DIVIDENDS, TAXES AND INVESTOR CLIENTELES

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B. Comm. University of Alberta, 1986

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF

THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE (BUSINESS ADMINISTRATION)

in

THE FACULTY OF GRADUATE STUDIES

FACULTY OF COMMERCE AND BUSINESS ADMINISTRATION

We accept this thesis as conforming

to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

May 1988

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Abstract

This thesis explores two facets of the Miller-Modigliani theorem; dividend irrelevance and value additivity. We explore these concepts in the capital market using a derivative asset recently introduced and a Black-Scholes option pricing model modified for different marginal tax rates. This technology was used to solve for the retention rates for dividend and capital gains implied in these instruments. These implied rates do not support the Miller-Modigliani hypothesis for Canada nor the United States. We find a significant, persistent premia on dividend income consistent with the clientele hypotheses in the literature.

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Acknowledgement

This thesis owes many debts none of which can likely ever be repaid in full. First and foremost I must acknowledge the patience and constant encouragement shown at all levels of development by Professor Gordon Sick. It was he who suggested this topic and whose many discussions during the project aided the essay in inumerable ways. Professors Zechner and Brander also provided helpful guidance and suggestions as the other members of the committee. I must also thank Professor Alan Kraus who provided me with a more lucid derivation of the tax-based option model in Chapter 2. The comments of the participants of the finance workshop held at U.B.C. are also greatly appreciated.

Finally, I must recognize the contributions made outside of school that provided the much-needed encouragement at all phases of my education. To this end, I would like to dedicate this thesis to my parents, Ray and Della Downie, and to my wife, Christina, who have borne the burden of my education at different but equally important stages of its development.

Chapter 1

Introduction

What should the individual investor do about dividends in his portfolio?

We don't know.

What should the corporation do about dividend policy?

We don't know.[1]

Prior to 1961, we thought we knew the answers to Black's two questions; investors preferred stocks that paid dividends because they were less risky. Corporations, therefore, should satisfy this demand by keeping the dividend payment high. However, in 1961, a seminal paper by Merton Miller and Franco Modigliani (M-M) [19] dramatically altered this perception. Their simple proposal that, in the absence of taxes and transaction costs, two firms identical in every respect, save dividend policies, must have the same share price, has forever changed the structure of the dividend-valuation controversy. This proposition implied that individual investors should be indifferent between those stocks that paid dividends and those which do not. Brealey, Myers, Sick and Whaley [4] call the "dividend controversy" one of the great unsolved problems in finance.

Until recently it has been impossible to design a conclusive empirical test to examine the M-M conjecture in an imperfect world. However, in the past few years, first in the United States and then in Canada, a new financial instrument has been introduced that will allow a more detailed examination of the dividend-valuation problem. Specifically

[1] Fischer Black, "The Dividend Puzzle," *The Journal of Portfolio Management,* (Winter 1976): 5–8

these assets combine the attributes of a long-lived option with a preferred share to "split" the dividend and capital gains of an ordinary common share into distinct entities. If we are in an M-M value additive world we should observe these two component part summing to the value of the underlying common.

Value additivity does not seem to hold in this case. We observe the value of the sum of the parts being greater than the value of the underlying common. This would imply that if M-M are correct, there are unexploited arbitrage opportunities available and therefore, investors are irrational. The other explanation is that there are certain "clienteles" of investors who prefer different forms of income for taxation reasons. If we observe implied taxation rates that differ amongst the component parts above, this would be consistent with clientele effects and investor rationality.

This thesis begins with a discussion of the M-M hypothesis and past empirical tests both in the U.S. and in Canada. We will argue that the likely cause of deviations from the M-M theory is differential taxation of capital gain and dividend income. At this point a pricing model based on the Black-Scholes [2] option pricing equation is used to conduct the empirical part of the thesis. The final chapter discusses the results and implications of the findings.

Chapter 2

Theory

2.1 Dividends and Share Value

In the era preceding the M-M proposition the consensus in finance was that high dividend payments were desirable attributes of stocks. The suggestion is that the higher the firm's dividend payout, the better off are its shareholders. Despite its intuitive appeal this notion was fallacious since it confused the firm's investment decision with its financing decision. The most commonly offered argument is often referred to as the "bird-in- the-hand" fallacy. Essentially this asserts that the investor who holds the dividend today faces less risk than one whose firm retains the dividend and reinvests it. Despite a lack of theoretical or empirical support this fallacy enjoys a wide following on the "street."[1]

The *middle of the road* is occupied by Miller and Modigliani. Their primary insight was to hold the investment policy of the firm constant and apply the arithmetic of accounting to its balance sheet. What they found was hardly surprising. From the assumption of an constant investment policy the "bird-in-the-hand" fallacy is easily refuted.

Essentially the M-M argument states that, in equilibrium, rational investors will

[1] Early (and influential) institutional support was provided by Benjamin Graham et. al., see B. Graham, D.L. Dodd and S. Cottle, *Security Analysis: Principles and Techniques 4th Edition,* McGraw-Hill Book Company, New York, 1962. Brennan [6] defanged the only serious academic advocacy of high dividend payout, presented in M.J. Gordon, "Dividends, Earnings and Stock Prices," *Review of Economics and Statistics*,41(May 1959) 99–105.

regard shares that pay dividends and those which do not (and thus accrue capital gains as the sole return to shareholders) as perfect substitutes for one another.

Another group of academics regard dividends as something investors wish to avoid because of the historical preferential treatment afforded capital gains versus dividend income. Brennan [5] first incorporated a weighted average tax rate into the Capital Asset Pricing Model and a similar approach was taken by Litzenberger and Ramaswamy [15] in the late 1970's. Central to both arguments was the theme that since investors should be indifferent in a world without taxes, if taxes are subsequently introduced *and* penalize one form of income more than another, we should observe lower prices (and, hence higher yields) for those assets for which the tax burden is highest. In the United States the historical taxation of dividends has been about twice that of capital gains. Canadian tax laws try to account for the double taxation of dividends through the gross-up and tax credit system which alleviates the U.S. problem to a great extent (see the following section on tax codes).

Even less is known about why firms pay dividends. If either of the two extreme cases above is true, we should observe all value- maximizing firms paying dividends (in the dividends are "good" world) or not paying dividends at all (in the dividends are "bad" world). Bhattacharya [1] has hypothesized that dividends provide the market with a signal of firm quality in a capital market with asymmetric information across managers and shareholders. An increased dividend payout may be seen as a signal to the market that the firm believes it has the higher cash flows required to maintain such a payout in the future, thus the market would re-evaluate its cash flow forecast for the firm and, hence the stock price would increase. What is less clear with this theory is why firms would go to the trouble and expense of such a signal, why wouldn't they just hold a news conference and announce the good news costlessly? A possible explanation for this is that either the firm cannot make public its information (if it were proprietary, for instance) or that the market "requires" that the signal be costly in order to be credible. A similar analysis has been applied to other valuation neutral firm actions such as stock splits and stock dividends.

2.1.1 The Tax Environment

Canada The introduction of capital gains taxation in 1972 also saw the arrival of the dividend tax credit in Canada. The intent of this credit was to eliminate the double taxation of dividends (first taxed at the corporate level and then the personal level). The taxation procedure begins by "grossing-up" the dividend, subtracting federal tax payable from the grossed-up dividend, subtracting a dividend tax credit from the federal tax payable and then calculating the provincial tax payable. This is illustrated below for three marginal federal tax rates.[2]

	Marginal Federal Tax Rates		
	6%	25%	36%
Dividend	100.00	100.00	100.00
Taxable or grossed-up dividend	133.33	133.33	133.33
Federal tax payable	8.00	33.25	48.00
less $22\frac{2}{3}\%$ DTC	-30.22	-30.22	-30.22
Net federal tax payable	-22.22	3.03	17.78
Net provincial tax (43.5% of federal tax)	-9.67	1.32	7.73
Total personal tax on dividend	-31.89	4.35	25.51
Capital gain	100.00	100.00	100.00
Taxable capital gain	50.00	50.00	50.00
Federal tax on capital gain	3.00	12.50	18.00
Provincial tax (43.5% of federal tax)	1.31	5.44	7.83
Total personal tax on capital gain	4.31	17.94	25.83

Table 2.1: Tax Implications for Dividends and Capital Gains - Canada

[2] from BMSW [4] p. 361

Table 2.1 shows that, given a marginal provincial tax rate of 43.5%, a Canadian investor with a marginal federal tax rate of approximately 36% should be indifferent between capital gains income and dividends. This was the system in place for the study period. The recently announced tax reform will reduce the gross-up to 25% and the dividend tax credit to $16\frac{2}{3}$% and gradually increase (to one) the fraction of capital gains that are taxable upon realization.

United States The pre-tax reform U.S. system taxed dividends as ordinary income but only half of the capital gain as (ordinary) income upon realization. Under the 1986 Reagan tax reform the entire capital gain is now taxed as ordinary income.

One important advantage capital gains enjoys in both countries is the so-called "tax timing" option. Tax timing allows the investor who holds an asset accruing capital gains to defer indefinitely the realization of the gain. This has two important consequences. The first is that the time value of money will reduce the present value of tax liabilities and, secondly, the holder may be able to realize the gain in the future when in a lower tax bracket. This option may make the investor who is supposedly indifferent (as is the 36% investor in Table 2.1) between the two forms of income, prefer capital gains in a multiperiod setting.

2.1.2 Empirical Evidence - Canada

The first rigorous examination of the dividend controversy in Canada was provided by Morgan [21] in 1980. In a test utilizing monthly data for Canadian stocks for the period between February 1968 and December 1977, Morgan found an overall positive dividend premium consistent with the models of Brennan and Litzenberger and Ramaswamy. However, when he divided the sample into two subsamples; one from 1968 to 1972 and the other, 1973 to 1977 to account for the change in the tax environment vis-a-vis capital gains taxation he was not able to refute the Miller-Modigliani assertion that dividends and capital gains are perfect substitutes for the latter period. Prior to the introduction of capital gains taxation in Canada there was a significant pre-tax premium accruing to investors for the bearing the burden of dividends.

The implications of the above research would logically provide a basis for the existence of tax-induced clienteles, or pockets of demand. These clienteles would be formed by groups of individuals who had a preference for one type of share over another. If this preference were pervasive and strong enough it would drive the prices of these assets to a premium over substitutes.

2.1.3 Empirical Evidence - United States

The dividend controversy has been most actively studied in the context of U.S. capital markets. Beginning with the first published [3] work by Black and Scholes [3] a series of articles have provided mixed evidence on whether dividend yield affects common stock prices and returns.

The two polar cases in this regard can be represented by the studies of Litzenberger and Ramaswamy [15] [16] and Black and Scholes [3] and Miller and Scholes [20]. These two groups essentially differ in two areas: the measurement of dividend yield and the definition of what period of time to measure this yield.

Early tests of the effects of dividend yield on stock prices utilized a modified Brennan [5] Capital Asset Pricing Model to test for a positively priced coefficient on the yield component. This was criticized by the Miller and Scholes and Black and Scholes group as measuring information effects as well as dividend effects (if there were such a thing.) For instance, if there were a change in investor expectations about the cash flows of

[3] BMSW cite the precedence of an unpublished paper by Brennan see [4] p. 373

the firm this would be impounded into prices and would cloud the price reaction of the dividend. Litzenberger and Ramaswamy [16] partly controlled for this and found that the market paid a positive premium to holders of stocks which pay dividends. This approach was again criticized for its definition of dividend yield. A 1982 study by Miller and Scholes [20] defined a "long-run" dividend yield and did not find the premium to be significant.

The above discussion serves to illustrate the state of the controversy as it stands today. Neither side has yet been able to prove its position empirically.

2.2 Tax Clienteles

As a result of the differential tax treatment afforded capital gains and dividend income one might expect individuals adversely affected by the taxation treatment imposed on one class of assets to specialize or "plunge" into those securities taxed in a less oppressive manner. Thus we would see investors whose marginal tax rate on dividends is high relative to that on capital gains holding stocks that have low or non-existent dividend payouts. Conversely, so-called "widows and orphans," those with lower relative marginal tax rates on dividends would hold the stocks of high payout firms. The behavior and strength of these two extreme positions can significantly impact asset pricing relationships.

Dybvig and Ross [10] define a clientele effect as "the notion that the behavior of different groups or *clienteles* are qualitatively different." They describe three different clientele situations:

1. *No Clientele Effects* - characterized by all agents seeing the same (shadow) prices for pre-tax cash flow and the same marginal tax rates in all states. In the example above the two dichotomous groups would be indistinguishable from one another

at the margin in every state of the world.

2. *Clientele Effects in Quantities (but not in prices)* - some agents plunge into tax-favorable assets but at least one agent (or representative individual) is marginal on all assets.

3. *Clientele Effects in Quantities and Prices* - no agent is marginal on all assets and thus no asset is a perfect substitute for another because of its tax consequences.

The above classifications provide a framework in which to study the dividend-valuation question. On one hand we see the Miller/Black and Scholes world where, even if clienteles exist (as in situation (2),) in aggregate there will be no price effect. Brennan and Litzenberger and Ramaswamy see the world as in (3). In this situation there is no marginal investor who is indifferent between those shares which do or do not pay dividends. Empirically, only situations (2) or (3) may be truly distinguishable from one another since we are only observing price information. Past empirical tests have so far been unable to unambiguously show that situation (3) prevails.

2.3 The "Stripped Common"

For the purposes of this essay a *"stripped common"* is a security that is "tailor made" for a group of investors. For instance, those investors whose marginal tax rate on capital gains is lower than that for dividends would prefer an asset that accrued capital gains and paid no dividends. The existence of differential taxation on capital gains and dividends may create "clienteles" of investors who prefer one type of share to another. The "stripped common" attempts to capitalize on these clienteles by creating pure dividend and pure capital gains instruments. In the 1960's this was done in a mutual fund setting by so-called "dual funds" which were subsequently studied in detail by

Ingersoll [11]. These funds tended to sell at a discount from net asset value but to a lesser extent than the closed-end mutual funds traditionally have. They were also quite unpopular due in large part to restrictive redemption characteristics. Since their inception in the late 1960's no new funds subsequently have been formed.

Recently there has been a resurgence of interest in this type of security in both Canada and the United States. Their appearance this time is in a different form than the "dual purpose" mutual fund. Instead of holding a variety of instruments; these funds hold only the common shares of a single firm. The structure is as follows: a holding company is formed which issues two types of shares, a preferred share which accrues no capital gains but pays a dividend [4] and a "capital" share which will pay no dividends but entitles the holder to exchange it and a set amount of money for the underlying common at a future date. The proceeds of the issue are used to purchase the common shares of a single, high quality firm and it is these shares that will generate the dividend income for the preferred shares and will be surrendered to the holder of the capital share at maturity. For example *RY Financial*, a Canadian company, had the following issue characteristics;

- RY Financial (RY) would hold the common equity of The Royal Bank of Canada common stock trading at $32.25 per share on June 9, 1986.
- From each share RY would issue a preferred share at a price of $25.00 which is lower than that of the common so that the yield is comparable to other preferred shares currently trading in the market.
- The capital share[5] is essentially a six year option expiring on June 30, 1992, to

[4] Usually these shares flow through all dividends from the underlying common. Some, however charge a nominal management fee which is reflected as the difference between the dividend on the underlying common and the preferred's dividend.

[5] see Appendix B for an excerpt from the prospectus.

buy the stock at a striking price equal to the face value of the preferred share above. The issue price of these was \$9.25.

- The total price of the repackaged securities is then \$34.25 or \$2.00 more than that of the common.

- The preferred share is retractable at the option of the holder on June 30, 1992 at

  $\min\{\$25.00 + \text{undistributed net earnings, net assets}/\#\text{of shares outstanding}\}$

The payoff to the capital share is shown in Figure 2.1. The terminal value of this share is analogous to a European call option on the underlying share with a striking price equal to the face value of the preferred share. One difference is that the holder receives the maximum of the terminal stock price *less* the face value of the preferred and \$1.00. Essentially, this assures that the option will always be exercised but is effectively ignored in the analysis since it does not play an important role in the pricing relationship.

In a M-M world we would expect the price of the two derived assets to sum to the value of the underlying common. The stylized facts point to a significant and persistent premium of the two over the underlying common.

2.4 Option Pricing

A *call* option is an asset that gives its owner the right to purchase an underlying share for a prespecified price called the *exercise* or *striking price* on or before a prespecified date.

The first satisfactory equilibrium pricing relationship for options was presented by Black and Scholes [2] in 1973. Their derivation relied on continuous trading to provide the rebalancing necessary to form a hedge portfolio that is invariant to small movements in the underlying stock. Since this portfolio will behave exactly as a call option, the

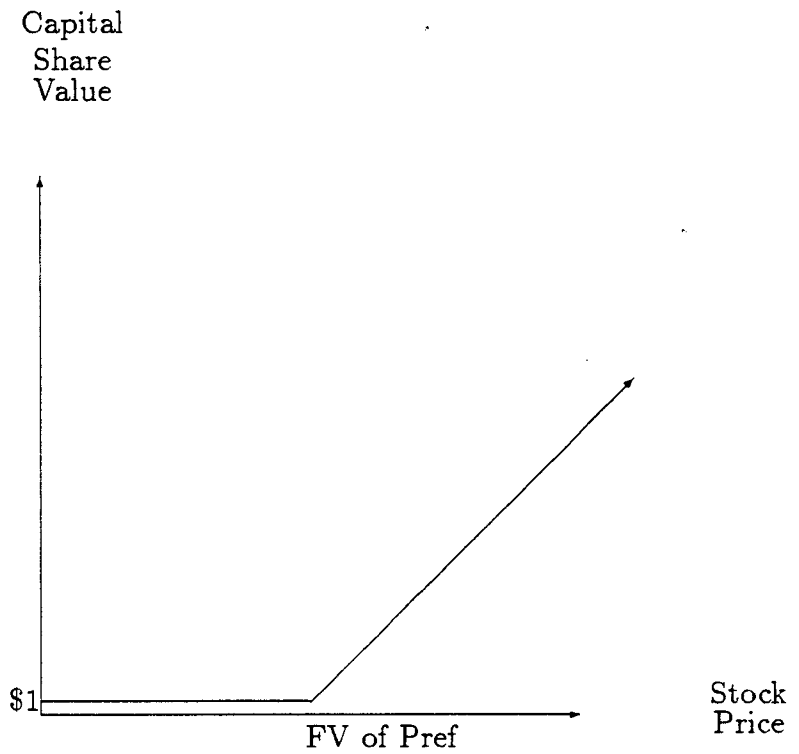

Figure 2.1: Payoff of the Capital Share at Maturity

"price" of the hedge portfolio must be the same as the option in equilibrium. Cox, Ross and Rubinstein [8] developed a simpler approach suggested by Sharpe in which the stock movements described by Black and Scholes could be couched in a binomial framework.Their methodology begins by defining the following:

- $S \equiv$ the current stock price;
- $C \equiv$ the value of the *call* option;
- $B \equiv$ the dollar amount of riskless bonds lent or borrowed;
- $1 + r \equiv$ the risk free return on B;
- $u \equiv$ the *proportion* of the upward price movement;

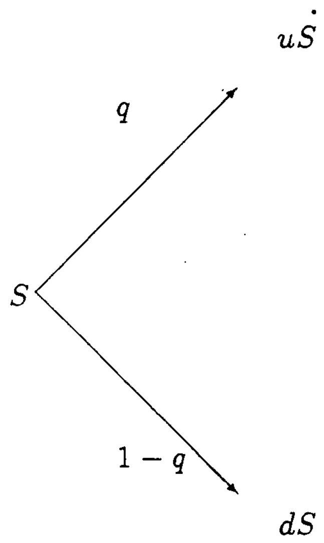

Figure 2.2: Binomial Stock Movement

- $d \equiv$ the *proportion* of the downward price movement;
- $\Delta \equiv$ the number of common shares in the hedge portfolio.

This approach is quite intuitive in that it only relies on simple algebra to construct the hedge portfolio instead of the more difficult stochastic calculus approach. First assume that the common stock will rise to a value of uS with probability q and fall to a level dS with probability $1-q$, as shown in Figure 2.2. This implies a terminal value of the call option of either C_u with probability q or C_d with probability $1-q$ where $C_u = \max[0, uS - K]$ and $C_d = \max[0, dS - K]$. This is shown in Figure 2.3 below.

We then wish to find values of Δ and B (shown in Figure 2.4) which, when combined with the common stock form a *replicating portfolio* that will provide the same payoff as the option.

Equating $C_u = uS\Delta + (1+r)B$ and $C_d = dS\Delta + (1+r)B$ to prevent riskless arbitrage we find the following relationship:

$$\Delta = \frac{C_u - C_d}{(u-d)S},$$

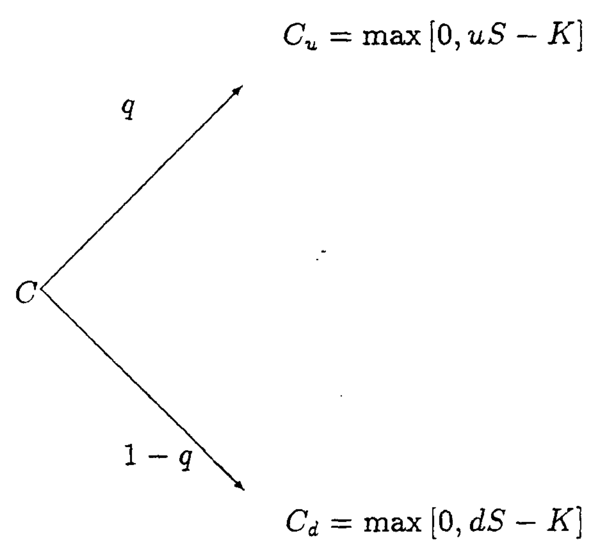

Figure 2.3: Call Value

$$B = \frac{uC_d - dC_u}{(u-d)(1+r)}$$

since, in equilibrium we require $C = S\Delta + B$ we can now solve for C.

$$C = \left[\left(\frac{(1+r)-d}{u-d}\right)C_u + \left(\frac{u-(1+r)}{u-d}\right)C_d\right] / (1+r).$$

Noting that if we define a new variable p as $p \equiv ((1+r)-d)/(u-d)$ then the above reduces to the simpler,

$$C = [pC_u + (1-p)C_d]/(1+r).$$

2.5 Taxes and Option Valuation – The Binomial Approach

This derivation is consistent with the Cox-Ross-Rubinstein formulation.

Define:

- $\delta \equiv$ the dividend yield (paid every period) as a constant fraction of the stock price;
- $\tau_d \equiv$ the marginal tax rate on dividend income;

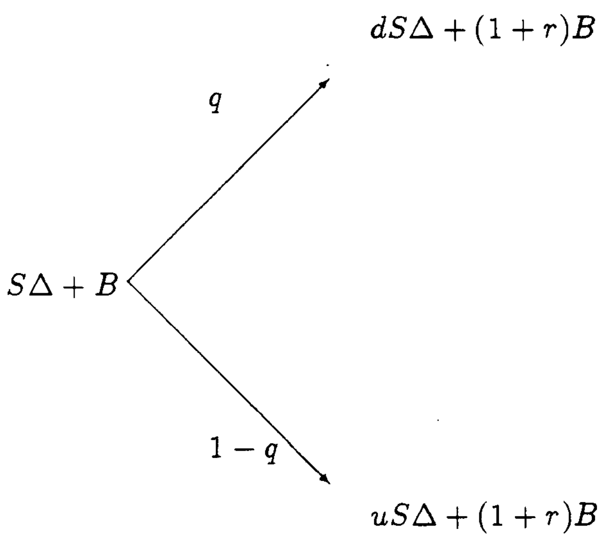

Figure 2.4: Replicating Portfolio

- $\tau_g \equiv$ the marginal tax rate on capital gains income;
- $\tau_b \equiv$ the marginal tax rate on interest income;
- $S_x \equiv$ the ex-dividend stock price; assumed equal to $S - \delta S$ in the absence of taxes.

Assume that all taxes are paid (accrued) continuously and that the full proceeds from short sales can be used by the investor.

The basic structure of the CRR derivation remains the same; we wish to find parameters Δ and B such that a portfolio of stock and riskless pure discount bonds will be invariant to small changes in the common stock and thus will provide the same payoff distribution as a call option written on that stock. Again, assume the stock can only move to a value of uS with probability q or to dS with probability $1-q$. The after-tax, ex-dividend, high state call value is now,

$$C_u - (C_u - C)\tau_g = B(1 + r(1 - \tau_b)) + \Delta uS\delta(1 - \tau_d)$$

$$+\Delta\left(\left(1-\delta\left(\frac{1-\tau_d}{1-\tau_g}\right)\right)uS(1-\tau_g)+S\tau_g\right)$$

With a new boundary condition,

$$C_u = \max\left[uS(1-\tau_g)\left(1-\delta\left(\frac{1-\tau_d}{1-\tau_g}\right)\right)-K,0\right]$$

Simplifying the above yields,

$$\begin{aligned}
C_u(1-\tau_g) &= -(\Delta S+B)\tau_g + B(1+r(1-\tau_b)) \\
&\quad +u\Delta S(1-\tau_g)+S\tau_g \\
&= B[(1-\tau_g)+r(1-\tau_b)]+u\Delta S(1-\tau_g) \\
C_u &= B\left[1+r\left(\frac{1-\tau_b}{1-\tau_g}\right)\right]+u\Delta S
\end{aligned}$$

Applying this to the down state we find,

$$C_u = B\left[1+r\left(\frac{1-\tau_b}{1-\tau_g}\right)\right]+u\Delta S$$

$$C_d = B\left[1+r\left(\frac{1-\tau_b}{1-\tau_g}\right)\right]+d\Delta S$$

$$\Rightarrow C_u - C_d = (u-d)\Delta S$$

$$\Delta = \frac{C_u-C_d}{(u-d)S}$$

$$B = \left(\frac{1}{1+r\left(\frac{1-\tau_b}{1-\tau_g}\right)}\right)\left[\frac{uC_d-dC_u}{u-d}\right]$$

The after-tax value of a call option is then

$$C = \frac{\left[\left(\frac{\left(1+r\left(\frac{1-\tau_b}{1-\tau_g}\right)\right)-d}{u-d}\right)C_u + \left(\frac{u-\left(1+r\left(\frac{1-\tau_b}{1-\tau_g}\right)\right)}{u-d}\right)C_d\right]}{\left(1+r\left(\frac{1-\tau_b}{1-\tau_g}\right)\right)}.$$

2.5.1 Option Pricing in Continuous Time

Cox, Ross and Rubinstein show that their binomial model can be made consistent with that of Black and Scholes when the number of price movements approaches infinity. Thus, in the B-S formulation, trades were assumed to be occurring every instant. In order to derive a continuous time model given the binomial framework presented above it is necessary to redefine some of the above variables so they are "compact" and infinitely divisible. Essentially this means that as we divide the period up into finer and finer intervals, the variables must divide up in a consistent manner (ie. a variable does not go to zero or "explode"[6].)

In particular, define

- $t \equiv$ time to expiration of the option;
- $n \equiv$ number of periods of length h prior to expiration;
- $h \equiv t/n$
- $\hat{r}^h = \left[\left(1 + r\left(\frac{1-\tau_b}{1-\tau_g}\right)\right)\right]^h$;
- $u = e^{\sigma\sqrt{h}}$;
- $d = 1/u = e^{-\sigma\sqrt{h}}$;

The binomial equation for the value of a call option then becomes

$$\left[\frac{\hat{r}^h - e^{-\sigma\sqrt{h}}}{e^{\sigma\sqrt{h}} - e^{-\sigma\sqrt{h}}}\right] C(e^{\sigma\sqrt{h}}S, t-h) + \left[\frac{e^{\sigma\sqrt{h}} - \hat{r}^h}{e^{\sigma\sqrt{h}} - e^{-\sigma\sqrt{h}}}\right] C(e^{-\sigma\sqrt{h}}S, t-h) - \hat{r}^h C(S,t) = 0$$

We can expand $C(e^{\sigma\sqrt{h}}, t-h), C(e^{-\sigma\sqrt{h}}, t-h), e^{\sigma\sqrt{h}}, e^{-\sigma\sqrt{h}}$, and $\hat{r}^h$ as Maclaurin Series and insert these into the binomial pricing equation. (See Appendix A for a detailed exposition of this step.)

[6] see R.C. Merton [18] for a rigorous discussion on the requirements for consistency and compactness as it relates to continuous time formulations

As $h \rightarrow 0$ this simplifies to the following linear differential equation,

$$1/2\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \ln \hat{r} S \frac{\partial C}{\partial S} - \frac{\partial C}{\partial t} - \ln \hat{r} C = 0$$

Along with the boundary condition $C = \max[0, S-K]$ this is solved by the Black-Scholes formula:

$$C = SN(x) - Ke^{-\hat{r}t} N(x - \sigma\sqrt{t})$$

where,

$$x \equiv \frac{\ln(S/K) + (\hat{r} + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

Now, substituting $S_x = Se^{-\delta(\frac{1-\tau_d}{1-\tau_g})t}$ for S (since we assume dividends are paid continuously at the rate δ and the option is European we can strip off the dividends before-hand) and $\hat{r} = \left(1 + r\left(\frac{1-\tau_b}{1-\tau_g}\right)\right)$, we arrive at a tax- adjusted Black and Scholes formula [7]

$$C = Se^{-\delta(\frac{1-\tau_d}{1-\tau_g})t} N(x) - Ke^{-r\left(\frac{1-\tau_b}{1-\tau_g}\right)t} N(x - \sigma\sqrt{t})$$

where

$$x = \frac{\ln\left(Se^{-\delta\left(\frac{1-\tau_d}{1-\tau_g}\right)t}/K\right) + (r\left(\frac{1-\tau_b}{1-\tau_g}\right) + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

[7]The term $N(x-\sigma\sqrt{t})$ can be interpreted as the probability of paying $K if, and only if, the option is in the money. Since the payoff to the Canadian capital shares at maturity are $\max[1, S-K]$ we should amend the model by adding

$$\$1e^{-r\left(\frac{1-\tau_b}{1-\tau_g}\right)}(1 - N(x - \sigma\sqrt{t}))$$

which is the probability of receiving $1 t periods from now. Note that because of the high degree of "moneyness" of the option this will be extremely small (ie. if $N(x - \sigma\sqrt{t}) \cong .9, t \cong 6$ years and $r\left(\frac{1-\tau_b}{1-\tau_g}\right) \cong .08$

$$\Longrightarrow \$1e^{-r\left(\frac{1-\tau_b}{1-\tau_g}\right)}(1 - N(x - \sigma\sqrt{t})) = \$0.06$$

2.6 Methodology

2.6.1 Data Set

The data set is comprised of observations on one U.S. and four Canadian firms. The U.S. data consists of daily prices for the underlying common shares, *Exxon,* the capital share (*Score*) and preferred (*Prime*) shares for the period January 1, 1986 to December 31, 1986. Daily returns and dividends were obtained from the *Centre for Research in Security Prices (CRSP)* daily returns file. These were used to reconstruct the daily price file which is unavailable at U.B.C.. The Canadian price observations were hand-collected using the *Financial Post* weekly closing prices for the period from issue up to and including September 30, 1987. These firms are listed in Table 2.2.

2.6.2 Inputs for the Black-Scholes Equation

The Black-Scholes equation normally requires only five inputs to calculate option prices; the current stock price, S, the exercise or striking price, K, the riskless rate of interest, r, the time to maturity t and the instantaneous volatility of the underlying common stock, σ. In addition, since all of the issues have stocks that pay dividends it is necessary to include the dividend yield, δ, as the sixth variable. Of these variables only the volatility need be estimated (although some assumptions about the payment and stability of the dividend yield are required.) The volatility estimate was calculated over a lagged window as follows:

$$\sigma_{i,t}^2 = \frac{\sum_{t=-N}^{0}(r_{i,t} - \bar{r}_i)^2}{N}$$

where N is the number of days in the window and $\bar{r}_i$ is the mean return over that period. The dividend yield is estimated by

$$\delta_{i,t} = \frac{\sum_{t=-N}^{0} DIV_{i,t}}{S_{i,t}}$$

As an estimate of the return on a riskless asset, the yield to maturity of a government bond with similar maturity characteristics and consistently traded throughout the sample period was used. For the Canadian data a *Canada 10 3/4* bond maturing in June of 1992 was used while a U.S. *9 7/8 Treasury Note* maturing in August of 1990 was used for pricing the Americus Trust for Exxon issue.

Sample Companies		
Name	Date of First Issue	Underlying Shares
"B" Corp	May 1986	Bell Canada Enterprises
RY Financial	June 1986	Royal Bank of Canada
Bancshare Portfolio Corporation	July 1986	The Bank of Montréal Canadian Imperial Bank of Comm. Royal Bank of Canada (price weighted)
BMO II and BMO NT	September 1986	Bank of Montréal
Americus Trust for Exxon Common	October 1985	Exxon Corp.

Table 2.2: Sample Set

Chapter 3

Results

3.1 Market Prices

As mentioned in Chapter 2, in a Miller-Modigliani world one would expect the market values of the capital share and the preferred share to sum to the value of the underlying common. As with the familiar anomaly concerning the discount on "closed-end" mutual funds; the M-M prediction does not hold in the "real world." However, instead of the parts summing to something less than the value of the whole, the derivative assets in this case combine to a total greater than that of the underlying asset. This is displayed in graphical form in Figures 3.5 to 3.9.

3.2 Theoretical Valuation Results

The tax-adjusted Black-Scholes model derived in Chapter 2 was used to calculate theoretical values for the capital shares. The following two subsections detail the influence of differing marginal tax rates on the postulated pricing relationship.

3.2.1 Capital Share Valuation with Equal Marginal Tax Rates

If our hypothesis about the influence of taxes on the valuation of capital shares by the market is correct we should observe M-M holding (at least within a band of transaction costs) when taxes are excluded by the valuation model. That is, if the world were characterized by marginal tax rates that were equivalent for all types of income ($\tau_g =$

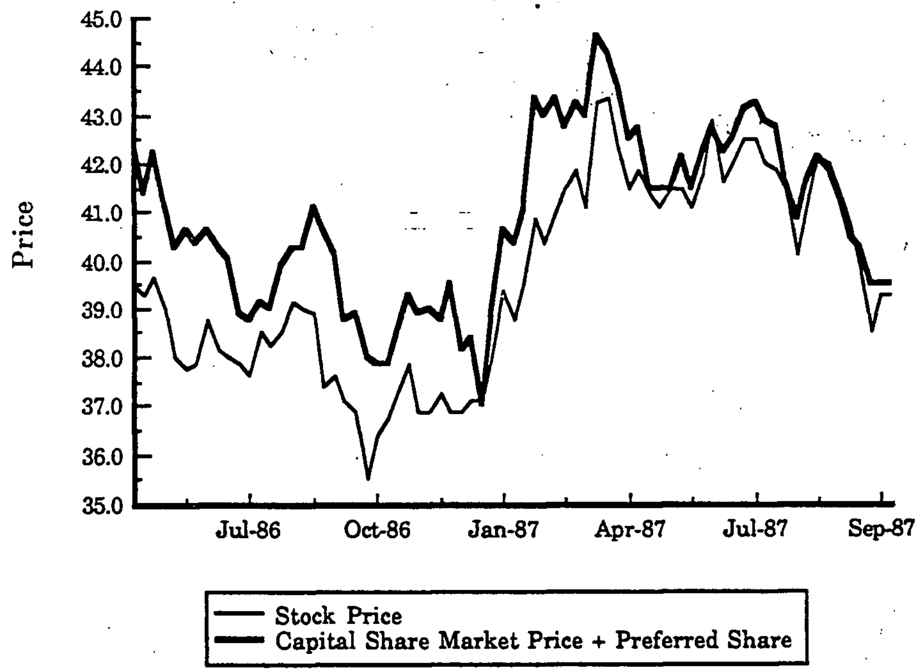

Figure 3.5: "B" Corp - Market Premium over M-M

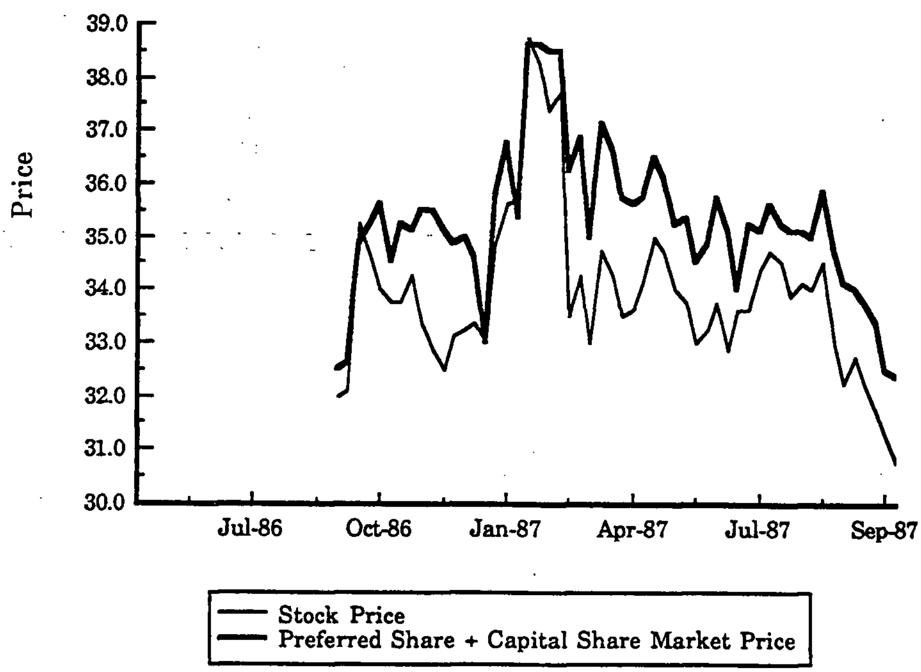

Figure 3.6: BMO Financial - Market Premium over M-M

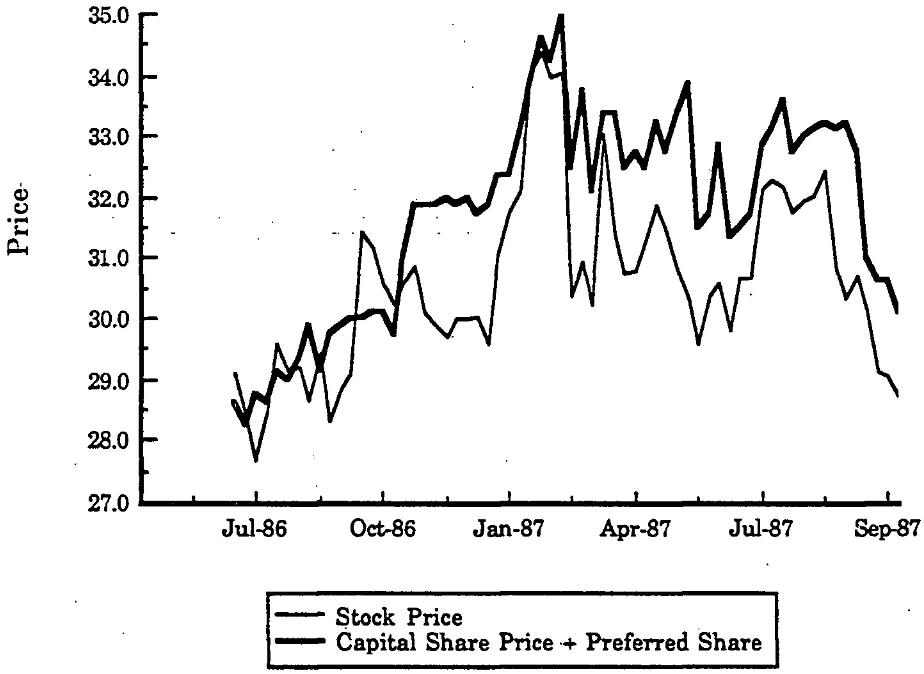

Figure 3.7: Bancshare Portfolio Corp. - Market Premium over M- M

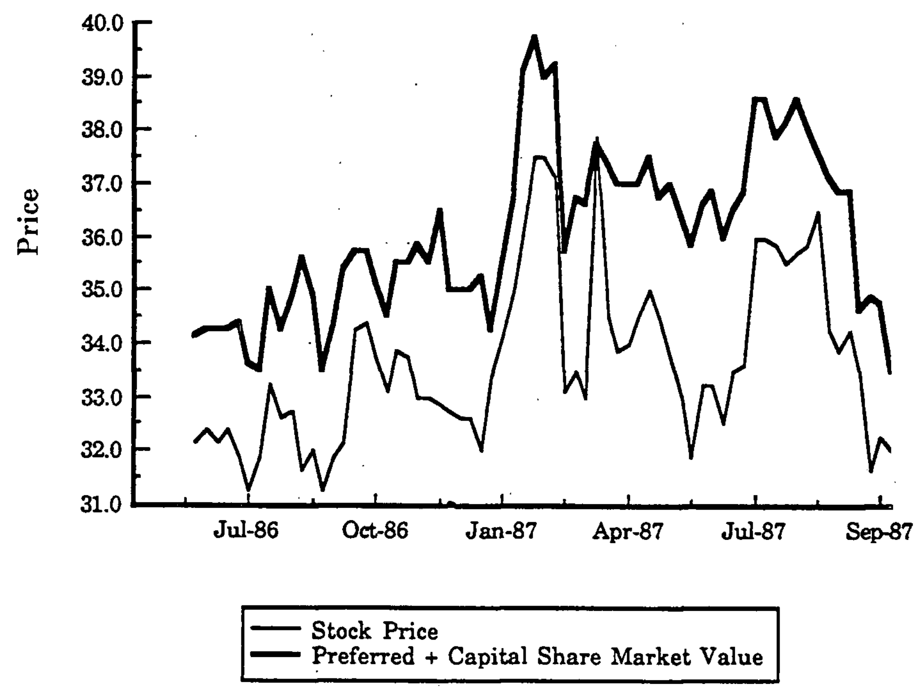

Figure 3.8: RY Financial - Market Premium over M-M

$\tau_d = \tau_b$) for at least one representative investor we should observe the sum of the *non tax-adjusted* theoretical capital share value and preferred share market price equaling the underlying stock price. Time series plots of these results are shown in Figures 3.10 to 3.14. A plot of the Capital Share values under no taxes against market prices is shown for each firm in Figures 3.15 to 3.19.

In general these figures lend support to our conjecture. When the capital share is valued as if all marginal tax rates are equal, the theoretical value (capital share + preferred share) seems to provide a closer fit to the actual stock price than does the sum of the market prices of he two assets. In most cases there appear to be systematic differences that persist throughout the pricing relationship (ie. the premium is persistent over the sample period.) This may be a result of a misspecification within the model such as the calculation of the dividend yield, δ, transaction costs, the omission of taxes or the variance of the common stock, σ^2.[1] Also, it would appear from these figures that nearly all of the premia is impounded in the capital share price. This suggests that the investor who purchases the capital share is willing to pay the premium in order to avoid the dividend (or receive capital gains). This supports the clientele effect described in Chapter 2 and provides an explanation as to why these premia persist over time. We do not, therefore, have to conclude that these are unexploited arbitrage profits and hence imply investors are irrational.

[1] Because of the nature of these two variables (they are not directly observable) sensitivity analysis was performed on the model by changing these parameters. Oddly enough, the capital share prices were *insensitive* to differing variance estimates. This is likely a result of the time to expiration and the "moneyness" of the option swamping the influence of the dispersion of the distribution of the underlying common share. Dividend yield changes of 10% or more altered prices significantly. Also, the proxy for the return on the riskless bonds used (yield to maturity on similarly lived government bonds) may also have an impact on the theoretical valuation although little can be done to find a substitute rate with more desirable properties.

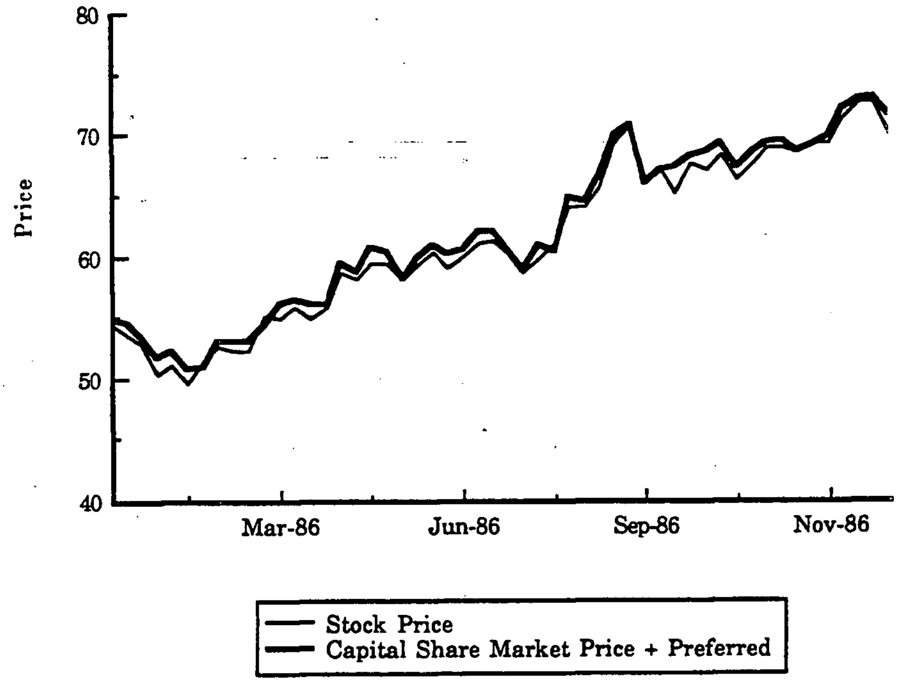

Figure 3.9: Americus Trust for Exxon Common Shares - Market Premium over M-M

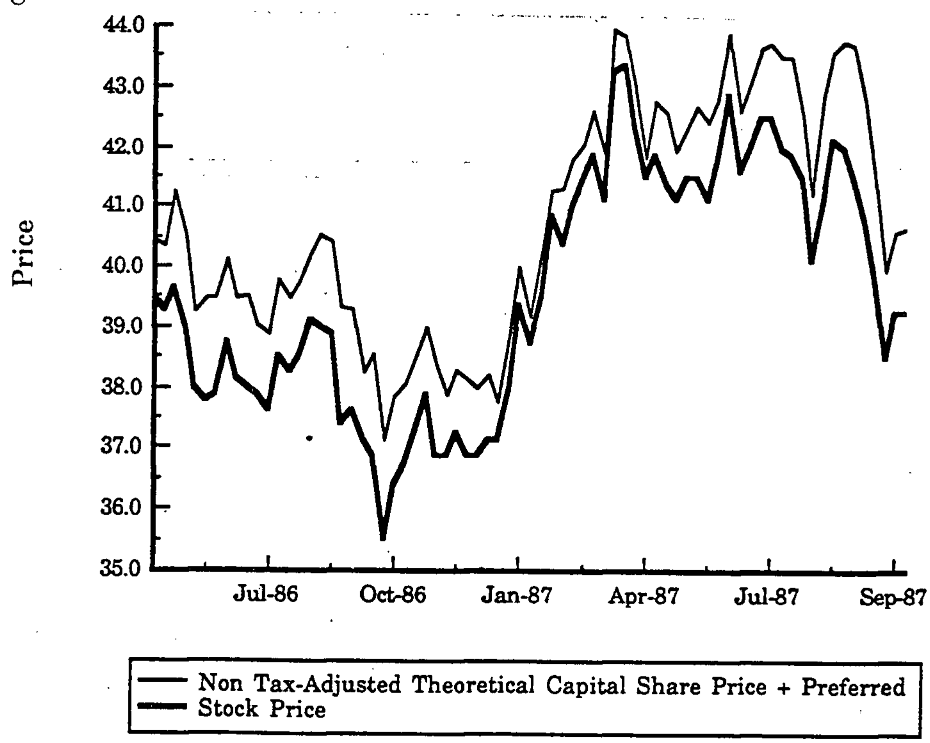

Figure 3.10: "B" Corp - Non Tax-Adjusted Theoretical Premium over M-M

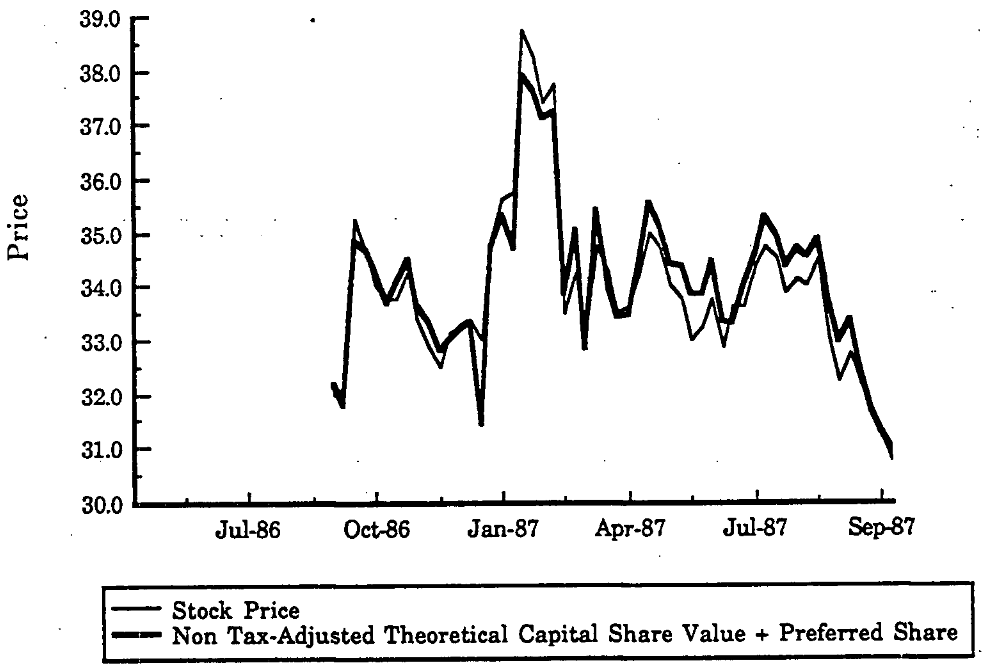

Figure 3.11: BMO Financial - Non Tax-Adjusted Theoretical Premium over M-M

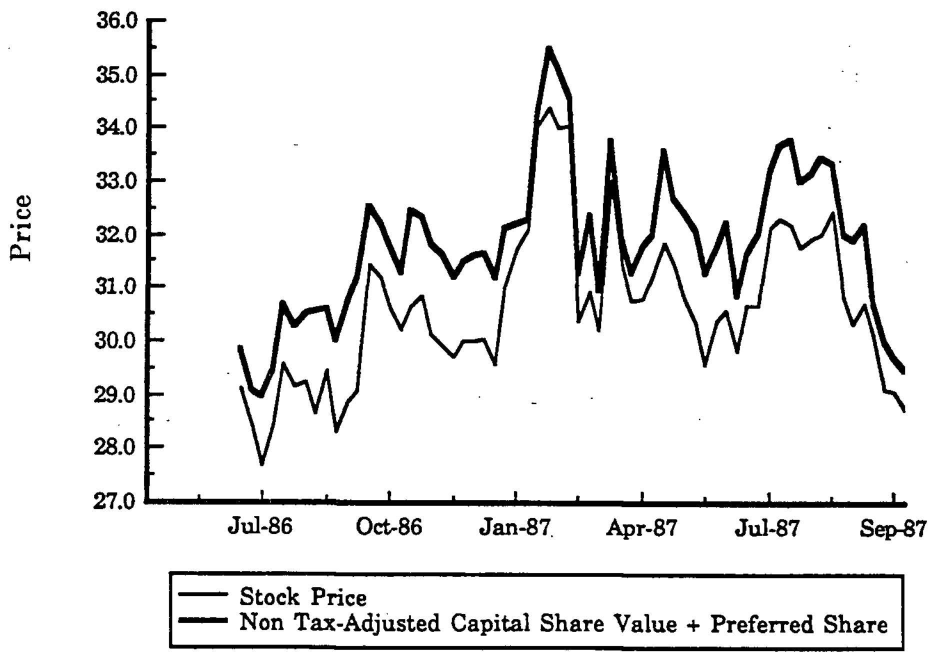

Figure 3.12: Bancshare Portfolio Corp. - Non Tax-Adjusted Theoretical Premium over M-M

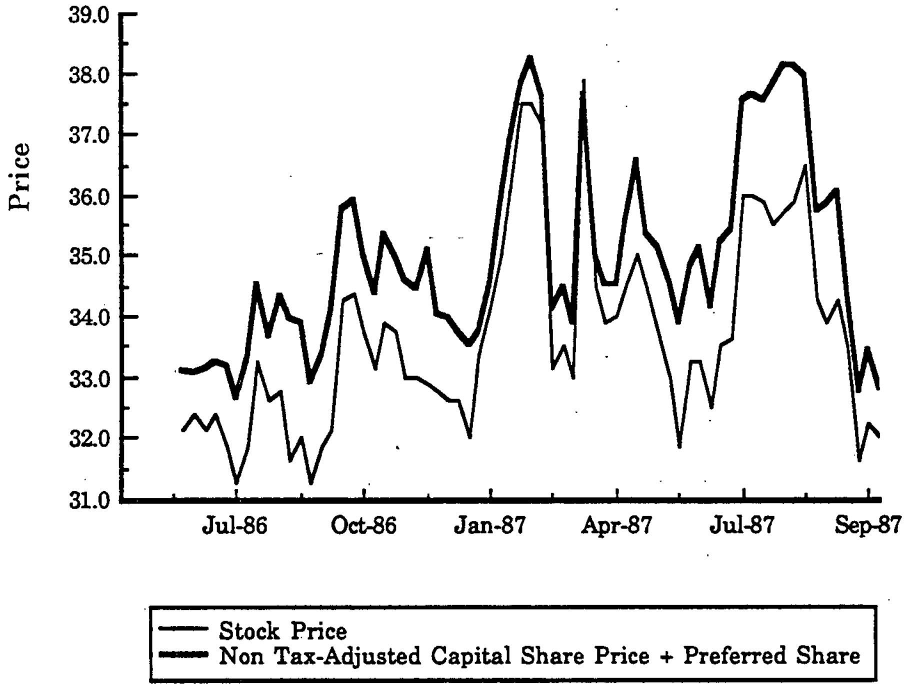

Figure 3.13: RY Financial - Non Tax-Adjusted Theoretical Premium over M-M

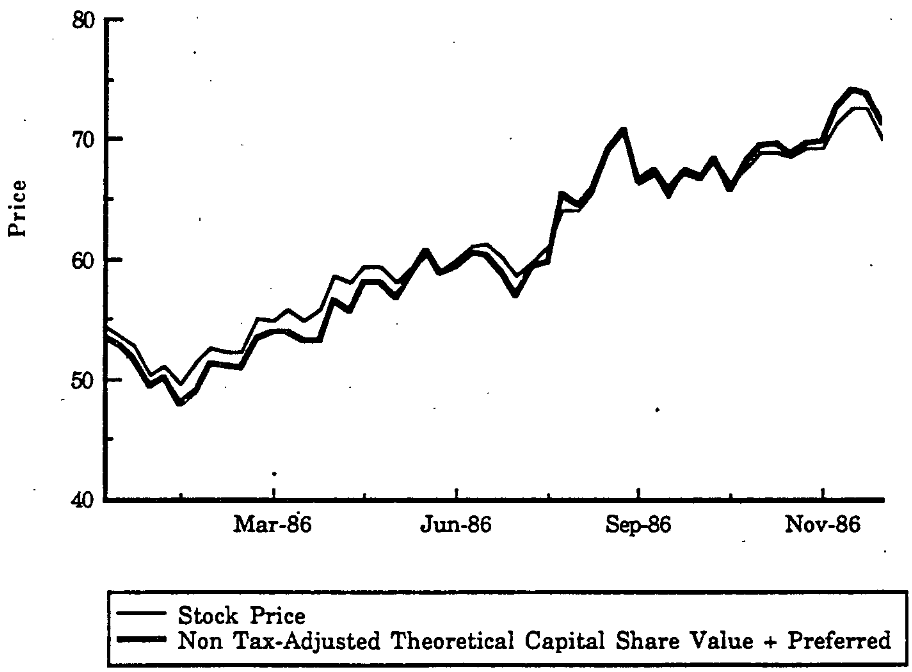

Figure 3.14: Exxon - Non Tax-Adjusted Theoretical Premium over M-M

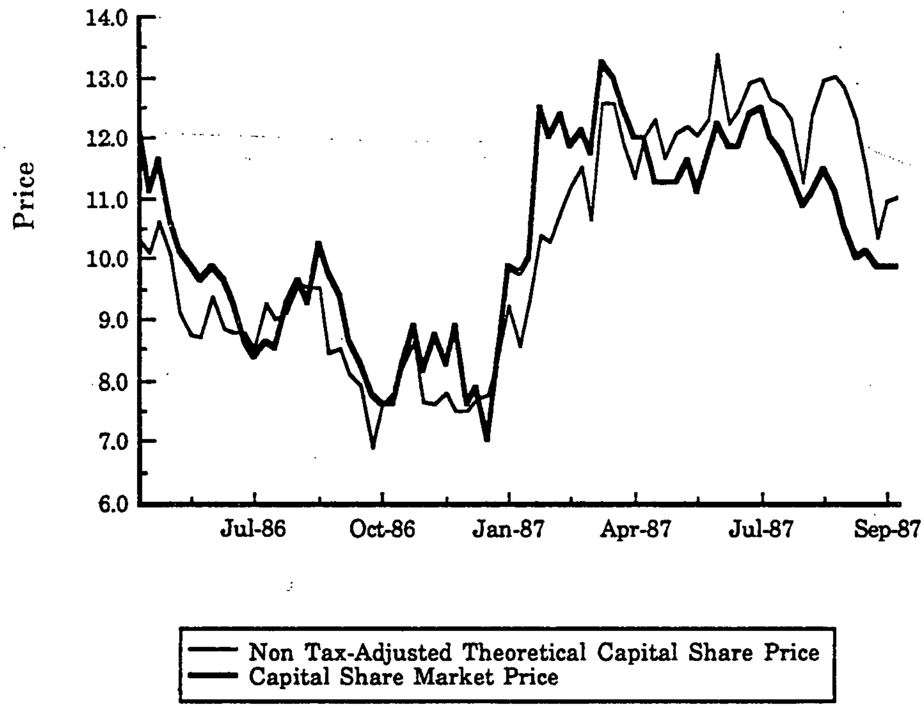

Figure 3.15: "B" Corp - Non Tax-Adjusted Theoretical Capital Share Value versus Market

