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CREDIT RATIONING WITH AN INDIVIDUAL SHORT-SIDE RULE: ESTIMATION FOR BUSINESS LOANS IN CANADA

by

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ABSTRACT

It is generally accepted that rationing occurs in loan markets with demand of some borrowers exceeding desired supply by lenders at prevailing interest rates. Previous empirical studies of credit rationing use established disequilibrium econometric methods to estimate structural models of business loan markets. This study argues that existing disequilibrium techniques are not suitable for analyzing credit rationing since they ignore features of loan markets emphasized in the theoretical literature. While recent theory distinguishes between equilibrium and disequilibrium categories of credit rationing, existing empirical work allows only the latter to exist. In addition the traditional empirical model does not derive loan equations from micro foundations despite a theoretical focus on loan determination at the individual borrower level. Instead, equations are constructed by assuming aggregate loan quantity corresponds to the minimum of aggregate supply and demand. These inconsistencies with theory suggest that estimates of rationing from the traditional model are unreliable.

Unlike traditional methods the empirical model developed in this study derives aggregate equations from a micro approach to loan determination. Individual loan sizes are determined by the minimum of borrower-specific supply and demand functions and explicit aggregation across all borrowers gives estimating equations with desired properties. An attractive feature of the new model is that it yields the first estimates of equilibrium credit rationing. This allowance for both equilibrium and disequilibrium rationing, together with the micro foundations, means that the proposed model provides greater consistency between theoretical and applied work than has been previously possible.

The new model is applied to the market for business loans from Canadian banks for the period 1968 to 1979. Results indicate that rationing is empirically significant as total rationing averages approximately one-third of aggregate flow demand for loans. Equilibrium rationing appears to be an important phenomenon since it exceeds disequilibrium rationing each period. However, intertemporal fluctuations in total rationing are caused primarily by changes in disequilibrium rationing. A comparison of the new and traditional models shows that rationing estimates are greater in the new approach with much of the difference attributable to the amount of equilibrium rationing in that model.

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CHAPTER 1

INTRODUCTION

1.1 Credit Rationing

Since the early 1950s there has been frequent controversy concerning the relative importance of price versus non-price allocation in loan markets. Numerous authors have identified characteristics of loan markets that allegedly prevent the interest rate from adjusting sufficiently to eliminate all excess demand for loans. This imperfect price flexibility means that some individuals experience unsatisfied demands at the rate of interest quoted for their loans. More formally, "credit rationing" occurs whenever an individual's desired loan demand is greater than the lender's actual supply at the interest rate set by a lender. It is necessary for lenders to use non-price criteria to determine whether a given customer is rationed or receives the loan size requested.

The existence of credit rationing appears to violate conventional economic analysis which emphasizes the allocative function of the price mechanism. In a competitive market non-price rationing would not occur in equilibrium since excess demand induces price increases until supply and demand are equalized. Similarly, rationing does not develop in standard analysis of monopoly pricing since the quantity traded is determined along the demand curve. Consequently, it must be considered why transactions may occur in loan markets at interest rates consistent with excess demand. Theoretical explanations of credit rationing, to be discussed in Chapter 2, must address the fundamental issue of the rationality of lenders allocating loans (at least in part) by non-price means.

If credit rationing does exist it has potential repercussions in several areas. The allocation of loanable funds among different types of borrowers may differ systematically according to whether loan decisions are guided by a pure price system or some combination of price and non-price factors. Therefore, credit rationing might influence the relative growth of different sectors in the economy by affecting the relative availability of loans to these sectors. The precise nature of these availability effects would depend on the particular non-price criteria used to decide which borrowers are rationed.

The possibility of rationing also poses interesting questions concerning the impact of a quantity constraint in one financial market on a firm's transactions in other financial and real markets. Potential borrowers may have access to several alternative sources of finance and adjust their demands among these sources if initial plans are not realized. As an example, a firm seeking to finance an investment project may try to obtain additional funds from non-bank financial intermediaries if it is rationed by a bank. The ultimate impact on real spending of rationing in the bank loan market would depend on the rationed customers' success in other financial markets and on the cost of such alternative financing.

The macroeconomic consequences of credit rationing have been noted in discussions of monetary policy effectiveness. The principle channel of monetary policy impact on real variables is often identified with interest rate effects.[1] Contractionary monetary policy which raises interest rates will decrease output and employment provided some component of aggregate demand is interest-sensitive. However, if credit rationing is an empirically significant phenomenon, it represents a second route for monetary policy to influence the real sector. If monetary contraction is accompanied by lagged adjustment of loan interest rates, the quantity of loans granted can be constrained by the lenders' willingness to supply

1. Alternative channels of monetary policy influence on the real sector are discussed in Park (1972).

loans rather than borrowers' demands at the current rate of interest.[2] This non-price rationing may decrease the aggregate volume of real expenditures by imposing financing constraints on activities of borrowers.

The addition of credit rationing to the list of monetary policy transmission mechanisms is significant for two reasons. Even if investment spending is not very interest-elastic, so that the interest rate channel is weak or non-existent, monetary policy could still have real effects by changing the volume of credit rationing. In addition, the speed with which monetary policy operates on real variables should depend on the strength of the rationing channel. Tucker (1968) notes that lagged adjustment of loan interest rates after contractionary monetary policy has two conflicting tendencies for investment demand. The immediate effect on investment through the interest rate channel is reduced since movement to higher interest rates has been delayed. However, as described previously, the partial adjustment of the loan interest rate creates non-price rationing which has the opposite effect of strengthening the response of investment spending to monetary policy. The relative strength of these two opposing forces influences the speed of output adjustment after contractionary monetary policy.[3,4]

Empirical studies are required to gain insights into such issues as the quantitative significance of rationing, the degree to which rationing in one loan market is offset by increased activity in other financial

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2. This point is considered by Park (1972) and Scott (1957a).
3. Tucker (1968) analyses the relationship between lagged interest rate adjustment and the speed of monetary policy effects with a dynamic IS-LM model. He concludes (p.83) that "if credit rationing has a significantly strong impact on investment demand, and if there is a significant market-clearing lag in the product market, then the economy will respond more rapidly to monetary contraction when the interest rate is sticky than when it behaves with extreme flexibility."
4. An indication that monetary authorities do perceive the operation of both interest rate and rationing channels is found in Rasminsky (1969), p. 14.

markets, and the ultimate impact of credit rationing on real expenditures. However, adequate investigation of these issues has been impeded by difficulties in obtaining a satisfactory empirical measure of credit rationing. In response to this deficiency the present study develops a methodology that can provide quantitative estimates of the volume of rationing. The model is then applied to the business loans market of Canadian chartered banks. Before proceeding to these objectives it will be beneficial to outline a priori evidence for the existence of credit rationing in Canada and describe the business loans market of Canadian banks.

1.2 A Priori Evidence

Several sample surveys contain information which suggests that credit rationing is an empirical phenomenon in Canada. The most comprehensive survey was undertaken as part of a recent study on the role of chartered banks in small business financing. Results of this survey, summarized in Hatch, Wynant, and Grant (1982), were derived from three sources of information. First, 400 small businesses responded to a questionnaire concerning their dealings with Canadian chartered banks. Secondly, interviews were carried out with 120 bank employees to determine the lenders' perspective on the loan process. Finally, 2,300 actual loan files for both small and large businesses were reviewed.

Analysis of the third information source indicated that on average firms in the sample had received approximately 90 per cent of the amounts requested on formal loan applications. However, the 10 per cent unsatisfied demand on these formal applications probably understates the overall magnitude of rationing. Interviews with bank branch managers suggested that about 25 per cent of all loan inquiries are rejected before reaching the formal application stage of the loan process. It is also possible that some loan sizes actually requested on formal applications were less than the amounts originally sought by the borrowers. Hence, the 10 per cent figure derived from actual applications may be regarded as a lower bound estimate of the prevalence of credit rationing. A final piece of evidence from the Hatch-Wynant-Grant study was a finding that one-third of all firms responding to the questionnaire had been denied a loan request at some time during the preceding three year period.[5]

Additional a priori indications of credit rationing in Canada may be inferred from comments of J.A. Galbraith who has been actively involved within the financial sector. Galbraith observed circumstances when interest rates did not adjust completely to changes in market conditions and non-price rationing was necessary.

> "When the monetary policy of the day restricts the lending resources of the banks, the banks must restrict their lending activities. In circumstances such as those faced by Canadian banks at the end of the 1960s, lending activity at the branches has to be curtailed. When lending rates are not raised sufficiently to discourage the demand for loans or when higher rates apparently fail to restrict demand to the available resources, instructions have to be communicated to the branches to curtail loans. A set of priorities is developed to be used as a guide by the branches."[6]

A statement by the Canadian Bankers' Association lists some of the non-price guidelines used by lenders to establish loan priorities during these periods.

> "... during periods when credit is less easily available a change in attitude must take place. Lending must become more selective. Several categories of loans come

5. Sears (1972) discusses some survey evidence on credit rationing from the early 1960s in Canada. Jaffee (1971), pp. 159-161, presents survey information from the United States.
6. Galbraith (1970), p. 255.

> under restriction, the severity of which depends on the tightness of the credit squeeze. For example, during a period of credit stringency all forms of lending related to speculative activity, such as trading in land and securities, come under early restriction. Applications for new or increased credit are also given close scrutiny. Certain types of programs, such as consumer instalment loans, are curtailed, with promotional activity being discontinued. New lending programs under consideration are postponed. Even borrowers with long-established lines of credit are asked to review their requirements under existing commitments if the liquidity position of the individual bank requires such action." [7]

1.3 Business Loans Market

The evidence presented in the previous section suggests that credit rationing does occur in Canada and that further empirical examination is warranted. As noted previously the principal focus of this study is an analysis of rationing in the market for business loans from Canadian chartered banks. Some basic details will establish the institutional background and relative importance of this financial market.

Activities of Canadian banks are governed by federal legislation under the Bank Act. In the 1967 revision of this legislation there were several major changes affecting the lending function of banks.[8] Prior to this revision there was a ceiling of 6% on the interest rate that could be charged on a loan by chartered banks. This maximum rate provision was removed in stages under the 1967 Bank Act until regulation of interest rates was eliminated completely by the beginning of 1968. A second regulatory change in 1967 increased the ability of chartered banks to engage in mortgage lending.

These Bank Act revisions represented a significant structural change having important implications for any empirical study of credit rationing.

7. Quoted in Galbraith (1970), pp. 253-254, from a presentation of the Canadian Bankers' Association to the Committee on Finance, Trade and Economic Affairs, September 1969.
8. These changes are discussed in Shearer, Chant and Bond (1984), p.361.

Comments of the Governor of the Bank of Canada during that period indicated a belief that the legislative ceiling on loan rates had increased the incidence of non-price rationing of business loans.

> "A good case could be made that in the past the legal restrictions on chartered bank lending rates backfired on the groups of borrowers they were intended to protect - such as small businessmen - because the banks were deprived of any profit incentive to make a serious effort to increase their access to bank credit ... It used to be the case that monetary restraint affected lending policies just about as much as it affected interest rates. Flexibility in the banks' lending rates and consequently in their deposit rates was limited by the Bank Act, and when the banks' liquidity was under downward pressure they were forced to adopt more selective lending policies, in other words to ration loans more closely."[9]

The legislative constraint on loan interest rates was a source of credit rationing unique to the pre-1968 period. In order to isolate the magnitude of rationing arising from non-legislative sources the time period selected for subsequent empirical work begins in 1968.

In nominal terms the value of business loans outstanding from chartered banks increased from $6.9 billion at the end of 1967 to $55.4 billion in 1980 (see Table 1). These loans are an important component of the banks' asset portfolios. Approximately 20% of total bank assets were held as business loans over the 1967-1980 period.

Table 2 indicates that chartered banks are the primary source of all loans to businesses in Canada. It is estimated that in 1979 approximately 80% of the total value of business loans from financial institutions were granted by chartered banks. This statistic suggests that a study of rationing by chartered banks should give a reliable indication of the overall degree of credit rationing confronting Canadian businesses.

9. Rasminsky (1969), pp. 13-14.

TABLE 1

Business Loans Outstanding from Chartered Banks
(millions of dollars, end of period values)

	Business Loans Outstanding	Total Bank Assets	Ratio of Business Loans to Total Assets
1967	6,929	31,669	.219
1968	7,589	36,746	.207
1969	8,654	42,632	.203
1970	8,900	47,307	.188
1971	11,068	54,428	.203
1972	13,461	63,222	.213
1973	17,135	79,754	.215
1974	20,568	97,015	.212
1975	23,228	108,378	.214
1976	28,218	126,403	.223
1977	31,323	150,477	.208
1978	34,441	189,100	.182
1979	44,866	229,440	.196
1980	55,385	281,244	.196

Source: Bank of Canada Review

TABLE 2

Business Loans Outstanding from Financial Institutions
(millions of dollars, end of 1979)

Chartered Banks	44,866
Trust Companies	276
Mortgage Loan Companies	173
Credit Unions	581
Financial Corporations	5,718
Financial Leasing Corporations	333
Business Financing Corporations	4,286
Total Business Loans	56,233

Institutions	Definitions of Business Loans
Chartered Banks	Business loans outstanding
Trust Companies	Other collateral business loans
Mortgage Loan Companies	Other collateral business loans
Credit Unions	Non-mortgage loans: commercial, industrial, and co-operative enterprises
Financial Corporations, Financial Leasing Corporations, Business Financial Corporations	Business loans: commercial; Retail sales financing: industrial and commercial; Wholesale financing

Sources: Bank of Canada Review, April 1980, Table 10.
Financial Institutions: Financial Statistics, April 1982, Statistics Canada, Tables 8, 16, 21, 23, 33, 42 and 46.

1.4 Contents

Recent theoretical explanations of credit rationing have emphasized a distinction between equilibrium and disequilibrium categories of rationing. Whereas equilibrium models explain why excess demand for loans may persist even at the equilibrium rate of interest, disequilibrium rationing is a transitory consequence of lagged adjustment of the interest rate to its equilibrium level.

