# EXPORT SUPPLY AND IMPORT DEMAND ELASTICITIES

By

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ABSTRACT

The aim of this thesis is to extend the empirical research which has been undertaken using the GNP function approach to measuring export supply and import demand responsiveness. Exports and imports are divided into several components and detailed sets of elasticities produced. In the second part of the thesis imperfect adjustment is allowed for in the GNP function model.

The GNP function framework treats imports as an input to the domestic technology while exports are an output. The aggregate technology can then be represented by a restricted profit function facilitating the derivation of net output supply elasticities. In this study the aggregate net outputs are exports, imports, labour and domestic sales supply. Capital is treated as a fixed input. Time-series of input-output data for Canada are used covering the period 1961 to 1980.

In the first model estimated, four export and four import components are included by the use of aggregator functions and a two-stage estimation process. The recently developed Symmetric Generalised McFadden functional form which permits imposition of the correct curvature conditions while retaining flexibility is used at both the aggregator and GNP function levels. The aggregate export own-price supply elasticity was found to be 1.67 in 1970 while the aggregate import own-price demand elasticity was -1.62. Increases in the prices of both imports and labour were found to decrease the supply of exports while exports were found to be complementary to the output of domestic sales supply. The demand for labour was found to be more elastic than in

earlier studies and a general trend towards increasing price responsiveness in the Canadian economy was observed. The own-price elasticities for the four export and four import components were stable and of reasonable magnitude. All the export and import components were found to be complementary.

To remove the assumption of separability, modelling was extended to two larger disaggregated Generalised McFadden GNP function models containing four export (import) components, aggregate imports (exports), labour and domestic sales as net outputs. Using this procedure more substitution between the export and import components was found.

A planning price model whereby the producers' notional price adjusts gradually to actual price changes indicated that imperfect adjustment is particularly important in the traded goods sector. Exports fully adjusted to price changes only over an extended period.

Finally, an adjustment costs model was estimated which indicated that the main effect of allowing for imperfect adjustment was on input use. Differences between long-run and short-run export supply and import demand responsiveness were relatively small. Considerable substitutability between labour and capital in the long-run was observed and since labour was also variable in the short-run this produced overshooting of labour demand. An increase in export prices thus caused a large short-run increase in labour demand but in the long-run the capital stock was increased and substituted for much of the short-run labour increase.

TABLE OF CONTENTS

| | |
|---|---|
| Abstract | ii |
| List of Tables | v |
| List of Figures | vii |
| Acknowledgement | viii |
| Chapter 1. INTRODUCTION | 1 |
| Chapter 2. PREVIOUS STUDIES | 5 |
| Chapter 3. A FLEXIBLE AGGREGATOR FUNCTION MODEL | 13 |
| 3.1 The GNP Function Framework | 13 |
| 3.2 Aggregator Functions | 20 |
| 3.3 Elasticities Produced | 26 |
| 3.4 Results | 27 |
| 3.5 Conclusions | 35 |
| Chapter 4. FLEXIBLE DISAGGREGATED MODELS | 48 |
| 4.1 The Generalised McFadden GNP Function | 49 |
| 4.2 Results | 52 |
| 4.3 Conclusions | 57 |
| Chapter 5. A PLANNING PRICE MODEL | 65 |
| 5.1 The Planning Price Approach | 65 |
| 5.2 Results | 69 |
| Chapter 6. AN ADJUSTMENT COSTS MODEL | 80 |
| 6.1 A Theoretical External Adjustment Costs Model | 80 |
| 6.2 An Econometric Adaptation | 84 |
| 6.3 Results | 90 |
| 6.4 Conclusions | 96 |
| Chapter 7. CONCLUSIONS AND FURTHER RESEARCH | 105 |
| 7.1 Further Research | 107 |
| Bibliography | 111 |
| Appendix 1. DATA | 116 |
| Appendix 2. PRIMAL VERSUS DUAL ESTIMATION | 129 |

## LIST OF TABLES


| | | |
|---|---|---|
| Table 3.1 | SGM Parameter Estimates | 37 |
| Table 3.2 | GNP Function Elasticities of Transformation | 38 |
| Table 3.3 | Export Supply Elasticities | 39 |
| Table 3.4 | Import Demand Elasticities | 40 |
| Table 3.5 | Labour Demand Elasticities | 41 |
| Table 3.6 | Domestic Sales Supply Elasticities | 42 |
| Table 3.7 | 1970 Export Aggregator Elasticities | 43 |
| Table 3.8 | 1970 Import Aggregator Elasticities | 43 |
| Table 3.9 | Export Component Own Supply Elasticities | 44 |
| Table 3.10 | 1970 Export Component Cross Supply Elasticities | 45 |
| Table 3.11 | Import Component Own Demand Elasticities | 46 |
| Table 3.12 | 1970 Import Component Cross Demand Elasticities | 47 |
| Table 4.1 | GM Parameter Estimates | 59 |
| Table 4.2 | Export Model Own Price Elasticities | 61 |
| Table 4.3 | 1970 Cross Elasticities - Export Model | 62 |
| Table 4.4 | Import Model Own Price Elasticities | 63 |
| Table 4.5 | 1970 Cross Elasticities - Import Model | 64 |
| Table 5.1 | Generalised Leontief Parameter Estimates | 76 |
| Table 5.2 | 1970 Net Output Supply Elasticities | 77 |
| Table 6.1 | Dynamic Parameter Estimates | 99 |
| Table 6.2 | 1965 Net Output Price Elasticities | 100 |
| Table 6.3 | 1970 Net Output Price Elasticities | 101 |
| Table 6.4 | 1978 Net Output Price Elasticities | 102 |
| Table 6.5 | Capital-Net Output Cross Elasticities | 103 |
| Table 6.6 | Scale and Technical Change Elasticities | 104 |
| Table A1.1 | Aggregate Price Indices | 122 |

Table A1.2 Aggregate Quantities in Millions of 1961 Dollars 123
Table A1.3 Export Component Price Indices 124
Table A1.4 Export Component Quantities in Millions of 1961 Dollars 125
Table A1.5 Import Component Price Indices 126
Table A1.6 Import Component Quantities in Millions of 1961 Dollars 127
Table A1.7 Adjustment Costs Model Capital Data 128
Table A2.1 MACE Data 144
Table A2.2 Cost Function Coefficients 146
Table A2.3 Fits and Tests 147
Table A2.4 Elasticities of Substitution 148
Table A2.5 Own Price Elasticities of Demand 150

# LIST OF FIGURES

Figure 5.1 Adjustment of Planning Prices 78

Figure 5.2 Adjustment of Quantities 79

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# 1. INTRODUCTION

Traditional empirical trade models have typically attempted to model export supply and import demand relations by the use of linear or log-linear functions of real income and the price of traded goods relative to the price of domestic substitutes. These models have assumed that exports, imports and domestic goods can be aggregated and have ignored much of the information available on the industrial structure of the economy. The use of single equation methods has further ignored much of the theoretical knowledge available on the properties of demand systems. The objective of this thesis is to extend the relatively small amount of empirical trade work which has been undertaken using models which integrate the supply of exports and demand for imports with the industrial structure of the economy and more closely approximate the well developed literature on trade theory.

The influence of the industrial structure of the economy on export supply and import demand is captured by using the GNP function framework first implemented by Kohli (1975, 1978). The GNP function framework treats imports as an input to the production technology and exports as an output of the technology thus enabling the derivation of an integrated system of supply and demand equations.

The responsiveness of export supply and import demand in the Canadian economy is characterised by a detailed set of elasticity estimates. These elasticities provide information on the response of the economy to changes in traded and non-traded goods prices and factor endowments. As such, the elasticities may be thought of as being analogous to the standard trade theory results except that they show a more complex set of responses as we move outside the standard 2x2x2 model and allow for joint production.

The elasticity estimates for exports and imports are likely to be of most interest in forecasting the effects of various exogenous changes. For instance, the effects of an across the board import tariff or export subsidy on the supply of exports and domestic outputs and import demand can be calculated (subject to the fixed factor supply). Similarly, the effect of such changes on the return to factors can be calculated. Alternatively, the effect of changes in factor prices (eg. due to favourable taxation treatment or increased unionisation) on the supply of the various outputs and exports, and import demand can be calculated. The elasticity estimates presented in this study will also be of interest to those constructing larger applied general equilibrium models of the Canadian economy.

The GNP function framework of Kohli is extended in a number of directions in this thesis. Firstly, exports and imports are each disaggregated into several components. While the usual GNP or restricted profit function approach rapidly becomes unmanageable as more output and input categories are allowed this can be facilitated by the use of aggregator functions as first applied by Fuss (1977). Furthermore, recently developed functional forms are used which have potentially superior curvature properties to the now traditional translog function. An alternative means of allowing for several export and import components explored is the use of larger disaggregated models which overcome the restrictive separability assumptions of the aggregator function approach but at the expense of not including the full set of net outputs in the one model.

The second avenue explored is the allowance for imperfect adjustment in the GNP function model. This is facilitated initially by the use of planning prices as developed by Woodland (1976, 1977). Modelling is then extended to an explicit costs of adjustment model as developed by Berndt, Fuss and Waverman (1977).

The data used in the study are time-series of input-output data for Canada covering the period 1961 to 1980. The data are available for 37 different industries but are aggregated to five output groups and six input groups for estimation of the GNP function models. The output groups are sales to domestic end-users and four types of exports while the input groups consist of four categories of imports, labour and capital. While this data set is limited to 20 observations ending in 1980 it has the advantage of being detailed, well developed and internally consistent for the entire period.

A brief review of previous empirical studies of export supply and import demand is presented in the following chapter of this thesis. The flexible aggregator function model and its results are presented in Chapter 3 followed by the larger export and import disaggregated models in Chapter 4. Chapters 5 and 6 contain presentations of the planning price and adjustment costs models, respectively. Finally, conclusions are drawn and areas for future research identified in Chapter 7. The data used in the study are described and listed in Appendix 1 while Appendix 2 contains the results of a study comparing primal and dual estimation routes.

## 2. PREVIOUS STUDIES

A brief review of previous studies of export supply and import demand responsiveness and their relationship to the underlying theory of international trade is presented in this chapter. In spite of the well developed literature on trade theory, it is only over the last decade and a half that both theoretical and empirical developments have enabled a more integrated approach to modelling actual export supply and import demand. A good review of the link between empirical studies and underlying trade theory can be found in Woodland (1982, Chapter 12).

Many traditional empirical studies concentrated on import demand and modelled imports as final goods not entering the domestic production sector. Assuming that imports are separable from other commodities demanded then import demand can be modelled as a function of import prices, the prices of other goods and domestic income. Import demand was often modelled as a log-linear relationship of these variables in spite of the fact that this cannot be derived from utility maximising behaviour except under very restrictive circumstances. Export demand can be modelled in an analogous manner using income and the price of other goods from the rest of the world. Typical of these studies was that of Houthakker and Magee (1969) in which demand functions for both imports and exports for 26 countries were estimated for the period 1951-66. For Canada an import price elasticity of -1.46 and income elasticity of 1.20 were obtained while the export price elasticity was -0.59 and the export income elasticity 1.41. In another study Rhomberg (1964) found the Canadian export demand elasticity to be around -2 and the import demand elasticity to be around -1 using a linear version of the demand relationship.

An alternative specification of the consumer demand model for imports was used by Gregory (1971). By assuming a CES utility function the logarithm of the ratio of import to domestic demand becomes a function of the logarithm of relative prices. An analogous procedure within a producer model was used by Alaouse, Marsden and Zeitsch (1977) to estimate the substitution between imported and domestic inputs to various Australian industries.

The CES function was also used by Hickman and Lau (1973) within an import allocation model. This type of model postulates that the quantity of a country's total imports is a CES function of the quantities of imports from each country. The import price index is also assumed to be a CES function and imports are sourced from countries on the basis of the cost minimisation principles implicit in the price index. The model is linearised and factors such as trends, expectations and adjustment lags are allowed for. Elasticities of substitution between imports from different countries in each import market are obtained and used in the derivation of aggregate export demand functions for each country. Using trade data for the period 1961-69 and an adaptive expectations dynamic model a short run elasticity of export demand for Canada of 0.59 was obtained. The corresponding long run elasticity was estimated to be 0.84.

The first empirical studies to model import demand within an integrated production sector model were those of Denny (1972) and Burgess (1974a, 1974b). These studies were also among the first to use flexible functional forms. Burgess (1974a) assumes separability of the transformation function and models output as a function of the inputs of imports, capital and labour. A set of translog share equations is estimated for the US for the period 1947-68. The elasticity of demand for imports ranges from -1.6 to -2.0. An increase in the price of imports was found to reduce the wage/rental price of capital ratio.

Burgess (1974b) models import demand by the use of a translog joint cost function. Imports are assumed to be an input into the aggregate production process. This assumption is justified by the argument that in many cases imports constitute intermediate inputs which have to undergo further processing before being supplied to the consumer. Even imports of consumer goods have to go through distribution and commercial channels before reaching final demand. Burgess uses a two output (consumer and investment goods), three input (labour, capital and imports) translog cost function for the U.S. for the period 1929-69 to model substitution possibilities. Imports were found to be substitutable with labour and complementary to capital. The own price elasticity of the demand for imports ranges from -0.51 to -0.66. Burgess (1976) uses a different functional form for the joint cost function and different output groups (durables and non-durables, and non-governmental services and structures) for the US for the shorter period 1948-69. Import demand elasticities range from -0.19 to -1.6 and imports were found to be substitutes for labour and complements to capital. These substitution relationships, however, assume that output levels and input prices are exogenously given.

An approach more in keeping with the neoclassical small country assumption in which the domestic country behaves as a price-taker in both export and import markets would have prices for outputs and imports and the quantities of factors exogenously given. Domestic export supply and import demand can then move in response to world prices subject to domestic factor endowments. This is the basis of the GNP function approach used by Kohli (1975, 1978). Imports are again treated as an input to the production technology and exports are treated as an output of the technology, ie. domestic consumers have no demand for export goods. This assumption may be justified by appealing to the fact that export goods typically proceed through different channels to those destined for domestic consumption. Kohli models the technology by the use of a translog restricted profit function which has consumption goods, investment goods, exports and imports as variable net outputs. Labour and capital inputs are assumed to be fixed with their prices adjusting endogenously. This is a similar representation to Samuelson's (1953-4) GNP function in trade theory. Supply equations for consumption goods, investment goods and exports, a demand equation for imports and inverse demand equations for labour and capital are derived from the GNP function. These were among the first studies to model domestic export supply subject to a given world price rather than modelling the rest of the world's demand for a country's exports.

Kohli (1978) estimated his model for Canadian data for the period 1949-72 and found the own-price elasticity of demand for imports to vary between -0.9 and -1.0 and the own price elasticity of supply of exports to vary between 1.5 and 2.2. Exports, consumption and investment goods were found to be substitutes in production and the wage rate was found to fall in response to an increase in import prices and rise in response to an increase in export prices. Capital rental prices responded in the opposite direction. Increases in investment goods and export prices increased the demand for imports while increases in consumption goods prices reduced import demand. Increases in capital stocks reduced both exports and imports. It should be noted that a model such as this will not necessarily produce results similar to the Rybczynski Theorem in traditional trade theory as joint production is allowed in the GNP function model. Also, the model is only partial equilibrium in that no balance of payments or exchange rate adjustment mechanisms are included and no explanation of the capital accumulation process is made. In the earlier study, Kohli (1975), an attempt was made to disaggregate imports and exports by the use of translog submodels but curvature conditions were not satisfied and the submodels did not perform well.

A recent application of the GNP function model is that of Diewert and Morrison (1986). In this study capital is treated as a fixed input and constant returns to scale are imposed. The economy's outputs are modelled as domestic sales and exports while its variable inputs are imports excluding petroleum, imports of petroleum, and labour. The normalised quadratic profit function is used which permits the correct curvature conditions to be imposed on the model with minimal cost to flexibility properties. The model is applied to US data for the period 1967-82. Export supply elasticities range from 0.52 to 0.60 while non-petroleum import demand elasticities range from -0.74 to -1.12. Petroleum import demand elasticities range from -0.14 to -0.87. Exports were found to be highly substitutable with domestic sales and highly complementary with labour. Non-petroleum imports were substitutable with labour while petroleum imports were complementary with domestic sales and exports. A relatively high own price elasticity of demand for labour of around -1.0 was obtained. A series of devaluation elasticities was also presented.

In order to model both the production and consumption sectors and their influence on export supply and import demand one has to move into the realm of general equilibrium models. A small scale general equilibrium model in which the consumption as well as the production sector is explicitly modelled as is the balance of payments mechanism is that of Clements (1980). The model is highly aggregated with only three goods (non-tradeables, importables and exportables). Unlike the Burgess and Kohli models, imports and exports are assumed to be perfect substitutes for domestic production. The model was estimated using U.S. data for the period 1952-71 but performed relatively poorly. A simulation of the effect of imposing a 10 per cent import tariff indicated that real exports would be 20 per cent lower in all periods and real imports would be 32 per cent lower after 12 years. Dynamics enter the model via intertemporal optimising behaviour by consumers.

Applied general equilibrium models have often been used in simulations of the effects of different trade policies and exogenous shocks on the supply of exports and demand for imports. Most applied general equilibrium modelling, however, has been on a "large scale" basis with many goods and industries. The objective of these studies has been to assess the impact of exogenous changes on the economy once all the flow on effects of the change have been worked through. This is done by comparing the post-shock solution of the complete model with the base solution. Typical of these larger scale models are those of Boadway and Treddenick (1978) of the Canadian economy and the ORANI model of Dixon, Parmenter, Ryland and Sutton (1977) of the Australian economy. Boadway and Treddenick find that the effect of reducing Canadian trade taxes to zero would be a small reduction in aggregate utility due to the role of trade taxes in exploiting monopoly power. The tariff was found to benefit tertiary industries and have an adverse impact on most manufacturing and primary industries while raising the wage/rental ratio.

The model of Harris (1984) represents the start of a new generation of general equilibrium models which incorporate recent developments in the fields of industrial organisation and trade theory. By allowing for internal economies of scale and product differentiation Harris finds that the results of simulations can differ markedly from those of models based on the neoclassical assumptions of constant returns to scale and perfect competition. For instance, the effect of a move to multilateral free trade on the Canadian economy was estimated as an 8.6 per cent gain in aggregate welfare using the model based on scale economies and product differentiation compared to only a 2.4 per cent welfare gain from the model based on neoclassical assumptions.

While large scale general equilibrium models are capable of producing more detailed results than many smaller scale models they are typically based on relatively simplistic functional forms to enable their implementation. Export supply and import demand elasticities are also usually assumed rather than estimated within the models. Furthermore, while recent developments in industrial organisation theory have opened up new areas of applied research, other recent developments in empirical techniques mean that many useful studies remain to be undertaken using models within the neoclassical framework. In particular, new means of incorporating many goods and factors using relatively flexible functional forms and models of imperfect adjustment will enable modellers to produce more detailed elasticity estimates. It is these avenues which are explored in this thesis within a neoclassical production sector model. Although the results will be of interest in their own right, they may also provide improved elasticity estimates for input to larger scale general equilibrium models.

# 3. A FLEXIBLE AGGREGATOR FUNCTION MODEL

In this Chapter the GNP function framework is elaborated and used to provide estimates of the responsiveness of export supply and import demand in Canada. By using aggregator functions several export and import components are included thus producing detailed sets of elasticities. Finally, the implications of these elasticity estimates for the effects of various policy changes and exogenous shocks are discussed.

## 3.1 The GNP Function Framework

The GNP function model assumes that the economy is made up of profit maximising firms operating under conditions of perfect competition in goods and factors markets. Output levels and mixes and import demands are chosen to maximise profits given output and import prices and available factor quantities. Factors are assumed to be mobile between firms with their market prices equal to their shadow prices. The aggregate technology is assumed to be characterised by constant returns to scale, free disposal, non-increasing marginal rates of substitution and transformation, and to be bounded from above for given finite factor endowments. The competitive equilibrium can then be represented as the solution to the problem of maximising GNP subject to the available technology, factor endowments and given output and import prices.

Exports are treated as an output of the production sector while imports are treated as an input. As noted in the preceding Chapter, treating imports as inputs to production may be justified by appealing to the fact that many imports are intermediate inputs and even those imports which are "final" consumer goods still have to go through distribution and retail channels before reaching the consumer. Treating exports as separate goods for which there is no domestic demand is not necessarily restrictive as domestic consumers may demand other goods from the production sector which are highly or even perfectly substitutable with the goods classified as exports. While this approach enables us to model export supply and import demand by concentrating on the production sector and not explicitly including the consumption sector which is usually difficult to model, the cost of this procedure is that the resulting model is partial equilibrium in nature. By holding all prices fixed, the model will be partly misspecified as not all the effects of exogenous changes will be captured. For instance, the price of domestic sales is taken as being exogenous and it is assumed that firms can sell any amount of domestic sales output at the existing price. Not all of the consumer income effect response to an exogenous price change will be captured as no allowance is made for the effect of changes in factor rewards on the price of domestic sales output. Also, no allowance is made for forces which would tend to eliminate disequilibrium in the balance of payments and no attempt is made at this stage to explain the process of capital accumulation.

Denoting the N variable net output quantities by the vector x (entries positive for outputs, negative for inputs), net output prices by the vector $p>>0$, the M fixed input quantities by the vector z, fixed input shadow prices by the vector w and the production possibility set by T, the technology can be represented by the following restricted profit (GNP) function:

$$(3.1) \qquad G(p;z) = \max_{x}. \{ p'x : (z;x) \text{ belongs to } T, p>>0 \}.$$

The restricted profit function (3.1) will be linearly homogeneous and convex in net output prices and monotonically increasing (decreasing) in the prices of variable outputs (inputs). It will be linearly homogeneous, concave and monotonically increasing in fixed input quantities. The properties of restricted profit functions are discussed in detail in Diewert (1973, 1974) while the GNP function in the trade theory context is described by Woodland (1982).

The model is also based on the small country assumption; i.e., that the home country is a price taker in both its import and export markets. The quantity of imports is then determined by domestic industry demand conditions while the quantity of exports is determined by domestic supply conditions. The small country assumption for Canada was tested by Appelbaum and Kohli (1979) who found they could not reject the price taking assumption for imports but found that it was rejected for Canadian exports.

If the restricted profit function is differentiable with respect to p then the net output supply functions can be derived by applying Hotelling's (1932) Lemma:

$$(3.2) \qquad x(p,z) = \nabla_p G(p;z)$$

Furthermore, if the restricted profit function is differentiable with respect to the fixed input quantities, z, then the inverse demand functions for the fixed inputs may be obtained by:

$$(3.3) \qquad w(p,z) = \nabla_z G(p;z)$$

In this study the aggregate capital stock is treated as the only fixed input. Profit is maximised each period subject to the capital stock available and so the process of capital accumulation is not modelled. Labour is treated as a variable input; i.e., producers choose how much labour they wish to employ at the given exogenous wage rate. With the existence of unemployment, this treatment of labour appears more plausible than the alternative of assuming that the labour stock is fully employed with the wage rate becoming an endogenous variable.

Constant returns to scale are also assumed with respect to the capital stock. The restricted profit function (3.1) can then be represented by a unit profit function which represents the maximum amount of revenue the economy can produce from one unit of capital. If the capital stock were increased by a given proportion then the economy's net revenue would increase by the same proportion. The assumption of constant returns to scale helps avoid the conceptual problems which can occur when aggregating over producers. However, as Blackorby and Schworm (1984) point out, the requirements for consistent aggregation and the existence of an aggregate technology are highly restrictive. Essentially there remains a trade-off between the requirements for consistent aggregation and the use of models sufficiently flexible to capture substitution possibilities.

The data used in this study are time-series of input-output data for the Canadian economy made available by Statistics Canada and covering the period 1961 to 1980. Initially four variable net outputs for the economy as a whole are identified;

- the quantity of domestic sales;
- the quantity of aggregate exports;
- (minus) the quantity of aggregate imports; and
- (minus) the quantity of labour.

Corresponding price indices are set equal to 1.0 in 1961 and the implicit quantities derived by dividing the value of the net output by the relevant price index. An aggregate capital price index is derived as a residual to equate the value of total outputs and total inputs under constant returns and the quantity of the capital stock rescaled so that the price index assumes the value of 1.0 in 1961. The elasticity estimates presented are invariant to this capital rescaling. The data are described in detail and listed in Appendix 1.

To implement the model empirically a functional form for the restricted profit function must be specified and estimation of the system of derived net output supply functions undertaken. The characteristics of the production technology and export and import responses are obtained from the calculation of various elasticities derived from the estimated profit function. These elasticities and their interpretation are discussed in Section 3.3 below.

Desirable characteristics of a functional form for the restricted profit function are that it be flexible (able to provide a second order approximation to an arbitrary twice continuously differentiable profit function), parsimonious (have the minimal number of free parameters required for flexibility), and consistent with the required theoretical properties of a profit function. While the translog and Generalised Leontief forms have become popular because of their flexibility and relative ease of implementation, they often suffer in empirical applications from failure to satisfy the required curvature properties at all (or any) of the observation points. In response to this problem, recent developments in functional forms have led to the development of functions which are flexible and easily verified as satisfying curvature conditions globally. If the curvature conditions are not satisfied they can be imposed with minimal cost to flexibility properties although non-linear regression techniques then have to be used.

The functional form for the unit profit function adopted in this study is the Symmetric Generalised McFadden (SGM) function of Diewert and Wales (1987). The 4-variable net output SGM unit profit function is given by;

$$(3.4) \quad G(p,K)/K = (1/2)\Sigma_{i=1}^{4}\Sigma_{j=1}^{4} s_{ij}p_ip_j/(\Sigma_{k=1}^{4} T_kp_k) + \Sigma_{i=1}^{4} b_{ii}p_i + \Sigma_{i=1}^{4} b_{it}p_it + b_{tt}(\Sigma_{i=1}^{4} C_ip_i)t^2$$

where time superscripts have been deleted and the $s_{ij}$, $b_{ii}$, $b_{it}$ and $b_{tt}$ are parameters to be estimated subject to;

$$(3.5) \quad s_{ij} = s_{ji} \qquad \text{for all } i,j; \text{ and,}$$

$$(3.6) \quad \Sigma_{i=1}^{4} s_{ij} = 0 \qquad \text{for } j=1,..,4.$$

The variable t is a time trend representing technical progress and the exogenous parameters $T_k$ and $C_i$ are set equal to the average net output quantity per unit of capital input quantity for k,i=1,..,4.

