THE PROBLEM-SOLVING STRATEGIES OF GRADE TWO CHILDREN: SUBTRACTION AND DIVISION

BY

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ABSTRACT

This study was aimed at discovering the differences in how children responded to word problems involving an operation in which they had received formal instruction (subtraction) and word problems involving an operation in which they have not received formal instruction. Nineteen children were individually interviewed and were asked to attempt to solve 6 subtraction and 6 division word problems. Their solution strategies were recorded, and analysed with respect to whether or not they were appropriate, as to whether or not they modeled the structure of the problem, and as to how consistent the strategies were, within problem types.

It was found that children tended to model division problems more often than subtraction problems, and also that the same types of errors were made on problems of both operations. It was also found that children were more likely to keep the strategies for the different interpretations separate for the operation in which they had not been instructed (division) than for the operation in which they had been instructed (subtraction). For division problems, the strategies used to solve one type of problem were seldom, if ever used to solve the other type of problem. For subtraction problems, children had more of a tendency to use the strategies for the various interpretations interchangeably.

In addition, some differences in the way children deal with problems involving the solution of a basic fact, and those involving the subtraction of 2-digit numbers, were found. The 2-digit open addition problems were solved using modeling strategies about half as often as any other problem type. The same types of errors were made for both the basic fact and the 2-digit problems, but there were more counting errors and more inappropriate strategy errors for the 2-digit problems, and more incorrect operations for the basic fact problems.

Finally, some differences were noted in the problem-solving behaviour of children who performed well on the basic fact tests and those who did not. The children in the low group made more counting errors, used more modeling strategies, and used fewer incorrect operations than children in the high group.

These implications for instruction were stated: de-emphasize drill of the basic facts in the primary grades, delay the formal instruction of the operations until children have had a lot of ex-

posure to word problem situations involving these concepts, use the problem situations to introduce the operations instead of the other way around, and leave comparison subtraction word problems until after the children are quite familiar with take away and open addition problems.

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CHAPTER ONE

THE PROBLEM

INTRODUCTION TO THE PROBLEM

In recent years there has been much concern regarding the problem-solving performance of our students. There is agreement among many mathematics educators and researchers that more attention should be given to helping students develop good problem-solving skills.The National Council of Teachers of Mathematics,(N.C.T.M., 1980, p.1) has recommended that "1. Problem solving be the focus of school mathematics in the 1980's", and that "2. Basic skills in mathematics be defined to encompass more than computational facility." The National Council of Supervisors of Mathematics (N.C.S.M., 1978) has developed a list of ten basic mathematics skills, and problem solving is an important one of them: "Learning to solve problems is the principal reason for studying mathematics" (N.C.S.M.,1978, p.148).

In spite of all this concern, students' performance in this area is not encouraging. Results of mathematics assessments and studies (Bidwell,1983; Carpenter, Kepner, Corbitt, Lindquist, and Reys,1980; Lindquist, Carpenter, Silver, and Matthews,1983; Robitaille and o'Shea,1985) indicate that students do not take the time to think through and analyse problems, but deal with them in a rote, algorithmic manner.

In contrast, young children seem to possess reasonably good problem-solving skills when they enter school. Studies involving young children (Bjonerud,1960; Carpenter, Hiebert & Moser, 1981,1983; Gibb, 1956; Good, 1979; Gunderson, 1955; Ibarra & Lindvall, 1982; Suydam & Weaver (Hughes,1979)) have shown that they are able to deal successfully with word problems involving operations for which they have received no instruction. As they go about solving such problems, young children show the ability to analyse the structure of the problem, and to represent it in a way that parallels the mathematical structure of the problem.

It would appear that somewhere along the line young children's thoughtful, analytical approach to solving problems starts to change. In a study done by Carpenter et al.(1983), it was found that after formal school instruction in the operations of subtraction and addition, Grade One children tend to use the same strategy (separating) for all three types of subtraction word problems (take away, comparison, and open addition). Before they had received formal school instruction on these operations, however, they tended to solve each problem type in a way that modeled the action in the problem. No definite conclusions were drawn as to the reason for this shift in strategies, but the fact that along with this shift children showed an increased tendency to choose an incorrect operation led the authors to surmise that this was not a beneficial change. In view of older children's apparent lack of ability to analyse the problem structure, and their failure to solve problems that are more complex than simple one-step problems, it is tempting to regard this shift of strategies as a first step toward a rote, algorithmic approach to solving problems. If instruction is responsible for this shift in strategies used to solve subtraction problems, it is not clear whether it is instruction in arithmetic computation in general, or specifically instruction in subtraction. It is also not clear whether students who have had some experience with computation would behave differently with problems dealing with a new operation than they would with problems that deal with an operation in which they have received instruction. If children are already experienced with one operation, will that influence the way in which they approach problems involving an operation in which they have not been instructed?

The purpose of this study is to examine children's problem-solving behaviour when faced with word problems involving an operation in which they have not been instructed and problems involving an operation in which they have not been instructed.

STATEMENT OF THE PROBLEM

This investigation is an attempt to examine the problem-solving behaviour of children who have had some experience with and instruction in computation. In particular, the study will examine their behaviour with one-step word problems involving an operation in which they have been instructed and problems involving an operation in which they have not been instructed. Are there differences in the ways in which children attempt to solve these two types of word problems, and if so, what are these differences? In addition, this study will look at what effect the size of numbers in the subtraction problems would have on children's problem-solving behaviour.

RESEARCH QUESTIONS

1. Do children behave differently with word problems involving an operation for which they have received formal school instruction (subtraction) and an operation in which they have received no formal school instruction (division) ?

a) What strategies do children use to solve word problems involving the three different interpretations of subtraction?

b) What strategies do children use to solve word problems involving the two different interpretations of division?

c) Are there any differences in the strategies used between the different interpretations for subtraction and division?

d) Does the strategy used model the action in the problem; are there any differences between subtraction and division in this regard?

e) Is there a tendency for children to make different types of errors on division problems than on subtraction problems?

2. For the subtraction problems, are there differences in the way children approach word problems involving the solution of a basic fact, and those involving the subtraction of 2-digit numbers?

a) Do these children differ in the number of modeling strategies they use for the basic fact problems and the 2-digit problems, and does the type of subtraction problem (take away, open addition, comparison) make a difference?

b) Is there a tendency for these children to make different types of errors in the basic fact problems than in the 2-digit problems and does the type of subtraction problem (take away, open addition, comparison) make a difference?

3. Do children who do well on the addition and subtraction basic fact tests approach word problems differently than children who perform poorly on the basic fact tests?

a) Do children who perform well on basic fact tests differ in the number of modeling strategies they use for the subtraction word problems , from children who perform poorly on basic fact tests?

b) Do the children who perform well on basic fact tests make different types of errors in the word problems than the children who perform poorly on basic fact tests?

DISCUSSION OF THE PROBLEM

Children, when they enter school, possess reasonably good problem-solving skills. This is a common finding of studies done with young children who have had no instruction in computation. This is in contrast to the studies and assessments that indicate that elementary children are approaching problem-solving in a rote, algorithmic manner. According to the results of these assessments, students cannot deal with extraneous information (Carpenter et al.,1980), cannot identify missing information needed to solve a problem (Carpenter et al.,1980), calculate to find solutions to problems that require no calculation (Bidwell,1983), and fail to consider whether or not their answers are reasonable (Bidwell,1983). It would appear that students view problem-solving as simply finding an operation to apply to the numbers in the problem, and then recording the result of the calculation.

In light of a study done by Carpenter et al.(1983) it would appear that this change begins to take place as early as Grade One. In this study, Grade One children were interviewed on addition and subtraction word problems before and after formal instruction in these two operations. It was found that, while the success rate on the problems did not change, the strategies used in solving the subtraction problems did change. Before instruction, the children used different strategies to solve the different types of subtraction word problems. Furthermore, the strategy chosen for each problem type modeled the action in the problem. After instruction, the children tended to solve all types of subtraction problems using a separating strategy.

There was no change in strategies for the addition problems, but there was a change in the types of errors made. In the first interview, 66% of the errors in the addition problems were due to the child giving one of the numbers in the problem as a response, while only 11% were due to the child using an incorrect operation. In the second interview, 52% of the errors were due to giving one of the numbers in the problem as a solution, and 43% were due to using the wrong operation. In the subtraction problems there were no instances of using the wrong operation in the first interview, and three in the second.

The results of Carpenter's study make it clear that after instruction a shift in strategies did occur. What is not clear is what was behind it. "It is not clear whether they understand that several strategies are possible and choose separating because it is the easiest or whether their eclectic approach has been replaced by a single unified interpretation of subtraction" (Carpenter et al., 1983, p.70). It was stated, however, that the children had been taught to look for the part-whole relationship in all word problems. If they had one part and the whole, they were to write a subtraction sentence, and if they had two parts they were to write an addition sentence. It was felt that this may have encouraged the children to view all subtraction problems as similar and to use a single strategy to solve them.

The increase in the choice of an incorrect operation from the first interview to the second was taken as evidence that the shift in strategies may not have been beneficial. It was seen as an indication that "some children may be beginning to view the solution of word problems as simply choosing the correct operation to perform on the numbers in the problem rather then analysing the semantics of the problem as they did for the direct modeling solutions" (Carpenter et al., 1983, p.70). It was suggested that premature instruction in sentence writing was a possible cause of children making a superficial analysis of the problem, leading to the use of the wrong operation.

It is very tempting to look at this and to assume that this is where the decline in problem-solving skills begins. But all that can be concluded from Carpenter's study is that the shift occurred. It is not known for sure that this shift means students are performing a superficial analysis of the problem, although the increase in the use of the wrong operation seems to point in that direction. It is also not known which factors influenced this change in behaviour. It may have been instruction in and emphasis on computation, the part-whole instruction on how to solve word problems, or possibly a mature concept of subtraction that allows students to understand that one strategy would cover all subtraction situations. It is not clear whether this is a temporary effect due to the newness of computation, or a permanent change. It also is not clear whether this change will affect the way children handle new types of problems.

Gibb's(1956) study provides some evidence that instruction in computation alone is not enough to produce this change in behaviour. She interviewed Grade Two children on their thoughts while solving subtraction problems, and found that they behaved much like Carpenter's pre-instruction Grade One children. She stated that "relatively young children with limited number experience gave different responses to problems involving three of the applications of subtraction" (Gibb, 1956, p.78). Aside from the fact that all of the Grade Two children had been taught the take-away method of subtraction, there is no information as to what their instructional experience had been. Because of this, it is difficult to make comparisons between this study and Carpenter's.

In this study, Grade Two children will be interviewed on subtraction and division problems with respect to observing how they behave with problems involving known operations and unknown operations. Learning how they behave in these situations, and relating this to their previous instructional experiences will hopefully add to the picture of young children learning to do mathematics.

LIMITATIONS OF THE STUDY

This study describes the responses of Grade Two children when presented with word problems involving an operation in which they had been formally instructed (subtraction) and word problems involving an operation in which they had not been formally instructed (division). Their responses were obtained in two individual interviews of approximately 20 minutes each.

Only 19 children were interviewed, and they came from two schools in one school district. No attempt was made to select a random sample. In fact, children were selected on the basis that their teachers had judged them to be average in mathematics. The results may have been different if Grade Two children in a more urban district had been chosen, or if all ability levels had been included.

SIGNIFICANCE OF THE STUDY

Problem solving has become " the focus of mathematics in the 1980's " (N.C.T.M., 1980, p.1) and " the principal reason for studying mathematics " (N.C.S.M., 1978, p.148). Concern has been expressed about students' lack of problem-solving skills, and workshops and conferences have produced a great many sessions on how to teach these skills. In recent years, mathematics journals and publications such as the Arithmetic Teacher, Mathematics Teacher, and Journal For Research In Mathematics Education have shown a tremendous increase in the number of articles dealing with problem solving. There have also been recent changes in the British Columbia mathematics curriculum which reflect this new emphasis on problem-solving skills. Educators are trying very hard to help our students become better problem-solvers, but in order to do this, we must understand what skills and resources children bring to the task, and the changes that occur as they receive formal instruction in mathematics. We already have a clear picture of how children solve problems prior to any formal instruction in arithmetic, and we have some information on how they behave after initial instruction in addition and subtraction. Providing information on how children with calculation experience approach problems involving operations with which they have no experience, will add to the picture of what problem-solving skills children have, and how they change.

CHAPTER TWO

A REVIEW OF THE LITERATURE

INTRODUCTION

There are some intriguing differences between studies done which describe young children's abilities in the area of problem-solving, and the studies and assessments done describing older children's difficulties dealing with problem solving. This study explores Grade Two children's problem solving behaviour when faced with word problems involving the subtraction and division operations (the former operation is one in which they have received formal school instruction while the latter is one in which they have not). The literature review will consist of a discussion of problem solving in general, a description of assessments and studies done on older children's problem solving behaviour, and a description of studies done on young children's problem-solving behaviour, with particular reference to a study done by Carpenter et al. (1983).

PROBLEM SOLVING

Problem solving in mathematics has been described in three ways - as a process, as a basic skill, and as a goal. We tend to think of problem solving only as a process, but let's look for a moment at its other aspects.