Current empirical studies of rationing may be criticized for neglecting to incorporate the theoretical interest in both equilibrium and disequilibrium credit rationing. Existing econometric evidence has been obtained from estimation of aggregate demand and supply equations for business loans. These two equations are derived from an "aggregate short-side rule" that assumes the observed aggregate loan volume is the minimum of aggregate demand and supply. Furthermore, these studies postulate that the equilibrium interest rate occurs at the intersection of the two aggregate functions. Taken together these two assumptions imply that the current method of estimating the volume of rationing focuses exclusively on disequilibrium rationing and ignores the equilibrium category.

The objective of this study is to reduce the current discrepancy between theory and econometric practice by estimating a model which combines equilibrium and disequilibrium credit rationing. In order to construct this unified model the aggregate short-side rule described above is replaced by an "individual short-side rule" found in the theoretical literature. The latter rule states that the loan size received by an individual is the minimum of borrower-specific loan demand and supply functions at the current interest rate.

The contents of the study are as follows. Chapters 2 and 3 summarize and critique theoretical explanations for the existence of credit rationing. Existing econometric models of disequilibrium rationing are presented in Chapter 4. Chapter 4 also evaluates these disequilibrium models using the theoretical individual short-side rule. Chapter 5 proposes a two-equation system that satisfies the stated objective of incorporating both rationing categories in a unified model. It is shown that parameter estimates from this system may be used to infer the separate magnitudes of equilibrium and disequilibrium rationing for each time period. Estimates of these quantities for chartered bank business loans are presented and discussed. Finally, Chapter 6 comments on potential applications of the model to the study of other issues related to credit rationing.

CHAPTER 2

Theories of Credit Rationing

Credit rationing occurs when a lender supplies a loan size that is less than borrower demand at the interest rate quoted for the loan. Conceivably this non-price allocation could have been replaced by a policy of raising interest rates until all excess loan demand is eliminated. The theoretical credit rationing literature explains the existence of rationing by identifying factors that might prevent the degree of interest rate flexibility necessary to remove all excess demand. Alternative theories may be differentiated according to the manner in which they rationalize this imperfect flexibility of loan rates. The various explanations can also be distinguished on the basis of their predictions concerning the characteristics of potential borrowers most likely to be rationed. These points are demonstrated in the survey of the theoretical literature which follows. This review will also clarify the concepts of equilibrium and disequilibrium credit rationing which are important in later empirical analysis.[1]

2.1 Availability Doctrine

Some of the earliest discussions of credit rationing originated with the availability doctrine. This doctrine was developed during the 1950s as an argument that monetary policy could be effective even if borrower behavior is insensitive to variations in interest rates. According to this viewpoint, contractionary monetary policy can decrease the volume of loans granted even if loan demand is perfectly inelastic, and the primary channel of monetary policy effectiveness is through changes in the availability of

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1. Baltensperger (1978) and Jaffee (1971) contain surveys of theoretical studies.

credit from lenders rather than the cost of borrowing.[2]

A representative theoretical exposition of the availability doctrine is given by Scott (1957a). He analyses the portfolio decision of a financial intermediary faced with the problem of allocating funds between government securities and private loans. Loans are assumed to be the higher risk asset since government securities are not subject to default risk on interest payments and repayment of principal. It is also assumed that investor utility depends on the mean and variance of portfolio yield. Scott shows that an open-market sale of government bonds by the central bank will raise expected return on the bonds and decrease the proportion of funds invested in private loans. Since it is assumed that expected return on loans is constant due to "the stickiness of customer loan rates,"[3] contractionary monetary policy reduces the quantity of loans supplied at the original loan interest rate by increasing the relative yield on government bonds.

The relationship between the availability doctrine and credit rationing can be demonstrated by interpreting Scott's result in terms of a supply-demand diagram for loans. An increase in expected yield on government bonds will shift the loan supply curve to the left at each interest rate for loans. If the initial loan rate occurred at the intersection of supply and demand, the leftward shift of the supply curve must reduce the quantity of loans transacted and cause non-price rationing if the loan rate remains unchanged as assumed by Scott. Rationing would develop even with price flexibility provided the loan rate did not move immediately to the new supply-demand intersection.

A major conclusion from Scott's analysis is that monetary policy can

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2. Roosa (1951) and Scott (1957b) provide general discussions of the availability doctrine.
3. Scott (1957a), p. 46.

be effective even if investment demand is interest-inelastic by reducing the quantity of loans supplied by lenders. Although credit rationing is given a role in this process, the availability doctrine is not considered an adequate theory of rationing. It has been criticized for not providing a theoretical explanation for the lender's failure to raise the loan rate to eliminate excess demand. For example, Jaffee (1971) states that Scott's assumption of rigidities in the loan rate "is more a statement of a necessary condition for his conclusion than an explanation."[4]

2.2 Customer Relationship Theories

Hodgman (1961) and Kane and Malkiel (1965) developed theories of rationing that consider the multi-dimensional nature of the relationship between an individual borrower and lender. In these customer relationship theories the incidence of rationing is related to factors such as the stability of the borrower-lender association and the profitability to the lender of any non-loan transactions.

Hodgman considers whether a loan applicant's demand deposit balances, by providing non-loan income to the lender, will affect that customer's success in obtaining credit. There is an incentive for banks to compete for profitable deposit accounts by offering loans to owners of these deposits at interest rates below the level charged non-depositors.[5] In a competitive market the structure of loan rates would adjust until the lender is indifferent between a loan to a borrower-depositor at a reduced interest rate and a loan to a non-depositor at a higher rate. Thus, if loan rates were competitively determined, an individual's deposits would

4. Jaffee (1971), p. 20.
5. Hodgman assumes an institutional setting where interest payments on demand deposits are prohibited by law. In this situation a bank regards a given depositor's balances as profitable if revenue from assets supported by those balances exceeds servicing costs on the account.

influence the level of the loan interest rate but would not be a source of non-price credit rationing.

A connection between deposit balances and credit rationing arises when Hodgman introduces pricing constraints into the loan process. He asserts that banks in an oligopolistic market limit inter-bank competition for deposits by implicitly agreeing not to grant any loans at rates below some minimum such as the prime rate. With this pricing constraint some customers are charged the prime rate but would have received a lower loan rate under competitive conditions due to their profitable deposit balances. Therefore, the bank's overall rate of return from all business of such individuals is above the general market return and these customers have preferred status. During periods of high demand loan requests from preferred borrowers are accommodated first and other individuals could be rationed. In principle a non-preferred customer could avoid rationing by paying a greater loan rate to match the bank's overall return from a preferred borrower. However, Hodgman argues that intertemporal considerations would induce lenders to use non-price rationing rather than raise interest rates immediately for non-preferred customers. In his opinion the former option would be less likely to cause these customers to shift their business to another bank.[6]

Kane and Malkiel's discussion parallels Hodgman's analysis and comes to similar conclusions. Refusal of a loan request could alienate the customer and lead to withdrawal of any deposits held with the lender. Since deposit variability reduces expected profit and raises lender risk,[7] loan applicants possessing large and stable deposit balances receive preferential treatment from the bank. If each bank has imperfect

6. Hodgman (1961), pp. 265-266.
7. This conclusion is obtained from a modified mean-variance model of portfolio allocation.

information on deposit characteristics of customers currently dealing with other banks, this information asymmetry among banks impedes the competitive pressures toward lower loan rates for borrower-depositors. Consequently, holders of large and stable deposits are compensated through preferential queuing during periods of excess loan demand rather than reduced interest rates on loans. Individuals without these favourable deposit characteristics would be the first to be denied loan requests. Kane and Malkiel speculate that this rationing would be a short-run result of lagged oligopolistic price adjustment after a shift in loan demand.

In the Hodgman and Kane-Malkiel studies large deposits provide protection from credit rationing because institutional or information problems prevent full compensation to holders of deposits through explicit loan or deposit interest rates. Blackwell and Santomero (1982) remove this pricing restriction and reach opposite conclusions regarding the link between deposits and the incidence of rationing. They analyse the case of a monopolistic lender that sets a profit-maximizing interest rate for each customer's loan. In this situation they argue that individuals with high deposit balances or strong intertemporal loan demand would be most susceptible to credit rationing since these characteristics reduce an individual's equilibrium loan rate. Evaluated at these individual rates, the lender's loss of profit from quantity rationing is lowest for borrowers with high deposits and intertemporal loan demand. If rationing does take place these customers would be rationed before other groups. However, Blackwell and Santomero only consider the incidence of unsatisfied loan demand under an assumption that it does exist. They do not explain the existence of rationing since the monopolist's decision to ration any customer is unprofitable at the individual rates of interest derived from their model.

2.3 Equilibrium Credit Rationing

Kane and Malkiel speculated that in the long-run loan interest rates would rise to levels that eliminated excess demands and the need for non-price rationing. In the short-run they believed rationing would occur if interest rates adjusted with a lag after an increase in loan demand. Subsequent theoretical studies have emphasized equilibrium credit rationing which refers to unsatisfied loan demands at the equilibrium rate of interest. The objective of these studies is to explain why rational lenders might be willing to set an equilibrium interest rate that is consistent with excess demand for at least some borrowers. An important implication of equilibrium rationing is that unsatisfied demands may be a permanent feature of loan markets and not merely a short-run consequence of incomplete interest rate movements. Some contributions in this area include Jaffee (1971), Koskela (1976, 1979a, 1979b), Jaffee and Russell (1976), Fried and Howitt (1980), and Stiglitz and Weiss (1981).

2.3.1 Jaffee (1971)

Jaffee examines the loan decision under an assumption that the lender can form expectations of each individual borrower's default risk.[8] In addition to this risk-screening capacity it is assumed implicitly that the lender knows each customer's demand curve. With these assumptions the existence of credit rationing is shown to depend on the nature of the price-setting regime. Jaffee initially considers a monopolistic lender that is able to charge a different interest rate for each customer's loan to reflect individual demand and risk characteristics. With this pricing system the monopolist's profit-maximizing loan is along the borrower's demand curve. Hence, equilibrium credit rationing would be non-existent

8. A formal presentation of Jaffee's model is contained in Chapter 3. Koskela (1976, 1979a, 1979b) uses a similar model.

since excess demand is zero at the lender's optimal (equilibrium) rate of interest.

A different conclusion is reached if there are constraints on a lender's ability or willingness to set different interest rates for different borrowers. In this regard Jaffee argues that it may be rational for banks to group customers into a limited number of borrower categories and charge each borrower within a given category an identical interest rate. One reason for this pricing scheme is that "the pressures of good will and social mores" tend to constrain banks from setting "widely different" rates for different customers. Furthermore, a comparatively simple borrower classification system and interest rate structure may promote coordinated collusive pricing and minimize competitive price-cutting in oligopolistic loan markets.[9] These factors could cause borrower classes to be established with limited differentiation of interest rates among customers.

Equilibrium rationing is shown to be profitable if the above factors induce a lender to charge an identical interest rate to each individual $i=1,\ldots,n$ in a given borrower category. At the group's equilibrium interest rate $\hat{R}$, which maximizes the lender's overall profit from all n customers, there will be some borrowers with demand greater than the lender's optimal supply at $\hat{R}$.[10] All borrowers in this position receive a supply-determined loan and experience equilibrium credit rationing since it would not be profitable for the lender to increase loan sizes to satisfy

9. Jaffee (1971), p. 48, elaborates on this point. "In order to prevent, or at least minimize, competitive underbidding of rates they would need tacit agreement as to the appropriate rate structure for customers, and thus a classification scheme based on readily verifiable objective criteria would appear as an efficient and effective device. Furthermore, to make the whole arrangement manageable, the number of different rate classes would have to be reasonably small."

10. The lender's optimal loan supply to an individual at alternative interest rates forms a loan offer curve. It is positively-sloped if the individual's probability of default increases with loan size.

their demands at $\hat{R}$. Default risk receives emphasis in the Jaffee model as the criterion determining which borrowers are rationed. Since default risk reduces the lender's desired supply at a given interest rate, the possibility that a borrower's demand exceeds supply at $\hat{R}$ is positively related to the individual's degree of default risk.

Jaffee's conclusion that constrained differentiation of interest rates causes credit rationing has been criticized by Azzi and Cox (1976) for disregarding the role of non-price terms of loan contracts. In response to excess loan demand lenders may adjust not only the explicit interest rate but also non-price terms such as equity, collateral, or compensating balance requirements. Azzi and Cox concluded that equilibrium credit rationing is not optimal for a lender unless there are constraints on the lender's selection of both interest rate and non-price terms for an individual's loan. While acknowledging that perfect discrimination of non-price terms among customers would eliminate rationing, Jaffee and Modigliani (1976) and Koskela (1976) argued that this condition is unlikely to be met for several reasons. In their opinions borrowers typically are limited in the amounts of equity and collateral they are able to provide. Furthermore, the same factors that limit interest rate differentiation also cause imperfect inter-customer differentiation of non-price terms. Consequently, the possibility of credit rationing is not removed by considering non-price elements of loan contracts.

2.3.2 Jaffee-Russell (1976)

A second type of equilibrium rationing model was proposed by Jaffee and Russell (1976) who consider a situation where lenders have imperfect information about the default risk characteristics of individual customers. By assumption, and in contrast with the previous model of Jaffee (1971), lenders cannot determine an individual's honesty (default risk) from a priori information. All borrowers are identical except for the degree of risk. "Honest" borrowers intend to repay their loans under all circumstances but "dishonest" borrowers default whenever this behavior raises their utility.[11] The lender's inability to screen individual customer risk means that both types of borrowers face the same loan supply schedule and are charged the same interest rate.[12]

The Jaffee-Russell model suggests two potential outcomes in a competitive market structure. Competitive pressures may lead to an equilibrium with all customers rationed at a loan size along the supply curve below the intersection of supply and demand. Both honest and dishonest individuals are rationed since the lender cannot distinguish between these groups in advance of actual loan defaults. There is also a possibility of instability under competitive conditions with entry and exit of lenders in response to short-run profits and losses. These competitive predictions differ from the pure monopoly case where the lender's profit-maximizing solution is on the loan demand curve and therefore precludes non-price rationing.

