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THE OPERATIONAL CURRICULA OF MATHEMATICS 8 TEACHERS IN BRITISH COLUMBIA

by

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A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF EDUCATION

in

THE FACULTY OF GRADUATE STUDIES
(Mathematics Education)

We accept this thesis as conforming to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA

January 1986

78

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Date 4 APRIL 1986

Abstract

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The purpose of this study was to describe the mathematics curricula as actually implemented by a sample of Mathematics 8 teachers in British Columbia. A survey of previous research indicated that knowledge about the mathematics subject matter which teachers present to their students and the interpretations which teachers give to that subject matter is sparse in spite of the importance such knowledge might have for the curriculum revision process, textbook selection, the identification of inservice education needs, and the interpretation of student achievement results.

The Mathematics 8 curriculum was divided into three content areas: arithmetic, algebra, and geometry. Within these content areas a total of 16 topics were identified as among the basic topics of the formal Mathematics 8 course. Four variables were identified as representing important aspects of a mathematics curriculum. The first of these, content emphasis, was defined as a function of the amount of time a teacher spent on each content area. The other three variables, mode of content representation, rule-orientedness of instruction, and diversity of instruction, were defined as functions of the content-specific methods teachers used to interpret the topics to their students.

Class achievement level and the primary textbook were identified as having strong potential relationships with a teacher's operational curriculum. These were used as background

variables in this study.

The data for this study were collected as part of the Second International Mathematics Study during the 1980/1981 school year. The sample consisted of 93 teachers who submitted five Topic-Specific Questionnaires throughout the school year regarding what they taught to one of their Mathematics 8 classes. Each class took a 40 item pretest at the beginning of the school year. The 27 classes with the highest class means were designated as "high achievement classes" for this study while the 27 classes with the lowest class means were designated as "low achievement classes."

Among the findings of this study were:

(1) Wide variation existed in the emphasis given by teachers to the three content areas with 60% giving at least one area light or very light emphasis.

(2) The median proportion of class time allocated for geometry was slightly higher than for algebra or arithmetic. However, teachers showed the most variation for this content area spending between 0% and 66% of their courses on geometry.

(3) In low achievement classes somewhat more time was spent on arithmetic and somewhat less time on geometry than in high achievement classes.

(4) Teachers using a text which placed more emphasis on a particular content area tended to spend more time on that content area in their classes.

(5) The mode of representation of mathematical content was

slightly more abstract than perceptual in general.

(6) The median mode of content repesentation varied substantially among topics.

(7) Teachers of low achievement classes tended to present mathematics in a slightly more abstract and rule-oriented way than teachers of high achievement classes.

(8) A weak positive association was found between the level of diversity in the textbook used and the level of diversity in the operational curricula of teachers using that textbook.

Table of Contents

List of Tables

List of Figures

Acknowledgement

I wish to thank David Robitaille, James Sherrill, and Walter Szetela who comprised my supervisory committee. They provided encouragement and direction throughout the research process and kept me on-task while I wrote this dissertation 700 kilometers from campus.

My wife Suree Dirks was unfailing in her help and understanding during my seven years as a doctoral student. I thank her and my two sons William and Robert most of all.

I. INTRODUCTION

The practice of criticizing the public school, both its programs and its products, has been a popular activity since the inception of that institution in its current form in North America over a century ago. The international "modern mathematics movement" of 1955-1975[1] was preceded by particularly strong criticism of school curriculum materials, teacher competence, and student achievement within that subject area (e.g., Bestor, 1953; Lynd, 1953; Smith, M., 1949). There is now evidence that criticism of school mathematics as well as science is once again increasing (Keitel, 1982; Usiskin, 1985). While a considerable body of knowledge about student mathematics achievement, both recent status and trends over time,[2] now exists, little is known about many aspects of mathematics classroom practice (NACOME, 1975). As Miles (1981) has noted:

> ...it seems clear that much more directly descriptive data are needed on matters of the most straightforward sort [including] the actual instructional modes being used by

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[1] As Howson (1982, p. 205) has noted, it is not possible to give fixed dates for this movement which actually encompasses many curriculum development and other activities with widely differing aims and orientations to both mathematics and education (Suydam & Osborne, 1977). The dates given here, however, do not differ widely from the various ones usually used in the literature (e.g., Suydam & Osborne, 1977; Steiner, 1980; Howson, Keitel, & Kilpatrick, 1981; Stebbins, 1978).

[2] (Robitaille & Sherrill, 1977) and (Robitaille, 1981) are examples of studies which have documented levels of student achievement in mathematics within British Columbia. O'Shea (1979) studied achievement trends within the same jurisdiction. The National Assessment of Educational Progress (NAEP) and the Assessment of Performance Unit (APU) have conducted similar survey research on a national level in the United States and the United Kingdom respectively.

> teachers....In most cases we do not have reliable, carefully sampled studies that would tell us, simply, what is really going on. (pp. 110-111, emphasis in original)

In particular, few studies have been conducted to investigate the curriculum that has been implemented by teachers in their classrooms even though it is widely recognized that what has been prescribed as an official curriculum may "bear little relationship to what actually goes on in the classroom" (Theisen, 1981, p. 7). Where studies have been conducted, it is not clear that the most important variables have been identified (Young, 1979). Such a lack of knowledge may explain why the modern mathematics movement was appraised as revolutionary in its impact on school practice at one time (NCTM, 1961; Seyfert, 1968), but was later criticized as "relatively insignificant" (Howson et al., 1981, p. 238) and finally as only a "minor perturbation" (Wheeler, 1982, p. 23). If the periodic criticisms of school mathematics programs are to be assessed and if curriculum development is to be carried out effectively, a means of describing mathematics curricula as implemented by teachers is needed as well as such descriptions themselves.

1. PURPOSE AND RATIONALE

This study had four major purposes:

(1) to develop and justify a framework for the description of a school mathematics curriculum;

(2) to use the variables defined as part of this framework to describe the operational curricula, curriculum-in-

use or implemented curriculum[3] of a sample of Mathematics 8 teachers in British Columbia during the 1980-81 school year (the year for which the necessary data are available);

(3) to evaluate partially the congruence of the operational curricula of these teachers with the formal curriculum of the curriculum guide and adopted textbooks as well as the ideal curriculum of mathematics educators;

(4) to generate hypotheses about the operational curricula of Mathematics 8 teachers in B. C.

There is a need for theory building in the general area of curriculum. In particular, no adequate theory of mathematics curriculum or instruction is available for confirmatory, hypothesis-testing investigations or to plan curriculum materials or classroom activities (Bauersfeld, 1979). This need for theory building was expressed by Mann (1975) as follows:

> I believe it is well known that there are no comprehensive theories about curriculum phenomena. But even such rudiments of theory as a limited set of explanatory propositions about selected curriculum phenomena, or disciplined efforts to suggest an approach to conceptualizing the events to which a theory might pertain, are quite limited in number. (p. 158)

Recognizing the limited development of curriculum theory generally and mathematics curriculum theory specifically, it became apparent that a reasonable theoretical goal of this study

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[3] The terms operational curriculum, curriculum-in-use, and implemented curriculum are used interchangeably in this study to refer to the actual curriculum of a teacher in a classroom.

would be the explication of a number of mathematics curriculum variables rather than the development of a complete theory of mathematics curriculum. The contributions which this study makes to the theory of mathematics curriculum have their foundations in the writings and research of Cooney (1976, 1980a, 1980b) and Goodlad (1979, 1983).

2. HISTORICAL ORIENTATIONS TO THE TERM CURRICULUM

The definition and scope of the term "curriculum" continues to be an unsettled issue within the field of education. While many variants are found in the literature, two basic formulations can be identified, each having a history which can be traced to the first quarter of this century. One definition employs an ends-means model in which curriculum constitutes the planned ends of the educational process with instruction as the means. Johnson's definition is typical of those within this category:

> ...it is here stipulated that curriculum is a structured series of intended learning outcomes. Curriculum prescribes (or at least anticipates) the results of instruction. (1967, p. 130)

In this definition, curriculum precedes instruction. Instruction is the instrumental process by which the curriculum or intended learning outcomes are transmitted to students. This formulation of the concept of curriculum has its origins in the work of Taylor, Bobbitt, and Thorndike (Howson et al., 1981, p. 85).

The second common orientation to the term curriculum emphasizes the actual experiences of persons in an educational setting (Brubaker, 1982). Stenhouse offered this characterization of what might be called the experiential formulation:

> ...the curriculum is not the intention or prescription but what happens in real situations. It is not the aspiration, but the achievement. The problem of specifying the curriculum is one of perceiving, understanding and describing what is actually going on in school and classroom. (1975, p. 2)

This association of curriculum with the lived experience of the classroom is rooted in the writing and practice of Dewey (Howson et al., 1981, p. 84). The recent debate regarding the extent to which modern mathematics has been implemented in typical school programs has illustrated that if the term curriculum is to be useful, it needs to encompass both intention and reality.

> In essence it seems to me that curriculum study is concerned with the relationship between the two views of curriculum--as intention and as reality. I believe that our educational realities seldom conform to our educational intentions. We cannot put our policies into practice... The central problem of curriculum study is the gap between ideas and aspirations and our attempts to operationalize them. (Stenhouse, 1975, pp. 2-3)

Attention to both the learnings intended by such documents as curriculum guides and embodied in textbooks, as well as the realities of the presentations of teachers and the activities and assimilations of students has characterized the work of Goodlad (e.g., Goodlad, 1979, 1983; Goodlad & Klein, 1970). His notion that curriculum should be perceived at several levels incorporates a concern for both intention and reality as central to curriculum. It was used as a basic conceptual framework for this study.

3. LEVELS OF CURRICULUM

The multi-level conception of curriculum used in the present study is similar to the several versions which have been proposed by Goodlad (1979) and used in his research. He identified four levels: the ideal curriculum, the formal curriculum, the operational curriculum, and the experiential curriculum.

The ideal or expert/professional level of the mathematics curriculum refers to a course of study proposed or produced by mathematics educators or other educational experts. Exemplars would include detailed recommendations for the content and methods of a particular course but they could be more general in nature. One example is the National Council of Teachers of Mathematics (NCTM) Agenda for Action (See Shufelt & Smart, 1983) which specified a broad outline for an ideal curriculum in mathematics. Ideal curricula are often specified in mathematics education methods texts. In this context ideal refers to ideas and academia and not to a best or perfect curriculum.

The second level, the formal curriculum, refers to curricula which have been formally or officially adopted within some legal jurisdiction such as a province or school district. Such curricula are represented by the contents of curriculum guides, approved textbooks, or other materials.

The third level, the operational curriculum, refers to a course of study as actually presented in the classroom by a teacher. The focus at this level is on the content and the interpretations given to that content by teachers. These presentations might mirror the contents of the textbook or might differ from it in some way. Unlike the previous two levels there are usually no written records of an operational curriculum. It was this level of curriculum that was investigated in this study.

The fourth level, the experiential curriculum, refers to the course of study actually received by an individual student. In a classroom of thirty students there could be thirty distinct experiential curricula with certain commonalities. As with the operational curriculum, written documentation of an experiential curriculum is unusual. Achievement tests provide measures of some of the effects of such a curriculum. Interview protocols have the potential of providing a more comprehensive view of the curriculum as received and interpreted by the student (e.g., Erlwanger, 1975).

No claim is made that the categorization of curricula into the four levels presented here provides a complete model for a theory of curriculum. It does, however, provide a way of making distinctions between courses of study and of recognizing curricula that otherwise might not be identified as such.

There are obvious connections among these four levels of curriculum. For example, the formal curriculum within a jurisdiction exerts a strong influence on the operational level. Indeed it has been claimed that this influence is especially strong in the case of mathematics (e.g., Goodlad, 1983). Likewise, the content covered and the interpretations given to that content in a teacher's operational curriculum will probably establish limits on the experiential curricula of most class members. The problem of identifying influences on any curriculum and measuring their strength is complex. Much of Goodlad's research has been in this area.[4]

4. THE COMPONENTS OF A CURRICULUM: CONTENT AND METHOD

It was noted above that there is no agreement as to whether the term curriculum should refer to intended learning outcomes, the reality of educational experience, or both. Likewise, there is no consensus within education as to the specific elements which might constitute a course of study or a curriculum so conceived. Howson (1979, p. 134) has argued that the identification of a mathematics curriculum with a syllabus or topic outline has impeded curriculum development. He identified aims for education and mathematics education, as well as content, methods, and assessment procedures as being central components of a mathematics curriculum. Goodlad and his associates (1979, p. 66) offered an even more extensive list of what they call "curriculum commonplaces": goals and objectives, materials, content, learning activities, teaching strategies,

[4] A growing body of research is emerging for each level of the curriculum in which attempts to investigate influences have been made. Stebbins (1978) and Quick (1978), for example, have both examined the influences of the ideal curriculum on the formal curriculum in the context of the curriculum reform movement of 1955-1975.

evaluation, grouping patterns, the use of time, and the use of space. Huebner (1976), on-the-other-hand, contended that the term curriculum should have a narrower, more focused set of referents. He argued that because curriculum has become concerned with so many facets of education it has lost its coherence, focus, and effectiveness (p. 156). He further asserted that "the nature of the student and the function of the teacher, examinations and school organizations" (p. 159), for example, should not be among the elements of the curriculum. Rather, the central components in his view are:

> (1) identification of those segments of culture...that can become the content of the course of study;
> (2) identification of the technologies by which this content can be made accessible or made present to particular individuals. (p. 160)

Huebner prescribed, then, that content and method should be the central foci of a curriculum and the central concerns of curriculum study. The position that curriculum should focus on content and method was adopted in this study for two reasons. First, these components were included explicitly or implicitly in, and were basic to, every formulation of the concept of curriculum which was examined as part of this study. Secondly, the curriculum commonplaces identified by Goodlad as well as the curriculum components identified by other authors can, in general, be described as aspects of or important to content, method or their interrelationships. The instrumental definition of method given by Huebner above was not, however, considered as adequate for this study.

Confrey (1981), in an essay on mathematics and curriculum, expands on Huebner's two categories or, more precisely, notes their interrelationship.

> I will add a third consideration which I think has been addressed inadequately and often, in fact, ignored by curriculum theorists; that is, the integral relationship between identifying educational content and deciding how to make it available to young people. It is the particulars of this relationship which I think are subject-matter specific, if not to a large extent concept specific, and hence must be undertaken with respect to one's subject matter. (p. 243)

It was the notion of method implied by the "third consideration" above that was used to conceptualize content-specific methods in this study. Thus, content-specific methods in this study were taken to refer to the ways content can be made meaningful to students such as the ways mathematical concepts can be interpreted. Teaching methods or instructional methods or technologies unrelated to specific content such as overhead projectors, programmed instruction, advance organizers, or clarity were not considered to be among the content-specific methods of a curriculum.

As an example of content-specific methods, consider Figure 1-1 which comes from a questionnaire used in the Second International Mathematics Study (SIMS), a project of the International Association for the Evaluation of Educational Achievement (IEA). This study is discussed in more detail in

Chapter 2 and Chapter 3. In the Figure 1-1, five interpretations of the mathematical concept of integers are shown. In this study, each of these interpretations was considered as a content-specific method of making the concept of integers available to students.

20. Extending the number ray to the number line:

I extended the number ray (0 and positive numbers) to the left by introducing direction as well as magnitude.

Ex: ----------------------------
-4 -3 -2 -1 0 1 2 3 4

-3 means 3 units to the <u>left</u> of 0.

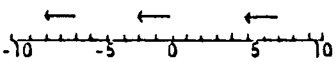
21. Extending the number system to find solutions to equations:

I discussed the need to extend the positive integers in order to find a solution to equations like __ + 7 = 5.

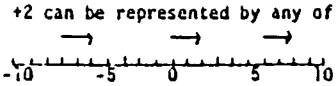
22. Using vectors or directed segments on the number line:

I defined an integer as a set of vectors (directed line segments) on the number line.

Ex: -2 can be represented by any of:



Ex: +2 can be represented by any of:



23. Defining integers as equivalence classes of whole numbers:

I developed the integers as equivalence classes of ordered pairs of whole numbers.

Ex: $\{(0,2),(1,3),(2,4),\ldots\} = {}^{-}2$

or $\{(a,b) \in W \times W : b = a + 2\} = {}^{-}2$

24. Using examples of physical situations:

I developed integers by referring to different physical situations which can be described with integers.

Ex: thermometer, elevation, money (credit/debit), sports (scoring), time (before/after), etc.

Figure 1- 1 - Five interpretations of the concept of integers.

5. TYPES OF MATHEMATICAL CONTENT

Within this study, mathematical content or subject matter was categorized into four types: facts, concepts, operations, and principles. This classification scheme was borrowed from Begle (1979) and is similar to that proposed by Cooney, Davis, and Henderson (1975). Although Begle did not define the terms "fact" and "concept", he did provide examples. He cited "two plus three equals five" and "7 X 8 = 56" as facts, referring to the first as an "arbitrary fact" and the second as "deducible." Cooney et al. (1975, p. 64) used the term "singular statements" for facts and described them as "statements about just one object" such as "2 is the only even prime number."

According to Cooney et al. (1975, p. 61) "a concept is knowledge of what something is." Skemp (1971) associated concepts with the processes of classification and abstraction and stated that:

> [An abstraction] is something learnt which enables us to classify; it is the defining property of a class. To distinguish between abstracting as an activity, and an abstraction as its end product, we shall hereafter call the latter a <u>concept</u>. (p. 22, emphasis in original)

Begle (1979, p. 7) listed rectangular array, fraction, and congruence among examples of mathematical concepts.

Begle's other two types of mathematical content were of a higher order than facts and concepts and he was able to provide definitions.

> An operation is a function which assigns mathematical objects to mathematical objects. Examples are:...counting,...,adding two numbers, [and] measuring the length of a line segment....
>
> A principle is a relationship between two or more mathematical objects: facts, concepts, operations, other principles. Any principle can be expressed as a mathematical theorem or axiom and every theorem or axiom expresses a principle (except for those which express a fact by stating the existence of a particular kind of mathematical object). (1979, p. 7)

In adopting Begle's classification of mathematical content, it was decided to use the word topic to refer to any particular fact, concept, operation, or principle. The term content area was used to refer to major groupings of mathematical topics.

6. MATHEMATICS CURRICULUM VARIABLES

From the basic curriculum components of content and method, four global variables were constructed in this study to characterize mathematics curricula and, in particular, operational mathematics curricula. These variables were among those originally suggested by Cooney (1980a) and McKnight (1980), and included: content emphasis, mode of content representation, rule-orientedness of instruction, and diversity of instruction.

6.1 Content Emphasis

In this study, the main components of a curriculum were specified to be content and content-specific methods. Although curriculum theorists differ as to what the concept of curriculum should include besides content, no one disputes that content is a fundamental part of a curriculum. Therefore, a necessary, though not sufficient, way of describing a curriculum is to list the topics or content areas included in that curriculum. Thus, an operational curriculum in which geometry, for example, is presented to students differs from an operational curriculum in which geometry is omitted. Some measure of what content is included, then, is needed to describe a curriculum.

Three content areas were identified for investigation: arithmetic, algebra, and geometry. The basis for selecting these particular collections of topics is discussed in Chapter 3. Since it was hypothesized that almost every teacher would include each of these content areas in his or her curriculum to some degree, it was necessary to define a variable which would quantify the amount of coverage each content area received in the operational curricula. For this reason the variable "content emphasis" was incorporated in the study. The way in which this variable was defined and measured is discussed in Chapter 3.

6.2 Mode Of Content Representation

The theoretical and empirical work of Bruner and Dienes in the area of mathematical concept formation and mathematics instruction has had a significant effect on research conducted in these areas and, to some extent, on school curriculum materials over the last 20 years (Resnick & Ford, 1981). A central construct in the theories of instruction which each of these researchers developed was the idea of the mode in which content is represented. Bruner identified three modes: enactive, iconic, and symbolic.

> Any domain of knowledge (or any problem within that domain of knowledge) can be represented in three ways: by a set of actions appropriate for achieving a certain result (enactive representation); by a set of summary images or graphics that stand for a concept without defining it fully (iconic representation); and by a set of symbolic or logical propositions drawn from a symbolic system that is governed by rules or laws for forming and transforming propositions (symbolic representation). (Bruner, 1966, pp. 44-45)

Bruner asserted that in teaching mathematics it is necessary to represent concepts first concretely (the enactive mode), then using diagrams or some other semi-concrete means of presentation (the iconic mode), and finally using the conventional or some other mathematical symbolism (the symbolic mode). Figure 1-2 illustrates this idea applied to quadratic expressions. The figure shows blocks which represent quadratic quantities along with the corresponding symbolism. The blocks themselves are concrete, while a picture of the blocks is semi-concrete. Bruner described the teaching sequence as follows:

> The object was to begin with an enactive representation of quadratics--something that could literally be "done" or built--and to move from there to an iconic representation, however restricted. Along the way, notation was developed and, by the use of variation and contrast, converted into a properly symbolic system. (1966, pp. 64-65)

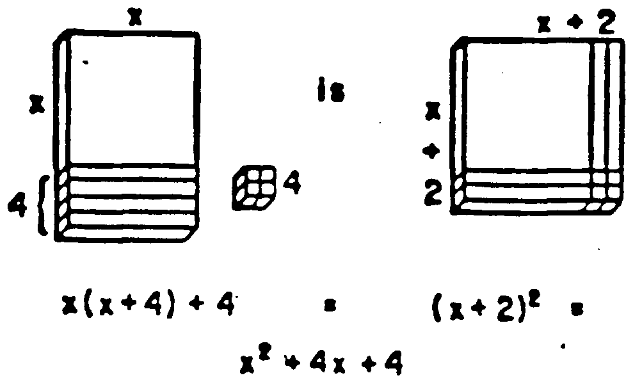


Figure 1- 2 - The concrete representation of quadratic expressions using blocks.

Dienes (1973; See also Dienes, 1960 and 1964) identified six stages in the process of learning mathematics. Within this process, he also asserted the necessity of representing content using each of the modes of representation identified by Bruner and in the same order. Dienes applied his teaching sequence to arithmetic concepts and also to such advanced topics as quadratics, logarithms, vectors, and functions.

Research has established a strong case for the use of concrete and semi-concrete representations of mathematical content before that content is represented symbolically (Suydam & Higgins, 1977) and this sequence is typically discussed in in detail in elementary methods texts (e.g., Heimer & Trueblood, 1977).

The theories of mathematics instruction formulated by Bruner and Dienes were based largely on the developmental psychology of Piaget and his classification of human development into four stages: sensori-motor, preoperational, the stage of concrete operations, and the stage of formal operations (Ginsburg & Opper, 1979). While, according to Piagetian theory, a child of between 11 and 15 should enter the stage of formal operations, and presumably be able to learn mathematics using symbolic representations alone, research has shown that this stage is frequently not reached until later, if at all (Ginsburg & Opper, 1979, p. 201). Research results in mathematics education have been consistent with this general psychological finding in that the use of concrete and semi-concrete representations of content has been shown to be beneficial to older children and adolescents as well as to younger children (Suydam & Higgins, 1977, p. 38). Skemp speculated that some form of progression from the concrete to the abstract may be required in learning mathematical ideas regardless of age.

> But it may well be the case that we all have to go, perhaps more rapidly than the growing child, through similar stages in each new topic which we encounter--that the mode of thinking available is partly a function of the degree to which the concepts have been developed in the primary system. One can hardly be expected to reflect on concepts which have not yet been formed, however well developed one's reflective system. So the "intuitive-before-reflective" order may be partially true for each new field of mathematical study. (Skemp, 1971, p. 66)

Bruner, in fact, asserted that even when it might seem possible to omit enactive and iconic representations of content, it could well be unwise to do so:

> For when the learner has a well-developed symbolic system, it may be possible to by-pass the first two stages. But one does so with the risk that the learner may not possess the imagery to fall back on when his symbolic transformations fail to achieve a goal in problem solving. (1966, p. 49)

Because of the prominence of the concept of mode of representation in theoretical discussions of mathematics learning and instruction as well as the supporting evidence of research studies, the variable "mode of content representation" was identified as an important curriculum variable and incorporated in this study.

6.3 Rule-Orientedness Of Instruction

Rules in mathematics are standard procedures or associations. They figure prominently in every branch of the subject (Beatley, 1954; Gordon, Achiman, & Melman, 1981). The school mathematics curriculum affords numerous examples of rules in connection with operations: for example, the rule of signs for integer multiplication, the "invert and multiply" rule for the division of fractions and the "transpose and change signs" rule for the solving of linear equations. Also, definitions of concepts can be considered as rules of association.

Despite the conspicuousness of rules in mathematics, an emphasis on rules in teaching to the exclusion or under-emphasis of other approaches to content has been decried as rote instruction. Skemp, for example, argues vigorously against what he calls the use of "rules without reasons" in teaching (Skemp, 1971, p. 17; Skemp, 1977, p. 20). He asserts that concepts must be introduced through examples rather than by definitions or rules.

> Concepts of a higher order than those which a person already has cannot be communicated to him by a definition, but only by arranging for him to encounter a suitable collection of examples. (1971, p. 32)

Teaching in which rules are emphasized has frequently been contrasted with teaching for understanding. An emphasis on the understanding[5] of mathematical ideas was one of the stated goals of the modern mathematics movement (Willoughby, 1968, p. 15; School Mathematics Study Group (SMSG), 1961, p. v). According to Callahan and Glennon (1975): "The 'new math' was intended to be more conceptually meaningful to the learners; rote, meaningless learning was to be de-emphasized" (p. 6). Price (1975) noted that a concern for the promotion of understanding in mathematics is not a new phenomenon:

> The problem is certainly not new and indeed reactions to the widespread learning of mathematics without some degree of understanding, that is "parrot fashion", "by rote" or "mechanically", have been going on for over a century. (p. 34)

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[5] The nature of mathematical understanding is a complex topic. See, for example, Backhouse, 1982; Byers & Herscovics, 1977; Skemp, 1977, 1982.

Because of the importance of rules in mathematics content and because of the stress placed on avoiding over-emphasis or premature introduction of rules by mathematics educators, the variable "rule-orientedness" was identified as an important curriculum variable and incorporated in this study.

6.4 Diversity Of Instruction

Cooney (1980a) and McNight (1980) argued that the diversity or variety of content presentations which teachers employ is an important variable in their operational curricula. McNight (1980) noted the prominent place of diversity or multiple embodiments in the learning and instructional theory of mathematics formulated by Dienes (See Dienes, 1960, p. 44) as did Resnick and Ford (1981, pp. 116-123). Cooney (1980a) implied a connection between diversity of presentations and teacher flexibility:

> One might expect different student outcomes for teachers with high variability than for those with medium or low variability. There is evidence in the literature to suggest that teachers who are more "flexible" are more effective. One aspect of being flexible is to be able to identify and utilize a number of instructional approaches. (p. 8)

Without necessarily accepting Cooney's association between diversity and flexibility, it would seem that diversity of approach is a curriculum variable which warrants investigation. In this study diversity within the operational curriculum was defined in terms of the number of ways teachers used to interpret and present mathematical ideas.

7. THE CONTEXTUAL VARIABLE: CLASS ACHIEVEMENT LEVEL

A mathematics curriculum is, in general, subject to many influences. These include psychological and sociological factors as well as changes within mathematics itself (Howson et al., 1981; Robitaille & Dirks, 1982). At the operational level, influences both internal and external to the classroom context may affect teachers' curriculum decisions. The content selected and the content-specific methods employed may be affected, for example, by the nature of the students in the class (Cooney, 1981).

One of the recommendations of the British Columbia Curriculum Guide(Curriculum Development Branch, 1978) is that teachers gear the depth of their courses and the approaches used to meet the needs of their students. Student needs in learning mathematics are related, at least in part, to their mathematical ability and prior achievement in the subject.

It was hypothesized in this study that differences might exist in the four curriculum variables between classes of high achievement and classes of low achievement. One might expect, for example, to find more stress on arithmetic and lower levels of abstraction in low achievement classes as compared to high achievement classes. Because it seemed reasonable to expect such differences, it was decided to analyze the four curriculum variables separately for different class achievement levels.

8. RESEARCH QUESTIONS

The operational curricula of B. C. Mathematics 8 teachers are explored in this study through the use of the curriculum and contextual variables discussed above. The following four research questions have been formulated to guide this inquiry:

(1) What patterns of content emphasis are present in the sample classes and how much variation exists?

(2) For each of the other three curriculum variables what levels are most common and how much variation exists for single topics, for each content area, and overall?

(3) Are there any differences in the distributions of each curriculum variable between the low and high achievement classes?

(4) To what degree do the descriptions of the operational curricula provided by this study coincide with or differ from the formal or ideal curricula?

II. REVIEW OF THE LITERATURE

The literature review is divided into two major sections. In the first section the North American literature on the operational curricula of secondary school mathematics teachers is discussed. This literature consists primarily of doctoral studies and components of large-scale evaluation projects which have had as one of their stated purposes the description of the content which secondary teachers incorporated in their courses and, more rarely, the content-specific approaches which teachers employed during instruction. While several of these studies have utilized interview and observational data (e.g., Stake & Easley, 1978a, 1978b), most, including the Second International Mathematics Study, have relied heavily upon teacher self-report data gathered via questionnaires. In the second section of the chapter the methodological issues surrounding questionnaires which are relevant to the SIMS project and the present study are reviewed.

1. THE TEACHING OF SECONDARY SCHOOL MATHEMATICS

Within the last decade the lack of comprehensive data on the beliefs, planning procedures and classroom practices of teachers of mathematics has been recognized. For example, in the United States the authors of the National Advisory Committee on Mathematics Education (NACOME) report (1975) asserted:

> The question "What goes on in the ordinary classroom in the United States?" is surely an important one, but in attempting to survey the status of mathematical education at "benchmark 1975," one is immediately confronted by the

> fact that a major gap in existing data occurs here. Appallingly little is known about teaching in any large fraction of U.S. classrooms. (p. 68)

The authors of the NACOME report noted the recent trend toward student assessment programs. They were concerned about the danger of formulating and the difficulty of refuting cause-and-effect explanations of unsatisfactory achievement results given "the vacuum of data on classroom processes" (p. 68). Similarly, Lanier (1978) asserted that "descriptive analyses of teachers planning for and instructing groups of learners in classrooms are obviously absent in mathematics education" (p. 7).

As it is used within the literature of educational research, the term "classroom processes" includes more than the curriculum-in-use in the classroom. Classroom management and teaching variables, for example, have been defined to describe aspects of the classroom process. Research in this area has advanced considerably since the NACOME report was written (See Rosenshine, 1982). However, if one focuses on knowledge of the operational curriculum--the content taught and how that content is approached--one finds that the situation at "benchmark 1985" within North America is only marginally advanced beyond the situation described ten years earlier within the United States. No attempt will be made in this literature review to summarize the research on teaching or any other area within the domain of classroom processes except for those studies which investigated the operational curriculum in some way.

1.1 Pre-NACOME Research In North America

Few research projects conducted prior to the NACOME study had, as a primary purpose, the description of the mathematics content teachers included in their courses or the content-specific approaches they employed. Most of the investigations in this area were conducted in connection with the academic year institutes and the summer institutes sponsored by the National Science Foundation (NSF) in the USA to upgrade the academic background of secondary mathematics and science teachers. In the main, these studies investigated the degree to which participants in NSF institutes introduced "modern" content into the courses they taught. Connellan (1962), for example, concluded that participants in Colorado tended to discuss such "modern" topics as set theory, the real number system, and non-Euclidean geometry in their courses more frequently than a matched group of control teachers. Similarly, Bradberry (1967) reported that over 70% of teachers participating in NSF programs in the Southeastern region of the USA who responded to her questionnaire agreed that they had "revised the course content they taught to include more up-to-date subject matter" (p. 2114A). Corbet (1976) reported that NSF participants in Kansas were more likely to introduce such topics as logic and sets, number theory, matrices, and transformation geometry than randomly selected Kansas mathematics teachers. Fields (1970), Martinen (1968), Roye (1968), Wiersma (1962), and Wilson (1967) each also concluded that NSF participants altered their curricula by introducing "modern" content following their experience in institutes. These studies did not, however, investigate the relative emphasis "modern" topics were given compared to "traditional" or other topics. Also, in none of these studies was the mathematics content specified at the level of particular concepts, operations, and principles.

While each of the studies cited above focused on the content implemented by NSF participants, rather than on content-specific or more general methods, several studies did make some mention of methodology or approach. Corbet (1976), for example, stated that:

> it could not be concluded from the data that the teaching of mathematics content courses had any effect upon the teaching methods of NSF participants, (p. 5206A)

In reaching a more negative conclusion, Connellan (1962) asserted that:

> the Academic Year Institute is not doing as much as it should in pointing out how traditional topics can be treated from a modern point of view. (p. 541)

Roye (1968), on-the-other-hand, concluded from his study that not only did NSF participants tend to teach new concepts in their courses, but also that they used "new curricular approaches and greater depth and detail in the subject or course taught" (p. 503A). Taken together these studies provide scant information about the approaches employed in teaching mathematical ideas.

Another group of American doctoral studies is more directly relevant to the subject of this literature review in that one of their stated purposes was to describe the content being taught within some jurisdiction (e.g., Alspaugh, 1966; Crawford, 1967; Dunson, 1970; Rudnick, 1963; Shetler, 1959). As with the previous group of studies, however, an overriding concern was the determination of what proportion of courses taught could be classified as "modern" as contrasted with "traditional." Thus, teachers and principals were surveyed for titles of courses taught and textbooks used and for their evaluations of how modern or traditional their courses were.

Alspaugh (1966) collected more detailed curriculum information than most of these researchers. He asked a sample of secondary teachers from Missouri whether or not each of a list of topics was included in the course they taught. On this basis, rather than by course title or the opinions of teachers, he rated the courses as modern or traditional. He concluded that 50% of the Algebra 1 courses and 7% of the Geometry courses in his sample had a "modern" curriculum.

Several of these studies also addressed questions of methodology. However, in each case "method" was conceived at the level of general teaching strategies or procedures. Thus, Alspaugh (1966) and Woods (1973) both concluded that a "show and tell" method of instruction predominated in which discussion of homework was followed by teacher explanation of new ideas which was in turn followed by the supervised study of new homework. Woods (1973) also investigated the use of team teaching, audio-visual materials, and the grouping of students. In none of these studies were teachers' methods of interpreting concepts or of teaching principles or operations the subject of investigation.

A few research projects can be identified in which some aspect of the mathematics operational curriculum in secondary schools was studied, but for which the primary focus was not the implementation of "modern mathematics." Neatrour (1969) and Smith, G. A. (1972) are of interest not only for this reason but also because they looked at the earlier secondary grades in contrast to most of the studies reviewed here. Neatrour (1969) sought to determine the geometry content included in the middle school curricula in 19 American states. To do this he investigated the 16 textbook series in use and also collected questionnaire data using a "scope and sequence form" from 156 teachers. He reported that, at the Grade 8 level, textbooks devoted an average of 32% of their pages to geometry content and that 79% of the teachers surveyed included at least 26 of 34 selected topics. He did not report the proportion of instructional time devoted to geometric topics however. Among his findings was a tendency for more geometric topics to be taught where a policy of multiple textbook adoption existed.

Smith, G. A. (1972), using questionnaires, addressed the following questions in his study of Los Angeles junior high school teachers:

> What topical content has recently been in use in seventh grade mathematics classrooms, which of these topics do teachers see as meriting special emphasis, and what topics do teachers desire to have included in the curriculum at this level? (p. 560A)

Among Smith's findings was the central place of the basic operations on whole numbers, decimals, and fractions at this level as well as the common inclusion of topics related to percent, exponential notation, and geometry. Smith also noted a trend toward combining traditional topics with modern approaches in instruction although it is not clear exactly what was meant by this.

The interpretations given by teachers to concepts in their operational curricula were not investigated in any of these studies; neither were the content-specific methods used to teach facts, operations, and principles. One global curriculum variable was a central focus of many of these studies. This variable dealt with how up-to-date school programs were and generally took on only two values--traditional or modern. The four curriculum variables defined in this study were not investigated by any of these researchers.

1.2 The International Study Of Achievement In Mathematics

The first large-scale cross-national evaluation project conducted by IEA dealt with secondary school mathematics at two levels, students 13 years of age and pre-university students.[1]

[1] At age 13 two groups were defined and tested: Population 1a consisting of all pupils 13 years old on the testing date and Population 1b consisting of all pupils at the grade level which contained the majority of 13 year olds. The pre-university level consisted of the grade(s) prior to university from which university students were normally drawn. Population 3a consisted of pupils who were studying mathematics as an integral part of their program; Population 3b consisted of pupils who were studying mathematics as a complemantary part of their program. (Husén, 1967a, p. 46)

The overall aim of this project, which will be referred to as the First Mathematics Study,

> ...was to relate certain social, economic and pedagogic characteristics of the different systems to the outcomes of instruction in terms of student achievement and attitudes. (Husén, 1975, p. 127)

While there were specific reasons for the choice of mathematics for IEA's first large-scale study, the study was really one of educational systems in general and not school mathematics.

> IEA regarded mathematics primarily as a surrogate for general school achievement and only secondarily as mathematics per se. Its analyses were therefore mainly aimed at providing information for policy makers and hence the chapter on school and system organization and social factors. It could be argued that more attention should have been given to data of interest to mathematics curriculum writers and teachers... (Postlethwaite, 1972, p. 102)

The place of the curriculum in the First Mathematics Study and the implications of this project for the curriculum field have been the subjects of debate. Bloom (1974) has asserted that this and subsequent "first round" IEA studies have documented differences in the "opportunity-to-learn" (OTL) curriculum content in the schools of various jurisdictions and have demonstrated the importance of the OTL variable as a predictor of achievement.[2] Freudenthal (1975), on-the-other-hand, has claimed that chief among the defects of the first round of IEA studies has been the neglect of the curriculum as a factor for accounting for achievement differences between and within countries.[3]

Content-specific methods were not investigated as such in the First Mathematics Study, but an attempt was made to investigate the prominence of "inquiry-centered methods" (Husén, 1967b, p. 148) as generally evidenced in the learning environment of each classroom. To determine the degree of inquiry in each classroom, students were asked to respond to items such as:

> My mathematics teacher wants pupils to solve problems only by the procedures he teaches.
> My mathematics teacher requires the pupils not only to master the steps in solving problems, but also to understand the reasoning involved. (Husén, 1967a, p. 116)

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[2] OTL variables within educational research are, in general, measures of content inclusion within instructional programs. In the First Mathematics Study OTL was operationally defined for each teacher and each achievement test item based on teacher assessments of the percentage of students who had "had an opportunity to learn this type of problem" (Husén, 1967b, p. 167). In the SIMS project teachers were also asked if the mathematics needed to answer each test item had been taught.

[3] Freudenthal (1975) was concerned that OTL might be perceived as an indicator of the importance of mathematics in the overall curriculum of each jurisdiction participating in the IEA project. He felt that OTL really measured only the opportunity students had to learn the material necessary for a particular set of test items and that these items might poorly represent the actual mathematics curricula of many countries. Specifically, he asserted that one-half to two-thirds of the mathematics program of the Netherlands was not represented by the IEA test items (p. 139).

Students were not asked about the methods used in teaching specific mathematical topics. The report focused on the importance of the inquiry variable as a predictor of achievement within and between countries and provided only summary measures of this variable at the country level.

In the First Mathematics Study, then, an input-output model of school achievement was used. The operational curricula of teachers were treated largely as part of the black box of the schooling process and was not the direct object of investigation (See Kilpatrick, 1972).