As a Goal

In the last decade, mathematics educators and researchers have given an increasing amount of attention to problem solving as one of the goals of teaching mathematics. "Learning to solve problems is the principal reason for studying mathematics" (N.C.T.M., 1978, p.147).It is of little value to learn mathematis skills in isolation, to not be able to use them on new or unfamiliar situations. What we should be doing is trying to help children see the relationship between daily activities and their mathematical structure. In this sense, problem solving is the goal in studying mathematics - it is the end product that we are aiming for. As Lester (1977) puts it "...problem solving has been said to be at the heart of all mathematics."

As a Basic Skill

Problem solving has also been referred to as a basic skill. The N.C.T.M. (1978), in its position statement on basic skills, listed problem solving as one of those ten basic skills. But what are basic skills? If by basic skills we mean skills that individuals need in order to function in our society, then problem solving is a basic skill. Children will be faced with a multitude of problems that they will have to deal with, from the time they are babies, all through life. Granted that many of the problems they will be called upon to solve in their lives will not be mathematical, the skills that they learn in solving mathematical problems will be of use to them in tackling other problem situations as well.

As a Process

Problem solving is the process an individual goes through in reaching the solution to a problem. It consists of the questions he asks himself, the way he interprets the data, and the strategies he employs to deal with the problem at hand. It is "...the process of applying previously acquired knowledge to new and unfamiliar situations" (N.C.T.M., 1978, p.148).

The problem-solving process does not end with a solution. "The learner should understand what he has done in solving the problem, and why what he did was appropriate for that problem." (Krulik, 1977, p.51) In other words, the problem-solving process includes understanding what the problem asks, understanding the information that is given, planning the method for solving it, carrying out the plan to reach a solution, and finally, looking back. In the looking back stage, the learner should reflect on the reasonableness of the solution that has been reached and also on the appropriateness of the plan that was employed.

In reviewing the literature on problem solving, it was found that authors tend to distinguish between word problems and non-routine or process problems (Krulik, 1977; Le Blanc, 1977; Burns, 1982). Some authors take the position that the typical word problems found in textbooks are not really problems for the children for whom they are intended (Krulik, 1977; Le Blanc, 1977; Suydam, 1980). For example, a typical word problem found in a Grade Four textbook would not be a problem for most Grade Four children, according to these authors. It would be an exercise, designed to reinforce a particular concept or operation presented in class. It might well be a problem for a Grade Two or Grade Three child, however. Other authors hold the view that, while solving word problems is a subset of problem solving, problem solving actually encompasses far more skills than are contained in just solving word problems. (Bruni, 1982; N.S.C.M., 1978;) "Developing problem solving skills is often equated with training children to solve arithmetic word problems, but problem solving has a much broader meaning....A real problem involves a question that can not immediately be answered; it requires some effort in making appropriate use of previously learned concepts and skills for its solution" (Bruni, 1982, p.10). The following definition by Krulik is a fairly typical definition of a non-routine problem situation:

> 1. The individual has a clear goal of which he is aware; in other words there is some personal involvement.
>
> 2. The individual's path toward the goal is blocked. His usual habitual responses do not remove the block; he experiences a sense of frustration.
>
> 3. The individual must deliberate carefully; he must think about how to remove the block. (Krulik, 1977, p.51)

Krulik's definition suggests that a problem situation exists only when the above conditions are met. Thus, the same situation may be a problem for one individual but not for another, or may be a problem for an individual at one time but not at a later time. Whether or not a problem exists depends on the person perceiving it; much depends on the skills and abilities an individual brings to the problem, her prior experience with similar problems, and her attitude toward such problems.

Kantowski (1980) separates problems into three types: 1. word problems 2. non-routine problems and 3. real problems, each at a different cognitive level. She considers word problems to be at the lowest cognitive level and real problems to be at the highest.

Word problems, according to Kantowski, fall into the category of "...an exercise stated in verbal form" (Kantowski, 1980, p.112) and "...are easily solved by application of algorithms that are a standard part of instruction" (Kantowski, 1980, p.112). She states that this type of problem involves three steps. "The student must recognize the structure of the verbal problem, select an appropriate algorithm, then correctly apply the algorithm." (Kantowski, 1980, p.114)

A non-routine problem is the type of situation, as defined by Krulik (1977), where the individual is blocked in his path to a solution, and must think carefully about how to remove the block. They are "...problems for which a problem solver knows no clear path to the solution and has no algorithm which can be directly applied to guarantee a solution" (Kantowski, 1980, p.114).

Real problems are a type of non-routine problem, but involve real-life situations, such as planning a hot-dog sale, effectively utilizing available space for a school function, or designing and building a pen for the class pet. In this type of situation, the problem solver must determine what information is necessary, go about getting it, and then deal with setting up a plan and carrying it out.

In considering the process involved in solving problems, it is interesting to note that most authors and researchers feel that it is important for students to have some framework within which to work, and almost all of these authors chose some form of Polya's (1957) four step model. Interestingly, this type of model was recommended for word problems as well as non-routine problems. In discussing non-routine problems, the following authors employed Polya's model as is: Leblanc, Proudfitt, and Putt (1980), Sowder (1980). Lester (1980) discusses a model very much like Polya's, but with six phases instead of four. He adds a step called Problem Awareness, which comes before Problem Comprehension. He also adds an additional step called Goal Analysis which comes before Plan Development. It appears that Lester's Goal Analysis step is similar to the second part of Polya's phase, Understanding the Problem. "In How to Solve It" Polya (1957) subdivides Understanding the Problem into two stages: Getting Acquainted, and Working for Better Understanding. In the second stage, Working for Better Understanding, he discusses considering the principal parts of the problem. "...consider them one by one, consider them in turn, consider them in various combinations, relating each detail to the other details and each to the whole of the problem. " (Polya, 1957, p.33) Lester's Goal Analysis involves "...identification of the component parts of a problem", and also "...an attempt to reformulate the problem so that familiar strategies and techniques can be used." (Lester, 1980, p.33)

In looking at articles dealing with word problems, one finds much the same type of framework recommended as for non-routine problems. Leblanc (1982) and Worth (1982) both use frameworks that correspond exactly to Polya's. Thornton and Bley (1982) give an elongated version of Polya's model, where the first four steps 1. Read the problem, underlining the key words if necessary 2. Picture what is happening 3. Use objects or draw a picture to illustrate the problem, if this helps, and 4. Think about what is being asked, are a breakdown of Polya's phase one, Understanding the Problem. The other three steps in Thornton and Bley's framework correspond to Devising a Plan, Carrying Out the Plan, and Looking Back. Krulik and Rudnick (1980) also recommend a similar framework: 1. Read 2. Explore 3. Select a Strategy 4. Solve 5. Review and Extend. Whether non-routine or word problems are being discussed the framework seems to be much the same. For word problems, Devising a Plan would simply consist of choosing the correct operation or operations and performing them in the correct order, but other than that the process seems to be much the same.

Since Polya's model is so widely accepted, I will use it in order to discuss the stages of successful problem solving activity. Polya's first step, Understanding the Problem, is very important and too often ignored in instructional settings. In this stage, the problem solver makes an attempt to understand what the problem is all about, and to restate it in his own terms, terms that have meaning for him. Here also, he makes sure that he knows what is wanted (i.e., what is the unknown?) and what information has been given. This is also the stage when the problem solver tries to work out how the relevant information interrelates. "Most importantly, this stage results in the formation of an internal representation of the problem within the individual" (Lester, 1980, p.33).

In the second stage, Devising a Plan, the problem solver reflects on various possible plans of attack. She asks herself questions such as a) Is there more than one way to do this problem? b) Have I ever solved a problem like this before? c) Can I solve part of the problem? d) Can I solve the problem using smaller numbers? It is here that she reflects on the strategies she knows and makes decisions about which ones might be most useful in this situation.

In the third step, Carrying Out the Plan, the problem solver applies the plan he has selected. Here, the problem solver should be constantly referring back to his plan, and evaluating whether or not he is applying it correctly. For example, making a table and looking for a pattern may be a very good strategy, but it will not work very well if a student misses the pattern due to a computational error. The other difficulty is that the student may forget the plan or become confused as he goes along.

At this point (i.e., having carried out the plan), many students consider the problem to be finished, but there is actually one more step, Looking Back. Here the student reflects on the steps she went through to solve the problem, evaluates the reasonableness of the solution, and attempts to relate this problem to some other problem. The student who has more experience solving different problems, using many different strategies will have a greater repertoire from which to draw than the less experienced student. This process of reflecting on the plan and the strategies used to arrive at a solution helps a student to see how the strategy works and how it could be used again in the future.

These four stages reflect the skills necessary to successful problem solving, and they form the basis for many recommended instructional frameworks.

Elementary School Children's Performance

Studies and assessments done on elementary school children's performance in the area of problem solving are not encouraging. According to the results of the second mathematics assessment of NAEP (Carpenter et al., 1980) students performed at a generally acceptable level in simple one-step word problems which involved choosing a single operation, but anything more complex presented difficulty. This assesment indicated that children were having particular difficulty with the first and second stages of Polya's model, Understanding the Problem and Devising a Plan. Students failed to appropriately identify the unknown, even though they performed the correct calculation. For example, on a question where the unknown was the number of buses needed to transport a given number of people, after performing the calculation only 19% of students rounded the fractional answer up to the next whole number. A full 39% either ignored the remainder or gave a mixed number as an answer. In another question where the unknown was the remainder to a division calculation, only 29% of the students recognized the remainder as the correct answer. Most of the students recognized division as the correct operation, however. "They simply apply whatever mathematical operation seems most appropriate to the numbers given in the problem" (Carpenter et al., 1980, p.427). As well as not being able to recognize the unknown, another difficulty was dealing with problems that had either missing or extraneous information. Many children, presented with problems containing extraneous information simply used all the numbers in some way. In a problem given to nine year olds, although 60% could solve the problem correctly, only 23% could identify the extra information that they did not need to use. When information was missing "...more than half of the 13-year olds and a third of the 17-year olds could identify what additional information would be needed to solve the problem (Carpenter et al., 1980, p.428). The results of this assessment also suggest that many students are failing to plan (Polya's second stage). Many students were able to use a correct strategy when the problem had one digit numbers but used a totally inappropriate strategy when a problem had small two digit numbers. "...the erroneous responses to a number of exercises that did not involve difficult calculations suggest that many students are not carefully analysing unfamiliar problems in order to devise a plan. Instead, they are attempting to apply a single operation to all the numbers in the problem" (Carpenter et al., 1980, p.430). This assessment also indicated that students are more successful at performing a calculation to get the answer than at making a reasonable estimate of the answer. As one way of checking on whether an answer is reasonable (the Looking Back stage) is to compare it to a reasonable estimate, this is a detriment to students. According to this assessment, "In general, respondents demonstrated a lack of even the most basic problem-solving skills" (Carpenter et al., 1980, p.47).

Results from the third mathematics assessment of NAEP (Montgomery et al., 1983) were no better. Again, students ran into difficulty with anything more complex than a routine one-step word problem. Performance of the thirteen year olds improved about 2% from the second assessment, but is still below an educationally acceptable level. In some calculator problems students were able to recognize the correct operation, but were not able to decide how to use the result of their calculations.

Bidwell's study on problem-solving performance as compared to computational performance (Bidwell, 1983) lends support to the NAEP findings. In this study, results showed that computational skills were better than problem-solving skills, and that students often failed to consider the reasonableness of their answers. There was also a tendency to ignore certain aspects of the problem, and to concentrate on something that appeared familiar. Many students put down calculation answers to problems that required no calculation at all, or were very easy to solve without calculating. According to Bidwell, this points to "...a strong urge to calculate at all costs to obtain a number to write down in the blank" (Bidwell, 1983, p.692).

The 1985 British Columbia Mathematics Assessment (Robitaille and O'Shea, 1985) is not any more encouraging than the rest. This assessment indicates that Grade Four students' overall performance was marginal in problem-solving skills, and Grade Seven students' performance was satisfactory overall. However, even though the performance of the Grade Seven students was satisfactory overall, they performed at only a marginal level on items which dealt with the problem-solving process rather than the correct answer.

These results seem to indicate that students are approaching problem solving in a rote, mechanical manner. They are not really learning to consider the problem at hand.

Young Children's Performance

In contrast, studies done concerning young children's problem-solving skills have indicated that young children are reasonably proficient at dealing with word problems. These studies tested children in individual interview situations and employed word problems involving arithmetic operations for which the children had received no instruction. Generally, it was found that although these children had received no instruction in the required operation, they solved the problems by using the knowledge they did have. That is, they made an effort to analyse the structure of the problem and then made a representation of the problem that paralelled this structure. Counting strategies were employed to finally solve the problem.

Siegler (1984) investigated the strategies used by 4- and 5-year olds while solving addition problems. Four strategies were identified - three overt and one covert. The overt strategies consisted of counting with fingers, counting (counting aloud without any obvious external referent), and fingers (looking at fingers without counting them aloud), while the covert strategy appeared to be a retrieval from memory. Counting and counting with fingers were the slowest strategies and retrieval from memory was the quickest one. It was observed that the children used the overt strategies only on the hardest problems. This was considered to be both adaptive and efficient. Adaptive because the overt strategies helped them to solve the problems successfully, and efficient because of the extra time involved in using the overt strategies. "By limiting the overt strategies to the hardest problems, children only expended large amounts of time on problems where the time was truly needed" (Siegler, 1984, p.5). These children not only had different strategies from which to choose, but they were able to use them in an efficient and adaptive way.

