Dynamic Models of Decision Making By Fishermen

By

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in

THE FACULTY OF GRADUATE STUDIES
(Commerce and Business Administration)

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
July 1986

ABSTRACT

This thesis examines the dynamic decision making behavior of fishermen. Two models are developed: (1) an intraseasonal model of vessel movement on the fishing ground during each season; and (2) an interseasonal model for investment decision making from year to year. Both decision models are driven by single economic objectives and the fisherman–decision maker is assumed to make rational choices to optimize the stated objective.

In this competitive market intraseasonal decisions are assumed to be made in the short–run to maximize the net operating income of each fishing enterprise. These decisions about where to fish to obtain the maximum return to fishing effort over the course of the season are modelled by a partially observable Markov decision process which incorporates the key elements of the problem facing each fisherman. The state space for this process is derived from total seasonal biomass. This aggregate description of the state space renders the problem practicable and solvable. The normative model is developed formally and applied to freezer trollers of the British Columbia commercial fishing fleet. Model results for average income and catch per troller, and seasonal fishing distribution over the fishing grounds reflect major tendencies in statistics arising from actual intraseasonal decisions made by this group of fishermen.

Interseasonal decisions concerning longer–term investment strategies are made in an environment which is highly variable from season-to-season. Extensive variability implies that economic survival is a primary consideration in the investment decision process. The investment decision making process is modelled as a probabilistic dynamic programming problem in discrete time. Investors are assumed to make rational decisions based on income expectations and subject to survivability conditions to maximize the net worth of the fishing enterprise at the end of a finite planning horizon. The formal analysis of the investment model is presented. The model is applied to all trollers of the British Columbia commercial fishing fleet. The pattern of actual investment by troller is simulated by tuning behavioral components of the investment model. These results provide insight into the behavioral basis of investment decision making by this group of fishermen.

This modelling framework has implications for planning and regulation in fisheries. Insight gained into the key factors behind fishermen's decisions can provide a basis for the development of strategic policies which anticipate fishermen's behavior and are aimed at stabilizing the economic viability of the fishing sector. The approach represents a movement away from reactive, short–term policies which have characterized fisheries regulation to date.

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ACKNOWLEDGEMENTS

Some sacrifices come with being a graduate student. Most are easily mitigated by a passion to understand and to contribute to understanding. Many have been borne by my partner Pat Tyrrell-Lane. Her acceptance of my many absences along with the emotional highs and lows of research has provided much needed support. She has contributed most to whatever may be worthwhile between these pages.

I have been very fortunate to have spent many hours in discussion with Ilan Vertinsky. His own passion to understand has been the prime motivating factor behind this thesis. His superior insight and direction have proved invaluable to me time and time again.

Shelby Brumelle has continually and freely offered his time and expertise to me, especially on occasion when I have strained his patience with my slowness to understand. A better product is the result of his careful scrutiny.

Gordon Munro, Don Ludwig, Colin Clark, and Marty Puterman, the remaining members of my Ph.D. Committee, have inspired this line of research. Their influence from both a personal and professional point of view is significant. I am and will continue to be proud to be a student of each of them.

Paul Macgillivray and Ron Giammarino played a major role in getting this project off the ground. I am indebted to them for their trust and friendship.

Finally, I extend my appreciation of their support to my family, who surely doubted that this work would ever be completed, and to Marg and Gordon Tanner, our friends and surrogate family, who have shared many important moments with us as well as having arranged field trips to beautiful Lesquiti Island.

Dynamic Models of Decision Making by Fishermen

1. Introduction

Canada's ocean fisheries have come under serious public scrutiny in recent years. Notably, the Commission on Pacific Fisheries Policy and the Task Force on Atlantic Fisheries both describe the deplorable economic conditions of the fishing sector. The universally identified major problem in the industry is stated as drastic overcapitalization.

Regulating bodies charged with maintaining the biological, economic and social viability of the fishery, historically have been placed in a conflicting position. Regulators have a mandate to protect the resource while at the same time preventing fishermen from engaging in cost spiralling investment tendencies which may seriously erode their economic viability and livelihood. While it may be stated that many biological objectives, i.e., the maintenance and enhancement of fish stocks, are being achieved, the economic and social costs incurred by the fishing sector have been mounting to a crescendo which has resulted in the current crisis situation.

One reason why regulation of Canada's fisheries has been less than successful in curtailling overcapitalization in fishing effort is its lack of emphasis on the cause of the problem. Current regulation typically treats the symptoms of the problem—mounting capital expenditures, increasing costs incurred by the fishing fleet and pressure on resource stocks—by limiting for example, the form and sizes of new capital investment by individual fishermen. However, this treatment fails to consider the cause of the problem–the behavior and motivation behind fishermen's decisions.

The purpose of this thesis is to present empirical–based models of decision making dynamics by fishermen. The motivation behind this research lies in the intuition that increased understanding of the dynamic decision behavior of fishermen will lead to more effective and *anticipatory* regulation. Without knowledge of the behavior of fishermen, fisheries management is confined to 'reacting to' changes induced by the collective action of fishermen. Reactive regulation characterizes much of the current regulatory policy in common property resources. There is growing need for more research directed at micro–based decision models of the kind proposed here.

In the analysis which follows, two aspects of dynamic decision making by fishermen are investigated:

1) intraseasonal decisions of when and where to fish; and

2) interseasonal decisions for new capital investment.

Intraseasonal decisions about when and where to fish throughout the fishing season are assumed to be based on the desire of fishermen to maximize seasonal net operating income. Accordingly, elements to be weighed in the intraseasonal decision process include the potential catch of fish, the cost of fishing effort, and the unit price of the catch in each fishing ground at each fishing period of the season. Fishermen will be attracted to fishing grounds which have higher relative catch potential, lower relative fishing costs, and higher relative unit prices per catch, *ceteris parebis*.

In order to model this decision process, it is necessary to develop representative seasonal dynamics describing the ongoing catch potential of the fishery, and the cost and price performances for the different fishing grounds. Seasonal cost and price dynamics are derivable from trends evident in historical data. Catch potential is modelled in two parts: stock dynamics and catchability.

Stock dynamics are modelled as a Markov chain which depends on total seasonal catchable biomass. The anadromous nature of Pacific salmon permit the use of this aggregate model (over all salmon species and all fishing grounds) as a simple representation of actual stock dynamics. Moreover, this aggregate representation of salmon stock dynamics is not incompatible with how experienced fishermen may intuitively view the system.

Secondly, since actual catches are not perfectly correlated with stock abundance then they must be described conditional on actual abundance. Inversely, observed catches provide only limited information about the actual state of abundance of the fishery. However, as the season progresses, more catches are made and thus more information is obtained about the overall state of seasonal abundance. This new information about catch potential is then used to make decisions about when and where to fish in upcoming periods of the season.

Dynamic decision processes which take place in this system are formulated as a Markov decision process with imperfect information about the actual state of the system. Decision policies for this problem are computed by the method of optimal control of the partially observable Markov process over a finite horizon due to Sondik(1971). Using these decision policies, distributions of seasonal net operating income earned by fishermen, and fishing effort and catch by fishing ground are generated by simulating the actual stochastic evolution of the system for the fishing fleet. The performance of the model in anticipating these key elements of the fishery is measured by comparing the model results with empirical data.

Interseasonal decision making is concerned with fishermen's investment decisions over time. Interseasonal investment decisions are assumed to be based on the desire of fishermen to maximize the economic value of their enterprises at the end of a finite planning period.

Investment potential in any single period depends on the fisherman's current financial and capital structure, and on the fisherman's current and expected future levels of net operating income. Moreover, since levels of the income random variable may vary appreciably from season to season (as is typical of most fisheries) then new investments may place the fisherman at considerable risk of solvency. This fact is evident from the difficulty many fishermen have in raising capital from conventional lenders.

Limitations on investment due to high income variability are incorporated into an investment decision model which makes investment contingent on continued survival in the fishing business to the end of a finite planning period. Each new investment generates a series of future financial liabilities and income enhancements over the planning period which in turn affect the ongoing Net Worth of the enterprise. It is with respect to these costs and benefits that fishermen are assumed to make their investment decisions.

To solve this problem, a discrete-time probabilistic dynamic programming formulation is developed to model the investment planning decisions of individual fishermen. Investment policies are then determined at each year with the objective of maximizing the expected economic Net Worth of the enterprise at the end of the planning period. Limitations to investment take the form of survivability constraints in each year of the dynamic programme.

Actual investments by fishermen are found by employing the derived investment policies in a simulation of the stochastic system. The performance of the investment model in anticipating the extent and type of new investments over time is measured by comparing the model generated results with empirical data.

The contribution of this research lies in the development, analysis, and application of the proposed intra and interseasonal decision making models for fishermen. The development of dynamic models of fishermen's decision making processes has implications for a number of regulatory policy questions, e.g., the effects of closures, area licensing, license limitations, investment trends, and the impact of enhancement programs on the evolution of the fishing fleet.

The empirical aspects of this study focus on a particular segment of the British Columbia commercial fishing fleet—salmon trollers. The salmon fishery is by far the most important fishery resource in both quantity and value on the Pacific coast (Pearse(1982), p.9). Trollers are the largest segment (in numbers of vessels) of the British Columbia commercial fleet.

A serious drawback to the development of empirical–based micro–models of the kind proposed here has been the absence of reliable data. The empirical basis of this research benefits from recently collected and previously unavailable data. In particular, detailed information on the performance of individual trollers over time provides useful comparisons of the modelling results with the representative data of the actual system. The database is comprised of historical data on landings by vessel, by species, by statistical zone; annual cash report (tax submission) data including investment and disinvestment by class of capital for individual troller operations; vessel characteristics including tonnage, length, vintage, and estimated value; aggregate catch statistics by zone, species, and gear type; and unpublished fisheries reports. All data has been disguised to protect the identity of the individuals.

The presentation of the modelling framework in this thesis is designed to illustrate its capabilities for a wider range of regulatory problems. Systematic exploration of the key uncontrollable and controllable elements in fishermen's decision processes will enable planners to anticipate various economic and social repercussions of their proposed programmes. Through the use of this kind of advance understanding the onerous task of regulating this complex environment may be made more strategic and less reactive.

2. Plan of Thesis

The remainder of the thesis is divided into four chapters. Chapter 3 presents a survey of the revelant literature related to the analysis of the decision making behavior of fishermen. This includes recent studies on the state of the fishery sector in Canada, stochastic models for fishery management, and models of investment behavior in fisheries.

Chapter 4 presents the intraseasonal decision making model. Chapter 5 details the interseasonal model. Each of these two chapters contains subsections which present:

i) the motivation behind the use of the particular model proposed. The arguments defending the normative and predictive purposes of each model are also presented.

ii) the formal development of the respective models, namely, the partially observable Markov decision process for the intraseasonal model, and the probabilistic dynamic programme with survivability constraints for the interseasonal investment model. Models assumptions are also discussed here.

iii) computational considerations for developing optimal decision rules for each model. Procedures, limitations, and restrictions on the computing of model solutions are also discussed.

iv) analysis of the results of each model. A discussion of the sensitivity of model results due to changes in key controllable and uncontrollable data elements is presented. Model results are also compared with corresponding actual statistics.

The final chapter summarizes each decision model and gives a discussion of wider perspectives for regulatory planning using the intra and interseasonal modelling framework. Extensions of the stochastic modelling approach are also discussed in this closing chapter.

3. Literature Review

This survey of the relevant literature for this research is divided into three main sections. The first section presents critiques on the current state of Canada's ocean fisheries. This material provides the background and motivation for the current research. Secondly, recent research related to the optimal control of Markov decision processes is discussed with emphasis on fisheries applications. This research provides the technical background for the intraseasonal partially observable Markov decision process developed in this thesis. The last section reviews analyses of investment in fisheries.

The tenuous state of Canada's ocean fisheries has prompted a number of thorough investigations aimed at revamping the industry. Munro(1980) examined the impacts of extended fisheries jurisdiction on the Newfoundland economy. In this study Munro cautions that the 'promise of abundance' anticipated from the 200 mile limit (Law of the Sea Convention, 1977) may nevertheless be dissipated by uncontrolled investment. In fact, significant capital expansion did occur in both the primary and secondary sectors of the fishery.

The consequence of this expansion coupled with cost escalation after 1977 eroded any benefits accruing from extended jurisdiction and plunged the east coast fishery into serious economic straits. By 1982 the Atlantic fisheries were on the verge of bankruptcy. In this year, the Task Force on Atlantic Fisheries chaired by Michael Kirby was given the mandate to recommend "how to achieve and maintain a viable Atlantic fishing industry with due consideration for the overall economic and social development of the Atlantic provinces" (Kirby(1982,p.3)).

What Kirby recommended was a sweeping reform of the east coast fisheries operations designed to grapple with the longstanding problems of the common property and the seasonal nature of the fishery, a low quality product, and poor management. The crisis of 1982 was further complicated by the overcapitalization which followed extended jurisdiction in 1977, and by the social and political traditions of the region.

In a concurrent study by the Institute for Research on Public Policy, Weeks and Mazany(1983) presented a plan for reducing the overcapacity of the Atlantic fisheries. Specific attention was paid to the social implications of their proposed strategy.

The situation on the Pacific coast parallelled that of the Atlantic coast. In the late seventies Canada's Pacific fisheries underwent large expansion primarily as a consequence of a series of high income years. Subsequently, prices fell and costs rose as part of the world-wide economic malaise. This phenomenon, together with growing concern about the precarious condition of many fish stocks (salmon, in particular) drove the industry toward economic disaster. In 1982 the Fleet Rationalization Committee was set up to discuss ways and means to reduce and prevent overcapitalization in the British Columbia fishing industry. The Committee represented the views and concerns of members of the fishing community and also acted as critic of the Royal Commission on the Pacific fisheries which had begun hearings in 1981.

The Commission on Pacific Fisheries Policy was given the task of finding ways to improve the conditions of Canada's Pacific fisheries. Commissioner Peter Pearse's Final Report(1982) presented a commerical quota licensing scheme for revamping the industry and halting the excessive capacity investment spiral. Capital reduction would initially be achieved through a buy-back plan after which the limited-entry quota system would be set up to prevent the fleet from expanding beyond the level required to harvest efficiently the annual catch. The licensing strategy was designed as an alternative to the myriad of regulations and harvesting restrictions currently in effect to control catch.

Scott and Neher(1981) declared that the current administrative system in Canada's fisheries is highly inefficient and results in increased real costs to the industry. They pointed to the excessive complexity of current regulation as the most significant cost overrun in the fisheries sector. A 'simplification of the rules' and the implementation of individual and exclusive fishing rights was recommended.

The Powell River Symposium, published in the Canadian Journal of Fisheries and Aquatic Sciences (1979) presented a series of papers on the regulation of Canada's commercial fisheries. Fraser(1979) gave examples which show that programmes such as licence limitations have not been totally successful because they merely react to the symptoms of the problem, e.g., increasing investment and pressure on fish stocks. Wilen(1979) called for the design and development of alternative regulatory programs in which policy outcomes are predictable. He focuses on what motivates individual fishermen in their investment decision making. Information about the impact of adjustment to price changes, income expectations, and cash availability on the average behavior of individual fishermen may be used to explain reactions to regulations in the past. Moreover, this information can be used to develop regulatory policies which anticipate investment behavior.

In his 1985 J.C. Stevenson Memorial Lecture, Hilborn(1985) called for a renewed effort at understanding the behavior of fishermen. He attributes the crisis in Canada's commercial fisheries in large part

> .....to poor understanding of the dynamics of fishermen, how they fish, and how they invest. I therefore argue that a major element of fisheries science should be the study of fishermen and fleet dynamics. (Hilborn(1985),p.3).

While relatively few papers specifically treat the decision making processes of fishermen, much research activity has been directed at the optimal control of stochastically varying aggregate fish stocks and fish dynamics *vis-à-vis* regulating these stocks. Primary emphasis is on renewable stock management and the structure of socially optimal fishing policies. The pioneering work of Clark(1976) provided a mathematical framework for modelling fisheries stock management in a deterministic setting. Clark(1985) examined risk and uncertainty in fisheries management. Mangel(1984) also treated the modelling of stochastic resource systems and the management of fluctuating resource stocks.

Adaptive management of natural systems undergoing uncertain dynamic changes has been investigated by Holling(1978). A case study on Pacific salmon management is included in this edition. Walters and Hilborn(1978) published a survey of dynamic optimization models dealing with uncertainty in ecological management. Surveys of stochastic modelling in fisheries include those by Andersen and Sutinen(1981) and Spulber(1982).

Among the approaches to stochastic fisheries modelling, Markov decision processes have been used to model uncertainty in the management of fish stocks. The recent survey by White(1985) attests to the increase in popularity of these models in fisheries as well as in other areas of application. In particular we note the following research. Mendelssohn(1979) analysed the consequences of the choice of grid size on policies from a Markov decision process in two Alaskan sockeye salmon stocks. The procedure leads to a better understanding of the robustness of fisheries escapement policies.

Ludwig and Walters(1982) showed that while stochastic effects on a given stock-recruitment relationship do not alter the conclusion of the corresponding deterministic theory, uncertainty about the stock-recruitment relationship itself can lead to large departures from the "optimal" escapements dictated by deterministic theory.

Mendelssohn and Sobel(1980) described in general terms the structure of optimal reinvestment and consumption decisions under uncertainty for single sector growth models including single fish species with structured age classes. In this context, 'reinvestment' in the fish stock denotes escapement and 'consumption' is fish harvest. The objective is to maximize the expected discounted utility. Both finite and infinite consumption horizons are examined. Sobel(1982,1981) used a game theory approach to characterize optimal policies for harvesting and escapement under a particular set of assumptions. The assumptions which are applied to fisheries management guarantee a stationary (period independent) optimal policy. Moreover, the assumptions imply that fishermen who behave according to the social optimal have no incentive to deviate from a stationary policy *vis-à-vis* their individual effort and investment strategies. Lovejoy(1983) used a policy bounding technique to a Markov decision process applied to the management of a stochastically varying, age structured fish population. Policies are developed for the Atlantic surf clam fishery.

In unpublished reports, Mendelssohn and Sondik(1979) and Sondik and Mendelssohn(1979) analysed the impact of information on the optimal management of renewable resources in an uncertain environment. In these studies special cases of a partially observable Markov decison process (POMDP) formulation is developed to examine the benefits and costs of perfect information versus no information, and delayed perfect information about the actual state of the system (i.e., stock abundance). The computation of optimal policies for the POMDP was first resolved by Sondik(1971). (See also Smallwood and Sondik(1973) and Sondik(1978).) Monahan(1982) provided an excellent review of POMDP theory, models, and algorithms. To date, POMDP models in the literature are primarily contrived problems used for the purposes of illustration. Applications of Markov decision processes or POMDP's in particular are not numerous (White(1985)).

The objective of the above stochastic models is to provide insight into the optimal escapement and harvesting policies for specific fish stocks. A few papers examine the decision making processes of fishermen who operate in the same stochastically varying environment. For example, Swierzbinski(1981) modelled fishermen's seasonal choices about what fishery to enter and how many fishing trips to make as a function of risk attitudes and profit expectations. A static model is applied to the herring fishery. A dynamic model with updating expectations is developed but not applied. Mangel and Clark(1982) modelled the effect of search by fishing vessels in reducing uncertainty in the location of schools of fish. Preliminary fishing and searching is followed by updating of the information and reallocation of the vessels.

In a similar approach to that of Mangel and Clark, Eales(1983) modelled searching behavior in the California shrimp fishery. Fishermen's decisions regarding the division of time between searching and fishing are updated continuously as a function of changing expected profits. Model results are compared to empirical data.

The chronic problem of overcapitalization in fishing has been well documented as the single most important issue in the survival of the fishing industry. However, as previously noted, considerably more attention has been attributed to aspects of stock management and fish dynamics. As Charles(1983a) stated, 'analytical studies of optimal fleet sizes and fisheries investment strategies have been rare.' Recently, fisheries investment strategies have received more research attention. Emphasis here is on the mathematical development of optimal fishing capacity policies for aggregate fishing fleets. In particular, Clark et al(1979) and Charles(1983a) solved the dual harvesting and investment optimal control problem in a deterministic setting. The problem of optimal aggregate investment in a fishery subject to random stock fluctuations has been studied by McKelvey(1979) for a multipurpose fleet and by Charles(1983b,1985). Charles(1983c) extended the previous analysis to consider the effect on optimal aggregate investment levels when the parameters of the stock-recruitment function are imperfect estimates. Similar issues related to optimal capacity for a developing fishery are studied by Clark et al(1985). The lack of alternative uses (irreversibility) of specialized fishing capital and its impact on optimal aggregate capacity and management of the fishery is taken up by Charles and Munro(1985).

A number of empirical analyses of investment in fisheries have appeared in the past. On the west coast, Blake(1971) examined the trends in over-expansion by the British Columbia salmon fleet prior to license limitation in 1969. In a follow-up study to the Sinclair Report(1960), Sinclair(1978) reviewed the state of the commercial fishing licensing scheme in British Columbia. A 1979 study (by Foodwest Research Consultants) reported on all sources of financing available to the primary and secondary sectors of the British Columbia fishing industry. In 1982 and 1983, Fisheries and Oceans Canada, Regional Planning and Economic Branch developed surveys of British Columbia commercial fishermen's earnings and expenses. The data was collected from an ongoing study initiated by the Fleet Rationalization Committee. Most recently, McMullan(1984) presented an interesting study on the historical investment climate and financial controls in British Columbia fisheries. Empirical results reveal that the government simultaneously engages in the restriction of fishing rights *and* the support of continued growth in fishing capacity.

On the east coast, Roy et al(1981) and Schrank et al(1980) investigated the efficiency of fishing capital in the Newfoundland groundfishery. Production functions were estimated based on actual data and then used to estimate the total costs and value of capturing the annual harvest. In other studies, Teltey et al(1984) examined the trends in increased investment by shrimp vessels in the Gulf of Mexico.

The above studies in fisheries investment deal with the theoretically optimal level of aggregate investment by an entire fishing fleet, or the actual trends of investment by the fleet. Little research has appeared in the literature which focuses on the investment behavior of individual fishermen. Of particular interest is the paper by Thompson et al(1973). In this study a stochastic dynamic programming formulation is used to model investment decisions by individual fishing firms. The dynamic model incorporates a condition which strictly quarantees survivability of the firm in all fishing periods. The survival model is applied to the case of shrimp fishing firms along the coast of Texas. The results are contrasted to investments made when survivability is not strictly considered. The research of this thesis extends this model by expanding the definition of the state variable by including different capital types, by elaborating on the financing of capital and on capital's income earning power effect. Moreover, the survivability condition is made flexible to the level of risk the fisherman decision maker is willing to assume. Revelant studies on risk in decision making are presented in Vinso(1979), Hanssman(1968) and Wilcox(1976).

Finally, we mention the following research on investment decision making by individual fishermen.

Bocksteal(1976) used a logit model analysis to examine the investment response of domestic fishermen to extended jurisdiction of the United States fishing industry. The model was successful in explaining changes to the capital stock of fishermen when gross revenues and capital costs were affected. More recently, Bocksteal and Opalach(1983) presented a discrete choice model of supply response under uncertainty and applied it to New England fishing firms making intraseasonal decisions about switching into other fisheries. Changes in expected returns and risk were shown to elicit switching indicative of actual behavior.

Wilen(1979) asserted the importance of modelling fisherman behavior 'for predicting, understanding and designing efficient regulation programmes.' More recently, Wilen(1981) used game theory to model investment decisions by fishermen vying for a share of annual total allowable catch. Wilen(1983) examined the effect on input/output configurations in analysing the benefits and costs of salmon enhancement in the British Columbia fishery. The need for the development of behavioral models of industry participants is stressed.

The above material presents the literature which is most pertinent to the work of this thesis. The major emphasis is on decision making and modelling in fisheries. Other publications related to general modelling approaches, technical details, and empirical data sources are found in the references.

4. Intraseasonal Decision Making

4.1 Motivation

Commercial trollers who fish for salmon off British Columbia's coast are faced with a similar set of problems throughout every fishing season. Briefly, their problem is one of deciding where along the coast to fish during each period of the season to catch the most fish with the highest return to fishing effort.

The unit components of return to fishing effort, i.e., the price and cost per unit catch, are in general knowable in advance of the upcoming periods of the season. However, the size of the catch in any zone of the fishery in each period of the season is not. This assertion is related to the fact that the actual abundance of catchable biomass is never directly observable. Moreover, even if the abundance of salmon stock was known explicitly, actual catch would still be random due to the selection process of the fishing technique. Consequently, when considering decisions to move among the zones of the fishery, estimates for catch potential must be taken into account as well as unit price and cost.

The anadromous nature of Pacific salmon provides useful information to fishermen in estimating seasonal stock abundance. For example, the seasonal run timings and migration patterns of major stocks of some salmon species are quite well known. Much research has been carried out into understanding salmon stock dynamics. The more experienced fisherman is able to rely to some extent on historical information regarding the sequential passage of salmon as they proceed from ocean to inland rivers and streams to natal spawning grounds. Moreover, the cyclical behavior of individual stock cohorts is well known among fishermen. As an illustration, differences in the 'odd' and 'even' year run cycles of pink salmon are common knowledge as is the regular four year abundant run of sockeye salmon to the Adams River of British Columbia.

Fishermen also acquire information about abundance (and hence catch potential) from their own fishing experience, the experience of other fishermen, or from the public policies of the Fisheries officers acting on behalf of Fisheries and Oceans Canada which monitors the fishing season on an ongoing basis. However, these estimates of abundance are never totally accurate. Thus the information available about the actual state of the system (i.e., the abundance of fish) is imperfect.

For individual fishermen the historical information about the typical abundance levels coupled with the anadromous behavior of the salmon provides the underlying process for interpreting the imperfect information about actual abundance levels. This information is useful for planning and for decision making during the fishing season.

Commercial salmon troll fishermen are currently free (with a few exceptions) to target on any species or to fish in any defined 'zones' along the coast throughout the season.† The problem for individual fishermen becomes one of evaluating at regular intervals over the course of the season the decision about staying and fishing in a particular zone or moving to a new zone to fish there. For example, if the overall seasonal abundance is evaluated as being 'low' relative to some average abundance, then some fishermen may be enticed to modify their fishing effort and strategy by fishing more or less often and in different fishing zones.

The commercial, perfectly competitive nature of the troll fishery (i.e., many small firms, homogeneous product, and exogenously defined prices) lends credence to the conjecture that intraseasonal decisions concerning movement among the fishing grounds are primarily economic–based. Throughout this analysis, fishermen are assumed to make rational decisions to maximize seasonal economic objectives such as market share, gross income, or net operating income.

In this analysis, we restrict our attention to income from fishing due exclusively to salmon catches. While the salmon fishery is predominant, troll fishermen also complement their fishing incomes by entering other fisheries, e.g., the herring fishery. However, due to the complexity of introducing other fishing alternatives and the lack of data, this intraseasonal analysis pertains only to the salmon fishery. Specifically, this approach provides a normative framework for modelling the intraseasonal decision process of fishermen.

As a normative formulation of the intraseasonal decision making process, let us consider that fishermen using their information sources implicitly define 'fuzzy' estimators to describe the state of the system (i.e., salmon abundance) throughout the season. For example, the descriptors 'good', 'mediocre', or 'poor', may be attributed to the abundance levels of salmon as a consequence of the observed return to effort of the fishery from catches made to this point in the season. Based on experience and related sources of information, the current estimate of seasonal abundance can be used to estimate the abundance levels of salmon in the various zones of the fishery for the next period. In turn, information from future catches can be used to update the seasonal abundance, and so on to the end of the season. Specifically, we assume that intraseasonal decisions are based on estimates of

1) the likelihood that the system will move from one state (e.g.,'good' overall abundance) to another state (e.g.,'poor' overall abundance) from period to period throughout the season;

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† We define seven zones along British Columbia's coast. These zones are aggregations of statistical areas used by Fisheries and Oceans Canada in the reporting of landings by species. See Hilborn and Ledbetter(1979) for a description of the zones and statistical areas.

2) the likelihood that actual observations about the system (e.g.,'high' or 'low' catches) which occur over the season depend on the actual state of abundance of the system; and

3) the relationship between expected catches and the associated benefits and costs in the different fishing zones throughout the season, i.e., unit prices and costs of effort.

The rational fisherman seeking to maximize his economic gain in the short–run (i.e., in each season) from salmon fishing will use this information to help determine his seasonal fishing policies. For example, salmon abundance levels are typically 'low' (relative to average seasonal abundance) in most zones during the early periods of each season. Accordingly, fishermen must first decide in what period and zone to begin the fishing season. This decision is based on the recognized historical behavior of the salmon runs, the expected catch per unit effort and the expected returns to effort from fishing in each period and zone. Moreover, catches in each period provide insight into the type of season which will follow.

Over the course of the season abundance levels change as the salmon make their way through the fishery and catches are taken. Fishermen use these additional pieces of information to modify and update their fishing strategies. If, for instance, catches are 'low' (relative to average seasonal catch levels) in the current period and zone, then this may signal a fisherman to move to a new zone to fish in the next period. Finally, as the season comes to a close, fishermen are signalled by historical data on typical season lengths, by late season catches, and by returns to fishing effort, as to when it is no longer beneficial to continue fishing this season.

It is however unrealistic to imagine that over a large fishery with many zones that fishermen actually update dynamically the catch potential (i.e., the estimate of salmon abundance) separately for each zone based on the current season's catches to date. It is more plausible to conjecture that catches by fishermen in a particular zone are used directly as a proxy for all zones, including those that are not fished in the current period. For example, a high catch in a particular period and zone may indicate to the fisherman that salmon are 'running well' this season across the entire fishery. This implies that the decision to move among zones should be primarily based on average zonal catchability, average zonal prices per catch, and average zonal costs of fishing. This perspective greatly simplifies the computational aspects of the proposed model.

The analysis of the empirical data (see Section 4.4) supports the simplified problem formulation. Briefly, we begin with an overall measure of seasonal salmon abundance. In each period of the season a proportion of this measure is actually present in the fishery. Information about salmon migration patterns, the timing of runs and and past fishing experience provide a priori estimates of the proportions applicable to each zone and period. When fishing occurs in some zone, the catch observations are used as imperfect measures of the overall seasonal abundance. These observations provide new information about actual overall seasonal abundance and the decision about where to fish in the next period is adjusted accordingly.

The presentation of the formal model in the following section procedes from (i) a description of the notation and elements of the general intraseasonal model; to (ii) a parsimonious description of abundance across all zones as suggested by the empirical results. The above intuitive description of intraseasonal decision making by fishermen, and their use of available information establishes the basis for a normative framework for modelling the decision making process within each season. The following section formally defines a model which incorporates the elements of the intraseasonal decision making environment of fishermen.

4.2 Formal Model

The intraseasonal decision problem may be viewed as a discrete–time dynamic control problem. The objective is to make periodic decisions to maximize the expected value of the seasonal net operating income in an environment with underlying stochastic fluctuations of changing stock abundance and catch observations. This problem is formulated as a particular case of a dynamic optimization process known as a Partially Observable Markov Decision Process (POMDP). (See Monahan(1982), Bertsekas(1976), and Smallwood and Sondik(1973).) In this section the formal elements of the finite horizon POMDP with discrete and countable sets for: the stages (decision periods over the season), the state space (the abundance of salmon), the signal or observation space (salmon catches), and the action or decision space (interzonal movement) are defined.

The following notational conventions are used: $\Pr\{\cdot\}$ denotes the probability of an event $\{\cdot\}$, $E\{\cdot\}$ is the expected value operator, $|A|$ denotes the number of elements (cardinality) in the finite set A, and '$\longleftarrow$' denotes the dynamic change in an element from one period to the next.

- Let $k \in K \equiv \{0, 1, \ldots, N\}$ denote the periods of the fishing season with $N \in I^{+}$ finite. k represents the stages of the decision process, i.e., the time periods for which the decision maker must decide on the next period's fishing policy.

- Let $z \in Z$ represent the zones of the fishery with $|Z| = N_Z$ where N_Z denotes the number of zones in the fishery and $N_Z \in I^{+}$ is finite.

- Let $\mathbf{X}_k$ be an N_Z–dimensional random variable defined on a sample space Ω. Let the random variable X_{kz}, an element of the vector $\mathbf{X}_k$, denote the actual (unobserved) state of the system at period k in zone z. X_{kz} represents the level of salmon abundance, i.e., the level of total (harvestable) biomass by weight at k in z. Assume that X_{kz} takes on only discrete values in the finite set $\{1, 2, \ldots, N_X\}$, where

N_X is constant for all k and all z. The discretization of the continuous abundance state space is an approximation to the actual state variable. The precise definition of the limits of the abundance levels in each zone is determined by the actual level of high to low salmon abundance throughout the season. The stochastic process $\{\mathbf{X}_k, k \in K\}$ describes the dynamics of salmon abundance over the season.

- The state–to–state dynamics of the process of the system between periods $k-1$ and k

$$\mathbf{X}_k \longleftarrow \mathbf{X}_{k-1}$$

is assumed to be a finite state Markov chain with nonstationary transition probabilities

$$p_{ijk} = \Pr\{\mathbf{X}_k = j \,|\, \mathbf{X}_{k-1} = i\} \tag{1}$$

where p_{ijk} is the probability of moving from state $\mathbf{X}_{k-1} = i$ in period $k-1$, to state $\mathbf{X}_k = j$ in period k and i, j are members of the set of N_Z–tuples, $\mathcal{N}$ where

$$\mathcal{N} = \{(1,1,\ldots,1),\ldots,(1,1,\ldots,N_X),\ldots,(N_X,N_X,\ldots,N_X)\},$$

and $|\mathcal{N}| = (N_X)^{N_z}$. The process of salmon abundance dynamics is completely described by the $|\mathcal{N}|\times|\mathcal{N}|$ probability transition matrix for each period k, $P_k = [p_{ijk}]$, and the initial probability distribution over $\mathcal{N}$ which is denoted by the vector

$$\pi(0) = (\pi_1(0), \pi_2(0), \ldots, \pi_{|\mathcal{N}|}(0)) \tag{2}$$

where $\pi_i(0) = \Pr\{\mathbf{X}_0 = i\}$, $i \in \mathcal{N}$.

- Let Y_k be a random variable which denotes the signals or observations of the actual state of the system at period k in zone z. These signals are the level of total catch by weight at each period k in zone z. It is assumed that effort spent fishing and the resulting catch in period k by a fisherman is within only a single zone of the fishery. The actual zone of catch is determined by the decisions in each period. These are described below.

  Assume that Y_k takes on only discrete values in the finite set $\mathcal{M} \equiv \{0, 1, 2, \ldots, N_Y\}$. As with the state space, this discretization of the continuous observation space is an approximation to actual catches. The precise definition of the limits of the catch levels is determined by the actual scale of high to low catches actually made throughout each season. The stochastic process $\{Y_k, k \in K\}$ is known as the *observation process* of the system.

- Information regarding the actual abundance, $\mathbf{X}_k$ is obtained when fishermen observe Y_k during fishing. Moreover, the probabilistic relationship between $\mathbf{X}_k$ (not observed) and Y_k (observed) is assumed to be known to the fisherman-decision maker. Define the state–to–observation function for each period k which relates observations Y_k in zone z to actual state $\mathbf{X}_k$ by the probabilistic relationship

$$q_{jlk}(z) = \Pr\{Y_k = l \mid \mathbf{X}_k = j, z\} \tag{3}$$

(assumed to be independent of all $\mathbf{X}_{k'}$, $k' \in K$ and $k' \neq k$) where $q_{ijk}(z)$ is the probability that catch level $Y_k = l$ will occur in period k at zone z given that the actual state of abundance is defined by $\mathbf{X}_k = j$ in period k; $l \in \mathcal{M}$; and $j \in \mathcal{N}$. The observation process is described by the signal or observation matrix for each period k, $Q_k(z) = [q_{jlk}(z)]$.

- The *core process* of the system may now be defined as the stochastic process $\{(\mathbf{X}_k, z_k), k \in K\}$ which describes the abundance of salmon, $\mathbf{X}_k$ and the zone of observation, z_k at period k.

- The decision maker is able to 'control' the core and observation processes by choosing the zone in which fishing is to occur in each period. Let A be a finite set which denotes all actions (i.e., all zone choices, z including the no fishing or idle alternative) available to decision makers in each period. Denote a particular decision in this set for period k by $a_k \in A$.

  Two distinct action space definitions may be described. First, $a_k \in A_1$ may be defined as the movement *to* a particular zone independent of the current zone from which the movement is made. Then, the set $A_1 \equiv Z \cup \{0\}$, where $a_k = 0$ denotes the no fishing option. Thus, $|A_1| = N_Z + 1$.

  Second, the action space $a_k \in A_2 \equiv A_1 \times A_1$ is defined as the movement from one zone to some other zone in the fishery. Thus, $a_k = a_k(i, j)$ is described as the decision taken in period k to move from the current zone i to zone j where $i, j \in A_1$. The cardinality of this augmented action space is $|A_2| = |A_1|^2 = [N_Z + 1]^2$. Section 4.4 discusses the computational aspects of these alternative action space definitions.

  The controllability of the core and observation processes through actions A_1 or A_2 implies that these processes depend on the choice of a_k in each period. Thus, if $(\mathbf{X}_k, z_k)$ is the current state of the core process and action a_k is chosen, then the core process moves to a new state $(\mathbf{X}_{k+1}, z_{k+1})$ with probability $p_{ijk+1}(a_k)$ where

$$p_{ijk+1}(a_k) = \begin{cases} p_{ijk+1}, & \text{for } a_k = z_{k+1}, a_k \in A_1; \\ p_{ijk+1}, & \text{for } a_k = (z_k, z_{k+1}), a_k \in A_2; \\ 0, & \text{otherwise.} \end{cases} \tag{3a}$$

  Similarly,

$$q_{jlk+1}(a_k) = \begin{cases} q_{jlk+1}, & \text{for } a_k = z_{k+1}, a_k \in A_1; \\ q_{jlk+1}, & \text{for } a_k = (z_k, z_{k+1}), a_k \in A_2; \\ 0, & \text{otherwise.} \end{cases} \tag{3b}$$

  Finally, let

$$P_k(a_{k-1}) = [p_{ijk}(a_{k-1})] \quad \text{and} \quad Q_k(a_{k-1}) = [q_{jlk}(a_{k-1})] \tag{4}$$

denote the probability transition matrix of the core process and the signal matrix of the observation process, respectively as a function of the actions chosen.

These relationships imply that information obtained about the estimated seasonal abundance levels from fishing in a particular zone and period are connected to abundance levels in future periods and other zones. To illustrate, if the early stages of runs of pink salmon to the Fraser River are high in abundance then it *may* be anticipated that the usual peak run period occurring later in the season will be more abundant than usual as well.

- Let $y_k \in \mathcal{M}$ denote the level of catch, Y_k observed at period k. And, let I_k be the vector of information accumulated from successive decisions and catches in the system up to and including time period k, with $I_k = (y_0, \ldots, y_k, a_0, \ldots, a_{k-1})$.

- Let the reward function $R_{k+1}(l, a)$ represent the immediate reward which results when action $a_k = a$ is taken and a catch of $y_{k+1} = l$ is made. Specifically, R_{k+1} is defined as the net operating income from fishing and is given by

$$R_{k+1}(l, a) = p_{k+1}(a) \cdot f(l, a) - c_{k+1}(a), \ \ k = 0, 1, \ldots, N-1 \tag{5}$$

where

$p_{k+1}(a)$ – average price per catch by weight in period $k+1$ for the zone defined by decision a;

$c_{k+1}(a)$ – average cost of fishing during period $k+1$ including fuel used during fishing, food, crew costs, ice, and bait, as well as the cost of moving associated with decision a based on interzonal distances.

$f(l, a)$ – the class mark of the level of catch by weight, $y_{k+1} = l$ in period $k+1$ for the zone defined by a where fishing takes place.

The expected net operating income $g_k(i, z, a)$ for decision $a_k = a$ when the actual abundance level at period k is $\mathbf{X}_k = i$, and zone z is the current zone is defined in (6). If the action space is given by A_2 and $a \notin \{z\} \times Z$ (representing the infeasibility of action $a \in A_2$ when the current zone is z) then $g_k(\cdot)$ is assigned a value of $-\infty$.

$$g_k(i, z, a) = E\{E\{R_{k+1}(l, a) \,|\, \mathbf{X}_{k+1}\} \,|\, \mathbf{X}_k = i, z\} = \sum_{j,l} R_{k+1}(Y_{k+1}, a) p_{ijk+1}(a) q_{jlk+1}(a) \tag{6}$$

where the random variable $\mathbf{X}_{k+1}$ may take on discrete values $j \in \mathcal{N}$ and Y_{k+1} may take on discrete values $l \in \mathcal{M}$. The problem may now be described as one of finding the controls or decision policies, $\mu(I_k)$ for all stages, k denoted by $\delta = \{\mu_0(I_0), \ldots, \mu_{N-1}(I_{N-1})\}$, where $\mu_k(I_k) \in A$ for $k = 0, 1, \ldots, N-1$

and the objective

$$J_\delta = E\{\sum_{k=0}^{N} g_k(\mathbf{X}_k, z_k, a_k)\} \tag{7}$$

is maximized.

Before solving this problem directly, it can be transformed using the sufficient statistic (Bertsekas(1976), p.120ff):

$$\pi_j(k) = \Pr\{\mathbf{X}_k = j \mid I_k\},\ j \in \mathcal{N}. \tag{8}$$

This is the conditional probability that the abundance level $\mathbf{X}_k = j$ occurs at period k, given information I_k. $\pi_j(k)$ may be considered as the new state variable of the transformed system. The sufficient statistic summarizes all the information that is necessary for controlling the original system in each period (Bertsekas(1976),p.122ff).

Since the state, control and observation spaces are all finite sets, then the transformation of the sufficient statistic from period $k-1$ to k may be written recursively using Bayes' formula (Hoel(1971),p.19ff). The sufficient statistic is defined in terms of the transfer function T_k as follows:

$$T_k(\pi \mid a)_j = \frac{q_{jlk}(a)\sum_i p_{ijk}(a)\pi_i}{\sum_{i,j} q_{jlk}(a)p_{ijk}(a)\pi_i} \tag{9}$$

for $k \in \{1, 2, \ldots, N\}$; $i, j \in \mathcal{N}$; $l \in \mathcal{M}$; and $a_{k-1} = a \in A$. Intuitively, Bayes' formula (9) examines the observed outcome of the fishing decision, the catch, and then asks what would be the probability that the catch was due to a particular cause, namely, an unobserved level of abundance. In this manner estimates of actual abundance are updated after every fishing period.

The $\pi_j(0)$ priors on the initial abundance levels at the start of the season are assumed to be known explicitly. This information is available to fishermen through pre-season fisheries outlooks such as the Commerical Fishing Guide, Proposed Fishing Plans and Stock Expectations provided by Fisheries and Oceans, Canada, or it may be based on the fisherman's own experiences.

Finally we may write the dynamic programming algorithm analogous to (7) above in terms of the new state variable $\pi(k) = (\pi_1(k), \ldots, \pi_{|\mathcal{N}|}(k))$ where $\sum_j \pi_j(k) = 1$ for all k. The recursive functional is

$$J_k(\pi, z) = \max_a \left[E\{g_k(\mathbf{X}_k, z_k, a) + J_{k+1}(T_k(\pi \mid a), z') \mid \pi(k)\}\right],\ k = 0, 1, \ldots, N-1 \tag{10}$$

where z' designates the zone of the fisherman in period $k+1$, and $J_{N+1}(\cdot) = 0$.

Equivalently, we may write (10) by expanding the expected value operator E to obtain:

$$J_k(\pi, z) = \max_a \left[\sum_i g_k(i, z, a)\pi_i(k) + \sum_{i,j,l} J_{k+1}(T_k(\pi \mid a), z')q_{jlk+1}(a)p_{ijk+1}(a)\pi_i(k)\right] \tag{11}$$

and $J_{N+1}(\cdot) = 0$.

The solution to the dynamic programming algorithm (11) above yields a decision rule (or control policy), $\mu_k^* = \mu_k(\pi(k))$, $k = 0, \ldots, N-1$ with the maximum expected net operating income from fishing of

$$J^* = E\{J_0(\pi)\}. \tag{12}$$

This completes the description of the notation and elements of the general POMDP model.

The 'size' of the general POMDP problem described above (as a function of the number of fishing zones, N_Z, the numbers of levels of the state and observation variables, N_X and N_Y respectively, the number of actions, $|A|$, and the number of periods, N) effectively renders the problem intractable for all reasonable values of the problem parameters. This is due to the fact that the sufficient statistic maps the state into6[CehN_X-unit simplex which is a continuous state space representation that must then be subdivided by the decision possibilities. In order to consider practical solutions in this setting, a reduction of the 'size' of the problem must be achieved. The results of the empirical analysis suggest that a more parsimonious representation of the state space may be used to make the problem tractable. Moreover, this representation more aptly describes the way in which fishermen view the system in which they make decisions.