Diewert and Wales (1987) show that the SGM form is flexible for a price vector $p^{\sim}$ satisfying $Sp^{\sim}=0_N$. While the non- symmetric Generalised McFadden function (analogous to the normalised quadratic form used by Diewert and Morrison (1986)) has superior flexibility properties in that it is not restricted to being flexible at just one point, the results obtained are sensitive to the choice of the numeraire good which plays an asymmetric role. This sensitivity is eliminated by use of the SGM form.

Differentiating the GNP function (3.4) with respect to the net output prices yields a domestic supply function, an export supply function, (minus) an import demand function and (minus) a labour demand function. The form of these net output supply functions is;

(3.7) $$x_i/K = \Sigma_{j=1}^{4} s_{ij}p_j/(\Sigma_{k=1}^{4} T_k p_k) - T_k(\Sigma_{k=1}^{4} \Sigma_{j=1}^{4} s_{kj}p_k p_j)/ 2(\Sigma_{k=1}^{4} T_k p_k)^2 + b_{ii} + b_{it}t + b_{tt}C_i t^2 + u_i \; ; \; i=1,..,4.$$

The variable net output quantity is divided by the quantity of the capital input to reduce heteroskedasticity problems and an error term is appended to each equation. The vectors of error terms for the observations are assumed to be independently distributed with a multivariate normal distribution with zero means and covariance matrix $\Omega$.

The estimating system consists of (3.7) subject to the restrictions (3.5) and (3.6). The profit function (3.4) is not included in the estimating system as it adds no new information. Maximum likelihood estimates of the system of equations (3.7) can be obtained by using the iterative Zellner technique available in the SYSTEMS command of SHAZAM (White 1978). If the matrix of estimated coefficients S is positive semi-definite then the restricted profit function can be shown to be globally convex in prices p.

If the estimated S matrix is not positive semi-definite then it can be reparameterised using a technique due to Wiley, Schmidt and Bramble (1973) to ensure global convexity. This technique replaces the matrix $S=[s_{ij}]$ by the product of a lower triangular matrix and its transpose:

(3.8) $S = AA'$ where $A = [a_{ij}]$ ; $i,j=1,..,4$; and $a_{ij}=0$ for $i<j$.

Using a result due to Lau (1978), Diewert and Wales (1987) show that this is a general way of imposing positive semi-definiteness. Using this procedure the coefficients in the first three rows and columns of S become;

(3.9) $$[s_{ij}]=\begin{bmatrix} a_{11}^2 & a_{11}a_{21} & a_{11}a_{31} \\ a_{11}a_{21} & a_{21}^2+a_{22}^2 & a_{21}a_{31}+a_{32}a_{22} \\ a_{11}a_{31} & a_{21}a_{31}+a_{32}a_{22} & a_{31}^2+a_{32}^2+a_{33}^2 \end{bmatrix} ;i,j=1,2,3.$$

The fourth row and column of S are obtained from the summing restrictions (3.6). The reparameterised system imposing curvature can be estimated by using the non-linear regression algorithm in SHAZAM.

## 3.2 Aggregator Functions

Aggregation of input and output components is a necessary part of any empirical study to ensure tractability but the cost of this procedure is usually a loss of information. There are three conditions under which aggregation will be consistent or not lose any of the available information. The first of these is Hicks aggregation where the prices of a group of goods always move in exact proportion. Aggregate price and quantity indices can then be formed which will behave as for a single good. Secondly, Leontief aggregation provides consistent aggregate quantity and price indices when the quantities of a group of goods always move in exact proportion. Clearly, Hicks and Leontief aggregation are based on strict conditions which are unlikely to be met in practice.

The third basis for aggregation provides a more general case by implying certain properties for the functional structure. This is the condition of homogeneous weak separability, originally due to Shephard (1953), which assumes the GNP function can be written as:

$$G(p,z) = \tilde{G}(R,V) \tag{3.10}$$

where $R=(R_1,\ldots,R_n,\ldots)$, $V=(V_1,\ldots,V_m,\ldots)$, $R_n=R_n(p_n)$, $V_m=V_m(z_m)$ and $p_n$, $z_m$ belong to $p,z$, respectively. $R_n(p_n)$ is a price index for the goods in group n while $V_m(z_m)$ is a quantity index for the fixed inputs in group m. The corresponding transformation function is:

$$T(x,z) = \tilde{T}(Y,V) = 0 \tag{3.11}$$

where $Y=(Y_1,\ldots,Y_n,\ldots)$ and $Y_n(x_n)$ is the corresponding quantity index which is assumed to be linearly homogeneous. We then have:

$$\begin{aligned} &\max_{x_n}.\ \{\ p_n x_n : Y_n(x_n) = Y_n\ \} \\ &\qquad = Y_n \max_{x_n/Y_n}.\ \{\ p_n x_n/Y_n : Y_n(x_n/Y_n)=1\} = Y_n R_n(p_n) \end{aligned} \tag{3.12}$$

where $R_n(p_n)$ is a revenue or aggregator function. Thus,

$$\begin{aligned} G(p,z) &= \max.\ \{\textstyle\sum_n p_n x_n : \tilde{T}(Y_1(x_n),\ldots,V)=0\ \} \\ &= \max.\ \{\textstyle\sum_n R_n(p_n) Y_n : \tilde{T}(Y,V)=0\ \} \end{aligned} \tag{3.13}$$

$$= G^{\sim}(R,V)$$

which is a valid GNP function in the aggregates to which Hotelling's Lemma and the standard GNP function properties can be applied (Woodland 1982, p.368).

The important implication of weak separability is that optimisation proceeds by a two-stage process. First, the optimal quantity of the aggregate is chosen and then the optimal mix of that aggregate quantity is chosen. The marginal rate of substitution between two components of one aggregate is independent of the quantities of the other aggregates. Thus, the mix of that aggregate is independent of both the level and the mix of the other aggregates. It is this aspect of weak separability which forms the basis of the use of aggregator functions as proposed by Fuss (1977) to accommodate many input (and output) components.

With the use of flexible functional forms the explicit incorporation of many inputs and outputs rapidly exhausts the available degrees of freedom and creates significant multicollinearity problems which are difficult, if not impossible, to overcome. The increased computational burden is also an important consideration. The use of aggregator functions permits the use of flexible functional forms for the GNP function at the aggregate level along with flexible aggregator functions. The response at the most disaggregated level can be obtained by:

(3.14) $$x_n = \partial G/\partial p_n = \partial G^{\sim}/\partial R_n . \partial R_n/\partial p_n$$

$$w_m = \partial G/\partial z_m = \partial G^{\sim}/\partial V_m . \partial V_m/\partial z_m .$$

The cost of this procedure is, of course, acceptance of the property of weak separability. An unfortunate implication of weak separability is that the substitutability of any two components within one aggregate with another aggregate is equal. Thus, imports of, say, tractors and hairpins might be assumed to be equally substitutable with the domestic sales aggregate.

In this study the aggregator function procedure is used to disaggregate total exports and total imports each into four components. The four export components are;

Group 1 : Agricultural and Forestry Products;

Group 2 : Minerals and Energy Products;

Group 3 : Motor Vehicles, Textiles and Electrical Products; and

Group 4 : Heavy Industrial and Service Products.

The four import components are;

Group 1 : Agricultural, Forestry and Service Products;

Group 2 : Metals and Energy Products;

Group 3 : Machinery, Electrical and Textile Products; and

Group 4 : Motor Vehicles, Chemicals and Other Products.

The four export and import groups were formed by aggregating input-output industries according to similarity of price movements over the 20 year period, the basis of Hicks aggregation. The composition of each of the components is explained and prices and quantities listed in Appendix 1.

Making use of the assumption that the $Y_n(x_n)$ functions are linearly homogeneous the following Symmetric Generalised McFadden unit revenue functions are used for the export and import aggregates;

$$(3.15)\quad R(p,X)/X = (1/2)\Sigma_{i=1}^{4}\Sigma_{j=1}^{4} s_{ij}p_ip_j/(\Sigma_{k=1}^{4} T_kp_k) + \Sigma_{i=1}^{4} b_{ii}p_i + \Sigma_{i=1}^{4} b_{it}p_it + b_{tt}(\Sigma_{i=1}^{4} C_ip_i)t^2$$

where time superscripts have again been deleted, X represents total exports (imports) and the $s_{ij}$, $b_{ii}$, $b_{it}$ and $b_{tt}$ are parameters to be estimated subject to;

$$(3.16)\qquad s_{ij} = s_{ji} \qquad \text{for all } i,j; \text{ and,}$$

$$(3.17)\qquad \Sigma_{i=1}^{4} s_{ij} = 0 \qquad \text{for } j=1,..,4.$$

The variable t is a time trend representing technical progress and the exogenous parameters $T_k$ and $C_i$ are set equal to the average export (import) component quantity per unit of total export (import) quantity for k,i=1,..,4.

Profit maximising behaviour implies that the export (import) component quantities per unit of total exports (imports) are given by;

$$(3.18)\quad x_i/X = \Sigma_{j=1}^{4} s_{ij}p_j/(\Sigma_{k=1}^{4} T_kp_k) - T_i(\Sigma_{k=1}^{4} \Sigma_{j=1}^{4} s_{kj}p_kp_j)/ 2(\Sigma_{k=1}^{4} T_kp_k)^2 + b_{ii} + b_{it}t + b_{tt}C_it^2 + u_i\ ;\ i=1,..,4.$$

The quantity of total exports (imports) X is derived as a Divisia index of the corresponding export (import) component quantities. Convexity in prices can be imposed on the aggregator functions by reparameterising the S matrix along the same lines as (3.8) and (3.9). The vectors of error terms are again assumed to be independently distributed with a multivariate normal distribution with zero means and covariance matrix $\Omega$.

By estimating the system (3.18) and substituting the estimated parameters in (3.15), an estimate of the aggregate unit price is obtained. A property of the two-stage optimisation procedure is that although the prices of the individual components of the aggregate are exogenous, the price of the aggregate itself is not exogenous because the choice of input and output mix will determine the aggregate price. Thus, to implement the procedure empirically an instrumental variable for the aggregate price is required. Fuss proposes the use of the estimated price of the aggregate obtained by substituting the parameters estimated in equations similar to (3.18) into the aggregator function. This is used as an instrumental variable for the aggregate price in the second stage of the estimation process. Fuss justifies the use of the estimated aggregate price as an instrumental variable in his case by appealing to the fact that the translog aggregator function is exact for the Divisia price index of the components as established by Diewert (1976).

The estimation procedure is thus to first estimate the unit quantity equations (3.18) for the export and import components. Next, the parameter estimates obtained in the first stage are substituted in (3.15) to obtain instrumental variables for the aggregate export and import prices. The second stage of the estimation procedure is the estimation of the net output supply equations (3.7) derived from the SGM restricted profit function using the instrumental variables for aggregate export and import prices. Application of this conditional estimation procedure produces estimates which are full information maximum likelihood (Fuss 1977). The role of weak separability can be seen from (3.15) where the instrument for each aggregate depends only on the prices of the components of that aggregate. In the absence of weak separability the prices of the other aggregates would also enter (3.15) and the above estimation procedure would not be consistent.

### 3.3 Elasticities Produced

The elasticities may be presented in either a scale invariant normalised form analogous to the Allen-Uzawa elasticities of substitution or in the standard net output price elasticity form. The scale invariant elasticities are a symmetric matrix of elasticities of transformation between net outputs (normalisations of $\partial x_n/\partial p_n$) given by:

$$ET = G.G_{pp}/(G_p.G_p) \tag{3.19}$$

where $G_p$ = diag. $\nabla_p G(p;z)$. The diagonal elements of ET are all non-negative.

The more familiar net output price elasticities represent the response of net output i's quantity to changes in net output j's price:

$$E_{ij} = d\ln x_i/d\ln p_j = s_j ET_{ij} \tag{3.20}$$

where $s_j$ is the share of net output j in restricted profit and the elasticities satisfy the following adding up restrictions;

$$\Sigma_{j=1}^{4} E_{ij} = 0. \tag{3.21}$$

Due to the maintained hypothesis of constant returns to scale the net output supply elasticities with respect to capital all take the value 1.0. This also means that the elasticities of complementarity and intensity normally obtained in profit function studies are not presented. These elasticities and various summation restrictions which apply to them are presented in Diewert (1974). For empirical applications of the translog restricted profit function and interpretations of the associated elasticities not in the trade model context see McKay, Lawrence and Vlastuin (1982, 1983).

While the elasticities above refer to aggregate exports and imports, and the second stage of the estimation procedure, two sets of elasticities are obtained for the individual components of exports and imports. In the case of cross-price elasticities between export components, for instance, from the first stage of estimation (equation (3.18)) we obtain cross-price supply elasticities given a fixed level of aggregate exports. By extending equation (3.14) we obtain cross-price supply elasticities between export components i and j subject to the constant fixed capital input quantity as follows:

$$E_{ij}{}^{K} = E_{ij}{}^{X} + s_j E_{XX}{}^{K} \tag{3.22}$$

where $E_{ij}{}^{X}$ is the cross-price elasticity between i and j given a constant level of aggregate exports, $s_j$ is the share of export j in total exports and $E_{XX}{}^{K}$ is the own-price elasticity of aggregate exports for a given fixed capital input level. By extending (3.22) to the import components price elasticities for all the export and import components are obtained which are directly comparable with the price elasticities for the other net output categories obtained from the second stage of estimation.

## 3.4 Results

Initial estimation of the linear systems in (3.18) and (3.7) produced coefficient matrices S which were not positive semi-definite for the export and import aggregators and the GNP function. In each case one eigenvalue of the S matrix was negative. Subsequent estimation was, therefore, undertaken using the non-linear reparameterised model imposing curvature. The results of these non-linear regressions and the corresponding asymptotic t-values are presented in Table 3.1. In each case the Davidson-Fletcher-Powell algorithm in the SHAZAM package was used and the systems converged from the default coefficient starting values of 1.0 within 200 iterations. Limited experimentation with different starting values produced the same parameter estimates. The low R-square value for the Group 3 equation in the import aggregator model is due to lack of variation in the dependent variable with the quantity ratio being almost constant for the entire period.

The scale invariant elasticities of transformation derived from the GNP function, the second stage of the estimation process, are presented in Table 3.2. The elasticities of transformation for import own demand are largest in magnitude followed by those for export own supply. The transformation elasticities for domestic sales own supply are particularly small indicating little price responsiveness for this output. The largest cross transformation elasticities are those between exports and imports indicating relative price sensitivity between these items. Of more interest, however, are the more easily interpreted conventional price elasticities. These price elasticities will now be discussed in turn for each of the four net output categories.

Export supply elasticities are presented in Table 3.3. The own-price elasticity of aggregate export supply increases from 1.26 to 2.29 over the period. In 1970 an export price increase of of 1 per cent would have brought forth an increase in total exports of 1.67 per cent. These findings are consistent with the relatively elastic export supply elasticities found by Kohli (1978) for Canada but larger than the comparable U.S. export supply elasticities of Diewert and Morrison (1986). A one per cent increase in the price of the inputs labour and imports would reduce export supply by 1.09 and 1.57 per cent, respectively, in 1970. An increase in the price of domestic sales of 1 per cent, on the other hand, would increase exports by approximately 1 per cent.

In the profit function context, two goods are Hicks (1946)-Allen (1938) substitutes if the cross partial derivative of the profit function with respect to their two prices is negative. Complementary goods have a positive second order price derivative. The elasticities presented in Table 3.3 are a second order price derivative of the profit function multiplied by the ratio of a price and a positive quantity. They will hence have the same sign pattern as the corresponding second order derivatives. Exports are thus substitutes for both imports and labour and complements for domestic sales.

From the import elasticites of demand presented in Table 3.4 it can be seen that the aggregate import own-price elasticities range from -0.98 to -2.40. In 1970 a 1 per cent increase in import prices due to, say, an across-the-board tariff would have reduced total import demand by 1.62 per cent. This elastic response of import demand to changes in the total import price is considerably higher than the earlier Canadian results of Kohli (where labour was treated as a fixed input) and also the U.S. results of Diewert and Morrison where the elasticity has a value closer to one. Import demand falls when labour prices increase. In 1970, 1 per cent increases in export and domestic sales prices would have increased import demand by 1.67 and 0.72 per cent, respectively. Import demand would have fallen by 0.78 per cent in response to a 1 per cent increase in labour prices. Since these elasticities are a second order price derivative multiplied by the ratio of a price to a negative quantity they will have the opposite sign to the corresponding second order price derivative. Consequently, imports are substitutes with domestic sales and complementary to labour.

A noticeable trend of increasing price responsiveness is apparent in the labour demand elasticities presented in Table 3.5. The own-price elasticity of labour demand increases from -0.21 in 1962 to -2.23 in 1980. If this result accurately reflects actual price responsiveness in the economy then there is a growing role for wage moderation in overcoming current unemployment problems. In 1970 a 1 per cent reduction in wages would have increased labour demand by 0.88 per cent. By 1980 the resulting increase in labour demand from a 1 per cent wage cut had more than doubled to 2.23 per cent. It is possible that the Canadian economy has become more price responsive and flexible in recent decades due to increasing openness in international trade and deregulation. The cross labour demand elasticities also show a pattern of increasing responsiveness highlighting the importance of other prices on labour demand as well. In 1970 a 1 per cent increase in export and domestic sales prices would have increased labour demand by 0.66 and 0.46 per cent, respectively. The positive value of the elasticity of labour demand with respect to the domestic sales price indicates that labour and domestic sales are substitutes.

The price responsiveness of domestic sales has also increased markedly over the period although this started from very low levels. As can be seen from Table 3.6, in 1962 a 1 per cent increase in the price of domestic sales would have had a negligible impact on the output of domestic sales but by 1980 such a price increase would have increased the quantity of domestic sales supply by 0.50 per cent. In 1970 domestic sales supply would have fallen by 0.12 and 0.31 per cent in response to 1 per cent increases in the prices of imports and labour, respectively. An increase in the export price would have increased domestic sales supply by 0.26 per cent reflecting the complementarity between exports and domestic sales.

Having derived these price elasticities it is of interest to examine their implications for the effects of various exogenous price changes on the economy. If import prices were to decrease by 10 per cent due to, say, a substantial trade barrier liberalisation or move to free trade then, using mid-point elasticities, imports would increase by 16 per cent, exports would increase by 11 per cent and labour demand would increase by

2 per cent. Domestic sales supply would fall slightly in response to the lowering of import prices. If Canadian export prices were to increase by 10 per cent due to, say, a substantial reduction in foreign trade barriers or an increase in world demand for Canadian products then exports would increase by 16 per cent, imports would also increase by 16 per cent and labour demand would increase by 6 per cent. Domestic sales supply would increase by about 2.5 per cent. It should be noted, however, that policy analysis should not be based too heavily on any one set of elasticity estimates due to likely sensitivity to the specification and data used and the failure to take account of all general equilibrium influences.

The price elasticities of supply for the four export components subject to a fixed aggregate export quantity are presented in Table 3.7 for the year 1970. These elasticities are derived from the aggregator function used in the first stage of the estimation process and so are not directly comparable with the GNP function elasticities which show the response of net outputs subject to the fixed capital input available. The export aggregator elasticities show that if the price of Agricultural and Forestry Products increased by 1 per cent then, to maintain a constant total export quantity, the quantity of Agricultural and Forestry Product exports would have to increase by 0.23 per cent and that of Minerals and Energy exports by 0.08 per cent. The quantities of Heavy Industrial and Service exports and Motor Vehicle, Textile and Electrical exports would fall by 0.23 and 0.16 per cent, respectively. In all cases the quantities of

Agricultural and Forestry exports and Minerals and Energy exports move together and in the opposite direction to those of Motor Vehicle, Textile and Electrical exports and Heavy Industrial and Service exports.

The corresponding import aggregator elasticities for 1970 are presented in Table 3.8. These elasticities show very little price responsiveness among import components to maintain a constant total quantity of imports but are also not comparable to the other elasticities presented nor readily interpreted. The elasticities show that if the price of Agricultural, Forestry and Service imports increased by 1 per cent there would be negligible falls in Agricultural, Forestry and Service imports and Vehicles, Chemical and Other imports to maintain a constant total import level. There would be offsetting negligible increases in Metals and Energy imports and Machinery, Electrical and Textile imports.

Of most interest are the export and import component elasticities derived from equation (3.22). These elasticities show the component response subject to a fixed aggregate capital input and are thus directly comparable with the other net output elasticities derived from the second stage of estimation. The export component own-price elasticities appear in Table 3.9 while the cross elasticities for 1970 are presented in Table 3.10. The price elasticities of supply for Agricultural and Forestry exports range from 0.62 to 0.86. Those for Motor Vehicle, Textile and Electrical exports range from 0.53 to 0.76. Minerals and Energy exports and Heavy Industrial and Service exports each exhibit slightly less price responsiveness with elasticities ranging from 0.40 to 0.77 and 0.41 to 0.64, respectively. The interesting result evident in the table of cross elasticities is that all the cross elasticities are positive. Hence, the four export components can be considered complementary in supply as an increase in the price of any one component will lead to increases in the quantities of all four export components subject to the fixed capital stock. This explains why the own-price elasticities of the components are all less than the elasticity of supply for total exports obtained from the GNP function. When the aggregate export price increases the prices of all four components effectively increase and hence compounding cross price effects come into play. If the price of just one component is increased then these compounding cross effects are not present. The other implication of these results is that if the price of one export component, say, Agricultural and Forestry Products, is reduced due to foreign trade barriers or dumping then the other export components will also be adversely affected.

Finally, import component own-price demand elasticities are presented in Table 3.11 and cross elasticities in Table 3.12. Agricultural, Forestry and Service imports exhibit the most price responsiveness with elasticities ranging from -0.36 to -0.75 while Machinery, Electrical and Textile imports exhibit the least responsiveness with a range of -0.27 to -0.41. Metals and Energy imports and Vehicle, Chemicals and Other imports exhibit intermediate responsiveness with ranges of -0.21 to -0.80 and -0.42 to -0.67, respectively. Again all cross import component elasticities are negative indicating a complementarity among the import groups. Hence, if a 10 per cent tariff had been placed on Machinery, Electrical and Textile imports in 1970 the imports of Machinery, Electrical and Textile Products would have fallen by 3.7 per cent, Agricultural, Forestry and Service imports would have fallen by 2.6 per cent and Metals and Energy imports and Vehicle, Chemicals and Other imports would have fallen by 3.1 per cent and 1.7 per cent, respectively.

## 3.5 Conclusions

These results illustrate the usefulness of the aggregator function approach in allowing the incorporation of several output and input categories within a GNP function model. They also illustrate the importance of recently developed functional forms such as the Symmetric Generalised McFadden in implementing the aggregator function model with the correct curvature requirements imposed. While the elasticities obtained from the model at the aggregate or second stage level are generally similar to comparable elasticities in other studies (eg. Kohli(1978)), they do exhibit some troublesome tendencies. The major anomaly present is the general trend towards rapidly increasing price responsiveness over time. This is particularly apparent for the labour demand elasticity. This tendency may be the consequence of shortcomings in the data, the imposition of curvature requirements on the model, or, of course, might be an accurate reflection of actual substitution possibilities. It may also be related to the failure to take account of declining capital capacity utilisation rates towards the end of the period. Another potential anomaly in the results is the complementarity observed between all export components and import components when responses are measured relative to a fixed capital input. To further examine the potential cause of these features of the results and in particular to ascertain the role of the separability assumption implicit in the aggregator function approach, the following Chapter of this thesis presents results for two flexible disaggregated models which do not use the aggregator approach.

## TABLE 3.1

## SGM PARAMETER ESTIMATES[1]

| Coefficient | Export Aggregator[2] | Import Aggregator[2] | GNP Function[3] |
|---|---|---|---|
| $a_{11}$ | -0.2652 (-3.62) | -0.0052 (-0.17) | 0.9169 (5.31) |
| $a_{12}$ | -0.0796 (-1.40) | 0.0523 (1.85) | -0.5440 (-6.86) |
| $a_{13}$ | 0.1826 (2.66) | 0.1318 (0.93) | -0.4119 (-4.88) |
| $a_{22}$ | -0.1057 (-1.58) | -0.0350 (-25.5) | 0.4221 (4.70) |
| $a_{23}$ | 0.1926 (1.42) | -0.0890 (-92.8) | -0.4512 (-2.67) |
| $a_{33}$ | -0.0095 (-0.00) | -0.0000 (-0.00) | -0.0000 (-0.00) |
| $b_{11}$ | 0.4095 (27.37) | -0.3486 (-28.78) | 0.5432 (18.19) |
| $b_{1t}$ | -0.1246 (-9.99) | 0.0104 (0.80) | 0.3497 (7.74) |
| $b_{tt}$ | 0.0094 (1.61) | 0.0174 (12.51) | -0.0633 (-3.36) |
| $b_{22}$ | 0.2999 (58.29) | -0.2394 (-62.93) | -0.3325 (-26.9) |
| $b_{2t}$ | -0.0691 (-10.06) | 0.0480 (14.78) | -0.1280 (-5.81) |
| $b_{33}$ | 0.1270 (5.45) | -0.2054 (-31.2) | -1.4233 (-68.3) |
| $b_{3t}$ | 0.1693 (8.15) | 0.0131 (2.47) | 0.0473 (0.70) |
| $b_{44}$ | 0.1692 (36.05) | -0.2277 (-21.08) | 2.3446 (46.1) |
| $b_{4t}$ | 0.0269 (3.76) | -0.0489 (-4.66) | 0.1341 (1.51) |
| $R^2$ Values | | | |
| Equation 1 | 0.7650 | 0.4052 | 0.7605 |
| Equation 2 | 0.8977 | 0.9007 | 0.9121 |
| Equation 3 | 0.7701 | 0.0166 | 0.9859 |
| Equation 4 | 0.7085 | 0.5948 | 0.4890 |
| Log Likelihood | 261.38 | 275.25 | 182.55 |

[1] Values in parentheses are assymptotic t-values.
[2] The coefficient subscripts and equation numbers 1,..,4 refer to Groups 1,..,4, respectively.
3 The coefficient subscripts and equation numbers 1,..,4 refer to Exports, Imports, Labour and Domestic Sales, respectively.

