ESSAYS ON THE MEASUREMENT OF WASTE
AND PROJECT EVALUATION

By

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ABSTRACT

Harberger's methodology for the measurement of deadweight loss is reformulated in a general equilibrium context with adopting the Allais-Debreu-Diewert approach and is applied to various problems with imperfect markets. We also develop second best project evaluation rules for the same class of economies.

Chapter 1 is devoted to the survey of various welfare indicators. We especially discuss the two welfare indicators due to Allais, Debreu, Diewert and Hicks, Boiteux in relation to Bergson-Samuelsonian social welfare function. We first show that these two measures generate a Pareto inclusive ordering across various social states, but they are rarely welfarist, so that both are unsatisfactory as Bergson-Samuelsonian social welfare functions. We next show that second order approximations to the Allais-Debreu-Diewert measure of waste can be computed from local information observable at the equilibrium, whereas second order approximations to the Hicks-Boiteux measure of welfare or to the Bergson -Samuelsonian social welfare function require information on the marginal utilities of income of households, which is unavailable with ordinal utility theory. Finally, we give a diagrammatic exposition of the two measures and their approximations to give an intuitive insight into the economic implications of the two measures.

Chapter 2 and Chapter 3 study an economy with public goods. In Chapter 2, we compute an approximate deadweight loss measure for the whole economy when the endogenous choice of public goods by the government is nonoptimal

and the government revenue is raised by distortionary taxation by extending the Allais-Debreu-Diewert approach discussed in Chapter 1. The resulting measure of waste is related to indirect tax rates, net marginal benefits of public goods, and the derivatives of aggregate demand and supply functions evaluated at an equilibrium. In Chapter 3, cost-benefit rules for the provision of a public good are derived when there exist tax distortions. We derive the rules as giving sufficient conditions for Pareto improvement, but we also discuss when these rules are necessary conditions for an interior social optimum. When indirect taxes are fully flexible but lump-sum transfers are restricted, we recommend a rule which generalized the cost-benefit rule due to Atkinson and Stern (1974) to a many-consumer economy. When both indirect taxes and lump-sum transfers are flexible, we suggest a rule which is based on Diamond and Mirrlees' (1971) productive efficiency principle. When only lump-sum transfers are variable, we obtain a version of the Harberger (1971)-Bruce-Harris (1982) cost-benefit rules.

Chapters 4 and 5 study an economy with increasing returns to scale in production and imperfect competition. In Chapter 4, we discuss a methodology for computing an approximate deadweight loss due to imperfect regulation of monopolistic industries by extending the Allais-Debreu-Diewert approach to incorporate the nonconvex technology. With the assumption of the quasi-concavity of production functions and fixed number of firms, we can derive an approximate deadweight loss formula which is related to markup rates of firms, and the derivatives of aggregate demand functions, factor supply and demand functions and the derivatives of marginal cost functions. We also discuss various limitations of our approach and the relation between our work and that of Hotelling (1938). In Chapter 5, we consider cost-benefit rules of a large project applicable in the presence of imperfect competition. We show that the index number approach due to Negishi (1962) and Harris (1978) can be extended to handle situations with imperfect competition.

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CHAPTER 1

APPLIED WELFARE ECONOMICS

The purpose of this chapter is to compare alternative criteria for social waste or welfare from several viewpoints and choose one criterion which suits our purpose best. In doing so, we present our basic strategies for the measurement of deadweight loss and discuss their pros and cons compared with other approaches to applied welfare economics.

In section 1, we introduce two criteria for measuring deadweight loss; that is, the Allais-Debreu-Diewert measure of social waste and the Hicks-Boiteux measure of social welfare. After explaining their intuitive meanings by illustrations we consider whether they can serve as Pareto-inclusive and individualistic (or welfarist) social welfare functions. We show that these measures are Pareto-inclusive, but not individualistic except when either Gorman's preference restriction is satisfied or the production possibilities set is linear. The Allais-Debreu-Diewert measure of waste is affected by the choice of the reference bundle of goods in terms of which the scale of efficiency loss is determined whereas the Hicks-Boiteux measure of welfare is affected by the choice of the optimal allocation of real income. This means that the former measure is affected by the valuation of each good for social efficiency while the latter is affected by the valuation of each individual in the measure of social welfare. Thus, the Allais-Debreu-Diewert measure is a pure efficiency waste measure whereas the Hicks-Boiteux measure shows a change of social welfare including both efficiency and equity aspects.

In the second section, this point is further elaborated by taking a second order approximation to the two measures when tax distortions prevail.

We show that the Allais-Debreu-Diewert measure is computable from the second order derivatives of expenditure functions and profit functions evaluated at the observed equilibrium while the Hicks-Boiteux measure or the Bergson-Samuelsonian social welfare function requires information on the difference between the inverse of the marginal utility of income and the marginal social importance to evaluate the equity loss. Since this information is not available with ordinal utility theory, it is difficult to use the Hicks-Boiteux measure in applied welfare economics. This provides the main reason why we use the Allais-Debreu-Diewert measure in this essay. The pros and cons of the approximation approach we adopt in this essay are next compared with an alternative influential approach, applied (or numerical) general equilibrium models. A numerical general equilibrium model computes the exact value of social welfare indicators by restricting the functional forms of production and utility to overly simple forms. Our approach, on the other hand, computes approximate values of social welfare indicators from more general functional forms and observable information. Finally, in light of the measurement of waste approach for welfare economics, we reconsider the theory of second best. Our conclusion here is that this theory is not a replacement for the measurement of deadweight loss, even though several positive results derived in second best theory are useful.

Finally, in section 3, in order to give insight into the economic implications of our approach, we give diagrammatic expositions of the two measures and their approximations for a one-consumer two-goods economy.

1-1. The Measure of Deadweight Loss

In a long series of papers on the measurement of deadweight loss (or 'welfare cost' or 'waste,' terms which are used interchangeably in this thesis) which includes Hotelling (1938), Hicks (1941-2), Allais (1943, 1977), Boiteux (1951), Debreu (1951, 1954), Harberger (1964, 1971), and Diamond-McFaddon (1974), two types of welfare criteria are chiefly used: the Allais-Debreu-Diewert measure of waste (the ADD measure hereafter) and the Hicks-Boiteux measure of surplus (the HB measure hereafter).[1]

Let us set up the model of our economy to discuss these two measures. There are H consumers having quasi-concave utility functions $f^h(x^h)$, $h = 1, \ldots, H$ defined over a translated orthant Ω^h where $x^h \equiv (x_1^h, \ldots, x_N^h)^T$ is a consumption vector of goods $1, \ldots, N$ by the hth consumer. The initial endowment vector of the hth consumer is given by $\bar{x}^h$, $h = 1, \ldots, H$. There are K firms and firm k produces y^k using the production possibilities set S^k, $k = 1, \ldots, K$. We can define the ADD measure in terms of a primal programming problem[2]:

$$(1) \qquad L_{ADD} \equiv r^0 \equiv \max_{r, x^h, y^k} \{ r : \Sigma_{h=1}^H x^h + \beta \cdot r \leq \Sigma_{k=1}^K y^k + \Sigma_{h=1}^H \bar{x}^h ;$$

$$f^h(x^h) \geq u_h^1, \; h = 1, \ldots, H; \; y^k \in S^k, \; k = 1, \ldots, K\},$$

where $\beta \equiv (\beta_1, \ldots, \beta_N)^T \geq 0_N$ is an arbitrarily chosen reference bundle of commodities. To interpret this problem we rewrite (1) in an alternative manner. The following notation is used. R_+^N is the N-dimensional nonnegative orthant. $\Sigma_{k=1}^K S^k$ is the direct sum of the production possibilities sets S^k. $S(u^1) \equiv \{x : \Sigma_{h=1}^H x^h \leq x; \; f^h(x^h) \geq u_h^1, \; h = 1, \ldots, H\}$ is the Scitovsky set corresponding to a utility allocation $u^1 \equiv (u_1^1, \ldots, u_H^1)$. Now (1) can be rewritten as

$$\text{(2)} \qquad \max_r \{r : \beta r \in Q \equiv \Sigma_{h=1}^{H}\bar{x}^h + \Sigma_{k=1}^{K}S^k - S(u^1)\}.$$

(2) has a straightforward iterpretation: maximize the scale of the reference set of goods in Q where Q is the set of goods producible from the aggregate production possibilities plus endowments which give consumers at least the utility vector u^1 when the goods are appropriately distributed. In Fig. 1 we depict the ADD measure, in a two goods economy where p^0 is a support price of the programming problem (1). p^0 is determined up to a multiplication by a positive number so that we can choose $p_1^0 = 1$; i.e., the optimal price of the first good is unity without loss of generality. Furthermore, we can choose the scale of β so that $p^{0T}\beta = 1$. Then, $r = (p^{0T}\beta)r$ equals AB since $p^{0T}\beta r$ is the difference between the value of production minus consumption evaluated at p^0. Note that the choice of the reference bundle β is crucial in the evaluation of the ADD measure (see Diewert (1985a;50)).[3]

Let us now turn to the HB measure. We begin from an attainable and socially optimal utility allocation $u^0 \equiv (u_1^0,\ldots,u_H^0)^T$. We also assume that there exists a price vector $p^0 = (p_1^0,\ldots,p_N^0)^T$ which supports the socially optimal allocation of resources. Then we can define the HB measure L_{HB} as follows:

$$\text{(3)} \qquad L_{HB} \equiv \Sigma_{h=1}^{H} m^h(p^0,u_h^0) - \Sigma_{h=1}^{H} m^h(p^0,u_h^1)$$

where we define the expenditure function:[4]

$$\text{(4)} \qquad m^h(p,u_h) \equiv \min_{x^h}\{p^Tx^h : f^h(x^h) \geq u_h\},$$

where $p \gg 0_N$ and $u_h \in$ Range f^h.

The measure L_{HB} defined by (3) can be interpreted as the sum of the negative of the equivalent variations obtained in moving from a socially optimal utility vector u^0 to the observed distorted utility vector u^1. The HB measure evaluated in units of the first good in a two good economy by choosing $P_1^0 = 1$ is illustrated in Fig. 2.

Generally, the desirable properties of the ordering of social states are summarized in the Bergson-Samuelsonian social welfare function (BSSWF hereafter). (See Samuelson (1956) for a discussion of the BSSWF and its properties listed below.) We first assume that the underlying social ordering is compatible with the Pareto partial ordering (i.e., if all individual utilities increase, then so does social welfare) so that the resulting BSSWF becomes <u>Pareto-inclusive</u>. Suppose also that the evaluation of social states is <u>individualistic</u> (or welfarist); i.e., the utility vector u prevailing at the state is the only information used in the evaluation. Also suppose that the evaluation takes the form of a continuous ordering of utility vectors. Then, Debreu's (1959;56) representation theorem is applied to get the BSSWF, W(u). Pareto-inclusiveness implies that W is monotone increasing in u.

Recalling that the ADD measure and the HB measure evaluate the states of the economy numerically, they generate orderings of the utility vectors where the utility vectors with smaller amounts of waste are ranked higher <u>given</u> the reference bundle β or the reference utility vector u^0.[5] It may, therefore, be interesting to ask whether these measures are Pareto-inclusive[6] and individualistic; i.e., whether they work as a kind of BSSWF. The first question may be answered easily. First, notice the definition (2) of the ADD measure. Suppose that u^a is preferred to u^b in terms of the Paretian partial ordering, then $S(u^a)$ is a subset of $S(u^b)$. Noting that production possibilities are fixed, $Q(u^a)$ is a subset of $Q(u^b)$ and hence $r(u^a) \leq r(u^b)$. In the case of the HB measure, Pareto inclusiveness directly follows from the nondecreasingness of the expenditure function with respect to u (see Diewert (1982;541)) and its definition (3).

The other question is more difficult to solve. The ADD measure $r = r(u,\beta)$ becomes a function of both u and β, so it cannot be individualistic; i.e., it is always affected by the choice of β, which is not related to individuals' welfare. We extend the concept of an 'individualistic' evaluation by saying that r is ordinally individualistic if and only if the ordering of utility vectors induced by r for given β is not affected by the choice of β. This definition is formalized as follows:

$$(5) \qquad r(u^a,\beta^a) \geq r(u^b,\beta^a) \text{ iff } r(u^a,\beta^b) \geq r(u^b,\beta^b) \quad \text{for all } \beta^a \geq 0_N \text{ and } \beta^b \geq 0_N.$$

The profit function π^k, $k = 1,\ldots,K$ is defined as

$$(6) \qquad \pi^k(p) \equiv \max_x \{p^T y : y \in S^k\},\ k = 1,\ldots,K,$$

where $p \gg 0_N$.

The regularity properties of the profit function are summarized in Diewert (1982;580-1).

We assume below that the production possibilities sets are convex and preferences are quasiconcave. Then, (1) is equivalent to the following dual max min problem:

$$(7)\qquad L_{ADD}(u,\beta) \equiv r(u,\beta) \equiv \max_r \min_{p\geq 0_N}\{r(1-p^T\beta)+\Sigma_{h=1}^{H}p^T\bar{x}^h+\Sigma_{k=1}^{K}\pi^k(p) - \Sigma_{h=1}^{H}m^h(p,u_h)\},$$

The proof is an application of the Uzawa (1958;34)-Karlin (1959;201) Saddle Point Theorem[7] (see Appendix II). If we further assume that r is twice continuously differentiable at the relevant values of u and β, then (5) is equivalent to requiring u to be <u>separable</u>[8] in $r(u,\beta)$; that is, $r(u,\beta)$ satisfies

$$(8)\qquad \partial\left(\frac{\partial r}{\partial u_i}\Big/\frac{\partial r}{\partial u_j}\right)\Big/\partial\beta_n = 0 \text{ for all } i,j = 1,\ldots,H \text{ and all } n = 1,\ldots,N.$$

We assume that the first order necessary conditions for the max min problem (7) are equalities and define the solution as (r^0,p^0). Then the well-known envelope theorem implies that $\partial r/\partial u_i = -\partial m^i(p^0,u_i)/\partial u_i$, $i = 1,\ldots,H$. Substituting it into (8) and using the relation: $\partial^2 m^i(p,u_i)/\partial u_i\partial p_m = [\partial x_m^i(p,y)/\partial y_i][\partial m^i(p^0,u_i)/\partial u_i]$ for $i = 1,\ldots,H$ and $m = 1,\ldots,N$, we have

$$(9)\qquad \Sigma_{m=1}^{N}[\partial x_m^i(p^0,y_i^0)/\partial y_i-\partial x_m^j(p^0,y_j^0)/\partial y_j](\partial p_m^0/\partial\beta_n) = 0$$

for all $i,j = 1,\ldots,H$ and all $n = 1,\ldots,N$, where $x_m^i(p,y_i)$, $i = 1,\dots,H$ is the ordinary demand function for the nth good for the ith consumer and $y_i^0 \equiv m^i(p^0,u_i)$. Conditions (9) are satisfied either if Gorman's (1953;73) restriction on preferences is satisfied; i.e., preferences are quasi-homothetic and their Engel curves are parallel to each other, (since the first term in the left-hand side of (9) is 0 for all i,j,m) or if the production possibilities sets are linear (since the second vector is 0 for all m and n). (9) has the following meaning: when we increase any one reference good β_n, then the scarcity of the nth good increases so that the system of shadow prices associated with (7) p^0 changes, and this change must be orthogonal to the difference of the gradients of the Engel curves for _any_ two consumers at the optimum. This condition does not seem to me to be satisfied globally except for the two cases above listed.

We now turn to the HB measure L_{HB}. By the same token as the ADD measure, $L_{HB}(u^1,u^0,p^0)$ is ordinally individualistic if and only if u^1 is separable in L_{HB}. Remember that u^0 is one Pareto optimal utility allocation and p^0 is its supporting price vector. Therefore p^0 is a function of u^0 (and other parameters of the general equilibrium) so that separability of u^1 is equivalent to the condition.

$$(10) \qquad \left(\frac{\partial L_{HB}}{\partial u_i^1} \Big/ \frac{\partial L_{HB}}{\partial u_j^1}\right) \Big/ \partial u_h^0 = 0 \text{ for all } i,j = 1,\dots,H \text{ and all } h = 1,\dots,H.$$

Using definition (3), $\dfrac{\partial L_{HB}}{\partial u_i^1} = -\,\partial m^i(p^0,u_i^1)/\partial u_i^1$. Substitute this into (10) and we find the following equivalent conditions:

$$(11) \qquad \Sigma_{m=1}^{M}[\partial x_m^i(p^0,y_i^0)/\partial y_i - \partial x_m^j(p^0,y_j^0)/\partial y_j](\partial p_m^0/\partial u_h^0) = 0$$

for all i,j = 1,...,H and all h = 1,...,H.

Conditions (11) seem analogous to (9), except for the difference between $\partial p_m^0/\partial \beta_n$ in (9) and $\partial p_m^0/\partial u_h^0$ in (11). The former is the change of the support prices of the Allais-Debreu-Diewert optimum with respect to an increase of the nth good in the reference bundle, while the latter is the change of the support prices of the reference Pareto optimal allocation with respect to an increase of the utility of the hth household. Therefore, as in the ADD measure, there does not seem to exist plausible conditions to guarantee the HB measure to be ordinally individualistic except for the two conditions cited above; i.e., Gorman's preference restriction or linear production possibilities.

Up to now we have learned that both measures are Pareto inclusive but not individualistic in general. The conditions necessary to make welfare prescriptions by the ADD measure ordinally individualistic are as stringent as those needed by the HB measure. However, the economic implications of the two measures are completely different. The ADD index measures pure technical efficiency in terms of the reference bundle of goods, and the HB index measures the loss of both efficiency and equity by indicating the monetary value of the difference between the social optimum and the observed equilibrium. Although based on pure efficiency considerations, using the ADD measure to rank social states means that implicitly it is being used as a measure of social welfare (instead of as just an estimate of the resource allocation waste of one observed equilibrium), and as I have shown, this method of valuing social states is affected by the choice of reference bundle of goods. Therefore, to add equity aspects to the ADD measure, we have to choose a reference bundle so that goods which are socially valuable are weighted more heavily. However, it is difficult to determine what these goods are, and what weights shall be attached to them. In contrast, the HB measure is a sum of money-metric scaling utility functions and it has a natural interpretation as a BSSWF, provided a reference price vector is fixed. Another drawback of the ADD measure is that it cannot be an appropriate welfare indicator if there is technological change (i.e., it is not welfarist in the sense that it depends on technological parameters). The HB measure is free of this defect, if the reference price vector is fixed (see Section 5.2)

Let us compare these measures from another viewpoint. Are these measures useful when the shadow price vector does not exist because of nonconvexities or externalities? We will show in the later chapters of this essay that the ADD measure is a very powerful tool to analyze deadweight loss under such market imperfections. It seems that we can also use the HB measure equally well to study deadweight loss in such circumstances. When we choose a reference Pareto optimal allocation u^0, we find both the optimal shadow prices p^0 for priced goods and the optimal demands q^0 of external goods or nonpriced goods. All we need is to compare the sum of the negative of the equivalent variations $m^h(p^0,q^0,u_h^0) - m^h(p^0,q^0,u_h^1)$, where $m^h(p,q,u^1)$ is a <u>restricted expenditure function</u> (see Diewert (1986;170-6)).

Note that the calculation of the two measures necessitates global computation of the optimal equilibrium which is very difficult to implement empirically. Therefore in this essay, we concentrate on the study of approximate measures of welfare. In the following section, we compare the approximate ADD measure and HB measure and discuss which one is more implementable in empirical research.

1-2. The Approximation Approach to the Measurement of Waste

This section is devoted to an introduction to our approximation approach to the measurement of waste. We first derive a second order approximation to the ADD measure of waste (1). This approximate measure depends on the economic environment and types of distortions. We assume initially that markets are complete, technologies are convex and that the only source of distortions is indirect taxes levied on consumers. Extensions of these assumptions are a main theme of the later chapters, so that we only work with the prototype model in this chapter. Given these assumptions, (1) is equivalent to (6). At this point, we use the concept of the <u>overspending function</u> B which will be fully utilized in this essay which is defined as

$$B(q,p,u) \equiv \Sigma_{h=1}^{H} m^h(q,u_h) - \Sigma_{h=0}^{H} q^T x^h - \Sigma_{k=0}^{K} \pi^k(p).$$

In Appendix I, B is restated with its economic interpretation and its useful properties are summarized. Using the definition (A.1), (7) may be rewritten concisely as follows:

$$(12) \qquad r^0 \equiv \max_r \min_{p \geq 0_N} \{r(1-p^T\beta) - B(p,p,u^1)\}.$$

Using the Uzawa (1958) - Karlin (1959) Theorem in reverse, (12) is also equivalent to:

$$(13) \qquad -\max_{p \geq 0_N}\{B(p,p,u^1) : p^T\beta \geq 1\}.$$

If (p^0,r^0) solves (12), p^0 solves (13) with r^0 being its associated Lagrangean multiplier.

In order to obtain a second order approximation to r^0, we assume: (i) (p^0,r^0) solves (12); (ii) the first order necessary conditions for (12) hold with equalities so that $p^0 \gg 0_N$; (iii) $B(q,p,u^1)$ is twice continuously differentiable at (p^0,p^0); (iv) Samuelson's (1947;361) strong second order sufficient conditions hold for (13) when the inequality constraints are replaced by equalities.

Let us consider the following system of equations in N+1 unknowns p and r which are functions of a scalar variable z, for $0 \leq z \leq 1$:

$$(14) \qquad -\nabla_q B(p(z) + tz,\ p(z),\ u^1) - \nabla_p B(p(z) + tz,\ p(z),\ u^1) - r(z)\beta = 0,$$

$$(15) \qquad 1 - p(z)^T\beta = 0.$$

When $z = 0$, (14) and (15) coincide with the first order conditions for (12) if $p(0) \equiv p^0$ and $r(0) \equiv r^0$. Suppose $p(1) \equiv p^1$ is the set of observed producer prices normalized by (15) in a tax-distorted equilibrium with indirect tax rates t. Setting $r(1) \equiv 0$, when $z = 1$ (14) is then the set of equations characterizing the equality of demand and supply in the tax-distorted equilibrium. If we assume that appropriate lump-sum transfers from the government to consumers are chosen, then there exist budget constraints for the H consumers compatible with (14) and (15). From these equations, it is also the case that satisfaction of the government budget constraint is implied.

Let us differentiate (14) - (15) totally with respect to z. We have

$$(16) \qquad - \begin{bmatrix} B_{qq}^{z} + B_{pp}^{z}, & \beta \\ \beta^{T} \quad , & 0 \end{bmatrix} \begin{bmatrix} p'(z) \\ r'(z) \end{bmatrix} = \begin{bmatrix} B_{qq}^{z} t \\ 0 \end{bmatrix},$$

where $q \equiv p + tz$ is the first set of arguments for B and $B_{ij}^{z} \equiv \nabla_{ij}^{2} B(p(z) + tz, p(z), u^{1})$ for $i,j = q,p,u$. Note that $B_{qp}^{z} \equiv 0_{N \times N}$. Also note that the left-hand side matrix of (16) is non-singular by assumption (iv) (see Diewert and Woodland (1977)). Therefore, using the differentiability assumptions (iii), by the Implicit Function Theorem there exist once continuously differentiable functions p(z) and r(z) at z close to 0 that satisfy (14) and (15). We show in Appendix III that the following equation is satisfied.

$$(17) \qquad - r'(z) = - zt^{T} B_{qq}^{z} (p'(z) + t).$$

We readily have

$$(18) \qquad r'(0) = 0$$

from (17). Using (17), it is shown in Appendix IV that the following equation follows.

$$(19) \qquad - r''(0) = - (p'(0) + t)^T B_{qq}^0 (p'(0) + t) - p'(0)^T B_{pp}^0 p'(0) \geq 0$$

where the inequality comes from the concavity of B with respect to q and p (see Appendix I). A second order Taylor approximation to the ADD measure is given by (noting that $r(1) \equiv 0$),

$$(20) \qquad L_{ADD} \equiv r(0) - r(1) \cong r(0) - (r(0) + r'(0) + \tfrac{1}{2} r''(0)) =$$

$$- \tfrac{1}{2}\{p'(0)^T B_{pp}^0 p'(0) + [p'(0) + t]^T B_{qq}^0 [p'(0) + t]\} \geq 0,$$

where we use (18) and (19). Equation (16) is used to compute $p'(0)$. Information we need to evaluate (20) is: (i) the set of indirect taxes t, (ii) the second order derivatives of the overspending function with respect to prices which equals the producers' aggregate substitution matrix and the consumers' aggregate compensated substitution matrix respectively, evaluated at the optimum equilibrium.

The remarkable advantage of this approximation approach is that it can be implemented from the derivatives of the overspending function evaluated at the optimum equilibrium, so that we need not know global functional forms for utility and production functions. However, as long as we must know the derivatives at the optimum as in (20), we must actually know the optimal prices so that we must compute the optimum or we must depend on some 'guessing' process about the values at the optimum. Harberger (1964) suggested replacing these (unobservable) derivatives by those which are evaluated at the observed distorted equilibrium, since they can be calculated using data prevailing at the observed equilibrium. This method can be justified more rigorously by Diewert's (1976;118) Quadratic Approximation Lemma which showed that the approximation

$$(21) \qquad L_{ADD} \equiv r(0) - r(1) \cong r(0) - \{r(0) + \tfrac{1}{2}r'(0) + \tfrac{1}{2}r'(1)\}$$

is also exact as the approximation (20) when the functional form is quadratic (see also Diewert (1985(b);238)). Evaluating (16) at z = 1 and using (17), we can show that $-r'(1)$ is identical with $-r''(0)$ in (19) except that all the relevant functions are evaluated at z = 1; i.e., at the observed equilibrium; $-r'(1)$ is nonnegative due to the semidefiniteness properties of the producer and consumer substitution matrices. Using also (18), we find

$$(22) \qquad L_{ADD} \cong -\tfrac{1}{2}\{p'(1)^T B_{pp}^1 p'(1) + [p'(1) + t]^T B_{qq}^1 [p'(1) + t]\} \geq 0.$$

This approximation uses only information observable at the prevailing equilibrium as Harberger originally required, so that it is highly valuable in empirical analysis.

The next task is to compute an approximation of the HB measure for the same economy and compare it with the approximation of the ADD measure. To begin with, we must clarify which reference optimal equilibrium to pick from a set of Pareto optimal allocations to calculate the HB measure or its approximations. According to Negishi's (1960) theorem, every competitive equilibrium is a solution of the maximum of a linear social welfare function $\Sigma_{h=1}^{H} \alpha^h f^h$ for some set of weights $\alpha \equiv (\alpha_1,\ldots,\alpha_H)^T$ given resource constraints and production possibilities of the economy, where it is assumed that f^h, h = 1,...,H are concave functions. In our model, this means that for some vector

α, a perfectly competitive equilibrium is a solution of the following programming problem:

$$(23) \qquad \mathrm{Max}_{x^h, y^k}\{\Sigma_{h=1}^{H}\alpha^h f^h(x^h) \text{ s.t. } \Sigma_{h=1}^{H} x^h \leq \Sigma_{k=1}^{K} y^k + \Sigma_{h=1}^{H} \bar{x}^h; \quad y^k \in S^k,$$

$$k = 1, \ldots, K\}.$$

Using the Uzawa-Karlin Saddle-point Theorem using the definition (4), (6) and (A.1), we can rewrite (23) as follows (the calculation is analogous to the derivation of (7) in Appendix II):

$$(24) \qquad \mathrm{Max}_u \ \mathrm{Min}_{p \geq 0_N} \ \{\alpha^T u - B(p,p,u)\}.$$

We assume that (i) (u^0, p^0) solves (24), (ii) the first order conditions for (24) hold with equality so that $p^0 \gg 0_N$, (iii) B is twice continuously differentiable at the optimum, and (iv) $B_{qq}^0 + B_{pp}^0$ is negative definite. From assumptions (i) and (ii), we find the first order conditions for (24) are:

$$(25) \qquad \alpha = \nabla_u B(p,p,u),$$

$$(26) \qquad - \nabla_p B(p,p,u) - \nabla_q B(p,p,u) = 0.$$

Condition (26) is the equality of demand and supply at the optimum while (25) is the rule to equate the marginal social importance of each person to the inverse of his marginal utility of income (see Negishi (1960)).[9] Note that the solution depends on α which is equivalent to picking a reference equilibrium. We have to pick one reference equilibrium from various competitive equilibria corresponding to various α. Varian (1974, 1976) persuasively discussed the welfare significance of the equal division equilibrium, which is a perfectly competitive equilibrium obtained from the equal division of initial endowments across individuals. Varian (1976), following the approach of Negishi (1960), also examined the relationship between his theory of fairness and more traditional welfare economics based on the concept of a social welfare function, which we followed in this section. By Negishi's theorem, the equal division equilibrium is also characterized as a solution to a nonlinear programming (23) for some choice of α. By finding this α and associated reference price vector p^0, we can find the HB measure.

We now compute the second order approximation to the HB measure around the optimal equilibrium in an analogous way as we computed the approximation to the ADD measure. First we construct a z-equilibrium:

$$(27) \qquad \nabla_u B(p(z) + tz,\ p(z),\ u(z)) = \alpha + \lambda z\ ;$$

$$(28) \qquad -\nabla_q B(p(z) + tz,\ p(z),\ u(z)) - \nabla_p B(p(z) + tz,\ p(z), u(z)) = 0.$$

When $z = 0$, (27) and (28) coincide with the first order conditions for the maximum of social welfare (25) and (26), if we define $u(0) \equiv u^0$ and $p(0) \equiv p^0$. When $z = 1$, (28) is a set of equations to show the market clearing conditions at the tax-distorted equilibrium, if $u(1) \equiv u^1$ and $p(1) \equiv p^1$ are the values prevailing at the observed distorted equilibrium. If we assume that the level of lump-sum transfers from the government to consumers are appropriately chosen, there exist budget constraints for consumers compatible with (27) and (28). (28) and these budget constraints imply the budget balance of the government. When $z = 1$, (27) quantifies the 'equity' distortions at the observed equilibrium; i.e., $-\lambda_h$ shows the difference between the marginal social importance of the hth person and the inverse of his marginal utility of income. It must be noted that both α and the marginal utility of income are not invariant to a monotone transformation of $f^h(x_h)$. However, they are adjusted proportionally so that (25) is valid. We must also adjust λ_h proportionally to h's marginal utility of income and α^h so that (27) is valid.