The following paragraphs describe the parsimonious representation of the state space. Details of the empirical results of this simplified model may be found in Section 4.4.

Assume that the transition of salmon abundance from period to period is representable by a simple model arising from the seasonal migration of all salmon stocks through the various zones of the fishery on their way to spawning grounds. This model may be described as follows:

$$X_{kz} = \nu_{kz} \sum_{z \in Z} X_{k-1z} \tag{12a}$$

where the ν_{kz} are constants relating the sum of salmon abundance over all zones in the previous period to the abundance in a particular zone in the next period. Now, let $X_k = X_{kz}$ be a simple random variable denoting the level of abundance in the zone of current fishing, z and defined on sample space Ω'. X_k is a collapsed representation of the actual state vector, $\mathbf{X}_k$. The model of (12a) implies that the state of salmon abundance in each zone of the fishery may be estimated from past information about salmon abundance derived primarily from fishing in a single zone during each period of the season. Accordingly, we need only concern ourselves with the state of abundance, X_{kz} in the zone, z of current fishing, since this zone provides information about the state of abundance in the other zones of the fishery for the upcoming period.

The set of factors ν_{kz} allows us to simplify the description of the core process by considering only the state of abundance in the single zone in which fishing takes place. The core process is now described by the couplet (X_k, z_k), where X_k is the unobserved portion of the state which occurs in zone z_k of the fishery. Thus, we may now write (1) simply as

$$p_{ijk} = \Pr\{X_k = j \mid X_{k-1} = i\} \tag{1'}$$

where $i, j \in \{1, 2, \ldots, N_X\}$. The core process is now describable by the $N_X \times N_X$ probability transition matrices for each period k, $P_k = [p_{ijk}]$, and the initial probability distribution

$$\pi(0) = (\pi_1(0), \pi_2(0), \ldots, \pi_{N_X}(0)). \tag{2'}$$

We may now write the expected net operating income (6) in terms of $X_k = i$ as follows

$$g_k(i, z, a) = E\{E\{R_{k+1}(Y_{k+1}, a) \mid X_{k+1}\} \mid X_k = i\} = \sum_{j,l} R_{k+1}(l, a) p_{ijk+1}(a) q_{jlk+1}(a) \tag{6'}$$

where $i, j \in \{1, 2, \ldots, N_X\}$, and the objective (7) to be maximized becomes

$$J_\delta = E\{\sum_{k=0}^{N} g_k(X_k, z_k, a_k)\} \tag{7'}$$

Finally, the simplified recursive functional (10) can be written

$$J_k(\pi, z) = \max_a [E\{g_k(X_k, z, a) + J_{k+1}(T_k(\pi \mid a), z') \mid \pi(k)\}], \; k = 0, 1, \ldots, N-1 \tag{10'}$$

with $J_{N+1}(\cdot) = 0$.

This completes the modification of the general POMDP model. All other aspects of the problem not specifically referred to remain unchanged in the simplified model formulation.

Finally, given a policy, δ which details the decisions to be made at each stage dependent on the actual value of the information vector, the sequence of events for the POMDP process can be described. Figure 1 illustrates the decision process which occurs at each period of the season.

The following itemizes the step by step procedure which recurs at each stage of this decision process:

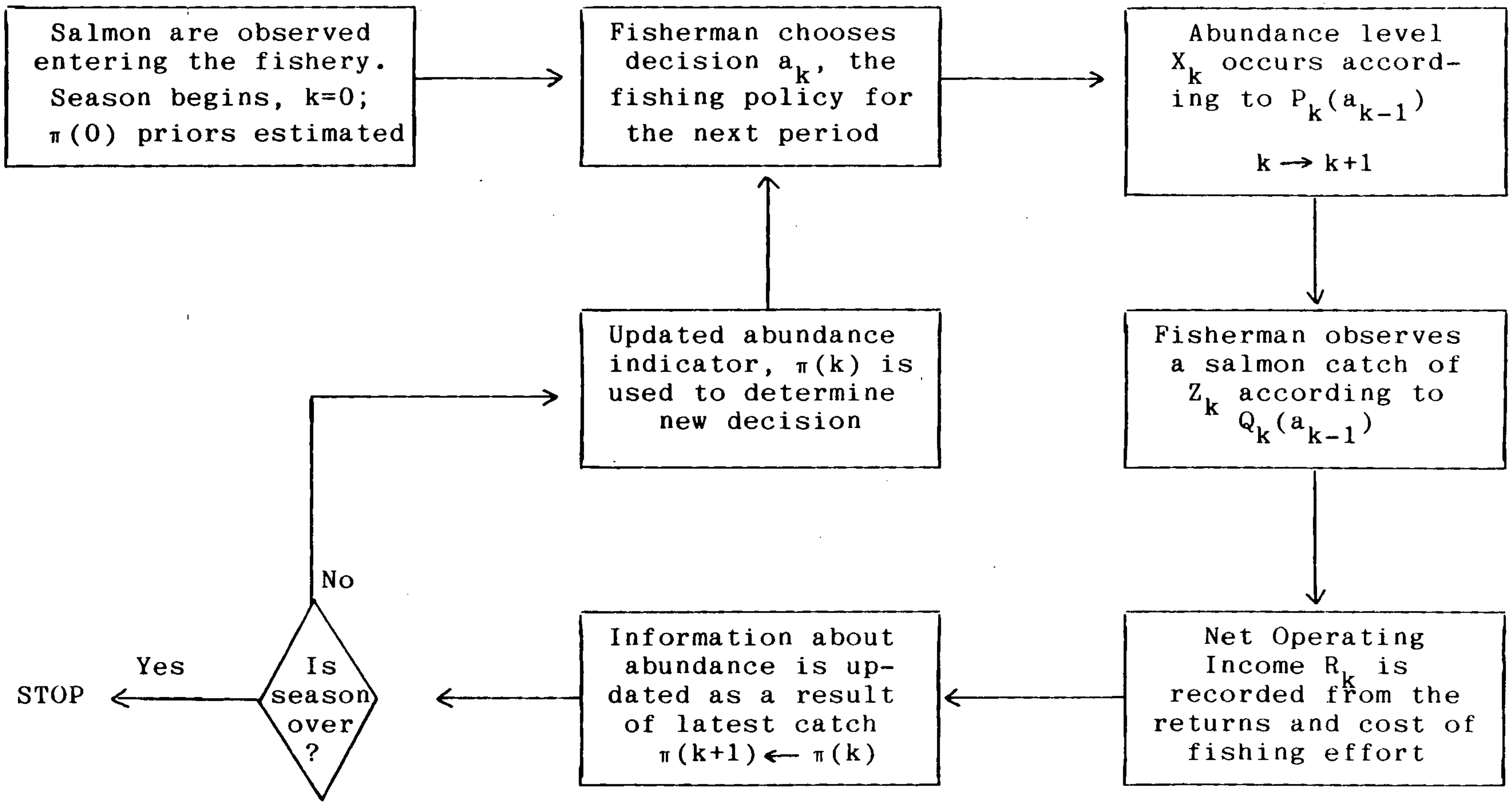


Figure 1 - The Intraseasonal Partially Observable Decision Process

Stage k, $k = 0, 1, \ldots, N$

START OF PERIOD

0. The fisherman begins the season in a Home Port Zone and estimates the initial abundance priors $\pi_i(0)$.

1. The abundance level state X_k occurs, generated from the probabilities $p_{ijk}(a_{k-1})$.

2. Catch level Y_k is observed from the particular zone defined by a_{k-1}, generated from probabilities $q_{jlk}(a_{k-1})$.

3. The net operating income from the revenues and costs of Y_k for decision a_{k-1} is generated from $R_k(Y_k, a_{k-1})$ defined in (5).

4. The information vector I_k and the sufficient statistic vector $\pi(k)$ are updated.

5. The fisherman decides on a fishing policy $a_k \in A$ for the upcoming period.

6. Proceed to the next stage, i.e., $k \longleftarrow k+1$. If k exceeds N the season is over then STOP; else proceed to the next period of the season at Step 1.

END OF PERIOD

This completes the formal statement of the POMDP model to be used in the analysis of intraseasonal decision making dynamics.

4.3 Empirical Development

The formal model of intraseasonal decision making presented in the previous section establishes a framework for discussion of the empirical development of the model applied to the freezer troller segment of British Columbia's commercial salmon fishing fleet. The following discussion treats each of the elements of the formal model relative to this application.

Freezer Trollers. Trolling is a method whereby fishing lines with lures attached are dragged through the water. Usually, trollers employ 6 to 8 weighted stainless steel fishing lines attached to poles with pulleys. The lines are reeled in and out on separate grudy spools with individually controlled clutch and brake mechanisms. Trollers move slowly through the water. When a fish strikes a line, it is hauled in, the fish released from the hook and the gear reset. In good fishing conditions, trollers may catch as much as 18 tonnes of salmon in a single day.

Freezer trollers are the fastest growing and the most capital intensive component of the troll fleet. Freezers have the capacity to stay longer on the fishing ground than other trollers without returning to dock to register their catch. At the same time their rapid freezing capability ensures a higher quality product and a higher price per unit weight than trollers with less sophisticated equipment. Freezer trollers are highly mobile vessels which are only minimally restricted by fishing area and fishing period regulations.

Fishing Season. The commercial salmon fishing season for trollers prior to 1984† was basically unrestricted (with minor exceptions) to area and time of fishing to legally licensed vessels (1984 Commercial Fishing Guide). The intensity of salmon fishing by trollers typically begins in March, increases to a peak in late July or early August and then declines until November or early December. In this analysis the salmon fishing season for freezers is assumed to begin in mid–March and end in mid–December.

Fishing Zones. The British Columbia coastline is divided into 30 statistical areas which are used by Fisheries and Oceans Canada in the reporting of all marine harvests (Hilborn and Ledbetter(1979)). All commercial landings are reported on sales slips as being made in one of these statistical areas. For commercial trollers the coastline is also divided into 7 larger aggregate zones (Archibald and Graham(1981)). These seven zones are defined in terms of their statistical areas in Table I. Figure 2 illustrates the areas and zones of the British Columbia coast.

† Troll restrictions have been on the increase for some time. In 1984 Fisheries and Oceans Canada for the first time initiated restrictions on the length of the troll season.

Zone	*Zone Description*	*Statistical Areas*
1	North Coast	1–5
2	Central Coast	6–11, 30
3	N.W. Vancouver Island	25–27
4	S.W. Vancouver Island	21–24
5	Johnstone Strait	12–13
6	Georgia Strait and Fraser River	14–19, 28, 29
7	Juan de Fuca Strait	20

Table I – British Columbia Coastal Fishing Zones †

†See also Figure 1 for a map of the British Columbia coast including Statistical Areas and Zones.

The intrafishery movement of freezer trollers for intraseasonal decision making purposes will be described in terms of the 7 zones defined in Table I and Figure 2.

State Dynamics. The underlying state dynamics or core process of the POMDP is the key component of this model. The state of the system is derived from a single aggregate random variable, the total seasonal harvestable biomass over all zones of the fishery. State dynamics are based on the proportion of this random variable which becomes available to the fishery throughout each period of the season.

The description of abundance level dynamics of Pacific salmon begins with an analysis of the dynamics of each of the 5 major species of salmon, namely chinook, sockeye, coho, pink, and chum. The end result is the estimation of the transition probabilities for state dynamics defined in (1′).

The procedure used in the development of state dynamics is a 6 step analysis which proceeds as follows:

1. Determine for the major stocks of each salmon species the primary seasonal movement (migration pattern) of the numbers of fish (pieces) through the zones of the fishery.

2. Given historical catches by period and final escapement to the spawning grounds in numbers of fish, reconstruct the runs by species to estimate the numbers of fish in each zone by period.

3. Determine average weight per piece by zone for each period of the season and for each species; apply average weights to the run reconstruction results for each species and add (over all species) to obtain estimates of the total abundance by weight in each zone of the total catchable biomass of salmon throughout the season.

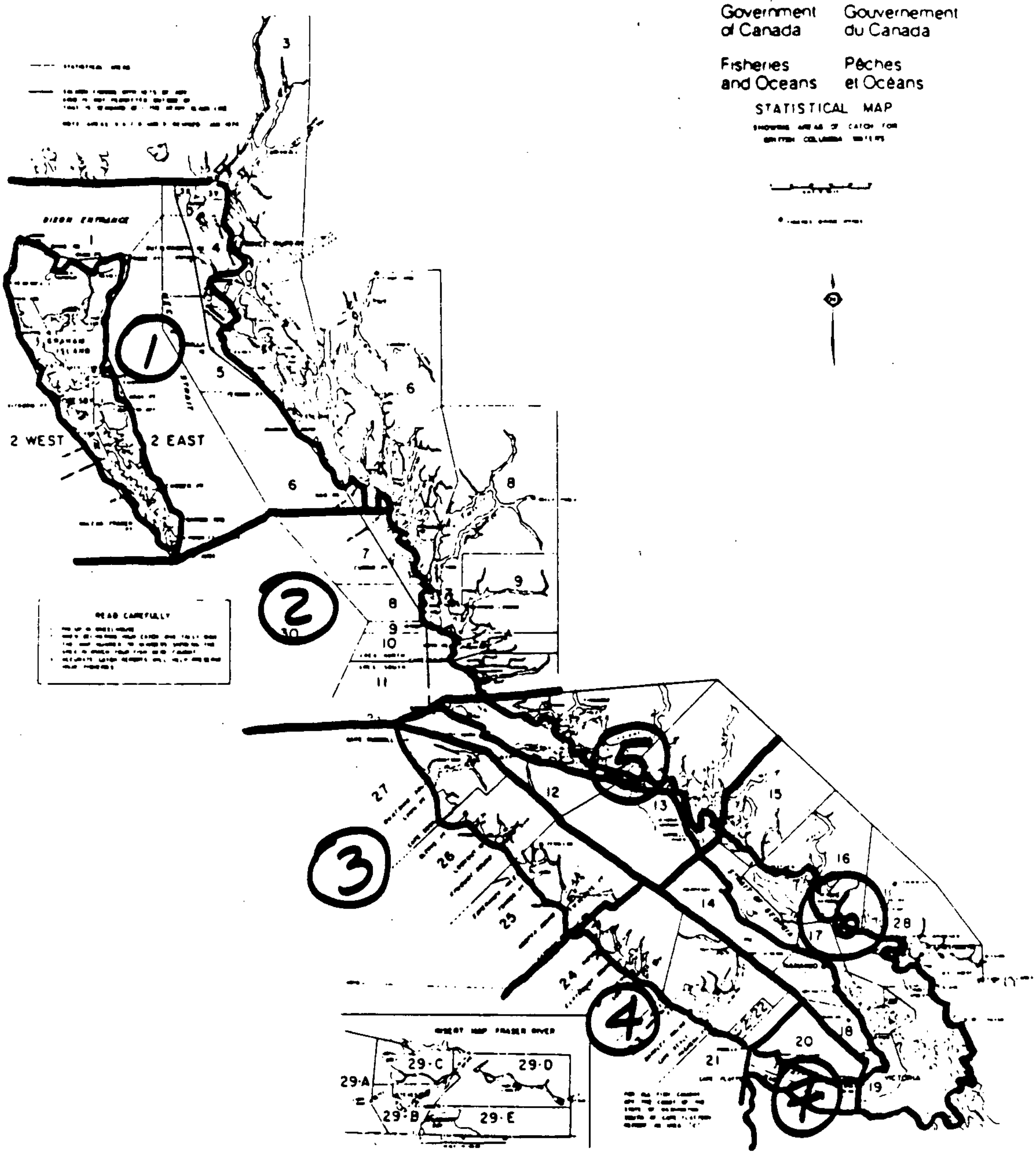


Figure 2 – British Columbia Coastal Fishing Zones

4. Carry out steps 1–3 for all years for which data is available; examine the performance of the inferred first order nonstationary Markovian model (1′) against the yearly total abundance data from Step 3.

5. Segregate the continously defined total abundance values from the yearly results of Step 3 into discrete abundance levels representative of the scale of high to low abundance levels in all zones.

6. Calculate the maximum likelihood estimators for the p_{ijk} defined in (1′) from the counts of the abundance levels determined in Step 5.

The details and results of each step of the 6 step procedure outlined above for describing stock abundance dynamics follows.

1. The recently released Fisheries and Oceans Canada Discussion Document entitled 'Pacific Region – Salmon Resource Management Plan' provides a voluminous and comprehensive picture of the state of salmon resources off British Columbia's coast. This document analyses the current and potential production capabilities of each of the 5 principal species of salmon. Furthermore, details are provided for the major stocks of each species. Major stocks of each species are identified by their unique migratory behavior to particular spawning zones, e.g., the Fraser River system, the Skeena River, etc. In particular, data is compiled on the migration patterns of each stock, the proportion of different stocks mixing at the same escapement zone, the usual periods of entry and exit from the fishery, and the peak abundance periods of each stock's run.†

In conjunction with this data, a number of studies (Anderson(1977), Aro and McDonald(1968), Houston et al(1976), Vernon et al (1964), Ledbetter and Hilborn(1981)) have been undertaken on particular stocks of salmon species in an effort to understand their migratory behavior. These studies are based primarily on the results of tagging programs. They present estimates of the proportion of seasonal total run of the stock through different areas of the fishery at each point in the season. As a consequence of tracking a stock in this manner, the length of time of passage from entry to exit throughout the different zones of the fishery can be estimated.

From the evidence of these studies, this analysis assumes a constant average passage time of two weeks between adjacent zones on the migration patterns for all stocks of all salmon species (Ledbetter and Hilborn(1981), and Vernon et al.(1964)). The two week average interzonal passage time also establishes

† On the one hand, the anadromous nature of Pacific salmon suggest that this kind of information may be reliable. However, it is recognized that the patterns of movements from sea to spawning ground of some stocks of some species, e.g., chinook, cannot be adequately predicted.

the number of periods, N into which each fishing season may be divided. Let each period of the season be two consecutive weeks long. The first period of the season begins on the third week of March and is identified as weeks numbers 11 and 12 of the 52 week year, or equivalently as $k = 1$. The last period of the season ending on the second week of December is weeks 49–50 or $k = 20 = N$.

The distribution of the total run abundance of each stock over each season is assumed to be triangularly distributed. This distribution depends only on the entry, exit and peak periods of the run. The triangular distribution compares well with the run times of the specific stocks studies referred to above (e.g., Ledbetter and Hilborn(1981)). Moreover, in the absence of specific information on the timing distribution of each salmon stock, this distribution may be considered as the representative family for all stocks of salmon (Law and Kelton(1982)). The parameters of the triangular distribution are obtained from the Discussion Document(1985). The assumed 2 week average zonal passage times and the proportion of different stocks at the same escapement zone, also obtained from the Discussion Document(1985), complete this first stage in the definition of the state dynamics. Table II 'Estimated Timing and Patterns of Migration– Pinks' presents the results of this first stage for the stocks of the pink salmon species. Migration patterns are assumed to begin outside the fishery (in the ocean) and end out of the fishery in a spawning ground adjacent to the final zone in the fishery. The percentages of each stock at the same final zone are also noted. Run timing periods are given in weeks of the 52 week year for stocks at the final zone along their respective migration patterns, i.e., the zone prior to escapement to the spawning ground.

2. Total catch and total escapement (in numbers of salmon) are estimated by Archibald and Graham(1983) for the period 1970–1979. Using their data, and assuming equal vulnerability of all stocks of each species and the same patterns of migration each season, the abundance of fish in pieces (prior to any catches) in a zone can be calculated by proceding backward from the spawning ground to entry to the fishery. This procedure is known as 'run reconstruction' (Hilborn(1983)). The results of run reconstruction for each species are estimates of the total abundance (in pieces of fish) in each zone at each period of the season. Formally, let $\hat{X}(i,j)$ be the matrix of abundance estimates (in pieces of salmon) for $i \in \{1,\ldots,N_S\}$, and $j \in \{1,\ldots,T\}$ where i is the species index with $N_S = 5$ different species, and j is the season index representing the years 1970-1979 with $T = 10$ (1979). Let the elements of $\hat{X}(i,j)$ be $\hat{x}_{kz}(i,j)$ with $k = 1,\ldots,N$, and $z = 1,\ldots,N_Z$ where z is the zone index for the $N_Z = 7$ zones defined in Table I, and $N = 20$ periods of the season. Table III presents run reconstruction results for the abundance estimates of pink salmon in each period and zone for the 1979 season.

3. The average weight of each salmon species by piece over the course of the season was determined from the actual landings reported by freezer trollers from 1971 to 1980. This data was obtained from the Fisheries and Oceans Canada sales slip database which records all salmon landings by vessel, including each

Stock	% Spawners Final Zone	Zone Paths †	Run Timing ‡ First	Peak	Last	Number of Weeks in the Fishery
1	85.0	O-1-S	23	31	40	2
2	15.0	O-2-1-S	30	33	40	4
3	90.0	O-2-S	28	31	39	2
4	15.0	O-2-5-S	30	35	37	4
5	15.0	O-2-5-6-S	32	36	40	6
6	100.0	O-3-S	24	34	40	2
7	15.0	O-3-2-5-S	28	35	37	6
8	15.0	O-3-2-5-6-S	30	35	40	8
9	100.0	O-4-S	25	34	40	2
10	60.0	O-4-7-6-S	29	36	40	6
11	60.0	O-4-7-6-5-S	31	35	37	8
12	10.0	O-1-2-S	25	33	39	4
13	10.0	O-1-2-5-S	27	35	37	6
14	10.0	O-1-2-5-6-S	29	36	40	8

Table II – Timing and Patterns of Migration - Pinks

†'O' denotes Ocean; 'S' denotes Spawning.

‡Run Timing periods are weeks of the 52 week year for stocks appearing at the Final Zone along their zone migration paths.

species captured, statistical area fished, date fished, total weight of catch, total pieces captured, and price received per unit weight.

For each species i, average seasonal weight (in kilograms) per piece for each zone, denoted $\bar{w}_z(i)$ is recorded. Next, each species average weight per piece by period, denoted $\bar{w}_k(i)$ is recorded. The overall average weight per piece for species i is denoted by $\bar{w}(i)$. Finally, the average weight per piece by zone and by period, $w_{kz}(i)$ is calculated by assuming independence between zones and periods, i.e., no average weight interaction term. Thus

$$w_{kz}(i) = \bar{w}_k(i) + \bar{w}_z(i) - \bar{w}(i), \; i = 1, \ldots, N_S \tag{13}$$

Let $W(i)$ be the matrix of average weights per piece for each species, i with elements $w_{kz}(i)$. Multiplying the corresponding elements of the matrices $W(i)$ and $\hat{X}(i,j)$ and summing across all species results in the matrix $B(j)$ of estimated total abundance of catchable salmon biomass by average weight for each of the

Run Reconstruction Results
by Zones and Biweekly Periods for 1979 Species: PINK

Weeks	Zone 1	2	3	4	5	6	7	Totals
11-12	0.	0.	0.	0.	0.	0.	0.	0.
13-14	0.	0.	0.	0.	0.	0.	0.	0.
15-16	0.	0.	0.	0.	0.	0.	0.	0.
17-18	0.	0.	0.	0.	0.	0.	0.	0.
19-20	0.	0.	0.	0.	0.	0.	0.	0.
21-22	0.	0.	0.	0.	0.	0.	0.	0.
23-24	44751.	0.	50677.	0.	0.	0.	0.	95428.
25-26	358005.	52153.	405417.	126737.	0.	0.	0.	942312.
27-28	716010.	532420.	810835.	1013899.	36459.	0.	94636.	3204255.
29-30	996854.	1756007.	1093279.	1955169.	332552.	61473.	757084.	6952415.
31-32	785737.	2775891.	630492.	2252785.	918906.	509214.	1459937.	9332961.
33-34	369495.	1863341.	80083.	1230346.	1429029.	1096689.	1682169.	7751151.
35-36	200192.	654137.	279.	153793.	903807.	1394491.	918707.	4225405.
37-38	82681.	248127.	139.	0.	117360.	771763.	114838.	1334908.
39-40	9610.	30569.	17.	0.	0.	96470.	0.	136666.
41-42	0.	0.	0.	0.	0.	0.	0.	0.
43-44	0.	0.	0.	0.	0.	0.	0.	0.
45-46	0.	0.	0.	0.	0.	0.	0.	0.
47-48	0.	0.	0.	0.	0.	0.	0.	0.
49-50	0.	0.	0.	0.	0.	0.	0.	0.
Totals	3563329.	7912644.	3071217.	6732728.	3738111.	3930099.	5027370.	33975296.

Table III - 1979 Run Reconstruction Results - Pinks study years $j = 1, 2, \ldots, 10$, i.e., 1970 to 1979. The matrix $B(j)$ has elements $b_{kz}(j)$ where

$$b_{kz}(j) = \sum_{i=1}^{N_S} w_{kz}(i)\hat{x}_{kz}(i,j) \tag{14}$$

This completes the third stage of the development of the state dynamics.

4. Next, the first order nonstationary transition model is tested on the ten years of amalgamated species abundance estimates, $B(j)$. The inferred model (1′) implies that the abundance of fish, i.e., the actual state of the system at period k of the season depends only on the state of the system at period $k-1$ and the current decision a_{k-1}. In other words, information about the actual state of the system at period $k-1$ is enough to describe the probability of the state of the system at period k. One representation of this process is found by considering the following model of state dynamics:

- Define the random variable M as the *total* availability of catchable biomass by weight for all zones over the entire season. Thus, define (ignoring the observations, j)

$$M = \sum_{z=1}^{N_Z} \sum_{k=1}^{N} b_{kz} \tag{15}$$

- Let $0 \leq t_k \leq 1$ be the fraction of biomass M which is available over all zones at period k of the season, where $\sum_k t_k = 1$ and

$$t_k = \sum_{z=1}^{N_Z} b_{kz}/M \tag{16}$$

- Let ν_{kz} be the factor relating the biomass at period k to the biomass available to zone z at period k, i.e.,

$$\nu_{kz} = \frac{b_{kz}}{t_{k-1}M} \tag{17}$$

  Equivalently,

$$b_{kz} = \nu_{kz} \sum_{z=1}^{N_Z} b_{k-1z} \tag{18}$$

- Finally, define the model with observations, $j, j = 1, 2, \ldots, 10$,

$$Y_{kzj} = \nu_{kz}\epsilon_{kzj} \tag{19}$$

  where Y, the dependent variable, is the matrix of factors relating the actual catchable biomass $b_{kz}(j)$ for zone z at period k to the total catchable biomass for all zones at period $k-1$, $\sum_z b_{k-1z}(j)$, i.e.,

$$Y_{kzj} = \frac{b_{kz}(j)}{\sum_z b_{k-1z}(j)}, \; j = 1, 2, \ldots, 10, \tag{20}$$

the ν_{kz} are constants for all j and ϵ_{kzj} are the multiplicative error terms. Thus, given (i) estimate $\hat{\nu}_{kz}$ for the zone and period factor, (ii) estimate $\hat{t}_{k-1}$, and (iii) estimated abundance of salmon, b_{kz} in zone z at period k, then the expected abundance in all zones for the succeeding periods may be determined through estimation of the random variable M, i.e.,

1) $\hat{M} = \hat{b}_{kz}/(\hat{\nu}_{kz}\hat{t}_{k-1})$

2) $\hat{b}_{lz} = \hat{\nu}_{lz}\hat{t}_{l-1}\hat{M}$, for $z = 1, \ldots, N_Z$ and for $l = k+1, \ldots, N$.

In this way, information about the actual state of the system in any zone at each period of the season is equivalent in this model to information about the single random variable M which drives the system. Estimating the actual salmon biomass, b_{kz} in the current period k and zone z yields estimates of the expected system dynamics thereafter through the matrix of factors ν_{kz}, and the vector t_k.

This representation of the system satisfies the assumption of the first order Markovian property. Letting $X_k = b_{kz}$ in the zone of current fishing, then we may write that

$$p_{ijk+1} = P\{X_{k+1} = j \mid X_k = i, X_{k-1}, \ldots, X_1\} = P\{X_{k+1} = j \mid X_k = i\} \tag{21}$$

which is the required form of the transition probabilities (1′).

Assuming the t_k are known constants for all seasons, then the hypothesis that the ν_{kz} are constant for all seasons can be tested by regressing Y on ν using model (19). We test the hypothesis that each vector ν_{kz}, $z = 1, \ldots, N_Z$ (denoted by ν_k) is a vector of constants for all seasons for which data is available. Expanding the multiplicative model (19) we obtain

$$Y_{kj} = \nu_k I_{N_z} \epsilon_{kj}, \; j = 1, \ldots, 10 \tag{22}$$

where I_{N_z} is the $N_Z \times N_Z$ identity matrix, and for a given j, Y_{kj} and ϵ_{kj} are vectors of N_Z elements. This model has at most N_Z parameters with 10 observations on each.† Now, taking logarithms yields the linear model

$$\log Y_{kj} = \log[\nu_k I_{N_z}] + \log \epsilon_{kj} \tag{23}$$

Figure 3(a)-(d) presents values for the vectors ν_k, $k = 3, 8, 13, 18$ for each observation year $j = 1, 2, \ldots, 10$ corresponding to the years 1970–1979. The least squares estimate of the log-linear regression (23) for each

† If salmon are not (appreciably) present in some zones during some periods of the season, then the catchable biomass is effectively zero in these zone–period pairs. In these cases the factors ν_{kz} are assigned zero values. Thus, there may be less than N_Z parameters to estimate for each period k.

ν_{kz} is also given. These plots depict the extent of season-to-season variability for the parameters of the model. Variability is low in the early and late periods of the season (Figures 3(a), 3(b) and 3(d)) when salmon runs are relatively smaller. There is increased variability during peak run periods occurring in the summer (Figure 3(c)).

A summary of results of the linear regression for all k periods and for $j = 1, 2, \ldots, 10$ are presented in Table IV. The agreement of the data to the model is indicated by the high r^2. Moreover, analysis of the error term in all regressions of (19) show no significant violation of the normality assumptions. From these results, it is assumed that the ν_{kz} are constants for all fishing seasons. Consequently, we describe salmon stock dynamics by the simple Markovian model (21) which depends on the single random variable, M.

Period k	Number of Parameters	r^2	# Parameters Nonsig @ 5% $H_0 : u = 0$
1	0	-	-
2	0	-	-
3	4	.997	none
4	6	.997	none
5	7	.996	none
6	7	.990	none
7	7	.998	none
8	7	.998	none
9	7	.980	none
10	7	.932	2
11	7	.721	1
12	7	.687	1
13	7	.708	2
14	7	.803	none
15	7	.898	1
16	7	.902	1
17	7	.928	1
18	7	.954	1
19	2	.968	none
20	1	.999	none
	Total=111	Ave=.913 Std Dev=.108	Total=10

Table IV – Summary of Regression Results

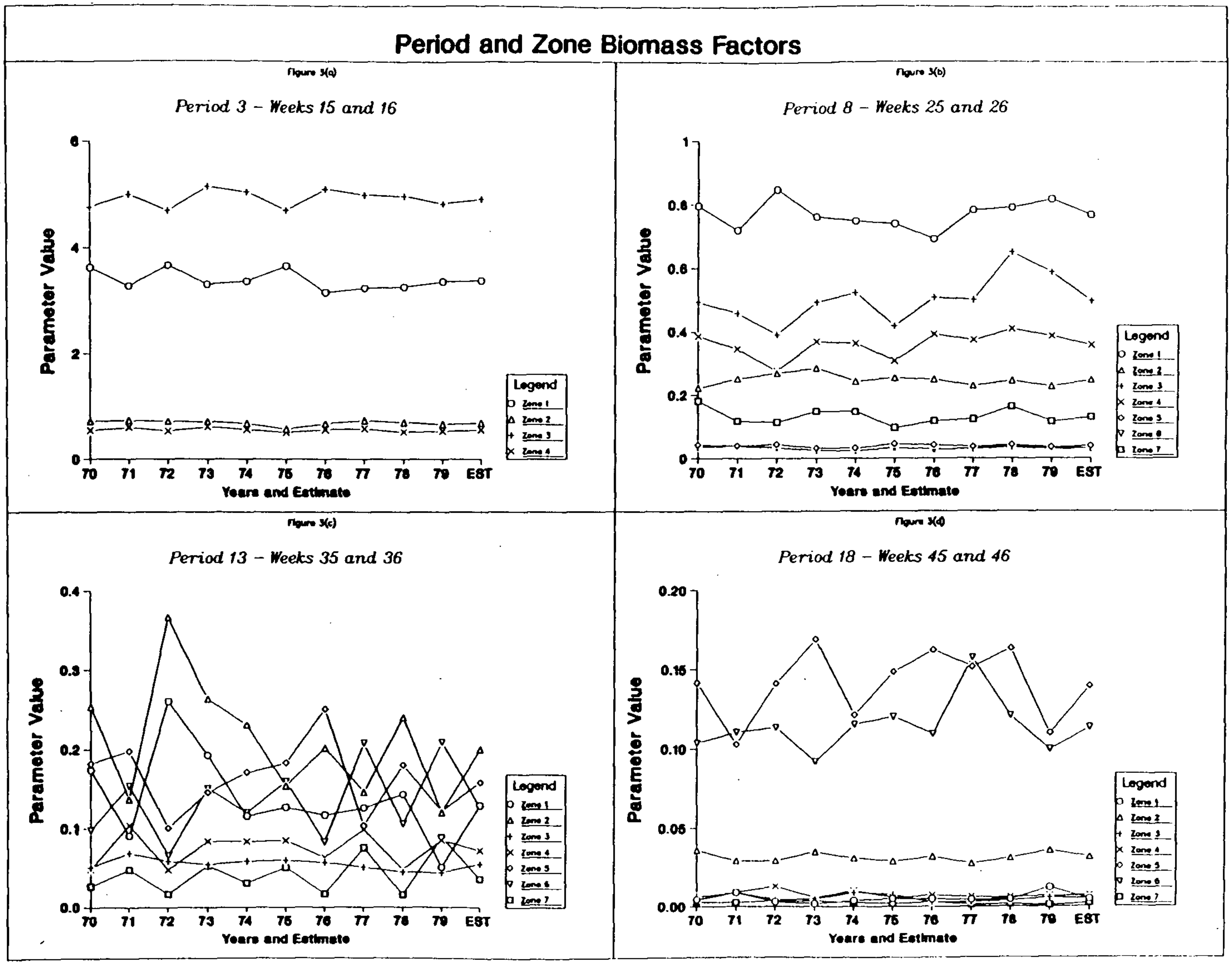


Figure 3 - Period and Zone Biomass Factors

5. The results of the previous step show that stock abundance dynamics may be considered as Markovian when the abundance is measured continuously (b_{kz}). However, in order to insure tractable solutions to the POMDP, it is necessary to define the state space of the system as a finite set (Bertsekas(1976),p.125ff). This requires the discretization of continuous measures of abundance. In a trade–off between tractability of the POMDP and the degree of discrimination of the actual decision makers, the number of states, N_X chosen for this analysis is three, ($N_X = 3$), e.g., 'low', 'medium', and 'high' levels of abundance. Class marks for the three levels of abundance are assigned according to the actual estimates of abundance by weight in each period over all zones for the 10 years 1970 to 1979.

6. The final stage in the development of the state dynamics is the estimation of the p_{ijk} in (1′) for the discrete state space. Maximum likelihood estimates $\hat{p}_{ijk}$ are calculated from the counts of the actual transitions from each discrete state to every other at each point in time k, $k = 1, \ldots, N$. The counts are determined from the 10 observation years. Alternatively, the counts may also be generated by estimating moments of abundance from the 10 observation years and then simulating B matrices using model (19). This procedure is used to define abundance scenarios for the testing of the sensitivity of model results to changes in the state dynamics of the system.

The maximum likelihood estimators for the transition probabilities are computed by the method of Anderson and Goodman(1957) for first order Markov chains. The results are sets of 3×3 transition matrices for each period to period movement over the season, i.e., $N - 1 = 19$ transitions. The specification of the dependence of the transitions probabilities, p_{ijk} on the actions, a_{k-1} of the decision maker are taken up in the next section. This completes the development of the state dynamics for the POMDP model in this application.

State–Observation Function. Once the levels of abundance have been established for each zone and period, their relationship to the actual recorded catches, i.e, observations of abundance, must be determined. This relationship is given by estimators for the state–to–observation probabilities q_{jlk} defined in (3). The procedure for developing these estimators is as follows:

1) Determine the actual distribution of catch by individual freezer trollers for each zone over the entire season, and for each period over all zones.

2) Test the empirical catch distributions by zone and period against families of theoretical probability distributions and determine the most representative model of the actual data.

3) Discretize catch by troller into levels of catch by zone.

4) For each zone, assign the most representative probability distribution model to each period and determine the probabilities of each level of catch occurring for the corresponding abundance level assigned in each observation year. Average the probabilities over all years to obtain the probabilities of the observation process, Q_k.

Details and results of each step toward the definition of the observation process are given below.

1. Actual total catch distributions by individual freezer trollers by zone and by period are obtained from the sales slip data for the years 1971–1980. Total catch is measured in kilograms summed over all salmon species landed. Periods and zones having negligible catch observations are ignored.

2. It is not unnatural to assume that the time between encounters of salmon on the lures of troll lines is exponentially distributed with parameter λ, i.e., the number of fish captured up to time t, $N(t)$ is a Poisson process. Moreover, in an analysis of actual catch rate data, Mangel and Clarke(1982) found it more appropriate in fitting the real data if λ itself were a random variable. In particular, they suggest that λ have a gamma distribution with parameters α and β. In this case $N(t)$, the number of fish captured up to time t, has a negative binomial (discrete) distribution with parameters α and $\frac{\beta}{1+\beta}$ (Ross(1976)). Since we measure catch continuously (by weight) then this suggests that a continuous analog of the discrete negative binomial distribution would be appropriate for actual catches.

In order to justify the above argument the empirical distributions for 4 zones and 6 monthly periods were tested for goodness–of–fit against seven theoretical distributions including the normal, Poisson, binomial, negative binomial, gamma, lognormal and exponential distributions. Table V presents the probabilities of the χ^2 test for goodness–of–fit of the data to each of these theoretical distributions. Acceptable levels of the χ^2 have probabilities exceeding 5%.

The gamma (continuous) distribution is at least as good as all other distributions in fitting the actual catch distributions followed closely behind by the negative binomial (discrete) distribution. The superiority of the gamma and the fact that the actual catch by weight is a continuous distribution suggests that the gamma family is the best representation of actual catch distribution. Moreover, the performance of the gamma and negative binomial distributions support the intuitive arguments presented above. For the purposes of this analysis the gamma distribution is used as the distribution of actual catch by individual fishermen in each zone and period of the season.

3. Analogous to the discretization procedure of the states, the zone observations are also discretized. Four levels of catch are assigned to approximate the actual levels of catch in each zone over the season. One level in each zone includes the zero catch value in connection with the no fishing option. Class marks for the remaining three levels (i.e., $N_Y = 3$) of catch by zone are assigned according to the catches in the 10

χ^2 Probabilities (%)

Month	N*	Normal	Poisson	Binomial	NegBin	Gamma	Lognorm	Exponl
5	50	23.90	40.23	5.23	–	6.98	0.23	2.90
6	70	5.46	10.81	5.94	43.21	49.53	3.39	39.42
7	160	0.12	0	0	37.64	70.60	3.23	0
8	130	0	0	0	2.49	2.67	0	0
9	90	0.25	0	0	40.26	40.39	2.13	59.45
10	140	0	0	0	41.31	42.14	0.08	49.30

Zone	N*	Normal	Poisson	Binomial	NegBin	Gamma	Lognorm	Exponl
1	110	22.68	0	0	14.85	5.07	0	0.68
2	40	8.83	4.86	0.25	9.38	11.37	0.05	16.82
3	100	0.09	0	0	67.06	73.22	0.70	5.17
4	150	0.46	0	0	0.65	0.42	0	0.43

Table V – Goodness of Fit Test Results

* N is the number of vessels reporting in a particular Month or Zone.

observation years. The right-ended class marks (i.e., the $f(y_{k+1}, a_k)$ in the reward function (5)) for each of the zones 1–7 are presented in Table VI.

Catch (in kilograms) by Level

Zone	*No Fishing*	*Low*	*Medium*	*High*
1	0	150	500	1300
2	0	200	600	1300
3	0	150	500	1400
4	0	150	500	1500
5	0	150	350	500
6	0	75	200	500
7	0	150	400	1000

Table VI – Discrete Catch Levels by Zone

4. Having determined the representative distribution and the discrete set of levels of catch by zone, the estimate of the probabilities q_{jlk} of each level of catch, l, occurring in period k and corresponding to the abundance level j is assigned for each zone. The estimates, $\hat{q}_{jlk}$ are obtained by taking the average of all corresponding estimates for the observation years 1971–1979. This completes the procedure for the state–to–observation function of the POMDP.

Reward Functional. The objective or reward functional for the POMDP model (5) is the maximization of the single criterion of net operating income. Like most single criterion optimization problems, this is a simplification of a complex multiple criteria decision making process. The choice of this particular criterion is a reasonable reflection of the objective of individuals in the competitive fishing environment. Nevertheless, it is recognized that different and/or conflicting objectives may lead to distinctly different modelled behavior. Other objectives, albeit single criterion ones, can be easily incorporated to study the sensitivity of the results, e.g., maximization of gross income or landed value, or maximization of total landings by weight.

The reward functional $R_{k+1}(y_{k+1}, a_k)$ is a function of three variables which depend on the decision, a_k made at the end of period k: (i) $p_{k+1}(a_k)$, average price per unit weight in period $k+1$; (ii) $f(y_{k+1}, a_k)$, catch by weight in period $k+1$; and (iii) $c_{k+1}(a_k)$, average cost of fishing during period $k+1$.

(i) Average price per unit weight data is obtained from the sales slip database and cross–referenced with the Fisheries and Oceans Canada 'Blue Sheets' (Preliminary Average Price Results by Species and Gear and Area) for the period 1971 to 1980. Where necessary, prices were transformed to dollars per kilogram and expressed in 1971 constant dollars using Statistics Canada Industry Selling Prices for Pacific Coast Salmon - Annual Average. Average prices for freezer troller salmon landings are calculated over all salmon species and computed for each zone and period of the season by dividing total real landed value of all salmon by the total kilograms of all salmon in each period and zone.

Analysis of the average price data for the 10 observation years (a) by zone over all periods of the season and (b) by month over all zones reveals no obvious time pattern of price behavior. However, a discriminant analysis revealed 3 distinct price scenarios by zone over all periods with a 73% correct jack–knifed classification over all observations. These price scenarios correspond to years of low, mean, and high salmon prices over the 10 year span. Moreover, the within season time pattern of each price scenario (with the exception of the 'low' scenario) exhibits a seasonal linear trend beginning with low prices and climbing to higher prices until season's end. Thus we may write the average price model for a particular scenario as

$$p_k(z) = \alpha(z) + \beta(z)k \tag{24}$$

where $\alpha(z)$ is the intercept and $\beta(z)$ is the slope or linear growth rate of the price for zone z over the season. The value of β defines the price scenario, i.e., the higher the β value, the higher the average prices (and vice versa). Assuming the same rate of linear growth for all zones, i.e., $\beta(z) = \beta$ then $\alpha(z)$ is computed from

$$\alpha(z) = \bar{p}(z) - \beta\bar{k}, \; z = 1, \ldots, N_Z \tag{25}$$

where $\bar{p}(z)$ is the average price per kilogram in zone z over the entire season. Table VII contains the scenario definition values, β and the average seasonal prices, $\bar{p}(z)$ for all $N_Z = 7$ zones. These values determine the matrix of average prices per kilogram for each zone and period of the season and for each price scenario.

Scenario Definition

	Low	*Mean*	*High*
β	0.03	0.07	0.10

Average Seasonal Prices

	Low	*Mean*	*High*
Zone 1	1.45	1.51	1.82
2	1.33	1.65	1.81
3	1.22	1.53	1.81
4	1.32	1.56	1.83
5	1.28	1.54	1.65
6	1.05	1.23	1.52
7	1.42	1.63	1.95

Table VII – Average Price Per Kilogram Parameters

(ii) Levels of catch by weight for each zone correspond to the discrete set of observations by zone defined in Table VI. The discrete function $f(y_{k+1}, a_k)$ is the set of right–ended class marks for the possible catch outcomes, y_{k+1} as a consequence of decision a_k (which defines the relevant fishing zone).

(iii) Average operating costs per zone were difficult to determine directly due to limited data availability. Freezer troller costs by zone for each period of the season are based on computation of the total seasonal real (1971=100) costs for the average freezer troller plus adjustments to account for period–to–period cost differences, zone cost differences and zone–to–zone moving costs.

Two sources of data were available for the computation of total seasonal operating costs incurred by freezer trollers. In a 1976 study in conjunction with the Sinclair Report(1978), Gislason(1976) reported on the itemized operating costs of classes of troll fishing vessels. In particular, he recorded estimates for labor, fuel, food ('variable operating expenses') and repair and maintenance, gear, insurance, licence and other costs ('fixed operating expenses') for freezer ('high line') trollers.