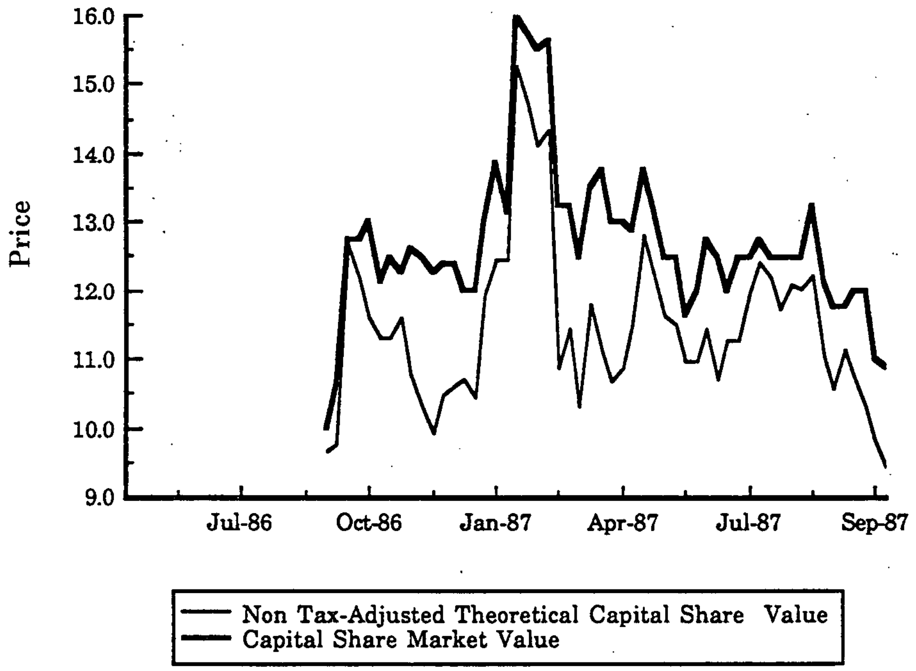

Figure 3.16: BMO Financial - Non Tax-Adjusted Theoretical Capital Share Value versus Market

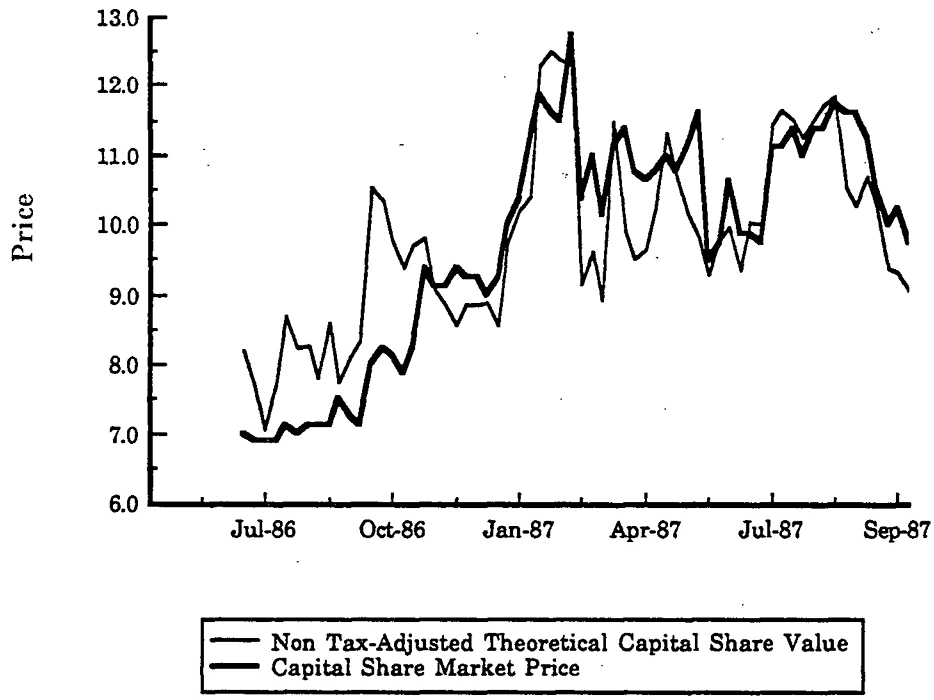

Figure 3.17: Bancshare Portfolio Corp. - Non Tax-Adjusted Theoretical Capital Share Value versus Market

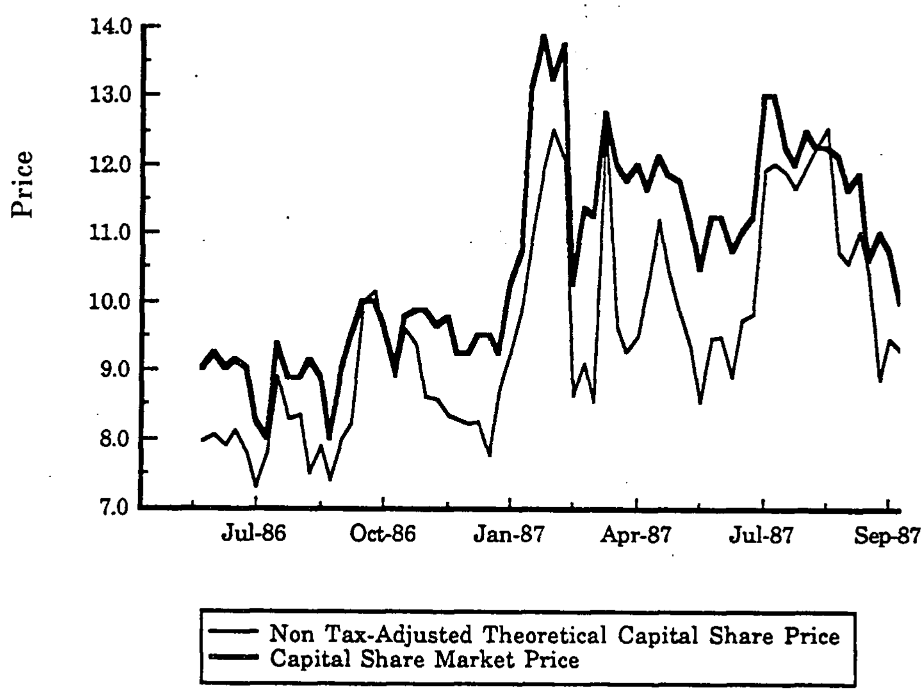

Figure 3.18: RY Financial - Non Tax-Adjusted Theoretical Capital Share Value versus Market

3.2.2 Capital Share Valuation with Differing Marginal Tax Rates

In order to estimate the tax rates that equated the theoretical and market values for the capital shares, a trial and error search technique analogous to Newton-Raphson was used. In order to simplify the procedure, the tax rate on capital gains income, τ_g, was fixed at zero and the other two rates varied. Thus the implied after tax retention rates, $1 - \tau_b$, and $1 - \tau_d$, are a fraction of the capital gains retention rate, $1 - \tau_g$. (See the model equation.) The results of ten separate runs are shown in Tables 3.3, 3.4, 3.5, 3.6 and 3.7.

The overall fit for all models was excellent. With the exception of the Americus Trust for Exxon Common Shares issue, all theoretical values priced within an average[2] of \$1. In most cases the fit for the tax-adjusted valuation appears [3] to be superior. This is shown graphically in Figures 3.20 to 3.24.

Goodness of fit between market capital share prices and those calculated with the tax-adjusted Black-Scholes model are plotted in Figures 3.20 to 3.24 and without the tax adjustment in Figures 3.15 to 3.19. The model appears to closely approximate the market value for the best-fitting tax rates, except for a few cases. These abberations may be a result of a persistent thin trading problem in most of the Canadian issues. Although the Canadian firms traded at least once per week, there were instances where only four round lots traded during the five day period between Monday and Friday. This could also be interpreted as an indication of a strong clientele effect. If the prices

[2]Measured as

$$\sqrt{\frac{\sum_{t=1}^{N}(MarketPrice_{it} - TheoreticalPrice_{it})^2}{N}}$$

where N is the number of observations for a given issue, i.

[3]No parametric or non-parametric statistics were calculated to test for significance in predictive power of one model over the other. Since these are actual prices, no standard tests were appropriate. However, it may be possible to use a non-linear estimation technique to calculate statistics but this is beyond the scope of this paper.

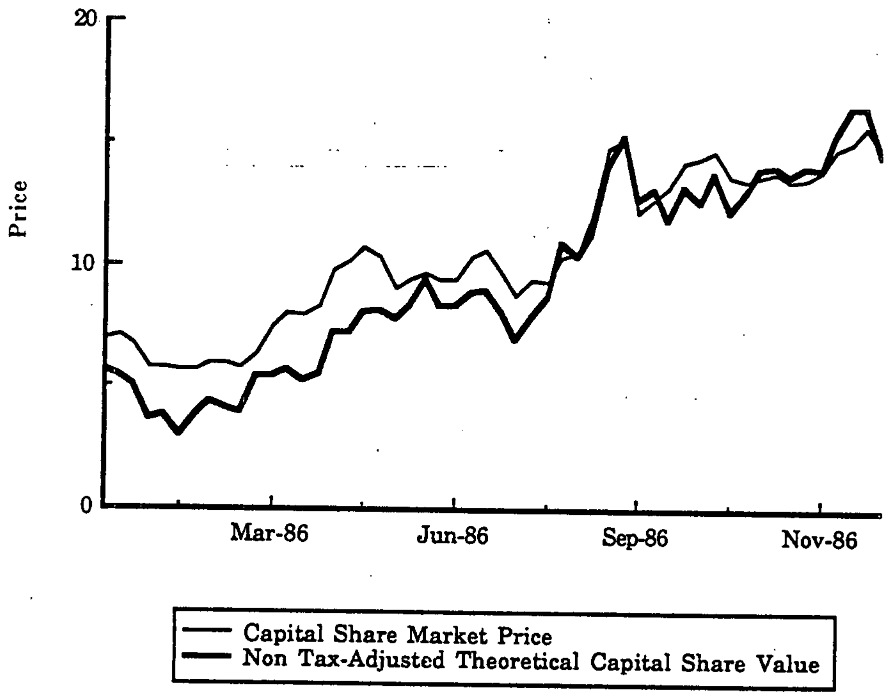

Figure 3.19: Exxon - Non Tax-Adjusted Capital Share Value versus Market

"B" Corp.				
Maturity: May 14, 1992; N = 73; K=\$30				
Iteration Number	$1-\tau_g$	$1-\tau_d$	$1-\tau_b$	Mean- Squared Deviation
1	1.00	1.00	1.00	0.9505
2	**1.00**	**0.80**	**0.80**	**0.8910**
3	1.00	0.80	1.00	1.9510
4	1.00	0.80	0.60	1.7690
5	1.00	1.00	0.80	1.8405
6	1.00	1.20	0.80	3.2535
7	1.00	0.90	0.80	1.1860
8	1.00	0.80	0.90	1.2859
9	1.00	0.80	0.70	1.1317
10	1.00	0.70	0.80	1.3339

Table 3.3: Implied Tax Rates - "B" Corp

BMO Financial				
Maturity: August 30, 1992; N = 53; K=\$22.50				
Iteration Number	$1-\tau_g$	$1-\tau_d$	$1-\tau_b$	Mean- Squared Deviation
1	1.00	1.00	1.00	1.3510
2	1.00	0.80	1.00	0.6780
3	1.00	0.80	0.80	1.0143
4	1.00	1.00	0.80	2.3405
5	1.00	1.20	1.00	2.6522
6	1.00	0.70	1.00	1.2376
7	1.00	0.90	1.00	0.7835
8	**1.00**	**0.80**	**0.90**	**0.6613**
9	1.00	0.80	1.10	1.0316
10	1.00	0.70	0.90	0.7938

Table 3.4: Implied Tax Rates - BMO Financial

Bancshare Portfolio Corporation				
Maturity: August 21, 1992; N = 64; K=\$21.50				
Iteration Number	$1-\tau_g$	$1-\tau_d$	$1-\tau_b$	Mean- Squared Deviation
1	1.00	1.00	1.00	0.9363 [4]
2	**1.00**	**0.80**	**0.80**	**0.9656**
3	1.00	0.80	1.00	1.6472
4	1.00	1.00	0.80	1.3257
5	1.00	1.10	1.00	1.0570
6	1.00	1.00	1.10	1.1100
7	1.00	0.80	0.70	0.9675
8	1.00	0.70	0.80	1.3052
9	1.00	1.00	0.70	1.7411
10	1.00	0.70	1.00	2.2308

Table 3.5: Implied Tax Rates - Bancshare Portfolio Corporation

RY Financial				
Maturity: May 30, 1992; N = 66; K=\$25				
Iteration Number	$1 - \tau_g$	$1 - \tau_d$	$1 - \tau_b$	Mean- Squared Deviation
1	1.00	1.00	1.00	1.3115
2	**1.00**	**0.80**	**0.95**	**0.7245**
3	1.00	0.80	0.80	1.1055
4	1.00	0.80	1.00	0.8121
5	1.00	1.00	0.80	2.3449
6	1.00	1.10	0.95	2.1688
7	1.00	0.80	1.10	1.2081
8	1.00	0.70	0.95	1.1012
9	1.00	0.80	0.70	1.6188
10	1.00	0.90	0.95	0.9885

Table 3.6: Implied Tax Rates - RY Financial

Americus Trust for Exxon Common Shares				
Maturity: September 20, 1990; N = 253; K=\$60				
Iteration Number	$1 - \tau_g$	$1 - \tau_d$	$1 - \tau_b$	Mean- Squared Deviation
1	1.00	1.00	1.00	1.5867
2	1.00	0.80	1.00	1.4001
3	1.00	1.00	0.80	2.6086
4	1.00	0.80	0.80	1.2972
5	1.00	0.80	0.70	1.7062
6	1.00	0.70	0.80	1.1662
7	1.00	0.70	0.70	1.1876
8	1.00	0.75	0.75	1.2386
9	1.00	0.60	0.80	1.6613
10	**1.00**	**0.70**	**0.75**	**1.1251**

Table 3.7: Implied Tax Rates - Americus Trust for Exxon Common Shares

are being moved for tax considerations we would expect the clienteles to be stationary over short periods of time; hence the thin trading.

The resulting implied retention rate for dividend income, $1 - \tau_d$, is less than unity for all firms in the sample. The Canadian firms' retention rates were all equal to 0.80, indicating that the *effective* marginal tax rate on dividend income is higher than that on capital gains. This implies that Canadian investors who buy these shares would theoretically require a twenty-five percent[5] premium on pretax dividend income in order to be indifferent between capital gains and dividends. In the U.S., because of a more punitive tax system vis-a-vis dividends and capital gains income the implied premium jumps to $\frac{1}{0.70} - 1 = 43\%$.

The data also showed some interesting similarities between Canadian capital share price behavior. These four firms each exhibited a large price jump during the middle of April, 1987. One possible explanation for this is speculation (or leaked information being disseminated) on the part of the market on the upcoming changes to the federal tax system. Specifically, the tax reform was to address alterations to the existing level of capital gains exemption which would have a profound impact here. A result that seems harder to explain is that we find implied tax rates on bond income, τ_b, that are lower than the implied marginal tax on dividend income, τ_d, in two of the Canadian issues (RY Financial and BMO Financial) and in the U.S. issue. The difference in the two rates for the Americus Trust for Exxon shares is only five basis points and is unlikely to be significant for a reasonable confidence interval. In the case of the Canadian issues it may be that the yield to maturity used in the study was a biased proxy for the return on a riskless, pure discount bond. This explanation does have some merit since we did not observe any implied marginal tax rates on bond income to

[5] $\frac{1-\tau_d}{1-\tau_g} = 0.80 \Rightarrow 1 - \tau_d = 0.80(1 - \tau_g)$, thus pretax income from capital gains need only be 80% of pretax income from dividend sources, or, alternatively, pretax dividend income must be 1.25 times greater than pretax capital gains.

be higher than the marginal tax rate on dividend income.

3.3 Conclusions and Future Research

This thesis has utilized a completely new technique to test the dividend irrelevancy argument from the perspective of the individual investor. The results support the arguments of Brennan and Litzenberger and Ramaswamy; specifically, that dividends are undesirable from an investor's perspective because of the preferential treatment afforded capital gains income by the tax system. From a Canadian perspective, this contradicts the Morgan results cited in Chapter 2 and may be a result of the small subset studied or simply, changes in the tax structure in the economy. Hence, for neither a Canadian nor a United States investor can we support the hypothesis that dividends and capital gains are perfect substitutes for one another.

Some puzzling questions remain. First, it is still not clear, given these results, why firms persist in their payment of dividends (or logically, why they don't issue two types of common shares like the stripped common) since we have shown that this subset of investors will pay relatively high premiums to avoid them. The answer to this might lay in the signalling hypothesis; dividends may provide the most cost effective means of information transmission in an imperfect capital market. The second, more difficult, question is the apparent absence of the "marginal investor" from this problem. This is the investor who would be marginal on pre-tax dividends and capital gains and could theoretically arbitrage away the premium. The reason for this can be found, I believe, in the market mechanism that drives this investor: arbitrage. In order to arbitrage, this investor has to form the proper replicating portfolio that provides the exact payoff distribution as the capital share at all times and in all states. This would involve (buying) constructing a portfolio like the one in Chapter 2 and short-selling the capital

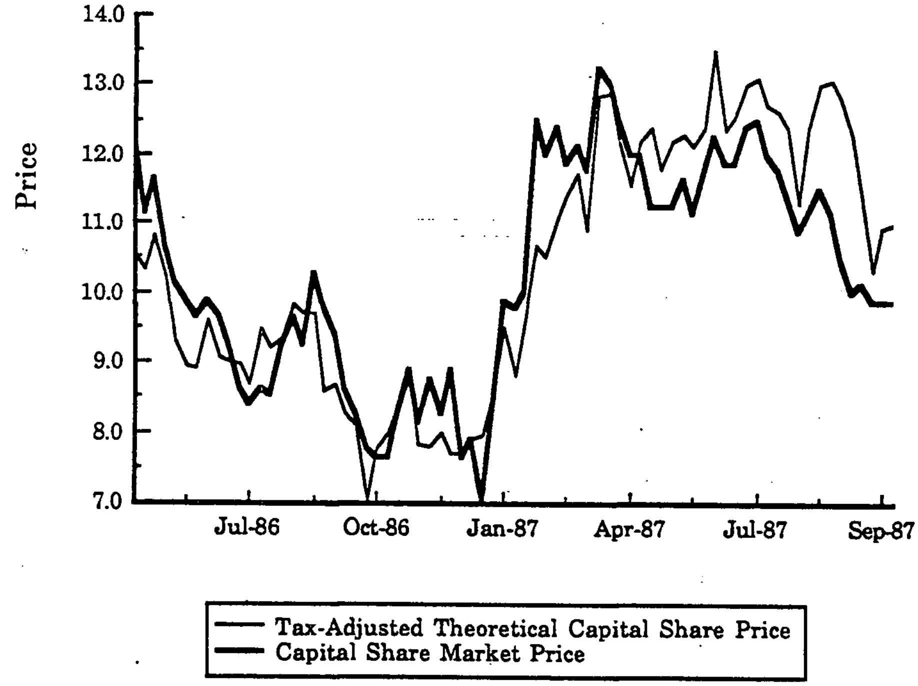

Figure 3.20: "B" Corp - Tax-Adjusted Theoretical Capital Share Value versus Market

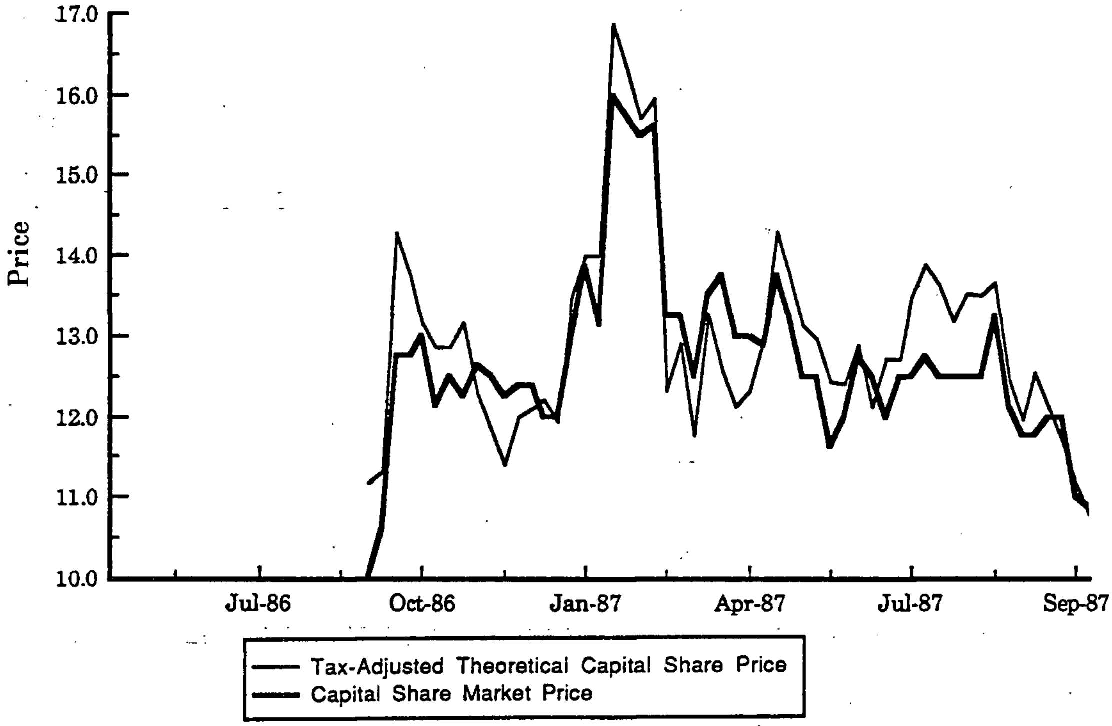

Figure 3.21: BMO Financial - Tax-Adjusted Theoretical Capital Share Value versus Market

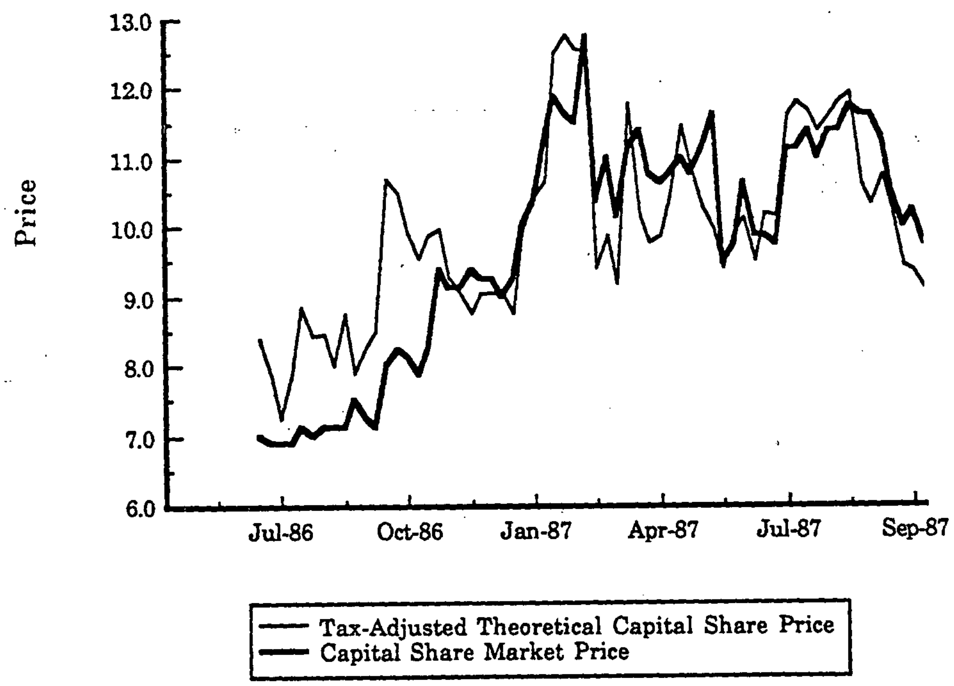

Figure 3.22: Bancshare Portfolio Corp. - Tax-Adjusted Theoretical Capital Share Value versus Market

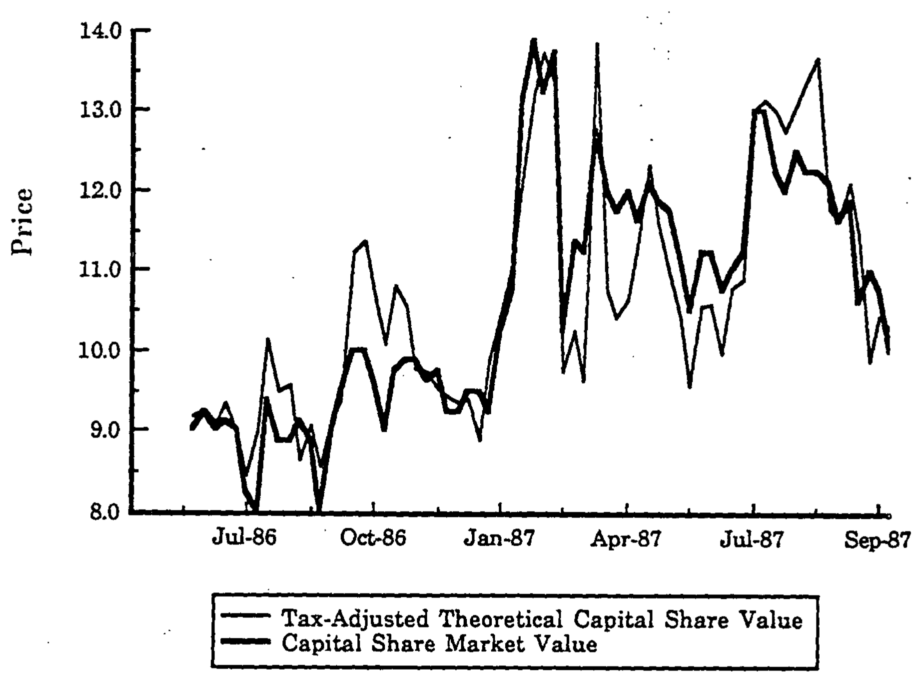

Figure 3.23: RY Financial - Tax-Adjusted Capital Share Value versus Market

share. Under restricted short selling and with the transaction costs necessary to adjust the hedge ratio, this may not be feasible nor cost effective.

In terms of future research, the field is wide open. The initial enthusiasm for these shares in Canada has since quelled and there have been no subsequent issues since the fall of 1986. The U.S. market for these instruments however, has expanded rapidly. There are now roughly twenty of these trusts traded on the American Stock Exchange issued by Americus Trust Shareholder Service. The differences in the number of new issues as well as the persistent thin trading problem for the Canadian firms may point to a limited demand for these in Canada, perhaps because a smaller fraction of the population of investors require such a shelter from the high taxes on dividends.

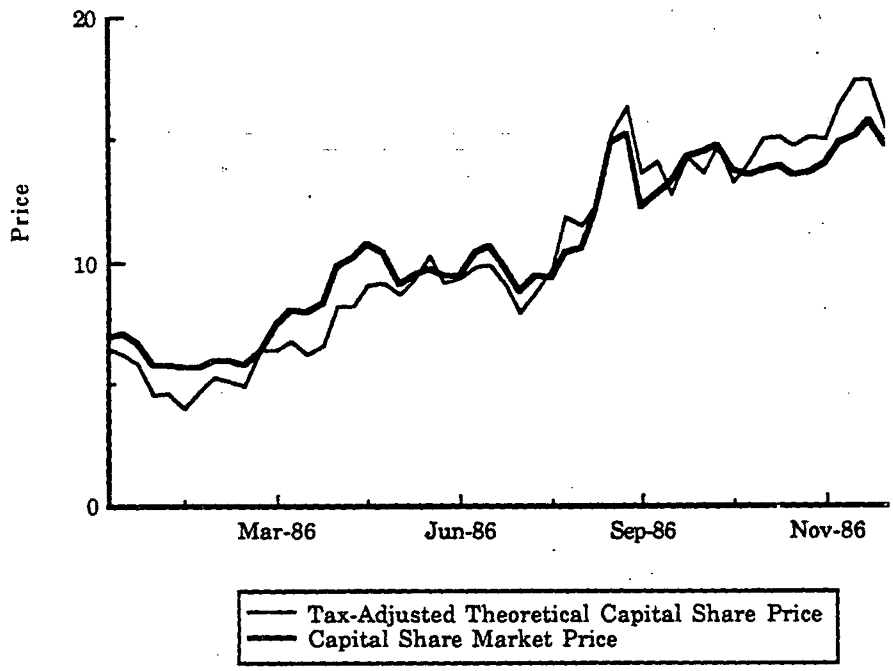

Figure 3.24: Exxon - Tax-Adjusted Capital Share Value versus Market

Appendix A

Proof of After-Tax Binomial Convergence in Continuous Time

Here we wish to prove that as $n \to \infty$ (or $h \to 0$), the binomial option pricing model converges to the Black-Scholes continuous time solution. After setting $u = e^{\sigma\sqrt{h}}$ and $d = e^{-\sigma\sqrt{h}}$, and $\hat{r} = 1 + r\left(\frac{1-\tau_b}{1-\tau_g}\right)$, the binomial equation from Chapter 2 is,

$$\left[\frac{\hat{r}^h - e^{-\sigma\sqrt{h}}}{e^{\sigma\sqrt{h}} - e^{-\sigma\sqrt{h}}}\right] C(e^{\sigma\sqrt{h}}S, t-h) + \left[\frac{e^{\sigma\sqrt{h}} - \hat{r}^h}{e^{\sigma\sqrt{h}} - e^{-\sigma\sqrt{h}}}\right] C(e^{-\sigma\sqrt{h}}S, t-h) - \hat{r}^h C(S,t) = 0, \tag{A.1}$$

where $C(e^{\sigma\sqrt{h}}S, t-h)$ is analogous to C_u and $C(e^{-\sigma\sqrt{h}}S, t-h)$ corresponds to C_d.

Expanding the next-period call option prices as Maclaurin Series, we find

$$C(e^{\sigma\sqrt{h}}S, t-h) = C(S,t) + C_S S(e^{\sigma\sqrt{h}}-1) - hC_t + \frac{1}{2}C_{SS}S^2(e^{\sigma\sqrt{h}}-1)^2 + O(h^2) + O(e^{\sigma\sqrt{h}}-1)^3$$

$$C(e^{-\sigma\sqrt{h}}S, t-h) = C(S,t) + C_S S(e^{-\sigma\sqrt{h}}-1) - hC_t + \frac{1}{2}C_{SS}S^2(e^{-\sigma\sqrt{h}}-1)^2 + O(h^2) + O(e^{-\sigma\sqrt{h}}-1)^3$$

(subscripts denote partial derivatives.) Next, by expanding $e^{\sigma\sqrt{h}}$, $e^{-\sigma\sqrt{h}}$ and $\hat{r}^h$ in a similar manner about $\sigma\sqrt{h} = 0$

$$e^{\sigma\sqrt{h}} = 1 + \sigma\sqrt{h} + \frac{\sigma^2 h}{2} + O(h^2) \tag{A.2}$$

$$e^{-\sigma\sqrt{h}} = 1 - \sigma\sqrt{h} + \frac{\sigma^2 h}{2} + O(h^2) \tag{A.3}$$

$$\hat{r}^h = 1 + h\ln\hat{r} + O(h^2) \tag{A.4}$$

where $\exists k \ni \left|\frac{O(h)}{h}\right| < k$ for large h

$$(\hat{r}^h - e^{-\sigma\sqrt{h}})C(e^{\sigma\sqrt{h}}S, t-h) + (e^{\sigma\sqrt{h}} - \hat{r}^h)C(e^{-\sigma\sqrt{h}}S, t-h)$$

$$-(e^{\sigma\sqrt{h}} - e^{-\sigma\sqrt{h}})\hat{r}^h C(S,t) = 0$$

Inserting A.2, A.3 and A.4 into A.1, yields

$$\left(h\ln\hat{r} + \sigma\sqrt{h} - \frac{\sigma^2 h}{2}\right)\left(C + C_S S\left(\sigma\sqrt{h} + \frac{\sigma^2 h}{2}\right) - hC_t\right.$$

$$\left. + \frac{1}{2}C_{SS}S^2\left(\sigma^2 h + \frac{\sigma^4 h^2}{4} + \sigma^3 h^{3/2}\right)\right)$$

$$+\left(\sigma\sqrt{h} + \frac{\sigma^2 h}{2} - h\ln\hat{r}\right)\left(C + C_S S\left(-\sigma\sqrt{h} + \frac{\sigma^2 h}{2}\right) - hC_t\right.$$

$$\left. + \frac{1}{2}C_{SS}S^2\left(\sigma^2 h + \frac{\sigma^4 h^2}{4} - \sigma^3 h^{3/2}\right)\right)$$

$$-(2\sigma\sqrt{h})(1 + h\ln\hat{r})C(S,t) = O(h^2) \tag{A.5}$$

Expanding A.5 and simplifying produces

$$C\left(-2\sigma h^{3/2}\ln\hat{r}\right) + C_S S\left(2\sigma\sqrt{h}\left(h\ln\hat{r} + \frac{\sigma^2 h}{2}\right) + \sigma^2 h\left(\sigma\sqrt{h}\right)\right)$$

$$-2hC_t\left(\sigma\sqrt{h}\right) + C_{SS}S^2\left(\left(\sigma^2 h + \frac{\sigma^4 h^2}{4}\right)\sigma\sqrt{h} + \sigma^3 h^{3/2}\left(h\ln\hat{r} - \frac{\sigma^2 h}{2}\right)\right) = O(h^2)$$

$$\Rightarrow -2\sigma h^{3/2} C\ln\hat{r} + 2C_S S\sigma h^{3/2}\ln\hat{r} - 2h^{3/2}\sigma C_t$$

$$+C_{SS}S^2\left(\sigma^3 h^{3/2} + \frac{\sigma^5 h^{5/2}}{4} + \sigma^3 h^{5/2}\ln\hat{r} - \frac{\sigma^5 h^{5/2}}{2}\right) = O(h^2)$$

Divide through by $2\sigma h^{3/2}$ and let $h \to 0$ to get

$$\frac{1}{2}\sigma^2 S^2 C_{SS} + \ln\hat{r} S C_S - C_t - (\ln\hat{r})C(S,t) = 0$$

Replacing $\hat{r}$ with $\left(1 + r\left(\frac{1-\tau_b}{1-\tau_g}\right)\right)$, S with $Se^{-\delta\left(\frac{1-\tau_d}{1-\tau_g}\right)}$ and solving leaves us with the tax-adjusted Black-Scholes equation shown in Chapter 2.

Appendix B

RY Financial Prospectus

The following describes the attributes of the preferred and capital shares as outlined in the prospectus for *RY Financial Corporation*. This is typical of the all of the Canadian issues. Americus Trust calls its installment receipts "Score" and the preferred shares, "Prime," they do not, however, have a provision for the payment to the holder of a Score if the value of the Exxon common share is less than the face value of the preferred share.

Preferred Shares

The holders of Preferred Shares will be entitled to receive cumulative preferential cash dividends equal to 100% of the Net Profits of the Company, which shall essentially equal the dividends received on the Royal Bank common shares held by the Company. The Preferred Shares will be retractable at the option of the holder on June 30, 1992. Reference is made to "Certain Provisions of Preferred Shares".

RY Instalment Receipts

The holder of RY Instalment Receipt will receive one Royal Bank common share on payment in full of the second instalment and therefore will ultimately be entitled to any capital appreciation in the market value of the Royal Bank common share. The first instalment is payable against delivery of a Royal Bank common share and will be equal to the lesser of: (a) $25.00

(the Preferred Share issue price); and (b) the Market Price (as defined) of a Royal Bank common share, less \$1.00.

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