CAPITAL MARKET EQUILIBRIUM AND DIFFERENTIAL INFORMATION: AN ANALYTICAL AND EMPIRICAL INVESTIGATION

by

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M.B.A., University of Windsor, 1980

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of DOCTOR OF PHILOSOPHY in The Faculty of Graduate Studies (Commerce and Business Administration)

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The University of British Columbia

October 1986

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ABSTRACT

This dissertation examines implications of models of differential information that formalize the following intuition: securities for which there is relatively little information are perceived as relatively more risky because of the higher uncertainty surrounding the exact parameters of their return distributions.

The testable implications of the theoretical model depend on the nature of the assumed economy. In a small market setting we find that the presence of differential information will lead investors to make an upward adjustment to the measure of systematic risk that they employ for the pricing of low information securities relative to its measurement without regard for differential information. Further, the model predicts that the adjustment will decrease at a decreasing rate as information increases. Lastly, in the face of such an adjustment, the typical estimate of beta employed by researchers is found to be upwardly biased. On the other hand, the model predicts that the size of the adjustment to beta at each information level falls with the degree of positive cross-correlation between the low and high information securities, and the lack of cross-correlation in the information patterns for the low information securities. Thus, in unrestricted economies, where these sources of diversification are available, the presence of differential information should have little or no impact on market equilibrium.

The implication that systematic risk ($\beta$) for low information firms should decline as information increases is confirmed with three independent samples. First, a sample of 142 newly-listed firms is used to examine the relationship between systematic risk and time following the listing date. Next, a sample of 348 NYSE
firms is employed to further explore this relationship. Finally, a sample of 140 newly-listed firms is used to examine the change in systematic risk at the time of the first annual earnings announcement. We also consider the suggestion that an association between information availability and both firm size and period of listing might provide a partial explanation for the firm size and period of listing anomalies. Our evidence suggests that this is unlikely where historical returns are used to estimate beta. Prior historical return associations tend to overstate beta at a point in time rather than understate it.
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CHAPTER ONE

INTRODUCTION

Banz [1981] conjectured that one possible explanation for the small firm effect is based on the concepts of estimation risk and information availability as examined by Klein and Bawa [1976, 1977]. Addressing this conjecture, Barry and Brown [1984, 1985] propose a "simple model" of differential information in the Capital Asset Pricing Model (CAPM) setting which allows for differences in the amount of information available for developing inferences about the parameters of the return distribution for alternative securities. Within the context of their models, both Klein and Bawa and Barry and Brown show that securities for which there is relatively less information are perceived as relatively more risky because of the higher uncertainty surrounding the exact parameters of their return distributions and consequently must provide correspondingly higher expected returns to clear the market.

For example, Barry and Brown [1985] argue,

"In an equilibrium setting, portfolios of low information securities will appear to earn positive abnormal returns if their betas are measured without regard to differential information and if their returns are consistent with a CAPM in which investors properly account for differential information."

They continue on to suggest that "... a researcher ignoring these risk perceptions will conclude that there are excess returns when in fact the returns are commensurate with risk."

The viability of these implications rests critically on the relation among three variables: (1) the estimate of beta used by researchers, (2) the estimate of beta measured without regard for differential information, and (3) the estimate of beta appropriately adjusted for the degree of differential information. A setting in which differential information exists can be characterized as one in which there is less
complete or less accurate information concerning a subset of firms. Alternatively, we might think of differential information in terms of a setting in which there are cross-sectional differences in the public availability of information potentially relevant for assessing common stock share values.

The purpose of this dissertation is to focus on the implications of the differential information model for estimation of the parameters of the return process. The questions that we address deal both with the relation that Barry and Brown develop between the unadjusted and adjusted measures of systematic risk, and with the implications of this relation for the measure estimated by the researcher. The remainder of the dissertation is organized as follows. Chapter 2 summarizes previous literature relevant to the research question. In Chapter 3, a model of the adjustment process is described, the relation among the three variables of interest is examined, and the robustness of these results to the assumed environment is considered. The testable implications of the model, data sets on which the model can be tested, and the empirical results for these samples are summarized in Chapter 4. Chapter 5 contains conclusions and a discussion of possible extensions.

In summary, we find that the testable implications of the differential information model for capital market equilibrium depend on the nature of the assumed economy. In a small market setting, the presence of differential information leads investors to make an upward adjustment to the measure of systematic risk that they employ for the pricing of low information securities relative to its measurement without regard for differential information. Further, this adjustment is found to decrease at a decreasing rate as information increases. Finally, in the face of such an adjustment, the typical estimate of beta employed by researchers (the measure resulting from a time-series regression) is found to be upwardly biased. On the other hand, we find, by relaxing the constraints on the economy to allow for multiple low and
high information securities as well as a generalized information structure, that the information effects may be diversifiable. In this case, the presence of differential information will have no impact on capital market equilibrium.

The results of our tests are consistent with the differential information model. Information risk appears to be partly nondiversifiable. For the samples that we examine, the adjustment process dissipates relatively quickly, however, essentially within several months' time. Where historical beta estimates are used to gauge investment performance, both our model and our empirical tests imply that information risk cannot provide an explanation for the firm size or period of listing anomalies. Historical estimates tend to overstate the beta risk existing at the end of the period.
CHAPTER TWO

ESTIMATION RISK AND CAPITAL MARKET EQUILIBRIUM:
A REVIEW OF RELEVANT LITERATURE

1. INTRODUCTION

Historically, portfolio choice has been viewed as a choice among alternative known probability distributions. The common practice has been to assume that the probability distribution of security returns belongs to a certain family of distributions (e.g., the normal distribution) and then treat the estimated parameter values as the true parameters when determining an individual’s optimal portfolio choice. In so doing, this approach ignores estimation risk. However, as Bawa, Brown, and Klein [1979] point out, the procedure explicitly recognizing estimation risk is appropriate under a set of reasonable axioms which have been expounded by von Neuman-Morgenstern and Savage; i.e., “... under the Savage extension of the von Neuman-Morgenstern axioms an investor maximizing expected utility will explicitly recognize estimation risk.”

2. ESTIMATION RISK AND PORTFOLIO CHOICE

By estimation risk it is meant formally that the probability distribution function, say $f$, is not completely known. This includes the situation in which the functional form of $f$ is unknown (nonparametric case) and the more common situation in which the functional form is presumed known but the value of the parameter vector, say $\theta$ (of finite dimension), is unknown. It has been shown that, with no information on the functional form of the distribution (nonparametric) and minimal prior information, there is no need to make a special correction for estimation risk (Bawa [1977, 1979]). However, in the parametric setting, with
some information on the functional form of the distribution, the traditional method of using point estimates (i.e., replacing the unknown value of the parameter vector, $\theta$, by a point estimate, $\hat{\theta}$, ) will be generally inappropriate (Klein, Rafsky, Sibley, and Willig [1978]).

To demonstrate the impact of estimation risk on the portfolio selection problem, we adopt the following simplistic example from Bawa, Brown, and Klein [1979]. The situation involves an investor who must choose between two risky securities with random returns, $\tilde{R}_j$ ($j=1,2$), where $\tilde{R}_j$ is normally distributed with unknown mean $\mu_j$ and known common variance $\sigma^2$. Further, it is assumed that, based on a random sample of $N_j$ observations on security j, the investor determines $\hat{\mu}_1 = \hat{\mu}_2$. If the investor ignores parameter uncertainty he will be indifferent between the two securities. However, if the investor incorporates estimation risk into his portfolio selection problem (i.e., he explicitly recognizes parameter uncertainty) then we find that this is no longer the case. Specifically, the investor will recognize that there is a distribution associated with $\tilde{R}_j$, the sampling distribution of the mean, and will assess the distribution of $\tilde{R}_j$ to be

$$\tilde{R}_j \sim N(\mu_j, \frac{N_j + 1}{N_j} \sigma^2)$$  \hspace{1cm} (2.1)

Thus, we see that the distribution which explicitly recognizes estimation risk, known as the predictive distribution (see Zellner [1971]), incorporates two components of risk. The first, $\sigma^2$, reflects the basic or inherent randomness in $\tilde{R}_j$, while the second, $\sigma^2/N_j$, reflects the estimation (or information) risk due to incomplete knowledge of the mean. This phenomenon is presented pictorially in Figure 1.

Preliminary consideration of the impact of parameter uncertainty (estimation risk) on the portfolio selection problem is provided by Kalymon [1971] and Barry [1974], among others. Kalymon finds that allowing for uncertainty about the mean
of the return distribution leads to an increase in portfolio variance. Extending
this work, Barry determines that each additional level of uncertainty (in order, no
uncertainty, then uncertainty about only the mean, and finally uncertainty about
both the mean and variance) serves to increase portfolio risk as measured by
variance. He also finds that while the mean-variance efficient set is unaffected,
the optimal portfolio will change as additional levels of uncertainty are introduced.
Finally, he notes that as the amount of information available for estimating the
parameters of the distribution increases, the decisions will converge.

Continuing this investigation, Klein and Bawa [1976] consider the effect of esti-
mation risk on optimal portfolio choice for the case of sufficient sample information
(the number of observations per security exceeds the number of securities) un-
der the assumptions that the joint distribution of security returns is normal and
the underlying parameters are unknown. They also find that the admissible set of
portfolios determined under the Bayesian approach (recognizing estimation risk)
is the same as under the traditional approach, but that an investor's optimal choice
will differ. Taking estimation risk into account, the optimal choice involves invest-
ing monotonically more in the riskless asset as the degree of (sample) information
decreases. Klein and Bawa [1977] advance this work to consider the situation of lim-
ited information and determine that, as a result of estimation risk, one would expect
risk averse investors to invest relatively more in those securities about which they
have the most information. In summary, Klein and Bawa find that the Bayesian
investor will tend to invest relatively more in those securities about which the most
information is available. To reduce estimation risk, a risk averse investor will tend
to reduce or even eliminate (asymptotically) diversification into those securities
which are characterized by low degrees of information.

Brown [1979] examines the impact of estimation risk on the traditional CAPM.
He shows that, with all investors having homogeneous beliefs, noninformative or
diffuse priors, and access to the same sample information, the "... composition of
the market portfolio, expected return on the market, and corresponding measures of
systematic risk are identical to their values under the assumption that investors take
unbiased sample estimates as the relevant true parameter values." 1 As pointed out
by Bawa, Brown, and Klein [1979], when combined with Klein and Bawa's results,
this implies that estimation risk causes risk-averse investors to behave as if they
exhibited additional risk aversion by investing more in riskless assets and less in the
market portfolio of risky assets.

Other applications of the Bayesian approach (explicitly recognizing estimation
risk) include Barry and Winkler [1976], who, assuming only an unknown mean
vector, adopt the estimation risk framework to consider the assumption of station-
arity. They demonstrate that, since as the degree of non-stationarity increases less
weight is given to all but the most recent observations, non-stationarity is analogous
to restricting the amount of information to a defined upper limit, say $n_L$. Thus,
non-stationarity will provide a lower support for risk at $(1 + 1/n_L)\Sigma$ (where $\Sigma$
is the known covariance matrix) and portfolio decisions will no longer converge to
the infinite information position.

3. A POSSIBLE EXPLANATION OF THE SMALL FIRM EFFECT

Barry and Brown [1985], in response to the conjecture by Banz [1981] that one

1 Support for this result is forthcoming from Alexander and Resnick [1985] who
counter Chen and Brown [1983] by demonstrating that the effect of estimation risk
on the tangency portfolio, using the market model, is not "... as substantive as
previously believed and in many situations can be safely ignored."
explanation for the 'small firm effect' involves differential information and the aforementioned estimation risk models of Klein and Bawa, develop what they term a "simple model" of differential information within the context of the CAPM setting. They consider a universe of S securities and assume that returns are multivariate normal with a known covariance matrix, but that investors have only imperfect knowledge about the value of the mean vector. The first $S_1$ securities are designated as high information securities, the remainder as low information securities. This designation is based on the premise that there are $N$ observed returns available from which to estimate the mean vector for the high information securities but only $n_2 < N$ observed returns for the low information securities. Finally, they assume that investors explicitly recognize parameter uncertainty, employing as the appropriate distribution for investment decisions, the predictive distribution. More precisely, the covariance matrix of returns they adopt is

$$\Sigma_P = \begin{pmatrix} h(N)\Sigma_{HH} & h(N)\Sigma_{HL} \\ h(N)\Sigma_{LH} & h(n_2)\Sigma_{LL} \end{pmatrix}$$

where

$$h(n) = 1 + \frac{1}{n}$$

Then, with $N = n_2 < \infty$ (i.e., with estimation risk but not differential information), they find, in a fashion similar to Brown [1979],

$$\beta_i(N, N) = \frac{h(N)\sigma_{im}}{h(N)\sigma_m^2} = \beta_i(\infty, \infty)$$

where $\beta_i(N, N)$ is the measure of systematic risk for security $i$ with $N$ observations available for both the high and low information securities and $\sigma_m^2$ is the variance.

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2 This term is used to denote the observed phenomenon that small firms have, on average, enjoyed higher risk adjusted returns than large firms where size is defined in terms of market capitalization. For further development, see Banz [1981], Reinganum [1981], Reinganum [1982], Roll [1981], and Keim [1983], among others.

3 For a development of the concept of differential information see Shores [1985] who examines the association between cross-sectional differential information and security returns surrounding annual earnings announcements.
of the market return. From this result they conclude that, with equal information, systematic risk is unaffected. However, with \( N > n_2 \), they determine

\[
\beta_H(N, n_2) = \frac{h(N)\sigma_{im}}{h(N)\sigma_m^2 + \gamma} < \beta_H(N, N), \quad \gamma > 0
\]

(2.4)

and conclude that "... high information securities have smaller betas under differential information than they would under equal estimation risk." They argue that, since the weighted average of the betas for all securities must be one, "... the portfolio of low information securities must have a beta that appears to be too low."

Finally, they suggest that

"... the portfolio of relatively low information securities will appear to earn abnormal returns if their betas are measured without regard to differential information and if their returns are consistent with a CAPM in which investors properly account for differential information"

and consequently

"... a researcher ignoring these risk perceptions will conclude that there are excess returns when, in fact, the returns are commensurate with risk."

At this point Barry and Brown turn their attention to the empirical realm. Employing relative period of listing (defined as the length of time for which a security has been listed) as a proxy for quantity of information, Barry and Brown [1984] determine the existence of an empirical regularity which they label the 'period of listing effect'. Their data set consists of all firms listed on the NYSE between 1930 and 1981 for a period of at least 61 months which have at least 21 months of data available on the CRSP monthly return file. To facilitate the analysis, the firms are cross-classified on the basis of size, period of listing, and systematic risk (beta) as estimated from the market model. They find that, when measured in terms of standardized excess returns, the size effect appears to be most pronounced among securities in the lowest period of listing group while the period of listing effect is most pronounced for the smallest size category. Further, they determine that the size effect, while associated with the period of listing effect, does not dominate
it. They conclude that "... there appear to be substantial returns associated with firms with low periods of listing and its does not appear that those returns can be attributed to firm size or risk."

Brown and Barry [1984] examine a possible misspecification due to nonstationarity of the market model during the period used to estimate beta. They estimate the market model for each security in the sample (again all NYSE firms listed between 1930 and 1981 for at least 61 months which have at least 21 months of data on the CRSP monthly return file available to estimate the market model), calculate one step ahead residuals, and then compute the recursive residual, $S_{it}$, for firm $i$ in month $t$. They argue that recursive residuals should have mean 0 and variance 1 if the market model is well-specified over the estimation and testing periods. The firms are then cross-classified on the basis of firm size and period of listing, and the "average" recursive residual for each class is calculated. On the basis of their sample, Brown and Barry conclude that the market model appears to be misspecified, noting that the most serious misspecification appears to be with the smallest firms, especially those listed for the least amount of time. In summary, they urge that caution must be used in interpreting excess returns that result from betas estimated by the market model.