Now differentiate (27) and (28) with respect to z;

$$(29) \qquad -\begin{bmatrix} B_{uu}^{z}, & B_{uq}^{z} \\ B_{qu}^{z}, & B_{qq}^{z} + B_{pp}^{z} \end{bmatrix} \begin{bmatrix} u'(z) \\ p'(z) \end{bmatrix} = \begin{bmatrix} -\lambda + B_{uq}^{z}t \\ B_{qq}^{z}t \end{bmatrix}$$

where $B_{ij}^{z} \equiv \nabla_{ij}^{2}B(p(z) + tz, p(z), u(z))$ for $i, j = q, p, u$. Note that $B_{up}^{z} = 0_{H \times N}$. Assumptions (iii) and (iv) guarantee, via the Implicit Function Theorem, that once continuously differentiable functions $u(z)$ and $p(z)$ satisfying (29) exist at z close to 0. Premultiplying $[0_H^T, p(z)^T]$ to both sides of (29) and using property (iii) of the overspending function in Appendix I, we can derive

$$(30) \qquad \Sigma_{h=1}^{H} \nabla_u m^h(p(z) + tz, u_h(z))\, u_h'(z)$$
$$= z[t^T B_{qq}^{z}(p'(z) + t) + t^T B_{qu}^{z} u'(z)],$$

analogously to the derivation of (17) in Appendix III. Evaluating (30) at $z = 0$, we get

(31) $$\Sigma_{h=1}^{H} \nabla_u m^h(p^0, u_h^0) u_h'(0) = 0.$$

Analogously to the derivation of (19) in Appendix IV, we next differentiate (30) with respect to z, and evaluate at $z = 0$ to obtain

(32) $$u'(0)^T B_{uu}^0 u'(0) + \Sigma_{h=1}^{H} \nabla_u m^h(p^0, u_h^0) u_h''(0) = t^T B_{qq}^0 (p'(0) + t) - p'(0)^T B_{qu}^0 u'(0).$$

Premultiplying (29) evaluated at $z = 0$ by $[0_H^T, p'(0)^T]$ and adding the resulting identity to (32), we have

(33) $$u'(0)^T B_{uu}^0 u'(0) + \Sigma_{h=1}^{H} \nabla_u m^h(p^0, u_h^0) u_h''(0) = - p'(0)^T B_{pp}^0 p'(0) - [p'(0) + t]^T B_{qq}^0 [p'(0) + t] \geq 0.$$

A second order Taylor approximation to the HB measure (3) at $z = 0$ is as follows:

(34) $$L_{HB} \cong - \Sigma_{h=1}^{H} \nabla_u m^h(p^0, u_h^0) u_h'(0) - \tfrac{1}{2}[u'(0)^T B_{uu}^0 u'(0) + \Sigma_{h=1}^{H} \nabla_u m^h(p^0, u_h^0) u_h''(0)].$$

Substituting (31) and (33) into (34) we have

(35) $$L_{HB} \cong -\tfrac{1}{2}\{p'(0)^T B_{pp}^0 p'(0) + [p'(0) + t]^T B_{qq}^0 [p'(0) + t]\} \geq 0.$$

To compute (35), we could again replace B_{ij}^0 by B_{ij}^1 in (35) and (29) since the B_{ij} are observable, again following Harberger.[10] It is interesting to compare (35) with the second order approximation to the gain in social welfare using the linear welfare function in moving to the optimum from the distorted equilibrium, $\Sigma_{h=1}^{H} \alpha^h [f^h(x^{h0}) - f^h(x^{h1})] \equiv L_L$. We find that

(36) $$\tilde{L}_L = \tilde{L}_{HB} + \tfrac{1}{2} u'(0)^T B_{uu}^0 u'(0) \geq \tilde{L}_{HB}$$

where the tilde shows it is an approximation of the original measure and the inequality comes from the positive semidefiniteness of B_{uu}^0, which is implied by the <u>concavity</u> of the utility functions. According to Varian (1976;257), the linear utility function does not count the problem of equity. Therefore, when moving from the equitable equilibrium to market distorted equilibrium, L_L only measures efficiency loss and does not evaluate its equity loss. In this sense, L_L may be taken as a lower bound of the welfare change.[11] However, (36) shows that $\tilde{L}_{HB}$ is even smaller than $\tilde{L}_L$. This is because, with diminishing marginal utility of income, increasing the inequality in terms of utility (or real income) holding the (weighted) sum of utility constant tends to increase the aggregate expenditure necessary to attain the reference utility allocation. This problem of inequity in the HB measure may not arise if we adopt money metric utility scaling so that $u_h \equiv m^h(p^0, u_h)$, $h = 1,\ldots,H$.[12] If this is the case, $\nabla_u B(p^0, u) = 1_H$ and $B_{uu}^0 = 0_{H \times H}$ so that $L_L = L_{HB}$ and $\tilde{L}_L = \tilde{L}_{HB}$. With this assumption we can regard the HB measure as summing the change of utilities of individuals; i.e., it is a utilitarian measure of welfare.

We now compare the empirical implementability of $\tilde{L}_{ADD}$ in (20) and (22) and $\tilde{L}_{HB}$ in (35). Though (20) and (35) look identical, their meanings are completely different. First, the substitution matrices are evaluated at distorted level of utilities in (20) while they are evaluated at optimal level of utilities in (35). Second, p'(0) is calculated from different sets of equations, (16) and (29). The first difference is inessential, since, as was already stressed, we replace these matrices with matrices evaluated at an observed distorted equilibrium. However, the second difference matters crucially. In (22), the substitution matrices, B_{pp}^{1}, B_{qq}^{1}, tax rates t and reference bundles β are all information required to compute p'(1) and hence (22). In (35), we need both the substitution matrices B_{pp}^{1}, B_{qq}^{1} and income effect matrices B_{qu}^{1}, tax rates t and the distributional distortion parameters λ so that the informational requirements are much higher. Though it is possible to calculate B_{pp}^{1} and B_{qq}^{1} from local information on ordinary demand curves and supply curves at the distorted equilibrium, we have to know the marginal utility of income $\nabla_u B(p^1+t,p^1,u^1)$ to compute λ from (27) or B_{qu}^{1} from ordinary demand curves. Even if we adopt the money metric scaling convention using the optimal prices, this does not give information on the marginal utility of income at the observed equilibrium, and this is what we really require. If we adopt money metric scaling at the observed distorted prices $p^1 + t$, then B_{qu}^{1} is easy to calculate since $\nabla_u B(p^1 + t,p^1,u^1) = 1_H$. In this case we also have $B_{uu}^{1} = 0_{H\times H}$. However, we still cannot compute λ from (27) since now we do not have $1_H \equiv \nabla_u B(p^0,p^0,u^0)$; i.e., α is not a vector of ones anymore. McKenzie (1983, chapter 3) studied the methodology for calculating the money metrics, and he correctly pointed out that the marginal utility of income is not an operational concept without knowing the utility function. His approach is based on normalizing the marginal utility of income at one price vector, but in our case, we have to know it at two sets of prices p and p + t, and we cannot normalize twice. Diewert (1984;36) already pointed out that his approximate HB measure depends on the hypothetical income vector at the optimum which is difficult to obtain. Though we adopted a different method of approximation, the same problem seems to occur by the measurement of marginal utility of income (more rigorously, the difference between the marginal social importance and the inverse of the marginal utility of income), instead of the measurement of hypothetical income. In light of these observations, we must conclude that the approximate HB measure lacks empirical operationality without a knowledge of the original utility functions whereas the ADD measure is free from this problem. Note that this criticism will also apply even if we compute the waste using the Bergson-Samuelsonian social welfare function. It is chiefly for this reason that we adopt the ADD measure as our welfare criterion. Needless to say, however, the informational advantage of using the ADD measure does not mean that it is a superior measure to either the HB measure or the BSSWF. As long as we can measure the difference between the weight of a linear BSSWF and the inverse of the marginal utility of income, the same type of analysis as is presented in Chapters 2 and 4 for the ADD measure can be carried out using the HB measure or a BSSWF.

We have compared the informational requirements for the approximations of the ADD and the HB measures to be empirically computable, and in this context we have found a remarkable property of the ADD measure: it is computable from local information on supply curves and ordinary demand curves at the observed equilibrium. A natural defect of our approximation approach is that the approximation might deviate from its true value considerably when the '<u>gap</u>' between two equilibria is large. The numerical general equilibrium approach by Shoven and Whalley (1972, 1973, 1977) chooses an alternative way to compute equilibria directly corresponding to various tax and expenditure policies so that a more exact welfare evaluation seems available. However, an obvious drawback of the numerical general equilibrium approach is that we must have information on global functional forms of utility and production functions. In contrast, our approximation approach requires only second order derivatives of these functions evaluated at the observed equilibrium. As an important corollary of this fact, our approximate measure can be derived from any set of flexible functional forms using information based on the observed equilibrium. On the contrary, in the numerical general equilibrium approach, very restrictive functional forms are adopted to make global computation possible, and these restrictions are easily rejected in econometric tests using more general functional forms (see Jorgenson (1984;140)). Moreover, the approximation approach does not involve any numerical computations that are more complicated than a single matrix inversion, whereas there are often substantial numerical difficulties involved in computing general equilibria. Therefore, these two competing programs have their own pros and cons so that it would be difficult to judge which one is universally superior to the other.[13]

The measurement of waste is prominently a practical subject. As is pointed out by Harberger (1964;58), the comparison of welfare measures is the only constructive way to give a policy prescription under the 'nth best' situation,[14] i.e., by comparing the amount of waste corresponding to various feasible policies we can give a ranking among them even if there are various other distortions. However, as long as we use approximations, we cannot avoid approximation errors which might cause erroneous policy assessment. For example, Green and Sheshinski (1979) pointed out that Harberger's triangle approximation may change considerably by changing the choice of approximation point. In this context, they criticized Feldstein (1978) who measured the net benefit of capital income tax reform by comparing Harberger's (1964) measure at two taxed equilibria. Green and Sheshinski noted that there exist differences between Feldstein's Harberger measure and a second order approximation of income gain evaluated at the initial tax equilibrium. A similar criticism also applies to Turunen (1986) who applied the approximate ADD measure for the numerical assessment of gains from tariff reform. It would be possible to derive Green-Sheshinski like approximate gains formula for tax reform which is a second order approximation to the change of the ADD measure evaluated at an initial tax equilibrium.[15] However, due to the complexity of the resulting formula, we have omitted this derivation. Therefore, this approximation error may lead to reversals in the true ranking of policies based on the exact amount of waste.

We have to admit a dilemma that we cannot get an exact welfare measure for various sets of policies either by approximation or by equilibrium computation while we have to reach some decision on the choice or reform of economic policies. In the second best theory approach originated by Lipsey and Lancaster (1956), recommendations for policies or their partial reforms are given using the programming method under the constraint that some of the optimality conditions are not met, or some of the instruments to attain the first best is restricted. This approach has successfully derived many interesting results in optimum taxation theory, piecemeal policy recommendations and cost-benefit analysis.[16] However, we have to note at least two basic drawbacks of this approach. First, in contrast to the first best solution, general second best solutions cannot be decentralized in a simple principle (see Guesnerie (1979)) so that the possibility of meaningful policy recommendations is quite restricted except under rather simplified second best situations as in an optimal taxation economy . Second, since most of the second best results depend on local necessary conditions for optimality, they suffer from theoretical criticisms from the viewpoint of general equilibrium theory. As is shown by Foster and Sonnenschein (1970) and Hatta (1977), multiple equilibria and instability can easily occur in a well-behaved economy with tax-distortions. Harris (1977) pointed out that the sufficiency of the necessary conditions for second best optimality depends on the third order derivatives so that the interpretation of these sufficiency conditions is not easy. In contrast, tax reform approach due originally to Meade (1955) avoids the problem by restricting its attention to the local area around the observed distorted equilibrium. Various authors, represented by Dixit (1975) and Hatta (1977), derived sufficiency conditions for welfare improvement by some policy changes. Unfortunately, these conditions depend on many restrictive assumptions. Especially, the assumption that the policy maker can change the set of taxes incrementally is often irrelevant, since its reform alternatives are discrete changes of taxation. By the same token, it is often the case that the reform alternatives are institutionally restricted to the ones which are short of fully satisfying the sufficient conditions. In these cases, this approach cannot tell anything about the ranking of policies, but our approach can. Furthermore, the sequence of local improvements may not converge to global optimum, but may stay on a local optimum or some stationary point. These problems seem to give limitations on the use of local optimality or improvement conditions for policy recommendations.

Considering these defects, we seem to be obliged to conform to a conventional view on second best; i.e., if conditions on propositions are met, implement the prescribed policy. If the actual economic situations do not coincide with the conditions, or we do not have enough information to judge whether it is actually the case, we cannot tell anything from the second best theory. Particularly, even if conditions are not met for positive second best propositions, this does not justify the status quo in any way, since even in this case, the deadweight loss of the economy could be too large to neglect. Following Harberger (1964), "The Economics of nth Best," to measure the deadweight loss associated with the economy's being in any given nonoptimal position is of high practical importance when we cannot know how to make the best of a bad situation.

1-3. A Diagrammatic Exposition

In this section we illustrate diagrammatically the ADD measure and the HB measure and their approximations using a simple model in order to clarify the intuitive content of the discussions in the previous section.

We assume that there is one good and one production factor (labour). One aggregate firm produces the good y using labour v according to the production function $y \leq g(v)$. We also assume that there is a single consumer who enjoys utility u from the consumption of the good x and leisure L by means of the utility function f(x,L). The initial endowment of labour is $\bar{v}$ and there is a zero endowment of the good.

We first specify the tax-distorted observed equilibrium. We choose labour as numeraire so that its price, w = 1. We assume that there is a specific tax t on the good levied for consumption so that its producer price is p whereas its consumer price is p + t. It is also assumed that the specific tax revenue is transferred to the consumer as a lump-sum transfer. Then, using the profit function $\pi(1,p)$ dual to $y \leq g(v)$ and the expenditure function dual to f(x,L), the observed equilibrium is characterized by the market clearing conditions for the good and labour;

$$(37) \qquad \nabla_p \pi(1,p) - \nabla_p m(1, p+t, u) = 0$$

$$(38) \qquad \nabla_w \pi(1,p) - \nabla_w m(1,p+t,u) + \bar{v} = 0.$$

We assume that (p^1,u^1) solves (37) and (38) uniquely. From the homogeneity properties of π and m, we can deduce

$$(39) \qquad \bar{v} + \pi(1,p^1) + t\nabla_p m,(1,p^1+t,u^1) = m(1,p^1+t,u^1);$$

i.e., the budget constraint of the representative individual is satisfied.

Now we define the ADD measure of waste in this simple model. We assume that the surplus of the economy is measured by the numeraire good, labour. Therefore, the general primal programming problem (1) and its dual (7) are simplified respectively in this model as follows:

(40) $\quad L_{ADD} \equiv \text{Max}_{y,v,L}\{\bar{v} - L - v : y \geq x,\ y \leq g(v),\ f(x,L) \geq u^1\}$

(41) $\quad \equiv \text{Min}_{p\geq 0}\ \{\pi(1,p) - m(1,p,u^1) + \bar{v}\}.$

We assume that $p = p^A > 0$ is a unique solution of the first order condition:

(42) $\quad \nabla_p\pi(1,p^A) - \nabla_p m(1,p^A,u^1) = 0.$

Therefore, (40) and (41) can be rewritten as

(43) $\quad \bar{v} + \nabla_w\pi(1,p^A) - \nabla_w m(1,p^A,u^1) = L_{ADD},$

(44) $\quad \bar{v} + \pi(1,p^A) = L_{ADD} + m(1,p^A,u^1).$

Note that (43) and (44) are equivalent by using (42) and the homogeneity properties of π and m. We can illustrate the ADD measure of waste diagrammatically in Fig. 3. The program (40) boils down to searching for a point where the horizontal length of the lens-shaped area formed by the production possibilities set and the indifference curve with $u = u^1$ is maximal. This maximum is characterized by an equal slope $1/p^A$ of the two curves.

In this simple example, we can also express the ADD measure of waste as a more familiar Hotelling-Harberger-like curvilinear triangle ABC in Fig. 4. This can be proven as follows. The area ABC is defined from Fig. 4 as

$$(45) \qquad ABC \equiv \int_{p^A}^{p^1+t} \nabla_p m(1,p,u^1)dp + \int_{p^1}^{p^A} \nabla_p \pi(1,p)dp - t\nabla_p m(1,p^1+t,u^1).$$

From this we have

$$ABC = m(1,p^1+t,u^1)-m(1,p^A,u^1)+\pi(1,p^A)-\pi(1,p^1)-t\nabla_p m(1,p^1+t,u^1),$$

$$= \bar{v} - m(1,p^A,u^1) + \pi(1,p^A) \qquad \text{(from (39))},$$

$$= L_{ADD} \qquad \text{(from (44))}.$$

In Fig. 4, we have also drawn two triangles ABC' and ABC". ABC' is a linear approximation to ABC using the slopes of the Hicksian demand curve and the supply curve at the optimum point whereas ABC" is a linear approximation to ABC using the slopes of the two curves at the distorted equilibrium. These two triangles correspond to the two approximations of the ADD measure of waste (20) and (22) in this simple example. To show this, let us first construct a z-equilibrium as in the previous section for this simple model as follows:

$$(46) \qquad \nabla_p \pi(1,p(z)) - \nabla_p m(1,p(z)+tz,u^1) = 0,$$

$$(47) \qquad \nabla_w \pi(1,p(z)) - \nabla_w m(1,p(z)+tz,u^1) + \bar{v} = r(z),$$

where $0 \leq z \leq 1$ and $p(0) \equiv p^A$, $p(1) \equiv p^1$, $r(0) \equiv L_{ADD}$ and $r(1) \equiv 0$. When $z = 0$, (46) and (47) correspond to (42) and (43), and when $z = 1$, they correspond to (37) and (38). Totally differentiating (46) and (47) with respect to z we can compute $r'(z)$. From this $r'(0)$, $r''(0)$ and $r'(1)$ are also computable so that we can calculate the two approximations to the ADD measure of waste (20) and (21) as follows (see Appendix V):

$$(48) \qquad \frac{t^2\Sigma_{pp}^{\ 0}\ S_{pp}^{\ 0}}{2(\Sigma_{pp}^{\ 0} - S_{pp}^{\ 0})} ,$$

$$(49) \qquad \frac{t^2\Sigma_{pp}^{\ 1}\ S_{pp}^{\ 1}}{2(\Sigma_{pp}^{\ 1} - S_{pp}^{\ 1})}$$

where $\Sigma_{pp}^{\ z} \equiv \nabla_{pp}^{\ 2}\, m(1,p(z)+tz,u^1)$ and $S_{pp}^{\ z} \equiv \nabla_{pp}^{\ 2}\pi(1,p(z))$ for $z = 0,1$.

As $AB = t$ and the height of the triangle ABC' is $(t\Sigma_{pp}^{\ 0}\ S_{pp}^{\ 0})/(\Sigma_{pp}^{0} - S_{pp}^{0})$, (48) equals the area ABC' while the height of ABC" is $(t\Sigma_{pp}^{\ 1}\ S_{pp}^{\ 1})/(\Sigma_{pp}^{1} - S_{pp}^{1})$ so that (49) equals the area ABC". (See Appendix V.) Note that the slope of AC' equals the slope of the demand curve at C while the slope of BC' equals the slope of the supply curve at the point C. Therefore, in this simple model, our triangular expression of the deadweight loss corresponds to that by Harberger (1964) except that we allowed for nonlinear production possibilities set.

We next turn to a diagrammatic interpretation of the HB measure of welfare and its approximations. For this purpose, we first have to find a price vector which supports the social optimum. In a single consumer economy, it may be defined as the price vector which corresponds to the utility maximum given resource and technology constraints. Therefore, it is the price solution (p^B, w^B) to the concave programming problem below:

$$(50) \qquad \mathrm{Max}_{x,L,y,v}\{f(x,L) : y \geq x,\ \bar{v} \geq v + L,\ y \leq g(v)\}$$

$$(51) \qquad \equiv \mathrm{Max}_u\ \mathrm{Min}_{p\geq 0, w\geq 0}\{u - m(w,p,u) + \pi(w,p) + w\bar{v}\}.$$

We assume that an interior optimum point (u^B, p^B, w^B) solves (51) uniquely with $p^B > 0$ and $w^B > 0$. It is a solution to the following first order necessary conditions for (51):

$$(52) \qquad \nabla_u m(w^B, p^B, u^B) = 1,$$

$$(53) \qquad \nabla_p \pi(w^B, p^B) - \nabla_p m(w^B, p^B, u^B) = 0,$$

$$(54) \qquad \bar{v} + \nabla_w \pi(w^B, p^B) - \nabla_w m(w^B, p^B, u^B) = 0.$$

As (53) and (54) are unchanged by a proportional change of w and p, we set $w^B = 1$. For this normalization, we can assume that (52) is always met by choosing a money-metric normalization of the utility function at the reference price $(1, p^B)$. Therefore we can delete (52) from the system and assume that (53) and (54) determine u^B and p^B from $w^B = 1$. Using this normalization, (53) and (54) imply the following budget constraint of the representative consumer for the optimum price vector $(1, p^B)$:

$$(55) \qquad \bar{v} + \pi(1, p^B) = m(1, p^B, u^B).$$

Now the HB measure of welfare (3) in this simple model may be defined as follows:

$$(56) \qquad L_{HB} \equiv m(1,p^B,u^B) - m(1,p^B,u^1).$$

Fig. 5. illustrates the HB measure for this simple model. This is nothing but a Hicksian compensating variation when moving from a tax-distorted equilibrium to a social optimum.

We also illustrate L_{HB} using a Hotelling-Harberger-like expression in Fig. 6. This figure is the same as Fig. 4 for the Hicksian compensated demand curve for the good $\nabla_p m(1,p,u^1)$ using the tax-distorted utility level u^1 and the supply curve $\nabla_p \pi(1,p)$. We also include the compensated demand curve for the good for the socially optimum utility level, $\nabla_p m(1,p,u^B)$. Fig. 6 corresponds to the case where the good is normal so that $\nabla_p m(1,p,u^B)$ is above $\nabla_p m(1,p,u^1)$. If the good is inferior, the former curve is below the latter curve and if the good changes from a normal to an inferior good then the two curves intersect. Our results below apply to all cases listed above. Using Fig. 6, the HB measure can be shown to be equal to the sum of two curvilinear triangles AFE and FBD. To show this, first note that L_{HB} can be decomposed as follows:

$$(57) \qquad L_{HB} = \{m(1,p^B,u^B) - m(1,p^1+t,u^1)\} + \{m(1,p^1+t,u^1) - m(1,p^B,u^1)\}.$$

Substituting (39) and (55) into the first term on the right-hand side of (57), L_{HB} may be further rewritten as

$$\text{(58)}\qquad L_{HB} = \{\pi(1,p^B)-\pi(1,p^1)-t\nabla_p m(1,p^1+t,u^1)\}+\{m(1,p^1+t,u^1)-m(1,p^B,u^1)\}.$$

However, the sum of the areas AFE and FBD, denoted as AFBCDE, is

$$\text{(59)}\qquad AFBCDE = \int_{p^B}^{p^1+t} \nabla_p m(1,p,u^1)dp + \int_{p^1}^{p^B} \nabla_p \Pi(1,p)dp - t\nabla_p m(1,p^1+t,u^1).$$

Performing the integration in (59) yields the expression in (58).

For this simple model, the triangles ABG and ABC" drawn in Fig. 6 correspond to the approximation to the HB measure where ABG corresponds to (35) and ABC" is its variant where observed information is used. This may be shown as follows. First, construct a z-equilibrium:

$$\text{(60)}\qquad \nabla_p \pi(1,p(z)) - \nabla_p m(1,p(z) + tz, u(z)) = 0,$$

$$\text{(61)}\qquad \bar{v} + \nabla_w \pi(1,p(z)) - \nabla_w m(1,p(z) + tz, u(z)) = 0,$$

where $0 \leq z \leq 1$ and $p(0) \equiv p^B$, $p(1) \equiv p^1$, $u(0) \equiv u^B$ and $u(1) \equiv u^1$. When $z = 0$, (60) and (61) correspond to (53) and (54) with $w^B = 1$ and when $z = 1$ they correspond to (37) and (38). Totally differentiating (60) and (61) with respect to z, we can derive $u'(z)$. From this $u'(0)$ and $u''(0)$ can also be computed so that we can calculate the second order approximation to the HB measure (34) as (see Appendix VI).

$$\text{(62)}\qquad \frac{t^2\Sigma_{pp}^0 \, S_{pp}^0}{2(\Sigma_{pp}^0 - S_{pp}^0)}$$

where $\Sigma_{pp}^{z} \equiv \nabla_{pp}^{2}m(1,p(z) + tz, u(z))$ and $S_{pp}^{z} \equiv \nabla_{pp}^{2}\pi(1,p(z))$ for $z = 0,1$. Note that Σ_{pp}^{0} and S_{pp}^{0} are different from the analogous expression in (48) since, in (62), the derivatives are evaluated at the socially optimum point $(1,p^B,u^B)$. As AB = t and the height of the triangle ABG is $(t\Sigma_{pp}^{0} S_{pp}^{0})/(\Sigma_{pp}^{0} - S_{pp}^{0})$, (62) equals the area ABG. If (62) is further approximated by replacing the derivatives Σ_{pp}^{0} and S_{pp}^{0} by those observable derivatives Σ_{pp}^{1} and S_{pp}^{1}, then this approximation is identical to (49) which is a suggested approximation of the ADD measure. (49) is illustrated in Fig. 6 as ABC", which is also shown in Fig. 4.

In this simple model, the ADD measure and the HB measure coincide if the two points C and D coincide in Fig. 6; i.e., the ADD optimum and the social optimum coincide. (48) and (62) (or ABC' in Fig. 4 and ABG in Fig. 6), which are second order approximations of the ADD measure, and the HB measure coincide if the curvatures of the compensated demand functions and the supply function at points C and D are the same. However, the further approximations to these functions depending on the derivatives of the supply and compensated demand functions at the observed equilibrium coincide for this simple model as the triangle ABC". It is, however, clear from the discussion of the previous section that this identity cannot go through for a general many-consumer model. Finally, all of these approximations coincide if $\nabla_p m(1,p,u^1) = \nabla_p m(1,p,u^B)$ for all p. This is the case where there is no income effect for the good and, in this case, the Marshallian consumer's surplus coincides with the ADD and the HB measures (see Hicks (1946;38-41)).

FOOTNOTES FOR CHAPTER 1

[1] These two measures were examined comparatively by Diewert (1981, 1984, 1985a).

[2] $x \gg 0_N$ means that each element of the vector x is strictly positive, $x \geq 0_N$ means that each element of x is nonnegative, and $x > 0_N$ means $x \geq 0_N$ but $x \neq 0_N$. A superscript T means transpose.

[3] Note also that the solution to (1) may not correspond to a Pareto optimal point. This does not, however, contradict the Pareto inclusiveness of the ADD measure which is discussed in this section.

[4] See Diewert (1982;554) for the regularity properties that must be satisfied by the functions m^h.

[5] Most welfare evaluation methods cannot even generate orderings. For example, the Kaldor (1939)-Hicks (1939)-Scitovsky (1941-2(a)) test is neither complete nor transitive (see Gorman (1955)). Aggregate Hicksian (1941-2) compensating and equivalent variations cannot be transitive if the base price is not fixed (see, for example, Mohring (1971;365-7) or Blackorby and Donaldson (1985;256-7)).

[6] A widely adopted welfare measure by Diamond and McFadden (1974) can be shown to be an equivalent variation where tax-distorted prices are base prices. Therefore, it cannot give a consistent ranking of utilities across various tax schemes even in a single-consumer economy, i.e., not Pareto inclusive. See Kay (1980) and Pazner and Sadka (1980).

[7] We also assume that the Slater constraint qualification condition applies in this economy; i.e., we require that a feasible solution for (1) exists that satisfies the first N inequality constraints strictly.

[8] The definitions of various concepts of separability and their economic applications are surveyed in Geary and Morishima (1973) and Blackorby, Primont and Russell (1978). Pages 52-61 of the latter book are important for our analysis.

[9] With appropriate lump-sum transfers across households, the budget constraints of individuals are satisfied and the government budget constraint is implied by them and (26). Combined with Negishi's theorem, the program (23) and its interpretation may be regarded as a restatement of the second fundamental theorem of welfare economics, due originally to Arrow (1951).

[10] It is difficult in this case to interpret this further approximation by adopting the Quadratic Approximation Lemma in the same manner as with the ADD measure. However, using the money-metric utility scaling adopted later, we can show that $-\frac{1}{2}\{p^1(1)^T B_{pp}^1 p'(1) + [p'(1)+t]^T B_{qq}^1 [p'(1)+t]\} + \frac{1}{2}u'(1)^T B_{uu}^1$ $u'(1)$ is also accurate for quadratic functions as (35) by this Lemma.

[11] If we assume that there exists a concave Bergson-Samuelsonian social welfare function which is maximized at the equal division equilibrium, we can show that the second order approximation to the difference of the BSSWF, evaluated at an optimum or distorted equilibrium $\tilde{L}_{BS}$, satisfies the inequality $\tilde{L}_{BS} \geq \tilde{L}_L$.

[12] The term money metric utility was introduced into economics by Samuelson (1974), but the concept dates back to McKenzie (1957). We assumed that $m^h(p^0,u^h)$ is strictly increasing in u^h. Its sufficient condition was given by Weymark (1985). We also assume that $m^h(p^0,f^h(x^h))$ is concave in x^h for the reference price p^0, but this is not guaranteed in general. See Blackorby and Donaldson (1986). Applications of money metrics to applied welfare economics are given by King (1983) and McKenzie (1983).

[13] Most computable general equilibrium models adopt neo-classical perfect market assumptions. However, Pigott and Whalley (1982) incorporated public goods and Harris (1984) introduced increasing returns to scale by allowing fixed costs in numerical general equilibrium models.

