FORMAL OPERATIONAL REASONING IN THE INTELLECTUALLY GIFTED

By

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ABSTRACT

This study addresses the problem of assessing the logical reasoning potential of intellectually gifted students as indicated by their ability to use the eight formal operational concepts defined by Inhelder and Piaget.

The performance of 39 tenth-grade and 26 twelfth-grade students previously identified as intellectually gifted was compared with that of 427 students representative of a cross-section of the school population at the same grade levels on a group-administered pencil-and-paper test of formal reasoning.

The unequal and disproportionate group numbers necessitated the use of a generalized least-squares regression model of analysis of variance. The gifted groups performed significantly better than the norm group at each grade level, the difference being greater for the higher grade students. Differences related to grade level, sex, and group were found when the total scores, representing both concrete and formal operational reasoning, were used as the dependent variable. However, the effects of grade level and sex failed to show significance when competence in the use of the formal operational concepts was more strictly defined by higher-order responses.

The subtests presented various degrees of difficulty, falling into three clearly defined levels which were similar for both gifted and norm group students.

Vocational information obtained from a sub-sample of the gifted twelfth-grade students revealed no real difference between the scores of arts-oriented and science-oriented students.

Low to moderate correlations between scores on the test of formal reasoning and I.Q. were found for the gifted group, the highest (0.53) being with verbal I.Q. at grade twelve.

The findings suggest that use of the test of formal reasoning provides information that is supplementary to that obtained from conventional intelligence tests, and which may prove useful in helping to identify and provide appropriate programs for intellectually gifted students.

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CHAPTER 1

STATEMENT OF THE PROBLEM

Educators have long been concerned with the problem of providing programs of appropriate levels of difficulty for students of varying levels of ability. Hunt (1961) suggested that knowledge of the stages of intellectual development proposed by Piaget is helpful in attacking this problem. According to Hunt, a student's propensity for learning is enhanced by matching the difficulty level of the program with the stage of development already attained by the student. Providing material that is suitably challenging so as to provoke the curiosity and maintain the motivation of the student while remaining within his or her learning capacity maximizes the likelihood of optimal progress. In order to provide this optimal match between student and program it is necessary to ascertain the levels at which students are capable of functioning and the levels of difficulty of the contents of programs that are available. Whether the material is appropriate or not will also depend upon the students' previous experience in that field (Brown, 1982).

Previous investigators have shown concern primarily with the relatively low proportions of subjects in the general population found to attain a formal operational level of reasoning in their high school and later adult years (Bart, 1979; Lawson, 1978; Shayer, 1980; Shayer, Adey and Wylam, 1981). This situation emphasizes the probable inappropriateness of the educational programs offered to these subjects with respect to the level of reasoning required to understand the concepts involved. Less attention has been given to the problems of the more able students. The difficulties encountered by students of above-average ability who are inappropriately matched with program content may be less obvious, but are no less acute, than those of their less able peers. Such difficulties could lead to boredom and withdrawal or behavioural problems (Feldhusen and Hoover, 1984; Fine and Pitts, 1980; Schauer, 1976).

Since the attainment of formal operational levels of thinking has not been found to be a universal characteristic of high school students (Dulit, 1972; Lawson, 1978; Lawson and Blake, 1976; Phillips, 1979; Ross, 1975; Shayer, 1979) or even of college students (Killian, 1979; McKinnon and Renner, 1971; Ross, Hubbel, Ross and Thompson, 1976), it is pertinent to enquire what proportion of students identified as intellectually gifted actually function at the formal operational level, and whether this applies to their performance in general or only in preferred areas of study.

If a program for intellectually gifted students is to offer material with abstract conceptual content, it should be ensured that the students participating in the program are able to understand such material. If students are unable to effectively employ abstract reasoning processes in dealing with course material, such a program may be rendered ineffective. On the other hand, students capable of abstract thinking may become disenchanted with, and fail to profit from, program material that is not sufficiently challenging to their acquired ability. An objective of the present study is to ascertain whether students already selected for a program of enrichment on the basis of having superior intellectual ability are able to apply the abstract reasoning processes implied by mastery of the eight schemata of formal operations.

The operational schemata are defined as "concepts which the subject potentially can organize from the beginning of the formal level when faced with certain kinds of data, but which are not manifest outside these conditions" (Inhelder and Piaget, 1958: p. 308). The eight specified schemata are:

1. The combinational operations;
2. Proportions;
3. Coordination of two systems of reference and the relativity of motion or acceleration;
4. The concept of mechanical equilibrium;
5. The notion of probability;
6. The notion of correlation;
7. Multiplicative compensations;
8. The forms of conservation which go beyond direct empirical verification.

In further definition, Inhelder and Piaget state: "These operational schemata consist of concepts or special operations (mathematical and not exclusively logical), the need for which may be felt by the subject when he tries to solve certain problems. When the need is felt, he manages to work them out spontaneously (or simply to understand - i.e., to re-work them in cases when academic instruction has already dealt with the relevant concepts). Before the formal level he is not able to do this." (1958: p. 308). The effective use of these concepts is therefore indicative of the level of mental functioning which Inhelder and Piaget term "formal operational".

Formal operational thought is characterized by the subordination of reality to possibility; that is, instead of deducing what may be possible from what actually exists, the formal operational thinker proceeds from what is possible to what is empirically real. To conceptualize an exhaustive set of possibilities, the subject uses a combinatorial system derived from the operations of propositional logic and mental forms of reversibility such as inversion and reciprocity. The formal thinker is able to make hypotheses from the data presented to him/her and then to reason deductively from these hypotheses to solve the problem.

The definition of the schemata presented by Inhelder and Piaget implies that the formal operational thinker has available the ability to use the specified concepts when occasion demands. The subject who is able to succeed when using some of the concepts, but not others, is not considered to be fully formal.

In the present study an attempt is made to ascertain the developmental levels attained by high school students who have been referred and screened for inclusion in a program of academic and creative enrichment on the basis of superior academic aptitude and attainment. This information can then be used to answer the following questions:

1. Do the students identified as intellectually gifted perform at a higher Piagetian level of conceptual development, as demonstrated by mastery of more formal operational schemata, than those not so identified?
2. Does this distinction apply to all students regardless of sex?
3. Does the distinction apply equally to students in arts- and science-oriented studies?

Answers to these questions should assist those planning programs for intellectually gifted students. Assessment of students' use of abstract concepts may indicate areas of specific strength and weakness for further investigation. Ensuing decisions may lead to the provision of programs of appropriate content or the promotion of abstract thinking ability in areas in which the ability is shown to be lacking.

CHAPTER 2

REVIEW OF THE PERTINENT LITERATURE

i) Previous Studies Involving Intellectually Superior Students and Formal Operations

A limited number of Piagetian studies involving the use of formal operational tasks and concepts have included subjects described as gifted, bright, or above average in intellectual ability. Using either group or individual methods of assessment, these studies have addressed issues such as the evidence for or against precocity, both within and between the recognized Piagetian stages (Carter and Ormrod, 1982; Karplus, 1979; Keating, 1975, 1976; Keating and Schaefer, 1975; Lovell and Shields, 1967; Ross, 1975, 1976; Sieber, 1978; Webb, 1974; Webb and Daurio, 1975; Yudin, 1966), the incidence of formal operational thinking among individuals of superior mental ability (Dulit, 1972; Martorano and Zentall, 1980; Roberge, 1976), effects, or lack of effects, of training on the attainment of higher level thinking (Bredderman, 1973), and differences between male and female subjects on Piagetian-type tests and tasks (Graybill, 1975). The consequences of differences in assessment procedures and content have also been addressed (Bart, 1971, 1972).

On the issue of precocity, conclusions reached by various investigators show a lack of consensus. Some findings support the contention that psychometrically bright students develop and consolidate formal operational thinking earlier and more completely than less gifted students, while other reports tend to refute the same contention. The apparent contradictions, however, become more reconcilable when consideration is given to the manner in which the conclusions were reached. Influential factors include the lack of standardization in testing procedures, the subjective interpretation of test responses, varying sample characteristics including numbers and age-spans of subjects together with differences in their home and school experiences, and the use of different tasks and criteria for judging subjects as successful or not.

Table I lists the formal operational tasks used in the studies involving intellectually gifted students and the schemata or concepts which these tasks are used to assess. It should be noted that only four of the eight formal operational schemata are specifically represented in studies to date.