1.3 The NACOME Study

Clarification of classroom practices was part of NACOME's terms of reference. However, only gross indicators of the operational curriculum were measured in the survey conducted under the auspices of that group.[4] Examples included such broad topic areas as probability, statistics, the metric system, and relations and functions, the copyright date and number of texts used by teachers in the sample, the total time devoted to mathematics instruction, and general information on teaching methods (NACOME, 1975, pp. 68-78; Price, Kelley, & Kelley, 1977). A second limitation of the NACOME Report was that while its discussion and conclusions covered grades K-12, the

[4] NACOME was appointed by the Conference Board of the Mathematical Sciences in May 1974 and was "directed to prepare an overview and analysis of U. S. school-level mathematical education--its objectives, current practices, and attainments." (NACOME, 1975, p. iii)

empirical survey was restricted to Grades 2 and 5. In fact, the discussion of the secondary school curriculum in the NACOME report is based on the listing of course titles and enrollments provided by a 1972-73 survey as well as inferences drawn from the content of items used by the National Assessment of Educational Progress (NAEP) in testing 13 and 17 year olds.

At the junior secondary school level "NACOME found no firsthand survey data that indicate relative emphasis of new and traditional...topics" (NACOME, 1975, p. 9) and relied solely on a content analysis of items in the NAEP test battery as well as several standardized test batteries. As a basis for inferences regarding what is actually being taught in classrooms such data are obviously weak especially since "the four most widely used batteries appear to be measuring quite different kinds of school programs" (NACOME, 1975, p. 10). The NACOME Report's validity for describing actual classroom curricula was probably limited to the grade levels at which empirical data were actually collected.

1.4 The National Science Foundation Studies

In 1978 the National Science Foundation published several volumes detailing the results of a study commissioned to investigate the lasting outcomes of the curriculum reform efforts which NSF had sponsored in science and mathematics education during the previous two decades (NSF, 1978, preface). The NSF study consisted of three separate sub-studies:

(1) Three literature reviews of research and historical data covering 1955-1975--one each for science education (Helgeson, Blosser, & Howe, 1977), mathematics education (Suydam & Osborne, 1977), and social studies education (Wiley & Race, 1978),

(2) A national survey of course offerings, enrollments and practices (Weiss, 1978),

(3) Eleven ethnographic case studies conducted at school sites throughout the U. S. (Stake & Easley, 1978a, 1978b).

The literature review in mathematics education addressed the question:

> What were and are current practices in mathematics education for curriculum, instruction, teacher education, performance of learners, and needs assessments during the twenty-year period beginning in 1955? (Suydam & Osborne, 1977, p. 3)

The researchers who conducted this extensive literature review also sought to determine the extent to which information about teaching practices had been utilized by decision-makers during this period.

The findings of what teachers teach and how they teach it are not impressive:

> It comes as a surprise to most people that there are actually relatively few studies which describe the actual classroom situation....In most studies in the classroom, the setting is described only generally. Comparisons are made with the "traditional" or "usual" classroom, as if everyone knew precisely what that was. (p. 54)

The specific findings that Suydam and Osborne cited dealt with classroom management and general methods rather than with what content was taught and how it was taught. Examples are the proportion of teacher talk in the average classroom, the amount of time spent on managerial duties, the pace of instruction, and the types of instructional materials utilized (pp. 54-56).

The second part of the NSF study, a national survey, was conducted by the Research Triangle Institute and reported by Weiss (1978). It did not provide significant new information regarding the operational curriculum. At the level of the formal curriculum data were collected regarding courses offered and their enrollments, textbooks used and, specifically, NSF materials used. Teachers were asked about their use of general methods such as lectures, discussions and learning contracts, and about audio-visual and other equipment such as overhead projectors, games and puzzles, and calculators. Teachers were also asked for the names of the courses they taught and the textbooks they used. Teachers were not asked anything about the content they taught except course titles.

The third component of the NSF project, eleven ethnographic case studies, was directed by Robert Stake and Jack Easley of the University of Illinois. In the synthesis of these case studies, Stake and Easley (1978b) contended that socialization of students was an overriding goal in the classrooms observed and that subject matter itself was used for aims of class control.[5] They noted that "subject matter that did not fit these aims got rejected, neglected, or changed into 'something that worked'" (p. 19:5). The observers did not, however, collect detailed information on the approaches or strategies used to teach mathematics. In fact, no uniform system of observations was employed across sites. While several illuminating examples of mathematics content being used for socialization were reported (Stake & Easley, 1978b, chapter 16), the contention that mathematics content itself is selected or distorted so that it serves as a vehicle for socialization was not convincingly supported or elaborated by this study (House & Taylor, 1978, pp. 11-13). It remains an interesting hypothesis regarding the operational curriculum, particularly since rules are a conspicuous part of mathematics as noted above, but one requiring more specific, comprehensive data for confirmation.

1.5 Studies Conducted Within British Columbia

Both the 1977 and 1981 British Columbia Provincial Mathematics Assessments included a component in which information was gathered from teachers regarding various aspects of their backgrounds, beliefs, and classroom practices. One of the volumes produced as a result of the 1977 Assessment dealt exclusively with what the authors termed "instructional practices" (Robitaille & Sherrill, 1977). The survey reported

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[5] By "socialization of students" Stake and Easley appear to mean the process whereby students come to accept the social norms of the school such as persistently trying one's best. (See Stake and Easley, 1978b, pp. 16:5 and 16:13.)

in that volume was similar to the NACOME study but represented an advance as a comprehensive source of knowledge regarding mathematics teaching practices in several respects:

(1) A more representative sample was specified and a higher return rate achieved,

(2) Data were collected at seven grade levels: 1, 3, 5, 7, 8, 10, and 12,

(3) Questions were asked regarding the inclusion of more specific topics.

In reference to the operational curriculum at the secondary level, teachers were asked to categorize learning outcomes on a five point scale of "not important" to "very important." Robitaille (1980) summarized the results:

> On the assessment questionnaire teachers were asked to rank order a list of some 20 objectives selected from the total list of objectives for their grade, as published in the official curriculum guide. Four of the five objectives rated most important by grade 8 teachers dealt with the computational skills of arithmetic; the fifth dealt with the ability to solve problems involving percentages. All but one of the geometry objectives were ranked among the lowest third of the entire list, and the only objective in the list which explicitly mentioned the term "sets" was ranked last, i.e., least important. A similar result was found among teachers of grade 10 mathematics. The three highest-ranked objectives at that level concerned computational skills. All of the objectives dealing in any way with "new mathematics" were given low rankings of importance relative to the others. (p. 302)

The operational curriculum in B. C. has only been partly specified by this survey, however. While teachers were asked to rank order learning objectives which involved specific content, they were not asked if they included corresponding insruction or other learning activities in their curricula. Further, while the learning objectives encompassed many of the concepts, operations, and principles which might be taught at this level, the content-specific methods used by teachers were not investigated.

A second study which investigated the operational mathematics curriculum of British Columbia teachers was the B. C. component of the IEA SIMS project which was conducted during the 1980-81 school year. This project, in which Grades 8 and 12 were investigated, is of note both for its longitudinal design and for the scope and detail of the data collected. At each level approximately 100 teachers were asked what content they taught both at the level of major topic areas and at the level of specific concepts, operations, and principles, as well as the amount of time spent on that content. At the Grade 8 level teachers were also asked what content-specific methods they used in their presentations. The findings, which are reported in Robitaille, O'Shea, and Dirks (1982), included the frequency with which teachers used various interpretations of such mathematical ideas as integers and the Pythagorean theorem and the number of class periods allocated to content areas and specific topics. As the principal author noted in the preface to the report, however, many other studies might be conducted using this rich data source since only a limited amount of data analysis had as yet been possible. For example, since each of the teacher questionnaires was analyzed separately, it was not possible to investigate each teacher's curriculum-in-use comprehensively in terms of the curriculum variables discussed in Chapter 1 of this dissertation.

One re-analysis of the B. C. SIMS data has been completed. Tam (1983) was interested in the mode of representation Mathematics 8 teachers used in presenting content and used the methods of Exploratory Data Analysis in her study. She concluded that teachers preferred abstract approaches over concrete approaches for most topics. An inspection of the box plots included in her report indicates considerable variation among teachers in their preferences, however. Possible associations between mode of representation and other factors such as class achievement were not explored in her study.

2. THE USE OF QUESTIONNAIRES IN RESEARCH

The operational curriculum of Mathematics 8 teachers in B. C. was investigated in this study by reanalyzing data collected as part of the B. C. SIMS project. These data included questionnaire self-reports of content taught, time allocated to content, and content-specific methods used in teaching. A reliance on self-reports was required since a reasonably comprehensive study of even a single teacher's operational curriculum would require nearly a full school year. Direct observations would have limited the study to two or three teachers at most, precluding the survey scope necessary to address the research questions. Self-reports from multiple interviews could have been conducted with perhaps eight or ten teachers. This sample size, however, would still have been inadequate particularly for exploring associations between the curriculum and class achievement level. Furthermore, the literature does not suggest greater validity for interview data in contrast to questionnaire data for questions of the type asked in this study (Conger, Conger, & Riccobono, 1976). By using questionnaires, detailed curriculum information was collected at five points in time from nearly 100 teachers over the course of a school year.

Surveys within educational research and descriptive surveys using questionnaire data have been criticized by some commentators.[6] Mouly (1978), in fact, has asserted that "probably no instrument of research has been more subject to censure than the questionnaire" (p. 188).[7] In addition, the validity of teacher reports of their own behavior has been called into question in one review (Hook & Rosenshine, 1979).

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[6] Sieber (1968) has documented a pervasive critical stance by the authors of educational research textbooks and other commentators from the beginning of this century to survey methods. He ascribed this attitude to two factors:

> (1) the identification of the questionnaire with the "school survey," which is a service rather than a research operation; and
> (2) the predominance of psychological approaches in educational research. (p. 275)

[7] While there may be a bias against survey research within education (Herriott, 1969, p. 1400), this methodology is the dominant research form within the social sciences (Orenstein & Phillips, 1978, p. 170) having achieved this status since the end of the Second World War (Szalai & Petrella, 1977, p. ix).

Because of these criticisms, the theoretical literature on the validity of questionnaire surveys is discussed below. Also reviewed is the research literature concerning questionnaire validity which is relevant to the instrumentation and design of the B. C. SIMS project. The validity studies which were conducted as part of the SIMS project are discussed in Chapter 3.[8]

2.1 Questionnaire Research: Theoretical Validity Principles

A questionnaire item is valid insofar as it elicits from a respondent the information intended by the investigator. This implies that response differences between individuals represent true differences of opinion, behavior, or other characteristics of the respondents who are being studied (Berdie & Anderson, 1974). The results of a questionnaire survey are valid only if individual items have elicited the intended information and an adequate return rate of questionnaires has been achieved.

Several threats to validity in research using questionnaires have been identified. If a questionnaire item does not communicate the meaning intended by the framer of the item, the validity of the response is in doubt. Further, if the

[8] Questions of validity occur in every area of educational research. They are not restricted to surveys using questionnaires. Standardized tests, for example, may not be valid indicators of student understanding, achievement, or ability (Davis & Silver, 1982; Krutetskii, 1976. p. 13). Rigorous, replicable laboratory experiments may lack ecological validity (Cole, Hood, & McDermott., 1979, p. 2). Observational research may produce invalid measures (Rowan, Bossert, & Dwyer, 1983, p. 25) and may suffer from observer subjectivity and hence lack validity (House & Taylor, 1978, pp. 11-13).

respondent is either unable or unwilling to provide the information solicited, validity is threatened. Even if the information received using questionnaires is valid, the validity of the survey itself is in doubt if, as noted above, the response rate has been low since nonrespondents may differ from respondents in some systematic way (Mouly, 1978, pp. 189-190.)

The literature dealing with questionnaire validity is not extensive. Because the response rate to surveys conducted by mail has typically been low,[9] much of this literature has focused on strategies for maximizing the return of questionnaires rather than other issues concerning validity. Reflecting the preoccupation with response rate, Nisbet and Entwistle (1970, p. 44) asserted that: "the percentage response is the most important consideration in evaluating a questionnaire study." Similarly, Mouly (1978, p. 189) and Herriott (1969, p. 1402) list the problem of nonreturns as the primary limitation of questionnaire surveys.

An examination of educational research texts (e.g., Ary, Jacobs, & Razavich, 1979; Best, 1981; Borg & Gall, 1979; Cohen & Manion, 1979; Mouly, 1978; Nisbet & Entwistle, 1970) and those few references devoted primarily to questionnaire construction and survey design (e.g., Berdie & Anderson, 1974; Dillman, 1978; Hyman, 1955) shows considerable emphasis on principles related to increasing questionnaire returns. Dillman (1978), for example, reviewed 16 guidelines that had been offered in the

[9] Rates below 50 percent have often been considered acceptable (Dillman, 1978 p. 2; Herriott, 1969, p. 6).

past to improve response rate and then explicated his own comprehensive system. Most authors of survey references do provide to some degree, however, criteria for designing surveys and constructing questionnaire items so that the information gathered will correspond to actual attitudes, perceptions, or behaviors. The criteria given typically deal with both the issues of respondent ability and willingness to provide information, although these categories are not always explicitly noted.

Respondent ability to answer questions is a function of both the knowledge of the respondents themselves and the questions being asked. Several principles and cautions are provided in the literature which deals with question construction. Borg and Gall (1979, p. 297), for example, recommended the use of clearly written, briefly stated items which are focused on a single idea and which avoid technical language. Berdie and Anderson (1974, pp. 36-48) advocated the use of items which are unambiguous, self-explanatory and which communicate something specific and require a minimum of "puzzling out." In addition, they noted that adequate response options must be provided. Lack of ambiguity requires that each item be meaningful to the respondents, that only one basic meaning be ascribed by all respondents, and that this be the meaning intended by the researcher (Berdie & Anderson, 1974; Orenstein & Phillips, 1978). For items involving self-reports of status or behavior, specificity is particularly important (Orenstein & Phillips, 1978, pp. 218-219). Questionnaire items which require a high degree of inference and interpretation by respondents would be particularly suspect according to the foregoing standards.

Obviously, a respondent is unable to provide a valid response to a questionnaire item if he or she does not possess the information being solicited. This would be the case if factual knowledge has been forgotten by the respondent or if he or she has no opinion about a particular issue. A respondent would also be unable to answer questions about his or her behavior or environment if the questions dealt with phenomena beyond his or her ability to perceive. A teacher, for example, might be able to perceive certain aspects of his or her behavior in the classroom and of the classroom learning environment but might be unable to perceive other aspects (Fraser, 1982).

Regarding willingness of respondents to answer questions accurately, apart from a basic willingness to participate and return the questionnaire, the authors of the theoretical literature are quite consistent in admonishing that biased, loaded, or emotional language will cause inacurate responses to questionnaire items. The comments of Borg and Gall (1979) are typical:

> ...it is very important that an effort be made to avoid biased or leading questions. If the subject is given hints as to the type of answer you would most prefer, there is some tendency to give you what you want. (p. 297)

Berdie and Anderson (1974) note further that many respondents may be unwilling to respond to hypothetical questions as well as what they term "why" questions, i.e., questions which are written to solicit reasons for attitudes or behavior.

Dillman (1978), in discussing his "Total Design Method" for survey research, offers two other theoretical guidelines for maximizing respondent willingness to answer questions accurately. First, "contamination by others" should be avoided. For example, teachers or students may collaborate in completing questionnaires if the situation permits and produce results that are not valid for all respondents. Secondly, Dillman (1978) advocates the availability of consultation and follow-up services to participants of surveys to increase their interest and motivation and thereby increase the validity of responses as well as the rate of questionnaire returns.

2.2 Questionnaire Research: Empirical Validity Studies

The number of empirical studies which have investigated the validity of questionnaire data is not large and those studies which have been conducted have not produced a comprehensive set of principles for the construction of questionnaire items. In fact, contradictory conclusions have been reached in these studies as to the general validity of surveys using questionnaires (Berdie & Anderson, 1974; Walsh, 1968). Typically questionnaires have been used in educational research when other methodologies such as observations or interviews have not been feasible due to high cost, or when a concern has existed that other methods might have reactive effects such as the presence of an observer changing the patterns of behavior of teachers or students. Thus, it is probably not surprising that

Berdie and Anderson (1974, p. 20) have noted that "owing to the nature of questionnaires, the ways to check the reliability and validity of questionnaire items are limited."

In those cases where validity studies have been conducted they have usually been adjuncts to studies designed for other purposes. For this reason Berdie and Anderson (1974) cautioned against accepting generalizations about questionnaire validity that go beyond the specific instruments which were used in the studies.

In several studies the validity of teachers' responses to questionnaire items has been the subject of investigation. The results largely support the principles which were discussed in the previous section of this chapter. In particular, the responses to questionnaires have shown the least validity when there has been reason to doubt either the ability or willingness of the subjects to respond.

As part of the National Longitudinal Study of the Class of 1972, Conger, Conger, and Riccobono (1976) reviewed the literature on the validity and reliability of survey research questionnaires. They were concerned in particular with how data collection procedures, item characteristics, respondent characteristics, and interactions among these factors might influence reliability and validity. On the basis of their literature review they concluded:

> Demographic characteristics and factual information about present behavior yield the highest validity (and

> reliability) coefficients,[10] respectively. Factual information on past behavior and evaluative or judgmental behavior yields the least stable data, with the latter representing the lowest response stability. Furthermore, validity of reports of past behavior may be moderated by the importance of the accomplishment. That is, past events which have low ambiguity and are significant to the respondent in terms of accomplishment...tend to have high rates of validity. (p. 10)

Further in their report Conger et al. (1976) stated even more emphatically that:

> The literature and reliability study are unequivocally consistent in the finding that contemporaneous, objective, factually oriented items are more reliable than subjective, temporally remote, or ambiguous items. The validity items are similarly consistent. (p. 31)

This assertion, based on a review of empirical research, provides guidelines which are quite similar to some of the typical theoretical admonitions discussed above, namely, that questions should be specific and unambiguous. What is of particular interest is the conclusion by Conger et al. (1976), cited above, that questionnaires can be used for behavioral self-reports, in cases where the items are designed to solicit information about behavior which is both reasonably recent and reasonably important to the respondent.

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[10] Conger, Conger, and Riccobono (1976) do not explicitly define high, moderate, and low levels of reliability and validity in their discussions perhaps due to "the variety of indices used to summarize the results" of studies investigating the reliability and validity of survey data (p. 4). However, they do report correlation coefficients, and at one point indicate that "moderate to moderately high coefficients" are in the $0.70 \le r \le 0.88$ range (p. 10).

Hook and Rosenshine (1979) reviewed nine studies in which questionnaire validity was investigated in conjunction with research on teaching. The conclusions they reached were mixed and offered only limited support for the use of questionnaires in this area of research.

> ...if a teacher answers a questionnaire on a variety of specific activities, we cannot assume that these reports correspond to actual practice....one is not advised to accept teacher reports of specific behaviors as particularly accurate. No slur is intended; teachers do not have practice in estimating their behavior and then checking against actual performance. There appears to be some value in teacher reports when behaviors are grouped into dimensions, but one has no way of knowing, a priori, which dimensions will correlate with actual practice. Finally, based on the two available studies on this topic, teacher reports used to classify teachers on a continuum such as traditional or informal, appear to be trustworthy. (pp. 9-10, emphasis in original)

In the studies reviewed by Hook and Rosenshine, teachers' responses to questionnaire items were inaccurate when observers coded their perceptions of the teachers' behaviors using the Flanders Interaction Analysis System, when the questionnaire items contained a bias towards preferred responses or when both conditions were present. In studies reported by Ehman (1970), Johnson, D. P. (1969), and Steele, House, and Kerins (1971), the Flanders system was used, and in each case it was found that teachers could not accurately estimate the percent of class time spent in teacher talk when their questionnaire responses were compared to the frequency count data of observers. It may be, however, that the teachers in these studies were unable to respond accurately because they could not perceive the "multiplicity of molecular events," as Walberg and Haertel (1980, p. 232) characterized them, which made up the class discussion and lecture time.

It should be noted that in research on teaching the term "specific behavior" has usually been used to refer to the very short events which are coded using interaction analysis systems and which may exceed the ability of teachers to perceive or recall. Thus, the questionnaire items used in research on teaching which are specific in this sense of the word have tended to produce invalid responses, while items which are specific in the more usual sense of the word have tended to produce valid responses in other studies.

In several of the studies reviewed by Hook and Rosenshine the willingness of the respondents to provide accurate answers can be questioned because of item bias. For example, in the study conducted by Goodlad and Klein (1970) teachers tended to over estimate their use of innovative teaching techniques. Squire and Applebee (1966) found that teachers over estimated their use of Socratic questioning and underestimated the time they spent lecturing. In both cases it can be argued that certain response options were perceived by teachers as preferred by the researchers.

In other studies the responses of teachers to questionnaire items about their curricula and classroom practices have been consistent with the judgement of observers. For example, in a study involving 37 teachers, Bennett (1976) found agreement between observers and teachers on the teaching style used in the classroom. Teaching style was measured by asking specific questions such as "Do you put an actual mark or grade on pupils' work?" of both teachers and observers (Bennett, 1976, p 168).

Kazarian (1978) and Marliave, Fisher, and Filby (1977) found that teachers were able to estimate and willing to report the amount of time allocated to instructional activities at levels judged acceptable by the researchers. Kazarian (1978) used agreement of teachers' questionnaire responses with teachers' interview responses as the criterion for questionnaire validity. Marliave et al. (1977), however, validated the responses of teachers to questionnaire items by comparing them to classroom observations.

The focus of Hardebeck's (1974) doctoral study was the validity of teachers' questionnaire reports of individualization of instruction. Classroom observations were used to validate self-reports. While teachers who were observed to do little individualizing did tend to over-report their individualizing behavior, the relationship between observations and self-reports was strong enough "so as to permit describing self-reported teacher practices of five aspects of individualization of instruction as being high, medium, or low" (p. 126A).

In the studies reviewed above, questionnaire items sometimes elicited accurate information from respondents according to criteria established by the researcher. Sometimes they did not. As Berdie and Anderson (1974) noted:

> ...the contradictory reports concerning questionnaire methods are not surprising, as they are based on results from different questionnaires used for different reasons with different people at different times. (p. 12)

Questionnaire data obtained from teachers showed the least validity in those studies in which they were asked to evaluate the percent of class time they talked (e.g., Steele et al., 1971), their degree of openness (e.g., Ehman, 1970), their use of innovative methods (e.g., Goodlad & Klein, 1970), and the like. In these cases either the ability of teachers to perceive the behaviors required to answer the questions or their willingness to appear out of touch with the latest educational fashion can be questioned. In cases where teachers were asked neutral questions about specific, yet perceivable classroom practices, the validity of questionnaire resonses was much better (e.g., Conger et al., 1976; Bennett, 1976; Marliave et al., 1977).

These results provide support for the validity of the type of items incorporated in the questionnaires used in the B. C. SIMS project. The literature also supports the position that any study in which questionnaires are used needs to have a component in which item validity is investigated. Several validity studies were conducted in conjunction with the SIMS project. These are discussed in Chapter 3.

III. RESEARCH DESIGN AND PROCEDURES

The methodology chapter has been divided into five major sections. In the first section, aspects of the B. C. SIMS project, the data source for this study, are discussed. Particular attention is given to the basic design of that study, the nature of the sample, and a description of the instruments used as well as their development and validation. The next three sections contain a discussion of the content areas and topics, the curriculum variables, and the contextual variable respectively which were incorporated in this study. In the final section the data analysis strategy, utilizing techniques of Exploratory Data Analysis (Tukey, 1977), is presented.

1. THE B. C. SIMS PROJECT

As noted earlier, British Columbia was one of the participants[1] in the SIMS project, the second survey of school mathematics organized by IEA. The B. C. SIMS report, (Robitaille, O'Shea, & Dirks, 1982), described the context of B. C. involvement:

> British Columbia's participation in the Second International Study of Mathematics was sponsored by the Learning Assistance Branch of the B. C. Ministry of Education. The project was undertaken as an adjunct to the 1981 Mathematics Assessment which was conducted during the same school year. Participation in the international study provided an opportunity to

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[1] Over 20 jurisdictions took part in the SIMS project. Most of these jurisdictions were countries, but there were exceptions such as British Columbia, Ontario, Flemish-speaking Belgium, and French-speaking Belgium.

> acquire important information about the teaching of mathematics and about students' performance which did not fall within the terms of reference of the Mathematics Assessment. (p. 1)

The curriculum was a key concept in the formulation of a framework for the SIMS project. As was stated in the B. C. report:

> The international study [was] a broadly-based, comparative investigation of the mathematics curriculum as prescribed, as taught, and as learned. For the purposes of the study, the mathematics curriculum may be viewed as consisting of three components or dimensions: the <u>intended</u> curriculum, the <u>implemented</u> curriculum, and the <u>attained</u> curriculum. (p. 5, emphasis in original)

The curriculum framework used in the SIMS project was very similar to the one adopted in this study. The three components mentioned correspond to the formal, operational, and experiential levels of curriculum respectively.

As noted in Chapter 2, the B. C. SIMS project involved substudies at both the Grade 8 level (referred to as Population A) and the Grade 12 level (referred to as Population B). The specific courses surveyed were Mathematics 8 and Algebra 12. The discussion which follows is restricted to the Grade 8 or Population A phase of this project.

1.1 Description Of SIMS Instrumentation

The instrumentation used in the B. C. SIMS project included pretests and posttests, teacher and student attitude scales, a teacher background questionnaire, and teacher "Classroom Process" questionnaires. Robitaille, O'Shea, and Dirks (1982) characterized the latter questionnaires as: "unique instruments designed to collect highly specific information from teachers regarding the methods they used in teaching specific topics in the curriculum" (p. 7).

Six classroom process instruments were used in the B. C. SIMS for Population A. One of these, the General Classroom Practices Questionnaire (GCPQ), solicited general information about classroom organization and management and the use of materials. The other five instruments, the Topic Specific Questionnaires (TSQs), were each directed at one of the following areas:

- Common and decimal fractions (the Fraction TSQ)
- Ratio, proportion, and percent (the Ratio TSQ)
- Algebra (formulas and equations) and integers (the Algebra TSQ)
- Geometry (the Geometry TSQ)
- Measurement (the Measurement TSQ)

The B. C. SIMS report categorized the aspects of classroom process dealt with by these five questionnaires as follows:

- resources such as textbooks, worksheets, and games used in teaching;
- specific subtopics taught;
- interpretations of specific concepts such as π
- content-specific methods and strategies such as the procedures used for teaching subtraction of integers;
- factors teachers perceived as influencing their choice of specific concept interpretations, methods, and strategies;
- time allocated to an entire topic and to individual subtopics;
- teacher' opinions regarding issues such as the need to justify for students the rules for multiplication of integers, or the place of calculators in teaching decimals. (pp. 28-29)

Measures for the four curriculum variables investigated in this study, which were discussed in Chapter 1 and which are defined operationally later in this chapter, were obtained using TSQ data. Measures of content emphasis came from TSQ questions about time allocations. An example taken from the Fraction TSQ is: "How many total class periods did you spend on teaching fractions? (Combine partial lessons where necessary.)"

Measures of the other curriculum variables, content representation level, rule-orientedness of instruction, and diversity of instruction, were obtained using TSQ data about the methods teachers used in interpreting specific concepts as well as teaching principles and operations (the third and fourth categories listed above). The content-specific methods for each topic are listed in Appendix A. Class achievement, the contextual variable investigated in this study, was defined using the SIMS Core pretest data as described later in this chapter. The test which was used is provided in Appendix B.

1.2 Sample Selection

Each of the jurisdictions which participated in the SIMS project had some latitude in defining the size and composition of the sample of classes selected for investigation (Robitaille, O'Shea, & Dirks, 1982, p. 8). The B. C. SIMS populations were defined and the samples selected as follows:

> For the B. C. sample, Population A was defined to include all students enrolled in regular Grade 8 classes in the public schools in the province as of September 1980. Excluded by this definition were the approximately 5% of the age cohort enrolled in independent schools as well as those following programs where the level of material covered was significantly below that prescribed for the Mathematics 8 course....
>
> In order to achieve a sample size of approximately 100 Grade 8 and Algebra 12 classes stratified according to geographic zone of the province and by school size, initial samples of 125 classes at each level were drawn. In most cases this resulted in the selection of no more than one class per school. Of the 125 classes, 105 were selected for initial contact and the remainder reserved for use when needed. In the spring of 1980, letters were sent from the Ministry of Education to all of the principals of the schools selected, soliciting their cooperation in the study and asking them to select a Mathematics 8 or Algebra 12 teacher or teachers at random from among the teachers available. In cases where it was not possible to make a random selection, the principals were asked to exercise their best judgement about which teacher or teachers to select.

1.3 Participation Rate And Instrument Return Rate

In 78 of the 105 classes selected in the Mathematics 8 sample, all of the student and teacher instruments were returned to the technical agency which administered the B.C. SIMS project.[2] In 13 of the remaining 27 classes selected, some of the tests and questionnaires were returned.[3] Twelve classes which were selected for participation were subsequently excluded.

> After the sample had been drawn and the materials for the study distributed to the schools, a problem developed in the Grade 8 sample which resulted in the loss of 12 of the 105 Mathematics 8 classes in the sample. In the schools in which these 12 classes were located, all of the Mathematics 8 classes established at the beginning of the school year were disbanded at the end of the first semester, and new classes were set up for the second half of the Mathematics 8 course. Since the students did not stay together but were distributed among several classes and teachers, it was not possible to include them in the study. Unfortunately, this problem was not identified until it was too late in the course of the study to obtain replacements for those classes. (Robitaille, O'Shea, & Dirks, 1982, pp. 9-10)

Thus, out of the 105 classes selected, 93 were suitable for participation. Of these eligible classes complete instrument returns were received from 84%, partial returns from 14%, and no returns from 2%.[4]

The return rate of the TSQs for the 93 eligible classes was

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[2] B. C. Research administered the technical aspects of the B. C. SIMS project. This included the printing and distribution of the tests and questionnaire as well as follow-up activities to maximize return rate.

[3] In the B. C. SIMS report the number of partial returns is given as 11 and the number of non-participants is given as 15 (p. 10). After the report was written, however, data were received from one of the classes which had been categorized as a non-participant. Also, one class was apparently miscategorized in the report since complete returns were eventually received from 78 rather than 79 classes.

[4] Labor disputes by school support staff, a postal strike, and teacher illness and transfer account for most of the unreturned questionnaires. (Robitaille, O'Shea, & Dirks, 1982, p. 10)

89%; 416 TSQs were returned out of the 465 distributed. In addition to the two non-participating teachers, three other teachers failed to complete a single TSQ. Thus, five teachers accounted for 25 of the 49 questionnaires which were not returned. The remaining 78 teachers returned all 390 of the TSQs they had been given.[5]

1.4 Representativeness Of The Achieved Mathematics 8 Sample

One goal of the B. C. SIMS project was to make inferences regarding a target population consisting of teachers of Mathematics 8 in B. C. This target population was not identical to the set of all teachers who taught Mathematics 8. In particular, no attempt was made to secure representative participation of teachers whose primary teaching load was outside of mathematics. The percent of teacher workload in mathematics for the Mathematics 8 B. C. SIMS sample and for the sample of Mathematics 8 teachers who participated in the 1981 B. C. Mathematics Assessment is shown in Table 1. While almost one quarter of the B. C. Assessment sample had 25% or less of their workload in mathematics, the corresponding figure for the B. C. SIMS project was only 6%.

Robitaille, O'Shea, and Dirks (1982) described the B. C. SIMS sample as follows:

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[5] On 14 of these 390 questionnaires it was indicated that the particular content dealt with by the questionnaire was not part of the curriculum as implemented by the teacher. Eight teachers omitted one of the TSQ topics and three teachers omitted two TSQ topics from their courses.

Table 3- 1 - Teacher Workload in Mathematics

Percent of Workload	1981 Mathematics Assessment Teachers		B.C. SIMS Teachers	
	Relative frequency (percent)	Cumulative frequency (percent)	Relative frequency (percent)	Cumulative frequency (percent)
0-25	23.9	23.9	5.6	5.6
26-50	17.6	41.5	14.6	20.2
51-75	12.6	54.1	16.9	37.1
76-100	45.9	100.0	62.9	100.0

> Teachers selected to participate in the international study at the Population A level were more likely to be mathematics specialists and were more experienced than the general population of teachers of secondary mathematics. Teachers of Mathematics 8 who were in the IEA sample had an average of 14 years of teaching experience, compared to 9 years for the population of teachers of secondary mathematics. There were also indications that the IEA teachers spent a greater proportion of their teaching time conducting mathematics classes than did the population of teachers of secondary mathematics. This latter finding is not surprising since principals were asked to select a teacher of mathematics to participate in the study. It is unlikely, in such circumstances, that they would have considered selecting a teacher who taught several subject areas, or whose specialization was in a field other than mathematics.

The comparison of teaching experience in the quote above is somewhat misleading since the mean of 14 years for the IEA sample refers to total teaching experience while the figure of 9 years actually refers to experience teaching mathematics only. Unfortunately, identical questions were not asked of the Mathematics 8 teachers in the SIMS and Assessment surveys so that actual comparisons of teaching experience are not possible.

Table 3- 2 - Years of Teaching Experience

Years of experience	1981 Mathematics Assessment: Teaching Mathematics — Relative frequency (percent)	Cumulative frequency (percent)	B.C. SIMS Sample: Teaching all subjects — Relative frequency (percent)	Cumulative frequency (percent)	Teaching Mathematics 8 — Relative frequency (percent)	Cumulative frequency (percent)
1- 2	18.5	18.5	3.4	3.4	18.0	18.0
3- 5	20.3	38.8	11.2	14.2	16.8	34.8
6-10	23.2	62.0	24.7	39.3	36.0	70.8
11-15	14.3	76.3	28.1	67.4	18.0	88.8
over 15	23.7	100.0	32.6	100.0	11.2	100.0

The percent of teachers with various levels of mathematics teaching experience who participated in the Assessment as well as levels of total teaching experience and Mathematics 8 teaching experience for the B. C. SIMS sample are shown in Table 3-2. While these data do not allow any exact comparisons, it can be noted that the two columns of cumulative frequencies for the B. C. SIMS teachers differ widely. Apparently the average number of years these teachers taught Mathematics 8 was considerably less than their average number of years of teaching. It is also possible, perhaps likely, that the average number of years these teachers had taught mathematics in general was less than their average of 14 years of total teaching experience. Thus, while the teachers in the B. C. SIMS sample were probably more experienced than the population of teachers who taught Mathematics 8 during the 1980-81 school year, five years is perhaps an overestimate of the difference between the average teaching experience of the two groups.

1.5 SIMS Validation: Instrument Development And Research

As noted in SIMS Bulletin Number 5 (IEA, 1980), concerns regarding the validity of the Topic-Specific Questionnaires "have been central to the development of the instruments" (p. 30). Bulletin Number 5 outlines the several phases of the validation process for these survey instruments.

In order for the Topic-Specific Questionnaires to have basic content validity it is necessary that the questionnaire items reflect the content which might be taught and the topic-specific methods which teachers might use. At this level validity can be established in part by expert opinion (Moser & Kalton, 1971, p. 356). The SIMS Topic-Specific Questionnaires were initially constructed at the University of Georgia in 1978 and were revised on several occasions by a Working Group composed of prominent members of the mathematics education community. They were reviewed further by the SIMS International Mathematics Committee.

At the second stage of the validation process, the reactions of classroom teachers to the SIMS questionnaires were obtained through limited pilot testing over a seven month period. Specifically,

> ...several experienced researchers in mathematics education volunteered to conduct in-depth interviews with classroom teachers concerning (a) the clarity and intention of the items, (b) the coverage of the instruments with respect to content and method, and (c) the time demands of the instruments. (IEA, 1980, p. 30)

On the basis of teacher reactions the questionnaires were subsequently revised and presented to the meeting of the IEA General Assembly in Paris in September 1979.

The objective of the third stage of the SIMS validation scheme was to assess the conformity of teacher self-reports of their operational curricula with the assessments of observers. The nature of the TSQ items made this a difficult task since a single curriculum question on one of the TSQs could take days or weeks to verify through assessments by observers. For example, if a teacher reported spending 20 days teaching integers, then at least 20 days would be required to validate the single Algebra TSQ question soliciting this information. Because of the time required to validate the TSQs through direct observation, it is not surprising that only two "small scale" validity studies of the SIMS Topic-Specific Questionnaires have been reported.

The first observational study was conducted at the University of Georgia and

> consisted of comparing the responses to two of the questionnaires (Fractions and Ratio) by teachers and by observers of the teachers' classes. Rather high agreement was found, but the study was too small to provide firm conclusions. (IEA, 1980, p. 30)

The criteria for asserting "rather high agreement" were not reported.

Flexer (1980) conducted an observational study of three eighth grade mathematics classrooms in Illinois using the Integer and the Ratio questionnaires. She specifically sought to investigate the validity of these instruments. Classroom observations were made over periods of between 3.5 and 9 weeks depending on the length of the instructional unit. After a topic corresponding to the content of a questionnaire had been taught by a participating teacher, that teacher completed the appropriate instrument in conformity with the SIMS research design. Flexer found correlations between observations and questionnaire responses of 0.83 on average. In reporting her research Flexer noted that despite these encouraging results the teachers in her study expressed concern over the length and complexity of the classroom process instruments. The questionnaires were later shortened and simplified in format by the Working Group.

A further validity study was conducted in B. C. by this researcher. Five Mathematics 8 teachers who had participated in the SIMS project were interviewed in June 1981 after the year long data collection period. The interviews of 80 to 100 minutes each had two components. In the first part the teachers were asked eight general questions regarding the appropriateness of the Topic-Specific Questionnaires for their courses. They were also asked if the items were clear and free from apparent bias and if the response categories provided were, in fact, adequate in their opinion. These teachers reported that the questionnaires were clear and highly relevant to their courses. In particular, none of the teachers reported using content-specific methods other than those listed on the questionnaires. This is not surprising considering the extensive piloting of these instruments which had taken place in B. C. The only criticism of the instruments was directed at those questions which asked them why they did or did not use each of the content-specific methods in their teaching. Teachers who participated in the Flexer study also singled out these questions for criticism and this is consistent with Berdie and Anderson's (1974) observation that "why" questions are in general not well suited to data collection using questionnaires.

In the second part of the interviews, teachers were asked questions from the Topic-specific Questionnaire which they had most recently completed. For the three teachers who had taught the questionnaire material within three weeks of the interviews, it was found that on average 9 percent of the subtopics which had been reported as taught on the questionnaires were reported as not taught during the interviews or vice versa. Similarly, 16 percent of the content-specific methods which had been reported as utilized on the questionnaires were reported as not utilized during the interviews or vice versa.[6] Time allocations

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[6] Each of these three teachers was asked questions from only one of the TSQs they had completed, specifically Algebra, Geometry, and Measurement. Overall, 75 questions were asked about subtopics and 99 questions about topic-specific methods to the three teachers interviewed.

reported during the interviews differed by an average of 13 percent from those which had been reported on the questionnaires. These findings were considered as supportive of the assumption that teachers had taken reasonable care in completing the Topic-Specific Questionnaires.

It should be noted that each B. C. teacher in the SIMS project participated in a one day orientation workshop. Also, contacts were made by mail and telephone when teachers did not return individual questionnaires soon after the probable topic completion dates which they had provided in September 1980. These factors may well have enhanced teacher care in completing the questionnaires in B. C.

2. MATHEMATICS 8 CONTENT INCLUDED IN THE STUDY

2.1 The Content Areas

This study was designed to investigate a substantial portion of the content which might be taught in Mathematics 8 classes. Three broad content areas--arithmetic, algebra, and geometry--were selected for study. These areas were identified both because they represent typical classifications of school mathematics content (e.g., Begle, 1979, pp. 14-15) and because of the prominence each area is given in the formal B. C. curriculum at this level.

