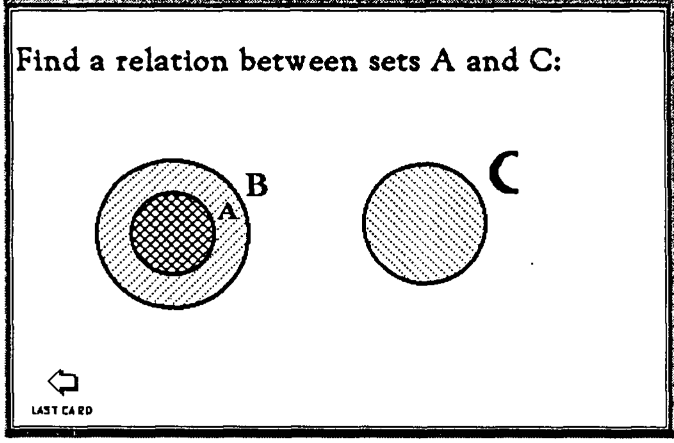
No Bs are Cs



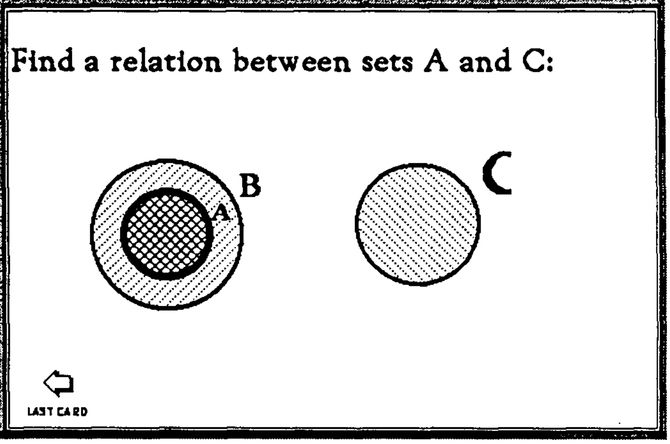
Find a relation between sets A and C:



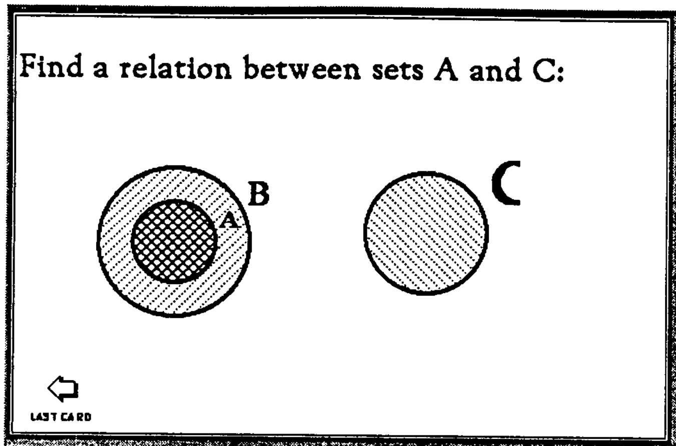
Find a relation between sets A and C:

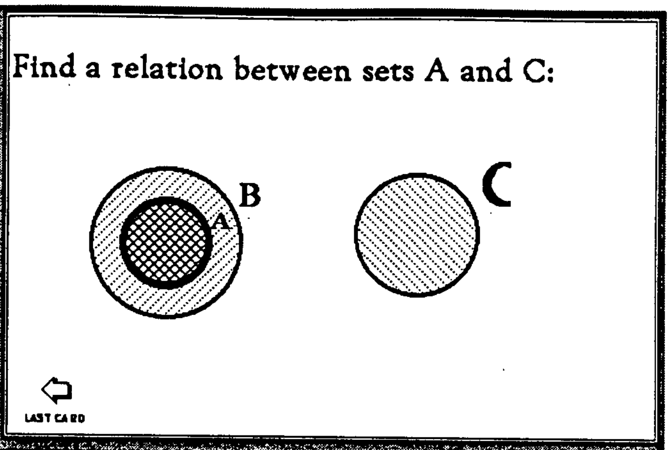


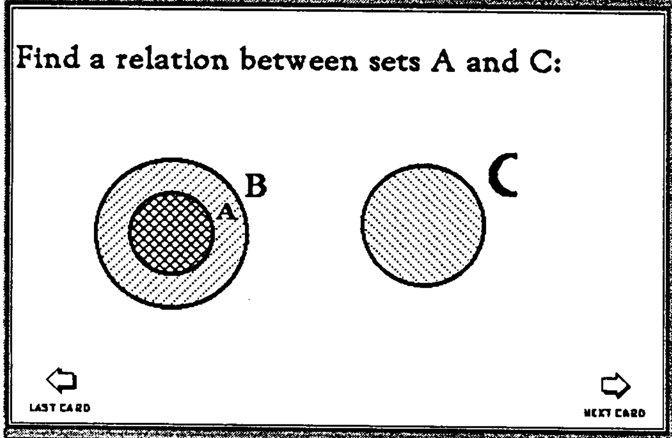
Find a relation between sets A and C:

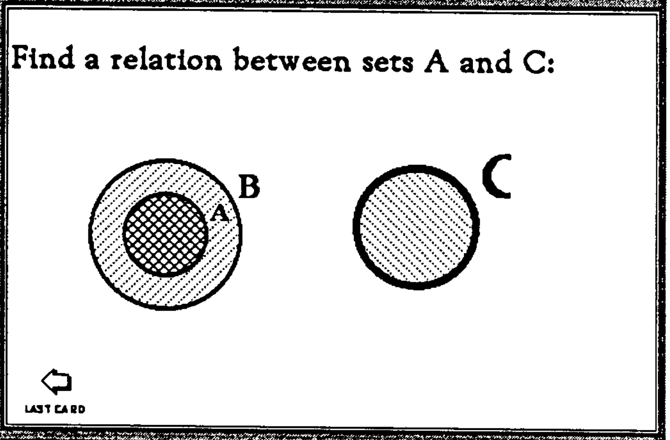


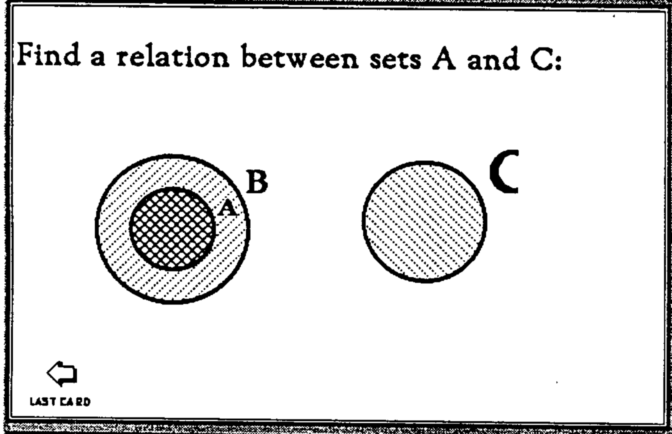
Find a relation between sets A and C:

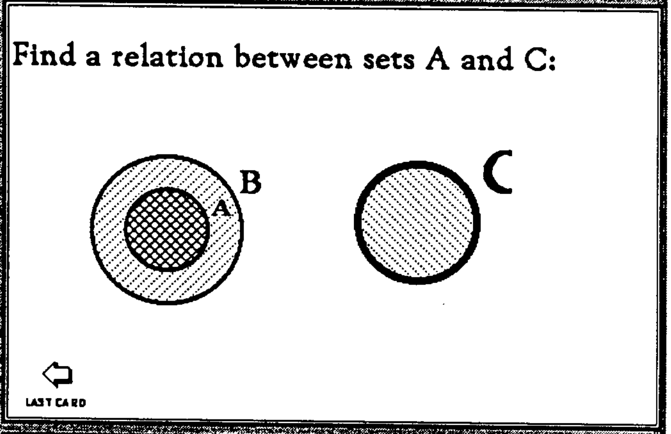

Find a relation between sets A and C:

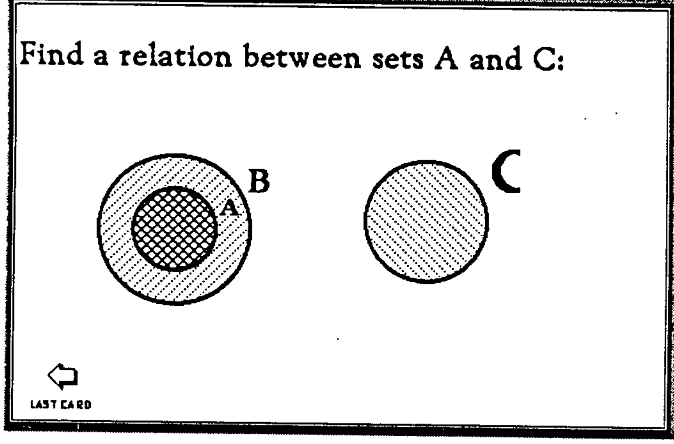

Find a relation between sets A and C:

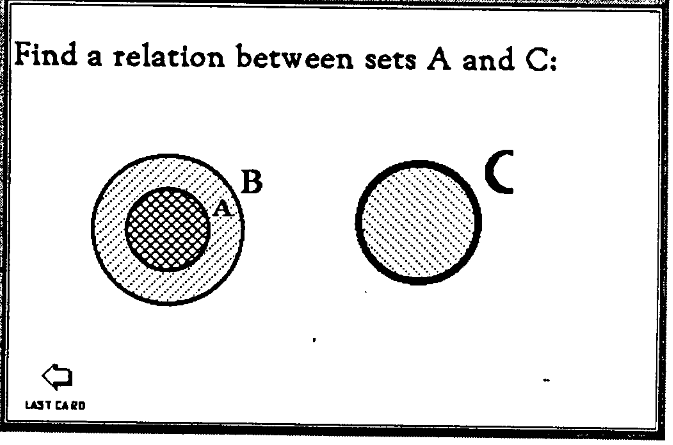

Find a relation between sets A and C:


Find a relation between sets A and C:

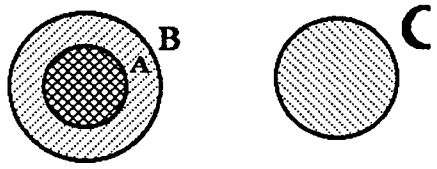

Find a relation between sets A and C:


Find a relation between sets A and C:


Find a relation between sets A and C:

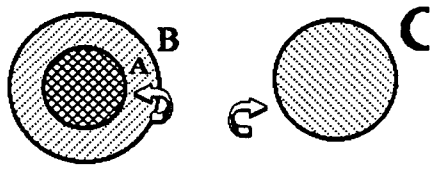

Find a relation between sets A and C:

Find a relation between sets A and C:



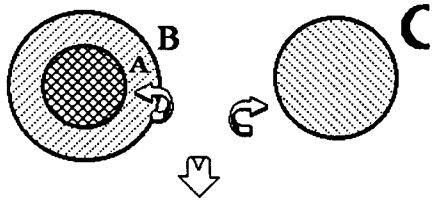
LAST CARD NEXT CARD

Find a relation between sets A and C:



LAST CARD NEXT CARD

Draw a conclusion:



No As are Cs

LAST CARD NEXT CARD

The conclusion we arrived at was:

No As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on worksheet #2 NOW.
Do the two answers match?
DO NOT change your original answer!!

LAST CARD NEXT CARD

<u>Example 2.</u>
Consider the following syllogism:

All As are Bs,
All Bs are Cs,

Conclusion ? ?

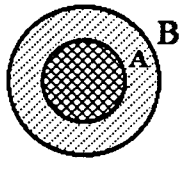
LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

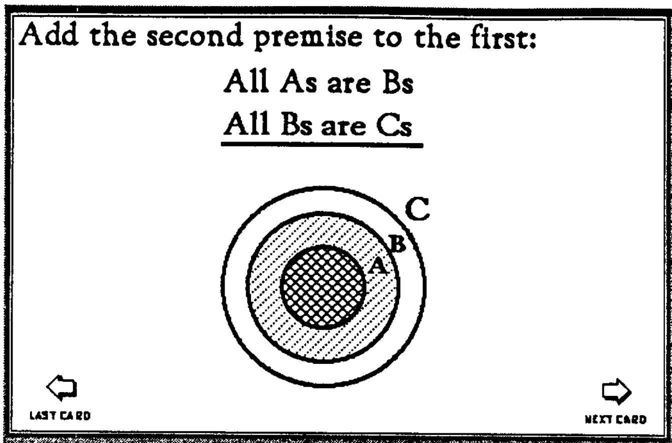
LAST CARD NEXT CARD

Look at the first premise:

<u>All As are Bs</u>



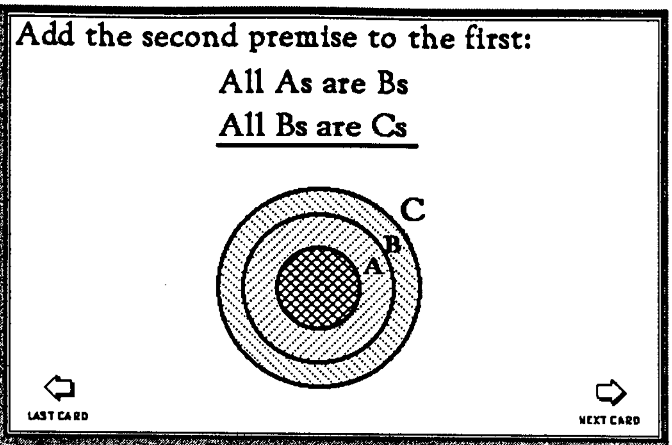
LAST CARD NEXT CARD



Add the second premise to the first:

All As are Bs

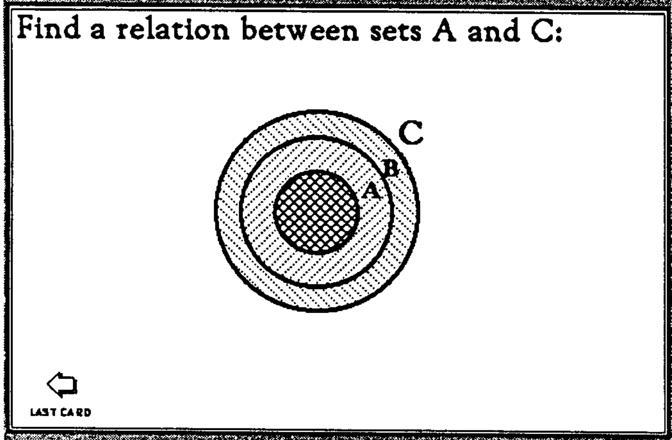
All Bs are Cs



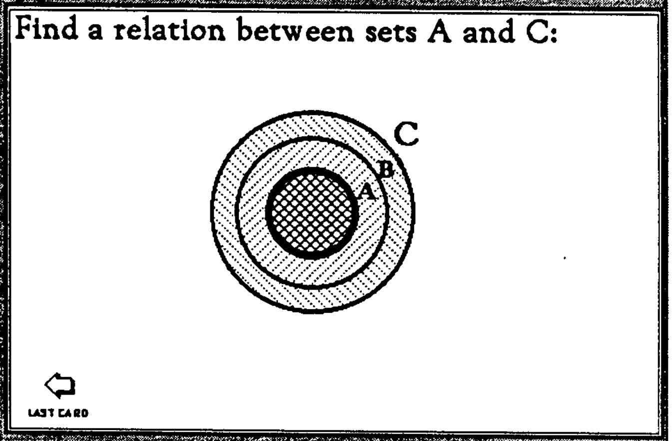
Add the second premise to the first:

All As are Bs

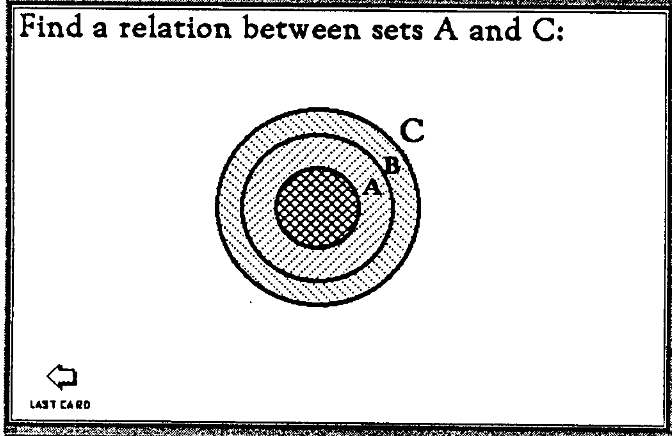
All Bs are Cs



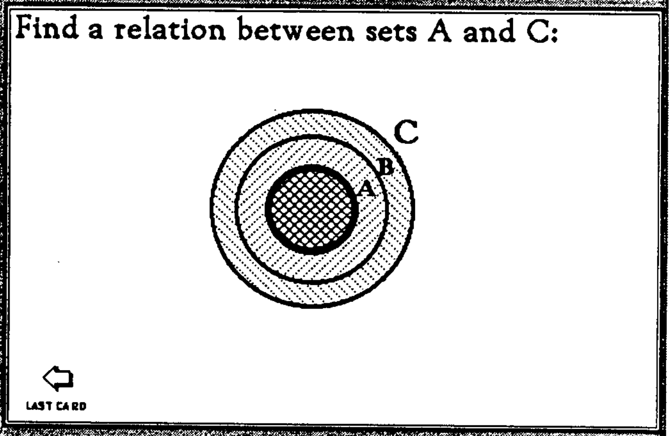
Find a relation between sets A and C:



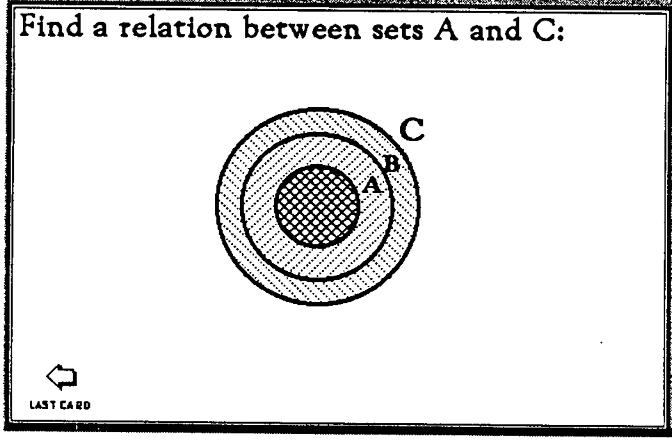
Find a relation between sets A and C:



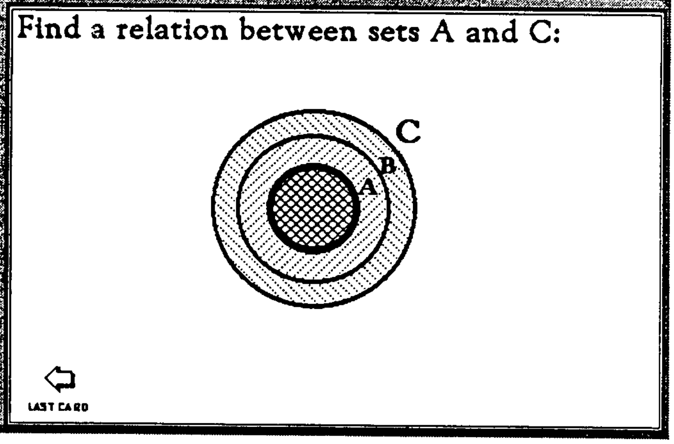
Find a relation between sets A and C:



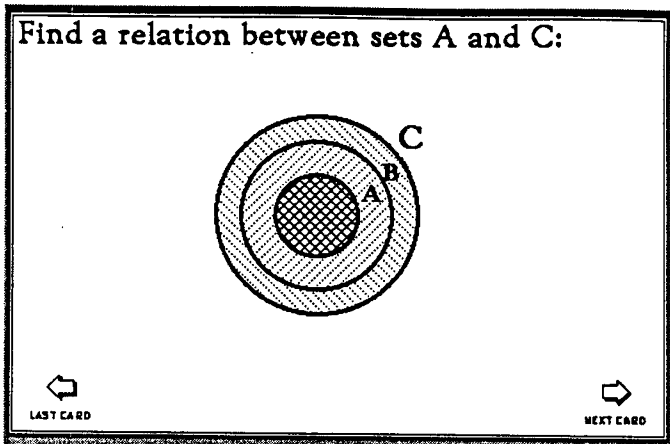
Find a relation between sets A and C:

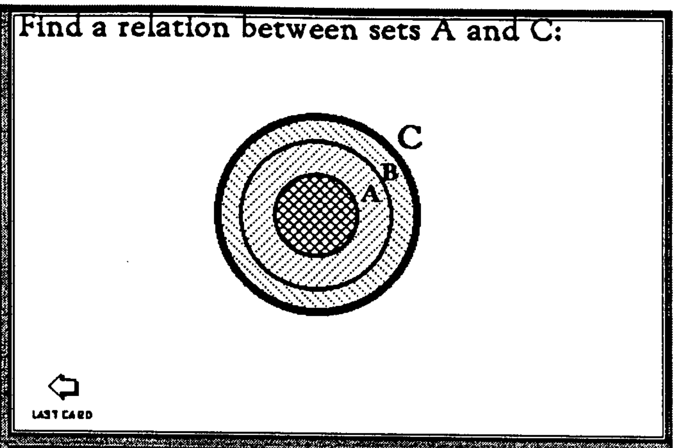


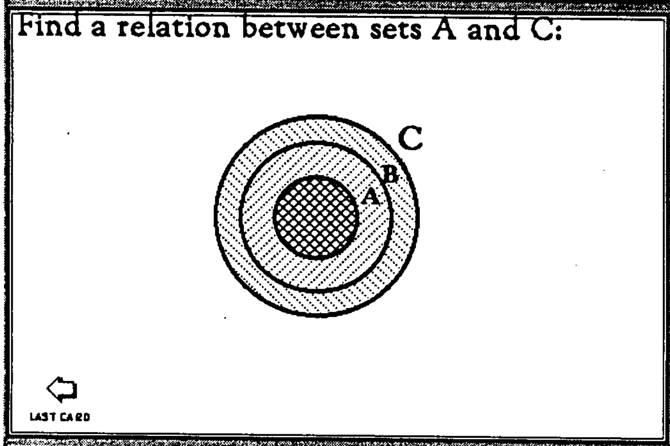
Find a relation between sets A and C:

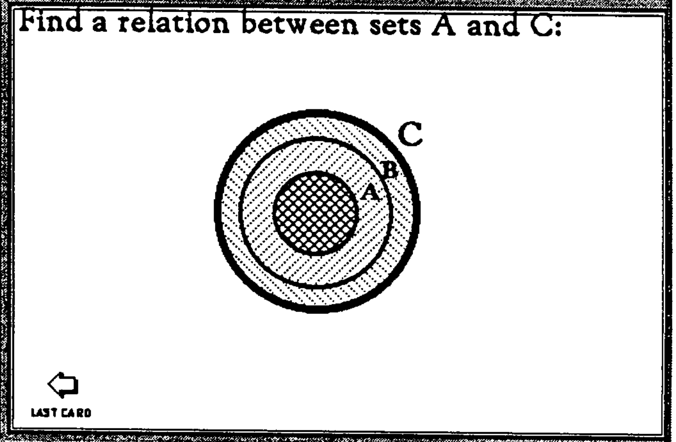


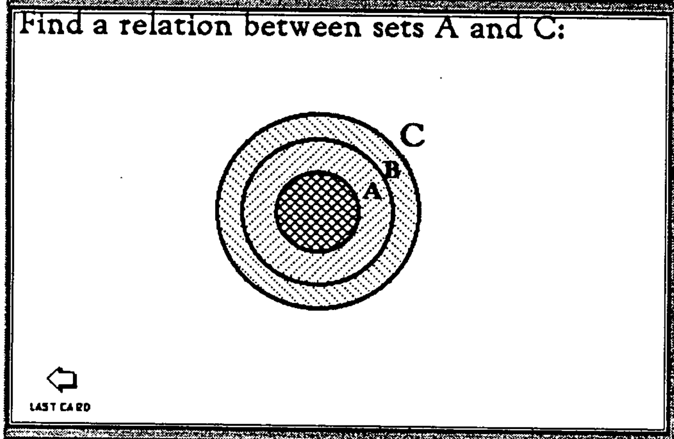
Find a relation between sets A and C:

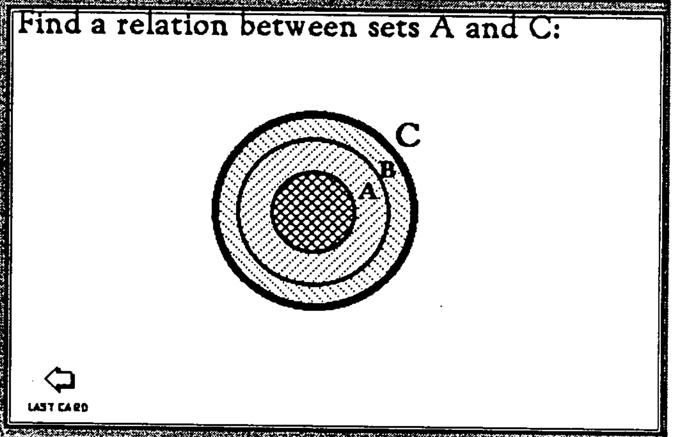

Find a relation between sets A and C:

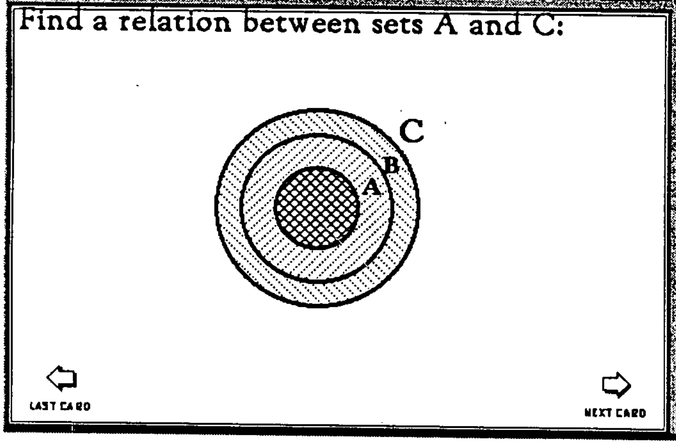

Find a relation between sets A and C:

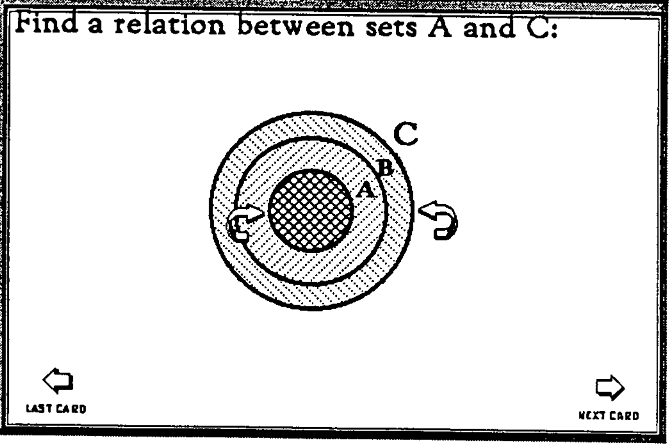

Find a relation between sets A and C:


Find a relation between sets A and C:

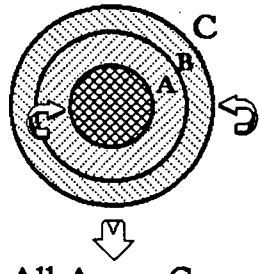

Find a relation between sets A and C:


Find a relation between sets A and C:


Find a relation between sets A and C:


Find a relation between sets A and C:

Draw a conclusion:



All As are Cs

LAST CARD NEXT CARD

The conclusion we arrived at was:

All As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on worksheet #2 NOW.
Do the two answers match?
DO NOT change your original answer!!

LAST CARD NEXT CARD

Example 3.
Consider the following syllogism:

Some As are Bs,
All Bs are Cs,

Conclusion ? ?