TABLE 3.2

GNP FUNCTION ELASTICITIES OF TRANSFORMATION[1]

| Year | $ET_{XX}$ | $ET_{XM}$ | $ET_{XL}$ | $ET_{XD}$ | $ET_{MM}$ |
|---|---|---|---|---|---|
| 1962 | 2.5643 | 2.8611 | 0.4974 | 0.1297 | 4.8283 |
| 1964 | 2.6340 | 3.0708 | 0.5896 | 0.2015 | 5.3129 |
| 1966 | 2.4235 | 2.7104 | 0.6382 | 0.2249 | 4.5329 |
| 1968 | 2.4597 | 2.5911 | 0.7703 | 0.2931 | 4.0931 |
| 1970 | 2.7651 | 2.7867 | 1.0177 | 0.4248 | 4.1704 |
| 1972 | 2.2500 | 1.9265 | 1.0549 | 0.3910 | 2.5755 |
| 1974 | 2.1969 | 2.0809 | 1.0900 | 0.4098 | 3.0177 |
| 1976 | 2.8083 | 2.4428 | 1.6868 | 0.6799 | 3.1997 |
| 1978 | 2.7395 | 2.6139 | 1.7616 | 0.7050 | 3.7070 |
| 1980 | 3.0694 | 3.4598 | 2.0327 | 0.8596 | 5.5805 |
| Year | $ET_{ML}$ | $ET_{MD}$ | $ET_{LL}$ | $ET_{LD}$ | $ET_{DD}$ |
| 1962 | 0.2177 | 0.1841 | 0.1660 | 0.0171 | 0.0075 |
| 1964 | 0.3209 | 0.2741 | 0.2095 | 0.0368 | 0.0163 |
| 1966 | 0.3224 | 0.2432 | 0.2701 | 0.0614 | 0.0209 |
| 1968 | 0.3702 | 0.2440 | 0.3840 | 0.1128 | 0.0380 |
| 1970 | 0.5060 | 0.3116 | 0.5729 | 0.2008 | 0.0752 |
| 1972 | 0.3429 | 0.1218 | 0.8337 | 0.3122 | 0.1169 |
| 1974 | 0.4371 | 0.1965 | 0.8794 | 0.3123 | 0.1115 |
| 1976 | 0.7022 | 0.3064 | 1.5577 | 0.6113 | 0.2402 |
| 1978 | 0.8485 | 0.4009 | 1.7039 | 0.6399 | 0.2424 |
| 1980 | 1.3446 | 0.7533 | 1.8792 | 0.6907 | 0.2684 |

[1] The subscripts X,M,L and D refer to Exports, Imports, Labour and Domestic Sales, respectively.

TABLE 3.3

EXPORT SUPPLY ELASTICITIES[1]

| Year | $E_{XX}$ | $E_{XM}$ | $E_{XL}$ | $E_{XD}$ |
|---|---|---|---|---|
| 1962 | 1.2666 | -0.9014 | -0.6381 | 0.2730 |
| 1964 | 1.3415 | -0.9977 | -0.7719 | 0.4281 |
| 1966 | 1.3406 | -0.9465 | -0.8850 | 0.4909 |
| 1968 | 1.4569 | -0.9731 | -1.1515 | 0.6677 |
| 1970 | 1.6657 | -1.0852 | -1.5704 | 0.9898 |
| 1972 | 1.4404 | -0.7445 | -1.5694 | 0.8736 |
| 1974 | 1.5666 | -0.8813 | -1.5647 | 0.8794 |
| 1976 | 1.9560 | -1.0483 | -2.3551 | 1.4475 |
| 1978 | 2.0125 | -1.1460 | -2.2722 | 1.4056 |
| 1980 | 2.2967 | -1.4871 | -2.4181 | 1.6085 |

[1] The subscripts X,M,L and D refer to Exports, Imports, Labour and Domestic Sales, respectively.

TABLE 3.4

IMPORT DEMAND ELASTICITIES[1]

| Year | $E_{MX}$ | $E_{MM}$ | $E_{ML}$ | $E_{MD}$ |
|---|---|---|---|---|
| 1962 | 1.4132 | -1.5212 | -0.2794 | 0.3874 |
| 1964 | 1.5639 | -1.7262 | -0.4201 | 0.5824 |
| 1966 | 1.4993 | -1.5830 | -0.4471 | 0.5308 |
| 1968 | 1.5347 | -1.5371 | -0.5534 | 0.5558 |
| 1970 | 1.6787 | -1.6240 | -0.7808 | 0.7261 |
| 1972 | 1.2333 | -0.9954 | -0.5101 | 0.2722 |
| 1974 | 1.4839 | -1.2781 | -0.6274 | 0.4217 |
| 1976 | 1.7014 | -1.3732 | -0.9804 | 0.6522 |
| 1978 | 1.9203 | -1.6252 | -1.0944 | 0.7993 |
| 1980 | 2.5888 | -2.3987 | -1.5996 | 1.4095 |

[1] The subscripts X,M,L and D refer to Exports, Imports, Labour and Domestic Sales, respectively.

TABLE 3.5

LABOUR DEMAND ELASTICITIES[1]

| Year | $E_{LX}$ | $E_{LM}$ | $E_{LL}$ | $E_{LD}$ |
|---|---|---|---|---|
| 1962 | 0.2457 | -0.0686 | -0.2129 | 0.0359 |
| 1964 | 0.3003 | -0.1043 | -0.2743 | 0.0782 |
| 1966 | 0.3531 | -0.1126 | -0.3745 | 0.1341 |
| 1968 | 0.4562 | -0.1390 | -0.5741 | 0.2569 |
| 1970 | 0.6131 | -0.1970 | -0.8839 | 0.4679 |
| 1972 | 0.6753 | -0.1325 | -1.2403 | 0.6974 |
| 1974 | 0.7773 | -0.1851 | -1.2624 | 0.6702 |
| 1976 | 1.1748 | -0.3013 | -2.1748 | 1.3013 |
| 1978 | 1.2941 | -0.3720 | -2.1978 | 1.2757 |
| 1980 | 1.5210 | -0.5780 | -2.2355 | 1.2925 |

1 The subscripts X,M,L and D refer to Exports, Imports, Labour and Domestic Sales, respectively.

TABLE 3.6

DOMESTIC SALES SUPPLY ELASTICITIES[1]

| Year | $E_{DX}$ | $E_{DM}$ | $E_{DL}$ | $E_{DD}$ |
|---|---|---|---|---|
| 1962 | 0.0641 | -0.0580 | -0.0219 | 0.0158 |
| 1964 | 0.1026 | -0.0891 | -0.0482 | 0.0346 |
| 1966 | 0.1244 | -0.0849 | -0.0852 | 0.0457 |
| 1968 | 0.1736 | -0.0916 | -0.1686 | 0.0866 |
| 1970 | 0.2559 | -0.1213 | -0.3099 | 0.1753 |
| 1972 | 0.2503 | -0.0471 | -0.4645 | 0.2612 |
| 1974 | 0.2922 | -0.0832 | -0.4483 | 0.2393 |
| 1976 | 0.4736 | -0.1315 | -0.8535 | 0.5114 |
| 1978 | 0.5180 | -0.1758 | -0.8254 | 0.4832 |
| 1980 | 0.6432 | -0.3238 | -0.8217 | 0.5023 |

1 The subscripts X,M,L and D refer to Exports, Imports, Labour and Domestic Sales, respectively.

TABLE 3.7

1970 EXPORT AGGREGATOR ELASTICITIES[1,2]

| | | With Respect to Price of | | | |
|---|---|---|---|---|---|
| | | AF | ME | VTE | HIS |
| | AF | 0.2238 | 0.0610 | -0.1383 | -0.1465 |
| Change in | ME | 0.0743 | 0.0646 | -0.1220 | -0.0169 |
| Quantity of | VTE | -0.1675 | -0.1213 | 0.2335 | 0.0553 |
| | HIS | -0.2282 | -0.0217 | 0.0711 | 0.1788 |

1 Export component response subject to a fixed quantity of aggregate exports.
2 Export components are Agricultural and Forestry Products (AF), Minerals and Energy Products (ME), Motor Vehicles, Textile and Electrical Products (VTE), and Heavy Industrial and Service Products (HIS).

TABLE 3.8

1970 IMPORT AGGREGATOR ELASTICITIES[1,2]

| | | With Respect to Price of | | | |
|---|---|---|---|---|---|
| | | AFS | MN | MET | VCO |
| | AFS | -0.0000 | 0.0002 | 0.0004 | -0.0006 |
| Change in | MN | 0.0004 | -0.0203 | -0.0429 | 0.0629 |
| Quantity of | MET | 0.0010 | -0.0505 | -0.1066 | 0.1561 |
| | VCO | -0.0009 | 0.0462 | 0.0975 | -0.1428 |

1 Import component response subject to a fixed quantity of aggregate imports.
2 Import components are Agricultural, Forestry and Service Products (AFS), Metals and Energy Products (MN), Machinery, Electrical and Textile Products (MET), and Vehicles, Chemicals and Other Products (VCO).

TABLE 3.9

EXPORT COMPONENT OWN SUPPLY ELASTICITIES[1,2]

| Year | $E_{AF}$ | $E_{ME}$ | $E_{VTE}$ | $E_{HIS}$ |
|---|---|---|---|---|
| 1962 | 0.6955 | 0.4377 | 0.5837 | 0.4122 |
| 1964 | 0.7395 | 0.4270 | 0.5344 | 0.4364 |
| 1966 | 0.6949 | 0.3997 | 0.5585 | 0.4392 |
| 1968 | 0.6147 | 0.4316 | 0.6888 | 0.4437 |
| 1970 | 0.6556 | 0.4966 | 0.7262 | 0.4881 |
| 1972 | 0.6399 | 0.3937 | 0.6425 | 0.4630 |
| 1974 | 0.6864 | 0.4815 | 0.5876 | 0.4693 |
| 1976 | 0.7630 | 0.5692 | 0.7312 | 0.5387 |
| 1978 | 0.7892 | 0.5582 | 0.7554 | 0.5492 |
| 1980 | 0.8594 | 0.7655 | 0.6631 | 0.6369 |

1 Export component response subject to fixed capital input available.
2 Export components are Agricultural and Forestry Products (AF), Minerals and Energy Products (ME), Motor Vehicles, Textile and Electrical Products (VTE), and Heavy Industrial and Service Products (HIS).

TABLE 3.10

1970 EXPORT COMPONENT CROSS SUPPLY ELASTICITIES[1,2]

| | | With Respect to Price of | | | |
|---|---|---|---|---|---|
| | | AF | ME | VTE | HIS |
| | AF | 0.6556 | 0.4930 | 0.3544 | 0.1628 |
| Change in | ME | 0.5061 | 0.4966 | 0.3707 | 0.2924 |
| Quantity of | VTE | 0.2643 | 0.3106 | 0.7262 | 0.3646 |
| | HIS | 0.2036 | 0.4103 | 0.5638 | 0.4881 |

1 Export component response subject to fixed capital input available.
2 Export components are Agricultural and Forestry Products (AF), Minerals and Energy Products (ME), Motor Vehicles, Textile and Electrical Products (VTE), and Heavy Industrial and Service Products (HIS).

TABLE 3.11

IMPORT COMPONENT OWN DEMAND ELASTICITIES[1,2]

| Year | $E_{AFS}$ | $E_{MN}$ | $E_{MET}$ | $E_{VCO}$ |
|---|---|---|---|---|
| 1962 | -0.6287 | -0.3410 | -0.3789 | -0.4765 |
| 1964 | -0.7009 | -0.3822 | -0.4113 | -0.5186 |
| 1966 | -0.6191 | -0.3287 | -0.3911 | -0.5255 |
| 1968 | -0.5530 | -0.3069 | -0.3593 | -0.5973 |
| 1970 | -0.5977 | -0.3348 | -0.3715 | -0.5900 |
| 1972 | -0.3631 | -0.2070 | -0.2851 | -0.4300 |
| 1974 | -0.4374 | -0.3613 | -0.2789 | -0.4569 |
| 1976 | -0.4528 | -0.3831 | -0.2766 | -0.5150 |
| 1978 | -0.5304 | -0.4153 | -0.3114 | -0.6240 |
| 1980 | -0.7551 | -0.8013 | -0.3978 | -0.6777 |

1 Import component response subject to fixed capital input available.
2 Import components are Agricultural, Forestry and Service Products (AFS), Metals and Energy Products (MN), Machinery, Electrical and Textile Products (MET), and Vehicles, Chemicals and Other Products (VCO).

TABLE 3.12

1970 IMPORT COMPONENT CROSS DEMAND ELASTICITIES[1,2]

| | | With Respect to Price of | | | |
|---|---|---|---|---|---|
| | | AFS | MN | MET | VCO |
| | AFS | -0.5977 | -0.3142 | -0.2645 | -0.4476 |
| Change in | MN | -0.5973 | -0.3348 | -0.3078 | -0.3842 |
| Quantity of | MET | -0.5967 | -0.3649 | -0.3715 | -0.2909 |
| | VCO | -0.5986 | -0.2682 | -0.1674 | -0.5900 |

1 Import component response subject to fixed capital input available.
2 Import components are Agricultural, Forestry and Service Products (AFS), Metals and Energy Products (MN), Machinery, Electrical and Textile Products (MET), and Vehicles, Chemicals and Other Products (VCO).

## 4. FLEXIBLE DISAGGREGATED MODELS

In order to compare the results obtained from the aggregator function model with those of a model not making use of the separability assumption, several flexible disaggregated models were investigated. Initial attempts to estimate a full model with the four export components, the four import components, domestic sales and labour as variable net outputs in the one model proved unsuccessful as the estimating system of ten equations would not converge. Attempts to economise on the number of parameters in the system by the use of semi-flexible functional forms as proposed by Diewert and Wales (1986) also proved to be unsuccessful. Semi-flexible functional forms, by reducing the size of the triangular matrices multiplied together to form the quadratic price coefficient matrix, reduce the total number of parameters in the system but at the expense of achieving less than full flexibility. In this case, however, even a four-column semi-flexible system would not converge. This would appear to further reinforce the tractability of the aggregator function procedure when dealing with many output and input categories, particularly when there is a limited number of observations available.

To further investigate the relationships between the export and import components and the other aggregate net outputs, two smaller disaggregated models were estimated. In the first of these the four export components were treated as net outputs along with aggregate imports, domestic sales and labour. In the second, the four import components, aggregate exports, domestic sales and labour were taken to be the net outputs. By examining these two disaggregated models it will be possible to gain more information on the relationships between the export components and the import components and on the stability of the estimated elasticities to changes in specification of the model.

## 4.1 The Generalised McFadden GNP Function

The GNP function framework outlined in the previous Chapter is again used in the models presented here. The same assumptions regarding profit maximising firms, perfect competition in goods and factor markets, and the characteristics of the aggregate technology set are made. Imports are again assumed to be an input to the production sector while exports are an output of the production sector not consumed domestically. Aggregate capital is again assumed to be the only fixed input and constant returns to scale are imposed with respect to aggregate capital. Seven net outputs are included in each model. In the first (second) model these are the 4 export (import) components, aggregate imports (exports), domestic sales and labour. Imports and labour quantities are again negative and the same data set as that of Chapter 3 is used.

On the basis of these assumptions the aggregate technology is represented by the following Generalised McFadden GNP function;

$$(4.1) \quad G(p,K)/K = (1/2) \sum_{i=1}^{6} \sum_{j=1}^{6} b_{ij}p_ip_j/p_7 + \sum_{i=1}^{7} b_ip_i + \sum_{i=1}^{7} b_{it}p_it + b_{tt}(\sum_{i=1}^{7} c_ip_i)t^2$$

where time superscripts have again been deleted and the $b_{ij}$ parameters satisfy the following symmetry restrictions;

$$(4.2) \qquad b_{ij} = b_{ji} \qquad \text{for all } i,j = 1,..,6.$$

The variable t is a time trend representing technical progress and the exogenous parameters $C_i$ are set equal to the average net output quantity per unit of capital input quantity for i=1,..,7.

As noted in the preceding Chapter, the Generalised McFadden (GM) restricted profit function is sensitive to the choice of the numeraire good (good 7 as specified in (4.1)). While the Symmetric Generalised McFadden (SGM) form used in Chapter 3 overcomes this sensitivity the non-symmetric GM form is more tractable when estimating a large model. In fact, in the present context estimation of an SGM model of this size with curvature imposed is precluded by the equation size constraint in the non-linear algorithm of the SHAZAM package. Domestic sales supply was used as the numeraire good in both the GM models estimated here. The GM form has the further slight advantage over the SGM form of not being limited to being flexible at just one price vector.

By applying Hotelling's Lemma the following set of net output supply equations is obtained;

$$(4.3) \quad x_i/K = \Sigma_{j=1}^{6} b_{ij}p_j/p_7 + b_i + b_{it}t + b_{tt}C_i t^2 + u_i; \; i=1,.,6;$$

$$(4.4) \quad x_7/K = -(1/2)\Sigma_{i=1}^{6}\Sigma_{j=1}^{6} b_{ij}p_i p_j/p_7^2 + b_7 + b_{7t}t + b_{tt}C_7 t^2 + u_7$$

The vectors of error terms for the observations are again assumed to be independently distributed with a multivariate normal distribution with zero means and covariance matrix $\Omega$. The estimating system thus consists of (4.3) and (4.4) subject to the symmetry restrictions (4.2).

If the matrix of estimated quadratic terms $B=[b_{ij}]$ is positive semi-definite then the restricted profit function is globally convex in prices (Diewert 1985). If B is not positive semi-definite then it can again be reparameterised using the Wiley, Schmidt and Bramble technique of replacing B by the product of a lower triangular matrix and its transpose;

(4.5) $B = AA'$ where $A = [a_{ij}]$ ; $i,j=1,..,6$ ; and $a_{ij}=0$ for $i<j$.

Estimation of the resulting system requires the use of non-linear regression techniques.

For simplicity of presentation, only the conventional net output supply elasticities derived from the estimated system are discussed in the following section. The conventional price elasticities are given by;

(4.6) $E_{ij} = d\ln x_i / d\ln p_j = DP_{ij}p_j/x_i$ ; $i,j=1,..,7$,

where $DP_{ij}$ is the second order price derivative of the restricted profit function and $x_i$ is the estimated quantity of net output i obtained from the system of net output supply equations (4.3) and (4.4). In the GM case the second order price derivatives are given by;

(4.7) $DP_{ij} = b_{ij}/p_7$ for $i,j=1,..,6$;

(4.8) $DP_{i7} = -\Sigma_{j=1}^{6} b_{ij}p_j/p_7^2$ for $i=1,..,6$; and

(4.9) $DP_{77} = \Sigma_{i=1}^{6} \Sigma_{j=1}^{6} b_{ij}p_ip_j/p_7^3$.

## 4.2 Results

Initial estimation of both the first (export) and second (import) models without curvature imposed produced systems which did not satisfy the convexity in prices property. In all cases the non-linear algorithm of SHAZAM was used with starting values of zero for the quadratic terms, and the constant and technology parameters set equal to values obtained from regressing these variables against the dependent variables. These starting values represent the polar Leontief case where there is no substitution between net outputs. The linear export model converged relatively quickly but three out of the six eigenvalues of the estimated B matrix were negative, indicating that the B matrix failed to be positive semi-definite. In the linear export model case three out of the seven estimated own-price elasticities had the wrong sign. Subsequent imposition of curvature requirements by reparameterising the B matrix produced slower convergence, a reflection of the degree to which curvature initially failed to be met. The log likelihood of the non-linear system was 423.48 compared to 443.74 for the linear system without curvature imposed.

The import model had two out of the six B matrix eigenvalues negative and two of the estimated own-price elasticities had the wrong sign. Imposition of curvature again led to slower convergence of the non-linear model from the Leontief starting values. The log likelihood of the non-linear system was 497.75 compared to 510.05 for the linear system without curvature imposed. The fact that the non-linear import model's log likelihood is closer to that of the linear model than is the case for the corresponding export model values reflects the fact that the import model came closer to meeting the curvature requirements and so imposition of curvature represents less of a restriction in the case of the import model. The non-linear parameter estimates for both models are presented in Table 4.1. Again some low equation R-square values are observed due to lack of variation in the quantity ratio dependent variables.

Own-price elasticity estimates for the export model are presented in Table 4.2. The own supply elasticities for exports of Agricultural and Forestry Products correspond closely to those obtained from the aggregator model of Chapter 3. The export model elasticities range from 0.70 to 0.90 compared to 0.63 to 0.85 for the corresponding aggregator elasticities. The elasticities of supply for exports of Minerals and Energy Products also correspond closely between the two models, ranging from 0.39 to 0.72 in the export model compared to 0.39 to 0.76 in the aggregator model.

The resemblance of the results breaks down, however, in the case of exports of Vehicles, Textile and Electrical Products. In the aggregator model these elasticities were stable and ranged from 0.53 to 0.76. In the export model, however, these elasticities are unstable and of unreasonable magnitude ranging from 1.84 to 10.58. These results are the major cause for concern in the export model's performance and are not a result of the imposition of curvature as elasticities of similar sign and magnitude were obtained in the linear model. Use of a different numeraire good in the model had little effect on these elasticities. This result may, in part, be due to the failure to take account of the influence of the Auto Pact which raised vehicle exports manyfold for given prices and technology. The export model also indicates more responsiveness for exports of Heavy Industrial and Service Products although the elasticities are of a reasonable order of magnitude ranging from 0.85 to 1.81 compared to 0.41 to 0.64 for the aggregator model. The export model indicates a trend of decreasing responsiveness for this export component whereas the aggregator model indicates a slight increase in responsiveness over the period.

The export model indicates a trend of increasing responsiveness for both domestic sales supply and labour demand. In the case of domestic sales supply, the export model indicates much greater responsiveness than the aggregator model with an elasticity range of 1.15 to 1.60 compared to 0.02 to 0.51 for the aggregator model. Labour demand elasticities are of similar magnitude between the two models at the end of the period although the export model indicates greater responsiveness at the beginning of the period. The export model's aggregate import demand elasticities follow a similar pattern to those of the aggregator model although they are on average somewhat higher than those of the aggregator model. Overall, then, the own-price elasticities for the export and aggregator models are largely in agreement with the exception of the third and fourth export components.

The important difference between the export and aggregator models becomes apparent in Table 4.3 where cross elasticities for the year 1970 are presented. Whereas the aggregator model indicated that all 4 export components were complementary, the export model indicates that exports of Agricultural and Forestry Products are substitutable with exports of Vehicles, Textile and Electrical Products which are in turn substitutable with exports of Minerals and Energy Products. Exports of Minerals and Energy Products are also substitutable with exports of Heavy Industrial and Service Products. The disaggregated export model thus gives a quite different impression of the relationships between the export components than does the restrictive aggregator model. This difference carries over to the relationships between the export components and the other aggregate net outputs. Exports of Vehicles, Textile and Electrical Products display the opposite relationship to aggregate imports, labour and domestic sales than do the other 3 export components in the export model and aggregate exports in the aggregator model.The relationships between aggregate imports, labour and domestic sales are the same in the export model as in the aggregator model.

Turning now to the import model, own net output price elasticities are presented in Table 4.4. The import model indicates greater price responsiveness for imports of Agricultural, Forestry and Service Products than does the aggregator model with a range of -0.71 to -1.89 compared to -0.36 to -0.76 for the aggregator model. Imports of Metals and Energy Products, on the other hand, exhibit less price responsiveness in the import model with elasticities ranging from -0.08 to -0.30 compared to -0.21 to -0.80 in the aggregator model. Imports of Machinery, Electrical and Textile Products and Vehicles, Chemicals and Other Products both exhibit considerably more price responsiveness in the import model than in the aggregator model but the elasticities are within reasonable bounds. The import model does, however, indicate considerably less price responsiveness for aggregate exports than does the aggregator model with elasticities less than half the size on average. The import model does indicate increasing price responsiveness for both domestic sales supply and labour demand as do both the export and aggregator models. The import model's labour demand elasticities coincide closely with those of the export model while its domestic sales supply elasticities lie approximately half way between those of the export and aggregator models.

The import model cross elasticities for the year 1970 are presented in Table 4.5. The cross elasticities indicate that all 4 import components are complementary with the exception of Vehicles, Chemicals and Other Products and Machinery, Electrical and Textile Products which are substitutable. These results are thus largely consistent with the aggregator model finding that all import components are complementary. The cross elasticities between the import components and the other aggregates as well as between aggregate exports, domestic sales and labour all indicate the same relationships as found in the aggregator model.

## 4.3 Conclusions

Examination of the disaggregated export and import models which do not make use of the separability assumption reveals some apparent advantages and disadvantages relative to the aggregator function model. The aggregator function model produces more stable estimates of the own-price elasticities for the export and import components, but less stable estimates of the price elasticities for domestic sales supply and labour demand. With the exception of the export model elasticity estimates for the third export component which are implausibly large and unstable, it is difficult to judge which specification produces the "best" or most accurate elasticity estimates. One feature which all three models agree upon, however, is that there has been a marked trend towards increasing price responsiveness of domestic sales supply and labour demand. In fact, the models indicate that wage rate policies would now have a major impact on the level of labour demand and, hence, on employment.

Another feature which the three models illustrate is that relatively small changes in specification can produce relatively large changes in the magnitude of various elasticities. This is best illustrated by the domestic sales supply elasticities in the three models. The conclusion one must draw from this is that not too much weight should be placed on any one set of estimates. Rather, a range of specifications should be tried to determine the stability of the results.

As expected, the major difference between the models comes in the area of cross elasticities. In particular, the export model results do not indicate complementarity between all the export components as found in the aggregator model. On the basis of these reults then, it would appear that the aggregator model has advantages in producing stable component own-price elasticity estimates over the larger disaggregated models which have difficulty producing stable estimates for all net outputs. On the other hand, the larger disaggregated models appear to have an advantage in detecting the relationships between the individual components. Which of the two approaches is used should take these considerations into account. If the main interest is in detecting the cross relationships then the larger disaggregated model would appear to be more suitable.