[14] In contributions collected in Harberger (1974), he applied his methodology in various policy assessments. Many studies use the HB measure or Marshallian consumer surpluses for the same purpose (see Currie-Murphy-Schmitz (1971)).

[15] Needless to say, both second order approximations as well as a mean value of the two first order derivatives are exact approximations for quadratic functions.

[16] We do not survey these studies in this paper. Excellent surveys were provided by Auerbach (1985;86-118), Mirrlees (1986) and Drèze and Stern (1986).

Appendices for Chapter 1

Appendix I: The Properties of the Overspending Function.

An overspending function, introduced into economics by Bhagwati, Brecher and Hatta (1983;608) summarizes the general equilibrium relations of an economy within one equation. It may be interpreted as the aggregate net expenditure of consumers facing prices q minus the aggregate profits of firms facing prices p. It inherits many useful properties of expenditure functions and profit functions which are exhibited in Diewert (1982). We collect several important properties for later use.

An overspending function is defined by

$$\text{(A.1)} \qquad B(q,p,u) \equiv \Sigma_{h=1}^{H} m^h(q,u_h) - \Sigma_{h=0}^{H} q^T \bar{x}^h - \Sigma_{k=0}^{K} \pi^k(p).$$

It has the following properties.

(i) B is concave with respect to p and q.

(ii) If B(q,p,u) is once continuously differentiable with respect to q and p at (q,p,u), then $\nabla_q B(q,p,u)$ is the aggregate net consumption vector and $-\nabla_p B(q,p,u)$ is the aggregate net production vector.

(iii) The following identities are valid for any (q,p,u) if B is twice continuously differentiable at (q,p,u):

$$\text{(A.2)} \qquad q^T B_{qq} = 0_N^T,$$

$$\text{(A.3)} \qquad q^T B_{qu} = (\nabla_u B)^T = (\partial m^1(q,u_1)/\partial u_1, \ldots, \partial m^H(q,u_H)/\partial u_H),$$

(A.4) $\quad p^T B_{pp} = 0_N^T,$

where $B_{ij} \equiv \nabla^2_{ij} B(q,p,u)$ for $i,j = q,p,u$.

Property (i) follows from the fact that an expenditure function is concave with respect to prices and a profit function is convex with respect to prices. Property (ii) is a straightforward consequence of Hotelling's (1932;594) lemma and the Hicks (1946;331)-Shephard (1953;11) lemma. Property (iii) is a consequence of the linear homogeneity of an expenditure function and a profit function with respect to prices.

Appendix II

In this Appendix, we show that (1) and (7) are equivalent given quasiconcave utility functions and convex production sets, provided the Slater constraint qualification holds. In (1), the set $\{x^h : f^h(x^h) \geq u_h^1\}$ is convex from the quasi-concavity of $f^h(x^h)$, S^k is also assumed to be convex and the inequalities are linear. Therefore, (1) is a concave programming problem and the Uzawa-Karlin Saddle Point Theorem is applicable. Rewrite (1) as:

$$\text{(A.5)} \quad r^0 \equiv \max_{r,x^h,y^k} \min_{p \geq 0_N} \{r + p^T[\textstyle\sum_{k=1}^{H} y^k + \sum_{h=1}^{H} \bar{x}^h - \sum_{h=1}^{H} x^h - \beta r] :$$

$$f^h(x^h) \geq u_h^1,\ h = 1,\ldots,H;\ y^k \in S^k,\ k = 1,\ldots,K\},$$

$$(A.6) \quad \equiv \max_r \min_{p \geq 0_N} \{r(1-p^T\beta) + \Sigma_{h=1}^H p^T\bar{x}^h + \Sigma_{h=1}^H [\max - p^T x^h : f^h(x^h) \geq u_h^1]$$

$$+ \Sigma_{k=1}^K [\max p^T y^k : y^k \in S^k]\}$$

$$\equiv \max_r \min_{p \geq 0_N} \{r(1 - p^T\beta) + \Sigma_{h=1}^H p^T\bar{x}^h + \Sigma_{k=1}^K \pi^k(p) - \Sigma_{h=1}^H m^h(p,u_h^1)\}$$

using the definitions (4) and (6).

Appendix III

In this Appendix, we derive (17). Premultiply both sides of (16) by $[p(z)^T,0]$. Using (15), we have:

$$(A.7) \qquad p(z)^T B_{qq}^z (p'(z) + t) + p(z)^T B_{pp}^z p'(z) = - r'(z).$$

From (A.2) and (A.4) evaluated at $(q,p,u) = (p(z) + tz,\ p(z),u^1)$, we have $p(z)^T B_{qq}^z = - zt^T B_{qq}^z$ and $p(z)^T B_{pp}^z = 0_N^T$. Substituting these equations into (A.7), we have (17).

Appendix IV

In this Appendix, we derive (19). Differentiate (17) with respect to z and evaluate at $z = 0$, and we have

$$(A.8) \qquad - r''(0) = -t^T B_{qq}^0 (p'(0) + t).$$

Next premultiply both sides of (16) evaluated at $z = 0$, by $[p'(0)^T,r'(0)]$. We obtain

$$(A.9) \qquad - p'(0)^T B_{qq}^0 (p'(0) + t) - p'(0)^T B_{pp}^0 p'(0) = 0.$$

Adding (A.8) and (A.9), we have (19).

Appendix V

Total differentiation of (46), (47) gives the following:

$$(A.10) \qquad \begin{bmatrix} S_{pp}^z - \Sigma_{pp}^z, & 0 \\ S_{wp}^z - \Sigma_{wp}^z, & -1 \end{bmatrix} \begin{bmatrix} p'(z) \\ r'(z) \end{bmatrix} = \begin{bmatrix} \Sigma_{pp}^z t \\ \Sigma_{wp}^z t \end{bmatrix}$$

where $\Sigma_{ij}^z \equiv \nabla_{ij}^2 m(1,p(z)+tz,u^1)$ and $S_{ij}^z \equiv \nabla_{ij}^2 \pi(1,p(z))$ for i,j, = p,w. Premultiplying both sides by [p(z),1], using the identities

$$S_{wp}^z + p(z)S_{pp}^z = 0, \; \Sigma_{wp}^z + (p(z)+tz)\Sigma_{pp}^z = 0,$$

we have

$$(A.11) \qquad r'(z) = zt\Sigma_{pp}^z(p'(z)+t).$$

Inverting the right-hand side matrix of (A.10), we have

$$p'(z) = (\Sigma_{pp}^z t)/(S_{pp}^z - \Sigma_{pp}^z).$$

Substituting it into (A.11), we have

(A.12) $\quad r'(z) = (zt^2 S_{pp}^{z}\Sigma_{pp}^{z})/(S_{pp}^{z}-\Sigma_{pp}^{z})$.

Using (A.12), we can compute the two approximations (20) and (21) which correspond to (48) and (49) respectively.

Now draw a perpendicular line from point C' to AB and define the cross point with AB, H. Then the height of the triangle is C'H. We have C'H = $-\Sigma_{pp}^{0}\cdot AH = S_{pp}^{0}\cdot(t-AH)$. From these two equations, we can solve C'H = $(t\Sigma_{pp}^{0}S_{pp}^{0})/(\Sigma_{pp}^{0}-S_{pp}^{0})$. The proof of ABC'' is perfectly analogous.

<u>Appendix VI</u>

Totally differentiating (60) and (61) with respect to z, we have

$$\text{(A.13)} \quad \begin{bmatrix} -\Sigma_{pu}^{z}, S_{pp}^{z}-\Sigma_{pp}^{z} \\ \\ -\Sigma_{wu}^{z}, S_{wp}^{z}-\Sigma_{wp}^{z} \end{bmatrix} \begin{bmatrix} u'(z) \\ \\ p'(z) \end{bmatrix} = \begin{bmatrix} \Sigma_{pp}^{z}t \\ \\ \Sigma_{wp}^{z}t \end{bmatrix}$$

where $\Sigma_{ij}^{z} \equiv \nabla_{ij}^{2}m(1,p(z)+tz,u(z))$, $S_{ij}^{z} \equiv \nabla_{ij}^{2}\pi(1,p(z))$ for i,j = w,p,u. We compute u'(z) by inverting the left-hand side matrix of (A.13). First, the determinant of the matrix D is

(A.14) $\quad D \equiv \Sigma_{wu}^{z}\,(S_{pp}^{z}-\Sigma_{pp}^{z}) - \Sigma_{pu}^{z}(S_{wp}^{z}-\Sigma_{wp}^{z})$.

Using the linear homogeneity properties of m and π,

$$\Sigma_{wu}^{z} + (p(z)+tz)\Sigma_{pu}^{z} = \nabla_u m(1,p(z)+tz,u(z)),$$

$$\Sigma_{wp}^{z}+(p(z)+tz)\Sigma_{pp}^{z} = 0, \quad S_{wp}+p(z)S_{pp}^{z} = 0,$$

we can rewrite (A.14) as

$$\text{(A.15)} \qquad D \equiv - \nabla_u m(1,p(z)+tz,u(z))(\Sigma_{pp}^{z}-S_{pp}^{z}) - zt\Sigma_{pu}^{z}S_{pp}^{z}.$$

The numerator of u'(z), defined as N, is given by

$$\begin{aligned}\text{(A.16)} \qquad N &\equiv \{S_{wp}^{z} - \Sigma_{wp}^{z}\}\ \Sigma_{pp}^{z}t - (S_{pp}^{z} - \Sigma_{pp}^{z})\ \Sigma_{wp}^{z}t \\ &= \{-p(z)S_{pp}^{z}+(p(z)+tz)\Sigma_{pp}^{z}\}\ \Sigma_{pp}^{z}t + (S_{pp}^{z}-\Sigma_{pp}^{z})(p(z)+tz)\Sigma_{pp}^{z}t \\ &= zt^{2}S_{pp}^{z}\Sigma_{pp}^{z}\end{aligned}$$

From (A.15) and (A.16) we have

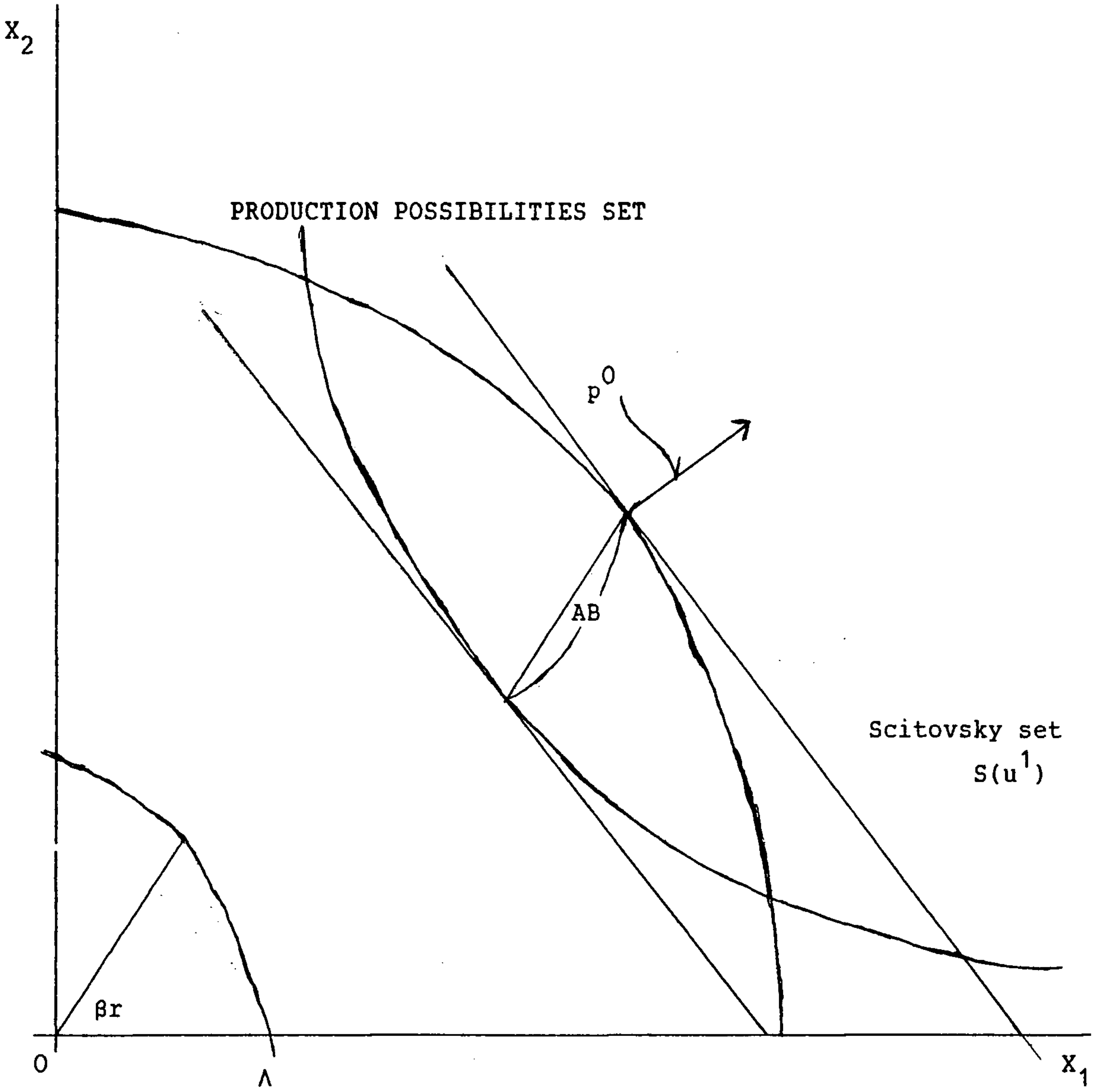
$$\text{(A.17)} \qquad u'(z) = -\{zt^{2}S_{pp}^{z}\Sigma_{pp}^{z}\}/\{\nabla_u m(1,p(z)+tz,u(z))(\Sigma_{pp}^{z}-S_{pp}^{z}) + zt\Sigma_{pu}^{z}S_{pp}^{z}\}.$$

From (A.17) we have,

$$u'(0) = 0 \text{ and } u''(0) = -\{t^{2}S_{pp}^{0}\Sigma_{pp}^{0}\}/\{\nabla_u m(1,p^{0},u^{0})(\Sigma_{pp}^{0}-S_{pp}^{0})\}.$$

Substituting them into (34), we have (62).

We can show analogously as Appendix V that (62) coincides with the area ABG.

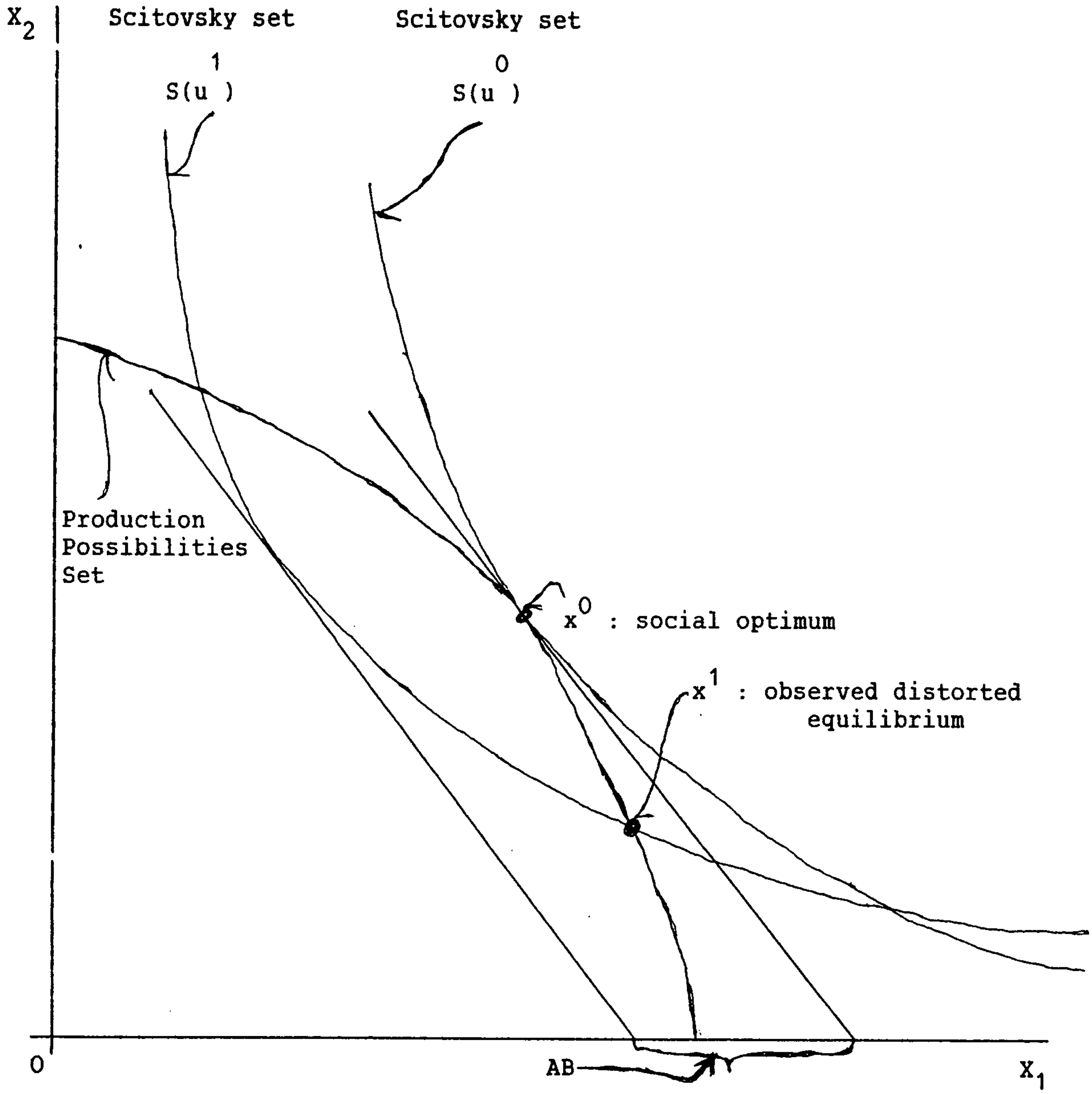


βr is the reference bundle

AB is the ADD measure

Fig. 1

The ADD Measure of Waste



AB is the HB measure

Fig. 2

The HB Measure of Welfare

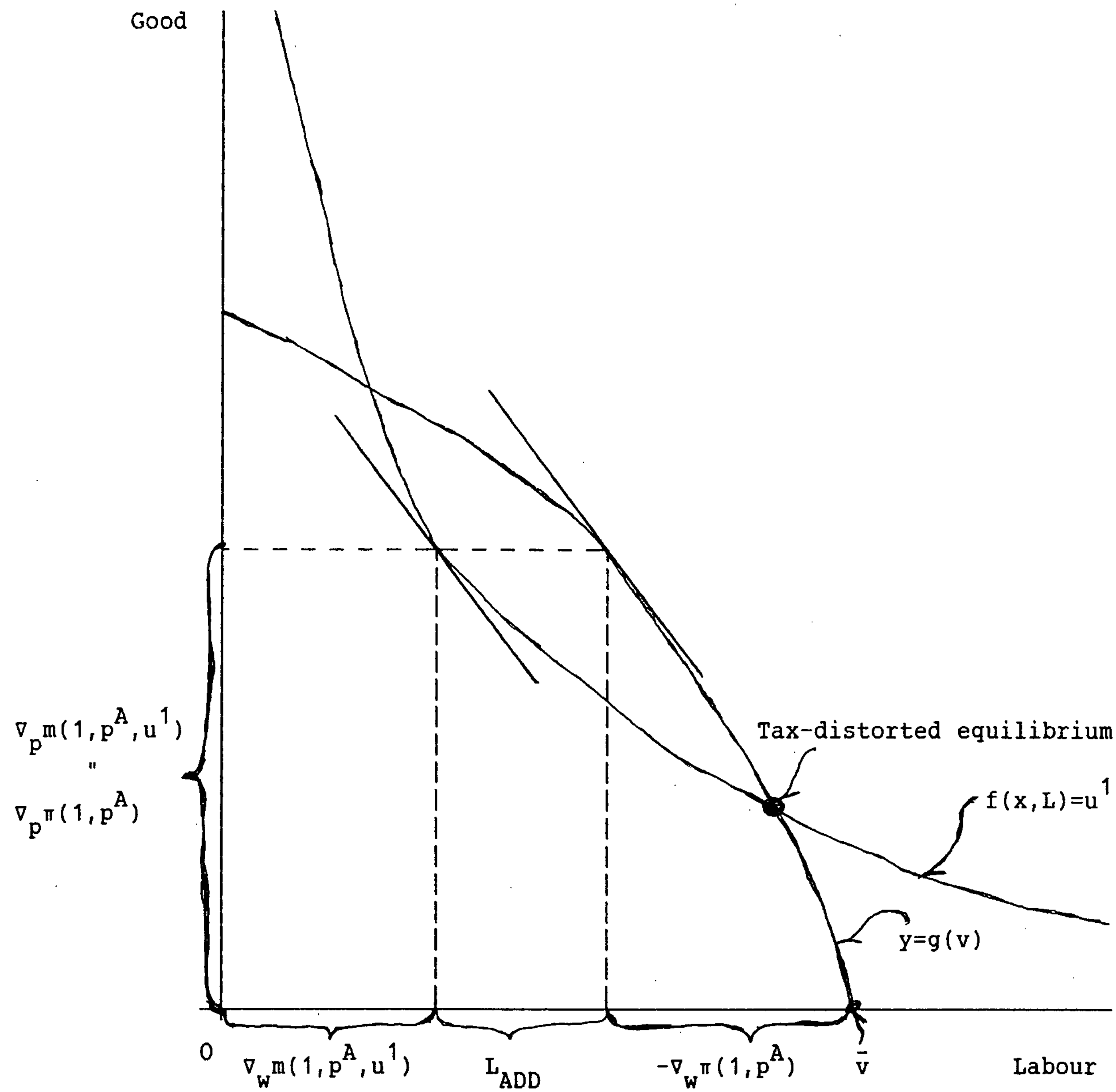


Fig. 3

The ADD Measure: A One-Consumer Two-Goods Economy

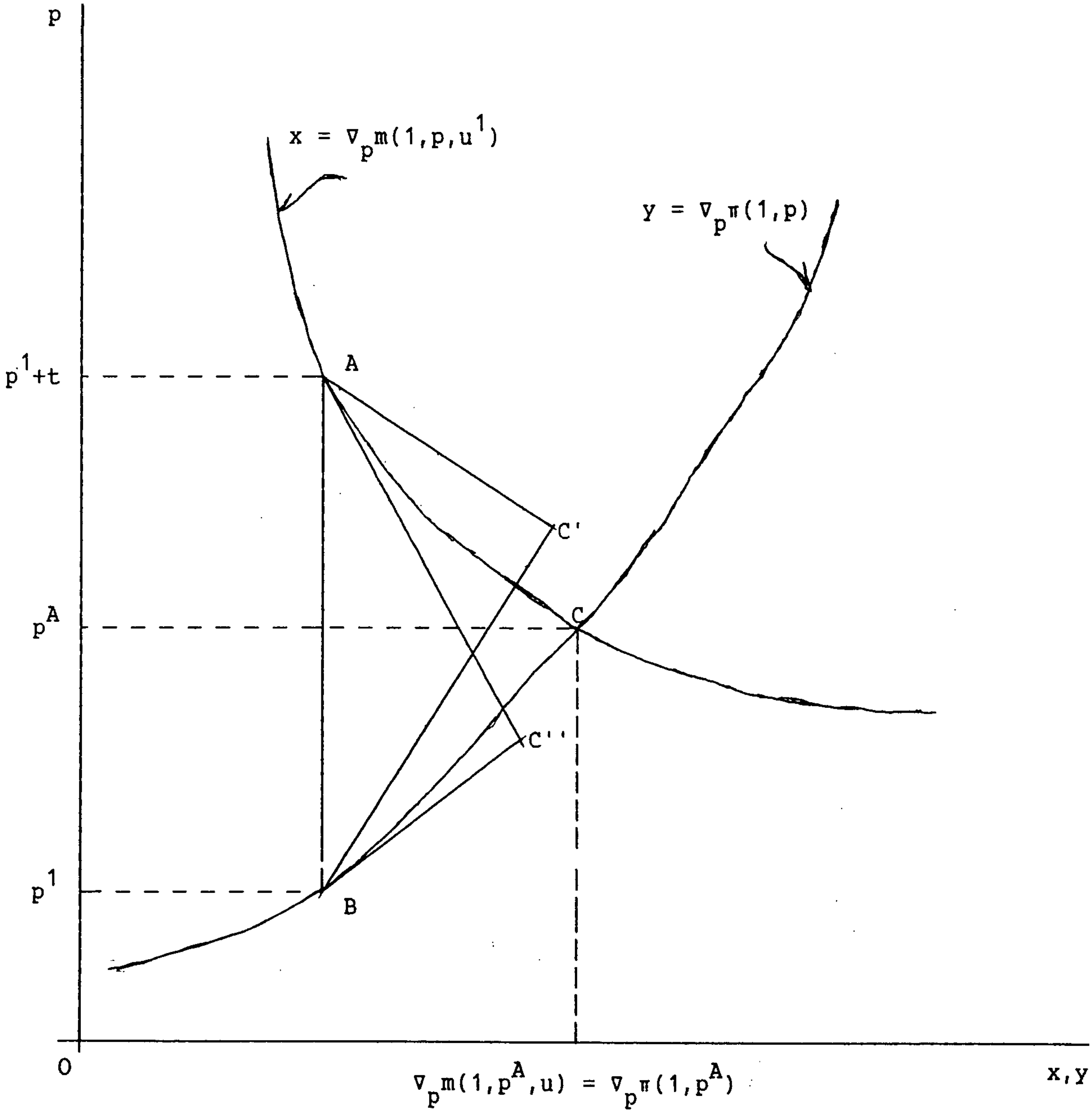


Fig. 4

The ADD Measure and its Approximations: A One-Consumer Two-Goods Economy

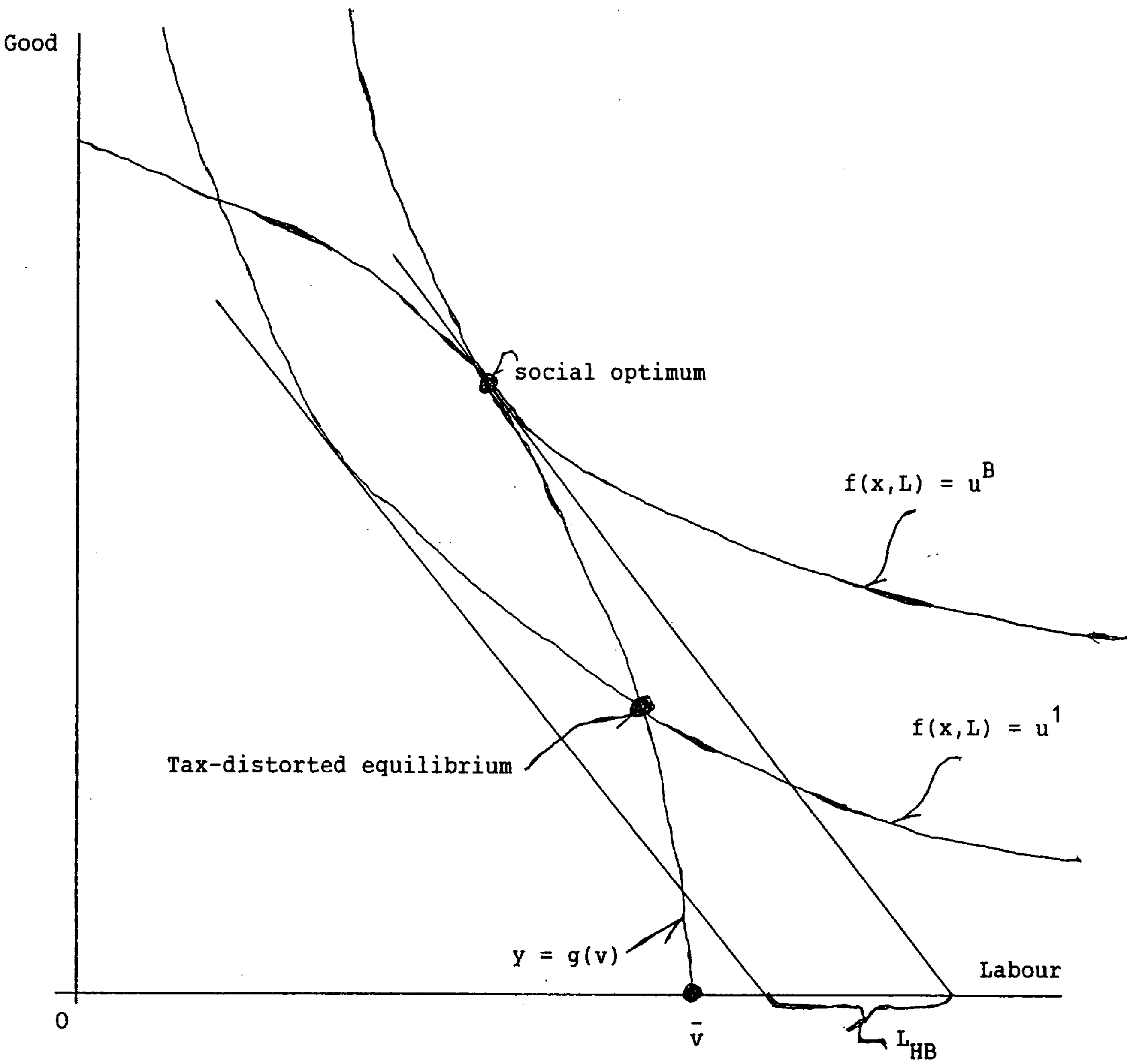


Fig. 5

The HB Measure: A One-Consumer Two-Goods Economy

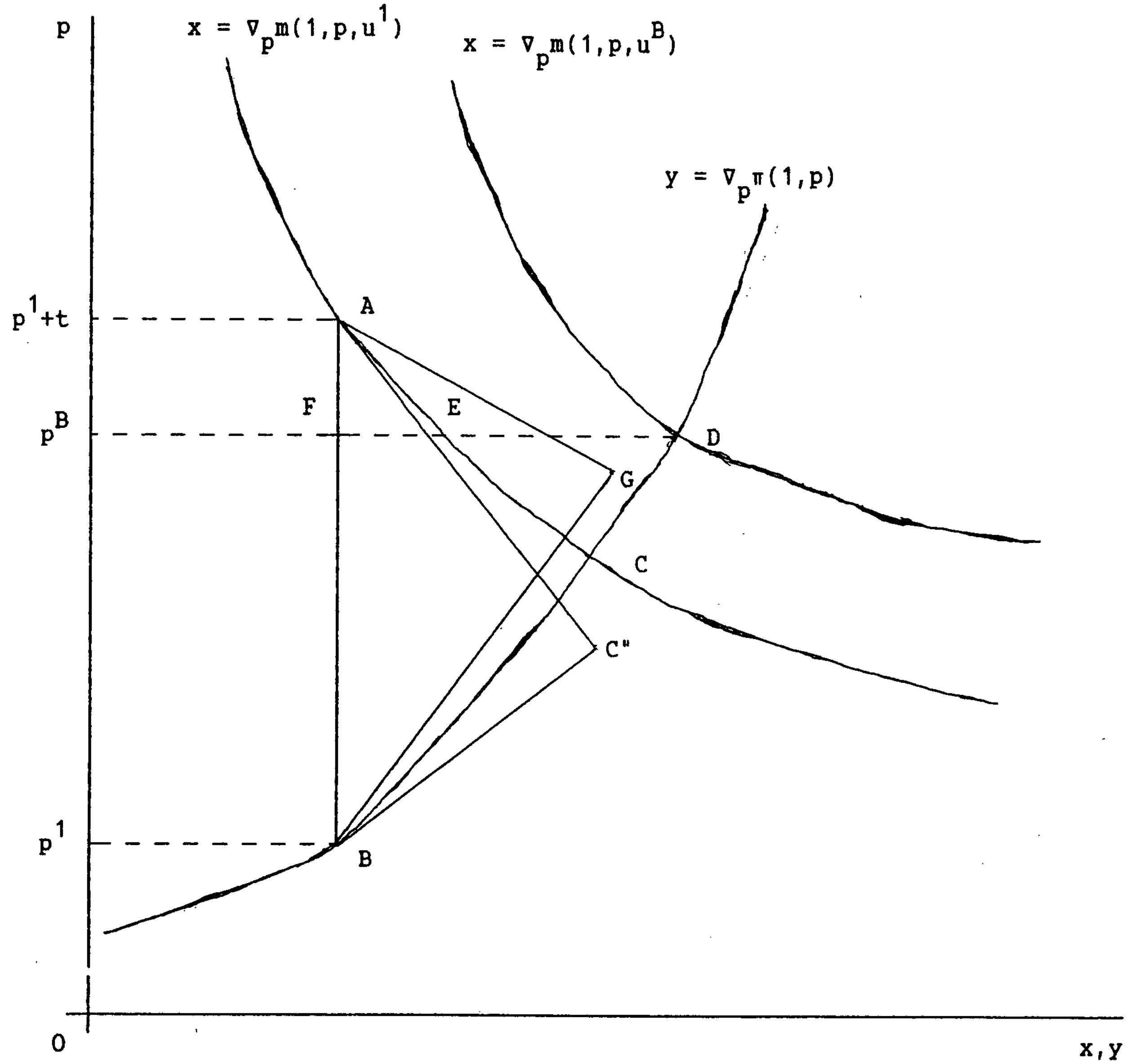


Fig. 6

The HB Measure and Its Approximations: A One-Consumer Two-Goods Economy

CHAPTER 2

THE MEASUREMENT OF WASTE IN A PUBLIC GOODS ECONOMY

2-1. Introduction

In the long history of the study on the measurement of deadweight loss in applied welfare economics, the waste due to indirect taxation has been the main concern of this literature. This section proposes a methodology for measuring the waste due to an externality, which seems to be an alternative and equally important situation involving a market failure. Though our methodology is applicable to other externalities, here we focus on the problem of public goods.