2.3.3 Stiglitz-Weiss (1981)

A number of studies have analysed equilibrium in markets where the quality of the good being traded varies with the price due to adverse

11. More specifically, a dishonest borrower defaults if the level of contractual (interest plus principal) payments exceeds that customer's cost of default. The model's conclusions are valid for any situation with uncertainty and imperfect information that causes the proportion of borrowers that default to increase with the size of contractual payments (Jaffee-Russell (1976), p. 657).
12. The loan supply schedule to an individual is positively-sloped (or possibly backward-bending) if the aggregate proportion of all borrowers that default increases with the size of contractual payments. This explanation of supply differs from the model of Jaffee (1971) where a positively-sloped supply curve is the result of an increase in the individual's probability of default at higher loan sizes.

selection effects.[13] It is concluded that when such a link exists between price and quality the equilibrium price does not necessarily coincide with the intersection of supply and demand schedules. Stiglitz and Weiss (1981) investigate whether similar behavior can explain the presence of excess demand for loans at the equilibrium interest rate.

In the Stiglitz-Weiss model default risk exists because returns from investment projects of borrowers are uncertain. By assumption lenders cannot screen customer risk directly since they cannot determine the degree of risk associated with a particular project. However, the interest rate may act as an indirect screening device if the riskiness of loans granted is known to increase with the loan rate. Stiglitz and Weiss demonstrate that this relationship can arise from two sources. As the interest rate increases a loan is no longer profitable for lower risk firms and they will not apply for loans.[14] Consequently, the higher interest rate shifts the overall mix of loan applicants toward higher risk firms. In addition to this adverse selection effect, each borrower's selection of investment projects may be affected unfavourably from the lender's perspective. If a firm is initially indifferent between two potential projects an increase in the loan rate would induce the firm to select the higher risk project.

Thus, by changing the mix of applicants and their choice of projects, an increase in interest rate can raise the general level of default risk in the lender's loan portfolio. As a result the lender's expected rate of return may reach a maximum at some interest rate and decline at higher interest rates. This non-monotonic relationship can cause equilibrium credit rationing since the equilibrium interest rate, set by lenders at the level that maximizes their expected return, may be below the intersection

13. An example of this type of study is Wilson (1980).
14. Stiglitz and Weiss measure risk by the mean-preserving spread criterion.

of the aggregate supply and demand schedules. Even with excess demand at this interest rate there would be no incentive for the lender to increase the loan rate since this action would raise the average risk on loans and lower expected lender return. Although direct screening has not been possible, risk has been controlled indirectly because borrower behavior responds to the lender's choice of interest rate.

The lender's inability to evaluate individual risk is the underlying cause of rationing in the Stiglitz-Weiss model. Therefore, if there is aggregate excess demand at the equilibrium interest rate the list of customers to be rationed cannot be determined on the basis of individual default risk as in the Jaffee (1971) case. The lender will arbitrarily ration some firms even though they appear to be identical to other applicants that do receive their loan requests.

2.3.4 Fried-Howitt (1980)

The equilibrium models of Jaffee, Jaffee-Russell, and Stiglitz-Weiss emphasize the relationship between default risk and credit rationing under various assumptions about the price-setting regime and the lender's ability to screen individual risk. Fried and Howitt (1980) take a different approach by considering a form of uncertainty other than default risk. They use implicit contract theory arguments to explain credit rationing as a possible result of an equilibrium risk-sharing agreement between a lender and its customers. In the absence of any agreements a borrower faces the risk of variations in the loan rate over time if the lender's cost of funds is a random variable. Given this intertemporal risk, borrowers might accept a higher average interest rate if the lender agrees to lower customers' risk by limiting fluctuations in the loan rate. Equilibrium contracts may emerge with credit rationing existing for some individuals if agents cannot change trading partners with zero cost. If switching costs were zero, so that borrowers could move costlessly among banks in search of a loan at the lowest possible "spot" rate, the lender would be unwilling to stabilize the interest rate and non-price rationing would not occur.[15]

2.4 Disequilibrium Credit Rationing

Equilibrium credit rationing has been defined as any excess demand that exists at the equilibrium rate of interest. The source of this rationing may originate from imperfect information on default risk by lenders (Jaffee-Russell, Stiglitz-Weiss) or constraints on inter-customer interest rate differentiation (Jaffee). There is a second source of rationing if the loan rate adjusts with a lag to a shift in supply or demand conditions. With non-instantaneous adjustment the quantity of rationing actually observed at a disequilibrium interest rate differs from the equilibrium rationing that would have occurred with complete adjustment. Any unsatisfied demand resulting from non-instantaneous movement to the long-run equilibrium loan rate may be defined as disequilibrium credit rationing. As the interest rate adjusts over time disequilibrium rationing approaches zero and in the limit all excess demand is attributed to equilibrium rationing.

Theoretical analysis of the causes of incomplete short-run movements in interest rates has been limited. Tucker (1968) examined some implications for the speed of monetary policy effects on the real sector but gave little consideration to the underlying source of imperfect flexibility. Lags in market-clearing were associated with loan market imperfections and slow-adjusting administered pricing by lenders. Jaffee's

15. Without switching costs a customer would borrow at an implicit agreement's fixed loan rate only when spot market rates, and the lender's cost of funds, were high relative to the fixed rate. Lender profit would be negative for these transactions so an agreement to stabilize the loan rate would be unprofitable for the lender.

(1971) analysis of disequilibrium rationing was also based on an argument that interest rate responses to disturbances tend to be lagged in oligopolistic loan markets.

A more rigorous analysis of the interest rate adjustment process is attempted by Koskela (1976, Chapter 6).[16] He examines optimal lender behavior when aggregate loan demand is stochastic and lump-sum costs are incurred by the lender each time the interest rate is changed. With these assumptions the interest rate is not altered after a shift in demand unless the benefits from adjustment exceed the lump-sum adjustment cost. Demand can fluctuate within some range (determined by the adjustment cost and other parameters of the model) without inducing a price change.

Further detail on disequilibrium rationing is presented in Chapter 3 where equilibrium and disequilibrium rationing are combined within a single model.

16. Koskela directly examines the optimal change in non-price terms of loan contracts but the same model could be used to explain interest rate variations.

CHAPTER 3

Imperfect Interest Rate Differentiation

Models of equilibrium credit rationing were discussed in Chapter 2 for the separate cases of perfect and imperfect screening of risk by lenders. Jaffee-Russell and Stiglitz-Weiss considered situations in which the lender has imperfect information concerning the default risk associated with an individual loan. This information deficiency can cause non-price rationing since it removes the lender's ability to discriminate among customers in its supply and interest rate behavior.

In contrast, Jaffee assumed that default risk of each potential borrower can be evaluated by the bank (perfect screening). His model was then based on the premise that a price-setting lender will engage in limited differentiation of interest rates among its customers. This imperfect differentiation was rationalized by references to moral or oligopolistic pricing considerations. Baltensperger (1978), after expressing dissatisfaction with moral or institutional rationalizations, called for alternative explanations of constrained differentiation in the perfect screening case. The present chapter is devoted to this issue. In particular, it is shown that even if moral or collusive factors were non-existent, a lender may decide to charge all customers (within a given borrower category) the same loan rate if there is imperfect a priori information on borrowers' loan demand curves. A common interest rate system may benefit the lender by avoiding bargaining problems which would develop with perfectly discriminating monopoly pricing. This explanation does not deny the potential importance of the sources of constrained inter-customer differentiation suggested previously. However, it does provide an additional justification that does not rely on moral or collusive reasons. This alternative perspective on credit rationing is discussed in section

3.4 after a formal presentation of the Jaffee (1971) model.[1]

3.1 Loan Offer Curve

The occurrence of credit rationing depends on the level of loan demand relative to desired lender supply at the prevailing rate of interest. This section analyses the lender's optimal supply behavior by considering a risk-neutral bank that faces n potential borrowers. Each of these customers seeks to finance an investment project using some combination of loans and equity. The end-of-period gross return from firm i's project is a random variable x_i defined by (2):

(1) $A_i = L_i + E_i \qquad i = 1,....,n$

(2) $\tilde{x}_i = \rho[A_i]\tilde{y}_i A_i \qquad i = 1,....,n$

with A = total investment in the project,

L = loan size,

E = equity,

x = random gross return,

y = random rate of return variable, and

$\rho[A]$ = non-random scale factor.

Gross return x is the product of total investment A and a random rate of return on investment denoted by ρy. The random variable y is multiplied by a non-random function $\rho[A]$ to allow the rate of return to vary systematically with the size of investment. The function $\rho[A]$ is assumed to have the following properties.

1. The following summary of the model is based on Jaffee (1971) and Koskela (1976, 1979a, 1979b).

$$(3)(a) \quad \rho'[A] = \frac{\delta\rho[A]}{\delta A} \leq 0$$

$$(b) \quad q[A] = \frac{\delta\rho[A]A}{\delta A} = \rho[A] + \rho'[A]A \geq 0$$

$$(c) \quad q'[A] = \frac{\delta q[A]}{\delta A} = 2\rho'[A] + \rho''[A]A \leq 0$$

Assumption (3)(a) asserts that there is either constant expected returns to scale ($\rho' = 0$) or decreasing returns ($\rho' < 0$). The remaining conditions ensure that the marginal return on investment $\delta\rho yA/\delta A = yq$ is non-negative (implied by (3)(b)) but non-increasing in A (implied by (3)(c)).

Due to the uncertainty of each borrower's total return the bank must consider the degree of default risk when calculating expected profit from a loan. It forms a subjective evaluation of the probabilities of different outcomes for the project's random rate of return y. These expectations are described by a density function $g_i(y)$.[2] It is assumed that there exist rates of return v_i and V_i, $0 \leq v_i \leq V_i < \infty$, such that

$$(4) \quad g_i(y) = 0 \quad \text{for} \quad y \leq v_i \quad \text{or} \quad y \geq V_i, \text{ and}$$

$$(5) \quad G_i[y] = 0 \quad \text{for} \quad y \leq v_i$$

$$G_i[y] = 1 \quad \text{for} \quad y \geq V_i$$

$$\text{with } G_i[m] = \int_v^m g_i(y)dy$$

$$= \text{cumulative density function evaluated at m.}$$

2. The lender's ability to perceive a density function $g_i(y)$ specific to each individual i=1,..,n indicates the perfect risk screening assumption.

The values v_i and V_i may be interpreted as the minimum and maximum rates of return (neglecting the scale adjustment ρ [A]) from the borrower's project.

The lender is interested in isolating the range of outcomes over which loan payments are defaulted. Borrower i defaults if gross returns are less than loan principal plus interest payments, that is default occurs if[3]

$$(6) \quad x = \rho yA < R_i L_i$$

with $R_i = (1 + r_i)$ = interest rate factor for borrower i, and

r_i = interest rate on borrower i's loan.

From (6) a critical rate of return (β) from the project can be calculated which is just sufficient to avoid any default on principal and interest payments.

$$(7) \quad \beta = \frac{R_i L_i}{\rho A} = \frac{R_i L_i}{\rho (L_i + E_i)}$$

At all rates of return y above the critical rate β there is no default. If y is less than β there is either partial or complete default by the borrower. The probability of realizing some default outcome is

$$G[\beta] = \int_v^\beta g(y)dy$$

3. Default condition (6) assumes there are no collateral terms or additional sources of revenue for loan payments. If collateral C is provided on the loan default is no longer synonymous with the project's return not covering loan payments. The default condition becomes $\rho yA+C<RL$ and the no-default rate of return is $\beta c=(R_i L_i - C)/\rho A$. The probability of default is reduced at given loan sizes but properties of offer curves described below are maintained when modifications to (6) are considered. For convenience (6) is used in subsequent analysis and borrower subscripts are included only for the interest rate and loan size.

Expected lender profit P_i^s on a loan to firm i may be represented by (8):

$$(8)\quad P_i^s = R_iL_i \int_{\beta}^{V} g(y)dy + \rho A \int_{v}^{\beta} yg(y)dy - IL_i$$

with I = (1 + j), and

j = lender's opportunity cost of the loan.

The initial two terms in (8) with integrals give the bank's expected revenue. The lender actually receives all contractual payments R_iL_i only if no default occurs, i.e. when $y \geq \beta$. Therefore, the no-default revenue R_iL_i is weighted by the probability of no default in the first term. The second component of (8) involves lender revenue in the event of default. It is assumed that the bank claims all gross returns ρyA of the firm when these returns are insufficient to meet loan payments, i.e. when $y < \beta$. Each default outcome is weighted by the appropriate probability to form the middle term in (8). The final term accounts for the lender's opportunity cost of supplying the loan.

A convenient form of the expected profit function is obtained by adding and subtracting $R_iL_i \int_{v}^{\beta} g(y)dy$ from (8) and noting that $\int_{v}^{V} g(y)dy = 1$ from (5).

$$(9)\quad P_i^s = R_iL_i - C_i(R_i, L_i)$$

with

$$(10)\quad C_i(R_i,L_i) = \int_v^{\beta} (R_iL_i - \rho yA)g(y)dy + IL_i$$

$$\equiv D + IL_i$$

This formulation isolates a cost function $C_i(R_i, L_i)$ which represents the lender's cost of granting a loan to borrower i. Cost at a given loan size depends on the opportunity cost I and expected default losses. Default losses equal to contractual payments R_iL_i minus realized gross returns ρyA are incurred by the lender whenever $y < \beta$. Therefore expected default losses are

$$D = \int_v^{\beta} (R_iL_i - \rho yA)g(y)dy$$

and total cost is $C_i(R_i,L_i) = D + IL_i$.