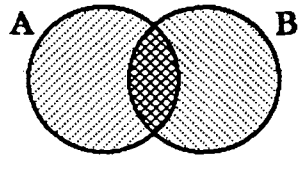
LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

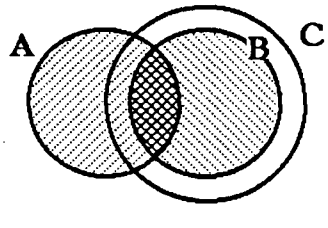
Some As are Bs



LAST CARD NEXT CARD

Add the second premise to the first:

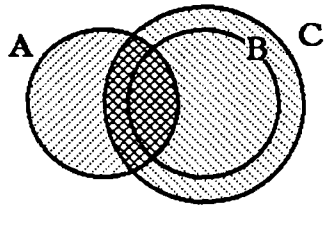
Some As are Bs
All Bs are Cs



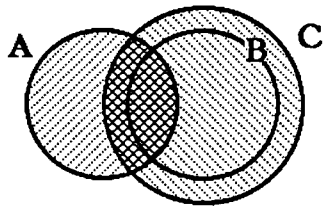
LAST CARD NEXT CARD

Add the second premise to the first:

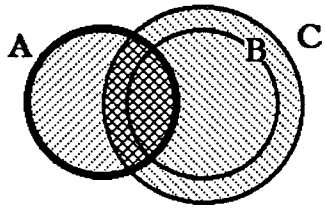
Some As are Bs
All Bs are Cs



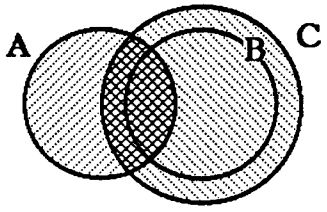
LAST CARD NEXT CARD



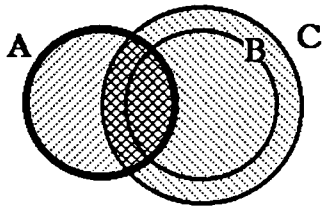
Find a relation between sets A and C:



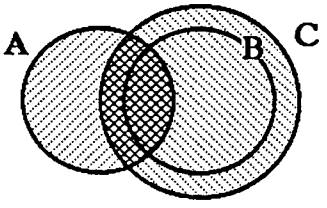
Find a relation between sets A and C:



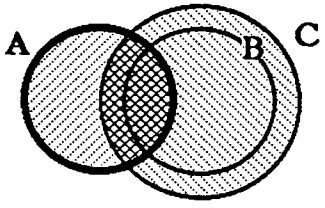
Find a relation between sets A and C:



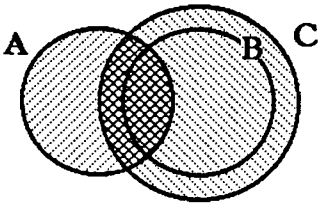
Find a relation between sets A and C:



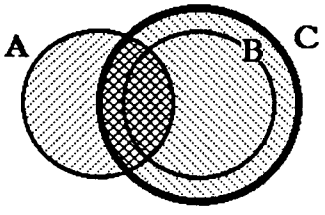
Find a relation between sets A and C:



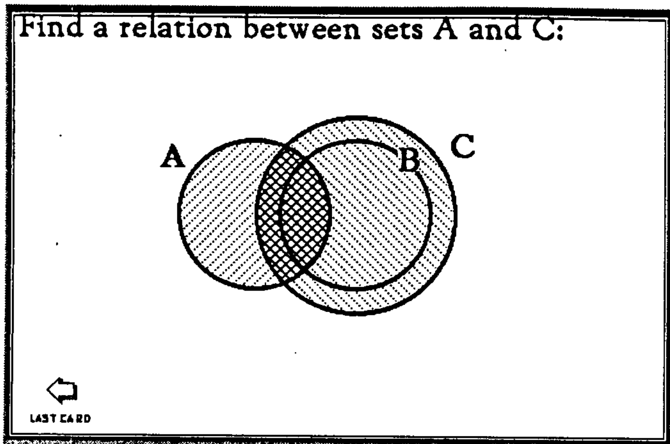
Find a relation between sets A and C:



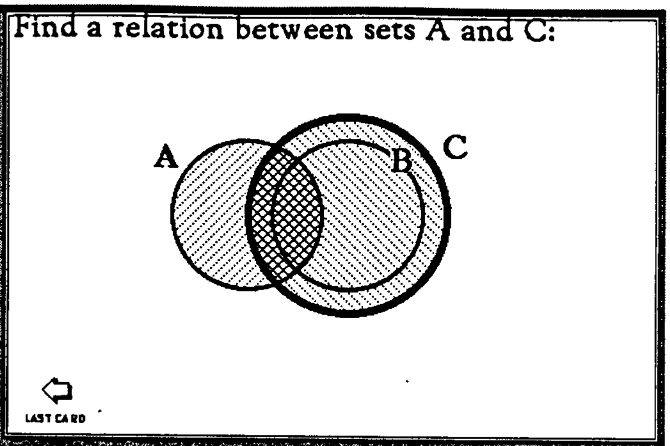
Find a relation between sets A and C:



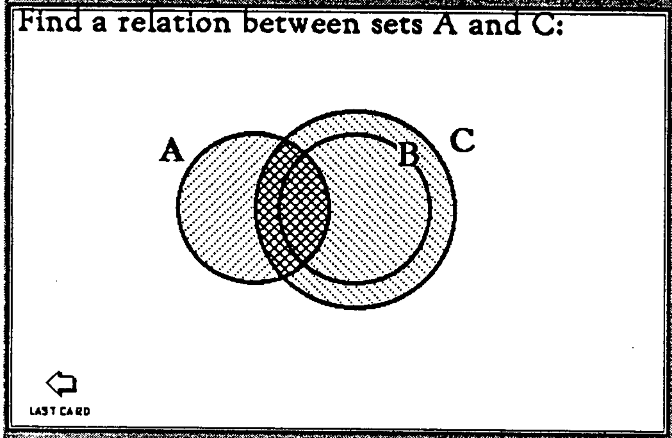
Find a relation between sets A and C:



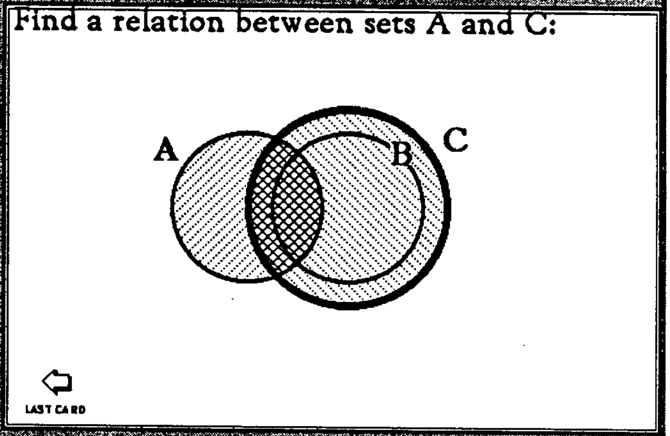
Find a relation between sets A and C:



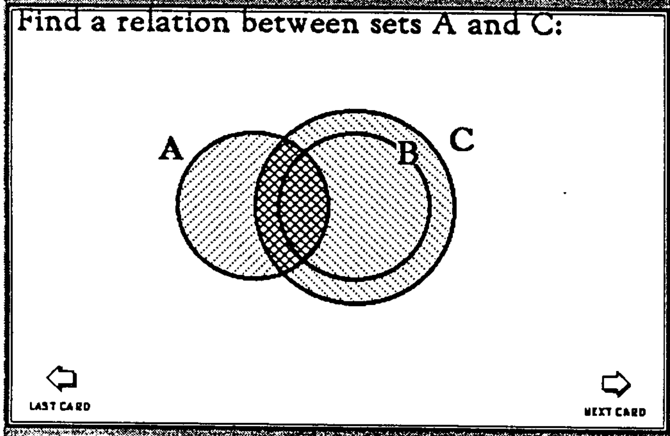
Find a relation between sets A and C:



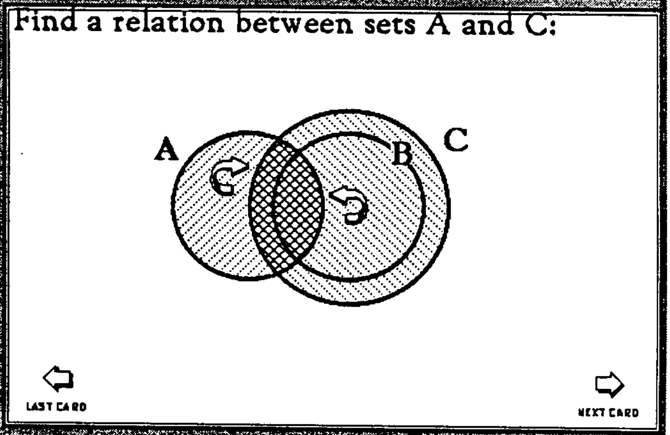
Find a relation between sets A and C:



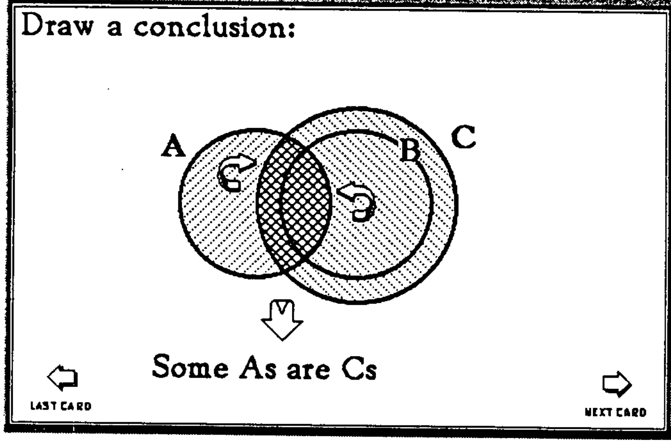
Find a relation between sets A and C:



Find a relation between sets A and C:

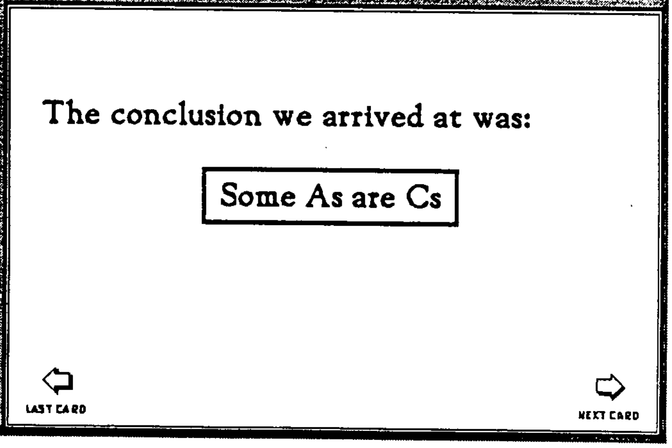


Find a relation between sets A and C:



Draw a conclusion:

Some As are Cs



The conclusion we arrived at was:

Some As are Cs

Write down this conclusion in the space on worksheet #2 NOW.
Do the two answers match?
DO NOT change your original answer!!

LAST CARD NEXT CARD

Example 4.
Consider the following syllogism:

No As are Bs,
All Cs are Bs,

Conclusion ? ?

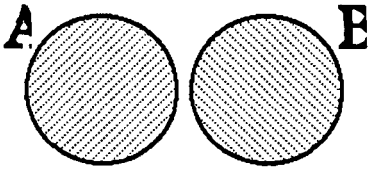
LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

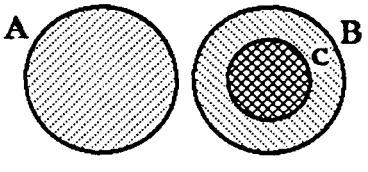
No As are Bs



LAST CARD NEXT CARD

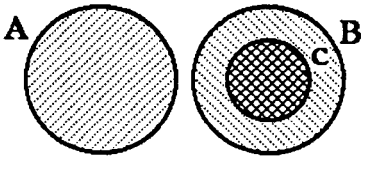
Add the second premise to the first:

No As are Bs
All Cs are Bs



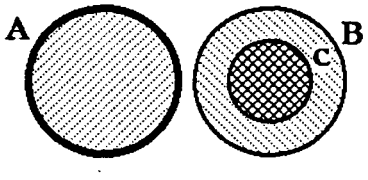
LAST CARD NEXT CARD

Find a relation between sets A and C:



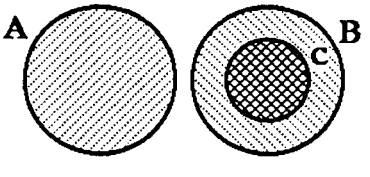
LAST CARD

Find a relation between sets A and C:

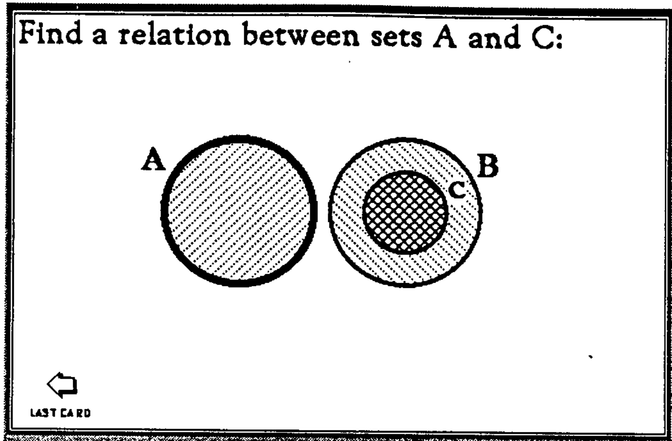


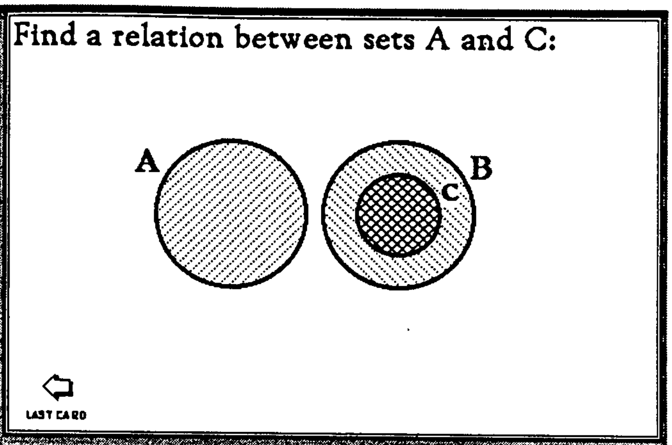
LAST CARD

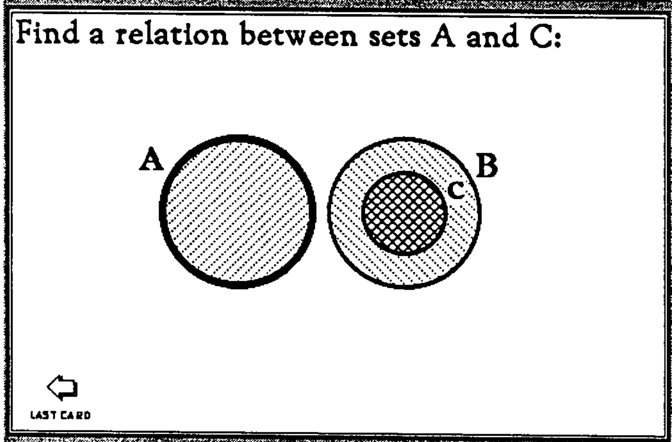
Find a relation between sets A and C:

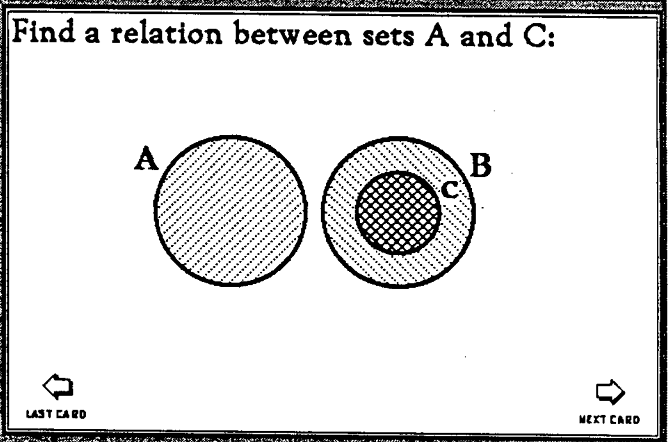


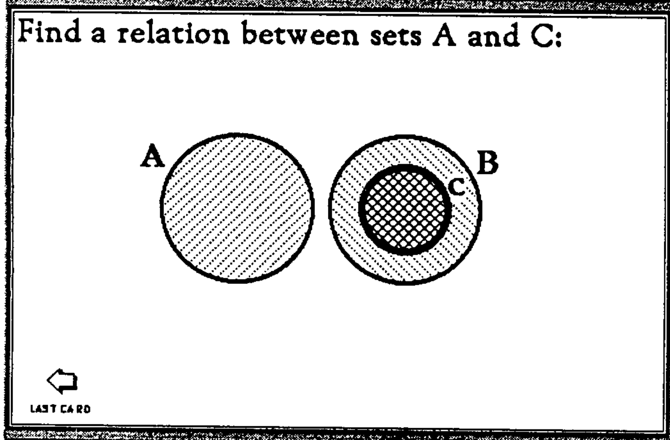
LAST CARD

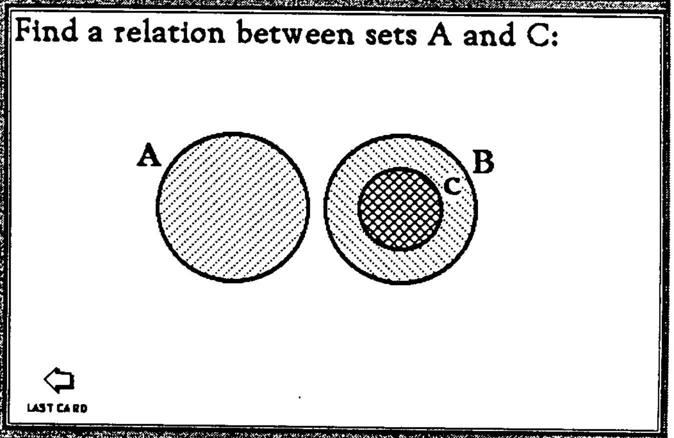

Find a relation between sets A and C:

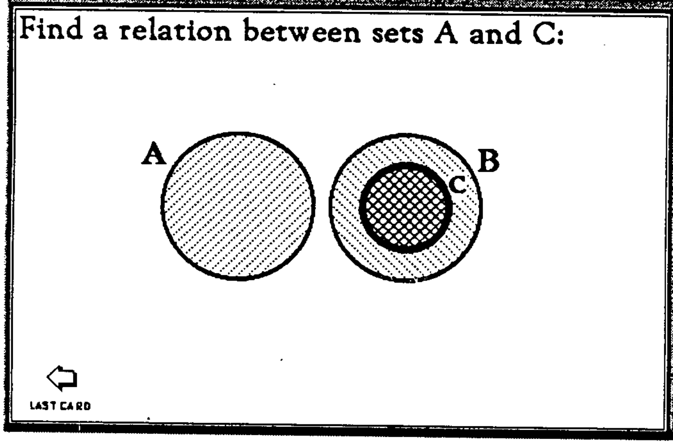

Find a relation between sets A and C:

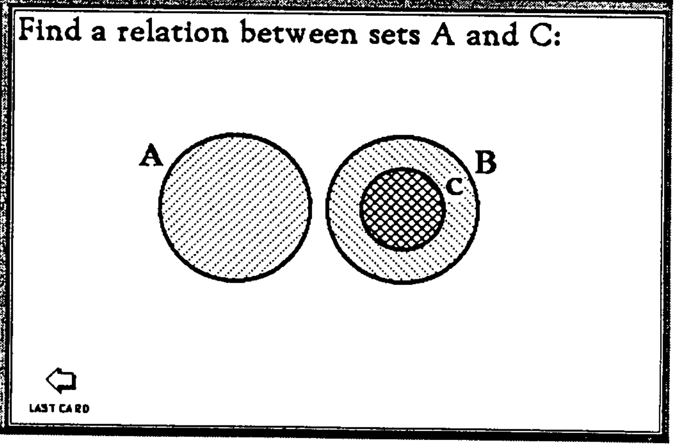

Find a relation between sets A and C:


Find a relation between sets A and C:

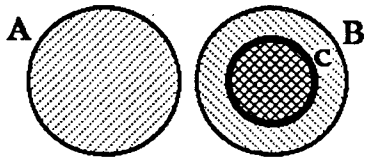

Find a relation between sets A and C:


Find a relation between sets A and C:


Find a relation between sets A and C:

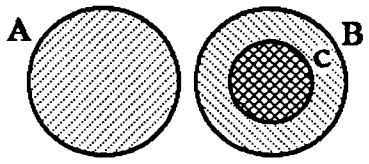

Find a relation between sets A and C:

Find a relation between sets A and C:



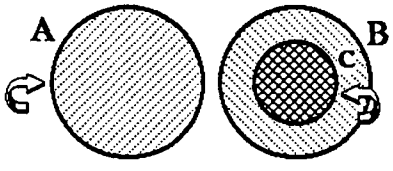
LAST CARD

Find a relation between sets A and C:



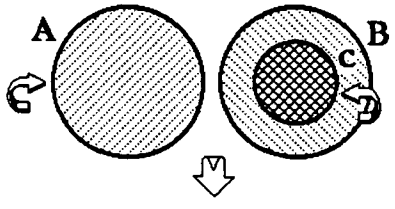
LAST CARD NEXT CARD

Find a relation between sets A and C:



LAST CARD NEXT CARD

Draw a conclusion:



No As are Cs

LAST CARD NEXT CARD

The conclusion we arrived at was:

No As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on worksheet #2 NOW.
Do the two answers match?
DO NOT change your original answer!!

LAST CARD NEXT CARD

<u>Example 5.</u>
Consider the following syllogism:

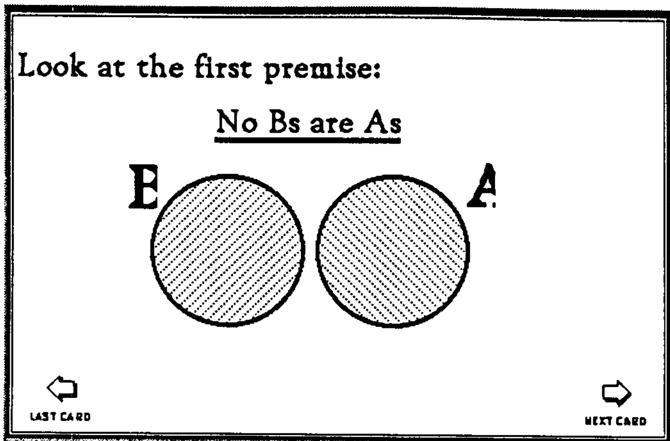
No Bs are As,
All Cs are Bs,

Conclusion ??

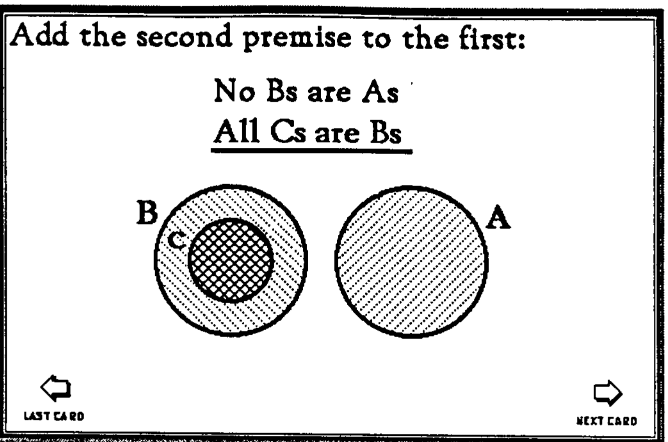
LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

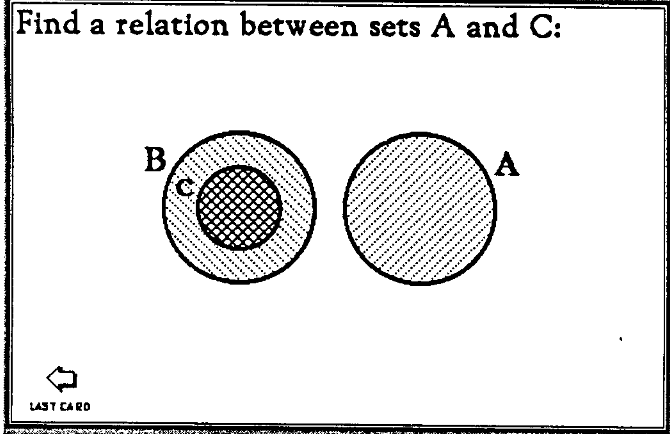
LAST CARD NEXT CARD

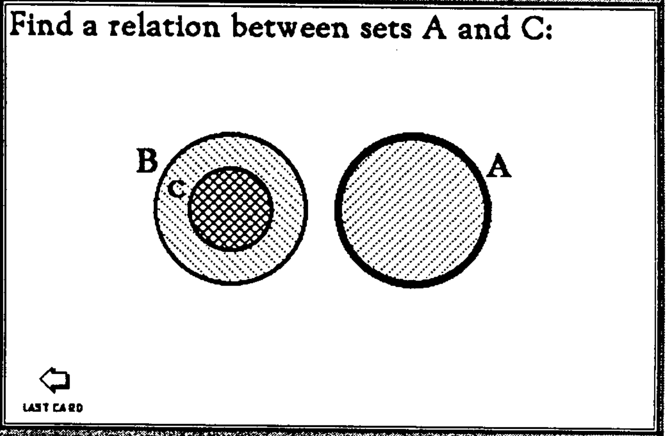

Look at the first premise:

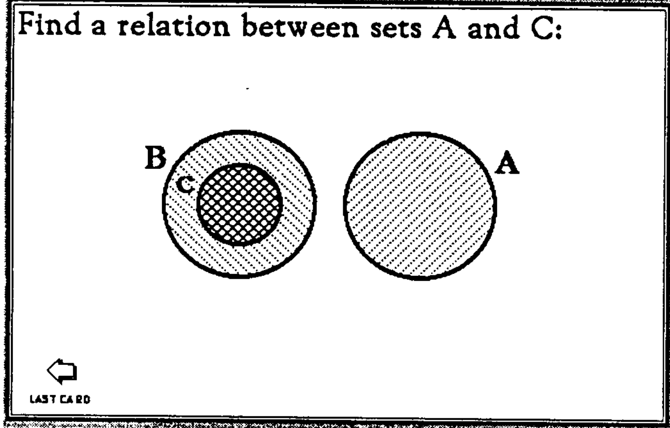
No Bs are As

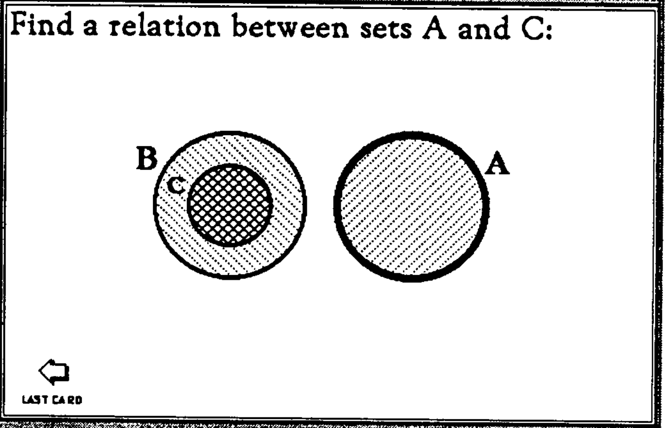

Add the second premise to the first:

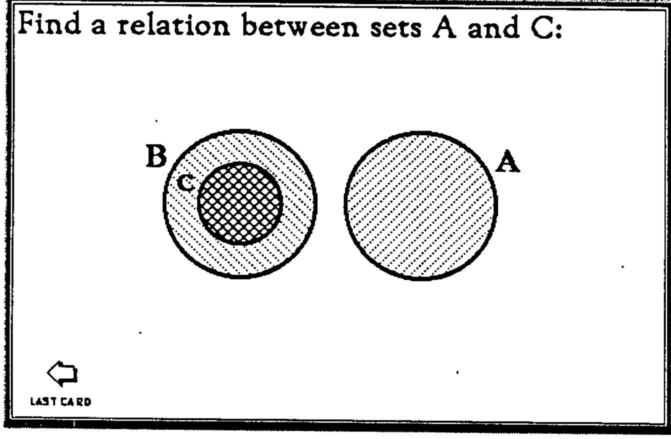
No Bs are As
All Cs are Bs

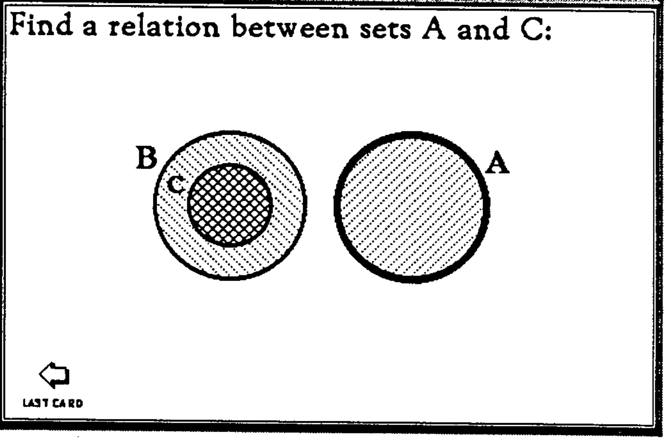

Find a relation between sets A and C:

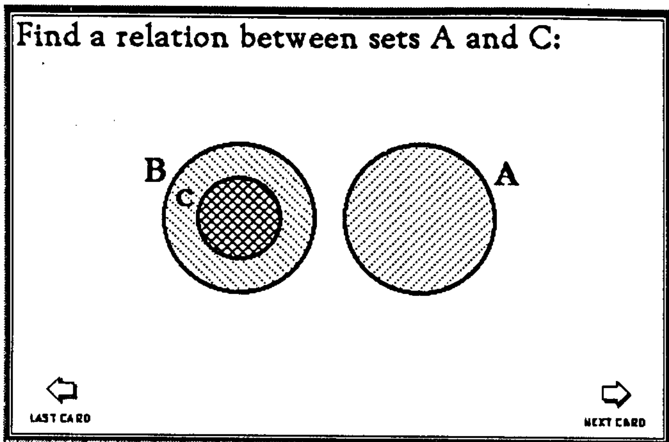

Find a relation between sets A and C:


Find a relation between sets A and C:

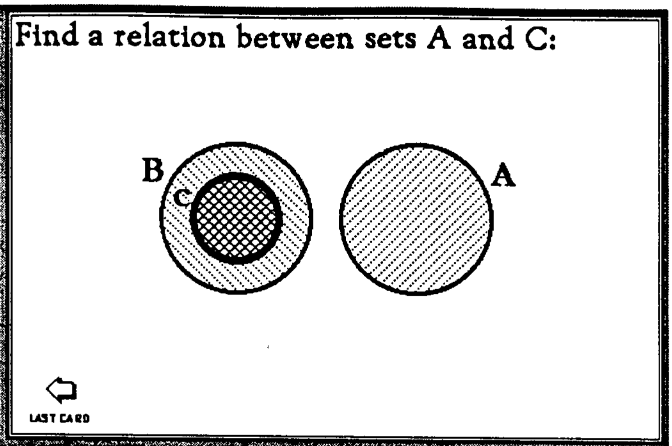

Find a relation between sets A and C:


Find a relation between sets A and C:

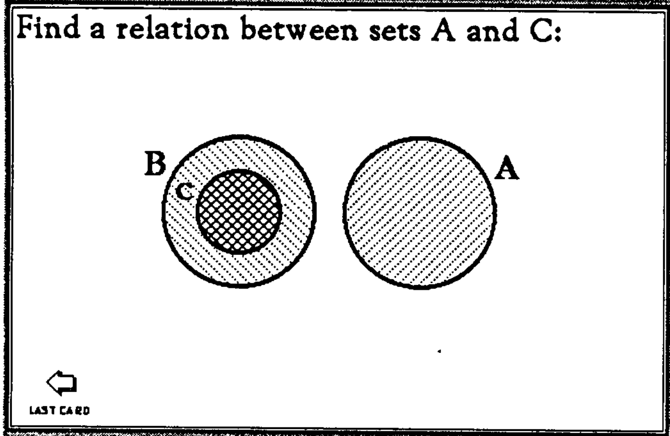

Find a relation between sets A and C:



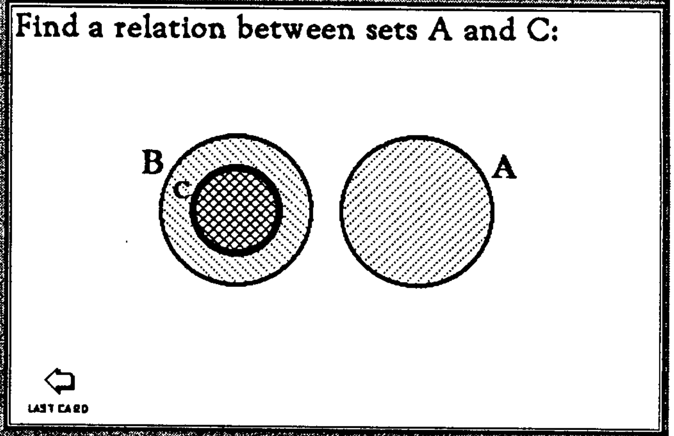
Find a relation between sets A and C:



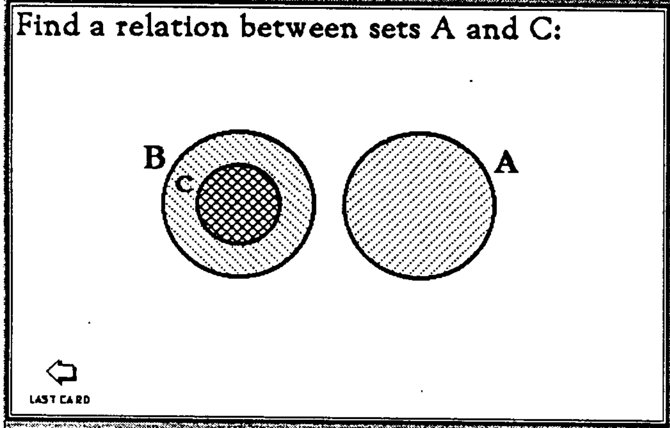
Find a relation between sets A and C:



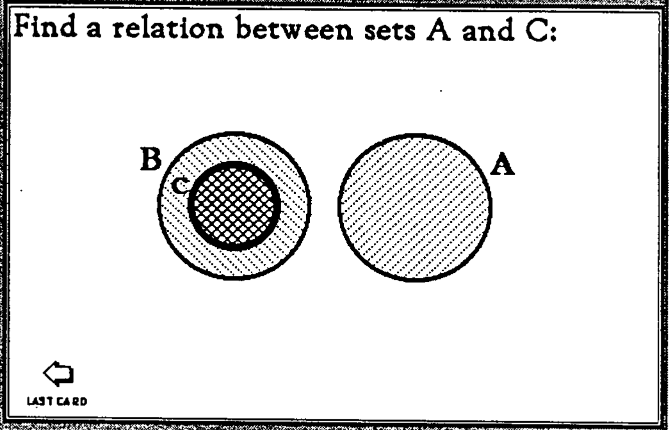
Find a relation between sets A and C:



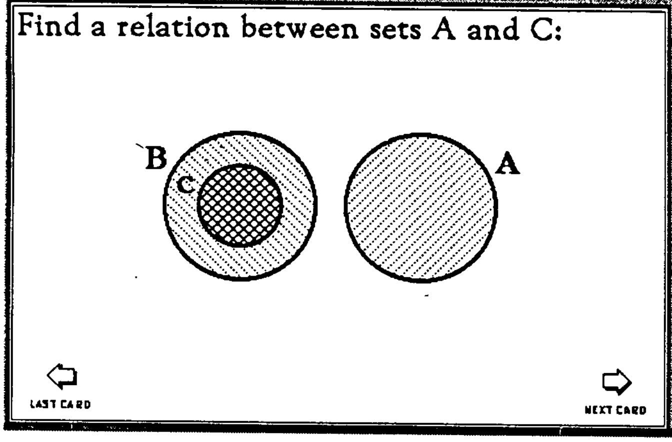
Find a relation between sets A and C:



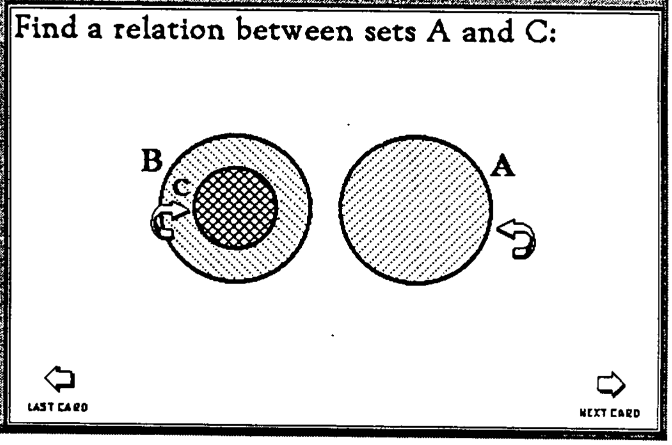
Find a relation between sets A and C:



Find a relation between sets A and C:

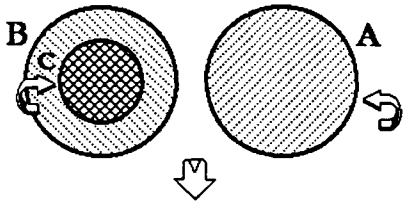


Find a relation between sets A and C:



Find a relation between sets A and C:

Draw a conclusion:



No As are Cs

LAST CARD NEXT CARD

The conclusion we arrived at was:

No As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on worksheet #2 NOW. Do the two answers match? DO NOT change your original answer!!

LAST CARD NEXT CARD

Example 6.
Consider the following syllogism:

All Bs are As,
Some Bs are Cs,

Conclusion ? ?

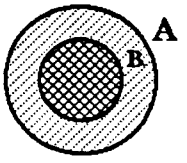
LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

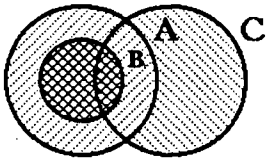
All Bs are As



LAST CARD NEXT CARD

Add the second premise to the first:

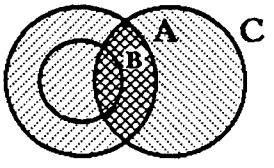
All Bs are As
Some Bs are Cs



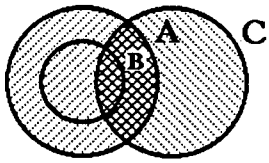
LAST CARD NEXT CARD

Add the second premise to the first:

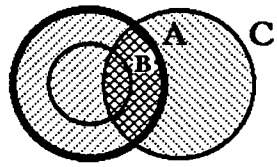
All Bs are As
Some Bs are Cs



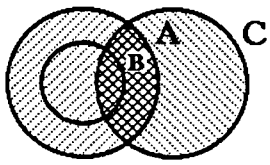
LAST CARD NEXT CARD



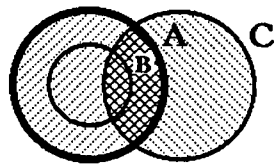
Find a relation between sets A and C:



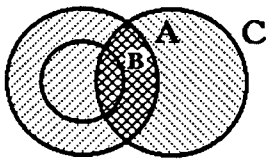
Find a relation between sets A and C:



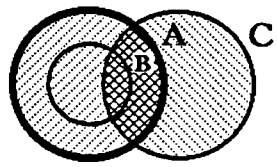
Find a relation between sets A and C:



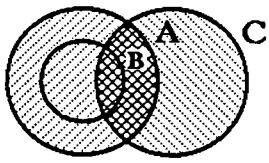
Find a relation between sets A and C:



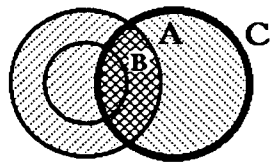
Find a relation between sets A and C:



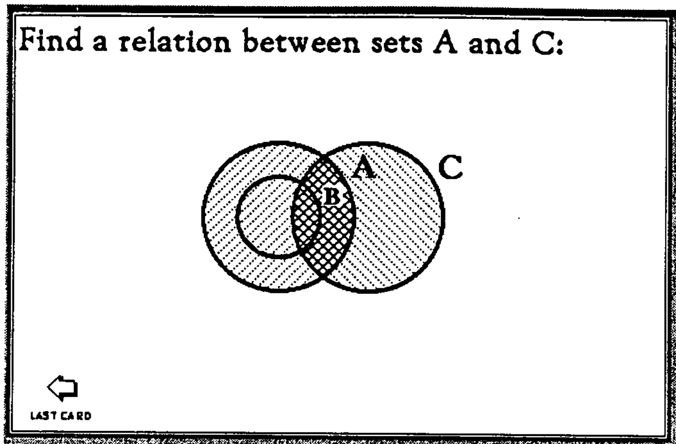
Find a relation between sets A and C:



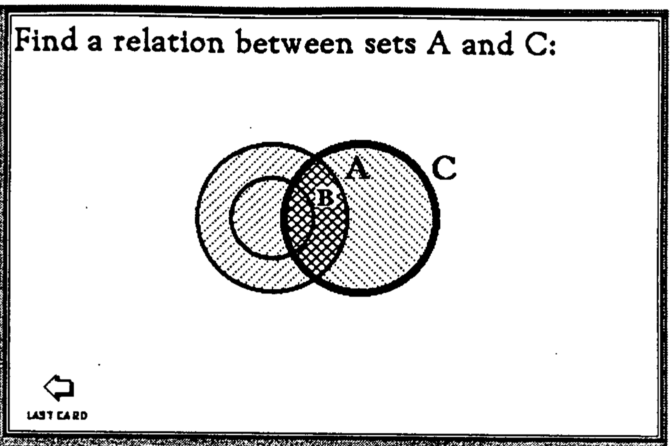
Find a relation between sets A and C:



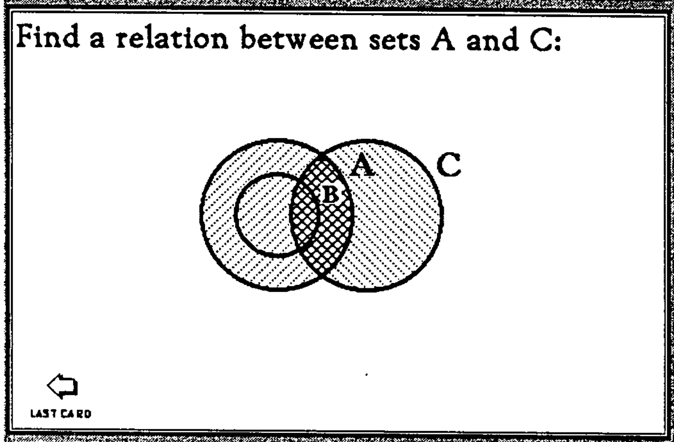
Find a relation between sets A and C:



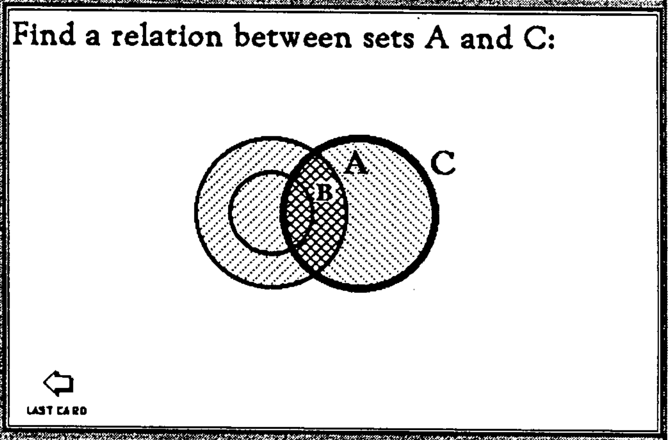
Find a relation between sets A and C:



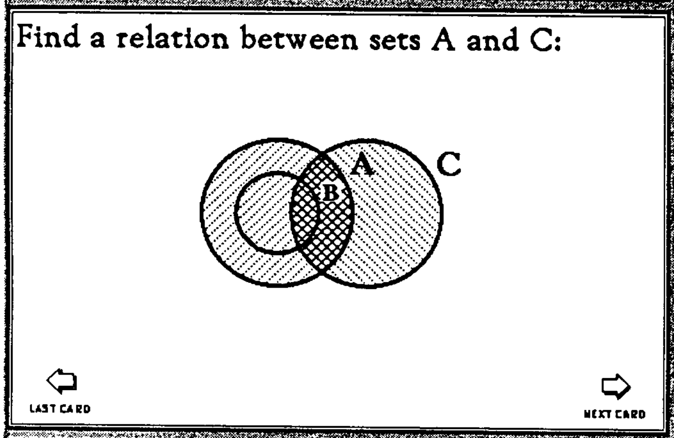
Find a relation between sets A and C:



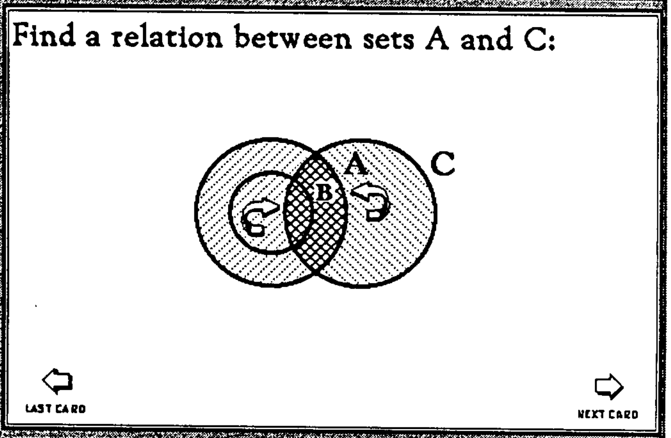
Find a relation between sets A and C:



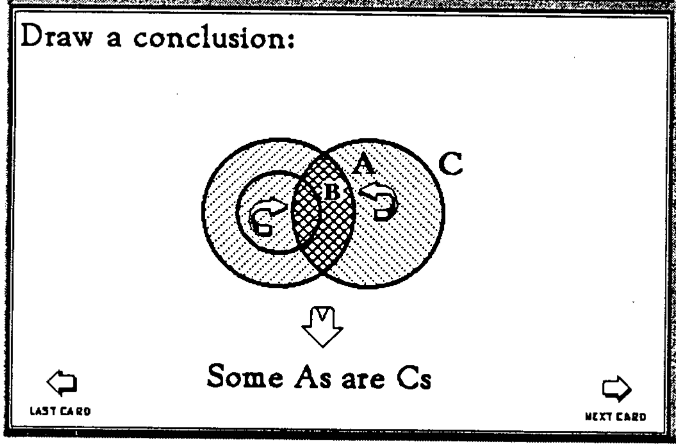
Find a relation between sets A and C:



Find a relation between sets A and C:

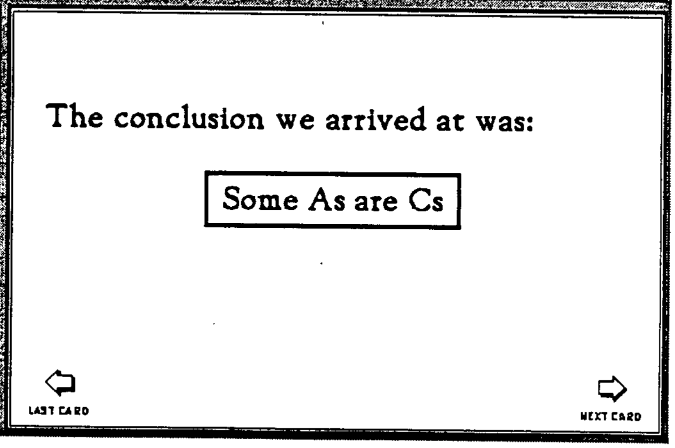


Find a relation between sets A and C:



Draw a conclusion:

Some As are Cs



The conclusion we arrived at was:

Some As are Cs

Write down this conclusion in the space on worksheet #2 NOW.
Do the two answers match?
DO NOT change your original answer!!

LAST CARD

NEXT CARD

Appendix D

Sample Representation Syllogism Instruction Module

For this research project, you will learn how to solve syllogisms.
Click on NEXT CARD to continue.

NEXT CARD

↓ = "is"

LAST CARD NEXT CARD

For example,

A is B

A
↓
B

LAST CARD NEXT CARD

and,

B is A

A
↑
B

LAST CARD NEXT CARD

The same rule applies for many As and Bs:

Many As are Bs

A A A
↓ ↓ ↓
B B B

LAST CARD NEXT CARD

and,

Many Bs are As

A A A
↑ ↑ ↑
B B B

LAST CARD NEXT CARD

⊥ = "is not"

LAST CARD NEXT CARD

For example,

A is not B

A
⊥
B

LAST CARD NEXT CARD

and,

B is not A

A
⊤
B

LAST CARD NEXT CARD

The same rule applies for many As and Bs:

Many As are not Bs

A A A
⊥ ⊥ ⊥
B B B

LAST CARD NEXT CARD

and,

Many Bs are not As

A A A
⊤ ⊤ ⊤
B B B

LAST CARD NEXT CARD

Let's review some statements and their representations. Click on NEXT CARD to continue.

LAST CARD NEXT CARD

All As are Bs

A
↓
B (B)

LAST CARD NEXT CARD

Also,

All Bs are As

A (A)
↑
B

LAST CARD NEXT CARD

No Bs are Cs

B B
⊥ ⊥
C C

LAST CARD NEXT CARD

Also,

No Cs are Bs

B B
⊤ ⊤
C C

LAST CARD NEXT CARD

Some As are Bs

A (A)
↓
B (B)

LAST CARD NEXT CARD

Also, Some Bs are As

A (A)
↑
B (B)

LAST CARD NEXT CARD

Some Bs are not Cs

B (B)
⊥ ↓
C C

LAST CARD NEXT CARD

Also, Some Cs are not Bs

B B
⊤ ↑
C (C)

LAST CARD NEXT CARD

Now, let us study the 5 steps needed to solve a syllogism: Click on NEXT CARD to continue.

LAST CARD NEXT CARD

In the following examples, write your answers to the syllogisms in the spaces provided on your worksheets. After completing each exercise, study the solution steps and compare your answer to the correct answer. Do the two answers match?

LAST CARD NEXT CARD

Example 1.
Consider the following syllogism:

All As are Bs,
No Bs are Cs,

Conclusion ? ?

LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

LAST CARD NEXT CARD

Some As are Bs

A (A)
↓
B (B)

LAST CARD NEXT CARD

Also, **Some Bs are As**

A (A)
↑
B (B)

LAST CARD NEXT CARD

Some Bs are not Cs

B (B)
⊥ ↓
C C

LAST CARD NEXT CARD

Also, **Some Cs are not Bs**

B B
⊤ ↑
C (C)

LAST CARD NEXT CARD

Now, let us study the 5 steps needed to solve a syllogism: Click on NEXT CARD *to continue.*

LAST CARD NEXT CARD

In the following examples, write your answers to the syllogisms in the spaces provided on your worksheets. After completing each exercise, study the solution steps and compare your answer to the correct answer. Do the two answers match?

LAST CARD NEXT CARD

Example 1.
Consider the following syllogism:

All As are Bs,
No Bs are Cs,

Conclusion ? ?

LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

All As are Bs

A
↓
B (B)

LAST CARD NEXT CARD

Look at the second premise:

No Bs are Cs

B B
⊥ ⊥
C C

LAST CARD NEXT CARD

Combine the premises

All As are Bs

A
↓
B (B)

LAST CARD

Combine the premises

All As are Bs

No Bs are Cs

A
↓
B (B)
⊥ ⊥
C C

LAST CARD NEXT CARD

Classify the paths between As and Cs

All As are Bs

No Bs are Cs

A
↓
B (B)
⊥ ⊥
C C

LAST CARD

Classify the paths between As and Cs

To see this step again, click here:

All As are Bs

No Bs are Cs

A
↓
B (B)
⊥ ⊥
C C

LAST CARD NEXT CARD

Draw a conclusion

A
⊥
C

Therefore, No As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on your worksheet #2 NOW. Do the two answers match? DO NOT change your original answer! !

LAST CARD NEXT CARD

Example 2.

Consider the next syllogism:

All As are Bs,
All Bs are Cs,

Conclusion ??

LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

All As are Bs

A
↓
B (B)

LAST CARD NEXT CARD

Look at the second premise:

All Bs are Cs

B
↓
C (C)

LAST CARD NEXT CARD

Combine the premises

All As are Bs

A
↓
B (B)

LAST CARD

Combine the premises

All As are Bs
All Bs are Cs

A
↓
B (B)
↓
C (C)

LAST CARD

Combine the premises

All As are Bs
All Bs are Cs

A
↓
B (B)
↓ ↓
C C (C)

LAST CARD NEXT CARD

Classify the paths between As and Cs

To see this step again, click here:

All As are Bs
All Bs are Cs

A
↓
B (B)
↓ ↓
C C (C)

LAST CARD NEXT CARD

Draw a conclusion

A
↓
C

Therefore, All As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on your worksheet #2 NOW. Do the two answers match? DO NOT change your original answer! !

LAST CARD NEXT CARD

Example 3.

Consider the following syllogism:

Some As are Bs,
All Bs are Cs,

Conclusion ? ?

LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

Some As are Bs

A (A)
↓
B (B)

LAST CARD NEXT CARD

Look at the second premise:

All Bs are Cs

B
↓
C (C)

LAST CARD NEXT CARD

Combine the premises

Some As are Bs

A (A)
↓
B (B)

LAST CARD

Combine the premises

Some As are Bs

All Bs are Cs

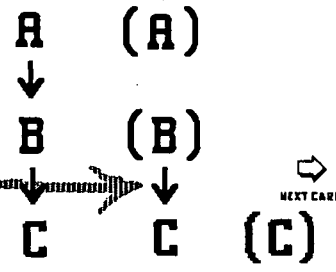
A (A)
↓
B (B)
↓
C (C)

LAST CARD

Combine the premises

Some As are Bs

All Bs are Cs



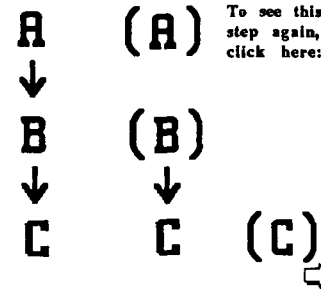
NEXT CARD

LAST CARD

Classify the paths between As and Cs

Some As are Bs

All Bs are Cs



To see this step again, click here:

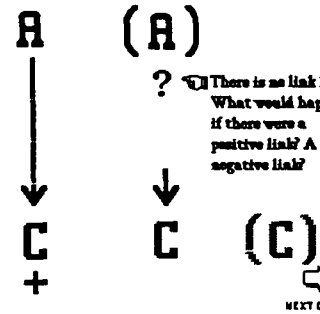
LAST CARD

NEXT CARD

Classify the paths between As and Cs

Some As are Bs

All Bs are Cs



There is no link here. What would happen if there were a positive link? A negative link?

LAST CARD

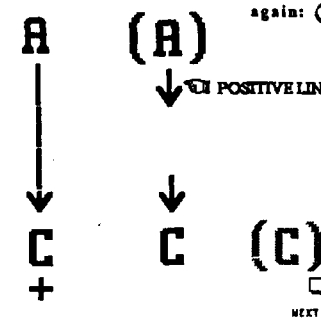
NEXT CARD

Classify the paths between As and Cs

To see again:

Some As are Bs

All Bs are Cs

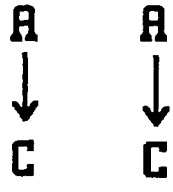


POSITIVE LINK.

LAST CARD

NEXT CARD

Draw a conclusion



Therefore, All As are Cs

LAST CARD

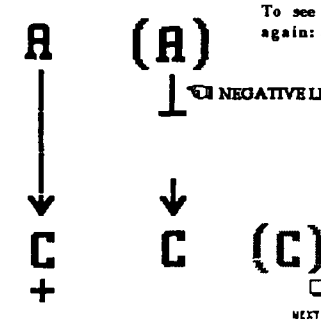
NEXT CARD

Classify the paths between As and Cs

To see again:

Some As are Bs

All Bs are Cs

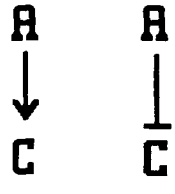


NEGATIVE LINK.

LAST CARD

NEXT CARD

Draw a conclusion



Therefore, Some As are Cs

LAST CARD

NEXT CARD

Look at the 2 conclusions we arrived at: All As are Cs and Some As are Cs. We must take the most general of the two as the valid conclusion. Since we can say Some As are Cs even if All As are Cs, then the conclusion must be:

Some As are Cs.

LAST CARD

NEXT CARD

Write down this conclusion in the space on your worksheet #2 NOW. Do the two answers match? DO NOT change your original answer! !

LAST CARD NEXT CARD

Example 4.