Bjonerud (1960), Good (1979), Ibarra & Lindvall (1982), and Suydam & Weaver (Hughes, 1979) have stated that kindergarten children who have had no instruction in addition and subtraction do well at addition and subtraction word problems. Children who were non-conservers of number relied more heavily on counting and were less likely to see and use similarities between problems, but both groups were able to deal successfully with these types of problems. Good (1979), and Hiebert, Moser, & Carpenter (1982) studied the effects of the cognitive abilities of number conservation, class inclusion, transitive reasoning, and information processing on children's ability to solve addition and subtraction word problems. They found that although "children who had developed a particular cognitive ability performed better on all problem types and under all problem conditions....the cognitive abilities do not appear to be prerequisites for solving the arithmetic problems" (Hiebert et al., 1982, p.94-96). The same appropriate strategies were used by children of high and low cognitive abilities, although less frequently by low ability children. Also, within each ability group, the relative distribution of these strategies is the same. These two studies (Good, 1979; Hiebert et al., 1982) seem to indicate that even though children who have not yet developed some of these cognitive skills may be less efficient in solving problems, all children have acquired a variety of strategies with which to work.

Klahr (1981) investigated young children's strategies in solving non-routine problems such as the Tower of Hanoi, and he concluded that "by the time children reach kindergarten, they appear to have acquired, without direct instruction, variations on many of the components of mature problem solving strategies" (Klahr, 1981).

Gunderson (1955) has found that Grade Two children who had no previous training in multiplication or division could solve such problems with the aid of concrete materials. Good (1979) also found that Grade Two children were able to handle multiplication and division problems in a concrete way, although it is not clear whether or not these children had any prior experience with the concepts. He also found, that of the Grade One children, 50% could handle the division problems and 60% could handle the multiplication problems.

Two studies done by Carpenter, Hiebert, & Moser (1981, 1983) show that before instruction in addition and subtraction, Grade One children use different strategies to solve different types of problems. "Prior to instruction most children possess the different strategies required to solve each problem type directly" (Carpenter et al., 1981, p.38). In the 1983 study, Carpenter et al. found that although these children exhibited a rich variety of problem-solving strategies before instruction in subtraction, after instruction they tended to resort to a separating strategy for all types of subtraction problems.

Gibb (1956) explored Grade Two children's approaches to solving different types of subtraction problems. She concluded that "...relatively young children with limited number experience gave different responses to problems involving three of the applications of subtraction" (Gibb, 1956, p.78).

SUMMARY

Some authors (Ball, 1986; Carpenter & Moser, 1982; Campbell, 1984) have suggested that we are instructing our students in a back-to-front manner. Instead of assuming that children must master the operations before they can solve word problems, we may be better off introducing the operations by means of the appropriate word problems. Because of young children's ability to successfully solve addition and subtraction word problems before they enter school "...verbal problems may give meaning to addition and subtraction and in this way could represent a viable alternative for developing addition and subtraction concepts in school" (Carpenter & Moser, 1982, p.9).

The results of these studies and assessments seem to indicate two things : 1. young children enter school with reasonably good problem-solving skills, and 2. children later exchange these skills for a rote, algorithmic approach. In particular, the study done by Carpenter et al. (1983) indicates that instruction in computation may be the cause of some of this shift in approach. The purpose of this study was to explore the ways in which children approach word problems involving an operation in which they had received instruction, and those involving an operation in which they had not received instruction.

CHAPTER THREE

DESIGN OF THE STUDY

INTRODUCTION

In this study, individual interviews were used to explore children's approaches to solving subtraction and division word problems. This chapter discusses the design of the study, and includes :

1. Procedures

2. Problems with the Data

3. Problems

4. Subjects

5. Research Design

PROCEDURES

This study used individual interviews to look at the ways in which children approach the solution of word problems. These interviews were audio-taped and handwritten notes were also kept on each interview. In particular, the study was concerned with primary children's approaches to word problems involving an operation in which they have received formal school instruction (subtraction) and word problems involving an operation in which they have received no formal school instruction (division). Twenty children from a rural area of British Columbia were individually interviewed in January of their Grade Two year. For all students, the instruction they received in subtraction in their Grade Two year consisted of the meaning of the operation and practice in determining the solutions to basic fact subtraction exercises. At the time of the study, the children had not been taught the 2-digit subtraction (or addition) algorithm, nor had they had any experience solving word problems. In Grade One, the children from both schools had had experience solving word problems orally, in a whole class situation. They had been encouraged to use counters to solve the problems, but had not been required to write down either the solution or the equation. Problem solving was done cooperatively as a group, so the children were not asked to tackle word problems independently. Up to the time of the study, these children had received no instruction in division, either in Grade One or Grade Two. The problems for the interview consisted of 6 division (3 partitive and 3 quotitive) and 6 subtraction (2 each of take-away, open addition, and comparison) word problems.

In each school the interviews were conducted in a supply room where paper and art supplies were housed. This spot was chosen because in these schools space was extremely limited, and the supply room was the room with the least likely number of interruptions and distractions. Each child was interviewed twice, for about 20 minutes each time. One child was dropped from the study because she was not available for the second interview. The problems were presented to each child in random order. Each problem was read to the child by the interviewer, and the problem, in written form, was placed so that the child could also read it if she chose. It was explained to the child that the problem would be re-read by the interviewer as often as the child considered necessary. It was also explained that the child could opt to read the problem on her own if that seemed easier. Multi-link cubes and pencil and paper were made available to the children. Many of the children claimed to have seen these counters before, and all of the children had used counters of some kind in their classrooms. If they did not indicate familiarity with the multi-link cubes, children were shown how they snapped and unsnapped and given a minute or two to play. It was explained to the children that these were things that they could use to help them solve the problem, and also to help them explain to the interviewer how they did it. If the children seemed to have trouble getting started on the problem, they were asked whether or not they thought that the cubes might help. If it was not clear what the children had done, or if what they had done seemed inappropriate to the problem, they were asked to explain what they had just done, and why they had done it. In some cases this led to the child spotting an error and correcting it, and in other cases it led to the discovery that a problem had been either misunderstood or remembered incorrectly. In the latter instance, the interviewer discussed with the child what the problem was about and what information was given in the problem. In many cases, even if it was clear what the child had done, she was asked for an explanation. The reason for this was to avoid questioning the child only when she had done something inappropriate. A child was not asked for an explanation if she had clearly explained what was being done as she went about solving the problem. It was felt that further questioning and explanations would be redundant.

PROBLEMS WITH THE DATA

There was some difficulty with some of the audio tapes, and as a result some information for six children was lost. The children who were affected this way represent a cross-section of the children involved in the study. Four were boys and two were girls, and even though five of them were from the same school, they were split 3-2 from two different classes. Three of the six scored less than 8 out of 30 on the basic fact subtraction test, and three scored 11 or above. According to the Grade One records, four of these children had been considered average at the end of Grade One, and two had been considered high. This compares to five high, four high-average, nine average, and one low-average for the 19 children in the study. For five of the children, six problems each were affected, and for one child two problems were affected. Lost information was distributed fairly evenly over the 12 problems.

Since handwritten notes were also kept, the loss of information was kept to a minimum. For the first five children, the method of solution was recorded by hand, but 24 out of 30 solutions were missing. Since the method of solution is what the study is mainly concerned with, this does not appear to be a serious loss. For the sixth child, two partitive problems were lost entirely. It is unfortunate that both the missing problems are division, and further, that they are both partitive. However, when the loss is viewed in terms of two partitive division problems lost out of a total of 57 in the study, again the loss does not appear to be a serious one.

In the discussion of the results, I have mentioned the lost tapes only when it seemed misleading not to do so. In other situations, I have simply made statements such as " Of the recorded solutions, x % were correct ", leaving the reader to understand that the unrecorded solutions are the ones lost on the defective tapes.

PROBLEMS

The children were asked to attempt to solve 12 problems, 6 for each operation. They were told that, while they were being asked to attempt to solve these problems, what was really wanted was an explanation of how they went about doing it.

Children were asked to attempt to solve two word problems for each of the three interpretations of subtraction. One of the word problems required the children to determine the solution of a basic fact, and the other problem involved the subtraction of 2-digit numbers. The basic facts used were : 12-5, 12-9, and 11-8. The 2-digit combinations were : 36-23, 35-12, and 27-12.

For the division problems, an attempt was made to choose combinations that were large enough so that the answer was not immediately available from a very well-known addition or subtraction fact, but small enough so that the use of counters or tallies did not become unwieldy. These combinations were : 21/7, 15/3, 12/2, 12/4, 12/3, and 24/4.

The word problems are found in the Appendix.

SUBJECTS

The children used in this study were drawn from three Grade Two classrooms in two elementary schools in the Fraser Valley. Letters were sent to the parents of these children explaining the purpose of the study and what would be required of the children. Written permission was received from the parents of 15 of the 20 children who had been selected, and parents of the other 5 children gave their consent in a telephone conversation with the interviewer. The children were interviewed twice, for about 20 minutes each time, on how the way they went about solving subtraction and division word problems. Of the twenty children who participated in the first interview, one child was dropped from the study because she was away for the second interview.

Classroom teachers were asked to select children for the study whom they considered to be in the average ability range in mathematics. They based their judgements on their observation of the children's oral and written performance in their classrooms. In addition to receiving the Grade Two teachers' assessments of the selected children as in the average ability range in mathematics, the interviewer was permitted look at these children's Grade One records in order to have access to the Grade One teacher's assessment of their mathematical ability. These records indicate that 14 of the 19 children selected for the study had been considered to be in the average ability range in mathematics at the end of Grade One. Of the 14, one child was considered to be low-average, nine were considered average, and four were considered high-average. The other five children chosen for the study had been considered to be of high ability in mathematics at the end of Grade One. As well as selecting children who were average in mathematics, there was some effort made to select children who were not too shy or reserved to talk to an interviewer.

Grade Two classroom teachers administered timed addition and subtraction basic facts tests to their classes. This was done after the completion of the study, and was used to compare children's knowledge of basic facts to their performance on the word problems. The marks ranged from 2 to 18 out of 30 on the subtraction test, and from 6 to 30 out of 30 on addition.

RESEARCH DESIGN

The investigation was designed to look at the ways in which Grade Two children approach word problems involving an operation in which they have received formal instruction (subtraction) and an operation in which they have not received formal instruction (division). Average children were chosen in an attempt to obtain a group of children who would be as typical as possible. Using a homogeneous group such as this minimizes the possibility of any differences being attributable to student ability levels, and increases the possibility that these differences are due to instruction in and experience with computation in certain operations. Due to the fact that it was the approach to solving problems, and not the ability to get the right answer that was being examined, an individual interview format was used. The strategies children used to solve the problems were classified as being appropriate (if correctly applied, an appropriate strategy will lead to a correct solution) or inappropriate (even if correctly applied, an inappropriate strategy will not lead to a correct solution). The appropriate strategies were then divided into modeling strategies (those strategies which followed the action in the problem) and non-modeling strategies. For example, the problem

It was Dad's turn to set the table for dinner. There were going to be 11 people for dinner but he only had 8 plates. How many more plates did he need?

is an open addition type of subtraction problem. An inappropriate strategy for this problem would be to add the given numbers in the problem together. Even if the addition was correctly performed, it would not lead to a correct solution for this problem. An appropriate, but non-modeling strategy would be to make a set of 11 and take away 8. This would produce a correct solution if properly done, but does not model what is happening in the problem. A modeling strategy would be to represent the set of 8 that exists in the problem, and to then add to it until the goal of 11 is reached. The extras that had to be added would then be counted, in response to the question " how many more are needed?".

The modeling and non-modeling categories were chosen in response to to Carpenter's discovery that, when dealing with subtraction word problems, the Grade One children he interviewed changed their approach from a modeling strategy before instruction to a non-modeling strategy after instruction. This, coupled with older children's poor problem-solving performance on standardized tests, prompts one to wonder whether the change from modeling to non-modeling strategies leads to, or is an indication of a very shallow analysis of the problem, which in turn would lead to poor problem-solving performance, especially on problems more complex than simple one-step problems.

Children were then grouped as to how they responded on certain tasks, with a view to looking for patterns in how they responded on other tasks.