Consider a government which collects revenue from both lump-sum and indirect taxation and provides public goods. This economy exhibits the waste due to a price distortion and to an incomplete market at the same time. As was already suggested by Harberger (1964;73), the deadweight loss of the whole economy depends on the difference between the social benefit and social cost of public goods in addition to the set of indirect taxes (or mark-up rates of noncompetitive firms). We derive approximations to the Allais-Debreu-Diewert measure of waste of this public good economy, and we show that the approximate deadweight loss can be expressed in terms of the derivatives of restricted expenditure functions and restricted profit functions evaluated at the observed equilibrium as long as we know the marginal benefits of public goods for consumers. In deriving the approximate waste, we need not assume local linearity of the production possibilities set as in Harberger (1964) and we need not assume restrictive functional forms for utility and production function as in the numerical or applied general equilibrium literature. The waste to be studied is due to the simultaneous existence of distortionary taxes and the nonoptimal provision of public goods. Needless to say, a simple sum of these two types of waste cannot even approximate the simultaneous loss measured in this section.

The next section is devoted to the description of our model of a public goods economy, while section 3 defines the Allais-Debreu-Diewert measure of waste in this economy. In section 4, we compute second order approximations to the ADD measure to gain more insight about the nature of the waste. We also interpret the empirical significance of the approximate ADD measure. In section 5, some drawbacks to our approximate ADD measures are discussed and a diagrammatic exposition of our analysis is presented.

2-2. The Model

Our model is similar to the one used in Section 1-1 and 1-2 except that we now introduce public goods into the model. There are N private goods which are traded at positive prices $p = (p_1,\ldots,p_N)^T$ and I public goods which affect both consumers' utilities and the production possibilities sets of firms. A quantity vector of public goods is denoted as $G = (G_1,\ldots,G_I)^T > 0_I$.

There are K profit maximizing private firms which produce goods and services by utilizing both private and public inputs using the production possibilities set S^k for $k = 1,\ldots,K$, i.e., if $(y^k, -G) \in S^k$, then the vector of net outputs $y^k = (y_1^k,\ldots,y_N^k)^T$ is producible by sector k using the vector of public goods G. The sector k <u>restricted profit function</u> π^k, which is dual to the production possibilities set S^k, is:

$$(1) \qquad \pi^k(p,G) = \max_y \{p^T y : (y, -G) \in S^k\}, \; k = 1,\ldots,K,$$

where $p \gg 0_N$. We assume that either G or an entrepreneurial factor is a limiting factor of production, so that S^k exhibits decreasing returns to scale when G is fixed. (See Meade (1952) for the definition of an 'unpaid factor' public input.)

The vector of public goods is produced by the government, k = 0, which has the production possibilities set S^0. If $(y,G) \in S^0$, the government produces G using the input vector y. If some component of y is positive, the government is jointly producing the corresponding private good with G. The government restricted profit function π^0, which is dual to S^0, is:

$$(2) \qquad \pi^0(p,G) = \max_y \{p^T y : (y,G) \in S^0 \},$$

where $p \gg 0_N$.[1]

Let us now look at the consumer side of our model. We assume that there are H individuals, h = 1,...,H, in the economy. The preference of individual h can be represented by a quasi-concave utility function f^h defined over a translated orthant in R^{N+I}, Ω^h. Define the individual h <u>restricted expenditure function</u> m^h, which is dual to f^h, for h = 1,...,H, by:

$$(3) \qquad m^h(p,G,u_h) = \min_x \{p^T x : f^h(x,G) \geq u_h, (x,G) \in \Omega^h\}$$

where $p \gg 0_N$ and $u_h \in$ Range f^h. We suppose that each individual h possesses nonnegative endowment vector of private goods, $\bar{x}^h \geq 0_N$, for h = 1,...,H. We also allow the government, which is h = 0, to have an initial endowment vector $\bar{x}^0 \geq 0_N$.

As in section 1-2, the government raises revenue by the set of indirect taxes $t = (t_1, \ldots, t_N)^T$ to provide the public goods. The government can also make a net transfer g_h to individual h. If $g_h < 0$, $-g_h$ is the amount of lump-sum tax collected from person h. Producers face prices $p \gg 0_N$ whereas consumers face $p + t \gg 0_N$ at the observed distorted equilibrium.

We use the overspending function defined by

$$(4) \qquad B(q,p,G,u) \equiv \Sigma_{h=1}^{H} m^h(q,G,u_h) - \Sigma_{h=0}^{H} q^T \bar{x}^h - \Sigma_{k=0}^{K} \pi^k(p,G)$$

to characterize our general equilibrium system. Diewert (1986;131-155, 170-176) showed that the properties of a profit function and an expenditure function are valid in their restricted functional form. This means that the properties of an overspending function (i) - (iii) listed in Appendix I to Chapter 1 are valid for (4). Diewert (1986) also showed that: (iv) a restricted profit function is concave with respect to G if the production possibilities set is convex and a restricted expenditure function is convex with respect to G, so that B is convex with respect to G, (v) $-\nabla_G m^h(q,G,u_h)$, for $h = 1,\ldots,H$, is the marginal benefit vector of consumer h for the public goods; $\nabla_G \pi^k(p,G)$ for $k = 1,\ldots,K$ is the marginal benefit vector of firm k for the public goods, and $-\nabla_G \pi^0(p,G)$ is a marginal cost vector for the public goods, so that $-\nabla_G B(q,p,G,u)$ shows the aggregate net benefit vector of public goods. From the linear homogeneity of B with respect to prices, the identity

$$(5) \qquad q^T B_{qG} + p^T B_{pG} = (\nabla_G B)^T$$

holds in addition to (1.A.2) - (1.A.4).

The system of equations characterizing the observed equilibrium is now stated in a fairly simple manner where α^{hk} is defined as the fraction of a firm k held by individual h, with $0 \leq \alpha^{hk} \leq 1$ for $h = 1,\ldots,H$ and $k = 1,\ldots,K$ and $\sum_{h=1}^{H}\alpha^{hk} = 1$ for $k = 1,\ldots,K$.

$$(6) \qquad m^h(p+t,G,u_h) = g_h + (p+t)^T\bar{x}^h + \sum_{k=1}^{K}\alpha^{hk}\pi^k(p,G),\ h = 1,\ldots,H,$$

$$(7) \qquad \nabla_q B(p+t,p,G,u) + \nabla_p B(p+t,p,G,u) = 0_N,$$

$$(8) \qquad -\nabla_G B(p+t,p,G,u) = d.$$

Here (6) shows the budget constraints for the H individuals and (7) shows the equality of demand and supply for goods $1,\ldots,N$. The government budget constraint is implied by (6) and (7). From the property (v), d in (8) defines the net marginal benefit vector of the public goods. If $d = 0_I$, (8) is consistent with the well-known Samuelson (1954)-Kaizuka (1965) conditions for the optimal provision of public goods. Therefore, $d \neq 0_I$ means that the public goods are not supplied optimally. We assume that the distortions parameter d arises because of the limited ability of the government to provide public goods efficiently.

We regard (6) - (8) as a general equilibrium system which determines $p_2,\ldots p_N,d,u$ and one component of t and g given the remaining components of t and g, with $p_1 = 1$ as numeraire and G fixed. We assume that an observed distorted equilibrium (u^1,p^1,G^1,t,d,g) exists.

2-3. An Allais-Debreu-Diewert Measure of Waste

An Allais-Debreu-Diewert measure of waste that was defined and discussed in Chapter 1 is now utilized to measure the waste due to the public good externalities.

Pick a nonnegative reference vector of private goods $\beta \equiv (\beta_1,\ldots,\beta_N)^T > 0_N^T$ and consider the following primal programming problem:

$$(9)\quad r^0 \equiv \max_{r,x^h,y^k,G.} \{r : \quad (i)\ \sum_{h=1}^{H} x^h + \beta r \leq \sum_{k=0}^{K} y^k + \sum_{h=0}^{H} \bar{x}^h \ ;$$
$$(ii)\ f^h(x^h,G) \geq u_h^1\ ;\ (x^h,G)\ \varepsilon\ \Omega^h,\ h = 1,\ldots,H;$$
$$(iii)\ (y^k,-G)\ \varepsilon\ S^k,\ k = 1,\ldots K;\ (y^0,G)\ \varepsilon\ S^0\ \}$$

where $u^1 = (u_1^1,\ldots,u_H^1)^T$ is the utility vector which corresponds to the observed distorted equilibrium defined in the previous section. The interpretation of $L_{ADD} \equiv r^0$ is discussed in Chapter 1 so that we will not repeat it here. For simplicity of computation, our reference bundle does not include public goods. Allais proposed to measure the waste in terms of a numeraire good, i.e., in our context $\beta = (1,0,\ldots,0)^T$. Debreu's coefficient of resource utilization model (which assumed that β was proportional to the economy's total endowment vector) is also consistent with our present model since we assumed that there were no endowments of public goods.

Given the level of G, (9) is a concave programming problem so that we can derive its dual equivalent problem:

$$(10)\qquad r^0 = \max_G[\max_r \min_{p\geq 0_N} \{r(1-p^T\beta) - B(p,p,G,u)\}].^2$$

(The process of derivation is analogous to that in Appendix 1-II.)

If G^0, r^0 and p^0 solve (10), then βr^0 is a measure of the resources that can be extracted from the economy while maintaining households at their distorted equilibrium utility levels and G^0 is a corresponding "optimal" level of public goods and p^0 is a vector of private goods prices which supports the efficient equilibrium. Note that in this "optimal" equilibrium, not only are public goods being provided efficiently, but also all commodity tax distortions have been removed.

Given the level of G, (10) may be rewritten by using the Uzawa-Karlin Saddle Point Theorem in reverse as

$$(11) \qquad r^0 = - \max_{p \geq 0_N} \{B(p,p,G,u^1) : p^T\beta \geq 1\}$$

where B is the overspending function defined by (4). If G^0, r^0 and p^0 solves (10), then p^0 solves (11) and r^0 is the associated Lagrangean multiplier for the constraint in (11). It is also the case that if G^0, r^0 and p^0 solve (10), then G^0 is the solution to the following unconstrained maximization problem:

$$(12) \qquad \max_G \{r^0(1-p^{0T}\beta) - B(p^0,p^0,G,u^1)\}.$$

Our expressions for the ADD measure, (10) and (11), present our basic approach to the measurement of deadweight loss. However, these abstract expressions do not indicate how the magnitude of the loss depends on the size of distortion parameters t and d. Furthermore, the global computation of

(10) is very difficult as was discussed in Chapter 1. Therefore, we turn to the computation of second order approximations to the ADD measure.

2-4. Second Order Approximations

To obtain a second order approximation to the loss measure, we require some stronger assumptions. Suppose that: (i) B is twice continuously differentiable with respect to q, p and G at the optimum of (10); (ii) $G^0 \gg 0_I$, $p^0 \gg 0_N$ so that the first order necessary conditions for the max min problem (10) hold with equality; (iii) Samuelson's (1947;361) strong second order sufficient conditions hold for (11) when the inequality constraints are replaced by equalities, and these conditions also hold for (12).

Consider the following system of equations in the N + I + 1 unknowns, p, G and r, regarded as functions of a scalar parameter z defined for $0 \leq z \leq 1$:

$$(13) \qquad \nabla_q B(p(z)+tz,p(z),G(z),u^1)+\nabla_p B(p(z)+tz,p(z),G(z),u^1)+\beta r(z) = 0_N,$$

$$(14) \qquad \nabla_G B(p(z)+tz,p(z),G(z),u^1) = -zd,$$

$$(15) \qquad 1 - p(z)^T\beta = 0.$$

When z = 0, define $p(0) = p^0$, $G(0) = G^0$ and $r(0) = r^0$. Then (13) - (15) become the first order conditions for the max min problem (10). Alternatively, when z = 1, define $p(1) = p^1$, $G(1) = G^1$ and $r(1) = 0$. Suppose that the reference waste bundle β satisfies the normalization

$$(16) \qquad p^{1T} \beta = 1,$$

by choosing the scale of β appropriately, which seems quite innocuous. Then, (13) - (15) coincide with (7), (8) and (16). Therefore, (13) - (15) characterizes the observed distorted equilibrium when $z = 1$. Note that when (7) and (8) are satisfied, (6) is also satisfied for the observed choice of g_h. Therefore, we can safely conclude that (13) - (15) maps the Allais-Debreu-Diewert reference equilibrium into the observed distorted equilibrium as z is adjusted from 0 to 1.

Differentiating the system (13) - (15) with respect to z and evaluating at $z = 0$, we obtain

$$(17) \qquad - \begin{bmatrix} B_{qq}^{0} + B_{pp}^{0}, & B_{qG}^{0} + B_{pG}^{0}, & \beta \\ B_{Gq}^{0} + B_{Gp}^{0}, & B_{GG}^{0} & , 0_I \\ \beta^T & , \quad 0_I^T & , 0 \end{bmatrix} \begin{bmatrix} p'(0) \\ G'(0) \\ r'(0) \end{bmatrix} = \begin{bmatrix} B_{qq}^{0} t \\ B_{Gq}^{0} t + d \\ 0 \end{bmatrix}$$

where the second order derivatives of the overspending function B_{ij}^{0}, i,j = q, p, G are evaluated at the optimum $z = 0$. The meaning of the B_{ij}^{0} are as follows: B_{qq}^{0} is an aggregate consumers' compensated substitution matrix whereas $- B_{pp}^{0}$ is an aggregate producers' substitution matrix evaluated at the optimum; B_{qG}^{0} shows the change of aggregate compensated demands with respect to an increase of public goods and $-B_{pG}^{0}$ shows the change of aggregate net supply of goods for firms with respect to an increase in the public good supply.

Now regard (17) as an identity in z, valid for z close to 0. Our assumptions (iii) introduced at the outset of this section imply that an inverse exists for the matrix on the left-hand side of (17). (See Diewert-Woodland (1977, Appendix I)). Hence, by the Implicit Function Theorem, there exist once continuously differentiable functions p(z), G(z) and r(z) which satisfy (13) - (15) in a neighbourhood of z = 0.

Premultiply both sides of (17) evaluated at z close to 0 by $[p(z)^T, 0_I^T, 0]$. Using identities, (1.A.2), (1,A.4), and (5) evaluated at the z-equilibrium, and then using (14) and (15) we get

$$(18) \qquad r'(z) = z[t^T B_{qq}^z (p'(z) + t) + t^T B_{qG}^z G'(z) + d^T G'(z)].$$

(The process for deriving (18) is similar to the one in Appendix 1-III.). From (18) we readily have

$$(19) \qquad r'(0) = 0.$$

Now differentiate (18) with respect to z, evaluate at z = 0, and adding the identity derived by premultiplying $[p'(0)^T, - G'(0)^T, 0]$ to both sides of (17), we find

$$(20) \qquad - r''(0) = G'(0)^T B_{GG}^0 G'(0) - p'(0)^T B_{pp}^0 p'(0) - [p'(0)+t]^T B_{qq}^0 [p'(0)+t].$$

(The derivation is analogous to the one in Appendix 1-IV.)

Note that the last two terms in the right-hand side of (20) are nonnegative because of the concavity of B with respect to prices. We also assume that B^0_{GG} is positive semidefinite; this assumption is satisfied if the production possibilities sets are all convex, but it is much milder than assuming global convexity in production. Intuitively, it means that the concavity of the utility functions outweighs any nonconvexity in aggregate production with respect to public goods in the neighbourhood of the optimum. Given this assumption, $-r''(0) \geq 0$ is implied.

The Allais-Debreu-Diewert measure of waste r(0) may be written as

$$(21) \qquad L_{ADD} = r(0) - r(1)$$

since $r(1) = 0$. A second order approximation to L_{ADD} is obtained by using a Taylor series expansion evaluated at $z = 0$,

$$(22) \qquad L_{ADD} \approx -\,[r'(0) + \tfrac{1}{2}r''(0)] = -\,\tfrac{1}{2}r''(0).$$

Using (19) we therefore have the following theorem.

<u>Theorem 1</u>

$$L_{ADD} \approx -\,\tfrac{1}{2}r''(0) \geq 0$$

where the inequality is valid from (20) and its following discussion. If r is quadratic, (22) provides an exact expression for L_{ADD}. To compute (22), use the expression for $r''(0)$ given in (20). The vectors of derivatives p'(0) and G'(0) in (20) can be calculated by inverting the matrix on the left-hand side of (17). Therefore, the information required to calculate the approximate ADD measure is the reference bundle β, the distortion parameters (t, d) and the second order derivatives of the overspending function evaluated at the optimum.

Let us scrutinize the informational requirements for computing (22) more carefully. The vectors t and β are directly observable. To know the vector d, we must know the consumers' marginal benefits from public goods evaluated at the observed consumer prices. This means that we must overcome the well-known preference revelation problem for public goods. Furthermore, to estimate the matrix B^0_{GG}, we need to know the derivatives of the net marginal benefits for public goods for both consumers and producers. To calculate the other second order derivatives of the overspending function, we need to know the first order derivatives of the net supply functions of firms for private goods and the compensated demand functions of consumers, which depend on both prices and public goods. Though the first set of functions is observable, the second set is not. It is well-known, however, that the compensated price elasticities can be computed from data on the ordinary demand functions using the derivatives with respect to both prices and income in the Slutsky equation. (See, for example, Diewert (1982;572).) Similarly, the derivatives of the compensated demand functions with respect to public goods can also be computed from market demand functions using 'Slutsky-like' equations (see Wildasin (1984;230)). The fact that we need information on the second order derivatives of the overspending function evaluated at the <u>optimum</u> considerably decreases the usefulness of (22), since these values are not observable (at the market distorted equilibrium) and, in general, are different from the values observed in the distorted equilibrium.

An alternative approach to approximating L_{ADD} can be developed using Diewert's (1976;118) Quadratic Approximation Lemma. This Lemma demonstrates that r(0) - r(1) can be appproximated by -(1/2)(r'(0) + r'(1)), with the approximation being exact if r is quadratic. Note that this approximation formula does not employ second-order derivatives of r.

Suppose that (17) is valid for z close to 1 (instead of our previous assumption that it is valid for z close to 0). Setting z = 1 in (18), we obtain that r'(1) is equal to the right-hand side of (20) evaluated at z = 1 instead of at z = 0. Using (19) and Diewert's Quadratic Approximation Lemma, we have the following corollary:

Corollary 1.1

(23) $$L_{ADD} \approx -(1/2)\, r'(1) \geq 0.$$

A desirable attribute of this approximation is that it only utilizes local information at the observed equilibrium.

We thus see that both of our approximations to the deadweight loss measure r^0 can be calculated from the derivatives up to second order of the overspending function evaluated at the reference equilibrium in the case of (22) and evaluated at the observed equilibrium in the case of (23). In particular, it is not necessary to make any assumptions concerning the functional form of B or place any restrictions on the values of observed economic variables, other than the general restrictions used in describing our model. On the contrary, to calculate r^0, as opposed to an approximation to r^0, it would be necessary to adopt specific (and possibly restrictive) functional forms in order to solve the max-min problem (10) globally.

2-5. Conclusion

This chapter has discussed the measurement of waste and its local approximations for an economy facing distortions due to indirect taxation and nonoptimal levels of public good production. Use has been made of the ADD measure defined in Chapter 1 and two local approximations to the exact measure were calculated. These approximations only required local information on an overspending function.

Figs. 7 - 9 illustrate the diagrammatic interpretation of the ADD measure of waste in a public goods economy and its approximations. Suppose that there is one private good and one public good. In Fig. 7, we have drawn an aggregate production possibilities set that transforms the private good into the public good and the indifference curve of the representative consumer corresponding to the utility level received at the observed distorted equilibrium. Though we cannot introduce a distortionary taxation in this one private good economy, the observed equilibrium is not optimal because of the distortionary provision of the public good, and it is expressed by the discrepancy of the marginal rate of substitution and the marginal rate of transformation at the equilibrium. By choosing the reference bundle to consist only of the private good, the ADD measure, as shown in Fig. 7, is a maximum surplus of the private good with holding the utility level of the consumer and satisfying the production possibilities set. The point where the surplus good is maximized is characterized by the equality of the marginal rate of substitution and the marginal rate of transformation. The ADD measure can be reinterpreted in a Hotelling-Harberger way in this simple model as in Fig. 8. The marginal benefit of the public good is the marginal rate of substitution at $u = u^1$ as a function of the amount of the public good, and the marginal cost of the public good is the marginal rate of transformation as a function of the amount of the public good. At the optimum they coincide, but the former is higher than the latter at the distorted equilibrium, and their discrepancy is denoted as d. We can show that the ADD measure of waste is shown as a curvilinear triangle ABC and that its two approximations (22) and (23) coincide with the triangles ABC' and ABC''. The derivation is analogous to Appendix V of chapter 1 for the interpretation of the tax loss as shown in Fig. 4. In Fig. 8, the approximations are rather accurate in comparison to the true amount of waste, but it is difficult to tell how well the approximations can approximate the true amount of waste in general. In Fig. 9, we show an example of one consumer economy with linear production possibilities set where two approximations can be quite inaccurate even in this simple model. Tsuneki (1987a) gives a more extensive discussion on this numerical example and concludes that the approximations can give at least an order of magnitude estimate of the true amount of waste and the approximations can work quite well as long as the optimum and the distorted equilibrium are not far apart.

To conclude this chapter: we can incorporate the choice of public goods by governments (which are used both by consumers and producers) in a traditional general equilibrium Harberger-type measurement of deadweight loss framework by adopting the Allais-Debreu-Diewert approach. Our approach is more general than Harberger's analysis in the sense that it allows for (i) the choice of flexible functional forms (instead of linear ones as in Harberger or CES-type ones in the numerical general equilibrium literature) for the production sectors and (ii) the loss due to indirect taxation and the nonoptimal provision of public goods is evaluated simultaneously.

FOOTNOTES FOR CHAPTER 2

[1] Assuming that there is a single government production sector involves no loss of generality. See Tsuneki (1987a) for more details.

[2] Formula (10) follows using definitions (1) - (3) and the Uzawa (1958)-Karlin (1959) Saddle Point Theorem. We assume that Slater's constraint qualification condition applies.