More recently, Fisheries and Oceans Canada (Pacific Division), Regional Planning and Economic Branch have produced comprehensive surveys of commercial salmon fishermen's incomes and expenses for the years 1982 and 1983. Operating costs are itemized (as in Gislason) for different categories of income earners (i.e., low 25% , medium 50% , high 25%) and for different gear types, e.g., trollers, gillnetters, seiners. Freezer trollers are primarily found in the 'high income, salmon trollers' category of these studies. The itemized average real operating costs per two week period for the three years 1976, 1982 and 1983 are found in Table VIII. On the basis of these results, real operating costs incurred while fishing in a particular zone (not including labor and the cost of interzonal movement) for each period of the season and from season to season is assumed to be constant, *ceteris parebis*.

Freezer Trollers Per Two Week Period

Item	*Year*			*Average*
	1976	1982	1983	
Labor†	15%	14.6%	10.5%	13.4%
Fuel	67	137	153	119
Food	83	89	80	84
Repair & Maint	204	137	157	166
Gear	87	68	79	78
Insurance	88	45	55	63
Licence Fees	11	24		
Miscellaneous	83	132	213*	154*
	—	—	—	—
Total Operating (less Labor)	623	632	737	664

Table VIII – Average Real Operating Costs

†Labor costs are expressed as a percentage of total landed value.

* Includes Licence Fees

Hilborn and Ledbetter(1979) postulate that there are cost differences due to fishing in different zones, e.g., due to the unsheltered conditions of Juan de Fuca relative to the Gulf of Georgia. However, while this has intuitive appeal, no data on actual operating costs differences among zones are readily available. For the purposes of this analysis a zone operating cost adjustment factor is provided to test the sensitivity of zone cost differences on optimal policies.

Period–to–period cost differences may be modelled as a function of the additional labor employed by trollers throughout the course of the season which reflects differences in fishing effort levels. For example, the early and late season (off–peak) periods generally employ less labor than the summertime peak run periods. The difference in labor employed will effect the operating cost accordingly. The periodic cost adjustment factor takes into account the effect these seasonal employment fluctuations have on periodic operating cost differences. All zones are assumed to be affected similarly.

Finally, interzonal moving costs are approximated by assuming they are proportional to the travel distance between zones. Adjacent zones are assumed to require one day of travel between them at a cost of one day's operating expense. Moreover, it is assumed that the average number of days spent fishing in each two week (14 day) period over the course of the season is 10 days (Gislason (1976)). This ensures that vessels have enough time to travel to any other zone without the added opportunity cost of lost average effort in any period. Table IX itemizes the components of the average cost calculation by zone and period.

This completes the development of the empirical elements for the POMDP intraseasonal decision making model.

Average Real Operating Costs Per Period

Total Average Operating Costs (less Labor @ 15% and Moving)=$ 500.00 per 2 week period

Total Average Daily Operating Costs = $ 50.00 per day of fishing

Zone Cost Adjustment Factors

Zone	1	2	3	4	5	6	7
Factor	1.05	1.02	1.00	1.02	1.00	.95	.90

Periodic Effort Adjustment Factors

Period	1	2	3	4	5	6	7	8	9	10
Factor	.5	.5	.5	.75	.75	1	1	1	1	1
Period	11	12	13	14	15	16	17	18	19	20
Factor	1	1	1	1	1	.75	.5	.5	.5	.5

Cost of Movement Proxy †

Zone	1	2	3	4	5	6	7
1	0	1	2	3	2	3	4
2	1	0	1	2	1	2	3
3	2	1	0	1	2	3	1
4	3	2	1	0	2	2	1
5	2	1	2	2	0	1	2
6	3	2	3	2	1	0	1
7	4	3	2	1	2	1	0

Table IX – Average Operating Cost Parameters

†a_{ij} – Travel time (in days) between Zone i and Zone j.

4.4 Computational Considerations

The POMDP formulation (Section 4.2) and the results of the empirical analysis (Section 4.3) for the components of this model suggest two distinct modelling approaches. These two approaches utilize the aggregated state definition which is contingent on the key random variable M, the total seasonal salmon biomass over all fishing zones. Differences in the two approaches arise from different definitions for the actions space, A.

1. The first model describes the state of the system explicitly in terms of the key random variable M, i.e.,

$$X_k = t_k M \tag{26}$$

Discretization of X will now depend on t_k and the random variable M only and will be independent of zone. Accordingly, (1′) may be written

$$p_{ijk+1} = P_{k+1}\{X_{k+1} = j \mid X_k = i\} \tag{27}$$

independent of the zone, z. The state–to–state transitions reflect the abundance level over all zones of the fishery together. Moreover, observations in each individual zone have the same information content about the overall abundance of salmon.

To illustrate, consider levels of the random variable, M the overall abundance level for the entire fishery and fishing season. If M is relatively low then the season is 'poor', etc. Now, assume that the key input to fishermen's intraseasonal planning is the overall measure of abundance, M. Fishermen obtain information about the kind of season it is after making observations through catch statistics. As the season progresses fishermen receive imperfect signals about M. These signals have equal information content about M in all zones at the same periods of the season. In other words, different catches of salmon in different zones in the same period provide probabilistically the same insight into the actual value of M for the season. Thus, as the season goes on, all fishermen have learned more about the overall state of the fishery independent of where they have fished. Their remaining fishing decisions reflect this information accordingly.

The action space, A_1 for this formulation is given by $A_1 = \{0, 1, \ldots, N_Z\}$. Thus, for decisions a_k at period $k = 0, 1, \ldots, N-1$, then $a_k = z$, a zone index, or $a_k = 0$, the no fishing (idle) alternative. Individual zone affects are captured by the state–to–observation function (3) and by the zone dependent reward functional (5). The costs of moving from zone to zone are approximated by assuming they are proportional to the interzonal distance from the vessel's prespecified 'home port zone' to the other fishing zones. This approximation assumes implicitly that fishermen return to their home ports at the end of each two week fishing period. For freezers trollers however, this assumption may not be true.

2. Alternatively, by expanding the action space the level of abundance in each zone may be explicitly considered. Letting

$$X_{kz} = \nu_{kz} t_k M \tag{28}$$

means that X_{kz} still depends on the single random variable M. Now, define the action $a_k(i,j) \in A_2$ as the movement from current zone i in period k to zone j in period $k+1$ where $i, j \in A_1$. For example, $a_k(0,2)$ denotes the decision to move from zone 0 (the 'no fishing' option) in period k to zone 2 in period $k+1$ to fish there. There are $N_Z + 1$ original alternative actions in any period, given the current zone of the fisherman. The expanded action space A_2 contains $|A_2| = (N_Z + 1)^2$ possible actions for this alternative formulation when all current zones are considered. However, at any decision point only a subset (cardinality $N_Z + 1$) of these total actions are feasible. (Eagle(1984) presented a similar action space structure in the application of a POMDP to searching on a grid where alternative actions in any period depended on the current location in the grid and its adjacent grid points.) This formulation has the advantage of incorporating explicitly the cost of moving from zone to zone. (In the first formulation this cost is approximated as a function of the home port zone.) Conversely, this formulation has increased dimensionality through the expansion of the control space.

Computation of solutions to the intraseasonal model were carried out using Sondik's One-Pass Algorithm for the optimal control of partially observable Markov processes over a finite horizon.† The One–Pass Algorithm is coded in double precision FORTRAN.

The dimensionality of the POMDP problem may be described in terms of the cardinality of the finite state, observation and action sets, i.e., N_X levels of abundance, N_Y catch levels, $|A|$ actions, and the total number of periods, N. The dimensionality of the two formulations proposed above are

Formulation 1. $N_X/N_Y/|A_1|/N = 3/4/8/20$

Formulation 2. $N_X/N_Y/|A_2|/N = 3/4/64/20$

Prior to computation of the optimal control policy, δ a scheme for reducing the action space at each decision point of the problem is developed for Formulation 1. Action elimination is based on analysis of the recursive functional (11) and the fact that each zone has the same transitional behavior over the season. Since information about the actual state of the system can be deduced equally well from any zone, then the

† The author acknowledges with thanks Edward Sondik for providing the original computer code for the One–Pass Algorithm.

differences among zones is fully attributable to the expected reward in each period. Consequently, if

$$g_k(i,u) < g_k(i,v) \tag{29}$$

for all $i \in \{1,\ldots,N_X\}$, and $u, v \in A_1$, then it follows that

$$E\{g_k(i,u)\} < E\{g_k(i,v)\}. \tag{30}$$

By Bellman's principle of optimality for dynamic programs, then the reward functional which results from employing action u must be strictly less than that employing action v, *ceteris parebis*, i.e.,

$$J(u) < J(v) \tag{31}$$

Thus, action elimination proceeds by eliminating all dominated actions $u \in A_1$ at each stage k where

$$g_k(i,u) < g_k(i,v) \tag{32}$$

for all i.

The solution to the POMDP results in a dynamic fishing policy over the periods of the season. The decision of where to fish in the next period is determined by choosing the action corresponding to the maximum value arising from a series of dot products. The vectors involved are the current value for the sufficient statistic, $\pi(k)$, and the l policy options vectors determined by the model and denoted by $\alpha_i, i = 1,\ldots,l$. At any stage when $l = 1$, only one option exists and the selection of the best action is trivial. When $l > 1$ then the selection of the best action signifies the fishing policy leading to the maximization of the expected reward functional over the remainder of the season.

To complete this section on the intraseasonal model, an example of fishing policy results from the POMDP of Model 1 are presented in Table X. To illustrate, note that in period 7, the policy states that the best action is to fish in Zone 2. However, in period 8 it may be best to fish in Zone 2,3 or 4 depending on the value of the sufficient statistic vector, the given α vectors corresponding to each option, and the result of the dot products $\pi \cdot \alpha_i$ at this stage.

The procedure for following the optimal decision policy over the course of the season is as follows:

1) Beginning with the prior distribution $\pi(0)$ of abundance at the start of the season and current observation, y_0 update the current sufficient statistic vector, $\pi_j(0)$ to obtain $\pi_j(1)$ using Bayes' formula (9).

2) Given the α vectors, calculate the dot products $\sum_j \pi_j(k) \cdot \alpha_{ji}, i = 1,\ldots,l$ for the l options at stage k of the policy.

Scenario Definition: Prices - MEAN; Abundance - ALL YEARS
Optimal Policy Results for this Season

Periods To Go	Time Period	Number of Options	Zone Option	α Vector 1	2	3
1	19	1	2	23.7998	23.7998	23.7998
2	18	1	2	89.8277	94.7780	102.2037
3	17	2	2	186.0580	224.0974	205.0777
3	17	2	4	155.7343	229.5432	192.6387
4	16	1	4	872.6531	874.6958	884.2856
5	15	2	3	1542.8350	1528.1250	1478.1865
5	15	2	4	1423.0735	1440.0208	1487.5334
6	14	1	4	1882.7109	1875.0195	1875.0195
7	13	2	4	2292.9856	2247.0173	2271.8398
7	13	2	2	2278.6501	2250.5022	2265.7021
8	12	1	3	2726.8979	2705.0183	2718.1458
9	11	1	2	3411.5164	3411.5164	3411.5164
10	10	1	4	4128.3945	4128.3945	4128.3945
11	9	1	4	4753.9844	4762.4682	4762.4687
12	8	3	3	5190.8594	5204.7773	5210.4922
12	8	3	2	5186.0977	5324.9414	5368.7930
12	8	3	4	5169.8867	5369.2109	5432.1602
13	7	1	2	5454.0547	5483.4609	5483.4609
14	6	1	4	5736.5625	5756.5195	5756.5195
15	5	1	2	5766.7148	5766.7148	5776.7148
16	4	1	2	5775.4102	5775.4102	5775.4102
17	3	1	8*	5775.4102	5775.4102	5775.4102
18	2	1	8*	5775.4102	5775.4102	5775.4102
19	1	1	8*	5775.4102	5775.4102	5775.4102

Table X – POMDP Optimal Policy Results (Base Case)

* Zone "8" denotes the idle or no fishing alternative.

3) Take the action μ_k^* which corresponds to the option $l^* = \arg\max_i\{\sum_j \pi_j(k) \cdot \alpha_{ji}, i = 1, \ldots, l\}$.

4) Proceed to the next stage, $k \leftarrow k + 1$, and update the sufficient statistic vector, $\pi_j(k)$.

Actual decision making using the optimal policy, δ is analysed by simulating the core and observation processes during a season for a number of fishermen–decision makers. System simulation results for fishermen decision makers and sensitivity analysis of the intraseasonal model are presented in the next section. This completes the formal analysis of the intraseasonal decision making model.

4.5 Analysis of Results

This section presents the results of the intraseasonal decision model corresponding to Formulation 1. The inputs and outputs of the computerized model are described along with the scenarios prepared to examine model sensitivity to changes in input data. Finally, average seasonal results for freezer trollers generated by the model are compared to actual values.

Model Inputs. The POMDP intraseasonal decision model uses the following sets of input data: (a) the State Transition Matrices; (b) the State–Observation parameters; (c) the Price Matrix; (d) the Cost Components. These input data parameters are described below in more detail. Section 4.3 provides details on the empirical development of all input data parameters.

(a) *State Transition Matrices.* These are the period to period probability transition matrices, P_k for the core process whose elements are defined in (1′). These matrices describe the probabilistic dynamics of the abundance of salmon throughout the season. The period to period transitions of the core process are described by 3×3 matrices for each period. Each transition matrix applies equally to every zone.

(b) *State–Observation Parameters.* These parameters define the relationship between the unobserved core process and the observation process. These data include:

   i. the observation levels class marks. These are the values for the function $f(y_{k+1}, a_k)$ of (5), the class mark for the level of catch $y_{k+1} \in \mathcal{M}$ measured in kilograms for period $k+1$ in the zone determined by the decision $a_k \in A$. The values for these parameters are given in Table VI.

   ii. the state–observation matrices. These are the matrices $Q_k(a_{k-1})$ which define the probabilistic relationship between actual abundance (the core process) and catch (the observation process). The elements of the 3×3 state–observation matrices for each zone and period are defined in (3).

(c) *Price Matrix.* This is the matrix of $p_{k+1}(a_k)$ values of (5), the average price per kilogram of salmon captured in each period $k+1$ for each zone of the fishery defined by decision $a_k \in A$.

(d) *Cost Components.* These data are the items which make up the values of $c_{k+1}(a_k)$ described in (5). These data include:

   i. the average total operating costs per day fishing by freezer trollers. Average costs for each two week period are assumed to be proportional to the number of fishing days (10) in each period.

ii. the zone cost adjustment factors. These are cost adjustment factors affecting average daily operating costs by freezer trollers in the different zones of the fishery.

iii. the periodic adjustment factors. These are the cost adjustment factors affecting average daily operating costs for the different periods of the fishing season. Average fishing effort is assumed to increase during the early part of each season and decrease during the later periods of the season.

iv. the cost of movement factors. These factors relate the cost of interzonal movement to the separation or distance of the zones one from another. The passage from one zone to any other adjacent zone is assumed to take one day and is costed at the average daily operating cost rate for the season (i. above). Table IX gives the cost component data items developed for this analysis.

This completes the description of the intraseasonal model inputs. Table XI presents the entire dataset for a scenario of 'Good' seasonal abundance and 'High' price levels.

Model scenarios are discussed in the following section.

Intraseasonal Model Scenarios. The sensitivity of the POMDP model results to changes in the input data is explored by defining a set of experiments or scenarios affecting specific data items. The scenario defines the 'frame of reference' under which intraseasonal decisions actually take place. Model scenarios are defined specifically for the two key uncontrollable elements of the system, namely, seasonal abundance and landed value prices.

Model scenarios are defined by specifying the following data items:

1. *Abundance Level.* Specify the set of state transition matrices for the core process corresponding to a level of the random variable M, the total seasonal abundance of all salmon in the fishery as in (15). In this analysis four different levels of abundance were selected based on actual estimates for M. These are:

   (i) Average – the annual average of all historical actual (estimated) seasonal abundance for the years 1971–1979. ($\bar{M}$=162M kg).†

   (ii) Good – the average actual (estimated) abundance for higher than average years. These included years 1972,1973, and 1978 with M values of 185M, 203M and 194M kg respectively.

   (iii) Mediocre – the average actual (estimated) abundance for the middle abundance years. These

† 'M kg' denotes Millions of kilograms.

Computerized Model Inputs

(a) State Transition Matrices, row-wise P (3×3) for each period 1–19

1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	-Period 1
1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	-Period 2
1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	-Period 3
0.90	0.10	0.00	1.00	0.00	0.00	1.00	0.00	0.00	-Period 4
0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	-Period 5
0.10	0.90	0.00	0.00	1.00	0.00	0.00	1.00	0.00	-Period 6
0.10	0.90	0.00	0.00	1.00	0.00	0.00	1.00	0.00	-Period 7
0.00	0.40	0.60	0.00	0.10	0.90	0.00	0.00	1.00	-Period 8
0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	-Period 9
0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	-Period 10
0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	-Period 11
0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	-Period 12
0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	1.00	-Period 13
0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.02	0.98	-Period 14
0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.14	0.86	-Period 15
0.00	1.00	0.00	0.00	1.00	0.00	0.00	0.86	0.14	-Period 16
0.50	0.50	0.00	0.00	1.00	0.00	0.00	1.00	0.00	-Period 17
1.00	0.00	0.00	0.86	0.14	0.00	1.00	0.00	0.00	-Period 18
1.00	0.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	-Period 19

(b) State-Observation Matrices, row-wise Q (3×3) for each period 1–19 and for each zone 1–7

0.96	0.04	0.00	0.85	0.15	0.00	0.75	0.25	0.00	
0.99	0.01	0.00	0.90	0.10	0.00	0.80	0.20	0.00	Q matrices by zone
0.95	0.05	0.00	0.85	0.15	0.00	0.75	0.25	0.00	for each period 1-19
0.93	0.07	0.00	0.83	0.17	0.00	0.72	0.28	0.00	
0.99	0.01	0.00	0.90	0.10	0.00	0.80	0.20	0.00	Zones are 1-7
0.98	0.02	0.00	0.90	0.10	0.00	0.80	0.20	0.00	-Period 1
0.99	0.01	0.00	0.90	0.10	0.00	0.80	0.20	0.00	

0.92	0.08	0.00	0.82	0.18	0.00	0.72	0.28	0.00	
0.96	0.04	0.00	0.90	0.10	0.00	0.80	0.20	0.00	
0.91	0.09	0.00	0.80	0.20	0.00	0.70	0.30	0.00	
0.86	0.13	0.01	0.75	0.20	0.05	0.70	0.20	0.10	-Period 2
0.96	0.04	0.00	0.90	0.10	0.00	0.80	0.20	0.00	
0.96	0.04	0.00	0.90	0.10	0.00	0.80	0.20	0.00	
0.97	0.03	0.00	0.90	0.10	0.00	0.80	0.20	0.00	

0.87	0.12	0.01	0.75	0.20	0.05	0.70	0.20	0.10	
0.93	0.07	0.00	0.90	0.10	0.00	0.80	0.20	0.00	
0.86	0.13	0.01	0.75	0.20	0.05	0.70	0.20	0.10	
0.80	0.18	0.02	0.70	0.20	0.10	0.65	0.25	0.10	-Period 3
0.91	0.08	0.01	0.80	0.15	0.05	0.75	0.15	0.10	
0.94	0.05	0.01	0.90	0.08	0.02	0.80	0.15	0.05	
0.93	0.07	0.00	0.90	0.10	0.00	0.80	0.20	0.00	

0.61	0.34	0.05	0.50	0.40	0.10	0.45	0.40	0.15	
0.69	0.29	0.02	0.60	0.35	0.05	0.50	0.40	0.10	
0.65	0.32	0.03	0.60	0.35	0.05	0.50	0.40	0.10	
0.53	0.31	0.10	0.50	0.40	0.10	0.45	0.45	0.10	-Period 4
0.72	0.21	0.07	0.60	0.30	0.10	0.55	0.35	0.10	
0.83	0.15	0.02	0.75	0.20	0.05	0.70	0.20	0.10	
0.70	0.24	0.06	0.65	0.25	0.10	0.60	0.25	0.15	

0.61	0.34	0.05	0.61	0.34	0.05	0.50	0.40	0.10	
0.69	0.29	0.02	0.69	0.29	0.02	0.50	0.40	0.10	
0.71	0.27	0.02	0.65	0.32	0.03	0.50	0.40	0.10	
0.53	0.38	0.09	0.53	0.37	0.10	0.45	0.40	0.15	-Period 5
0.72	0.21	0.07	0.72	0.21	0.07	0.60	0.30	0.10	
0.83	0.15	0.02	0.83	0.15	0.02	0.70	0.20	0.10	
0.70	0.24	0.06	0.70	0.24	0.06	0.60	0.30	0.10	

0.15	0.51	0.34	0.37	0.48	0.15	0.30	0.50	0.20	
0.39	0.52	0.09	0.41	0.49	0.10	0.40	0.50	0.10	
0.23	0.56	0.21	0.38	0.50	0.12	0.40	0.50	0.10	
0.38	0.50	0.12	0.30	0.47	0.23	0.20	0.50	0.30	-Period 6
0.50	0.35	0.15	0.49	0.35	0.16	0.35	0.45	0.20	
0.69	0.25	0.06	0.69	0.25	0.06	0.50	0.35	0.15	
0.47	0.36	0.17	0.47	0.36	0.17	0.35	0.45	0.20	

0.45	0.45	0.10	0.37	0.48	0.15	0.40	0.46	0.14	
0.50	0.45	0.05	0.41	0.49	0.10	0.35	0.55	0.10	
0.60	0.35	0.05	0.38	0.50	0.12	0.35	0.52	0.13	
0.40	0.50	0.10	0.40	0.47	0.13	0.24	0.45	0.31	-Period 7
0.50	0.35	0.15	0.49	0.35	0.16	0.40	0.40	0.20	
0.69	0.25	0.06	0.69	0.25	0.06	0.55	0.35	0.10	
0.52	0.42	0.06	0.47	0.36	0.17	0.40	0.35	0.25	

0.25	0.45	0.30	0.46	0.51	0.03	0.12	0.40	0.48	
0.20	0.45	0.35	0.28	0.58	0.14	0.12	0.45	0.43	
0.30	0.40	0.30	0.13	0.49	0.38	0.17	0.40	0.43	
0.25	0.40	0.35	0.27	0.51	0.22	0.11	0.32	0.57	-Period 8
0.20	0.37	0.43	0.34	0.46	0.20	0.16	0.34	0.50	
0.49	0.31	0.20	0.95	0.05	0.00	0.40	0.35	0.25	
0.35	0.50	0.15	0.35	0.48	0.17	0.27	0.29	0.44	

0.30	0.50	0.20	0.25	0.50	0.25	0.16	0.41	0.43	
0.20	0.45	0.35	0.16	0.39	0.45	0.14	0.46	0.40	
0.30	0.50	0.20	0.25	0.50	0.25	0.17	0.40	0.43	
0.30	0.50	0.20	0.25	0.50	0.25	0.13	0.34	0.53	-Period 9
0.25	0.40	0.35	0.18	0.35	0.47	0.18	0.35	0.47	
0.55	0.40	0.05	0.46	0.32	0.22	0.46	0.32	0.22	
0.40	0.30	0.30	0.30	0.40	0.30	0.28	0.32	0.40	

0.15	0.40	0.45	0.10	0.37	0.53	0.13	0.43	0.44	
0.10	0.50	0.40	0.08	0.42	0.50	0.13	0.42	0.45	
0.20	0.30	0.50	0.20	0.30	0.50	0.19	0.32	0.49	
0.70	0.30	0.00	0.65	0.34	0.01	0.14	0.29	0.57	-Period 10
0.30	0.40	0.30	0.20	0.35	0.45	0.17	0.33	0.50	
0.60	0.40	0.00	0.49	0.43	0.08	0.43	0.34	0.23	
0.60	0.30	0.10	0.40	0.40	0.20	0.30	0.28	0.42	

0.20	0.50	0.30	0.12	0.45	0.43	0.13	0.43	0.44	
0.20	0.40	0.40	0.06	0.35	0.59	0.13	0.42	0.45	
0.40	0.40	0.20	0.27	0.27	0.46	0.19	0.32	0.49	
0.40	0.40	0.20	0.23	0.32	0.45	0.14	0.29	0.47	-Period 11
0.40	0.50	0.10	0.30	0.47	0.23	0.17	0.33	0.50	
0.60	0.30	0.10	0.50	0.35	0.15	0.43	0.34	0.23	
0.70	0.30	0.00	0.63	0.35	0.02	0.30	0.28	0.42	

0.45	0.45	0.10	0.62	0.36	0.02	0.40	0.42	0.18	
0.40	0.45	0.15	0.47	0.46	0.07	0.45	0.42	0.13	
0.50	0.45	0.05	0.27	0.49	0.24	0.23	0.39	0.38	
0.50	0.40	0.10	0.42	0.45	0.13	0.34	0.38	0.28	-Period 12
0.60	0.40	0.00	0.52	0.37	0.11	0.51	0.34	0.15	
0.80	0.20	0.00	0.98	0.02	0.00	0.69	0.24	0.07	
0.92	0.08	0.00	0.52	0.39	0.09	0.43	0.38	0.16	

0.40	0.45	0.15	0.40	0.45	0.15	0.35	0.43	0.22	
0.40	0.40	0.20	0.34	0.47	0.19	0.30	0.48	0.22	
0.25	0.48	0.27	0.26	0.49	0.25	0.24	0.46	0.30	
0.30	0.40	0.30	0.29	0.42	0.29	0.25	0.41	0.34	-Period 13
0.50	0.40	0.10	0.47	0.35	0.18	0.45	0.33	0.22	
0.90	0.10	0.00	0.75	0.18	0.07	0.65	0.24	0.11	
0.92	0.08	0.00	0.39	0.39	0.22	0.37	0.32	0.31	

0.40	0.40	0.20	0.42	0.44	0.14	0.44	0.36	0.20	
0.40	0.45	0.15	0.40	0.45	0.15	0.37	0.41	0.22	
0.25	0.48	0.27	0.32	0.47	0.21	0.30	0.40	0.30	
0.41	0.46	0.13	0.28	0.43	0.29	0.27	0.42	0.31	-Period 14
0.50	0.40	0.10	0.46	0.36	0.18	0.41	0.35	0.24	
0.80	0.20	0.00	0.76	0.19	0.05	0.66	0.26	0.08	
0.60	0.30	0.10	0.53	0.32	0.15	0.51	0.24	0.25	

0.40	0.43	0.17	0.43	0.42	0.15	0.45	0.35	0.20	
0.40	0.45	0.15	0.40	0.45	0.15	0.33	0.44	0.23	
0.28	0.43	0.29	0.33	0.44	0.33	0.31	0.38	0.31	
0.34	0.40	0.26	0.30	0.40	0.30	0.25	0.38	0.37	-Period 15
0.70	0.30	0.00	0.47	0.31	0.22	0.40	0.35	0.25	
0.80	0.20	0.00	0.71	0.22	0.07	0.62	0.24	0.14	
0.39	0.36	0.25	0.56	0.29	0.15	0.50	0.30	0.20	

0.47	0.36	0.17	0.46	0.36	0.18	0.40	0.41	0.19	
0.37	0.48	0.15	0.46	0.40	0.14	0.40	0.41	0.19	
0.43	0.37	0.20	0.40	0.37	0.23	0.37	0.32	0.31	
0.30	0.40	0.30	0.38	0.35	0.27	0.29	0.43	0.28	-Period 16
0.50	0.30	0.20	0.52	0.28	0.20	0.45	0.28	0.27	
0.37	0.37	0.26	0.71	0.21	0.08	0.69	0.20	0.11	
0.56	0.29	0.15	0.63	0.25	0.12	0.34	0.39	0.27	

0.93	0.07	0.00	0.73	0.24	0.03	0.70	0.20	0.10	
0.99	0.01	0.00	0.82	0.18	0.00	0.70	0.20	0.10	
0.92	0.08	0.00	0.68	0.29	0.03	0.60	0.30	0.10	
0.88	0.12	0.00	0.62	0.33	0.05	0.50	0.40	0.10	-Period 17
1.00	0.00	0.00	0.87	0.12	0.01	0.87	0.13	0.00	
0.97	0.03	0.00	0.93	0.07	0.00	0.90	0.10	0.00	
0.90	0.10	0.00	0.79	0.17	0.04	0.65	0.25	0.10	

0.86	0.13	0.01	0.80	0.15	0.05	0.75	0.20	0.05	
0.94	0.06	0.00	0.90	0.10	0.00	0.80	0.20	0.00	
0.85	0.15	0.00	0.75	0.20	0.05	0.70	0.20	0.10	
0.79	0.20	0.01	0.65	0.30	0.05	0.60	0.30	0.10	-Period 18
0.97	0.03	0.00	0.95	0.05	0.00	0.90	0.10	0.00	
0.98	0.02	0.00	0.95	0.05	0.00	0.90	0.10	0.00	
0.88	0.11	0.01	0.80	0.15	0.05	0.75	0.20	0.05	

```
0.97 0.03 0.00 0.95 0.05 0.00 0.90 0.10 0.00
1.00 0.00 0.00 1.00 0.00 0.00 1.00 0.00 0.00
0.97 0.03 0.00 0.95 0.05 0.00 0.90 0.10 0.00
0.94 0.06 0.00 0.90 0.10 0.00 0.90 0.06 0.04 -Period 19
1.00 0.00 0.00 1.00 0.00 0.00 1.00 0.00 0.00
1.00 0.00 0.00 1.00 0.00 0.00 1.00 0.00 0.00
0.97 0.03 0.00 0.95 0.05 0.00 0.90 0.10 0.00
```

```
150.  500.  1300.
200.  600.  1300.
150.  500.  1400.      Class marks for observations
150.  500.  1500.      of catch (in kg) for each zone
150.  350.  500.
 75.  200.  500.
150.  400.  1000.
```

(c) Seasonal Price Matrix

```
HIGH Prices,GOOD Abundance
1.47 1.46 1.46 1.48 1.30 1.17 1.60
1.47 1.46 1.46 1.48 1.30 1.17 1.60     Two--way layout for prices of
1.52 1.51 1.51 1.53 1.35 1.22 1.65     catch (in real 1971=100 dollars)
1.57 1.56 1.56 1.58 1.40 1.27 1.70     by zone and by period.
1.62 1.61 1.61 1.63 1.45 1.32 1.75
1.67 1.66 1.66 1.68 1.50 1.37 1.80     Prices are in $/kg averaged
1.72 1.71 1.71 1.73 1.55 1.42 1.85     over all salmon species.
1.77 1.76 1.76 1.78 1.60 1.47 1.90
1.82 1.81 1.81 1.83 1.65 1.52 1.95
1.87 1.86 1.86 1.88 1.70 1.57 2.00
1.92 1.91 1.91 1.93 1.75 1.62 2.05
1.97 1.96 1.96 1.98 1.80 1.67 2.10
2.02 2.01 2.01 2.03 1.85 1.72 2.15
2.07 2.06 2.06 2.08 1.90 1.77 2.20
2.12 2.11 2.11 2.13 1.95 1.82 2.25
2.17 2.16 2.16 2.18 2.00 1.87 2.30
2.22 2.21 2.21 2.23 2.05 1.92 2.35
2.27 2.26 2.26 2.28 2.10 1.97 2.40
2.32 2.31 2.31 2.33 2.15 2.02 2.45
```

(d) Cost Components

```
525.00 510.00 500.00 510.00 500.00 475.00 450.00 Zone Cost/pd
.50 .50 .50 .75 .75 .751.0 1.0 1.0 1.0 1.0 1.0 1.0 1.0 .75 .50 .50 .50 .50
```

```
0 1 2 3 2 3 4
1 0 1 2 1 2 3
2 1 0 1 2 3 2     Matrix of pseudo distances from zone to zone
3 2 1 0 2 2 1     to be used in the calculation of the moving
2 1 2 2 0 1 2     cost proxy.
3 2 3 2 1 0 1
4 3 2 1 2 1 0
```

Table XI – POMDP Input Data

included years 1971,1974,1976,1977, and 1979 with M values of 143M, 166M, 160M, 152M and 147M kg respectively.

(iv) Poor – the average actual (estimated) abundance for the poor abundance years. 1975 was chosen as the only poor abundance year having an M value of 107M kg.

2. *Price Level.* Specify one of three levels of price corresponding to 'Low', 'Mean', or 'High' prices for landed salmon by weight.

The Base Case for the sensitivity analysis is defined as having 'Average' abundance transitional behavior and 'Mean' price level.

The analysis of the intraseasonal model results begins with a specification of the scenario for abundance and price levels. The corresponding inputs (see Table XI) are used in the computer model to generate the POMDP fishing policy parameters. (See Table X for the fishing policy parameters for the Base Case.) Once the season's fishing policy has been determined, the stochastic system (corresponding to the defined scenario) is simulated for 20 similar seasons with 50 freezer trollers in each season.

During each season the core process evolves according to the state transition matrices of the scenario. Each vessel then applies the POMDP fishing policy at each period of the season depending on the random catch levels observed and the updated state probabilities to this period. Catches occur during each period fished according to the state–observation parameters. The distribution of catch by zone and by period are recorded for each freezer troller in each of the simulated seasons.

For example, consider the Base Case fishing policy of Table X. No fishing occurs until period 4 of the season. In periods 4 and 5 zone 2 is fished. Then in period 6 switching occurs to zone 4. The policy returns the Base Case fisherman to zone 2 in period 7. In period 8 either of zones 2,3 or 4 may be fished depending on the results of the season to this point. Fishing occurs in the remaining periods in zones 2,3 and 4. The season ends with fishing in zone 2 during the final periods 18 and 19.

Simulation statistics generated include freezer troller averages for salmon catch, gross income from salmon fishing, total operating costs, and net operating income by zone and by period. Statistics on the distributions of each of the four seasonal averages for all 1000 (= 20 × 50) trials are also produced in a separate analysis. Simulation results for the Base Case scenario are found in Table XII.

Intraseasonal Model Sensitivity Results. Figures 4 and 5 present the intraseasonal model results in

Scenario Definition: Prices - MEAN; Abundance - ALL YEARS

SIMULATION RESULTS FOR 50 VESSELS DURING 20 SEASONS†

Report by Zone:

	ZONES							
Item	1	2	3	4	5	6	7	TOTALS
Catch (kg)	0.00	2451.52	1369.60	6001.11	0.00	0.00	0.00	9822.24
Gross Inc ($)	0.00	4013.48	2293.25	9702.41	0.00	0.0	0.00	16009.14
Oper Cost ($)	0.00	3125.86	1393.78	5927.69	0.00	0.0	0.00	10447.33
Net Op Inc ($)	0.00	887.15	899.21	3779.44	0.00	0.00	0.00	5565.79

Report by Period:

	PERIODS						
Item	1	2	3	4	5	6	7
Catch (kg)	0.00	0.00	0.00	333.28	332.88	624.63	495.94
Gross Inc ($)	0.00	0.00	0.00	489.90	499.26	905.62	778.66
Oper Cost ($)	0.00	0.00	0.00	494.23	495.58	633.08	677.70
Net Op Inc ($)	0.00	0.00	0.00	-4.29	3.70	272.62	100.91

Report by Period: (cont'd)

	PERIODS						
Item	8	9	10	11	12	13	14
Catch (kg)	964.19	983.84	1011.94	865.69	761.81	781.84	715.83
Gross Inc ($)	1465.40	1524.91	1603.84	1480.41	1241.77	1321.27	1238.36
Oper Cost ($)	882.74	891.67	904.24	782.99	786.18	861.11	848.75
Net Op Inc ($)	582.70	633.17	704.54	697.35	455.4	460.10	389.60

Report by Period: (cont'd)

	PERIODS					
Item	15	16	17	18	19	TOTALS
Catch (kg)	607.82	625.98	294.09	224.00	200.00	9823.72
Gross Inc ($)	1051.50	1126.69	520.47	407.59	357.97	16013.61
Oper Cost ($)	607.66	500.43	409.50	341.61	334.15	10451.59
Net Op Inc ($)	443.81	626.21	110.97	66.03	23.80	5565.66

Table XII – Intraseasonal Model Simulation Results (Base Case)

†Reported results are averages per troller vessel.

graphical form for the 12 different scenarios of the sensitivity analysis. The figures illustrate the results of the simulation statistics for salmon average seasonal: catch (Figure 4(a)), gross income (Figure 4(b)), operating costs (Figure 5(a)), and net operating income (Figure 5(b)) by freezer trollers and for each scenario. The table of values corresponding to these figures are given in Table XIII.

A number of general points can be made about the POMDP model results for the different scenarios analysed and presented in Figures 4 and 5 and Table XIII. In particular, the results are relatively insensitive to changes in abundance levels, *ceteris parebis*. Conversely, changes in the price level have a much more

Freezer Troller Seasonal Average Values

Scenario Definition *Abundance*	*Price*	*Actual Years*	*Catch (kg)*	*Gross Income ($)*	*Operating Costs ($)*	*Net Operating Income ($)*
10yr Ave	Mean‡	–	9824	16018	10452	5567
10yr Ave	High	–	10100	19317	11235	8082
10yr Ave	Low	–	8369	11696	8190	3505
Good	Mean	72	10030	16407	10557	5849
Good	High	73,78	10319	19778	11357	8421
Good	Low	–	8463	11832	8210	3621
Mediocre	Mean	79	9627	15767	10261	5506
Mediocre	High	76	9743	18717	10756	7960
Mediocre	Low	71,74,77	8318	11624	8179	3444
Poor	Mean	–	9148	15120	9944	5175
Poor	High	–	9456	18170	10611	7558
Poor	Low	75	7898	10997	7920	3077

Table XIII – Intraseasonal Model Sensitivity Analysis†

†All results are based on simulation of the scenarios for 20 similar seasons and for 50 freezer trollers on each season.

* Scenarios applicable to the actual (estimated) abundance and prices for the years 1971–1979.

‡Base Case scenario definition. See also Tables X and XII for Base Case fishing policies and simulation results.

significant impact on the simulation statistics results. Zones 5–7 do not appear as alternative zones to fish in any of the scenarios analysed. Of the seven zones of the fishery, only zones 1–4 are ever fished by the model decision makers.

More fishing takes place when prices are high and total fishing effort decreases as prices fall from high to low, *ceteris parebis*. When prices are high, fishing begins in period 3 of the season and continues until the final period 19. During the season zones 1–4 are fished. Prices at their 'Mean' levels induce fishermen to begin fishing one period later in period 4. Fishing continues until the final period covering zones 2–4 only. When prices are low, fishing does not begin until period 6 and ends in period 18. During these periods fishing takes place primarily in zones 1,3 and 4. The distribution of average fishing performance by freezer

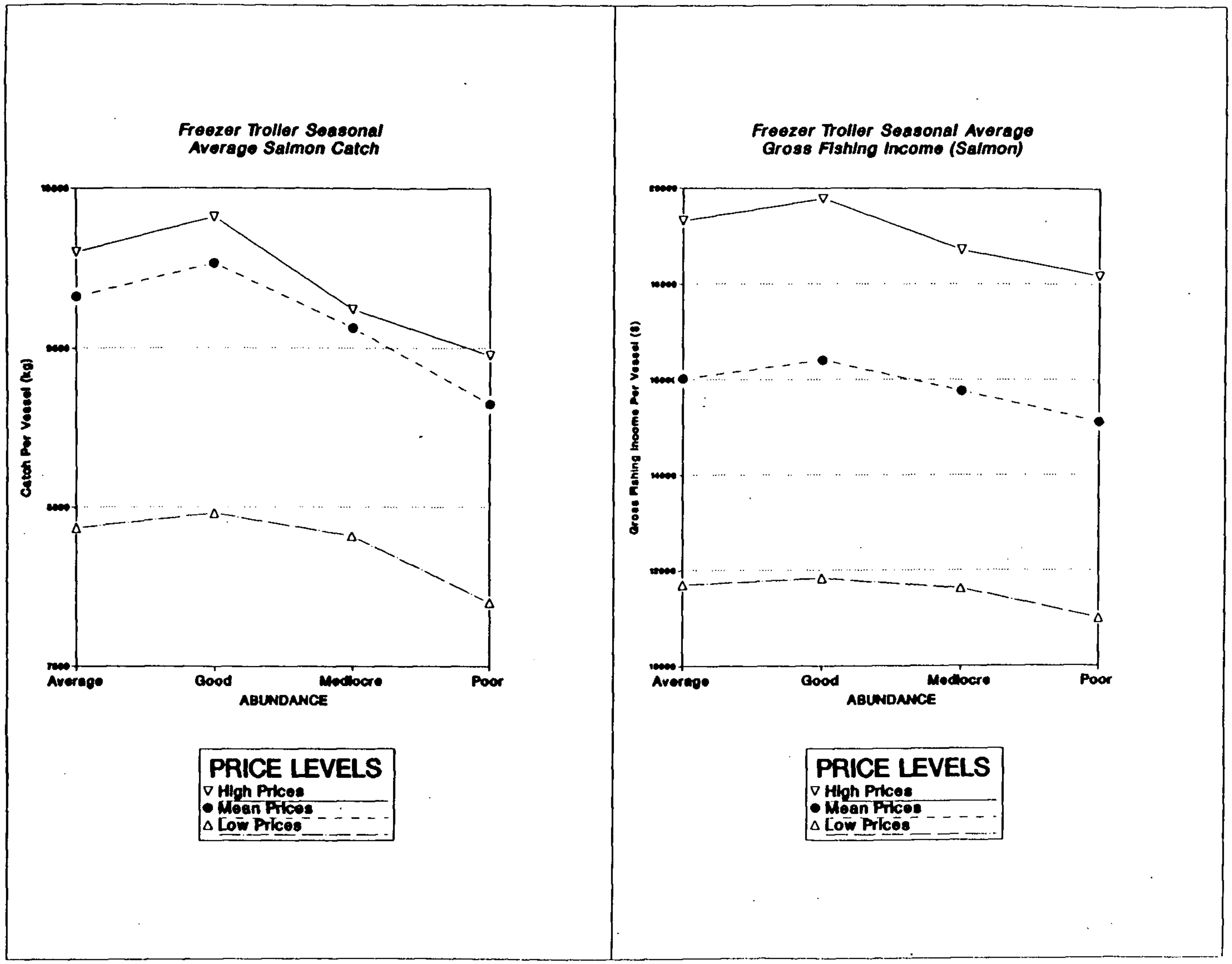
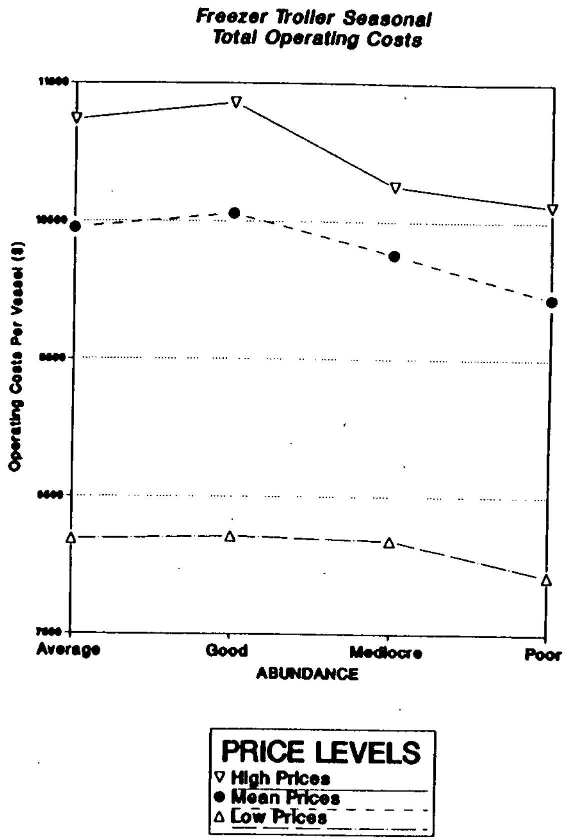


Figure 4 – Freezer Troller Average Catch and Gross Income



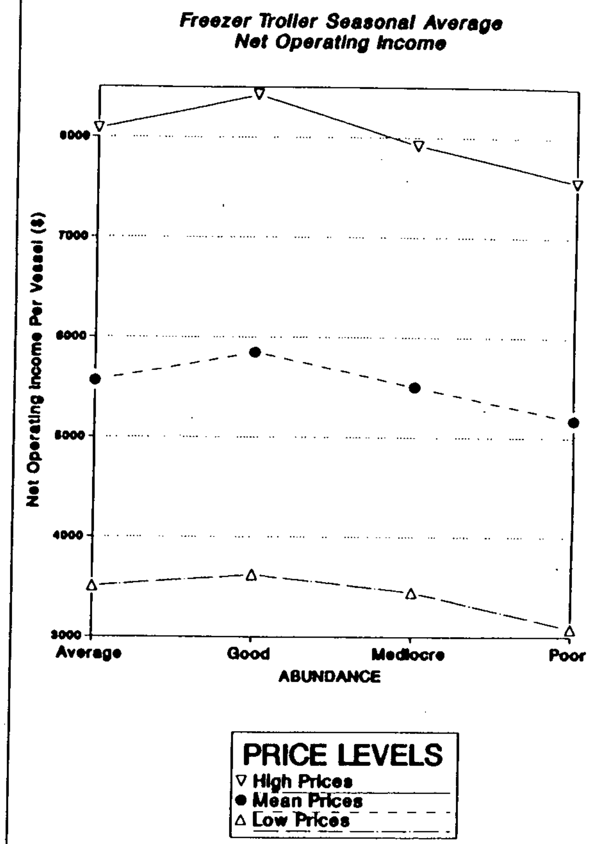


Figure 5 – Freezer Troller Average Cost and Net Income

trollers into zones and periods is analysed in the next section which compares model results to actual data.