## TABLE 4.1

## GM PARAMETER ESTIMATES

| Coefficient | Export Model[1] | Import Model[2] | Coefficient | Export Model[1] | Import Model[2] |
|---|---|---|---|---|---|
| $a_{11}$ | 0.3784 | 0.6077 | $a_{55}$ | 0.0360 | -0.1328 |
| $a_{21}$ | 0.1884 | -0.2257 | $a_{65}$ | 0.1165 | -0.1268 |
| $a_{31}$ | -0.3349 | -0.0543 | $a_{66}$ | 0.0123 | 0.1616 |
| $a_{41}$ | 0.0774 | -0.1761 | $b_1$ | 0.6301 | 1.0301 |
| $a_{51}$ | -0.4376 | -0.1101 | $b_2$ | 0.5588 | -0.2915 |
| $a_{61}$ | -0.8882 | -0.9145 | $b_3$ | -1.1873 | -0.1006 |
| $a_{22}$ | -0.1849 | 0.2669 | $b_4$ | 0.0765 | -0.2559 |
| $a_{32}$ | 0.7611 | 0.0119 | $b_5$ | -0.8355 | -0.1322 |
| $a_{42}$ | 0.1562 | -0.1131 | $b_6$ | -2.8092 | -2.3840 |
| $a_{52}$ | 0.1212 | -0.0286 | $b_7$ | 3.4573 | 2.6636 |
| $a_{62}$ | 0.2803 | -0.2785 | $b_{1t}$ | 0.1038 | 0.5178 |
| $a_{33}$ | -0.2829 | -0.0681 | $b_{2t}$ | 0.0680 | -0.0929 |
| $a_{43}$ | -0.0820 | -0.0622 | $b_{3t}$ | 0.2045 | -0.0272 |
| $a_{53}$ | 0.5151 | -0.1998 | $b_{4t}$ | 0.1827 | -0.0605 |
| $a_{63}$ | -0.1768 | 0.5283 | $b_{5t}$ | -0.2679 | -0.0983 |
| $a_{44}$ | -0.3573 | -0.3628 | $b_{6t}$ | -0.3094 | -0.4292 |
| $a_{54}$ | 0.0385 | 0.2103 | $b_{7t}$ | 0.8264 | 1.0296 |
| $a_{64}$ | 0.6002 | 0.0484 | $b_{tt}$ | -0.1439 | -0.1583 |
| **$R^2$ Values** | | | | | |
| Equation 1 | 0.1190 | 0.8147 | Equation 5 | 0.9349 | 0.9550 |
| Equation 2 | 0.1243 | 0.9316 | Equation 6 | 0.9795 | 0.9802 |
| Equation 3 | 0.9322 | 0.6754 | Equation 7 | 0.7499 | 0.6070 |
| Equation 4 | 0.9475 | 0.7721 | | | |

TABLE 4.1 (CONTINUED)

1 The subscripts and equation numbers 1,..,7 refer to Aricultural and Forestry Product Exports, Minerals and Energy Product Exports, Vehicles, Textile and Electrical Product Exports, Heavy Industrial and Service Product Exports, Aggregate Imports, Labour and Domestic Sales, respectively.

2 The subscripts and equation numbers 1,..,7 refer to Aggregate Exports, Agricultural, Forestry and Service Product Imports, Metals and Energy Product Imports, Machinery, Electrical and Textile Product Imports, Vehicles, Chemical and Other Product Imports, Labour and Domestic Sales, respectively.

TABLE 4.2

EXPORT MODEL OWN PRICE ELASTICITIES[1]

| Year | $E_{AF}$ | $E_{ME}$ | $E_{VTE}$ | $E_{HIS}$ | $E_M$ | $E_L$ | $E_D$ |
|---|---|---|---|---|---|---|---|
| 1962 | 0.7147 | 0.4636 | 10.5813 | 1.8161 | -2.6624 | -0.9502 | 1.0416 |
| 1964 | 0.7087 | 0.4414 | 6.9753 | 1.5493 | -2.5862 | -0.9960 | 1.1553 |
| 1966 | 0.6961 | 0.4204 | 4.9657 | 1.3808 | -2.0776 | -1.0908 | 1.1460 |
| 1968 | 0.7255 | 0.4267 | 3.2552 | 1.2787 | -1.8132 | -1.2627 | 1.2462 |
| 1970 | 0.7557 | 0.4459 | 2.5020 | 1.2045 | -1.6876 | -1.4554 | 1.3646 |
| 1972 | 0.7176 | 0.3913 | 2.0495 | 1.0465 | -1.0762 | -1.6191 | 1.3573 |
| 1974 | 0.7559 | 0.4240 | 2.4006 | 1.0000 | -1.2253 | -1.5926 | 1.2052 |
| 1976 | 0.8024 | 0.5132 | 1.9087 | 1.0106 | -1.2115 | -1.9671 | 1.4234 |
| 1978 | 0.8466 | 0.6023 | 1.8459 | 0.8972 | -1.3623 | -2.1072 | 1.5198 |
| 1980 | 0.9023 | 0.7216 | 1.9225 | 0.8451 | -1.7266 | -2.2312 | 1.6016 |

[1] The subscripts AF, ME, VTE, HIS, M, L and D refer to Aricultural and Forestry Product Exports, Minerals and Energy Product Exports, Vehicles, Textile and Electrical Product Exports, Heavy Industrial and Service Product Exports, Aggregate Imports, Labour and Domestic Sales, respectively.

TABLE 4.3

1970 CROSS ELASTICITIES - EXPORT MODEL[1]

| Change in Quantity of: | With Respect to Price of: AF | ME | VTE | HIS | M | L | D |
|---|---|---|---|---|---|---|---|
| AF | 0.756 | 0.384 | -0.570 | 0.151 | -0.988 | -2.439 | 2.701 |
| ME | 0.447 | 0.446 | -1.089 | -0.088 | -0.743 | -1.887 | 2.914 |
| VTE | -0.482 | -0.661 | 2.502 | 0.432 | 0.400 | 2.930 | -4.990 |
| HIS | 0.219 | -0.109 | 0.741 | 1.205 | -0.600 | -2.312 | 0.856 |
| M | 0.522 | 0.337 | -0.250 | 0.218 | -1.688 | -1.551 | 2.410 |
| L | 0.280 | 0.186 | -0.398 | 0.183 | -0.338 | -1.455 | 1.542 |
| D | 0.204 | 0.189 | -0.445 | 0.045 | -0.345 | -1.012 | 1.365 |

[1] The labels AF, ME, VTE, HIS, M, L and D refer to Aricultural and Forestry Product Exports, Minerals and Energy Product Exports, Vehicles, Textile and Electrical Product Exports, Heavy Industrial and Service Product Exports, Aggregate Imports, Labour and Domestic Sales, respectively.

TABLE 4.4

IMPORT MODEL OWN PRICE ELASTICITIES[1]

| Year | $E_X$ | $E_{AFS}$ | $E_{MN}$ | $E_{MET}$ | $E_{VCO}$ | $E_L$ | $E_D$ |
|---|---|---|---|---|---|---|---|
| 1962 | 0.7115 | -1.6126 | -0.1332 | -3.4082 | -1.8418 | -0.9207 | 0.4987 |
| 1964 | 0.6448 | -1.8946 | -0.1296 | -3.0592 | -1.5716 | -0.9680 | 0.5511 |
| 1966 | 0.5795 | -1.5893 | -0.1170 | -2.5197 | -1.2522 | -1.0612 | 0.5891 |
| 1968 | 0.5418 | -1.3762 | -0.1084 | -2.3039 | -1.0780 | -1.2217 | 0.6746 |
| 1970 | 0.5146 | -1.3613 | -0.1055 | -2.1502 | -0.9290 | -1.3998 | 0.7818 |
| 1972 | 0.4517 | -0.7099 | -0.0824 | -1.6090 | -0.7101 | -1.5604 | 0.8015 |
| 1974 | 0.4688 | -0.8012 | -0.1447 | -1.7685 | -0.6626 | -1.6089 | 0.8227 |
| 1976 | 0.4716 | -0.7293 | -0.1529 | -1.6312 | -0.6549 | -1.9516 | 0.9877 |
| 1978 | 0.4842 | -0.7921 | -0.1758 | -1.6337 | -0.7798 | -2.0249 | 1.0130 |
| 1980 | 0.5059 | -0.8581 | -0.3010 | -1.6102 | -0.9053 | -2.0682 | 1.0286 |

1 The subscripts X, AFS, MN, MET, VCO, L and D refer to Aggregate Exports, Agricultural, Forestry and Service Product Imports, Metals and Energy Product Imports, Machinery, Electrical and Textile Product Imports, Vehicles, Chemical and Other Product Imports, Labour and Domestic Sales, respectively.

TABLE 4.5

1970 CROSS ELASTICITIES - IMPORT MODEL[1]

| Change in Quantity of: | With Respect to Price of: X | AFS | MN | MET | VCO | L | D |
|---|---|---|---|---|---|---|---|
| X | 0.515 | -0.264 | -0.050 | -0.139 | -0.095 | -1.088 | 1.122 |
| AFS | 1.107 | -1.361 | -0.137 | -0.072 | -0.142 | -1.497 | 2.102 |
| MN | 0.412 | -0.211 | -0.106 | -0.145 | -0.245 | -0.181 | 0.531 |
| MET | 1.377 | -0.170 | -0.176 | -2.150 | 0.542 | -2.568 | 3.145 |
| VCO | 0.530 | -0.188 | -0.167 | 0.305 | -0.929 | -0.335 | 0.785 |
| L | 0.448 | -0.147 | -0.009 | -0.107 | -0.025 | -1.400 | 1.240 |
| D | 0.301 | -0.134 | -0.018 | -0.085 | -0.038 | -0.808 | 0.782 |

[1] The labels X, AFS, MN, MET, VCO, L and D refer to Aggregate Exports, Agricultural, Forestry and Service Product Imports, Metals and Energy Product Imports, Machinery, Electrical and Textile Product Imports, Vehicles, Chemical and Other Product Imports, Labour and Domestic Sales, respectively.

# 5. A PLANNING PRICE MODEL

As in other areas of economics it is important to allow for imperfect adjustment in the GNP function model. In fact, imperfect adjustment is likely to be particularly important in regard to traded goods due to the relatively long lags involved between the decision to buy or sell a good internationally and its ultimate delivery to the end-user. The J-curve effect whereby a devaluation leads to an initial worsening of the trade balance but then to a longer-term improvement is an important example of the role of slow adjustment in the traded goods sector. Its explanation, however, requires a more sophisticated model than those presented here. Many Canadian exports are also of primary products which have long lead times between the decision to increase supply of, say, a particular mineral and the time when that supply is available for sale. As a result, GNP function models which assume instantaneous adjustment are likely to miss much of the underlying dynamics at work in the economy's traded goods sector. This Chapter and the following one of this thesis present the results of models which attempt to include dynamics and imperfect adjustment within the GNP function framework.

## 5.1 The Planning Price Approach

An approach to modelling imperfect adjustment which has received little attention is the use of "planning prices" as developed by Woodland (1976,1977). Under this approach producers do not adjust fully to current prices within the observation period. Instead they adjust fully within the period to planning prices which in turn adjust gradually to actual prices. This behaviour may be interpreted in one of two ways. Firstly, firms may have to commit themselves to input decisions before current prices are known or even if current prices are known the firm may wish to wait and see if price changes are permanent before fully adjusting to a new current price. This may be likened to a partial adjustment process whereby producers adjust only part-way towards a new price in the current period depending on their expectations of future price movements. Either way, planning prices will adjust to actual prices only gradually.

An alternative interpretation is that the use of planning prices is a dual representation of a quantity adjustment path. For instance, if input prices change to a new level and then remain at that level then producers faced with adjustment costs and quasi-fixed inputs will gradually change their input mix to approach the new optimal quantities if the adjustment path is stable. Thus, it may not be possible or profitable to fully adjust capital, particularly that in the form of buildings, in the current period. Rather, capital would be increased towards its new optimal level over a number of periods. If producers are technically efficient then the quantity adjustment path will follow the boundary of the transformation frontier. However, corresponding to each point on the boundary of the transformation frontier there will be a normal vector of prices for which that quantity decision is optimal. Hence, a planning price path which approaches the new price vector will be a dual representation of, and observationally equivalent to, an optimal quantity adjustment path.

The planning price approach has the advantage, over early attempts to model quantity adjustment paths, of automatically ensuring technical efficiency at each point. It has the disadvantage though that an adjustment relationship of planning to actual prices must be specified to make the approach operational. This introduces a degree of arbitrariness.

In this application the following adaptive price adjustment model is used:

(5.1) $$q_{it} - q_{i,t-1} = D_i(p_{it} - q_{i,t-1})$$

where the $q_{it}$ are planning prices and $p_{it}$ actual prices. If $D_i=1$ then adjustment of planning to actual prices is instantaneous. For the adjustment process to be stable the adjustment parameters $D_i$ must lie in the interval (0,2). If $D_i$ is in the range (0,1) then adjustment to the new price is monotonic while it is cyclical if $D_i$ is in the range (1,2). To make this mechanism implementable the following version is estimated:

(5.2) $$q_{it} = D_i\Sigma_{j=0}^{t-1} (1-D_i)^j p_{t-j} + (1-D_i)^t q_{i0}$$

where the base period planning price $q_{i0}$ is treated as a parameter and estimated along with the adjustment coefficient $D_i$. Embedding this price relationship within a standard functional form for the GNP function means that non-linear regression techniques must be used. The Davidson-Fletcher-Powell non-linear algorithm in the SHAZAM package was again used.

The planning price model estimated uses a unit Generalised Leontief restricted profit function. Given the computational complexity of the estimation procedure, use of the SGM or Generalised McFadden forms would be prohibitive, particularly given the size constraint on each equation in the non-linear SHAZAM facility. The translog form is not suited to the planning price procedure because the dependent variables of the share equations contain the planning price terms which are not known before estimation.

Estimation of a four-variable net output model analogous to the second stage GNP function model in Chapter 3 imposing constant returns to scale with respect to the aggregate capital input was not possible using the SHAZAM package due to the non-linear equation size restriction. Consequently, labour and capital were aggregated into a single fixed input and constant returns to scale imposed with respect to this aggregate fixed input. The remaining 3 variable net output categories were, thus, the quantity of exports, (minus) the quantity of imports and the quantity of domestic sales.

The 3 variable net output unit Generalised Leontief restricted profit function is given by:

$$(5.3) \qquad G(p,Z)/Z = \Sigma_{i=1}^{3}\Sigma_{j=1}^{3} b_{ij}q_i^{1/2}q_j^{1/2} + \Sigma_{i=1}^{3} b_{ii}q_i + \Sigma_{i=1}^{3} b_{it}q_it + \Sigma_{i=1}^{3} b_{itt}q_it^{2}$$

where time subscripts have been deleted, Z is the aggregate fixed input and the $q_i$ are planning prices as given by (5.2). The parameters $b_{ij}$ satisfy the following symmetry restriction;

$$(5.4) \qquad b_{ij} = b_{ji} \qquad \text{for all } i,j = 1,2,3.$$

The net output supply equations derived from (5.3) by differentiating with respect to prices are;

$$(5.5) \qquad x_i/Z = \Sigma_{j=1}^{3} b_{ij}(q_j/q_i)^{1/2} + b_{ii} + b_{it}t + b_{itt}t^{2};\ i=1,2,3.$$

The estimating system thus consists of (5.5) where the planning prices are given by (5.2). The parameters of the net output supply equations ($b_{ij}$, $b_{ii}$, $b_{it}$ and $b_{itt}$ ), the planning price adjustment coefficients ($D_i$) and the base period planning prices ($q_{i0}$) are all chosen simultaneously to maximise the concentrated likelihood function of (5.5).

Estimation of this model enables tests to be carried out of the validity of the instantaneous adjustment model normally used by testing whether $D_i=1$ for $i=1,2,3$. The relationship between planning and actual prices, and instantaneous and imperfect quantity adjustment paths, will be plotted by tracking the effects of simulated price increases. One would expect the planning prices for exports and imports to lag behind actual prices (ie. $0<D_i<1$) due to the lags involved between producer decisions and delivery dates.

## 5.2 Results

The maximum likelihood parameter estimates for the Generalised Leontief models using actual prices and planning prices are both presented in Table 5.1. Both estimated profit functions are positive at all observation points and satisfy the curvature requirements of being convex in prices at all observation points. The gradients with respect to prices have the correct signs and so both estimated profit functions are well behaved. The non-linear model was estimated using the linear estimates of the instantaneous adjustment model as starting values for the price and technology parameters in the planning price model along with values of 0.8 and 1.0 for the adjustment coefficients and base period planning prices, respectively.

The first result of interest to be examined is whether the two models are significantly different; i.e., are the adjustment coefficients in the planning price model significantly different from 1.0 indicating that imperfect adjustment is of importance. The hypothesis that all adjustment coefficients are equal to unity (subject to the base period planning prices being unrestricted) may be tested by use of the likelihood ratio test. The test statistic has a value of 43.58 (twice the difference between the two log likelihood values) compared to a 1 per cent critical Chi-square value of 11.34 with 3 degrees of freedom. Consequently, the hypothesis of instantaneous adjustment is strongly rejected by the model. This indicates that it is important to allow for imperfect adjustment when modelling production sector activities. This result is not unexpected but it remains to establish whether the planning price model provides reasonable estimates of the imperfect adjustment process.

As the base period planning prices are estimated in this model examination of the estimated base period values and the relationship between actual prices and the estimated planning price series provides one method of checking the reasonableness of the model. In the estimated model the base period planning price refers to the planning price for the year 1960. While the input-output data are only available from 1961 onwards it is reasonable to assume that the actual prices prevailing in 1960 would be close to and probably slightly below the 1961 price index values of 1.0. The base period planning price estimated for imports is indeed 0.95 which seems very close to what might be expected a priori. The estimate of 0.64 for the base period domestic sales planning price is somewhat below what might be expected. The estimate of 1.89 for the export base period planning price appears to be unreasonable, being considerably higher than the actual export prices likely to have prevailed prior to 1961.

Comparisons of the actual price and estimated planning price series for the observation period tend to confirm these impressions. The planning price series for domestic sales closely follows but lags slightly behind the actual price series, ranging from 0.92 to 2.85 compared to the actual price range of 1.00 to 2.94. Import planning prices also follow but lag further behind actual import prices, ranging from 0.97 to 2.94 compared to the actual price range of 1.00 to 3.74. Export planning prices, however, bear less resemblance to actual export prices, being higher than actual prices for the first half of the period and lower than actual prices for the second half of the period. These comparisons would appear to indicate that less reliance can be placed on the model's results with regard to exports than its predictions for both imports and domestic sales.

The parameter estimates of most interest in the model are those of the planning price adjustment coefficients. Using one interpretation, these parameters indicate how quickly planning prices change when there is a change in the actual price. The three estimated parameters all lie in the range (0,2) required for stability of the adjustment process. Furthermore, they all lie in the range (0,1) indicating that adjustment in all three cases is monotonic rather than cyclical. As expected, the adjustment coefficient for domestic sales is closer to unity than those for the two traded goods indicating that domestic sales supply is quicker to respond to actual price changes than is export supply and import use. Indeed, starting from a position of long run equilibrium where the initial actual and planning prices are equal, an increase in the actual price of domestic sales of ten per cent would lead to an increase in the planning price of 7.7 per cent in the first period. The adjustment of the planning prices to the actual price changes under these conditions is graphed in Figure 5.1. In the case of domestic sales the planning price approaches the new actual price relatively quickly with the adjustment effectively being complete within 5 years.

In the case of imports the import planning price would increase 3.4 per cent in the first year in response to a 10 per cent increase in the actual price of imports. This slower adjustment is likely due to the longer order and delivery lags associated with imported purchases. The time elapsed between the initial price increase and the effective adjustment of the import planning price is 10 years. The export price adjustment coefficient is relatively small indicating that adjustment of the planning price to the actual price is very sluggish indeed. In fact, the planning price would only increase 0.7 per cent the first year in response to a 10 per cent increase in the actual export price. Even after 15 years only two-thirds of the adjustment would have taken place. While this very sluggish adjustment of exports may be realistic for ventures such as bringing a new mine on stream or expanding cropping into virgin, uncleared land, it seems less plausible for the output of manufacturing exports and increasing production from existing mines and agricultural land.

The alternative interpretation of the model is that the planning prices are simply part of the dual representation of a quantity adjustment path. The adjustment path will depend on the initial prices, the magnitude of the price changes and the characteristics of the GNP function. Within this context it is difficult to interpret the individual adjustment coefficients directly in terms of their implications for the quantity adjustment paths. To gain a better understanding of the quantity adjustment process a series of simulations were carried out. The price of each net output was in turn assumed to increase by 10 per cent and then remain at this higher level. The effects of these price changes on net output supply were simulated subject to the initial period technology level and a constant level of the aggregate fixed input. The results of these simulations are presented in Figure 5.2. The quantity adjustment paths are monotonic in each case which follows from the monotonic rather than cyclical adjustment of the three planning prices.

An increase in the price of domestic sales leads to increases in domestic sales supply and import use and to a decrease in exports. Adjustment is effectively complete in five years with most of the adjustment occurring in the first three years. An increase in import prices leads to a large fall in import demand and falls in supply of the two outputs. Adjustment is effectively complete in ten years with most of the adjustment having taken place in the first five years. An increase in export prices leads to increases in export supply and import demand and a fall in domestic sales supply. Adjustment is very sluggish, however, with the process still being incomplete after fifteen years.

Finally, the net output supply elasticities obtained from the two models are presented in Table 5.2. It should be noted that the instantaneous adjustment model elasticities represent the one period response to a change in actual prices while the planning price model elasticities represent the response to a change in the planning price, not the actual price. Accordingly, the planning price model own elasticities are all substantially larger than the corresponding instantaneous adjustment elasticities because more adjustment is allowed for in the planning price case. The cross elasticities are also larger in the planning price case with the exception of that between exports and imports which remains approximately constant. Exports and domestic sales supply are substitutes in the planning price case but slight complements in the instantaneous adjustment case. The instantaneous adjustment elasticities sign pattern corresponds to that of the instantaneous adjustment GNP function of Chapter 3 although the two sets of elasticities show responses subject to different conditions being held fixed.

In conclusion, then, the planning price model has served to demonstrate the importance of allowing for imperfect adjustment when modelling production sector response. It appears that imperfect adjustment is more important for traded goods sectors than for those supplying domestic sales. The model appears to present plausible results for import demand and domestic sales supply but may overstate the importance of imperfect adjustment in the case of export supply. To further investigate the role of imperfect adjustment in export supply and import demand a more sophisticated model of imperfect adjustment is presented in the following chapter.

TABLE 5.1

GENERALISED LEONTIEF PARAMETER ESTIMATES[1,2]

| Coefficient | Instantaneous Adjustment | Planning Price Model |
|---|---|---|
| $b_{DD}$ | 1.0320 (2.10) | 2.1937 (8.37) |
| $b_{DM}$ | -0.1287 (-0.96) | -0.1668 (-1.44) |
| $b_{DX}$ | 0.0241 (0.04) | -0.7681 (-4.68) |
| $b_{MM}$ | 0.2807 (5.70) | 0.4843 (3.95) |
| $b_{MX}$ | -0.2825 (-1.70) | -0.3159 (-2.15) |
| $b_{XX}$ | 0.4468 (0.62) | 0.9386 (4.95) |
| $b_{Dt}$ | 0.2417 (3.72) | -0.0530 (-0.56) |
| $b_{Mt}$ | -0.0836 (-4.24) | -0.1908 (-2.31) |
| $b_{Xt}$ | 0.2081 (2.90) | 0.3971 (4.93) |
| $b_{Dtt}$ | -0.0609 (-1.76) | -0.0414 (-1.79) |
| $b_{Mtt}$ | 0.0123 (1.21) | 0.0241 (1.18) |
| $b_{Xtt}$ | -0.0466 (-1.18) | 0.0101 (0.31) |
| $q_{D0}$ | | 0.6397 (2.23) |
| $q_{M0}$ | | 0.9595 (6.33) |
| $q_{X0}$ | | 1.8897 (4.48) |
| $D_D$ | | 0.7683 (1.35) |
| $D_M$ | | 0.3403 (6.33) |
| $D_X$ | | 0.0666 (59.71) |
| $R^2$ Values | | |
| Equation D | 0.9216 | 0.9565 |
| Equation M | 0.8494 | 0.9819 |
| Equation X | 0.9216 | 0.9884 |
| Log Likelihood | 186.28 | 208.07 |

TABLE 5.2 (CONTINUED)

[1] The subscripts and equation labels D, M and X refer to Domestic Sales, Imports and Exports, respectively.
[2] Values in parentheses are asymptotic t-values.

TABLE 5.2

1970 NET OUTPUT SUPPLY ELASTICITIES[1]

Instantaneous Adjustment Model

| | | With Respect to Price of: | | |
|---|---|---|---|---|
| | | X | M | D |
| Change in | X | 0.4236 | -0.4613 | 0.0377 |
| Quantity of: | M | 0.7445 | -1.0934 | 0.3488 |
| | D | 0.0106 | -0.0610 | 0.0504 |

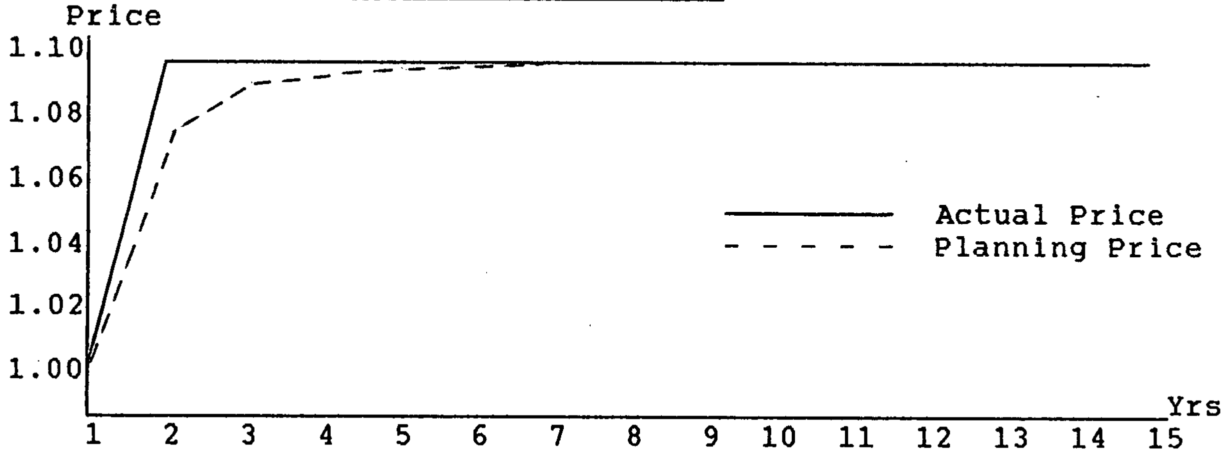
Planning Price Model

| | | With Respect to Planning Price of: | | |
|---|---|---|---|---|
| | | X | M | D |
| Change in | X | 1.4674 | -0.4362 | -1.0312 |
| Quantity of: | M | 0.9342 | -1.3874 | 0.4531 |
| | D | -0.3832 | -0.0786 | 0.4619 |

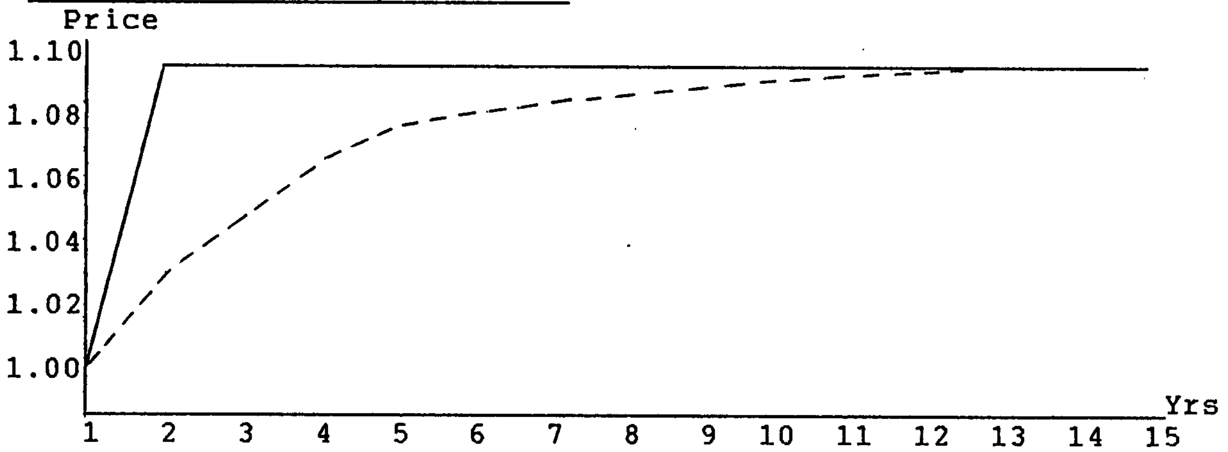
[1] The letters X, M and D refer to Exports, Imports and Domestic Sales, respectively.