CHAPTER FOUR

FINDINGS

INTRODUCTION

The findings of this study constitute an exploration of the way in which children solve word problems, and the possible effect of instruction on their method of solution. Specific strategies used by the children, both appropriate and inappropriate, and the types of errors made, are discussed in this chapter. Comparisons are made between children's behaviour with subtraction word problems and division word problems. Comparisons are also made between children's behaviour with subtraction problems involving basic facts and those problems involving 2-digit numbers

The chapter is organized around the research questions. After each question a discussion of the children's behaviour, relative to that question, is presented. Also in this chapter, specific strategies (appropriate and inappropriate) that children used to attempt to solve subtraction and division word problems are discussed. Three types of subtraction problems were identified: take away, open addition, and comparison. The two types of division problems that were identified were partitive and quotitive. Strategies used to attempt to solve subtraction problems were categorized as: separating, counting back, counting up, using a known addition fact, using the wrong operation, guessing, and choosing one of the numbers given in the problem. Strategies used to attempt to solve the two types of division problems were categorized as: sharing, separating sets (repeated subtraction), skip counting, using a known addition fact, using the wrong operation, guessing, and choosing a number given in the problem. A brief explanation of the different types of subtraction and division problems, and of the strategies used to solve them, follows.

SUBTRACTION PROBLEMS

Take Away

The action in a take away problem is subtraction. There is an initial quantity, from which some quantity is removed. An example of this type of problem would be:

> Miss Jones had 12 stickers on her desk in the morning. By recess she had handed out 5 of them. How many stickers were on her desk then?

Open Addition

In an open addition problem the action is additive. There is an initial quantity, and a larger quantity, which is a goal to be reached. The problem is to increase the smaller quantity in order to reach the goal. An example of an open addition problem would be:

> Tonight it's Dad's job to set the table for dinner. He needs 11 plates but can only find 8. How many more plates does he need?

Comparison

Comparison problems look at two quantities and ask for the difference in size. No attempt is made to increase one to the size of the other. An example of a comparison problem would be:

> Mr. Smith's class was asked to bring buttons to school for an art project. Jason has collected 23 buttons and Elizabeth has collected 36. How many more buttons has Elizabeth collected?

DIVISION PROBLEMS

Partitive

In a partitive problem, there is a sharing action. The unknown quantity is the size of each group. An example of a partitive problem would be:

> Tony and David found 12 tickets for video games. The 2 boys shared the tickets equally. How many tickets did each boy get?

Quotitive

In a quotitive problem, the action is subtractive. The size of each group is given, and the unknown quantity is the number of groups. An example of a quotitive problem would be:

> A group of 24 people decided to rent toboggans for the day. Each toboggan could hold 4 people. How many toboggans did they need?

An explanation of the strategies used to solve these types of problems follows.

APPROPRIATE STRATEGIES

Separating

Given a subtraction problem such as

> It was Dad's turn to set the table for dinner. There were going to be 11 people for dinner but he only had 8 plates. How many more plates did he need?

a child who used a separating strategy would count out a set of 11 (the larger set) and then separate 8 from it (the smaller set). This is an appropriate strategy for all types of subtraction problems, and it is a modeling strategy for the take away and comparison problems.

Counting Back

Given a problem such as

> Miss Jones had 12 stickers on her desk. By recess she had handed out 5 of them. How many stickers were on her desk then?

a child who counted back would start with the larger quantity (12), either in his head or by making a pile of counters, and then count backwards, the number of steps back being determined by the smaller quantity (5).

eg. 12-11,10,9,8,7.

1, 2,3,4,5.

This strategy is appropriate for all subtraction types, and is a modeling strategy for take away and comparison problems.

Counting Up

To use a counting up strategy, the child would start with the smaller number in the problem, and then count up until she reached the larger number. The child would keep track of how many increments she counted up, and that number would be the answer to the problem. For example, in the problem

> It was Dad's turn to set the table for dinner. There were going to be 11 people for dinner, but he only had 8 plates. How many more plates did he need?

a child who used a counting up strategy would count

8-"9, 10, 11".

1, 2, 3.

Since there were three increments counted up, 3 would be the solution to the problem. This is considered a modeling strategy for both the open addition and comparison problems.

Using a known addition fact

Given a problem such as

> It was Dad's turn to set the table for dinner. There were going to be 11 people for dinner, but he only had 8 plates. How many more plates did he need?

a child who used a known addition fact to solve this problem would say, " I know that 3 is the answer because I know that 8+3=11. " This strategy was used only for the open addition problems and one of the partitive problems, and was considered a modeling strategy for those problems.

Sharing

Given a problem such as

> There are 21 children in our class and we want to make 3 teams for a relay race. If all the teams must be the same size, how many children will be on each team?

a child who used a sharing strategy would put the large set of 21 counters into 3 equal groups. Children sometimes did this one counter at a time (one for you, one for you, and one for you,then start again) and sometimes they used two or more counters at a time. This is an appropriate strategy for both partitive and quotitive division problems, but it is a modeling strategy only for the partitive problems.

Separating Sets

Given a problem such as

> Ann is helping to make toy cars. Her job is to put the wheels on. She has a box with 12 wheels in it. If each car needs 4 wheels, how many cars can she make?

a child who was separating sets would make a set of 12, and then separate from it sets of 4. The number of sets of 4 that could be separated would be the solution. This is an appropriate strategy for both partitive and quotitive division problems, but is a modeling strategy only for the quotitive problems.

Skip Counting

Given a problem such as

> A group of 12 friends wanted to go on a ride at the fair. The attendant said they had to ride 3 in a seat. How many seats did they need?

a child who used skip counting would count by fours up to 12. The number of counts it took to get to 12 would be the answer. This is an appropriate strategy for both types of division problems, but is a modeling strategy only for the quotitive problems.

INAPPROPRIATE STRATEGIES

Using the wrong operation

Given a problem such as

> Mary planted some tulips and daffodils in her flower garden. She wanted to know which flowers would grow faster. So far, 12 daffodils and 9 tulips have come up. How many more daffodils than tulips are there?

children who used the wrong operation simply took the two numbers in the problem and added them together instead of subtracting. For division problems, some children who used the wrong operation either added or subtracted.

Guessing

Occasionally, a child simply did not know what to do to even start working on the problem, or worked on it a little and then became confused. At that point, some children resorted to random guessing.

Choosing a number given in the problem

Given a problem such as

> Mary planted some tulips and daffodils in her flower garden. She wanted to know which flowers would grow faster. So far, 12 daffodils and 9 tulips have come up. How many more daffodils than tulips are there?

a child would occasionally choose either the 9 or the 12 given in the problem as a solution.

PRESENTATION OF THE FINDINGS

The research questions and the discussion of children's responses relative to each question follows.

> Do children behave differently with word problems involving an operation in which they have received formal school instruction (subtraction) and an operation in which they have received no formal school instruction (division?)

The first general research question deals with three ways in which children's responses in solving word problems can vary. These three areas are : a) the variety of strategies used, b) the frequency of strategies used which model the action in the problem, and c) the types of errors made. In order to deal with these three areas, the first question was broken down into five sub-questions. Three of these sub-questions deal with the variety of strategies used, one deals with modeling strategies, and one deals with the types of errors made.

Research Question 1(a)

> What strategies do children use to solve word problems involving the three different interpretations of subtraction (take away, comparison, open addition)?

Nineteen children were asked to attempt to solve word problems involving the take away, open addition, and comparison interpretations of subtraction. These six problems are given in Appendix A. The results from the two take away problems are given in Table 1.

I have used tables such as Table 1 in order to display the results of the various parts of each question. Since many tables will be used in this chapter, I will use Table 1 as an example and discuss briefly how to read them.

The purpose of Table 1 is to show the strategies children used to solve the take away problems. The heading for the first column is Strategy. Under this heading are the number of children who used a separating strategy for both take away problems (separate twice), children who used a separating strategy for one of the problems (separate once), and children who did not use a separating strategy for either problem (no separating). The second column is headed Number of Children. This column shows how many children fall into each category. As the table shows, 13 children separated for both take away problems, three children separated for one of them, and three children did not separate for either problem. The third column is again entitled Strategy. It shows the other strategies used by children who did not separate for both problems. Of the three children who separated for one of the take away problems, two used an inappropriate strategy and one used a counting back strategy on the other problem. Following the three children who did not separate for either problem, two of them used a counting back strategy for both problems, and the third child used an inappropriate strategy for one problem and explained the 2-digit subtraction algorithm for the other problem. An examination of this table shows that, for the take away problems, separating was by far the most frequently used strategy. Of the 19 children, 13 used this strategy exclusively. Separating accounted for 29 of the 35 appropriate strategies used.

Table 1
Strategies Used For Take Away Probems

<table>
<tr><th>Strategy</th><th>Number of Children</th><th>Strategy</th><th>Number of Children</th></tr>
<tr><td>Separate twice</td><td>13</td><td colspan="2"></td></tr>
<tr><td rowspan="2">Separate once</td><td rowspan="2">3</td><td>Inappropriate</td><td>2</td></tr>
<tr><td>Count Back</td><td>1</td></tr>
<tr><td rowspan="3">No Separating</td><td rowspan="3">3</td><td>Count Back twice</td><td>2</td></tr>
<tr><td>Explanation of 2-digit algorithm</td><td>1</td></tr>
<tr><td>Inappropriate</td><td>1</td></tr>
</table>

The results from the two questions on open addition word problems are given in Table 2.

Table 2
Strategies Used for Open Addition Problems

<table>
<tr><th>Strategy</th><th>Number of Children</th><th>Strategy</th><th>Number of Children</th><th>Strategy</th><th>Number of Children</th></tr>
<tr><td>Count up twice</td><td>4</td><td colspan="4"></td></tr>
<tr><td rowspan="3">Count up once</td><td rowspan="3">13</td><td>Separate</td><td>7</td><td rowspan="3" colspan="2"></td></tr>
<tr><td>Known Addition Fact</td><td>2</td></tr>
<tr><td>Inappropriate</td><td>4</td></tr>
<tr><td rowspan="2">No Counting Up</td><td rowspan="2">2</td><td rowspan="2">Separate once</td><td rowspan="2">2</td><td>Known Addition Fact</td><td>1</td></tr>
<tr><td>Inappropriate</td><td>1</td></tr>
</table>

As Table 2 shows, most of the children interviewed used a mixture of strategies for the open addition problems. Counting up, the modeling strategy for open addition, was used by more children than any other single strategy, but was not as consistently used for open addition problems as separating was used for take away problems. Separating was the second most frequently used strategy for open addition problems. Counting up was used at least once by seventeen children, and separating was used at least once by nine. Counting up accounted for 20 (59%) of the 34 appropriate strategies used, while separating accounted for 9 (26%). It appears that, while counting up was used by the largest number of children, separating was also an influential strategy for this type of problem.

Table 3
Strategies Used for Comparison Problems

Strategy	Number of Children	Strategy	Number of Children
Count up twice	7		
Separate twice	1		
Inappropriate twice	3		
Count up once	5	Separate	4
		Inappropriate	1
Separate once	3	Known Addition Fact	1
		Inappropriate	2

Table 3 displays the results of the two comparison problems. In looking at this table, three patterns emerge. Firstly, counting up was the strategy used the most frequently for comparison problems. Seven children used it exclusively. Counting up was used by 12 children for at least one of the problems, and this strategy accounted for 19 (66%) of the 29 appropriate strategies used. Secondly, in spite of the fact that counting up was the strategy that children used most often, separating was also an influential strategy for comparison problems. Eight of the children used a separating strategy for at least one of the problems, and this strategy accounted for 9 (31%) of the appropriate strategies used. Thirdly, there appeared to be more of a tendency for children to use a single strategy for the comparison problems than for the open addition problems. Eight (61%) of the 13 children who used appropriate strategies for both comparison problems used a single strategy, compared to 4 out of 14 children for open addition problems.

In summary, for each type of subtraction word problem (take away, comparison, open addition) there was one strategy that was used more than any other. For take away problems separating was used almost exclusively. For comparison and open addition problems counting up was the main strategy, with separating also being used frequently. Most of the children interviewed chose one appropriate strategy and stayed with it for the take away problems, but for the open addition problems the reverse was true. Almost half of the children interviewed used a single appropriate strategy for the comparison problems.

Research Question 1(b)

> What strategies do children use to solve word problems involving the two different interpretations of division?

Nineteen children were asked to attempt to solve six division word problems, three each involving the partitive and quotitive interpretations. The results of their responses are given in Tables 4 and 5.

As Table 4 shows, sharing was by far the most frequently used strategy for the partitive problems. All of the children used it for at least one of the problems, and it was a strong preference for 14 children, who shared for at least two out of three problems. Sharing accounted for 37 (79%) of the 47 appropriate strategies used, while the next most popular strategy, using a known addition fact, accounted for only 8 (17%).

Table 5 displays the results of the three quotitive problems. Again it is apparent that one strategy, separating sets, is by far the most frequently used strategy for these problems. It was a strong preference for 12 of the children, who used it for two or more of the three quotitive problems. This strategy accounted for 35 (66%) of the appropriate strategies used. However, skip counting was also used more than occasionally, and in fact, was used exclusively by three children. This strategy accounted for 13 (25%) of the appropriate strategies used.