Reference Number	Task	Schema/Concept Assessed
1	Combination of chemicals	Combinatorial operations†
2	Equilibrium in the balance	Proportions†
3	Paper clips (Mr. Short & Mr. Tall)	Proportions†
4	Projection of shadows	Proportions†
5	Communicating vessels	Mechanical equilibium†
6	Conservation of motion on a horizontal plane	Forms of conservation beyond direct verification†
7	Oscillation of a pendulum	Exclusion of variables
8	Flexibility of rods	Separation of variables
9	Equality of angles of incidence & reflection	Reciprocal implication
10	Law of floating bodies	Elimination of contradiction

† formal operation schema

Table I: <u>Formal operational tasks and the concepts or schemata assessed</u>

Carter and Ormrod (1982) compared the performances of a group of intellectually gifted children (I.Q.'s of 130^{+}) with those of a group of children of normal intelligence (I.Q.'s from 90 to 115) in the age range of ten to fifteen years. A paper-and-pencil multiple-choice test was used consisting of thirty Piagetian-type items which covered the stages of both concrete and formal operations and included items on combinatorial logic, probability, proportional reasoning, and control of variables. Results, which support the contention of within-stage precocity, indicated that the gifted children attained the stage of formal operations at the age of approximately thirteen years whereas the children of normal ability were still in the transitional stage even at the age of fifteen years. The mean scores of the gifted children were significantly higher than those of the children of normal ability at each age level. In view of the fact that, in this study, some gifted children attained formal operations at an earlier age than their less gifted counterparts, the authors recommend the use of Piagetian assessment to determine the cognitive levels of students, and to appropriately structure the curriculum according to individual needs. They also suggest that Piagetian assessment may be helpful in identifying gifted children "from special populations" who are often missed by traditional psychometric aptitude tests.

In studying the proportional reasoning performance of 14 "gifted" fourth-grade students and 35 "selected" fifth-grade students at the laboratory school of Shanghai Teachers' University, Karplus (1979) found 76 per cent of the sample to be completely successful on his paper clips task (3).* Further supporting the contention of precocity, this figure was compared with only 20 percent of unselected upper middle class American students in grade eight, and 53 per cent in grades eleven and twelve, who had successfully completed the same task. The superior performance of the young Shanghai students on this one scientific experimental task is more easily explained when viewed in the light of the students' academic background and presumably, therefore, their science-oriented educational experience.

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* Numbers in parenthesis refer to tasks listed in Table I.

Keating (1975,1976) found that, in spite of the two-year age difference, fifty bright fifth-grade boys (scoring at the 98th and 99th percentile levels on the arithmetic section of the Iowa Test of Basic Skills) performed significantly better than fifty average seventh-grade boys (scoring between the 45th and 55th percentiles) on a number of Piagetian formal operational tasks (2,7,10). There were no significant differences in the performance of bright fifth-grade and bright seventh- grade subjects on the same measures. However, as might have been expected, average seventh-grade subjects significantly outperformed average fifth- grade subjects. Between seventy and eighty per cent of the bright subjects, both in grade five and grade seven, were successful in all the Piagetian tasks. In contrast, no more than fifty per cent of average seventh-grade students were successful in any of the tasks. The author suggests that "precocity across stages is clearly present, but perhaps not as pronounced as that within stages" (1976, p. 98).

In a comparable study using the same Piagetian tasks, but with bright and average twelve- and thirteen-year-old girls instead of boys in their sample, Keating and Schaefer (1975) replicated the finding of precocity and, summarizing the two studies, concluded that "ability is clearly the important variable, even to the extent that the younger bright students evidenced more formal reasoning than older average students, both for the boys and the girls" (p. 531).

Evidence supporting precocity also is provided by Sieber (1978) who matched 33 "bright" girls in grades five through eight with 33 who were average in intelligence as measured by performance on the WISC-R. Using both concrete and formal operational tasks (1,2), it was found that I.Q. predicted precocity both within and between stages. The bright subjects consolidated operations within a stage better than did the subjects of average intelligence.

In contrast with the above studies in which the authors have found evidence to support the argument for precocity either within a stage (Carter and Ormrod, 1982; Keating and Schaefer, 1975) or between stages (Karplus, 1979), other investigators have failed to discover such evidence.

Lovell and Shields (1967) found that only 10 per cent of the responses from 50 subjects aged from eight to ten years, and with WISC Verbal I.Q.'s of 140^{+}, and, therefore, mental ages of at least eleven to fourteen years, were at the formal operational level on Piagetian tasks of logical thinking (1,2,7).

Further evidence against precocity is presented by Webb (1974) who studied the performances of 25 children aged between six and eleven years and having I.Q.'s of 160^{+}. Webb found that all the children were able to pass three tests of concrete operations but only the four oldest boys could pass the two tests of formal operational thinking (2,10). The author suggested that, below the age of eleven, performance on Piagetian tasks is more closely related to chronological than to mental age expectations, and stated that there was "essentially no precocity on formal operations associated with high I.Q.'s" (p. 299).

In a later study, using three formal operational tasks (2,5,7), Webb and Daurio (1975) included children up to the age of fourteen years with I.Q.'s of approximately 150. They found that high ability in these youths did carry over into formal operations, and concluded that, by the time they were thirteen or fourteen, those youngsters were "almost certainly better than the average adult population" (p. 10).

In both the Webb (1974) and Lovell and Shields (1967) studies the subjects were somewhat younger than those in the Keating (1975,1976) studies. This may help to account for the apparent discrepancy in the conclusions reached. It would appear that success in using the formal operational concepts and schemata is not to be expected, even in intellectually gifted students, before the age of about ten years. Factors other than psychometric intelligence, and probably maturational or experiential in nature, appear to be involved in the acquisition of formal operational competence.

Using multiple-choice items based on three Piagetian tasks (2,4,9), and contained in Tisher's (1971) questionnaire, Ross (1975,1976) compared the performances of students in a high track with those of students in a moderate track at both the sixth- and the tenth-grade levels. Only 22 per cent of high track sixth-graders, who scored significantly higher than their moderate track peers, were classified as formal operational. 40 per cent of moderate track and 43 per cent of high track tenth-graders were judged to be formal operational. Evidence to support precocity would require higher proportions of high track students, more especially those at the higher grade level, to be performing at a formal operational level. Failure to do so may be consequent upon the sample selection procedure, which was based on teachers' ratings of students' scholastic achievement rather than on a measure of academic ability such as I.Q.

In summary, while intellectual giftedness does not appear to guarantee the early appearance of formal operations (Lovell and Shields, 1967; Webb, 1974), the acquisition and consolidation of forml operational competence seems to have a higher probability of occurrence in students of higher intelligence (Dulit, 1972; Keating, 1975), whereas those of average intelligence are equally likely to perform at a concrete operational level throughout their adolescent and adult years (Shayer and Adey, 1981).

The proportion of intellectually gifted adolescent or pre-adolescent students classified as formal operational by various investigators in the studies under review ranges from as low as 22 to as high as 80 per cent. The narrow range of tasks used and concepts assessed precludes a direct comparison of these results. No more than four tasks were used in any one study, and the tasks or items relate to only four of the eight operational schemata.

In a study which involved 10 males and 10 females from each of grades five through eight, all having I.Q.'s above 110, Roberge (1976) found that no more than 45 per cent of the grade eight subjects were consistently successful in combinatorial and conditional reasoning tasks. From the lack of correlation between scores obtained on the two forms of reasoning, it was concluded that "evidence seems to contradict Piaget's contention that an integration of formal operational competencies occurs during the period from 11-12 to 14-15 years of age" (p. 564).

Dulit (1972) included 23 "gifted older adolescents" aged between sixteen and seventeen years in a study which also involved normal younger and older adolescents and adults. The gifted students were selected for "very superior academic aptitude and performance, especially in the sciences" and had I.Q.'s in the 130 to 140 range. Using paper-and-pencil simulations of two Piagetian formal operational tasks (1,4), over half of the gifted subjects was judged to be fully formal operational. In contrast, no more than one-third of the normal older adolescents and adults was classified in that category. In each analysis, and for both gifted and normal subjects, significantly more males than females were classified at the highest level.

Martorano and Zentall (1980) used a separation of variables task (8) with eight-, ten-, and thirteen-year-old subjects having mean I.Q.'s of over 120. The authors classified as formal operational the performances of 22, 50 and 78 per cent of the three age groups. These percentages are in accord with predictions that may be made when using mental age level guidelines.

Bredderman (1973) identified 27 students in grades five and six (ages ten to thirteen years) as being unable to control variables (8). The students' I.Q.'s ranged from 98 to 141, the average being 117. The sample, therefore, contained some gifted children. After a period of training involving external reinforcement or cognitive conflict, the treatment groups did only slightly better than the control group on a post-test. The scores of both treatment and control groups were almost identical on a retention test taken one month later. According to the scores obtained in the retention test, nearly half of the students, with a mean age of 11.8 years, was classified as "late formal" (Piagetian stage IIIB), a level not normally reached until the age of fourteen or fifteen (Inhelder and Piaget, 1958). The success of these pre-adolescents of average to superior intelligence on the control of variables task may be attributed to a number of factors including maturation, experience, and practice. Considering the extent of progress made in so short a time by both experimental and control groups, the most obvious explanation would appear to be practice in, or exposure to, the control of variables task.