For the purposes of this study the formal B. C. curriculum at the Grade 8 level was defined to consist of those mathematical topics which were either:

(1) explicitly specified in the Provincial Mathematics Curriculum Guide, or

(2) contained within a chapter of any of the three prescribed texts for Mathematics 8 which also contained some topics which were specified in the Curriculum Guide.

(3) contained in the first two-thirds of any of the prescribed texts.

The second and third criteria were included because it is assumed that prescribed texts carry messages to teachers as to what should be taught which may be as strong as the specifications of the Curriculum Guide. The "first two-thirds" stipulation of the third criterion is somewhat arbitrary. It reflects an assumption that teachers tend to view the latter portions of a textbook as optional. The three content areas which were investigated in this study each contain a substantial amount of content relevant to the formal curriculum of Mathematics 8 as defined above, both in terms of topics listed in the Curriculum Guide and in terms of prescribed textbook emphasis.

The scope of the terms arithmetic, algebra, and geometry as used here was determined not only by the mathematical topics which fall under these categories and appear within the formal curriculum for Mathematics 8 but also by the inclusion of particular topics on the questionnaires used for the B. C. SIMS project. Since the data from that project were re-analyzed for this study, a few topics which might be expected to be included within the three content areas at this grade level were omitted. For example, the topics of square roots and scientific notation were not incorporated in the SIMS questionnaire and thus could not be included in arithmetic in this study though they are part of the formal B.C. curriculum.

2.2 The Specific Mathematics Topics

Within each of the three content areas five or six concepts, operations, and principles were selected for investigation. The topics selected were both important topics within the formal curriculum of Mathematics 8 and included in the B. C. SIMS instrumentation. To be considered as important content in the formal curriculum each topic had to satisfy one or both of the following criteria:

(1) the topic was listed as "core" content in the Curriculum Guide at this level, or

(2) the topic was contained in the first two-thirds of each of the three prescribed texts.

The topics which were included in this study are listed below according to content area. The SIMS TSQ which contained items about the content-specific methods teachers used in presenting these topics is indicated in parentheses. Each topic satisfied one or both criteria for important content as specified above.

Arithmetic

the concept of fractions (Fraction TSQ)

the addition of fractions (Fraction TSQ)

the concept of decimals (Fraction TSQ)

operations with decimals (Fraction TSQ)

the concept of proportions (Ratio TSQ)

Algebra

the concept of integers (Algebra TSQ)

the addition of integers (Algebra TSQ)

the subtraction of integers (Algebra TSQ)

the multiplication of integers (Algebra TSQ)

the concept of formulas (Algebra TSQ)

solving linear equations (Algebra TSQ)

Geometry

the triangle angle sum theorem (Geometry TSQ)

the Pythagorean theorem (Geometry TSQ)

the concept of π (Measurement TSQ)

the area of a parallelogram (Measurement TSQ)

the volume of a rectangular prism (Measurement TSQ)

For each of the mathematical topics listed above the teaching methods incorporated in the SIMS TSQs were assumed to include the most common alternatives which teachers might use in their presentations. This assumption is warranted as the questionnaires were developed and refined by a panel of mathematics educators, as noted earlier, and were extensively piloted, a process which involved considerable input from teachers in B. C. and elsewhere.

3. MEASUREMENT OF THE CURRICULUM VARIABLES

3.1 Content Emphasis

The emphasis given to each of the three content areas: arithmetic, algebra, and geometry was measured in terms of the number of class periods spent on that content. Since there was variation in the number of periods teachers had available for their courses, the measure of emphasis which each content area received was defined in this study as the proportion of time allocated to a particular topic relative to the total time allocated for all three areas. Thus, for each teacher and each content area the level of content emphasis could take on values between zero and one inclusive. The sum of content emphasis measures for the three areas was one for each teacher.

In order to facilitate the discussion of the findings of this study, five levels of content emphasis were defined prior to data analysis: very heavy emphasis, heavy emphasis, moderate emphasis, light emphasis, and very light emphasis. The letter C was used to represent the content emphasis variable. The following values for C were associated with the five specified levels:

$0.66 < C \leq 1.00$: very heavy emphasis

$0.50 < C \leq 0.66$: heavy emphasis

$0.25 \leq C \leq 0.50$: moderate emphasis

$0.17 \leq C < 0.25$: light emphasis

$0.00 \leq C < 0.17$: very light emphasis

For each teacher there were actually three separate content emphasis variables, one for each content area. For example, suppose a teacher allocated 36 periods to arithmetic, 36 periods to algebra, and 18 periods to geometry. Then the emphasis score for arithmetic was 0.40, the emphasis score for algebra was also 0.40, and the emphasis score for geometry was 0.20.

The intervals specified above were constructed by noting that equal emphasis on all three content areas by a teacher would result in three content emphasis values of 1/3 for that teacher. Very heavy content emphasis was defined as twice the equal emphasis value or greater, while heavy emphasis was defined as between one-and-one-half and two times the equal emphasis value. Given that one area received very heavy emphasis by a teacher according to the foregoing, the sum of the other two content emphasis values for that teacher could not exceed 1/3. The criterion for very light emphasis was stipulated as a value less than one-half of this remaining amount or less than 1/6. Similarly, the interval associated with light emphasis was formulated in relation to that defined for heavy emphasis, i.e., $1/2(1 - 1/2) = 1/4$ and $1/2(1 - 2/3) = 1/6$ were taken as cutoff values. The remaining interval spanning the equal emphasis value of 1/3 was defined as indicating moderate emphasis.

3.2 Mode Of Content Representation

As part of the development of the SIMS Classroom Process Questionnaires, Cooney (1980a) classified the content specific methods or approaches to most of the mathematical topics which were contained in those instruments as either perceptual or abstract. In particular, the approaches associated with 14 of the 16 topics investigated in this study, the exceptions being proportions and solving linear equations, were classified by Cooney, and reviewed by members of the Classroom Process Questionnaire Working Group, who used the following definitions:

> A perceptual treatment of the content relies on concrete materials, diagrams or pictures or derives its meaning from the environment, experiential activities or some sort of perceptual activity. An abstract treatment of the content relies on explanations which are symbolic in nature and derives its meaning from other mathematical content. (Cooney, 1980a, p. 20)

In this study Cooney's definitions as stated above were used to classify the content-specific methods for teaching arithmetic and algebraic topics into abstract and perceptual categories. For geometric topics Cooney's definitions were modified so that the use of a diagram or picture in and of itself was not sufficient for a teaching method to be clasified as perceptual. Rather, a method used for teaching a geometric topic was classified as perceptual only when an actual physical activity was involved or a diagram was used which suggested some physical activity. Otherwise, the method was classified as abstract. In quantifying this variable for each topic and content area, the measure of interest was the ratio of abstract to total methods.

Teachers were asked in the SIMS Classroom Process questionnaires whether they emphasized, used without emphasis, or did not use the approaches listed for each mathematical topic. In order that emphasized approaches receive more weight than those which were used only, the emphasized approaches were given a double weight in this ratio. Thus, L, the level of abstraction used by a teacher in presenting a particular topic, could take on values between zero (no abstract approaches) and one (all approaches abstract) inclusive.

As with the content emphasis scale, five levels of content representation were defined prior to data analysis to facilitate the discussion of the findings of this study: highly abstract, somewhat abstract, balanced, somewhat perceptual, and highly perceptual. The following values for L were associated with the five specified levels:

$0.80 < L \leq 1.00$: highly abstract

$0.60 < L \leq 0.80$: somewhat abstract

$0.40 \leq L \leq 0.60$: balanced

$0.20 \leq L < 0.40$: somewhat perceptual

$0.00 \leq L < 0.20$: highly perceptual

Since there seemed to be no compelling reason for doing otherwise, the five intervals were chosen of equal length.

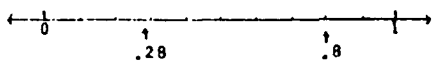
Decimals

The interpretations given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. *Emphasized (used as a primary explanation, referred to extensively or frequently)*
2. *Used but not emphasized*
3. *Not used*

51. A decimal as the coordinate of a point on the number line.



$.28 < .8$

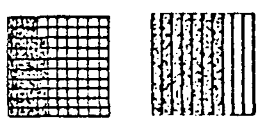
___1 ___2 ___3

52. A decimal as another way of writing a fraction.

$0.17 = \frac{17}{100}$ $0.8 = \frac{8}{10}$

___1 ___2 ___3

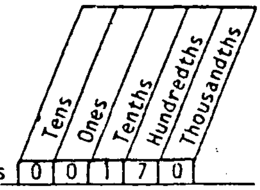
53. A decimal as part of a region.



0.38 0.7

___1 ___2 ___3

54. A decimal as an extension of place value.



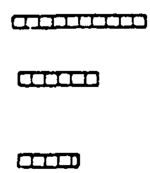
___1 ___2 ___3

55. A decimal as a series

$0.243 = \frac{2}{10} + \frac{4}{100} + \frac{3}{1000}$

___1 ___2 ___3

56. A decimal as a comparison



___1 ___2 ___3

Figure 3- 1 - Content-specific methods for teaching the concept of decimals.

Figure 3-1 is taken from the Fraction TSQ. It shows the six teaching approaches included in that instrument for the concept of decimals. Of these, options 51, 53, and 56 were considered as perceptual, the others as abstract. As an example of the computation of L, suppose that a particular teacher emphasized the abstract approach given by option 54 and used without emphasis the abstract approach given by option 52 as well as the perceptual approach given by option 53. For this teacher the value of L for this topic would be computed as follows:

Emphasized	54	(abstract)	= 2
Used	52	(abstract)	= 1
Used	53	(perceptual)	= 1

$$L = \frac{2+1}{2+1+1} = 0.75$$

Thus, according to the foregoing definition this teacher's instruction was somewhat abstract for the concept of decimals.

3.3 Rule-Orientedness Of Instruction

If a concept is defined formally but no further interpretation provided or if an operation or principle is stated as a rule with no interpretation or justification, then the mathematical idea in question is being presented in what might be called a rule-oriented way as discussed earlier. For eight of the 16 mathematical topics included in this investigation Cooney (1980a) identified one of the alternate approaches as clearly a rule, e.g., the rule of signs for integer multiplication. These topics were: operations with decimals, addition of integers, subtraction of integers, multiplication of integers, the Pythagorean theorem, the concept of π, the area of a parallelogram, and the volume of a rectangular prism.

Conceivably, for some topics teachers might:

(1) Present rules without any conceptual development,
(2) Present rules together with one or more conceptual approaches, or
(3) Develop a mathematical concept without an explicit statement of rules.

Cooney (1980a, p. 29) in discussing this variable hypothesized that a highly rule-oriented teacher might effectively promote computational skills in students but at the expense of higher order outcomes such as problem solving. Alternately, one might expect that some teachers would be more rule-oriented for review areas than for new content.

Table 3-3 defines values for the rule-orientedness, R, of a teacher on a particular topic. This measure was used to quantify the rule-orientedness of a teacher's operational curriculum. The value of R is a function of (1) whether the presentation of rules was emphasized, used without emphasis, or not used in instruction, and (2) whether other approaches were emphasized, used without emphasis, or not used in instruction. The higher the value of R the stronger the emphasis on rules in instruction.

As with the content emphasis and level of representation variables, five levels of rule-orientedness were defined prior to data analysis: highly rule-oriented, somewhat rule-oriented, balanced, non-rule-oriented, highly non-rule-oriented. The

Table 3- 3 - Defined Values for Rule-Orientedness

Rule approach / Any other approach	Emphasized	Used	Not Used
Emphasized	0.50	0.25	0.00
Used	0.75	0.50	0.25
Not Used	1.00	0.75	----

following values for R were associated with the five specified levels:

$0.80 < R \leq 1.00$: highly rule-oriented

$0.60 < R \leq 0.80$: somewhat rule-oriented

$0.40 \leq R \leq 0.60$: balanced

$0.20 \leq R < 0.40$: somewhat non-rule-oriented

$0.00 \leq R < 0.20$: highly non-rule-oriented

For a particular teacher and topic, R takes on values of 1.00, 0.75, 0.50, 0.25, and 0.00 only, each of the above categories containing exactly one of these five values. Related measures found by aggregating across topics and/or teachers can, however, attain other values and hence the necessity of intervals. For example, if a teacher had rule-orientedness scores of 0.75 on six topics and scores of 0.25 on two topics, then that teacher's overall measure for R for those eight topics would be 0.625, the arithmetic average.

Figure 3-2 is taken from the Geometry TSQ. It shows the seven teaching approaches in that instrument for the Pythagorean theorem. Option 69 specifies the presentation of a rule and was

Several methods for teaching the Pythagorean Theorem are given below. CHECK the response code which describes the treatment of each method in your class.

RESPONSE CODES:

1. *Used as a primary method of explanation*
2. *Used but not as a primary means of explanation*
3. *Not used*

67. I presented my students with a variety of right triangles and had them measure and record the lengths of the legs and hypotenuse. The pattern was discussed and then we stated the property.

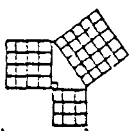
Ex:

leg	leg	hypotenuse
3	4	5
9	12	15

$3^2 + 4^2 = 5^2$
$9^2 + 12^2 = 15^2$
$\therefore a^2 + b^2 = c^2$

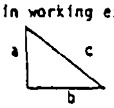
___1 ___2 ___3

68. I used diagrams like the following to show that, in a right triangle $a^2 + b^2 = c^2$



___1 ___2 ___3

69. I gave my students the formula $a^2 + b^2 = c^2$ and had them use it in working examples.



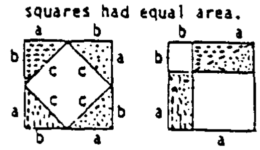
___1 ___2 ___3

70. The theorem was presented in the context of a historical account of Pythagoras and Euclid.

___1 ___2 ___3

71. I presented an informal area argument using physical, e.g. geoboards, or pictoral models.

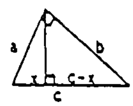
Ex: I showed that the two squares had equal area.



___1 ___2. ___3

72. I presented a formal deductive "algebraic" argument.

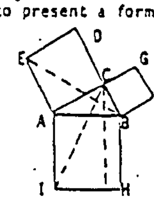
Ex: Using similar right triangles, proportions can be set up to yield $a^2 + b^2 = c^2$



___1 ___2 ___3

73. I presented a formal deductive argument using area.

Ex: This figure is sometimes used to present a formal proof.



___1 ___2 ___3

Figure 3- 2 - Content-specific methods for teaching the Pythagorean theorem.

designated as the rule approach. To illustrate the determination of R, suppose that a particular teacher emphasized this approach but also used without emphasis the approach given by option 71. The appropriate value of R for this teacher's presentations of this topic would be 0.75. Thus, according to the foregoing definition this teacher's instruction was somewhat rule-oriented for the Pythagorean theorem.

3.4 Diversity Of Instruction

In this study the number of approaches which a teacher employed in presenting a mathematical concept, operation, or principle was used as the basis for quantifying the diversity with which mathematical topics were approached within the operational curricula. For each of the 16 mathematical topics included in this study the measure of the diversity employed by a teacher in presenting a particular topic, D, was taken to be the number of approaches emphasized plus one-half multiplied by the number of approaches used without emphasis.

Unlike the other three curriculum variables investigated in this study, the diversity measure did not take on values of from 0.00 to 1.00 inclusive. If a teacher indicated inclusion of one of the topics investigated in this study in his or her operational curriculum, the related diversity measure was at least 0.50. This value would occur in the case of use without emphasis of a single approach to a topic and was the minimum value for D. If a topic was not taught no diversity measure was computed. The maximum value for this variable was dependent upon the number of approaches given for a particular topic in the Topic Specific Questionnaires which varied between three and ten. Thus, the maximum value for D varied between 3.0 and 10.0. While the diversity measure could have been standardized across topics by making it a proportion of the number of approaches listed in the questionnaires for each topic, the resulting scale would not have adequately reflected actual diversity in instruction. For example, if proportions were used, the emphasis of five of the ten listed fraction interpretations, and exclusion of the others, would have resulted in the same diversity measure as the emphasis of two of the four listed proportion interpretations, and exclusion of the others. Assuming, as was done in this study, that the alternative approaches to topics given in the Topic Specific Questionnaires include nearly all of actual approaches used by teachers, the first example should have resulted in a higher diversity value than the second. Using the scale as defined, the resulting values for D in these examples are 5.0 and 2.0 respectively.

As with the other variables, levels of diversity were defined prior to data analysis to facilitate discussion. These levels were as follows:

$D \geq 3.00$: high diversity

$1.50 < D < 3.00$: moderate diversity

$0.50 \leq D \leq 1.50$: low diversity

Thus, if a teacher emphasized three or more approaches to a mathematical idea, the instruction was characterized as highly diversified for that topic in this study. Alternately, the emphasis of two approaches and the use of two others resulted in the same characterization. At the other extreme, if a teacher emphasized only one approach to an idea in instruction and used at most one other approach without emphasis, the instruction was characterized as showing a low level of diversity for that topic.

Despite the reasonableness of these categories as defined, it should be kept in mind that the differences in the number of approaches given for the various topics as noted above make comparisons of diversity between teachers or groups of teachers for the same topic or group of topics less problematic than comparisons between topics across teachers.

4. THE CONTEXTUAL VARIABLE: CLASS ACHIEVEMENT

Each of the curriculum variables investigated in this study: content emphasis, content representation level, rule-orientedness of instruction, and diversity of instruction, was examined with reference to class achievement. Unlike the case of the curriculum variables, however, the categories for class achdievement were identified after rather than before a preliminary analysis of the data and on the basis of naturally occurring variation.

Class means on the 40 item SIMS pretest were used to designate each class in the study as low achievement, middle achievement, or high achievement.[7] The low achievement group consisted of the 29 lowest scoring classes. Their class means varied between 10.57 and 15.83 and had a mean value of 14.04. The high achievement group consisted of the 29 highest scoring

[7] Each student took the Core Pretest at the beginning of the course and an identical Core Posstest plus one of four rotated test forms at the end of the course.

classes. Their class means varied between 19.81 and 30.71 and had a mean value of 23.48.

5. DATA ANALYSIS

The rationale for this study rests on the assumption that the operational curricula of secondary mathematics teachers are important educational phenomena which warrant disciplined inquiry. Furthermore, it was assumed that no adequate, global theory of mathematics curriculum and instruction is currently available thus implying that definitive, hypothesis testing studies are premature at this point. By conceptualizing curriculum as involving mathematical concepts, operations, and principles and the content-specific methods which are used in presenting these ideas, it was possible to define quantitative descriptors of mathematics curricula. In pursuing a quantitative approach to describing curriculum-in-use and the relationships between curriculum-in-use and class achievement level, methods of Exploratory Data Analysis (EDA) were utilized. In this section the reasons for using EDA will be briefly outlined and those EDA techniques employed in this investigation are identified.

EDA is a body of statistical techniques developed by John Tukey (Erickson & Nosanchuk, 1977, p. v) who characterized it as involving:

> ...looking at data to see what it seems to say. It concentrates on simple arithmetic and easy-to-draw pictures. It regards whatever appearances we have recognized as partial descriptions, and tries to look beneath them for new insights. Its concern is with

appearance, not with confirmation. (Tukey, 1977, p. v)

Leinhardt and Leinhardt (1980) have noted the relevance of EDA to educational research:

> ...EDA is especially important to educational research, where many of the variables studied and data collected are not brought into analyses because well-verified, substantive theory demands their inclusion. Rather, variables are often included in a study because investigators "feel" they ought to be, because they are "convenient" to use, their measures have been recorded in some assumedly "reasonable" manner. Nor do the data always derive from scientifically designed random experiments. It is precisely in such ad hoc empirical research that EDA can be used to its greatest advantage because it is here that an open mind is an absolute necessity: The analyst rarely has the support of theoretically based expectations, and the real task confronting the data analyst is to explore--to search for ideas that make sense of the data. (p. 87)

In this study two of the basic techniques of EDA were employed: stem-and-leaf plots and box-and-whisker plots, with an emphasis on the latter form of data display. The following were produced and are presented in Chapter 4.

(1) Plots of content emphasis for all teachers and separately for the teachers of low and high achievement classes.

(2) Plots of rule-orientedness, level of representation, and diversity for the topics for which each of these variables are defined, across each of the content areas, and overall. Plots were made for all teachers and separately for teachers of low and high achievement classes.

IV. THE RESULTS OF THE STUDY

1. DESCRIPTION OF THE GRAPHICAL DISPLAYS

The results of this study are presented in part using the stem-and-leaf plots and the boxplots of Exploratory Data Analysis. In a stem-and-leaf plot all of the values in a given data set are retained in a display which is similar to a rotated histogram. In a boxplot the distribution of the data is shown using five summary statistics as well as any outliers which may occur in the data. The following discussion explains these graphical techniques and the associated terminology in the context of the curriculum variables examined in this study.

1.1 The Stem-and-Leaf Plot

To construct a stem-and-leaf plot each data value is first split at the last pair of adjacent digits. For example, a value of 22.9 in a data set would appear as 22|9 and every other value in the set would be similarly split between the ones digit and the tenths digits.

The emphasis which a teacher gave to a particular content area, C, was defined numerically as the proportion of time allocated to that content area relative to the time allocated to all three content areas. The possible values for this variable were thus between 0.0 and 1.0 inclusive. In constructing the stem-and-leaf plots for this variable each content emphasis score was first rounded to the hundredths digit. For the content area of aithmetic these scores ranged from a low of 0.00

to a high of 0.66. On a stem-and-leaf plot these two values appear as 0|0 and 6|6. The leading digits of the data values form the "stem" of the plot. To construct the display these values are written in a vertical column which is followed by a vertical line. The display is completed by writing down the trailing digit (the "leaf") of each data value on the line corresponding to its leading digit. The leaves are written in numerical order on each line.

```
 0 0
 1 5689
 2 0002345566778889
 3 000112334445555666777888889
 4 0011122333344567
 5 11477
 6 6
 7
 8
 9
10

1|5 represents 0.15
N=70
```

```
 0|0
 1|56
  |
 1|89
 2|000234
  |
 2|55667788889
 3|000112334445555666777888889
 4|0011122333344567
 5|
  |
 5|11477
 6|6
  |
 6|
 7|
 8|
 9|
10|
```

Figure 4- 1 - Distribution of arithmetic emphasis scores (illustrative stem-and-leaf plot).

In Figure 4-1 the content emphasis scores for arithmetic are shown. The plot at the left is a standard stem-and-leaf display. The plot at the right is a modified display showing the distribution of content emphasis scores for arithmetic into the five specified levels:

0.66 < C ≤ 1.00 very heavy emphasis

0.50 < C ≤ 0.66 heavy emphasis

0.25 ≤ C ≤ 0.50 moderate emphasis

0.17 ≤ C < 0.25 light emphasis

0.00 ≤ C < 0.17 very light emphasis

Note that it is necessary to write several of the leading digits twice in the stem since, for example, 0.16 and 0.18 are in different levels. Modified stem-and-leaf plots will be used throughout this chapter in place of the standard plots since they provide additional information while still showing the basic distribution of scores.

highly perceptual		0		
		1		
somewhat perceptual		2		
		3	3	
		4		
balanced	9972	4	377899	
	86222220	5	33588	
	0	6		
somewhat abstract	8511	6	123337799	
	75322210	7	2	
		8		
highly abstract	995	8	2	
		9	012	
$N_L=28$		10	00	$N_H=28$
3|3 represents 0.33	low achievement		high achievement	

Figure 4- 2 - Distribution of mode of representation scores for arithmetic for low and high achievement classes.

To compare the distributions of scores for low and high achievement classes on one of the curriculum variables, back-to-back stem-and-leaf plots are used. Using this display, the leaves for low achievement classes appear to the left of the stem while the leaves for high achievement classes appear to the right of the stem. To illustrate this plot, the mode of representation scores for arithmetic content are shown for classes of low and high achievement in Figure 4-2. The plot is a modified version showing the distribution of mode of representation scores using the five specified levels:

$0.80 < L \leq 1.00$ highly abstract

$0.60 < L \leq 0.80$ somewhat abstract

$0.40 \leq L \leq 0.60$ balanced

$0.20 \leq L < 0.40$ somewhat perceptual

$0.00 \leq L < 0.20$ highly perceptual

In constructing stem-and-leaf plots for the diversity of instruction variable, it was necessary to spread out the data by using two lines for each stem. On one line the digits 0-4 which occurred as leaves were written; on the other line the digits 5-9 were written. In Figure 4-3 the distribution of overall diversity scores for arithmetic is shown. Each overall score represents the average number of teaching methods a teacher used in presenting each arithmetic topic. Note that 1|3 represents 1.3 and not 0.13 as it would in the plots for the other variables. The plot is a modified version of the standard

	Stem	Leaves
low diversity	0	
	1	33
	1	5
moderate diversity	1	7889
	2	000011112222333333444
	2	5555556666677788899999
high diversity	3	000001111222234444
	3	5566667779999
	4	022
	4	5
	5	
	5	
	6	1
	6	N=85

1|3 represents 1.3

Figure 4- 3 - Distribution of overall diversity scores for arithmetic

display showing the distribution of scores into the three specified levels:

$D \geq 3.00$ high diversity

$1.50 < D < 3.00$ moderate diversity

$0.50 \leq D \leq 1.50$ low diversity

1.2 The Boxplot

The construction of a boxplot from a set of ordered data is easily carried out by sorting and counting. The description of this process as well as the interpretation and comparison of boxplots, however, requires the use of some technical terminology. The definitions which follow presuppose that a given set of N observations is arranged into ascending order. The position of a data value refers to its place within this ordering.

Lower Extreme: the least data value.

Upper Extreme: the greatest data value.

Upward Rank: the position of a data value counting upward from the lower extreme.

Downward Rank: the position of a data value counting downward from the upper extreme.

Depth: the smaller of the upward and downward ranks of a given data value.

Median: the data value whose depth is $(n+1)/2$. If the depth of the median is not an integer, the median is determined by interpolating between the two data values whose depths are nearest the depth of the median.

Lower Fourth (F_L) and Upper Fourth (F_U): the data values whose upward and downward ranks respectively are given by the following equation:

$$\text{depth of fourths} = \frac{[\text{depth of median}] + 1}{2}$$

where [X] stands for the largest integer not exceeding X. If the depth of the fourths is not an integer, the lower and upper fourths are determined by interpolating between the data values whose depths are nearest the depth of the fourths.

Fourth-spread or F-spread (D_F): the number determined by subtracting the lower fourth from the upper fourth.

Lower Outlier Cutoff: the value of $F_L - 1.50D_F$.

<u>Upper Outlier Cutoff</u>: the value of $F_U + 1.50D_F$.[1]

<u>Outlier</u>: any data value which is less than the lower outlier cutoff or greater than the upper outlier cutoff.

The construction of a boxplot will be illustrated using the data given in the stem-and-leaf plot of Figure 4-1. Since n = 70, the median has a depth of 35.5. Its value is 0.35. The depth of the fourths is 18.

The lower and upper fourths are 0.28, and 0.41 respectively. The plot is begun by drawing a rectangle or box using the fourths to determine two sides. The box thus shows the location of the central 50% of the data. The F-spread, 0.13, is the length of the box. The median is indicated by a segment within the box.

The outlier cutoffs are determined next. In this case the values are 0.28 - (1.5)x(0.13) rounded to 0.09, and 0.41 + (1.5)x(0.13), rounded to 0.61. Two tails or "whiskers" are drawn from the box. The lower tail is drawn to the greatest data value not less than the lower cutoff, in this case 0.15. The upper tail is drawn to the greatest data value not exceeding the upper cutoff, in this case 0.57. There are two outliers, 0.00 and 0.66. These are indicated by Xs on the plot. If either outlier value had occurred more than once in the data set, the number of occurrences would have been indicated in

---

[1] For normally distributed data slightly less than 0.7% of the observations would be outliers using these standard definitions for outlier cutoffs.(Hoaglin, 1983, p.40)

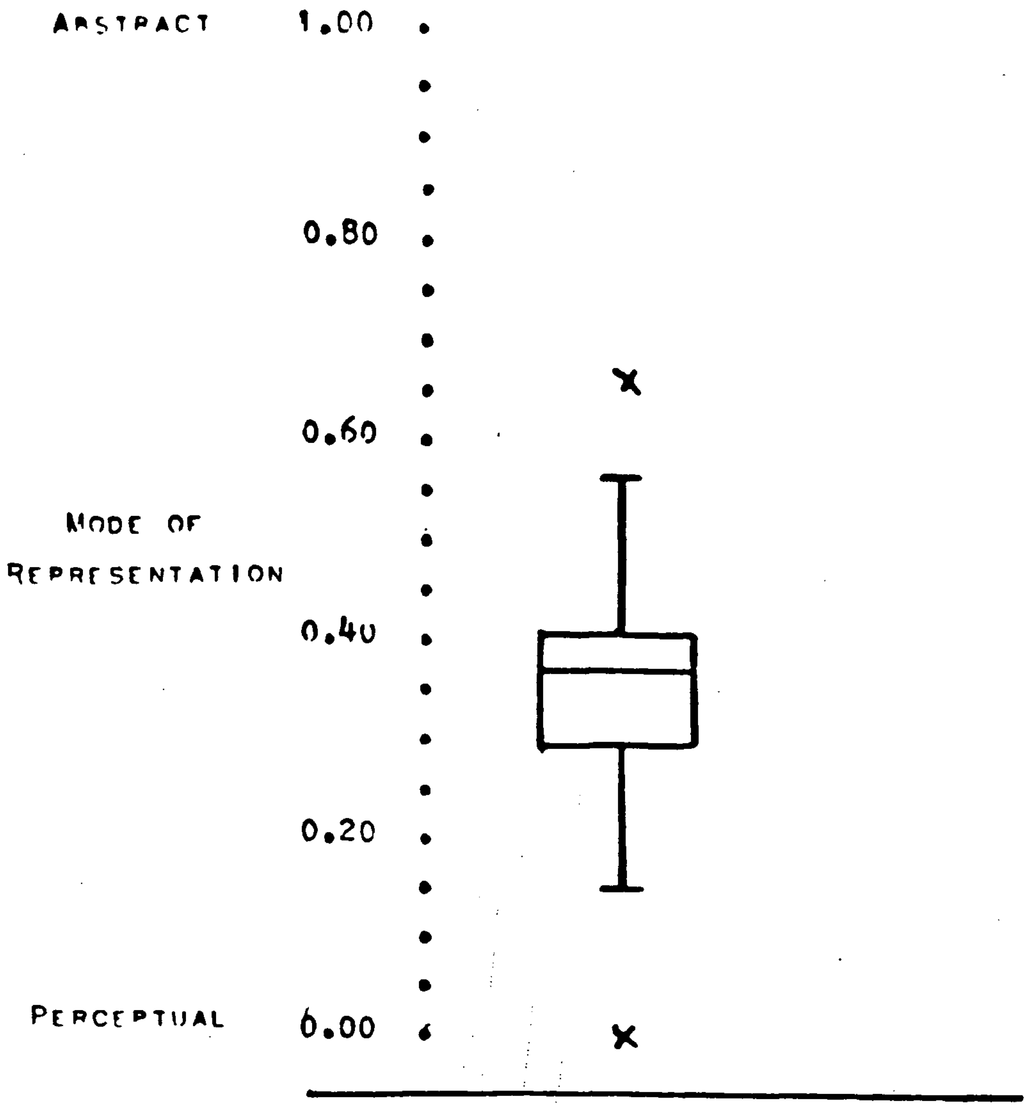


Figure 4- 4 - The distribution of arithmetic emphasis scores (illustrative boxplot).

parentheses after the X. The completed boxplot is shown in Figure 4-4.

2. CONTENT EMPHASIS

2.1 The Emphasis Given To Arithmetic

Figure 4-5 is a stem-and-leaf plot which shows the proportion of time each teacher spent on arithmetic in his or her class. It is a modified version of Figure 4-1. The distribution is close to normal in form and has a median value of 0.35. The lower fourth is 0.28, the upper fourth is 0.41, and the fourth-spread is 0.13. Thus, about half of the teachers spent between 28% and 41% of their time on this review area of Mathematics 8.

very light emphasis	0	0
	1	56
light emphasis	1	89
	2	000234
moderate emphasis	2	55667788889
	3	000112334445555667778888889
	4	0011122333344567
	5	
heavy emphasis	5	11477
	6	6
very heavy emphasis	6	
	7	
	8	N=70
	9	
	10	

Figure 4- 5 - Distribution of arithmetic emphasis scores.

Three of the teachers in this study had content emphasis scores for arithmetic which were in the very light emphasis category. One of these teachers spent no time at all on this area. An additional eight teachers had scores within the light emphasis category. Altogether 11 teachers devoted less than 25% of their operational curriculum to arithmetic.

No teacher gave arithmetic very heavy emphasis. Six teachers, however, did give arithmetic heavy emphasis by spending over 50% of their instructional class time on that area. The remaining 53 teachers,[2] 76% of the total, gave arithmetic moderate emphasis. These teachers thus allocated between one-quarter and one-half of their instructional time to arithmetic content.

Since student assessment results have not always been satisfactory in arithmetic for Grade 8 students (e.g., Robitaille, 1981,pp. 134-142), one can argue that arithmetic should continue to be taught to students at this grade level. However, it is not clear that instruction in these topics for all students is necessary or desirable. One might expect that differences in the amount of time teachers give to arithmetic would be related to the achievement level of the class with low achievement classes receiving more review of arithmetic content than high achievement classes. There was, in fact, some tendency in this direction in the operational curricula which were investigated.

The distributions of arithmetic emphasis scores for the low

---

[2] The data for this variable consist of 70 scores. There were more missing data for this variable than for the other curriculum variables. This was due to the fact that a teacher could not have a score on this variable unless he or she returned all five TSQs and in each case provided a response for the time allocation items.

and high achievement classes are shown separately in Figure 4-6. Although only four teachers of low achievement classes spent under 30% of their instructional time on arithmetic, ten teachers of high achievement classes did so. While three teachers of low achievement classes devoted over 50% of their courses to arithmetic, no teacher of a high achievement class spent that much time on that area.

very light emphasis		0	
	6	1	
light emphasis	9	1	8
	0	2	00234
moderate emphasis	7	2	5669
	8877554443321	3	0016678
	6330	4	1134457
		5	
heavy emphasis	71	5	
	6	6	
very heavy emphasis		6	
		7	
		8	
	$N_L=24$	9	$N_H=24$
		10	
	low achievement classes		high achievement classes

Figure 4- 6 - Distribution of arithmetic emphasis scores for low and high achievement level classes.

The expected differences in arithmetic emphasis for low and high achievement level classes were found when extreme cases were considered. Otherwise, the distributions were similar. In 75% of each group of classes the emphasis of arithmetic was moderate. The median values of emphasis for arithmetic were 0.35 and 0.31 for the low and high achievement classes respectively. Thus, while more time was spent on arithmetic content in the median low achievement level class than in the median high achievement level class, the difference was slight.[3]

These results provide some cause for concern, particularly with regard to the amount of time that was allocated to arithmetic in most high achievement level classes. It is arguable that too much time was spent teaching arithmetic in these classes. It is not clear, however, on what basis these teachers decided to allocate this much time to review material. It is possible that they were not aware of the achievement level of their classes.[4] It is also possible that the inclusion of arithmetic topics in the authorized texts was an influential factor in teachers' decisions regarding content selection. A third possibility is that these teachers believed that an extensive review of arithmetic would further enhance performance and retention for their high achievement level classes. In any event, the finding that in most high achievement level classes arithmetic received the same level of moderate emphasis that it received in most low achievement level classes could indicate that there is a need to specify options in the formal curriculum for classes of low and high achievement in this area and to

---

[3] For all classes the correlation between the class Core Pretest mean and the content emphasis score for arithmetic was -0.23. This indicates a weak tendency for more arithmetic instruction in lower achievement classes.

[4] Subsequent analysis of the B.C. SIMS data has shown a strong positive association between these teachers' perception of the achievement level of their classes and achievement level based on core pretest scores. Further discussion of this issue is provided in Chapter

provide the necessary instructional materials.

2.2 The Emphasis Given To Algebra

Figure 4-7 shows the proportion of time each teacher in this study allocated to algebra content. This distribution has a median of 0.29, a lower fourth of 0.24 and an upper fourth of 0.35. Each of these values is somewhat lower than the corresponding value for arithmetic indicating less emphasis in the implemented curriculum on algebra than arithmetic. The fourth-spread is 0.11 indicating just slightly less than the level of variation which was present in the arithmetic emphasis scores for the middle half of the distribution.

very light emphasis	0	
	1	66
light emphasis	1	889
	2	0122222333334
moderate emphasis	2	56667777788888899
	3	00001111222344555567889
	4	12233336788
	5	0
heavy emphasis	5	
	6	
very heavy emphasis	6	
	7	
	8	
	9	N=70
	10	

Figure 4- 7 - Distribution of algebra emphasis scores.

Most teachers, 52 out of 70 or 74%, gave moderate emphasis to algebra in their operational curricula. No teacher gave this content area heavy or very heavy emphasis. On-the-other-hand, 16 teachers gave algebra light emphasis and two teachers gave it very light emphasis. Algebra was, however, part of every operational curriculum receiving no less than 16% of the instructional time.

It is not clear that one should expect substantial differences in the amount of time given to algebra in high achievement classes as compared to low achievement classes. One might expect that those teachers of high achievement level classes who spent relatively little time reviewing arithmetic would have spent a correspondingly larger amount of time on algebra, geometry or other content such as probability. Figure 4-8 shows the distributions of algebra emphasis scores for the achievement groups separately. Although the distributions are not identical, they are certainly similar. The median proportion of time given to algebra in low achievement classes was 0.31 compared to 0.29 in high achievement classes. In both cases the lower fourth is 0.27. In five classes in each group, algebra received light emphasis.

The major difference between the two achievement distributions involves the number of classes in which over 40% of the instructional time was devoted to algebra. For low achievement classes this number was eight, for high achievement classes it was two. As a result, the upper fourth for low achievement classes is 0.43 compared to 0.32 for high achievement classes. Thus, the hypothesis that high achievement classes might show a stronger tendency to emphasize algebra was

very light emphasis		0		
		1	6	
light emphasis		1		
	32222	2	0134	
moderate emphasis	998776	2	6778888	
	76220	3	0001112355	
	88633322	4	17	
		5		
heavy emphasis		5		
		6		
very heavy emphasis		6		
		7		
		8		
$N_L=24$		9		$N_H=24$
		10		
	low achievement classes		high achievement classes	

Figure 4- 8 - Distributions of algebra emphasis scores for low and high achievement classes.

not borne out. The median, upper fourth, and lower fourth were all higher for the low achievement class scores than for the high achievement class scores.[5]

2.3 The Emphasis Given To Geometry

Figure 4-9 shows the proportion of time each teacher spent teaching geometry content. This distribution has a median of 0.36 and lower and upper fourths of 0.28 and 0.44 respectively. Thus, about half of the teachers spent between 28% and 44% of their time on geometry, an area that included topics which are

[5] For all classes the correlation between the class Core pretest mean and the content emphasis score for algebra was -0.15. This indicates a very weak tendency for more algebra instruction in lower achievement classes.

not part of the formal curriculum in prior grades.

Five of the teachers gave very light emphasis to geometry. Of these, two spent no time at all teaching geometry. An additional 10 teachers gave light emphasis to this content area.

None of the content emphasis scores for geometry were within the very heavy emphasis category. Seven scores, however, were within the heavy emphasis category.

very light emphasis	0	0077
	1	1
light emphasis	1	7889
	2	233444
moderate emphasis	2	57888999
	3	003335555666677778899
	4	111233344456677779
	5	0
heavy emphasis	5	12355
	6	46
very heavy emphasis	6	
	7	
	8	
	9	N=70
	10	

Figure 4- 9 - Distribution of geometry emphasis scores.