Table 4
Strategies Used for Partitive Problems

<table>
<tr><th>Strategy</th><th>Number of Children</th><th>Strategy</th><th>Number of Children</th><th>Strategy</th><th>Number of Children</th></tr>
<tr><td>Share 3 times</td><td>4</td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="2">Share twice</td><td rowspan="2">10</td><td>Inappropriate</td><td>4</td><td></td><td></td></tr>
<tr><td>Known Addition Fact</td><td>6</td><td></td><td></td></tr>
<tr><td rowspan="3">Share once</td><td rowspan="3">5</td><td>Inappropriate twice</td><td>2</td><td></td><td></td></tr>
<tr><td>Known Addition Fact</td><td>2</td><td>Skip count once</td><td>2</td></tr>
<tr><td>Lost 2 problems</td><td>1</td><td></td><td></td></tr>
</table>

Table 5
Strategies Used for Quotitive Problems

<table>
<tr><th>Strategy</th><th>Number of Children</th><th>Strategy</th><th>Number of Children</th><th>Strategy</th><th>Number of Children</th></tr>
<tr><td>Separate sets 3 times</td><td>8</td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="3">Separate sets twice</td><td rowspan="3">4</td><td>Skip count</td><td>t</td><td></td><td></td></tr>
<tr><td>Incorrect operation</td><td>1</td><td></td><td></td></tr>
<tr><td>Share</td><td>1</td><td></td><td></td></tr>
<tr><td rowspan="2">Separate sets once</td><td rowspan="2">3</td><td>Inappropriate twice</td><td>1</td><td></td><td></td></tr>
<tr><td>Inappropriate once</td><td>2</td><td>Skip count once</td><td>2</td></tr>
<tr><td rowspan="2">No Separating sets</td><td rowspan="2">4</td><td>Skip count 3 times</td><td>3</td><td></td><td></td></tr>
<tr><td>Share twice</td><td>1</td><td>Inappropriate once</td><td>1</td></tr>
</table>

In summary, for both the partitive and the quotitive word problems, there was one strategy that was more frequently used than any other. Futhermore, for both the partitive and quotitive word problems, most children (14 and 15 respectively) displayed a strong preference for one strategy (using it two or more times).

Research Question 1(c)

> Are there any differences in the strategies used between the different interpretations for subtraction and division?

As tables 1,2,3,4 and 5 illustrate, each of the five types of word problems (two subtraction and three division) has one main strategy that is used more than any other. When looking at the results of the subtraction problems it becomes clear that separating, the most frequently used strategy for the take away problems, was also an influential strategy for both the open addition and comparison word problems. Although counting up was the main strategy for both the open addition and comparison word problems, separating accounted for 26% and 31% of the appropriate strategies, respectively. For the division word problems, however, Tables 4 and 5 show that the main strategy used for one type of word problem is seldom used for the other type. Sharing, the main strategy used for the partitive word problems, was used by only two children for the quotitive problems, and separating sets, the main strategy for the quotitive word problems was never used for the partitive problems.

It appears that children have a greater tendency to use different strategies for the different types of division word problems than they do for subtraction word problems. There was very little overlap between the main strategies for the partitive and quotitive word problems, but the main strategy for the take away word problems was a commonly used one for both open addition and comparison word problems.

Research Question 1(d)

> How often does the strategy used model the action in the problem, and is there a difference between division and subtraction word problems in this regard?

Tables 6 and 7 summarize the number of children using modeling strategies for the subtraction and division word problems. Tables 8 and 9 show the number of children using modeling strategies for the various interpretations of each operation.

Table 6
Modeling Strategies for Subtraction Problems

Number of Children	Number of Modeling Strategies	Un-Modeled Problems	
		Number of Children	Strategy Used On Other Problems
5	6		
7	5	1	Inappropriate
		6	Appropriate
3	4	3	Inappropriate once Appropriate once
2	3	2	Inappropriate 3 times
2	2	1	Inappropriate 4 times
		1	Inappropriate 4 times/ Appropriate once

Table 7
Modeling Strategies for Division Problems

<table>
<tr><th rowspan="2">Number of Children</th><th rowspan="2">Number of Modeling Strategies</th><th colspan="2">Un-Modeled Problems</th></tr>
<tr><th>Number of Children</th><th>Strategy Used On Other Problems</th></tr>
<tr><td>8</td><td>6</td><td></td><td></td></tr>
<tr><td rowspan="2">7</td><td rowspan="2">5</td><td>2</td><td>Appropriate</td></tr>
<tr><td>5</td><td>Inappropriate</td></tr>
<tr><td rowspan="2">2</td><td rowspan="2">3</td><td>1</td><td>Inappropriate twice/
Appropriate once</td></tr>
<tr><td>1</td><td>Lost 2 problems/
Appropriate once</td></tr>
<tr><td rowspan="2">2</td><td rowspan="2">2</td><td>1</td><td>Inappropriate 4 times</td></tr>
<tr><td>1</td><td>Inappropriate twice/
Appropriate once</td></tr>
</table>

In looking at Tables 6 and 7 it becomes apparent that, while there were more modeling strategies used for division problems, the difference between the numbers of modeling strategies used for subtraction and division was slight. Children used modeling strategies for 76% of the subtraction problems and 81% of the division problems. A look at Tables 8 and 9, however, shows that when the numbers of modeling strategies used is broken down into problem types for each operation, there is a difference between subtraction and division. As Table 8 shows, the take away problems were modeled most often (90% of the time), followed by the comparison problems (76% of the time), while the open addition problems were modeled only 63% of the time. This is a 27% difference between the number of modeling strategies used for the take away and the open addition problems. For the division problems however, there was little difference between the two types of problems, in terms of modeling strategies. The partitive problems were modeled 79% of the time, and the quotitive problems 84% of the time, only a 5% difference.

Table 8
Modeling Strategies for Take Away, Comparison and Open Addition Problems

Problem type	Model twice	Model Once		No Modeling	
		Number of children	Strategies Used on Other Problems	Number of children	Strategies Used on Other Problems
Take Away	16	2	Inappropriate once	1	Explain 2-digit algorithm/ inappropriate
Com-parison	13	3	Inappropriate once	3	Inappropriate twice
Open Addition	6	8	Separate	1	Separate/ Inappropriate
		4	Inappropriate		

Table 9
Modeling Strategies Used for Partitive and Quotitive Problems

Problem Type	Model 3 times	Model Twice		Model Once		No Modeling	
		Number of children	Strategy Used on Other Problem	Number of children	Strategies Used on Other Problems	Number of children	Strategies Used
Partitive	10	4	Inap-propriate	2	Inap-propriate twice	0	
		2	Skip count	1	Lost 2 problems		
Quotitive	13	1	Inap-propriate	1	Inap-propriate twice	1	Share twice /Inap-propriate
		3					

When they did not use modeling strategies, children behaved in much the same way for both operations. For the subtraction problems, 63% of the non-modeling strategies were inappropriate, and 37% were appropriate. For the division problems, 68% were inappropriate and 32% were appropriate. Looking at the breakdown of non-modeling strategies into problem types, again there is a difference between subtraction and division word problems. For the take away and comparison subtraction word problems, most of the non-modeling strategies were inappropriate (75% and 100%, respectively). But for the open addition problems, the reverse was true. Only 34% of the non-modeling strategies used for the open addition problems were inappropriate. For both the partitive and quotitive division problems, the non-modeling strategies were mostly inappropriate (80% and 64% respectively).

In summary, children used a similar number of modeling strategies for both subtraction and division word problems. However, for the subtraction problems, the number of modeling strategies varied greatly depending on the problem type, while for the division problems there was little variation between the two types. Also, when they were not using modeling strategies, children behaved in much the same way on the subtraction and division word problems. Here again children's behaviour on the subtraction problems depended on the problem type, while their behaviour on the division problems behaviour was consistent for both types.

Research Question 1(e)

> Is there a tendency for children to make different types of errors in division word problems than in subtraction word problems?

Errors were labelled as counting errors (where a child simply makes a mistake in counting), counting up to the wrong quantity (where a child is asked, for example, to find how many more to make 12 if we already have 8, and counts from 8 to 13 instead of to 12), using the wrong operation (the child adds, for example, instead of subtracting), pick a number in the problem (the child chooses one of the numbers given in the problem as a solution), guess (the child guesses randomly), and no response (the child simply could not start on it at all). All types of errors, with the exception of no response, were made for both operations.

Other than the no response type, every type of error was made for each of the operations. More children committed counting, wrong operation, pick a number given in the problem, and count up to the wrong initial quantity errors for subtraction problems than for division. More children made guessing errors for division than subtraction problems. The largest difference, however, was in the number of children who made errors in counting. Fourteen children made subtraction counting errors, for a total of 22 errors, and five children made counting errrors in division problems, for a total of 5 errors.

The errors that have been identified can be put into three categories - 1) counting type errors(counting and counting up to the wrong quantity), 2) wrong operation, and 3) inappropriate strategies, other than wrong operation (pick a number in the problem, no response, guess). With the counting types of errors it is possible for a child to have a very good idea of what the problem is about and how to go about solving it, but simply make a computational mistake. If a child is using a wrong operation or other inappropriate strategy to solve a problem, it is not likely that he comprehends the problem. A child who uses inappropriate strategies such as guessing, pick a number in the problem, or no response, does not appear to have chosen a deliberate method of solution and followed it through, but rather behaved in a random way. The child who uses a wrong operation, however, has definitely selected a method of solution (although it is inappropriate) and acted upon it.

The types of errors made for the subtraction and division problems are displayed in Tables 10, 11, and 12. Table 10 displays the incorrect operation errors for both operations, Table 11 displays the counting errors, and Table 12 displays the errors due to using an inappropriate strategy.

As Table 10 shows, there were two more children who used an incorrect operation for subtraction than for division. All of the children who used an incorrect operation for division performed successfully on the other five problems, but three out of five children who used an incorrect operation on the subtraction problems used at least two inappropriate strategies for the other five problems. It appears that children tend to use an incorrect operation for subtraction slightly more than for division problems. Also, children who use an incorrect operation for one subtraction problem are more likely to use inappropriate strategies for the other problems than childen who use an incorrect operation for one division problem.

Table 10
Behaviour of Children Who Used An Incorrect Operation

Operation	Number of Children	Incorrect Operations Per Child	Other Problems	
			Number of Children	Type of Strategy Used
Subtraction	5	1	2	Model 4 times/Appropriate once
			2	Model 3 times/ Inappropriate twice
			1	Model twice/ Inappropriate 3 times
	1	3	1	Model twice/Appropriate once
Division	3	1	3	Model 5 times
	1	3	1	Model twice/ Inappropriate once

Table 11 displays counting errors for both operations, and as indicated, there were many more children who made counting errors for subtraction problems than for division problems (16 and 6, respectively).

There were 27 counting errors for subtraction problems and 6 for division problems. However, half of the subtraction problems dealt with 2-digit calculations, which can be expected to have a higher number of counting errors than problems dealing with basic facts. Six children made one counting error each on basic fact problems, while 14 children made a total of 21 errors on 2-digit problems. The comparison of division problems to basic fact problems is more similar than the comparison of division problems to all subtraction problems. However, basic fact subtraction problems still incurred some type of counting error on 12% of the problems, while the same type of error was made on 6% of the division problems.

It is also evident from Table 11 that making counting errors is not an indication of serious difficulty in dealing with that particular problem. All the children who made counting errors used either a modeling or an appropriate non-modeling strategy for the problem with the error. Overall,

Table 11
Behaviour of Children Who Made A Counting Error

Operation	Number of Children	Counting Errors Per Child	Number of Children	Strategy Used For Problem With The Error	Other Problems	
					Number of Children	Type of Strategy
Subtraction	8	1	6	model	4	model 5 timest
					2	model 4 times/ inappropriate once
			2	appropriate but non-modeling	1	model twice/ inappropriate once
					1	model once/ inappropriate 4 times
	5	2	5	model both problems	1	model 4 times
					2	model 3 times/ appropriate but non-modeling once
					1	model 3 times/ inappropriate once
					1	model once/ inappropriate 3 times
	3	3	2	model 3 times	1	model 3 times
			1	model twice appropriate but non-modeling once	2	model twice/appropriate but non-modeling once
Division	6	1	5	model	5	model 5 times
			1	appropriate but non-modeling	1	model once/ appropriate but non-modeling

it appears children tend to make more counting errors on subtraction problems than division problems, but making these errors does not appear to be an indication of misunderstanding the problem, for either operation.

Table 12 displays the errors due to using an inappropriate strategy. As indicated, children made almost the same number of this type of error for subtraction problems as for division problems (7 and 8, respectively). They also tended to behave in much the same way for the remaining subtraction and division problems, using modeling strategies for 71% and 70% of the remaining problems. The remaining 29% of the subtraction problems without inappropriate strategy errors were attempted using the wrong operation. The remaining 30% of the division problems without inappropriate strategy errors were split between using the wrong operation and using an appropriate but non-modeling strategy (18% and 12%, respectively).

Table 12
Behaviour of Children Who Made Inappropriate Strategy Errors

Operation	Number of Children	Errors per Child	Other Problems	
			Number of Children	Type of Strategy
Subtraction	2	2	2	model 3 times/ incorrect operation once
	1	3	1	model twice/ incorrect operation once
Division	3	1	2	model 5 times
			1	model twice/incorrect operation 3 times
	1	2	1	model twice/appropriate but non-modeling twice
	1	3	1	model 3 times

Summary of Research Question #1

> Do children behave differently with word problems involving an operation in which they have received formal school instruction (subtraction) and an operation in which they have received no formal school instruction (division)?