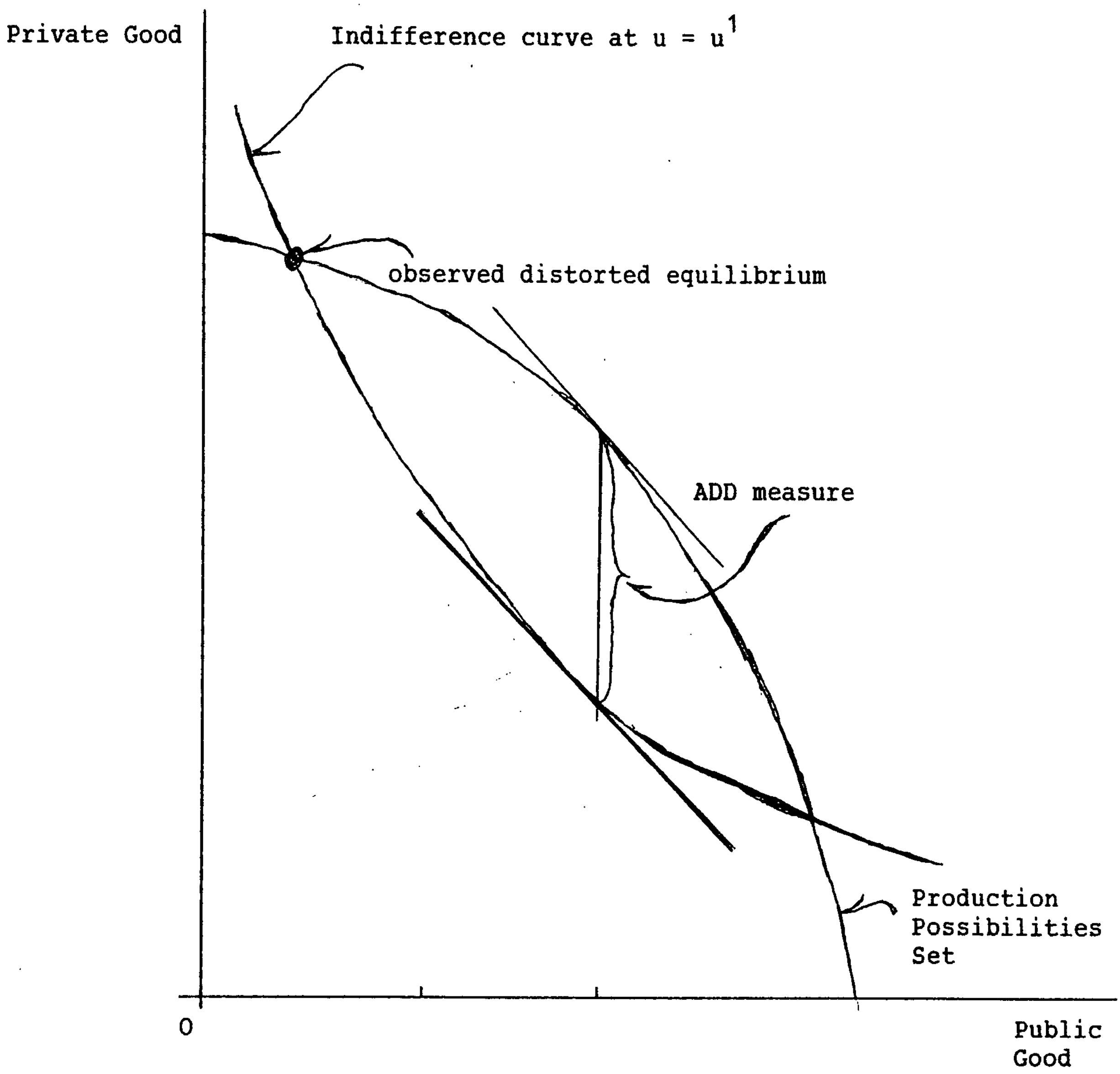


Fig. 7

The ADD Measure in a Public Goods Economy

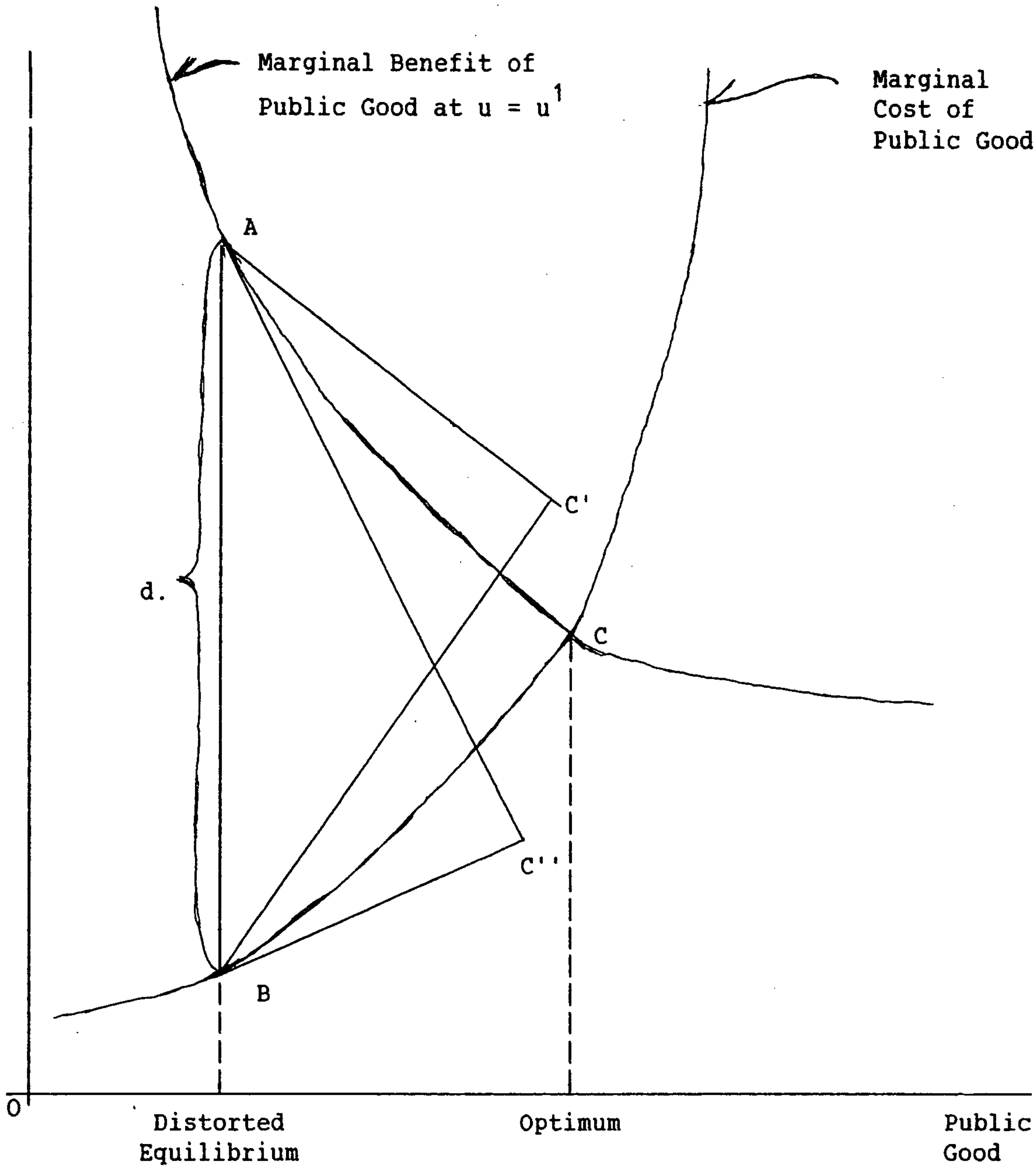


Fig. 8

The ADD Measure and its Approximations in a Public Goods Economy

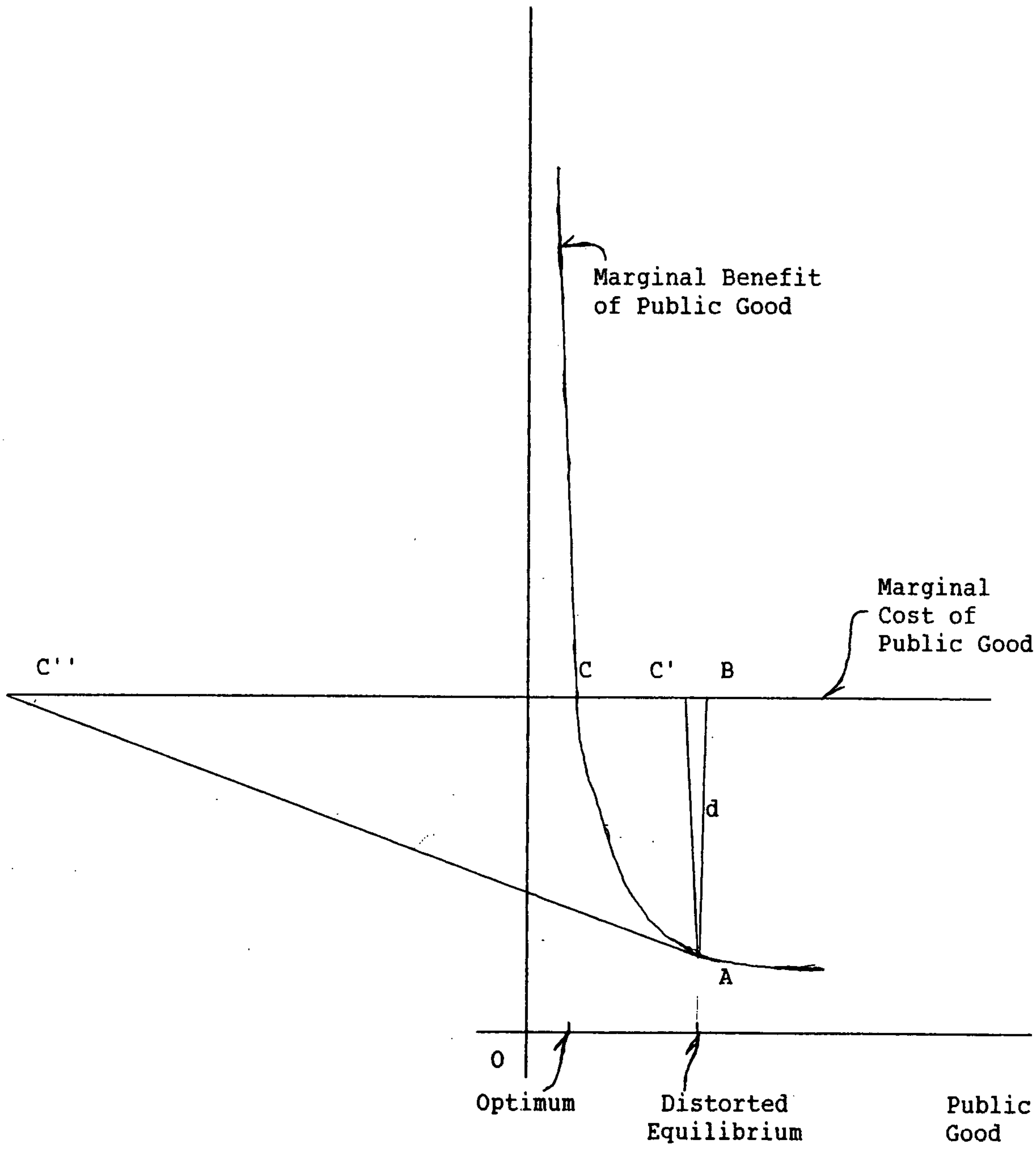


Fig. 9

An Example where Approximations of the ADD Measure are Inaccurate

CHAPTER 3

PROJECT EVALUATION RULES FOR THE PROVISION OF PUBLIC GOODS

3-1. Introduction

The theory of the provision of a public good with distortionary taxation, first set forth by Pigou (1947), maintains that the Samuelsonian (1954) rule to equate the sum of marginal benefits to its marginal cost cannot be an appropriate rule for the maximum of social welfare.

A main objective of the present chapter is to formulate some cost-benefit rules for the provision of a public good which definitely improves the welfare of all the individuals within the economy. This means that our approach considers sufficient conditions for the existence of a Pareto improvement when the public good is provided in a distortionary fashion, and (i) indirect tax rates, (ii) indirect taxes rates and lump-sum transfers, (iii) lump-sum transfers, are allowed to vary with the provision of public good.

In case (i), we suggest a Generalized Pigovian Rule which is a many-person generalization of the second-best optimality condition for the public good provision which is due to Atkinson and Stern (1974), while in cases (ii) and (iii) we suggest a Generalized Samuelsonian Rule and a Modified Harberger-Bruce-Harris Rule, where all of them more or less differ from the Samuelsonian rule.

Since our rules are valid when the equilibrium is away from the second-best optimum, our approach contrasts with the previous literature on project evaluation rules for public goods by Stiglitz and Dasgupta (1971), Atkinson and Stern (1974), Diamond (1975), Atkinson and Stiglitz (1980) and King

(1986). They analyze the first order necessary conditions for (interior) second best social welfare optima. Another objective of this chapter is, however, to reconcile these two apparently different approaches and to tie together strands of previous discussions within our framework.[1] We show that the cost-benefit rules in this chapter are valid both as necessary and sufficient conditions if the manipulable taxation scheme is optimized.

After describing our model in the next section, 3-3 studies an economy where distortionary commodity taxes are used to finance the provision of public goods; lump-sum transfers are not available. Atkinson and Stern's (1974) cost-benefit rule for public goods provision, which generalized the result in Pigou's (1947) pioneering study, is extended to a heterogeneous-consumers' economy in this section. In Atkinson and Stern's (1974) model, the marginal utility of income does not equal the marginal social cost of raising one dollar by indirect taxation; this difference arises because there is a welfare cost due to indirect taxation and there is an income effect due to taxation on tax revenue. The first distortion is emphasized by Pigou, but the second one is neglected by him. When we extend the Atkinson and Stern result to a heterogeneous-consumers' economy, two differences arise. First, the income effect of taxation is the sum of individual income effects with the hth weight being the share of tax revenue paid by the hth individual. Second, the change in the income distribution that results from increased taxation affects the social cost of taxation; e.g., if the tax is levied on people with high social importance, the social cost of taxation will be higher.

Section 3-4 discusses the cases where lump-sum transfers are available to finance an increased supply of public goods. If we can perturb both indirect tax rates and lump-sum transfers at the same time, a generalization of a traditional Samuelsonian rule, generalized Samuelsonian rule applies for the project evaluation. However, when there exists unchangeable indirect tax distortions, we derive a Modified Harberger-Bruce-Harris rule for evaluating the public good. This approach proceeds by using the lump-sum tranfers to keep everyone on their initial indifference curves when the supply of a public good is increased. The induced change in the net supply of private goods is then evaluated using Harberger's generalized weighted-average shadow prices for fixed indirect tax distortions.

Since we adopted the approach of searching for sufficient conditions for a Pareto improvement, our cost-benefit rules can be implemented with knowledge of the initial demand and supply vectors and of the derivatives of the aggregate demand and supply functions evaluated at the observed equilibrium value, as long as preferences for public goods can be determined. Our approach may be contrasted with an alternative approach which searches for necessary conditions for an interior welfare optimum. In this approach, the cost-benefit rules depend on the derivatives of the aggregate demand and supply functions evaluated at the optimum point.

3-2. The Model

The model we utilize in this chapter is identical with the one we used in the previous chapter to characterize the observed distorted equilibrium, (2.6) - (2.8). We assume for simplicity that profit income is completely taxed away, following Diamond and Mirrlees (1971). This assumption can be relaxed by assuming that the entrepreneurial factors are additional commodities (see Diewert (1978) and Dixit (1979)). With these assumptions, we can restate (2.6) and (2.7) as follows:

$$(1) \qquad m^h(p+t,G,u_h) = g_h + (p+t)^T\bar{x}^h, \quad h = 1,\ldots,H,$$

$$(2) \qquad \nabla_q B(p+t,p,G,u) + \nabla_p B(p+t,p,G,u) = 0_N.$$

We assume that (1) and (2) determine $\tilde{p} = (p_2,\ldots,p_N)^T$, $u \equiv (u_1,\ldots,u_H)^T$ and t_1 endogenously given $p_1 = 1$, $\tilde{t} = (t_2,\ldots,t_N)^T$, $g \equiv (g_1,\ldots,g_H)^T$ and $G \equiv (G_1,\ldots,G_N)^T$. Note again that by Walras' law (1) and (2) imply the budget constraint of the government is satisfied. When the equality in (2) is replaced by the inequality ($\leq$) by assuming free disposal, we call it "the inequality version of (2)."

The Pigovian cost-benefit problem we study in this chapter is simply a comparative statics exercise in which at the initial observed equilibrium we perturb G and some of the available tax variables. We assume that G is a scalar (or alternatively, we assume that only the production of the first public good is varied while the other public goods are held fixed). Three alternative rules are derived depending on which taxation instruments we can change. Differentiating (1) and (2) totally, assuming that

$$(3) \qquad \partial m^h(p+t,G,u_h)/\partial u_h = 1, \quad h = 1,\ldots,H,$$

which is implied by money metric utility scaling (see Samuelson (1974)), we obtain:

$$\begin{bmatrix} I_H \\ \\ B_{qu} \end{bmatrix} du = \begin{bmatrix} & -\tilde{X} \\ \\ -B_{q\tilde{q}} & -B_{p\tilde{p}} \end{bmatrix} d\tilde{p} + \begin{bmatrix} -X_1 \\ \\ -B_{qq_1} \end{bmatrix} dt_1 + \begin{bmatrix} -\tilde{X} \\ \\ -B_{q\tilde{q}} \end{bmatrix} d\tilde{t}$$

(4)

$$+ \begin{bmatrix} I_H \\ \\ 0_{N*H} \end{bmatrix} dg + \begin{bmatrix} W \\ \\ -B_{qG} - B_{pG} \end{bmatrix} dG$$

where the net demand matrix of consumers is:

$$X \equiv [X_1, \tilde{X}\ ] \text{ (H*N matrix, with } X_1, H*1 \text{ and } \tilde{X}\ , H*(N-1))$$

where the hth row shows the net demand vector of the hth consumer and

$$W \equiv (W_1, \ldots, W_H)^T = (-\nabla_G m^1(p+t, G, u_1), \ldots,\ -\nabla_G m^H(p+t, G, u_H))^T$$

is a vector of the marginal benefits of the public good for the consumers. The scalar

$$MC = - \Sigma_{k=0}^{K} \nabla_G \pi^k(p, G)$$

is the net aggregate marginal cost of the public good, I_H is an $H*H$ unit matrix and 0_{N*H} is an $N*H$ matrix consisting of zeros. All the derivatives of the overspending function B are evaluated at the observed equilibrium point $(p+t, p, G, u)$.

Throughout the chapter, we assume that

(5) $\quad B_{\tilde{p}\tilde{p}} = \nabla^2_{\tilde{p}\tilde{p}} B(p+t,p,G,u)$ is negative definite.

We express (3) in a different way for later use:

(6) $\quad A du = B_1 d\tilde{p} + B_2 dt_1 + B_3 d\tilde{t} + B_4 dg + B_5 dG.$

When we refer to "the inequality version of (6)" we mean that the H+1,..., H+Nth equalities in (6) are replaced by inequalities ($\leq$). This case utilizes the assumption that an excess supply of goods can be freely disposed. We assume that $[A,-B_1,-B_2]^{-1}$ exists, so that we can locally solve for u, $\tilde{p}$, and t_1 as functions of the exogenous variables, using the Implicit Function Theorem. This analytical technique closely follows Diewert (1983b).

Finally, we have to define our welfare criteria. In a many-consumer economy, we have to distinguish between two criteria for a welfare improvement. The first criterion is the strict Pareto criterion. A strict Pareto improvement occurs if each person's utility is increased. The second criterion makes explicit use of the social welfare function $\beta^T u = \Sigma_{h=1}^{H} \beta_h^T u_h$ where $\beta > 0_H$. The linear function $\beta^T u$ can be thought of as a local linear approximation to a general quasiconcave social welfare function evaluated at the initial utility vector u. In this chapter, we consider a differential effect of the various sets of tax-expenditure instruments with respect to social welfare. If a set of available instruments is fully perturbed with satisfying (4) and $du \gg 0_H$ occurred, then we define it as a differentially strict Pareto improvement. If available tools are fully perturbed with satisfying (4) and $\beta^T du > 0$ occurred, then we define it as a differentially strict welfare improvement. (These definitions follow Diewert (1983b).) Obviously, a differentially strict Pareto improvement (i.e., $du \gg 0_H$) is a differentially strict welfare improvement for any nonnegative, but nonzero, utility weight vector β. Therefore, if we can find a sufficient condition for the existence of a differentially strict Pareto improvement, then there exists a differentially strict welfare improvement as well. Note also that $\beta^T du > 0$ implies the improvement of social welfare $\beta^T u$ in a local sense but the opposite is not true in general, since it is possible that there exists an inflexion point of $\beta^T u$ with respect to the set of instruments so that the improvement of social welfare occurs even if $\beta^T du = 0$. The same argument applies for the change of individual utility. We also define β-optimality with respect to some set of instruments as an equilibrium in which $\beta^T u$ is maximized with respect to the instruments.

3-3. Pigovian Rules Reconsidered

Most papers on cost-benefit rules for public goods provision follow the Pigovian tradition and suppose that the government can vary indirect tax rates t simultaneously with changes in the production of the public good $dG \geq 0$; however, lump-sum transfers g are fixed. Atkinson and Stern (1974) gave the most elegant formula for such a cost-benefit rule by assuming that all consumers have identical preferences and wealth. The purpose of this section is to extend their formula, which we call a Generalized Pigovian Rule to a heterogeneous-consumers' economy and to compare the economic implications of this new rule with that of Atkinson and Stern. We state our main theorem in this section as follows:

Theorem 3.1

Suppose that public good production is irreversible so that $dG \geq 0$[2] and the government can perturb $\tilde{t}$ arbitrarily.

Suppose also that

(7) there is no solution a_u to $a_u > 0_H$ and $a_u^T X = 0_N^T$

and that the indirect tax revenue $R = t^T \nabla_q B$ is nonzero.

Then if all for $\gamma > 0_H$ for which no differentially strict improvement of $\gamma^T u$ with respect to indirect tax rates exists,

$$(8)\quad \gamma^T W\left(1 + \frac{t^T B_{qq} t}{R} - \Sigma_{h=1}^{H} t^T S_{qu}^{h} \frac{R^h}{R}\right) / \Sigma_{h=1}^{H} \gamma^h \left(\frac{R^h}{R}\right) > MC - \Sigma_{h=1}^{H} t^T (\partial x^h(p+t, G, I_h)/\partial G)$$

is satisfied where $R^h = t^T x^h$ is the amount of indirect tax revenue paid by hth person and

$$(9)\quad S_{qu}^{h} \equiv \partial x^h(p+t, G, u_h)/\partial u_h = \partial x^h(p+t, G, I_h)/\partial I_h, \quad h = 1, \ldots, H$$

is a vector of income effects for the hth individual, then there exists a differentially strict Pareto improvement $du \gg 0_H$.

(ii) If the pre-project equilibrium is β-optimal with respect to the choice of $\tilde{t}$, and if

$$(10) \qquad \beta^T W(1 + \frac{t^T B_{qq} t}{R} - \Sigma_{h=1}^{H} t^T s_{qu}^{h} \frac{R^h}{R}) / \Sigma_{h=1}^{H} \beta^h (\frac{R^h}{R})$$

$$> MC - \Sigma_{h=1}^{H} t^T (\partial x^h(p+t, G, I_h)/\partial G)$$

is satisfied, then (10) is a necessary condition for a differentially strict increase in social welfare $\beta^T du > 0$.

PROOF:

(i) A Pareto improvement with $dG \geq 0$ exists if and only if

(11) there exist $dG \geq 0$, du, $d\tilde{t}$ such that $du \gg 0_H$ and the inequality version of (6) is satisfied with $dg \equiv 0_H$.

Applying Motzkin's Theorem (see Appendix I), this is equivalent to

(12) there does not exist an $a^T = [a_u^T, a_1, \tilde{a}^T]$ such that $[a_1, \tilde{a}^T] \geq 0_N^T$, $a^T[B_1, B_2, B_3] = 0_{2N-1}^T$, $a^T A > 0_H^T$, and $a^T B_5 \leq 0$.

If a Pareto improving indirect tax perturbatin is possible, then (11) is always satisfied with $dG = 0$ and the problem is vacuous. Therefore, we assume that such an improvement does not exist. Then there exists an a^T =

$[a_u^T, a_1, \tilde{a}^T]$ such that $[a_1, \tilde{a}^T] \geq 0_N$, $a^T[B_1, B_2, B_3] = 0_{2N-1}^T$, $a^T A > 0_H^T$. For any a that satisfies this condition, we define $\gamma^T = a^T A$. We would like to show that for such a, (12) is satisfied. Suppose (8) holds, and also suppose, contrary to the theorem, a solution to (12) exists. Subtracting $a^T B_3 = 0_{N-1}^T$ from $a^T B_1 = 0_{N-1}^T$, using the identity (1.A.4) and using the supposition (5), we have (see Appendix II)

$$(13) \qquad \tilde{a} = a_1 \tilde{p}.$$

Using (13), the identity (2.5), and the definitions of W and MC, we get

$$(14) \qquad a^T B_5 = (a_u^T + a_1 \, 1_H^T)W - a_1 MC + a_1 t^T B_{qG}.$$

Suppose that $a_1 = 0$. Then, $\tilde{a} = 0_{N-1}$ from (13). Therefore, $a^T[B_1, B_2] = 0_N^T$ implies $a_u X = 0_N^T$. Furthermore, since $a^T A > 0_H^T$, we have $a_u > 0_H$. This contradicts the supposition (7), so that (12) is satisfied. Suppose that $a_1 > 0$. Now postmultiply t_1 and $\tilde{t}$ to $a^T B_2 = 0$ and $a^T B_3 = 0_{N-1}^T$ respectively, add them together and using (13) and (1.A.2) we have (see Appendix III)

$$(15) \qquad \Sigma_{h=1}^{H} \frac{a_u^h}{a_1} \frac{R^h}{R} = t^T B_{qq} t/R.$$

We also have $a^T A = \gamma^T \equiv (\gamma^1, \ldots, \gamma^H)$ so using (13) and (1.A.3) and (3), we can show that (see Appendix III)

$$(16) \qquad \frac{a_u^h}{a_1} = \frac{\gamma^h}{a_1} - 1 + t^T S_{qu}^{\ h}, \qquad h = 1,\ldots,H.$$

Substituting (16) into (15), we find

$$(17) \qquad \frac{1}{a_1} = (1 + \frac{t^T B_{qq} t}{R} - \Sigma_{h=1}^{H} t^T S_{qu}^{\ h} \frac{R^h}{R}) / \Sigma_{h=1}^{H} \gamma^h (\frac{R^h}{R}).$$

Now substituting (16) and (17) into (14) and using the Slutsky-like equation by Wildasin (1984;230),[3] we get

$$(18) \qquad a^T B_5 = a_1 \{\gamma^T W (1 + \frac{t^T B_{qq} t}{R} - \Sigma_{h=1}^{H} t^T S_{qu}^{\ h} \frac{R^h}{R}) / \Sigma_{h=1}^{H} \gamma^h (\frac{R^h}{R})$$

$$- MC + \Sigma_{h=1}^{H} t^T (\partial x^h (p+t, G, I_h) / \partial G)\}.$$

Therefore, (8) implies $a^T B_5 > 0$ and we have a contradiction.

(ii) If $\tilde{t}$ is chosen optimally at the pre-project equilibrium then it is a solution to the problem:

$$(19) \qquad \max_{u,p,\tilde{t}} \{\beta^T u : \text{(1) and the inequality version of (2) are satisfied}\}.$$

The first order Kuhn-Tucker conditions for (19) are:

(20) there exist $[a_1, \tilde{a}^T] \geq 0_N^T$, such that $a^T A = \beta^T$, $a^T[B_1, B_2, B_3] = 0_{2N-1}^T$

since the Mangasarian-Fromovitz constraint qualification conditions are implied by the existence of $[A, -B_1, -B_2]^{-1}$ (see Mangasarian (1969;172-3)).

A differentially strict improvement in social welfare exists if and only if (11) is satisfied with $du \gg 0_H$ replaced by $\beta^T du > 0$. Its dual condition is given by replacing $a^T A > 0_H^T$ in (12) by $a^T A = \beta^T$. This dual condition, and (20) imply $a^T B_5 > 0$, which is equivalent to (10) using the argument to establish (8) from (12). Q. E. D.

We now have to consider the economic implications of Theorem 3.1. The assumption that indirect tax revenue is nonzero is standard in the optimal tax literature. Assumption (7) is more subtle, but it may well be justified, since it is implied by the existence of a Diamond and Mirrlees' good (1971;23). More generally, (7) is the condition for the existence of Pareto improving price changes ignoring production constraints, and equivalently there exists a Hicksian composite good in net demand (or net supply) by all consumers. Then, lowering (raising) the price of the Hicksian good makes all consumers better off (see Weymark (1979)). With these assumptions, we may call (8) and (10) the Generalized Pigovian Rules (GPR hereafter) or the many-person Pigovian rules for the provision of public goods. There are several interesting interpretations of these two formulae.

Let us first consider the relation between (8) and (10). Obviously, the only difference between the two formulae is that we must consider any semipositive utility weight vector for which a social welfare improving tax perturbation does not exist in the former, while we specify the weight β in the latter. This may be explained as follows. We first assume that indirect taxes are set so that we cannot make a differentially strict Pareto improvement with $dG = 0$. Otherwise, the problem is trivial. However, once the indirect taxes are set in this manner, there exists at least one weight vector γ (and probably many) so that the indirect taxes are set such that increasing $\gamma^T u$ is impossible (see Dixit (1979, 152)). Therefore we can use the first order necessary conditions of the maximal social welfare $\gamma^T u$ with respect to indirect taxes, and hence the rest of the problem is an extension of Atkinson and Stern's (1974) result on socially optimal provision of public good with optimal taxes to a many-consumer economy. Furthermore, if we specify $\gamma = \beta$ assuming that the economy is at the β-optimum, then we can get (10).

We now discuss how to extend the Atkinson and Stern's (1974, 122) cost-benefit rules to a many-consumer economy. With taxes set optimally, i.e., at a β-optimum, (14) has the following interpretation.

At the β-optimum, a_u^h, $h = 1,\ldots,H$, and a_1 are the Lagrange multipliers from the programming (19). As $d(\beta^T u)/dg_h = a_u^h$, a_u^h is a net benefit of giving hth person one unit of numeraire good by raising the indirect taxes. It is also the case that $d(\beta^T u)/d\bar{x}_h^1 = a_1$, a_1 is the social gain of the society to have one more unit of the numeraire good (so that indirect taxes are reduced). Therefore, $(a_u^h + a_1)/a_1$ is a gross benefit in terms of social value of the numeraire of giving hth person one unit of numeraire good, and hence it is Diamond's (1975;341) social marginal utility of income α_h, $h = 1, \ldots,H$. Therefore, we can rewrite $a^T B_5 > 0$ using the definition of α_h, $h = 1, \ldots,H$ it is equivalent to

$$(21) \qquad \Sigma_{h=1}^{H}\alpha_h W_h > MC - t^T B_{qG},$$

where the left-hand side is the social value of the public good while the right-hand side is the net social cost of the public good both measured in terms of the social value of numeraire.

α^h can also be rewritten from $a^T A = \beta^T$ using (13) as

$$(22) \qquad \alpha^h = \frac{\beta^h}{a_1} + t^T S_{qu}^{\ h}, \quad h = 1,\ldots,H$$

which coincides with Diamond's (1975;341) original formula. By substituting (22) into (21), we have

$$(23) \qquad \Sigma_{h=1}^{H}\left(\frac{\beta^h}{a_1} + t^T S_{qu}^{\ h}\right)W^h > MC - t^T B_{qG}.$$

Using Wildasin's (1984;231) Slutsky-like equation for public goods (see footnote 3), (23) may be further rewritten as

$$(24) \qquad (\Sigma_{h=1}^{H}\beta^h W^h)/a_1 > MC - t^T(\partial\Sigma_{h=1}^{H}x^h(p+t,G,I_h)/\partial G).$$

The left-hand side of (24) is the weighted sum of the marginal willingness to pay for public goods discounted by the shadow cost of raising one dollar by indirect taxation. The right-hand side is the marginal cost of the public good minus the complementarity effect of public goods provision which means the effect of public good provision on tax revenue due to the complementarity between public and private goods. Therefore, (24) extends the formula (3) of Atkinson and Stern (1974;122) to a many consumer context. What was emphasized by Atkinson and Stern was that $1/a_1$ may not necessarily be smaller than unity, in spite of Pigou's (1947;34) conjecture. This may also be seen from our formula (17) for $1/a_1$. There are two main differences between our formula and theirs. First, the revenue effect of taxation $\Sigma_{h=1}^{H} t^T S_{qu}^{h} R^h/R$ is a weighted sum of the individual revenue effect $t^T S_{qu}^{h}$ where the hth weight is the share of total taxes paid by the hth individual. When there is only one person this expression is simply $t^T S_{qu}$ (see Atkinson and Stern (1974;123)). Second, in a many consumer context one also has distributional effects to consider. Raising one dollar by taxation involves changing the distribution of income proportionately to the tax shares of individuals. This distributional effect is reflected in the term $\Sigma_{h=1}^{H} \beta^h (R^h/R)$. If the tax is levied on people with high social importance β^h, then this expression increases as does a_1; i.e., the social cost of raising one dollar is higher because of the increase of social inequity. These concerns are summarized in the GPR (10). To see the distributive concern in (10) more fully, we define the covariance term following Feldstein (1972);

$$(25) \qquad \varphi_G \equiv \Sigma_{h=1}^{H} \frac{\beta^h}{\bar{\beta}} \frac{W^h}{\bar{W}} \, / \, H,$$

$$(26) \qquad \varphi_R \equiv \Sigma_{h=1}^{H} \frac{\beta^h}{\bar{\beta}} \frac{R^h}{\bar{R}} \, / \, H.$$

where $\bar{\beta}$, $\bar{R}$ and $\bar{W}$ are defined as $\bar{\beta} \equiv \Sigma_{h=1}^{H} \beta^h/H$, $\bar{R} = R/H$ and $\bar{W} \equiv \Sigma_{h=1}^{H} W^h/H$. As the correlation between the social importance and the distribution of marginal willingnesses to pay or of tax burdens increases, φ_G and φ_R increase. Substituting (25) and (26) into (10) yields:

$$(27) \qquad \Sigma_{h=1}^{H} w^h \left(\frac{\varphi_G}{\varphi_R}\right) \left(1 + \frac{t^T B_{qq} t}{R} - \Sigma_{h=1}^{H} t^T s_{qu}^{h} \frac{R^h}{R}\right) > MC - t^T(\partial x^h(p+t,G,I_h)/\partial G).$$

This formula explicitly shows the importance of distributional concern in a many-person GPR by the term $\frac{\varphi_G}{\varphi_R}$. If the distribution of the public goods benefits are regressive or the distribution of the tax burden is progressive, the social welfare of the public good must be valued higher than the simple sum of the marginal willingnesses to pay.