As with the other two content areas, most of the geometry emphasis scores were in the moderate category; 48 of the 70 scores (69%). This value is somewhat below the corresponding values for arithmetic and algebra.

As with algebra, it was not clear whether to expect teachers of low achievement classes to spend more or less time on geometry than teachers of high achievement classes. One might expect less emphasis on geometry in low achievement classes due to a greater stress on arithmetic. Alternately, one might expect more emphasis on geometry in low achievement classes due to the possibilities for student exploration and the use of concrete materials in teaching this content area.

In fact, teachers of low achievement classes tended to spend less time on geometry than teachers of high achievement classes as shown in Figure 4-10. Both distributions appear approximately normal and appear to have similar spreads but differing central values. The median value for the low achievement classes is 0.32 compared to 0.40 for the high achievement classes. The lower fourths of the two distributions are 0.23 and 0.30; the upper fourths are 0.39 and 0.47. While the content emphasis in nine low achievement classes was light or very light, this was true in only two high achievement classes. While geometry received heavy emphasis in four high achievement classes, it received this degree of emphasis in only one low achievement class. All of these descriptive statistics reinforce the differing visual features of the two distributions.[6]

The greater emphasis which geometry received in high achievement classes compared to the emphasis it received in low achievement classes may represent an undesirable state of affairs. Geometry approached in an informal and experiential

---

[6] For all classes the correlation between the class Core pretest mean and the content emphasis score for geometry was +0.28. This indicates a weak tendency for more geometry instruction in higher achievement classes.

very light emphasis	70	0	7	
		1		
light emphasis	987	1	8	
	4432	2		
moderate	885	2	899	
	9877665	3	0035668	
	7442	4	11334779	
		5		
heavy emphasis	5	5	123	
		6	4	
very heavy emphasis		6		
		7		
		8		
$N_L=24$		9		$N_H=24$
		10		
	low achievement classes		high achievement classes	

Figure 4-10 - Distributions of geometry emphasis scores for low and high achievement level classes.

manner is probably just as important, if not more important, for the low as for the high achievement student. If so, strategies which could reduce the time needed to review and extend arithmetic content with low achievement classes, such as a greater use of calculators, need to receive more consideration.

2.4 Comparisons Among The Content Areas

Figure 4-11 shows the distributions of emphasis scores for the three content areas. Boxplots have been used to facilitate comparisons. As noted above, the median time allocations were: arithmetic, 0.35; algebra 0.29; and geometry, 0.36. While the algebra distribution has the lowest median value, it also has the lowest spread indicating that somewhat more uniformity in emphasis occured for algebra than for the other two areas. In contrast, the geometry distribution shows the greatest spread. The F-spread values for arithmetic, algebra, and geometry are 0.13, 0.11, and 0.16 respectively. Despite these differences, however, the overall patterns of emphasis for the three content areas are strikingly similar.

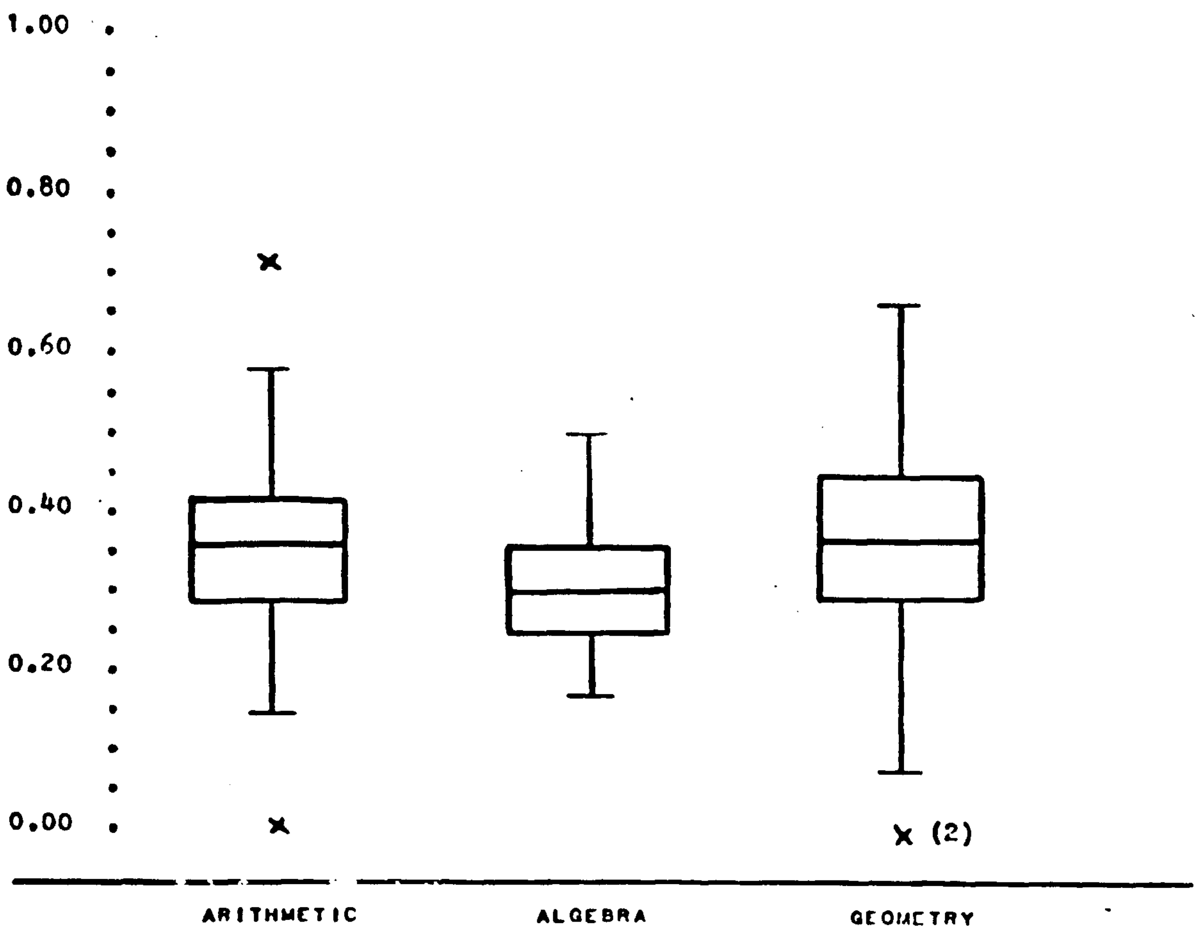


Figure 4-11 - Distribution of content emphasis scores.

In Table 4-1 are the percent of teachers whose content emphasis scores fell into each of the five categories for each content area. For each area the majority of teachers provided a moderate level of emphasis. No teacher gave a content area very heavy emphasis and only 5% of all scores were within the very light range. The corresponding values for heavy and light emphasis were 6% and 16% respectively. Despite the preponderance of moderate emphasis scores, 60% of all teachers surveyed gave at least one area light or very light emphasis.

Table 4- 1 - Percent of Teachers Scoring Within Each Level of Content Emphasis for Each Content Area

Content Area	Content Emphasis Distribution (% of scores in Category)					Median Score
	Very Light	Light	Moderate	Heavy	Very Heavy	
Arithmetic	4.3	11.4	75.7	8.6	0.0	0.35
Algebra	2.9	22.9	74.3	0.0	0.0	0.29
Geometry	7.1	14.3	68.6	10.0	0.0	0.36

Figure 4-12 shows the distributions of content emphasis scores for low and high achievement classes for each content area. As noted above, geometry received greater emphasis in high achievement classes than in low achievement classes. Both arithmetic and algebra received greater emphasis in low achievement classes.

The fact that the distribution of algebra emphasis scores for high achievement classes has relatively short tails as well as a relatively small F-spread of 0.05 means that the scores show less variation from the median than cases where distributions have longer tails and larger F-spreads. With the exception of the three outliers, all of the scores are rather tightly bunched. In fact, none of these three values would be outliers within any of the other distributions.

This shows further the relative uniformity among the teachers of high achievement classes in their time allocations to algebra. In contrast, the distribution of arithmetic emphasis scores for low achievement classes while having a small F-spread has relatively long tails. This means that while the middle half of the teachers in this group emphasized arithmetic rather uniformly, there was relatively high diversity among the other half of the teachers in this group. Each of the other four distributions shown in Figure 4-12 has an F-spread of either 0.16 or 0.17. Thus, there was much more variation among the middle half of the teachers in these cases than in the two already discussed. The geometry emphasis distributions for both low and high achievement classes show the greatest overall variation since the F-spreads are realatively large and the tails at both ends are considerably longer than is the case for the other distributions. Thus, even when class achievement is taken into consideration, one finds that the least uniformity occured regarding how much time should be spent teaching geometry.

2.5 Content Emphasis Of Teachers And Textbooks

It has been asserted fairly frequently that school mathematics instruction is textbook oriented in that a text is usually used and closely followed (e.g., Begle, 1973; Fey, 1979). One aspect of this study was to explore the strength of the link between this component of the formal curriculum and the operational curriculum of the classroom. Specifically, were

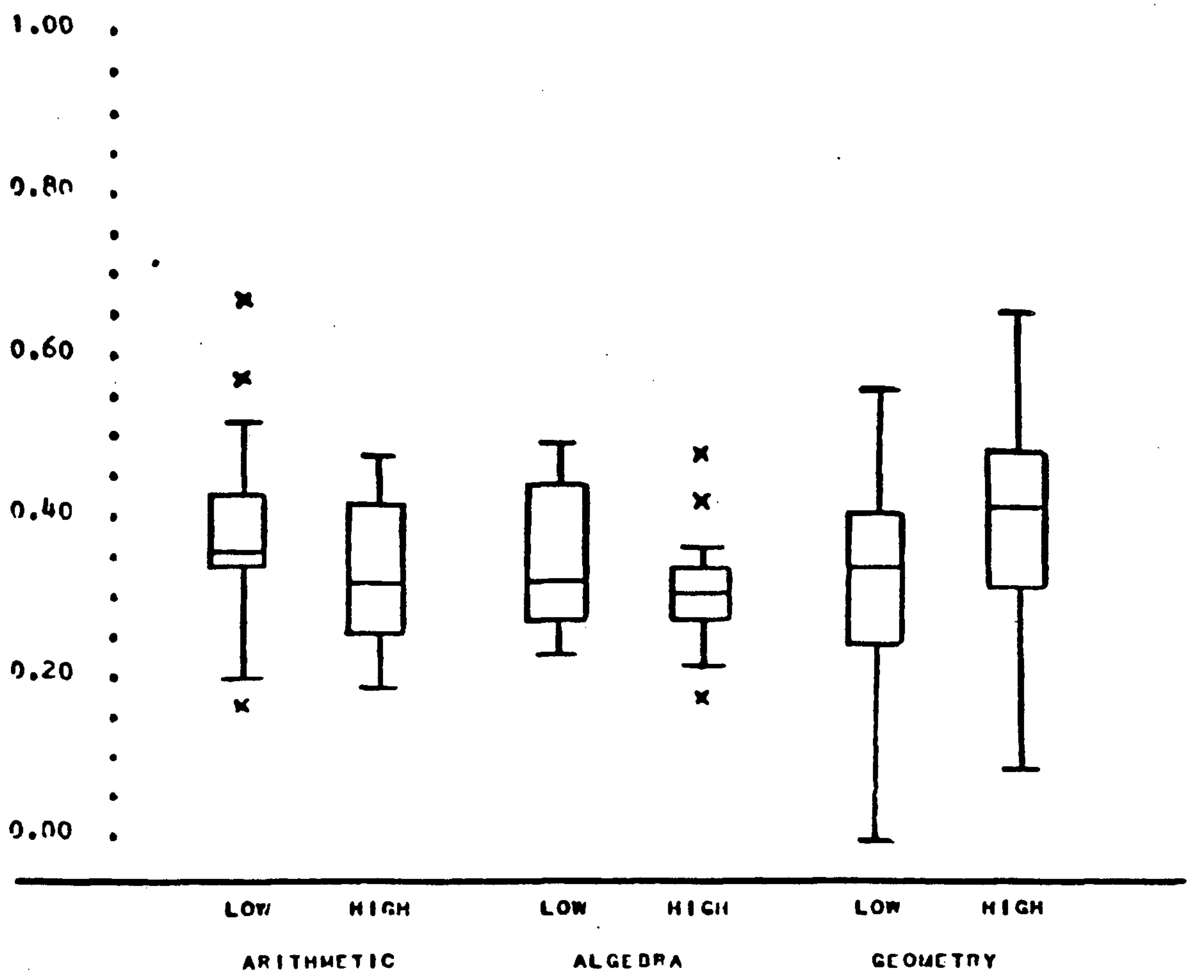


Figure 4-12 - Distributions of content emphasis scores for low and high achievement level classes.

differing emphases in textbooks reflected in the instruction of teachers using those books?

The B.C. Mathematics Curriculum Guide authorizes the use of three textbooks for Mathematics 8. In practice, two of these books are used with about the same degree of frequency by teachers while the third book is seldom used as the basic text (Robitaille, 1981, p. 244). The two widely used books are <u>Mathematics II</u> (Sobel & Maletsky, 1971) and <u>School Mathematics 2</u>

(Fleenor, Eicholz, & O'Daffer, 1975). Table 4-2 shows the percent of each textbook devoted to arithmetic, algebra, geometry, and other content.[7] Although "other content" includes a sizeable proportion of the content of each textbook, it consists primarily of the material at the end of each book. Moreover, most of this material is in areas such as trigonometry and probability which are not part of the formal curriculum at this grade level. An inspection of Table 4-2 shows that the two

Table 4- 2 - Percent of the Commonly Used Textbooks Devoted to Each Content Area.

	Content Area			
Textbook	Arithmetic	Algebra	Geometry	Other
School Mathematics 2	28.1	16.0	22.0	33.9
Mathematics II	18.2	16.3	39.0	26.5

texts differ markedly in the emphasis given to arithmetic and geometry while they provide nearly identical emphasis to algebra. Arithmetic receives 54% more emphasis in School Mathematics 2 than in Mathematics II while geometry receives 77% more emphasis in Mathematics II than in the other text. In contrast to these large differences, algebra receives just 2% more emphasis in Mathematics II.

[7] These percents were determined by first categorizing each page of a text which contained mathematical content according to the content area with which it dealt. The total number of pages devoted to arithmetic in a text, for example, compared to the total number of pages in that text which contained mathematical content was then used as the percent of that text devoted to arithmetic.

Figure 4-13 displays the distributions of content emphasis scores separately according to the basic textbook used in each class. The median values for the algebra emphasis distributions are almost identical at 0.30 and 0.28. The medians for the arithmetic distributions differ much more. For classes which used <u>School Mathematics 2</u> the median for arithmetic is 0.40 while for classes which used <u>Mathematics II</u> the median is 0.29. This difference is consistent with the difference in emphasis in the books themselves. The difference between the medians of the two distributions of geometry emphasis scores is similarly consistent with the difference in emphasis of geometry in the two texts. For classes which used <u>School Mathematics 2</u> the median for geometry is 0.28 while for classes which used <u>Mathematics II</u> the median is 0.41.

These results can be interpreted as supporting the hypothesis that the content of the formal curriculum as embodied by a textbook has an observable influence on the operational curricula of teachers. The two books which were used as basic texts contained virtually the same number of pages of algebra material and the distributions of algebra emphasis scores for the groups of teachers using each book were very similar. Likewise, the differences in emphasis of arithmetic and geometry content in the books were associated with consistent differences between the operational curricula of the teachers using the books.

The association which was found between textbook emphasis of content and teacher emphasis of content can also be

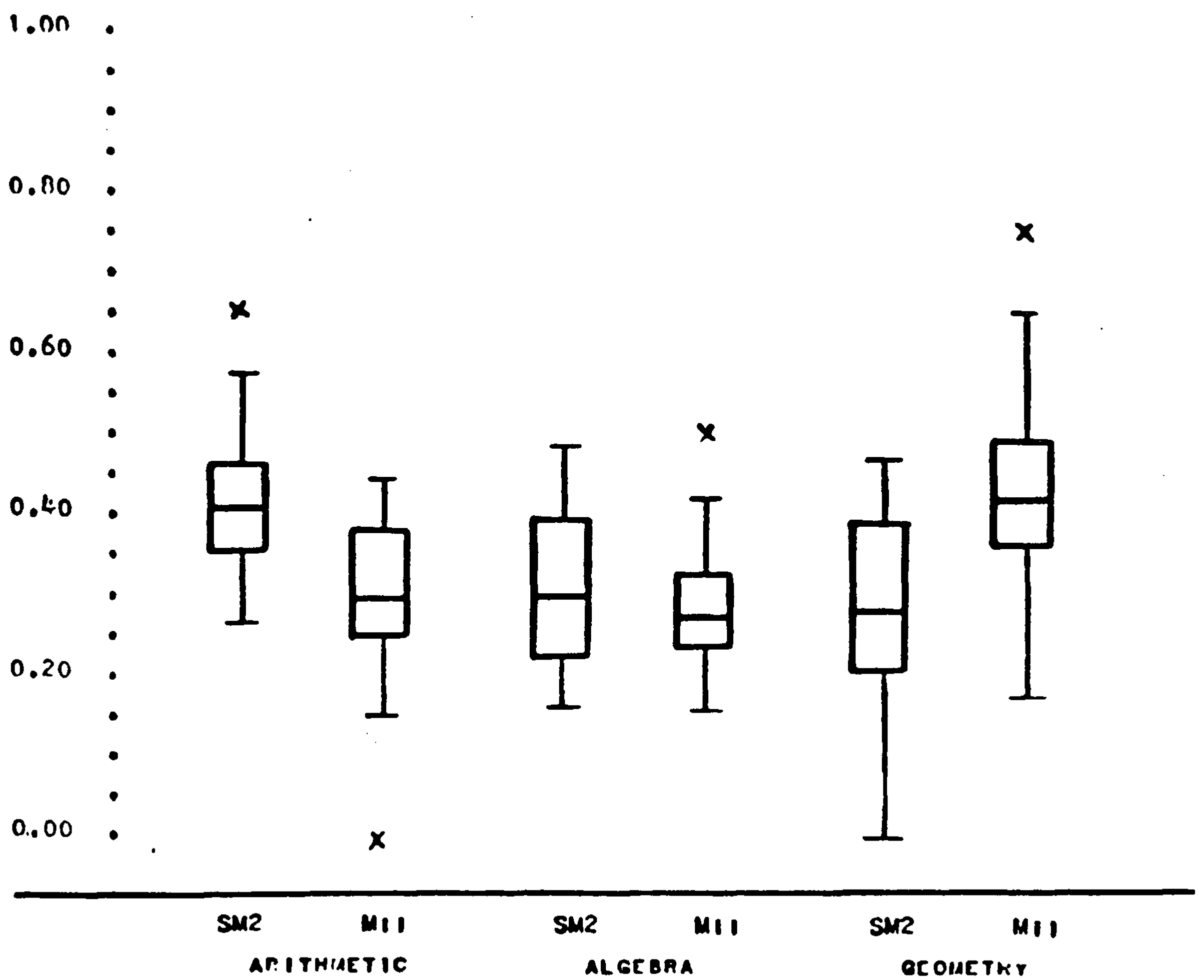


Figure 4-13 - Distributions of content emphasis scores for classes using each of the commonly used textbooks.

interpreted as supporting the validity of the self-reports of the teachers in this study. If one takes as a given that the operational curricula will be strongly influenced by the coverage given to content in the textbooks, then the self-reports of teachers are valid only insofar as there is a correspondence between the content emphasis teachers reported and the emphasis given to content in the books they used. For the teachers who participated in this study such a correspondence did exist.

3. MODE OF CONTENT REPRESENTATION

The content representation variable was defined so that a score of 0.0 indicates the use of only perceptual approaches to content in teaching a topic while, at the other extreme, a score of 1.0 indicates a reliance on only abstract approaches. In Table 4-3 the percentage of teachers whose mode of representation scores are in each of three categories[8] is given for each topic, each content area and overall. The content area and overall scores were obtained by averaging the appropriate topic scores for each teacher. The table also contains median content representation scores as well as the proportion of the teaching methods contained in the TSQs which were classified as abstract.

The median content representation score across all topics is 0.57 indicating that overall teachers used abstract approaches to their course content somewhat more frequently than perceptual approaches. Slightly over one-third of the overall scores are within the abstract representation category, with 63% in the balanced category. Only 2% of the scores are below 0.40 and thus classified as indicating a perceptual orientation to content. The distribution of overall content representation scores is shown in Figure 4-14. It is nearly normal in form and contains two scores in the highly abstact category. These

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[8] The somewhat perceptual and highly perceptual categories of content representation are combined into a single perceptual category for this table. The somewhat abstract and highly abstact categories are similarly combined. The original categorization is used in the stem-and-leaf plots which follow.

Table 4- 3 - Mode of Content Representation Scores

Topic or Area	Mode of Representation (% of Scores in Category)			Proportion of Abstract Methods in TSQs	Median Representation Score
	Perceptual	Balanced	Abstract		
Fractions	32.1	55.1	12.8	0.60	0.44
Fraction Addition	48.1	32.0	20.0	0.34	0.40
Decimals	0.0	23.2	76.8	0.50	0.71
Decimal Operations	0.0	1.2	98.8	0.67	1.00
Arithmetic Topics	2.4	36.5	61.2	0.52	0.62
Integers	90.0	10.0	0.0	0.40	0.25
Integer Addition	65.0	32.5	2.5	0.33	0.33
Integer Subtraction	1.3	27.5	71.2	0.67	0.67
Integer Multiplication	1.2	13.6	85.2	0.75	0.80
Formulas	3.9	42.9	53.2	0.40	0.67
Algebraic Topics	0.0	84.0	16.0	0.52	0.54
Angle Sum Theorem	90.4	8.2	1.4	0.29	0.0
Pythagorean Theorem	61.4	21.4	17.1	0.50	0.33
π	10.1	21.7	68.1	0.43	0.83
Area of a Parallelogram	7.2	15.9	76.8	0.63	0.71
Volume of a Prism	27.2	15.9	56.5	0.33	0.67
Geometric Topics	25.9	46.9	27.2	0.45	0.54
All Topics	2.3	63.2	34.5	0.49	0.57

teachers used abstract methods over four times as frequently as perceptual methods. No scores in the highly perceptual category are contained in the distribution.

An inspection of scores at the topic level shows that several topics such as integers and the angle sum theorem were typically presented using perceptual modes of representation almost exclusively. Other topics, such as the concept of π and decimal operations, were presented very abstractly. These results are examined in more detail in the sections that follow. Boxplots are used so that the distributions of scores can be compared.

highly perceptual	0	
	1	
somewhat perceptual	2	
	3	26.
balanced	4	12223344577789
	5	00000111122222233334455666677788889
	6	000000
somewhat abstract	6	1111122333455555678899
	7	001238
	8	
highly abstract	8	19
	9	N=87
	10	

Figure 4-14 - Distribution of content representation scores averaged over all topics.

3.1 Mode Of Content Representation For Arithmetic

In Figure 4-15 boxplots of the distributions of content representation scores for the four arithmetic topics for which this variable was defined are shown.[9] One of the plots does not appear to be a boxplot at all because 71 of the 81 scores are 1.00 causing a degenerate plot of Xs. There are, however, two basic patterns of content representation for the four arithmetic topics.

Overall, teachers relied more heavily on perceptual methods than abstract methods for fractions and addition of fractions. For each of these topics 75% of the content representation scores are at or below 0.50, the value indicating an exact

[9] In Appendix A the teaching methods given in the TSQs for the 16 topics examined in this study are given. Also included is a listing of which methods were considered perceptual and which were considered abstract for the 14 topics for which the mode of representation variable was defined.

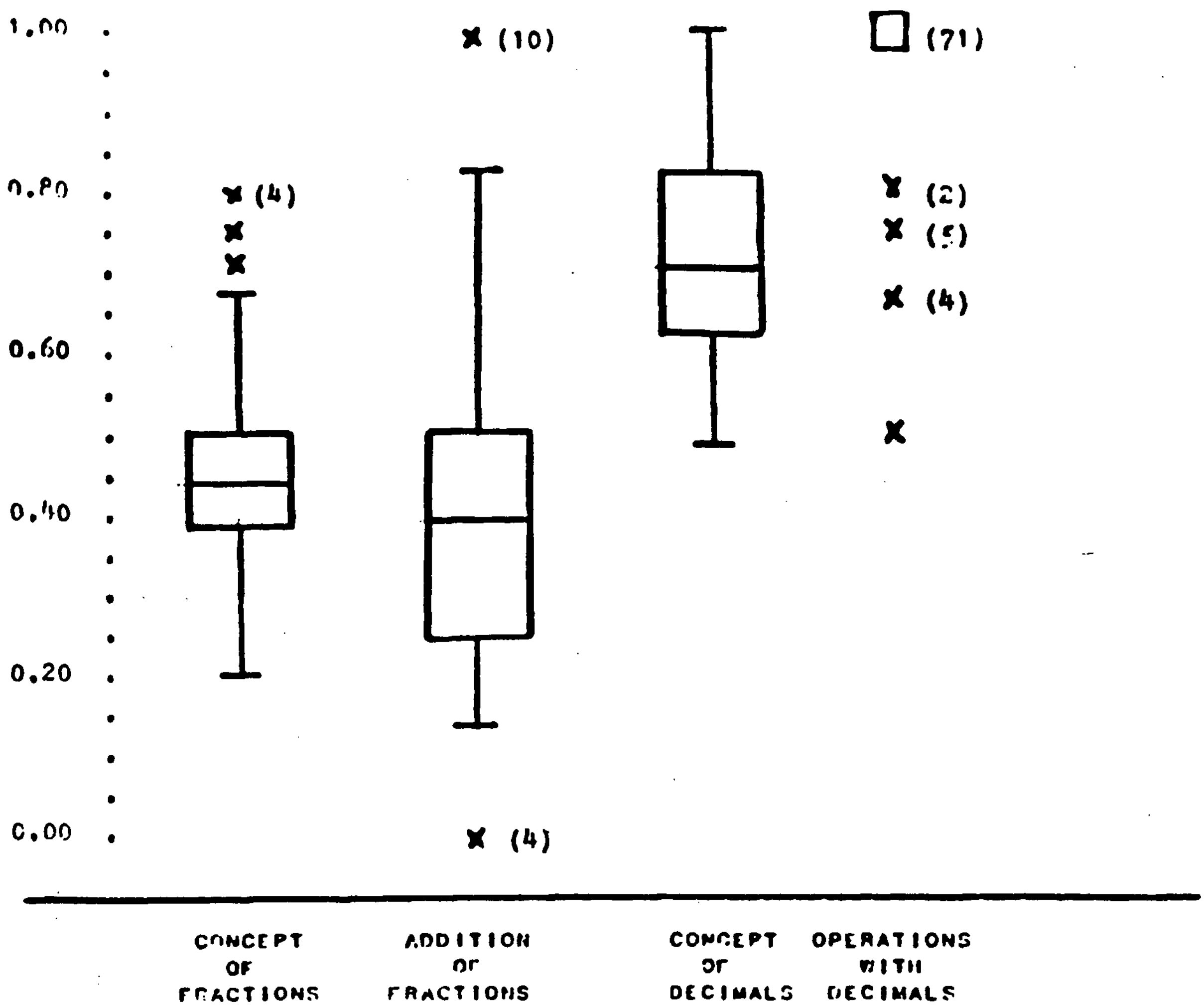

Figure 4-15 - Distributions of the content representation scores for the arithmetic scores.

balance between the two types of representation. In each case over 30% of the scores are within the perceptual categories and 20% or less are within the abstact categories. The tendency toward the use of perceptual methods by teachers was strongest for addition of fractions. The least consensus occured for this topic, however, as the F-spread is the largest and outlier scores of both 0.00 and 1.00 are present in the distribution.

Decimals and operations with decimals were both treated in an abstract manner in most cases. For both topics 75% or more of the scores are within one of the two abstract categories. The tendency toward an abstact representation of content was strongest for operations with decimals with 86% of the scores within the highly abstract range.

The arithmetic content is review material at this grade level. The abstract treatment given by teachers to the two decimal topics reflects this fact. Many teachers, on-the-other-hand, apparently felt that students still needed perceptual representations of the fraction material in spite of its review nature. Although traditionally fractions are introduced earlier than decimals in the elementary grades, teachers may believe that the difficulty many students have with fractions requires more frequent enactive and iconic representations than is the case for decimals even at the Grade 8 level.

The mean of the content representation scores for the five arithmetic topics for each teacher was taken as the overall content representation score for arithmetic for that teacher. The distribution of these scores is shown in Figure 4-16.

Out of 85 scores, 31 are in the balanced category and 38 are in the somewhat abstract category. Only two scores are in the perceptual categories, while 14 scores are in the highly abstract category. The median of the distribution is 0.64; the lower and upper fourths are 0.55 and 0.72.

Although more perceptual methods were used by most teachers for two of the four arithmetic topics, the other two topics were dealt with in an abstract way very uniformly by teachers. The effect of this is that the average scores are almost all in the

highly perceptual	0	
	1	
somewhat perceptual	2	
	3	33
balanced	4	12377789999
	5	022222335567788888
	6	00
somewhat abstract	6	11122233344567788888999
	7	011122222234577
	8	0
highly abstract	8	23599999
	9	0123 N=85
	10	00

Figure 4-16 - Distribution of overall content representation scores for arithmetic.

balanced or the two abstract categories. Thus, across all arithmetic topics over one-third of the teachers in the sample took a balanced approach between perceptual and abstract methods. Almost all of the other teachers represented content more frequently in an abstract manner than in a perceptual manner.

3.2 Mode Of Content Representation For Algebra

Boxplots of the distributions of the content representation scores for the five algebraic topics are shown in Figure 4-17. As with the arithmetic topics, two patterns are evident.

The concept of integers and the addition of integers were taught by most teachers using predominantly perceptual methods. This tendency was especially strong for the concept of integers; all but 10% of the scores for this topic were below 0.40. For each of these two topics over 18% of the teachers used no abstract methods at all.

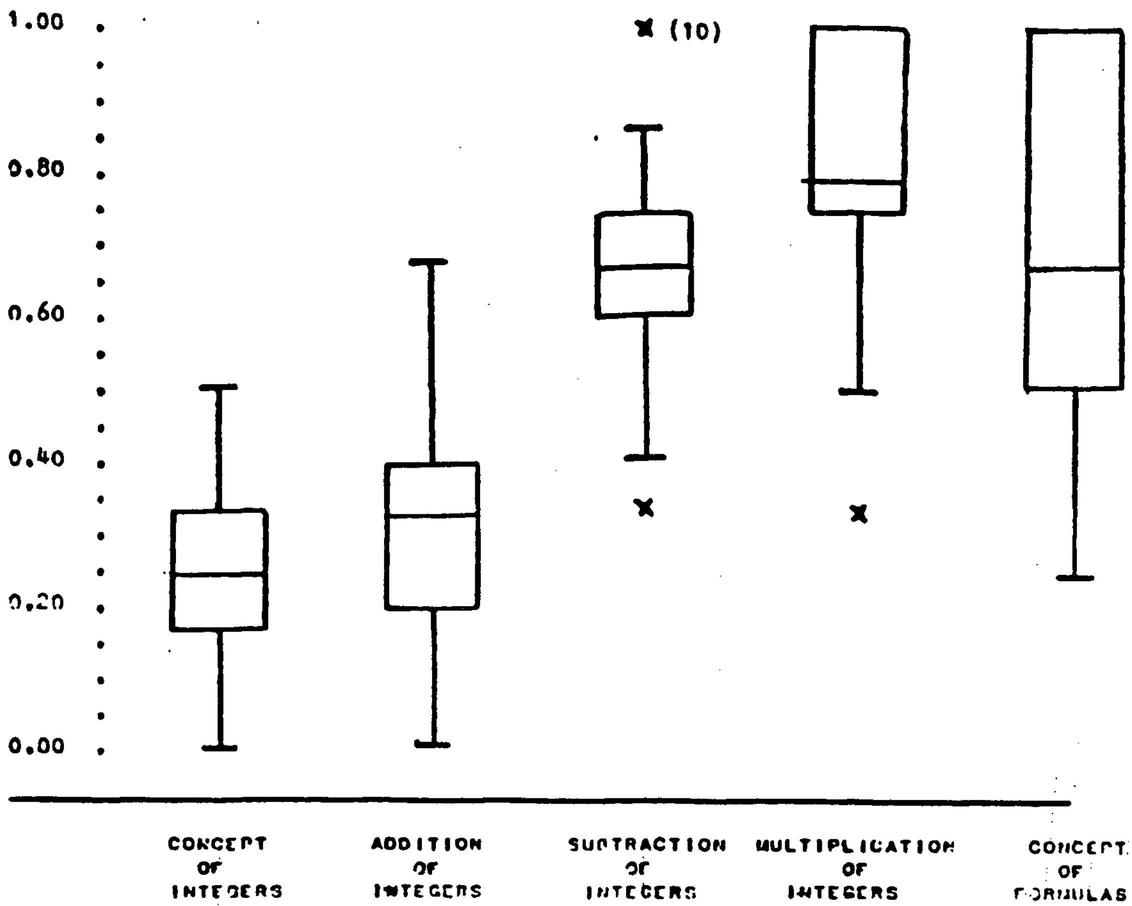

Figure 4-17 - Distributions of content representation scores for the algebraic topics.

The other two operations with integers which were investigated as well as the concept of formulas were presented by most teachers using more abstract than perceptual methods. No more than 10% of the scores for each of these distributions were below 0.50.

Teachers were generally not consistent in the type of representation with which the four integer topics were presented. Typically, the concept of integers and the operation of addition of integers were taught with a perceptual orientation. Apparently, most teachers believed that the operations of subtraction and multiplication of integers could then be presented in a largely abstract manner, as that was the usual approach.

For each teacher the mean of the content representation scores for the five algebraic topics was taken as the overall content representation score for algebra for that teacher. The distribution of these scores is shown in Figure 4-18.

```
highly perceptual     0|
                      1|
somewhat perceptual   2|
                      3|
balanced              4| 022344444555566777777889
                      5| 000000000022233444445555666667777778888899
                      6| 00
somewhat abstract     6| 23333444588
                      7| 5
                      8|
highly abstract       8|
                      9|                  N=81
                     10| 0
```

Figure 4-18 - Distribution of overall content representation scores for algebra.

Out of 81 scores, 68 are in the balanced category and 12 are in the somewhat abstract category. The remaining score is in the highly abstract category. The median for the distribution is 0.54; the lower and upper fourths are 0.47 and 0.58. Thus, a large majority of teachers, 84%, presented algebraic content using a roughly equal balance of perceptual and abstract methods. All of the other teachers relied more heavily on abstract than on perceptual methods.

While most teachers did present algebra content using a balance of both types of methods, it should be emphasized that this was not true at the level of individual topics. As was noted above, two topics tended to be presented perceptually, three abstractly. In fact, only 25% of the content representation scores for the five algebraic topics are in the balanced category. The effect of a perceptual representation of some topics by teachers and an abstract representation of others was a balanced overall representation for most teachers, however.

3.3 Mode Of Content Representation For Geometry

Boxplots for the distributions of the content representation scores for the five geometric topics are shown in Figure 4-19.

As with the arithmetic and algebraic topics there were substantial differences among the typical levels of abstraction with which each geometric topic was presented. At one extreme, the angle sum theorem for triangles and the Pythagorean theorem were usually taught perceptually; 90% and 61% of the scores for these topics respectively are below 0.40. At the other extreme, the concept of π and the area of a parallelogram were usually taught abstractly; 68% and 77% of the scores for these topics respectively are over 0.60. The fifth geometric topic, the

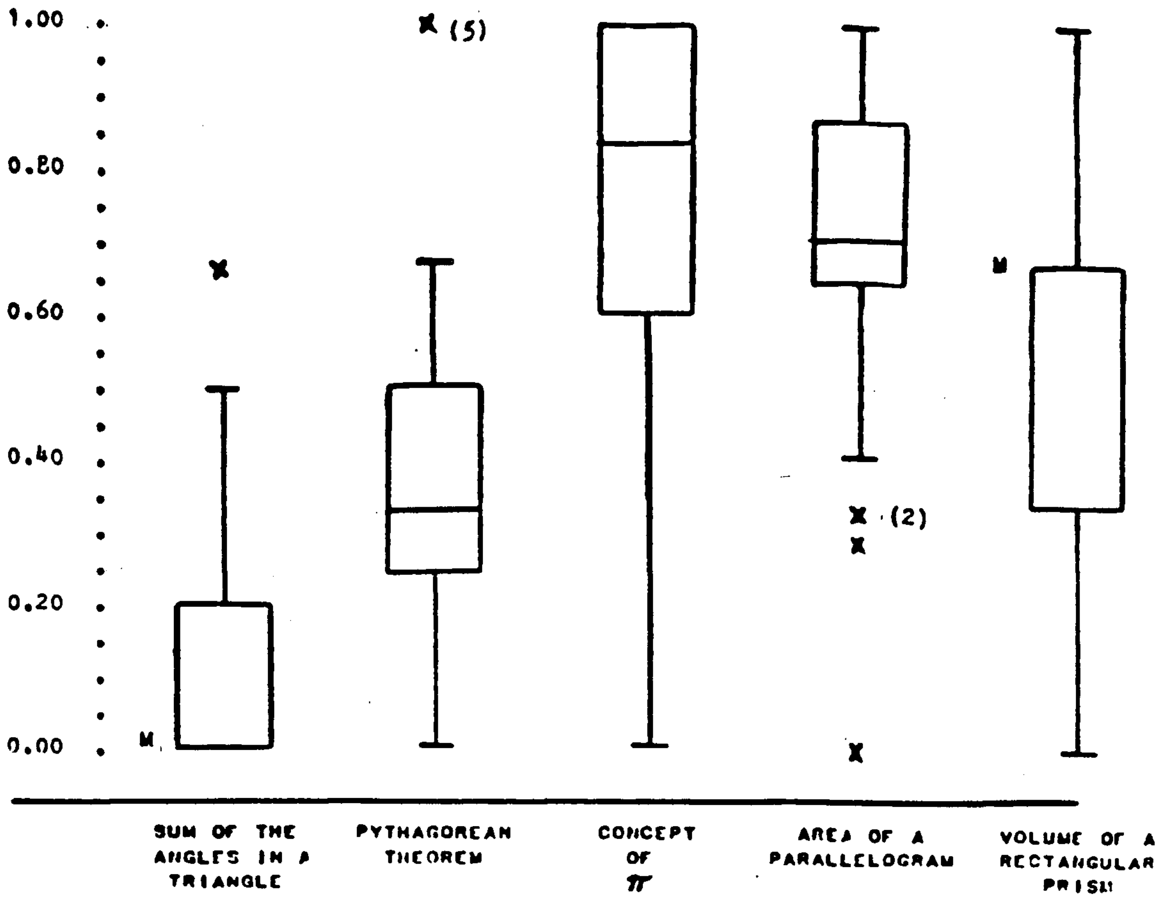

Figure 4-19 - Distributions of content representation scores for the geometric topics.

volume of a rectangular prism, was also dealt with abstractly by a majority of teachers. In contrast to the previous two topics, however, this topic was also taught with an emphasis on perceptual methods by a substantial number of teachers.

For each teacher the mean of the content representation scores for the five geometric topics was taken as the overall content representation score for geometry for that teacher. The distribution of these scores is shown in Figure 4-20.