What differences are there in the ways in which primary children approach one-step word problems involving an operation in which they have received formal school instruction (subtraction) and an operation in which they have received no formal school instruction (division)?

For both operations (subtraction and division), each of the interpretations had one main strategy used by more children than any other. The two interpretations of division overlapped very little with respect to these strategies. Sharing, the main strategy for partitive problems, was used very little for quotitive problems, and separating sets, the main strategy for quotitive problems, was not used at all for partitive problems. There was more overlap in the strategies utilized for the three interpretations of subtraction, however. The main strategy for both open addition and comparison problems was counting up. This strategy was not used at all for the take away problems. However, separating, the main strategy for take away problems, was an influential strategy for both the open addition and comparison word problems. This strategy accounted for 26% and 31% of the appropriate strategies used for the open addition and comparison word problems, respectively.

There was little difference in the number of modeling strategies used for the subtraction and division word problems (76% and 81%, respectively). However, for the subtraction problems, there was a great variation in the number of modeling strategies, depending on the problem type (take away - 90%, comparison - 76%, and open addition - 63%). This was not the case for the division problems. When they were not using modeling strategies, children used inappropriate strategies for 63% of the remaining subtraction problems (75% of the take away problems, 100% of the comparison problems, and 36% of the open addition problems) and 68% of the remaining division problems.

The children made the same types of errors on both division and subtraction problems. They had a tendency to make more counting errors on the subtraction problems, but this was partly due to the use of 2-digit numbers in half of the subtraction problems. There were a few more children who used an incorrect operation for subtraction problems than for division problems, but the difference was small. All the children who used an incorrect operation for one of the division problems performed successfully on the other five division problems, but three out of five children who used an incorrect operation for the subtraction problems used an inappropriate strategy for at least two of the other five problems. It appears that using an incorrect operation for one subtraction problem is more likely to be an indication of general difficulty than using an incorrect operation for one division problem.

About the same number of children used inappropriate strategies for subtraction and division problems (3 and 4, respectively). There were a total of seven inappropriate strategies for subtraction and eight for division problems. Children who behaved in this way also behaved similarly to each other on the other problems.

The second general problem area was:

> For the subtraction problems, are there differences in the way children approach word problems involving the solution of a basic fact, and those involving the subtraction of 2-digit numbers?

This question was broken down into two sub-questions, one dealing with modeling strategies and the other dealing with errors. Each sub-question will be discussed separately, and then the entire general problem area will be summarized.

<u>Research Question 2(a)</u>

> Do these children differ in the number of modeling strategies that they use for the basic fact subtraction problems, and the 2-digit subtraction problems?

The children were presented with six subtraction word problems, two each of open addition, take away, and comparison. For each of these three interpretations of subtraction, children were asked to solve one problem involving a basic subtraction fact, and one involving 2-digit numbers. They solved or attempted to solve three word problems involving basic facts and three word problems involving 2-digit numbers.

Table 13 shows how many children modeled all three basic fact problems, how many children modeled two of them, and how many children modeled only one of them. Table 13 also shows what types of strategies they used when they did not use modeling strategies. It also displays the same information for the 2-digit problems. Table 14 indicates the number of children modeling each of the six subtraction problems.

As Table 13 illustrates, children were more likely to model basic fact subtraction problems than 2-digit problems (18 children modeled 2 or more basic fact problems and 15 children modeled 2 or more 2-digit problems). When they did not use modeling strategies, more children used inappropriate strategies for basic fact problems than for the 2-digit problems. Seven children used inappropriate strategies for basic fact problems, with these inappropriate strategies comprising 64% of the non-modeling strategies. Five children used inappropriate strategies for 2-digit problems. These inappropriate strategies comprised 44% of the non-modeling strategies.

Table 14 indicates the number of modeling strategies used was fairly consistent for five out of the six subraction problems. The exception was the 2-digit open addition problem, which was solved using a modeling strategy about half as often as any of the other problems. However, as indicated by Table 13, of the non-modeling strategies used, both types of open addition problems had a higher percentage of appropriate strategies than either take away or comparison problems.

It appears, from the information given in Tables 14 and 15 that although the size of the numbers seems to have an effect on children's ability to model 2-digit open addition problems, it does not have an effect on the number of appropriate strategies used. Approximately the same number of appropriate strategies were used for 2-digit problems as for basic fact problems.

Table 13
Modeling Strategies for Basic Fact Problems

<table>
<tr><th>Problem Type</th><th>Number of Modeling Strategies</th><th>Number of Children</th><th>Strategy</th><th>Number of Children</th></tr>
<tr><td rowspan="5">Basic Fact</td><td>3</td><td>11</td><td></td><td></td></tr>
<tr><td rowspan="2">2</td><td rowspan="2">7</td><td>Inappropriate</td><td>6</td></tr>
<tr><td>Appropriate but non-modeling</td><td>1</td></tr>
<tr><td>1</td><td>0</td><td></td><td></td></tr>
<tr><td>0</td><td>1</td><td>Inappropriate 3 times</td><td>1</td></tr>
<tr><td rowspan="5">2-Digit</td><td>3</td><td>7</td><td></td><td></td></tr>
<tr><td>2</td><td>8</td><td>Appropriate but non-modeling</td><td>8</td></tr>
<tr><td rowspan="2">1</td><td rowspan="2">3</td><td>Inappropriate twice</td><td>2</td></tr>
<tr><td>Inappropriate once/ Appropriate but non-modeling once</td><td>1</td></tr>
<tr><td>0</td><td>1</td><td>Inappropriate 3 times</td><td>1</td></tr>
</table>

Table 14
Modeling Strategies for Each Subtraction Problem

Problem Type	Take Away	Comparison	Open Addition
Basic Fact	17	14	16
2-Digit	17	15	8

To summarize, the size of the numbers seems to have a slight effect on how consistently children model subtraction problems. More children modeled all three basic fact problems than all three 2-digit problems (11 and 7, respectively). The same held true for modeling two or more problems (18 and 15, respectively). Number size appeared to have a negative effect on children's ability to model 2-digit open addition problems but had no effect on the number of appropriate strategies used.

Table 15
How Children Responded When They Failed to Model

Problem Type	Take Away		Comparison		Open Addition	
	Number of Children who failed to model	Strategy Used	Number of Children who failed to model	Strategy Used	Number of Children who failed to model	Strategy Used
Basic Fact	2	Incorrect Operation	3	Incorrect operation	1	Separate
					1	Incorrect Operation
			2	Pick a number in the problem	1	Pick a number in the problem
2-Digit	1	Explanation of algorithm	2	Guess	8	Separate
					1	Guess
					1	Pick a number in the problem
	1	Pick a number in the problem	2	Pick a number in the problem	1	Incorrect operation

Research Question 2(b)

> Is there a tendency for these children to make different types of errors in the 2-digit subtraction problems than in the basic fact subtraction problems?

The five types of errors made on the subtraction problems were counting, counting up to the wrong quantity, using the wrong operation, picking a number given in the problem, and guessing. These errors can be put into three categories - counting type errors (counting and counting up to the wrong quantity), wrong operation, and inappropriate strategies other than wrong operation (guessing, picking a number given in the problem). The frequency of these errors is shown in Table 16.

Table 16
Errors Made for Basic Fact and 2-digit Subtraction Problems

Error Type	Basic Fact			2-Digit		
	Take Away	Comparison	Open Addition	Take Away	Comparison	Open Addition
Incorrect Operation	2	3	1	0	1	1
Counting	3	0	3	9	6	6
Inappropriate	0	2	1	1	3	2

All three kinds of errors (counting type errors, wrong operation, and other inappropriate strategies) were made for both the basic fact and the 2-digit problems. However, there were differences in the number of each type of error made. Not surprisingly, for each type of subtraction problem there were at least twice as many counting type errors committed for the 2-digit as for the basic fact problems. There were six errors of this type made for the basic fact problems and 15 for the 2-digit problems. Counting type errors accounted for 65% of the errors made for the 2-digit problems and 40% of the errors made for the basic fact problems. It is not unexpected to see a larger number of counting errors in problems with larger numbers, since more counting is required. Longer counting sequences provide more opportunity to make a counting error.

While more children committed counting errors on the 2-digit problems than on the basic fact problems, the reverse was true for using an incorrect operation. Children made two incorrect operation errors on the 2-digit problems and six on the basic fact problems. This type of error accounted for 40% of the errors made on the basic fact problems, but only 9% of the errors made on the 2-digit problems.

There were more inappropriate strategies (other than an incorrect operation) used for the 2-digit problems than for the basic fact problems. In fact, there were twice as many errors for this type made for the 2-digit problems. These errors accounted for 20% of all the errors made on the 2-digit problems.

In summary, it appears that an incorrect operation was used by more children on basic fact problems than on 2-digit problems. This is in contrast to the other types of errors which were committed more often for the 2-digit problems than for the basic fact problems.

Summary of Research Question #2

There were six subtraction problems - two take away, two comparison, and two open addition. Each interpretation had one problem involving the calculation of 2-digit numbers and one problem involving basic subtraction facts. For the take away and comparison interpretations, the number of children using modeling strategies for the basic fact and the 2-digit problems is about the same. For the open addition problems, however, there were about half as many modeling strategies used for the 2-digit problems as for the basic fact problems. Eight out of 11 open addition problems that were not done using a modeling strategy were done using an appropriate strategy, however. Children used the same number of appropriate strategies for basic fact and 2-digit problems of each interpretation (take away - 17, 18; comparison - 14, 15; open addition - 16, 16, respectively).

The three types of errors (counting type errors, wrong operation, and other inappropriate strategies) were committed on both the basic fact and the 2-digit problems. Counting type errors were committed more than three times as often for 2-digit problems as for basic fact problems, and inappropriate strategies were used twice as often for 2-digit problems. Three times as many children used an incorrect operation for the basic fact problems as for the 2-digit problems. Although all three error types were made on both 2-digit and basic fact problems, certain errors were more common with one type of problem than the other.

The third general problem area was:

> Do children who do well on the addition and subtraction basic fact tests approach word problems differently than children who do not perform as well on basic fact tests?

This question was broken down into two sub-questions, one dealing with modeling strategies and one dealing with errors. Each sub-question will be discussed separately, and then the entire general problem area will be summarized.

Research Question 3(a)

> Do children who perform well on the basic fact tests differ in the number of modeling strategies that they use for the subtraction word problems, from the children who perform poorly on the basic fact tests?

The children were divided into two groups on the basis of their score on a subtraction basic fact test. The scores tended to divide themselves naturally, with seven representing the top of the low group, and ten representing the bottom of the high group. The scores ranged from 2 to 7, consecutively, then a gap, followed by three scores of 10, and one each of 11, 13, 15, and 18. There were 10 children who scored 7 or less out of 30, and seven children who scored 10 or more. The low group had an average mark of 4.8 and the high group had an average mark of 10.9 on the subtraction test. Of the 10 children who scored 7 or less on the subtraction basic fact test, seven of them scored less than 19 out of 30 on the addition basic fact test. The average mark for this group on the addition test was 16.6 out of 30. Of the seven children who scored 10 or more on the subtraction test, only two scored less than 19 out of 30 on the addition test. The average mark for this group on the addition test was 22.5 out of 30.

Tables 17 and 18 show the modeling strategies used by children in the low and high groups for the subtraction problems. Table 19 shows children's consistency in the appropriate strategies they used for the subtraction problems. The tables are discussed in the following section.

Table 17
Modeling Strategies for Subtraction Problems (High Group)

Problem Type	Model twice	Model Once		No Modeling	
		Number of Children	Strategy Used On Other Problem	Number of Children	Strategies Used on Other Problem
Take Away	4	2	Inappropriate	1	Inappropriate once/ appropriate but non-modeling
Comparison	4	2	Inappropriate	1	Inappropriate twice
Open Addition	3	2	Appropriate but non-modeling	1	Inappropriate once/ appropriate but non-modeling once
		1	Inappropriate		

Table 18
Modeling Strategies for Subtraction Problems (Low Group)

Problem Type	Model twice	Model Once		No Modeling	
		Number of Children	Strategy Used On Other Problem	Number of Children	Strategies Used on Other Problem
Take Away	10	0		10	
Comparison	8	0		2	Inappropriate twice
Open Addition	3	4	Appropriate but non-modeling	0	
		3	Inappropriate		

There were ten children in the low group and seven children in the high group. All of the low group and 57% of the high group were able to model both take away problems, 80% of the low group and 57% of the high group modeled both comparison problems, and 30% of the low group and 43% of the high group modeled both open addition problems. It appears for both take away and comparison problems the low group has a greater tendency to use modeling strategies than the high group. For the open addition problems, more children in the high group than the low group modeled both problems. However, the difference was not as great for this interpretation as it was for the take away and comparison problems. When one considers appropriate strategies rather than only modeling strategies, the percentages of the two groups are closer. The percentages of appropriate strategies used by the low and the high groups are as follows: 1) for the take away problems, 100% and 79%, respectively, 2) for the comparison problems, 80% and 71% , respectively, and 3) for the open addition problems, 85% and 86%, respectively.