FIGURE 5.1
ADJUSTMENT OF PLANNING PRICES

a) 10% Increase in Domestic Sales Price




b) 10% Increase in Import Price




c) 10% Increase in Export Price

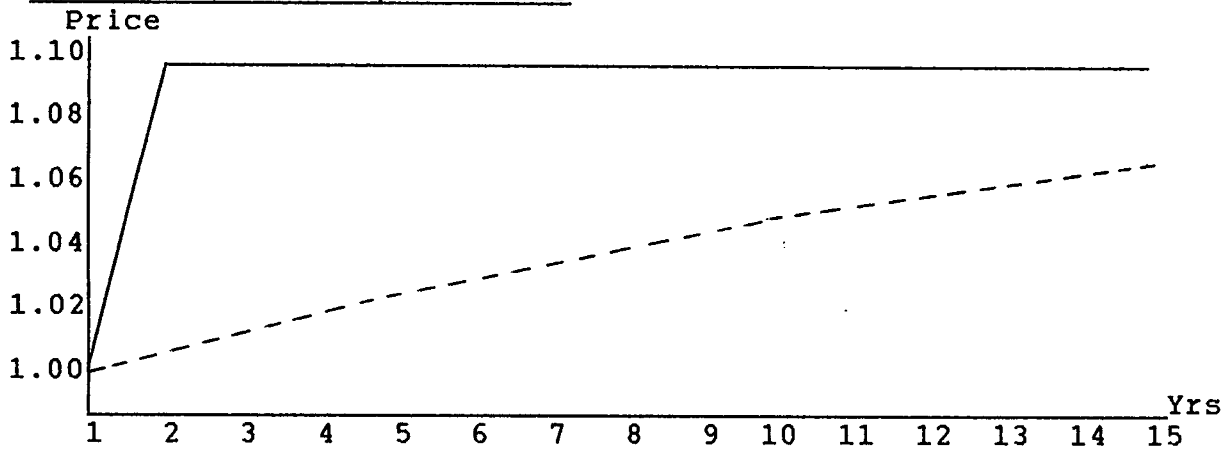
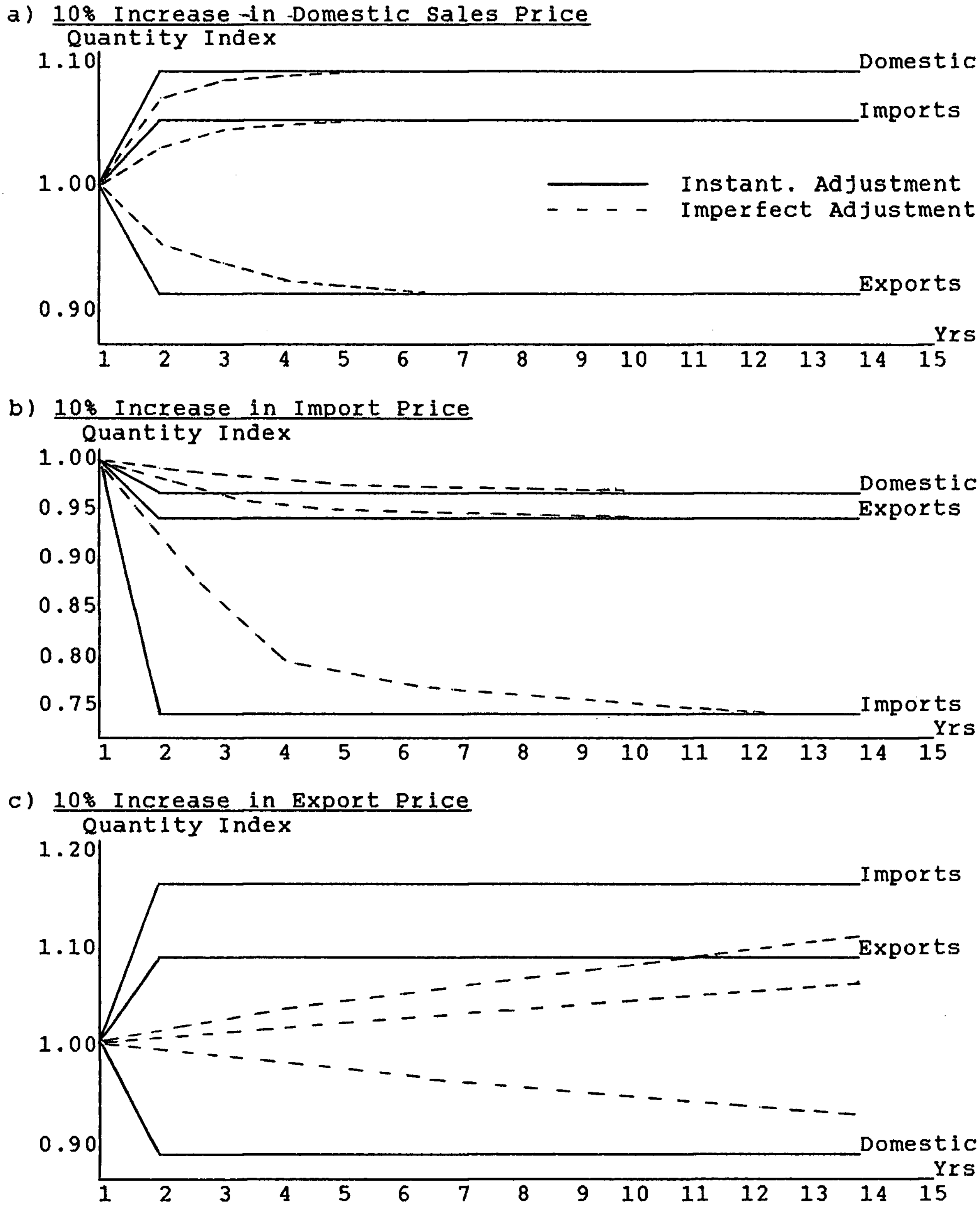


FIGURE 5.2
ADJUSTMENT OF QUANTITIES



## 6. AN ADJUSTMENT COSTS MODEL

An approach to modelling imperfect adjustment which has proven to be more popular than the planning price model outlined in the previous Chapter is the development of models which explicitly allow for costs of adjustment within an optimising framework. The aim of these models can be described as modelling short and long run factor demands within a unified framework where the (costly) rate of adjustment of quasi-fixed factors is an endogenous choice variable and factor demands are interrelated. Shocks which result in all factor demands being out of long-run equilibrium are consistent with the models although short-run equilibrium is maintained at all times, ie. the phenomenon of "overshooting" of short-run demands for variable factors is allowed for. At the same time output feasibility is maintained and the Le Chatelier principle is satisfied whereby long-run own-price elasticities are greater in absolute value than the corresponding short-run own-price elasticities. By using an empirical model derived from such an adjustment costs model it should be possible to gain a better understanding of the role of imperfect adjustment in the GNP function framework. A theoretical adjustment costs model is briefly outlined in the following section and an econometric adaptation is then presented.

### 6.1 A Theoretical External Adjustment Costs Model

The theoretical model used in this application is similar to that of Berndt, Fuss and Waverman (1977, Chapter 4) and this section draws on their presentation. The Berndt, Fuss and Waverman model is in turn derived from Lucas' (1967) model of external adjustment costs where quasi-fixed factors (denoted by z) are fixed in the stort-run but can be varied over time subject to positive, increasing marginal costs of adjustment. The marginal adjustment costs are denoted by $C_m(\dot{z}_m)$ where $\dot{z}_m = dz_m/dt$ and where;

(6.1) $C_m(0) = 0$ , $C_m'(\dot{z}_m) > 0$ , $C_m''(\dot{z}_m) > 0$ ; m=1,..,M.

Firms are assumed to know the prices of variable net outputs with certainty and have static expectations regarding those prices.

Adjustment costs are external in the sense that current production is unaffected by changes in the quasi-fixed factors although future production is affected. Time paths for variable net outputs and fixed inputs are chosen to maximise the present value of net receipts given the initial stock of quasi-fixed inputs. The present value of net receipts is;

(6.2) $V(0) = \int_0^{\infty} e^{-rt}R(t)\, dt$

where r is the appropriate discount rate and R(t) is the value of net receipts.

Using a restricted profit function which satisfies the standard properties of Hotelling's Lemma, convexity in p and concavity in z and denoting the restricted profit function by H(p,z), the revenue function (R(t)) can be written as;

(6.3) $R(t) = H(p(t),z(t)) - \sum_{m=1}^{M} C_m(\dot{z}_m(t))$.

The first order condition with respect to the quasi-fixed inputs from the maximisation of the present value of net receipts is now given by;

(6.4) $H_{z_m} - rC_m'(\dot{z}_m) + C_m''(\dot{z}_m)\ddot{z}_m = 0$ ; m=1,..,M.

A stationary solution for the quasi-fixed inputs exists when $\dot{z}_m = \ddot{z}_m = 0$ and hence satisfies;

(6.5) $H_{z_m}(p,z) - rC_m'(0) = 0.$

This condition can be shown to be equivalent to the requirement that in a steady state the marginal value product of the quasi-fixed input equals its marginal accumulation cost. The steady state net supplies of the variable net outputs can be obtained by substituting the steady state values of the quasi-fixed inputs into the restricted profit function.

Lucas shows that this model can be linked to the ad hoc partial adjustment literature (where actual stocks are adjusted part-way towards some optimal level each period) by the short-run demand for quasi-fixed inputs derived from (6.4) and (6.5) being an approximate solution to a linear differential equation system. In the case of one quasi-fixed input the differential equation system to which (6.4) and (6.5) are an approximate solution reduces to;

(6.6) $\dot{z} = B^{\sim}(z^{\sim}(t) - z(t))$

where $z^{\sim}$ is the steady state quasi-fixed input level and $B^{\sim}$ is an endogenous adjustment parameter given by;

(6.7) $B^{\sim} = -(1/2)\ [r - \{r^2 - 4H''(z^{\sim})/C''(0)\}^{1/2}].$

Since H is concave in z and $C''(0)>0$, $B^{\sim}$ must lie between zero and one so that the actual stock approaches the steady state stock monotonically. Further, as marginal adjustment costs increase rapidly then $B^{\sim}$ approaches zero and no adjustment occurs. The adjustment parameter is also affected by the curvature of the restricted profit function and the interest rate (a decrease in the interest rate acts to increase the adjustment speed). As a result, the adjustment parameter is determined within the economic model.

Before this model can be implemented empirically the zero depreciation assumption has to be relaxed, the units of the cost of adjustment function must be specified and functional forms have to be chosen for the restricted profit and adjustment cost functions. To simplify the presentation, it will be assumed that there is only one quasi-fixed input. To allow for non-zero depreciation it will be assumed that the quasi-fixed input depreciates exponentially at the rate d. We then have;

$$(6.8) \qquad \dot{z} = y - dz$$

where y is the gross addition to the stock of z. We now specify $C(\dot{z})$ as;

$$(6.9) \qquad C(\dot{z}) = qy + qD(\dot{z})$$

where q is the asset price of the quasi-fixed input.

Using (6.9) the cost of not changing the level of the quasi-fixed input is now the cost of depreciation and adjustment costs are specified in terms of the fixed input's asset price. The present value of net receipts function (6.2) now becomes;

$$(6.10) \quad V(0) = \int_0^{\infty} e^{-rt}\{H(t) - qdz - qD(\dot{z}) - q\dot{z}\}\, dt$$

noting that $y=\dot{z}+dz$. Integrating the last term in (6.10) by parts produces;

$$(6.11) \quad \int_0^{\infty} e^{-rt}q\dot{z}\, dt = \int_0^{\infty} e^{-rt}rqz\, dt - qz(0).$$

Substituting this in (6.10) yields;

$$(6.12) \quad qz(0) + V(0) = \int_0^{\infty} e^{-rt}\{H(t) - uz - qD(\dot{z})\}\, dt$$
$$= \int_0^{\infty} W(0)\, dt$$

where $u=q(r+d)$ is the user cost of the quasi-fixed input.

Maximising the present value of net receipts is now equivalent to maximising the right hand side of (6.12) since $qz(0)$ is an initial condition. The first order condition for this maximisation problem now becomes;

(6.13) $\quad dW(0)/dz - d\{dW(0)/d\dot{z}\}/dt = 0$

which is;

(6.14) $\quad H_z - u - rqD'(\dot{z}) + qD''(\dot{z})\ddot{z} = 0.$

The steady state solution must then satisfy;

(6.15) $\quad H_z(p,z) - u - rqD'(0) = 0.$

The adjustment parameter $B^{\sim}$ is now given by;

(6.16) $\quad B^{\sim} = -(1/2)\ [r - \{r^2 - 4H_{zz}(p,z)/(qD''(0))\}^{1/2}].$

This completes the theoretical external adjustment costs model and the next task is to specify an implementable version of the model.

## 6.2 An Econometric Adaptation

In this application the external adjustment costs model is used in conjunction with the four-net output, one-fixed input aggregate model of Chapter 3 to estimate short, intermediate and long-run responses of net outputs to changes in prices. The four net outputs are aggregate exports, aggregate imports, labour and domestic sales. Capital is again treated as the sole fixed input. The same assumptions regarding profit maximising firms, perfect competition in goods and factor markets, and the characteristics of the aggregate technology set are made. Imports are again assumed to be an input to the production sector while exports are an output of the production sector not consumed domestically. The same data set is used as in Chapter 3 except that the price of capital is now derived explicitly rather than as a residual. The capital asset price, user cost and quantity are described and listed in Appendix 1 along with the discount rate.

The aggregate technology is now represented by the following Generalised McFadden or biquadratic restricted profit function as specified by Diewert (1985);

$$(6.17) \quad H(p,K) = \Sigma_{i=1}^{4} b_i p_i + (1/2)\, \Sigma_{i=1}^{3}\Sigma_{j=1}^{3} b_{ij} p_i p_j / p_4 + \Sigma_{i=1}^{4} b_{iK} p_i K + \Sigma_{i=1}^{4} b_{it} p_i t + (1/2)\, b_{KK}(\Sigma_{n=1}^{4} p_n) K^2 + b_{Kt}(\Sigma_{n=1}^{4} p_n) Kt + (1/2)\, b_{tt}(\Sigma_{n=1}^{4} p_n) t^2$$

where the $b_{ij}$ parameters again satisfy the following symmetry restrictions;

$$(6.18) \quad b_{ij} = b_{ji} \qquad \text{for all } i,j = 1,2,3.$$

A time trend is again used to represent the technology index. It should be noted that this specification of the Generalised McFadden GNP function differs from those used earlier in this thesis in that constant returns to scale are not imposed with respect to the fixed capital input in order to obtain more information on the capital adjustment process and, following Diewert's (1985, p.90) suggestion, the weights on the prices in the second order capital and technology terms are set equal to the inverse of the base period prices.

By applying Hotelling's Lemma the following set of net output supply equations is obtained;

$$(6.19) \quad x_i = b_i + \Sigma_{j=1}^{3} b_{ij} p_j / p_4 + b_{iK} K + b_{it} t + (1/2)\, b_{KK} K^2 + b_{Kt} Kt + (1/2)\, b_{tt} t^2 \qquad ;\ i=1,2,3;$$

$$(6.20) \quad x_4 = b_4 - (1/2)\, \Sigma_{i=1}^{3}\Sigma_{j=1}^{3} b_{ij} p_i p_j / p_4 + b_{4K} K + b_{4t} t$$

$$+ (1/2)\ b_{KK}K^2 + b_{Kt}Kt + (1/2)\ b_{tt}t^2.$$

If the matrix of estimated quadratic terms $B=[b_{ij}]$ is positive semi-definite then the restricted profit function is globally convex in prices (Diewert 1985). If B is not positive semi-definite then it can again be reparameterised using the Wiley, Schmidt and Bramble technique of replacing B by the product of a lower triangular matrix A and its transpose;

(6.21) $B = AA'$ where $A = [a_{ij}]$ ; $i,j=1,2,3$ ; and $a_{ij}=0$ for $i<j$.

In this specification of the Generalised McFadden GNP function concavity in the quantity of the fixed capital input requires that $b_{KK}$ be negative. If the estimated $b_{KK}$ parameter is non-negative then concavity in the capital input quantity can be imposed by replacing $b_{KK}$ by the term $-a_{KK}^2$ where $a_{KK}$ is a parameter to be estimated.

To capture costs of adjustment a quadratic approximation is used;

(6.22) $D(\dot{K}) = (1/2)\ d_{KK}\dot{K}^2.$

Using (6.22) and the restricted profit function (6.17) the first order condition with respect to the fixed input (6.14) is;

(6.23) $\Sigma_{i=1}^{4}\ b_{iK}p_i + b_{KK}(\Sigma_{i=1}^{4}\ p_i)K + b_{Kt}(\Sigma_{i=1}^{4}\ p_i)t - u_K$

$- rq_K(d_{KK}\dot{K}) + q_K d_{KK}\ddot{K} = 0.$

From (6.15) the steady state level of the capital input is now;

(6.24) $K^{\sim} = -(\Sigma_{i=1}^{4}\ b_{iK}p_i + b_{Kt}(\Sigma_{i=1}^{4}\ p_i)t - u_K)/(b_{KK}\Sigma_{i=1}^{4}\ p_i).$

It is further assumed that current period production activity is affected by the capital stock existing at the beginning of the period and that changes made to the capital stock in the current period only affect next period's production. This enables the following discrete approximation to the flexible accelerator formulation to be used;

(6.25) $$K_t - K_{t-1} = B^{\sim}(K^{\sim}_t - K_{t-1})$$

where;

(6.26) $$B^{\sim} = -(1/2)\ [r - \{r^2 - 4b_{KK}\Sigma_{i=1}^{4}\ p_i/(q_K d_{KK})\}^{1/2}].$$

The complete estimating system is now given by the four net output equations and the capital adjustment equation;

(6.27) $$x_i = b_i + \Sigma_{j=1}^{3}\ b_{ij}p_j/p_4 + b_{iK}K + b_{it}t + (1/2)\ b_{KK}K^2 + b_{Kt}Kt + (1/2)\ b_{tt}t^2 + e_i \qquad ;\ i=1,2,3;$$

(6.28) $$x_4 = b_4 - (1/2)\ \Sigma_{i=1}^{3}\Sigma_{j=1}^{3}\ b_{ij}p_ip_j/p_4 + b_{iK}K + b_{it}t + (1/2)\ b_{KK}K^2 + b_{Kt}Kt + (1/2)\ b_{tt}t^2 + e_4$$

(6.29) $$K - K_{-1} = -(1/2)\ [r - \{r^2 - 4b_{KK}\Sigma_{i=1}^{4}\ p_i/(q_K d_{KK})\}^{1/2}][(\Sigma_{i=1}^{4}\ b_{iK}p_i + b_{Kt}(\Sigma_{i=1}^{4}\ p_i)t - u_K)/(b_{KK}\Sigma_{i=1}^{4}\ p_i) - K_{-1}] + e_K.$$

The vectors of error terms for the observations are again assumed to be independently distributed with a multivariate normal distribution with zero means and covariance matrix $\Omega$.

From the parameters of the estimating system a set of elasticities can be derived which completely describes the dynamic relationships within the estimated system. In the case of variable net outputs, elasticities which characterise the short, intermediate and long-run response of net output supply to the prices of net outputs are produced. The short-run response represents the response which occurs in less than one period to a price change when capital input levels have not changed. The intermediate-run response represents the response after one period has elapsed and the capital input has partially adjusted.

The long-run response represents the complete response after capital input levels have fully adjusted to their new steady state levels.

The short-run net output price elasticities are given by;

(6.30) $ES_{ij} = (d \log x_i/d \log p_j)|_{K=K_{-1}} = (p_j/x_i)(\partial x_i/\partial p_j)$.

In the Generalised McFadden case this produces;

(6.31) $ES_{ij} = (p_j/x_i)(b_{ij}/p_4)$ ; $i,j=1,2,3$;

(6.32) $ES_{i4} = (p_4/x_i)(-\Sigma_{j=1}^{3} b_{ji}p_j/p_4^2)$ ; $i=1,2,3$;

(6.33) $ES_{44} = (p_4/x_4)(\Sigma_{i=1}^{3}\Sigma_{j=1}^{3} b_{ij}p_ip_j/p_4^3)$.

The long-run net output price elasticities are given by;

(6.34) $EL_{ij} = (d \log x_i/d \log p_j)|_{K=K^{\sim}}$

$$= (p_j/x_i)[(\partial x_i/\partial p_j) + (\partial x_i/\partial K^{\sim})(\partial K^{\sim}/\partial p_j)]$$

This produces in the Generalised McFadden case;

(6.35) $EL_{ij} = ES_{ij} + [(b_{iK} + b_{KK}K + b_{Kt}t)(\Sigma_{n=1}^{4} b_{nK}p_n - u_K - b_{jK}\Sigma_{n=1}^{4} p_n)p_j / \{b_{KK}x_i (\Sigma_{n=1}^{4} p_n)^2\}]$ ; $i,j=1,..,4$.

The intermediate-run elasticities after one period's capital stock adjustment has taken place are given by;

(6.36) $EI_{ij} = ES_{ij} + [(b_{iK} + b_{KK}K + b_{Kt}t)(\Sigma_{n=1}^{4} b_{nK}p_n - u_K - b_{jK}\Sigma_{n=1}^{4} p_n)p_j/\{b_{KK}x_i(\Sigma_{n=1}^{4} p_n)^2\}]B1$ ; $i,j=1,..,4$.

where;

(6.37) $B1 = B^{\sim}(1 + dB^{\sim}/dp_j)$

$$= B^{\sim}[1 - b_{KK}/\{(r^2 - 4b_{KK}\Sigma_{i=1}^{4} p_i/(q_K d_{KK}))^{1/2} q_K d_{KK}\}]$$

It can be shown that the own-price elasticities satisfy the Le Chatelier principle if the restricted profit function is well behaved and the absolute value of the long-run elasticity will exceed that of the intermediate-run elasticity which in turn exceeds that of the short-run elasticity. With regard to cross-price elasticities the phenomenon of "overshooting" is allowed whereby short or intermediate-run cross-elasticities may exceed the magnitude of the corresponding long-run cross-elasticity. This may arise due to an output price increase. The response to this will be an increase in output supply but in the short-run this will have to be achieved by using some variable inputs more intensively due to the fixity of capital. In the long-run when the capital input has been increased the variable inputs may be used less intensively and so the long-run cross-elasticity may be less than the short-run cross-elasticity.

By definition all short-run elasticities involving either the price or quantity of capital are zero. However, intermediate and long-run elasticities between net output quantities and the user cost of capital can be obtained. The long-run elasticity between net output quantities and the user cost of capital is given by;

(6.38) $EL_{iu} = (u_K/x_i)(\partial x_i/\partial u_K)|_{K=K^{\sim}}$

$= (u_K/x_i)[(b_{iK} + b_{KK}K + b_{Kt}t)/(b_{KK}\Sigma_{j=1}^{4} p_j)]$ ; $i=1,.,4$.

The corresponding intermediate-run elasticity is given by;

(6.39) $EI_{iu} = B^{\sim}EL_{iu}$.

The long and intermediate-run response of capital input levels to changes in net output prices can also be obtained. The long-run cross-elasticity is given by;

(6.40) $EL_{Ki} = (p_i/K)(dK^{\sim}/dp_i)$ ; $i=1,..,4$;

$$= (p_i/K)[(\Sigma_{j=1}^{4} b_{jK}p_j - u_K - b_{iK}\Sigma_{j=1}^{4} p_j)/\{b_{KK}(\Sigma_{j=1}^{4} p_j)^2\}].$$

The corresponding intermediate-run elasticity is given by;

(6.41) $EI_{Ki} = B1.EL_{Ki}$ ; $i=1,..,4.$

Finally, elasticities can be derived which provide information on the response of profits to changes in scale and technology. The elasticity for returns to scale is;

(6.42) $E_{RTS} = [(dH(p,K)/dK).K]/H(p,K).$

This elasticity shows the percentage change in profits following a one percent change in the capital input. If its value is greater than one then there is increasing returns to scale.

The technical change elasticity is given by ;

(6.43) $E_{TC} = (dH(p,K)/dt)/H(p,K).$

If this elasticity is negative there is technical regress as an increase in technology acts to reduce profits.

From this set of elasticities it should be possible to gain a better understanding of the importance of allowing for the dynamics of the adjustment process within the GNP function model.

## 6.3 Results

Initial estimation of the system (6.27)-(6.29) produced results which failed to meet convexity of the restricted profit function in prices but which were concave in the quantity of the capital input. Subsequent estimation was therefore of the reparameterised model imposing price convexity using (6.21). The resulting maximum likelihood parameter estimates and their asymptotic t-values are presented in Table 6.1. Concavity of the restricted profit function in the quantity of capital input was again satisfied as indicated by the negative value of $b_{KK}$ which is significantly different from zero. The adjustment costs function parameter $d_{KK}$ is positive and significantly different from zero as required by economic theory. The estimated dependent variables track the actual dependent variable values well in all cases and the equation R-square values are reported in Table 6.1.