This distribution appears roughly normal. Out of 81 scores

highly perceptual	0	06
	1	0
somewhat perceptual	2	0589
	3	2333333366689
balanced	4	0013666677888
	5	00022334456666788889999
	6	000
somewhat abstract	6	1222333558899
	7	1558
	8	
highly abstract	8	27
	9	0
	10	00

N=81

Figure 4-20 - Distribution of overall content representation scores for geometry.

39 are in the balanced category, while 17 are in the somewhat perceptual and 17 are in the somewhat abstract categories. Three scores are in the highly perceptual category and five in the highly abstract category. Thus, slightly less than half of the teachers relied about equally on perceptual and abstract methods in teaching geometry. The remaining teachers were almost evenly split between those who put more emphasis on perceptual methods and those who put more emphasis on abstract methods.

3.4 Comparisons Among The Topics And Content Areas

Substantial differences exist among the 15 topics regarding the mode of content representation employed by teachers. In Figure 4-21 the distributions for this variable are shown with topics identified by content area. At one extreme, an arithmetic topic has a median level of content representation of 0.00, while at the other extreme a geometry topic has a median level of 1.00. Since topics from all three content areas have both low and high median values, there does not appear to be any strong association between the content area and the median value of the distribution when topics are considered individually.

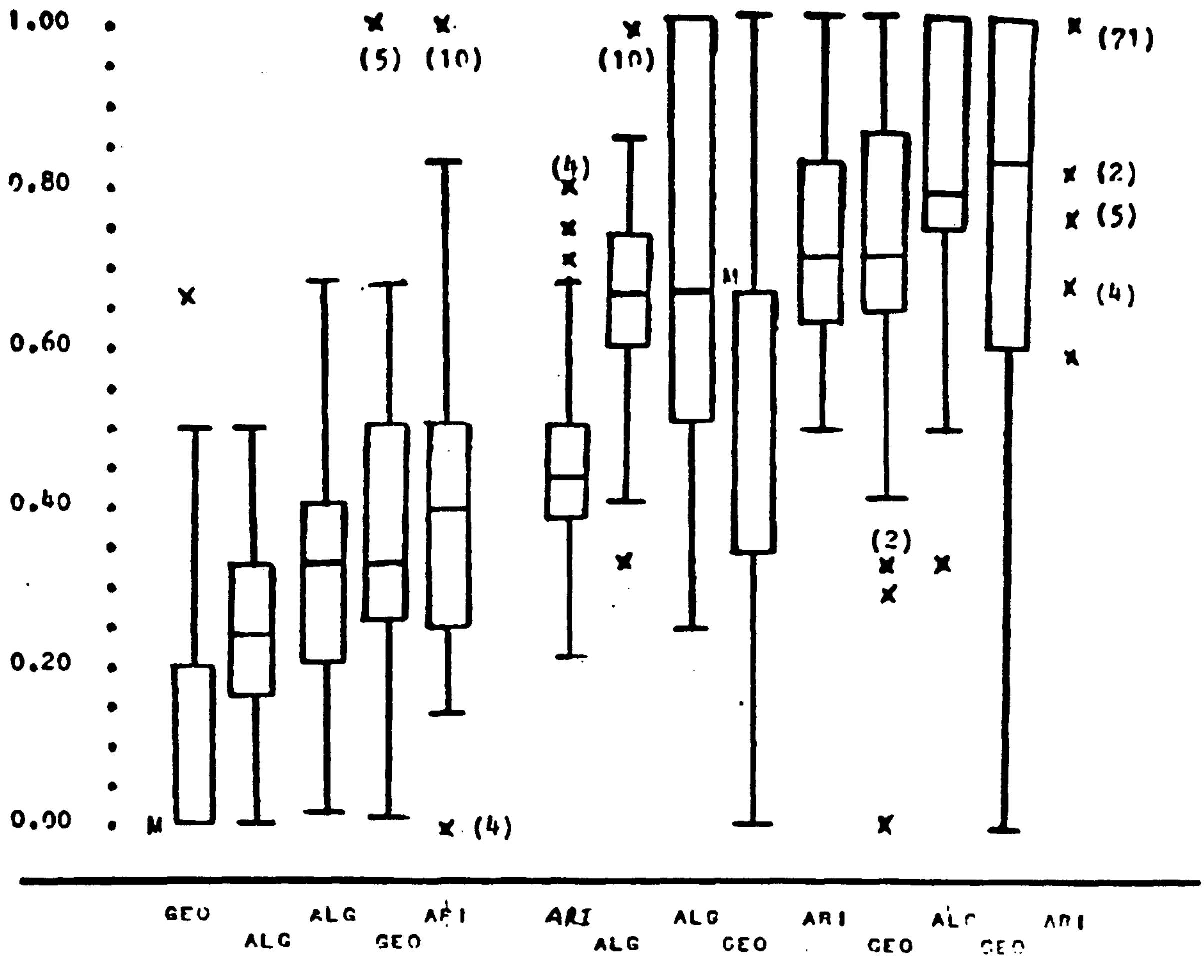

Figure 4-21 - Distributions of content representation scores for 14 topics.

When the content representation scores are averaged for each teacher both for topics within a content area and across all topics, the distributions of scores vary much less than was true for the distributions at the topic level. These distributions are shown in Figure 4-22. In contrast to the very

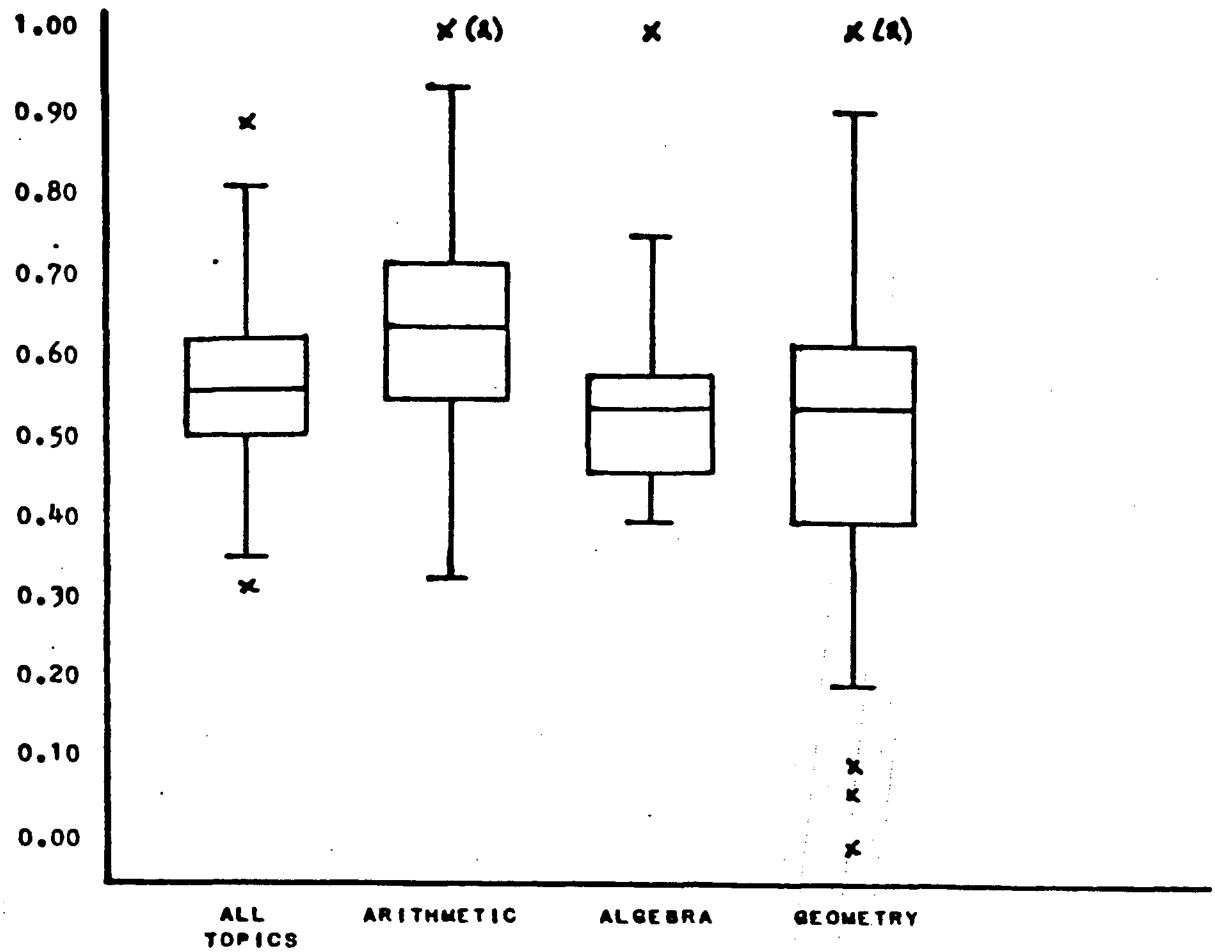

Figure 4-22 - Distributions of content representation scores for each content area and across all topics.

noticeable differences which existed among the distributions at the topic level, the distributions for the three content area scores and the overall score are more nearly similar. The algebra and geometry distributions, for example, both have identical median values of 0.54. Thus, for these content areas the median teacher used just slightly more abstract than perceptual representations of content. The median value for all topics was somewhat higher at 0.57.

While teachers typically balanced perceptual and abstract approaches to algebra and geometry content, they dealt with arithmetic content in a slightly more abstract manner. The median score for the arithmetic content distribution is 0.64, a value in the somewhat abstract category. The upper and lower fourths are also higher for the arithmetic distribution. Since the arithmetic content was largely review material, differences are not surprising.

Another difference that can be noted among the distributions is the degree of variation in the scores. In particular, in comparing the middle 50% of the distributions the largest variation occurs for geometry with an F-spread of 0.22. The F-spreads of the arithmetic, algebra and overall distributions are 0.17, 0.11, and 0.12 respectively. The tails of the geometry distribution extend further than those of the other distributions and the geometry distribution contains the largest number of outliers. These results provide further indications that these teachers showed the least uniformity in their mode of content representation for geometry.

A further comparison of the way in which teachers represented content in the three areas can be gained by a reinspection of Table 4-3. Almost no teachers dealt with arithmetic or algebra perceptually. Arithmetic was presented abstractly by a majority of teachers, while a large minority balanced perceptual and abstract presentations. A large majority of teachers used a balanced approach for algebra content. While almost half of the teachers also balanced perceptual and abstract approaches to geometry, the remainder were about evenly split between those whose preference was for perceptual methods and those whose preference was for abstract methods.

3.5 Achievement Level Comparisons

In Figure 4-23 separate distributions of content representation scores are shown for low and high achievement level classes over all topics and for the content areas. It might be expected that perceptual methods would be used more frequently in low achievement classes; such was not the case, however. The median for the overall distribution is 0.58 for low achievement classs and 0.54 for high achievement classes. The lower and upper fourths are also slightly higher for the low achievement classes.

An inspection of the distributions of scores for the content areas does not show a consistent tendency for more abstract presentations in low achievement classes. For algebra and geometry the median content representation score is slightly higher for low achievement classes indicating a more abstract presentation. For arithmetic, however, the median score is slightly higher for the high achievement classes.

Although the finding that, overall, teachers presented

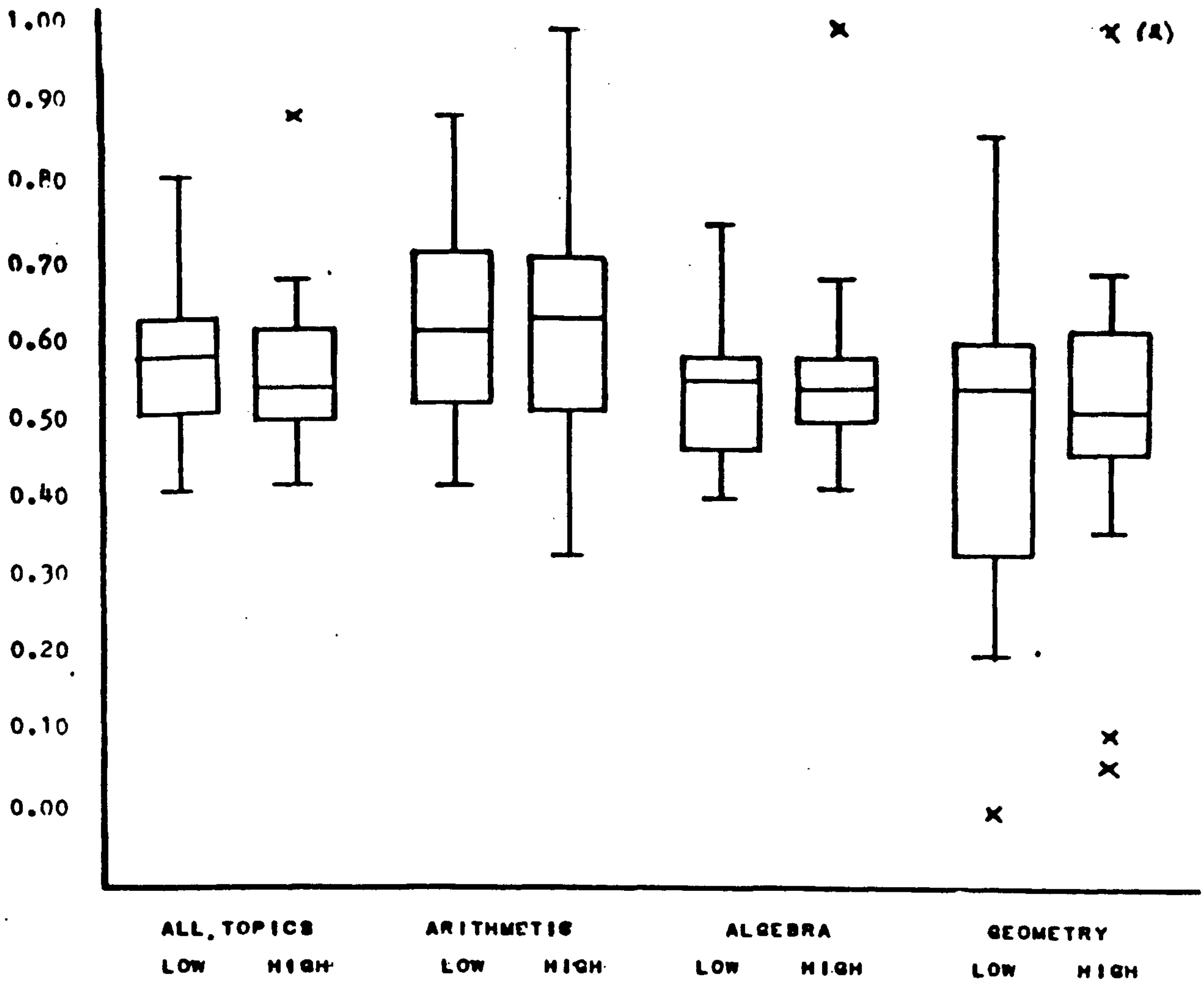

Figure 4-23 - Distributions of content representation scores for low and high achievement level classes.

content in a slightly more abstract way to low achievement classes than to high achievement classes was not replicated for all content areas, it is consistent with the findings of another recent study. Crosswhite et al. (1985) using SIMS data collected in the United States reported that "instruction tended to be more symbolic with remedial classes than with other types of classes and tended to be more symbolic when reviewing content than when covering new subject matter." (p. 24)

Although the differences found in the way content was represented to low and high achievement classes were not large, this result is disturbing. One can speculate, for example, that while the low achievement student might profit at least as much as the high achievement student from perceptual methods, differences in the motivation and behavior patterns of these types of students might mitigate against a stronger perceptual orientation in low achievement classes.

3.6 Content Representation Of Teachers And Textbooks

The two major textbooks used in Mathematics 8 classrooms stress abstract approaches to content somewhat more often than perceptual approaches. Table 4-4 shows the proportion of abstract TSQ methods to total TSQ methods that appear in each text for the three content areas and overall. Except for the treatment of geometry in *School Mathematics 2* more abstract methods are included in the texts than perceptual methods in each case. The treatment of geometry in *School Mathematics 2* includes an equal number of abstract and perceptual methods. Overall, *Mathematics II* contains the greater proportion of abstract methods. In that text 62% of the methods are abstract compared to 55% for *School Mathematics 2*. The treatment of algebra and geometry content is likewise more abstract in *Mathematics II*. Only for arithmetic content does *School Mathematics 2* contain a slightly greater proportion of abstract methods than the other text.

Figure 4-24 shows the distributions of content

Table 4- 4 - Proportion of Abstract TSQ Methods to Total TSQ Methods in the Textbooks

	Mathematics II	School Mathematics 2
Arithmetic	0.60	0.63
Algebra	0.60	0.54
Geometry	0.67	0.50
All topics	0.62	0.55

representation scores for each content area and overall separately for users of the two textbooks. In each case the median teacher used more abstract than perceptual methods and for users of both texts presentations were more abstract for arithmetic than for the other content areas. For users of Mathematics II the median content representation scores are 0.68, 0.53, and 0.51 for arithmetic, algebra, and geometry respectively. For School Mathematics 2 the corresponding values are 0.63, 0.55, and 0.56. The way content was typically represented by teachers in instruction and the way it was represented in the textbooks are consistent in that in both instances slightly more abstract methods occurred than perceptual methods. The median content representation score across all topics for users of School Mathematics 2 is 0.57 compared with a proportion of 0.55 abstract methods in the textbook itself. Similarly, the median content representation score for users of Mathematics 11 is 0.55 and the proportion of abstract methods in that textbook is 0.62.

Although median content representation scores and the two

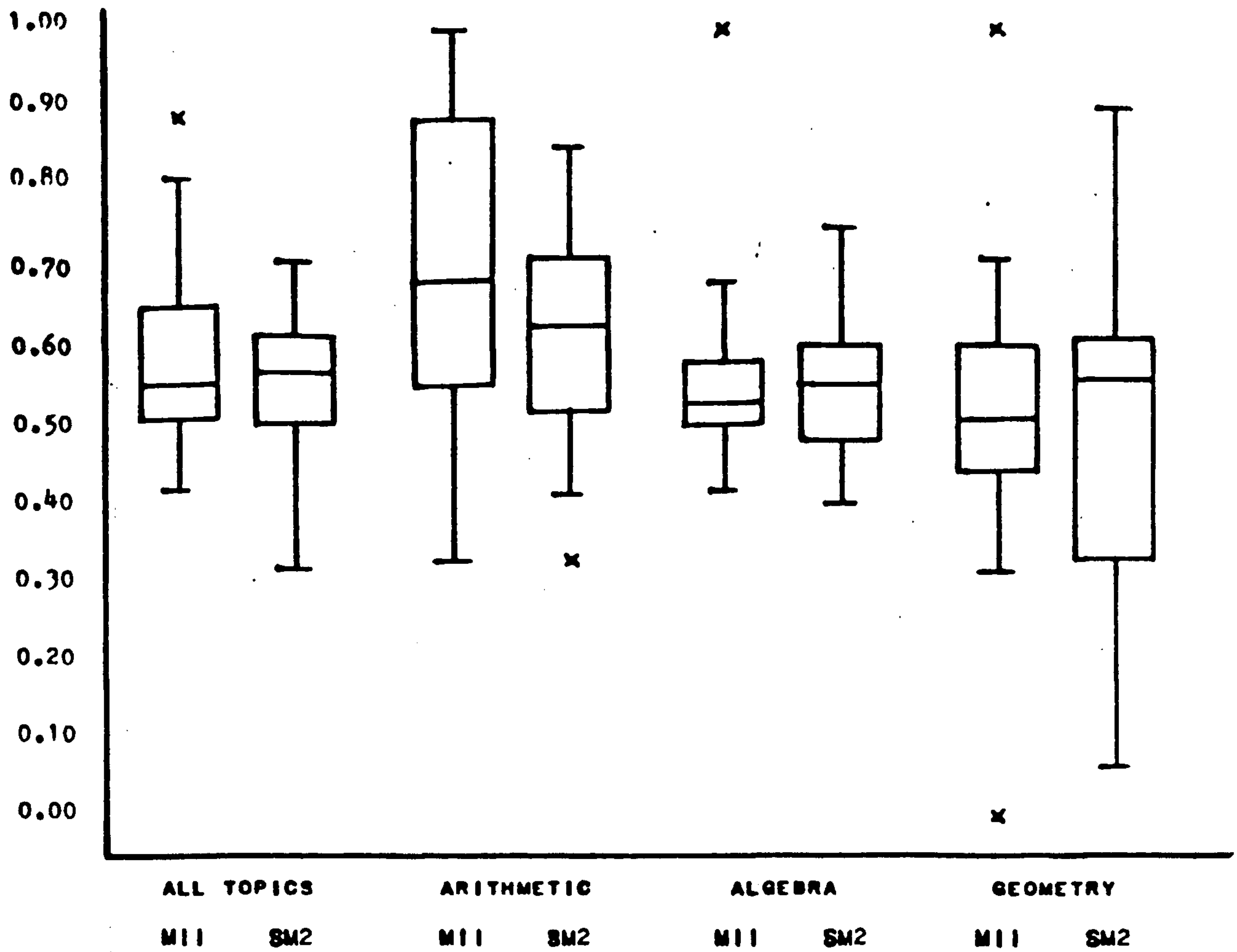

Figure 4-24 - Distributions of Content Representation Scores for Classes Using Each of the Commonly Used Textbooks.

textbooks all showed a greater stress on abstract than perceptual methods, the differences between the content representation scores of the users of the two textbooks were not consistent with the differences between the textbooks themselves. Thus, while Mathematics II contained a slightly more abstract treatment of content than the other text, the users of Mathematics II displayed a slightly less abstract orientation in their classroom presentations than the users of the other text. This inconsistency might be a result of the small differences involved. Alternately, it might be the case that the contents of a textbook influence teachers' choices of the topics they will teach and the emphasis they will give to content areas more strongly than their choices of how they will represent content during instruction.

4. RULE-ORIENTEDNESS OF INSTRUCTION

Table 4-5 shows the percentage of teachers whose rule-orientedness scores are in each of three categories[10] for each topic for which this variable was defined, for the content areas of algebra and geometry, and overall. Median scores are also included in the table. The content area and overall scores were determined for each teacher by averaging the scores of the appropriate topics for that teacher.

The median overall rule-orientedness score is 0.47. Thus, teachers typically explained concepts, operations, and principles just slightly less often by stating a computational rule, a definition or a theorem followed by examples than by using a physical interpretation, the investigation of a pattern or some other non-rule-oriented method. Just over half of the overall scores, 52%, are in the balanced category while 26% are in the non-rule-oriented category and 22% are in the rule-oriented category. Thus, almost a quarter of the teachers

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[10] For this table the highly non-rule-oriented and somewhat non-rule-oriented categories have been combined into a single non-rule-oriented category. Similarly, the highly rule-oriented and somewhat rule-oriented categories have been combined into a single rule-oriented category.

placed strong emphasis on rules in explaining mathematical ideas while over a quarter placed weak emphasis on rules.

Table 4- 5 - Rule-Orientedness of Instruction Scores

Topic Area	Rule-Orientedness distribution (% of Scores in Category)			
	Non-Rule Oriented	Balanced	Rule Oriented	Median Rule Orientedness Score
Decimal Operations	7.2	21.4	71.4	0.75
Integer Addition	47.6	45.0	7.6	0.50
Integer Subtraction	22.5	40.0	37.5	0.50
Integer Multiplication	67.9	18.5	13.5	0.25
Algebraic Topics	48.8	37.2	14.0	0.42
Pythagorean Theorem	52.8	25.0	22.3	0.25
π	27.8	40.3	32.0	0.50
Area of a Parallelogram	29.1	38.9	31.9	0.50
Volume of a Prism	25.7	21.6	52.7	0.75
Geometric Topics	26.8	40.2	32.9	0.50
All Topics	26.4	51.7	21.8	0.47

The distribution of overall rule-orientedness scores is shown in the stem-and-leaf plot of Figure 4-25. Out of 87 scores, only a single value is in the highly non-rule-oriented category and no scores are in the highly rule-oriented category. The other scores are distributed nearly normally from a low of 0.20 to a high of 0.78. Thus, while teachers showed considerable variation in the amount of stress they put on rules in their implemented curricula, virtually none relied almost totally on rules or excluded rules altogether.

4.1 <u>Rule-Orientedness In Teaching Arithmetic</u>

The rule-orientedness variable was defined for a single arithmetic topic, namely decimal operations. Of the three methods listed for this topic in the Fraction TSQ, the option: "Related operations with decimals to operations with whole

highly non-rule-oriented	0	0
	1	
somewhat non-rule-oriented	2	012288888
	3	1144445688889
balanced	4	0011111124444456667777
	5	00000033333566666778999
	6	
somewhat rule-oriented	6	1333466668999
	7	125558
	8	
highly rule-oriented	8	
	9	N=87
	10	

Figure 4-25 - Distribution of rule-orientedness scores averaged over eight topics.

numbers, teaching rules for placing the decimal point" was classified as the rule approach.[11]

This topic was taught by 52% of the teachers in a somewhat rule-oriented way and by 19% of the teachers in a highly rule-oriented way. Twenty-one percent of the teachers relied on the rule approach and other methods equally, while an additional 7% taught this topic in a somewhat or highly non-rule-oriented manner. An inspection of Table 4-5 shows that for no other topic was the total percent of scores in the two rule-oriented categories so high. On this basis, it can be stated that the arithmetic topic decimal operations was treated in a more rule-oriented way than any of the algebraic or geometric topics. While these comparative figures are probably to be expected given the review nature of arithmetic at this grade level, it is

[11] In Appendix B the rule option is listed for each of the eight topics.

not necessarily the case that a strong reliance on rules in teaching operations with decimals is desirable. On a SIMS Test item which required that students be able to estimate the answer to a multiplication of decimals problem the results were poor "indicating that students may be applying a mechanical process rather than dealing with quantities with understanding" (Robitaille, O'Shea, & Dirks, 1982, p. 98). Perhaps if instruction had been less strongly rule-oriented, student understanding and achievement might have been higher.

4.2 Rule-Orientedness In Teaching Algebra

Figure 4-26 shows boxplots of the distributions of rule-orientedness scores for the three algebraic topics. No two plots are particularly similar overall, although the median value for both addition and subtraction of integers is 0.50. The plots show, however, that for addition most teachers who did not stress rules and other methods equally, tended to emphasize non-rule methods. For subtraction the tendency was to stress rules more frequently.

Multiplication of integers was the third algebraic topic for which teachers' stress on rules was investigated. Of the four methods given in the algebra TSQ for this topic, the option: "No development--students were given rules" was classified as the rule approach. An inspection of Table 4-5 shows that the majority of teachers put more stress on methods other than rules for multiplication of integers. The wording of the rule option which seems to preclude the use of other methods

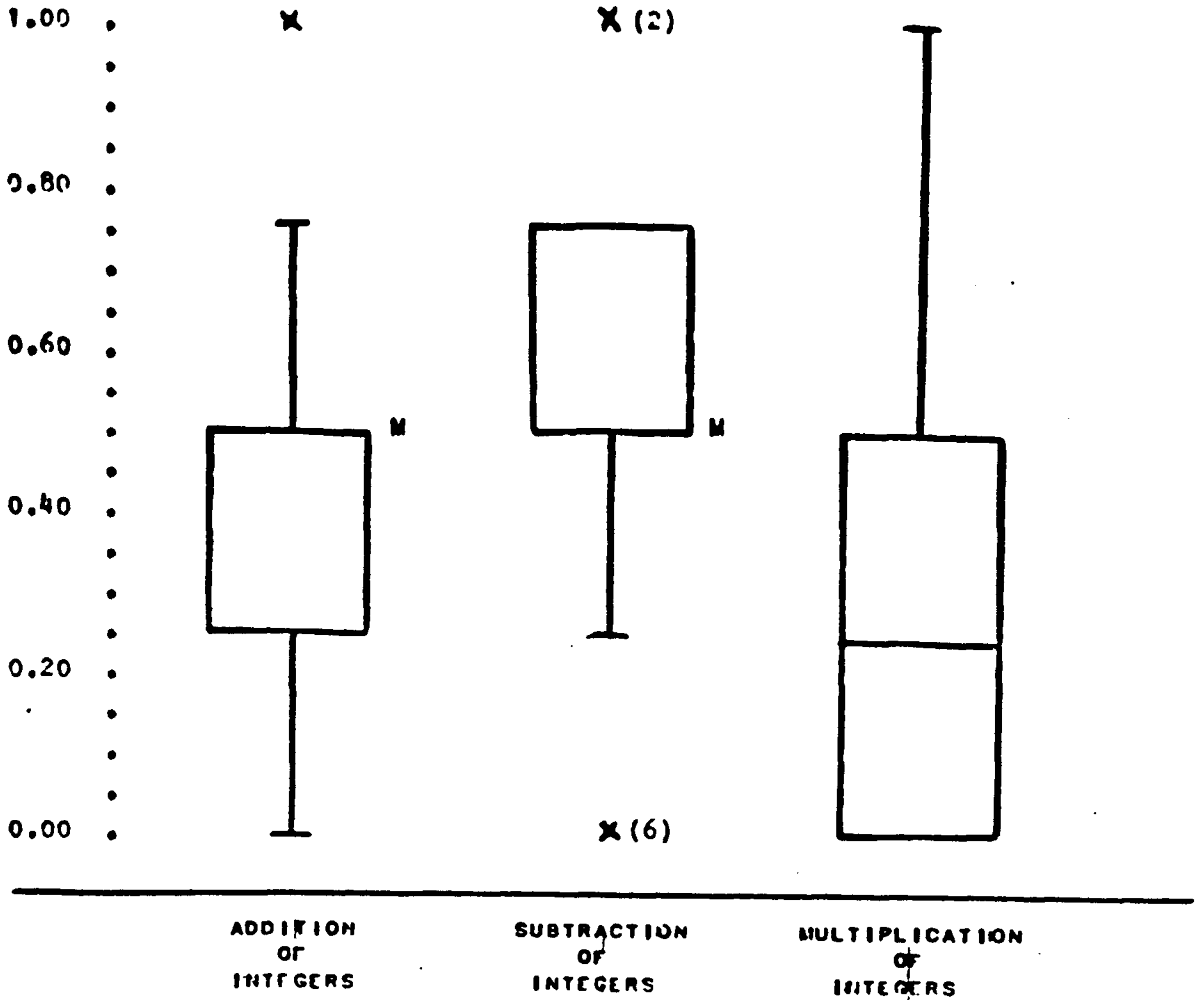

Figure 4-26 - Distributions of rule-orientedness scores for algebraic topics.

is unfortunate, however, and casts some doubt on the validity of the results for this topic. Only 13% of the scores are in the two rule-oriented categories and only 19% are in the balanced category. Over two-thirds of the scores, 68%, are in the two non-rule-oriented categories.

The mean of the rule-orientedness scores for the three algebraic topics for each teacher was taken as the overall rule-orientedness score for algebra for that teacher. The distribution of tese scores is shown in Figure 4-27.

highly non-rule-oriented	0	888
	1	7777777777
somewhat non-rule-oriented	2	5555555
	3	3333333333333333333333
balanced	4	222222222222
	5	00000000000008888888
	6	
somewhat rule-oriented	6	77777
	7	555555
	8	
highly rule-oriented	8	3
	9	N=86
	10	

Figure 4-27 - Distribution of rule-orientedness scores for algebra.

Out of 86 scores, 32 are in the balanced category. Nearly as many scores, 29, are in the somewhat non-rule-oriented category, while 13 scores are in the highly non-rule-oriented category. Of the remaining 12 scores, 11 are in the somewhat rule-oriented category and only 1 is in the highly rule-oriented category. The median of the distribution is 0.42; the lower and upper fourths are 0.33 and 0.50.

Thus, when all three operations with integers are considered, almost half of the teachers, 49%, stressed non-rule-oriented methods more than rule-oriented methods in their implemented curricula. Somewhat fewer, 37%, put equal stress on rules and other content-specific methods. Only 14% of the sample put more emphasis on rules than on alternative methods. Given that these operations are probably new to students at this grade level, this stress on methods other than rules is probably desirable. Students, in fact, showed their greatest improvement between the Core Pretest and Core Posttest on a multiplication of integers item (Robitaille, O'Shea, and Dirks, 1982, p. 96).

4.3 Rule-Orientedness In Teaching Geometry

Figure 4-28 shows boxplots of the distributions of rule-orientedness scores for the four geometric topics. In each case the lower tail extends to 0.00 and the lower fourth is 0.25. Each distribution except the one for the Pythagorean theorem has an upper fourth of 0.75 and an upper tail which extends to 1.00. These plots are very similar to each other and show graphically the wide variation between teachers in their relative emphasis of rules for each topic.

Although the distributions are similar, there are differences in the median values. For the Pythagorean theorem the median is 0.25, for the volume of a rectangular prism it is 0.75. Thus, the greatest difference in the emphasis given to rules occured between the Pythagorean theorem and the volume of a rectangular prism. Most teachers tended not to emphasize the statement of the rule itself in teaching the former topic but instead tended to emphasize methods which provided justification or interpretation. When teaching the latter topic, however, the more common tendency was to put stress on the rule itself. Based on poor performance by students on a SIMS test item involving volume, one can speculate that teaching the calculation of volumes as an exercise in substituting values into a formula does not ensure student understanding of volumetric concepts (Robitaille, O'Shea, and Dirks, 1982, p.

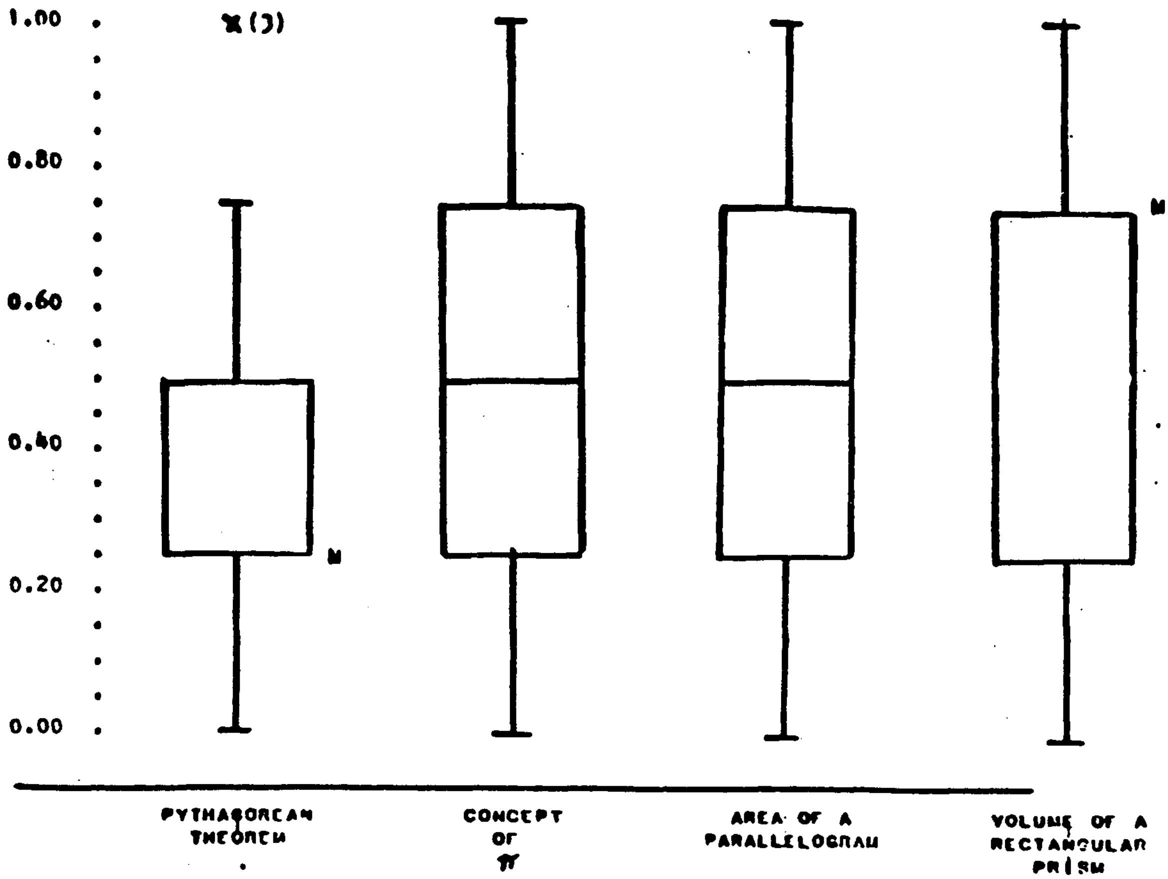

Figure 4-28 - Distributions of rule-orientedness scores for geometric topics.

108).

The mean of the rule-orientedness scores for the four geometric topics for each teacher was taken as the overall rule-orientedness score for geometry for that teacher. The distribution of these scores is shown in Figure 4-29.

Out of 81 scores, 33 are in the balanced category, 10 are in the somewhat non-rule-oriented category, 11 are in the highly non-rule-oriented category, 23 are in the somewhat rule-oriented category, and 4 are in the highly rule-oriented category. The median for the distribution is 0.50; the lower and upper fourths

highly non-rule-oriented	0	06688
	1	339999
somewhat non-rule-oriented	2	55555
	3	11188
balanced	4	2444444
	5	00000000000000000666666666
	6	
somewhat rule-oriented	6	3333377999999
	7	5555555555
	8	
highly rule-oriented	8	1338
	9	N=81
	10	

Figure 4-29 - Distribution of rule-orientedness scores for geometry.

are 0.38 and 0.67.

Thus, when all four geometric topics are considered, wide variation among teachers in their stress of rules is still present just as it was at the individual topic level. The percent of scores in the two non-rule-oriented categories, the balanced category and the two rule-oriented categories are 26%, 41%, and 33% respectively. Thus, the largest number of teachers put equal emphasis on rules and other content-specific methods. Of the remaining teachers, slightly more stressed rules than stressed alternative approaches to content.

4.4 Comparisons In Rule-Orientedness For Topics And Content Areas

In Figure 4-30 the distributions of rule-orientedness scores for the eight topics for which this variable was defined are shown identified by content area. These distributions are not identical. In particular, two have medians of 0.25, four have medians of 0.50, and two have medians of 0.75. Also, the F-spread of four distributions is 0.25, while the F-sread of the other four distributions is 0.50. There does not appear to be any strong association between the content area and the median value of F-spread of the distribution.