Before closing this section, we should mention the relation between our model and the recent work by King (1986). Our formula (21) with the interpretation of α^h by (22) is obviously indentical with his formula (31) in King (1986;281) so that it is possible to interpret (21) in his way. His result is more general than ours in the sense that he is not assuming the Pareto efficient indirect taxation, but our approach is more complete than his in the sense that he is not deriving the explicit formula and interpretation of the shadow price of government revenue like (17) of ours, for it utterly depends on the arbitrary structure of indirect taxation in his model.

3-4. Cases Where Lump-sum Transfers Are Available

In contrast to the previous section where lump-sum transfers cannot be changed, the conventional Samuelsonian project evaluation rule which equates the sum of the marginal willingnesses to pay with the marginal cost of the public good has a strong intuitive appeal when lump-sum taxes are available for financing the public good. We show in this section that the Samuelson rule is appropriate with some generalizations if both indirect taxes and lump-sum transfers are variable, whereas it is not appropriate if there exits unchangeable distortions due to indirect taxation.

Let us first consider the case where we can change indirect taxes and lump-sum tranfers at the same time.

Theorem 3.2

(i) Suppose that the government can change $\tilde{t}$ and g when G is increased, i.e., $dG \geq 0$. If,

$$\text{(28)} \qquad \Sigma_{h=1}^{H} W_h + t^T B_{qG} > MC,$$

then there exists a strict Pareto improvement $du \gg 0_H$.

(ii) Suppose that $\tilde{t}$ and g are chosen so that the pre-project equilibrium is a β-optimum. Then (28) is also necessary for the existence of a differentially strict increase of social welfare $\beta^T du > 0$.

PROOF:

(i) A sufficient condition for the existence of a Pareto improvement with $dG \geq 0$ is:

(29) there exists $dG \geq 0$, $d\tilde{t}$, dg, such that the inequality version of (6) is satisfied with $du \gg 0_H$.

By Motzkin's Theorem, this is equivalent to:

(30) there does not exist an $a^T \equiv [a_u^T, a_1, \tilde{a}^T]$ such that:
$[a_1, \tilde{a}^T] \geq 0_N^T, a^T[B_1, B_2, B_3, B_4] = 0_{2N+H-1}^T$, $a^T A > 0_H^T$, and $a^T B_5 \leq 0$.

The argument used to show the equivalence of (29) and (30) is similar to that used to show the equivalence of (11) and (12) in Appendix I, so is omitted. Suppose (28) holds, but also suppose, contrary to the theorem, a solution to (30) exists. The conditions $a^T B_4 = 0_H^T$ implies $a_u = 0_H$. In the proof of Theorem 3.1, it is shown that $a^T[B_1, B_3] = 0_{2N-2}^T$ implies (13). We can rewrite $a^T B_5$ by using $a_u = 0_H$ and (13) as

(31) $$a^T B_5 = a_1 p^T(-B_{pG} - B_{qG}) = a_1 \left(\Sigma_{h=1}^H W_h + t^T B_{qG} - MC\right).$$

If $a_1 > 0$, then $a^T B_5 > 0$ by (28), a contradiction.
If $a_1 = 0$, $\tilde{a} = 0_{N-1}$ from (13). Therefore, $a^T A = 0_H^T$ and again we have a contradiction.

(ii) If $\tilde{t}$ and g are optimally chosen at the pre-project equilibrium, then they are a solution to the problem:

(32) $\max_{u,\tilde{p},t,g}\{\beta^T u$: (1) and the inequality version of (2) are satisfied$\}$.

The first order Kuhn-Tucker conditions for (32) are:

(33) there exists $[a_1, \tilde{a}^T] \geq 0_N^T$, such that $a^T A = \beta^T$,

$$a^T [B_1, B_2, B_3, B_4] = 0_{2N+H-1}^T.$$

Suppose (33) is satisfied but (28) is not. The argument following (30) then establishes that $a^T B_5 \leq 0$, so (30) is not satisfied. Consequently, (28) is also necessary for the existence of a differentially strict increase of social welfare $\beta^T du > 0$ at a β-optimum for $\tilde{t}$ and g. Q. E. D.

To understand the implications of the Generalized Samuelsonian Rule (GSR hereafter) (28), let us assume that indirect taxes and transfers are set Pareto efficiently, so that we cannot make a differentially strict Pareto improvement with dG = 0. Pareto efficient indirect taxes and transfers imply that the economy is in first best. It is well-known that the proportional commodity tax rates $t = \theta(p+t)$ for some real number θ is first best with some appropriate lump-sum transfers. Substituting this relation into (28) and using $(p+t)^T B_{qG} = - \Sigma_{h=1}^{H} W_h$, (28) equals

(34) $(1-\theta) \Sigma_{h=1}^{H} W_h > MC.$

This means that the sum of marginal willingnesses to pay for the public good deflated by θ (which is a ratio between producer and consumer prices) must be compared with the marginal cost. Needless to say, if $\theta = 0$ so that there are no indirect taxes, then the Samuelsonian rule applies.

Though proportional indirect taxes are always first best, there may exist some other first best taxes depending on the structure of the economy. For example, if there is no room for technological substitutability among private goods so that $B_{pp} = 0_{N\times N}$, then any indirect taxes can be β-optimal with apropriate lump-sum transfers (see Diewert (1978)). It is obvious in this case that the use of the simple Samuelsonian rule is erroneous and we have to use the GSR (28).

We now move to an alternative case where we can perturb g and G while holding the commodity tax distortions $\tilde{t}$ fixed. We call the resulting rule within the following proposition, a Modified Harberger-Bruce-Harris Rule (MHBHR hereafter), since it is an application of Harberger (1971) and Bruce-Harris (1982) to a project evaluation approach to the production of a public good (see also Diewert (1983b)).

<u>Theorem 3.3</u>

(i) Suppose that the government can change only the transfer vector g when G is increased; i.e., $dG \geq 0$. Then, the Modified Harberger-Bruce-Harris Rule[4] is

$$(35) \qquad (p + \varepsilon)^T [-B_{PG} - B_{qG}] > 0$$

where

$$(36) \qquad \varepsilon^T \equiv [0, \tilde{\varepsilon}^T] = [0, t^T B_{q\tilde{q}} (B_{\tilde{q}\tilde{q}} + B_{\tilde{p}\tilde{p}})^{-1}].$$

Condition (35) is sufficient for the existence of a Pareto improvement $du \gg 0_H$.

(ii) If in the tax-distorted pre-project economy, g was chosen optimally, then (35) is also a necessary condition for a small increase in public good production to lead to a differentially strict welfare improvement.

PROOF:

(i) A sufficient condition for a Pareto improvement is:

(37) there exists dg and $dG \geq 0$ such that the inequality version of (6) is satisfied and $du \gg 0_H$.

Condition (37) is equivalent to the following Motzkin dual condition:

(38) there does not exist $[a_u^T, a_1, \tilde{a}^T] = a^T$ such that
$a^T A > 0_H^T$, $a^T B_5 \leq 0$, $a^T [B_1, B_2, B_4] = 0_{N+H}^T$, $[a_1, \tilde{a}_0^T] \geq 0_N^T$.

Suppose (35) holds, but also suppose that, contrary to the theorem, a solution to (38) exists. The conditions $a^T B_4 = 0_H^T$ imply $a_u = 0_H$. Hence we can rewrite $a^T B_1 = 0_{N-1}^T$ as:

(39) $\tilde{a} = a_1(\tilde{p} + \tilde{\varepsilon})$

using (1.A.2) and (1.A.4) (see Appendix IV). Using $a_u = 0_H$, (36) and (39),

$$a^T B_5 = a_1 (p + \varepsilon)^T[-B_{pG} - B_{qG}].$$

If $a_1 > 0$, (35) implies $a^T B_5 > 0$, a contradiction. If $a_1 = 0$, $\tilde{a} = 0_{N-1}$ from (39). With $a_u = 0_H$ we have $a^T A = 0_H^T$, and again we have a contradiction.

(ii) If g is optimally chosen at the pre-project equilibrium, g is a solution to the problem.

(40) $\max_{u,\tilde{p},t_1,g}\{\beta^T u$; (1) and the inequality version of (2) are satisfied$\}$.

The first order Kuhn-Tucker conditions for (40) are:

(41) there exists $[a_1, \tilde{a}^T] \geq 0_N^T$, such that $a^T A = \beta^T, a^T[B_1, B_2, B_4] = 0_{N+H}^T$.

Suppose (41) is satisfied but (35) is not. The argument following (38) then establishes that $a^T B_5 \leq 0$, so (38) is not satisfied. Consequently, (35) is also necessary for a differentially strict increase of social welfare at a β-optimum for g. Q. E. D.

The economic intuition behind the two Propositions in this section is as follows. Given the pre-project levels of utility, increasing the provision of the public good permits a reduction in the consumption of private goods but requires additional inputs for the increased public good production. By appropriately offsetting the marginal benefits of the public good (i.e., the externality) by changing lump-sum transfers to keep consumers at their original utility levels, it is only necessary to evaluate the resulting change in the quantities of the private goods by appropriate shadow prices. If the vector of tax rates $\tilde{t}$ is variable, the production price vector is the appropriate shadow price vector. See (31) behind a GSR (28) . This result is a version of the production efficiency theorem in Diamond and Mirrlees (1971). If $\tilde{t}$ is fixed, a MHBHR (35) must be adopted which uses a Harberger-Bruce-Harris shadow price vector.

3-5. Conclusion

Our present chapter has derived project evaluation formulae for the provision of public goods in various second-best situations. We considered three cases. (1) the case where indirect tax rates can be varied; (2) the case where both lump-sum transfers and indirect tax-rates can be varied (3) the case where lump-sum transfers are varied. We showed that the the use of a GPR, a GSR and a MHBHR are suggested for cases (1), (2) and (3) respectively. Our basic point is that project evaluation rules must vary depending on what instruments we can change when we alter the supply of public goods.

We have to note that there are severe limitations in utilizing our cost-benefit rules; i.e., we have ignored the preference revelation problem for public goods in measuring the marginal willingnesses to pay W for consumers. Once this difficulty is overcome, our rules can be implemented by using only local information observable at the pre-project equilibrium, that is, the level of taxes, public goods, prices, incomes, and the first order derivatives of the ordinary demand functions and the net supply functions for private goods (which depend on both prices and public goods). Note that B_{qq} and B_{qG} can be computed from ordinary demand functions as we pointed out in 2.4. Note further that information on W is necessary to use the MHBHR (35) as we need to compute B_{qG} from data on the ordinary demand functions. Therefore, this rule is also vulnerable to the free-rider problem.

We have shown that it is fairly easy to obtain sufficient conditions for the existence of a small Pareto improvement corresponding to an increase in public goods production, given that various taxation instruments are available. It seems that this approach is more useful compared to the traditional approach which derives the first order conditions for an interior second-best welfare optimum. Our results also show that conventional cost-benefit rules for the provision of public goods are not always correct.

FOOTNOTES FOR CHAPTER 3

[1] Our approach draws on the methodology found in the tax reform literature, e.g., Guesnerie (1977), Diewert (1978), Dixit (1979) and Weymark (1979), and the project evaluation study by Diewert (1983b). Wildasin (1984) also worked with a framework similar to ours, but his paper has various restrictive assumptions; e.g., only one commodity tax rate is variable and all other goods are untaxed.

[2] If we evaluate a possible reduction in the production of the public good $dG \leq 0$, all we need is to reverse the direction of the inequalities in the cost-benefit rule. The proof is straightforward and hence may be omitted. The same comment applies to all cost-benefit formulae in this chapter.

[3] It is given by

$$\partial x^h(p+t,G,I_h)/\partial G = \{\partial x^h(p+t,G,u_h)/\partial G\} + W_h S_{qu}^h.$$

Premultiplying by t^T and in summation over h, we have

$$\Sigma_{h=1}^{H} t^T(\partial x^h(p+t,G,I_h)/\partial G) = t^T B_{qG} + \Sigma_{h=1}^{H} t^T W_h S_{qu}^h,$$

which is used to derive (18).

[4] If $t = 0_N$ so that there are no pre-existing tax distortions, then the Modified Harberger-Bruce-Harris Rule is identical with the traditional Samuelsonian rule. The proof is straightforward.

APPENDICES FOR CHAPTER 3

Appendix I: The derivation of (12).

Motzkin's Theorem is as follows:

Either $Ex \gg 0$, $Fx \geq 0$, $Gx = 0$ has a solution x where E is a nonvacuous matrix, F and G are matrices and x is a vector or $v^{1T}E + v^{2T}F + v^{3T}G = 0^T$, $v^1 > 0$, $v^2 \geq 0$ has a solution where v^1, v^2 and v^3 are vectors, but not both. See Mangasarian (1969).

We now apply it to rewrite (11). B_4dg can be dropped from (6), for $dg \equiv 0_H$. Decompose A, B_i (i=1,2,3,5) between A*, B_i^*, which are the top H rows and A**, B_i^{**}, which are the bottom N rows. Define

$$x = [d\tilde{u}^T, d\tilde{p}^T, dt_1, d\tilde{t}^T, dG]^T,$$

$$E = [I_H, 0_{H\times(2N+H)}],$$

$$F = \begin{bmatrix} -A_1^{**}, & B_1^{**}, & B_2^{**}, & B_3^{**}, & B_5^{**} \\ e_{2N+H}^T & & & & \end{bmatrix}$$

where e_{2N+H} is a unit vector with unity in 2N+Hth row, and $G = [-A_1^*, B_1^*, B_2^*, B_3^*, B_5^*]$.

Then, the primal condition of the Motzkin's Theorem is identical with (11). Defining v_1^T(1×H row vector), $v_2^T = [a_1, \tilde{a}^T, v]$ where v is a scalar, $v_3^T = a_u^T$, the dual condition is:

there is no solution v_1, a_1, $\tilde{a}$, v, a_u such that

$$v_1 > 0,\ a_1 \geq 0,\ \tilde{a} \geq 0,\ v \geq 0,$$

$$v_1^T - a^T A = 0,\ a^T[B_1, B_2, B_3] = 0_{2N-1}^T,\ a^T B_5 + v = 0,$$

which is in turn identical with (12).

<u>Appendix II</u>: The derivation of (13).

Subtracting $a^T B_3 = 0_{N-1}^T$ from $a^T B_1 = 0_{N-1}^T$, we have

(A.1) $$- a_1 B_{p_1\tilde{p}} - \tilde{a}^T B_{\tilde{p}\tilde{p}} = 0_{N-1}^T.$$

From (1.A.4),

(A.2) $$B_{p_1\tilde{p}} + \tilde{p}^T B_{\tilde{p}\tilde{p}} = 0_{N-1}^T$$

where $p_1 = 1$. Substituting (A.2) into (A.1), we have

(A.3) $$(a_1\tilde{p}^T - \tilde{a}^T)\ B_{\tilde{p}\tilde{p}} = 0_{N-1}^T.$$

By assumption (5), $B_{\tilde{p}\tilde{p}}$ is nonsingular, which implies (13).

Appendix III: The derivation of (15) and (16)

Postmultiply t_1 and $\tilde{t}$ to $a^T B_2 = 0$ and $a^T B_3 = 0_{N-1}^T$, and adding them together, we have

$$\text{(A.4)} \qquad a_u^T X t + [a_1, \tilde{a}^T] B_{qq} t = 0.$$

Substituting (13) and rewriting the first term of (A.4), we get

$$\text{(A.5)} \qquad \Sigma_{h=1}^{H} a_u^h R^h + a_1 p^T B_{qq} t = 0.$$

Substituting the identity (1.A.2) into the second term of (A.5), and rearranging terms, we get (15). We can rewrite $a^T A = \gamma^T$ as

$$\text{(A.6)} \qquad a_u^T + [a_1, \tilde{a}^T] B_{qu} = \gamma^T.$$

Substituting (13) into (A.6), we have

$$\text{(A.7)} \qquad a_u^T + a_1 p^T B_{qu} = \gamma^T.$$

However, from (1.A.3) and (3) we get

$$\text{(A.8)} \qquad (p + t)^T B_{qu} = 1_H^T .$$

Substituting (A.8) into (A.7) we have

$$\text{(A.9)} \qquad a_u^T = \gamma^T - a_1 \; 1_H + a_1 t^T B_{qu}.$$

From the definition of B_{qu} and S_{qu}^{h} in (9), (A.9) is identical with (16).

Appendix IV: The derivation of (39).

$a^T B_1 = 0_{N-1}^T$ can be rewritten as

$$\text{(A.10)} \qquad -a_1[B_{q_1\tilde{q}} + B_{p_1\tilde{p}}] - \tilde{a}^T[B_{\tilde{q}\tilde{q}} + B_{\tilde{p}\tilde{p}}] = 0_{N-1}^T$$

using $a_u = 0_H$. From (1.A.4) and (1.A.2), we get

$$\text{(A.11)} \qquad B_{p_1\tilde{p}} + \tilde{p}^T B_{\tilde{p}\tilde{p}} = 0_{N-1}^T$$

and

$$\text{(A.12)} \qquad t^T B_{q\tilde{q}} + B_{q_1\tilde{q}} + \tilde{p}^T B_{\tilde{q}\tilde{q}} = 0_{N-1}^T,$$

respectively. Therefore, adding up (A.11) and (A.12), we have

$$\text{(A.13)} \qquad B_{q_1\tilde{q}} + B_{p_1\tilde{p}} = - t^T B_{q\tilde{q}} - \tilde{p}^T B_{\tilde{q}\tilde{q}} - \tilde{p}^T B_{\tilde{p}\tilde{p}}$$

Substituting (A.13) into (A.10),

$$\text{(A.14)} \qquad a_1[t^T B_{q\tilde{q}} + \tilde{p}^T B_{\tilde{q}\tilde{q}} + \tilde{p}^T B_{\tilde{p}\tilde{p}}] = \tilde{a}^T[B_{\tilde{q}\tilde{q}} + B_{\tilde{p}\tilde{p}}].$$

Inverting the matrix $[B_{\tilde{q}\tilde{q}} + B_{\tilde{p}\tilde{p}}]$, and using definition (36), (39) follows.

CHAPTER 4

INCREASING RETURNS, IMPERFECT COMPETITION AND THE MEASUREMENT OF WASTE

4-1 Introduction

In the presence of increasing returns to scale in production, it is well-known that Pareto optimal equilibria may not be decentralized through perfect competition and moreover, imperfect competition prevails frequently. Therefore, both positive and normative analysis of resource allocation with increasing returns to scale becomes an important topic in applied welfare economics. The normative problem of developing mechanisms to support Pareto optima in the presence of increasing returns to scale has been discussed by many authors, including Arrow and Hurwicz (1960), Guesnerie (1975) and Brown and Heal (1980). The second best pricing problem of public utilities facing a revenue constraint is discussed by the optimal pricing and taxation literature beginning with Boiteux (1956). There have been numerous positive analyses of oligopolistic markets in the vast literature on strategic interactions among incumbent firms or among incumbent firms and potential entrants. Moreover, there is a large literature on Chamberlinian (1962) monopolistic competition. In contrast, the measurement of waste due to imperfect competition with increasing returns to scale is a relatively less developed area, although the important seminal paper by Hotelling (1938) dealt with this topic. The aim of this chapter is to consider this measurement of waste problem.

Let us first review the problem discussed by Hotelling (1938) and list the points which seem to call for extensions. First, Hotelling claimed that first best optimality is characterized by the marginal cost principle, i.e., the price of the product should equal its marginal cost, for increasing returns to scale firms. However, it was pointed out by Arrow and Hurwicz (1960) that this solution is not necessarily optimal with a general nonconvex technology, and Silberberg (1980) pointed out that Hotelling (1938) is actually not proving the optimality of marginal cost pricing. Therefore, in the literature on the measuremnt of deadweight loss, which includes Debreu (1954), Harberger (1964) and Diewert (1981, 1983(a), 1985(a)) in order to avoid this difficulty it is assumed that all firms have a convex technology. Therefore, in order to compute the deadweight loss, we first characterize the optimality in nonconvex economy rigorously. Second, Hotelling's (1938) measure of waste does not seem to be correct in a general equilibrium sense, and furthermore, requires the computation of an optimum equilibrium which necessitates global information on consumer preferences and technology, so that we would like to derive a measure of waste which can be evaluated using only local information on preferences and technology, so that the measure is more useful in empirical research on the measurement of waste.

In this chapter, we show that these problems can be solved in a satisfactory way, at least in our simplified model.

Our findings in this chapter may be summarized as follows. We can derive a Hotelling-Harberger type general equilibrium approximate deadweight loss measure due to imperfect competition allowing for quite general differentiable functional forms for production and utility functions, including production functions that exhibit increasing returns to scale. This approximate measure can be implemented from local information up to the second order obtained at an observed distorted equilibrium. There are different waste measures depending on the types of increasing returns to scale, since the characterization of the optimum depends on these types of increasing returns to scale.

In the next section, we construct a model employing the assumptions that production functions are quasiconcave, factor markets are competitive, and the number of firms in one production sector is fixed. We characterize the imperfectly competitive general equilibrium by a system of equations. In 4-3, we derive an Allais-Debreu-Diewert measure of waste with increasing returns to scale and show that the corresponding optimum equilibrium is characterized by the marginal cost principle. In 4-4, we compute a second order approximation to the ADD loss measure, discuss its informational requirements, and show how our measure generalizes Hotelling's original approach and other works on deadweight loss which assume technologies are convex. We also discuss various relaxations of our assumptions, and limitations on applying our approach to empirical studies of various market imperfections. Section 4-5 concludes with a diagrammatic interpretation of our approximate measures.

4-2. The Model

We assume that there are N goods in the economy, where the corresponding price vector is $p \equiv (p_1,\ldots,p_N)^T \gg 0_N$, and that only sector n produces the nth good for $n = 1,\ldots,N$ by combining the other goods and M nonproducible factors. This vector of primary factors has the vector of factor prices $w \equiv (w_1,\ldots,w_M)^T \gg 0_M$.

Each production unit is assumed to have a quasi-concave production function $f^n(x_1,\ldots,x_N, v_1,\ldots,v_M)$; that is, for a given level of output y_n, marginal rates of technical substitution between inputs are diminishing.[1] This assumption is weaker than global convexity in production; the possibility of increasing returns to scale is allowed for when we change the level of output in this characterization.[2] We define the sector n cost function $C^n(p,w,y_n)$ as

$$(1) \qquad C^n(p,w,y_n) \equiv \min_{x \geq 0_N, v \geq 0_M} \{p^T x + w^T v : f^n(x,v) \geq y_n\},\ n = 1,\ldots,N.$$

C^n is identical to the expenditure function m^h defined by (1.4), except that the utility level is replaced by the production level. We assume that the regularity conditions listed in Diewert (1982;554) are satisfied. There are H households in this economy and their demands are characterized in terms of the expenditure functions

$$(2) \qquad m^h(p,w,u_h) \equiv \min_{a,b}\{p^T a + w^T b : f^h(a,b) \geq u_h, (a,b) \in \Omega^h\},\ h = 1,\ldots,H,$$

where Ω^h is a (translated) orthant of R^{N+M} defined as in (1.4). We assume that the hth household holds the vector of initial endowments $\bar{Y}^h \equiv (\bar{a}^{hT}, \bar{b}^{hT})^T$.

To characterize the general equilibrium, we utilize the overspending function B defined by:

$$(3) \qquad B(y,p,w,u) \equiv \Sigma_{h=1}^{H}\{m^h(p,w,u_h) - p^T\bar{a}^h - w^T\bar{b}^h\} - \Sigma_{n=1}^{N}\{p_n y_n - C^n(p,w,y_n)\},$$

where $y \equiv (y_1,\ldots,y_N)^T$ and $u \equiv (u_1,\ldots,u_H)^T$.

Compared with the overspending functions in previous chapters, consumers and producers are facing the same prices in (3) so that we no longer have two sets of prices as arguments. Definition (3) may be simplified by defining $Q \equiv (p^T, w^T)^T$ as follows:

$$(4) \qquad B(y,Q,u) \equiv \Sigma_{h=1}^{H}\{m^h(Q,u_h) - Q^T\bar{Y}^h\} - \Sigma_{n=1}^{N}\{p_n y_n - C^n(Q,y_n)\}.$$

In the same manner as we derived the properties of an overspending function in Appendix I of chapter 1, we can easily derive the following properties for the new overspending function: (i) B is concave with respect to Q; (ii) if B is once continuously differentiable with respect to prices, $\nabla_Q B(y,Q,u)$ equals the vector of excess demands; (iii) B is linearly homogeneous with respect to prices. From this, the resulting identities are satisfied:

$$(5) \qquad Q^T B_{QQ} = 0_{N+M}^T,$$

and

$$(6) \qquad Q^T B_{Qy} = \nabla_y B(y,Q,u)^T$$

where $B_{QQ} \equiv \nabla_{QQ}^2 B(y,Q,u)$ and $B_{Qy} \equiv \nabla_{Qy}^2 B(y,Q,u)$. Note that $-\nabla_y B(y,Q,u)$ is a vector whose ith component is the difference between the price and marginal cost of the ith good.

Now using the above relations, we characterize the general equilibrium (y^1, Q^1, u^1) where $Q^1 \equiv (p^{1T}, w^{1T})^T$ as follows:

$$(7) \qquad m^h(Q^1,u_h^1) = Q^{1T}\bar{Y}^h + \Sigma_{n=1}^{N}\alpha^{hn}\{p_n^1 y_n^1 - C^n(Q^1,y_n^1)\} + g_h, \quad h = 1,\ldots,H,$$

(8) $$-\nabla_y B(y^1,Q^1,u^1) = t,$$

(9) $$-\nabla_Q B(y^1,Q^1,u^1) = 0_{N+M}$$

where α^{hn} is the share of the nth firm held by the hth individual and $\Sigma_{h=1}^{H}\alpha^{hn} = 1$, for $n = 1,\ldots,N$. The number g_h, $h = 1,\ldots,H$, shows the net lump-sum transfer given to the hth individual and $t \equiv (t_1, \ldots, t_N)^T$ where t_n is the monopolistic mark-up imposed by firm n on his sales.

We can show that (7) and (9) imply that the sum of the transfers g_h, $h = 1,\ldots,H$, equals zero. The equations in (7) are the budget constraints of the H individuals. The equations in (8) state that the difference between the price of the ith good and its marginal cost is equal to the mark-up t_i. For perfectly competitive firms $t = 0_N$, but with imperfect competition we expect $t \gg 0_N$. With increasing returns to scale, firms must charge prices larger than their marginal costs in order to attain nonnegative profits. This does not necessarily mean that the monopolistic markup is fixed for monopolists. We just define t ex-post at the equilibrium as the difference between consumer prices and marginal costs. Noting that $\nabla_Q B$ equals the vector of excess demands, the equations in (9) are the market clearing conditions for the equilibrium. Therefore, (7) to (9) characterize an imperfectly competitive general equilibrium, as elaborated by Negishi (1960-1), Arrow and Hahn (1971, Ch. 6) and Roberts and Sonnenschein (1977).

4-3. The Allais-Debreu-Diewert Measure of Waste

Let us first take an N + M dimensional nonnegative reference bundle of goods and factors $\lambda \equiv (\alpha^T,\beta^T)^T \geq 0_{N+M}$ and each consumer's utility level u_h^1,

$h = 1,\ldots,H$, in the imperfectly competitive equilibrium, and consider the following primal planning problem:

$$(10) \qquad r^0 \equiv \max_{a^h, b^h, y_n, x^n, v^n} \{r : \quad \text{(i)} \ \Sigma_{h=1}^{H} a^h + \Sigma_{n=1}^{N} x^n + \alpha r \leq y + \Sigma_{h=1}^{H} \bar{a}^h,$$

$$\text{(ii)} \ \Sigma_{h=1}^{H} b^h + \Sigma_{n=1}^{N} v^n + \beta r \leq \Sigma_{h=1}^{H} \bar{b}^h,$$

$$\text{(iii)} \ f^h(a^h, b^h) \geq u_h^1, (a^h, b^h) \varepsilon \Omega^h \ h=1,\ldots,H,$$

$$\text{(iv)} \ f^n(x^n, v^n) \geq y_n, \ n = 1,\ldots,N\}.$$

The solution to (10) defines the ADD measure of waste $L_{ADD} \equiv r^0$. Problem (10) may be interpreted as maximizing the number of multiples r of the given reference bundle λ that can be obtained while maintaining consumers' utilities at u_h^1, $h = 1,\ldots,H$, and satisfying the materials balance and technology constraints. We assume that a finite maximum exists for (10).