Table 19
Consistency of Strategies for Subtraction Problems (High and Low Groups)

	Take Away		Comparison		Open Addition	
	High Group	Low Group	High Group	Low Group	High Group	Low Group
Single Appropriate Strategy	4 (57%)	9 (90%)	3 (43%)	4 (40%)	1 (14%)	3 (30%)
Two Different Appropriate Strategies	0	1 (10%)	1 1 (14%)	4 (40%)	4 (57%)	4 (40%)
Appropriate/ Inappropriate Strategy Mixture	3 (43%)	0	1 (14%)	0	2 (29%)	3 (30%)
Two Inappropriate Strategies	0	0	2 (29%)	2(20%)	0	0

It also appears that the low group has slightly more of a tendency to use a single strategy for each interpretation than the high group. For the take away problems 90% of the low group used a single appropriate strategy, while 57% of the high group did so. For comparison problems, it was 40% for the low group and 43% for the high group, and for open addition the percentages were 30% and 14%, respectively.

It appears that, as well as using fewer modeling strategies than the low group, the high group is more likely to use a single appropriate strategy for an interpretation. Except for the open addition problems, the low group also used more appropriate strategies than the high group.

Table 20
Modeling Strategies for Division Problems (High Group)

Problem Type	Model Three Times	Model Twice		Model Once		No Modeling	
		Number of Children	Strategy Used On Other Problem	Number of Children	Strategies Used On Other Problem	Number of Children	Strategies Used
Partitive	2	2	Skip Count	1	Inappropriate twice	0	
		2	Inappropriate				
Quotitive	4	2	Inappropriate	2	Inappropriate twice	0	

Table 21
Modeling Strategies for Division Problems (Low Group)

Problem Type	Model Three Times	Model Twice		Model Once		No Modeling	
		Number of Children	Strategy Used On Other Problem	Number of Children	Strategies Used On Other Problem	Number of Children	Strategies Used
Partitive	6	2	Inappropriate	1	Inappropriate twice	0	
Quotitive	7	2	Inappropriate	0		0	
		1	Share				

As Tables 20 and 21 show, children in the low group were more likely to model the division problems than children in the high group. The low group modeled 85% of the partitive problems and 90% of the quotitive problems, while the high group modeled 71% of the partitive problems and 81% of the quotitive problems. Six out of nine children in the low group modeled all three of the partitive problems (one child's first and second partitive problems were lost on the defective tape), and seven out of ten modeled all three of the quotitive problems. Two out of seven children from the high group modeled all the partitive and four out of seven modeled all of the quotitive problems.

All three of the partitive problems were modeled by 66% of the low group and 29% of the high group. 70% of the low group and 57% of the high group modeled all of the quotitive problems. For both quotitive and partitive types of division problems, more children in the low group modeled all the problems of a particular type than children in the high group.

As Table 22 shows, children in the low group were also more likely to use a single appropriate strategy than children in the high group.

Table 22
Consistency of Strategies for Division Problems (High and Low Groups)

	Partitive		Quotitive	
	High Group	Low Group	High Group	Low Group
One Single Appropriate Strategy	0	4 (45%)	3 (43%)	6 (60%)
Mixture of Three Appropriate Strategies	4 (57%)	2 (22%)	1 (14%)	2 (20%)
Appropriate/ Inappropriate Stragtegies	3 (43%)	3 (33%)	3 (43%)	2 (20%)

Of the children in the low group, 45% used a single appropriate strategy for all three partitive problems, while none of the high group did so. For the quotitive problems, 60% of the low group and 43% of the high group used a single appropriate strategy. As well as using one single appropriate strategy more often, children in the low group used a larger percentage of appropriate strategies than children in the high group. The low group used appropriate strategies for 86% of the partitive and 93% of the quotitive problems, while the high group used appropriate strategies for 81% of the problems of each type.

It appears that children who know their subtraction basic facts better, use fewer modeling strategies, slightly fewer appropriate strategies, and more inappropriate strategies for both operations and all interpretations except open addition.

<u>Research Question 3(b)</u>

> Do the children who perform well on basic fact tests make different types of errors on the subtraction word problems than the children who perform poorly on basic fact tests?

The five errors that were identified for subtraction problems were counting , counting up to the wrong quantity, using the wrong operation, picking a number given in the problem as a solution, and guessing. These errors were grouped into three categories - counting type errors (counting, counting up to the wrong quantity), using the wrong operation, and other inappropriate strategies (picking a number in the problem, guessing).

Table 23
Errors on Subtraction Problems for the High and Low Groups

Type of Error	High Group		Low Group	
	Number of Children	Errors per Child	Number of Children	Errors per Child
Counting	1 4	3 1	2 5 3	3 2 1
Incorrect Operation	1 2	3 1	2	1
Inappropriate Strategies	1 1	3 1	2 1	2 1

Tables 23 and 24 display information on the number and type of errors made on the subtraction problems and how the high and the low groups compare.

As Table 24 shows, for each subtraction interpretation, children in the low group made consistently more counting errors than children in the high group. Counting type errors accounted for 73% of all the errors made by the low group and 44% of all the errors made by the high group.

Table 24
Errors for Three Types of Subtraction Problems

Type of Error	Take Away		Comparison		Open Addition	
	High	Low	High	Low	High	Low
Counting	3	9	1	5	3	5
Incorrect Operation	2	0	2	1	1	1
Inappropriate Strategies	1	0	2	3	1	2

While the low group made more counting errors than the high group, the reverse was true for errors involving an incorrect operation. This type of error accounted for 8% of all the errors made by the low group and 31% of all the errors made by the high group.

Of the three types of errors made on the subtraction problems, the performance of the high and low group was most similar with respect to errors involving inappropriate strategies (such as guessing or choosing a number in the problem). Errors of this type accounted for 19% of all the errors made by the low group and 25% of all the errors made by the high group.

Tables 25 and 26 display information on the numbers and type of errors made on the division problems, and how the high and the low groups compare.

For the most part, children made the same types of errors, and the same difference existed between the high and low group, for the division problems as for the subtraction. One exception to this is the amount of counting errors made for the division problems. For the subtraction problems, children in the low group made twice as many counting errors per child as the high group, but for the division problems this was not the case. For the division problems, two children in the high group and three children in the low group made one counting error each.

Table 25
Errors on Division Problems for High and Low Groups

Type of Error	High Group		Low Group	
	Number of Children	Errors per Child	Number of Children	Errors per Child
Counting	2	1	3	1
Incorrect Operation	2 1	1 3	1	1
Inappropriate Strategies	1 1	1 2	2 1	1 3

Table 26
Errors for Partitive and Quotitive Division Problems

Type of Error	Partitive		Quotitive	
	High	Low	High	Low
Counting	2	1	0	1
Incorrect Operation	2	0	3	1
Inappropriate Strategies	2	4	1	1

There were only 19 errors made on the division problems as compared to 42 on the subtraction problems, but of those 19, 12 errors were made on the partitive problems and only 7 on the quotitive. Although the numbers for each type of error are small, more children consistently made errors on the partitive problems than on the quotitive. There was very little difference in the total number of errors made by children in the low and high groups. In fact, the low group made nine errors and the high group made ten. Also, the number of each type of error made was much the same for both groups. The one exception to this was errors made using an incorrect operation. There was one such error made by one child in the low group, and five such errors made by three children in the high group. This is similar to the situation for the subtraction problems, in which two children in the low group each made one such error, and three children in the high group made a total of five such errors.

Summary of Research Question #3

For the subtraction problems, the children in the low group used more modeling strategies, slightly more appropriate strategies, and less incorrect operations than children who performed better on the basic fact tests. The only exception to this is the open addition problems. The picture is much the same for the division problems, with children in the low group using more modeling strategies, about the same number of appropriate strategies, and less incorrect operations than the children in the high group.

All types of errors (counting type errors, using the wrong operation, and other inappropriate strategies) were made by both low and high groups, for both operations. For the subtraction problems, the low group made more counting errors than the high group, and for both operations the high group used an incorrect operation more often than the low group. Other inappropriate strategies were used similarly by both low and high groups.

CHAPTER FIVE

SUMMARY OF MAJOR FINDINGS

INTRODUCTION

This chapter will review the objectives of the study, the findings, the educational implications, and give some recommendations for further research into children's approaches to solving word problems.

CONCLUSIONS AND DISCUSSION

The primary objective of this study has been to explore the ways in which Grade Two children attempt to solve word problems involving an operation in which they have received formal instruction (subtraction) and an operation in which they have received no formal instruction (division). This was done through the use of audio-taped individual interviews. The study was directed towards answering the following research questions:

1. Do children behave differently with word problems involving an operation in which they have received formal school instruction (subtraction) and an operation in which they have received no formal school instruction (division)?

2. For the subtraction problems, are there differences in the way children approach word problems involving the solution of a basic fact, and those involving the subtraction of 2-digit numbers?

3. Do children who do well on addition and subtraction basic fact tests approach word problems differently than children who perform poorly on basic fact tests?

A discussion of the findings and conclusions for each major question follows.

Conclusions and Discussion for Research Question #1

> Do children behave differently with word problems involving operations for which they have received formal school instruction and operations for which they have not received formal school instruction?

The data that has been collected suggests that, for the children in this study at least, there are some differences in the ways in which they deal with problems involving these two types of operations. For both operations, each interpretation of that operation has one main strategy that is used more than any other. Other strategies were also used, but to a lesser degree. For the division problems, the sharing strategy used to solve the partitive type of problems was seldom used for the quotitive problems and vice versa. In fact, separating sets, the main strategy for quotitive problems was never used for partitive problems. The children tended to use strategies for one type of problem that were quite distinct from strategies used for the other type. For the most part, the strategies that were used tended to model the structure of the problem. Children appeared to have analysed the structure of the problem, and then used a strategy that modeled that structure. For subtraction, on the other hand, children had more of a tendency to use the strategies for the various interpretations interchangeably. Separating was used almost exclusively for the take away problems, and counting up was the main strategy used for both open addition and comparison problems. Although counting up was the main strategy, separating was used for a large minority of both the open addition and comparison problems (26% and 31% of the appropriate strategies, respectively). Since subtraction was the operation in which children have been formally instructed, and with which they have computational experience, this seems to suggest children are more likely to keep strategies for the different interpretations separate for operations in which they have not been instructed. It also suggests that children may pay more attention to the structure of the problem and take more time to analyse the problem when it involves an operation with which they are unfamiliar. Part of the reason for this tendency to keep the strategies for the two different division interpretations separate, may lie in the nature of the partitive and quotitive meanings of division. These two interpretations and the strategies that model the structures of these types of word problems are very distinct. A modeling strategy for one interpretation does not in any way appear to be an appropriate strategy for the other type of division problem. It does not appear to fit the problem at all.

Also, children with some experience in an operation may recognize the problem, in a general way, as something they have dealt with before (or similar to something they have dealt with before) in a subtraction (or division) situation. Because they already recognize (or think they recognize) the situation in a general way, they then simply choose a strategy that fits the operation, rather than looking at the problem in a more analytical way. With a problem involving an operation in which they have not been instructed, children have only the structure of the problem itself as a guide to how to proceed. This being the case, they are very likely to use a strategy that models that structure.

Another aspect of children's behaviour with word problems is the errors they make. Overall, children made the same types of errors on the division problems as they did on the subtraction problems. There were many more counting errors made on the subtraction problems than on the division problems, but most of those errors were made on the 2-digit problems. Since it seems reasonable that children would make more counting errors when dealing with larger numbers, this is to be expected. There was a slight difference in the number of children who used an incorrect operation (three for division and five for subtraction) but the real difference came in the way these children behaved on the remaining problems. All of the children who used an incorrect operation for a division problem performed successfully on the remaining five division problems, but three out of five children who used an incorrect operation for a subtraction problem used an inappropriate strategy for at least two of the other five subtraction problems. For some reason, using an incorrect operation for a subtraction problem was far more likely to indicate general difficulty in dealing with word problems involving that operation than using an incorrect operation for a division problem. In fact, using an incorrect operation for a division problem was not an indication of general difficulty with division problems at all.

There are also some differences in the ways in which children attempt to solve the three types of subtraction problems. The take away problems were almost all solved using a modeling strategy (89%), while the comparison problems were solved similarly 76% of the time, and open addition problems 63%. At first glance it may appear that open addition problems posed the most difficulty to the students, as they were modeled the least number of times. However, a look at some other responses suggests that this is not the case. While comparison problems were modeled more often than open addition problems, the total number of appropriate strategies (includes modeling and appropriate but non-modeling strategies) used was greater for the open addition problems (76% and 87%, respectively). Open addition problems also produced more correct solutions than comparison problems. All types of errors were made for all three types of subtraction problems, but more errors involving inappropriate strategies were made on comparison problems than on the other two types combined. An incorrect operation was used twice as often for comparison problems as for either open addition or take away problems. Also, when children were solving open addition and take away problems, they tended to choose an approach and act on it fairly promptly, but this was not the case with the comparison problems. After being read a comparison problem, children often asked to have the problem re-read (many times more than once) or questioned some of the information given in the problem. They also often asked to have the question clarified. This exchange of information occurred either before a child started on the problem, or in some cases, after a child was asked to explain what he had done to solve the problem. There were 15 occasions when such a discussion occurred for comparison problems, two for open addition problems and none for take away problems. After talking about the problem and clarifying the information that was given and what was wanted, 10 of the 15 comparison problems were solved using an appropriate strategy, and seven of those yielded correct answers. The large number of times that children felt it was necessary to have comparison problems clarified makes it clear that they are much more difficult for children to analyse than either take away or open addition problems.