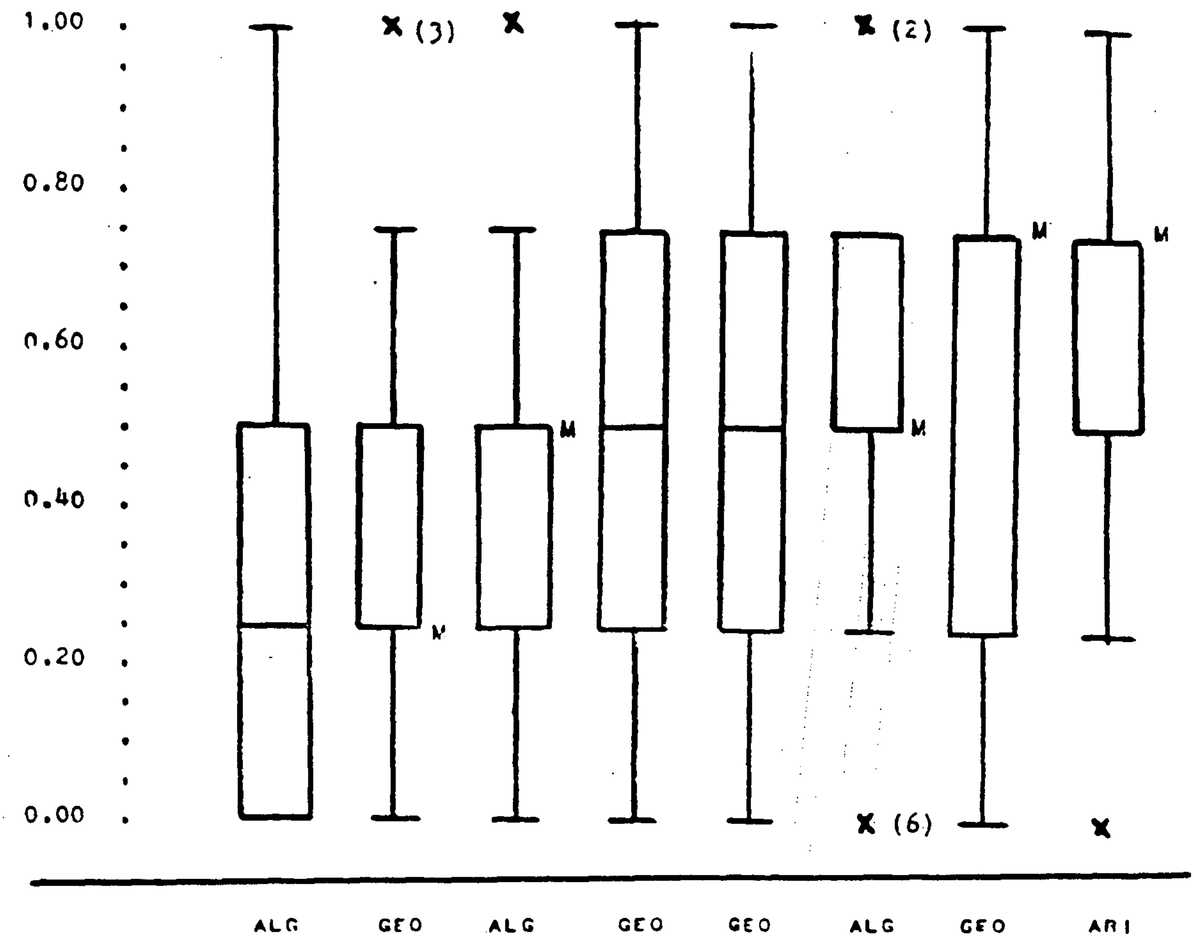

Figure 4-30 - Distributions of rule-orientedness scores for eight topics.

In spite of the differences among these distributions, they are not as different from each other as was the case for the distributions of content representation scores at the topic level. In particular, in each of the rule-orientedness distributions considerable variation between scores exists. In four of the eight distributions, the difference between the outlier cutoffs is 1.00 and in three others it is 0.75.

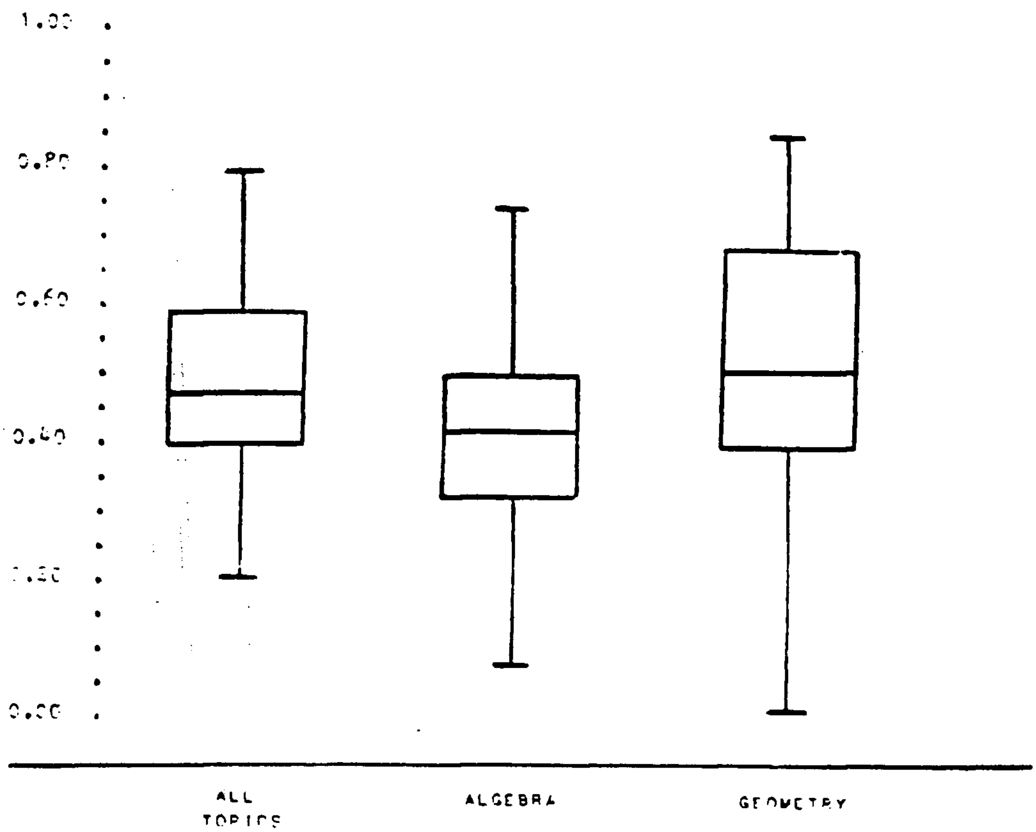

Figure 4-31 - Distributions of rule-orientedness scores for each content area and overall.

When the rule-orientedness scores are averaged for each teacher both for topics within a content area and across all topics, the resulting distibutions still show considerable variation between scores. These distributions are shown in Figure 4-31.

An inspection of these distributions shows that geometry was treated in a more rule-oriented manner than was the case for algebra. The lower fourth, median, and upper fourth of the algebra distribution are 0.33, 0.42 and 0.50. The corresponding values for the geometry distribution are 0.38, 0.50 and 0.67. The F-spreads of the algebra and geometry distributions are 0.17 and 0.29 showing more variation in the stress given to rules in geometry than in algebra.

4.5 Achievement Level Comparisons

In Figure 4-32 boxplots are used to compare the distributions of rule-orientedness scores separately for low and high achievement classes. Plots are given for algebra scores, geometry scores, and overall scores.

The plots show the similar emphasis which rules received in both types of classes. For all topics the stress placed on rules was slightly greater in low achievement classes. The median of the distribution for low achievement classes is 0.46 compared with 0.44 for high achievement classes. For Algebra content, however, the median score is higher for high achievement classes than for low achievement classes.

For algebra content the rule-orientedness scores are more frequently in the two rule-oriented categories for low

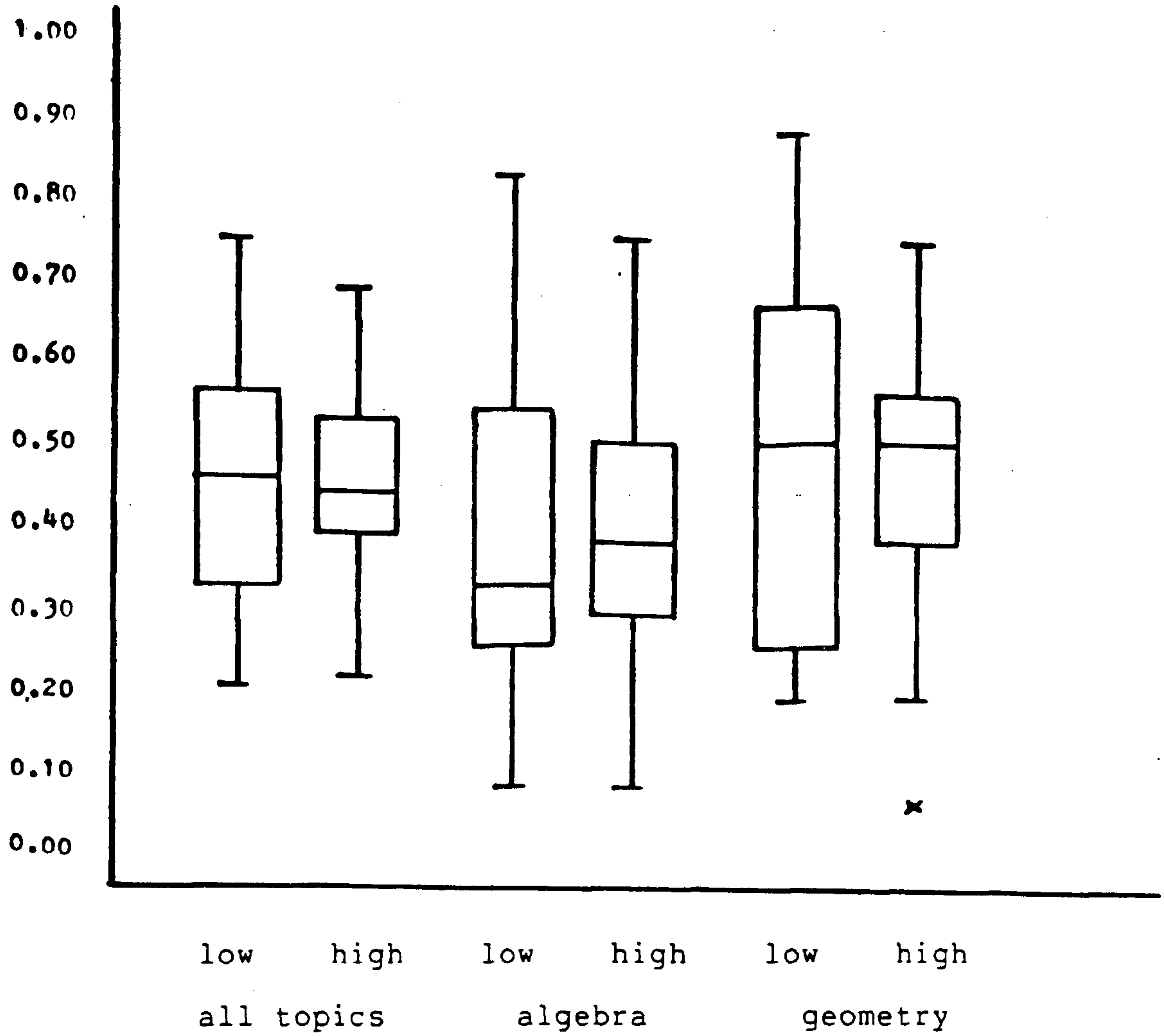

Figure 4-32 - Distributions of rule-orientedness scores for low and high achievement level classes.

achievement classes than for high achievement classes. While there are six scores above 0.60 for low achievement classes, there is a single score above this value for high achievement classes, the scores are also most frequently in the non-rule-oriented categories for low as compared to high achievement classes. The difference is small, however, with 16 scores below 0.40 for low achievement classes compared to 14 scores for high achievement classes. Thus, the most noticeable contrast between the two groups is the greater frequency with which rules were stressed in low achievement classes.

For geometry content the two distributions have identical medians of 0.50. The distributions differ, however, in the extent to which scores vary from this central value. Considerably more variation from the median occurs for the distribution of scores for low achievement classes. The F-spread for that distribution is 0.42 while the corresponding value for the distribution of scores for high achievement classes is only 0.18. For low achievement classes, eight scores are in either the highly rule-oriented or the highly non-rule-oriented categories. For high achievement classes the corresponding number of scores is two. Thus, it was more likely in low achievement classes than in high achievement classes that rules would either receive a great deal of stress or very little stress.

5. DIVERSITY OF INSTRUCTION

Table 4-6 shows the percentage of teachers whose diversity of instruction scores are in the low, moderate and high diversity categories for each topic, each content area, and overall. The table also shows median diversity scores as well as the number of teaching methods listed in the TSQs for each topic. The content area scores represent the average number of methods used by teachers for those topics in each content area. Similarly, the overall scores represent the average number of methods used by teachers across all 16 topics.

The median diversity score over all topics is 2.5. Such a

Table 4- 6 - Diversity of Instruction Scores

Topic or Area	No. of TSQ Methods	Diversity Score Distribution (% of Scores in Category) Low	Moderate	High	Median Diversity Score
Fractions	10	1.3	11.5	87.2	5.0
Fraction Addition	8	40.0	29.3	30.7	2.0
Decimals	6	1.2	23.2	75.6	3.3
Decimal Operations	3	80.7	18.1	1.2	1.5
Proportions	4	17.1	50.0	32.9	2.5
Arithmetic Topics	(6.2)	3.5	55.3	41.2	2.6
Integers	5	17.5	43.8	38.8	2.5
Integer Addition	3	33.8	55.0	11.3	2.0
Integer Subtraction	6	17.5	20.0	62.5	3.0
Integer Multiplication	5	20.0	56.0	24.0	2.0
Linear Equations	5	7.4	34.6	58.0	3.0
Formulas	5	37.8	36.5	25.7	2.0
Algebraic Topics	(4.8)	1.2	80.2	18.5	2.5
Angle Sum Theorem	8	35.6	45.2	19.2	2.0
Pythagorean Theorem	7	28.6	54.2	17.1	2.0
π	7	31.9	44.9	23.2	2.0
Area of a Parallelogram	8	20.3	26.1	53.6	3.0
Volume of a Prism	3	81.2	18.8	0.0	1.5
Geometric Topics	(6.6)	14.8	71.6	13.6	2.0
All Topics	(93)	1.1	78.2	20.7	2.5

score could be attained by the emphasis of two methods and the use without emphasis of one method, by the emphasis of one method and the use without emphasis of three methods, or by several other combinations of emphasizing and using content specific methods. The distribution of overall diversity scores is shown in the stem-and-leaf plot of Figure 4-33. The distribution has three outliers consisting of one exceptionally low score of 1.2, the only overall diversity score in the low category, and two relatively high scores of 4.0 and 5.3. The latter score could be attained through the emphasis of over five methods for each topic.

low diversity	0	
	1	2
	1	
moderate diversity	1	88999
	2	0000000001111111222223333333334444444
	2	55555556667777777888888999
high diversity	3	000002334
	3	6777777
	4	0
	4	
	5	3
	5	

N=87

Figure 4-33 - Distribution of diversity scores averaged over all topics.

Almost four-fifths of the overall diversity scores were between 1.8 and 2.9 inclusive and were thus in the moderate category. All of the other scores (except for the one lower outlier) were 3.0 or higher and were thus in the high diversity category. Almost all of the teachers typically, then, taught the concepts, operations, and principles in their curricula through the use of several approaches on average. About one-fifth of the sample employed enough methods to be considered highly diverse in their instruction.

5.1 Diversity In Teaching Arithmetic

The mean of the diversity scores for the five arithmetic topics for each teacher was taken as the diversity score for arithmetic for that teacher. The distibution of these scores is shown in Figure 4-34.

low diversity	0		
	1	33	
	1	5	
moderate diversity	1	7889	
	2	0000111122223333333444	
	2	5555556666677778899999	
high diversity	3	0000011111222223444	
	3	5566667779999	
	4	022	
	4	5	
	5		
	5		
	6	1	N=85
	6		

Figure 4-34 - Distribution of diversity scores for arithmetic.

Out of 85 scores, 47 are in the moderate diversity category, 3 are in the low diversity category and 35 are in the high diversity category. The median of the distribution is 2.7; the lower and upper fourths are 2.3 and 3.3. The only outlier is the extremely high score of 6.1.

Thus, in teaching arithmetic topics, somewhat over one-half of the teachers chose to present a moderate number of content-specific methods on average. Slightly over two-fifths of the teachers showed a greater degree of diversity in their instruction with scores in the high range. Very few teachers presented so few methods that their arithmetic diversity scores were in the low range.

Figure 4-35 shows boxplots of the distributions of diversity scores for the arithmetic topics. The most diversity was shown for the concept of fractions. The mean for the distribution of scores for this topic is 5.0; the lower and upper fourths are 4.0 and 5.5. All of these values are within the high diversity range. Altogether, 87% of the teachers showed a high degree of diversity when teaching this topic. This was the largest percentage of scores in this category for all 16 topics. Teachers also, in general, showed high diversity when teaching the concept of decimals. The mean for that topic is 3.3 and the lower and upper fourths are 3.0 and 4.0. For this topic, 76% of the scores are within the high diversity range.

The median diversity scores for two topics, the addition of fractions and the concept of proportions, are within the moderate range being 2.0 and 2.5 respectively. The lower and upper fourths for addition of fractions are 1.5 and 3.0. For the concept of proportions the corresponding values are 2.0 and 3.0.

The fifth topic, operations with decimals, was taught with much less diversity than the other four topics. The median score for this topic is 1.5; the lower and upper fourths are 1.0 and 1.5. All of these values are within the low diversity range. In all, 81% of the teachers showed low diversity in

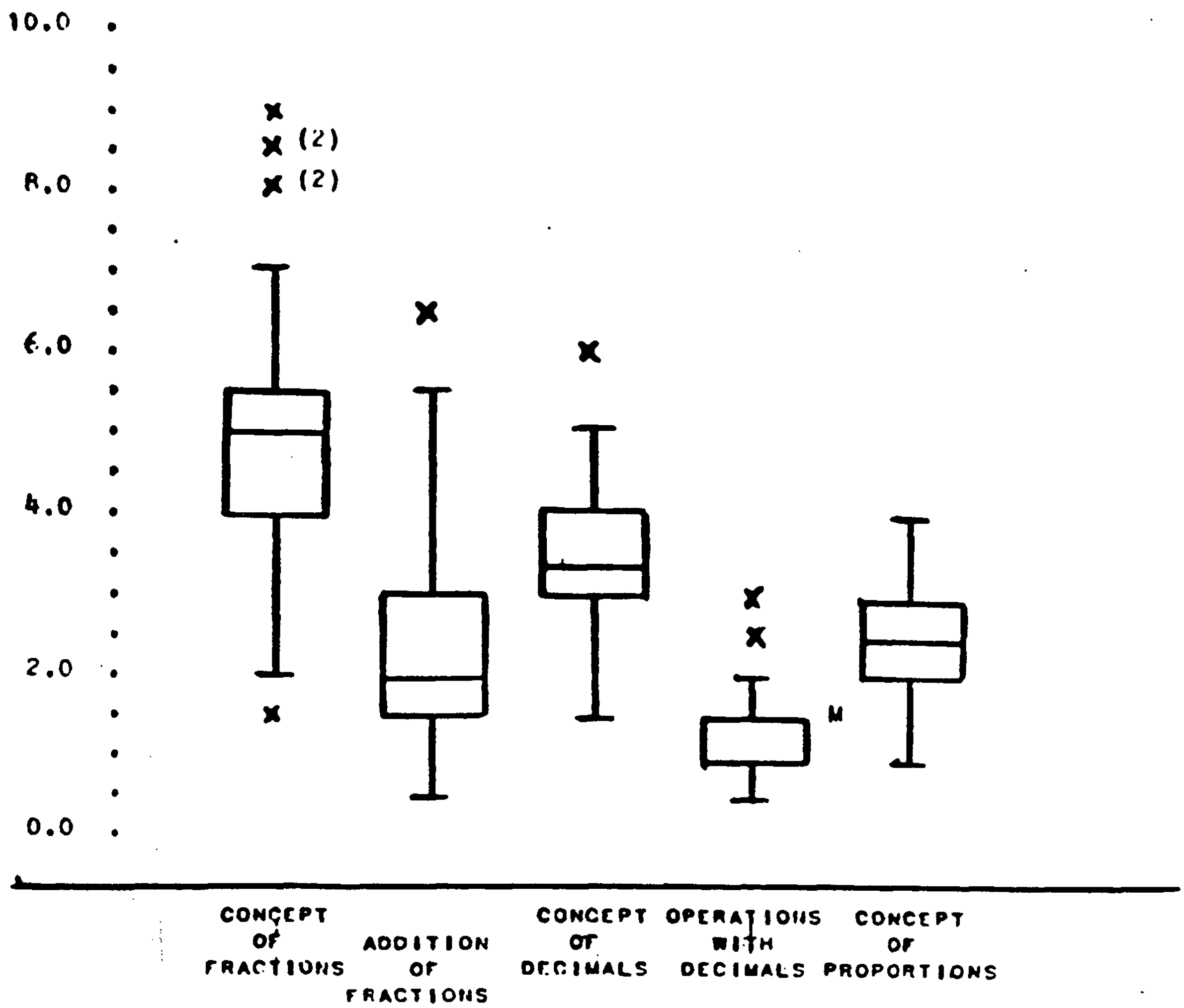

Figure 4-35 - Boxplots of the distributions of diversity scores for arithmetic topics.

teaching operations with decimals.

For the five arithmetic topics there was a strong positive association between the number of methods used in presenting a topic and the number of available methods. The correlation between the median diversity score and the number of methods listed in the TSQs is 0.74. Teachers did not follow this trend in teaching the addition of fractions, however. While eight methods were listed in the Fraction TSQ for this topic, the median diversity score was only 2.0. One explanation for the fact that teachers showed the most diversity in presenting the concepts of fractions, decimals and proportions and the least diversity in presenting the operation of addition of fractions and operations with decimals is that, perhaps, these teachers felt that while arithmetic concepts should be presented with considerable diversity, arithmetic operations are better taught using fewer methods even if many are available.

5.2 Diversity In Teaching Algebra

The mean of the diversity scores for the six algebraic topics for each teacher was taken as the diversity score of algebra for that teacher. The distribution of these scores is shown in Figure 4-36.

low diversity	0	
	1	3
	1	
moderate diversity	1	677888999
	2	000000011111222223333333334
	2	55555556666777788888888888999
high diversity	3	0000223333
	3	55667
	4	N=81
	4	

Figure 4-36 - Distribution of diversity scores for algebra.

Out of 81 scores, 65 are in the moderate diversity range, one is in the low diversity range, and 15 are in the high diversity range. The median of the distribution is 2.5; the lower and upper fourths are 2.1 and 2.8. These results are similar to those that were obtained for arithmetic. For algebra even a larger percentage of scores, 80%, are within the moderate diversity range and a correspondingly smaller percentage of scores, 19%, are within the high diversity range. As with arithmetic, very few teachers showed low overall diversity for algebra.

Figure 4-37 shows boxplots of the distributions of diversity scores for the algebraic topics. These distributions differ less markedly from each other than was the case for the arithmetic topics. While the medians for arithmetic topics ranged from 1.5 to 5.0, for algebraic topics the range is form 2.0 to 3.0.

The greatest mean diversity for algebraic topics occurred for subtraction of integers and solving linear equations. In both cases the median diversity score is 3.0. The lower fourth in both cases is 2.5. The upper fourth for subtraction of integers is 3.8, while for solving linear equations it is 3.5. Thus, in terms of diversity these topics were treated in a very similar fashion.

The concept of integers has the third highest median diversity value for this group of topics at 2.5. The lower and upper fourths are 2.0 and 3.0.

Teachers tended to show the least diversity within algebra when they taught addition and multiplication of integers and the concept of formulas. In each case the median diversity score is 2.0. For multiplication of integers the middle 50% of the scores vary little from this value with a lower fourth of 2.0

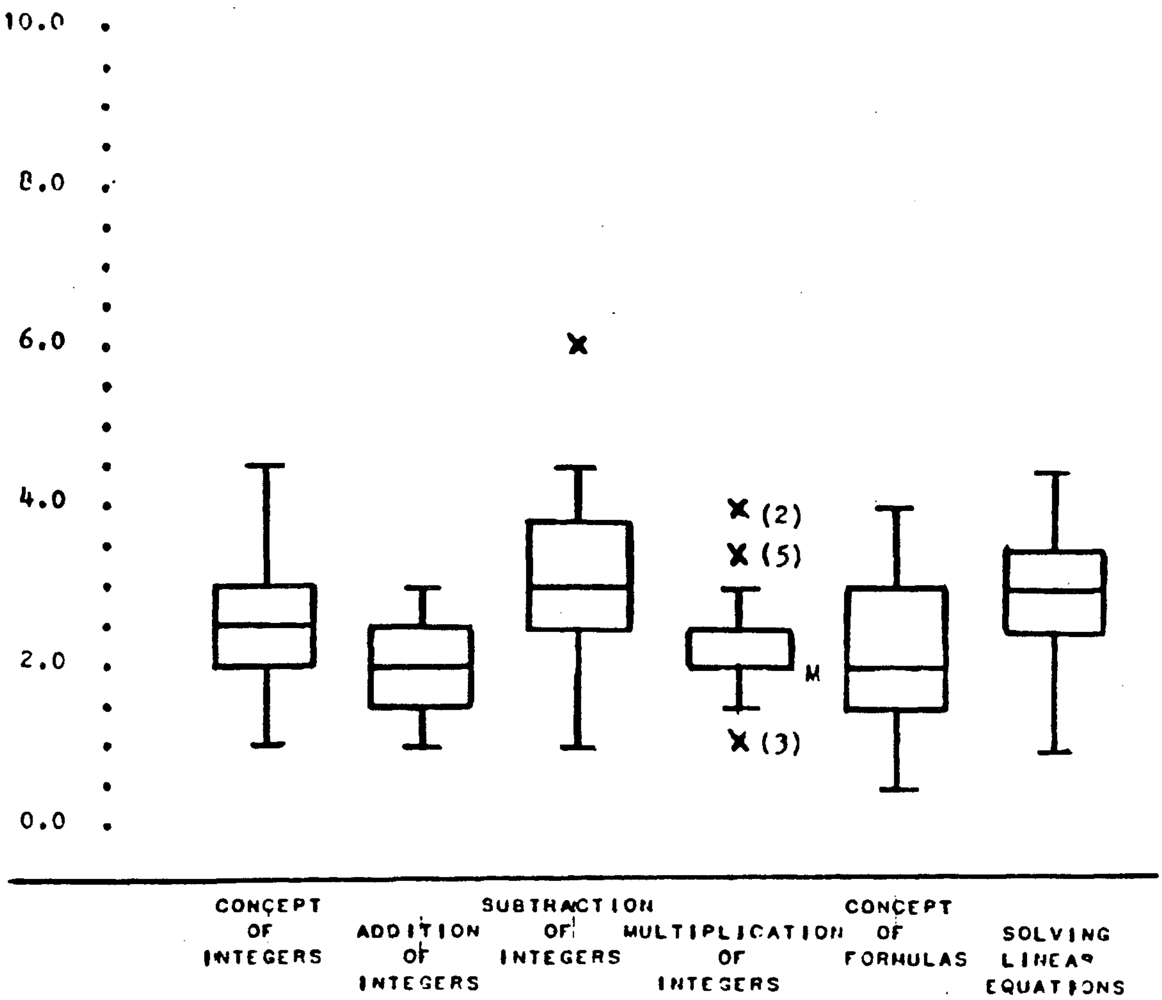

Figure 4-37 - Boxplots of the distributions of diversity scores for algebraic topics.

and an upper fourth of 2.5. For addition of integers the corresponding values are 1.5 and 2.5 while for the concept of formulas they are 1.5 and 3.0.

The association between the number of teaching methods available to teachers for each topic and the number which they actually used was not as strong for algebra as it was for arithmetic. The correlation between the number of TSQ methods and the median diversity score for each topic was 0.59 compared to 0.74 for arithmetic. Also, teachers did not consistently use more approaches for algebraic concepts than for algebraic operations as had been the case for arithmetic.

5.3 Diversity In Teaching Geometry

As with the other content areas, the mean of the diversity scores for the five geometric topics for each teacher was taken as the diversity score for geometry for that teacher. The distribution of these scores is shown in Figure 4-38.

low diversity	0	5
	1	233344
	1	55555
moderate diversity	1	6666777777778888899999
	2	0000000111112233333334444
	2	55566778889
high diversity	3	033
	3	566779
	4	03
	4	

N=81

Figure 4-38 - Distribution of diversity scores for geometry.

Out of 81 scores for this variable, 58 are in the moderate diversity range. This is similar to the results for arithmetic and algebra. Unlike the other two content areas for which the remaining diversity scores were almost all in the high diversity range, however, the remaining diversity scores for geometry are almost equally split between the low and high diversity categories. Of the 23 scores, 12 are in the low diversity category and 11 are in the high diversity category.

The median of the geometry distribution is 2.0. This score could be attained by the emphasis of just two content-specific methods for each topic. The lower and upper fourths of the distribution are 1.7 and 2.5. Each of these values is less than the corresponding values for both arithmetic and algebra. Thus, teachers showed considerably less diversity in teaching geometric topics than was the case for the other two content areas. This is in spite of the fact that more content-specific methods appear to be available for the geometric topics than for the topics in the other content areas. An average of 6.6 methods were listed in the TSQs for each geometric topic compared to 6.2 and 4.8 methods for the arithmetic and algebraic topics respectively.

Figure 4-39 shows boxplots of the distributions of diversity scores for the geometric topics. The first three of these plots appear almost identical. The medians, lower fourths, and upper fourths are 2.0, 1.5 and 2.5 in each case.

For each of these topics, the triangle angle sum theorem, the Pythagorean theorem, and the concept of π, low diversity scores occurred more frequently than high diversity scores. This is of interest because of the relatively large number of teaching options available for the topics. The Geometry TSQ listed eight methods for the triangle angle sum theorem and seven methods for the Pythagorean theorem. The Measurement TSQ listed seven methods for the concept of π . Very few teachers were familiar with or chose to use several of the content specific methods for each of these topics. During the interviews which this researcher conducted, teachers indicated

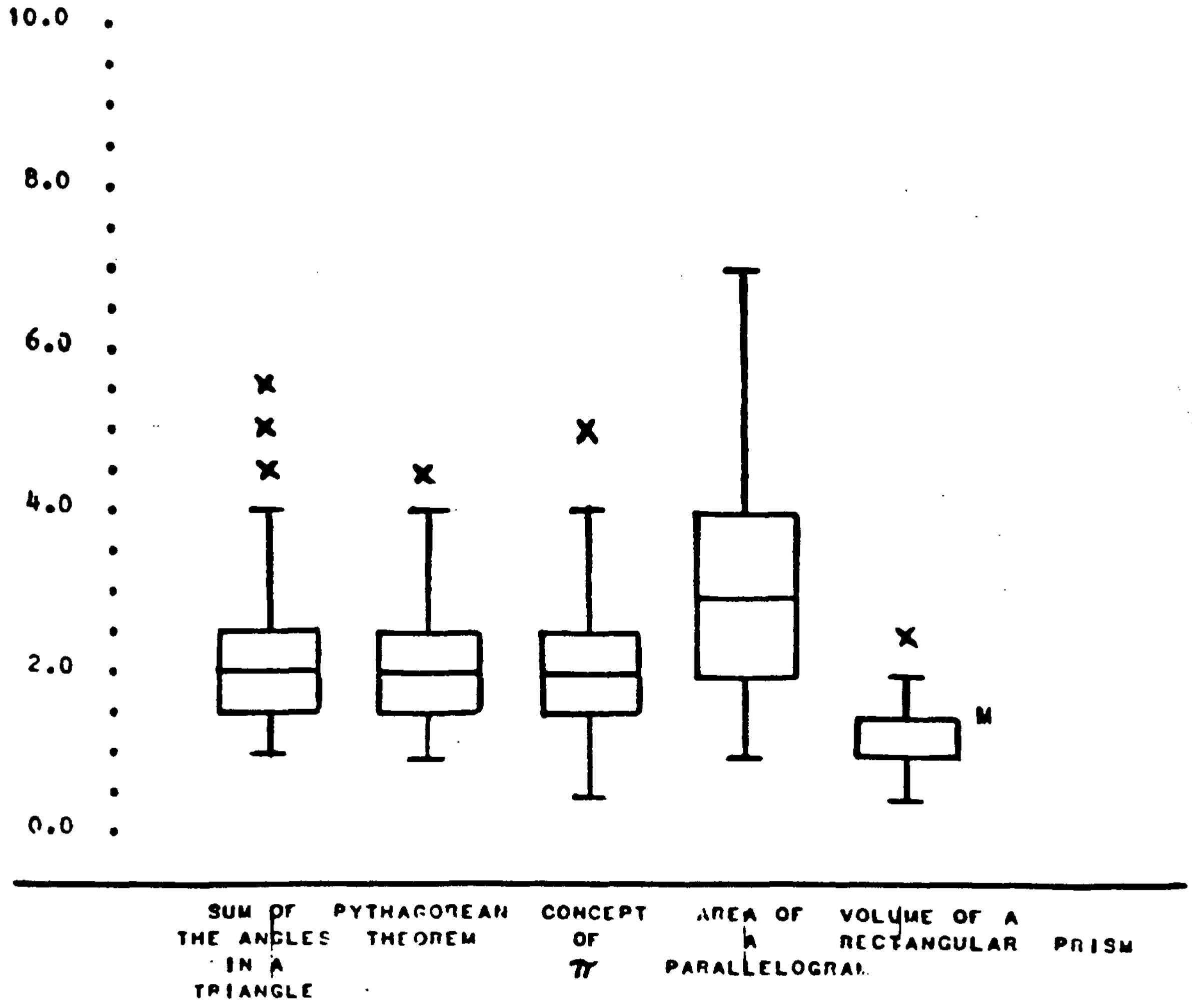

Figure 4-39 - Boxplots of the distributions of diversity scores for geometric topics.

that they were not familiar with several of the teaching methods given in the TSQs for geometry but intended to use them in their teaching in the future. The fact that for these three topics the diversity scores tend to be low in spite of a relatively large number of options listed in the TSQs further supports the contention that teachers completed the questionnaires in a conscientious manner.

Of the remaining two geometric topics, one was approached with more diversity and one with less diversity than the three topics discussed above. The area of a parallelogram was presented with the greatest diversity of the geometric topics. The median of the distribution of scores for this topic is 3.0, the only median of a geometric topic in the high diversity range. The lower and upper fourths are 2.0 and 4.0. For the volume of a rectangular prism, in contrast, the median is 1.5 and the lower and upper fourths are 1.0 and 1.5, all values in the low diversity range.

As noted, the number of methods teachers used for geometric topics was less than might be expected based on the number of methods available and the number of methods used for teaching topics within other content areas. The correlation between available methods and the median diversity scores for the geometric topics was nonetheless quite high at 0.70.

5.4 Comparisons In Diversity Between Content Areas And Topics

The distributions of diversity scores for the three content areas and overall topics are shown in Figure 4-40. The boxplots show the similarity between the four distributions. The most noticeable difference among them is the lower median and lower fourths for geometry. As noted earlier, the median diversity score for geometry is 2.0 while the median values for arithmetic and algebra are 2.7 and 2.5 respectively. For the distribution of overall diversity scores the median is 2.5. One possible implication of this result is that teachers may need more inservice training in the area of geometry in order to learn

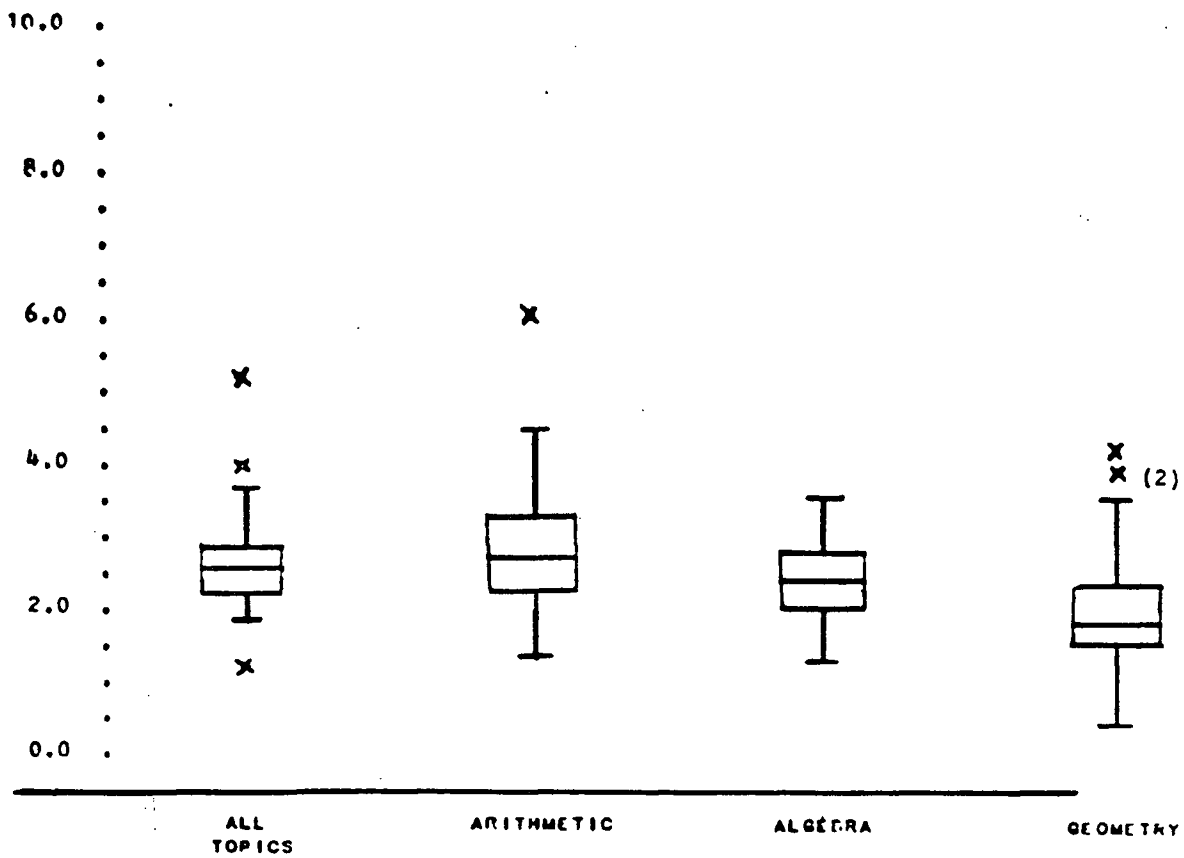

Figure 4-40 - Distribution of diversity scores for each content area and across all topics.

more content-specific teaching methods than they do in the areas of arithmetic and algebra.

In Figure 4-41 the distributions of diversity scores for the 16 topics are shown identified by content area with the number of methods listed for each in the TSQs included in parentheses. Teachers apparently tended to present arithmetic topics with a greater diversity of methods than algebraic or geometric topics. This could reflect a greater degree of familiarity with arithmetic topics on the part of the teachers.