Equations (9) and (10) may be used to determine the lender's optimal loan supply to firm i at a given interest rate. Expected profit is maximized at a given R by differentiating (9) with respect to L_i and setting the result equal to zero.

$$(11)\quad \frac{\partial P_i^s}{\partial L_i} = R - \frac{\partial C_i}{\partial L_i} = 0$$

with $$\frac{\partial C_i}{\partial L_i} = \int_v^{\beta} (R_i - qy)g(y)dy + I$$

$$\equiv D' + I,$$

$$D' = \frac{\partial D}{\partial L_i} = \text{marginal default loss, and}$$

q as defined by (3)(b)

The locus of optimal loan sizes at different interest rates, implicitly given by (11), is defined as the bank's optimal loan offer curve $L_i^S(R_i)$ to borrower i. It indicates the lender's desired loan at each interest rate independent of any demand constraint. The characteristics of an offer curve are described below where $\delta\beta / \delta R_i > 0$, $\delta\beta/\delta L_i > 0$, and $(q\beta - R_i) < 0$. [4]

(12)(a) $L_i^S = 0 \qquad \text{for } R_i < I$

(b) $0 \leq L_i^S \leq \frac{v\rho E}{I-v\rho} \qquad \text{for } R_i = I \text{ and } I-v\rho > 0$

(c) $$\frac{\partial L_i^S}{\partial R_i} = \frac{-(1-G[\beta]+(q\beta-R_i)\frac{\delta\beta}{\delta R_i}g(\beta))}{(q\beta-R_i)\frac{\delta\beta}{\delta L_i}g(\beta)+q'\int_v^\beta yg(y)dy} \qquad \text{for } R_i>I$$

$$> 0 \text{ if } 1-G[\beta] + (q\beta-R_i)\frac{\delta\beta}{\delta R_i}g(\beta) > 0$$

(d) $$\frac{\delta L_i^S}{\delta I} = \frac{1}{(q\beta-R_i)\frac{\delta\beta}{\delta L_i}g(\beta)+q'\int_v^\beta yg(y)dy} < 0$$

(e) $\lim_{R_i\to\infty} L_i^S = N$ where N is finite

4. These properties are discussed in Appendix 1. Analysis could be extended to consider the impact on offer curves of risk aversion and portfolio diversification principles. With risk aversion offer curves are obtained by the lender maximizing utility $U = U(P^S,\sigma^2)$ where P^S and σ^2 are the mean and variance of lender profit, $U_1 = \partial U/\partial P^S > 0$, $U_2 = \partial U/\partial\sigma^2 < 0$, and the lender's balance sheet constraint is substituted into the definitions of P^S and σ^2. The offer curve for borrower i is defined by $U_1\partial P^S/\partial L_i + U_2\partial\sigma^2/\partial L_i = 0$. Since $\partial P^S/\partial L_i$ (defined by (11)) and $\partial\sigma^2/\partial L_i$ depend on individual i's risk characteristics borrower-specific offer curves also exist with risk aversion. Following Jaffee and Koskela, the following discussion considers the risk-neutral case.

The loan offer curve to individual i depicted in Figure 1 is based on the properties summarized by (12). The horizontal segment at $R_i = I$ corresponds to all loan sizes for which there is no possibility of default losses for the bank. Along this segment of L_i^S the critical no-default rate of return β defined by (7) is less than the investment project's minimum rate of return v. Consequently, from (11) the offer curve reduces to $R_i = I$ over this range. The positively-sloped portion above I represents loan sizes with positive default risk.[5] An increase in the lender's opportunity cost would shift the offer curve to the left according to (12)(d). Finally, offer curves to different individuals will vary due to differences in individual default risk. An increase in default risk at given loan sizes raises $\partial C_i/\partial L_i = D' + I$ and shifts the offer curve upward. Therefore, the offer curve to a high-risk borrower is above the offer curve to a lower-risk customer.

Sections 3.2 and 3.3 use offer curves to examine the possibility of credit rationing under various pricing systems.

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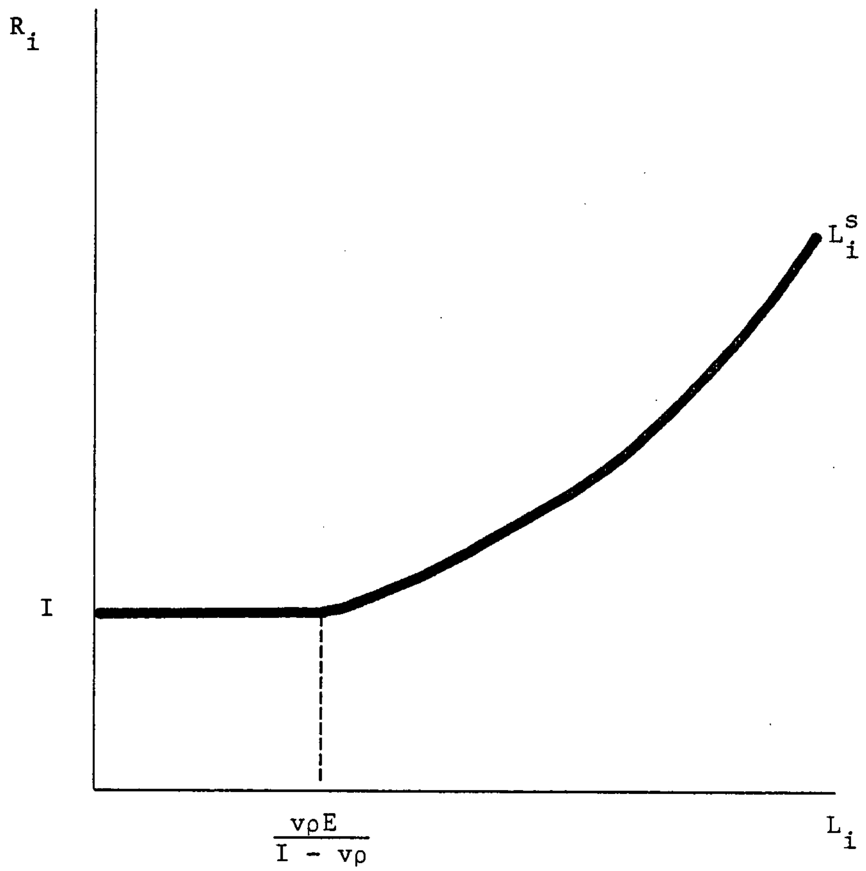
5. From (12)(c) the offer curve may be backward-bending over some range if the individual's probability of default $G[\beta]$ and the term $g(\beta)$ become sufficiently large as loan size increases.

FIGURE 1: Loan Offer Curve to Borrower i



3.2 Discriminating Monopoly Pricing

A lender acts as a discriminating monopolist if it can set a different interest rate for each loan. Interest rates will vary among customers to exploit individual demand conditions and compensate for differences in default risk. Subject to a constraint that an individual's loan cannot exceed desired demand at the interest rate charged, the discriminating monopolist chooses the interest rate and loan size which maximize expected profit P_i^s. The optimization problem is

$$(13) \quad \max_{\{R_i, L_i\}} \quad P_i^s = R_i L_i - C_i(R_i, L_i)$$

$$\text{subject to } L_i - L_i^d(R_i) \leq 0 \qquad i = 1,\ldots,n$$

with $C_i(R_i, L_i)$ = lenders cost function for granting loans to borrower i (defined by (10)), and

$L_i^d(R_i)$ = borrower i's loan demand function with $\delta L_i^d / \delta R_i < 0$

The Lagrangian functions are

$$Z_i = R_i L_i - C_i(R_i, L_i) - \lambda_i (L_i - L_i^d(R_i)) \qquad i = 1,\ldots,n$$

where λ_i is a Lagrangian multiplier. Kuhn-Tucker conditions for a maximum are:

$$(14)(a) \quad \frac{\delta Z_i}{\delta R_i} = L_i - \frac{\delta C_i}{\delta R_i} + \lambda_i \frac{\delta L_i^d}{\delta R_i} = 0 \qquad i = 1,\ldots,n$$

$$(b) \quad \frac{\delta Z_i}{\delta L_i} = R_i - \frac{\delta C_i}{\delta L_i} - \lambda_i \leq 0 \qquad i = 1,\ldots,n$$

$$(c) \quad \lambda_i \geq 0 \qquad i = 1,\ldots,n$$

(d) $L_i - L_i^d(R_i) \leq 0 \quad i = 1,\ldots,n$

(e) $\lambda_i(L_i - L_i^d(R_i)) = 0 \quad i = 1,\ldots,n$

If loan size L_i is positive then it follows from (14)(a) that $\lambda_i>0$ since $L_i - \delta C_i/\delta R_i = L_i(1 - G[\beta]) \geq 0$ and $\delta L_i^d/\delta R_i < 0$. From condition (14)(e) if $\lambda_i > 0$ then $L_i=L_i^d(R_i)$. Thus, a discriminating monopolist would not ration any borrowers since the optimal loan size to any borrower i is always determined along the demand curve $L_i^d(R_i)$. This result demonstrates Jaffee's conclusion that equilibrium rationing is never profitable for a lender that is able to charge different interest rates to all customers.

3.3 Rationing with Constrained Differentiation

As described in Chapter 2 Jaffee shows that equilibrium credit rationing is profitable for the lender if it is constrained to charge heterogeneous customers the same rate of interest. He considers a bank that maximizes expected profit by selecting optimal loan sizes for n customers and a common interest rate $\hat{R}$ for each of these n loans.[6] Realized loan size can be no greater than the borrower's desired demand at a given interest rate. The bank's decision problem is

$$(15) \quad \max_{\{R,L_1,\ldots,L_n\}} P^s = \sum_{i=1}^{n} P_i^s = \sum_{i=1}^{n} (RL_i - C_i(R,L_i))$$

$$\text{subject to } L_i - L_i^d(R_i) \leq 0 \qquad i = 1,\ldots,n$$

6. The rationality of this behavior was discussed in Chapter 2 and is re-examined in 3.5. The following analysis directly concerns a single group of n potential borrowers. However, the same conclusions hold if lenders establish a larger number of customer categories and charge each member within a given category the same interest rate (see Jaffee (1971), pp. 45-47).

Total expected profit of the lender P^s is the summation of expected profits from individual loans as defined by (9). Borrower heterogeneity is introduced through inter-customer differences in loan demand and default risk. The Lagrangian function for (15) is

$$Z = \sum_{i=1}^{n} (RL_i - C_i(R,L_i)) - \sum_{i=1}^{n} \lambda_i (L_i - L_i^d(R))$$

and the Kuhn-Tucker conditions for a maximum are:

$$(16)(a)\quad \frac{\delta Z}{\delta R} = \sum_{i=1}^{n} \left(L_i - \frac{\delta C_i}{\delta R} + \lambda_i \frac{\delta L_i^d}{\delta R}\right) = 0$$

$$(b)\quad \frac{\delta Z}{\delta L_i} = R - \frac{\delta C_i}{\delta L_i} - \lambda_i \leq 0 \qquad i = 1,\ldots,n$$

$$(c)\quad \lambda_i \geq 0 \qquad i = 1,\ldots,n$$

$$(d)\quad L_i - L_i^d(R) \leq 0 \qquad i = 1,\ldots,n$$

$$(e)\quad \lambda_i(L_i - L_i^d(R)) = 0 \qquad i = 1,\ldots,n$$

This set of Kuhn-Tucker conditions is identical to the preceding conditions for a discriminating monopolist with the exception of (16)(a) which defines the optimal value of the group's common interest rate $\hat{R}$.

The profitability of equilibrium credit rationing is now shown for the common interest rate system. It is profitable for the lender to ration customer i at the equilibrium interest rate $\hat{R}$ only if the marginal cost of the loan exceeds $\hat{R}$ when loan size is evaluated along the demand curve, that is,

$$(17)\quad \hat{R} - \frac{\delta C_i(L_i^d(\hat{R}))}{\delta L_i} < 0$$

is a necessary condition for equilibrium rationing. The possibility of rationing is seen from (18) which is obtained by substituting (16)(b) into (16)(a), imposing $L_i = L_i^d$, and noting that $\delta C_i/\delta R_i = L_i G[\beta]$.

$$(18)\quad \sum_{i=1}^{n} (L_i - \frac{\delta C_i}{\delta R} + \lambda_i \frac{\delta L_i^d}{\delta R})$$

$$= \sum_{i=1}^{n} (L_i^d(1-G[\beta]) + (R - \frac{\delta C_i}{\delta L_i}) \frac{\delta L_i^d}{\delta R}) = 0$$

The expression after the summation sign in (18) is positive for some potential borrowers and negative for others since the total summation across all n customers is zero. Some individuals with positive values will have combinations of demand and risk parameters such that $\hat{R} - \delta C_i/\delta L_i < 0$ at the loan sizes given by their demand curves at $\hat{R}$. Thus, necessary condition (17) for equilibrium rationing is satisfied for some borrowers when banks set an identical interest rate for a group of customers.

The solution to (16)(a) - (16)(e) implies that quantity determination follows a switching rule. If the demand constraint is binding ($\lambda_i > 0$) then $L_i = L_i^d$ from (16)(e). If the demand constraint is not binding, so that $\lambda_i = 0$ and $L_i < L_i^d$, then from (16)(b) $R = \delta C_i/\delta L_i$ determines loan quantity. In this event the loan size is along the lender's offer curve ($L_i = L_i^s$) since $R = \delta C_i/\delta L_i$ defines L_i^s from (11). These two results form an individual short-side rule (19) which states that customer i's loan size is the minimum of borrower-specific demand and offer curves at the quoted interest rate.

$$(19)\quad L_i = \min(L_i^d, L_i^s)$$

All individuals with supply-determined loans at the current interest rate are rationed with $L_i = L_i^s < L_i^d$. Equilibrium rationing occurs for those customers with $L_i^s = \min(L_i^d, L_i^s)$ at the equilibrium interest rate $\hat{R}$ implied by (16)(a).

Insight into the incidence of equilibrium rationing is obtained from the necessary condition (17) and the expression for marginal cost:

$$\frac{\delta C_i}{\delta L_i} = \int_v^\beta (R - qy)\ g(y)dy + I$$

$$\equiv D' + I \quad \text{from (11)}$$

Any borrower i with marginal cost $\delta C_i/\delta L_i$ above $\hat{R}$ at loan size $L_i = L_i^d(\hat{R})$ is rationed in equilibrium. Since $\delta C_i/\delta L_i$ is positively related to the default risk measure D', it is evident that higher-risk individuals have a greater likelihood of rationing at the common interest rate $\hat{R}$. The role of default risk as a determinant of rationing status is illustrated by Figure 2. Figure (2)(a) contains loan offer curves for two individuals with identical loan demand but different default risk. By assumption, customer 1 has higher risk than customer 2 ($\delta C_1/\delta L_1 > \delta C_2/\delta L_2$ at all loan sizes) so offer curve L_1^s is above L_2^s. At $\hat{R}$ short-side rule (19) indicates that the high-risk customer 1 is rationed with loan size L_1 but the lower-risk borrower 2 is unrationed at the demand-determined quantity L_2. Figure (2)(b) presents the polar case of a customer with zero default risk at all loan sizes. The lender's marginal cost is constant for this case so the offer curve to customer 3 is horizontal at R = I. With $\hat{R} > I$ this risk-free borrower is not rationed since the realized loan size is along the demand curve. In summary, at equilibrium interest rate $\hat{R}$

$$L_1 = L_1^s(\hat{R}) = \min(L_1^d, L_1^s)$$
$$L_2 = L_2^d(\hat{R}) = \min(L_2^d, L_2^s)$$

$$L_3 = L_3^d(\hat{R}) = \min(L_3^d, L_3^s)$$

and total equilibrium credit rationing (ER) is

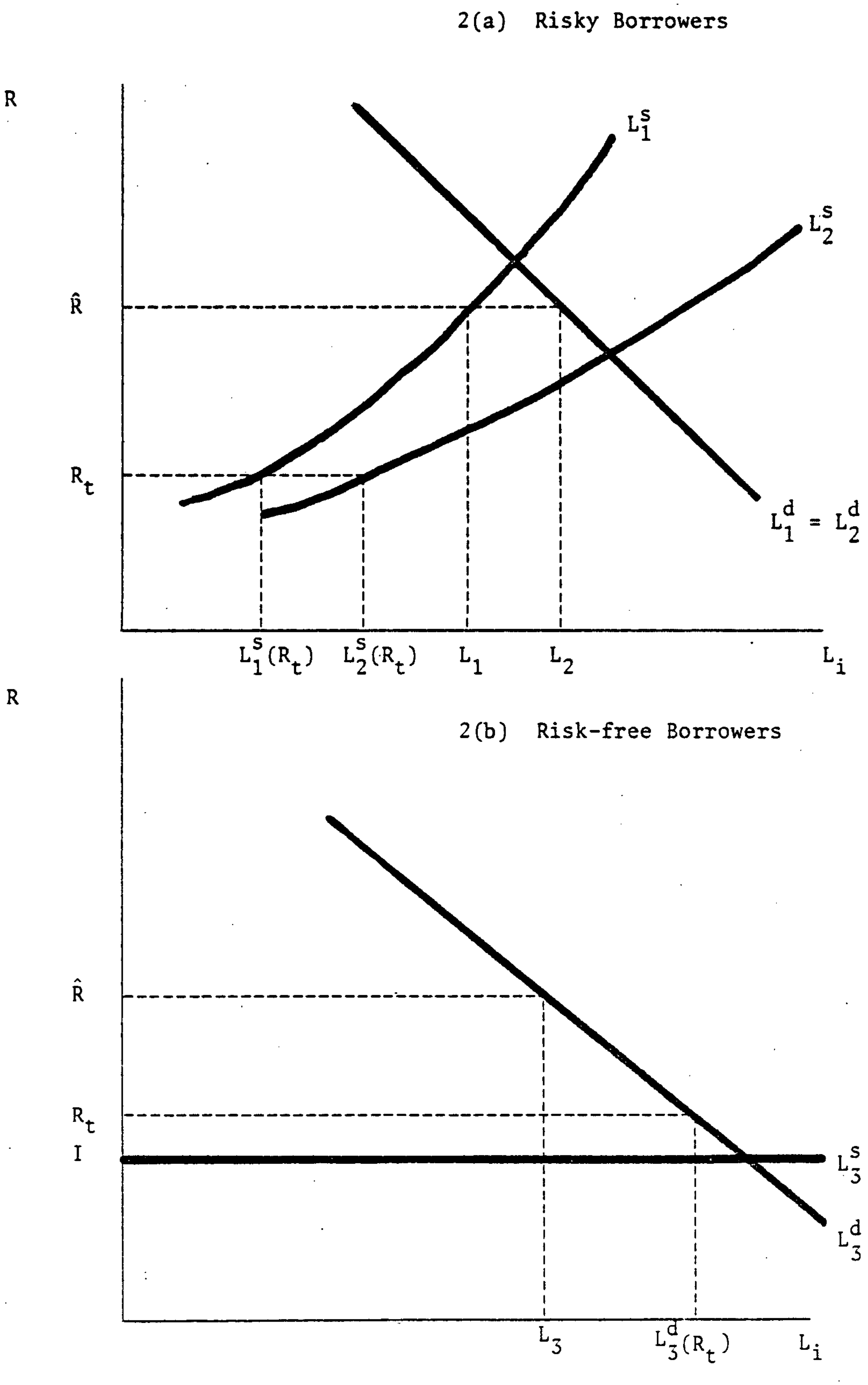
$$(20) \qquad ER = L_1^d(\hat{R}) - L_1 \cdot$$

An important comparative statics prediction of Jaffee's model is that there is no systematic relationship between the general level of interest rates and the total magnitude of equilibrium rationing. An increase in the lender's opportunity cost I has two effects with conflicting repercussions for the amount of rationing. Individual loan offer curves shift to the left (from (12)(d)) and total unsatisfied demand increases at the initial equilibrium interest rate. However, the equilibrium interest rate will rise after an increase in opportunity cost, and this adjustment in the loan rate tends to offset the initial positive impact on rationing from the shifts in offer curves. As a result the net change in equilibrium rationing is indeterminate. An increase in aggregate loan demand also has ambiguous effects on equilibrium rationing since the higher demand is accompanied by an increase in $\hat{R}$.

The preceding comparative statics results suggest that intertemporal variations in rationing are not related systematically to changes in equilibrium rationing. Instead, the primary source of these variations is disequilibrium rationing (DR) which is the difference between total rationing (TR) at the current interest rate and that which would have occurred at the equilibrium rate $\hat{R}$, i.e. DR = TR - ER.[7] This conclusion is shown using the three borrowers considered in Figure 2. Suppose the current period's interest rate R_t is below the equilibrium rate $\hat{R}$. Whereas only customer 1 would be rationed at $\hat{R}$, both risky borrowers in Figure 2(a)

7. Section 2.4 discussed reasons why the interest rate may not adjust immediately to its equilibrium level.

FIGURE 2: Default Risk and Rationing Status



now have excess demand at R_t and total rationing during the current period is

$$(21)\ TR = (L_1^d(R_t) - L_1^s(R_t)) + (L_2^d(R_t) - L_2^s(R_t)).$$

Examination of Figure 2 together with (20) and (21) indicates that total rationing exceeds equilibrium rationing when the interest rate is below its equilibrium value. This result signifies that disequilibrium rationing is positive when $R_t < \hat{R}$ since DR = TR - ER. However, as R rises toward $\hat{R}$ over time disequilibrium rationing approaches zero and total rationing falls toward the level of equilibrium rationing given by (20).

It is also apparent from Figure 2 that when the current rate of interest is above $\hat{R}$ actual rationing at R_t is less than the quantity of equilibrium rationing that would have occurred at $\hat{R}$. Therefore, disequilibrium rationing is negative with $R_t > \hat{R}_t$, since at the higher current rate fewer borrowers would be rationed and those borrowers that continue to be rationed would have smaller excess demands.

The preceding conclusions are summarized below.

Proposition 1: There is no systematic relationship between the absolute levels of interest rates or aggregate loan demand and the magnitude of equilibrium credit rationing.

Proposition 2: (a) Intertemporal variations in total rationing are due primarily to variations in disequilibrium rationing.

(b) Disequilibrium rationing is positive (negative) when the current interest rate R_t is less than (greater than) the equilibrium rate $\hat{R}$, and the absolute value of DR is positively related to the difference $(\hat{R} - R_t)$.

Finally, the incidence of disequilibrium rationing depends on default risk. In the example of Figure 2 the risk-free borrower 3 is still unrationed at R_t but borrower 2, who would not be rationed at $\hat{R}$, now has excess demand at the disequilibrium rate R_t. This result is generalized as Proposition 3.

Proposition 3: As the current interest rate falls relative to the equilibrium rate, the resulting increase in rationing is distributed among customers on the basis of default risk (ceteris paribus), with low risk borrowers the least likely to experience unsatisfied demand.

3.4 Iso-Profit Curves

Characteristics of iso-profit curves for borrowers and lenders will now be presented for later use in section 3.5. An iso-profit curve of the lender, denoted by Π^S, consists of all combinations of borrower i's loan size and interest rate yielding a constant expected profit to the lender. The slope of an iso-profit curve is found by setting expected profit (8) equal to a constant and differentiating. Along the iso-profit curve Π^S,

$$(22)\quad \left.\frac{\delta R_i}{\delta L_i}\right|_{\Pi^S} = -\frac{\delta P_i^S/\delta L_i}{\delta P_i^S/\delta R_i} = \frac{-\left(R_i(1 - G[\beta]) + q\int_v^\beta yg(y)dy - I \right)}{L_i(1 - G[\beta])}$$

The following propositions describe the lender's iso-profit map.[8]

8. These propositions are proven in Appendix 2.

Proposition 4: An iso-profit curve of the lender is positively-sloped to the right of the offer curve L_i^s and negatively-sloped to the left of L_i^s.

Proposition 5: Expected lender profit increases along an offer curve L_i^s as the interest rate increases. Therefore, iso-profit curves intersecting L_i^s at successively higher interest rates represent successively greater levels of expected lender profit.

Iso-profit curves of the borrower are derived in a similar manner. Borrower profit from the investment opportunity is uncertain since end-of-period gross return defined by (2) is unknown.

(2) $x = \rho[A]yA$

The firm's expectations for the random rate of return y from the project are described by a density function h(y).[9] It is assumed that there exist rates of return k and K, $0 \leq k \leq K \leq \infty$, such that

(23) $h(y) = 0$ for $y \leq k$ or $y \geq K$, and

(24) $H[y] = 0$ for $y \leq k$

$H[y] = 1$ for $y \geq K$

$$\text{with } H[m] = \int_k^m h(y)dy$$

= cumulative density function evaluated at m.

9. The borrower's expectations represented by h(y) may differ from those of the lender g(y).

These expectations enter into the firm's expected profit function (25). If default on the loan does not occur, with y greater than or equal to the no-default rate of return β, borrower profit equals gross returns ρyA minus the contractual loan payments $R_i L_i$. Each no-default result is weighted by the appropriate probability given by h(y). If the borrower does default with $y<\beta$ it is assumed that the lender receives all returns from the investment project. Therefore, borrower i's expected profit P_i^d is

$$(25)\quad P_i^d = \int_{\beta}^{K} (\rho yA - R_i L_i) h(y) dy$$

The borrower maximizes expected profit at a given interest rate by choosing an optimal loan size. From (25) the first-order condition is

$$(26)\quad \frac{\delta P_i^d}{\delta L_i} = \int_{\beta}^{K} (qy - R_i) h(y) dy = 0$$

with q defined by (3)(b). The solution to (26) at alternative interest rates provides borrower i's loan demand curve $L_i^d(R_i)$. Along L_i^d

$$(27)\quad \frac{\delta L_i^d}{\delta R_i} = \frac{1 - H[\beta] + (q\beta - R_i) \frac{\delta \beta}{\delta R_i} h(\beta)}{q' \int_{\beta}^{K} yh(y)dy - (q\beta - R_i) \frac{\delta \beta}{\delta L_i} h(\beta)} .$$

An iso-profit curve of the borrower Π^d is a locus of interest rates and loan sizes that give a constant level of expected profit to borrower i. The slope of this curve is found by equating (25) to a constant and differentiating the resulting expression.

$$(28)\quad \left.\frac{\delta R_i}{\delta L_i}\right|_{\Pi^d} = - \frac{\delta P_i^d/\delta L_i}{\delta P_i^d/\delta R_i}$$

$$= \frac{q\left(\int_{\beta}^{K} yh(y)dy - R_i(1 - H[\beta]\,)\right)}{L_i(1 - H[\beta]\,)}$$

Propositions 6 and 7 describe the borrower's iso-profit map.[10]

Proposition 6: An iso-profit curve of the borrower is positively-sloped to the left of the loan demand curve L_i^d and negatively-sloped to the right of L_i^d.

Proposition 7: Expected profit of borrower i increases as the interest rate falls along L_i^d. Therefore, borrower iso-profit curves intersecting L_i^d at successively lower interest rates represent successively greater levels of expected borrower profit.

3.5 Imperfect Differentiation and Demand Uncertainty

Sections 3.1 - 3.3 developed previous arguments that indicate non-price rationing is profitable for a lender if there are moral or collusive constraints on the lender's ability to price discriminate. An implicit assumption of this literature is that the lender has perfect information about each borrower's loan demand. The current section suggests that limited interest rate differentiation may be an endogenous outcome of the model once this information assumption is relaxed.

Iso-profit curves of the lender and borrower i are combined in Figure 3. Slope characteristics are inferred from Propositions 4 and 6 while

10. These propositions are proven in Appendix 2.

Propositions 5 and 7 indicate that $\Pi^{s}_{R*} > \Pi^{s}_{\hat{R}}$ and $\Pi^{d}_{R*} < \Pi^{d}_{\hat{R}}$. Point A is the conventional equilibrium for discriminating monopoly pricing discussed in section 3.2. Subject to the demand constraint $L_i \leq L_i^d(R_i)$ the lender reaches the highest attainable iso-profit curve at the loan contract given by the tangency of Π^{s}_{R*} and L_i^d. At the discriminating interest rate R_i^* there is no rationing since the lender's profit-maximizing loan size $L_i^d(R_i^*)$ is demand-determined. If instead the lender charges a group of borrowers a common interest rate $\hat{R}$, the borrower of Figure 3 is rationed at point B since $L_i^s = \min(L_i^d, L_i^s)$ at $\hat{R}$.

Figure 3 illustrates that a common interest rate policy can induce credit rationing but the rationality of this pricing strategy could be challenged. Comparison of points A and B shows that movement from individual discrimination to uniform pricing reduces the lender's expected profit from Π^{s}_{R*} to $\Pi^{s}_{\hat{R}}$. This conclusion is repeated for every borrower i = 1, ... , n since the lender reaches the highest attainable iso-profit curves only at the interest rates R_i^* (i=1, ... ,n). Therefore, total expected profit P^s has decreased relative to the individual discrimination case with

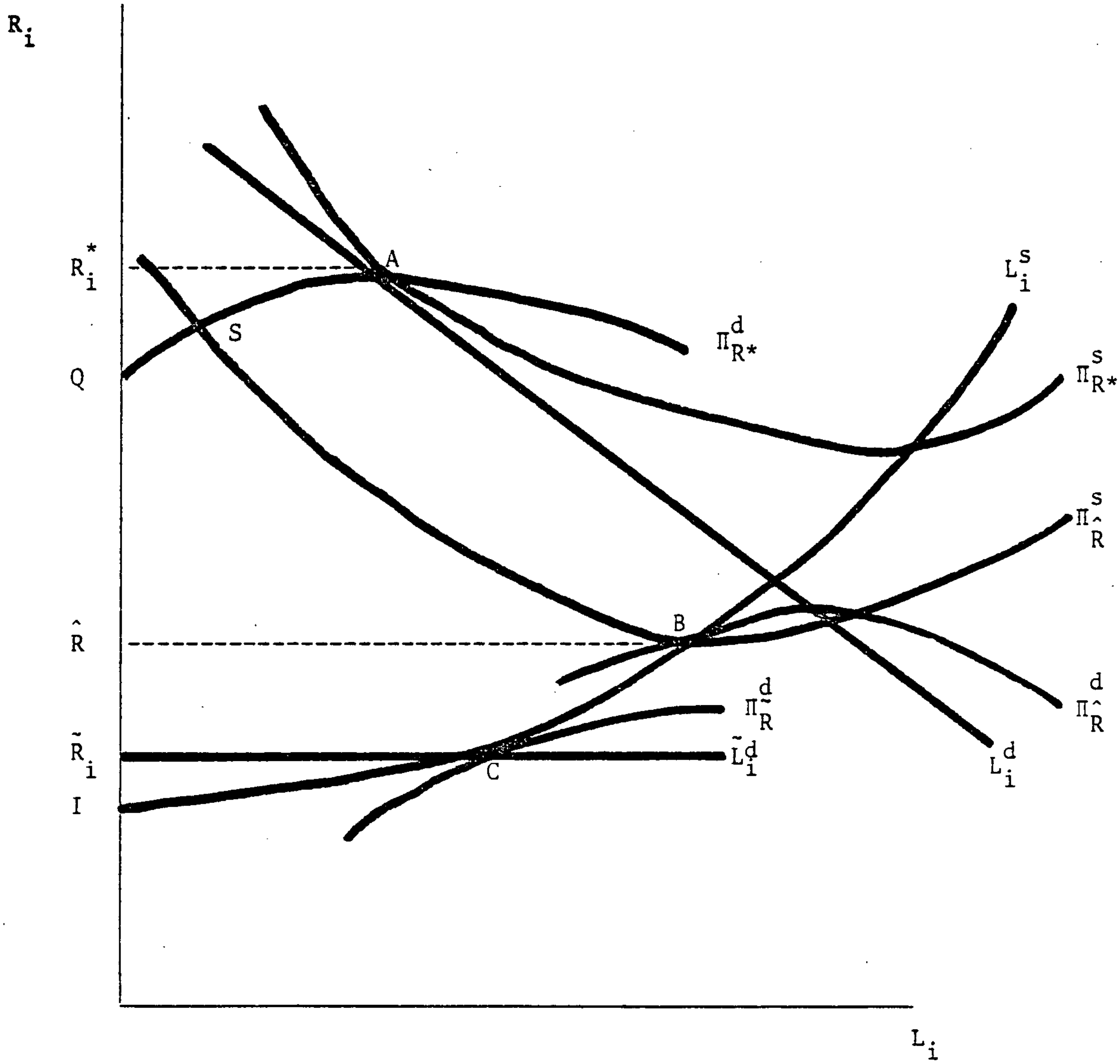
$$P^s(R^*) = \sum_{i=1}^{n} P_i^s(R_i^*) > P^s(\hat{R}) = \sum_{i=1}^{n} P_i^s(\hat{R})$$

Based on points A and B the common interest rate policy appears to be irrational in the absence of oligopolistic pricing arguments. However, it is not certain that the actual outcome with discriminatory pricing would be A in Figure 3. The Jaffee model implicitly assumes

(a) the lender has information on each borrower's demand curve, and

(b) the demand curve perceived by the lender is the schedule L_i^d implied by (26), which gives the borrower's profit-maximizing loan sizes at given interest rates.

FIGURE 3: **Potential Equilibrium Positions with Individual Demand Uncertainty**



With respect to (a), the information on loan demand could be obtained in several ways. The lender might ask each borrower for desired loan quantities at various interest rates. Alternatively, loan demand could be inferred from observed customer behavior in current and previous periods. In either case it should be noted that the lender's perception of demand relies on information gathered from the borrower. An important issue is whether borrower behavior, and therefore the perceived demand curve, is likely to be independent of the pricing system established by the lender. In fact, there is reason to believe that a borrower would prefer not to reveal the conventional demand function L_i^d to the lender under a system of individual price discrimination. If this is true then point A in Figure 3 may not be the actual solution for discriminatory pricing.

This argument and its implications for non-price rationing are seen from Figure 3. Suppose borrower i could select any combination of interest rate and loan size subject only to a restriction that loan size could not exceed the offer curve L_i^s. The highest attainable iso-profit curve would be reached at C where iso-profit curve $\Pi_{\tilde{R}}^d$ is tangent to L_i^s. Having identified C as the borrower's optimal loan contract, it is instructive to determine conditions under which this outcome would be realized. As shown earlier C is not the equilibrium when the lender maximizes expected profit subject to $L_i^d(R_i)$. With individual price discrimination and marginal revenue equals marginal cost pricing, C is chosen by the lender only if it is believed that demand is an infinitely elastic schedule $\tilde{L}_i^d$ at interest rate $\tilde{R}_i$. Therefore, there is an incentive under discriminatory pricing for the borrower to behave according to the pseudo-demand function $\tilde{L}_i^d$ rather than the conventional relationship L_i^d since a higher iso-profit curve is achieved ($\Pi_{\tilde{R}}^d > \Pi_{R^*}^d$) if $\tilde{L}_i^d$ is the perceived demand curve.

To this point it has been assumed that the bank passively accepts whatever demand curve is revealed by the customer and price discriminates subject to this function. However, as Figure 3 illustrates, the borrower's decision to reveal $\tilde{L}_i^d$ rather than L_i^d has reduced lender profit by moving the lender from Π^s_{R*} at A to a lower iso-profit curve Π^s_R (not shown in Figure 3) through C. It is unlikely that the bank would not perceive this adverse possibility and anticipate the borrower's efforts to conceal actual willingness to pay. One potential counter-response of lenders would be to maintain individual price discrimination but abandon the passive optimization subject to the initial demand expressed by the borrower. A bilateral bargaining process could develop in which the lender attempts to avoid unfavourable outcomes such as C by acquiring information on actual ability to pay as described by L_i^d. According to this approach, the lender's optimum at A is unknown to the lender at the start of negotiations and is attained only if perfect information on demand is collected during negotiations. The borrower, in contrast, tries to exploit the initial asymmetry in information by bargaining for an outcome as close to iso-profit curve Π^d_R as possible. With such a bargaining process it appears that the eventual equilibrium would be indeterminate under individual price discrimination and demand uncertainty. The conventional solution at A and the borrower's optimum at C may be regarded as polar cases that maximize expected profit of the lender and borrower respectively.

An alternative lender response to strategic borrower behavior would be to replace individual price discrimination with the system whereby a group of borrowers is charged an identical rate $\hat{R}$. The consequences of this practice can be examined by modifying analysis in section 3.3 to incorporate demand uncertainty. For simplicity it is assumed that the lender has no a priori knowledge of individual demand curves but can form expectations of demand by a representative individual from the group. The previous assumption that the lender can screen default risk of each customer is maintained. In these circumstances a two-stage lender optimization might circumvent the problem of demand revelation observed under individual price discrimination. In the first stage the common interest rate $\hat{R}$ is set before borrowers express their demands. Once $\hat{R}$ is announced borrowers reveal loan demands and actual loan sizes are determined.

At the preliminary stage of optimization the lender forms its expectations of demand by a representative individual and determines the optimal interest rate. The maximization problem is

$$(15)' \quad \max_{\{R, L_1^e, \ldots, L_n^e\}} \quad P^s = \sum_{i=1}^{n} (RL_i^e - C_i(R, L_i^e))$$

$$\text{subject to } L_i^e - L^{d,e}(R) \leq 0 \qquad i = 1, \ldots, n$$

with L_i^e = lender's ex ante expectations of borrower i's loan size, and

$L^{d,e}$ = lender's expectations of demand by a representative individual from a given borrower category.

The Lagrangian function is

$$Z = \sum_{i=1}^{n} (RL_i^e - C_i(R, L_i^e)) - \sum_{i=1}^{n} \lambda_i (L_i^e - L^{d,e}(R))$$

and Kuhn-Tucker conditions for a maximum are

$$(29)(a) \quad \frac{\delta Z}{\delta R} = \sum_{i=1}^{n} (L_i^e - \frac{\delta C_i}{\delta R} + \lambda_i \frac{\delta L^{d,e}}{\delta R}) = 0$$

$$(b) \quad \frac{\delta Z}{\delta L_i^e} = R - \frac{\delta C_i}{\delta L_i^e} - \lambda_i \leq 0 \qquad i = 1, \ldots, n$$

$$(c) \quad \lambda_i \geq 0 \qquad i = 1, \ldots, n$$

$$(d) \quad L_i^e - L^{d,e}(R) \leq 0 \qquad i = 1, \ldots, n$$

(e) $\quad \lambda_i(L_i^e - L^{d,e}(R)) = 0 \qquad i = 1,\ldots,n.$

Equation (29)(a) defines the equilibrium interest rate $\hat{R}$ and (29)(b) defines the loan sizes expected by the lender before actual demands are known. In contrast to individual price discrimination, it is evident from (29)(a) that borrower i's interest rate is independent of the demand eventually expressed by this individual. If the interest rate is given to the individual then the profit-maximizing loan demand is determined by the demand function L_i^d defined by (26). Consequently, if the lender establishes a common interest rate policy there is no incentive for the borrower to conceal the conventional demand function L_i^d in an attempt to reach a higher iso-profit curve. This conclusion demonstrates that after $\hat{R}$ is announced borrower i will submit a request to the lender for the loan size given by $L_i^d(\hat{R})$.

At the second stage of optimization the lender has knowledge of actual demands and selects loan sizes that maximize expected profit from each borrower subject to $R_i = \hat{R}$ and $L_i \leq L_i^d(\hat{R})$ for all i = 1, ... , n. The remaining problems to be solved are

$$(30) \quad \max_{\{L_i\}} Z_i = \hat{R}L_i - C_i(\hat{R},L_i) - \lambda_i(L_i - L_i^d(\hat{R}))$$

for all i = 1, ... , n. The solutions to (30) are consistent with previous conclusions in 3.3 for a common interest rate system. Borrower i's loan size is determined by the short-side rule $L_i = \min(L_i^d, L_i^s)$ and any customer with $L_i^s(\hat{R}) = \min(L_i^d(\hat{R}), L_i^s(\hat{R}))$ is rationed at the equilibrium interest rate $\hat{R}$.

The potential advantage to the lender from the common interest rate policy is seen from Figure 3. At $\hat{R}$ the lender reaches iso-profit curve $\Pi_{\hat{R}}^s$ which is less favourable than the level Π_{R*}^s attained in standard analysis of individual discrimination. However, with imperfect information on borrowers' demand curves the relative profitability of the two pricing regimes should not be assessed by comparing points A and B. The outcome at A assumes that demand schedule L_i^d is known by the lender. If this assumption is removed, a borrower facing price discrimination attempts to convey pseudo-demand conditions and benefits from this behavior as long as the eventual bargaining solution lies below iso-profit curve Π_{R*}^d. From the lender's perspective the profitability of $R = \hat{R}$ depends on a comparison of the uncertain bargaining solution under discrimination and the $\hat{R}$ solution at B. A common interest rate policy improves lender profit from borrower i if it has avoided a discriminatory loan contract within area IQSB where $\Pi^s < \Pi_{\hat{R}}^s$. However, due to the uncertainty of bargaining outcomes with individual price discrimination, the relative profitability of the two pricing regimes is ambiguous with [11]

$$(31) \qquad P^s(\hat{R}) = \sum_{i=1}^{n} P_i^s(\hat{R}) \gtrless \tilde{P}^s(R^*) = \sum_{i=1}^{n} \tilde{P}_i^s(R_i^*)$$

where $P^s(\hat{R})$ = total expected lender profit with a common interest rate $\hat{R}$, and

$\tilde{P}^s(R^*)$ = total expected lender profit with individual price discrimination and imperfect information on individual loan demand schedules.

11. The profit functions in (31) ignore any costs to the lender of processing loan applications. It is expected that these costs would be greatest with individual price discrimination since the lender must allocate time and resources to gain information about borrowers' underlying demand conditions. With $R_i = \hat{R}$ there is no incentive for borrowers to withhold this information so processing costs should be smaller. If processing costs are added to (31) the relative profitability of a common interest rate policy should improve.

The preceding argument suggests the possibility that it may be profitable for a bank to set a common loan rate for a group of borrowers if there is imperfect information on individual demand curves. As shown in 3.3 equilibrium credit rationing can occur when different customers are charged a uniform interest rate. Thus, the existence of equilibrium rationing in the perfect risk-screening case may not depend on moral or collusive constraints on interest rate differentiation.

The above explanation for rationing shares a common theme with the Stiglitz-Weiss (1981) analysis of imperfect risk-screening. In each example the lender reacts to an information deficiency by establishing loan terms that have desired effects on borrower behavior. Despite the inability to screen individual risk in the Stiglitz-Weiss model, a lender could induce favourable borrower actions and control risk through an appropriate choice of the level of interest rate. Similarly, in the above context of unknown individual demand schedules, the lender's selection of a uniform pricing regime over price discrimination may be a logical response to the information problem. By adopting uniform pricing the lender structures the loan process to remove the borrower's incentive to exploit a superior information position.

Chapters 2 and 3 have presented various explanations for the non-price rationing believed to exist in loan markets. This theory is used in the following chapter to assess the theoretical merit of existing empirical models of credit rationing.

Chapter 4

Econometric Studies of Disequilibrium Rationing

It would be useful to have information on the empirical significance of credit rationing for a number of reasons as discussed in Chapter 1. However, comprehensive direct measures are not available since the actual volume of unsatisfied loan demand is not recorded. The sample survey of Hatch-Wynant-Grant (1982) provides a partial indication of the importance of credit rationing in Canada but it is incapable of resolving many questions, such as:

(a) the aggregate volume of rationing and the pattern of intertemporal variations in this total, and

(b) the relative magnitudes of equilibrium and disequilibrium rationing in each period.

Given the limited information obtainable from direct sources several indirect methods have been developed to test for the existence of credit rationing. One approach attempts to examine the issue with the aid of proxy variables believed to be correlated with the unobservable actual level of rationing. A second approach uses disequilibrium econometric techniques to estimate structural loan supply and demand equations and obtain information on rationing. This chapter discusses and evaluates these alternative methods.

4.1 Proxy Method

The proxy method, developed by Jaffee (1971) and subsequently followed by Rimbara and Santomero (1976), constructs a measure of rationing to serve as a dependent variable for tests of the Jaffee theoretical model. The ideal measure CR would be the ratio of total rationing to demand of risky firms.[1]

1. The definition of rationing in (1) depends only on D_1 and L_1 since the model predicts risk-free customers are never rationed.

$$(1) \quad CR = \frac{D_1 - L_1}{D_1}$$

where D_1 = total demand of risky firms, and

L_1 = actual volume of loans granted to risky firms

Since excess demand $(D_1 - L_1)$ is unobservable some proxy related to the actual measure CR must be identified. According to Propositions 2 and 3 of Chapter 3, the level of disequilibrium (and total) rationing is positively correlated with the proportion of total loans granted to risk-free borrowers. Therefore, CR_p is proposed as a proxy for CR.

$$(2) \quad CR_p = \frac{L_2}{L_1+L_2}$$

where L_2 = volume of loans granted to risk-free firms.

After noting that loans to risk-free firms are demand-determined $(L_2 = D_2)$ in the theoretical model, the relationship between the proxy CR_p and CR can be derived from (1) and (2).

$$(3) \quad CR_p = \frac{1}{1 + \frac{D_1}{D_2}(1 - CR)}$$

From (3) it is seen that the proxy is positively related to the actual measure CR.[2] In order to make the proxy operational the volume of loans to risk-free firms is represented by available data on loans made at the prime interest rate.

Time series data for CR_p can be used to test several predictions from the Jaffee model. In particular, the model predicts that variations in rationing are due primarily to changes in disequilibrium rationing, and

2. One problem with the proxy is that it can vary independently of CR if there is a change in the relative demand D_1/D_2 of the risky and risk-free borrowers.

that disequilibrium rationing depends on the difference between the equilibrium and current interest rates. These concepts may be tested with regression equation (4) which uses rationing measure CR_p as the dependent variable.

$$(4) \quad CR_p = a_0 + a_1(\hat{R}_t - R_t) + \varepsilon$$

where CR_p = proportion of total loans granted at the prime interest rate,

$\hat{R}_t$ = equilibrium interest rate (measured as an average across all borrowers),

R_t = current (average) interest rate, and

ε = random error term.

If the current interest rate is below (above) the equilibrium there will be positive (negative) disequilibrium rationing. Therefore, a positive relationship is expected between the rationing proxy CR_p and $(\hat{R}_t - R_t)$. The regression equation is completed by specifying determinants of the equilibrium loan rate $\hat{R}_t$. Portfolio theory suggests that the equilibrium return on loans will equal the yield from any other asset held by the lender after adjustment for other factors such as liquidity and risk. Thus, $\hat{R}_t$ may be specified as a function of the yield on treasury bills and other variables which attempt to adjust for liquidity differences between loans and treasury bills.

Jaffee's application of equation (4) to the commercial loans market of U.S. commercial banks gave results that were consistent with the underlying theory. With the estimated coefficient for a_1 having the expected positive sign and statistically significant, risk-free borrowers received a higher proportion of total loans granted as $(\hat{R}_t - R_t)$ increased. This finding suggests that disequilibrium rationing does exist and that the incidence of rationing is related to default risk. However, despite this evidence on existence, the proxy method does not provide measures of either the total volume of credit rationing or the relative significance of the equilibrium and disequilibrium components.

4.2 Structural Estimation

Econometric techniques have been developed for estimating supply and demand schedules in markets which may be in disequilibrium.[3] Laffont-Garcia (1977), Sealey (1979), and Ito and Ueda (1981) use these methods to examine rationing in the business loans markets of various countries. Given their hypothesis that the loan rate may not adjust to equate supply and demand within the current period, a short-side rule must be specified to determine the quantity transacted when the interest rate differs from the market-clearing level. Each of the above studies assumes that the observed market quantity is the minimum of notional aggregate demand and aggregate supply at the current interest rate:

$$(5) \quad L_t = \min (L_t^d, L_t^s)$$

where L_t = observed value of total loans at time t,

L_t^d = notional aggregate demand, and

L_t^s = notional aggregate supply

These authors then specify arguments in L^d and L^s and combine these schedules with an interest rate equation to estimate structural supply and demand parameters and examine disequilibrium behavior. For example, Ito and Ueda consider the following system in conjunction with (5).

$$(6) \quad L_t^d = \alpha_1 X_t + \alpha_2 R_t + u_{1t}$$

3. Quandt (1982) surveys the literature on econometric disequilibrium models.

$$(7) \quad L_t^s = \beta_1 Z_t + \beta_2 R_t + u_{2t}$$

$$(8) \quad R_t = \mu R_{t-1} + (1 - \mu) R_t^* \qquad 0 \leq \mu \leq 1$$

where X, Z = vectors of predetermined variables,

R_t = observed interest rate at time t,

R_t^* = equilibrium interest rate at time t, and

u_{1t}, u_{2t} = error terms.

Equation (8) is a partial price adjustment mechanism in which the current interest rate is a weighted average of the preceding period's value and the equilibrium rate for the current period. The parameter μ is inversely related to the degree of interest rate flexibility. If $\mu = 0$ the interest rate adjusts completely to its equilibrium R_t^* within the current period such that the loan market is in equilibrium each period. If $0 < \mu < 1$ there is partial adjustment in the short-run and disequilibrium prevails in the loan market throughout the period of transition. The equilibrium interest rate R_t^* is determined simply by the intersection of the aggregate demand and supply schedules. From (6) and (7),[4]

$$(9) \quad R_t^* = \frac{1}{(\beta_2 - \alpha_2)} (\alpha_1 X - \beta_1 Z + u_1 - u_2) .$$

Bowden (1978) proposed (8) as an alternative to the usual adjustment scheme (10) where the change in price is proportional to excess demand:

$$(10) \quad \Delta R_t = \lambda (L_t^d - L_t^s) \qquad 0 \leq \lambda \leq \infty$$

4. For convenience time subscripts will be deleted from all variables other than interest rates.

One criticism of (10) is that the size of the adjustment speed parameter λ depends on the units of measurement for the interest rate and quantity variables.[5] In contrast, the parameter μ in (8) is independent of the units of measurement which makes it an easily interpreted measure of the degree of price flexibility.

Equations (6) and (7) cannot be estimated in their present form since (5) indicates that one of the dependent variables is unobservable when the market is in disequilibrium. However, estimating equations for aggregate supply and demand schedules can be derived from (5)-(9). During a period of aggregate excess demand, identified by a rising interest rate, the short-side principle (5) states that the observed quantity is aggregate supply. Although demand is not observed directly it equals the realized quantity plus excess demand. Therefore,

$$\left.\begin{aligned} &(11)(a) \quad L^s = L \\ &\quad\;\;(b) \quad L^d = L + (L^d - L^s) \end{aligned}\right\} \text{ if } R_t > R_{t-1}$$

After rearranging (11)(b) and expressing the excess demand component in terms of (6) and (7),

$$\begin{aligned}(12)\ L &= L^d - ((\alpha_2 - \beta_2)R_t + \alpha_1 X - \beta_1 Z + u_1 - u_2) \\ &= L^d + (\alpha_2 - \beta_2)(R_t^* - R_t)\end{aligned}$$

using (9). It can also be shown from (8) that

$$(13)\ (R_t^* - R_t) = \frac{\mu}{(1-\mu)} (R_t - R_{t-1}).$$

5. Laffont-Garcia and Sealey use the traditional adjustment mechanism (10).

Finally, the estimating equation for demand during excess demand periods is obtained by substituting (6) and (13) into (12).

$$(14)\ L = \alpha_1 X + \alpha_2 R_t + \frac{\mu}{(1-\mu)}(\alpha_2 - \beta_2)(R_t - R_{t-1}) + u_1$$

In (14) the observed change in interest rate acts as an indicator of unobserved excess demand with rationing an increasing function of $(R_t - R_{t-1})$ for a given adjustment speed parameter μ.

For a period with aggregate excess supply, identified by a falling interest rate, the short-side rule indicates that the quantity exchanged is aggregate demand and aggregate supply equals the observed quantity plus excess supply.

$$\left.\begin{array}{ll} (15)(a) & L^d = L \\ \quad (b) & L^s = L + (L^s - L^d) \end{array}\right\} \text{ if } R_t < R_{t-1}$$

By following similar steps to those used in the excess demand case, (16) is derived as the estimating equation for supply in the presence of market excess supply.

$$(16)\ L = \beta_1 Z + \beta_2 R_t + \frac{\mu}{(1-\mu)}(\beta_2 - \alpha_2)(R_t - R_{t-1}) + u_2$$

Since the interest rate change $(R_t - R_{t-1})$ appears in the demand (supply) estimating equation only when there is aggregate excess demand (supply), the general forms for these equations are:[6]

6. The estimating equations have a structure similar to (17) and (18) when the conventional price adjustment mechanism (10) is used in place of the Bowden process. These alternative equations can be derived as

$$(17)'\ L = \alpha_1 X + \alpha_2 R_t - \frac{1}{\lambda}\Delta R^+ + u_1$$

$$(18)'\ L = \beta_1 Z + \beta_2 R_t + \frac{1}{\lambda}\Delta R^- + u_2$$

$$(17)\ L = \alpha_1 X + \alpha_2 R_t + \frac{\mu}{(1-\mu)} (\alpha_2 - \beta_2)\ \Delta R_t^+ + u_1$$

$$(18)\ L = \beta_1 Z + \beta_2 R_t + \frac{\mu}{(1-\mu)} (\beta_2 - \alpha_2)\ \Delta R_t^- + u_2$$

where

$$\Delta R_t^+ = \begin{cases} R_t - R_{t-1} & \text{if } R_t \geq R_{t-1} \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta R_t^- = \begin{cases} R_t - R_{t-1} & \text{if } R_t < R_{t-1} \\ 0 & \text{otherwise} \end{cases}$$

Information concerning the structural demand and supply functions is obtained from parameter estimates of α_1, α_2, β_1, and β_2 from the system defined by (17) and (18). The hypothesis that the loan market is always in equilibrium is tested under the null hypotheses $\mu = 0$.[7] For the case $\mu = 0$ the disequilibrium model (17) and (18) collapses to a standard equilibrium model with the observed loan quantity attributed to the intersection of L^d and L^s.

This structural approach meets one of the desired characteristics of an empirical model by permitting the volume of credit rationing to be estimated as the difference $(L^d - L^s)$ for those periods when the current interest rate is below equilibrium. For example, if the present loan rate is R_t the magnitude of excess demand would be (b-a) in Figure 4. Despite this improvement over the proxy method, the model of Laffont-Garcia et al

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7. Previous studies have reached conflicting conclusions regarding the prevalence of credit rationing. Laffont-Garcia (1977), p. 1198 concluded that in the Canadian business loans market "the main feature was a downward sluggishness of prices leading to an essentially demand-determined market". Sealey (1979), p. 697 found that in the U.S. "business loans are essentially supply determined with intermittent periods of demand determination". Ito-Ueda, using the formal test involving the parameter μ, accepted the equilibrium hypothesis for the U.S. but rejected the equilibrium hypothesis for Japan.