Figure 4(a) illustrates the differences in average freezer troller seasonal catch to changes in abundance and prices. Relative to the Base case scenario, the range in the percentage difference of catch statistics is -20% (Poor abundance, Low prices) to +4% (Good abundance, High prices). Model catch levels differ only slightly for all abundance scenarios when prices are either high or mean (range is 9148kg to 10319kg). Catches for high prices are 3% above those for mean prices within each abundance level. When the price level is low, effort is more restricted and average catch levels fall by 15% of those for mean prices. Catch levels for low prices fall by approximately 600kg or 7% of the catch across abundance levels. Catch levels are highest when abundance is good, followed by mediocre and poor abundance catches, for the same price levels. Good abundance means a 2% increase in the Base Case catch level; mediocre abundance has catch levels 2% below Base Case values; poor abundance catches are 6% below Base Case levels, *ceteris parebis.*

Gross income from salmon landings, Figure 4(b) exhibits similar tendencies to salmon catches, however, these tendencies are exaggerated as a consequence of the direct effect of differences in price levels. For example, the range in the percentage difference of gross income relative to the Base Case is -31% (Poor abundance, Low prices) to +25% (Good abundance, High prices). This represents a difference of $9000 in gross income. Relative to the Base Case results, good, mediocre, and poor abundance levels realize +2%, -2% and -6% differences on the gross income statistic respectively for corresponding price levels. Within abundance levels, high prices have +20% higher gross incomes than mean prices, while low prices are 20% below mean price gross income levels.

Figure 5(a) illustrates the results for the operating cost statistic. Differences in the average value of this item are a result of differences in the number of periods fished and the different zones fished in each scenario. As such, the results are most sensitive to price levels and relatively insensitive to differences in abundance. The percentage differences in operating costs relative to the Base Case across abundance levels are at most 4% for corresponding price levels. However, low price levels result in a 20% decrease in operating costs in comparison to mean price levels within same abundance levels. This is a consequence of fishing less often with fewer interzonal moves when prices are low. The increase in fishing effort when prices are high, *ceteris parebis* translates into a 7% increase in operating cost levels. Average operating costs range between $7920 for the Poor-Low scenario to a high of $11357 for the Good-High scenario, a range of $3437.

Finally, Figure 5(b) presents the net operating statistic results for each scenario. This is the simple difference between the gross income statistic and the operating cost statistic. Net operating income ranges between $3077 (Poor-Low) and $8421 (Good-High) a difference of $5344. With respect to the abundance levels, net operating income for the good, mediocre and poor scenarios differ by +3%, -1% and -8% in comparison to Base case values for corresponding price levels. With respect to price levels, high prices mean a 45% increase in average net operating income over mean price levels, *ceteris parebis*. Low prices results in a 38% decline in net operating income relative to the mean price level within same abundance levels.

Summarizing, the most important variable affecting the results of fishermen's intraseasonal decisions is price. Differences in abundance levels appear to have little effect on intraseasonal policies and their economic consequences. One explanation for this is the high degree of aggregation used in describing the core process of this system. A more refined definition of states of abundance, while incurring more computational complexity, may reveal more sensitivity to this stochastic element of the system.

The crucial role of salmon prices on the decisions and average results for modelled freezer trollers suggests that this element of the system be considered as a key strategy variable. In this context price may be viewed as a means of 'controlling' intraseasonal decisions by fishermen and their consequences. This topic is pursued further in Chapter 6 of this thesis.

This completes the results of the intraseasonal model sensitivity analysis for the scenarios on abundance and prices. The section which follows compares the model statistics for catch and gross income from salmon with their actual values for the period 1971–1979.

Model versus Actual Results. This subsection compares model generated statistics with actual values for each of the years 1971 to 1979 for which empirical data is available. Based on the empirical data, each year is assigned a scenario for abundance and price (see Table XIII for these 'Actual Years' assignments). Model results are then assigned to each year according to the year's scenario definition. The statistics to be compared are seasonal average freezer troller salmon catches and gross fishing income from salmon landings. (Actual data for operating cost and net operating income attributable to salmon only is not available.)

Figure 6 presents in graphical form a comparison of model versus actual results for catch and gross income.

With the exception of 1972†, average annual freezer troller statistics predicted by the model compare well with actual values. The correlation for average catch is 0.10 (0.60 excluding 1972) over the 9 year period, while the correlation for gross income is 0.58 (0.87 excluding 1972). The Mean Absolute Deviation (MAD) is 2054kg/year (1463kg/year excluding 1972) for catch over this period. The MAD for gross income

† Actual results for 1972 are difficult to explain. However, it was in this year that new regulations were applied to the replacement of vessels. This required the redefinition of many vessels and may account for the understatement of fishing statistics for freezer trollers in this year.

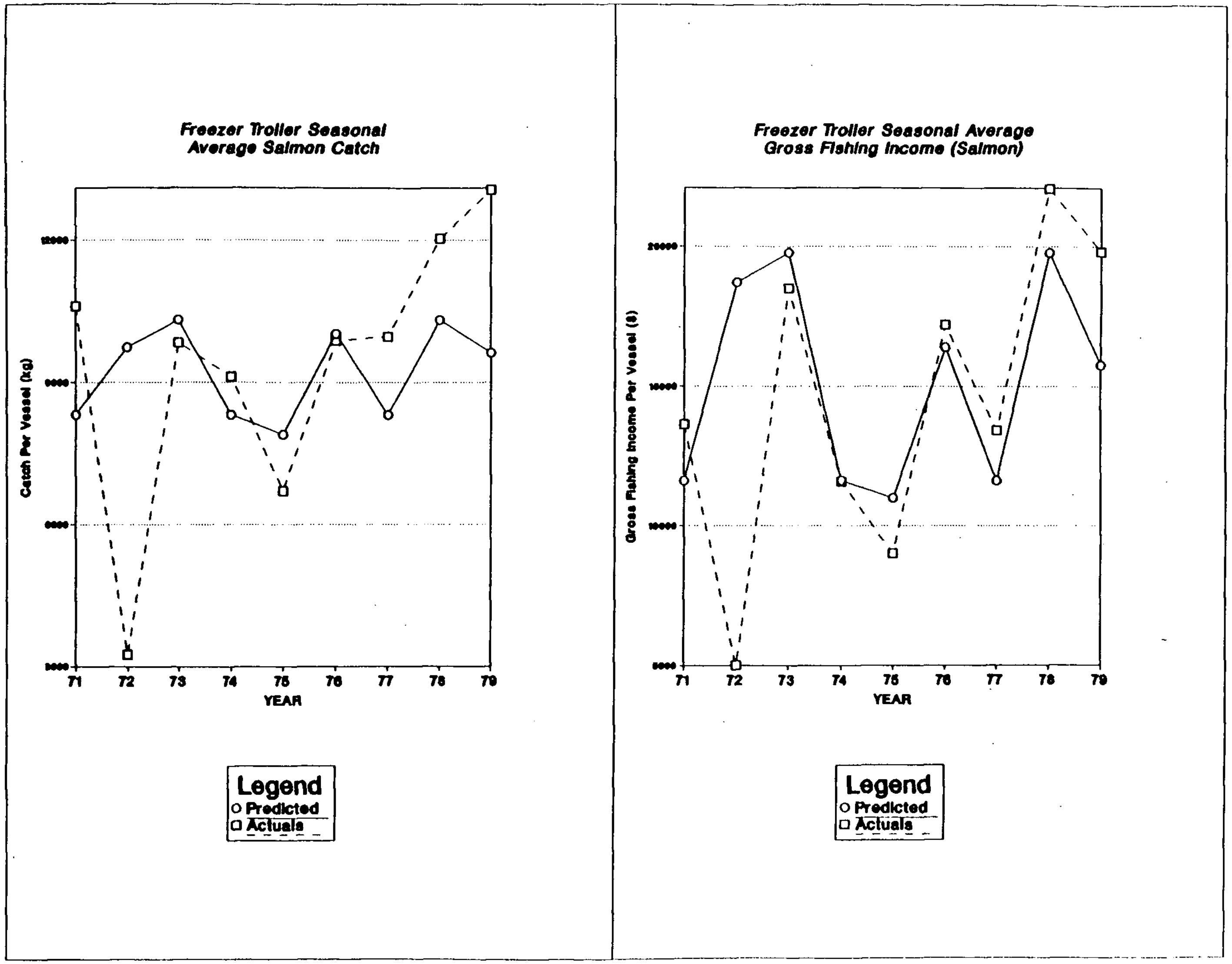


Figure 6 – Freezer Troller Average Catch and Gross Income Predicted versus Actuals, 1971–1979

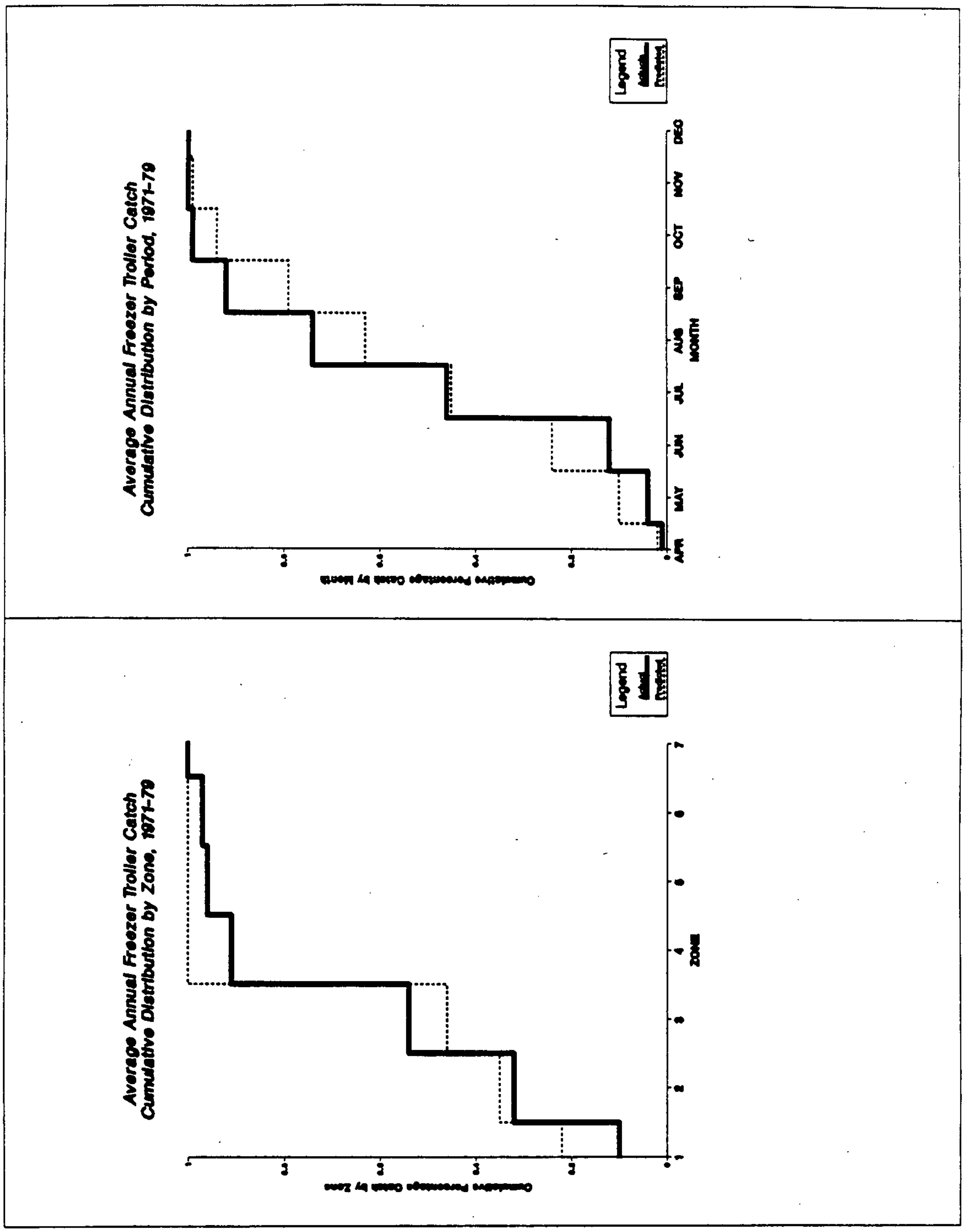


Figure 7 – Freezer Troller Average Catch Distributions
by Zone and Period

is \$2929/year (\$1870/year excluding 1972). These results indicate that the model statistics are reasonable predictors of the actual values for freezer trollers.

The zonal and periodic results of the simulation analysis (see Table XII), may also be compared to actual distributions by zone and period. Figure 7 displays a comparison of average annual distributions by zone (Figure 7(a)), and by month (Figure 7(b)) for the average seasonal freezer troller salmon catch by weight. (Model results for each two week period are aggregated to obtain monthly catches. These results may then be compared to the actual data which is recorded on a monthly basis.)

Average annual distribution percentages are computed for the 9 years 1971–1979. The results reveal that the average annual distributions by zone and by month predicted by the model agree favorably with the actual distributions. In defence of this statement, we cite a correlation between predicted and actual percentage distributions by zone of 0.75 and that by month of 0.84. The mean absolute percentage deviation by zone is 8% per zone while the comparable figure for the monthly distribution of catch is 7% per month.

This completes the analysis of results for the intraseasonal model. Implications and perspectives for planning and designing regulatory policies based on the results of this model are discussed in Chapter 6 of this thesis.

5. Interseasonal Decision Making

5.1 Motivation

The most serious problem facing Canada's ocean fisheries is overcapitalization of the fishing fleet. Economic theorists have long acknowledged overcapitalization (and associated rent dissipation) as a consequence of the common property nature of the fishing activity. Nevertheless, regulators' attempts to control this problem in practice cannot be termed successful. For example, in 1969 the federal government introduced a licence limitation scheme for the British Columbia salmon fishery known as the 'Davis Plan'. This plan effectively froze the number of vessels by licensing only those that showed a dependence on the salmon fishery. As Pearse states,

> Today, after more than a decade of restrictive licensing, the number of vessels in the salmon fleet is smaller, the fleet's structure has changed significantly, and the vessels are much improved in technical sophistication and safety. But the plan has clearly failed in its main purpose, which was to control and reduce excessive *capacity*. Investment in fishing power continued as the value of the catch increased, and the capacity of the fleet, already excessive when the program began, doubled or perhaps trebled. (Pearse(1982),p.79.)

A by–product of the current situation is that fishing equipment has become more efficient and more specialized than ever before. As well, the skills of fishermen have likewise become more highly specialized and, consequently, more limited to fishing activities. For example, the limited entry regulation has concentrated salmon fishing in the hands of a particular group of individuals whose primary source of income comes from the fishing activity. The opportunity costs of these individuals is low (relative to individuals in other sectors) which accounts for their fundamental reliance on fishing income. The thin trading of fishing rights (i.e., licenses) is further evidence of the primary dependence of individuals on their fishing activities and reflects their intention to survive in the fishing business.

To understand the decision making process of fishermen with regard to capital investment, it is necessary to describe the environment in which these investment decisions are made. The variability of the economic situation faced by fishermen substantiates the need for consideration of survival as a primary component in investment decision making. This variability comes from three major sources:

1. Natural and historical biological variations in stock dynamics (recruitment and survival), e.g., the domination of particular cohorts of a species such as Adams River sockeye.

2. Biological variations in stock dynamics due to oceanographic influences, pollution, over–exploitation of individual stocks, or destruction of habitat.

3. Economic fluctuations caused by changes in exogenous world price levels, reappearance or disappearance of markets (e.g., a recent botulism scare in Europe had a drastic effect on exports of British Columbia salmon), and fluctuations due to the general business cycle.

The consequence of the above discussion is that assuming the fundamental desire to survive, fishermen must be more wary of the downside risk of becoming insolvent than the upside risk of making large economic gains in a single season. As Thompson et al(1973) argue, fishermen may be considered to have an asymmetric attitude toward risk. In other words, fishermen are expected to be risk averse to poor seasonal earnings, becoming decreasingly less risk averse to higher than anticipated earnings. As an illustration, it is observed that fishermen make more conservative decisions in poor harvest years (e.g., 'movers' reduce operating costs by being less mobile, 'stayers' may fish less often to reduce costs, or diversify their catches in other fisheries) than in good harvest years (i.e., 'movers' are attracted to higher yield zones, 'stayers' fish more regularly). While it is acknowledged that there are conflicting behavioral theories about fishermen (Wadel (1972)), planning for survivability provides a reasonable basis for longer–term decision making in this highly variable business environment.

The variable envionment, the restricted size of troller operations, and the current overcapacity of the fleet, ultimately mean that there is significant capital rationing to trollers by financial institutions. Troller fishermen endeavoring to invest back into the fishery have limited capacity to raise funds. Riskiness of the enterprise and scale of operation are the primary constraining forces to financing new capacity. Moreover, this constraint compels fishermen desiring to reinvest to structure their financing in specific ways.

The following section presents a formal model of investment dynamics by fishermen seeking to increase the value of their operations with primary emphasis on survival. The normative model of investment dynamics is applied to the troller segment of the British Columbia commercial fishing fleet. The availability of actual data on a subset of all trollers permits the comparison of model results with actual investment decisions recorded annually. By analysing model sensitivity the behavioral conditions under which actual investments were made in the past can be conjectured by tuning the model to match past investment trends. Better understanding of this behavior is a major step away from 'reactive' and short–term regulation strategies. Moreover, it promotes the design of longer–term policies which incorporate and anticipate the behavior of fishermen. Herein lies the importance of the interseasonal investment model.

The development of the formal interseasonal investment model begins with the presentation of a dynamic decision model with no survivability restrictions. Investment policies are derived using a dynamic programming model. Linear and nonlinear (quadratic) functions are examined in the dynamic programming equation. Next, the unrestricted (with respect to survivability) model is revised to include constraints on investment due to survivability considerations. Different forms of the survivability constraint are examined including linear and quadratic functional forms. Finally, algorithms are presented which produce investment policies under each of the linear and quadratic survivability constraints.

5.2 Formal Model

The notation used in this section is defined independently from that of Section 4.2 for the intraseasonal POMDP model.

5.2.1 Unconstrained Model. This section defines the investment model with no conditions on continued survivablity for the fisherman-investor. The investment decision model with survivability constraints will be discussed as an extension to the unconstrained model.

Let $t \in \{0, 1, \ldots, T\}$ be the periods (i.e., years) of the investment planning horizon, with $t = 0$ being the initial period and $t = T \in I^+$ the final period in the horizon. These are the stages of this decision process, i.e., the time points at which the decision maker must decide on an investment policy. Each investment decision point is assumed to occur at the end of each fishing season, t.

Let NW_t denote the Net Worth (or Owner's Equity) of the fishing enterprise in real terms (i.e., discounted market values) at the end of fishing season t. NW_t is defined as the difference between assets and liabilities of the fishing enterprise at time t,

$$NW_t = (CA_t - CL_t) + (LTA_t - LTL_t) \tag{1}$$

where

CA_t – Current Assets of the fishing enterprise at the end of season t including cash available after taxes, interest and principal on debt, and family living expenses; accounts receivable; extended short–term credit; savings instruments and rebates. Current Assets are assumed to be convertible to cash quickly at no expense.

LTA_t – Long–Term Assets of the fishing enterprise at the end of season t including all working or fixed assets, e.g., appraised value of vessel, gear and electronics as well as other equipment associated with fishing. (See the 1982 Fishermen's Tax Guide.) Long–term assets require an extended period of time before conversion to cash at appraised values.

CL_t – Current Liabilities of the fishing enterprise at the end of season t, including cash deficiencies, accounts payable and short–term loans.

LTL_t – Long–Term Liabilities of the fishing enterprise at the end of season t, including mortgages on fishing equipment, and long–term loans.

The accumulation of business profits and increases in the value of assets increase Net Worth. On the other hand, fluctuations in market values of fishing capital causes annual shifts in asset values and accordingly, Net Worth. Long–term growth in Net Worth is a reflection of a viable operation and is a desirable criterion for making investment decisions. Falling Net Worth is a signal for required changes in the structure of the operation. When Net Worth is less than zero, liabilities exceed assets and the fishing enterprise is in danger of being declared insolvent or bankrupt. Survival of the business is contingent on continued positive Net Worth.

Let $s_k^i \geq 0$ denote a new investment (measured in constant dollar values) in capital of type i at period k, where $i \in S$ represents a single capital type from the set of investment classifications S with $|S| = N_S$ finite, and $k \in K \equiv \{0, 1 \ldots, T-1\}$ is the investment time period index. Moreover, assume that s_k^i is a bounded investment for each type i and for all periods k, with $0 \leq s_k^i \leq U^i,\ k \in K$, where $U^i \geq 0$ represents the largest investment of capital of type i which may be made in any one period k. The bounds on s_k^i are established in part by the limits on the borrowing capacity of all fishermen as a single risk group, the ability of individual fishermen to raise capital in any period, and the investment opportunities of each capital type.

Let $s_k = \sum_{i \in S} s_k^i$ represent the total constant dollar value of new investment in all categories at time k. Assume that each investment of type i has an associated financing structure which fishermen use to acquire the funds necessary to procure capital assets of each type. For example, assume that funds for an investment of type i are raised $\beta_i(LTL)100\%$ by long–term bank loans, $\beta_i(CL)100\%$ by cash and/or short-term borrowing, and $\beta_i(LTA)100\%$ by disposing of assets with $\sum_l \beta_i(l) = 1$. Moreover, once an investment s_k^i has been secured, its financing generates future deterministic schedules (assuming continued solvency of the enterprise) of related interest and principal payments as well as depreciation expense (amortizaton or capital cost allowance (CCA)) over the life of the asset purchased. Accordingly, each new investment, s_k^i (assuming no other investments are made after period k) will have a known cost effect on NW_t. Thus, we may write $NW_t = NW_t(s_k),\ t = k+1, \ldots, T$.

We develop the dynamic relationship for $NW_t(s_k)$ by considering the effect of new investment s_k on the two elements of (1), $(CA_t - CL_t)$ and $(LTA_t - LTL_t)$. Define for each period t, $k+1 \leq t \leq T$:

- The first term in the definition of NW_t from (1) may be written as

$$CA_t(s_k) - CL_t(s_k) = \left[1 + r(1 - TR)\right]\left[CA_{t-1}(s_k) - CL_{t-1}(s_k)\right] +$$

$$NI_t(s_k) - I_t(s_k) - RP_t(s_k) - TR\left[NI_t(s_k) - I_t(s_k) - D_t(s_k)\right] \tag{2}$$

with $CA_k(s_k) - CL_k(s_k) = CA_k - CL_k - F_k(CL)$.

In words, the first term of (1) in the definition of NW_t is given by (2), the cash carryover from the previous year plus the net operating income earned in the current year less debt payments owing (for interest and principal) and taxes payable. Each of the elements of (2) are defined as follows:

r – annual real interest rate received (if $CA_{t-1} > CL_{t-1}$), or paid (if $CA_{t-1} < CL_{t-1}$) on the cash account. For simplicity cash balances are assumed to be invested in, or financed by, short-term instruments at the same rate of interest, i.e., $r = r_{CL} = r_{CA}$.

NI_t – Actual Net Operating Income (Gross Income minus Operating Costs) earned by individual fishermen in a season measured in constant dollars. This is the key random variable of the investment decision process.

I_t – Interest expense (on debt) in constant dollars

D_t – Depreciaton expense (for tax purposes) in constant dollars

TR – Income tax rate (assumed to be constant over the planning period).

RP_t – Principal repayment on debt outstanding in constant dollars

$F_k(CL)$ – Amount of total new investment, s_k financed by Cash and/or Short-Term Debt at the time of investment at the end of period k in constant dollars.

- The second term in the definition of NW_t from (1) is

$$LTA_t(s_k) - LTL_t(s_k) = LTA_{t-1}(s_k) - LTL_{t-1}(s_k) \tag{3}$$

with $LTA_k(s_k) = LTA_k + s_k - F_k(LTA)$ and $LTL_k(s_k) = LTL_k + F_k(LTL)$

$F_k(LTA)$ – Amount of total new investment s_k financed by disposing of Long-Term Assets, LTA_k available at the time of investment in constant dollars.

$F_k(LTL)$ – Amount of total new investment, s_k financed by Long-Term Debt, LTL_k at the time of investment in constant dollars.

Since we require that all new investments be 100% financed then

$$s_k = F_k(LTA) + F_k(CL) + F_k(LTL). \tag{4}$$

This also implies that the year end cash balance, $CA_t - CL_t$ is equivalent to the difference between successive Net Worth values, $NW_t - NW_{t-1}$. Either of these expressions will be used in this analysis to denote year end cash on which short–term interest or financing will apply.

Define the set of financing parameters for each investment type i by the $N_S \times 3$ matrix, B. Row i of B may be written $(\ \beta_i(LTA)\ \beta_i(CL)\ \beta_i(LTL))$ for each row $i \in \{1, 2, \ldots, N_S\}$ where $\beta_i(\cdot)$ denotes the share of investment of type i financed by the disposal of long–term assets or by increasing short or long–term debt. The investment financing parameters, B are assumed to apply to all investment decision makers in every period with

$$\beta_i(LTA) + \beta_i(CL) + \beta_i(LTL) = 1 \tag{5}$$

for all i. The parameters, $F_k(l)$ can be written explicitly in terms of B as follows

$$F_k(l) = \sum_{i \in S} \beta_i(l) s_k^i \text{ for } l \in \{LTA, CL, LTL\}. \tag{6}$$

We now define formal relationships for each of the components of (2), namely, I_t, RP_t, D_t, and NI_t as explicit functions of the investment decision variable, s_k^i. In order to establish interest, principal payments, and depreciation explicitly as a function of s_k^i, we define the following parameters:

- Let L_m denote the (tax) amortization period for non-physical investments of type $m \in M \subset \{1, \ldots, N_S\}$. Non–physical assets include fishing rights and licences issued by the government. The amortization period L_m is the parameter which defines the straight–line depreciation schedule for newly acquired non–physical assets of type m. The straight–line depreciation amount is $100/L_m\%$ of the original asset value per year for L_m years.

- Let c_p denote the annual capital cost allowance (CCA) rate for physical investments of type $p, p \in M^c$, where M^c denotes the complement of the set M defined previously. The depreciation schedule for physical assets with specified CCA rate is calculated by applying the rate to the remaining balance for the pool of assets in each CCA class.

- Let r_l denote the annual real rate of interest for debt financing instruments $l = CL, LTL$ purchased in any period t of the planning horizon. For simplicity, short–term interest rates on cash received or borrowed are denoted by r, i.e., $r = r_{CA} = r_{CL}$. (The interest rate for long–term assets, LTA is not defined.)

- Let a_l denote the term for the financing instruments $l = CL, LTL$, purchased in any period t of the planning horizon. By definition, $a_{CL} < a_{LTL}$. All lending of funds is assumed to be for the short-term. The short–term borrowing and/or lending terms are assumed to be for one year, i.e., $a_{CA} = a_{CL} = 1$.

Given new investments, s_k^i, $i = 1, \ldots, N_S$ in period k and either L_m or c_p (whichever value is appropriate according to the tax regulations for each capital type i) the depreciation schedule, $D_t(s_k)$ is deterministic out to period T and explicit in terms of the decision variables s_k^i. Formally, we have

$$D_t(s_k) = D_t(0) + \sum_{i \in S} d_t^i c_i \tag{7}$$

where

$$d_t^i = \begin{cases} (1-c_i)^{t-k-1} s_k^i, \; t = k+1, \ldots, T & \text{if } i \text{ is a physical asset,} \\ s_k^i, \; c_i = 1/L_i, \; t = k+1, \ldots, k+L_i & \text{if } i \text{ is a nonphysical asset} \end{cases} \tag{8}$$

and $D_t(0)$ is the total depreciation in period t arising from existing capital.

Equivalently, we may define the following recursive relationship with $d_k^i = 0$ and $s_t^i = 0$, for $t = k+1, \ldots, T$

$$d_t^i = \begin{cases} d_{t-1}^i(1-c_i) + s_{t-1}^i, & \text{if } i \text{ is a physical asset,} \\ d_{t-1}^i + s_{t-1}^i, \; c_i = 1/L_i, \; t = k+1, \ldots, k+L_i & \text{if } i \text{ is a nonphysical asset.} \end{cases} \tag{9}$$

Given the starting principal instrument value $F_k(l)$ at time k, the interest rate r_l, and financing term a_l, $l = CL, LTL$, it is straightforward using annuities (i.e., uniform year end annual payments) to calculate the deterministic schedule of interest expense payments, $I_t(s_k)$ and principal repayments, $RP_t(s_k)$ from period $k+1$ out to T which arise from investment s_k. Writing I_t and RP_t explicitly in terms of investments s_k^i yields

$$I_t(s_k) = I_t(0) + \sum_{i \in S} \sum_{l} \beta_i(l) s_k^i \left(\frac{r_l(1-(1+r_l)^{t-k-1-a_l})}{1-(1+r_l)^{-a_l}} \right), \tag{10}$$

and

$$RP_t(s_k) = RP_t(0) + \sum_{i \in S} \sum_{l} \beta_i(l) s_k^i \left(\frac{r_l(1+r_l)^{t-k-1-a_l}}{1-(1+r_l)^{-a_l}} \right), \; t = k+1, \ldots, k+a_l; \; l = CL, LTL \tag{11}$$

where $I_t(0)$ and $RP_t(0)$ are the total outstanding interest and principal repayment in period t from existing debt.

Using (10) and (11) we may define the recursive relationship for z_t^i, the sum of interest and repayment, and $rp_t^i(l)$, the repayment portion only for new investment of type i capital at period k:

$$z_t^i = z_{t-1}^i + s_{t-1}^i \sum_{l} \beta_i(l) \frac{r_l}{1-(1+r_l)^{-a_l}}, \; t = k+1, \ldots, k+a_l; \; l = CL, LTL \tag{12}$$

and

$$rp_t^i(l) = rp_{t-1}^i(l)(1+r_l) + s_{t-1}^i\beta_i(l)\frac{r_l(1+r_l)^{-a_l}}{1-(1+r_l)^{-a_l}},\ t = k+1,\ldots,k+a_l;\ l = CL, LTL \tag{13}$$

Finally, we define the effect of new investment s_k^i on the random variable $NI_t \in$ NI, the seasonal net operating income. This variable is the key probabilistic component in the investment model. Define NI_t to be the average seasonal net operating income earned by fishermen of the troll fleet in season t. (The actual net operating income earned by individual fishermen, $NI_t(s_k)$ is assumed to be a function of NI_t in each season.) Moreover, NI_t accounts for season-to-season variation in earnings which affect all fishermen. Within season variation is assumed to be dependent only on the current capital structures of the fishing enterprises. Let NI_t be an independent and identically distributed random variable with known probability distribution function F_{NI} for all $t = 0, 1, \ldots, T$.

Now, assume that each investment of type i has a particular effect on gross earnings and variable costs in future periods relative to their current levels. We define the investment-income effect function (constant for all periods)†

$f(s_k^i)$ = The % shift in adjusted Net Operating Income per investment in type i assets

Consider the effect on the net operating income at period $k+1$ immediately following a new investment s_k at the end of period k. Let

$$NI_{k+1}(s_k) = NI_{k+1}(0) + \sum_{i\in S} f(s_k^i)s_k^i[NI_{k+1} - NI_L],\ k = 0, 1, \ldots, T-1 \tag{14}$$

where

$$NI_{k+1}(0) = NI_L + \sum_{i\in S} y_k^i\rho_i[NI_{k+1} - NI_L] \tag{15}$$

and NI_L is a real-valued constant ($NI_L < 0$), equal to the lowest realizable value of the random variable NI. Equation (14) denotes the net operating income earned when there is a non-zero investment, s_k. The second term on the right hand side of (14) denotes the adjustment in net operating income in period $k+1$ due to the new investment s_k through the investment-income effect function $f(s_k^i)$. The first term on the right hand side of (14) denotes the net operating income earned, $NI_{k+1}(0)$ when there is no new investment at the end of period k, i.e., $s_k = 0$. In (15), $NI_{k+1}(0)$ defines a shift in the distribution of the adjusted average seasonal net operating income random variable, $NI_{k+1} - NI_L$ through the parameters y_k^i and ρ_i.

† The range of the random variable NI includes the region about 0, i.e., NI_t may be either positive or negative. To avoid problems with applying the income effects function, $f(s_k^i)$ when NI_t is in the neighborhood zero, the random variable is first shifted upward by its minimum value denoted, $NI_L < 0$ before applying the factor $f(s_k^i)$. Adjusted NI is thus equal to $NI_t - NI_L$.

The parameter y_k^i denotes the net operating income earning power of a fishing operation at the end of period k. The value of y_k^i is assumed to depend on the enterprise's capital structure by type of capital, i at the end of period k. This parameter captures the differences between individual fishing operations by attributing earning power to vessel capital configurations relative to the average configuration ($\sum_i y_k^i = 1$). For example, vessels with greater hull capacity, gross tonnage, and more electronics and gear are assumed to have a larger net operating income earning power than other vessels with less tonnage, gear, etc, *ceteris parebis*.

The parameter ρ_i is the annual discount factor on the net operating income earning power for all capital of type i. The discount factor accounts for the rate of technological improvements, learning, and the deterioration of the real earning power of capital equipment.

New investments are assumed to have an initial effect on the net operating earning power of the enterprise through the function $f(s_k^i)$. The investment-income effect function, $f(s_k^i)$ is interpreted as a direct net operating income adjustment through altering operating costs or catch revenues. The particular form of $f(s_k^i)$ and its basis in empirical data is discussed later in Section 5.4.

The initial income effect of $f(s_k^i)$ in succeeding periods is also discounted by the annual factor ρ_i, i.e., the initial effect is added to the current earning power of the enterprise by updating y_t^i whenever a new investment takes place. In general then, $NI_t(s_k)$, the revised net operating income for $t = k+1, \dots, T$ which results from a new investment at the end of period k, is as follows

$$NI_t(s_k) = NI_{k+1}(0) + \sum_{i \in S} f(s_k^i) s_k^i \rho_i^{t-k-1} [NI_t - NI_L], \; t = k+1, \dots, T. \tag{16}$$

Equivalently, we define the recursive relationship

$$NI_t(s_k) = NI_L + \sum_{i \in S} y_t^i [NI_t - NI_L] \tag{17}$$

where

$$y_t^i = y_{t-1}^i \rho_i + f(s_{t-1}^i) s_{t-1}^i \tag{18}$$

The initial net operating income earning power values by type of capital, y_0^i, the discount factors, ρ_i, and the parameters of the investment-income effects function, $f(s_t^i)$ are derived from empirical data. Derivation procedures and the resulting parameter values are presented and discussed in Section 5.4 on the empirical development of the investment model.

This completes the definitions of the elements of NW_t as explicit functions of the investment decision variables s_k^i. We now examine the overall impact on NW_t as a result of new investment decisions.

Initially, let us assume that from period k with net worth NW_k (known) no new investments are made out to the end of the planning horizon. Thus, we have $s_t = 0$, for $t = k, k+1, \ldots, T-1$. $NW_t(0)$ may be written explicitly in terms of its components using (2) and (3) as follows

$$NW_t(0) = NW_k + \sum_{j=k+1}^{t} \{NI_j(0)(1-TR) - I_j(0) - RP_j(0)$$

$$+TR[D_j(0) + I_j(0)] + r(1-TR)[CA_{j-1}(0) - CL_{j-1}(0)]\}. \tag{19}$$

Equivalently, by considering the contribution to cash carryover from period to period as in (3) we may write

$$NW_t(0) = NW_k + \sum_{j=k+1}^{t} [1 + r(1-TR)]^{t-j} \{NI_j(0)(1-TR) - I_j(0) - RP_j(0)$$

$$+TR(D_j(0) + I_j(0))\} + ([1 + r(1-TR)]^{t-k} - 1)[CA_k(0) - CL_k(0)], \; t = k+1, \ldots, T. \tag{20}$$

With the exception of $NI_j(0)$, all the elements in this expression are known since they arise from the existing –and unchanging– capital configuration of the enterprise at $t = k$. We note that $NI_j(0)$ depends on the random variable $NI_t \in \mathrm{NI}$ and on the ongoing net operating income earning power of the fishing operation as in (15). The impact of this 'no investment' strategy on the terminal net worth of the enterprise, NW_T is

$$NW_T(0) = NW_k + \sum_{j=k+1}^{T} [1 + r(1-TR)]^{T-j} \{NI_j(0)(1-TR) - I_j(0) - RP_j(0)$$

$$+TR[D_j(0) + I_j(0)]\} + ([1 + r(1-TR)]^{T-k} - 1)[CA_k(0) - CL_k(0)]. \tag{21}$$

Similarly, we can write $NW_T(s_k)$, the net worth of the enterprise at period T when there is new investment at period k, $s_k > 0$, and $s_t = 0$, $t = k+1, \ldots, T-1$. Analogous to (21) we may write

$$NW_T(s_k) = NW_k + \sum_{j=k+1}^{T} [1 + r(1-TR)]^{T-j} \{NI_j(s_k)(1-TR) - I_j(s_k) - RP_j(s_k)$$

$$+TR(D_j(s_k) + I_j(s_k))\} + ([1 + r(1-TR)]^{T-k} - 1)[CA_k(0) - CL_k(0)]. \tag{22}$$

Now, writing $D_j(s_k)$, $I_j(s_k)$, $RP_j(s_k)$ and $NI_j(s_k)$ explicitly in terms of s_k from (9)–(11),(16) above, and simplifying yields the impact of new investment s_k on terminal net worth, NW_T:†

$$NW_T(s_k) = NW_T(0) + \sum_{j=k+1}^{T} [1 + r(1-TR)]^{T-j} \sum_{i \in S} s_k^i \{f(s_k^i)\rho_i^{j-k-1}[NI_j - NI_L](1-TR)$$

† N.B. For non–physical capital, depreciation for tax purposes is the straight–line method. For simplicity, throughout this analysis it is assumed that for these capital types the term $(1-c_i)^{j-k-1}$ (as in (23)) equals 1 for all j and k, and $c_i = 1/L_i$ as described in (9).

$$-\sum_{l}\beta_i(l)\frac{r_l}{1-(1+r_l)^{-a_l}}+TR\big[(1-c_i)^{j-k-1}c_i+\sum_{l}\beta_i(l)r_l\frac{1-(1+r_l)^{j-k-1-a_l}}{1-(1+r_l)^{-a_l}}\big]\} \tag{23}$$

The objective function of the investment problem is given by the single-valued objective of maximizing expected real net worth of the enterprise at the end of the planning horizon, i.e.,

$$\max_{s_k,\ k=0,1,\ldots,T-1} E\{NW_T(s_k)\}. \tag{24}$$

In the expression for $NW_T(s_k)$ in (23) above note that $NW_T(0)$ is a constant independent of the decision variables. Accordingly, the objective (24) is equivalent to maximizing the second term in the right-hand side of (23). Thus, we may consider as equivalent to (24) the maximization of the expected contribution to terminal net worth by each new investment at period k. Thus,

$$\max_{s_k,\ k=0,1,\ldots,T-1} E\{\sum_{k=0}^{T-1}\Delta NW_T(s_k)\} = \max_{s_k,\ k=0,1,\ldots,T-1}\sum_{k=0}^{T-1}E\{\Delta NW_T(s_k)\}. \tag{25}$$

Since $\Delta NW_T(s_k) = NW_T(s_k) - NW_T(0)$, then from (23)

$$\Delta NW_T(s_k) = \sum_{j=k+1}^{T}[1+r(1-TR)]^{T-j}\sum_{i\in S}s_k^i\{f(s_k^i)\rho_i^{j-k-1}[NI_j - NI_L](1-TR)$$

$$-\sum_{l}\beta_i(l)\frac{r_l}{1-(1+r_l)^{-a_l}}+TR\big[(1-c_i)^{j-k-1}c_i+\sum_{l}\beta_i(l)r_l\frac{1-(1+r_l)^{j-k-1-a_l}}{1-(1+r_l)^{-a_l}}\big]\}. \tag{26}$$

Simplifying, we may write

$$E\{\Delta NW_T(s_k)\} = \sum_{i\in S}s_k^i\big[f(s_k^i)v_T^i(k) - \gamma_T^i(k)\big],\ k=0,1,\ldots,T-1 \tag{27}$$

where $f(s_k^i)v_T^i(k)$ is the expected terminal benefit per unit of investment of type i, s_k^i out to the end of the planning period with

$$v_T^i(k) = \sum_{j=k+1}^{T}[1+r(1-TR)]^{T-j}\rho_i^{j-k-1}[E\{NI\} - NI_L](1-TR) \tag{28}$$

and $\gamma_T^i(k)$ is the deterministic unit cost out to the end of the planning period of financing the new investment of type i, s_k^i out to the end of the planning horizon with

$$\gamma_T^i(k) = \sum_{j=k+1}^{T}[1+r(1-TR)]^{T-j}\{\sum_{l}\beta_i(l)\frac{r_l}{1-(1+r_l)^{-a_l}}(1-TR(1-(1+r_l)^{j-k-1-a_l}))$$

$$-TR\big[(1-c_i)^{j-k-1}c_i\big]\} \tag{29}$$

The unconstrained investment model yields solutions which are independent of the actual economic state of the fishing enterprise. Simple optimal decision rules can be computed for all periods at the beginning of the planning horizon. The characterization of the solutions depends on the form of the investment-income effect function, $f(s_k^i)$. Two forms of this function are examined:

(i) **Linear Investment–Income Effect Model** – $f(s_k^i) = p^i$

(ii) **Nonlinear Investment–Income Effect Model** – $f(s_k^i) = q_0^i + q_1^i s_k^i$

(i) The linear form of the investment-income effect function is independent of s_k^i and consequently the solution to (25) above is the 'greedy' solution. For some k in the planning horizon the optimal investment s_k^{i*} is†

$$s_k^{i*} = \begin{cases} U^i, & \text{if } p^i v_T^i(k) - \gamma_T^i(k) > 0, \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

This form of the function presumes that the expected contribution to terminal net worth of a marginal unit of investment intype i capital at time k is constant. Thus, when this contribution is positive, the decision maker invests up to the maximum amount, U^i.

(ii) The nonlinear form of the investment-income effect function assigns a decreasing expected contribution to terminal net worth of the marginal unit of investment in each type of capital. This notion is analogous to the economic concept of diminishing returns. The form of the optimal investment in each period in this case is found by maximizing (25). First order necessary conditions result in the optimal investment value s_k^{i*} with

$$s_k^{i*} = \frac{\gamma_T^i(k) - q_0^i v_T^i(k)}{2q_1^i v_T^i(k)}, \quad i = 1, 2, \ldots, N_S; \text{ and } k = 0, 1, \ldots, T-1 \tag{31}$$

where $v_T^i(k)$ is the normalized expected benefit to Net Worth at time T due to new investment s_k^i (as in (28)). For the nonlinear model s_k^i is unrestricted with respect to sign. Thus, $s_k^i < 0$ implies that a disposal of existing assets of type i take place. In this analysis, we restrict ourselves to the case where $s_k^i \geq 0$ and allow disposals to take place only in conjunction with a net positive new investment through the B matrix of financing parameters. This restriction is consistent with the thin trading of the market for fishing capital.

† It is assumed that the event $p^i v_T^i(k) = \gamma_T^i(k)$ does not invoke a positive investment. In fact, the choice of s_k^i is immaterial in this unique situation.

5.2.2 Survivability Constrained Model. The dynamic model described above determines the optimal investment portfolio for a rational decision maker who desires to maximize the expected contribution to terminal net worth. As such, the model is independent of the limitations to investment imposed by lending institutions and by investors themselves due to high income variability. We now incorporate these limitations by explicitly considering the risk of bankruptcy, $NW_t \leq 0$ in which each investment places the decision maker. Specifically, we introduce the risk of survival of the enterprise over the planning horizon into the investment decision process. Risk of survivability is quantified by the condition that Net Worth will 'most likely' exceed zero for all interim periods of the planning horizon. This restriction is included as a constraint in the dynamic model for survival.