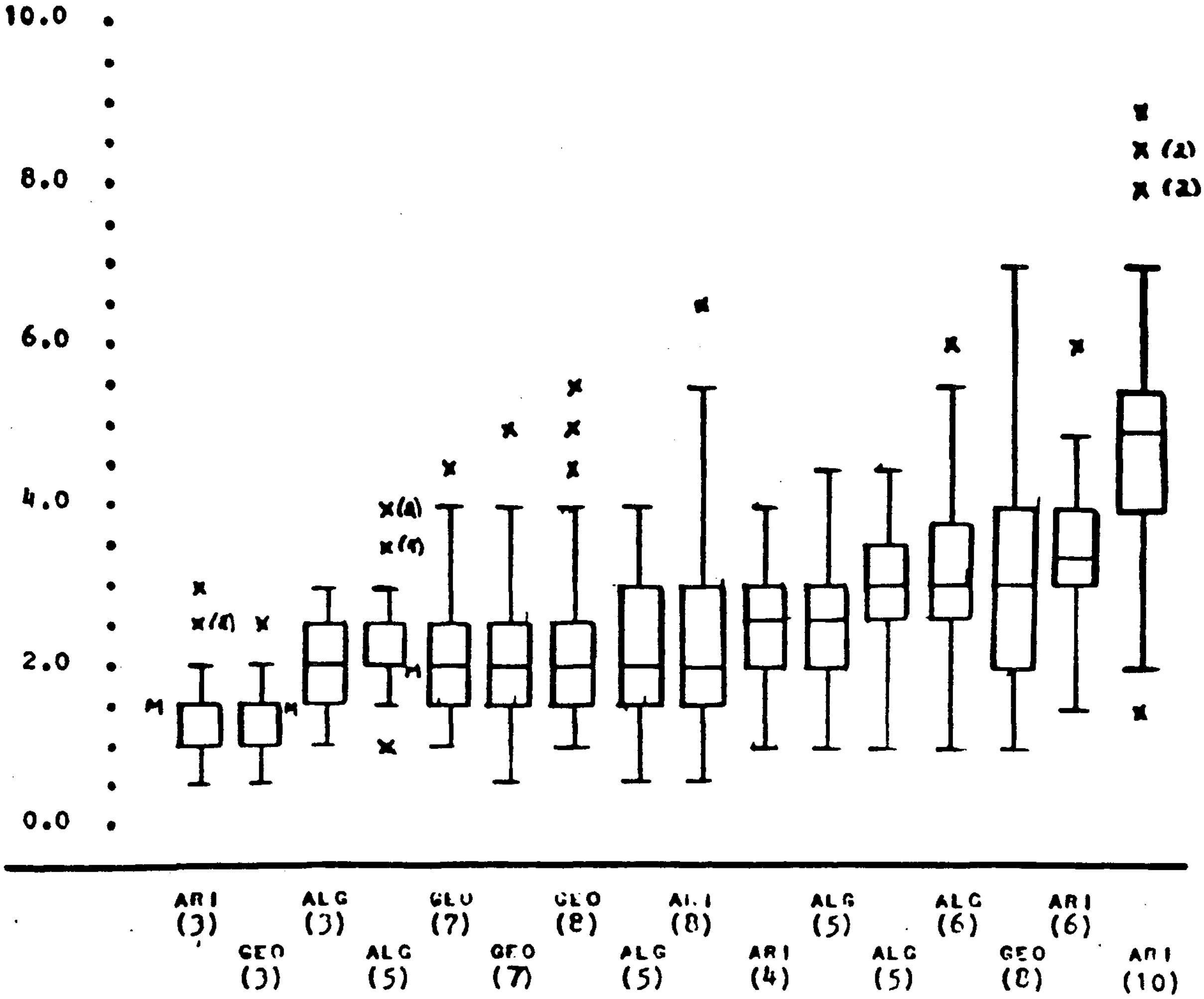

Figure 4-41 - Distributions of diversity scores for the 16 topics.

Figure 4-41 also shows that, with the exception of three geometric topics, teachers tended to use more methods in their teaching when more alternatives were actually available to them. The correlation between the median diversity score for each of the 16 topics and the number of methods listed in the TSQs for that topic is 0.59.

5.5 Achievement Level Comparisons

In Figure 4-42 boxplots are used to compare the distributions of diversity scores of low and high achievement classes for each content area and over all topics. These plots show that there was no apparent association between class achievement level and the number of methods teachers used in presenting mathematical topics. The median values for each of the four pairs of distributions differ from each other by no more than 0.1. Teachers of low achievement classes, for example, have a median diversity score of 2.1 for geometry while the corresponding value for teachers of high achievement classes is 2.2.

Greater differences between the distributions exist for variation as measured by F-spread, but these differences are not consistent for all content areas. Overall, teachers of low achievement classes show more uniformity in their diversity scores than teachers of high achievement classes. The F-spread of the distribution of overall diversity scores for low achievement classes is 0.7 compared to 0.9 for high achievement classes. For arithmetic content the F-spreads for low and high achievement classes are 1.0 and 1.4 respectively. For geometry, however, the F-spread is greater for the distribution of high achievement level classes and for algebra the F-spreads of the two distributions are the same.

Thus, it would appear that teachers did not in general base their decisions as to how many methods would be optimal in presenting the topics in their curricula on the achievement

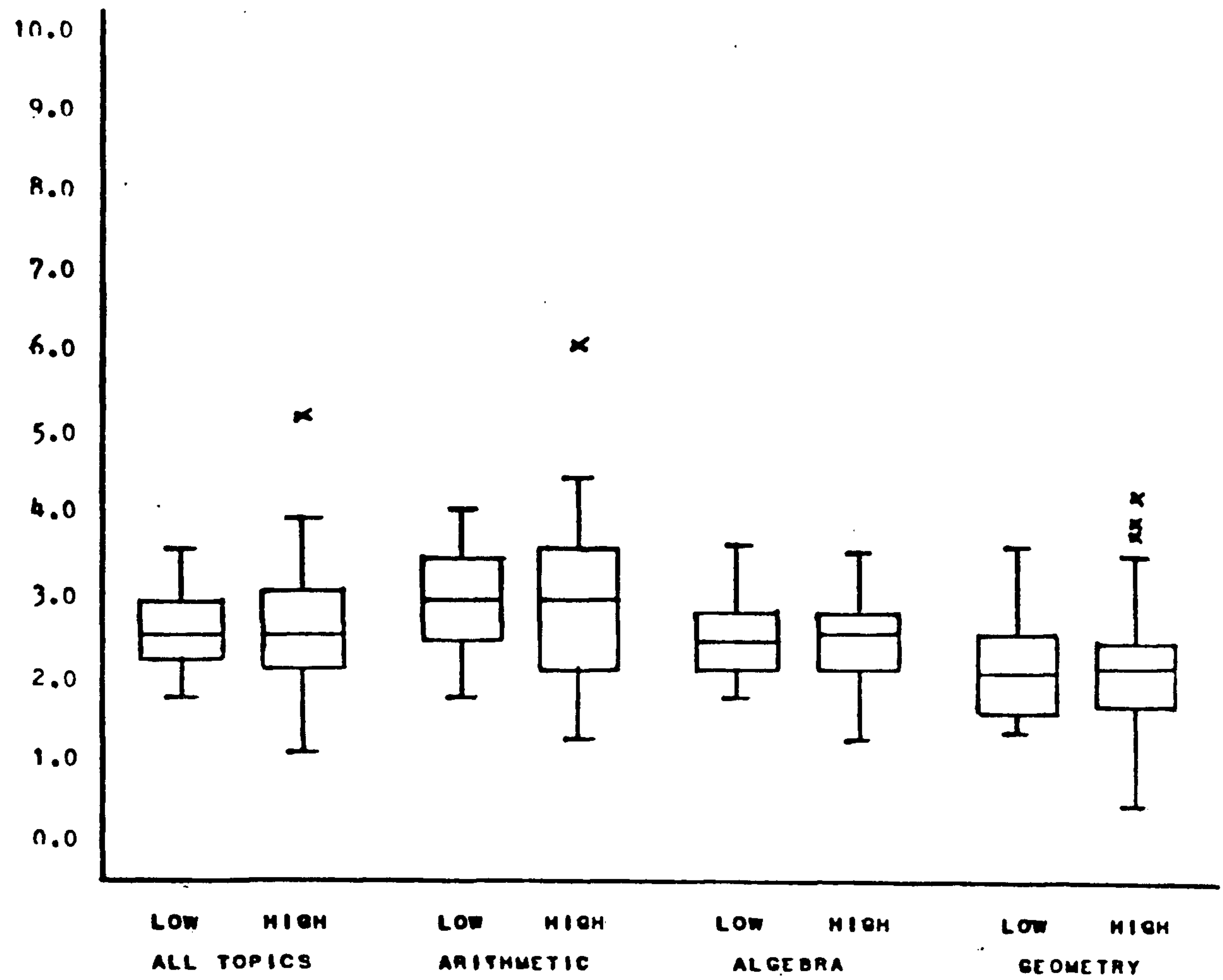

Figure 4-42 - Distributions of diversity scores for low and high achievement level classes.

level of their classes. Alternately, they may have felt that the same number of content-specific methods was appropriate for both low and high achievement classes.

5.6 <u>Diversity Scores Of Teachers And Textbooks</u>

When the content emphasis scores of teachers using each of the two major textbooks were analyzed separately, an association was found between the relative emphasis of the content areas in the textbooks and the emphasis given by teachers to content areas in their implemented curricula. A similar, though weaker, positive association was found between the number of methods included in each textbook for groups of topics and the average number of methods employed by teachers using each textbook.

Table 4-7 shows the number of content-specific methods listed in the TSQs for the 16 topics which appeared in the two Mathematics 8 textbooks. The table also shows the number of methods contained in each textbook for two groups of topics. Topic Group 1 consists of those topics for which more TSQ methods were found in Mathematics II than in School Mathematics 2 while Topic Group 2 consists of those topics for which more methods were found in School Mathematics 2.[12]

Table 4- 7 - Number of TSQ Methods Contained in the Commonly Used Textbooks for Selected Topics

Textbook	All Topics	Topic Group 1	Topic Group 2
Mathematics II	31	16	4
School Mathematics 2	36	11	11

In Figure 4-43 boxplots are used to compare the average diversity scores of the teachers using each textbook for the three groups of topics. The two distributions of diversity scores for all topics are nearly identical. The teachers using School Mathematics 2 had the higher median score, however, 2.5 compared to 2.3. This corresponds to a somewhat greater number of total teaching methods in School Mathematics 2 for the 16

[12] Topic Group 1 includes integer subtraction, solving linear equations, decimals and the Pythagorean theorem. Topic Group 2 consists of fractions, decimal operations, the concept of π, and the volume of a rectangular prism.

topics.

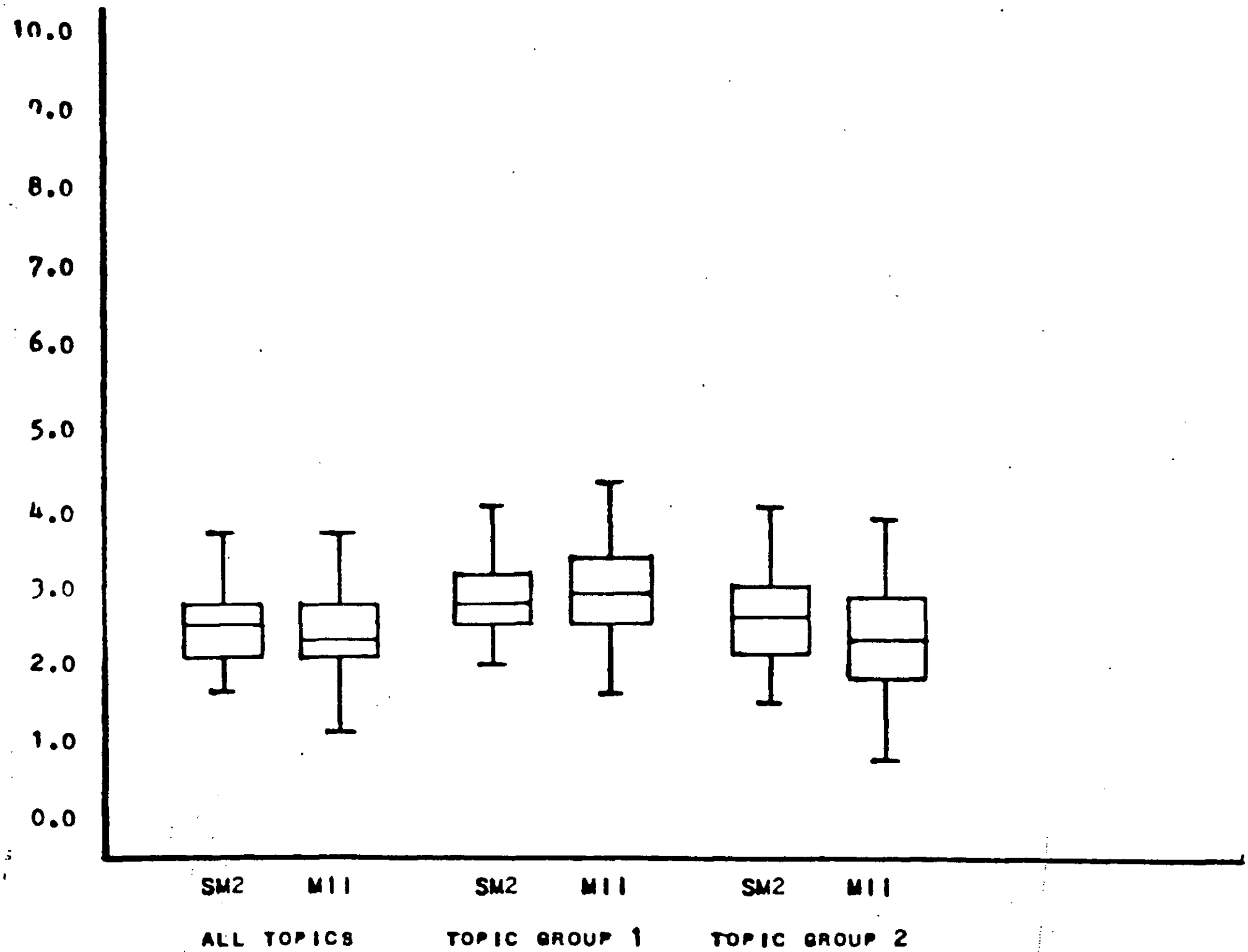

Figure 4-43 - Distributions of average diversity scores for the groups of topics for users of each textbook.

For Topic Group 1, the median score of those teachers using Mathematics II is 2.9 compared to 2.8 for teachers using School Mathematics 2. The upper fourths for the two distributions are 3.4 and 3.1 respectively. These figures are consistent with the fact that 16 methods are contained in the former textbook for this group of topics, compared to 11 methods contained in the latter text.

For Topic Group 2, only four teaching methods are contained in Mathematics II compared to 11 methods in School Mathematics 2. For this group of topics, the teachers using School Mathematics 2 showed more diversity in their instruction than the teachers using the other text. The median diversity score for the teachers using School Mathematics 2 is 2.6 compared to a median score of 2.3 for the teachers using the other book. The lower and upper fourths were also higher for the teachers using School Mathematics 2.

Although differences between the distributions of diversity scores for users of the two textbooks were not large, the pattern was consistent. When more methods were contained in one textbook for a group of topics, teachers tended to use more methods in teaching those topics.

V. CONCLUSIONS

1. SUMMARY OF THE RESULTS

The implemented curricula of Mathematics 8 teachers which were examined in this study had, almost universally, certain commonalities with respect to content. Nearly all of these curricula included arithmetic, algebra, and geometry. Only three teachers reported omitting one of these content areas altogether. Furthermore, the mean relative emphasis given each area was nearly the same with algebra receiving somewhat less class time than arithmetic or geometry. On average, teachers devoted 29% of their time to algebraic topics, as compared with 35% for arithmetic topics and 36% for geometric topics. Thus, not only were each of the content areas almost always part of the implemented curricula, their average degrees of emphasis within these curricula were nearly equal.

Despite these similarities, it cannot be said that there was a common curriculum in terms of content emphasis in the classes which were studied. The range of emphasis scores within each content area was large with the most diversity occurring for geometry. Two teachers, for example, spent only 7% of their class time on geometry while a third devoted 66% to that content area. Considering only the middle 50% of the content emphasis scores in each case, teachers differed by 13% in the percent of class time given to arithmetic, by 9% in the percent of class time given to algebra, and by 16% in the percent of class time given to geometry. Including all non-zero values, the range of content emphasis scores expressed as percents was 51% for arithmetic, 34% for algebra, and 59% for geometry. Thus, there was far from a common curriculum in the courses studied with wide variation in the emphasis given to the three content areas.

The proportion of scores within each category of content emphasis further demonstrates both the similarities and differences in the implemented curricula as measured by this variable. For arithmetic, algebra, and geometry 76%, 74%, and 69% of the respective emphasis scores were within the moderate range. Thus, for each content area most teachers provided a moderate degree of content emphasis. On-the-other-hand, 60% of the teachers in this study gave light or very light emphasis to at least one content area. Thus, most teachers did not give moderate emphasis to all content areas.

The differences in content emphasis among teachers were the strongest for geometry. Although this content area had the highest mean emphasis score, 0.36, it is also the case that a larger percentage of teachers, 31%, gave this area light or very light emphasis than was true for the other content areas.

Due to the sequential nature of mathematics and the spiral approach incorporated in many textbooks and suggested by the B. C. Curriculum Guide, this level of variation in content emphasis may not be desirable. Especially with respect to geometry this diversity within the Mathematics 8 curriculum may make assumptions made by Mathematics 9 teachers regarding prior learning of content quite problematic.

Overall and for each content area, teachers showed a slight preference for abstract as compared with perceptual teaching methods. The median overall mode of representation score was 0.57. For arithmetic, algebra, and geometry the median scores were 0.64, 0.54, and 0.54 respectively. Slightly over one-third of the teachers in this study could be classified as abstract in their presentations to students based on their overall content representation scores. Almost all of the other teachers could be classified as showing a balance between abstract and perceptual teaching methods.

Of the three content areas, arithmetic was most usually dealt with abstractly by teachers. About one-third of the teachers balanced their instruction between perceptual and abstract methods for arithmetic while almost all of the other teachers showed a clear preference for abstract methods for this review content area.

Most teachers, 84% of the sample, balanced their instruction for algebra content. The remainder favored abstract methods.

Teachers showed the least uniformity in their mode of content representation for geometry. While 48% balanced abstract and perceptual methods in teaching geometry to students, 27% clearly favored abstract methods while almost as many, 25%, clearly favored perceptual methods. Geometry was the only content area for which an appreciable number of teachers put a definite stress on perceptual approaches to content across all topics.

Wide variation existed among topics in the mode of representation typically used during instruction. Some topics were most often taught using perceptual methods while other topics were most often taught using abstract methods. For four topics the median representation score was below 0.40 and hence within the perceptual range. Two of the topics are algebraic, two geometric. These topics were integers, integer addition, the angle sum theorem, and the Pythagorean theorem. The median representation score was not in the perceptual range for any arithmetic topic.

The majority of topics, eight out of the 14 for which this variable was defined, were usually taught abstractly. For each of these topics the median representation score was above 0.60. The arithmetic topics which were taught with this level of abstraction were decimals and decimal operations. The corresponding algebraic topics were integer subtraction, integer multiplication, and formulas while the corresponding geometric topics were the concept of π, the area of a parallelogram, and the volume of a prism.

For only two topics was the median representation score within the balanced range. These topics, fractions and fraction addition, each involved arithmetic content.

An inspection of the mode of representation scores for the three content areas as a whole would lead one to the conclusion that the two areas which contained mostly new material, algebra and geometry, were usually taught with a nearly equal balance between perceptual and abstract methods while the review area, arithmetic, was taught with an emphasis on abstract methods.

While true in a general sense, it is also true that this pattern was not evident at the level of specific topics. Four topics were taught quite perceptually by most teachers while eight topics were taught quite abstractly by most teachers.

Overall, the teachers in this study tended to show a slight preference for non-rule-oriented approaches to content over rule-oriented approaches. Averaged over the eight topics for which this variable was defined, the median rule-orientedness score was 0.47. Fifty-two percent of the overall rule-orientedness scores were in the balanced category, while 26% were in the two non-rule-oriented categories, and 22% were in the two rule-oriented categories. Thus, over a quarter of the teachers in this study showed a clear tendency to emphasize approaches to content other than the statement of rules while almost as many showed a clear tendency to emphasize statements of mathematical rules.

Rules were emphasized strongly for the one arithmetic topic for which this variable was defined. The median rule-orientedness score for this topic, decimal operations, was 0.75. Over 70% of the teachers in this study put more emphasis on the actual rules for decimal point placement during instruction than on approaches which provide reasons for the placement of the decimal point.

Overall, teachers put more emphasis on rules when teaching geometric topics than when teaching algebraic topics. For algebraic content the median score was 0.42, with 49% of the scores in the non-rule-oriented categories, 37% in the balanced category, and 14% in the rule-oriented categories. For geometric content the median score was 0.50 with 27% of the scores in the non-rule-oriented categories, 40% in the balanced category, and 33% in the rule-oriented category. Thus, while one-third of the teachers emphasized rules such as statements of definitions and theorems when teaching geometry, less than half as many teachers put a similar heavy emphasis on algebraic rules such as the rules for signed numbers.

The differences between the rule-orientedness scores measured at the level of individual topics and the rule-orientedness scores measured at the level of the content areas were smaller than the corresponding differences for the mode of content representation variable. For four of the eight topics: integer addition, integer subtraction, the concept of π, and the area of a parallelogram the median rule-orientedness score was 0.50. This compares with medians of 0.42 and 0.50 for algebraic and geometric content, respectively, as noted above.

Teachers put relatively heavy emphasis on rules for decimal operations and the area of a parallelogram. In each case the median rule-orientedness score was 0.75. In contrast, teachers put relatively light emphasis on rules for integer multiplication and the Pythagorean theorem. In each case the median rule-orientedness score was 0.25.

Across all 16 topics, almost 80% of the teachers showed moderate diversity in their use of teaching methods while over 20% showed high diversity. Only one percent of the sample showed low diversity as defined in this study.

Teachers showed almost identical median levels of diversity in teaching arithmetic and algebra. For arithmetic content the median diversity score was 2.6, while for algebraic content it was 2.5. More diversity scores were in the high diversity category for arithmetic as compared with algebra, 41% compared with 19%. In both cases nearly all of the other scores were in the moderate diversity category.

Teachers showed substantially less diversity in their teaching of geometry than in their teaching of the other two content areas. The median diversity score for geometric content was 2.0. Fifteen percent of the scores were in the low diversity category, 72% were in the moderate category and 14% were in the high category.

As was the case for the mode of content representation and rule-orientedness variables, there were marked differences among the median scores for the diversity variable at the topic level. For two topics, decimal operations and the volume of a prism, the median score was in the low diversity category. In both cases over 80% of the teachers showed low diversity when teaching the topic.

Teachers showed high diversity in teaching five topics, two from arithmetic, two from algebra, and one from geometry. These topics were fractions, decimals, integer subtraction, linear equations, and the area of a parallelogram. The most diversity was shown by teachers in approaching fractions. The median diversity score for that topic was 5.0. For the other four topics the median score was between 3.0 and 3.3 inclusive.

Teachers showed moderate diversity in their approach to the remaining nine topics. In each case the median score was between 2.0 and 2.5 inclusive.

Differences were found between the implemented curricula in classes in which the overall student achievement level was low compared with classes in which the overall student achievement level was high. These differences, however, were in general quite small. This is surprising because the differences in mathematical achievement between the two groups of classes were quite large. Also, the B. C. Curriculum Guide (1978) states that teachers should provide "differences in approach, depth and rate of learning" through the use of the multiple-text adoption (p.3)[1]. Thus, one might have expected greater differences in instruction between the two groups of classes than were actually found to occur.

In general, students in low achievement classes were taught less geometry, more arithmetic, and slightly more algebra than their counterparts in high achievement classes. The correlations between class achievement as measured by the SIMS Core Pretest and the content emphasis scores for the three content areas were +0.28 for geometry, -0.23 for arithmetic, and -0.15 for algebra.

The differences in content emphasis for arithmetic and geometry were particularly noticeable when scores outside of the

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[1] Response by teachers to the GCPQ indicated that very few classrooms in this study were organized so that individualization occured within classrooms.

moderate emphasis category were considered. The only three classes in which arithmetic received heavy emphasis were low achievement classes while six of the nine classes in which arithmetic received light or very light emphasis were high achievement classes. For geometry this pattern was reversed. Four out of five classes in which geometry received heavy emphasis were high achievement classes while nine out of 11 classes in which geometry received light or very light emphasis were low achievement classes.

Across all content, teachers taught somewhat more abstractly in low achievement than in high achievement classes. The median mode of content representation score was 0.58 for low achievement classes compared with 0.54 for high achievement classes. Although this is a small difference, this finding is of interest for two reasons. First, it is consistent with recent research conducted in the United States as noted previously. Second, this finding is counter to what might be expected based on the ideal curriculum of mathematics educators in which the slower student might be expected to require more concrete representations of content than the more able student.

Overall, there was a slight tendency for rules to receive heavier emphasis in low achievement classes than in high achievement level classes. Rules received more emphasis in low achievement level classes for geometry content considered separately. Although the median rule-orientedness score for low achievement classes was not greater than the corresponding value for high achievement level classes for algebra content, six out of seven scores which were in the two rule-oriented categories for algebra were scores for low achievement classes. For the single arithmetic topic for which the rule-orientedness variable was defined no achievement level differences existed. This topic was usually taught with a heavy emphasis on rules in both low and high achievement classes.

In general students in low achievement classes tended to be taught more arithmetic and algebra and less geometry than students in high achievement classes. These students also tended to be taught somewhat more abstractly and with a very slightly greater emphasis on rules. Classes of both achievement levels tended to be taught with the same number of content specific teaching methods.

A fairly strong relationship was found between the emphasis which a content area receives in a textbook and the emphasis which that content area received in the implemented curriculum of a teacher using that textbook. <u>School Mathematics 2</u> contains 54% more arithmetic content than <u>Mathematics II</u> and the median content emphasis score for arithmetic was 38% higher for teachers using the former book. Similarly, <u>Mathematics II</u> contains 77% more geometry content than <u>School Mathematics 2</u> and the median content emphasis score for geometry was 46% higher for teachers using the former book. Algebra receives approximately the same amount of emphasis in both textbooks and the median content emphasis scores for teachers using the two books were nearly equal.

There was, then, a close association between the content emphasis of the formal B.C. Mathematics 8 curriculum as indicated by the number of pages the textbooks devoted to various content areas and the emphasis the teachers in this study gave to those content areas. Thus, it would seem that a necessary condition for influencing teachers to emphasize a content area more heavily is to select a textbook which provides an appropriate amount of emphasis for that content area. It should be noted, however, that the presence of a content area in a textbook in and of itself is no guarantee that teachers will teach that content. Probability and statistics content, for example, is included in both textbooks. However, this material is contained in chapters near the end of the textbooks and this content area is not included in the B. C. Curriculum Guide (1978) for Mathematics 8. The teachers in this study were asked in the GCPQ how much time they would spend teaching probability and statistics and indicated that they would devote a median of only 4% of their instructional time to this content area. Most commonly, teachers reported that they would spend no time at all teaching probability and statistics. Thus, the presence of probability and statistics chapters near the end of the commonly used texts did not influence most teachers to teach this material.

The association between the other curriculum variables examined in this study and the contents of the two commonly used textbooks were weaker than was the case for the content emphasis variable. This is probably true at least in part because the two texts differ much less with respect to these other variables than they differ with respect to content emphasis.

The teachers in this study represented mathematical content to their students more frequently in an abstract mode than in a perceptual mode. The median content representation score across all topics was 0.57. The two textbooks also represent content more frequently in an abstract than in a perceptual manner. In Mathematics II 62% of the content specific methods are abstract while in School Mathematics 2 55% of the methods are abstract. Thus, there was a consistency between the formal curriculum as embodied in the textbooks and the implemented curricula of teachers. The difference in abstraction between the two textbooks was not associated with a similar difference between the implemented curricula of the users of the textbooks, however. The users of the less abstract text, School Mathematics 2 were actually slightly more abstract in their presentations to students than were the users of the other text.

Because of these mixed findings, it is difficult to hypothesize what effect, if any, a change in the level of abstraction of the textbooks might have on the implemented curricula. It is possible that while teachers are influenced in the amount of time they spend on a content area by the amount of coverage given to that area by the textbook they are using, they are not similarly influenced in the content specific methods they choose to use to represent that content by the methods contained in the textbook. Alternately, it is possible that a relationship may exist and that the adoption of a more perceptually oriented textbook might result in a greater utilization of perceptual methods of representing content by teachers.

A weak positive association was found between the number of methods teachers employed in teaching the 16 topics examined in this study and the number of methods contained in the textbook they used. Overall, <u>Mathematics II</u> contains 16% more teaching methods than <u>School Mathematics 2</u> for these topics and the median diversity of instruction score for teachers using the former text was 9% higher than for teachers using the latter text. For four topics (Topic Group 2), <u>School Mathematics 2</u> provides more instructional options for teaching containing almost three times as many teaching methods as the other text. For those topics, teachers using <u>School Mathematics 2</u> had a median diversity score 13% higher than users of the other text. These findings provide some evidence that teachers may be influenced in the number of approaches they use for the mathematical topics they teach by the number of approaches contained in the textbook they use. The relatively small difference in methods used by teachers for Topic Group 2 in spite of the large difference in the number of methods contained in the two texts indicates, however, that this influence may be small.

2. IMPLICATIONS FOR PRACTICE

Many of the assertions which are made in this section go beyond the data presented in this study. Inferences are drawn not only from the results of the study itself but also from the author's experience as a teacher as well as his interpretation of the ideal curricula contained in the mathematics education literature. This has been done because the section deals specifically with implications for practice and requires additional knowledge beyond that gained from this study. It is anticipated that some readers will reach conclusions different from those of the author based on this study as well as their own experiences and thereby arrive at implications for practice different from those outlined below.

One important finding of this study is that for all of the curriculum variables which were examined considerable variation existed among teachers. In particular, teachers did not follow a single pattern of content emphasis in their courses with the widest variation existing for geometry. Whether or not the degree of variation observed is desirable is a matter for discussion. Certainly, when teachers plan their courses on the basis of an assumed curricular background of students, such a lack of uniformity may be problematic. If so, a more detailed and prescriptive curriculum guide might be needed although the relationship between the contents of such a document and teacher practice is not known. A single text adoption might also increase uniformity in implemented curricula. Alternately, a method of deciding which of several texts is appropriate for any particular class might provide a more rational basis for having different mathematics curricula at the Grade 8 level.

Although mathematics deals inherently with abstractions, mathematical ideas, at least those included in the formal Mathematics 8 curriculum, can be represented quite concretely. Particularly when students are introduced to mathematical concepts, operations, and principles for the first time it may be advisable for perceptual representations to predominate over abstract representations. Although the majority of teachers balanced their approaches to algebraic and geometric content, a significant number relied heavily on abstract methods. Also, a majority of teachers dealt with three of the algebraic topics and three of the geometric topics in an abstract manner. Inservice training to make teachers aware of the mode of representation variable and to expose them to more perceptual teaching methods for selected topics might increase the number of perceptual methods they present to students.

The results of this study indicate (to me) that overall teachers are probably putting a reasonable emphasis on rules during instruction. However the median rule-orientedness scores varied considerably among topics. Rules should probably be more heavily emphasized for review topics than for new materials. Inservice training to make teachers aware of the rule-orientedness variable might increase the probability that teachers would follow a more consistent pattern in emphasizing rules.

Overall, teachers seemed to provide their students with a reasonable diversity of approaches to the topics they taught. However, a number of topics were taught with low diversity by a significant number of teachers and in several cases by a majority of teachers. The median diversity scores were particularly low for geometric topics. Inservice training to make teachers aware of additional methods for teaching the topics which were frequently dealt with with low diversity may improve this situation

Some evidence of associations between textbook contents and the implemented curricula of teachers was provided by this study. A fairly strong positive association was found between the way content areas were emphasized in textbooks and the content emphasis scores of teachers using those textbooks. Also, there was evidence that teachers used greater diversity in teaching topics when more content specific teaching methods were contained in the book they were using. Also, the overall level of abstraction that teachers used in presenting mathematical topics was nearly the same as the level of abstraction with which those topics were dealt with in the two textbooks. While the differences in level of abstraction in the textbooks was not associated with similar differences in the instruction of teachers using those textbooks, this is probably true because of the similarity of the two textbooks in their mode of representing content.

The associations found between the formal curriculum of textbooks and the implemented curriculum of teachers do not necessitate a cause and effect relationship. However, it can be hypothesized that such a relationship does exist and that the contents of a textbook in terms of the four curriculum variables will influence teachers' implemented curricula. Thus, it can be speculated that if it is desired that teachers put more emphasis on geometry, for example, or use more perceptual teaching methods, then a textbook consistent with these goals should be selected as part of the formal curriculum.

It was found in this study that the relationship between a teacher's implemented curriculum and the achievement level of the class to be taught is not a strong one. It can be asserted that a class with a large number of low achievement students requires more perceptual approaches to content than a class with a small number of low achievement students. The results of this study do not support the hypothesis that this occurs in practice but rather that, if anything, the opposite occurs.

While low achievement students may require more perceptual methods than high achievement students, they may also require that a stronger emphasis be put on the rules of mathematics. Very weak evidence was found that some differentiation of this sort by class achievement does occur.

The clearest relationship between class achievement and the implemented curriculum existed for the content emphasis variable. In general, low achievement classes received more arithmetic and algebra instruction and less geometry instruction than high achievement classes. While more stress on arithmetic in low achievement classes is probably appropriate, it may be that this is happening too much at the expense of geometry.

Also, while the median content emphasis score for low achievement classes is higher than the corresponding score for high achievement classes, both distributions of scores contain considerable variation. Thus, there appear to be many low achievement classes in which arithmetic is emphasized too little and many high achievement classes in which this content area is emphasized too much.

In spite of the differences in the implemented curricula of low and high achievement classes noted above, a major finding of this study is that two classes of markedly different achievement levels may have nearly identical curricula or may, in fact, differ in ways opposite to what might be thought desirable. While the B. C. Curriculum Guide does indicate that teachers should organize their classes to meet their students' needs, it may be that teachers will require more assistance to realize this goal. More specification and differentiation by achievement may be required within the formal curricula if one is to expect substantial curricular differences for classes of different achievement levels.

3. SIGNIFICANCE OF THE STUDY

This study is significant for several reasons. First, it represents an initial attempt to describe implemented curricula using the four mathematics curriculum variables which were incorporated in this study and to display and analyze the results using basic techniques of Exploratory Data Analysis. This represents a more detailed method of investigating the curricular aspect of teaching practice than has been the case previously.

Besides the methodological contribution made by this study, this research provides results which may be useful in planning curriculum revision strategies and in establishing a benchmark from which curriculum change can be measured. Information over time similar to that presented here should provide a sounder basis for assessing the curriculum that is reaching students and for comparing this curriculum to those of the past.

4. LIMITATIONS OF THE STUDY

A limitation of this study is that it utilized instrumentation which was not specifically constructed to collect data about the curriculum variables which were examined in the study. The TSQ and GCPQ instruments were designed as part of the SIMS project to provide data about the implemented curricula within various international jurisdictions and the teaching methods which were contained in the TSQs for each mathematical topic were quite comprehensive. However, the wording of some of the content specific methods given in TSQ items was such that it was not possible to investigate each curriculum variable for every topic. For example, for the angle sum topic one item contained the rule approach "I told my students that the sum of the measures of the angles of a triangle is 180 degrees." Unfortunately, this item included additional wording so that it was not possible to say that a teacher who utilized the teaching method contained in that item was, strictly speaking, presenting the mathematical idea in question by just giving students a rule. Because of similar problems of wording it was only possible to define the rule-orientedness variable for half of the topics which were studied.

The fact that this study involved the re-analysis of data collected for another study meant that several topics which were part of the formal B.C. Mathematics 8 curriculum could not be examined. Square roots, scientific notation, and the translation of English phrases into mathematical expressions are examples of topics which could not be included in this study because they were not included in the TSQs.

A further possible limitation of this study is related to teachers' assessments of the achievement levels of their classes. In this study associations between implemented curricula and class achievement were examined. Associations between implemented curricula and teacher perception of class achievement were not examined. If teachers' perceptions differed markedly from actual class achievement, some results of this study may be misleading.

To determine the congruence between class achievement and teacher perception, data which were collected as part of the B.C. SIMS project were used. Each teacher was asked to estimate the number of students in his or her class who fit into each of three categories of mathematical achievement for the province: 1) top third, 2) middle third, and 3) bottom third. A fourth category, "unable to judge," was also included. A measure of teacher perception of class achievement was determined for each teacher by subtracting the number of students reported in category three from the number of students reported in category one and dividing this quantity by the sum of the number of students in categories one, two, and three. The resulting quantity could take on values from -1 to +1 with -1 indicating that the teacher perceived every student as low achievement (bottom third) and +1 indicating that the teacher perceived every student as high achievement (top third).

This measure of teacher perception of class achievement was then correlated with the measure of actual class achievement which was used in this study, the Core Pretest score. The resulting correlation was +0.65. While one must still use caution in equating teacher perception of class achievement with actual class achievement when interpreting the results of this study, the rather high positive correlation obtained indicates that the teachers in this study did, in general, assess the achievement level of their classes rather accurately.

5. SUGGESTIONS FOR FUTURE RESEARCH

This study provides a survey of the implemented Mathematics 8 curriculum in British Columbia as it existed in the 1980/1981 school year. In order to assess changes in the implemented curriculum over time replications of this study could be undertaken on a periodic basis. The data collection instruments could be refined to more accurately measure the curriculum variables and the questionnaires could be expanded to include additional topics and content areas such as probability and statistics. Also, instrumentation could be developed for other grade levels.

Because of the survey nature of this research it was only possible to examine associations and not to establish causal relationships. Future research could examine how manipulations of the formal curriculum might influence the curriculum as implemented in the classroom. For example, curriculum materials could be selected or produced as part of a curriculum revision and randomly assigned to some B.C. classrooms. If these new materials contain a different pattern of content emphasis than the old materials, then the implemented curricula of classes using both the old and new materials could be examined. In this way the strength of the link between the content in the formal and implemented curricula could be investigated. In a similar way the influence of the mode of content representation, rule-orientedness and diversity within the formal curriculum on the implemented curriculum could be investigated.

Wide variation among the implemented curricula which were examined in this study existed for each of the curriculum variables. It is not clear why such variations exist. Further research on teacher decision-making in curriculum implementation and selection is needed. In this regard the relationship between the degree of specificity within the formal curriculum and the amount of variation in content emphasis should be investigated. The formal curriculum in British Columbia for Mathematics 8 allows teachers considerable flexibility in planning their courses. In other jurisdictions the formal curriculum is more precisely specified. The degree to which this specificity tends to produce a more uniform implemented curricula is unknown and warrants further investigation.

The evidence of this study supports the hypothesis that class achievement level is not an especially influential factor in teacher decision making regarding curriculum for Mathematics 8 courses. In particular no strong associations were observed between mode of content representation, rule-orientedness of instruction, and diversity of instruction on the one hand and class achievement level on the other hand. Since it would seem that some differences in curriculum might be desirable for classes of differing achievement levels, this hypothesis warrants further investigation.

Although the teachers who participated in this study in general used both perceptual and abstract teaching methods, it is not clear if they usually or ever sequenced their presentations in accordance with the enactive, iconic, symbolic model of Bruner (1966) and Dienes (1960, 1964, 1973). This is another area requiring further research.

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VI. APPENDIX A - TOPICS FROM THE SIMS TOPIC SPECIFIC QUESTIONNAIRES USED IN THIS STUDY

The mode of representation variable was defined for the following 14 topics. Item numbers are given for the methods which were classified as perceptual and abstact for each topic.

	Topic	Perceptual Methods	Abstract Methods
1.	Fractions	21,22,23,27,28,30	24,25,26,29
2.	Addition of Fractions	31,32,33,37,38	34,35,36
3.	Decimals	51,53,56	52,54,55
4.	Operations with decimals	59	57,58
6.	Integers	20,22,24	21,23
7.	Addition of integers	25,27	26
8.	Subtraction of integers	28,32	29,30,31,33
9.	Multiplication of integers	36	34,37,38
10.	Formulas	45,47,48	44,46
12.	Angle sum theorem	59,61,62,63,66	60,64
13.	Pythagorean theorem	67,68,71	69,72,73
14.	Concept of π	48,50,53,54	49,51,52
15.	Area of a parallelogram	56,57,59	55,58,60,61,62
16.	Volume of a prism	64,65	63

Topics 5 and 11 are the concept of proportions and solving linear equations respectively. The mode of representation variable was not defined for these two topics.

The rule-orientedness variable was defined for the following eight topics. An item number is given for the method which was considered the rule approach for each topic.

	Topic	Rule Method
4.	Operations with decimals	58
7.	Addition of integers	26
8.	Subtraction of integers	33
9.	Multiplication of integers	38
12.	Pythagorean theorem	69
14.	Concept of π	49
15.	Area of a parallelogram	55

TOPICS 1-16, SIMS TOPIC SPECIFIC QUESTIONNAIRE

Leaves 197-212 not filmed; permission not obtained.