The values of $B^{\sim}$, the partial adjustment coefficient for the capital stock, produced by the estimated system are stable and range from 0.07 to 0.09, taking a value of 0.0727 in 1970. This means that the actual capital stock adjusts only seven to nine per cent towards its steady state value in one year. This implies relatively slow adjustment of the capital stock. As a result one would expect substantial differences between the estimated short-run and long-run own-price elasticities for net outputs which are either strongly substitutable or strongly complementary to capital. For net outputs which are relatively independent of capital (ie. those which are either weakly substitutable or weakly complementary with capital) the slow adjustment of capital need not imply substantial differences between short-run and long-run own-price elasticities.

Due to the large volume of output produced by this model, detailed elasticity estimates are presented only for three of the nineteen years for which output is obtained. Net output price elasticities for the year 1965 near the beginning of the time-series are presented in Table 6.2. Those for the year 1970 near the middle of the time-series are presented in Table 6.3 while those for 1978, a year near the end of the time-series, appear in Table 6.4.

The estimated short-run export own-price elasticities are of similar magnitude to those obtained in the aggregate model of Chapter 3. The significant difference is that in this case the export own-price elasticity tends to decrease over time. This is likely due to the fact that constant returns are not being imposed here and the dependent variable in the estimating equation is the gross export quantity rather than export quantity per unit of capital input quantity. This would appear to indicate that more confidence can be placed in mid-point estimates. The estimated long-run export own-price elasticities are somewhat larger than the short-run elasticities but not to the extent that may have been expected beforehand. For instance, in 1970 the long-run elasticity was 1.58 compared to a short-run elasticity of 1.51. This would appear to indicate that the relationship between export supply and capital is not strong. As expected from the small value of the estimated $B^{\sim}$ adjustment coefficient the intermediate-run elasticity is close to the short-run elasticity value.

The export cross-elasticities produce a more interesting set of results. While exports fall in response to an increase in the price of imports the fall is slightly less in the long-run than it is in the short-run indicating that some substitution away from imports and towards capital occurs in the long-run. This effect is most pronounced in the case of the labour cross-price elasticity. While exports fall in response to an increase in labour prices the reduction in the long-run is only one fifth what it is in the short-run indicating substantial substitution away from labour and towards capital. This trend is even evident in the intermediate-run and represents a classic case of "overshooting". It would also appear to indicate that the relationship between labour and capital is relatively strong. Cross-price elasticities with respect to the domestic sales price are small but negative (being near zero in the short-run) indicating that exports are reduced slightly following an increase in the domestic sales price. This is the only cross-price elasticity which differs in sign from the corresponding elasticities obtained in Chapter 3.

Import short-run own-price elasticities are also of similar magnitude to those obtained in Chapter 3. The elasticities tend to decline in magnitude during the first half of the period and then stabilise during the second half. There is very little difference between the estimated long-run and short-run import own-price elasticities indicating that imports and capital are almost independent. Again a larger difference between short-run and long-run elasticities is evident in the import cross-price elasticities. Some overshooting occurs in response to an export price increase with import usage increasing less in the intermediate and long-run than it does in the short-run. Following an increase in labour prices, however, import usage drops more in the long-run than it does in the short-run indicating further substitution away from imports as the capital stock increases. Import usage is increased more in the long-run than it is in the short-run following an increase in the domestic sales price.

The estimated short-run labour own-price elasticities are all relatively stable around the value of -1.0 which is near the mid-point value obtained in Chapter 3. As expected from the results above, the long-run labour own-price elasticity is substantially larger than the short-run elasticity, averaging around -1.50. This indicates that labour and capital are strongly substitutable with the usage of labour declining considerably more in the long-run following an increase in labour prices once the capital stock has been increased. Again considerable overshooting is evident in the usage of labour following an increase in export prices with the increase in labour use being approximately half in the long-run what it is in the short-run. Labour usage declines slightly more in the long- run than in the short-run following an increase in import prices. Labour usage increases considerably more in the long-run than it does in the short-run, however, following an increase in the domestic sales price. This would appear to indicate that domestic sales production is relatively intensive in its use of labour, a plausible result given the importance of service industries in domestic sales production.

The estimated domestic sales short-run own-price elasticity is somewhat higher than that obtained in Chapter 3 and stable around the value of 0.85. The long-run own-price elasticity is only slightly larger, again indicating a relatively weak relationship between domestic sales supply and capital. Domestic sales supply drops slightly following an increase in export prices but the cross elasticity values are close to zero in both the short and long-run. Domestic sales supply also falls to a small degree following an increase in import prices with the short and long-run cross elasticities being approximately equal. Larger falls in domestic sales supply occur following an increase in labour prices with the fall being larger in the long-run.

Intermediate and long-run cross-price elasticities between net output and capital prices and quantities are presented in Table 6.5. As expected from the small value of $B^{\sim}$, an increase in the user cost of capital has a negligible impact on net output supply in the intermediate-run. In the long-run, however, an increase in the user cost of capital acts to decrease the supply of exports and slightly increase the supply of domestic sales output, thus confirming that domestic sales output is relatively labour intensive while export supply is relatively capital intensive. Increases in the user cost act to increase the usage of the other two inputs, labour and imports.

The effect on the quantity of the capital stock of an increase in net output prices is also near zero in the intermediate-run due to the small value of the partial adjustment coefficient $B^{\sim}$. In the long-run, however, an increase in export prices increases the capital stock while an increase in the domestic sales price actually decreases the capital stock. As expected an increase in import prices has a negligible effect on the capital stock even in the long-run indicating that capital and imports are almost independent. The strongest relationship is between capital and labour with an increase in labour prices leading to a substantial increase in the capital stock in the long-run.

Finally, the returns to scale and technical change elasticities are presented in Table 6.6. The returns to scale elasticities are calculated in the neighbourhood of existing capital stock levels and indicate that there are increasing returns to scale and increase in value over time. The technical change elasticites, however, indicate that there is initially technical progress but then technical regress towards the end of the period. There, hence, appears to be some difficulty distinguishing the influences of returns to scale and technical change with the increasing value of the returns to scale elasticities capturing some of the effects likely due to technical change. This would account for the declining and negative values of the technical change elasticities towards the end of the period and the high values of the returns to scale elasticities. To the extent that there are increasing returns to scale, however, this represents a contradiction of the neoclassical assumptions of the model as a competitive equilibrium will not exist with increasing returns to scale.

## 6.4 Conclusions

The adjustment costs model presented in this Chapter has the advantage of incorporating the adjustment process of the quasi-fixed input capital as the solution to an explicit dynamic optimisation problem where the rate of adjustment of the quasi-fixed input is endogenously determined. Output feasibility is maintained while overshooting of variable input demands is allowed. The short, intermediate and long-run elasticities produced by the model help to increase our understanding of the dynamic adjustment process within the GNP function model.

While the results may indicate less divergence between short and long-run elasticities for the exports and imports of the traded sector they do point to other important areas of adjustment in the dynamic process. Exports turn out to be only weakly related to capital as is domestic sales supply while import demand is almost independent of capital. As a result the difference between short and long-run own-price elasticities is negligible for imports and small for exports and domestic sales supply.

The important effect of allowing for dynamics in this model is on input usage, particularly that of labour. While a short-run increase in export supply is brought about by using more labour in conjunction with the fixed capital input, in the long-run the capital stock is increased and substituted for the increased labour usage. This also happens to a lesser extent with import usage following an export price increase. Hence, while allowing for imperfect adjustment appears to make little difference to actual export supply it does make a significant difference to the input usage which goes into producing that export supply. Another aspect of the adjustment process highlighted by the dynamic model is the importance of the labour intensiveness of domestic sales supply. An increase in domestic sales prices actually acts to decrease the steady state capital stock and hence reduces export supply.

While the adjustment costs model may appear to downplay the importance of allowing for imperfect adjustment on export supply and import demand relative to the results of the planning price model of the preceding Chapter, it must be remembered that the two models have quite different assumptions and hence the results are not directly comparable. In the planning price model capital and labour are aggregated together and treated as a fixed input. Hence the major avenue of dynamic adjustment indicated by the adjustment costs model, namely the adjustment of labour input usage as the capital stock is increased, is precluded in the planning price model. The effect of this in the planning price model is highlighted by the much greater increase in import use following an export price increase, given that both labour and capital inputs are fixed for the duration of the simulation. Furthermore, the adjustment costs model views the shift of the production frontier as the only source of dynamics, while the planning price model considers adjustment along the production frontier following changes in net output prices. In principle, both forms of adjustment could be considered simultaneously. As a result more experimentation with different dynamic models and sets of assumptions will be necessary before the dynamic process is fully understood. To fully understand phenomena such as the J-curve effect it will be necessary to model imperfect adjustment on the demand side within a general equilibrium context and to allow for endogeneity with respect to import and export prices.

TABLE 6.1

DYNAMIC PARAMETER ESTIMATES[1]

| Coefficient | Estimate | t-value | Coefficient | Estimate | t-value |
|---|---|---|---|---|---|
| $b_X$ | -0.1251 | -0.1440 | $a_{ML}$ | -1.3082 | -4.7518 |
| $a_{XX}$ | -1.4685 | -4.7798 | $b_{MK}$ | 0.0527 | 1.4400 |
| $a_{XM}$ | 0.8951 | 5.7610 | $b_{Mt}$ | -1.0516 | -4.7532 |
| $a_{XL}$ | 0.3249 | 0.7640 | $b_L$ | -4.2531 | -4.4495 |
| $b_{XK}$ | 0.0758 | 1.2022 | $a_{LL}$ | -0.0000 | -0.0000 |
| $b_{Xt}$ | 0.4740 | 0.8415 | $b_{LK}$ | 0.1486 | 4.3780 |
| $b_{KK}$ | -0.0034 | -2.1381 | $b_{Lt}$ | -2.0457 | -8.2437 |
| $b_{Kt}$ | 0.0334 | 2.2309 | $b_D$ | 4.1866 | 4.2798 |
| $b_{tt}$ | -0.4001 | -2.4949 | $b_{DK}$ | -0.0175 | -0.2227 |
| $b_M$ | -0.9898 | -2.5164 | $b_{Dt}$ | 3.0128 | 5.0193 |
| $a_{MM}$ | -0.2811 | -0.8718 | $d_{KK}$ | 1.6831 | 2.3937 |

$R^2$ Values

| | | | |
|---|---|---|---|
| Equation X | 0.9795 | Equation D | 0.9842 |
| Equation M | 0.9851 | Equation K | 0.4667 |
| Equation L | 0.9701 | | |

Log Likelihood 141.80

[1] The subscripts and equation labels X, M, L, D, K and t refer to aggregate exports, aggregate imports, labour, domestic sales, capital and technology, respectively.

TABLE 6.2

1965 NET OUTPUT PRICE ELASTICITIES[1]

| Elasticity | Short-run | Intermediate-run | Long-run |
|---|---|---|---|
| $E_{XX}$ | 2.2607 | 2.2692 | 2.3737 |
| $E_{XM}$ | -1.6610 | -1.6584 | -1.6265 |
| $E_{XL}$ | -0.5535 | -0.5221 | -0.1342 |
| $E_{XD}$ | -0.0462 | -0.0634 | -0.2752 |
| $E_{MX}$ | 2.5377 | 2.5291 | 2.4236 |
| $E_{MM}$ | -2.0484 | -2.0511 | -2.0833 |
| $E_{ML}$ | -1.4070 | -1.4387 | -1.8306 |
| $E_{MD}$ | 0.9177 | 0.9351 | 1.1491 |
| $E_{LX}$ | 0.2206 | 0.2115 | 0.0998 |
| $E_{LM}$ | -0.3670 | -0.3698 | -0.4039 |
| $E_{LL}$ | -0.9296 | -0.9632 | -1.3778 |
| $E_{LD}$ | 1.0760 | 1.0944 | 1.3208 |
| $E_{DX}$ | -0.0114 | -0.0132 | -0.0346 |
| $E_{DM}$ | -0.1487 | -0.1492 | -0.1557 |
| $E_{DL}$ | -0.6682 | -0.6746 | -0.7542 |
| $E_{DD}$ | 0.8282 | 0.8318 | 0.8752 |

1 The subscripts X, M, L and D refer to aggregate exports, aggregate imports, labour and domestic sales, respectively.

TABLE 6.3

1970 NET OUTPUT PRICE ELASTICITIES[1]

| Elasticity | Short-run | Intermediate-run | Long-run |
|---|---|---|---|
| $E_{XX}$ | 1.5052 | 1.5105 | 1.5779 |
| $E_{XM}$ | -1.0609 | -1.0600 | -1.0479 |
| $E_{XL}$ | -0.4678 | -0.4404 | -0.0934 |
| $E_{XD}$ | 0.0236 | 0.0099 | -0.1625 |
| $E_{MX}$ | 1.6559 | 1.6502 | 1.5779 |
| $E_{MM}$ | -1.2823 | -1.2833 | -1.2962 |
| $E_{ML}$ | -1.1654 | -1.1948 | -1.5666 |
| $E_{MD}$ | 0.7918 | 0.8064 | 0.9911 |
| $E_{LX}$ | 0.1955 | 0.1884 | 0.0981 |
| $E_{LM}$ | -0.3121 | -0.3134 | -0.3296 |
| $E_{LL}$ | -1.0459 | -1.0826 | -1.5473 |
| $E_{LD}$ | 1.1624 | 1.1807 | 1.4116 |
| $E_{DX}$ | 0.0063 | 0.0053 | -0.0069 |
| $E_{DM}$ | -0.1357 | -0.1359 | -0.1381 |
| $E_{DL}$ | -0.7441 | -0.7491 | -0.8122 |
| $E_{DD}$ | 0.8735 | 0.8760 | 0.9073 |

1 The subscripts X, M, L and D refer to aggregate exports, aggregate imports, labour and domestic sales, respectively.

TABLE 6.4

1978 NET OUTPUT PRICE ELASTICITIES[1]

| Elasticity | Short-run | Intermediate-run | Long-run |
|---|---|---|---|
| $E_{XX}$ | 0.9846 | 0.9873 | 1.0238 |
| $E_{XM}$ | -0.6372 | -0.6371 | -0.6366 |
| $E_{XL}$ | -0.3216 | -0.3051 | -0.0840 |
| $E_{XD}$ | -0.0258 | -0.0336 | -0.1382 |
| $E_{MX}$ | 1.0752 | 1.0722 | 1.0327 |
| $E_{MM}$ | -0.7644 | -0.7645 | -0.7651 |
| $E_{ML}$ | -0.7953 | -0.8132 | -1.0531 |
| $E_{MD}$ | 0.4845 | 0.4930 | 0.6065 |
| $E_{LX}$ | 0.1823 | 0.1771 | 0.1087 |
| $E_{LM}$ | -0.2671 | -0.2672 | -0.2682 |
| $E_{LL}$ | -1.0247 | -1.0557 | -1.4707 |
| $E_{LD}$ | 1.1095 | 1.1242 | 1.3205 |
| $E_{DX}$ | -0.0096 | -0.0101 | -0.0179 |
| $E_{DM}$ | -0.1064 | -0.1064 | -0.1065 |
| $E_{DL}$ | -0.7255 | -0.7290 | -0.7758 |
| $E_{DD}$ | 0.8414 | 0.8431 | 0.8652 |

1 The subscripts X, M, L and D refer to aggregate exports, aggregate imports, labour and domestic sales, respectively.

TABLE 6.5

CAPITAL - NET OUTPUT CROSS ELASTICITIES[1]

| Year | Elasticity | | | |
|---|---|---|---|---|
| | $EI_{XU}$ | $EI_{MU}$ | $EI_{LU}$ | $EI_{DU}$ |
| 1965 | -0.0251 | 0.0254 | 0.0268 | 0.0051 |
| 1970 | -0.0199 | 0.0214 | 0.0267 | 0.0036 |
| 1978 | -0.0114 | 0.0124 | 0.0215 | 0.0024 |
| | $EL_{XU}$ | $EL_{MU}$ | $EL_{LU}$ | $EL_{DU}$ |
| 1965 | -0.3378 | 0.3413 | 0.3611 | 0.0693 |
| 1970 | -0.2742 | 0.2938 | 0.3672 | 0.0499 |
| 1978 | -0.1650 | 0.1791 | 0.3097 | 0.0349 |
| | $EI_{KX}$ | $EI_{KM}$ | $EI_{KL}$ | $EI_{KD}$ |
| 1965 | 0.0130 | 0.0040 | 0.0481 | -0.0263 |
| 1970 | 0.0083 | 0.0015 | 0.0425 | -0.0211 |
| 1978 | 0.0047 | 0.0001 | 0.0283 | -0.0134 |
| | $EL_{KX}$ | $EL_{KM}$ | $EL_{KL}$ | $EL_{KD}$ |
| 1965 | 0.1728 | 0.0528 | 0.6413 | -0.3502 |
| 1970 | 0.1127 | 0.0202 | 0.5798 | -0.2881 |
| 1978 | 0.0671 | 0.0010 | 0.4072 | -0.1926 |

[1] The subscripts X, M, L, D, U and K refer to aggregate exports, aggregate imports, labour, domestic sales, the user cost of capital and capital input quantity, respectively.

TABLE 6.6

SCALE AND TECHNICAL CHANGE ELASTICITIES

| Year | $E_{RTS}$ | $E_{TC}$ |
|---|---|---|
| 1962 | 1.1229 | 0.0926 |
| 1964 | 1.1866 | 0.0593 |
| 1966 | 1.2817 | 0.0380 |
| 1968 | 1.4217 | 0.0193 |
| 1970 | 1.5936 | 0.0044 |
| 1972 | 1.6490 | -0.0032 |
| 1974 | 1.8280 | -0.0123 |
| 1976 | 1.9710 | -0.0193 |
| 1978 | 1.9361 | -0.0192 |
| 1980 | 1.9495 | -0.0216 |

# 7. CONCLUSIONS AND FURTHER RESEARCH

The results presented in this thesis have extended our understanding of the roles of disaggregation and imperfect adjustment in the GNP function approach to measuring the responsiveness of export supply and import demand to price changes. They have also pointed to a number of directions in which further research needs to be undertaken. Combination of the aggregator function method of including several export and import categories with the use of the recently developed Generalised McFadden functional form which allows imposition of the correct curvature conditions has proved to be an effective means of obtaining detailed sets of elasticities characterising export supply and import demand responsiveness.

At the aggregate level export supply responsiveness was found to be of a similar magnitude to that found by Kohli (1978) in an earlier Canadian study, while import responsiveness was found to be somewhat higher in this study. Increases in the prices of both imports and labour were found to decrease the supply of exports while exports were found to be complementary with the output of domestic sales supply. The demand for labour was found to be more elastic than in most earlier studies and a general trend towards increasing price responsiveness within the Canadian economy was observed.

At the disaggregated level the own-price elasticities produced for the export and import components were generally stable and of reasonable magnitude. The disadvantage of the aggregator function approach, however, is that it relies on the restrictive assumption of separability of the production structure and all export and import components, respectively, were found to be complementary with each other subject to the fixed capital input available. Extension of modelling to larger disaggregated models which contained four export (import) components along with aggregate imports (exports) produced a different impression of the cross relationships between export (import) components with some substitution becoming evident. The larger disaggregated models tended, however, to produce less stable estimates of the component own-price elasticities in some instances.

The major conclusion, then, regarding disaggregation is that the aggregator function approach, when combined with flexible functional forms which permit curvature imposition, is an effective means of obtaining information on the responsiveness of export and import components to own-price changes. The aggregator function approach appears to be less suited to providing information on cross relationships among components. Larger disaggregated models, on the other hand, appear to have a relative advantage in providing information on cross relationships.

Combining the planning price method of allowing for imperfect adjustment with the GNP function framework produced results which indicated that imperfect adjustment was particularly important in the traded goods sector with exports in particular taking an extended period to fully adjust to price changes. Extension of modelling to a more sophisticated adjustment costs model, however, produced a different impression of the adjustment process. Both export supply and import demand responses were found to be little different in the long-run to those of the short-run. Important differences were found, however, in the composition of input usage over time with a strong substitutability between capital and labour in the long-run. Since capital was fixed in the short-run but variable in the long-run, while labour was variable in the short-run, the adjustment costs model indicated considerable overshooting with respect to labour demand. This avenue of adjustment was not available in the planning price model where both labour and capital were treated as fixed in both the short and long-run.

The major conclusions, then, from these results are that it is important to allow for imperfect adjustment to gain a fuller understanding of export supply and import demand response but that the results obtained are likely to be sensitive to the assumptions made. Consequently, more experimentation with different models and sets of assumptions will be necessary before the dynamic process is fully understood.

## 7.1 Further Research

At the most general level, an important area which warrants further research is extension of the GNP function framework to make it more general equilibrium in nature. This would involve explicit modelling of the domestic consumption sector and allowance for exchange rate adjustment to maintain balance of payments equilibrium. The work of Clements (1980) represents a promising start in this direction.

With regard to the specific results of this thesis an obvious area for further research is experimentation with different functional forms to assess the robustness of the results. This would be particularly interesting in examining whether the trend towards increasing price responsiveness found in the aggregator function and disaggregated models is replicated with other flexible forms permitting curvature imposition such as the Generalised Barnett (Diewert and Wales 1987). Extension of the work to different data sets may permit successful estimation of semi-flexible functional forms as proposed by Diewert and Wales (1986). This procedure would have the advantage of including all the variable net output components while not relying on the separability assumption.

An important area of input adjustment which has received little attention is allowance for variable capacity utilisation rates of the fixed inputs. In all the models presented here the capital stock is assumed to be utilised at a constant rate. Although the adjustment costs model attempts to allow for adjustment of the capital stock over time within an optimising framework no allowance is made for changes in capacity utilisation. In reality changes in capacity utilisation of capital and other quasi-fixed inputs such as some labour types will represent an important response to output fluctuations and it is correspondingly important that they be modelled in applied work. One approach to allowing for quasi-fixity and variable utilisation is that of Helliwell and Chung (1986). Epstein and Denny (1980) attempt to endogenise utilisation and depreciation decisions as solutions to optimising problems.

Recent advances in industrial organisation theory have increased interest in relaxing the perfect competition assumption present in most applied work. A relatively straight-forward extension of the material in this thesis would involve testing the validity of the perfect competition price-taking assumption along the lines of Appelbaum (1979).

Many avenues exist for extending the material presented in this thesis on imperfect adjustment. A simple first step might be extension of the planning price model to include labour as a variable net output, leaving capital as the sole fixed input. This would go some way towards making the model more comparable to the adjustment costs model although as currently specified the fixed input remains fixed in the long-run in the planning price model. More information on the fixed input could be obtained by explicitly estimating a "planning shadow price" equation for the fixed input.

A number of variations can be made to the adjustment costs model presented in Chapter 6. Adjustment costs can be made internal in the sense that a change in the stock of the quasi-fixed input affects current production as well as future production. The model could be extended along the lines of Morrison and Berndt (1981) to include two quasi-fixed factors. The assumption of static price expectations could be relaxed and a version consistent with rational expectations estimated along the lines of Morrison (1985).

There are also alternative models of the dynamic adjustment process which should be investigated. A framework for testing restrictions within a flexible dynamic system is developed by Anderson and Blundell (1983). An alternative model of the dynamic adjustment process derived explicitly from the solution to an optimisation problem which is also consistent with rational expectations is that of Pindyck and Rotemberg (1983a, 1983b). Further experimentation with these and other models will add to our understanding of the dynamic adjustment process and its impact on export supply and import demand.

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# APPENDIX 1

## DATA

The data used in this study are derived from a 20 year time series of annual input-output data made available by Statistics Canada to UBC in 1984. The data consist of current and constant dollar series for 37 industrial classifications. The methodology used by Statistics Canada in preparing its input-output data is described in detail in Dominion Bureau of Statistics (1969) and Statistics Canada (1983). Lal (1982) reviews some of the methodological problems encountered in reconciling input-output data with the National Accounts data and compares the Canadian input-output tables to those of other countries.

For each industry data is available for 8 primary inputs, 36 interindustry inputs and 2 outputs. The primary inputs consist of competitive and non-competitive imports, inputs purchased from Crown corporations and government bodies, and 5 durable inputs; inventories of raw materials, inventories of finished goods, machinery and equipment capital, construction capital and land. Outputs of each industry are classified according to end use, either domestic sales or exports. In addition to these variables, several financial variables are also available for each industrial classification. To more closely approximate the actual prices faced by producers these financial variables were distributed to other input and output categories. Given the lack of detail available on the financial variables this procedure often involved relatively ad hoc methods. For instance, commodity indirect taxes were distributed across the 36 intermediates and 2 import components proportionate to each commodity's share of total intermediate input value. Other indirect taxes were analogously distributed across construction capital and land while subsidies and royalties were, respectively, added to and subtracted from domestic output.

The constant dollar series are available for the periods 1961 to 1971 using 1961 as the base year and 1971 to 1980 using 1971 as the base year. The two series were spliced using the overlapping year 1971 and based in 1961. A price series was obtained by dividing the nominal dollar series by the constant dollar series. The constant dollar series then serves as an implicit quantity index. The resulting industry data is listed and its construction described in Ostensoe (1986). Other recent applications using the data set are Cas, Diewert and Ostensoe (1986) and Diewert and Ostensoe (1986).

In this study the data were aggregated over the 37 industries by the use of Divisia indices in the SHAZAM package. The discrete Divisia index procedure has the advantage that it is superlative, being exact for the flexible translog aggregator function (Diewert 1976). The domestic sales, aggregate export and labour hours variables were taken directly from the input-output data aggregated to the economy level. The aggregate import variable was obtained by aggregating the competitive and non-competitive import categories, the distinction being considered neither reliable nor useful. Finally, an aggregate capital stock quantity series was obtained by aggregating the 5 durable input categories listed earlier for the 37 industries using capital stock prices as weights. Since constant returns to scale are imposed in this study the user cost of capital was derived as a residual to equate the values of outputs and inputs. After adding the values of domestic sales and exports and subtracting the values of labour and imports, the resulting residual value was divided by the capital quantity series to obtain a price index for the user cost of capital. Subsequent rescaling made the value of the price index 1.0 in 1961 and price times quantity equal to the residual value. The price and quantity series used for the four net outputs (aggregate exports, aggregate imports, domestic sales and labour) and the fixed input capital are listed in Tables A1.1 and A1.2, respectively.

The technology index used throughout this thesis was a time trend ranging from a value of 0.1 in 1961 to 2.0 in 1980. This scaling was chosen so that the squared value of the index was of the same maximum order of magnitude as the price index series.