In attempting to determine the possible differences between males and females in their development of formal operational thinking skills, Graybill (1975) administered four Piagetian tasks (1,8,9,10) to three pairs of boys and girls at each of the ages nine, eleven, thirteen, and fifteen years taken from "above-average ability and achievement groups" and matched for age, I.Q., school achievement, and socio-economic status. From the age of eleven onwards the boys performed significantly better than the girls on all tasks. While boys' responses began to be classified as formal operational at age thirteen, no girl in the study attained a standard that could be so designated.

In the study reviewed above, Ross (1976) also found that male subjects in grades six and ten scored significantly higher than their female counterparts on three formal operational tasks (2,4,9) which form the basis for Tisher's questionnaire. Furthermore, Keating and Schaefer (1975) had demonstrated that bright eleven-year-old boys scored significantly higher than comparably bright twelve-year-old girls on three Piagetian tasks (2,7,10) not identical to those administered by Ross. The results from this study, together with those from previous studies (Dulit 1972; Stanley, Keating and Fox, 1974), strengthen the argument, at least when using content relating to the physical sciences, that "the sex difference in high-level reasoning is greater at high levels of ability" (Keating and Schaefer, 1975: p. 532).

Bart (1971,1972) examined the performances of 30 "scholastically above-average" male and female students at each of the thirteen-, sixteen-, and nineteen-year-old age levels on four standard Piagetian formal operational tasks (2,4,6,7) and three multiple-choice formal reasoning tests in the content areas of biology, history, and literature. Correlations between the mean formal reasoning test scores and the total task scores were moderate, being .33, .62, and .57 for the thirteen-, sixteen-, and nineteen-year-old groups respectively. Factor analysis yielded a substantial formal operational factor and a secondary factor which separated the tasks from the tests. In this study the effects of content and form of testing are confounded; that is, the moderate correlations between task and test scores may be attributed to differences in content of the questions asked, to the difference between a flexible interview procedure and set multiple-choice responses, or to a combination of these factors. The group administration of the present study is more directly comparable to achievement test procedures in common use. Form of testing being held constant, therefore, the effect of the content factor should be more easily observable and the relative effectiveness of the test for use with science- and arts-oriented subjects, for example, more readily determined.

ii) <u>The Comparability of Psychometric and Piagetian Measures</u>

Selection procedures for identifying intellectually gifted students have typically employed conventional psychometric measures such as intelligence and academic attainment tests. Intelligence tests contain items which assess competencies in such diverse areas as perception, memory, reasoning, and general knowledge. Piagetian measures, on the other hand, focus on a narrower range of competencies, namely those of logical thinking. When both traditional and Piagetian measures are used in assessing either the capabilities of students or the conceptual demands of tasks, it is pertinent to ask whether or not the scales which the two methods employ are directly comparable, and what additional information is contributed by the inclusion of a Piagetian measure in the assessment battery.

Most investigators who have addressed these questions have limited their inquiries to subjects who were expected to be reasoning at the concrete operational level (Glass and Stephens, 1980; Humphreys and Parsons, 1979; Stephens, McLaughlin, Miller and Glass, 1972). The findings reach a general consensus that, while not measuring identical components, Piagetian and traditional intelligence tests are at least both strongly influenced by a general factor (Carroll, Kohlberg, and DeVries, 1984). That the applicability of these findings can be extended to formal operational measures is suggested by the latter's moderate but significant correlations with traditional intelligence test scores (Cloutier and Goldschmid, 1976; Flexer and Roberge, 1980), and with achievement test scores (Keating and Schaefer, 1975; Roberge and Flexer, 1984) for students in grades five through eight, that is, in the age range at which formal operations are expected to emerge.

Although her sample did not contain subjects beyond the age of twelve years, Kuhn (1976) found a lower correlation between psychometric and Piagetian assessments for children approaching the formal operational stage than for those in the concrete operational stage. In a later paper, Kuhn (1979) suggested that the difference between the two methods of assessment was essentially that between quantitative and qualitative measurement. The objective of formal reasoning assessment, she contends, is to determine the individual's degree of competency in certain reasoning strategies rather than the extent of the individual's knowledge across various areas of functioning.

A similar distinction is made by Elkind (1981) who conceives of intelligence as "adaptive capacity resulting from the interaction of invariant mental processes (traits) and variable mental organizations (forms)" (p. 109). Elkind proposes that intelligence test performance be reported as both a score (the trait dimension) and a description of the subject's form level. In the context of the present study, the subject's overall cognitive level on the test represents the form component of the assessment process, while the number of subtests or schemata mastered provides a trait-like measure of the subject's performance.

Fischer and Pipp (1984) suggest that Piagetian and psychometric approaches to assessment are compatible; the former being largely concerned with determining an individual's optimal level of functioning, whereas the latter emphasizes levels of skill acquisition. Optimal levels are considered to be primarily age-related and to set limits on the development of skill acquisition processes within each structural level. This hypothesis holds implications for the extent to which acceleration in cognitive development is possible for individuals of above-average intelligence. Such individuals may be expected to make rapid progress in acquiring skills within each level, but to show little evidence of advancing beyond the optimal level appropriate to their ages.

CHAPTER 3

DESIGN OF THE INQUIRY

(i) Subjects

The target group consisted of 39 tenth-grade and 26 twelfth-grade students previously identified as intellectually gifted, and screened for possible inclusion in a program for academic and creative enrichment (PACE) in a school district on Vancouver Island, British Columbia.

PACE is intended to be a support program for students with exceptional learning abilities. The program takes the form of one course elective in a student's eight course schedule.

General requirements for placement in the program include

(a) high grades in the core academic subjects,

(b) a high level of task commitment, and

(c) a personal commitment to participate in a challenging program.

Additionally, students in grades seven through ten are required to have intelligence quotients at or above the 95th percentile. Those with IQ's at or above the 97th percentile are accepted into the program regardless of academic performance and demonstrated task commitment. Students entering in grades eleven or twelve are expected to satisfy the three requirements listed above, with the possible exception of students having IQs at the 99th percentile, who may be admitted to, or maintained in, the program without the prerequisite high level of academic performance. Otherwise, students in the program who are not achieving a sufficiently high academic standard may be asked to withdraw.

The intellectually gifted students in this study who participated in the program had entered at various stages of their school careers ranging from grade seven to grade twelve.

Further data that were available on the gifted group included IQs (WISC-R or WAIS-R) and grade scores in the academic subjects (English, Mathematics, Science, and Social Studies) obtained over the period of high-school attendance. For the twelfth-grade students completing the PACE program information obtained from personal interview on further education and career intentions was also made available. This information enabled the students to be classified as either arts- or science-oriented. The arts-oriented group consisted of 5 males and 7 females, while the science-oriented group contained 9 males and 3 females.

The comparison group consisted of 234 tenth-grade and 193 twelfth-grade students attending a large comprehensive high school located in a small city on the coast of Maine.

Although somewhat different geographically and culturally, the target (gifted) and comparison (norm) groups were both from largely middle-class, English-speaking, suburban communities. A considerable number of students in both groups were members of the families of military personnel, and the majority tended to be achievement-oriented. A similar proportion from each high-school population went on to higher educational institutions.

(ii) The Test Instrument

The Arlin Test of Formal Reasoning (ATFR; Arlin, 1984) is a group-administered written test in a four-response multiple-choice format designed to be used in middle or high schools. Item readabilities range from grade five to grade seven as measured by Chall's formula.

The four-fold purpose of the test is:

1. to obtain a general assessment of students' levels (or stages) of cognitive development.
2. to yield specific subtest scores with reference to each of Inhelder and Piaget's (1958) eight formal schemata which can be used diagnostically by teachers in instructional planning.
3. to be used in conjunction with other instruments for screening students for programs for the gifted and for early admission to special science and mathematics classes.
4. to assess the logical reasoning skills of students with reading and other learning disabilities separately from general achievement and intelligence tests.

The first three stated aims make the ATFR an appropriate instrument to use in the assessment of intellectually gifted students. Knowledge of the overall cognitive levels of students will help in determining whether the students identified as intellectually gifted are also those who have mastered a greater number of formal operational schemata and are able to benefit from a program which demands the use of abstract concepts. The relative strengths and weaknesses indicated by subtest scores should help in grouping students and in defining the areas in which instruction may most profitably be directed for a particular group. The comparative performances of males and females, and of science-oriented and arts-oriented students, should assist in identifying the population for which the ATFR is most appropriate to use in screening and selection procedures for inclusion in special programs.

The test consists of 32 items organized into eight sub-tests, each sub-test representing one of the eight formal operational schemata, mastery of which indicates an individual's access to the formal operational stage of reasoning (Inhelder and Piaget, 1958). The eight sub-tests are: volume, correlations, probability, combinations, proportions, momentum, mechanical equilibrium, and frames of reference.