The survivability constraint is defined by

$$P\{NW_t \leq 0\} \leq p_0 \text{ for all } t \tag{32}$$

where p_0 is given exogenously and related to the risk nature of the individual fishermen. The most restrictive case has $p_0 = 0$. In this case, investments will not be undertaken if there is the slightest possibility that insolvency ($NW_t \leq 0$) may occur at any time as a result of the investment. At the opposite extreme (when p_0 approaches 1) then all investments which have a positive expected contribution to Net Worth will be undertaken regardless of the ensuing risk of survival in each period. This second case reduces to the unconstrained investment decision making model discussed in the previous subsection (5.2.1).

The survival or solvency condition, $NW_t > 0$ can be written in terms of the random variable NI by expanding NW_t in (32) using (23) and (18),(19),(28),(29). Thus, $NW_t > 0$ is equivalent to

$$NI_t > \frac{1}{(1-TR)}\left(\frac{\sum_{i\in S} s_k^i[\gamma_t^i(k) - g_{t-1}^i(k)] - w_{t-1} + C_t + H_t}{\sum_{i\in S}(y_k^i \rho_i^{t-k} + f(s_k^i)\rho_i^{t-k-1}s_k^i)}\right), \; t = k+1, \ldots, T \tag{33}$$

where $g_k^i(k) = 0$, $C_t = I_t(0) + RP_t(0) - TR[D_t(0) + I_t(0)]$ and

$$H_t = (1-TR)NI_L[\sum_{i\in S} \rho_i^{t-k-1}(y_k^i \rho_i + f(s_k^i)s_k^i) - 1].$$

- The term w_t is defined as the net worth of the enterprise at period t after realizing the lowest level of net operating income anticipated under the survival constraint (32) for all periods $t = k+1, \ldots, T$ and assuming no investment at the end of period k and for all periods to the end of the planning horizon. The lowest level of net operating income anticipated under (32) for a given value of p_0 is determined by taking the inverse of the probability distribution function, F_{NI}, i.e., $F_{NI}^{-1}(p_0)$. Given the net worth of the enterprise at the time of investment, NW_k, the cash balance $CA_k(0) - CL_k(0)$, and $F_{NI}^{-1}(p_0)$, then we may write w_t using (17) and (22) as follows

$$w_t = NW_k + \sum_{j=k+1}^{t} [1 + r(1-TR)]^{t-j}\{(1-TR)[(F_{NI}^{-1}\{p_0\} - NI_L)\sum_{i\in S} \rho_i^{j-k} y_k^i + NI_L]$$

$$-C_j\} + ([1 + r(1 - TR)]^{t-k} - 1)[CA_k(0) - CL_k(0)] \tag{34}$$

- The term $g_t^i(k)$ is defined as the sum of the after-tax benefits from new investments s_k^i when the minimum level of net operating income anticipated under the survival constraint, $F_{NI}^{-1}\{p_0\}$ is realized for all periods, t of the planning horizon following period k, $t = k+1, \ldots, T$. Formally,

$$g_t^i(k) = \sum_{j=k+1}^{t} [1 + r(1 - TR)]^{t-j} f(s_k^i)\rho_i^{j-k-1}[F_{NI}^{-1}\{p_0\} - NI_L](1 - TR). \tag{35}$$

The expression (33) describes the most conservative relationship between NI_t and s_k given p_0. In other words, the decision maker visualizes the lowest anticipated result for the random variable, NI occurring in the future in which to construct a survivable investment policy for the planning horizon.

Now, from (33) above, and taking probabilities, we may write

$$\Pr\{NW_t(s_k) \le 0\} = \Pr\left\{ NI_t \le \frac{1}{(1-TR)} \left(\frac{\sum_{i\in S} s_k^i[\gamma_t^i(k) - g_{t-1}^i(k)] - w_{t-1} + C_t + H_t}{\sum_{i\in S}(y_k^i\rho_i^{t-k} + f(s_k^i)\rho_i^{t-k-1}s_k^i)} \right) \right\} \tag{36}$$

Using the survivability constraint (32) we have that

$$\Pr\left\{ NI_t \le \frac{1}{(1-TR)} \left(\frac{\sum_{i\in S} s_k^i[\gamma_t^i(k) - g_{t-1}^i(k)] - w_{t-1} + C_t + H_t}{\sum_{i\in S}(y_k^i\rho_i^{t-k} + f(s_k^i)\rho_i^{t-k-1}s_k^i)} \right) \right\} = F_{NI}\{\cdot\} \le p_0. \tag{37}$$

The above condition (37) can be written explicitly in terms of the s_k^i by taking the inverse of the probability functions, thus

$$\frac{1}{(1-TR)} \left(\frac{\sum_{i\in S} s_k^i[\gamma_t^i(k) - g_{t-1}^i(k)] - w_{t-1} + C_t + H_t}{\sum_{i\in S}(y_k^i\rho_i^{t-k} + f(s_k^i)\rho_i^{t-k-1}s_k^i)} \right) \le F_{NI}^{-1}\{p_0\}. \tag{38}$$

Clearing the denominator and simplifying using (34) and (35) we obtain

$$\sum_{i\in S} s_k^i[\gamma_t^i(k) - g_t^i(k)] \le w_t \tag{39}$$

which is an alternate form of the original survivability constraint (32) written in terms of the decision variables and assuming the worst anticipated realizations of the random variable NI_t. Equivalently, we write (39) as follows

$$\sum_{i\in S} s_k^i A_t^i(k) \le B_t(k) + NW_k + \sum_{i\in S} G_t^i(k) y_k^i + CB_t(k), \ t = k+1, \ldots, T \tag{40}$$

where

- $G_t^i(k)y_k^i$ – analogous to (28), this is the sum of the after-tax benefit derived from existing capital of type i at period k when the minimum level of net operating income anticipated under the survivability constraint, $F_{NI}^{-1}\{p_0\}$ is realized for all periods from $k+1$ to t and

$$G_t^i(k) = \sum_{j=k+1}^{t} [1 + r(1 - TR)]^{t-j} \rho_i^{j-k} [F_{NI}^{-1}\{p_0\} - NI_L](1 - TR) \tag{41}$$

- $CB_t(k)$ – the total interest amount carried forward to period t on the existing cash on hand at the end of period k, and

$$CB_t(k) = ([1 + r(1 - TR)]^{t-k} - 1)[CA_k(0) - CL_k(0)] \tag{42}$$

- $A_t^i(k)$ – the net negative contribution to NW_t per unit of new investment, s_k^i when the net operating income realized is the lowest anticipated level from period $k+1$ to t, and

$$A_t^i(k) = \gamma_t^i(k) - g_t^i(k) \tag{43}$$

- $B_t(k)$ – the contribution to NW_t from all outstanding liabilities, C_j from period $j = k+1$ to $j = t$ including the net income shift factor, NI_L, and

$$B_t(k) = \sum_{j=k+1}^{t} [1 + r(1 - TR)]^{t-j} [NI_L(1 - TR) - C_j] \tag{44}$$

The right hand side of (40) may be interpreted as the net contribution to NW_t visualizing the worst anticipated scenario independent of the new investment s_k. $A_t^i(k)$ may be interpreted as the unit cost of new investment s_k^i visualizing the worst anticipated income scenario. Thus, (40) implies that if the benefits exceed the costs when the lowest anticipated level of net operating income is realized then the investment is deemed to be '$1 - p_0$ survivable.' This is expressed formally in the following lemma.

Lemma 1. *s_k is a '$1 - p_0$ survivable' investment if and only if (40) is true, i.e.,*

$$\sum_{i \in S} s_k^i A_t^i(k) \leq B_t(k) + NW_k + \sum_{i \in S} G_t^i(k) y_k^i + CB_t(k), \quad t = k+1, \ldots, T.$$

Proof. (By contradiction.) Assume that s_k is not $1 - p_0$ survivable if $\sum_{i \in S} s_k^i A_t^i(k) \leq w_t$, then

$$\Pr\{NW_T > 0\} < 1 - p_0, \quad t = k+1, \ldots, T.$$

Equivalently,

$$\Pr\{NW_T \leq 0\} > p_0, \quad t = k+1, \ldots, T.$$

From (37),

$$\Pr\left\{NI_t \le \frac{1}{(1-TR)}\left(\frac{\sum_{i\in S} s_k^i\left[\gamma_t^i(k) - g_{t-1}^i(k)\right] - w_{t-1} + C_t + H_t}{\sum_{i\in S}(y_k^i \rho_i^{t-k} + f(s_k^i)\rho_i^{t-k-1} s_k^i)}\right)\right\} > p_0,\ t = k+1,\ldots,T$$

$$\frac{1}{(1-TR)}\left(\frac{\sum_{i\in S} s_k^i\left[\gamma_t^i(k) - g_{t-1}^i(k)\right] - w_{t-1} + C_t + H_t}{\sum_{i\in S}(y_k^i \rho_i^{t-k} + f(s_k^i)\rho_i^{t-k-1} s_k^i)}\right) > F_{NI}^{-1}\{p_0\},\ t = k+1,\ldots,T.$$

Equivalently from (39),

$$\sum_{i\in S} s_k^i\left[\gamma_t^i(k) - g_t^i(k)\right] > w_t,\ t = k+1,\ldots,T.$$

Or,

$$\sum_{i\in S} s_k^i A_t^i(k) > B_t(k) + NW_k + \sum_{i\in S} G_t^i(k) y_k^i + CB_t(k),\ t = k+1,\ldots,T$$

which is a contradiction. Thus, s_k is $1 - p_0$ survivable.

Assume that

$$\sum_{i\in S} s_k^i A_t^i(k) > B_t(k) + NW_k + \sum_{i\in S} G_t^i(k) y_k^i + CB_t(k),\ t = k+1,\ldots,T.$$

Then,

$$\sum_{i\in S} s_k^i\left[\gamma_t^i(k) - g_t^i(k)\right] > w_t,\ t = k+1,\ldots,T.$$

Equivalently,

$$\frac{1}{(1-TR)}\left(\frac{\sum_{i\in S} s_k^i\left[\gamma_t^i(k) - g_{t-1}^i(k)\right] - w_{t-1} + C_t + H_t}{\sum_{i\in S}(y_k^i \rho_i^{t-k} + f(s_k^i)\rho_i^{t-k-1} s_k^i)}\right) > F_{NI}^{-1}\{p_0\},\ t = k+1,\ldots,T.$$

From (37),

$$\Pr\left\{NI_t \le \frac{1}{(1-TR)}\left(\frac{\sum_{i\in S} s_k^i\left[\gamma_t^i(k) - g_{t-1}^i(k)\right] - w_{t-1} + C_t + H_t}{\sum_{i\in S}(y_k^i \rho_i^{t-k} + f(s_k^i)\rho_i^{t-k-1} s_k^i)}\right)\right\} > p_0,\ t = k+1,\ldots,T.$$

Finally,

$$\Pr\{NW_T \le 0\} > p_0,\ t = k+1,\ldots,T$$

which is equivalent to

$$\Pr\{NW_T > 0\} < 1 - p_0,\ t = k+1,\ldots,T.$$

But, this is a contradiction since s_k is $1 - p_0$ survivable. Therefore,

$$\sum_{i\in S} s_k^i A_t^i(k) \le B_t(k) + NW_k + \sum_{i\in S} G_t^i(k) y_k^i + CB_t(k),\ t = k+1,\ldots,T$$

as required. ■

Examination of the benefit series of the right hand side of (35) and of the cost series (29) provides insight into the dynamic behavior of the unit cost of new investments, $A_t^i(k), t = k+1, \ldots, T$. Under two reasonable conditions the following lemma proves $A_t^i(k)$ to be greater than zero for all t and k. The first condition requires that the longest financing term on debt instruments, a_l be at least as long as the planning horizon, T. In other words, T is less than or equal to the term of the longest financing instrument (i.e., long–term debt). Moreover, it is natural to assume that fishermen's planning periods are determined by their financial obligations in the future. The second condition assumes that in the first year of a new investment in type i capital, the per dollar (after tax) financing cost (interest plus repayment less depreciation) exceeds the per dollar increase in after tax adjusted net operating income assuming the worst anticipated scenario is realized. For reasonable levels of p_0 (e.g., less than 0.2) this assumption is a logical one to make.

Lemma 2. *Under the condition* $a_l \geq T - k - 1$ *for at least one* $l \in \{CL, LTL\}$ *and*

$$\beta_i(l)\frac{r_l}{1-(1+r_l)^{-a_l}}(1 - TR(1-(1+r_l)^{-a_l})) - TRc_i > f(s_k^i)[F_{NI}^{-1}\{p_0\} - NI_L](1-TR),$$

for all $l = CL, LTL$, *then*

$$A_t^i(k) > 0,\ i = 1, \ldots, N_S;\ k = 0, 1, \ldots, T-1;\ t = k+1, \ldots, T.$$

Proof. (By induction and contradiction.) For $t = k+1$ and some i assume

$$A_{k+1}^i(k) = \gamma_{k+1}^i(k) - g_{k+1}^i(k) \leq 0$$

then $\gamma_{k+1}^i(k) \leq g_{k+1}^i(k)$. Expanding using (29) and (35) yields

$$\sum_l \beta_i(l)\frac{r_l}{1-(1+r_l)^{-a_l}}(1 - TR(1-(1+r_l)^{-a_l})) - TRc_i \leq f(s_k^i)[F_{NI}^{-1}\{p_0\} - NI_L](1-TR)$$

which violates the stated condition. Thus, $A_t^i(k) > 0,\ i = 1, \ldots, N_S;\ k = 0, 1, \ldots, T-1;\ t = k+1$.

Now, suppose that $A_n^i(k) > 0$ for $k+1 < n < T$, and for some i assume

$$A_{n+1}^i(k) = \gamma_{n+1}^i(k) - g_{n+1}^i(k) \leq 0$$

then

$$\sum_l \beta_i(l)\frac{r_l}{1-(1+r_l)^{-a_l}}(1 - TR(1-(1+r_l)^{n-k-a_l})) - TR(1-c_i)^{n-k}c_i$$
$$-f(s_k^i)\rho^{n-k}[F_{NI}^{-1}\{p_0\} - NI_L](1-TR) + (1+r(1-TR))(\gamma_n^i(k) - g_n^i(k)) \leq 0.$$

Assuming $a_l > n-k$ for at least one l, and $c_i and \rho_i \leq 1$ then

$$\sum_l \beta_i(l)\frac{r_l}{1-(1+r_l)^{-a_l}}(1 - TR(1-(1+r_l)^{n-k-a_l})) - TR(1-c_i)^{n-k}c_i > \gamma_{k+1}^i(k)$$

and

$$-f(s_k^i)\rho^{n-k}[F_{NI}^{-1}\{p_0\} - NI_L](1-TR) < g_{k+1}^i(k).$$

Accordingly we may write

$$\gamma_{k+1}^i(k) - g_{k+1}^i(k) + (1 + r(1-TR))(\gamma_n^i(k) - g_n^i(k)) \le 0$$

which cannot be true since $A_{k+1}^i > 0$ and $A_n^i > 0$. Therefore, $A_{n+1}^i(k) > 0,\ k+1 < n < T$. ■

From Lemma 1 we know that s_k is $1-p_0$ survivable if and only if the survivability condition holds for all intermediate periods from $k+1$ to T. Now, if it can be demonstrated that $A_t^i(k)$ is nondecreasing as t increases, and that the right hand side of the Lemma 1 inequality is nonincreasing as t increases, then the survivability constraint (40) can be simplified. Under these conditions, the survivability constraint at the terminal period of the planning horizon, $t = T$ is a dominant restriction which guarantees $1-p_0$ survivability at each intermediate period from $k+1$ to T. The following lemma proves the desired result.

Lemma 3. *Under the condition that*

$$F_{NI}^{-1}(p_0) < \frac{1}{\sum_{i\in S} y_{t-1}^i \rho_i (1-TR)} \min\{NI_L[\sum_{i\in S} y_{t-1}^i \rho_i - 1](1-TR) + C_t,$$

$$NI_L[\sum_{i\in S} y_{t-1}^i \rho_i - 1](1-TR) + C_t - r(1-TR)[CA_k(0) - CL_k(0)]\}$$

for all t, then s_k is '$1-p_0$ survivable' if and only if

$$\sum_{i\in S} s_k^i A_T^i(k) \le B_T(k) + NW_k + \sum_{i\in S} G_T^i(k) y_k^i + CB_T(k),\ k = 0, 1, \ldots, T-1.$$

Proof. (By contradiction.) Assume that $A_{t+1}^i(k) < A_t^i(k),\ t = k+1, \ldots, T-1$, then

$$\gamma_{t+1}^i(k) - g_{t+1}^i(k) < \gamma_t^i(k) - g_t^i(k).$$

Equivalently,

$$\sum_l \beta_i(l) \frac{r_l}{1-(1+r_l)^{-a_l}} (1 - TR(1 - (1+r_l)^{t-k-a_l})) - TR(1-c_i)^{t-k} c_i$$

$$-f(s_k^i)\rho^{t-k}[F_{NI}^{-1}\{p_0\} - NI_L](1-TR) + (1 + r(1-TR))(\gamma_t^i(k) - g_t^i(k)) < \gamma_t^i(k) - g_t^i(k).$$

Since $a_l > t-k$ by assumption, and $c_i, \rho_i < 1$ then

$$\sum_l \beta_i(l) \frac{r_l}{1-(1+r_l)^{-a_l}} (1 - TR(1 - (1+r_l)^{t-k-a_l})) - TR(1-c_i)^{t-k} c_i > \gamma_{k+1}^i(k)$$

and

$$f(s_k^i)\rho^{t-k}[F_{NI}^{-1}\{p_0\} - NI_L](1-TR) < g_{k+1}^i(k).$$

Accordingly we may write

$$\gamma_{k+1}^i(k) - g_{k+1}^i(k) + (1 + r(1-TR))(\gamma_t^i(k) - g_t^i(k)) < \gamma_t^i(k) - g_t^i(k)$$

which cannot be true since $A_t^i > 0$ for all t by Lemma 2. Therefore, $A_{t+1}^i(k) \geq A_t^i(k)$, $k+1 < t < T$ and $A_t^i(k)$ is nondecreasing as t increases.

Assume $w_{k+1} > w_k$, then from (39) and (40),

$$B_{k+1}(k) + NW_k + \sum_{i \in S} G_{k+1}^i(k) y_k^i + CB_{k+1}(k) > NW_k.$$

Simplifying and expanding using (41)-(44) we obtain

$$NI_L(1-TR) + \sum_{i \in S} y_{t-1}^i \rho_i [F_{NI}^{-1}(p_0) - NI_L](1-TR) - C_{k+1} + r(1-TR)[CA_k(0) - CL_k(0)] > 0.$$

Or,

$$\sum_{i \in S} y_{t-1}^i \rho_i F_{NI}^{-1}(p_0)(1-TR) > NI_L[\sum_{i \in S} y_{t-1}^i \rho_i - 1](1-TR) + C_{k+1} - r(1-TR)[CA_k(0) - CL_k(0)]$$

which contradicts the condition. Therefore, $w_{k+1} \leq w_k$.

Now, assume $w_n \leq w_{n-1} \leq \cdots \leq w_k$, $k+1 < n < T$, and assume that $w_{n+1} > w_n$ then from (39) and (40),

$$B_{n+1}(k) - B_n(k) + \sum_{i \in S} y_k^i (G_{n+1}^i(k) - G_n^i(k)) + CB_{n+1}(k) - CB_n(k) > 0.$$

Expanding using (41)-(44) yields

$$NI_L(1-TR) + \sum_{i \in S} y_{t-1}^i \rho_i^{n+1-k} [F_{NI}^{-1}(p_0) - NI_L](1-TR)$$

$$-C_{n+1} + r(1-TR)[B_n(k) + \sum_{i \in S} y_k^i G_n^i(k) + CB_n(k)] > 0.$$

Or,

$$\sum_{i \in S} y_{t-1}^i \rho_i^{n+1-k} F_{NI}^{-1}(p_0)(1-TR) > NI_L[\sum_{i \in S} y_{t-1}^i \rho_i^{n+1-k} - 1](1-TR) + C_{n+1} - r(1-TR)[w_n(0) - w_k(0)]$$

which contradicts the condition since $w_n \leq w_k$. Therefore, $w_{n+1} \leq w_n$, $k+1 < n < T$ and so the right-hand side of the Lemma 1 inequality is nonincreasing as t increases. ■

The dynamic programming model with survivability constraints may now be stated formally. The problem may be formulated as a constrained discrete time probabilistic dynamic program. In this framework we use the following standard notation (see Bertsekas(1976),pp.28ff):

- Let $(NW_t, y_t, d_t, rp_t, z_t, cb_t)$ be the state vector of the system at time t, $t = 0, 1, \ldots, T-1$.
- As a consequence of the recursive relationships defined in (9),(12),(13),(18), and (23) for depreciation, interest expense and repayment, repayment of principal on loans, net operating income, and net worth respectively, then the state dynamics equations may be written as follows

$$\begin{aligned}
\text{Net Worth}: \quad NW_t &= NW_{t-1} + NI_L(1-TR) + \sum_{i=1}^{N_S}\{y_t^i[NI_t - NI_L](1-TR) \\
&\quad -z_t^i + TR[d_t^i c_i + z_t^i - \sum_l rp_t^i(l)]\} + cb_t - C_t \qquad (45) \\
\text{Income Effects}: \quad y_t^i &= y_{t-1}^i \rho_i + s_{t-1}^i f(s_{t-1}^i) \\
\text{Depreciation}: \quad d_t^i &= d_{t-1}^i(1-c_i) + s_{t-1}^i \\
\text{Interest Plus Repayment}: \quad z_t^i &= z_{t-1}^i + s_{t-1}^i \sum_l \beta_i(l) \frac{r_l}{1-(1+r_l)^{-a_l}} \\
\text{Repayment}: \quad rp_t^i(l) &= rp_{t-1}^i(l)(1+r_l) + s_{t-1}^i \beta_i(l) \frac{r_l(1+r_l)^{-a_l}}{1-(1+r_l)^{-a_l}} \\
\text{Cash Balance}: \quad cb_t &= r(1-TR)(NW_{t-1} - NW_{t-2})
\end{aligned}$$

  where cb_t is the interest earned or expensed to the end of period t on the cash balance at the end of period $t-1$. The decision or control variables are $s_t^i, i \in S$ and $NI_t \in$ NI , the random variable of the system. NI_t is characterized by the probability distribution function, F_{NI}. NI_t is defined as the net operating income for a vessel of average configuration in period t. NI_t is independently and identically distributed for all t, $t = 0, 1, \ldots, T$.

- The objective of this problem is to find the feasible investment decision rule, $\pi = \{s_0, s_1, \ldots, s_{T-1}\}$ which maximizes the reward functional

$$J^\pi(NW_0, y_0, d_0, rp_0, z_0, cb_0) = \max E\{\sum_{t=0}^{T-1} \Delta NW_T(NW_t, y_t, d_t, rp_t, z_t, cb_t)\} \qquad (46)$$

  subject to the survivability constraint (40) and the state dynamics (45). Figure 8 presents a schematic of the dynamic investment decision process.

The following sequence of events takes place once an investment decision rule π has been established:

Stage t, $t = 0, 1, \ldots, T-1$

START OF STAGE

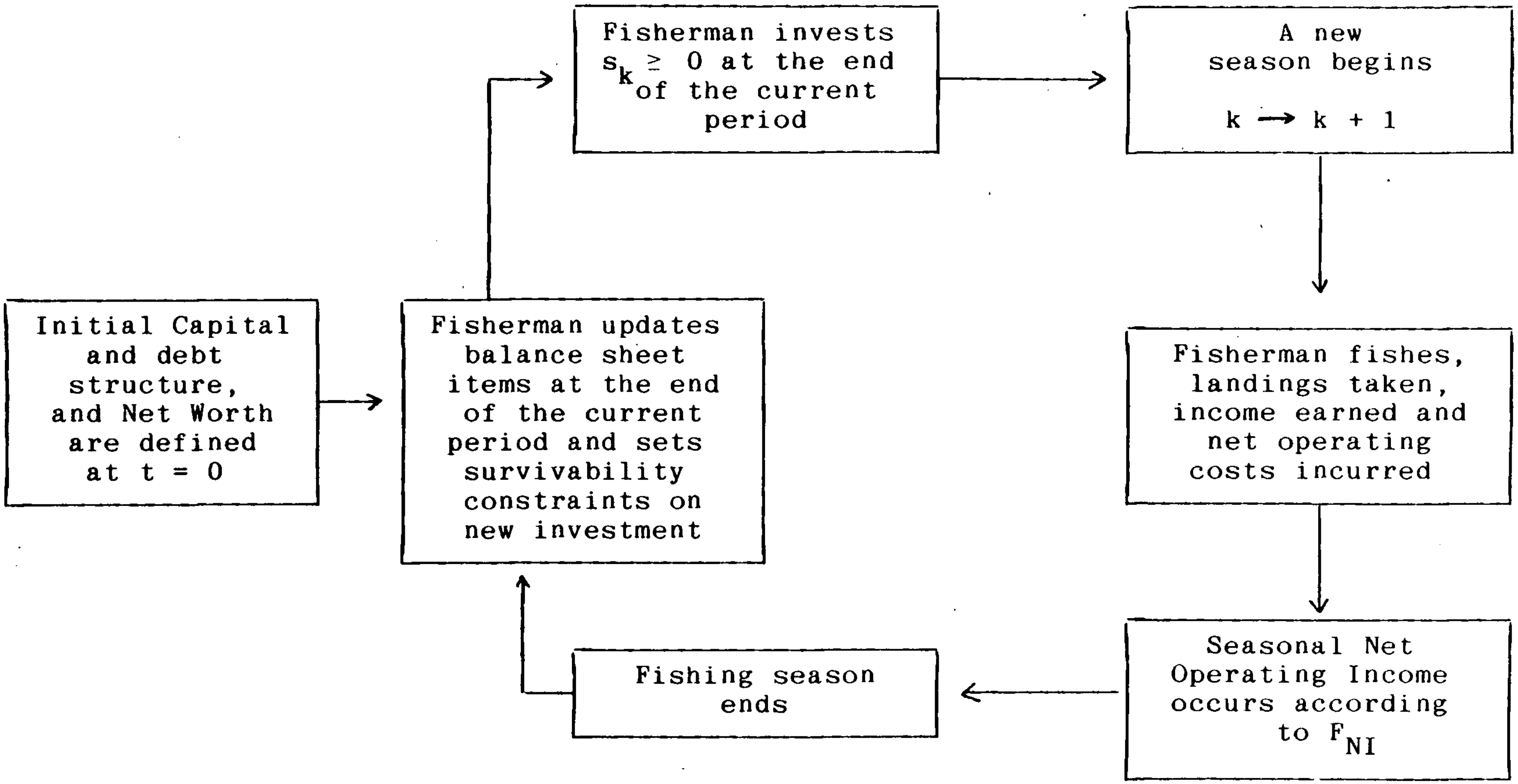


Figure 8 - The Interseasonal Decision Process

Figure 8 - The Interseasonal Decision Process

1) Given NI_t, the decision maker updates the balance sheet, NW_t. Limitations on new investment s_t^i due to the survivability constraints derived from p_0 are established.

2) The decision maker invests s_t according to the control law π previously determined and the constraints on each s_t^i.

3) Fishing season $t+1$ begins, landings are taken, income earned and operating costs incurred through the realization of the random variable NI_t.

4) The fishing season ends; $t \longleftarrow t+1$. Return to Step 1 if $t < T$ else STOP.

END OF STAGE

The solution procedure for the dynamic program is carried out using backward recursion. The recursion is written explicitly as follows:

$$J_k^{\pi}(NW_k, y_k, d_k, rp_k, z_k, cb_k) = \max_{s_k^i} E\Big\{\sum_{i\in S} s_k^i\Big(\sum_{j=k+1}^{T}[1+r(1-TR)]^{T-j}\cdot$$

$$f(s_k^i)\rho_i^{j-k-1}[NI_j - NI_L](1-TR) - \gamma_T^i(k)\Big) + J_{k+1}^{\pi}(NW_k, y_k, d_k, rp_k, z_k, cb_k)\Big\},\ k = 0, 1, \ldots, T-1 \quad (47)$$

and $J_T^{\pi}(\cdot) = 0$. Explicit solutions to the problem with survivability constraints may be derived for each of the investment-income effect functions $f(s_k^i)$ described earlier. The following paragraphs provide the basis for the development of algorithms which may be used to determine dynamic investment policy solutions.

(i) Linear Model. The first subproblem in the backward recursion of the dynamic program (47) considers the last investment decision in the planning period. This problem is expressed by

$$J_{T-1}^{\pi} = \max_{s_{T-1}^i} E\Big\{\sum_{i\in S} s_{T-1}^i\big(p^i[NI_T - NI_L](1-TR) - \gamma_T^i(T-1)\big)\Big\} \quad (P_1)$$

subject to the survivability constraint for the last period,

$$\sum_{i\in S} s_{T-1}^i A_T^i(T-1) \le B_T(T-1) + NW_{T-1} + \sum_{i\in S} G_T^i(T-1) y_{T-1}^i + CB_T(T-1)$$

$$s_{T-1}^i \ge 0,\ i = 1, \ldots, N_S.$$

The problem P_1 is characterized by a linear objective function and a single linear constraint in the decision variables $s_{T-1}^i, i = 1, \ldots, N_S$. Its solution is determined by first calculating the N_S values for the ratios of the objective function coefficient to constraint coefficient for each decision variable, s_{T-1}^i. The solution assigns the maximum amount to the decision variable with the highest ratio value. All other decision variables are assigned a value of zero. This solution to the linear program (commonly referred to as the knapsack problem) is known as the 'greedy solution'. Formally, letting $u^i_{T-1} = p^i[E\{NI_T\} - NI_L](1-TR) - \gamma^i_T(T-1)$, then the first subproblem may be written as

$$J^\pi_{T-1} = \max \sum_{i \in S} s^i_{T-1} u^i_{T-1} \tag{P_2}$$

subject to the survivability constraint from Lemma 3:

$$\sum_{i \in S} s^i_{T-1} A^i_T(T-1) \le B_T(T-1) + NW_{T-1} + \sum_{i \in S} G^i_T(T-1) y^i_{T-1} + CB_T(T-1)$$

$$s^i_{T-1} \ge 0, \; i = 1, \ldots, N_S.$$

Assuming that investments of any capital type i may be arbitrarily large, then for $k = T-1$ the solution to this problem is to invest only in type j capital where

$$j = \arg\max_i \{ \frac{u^i_k}{A^i_T(k)} \} \tag{48}$$

and the size of the investment is

$$s^{j*}_k = \begin{cases} \frac{B_T(k) + NW_k + \sum_{i \in S} G^i_T(k) y^i_k + CB_T(k)}{A^j_T(k)}, & \text{if } u^j_k > 0; \\ 0, & \text{otherwise.} \end{cases} \tag{49}$$

Also $s^{i*}_k = 0, i \ne j$, with

$$u^*_k = \begin{cases} u^j_k, & \text{if } s^i_k > 0; \\ 0, & \text{if } s^i_k = 0 \end{cases} \tag{50}$$

and $A^*_T(k) = A^j_T(k)$. Thus $J^\pi_{T-1} = s^{j*}_{T-1} u^*_{T-1}$.

The next subproblem in the recursion (47) may now be written as follows

$$J^\pi_{T-2} = \max_{s^i_{T-2}} E\Big\{ \sum_{i \in S} s^i_{T-2} \Big(\sum_{j=T-1}^{T} [1 + r(1-TR)]^{T-j} \cdot$$

$$p^i \rho_i^{j-T+1} [NI_j - NI_L](1-TR) - \gamma^i_T(T-2) \Big) + s^{j*}_{T-1} u^*_{T-1} \Big\} \tag{P_3}$$

subject to the survivability constraint,

$$\sum_{i \in S} s^i_{T-2} A^i_T(T-2) \le B_T(T-2) + NW_{T-2} + \sum_{i \in S} G^i_T(T-2) y^i_{T-2} + CB_T(T-2)$$

$$s^i_{T-2} \ge 0, \; i = 1, \ldots, N_S.$$

Now, since s^{j*}_{T-1} (49) is a function of the state variables then P_3 may be rewritten in terms of the state dynamics equations (45) and $s^i_{T-2}, i = 1, \ldots, N_S$. Moreover, because u^*_{T-1} is a constant and s^{j*}_{T-1} is everywhere (in (45)) a linear function in s^i_{T-2} then this subproblem can be written equivalently as

$$J^\pi_{T-2} = \max \sum_{i \in S} s^i_{T-2} u^i_{T-2} \tag{P_4}$$

where

$$u^i_{T-2} = v^i_T(T-2)p^i - \gamma^i_T(T-2) + \frac{u^*_{T-1}}{A^*_T(T-1)}\{[1+r(1-TR)]^2[v^i_T(T-1)p^i - \gamma^i_T(T-1)] + G^i_T(T-1)p^i\}$$

subject to the survivability constraint,

$$\sum_{i\in S} s^i_{T-2}A^i_T(T-2) \le B_T(T-2) + NW_{T-2} + \sum_{i\in S} G^i_T(T-2)y^i_{T-2} + CB_T(T-2)$$

$$s^i_{T-2} \ge 0,\ i = 1,\ldots,N_S.$$

The form of the problem P_4 is exactly analogous to that of P_2, the knapsack problem. At this point it seems intuitive to suppose that the form of subsequent subproblems in the backward recursion yields similar separable knapsack problems. This idea provides the basis for the following theorem.

Theorem. *Optimal solutions for the reward functional J^π_k for the linear investment-income effects function are found by solving at each stage k, $k = 0,1,\ldots,T-1$ the problem*

$$J^\pi_k = \max_{s^i_k}\{\sum_{i\in S} s^i_k u^i_k\},\ k = 0,1,\ldots,T-1 \qquad (P_5)$$

where

$$u^i_k = v^i_T(k)p^i - \gamma^i_T(k) +$$
$$\frac{u^*_{k+1}}{A^*_T(k+1)}\{[1+r(1-TR)]^{T-k}[v^i_T(T-1)p^i - \gamma^i_T(T-1)] + G^i_T(k+1)p^i\} \qquad (51)$$

with $u^*_T = 0$.

Proof.(By induction.) For $k = T-1$, J^π_{T-1} is given by problem P_1 above which has the equivalent and required form P_2. For $k = T-2$, J^π_{T-2} is given by problem P_3 above which simplifies to the required form P_4. Now, suppose J^π_{k+1} has the required form P_5 for $0 < k < T-1$. Then from the dynamic programming recursion (47), J^π_k is written as

$$J^\pi_k = \max_{s^i_k} E\Big\{\sum_{i\in S} s^i_k\big(\sum_{j=k+1}^{T}[1+r(1-TR)]^{T-j}p^i\rho_i^{j-k-1}[NI_j - NI_L](1-TR) - \gamma^i_T(k)\big) + J^\pi_{k+1}\Big\} \qquad (52)$$

where $J^\pi_{k+1} = s^{j*}_{k+1}u^*_{k+1}$. Since u^*_{k+1} is a constant and s^{j*}_{k+1} is a linear function of state variables (49), then s^{j*}_{k+1} may be written in terms of $NW_k, y^i_k, d^i_k, z^i_k, rp^i_k(l), i = 1,\ldots,N_S$ and cb_k, and the decision variables $s^i_k, i = 1,\ldots,N_S$ using the state dynamics equations (45). Now, since all equations (45) are linear expressions in s^i_k then s^{j*}_{k+1} can be expressed in the general linear form:

$$s^{j*}_{k+1} = \alpha_0 + \alpha_1 NW_k + \sum_i(\alpha^i_2 y^i_k + \alpha^i_3 d^i_k + \alpha^i_4 z^i_k + \sum_l \alpha^{i,l}_5 rp^i_k(l)) + \alpha_6 cb_k + \sum_i \alpha^i_7 s^i_k \qquad (52a)$$

Substitution of this expression into (52) yields

$$J_k^\pi = \max_{s_k^i}\left\{C + \sum_{i\in S} s_k^i\Big(\sum_{j=k+1}^{T}[1+r(1-TR)]^{T-j}p^i\rho_i^{j-k-1}[NI_j - NI_L](1-TR) - \gamma_T^i(k) + \alpha_7^i\Big)\right\} \quad (52b)$$

where C is a constant independent of the s_k^i and the multiplier of s_k^i in (52b) (in rounded brackets) is the constant u_k^i. Equivalently, for the purposes of maximizing with respect to $s_k^i, i = 1, \ldots, N_S$, J_k^π can be written as

$$J_k^\pi = \max_{s_k^i}\{\sum_{i\in S} s_k^i u_k^i\}$$

which is the required form. ∎

The above theorem and proof depends on the assumption that investments of any capital type may be arbitrarily large. This implies that investments which are optimal to the linear program at each stage will be in one type of capital only. This assumption excludes the case where feasible investments by type have explicit upper bound constraints, i.e., $s_k^i \leq U^i$. In this case, the solution to the linear program (where the upper bound constraints are binding) may result in investments in more than one type of capital at each stage. When the dynamic programming model explicitly considers the N_S upper bound constraints, then a maximum of $N_S + 1$ different solutions may be characterized at each stage of the problem. These solutions include all the possibilities that the upper bound constraints are binding or not binding. Each of the solution forms must be recorded and carried forward in the backward recursion since it cannot be known immediately which characterization is valid at each stage. For long planning horizon problems this procedure becomes computationally unmanageable. For simplicity, in this analysis the linear model results are based on the assumption that investments of each type may be arbitrarily large.

Since at every stage the optimal solution yields a nonzero investment in at most a single investment type, the total new investment in period k, s_k is thus limited to investment in the single class, j which has the highest per dollar contribution to NW_T after accounting for survivability. At each stage in the backward recursion, s_k^{j*} is a function of the state variables NW_k, y_k^j, and $CB_T(k)$ and the assigned value of $E\{NI\}$. Thus, s_k^{j*} cannot be known explicitly at each stage. However, as the process procedes forward in actual time, the state and random variables become known and the optimal new investments, s_k^{j*} can be determined uniquely using (49). This aspect of the decision making process is intuitively pleasing. Actual decisions are made based on current information about the economic position of the enterprise.

(ii) Nonlinear Model. The nonlinear (quadratic) investment-income effects model does not permit the direct calculation of closed form solutions for the decision variables, s_k^i. The approach here is to devise a heuristic procedure to develop estimates for the optimal investments to be made over time. As in the case of the linear model, the linearity of the state dynamics equations (45) allows for the separable treatment of the investment decision subproblems at each stage of the backward recursion. The reward functional J_k^π (47) for the nonlinear model which determines the investment policy at each stage as a function of the state variables may be written as follows

$$J_k^\pi(\cdot) = \max_{s_k^i}\{\sum_{i\in S}[s_k^i b_k^i + e_k^i(s_k^i)^2] + C\},\ k = 0, 1, \ldots, T-1 \tag{53}$$

where

$$b_k^i = v_T^i(k)q_0^i - \gamma_T^i(k) +$$
$$D_{k+1}\{[1 + r(1-TR)]^{T-k}[v_T^i(T-1)q_0^i - \gamma_T^i(T-1)] + q_0^i G_T^i(k+1)\} \tag{54}$$

$$e_k^i = v_T^i(k)q_1^i + D_{k+1}\{([1 + r(1-TR)]^{T-k} v_T^i(T-1) + q_1^i G_T^i(k+1)\} \tag{55}$$

and C is a constant independent of the decision variables s_k^i. The values s_k^{i*} and D_{k+1} at each stage are determined by solving the following nonlinear program

$$Z = \max \sum_{i\in S}(s_k^i b_k^i + e_k^i(s_k^i)^2) \tag{56}$$

subject to the nonlinear (i.e., order $(s_k^i)^2$) survivability constraint from Lemma 3,

$$\sum_{i\in S} s_k^i A_T^i(k) \le B_T(k) + NW_k + \sum_{i\in S} G_T^i(k)y_k^i + CB_T(k)$$

$$s_k^i \ge 0,\ i = 1, \ldots, N_S.$$

The solution to this problem may be found by the method of Lagrange. The resulting solution for s_k^{i*} in terms of the Lagrangian mulitiplier, λ from the nonlinear survivability constraint is

$$s_k^{i*} = \frac{-b_k^i - \lambda\mu_1^i}{2(e_k^i + \lambda\mu_2^i)},\ i = 1, \ldots, N_S \tag{57}$$

where μ_1^i and μ_2^i are respectively the constant and s_k^i terms of $A_T^i(k)$. For given values of NW_k, y_k^i, and $CB_T(k)$, and $E\{NI\}$ the unique value of λ is determined by substituting the expression (57) into the nonlinear survivability constraint of Lemma 3 and then solving with the constraint expressed as an equality. For simplicity, this procedure assumes that the survivability constraint is binding in the optimal solution to the nonlinear (quadratic) program (56) at each stage. In this case, λ will have a non-zero value in accordance with the Kuhn–Tucker conditions. However, if the survivability constraint is not binding, then λ is zero-valued in the optimal solution. Thus, two different solutions are characterized at each stage of the problem corresponding to $\lambda = 0$ and $\lambda > 0$. Each of these possible solutions is recorded and carried forward at each stage of the backward recursion.

Now, since NW_k, y_k^i and $CB_T(k)$ are unknown at each stage of the backward recursion, we need to express λ as an explicit function of these state variables. Letting

$$x_k = NW_k + \sum_{i\in S} G_T^i(k)y_k^i + CB_T(k) \tag{58}$$

we approximate $\lambda(x_k)$ by the first degree (linear) interpolating polynomial†

$$\lambda = \alpha_0 + \alpha_1 x_k \tag{59}$$

which minimizes the mean-square error $\sum_{i=1}^{n}(\lambda_i - \alpha_0 - \alpha_1 x_{ki})^2$ for n pairs of values for λ and x_k. The value for each λ based on this model (given a set of data dependent values for x_k) is found by an iterative procedure. Pairs of λ and x_k are generated until stable estimators of α_0 and α_1 are found. Thus, the decision variables (57) can be written in terms of the linear approximation model (59) for λ. Consequently, the objective function Z of the nonlinear program (56) may also be written in terms of the estimated parameters so that $Z = Z(x_k)$.

Finally, the function $Z(x_k)$ is expanded as a Maclaurin series (i.e., a Taylor series expansion about zero) to yield

$$Z(x_k) = Z(0) + Z'(0)x_k + R(x_k^2) \tag{60}$$

where $R(x_k^2)$ is the error term arising from all terms in x_k of powers greater than or equal to two. Assigning $D_k = Z'(0)$ completes the recursion. We also note that $D_k > 0$ since Z is an increasing function of x_k. Once D_k is determined, then the terms b_{k-1}^i and e_{k-1}^i may be calculated at the next stage of the backward recursion from (54) and (55) respectively. If $b_k^i < 0$, then s_k^{i*} is assigned a value of zero.

In contrast to the linear investment-income effects model, the new investment, s_k may be distributed across more than one type of capital. Thus, more realistically, model generated investment strategies in any period may include a mix of different capital types. Since s_k^{i*} (57) is a function of the state variables NW_k, y_k^i, and $CB_T(k)$, and the assigned value of $E\{NI\}$ through the multiplier, λ then it cannot be known explicitly by the backward recursion process. However, as the process moves forward in time, the state variables become known. Thus, a new investment, s_k^i of capital type i can be determined uniquely given estimates for the parameters α_0 and α_1 of the linear model (59) for λ. As for the linear investment model, investment decisions are made based on the actual position of the decision maker at the time the decision is made. These decisions depend explicitly on the stochastic elements of the system which are realized and updated at the end of each fishing season.

† In general, λ may be approximated by an nth degree polynomial in x_k. The linear model (59), or first degree polynomial, simplifies the recursion and permits, after lengthy algebraic manipulation, the expression of J_k^{π} as a simple function of x_k. Moreover, for the data used in this analysis, the linear model (59) appears to be a reliable estimate of the actual function, $\lambda(x_k)$.

5.3 Computational Considerations.

The linear and nonlinear dynamic programming models described above lead directly to the following algorithms for computing the investment decision rules.

(i) Linear Model:

0. Set $k = T - 1$.

1. Calculate the coefficients of the linear objective function, u_k^i, $i = 1, \ldots, N_S$ given $A_T^*(k+1)$ and u_{k+1}^* as in (47a). ($u_T^* = 0$).

2. Calculate the survivability constraint coefficients, $A_T^i(k)$, $i = 1, \ldots, N_S$ from (43).

3. Determine j from the solution to the linear program as in (48). Calculate the investment policy state variables multipliers, i.e., the constant term $B_T(k)/A_T^j(k)$ and the multipliers of NW_k, $y_k^i, i = 1, \ldots, N_S$, and $CB_T(k)$, namely $1/A_T^j(k), G_T^i(k)/A_T^j(k), i = 1, \ldots, N_S$ and $1/A_T^j(k)$ respectively from (49).