The four export and import component categories were obtained by aggregating the exports and imports of industries which had similar export and import price patterns over the 20 year period. Some effort was also made to keep similar industries together. The composition of the four export component groups by input-output industries (and corresponding I-O industry numbers) is as follows;

Export Group 1 : Agricultural and Forestry Products

1) Agriculture and Fishing
2) Forestry
4) Food and Beverages
7) Leather
11) Woods
13) Paper and Allied

Export Group 2 : Minerals and Energy Products

| | |
|---|---|
| 3) Mines, Quarries and Oil Wells | 21) Petroleum and Coal |
| 15) Primary Metals | 33) Electric Power |

Export Group 3 : Motor Vehicles, Textile and Electrical Products

| | |
|---|---|
| 5) Tobacco | 14) Printing and Publishing |
| 6) Rubber and Plastic | 18) Transport Equipment |
| 8) Textiles | 19) Electrical Equipment |
| 9) Knitting Mills | 26) Railway Transport |
| 10) Clothing | 32) Telephones |
| 12) Furniture and Fixtures | 34) Gas Distribution |

Export Group 4 : Heavy Industrial and Service Products

| | |
|---|---|
| 16) Metal Fabricating | 28) Motor Transport |
| 17) Machinery | 29) Urban Transport |
| 20) Non-metallic Minerals | 30) Storage |
| 22) Chemicals | 31) Broadcasting |
| 23) Miscellaneous Manufacturing | 35) Trade |
| 25) Air Transport | 36) Finance, Insur. & Realty |
| 27) Water transport | 37) Commercial Services. |

The price and implicit quantity series for the four export components are listed in Tables A1.3 and A1.4, respectively.

The I-O industrial composition of the four import components is as follows;

Import Group 1 : Agricultural, Forestry and Service Products

| | |
|---|---|
| 1) Agriculture and Fishing | 26) Rail Transport |
| 2) Forestry | 27) Water Transport |
| 4) Food and Beverages | 28) Motor Transport |
| 7) Leather | 29) Urban Transport |

11) Woods
13) Paper and Allied
14) Printing and Publishing
24) Construction
25) Air Transport
30) Storage
35) Trade
36) Finance, Insur. & Realty
37) Commercial Services

<u>Import Group 2 : Metals and Energy Products</u>

3) Mines, Quarries and Oil Wells
15) Primary Metals
16) Metal Fabricating
21) Petroleum and Coal
33) Electric Power
34) Gas Distribution

<u>Import Group 3 : Machinery, Electrical and Textile Products</u>

5) Tobacco
6) Rubber and Plastics
8) Textiles
9) Knitting Mills
10) Clothing
12) Furniture and Fixtures
17) Machinery
19) Electrical Equipment
31) Broadcasting
32) Telephones

<u>Import Group 4 : Vehicles, Chemicals and Other Products</u>

18) Transport Equipment
20) Non-metallic Minerals
22) Chemicals
23) Misc. Manufacturing

The price and implicit quantity series for the four import components are listed in Tables A1.5 and A1.6, respectively.

In the adjustment costs model of Chapter 6 constant returns to scale with respect to capital are not imposed. As a result an explicit asset price and user cost for capital have to be derived. In this case the asset price of capital was derived directly from the input-output data. The asset prices for the five durable inputs were aggregated using a Divisia index to form an aggregate capital asset price.

Derivation of a user cost for capital presents more problems. Using a nominal discount rate tends to produce rapidly increasing user cost series when capital gains are not allowed for while using a real discount rate produces negative user costs in more recent years. To overcome this problem the same ad hoc approach as used by Ostensoe (1986) was used whereby a weighted average of the nominal and real discount rates is used along with appropriate depreciation rates to produce a relatively stable, non-negative user cost series. The nominal interest rate was taken as the unweighted average of the 90 day corporate paper rate taken at monthly intervals. The real interest rate was constructed by subtracting the percentage change in the consumer price index from the nominal interest rate. The discount rate used in deriving the user cost of capital was obtained by placing a weight of 0.68 on the real interest rate and 0.32 on the nominal interest rate. The disaggregated data from which the user cost series was obtained are listed in detail in Ostensoe (1986). For consistency, the same weighted average discount rate was used in the estimating system of Chapter 6. The asset price, user cost and quantity series for capital are presented in Table A1.7 along with the weighted average discount rate.

TABLE A1.1

AGGREGATE PRICE INDICES

| Year | Exports | Imports | Dom. Sales | Labour | Capital |
|---|---|---|---|---|---|
| 1961 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 1962 | 1.0204 | 1.1960 | 1.0071 | 1.0350 | 1.0667 |
| 1963 | 1.0239 | 1.2399 | 1.0250 | 1.0747 | 1.1129 |
| 1964 | 1.0340 | 1.2723 | 1.0370 | 1.1101 | 1.1654 |
| 1965 | 1.0559 | 1.2728 | 1.0603 | 1.1684 | 1.2191 |
| 1966 | 1.0853 | 1.2957 | 1.1054 | 1.2664 | 1.2711 |
| 1967 | 1.1078 | 1.3132 | 1.1437 | 1.3564 | 1.2405 |
| 1968 | 1.1319 | 1.3150 | 1.1730 | 1.4581 | 1.3114 |
| 1969 | 1.1653 | 1.3447 | 1.2216 | 1.5748 | 1.3680 |
| 1970 | 1.2053 | 1.3939 | 1.2759 | 1.6931 | 1.3510 |
| 1971 | 1.2230 | 1.2108 | 1.3343 | 1.8360 | 1.4353 |
| 1972 | 1.2751 | 1.2494 | 1.3961 | 1.9738 | 1.5392 |
| 1973 | 1.4167 | 1.3670 | 1.5140 | 2.1892 | 1.7841 |
| 1974 | 1.7102 | 1.8005 | 1.7390 | 2.4937 | 1.9934 |
| 1975 | 1.9456 | 2.0591 | 1.9653 | 2.8539 | 2.1016 |
| 1976 | 2.0819 | 2.1346 | 2.1110 | 3.2378 | 2.2778 |
| 1977 | 2.2671 | 2.3861 | 2.2532 | 3.4999 | 2.3690 |
| 1978 | 2.5106 | 2.6657 | 2.4130 | 3.7063 | 2.5881 |
| 1979 | 2.8651 | 3.0923 | 2.6024 | 3.9336 | 2.9382 |
| 1980 | 3.2431 | 3.7384 | 2.9404 | 4.4886 | 3.1815 |

TABLE A1.2

AGGREGATE QUANTITIES IN MILLIONS OF 1961 DOLLARS

| Year | Exports | Imports | Dom. Sales | Labour | Capital |
|---|---|---|---|---|---|
| 1961 | 6445.8 | 4272.8 | 29988.3 | 18747.4 | 13414.0 |
| 1962 | 6840.8 | 3929.6 | 32298.7 | 19403.0 | 13806.3 |
| 1963 | 7550.8 | 4117.6 | 33783.9 | 19811.5 | 14345.1 |
| 1964 | 8664.6 | 4581.1 | 35865.0 | 20604.1 | 14971.4 |
| 1965 | 8989.3 | 5038.3 | 39067.5 | 21721.4 | 15686.2 |
| 1966 | 10251.3 | 5607.1 | 41243.2 | 22439.1 | 16547.3 |
| 1967 | 11047.8 | 5777.0 | 41578.5 | 22568.9 | 17407.0 |
| 1968 | 12866.7 | 6416.4 | 42976.8 | 22459.5 | 18139.0 |
| 1969 | 13743.4 | 7097.8 | 45548.1 | 22984.0 | 18943.8 |
| 1970 | 14960.6 | 6939.6 | 44712.8 | 22782.9 | 19862.8 |
| 1971 | 15672.1 | 8923.1 | 47519.2 | 23056.7 | 20507.3 |
| 1972 | 16891.3 | 10010.0 | 50745.1 | 23808.4 | 21365.4 |
| 1973 | 19083.3 | 11191.2 | 54886.0 | 25095.6 | 22360.1 |
| 1974 | 19287.3 | 11883.3 | 57993.5 | 26230.5 | 23591.0 |
| 1975 | 17699.2 | 11169.1 | 58963.9 | 26236.0 | 24954.1 |
| 1976 | 19292.2 | 11688.1 | 61903.0 | 26563.6 | 26291.6 |
| 1977 | 20802.9 | 11825.6 | 62552.0 | 26848.2 | 27827.7 |
| 1978 | 22603.7 | 12499.1 | 64277.8 | 27685.4 | 29333.9 |
| 1979 | 23949.6 | 13296.2 | 67519.4 | 28674.0 | 30773.8 |
| 1980 | 25055.1 | 12613.9 | 67671.5 | 29153.3 | 32132.0 |

## TABLE A1.3

## EXPORT COMPONENT PRICE INDICES

| Year | Agricultural & Forestry Products | Minerals & Energy Products | MVs, Textiles & Electrical Products | Heavy Ind. & Services Products |
|---|---|---|---|---|
| 1961 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 1962 | 1.0347 | 1.0212 | 0.9821 | 1.0099 |
| 1963 | 1.0458 | 1.0228 | 0.9683 | 1.0111 |
| 1964 | 1.0548 | 1.0356 | 0.9613 | 1.0324 |
| 1965 | 1.0764 | 1.0685 | 0.9645 | 1.0526 |
| 1966 | 1.1200 | 1.1013 | 0.9750 | 1.0667 |
| 1967 | 1.1420 | 1.1275 | 0.9852 | 1.0989 |
| 1968 | 1.1718 | 1.1480 | 1.0006 | 1.1308 |
| 1969 | 1.2218 | 1.1759 | 1.0236 | 1.1623 |
| 1970 | 1.2336 | 1.2583 | 1.0509 | 1.2042 |
| 1971 | 1.2518 | 1.2418 | 1.0764 | 1.2482 |
| 1972 | 1.3629 | 1.2697 | 1.0996 | 1.2950 |
| 1973 | 1.6445 | 1.4499 | 1.1361 | 1.3765 |
| 1974 | 1.9986 | 1.9430 | 1.2625 | 1.6014 |
| 1975 | 2.1906 | 2.3470 | 1.4058 | 1.8181 |
| 1976 | 2.2353 | 2.6105 | 1.5081 | 1.9618 |
| 1977 | 2.3731 | 2.9656 | 1.6420 | 2.0917 |
| 1978 | 2.6363 | 3.3414 | 1.8140 | 2.2630 |
| 1979 | 3.0640 | 3.9620 | 2.0077 | 2.5050 |
| 1980 | 3.3090 | 4.7162 | 2.2520 | 2.8385 |

TABLE A1.4

EXPORT COMPONENT QUANTITIES IN MILLIONS OF 1961 DOLLARS

| Year | Agricultural & Forestry Products | Minerals & Energy Products | MVs, Textiles & Electrical Products | Heavy Ind. & Services Products |
|---|---|---|---|---|
| 1961 | 2763.9 | 1853.1 | 687.8 | 1141.0 |
| 1962 | 2767.9 | 2053.5 | 831.1 | 1191.3 |
| 1963 | 3074.8 | 2130.5 | 968.5 | 1383.2 |
| 1964 | 3502.0 | 2379.5 | 1215.0 | 1582.2 |
| 1965 | 3455.6 | 2477.3 | 1376.4 | 1707.4 |
| 1966 | 3700.7 | 2567.8 | 2127.4 | 1948.9 |
| 1967 | 3320.9 | 2802.2 | 3037.9 | 2087.3 |
| 1968 | 3448.6 | 3227.6 | 4230.8 | 2285.3 |
| 1969 | 3471.1 | 3129.7 | 4983.0 | 2575.4 |
| 1970 | 3788.9 | 3716.3 | 5075.1 | 2780.7 |
| 1971 | 4072.4 | 3617.7 | 5490.1 | 2938.4 |
| 1972 | 4306.0 | 3880.6 | 5914.6 | 3273.0 |
| 1973 | 4748.4 | 4572.1 | 6803.6 | 3536.2 |
| 1974 | 4834.7 | 4615.0 | 6594.8 | 3765.5 |
| 1975 | 4187.7 | 4034.8 | 6607.2 | 3577.6 |
| 1976 | 4789.9 | 3980.2 | 7614.8 | 3864.9 |
| 1977 | 5343.8 | 4025.5 | 8365.9 | 4209.8 |
| 1978 | 5726.4 | 4135.5 | 9267.5 | 4870.5 |
| 1979 | 5955.1 | 4642.0 | 8836.5 | 5684.3 |
| 1980 | 6385.3 | 5209.8 | 8126.7 | 6079.3 |

TABLE A1.5

IMPORT COMPONENT PRICE INDICES

| Year | Ag., Forestry & Service Products | Metals & Energy Products | Machinery,Elec. & Textile Products | Vehicles, Chem. &Other Products |
|---|---|---|---|---|
| 1961 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 1962 | 1.3568 | 1.1344 | 1.0708 | 1.0642 |
| 1963 | 1.4575 | 1.1375 | 1.0836 | 1.0872 |
| 1964 | 1.5280 | 1.1587 | 1.0834 | 1.1017 |
| 1965 | 1.5174 | 1.1678 | 1.0835 | 1.1093 |
| 1966 | 1.5504 | 1.1940 | 1.0837 | 1.1332 |
| 1967 | 1.5516 | 1.2223 | 1.0915 | 1.1685 |
| 1968 | 1.5457 | 1.2270 | 1.0903 | 1.1783 |
| 1969 | 1.5768 | 1.2563 | 1.1103 | 1.2102 |
| 1970 | 1.6638 | 1.3221 | 1.1235 | 1.2316 |
| 1971 | 1.2443 | 1.2147 | 1.0643 | 1.1901 |
| 1972 | 1.3027 | 1.2526 | 1.0754 | 1.2208 |
| 1973 | 1.4509 | 1.3997 | 1.1803 | 1.2847 |
| 1974 | 1.7726 | 2.5983 | 1.3946 | 1.4764 |
| 1975 | 1.9386 | 3.1224 | 1.5062 | 1.7539 |
| 1976 | 1.9853 | 3.2792 | 1.5800 | 1.8102 |
| 1977 | 2.2172 | 3.6450 | 1.7327 | 2.0544 |
| 1978 | 2.4396 | 3.9983 | 1.9135 | 2.3807 |
| 1979 | 2.7455 | 5.0706 | 2.1486 | 2.6856 |
| 1980 | 3.0906 | 7.2636 | 2.3489 | 3.0792 |

TABLE A1.6

IMPORT COMPONENT QUANTITIES IN MILLIONS OF 1961 DOLLARS

| Year | Ag., Forestry & Service Products | Metals & Energy Products | Machinery, Elec. & Textile Products | Vehicles, Chem. &Other Products |
|---|---|---|---|---|
| 1961 | 1821.8 | 923.9 | 773.7 | 753.5 |
| 1962 | 1431.3 | 885.4 | 777.0 | 865.9 |
| 1963 | 1426.1 | 958.0 | 832.5 | 952.1 |
| 1964 | 1548.8 | 1066.3 | 948.9 | 1087.9 |
| 1965 | 1655.8 | 1140.5 | 1039.1 | 1300.7 |
| 1966 | 1832.5 | 1196.0 | 1202.2 | 1494.5 |
| 1967 | 1915.7 | 1158.8 | 1214.4 | 1602.3 |
| 1968 | 1963.7 | 1286.9 | 1260.4 | 2078.5 |
| 1969 | 2175.4 | 1352.4 | 1448.2 | 2319.4 |
| 1970 | 2139.7 | 1416.5 | 1404.3 | 2162.0 |
| 1971 | 3147.8 | 1707.7 | 1649.9 | 2568.4 |
| 1972 | 3501.9 | 1839.6 | 1954.3 | 2898.7 |
| 1973 | 3831.3 | 2100.7 | 2118.8 | 3345.5 |
| 1974 | 4124.6 | 2162.1 | 2262.1 | 3598.3 |
| 1975 | 3901.7 | 2058.1 | 2053.7 | 3372.7 |
| 1976 | 4139.1 | 1958.6 | 2165.8 | 3804.6 |
| 1977 | 4193.2 | 1899.6 | 2154.3 | 4022.4 |
| 1978 | 4455.7 | 1956.5 | 2364.8 | 4242.8 |
| 1979 | 4773.7 | 2160.2 | 2755.9 | 4146.1 |
| 1980 | 4798.2 | 2069.4 | 2736.8 | 3528.8 |

TABLE A1.7

ADJUSTMENT COSTS MODEL CAPITAL DATA

| Year | Asset Price | User Cost | Capital Quantity[1] | Discount Rate |
|---|---|---|---|---|
| 1961 | 1.0000 | 0.0998 | 10.3960 | 0.0510 |
| 1962 | 1.0152 | 0.1012 | 10.7000 | 0.0501 |
| 1963 | 1.0395 | 0.0956 | 11.1176 | 0.0414 |
| 1964 | 1.0731 | 0.0977 | 11.6030 | 0.0405 |
| 1965 | 1.1250 | 0.0975 | 12.1570 | 0.0361 |
| 1966 | 1.1858 | 0.0977 | 12.8243 | 0.0321 |
| 1967 | 1.2299 | 0.1009 | 13.4906 | 0.0320 |
| 1968 | 1.2478 | 0.1186 | 14.0579 | 0.0458 |
| 1969 | 1.3084 | 0.1278 | 14.6816 | 0.0486 |
| 1970 | 1.3582 | 0.1240 | 15.3939 | 0.0414 |
| 1971 | 1.4393 | 0.1073 | 15.8934 | 0.0240 |
| 1972 | 1.5330 | 0.1033 | 16.5584 | 0.0170 |
| 1973 | 1.7104 | 0.1039 | 17.3293 | 0.0126 |
| 1974 | 2.0089 | 0.0950 | 18.2833 | 0.0011 |
| 1975 | 2.2797 | 0.1171 | 19.3397 | 0.0059 |
| 1976 | 2.5141 | 0.1747 | 20.3762 | 0.0264 |
| 1977 | 2.7302 | 0.1853 | 21.5668 | 0.0245 |
| 1978 | 2.9436 | 0.2474 | 22.7341 | 0.0427 |
| 1979 | 3.2096 | 0.2902 | 23.8500 | 0.0507 |
| 1980 | 3.4800 | 0.3251 | 24.9026 | 0.0540 |

[1] In tens of billions of 1961 dollars.

# APPENDIX 2

# PRIMAL VERSUS DUAL ESTIMATION

## A2.1. Introduction

Applied researchers are interested in modelling the production structure of industries and economies as an input to larger econometric models and to simple policy analyses. The characteristics of the production structure are usually summarised by estimates of the elasticities of substitution and price elasticities of demand between the various inputs.

Advances in the field of flexible functional forms have made available flexible forms such as the translog and Generalised Leontief (GL) which are able to model the substitution possibilities between inputs much more accurately than tradidional forms such as the Cobb-Douglas and Constant Elasticities of Substitution models. In the case of the Cobb-Douglas form all elasticities of substitution are restricted to the value one. The CES form restricts the elasticities between all pairs of factors to be equal - a significant disadvantage when there are more than two inputs. The flexible functional forms, on the other hand, are able to approximate an arbitrary cost function up to the second-order terms and hence to approximate an arbitrary matrix of substitution elasticities.

The Cobb-Douglas and CES forms are, however, self-dual which means that both the production function and the cost function are members of the same family of functional forms. Hence the choice of whether to model the production structure by the primal (production function) or dual (cost function) route is in theory of no significance. In practice, however, differences in estimated elasticities would be observed due to differences in the behavioural implications of the stochastic specification (Burgess 1975). In the case of the flexible functional forms mentioned, however, the choice between primal and dual estimation routes is no longer trivial since these functional forms are not self-dual, ie. a translog cost function does not have as its equivalent in the primal representation a translog production function. Hence the choice between primal and dual forms will not only imply differences in the elasticity estimates due to different stochastic specifications but also due to different underlying production structures being modelled.

In spite of the different implications of choosing to derive elasticity estimates from the primal or dual form, which approach is adopted appears to depend primarily on the biases of the individual researcher. Some authors consistently use the dual approach while others remain with the traditional primal approach. Little work has been done to compare the performance and magnitude of elasticity estimates from primal and dual flexible functional forms. One exception is that of Burgess (1975) where translog production and cost function elasticities were derived for the same data and found to give significantly different information regarding substitution possibilities.

Fisher and Chung (1986) have recently calculated elasticity estimates from translog and Generalised Leontief production functions for three inputs (capital, labour and energy) using aggregate Canadian data for the period 1954 to 1982. The data is that used in the MACE macro econometric model (Helliwell, MacGregor and Padmore, 1984). This work is extended in this Appendix where elasticity estimates are derived from translog and GL cost function models using the same data and compared with those derived by Fisher and Chung from the primal specification. The cost function estimation is extended to the recently developed Symmetric Generalised McFadden (SGM) functional form developed by Diewert and Wales (1987). This form has significant advantages over earlier flexible functional forms in regard to satisfying global curvature conditions. Elasticity estimates are found to be quite sensitive to the specification used.

The methodology used in the comparison of primal and dual estimation is outlined in the following section while the results are presented in the third section. Finally, conclusions are drawn in the fourth section.

## A2.2. Methodology

The translog and GL production function estimation is summarised below. More details can be found in Fisher and Chung (1986). A detailed discussion of the properties of the translog, GL and SGM cost functions can be found in Diewert and Wales (1987).

### A2.2.1 The Translog Form

The 3-factor [capital (K), labour (L) and energy (E)] translog production function is given by;

$$\begin{aligned}
\ln Q = {} & a_Q + a_K\ln K + a_L\ln L + a_E\ln E + a_t t + (1/2)a_{KK}(\ln K)^2 \\
& + (1/2)a_{KL}\ln K\ln L + a_{KE}\ln K\ln E + (1/2)a_{LL}(\ln L)^2 + \\
& a_{LE}\ln L\ln E + (1/2)a_{EE}(\ln E)^2 + (1/2)a_{tt}t^2 + a_{Kt}t\ln K + \\
& a_{Lt}t\ln L + a_{Et}t\ln E
\end{aligned} \tag{A2.1}$$

where Q is output, T is a time trend and symmetry has been imposed. The translog production function exhibits constant returns to scale if:

$$\begin{aligned}
& a_K + a_L + a_E = 1 \\
& a_{KK} + a_{KL} + a_{KE} = 0 \\
& a_{KL} + a_{LL} + a_{LE} = 0 \\
& a_{KE} + a_{LE} + a_{EE} = 0 \\
& a_{Kt} + a_{Lt} + a_{Et} = 0
\end{aligned} \tag{A2.2}$$

If the test for constant returns to scale (A2.2) is accepted and profit maximisation is assumed then the production function (A2.1) can be estimated along with the following cost share equations:

$$\begin{aligned}
S_K &= a_K + a_{KK}\ln K + a_{KL}\ln L + a_{KE}\ln E + a_{Kt}t \\
S_L &= a_L + a_{KL}\ln K + a_{LL}\ln L + a_{LE}\ln E + a_{Lt}t
\end{aligned} \tag{A2.3}$$

The third share equation is excluded since the three factor shares must sum to unity. The test for neutral technical change is:

$$a_{Kt} + a_{Lt} + a_{Et} = 0 \tag{A2.4}$$

Elasticities of substitution are derived from the bordered Hessian of the production function, G, as follows:

$$ES_{ij} = |G_{ij}|/|G| \tag{A2.5}$$

where $|G_{ij}|$ is the i,j cofactor of G. Own price elasticities of demand are derived as follows:

(A2.6) $ED_{ij} = S_j ES_{ij}$

The translog cost function may be represented as follows:

$$\begin{aligned} \ln C(p,Q,t) = {} & a_0 + \Sigma_{i=1}^{3} a_i \ln p_i + a_Q \ln Q + a_t t + (1/2)a_{tt}t^2 + \\ (A2.7) \qquad & (1/2)\Sigma_{i=1}^{3}\Sigma_{j=1}^{3} a_{ij}\ln p_i \ln p_j + \Sigma_{i=1}^{3} a_{iQ}\ln p_i \ln Q + \\ & \Sigma_{i=1}^{3} a_{it} t \ln p_i + (1/2)a_{QQ}\ln Q \ln Q + a_{Qt} t \ln Q \end{aligned}$$

where $a_{ij}=a_{ji}$ for all i,j; $\Sigma_{i=1}^{3} a_i=1$, $\Sigma_{j=1}^{3} a_{ij}=0$ for i=1,2,3; $\Sigma_{i=1}^{3} a_{iQ}=0$ and $\Sigma_{i=1}^{3} a_{it}=0$.

By applying Shephard's Lemma cost share equations are obtained:

(A2.8) $S_i(p,Q,t) = a_i + \Sigma_{j=1}^{3} a_{ij}\ln p_j + a_{iQ}\ln Q + a_{it}t$; i=1,2,3.

Again the third share equation is excluded so that the estimating system consists of (A2.7) and (A2.8). The following test for constant returns to scale can be made;

(A2.9) $a_Q = 1$; $a_{iQ} = 0$, i=1,2,; $a_{QQ} = 0$; $a_{Qt} = 0$.

The test for neutral technical change is;

(A2.10) $a_t = 0$; $a_{it} = 0$, i=1,2; $a_{Qt} = 0$; $a_{tt} = 0$.