Ibbotson [1977], in documenting positive initial performance for a sample of newly listed firms, also found evidence of parameter instability. Specifically, he found that the "... systematic risks of new issues are greater than the systematic risk of the market, and the systematic risks of securities are not stable in that they drop as issues become seasoned."

Lastly, although not addressing the small firm effect, Williams [1977], using a continuous time model, develops predictions similar to those of Barry and Brown [1985]. He initially demonstrates, assuming that the means, variances, and covari-
ances of returns on risky assets must be estimated using available information, that investors can, in this setting, estimate with complete accuracy unknown variances and covariances, but not means; i.e., "... investors generally cannot in finite time observe without error true expected returns on risky securities." He then argues that the investor will, in the absence of costs to acquiring and processing information, continuously update his posterior distribution over the means, and consequently, his predictive distribution over the uncertain returns on risky securities. Thus, the investor is seen to replace the true means with the corresponding estimated means, and the true covariance matrix, $\Sigma$, with the corresponding matrix from the predictive distribution, $\Sigma / \tau_i(t)$ where $\tau_i(t) \leq 1$, when making his portfolio choice.

Moving to consider the impact of this uncertainty on the CAPM (specifically the security market line (SML)), Williams notes that the adjustment to the variance, $\tau_i(t)$, will reduce the investor's position in each risky asset ($\tau_i(t) \leq 1$). In addition, as the investor accumulates progressively more and more information, the position will converge to the usual portfolio of risky securities. Further, he finds, within the context of his model, that "... the SML specifies a linear trade-off in the market between expected returns and perceived risk, where beta no longer remains a complete measure of risk." The additional risk terms arise from the investor's efforts to protect himself against uncertain future adjustments in the estimates of the means. In effect, the uncertainty "... induces investors to adjust personal portfolios to hedge against the risk from unpredictable shifts over time in subjective estimates of unknown means." Thus, Williams predicts, in a fashion which is consistent with Barry and Brown, that limited information will affect the measure of risk used for the pricing of securities in the CAPM setting. His analysis also implies that the

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*4 This covariance matrix is of a form similar to the one advanced by Barry and Brown [1985] - see equation (2.2).*

11
magnitude of the adjustment to systematic risk will depend on the degree to which the risk from unpredictable shifts in estimates of the means is common among all securities. In closing, Williams points out that one empirical implication of this result is that the relation between excess returns and the associated betas will, in general, have a non-zero intercept.
CHAPTER THREE

THE ECONOMY

1. INTRODUCTION

As argued in Chapter One, there are three important beta estimates: (1) the estimate of beta measured without regard to differential information, (2) the estimate of beta appropriately adjusted for the degree of differential information, and (3) the estimate used by researchers to gauge investment performance. The purpose of this chapter is to focus on the implications of the differential information model for the estimation of these parameters. In Section 2 we provide a framework within which the estimates can be developed and compared explicitly. In Section 3 we consider the model within a small economy. The degree to which the results found in the small economy are dependent on its structure is explored by relaxing the constraints on the number of securities and on the information structure in Section 4. Finally, a summary is provided in Section 5.

2. THE MODEL

In the model that we consider (see Appendix A for a formal development), terminal cash flows are assumed to have a known covariance with the market cash flows. The mean cash flow is uncertain but investors possess a prior on its value. Market participants determine their asset demands in accordance with the single-period CAPM. The market clears when the price per dollar of future expected cash flows provides an expected return consistent with the CAPM (see equation (A.8) in Appendix A). The estimate entering the pricing equation is a posterior based on the known covariance of the future cash flows and an adjustment for the uncertainty in expected cash flows; i.e., explicit recognition is provided parameter uncertainty
or estimation risk (see equations (A.18) and (A.19) in Appendix A).

Differential information is introduced by providing for a situation in which the level of precision of the available information set is lower for some securities than for others; i.e., sample observations are missing for the securities with lower levels of information precision. The investors are assumed to formulate their beliefs on the basis of \( N_H \) sample observations for those securities designated as high information securities and \( N_L \) observations for those securities designated as low information securities, where \( N_H > N_L \). We adopt the monotonic or nested data pattern considered by Anderson [1957], Lord [1955a], and Morrison [1971] (see equation (A.13) in Appendix A).

In this setting, we determine, in a manner consistent with Barry and Brown [1985] and Williams [1977], that estimation risk does matter when the extent to which there is estimation risk varies across securities (see equations (A.20) and (A.21) in Appendix A). The existence of differential information leads investors to augment the measure of systematic risk that they employ for the pricing of low information securities to compensate for relative information risk.

3. A SMALL ECONOMY

The purpose of this section is to examine, within the confines of a small or restrictive economy, the adjustment process envisioned for beta. The economy is assumed to consist of but one high information security, \( S_H \), and one low information security, \( S_L \).

3.1. Beta Appropriately Adjusted for Differential Information

Define \( \beta_L^{\infty} \) as the beta of the low information security based solely on the known covariance with the market portfolio, i.e., the beta given full information. It
is the beta value that investors would use if there were uncertainty regarding only the actual cash flow for the security, but the expected cash flow was certain (i.e., infinite information). This value can be expressed as the beta of the cash flow, $\beta_{CL}^{CF}$, divided by the market clearing price of the low information security (given that investors know this true value), $P_L^\infty$, and multiplied by the market portfolio price, $P_M$ (see equation (B.18) in Appendix B) i.e.,

$$\beta_L^{R\infty} = \frac{P_M}{P_L^\infty} \beta_L^{CF}$$

(3.1)

Now let $P_L^\succ$ and $\beta_L^{R\succ}$ denote the adjusted price and beta of the security after investors penalize the security for differential information risk. These values reflect the fact that the information set is larger for some securities than for others. Then

$$\beta_L^{R\succ} = \frac{P_M}{P_L^\succ} \beta_L^{CF\succ}$$

(3.2)

Combining (3.1) and (3.2), we find (since the price of the market portfolio is unaffected by the presence of differential information - see footnote 9 in Appendix B)

$$\beta_L^{R\succ} = \beta_L^{R\infty} \left( \frac{P_L^\infty}{P_L^\succ} \frac{\beta_L^{CF\succ}}{\beta_L^{CF\infty}} \right)$$

(3.3)

The term multiplied by $\beta_L^{R\infty}$ on the right hand side of (3.3) can be interpreted as a measure of the sensitivity of the beta adjusted for differential information to the full information beta. In turn, it can be shown that this measure converges to one at a decreasing rate as the amount of information upon which $\beta_L^{CF\succ}$ and $P_L^\succ$ depend increases.

To demonstrate, we turn to the general form for the market clearing price. From equation (A.8) in Appendix A we have

$$P_L = E(CF_L) - [E(CF_M) - P_M] \beta_L^{CF}$$

(3.4)
The measure of systematic risk employed by investors in the pricing formula is dependent on the nature of the information environment, with

\[ \beta_L^{CF} = \beta_L^{CF\infty} + b_L \]

where \( b_L > 0 \) and is a decreasing function of two variables, namely (1) the correlation between the high and low information securities' cash flows, \( \rho_{HL} \), and (2) the relative levels of information precision associated with the securities, \( 1/N_L - 1/N_H \) (see equations (B.4) through (B.10) in Appendix B). The former represents investors' abilities to infer or "fill-in" the missing information for the low information security from the information available about the high information security while the latter reflects the direct impact of the missing data pieces on systematic risk.

In addition, the magnitude of the adjustment term, \( b_L \), falls at a decreasing rate as information increases (see equations (B.7), (B.8), and (B.9) in Appendix B). Figure 2 shows this relationship graphically. We also see, from (3.4), that \( P_L^\gamma \) will increase proportionately with any decrease in \( \beta_L^{CF} \). Thus, the fractions \( \beta_L^{CF} / \beta_L^{CF\infty} \) and \( P_L^{CF\infty} / P_L^\gamma \) will both converge to one at a decreasing rate as information increases.

Since the analysis is in a two date framework, the convergence is a cross-sectional concept. Across economies that differ only by the amount of information available for the low information security, the difference between the beta estimates with and without regard for differential information should decrease at a decreasing rate as information increases.

The basic logic that drives the foregoing result applies to a time-series of returns for a single security as well (see equations (B.14) through (B.23) in Appendix B). While the form of the adjustment over time is the same as the form of the adjustment across economies with increasing information, the parameters are different.

\footnote{This assumes that beta is positive}
Assuming that information increases in time, we find that investors, even knowing that information will increase, penalize low information securities for their current information deficiency. In particular, expectations about cash flows in the current period must reflect only knowable information at the beginning of the period. Thus, firms with low information will appear more risky, but the risk will diminish at a decreasing rate as time passes and the precision of the expectation increases.

3.2. A Researcher’s Estimate of Beta

Finally, we seek to establish the relation of the beta estimated by researchers to the adjusted measure of systematic risk, $\beta^{R>}_t$, and to the unadjusted measure, $\beta^{R>}_L$. Define $\beta^{R>}_t$ as the adjusted beta for security $S_L$ at time $t$ (i.e., at information level $N^t_L$) and $\hat{\beta}^{OLS}_t$ as the measure researchers estimate based on the sequence of return observations up to time $t$ (to information level $N^t_L$). One can view $N_L$ as an increasing function of time, the idea being that more information becomes available with the passage of time, say in the form of observations of cash flows from accounting or other reports (one observation per unit time). We then find that $\hat{\beta}^{OLS}_t$ overstates $\beta^{R>}_t$, as well as $\beta^{R>}_L$ (see equations (B.25) through (B.29) in Appendix B). In addition, as $N^t_L \rightarrow \infty$ all $\beta$ values converge to $\beta^{R>}_L$. This result is not unexpected since $\hat{\beta}^{OLS}_t$ is, in effect, a weighted average of previous information positions while $\beta^{R>}_t$ reflects only the current information position. Thus, a researcher who bases his beta estimate on a sequence of returns over time with information increasing would tend to overestimate $\beta^{R>}_t$, and consequently observe the average return to the security to lie below his CAPM prediction.

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2 The significance of this overstatement, both economically and statistically, will depend on the estimation period. If, for example, there is relatively little information available about the firm, beta should experience a sizeable drop over the estimation period. As a result, the overstatement may be significant. On the other hand, if there is a large amount of information available about the firm, beta can be expected to decline only a small amount over the estimation period.
Barry and Brown [1984,1985] argue that betas "... measured without regard for differential information" will be too low for low information securities. In our model the beta so measured is \( \beta^{R \infty}_L \). A sample of prior returns would not be measured without regard for differential information because the price process provides the adjustment. A sample of prior returns would tend to overstate \( \beta^{R>}_L \) because prior returns are taken from periods with less information. To measure beta without regard for differential information would appear to require an examination of cash flow or accounting earnings data where a market price adjustment is avoided.

In this implication, our model is contrary to the intuition offered by Barry and Brown [1984] who use historical beta estimates to document the period of listing effect. That the period of listing anomaly exists when historical betas are used implies that it is not caused by differential information of the sort we envisage. While the underlying model of Barry and Brown is consistent with ours, we would suggest that the historical beta is an inappropriate proxy for beta estimated without regard for differential information. It is influenced by the price formation process in equilibrium.

4. THE EFFECTS OF DIVERSIFICATION

The results of the previous section apply to an economy with a single low and a single high information security. We find that investors will, because of the presence of differential information, require higher expected returns for low information securities to compensate for the differential information risk (relative to the infinite information setting). Where multiple low and high information securities exist the effects of differential information are reduced (see Appendix C). There are two sources of risk reduction. The first is through an inference of information based on what is known about related securities. As previously noted, a low information
security which is highly correlated with a high information security effectively reflects the a great deal of the higher information. With many cross-correlated high information securities the market is able to infer or “fill-in” the missing information for the low information securities from its counterpart for the high information securities (see equations (C.4), (C.13), and (C.14) in Appendix C). The remaining adjustment for differential information is diminished, and any sensitivity to period of listing or other proxies for firm-specific information would be correspondingly reduced.

The second source of differential information risk reduction is through diversification across low information securities. As noted by Reinganum and Smith [1983], “... to the extent this risk is idiosyncratic rather than systematic it is diversifiable.” We find that if the sources of the uncertainty surrounding expected cash flows are largely uncorrelated across a sample of low information securities, portfolio formation will reduce the requirement for adjustments to beta (see equations (C.1) through (C.9) in Appendix C). The adjustment term arises because data pieces (information) are missing (the content of the missing pieces is unimportant). If the address or identity of these missing pieces is only partially correlated across low information securities, the differential information risk can be reduced by holding a large number of low information securities.3

5. SUMMARY

In summary, we show that, for a given market structure comprised of a given number of securities and a known covariance matrix of cash flows4, the beta ad-

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3 This result is consistent with Williams [1977] who suggests that, within the context of his model, “... the magnitude of the adjustments to systematic risk will depend on the degree to which the risk from unpredictable shifts in estimates of the means is common among all securities.”

4 Barry and Brown [1985] argue that relaxation of this assumption merely acts
justment declines at a decreasing rate as information increases. The size of the adjustment at each given information level falls with the number of high and low information securities, the degree of positive cross-correlation with the high information securities, and the lack of cross-correlation in information patterns for the low information securities. In restricted economies, with one or few low and high information securities, neither source of diversification is available.
CHAPTER FOUR

EXPERIMENTAL DESIGN AND EMPIRICAL RESULTS

1. OVERVIEW

The model of differential information risk described in Chapter 3 suggests a time series process for security returns that is non-linear in the market return. For low information securities, the return sensitivity to the market return (beta) should decline as the amount of information increases. In addition, beta coefficients based on historical return series should overestimate the beta estimate that correctly adjusts for differential information at a point in time, if information is increasing in time.

The empirical investigation will address the relationship between the beta coefficients and quantity of information from two perspectives. The first adopts time or relative period of listing (Barry and Brown [1984]) as a proxy for quantity of information. If information is increasing in time, we would expect the extent of any adjustment to beta to be inversely related to period of listing. We would also expect that the estimate of beta based on the historical return series will overestimate the point estimate of beta at a given point in time. The second considers quantity of information from the perspective of an information release. If a public release is perceived by investors to convey unanticipated information it should lead to an increase in the quantity of available information. The adjusted beta would then be expected to drop at the time of the information event. In Section 2 we propose and discuss a procedure for estimating the adjusted beta at a point in the information sequence. In Section 3 we identify and discuss the data sets on which the investigations will be based. The results are presented in Section 4. A summary
is provided in Section 5.

2. THE MODEL

The differential information model predicts that the measure of systematic risk appropriately adjusted for differential information will depend on quantity of information. While the covariability between firm and market cash flows is assumed known and constant, an adjustment for relative information risk will enter through the pricing process. As a result, historical returns will reflect the appropriate adjustment at each information level. In addition, the model predicts that the adjusted beta will decrease at a decreasing rate as information increases. Assuming that information is increasing in time, we see that it will be difficult to estimate an adjusted beta at a point in time for a single security because only one observation pair exists at each information level. Further, for single low information securities, the market model lacks sufficient explanatory power to provide much hope of rejecting the hypothesis that information risk is diversifiable. An alternative approach is, then, to aggregate across a temporally separated sample of low information firms, each at the same information level. For this purpose we adopt a procedure similar to Ibbotson’s [1975] RATS(N,N) model which will, in our economic model, allow for an estimate of the adjusted beta at a point in the information sequence.