4. Assign $A_T^*(k)$ and u_k^* appropriately as in (50).

5. Set $k = k - 1$.

6. If $k < 0$ then STOP; else GO TO Step 1.

(ii) Nonlinear Model:

0. Set $k = T - 1$.

1. Calculate the coefficients of the quadratic objective function, b_k^i, e_k^i, $i = 1, \ldots, N_S$ given D_{k+1} from (54) and (55). ($D_T = 0$).

2. Calculate D_k, from b_k^i, e_k^i, the survivability coefficients, $A_T^i(k)$, and the Maclaurin series for Z as in (60).

3. Determine the new investments s_k^{i*} from (57) for $i = 1, 2, \ldots, N_S$.

4. Set $k = k - 1$.

5. If $k < 0$ then STOP; else GO TO Step 1.

Given the derived investment decision rule or control law, the actual investment strategy proceeds by the movement of the decision process from one decision point to another over time. The actual investments in each capital type over time depends on (i) the initial position of the firm denoted by $(NW_0, y_0, d_0, rp_0, z_0, cb_0)$ and (ii) the expectations of and the actual realizations of the random variable NI in each year over the planning horizon.

The computational algorithms for the linear and nonlinear investment-income effect models are coded in double precision FORTRAN. For the linear model, the optimal investment policy for each period k of the planning horizon is calculated in terms of the current state variables NW_k, y_k^i, and $CB_T(k)$. Sample results are presented in Table XIV(a). This table presents the constant and linear multipliers of the state variables from Step 3 of the linear model algorithm for each period. Given the actual values of the state variables at any period, the new investment can be calculated directly using (49). To illustrate, suppose that $NW(1) = 20$, (i.e., a net worth at period 1 of \$20,000(1971)), $cb(1) = 0$, and $y_1^i = 0.1, i = 1,2,3,4$. Using Table XIV(a), the investment in type 3 capital at the end of period 1 is -103.794+20(4.597)+.1(46.587+36.379+28.665+46.587)+0, or 3.968 thousands of 1971 dollars. The investment in other years of the planning period depends on the current values for the state variables and the policy parameters of the table in a similar manner.

Solutions to the nonlinear model are developed using a Newton–Raphson iterative method to solve for λ, the Lagrange multiplier (59) given a set of representative trial values for x_k based on the actual data. For all test problems the iterative method converged to a λ value after at most 15 iterations for each trial value x_k. The x_k and λ pairs were then used to estimate the linear regression parameters α_0 and α_1 of (59). These values were then used in (56) to estimate the coefficients of Z in terms of x_k for the MacLaurin series expansion (60). The resulting investment policy for each period k of the planning horizon is presented as a function of the current value of x_k (58), representing the sum of the state variables NW_k, y_k^i, and $CB_T(k)$. Sample results for the nonlinear investment model are presented in Table XIV(b). The policy described in terms of x_k takes the form of a ratio with x_k appearing in the numerator and denominator. This expression assumes that the survivability constraint is binding. If the survivability constraint is not binding, then the investment policy is given by the constant which appears after OR in Table XIV(b).

To illustrate, suppose that based on the structure of the firm in period 1,(58) yields $x_1 = 20$. From Table XIV(b) and assuming that the survivability constraint is binding then the investment at the end of period 1 in type 3 capital is given by [.00443-.00229(20)]/[-.00647+.0000237(20)], or 6.900 thousands of 1971 dollars. Model investment in other periods and for other capital types is determined similarly depending on the value of the state variables and the given policy multipliers.

(a)

DYNAMIC PROGRAMMING RESULTS: Linear Model (Base Case) †

PD k	PDS TO GO	INVEST TYPE	RETURN/ DOLLAR	Constant	95.0% SURVIVABLE INVESTMENT NW(k)	y(1)	y(2)	y(3)	y(4)	cb(k)
4	1	3	-0.047							
3	2	3	+0.003	-97.780	7.709	38.675	33.238	28.568	38.675	7.709
2	3	3	+0.019	-110.485	6.116	46.259	37.877	31.128	46.259	6.116
1	4	3	+0.010	-103.794	4.597	46.587	36.379	28.665	46.587	4.597
0	5	3	-0.029							

(b)

DYNAMIC PROGRAMMING RESULTS: Nonlinear Model (Base Case) †

PERIOD k	PERIODS TO GO	INVESTMENT TYPE	95.0% SURVIVABLE INVESTMENT
3	2	3	$\frac{-0.586E-04 + -0.306E-01 * X(3)}{-0.406E-02 + 0.336E-03 * X(3)}$ OR 0.213E+01‡
2	3	2	$\frac{0.129E-02 + -0.444E-02 * X(2)}{-0.737E-03 + 0.887E-05 * X(2)}$ OR 0.106E+02
2	3	3	$\frac{-0.880E-02 + -0.568E-02 * X(2)}{-0.547E-02 + 0.658E-04 * X(2)}$ OR 0.378E+01
1	4	2	$\frac{-0.325E-02 + -0.171E-02 * X(1)}{-0.920E-03 + 0.336E-05 * X(1)}$ OR 0.796E+01
1	4	3	$\frac{0.443E-02 + -0.229E-02 * X(1)}{-0.647E-02 + 0.237E-04 * X(1)}$ OR 0.146E+00
0	5	1	$\frac{-0.261E-03 + -0.144E-03 * X(0)}{-0.281E-02 + 0.618E-07 * X(0)}$ OR 0.208E+01

Table XIV – Investment Models Policy Results

†All values are in thousands of real 1971 dollars.

‡Investment policies written in terms of X(k) assume that the survivability constraint is binding. If the constraint is not binding then the investment amount following the 'OR' applies.

5.4 Empirical Development

The purpose of this section is to describe the empirical aspects of the investment model with survivability constraints.

Objective Function. Maximizing net worth is used in this model as the measure of the economic value of the fishing enterprise. A high relative Net Worth indicates a successful operation, low relative Net Worth is a reflection of a less efficient operation. It is natural to think that individual fishing enterprises desire to have higher Net Worth, *ceteris parebis* (Smith(1975)).

Net Worth is also an indication of the solvency of the fishing operation. If Net Worth does not exceed zero, then the operation may not be capable of repaying its debtors in the current period. The business may be declared bankrupt if its debtors decide to foreclose and the courts deem that the business will not likely be able to meet the debtors claims at any time in the future.† Accordingly, maximizing expected Net Worth greater than zero is analogous to minimizing the event of foreclosure. Given the sources and extent of variability in fisheries, the Net Worth criterion is felt to be a valid operating condition for decision makers who seek primarily to survive as fishermen.

Sources of Variability. Empirical evidence on the economic variability of fishing abound. This may be illustrated by analysing year–to–year variations in: individuals' landed values and earnings shares; prices by weight for different species; landings by species (e.g., even and odd year pink salmon). Figure 9 presents a box-and-whisker display (Tukey(1977)) of the distribution of real gross fishing income by individual trollers for the period from 1970 to 1983. Figure 10 gives the mean and seasonal range of actual real (1971=100) prices per kilogram landed of four major salmon species for the period 1970 to 1983. The independence of each of many random, exogenous effects adds directly to the overall variability of earnings in the fishing operation.

Investment Classifications. The nature of the fishing business together with the methods of accounting for fishing investment for tax purposes result in the classification of investment into four major investment types, i.e., $N_S = 4$. These are classified and described as follows:

The investment classifications above may be further characterized according to their designation as physical or non–physical assets, and their associated depreciation treatment for tax purposes. Physical assets are depreciated for tax purposes using their Capital Cost Allowance (CCA) rate. Non–physical assets are depreciated for tax purposes on a straight-line basis over their specified accounting lives. The following table gives these characterizations for each investment class.

The assets represented by the four investment classifications are all accounted for as Long-Term Assets on the balance sheet of the fishing operation. As well, it is useful to point out that the relative size of typical investments in each classification and the frequency with which each type of investment occurs is different. For example, actual investments in Eligible Capital (licenses) usually represent a major outflow of funds and

† The legal claim that a firm cannot be expected to pay off its debtors at any future time is necessarily situational. Accordingly, legal grounds for bankruptcy may vary substantially from case to case. In this context, the condition that Net Worth be less than zero is a necessary but not sufficient condition for bankruptcy. Sufficient conditions may be expressed in terms of some level of Net Worth less than zero, or in conjunction with other criteria such as the ratio of assets to liabilities, etc.

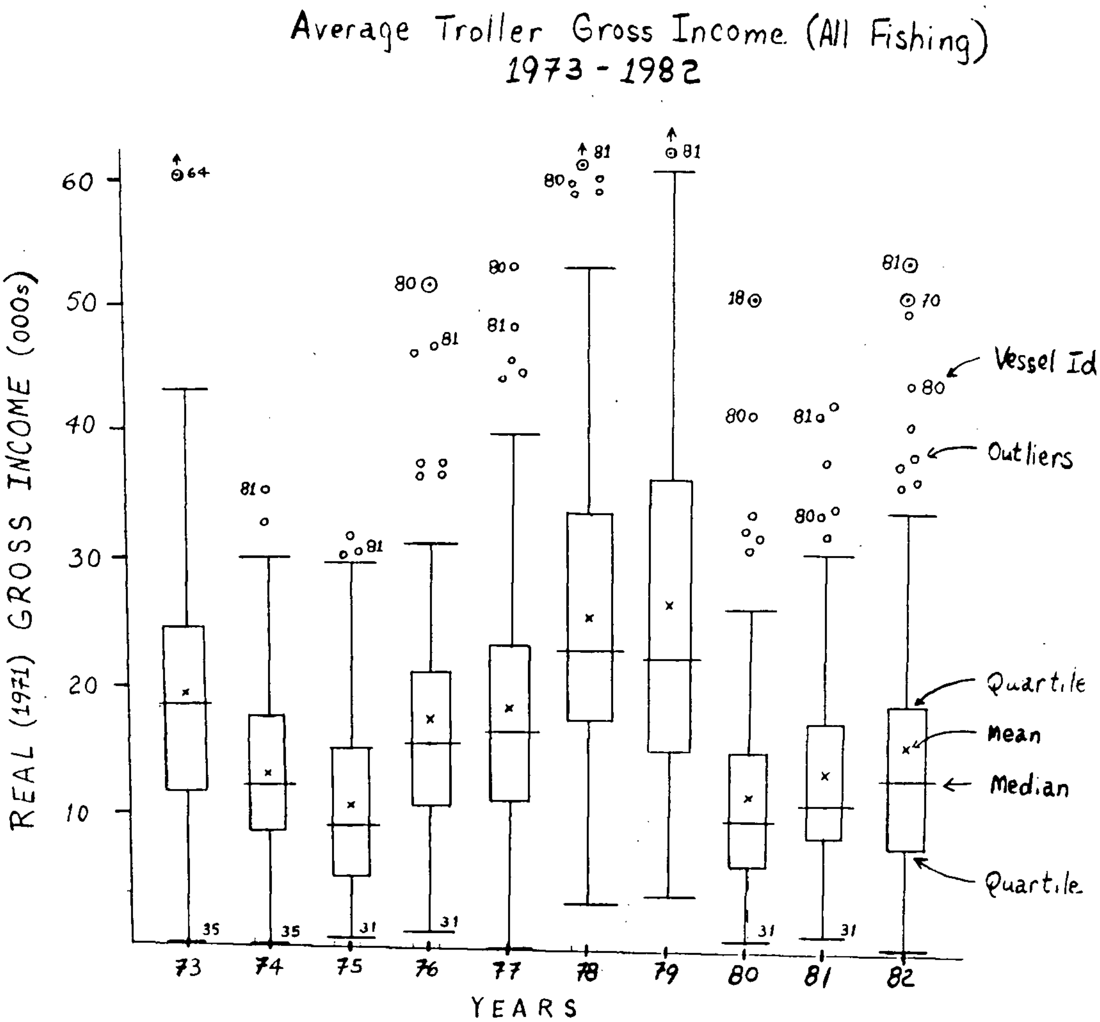


Figure 9 – Troller Gross Income Distribution 1973–1982

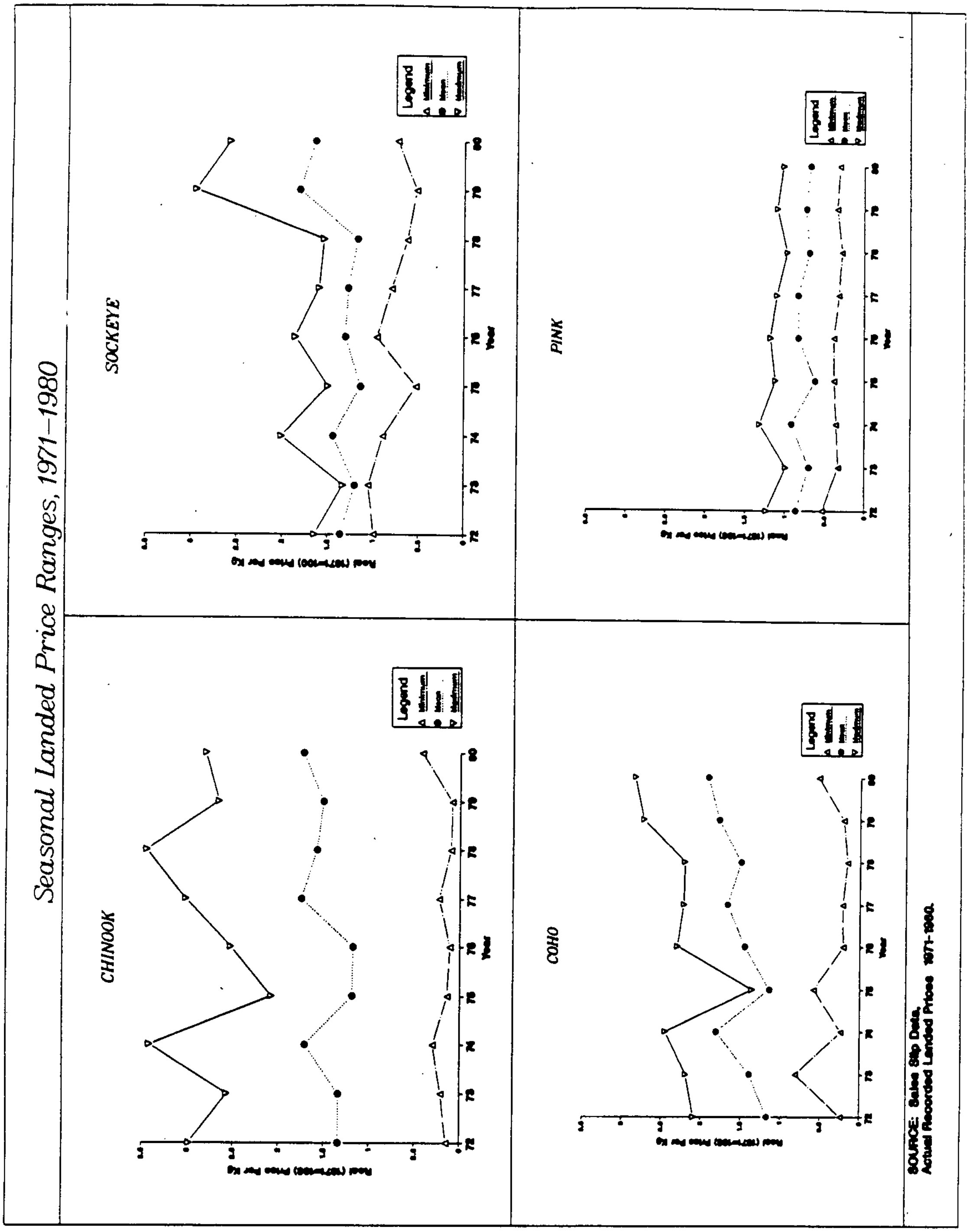


Figure 10 – Seasonal Landed Price Ranges 1971–1980

i	*Investment Classification*	*Description*
1.	Eligible Capital	Fishing license and other Governmental Rights (see 1982 Fisherman's Tax Guide, p.11).
2.	Vessels	Boats and major component parts, e.g., engines, furniture, fixtures, hydraulics.
3.	Gear/Electronics	Accessories to the operation, including drills, electric motor and engines, lines, ice machines, pumps, radio equipment, radar, tools over $200, welding equipment.
4.	Other Equipment	Equipment used as part of the fishing operation, e.g., cars, trucks, trailers, structures (buildings, wharves), chain saws, outboard motors.

Table XV – Investment Classifications

(Source: Fisherman's Income Tax Guide, Revenue Canada, Taxation, 1982.)

Type *i*	*Investment Classification*	*Asset Type*	*Depreciation Annual Rate*	*Expected Life (Yrs)*
1.	Eligible Capital	Non-Physical	10%	10
2.	Vessel	Physical	15%	30
3.	Gear/Electronics	Physical	20%	10
4.	Other Equipment	Physical	30%	5

Table XVI – Investment Class Tax Treatment

(Source: Fisherman's Income Tax Guide, Revenue Canada, Taxation, 1982.)

as such occur infrequently for most individual fishermen. (The trading and purchasing of licenses is a thinly traded market.) On the other hand, actual Gear/Electronics investment require much less of an outlay of funds and are purchased more often. Table XVII provides summary statistics on actual investments by 100 trollers for the period 1973 to 1982.

Financing Structure. Information on the relative size and frequency of investment in the different classes provides a glimpse of the financing structure which fishing enterprises use to procure capital. Moreover, regulation requirements (e.g., on the size and replacement of vessels, and gear restrictions) determine in part the ability of the firm to acquire new capital.

Fishermen are assumed to finance new investment from three major sources, namely by (i) increasing

Type *i*	*Investment Classification*	*Average Nonzero Investment*	*Maximum Nonzero Investment (Year)*	*Average Percent of Investors*
1	Eligible Capital	14160	49025 (1979)	7.55%
2	Vessel	13090	209107 (1979)	41.00%
3	Gear/Electronics	1626	15508 (1979)	43.67%
4	Other Equipment	2719	25714 (1978)	21.11%

Table XVII – Troller Actual Investment Statistics 1973–82 †

†All investment amounts are in real (1971=100) dollars.

long-term debt, e.g., taking out a bank loan; (ii) increasing short-term debt and/or decreasing savings, e.g., borrowing from a friend, cashing in on a term deposit, or withdrawing from a savings account; and (iii) disposing of some existing assets, e.g., selling an old vessel to build a new one. The propensity to invest depends on the ability of the fisherman to gather sufficient funds from each of these sources in order to finance the desired investment. Conversely, the set of investments which are feasible to the fisherman depend on the funds arising from these sources.

Although specific data on how fishermen actually apportion the financing of new investment is not available, the analysis of individual troller data recorded for tax submission purposes provides insight into the average way in which fishermen proportionately finance investments of each capital type. Table XVIII gives an example of the Annual Cash Report data for a single troller (among the 100 trollers in the database) over the period 1973 to 1982.

This data is used to estimate the proportion of new investment which is accompanied by capital disposals of the same capital class. The remaining proportion of new investment is assumed to be financed by long-term debt, and short-term debt and/or cash. The proportions for the debt items are estimated from year-to-year changes in interest payments available from annual survey data of British Columbia Commercial Salmon Fishermen's Earnings produced by Fisheries and Oceans, Canada.

The lack of specific data related to how fishermen actually finance new investments limits the justification of this empirical component of the investment model. Accordingly, the following estimates represent only average values for the parameters of the B matrix presented in Section 5.2.

This table provides the estimates for the time independent values for the matrix of financing parameters, B defined in Section 5.2. These financing structure parameters are used throughout the analysis of results

INVESTMENT STUDY

	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982
Vessel #	23									
Gear Category	TR-SN+									
Total Fishing Income	15,349	32,255	21,960	53,673	60,162	69,922	78,318	99,389	122,372	124,122
Net Income from Fishing	5,716	9,631	1,236	27,471	33,484	52,193	33,700	5,722	12,864	54,425
Other Income	511	523	2,753	1,764	(746)	(3,101)	8,850	18,651	14,571	27,235
Income Tax net of CPP Payable	960	116	214	3,314	6,077	13,002	0	0	2,490	14,372
Net Cash Income	5,267	10,084	3,865	25,941	26,661	36,090	42,550	24,591	24,935	67,331
Repair & Maintenance Expense	1,659	2,485	3,797	5,689	2,547	6,622	3,465	7,189	6,353	6,392
Gear Expense	1,421	4,661	2,683	2,960	3,155	4,150	2,469	3,270	3,552	1,542
Total R & M and Gear	3,080	7,146	6,486	8,649	5,702	10,772	5,934	10,259	9,905	7,934
I.T.C. - Created	0	0	0	166	48	741	15,410	4,115	0	0
- Claimed	0	0	0	185	46	741	0	0	6,302	9,434
Licence Additions	22,020	0	0	0	0	37,500	52,000	0	0	0
Licence Disposals	0	0	0	0	0	0	0	0	0	10.500
Licence Class	A					A,H	A			A
Vessel Additions (Class 7)	32,759	2,956	350	0	669	13,028	215,907	55,785	0	0
Vessel Disposals	0	0	0	0	0	0	0	0	0	106.414
Vessel Additions (Part XVII)	0	0	0	0	0	0	0	0	0	0
Vessel Additions (Rep. 1101)	0	0	0	0	0	0	0	0	0	0
Vessel Disposals (Both)	0	0	0	0	0	0	0	0	0	0
Gear/Electronics - Additions	10,305	1,213	0	4,522	698	2,442	4,241	3,000	0	0
- Disposals	0	0	0	0	0	0	0	0	0	0
Other Additions	0	0	0	0	1,600	0	0	0	0	0
Other Disposals	0	0	0	0	0	0	0	0	0	0
Gear (Inventory) - Additions	0	0	0	0	0	4,150	2,469	3,270	0	0
- Disposals	0	0	0	0	0	0	0	0	0	0
TOTAL ADDITIONS	65,104	4,169	350	4,522	2,767	57,120	272,617	61,856	0	0
TOTAL DISPOSALS	0	0	0	0	0	0	0	0	0	116,914

Subsidy Vessel (Y/N)	N
Gear Category - Jan. 1, 1973	TR-S
Fishing Category Change	1979
Previous Gear Type	

Table XVIII – Troller Annual Cash Report Data

Type *i*	*Investment Classification*	*Short-term*	*Long-term*	*Disposals*
1.	Eligible Capital	10%	65%	25%
2.	Vessel	15%	60%	25%
3.	Gear/Electronics	15%	80%	5%
4.	Other Equipment	15%	80%	5%

Table XIX – Average Sources of Financing

of the investment model which is discussed in Section 5.4.

To complete the definition of the financing structure for fishermen's investment decisions the real rates of interest and the length of the terms for the sources of financing identified above are defined. These are based on the historical data for the fishing fleet of British Columbia. Sources for this data are Financing in the B.C. Fishing Industry (1979), McMullan(1984), and Fisheries Improvement Loans Act Annual Reports, 1970–1983.

1. *Cash.* Cash or Current Asset items are all assumed to be demand notes 100% refundable (principal plus interest) with no penalty.

2. *Short–Term Debt.* These items are Current Liabilities which are assumed to be 100% payable (principal plus interest) at the end of their term. The terms of these loans are for one year at an average real rate of interest of 2.0%. Table XX presents the actual real short-term interest rates for 1970 to 1983 as compiled in the *Economic Review* of the Department of Finance, Canada.

3. *Long–Term Debt.* These items are Long–Term Liabilities whose payments (principal plus interest) are payable in equal year–end amounts over the term of the loan. The actual terms of these loans vary between five and ten years at an average rate of interest of 3.7%. This analysis assumes a term of seven years for long-term debt at a rate of 4.0% per annum. Table XX also presents the actual real long-term interest rates for 1970 to 1983 as compiled in the *Economic Review* of the the Department of Finance, Canada.

4. *Disposals.* Disinvestments in Long–Term Assets items are assumed to be 100% refundable at existing market values. All disposals are assumed to be connected with a corresponding new investment of the same type, i.e., disposals occur only when an investment takes place. (Actually, net disinvestment does occur. For example, 1981 was a year of net capital disinvestment by trollers.) Moreover, disposals are assumed to reduce the total financing requirements for the new investment.

Investment Effects on Income. The key random component in the investment model is the variable

Year	*Short Term Rates‡*	*Long Term Rates††*
1970	3.9%	5.8%
1971	1.7	4.7
1972	0.3	2.2
1973	-0.1	1.1
1974	-0.4	0.9
1975	2.9	-0.4
1976	1.7	3.5
1977	-0.5	1.5
1978	-0.1	1.8
1979	2.9	4.7
1980	2.9	5.1
1981	5.9	7.8
1982	3.3	6.0
1983	3.6	6.4
Average	2.0%	3.7%

Table XX – Real Rates of Interest 1970-83 †

†Sources: *Economic Review*, Department of Finance, Canada, April 1984, and *The Consumer Price Index*, Statistics Canada, Monthly, 62–001.

‡Derived by computing the difference between the average annual nominal Prime Corporate Paper (90 days) rate and the Consumer Price Index ('All Items', year-over-year change).

††Average Annual Fisheries Improvement Loans Act Rates, derived by computing the difference between the average annual nominal Bank Prime Lending rate and the Consumer Price Index ('All Items', year-over-year change).

NI_t, the average per vessel Net Operating Income from season t. The impact on actual Net Operating Income due to new investment s_k^i is modelled by the function, $f(s_k^i)$ and the discounting factor, ρ_i. The discount factors, ρ_i, and the parameters of the $f(s_k^i)$ function (p^i for the linear model and q_0^i, q_1^i for the nonlinear model) for $i = 1, 2, 3, 4$ were derived from data on troller vessel attributes and actual troller investments by vessel for the period 1973 to 1982. The following presents the steps of the parameter estimation procedure:

1. Separate vessel attributes from the 1972 troller attributes database into the capital classifications for eligible capital (i.e., licences), vessels, gear and electronics, and other equipment.

2. Determine estimates of the coefficient of variation for each attribute of each capital class over all troll vessels in the set; weight each attribute by its contribution to the sum of the reciprocal of the coefficients of variation for all attributes in the same capital class.

3. For all 100 trollers for which actual annual investment data is available (from the Annual Cash Reports database, Table XVIII), compute

$$y_t = \sum_{i \in S} y_t^i = \frac{NI_t(s) - NI_L}{NI_t - NI_L}, \quad t = 1973, \ldots, 1982 \tag{61}$$

using (17). (Note that $E\{NI_t(s)\} = NI_t$ and so $E\{y_t\} = 1$.)

In (17), $NI_t(s)$ is the actual (real) reported Net Operating Income for the year t, NI_t is the average actual (real) Net Operating Income for all 100 trollers in year t, and NI_L, the shift factor, is the lowest (real) Net Operating Income reported by trollers between 1973 and 1982. Thus, y_t is a shifted and normalized Net Operating Income variable.

4. Determine estimates for the weighting factors by capital type i in the sum y_t of (61) by regressing y_{1973} (the dependent variable) from 3 above against capital attributes (the independent variable) for eligible capital, vessels, gear and electronics, and other equipment as determined in 2 above. Class attributes from the attribute database were paired with y_{1973} values from the investment database by assuming the same probability distributions between the two unconnected databases. This establishes best estimates for y^i_{1973}, $i = 1, 2, 3, 4$ for all 100 trollers of the Annual Cash Report database.

5. Using (18) and the values for the independent variables y^i_{1973} (from 4 above) and s^i_{t-1} (from the Annual Cash Report database) for all 100 trollers, the model

$$y_t = \sum_{i \in S} (y^i_{t-1}\rho_i + f(s^i_{t-1})s^i_{t-1}) + \epsilon_t, \; t = 1974, \ldots, 1982 \tag{62}$$

was used to estimate the discount factors, ρ_i and the parameters of the $f(s^i_k)$ functions by ordinary least squares minimization. Minimization was carried out by the method of steepest descent (Gottfried and Weisman(1973)).

Details of the step-by-step procedure outlined above follow.

1. The 1972 (end of year) troller attributes database includes information about vessel value, vessel capital items, vessel licence, and vessel owner's data. Among 35 separate attributes for a set of $N = 1435$ trollers, selections were made and placed into capital classifications for: licence-related (eligible capital), vessel-related, and gear and electronics-related attributes. (There are no discernable attributes in the attributes database related to the 'Other Equipment' capital classification.) Table XXI contains the list of specific attributes of the troller attribute database assigned to each capital classification.

2. The estimates of the coefficient of variation, $\hat{\delta}_{ij}$ for each attribute j of capital classification i was calculated using all cases of the attributes database. Now, let ω_i be defined as

$$\omega_i = \sum_j 1/\hat{\delta}_{ij}, \; i = 1, 2, 3 \tag{63}$$

Type	*Investment Classification*	*Assigned Attributes*
1	Eligible Capital	Fishing Code (Licence 'Tab' Specification)
2	Vessels	Year Built and Rebuilt ('Age'); Estimated Length (in feet); Net and Gross Tonnage (in Imperial Units); Engine Type (gas or deisel); Horsepower; Estimated Boat Value
3	Gear/Electronics	Numbers of: Radiophones, Lorans, Echosounders, Direction Finders, Radar, Autopilot, and Sonar on the Vessel
4	Other Equipment	Nil

Table XXI – Troll Attributes by Capital Class

where j is the index of an attribute in capital class i. Then,

$$\omega_{ij} = \frac{1/\hat{\delta}_{ij}}{E\{x_j\}\omega_i} \tag{64}$$

is the weight assigned to attribute j of capital class i where $E\{x_j\}$ is the average value of attribute j. (Note that $\sum_j E\{x_j\}\omega_{ij} = 1$ for all i.)

Finally, for each troller all attributes j within each capital class i are consolidated into a single value by weighting the attribute value x_j by ω_{ij} and summing over all j. The result is a single measure for each capital class denoted by Ω_i where

$$\Omega_i = \sum_j x_j\omega_{ij}, \ i = 1, 2, 3. \tag{65}$$

(Note that $E\{\Omega_i\} = 1$ for all i.)

This approach captures variability within each capital class through the weighted variability of the attributes which belong to the class. The reciprocal of the coefficient of variation weighting assigns highest weight to those attributes with least variation relative to its mean. Such attributes are deemed to be more indispensible to the troll fishing activity as a result of their observed constancy over the troller fleet (e.g., length of vessel, horsepower). Conversely, low weights are assigned to attributes with the greatest variation relative to the mean. These attributes are not deemed to be crucial to the troll fishing activity *per se* by virtue of their higher variation among the fleet. Individual variations of these attributes are weighted lower (e.g., gross tonnage, boat value).

3. The Annual Cash Report database provides actual annual Net Operating Income values, $NI_t(s)$ for 100 trollers of the commercial salmon fishing fleet over the period $t = 1973, \ldots, 1982$. The shift parameter,

NI_L is determined from this database item ($NI_L = -\$6098.(1971)$), as is the actual average per vessel Net Operating Income value in each year, NI_t. The variable y_t, $t = 1973, \ldots, 1982$ is computed using these values for each of the 100 trollers from model (61) as previously described.

4. Model (17) is used to assign relative weighting to each capital class attribute, Ω_i. The variable y_{1973} is treated as the dependent variable in a multiple linear regression against the independent variables, Ω_i, $i = 1, 2, 3$. However, the attribute database ($N = 1435$ trollers) from which the Ω_i were derived, and the Annual Cash Report database ($N = 100$ trollers) from which the y_{1973} values were derived are not logically linked databases. Accordingly, a procedure for pairing y_{1973} with Ω_i values for all 100 trollers is required. This procedure involves breaking the attribute database into 100 classes of equal size (14 trollers per class). The observation of the class mark (the eighth ranked observation in each class) was then assigned to the corresponding class (of size 1 each) in the Annual Cash Report database.

The regression model which results is

$$y_{tn} = \phi_1 \Omega_{1n} + \phi_2 \Omega_{2n} + \phi_3 \Omega_{3n} + \phi_4 + \epsilon_n, \; n = 1, \ldots, 100 \tag{66}$$

where the ϕ_i, $i = 1, 2, 3, 4$ are the weights assigned to each capital class. Note that all trollers are assumed to have the same (constant) weight for 'Other Equipment', ϕ_4 in their capital structure prior to new investments at the end of the 1973 fishing season. The results of the multiple linear regression are contained in Table XXII.

The squared correlation coefficient is high, ($r^2 = 0.9928$) and all parameters are significantly different from zero. Examination of the residuals reveals no evidence of model misspecification. The regression results establish an initial configuration description

$$y^i_{1973} = \hat{\phi}_i \Omega_i, \; i = 1, 2, 3, 4 \tag{67}$$

at the end of 1973 (prior to new investment taking place for the 1974 season) for all 100 trollers of the Annual Cash Report database.

5. Having established initial capital configurations by class in the initial year (1973), y^i_{1973}, $i = 1, 2, 3, 4$, the components of the recursive model (62) are now completed. This includes the dependent variables y_t from 3 above, and the actual annual investments by class s^i_{t-1} from the Annual Cash Report database.

The parameters of the $f(s^i_k)$ functions and the ρ_i are determined by minimizing the sum of the squared errors, ϵ as in the ordinary least squares approach of linear regression. In particular, for the linear model we

```
PAGE 2 BMDP1R Normalized Net Income versus Capital Types

    BASED ON INPUT FORMAT SUPPLIED      1 RECORDS READ PER CASE
    NUMBER OF CASES READ ............................     100

    VARIABLE       MEAN        STANDARD          COEFFICIENT       MINIMUM        MAXIMUM
                               DEVIATION         OF VARIATION

      1 NIDEP    1.00005       0.33118            0.33117          0.49100        2.19100
      2 WV       0.93966       0.41422            0.44082          0.29600        3.41000
      3 WGE      1.01653       0.69105            0.67981          0.0            3.51800
      4 WL       1.05360       0.16849            0.15991          0.65020        1.51745

    DEPENDENT VARIABLE ...................................     1 NIDEP
    TOLERANCE ............................................   0.0100
    ALL DATA CONSIDERED AS A SINGLE GROUP

    MULTIPLE R               0.9964             STD. ERROR OF EST.        0.0285
    MULTIPLE R--SQUARE       0.9928

    ANALYSIS OF VARIANCE

                         SUM OF SQUARES     DF     MEAN SQUARE      F RATIO    P (TAIL)
    REGRESSION               10.7805         3         3.5935      4416.195     0.0000

                                                       STD. REG
       VARIABLE       COEFFICIENT      STD. ERROR     COEFF        T      P(2 TAIL) TOLERANCE

    INTERCEPT           0.31113
    WV          2       0.35145        0.00938       0.440    37.486    0.0        0.54497
    WGE         3       0.31705        0.00549       0.662    57.741    0.0        0.57085
    WL          4       0.03453        0.01816       0.018     1.902    0.0602     0.87832

    NUMBER OF INTEGER WORDS OF STORAGE USED IN PRECEDING PROBLEM       744
    CPU TIME USED        0.273 SECONDS
```

Table XXII – 1973 Capital Configuration Regression Results

seek to find parameters p^i and ρ_i, $i = 1, 2, 3, 4$ which minimize

$$\sum_{t,n}(y_{tn} - \sum_{i=1}^{4}[y^i_{t-1n}\rho_i + p^i s^i_{t-1}])^2, \; t = 1974, \ldots, 1982; \; n = 1, \ldots, 100. \tag{68}$$

And, for the nonlinear model we seek parameters q^i_0, q^i_1, and ρ_i, $i = 1, 2, 3, 4$ which minimize

$$\sum_{t,n}(y_{tn} - \sum_{i=1}^{4}[y^i_{t-1n}\rho_i + q^i_0 s^i_{t-1} + q^i_1(s^i_{t-1})^2])^2, \; t = 1974, \ldots, 1982; \; n = 1, \ldots, 100. \tag{69}$$

Parameter estimates were determined by minimizing (68) and (69) using the method of steepest descent. Near the minimum point, the steepest descent procedure converged slowly. An iterative ('stop and go') procedure was used to speed up the convergence of the steepest descent method when this occurred. The values of all components of the gradient function at the final stopping point for both linear and nonlinear models did not exceed 0.01. The resulting parameter estimates are found in Table XXIII. Of interest is the fact that license investments have an initial negative effect on (transformed) net operating income. However, earnings effects in future years are augmented since the discounting factor for licenses exceeds 1. These results may be explained by the presence of an initial learning effect due to a shift to new fishing grounds or new fishing techniques used in the capture of different species. It is also interesting to note that the density effects in the nonlinear model all have negative parameter values. This is indicative of investment capital density dependence and diminishing returns to marginal investment. The parameter estimates of Table XXIII are used in the subsequent analysis of the investment model (Section 5.4).

A. Linear Model

Type i	*Investment Classification*	*Discount Factor,* $\hat{\rho}^i$	*Initial Factor,* $\hat{p}^i$
1	Eligible Capital	1.141	-0.139
2	Vessels	0.903	0.001
3	Gear/Electronics	0.814	1.680
4	Other Equipment	1.108	0.345

B. Non Linear Model

Type i	*Investment Classification*	*Discount Factor,* $\hat{\rho}^i$	*Initial Effect,* $\hat{q}_0^i$	*Density Effect,* $\hat{q}_1^i$
1	Eligible Capital	1.156	-0.205	-0.039
2	Vessels	0.900	0.183	-0.189
3	Gear/Electronics	0.795	2.680	-1.580
4	Other Equipment	1.104	0.346	-3.250

Table XXIII – Income Effects Parameter Estimates †

†All Initial Effect and Density Effect parameter estimates are in terms of (transformed) net operating income effects per hundred thousand dollars of real (1971=100) new investment.

Survivability Conditions. The survivability condition (32) requires the specification of a maximum probability that Net Worth in any period to the end of the planning horizon be less than zero. The precise value of this probability will depend on the risk behavior of individual fisherman. For example, if a fishermen is relatively new to the fishing activity and has a young family to support, then p_0 may be low relative to an older and more independent fishermen, *ceteris parebis*. This interpretation of p_0 allows the model to be "tuned" to different risk situations thereby enabling an interpretation of endogenous risk on investment policies. A set of different p_0 values ranging from loose constraints on investment due to survivability ($p_0 = 0.10$) to tightly constrained investment ($p_0 = 0.0$) are used in the analysis of the results (Section 5.5) of this model to explore the impact of this parameter on investment.

Planning Horizon. The determination of the length of the planning horizon actually used by fishermen can be surmised by considering the period over which cash flow information is knowable. On the one hand, there is evidence that fishermen are probably very myopic *vis-à-vis* their expectations of future income streams. (Pàlsson and Durrenburger(1982).) The random nature of incomes seriously restricts planning based on this item beyond periods of five years. On the other hand, investment financing gives specific information on required cash outflows to a maximum of ten years in the future corresponding to long-term financing committments. Differences in the length of the horizon result in net differences in investment policies. The consequences of changing horizon length on model results are investigated by analysing results using planning horizons of between three and seven years. The results of this analysis are presented in the following section.

This completes the analysis of the empirical elements of the investment model.

5.5 Analysis of Results

This section presents the results of the investment model algorithms discussed previously. The specific inputs to the computerized algorithms are described. Also presented are the input scenarios which were prepared to examine the sensitivity of the model results to changes in the input data. The model generated results for average investment by trollers for linear and nonlinear (investment–income effects) models are summarized and the sensitivity of the model to key input parameters is discussed. Finally, the results of the investment model are compared to the actual observed investment by trollers.

Model Inputs. The investment decision models are driven by four sets of input parameters. These are: (a) the Fixed Global Parameters; (b) the Fixed Financial Parameters; (c) the Troller Specific Parameters; and (d) the Troller Grouping Parameters. The following paragraphs describe each of these input parameter sets in more detail.

(a) *Fixed Global Parameters.* These are the nonfinancial input data which are constant for all investment problems analysed. These data include:

   i. the number of investment classifications, or capital types, N_S. (Refer to Section 5.4 for class definitions.)

   ii. the upper bounds on single period real (1971=100) investment by type. These values are based on the observed maximum troller investments by type for the period 1973–1982.

   iii. the net operating income shift factor, NI_L representing the lowest real net operating income realizable by trollers in a single season. This value is based on the observed troller minimum annual net operating income by trollers for the period 1973–1982.

(b) *Fixed Financial Parameters.* These are the financial input data which are constant for all investment problems analysed. These data include:

   i. the annual income tax rate.

   ii. the annual rate of depreciation for each capital type. These data are comprised of the capital cost allowance rates for physical assets and the straight-line depreciation rate for non-physical assets. (See also Section 5.4 for a discussion on investment class tax treatment.)

   iii. the annual rates and terms for loan financing. These data include the annual real (1971=100) rate of interest and the terms for short and long term financing arrangements. (See also Section 5.4 for financing structure details.)

iv. the matrix of financing parameters, B. These data include the proportions for financing new investment by type among short-term and long-term loans, and disposals. (See also Section 5.4 for a discussion on financing treatment.)

(c) *Troller Specific Parameters.* These are the input data which describe the initial position of the troller and provide the frame of reference for future investment decisions. Variations on these inputs permit the exploration of many possible investment scenarios. These data include:

i. the length of the planning period in years. This input determines the future years for which investment decision policies are prepared based on the initial position and economic outlook of the operation at the start of the planning period.

ii. the initial capital structure of the troller operation, namely, the current cash and net worth positions, the stream of outstanding debt payments (interest and repayment) and the tax depreciation schedule (assuming no new future investments) over the planning period, and the initial net operating income earning power of each operation.

iii. the investment-income effects parameters. For the linear model, these data are the annual discount factors, ρ_i, and the initial income earning effect factors, p^i for each capital type i. For the nonlinear model, these data are the annual discount factors, ρ_i, and the linear and quadratic initial effect factors, q_0^i and q_1^i, respectively. (See Section 5.4 for details on the derivation of these parameters.)

iv. the survivability (or risk) factor, p_0. This input determines the constraint on investment over the planning period due to the risk the decision maker places on continued survivability in the troll fishing business.

v. the annual net operating income anticipated by the decision maker for each year of the planning period.

(d) *Troller Grouping Parameters.* These data identify a subset of the troller database for which development of average troller investment policies is to be undertaken. The 100 trollers of the Annual Cash Report Database (Table XVIII) are divided into four groups for the analysis of their separate investment strategies. These groups are defined as follows:

i. all 100 trollers (100% sample of the troller database). Model results for this group of trollers are representative of overall average troller investment performance.

ii. the low 25% total fishing income trollers each season (25% sample from the 100 troller database.)

Model results for this group of trollers are representative of average investment decisions made by low fishing income trollers.

iii. the middle 50% total fishing income trollers each season (50% sample from the 100 troller database.) Model results for this group of trollers are representative of average investment decisions made by trollers who earn medium level incomes.

iv. the high 25% total fishing income trollers each season (25% sample from the 100 troller database.) Model results for this group of trollers are representative of average investment decisions made by high income earning trollers.

The grouping of trollers into these Total (or Gross) Annual Fishing Income categories has been used elsewhere in the description of troller financial performance. (For example, see also the report of the Fleet Rationalization Committee and the 1982 and 1983 surveys on British Columbia commercial fishermen's earnings.) These groupings provide a means of comparison with existing results.

This completes the description of model inputs for the investment models. Table XXIV presents a summary of investment model inputs for the linear and nonlinear base case scenarios (to be defined in the next subsection).

Investment Model Scenarios. The troller specific data inputs and the troller grouping data inputs provide for the exploration of a wide range of investment model scenarios. These scenarios define the frame of reference under which investment policies are derived and investment decisions actually take place.

In this analysis investment model scenarios are defined by specifying the following data items:

1. *Troller Group.* Specify one of the four groups described in model input item (d) above. This item automatically determines the initial average capital position assigned to all individual trollers of the group. Moreover, all subsequent data items in the scenario definition pertain equally to all members of the group. All trollers of the specified group are assigned the group average initial cash, net worth, debt and tax depreciation schedules. Lack of troller specific information on these items does not permit individual assignment to individual trollers. However, the initial net operating income earning factors, y_{1973} are estimated and assigned to each troller as described in Section 5.4.

2. *Investment-Income Effects.* Specify either the linear or nonlinear model and the corresponding discount and initial effects parameters.

3. *Planning Period Length.* Specify the length of the planning period in years. In this analysis planning periods of 3, 5 and 7 years are examined.