We can also derive a dual expression to (10) as follows. First, let us fix $y = (y_1,\ldots,y_N)^T$. From the definition of quasi-concavity, the sets $f^n(x^n,v^n) \geq y_n (n = 1,\ldots,N)$ are convex sets belonging to R_+^{N+M}. Then, the remaining programming problem becomes a concave programming so that we can rewrite (10) using the Uzawa (1958)-Karlin (1959) Saddle Point Theorem[3] as

$$(11) \qquad r^0 = \max_{y \geq 0_N} [\max_r \min_{Q \geq 0_{N+M}} \{r(1-Q^T\lambda) - B(y,Q,u^1)\}]$$

using definitions (1) (2) and (4), where Q is the vector of Lagrangean multipliers associated with the resource constraints, (i) and (ii). The max-min problem within the squared bracket of (11) can be rewritten using the Uzawa-Karlin Theorem in reverse as

$$(12) \qquad -\max_{Q \geq 0_{N+M}} \{B(y,Q,u^1) \text{ s.t. } Q^T\lambda \geq 1\}.$$

For the given level of y, the solution of the max-min problem within (11) becomes r(y) and Q(y) which are functions of y. Then (11) can also be written as

$$(13) \qquad r^0 = \max_{y \geq 0_N} \{r(y)(1-Q(y)^T\lambda) - B(y,Q(y),u^1)\}.$$

The global programming problem (10) and (11) define the ADD measure of waste when the observed utilities are u^1, but it is difficult to compute r^0 using this approach since we need global information on preferences and technologies. To get more insight about the amount of waste in relation to the degree of monopoly, and bridge the gap between conventional deadweight loss measures and our ADD measure, we derive a second order approximation to the ADD measure of waste. For this purpose, we have to strengthen our assumptions as follows:
(i) (y^0,r^0,Q^0) solves (11) with $y^0 \gg 0_N$, $Q^0 \gg 0_{N+M}$ so that the first order conditions for (11) hold with equality;[4] (ii) the expenditure functions m^h, $h = 1,\ldots,H$, are twice continuously differentiable with respect to Q at (Q^0,u_h^1); (iii) the cost functions C^n, $n = 1,\ldots,N$, are twice continuously differentiable at (Q^0,y_n^0); (iv) Samuelson's (1947) strong second order conditions hold for the two problems (12) and (13) when the inequality constraint in (12) is replaced by an equality.

The regularity condition (i) implies that there are no free goods and all firms are useful. Conditions (ii) and (iii) are differentiability assumptions, which are natural for a local analysis such as ours. Condition (iv) is an assumption which guarantees that the maximum of the planning problem (10) is locally unique. Our regularity conditions on (12) imply the bordered Hessian

$$(14) \qquad C^0 \equiv \begin{bmatrix} -B^0_{QQ}, & -\lambda \\ -\lambda^T, & 0 \end{bmatrix} \text{ is positive definite}$$

where $B^0_{QQ} = \nabla^2_{QQ} B(y^0, Q^0, u^1)$ and the superscript 0 means that the derivatives are evaluated at $z = 0$. By defining

$$(15) \qquad A^0 \equiv -B^0_{yy}$$

and

$$(16) \qquad B^0 = [-B^0_{yQ}, 0_N]$$

where the superscript 0 means that B^0_{yy} and B^0_{yQ} are evaluated at the optimum, our condition in (iv) is equivalent to the following condition:

$$(17) \qquad A^0 - B^0(C^0)^{-1}B^{0T} \text{ is negative definite.}$$

The condition (17) is much weaker than assuming marginal costs are increasing, which requires A^0 to be negative definite, for C^0 is positive definite by (14). By merely requiring condition (iv), we are admitting the possibility of a downward sloping marginal cost curve, which follows the spirit of Hotelling (1938;255-6).[5]

It follows from assumption (i) that an interior solution exists to (11). The first order conditions are given by:

(18) $$-\nabla_y B(y^0,Q^0,u^1) = 0_N,$$

(19) $$-\lambda r^0 - \nabla_Q B(y^0,Q^0,u^1) = 0_{N+M},$$

(20) $$1 - Q^{0T}\lambda = 0,$$

where (18) is a marginal cost pricing principle for monopolistic firms, (19) are resource balance equations for goods and factors with $\lambda r^0 \geq 0_N$ being the vector of surplus goods and factors, and (20) is a normalization rule for the optimal prices.

4-4. Second Order Approximations

Now comparing the market equilibrium conditions and the first order conditions for the optimum, we construct a z-equilibrium which depends on a scalar parameter z ($0 \leq z \leq 1$);

(21) $$-\nabla_y B(y(z),Q(z),u^1) = tz,$$

(22) $$-\nabla_Q B(y(z),Q(z),u^1) - \lambda r(z) = 0_{N+M},$$

(23) $$1 - Q(z)^T\lambda = 0.$$

If we define $(y(0), Q(0), r(0)) \equiv (y^0, Q^0, r^0)$, then (21) - (23) coincide with the optimality conditions (18) - (20) when $z = 0$. In contrast, if we define $(y(1), Q(1), r(1)) \equiv (y^1, Q^1, 0)$, then (21) and (22) coincide with (8) and (9) respectively when $z = 1$. In this case, (7) is also satisfied for an appropriate choice of transfers g_h, $h = 1,\ldots,H$. From condition (23) at $z = 1$, we also assume that the market prices satisfy the normalization,

$$(24) \qquad 1 = Q^{1T}\lambda,$$

by choosing the scale of λ appropriately. Thus we can conclude that the z-equilibrium (21) - (23) maps the optimal equilibrium into the imperfectly competitive equilibrium as z is adjusted from zero to one.

Equation (21) that maps the marginal cost pricing condition (18) into the monopolistic markup equilibrium condition (8) may seem unnatural because markups are decreasing linearly, but the change in t may be nonlinear depending on the behaviour of monopolists. Even in the case of tax-distortions, however, it is possible to choose some nonlinear path of the change of tax rates as the equilibrium is adjusted and the resulting magnitude of waste depends on this choice of path. (This problem is also related to the traditional problem of path independence in consumers' surplus analysis.) As it is difficult to overcome this arbitrariness within our framework, we have just assumed that there is a uniform reduction of monopoly distortions.

The main theorem in this section is as follows:

Theorem 1: A second order approximation to the ADD measure of waste (10) is given by

$$(25) \qquad -\tfrac{1}{2}t^T(A^0 - B^0(C^0)^{-1} B^{0T})^{-1}t > 0.$$

PROOF: Differentiate (21) - (23) with respect to z and we have

$$(26) \qquad \begin{bmatrix} A^z, & B^z \\ & \\ B^{zT}, & C^z \end{bmatrix} \begin{bmatrix} y'(z) \\ Q'(z) \\ r'(z) \end{bmatrix} = \begin{bmatrix} t \\ 0_{N+M} \\ 0 \end{bmatrix},$$

where A^z, B^z, C^z are the matrices A^0, B^0 and C^0 defined by (15), (16) and (14) evaluated at z, rather than 0.
Premultiplying (26) by $[0_N^T, Q(z)^T, 0]$, we have

$$(27) \qquad -Q(z)^T B^z_{QY} y'(z) - Q(z)^T B^z_{QQ} Q'(z) - Q(z)^T \lambda r'(z) = 0.$$

Substituting (5), (6), and (23), and then (21) into (27), we obtain

$$(28) \qquad r'(z) = zt^T y'(z).$$

Noting that $r(1) \equiv 0$, by using a Taylor series expansion the ADD measure of waste $L_{ADD} = r^0 = r(0) - r(1)$ can be approximated by

$$(29) \qquad r^0 - r^1 \cong r^0 - \{r^0 + r'(0) + \tfrac{1}{2}r''(0)\} = - r'(0) - \tfrac{1}{2}r''(0).$$

However, from (28), $r'(0) = 0$ and $r''(0) = t^T y'(0)$. Therefore, we have

(30) $\qquad r^0 - r^1 \cong -\tfrac{1}{2}t^T y'(0).$

Evaluating (26) at z = 0 and inverting the left-hand side matrix yields $y'(0) = (A^0 - B^0(C^0)^{-1}B^{0T})^{-1}t$. Substituting this expression into (30), the result (25) follows. The inequality in (25) follows from (17). Q. E. D.

The formula (25) gives a general formula of deadweight loss applicable to either a convex or nonconvex economy. This formula is identical to the Debreu (1954)-Diewert (1985a) approximate deadweight loss formula when the technologies are convex. However, the converse is not true, since the optimal shadow price (or intrinsic price to use Debreu's (1951, 1954) term) may not exist with increasing returns to scale. This problem is overcome by our two-stage optimization procedure (11) for the characterization of the optimum, an approach which was suggested by Arrow and Hurwicz (1960) and Guesnerie (1975). Our resulting approximate loss formula (25) is calculated not from the derivatives of supply functions, but from the derivatives of restricted factor demand functions and marginal cost functions evaluated at the optimal level of output.

As our work is preceded by Hotelling (1938), it is important to discuss his work in relation to ours. Hotelling's contributions in this paper are known to be that (i) he showed the optimality of the marginal cost pricing principle of the regulated firms, and that (ii) he derived the approximate deadweight loss formula deviating from the optimality above.

For the first point, Silberberg (1980) pointed out that Hotelling's proof is not a valid one. In section 3 we gave a rigorous proof based on programming that the marginal cost pricing principle is necessary for optimality if technologies are quasi-concave. As Arrow and Hurwicz (1960) showed, this condition is not necessary for general nonconvex technology. Suppose that we alternatively consider Guesnerie's (1975) type 3 firm; that is a firm's technology is convex if some input is given. By applying results in Guesnerie (1975;12-13), it is straightforward to show that optimality is characterized by the competitive maximization of 'restricted' profit given the level of the input which causes the increasing returns to scale, and by the equality of the marginal value product with the factor price. (See also Aoki (1971).)[6] For the second point, Hotelling (1938;254) derived a similar deadweight loss measure to our formula (30). Similar to his first point, however, the derivation of his loss measure lacks true general equilibrium considerations and cannot be valid despite his own conjecture. (See Tsuneki (1987b)). Furthermore, even if we interpret his measure as in (30), it is not useful without knowing how to compute y'(0) using (26) via the Implicit Function Theorem. We must also note that our approach for the measurement of waste can be applied to an economy including type-3 firms which was not considered by Hotelling (1938). As we have seen, we can derive first order necessary conditions for the optimality. Then, comparing the optimum with a market equilibrium which includes mark-up rates in either product or factor markets, we can derive a deadweight loss measure using the methodology employed above. However, the resulting approximate measure is different from (25). What matters now are derivatives of restricted profit functions, given the input that causes the nonconvexity, instead of the derivatives of cost functions.

The drawback of our approach is that these derivatives are not observable at the distorted observed equilibrium. It is somewhat overcome by the following corollary:

<u>Corollary 1.1</u>: The approximate ADD measure

$$(31) \qquad -\tfrac{1}{2}t^T(A^1-B^1(C^1)^{-1}B^{1T})^{-1}t$$

is also accurate for quadratic functions as (25).

<u>PROOF</u>: According to Diewert's (1976;118) Quadratic Approximation Lemma, both $-(r'(0) + \tfrac{1}{2}r''(0))$ and $-\tfrac{1}{2}(r'(0) + r'(1))$ give the exact value of $r(0) - r(1)$ if r is quadratic. The former was adopted to derive (25). Now using the latter approximation and using (28) we have

$$(32) \qquad r^0 - r^1 \cong -\tfrac{1}{2}t^T y'(1).$$

Evaluating (26) at $z = 1$, computing $y'(1)$ by inverting the left-hand side matrix and substituting it into (32), we get (31). Q. E. D.

The remarkable property of (31) is that we can compute the deadweight loss of the economy from the local derivatives of demand and supply (cost) functions evaluated at the observed equilibrium. One important consequence of this observation, is that (31) can be computed using flexible functional forms for utility and production functions, so that we need not assume restrictive functional forms to calculate the global optimum point, as is usual in the numerical general equilibrium literature.

To derive our approximate loss formulae (25) and (31), we maintained several restrictive assumptions. The assumption of competitive factor markets can be dropped by introducing mark-up rates on factor prices, even though the resulting formulae become more complicated. The assumption that the production functions must be quasi-concave was required to guarantee the optimality of the marginal cost principle, and we already discussed how to extend our approach when we dropped the assumption.

We have assumed that each industry is monopolized. It is easy to extend the result to the case of an oligopolistic industry if we know the mark-up rates of firms and that the number of firms within one industry is fixed for all industries. However, it is difficult to introduce entry-exit behaviour, since the first order social optimality and market equilibrium conditions for incumbents and entrants are characterized by inequalities rather than equalities, for the number of firms changes discontinuously as equilibria are adjusted from the observed equilibrium to the optimum as is shown in the limit pricing literature. Therefore, it is difficult to apply our approach based on the Implicit Function Theorem.[7] The only case with entry that we can deal with within our framework is a Chamberlinian (1962) monopolistic competition with each product produced by homogeneous producers with respect to market shares, product qualilty and technology. Suppose also that the number of firms is continuous. Then, the long-run equilibrium is characterized by the zero-profit conditions of firms, i.e. equalities where the number of firms is also endogenous, and Chamberlinian excess capacities cause deadweight loss. The optimality conditions are characterized by the marginal cost pricing principle and the optimum number of firms is determined at the point where the marginal cost equals average cost. However, this model may be incomplete as a monopolistic competition model, since product diversity is exogenous in our model. To make it endogenous, we must work with much more simplified models, as adopted in Spence (1976), and Dixit and Stiglitz (1977).

4-5. Conclusion

This chapter has reconsidered the methodology for the measurement of waste due to imperfect competition in the presence of increasing returns to scale. We noted Hotelling's (1938) confusion about the optimality of the marginal cost principle and the derivation of his deadweight loss formula and rederived his formula as (30). The drawback to Hotelling's measure (30) is that it cannot be computed without finding the optimum beforehand. This drawback was corrected by our measure (25) and (31) where we required only local information in order to measure the deadweight loss. In particular, for the loss measure defined by (31), only information observable at the distorted equilibrium is required to measure the dead loss.

Fig. 10 shows a single-consumer economy with one good y and one non-producible production factor v, labour for example. The production possibility set OA exhibits increasing returns to scale, so that a competitive equilibrium cannot exist. However, an imperfectly competitive equilibrium $M \equiv (y_M, v_M)$ can exist where the marginal rate of substitution between the good and labour in consumption is different from the rate of substitution in production. The ADD optimum point $D \equiv (y_0, v_0)$ is a point where surplus labour is maximized given the utility level at the observed distorted equilibrium where the reference bundle β consists only of labour. The point D is characterized by the equality of the marginal rates of substitution in consumption (or marginal benefit of the good) and the marginal rates of substitution in production (or marginal cost of the good). Fig. 11 shows these two curves as MB and MC. The true amount of deadweight loss is shown by the curvilinear triangle ABC while the approximate measure (25) is shown by the triangle ABC' and (31) is shown by ABC''. The proof that ABC, ABC',

ABC'' really correspond to (11), (25), (31) for this simple economy is analogous to the derivation and construction of (1.45), (1.48), (1.49) in Chapter 1.

Given the limitations and assumptions listed within the chapter, we can apply our generalized Hotelling's measure to various models of imperfect competition and to publicly regulated markets when increasing returns to scale are present. We hope that the theoretical foundation provided here for Hotelling's measure will stimulate future empirical research and policy evaluation using it.

FOOTNOTES FOR CHAPTER 4

[1] For the production function $f^n(x_1,\ldots,x_N,v_1,\ldots,v_M)$, we assume that $\partial f^n/\partial x_n \equiv 0$. Therefore, the cost function C^n dual to f^n has the derivative $\partial C^n/\partial p_n \equiv 0$.

[2] For example, the increasing returns to scale technology obtained by combining a convex production possibilities set with a large fixed cost can be dealt with within our framework. (See Negishi (1962).) Aoki (1971) also used a similar technological assumption to the one adopted here.

[3] To apply the theorem, we need to assume that the Slater constraint qualification condition holds; that is, we assume that a feasible solution for (10) exists that strictly satisfies the first N+M inequality constraints.

[4] With increasing returns to scale, a local optimum that satisfies the first order conditions may not be globally optimal. We assume that (r^0,y^0,p^0,w^0) is a global optimum.

[5] Increasing returns to scale is usually defined as a more than proportionate increase of output when all the inputs are proportionately increased. Baumol, Panzar and Willig (1982;18-21) propose a weaker notion of increasing returns to scale, i.e., decreasing average cost, and showed that it is implied by decreasing marginal cost.

[6] Arrow and Hurwicz (1960) and Arnott and Harris (1976) gave examples where cost minimization and the marginal cost principle result in productive inefficiency in a type-3 economy.

[7] According to the recent study of contestable markets by Baumol, Panzar and Willig (1982), these strategic aspects are immaterial when the fixed cost is not sunk. Since a natural monopoly must set the price equal to its

average cost for a sustainable equilibrium, the mark-up rates t_n equal the difference between the average cost and marginal cost, so that our approach is applicable.

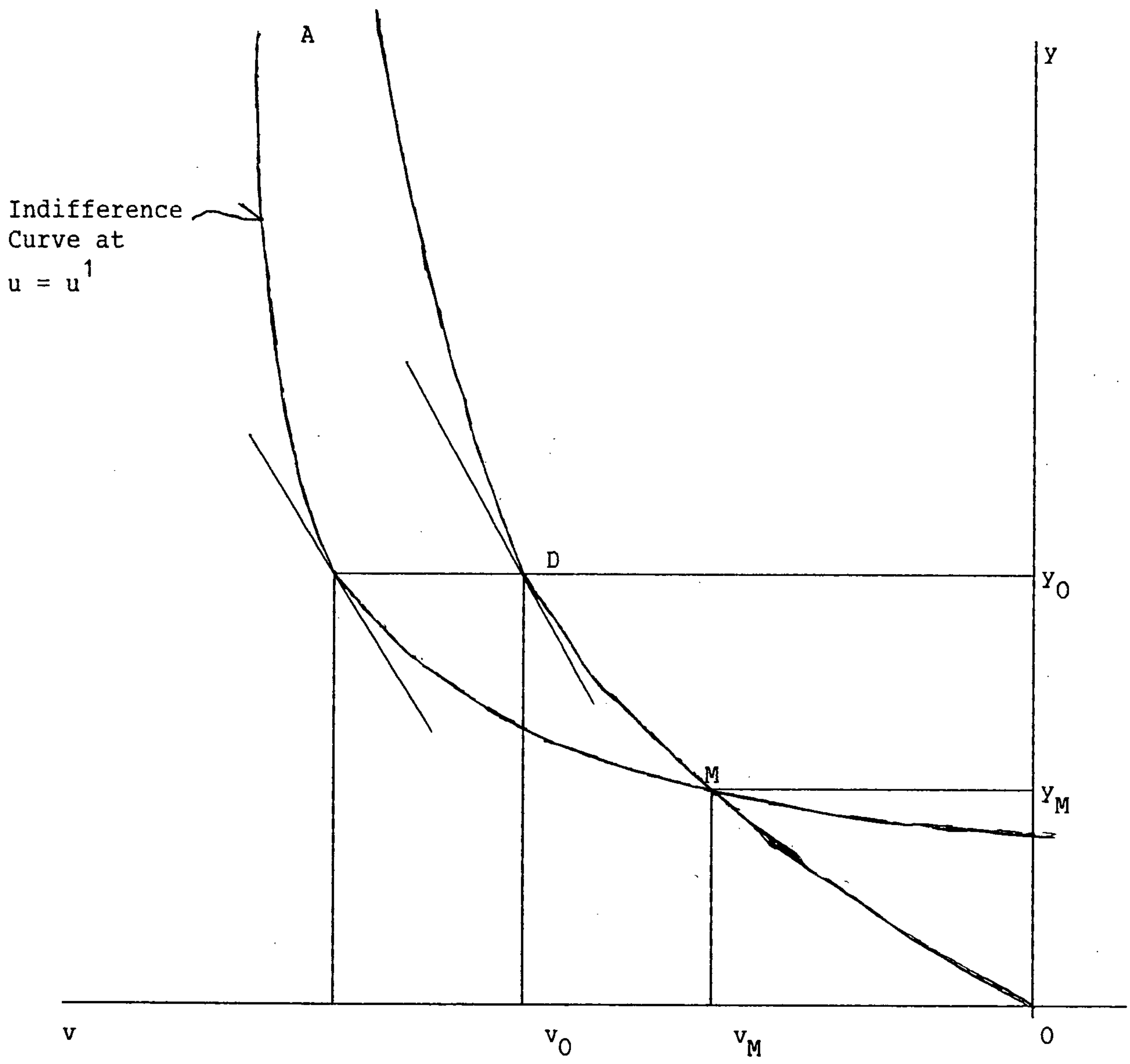


Fig. 10

The ADD Measure with Increasing Returns to Scale

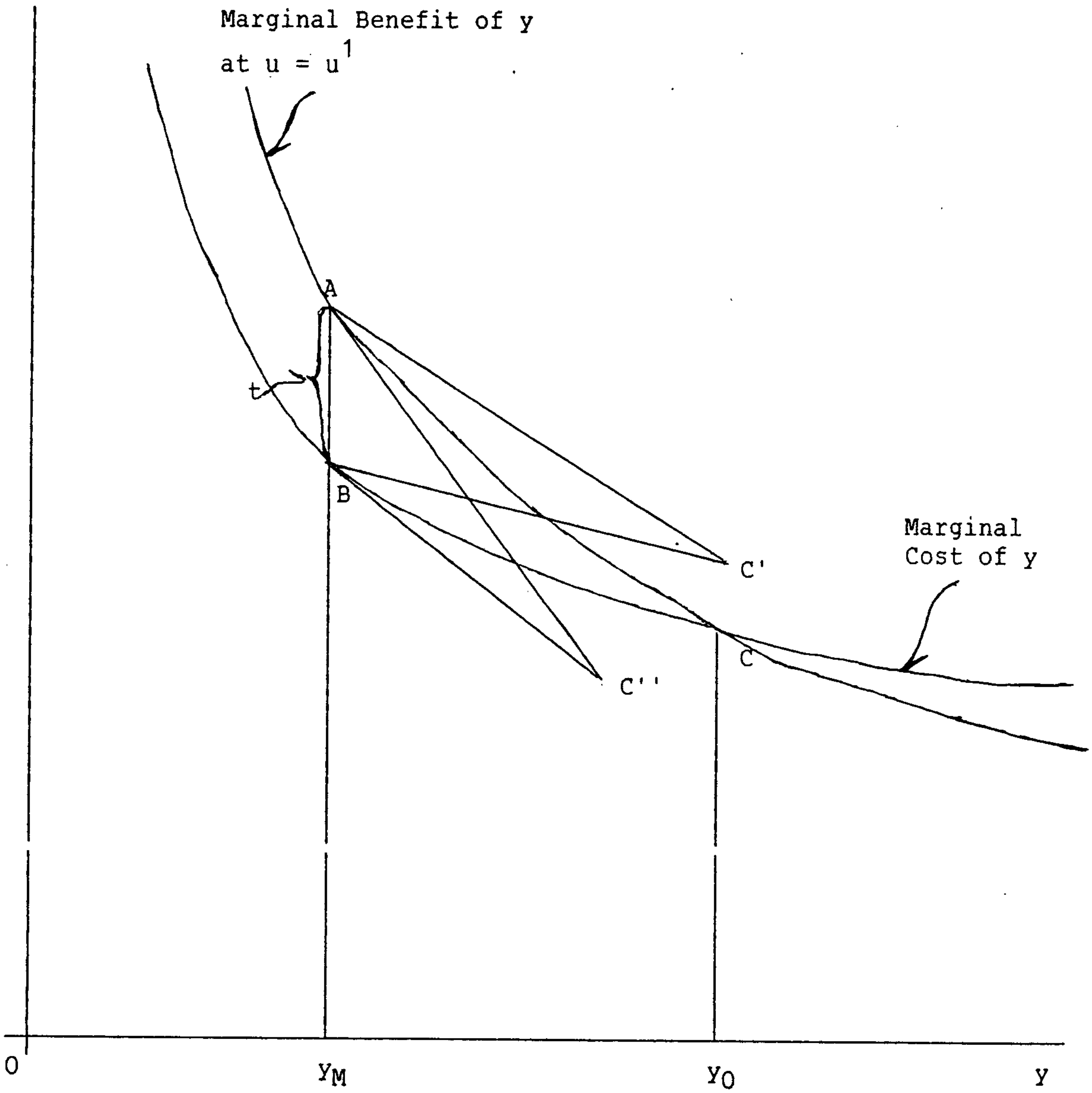


Fig. 11

The ADD Measure and its Approximations with Increasing Returns to Scale

CHAPTER 5

PROJECT EVALUATION RULES FOR IMPERFECTLY COMPETITIVE ECONOMIES

5-1 Introduction

In this chapter, we are interested in evaluating the net benefit of introducing a new technology in the presence of pre-existing distortions. This problem, we call project evaluation, may be defined as follows. Given a pre-project general equilibrium where consumers and firms follow some behavioural rules and demand and supply are equal, consider introducing a net output vector, called a project. Both consumers and firms adjust to this change and the economy moves to a post-project equilibrium. Project evaluation means to determine whether the project increased or decreased social welfare. The evaluation of a small project when there is perfect competition with tax distortions was surveyed by Diewert (1983b) and we applied his approach to evaluate the benefit of public goods when there are tax distortions in Chapter 3. Therefore, a natural way to proceed seems to be to extend this approach to the evaluation of a small project in an imperfectly competitive economy. However, this approach may not be as promising as it looks at first. Commenting on Davis and Whinston's (1965) use of a perceived demand curve in the second best theory of imperfect competition, Negishi (1967) pointed out that the second best policy of a public firm is indeterminate unless the perceived demand curves of the imperfect competitors are known. Therefore, we have to follow a different avenue.

In project evaluation, it is often the case that a new project has effects which are too large to be approximated by differential changes so that a shadow-pricing approach must be given up. Project evaluation rules for large projects have been studied by Negishi (1962) and Harris (1978) for the case of perfect competition with an increasing returns to scale technology due to a large fixed cost. Negishi (1962) studied the welfare implications of the entry of a new firm which is either a perfect competitor but has a large fixed cost technology or is the only firm which deviates from perfect competition. Some of Negishi's results were extended and some new rules were developed by Harris (1978), who also considered economies with distortionary taxation and public goods. However, Harris (1978) kept Negishi's (1962) assumptions about perfect competition and a convex technology with a fixed cost.

The purpose of this chapter is to extend the Negishi and Harris results to an imperfect market economy. This extension to an imperfectly competitive economy may be important considering the above mentioned indeterminacy of the optimum policy when there is imperfect competition.

Our results in this chapter may be summarized as follows. First, the Harris and Negishi results hold even if the assumption of a convex technology with a large fixed cost is replaced by general nonconvex technology, provided it is assumed that pre and post-project equilibria exist. Second, some of the extensions of Negishi's (1962) results by Harris (1978) depend on an implicit weakening of the criterion for welfare improvement made by Harris compared with Negishi's original welfare criterion. Thirdly, but most importantly, most of their rules can be applied in imperfectly competitive economies generally, again as long as pre and post-project equilibria are assumed to exist.

In the next section, we discuss welfare criteria for cost-benefit analysis. We discuss some confusion which exists concerning the use of the compensation principle and show that the criterion adopted by Negishi (1962) for the acceptance of a project is more strict than that by Harris (1978). We cannot judge which criterion is superior to the other. However, when we develop project evaluation rules, we simply have to be explicit on which criterion each rule is based. After presenting the model in section 3, in section 4 we reconsider the rules listed by Harris (1978), which include Negishi's (1962) original rules, and show that most of them are applicable in an imperfectly competitive economy. Economic implications and informational requirements for extending project evaluation rules to imperfectly competitive environments are also discussed. Section 5 concludes.

5-2. Compensation Criteria for Cost-Benefit Analysis Reconsidered

In chapter 1, we analyzed the properties of the ADD measure and the HB measure as a social welfare function. It was shown that they are consistent with the Pareto quasiordering, but they are not typically welfarist. We also suggested the use of deadweight loss measures for policy evaluation; alternative policies are ranked by the associated level of deadweight loss. The crucial assumption for using this procedure was that production possibilities sets remain unchanged by these policies. Therefore, this approach works for changes in tax or regulation policies with a fixed technology. We should note, however, that it is impossible to compare the values of these measures when production possibilities sets are changed by the introduction of new projects, since these measures do not satisfy welfarist assumptions. For example, the definition of Pareto optimal allocations takes as given the production possibilities sets. Consider a Pareto optimal allocation $\bar{a}$. Suppose there is a change in technology which permits the attainment of a new allocation $\hat{a}$ which is preferred by all consumers to $\bar{a}$. Using the ADD measure, $\hat{a}$ is always measured to exhibit at least as much deadweight loss as $\bar{a}$, as all Pareto optimal allocations have ADD measures of zero. As a consequence, it is inappropriate to use this deadweight loss measure when the technology is not fixed. The optimal reference equilibrium on which the HB measure is based depends on technology so if the introduction of a new project changes the technology, then a unique reference price cannot be determined to calculate the HB measure. Therefore, the topic of this chapter, the evaluation of a new project, necessitates an alternative criterion for social welfare and especially the problem of utility comparison. We recommend two criteria; the first is Bergson Samuelson social welfare function and the other is Hicks-Kaldor compensation principle.[1] In either criterion, we show in this chapter that our project evaluation criteria can be related to aggregate quantities of individual consumption bundles.

One natural way to proceed is to suppose that there exists a Pareto-inclusive social welfare function by assuming that either there exists an omniscient planner who distributes income optimally at any point (see Samuelson (1956)) or consumers' preferences satisfy Gorman's (1953) restriction of quasi-homotheticity. It is obvious that a Bergson-Samuelsonian social welfare function can serve as a welfare indicator to evaluate the states corresponding to different technologies in a consistent manner. Unfortunately, it is difficult to come to a consensus as to what an appropriate functional form for the social welfare function is. Also the assumption of quasihomothetic preferences is empirically restrictive.

An alternative method traditionally adopted for the evaluation of projects is the Kaldor (1939)-Hicks (1939, 1940) compensation principle, which states that a move from one state to another should be made if a potential Pareto improvement can be made.[2] However, there are several versions of the compensation principle and we have to be careful in distinguishing their different meanings. We suppose that two states of the economy (z, Y^0, x^0) and (z, Y^1, x^1) are compared where z is a vector of initial resources which is fixed, Y^i is an aggregate production possibilities set in state i = 0,1, and x^i is an aggregate consumption bundle in state i = 0,1. We also define the utility levels for the H households $u \equiv (u_1, \ldots, u_H)^T$ and the Scitovsky sets $S(u^i)$ for i = 0,1 corresponding to the utility functions f^h; the utility functions are assumed to be continuous from above, quasiconcave, and nonsatiated. The Scitovsky set for period i is defined as $S(u^i) \equiv \{x : \Sigma_{h=1}^H x^h \leq x,\ f^h(x^h) \geq u_h^i,\ h = 1, \ldots, H\}$, where $u^i \equiv (u_1^i, \ldots, u_H^i)^T$. Our assumptions ensure that $S(u^i)$is convex (see Scitovsky (1941-2(b)). Now we can define the four types of compensation test.