For the estimated cost function to satisfy the requirements of economic theory it must be concave in prices at all the observation points. In the translog case this requires the following matrix to be negative semi-definite at each observation point (Diewert and Wales 1987);

$$(A2.11) \qquad \begin{bmatrix} a_{KK}-S_K+S_K^2 & a_{KL}+S_KS_L & a_{KE}+S_KS_E \\ a_{KL}+S_KS_L & a_{LL}-S_L+S_L^2 & a_{LE}+S_LS_E \\ a_{KE}+S_KS_E & a_{LE}+S_LS_E & a_{EE}-S_E+S_{EE}^2 \end{bmatrix}$$

Allen-Uzawa elasticities of substitution are given by:

(A2.12) $ES_{ii} = (a_{ii} + S_i^2 - S_i)/S_i^2$

$$ES_{ij} = a_{ij}/(S_iS_j) + 1$$

Partial price elasticities are given by:

$$ED_{ii} = S_iES_{ii} \quad ; \quad ED_{ij} = S_jES_{ij}. \tag{A2.13}$$

A2.2.2 The Generalised Leontief Form

The primal GL system estimated by Fisher and Chung is:

$$\begin{aligned} Q = {} & a_Q + a_KK^{1/2} + a_LL^{1/2} + a_EE^{1/2} + a_tt + a_{KK}K + \\ & 2aKL(KL)^{1/2} + 2aKE(KE)^{1/2} + aLLL + aLE(LE)^{1/2} + \\ & aEEE + (1/2)attt^2 + aKttK + aLttL + aEttE \end{aligned} \tag{A2.14}$$

$$pK/pQ = aKK + 1/2aKK^{-1/2} + aKL(L/K)^{1/2} + aKE(E/K)^{1/2} + aKtt$$

$$pL/pQ = aLL + 1/2aLL^{-1/2} + aKL(K/L)^{1/2} + aLE(E/L)^{1/2} + aLtt$$

$$pE/pQ = aEE + 1/2aEE^{-1/2} + aKE(K/E)^{1/2} + aLE(L/E)^{1/2} + aEtt$$

The test for constant returns to scale is:

$$a_Q = a_K = a_L = a_E = a_t = a_{tt} = 0 \tag{A2.15}$$

The GL cost function as specified by Diewert and Wales can be represented as:

$$\begin{aligned} C(p,Q,t) = {} & \Sigma_{i=1}^{3}\Sigma_{j=1}^{3} a_{ij}(p_ip_j)^{1/2}Q + \Sigma_{i=1}^{3} a_ip_i + \\ & \Sigma_{i=1}^{3} a_{it}p_itQ + a_t(\Sigma_{i=1}^{3} A_ip_i)t + \\ & a_{QQ}(\Sigma_{i=1}^{3} B_ip_i)Q^2 + a_{tt}(\Sigma_{i=1}^{3} C_ip_i)Qt^2 \end{aligned} \tag{A2.16}$$

where $a_{ij}=a_{ji}$, for $i,j=1,2,3$ and $A_i$, $B_i$, and $C_i$ are exogenously specified constants. Since all the coefficients of the cost function would appear in the input demand functions derived by the use of Shephard's Lemma, the cost function and input demand functions cannot be estimated as a total system. Consequently, in this application the following set of input-output coefficient equations was estimated:

$$\text{(A2.17)}\quad x_i(p,Q,t)/Q = \Sigma_{j=1}^{3} a_{ij}(p_j/p_i)^{1/2} + a_iQ^{-1} + a_{it}t + a_tA_it/Q + a_{QQ}B_iQ + a_{tt}C_it^2$$

Input-output coefficient equations were estimated rather than input demand equations to reduce the problem of heteroskedasticity.

The test for constant returns to scale is:

$$\text{(A2.18)}\qquad a_i = 0,\ 1=1,2,3;\ a_t = 0;\ a_{QQ} = 0.$$

The test for the cost function not being dependent on time is:

$$\text{(A2.19)}\qquad a_{it} = 0,\ i=1,2,3;\ a_t = 0;\ a_{tt} = 0.$$

Concavity of the cost function was examined by determining whether the matrix of second order price derivatives was negative semi-definite at each observation point. Elasticities of substitution were calculated using the original definition:

$$\text{(A2.20)}\qquad ES_{ij} = CC_{ij}/(C_iC_j)$$

where $C_i$ and $C_{ij}$ are, respectively, the first and second order price derivatives of the cost function. Partial price elasticities were then computed using equation (A2.13).

A2.2.3 The Symmetric Generalised McFadden Form

A problem often encountered by empirical studies using the dual specification is that the estimated cost function is not concave in prices. This renders the estimated elasticities suspect as they are not derived from a cost function which satisfies the basic requirements of economic theory. While it is possible to force the translog form to be concave this destroys the translog's flexibility properties. The SGM form developed by

Diewert and Wales allows us to ensure global concavity in prices with minimal loss of flexibility properties. The SGM cost function is given by:

$$C(p,Q,t) = g(p)Q + \Sigma_{i=1}^{3} a_{ii}p_iQ + \Sigma_{i=1}^{3} a_ip_i + \Sigma_{i=1}^{3} a_{it}p_itQ + a_t(\Sigma_{i=1}^{3} A_ip_i)t + a_{QQ}(\Sigma_{i=1}^{3} B_ip_i)Q^2 + a_{tt}(\Sigma_{i=1}^{3} C_ip_i)Qt^2 \tag{A2.21}$$

where $g(p) = (1/2)\ p'Sp/(T'p)$

and the $A_i$, $B_i$ and $C_i$ are exogenously given.

Again this cost function cannot be estimated along with its input demand equations so the following set of input-output coefficient equations was used for estimation:

$$x_i/Q = \Sigma_{j=1}^{3}s_{ij}p_j/(\Sigma_{k=1}^{3}T_kp_k) - T_i(\Sigma_{k=1}^{3}\Sigma_{j=1}^{3}s_{kj}p_kp_j)/2(\Sigma_{k=1}^{3} T_kp_k)^2 + a_{ii} + a_iQ^{-1} + a_{it}t + A_it/Q + B_iQ + C_i\ t^2 + u_i;\ i=1,2,3. \tag{A2.22}$$

where $s_{ij}=s_{ji}$ and $\Sigma_{i=1}^{3} s_{ij} = 0$ for $i,j=1,2,3$.

Using the specification (A2.22) the $a_t$, $a_{QQ}$ and $a_{tt}$ in (A2.21) are set to unity to produce a more flexible form. The $T_i$ in (A2.22) are set equal to the sample midpoint quantities for the relevant inputs. The tests for constant returns to scale and the cost function not being dependent on time are again given by (A2.18) and (A2.19), respectively. If the $[S_{ij}]$ matrix is negative semi-definite then the cost function is globally concave. If it is not, the S matrix can be made negative semi-definite without losing the function's flexibility properties.

The elasticities of substitution were again calculated using equation (A2.20) while the price elasticities of demand were in turn derived using equation (A2.13).

## A2.3. Results

Detailed results for the translog and GL production function estimation can be found in Fisher and Chung. Only the results for the cost function estimation are presented in detail in this Appendix.

In undertaking the cost function estimation the data were converted to price indices having a value of 1.0 for the first observation and corresponding implicit quantities for each of the inputs obtained by dividing the input value by the relevant price index. In the case of output the quantity was normalised to have a value of 1.0 for the first observation. The translog form was found to be invariant in terms of fit and elasticities obtained to the scaling of the data but the GL form was found to be quite sensitive to scaling. The data used are listed in Table A2.1.

Fisher and Chung report results for two capital price specifications. One has a constant real opportunity cost component in deriving the user cost while the other has a real opportunity cost component which varies over time. Since there appears to be no theoretical justification for imposing a constant real opportunity cost of capital the results reported here are those for the time-varying capital price.

Conclusive evidence of autocorrelation was found in all three dual estimating systems. In the case of the translog cost function this was corrected for by use of the AUTO option which imposes a constant value of the autocorrelation coefficient across equations. In the GL and SGM cases autocorrelation could not easily be corrected for but did not appear to be a serious problem with the scatter of residuals showing no marked pattern with the exception of the capital equations where a cyclical trend in the residuals could be discerned. This is not surprising given the likely misspecification of the capital equation due to the failure to take account of inventory, utilisation rate, and other considerations which affect the most durable input.

The estimated coefficients for the translog, GL and SGM cost functions and corresponding asymmtotic t-values are presented in Table A2.2 for the time-varying capital price data. Details of the fit of the functions and the tests for constant returns to scale (CRTS) and neutral technical change (NTC) are presented in Table A2.3. The translog cost function appears to give the best fit to the data, having a log likelihood value of 421. In terms of fit the SGM system appears to perform better than the GL. In comparing the results of the tests for CRTS the first major difference between the primal and dual systems becomes apparent. In all three dual systems the assumption of CRTS is very strongly rejected whereas it is accepted in the translog production function tests. It should be noted, however, that the translog production function test for CRTS is a relatively weak one as it is conducted on the OLS estimate of the production function alone whereas the other tests are conducted within the complete estimating systems. The assumption of NTC is accepted for both primal systems but is decisively rejected for all three dual systems.

Fisher and Chung find the translog and GL primal systems to satisfy the necessary curvature conditions at all observation points. Similarly, the translog dual system is concave in input prices at all observation points with the matrix (A2.11) being negative semi-definite at each point. The GL dual system, however, does not satisfy concavity at all observation points with the matrix of second-order price derivatives not being negative semi-definite for 12 of the 29 observations. In contrast the S matrix in the SGM case was found to be negative semi-definite and hence the estimated cost function is globally concave. Given the failure of the GL system to satisfy concavity at all the observation points the derived elasticities must be treated with suspicion.

The Allen-Uzawa elasticities of substitution between each pair of the three inputs are presented in Table A2.4 for 8 of the 29 observations. In the case of capital-labour and capital-energy substitution the translog cost function elasticities generally indicate much less scope for substitution than does the translog production function. The capital-labour elasticities from the cost function are only one fifth the magnitude of those from the production function. Labour-energy elasticities are approximately the same for both the primal and dual sources but while this is the highest of the translog cost function elasticities, the greatest scope for substitution as indicated by the translog production function is between capital and labour. Hence the details of the technology conveyed by the primal and dual translog estimates differ greatly.

The GL dual estimates again generally indicate less scope for substitution between inputs than do the primal GL estimates. The capital-labour elasticities for the dual GL system indicate that there is almost negligible scope for substitution. The GL cost function indicates the most scope for substitution between labour and energy. The GL primal estimates indicate that capital and energy are complements and this finding is reinforced by the dual GL estimates, contrary to the translog results. While the dual GL substitution elasticities are relatively stable they must be treated with suspicion due to the failure of the concavity requirement.

The dual SGM elasticities are of similar magnitude to the translog dual estimates for capital-labour substitution and indicate the greatest scope for substitution between labour and energy as did the dual translog estimates. In the case of capital-energy substitution, however, the SGM indicates a change through time from slight substitutability to no relationship to one of increasing complementarity.

From the substitution elasticities, then, one must conclude that the impression of the technology characteristics conveyed is very sensitive not only to the choice between primal and dual estimation routes but also to the choice of functional form. With the exception of the dual GL estimates which can be ruled out due to concavity violations there is little to indicate which set of estimates (translog or SGM, primal or dual) should be preferred.

Moving to Table A2.5 own price elasticities of demand for each of the three inputs are presented for the same observations. As indicated by the substitution elasticities the dual translog and GL price elasticities generally indicate much less price responsiveness. The dual translog capital and labour elasticities indicate minimal response of input demand to own price changes. The energy own price elasticities from the primal and dual translog sources are approximately equal. The dual GL capital price elasticities are implausibly small and start off being positive, further evidence that this set of elasticities must be treated with suspicion. Labour own price elasticities from the GL cost function are so small as to indicate huge wage reductions being necessary to alleviate even small unemployment levels.

The dual SGM own price elasticity estimates are extremely close to the dual translog estimates in all three cases. This result is reassuring as the two dual forms which perform best in terms of fit and curvature requirements produce similar own price elasticities. The only major difference between the dual translog and SGM results is the SGM finding that capital and energy are increasingly complementary whereas the translog finds them to be substitutes. There is conflicting evidence from other empirical studies as to whether capital and energy are in fact complementary or substitutable.

A2.4. Conclusions

The findings of this study support the earlier findings of Burgess which indicate that the choice between primal and dual estimation routes is not a trivial one. Rather, quite different impressions of the production technology will be obtained depending on which estimation route is adopted. Furthermore, the information obtained on the technology appears to be quite sensitive to the particular functional form chosen. While some sets of results can be discarded because they fail to satisfy curvature conditions the choice between remaining options is largely arbitrary. For instance, while the translog and SGM cost functions produce similar elasticitiy estimates for most inputs and both satisfy curvature conditions, they predict very different relationships between capital and energy. The translog cost function has this pair as substitutes while the SGM cost function has them as complements. This serves to highlight the sensitivity of results to the estimation choices made. Furthermore, it highlights the relative lack of robustness of elasticity estimates used as input to larger econometric models.

Finally, it may be useful to consider criteria which should be used when deciding to model the production sector by primal or dual means. At the most basic level the choice should depend on one's view of which variables are truly exogenous to the firm. If prices faced are beyond the firm's control but the firm has control of its input and output decisions then the dual model would seem to be more appropriate. If, on the other hand, the firm is committed to certain quantity levels and is prepared to accept whatever price clears the market given its quantity levels then the primal model may be more appropriate.

At a more practical level, however, it should also be recognised that the primal and dual models have different comparative advantages in modelling and predicting certain variables. For instance, if one is interested mainly in forecasts of output levels then the primal model is likely to give more accurate results. If one is interested in cost levels, however, the dual model is likely to be more accurate and, hence, appropriate. Whichever choice is made and for whatever reasons the important point to bear in mind is that the results obtained will be sensitive to the method of estimation (primal or dual, choice of functional form, etc.) and that some experimentation with different estimation methods may be appropriate to determine the robustness of the results obtained.

## TABLE A2.1

## MACE DATA

| Year | Output Quantity | Capital Price | Capital Quantity | Labour Price | Labour Quantity | Energy Price | Energy Quantity |
|---|---|---|---|---|---|---|---|
| 1954 | 1.0000 | 1.0000 | 5.8912 | 1.0000 | 15.3590 | 1.0000 | 2.1269 |
| 1955 | 1.0945 | 1.0481 | 6.1646 | 1.0253 | 15.7400 | 0.9951 | 2.4860 |
| 1956 | 1.1864 | 1.1287 | 6.4954 | 1.0936 | 16.3890 | 0.9604 | 2.7820 |
| 1957 | 1.2137 | 1.1950 | 6.8532 | 1.1440 | 16.8338 | 0.9825 | 3.0051 |
| 1958 | 1.2447 | 1.2468 | 7.1946 | 1.1727 | 16.7562 | 0.9857 | 3.1070 |
| 1959 | 1.2892 | 1.2767 | 7.5389 | 1.2087 | 17.2674 | 0.9050 | 3.4664 |
| 1960 | 1.3233 | 1.3157 | 7.8683 | 1.2464 | 17.5751 | 0.8753 | 3.7356 |
| 1961 | 1.3555 | 1.3546 | 8.1408 | 1.2802 | 17.8565 | 0.8755 | 3.9108 |
| 1962 | 1.4357 | 1.4112 | 8.3940 | 1.3228 | 18.3813 | 0.8681 | 4.1749 |
| 1963 | 1.5168 | 1.4630 | 8.6693 | 1.3692 | 18.8528 | 0.8608 | 4.4541 |
| 1964 | 1.6099 | 1.5026 | 9.0063 | 1.4312 | 19.5810 | 0.8848 | 4.8503 |
| 1965 | 1.7143 | 1.5377 | 9.4273 | 1.5165 | 20.3785 | 0.8707 | 5.2459 |
| 1966 | 1.8328 | 1.5859 | 9.9122 | 1.6248 | 21.2819 | 0.8747 | 5.6338 |
| 1967 | 1.8860 | 1.6306 | 10.4008 | 1.7459 | 21.9057 | 0.8888 | 6.0376 |
| 1968 | 1.9953 | 1.6838 | 10.8410 | 1.8373 | 22.3240 | 0.9091 | 6.5599 |
| 1969 | 2.0953 | 1.7787 | 11.2888 | 2.0112 | 23.0340 | 0.8969 | 6.9819 |
| 1970 | 2.1458 | 1.8797 | 11.7445 | 2.1442 | 23.2874 | 0.9079 | 7.4121 |
| 1971 | 2.2950 | 1.9572 | 12.2232 | 2.2969 | 23.8680 | 0.9503 | 7.7892 |
| 1972 | 2.4190 | 2.0465 | 12.7750 | 2.4786 | 24.5353 | 0.9571 | 8.4472 |
| 1973 | 2.6089 | 2.1812 | 13.4246 | 2.7204 | 25.7767 | 1.0223 | 8.7976 |
| 1974 | 2.7222 | 2.4083 | 14.1568 | 3.1314 | 26.8543 | 1.2509 | 9.3174 |
| 1975 | 2.7996 | 2.5916 | 14.8831 | 3.5865 | 27.3125 | 1.4768 | 9.2115 |
| 1976 | 2.9713 | 2.6789 | 15.6112 | 4.1175 | 27.8397 | 1.6554 | 9.7194 |

TABLE A2.1 (CONTINUED)

| Year | Output Quantity | Capital Price | Capital Quantity | Labour Price | Labour Quantity | Energy Price | Energy Quantity |
|---|---|---|---|---|---|---|---|
| 1977 | 3.0418 | 2.7352 | 16.3232 | 4.4615 | 28.3814 | 1.8997 | 10.1380 |
| 1978 | 3.1701 | 2.8898 | 16.9430 | 4.7199 | 29.3694 | 2.0830 | 10.4261 |
| 1979 | 3.2782 | 3.2439 | 17.5675 | 5.1023 | 30.5767 | 2.3172 | 10.7488 |
| 1980 | 3.3104 | 3.8424 | 18.2553 | 5.5827 | 31.5144 | 2.6869 | 11.0033 |
| 1981 | 3.4178 | 4.7141 | 18.9659 | 6.2671 | 32.3699 | 3.4063 | 10.8558 |
| 1982 | 3.2756 | 5.4384 | 19.5231 | 6.9630 | 31.2575 | 3.9482 | 10.9788 |

# TABLE A2.2

## COST FUNCTION COEFFICIENTS

| Translog | | | Gen. Leontief | | | Sym. Gen. McFadden | | |
|---|---|---|---|---|---|---|---|---|
| $a_0$ | 3.14 | (248.3) | $a_{KK}$ | 1.22 | (2.1) | $s_{KK}$ | -2.45 | (-0.4) |
| $a_K$ | 0.26 | (120.2) | $a_{KL}$ | 0.35 | (1.3) | $s_{KL}$ | 19.77 | (1.3) |
| $a_L$ | 0.64 | * | $a_{KE}$ | -0.37 | (-2.6) | $s_{KE}$ | 3.68 | * |
| $a_E$ | 0.10 | (50.9) | $a_{LL}$ | 5.86 | (7.7) | $s_{LL}$ | -78.88 | (-3.5) |
| $a_Q$ | 0.16 | (1.3) | $a_{LE}$ | 1.67 | (9.9) | $s_{LE}$ | 59.12 | * |
| $a_t$ | 0.03 | (4.8) | $a_{EE}$ | 1.09 | (1.6) | $s_{EE}$ | -62.80 | * |
| $a_{KK}$ | 0.16 | (36.4) | $a_K$ | 4.80 | (8.9) | $a_{KK}$ | 0.25 | (0.2) |
| $a_{KL}$ | -0.14 | * | $a_L$ | 7.29 | (8.9) | $a_K$ | 5.24 | (9.2) |
| $a_{KE}$ | -0.02 | (-7.4) | $a_E$ | -0.15 | (-0.3) | $a_{Kt}$ | -0.12 | (-0.2) |
| $a_{LL}$ | 0.17 | * | $a_{Kt}$ | 0.001 | (0.03) | $A_K$ | 0.41 | (1.9) |
| $a_{LE}$ | -0.03 | * | $a_{Lt}$ | -0.11 | (-2.8) | $B_K$ | 0.12 | (0.2) |
| $a_{EE}$ | 0.05 | (14.1) | $a_{Et}$ | -0.09 | (-2.9) | $C_K$ | 0.004 | (3.5) |
| $a_{KQ}$ | -0.11 | (-6.9) | $a_t$ | 0.66 | (7.3) | $a_{LL}$ | 6.31 | (1.0) |
| $a_{LQ}$ | 0.12 | * | $a_{QQ}$ | -0.80 | (-1.6) | $a_L$ | 9.40 | (1.6) |
| $a_{EQ}$ | -0.01 | (-1.1) | $a_{tt}$ | 0.007 | (4.5) | $a_{Lt}$ | 0.20 | (0.6) |
| $a_{Kt}$ | 0.006 | (10.0) | | | | $A_L$ | -0.04 | (-0.3) |
| $a_{Lt}$ | -0.008 | * | | | | $B_L$ | -0.61 | (-0.9) |
| $a_{Et}$ | 0.002 | (4.4) | | | | $C_L$ | -0.003 | (-0.4) |
| $a_{QQ}$ | 0.82 | (2.9) | | | | $a_{EE}$ | 4.27 | (2.0) |
| $a_{Qt}$ | -0.03 | (-1.9) | | | | $a_E$ | -1.11 | (-0.5) |
| $a_{tt}$ | 0.001 | (1.1) | | | | $a_{Et}$ | -0.01 | (-0.1) |
| | | | | | | $A_E$ | 0.04 | (0.4) |
| RHO | 0.56 | | | | | $B_E$ | -1.04 | (-2.7) |
| | | | | | | $C_E$ | 0.002 | (0.7) |

Asymptotic t-values in parentheses
* Coefficient derived from summation restrictions

TABLE A2.3

FITS AND TESTS

| | Translog | | GL | | SGM |
|---|---|---|---|---|---|
| | Primal | Dual | Primal[1] | Dual | Dual |
| Log Likelihood | 310.99 | 421.55 | 128.30 | 120.83 | 134.60 |
| Test for CRTS | 5.40[2] | 114.34[2] | 32.08[3] | 129.36[2] | 126.00[3] |
| Test for NTC | 3.18[4] | 75.50[2] | 9.80[5] | 78.46[2] | 112.60[3] |
| Concavity Violations | 0 | 0 | 0 | 12 | 0 |

[1]All GL primal results are not corrected for autocorrelation

[2]Critical Chi-square (0.99) = 15.09

[3]Critical Chi-square (0.99) = 16.81

[4]Critical Chi-square (0.99) = 9.21

[5]Critical Chi-square (0.99) = 11.34

## TABLE A2.4

## ELASTICITIES OF SUBSTITUTION

| | Translog | | Gen. Leontief | | SGMcFadden |
|---|---|---|---|---|---|
| | Primal | Dual | Primal | Dual | Dual |
| | | | Capital-Labour | | |
| 1954 | 1.00 | 0.16 | 1.50 | 0.05 | 0.13 |
| 1958 | 0.99 | 0.19 | 1.66 | 0.05 | 0.16 |
| 1962 | 0.99 | 0.22 | 1.74 | 0.05 | 0.19 |
| 1966 | 0.99 | 0.21 | 1.82 | 0.05 | 0.21 |
| 1970 | 0.99 | 0.21 | 1.91 | 0.05 | 0.22 |
| 1974 | 0.99 | 0.17 | 1.99 | 0.06 | 0.23 |
| 1978 | 0.99 | 0.09 | 2.08 | 0.06 | 0.21 |
| 1982 | 1.00 | 0.18 | 2.17 | 0.06 | 0.20 |
| | | | Capital-Energy | | |
| 1954 | 0.46 | 0.28 | -0.36 | -0.34 | 0.17 |
| 1958 | 0.42 | 0.29 | -0.60 | -0.30 | 0.04 |
| 1962 | 0.39 | 0.27 | -0.82 | -0.27 | -0.08 |
| 1966 | 0.38 | 0.21 | -0.94 | -0.28 | -0.16 |
| 1970 | 0.36 | 0.16 | -1.12 | -0.28 | -0.24 |
| 1974 | 0.36 | 0.14 | -1.21 | -0.29 | -0.28 |
| 1978 | 0.38 | 0.17 | -1.34 | -0.28 | -0.27 |
| 1982 | 0.41 | 0.41 | -1.52 | -0.23 | -0.14 |

TABLE A2.4 (CONTINUED)

| | Translog | | Gen. Leontief | | SGMcFadden |
|---|---|---|---|---|---|
| | Primal | Dual | Primal | Dual | Dual |
| | | | Labour-Energy | | |
| 1954 | 0.56 | 0.56 | 0.99 | 0.61 | 1.06 |
| 1958 | 0.54 | 0.52 | 1.04 | 0.57 | 0.88 |
| 1962 | 0.53 | 0.48 | 1.09 | 0.57 | 0.72 |
| 1966 | 0.53 | 0.47 | 1.10 | 0.59 | 0.64 |
| 1970 | 0.52 | 0.45 | 1.14 | 0.60 | 0.54 |
| 1974 | 0.53 | 0.49 | 1.19 | 0.61 | 0.55 |
| 1978 | 0.56 | 0.57 | 1.31 | 0.57 | 0.60 |
| 1982 | 0.59 | 0.58 | 1.44 | 0.55 | 0.67 |

TABLE A2.5

OWN PRICE ELASTICITIES OF DEMAND

| | Translog | | Gen. Leontief | | SGMcFadden |
|---|---|---|---|---|---|
| | Primal | Dual | Primal | Dual | Dual |
| | | | Capital | | |
| 1954 | -0.57 | -0.13 | -0.91 | 0.002 | -0.10 |
| 1958 | -0.61 | -0.14 | -0.99 | -0.000 | -0.11 |
| 1962 | -0.68 | -0.16 | -1.05 | -0.004 | -0.11 |
| 1966 | -0.70 | -0.15 | -1.06 | -0.007 | -0.12 |
| 1970 | -0.67 | -0.15 | -1.07 | -0.011 | -0.12 |
| 1974 | -0.68 | -0.12 | -1.09 | -0.013 | -0.13 |
| 1978 | -0.62 | -0.08 | -1.18 | -0.012 | -0.11 |
| 1982 | -0.68 | -0.15 | -1.33 | -0.007 | -0.10 |
| | | | Labour | | |
| 1954 | -0.38 | -0.10 | -0.52 | -0.07 | -0.13 |
| 1958 | -0.35 | -0.10 | -0.56 | -0.07 | -0.13 |
| 1962 | -0.32 | -0.11 | -0.57 | -0.07 | -0.12 |
| 1966 | -0.31 | -0.10 | -0.60 | -0.07 | -0.12 |
| 1970 | -0.32 | -0.10 | -0.64 | -0.07 | -0.11 |
| 1974 | -0.32 | -0.09 | -0.67 | -0.07 | -0.11 |
| 1978 | -0.33 | -0.08 | -0.68 | -0.07 | -0.11 |
| 1982 | -0.32 | -0.12 | -0.66 | -0.08 | -0.14 |

TABLE A2.5 (CONTINUED)

| | Translog | | Gen. Leontief | | SGMcFadden |
|---|---|---|---|---|---|
| | Primal | Dual | Primal | Dual | Dual |
| | | | Energy | | |
| 1954 | -0.36 | -0.42 | -0.46 | -0.31 | -0.73 |
| 1958 | -0.44 | -0.40 | -0.49 | -0.28 | -0.63 |
| 1962 | -0.44 | -0.37 | -0.47 | -0.27 | -0.52 |
| 1966 | -0.46 | -0.36 | -0.44 | -0.29 | -0.47 |
| 1970 | -0.46 | -0.33 | -0.42 | -0.30 | -0.39 |
| 1974 | -0.41 | -0.35 | -0.43 | -0.32 | -0.41 |
| 1978 | -0.42 | -0.42 | -0.47 | -0.32 | -0.47 |
| 1982 | -0.44 | -0.46 | -0.59 | -0.27 | -0.47 |