Consider the following syllogism:

No As are Bs,
All Cs are Bs,

Conclusion ? ?

LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

No As are Bs

A A
⊥ ⊥
B B

LAST CARD NEXT CARD

Look at the second premise:

All Cs are Bs

B (B)
↑
C

LAST CARD NEXT CARD

Combine the premises

No As are Bs

A A
⊥ ⊥
B B

LAST CARD

Combine the premises

No As are Bs
All Cs are Bs

A A (B)
⊥ ⊥
B B
↑ ↑
C C

LAST CARD

Combine the premises

No As are Bs
All Cs are Bs

A A A
⊥ ⊥ ⊥
B B (B)
↑ ↑
C C

LAST CARD NEXT CARD

Classify the paths between As and Cs

To see again:

No As are Bs

All Cs are Bs

```
A     A     A
⊥     ⊥     ⊥
B     B    (B)
↑     ↑
C     C
```

LAST CARD

Draw a conclusion

```
A     A
|     |
C     C
```

Therefore, No As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on your worksheet #2 NOW. Do the two answers match? DO NOT change your original answer! !

LAST CARD NEXT CARD

Example 5.

Consider the following syllogism:

No Bs are As,
All Cs are Bs,

Conclusion ? ?

LAST CARD NEXT CARD

On your worksheet #2, write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

No Bs are As

```
A     A
T     T
B     B
```

LAST CARD NEXT CARD

Look at the second premise:

All Cs are Bs

```
B    (B)
↑
C
```

LAST CARD NEXT CARD

Combine the premises

No Bs are As

```
A     A
T     T
B     B
```

LAST CARD

Combine the premises

No Bs are As

All Cs are Bs

A A
T T
B B (B)
↑ ↑
C C

LAST CARD

Combine the premises

No Bs are As

All Cs are Bs

A A A
T T T
B B (B)
↑ ↑
C C

NEXT CARD

LAST CARD

Classify the paths between As and Cs

To see again:

No Bs are As

All Cs are Bs

A A A
T T T
B B (B)
↑ ↑
C C

LAST CARD

Draw a conclusion

A A
| |
C C

Therefore, No As are Cs

LAST CARD

NEXT CARD

Write down this conclusion in the space on your worksheet #2 NOW. Do the two answers match? DO NOT change your original answer! !

LAST CARD

NEXT CARD

Example 6.

Consider the following syllogism:

All Bs are As,
Some Bs are Cs,

Conclusion ? ?

LAST CARD

NEXT CARD

On your worksheet #2, write down your conclusion NOW!

LAST CARD

NEXT CARD

Look at the first premise:

All Bs are As

A (A)
↑
B

LAST CARD

NEXT CARD

Look at the second premise:

Some Bs are Cs

B (B)
↓
C (C)

LAST CARD NEXT CARD

Combine the premises

All Bs are As

A (A)
↑
B

LAST CARD

Combine the premises

All Bs are As
Some Bs are Cs

A (A)
↑
B (B)
↓
C (C)

LAST CARD

Combine the premises

All Bs are As
Some Bs are Cs

A A (A)
↑ ↑
B (B)
↓
C (C)

LAST CARD NEXT CARD

Classify the paths between As and Cs

All Bs are As
Some Bs are Cs

A A (A)
↑ ↑
B (B)
↓
C (C)

To see again:

LAST CARD NEXT CARD

Classify the paths between As and Cs

All Bs are As
Some Bs are Cs

A A (A)
↓ ↑
C (C)
+

? There is no link here. What would happen if there were a positive link? A negative link?

LAST CARD NEXT CARD

Classify the paths between As and Cs

All Bs are As
Some Bs are Cs

A A (A)
↓ ↑
C (C)
+

To see again:

↓ POSITIVE LINK.

LAST CARD NEXT CARD

Draw a conclusion

A A
↓ ↓
C C

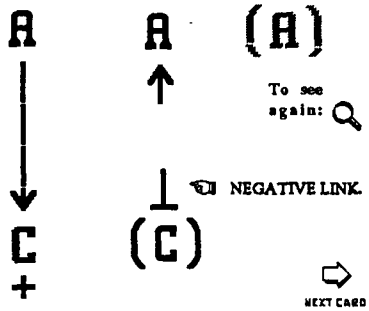
Therefore, All As are Cs

LAST CARD NEXT CARD

Classify the paths between As and Cs

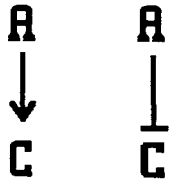
All Bs are As

Some Bs are Cs



LAST CARD NEXT CARD

Draw a conclusion



Therefore, Some As are Cs

LAST CARD NEXT CARD

Look at the 2 conclusions we arrived at: All As are Cs and Some As are Cs. We must take the most general of the two as the valid conclusion. Since we can say Some As are Cs even if All As are Cs, then the conclusion must be:

Some As are Cs.

LAST CARD NEXT CARD

Write down this conclusion in the space on your worksheet #2 NOW. Do the two answers match? DO NOT change your original answer! !

LAST CARD NEXT CARD

Appendix E

Non-visual Syllogism Instruction Module

For this research project, you will learn how to solve syllogisms.

NEXT CARD

What is a syllogism?

A syllogism is a word problem that has:
a) two clues; and
b) a solution.
Click on NEXT CARD for an example.

LAST CARD NEXT CARD

Here is an example of a syllogism:

Clue #1: All As are Bs
Clue #2: No Bs are Cs
Solution: No As are Cs.

LAST CARD NEXT CARD

We will refer to the "clues" as "statements" and the correct solution as the "conclusion". Therefore, this syllogism takes the form of:

Statement #1: All As are Bs
Statement #2: No Bs are Cs
Conclusion: No As are Cs.

LAST CARD NEXT CARD

Statement #1 always involves As and Bs: All As are Bs

Statement #2 always involves Bs and Cs: No Bs are Cs

The Conclusion always expresses a relationship between As and Cs: No As are Cs.

LAST CARD NEXT CARD

Here is a problem to solve:

Clue #1: All As are Bs
Clue #2: No Bs are Cs
Solution: ?

What is the relationship between the As and the Cs? Click on NEXT CARD to find out.

LAST CARD NEXT CARD

The correct solution for this syllogism is:
"No As are Cs".

You will learn how to use the two given clues to figure out the correct solution.

LAST CARD NEXT CARD

Let's learn how to solve various syllogisms! But first, let's start with a few basics.

LAST CARD NEXT CARD

DEFINITION

A group of objects is called a set. The objects making up a set belong to the set. These items which belong to the set are called members.

LAST CARD NEXT CARD

DEFINITION (con'd)

From now on, we will be talking about sets A, B, and C. The members of set A are As, the members of set B are Bs, and the members of set C are Cs.

LAST CARD NEXT CARD

Let's look at some statements and see how they can be represented:

LAST CARD NEXT CARD

Statement: All As are Bs

EXPLANATION:

Each member in set A is associated with a member in set B. In other words, each A is a B. Even if there are more Bs than As, those extra Bs don't count because still all As are Bs.

LAST CARD NEXT CARD

Statement: No Bs are Cs

EXPLANATION:

Here it is stated that each member in set B is not associated with a member in set C. It is important to remember that each B can be anything, but not C. The size of sets B and C does not matter because there are still no Bs that are Cs.

LAST CARD NEXT CARD

Statement: Some As are Bs

EXPLANATION:

There are 2 cases:
Case 1) Some As are Bs
There are some members of set A that are Bs regardless of the size of the sets. Also, even if there are more As than Bs, still some As can be Bs.

LAST CARD NEXT CARD

Statement: Some As are Bs

EXPLANATION (con'd):

Case 2): All As are Bs
In the case that all As are Bs, we can take some of those As and say they are Bs.

LAST CARD NEXT CARD

Statement: Some Bs are not Cs

EXPLANATION:

There are 2 cases:
Case 1): Some Bs are not Cs
There are some members of set B that are not Cs, regardless of the size of the sets. Specially if there are more Bs than Cs, those extra Bs are the ones that are not Cs.

LAST CARD NEXT CARD

Statement: **Some Bs are not Cs**

EXPLANATION (con'd):

Case 2): No Bs are Cs
In the case that No Bs are Cs, we can take some of those Bs and say they are not Cs.

LAST CARD NEXT CARD

Let's look at the steps needed to solve a syllogism:
Click on NEXT CARD to continue.

LAST CARD NEXT CARD

In the following examples, write your answers to the syllogisms in the spaces provided on your worksheets. After completing each exercise, study the solution steps and compare your answer to the correct answer. Do the two answers match?

LAST CARD NEXT CARD

Example 1.

Consider the following syllogism:

All As are Bs,

No Bs are Cs,

Conclusion?

LAST CARD NEXT CARD

On your worksheet #2 , write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

All As are Bs

Each member of set A is a B. All As are Bs.

LAST CARD NEXT CARD

Look at the second premise:

No Bs are Cs

Each member of set B is not a C. No Bs are Cs.

LAST CARD NEXT CARD

Combine the premises:

All As are Bs

No Bs are Cs

We know that each A is a B, and no B can be a C, so this leads us to the conclusion that no As are Cs.

LAST CARD NEXT CARD

Look at the conclusion(s):

We have come to 1 conclusion:
1) No As are Cs.
Therefore, the valid conclusion is:

No As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on worksheet #2 NOW.
Do the two answers match?
DO NOT change your original answer!!

LAST CARD NEXT CARD

Example 2.

Consider the next syllogism:

All As are Bs,
All Bs are Cs,

Conclusion?

LAST CARD NEXT CARD

On your worksheet #2 , write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

All As are Bs

Each member of set A is a B. All As are Bs.

LAST CARD NEXT CARD

Look at the second premise:

All Bs are Cs

Each member of set B is a C. All Bs are Cs.

LAST CARD NEXT CARD

Combine the premises:

All As are Bs
All Bs are Cs

We know that each A is a B, and we also know that each B is a C, therefore this leads us to conclude all As are Cs.

LAST CARD NEXT CARD

Look at the conclusion(s):

We have come to 1 conclusion:
1) All As are Cs.
Therefore, the valid conclusion is:

All As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on worksheet #2 NOW.
Do the two answers match?
DO NOT change your original answer!!

LAST CARD NEXT CARD

Example 3.

Consider the next syllogism:

Some As are Bs,

All Bs are Cs,

Conclusion?

LAST CARD NEXT CARD

On your worksheet #2 , write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

Some As are Bs

Remember cases 1 and 2 for the premise Some As are Bs. We'll look at one at a time.

LAST CARD NEXT CARD

Let us consider case 1 for now:

Some As are Bs

There are some As that are Bs and some that are not Bs.

LAST CARD NEXT CARD

Look at the second premise:

All Bs are Cs

Each member of set B is a C. All Bs are Cs.

LAST CARD NEXT CARD

Combine the premises:

Some As are Bs

All Bs are Cs

If some As are Bs, and every B is a C, this means that the As that are Bs must also be Cs. Not all As are Bs, so we conclude that Some As are Cs.

LAST CARD NEXT CARD

Premise 1

Now, let us consider case 2:

Some As are Bs

We must consider the case where All As are Bs, because from this we can correctly state that some As are Bs.

LAST CARD NEXT CARD

Remember the second premise:

All Bs are Cs

Each member of set B is a C. All Bs are Cs.

LAST CARD NEXT CARD

Combine the premises:

Some As are Bs

All Bs are Cs

If some As are Bs, and every B is a C, this means that the As that are Bs must also be Cs. In this case, all As are Bs, so we conclude that All As are Cs

LAST CARD NEXT CARD

Look at the conclusion(s):

We have come to 2 conclusions:
1) Some As are Cs and
2) All As are Cs.
Because we can also say that some As are Cs if all As are Cs, the most general conclusion is
Some As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on worksheet #2 NOW.
Do the two answers match?
DO NOT change your original answer!!

LAST CARD NEXT CARD

Example 4.

Consider the next syllogism:

No As are Bs,

All Cs are Bs,

Conclusion?

LAST CARD NEXT CARD

On your worksheet #2 , write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

No As are Bs

Each member of set A is not a B. No As are Bs.

LAST CARD NEXT CARD

Look at the second premise:

All Cs are Bs

Note that this premise is a reversal of All Bs are Cs. Each member of set C is a B. All Cs are Bs.

LAST CARD NEXT CARD

Combine the premises:

No As are Bs

All Cs are Bs

If no As are Bs, and every C is a B, this means that all the Cs that are Bs cannot be As. So we can conclude that No As are Cs.

LAST CARD NEXT CARD

Look at the conclusion(s):

We have come to 1 conclusion:
1) No As are Cs.
Therefore, the valid conclusion is:

No As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on worksheet #2 NOW.
Do the two answers match?
DO NOT change your original answer!!

LAST CARD NEXT CARD

Example 5.

Consider the next syllogism:

No Bs are As,

All Cs are Bs,

Conclusion?

LAST CARD NEXT CARD

On your worksheet #2 , write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

No Bs are As

Each member of set B is not an A. No Bs are As. Note the reversal in the premise.

LAST CARD NEXT CARD

Look at the second premise:

All Cs are Bs

Note that this premise is a reversal of All Bs are Cs.Each member of set C is a B. All Cs are Bs.

LAST CARD NEXT CARD

Combine the premises:

No Bs are As

All Cs are Bs

If no Bs are As, and every C is a B, this means that all the Cs that are Bs cannot be As. So we can conclude that No As are Cs.

LAST CARD NEXT CARD

Look at the conclusion(s):

We have come to 1 conclusion:
1) No As are Cs.
Therefore, the valid conclusion is:

No As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on worksheet #2 NOW.
Do the two answers match?
DO NOT change your original answer!!

LAST CARD NEXT CARD

Example 6.

Consider the next syllogism:

All Bs are As,

Some Bs are Cs,

Conclusion?

LAST CARD NEXT CARD

On your worksheet #2 , write down your conclusion NOW!

LAST CARD NEXT CARD

Look at the first premise:

All Bs are As

Note that this premise is a reversal of All As are Bs. Remember the difference between the two. Each member of set B is an A. All Bs are As.

LAST CARD NEXT CARD

Look at the second premise:

Some Bs are Cs

Remember cases 1 and 2 for the premise Some Bs are Cs. We'll look at one at a time.

LAST CARD NEXT CARD

Let us consider case 1 for now:

Some Bs are Cs

There are some Bs that are Cs and some that are not Cs.