(1) 1 R_{KSP} 0 (read state 1 is preferred to state 0 by the Kaldor strong principle) iff $x^1 \in S(u^0)$.

(2) 1 R_{KWP} 0 (read state 1 is preferred to state 0 by the Kaldor weak principle) iff $z + Y^1$ intersects with $S(u^0)$.

(3) 1 R_{HSP} 0 (read state 1 is preferred to state 0 by the Hicks strong principle) iff $x^0 \notin S(u^1)$.

(4) 1 R_{HWP} 0 (read state 1 is preferred to state 0 by the Hicks weak principle) iff $z + Y^0$ and $S(u^1)$ are disjoint.

What is called Scitovsky's (1941-2(a)) double criterion is that 0 is preferred to 1 iff both the Hicks and Kaldor criteria are met in either weak or strong form. The following two propositions are obvious.

PROPOSITION 1: If $1 \; R_{KSP} \; 0$, then $1 \; R_{KWP} \; 0$, but not vice versa.

PROPOSITION 2: If $1 \; R_{HWP} \; 0$, then $1 \; R_{HSP} \; 0$, but not vice versa.

Negishi (1962;88) wrote that if $(z+Y^1)$ and $S(u^0)$ are disjoint, i.e., $0 \; R_{HWP} \; 1$, then state 1 is not recommended. In page 89, he wrote if $z + Y^0$ and $S(u^1)$ are disjoint, i.e., $1 \; R_{HWP} \; 0$ then state 1 is preferred to 0. Therefore, we may conclude that Negishi adopted the Hicks weak compensation criterion (4) as his project acceptance criterion. In contrast, Harris (1978;412) suggested that if $x^1 \notin S(u^0)$ then 0 is preferred to 1 and that on p. 414, if $x^0 \notin S(u^1)$ then 1 is preferred to 0. Therefore, he utilized Hicks strong compensation criterion (3). By Proposition 2, we deduce that if project 1 is accepted by Negishi's criterion, 1 is also accepted by Harris' criterion, but not vice versa. In economic terms, state 1 meets Harris' acceptance criterion if the pre-project aggregate consumption bundle x^0 cannot be redistributed so as to make everyone as well off as u^1, whereas state 1 meets Negishi's acceptance criterion if everyone cannot be made as well off as u^1 even when the best production plans and income distribution policy are executed using the initial endowment z and technology Y^0.

These two project rules are equivalent under the following condition.

PROPOSITION 3: We define perfect competition as an equilibrium where there exits a price vector that equilibrates the markets and consumers are maximizing utilities and producers are maximizing profits given the prices. If consumers' preferences are quasiconcave and quasihomothetic[3] and state 0 is perfectly competitive, then $1\ R_{HSP}\ 0$ implies $1\ R_{HWP}\ 0$, i.e., HSP and HWP are equivalent.

PROOF: From definition (3), $x^0 \notin S(u^1)$. If state 0 is perfectly competitive, there exists $p^0 \geq 0_N$ such that $p^{0T}x \geq p^{0T}x^0$ for all $x \in S(u^0)$ and $p^{0T}x^0 \geq p^{0T}\ y$ for all $y \in Y^0 + z$. (See Debreu (1951, 1959)).

Since the Gorman aggregation conditions are met, $x^0 \notin S(u^1)$ implies $x^1 \in S(u^0)$ and $S(u^1)$ is a subset of $S(u^0)$, but x^1 is not on the boundary of $S(u^0)$. Therefore, $p^{0T}x > p^{0T}x^0 \geq p^{0T}y$ for all $y \in Y^0 + z$ and for all $x \in S(u^1)$. Therefore, $S(u^1)$ and $Y^0 + z$ are disjoint and from definition (4) $1\ R_{HWP}\ 0$ follows. Q. E. D.

Therefore, for perfectly competitive economies in which the Gorman aggregation conditions hold, the two Hicksian criteria are equivalent. If these two conditions are not met, Negishi's criterion is stronger than that of Harris, so that we have to clarify whether a project acceptance rule is based on the Negishi or Harris criterion.

5-3. The Model

We now sketch the model of Harris (1978). There are N goods n = 1,...N, and H consumers, h = 1,...,H. Consumers' preferences are represented by utility functions $f^h(x_h)$ where $x_h \in \Omega^h$, a transformed orthant R_+^N. As for production, an aggregate closed production set and a net output vector belonging to it for the private firm sector are denoted by Y and y. A new firm introduced by the government has an operating technology and a net output vector G and g, respectively. We do not assume anything about the properties of the technologies Y and G except closedness (which is harmless from an empirical point of view). We assume the existence of equilibria as follows.

A before-project equilibrium is defined as an H + 2 tuple in R^N, $\{p^0, y^0, (x_1^0, \ldots, x_H^0)\}$ such that

(5) $f^h(x_h^0) \geq f^h(x_h)$ for all $x_h \in$ {budget constraint for h under p^0}, for $h = 1, \ldots, H$,

(6) $y^0 \in Y$

(7) $x^0 \equiv \Sigma_{h=1}^{H} x_h^0 = y^0 + z$.

In the same way, we define an after-project equilibrium to be an H + 3 tuple in R^N, $\{p^1, g^1, y^1, (x_1^1, \ldots, x_H^1)\}$ such that

(8) $f^h(x_h^1) \geq f^h(x_h)$ for all $x_h \in$ {budget constraint for h under p^1} for $h = 1, \ldots, H$,

(9) $y^1 \in Y^1$

(10) $g^1 \in G^1$

(11) $x^1 \equiv \Sigma_{h=1}^{H} x_h^1 = y^1 + g^1 + z.$

These two definitions seem to incorporate the minimum requirements for an imperfectly competitive equilibria studied by Negishi (1961-2), Arrow and Hahn (1971, Ch. 6), and Roberts and Sonnenschein (1977); i.e., (i) price taking behaviour of consumers, (ii) feasibility of equilibrium production, and (iii) equality of demand and supply. The main result in this chapter is that the existence of before and after-project equilibria is sufficient for the validity of most of the project evaluation rules developed by Negishi and Harris.

5-4. Project Evaluation Rules

Using the Hicksian strong compensation criterion, Harris' two main project evaluation rules (using his numbering of the rules) can be restated within our framework as follows.

Rule 2: A sufficient condition to reject a proposed project is that the project have a net value at before-project prices which is less than the change in the profits on all other production activities evaluated at before-project prices; i.e., the rejection criterion is $p^{0T}g^1 < p^{0T}(y^0 - y^1)$.

Rule 4: A sufficient condition to accept a project is that minus the profits (or minus the net value) of the project at post-project prices be less than the change in profits in the rest of the economy at after-project prices; i.e., the acceptance criterion is $-p^{1T}g^1 < p^{1T}(y^1 - y^0)$.

PROOF: Substituting the resources constraints (7) and (11) into Rule 2 and Rule 4, we find that Rule 2 is equivalent to $p^{0T}x^0 > p^{0T}x^1$, and Rule 4 is equivalent to $p^{1T}x^1 > p^{1T}x^0$. As the Scitovsky sets $S(u^0)$ and $S(u^1)$ are convex sets, and x^0 and x^1 belong to the boundary of $S(u^0)$ and $S(u^1)$, respectively from (5) and (8), Rule 2 implies $x^1 \notin S(u^0)$ and Rule 4 implies $x^0 \notin S(u^1)$. Now applying the Hicks strong compensation principle (3), Rule 2 gives a sufficient condition for state 0 to be preferred to 1, and Rule 4 gives a sufficient condition for state 1 to be preferred to 0. Q. E. D.

We proved Harris's two main rules without making any assumptions concerning market structure. We assumed only that markets clear and consumers are price takers. In particular, we did not assume that either the private production sector or the government optimizes. Therefore, Harris' rules have a very broad applicability.

From Proposition 3, Rule 2 is valid for Hicks' weak compensation principle if the Gorman aggregation conditions for consumers' preferences are met and the after-project equilibrium is perfectly competitive. Similarly, Rule 4 is valid for Hicks' weak compensation principle if the aggregation of consumers' preferences conditions are met and the before-project equilibrium is perfectly competitive.[4] This is the reason why Negishi (1962;91) assumed that Gorman's preference restrictions were met in his demonstration of the validity of Rule 4.

Harris (1978) restated Negishi's (1962) main two rules, Rule 1 and Rule 3 in Harris' numbering, as follows: Referring to Hicks weak compensating principle

Rule 1: A sufficient condition to reject a proposed project is that it is impossible for the project to show a nonnegative net value at before-project equilibrium prices; i.e., the rejection criterion is $p^{0T}g^{1} < 0$.

Rule 3. A sufficient condition to accept a project is that the project show positive profits at post-project prices; i.e., the acceptance criterion is $p^{1T}g^{1} > 0$.

In general, unless the economy is competitive, except for the new firm introduced by the government, Rule 1 and Rule 3 are invalid (see Negishi (1962)).[5] However, if the competition assumption is met, then Rule 1 implies Rule 2 and Rule 3 implies Rule 4 as the price-taking assumptions for firms imply $p^{0T}y^{0} \geq p^{0T}y^{1}$ and $p^{1T}y^{1} \geq p^{1T}y^{0}$. Obviously, other implications cannot be valid in general. This means that the Harris rules are more complete than Negishi's for competitive economies (Harris (1978;413)). Stated another way, some project accepted by Harris' Rule 4 may not be accepted by Negishi's Rule 3 and some project rejected by Harris' Rule 2 may not be rejected by Negishi's Rule 1. This indeterminacy of Rules 1 and 3 comes partly from the fact that Negishi adopted Hicks' weak principle as his welfare criterion, which is more indeterminate than the Hicks strong principle adopted by Harris, but chiefly it is because the profitability criterion is a less exact estimate of the social welfare change than the index number approach used in Rule 2 and Rule 4.

The drawback of Rule 2 and Rule 4 seems to be their more demanding informational requirements, i.e., as long as the economy is competitive, the informational requirements for implementing Rules 1 and 3 seem less onerous than for Rules 2 and 4. In Rule 1, we only need to know the production possibilities set of the public agency. In Rule 3, after-project prices must be somehow predicted. However, in Rules 2 and 4, after-project output levels of the rest of the economy are also required, and this is difficult to obtain ex ante. In summary, Rules 1 and 3 show the first-best significance of a new technology in terms of its profitability. They are almost always less exact than Rule 2 and Rule 4, and cannot tell us anything in second best conditions when we do not have perfect competition. Rule 2 and Rule 4 are valid in both first best and second best conditions, and in a second best, it evaluates the improvement of technical efficiency and market efficiency at the same time.

5-5. Conclusion

Harris discussed thirteen project evaluation rules in his paper. Excluding Rule 1 and Rule 3, which we have discussed, and Rule 8, which is analogous to Rule 1 in the tax-distorted economy, all of his rules are effective for non-competitive market structures, because the proofs of all of them are similar to the proofs of Rule 2 and Rule 4, or they are contrapositives of other rules. In particular, the satisfaction of Samuelsonian conditions is not necessary to prove Harris' Rules 11, 12 and 13, which give cost-benefit rules for supplying a public input. Although profitability of a new project has a normative meaning only in first-best situations where the usual marginal conditions hold, the application of index number theorems due to Hicks (1940, 1941-2) and Samuelson (1950) are fruitful even in a second best economy.

FOOTNOTES FOR CHAPTER 5

[1] Note also that, in general, the relationship between the compensation principle and the sum of equivalent or compensating variations is ambiguous. See Boadway (1974), Smith and Stevens (1975), Foster (1976) and Boadway (1976).

[2] In chapter 3, we considered a sufficient condition for the existence of a Pareto improvement. This discussion may be related to the compensation principle. See Bruce and Harris (1982).

[3] Quasi-homotheticity is satisfied if Engel curves are straight lines and they are parallel for all consumers. See Gorman (1953). Alternatively, we can think of the case where income distribution is always optimized with respect to a Bergson-Samuelsonian social welfare function. In this case, Bergson's social indifference surfaces do not intersect and convex to the origin if utility functions are concave and a social welfare function is quasi-concave (see Gorman (1959) and Negishi (1963)). Replacing the Scitovsky set with the better set of Bergson's indifference surface, the rest of the discussion goes through. When we mention Gorman's restrictions on preferences, we can also allow for this alternative case.

[4] Harris (1978;410) pointed out that his welfare criterion is consistent with an ordering based on the Bergson-Samuelsonian social welfare function, if it exists. Referring to Proposition 3, Negishi's welfare criterion may not be consistent with such a social welfare function, if the assumption of perfect competition is dropped.

[5] More exactly, Rule 1 applies even if the after-project equilibrium is imperfectly competitive and Rule 3 applies even if the before-project

equilibrium is imperfectly competitive, provided that the equilibria exist. This is obvious from the proofs of these rules by Negishi (1962).

REFERENCES

Allais, M. (1943). A la recherche d'une discipline économique, Tom I, Paris: Imprimerie Nationale.

Allais, M. (1977). "Theories of General Economic Equilibrium and Maximum Efficiency," pp. 129-201 in Equilibrium and Disequilibrium in Economic Theory, E. Schwödiauer ed., Dordrecht: D. Reidel.

Aoki, M. (1971). "Investment Planning Process for an Economy with Increasing Returns." Review of Economic Studies 38, 273-80.

Arnott, R., and R. G. Harris. (1976). "Increasing Returns and Decentralization Rules for the Public Sector." Queen's University Discussion Paper No. 22.

Arrow, K. J. (1951). "An Extension of the Basic Theorems of Classical Welfare Economics." pp. 507-532 in Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability. J. Neyman, ed. Berkeley: University of California Press.

Arrow, K. J., and F. H. Hahn. (1971). General Competitive Analysis. San Francisco: Holden Day.

Arrow, K. J., and L. Hurwicz. (1960). "Decentralization and Computation in Resource Allocation." pp. 34-104 in Essays in Economics and Econometrics. R. Pfouts, ed. University of North Carolina Press.

Atkinson, A. B., and N. H. Stern. (1974). "Pigou, Taxation and Public Goods." Review of Economic Studies, 41, 119-128.

Atkinson, A. B., and J. E. Stiglitz. (1980). Lectures on Public Economics. New York: McGraw-Hill.

Auerbach, A. J. (1985). "The Theory of Excess Burden and Optimal Taxation." pp. 61-127 in Handbook of Public Economics Vol. 1. A. J. Auerbach, and M. S. Feldstein, eds. Amsterdam, North-Holland.

Baumol, W. J., J. C. Panzar, and R. D. Willig. (1982). Contestable Markets and the Theory of Industry Structure. New York: Harcourt Brace and Jovanovich.

Bhagwati, J. N., R. A. Brecher, and T. Hatta (1983). "The Generalized Theory of Transfer and Welfare: Bilateral Transfers in a Multilateral World." American Economic Review 73, 606-618.

Blackorby, C., and D. Donaldson. (1985). "Consumers' Surpluses and Consistent Cost-Benefit Tests." Social Choice and Welfare 1, 251-262.

Blackorby, C., and D. Donaldson. (1986). "Money-Metric Utility: A Harmless Normalization?" U.B.C. discussion paper, 86-9.

Blackorby, C., D. Primont, and R. R. Russell. (1978). Duality, Separability and Functional Structure: Theory and Economic Applications. New York: Elsevier North-Holland.

Boadway, R. W. (1974). "The Welfare Foundations of Cost-Benefit Analysis." Economic Journal 84, 426-439.

Boadway, R. W. (1976). "The Welfare Foundations of Cost-Benefit Analysis--A Reply." Economic Journal 86, 359-361.

Boiteux, M. (1951). "Le 'revenue distruable' et les pertes économiques." Econometrica 19, 112-133.

Boiteux, M. (1956). "Sur la gestion des monopoles publics astreints à l'équilibre budgétaire." Econometrica 24, 22-40. (Translation in, Journal of Economic Theory 3, 219-240.)

Brown, D. J., and G. Heal. (1980). "Two-Part Tariffs, Marginal Cost Pricing and Increasing Returns in a General Equilibrium Model." Journal of Public Economics 13, 25-49.

Bruce, N., and R. G. Harris. (1982). "Cost-Benefit Criteria and the Compensation Principle in Evaluating Small Projects." Journal of Political Economy 90, 755-775.

Chamberlin, E. H. (1962). The Theory of Monopolistic Competition, 8th edition. Harvard University Press.

Currie, J. M., J. A. Murphy, and A. Schmitz. (1971). "The Concept of Economic Surplus and Its Use in Economic Analysis." Economic Journal 81, 741-799.

Davis, O. A., and A. B. Whinston. (1967). "Piecemeal Policy in the Theory of Second Best." Review of Economic Studies 34, 323-331.

Debreu, G. (1951). "The Coefficient of Resource Utilization." Econometrica 19, 273-291.

Debreu, G. (1954). "A Classical Tax-Subsidy Problem." Econometrica 22, 14-22.

Debreu, G. (1959). Theory of Value. New York: John Wiley.

Diamond, P. A. (1975). "A Many-Person Ramsey Tax Rule." Journal of Public Economics 4, 227-244.

Diamond, P. A., and D. L. McFadden. (1974). "Some Uses of the Expenditure Function in Public Finance." Journal of Public Economics 3, 3-21.

Diamond, P. A., and J. A. Mirrlees. (1971). "Optimal Taxation and Public Production I: Production Efficiency and II: Tax Rules." American Economic Review 61, 8-27 and 261-278.

Diewert, W. E. (1976). "Exact and Superlative Index Numbers." Journal of Econometrics 4, 115-145.

Diewert, W. E. (1978). "Optimal Tax Perturbations." Journal of Public Economics 10, 139-177.

Diewert, W. E. (1981). "The Measurement of Deadweight Loss Revisited." Econometrica 49, 1225-1244.

Diewert, W. E. (1982). "Duality Approaches to Microeconomic Theory." pp. 535-599 in Handbook of Mathematical Economics Vol. 2, K. J. Arrow and M. D. Intriligator, eds. Amsterdam: North-Holland.

Diewert, W. E. (1983a). "The Measurement of Waste Within the Production Sector of an Open Economy." Scandinavian Journal of Economics 85, 159-179.

Diewert, W. E. (1983b). "Cost-Benefit Analysis and Project Evaluation: A Comparison of Alternative Approaches." Journal of Public Economics 22, 265-302.

Diewert, W. E. (1984a). "The Measurement of Deadweight Loss in an Open Economy." Economica 51, 23-42.

Diewert, W. E. (1984b). "On the Relationship Between the Measurement of Deadweight Loss and the Measurement of Project Benefits." U.B.C. Discussion Paper 84-32.

Diewert, W. E. (1985a). "The Measurement of Waste and Welfare in Applied General Equilibrium Models." pp. 42-103 in New Developments in Applied General Equilibrium Analysis. J. Pigott and J. Whalley eds. Cambridge: Cambridge University Press.

Diewert, W. E. (1985b). "A Dynamic Approach to the Measurement of Waste in an Open Economy." Journal of International Economics 19, 213-240.

Diewert, W. E. (1986). The Measurement of Economic Benefits of Infrastructure Services. Lecture Notes in Economics and Mathematical Systems 278, Berlin: Springer Verlag.

Diewert, W. E., and A. D. Woodland. (1977). "Frank Knight's Theorem in Linear Programming Revisited." Econometrica 45, 375-398.

Dixit, A. K. (1975). "Welfare Effects of Tax and Price Changes." Journal of Public Economics 4, 103-123.

Dixit, A. K. (1979). "Price Changes and Optimum Taxation in a Many-Consumer Economy." Journal of Public Economics 11, 143-157.

Dixit, A. K., and J. E. Stiglitz. (1977). "Monopolistic Competition and Optimum Product Diversity." American Economic Review 67, 297-308.

Drèze, J. P., and N. H. Stern. (1986). "The Theory of Cost-Benefit Analysis," forthcoming in Handbook of Public Economics, A. J. Auerbach and M. S. Feldstein, eds. Amsterdam, North-Holland.

Feldstein, M. S. (1972). "Distributional Equity and the Optimal Structure of Public Prices." American Economic Review 62, 32-36.

Feldstein, M. S. (1978). "The Welfare Cost of Capital Income Taxation." Journal of Political Economy 86, S29-51.

Foster, E. (1976). "The Welfare Foundations of Cost-Benefit Analysis--A Comment." Economic Journal 86, 353-358.

Foster, E., and H. Sonnenschein. (1970). "Price Distortions and Economic Welfare." Econometrica 38, 281-297.

Geary, P. T., and M. Morishima. (1973). "Demand and Supply under Separability." pp. 87-147 in Theory of Demand: Real and Monetary. M. Morishima and others. Oxford: Clarendon.

Gorman, W. M. (1953). "Community Preference Fields." Econometrica 21, 63-80.

Gorman, W. M. (1955). "The Intransitivity of Certain Criteria Used in Welfare Economics." Oxford Economic Papers NS7, 25-35.

Gorman, W. M. (1959). "Are Social Indifference Curves Convex?" Quarterly Journal of Economics 73, 485-496.

Green, J., and E. Sheshinski. (1979). "Approximating the Efficiency Gain of Tax Reforms." Journal of Public Economics 11, 179-195.

Guesnerie, R. (1975). "Pareto Optimality in Non-Convex Economies." Econometrica 43, 1-30.

Guesnerie, R. (1977). "On the Direction of Tax Reform." Journal of Public Economics 7, 179-202.

Guesnerie, R. (1979). "General Statements on Second-Best Pareto Optimality." Journal of Mathematical Economics 6, 169-194.

Hammond, P. (1986). "Project Evaluation by Potential Tax Reform." Journal of Public Economics 30, 1-36.

Harberger, A. C. (1964). "The Measurement of Waste." American Economic Review 54, 58-76.

Harberger, A. C. (1971). "Three Basic Postulates for Applied Welfare Economics: An Interpretive Essay." Journal of Economic Literature 9, 785-797.

Harberger, A. C. (1974). Taxation and Welfare. Boston: Little, Brown and Company.

Harris, R. G. (1976). "A Note on Convex-Concave Demand Systems with an Application to the Theory of Optimal Taxation." Queen's University discussion paper No. 197.

Harris, R. G. (1978). "On the Choice of Large Projects." Canadian Journal of Economics 11, 404-423.

Harris, R. G. (1984). "Applied General Equilibrium Analysis of Small Open Economies and Imperfect Competition." American Economic Review 74, 1016-1032.

Hatta, T. (1977). "A Theory of Piecemeal Policy Recommendations." Review of Economic Studies 44, 1-21.

Hicks, J. R. (1939). "The Foundation of Welfare Economics." Economic Journal 49, 696-711.

Hicks, J. R. (1940). "The Valuation of Social Income." Economica NS7, 105-124.

Hicks, J. R. (1941-2). "Consumers' Surplus and Index Numbers." The Review of Economic Studies 9, 126-137.

Hicks, J. R. (1946). Value and Capital, second edition, Oxford: Clarendon Press.

Hotelling, H. (1932). "Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions." Journal of Political Economy 40, 577-616.

Hotelling, H. (1938). "The General Welfare in Relation to Problems of Taxation and of Railway and Utility Rates." Econometrica 6, 242-269.

Jorgenson, D. W. (1984). "Econometric Methods for Applied General Equilibrium Analysis." pp. 139-203 in Applied General Equilibrium Analysis. H. E. Scarf and J. B. Shoven eds. Cambridge: Cambridge University Press.

Kaizuka, K. (1965). "Public Goods and Decentralization of Production." Review of Economics and Statistics 47, 118-120.

Kaldor, N. (1939). "Welfare Propositions and Interpersonal Comparisons of Utility." Economic Journal 49, 549-552.

Karlin, S. (1959). Mathematical Methods and Theory in Games, Programming and Economics. Reading, Mass.: Addison-Wesley.

King, M. A. (1983). "Welfare Analysis of Tax Reforms Using Household Data." Journal of Public Economics 21, 183-214.

King, M. A. (1986). "A Pigovian Rule for the Optimum Provision of Public Goods." Journal of Public Economics 30, 273-291.

Kay, J. A. (1980). "The Deadweight Loss from a Tax System." Journal of Public Economics 13, 111-119.

Lipsey, R. G., and K. J. Lancaster. (1956). "The General Theory of Second Best." Review of Economic Studies 24, 11-32.

Mangasarian, O. L. (1969). Nonlinear Programming. New York: McGraw-Hill.

McKenzie, G. W. (1983). Measuring Economic Welfare: New Method. London: Cambridge University Press.

McKenzie, L. W. (1957). "Demand Theory Without a Utility Index." Review of Economic Studies 24, 185-189.

Meade, J. E. (1952). "External Economies and Diseconomies in a Competitive Situation." Economic Journal 62, 54-67.

Meade, J. E. (1955). Trade and Welfare. Oxford: Oxford University Press.

Mirrlees, J. (1986). "The Theory of Optimal Taxation." pp. 1197-1249 in Handbook of Mathematical Economics Vol. 3, K. J. Arrow and M. D. Intriligator, eds. Amsterdam: North-Holland.

Mohring, H. (1971). "Alternative Welfare Gain and Loss Measures." Western Journal of Economics 9, 349-368.

Negishi, T. (1960), "Welfare Economics and Existence of an Equilibrium for a Competitive Economy." Metroeconomica 12, 92-97.

Negishi, T. (1960-1). "Monopolistic Competition and General Equilibrium." Review of Economic Studies 28, 196-201.

Negishi, T. (1962). "Entry and the Optimal Number of Firms." Metroeconomica 14, 86-96.

Negishi, T. (1963). "On Social Welfare Function." Quarterly Journal of Economics 77, 156-158.

Negishi, T. (1967). "Perceived Demand Curve in the Theory of Second Best." Review of Economic Studies 34, 316-317.

Negishi, T. (1978). General Equilibrium Theory and International Trade, Amsterdam: North-Holland.

Pazner, E. A., and E. Sadka. (1980). "Excess-Burden and Economic Surplus as Consistent Welfare Indicators," Public Finance 35, 439-449.

Pigott, J., and J. Whalley. (1982). "General Equilibrium with Public Production and Public Provision: Computation, and Results for the U.K. Case." pp. 1-26 in Public Production. Zeitschrift für Nationalökonomie, Supplementum 2, D. Bös, R. A. Musgrave and J. Wiseman, eds. Vienna: Springer Verlag.

Pigou, A. C. (1947). A Study in Public Finance. Third edition. London: Macmillan.

Ramsey, F. P. (1927). "A Contribution to the Theory of Taxation." Economic Journal 37, 47-61.

Roberts, D. J., and H. Sonnenschein. (1977). "On the Foundations of the Theory of Monopolistic Competition." Econometrica 45, 101-113.

Samuelson, P. A. (1947). Foundations of Economic Analysis, Cambridge, Mass.: Harvard University Press.

Samuelson, P. A. (1950). "Evaluation of Real National Income." Oxford Economic Papers NS2, 1-29.

Samuelson, P. A. (1954). "Pure Theory of Public Expenditure." Review of Economics and Statistics 36, 387-389.

Samuelson, P. A. (1956). "Social Indifference Curves." Quarterly Journal of Economics 70, 1-22.

Samuelson, P. A. (1974). "Complementarity--An Essay on the 40th Anniversary of the Hicks-Allen Revolution in Demand Theory." Journal of Economic Literature 4, 1255-1289.

Scitovsky, T. (1941-2(a)). "A Note on Welfare Propositions in Economics." Review of Economic Studies 9, 77-88.

Scitovsky, T. (1941-2(b)). "A Reconsideration of the Theory of Tariffs." Review of Economic Studies 9, 89-110.

Shephard, R. W. (1953). Cost and Production Functions. Princeton: Princeton University Press.

Shoven, J. B., and J. Whalley. (1972). "A General Equilibrium Calculation of the Effects of Differential Taxation of Income from Capital in the U.S." Journal of Public Economics 1, 281-321.

Shoven, J. B., and J. Whalley. (1973). "General Equilibrium with Taxes: A Computational Procedure and an Existence Proof." The Review of Economic Studies 40, 475-489.

Shoven, J. B., and J. Whalley. (1977). "Equal Yield Tax Alternatives--General Equilibrium Computational Techniques." Journal of Public Economics 8, 211-224.

Silberberg, E. (1980). "Harold Hotelling and Marginal Cost Pricing." American Economic Review 70, 1054-1057.

Smith, B., and F. Stephen. (1975). "The Welfare Foundations of Cost-Benefit Analysis: Comment." Economic Journal 85, 902-905.

Spence, A. M. "Product Selection, Fixed Costs and Monopolistic Competition." Review of Economic Studies 43, 217-235.

Stiglitz, J. E., and P. S. Dasgupta. (1971). "Differential Taxation, Public Goods and Economic Efficiency." Review of Economic Studies 38, 151-174.

Tsuneki, A. (1985). "On the Choice of Large Projects: A Generalization." Canadian Journal of Economics 18, 660-664.

Tsuneki, A. (1987a). "The Measurement of Waste in a Public Goods Economy." Journal of Public Economics, forthcoming.

Tsuneki, A. (1987b). "Harold Hotelling's General Equilibrium Welfare Economics." Mimeographed.

Turunen, A. H. (1986). "A Note on the Measurement of Production Gain Due to a Tariff Reform in a Small Open Economy." Working Paper No. 86-1, Center for Economic Research, University of Texas at Austin.

Uzawa, H. (1958). "The Kuhn-Tucker Theorem in Concave Programming." pp. 32-37 in Studies in Linear and Nonlinear Programming. K. J. Arrow, L. H. Hurwicz and H. Uzawa, eds. Stanford, California: Stanford University Press.

Varian, H. R. (1974). "Equity, Envy and Efficiency." Journal of Economic Theory 9, 63-91.

Varian, H. R. (1976). "Two Problems in the Theory of Fairness." Journal of Public Economics 5, 249-260.

Weymark, J. A. (1979). "A Reconciliation of Recent Results in Optimal Taxation Theory." Journal of Public Economics 12, 171-189.

Weymark, J. A. (1985). "Money-Metric Utility Functions." International Economic Review 26, 219-232.

Wildasin, D. E. (1984). "On Public Good Provision with Distortionary Taxation." Economic Inquiry 22, 227-243.