The volume sub-test examines an individual's ability to consider two or more dimensions in solving a problem which requires mastery of the concept of multiplicative compensations.

The correlations sub-test requires an understanding of the strengths of relationships between variables.

The probability sub-test examines an individual's understanding of the relationship between confirming and possible cases such as occurs in calculating the odds in games of chance.

Combinations sub-test items require the generating of all possible combinations of a given number of elements.

Proportions sub-test items measure the ability to discover the equality of two ratios which form the proportion.

The momentum sub-test items involve deduction and verification of certain conservations by observing their effects and thus inferring their existence.

Mechanical equilibrium requires the ability to simultaneously make the distinction and the coordination of two complementary forms of reversibility: reciprocity and inversion.

Frames of reference requires the ability to coordinate two systems, each involving a direct and an inverse operation, but with one of the systems in a relation of compensation or symmetry in terms of the other.

The examinee receives a score for each sub-test ranging from 0 to 4. A sub-test score of 3 or 4 signifies the examinee's competence in using that particular sub-test schema* (Arlin, 1984). An overall test score is obtained from which the examinee's cognitive level is determined (Table 2). The test can be either machine- or hand-scored.

Table 2: Ranges of ATFR total scores corresponding to each cognitive level

Total Score	Cognitive Level
0 - 7	Concrete
8 - 14	High Concrete
15 - 17	Transitional
18 - 24	Low Formal
25 - 32	High Formal

No rigid time limit is set for completion of the test but field testing has shown that forty-five minutes is sufficient time for most students, and extra time is allowed for those unable to complete the test within the forty-five minute period.

Mean sub-test and total scores, together with standard deviations, reliabilities, and standard errors, are available by grade level for almost 3,500 students in grades six through twelve from schools in the States of California, Maine, New York, and Washington.

In a multitrait-multimethod validity study (Arlin, 1982), 38 military recruits under twenty years of age were selected from 244 subjects who took

* For the purpose of conciseness, the number of subtests in which a student demonstrates competence, by scoring 3 or 4, will be termed the student's mastery score.

the group test. These 38 subjects were individually administered a test consisting of six Piagetian formal operational tasks which represented the traits. The two methods in the matrix were the group and individual forms of assessment. Validity coefficients for the six operational schemata ranged from .55 to .74. In contrast, the inter-task correlations for the individually administered test scores ranged from .02 to .55 and, for the six scores of the group sub-tests, from .11 to .47. These findings support the construct validity of the ATFR, indicating that relationships between scores for the same schema, measured by the group and individual methods, are higher than those between the scores for the different schemata measured by the same method.

(iii) Procedure

The ATFR was group-administered to both gifted and norm group students during regular class sessions by trained examiners in the Spring and early Summer of 1986.

Raw data, in the form of students' responses to items in the ATFR recorded on individual answer sheets, were collected and processed by computer to obtain subtest scores, a total score, and a cognitive level for each student. Mastery scores were calculated in the manner described above.

(iv) Method of Analysis

Means and standard deviations of ATFR total, subtest, and mastery scores were obtained for males and females in both gifted and norm groups at each grade level.

Separate three-way analyses of variance were conducted with ATFR total, mastery, and subtest scores as the dependent variables to determine any real differences relating to grade level, sex, group, or any interactions of these effects.

The unbalanced nature of the factorial design in this study makes a straightforward analysis of variance inappropriate. When cell frequencies are unequal and disproportionate, there is a correlation among the factors. This makes it difficult to determine the magnitude of the separate effects that each factor has on the criterion because the total sum of squares cannot be decomposed into a series of additive components to permit analysis of the effects. For this reason, a generalized least-squares regression approach (Berenson, Levine, and Goldstein, 1983) was used.

In the complete least-squares or general linear model employed in the study each effect, whether it be main effect or interaction, is estimated while controlling for possible relationships with all other effects in the model.

For ATFR total, subtest, and mastery scores, the differences between the means of arts- and science-oriented groups of students completing the PACE program were tested for statistical significance.

Product-moment correlations (Pearson "r") were obtained between total and mastery ATFR scores and Verbal, Performance, and Full Scale IQs for the gifted group students.

(v) Hypotheses

The following hypotheses are made:

1. Students identified as intellectually gifted obtain (a) higher total scores, and (b) higher mastery scores in the ATFR, than students not so identified.

2. Male students obtain (a) higher total scores, and (b) higher mastery scores in the ATFR, than female students in the same group and grade.
3. Students who are oriented towards science and mathematics obtain (a) higher total scores, and (b) higher mastery scores in the ATFR, than students oriented towards the arts.

CHAPTER 4

STATEMENT OF RESULTS

Four sets of scores were obtained from students' responses to items in the ATFR: total scores, mastery scores, subtest scores, and cognitive levels. Each student's performance in the test yields eight subtest scores which sum to provide a total score. From the total score the student's cognitive level is determined according to Table 2 (p. 21). The number of subtests in which a score of 3 or 4 (maximum) is obtained constitutes the student's mastery score (Arlin, 1984).

The four sets of scores will be reported and analyzed in the present chapter in order to provide answers to the questions posed in Chapter 1, and to support the acceptance or rejection of the hypotheses proposed in Chapter 3.

(i) Analysis of Total Scores

Means and standard deviations of the total scores of gifted and norm group males and females at each grade level are shown in Table 3. The total scores of the grade ten gifted students ranged from 11 to 28 (out of a possible 32) with a mean of 19.82 (S.D. = 3.97), while the scores of the grade twelve gifted students ranged from 17 to 30 with a mean of 22.73 (S.D. = 3.49). The total scores of the grade ten norm group students ranged from 3 to 30 with a mean of 15.92 (S.D. = 4.83) and those of the grade twelve norm group students ranged from 5 to 30 with a mean of 17.05 (S.D. = 5.85).

In order to test hypotheses 1(a) and 2(a) (pp. 23, 24) the total scores of the gifted and norm group students were tested for significant differences related to grade, sex, and group ("gifted" or "norm").

Table 3. Means and standard deviations of ATFR total scores for gifted and norm group males and females at each grade level.

Grade	Group	Sex	N	Mean	Standard Deviation
10	G	M	19	20.37	4.45
10	G	F	20	19.30	3.48
10	G	M+F	39	19.82	3.97
10	N	M	118	16.26	4.99
10	N	F	116	15.58	4.66
10	N	M+F	234	15.92	4.83
12	G	M	16	23.63	3.79
12	G	F	10	21.30	2.50
12	G	M+F	26	22.73	3.49
12	N	M	105	17.90	5.86
12	N	F	88	16.03	5.71
12	N	M+F	193	17.05	5.85

G = Gifted M = Male
N = Norm F = Female

Table 4. Generalized least-squares analysis of variance of total scores in the ATFR

Source of Variation	Sum of Squares	Degrees of Freedom	Variance Estimate
Grade (A)	178.47	1	6.82 **
Sex (B)	116.77	1	4.46 *
Group (C)	1,167.80	1	44.63 **
A B	19.68	1	0.75
A C	32.90	1	1.26
B C	2.32	1	0.09
A B C	0.02	1	0.00
Explained	1,674.84	7	9.14 **
Residual	12,665.57	484	
Total	14,340.41	491	

* $p < .05$
** $p < .01$

The generalized least-squares analysis of variance of the total scores (Table 4) reveals significant differences for the three main effects: grade and group ($p < .01$), and sex ($p < .05$). There are no significant interactions. The gifted group outperformed the norm group at both grade ten and grade twelve levels, and in the cases of both male and female students. Grade twelve students scored higher than grade ten students in both gifted and norm groups. However, the grade ten gifted students outperformed those in the grade twelve norm group.

Sex-related differences at each grade level, and in both gifted and norm groups, are in favour of the males. Only across groups is a superiority in female students' scores observed. Gifted girls scored higher than norm group boys, irrespective of grade level.

These results support the acceptance of hypotheses 1(a) and 2(a): namely, that students identified as intellectually gifted obtain higher ATFR scores than those not so identified; and that male students obtain higher total scores than female students in the same group and grade.

(ii) Composition of Cognitive Levels

Supplementary to the above analysis, and in order to determine whether or not more gifted than norm group students and more males than females reach the higher cognitive levels, the percentages of gifted and norm group males and females within each cognitive level were evaluated (Table 5).