LAST CARD NEXT CARD

Combine the premises:

All Bs are As

Some Bs are Cs

If all Bs are As, and some Bs are Cs, this means that some of the As that are Bs are also Cs. Therefore, we can conclude that

Some As are Cs

LAST CARD NEXT CARD

Premise 2

Now, let us consider case 2:

Some Bs are Cs

We must consider the case where All Bs are Cs, because from this we can correctly state that some Bs are Cs.

LAST CARD NEXT CARD

Remember the first premise:

All Bs are As

Each member of set B is an A. All Bs are As.

LAST CARD NEXT CARD

Combine the premises:

All Bs are As

Some Bs are Cs

If all Bs are As, and "all" Bs are Cs, this means that some of the As that are Bs are also Cs. If there are fewer Cs than As, then Some As are Cs. If there are more Cs than As, then All As are Cs.

LAST CARD NEXT CARD

Look at the conclusion(s):

We have come to 2 conclusions:

1) Some As are Cs and

2) All As are Cs.

Because we can also say that some As are Cs if all As are Cs, the most general conclusion is

Some As are Cs

LAST CARD NEXT CARD

Write down this conclusion in the space on worksheet #2 NOW.

Do the two answers match?

DO NOT change your original answer!!

LAST CARD NEXT CARD

Appendix F

Venn Diagram Notation Worksheet

Worksheet 1 Example 1:

Statement: **All Bs are As**

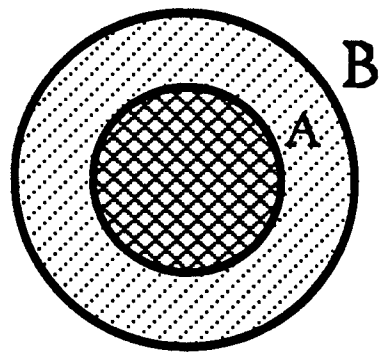
Representation:

Show your work:

Worksheet 1 Example 2:

Statement:

Representation:



Show your work:

Worksheet 1 Example 3:

Statement: **All Bs are Cs**

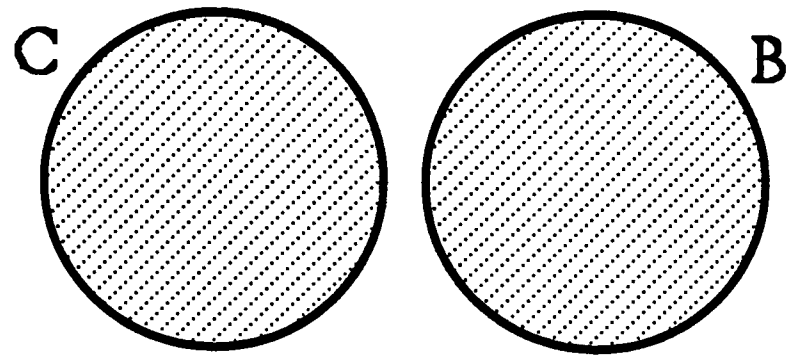
Representation:

Show your work:

Worksheet 1 Example 4:

Statement:

Representation:



Show your work:

Worksheet 1 Example 5:

Statement: **Some Bs are As**

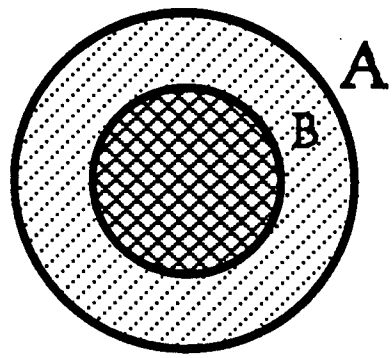
Representation:

Show your work:

Worksheet 1 Example 6:

Statement:

Representation:

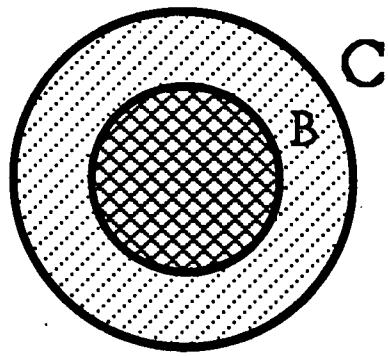


Show your work:

Worksheet 1 Example 7:

Statement:

Representation:



Show your work:

Worksheet 1 Example 8:

Statement: **Some Cs are Bs**

Representation:

Show your work:

Worksheet 1 Example 9:

Statement: **Some As are not Cs**

Representation:

Show your work:

Worksheet 1 Example 10:

Statement: **Some As are not Bs**

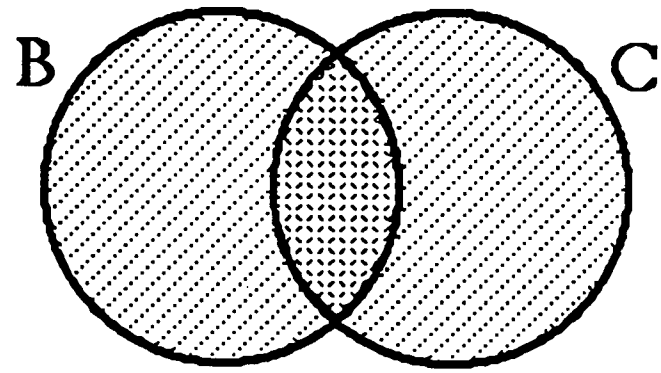
Representation:

Show your work:

Worksheet 1 Example 11:

Statement:

Representation:



Show your work:

Worksheet 1 Example 12:

Statement: **Some Bs are Cs**

Representation:

Show your work:

Appendix G

Sample Representation Notation Worksheet

Worksheet 1 Example 1:

Statement: **A is B**

Representation:

Show your work:

Worksheet 1 Example 2:

Statement:

Representation:

A

↑

B

Show your work:

Worksheet 1 Example 3:

Statement: **C is not B**

Representation:

Show your work:

Worksheet 1 Example 4:

Statement:

Representation:

A

↓

B (B)

Show your work:

Worksheet 1 Example 5:

Statement: **All Bs are Cs**

Representation:

Show your work:

Worksheet 1 Example 6:

Statement:

Representation:

B	B
T	T
C	C

Show your work:

Worksheet 1 Example 7:

Statement:

Representation:

A (A)

↓

B (B)

Show your work:

Worksheet 1 Example 8:

Statement: **Some Cs are Bs**

Representation:

Show your work:

Worksheet 1 Example 9:

Statement: **Some As are not Bs**

Representation:

Show your work:

Worksheet 1 Example 10:

Statement:

Representation:

B	B
T	↑
C	(C)

Show your work:

Worksheet 1 Example 11:

Determine the path between A and C:

Is the path positive or negative?:

Your answer:

Show your work:

Worksheet 1 Example 12:

Determine the path between A and C:

Is the path positive or negative?:

Your answer:

Show your work:

Appendix H

Syllogism Instruction Worksheet (all methods)

Worksheet on Syllogisms

For each of the following syllogisms, write down, in the space provided, what you think is the correct conclusion. Choose your answer from the ones listed below:

a) All As are Cs
b) No As are Cs
c) Some As are Cs
d) Some As are not Cs

Then, after completing each exercise on the computer, write down the correct answer. Do the two conclusions match?

Worksheet 2 Example 1:

Syllogism:

All As are Bs
No Bs are Cs

Your conclusion:

Correct conclusion:

Show your work:

Choose from:
a) All As are Cs
b) No As are Cs
c) Some As are Cs
d) Some As are not Cs

Worksheet 2 Example 2:

Syllogism:

All As are Bs
All Bs are Cs

Your conclusion:

Correct conclusion:

Show your work:

Choose from:
a) All As are Cs
b) No As are Cs
c) Some As are Cs
d) Some As are not Cs

Worksheet 2 Example 3:

Syllogism:

Some As are Bs
All Bs are Cs

Your conclusion:

Correct conclusion:

Show your work:

Choose from:
a) All As are Cs
b) No As are Cs
c) Some As are Cs
d) Some As are not Cs

Worksheet 2 Example 4:

Syllogism:

No As are Bs
All Cs are Bs

Your conclusion:

Correct conclusion:

Show your work:

Choose from:
a) All As are Cs
b) No As are Cs
c) Some As are Cs
d) Some As are not Cs

Worksheet 2 Example 5:

Syllogism:

No Bs are As
All Cs are Bs

Your conclusion:

Correct conclusion:

Show your work:

Choose from:
a) All As are Cs
b) No As are Cs
c) Some As are Cs
d) Some As are not Cs

Worksheet 2 Example 6:

Syllogism:

All Bs are As
Some Bs are Cs

Your conclusion:

Correct conclusion:

Show your work:

Choose from:
a) All As are Cs
b) No As are Cs
c) Some As are Cs
d) Some As are not Cs

Appendix I

Venn Diagram Notation Test

TEST ITEM #1

Answer each question to the best of your ability using the statements and representations you learned. Please show your work.

Statement: **All Bs are Cs**

Representation:

Show your work:

TEST ITEM #2

Statement: **No Bs are As**

Representation:

Show your work:

TEST ITEM #3

Statement: **Some Bs are Cs**

Representation:

Show your work:

TEST ITEM #4

Statement: **Some Bs are not As**

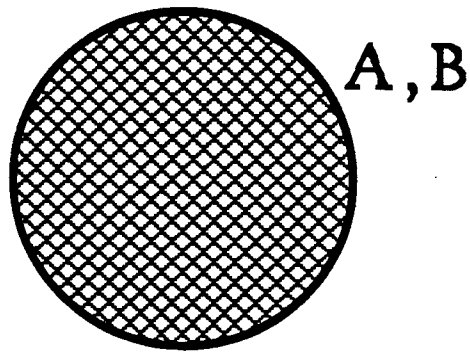
Representation:

Show your work:

TEST ITEM #5

Statement:

Representation:

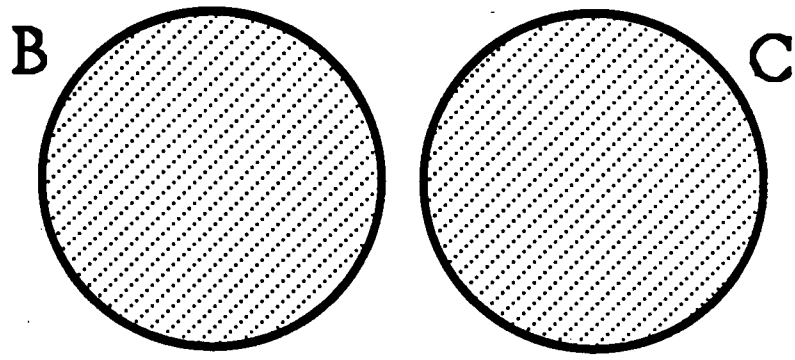


Show your work:

TEST ITEM #6

Statement:

Representation:

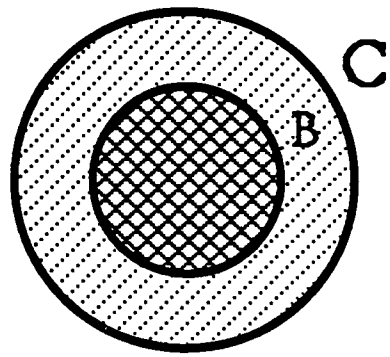


Show your work:

TEST ITEM #7

Statement:

Representation:

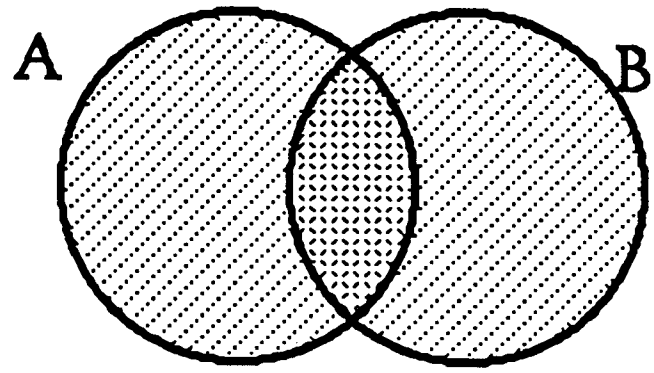


Show your work:

TEST ITEM #8

Statement:

Representation:



Show your work:

Appendix J

Sample Representation Notation Test

TEST ITEM #1

Answer each question to the best of your ability using the statements and representations you learned. Please show your work.

Statement: **A is B**

Representation:

Show your work:

TEST ITEM #2

Statement:

Representation:

B

T

C

Show your work:

TEST ITEM #3

Statement: **All Bs are Cs**

Representation:

Show your work:

TEST ITEM #4

Statement:

Representation:

A

↓

B (B)

Show your work:

TEST ITEM #5

Statement: **No Bs are As**

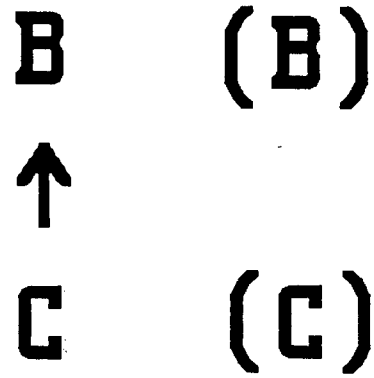
Representation:

Show your work:

TEST ITEM #6

Statement:

Representation:



Show your work:

TEST ITEM #7

Statement: **Some Bs are Cs**

Representation:

Show your work:

TEST ITEM #8

Statement:

Representation:

A	(A)
⊥	↓
B	B

Show your work:

Appendix K

Posttest

TEST ITEM #1

Instructions: For each syllogism, study the given premises. Using the method which you were taught, solve each exercise by arriving at a valid conclusion. Please show your work.

Syllogism: All As are Bs
All Cs are Bs

Conclusion:

Show your work:

TEST ITEM #2

Syllogism:

All Bs are As
No Cs are Bs

Conclusion:

Show your work:

TEST ITEM #3

Syllogism: Some Bs are As
All Bs are Cs

Conclusion:

Show your work:

TEST ITEM #4

Syllogism:

All Bs are As
Some Cs are Bs

Conclusion:

Show your work:

TEST ITEM #5

Syllogism:

No As are Bs
All Cs are Bs

Conclusion:

Show your work:

TEST ITEM #6

Syllogism:

Some As are Bs
All Bs are Cs

Conclusion:

Show your work:

TEST ITEM #7

Syllogism:

All Bs are As
Some Bs are Cs

Conclusion:

Show your work:

TEST ITEM #8

Syllogism:

All As are Bs
No Bs are Cs

Conclusion:

Show your work: