COMPUTER IMAGE BASED SCALING OF LOGS

by

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B.A.Sc.(EE), The University of Toronto, 1984

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTERS OF APPLIED SCIENCE

in

THE FACULTY OF GRADUATE STUDIES
ELECTRICAL ENGINEERING

We accept this thesis as conforming
to the required standard

THE UNIVERSITY OF BRITISH COLUMBIA
October, 1987

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Abstract

Individual log scaling for the forest industry is a time consuming operation. Presented here are the design and prototype test results of an automated technique that will improve on the current speed of this operation, while still achieving the required accuracy. This is based on a technique that uses a television camera and graphics monitor to enable the operator to spot logs in images, which an attached processor can automatically scale. The system must be first calibrated however. Additional to the time savings are the advantages that the accuracy will be maintained, if not improved, and the operation may now be performed from a sheltered location.
## Contents

Abstract

List of Tables

List of Figures

Acknowledgements

1 Introduction

2 Overview of the Problem and Related Work

2.1 Log Scaling Background

2.2 Problem Formulation

2.3 Practical Considerations

2.4 Related Works

2.4.1 Scope of the Survey

2.4.2 Modelling

2.4.3 Information Acquisition Techniques

3 Development of the Scaling Method

3.1 Camera Modelling

3.2 Camera Calibration
3.3 Log Recognition ........................................... 35
3.4 Log Scaling Calculations ................................. 35
3.5 Sources of Error ........................................... 48
3.6 Simulation ............................................... 49
3.7 Simplifying Assumptions ................................. 52
3.8 Advantages and Disadvantages of the System ......... 54

4 Experiments .................................................. 56
4.1 Equipment ................................................. 56
4.2 Calibration .................................................. 57
  4.2.1 Calibration Procedure ................................. 57
  4.2.2 Calibration Observations ............................. 60
  4.2.3 Calibration Results ................................... 61
4.3 Scaling ....................................................... 68
  4.3.1 Procedure .............................................. 68
  4.3.2 Scaling Results ........................................ 70
  4.3.3 Scaling Conclusions ................................. 78

5 Conclusions .................................................. 80

6 Recommendations ........................................... 82

References .................................................... 87

A Experimental Results - Simulation ....................... 90

B Experimental Results - Calibration ....................... 100

C Experimental Results - Scaling .......................... 124
List of Tables

4.1 Summary of a Sample of the Calibration Test Results ................. 67
4.2 Summary of the Scaling Tests from the Calibration Models - Radius 75
4.3 Summary of the Scaling Tests from the Calibration Models - Length 77
List of Figures

3.1 Pin-Hole Camera Model ................................................. 21
3.2 Two-Plane Camera Calibration ........................................... 22
3.3 Perspective Transformation ............................................. 28
3.4 Image Output Device Transformation ................................. 29
3.5 Log Scaling Geometry .................................................. 37
3.6 Log Scaling Simulation Results ....................................... 51

4.1 Laboratory Scaling Apparatus ........................................... 58
4.2 Mean RMS Error, $\Phi$, for the Calibration Process ................. 62
4.3 Standard Deviation of $\Phi$, $\sigma_{\Phi}$, for the Calibration Process .... 62
4.4 Result Distribution for $\theta_x$ - Test Image .......................... 64
4.5 Result Distribution for $\theta_x$ - Six Images .......................... 64
4.6 Result Distribution for $h_0$ - Test Image ......................... 65
4.7 Result Distribution for $h_0$ - Six Images ............................ 65
4.8 Result Distribution for $Y_0$ - Test Image ............................ 66
4.9 Result Distribution for $Y_0$ - Six Images ............................ 66
4.10 Log Scaling Radius Distribution - Centre of the Image .......... 71
4.11 Log Scaling Radius Distribution - Corners of the Image ........ 71
4.12 Log Scaling Radius Distribution - Corners of the Image ........ 72
4.13 Log Scaling Radius Distribution - Corner Model Fit to the Centre 74
4.14 Log Scaling Radius Distribution - Corner Model Fit to the Opposite Corner ........................................... 74
4.15 Log Scaling Length Distribution - Centre of the Image .......................................................... 75
4.16 Radius Scaling Accuracy - All Tests Combined ................................................................. 76
4.17 Length Scaling Accuracy - All Tests Combined ................................................................. 76

A.1 Simulation Results: Radius Error vs. Log Angle ............... 95
A.2 Simulation Results: Standard Deviation vs. Log Angle .......... 95
A.3 Simulation Results: Radius Error vs. Log Radius ........... 96
A.4 Simulation Results: Standard Deviation vs. Log Radius ...... 96
A.5 Simulation Results: Radius Error vs. Normal Distance ........ 97
A.6 Simulation Results: Standard Deviation vs. Normal Distance ... 97
A.7 Simulation Results: Radius Error vs. Diagonal Distance ....... 98
A.8 Simulation Results: Standard Deviation vs. Diagonal Distance ... 98
A.9 Simulation Results: Radius Error vs. Zoom Limit .............. 99
A.10 Simulation Results: Standard Deviation vs. Zoom Limit ...... 99

B.1 Calibration Results: Convergence of $\Phi$ - Test Image .......... 103
B.2 Calibration Results: Convergence of $\sigma_\Phi$ - Test Image .......... 103
B.3 Calibration Results: $fM_\psi$ Estimation Histogram - Test .......... 104
B.4 Calibration Results: $fM_\psi$ Estimation Histogram - Centre .......... 104
B.5 Calibration Results: $fM_\psi$ Estimation Histogram - Corner .......... 105
B.6 Calibration Results: $fM_\psi$ Estimation Histogram - Total .......... 105
B.7 Calibration Results: $Mratio$ Estimation Histogram - Test .......... 106
B.8 Calibration Results: $Mratio$ Estimation Histogram - Centre .......... 106
B.9 Calibration Results: $Mratio$ Estimation Histogram - Corner .......... 107
B.10 Calibration Results: $Mratio$ Estimation Histogram - Total .......... 107
B.11 Calibration Results: $\theta_e$ Estimation Histogram - Test ............... 108
B.12 Calibration Results: $\theta_e$ Estimation Histogram - Centre ............... 108
B.13 Calibration Results: $\theta_e$ Estimation Histogram - Corner ............... 109
B.14 Calibration Results: $\theta_e$ Estimation Histogram - Total ............... 109
B.15 Calibration Results: $\theta_v$ Estimation Histogram - Test ............... 110
B.16 Calibration Results: $\theta_v$ Estimation Histogram - Centre ............... 110
B.17 Calibration Results: $\theta_v$ Estimation Histogram - Corner ............... 111
B.18 Calibration Results: $\theta_v$ Estimation Histogram - Total ............... 111
B.19 Calibration Results: $\theta_e$ Estimation Histogram - Test ............... 112
B.20 Calibration Results: $\theta_e$ Estimation Histogram - Centre ............... 112
B.21 Calibration Results: $\theta_e$ Estimation Histogram - Corner ............... 113
B.22 Calibration Results: $\theta_e$ Estimation Histogram - Total ............... 113
B.23 Calibration Results: $h_0$ Estimation Histogram - Test ............... 114
B.24 Calibration Results: $h_0$ Estimation Histogram - Centre ............... 114
B.25 Calibration Results: $h_0$ Estimation Histogram - Corner ............... 115
B.26 Calibration Results: $h_0$ Estimation Histogram - Total ............... 115
B.27 Calibration Results: $v_0$ Estimation Histogram - Test ............... 116
B.28 Calibration Results: $v_0$ Estimation Histogram - Centre ............... 116
B.29 Calibration Results: $v_0$ Estimation Histogram - Corner ............... 117
B.30 Calibration Results: $v_0$ Estimation Histogram - Total ............... 117
B.31 Calibration Results: $X_0$ Estimation Histogram - Test ............... 118
B.32 Calibration Results: $X_0$ Estimation Histogram - Centre ............... 118
B.33 Calibration Results: $X_0$ Estimation Histogram - Corner ............... 119
B.34 Calibration Results: $X_0$ Estimation Histogram - Total ............... 119
B.35 Calibration Results: $Y_0$ Estimation Histogram - Test ............... 120
B.36 Calibration Results: $Y_0$ Estimation Histogram - Centre ............... 120
B.37 Calibration Results: $Y_0$ Estimation Histogram - Corner ........ 121
B.38 Calibration Results: $Y_0$ Estimation Histogram - Total ........ 121
B.39 Calibration Results: $Z_0$ Estimation Histogram - Test ........ 122
B.40 Calibration Results: $Z_0$ Estimation Histogram - Centre ....... 122
B.41 Calibration Results: $Z_0$ Estimation Histogram - Corner ....... 123
B.42 Calibration Results: $Z_0$ Estimation Histogram - Total ....... 123

C.1 Scaling Results: Radius Distribution for Image Centre - Test Model 127
C.2 Scaling Results: Radius Distribution for Image Corners - Test Model 127
C.3 Scaling Results: Radius Distribution from All Images - Test Model . 128
C.4 Scaling Results: Radius Distribution for Corner Images - Same Corner Models .................................................. 128
C.5 Scaling Results: Radius Distribution for Centre Image - Corner Models ......................................................... 129
C.6 Scaling Results: Radius Distribution for Corner Images - Opposite Corner Models .............................................. 129
C.7 Scaling Results: Radius Distribution for All Images Combined ... 130
C.8 Scaling Results: Length Distribution for Image Centre - Test Model 131
C.9 Scaling Results: Length Distribution for Image Corners - Test Model 131
C.10 Scaling Results: Length Distribution from All Images - Test Model . 132
C.11 Scaling Results: Length Distribution for Corner Images - Same Corner Models .................................................. 132
C.12 Scaling Results: Length Distribution for Centre Image - Corner Models ......................................................... 133
C.13 Scaling Results: Length Distribution for Corner Images - Opposite Corner Models .............................................. 133
C.14 Scaling Results: Length Distribution for All Images Combined ... 134
C.15 Scaling Results: Radius of Log at 40° Angle to the Image Plane - Test Model ..................................................... 135
C.16 Scaling Results: Length of Log at 40° Angle to the Image Plane - Test Model ............................................. 135
C.17 Scaling Results: Radius of Smaller Cylinder - Test Model ........... 136
C.18 Scaling Results: Length of Smaller Cylinder - Test Model .......... 136
C.19 Scaling Results: Radius of Real Log - Test Model ................. 137
C.20 Scaling Results: Length of Real Log - Test Model ................. 137
Acknowledgements

I would like to thank my supervisor, Dr. Peter Lawrence, for the guidance and encouragement that he provided throughout this project. His technical advice and opinions have been crucial to the outcome of this work.

I would also like to thank the fellow graduate students that provided extremely valuable technical and moral support, in particular James Reimer, Derek Hutchinson, Joe Poon and Nader Riahi.

This research would not have been possible without the financial support of the Natural Sciences and Engineering Research Council.
Chapter 1

Introduction

The forest industry in British Columbia currently employs large, relatively flat fields as sorting yards where, amongst other things, delimbed logs are brought in to be measured. A group of logs are laid out, roughly parallel to each other, on the surface of the sort yard. There are numerous formulae for computing the volume of a log, based on the length and radius, in use in industry today. The process of determining the log volume is called scaling. One process of scaling these logs involves a scaler with a calibrated stick who measures the dimensions of the log.

The measurement of length and diameter of a log is a relatively slow technique and thus the goal of this project was to investigate a more automated process. At the same time, the accuracy (modelling of reality) and precision (repeatability) of the measurements must not suffer.

This problem, as it turns out, is fairly complicated if one turns to computational vision techniques using cameras and signal processing algorithms. The problem is also not very amenable to laser-based or other active sensing techniques. Initially, a survey of possible solutions to this industrial problem is presented.

The second section of this report contains a new design, based on satisfying the problem constraints. This consists of the considerations which led to the design equations, and those decisions which were made about the actual process and the
human interface. A simulation was performed of the situation, using some idealizations. This simulation worked well enough to warrant testing of a prototype.

The final major section of the body of this report describes the experimental work that was performed with a laboratory prototype in order to verify design theory, justify the maintenance of accuracy with a substantial time savings, and provide insight into future work on this topic. In addition, this insight led to a section of practical recommendations based on this 'hands-on' experience.

Appendices are also included which contain more complete listings of the results determined experimentally.
Chapter 2

Overview of the Problem and Related Work

2.1 Log Scaling Background

In the logging industry, sort yards are used as intermediate destinations for cut trunks on their way to the mills. These large and relatively flat fields are generally located near water so that the logs may be brought in by truck or water (without limbs, but with bark) in order to be bundled into booms for water-based shipment to a saw-mill. In these sort yards, each individual log is graded according to its quality and species so that proper milling may take place [37]. Further, the sort yard is required to measure the volume of the logs in order to calculate payments (to the Crown and logging contractors) and as an industrial measurement technique (for monitoring resource flow, as well as the unit productivity)[29]. Thus, the process becomes one of tremendous financial importance when considering the value of annual production in British Columbia from this primary industry.

Under the current system, log volumes are measured either by weighing whole truckloads and making gross assumptions about the uniformity of the load moisture content and species, or by the more accurate method involving individual log, stick scaling [29]. In yards implementing this latter technique, qualified log scalers actu-
ally measure the individual pieces with the aid of a graduated stick (1-2 m long). While the former method is quicker and statistically correct over a long period of time, it does lead to errors due to the assumptions being made [29].

The individual log scaling technique provides more accurate values for the diameters of the end faces and the length. However, this process is seen as being somewhat time consuming and a bottle-neck to the overall productivity of the logging industry [29]. As the size of the individual pieces being harvested drops in attempts to stretch productivity in areas of diminishing returns, this bottle-neck becomes even more predominant, as more scaling will be required to process similar volumes.

2.2 Problem Formulation

With the foregoing discussion in mind then, it becomes immediately apparent that the alleviation of the log scaling time bottle-neck without a loss in accuracy would be a valuable contribution to the industry. To be more technically specific, it is desired to find, by means of automation, a method of determining the length and radius (or diameter) of a log. The ground, which is going to be composed of grass, dirt and bark, will be of varied colour, but may be assumed to be planar. The logs themselves may be modelled as tapered cylinders (hence of a characteristic length and radius) in order to extract the volume measurement. No rigorous surface or volume calculation is requested as the formulae currently employed to calculate the size of the log are dependent on length and diameter measurements only. This is not to say that the volume could not be calculated more directly, just that, by the current industrial standards, no need exists.

As described in the “Forestry Handbook of B.C.” [37], there are primarily two scaling formulae used. The first of these is the B.C. Cubic Scale, which measures “the actual solid wood contents of a log in cubic feet without deducting for slabs, edgings, or saw kerf” [37]. This is a fairly straightforward method that requires the
two end diameters and the length:

\[ V = \frac{(A_1 + A_2)}{2} l \]  \hspace{1cm} (2.1)

where:

- \( V \) is the log's cubic scale volume
- \( A_1, A_2 \) are the end face areas determined from \( d_1, d_2 \)
- \( l \) is the length of the log, measured to within 10 cm
- \( d_1, d_2 \) are the two end face diameters, measured inside the bark to the nearest inch (2.54 cm)

The other commonly used scale is the B.C. Board Foot Scale, which measures "the number of inch-thick boards that may be cut from logs of various lengths and diameters" [37]. This one is not as intuitively appealing a measure, but rather is more practically oriented. The formula for it is correspondingly more complex.

\[ V = \frac{\pi}{4} (D - \frac{1}{2})^2 l \left( \frac{1}{12} \frac{8}{11} \right) \]  \hspace{1cm} (2.2)

where:

- \( D \) is the top diameter, measured inside the bark to the nearest inch

There are some important facts that should be noticed from the above two formulae. Both volume calculations are based on the length and the square of the radius, as they are cylindrical-type modellings. The first one is a tapered cylinder, while the second formula makes no allowance for taper in logs of up to forty feet in length. For the purpose of automating log scaling, the outside diameters could be measured and a constant correction for the bark could then be subtracted.

Finally, it is noticed that while the length accuracy is not very stringent, that of the diameter, which is both easier to measure and more dominant in the volume
formulae, is required to within one inch. While the diameters are used here for ease of measurement, these formulae could both be thought of in terms of the end face radii instead, if this should prove to be a more relevant way of thinking. The accuracy desired would then become one-half inch (1.27 cm) for this measurement.

2.3 Practical Considerations

Even without any preconceptions as to what form of automation might be employed for the log scaling process, several practical considerations should be stated.

Log sorting yards tend to be relatively busy places. A great deal of heavy machinery is constantly on the move as shipments of logs are being simultaneously brought in, scaled, graded and parcelled for shipment to the saw mill. This makes for work conditions where each individual involved must be constantly on the alert for the safety of themselves and everyone else around. The log scalers are no exception to this. It is probable that automating their task, and possibly also removing them from the scene (on foot) at least part of the time, would prevent inaccuracies in measurements due to haste or distraction. Someone will still be required to grade and mark the logs on foot, but without the burden of a measuring stick. Thus, an ideal automation scenario would see the log scaling operator located away from traffic (and the natural elements as well).

The log scaling apparatus itself should preferably be a single unit device. It should be no more obtrusive a factor in the sort yard than the scalers on foot currently are and, in fact, it would be a great advantage to everyone involved if it could be placed somewhere out of the way or attached to existing equipment.

Finally, the log scaling apparatus should be sturdy and accurate enough to withstand weather elements (rain, snow, fog, wind). It would be advantageous if the device was easily protected, both while in use and when not in use. Further, it should ideally have little in the way of moving parts.
2.4 Related Works

2.4.1 Scope of the Survey

A survey of works that might provide useful direction for solving this problem is a wide ranging one, encompassing the fields of computer science, electrical and civil engineering, as well as forestry. While the goal of the project is to contribute a practical method of solving an industrial problem, there is theoretical literature of equal importance in assessing its possible impact on the solution.

The first step in the analysis of this problem is that of bringing the real world into a more digestible form by modelling. While some of this was given already in the problem formulation, a statement and previous work in this field is initially reviewed here.

Next, the crux of the problem, that of sensing the input data is attacked. Literature is surveyed on various means of acquiring the information necessary to produce the log dimensions. Sensors may be categorized as either active or passive [15]. Active sensors rely on being able to transmit and receive a beam of energy (light, sound, etc.), where the desired information is somehow encoded in the relation between the two beams. Passive sensors rely solely on ambient conditions in order to derive information.

2.4.2 Modelling

As was mentioned in Section 2.2, for the purpose of accounting for the amount of usable product that passes through a log sort yard, logs are assumed to be fundamentally cylindrical. From there, different formulae are based on different assumptions about the nature of the departure from this ideal modelling and the amount of wood that is actually usable within the derived volume.

Many computational vision techniques have been proposed for the extraction of
three dimensional surfaces from two dimensional images. Among these is a modelling system, very close to that suggested above, based on what is known as a generalized cylinder. This technique was first proposed by T.O. Binford [3] in 1971 and may also be referred to as that of generalized cones [18,2].

One of the earliest applications of generalized cylinders was implemented by Agin and Binford in 1973 [1]. In their experimental apparatus, a laser beam was deflected through a glass rod to achieve a horizontal beam of light incident on the subject. Using a television camera to spatially sample the scene and the process of triangulation, the scene was scanned and non-zero brightness points were stored. This data was processed to perform line detection and the axes of the generalized cylindrical representations were extracted from the image. When the software had located a part of a cylinder, it created a characterization of the cross-section there and attempted to extend the cylinder in both directions. They proposed the scenario where the outline was defined as:

\[ RADIUS(n) = RADIUS(0) + M \cdot n \]  

(2.3)

where \( RADIUS(0) \) and \( M \) are parameters of the function, and \( n \) corresponds to the order of points along the axis” [1].

This could very easily be a modelling equation for a log, seen to be the frustum of a cone. In practice, the algorithm utilized to trace out each possible cylinder proceeded initially to arrive at cross-sectional planes normal to the points on the axis. Then, for each plane, points were located on the actual surface of the object and the diameter of each resulting cylindrical slice was calculated. A linear radius function was then fit to the set of values obtained as the diameters. Conical cross-sections resulting from this function were then retro-fit to the surface data to justify the validity of the original fit. Possible problems arose when the curve fit to the diametric data was not very good, as this tended to become a divergent process. The ends of the cylinder were particularly susceptible to this problem and they were, therefore, tested appropriately [1].
In actual operation, this system required some operator control. For example, while all of the initial cylinder groupings and rankings (according to the length-to-width ratios for their likelihood of being cylindrical) were automatically performed, the operator must still specify which of these groups are to be analyzed further. The cylinder-tracing algorithm presented above was then run and the operator was required to decide on the success or failure of the operation.

The above work was also utilized as an input medium for some work on “Structured Descriptions of Complex Objects” by Nevatia and Binford [25] in order to develop specific object recognition techniques for robotic applications.

Generalized cylinders may be used to represent or approximate a three-dimensional object or any part thereof. As an example of their applicability, generalized cylinders may be used for a hierarchical recognition system for some three-dimensional objects characterizing say, the human body, with the individual models being used for each of the head, trunk, arms and legs [19]. This sort of recognition was also performed on artificially generated, three-dimensional data by Soroka and Bajcsy [32]. The end goal of their work was tomographic reconstruction however. Another three-dimensional, model-driven recognition system was developed by Shani in 1981 [28]. In this system, generalized cylinders were used as geometric models for the abdominal anatomy.

Model descriptions for generalized cones may take on a few different forms when parameterized. One such representation, that of a linear radius function, has already been presented. This is a relatively simple reduction of information done by assuming a circular cross-section that is perpendicular to the axis. While it is not necessary to restrict oneself to thinking solely of a circular template sweeping out the volume of a generalized cone, it is certainly the simplest and most mathematically tractable. For the purpose of log scaling, this reduction in information is both attractive and consistent with current practice. The simplest case of this linear radius model is obviously that of a constant radius cone - a cylinder. The radius of the cross-section does not necessarily have to be an analytic function at all,
however. With the example of log scaling, it could conceivably become a sampled function of arc length along the principle axis.

It also becomes necessary to describe the individual object's principle axis. One choice would be as three, linearly independent functions of the arc length, such as:

\[
\vec{a}(s) = \begin{pmatrix} x(s) \\ y(s) \\ z(s) \end{pmatrix}
\]

(2.4)

where:

- \( s \) is the arc length along the principle axis
- \( \vec{a}(s) \) is the object's principle axis [2]

In this manner, a Cartesian coordinate system is defined and the projection function of the object's cross-section is determined along each of the axes. Further, this idea can expand to include, by definition, the ability for the cross-sectional template (and principle axes) to rotate as it moves through space, as is the case with a screw.

In contrast, the simplest case of axis representation involves a linear assumption [36]. For this situation, it may then be characterized by means of a point in the image plane, an angle relative to the horizontal (\( \delta \)) and an angle relative to the line of sight (\( \sigma \)). These may be found by various fitting techniques dealing with such things as the symmetry of the surface normal data about a proposed axis. In a work by Walker and Kanade, the object's coordinates were represented by two parameters: \( s \) and \( t \). \( s \) is defined to be the normalized distance along the cylinder's axis and \( t \) is defined to be the normalized distance (angular, in the right-handed sense) around the axis in the normal plane, starting at the point on the far side of the cylinder that is coplanar with both the axis and the line of sight [36].

A somewhat different manner of parametrically representing the principle axis is that of a "Frenet frame". This system uses points along the object's axis as the
origins of local coordinate systems and defines these coordinate systems with the aid of three orthogonal unit vectors: $\xi$, $\nu$ and $\zeta$. $\xi$ is the unit vector that is tangent to the principle axis, $\nu$ is defined to be in the direction of the centre of curvature, and $\zeta$ is called the centre of torsion of the axis. While these three vectors are a more meaningful description of the axis' activity in three dimensions, they do have their drawbacks. For one, the centre of torsion of the axis is determined as the vector which is orthogonal to both the tangent and the radius of curvature. When the curvature approaches zero (which is a quite common occurrence), the torsion is ill-defined. Further, if one is to allow the cross-sectional template to twist as it passes along the axis, then an additional parameter is required to deal with this [2].

On the positive side for the use of generalized cylinders is their modularity when being used to represent a portion of an object. In addition, cross-sectional area and object volume may be analytically derived from the extracted model.

There are further problems not yet mentioned that arise from this technique however. If the axis of the cylinder curves sharply, then the volume and surface area calculations will be in error due to an overlap of successive slices. Further, if one must match the cylinder derived from the image to a model, then the axis should, in general, be parallel to the image plane for optimal shape extraction. Should any other orientation occur, then a foreshortening effect will result. The perspective projection of objects onto an image plane causes those farther away to be smaller than those which are closer. This can result in the far end of the log appearing smaller than the near end. Additionally, the overall length of the projection varies and will be maximized only when the log axis is parallel to the image plane. This could possibly be detected from clues provided by a cooperative process [20], but in general, will lead to an ambiguity in the image to three-dimensional model transformation.
2.4.3 Information Acquisition Techniques

As has been mentioned, data acquisition techniques for this problem may be grouped into two categories: active and passive [15]. Active techniques are those which rely on being able to receive a transmitted beam of energy from the log target, the desired information being somehow encoded in that beam. This includes methods utilizing lasers, structured lighting and ultrasound. On the other side of these are techniques which depend solely on ambient conditions. Cameras are the most common transducers for this situation, possibly also utilizing optical filters.

Active sensing is not new to the logging industry. Lasers have been used in saw mills to determine the diameter of a log moving along a conveyor belt [10,35]. This is done by detecting a broken beam of laser-generated light as the log passes between the source and the detectors. The reason that this can be done is that the conveyor is located in a controlled industrial environment where the geometry of the measuring system can be fixed and the laser light intensity may be readily detected amid the ambient light. These conditions, primarily the latter, impose a severe restriction on most light-oriented, data acquisition systems. However this restriction is not so severe that lasers have not been a popular data acquisition tool for other tasks in the past. A good treatment of their applications to various industrial tasks may be found in Harry [11] and Maurer [22].

Civil engineering has seen lasers used in the outdoors with great success [11,30,27,38]. For surveying purposes, electronic distance metering equipment (laser range finders) are available that are able to measure distances that typically run up to ten kilometres with a standard deviation of 5mm + 5ppm. In addition, they may be mounted on highly accurate optical or electronic theodolites in order to determine the angles of the measurements. If measurements of this type could be made in a sort yard, then any linear dimensions of the logs could be extracted from two distance measurements and the angle between them. Unfortunately, it is not that simple. The bulk of this equipment is designed such that the laser beam is reflected off of a flat prism face (or a bank of prisms for farther distances) and
received at the source. In this manner, there is sufficient received power to isolate the monochromatic (usually infrared) light from the broad-spectrum, atmospheric light.

Laser range finders operate based on a number of different techniques depending on the application and measuring distance. For short, high-accuracy work, such as machine tool calibration, interferometers are used to receive the incident light (directly or off of a mirror) and determine the distance measurement [13]. This is highly unsuitable for the log sort yard, as it would be far too labour intensive to spot each measured point with the receiver.

For longer distances, where the error may be of a higher absolute value, the most common technique, as with the surveying equipment, is to modulate the outgoing light intensity and perform phase detection on the received signal [15,11]. The difficulties, as were alluded to above, are that the received power will be insufficient off of a log target and that it is too time consuming to place a reflector at each of the desired measurement points.

A further long distance technique that has been implemented involves the measurement of the time of flight of a pulse of laser light off of an adequately reflective target [11,15]. This is known as lidar and requires highly sensitive and fast photodetectors combined with state-of-the-art electronics to be able to find the return pulse and accurately measure the time lag amid pulse dispersion and ambient light noise. In addition to this, the same limitations occur as were mentioned for the phase modulation scheme.

Laser range finders have been used many times before in controlled environments for the purpose of generating a range image, where each element of the image is a range value rather than a light intensity level. [26,15].

With the case of a known scanning-laser position and a planar lateral effect photodiode, range images may also be calculated by triangulation [9].
Laser range finders offer the possibility of a variety of techniques for determining individual distances by active sensing. Still, problems exist with the use of this technology for this application. The logs (and their background of grass and mud) are very poor reflectors and will not return sufficient power to overcome ambient light. To place a reflector at each desired measurement location would be far too labour intensive. In addition, while the low-end laser range finder is relatively inexpensive, it would also be ineffective. The increases in cost towards more sophisticated equipment are quite substantial if the unit is to be outfit with scanning mirrors, planar photodiodes, time of flight measuring electronics, or any other equipment that would be required to perform the actual task.

Other active methods of range finding have also been used in industrial situations. A more common of these for eyesight range measurements is ultrasound. However, ultrasonic beams are not directive enough to be able to make the pinpoint measurements [15,31].

Structured lighting provides another example, where the use of a light pattern, such as stripes [1,2,14,39] can provide sufficient depth and orientation clues to a television camera for a post-processor to deduce the desired information.

In light striping, a plane of light is scanned across the scene that the camera is viewing and the illuminated image coordinates for each scan position are stored. As the three-dimensional coordinates of the light plane are known, and each of these stored image coordinates transforms to a line of sight in the real-world, the intersection of the above information leads to each of the real-world points on the surface of an object shown in the image. This is a triangulation technique and the distance between the source and camera is referred to as the baseline. While each stripe also adds surface continuity information to the image, problems arise where concavities in the image occur. If the concavity is deep enough, the camera and the light source will not both be able to see into it, however this is not a severe problem in dealing with cylinders. This technique is also less expensive than most of the other active sensing schemes. The limiting factor here however, as with the lasers,
is that the structured light will not be visible enough to the camera in daylight.

Much the same could be said of any variation on the above structured lighting technique. The feasibility of these were also rejected by Clark [6], in dealing with the log scaling problem.

Passive data acquisition techniques offer alternatives that should be more feasible for an industrial problem that is out of doors in a log sort yard environment.

The volume scaling of stacked pulpwood was automated by Miller and Tardiff, who photographed end views of uniform length, evenly piled pulpwood [12]. This photograph was viewed with a 729 line television camera. Binary thresholding was then applied to the camera image. This is the technique of classifying those pixels as '1' that possess a grey-scale (intensity) value above some predetermined value, and '0' otherwise. Following this, the number of '1' pixels, indicating the presence of a log end face at that point in the image, was electronically counted to arrive at a volume figure. This was estimated to be within 2% for these uniform loads.

One study on automating the log scaling process was performed by Demaerschalk, Cottell and Zobeiry in 1980 [7]. With the use of a two-camera, data acquisition system, their technique involved photographing an end and a side view of the logs while still on the truck. They investigated a system to improve on weigh scaling by enlarging these two views and measuring the logs directly from the images. In this manner, weigh scaling could be replaced by stick scaling a sample of incoming logs and applying a regression technique to estimate the total volume and species volume. This study concluded that the efficiency of total volume prediction could be increased thus.

A very similar problem was studied by Clark [5,6], who investigated the possibility of using stereo vision techniques to automate the stick scaling process. In his 1985 Ph.D. dissertation, he provided a theoretical design of a system based on the Marr-Poggio stereo matching algorithm [19,20].
In this system, he sought to provide a method of edge detection in order to derive the outline of the logs for recognition and subsequent measurement purposes. A basic difficulty which complicated this problem is that the surface which the logs are lying on (the background in a camera image) is going to be open ground which will be composed of mud, wood pieces, etc. This is unpredictably close in appearance to the actual logs themselves. Binary thresholding failed for the reason that the logs did not have a uniform distribution, but rather are highly textured.

He too precluded the ideas that involved control over either the lighting on the logs or the characteristics of the surfaces of the logs (or their background), as these techniques were not practical for the open environment of a logging sort yard.

The conclusion eventually reached was that a computational vision method of edge detection was required for this task. This is a process whereby the occluding contours of objects in the image are extracted, thus reducing the scene to a simpler line drawing with more tractable information for this end goal. In aid of this technique is the fact that logs are simply connected (i.e., they can be assumed to contain no holes). It was discovered from there that the problem was not all that simple because the two-dimensional spatial filtering operator (a Laplacian of a Gaussian - $\nabla^2G$) was producing many extraneous edges (corresponding to zero-crossings in the filtered image). This filter was chosen because it performs both a derivative function on the image, thus extracting sharp discontinuities (the Laplacian), and an adjustable bandpass function, thus providing it with a tunable edge frequency analyzer (the Gaussian). It has been argued that the $\nabla^2G$ filter, which is a rotationally symmetric function, is an optimum choice for this purpose [19].

It was at this point that the concept of stereo vision entered into the picture, as it may take advantage of the rich image texture of both the logs and the irregular background to assemble disparity maps of the two-dimensional images. Stereo vision uses a triangulation technique, where identical points in two images are matched and their displacement relative to each other (the disparity) is taken as inversely proportional to the normal distance from the cameras to the world point. The
surfaces of the logs are closer to the cameras than the surface that the logs are lying on, thus enabling the occluding contours to be readily extracted from the depth values (by thresholding these values). While the depth values acquired may also be determined by means of other commonly used industrial methods, these have been pointed out to require an environment that may be tightly constrained.

For Clark's proposal, experimental apparatus consisting of two television cameras suspended from a horizontal track above the ground was set up. The dimensions of the apparatus involved a trade-off between having the image disparities large enough to provide a useful measurement, and small enough to allow the matching algorithm to succeed. While it was intended to use a hierarchical system of filtering (four different scales for the filters to analyze in succession, coarse to fine), only one level was implemented. In fact, it was recommended that scale space techniques, which are those implementing a continuously ranging filter scale factor, would remove false stereo matching and allow for the optimum accuracy.

Following the disparity map thresholding, a filling in technique was used to eliminate the holes in the log image and isolate the occluding contour of the target object. This was done by scanning the thresholded image and setting all pixels to one that have at least one pixel on both sides (within a fifteen pixel range) that is set to one. This scanning was repeatedly run for five different angular directions between zero and ninety degrees.

From this occluding contour and an application of a discrete version of Green's theorem, the centroid and axes of the ellipse that will produce equivalent moments of inertia were computed. Then, with the assumption that the log's axis is perfectly straight, the major and minor axes of the ellipse were used as the directions along which the length and width were calculated, respectively.

Using this proposed technique, an estimated volume accuracy of 10% was arrived at, although errors as high as 20% were exhibited for some volume calculating formulae. Aside from these general estimates, no mention was given of the actual
values derived for the length and width of the log, and their comparison to reality. This would have de-coupled these measurements and provided a slightly more useful yardstick by which to gauge the accuracy of the technique. This would also have shed more light on whether the assumption about a straight log axis is true or not, although essentially this same assumption is being used in practice with the current manual technique.

A further source of error arises with the use of a stereo technique at all. One of the disadvantages of stereo matching is that it suffers at hidden edges; i.e., those points in the image which are visible to one camera, but not to the other. In this case, there is no stereo match and any attempt to do so will lead to error. This exact situation will occur at the rounded, non-uniform, log edges, as the rays that project from the occluding edges back to the image planes do not actually touch the logs at the points on their surface that are common to both of the images.

Finally, the above algorithm is very computationally intensive, especially if more than one filtering resolution is required, as it currently appears. While it could be sped up considerably with the aid of parallel architecture hardware for the filtering, and a pipelined system of handling each of the filter resolutions simultaneously, this will still quite likely result in a delay for the operator in awaiting the measurements. A further concern is the cost of the required hardware, which would be quite substantial.

Other computational vision techniques exist for object recognition from camera images [20,2,14], based on such things as motion, shading or texture. Some have even designed systems based on a cooperation between several of these techniques in order to identify objects from models. Unfortunately, these are computationally sophisticated and would require expensive hardware to attempt to implement with any realistic turnaround time.

The difficulty with the ideas discussed so far is that they either require a constrained environment in the case of most of the active sensors, or spend too long
performing object recognition in the case of the passive techniques. One method of overcoming these hurdles is to use a passive technique (television camera) which creates a situation where the operator can quickly perform the recognition. This allows for an immediate calculation of the measurements. The next section will lay out a system of this nature, beginning with a mathematical analysis of the transducer itself, the camera.
Chapter 3

Development of the Scaling Method

This chapter presents the theory behind the design of the log scaling system to be proposed. It uses, as its means of data acquisition, a single television camera. As far as the computational power required to do the processing is concerned, only a small amount of on-line calculations are required. The hurdle of object recognition is overcome by prompting the operator for a few easily found points on a graphics monitor.

Initially, the camera will be modelled mathematically in order to determine the relationship between the image plane and the real world. Then, the basic algorithm and physical layout will be described as it was proposed for a simulation. Finally, results of this simulation will be discussed.

3.1 Camera Modelling

The simplest and most commonly used model of a camera is that known as a “pin-hole” model [21,33]. As shown in Figure 3.1, all lines of sight in the pin-hole model pass through a single point (lens centre) before intersecting with the image plane. In the image plane, which is a focal length in distance \( f \) behind the lens
Figure 3.1: Pin-Hole Camera Model

centre, objects in the real world are inverted. For this reason, the situation is generally treated as if the image plane were actually in front of the lens centre by a distance equal to \( f \). Here, objects do not appear inverted, but the world-to-image transformation is the same. The transformation from world coordinates to image coordinates has the effect of shrinking the \( x \) and \( y \) dimensions by a factor equal to the depth, \( z \), divided by the focal length, \( f \).

\[
\begin{align*}
x_c &= \frac{f \cdot x_w}{z_w} \\
y_c &= \frac{f \cdot y_w}{z_w}
\end{align*}
\]

If homogeneous coordinates are being used, this could also be represented by a transformation matrix, however the treatment used here will not describe it strictly as such.
This model requires seven parameters to fully define orientation (three angular degrees of freedom), location (three positional degrees of freedom) and the focal length.

Another model exists that is more general than the above linear model. This is the two plane model [4,21]. Instead of having all of the lines of sight project to a central point, this model associates all image points with a pre-determined line of sight that need not pass through any other point in particular. The two plane model is actually so named because the technique of deriving the lines of sight involves measuring points in each of two planes that correspond to the same image point (refer to Figure 3.2). Not every image point need be measured. Rather, a regular grid of points is calibrated and the remainder of the lines of sight are determined by an interpolation technique. Martins, et. al. [21] did this using linear, quadratic and spline interpolation schemes with results that verified an improvement in accuracy over the strict pin-hole model for a variety of different camera/lens combinations.
This technique was used by Chen, et. al. [4] in 1980 with an 8 x 8 grid of calibration data. They found that, for ten image test points, the reverse process of calculating the two points in the calibration planes that correspond to a given image point (strictly from the interpolation formulae) produced points that, when viewed in the image, differed by an average of 0.2 pixels.

This figure is quite good, however the calibration process involved beforehand is quite extensive and time-consuming. An operator in the field would have great difficulty in achieving it. It could be done in a laboratory, prior to field installation of the equipment, but might then be subject to such factors as temperature change, change resulting from any movement or blows during shipping, and parameter drift with time.

The calibration process itself was carried out with the aid of a robot arm capable of 0.001 inch placement accuracy. If the sampling grid was reduced to only 6 x 6, the average error rose to about 0.6 pixels, and similarly to a full pixel when the grid was 2 x 2. Thus, this process is neither simple nor quick enough for field calibration, although it may become necessary to do it in a laboratory before installation should the pin-hole model not be accurate enough.

3.2 Camera Calibration

The pin-hole model, being the simplest and most commonly used, shall be the one selected for the initial design with the awareness that it is merely a good linearization of the situation. Greater accuracy could possibly be attained at the expense of time, a loss of flexibility and any cost of more precise calibration equipment than that described herein.

The task of rigorous camera calibration using this camera model was performed by Sobel [33]. His system was designed to allow a computer controlled television camera to guide robot manipulators.
In order to calibrate a television camera, one must first be able to analyze all of its internal and external parameters. In addition to the six degrees of freedom specifying the world-coordinate position and orientation of the camera in some reference position, Sobel, described the camera as having variable PAN (rotation about the unrotated, world-coordinate y-axis) and TILT (rotation about the once rotated, world-coordinate z-axis). SWING (rotation about the twice rotated, world-coordinate z-axis) was held constant. This introduced two further parameters to be solved for, as each of PAN and TILT were assumed to be measured by linearly varying potentiometers. In addition, his camera lens centre was specified by three more parameters (a vector from the centre of rotation of the camera to the lens centre) rather than just the three given above (the point), as this point itself will not be stationary while the camera pans and tilts. This brings the total number of parameters determining the external position of the model to be found up to eleven.

The internal geometry of this camera also required derivation. The image reference (centre) coordinates needed to be determined, as did the ratio of the quantization factors \( M_{ratio} = M_x/M_y \) which scaled the x and y values from the image plane to the quantized video output (see Figure 3.4). Finally, both the zoom and the focus of the camera were variable. The product of the focal length and the vertical quantization factor \( fM_y \) was linearly dependent on the potentiometer measuring the focal control \( (k_1(focus) + k_2) \) and hyperbolically dependent on that measuring the zoom control \( (c_1 + c_2/(zoom) \). This led to four more values to be determined, bringing the total up to eighteen for this particular situation.

Analytic equations for each of the horizontal and vertical image coordinates may be derived that are functions of the world coordinates and each of the four potentiometer settings. To determine these equations, which are not linear, the above eighteen parameters had to be determined.

A system of two equations of eighteen parameters each should theoretically require nine data sets, each consisting of the real-world coordinates of a point, the associated potentiometer readings and the resulting image coordinates. In fact, to
produce sufficient, independent information in order to solve for the parameters, ten data sets spread appropriately over a total of four images were required as a minimum.

Since the model used for the camera is a simplification of reality, the more information that can be applied to the optimization of this model via its parameters, the better. It is best to provide enough calibration data to not only solve for the required parameters, but also to exercise as much of the input (world, image and potentiometer reading) space as possible in doing so. In this way, a regression technique will better fit the model to reality.

In reconsidering the original problem of the log sort yard, an additional factor also becomes important in the design of a camera-based system. For the system to remain flexible, it should be possible to quickly and easily calibrate it in a field situation, by an operator with a minimal of training. While the above model calibration may still seem quite daunting, it will be shown to be feasible with the aid of certain simplifications.

For each world coordinate point located, a transformation, $P$, is involved that converts from world coordinates to camera coordinates [12]. This may first be described in homogeneous coordinates as rotations about the $y$-, $x$- and $z$-axes, in that order (analogous to the operations of pan, tilt and swing). The rotation angles of the transformation are the negative values of those angles, $\theta_z, \theta_y, \theta_x$, that describe the orientation of the camera in world coordinates. $\theta_x, \theta_y$ and $\theta_z$ are derived in the real-world as the angles that are required to map the world-coordinate axes onto the orientation of the camera coordinate axes.

\[
P' = Rot_y(-\theta_y) \cdot Rot_z(-\theta_z) \cdot Rot_x(-\theta_x)
\]

\[
= \begin{pmatrix}
\cos \theta_y & -\sin \theta_y & 0 \\
0 & 1 & 0 \\
\sin \theta_y & \cos \theta_y & 0
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta_z & \sin \theta_z & 0 \\
0 & -\sin \theta_z & \cos \theta_z & 0
\end{pmatrix}
\begin{pmatrix}
\cos \theta_z & \sin \theta_z & 0 & 0 \\
-sin \theta_z & \cos \theta_z & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\] (3.3) (3.4)
\[ P' \] is the rotation transformation portion of \( P \), the world-to-camera transformation matrix

\[ Rot_i(\theta_j) \] is a homogeneous, rotation transformation matrix, describing a right-handed rotation, \( \theta_j \), about the \( i \)-axis \([12,2]\)

Following this, a translation transformation is applied in order to shift the coordinates to a camera frame of reference. The translation components consist of the negative of the camera's world coordinates. This makes the entire homogeneous transformation equal to the following:

\[
P = P' \cdot \text{Trans}(-X_0, -Y_0, -Z_0) = Rot_y(-\theta_y) \cdot Rot_z(-\theta_z) \cdot Rot_z(-\theta_z) \cdot \text{Trans}(-X_0, -Y_0, -Z_0)
\]

\[
\begin{bmatrix}
\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
-X_0 & -Y_0 & -Z_0 & 1
\end{array}
\end{bmatrix}
\]

where:

\[
\begin{align*}
P &= \begin{pmatrix}
cos\theta_y & sin\theta_y & 0 & 0 \\
0 & cos\theta_z & sin\theta_z & 0 \\
-sin\theta_y & -cos\theta_y & cos\theta_z & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\end{align*}
\]

\[
\begin{pmatrix}
cos\theta_z & sin\theta_z & 0 & 0 \\
-sin\theta_z & cos\theta_z & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}
\]

\[
\begin{align*}
P' &= \begin{pmatrix}
cos\theta_y & sin\theta_y & -cos\theta_z & sin\theta_y & 0 \\
0 & cos\theta_z & sin\theta_z & 0 & 0 \\
sin\theta_y & -cos\theta_y & cos\theta_z & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
&= \begin{pmatrix}
cos\theta_y \cos\theta_z - sin\theta_z \sin\theta_y \sin\theta_z & cos\theta_y \sin\theta_z + sin\theta_z \sin\theta_y \cos\theta_z & -cos\theta_z \sin\theta_y & 0 \\
-sin\theta_z \cos\theta_z & cos\theta_z & 0 & 0 \\
\sin\theta_y \cos\theta_z + \sin\theta_z \cos\theta_y \sin\theta_z & \sin\theta_y \sin\theta_z - \sin\theta_z \cos\theta_y \cos\theta_z & \cos\theta_z \cos\theta_y & 0 \\
0 & -X_0 & -Y_0 & -Z_0 & 1
\end{pmatrix}
\end{align*}
\]

\[ (3.5) \]

\[ (3.6) \]

\[ (3.7) \]

\[ (3.8) \]

\[ (3.9) \]

\[ (3.10) \]
\( X_0, Y_0, Z_0 \) are the camera's \( x, y, z \) coordinates in the world frame of reference.

\( \text{Trans}(d_x, d_y, d_z) \) is a homogeneous transformation matrix producing a translation by the vector \((d_x, d_y, d_z)^t\) [12,2].

For the purpose of deriving analytic expressions with a minimal number of parameters, the above homogeneous relations were expanded out when the complete system transformation equations were required:

\[
\begin{pmatrix}
  u \\
v \\
t \\
1
\end{pmatrix} = \begin{pmatrix} X & Y & Z & 1 \end{pmatrix} \cdot P \\
= \begin{pmatrix} X & Y & Z & 1 \end{pmatrix} \cdot \text{Rot}_y(-\theta_y) \cdot \text{Rot}_z(-\theta_z) \cdot \text{Rot}_z(-\theta_z) \cdot \text{Trans}\left(-X_0, -Y_0, -Z_0\right)
\]

(3.12)

\[
u = X(\cos \theta_y \cos \theta_z - \sin \theta_x \sin \theta_y \sin \theta_z) + Y(-\cos \theta_x \sin \theta_z) + Z(\sin \theta_y \cos \theta_z + \sin \theta_x \cos \theta_y \sin \theta_z) - X_0
\]

(3.13)

\[
v = X(\cos \theta_y \sin \theta_z + \sin \theta_x \sin \theta_y \cos \theta_z) + Y(\cos \theta_x \cos \theta_z) + Z(\sin \theta_y \sin \theta_z - \sin \theta_x \cos \theta_y \cos \theta_z) - Y_0
\]

(3.14)

\[
t = X(-\cos \theta_z \sin \theta_y) + Y(\sin \theta_z) + Z(\cos \theta_z \cos \theta_y) - Z_0
\]

(3.15)

where:

\( u, v, t \) are the \( x, y, z \) coordinates in the camera-based coordinate system.

\( X, Y, Z \) are the \( x, y, z \) coordinates in the world coordinate system.

\( \theta_x, \theta_y, \theta_z \) are the rotation angles that transform the world coordinate axes into the camera's orientation.

\( X_0, Y_0, Z_0 \) are the translations required to shift the world coordinate origin onto that of the camera.
Left-handed coordinate systems were used for both of the coordinate systems in order to keep the depth, \( t \), in the camera-based coordinates positive, while avoiding having to invert one of the axes in the transformation.

Next, a transformation exists that maps a point in the camera-based coordinate system to the image plane via its line of sight. This is the perspective transformation:

\[
\begin{align*}
    u' &= (\frac{f}{t})u \\
    v' &= (\frac{f}{t})v
\end{align*}
\]

where:

\( u', v' \) are the \( x \) and \( y \) image transducer plane coordinates, as shown in Figure 3.3.
Figure 3.4: Image Output Device Transformation

The output image from the system is not a continuous image, but rather a spatially-sampled graphic. This leads to one more transformation, as shown below and in Figure 3.4

\begin{align*}
h &= M_x u' + h_0 = M \text{ratio}(f M_y) \left( \frac{u}{l} \right) + h_0 \quad (3.18) \\
v &= M_y v' + v_0 = (f M_y) \left( \frac{v}{l} \right) + v_0 \quad (3.19)
\end{align*}

Thus, from the original world-coordinates, the output image may be calculated if ten parameters are known:

- \(f M_y\), the focal length/vertical quantization factor product
- \(M \text{ratio} = M_x / M_y\), the quantization factor ratio
- \(\theta_x, \theta_y, \theta_z\), the angles that transform the world coordinate axes into the camera coordinate axes
• $h_0, v_0$, the output image coordinates when $u = 0$ and $v = 0$, respectively

• $X_0, Y_0, Z_0$, the camera's location in world coordinates

An advantage of this model is the need for inversion of the world-to-image transformation. By inverting Equations 3.18 and 3.19, $(\frac{h}{t})$ and $(\frac{v}{t})$ may be found from $h$ and $v$. By setting $t$ arbitrarily to unity and reversing the initial translation and rotation, line of sight vector components arising from image points may be calculated.

As can be seen from the analytic expressions derived above, the relations that are to be used to determine the above parameters are quite non-linear. If one thinks of this problem as that of finding the peak (optimum) in a ten-dimensional, multi-modal surface, then the difficulty of the problem can be better understood, and it becomes apparent that a standard least squares technique is not applicable.

In attempting to model the movements of celestial bodies, Gauss was confronted with a similar type of optimization problem in the nineteenth century \[8,17\]. His relations too involved non-linear, trigonometric terms, not unlike those expressions discussed above. The technique derived to solve situations of this nature is named for him - the Gaussian least squares, differential correction, parameter estimation technique.

This is still a least squares regression technique, but operates on a linearizing assumption to overcome the barrier. A standard, least squares technique seeks to minimize the squared error sum directly. This method seeks to minimize the predicted sum of the squared residuals as the parameter estimates are varied. While a more complete description of this (with an example similar to this situation) may be found in Junkins \[17\], a short presentation of the theory, as applied here, will be given, as follows.

It is known that the image measurements may be modelled as a function of the
world data:
\[ \tilde{y} = \begin{pmatrix} h \\ v \end{pmatrix} = \begin{pmatrix} f_h(X,Y,Z) \\ f_v(X,Y,Z) \end{pmatrix} = f(\tilde{\beta}) \]  

where:

\( \tilde{y} \) is the vector of image plane coordinates

\( h, v \) are the horizontal and vertical image plane coordinates, respectively

\( f_h, f_v \) are functions involving the ten camera parameters to be determined

\( \tilde{\beta} \) is the vector of camera parameters

As this system of equations is non-linear, the least squares technique of taking the weighted, pseudo-inverse will not work directly. Instead, it must be first assumed that a reasonably good, starting estimate of the parameters is available. This is no problem for the log scaling situation, as most of the parameters are directly measurable to a reasonable degree of accuracy.

The sum of the squared residuals, \( \phi \), is derived from the difference between the measured image data, \( \tilde{y} \), and that calculated from the transformation equations, \( \tilde{y}_c \):

\[ \phi_c = \Delta\tilde{y}_c^t W \Delta\tilde{y}_c = (\tilde{y} - \tilde{y}_c)^t W (\tilde{y} - \tilde{y}_c) \]  

where:

\( \tilde{y} \) is the vector of measured image coordinates, \( \tilde{h}_i \) and \( \tilde{v}_i \)

\( \tilde{y}_c \) is the vector of corresponding image coordinates calculated from the world coordinate input and the transformation equations, \( f(\tilde{\beta}) \)

\( W \) is a weighting matrix related to the accuracy of the measurements

Using a linearizing assumption (the first term of the Taylor series), the residual vector for that situation where there is a local variation in the parameters, \( \Delta\tilde{\beta} \), about the current estimates, \( \tilde{\beta}_c \), can be predicted:
\[ \Delta \tilde{y}_p = \Delta y_c - A \Delta \beta \]  

(3.22)

where:

\( \Delta y_c \) is the current residual vector

\( \Delta \tilde{y}_p \) is the linearly predicted residual vector

\( A = \frac{\partial f}{\partial \beta} (\tilde{\beta}_c) \) is the partial derivative matrix of the transformation functions with respect to the camera parameters, evaluated at the current estimates for the parameters

Now the situation is exactly analogous to that of standard least squares. With the weighting set equal to the identity matrix, as the measurements are all taken to the same accuracy, the pseudo-inverse situation is derived as follows.

The square of the linearly-predicted residuals is:

\[ \phi_p = (\Delta \tilde{y}_p)^t \Delta \tilde{y}_p = (\Delta y_c - A \Delta \beta)^t (\Delta y_c - A \Delta \beta) \]  

(3.23)

Taking the derivative and setting it equal to zero will minimize this expression:

\[ \frac{\partial \phi_p}{\partial \Delta \beta} = -2 A^t \Delta \tilde{y}_c + 2 A^t A \Delta \beta = 0 \]  

(3.24)

Hence:

\[ \Delta \beta = (A^t A)^{-1} A^t \Delta \tilde{y}_c \]  

(3.25)

Therefore, the predicted sum of the square of the residuals may be minimized with the application of the parameter change, \( \Delta \beta \). This is based on the linear assumption. Turning the above into an iterative process, the parameter estimates may be improved until the model optimum is achieved or the desired accuracy is reached - all provided that the initial parameter estimates are good enough that the first derivatives will lead them to the local optimum that is desired.
Computing the inverse of a matrix is prone to numerical errors [17]. Fortunately, other techniques exist for solving the above equation. One of these is known as Householder reduction. It is a process by which the matrix $A$ is reduced to upper triangular form by means of application of a series of Householder transforms [17].

Back ing up a step, it is clear that (analogous to the standard least square technique) $\Delta \beta$ is a solution of:

$$A \Delta \beta = \Delta y_c$$

(3.26)

where:

$A, \Delta y_c$ are known

$A$ may be reduced to upper triangular form and the same transformations may be applied to $\Delta y_c$, leading to:

$$R \Delta \beta = \bar{C}$$

(3.27)

where:

$R = QA$ is the upper triangular version of $A$

$\bar{C} = Q \Delta y_c$ is the corresponding measurement vector

$Q$ is the lower triangular matrix containing the Householder transforms

Therefore, $\Delta \beta$ may be solved for by simple back substitution, starting with its last element.

To improve upon the estimates for the image centre coordinates before the main optimization process began, a further adjustment was made. This involved adding the mean error of the initial calculated image coordinate estimates back to the values of $h_0$ and $v_0$. In this way, a constant correction can initially be added to the constant offsets in each of the image transformation equations.

$$h_0 \leftarrow h_0 + \frac{1}{n} \sum_{i=0}^{n-1} (h_c(i) - \tilde{h}(i))$$

(3.28)
\[ v_0 \leftarrow v_0 + \frac{1}{n} \sum_{i=0}^{n-1} (v_c(i) - \tilde{v}(i)) \]  

(3.29)

where:

\( n \) is the number of measurements involved in the calibration. Each image point consists of two measurements \((h, v)\)

\( \hat{h}(i), \tilde{v}(i) \) are the measured image coordinates

\( h_c(i), v_c(i) \) are the image coordinates computed from the initial camera transformations

The calibration technique described was then iterated. For each iteration, the value of \( \phi_p \) was tested against a minimum threshold for termination unless an iteration limit was reached. This figure, when divided by the number of measurements, led to an estimate of the convergence behaviour of the model parameters. The parameters thus derived were written out to a disk file for the scaling process to utilize, in determining the inverse camera transforms.

While the value of \( \phi_p \) itself is dependent upon the number of measurements used for the minimization, another value, derived from it, provides a more useful figure of merit. This value, to be called \( \Phi \), is a root-mean-square (rms) error for the coordinate measurements used to fit the model.

\[ \Phi = \sqrt{\frac{1}{n} (\phi_p)} \]  

(3.30)

where:

\( n \) is the number of measurements

The value of \( \Phi \) indicates an rms number of pixels that the model will be in error for the calculation of an image plane coordinate value \((h, v)\), given the actual world coordinates of the imaged point.
3.3 Log Recognition

With a camera that has been calibrated, the problem now posed is that of finding the log in the image. Here is where the operator is required. With the use of a graphics screen display of the camera image, the operator will easily be able to visually discern the log in the image. A light pen may then be used to quickly find a few simple features from which, with the use of the inverse camera transform, a floating-point processor may calculate the desired real-world measurements.

An operator is required for any sort of log scaling implemented because of the grading and species identification tasks. By using an operator to perform quick and simple operations in scaling the log, it can be better guaranteed that the operation is proceeding without error, as the attention will be directed to the task at hand. As a result of having to look only at a graphics screen attached to a remote camera, this operation may be located in a shelter away from any noise and discomfort caused by the weather, and the large machinery operating in the vicinity. This step too should increase the reliability of the measurement process.

By performing the object recognition manually, a tremendous computational burden is alleviated, thus lowering the cost of the system and the time required to make measurements.

3.4 Log Scaling Calculations

The idea, then, would be to arrange it such that the salient features that the operator must locate are both simple to find and small in number. Reverting to the cylindrical model of a log, it would be simplest to have the operator select the four corners of the log’s projection onto the image plane. These points could then be used to derive the length and radius of the equivalent cylinder that would fit between the four lines of sight that result from inverting the camera model transformation.
An algorithm for computing this has been derived and it will now be presented, based on the situation of Figure 3.5.

The two image points that are found on the underside of the log's projection in the image plane correspond to lines of sight that are tangent to the presumed cylindrical shape. These lines form a plane which passes through the camera's lens centre:

\[ A_1 x + B_1 y + C_1 z + D_1 = 0 \]  \hspace{1cm} (3.31)

Similarly the two points on the topside of the log's image projection lead to an upper tangential plane:

\[ A_2 x + B_2 y + C_2 z + D_2 = 0 \]  \hspace{1cm} (3.32)

It is assumed that the ground plane is described as:

\[ A_G x + B_G y + C_G z + D_G = 0 \]  \hspace{1cm} (3.33)

The log description may then be derived as the largest cylinder which will fit between these three planes and truncated at each of the end points.

The unit normal vectors for each of these planes may be derived from their first three coefficients.

\[ \hat{n}_1 = \frac{1}{\sqrt{A_1^2 + B_1^2 + C_1^2}} \begin{pmatrix} A_1 \\ B_1 \\ C_1 \end{pmatrix} \]  \hspace{1cm} (3.34)

\[ \hat{n}_2 = \frac{1}{\sqrt{A_2^2 + B_2^2 + C_2^2}} \begin{pmatrix} A_2 \\ B_2 \\ C_2 \end{pmatrix} \]  \hspace{1cm} (3.35)

\[ \hat{n}_G = \frac{1}{\sqrt{A_G^2 + B_G^2 + C_G^2}} \begin{pmatrix} A_G \\ B_G \\ C_G \end{pmatrix} \]  \hspace{1cm} (3.36)

where:

\[ \hat{n}_1, \hat{n}_2, \hat{n}_G \] are the unit normal vectors for the three planes
Figure 3.5: Log Scaling Geometry
This constraint is applied in order to ensure that all of the vectors point in the “up” direction (positive y-component) in Figure 3.5.

For each of these planes, the points of intersection with the log cylinder form a line. These lines are parallel to the axis of the cylinder and at a distance equal to the radius, \( r \). The vectors from each of these lines to the cylinder axis at normal incidence are:

\[
\vec{n}_1 = r\hat{n}_1 = \frac{r}{\sqrt{A_1^2 + B_1^2 + C_1^2}} \begin{pmatrix} A_1 \\ B_1 \\ C_1 \end{pmatrix} \\
\vec{n}_2 = -r\hat{n}_2 = -\frac{r}{\sqrt{A_2^2 + B_2^2 + C_2^2}} \begin{pmatrix} A_2 \\ B_2 \\ C_2 \end{pmatrix} \\
\vec{n}_G = r\hat{n}_G = \frac{r}{\sqrt{A_G^2 + B_G^2 + C_G^2}} \begin{pmatrix} A_G \\ B_G \\ C_G \end{pmatrix}
\]

These lines may be represented parametrically by three linearly dependent variables, as follows:

\[
\vec{I}_1 = x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + y_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z_1 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\
\vec{I}_2 = x_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + y_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \\
\vec{I}_G = x_G \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + y_G \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z_G \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
\]

where:

each set \( \{x_i, y_i, z_i; i = 1, 2, G\} \), are linearly dependent parameters that describe points along the lines of intersection

\( \vec{I}_1, \vec{I}_2, \vec{I}_G \) are the equations for the lines of intersection between the planes and the cylinder
Knowing the equations of the planes applies one constraint on the above equations, thus:

\[
\begin{align*}
\vec{I}_1 &= x_1 \left( \begin{array}{c} 1 \\ -\frac{A_1}{B_1} \\ 0 \end{array} \right) + z_1 \left( \begin{array}{c} 0 \\ -\frac{C_1}{B_1} \\ 1 \end{array} \right) + \left( \begin{array}{c} 0 \\ -\frac{D_1}{B_1} \\ 0 \end{array} \right) \\
\vec{I}_2 &= x_2 \left( \begin{array}{c} 1 \\ -\frac{A_2}{B_2} \\ 0 \end{array} \right) + z_2 \left( \begin{array}{c} 0 \\ -\frac{C_2}{B_2} \\ 1 \end{array} \right) + \left( \begin{array}{c} 0 \\ -\frac{D_2}{B_2} \\ 0 \end{array} \right) \\
\vec{I}_G &= x_G \left( \begin{array}{c} 1 \\ -\frac{A_G}{B_G} \\ 0 \end{array} \right) + z_G \left( \begin{array}{c} 0 \\ -\frac{C_G}{B_G} \\ 1 \end{array} \right) + \left( \begin{array}{c} 0 \\ -\frac{D_G}{B_G} \\ 0 \end{array} \right)
\end{align*}
\] (3.43) (3.44) (3.45)

Therefore, knowing the equations for the three lines parallel to the log axis and the normal vectors from these lines to it, three equivalent descriptions of the infinite line containing the log's principle axis may be determined.

\[
\begin{align*}
\vec{R}_1 &= \vec{I}_1 + \vec{n}_1 = x_1 \left( \begin{array}{c} 1 \\ -\frac{A_1}{B_1} \\ 0 \end{array} \right) + z_1 \left( \begin{array}{c} 0 \\ -\frac{C_1}{B_1} \\ 1 \end{array} \right) + \left( \begin{array}{c} 0 \\ -\frac{D_1}{B_1} \\ 0 \end{array} \right) + \frac{r}{P_1} \left( \begin{array}{c} A_1 \\ B_1 \\ C_1 \end{array} \right) \\
\vec{R}_2 &= \vec{I}_2 + \vec{n}_2 = x_2 \left( \begin{array}{c} 1 \\ -\frac{A_2}{B_2} \\ 0 \end{array} \right) + z_2 \left( \begin{array}{c} 0 \\ -\frac{C_2}{B_2} \\ 1 \end{array} \right) + \left( \begin{array}{c} 0 \\ -\frac{D_2}{B_2} \\ 0 \end{array} \right) - \frac{r}{P_2} \left( \begin{array}{c} A_2 \\ B_2 \\ C_2 \end{array} \right) \\
\vec{R}_G &= \vec{I}_G + \vec{n}_G = x_G \left( \begin{array}{c} 1 \\ -\frac{A_G}{B_G} \\ 0 \end{array} \right) + z_G \left( \begin{array}{c} 0 \\ -\frac{C_G}{B_G} \\ 1 \end{array} \right) + \left( \begin{array}{c} 0 \\ -\frac{D_G}{B_G} \\ 0 \end{array} \right) + \frac{r}{P_G} \left( \begin{array}{c} A_G \\ B_G \\ C_G \end{array} \right)
\end{align*}
\] (3.46) (3.47) (3.48)

where:

\[
\begin{align*}
P_1 &= \sqrt{A_1^2 + B_1^2 + C_1^2} \\
P_2 &= \sqrt{A_2^2 + B_2^2 + C_2^2} \\
G &= \sqrt{A_G^2 + B_G^2 + C_G^2}
\end{align*}
\]

\(\vec{R}_i\) are the infinite lines containing the log's principle axis

\(A_i, B_i, C_i, D_i\) are the constant coefficients of the known plane equations
$x_i, z_i$ are linearly dependent parameters which describe points along the lines of intersection and the log’s axis

The following equalities must then hold:

\[
x_1 + \frac{rA_1}{P_1} = x_2 - \frac{rA_2}{P_2} = x_G + \frac{rA_G}{G} \tag{3.49}
\]

\[
\begin{align*}
\frac{-A_1}{B_1} x_1 - \frac{C_1}{B_1} z_1 - \frac{D_1}{B_1} + \frac{rB_1}{P_1} &= \frac{-A_2}{B_2} x_2 - \frac{C_2}{B_2} z_2 - \frac{D_2}{B_2} - \frac{rB_2}{P_2} \\
&= \frac{-A_G}{B_G} x_G - \frac{C_G}{B_G} z_G - \frac{D_G}{B_G} + \frac{rB_G}{G} \tag{3.50}
\end{align*}
\]

\[
\begin{align*}
z_1 + \frac{rC_1}{P_1} &= z_2 - \frac{rC_2}{P_2} \\
&= z_G + \frac{rC_G}{G} \tag{3.51}
\end{align*}
\]

Therefore:

\[
\begin{align*}
\frac{-A_1}{B_1} x_1 - \frac{C_1}{B_1} z_1 - \frac{D_1}{B_1} + \frac{rB_1}{P_1} &= \frac{-A_G}{B_G} (x_1 + \frac{rA_1}{P_1} - \frac{rA_G}{G}) - \frac{C_G}{B_G} (z_1 + \frac{rC_1}{P_1} - \frac{rC_G}{G}) - \frac{D_G}{B_G} + \frac{rB_G}{G} \tag{3.52}
\end{align*}
\]

\[
\begin{align*}
x_1 &= z_1 \left( \frac{B_1 C_G - B_G C_1}{A_1 B_G - A_G B_1} \right) + \\
&\quad r \left( \frac{B_1}{P_1} \left( \frac{A_1 B_G - A_G B_1}{A_1 B_G - A_G B_1} \right) \right) \left( A_1 A_G + B_1 B_G + C_1 C_G - GP_1 \right) + \\
&\quad \left( \frac{B_1 D_G - B_G D_1}{A_1 B_G - A_G B_1} \right) \tag{3.53}
\end{align*}
\]

Substituting back into Equation 3.46, the three orthogonal components of $\mathbf{R}_1$ may be solved for with only one parameter:

\[
\mathbf{R}_{1z} = z_1 \left( \frac{B_1 C_G - B_G C_1}{A_1 B_G - A_G B_1} \right) + \\
\quad r \left( \frac{B_1}{P_1} \left( \frac{A_1 B_G - A_G B_1}{A_1 B_G - A_G B_1} \right) \right) \left( A_1 A_G + B_1 B_G + C_1 C_G - GP_1 \right) + A_1 \tag{3.54}
\]

40
Therefore:
\[
\tilde{R}_{1z} = z_1 \left( \frac{B_1 C_G - B_G C_1}{A_1 B_G - A_G B_1} \right) + \\
\frac{r}{P_1} \left( \frac{1}{A_1 B_G - A_G B_1} \right) \left( B_G \left( A_1^2 + B_1^2 \right) - B_1 (G P_1 - C_1 C_G) \right) + \\
\left( \frac{B_1 D_G - B_G D_1}{A_1 B_G - A_G B_1} \right) \tag{3.55}
\]

\[
\tilde{R}_{1y} = z_1 \left( -\frac{A_1}{B_1} \left( \frac{B_1 C_G - B_G C_1}{A_1 B_G - A_G B_1} \right) - \frac{C_1}{B_1} \right) + \\
\frac{r}{P_1} \left( \frac{A_1}{B_1} \left( \frac{B_1}{A_1 B_G - A_G B_1} \right) \left( A_1 A_G + B_1 B_G + C_1 C_G - G P_1 \right) + B_1 \right) - \\
\left( \frac{A_1}{B_1} \left( \frac{B_1 D_G - B_G D_1}{A_1 B_G - A_G B_1} \right) - \frac{D_1}{B_1} \right) \tag{3.56}
\]

Therefore:
\[
\tilde{R}_{1y} = z_1 \left( \frac{A_G C_1 - A_1 C_G}{A_1 B_G - A_G B_1} \right) + \\
\frac{r}{P_1} \left( \frac{1}{A_1 B_G - A_G B_1} \right) \left( -A_G \left( A_1^2 + B_1^2 \right) + A_1 (G P_1 - C_1 C_G) \right) + \\
\left( \frac{A_G D_1 - A_1 D_G}{A_1 B_G - A_G B_1} \right) \tag{3.57}
\]

And:
\[
\tilde{R}_{1z} = z_1 (1) + \frac{r}{P_1} (C_1) \tag{3.58}
\]

In a similar manner, the expressions for the three components of \( \tilde{R}_2 \) may be derived:
\[
-\frac{A_2}{B_2} x_2 - \frac{C_2}{B_2} z_2 - \frac{D_2}{P_2} - \frac{r B_2}{B_G} = -\frac{A_G}{B_G} \left( x_2 - \frac{r A_2}{P_2} - \frac{r A_G}{G} \right) - \frac{C_G}{B_G} \left( z_2 - \frac{r C_2}{P_2} - \frac{r C_G}{G} \right) - \frac{D_G}{B_G} + \frac{r B_G}{G} \tag{3.59}
\]
\[
x_2 = z_2 \left( \frac{B_2 C_G - B_G C_2}{A_2 B_G - A_G B_2} \right) + \\
\frac{r}{P_2} \left( \frac{-B_2}{A_2 B_G - A_G B_2} \right) \left( A_2 A_G + B_2 B_G + C_2 C_G + G P_2 \right) + \\
\left( \frac{B_G D_2 - B_2 D_G}{A_G B_2 - A_2 B_G} \right) \tag{3.60}
\]
The three orthogonal components of $\tilde{R}_2$ are thus:

$$
\tilde{R}_{2z} = z_2 \left( \frac{B_2C_G - B_G C_2}{A_2B_G - A_G B_2} \right) + \frac{r}{P_2} \left( \frac{-B_2}{A_2B_G - A_G B_2} \right) \left( A_2A_G + B_2B_G + C_2C_G + GP_2 - A_2 \right) + \frac{B_G D_2 - B_2 D_G}{A_G B_2 - A_2 B_G} \right)
$$

Therefore:

$$
\tilde{R}_{2z} = z_2 \left( \frac{B_2C_G - B_G C_2}{A_2B_G - A_G B_2} \right) + \frac{r}{P_2} \left( \frac{-1}{A_2B_G - A_G B_2} \right) \left( B_G \left( A_2^2 + B_2^2 \right) + B_2 \left( GP_2 + C_2C_G \right) \right) + \frac{B_G D_2 - B_2 D_G}{A_2B_G - A_G B_2} \right)
$$

$$
\tilde{R}_{2y} = z_2 \left( -\frac{A_2}{B_2} \left( \frac{B_2C_G - B_G C_2}{A_2B_G - A_G B_2} \right) - \frac{C_2}{B_2} \right) + \frac{r}{P_2} \left( -\frac{A_2}{B_2} \left( \frac{-B_2}{A_2B_G - A_G B_2} \right) \left( A_2A_G + B_2B_G + C_2C_G + GP_2 - B_2 \right) \right) - \frac{A_2}{B_2} \left( \frac{B_G D_2 - B_2 D_G}{A_G B_2 - A_2 B_G} \right) \right)
$$

Therefore:

$$
\tilde{R}_{2y} = z_2 \left( \frac{A_G C_2 - A_2 C_G}{A_2 B_G - A_G B_2} \right) + \frac{r}{P_2} \left( \frac{1}{A_2 B_G - A_G B_2} \right) \left( A_G \left( A_2^2 + B_2^2 \right) + A_2 \left( GP_2 + C_2 C_G \right) \right) + \frac{A_G D_2 - A_2 D_G}{A_2 B_G - A_G B_2} \right)
$$

And:

$$
\tilde{R}_{2z} = z_2 \left( 1 + \frac{r}{P_2} \left( -C_1 \right) \right)
$$

Therefore, the equations for the line containing the axis of the cylinder may be reduced to equivalent forms involving only one parameter and the unknown radius.
\[ r: \]

\[ \tilde{R}_1 = z_1 \begin{pmatrix} v_1 \\ v_2 \\ 1 \end{pmatrix} + \frac{r}{P_1} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} + \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} \]  \quad (3.66)

where:

\[ v_1 = \frac{B_1 C_G - B_0 C_1}{A_1 B_0 - A_0 B_1} \]
\[ v_2 = \frac{A_0 C_1 - A_1 C_G}{A_1 B_0 - A_0 B_1} \]
\[ \alpha_1 = \frac{1}{A_1 B_0 - A_2 B_1} (B_G (A_1^2 + B_1^2) - B_1 (G P_1 - C_1 C_G)) \]
\[ \alpha_2 = \frac{1}{A_1 B_0 - A_2 B_1} (-A_G (A_1^2 + B_1^2) + A_1 (G P_1 - C_1 C_G)) \]
\[ \alpha_3 = C_1 \]
\[ \beta_1 = \frac{B_1 D_G - B_0 D_1}{A_1 B_0 - A_0 B_1} \]
\[ \beta_2 = \frac{A_0 D_1 - A_1 D_G}{A_1 B_0 - A_0 B_1} \]
\[ \beta_3 = 0 \]

\[ \tilde{R}_2 = z_2 \begin{pmatrix} v_4 \\ v_5 \\ 1 \end{pmatrix} + \frac{r}{P_2} \begin{pmatrix} \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{pmatrix} + \begin{pmatrix} \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix} \]  \quad (3.67)

where:

\[ v_4 = \frac{B_2 C_G - B_1 C_2}{A_2 B_0 - A_1 B_2} \]
\[ v_5 = \frac{A_1 C_2 - A_2 C_G}{A_2 B_0 - A_1 B_2} \]
\[ \alpha_4 = \frac{-1}{A_2 B_0 - A_2 B_2} (B_G (A_2^2 + B_2^2) + B_2 (G P_2 + C_2 C_G)) \]
\[ \alpha_5 = \frac{1}{A_2 B_0 - A_2 B_2} (A_G (A_2^2 + B_2^2) + A_2 (G P_2 + C_2 C_G)) \]
\[ \alpha_6 = -C_2 \]
\[ \beta_4 = \frac{B_2 D_G - B_1 D_2}{A_2 B_0 - A_1 B_2} \]
\[ \beta_6 = \frac{A_2 D_2 - A_2 D_1}{A_2 B_G - A_2 B_1} \]

\[ \beta_6 = 0 \]

A similar equation could also be derived for \( R_G \), but it is not necessary.

These equations are in the form of a fixed point, which is a function of the constant, \( r \), and a vector. As the vector portions must all have the same orientation and their \( z \)-components are equal to unity, their \( x \)- and \( y \)-components must also be the same. That is:

\[ v_1 = v_4 \quad (3.68) \]
\[ v_2 = v_6 \quad (3.69) \]

By equating two of the three line expression components, \( R_x \) and \( R_z \), an analytic expression for the radius may be derived.

\[ z_1 v_1 + \frac{r \alpha_1}{P_1} + \beta_1 = z_2 v_4 + \frac{r \alpha_4}{P_2} + \beta_4 \quad (3.70) \]

Therefore:

\[ z_1 = \frac{1}{v_1} \left( z_2 v_4 + r \left( \frac{\alpha_4}{P_2} - \frac{\alpha_1}{P_1} \right) + (\beta_4 - \beta_1) \right) \quad (3.71) \]

Also:

\[ z_1 + \frac{r \alpha_3}{P_1} + \beta_3 = z_2 + \frac{r \alpha_6}{P_2} + \beta_6 \quad (3.72) \]

Therefore:

\[ z_1 = z_2 + r \left( \frac{\alpha_6}{P_2} - \frac{\alpha_3}{P_1} \right) + (\beta_6 - \beta_3) \quad (3.73) \]

Equating 3.71 and 3.73 leads to:

\[ z_2 \frac{v_4}{v_1} + \frac{r}{v_1} \left( \frac{\alpha_4}{P_2} - \frac{\alpha_1}{P_1} \right) + \frac{1}{v_1} (\beta_4 - \beta_1) = z_2 + r \left( \frac{\alpha_6}{P_2} - \frac{\alpha_3}{P_1} \right) + (\beta_6 - \beta_3) \quad (3.74) \]

But \( v_1 = v_4 \), therefore:

\[ r \left( \left( \frac{\alpha_4}{P_2} - \frac{\alpha_1}{P_1} \right) + v_1 \left( \frac{\alpha_3}{P_1} - \frac{\alpha_6}{P_2} \right) \right) = v_1 (\beta_6 - \beta_3) - (\beta_4 - \beta_1) \quad (3.75) \]
Therefore:

\[ r = \frac{v_1(\beta_6 - \beta_3) + (\beta_1 - \beta_4)}{v_1\left(\frac{\alpha_2}{P_1} - \frac{\alpha_4}{P_2}\right) + \left(\frac{\alpha_4}{P_2} - \frac{\alpha_4}{P_1}\right)} \]  

(3.76)

This may, by back substitution for some of the symbols, be reduced to:

\[ r = \frac{N}{D} \]  

(3.77)

where:

\[ N = P_2[D_G(A_2B_1 - A_1B_2) + D_1(A_GB_2 - A_2B_G) + D_2(A_1B_G - A_GB_1)] \]

\[ D = (A_2B_1 - A_1B_2)(P_2G + C_2C_G) + (A_GB_2 - A_2B_G)(P_1C_1 + C_2C_1) - (A_1B_G - A_GB_1)(P_2^2 - C_2^2) \]

Thus, with the above formula, the radius may be calculated with merely nineteen floating-point multiplications and ten floating-point additions. Therefore, by picking out the four corners of the projection of a cylinder (log) onto an image plane, the radius of the cylinder may be derived. This assumes the initial knowledge of the inverse camera transformation and the ground plane.

Following the calculation of the radius, the length too must be derived. The infinite line that contains the axis is fully known, with the solution for the radius given above. What remains is to find out where along this cylinder the lines of sight touch.

One of the lines of sight that is tangent to the bottom side of the log may be described by:

\[ \tilde{L}_{11} = \kappa \begin{pmatrix} \gamma_{11} \\ \gamma_{12} \\ \gamma_{13} \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} \]  

(3.78)

where:

\[ \kappa \] is the varying parameter of the line
\( \gamma_i \) are the three, orthogonal direction components of the line of sight corresponding to the selected image point, as derived from the inverse camera transformation

\[ X_0, Y_0, Z_0 \] is the camera’s origin, in world coordinates

Where this line touches the cylinder, \( \kappa \) takes on a fixed (but as yet unknown) value, \( \kappa_{11} \). By projecting along a radial vector of the cylinder, an axial point is reached that determines where this line of sight truncates this cylinder. Here:

\[
\begin{align*}
\kappa_{11} \begin{pmatrix} \gamma_{11} \\ \gamma_{12} \\ \gamma_{13} \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} + \frac{r}{P_1} \begin{pmatrix} A_1 \\ B_1 \\ C_1 \end{pmatrix} &= z_1 \begin{pmatrix} v_1 \\ v_2 \\ 1 \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} \\
\end{align*}
\]

(3.79)

where:

\( \kappa_{11} \) is an unknown constant which determines the tangent point of the line of sight with the cylinder surface

\( \eta_i = \frac{r}{P_1} \alpha_i + \beta_i \), as determined in equation 3.66

\( \kappa_{11} \) may be solved for by equating the \( x \)- and \( z \)-components of the above expressions.

\[
\begin{align*}
\kappa_{11} \gamma_{11} + X_0 + \frac{r A_1}{P_1} &= z_1 v_1 + \eta_1 \\
z_1 &= \frac{1}{v_1} \left( \kappa_{11} \gamma_{11} + X_0 + \frac{r A_1}{P_1} - \eta_1 \right) \\
\end{align*}
\]

(3.80)

(3.81)

Substituting into the \( z \)-component expression from above:

\[
\begin{align*}
\kappa_{11} \gamma_{13} + Z_0 + \frac{r C_1}{P_1} &= \frac{1}{v_1} \left( \kappa_{11} \gamma_{11} + X_0 + \frac{r A_1}{P_1} - \eta_1 \right) + \eta_3 \\
\end{align*}
\]

(3.82)

Therefore:

\[
\kappa_{11} = \frac{1}{v_1 \gamma_{13} - \gamma_{11}} \left( \left( X_0 + \frac{r A_1}{P_1} - \eta_1 \right) - v_1 \left( Z_0 + \frac{r C_1}{P_1} - \eta_3 \right) \right) \\
\]

(3.83)
Similarly, $\kappa_{12}$, the constant corresponding to the other bottom intersection with the cylinder may be determined with the aid of the other line of sight, given by:

$$L_{12} = \kappa \begin{pmatrix} \gamma_{14} \\ \gamma_{15} \\ \gamma_{16} \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}$$

(3.84)

The length of this log segment, as determined from the bottom lines of sight, is therefore given by the distance between the two points of intersection with the cylinder.

$$l' = \left| \kappa_{12} \begin{pmatrix} \gamma_{14} \\ \gamma_{15} \\ \gamma_{16} \end{pmatrix} - \kappa_{11} \begin{pmatrix} \gamma_{11} \\ \gamma_{12} \\ \gamma_{13} \end{pmatrix} \right|$$

(3.85)

where:

$l'$ is the length of the cylinder determined from the bottom lines of sight

It is unlikely that the points that the operator has picked out from the image are exactly diametrically opposed on the log surface, an averaging of the length, as determined from both the top and bottom planes would be more accurate. In this case, the length becomes:

$$l = \frac{1}{2} l' + \frac{1}{2} \left| \kappa_{22} \begin{pmatrix} \gamma_{24} \\ \gamma_{25} \\ \gamma_{26} \end{pmatrix} - \kappa_{21} \begin{pmatrix} \gamma_{21} \\ \gamma_{22} \\ \gamma_{23} \end{pmatrix} \right|$$

(3.86)

where:

$\kappa_{2i}$ are the constants determining where, on the top lines of sight, intersection with the cylinder occurs, as determined in the same manner as $\kappa_{11}$ and $\kappa_{12}$

$\gamma_{2i}$ are the constants describing the vectors of the top lines of sight
3.5 Sources of Error

These two analytic equations involving the length and radius of the log form the model description to be used and provide the desired measurements directly. The errors involved in these calculations may be classified into three types.

While the length is a function of the calculated radius, it is the accuracy of the radius that is more of a concern. The radius will be much smaller than the length and, therefore its measurement will be more limited by the resolution of the imaging system. What will be called here a type 1 error arises from the spatial quantization of the image. This will add a fixed-sized term to the error budget of linear measurements that are derived from a subtended angle in the real-world. The smaller the measurement, the higher the error becomes as a fraction of it.

The same could be said of a type 2 error, arising from the feature point not being located exactly by the operator, due to such factors as haste, image noise or partial obscurity by grass or mud.

While the above error situations will also affect the length calculation, the effect is not as severe because the length value is going to be on the order of ten to one hundred times that of the radius. The length measurement will be affected more by such things as errors in the radius calculation and errors in the assumption of a straight axis. The former of these problems is unavoidable, however the latter may be at least partially remedied by allowing the operator to locate particularly bent logs as a series of straight cylindrical segments, each with a radius and length. The overall length and radius may then be determined as the sum and weighted sum respectively, of those of the individual segments.

Finally, type 3 errors will be introduced as a result of the fact that the models used are not perfect representations of reality. This includes the pin-hole model for the camera, the cylindrical model for the log, and the planar assumption for the ground. All three are intuitively appealing and mathematically tractable however,
and it was not felt that this type of error would be too large.

3.6 Simulation

In order to verify that in fact useful information about logs could be derived from a single-image system at a reasonable distance, a simulation based on just this was performed. It consisted of a program which fabricated a 512 x 512 pixel, log image.

At the outset of the simulation, the operator is prompted for the log’s length, radius, location and angle relative to the camera, the ground slope, and the camera’s height off of the ground. Using an ideal, pin-hole camera model, the software generated the image on a graphics screen that this camera would see. Potential errors of types 1 and 2 were incorporated in the simulation, while type 3 errors were not.

The operator was prompted to pick out the log corners with the aid of a movable cursor and combined keyboard/knob box input. This allowed control, not only of the cursor motion, but of the ability to re-draw the image following panning, tilting or zooming (up to 200mm) of the camera.

The result of all of this was a radius figure arrived at by the derived equations. This figure turned out to be quite accurate through a number of trials (see Figure 3.6). The average error was only about 0.2 cm for a log located fifteen metres from a camera placed five metres above the ground. For a more complete description of results that were obtained with the aid of this simulation software, refer to Appendix A.

As can be seen from the results of Figure 3.6, and expanded upon in Appendix A, some trends are prevalent for this measuring scheme. The error tended to worsen as the log angle increased. The optimum angle for measurements is to have the principle axis parallel to the image plane (0°). For situations other than this, there is a practical difficulty in locating the “corners”, as the projection is no longer
essentially square, but becomes rounded at the ends.

When the radius of the log was varied, no clear trend resulted. It is expected that the absolute magnitude of the error in a real system will not be a function of radius (at a constant distance), but that it will remain constant. This follows from the fact that this error will arise from spatial quantization and mistakes in locating the exact pixel closest to the desired feature. Both of these errors will lead to inaccuracies that are independent of the other feature locations and therefore will not be tied to the size of the log itself.

The simulation allowed the camera model to zoom in on the fabricated scene up to a certain limit. This allowed for greater accuracy as both of the quantization and mis-location errors were decreased at the maximum zoom position (focal length). This trend is quite clearly seen from the experimental results.

Finally, the distance of the target from the camera played a role in the accuracy of the measurement system. This too would be expected, as linear errors in the image plane correspond to angular errors in the real world. The farther away that the log is, the larger the distance between the two arms of this angle become. The magnitude of the error in radial measurements should vary linearly with distance. Despite all of this, the results were certainly accurate enough to continue the development.

While this simulation does provide a reasonable assurance that the derived calculations will be accurate enough under ideal conditions, it makes no allowance for errors introduced by the camera model. For this, an actual test apparatus is required, the experimentation and discussion of which is the basis for the next chapter. Although the pin-hole model was used to produce the results of the simulation, it was also used to generate the original image, therefore no conclusion could yet be drawn on its viability. However, as it is by far the simplest and most commonly used approximation, it was implemented in the test apparatus as, at worst, a starting point.
Figure 3.6: Effect of Log Angle and Log Radius on the Scaling Accuracy of a Simulated Log; radius = 10 cm, distance to log from camera base = 15 m, height of camera above base = 5 m
3.7 Simplifying Assumptions

In the camera calibration phase, the parameters of a model are found such that the world-to-image transformation is known and the image-to-world coordinate transformation may be determined to within one degree of freedom (the line of sight). Using a pin-hole camera model, where all lines of sight must pass through the centre of the lens, a model may be optimized to fit a set of test data. This test data consists of sets of known, world-coordinate points, corresponding image points and the values of any camera variables.

It would require up to eighteen parameters to completely specify a pin-hole camera model [33]. However, certain simplifications will allow for a reduction in this number and a corresponding decrease in the complexity of the calibration and the model's inverse.

If the camera can be assumed, to a good approximation, to rotate about its lens centre (which could be arranged with a carefully designed apparatus), then three degrees of freedom are eliminated from the external geometry parameters. As the logs are not going to be located in the near field of the camera's vision, they may be imaged with a camera set up with constant zoom and focus, as these are only required to vary when the deviation in the depth of field is a large proportion of the depth. This would eliminate a non-linear equation and three degrees of freedom in the internal geometry.

For the test apparatus, the pan and tilt functions of the camera will be held constant, thus eliminating two further external degrees of freedom and bringing the total number of parameters down to ten. In the field, the change in these angles may be measured to a very high degree of accuracy. For example, electronic theodolites used for surveying, which utilize an opto-electronic scanning system of a precise, graduated ring can achieve accuracies of better than 0.5 seconds (0.15mgon) [38]. This exceeds the quantization accuracy of the image resolution itself.
Thus, what remains is a camera with three orientational \((\theta_x, \theta_y, \theta_z)\) and three positional \((X_0, Y_0, Z_0)\) degrees of freedom. There are also four internal parameters consisting of the centre coordinates of the image \((h_0, v_0)\), the ratio of the resolution scale factors \((M_{ratio} = M_x/M_y)\), and the focal length/vertical resolution scale factor product \((fM_y)\).

To solve for these parameters, only one image is required, however, an absolute minimum of five, real-world points are required in that image. These may be pointed out on the graphics screen by the operator in the same manner as was suggested for the scaling feature extraction. At least one of these points should not be co-planar so as to introduce sufficiently independent information, and the data should ideally span the full range of the camera’s image space. Clearly, a regression technique involving as many points as is possible would be the best means to optimize the camera’s model. Still, this operation may be carried out quickly in the field by a user with little training, if the equipment is in a well-designed framework.

Having solved for the camera parameters, the operator can then scale logs. This is done by aiming the camera at the logs by remote control, and picking out, with the aid of a light pen, the end corners of their projections onto the image plane, as displayed on a local graphics screen. In addition, logs that deviate from the straight cylindrical model used may be scaled as a series of straight segments by picking out diametrically opposed break points where curvature becomes noticable.

This scenario allows the operator to perform these measurements in a quick and relatively comfortable setting. Most importantly, the measuring time will be greatly shortened. The calculations are simple and the processor could perform them rapidly. As with any new equipment, there would be a period of getting used to it, but with a bit of practice and experience in situations such as spotting edges occluded by grass and the effects of some small degree of foreshortening, this system could be very accurate.
3.8 Advantages and Disadvantages of the System

One major advantage of this system is speed. This technique for individual log measurement can scale a log in less than ten seconds. It would take little more than the length of time for the operator to touch the screen four times with a light pen in order to scale a log. Compared with just the time that it takes to walk from one end of a log to the other, this will be a savings.

Another major advantage of this system is its simplicity. It requires nothing more sophisticated than a high resolution imaging system. The processing power required is not great. The actual processes involved are very simple to perform - even more so once a little practice has been gained.

The accuracy of the measurements made with a system of this nature are, at least from the simulation results, able to meet the radius accuracy set out for stick scaling.

Relative to the other possible automation techniques looked at, this system will be inexpensive. No sophisticated timing electronics or opto-electronics are required. Only one television camera is required and the processor only need be able to do some simple, floating-point calculations. There are few moving parts, thus, also increasing the reliability of the equipment, especially since it is in an outdoor environment.

Further along the lines of reliability, this equipment may be portable, as the calibration is quick enough that it may be done every time that set-up occurs. Portability means that the apparatus need not be exposed to the elements when not in use.

Portability also means that the same equipment may scale at several locations in the same or even different sort yards, if desired. For that matter, it could be taken anywhere to scale logs that these measurements would be useful.
Finally, this system could isolate the user from the outdoor setting of the log sort yard, allowing a more comfortable work environment for the scaling process.

As with most things, there are drawbacks to this design. The fact that it does require operator input leaves room for human error and some time in the interaction process. Additionally, the system will take some getting used to, as judgement may be required for partial occlusion of a bottom edge by grass or rounded end segments with no clear-cut corner in the projection. These factors must be evaluated in the field.

The main drawbacks however, deal with the errors involved. This error primarily results from the assumptions made. The ground is not perfectly flat, although log sort yards are reasonably so. The logs are not perfectly cylindrical, although it is felt that with the ability to break any especially crooked logs down into a series of smaller log segments, this can be mitigated. Finally, any practical camera model is going to be only an approximation to reality. The price that is paid for the simplicity of the pin-hole model is that it quite likely leads to the most error. Just how much error this and other factors introduce to corrupt the log scaling measurements will be discussed in Chapter 4.
Chapter 4

Experiments

The purpose of this section is to describe a series of experiments that took place to verify and analyze the design of the automated log scaling technique described previously.

4.1 Equipment

The hardware used for the processing of both phases of the test consisted of a 1024 x 768 pixel colour graphics screen (HP98700) attached to an HP9050 computer. The processes were called and controlled from a standard display terminal, although the cursor used for locating the feature points was manipulated with the aid of two, optically-encoded knobs. The 480 x 512 pixel images, which were displayed in seven-bit grey-scale intensities, were generated by a Dage television camera attached to an Imaging Technology IP512 frame grabber board.
4.2 Calibration

4.2.1 Calibration Procedure

The calibration portion of the experiment was performed very similarly to that described in Chapter 3.

The television camera was hung from the ceiling of a laboratory at an angle of roughly thirty degrees to the horizontal. A calibration object, a cube, was put into the field of view of the camera and the known world coordinate points of its vertices were used as input data (see Figure 4.1).

The cube was chosen for this because it provided a set of distinct and attached points that had easily calculated positions relative to each other. Other objects, or for that matter, any set of available world coordinate points could be used however.

The calibration technique began with actual measurements of the six external parameters. \( fM_y \) and \( Mratio \) were both estimated based on a knowledge of the imaging system being used. The image centre, \( h_0 \) and \( v_0 \), may be seen as constant (or dc) offsets in the graphic output coordinates and the initial estimates were set to half the number of pixels in the horizontal and vertical outputs, respectively.

Once set up, the physical procedure was similar to the scaling simulation. The user was provided with the image of the calibration object and then asked to locate the relevant feature points with the aid of knob box cursor controls and keyboard data input. At each point location, the world coordinates were entered, either from the keyboard or a previously created text file. As there were no camera parameters that varied, the two image coordinates and the three world coordinates alone formed an input data set for each point.

After the seven visible vertices of the cube were located and entered, the calibration routines were invoked. This performed the least squares fit of the input
Figure 4.1: Laboratory Scaling Apparatus
data to the pin-hole camera model.

For the calibration tests, six images were used. Each one was an image of the same cube, however, it was placed in six different locations. For the first case, referred to from here on as the test case, the cube (which was 38 cm on edge) was centred in the image space at a distance from the camera that was so close that it filled most of the image. This provided information to the regression process about as much of the image space as possible, and should have been the "best" model that could be derived, as a result.

A second image, referred to as the centre image, was taken in which the cube was centred in the image plane, however at a greater distance (about 4.5 m) from the camera. This meant that information was provided to the camera about only the centre portion (about 10%) of the image plane. This is where the imaging system is the most linear and may best be described by the pin-hole model [21]. However, the fit derived by calibrating the camera with this image was not as useful for other portions of the image space.

Finally, the cube was placed in each of the four corners of the image space. These are the least linear portions of the imaging system and these models should, therefore, have been the least useful about other portions of the image space, in all likelihood.

For each of these six images described, the calibration procedure was repeated five times. The repetition was performed in order to ensure that the model fit was not dependent on extremely accurate feature point spotting, and should provide a feel for the precision (repeatability) of this operation, averaging out any particularly sloppy trials.
4.2.2 Calibration Observations

It was at this time that it was observed that all of the objects of interest in the image were distinct and clear, without any variation of the zoom or focus controls. This verified the lack of having to calibrate these also.

The calibration process, as described previously, led to a problem. While there are ten parameters to be optimized, they are not independent. In fact, there turned out to be four pairs of dependent parameters amongst them. \( X_0 \) and \( Y_0 \) were dependent on \( h_0 \) and \( v_0 \), respectively. The scale factor, \( fM_y \), was dependent on the distance along the camera's normal axis, \( Z_0 \). Finally, two of the angles, \( \theta_y \) and \( \theta_z \), were not independent of each other.

The problem with this is that, in trying to optimize the model to fit all of these parameters simultaneously, the calibration technique did not arrive at a unique solution. Instead, it would converge on one of an infinite number of solutions which locally minimized the sum of the square of the residuals, and it became a highly data dependent process.

Fortunately, a technique was developed which remedied this problem. Most of the parameters being optimized for were directly measurable and very good estimates could be obtained for their values.

The calibration was broken down into two stages. In stage 1, one of each of the four pairs of dependent parameters \((\theta_y, h_0, v_0, Z_0)\) was held constant. These were considered to be quite well known from direct measurement \((\theta_y, Z_0)\) or the initial equation offset process carried out \((h_0, v_0)\), as mentioned in Section 3.2. The remaining six parameters in the model were then allowed to vary by means of the described least squares technique. This generally reduced the sum of the squares of the residual errors down two orders of magnitude. Following this, the other four dependent parameters of the model were fixed at the values derived from stage 1 and optimization continued on six free camera parameters again. Stage 2 reduced
the sum of the squares of the residuals by only about 25%. This stage could be thought of as almost a fine tuning, however it proved extremely valuable also in increasing the repeatability of the solutions obtained, as borne out partly by the decrease in the standard deviation of the rms error, $\sigma_\Phi$, over a number of trials of the same test.

Two variations of this two stage calibration technique were also tried. First, if a different set of variables was held constant for each successive iteration (instead of iterating one set to minimization, and then the other), the solution oscillated for a short period before settling on a value that did not quite minimize the value of the sum of the squares of the residuals. Finally, if a third stage, in which the same parameters are held constant as in stage 1, was performed, no improvement was derived. Thus these efforts were abandoned in favour of the two stage method above.

\subsection*{4.2.3 Calibration Results}

With this technique in mind, some results may be seen in Figures 4.2-4.9 and a more complete list of results may be found in Appendix B.

Figures 4.2-4.3 show the value of the mean and standard deviation of $\Phi$ as the calibration from the test image was repeated over five trials. The test image involved the seven visible vertices of a cube that spanned a great deal of the image space. The starting estimates for $X_0, Y_0, Z_0, \theta_x, \theta_y$ and $\theta_z$ were determined by linear measurements (with the aid of a tape measure) that had an estimated accuracy of one inch (2.54 cm). The method for determining the starting estimates for $h_0$ and $v_0$ was described in Section 3.2. $fM_\Phi$ was estimated using some "ballpark" figures for the focal length, aperture and image pixel density, as was $Mratio$. These last two parameters were seen as being the least accurate of the starting parameter estimates.

From the logarithmic plots of the rms error, the convergence behaviour of the
Figure 4.2: RMS Error, $\Phi$, for the Camera Calibration; test image, averaged over five trials.

Figure 4.3: Standard Deviation of the RMS Error, $\sigma_{\Phi}$, for the Camera Calibration; test image, averaged over five trials.
calibration may be seen. Convergence for both stages required only two to three iterations.

Figures 4.4 and 4.5 illustrate the statistical distribution of one of the parameters, \( \theta_z \), through a number of calibration trials. \( \theta_z \) was not fixed in any stage of the calibration. In Figure 4.4, \( \theta_z \) is taken from the calibration based on the test image data. This was expected to yield the best fit for the values of each of the parameters.

Additionally, five other images were used to calibrate the same camera, placing a smaller version of the cube in each of the four corners of the image and in the centre. For each of these test images, the calibration was repeated.

The combined results of \( \theta_z \) for all six of the images is shown in Figure 4.5. While the mean remains almost the same, the standard deviation has risen by a factor of four. This indicates that the model calibration is most repeatable (precise) when a spanning set of test data is used.

\( h_0 \) was held constant in stage 1 only. The plots for the same two sets of tests just discussed are shown in Figures 4.6 and 4.7. While the same two observations can also be made here, Figure 4.7 brings out even better the variation in a parameter's value when different calibration data is used. Here, distinct clusters occur around false means located ±20 pixels (±3.8% of 480 pixels) away from the true mean. This is a side effect of the camera model being a linearization of reality. Locating all of the calibration data into one corner of the image will tend to skew the results obtained by the optimization.

These conclusions are once more borne out by Figures 4.8 and 4.9, which histogram results of the calibration of \( Y_0 \), which was held constant only in stage 2. Here, the mean is roughly the same, while the standard deviation rises by two orders of magnitude.

The results just presented describe a situation where a camera was successively calibrated with data that spanned either the entire image space or a distinct subset.
Figure 4.4: Result Distribution for $\theta_z$; test image calibration, 5 trials

Figure 4.5: Result Distribution for $\theta_z$; total from all 6 image calibration test, 30 trials
Figure 4.6: Result Distribution for $h_0$; test image calibration, 5 trials

Figure 4.7: Result Distribution for $h_0$; total from all 6 image calibration test, 30 trials
Figure 4.8: Result Distribution for $Y_0$; test image calibration, 5 trials

Figure 4.9: Result Distribution for $Y_0$; total from all 6 image calibration test, 30 trials
Sample Calibration Result Statistics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>% of F.S./Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_x ) (test image)</td>
<td>-0.6642 rad</td>
<td>0.0028 rad</td>
<td>0.04% of 2\pi</td>
</tr>
<tr>
<td>( \theta_x ) (total)</td>
<td>-0.6682 rad</td>
<td>0.0111 rad</td>
<td>0.17% of 2\pi</td>
</tr>
<tr>
<td>( h_0 ) (test image)</td>
<td>520.8 pixels</td>
<td>6.664 pixels</td>
<td>0.17% of mean</td>
</tr>
<tr>
<td>( h_0 ) (total)</td>
<td>524.3 pixels</td>
<td>16.48 pixels</td>
<td>3.1% of mean</td>
</tr>
<tr>
<td>( Y_0 ) (test image)</td>
<td>2.481 m</td>
<td>0.0003 m</td>
<td>0.01% of mean</td>
</tr>
<tr>
<td>( Y_0 ) (total)</td>
<td>2.488 m</td>
<td>0.0214 m</td>
<td>0.86% of mean</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of a Sample of the Calibration Test Results

A summary of the above statistics may be seen in table 4.1.

Each of the six camera models derived was repeatable, although more so for the spanning set of data (the test image). Those model parameters derived from either just the centre of the image (the centre image) or just a corner of the image plane (the corner images) tend to have their mean calibration value located at a slightly different value than that derived from the more spanning data. The values for these parameters derived from the test image can be seen to be, not necessarily the exact physical values, but rather the mean of the values that would be determined from each of the less spanning information sets (calibration images). This was expected, as the regression technique is fitting a linear model to a slightly, non-linear system.

The value of the rms error, \( \Phi \), at convergence for the test image is 2.1388 pixels, with a standard deviation, \( \sigma_\Phi \), of 0.4634 pixels. This indicates the rms error in horizontal or vertical image coordinate values that could be expected from the camera transformation equations. The horizontal scale was 480 pixels at full scale. Thus, the rms error turned out to be 0.44% of it. The low standard deviation points
to the high repeatability of the model derived from the tests.

These results along with those included in appendix B, may be seen as a partial justification of the calibration technique. However, what still lacks is some sense of how accurate this technique is when coupled to the log scaling calculation routines. This will be the subject of the next subsection. For this, all thirty sets of camera parameters (derived from the six images, tested five times apiece) will be used to scale a cylinder.

4.3 Scaling

4.3.1 Procedure

In order to test the accuracy of both an operator at locating points and the camera model used, cylindrical objects were imaged and scaled by the same apparatus as that used for the calibration. The process and calculation algorithms were exactly those presented in Chapter 3 for the radius and length of the objects. Cylindrical objects were used for these tests instead of real logs because they have readily determinable dimensions that provide a direct measurement of how accurate this system is. This provides a real-world situation where type 1 and 2 errors are present. Type 3 errors are also present, but only in the form of camera model non-idealities.

Finally, an actual log was imaged and scaled in order to include type 3 errors that arise from the cylindrical log assumption. The only error not included is that type 3 error arising from the log sort yard not being perfectly flat. This, as well as further tests on actual logs should be done in field tests.

It is known that a model fit to the camera may be derived, and it has been shown that an algorithm exists that will calculate the dimensions of a log, given an ideal, camera model. What remained next to be seen was whether these two may be combined to accurately scale logs with a real-world camera situation.
In the last section, the extraction of thirty sets of camera parameters from six different images was performed with the aid of a regression technique. Some of these derived models were not as complete a description of the entire image space as others. All of them were then used to scale a cylinder of known dimensions in order to provide conclusive evidence about system accuracy.

The images used for scaling were taken by placing a cylinder with a radius of 12.86 cm and a length of 50.48 cm in view of the camera. Five different images of this object were used, with the cylinder being viewed in each of the four corners of the image and once in the centre. Thus, if a particular set of the camera parameters was derived from only a portion of the image plane, then this model’s scaling accuracy in that same section (or any other section) can be isolated. Similarly, the parameters calibrated from a more spanning set of data can be tested on various portions of the image space.

Each of the tests was run five times, as with the simulation, in order to verify its precision and allow for quantization error and mis-location of the feature points due to human limitations. Both the length and the radius were calculated, using the design equations. The radius is the most important of the calculations made however. The length is derived as a function of the radius, and so is partially dependent on its accuracy. The radius is by far the smaller of the two measurements, thus the same magnitude of error will be a larger proportion of it. With the current stick scaling process, the radius may be determined by one simple measurement, whereas the length must be determined by going end-over-end with the stick, which is inherently less accurate. Finally, in calculating the size of the log, the formulae used are dependent on the square of the radius, while the length is only a linear factor.
4.3.2 Scaling Results

Figures 4.10 and 4.11 display the results of scaling with the parameters derived from the test case (where the calibration input information spans the entire image plane). In 4.10, the target was located in the centre of the image and the mean of the results was correct to five digits. Just as importantly, the standard deviation of the measurements was only 0.15 cm. The full scale value for the number of horizontal pixels in the image is 480. This image size corresponds, at the depth of the target, to 2.432 m in the world. Thus, the standard deviation of the measurements, indicating the precision of them, was only 0.06% of the full scale, or 0.29 pixels. This figure is very favourable, especially when considered with the mean. It is under quite favourable conditions however.

For the measurements of Figure 4.11, the same object was placed in each of the four corners of the image and the test model was used again. For this case, both the error and standard deviation of the mean radius got worse due to the non-linearity of the image transducer. Still, the mean radius is only in error by less than 0.4 cm (0.16% of full scale = 0.77 pixels).

When each of the 20 calibration parameter sets derived from the 4 corner images were used to scale the "log" located in the same corner as the respective calibration model, the results shown in Figure 4.12 were derived. The distribution of results for this case is quite accurate, as would be expected. The calibration model has been fit, as accurately as possible to this space. The mean radius is out by an equivalent of 0.06 pixels, with a standard deviation of 0.71 pixels.

Figure 4.13 shows the histogram of results of scaling in the centre of the image with all of those parameter sets derived from the corners, combined with the results from scaling the object in each of the four corners with camera parameters derived from just the centre of the image. Here, the mean radius is still very accurate, but the distribution of results has flattened out somewhat, with results occurring ±1 cm from the mean. The standard deviation of the mean radius has risen to an
Figure 4.10: Log Scaling Radius Distribution for a Log Placed in the Centre of the Image; *test* model parameters, actual radius = 12.86 cm, 5 trials

Figure 4.11: Log Scaling Radius Distribution for Logs Placed in the 4 Corners of the Image; *test* model parameters, actual radius = 12.86 cm, 5 trials × 4 Images
Figure 4.12: Log Scaling Radius Distribution for Logs Placed in the 4 Corners of the Image; same corner model parameters, actual radius = 12.86 cm, 5 trials x 4 Images
Another "worst case" situation uses the parameters from the corner models for scaling a log in the opposite corner from that in which they were derived (see Figure 4.14). While the mean radius is closer to 12.86 cm, the standard deviation of the mean is higher than any of the other cases, indicating that the result of any one scaling operation is more likely to be in error. Still, the results are not that bad, when one considers that this is essentially a worst case situation for a simple linear model.

As expected, the length was not as accurate as the radius. Figure 4.15 shows the length histogram that corresponds to the standard tests of Figure 4.10. The mean length is in error by 1.12 cm (2.21 pixels), with a standard deviation of 0.89 cm (1.76 pixels). This may sound like a great deal, compared to the accuracy of the radius exhibited above, however it is not that bad compared to the error from stick scaling.

Figures 4.16 and 4.17 show the radius and length result distributions for all of the tests carried out. The test calibration parameter set was used to scale a cylinder in the centre of the image and in each of the four corners of the image plane (25 tests total). The set of parameters derived from strictly the centre portion of the image plane was used to scale the cylinder in each of the four corners (20 tests total), and each of the corner parameter sets were used in the same corner, in the centre and in the opposite corner (60 tests total). This provided a good collection of results of not optimal conditions. Still, the mean radius is within 0.6mm (0.12 pixels) of the true value, with a standard deviation of 0.5234 cm (1.03 pixels) The same distribution of measurements for the length illustrates a mean that is high by about 0.58 cm (1.14 pixels) with a standard deviation of 1.24 cm (2.45 pixels). A tabular summary of the above scaling tests may be seen in Tables 4.2 and 4.3.

Some further tests were also run on other objects using the test calibration data, which was deemed to be the best fit over the entire image. A small cylinder of
Figure 4.13: Log Scaling Radius Distribution for Logs Placed in the 4 Corners of the Image and in the Centre; \textit{corner} model parameters for the centre image, \textit{centre} model parameters for the corner images, actual radius = 12.86 cm, 5 trials \times 4 corner images + 20 trials \times 1 centre image.

Figure 4.14: Log Scaling Radius Distribution for Logs Placed in the 4 Corners of the Image; opposite \textit{corner} model parameters for each of corner image, actual radius = 12.86 cm, 5 trials \times 4 images.
Figure 4.15: Log Scaling Length Distribution for a Log Placed in the Centre of the Image; test model parameters, actual length = 50.48 cm, 5 trials

<table>
<thead>
<tr>
<th>Model Source</th>
<th>&quot;Log&quot; Position</th>
<th>Mean Error</th>
<th>St. Deviation</th>
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<td></td>
<td></td>
<td>cm</td>
<td>cm</td>
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<tr>
<td></td>
<td></td>
<td>pixels</td>
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<td>all 6 images</td>
<td>all 5 places</td>
<td>-.05</td>
<td>.52</td>
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Table 4.2: Summary of the Scaling Tests from the Calibration Models - Radius
Figure 4.16: Log Scaling Radius Distribution - All Tests Combined; all 6 parameter models, all 5 images, actual radius = 12.86 cm, 105 trials

Figure 4.17: Log Scaling Length Distribution - All Tests Combined; all 6 parameter models, all 5 images, actual length = 50.48 cm, 105 trials
### Scaling Length Result Statistics

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<thead>
<tr>
<th>Model Source</th>
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<th>Mean Error cm</th>
<th>St. Deviation cm</th>
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Table 4.3: Summary of the Scaling Tests from the Calibration Models - Length

radius 2.54 cm and length 13.65 cm was scaled. The result for the mean radius was 2.9436 cm (mean radius error = 0.80 pixels) with a standard deviation of 0.3062 cm (0.60 pixels). Both of these results are quite similar in magnitude to those derived for the larger object, attesting further to the fact that the magnitude of the error is independent of the size of the object, down to a certain point anyway.

Not so accurate results were obtained in scaling the original cylinder when it was placed at an angle of about 40° to the image plane. The mean of the results was high by 2.33 cm (4.60 pixels) for the radius and low by 2.23 cm (4.40 pixels) for the length. This decrease in accuracy is primarily attributed to a human error in properly spotting the end points, which should be reduced with practice. The large error in the radius also would have had an effect on the length calculations.

Lastly, a real log was scaled. The mean radius of its end points was measured (to within 0.1 cm) to be 4.20 cm, while that measured by the log scaling system was 4.08 cm (mean radius error = 0.24 pixels). The length (measured to within 0.25 cm) of the log was 1.017 m, while that measured by the automation was 1.034 m (3.36
pixels). As could be predicted, the results for a real log are in greater disagreement with the design calculations than those for any of the strictly cylindrical objects due to this type 3 modelling error. It is felt, however, that this departure does not corrupt the accuracy by so much as to invalidate the technique.

While all of the previous tests were performed by the author, a subsequent series of tests were performed on two other individuals. They had neither seen the scaling tests before, nor knew the actual dimensions of the test cylinder. Each of the individuals used those 5 models derived from the test calibration images to scale the cylinder in the centre of the image. The mean value for the radius derived from the ten trials was 12.6615 cm, with a standard deviation of 0.3530 cm. Thus, their mean radial error is 0.39 pixels, with a standard deviation of 0.70 pixels. These figures are quite comparable with those derived by the author, and lead to the conclusion that no personal bias has artificially improved the accuracy or precision of the results determined experimentally.

4.3.3 Scaling Conclusions

All of these results are sufficiently accurate to lead to the conclusion that this system does provide an accurate enough result, given that the logs are not going to be placed at too large an angle with the image plane. Using the results of the tests shown in Figure 4.16, the mean is convergent on the true value of the radius. This is true of the weigh scaling technique currently employed [7], and quite likely also of the stick scaling technique. However, the precision here is in the pixel range ($\sigma_r = 1.03$ pixels for all of the tests combined). The expected value of the error term contributed by the quantization error would be 0.25 pixels, which leaves approximately 0.78 pixels of error to the type 2 and 3 errors.

The calibration procedure provides a linearized model of the image extraction system, so that feature points spotted fairly quickly by a human operator will lead to the measurement of the length and radius of a target cylinder (log). The computer
system used for these calculations required 1 second to perform them, once the features had been located, so that the entire operation is going to produce a savings in time over stick scaling.
Chapter 5

Conclusions

The scaling system developed in this thesis utilizes a simple, camera-based, imaging system, attached to a high-resolution graphics screen with a floating-point processor. Instead of having the log scaler walk around the logs with a measuring stick, the desired dimensions (and, in fact, an entire, cylindrical fit) may be derived from camera image data.

Two phases are required in the implementation operation. The first of these is the camera calibration, which is able to define the parameters of a simple camera model from non-linear, real-world data. This requires a Gaussian least square regression technique that minimizes the linearly-predicted residuals to overcome the non-linearities. This calibration process was seen to repeatably converge and produce parameter value distributions that were precise. The greater the portion of the image space spanned by the input data, the better the overall fit of the model. The rms error of the world-to-camera transformation equations was 2.14 pixels, with a standard deviation of 0.46 pixels.

The second phase of the process is the scaling operation in which the operator picks out simple features of the log from an image. This is quick (less than 10 seconds with light pen implementation), simple, accurate and precise, as determined herein. Statistically, the correct value for the radius of the target was achieved in the mean,
with a standard deviation of only 1.03 pixels. The length was slightly high in the mean, due to difficulty in spotting the exact end point of the object, and carry through error from the radius calculation. The standard deviation of the length was just less than 1.24 cm (2.45 pixels), which is still at least as accurate as what could be determined by the current stick scaling technique.

Thus, with this system analyzed for its accuracy, and precision, the next stage should be field tests before full implementation. One major strength of this system is its simplicity, as no complicated manual operations or sophisticated electronics are involved. Another predominant advantage is the speed with which a log may now be scaled (less than 10 seconds) accurately. In addition, the operator may now perform this task from a closed hut or the cab of a vehicle. Reports based on this information could easily be generated using the same processor and an attached printer. This step would save a little more time and any possible transcription error. Additionally, having this computing power available could allow for direct communication with a data base.
Chapter 6

Recommendations

Having analyzed and verified this log scaling system under laboratory prototype conditions, the next stage is that of field trials. This involves installing an actual imaging system in a log sort yard and testing it there for such things as accuracy (as compared with these previous results), precision, time savings and operator acceptance. While the first three have been examined already, the final point is worthy of mention, as it may be the human interface that ultimately limits this system. A recommended design scenario will now be presented that satisfies these requirements best.

For this system, a high-resolution camera, frame grabber and graphics screen will be required to present the scene to the operator. In addition, a floating-point processor and a light pen will be needed for the actual scaling process.

This structure should be sturdy enough to be unaffected by wind. If it is mounted on, for example, the top of the cab of a pick-up truck, then it becomes portable. This means that the scaling may be done at various locations within the sort yard, or for that matter, anywhere that they are desired. Portability also means that the device may be stored in a dry place where deterioration of the equipment is minimized. The camera will have to be calibrated each time that the vehicle is moved. The mounting structure should be of well-known dimensions in order to
provide accurate starting estimates of the camera's position for the calibration procedure. In addition, some accurate means, such as optical encoders, may be used to measure changes in the pan and tilt of the camera, once mounted.

The calibration procedure may be broken down in order to improve its reliability. Using a laboratory test jig, the external position and orientation of the camera and the calibration data may be made constant to whatever accuracy is desired. The internal parameters may then be calibrated to provide extremely good starting estimates for the field. In fact, these values may be accurate enough to never require further adjustment.

The external parameters of the camera may be fixed also by the careful design of a mounting structure. If it is not convenient or simple enough to mount the camera so that it rotates about its lens centre, then the vector arm between the lens centre and the centre of rotation will have to be calculated or calibrated (once) and included in the design equations as a further constant transformation of coordinate frames. This may be done when the mounting structure is being designed and need not be updated, unless the lens centre is changed (remember that zoom and focus are held constant).

Having derived accurate, starting estimates for the imaging system's internal parameters in a laboratory and the external parameters by careful design and construction, the calibration of the two need not be done simultaneously. Each time that the imaging system is moved, the external calibration will have to be performed, however the internal calibration need not occur each time. In fact, the only time that this need be done subsequently is if the measurements are becoming suspect or if a factor, such as a temperature change, may have affected them.

The internal calibration may be achieved by having a set or test position into which the camera snaps. In this position, if the camera is looking at a portion of its own mounting structure or the vehicle upon which it is located, fixed points with exact positions relative to the camera may be placed in sight for calibration.
purposes. If these points are painted spots, then the operator may perform the internal calibration by locating them, just as was done in the simulation. If they are lights, then their positions may be automatically derived by locating the centre of mass of the intensity peaks in the image. This may be a needless expense in both monetary and computational terms however. The world coordinates of these marked points are a part of the design structure too, and their values may be contained within the software so that this operation is quick. The calibration software will only need one stage for this, as the external parameters are considered a constant here.

Having calibrated the internal parameters, they may be held constant while the external parameters are likewise found in the field by optimizing the model over some test data. This test data is not going to be as accurate as that used for the internal calibration, as these points will have to be located independent of the accurately designed, supporting structure, and relative to the ground plane.

The points used to do the external calibration may be placed in a number of ways. The placement of the origin is somewhat arbitrary. As the ground is not a perfectly flat plane, the farther apart that the calibration points are located (within the direction parallel to the ground), the better that the averaging effect will be on its discontinuities. One technique for providing some calibration data would be to have a set of fixed length sticks joined, both top and bottom, by fixed length string. These could be stuck in the ground in a regular pattern, such as a triangle. This would provide two fixed points per stick, one of them on the ground and one at the top of each stick. If this pattern were laid out and some information about its position were entered, then the operator could calibrate the camera's external position.

Laying out the external calibration points on the ground has the advantage that they may be farther apart and, thus, better average any unevenness in the ground. It does require some time whenever the camera has been moved however. An alternative to this would be to mount the points (painted spots or lights) on
another vehicle, such as that which transports the logs in the first place, or that which places them on the ground. These points may be fixed and their positions well-known, relative to some fairly arbitrary origin.

Once calibrated, scaling is just as simple as that process described for the experiments here. The log placement is somewhat constrained and recommendations may be made based on the results of the simulation. As the zoom and focus are to be a constant, the logs will have to be placed beyond a certain depth, although this will likely be satisfied by the camera-to-ground distance by itself. The logs will have to be placed on the ground such that one does not obscure the view to any others from the camera. This will entail a minimum distance between their locations on the ground, based on the camera’s height and the maximum distance that they may be placed relative to it. Optimally, the log axes will be parallel to the image plane, so that the ends are simple for the operator to pick out, and parallel to each other so that they may be packed as close to each other as possible.

The logs will be constrained to be within a certain distance of the imaging system in order to maintain the accuracy. This will be a function of the resolution of the imaging system, and possibly the log’s minimum radius.

The operator should be able to light up chosen pixels with a light pen, selecting or de-selecting points with the aid of single buttons or simple keyboard entries. Once a log has been completely spotted, it might be useful to see its outline drawn on the screen over top of the image in order to verify the fit. This might, however, require too much computing time to be invoked automatically, but could be a useful option. If the fit is unsuitable, then points may be moved.

The camera will not likely be able to see both ends of the logs at once. Therefore, the scaling technique, once a group of logs has been laid out, should consist of looking at one end of the logs first, storing all of the end points in a set order, and then doing the same for the other end. All of the end points on one side will likely be fit into a single image frame, although again, they need not. The amount of
panning and tilting (both of which could be joystick controlled) required should be minimized though, for time considerations.

As there is further sorting of the logs that must be performed on foot, a printed report, including the information just derived would be a useful by-product of the scaling process. This would lessen the paperwork a little and allow the operator to just fill in the remaining blanks as the logs are surveyed on foot for knots, species, etc.

The user interface for this system shall, as expected, be as simple as possible. No computer knowledge should be necessary and as little of the calibration input information as possible should be manually entered.

For the purpose of training operators, some test cylinders and pre-scaled logs should be available. The process of accurately locating the actual end points is not a difficult one, but accuracy may be improved if one has some standard to compare the results against at the start. If scaling is practised on these test objects in a variety of positions for a while, the difficulties encountered from spotting the rounded edges and estimating corners that may be partially obscured by grass may be overcome.
References


[27] Pentax Electronic Distance Meter Specifications: a. EDM Theodolite PX-06D/PX-10D, b. EDM PM-81, Asahi Precision Co. Ltd., Tokyo.


[31] Sonic Tape Specifications: FCO1 Sonic Tape, Sonic Tape PLC.


Appendix A

Experimental Results - Simulation

In chapter 3, a description of a simulation was presented. It was designed to be a tool that would verify the design equations, to provide some insight into how well a system like this would work with 512 x 512 pixel images, and to assess its drawbacks.

For the simulation to run, the operator was queried for some input parameters with which the scaling scene would be generated. These were:

- log radius
- log length
- x- and z-coordinates of the log’s centre of mass
- angles that the ground plane makes with the x- and z-axes
- height of the camera off of the ground

As a reference case, a log was ‘placed’ 15 metres away from the camera, which was raised 5 metres off of the ground \((x = 0, y = 5, z = 0)\). This log was 10 centimetres in radius. A test was conducted where this log was scaled ten times in
succession. The mean radius for all of these tests was 10.2190 cm, and the standard deviation was 0.1086 cm.

This result verified the design equations, and was to stand as the reference case for further tests. The next step was then to vary some of the input parameters, in order to determine whether this accuracy just achieved could be affected.

Each time that a parameter was fixed to a new value, it was tested five times in succession in order to try to overcome the effects caused by difficulty in locating the exact feature point and quantization round-off. The measurements were conducted at a reasonable pace (less than five seconds per point), in order to avoid false accuracy arising from spending an excessive amount of time locating the features and any possible ‘learning’ of where the best results were obtained from any particular cursor placement. It was, nonetheless, noticed that as time progressed, the spotting process became slightly faster and more accurate as the proper points became easier to recognize with the eye, and familiarity with the process controls increased.

Figures A.1- A.10 show the results obtained from five experiments in graphical form. For each test, the mean error of the radius, $r$, the standard deviation of this mean, $\sigma_r$, are plotted as a function of the control variable tested. As well, the mean of both of these figures for each complete experiment is drawn in to provide a better, visual idea of whether the accuracy (error) or precision (standard deviation) of the measurements are dependent on the control variable.

The error plots for each indicate just how accurate the radius measurements made with the use of the design equations are. The standard deviation plots, on the other hand, indicate more how reliable the error plots are. If the standard deviation figure increases, it can be expected that the error measurements are less reliable and subject to greater fluctuation.

It is not expected that the plots will conform to any simply-described, analytic relations. In addition to the statistical variation arising from repeated testing, it is felt that the errors involved are going to be the result of a combination of factors.
The control variable may not even become one of these factors until it exceeds a certain threshold, below which the error it generates is not noticeable above the error "noise floor". The "noise floor" would be considered to be that error which is essentially constant due to such things as quantization noise and statistical errors resulting from the repetition of the spotting operation. In addition to the spatial quantization of the image plane, it should be pointed out that the log itself was generated as an eighty-sided polygon, rather than as a pure cylinder. While this did not provide too perceptible an error to the eye, it may have been a further quantization type of error.

Figures A.1 and A.2 deal with the angle that the log's axis makes with the plane $x = 0$. As this angle is increased, only a slight overall decrease in accuracy is exhibited as a trend. On the other hand, the precision of the results is dramatically altered at angles above $40^\circ$. This results primarily from the difficulty in spotting the cursor over what are perceived to be the "corners" of the log in the image plane at the rounded ends. It would be recommended, as a result, that the logs being scaled in the sort yard be placed sufficiently parallel to the image plane in order to avoid this.

The error, as a function of the log radius, did not exhibit a clear trend, as shown in Figures A.3 and A.4. This leads to the conclusion that this error is roughly a constant. As was mentioned in chapter 4, the spotting process of each of the feature points is independent of each of the others. Therefore, the proximity of these points to each other has no effect on the accuracy or precision. Should they, for some reason, be so close to each other that the ensuing calculations are subject to numerical errors, or the resolution of the image is insufficient to provide any real accuracy, then the radius would become a factor. However, this is unlikely for logs of an economically viable size located sufficiently close to the imaging device. If this does become a problem, then a higher resolution imaging system, or a greater zoom capability will be required.

Figures A.5- A.8 deal with the proximity of the log to the image plane. These
cases present very similar information to those derived above for the varying radius case, and in fact a great deal of the above paragraph could be repeated here. What these experiments do bring out more is that the resolution of the imaging system is a factor in determining the accuracy of the measurements. For example, if the actual corner point to be spotted is exactly half-way between the two pixels that are nearest, then the quantization error subtends an angle that corresponds to one-half pixel in the image plane, as seen from the focal point. The arms of this angle are much farther apart at fifteen or twenty metres, however, and lead to a resolution error. Similarly, if the operator spots a feature point in error by \( n \) pixels, that will contribute to a greater absolute measurement error at longer target distances.

In A.5 and A.6, the log's centre is moved along a line at \( z = 0 \), while in A.7 and A.8, this point was moved along a line at \( z = 2z \), removing some of the symmetry from the corner spotting process. The log's axis was always held parallel to the image plane however. In both cases, the accuracy and precision of the measurements decreased as the log target was located further away. This result indicates that there will be an upper bound to the distance that logs may be away from the camera. This limit will be dictated by the resolution of the imaging system and the maximum focal length of the camera.

Finally, in Figures A.9 and A.10, the maximum focal length allowed (by the zoom operation) was varied. The lower that this value was constrained to be, the higher the mean error and standard deviation of the radius measurement tests were. This relates very closely to the above arguments for a higher resolution imaging system, as increasing the distance from the focal point to the image plane effectively spreads the same number of pixels over a smaller, real-world viewing area. For a real system, there will be a limit as to how much zoom should be allowed, however, as having a too magnified view of the scene will mean that more time will have to be spent panning and tilting the camera around in search of the feature points. Further, it might have some effect on just how close the logs can get to the camera before they become out of focus.
The above is an analysis of the errors arising from scaling logs with an ideal pin-hole camera model. All of these comments and recommendations could equally well apply to a real-world situation. As this has decoupled the effects of errors of the camera modelling process from the scaling process, distinct trends were noticeable and conclusions drawn. However, the error values themselves are optimistic, as only the use of an actual imaging system will be able to provide a complete system error description.
Figure A.1: Simulation Results: Mean Log Radius Error vs. Log Angle with respect to the Image Plane; radius = 10 cm, distance to log from camera base point = 15 m, camera height above base point = 5 m

Figure A.2: Simulation Results: Standard Deviation of the Mean Log Radius Error vs. Log Angle with respect to the Image Plane; radius = 10 cm, distance to log from camera base point = 15 m, camera height above base point = 5 m
Figure A.3: Simulation Results: Mean Log Radius Error vs. Log Radius with respect to the Image Plane; distance to log from camera base point = 15 m, camera height above base point = 5 m

Figure A.4: Simulation Results: Standard Deviation of the Mean Log Radius Error vs. Log Radius with respect to the Image Plane; distance to log from camera base point = 15 m, camera height above base point = 5 m
Figure A.5: Simulation Results: Mean Log Radius Error vs. Distance to the Log along the Line $z = 0$; radius = 10 cm, camera height above base point = 5 m

Figure A.6: Simulation Results: Standard Deviation of the Mean Log Radius Error vs. Distance to the Log along the Line $z = 0$; radius = 10 cm, camera height above base point = 5 m
Figure A.7: Simulation Results: Mean Log Radius Error vs. Distance to the Log along the Line $x = 2z$; radius = 10 cm, camera height above base point = 5 m

Figure A.8: Simulation Results: Standard Deviation of the Mean Log Radius Error vs. Distance to the Log along the Line $x = 2z$; radius = 10 cm, camera height above base point = 5 m
Figure A.9: Simulation Results: Mean Log Radius Error vs. Maximum Allowed Focal Length (Zoom); radius = 10 cm, distance to log from camera base point = 15 m, camera height above base point = 5 m

Figure A.10: Simulation Results: Standard Deviation of the Mean Log Radius Error vs. Maximum Allowed Focal Length (Zoom); radius = 10 cm, distance to log from camera base point = 15 m, camera height above base point = 5 m
Appendix B

Experimental Results - Calibration

In Chapter 4, a description of the results of some tests run to analyze the camera calibration process were described. This section will include a complete listing of those results.

The mean value for the rms error, $\phi$, as a function of the number of iterations carried out is shown in Figure B.1, and the standard deviation of this value, $\sigma_\phi$, is shown in Figure B.2. The image used for those tests consisted of a cube with seven visible vertices that covered a large portion of the image plane (the test image). In the first stage of the calibration, the rms error dropped from a value of over 24 pixels to a value around 2.5 pixels. This drop occurred very sharply (2 iterations) and the standard deviation similarly exhibited a sharp drop at the same time. After two iterations, the first stage exhibited no further optimization of the parameters. Stage 2 produced more modest improvements, with a decrease in the rms error plot down to about 2.14 pixels. The standard deviation of this value over all of the tests decreased to a value of around 0.46 pixels, indicating that the result was quite consistent and not overly dependent on the spotting process involving the operator.

This is not to say that the optimization was independent of the image used to calibrate the camera. The image used for these plots contained information about a
great deal of the image plane and, thus, provided the best model fit to reality. While
the actual camera parameters that one might physically measure might be slightly
different than those derived here, a model based on them would not be as accurate
on the whole as the model derived by this calibration. This calibration technique
fits model-based coordinate calculations to real-world data. Any portion of the
image space which is not within the span of the input space will not necessarily be
well described by the model.

It was for the purpose of illustration of this fact that a group of five further
images were taken and the calibration tests re-run. In these images, a smaller
version of the cube was located in each of the four corners of the image and the
centre, thus intentionally only calibrating a portion of the image space. This tended
to skew the results of the model fit to values that, while being just as accurate (if
not more so) for that portion of the image, were not as conclusive for the rest if the
image space.

Figures B.3-B.42 show histograms of the values derived from these tests for all
ten parameters. In the first plot for each of the camera parameters, the distribution
of the parameter values derived from the test image are shown. These values are
closest to those desired from the optimization and this is further borne out by the
fact that the mean of their values is generally very close to the mean derived from
the aggregate of all of the tests performed here (shown in the fourth plot for each
parameter).

The second and third plots show the distribution of values derived for each of
the parameters under more limited conditions. The image used for the second plot
contained the scaled down cube placed in the centre of the image. This is where
the model is the most linear. It is not the best fit to the model however, due to
the non-linearity towards the edge of the image. The third image shows the results
obtained for each parameter upon calibration of the camera with just data from the
corners of the image plane.
The results here show a calibration process fitting a simple, linear model of a camera to a non-linear, real-world situation with consistent results. The residual behaviour converges quickly and repeatably. The values for individual parameters derived produce the best fit of the camera model to the information provided to it.

The question of whether this is accurate enough is best answerable by using the parameter sets derived from here for the scaling process. This is the subject of Section 4.3.3 and appendix C.
Figure B.1: RMS Error, $\Phi$, Convergence Behaviour Using Data from the Test Image; 7 feature points per image, 5 trials

Figure B.2: Standard Deviation of the RMS Error, $\sigma_\Phi$, Convergence Behaviour Using Data from the Test Image; 7 feature points per image, 5 trials
Figure B.3: Result Distribution for $fM_v$ Estimation Using Data from the *Test* Image; 7 feature points per image, 5 trials

Figure B.4: Result Distribution for $fM_v$ Estimation Using Data from the *Centre* Image; 7 feature points per image, 5 trials
Figure B.5: Result Distribution for $fM_y$ Estimation Using Data from the Corner Images; 7 feature points per image, 20 trials

Figure B.6: Result Distribution for $fM_y$ Estimation Using Data from all 6 Images; 7 feature points per image, 30 trials
Figure B.7: Result Distribution for $M_{\text{ratio}}$ Estimation Using Data from the Test Image; 7 feature points per image, 5 trials

Figure B.8: Result Distribution for $M_{\text{ratio}}$ Estimation Using Data from the Centre Image; 7 feature points per image, 5 trials
Figure B.9: Result Distribution for \( M_{ratio} \) Estimation Using Data from the Corner Images; 7 feature points per image, 20 trials

Figure B.10: Result Distribution for \( M_{ratio} \) Estimation Using Data from all 6 Images; 7 feature points per image, 30 trials
Figure B.11: Result Distribution for $\theta_x$ Estimation Using Data from the Test Image; 7 feature points per image, 5 trials

Figure B.12: Result Distribution for $\theta_x$ Estimation Using Data from the Centre Image; 7 feature points per image, 5 trials
Figure B.13: Result Distribution for $\theta_z$ Estimation Using Data from the Corner Images; 7 feature points per image, 20 trials

Figure B.14: Result Distribution for $\theta_z$ Estimation Using Data from all 6 Images; 7 feature points per image, 30 trials
Figure B.15: Result Distribution for $\theta_y$ Estimation Using Data from the Test Image; 7 feature points per image, 5 trials

Figure B.16: Result Distribution for $\theta_y$ Estimation Using Data from the Centre Image; 7 feature points per image, 5 trials
Figure B.17: Result Distribution for $\theta_v$ Estimation Using Data from the Corner Images; 7 feature points per image, 20 trials

Figure B.18: Result Distribution for $\theta_v$ Estimation Using Data from all 6 Images; 7 feature points per image, 30 trials
Figure B.19: Result Distribution for $\theta_z$ Estimation Using Data from the Test Image; 7 feature points per image, 5 trials

Figure B.20: Result Distribution for $\theta_z$ Estimation Using Data from the Centre Image; 7 feature points per image, 5 trials
Figure B.21: Result Distribution for $\theta_z$ Estimation Using Data from the Corner Images; 7 feature points per image, 20 trials

Figure B.22: Result Distribution for $\theta_z$ Estimation Using Data from all 6 Images; 7 feature points per image, 30 trials
Figure B.23: Result Distribution for $h_0$ Estimation Using Data from the Test Image; 7 feature points per image, 5 trials

Figure B.24: Result Distribution for $h_0$ Estimation Using Data from the Centre Image; 7 feature points per image, 5 trials
Figure B.25: Result Distribution for $h_0$ Estimation Using Data from the Corner Images; 7 feature points per image, 20 trials

Figure B.26: Result Distribution for $h_0$ Estimation Using Data from all 6 Images; 7 feature points per image, 30 trials
Figure B.27: Result Distribution for $v_0$ Estimation Using Data from the Test Image; 7 feature points per image, 5 trials

Figure B.28: Result Distribution for $v_0$ Estimation Using Data from the Centre Image; 7 feature points per image, 5 trials
Figure B.29: Result Distribution for $v_0$ Estimation Using Data from the Corner Images; 7 feature points per image, 20 trials

Figure B.30: Result Distribution for $v_0$ Estimation Using Data from all 6 Images; 7 feature points per image, 30 trials
Figure B.31: Result Distribution for $X_0$ Estimation Using Data from the Test Image; 7 feature points per image, 5 trials

Figure B.32: Result Distribution for $X_0$ Estimation Using Data from the Centre Image; 7 feature points per image, 5 trials
Figure B.33: Result Distribution for $X_0$ Estimation Using Data from the Corner Images; 7 feature points per image, 20 trials

Figure B.34: Result Distribution for $X_0$ Estimation Using Data from all 6 Images; 7 feature points per image, 30 trials
Figure B.35: Result Distribution for $Y_0$ Estimation Using Data from the Test Image; 7 feature points per image, 5 trials

Figure B.36: Result Distribution for $Y_0$ Estimation Using Data from the Centre Image; 7 feature points per image, 5 trials
Figure B.37: Calibration Results: $Y_0$ Estimation Histogram - Corner] Result Distribution for $Y_0$ Estimation Using Data from the Corner Images; 7 feature points per image, 20 trials

Figure B.38: Calibration Results: $Y_0$ Estimation Histogram - Total Result Distribution for $Y_0$ Estimation Using Data from all 6 Images; 7 feature points per image, 30 trials
Figure B.39: Calibration Results: $Z_0$ Estimation Histogram - Test] Result Distribution for $Z_0$ Estimation Using Data from the Test Image; 7 feature points per image, 5 trials

Figure B.40: Calibration Results: $Z_0$ Estimation Histogram - Centre] Result Distribution for $Z_0$ Estimation Using Data from the Centre Image; 7 feature points per image, 5 trials
Figure B.41 Calibration Results: $Z_0$ Estimation Histogram - Corner Result Distribution for $Z_0$ Estimation Using Data from the Corner Images; 7 feature points per image, 20 trials

Figure B.42: Calibration Results: $Z_0$ Estimation Histogram - Total Result Distribution for $Z_0$ Estimation Using Data from all 6 Images; 7 feature points per image, 30 trials
Appendix C

Experimental Results - Scaling

This section contains a more complete listing of the results derived from scaling a test cylinder (radius = 12.86 cm, length = 50.48 cm), using those parameters derived from the calibration process of appendix B.

In Figures C.1- C.3, those parameters determined from the test image (which contained information about most of the image plane) were used to scale the object. In C.1, the object was located in the centre of the image plane and the results are correspondingly good. The mean radius is as accurate as could ever be hoped, while the standard deviation of the results (which is perhaps more telling in this case) is only about 0.15 cm. This corresponded to 0.06% of full scale, or 0.29 pixels.

When the same camera model was used to scale the object in each of the four corners of the image, the result was not as good. This portion of the image is the least linear which leads to a decrease in both accuracy and precision [21]. Figure C.3 shows the aggregate distribution for all of the measurements with this model.

In Figure C.4, the twenty data sets derived from the four corners of the image space are used to scale the object in the same corner of the image space that the model was derived for. This led to quite accurate results. When these same models were used to scale the object, first in the centre of the image (Figure C.5), and then in the opposite corner of the image space (Figure C.6), the results were degraded.
Figure C.5 also contains the results of using the model derived from solely the centre portion of the image on objects located in each of the four corners.

Finally, Figure C.7 shows the radius measurement distribution for all of the data. The mean is in error by less than 0.06 cm (0.12 pixels), while the standard deviation is slightly more than 0.5 cm (1.03 pixels).

The same seven result distributions are shown for the length calculations in Figures C.8- C.14. The accuracy achieved is not nearly as good in absolute value as that of the radius, but is still quite acceptable. A consistently high bias on the calculation of the length would tend to point to a human error in estimating the exact end points of the targets profile or a camera modelling error in the scale factor of one of the image’s axes. Still, the mean length for all of the tests is within 0.6 cm (1.14 pixels) of reality.

The precision of the results indicates that the distribution of the calculations has a standard deviation of 1.2398 cm (almost one-half inch). Considered with the fairly accurate mean length derived, one can see that this will quite likely be more accurate than what one could do for a log in a sort yard with a measuring stick. A 50 cm target may be simple enough to scale with a measuring stick, but it must be remembered that (as determined in the simulation portion of the test results) the measurements of each feature point are largely independent of the proximity of each of the others. Therefore, the accuracy and precision derived for the cylinder here are going to be independent of its size.

When the same cylinder was placed at about a 40° angle to the image plane, the scaling accuracy decreased substantially. The mean measured radius was high by 2.33 cm (see Figure C.15), while the length was low by 2.23 cm (see Figure C.16). It is felt that the only reason for this is the difficulty in locating the end points on the rounded log projection. While this magnitude of error, when combined with real-world non-idealities, may lead to an inadequate solution, it can be simply prevented by restricting the placement of the logs such that they are reasonably parallel to
the image plane, as was suggested in the simulation results.

A smaller object (radius = 2.54 cm, length = 13.65 cm) was scaled to test that the magnitude of the error would remain largely independent of the size of the object. The results, which have an accuracy and precision on par with the standard results, verifies this in Figures C.17 and C.18.

Finally, a real log was measured, as shown by the distributions in Figures C.19 and C.20. The actual measured values for the mean end point radius and length were 4.24 cm (±0.1 cm) and 1.017 m (±0.25 cm). These values are less accurate due to the inclusion of a non-cylindrical object to be scaled with a cylindrical model. The mean error in the radius and the standard deviation of this value are 0.24 pixels and 3.36 pixels, respectively.
Figure C.1: Distribution of the Log Radius Scaling Experiments; cylinder in the 
centre of the image, test model, 5 trials

Figure C.2: Distribution of the Log Radius Scaling Experiments; cylinder in the 
corners of the image, test model, 20 trials
Figure C.3: Distribution of the Log Radius Scaling Experiments; cylinder in the corners (4) and the centre (1) of the image, test model, 25 trials

Figure C.4: Distribution of the Log Radius Scaling Experiments; cylinder in the corners of the image, matching corner models, 20 trials
Figure C.5: Distribution of the Log Radius Scaling Experiments; cylinder in the centre of the image with corner models and in the corners of the image, 40 trials

mean = 12.5652 cm
st. dev. = 0.5060 cm

Figure C.6: Distribution of the Log Radius Scaling Experiments; cylinder in the corners of the image, opposite corner models, 20 trials

mean = 12.7587 cm
st. dev. = 0.6549 cm
Figure C.7: Distribution of the Log Radius Scaling Experiments; aggregate of all of the experiments, 105 trials.
Figure C.8: Distribution of the Log Length Scaling Experiments; cylinder in the centre of the image, test model, 5 trials

Figure C.9: Distribution of the Log Length Scaling Experiments; cylinder in the corners of the image, test model, 20 trials
Figure C.10: Distribution of the Log Length Scaling Experiments; cylinder in the corners (4) and the centre (1) of the image, test model, 25 trials

Figure C.11: Distribution of the Log Length Scaling Experiments; cylinder in the corners of the image, matching corner models, 20 trials
Figure C.12: Distribution of the Log Length Scaling Experiments; cylinder in the centre of the image with corner models and in the corners of the image, 40 trials

Figure C.13: Distribution of the Log Length Scaling Experiments; cylinder in the corners of the image, opposite corner models, 20 trials
Figure C.14: Distribution of the Log Length Scaling Experiments; aggregate of all of the experiments, 105 trials
Figure C.15: Distribution of the Log Radius for a Log at 40° to the Image Plane; test model, 5 trials

Figure C.16: Distribution of the Log Length for a Log at 40° to the Image Plane; test model, 5 trials
Figure C.17: Distribution of the Log Radius for a Smaller Cylinder; test model, 5 trials, actual radius = 2.54 cm

Figure C.18: Distribution of the Log Length for a Smaller Cylinder; test model, 5 trials, actual length = 13.65 cm
Figure C.19: Distribution of the Real Log Radius; test model, 5 trials, actual mean end radius = 4.20 cm

Figure C.20: Distribution of the Real Log Length; test model, 5 trials, actual mean length = 1.017 m