In summary, children modeled comparison problems more often, but also attempted them using a greater number of incorrect operations than they did with open addition problems. There were more correct solutions and more appropriate strategies used for open additon problems than for comparison. And finally, children felt it necessary to have comparision problems clarified far more often than open addition problems. This seems to indicate that, even though children do not use modeling strategies for the open addition problems as often as for comparison problems, open addition problems are dealt with more easily and more successfully. Perhaps the fact the comparison problems seemed more difficult to analyse accounted for the larger number of modeling strategies used. Following the action in the problem one step at a time may be a way to cope with a problem that seems difficult to analyse.

Conclusions and Discussion for Research Question #2

> For the subtraction problems, are there differences in the way children approach word problems involving the solution of a basic fact, and those involving the subtraction of 2-digit numbers?

According to the data collected in this study, there are some differences in the way children deal with problems involving the solution of a basic fact, and those involving the subtraction of 2-digit numbers. The number of modeling strategies used for the basic fact problems of all types and the 2-digit take away and comparison problems was similar, but the 2-digit open addition problems were solved using a modeling strategy about half as many times as any other problem type. Eight of the eleven children who did not use a counting up (modeling) strategy for the 2-digit open addition problem used an appropriate but non-modeling strategy for this problem. Children made more counting errors and produced fewer correct answers for the 2-digit problems, but their method of solution was just as appropriate for one type as for the other. It appears, then, that children use appropriate strategies for the 2-digit open addition problems as often as they do for the basic fact open addition problems , but that almost half the time children choose not to model the 2-digit problems. Perhaps, when the children were working with the basic fact open addition and comparison problems they found the numbers fairly easy to count up in their head (the smaller addend was always the one wanted), and so had a tendency to use a counting up strategy or make use of an additon fact that they already knew. Then, when they encountered larger numbers which made the counting up strategy no more efficient than separating, children chose to use the more familiar separating strategy. This is supported by the fact that, of the appropriate strategies used for the 2-digit open addition and comparison problems, 50% and 40% respectively, were separating strategies. For the basic fact open addition and comparison problems, only 6% and 21% of the appropriate strategies used were separating strategies. The counting up strategy would possibly be easier and more efficient than separating, when basic fact problems are involved. Instead of counting three sets (first the whole set, then counting a part to be removed, and finally counting what is left) the child really only needs to count two sets for a counting up strategy. He first counts the known part of the whole set, then continues on up to the whole set, keeping track of the remaining counts in some way. He then counts the amount that was kept track of. This strategy becomes more cumbersome as the difference between the numbers gets larger. The keeping-track procedure gets more difficult, thus, if a child is to use this method he will probably have to use counters and make two distinct sets. That is, he will count out the given part of the whole (making a set), and continue counting on to the total given (making a second set). The child would then have to count the second set to get the solution. This is no less complicated than separating. It would be reasonable for children who had a clear understanding of the problem's structure to use the counting up method for the small numbers, but to return to the more familiar separating strategy for the 2-digit numbers. Out of 13 children who used a counting up strategy for the basic fact open addition problem, six made the shift to a separating strategy for the 2-digit problem. Of the 10 children who used a counting up strategy for the basic fact comparison problem, only two made the shift to a separating strategy for the 2-digit problem. Seven children counted up for both comparison problems, while only four children counted up for both open addition problems. It is not known whether the low number of modeling strategies used for the 2-digit open addition problems is a negative indicator or not. However, there were many other differences between the open addition and comparison problems that would indicate that the open addition problems were more easily analysed and understood. First, fewer inappropriate strategies were used for open addition problems than for comparison problems. In particular, there was less use of an incorrect operation with the open addition problems. Secondly, children gave more correct responses to the open addition problems. Thirdly, it was clear that children needed much clarification of the comparison problems before they could even start on them. This was not the case for open addition problems. In view of these other differences, it is tempting to regard this shift from a counting up strategy for the basic fact to a separating strategy for 2-digit open addition problems as an indicator that the children understand the problem well enough to apply whatever strategy is easiest or more efficient.

In addition to some difference in the number of modeling strategies used for 2-digit and basic fact problems, there were also some differences in the types of errors made. Although all types of errors were made for both basic fact and 2-digit problems, there was a difference in the frequency of these errors for each type of problem. Not surprisingly there were 3.5 times as many counting errors for 2-digit problems as for basic fact problems. When they had larger numbers to deal with, and more counting to do, there was more opportunity for children to make counting errors. The surprising difference was in the number of incorrect operations used. Children were much more likely to use an incorrect operation on the basic fact problems than on the 2-digit problems. There were six occasions when an incorrect operation was used for the basic fact problems and only two occasions for the 2-digit problems. Other inappropriate strategies, such as guessing, were used twice as often for 2-digit problems as for basic fact problems. Here, it would appear that the larger numbers influenced some children to feel that the problem was out of their depth, and so they resorted to strategies such as guessing or choosing one of the numbers given in the problem. However, the larger number of incorrect operations used for basic fact problems is puzzling. The six occasions when incorrect operations were used for basic fact problems represent four children, and the two occasions when incorrect operations were used for 2-digit problem represent two children. This means that there are more children using more incorrect operations for basic fact problems than for 2-digit problems. Perhaps, because of the perceived difficulty of the 2-digit problems (on account of the larger numbers), children were less sure of themselves and so were more careful when it came to analysing the problem. With basic fact problems, the children may have perceived them as easier, and because of this made a more superficial analysis of the problem. This superficial analysis could lead a child to carelessly choose the wrong operation.

Conclusions and Discussion for Research Question #3

> Do children who do well on the addition and subtraction basic fact tests approach word problems differently than children who perform poorly on the basic fact tests?

The data indicates that there are some differences in the problem solving behaviour of children who perform well on basic fact tests and those who do not. First of all, for both subtraction and division word problems, more children in the low group (the children who scored poorly on the basic fact tests) modeled all of the problems than children in the high group. The reason for this is not clear. It may be that children who have been successful at memorizing the basic facts and have been rewarded for it, are now computationally oriented. Perhaps they are primed to look for a possible computation and perform it, thus reducing the amount of times when they will use a strategy that models the action in the problem. The other possibility is that children who know their basic facts better are also quicker to see how the numbers in the problem relate to each other. From there it is a short step to recognizing which calculation would be applicable, and then apply it.

Secondly, for both types of problems, the children in the low group used fewer incorrect operations than the children in the high group. Of the ten children in the low group, two used one incorrect operation each for the subtraction problems, and one used one incorrect operation for division problems. Of the seven children in the high group, two used one incorrect operation each and one used three incorrect operations for the subtraction problems. The same distribution applied for the division problems.

Another difference regarding errors was the fact that for the subtraction problems, children in the low group made almost twice as many counting errors per child as children in the high group did. For the division problems the number of counting errors was similar for both groups. Since the subtraction problem set was made up of three basic fact and three 2-digit problems, it would seem likely that the inclusion of problems with larger numbers affected the number of counting errors in the low group. In fact, in both the high group and the low group there were far more counting errors for the 2-digit problems than for the basic fact problems. In the low group there were 14 and five counting errors for the 2-digit and basic fact problems respectively, while in the high group the numbers were six and one respectively. It seems that children who know the number facts better simply make fewer errors in counting. As for the greater number of incorrect operations used by the high group, it is possible that the children who have been successful at memorizing the basic facts have a greater tendency to become computationally oriented than the others do. It is possible that their experience and success with computation has fostered the outlook that finding a calculation to perform and performing it is the road to success in mathematics. Perhaps there is a greater tendency on the part of some of these children to simply look for a calculation to perform, without giving much thought to the structure of the problem.

IMPLICATIONS FOR INSTRUCTION

This study has been exploratory in nature, and the reasons for the differences in children's behaviour with various types of word problems were not always clear. In spite of this, I think the study has some suggestions for instruction in the primary grades.

First of all, it seems that we should de-emphasize drill of the basic facts in the primary grades. Assessments and studies done with older children have indicated that one weakness they have with problem solving is their failure to analyse the problem. (Carpenter et al., 1980; Montgomery et al., 1983; Bidwell, 1983) Also studies done with young (pre-school or kindergarten) children indicate that these children are able to deal successfully with word problems involving operations that they have not been taught. (Siegler, 1984; Klahr, 1981; Carpenter et al., 1983) In solving these problems young children displayed the ability to analyse the problem, and to use a strategy that paralleled its mathematical structure. In this study, children displayed the ability to successfully model division problems before they had received any formal instruction in that operation. In fact, they modeled the division problems more often than the subtraction problems. Also, children who had not been as successful at memorizing basic subtraction and addition facts had more of a tendency to use a strategy that modeled the structure of the problem than the children who had been more successful at learning the basic facts. This could be an indication that an emphasis placed too early on computation, particularly drill activities, encourages children to take an approach to solving problems that is more rote and algorithmic than analytical.

Along with de-emphasizing computational drill in the primary grades, it may also be wise to delay the formal instruction of the operations until the children have had much exposure to word problem situations involving these concepts. Some authors (Bell, 1985; Carpenter & Moser, 1985; Campbell, 1984) are of the opinion that we are instructing our children in a back-to-front manner. They feel we should be using familiarity with word problems to introduce an operation instead of the other way around. This study indicates that a) when they are dealing with word problems involving an operation in which they have not been formally instructed, children tend to use more modeling strategies and be more analytical than when they are dealing with an operation in which they have been instructed, and b) children who were more successful at learning the addition and subtraction basic facts used fewer modeling strategies and more incorrect operations than children who were less successful at learning them. In view of these indications, the suggestion for delay-ing the formal introduction of computation and previously spending more time doing oral problem solving, either as a whole class or in small groups, seems to be a good suggestion.

The last suggestion for instruction of word problems involves the three types of subtraction problems. It may appear, from the number of modeling strategies used, that the comparison problems were handled quite well. In fact, the data in this study indicates that comparison problems were actually much more difficult for children to analyse than either the take away or the open addition problems. This was made clear by the large number of times children asked for clarification of the comparison problems, as opposed to the two occasions when children asked for clarification of the open addition problems. In most cases the children were persistent in seeking clarification, and did not stop until the problem was clear to them. In light of this, the apparent competent handling ofthe problems, and the number of modeling strategies used is not surprising. Since they are so difficult to analyse, comparison problems should be formally taught last, and only after a) the children are comfortable with open addition and take away problems, and b) the children have been exposed to these problems in many informal situations. If we want our children to feel comfortable and confident that they are able to successfully analyse and solve word problems, then we must give them challenging tasks that are still within their range of capability.

SUGGESTIONS FOR FURTHER RESEARCH

Based upon the findings and conclusions of this study, the following are suggestions for further research.

1. Conducting further research on children's responses to word problems involving operations which have been taught to the children and those which have not been taught, using a larger sample of children from a variety of ability levels, and a variety of environments.

2. Doing a study comparing classes who have had a lengthy period of solving problems before they were formally introduced to computation, with classes who were formally introduced to computation early and then taught to solve problems using those computations.

3. Looking more closely at the problem solving strategies of children who are good at computation and those who are not. Do children who are good at computation consistently use fewer model-

ing strategies than children who are not as good? Is there evidence that children who are good at computation analyse the problems less thoroughly than children who are not as good?

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APPENDIX

INTERVIEW PROBLEMS

1. Tony and David found 12 tickets for video games. The 2 boys shared the tickets equally. How many tickets did each boy get?

2. There are 21 children in our class and we want to make 3 teams for a relay race. If all the teams must be the same size, how many children will be on each team?

3. Mom bought 15 little plastic toys at the store. She gave them to her 3 children to share equally. How many toys did each child get?

4. Anne is helping to make toy cars. Her job is to put the wheels on. She has a box with 12 wheels in it. If each car needs 4 wheels, how many cars can she make?

5. A group of 12 friends wanted to go on a ride at the fair. The attendant said they had to ride 3 in a seat. How many seats did they need?

6. A group of 24 people decided to rent toboggans for the day. Each toboggan could hold 4 people. How many toboggans did they need?

7. Mr. Smith's class was asked to bring buttons to school for an art project. Jason has collected 23 buttons and Elizabeth has collected 36 buttons. How many more has Elizabeth collected?

8. Mary planted some tulips and daffodils in her flower garden. She wanted to know which flowers would grow faster. So far, 12 daffodils and 9 tulips have come up. How many more daffodils than tulips are there?

9. For his birthday party Sean decorated the room with 35 balloons. At the end of the party he gave 12 of them to his friends. How many balloons did he keep?

10. Janet wants to buy a game that costs 27 dollars. So far, she has saved 12 dollars. How much more does she need to save?

11. Miss Jones had 12 stickers on her desk in the morning. By recess she had handed out 5 of them. How many stickers were on her desk then?

12. Tonight it's Dad's job to set the table for dinner. He needs 11 plates but can only find 8. How many more plates does he need?