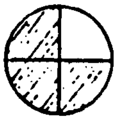
APPENDIX B - SIMS CORE PRETEST FOR MATHEMATICS 8

Leaves 213-28 not filmed; permission not obtained.

Topic 1 (Arithmetic) - Concept of Fractions

21. Fractions as part of regions:

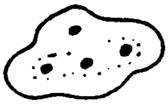
$\frac{3}{4}$ means



___ 1 ___ 2 ___ 3

22. Fractions as part of a collection:

$\frac{3}{4}$ means



___ 1 ___ 2 ___ 3

23. Fractions as the coordinates of points on the number line:

___ 1 ___ 2 ___ 3

24. Fractions as quotients:

$\frac{3}{4}$ means "3 divided by 4"

___ 1 ___ 2 ___ 3

25. Fractions as decimals:

$\frac{3}{4} = 0.75$

___ 1 ___ 2 ___ 3

26. Fractions as repeated addition of a unit fraction.

$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

___ 1 ___ 2 ___ 3

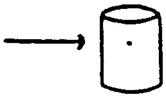
27. Fractions as ratios:

$\frac{3}{4}$ means

___ 1 ___ 2 ___ 3

28. Fractions as measurements:
this container holds

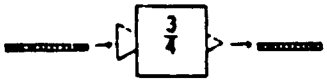
$\frac{3}{4}$ L



this stick is $\frac{3}{4}$ cm

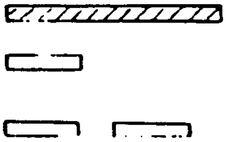
___ 1 ___ 2 ___ 3

29. Fractions as operators:



___ 1 ___ 2 ___ 3

30. Fractions as comparisons



unit rod

$\frac{1}{3}$ rod

$\frac{2}{3}$ rod

___ 1 ___ 2 ___ 3

Topic 2 (Arithmetic) - Addition of Fractions

31. The sum of two fractions as the union of two regions

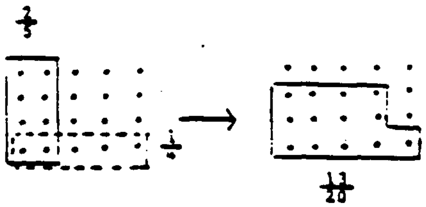
Ex. $\frac{2}{3} + \frac{1}{4}$ as

___1 ___2 ___3

32. The sum of two fractions as combination of fractional parts of a collection

Ex. $\frac{2}{5} + \frac{1}{4}$ as

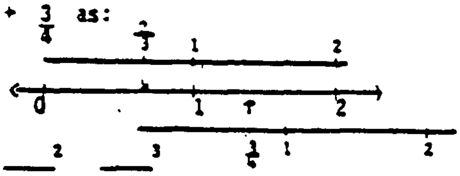
(Note: the collection consists of 20 dots)



___1 ___2 ___3

33. The sum of two fractions on the number line

Ex. $\frac{2}{3} + \frac{3}{4}$ as:



___1 ___2 ___3

34. The sum of fractions as the sum of two quotients

Ex. $\frac{2}{3} + \frac{3}{4}$ as (2 ÷ 3) + (3 ÷ 4)

Since 2 ÷ 3 = 8 ÷ 12
And 3 ÷ 4 = 9 ÷ 12
(8 ÷ 12) + (9 ÷ 12) = (8 + 9) ÷ 12,
= 17 ÷ 12

___1 ___2 ___3

35. The sum of two fractions as the sum of two decimals.

Ex. $\frac{3}{4} + \frac{2}{5} = 0.75 + 0.40$
$= 1.15$

___1 ___2 ___3

36. The sum of two fractions using fractions as repeated addition of the unit fractions

Ex. $\frac{2}{5} + \frac{4}{5}$

$= (\frac{1}{5} + \frac{1}{5}) + (\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5})$

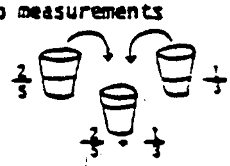
$= \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

$= \frac{6}{5}$

___1 ___2 ___3

37. The sum of two fractions as a combination of two measurements

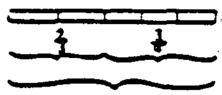
Ex. $\frac{2}{5} + \frac{1}{3}$ as



___1 ___2 ___3

38. The sum of two fractions as joining two segments

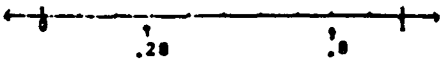
Ex. $\frac{2}{3} + \frac{3}{4}$ as



___1 ___2 ___3

Topic 3 (Arithmetic) - Concept of Decimals

51. A decimal as the coordinate of a point on the number line.



.28 < .8

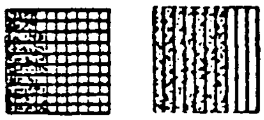
___ 1 ___ 2 ___ 3

52. A decimal as another way of writing a fraction.

$0.17 = \frac{17}{100}$ $0.8 = \frac{8}{10}$

___ 1 ___ 2 ___ 3

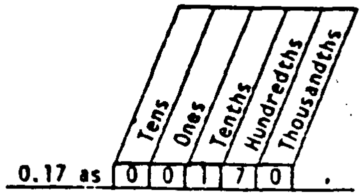
53. A decimal as part of a region.



0.38 0.7

___ 1 ___ 2 ___ 3

54. A decimal as an extension of place value.



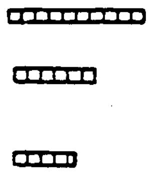
___ 1 ___ 2 ___ 3

55. A decimal as a series

$0.243 = \frac{2}{10} + \frac{4}{100} + \frac{3}{1000}$

___ 1 ___ 2 ___ 3

56. A decimal as a comparison



unit rod

0.6

0.45

___ 1 ___ 2 ___ 3

Topic 4 (Arithmetic) - Operations with Decimals

57. Related operations with decimals to operations with fractions.

Ex. $0.7 \times 0.6 =$

But $0.7 = \frac{7}{10}$ and $0.6 = \frac{6}{10}$

So $0.7 \times 0.6 = \frac{7}{10} \times \frac{6}{10}$

$= \frac{42}{100}$

Therefore $0.7 \times 0.6 = 0.42$

___1 ___2 ___3

58. Related operations with decimals to operations with whole numbers, teaching rules for placing the decimal point.

Ex. $1.38 \times 5.2 =$ ____

Since
$$\begin{array}{r} 138 \\ \times\ 52 \\ \hline 276 \\ 690 \\ \hline 7176 \end{array}$$

$\underset{2\,places}{1.38} \times \underset{1\,place}{5.2} = \underset{3\,places}{7.176}$

___1 ___2 ___3

59. Used concrete materials to illustrate operations with decimals.

Ex. $3.47 + 2.13 =$ ____

Using rods I demonstrated that

3.47 m

and

2.13 m

makes

5.60 m

___1 ___2 ___3

Topic 5 (Arithmetic) - Concept of Proportions

26. Proportions as equivalent ratios:

    Ex. 12 heartbeats per 10 seconds is the same as 72 beats per min.

    ___1 ___2 ___3

27. Proportions as equivalent comparisons:

    Ex. 9 red cars to 12 blue ones is the same as 3 to 4.

    ___1 ___2 ___3

28. Proportions as equivalent fractions:

    Exs. i) 1/3 = 4/12

ii)

$\frac{3}{2}$	$\frac{3 \times 2}{2 \times 2}$	$\frac{3 \times 3}{2 \times 3}$	$\frac{3 \times 4}{2 \times 4}$
$\frac{3}{2}$	$\frac{6}{4}$	$\frac{9}{6}$	$\frac{12}{8}$

    ___1 ___2 ___3

29. Proportions as equivalent quotients:

    Ex. 3:4 and 9:12 Since $3 \div 4 = 0.75$ and $9 \div 12 = 0.75$. the quotients are equal; so 3:4 and 9:12 are equivalent.

    ___1 ___2 ___3

Topic 6 (Algebra) - Concept of Integers

20. Extending the number ray to the number line:

I extended the number ray (0 and positive numbers) to the left by introducing direction as well as magnitude.

Ex: --------------------------
-4 -3 -2 -1 0 1 2 3 4

-3 means 3 units to the <u>left</u> of 0.

___1 ___2 ___3

21. Extending the number system to find solutions to equations:

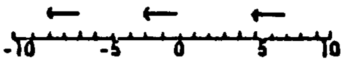
I discussed the need to extend the positive integers in order to find a solution to equations like __ + 7 = 5.

___1 ___2 ___3

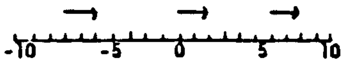
22. Using vectors or directed segments on the number line:

I defined an integer as a set of vectors (directed line segments) on the number line.

Ex: -2 can be represented by any of:



Ex: +2 can be represented by any of:



___1 ___2 ___3

23. Defining integers as equivalence classes of whole numbers:

I developed the integers as equivalence classes of ordered pairs of whole numbers.

Ex: $\{(0,2),(1,3),(2,4),\ldots\} = {}^{-}2$

or $\{(a,b) \in W \times W : b = a + 2\} = {}^{-}2$

___1 ___2 ___3

24. Using examples of physical situations:

I developed integers by referring to different physical situations which can be described with integers.

Ex: thermometer, elevation, money (credit/debit), sports (scoring), time (before/after), etc.

___1 ___2 ___3

Topic 7 (Algebra) - Addition of Integers

25. **Addition on the number line:**

    I used the number line to add integers.

    ____1 ____2 ____3

26. **Addition by rules:**

    I used rules to add integers.

    Ex: If both addends have the same sign, the sum is found by adding their numerical (absolute) values and adjoining the common sign.

    ____1 ____2 ____3

27. **Use of physical situations:**

    I used physical situations to add integers.

    Ex: In climbing out of the Dead Sea Valley, the car started at an elevation of -643 feet and climbed 432 feet to an elevation of ______ feet.

    ____1 ____2 ____3

Topic 8 (Algebra) - Subtraction of Integers

28. Subtraction as addition of opposites:

    I used the number line to subtract integers by starting at the minuend and going the number of units indicated by the subtrahend but in the direction opposite of its sign.

    ___1 ___2 ___3

29. Subtraction as inverse of addition:

    I used the inverse relation between addition and subtraction to subtract integers.

    Ex: $^{+}4 - {}^{-}3 =$ ____
    Solve $^{+}4 =$ ____ $+ {}^{-}3$

    ___1 ___2 ___3

30. Subtraction by rules:

    I used rules to subtract integers.

    Ex: To subtract an integer, add its opposite.

    ___1 ___2 ___3

31. Subtraction as a number of units:

    I extended the meaning of subtraction of whole numbers (i.e. $y - x$ means the number of units from x to y) to integers.

    Ex: $^{+}4 - {}^{-}3$ means the number of units from $^{-}3$ to $^{+}4$.

    ___1 ___2 ___3

32. Subtraction as distance:

    I used the number line to subtract integers by finding the number of units (or distance) from the subtrahend to the minuend.

    ___1 ___2 ___3

33. Subtraction as "what must be added":

    I interpreted subtraction to mean "what must be added" to the subtrahend to get the minuend.

    Ex: $^{+}4 - {}^{-}3 =$ ____ means "what must be added to $^{-}3$ to get $^{+}4$".

    ___1 ___2 ___3

Topic 9 (Algebra) - Multiplication of Integers

34. Development by use of repeated addition:

I developed the concept of multiplication by appealing to repeated addition, e.g., $4 \times {}^-3 = {}^-3 + {}^-3 + {}^-3 + {}^-3 = {}^-12$

35. Development by the extension of properties of the whole number system:

I developed the concept of multiplication of integers by using the commutative, associative, and distributive properties to justify the products, e.g.,

${}^-4 \times {}^-3 = \underline{\qquad}$

$$\begin{aligned} \text{But } 0 &= ({}^-4 + {}^+4) \times {}^-3 \\ &= ({}^-4 \times {}^-3) + ({}^+4 \times {}^-3) \\ &= ({}^-4 \times {}^-3) + {}^-12 \end{aligned}$$

Hence ${}^-4 \times {}^-3 = {}^+12$

36. Development by use of physical situations:

I developed the concept of multiplication of integers by appealing to physical situations that might illustrate the product of positive and negative numbers, e.g., A refrigerator is cooling at a rate of 4° per minute. Its thermometer is at 0°. What will be its temperature 4 minutes from now?

37. Development by use of patterns:

I developed the concept of multiplication of integers by appealing to patterns of products, e.g.,

$$\begin{array}{llll} {}^+4 & \times & {}^-3 & = {}^-12 \\ {}^+3 & \times & {}^-3 & = {}^-9 \\ {}^+2 & \times & {}^-3 & = {}^-6 \\ {}^+1 & \times & {}^-3 & = {}^-3 \\ 0 & \times & {}^-3 & = 0 \\ {}^-1 & \times & {}^-3 & = 3 \\ {}^-2 & \times & {}^-3 & = 6 \end{array}$$

38. No development -- students were given rules:

I did not develop the facts for multiplication of integers by using any of the above methods. I instead gave them rules similar to the following.

If the signs are alike, the answer is positive. If they are different the answer is negative. If one factor is zero, the answer is zero.

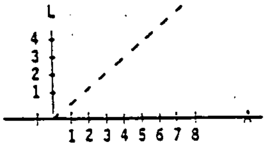
Topic 10 (Algebra) - Concept of Formulas

44. Presenting formulas and explaining the meaning of the terms in the formulas:

Ex: Formula: $A = \frac{1}{2} bh$
<u>A</u> stands for the area of a triangle
<u>b</u> stands for the base of a triangle
<u>h</u> stands for the height of a triangle

45. Having the students inspect graphs and find formulas to express the relationships portrayed by the graph:

Ex:



$A = 2 \times L$

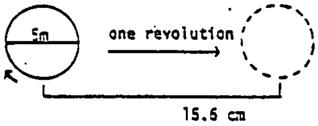
46. Providing data from which formulas or equations are developed:

x	y
0	0
1	3
2	5
3	7
4	9
5	11

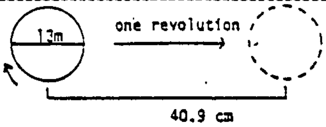
Hence $y = 2x + 1$

47. Having students collect data on related variables and formulate the relationship between the variables:

Ex.



Ratio: $\frac{15.6}{5} = 3.12$

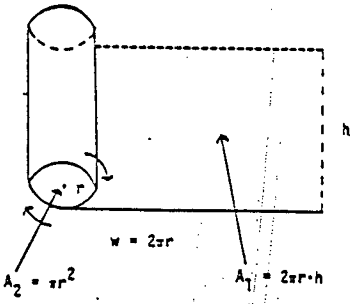


Ratio: $\frac{40.9}{13} = 3.15$

Hence $\frac{c}{d} = 3.1$, So $C = 3.1d$

48. Having students create new formulas based on known, simpler formulas:

Ex. Create formula for surface area of a cylinder based on formulas for area of the rectangle and the circle.



So, surface area $= 2\pi rh + 2\pi r^2$

$SA = 2\pi r (h + r)$

Topic 11 (Algebra) - Solving Linear Equations

39. Using properties of equality with operations with numbers:

Ex: $7x + 5 = 40$
$7x + 5 - 5 = 40 - 5$
(Subtract 5 from both sides)
$7x = 35$
(arithmetic fact)
$\frac{7x}{7} = \frac{35}{7}$
(divide both sides by 7)
$x = 5$

___1 ___2 ___3

40. Using inverse operations with numbers:

Ex: $7x + 5 = 40$
$7x + 5 - {}^{-}5 = 40 + {}^{-}5$
(add the inverse of 5 to both sides)
$7x = 35$
$\frac{1}{7} \cdot (7x) = \frac{1}{7} \cdot 35$
(multiply both sides by the reciprocal of 7)
$x = 5$

___1 ___2 ___3

41. Using arithmetical reasoning:

Ex: Given $7x + 5 = 40$

What number increased by 5 is 40 (__ + 5 = 40)? Since the number is 35, then 7 times what number gives 35 (7 x __ = 35)? The solution is 5.

___1 ___2 ___3

42. Using trial and error:

Ex: Given $7x + 5 = 40$

Try $x = 4$. But $7(4) + 5 = 33$. So try $x = 5$, as x needs to be larger. $7(5) + 5 = 40$. So, $x = 5$.

___1 ___2 ___3

43. Given $7x + 5 = 40$

Example Rules

-- collect all constant terms on one side of the equation and all variable terms on the other.
$7x = 40 - 5$
-- combine like terms.
$7x = 35$
-- divide by the coefficient of x.
$x = 5$

___1 ___2 ___3

Topic 12 (Geometry) - Angle Sum Theorem

59. My students measured the angles of a triangle and added the measures to discover that the sum of the measures is 180°.

___ 1 ___ 2 ___ 3

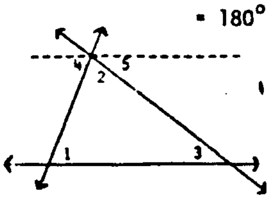
60. I drew a line through a vertex parallel to the opposite side and used alternate interior angles to show that the sum of the angles of a triangle is 180°.

Ex:

In the figure $\angle 1 = \angle 4$ and $\angle 3 = \angle 5$,

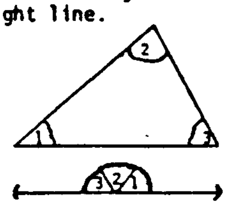
So $\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 2 + \angle 5$

$= 180°$



___ 1 ___ 2 ___ 3

61. My students cut the angles off a triangle and arranged them on a straight line.

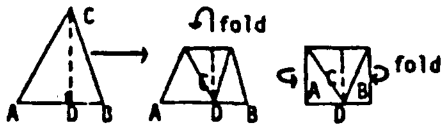


___ 1 ___ 2 ___ 3

62. I told my students that the sum of the measures of the angles of a triangle is 180° and had them verify it by measuring the angles and adding the measures.

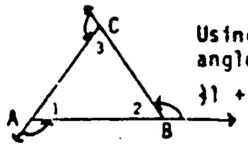
___ 1 ___ 2 ___ 3

63. I had my students verify the relationship by paper folding.



___ 1 ___ 2 ___ 3

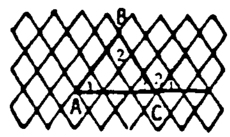
64. I used the fact that (as illustrated in the figure) in traveling AB, BC, CA, a complete revolution (360°) is swept.



Using this and angle supplements, $\angle 1 + \angle 2 + \angle 3 = 180°$

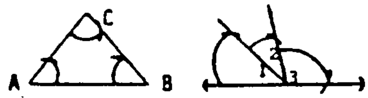
___ 1 ___ 2 ___ 3

65. Using tessellations perhaps from the real world, I identified three angles at a point (C) congruent with three angles in a triangle (ABC) embedded in the tessellation.



___ 1 ___ 2 ___ 3

66. A ruler and compass construction was used to show the relationship.



$\angle A = \angle 1$ $\angle B = \angle 2$ $\angle C = \angle 3$

___ 1 ___ 2 ___ 3

Topic 13 (Geometry) - Pythagorean Theorem

67. I presented my students with a variety of right triangles and had them measure and record the lengths of the legs and hypotenuse. The pattern was discussed and then we stated the property.

Ex:

leg	leg	hypotenuse
3	4	5
9	12	15

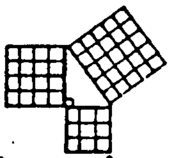
$3^2 + 4^2 = 5^2$

$9^2 + 12^2 = 15^2$

$\therefore a^2 + b^2 = c^2$

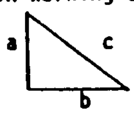
___ 1 ___ 2 ___ 3

68. I used diagrams like the following to show that, in a right triangle $a^2 + b^2 = c^2$



___ 1 ___ 2 ___ 3

69. I gave my students the formula $a^2 + b^2 = c^2$ and had them use it in working examples.



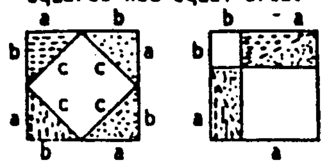
___ 1 ___ 2 ___ 3

70. The theorem was presented in the context of a historical account of Pythagoras and Euclid.

___ 1 ___ 2 ___ 3

71. I presented an informal area argument using physical, e.g. geoboards, or pictoral models.

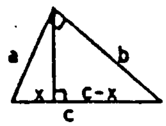
Ex: I showed that the two squares had equal area.



___ 1 ___ 2 ___ 3

72. I presented a formal deductive "algebraic" argument.

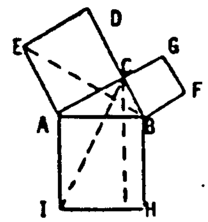
Ex: Using similar right triangles, proportions can be set up to yield $a^2 + b^2 = c^2$



___ 1 ___ 2 ___ 3

73. I presented a formal deductive argument using area.

Ex: This figure is sometimes used to present a formal proof.



___ 1 ___ 2 ___ 3

Topic 14 (Geometry) - Concept of π

48. I had my students measure and find the ratio of the circumference to the diameter of a number of circular objects, and approximate $\frac{c}{d}$ for any circle.

___ 1 ___ 2 ___ 3

49. I told my students that $\pi = \frac{22}{7}$ or 3.14.

___ 1 ___ 2 ___ 3

50. My students estimated the value of π using Buffon's Needle Problem.

___ 1 ___ 2 ___ 3

51. I presented a chart relating the values of the circumference to that of the diameter of various circles like the following:

C	44	28	37
d	14.0	8.9	11.8

I asked the students to find the ratio of the circumference to the diameter for each circle and generalized that $\frac{c}{d} = 3.14$.

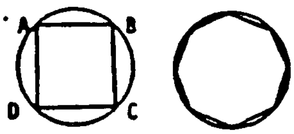
___ 1 ___ 2 ___ 3

52. I told my students that π is an irrational number obtained as the result of dividing the circumference of any circle by its diameter.

___ 1 ___ 2 ___ 3

53. I had my students use regular polygons inscribed in a circle to obtain successive approximations of π.

Ex.



Using square ABCD, $\pi \doteq 2.75$
Using the octagon, $\pi \doteq ?$
and so on, to show that π approaches 3.14 as the number of sides of the polygons increases.

___ 1 ___ 2 ___ 3

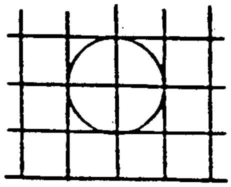
54. I introduced π as the area of a circle of radius 1.

Ex. Using successive approximations to the area of the unit circle, I showed that:

$2 <$ area of circle < 4

OR

$2 < \pi < 4$



Using a finer grid, I showed that:

$\frac{68}{25} <$ area of circle $< \frac{88}{25}$ OR

$2.72 < \pi < 3.52$

Using still a finer grid, I showed that:

$\frac{288}{100} <$ area of circle $< \frac{344}{100}$ OR

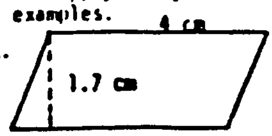
$2.88 < \pi < 3.44$ and so on.

___ 1 ___ 2 ___ 3

Topic 15 (Geometry) - Area of a Parallelogram

55. I presented the formula $A = b \times h$ and demonstrated how to apply it by means of examples.

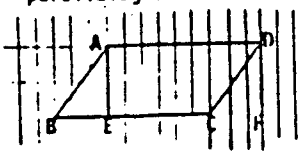
Ex.



$A = 4\ cm \times 1.7\ cm = 6.8\ cm^2$

___ 1 ___ 2 ___ 3

56. I presented a parallelogram on a grid (or a geoboard) like the one below (parallelogram ABCD), and had the students relate the number of square units inside ABCD to the base and altitude of the parallelogram.

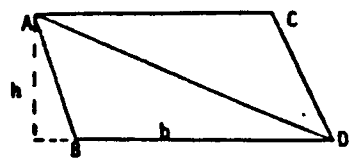


___ 1 ___ 2 ___ 3

57. I presented a parallelogram on a grid (or a geoboard) like the one shown above and had the students count the square units inside triangles ABE and CDF. Then I had them relate the area of ABCD to that of rectangle BEFC based on the congruence of $\triangle$ ABE and $\triangle$ BCF.

___ 1 ___ 2 ___ 3

60. I partitioned the parallelogram by a diagonal into two congruent triangles.



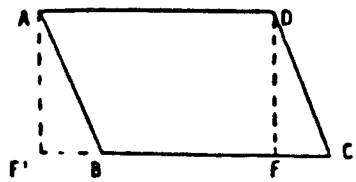
Then the area of $\triangle$ ABD is $\frac{1}{2}$ bh and the area of the parallelogram is then bh.

___ 1 ___ 2 ___ 3

58. I derived the formula $A = b \times h$ by comparing the area of the parallelogram to that of a related rectangle of equal dimensions.

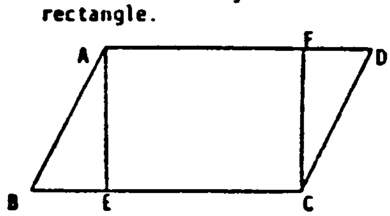
___ 1 ___ 2 ___ 3

59. I gave the student a parallelogram like the one below, and asked them to cut off triangle FDC and to use this to form a rectangle (AF'FD). The students then related the formula for the area of the rectangle to the area of the parallelogram.



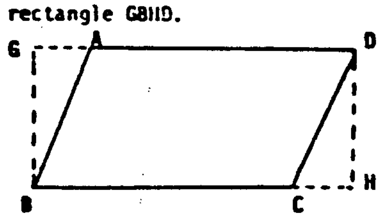
___ 1 ___ 2 ___ 3

61. I partitioned the parallelogram ABCD into $\triangle$ ABE, $\triangle$ CDF and rectangle AECF so that the area of the parallelogram is obtained by adding the areas of the two triangles and the rectangle.



___ 1 ___ 2 ___ 3

62. I obtained the area of the parallelogram by subtracting the areas of $\triangle$ ABG and $\triangle$ DCH from the area of the rectangle GBHD.

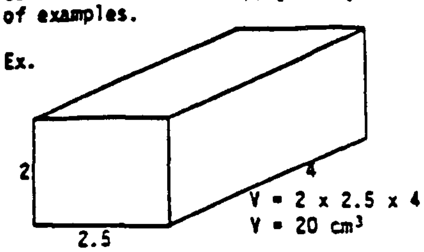


___ 1 ___ 2 ___ 3

Topic 16 (Geometry) - Volume of a Prism

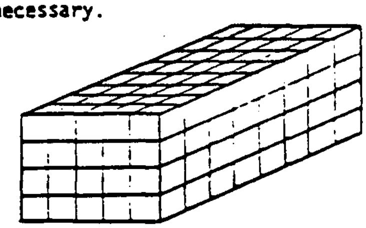
63. I presented the formula $V = l \times W \times h$ or $V =$ (area of base) $\times$ height and demonstrated how to apply it by means of examples.

Ex.



64. I presented a physical model of a right rectangular prism (box) with its faces marked off in square units, as illustrated below. I had students generate the formula by relating the number of cubic units contained in the prism to the dimensions of the box, giving hints only if necessary.

Ex.



65. I provided my students with unit cubes and asked them to build rectangular prisms of specified dimensions. I asked them to relate the number of unit cubes used to the given dimensions, giving hints only if necessary.

APPENDIX B - SIMS CORE PRETEST FOR MATHEMATICS 8

1. 2 metres + 3 millimetres is equal to

    A. 2.0003 metres

    B. 2.003 metres

    C. 2.03 metres

    D. 2.3 metres

    E. 5 metres

2. $\frac{1}{5}$ =

    A. 0.20%

    B. 2%

    C. 5%

    D. 20%

    E. 25%

3. If $5x + 4 = 4x - 31$, then x is equal to

    A. -35

    B. -27

    C. 3

    D. 27

    E. 35

4. Four 1-litre bowls of ice cream were set out at a party. After the party, 1 bowl was empty, 2 were half full, and 1 was three quarters full. How many litres of ice cream had been EATEN?

    A. $3\frac{3}{4}$

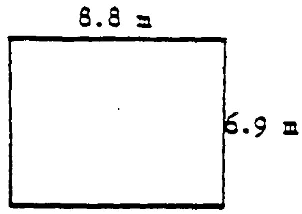
    B. $2\frac{3}{4}$

    C. $2\frac{1}{2}$

    D. $1\frac{3}{4}$

    E. None of these

5.



Which of the following is the closest approximation to the area of the rectangle with measurements given?

A. 48 m^2

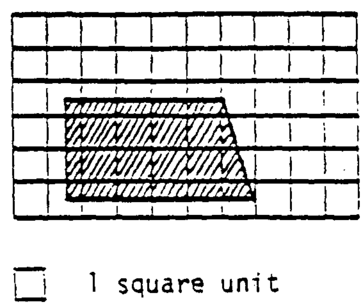
B. 54 m^2

C. 56 m^2

D. 63 m^2

E. 72 m^2

6.



The area of the shaded figure, to the nearest square unit, is

A. 23 square units

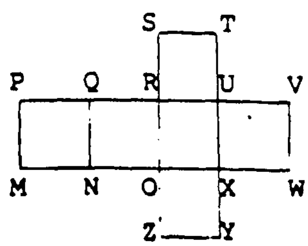
B. 20 square units

C. 18 square units

D. 15 square units

E. 12 square units

7.



The diagram shows a cardboard cube which has been cut along some edges and folded out flat. If it is folded to again make the cube, which two corners will touch corner P?

A corners Q and S

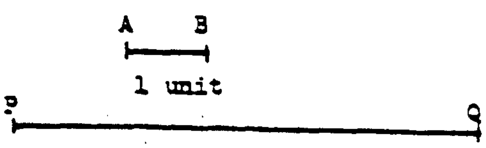
B corners T and Y

C corners W and Y

D corners T and V

E corners U and Y

8.



The length of $\overline{AB}$ is 1 unit. Which is the best estimate for the length of $\overline{PQ}$?

A 2 units

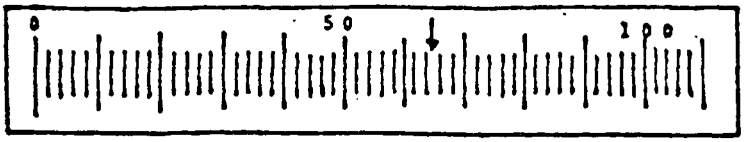
B 6 units

C 10 units

D 14 units

E 18 units

9.



On the above scale the reading indicated by the arrow is between

A. 51 and 52

B. 57 and 58

C. 60 and 62

D. 62 and 64

E. 64 and 66

10. A solid plastic cube with edges 1 centimetre long weighs 1 gram. How much will a solid cube of the same plastic weigh if each edge is 2 centimetres long.

A. 8 grams

B. 4 grams

C. 3 grams

D. 2 grams

E. 1 gram

11. On a number line two points A and B are given. The coordinate of A is -3 and the coordinate of B is +7. What is the coordinate of the point C, if B is the midpoint of the line segment $\overline{AC}$?

A. -13

B. $\frac{1}{2}$

C. +2

D. +12

E. +17

12. A painter is to mix green and yellow paint in the ratio of 4 to 7 to obtain the colour he wants. If he has 28 litres of green paint, how many litres of yellow paint should be added?

A. 11

B. 16

C. 28

D. 49

E. 196

13. If P = LW and if P = 12 and L = 3, then W is equal to

A. $\frac{3}{4}$

B. 3

C. 4

D. 12

E. 36

14. A model boat is built to scale so that it is $\frac{1}{10}$ as long as the original boat. If the width of the original boat is 4 metres, the width of the model should be:

A. 0.1 metre

B. 0.4 metres

C. 1 metre

D. 4 metres

E. 40 metres

15. The value of 0.2131 x 0.02958 is approximately

A. 0.6

B. 0.06

C. 0.006

D. 0.0006

E. 0.00006

16. (-2) x (-3) is equal to

A. -6

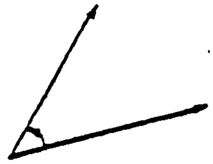
B. -5

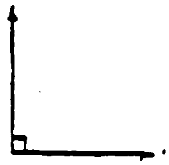
C. -1

D. 5

E. 6

17. Which of the indicated angles is ACUTE?

A. 

B. 

C. 

D. 

E. 

18. If $\frac{4x}{12} = 0$, then x is equal to

A. 0

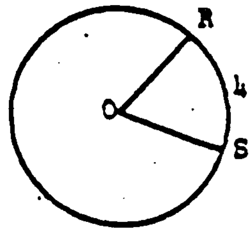
B. 3

C. 8

D. 12

E. 16

19.



The length of the circumference of the circle with centre at O is 24 and the length of arc RS is 4. What is the measure in degrees of the central angle ROS?

A. 24

B. 30

C. 45

D. 60

E. 90

20. In a discus-throwing competition, the winning throw was 61.60 metres. The second place throw was 59.72 metres. How much longer was the winning throw than the second place throw?

A. 1.12 metres

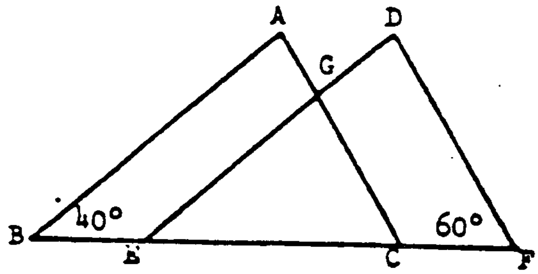
B. 1.88 metres

C. 1.92 metres

D. 2.12 metres

E. 121.32 metres

21.



In the above diagram, triangles ABC and DEF are congruent, with BC = EF. What is the measure of angle EGC?

A. 20°

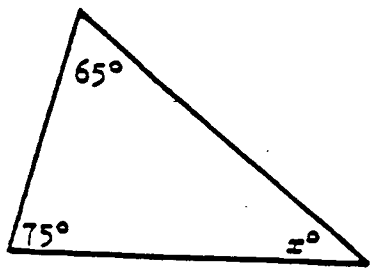
B. 40°

C. 60°

D. 80°

E. 100°

22.



x is equal to

A. 75

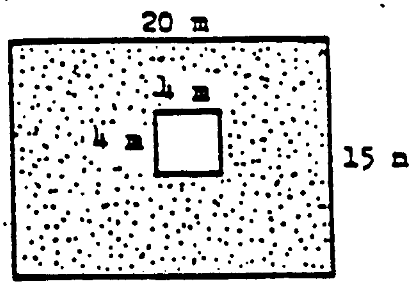
B. 70

C. 65

D. 60

E. 40

23.



A square is removed from the rectangle as shown. What is the area of the remaining part?

A. 316 m^2

B. 300 m^2

C. 284 m^2

D. 80 m^2

E. 16 m^2

24. Cloth is sold by the square metre. If 6 square metres of cloth cost \$4.80, the cost of 16 square metres will be

A. \$12.80

B. \$14.40

C. \$28.80

D. \$52.80

E. \$128.00

25. The air temperature at the foot of a mountain is 31 degrees. On top of the mountain the temperature is -7 degrees. How much warmer is the air at the foot of the mountain?

A. -38 degrees

B. -24 degrees

C. 7 degrees

D. 24 degrees

E. 38 degrees

26. 0.40 x 6.38 is equal to

A. .2552

B. 2.452

C. 2.552

D. 24.52

E. 25.52

27. A shopkeeper has x kg of tea in stock. He sells 15 kg and then receives a new lot weighing $2y$ kg. What weight of tea does he now have?

A. $x - 15 - 2y$

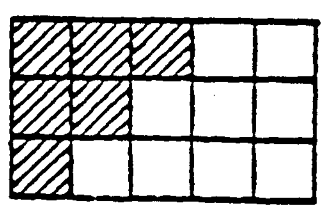
B. $x + 15 + 2y$

C. $x - 15 + 2y$

D. $x + 15 - 2y$

E. None of these

28.



In the figure the little squares are all the same size and the area of the whole rectangle is equal to 1. The area of the shaded part is equal to

A. $\frac{2}{15}$

B. $\frac{1}{3}$

C. $\frac{2}{5}$

D. $\frac{3}{8}$

E. $\frac{1}{2}$

29. The distance between two towns is usually measured in

A. millimetres

B. centimetres

C. decimetres

D. metres

E. kilometres

30. 0.00046 is equal to

A. 46×10^{-3}

B. 4.6×10^{-4}

C. 0.46×10^{3}

D. 4.6×10^{4}

E. 46×10^{5}

31. $\frac{2}{5} + \frac{3}{8}$ is equal to

A. $\frac{5}{13}$

B. $\frac{5}{40}$

C. $\frac{6}{40}$

D. $\frac{16}{15}$

E. $\frac{31}{40}$

32. $7\frac{3}{20}$ is equal to

A. 7.03

B. 7.15

C. 7.23

D. 7.3

E. 7.6

33. In a school of 800 pupils, 300 are boys. The ratio of the number of boys to the number of girls is

A. 3 : 8

B. 5 : 8

C. 3 : 11

D. 5 : 3

E. 3 : 5

34. What is 20 as a percent of 80?

A. 4%

B. 20%

C. 25%

D. 40%

E. None of these

35. The sentence "a number x decreased by 6 is less than 12" can be written as the inequality

A. $x - 6 > 12$

B. $x - 6 \geq 12$

C. $x - 6 < 12$

D. $6 - x \geq 12$

E. $6 - x < 12$

36. 30 is 75% of what number?

A. 40

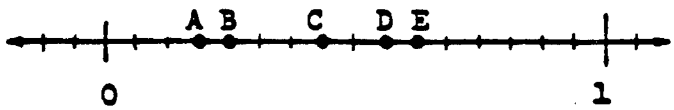
B. 90

C. 105

D. 225

E. 2250

37. Which of the points A, B, C, D, E on this number line corresponds to $\frac{5}{8}$?



A. point A

B. point B

C. point C

D. point D

E. point E

38. 20% of 125 is equal to

A. 6.25

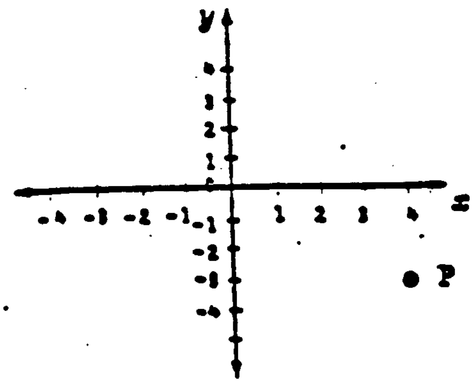
B. 12.50

C. 15

D. 25

E. 50

39.



What are the coordinates of point P?

A. (-3,4)

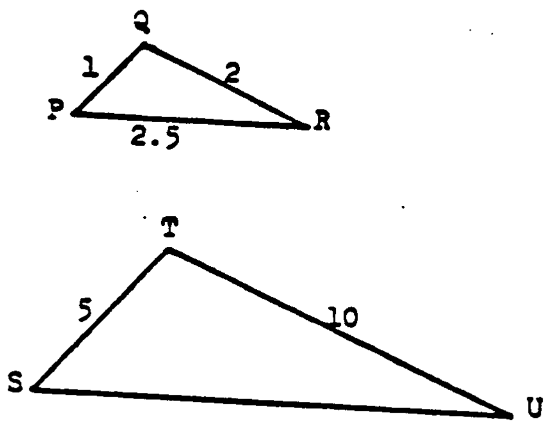
B. (-4,-3)

C. (3,4)

D. (4,-3)

E. (-4,3)

40



Triangles PQR and STU are similar. How long is $\overline{SU}$?

A. 5

B. 10

C. 12.5

D. 15

E. 25