The percentage of gifted students within a formal level; that is, classified as high formal or low formal, was greater than the percentage of norm group students at each grade level. The grade ten gifted was also superior to the grade twelve norm group, 69 percent of the former compared to 48 percent of the latter attaining a formal level. There were also

Table 5. Percentages of male and female students within each cognitive level

Grade	Group	Sex	N	CONC	HICN	TRAN	LOFM	HIFM
10	G	M	19	0	11	16	58	16
10	G	F	20	0	10	25	60	5
10	G	M+F	39	0	10	21	59	10
10	N	M	118	5	30	25	36	5
10	N	F	116	3	42	23	29	3
10	N	M+F	234	4	36	24	32	4
12	G	M	15	0	0	7	53	40
12	G	F	11	0	0	9	82	9
12	G	M+F	26	0	0	8	65	27
12	N	M	105	4	24	13	47	12
12	N	F	88	6	38	20	28	8
12	N	M+F	193	5	30	17	38	10

G = Gifted
N = Norm
M = Male
F = Female

CONC = Concrete
HICN = High Concrete
TRAN = Transitional
LOFM = Low Formal
HIFM = High Formal

sex-related differences, more males than females scoring as formal at each grade level and in both gifted and norm groups. Yet it should be noted that the difference between the sexes lessened in the case of the gifted but increased for the norm group between grade ten and grade twelve. 93 percent and 91 percent of twelfth grade gifted males and females respectively attained a formal level in contrast to 59 percent and 36 percent of males and females respectively in the twelfth grade norm group. However, when only differences in the numbers of students attaining the highest level (high formal) are considered, the relative positions of gifted and norm group males and females are reversed. 40 percent of gifted males in comparison to 9 percent of gifted females attained the high formal level in grade twelve. In contrast, only 12 percent of males and 8 percent females in the twelfth grade norm group reached the high formal level. This reversal is brought about by the high percentage (82 percent) of gifted females reaching the low formal level by grade twelve yet, all excepting 9 percent, failing to advance to the high formal level. The gifted males showed no such lack of advancement. This finding will be discussed and a possible reason for it suggested in Chapter 5.

(iii) Analysis of Mastery Scores

Means and standard deviations of mastery scores for gifted and norm group male and female students at each grade level are shown in Table 6.

Table 6. Means and standard deviations of mastery scores for gifted and norm group males and females at each grade level.

Grade	Group	Sex	N	Mean	Standard Deviation
10	G	M	19	4.47	1.63
10	G	F	20	4.20	1.29
10	G	M+F	39	4.33	1.47
10	N	M	118	2.91	1.71
10	N	F	116	2.79	1.56
10	N	M+F	234	2.85	1.64
12	G	M	16	5.25	1.26
12	G	F	10	4.50	1.21
12	G	M+F	26	4.96	1.26
12	N	M	105	3.50	1.91
12	N	F	88	2.95	1.76
12	N	M+F	193	3.25	1.86

G = Gifted   M = Male
N = Norm   F = Female

Grade ten students in the gifted group mastered between 1 and 7 subtests with a mean of 4.33 (S.D. = 1.47). The range of subtests mastered by grade twelve gifted group students was from 3 to 7 with a mean of 4.96 (S.D. = 1.26). Numbers of subtests mastered by norm group students covered the entire range (0 to 8) with means of 2.85 (S.D. = 1.64) and 3.25 (S.D. = 1.86) for the grade ten and grade twelve groups respectively.

In order to test hypotheses 1(b) and 2(b) (pp. 23, 24) the mastery scores of the gifted and norm group students were tested for significant differences related to grade, sex, and group.

Table 7. Generalized least-squares analysis of variance of mastery scores

Source of Variation	Sum of Squares	Degrees of Freedom	Variance Estimate
Grade (A)	11.12	1	3.84
Sex (B)	9.40	1	3.24
Group (C)	129.47	1	44.68 **
A×B	2.75	1	0.95
A×C	0.33	1	0.11
B×C	0.43	1	0.15
A×B×C	0.01	1	0.00
Explained	178.67	7	8.81 **
Residual	1,402.51	484	
Total	1,581.18	491	

** $p < .01$

The generalized least-squares analysis of variance of mastery scores (Table 7) shows a significant difference ($p < .01$) between gifted and norm group students' scores. Gifted group students mastered more subtests than norm group students in the cases of both males and females, and at each grade level. The tenth-grade gifted group also outperformed the twelfth-grade norm group. There are no other significant differences, either for the main effects (grade and sex) or for any of the interactions.

These results support the acceptance of hypothesis 1(b): that gifted students obtain higher mastery scores than norm group students, and the rejection of hypothesis 2(b): that males obtain higher mastery scores than females of the same group at each grade level. The results of this analysis also stand in contrast to those of the previous analysis of total scores in which all three main effects showed significant differences. This finding will be discussed in Chapter 5.

(iv) Correspondence Between Mastery Scores and Cognitive Levels

Frequencies of mastery scores obtained by students at each cognitive level are shown in Table 8. Most students classified as high formal mastered either six or seven subtests. It may be seen that no gifted student mastered all eight subtests although three male students in the norm group were able to do so. The majority of students classified as low formal mastered four or five subtests. Most students in the transitional level mastered two or three subtests, while the majority of those classified as high concrete mastered one or two. The students in the norm group who were classified as concrete showed mastery in no more than one subtest. The most notable exceptions to this pattern included a gifted girl in grade ten who, while still classified as high concrete, mastered four subtests. In contrast, four students in the norm group, and three in the gifted, classified as low formal mastered only two. Comparing the scores of three norm group students who were classified as transitional, each of the two boys, one from each grade, mastered only one subtest whereas a grade ten girl showed mastery in five.

Table 8. Frequencies of mastery scores of students within each cognitive level

Group	Grade	Cognitive Level	Mastery Scores								
			0	1	2	3	4	5	6	7	8
Gifted	10	CONC									
		HICN		1	1	1	1				
		TRAN			2	4	2				
		LOFM			3	7	6	7			
		HIFM						2	2		
Gifted	12	CONC									
		HICN									
		TRAN				2					
		LOFM				1	8	6	2		
		HIFM							3	4	
Norm	10	CONC	7	2							
		HICN	5	38	36	5					
		TRAN		1	15	32	7	1			
		LOFM			1	15	34	17	9		
		HIFM						1	5	1	2
Norm	12	CONC	5	4							
		HICN	5	22	24	7					
		TRAN		1	10	18	3				
		LOFM			3	12	30	21	8		
		HIFM							10	9	1

CONC = Concrete
HICN = High Concrete
TRAN = Transitional
LOFM = Low Formal
HIFM = High Formal

(v) Analysis of Subtest Scores

To further explore the usefulness of the ATFR in differentiating between gifted and norm group students, the subtest scores of the two groups were tested for significant differences related to grade, sex, and group.

Means and standard deviations of the ATFR subtest scores of gifted and norm group males and females at each grade level are shown in Table 9.

Table 9. Means and standard deviations (in parentheses) of ATFR subtest scores for gifted and norm group males and females at each grade level

Group	N	Vol.	Prob.	Frames.	Prop.	Comb.	Corr.	Mom.	Mech.
10 Gift M	19	2.89 (1.33)	3.26 (0.73)	2.74 (1.10)	2.58 (1.26)	2.37 (1.26)	3.47 (1.02)	1.74 (1.37)	1.32 (1.20)
10 Norm M	118	2.36 (1.36)	2.68 (1.15)	2.15 (1.20)	2.16 (1.30)	1.24 (1.10)	3.20 (1.13)	1.14 (1.06)	1.33 (0.98)
10 Gift F	20	2.25 (1.68)	3.20 (0.83)	2.50 (1.05)	2.80 (1.11)	2.25 (1.41)	3.70 (0.47)	1.45 (1.28)	1.15 (1.09)
10 Norm F	116	2.44 (1.32)	2.38 (1.12)	1.89 (1.16)	1.66 (1.26)	1.34 (1.11)	3.28 (0.99)	1.09 (1.10)	1.51 (1.13)
12 Gift M	16	3.06 (1.34)	3.63 (0.50)	3.25 (1.13)	3.50 (0.82)	2.50 (0.73)	3.50 (0.89)	2.31 (1.20)	1.88 (1.15)
12 Norm M	105	2.61 (1.46)	2.94 (1.00)	2.32 (1.24)	2.50 (1.38)	1.54 (1.15)	3.25 (1.03)	1.30 (1.17)	1.43 (1.12)
12 Gift F	10	2.30 (1.16)	3.50 (0.71)	2.90 (0.99)	3.30 (1.06)	1.90 (0.99)	3.40 (1.27)	1.90 (0.99)	2.10 (1.10)
12 Norm F	88	2.72 (1.23)	2.60 (1.18)	1.74 (1.15)	1.64 (1.33)	1.63 (1.16)	3.19 (1.18)	1.13 (1.20)	1.40 (1.22)

The generalized least-squares analyses of the variances are presented in Appendix 1. Tests of fit to the linear model yield F-values which fail to reach significance ($p < .05$) in the cases of three subtests: Volume, Correlations, and Mechanical Equilibrium. No further interpretation of the analyses for these tests is made. There are significant differences (d.f. = 1,484; $p < .01$) between gifted and norm group students' scores in the remaining five subtests: Probability (F = 25.65), Frames of Reference (F = 25.80), Proportions (F = 35.37), Combinations (F = 27.61), and Momentum (F = 18.88). Grade level also is significant in the Proportions subtest ($F_{1,484} = 6.05$; $p < .05$), and sex is significant in Frames of Reference ($F_{1,484} = 4.94$; $p < .05$). Differences are in favour of gifted, twelfth grade, and male students for group, grade, and sex respectively. A group-by-sex interaction also is significant in the Proportions Subtest ($F_{1,484} = 3.86$; $p < .05$). Norm group males scored higher than norm group females, while gifted group females scored higher than gifted group males at the tenth-grade level.

(vi) Order of Difficulty in ATFR Subtests

To assist in interpreting subtest score profiles, differences in subtest difficulty are obtained from the mean scores of gifted and norm group males and females at each grade level. The order of difficulty for each subgroup is shown in Table 10. A consistent pattern emerges.

Table 10. Descending order of difficulty of subtests derived from mean scores

Grade	Group	Sex	N	Vol.	Prob.	Frames.	Prop.	Comb.	Corr.	Mom.	Mech.
10	G	M	19	6	7	5	4	3	8	2	1
10	G	F	20	3	7	5	6	3	8	2	1
10	G	M+F	39	4	7	5	6	3	8	2	1
10	N	M	118	6	7	4	5	2	8	1	3
10	N	F	116	7	6	5	4	2	8	1	3
10	N	M+F	234	6	7	5	4	2	8	1	3
12	G	M	16	4	8	5	6	3	6	2	1
12	G	F	10	4	8	5	6	1	7	1	3
12	G	M+F	26	4	8	5	6	3	7	2	1
12	N	M	105	6	7	4	5	3	8	1	2
12	N	F	88	7	6	5	4	3	8	1	2
12	N	M+F	193	6	7	4	5	3	8	1	2

G = Gifted
N = Normal
M = Male
F = Female

Vol. = Volume
Prob. = Probability
Frames = Frames of Reference
Prop. = Proportions
Comb. = Combinations
Corr. = Correlations
Mom. = Momentum
Mech. = Mechanical Equilibrium

Qualitative analysis suggests that the subtests may be grouped into three levels of difficulty, as follows:

Most difficult	Momentum Mechanical Equilibrium Combinations
Intermediate in difficulty	Frames of Reference Volume Proportions
Least difficult	Probability Correlations

The lowest mean scores were obtained in Momentum, Mechanical Equilibrium, and Combinations. The highest mean scores were obtained in Probability and Correlations. Scores intermediate in value were obtained in the three remaining subtests: Frames of Reference, Volume, and Proportions.

When the subtests are ranked in order of the percentages of students attaining mastery in each subtest, the same order of levels of difficulty is observed (Table 11).

Table 11. Percentages of male and female students in the gifted and norm groups at each grade level attaining mastery in each subtest

	Grade 10		Grade 12		Total	Order of Difficulty
	M	F	M	F		
Gifted N =	19	20	16	10	65	
Vol.	53	50	63	30	51	4
Prob.	79	80	100	90	86	7
Frames.	68	55	75	70	66	5
Prop.	58	55	81	80	66	5
Comb.	58	40	50	30	46	3
Corr.	84	100	88	80	89	8
Moment.	37	30	44	30	35	2
Mech.	11	10	25	40	19	1
Norm. N =	118	116	105	88	427	
Vol.	42	41	50	56	46	6
Prob.	55	49	67	55	56	7
Frames.	40	35	50	23	38	5
Prop.	35	27	51	25	35	4
Comb.	12	16	21	20	17	3
Corr.	80	81	79	81	80	8
Moment.	15	12	19	17	16	1
Mech.	13	18	14	20	16	1

(vii) Differences in the Scores of Arts- and Science-Oriented Students

In order to test hypotheses 3(a) and 3(b), differences in the means of total, subtest, and mastery scores of science- and arts-oriented students in the gifted group were tested for significance.

The complete lists of total, subtest, and mastery scores of gifted, twelfth-grade, arts- and science-oriented students are presented in Appendix 2. Means and standard deviations, together with differences between means and values of "t" obtained by testing the differences for significance ($p < .05$), are shown in Table 12. Although differences in

Table 12. Means and standard deviations of ATFR total, subtest, and mastery scores for arts-oriented and science-oriented gifted students in Grade 12 with values of "t" from tests of differences in group means

	Arts-Oriented (N = 12)		Science-Oriented (N = 12)			
	Mean	S.D.	Mean	S.D.	Difference	Value of 't'
Total	21.75	2.65	23.42	3.38	-1.67	1.35
Volume	2.75	1.09	2.58	1.32	0.17	0.34
Probability	3.42	0.64	3.67	0.47	-0.25	1.14
Frames of Reference	3.08	0.95	3.08	1.19	0.00	0.00
Proportions	3.25	1.01	3.67	0.62	-0.42	1.27
Combinations	2.08	0.86	2.42	0.86	-0.34	0.97
Correlations	3.17	1.14	3.58	1.61	-0.41	0.72
Momentum	2.00	1.08	2.33	0.85	-0.33	0.83
Mechanical Equilibrium	2.00	1.00	1.75	1.09	0.25	0.60
Mastery	4.67	1.03	5.17	1.34	-0.50	1.02

Value of "t" required for significance (22 d.f.; $p < .05$) = 2.07

five of the eight subtests are in favour of the science-oriented students, none proves to be significant. Differences in total and mastery scores also fail to show significance. These results support the rejection of hypothesis 3: that science-oriented students obtain higher total and mastery scores than arts-oriented students. This finding should be regarded as tentative in view of the low number and restricted range of students involved in this part of the study. To generalize beyond twelfth-grade gifted students also would require more comprehensive replication.

(viii) Correlations of ATFR Scores with IQ

The relationship between ATFR scores and IQ was explored to ascertain the former's usefulness or redundancy in identifying gifted students. High correlations would suggest that the two measures are tapping similar forms of ability. Low correlations would tend to support the use of both measures in assessing different forms of ability.

Wechsler Verbal, Performance, and Full Scale IQs, obtained from the WISC-R or WAIS-R, for the gifted group students, together with their ATFR total and mastery scores, are presented in Appendix 3. Correlations (Pearson "r") of ATFR scores with IQs at each grade level are shown in Table 13. Correlations with the Full Scale IQs were higher for the

Table 13. Correlations (Pearson "r") of ATFR total and mastery scores with Wechsler Verbal, Performance and Full Scale IQs for the gifted group students at each grade level

Grade 10 (N = 25)

	Verbal IQs	Performance IQs	Full Scale IQs
Total scores	0.16	0.10	0.24
Mastery scores	0.03	0.12	0.13

Grade 12 (N = 20)

	Verbal IQs	Performance IQs	Full Scale IQs
Total scores	0.53	0.05	0.29
Mastery scores	0.52	0.07	0.39

twelfth-grade than for the tenth-grade students. The highest correlations were between Verbal IQ and the ATFR scores at grade twelve (0.53 and 0.52 for total and mastery scores respectively). The lowest correlations were between Performance IQ and the ATFR scores, also at grade twelve, (0.05 and 0.07 for total and mastery scores respectively).

The generally low correlations between IQs and ATFR scores are illustrated by the following three cases:

1. A tenth-grade male student who had a Full Scale IQ of 133 (V.IQ = 127, P.IQ = 131), and was ranked second highest of the tenth-grade students, achieved mastery in only two subtests and was classified as high concrete.

2. A tenth-grade female student who had a Full Scale IQ of 116 (V.IQ = 118, P.IQ = 109), and was ranked lowest in the grade ten list, achieved mastery in six subtests and was classified as low formal.

3. A twelfth-grade female student with a Full Scale IQ of 124 (V.IQ = 103, P.IQ = 142) was ranked lowest in Verbal IQ and second highest in Performance IQ. She was classified as low formal but achieved mastery in only three subtests, which was less than any other gifted grade twelve student achieved.

CHAPTER 5

DISCUSSION

The purpose of the study was to investigate the effectiveness of a group test of formal operational reasoning in providing information that would assist in planning appropriate programs for students identified as intellectually gifted.

After describing the limitations imposed upon generalization of the results to the intellectually gifted population, the findings will be discussed and related to previous research. Implications for both education and future research will be drawn.

(i) Limitations of Present Findings

1. Assessment by means of the ATFR is confined to students' ability to effectively use the eight formal operational schemata which may be viewed as latent potentialities elicited by certain situations in the environment (Inhelder and Piaget, 1958).

The schemata are less abstract and more highly specialized than other propositional operations. When their effective use is taken as the criterion for assessment as formal operational, both false positives and false negatives are likely. An individual's use of the schemata may or may not be elicited by the given stimuli and may or may not generalize to other areas of thought (Piaget, 1972). Evidence to date is inconclusive that effective use of the schemata is necessary or sufficient for success in propositional reasoning or logical thought in other content areas.

2. The subjects were identified as intellectually gifted mainly on the basis of high IQ and academic performance rather than for creativeness or originality; that is, they were selected on evidence of convergent rather than divergent thinking ability. While it has been found that individuals who are high in divergent thinking ability are almost always high in IQ also, the reverse is not always the case (Guilford, 1975).

It may be that students who excel in divergent thinking would also show superior formal reasoning skills, and that higher correlations would be obtained between scores on the ATFR and a test of creativity than were found with IQ. This remains to be explored through further research.

3. The gifted and norm groups were taken from different, although comparable, populations. Differences in performance may be attributable to subtle differences related to school location and educational practices.

4. Although intelligence is considered a relatively stable trait, recent IQ scores (obtained within the past three years) were available only for a limited number of gifted students. Generalization of the findings concerning the relationship of performance on the ATFR to IQ is restricted to the gifted population defined primarily on IQ and achievement parameters.

5. Generalization of findings related to science- and arts-oriented students is confined to twelfth-grade gifted students; that is, to those who successfully completed the PACE program or possibly to students with similar IQ and achievement profiles.

(ii) Group Differences

The finding of superior performance in tasks measuring formal operational reasoning by individuals of high IQ has been reported previously (Carter and Ormrod, 1982; Dulit, 1972; Keating, 1975, 1976; Seiber, 1978). In the present study over 90 percent of the gifted group demonstrated competence in the use of the formal operational schemata by grade twelve. This was not so in the case of the norm group, less than half of which attained a formal level. Similar findings of low incidence of formal operational reasoning in the general population have been reported (Dulit, 1972; Shayer, Adey, and Wylam, 1981). The superiority of the gifted group was evident at the grade ten level and became greater at grade twelve. The finding that the twelfth-grade gifted students showed evidence of more formal operational competence than those in grade ten, while such progress was less evident in the norm group, suggests that the latter tend to reach a ceiling in their conceptual development at an earlier age.

As in the present study, Keating and Schaefer (1975) found that younger bright students perform significantly better than older average students, although their sample was somewhat younger. Chronological age appears to set certain limits on the appearance of precocity in formal operational reasoning (Lovell and Shields, 1967; Webb, 1974). It would seem that investigators would witness more success in finding precocity if they focussed on an older age range of gifted individuals.

The acceptance of hypotheses 1(a), 1(b), and 2(a) lends tentative support to the argument favouring the provision of special programs for intellectually gifted students. Material requiring the understanding of higher-order concepts and abstract propositions can be effectively introduced into programs for students who use a less empirical-intuitive and more hypothetical-deductive approach to problem solving.

(iii) Sex Differences

The superiority of male students' scores over those of females in formal operational reasoning tests has been reported in previous investigations (Dulit, 1972; Graybill, 1975; Keating and Schaefer, 1975; Ross, 1976; Stanley, Keating, and Fox, 1974). Differences in the total scores of male and female students in this study were significant ($p > .05$) being greater for the gifted group and at the higher grade level. It is noteworthy that, unlike the difference between total scores, the difference between mastery scores of males and females, while being in favour of the former, failed to reach statistical significance. Mastery scores present a purer assessment of students' competence in the use of the formal operational schemata than the more global total scores or cognitive levels. Responses involving lower level reasoning receive some weight when assessment is by total score, whereas in recording mastery scores, recognition is given only to higher level reasoning. Only subtest scores of three or four contribute to the mastery score, lower subtest scores being discounted.

Although most differences between males' and females' scores in the five subtests which fit the linear model were in favour of the males, in only one subtest (Frames of Reference) did the value reach significance ($p < .05$). One could speculate that the female students were deterred from optimal performance by the spatial relationships seen to be necessary for success in this subtest.

A possible reason for the high percentage (91 percent) of gifted females failing to attain the high formal level by grade twelve in comparison to males (60 percent) may be found in the content of the curriculum that is usually followed by girls. As was discovered when students were interviewed about their career intentions, girls are less likely than boys to proceed to advanced mathematics and science courses. Hence they are less likely to be familiar with the concepts underlying the formal operational tasks. This is a tentative suggestion which requires confirmation by further research. It is possible that the female students have the competence in formal operational reasoning but that it is not translated into as high an overall performance score as is that of the males. Reasons for this may be found in a lack of interest or motivation and less appropriate curricular experience.

(iv) Subtest Differences

The fact that all the differences in subtest scores that proved to be significant were in the expected direction but few in number, suggests limitations in the power of the test, possibly by reason of the small number of items in each subtest. This likelihood is increased by the fact that the Proportions subtest in which more significant differences were found than in any other subtest, was the one in which the explained variance occupied a larger portion of the total variance than in any other subtest.

Some subtests proved very difficult, even for the gifted students: for example, Mechanical Equilibrium was mastered by only 9 percent of the group. Other subtests presented relatively little difficulty: for example, Correlations was mastered by 89 percent. Such fluctuations in students' performance provide evidence which fails to support the unitary nature of formal operational reasoning, (Bart, 1971; Arlin, 1981).

The subtests which differentiated most clearly and consistently between gifted and norm groups were Probability, Frames of Reference, Proportions, Combinations, and Momentum. This information may be utilized in proposing a short-form of the ATFR with fewer subtests but more items in each subtest retained in order to improve the test's reliability.

From individual or group composite subtest profiles, students' "goodness of fit" to the formal operational model could be determined; that is, the degree of correspondence of the order of difficulty encountered by the students to that generally encountered. Extreme divergence from the general pattern could serve to prompt further investigation through alternative channels of assessment.

Assessment by means of the ATFR would seem especially appropriate for matching the material presented to learners with their demonstrated competence in a variety of areas of concept development. Students may be grouped according to their mastery scores, or the pattern of subtests mastered, and offered programs suited to the patterns of their abilities, capitalizing on strengths and remediating or compensating for areas of weakness. This suggestion is applicable to both gifted and non-gifted students since the same general pattern of concept acquisition was shown to apply to both groups.

(v) Differences in Orientation and I.Q.

Since the scores of science- and arts-oriented students were not significantly different, no support is given to the contention that the reasoning abilities necessary for successful formal operational performance are prerequisite to mastering science and mathematics rather than social studies or language courses, (Bart, 1979). It would seem that a high cognitive level is equally advantageous, whatever the subject-orientation. Although no difference is discernible in the quality of the responses of arts- and science-oriented students, it may be that formal operational reasoning will manifest itself earlier for all indivduals in the area of physical science than in other content areas.

The higher correlations between ATFR scores and IQ at the grade twelve level, and the differences between correlations with Verbal and Performance IQs suggest that high IQ plays a more important role in determining formal operational performance at the higher grade level; and that factors underlying success in using the formal operational schemata have a strong verbal component (Bart, 1971). This, in turn, lends support to the practical use of the ATFR when predicting scholastic success.

The rejection of hypotheses 2(b), 3(a), and 3(b) attest to the lack of bias in the ATFR with regard to differences between males and females or between arts- and science-oriented students in assessing their use of the formal operational schemata.

(vi) Implications for Future Research

1. The differences found between gifted and norm group students in grades ten and twelve suggest that the investigation of such differences could profitably be extended into the lower grades in attempting to discover at which grade level the difference first becomes evident, making differential programming appropriate.

2. The norm group contained a cross-section of the entire school population at the grade levels examined. It would be reasonable to expect a wide range of ability and achievement in the group. This is reflected in the frequencies of mastery scores (Table 8, pg. 32). Three norm group students in grade ten, and one in grade twelve, achieved higher mastery scores than any gifted group student in the same grade. The norm group, therefore, may include some gifted individuals. When a sufficient number of students is available from the referred population it should be possible to compare the performances of selected and unselected students. This would more clearly demonstrate the contribution that the ATFR could make as a screening instrument.

3. Inferences drawn from the comparison of scores of science- and arts-oriented students should be viewed with caution due to the limited number and restricted range of subjects involved. A more comprehensive study including members of the norm group and students from different grade levels may provide information indicating the range of individuals for whom the ATFR is an effective, non-biased assessment tool.

(vii) Summary and Conclusions

Level of intelligence has been found to be more important than grade level or sex in determining an individual's standard of performance in formal operational reasoning tasks (Carter and Ormrod, 1982; Keating and Schaefer, 1975). This finding is supported by the results of the present study. Although within both gifted and norm groups males scored higher than females and students in grade twelve scored higher than those in grade ten, nevertheless, gifted students who were both female and in grade ten performed better than norm group students who were male and in grade twelve.

The primary objective of this study was to examine ways in which the ATFR could be utilized in planning appropriate programs for intellectually gifted students. Initially it was pertinent to ask whether or not students' performance in the ATFR contributes information supplementary to that obtained through traditional psychometric measures; if so, what the nature of this additional information might be, and how it could be used in program planning. Why is the information especially appropriate for planning programs to meet the needs of intellectually gifted students? On examining the content of the items in the test, many of which appear closely linked to principles commonly used in mathematics and physical science courses, it may also be asked whether or not the test is biased in favour of particular sections of the population. For example, do the test items favour males rather than females, or students in science courses rather than those in arts? The present study was designed to assist in providing answers to questions such as these.

Briefly, from the results already discussed it may be inferred that, although there is some overlap between the measures, the ATFR is assessing attributes which differ from those measured by conventional IQ tests. The additional information is qualitative rather than quantitative, and indicates areas of concept development in which competence may be demonstrated under appropriate conditions; that is, an optimal level of functioning.

Students of high intellectual ability are more likely to display formal operational competence than students of average ability, but evidence suggests that some individuals capable of benefitting from working with advanced conceptual material are presently missed by traditional screening procedures.

Samples of the students' performance in using the formal operational schemata may indicate areas of strength and weakness in concept development. From this information, certain concepts may be selected for emphasis when preparing programs and materials of appropriate content and levels of difficulty.

From the range of cognitive levels and potential for further development shown by the gifted students a wider choice of program planning for these students is shown to be warranted.

The evidence presented here indicates the ATFR to have a potential contribution to make in the assessment of intellectually gifted students and to be without obvious bias with regard to sex and subject-orientation.

The ATFR could advantageously be used with a variety of measures of IQ, achievement, creativity and interests in assessing students' educational potential. This battery could be used to provide teachers with useful information about the abilities of students to master specific topics in the curriculum.

It appears that while gifted students show evidence of more developed formal reasoning by grade twelve than the typical school population, a number of questions remain as to their acquisition of the eight formal concepts described by Inhelder and Piaget (1958) and the extent to which their use of these concepts is a function of their participation in programs designed for the gifted and talented.

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Appendix 1

Generalized Least-Squares Analyses of Variance of ATFR Subtest Scores.

(a) Tests of Fit to the Linear Model.

Test	Source of Variation	Sum of Squares	D.F.	Mean Square	F
Volume	Explained	17.39	7	2.48	1.34 ns
	Residual	893.09	484	1.85	
	Total	910.48	491	1.85	
Probability	Explained	49.27	7	7.04	6.14 **
	Residual	554.94	484	1.15	
	Total	604.21	491	1.23	
Frames of Reference	Explained	59.96	7	8.57	6.21 **
	Residual	667.36	484	1.38	
	Total	727.32	491	1.48	
Proportions	Explained	118.98	7	17.00	10.22 **
	Residual	804.69	484	1.66	
	Total	923.67	491	1.88	
Combinations	Explained	55.41	7	7.92	6.17 **
	Residual	621.00	484	1.28	
	Total	676.41	491	1.38	
Correlations	Explained	6.61	7	0.95	0.84 ns
	Residual	543.35	484	1.12	
	Total	549.96	491	1.12	
Momentum	Explained	33.72	7	4.82	3.67 **
	Residual	634.53	484	1.31	
	Total	668.25	491	1.36	
Mechanical Equilibrium	Explained	11.45	7	1.64	1.33 ns
	Residual	597.20	484	1.23	
	Total	608.65	491	1.24	

** $p < .01$
ns not significant

Appendix 1

(b) Further Analyses of Subtests Retained

Test	Source of Variation	Sum of Squares	F (1,484 d.f.)
Probability	Grade (A)	4.36	3.80
	Sex (B)	2.26	1.97
	Group (C)	29.41	25.65 **
	A×B	0.04	0.03
	A×C	0.10	0.09
	B×C	0.67	0.59
	A×B×C	0.00	0.00
Frames of Reference	Grade (A)	2.89	2.09
	Sex (B)	6.81	4.94 *
	Group (C)	35.58	25.80 **
	A×B	0.62	0.45
	A×C	2.62	1.90
	B×C	0.23	0.17
	A×B×C	0.14	0.10
Proportions	Grade (A)	10.06	6.05 *
	Sex (B)	6.04	3.63
	Group (C)	58.80	35.37 **
	A×B	2.03	1.22
	A×C	3.96	2.38
	B×C	6.42	3.86 *
	A×B×C	0.01	0.01
Combinations	Grade (A)	0.47	0.36
	Sex (B)	0.95	0.74
	Group (C)	35.43	27.61 **
	A×B	0.82	0.64
	A×C	2.18	1.70
	B×C	2.67	2.08
	A×B×C	0.71	0.56
Momentum	Grade (A)	4.95	3.78
	Sex (B)	2.90	2.21
	Group (C)	24.76	18.88 **
	A×B	0.20	0.15
	A×C	2.25	1.72
	B×C	0.70	0.54
	A×B×C	0.00	0.00

* $p < .05$
** $p < .01$

Appendix 2

ATFR Total, Subtest, and Mastery Scores of Arts-Oriented and Science-Oriented Gifted Students in Grade 12

ARTS:

		Total	Vol.	Prob.	Frames	Prop.	Comb.	Corr.	Mom.	Mech.	Mastery
004	F	22	2	3	3	4	2	4	2	2	4
005	F	18	3	3	3	3	1	1	1	3	5
006	F	24	2	4	3	4	2	4	3	2	5
007	M	18	0	3	3	4	2	4	1	1	4
008	M	23	4	4	4	2	2	2	2	3	4
015	F	22	2	4	2	4	3	4	2	1	4
017	M	26	4	3	4	4	3	3	4	1	7
018	F	23	4	4	4	3	3	1	3	1	6
095	M	24	4	3	2	4	3	4	2	2	5
020	F	17	2	4	1	2	0	4	0	4	3
022	M	23	2	4	4	4	2	3	3	1	5
069	F	21	4	2	4	1	2	4	1	3	4

SCIENCE:

		Total	Vol.	Prob.	Frames	Prop.	Comb.	Corr.	Mom.	Mech.	Mastery
001	M	22	4	4	4	3	1	4	2	2	0
002	M	17	1	3	1	2	2	4	3	3	1
009	M	27	2	4	4	4	3	4	4	4	2
010	M	21	4	4	2	3	2	4	1	1	1
011	M	28	4	4	4	4	4	4	2	2	2
014	M	27	4	4	4	4	3	4	2	2	2
016	F	24	2	4	3	4	2	4	2	2	3
019	M	21	2	3	4	4	2	4	1	1	1
021	F	19	2	3	2	4	1	4	2	2	1
023	M	27	2	4	4	4	3	4	3	3	3
063	F	23	0	4	4	4	3	4	3	3	1
066	M	25	4	3	1	4	3	3	3	3	4

Vol. - Volume
Prob. - Probability
Frames - Frames of Reference
Prop. - Proportions
Comb. - Combinations
Corr. - Correlations
Mom. - Momentum
Mech. - Mechanical Equilibrium

Appendix 3

Wechsler Verbal, Performance, and Full Scale IQs with ATFR Total and Mastery Scores of Gifted Group Students

I.D.	Verbal I.Q.	Performance I.Q.	Full Scale I.Q.	ATFR Total	Mastery Scores
Grade 10 (N=25)					
026	118	117	120	17	3
027	127	118	126	16	3
028	136	121	132	19	3
029	119	142	134	21	4
030	103	136	121	22	5
031	127	131	133	13	2
032	111	123	118	11	1
034	128	130	133	28	7
035	136	121	132	22	4
036	117	121	121	23	6
039	122	129	128	16	4
041	137	117	130	22	6
045	123	123	126	16	2
046	111	141	128	21	6
047	122	121	124	23	5
049	119	126	125	22	5
050	133	111	125	24	6
053	113	136	127	20	5
055	120	123	124	14	4
056	114	131	125	19	4
057	118	109	116	24	6
060	119	129	126	24	6
061	122	121	124	18	4
065	122	135	132	23	6
072	117	114	118	13	3
Grade 12 (N=20)					
001	123	133	132	22	5
004	126	108	121	22	4
005	117	128	124	18	5
006	131	114	129	24	5
008	111	106	109	23	4
009	142	117	133	27	6
010	119	118	120	21	4
011	140	132	135	28	6
012	131	117	127	19	4
013	117	111	116	30	7
014	135	123	132	27	6
015	107	112	110	22	4
016	136	136	141	24	5
017	131	132	136	26	7
018	124	103	116	23	6
019	124	106	118	21	4
021	103	142	124	19	3
022	123	147	140	23	5
023	140	123	135	27	7
063	113	131	124	23	6