Suppose we have a sample of low information securities, each with information level $N^t_L$ (where the superscript $t$ refers to event or information time), observed at different points in calendar time. Run the regression

$$R_{jt} = \alpha_t + \beta_t R_{Mjt} + \epsilon_{jt} \quad j = 1, J \quad (4.1)$$

For each security there is a realized return, $R_{jt}$, and a contemporaneous market return, $R_{Mjt}$. Across securities the calendar time periods differ so that market returns differ and no cross-correlation in security returns is present.

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To see that this regression provides an estimate of beta appropriately adjusted for the degree of differential information for the portfolio of low information securities at information level \( N_L^t, \beta_L^{R^2} \), picture the process under the assumption that all the securities have the same underlying cash flow process and \( \beta_L^{R^\infty} \). Because the firms are all of the same information level, \( N_L^t \), they all have the same information risk adjustment inherent in their prices. Their adjusted betas should be the same, and hence, the cross-sectional regression involves a sample of returns from the same process. Least squares regression implies that the regression \( \hat{\beta}_t \) is an unbiased estimate of \( \beta_L^{R^2} \) (see Appendix D). It is not an estimate of \( \beta_L^{R^\infty} \) because of the price adjustment which scales the covariance of returns for relative information risk. The series of cross-sectional regressions with time increasing with each cross-section would provide a series of adjusted betas that should conform to the pattern in Figure 2 if information is linear in time, or demonstrate a sharp drop at the time of the information event if information arrives in a lump-sum fashion (see Figure 5). Under the null hypothesis that differential information risk is diversifiable, the series of adjusted betas should show no tendency to decline as information increases.

3. DATA

3.1. Sample Selection - Period of Listing Perspective

To test the hypothesis that the series of adjusted betas declines in a non-linear fashion with time, we adopt, as our primary sample, the following. The data consist of firms which were included in the SEC listing of initial public offerings for 1981;¹

¹ The year 1981 was selected because of the large number of new listings that occurred that year. For example, the SEC listing of initial public offerings included 512 firms for 1981, while the next closest year during the period 1970 through 1982 was 1980 with 256 new listings. One potential problem associated with this sample relates to Ritter [1984] who found that, for the 15 month period commencing with
for which prospectuses could be obtained; and for which complete daily price data for the first 25 days, and month end price data for the first 20 months of trading are available in the Daily Stock Price Record, a publication of Standard and Poor's Corp. It was additionally required that trading volume be non-zero for each of the first 25 trading days and for each of the first 20 months. These conditions led to a sample of 142 firms. As indicated in Tables I and II, the sample has a broad base, both in terms of market value and in terms of industry classification. Table III represents a frequency distribution of the initial trading date by month.

The sample selection criteria were established to maximize the likelihood of detecting a differential information effect in the data (if present). To begin, we single out new listings because the longevity of such an effect is, ex ante, unknown. In addition, the uncertainty surrounding the relation between time and information leads us to consider both daily and monthly returns. The CRSP value weighted index is used as an index of market returns. The initial price of each issue is taken from its offering prospectus. Returns are calculated using the mid-point between bid and ask prices. While Ibbotson [1975] suggests that bid quotes tend to be more accurate than ask quotes because ask quotes may reflect a potential dealer short position, we adopt the mid-point between the bid and ask quotes because the initial offering price results from a transaction.

January of 1980, the mean return to initial public offerings for the first trading day was almost 3 times the average for the period 1977 through 1982. 31 firms out of the sample of 142 firms come from this 15 month window. While there is no a priori reason why this phenomenon should affect systematic risk, as a check, we also studied the remaining sample of 111 firms, finding relative magnitudes substantially unaltered.

2 The value weighted index was used to maintain comparability with Banz [1981] and Barry and Brown [1984]. As a check, we also studied the effect of using an equal weighted index and found that relative magnitudes were unaltered.

3 All prices are adjusted for dividends, stock dividends, and stock splits.

4 We also considered returns calculated from bid prices. This change did not alter our findings.
We also consider a subset of the data used by Barry and Brown [1984]. This supplemental sample consists of the 348 firms for which complete data are available on both the CRSP daily and monthly returns files for a period of at least 100 months, and for which the date of initial data availability on the two return files coincides. The latter constraint is established to allow for comparisons between the results of the supplemental sample and the results of the sample of newly-listed securities. Again the CRSP value weighted index is used as an index of market return.

3.2. Sample Selection - Information Release Perspective

To test the hypothesis that the series of adjusted betas drops at the time of an information release we focus on the first annual earnings announcement made by a sample of newly-listed firms. This particular information release was selected because it is a standard release required of all firms, and because the unanticipated component of the annual earnings announcement in general, has documented information content (see, for example, Ball and Brown [1968]). One drawback to the selection of a homogeneous information event like the first annual earnings announcement, is that it will occur at different points in time relative to the initial date of listing for different firms. If indeed information is increasing in time (as conjectured by Barry and Brown [1984]) then the importance of the announcement (i.e., its impact on systematic risk) would be expected to diminish, on average, as the time between listing and the announcement increases. An alternative would have been to focus on the first information release by the firm subsequent to listing. This would, however, involve information events of a dissimilar nature, some rep-

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5 The tests reported for this section were also reported for the subset of firms for which the annual earnings announcement was the first press release in the Wall Street Journal.
resenting a major informational contribution, others a negligible contribution. The former approach has been selected because of the ability to measure and control for the time dimension.  

The sample was developed in the following manner. 350 firms were randomly selected from the SEC listing of initial public offerings, 50 from each of the seven years 1976 through 1982. This sample was then further constrained by the conditions that the first post-issue annual earnings announcement date be available in the Wall Street Journal Index, that complete price data be available in the Daily Stock Price Record for a period of 30 days surrounding the first annual earnings announcement date, and that trading volume be non-zero on each of these 30 days. These conditions led to a sample of 140 firms (with approximately the same attrition rate attributable to each constraint). Table IV represents a frequency distribution of the initial trading date by year for this sample. A frequency distribution of the number of trading days from initial date of issue to the earnings announcement date is provided in Table V. Following the arguments of Ibbotson [1975] discussed in the previous subsection, daily returns are calculated using bid prices (since the analysis no longer involves the initial offering price, the use of bid prices does not represent an inconsistency).

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6 The correlation coefficient between the number of trading days from the date of issue to the date of the first annual earnings announcement and the number of Wall Street Journal articles preceding the announcement date is 0.587. Partitioning on the basis of time should, therefore, capture a significant portion of the influence of other information events.

7 This sample, which is separate and distinct from the sample in the previous subsection, contains only 17 firms from the 15 month period discussed by Ritter [1984] and should, therefore, be relatively unaffected by the 'Hot Issues' phenomenon.
4. RESULTS

4.1. Empirical Results - Period of Listing Perspective

Figure 3 shows a plot of twenty-five cross-sectional beta estimates corresponding to the first twenty-five days of trading for the sample of 142 newly-listed companies. The betas are fit in a linear regression against time in days, the inverse of time in days, and the log of time in days. Since the relationship between information and time is arbitrary, we consider all three models as a check against sensitivity to model specification. If information is roughly proportional to time, the model with the inverse of time is the most appealing theoretically. All of the models give the impression of a negative relation between the cross-sectional beta estimate and time. They are reported in Panel A of Table VI. The relation between beta and the inverse of time is the most significant, with a t-value for the inverse of time of 8.19.\(^8\) This relation has an intercept of 0.92 and a slope of 3.20. Standard diagnostic tests fail to reject the hypothesis that the residuals are normal and serially independent.\(^9\)

Since the issue of nonsynchronous or infrequent trading arises with daily data (see Scholes and Williams [1977] and Dimson [1979]), we also implement an adjustment to the cross-sectional beta estimates based on the Scholes-Williams approach.\(^10\)

\(^8\) For the 111 firms in the sample which do not come from the 'Hot Issues' time period documented by Ritter [1984], the t-value for the inverse of time is 5.31.

\(^9\) The test statistic from the Chi-Square goodness of fit test for normality of the residuals is 2.001 with 2 degrees of freedom. The first-order autocorrelation coefficient for the residuals is -0.127, and the Durbin-Watson test statistic is 2.148.

\(^10\) The adjusted estimate of systematic risk, \(\hat{\beta}_t'\), is

\[
\hat{\beta}_t' = \frac{\hat{\beta}_t^- + \hat{\beta}_t + \hat{\beta}_t^+}{1 + 2\hat{\rho}_M}
\]

where \(\hat{\beta}_t^-\), \(\hat{\beta}_t\), and \(\hat{\beta}_t^+\) are the OLS estimates of beta using as the independent variables, the realised market return for the day before, the day of, and the day after the period of interest, and where \(\hat{\rho}_M\) is the estimation period value for the first order autocorrelation coefficient of the market return.
The results of the three models based on this adjustment are found in Panel B of Table VI. The t-value for the model based on the inverse of time is now 8.75. Thus, infrequent trading does not appear to explain the time trend that we observe in cross-sectional betas. A plot of the adjusted cross-sectional beta estimates is also found in Figure 3.

Panel A of Table VII presents the results for the same three models with the dependent variable changed to the cross-sectional beta estimates corresponding to the first 20 months of trading for the sample of 142 newly-listed companies, and with time measured in months. Once again, the results give the impression of a negative relation. The t-value for the inverse of time is 2.25, while the t-values for time and the log of time are -3.52 and -3.26, respectively. Here also, standard diagnostic tests fail to reject the hypothesis that the residuals are normal and serially independent.11 Figure 4 provides a plot of the cross-sectional beta estimates corresponding to the first 20 months of trading. Interestingly, the model with the inverse of time does not provide the best fit. This can perhaps be explained by Ibbotson’s [1975] observation that the beta estimate for the first month is downward biased since, unlike the return on the market, the first month’s returns for the newly listed securities do not typically reflect entire calendar month’s returns. Consequently, we also consider a revised cross-sectional beta estimate for month 1. The revision was made by calculating the market return corresponding to the period that each security had been trading from the CRSP daily return file. This revision led to an increase in the beta estimate for month 1 from 1.81 to 2.75 (beta estimates for subsequent months are unaffected by the revision). The results for the models based on the revision are presented in Panel B of Table VII.

11 The test statistic from the Chi-Square goodness of fit test for normality of the residuals is 2.008 with 2 degrees of freedom. The first-order autocorrelation coefficient is 0.101, and the Durbin-Watson test statistic is 1.678.
The results in Tables VI and VII reject the hypothesis that differential information risk is perceived by market participants as diversifiable. Apparently a penalty is imposed on newly-listed companies for information risk. Based on these data, however, the differential information risk is largely eliminated within the first several months of trading. These results are consistent with those of Ibbotson [1975] who concludes, based on monthly return data for a sample of newly-listed companies, that betas appear to decline with seasoning, with the largest values for beta observed in the first two months.\textsuperscript{12}

The differential information model also predicts that if information is linear in time, an estimate based on a sample of historical return observations will tend to overstate beta relative to its value appropriately adjusted for the degree of differential information. Extrapolating from the fitted model based on the inverse of time, the estimate of $\beta_{L}^{R_{\infty}}$ based on daily data is 0.92 (0.97 with the Scholes-Williams adjustment) and 1.15 based on monthly data (1.13 with the revision to the first month). From the daily data, at time equal to the end of our observations, $t = 26$, the estimate of $\beta_{L26}^{R_{\infty}}$ based on the same fitted model would be 1.04 (1.10 with the adjustment). Had a researcher used the first 25 trading days to compute an estimate of beta for each firm from the market model, the average beta, $\hat{\beta}_{OLS}^{26}$, would have been 1.54. Similarly, from the monthly data, the estimate of $\beta_{L21}^{R_{\infty}}$ would be 1.29 (1.31 with the revision) based on the fitted model with the inverse of time while a researcher, using the first 20 months to estimate a beta for each firm from the market model, would determine the average beta, $\hat{\beta}_{OLS}^{21}$, to be 1.46. Thus the

\textsuperscript{12} Ibbotson [1975] considered the relationship between systematic risk and time measured in months over the first 60 months from date of issue for securities issued between 1960 and 1969. He found, for two distinct samples, the coefficients of time to be -0.010 and -0.056 with t-values of -2.68 and -5.35 respectively, and concluded that systematic risk declines with seasoning. Our analysis provides a possible explanation for his results.
researcher would have overestimated the point estimate of beta for day 26, $\beta_{L26}^R$, and for month 21, $\beta_{L21}^R$, as the model predicts. Attempts to use historical betas for abnormal return calculations would thus tend to understate average abnormal returns for low information firms. If the CAPM benchmark were used and the required excess return on the market portfolio is about 10% per year, the underestimation in average abnormal returns for the firms in our sample would be about 5% per year based on the daily data (1.54 - 1.04).

Support for the conclusions drawn from the sample of newly-listed companies follows from the supplemental sample of 348 NYSE firms. The relations between the cross-sectional beta estimates and variants of time (as before time, log time, and the inverse of time) for this subset of the data used by Barry and Brown [1984] are reported in Table VIII. Panel A presents the results for daily betas corresponding to the first 100 days of data availability on the CRSP daily return file. The t-value for the inverse of time is 2.09 while the t-value for log time is -2.22. Since the first day of data availability on CRSP does not necessarily correspond to the first day of trading for the security, the appropriate form of the model is not clear.\(^{13}\)

Analogous results for monthly cross-sectional beta estimates corresponding to the first 20 months of data availability on the CRSP monthly return file are found in Panel B of Table VIII. All coefficients have the predicted signs consistent with an inverse relationship between cross-sectional beta estimates and time. However, the coefficients are not found to be significant. The t-value for the inverse of time is 1.03 while the t-value for log time is -1.21. This lack of significance suggests that the process does not extend for the length of time that these securities have been listed.

\(^{13}\) The procedure for listing on the NYSE is a somewhat lengthy process, with approximately four weeks lapsing between the date of application and approval for listing and a further 30 days after approval of the application before trading can commence.
trading. This is consistent with the apparent duration of the differential information effect found in the sample of newly-listed companies.

Nevertheless, researchers would tend to overestimate the point estimate of beta at the end of the observation period by using historical return data. Average beta estimates for day 101, $\hat{\beta}^{101}_{OLS}$, and for month 21, $\hat{\beta}^{21}_{OLS}$, based on the simple market model and computed from the first 100 trading days and the first 20 trading months, respectively, would have been 1.01 and 1.21, while extrapolating from the fitted models based on the inverse of time, the estimates of $\beta_{L101}^{R^>}$ and $\beta_{L21}^{R^>}$ would be 0.92 and 1.12, respectively.

4.2. Empirical Results - Information Release Perspective

The differential information model of Chapter 3 suggests that the adjustment to beta is linear in the inverse of the quantity of available information. If relative period of listing is an appropriate proxy for quantity of information (as suggested by Barry and Brown [1984] and by the results of the previous subsection) then an instantaneous increase in the quantity of information resulting from an information release by the firm, an increase which is independent from and outside of the assumed time pattern of information accumulation, will lead to a shift in the parameters of the relation between the adjustment to beta and the inverse of time at the point in time when the information release occurs. The relation can be written as follows:

$$\beta_t \propto \begin{cases} \frac{1}{t} & \text{if } t < t^*; \\ \frac{1}{t + \delta} & \text{if } t \geq t^* \end{cases}$$

(4.2)

where $t^*$ represents the point in time when the information release occurs and

14 The release, at a single point in time, of an information document such as the first annual earnings report, will represent, if perceived by investors as making a contribution to the available information set, a discontinuity in an information accumulation pattern which is (assumed to be) linear in time.
\( \delta \) represents the time equivalent of the quantity of information contained in the information release. This phenomenon is presented pictorially in Figure 5. The information release causes a discrete jump in the information accumulation pattern (Panel A), a jump which translates into a shift in the adjustment to beta at the point in time when the information release occurs (Panel B). Effectively, a segment of the curve has been removed with the result that quantity of information and time are no longer synchronized. Adopting the inverse of time as the independent variable, we find that the parameters of the relation between beta and the inverse of time are altered relative to the pre-release relation (Panel C). That is, we postulate the existence of two relations

\[
\beta_t = \begin{cases} 
\psi_{01} + \psi_{11}(1/t) + \epsilon_1 & \text{if } t < t^*; \\
\psi_{02} + \psi_{12}(1/t) + \epsilon_2 & \text{if } t \geq t^*
\end{cases}
\]  

(4.3)

Theoretically, we would expect the slope of the relation to decrease and the intercept to increase with the information release.

To examine the impact of an information release on the adjustment to beta, we therefore compare the relation between the cross-sectional beta estimates and the inverse of time from the before announcement period, regime 1, with its counterpart from the after announcement period, regime 2 (see Quandt [1958,1960,1972], Chow [1960], and Johnson [1972]). The tests of the null hypothesis that no switch in regimes has taken place will involve not only a test of the homogeneity of the relations, but also tests of differences in the slopes and intercepts from the before and after periods. The following model is adopted for this purpose

\[
\beta_t = \psi_o + \psi_1(1/t) + \psi_2X + \psi_3(1/t)X + \epsilon
\]  

(4.4)

where \( X = 0 \) for the before announcement period and \( X = 1 \) for the after announcement period.\(^{15}\) Since, as noted by Quandt [1958], the power of the tests is related to

\(^{15}\) Neter and Wasserman [1974] demonstrate that this formulation yields the same
the ability to determine the true switching point, we eliminate the observations from day -2 through day +2 (the central observations). It is hoped that in this manner we can reduce the probability of contamination of one regression with observations from the other regime. Finally, since the model also suggests that the magnitude of a shift in the cross-sectional betas at the time of the information release will depend on the relative importance of the release (i.e., on both the quantity of information available prior to the release and the incremental quantity contributed by the release - see equations (B.7) and (B.8) in Appendix B), the sample of 140 newly-listed firms is partitioned on the basis of the number of trading days from the date of issue to the annual earnings announcement date.\textsuperscript{16} This time dimension (T) is organized into three classes, with T=1 corresponding to those firms listed for the shortest time before the announcement date.

Panel A of Table IX reports the results of the tests for homogeneity of the complete relation. \textsuperscript{17} For the overall sample of 140 firms, the test ratio, $F(2,22)$, is 6.37. At the 1% level of significance the critical value of $F(2,22)$ is 5.72. Based results as fitting separate regressions for the pre- and post-announcement periods. By setting $X = 0$ in equation (4.4) we have $E(\beta_t) = \psi_o + \psi_1 (1/t)$ while with $X = 1$

\[ E(\beta_t) = (\psi_o + \psi_2) + (\psi_1 + \psi_3)(1/t) \]

\textsuperscript{16} Other possible partitions include excess market return, number of articles in the Wall Street Journal, and firm size. For example, Atiase [1985] documents an inverse relation between firm size and excess return following the annual earnings announcement. The annual earnings announcement might, therefore, be more important for small firms and consequently, have a greater impact on their betas.

\textsuperscript{17} The test statistic for overall model homogeneity is

\[ F = \frac{(SSE(R) - SSE(F))/2}{SSE(F)/(n_1 + n_2 - 4)} \]

where $SSE(R)$ is the error sum of squares for the regression based on the combined data set (observations from regime 1 and regime 2 are pooled), $SSE(F)$ is the sum of the error sum of squares for the regressions run separately for the pre- and post-announcement periods (i.e., for the model in equation (4.4)) and $n_1$ and $n_2$ are the pre- and post-announcement sample sizes.
on this test, we reject the hypothesis of a common relation.\textsuperscript{18} The results suggest a shift in the relation between the cross-sectional beta estimates and the inverse of time at the date of the first annual earnings announcement. The t-values for $\psi_2$ and $\psi_3$ (i.e., for the differences between the pre- and post-announcement intercepts and slopes) are presented in Panel B of the same table. The t-value for $\psi_3$ for the overall sample is -1.22. Since we do not reject the hypothesis of a common regression slope, we also test for differential intercepts. The t-value for $\psi_2$ is 1.85. At the 5\% level of significance the critical value of $t$ for a one-tailed test with 24 degrees of freedom is 1.71. The signs of the tests for differences in the slopes and intercepts suggest that, as predicted, the slope of the relation between the cross-sectional beta estimate and the inverse of time has decreased at the time of the announcement while the intercept has increased. Since the shortest time lapse between the date of issue and the date of the first annual earnings announcement for the sample of 140 newly-listed firms is 29 trading days, the lack of significance associated with the test for a difference in slopes is consistent with the duration of the differential information effect found in the previous section.

We also note that the shift appears to be more pronounced among securities in the shorter period of listing classes.\textsuperscript{19} This, also, is consistent with the results of the previous subsection where the cross-sectional beta estimate was found to decline with seasoning.

The results of the tests in this subsection serve to reject the hypothesis that

\textsuperscript{18} These results are found to be unaffected by the adjustment for infrequent trading discussed in the previous section.

\textsuperscript{19} The first annual earnings announcement was the first post-issue article appearing in the Wall Street Journal for 53 of the 140 firms in the sample. For this subset (which includes 35 of the firms from the first period of listing class), the test statistic for overall model homogeneity is $F = 7.41$, while the t-values for $\psi_2$ and $\psi_3$ (for the differences in intercept and slope, respectively) are 2.54 and -1.49. As to be expected, these results are stronger than for the overall sample.
differential information risk is perceived by market participants as diversifiable. The penalty imposed by investors for information risk is apparently reduced at the time of an information release. Further, the magnitude of the reduction appears to be inversely related to the number of trading days between the date of initial issue and the date of the information release.

5. SUMMARY

The empirical results presented in this chapter are consistent with the predictions of the differential information model of Chapter 3. It would appear that investors penalize securities for differential information risk in their pricing decisions. The magnitude of the penalty is found to inversely related to period of listing, and to decline at the date of the first annual earnings announcement by newly-listed firms. In addition, researchers, basing their estimates of systematic risk on historical return data, apparently overestimate beta appropriately adjusted for the level of differential information risk.
This study investigates the relation among three beta estimates: (1) the estimate of beta measured without regard to differential information, (2) the estimate of beta appropriately adjusted for the degree of differential information, and (3) the estimate of beta used by researchers to gauge investment performance. In order to develop insights into this relation we employ a model of differential information within the CAPM setting. We then consider the predictions of the model in light of empirical evidence.

The testable implications of the theoretical model depend on the nature of the assumed economy. In a small or restricted market setting consisting of but one high and one low information security, we find (as Barry and Brown [1985] do) that the presence of differential information will lead investors to make an upward adjustment to the measure of systematic risk that they employ for the pricing of low information securities relative to its measurement without regard for differential information. Further, the model predicts that the adjustment will decrease at a decreasing rate as information increases. Lastly, in the face of such an adjustment, the typical estimate of beta employed by researchers is found to be upwardly biased (assuming that information is increasing in time). On the other hand, the model predicts that the size of the adjustment to beta at each information level falls with the number of high and low information securities, the degree of positive cross-correlation between the low and high information securities, and the lack of cross-correlation in the information patterns for the low information securities. Thus, in unrestricted economies, where these sources of diversification
are available, the presence of differential information should have little or no impact on capital market equilibrium.

The empirical investigation was conducted from two different perspectives. The first adopted time or relative period of listing as a simple proxy for quantity of information. The second considered changes in the quantity of information from the perspective of an information release. In each case the results serve to reject the hypothesis that differential information risk is perceived by market participants as diversifiable. However, based on the data, the differential information adjustment process does not appear to have a significant impact after the first several months of trading.

For a sample of newly-listed firms, we observe a significant and positive association between systematic risk and the inverse of time, both for daily and monthly data. We also find, for this sample of newly-listed firms, that estimates of beta based on the historical return series consistently overstate beta at a point in time relative to the value predicted by the model with the inverse of time. Further evidence of these relations is provided by a supplemental sample of seasoned firms trading on the NYSE. Again, we find a positive association between systematic risk and the inverse of time (although far less significant than for the newly-listed firms), and a tendency to overestimate beta by using historical return data.

For a second and separate sample of newly-listed firms, we observe a significant shift in the relation between systematic risk and the inverse of time at the time of an information release (the first annual earnings announcement). In addition, the magnitude of the switch appears to be inversely related to the number of trading days between the date of initial issue and the date of the information release.

These results, both individually and collectively, provide support for the differential information model. They suggest that systematic risk will decline at a
decreasing rate as the quantity of available information increases. They also suggest that researchers will, by employing historical return data, overestimate the point estimate of beta at any given point in time.

Our model and empirical results are contrary to the intuition offered by Barry and Brown [1984,1985] who use historical beta estimates to document the period of listing effect. Barry and Brown [1985] suggest that

"... the portfolio of relatively low information securities will appear to earn abnormal returns if their betas are measured without regard to differential information and if their returns are consistent with a CAPM in which investors properly account for differential information"

and consequently

"... a researcher ignoring these risk perceptions will conclude that there are excess returns when, in fact, the returns are commensurate with risk."

Our model is in agreement with the relation that they develop between the unadjusted and adjusted measures of systematic risk. However, within the context of our model where the price process provides the adjustment, we find that a sample of prior returns would not be measured without regard for differential information, and that the typical estimate of beta employed by the researcher is, therefore, upwardly biased (the predictions are supported by the empirical results). Thus, the results suggest that attempts to use historical betas (i.e., those based on the historical return series) will tend to understate average abnormal returns for low information firms. If, for example, small firms are assumed to be low information firms, typically, then this would imply that the small firm effect is perhaps even more pronounced than currently believed. ¹

Our results might also be seen as providing an explanation of Ibbotson's [1977] findings. Ibbotson documented positive initial performance for a sample of newly listed firms with aftermarket efficiency. He concluded that the results indicate that

¹ There remains the question of whether market-line deviations are meaningful from a performance perspective - see for example, Korkie [1986].
new issue offerings are underpriced. He also found that the "... systematic risks of new issues are greater than the systematic risk of the market, and the systematic risks of securities are not stable in that they drop as the issues become seasoned."²

One limitation of the formal analysis is the assumption of a known covariance matrix of returns. It is not clear from the model what impact a relaxation of this assumption would have on the diversification results for the large or unrestricted economy. An interesting extension is, therefore, to model the differential information economy with an uncertain covariance matrix, as well as an uncertain mean vector. We conjecture that the reduced ability, on the part of investors, to infer or "fill-in" the missing data pieces for the low information securities, will serve to prolong the adjustment process.

Possible empirical extensions include estimating the measure of beta without regard for differential information through cash flow or accounting earnings data. It might be interesting to establish its relation to the other two estimates considered empirically. In addition, the use of alternative proxies to control for prior information quantity in the investigation of the reaction around the first earnings announcement might prove fruitful in understanding how investors evaluate information.

In summary, the results of our tests are consistent with the differential information model. Information risk appears to be partly nondiversifiable. For the samples we examine, the adjustment process dissipates relatively quickly, however, essentially within several months' time. Where historical betas are used to gauge investment performance, both our model and our empirical tests imply that information risk cannot provide an explanation for the firm size or period of listing anomalies. Historical estimates tend to overstate the beta risk existing at the end

² An alternative explanation is offered by, among others, Rock [1986].
of the period.
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APPENDIX

APPENDIX A

The Model

The securities market is assumed to consist of a riskless bond, B, and J+K risky securities, \((S_1, \ldots, S_J, \ldots, S_{J+K})\). Without loss of generality, these securities are rank ordered on the basis of their available information sets (this concept to be defined shortly) with the first J securities designated as high information securities and the remaining K as low information securities. There are I market participants, called traders or investors, who are assumed to be risk averse, expected utility maximizers with a utility for wealth implied by constant absolute risk aversion. Trader i's utility for wealth, \(W_i\), is, therefore, given by the negative exponential utility function

\[
U_i(W_i) = -\exp(\gamma_i W_i) \quad \gamma_i > 0 \quad i = 1, \ldots, I
\]  

The investors' decision problem in this single period economy involves allocating present wealth between the risky securities and the riskless bond with the intention of maximizing welfare, \(U_i\). Each security is viewed, from the investors' perspective, as paying a distributing dividend, \(\tilde{CF}_j\), at the end of the investment period.

The numeraire in the market is the return on the bond which all investors know to be fixed at one. The joint distribution of terminal firm values or cash flows, \(\tilde{CF}_j\), \((j = 1, \ldots, J + K)\), for the risky securities is assumed to be multivariate normal with mean vector \(\mu\) and covariance matrix \(\Sigma\) where \(\mu\) and \(\Sigma\) can be partitioned on the basis of high and low information securities as in (A.2)

\[
\mu = \begin{pmatrix} \mu_J \\ \mu_K \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} \Sigma_{JJ} & \Sigma_{JK} \\ \Sigma_{JK} & \Sigma_{KK} \end{pmatrix}
\]

(A.2)

It is assumed that all investors have identical, accurate assessments of \(\Sigma\) but are
unable to resolve all of the uncertainty surrounding $\bar{\mu}$ (investors possess imperfect knowledge about the values of $\bar{\mu}$). ¹ This assumption will allow for an examination of the implications of differential information for capital market equilibrium independent of any obscuring or confounding effects that information might have on the elements of the covariance matrix. Barry and Brown [1985] provide support for such an assumption, noting that their "... principal conclusions were not compromised by a relaxation of this assumption." ² Each investor is therefore assumed to approach $\bar{\mu}$ as having a multivariate normal distribution with parameters $M_\mu$ and $\Sigma_\mu$.

In this setting, maximizing expected utility is equivalent to maximizing the

¹ This is in keeping with Williams [1977] who describes a scenario in which the variance of return on a security is rapidly learned whereas the mean return is learned only gradually.

² Barry and Brown's argument is based on work by Klein and Bawa [1977], who show, in a setting with no differential information, that the predictive distribution after $N > J + K$ observations is Student-t with $N - (J + K)$ degrees of freedom, mean $M_\mu$, and covariance matrix

$$\Sigma^* = \left( \frac{N - 1}{N - (J + K)} \right) \left( \frac{N + 1}{N} \right) \hat{\Sigma}$$

where $\hat{\Sigma}$ is the sample covariance matrix. Thus, with $\Sigma$ replaced by its counterpart $\hat{\Sigma}$, we find, on average, that the value of $\Sigma_P$ is modified to

$$\Sigma_P = \left( \frac{N - 1}{N - (J + K)} \right) \left( \frac{N + 1}{N} \right) \Sigma > \left( \frac{N + 1}{N} \right) \Sigma$$

Consequently, allowing for an unknown covariance matrix merely acts to exacerbate the effects of information (estimation) risk.
expression $E(\tilde{W}_i) - (\gamma_i/2)\sigma_i^2(\tilde{W}_i)^3$ i.e., the relevant arguments are the first two moments of the distribution for terminal wealth, $E_i(\tilde{W}_{i1})$ and $\sigma_i^2(\tilde{W}_{i1})$, where utility is increasing in $E_i(\tilde{W}_{i1})$ and decreasing in $\sigma_i^2(\tilde{W}_{i1})$ (the objective function is assumed to be differentiable at least once in each of its arguments). The investor's problem is given by

$$\max E[U_i(\tilde{W}_i)]$$

subject to

$$W_{i0} = \sum_{j=1}^{J+K} w_{ij} P_{jo} + w_{io}$$

where

$W_{i0} =$ current $(t_0)$ wealth

$P_{jo} =$ current $(t_0)$ equilibrium market price of firm $j$

$w_{ij} =$ fraction of firm $j$ demanded by investor $i$

$w_{io} =$ fraction of the riskless bond demanded by investor $i$

Since each investor's final wealth will be a linear mixture of the final security values, we then have

$$\gamma_i\tilde{W}_i \sim N(\gamma_i\tilde{W}_i, \gamma_i^2\sigma_i^2(\tilde{W}_i))$$

Consequently

$$E[U_i(\tilde{W}_i)] = \int_{-\infty}^{+\infty} -\exp(-\gamma_i\tilde{W}_i) \frac{1}{\sigma_i(\tilde{W}_i)\sqrt{2\pi}} \exp \left[ -\frac{(\tilde{W}_i - \tilde{W}_i)^2}{2\sigma_i^2(\tilde{W}_i)} \right] d\tilde{W}_i$$

Thus, we find

$$E[U_i(\tilde{W}_i)] = -\exp(-\gamma_i[\tilde{W}_i - \frac{\gamma_i}{2}\sigma_i^2(\tilde{W}_i)])$$

Finally, since $\tilde{W}_i - (\gamma_i/2)\sigma_i^2(\tilde{W}_i)$ is a monotone increasing function of the expression above, we have

$$\arg\max E[U_i(\tilde{W}_i)] = \arg\max[\tilde{W}_i - \frac{\gamma_i}{2}\sigma_i^2(\tilde{W}_i)]$$
and

$$E_t(W_{i1}) = \sum_{j=1}^{J+K} w_{ij} E(CF_j) + w_{io} \quad (A.5)$$

$$\sigma_t^2(W_{i1}) = \sum_{j=1}^{J+K} \sum_{k=1}^{J+K} w_{ij} w_{ik} \text{cov}(CF_j, CF_k) \quad (A.6)$$

Solving the maximization problem for $w_{io}$ and $w_{ij}$ ($j = 1, \ldots, J + K$), invoking the market clearing conditions

$$\sum_{i=1}^{J+K} w_{ij} = 1 \quad j = 1, \ldots, J + K \quad (A.7)$$

$$\sum_{i=1}^{J+K} w_{io} = 0$$

the equilibrium price for security $S_j$ ($j = 1, \ldots, J + K$), given that the riskless bond has a return of one, is (see Fama [1976] for proof)

$$P_{jo} = E(CF_j) - [E(CF_M) - P_{M0}] \beta_j^{CF} \quad j = 1, \ldots, J + K \quad (A.8)$$

where

$$\beta_j^{CF} = \frac{\text{cov}(CF_j, CF_M)}{\sigma^2(CF_M)} \quad (A.9)$$

$$P_M = \sum_{j=1}^{J+K} P_j$$

$$E(CF_M) = \sum_{j=1}^{J+K} E(CF_j)$$

Thus, the current equilibrium price of each risky security, $P_{jo}$, depends exclusively on the parameters of the distribution for $CF_j$. It is to the development of these parameters and the concept of differential information that we now turn our attention.
In general terms, investors, starting with initial beliefs characterized by a low information or diffuse probability density function, seek to acquire sample information for the purpose of reducing uncertainty. On the basis of information gathered up to a given point in time, say \( t_o \), investors formulate beliefs about the distribution for \( \bar{\mu} \). They then transform these beliefs into beliefs about \( \widetilde{CF} \) (see Zellner [1971]), since it is on the basis of the parameters for the distribution of \( \widetilde{CF} \) (the so-called predictive distribution) that portfolio decisions will be made (it is from this distribution that investors perceive the next payoff will develop). Thus, at \( t_o \) investors will hold the identical prior beliefs that

\[
\bar{\mu} \sim N(M_{\mu o}, \Sigma_{\mu o}) \tag{A.10}
\]

and consequently

\[
\widetilde{CF} \sim N(\mu_{Po}, \Sigma_{Po}) \tag{A.11}
\]

More formally, the information acquired by each investor arrives as a signal, \( \bar{\mu} \), which communicates the true asset values, \( \mu \), perturbed by some noise, \( \bar{\epsilon} \) i.e.,

\[
\bar{\mu} = \mu + \bar{\epsilon} \tag{A.12}
\]

Each noise term, \( \bar{\epsilon} \), is a random variable which has a multivariate normal distribution with mean zero and a covariance matrix of the form of \( \Sigma_{\mu} \). Obviously, the greater the precision of the signal, the more valuable the information conveyed by it will be to its recipients. The precision of a signal might, therefore, be thought of as a measure of information value, and if the precision of a signal about firm \( j \) (\( j=1,\ldots,J+K \)) is represented by \( N_j \), the difference \( N_j - N_k \) as a measure of the informational difference between securities \( S_j \) and \( S_k \).

The acquisition of information is characterized as a sample generated by the observation of some predetermined number (\( N_j \)) of random variables \((y_{j1}, \ldots, y_{jN_j})\)
about firm j, each of which is independent and identically distributed. Each data piece is interpreted by the investor to represent an outcome drawn from the asset value distribution. The signal acts as a summary statement, presenting the aggregate information content of the data pieces acquired by the investor. Note that while we may be explicitly measuring the quantity of information (or data pieces) through $N_j$, it is perhaps more accurate to think of $N_j$ as a summary measure for all aspects of the information, most importantly its precision. This measure of information quantity, $N_j$, should, therefore, be seen as continuous in nature ($N_j \in \mathbb{R}^+$) as opposed to attaining integer values exclusively. Support for this position can be found from Raiffa and Schlaifer [1961] who point out that there is an ‘equivalent sample information’ interpretation of Bayesian prior and posterior distributions for parameters of many common processes such as the multinormal.

A situation of differential information (i.e., a situation in which the level of precision of the available information set is lower for some securities than for others) can then be characterized as a situation in which there is missing data for the securities with lower levels of information precision. Unfortunately, as noted by Morrison [1971], “Maximum likelihood estimation of the multinormal mean vector and covariance matrix from a data matrix with a general random pattern of missing values leads to systems of nonlinear equations and their associated problems in numerical analysis.” We, therefore, adopt the monotonic or nested data pattern (see (A.13) following) considered by Anderson [1957], Lord [1955a], and Morrison [1971] (among others), since in this setting calculation of the maximum likelihood estimates of both $\mu$ and $\Sigma$ is possible. The investor is assumed to formulate his beliefs about the parameters of the distribution for $\bar{CF}$ on the basis of $N_H$ sample observations for $S_j \in (S_1, ..., S_J)$ and $N_L$ sample observations for $S_k \in (S_{J+1}, ..., S_{J+K})$ where $N_H > N_L$ (that is, $N_H - N_L$ observations on $S_k$ are missing) i.e., the data
are

$$
\begin{pmatrix}
  y_{1,1} & \cdots & y_{1,N_L} & \cdots & y_{1,N_H} \\
  \vdots & \ddots & \vdots & & \vdots \\
  y_{J,1} & \cdots & y_{J,N_L} & \cdots & y_{J,N_H} \\
  x_{J+1,1} & \cdots & x_{J+1,N_L} & \cdots & x_{J+1,N_H} \\
  \vdots & \ddots & \vdots & & \vdots \\
  x_{J+K,1} & \cdots & x_{J+K,N_L} & \cdots & x_{J+K,N_H}
\end{pmatrix}
$$

(A.13)

Because of the monotonic pattern, the likelihood of the sample can be written as

(see Anderson [1957] and Morrison [1971])

$$
L(\mu, \Sigma) = \left[ \prod_{\alpha=1}^{N_L} f(y_{1,\alpha}, \ldots, y_{J,\alpha}, x_{J+1,\alpha}, \ldots, x_{J+K,\alpha} | \mu, \Sigma) \right] \left[ \prod_{\alpha=N_L+1}^{N_H} f(y_{1,\alpha}, \ldots, y_{J,\alpha} | \mu_J, \Sigma_{JJ}) \right]
$$

(A.14)

Taking the logarithm of (A.14) and maximizing, we find that the estimates of the parameters of the distribution are (assuming \( \Sigma \) known) given by

$$
M_\mu = \left( \frac{1}{N_L} \sum_{\alpha=1}^{N_L} x_{\alpha} + \Sigma_{JJ}^{-1} \Sigma_{JK} \left[ \frac{1}{N_H} \sum_{\alpha=1}^{N_H} y_{\alpha} - \frac{1}{N_L} \sum_{\alpha=1}^{N_L} y_{\alpha} \right] \right)
$$

(A.15)

and

$$
\Sigma_\mu = \frac{1}{N_H} \Sigma + \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \Omega
$$

(A.16)

where

$$
\Omega = \begin{pmatrix}
  0 & 0 \\
  0 & \delta_{kk'}
\end{pmatrix}
$$

(A.17)

with

$$
\delta_{kk'} = \rho_{kk'} \sigma_k \sigma_{k'} g_{kk'}
$$

and \( g_{kk'} \) the elements of the matrix \( [1 - (1/|C_{11}| \rho_{kk'})C_{12}'C_{11}^{-1}C_{12}] \) (C is the correlation matrix corresponding to the covariance matrix \( \Sigma \)). The impact of differential information is reflected exclusively in the second component of (A.16),
\[ (1/N_L - 1/N_H)\Omega. \]

Now, while the investor holds beliefs as to the parameters of the distribution for \( \mu \), portfolio decisions will, as noted, be based on the parameters of the predictive distribution (the distribution for \( \bar{C} \) as perceived by investors). Appealing to Zellner [1971],\(^5\) the parameters of this distribution are

\[ \mu_P = M\mu \] (A.18)

and

\[ \Sigma_P = \frac{N_H + 1}{N_H} \Sigma + \left( \frac{1}{N_L} - \frac{1}{N_H} \right)\Omega \] (A.19)

Equation (A.19) verifies that the investor will, as expected, add an amount at least equal to \( (1/N_H)\sigma_j^2 \) to the risk associated with each security in the face of limited information. This additional risk component is introduced by the investor to compensate for the presence of estimation (or information) risk. The magnitude of the adjustment associated with the risk of a low information security is augmented by an additional risk component, \( (1/N_L - 1/N_H)\Omega \), which is designed to compensate for relative levels of information availability.

\(^4\) The form of \( \Omega \) is consistent with Kalymon [1971], who shows, within the context of his model, that if there are \( n_i \) observations on \( S_i \) and \( n_j \) observations on \( S_j \), the posterior covariance is calculated from the known covariance by \( \sigma_{ij}' = \sigma_{ij}/\max(n_i, n_j) \)

\(^5\) Zellner shows that if the signal, \( \bar{y} \) can be written in the form

\[ \bar{y} = M + \tilde{\epsilon} \]

where \( \tilde{\epsilon} \) is multivariate normal with mean zero and variance \( \Sigma \), and is independent of \( M \), then the parameters of the predictive distribution for the as yet unobserved observation, \( \bar{Y} \), are given by

\[ \mu_P = E(\bar{Y}|y) = E(M|y) = \hat{M} \]

\[ \Sigma_P = \text{var}(\bar{Y}|y) = \text{var}(\hat{M}|y) + \text{var}(\tilde{\epsilon}) \]
Thus, we have a general formulation which explicitly recognizes the existence of differential information, and are therefore in a position to examine its equilibrium implications. The examination will focus exclusively on $\beta_j^{CF}$ because it is the relevant measure of risk for the pricing of securities and because, as can be seen from (A.18), the explicit recognition of differential information does not alter $E(\widehat{CF}_j)$. For purposes of notational simplicity, we employ the superscript "∞" to denote a derivation which occurs in an environment where there is an infinite amount of information available about each security (no estimation risk), the superscript "=" when there is limited information available about the securities (the environment includes estimation risk) but the amount of information is the same for all securities, and finally the superscript "＞" when differential information is present.

Starting with an equal information setting (estimation risk but no differential information), we find that accounting for information (or estimation) risk leads to an equilibrium asset pricing model that is observationally equivalent to one derived ignoring estimation risk considerations i.e., with $N_L = N_H$

$$
\beta^{CF \infty} = \frac{N_H + 1}{N_H} \sum w = \beta^{CF \infty}
$$

This result, which is consistent with Brown [1979], is as expected because the degree of information risk associated with each security is the same. That is, while the absolute risk associated with each security has increased, relative riskness remains unchanged, and since the systematic risk of the market, $\beta_M^{CF}$, is anchored at one by construction, there can be no movement in the equilibrium measures of risk. However, by allowing for the presence of differential information a radically different result is obtained. In this setting the appropriate measures of systematic risk become
A comparison of (A.20) with (A.21) then leads us to conclude, as did Barry and Brown [1985], that there would appear to be a downward adjustment to the measures of systematic risk associated with high information securities and that there must, therefore, be a portfolio of low information securities for which the existence of differential information will lead to an upward revision in $\beta^{CF}$. This result derives because securities for which the available information sets differ, will be affected to different degrees by estimation (information) risk. The magnitude of this “implied $\beta$ adjustment” will depend on the nature of the differential information terms in (A.21), $(1/N_L - 1/N_H)\Omega$.

In summary, the only risk of importance for the pricing of securities, even in the face of estimation risk or differential information, would appear to be systematic risk as measured by $\beta^{CF}_j$. The existence of differential information does, however, appear to affect the level of this risk employed by rational investors in the pricing of securities. In fact, a comparative examination of (A.20) and (A.21) suggests that in determining the appropriate level of systematic risk investors will engage in a two stage process. The first stage is to develop the measure of $\beta^{CF}$ which obtains in a world without differential information. The second stage is to augment this measure in a linear fashion by adding (or subtracting as appropriate) an amount to compensate for the relative level of information (estimation) risk associated with each security. The measure of $\beta$ employed by investors in the pricing of securities,

\[
\beta^{CF} = \frac{\frac{N_H}{N_H-1} \sum w + \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \Omega w}{\frac{N_H}{N_H-1} w' \sum w + \left( \frac{1}{N_L} - \frac{1}{N_H} \right) w' \Omega w}
\]  

(A.21)

6 Differential information increases the variance of the market relative to the equal information setting, but leaves the covariance term unaltered for the high information securities (see the form of $\Omega$ in (A.17)).

7 The weighted average of all measures of systematic risk must be unity in both its adjusted and unadjusted forms. This result assumes that betas are positive.
denoted $\beta^{CF^>}$, would, therefore, include a component to compensate for market (or undiversifiable) risk, $\beta^{CF^=}$, and a component to compensate for information risk, $\beta^{CF^>} - \beta^{CF^=}$.
APPENDIX B

A Small Economy

In a small economy consisting of but one high information security, \( S_H \), and one low information security, \( S_L \), the parameters of the predictive distribution are (from (A.15) and (A.16))

\[
\mu_P = \left( \frac{1}{N_L} \sum_{a=1}^{N_L} \alpha_a - \frac{\rho_{HL} \sigma_L}{\sigma_H} \left[ \frac{1}{N_L} - \frac{1}{N_H} \right] \sum_{a=1}^{N_L} \alpha_a - \frac{1}{N_H} \sum_{a=N_L+1}^{N_H} \alpha_a \right)
\]

and

\[
\Sigma_P = \frac{N_H + 1}{N_H} \begin{pmatrix} \frac{\sigma_H^2}{\rho_{HL} \sigma_H \sigma_L} & \rho_{HL} \sigma_H \sigma_L \\ \rho_{HL} \sigma_H \sigma_L & \frac{\sigma_L^2}{1 - \rho_{HL}^2} \end{pmatrix} + \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \begin{pmatrix} 0 & 0 \\ 0 & (1 - \rho_{HL}^2) \sigma_L^2 \end{pmatrix}
\]

and the appropriate measures of systematic risk are

\[
\beta_H^{CF} = \beta_H^{CFm} \left[ 1 - \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \frac{\omega^2_L (1 - \rho_{HL}^2) \sigma_L^2}{\sigma^2 (CF_M)^2} \right]
\]

and

\[
\beta_L^{CF} = \beta_L^{CF} = + \frac{\beta_H^{CFm} \left( \frac{1}{N_L} - \frac{1}{N_H} \right) w_H w_L (1 - \rho_{HL}^2) \sigma_L^2}{\sigma^2 (CF_M)^2}
\]

In this setting the elements of \((1/N_L - 1/N_H)\Omega\) are given in the second term of (B.2). Since the adjustments to systematic risk found in (B.3) and (B.4) are the direct result of this differential information risk component, we see that there are two parameters which are fundamental to the relationship between differential information and capital market equilibrium. They are fundamental because the existence of a differential information setting is dependent on the values they attain. These parameters are: (1) the correlation between the high information security, \( S_H \), and the low information security, \( S_L \), as given by \( \rho_{HL} \), and (2) the relative levels of precision associated with the securities as measured by \( 1/N_L - 1/N_H \). Other
parameters found in the “implied $\beta$ adjustment” terms are of secondary importance to this study because they have no relationship with differential information. In addition, we note

$$\beta_L^{CF^*} - \beta_L^{CF^=} = \frac{-w_H}{w_L}(\beta_H^{CF^*} - \beta_H^{CF^=})$$ (B.5)

and restrict our attention to the adjustment for the low information security. To facilitate the analysis, we assume that unadjusted measures of market risk are exogenously specified.

Starting our examination of the characteristics of the “implied $\beta$ adjustment” with $\rho_{HL}$, we find that if the low information security is perfectly correlated with the high information security $S_H$ (i.e., $\rho_{HL} = 1$) then the “implied $\beta$ adjustment” disappears. In this setting investors are able to accurately infer or “fill in” the missing data pieces from their counterparts for the high information security. More generally

$$\frac{\partial(\beta_L^{CF^*} - \beta_L^{CF^=})}{\partial \rho_{HL}} = -2\rho_{HL}\beta_H^{CF^=}(\frac{1}{N_L} - \frac{1}{N_H})w_Hw_L\sigma^2(CF_M)^= < 0$$ (B.6)

Thus, as the correlation between $S_H$ and $S_L$ increases, the “implied $\beta$ adjustment” term will decrease, reaching a minimum of 0 since $\rho_{HL} \leq 1$. This result reflects the investors’ increased ability to infer the missing data pieces.

Moving to consider $1/N_L - 1/N_H$, we find, assuming that information (as measured by $N_L$ and $N_H$) is increasing and differentiable at least once in time, $t$, and that all other parameters in the adjustment term are independent of the information measure

$$\frac{\partial(\beta_L^{CF^*} - \beta_L^{CF^=})}{\partial t} = \beta_Hw_Hw_L(1-\rho_{HL})\sigma^2(CF_M)^= \left(\frac{\partial(\frac{1}{N_L} - \frac{1}{N_H})}{\partial t}\right)$$ (B.7)

where

$$\frac{\partial(\frac{1}{N_L} - \frac{1}{N_H})}{\partial t} = \frac{-1}{N_L^2}\frac{\partial N_L}{\partial t} + \frac{1}{N_H^2}\frac{\partial N_H}{\partial t}$$ (B.8)
If the rate of information flow is the same for both the high and low information securities, then, since $N_L < N_H$, we can conclude that the magnitude of the adjustment is a decreasing function of time (and of information). Additionally, if the rate at which information is received remains constant over time

$$\frac{\partial^2(\beta_L^{LF} - \beta_L^{FM})}{\partial t^2} > 0$$  \hspace{1cm} (B.9)

We, therefore, conclude that as the degree of differential information decreases, the magnitude of the "implied $\beta$ adjustment" not only decreases, but does so at a decreasing rate i.e., the magnitude of the adjustment behaves in a non-linear fashion relative to the degree of differential information (see Figure 2). Note that the form of the results described in equations (B.7) through (B.9) is identical to that which would obtain if we assume that $N_H = \infty$ and perform the analysis relative to movements in $N_L$.

Having established the behavioural characteristics of the "implied $\beta$ adjustment", we now move to consider how this adjustment term will impact on capital market equilibrium. For purposes of notational simplicity, the adjustment term is defined as follows

$$\beta_L^{LF} - \beta_L^{FM} = b_L(\frac{1}{N_L} - \frac{1}{N_H}, \rho_{HL})$$  \hspace{1cm} (B.10)

The analysis will start with an examination of the impact of $b_L$ on equilibrium prices and expected returns (making use of a sequence of three increasingly realistic economies), and then turn to consider how it might be reflected in market measures of systematic risk.

8 More generally, if

$$\frac{\partial N_H}{\partial t} < \frac{N_H^2}{N_L^2} \frac{\partial N_L}{\partial t}$$

the adjustment term will be a decreasing function of information.
To begin, we see, from (A.8) and (B.4), that the equilibrium price of the low information security in a single period economy where investors explicitly recognize differential information is lower than it would be in an identical economy where differential information is ignored 9 i.e.,

\[
P_{ko} - P_{ko} = |E(\widetilde{CF_M}) - P_{M0}| b_L > 0 \tag{B.11}
\]

and consequently

\[
E(R_L^\text{>}) = \frac{E(\widetilde{CF}_{L})) - P_L^\text{>}}{P_L^\text{>}} > \frac{E(\widetilde{CF}_{L})) - P_L^\text{=}}{P_L^\text{=}} = E(R_L^\text{=} \tag{B.12}
\]

where \( E(R_L) \) is the expected return for \( S_L \). Similarly, looking across single period economies that differ only by the amount of information available for the low infor-

9 This result is dependent on the price of the market being the same in both economies. To demonstrate, we appeal to (A.8) and (B.18). In general

\[
P_k = E(\widetilde{CF}_k) - \beta_k \left[ \frac{E(\widetilde{CF}_M)}{P_M} - 1 \right] \quad k = H, L
\]

where

\[
\beta_k = \frac{\text{cov}(\widetilde{CF}_k, R_M)}{\sigma^2(R_M)}
\]

Thus

\[
P_M^\text{>} = w_H P_H^\text{>} + w_L P_L^\text{>} = w_H E(\widetilde{CF}_H) + w_L E(\widetilde{CF}_L) - \frac{E(\widetilde{CF}_M)}{P_M^\text{>}} + 1
\]

since \( w_H \beta_H^\text{>} + w_L \beta_L^\text{>} = \beta_M^\text{>} = 1 \).

Similarly

\[
P_M^\text{=} = w_H E(\widetilde{CF}_H) + w_L E(\widetilde{CF}_L) - \frac{E(\widetilde{CF}_M)}{P_M^\text{=}} + 1
\]

and consequently

\[
P_M^\text{>} - P_M^\text{=} = E(\widetilde{CF}_M) \left( \frac{P_M^\text{>} - P_M^\text{=}}{P_M^\text{>}, P_M^\text{=}} \right)
\]

This result is satisfied when either \( P_M^\text{>} - P_M^\text{=} = 0 \) or when \( P_M^\text{>}, P_M^\text{=} = E(\widetilde{CF}_M) \). We therefore conclude that \( P_M^\text{<} = P_M^\text{=} \).
mation security, we find that price will increase (and expected return decrease) following a non-linear path as \( N_L \) increases. In fact, from (A.8), (B.7), and (B.8),

\[
\frac{\partial P_L^>}{\partial N_L} = -[E(CF_M) - P_M] \frac{\partial \beta^{CF>}_L}{\partial N_L} > 0
\]  

(B.13)

Finally, addressing a multiperiod economy in which investors explicitly recognize the inter-related nature of consecutive periods, we once again find that differential information will negatively impact on the price of the low information security.\(^{10}\) To demonstrate, consider a two period economy with the following structure. Trading occurs at two points in time. At the beginning of the first period, time \( t_0 \), security demands are based on the information set with which investors are initially endowed (measure \( N^o_L \)). Information about first period operations is released at the end of period one, following which a second round of trading takes place (at time \( t_1 \)). Investor demands at \( t_1 \) will be based on the revised information set (measure \( N^1_L > N^o_L \)). Finally, consumption occurs at the end of the second period (time \( t_2 \)).

This setting can be likened to the situation where the firm's distributing dividend, \( CF_{L2} \), is the sum of its earnings for each of the two periods, say \( EPS_{L1} \) and \( EPS_{L2} \). Here the release of an earnings report at the end of period one will provide investors with additional information on which to base their investment decisions at \( t_1 \). In this economy we find, from (A.8),

\[
P_{L1}^> = E^1(CF_{L2}) - [E^1(CF_{M2}) - P_{M1}]\beta^{CF>}_{L1}
\]  

(B.14)

and similarly

\[
P_{L0}^> = E^0(P_{L1}) - [E^0(P_{M1}) - P_{M0}]\beta^{CF>}_{L0}
\]  

(B.15)

where \( E^t(\cdot) \) is the expectations operator from the perspective of time \( t \). Transform-
ing to returns (see for example Bowman [1979]), (B.14) and (B.15) become

\[ P_{L1} > \frac{E^1(CF_{L2})}{1 + E(R_M)\beta_{L1}^{R>}} \]  
\[ (B.16) \]

\[ P_{Lo} > \frac{E^o(CF_{L1})}{1 + E(R_M)\beta_{Lo}^{R>}} \]  
\[ (B.17) \]

where

\[ \beta_{L}^{R>} = \frac{\text{cov}(R_L, R_M)}{\sigma^2(R_M)} = \frac{P_M}{P_L} \beta_{L}^{CF>} \]  
\[ (B.18) \]

\[ E(R_M) = \frac{E(CF_{M2}) - P_{M1}}{P_{M1}} = \frac{E(P_{M1}) - P_{Mo}}{P_{Mo}} \]  
\[ (B.19) \]

However, from Degroot [1970]

\[ M_{\mu} = (\Sigma^{-1}_{\mu} + \Sigma^{-1}_{\mu a})[\Sigma^{-1}_{\mu o} M_{\mu o} + \Sigma^{-1}_{\mu a} M_{\mu a}] \]  
\[ (B.20) \]

where \( M_{\mu t} \) and \( \Sigma_{\mu t} \) are the investors' assessments of the parameters for \( \tilde{\mu} \) at time \( t \), and \( M_{\mu a} \) and \( \Sigma_{\mu a} \) are the analogous parameters for the information release. Consequently, with the two periods perceived as being identical

\[ E^o[E^1(CF_{L2})] = E^o(CF_{L2}) \]  
\[ (B.21) \]

Appealing to (B.21), and substituting (B.16) into (B.17), leaves

\[ P_{Lo} > \frac{E^o(CF_{L2})}{[1 + E(R_M)\beta_{Lo}^{R>}] [1 + E(R_M)\beta_{L1}^{R>}]} \]  
\[ (B.22) \]

Finally, noting

\[ \frac{\partial \beta_{L}^{R>}}{\partial N_L} = \frac{\partial (\frac{P_M}{P_L} \beta_{L}^{CF>})}{\partial N_L} = P_M \left[ -\beta_{L}^{CF>} \frac{\partial P_L}{P_L^2} \frac{\partial N_L}{\partial N_L} + \frac{1}{P_L} \frac{\partial \beta_{L}^{CF>}}{\partial N_L} \right] < 0 \]  
\[ (B.23) \]

we see that the price discount in the first period is greater than it would have been if investors had treated the available information set at \( t_o \) as being characterized by \( N_{L1}^1 \).
Thus, the existence of differential information will lead, in all situations, to a reduction in price which is commensurate with the precision of the information held at the given point in time. In addition, this result is wholly dependent on the adjustment factor $b_L$, and will therefore follow a non-linear pattern, paralleling that of $b_L$ (i.e., as $N_L$ increases, the magnitude of the discount will decrease, and price will asymptotically approach its infinite information position).

Turning to consider how market measures of systematic risk, $\beta_L^{RF}$, will reflect the presence of differential information, we note that, within the context of our economy, investors rely on accounting or financial data to arrive at an estimate of the systematic risk associated with any given security. They then, in incorporating the existence of differential information into their decision models, add an adjustment factor to compensate for relative information risk, and use as the appropriate measure of systematic risk for the pricing of the low information security, $S_L$,

$$\beta_L^{RF} = \beta_L^{RF^*} + b_L\left(\frac{1}{N_L} - \frac{1}{N_H}, \rho_{HL}\right)$$  \hspace{1cm} (B.24)

Assuming that the covariability of the underlying process remains constant, $N_L$ increases over time (i.e. $b_{Lt'} < b_{Lt}$ if $t' > t$), and $\beta_L^{RF^*}$ is independent of $N_L$, we find that the appropriate measure of risk for pricing the low information security at any point in time, say $T$, (i.e., at any level of $N_L$) as reflected in the return series, $\beta_L^{RF^*}$, will be less than the value that would be obtained as an estimate of $\beta_L^{RF^*}$ from the historical returns up to the given point in time, $\hat{\beta}_{OLS}^{T}$ (e.g., from OLS regression) i.e., at time $T$

$$\hat{\beta}_{OLS}^{T} = \frac{P_{MT}}{\sigma^2(R_M)} \sum_{t=1}^{T} \left[ \left( R_F + \frac{1}{P_{Lt}}(\beta_L + b_{Lt})(R_{Mt} - R_F) \right) - \left( R_F + \frac{1}{T} \sum_{t=1}^{T} \frac{1}{P_{Lt}}(\beta_L + b_{Lt})(R_{Mt} - R_F) \right) \right] \left( R_{Mt} - \bar{R}_M \right)$$  \hspace{1cm} (B.25)
and assuming independence between the market return and the adjustment for differential information i.e.,

\[
E\left( \frac{\beta_L + b_{Lt}}{P_{Lt}} R_{Mt} \right) = \left( \frac{1}{T} \sum_{t=1}^{T} \frac{\beta_L + b_{Lt}}{P_{Lt}} \right) R_M. \tag{B.26}
\]

we find (B.25) reduces, in expectation, to

\[
\hat{\beta}_{OLS}^T = \frac{P_{Mt}}{\sigma^2(R_M)} \sum_{t=1}^{T} \frac{1}{P_{Lt}} (\beta_L + b_{Lt})(R_{Mt} - R_M)^2 \tag{B.27}
\]

On the other hand, the appropriate measure of risk at T is given by

\[
\beta_{LT}^{R >} = \frac{P_{MT}}{\sigma^2(R_M)} \left( \frac{\beta_L + b_{LT}}{P_{LT}} \right) \sum_{t=1}^{T} (R_{Mt} - R_M)^2 \tag{B.28}
\]

Finally, because \( b_{Lt} > 0 \) \( \forall t \in [0, T] \) and \( b_{Lt} \) is a decreasing function of \( t \), we have, comparing (B.27) and (B.28)

\[
\hat{\beta}_{OLS}^T - \beta_{LT}^{R >} \propto \frac{1}{T} \sum_{t=1}^{T} \frac{b_{Lt}}{P_{Lt}} - \frac{b_{LT}}{P_{LT}} > 0 \tag{B.29}
\]

This result is not unexpected since \( \hat{\beta}_{OLS}^T \) is, in effect, a weighted average of previous information positions while \( \beta_{LT}^{R >} \) only reflects the current information position. We may, therefore, conclude that if investors employ accounting or financial data to estimate \( \beta_{LT}^{CF >} \) then we would expect \( \hat{\beta}_{OLS} \) to exceed \( \beta_{LT}^{R >} \) at any point in time.

Interestingly, the relationship between \( \hat{\beta}_{OLS}^T \) and \( \beta_{LT}^{CF >} \) described in (B.29) is the opposite of the one conjectured by Barry and Brown [1985]. Barry and Brown suggest that researchers may estimate a measure of systematic risk for the low information security which is lower than the measure which appropriately reflects the differential information adjustment. Their result will, however, only derive in two somewhat unlikely settings. The first requires that investors rely exclusively on the historical return series for their estimates, and employ, as the appropriate measure of systematic risk for the pricing of securities, the following

\[
\beta_{LT}^{CF >} = \frac{P_{Lt}}{P_{Mt}} \hat{\beta}_{OLS} + b_{Lt}(N_L, \rho_{HL}) \tag{B.30}
\]

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In this case $\beta_{ Lt}^{R^2}$ exceeds $\hat{\beta}_{OLS}$ at every point in time. Unfortunately, this setting makes the assumption that investors will compound the information effect by making an additional adjustment every period ($\hat{\beta}_{OLS}$ will reflect all previous information adjustments through the return series). The second setting in which Barry and Brown's conjecture holds is the setting where information decreases, rather than increases, over time. Here we also find that $\beta_{ Lt}^{R^2}$ will exceed $\beta_{OLS}$ at all points.

In summary, within the confines of a small economy, the presence of differential information has a significant effect on the nature of the capital market equilibrium which develops. Its primary or direct impact is on the measure of systematic risk employed by investors for the pricing of securities. The magnitude of this effect is a decreasing function of two important variables, namely the coefficient of correlation between the high and low information securities, $\rho_{HL}$, and the relative level of differential information, $1/N_L - 1/N_H$. Differential information also indirectly affects price and expected return through its impact on systematic risk. Finally, the analysis suggests that for the low information security, use of the historical return series will provide researchers with an upwardly biased estimate of systematic risk.
APPENDIX C

An Unrestricted Economy

The purpose of this analysis is to examine how well the results in Appendix B withstand a relaxation of the assumptions underlying the small economy. We start by allowing for an unlimited number of low information securities, then relax the structure of the information environment, and finally allow for an unlimited number of high information securities.

With an unlimited number of low information securities in the economy, we find, from (A.15), that the first moment of the predictive distribution for any low information security, say $S_k$, is

$$\mu_k = \frac{1}{N_L} \sum_{\alpha=1}^{N_L} \xi_{k\alpha} - \frac{1}{\sigma_1} \left[ \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \sum_{\alpha=1}^{N_L} \eta_{1\alpha} - \frac{1}{N_H} \sum_{\alpha=N_L+1}^{N_H} \eta_{1\alpha} \right]$$ (C.1)

and consequently, the elements of the low information partition of $\Sigma_F$ (the predictive distribution - see (A.16) ) for $S_k$ are

$$\text{var}(\tilde{CF}_k) = \frac{N_H + 1}{N_H} \sigma_k^2 + \frac{1}{N_L} \frac{1}{N_H} (1 - \rho_{1k}^2) \sigma_k^2$$ (C.2)

and

$$\text{cov}(\tilde{CF}_k, \tilde{CF}_{k'}) = \frac{N_H + 1}{N_H} \rho_{kk'} \sigma_k \sigma_{k'} + \frac{1}{N_L} \frac{1}{N_H} (1 - \rho_{1k} \rho_{1k'}) \rho_{kk'} \sigma_k \sigma_{k'}$$ (C.3)

The appropriate measure of systematic risk for $S_k$ is then

$$\beta_k^{CF} = \beta_k^{CF} + \frac{\left( \frac{N_L}{N_L} - \frac{N_H}{N_H} \right)}{\sigma^2(CF)} \left[ w_k \sigma_k^2 (1 - \rho_{1k}^2) \right] + \sum_{\alpha=2}^{K+1} \sum_{\alpha \neq k} w_{\alpha} \sigma_k \sigma_\alpha (\rho_{k\alpha} - \rho_{1k} \rho_{1\alpha})$$ (C.4)
Thus, simply allowing for additional low information securities in the economy is not sufficient to eliminate $b_L$.\footnote{While the variance component of the adjustment term will disappear as the number of low information securities increases (and $w_k$ becomes small), the covariance component will not be materially affected.} Such a result is, however, not totally unexpected since it is the fact that data pieces are missing and not the content of the missing pieces which leads to the "implied $\beta$ adjustment". In the economy, as developed, the same data pieces are missing for each low information firm (see (A.13)). This is not to say that the content of the missing data pieces is the same for all firms, but rather that the missing pieces are aligned. Thus, investors, even by including a large number of low information securities in their portfolios, will be unable to eliminate the differential information risk.

On the other hand, as noted by Reinganum and Smith [1983], "... to the extent this risk is idiosyncratic rather than systematic, it is diversifiable". Perhaps then, in an economy where the differential information risk is firm specific, it will not be priced. Pursuing this conjecture, we find, from Lord [1955a,b], that when the missing data pieces are not aligned (i.e. the information risk is firm specific) the covariance matrix of the predictive distribution is given by

$$\Sigma_P = \frac{N_H}{N_H} \Sigma + \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \Omega^*$$  \hspace{1cm} (C.5)

with $\Omega^*$, the component of the covariance matrix reflecting the impact of differential information, identical to its counterpart, $\Omega$, from (A.17), except that the off-diagonal elements of $\delta_{kk'}$ are now zero i.e., $\delta_{kk'} = 0$ if $k \neq k'$. In this extreme case $b_L = 0$ i.e., this is no "implied $\beta$ adjustment". To illustrate, consider an economy with one high information security and two low information securities. The data matrix under consideration is (in contrast to (A.13)) given by (assuming...
without loss of generality \( N_H = 2N_L \)

\[
\begin{pmatrix}
y_{1,1} & \cdots & y_{1,N_L} & y_{1,N_L+1} & \cdots & y_{1,N_H} \\
x_{2,1} & \cdots & x_{2,N_L} & - & \cdots & - \\
- & \cdots & - & x_{3,N_L+1} & \cdots & x_{3,N_H}
\end{pmatrix}
\]  

(C.6)

Appealing to (C.1), we have, for the element of \( \Sigma_F \)

\[
\text{var}(CF_k) = \frac{N_H + 1}{N_H} \sigma_k^2 + \left( \frac{1}{N_L} - \frac{1}{N_H} \right)(1 - \rho_{1k}^2)\sigma_k^2 
\]  

(C.7)

\[
\text{cov}(CF_k, CF_{k'}) = \frac{N_H + 1}{N_H} \rho_{kk'}\sigma_k\sigma_{k'} 
\]  

(C.8)

Thus, the appropriate measure of systematic risk is

\[
\beta_k^{C_F} = \beta_k^{CF} = \frac{\sigma_k}{\sigma_{(CF_M)}} \omega_k \sigma_k^2(1 - \rho_{1k}^2) 
\]  

(C.9)

and we find, with a large number of low information firms, investors are able to eliminate the effect of differential information on systematic risk.\(^\text{12}\)

We, therefore, see that at the one extreme, when the same data pieces are identified as missing for all low information securities, investors will make an adjustment to the measure of systematic risk they use for pricing securities, but, at the other extreme, where there is no overlap in the missing pieces, investors will be able to eliminate any risk associated with the existence of differential information. It is, however, more likely that the degree to which the missing data pieces overlap is somewhere between these two extremes. It is then to be expected that investors are able to reduce the impact of differential information risk on systematic risk through the holding of many low information securities, but that the reduction will not be complete.

Finally, as we have already seen, the magnitude of \( b_L \) is inversely related to the investors' abilities to infer the missing data pieces from their counterparts for

\(^{\text{12}}\) As \( K \to \infty \) \( \omega_k \to 0 \)
the high information security. Perhaps then, if enough high information securities are present, the "implied $\beta$ adjustment" will disappear. To address this conjecture we will allow for an expansion of the economy to include two high information securities.

For an economy in which there are two high information securities, $S_1$ and $S_2$, we find, from (A.15), that the expected payoff on a low information security, $S_k$, is given by

$$
\mu_k = \frac{1}{N_L} \sum_{\alpha=1}^{N_L} X_{k\alpha} - \left( \frac{1}{1 - \rho_{12}^2} \right) \left( \frac{\sigma_k}{\sigma_1} (\rho_{1k} - \rho_{12} \rho_{2k}) \right) \left[ \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \sum_{\alpha=1}^{N_L} y_{1\alpha} - \frac{1}{N_H} \sum_{\alpha=N_L+1}^{N_H} y_{1\alpha} \right]
$$

$$
+ \frac{\sigma_k}{\sigma_2} (\rho_{2k} - \rho_{12} \rho_{1k}) \left[ \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \sum_{\alpha=1}^{N_L} y_{2\alpha} - \frac{1}{N_H} \sum_{\alpha=N_L+1}^{N_H} y_{2\alpha} \right]
$$

(C.10)

and consequently, the elements of $\Sigma_F$ are

$$\text{var}(CF_k) = \frac{N_H + 1}{N_H} \sigma_k^2 + \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \left( 1 - \frac{\rho_{1k}^2 + \rho_{2k}^2 - 2 \rho_{12} \rho_{1k} \rho_{2k}}{1 - \rho_{12}^2} \right) \sigma_k^2
$$

(C.11)

and

$$\text{cov}(CF_k, CF_{k'}) = \frac{N_H + 1}{N_H} \rho_{kk'} \sigma_k \sigma_{k'} + \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \left[ 1 - \frac{\rho_{1k} \rho_{1k'} + \rho_{2k} \rho_{2k'} - \rho_{12} (\rho_{1k} \rho_{2k'} + \rho_{1k'} \rho_{2k})}{\rho_{kk'} (1 - \rho_{12}^2)} \right] \rho_{kk'} \sigma_k \sigma_{k'}
$$

(C.12)

The appropriate measure of systematic risk is then

$$\beta_k^{CF^*} = \beta_k^{CF^*} + \left( \frac{1}{N_L} - \frac{1}{N_H} \right) \left[ \sum_{\alpha=1}^{N_L} \left( \rho_{k\alpha} - \rho_{1k} \rho_{1\alpha} \right) - \frac{(\rho_{2k} - \rho_{12} \rho_{1k})(\rho_{2k} - \rho_{12} \rho_{1k})}{1 - \rho_{12}^2} \right]
$$

(C.13)

Finally, comparing (C.4) with (C.13) we find that the existence of a second high information security will serve, in a setting with multiple low information securities,
to change the "implied $\beta$ adjustment" (relative to the setting with only one high information security) by an amount equal to

$$\sum_{\alpha=3}^{K+2} w_k \sigma_{\alpha} \sigma_k \frac{(\rho_{2k} - \rho_{12}\rho_{1k})(\rho_{2\alpha} - \rho_{12}\rho_{1\alpha})}{1 - \rho_{12}^2}$$

Thus, if $\rho_{2k}/\rho_{1k} > \rho_{12}$ $\forall k \in K$, there will be a reduction in the magnitude of the adjustment. This condition is intuitively straightforward, merely providing for a setting in which investors are able to draw inferences about the missing data pieces from the second high information security in addition to those drawn from the first high information security. This result suggests that the addition of partially correlated high information securities to the market will allow investors to improve their estimates of the missing data pieces for low information securities, leading to a reduction in the perceived information deficiency, and a resultant reduction in the magnitude of the "implied $\beta$ adjustment", $b_k$. Thus, in a large market with an unrestricted number of high information securities, say $J$, we would expect that as $J \to \infty b_k \to 0$. That is to say, we would expect that the presence of a large number of partially correlated high information securities, each making a positive marginal information contribution, would allow investors to effectively infer the missing data pieces for the low information securities.

In summary, it appears that in an unrestricted market (with many high and many low information securities and a generalized information structure) investors may be able to effectively eliminate differential information and operate as if it does not exist. That is not to say that the available information sets for all firms are identical, but rather, that investors may be able to eliminate the differences through a process of inference or through diversification. In either case, the so-called differential information risk will not be priced.
APPENDIX D

Cross-Sectional Beta Estimation

The differential information model presented in Appendix A predicts that the measure of systematic risk appropriately adjusted for differential information will depend on quantity of information. Further, the model predicts that this adjusted beta will decrease at a decreasing rate as information increases. Assuming that information is increasing in time, we see that it will be difficult to estimate an adjusted beta at a point in time for a single security because only one observation pair exists at each information level. An alternative approach is, then, to aggregate across a temporally separated sample of low information firms, each at the same information level. For this purpose we adopt a procedure similar to Ibbotson’s [1975] RATS(N,N) model which will, in our economic model, allow for an estimate of the adjusted beta at a point in the information sequence. Run the regression

\[ R_{jt} = \alpha_t + \beta_t R_{Mj,t} + \epsilon_{jt} \quad j = 1, J \]  

where \( t \) refers to event or information time (i.e., \( t \) proxies for information level), \( R_{jt} \) is the realized return to security \( j \) at information level \( t \) (time \( t \)), and \( R_{Mj,t} \) is the contemporaneous market return. There will be no cross-correlation in security returns because calendar time periods will differ across securities.

Least squares regression implies that \( \hat{\beta}_t \) is an unbiased estimate of beta appropriately adjusted for differential information for the sample of low information firms, \( \beta^{R>}_{Lt} \). To see this, consider a sample of returns, \( R_{jt} \), each drawn from a distribution with unique mean, \( \mu_{jt} \), and covariance with the market return, \( C_{jt} \). Note, \( C_{jt} \) is the return covariance which is equal to the cash flow covariance divided by price (see equation (B.18) in Appendix B). The sample covariance of the \( R_{jt} \) with
pairwise drawings of the market return, $R_{M,t}$, has an expected value equal to the average covariance, $\bar{C}_t$. It is assumed that the $\mu_{jt}$ are uncorrelated with $R_{M,t}$ in expectation (that high mean firms are no more likely to be paired with high market returns than with low market returns). To demonstrate, write the sample covariance as

$$\hat{C}_t = \frac{1}{J-1} \sum_{j=1}^{J} [R_{jt} - \bar{R}_t][R_{M,t} - \bar{R}_{M,t}]$$

$$= \frac{1}{J-1} \sum_{j=1}^{J} [(R_{jt} - \mu_{jt}) + (\mu_{jt} - \bar{R}_t)][(R_{M,t} - \mu_{Mt}) + (\mu_{Mt} - \bar{R}_{M,t})]$$

$$= \frac{1}{J-1} \sum_{j=1}^{J} [(R_{jt} - \mu_{jt})(R_{M,t} - \mu_{Mt}) + (R_{jt} - \mu_{jt})(\mu_{Mt} - \bar{R}_{M,t}) + (\mu_{jt} - \bar{R}_t)(R_{M,t} - \mu_{Mt}) + (\mu_{jt} - \bar{R}_t)(\mu_{Mt} - \bar{R}_{M,t})]$$

Then

$$E(\hat{C}_t) = \frac{1}{J-1} \left[ \sum_{j=1}^{J} C_{jt} - \sum_{j=1}^{J} \frac{1}{J} C_{jt} - \sum_{j=1}^{J} \frac{1}{J} C_{jt} + \sum_{j=1}^{J} \frac{J}{J^2} C_{jt} \right]$$

$$= \frac{1}{J-1} (J-1) \bar{C}_t$$

$$= \bar{C}_t$$

The sample beta coefficient is the sample covariance divided by the sample variance of the market returns and, therefore, has an expected value equal to the average beta of the sample. Thus, the cross-sectional beta estimation procedure leads to an unbiased estimate of beta appropriately adjusted for differential information for a portfolio of low information securities at a given information level (point in time).

The regression (D.1) can be given a random coefficient interpretation. In random coefficient models, OLS regression can be subject to heteroscedasticity that serves to bias the standard error of the coefficients even though the coefficients themselves are unbiased. In our application, we avoid the requirement that the
standard errors of the coefficients be estimated from the cross-sectional regression. Our hypotheses relate to the time series properties of the cross-sectional coefficients and the standard errors of the coefficients are estimated through the aggregation over time in a manner similar to the intuition underlying the procedures in Fama and MacBeth [1973].
## TABLE I

FREQUENCY DISTRIBUTION OF MARKET VALUE AFTER OFFER

FOR 142 NEWLY-LISTED FIRMS+

<table>
<thead>
<tr>
<th>Market Value</th>
<th>Number of Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 10M</td>
<td>26</td>
</tr>
<tr>
<td>10M - 20M</td>
<td>43</td>
</tr>
<tr>
<td>20M - 50M</td>
<td>37</td>
</tr>
<tr>
<td>50M - 100M</td>
<td>18</td>
</tr>
<tr>
<td>Over 100M</td>
<td>18</td>
</tr>
<tr>
<td>total</td>
<td>142</td>
</tr>
</tbody>
</table>

+ Based on a sample of 142 newly-listed firms from the SEC listing of initial public offerings for 1981
TABLE II

FREQUENCY DISTRIBUTION OF S.I.C. INDUSTRY CLASSIFICATIONS
FOR 142 NEWLY-LISTED FIRMS*

<table>
<thead>
<tr>
<th>Division</th>
<th>Industry Group</th>
<th>Number of Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>AGRICULTURE, FORESTRY, AND FISHING</td>
<td>3</td>
</tr>
<tr>
<td>B</td>
<td>MINING</td>
<td>25</td>
</tr>
<tr>
<td>D</td>
<td>MANUFACTURING</td>
<td>65</td>
</tr>
<tr>
<td>E</td>
<td>TRANSPORTATION, COMMUNICATIONS, ELECTRIC, GAS, AND SANITARY SERVICES</td>
<td>6</td>
</tr>
<tr>
<td>F</td>
<td>WHOLESALE TRADE</td>
<td>1</td>
</tr>
<tr>
<td>G</td>
<td>RETAIL TRADE</td>
<td>8</td>
</tr>
<tr>
<td>H</td>
<td>FINANCE, INSURANCE, AND REAL ESTATE</td>
<td>2</td>
</tr>
<tr>
<td>I</td>
<td>SERVICES</td>
<td>32</td>
</tr>
</tbody>
</table>

total 142

+ Based on a sample of 142 newly-listed firms from the SEC listing of initial public offerings for 1981
<table>
<thead>
<tr>
<th>Month</th>
<th>Number of Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>JANUARY</td>
<td>6</td>
</tr>
<tr>
<td>FEBRUARY</td>
<td>7</td>
</tr>
<tr>
<td>MARCH</td>
<td>18</td>
</tr>
<tr>
<td>APRIL</td>
<td>25</td>
</tr>
<tr>
<td>MAY</td>
<td>20</td>
</tr>
<tr>
<td>JUNE</td>
<td>22</td>
</tr>
<tr>
<td>JULY</td>
<td>7</td>
</tr>
<tr>
<td>AUGUST</td>
<td>6</td>
</tr>
<tr>
<td>SEPTEMBER</td>
<td>5</td>
</tr>
<tr>
<td>OCTOBER</td>
<td>7</td>
</tr>
<tr>
<td>NOVEMBER</td>
<td>11</td>
</tr>
<tr>
<td>DECEMBER</td>
<td>8</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td><strong>142</strong></td>
</tr>
</tbody>
</table>

+ Based on a sample of 142 newly-listed firms from the SEC listing of initial public offerings for 1981
TABLE IV

FREQUENCY DISTRIBUTION OF YEAR OF INITIAL ISSUE
FOR 140 NEWLY-LISTED FIRMS+

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976</td>
<td>18</td>
</tr>
<tr>
<td>1977</td>
<td>18</td>
</tr>
<tr>
<td>1978</td>
<td>23</td>
</tr>
<tr>
<td>1979</td>
<td>16</td>
</tr>
<tr>
<td>1980</td>
<td>15</td>
</tr>
<tr>
<td>1981</td>
<td>26</td>
</tr>
<tr>
<td>1982</td>
<td>24</td>
</tr>
<tr>
<td>total</td>
<td>140</td>
</tr>
</tbody>
</table>

+ Based on a sample of 140 newly-listed firms from the SEC listing of initial public offerings for the period 1976-1982
TABLE V

FREQUENCY DISTRIBUTION OF NEWLY-LISTED FIRMS BY NUMBER OF TRADING DAYS FROM DATE OF ISSUE TO EARNINGS ANNOUNCEMENT+

<table>
<thead>
<tr>
<th>Number of Trading Days</th>
<th>Number of Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 20</td>
<td>0</td>
</tr>
<tr>
<td>21 - 40</td>
<td>8</td>
</tr>
<tr>
<td>41 - 60</td>
<td>13</td>
</tr>
<tr>
<td>61 - 80</td>
<td>3</td>
</tr>
<tr>
<td>81 - 100</td>
<td>6</td>
</tr>
<tr>
<td>101 - 120</td>
<td>7</td>
</tr>
<tr>
<td>121 - 140</td>
<td>8</td>
</tr>
<tr>
<td>141 - 160</td>
<td>17</td>
</tr>
<tr>
<td>161 - 180</td>
<td>22</td>
</tr>
<tr>
<td>181 - 200</td>
<td>19</td>
</tr>
<tr>
<td>201 - 220</td>
<td>24</td>
</tr>
<tr>
<td>221 - 240</td>
<td>3</td>
</tr>
<tr>
<td>241 - 260</td>
<td>2</td>
</tr>
<tr>
<td>261 - 280</td>
<td>5</td>
</tr>
<tr>
<td>281 - 300</td>
<td>3</td>
</tr>
<tr>
<td>total</td>
<td>140</td>
</tr>
</tbody>
</table>

+ Based on a sample of 140 newly-listed firms from the SEC listing of initial public offerings for the period 1976-1982
TABLE VI

FITTED MODELS OF THE CROSS-SECTIONAL BETA RELATIONSHIP FOR NEWLY-LISTED FIRMS - DAILY DATA+

Panel A: Unadjusted

<table>
<thead>
<tr>
<th></th>
<th>$\psi_0$</th>
<th>$t_{\psi_0}$</th>
<th>$\psi_1$</th>
<th>$t_{\psi_1}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>2.194</td>
<td>8.369</td>
<td>-0.061</td>
<td>-3.434*</td>
<td>0.310</td>
</tr>
<tr>
<td>Log Time</td>
<td>3.022</td>
<td>9.856</td>
<td>-0.696</td>
<td>-5.585*</td>
<td>0.557</td>
</tr>
<tr>
<td>1/Time</td>
<td>0.918</td>
<td>9.277</td>
<td>3.199</td>
<td>8.190*</td>
<td>0.734</td>
</tr>
</tbody>
</table>

Panel B: Adjusted for Nonsynchronous Trading

<table>
<thead>
<tr>
<th></th>
<th>$\psi_0$</th>
<th>$t_{\psi_0}$</th>
<th>$\psi_1$</th>
<th>$t_{\psi_1}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>2.394</td>
<td>9.880</td>
<td>-0.080</td>
<td>-4.885*</td>
<td>0.509</td>
</tr>
<tr>
<td>Log Time</td>
<td>3.299</td>
<td>12.382</td>
<td>-0.836</td>
<td>-7.720*</td>
<td>0.722</td>
</tr>
<tr>
<td>1/Time</td>
<td>0.967</td>
<td>8.180</td>
<td>3.432</td>
<td>8.746*</td>
<td>0.768</td>
</tr>
</tbody>
</table>

* Significant at the $\alpha=0.01$ level of significance.

+ Based on a sample of 142 unseasoned issues

Results are for the regression

$$\beta_t = \psi_0 + \psi_1 X_t + \epsilon_t$$

where

$\beta_t$ = estimated cross-sectional beta for trading day $t$

$\psi_0$ = component of $\beta_t$ unrelated to predictor variable

$X_t$ = predictor variable (time, log time, or 1/time)

$\psi_1$ = contribution of predictor variable to $\beta_t$, and

$\epsilon_t$ = error term
**TABLE VII**

**FITTED MODELS OF THE CROSS-SECTIONAL BETA RELATIONSHIP FOR NEWLY-LISTED FIRMS - MONTHLY DATA**

<table>
<thead>
<tr>
<th>Panel A: Unadjusted</th>
<th>X_t</th>
<th>( \psi_o )</th>
<th>t_{\psi_o}</th>
<th>( \psi_1 )</th>
<th>t_{\psi_1}</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>2.415</td>
<td>8.701</td>
<td>-0.079</td>
<td>-3.518*</td>
<td>0.421</td>
<td></td>
</tr>
<tr>
<td>Log Time</td>
<td>3.303</td>
<td>5.947</td>
<td>-0.749</td>
<td>-3.261*</td>
<td>0.385</td>
<td></td>
</tr>
<tr>
<td>1/Time</td>
<td>1.152</td>
<td>5.126</td>
<td>2.847</td>
<td>2.245**</td>
<td>0.229</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Adjusted</th>
<th>X_t</th>
<th>( \psi_o )</th>
<th>t_{\psi_o}</th>
<th>( \psi_1 )</th>
<th>t_{\psi_1}</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>2.457</td>
<td>12.586</td>
<td>-0.072</td>
<td>-4.738*</td>
<td>0.555</td>
<td></td>
</tr>
<tr>
<td>Log Time</td>
<td>3.500</td>
<td>10.094</td>
<td>-0.783</td>
<td>-5.542*</td>
<td>0.630</td>
<td></td>
</tr>
<tr>
<td>1/Time</td>
<td>1.128</td>
<td>8.532</td>
<td>3.795</td>
<td>4.965*</td>
<td>0.578</td>
<td></td>
</tr>
</tbody>
</table>

* Significant at the \( \alpha=0.01 \) level of significance.

** Significant at the \( \alpha=0.05 \) level of significance.

* Results are for the regression

\[
\beta_t = \psi_o + \psi_1 X_t + \epsilon_t
\]

where

- \( \beta_t \) = estimated cross-sectional beta for trading month t
- \( \psi_o \) = component of \( \beta_t \) unrelated to predictor variable
- \( X_t \) = predictor variable (time, log time, or 1/time)
- \( \psi_1 \) = contribution of predictor variable to \( \beta_t \), and
- \( \epsilon_t \) = error term
### TABLE VIII

**FITTED MODELS OF THE CROSS-SECTIONAL BETA RELATIONSHIP FOR NYSE FIRMS**

Panel A: Daily Data

<table>
<thead>
<tr>
<th></th>
<th>$\psi_0$</th>
<th>$t_{\psi_0}$</th>
<th>$\psi_1$</th>
<th>$t_{\psi_1}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>0.999</td>
<td>26.650</td>
<td>-0.001</td>
<td>-1.952</td>
<td>0.028</td>
</tr>
<tr>
<td>Log Time</td>
<td>1.111</td>
<td>13.730</td>
<td>-0.047</td>
<td>-2.216**</td>
<td>0.038</td>
</tr>
<tr>
<td>1/Time</td>
<td>0.912</td>
<td>42.390</td>
<td>0.564</td>
<td>2.088**</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Panel B: Monthly Data

<table>
<thead>
<tr>
<th></th>
<th>$\psi_0$</th>
<th>$t_{\psi_0}$</th>
<th>$\psi_1$</th>
<th>$t_{\psi_1}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>1.255</td>
<td>13.650</td>
<td>-0.008</td>
<td>-1.194</td>
<td>0.073</td>
</tr>
<tr>
<td>Log Time</td>
<td>1.330</td>
<td>8.910</td>
<td>-0.076</td>
<td>-1.209</td>
<td>0.078</td>
</tr>
<tr>
<td>1/Time</td>
<td>1.106</td>
<td>17.140</td>
<td>0.384</td>
<td>1.029</td>
<td>0.056</td>
</tr>
</tbody>
</table>

** Significant at the $\alpha=0.05$ level of significance.

+ Results are for the regression

$$\beta_t = \psi_o + \psi_1 X_t + \epsilon_t$$

where

$\beta_t$=estimated cross-sectional beta for trading day or month $t$

$\psi_o$=component of $\beta_t$ unrelated to predictor variable

$X_t$=predictor variable ( time, log time, or 1/time )

$\psi_1$=contribution of predictor variable to $\beta_t$, and

$\epsilon_t$=error term
TABLE IX

TESTS FOR DIFFERENCES BETWEEN PRE- AND POST-ANNOUNCEMENT RELATIONS BETWEEN CROSS-SECTIONAL BETA AND THE INVERSE OF TIME +

Panel A: Test of Overall Model Homogeneity

<table>
<thead>
<tr>
<th>Period of Listing Class (T)</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.374*</td>
</tr>
<tr>
<td>2</td>
<td>4.979**</td>
</tr>
<tr>
<td>3</td>
<td>2.124</td>
</tr>
<tr>
<td>F Ratio</td>
<td>6.374*</td>
</tr>
</tbody>
</table>

Panel B: Tests for Differences in Intercept and Slope + +

<table>
<thead>
<tr>
<th>Period of Listing Class (T)</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.479*</td>
</tr>
<tr>
<td>2</td>
<td>1.817**</td>
</tr>
<tr>
<td>3</td>
<td>0.971</td>
</tr>
<tr>
<td>( t_{\psi_2} )</td>
<td>1.845**</td>
</tr>
<tr>
<td>( t_{\psi_3} )</td>
<td>-1.217</td>
</tr>
</tbody>
</table>

* Significant at the \( \alpha=0.01 \) level of significance.

** Significant at the \( \alpha=0.05 \) level of significance.

+ Based on a sample of 140 unseasoned issues

++ Results are based on the parameters of the relation

\[
\beta_t = \psi_0 + \psi_1 (1/t) + \psi_2 X + \psi_3 (1/t)X + \epsilon
\]

where

- \( \beta_t \) = estimated cross-sectional beta for trading day \( t \)
- \( X = \begin{cases} 0 & \text{if } t < t^*; \\ 1 & \text{if } t \geq t^* \end{cases} \)
- \( t^* \) = point in time when information release occurs.
FIGURE 1

THE PREDICTIVE DISTRIBUTION

where

\[ \hat{\mu} = \text{estimated mean return, } \hat{\mu} \sim N(\mu, \sigma^2/n) \]

\[ \sigma^2 = \text{known variance of the return distribution} \]

\[ n = \text{information level (number of available observations)} \]

estimated return distribution ignoring estimation risk

extreme return distributions recognizing estimation risk
FIGURE 2

HYPOTHESIZED RELATIONSHIP BETWEEN BETA AND INFORMATION

\[ \beta^{R\infty}_L = \text{beta measured without regard for differential information} \]
\[ \beta^{R*}_L = \text{beta appropriately adjusted for differential information} \]
\[ \hat{\beta}_{OLS} = \text{beta estimated by researchers from prior return data} \]
\[ N^t_L = \text{information level of low information firms at time } t \]
\[ \bullet = \text{researchers' estimate of beta at } N^t_L \ (i.e., \text{at time } t) \]
FIGURE 3

CROSS-SECTIONAL BETA OVER THE INTERVAL FROM DAY 1 TO DAY 25 AFTER THE DATE OF INITIAL LISTING*

Legend

ACTUAL
ADJUSTED
PREDICTED

* Cross-sectional betas are based on regression (4.1) in the text

** Adjusted values are based on the Scholes-Williams adjustment for infrequent trading

*** Predicted values are based on the estimated relationship between the observed cross-sectional betas and the inverse of time (see Panel A of Table 6 for the estimated parameters)
FIGURE 4

CROSS-SECTIONAL BETA OVER THE INTERVAL FROM MONTH 1 TO MONTH 25 AFTER THE DATE OF INITIAL LISTING*

* Cross-sectional betas are based on regression (4.1) in the text

** Predicted values are based on the estimated relationship between the observed cross-sectional betas and the inverse of time (see Panel A of Table 7 for the estimated parameters)
FIGURE 5

HYPOTHESIZED IMPACT OF AN INFORMATION RELEASE AT TIME $t^*$ ON BETA

Panel A

Panel B

Panel C

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