Investment Model Inputs

(a) *Fixed Global Parameters*

i. Number of Investment Types, $N_S = 4$

ii. Investment Upper Bounds

Type	1	2	3	4
Amount ($)	40,000.	100,000.	15,000.	20,000.

iii. Lowest Realizable Income, $NI_L = -\$6098.00$

(b) *Fixed Financial Parameters*

i. Annual Income Tax Rate, $TR = 0.50$

ii. Rates of Depreciation

Type	1	2	3	4
CCA/Str Line	0.10	0.15	0.20	0.30

iii. Rates and Terms of Financing

Sources of Financing	Real Rates (%)	Term (Years)
Short Term	2.0	1
Long Term	4.0	7

iv. Financing Matrix, B

Type	1	2	3	4
Short Term	0.12	0.10	0.14	0.15
Long Term	0.63	0.65	0.81	0.80
Disposals	0.25	0.25	0.05	0.05

(c) *Troller Specific Parameters*

i. Length of the Planning Period, $T = 5$ years

ii. Survivability Factor, $p_0 = 0.05$

iii. Initial Capital Structure

Year	1973	1974	1975	1976	1977
Depreciation	10,000.	8,000.	6,400.	5,120.	4,096.
Interest Owing	1,833.	1,660.	1,480.	1,293.	1,098.
Repayment Owing	4,331.	4,504.	4,685.	4,872.	5,067.

iv. **Investment--Income Effects Parameters**

Linear Model

Type	1	2	3	4
Discount Factors, ρ	1.000	0.903	0.814	1.000
Initial Factor, p	0.700	0.800	1.600	0.345

Nonlinear Model

Type	1	2	3	4
Discount Factors, ρ	1.000	0.886	0.772	1.000
Initial Factor, q_0	1.000	1.100	1.680	0.346
Density Factor, q_1	-0.039	-0.189	-1.580	-3.250

v. **Anticipated Net Operating Income: 'Average' Level**

Year	1973	1974	1975	1976	1977
Income ($)	8,000.	8,000.	8,000.	8,000.	8,000.

(c) *Troller Grouping Parameters*

All 100 Trollers - 100% Sample of Troller Database

Table XXIV – Investment Model Inputs

4. *Survivability Factor, p_0.* Specify the value of the survivability or risk factor, where $0 \leq p_0 < 1$. In this analysis p_0 values ranged between 0.0 and 0.10 (or equivalently, 0.0% and 10.0%).

5. *Anticipated Annual Net Operating Income.* Specify the schedule of anticipated annual net operating income for each year of the planning period. Based on the empirical data, this value may range in any year from a low of 0 to a high of almost 20 thousand (real) dollars. In this analysis empirically based values were used for pessimistic, average, optimistic and actual (informed) annual net operating income levels for each troller group.

Table XXV summarizes the investment model scenarios definitions used in this analysis.

The analysis of investment model results begins with the specification of the scenario. These inputs are then used to generate the dynamic programming investment policy (e.g., as presented in Tables XIV)

INPUT ITEMS

Troller Group	*Investment-Income Effects*	*Planning Period*	*Survivability Factor, p_0*	*Anticipated Net Operating Income*
All Trollers *	Linear Model	3 years	0.00	Pessimistic
Low Earners		5 years *	0.05 *	Average *
Middle Earners	Nonlinear	7 years	0.10	Optimistic
High Earners				Actuals

Table XXV – Investment Model Scenario Definitions

* Denotes Base Case scenario values. See also Tables XXIV and XXVI.

for each year of the planning period beginning in 1973. This investment policy is assumed to be followed by each troller of the group defined in the scenario. Next, the policy results are used in a simulation of the actual fishing seasons beginning in 1973 for each troller of the group. The troller's initial position is established (from the troller specific data) and an initial investment is made using the dynamic programming derived policy for the group. The seasonal net operating income for the 1973 season following the initial (beginning of year) investment is simulated based on the revised earning power of the troller (due to the new investment) and a random disturbance related to the 1973 income for the group. The troller's cash and net worth position are updated and the investments for the next period are established using the policy for 1974 and the updated position of the troller. The procedure continues for each troller until the end of the planning period.

For the purpose of the analysis of model sensitivity no updating of the derived investment policy takes place as time evolves. Instead, the derived policy at the initial starting point is assumed to apply to each period over the planning period. It is implicitly assumed that at the end of the planning period either a new policy is determined (given the status of the operation at that point) or the operation ceases, e.g., is sold out. Specifically, the decision maker does not revise the net operating income anticipated for the remaining years of the planning period. The revision of investment policies and updating of income expectation based on the most recently observed average income levels during the planning period is taken up later in this section.

Results reported for each scenario are the average troller investment by type and the average troller cash and net worth positions for the specified troller group at each year of the planning period. Table XXVI presents these results for the linear and nonlinear models using the base case scenario defined in Table XXV. The following paragraphs present the results of the investment model scenario analyses for the linear and nonlinear investment-income effects models.

Average Values for 100 Trollers†

(a) Linear Model Results

Simulated Year	*Type 3 Investment Gear/Electronics*	*Net Operating Income*	*Year End Net Worth*	*Year End Cash Balance*
1972	0.000		20.000	0.000
1973	0.000	11.388	28.464	5.446
1974	3.944	5.926	29.570	6.552
1975	4.337	3.526	28.384	5.366
1976	4.230	8.541	28.716	5.698
1977	0.000	7.664	28.375	5.357
Annual Averages	2.502	7.500	28.702	5.684

(b) Nonlinear Model Results

Simulated Year	*Investment Type* * 1	2	3	4	*Net Operating Income*	*Year End Net Worth*	*Year End Cash Balance*
1972						20.000	0.000
1973	0.976	0.000	0.000	0.000	11.356	28.563	5.270
1974	0.000	5.477	0.070	0.000	5.910	29.571	6.278
1975	0.000	8.343	3.061	0.000	3.470	27.475	4.183
1976	0.000	0.000	1.816	0.000	8.111	27.670	4.377
1977	0.000	0.000	0.000	0.000	7.628	27.027	3.734
Annual Averages	0.195	2.764	0.989	0.000	7.500	28.061	4.768

Table XXVI – Investment Model Results‡

‡The scenario definition for these results is given in Table XXV.

†All values are in thousands of real (1971=100) dollars.

* The definition of each investment type is reported in Section 5.4.

LINEAR MODEL SENSITIVITY RESULTS.

In the development of the investment algorithm for the linear form of $f(s_k^i)$, the investment–income effects function, it was assumed that investments of any type could be arbitrarily large. The result of this simplifying assumption was that linear model investments in any period are made in only one type of capital. The consequence of this is that model results could exceed the empirical upper bounds implicit to each capital type. In such cases those investments were limited to be equal to a value less than the empirical upper bound. This model upper bound was set to reflect more closely the size of average investment behavior by capital type.

The simplifying assumptions and adjustments to the linear model mean that the subsequent results for this model do not compare well with actual investment data. Nevertheless, the sensitivity of the linear model results to key input parameters is instructive for understanding the impact of these inputs on actual investment.

Figure 11 presents linear model investment results in graphical form for selected scenarios. The planning period for all illustrated scenarios is 5 years beginning in 1973 and the troller group is the set of all 100 trollers. Each figure illustrates investment results for four levels of anticipated annual net operating income. Figure 11(a) has a survivability factor value, $p_0 = 0.0$ while Figure 11(b) has a survivability factor value, $p_0 = 0.1$. Model generated investments for all scenarios occur only in one capital type, namely, Gear and Electronics (type 3).† The following observations are based on an examination of the results of Figure 11.

Figure 11 shows that investment on average increases when p_0 increases, *ceteris parebis*. However, the size of the increase is small in relation to the change in the survivability factor. Moreover, the pattern of average investment over time for the different p_0 values is very similar. Consequently, investments appear to be relatively insensitive to changes in p_0.

Investment patterns and average amounts are much more sensitive to changes in the anticipated annual net operating income values. When income levels anticipated in future years are at or below the current level of outstanding obligations (interest, repayment, and taxes) no model investment takes place in any period. Beyond this cut–off income value, investment patterns change rapidly. For the anticipated income scenarios of Figure 11, 'pessimistic' values are 94% of 'average' values ($7500 versus $8000). Anticipated annual income values of $7000 and below resulted in no model investments for any p_0 value up to 10%, *ceteris parebis*. It is significant to note from Figure 11 that the small increase in anticipated income from 'pessimistic' levels to 'average' levels results in roughly 3 times the total average investment spread over the

† For the linear model, all scenarios resulted in only type 3 capital investments. This is a consequence of the constant and linearly dominant discount and investment-income effect parameters for type 3 capital.

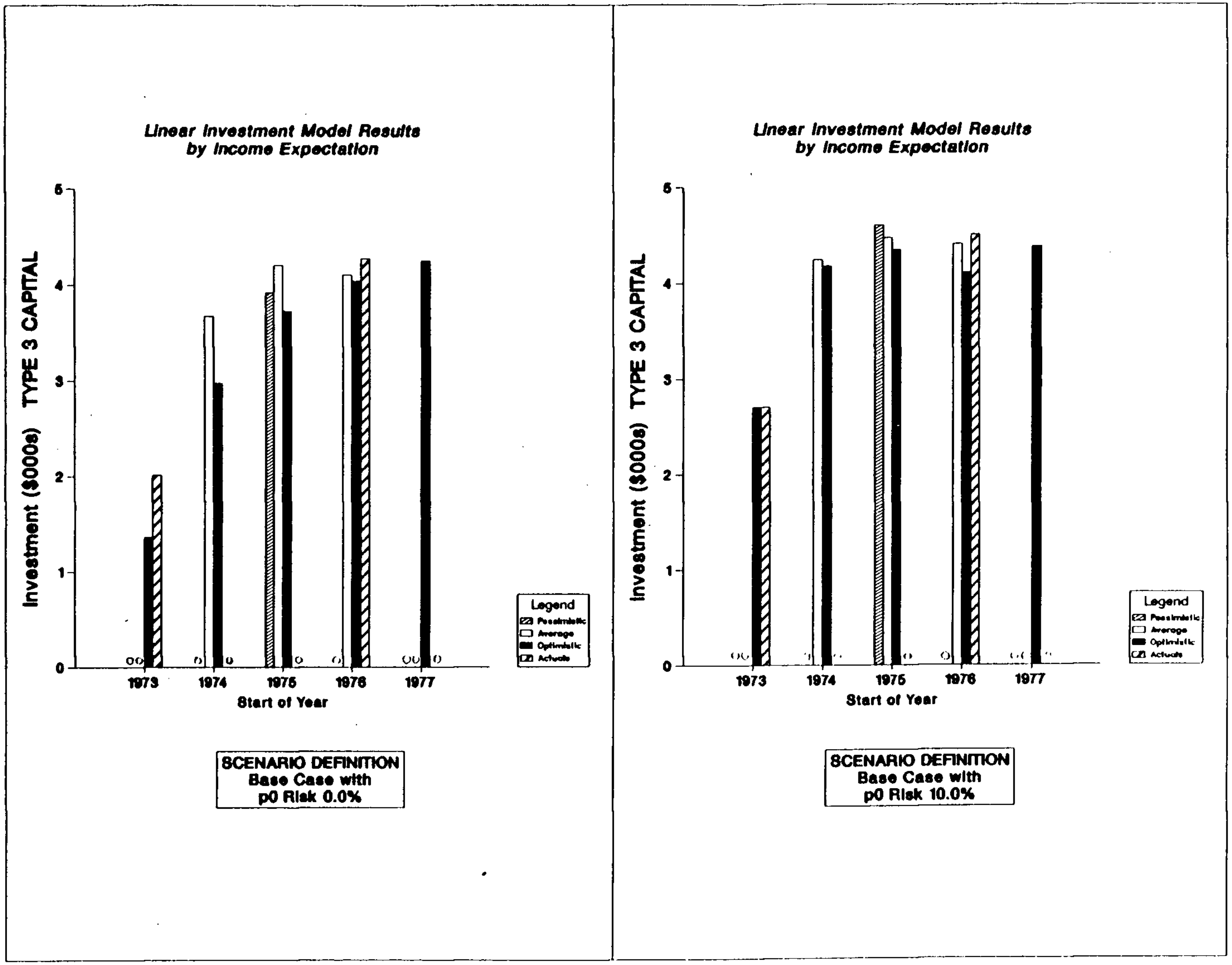


Figure 11 – Linear Investment Model Results by Income Expectation

5 year planning period.

In particular, pessimistic income expectations, or 'low' anticipated net operating incomes (i.e., relative to actual average annual net operating income levels for trollers over the period 1970 to 1980) result in a single investment in simulated year 1975 (for both p_0 values). The two years prior to 1975 are relatively 'good' income seasons. (See also Figure 9.) During these years, trollers with low income expectations actually improve their average net worth position (by making no new investments and increasing their cash balances), until at the start of 1975 their current positions have improved enough that it becomes attractive to invest. (For $p_0 = 0.1$ or 10%, this investment represents the largest single period average investment of all Figure 11 scenarios.) Since 1975 is a relatively poor income year average net worth actually declines over the year and no new investments are made thereafter out to the end of the planning period.

When anticipated income is increased to the average value for the period 1970–1980, ($8000 per year in real terms), the average position of trollers at the beginning of 1973 does not make any investment attractive. However, following the high income year of 1973, investment becomes attractive at roughly equivalent average amounts for 1974 through 1976. New investments and fluctuating incomes over this period (see also Figure 9) maintain the average cash and net worth positions of the group at 1973 levels or slightly higher. Finally, the increased financing load of the new investments to 1976 and the high cost of initial year financing of new investments no longer make investment attractive for the last year of the planning period.

As optimistic income levels are reached ($15000 per year) investment increases still further and average values approach the preset upper limit on type 3 investments in each period. Increased investment resulting from more optimistic future income expectations actually leads to decreased average cash and net worth positions relative to the less optimistic income expectations. This is a consequence of the average operation being required to service more debt.

When actual average net operating income values for each year of the planning period are assumed known in advance, a different pattern and level of investment occur. Model investment now occurs only prior to the 1973 and 1976 simulated seasons which correspond to the highest actual average net operating income years of the planning period, 1973–1977. (See also Figure 9.) However, the average levels of cash and net worth are not appreciably better than those achieved under the pessimistic income scenarios. Of interest here is the fact that the actual average net operating income for the period 1973–1977 is approximately equal to that assumed under the pessimistic scenario ($7410 versus $7500).

Changing the length of the planning period also effect average model investments. In general, the shorter the planning period, the earlier investments occur, and the larger these investments tend to be in average annual terms. This result is a consequence of the less restrictive survivability constraint on investment over all years of the planning period. Higher investments in the shorter planning periods mean larger debt loads and lower average annual cash and net worth positions. Investment in longer planning periods is initially delayed until the terminating year is closer (and, consequently, the survivability constraints become less restrictive). If all investors behave accordingly, then the result is higher average annual cash and net worth positions in comparison to those made under shorter planning periods.

The size of model investments differ according to the particular troller group under study, although the time pattern of investments remains the same. Low income earners invest less than higher income earners. This is a consequence of this group's lower initial average net worth position (making them more vulnerable to bankruptcy) and in spite of anticipated income ('average' for all trollers) which is high to members of this group. Not surprisingly, middle income earners perform according to the average for all trollers. For this group the initial average net worth position and their average annual net operating income is slightly below the overall average. As a consequence, the 'average' anticipated income level assumed is more attractive to this group which translates to higher investment (relative to that for all trollers). Conversely, higher income earners do not view the 'average' anticipated income as very attractive to investment since their actual average net operating income level is much higher. Investment by this group occurs more because of their advantageous initial average net worth position and is only slightly above the overall average investment levels for all trollers.

Table XXVII summarizes the linear investment model sensitivity analysis results relative to the base case scenario results presented in Table XXVI. The base case assumes an 'average' level of anticipated income, a 5 year planning period, a p_0 value of 0.05 (or 5%) and all 100 trollers. Reported results are in terms of the percentage differences in average annual: investment, net operating income, net worth and cash balance positions versus the corresponding base case results.

NONLINEAR MODEL SENSITIVITY RESULTS.

The investment algorithm for the nonlinear (quadratic) form of the investment-income effects function allows for simultaneous model investments in more than one type of capital. Furthermore, no artificial bounds on investment by capital type is required for the nonlinear model since the concave quadratic function combined with the survivability constraints limit model investments by type to empirically justifiable ranges. These relaxations (*vis-à-vis* the linear model discussed above) render the nonlinear model more realistic. Consequently, the ensuing results are more directly comparable to actual investment behavior. The comparison of model and actual investments will involve tuning key model components. This comparison will be facilitated by first investigating the sensitivity of the nonlinear model results to changes in the key input data.

Summary of Results †

Input Variable	*Variable Value*	*Ave Annual Investment*	*Average Annual Net Op Income*	*Ave Ending Net Worth*	*Ave Ending Cash Balance*
Planning Period	3 years	+17.9%	-6.7%	-16.7%	-11.1%
	5 years *				
	7 years	-30.5%	+20.3%	+13.7%	+38.4%
p_0	0.00	-13.0%	0	+0.8%	+4.1%
	0.05 *				
	0.10	+27.9%	0	-0.1%	-0.7%
Income Anticipd	Pessimistic	-64.6%	0	+3.7%	+14.8%
	Optimistic	+49.3%	0	-3.4%	-17.4%
	Actual	-47.3%	0	+1.7%	+8.9%
	Average *				
Troller Group	Low Earners	-48.1%	-70.4%	-62.9%	-92.4%
	Middle	+10.9%	-7.9%	-5.4%	+1.3%
	High	+17.1%	+81.1%	+83.1%	+83.1%
	All Trlers *				

Table XXVII – Linear Model Sensitivity Analysis

†All values are percentage differences reported relative to the Base Case results for single variable input changes in the Linear Model reported in Table XXVI.

* Denotes Base Case variable values. See also Tables XXIV–XXVI.

Figures 12-15 present nonlinear investment model results in graphical form for a subset of the scenarios given in Table XXV. As for the linear model sensitivity analysis, the nonlinear model results are discussed relative to the base case scenario defined by the data of Table XXIV, and the nonlinear model base case results given in Table XXVI.

The following observations are made from examining Figures 12-15 and the nonlinear model sensitivity analysis relative to the base case results as presented in Table XXVIII.

The nonlinear model results illustrated in Figures 12-15 show no new investment for type 4 capital ('Other Equipment'). Examination of the investment-income effects parameters (Table XXIV) reveals that type 4 capital is dominated by all other capital types. The high negative q_1^4 value and the low positive q_0^4 value relative to the corresponding values of the other types demonstrate this domination.

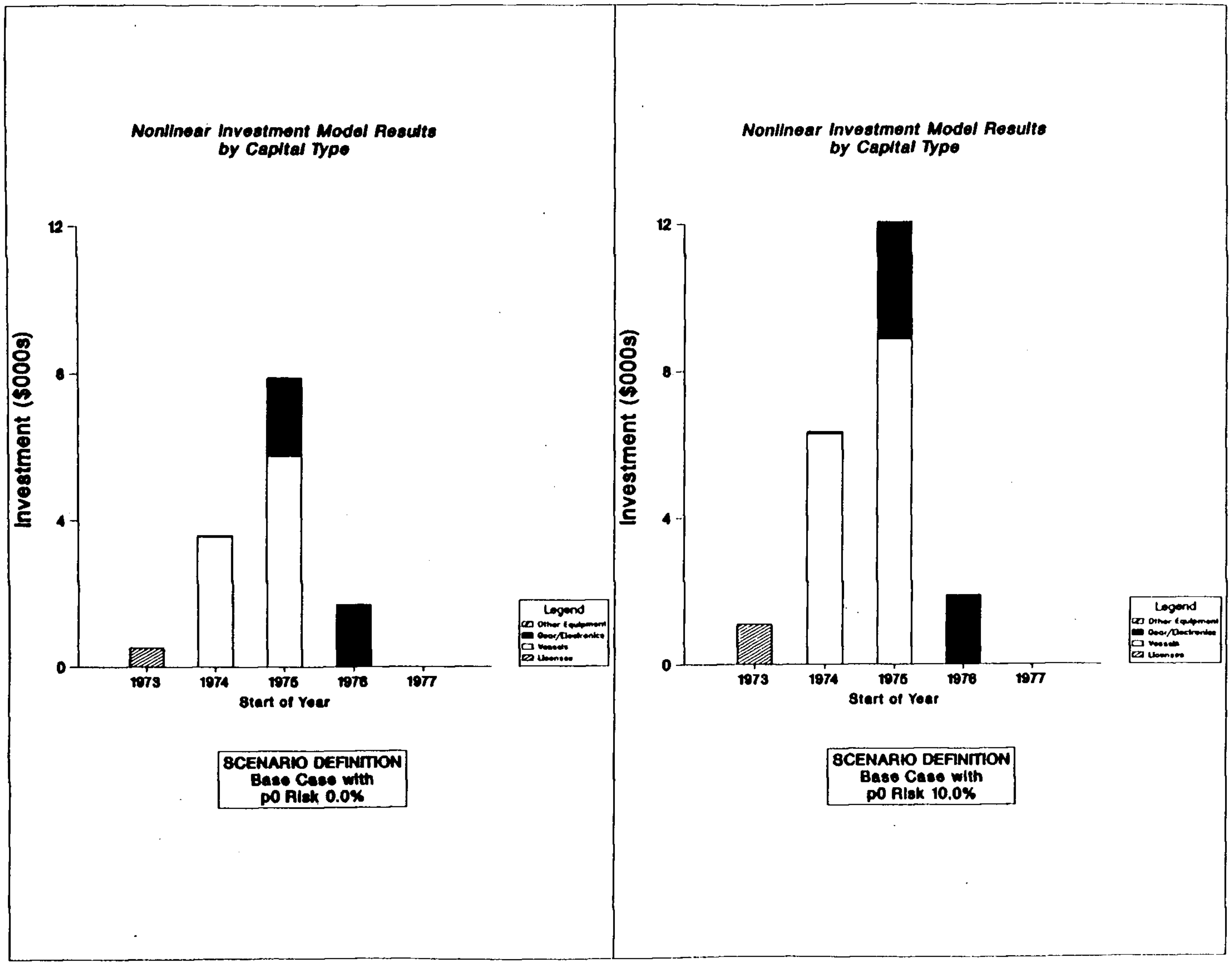


Figure 12 - Nonlinear Investment Model Results by Capital Type
- Risk Level Scenarios

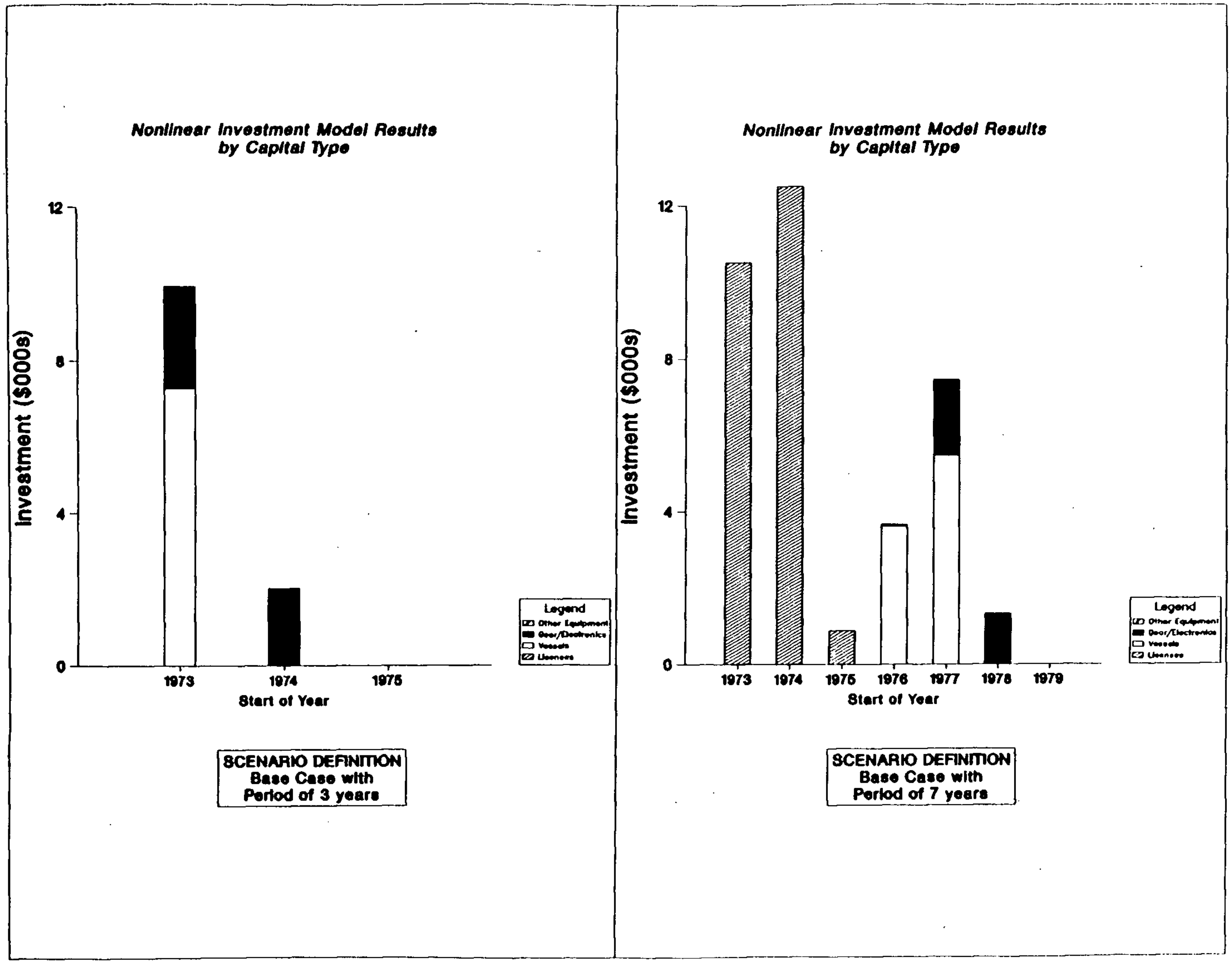


Figure 13 – Nonlinear Investment Model Results by Capital Type
– Horizon Length Scenarios

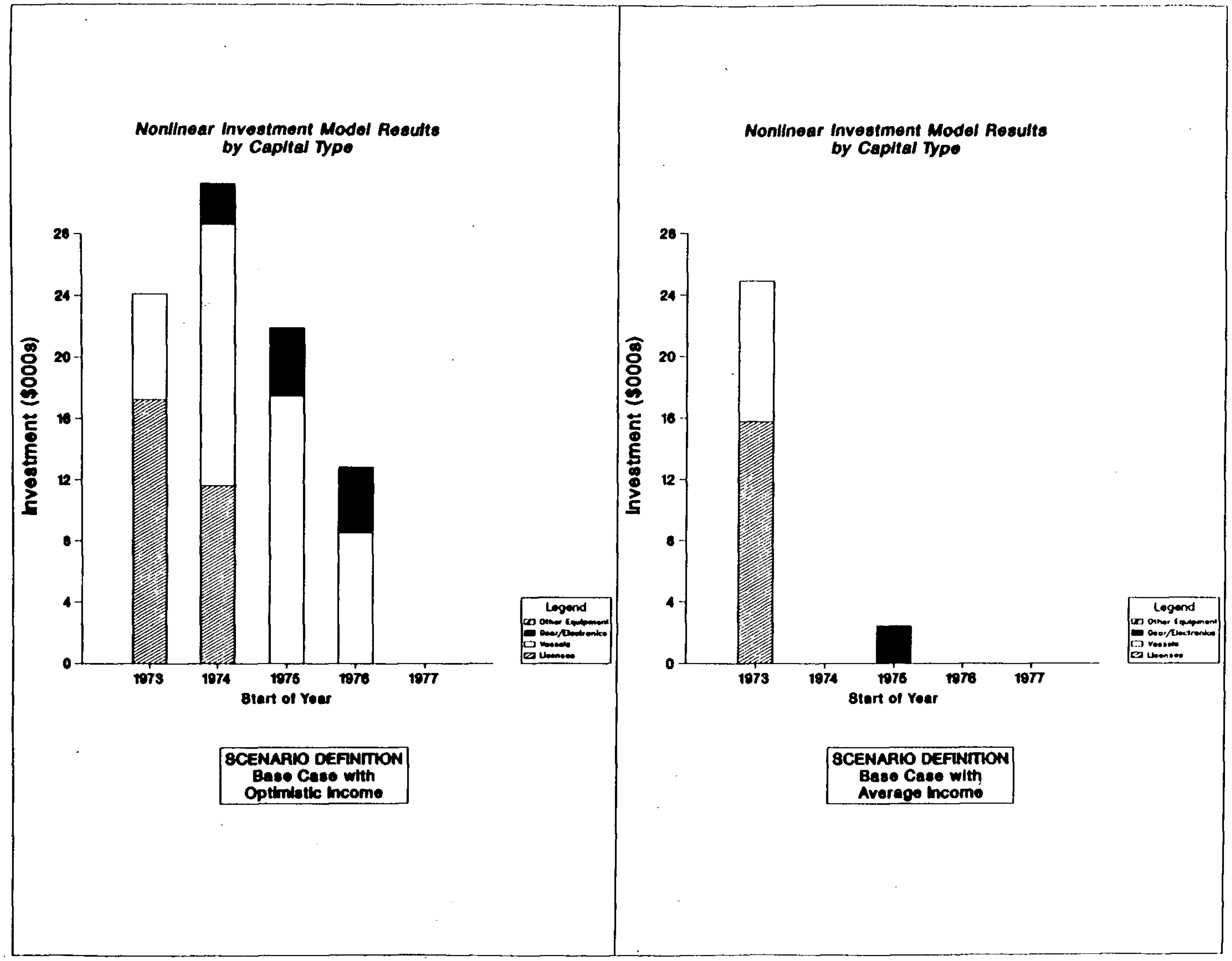


Figure 14 – Nonlinear Investment Model Results by Capital Type
– Income Expectation Scenarios

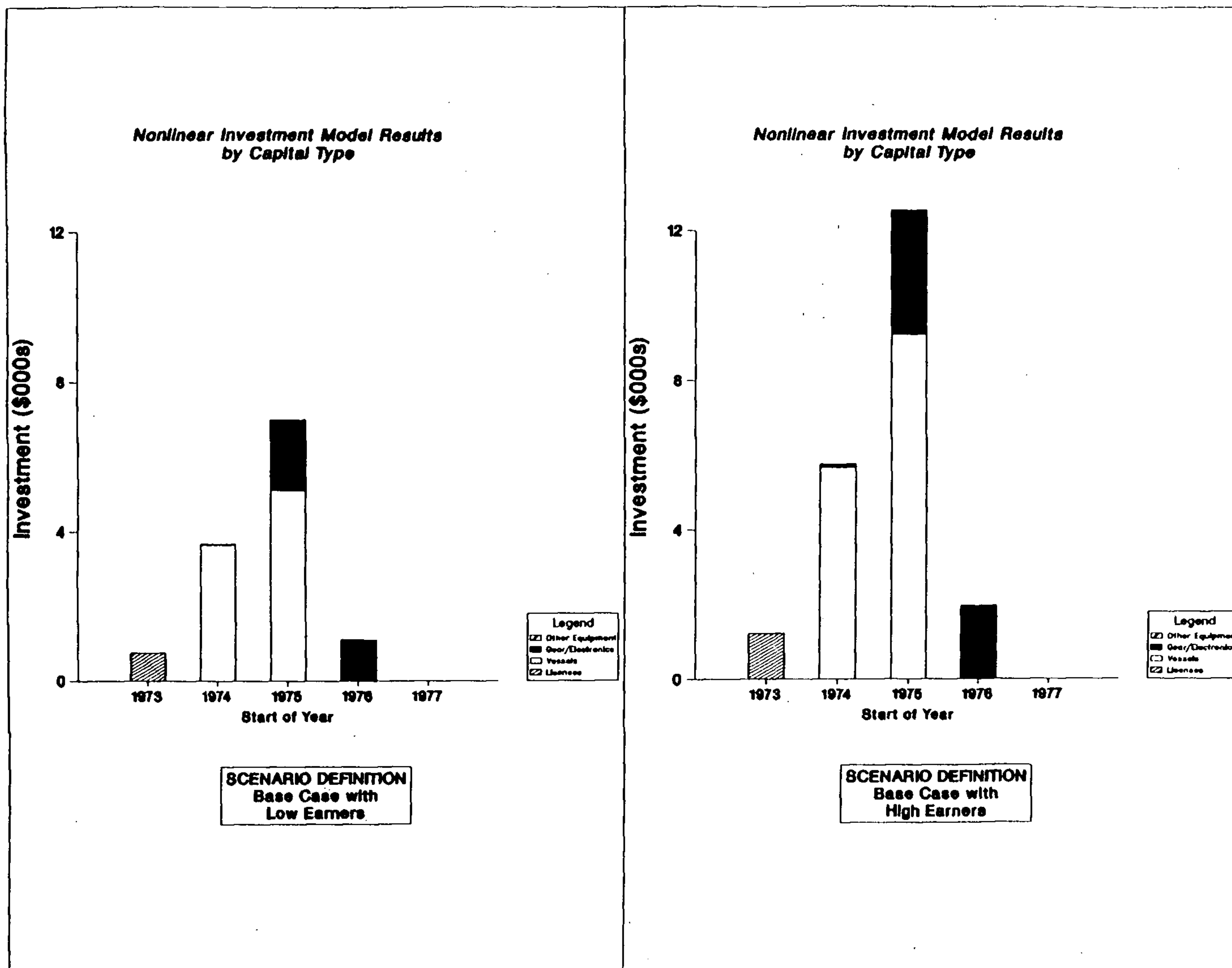


Figure 15 – Nonlinear Investment Model Results by Capital Type
– Troller Group Scenarios

Summary of Results †

Input Variable	*Variable Value*	*Ave Annual Investment* 1	2	3	4	*Ave Net Opg Inc*	*Ave Net Worth*	*Ave Cash Balance*
Planning	3 years	-100.0	-12.5	58.5	–	-6.7%	- 8.7%	-11.6%
	5 years *							
Period	7 years	1653.3	-53.0	-51.1	–	+20.3%	+10.5%	-101.4%
p_0	0.00	-46.8	-32.9	-21.7	–	0	2.5%	14.5%
	0.05 *							
	0.10	12.1	9.6	4.1	–	0	-0.6%	-3.5%
Income	Pessimistic	-100.0	-98.2	-61.3	–	0	7.7%	45.2%
Anticipd	Optimisitic	2852.0	260.1	130.1	–	0	-43.9%	-893.5%
	Actual	1517.0	-33.6	-50.7	–	0	-14.7%	-86.6%
	Average *							
Troller	Low	-23.6	-36.9	2.4	–	-70.4%	-64.6%	-101.5%
Group	Middle	36.5	11.3	6.6	–	-7.9%	-9.6%	11.4%
	High	25.0	7.6	8.5	–	+81.1%	75.3%	93.4%
	All Trlers *							

Table XXVIII – Nonlinear Model Sensitivity Analysis

†All values are percentage differences reported relative to the Base Case results for single variable input changes in the Nonlinear Model reported in Table XXVI.

* Denotes Base Case variable values. See also Tables XXIV–XXVI.

In all scenarios of the figures no investment takes place in the final year of any planning period. Investments in this final year do not contribute to increasing the expected net worth to the end of the planning period. In general, the additional earning power in the first year after a new investment is outweighed by the higher cost of financing the investment in its first year. This negative effect on expected net worth discourages investment in the last year before the end of the planning period.

It is significant to note from Figures 12-15 that new model investment in type 1 capital (eligible capital–licenses) occurs only in the earlier years of all planning periods. This is also a consequence of the investment-income effects parameters for licenses (Table XXIV). The high relative discount factor, ρ_1 and the more positive initial effects parameters, q_0^1 and q_1^1 results in no deterioration of the earning power effect for license capital. Consequently, the benefits to net worth are realized over a longer period making these investments attractive at a certain distance from the end of the planning period. When the planning period is short, e.g., 3 years, license investments do not have enough time to build up benefits on the terminating net worth to warrant investment (Figure 15). However, for longer planning periods, e.g., 7 years, license investments dominate all other types in the beginning years (Figure 15). In actuality, the purchase of new licenses often means that a change is required in fishing operations and skills, i.e., new areas are now open to fishing, or new fisheries are now available (e.g., herring). In such cases it takes some time before fishermen become efficient, to learn the new skills, or to familiarize themselves with new fishing grounds. Once this initial learning period has passed the net effect on the operation is a positive economic one.

The largest model investments of Figures 12-15 occur in type 2 capital, vessels. Moreover, investments in vessels occur in all scenarios analysed. In comparing the parameters of the investment-income effects model vessels have a more positive initial effect (q_0^2, q_1^2) than licenses, but this effect is more heavily discounted (ρ_2). Thus, vessel investments make a larger and more immediate impact on net worth. A consequence of this is higher investment in the interim years of each planning period.

The pattern of model investment in type 3 capital, gear and electronics is likewise a function of its investment-income effects parameters. From examining Figures 12-15, these investments make a more immediate impact on net worth since they occur near the end of each planning period. This is a result of higher initial effects on income earning power (through q_0^3 and q_1^3) tempered by faster deterioration over time through ρ_3 (relative to ρ_1 and ρ_2).

The above discussion points out the key role of the parameters q_0, q_1, and ρ from the investment-income effects model. Moreover, if these values change then investment results will be affected accordingly.

As was the case for the linear model sensitivity results, changing the survivability factor, p_0 only slightly effects model investment. For example, Figure 12 shows that the type and patterns of investment remain the same as p_0 ranges from 0.0 to 0.1. However, the size of investments increases as p_0 increases reflecting the greater willingness to invest by decision makers with higher risk positions. The effects on the average annual cash and net worth positions due to differences in p_0 alone are not dramatic (Table XXVIII).

Figure 13 indicates that investment is highly sensitive to changes in the anticipated level of annual net operating income over the planning period. As for the linear model, when income expectations exceed a minimum value (which is approximated by the level of annual obligations outstanding) then new investment takes place. At higher levels of anticipated income a significant increase in investment occurs. (See also Table XXVIII.) Moreover, average annual cash and net worth positions fall with higher income expectations, *ceteris parebis*.

When anticipated income is set to their actual average values over the planning period the pattern of investment is very different. In this case investments occur at the beginning of only 2 years, 1973 and 1976. These years are the years of highest actual average income over the planning period. Investment is significantly increased in licenses but cut back in vessels and gear and electronics relative to base case investments (Table XXVIII). The result is a marginal decrease in the average annual net worth position of the troller group *vis-à-vis* the base case values.

Illustrated in Figure 14 are the investment patterns for different troller groups, namely, low and high fishing income earners. The investment patterns displayed here are comparable to those of Figure 12. As well, the observations made for the linear model investment by group results also apply here. Low earners being affected by a less advantageous net worth position, invest less than middle or high earners in spite of the attractive anticipated income to the members of this group. Conversely, high income earners (as a consequence of their more secure net worth positions) invest more in spite of the less attractive anticipated income level to the trollers of this group.

Finally, Figure 15 presents the results of scenarios with planning periods of 3 and 7 years in length. When the shift in years relative to the base case scenario (5 year planning period) is accounted for, the pattern of model investment is similar among all planning periods. Specifically, the 3 year scenario investments are similar to that of the last three years of the base case results (see for example Figure 12). The final 5 years of the 7 year planning period investments are likewise similar to the 5 year base case results. In the 7 year scenario the initial 2 years have significant investments in license capital reflecting the greater benefits to expected net worth at the terminating year of this longer planning period.

The following subsection uses the insights of the sensitivity analyses to model the actual investment behavior by trollers over the period 1973–1982.

Model versus Actual Results. This subsection compares model generated average annual investment per troller with actual values for each year from 1973 to 1982. Actual average annual investment by trollers is obtained from the troller annual cash report database (Table XVIII) and is illustrated in Figures 16-19. Average annual investment is recorded by capital type for each of the four troller groups: all 100 trollers (Figure 16); low income earners (Figure 17); middle income earners (Figure 18); high income earners (Figure 19).

Model results for each of the troller groups were obtained by first 'tuning' parameter values of the input data. In particular, the investment-income effects parameters were initially set equal to the empirically derived nonlinear model values (as given in Table XXIII); the planning period was fixed at four years; the survivability factor, p_0 was set to 5.0%; and the anticipated annual net operating income for each year of the first planning period (beginning in 1973) was set to $8500. Investment policy updating was assumed to occur after every year. Average actual net operating income for the current year was averaged with the current anticipated net operating income value (at the beginning of the same year) to result in a new anticipated annual net operating income value for the upcoming years. This new value was then used to determine a new investment policy. Annual updating and new policy development continued from 1973 to 1982.

Next, the results of model generated average annual investment per troller from the above procedure were compared directly with the actual values for the period 1973 to 1982. Since model investment differences from year to year depended on the updating of the anticipated net operating income, then discrepancies between model versus actual average investment could be corrected by adjusting model parameters to replicate the actual investment response. The investment effect of changes to the anticipated net operating income was tuned by alternatively varying the parameters of the financing matrix, B, and the q_0^i values of the investment-income effects function. Decreasing q_0^i decreases investment in type i capital, *ceteris parebis* and dampens the impact of changes to the anticipated net operating income. Decreasing the short-term and/or long-term financing requirements of capital of type i causes increased investment in this capital. Moreover, it was found that adjustments to these parameters for each capital type i resulted in roughly independent investment effects for small perturbations, i.e., the investments generated by the model for type i capital which did not undergo parameter adjustment were essentially unchanged. Accordingly, adjustments were made alternatively to these input values for each capital type until the model generated results approached those for the actual investment values from 1973 to 1982.

Other procedures for 'tuning' model results to actual values can also be explored. For example, different procedures can be used to update anticipated annual net operating income values, including n-period moving average calculations, constant income values (i.e., no updating), average growth rate values, or specific patterns of income. As well, other model inputs may be varied such as the length of the planning period, the

Figure 16

Actual Average Troller Investment by Capital Type

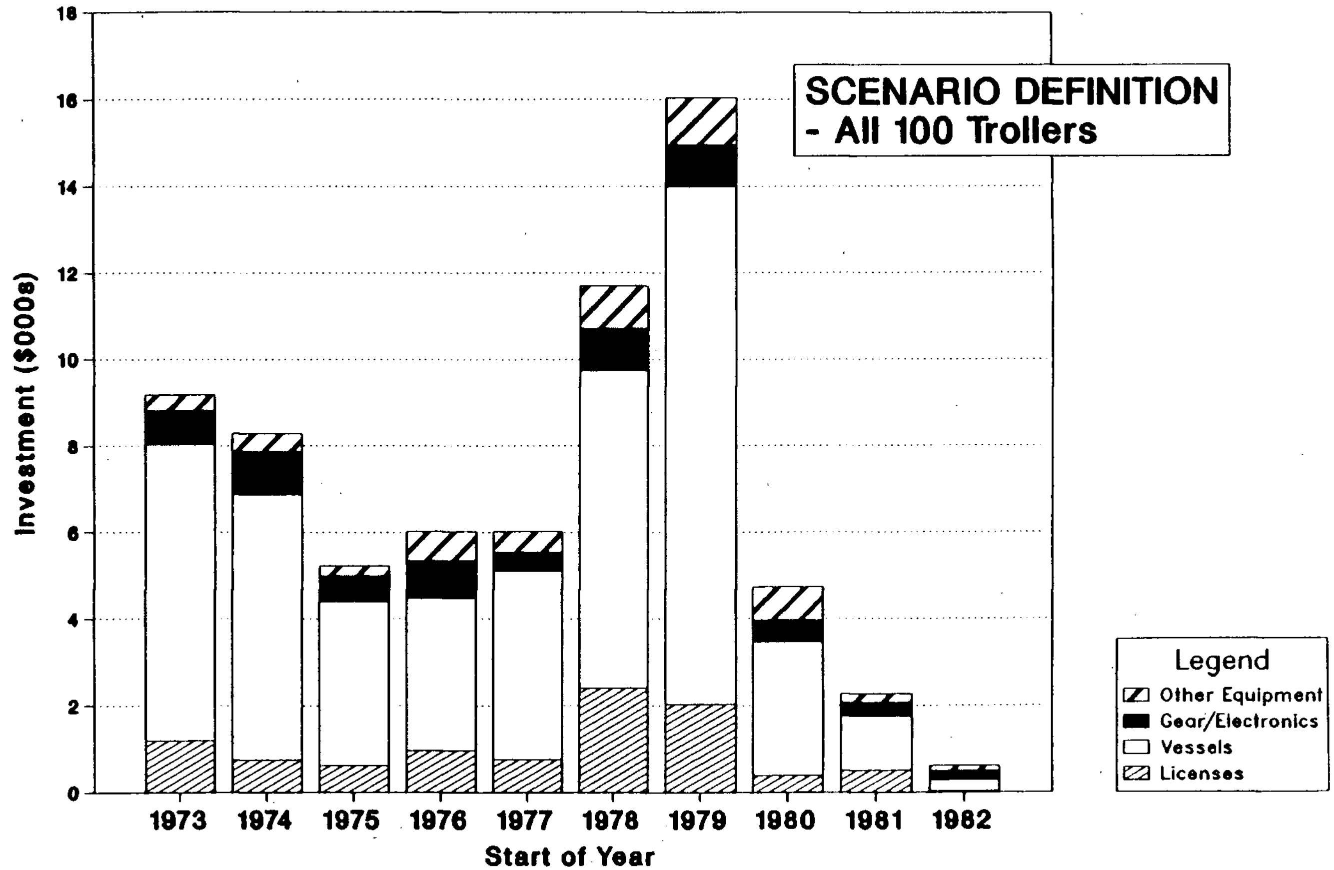


Figure 16 - Actual Average Troller Investment by Capital Type - All 100 Trollers

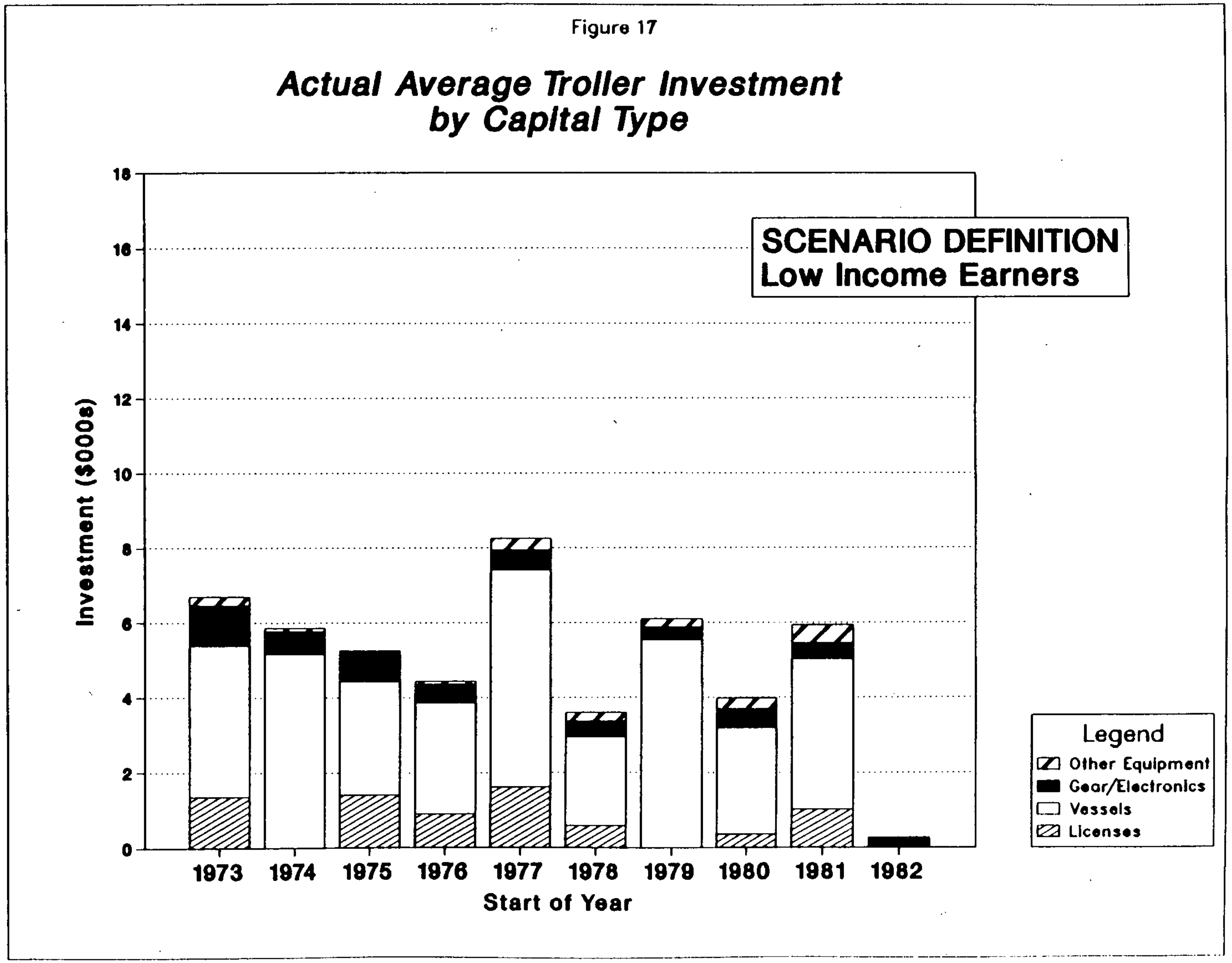


Figure 17 – Actual Average Troller Investment by Capital Type – Low Income Earners

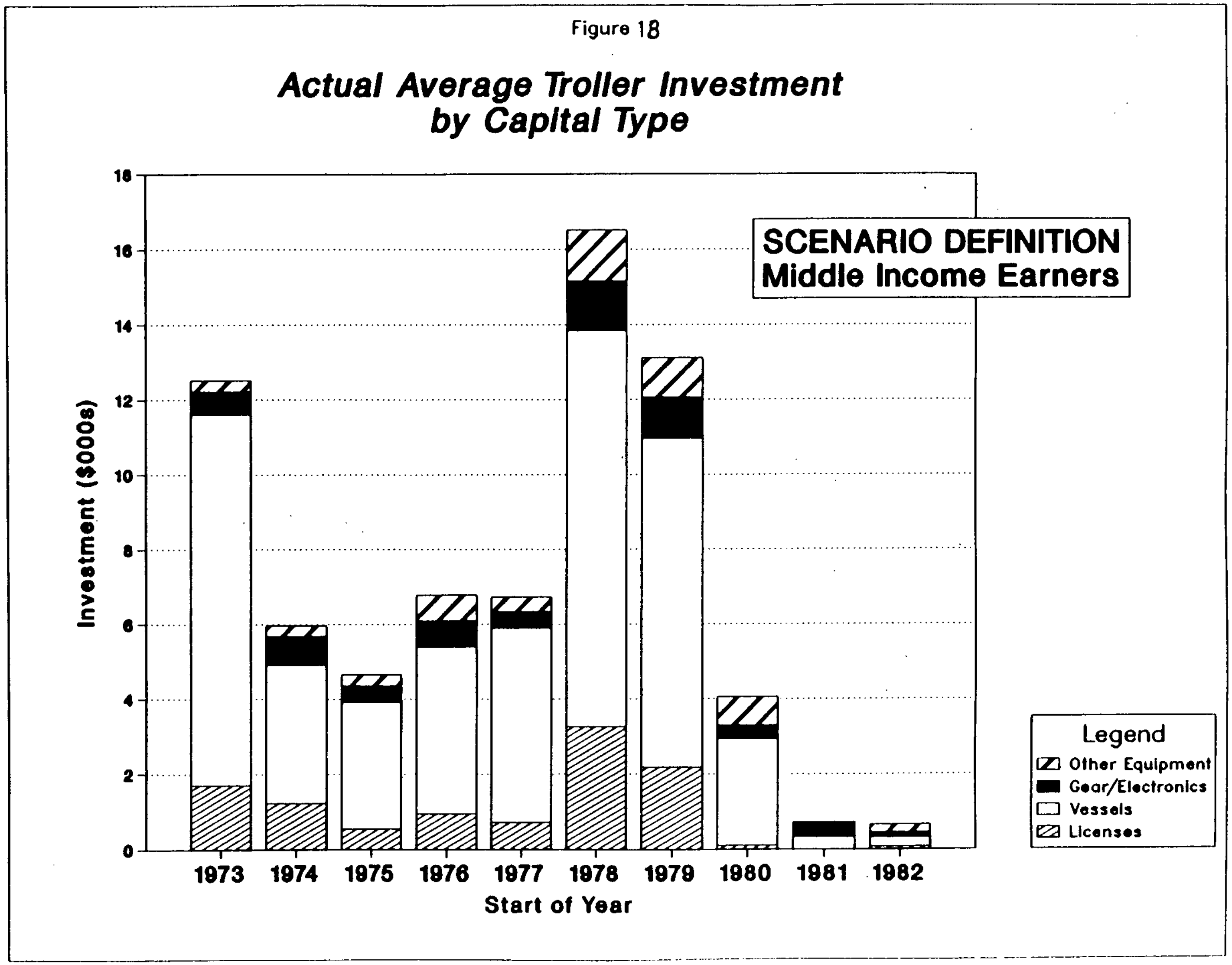


Figure 18 – Actual Average Troller Investment by Capital Type – Middle Income Earners

Figure 19

Actual Average Troller Investment by Capital Type

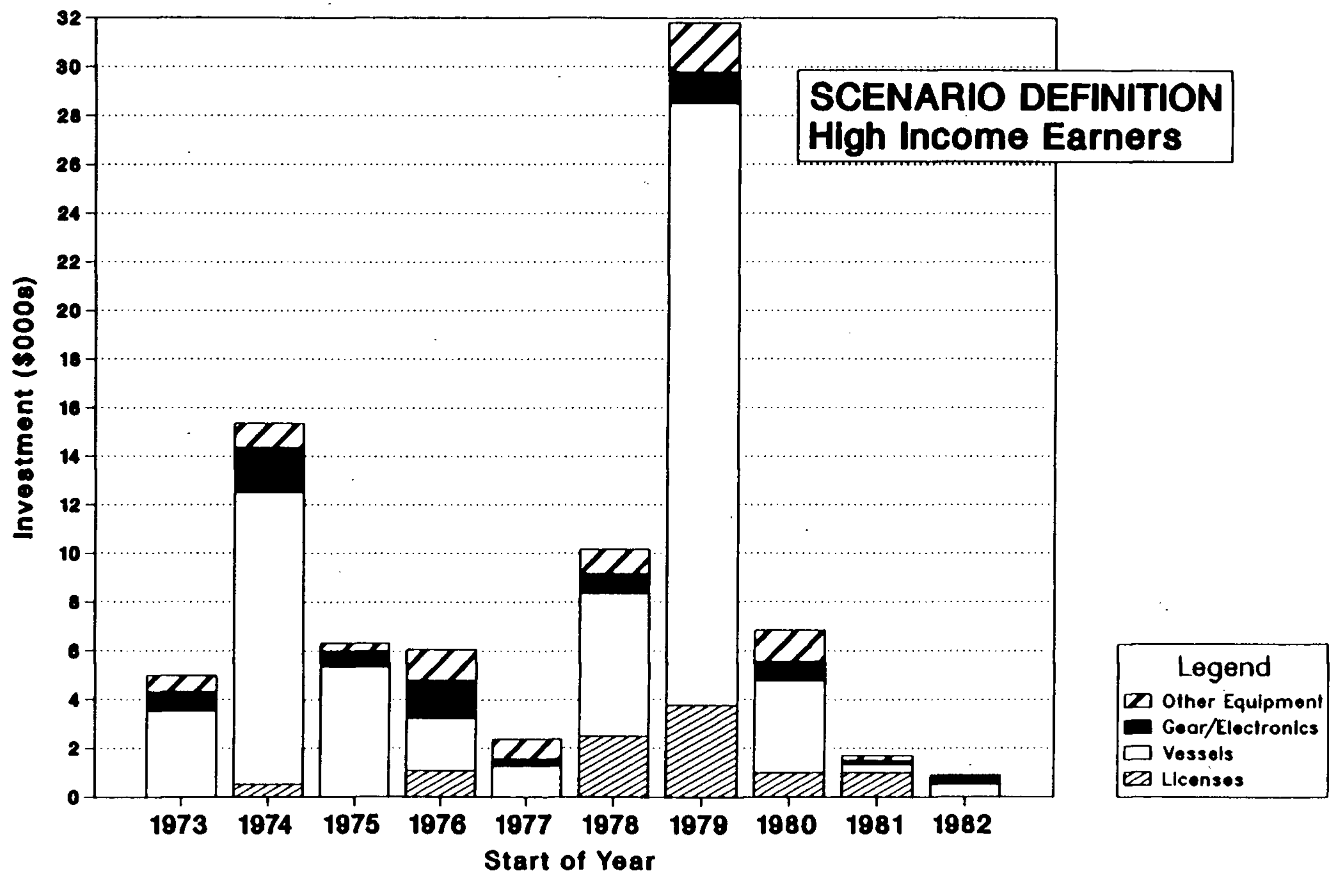


Figure 19 – Actual Average Troller Investment by Capital Type – High Income Earners

survivability factor, or the investment-income effects parameters. Alternatively, a formal analysis may be undertaken to determine explicitly the conditions and extent of input data changes on the model results. The procedure chosen here to demonstrate the tuning of model results is based on the analysis of the sensitivity of the investment model to changes in inputs as presented previously. The results of the comparison of model versus actual investment illustrates the ability of the model to replicate the real system through an appropriate tuning process.

Model generated average troller investments predicted for each of the four troller groups over the period 1973 to 1982 are presented graphically in Figure 20-23. These figures show the results of the tuned models for each troller group analogous to the actual average troller investment presented in Figures 16-19.

For the 'All 100 Trollers' group (Figures 16 and 20), actual total investment (summed over all capital types) compares favorably with predicted model values. The correlation between actual and model results is $r = 0.86$. The annual Mean Absolute Deviation (MAD) is $2,300/year for actual average totals of $7,000/year. Comparison of investment by each capital type for this troller group shows that model investments are biased downward for type 1 capital (eligible capital - licences) and type 2 capital (vessels). Type 3 investments (gear/electronics) have slightly positive bias while type 4 investments (other equipment) are essentially unbiased over the 10 year period. The MAD by capital type is largest for type 2 capital ($1,800/year), in which actual investment is greatest. MAD statistics for types 1,3, and 4 respectively are $449, $728, and $264 per year.

The comparison of middle income earners actual versus model results (Figures 18 and 22) are similar to the all trollers group comparison. The correlation for total average annual investment is $r = 0.95$ with a MAD of $2,200/year on actual average total investments of $7,100/year. Analysis of investment by each capital type shows a slight negative bias in model results compared to actuals. (This bias could be alleviated by further tuning of the model corresponding to this group.) MAD values for each capital type 1–4 are respectively $486, $1,372, $539, and $225 per year.

The tuning process for high and low income earners was less successful than for the all trollers and the middle income earners groups. For high income earners the anticipated net operating income was initially set at $12,000/year which is in line with the actual net operating incomes of trollers in this group. The correlation for total annual troller investment is $r = 0.62$ with an associated MAD of $6,800/year on average total investments of $8,700/year. The largest discrepancy occurred in type 2 investments (MAD=$5,110/year) where actual investments are abnormally high in 1974 and 1979. Attempts at tuning type 2 model investments failed to replicate these anomalous years. MAD values for type 1,3, and 4 capital are $602, $1260, and $1051 per year respectively. A positive bias in model investment occurs for both type 3 and 4 capital

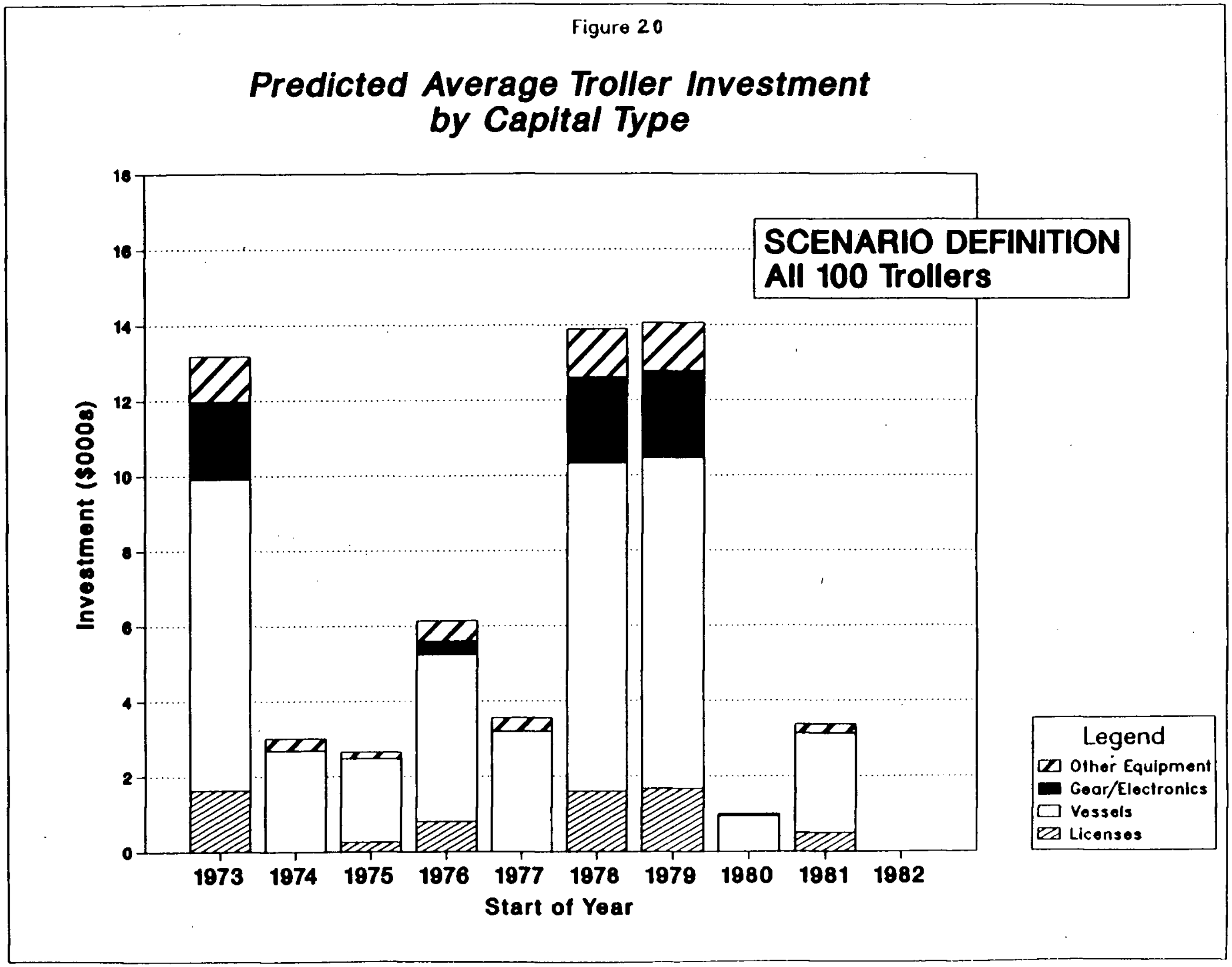


Figure 20 - Predicted Troller Investment by Capital Type
- All 100 Trollers

Figure 21

Predicted Average Troller Investment
by Capital Type

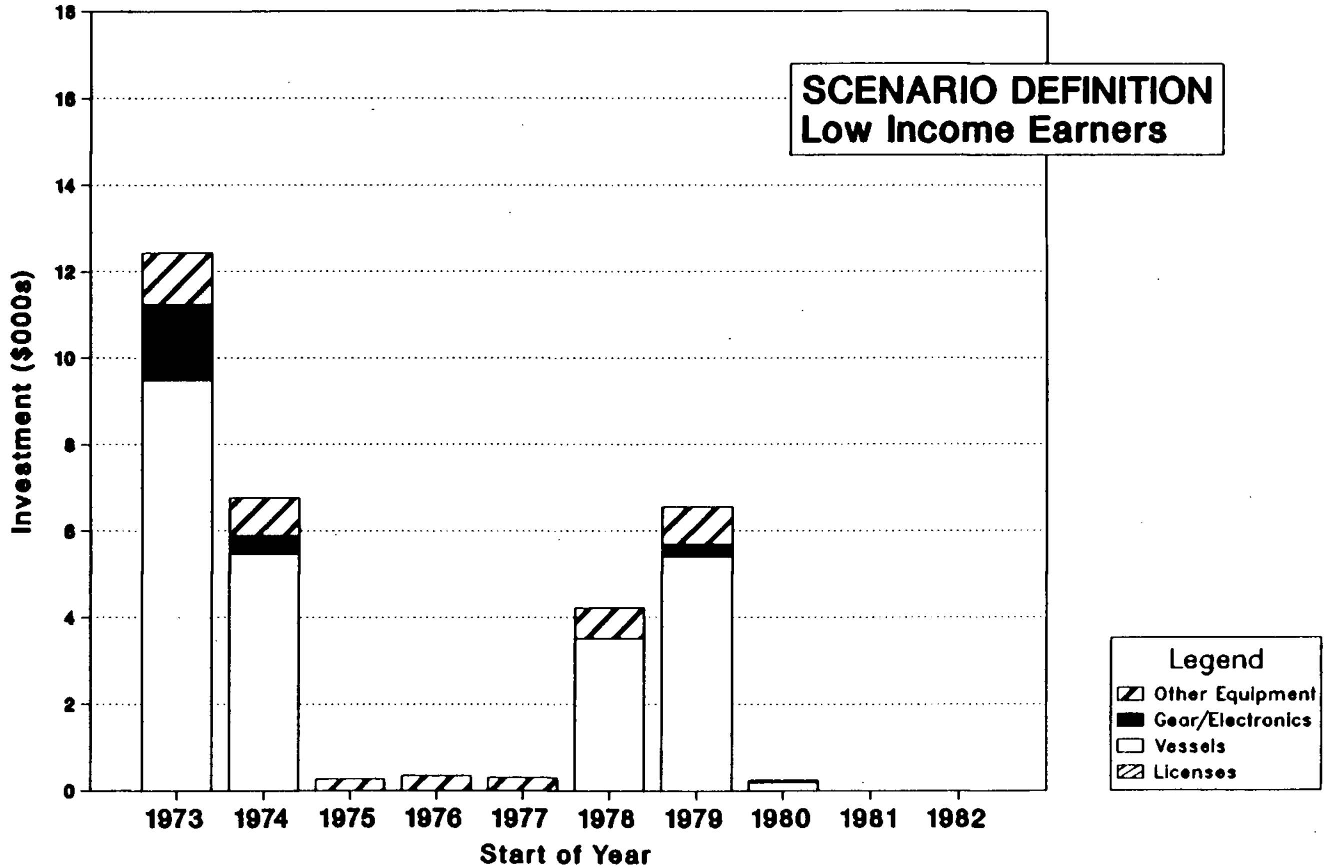


Figure 21 - Predicted Troller Investment by Capital Type
- Low Income Earners

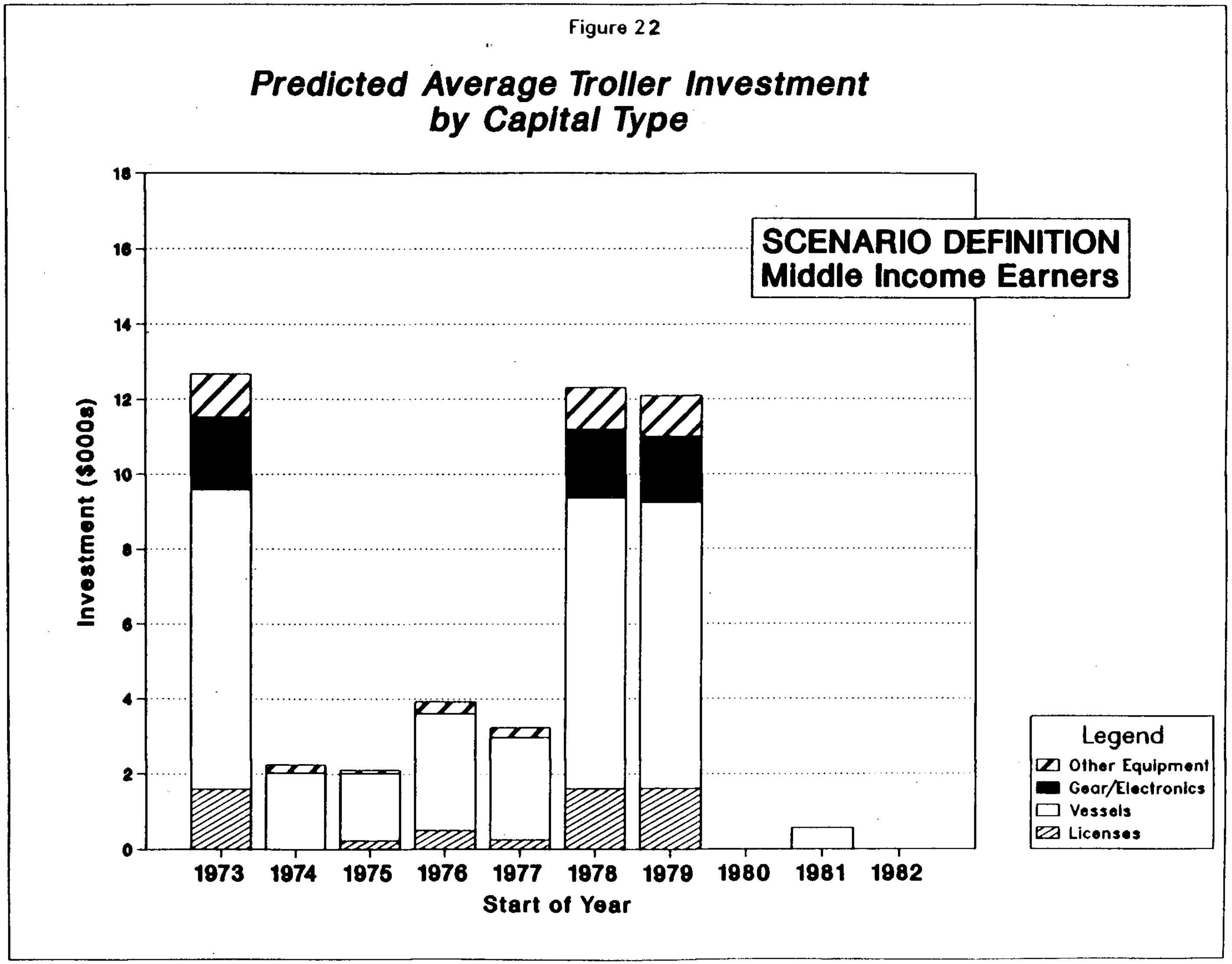


Figure 22 - Predicted Troller Investment by Capital Type - Middle Income Earners

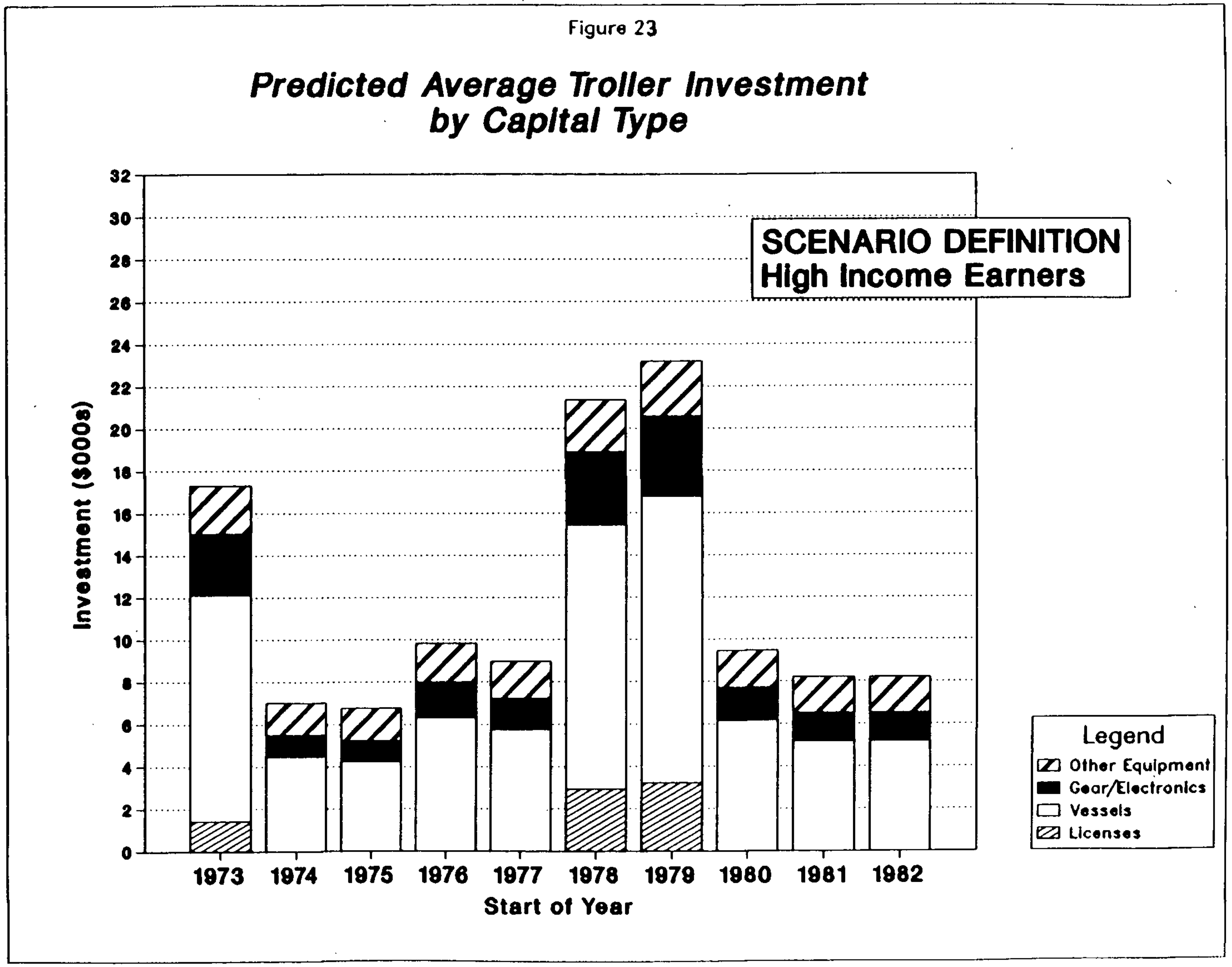


Figure 23 – Predicted Troller Investment by Capital Type
– High Income Earners

over the 10 year period.

Actual average investment by low income trollers shows an uncharacteristic smoothness over the years 1973 to 1982 (Figure 17) relative to other troller groups. This suggests that investment by this group depends less on fluctuations in actual income. For example, average net operating income in 1975 for low income trollers was less than \$50. This amount certainly does not cover average outstanding debt for 1975 estimated at \$3,000 per troller. Yet, average actual investments in 1975 and 1976 remained at roughly equivalent levels amounting to over \$4,000/year. Only in 1982 does actual investment differ substantially from that of all previous years.

As a consequence, tuning the model for this group yielded less satisfactory results. Initially the anticipated net operating income was set at \$3,000/year for trollers of this group which approximates their actual income level. The correlation for total average annual investment is $r = 0.36$ with a MAD of \$3,400/year corresponding to actual total investments of \$5,000/year. Most of the discrepancy occurs for type 2 capital. These are the largest actual investments and are maintained at a constant level of approximately \$4,000/year from 1973 to 1981 (Figure 17). Model investments however (Figure 21) fluctuate as a function of actual and updated anticipated incomes. MAD values by capital type are respectively \$725, \$2546, \$427, and \$424 per year for this troller group.

Finally, from the comparison of actual and model investments insight may be acquired about the actual decision processes of the different troller groups. The above results suggest that each troller group behaves differently with respect to investment decision making. For example, low earners appear to be overly optimisitic about future earnings. (This could be incorporated into the investment model by redefining the updating procedure for anticipated net operating income.) They do not adjust their investment strategies to take account of the actual swings in fishing income. Accordingly, they may build up debt loads which could become critical after a series of low income years. In fact, the expected net worth for an average low income earning troller who follows the actual investment strategy of Figure 17 approaches zero by 1981. This precarious situation actually resulted in the liquidation by bankruptcy of many trollers during this period.

On the other hand, actual investment by high income trollers follows a pattern of large investment in a single year followed by a series of years with no substantial investment. Peak investment periods (e.g., 1974 and 1979 of Figure 19) are followed by sharply reduced investment years. It is interesting to note that the investment peaks occur after a sustained period of relatively good income years, e.g., substantial investment takes place in 1974 after good incomes from 1972 to 1974; likewise, the 1979 investment spike occurs after good incomes were experienced from 1977 to 1979. Based on this observation, high income earners appear to be more cautious about anticipating future incomes. But, when income expectations are reinforced by a series of good years this 'caution' is translated into an investment splurge. (This reaction could be incorporated into the investment model by defining the updating procedure for anticipating net operating income accordingly.) From the results of the investment model, this actual strategy contributes to reducing the expected net worth of the average high income earning troller at the end of 1982. A smoother strategy, as developed by the investment model results in an improved expected net worth position at the end of 1982.

While implications about the actual behavior can be drawn from model versus actual results, actual verification of these implications can only be determined by 'getting inside the heads' of the fisherman decision maker. To this end, there can be no substitute for interviewing, querying, and surveying individual fishermen. The role of a decision making model of the kind proposed here provides a focus for the collection of more practical empirical data. With more data, firmly grounded in a better understanding of the behavioral aspects of investment through the modelling analysis, we will be in a much improved position to evaluate the impact of proposed new schemes aimed at stabilizing and ensuring a viable future for the fisheries.

This completes the analysis of the results for the interseasonal investment decision model. This model's implications for planning and designing regulatory policies are discussed further in the concluding chapter which follows.

6. Perspectives for Planning and Extensions

6.1 Intraseasonal Model

The intraseasonal model begins with a simplified description of salmon stock dynamics. The model incorporates uncertainty in the decision process with respect to two areas: uncertainty about actual stock abundance and uncertainty in salmon catches. The dynamics of decision making are embodied in the model through Bayesian updating of imperfect abundance information throughout the season. Finally, decisions are assumed to be economic-based through the seasonal price and cost structure of the fishery.

The model of the intraseasonal decision making process of fishermen is presented as a normative framework for explaining the within season dynamics of individual fishermen. Moreover, the model provides information on the seasonal fleet dynamics and earning power of fishermen. The determination of fishermen's earning power is important since earnings are directly attributable to investment potential in future years.

As a normative model the POMDP formulation has intuitive appeal in this application. It incorporates the important informational and stochastic elements of the actual environment in which fishermen's intraseasonal decisions are made. Practically however, it is unlikely that individual decision makers actually develop adaptive strategies in the style of the POMDP. Nevertheless, the results for a subset of experienced fishermen operating freezer trollers in the salmon fishery suggest that major aspects of the actual system can be successfully reproduced. For example, the distribution of total seasonal catch and effort by zone, and the average level of gross income from salmon fishing under different conditions for abundance and prices closely approximate actual statistics under like conditions. Since income from fishing may not be restricted to salmon only, the use of this particular application of the model as an overall predictor of income is limited. For instance, some freezer trollers are also licensed for the short but often lucrative herring fishery. The consequences of participating in this and other fisheries affect average seasonal income.

The sensitivity analysis results for this model (Section 4.5) reflect key aspects of the system in which fishermen's decisions take place. The anadromous nature of migrating salmon and the current method of regulation of salmon stocks by 'total allowable catch' or 'constant escapement' policies contributes to diminishing variation in stock abundance levels. The result is a relatively stable level of abundance and catch from year to year. As the model illustrates, stock abundance (and catch) are not major determinants of fishermen's intraseasonal decisions or of their incomes. This is especially valid in a fishery with limited entry regulations such as the British Columbia commercial salmon fishery.

The sensitivity analysis also shows that landed value prices are a much more important determinant of fishermen's decisions and incomes. Fishing effort and income are directly affected by differences in prices for landed value salmon. Conversely, by controlling prices, the effort and income results of fishermen's intraseasonal decisions can be affected accordingly. Model results suggest that schemes such as taxes on the landed value of salmon (as recommended by Pearse(1982), p.93ff) could be an effective means of regulating effort in the limited entry fishery. A landed value tax, or royalty on salmon landings has the direct affect of altering the unit price of salmon landings. Moreover, the intraseasonal model could be used to quantify the expected impact on fishing effort and fishermen's incomes from such a regulation scheme.

Other useful information for planning purposes may be explored within the framework of the intraseasonal model. For example, the affects of area licensing schemes (the restriction of salmon fishing to particular zones of the fishery) can be examined by limiting the members of the action set, A of the POMDP. Comparison of the restricted and unrestricted effects on fishermen's incomes provides a measure of the value of zonal fishing rights to the fishermen. This information may be used to establish a fee structure for area licensing schemes based on fishermen's 'willingness to pay.'†

Similarly, restrictions on the length of the season can be examined by restricting fishing to particular periods of the season. The effects on fishermen's intraseasonal decisions about when and where to fish and seasonal income can be determined by comparing the results of the decisions generated by the model with and without length of season restrictions.

The information content of the POMDP can also be examined by calculating the expected results of the model strategies under different assumptions concerning the observations fishermen make about the actual state of the system (i.e., level of abundance). Consider the three cases:

1) Perfect Information – observations uniquely identify the actual abundance of the system at each time period. Let J_p^* denote the expected value of the reward functional assuming perfect information about the actual state is available from each observation.

2) Imperfect Information – observations provide imperfect information about the actual abundance of the system at each time period. This is the usual POMDP case as examined in Chapter 4 of this

† Quantification of 'willingness to pay' is conditional on there being no resulting externalities, e.g., crowding on the fishing ground so that effort is hindered as a result of the distribution of fishermen on the fishing ground.

thesis. Let J^* denote the expected value of the reward functional for the POMDP under imperfect information.

3) No Information – observations provide no additional information about the actual abundance of the system at each time, or it may be assumed that the observations are not used to infer the actual abundance of the system. The solution to this problem, denoted by J_0^*, is known as the *open-loop value.*

The expected value of perfect information is measured by taking the difference between J_p^* and J_0^*:

$$\text{Expected Value of Perfect Information } = J_p^* - J_0^*.$$

The expected cost of the imperfect observations on the system can be measured by comparing the differences in the strategies resulting from (1) and (2) above and by taking the difference between J_p^* and J^*:

$$\text{Expected Cost of Imperfect Information } = J_p^* - J^*.$$

Finally, the expected value of the information obtained from the actual observations is measured by comparing the difference in strategies between (2) and (3) and by taking the difference:

$$\text{Expected Value of Information } = J^* - J_0^*.$$

It follows that

$$J_0^* \leq J^* \leq J_p^*.$$

Information of this kind can be used for example to quantify the benefits of stock assessment procedures which lead to improved observations on the actual abundance levels. As the observations improve, J^* increases resulting in lower costs of imperfect signals, higher value of information and higher seasonal incomes to fishermen. Stock assessment policies can be evaluated in this manner by comparing stock assessment costs with its overall benefits to fishermen.

Technically, the POMDP algorithm is effectively restricted by the dimensionality of problems. In particular, the size of the state space is the most serious restricting factor. As the size of the state space increases interpretational and computational problems arise. The development of a more flexible algorithm for this problem would facilitate the modelling and analysis of more complex systems. Heuristic procedures might also be useful in this context.

Another extension of the intraseasonal model is related to the management of factory trawlers. These vessels have been proposed for fisheries in Canada's North Atlantic fishing grounds. The within season decision process involves the determination of when the trawler should stop fishing and report to shore to unload its cargo of processed fish products for final delivery to the marketplace. Assuming a single economic objective, e.g., maximize net income, the problem is one of determining the optimal stopping points subject to changing returns to effort, stock dynamics, and time varying prices and costs. This within season optimal stopping problem may be formulated as a stochastic dynamic programming problem. Inputs to the problem are similar to those for the POMDP. Outputs establish an optimal policy for fishing and unloading fish based on the ongoing random events which occur throughout the season.

6.2 Interseasonal Model

The interseasonal model begins with a simplified description of the net worth of the fishing enterprise. Different capital types are defined with associated financing arrangements. The key component of the model is the definition of functions relating investment and income. The risk of survivability is explicitly considered in a dynamic framework where updating of investment strategies may take place. Behavioral elements of the model are tuned to investigate aspects of actual investment trends.

The interseasonal model of investment decison making is the more important of the two models discussed in this thesis. The importance of the model is in its ability to anticipate actual investment trends in fisheries.

The sensitivity analysis of this model illustrates the key role of income expectations on investment decisions. In fact, the evolution of the British Columbia salmon fishery since 1969 and licence limitation can be viewed as a series of modulating expectations which parallel investment trends. The 1969 Davis Plan also contained provisions for a buy–back programme. As a result, salmon fishing became more concentrated and the net worth of the remaining enterprises became immediately more valuable than before. High prices for salmon in the early 70's, the emergence of the roe herring fishery, and 'loose' regulation, e.g., for vessel replacement rules, contributed to significant new investment buoyed by high future income expectations. Coincidentally, these events caused the termination of the buy–back programme due to sharply increasing vessel values and limited funds.

In 1975 a poor harvest aggravated by a fishermen's strike clouded income expectations and investment fell off. By 1977 real prices were approaching pre-1975 levels. As well, a comprehensive salmonoid enhancement programme (SEP) was implemented and income expectations rose once again. Record high fish sales in 1978 and 1979 resulted in extraordinary new investment which was followed by ever–stricter vessel regulations. Late in the 1979 season however, prices began to fall off sharply.

From 1980 to 1982 market demand for salmon and herring fell dramatically as part of the world–wide economic malaise. The high income expectations of the late 70's were not realized and consolidation occurred with net new investment in 1982 falling below zero. The plight of fishermen during this 'down' period for fish prices precipitated the formation of the Pearse Commission in 1981. In recent years fish markets have recovered and a new round of high expectations for the future and increased investment is now taking place in the commercial fisheries in British Columbia.

Based on the observations about the past and investment model results, it is clear that if income expectations could be 'controlled' then investment would be affected directly. Moreover, the variability of income from year to year suggests that income expectations depend heavily on the current period's actual income. Accordingly, by controlling current income, income expectations are also affected. To this end, a landed value tax, as suggested for controlling intraseasonal decisions, could likewise be an efficient mechanism for achieving control of investment decisions. The interrelated nature of the intraseasonal and the interseasonal decision processes make this price regulation scheme a natural one in this setting. The investment model can be used to quantify the expected impact of this regulation scheme on fishermen's investment decisions and the results on these decisions, e.g., on the economic value of their fishing operations under various conditions.

Capital investment in the model is restricted by the classification of investments for tax purposes. Each class includes investments which may or may not be directly related to increased fishing power. For example, some investments are directed at increasing safety and comfort during the fishing activity. These investments may not contribute directly to increased earning power. In fact, such investments may even decay current earnings. The rationale for these investments is not explicitly captured in this model. Moreover, under current means of accounting for investment, it is impossible to identify the precise designation and use of purchased capital. More comprehensive databases are required before this issue can be adequately addressed.

Limitations on financing possibilities and on the amount of new investment in each period are based on empirical information as well as defined policy regulations, e.g., Fisheries Improvement Loans Act. These restrictions may not necessarily apply to all fishermen. As well, the investment possibilities available in the model are limited. The actual range of financing opportunities both inside and outside the fishery are not fully considered here.

Finally, the investment model is designed for modelling the decisions of fishermen within a particular class of capital, e.g., trollers. The consequences of major upgrading investments, for example whereby the fishermen moves to a new category of capital are not captured in the current analysis. The move from a troller to a salmon seiner for instance takes the fishermen out of one homogeneous investment group and into another which may have different income and future investment characteristics. The approach in this analysis is to treat each of these groups separately and to model their within group investment behavior only.

In spite of these shortcomings of the investment model, the empirical results for trollers of the British Columbia commercial fishing fleet give an indication that the model can be useful in predicting investment trends and potential under varying conditions. As an extension of this model, the investment potential of the secondary (processing) sector of the fishery can be similarly examined. The dynamic investment behavior also takes place in a related environment where survivability is a primary consideration. To accommodate the modelling of the processing sector, data on capital costs, investment possibilities, financing, and net operating income would be required.

6.3 Other Extensions

The framework of intra and interseasonal decison making models is ultimately designed to provide useful, quantitative information to policy makers faced with regulating the fishery. Using the modelling framework established here a comprehensive model of the fishery can be sketched. The intraseasonal model could theoretically incorporate all important fisheries as decision alternatives available to fishermen throughout the season. This would provide results for income from all fishing sources. These aggregate income results could then be used as inputs to the interseasonal investment decision model to determine the ongoing expected investment of the fishermen under study. This procedure defines an explicit link between the intra and the interseasonal decision models. This link is intuitive to the actual decision processes of fishermen who explicitly link up their own intraseasonal and interseasonal decisions. An overall picture of the fishery could be obtained by weighting and aggregating the different groups of fishermen, e.g., by type of vessel, or by behavioral assumptions. In this manner a composite scenario of decision making in the fishery could be modelled.

Finally, the modelling exercise could be extended to include the intraseasonal and interseasonal decisions of both the primary (fishing) and the secondary (processing) sectors of the fishery. This linked modelling framework has been suggested elsewhere (Silvert(1982)) but has not been developed. Initial considerations for this research would be in fisheries which are carried out by specialized fishermen and processors who earn the majority of their income from this particular fishery. Examples of fisheries of this kind include the northern cod fishery of the Atlantic coast and the scallop fishery off Georges' Bank.

As an illustration of the findings of this modelling analysis, a landed value tax is suggested as one means of regulating a fishery whose harvesting units are highly sensitive to changes in the price of the resource. The models have been presented as vehicles for quantifying the results of rational decision making processes under varying conditions including different regulatory schemes. The practicality of this model is in its attempt to understand and incorporate the fisherman at the centre of the decison making process affecting his/her own destiny in the fishery. The political and social costs of different regulation schemes are not dealt with in any way (although it could be argued that model results could be used to quantify these associated costs as well). As has been pointed out elsewhere (e.g., Pearse(1982), Kirby(1982)), harsh measures must be taken to ensure the continuation and strength of Canada's ocean fisheries within our social fabric. The longer we delay, the more difficult it becomes to recoup the loses. This research is motivated by the great potential for a healthy and thriving Canadian fishery.

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