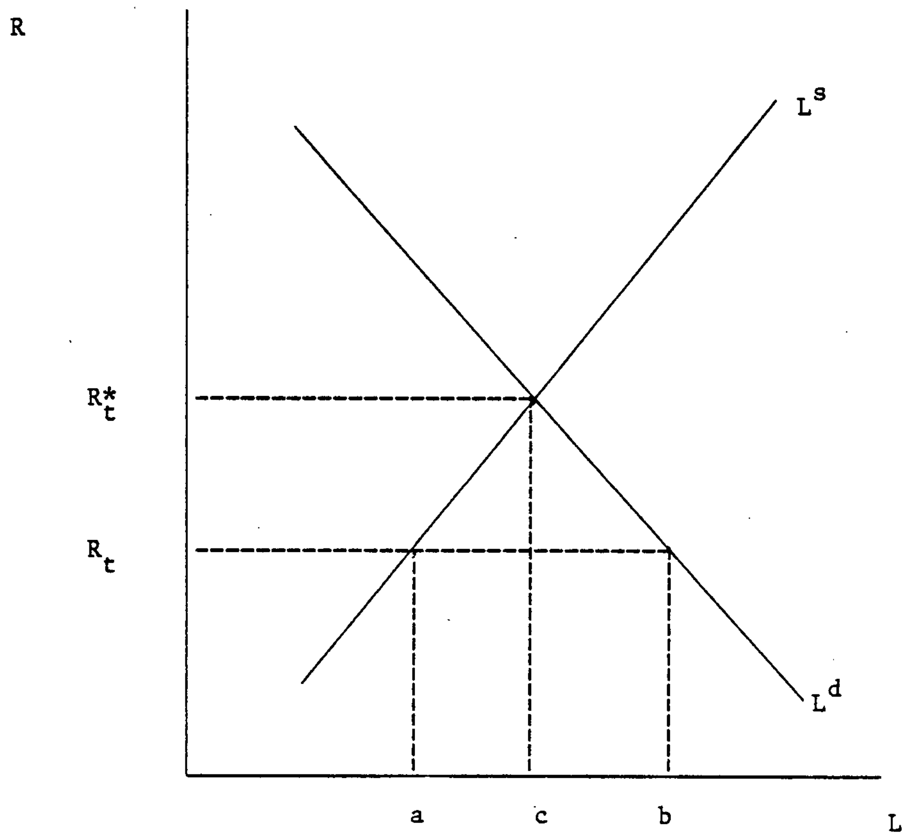
is inadequate since it arbitrarily excludes the possibility of equilibrium rationing. It is recalled that two major assumptions were used to derive the model. The aggregate short-side rule assumes that the observed market quantity is the minimum of aggregate demand L^d and supply L^s at the current rate of interest. In addition, the equilibrium rate occurs at the intersection of the two aggregate schedules. These assumptions have two implications:

(a) rationing can occur only when the current interest rate is below its equilibrium, and

(b) there is no rationing at the equilibrium interest rate since the equilibrium quantity (c in Figure 4) must correspond to the intersection of L^d and L^s.

These conclusions demonstrate that previous empirical studies focus exclusively on disequilibrium rationing and make no allowance for the equilibrium rationing discussed in the theoretical literature.

FIGURE 4: Credit Rationing with $L = \min(L^d, L^s)$



4.3 Aggregate Rationing with an Individual Short-side Rule

The objective of this study is to reduce the present inconsistency between theoretical and applied studies by estimating a general model incorporating both equilibrium and disequilibrium credit rationing. It would be desirable to construct an aggregate empirical model from an explicit micro explanation of loan markets. Jaffee's (1971) theory, presented in Chapters 2 and 3, was selected to represent these micro-foundations and serve as a basis for critiquing the theoretical validity of the disequilibrium models of Laffont-Garcia and others.[8] A distinguishing feature of the Jaffee model is that an individual's loan size is the minimum of that borrower's demand and supply (offer) curves:

$$(19)\quad L_i = \min(L_i^d,\ L_i^s)$$

The implications of short-side rule (19) were not taken into account by the structural models of disequilibrium rationing. After deriving potential estimating equations for aggregate demand and supply functions from this individual short-side rule, it will be shown that the aggregate short-side principle (5) utilized by previous researchers is valid only under very restrictive conditions.

Consider the following specification.

$$(19)\quad L_i = \min(L_i^d,\ L_i^s)$$

$$(20)\quad L = \sum_{i=1}^{n} L_i = \sum_{i=1}^{n} \min(L_i^d,\ L_i^s)$$

8. It should be noted that Jaffee, unlike Jaffee-Russell (1976) or Stiglitz-Weiss (1981), assumes lenders can form expectations of default risk for each customer individually.

$$(21) \quad L^d = \sum_{i=1}^{n} L_i^d = \sum_{i=1}^{n} (\alpha_{1i}X_i + \alpha_{2i}R_t + u_{1i})$$

$$(22) \quad L^s = \sum_{i=1}^{n} L_i^s = \sum_{i=1}^{n} (\beta_{1i}Z_i + \beta_{2i}R_t \pm u_{2i})$$

$$(23) \quad R_t = \mu R_{t-1} + (1-\mu)\hat{R}_t \qquad 0 \leq \mu \leq 1$$

with L = observed aggregate volume of loans,

L^d = notional aggregate demand,

L^s = notional aggregate supply,

X_i, Z_i = vectors of predetermined variables in customer i's demand and supply functions,

u_{1i}, u_{2i} = error terms, and

$\hat{R}_t$ = equilibrium interest rate

This system differs from the conventional aggregate model in several important respects. Firstly, notional aggregate demand and supply schedules are expressed explicitly as summations across n customers. In many markets it would not be realistic to define a seller's supply function for each individual but, following Jaffee (1971), individual loan supply curves are justified given inter-customer variations in default risk and risk-screening by lenders. Secondly, according to the individual short-side rule (19) the aggregate quantity observed (20) is the sum of n individual minimums, rather than the minimum of aggregate demand and supply as assumed by the econometric studies of disequilibrium rationing. Equation (23) restates the partial price adjustment mechanism which represents the current interest rate as a weighted average of last period's value and the current equilibrium rate.

Equations (16)(a)-(16)(e) in Chapter 3 suggested that during a given period some borrowers receive loans on their demand functions L_i^d while others are rationed along supply functions L_i^s. Partition the former group to a set D and the latter to set S. Notional aggregate demand may then be defined as

$$L^d = \sum_{i\varepsilon D} L_i^d + \sum_{i\varepsilon S} (L_i^s + (L_i^d - L_i^s))$$

$$\text{where} \begin{cases} i\varepsilon D & \text{if } L_i^d = \min(L_i^d, L_i^s) \\ i\varepsilon S & \text{if } L_i^s = \min(L_i^d, L_i^s) \end{cases}$$

Since $L_i = \min(L_i^d, L_i^s)$, $\sum_{i\,\varepsilon\, D} L_i^d$ and $\sum_{i\varepsilon\, S} L_i^s$ are the observable loan quantities for the two sets of borrowers. Therefore,

$$\begin{aligned} (24)\quad L^d &= \sum_{i\varepsilon D} L_i^d + \sum_{i\varepsilon S} L_i^s + \sum_{i\varepsilon S} (L_i^d - L_i^s) \\ &= L + \sum_{i\varepsilon S} (L_i^d - L_i^s) \end{aligned}$$

with $\sum_{i\,\varepsilon\, S} (L_i^d - L_i^s) > 0$ equal to total excess demand of rationed borrowers. After substituting for L_i^d and L_i^s from (21) and (22), and solving for the interest rate r_i corresponding to the intersection of individual i's demand and supply functions, (24) may be expressed as

$$(25)\quad L^d = L + \sum_{i\varepsilon S} (\beta_{2i} - \alpha_{2i})(r_{it} - R_t)$$

$$\begin{aligned} \text{where } r_{it} &= \frac{1}{(\beta_{2i} - \alpha_{2i})} (\alpha_{1i}X_i - \beta_{1i}Z_i + u_{1i} - u_{2i}), \\ &(r_{it} - R_t) > 0 \quad \text{for all } i\varepsilon S, \text{ and} \\ &\beta_{2i} > 0, \quad \alpha_{2i} < 0. \end{aligned}$$

If the equilibrium interest rate $\hat{R}_t$ is added and subtracted from $(r_{it} - R_t)$, the excess demand term in (25) is expanded to

$$(26) \quad \sum_{i \varepsilon S} (\beta_{2i} - \alpha_{2i})(r_{it} - \hat{R}_t) + \sum_{i \varepsilon S} (\beta_{2i} - \alpha_{2i})(\hat{R}_t - R_t) =$$

$$\sum_{i \varepsilon S} (\beta_{2i} - \alpha_{2i})(r_{it} - \hat{R}_t) + \sum_{i \varepsilon S} (\beta_{2i} - \alpha_{2i})\frac{\mu}{(1-\mu)} (R_t - R_{t-1})$$

using (13). Finally, from (25) and (26) notional aggregate demand may be written as (27) when quantity determination is governed by the individual short-side rule.

$$(27) \quad L^d = L + \sum_{i \varepsilon S} (\beta_{2i} - \alpha_{2i})(r_{it} - \hat{R}_t)$$

$$+ \sum_{i \varepsilon S} (\beta_{2i} - \alpha_{2i}) \frac{\mu}{(1-\mu)} (R_t - R_{t-1})$$

Total excess demand of rationed borrowers, given by the two summations over $i \varepsilon S$ in (27), contains both equilibrium and disequilibrium rationing. Unlike the model of section 4.2 used by Laffont-Garcia et al, the realized loan quantity does not correspond to demand if the loan market is in equilibrium. If $\mu = 0$ so that the interest rate adjusts completely to equilibrium within the current period, or if there has been full adjustment to past disturbances with $R_t = R_{t-1}$, disequilibrium rationing does not exist and the second summation is zero. Nevertheless, if individual i's demand and supply curves intersect at an interest rate r_{it} greater than the equilibrium rate $\hat{R}_t$, then $(\beta_{2i} - \alpha_{2i})(r_{it} - \hat{R}_t)$ is the magnitude of excess demand this borrower would experience at $\hat{R}_t$. The first summation in (27) would give the total amount of equilibrium credit rationing.

Similarly, the notional aggregate supply schedule consistent with (19) can be derived as

$$(28)\quad L^s = L + \sum_{i\varepsilon D} (\alpha_{2i} - \beta_{2i})(r_{it} - \hat{R}_t) + \sum_{i\varepsilon D} (\alpha_{2i} - \beta_{2i}) \frac{\mu}{(1-\mu)}(R_t - R_{t-1})$$

using the steps outlined above. Desired lender supply to a borrower may be greater than the individual's demand at the current interest rate. Since realized loan size cannot exceed desired demand, these relationships are characterized by excess supply. The summations in (28) represent total excess supply to unrationed borrowers $i\varepsilon D$.

In order to facilitate comparison, the systems derived from the alternative short-side principles $L = \min(L^d, L^s)$ and $L_i = \min(L_i^d, L_i^s)$ are restated as A. and B. respectively.

A. Aggregate Short-side Rule System

$$(5)\quad L = \min(L^d, L^s)$$

$$(17)\quad L = \alpha_1 X + \alpha_2 R_t + \frac{\mu}{(1-\mu)} (\alpha_2 - \beta_2)\Delta R_t^+ + u_1$$

$$(18)\quad L = \beta_1 Z + \beta_2 R_t + \frac{\mu}{(1-\mu)} (\beta_2 - \alpha_2) \Delta R_t^- + u_2$$

where ΔR_t^+ and ΔR_t^- are as defined above.

B. Individual Short-side Rule System

$$(20)\quad L = \sum_{i=1}^{n} L_i = \sum_{i=1}^{n} \min(L_i^d, L_i^s)$$

$$(29)\quad L = \sum_{i=1}^{n} (\alpha_{1i}X_i + \alpha_{2i}R_t + u_{1i}) - \sum_{i\varepsilon S} (\beta_{2i} - \alpha_{2i})(r_{it} - \hat{R}_t)$$

$$= \sum_{i\varepsilon S} (\beta_{2i} - \alpha_{2i}) \frac{\mu}{(1-\mu)} (R_t - R_{t-1}) \qquad \text{(from (21), (27))}$$

$$(30)\quad L = \sum_{i=1}^{n} (\beta_{1i}Z_i + \beta_{2i}R_t + u_{2i}) - \sum_{i\varepsilon D} (\alpha_{2i} - \beta_{2i})(r_{it} - \hat{R}_t)$$

$$- \sum_{i\varepsilon D} (\alpha_{2i} - \beta_{2i}) \frac{\mu}{(1-\mu)} (R_t - R_{t-1}) \qquad \text{(from (22), (28))}$$

A major dissimilarity between the two systems concerns the treatment of equilibrium credit rationing. As detailed previously, the traditional system A. abstracts from the positions of individual borrowers and precludes the existence of equilibrium rationing. In contrast, system B. is based on the underlying supply and demand functions of individual customers. If lenders face constraints on inter-customer interest rate differentiation, the equilibrium rate generally will not coincide with the intersection of an individual's supply and demand functions. Consequently, some borrowers are rationed at the equilibrium rate of interest in system B.

The alternative systems differ significantly in another important respect. According to the aggregate short-side rule system, the observed market quantity <u>always</u> lies on at least one of the aggregate demand and supply schedules. However, examination of (29) and (30) indicates that L is positioned on L^d or L^s only under extreme conditions; during the current period either all individuals belong to the unrationed set D or all must belong to the rationed set S.[9] The theory associated with the individual short-side rule suggests instead that in all periods there will be some customers in each of these sets. Given this predicted diversity in rationing status, it is readily shown that realized market quantity belongs to neither L^d nor L^s, but rather is less than both notional functions.

With $L = \sum_{i=1}^{n} L_i = \sum_{i=1}^{n} \min(L_i^d, L_i^s)$, it follows immediately that

$$L = \left(\sum_{i\varepsilon D} L_i^d + \sum_{i\varepsilon S} L_i^s\right) < L^d = \sum_{i=1}^{n} L_i^d$$

since $L_i^s < L_i^d$ for all $i\varepsilon S$, and

$$L = \left(\sum_{i\varepsilon D} L_i^d + \sum_{i\varepsilon S} L_i^s\right) < L^s = \sum_{i=1}^{n} L_i^s$$

since $L_i^d < L_i^s$ for all $i\varepsilon D$.

Hence,

$$(32)\quad L < \min(L^d, L^s) \text{ with } L_i = \min(L_i^d, L_i^s) \quad .$$

This last result indicates that the observed quantity understates both aggregate demand and supply in all periods. Consequently, existing econometric studies which utilize short-side rule (5) misspecify these aggregate relationships and give biased estimates for parameters (including the speed of interest rate adjustment).

Equations (29) and (30) from system B. have been useful in illustrating the deficiencies of the current method of estimation. However, (29) - (30) cannot be estimated since these equations are not identified. If the definition of r_{it} is substituted into (29) and (30), it is seen that

9. This requirement restricts the current interest rate to be either above the intersection of individual supply and demand functions for every borrower or, alternatively, below this intersection for every borrower.

each equation contains an identical list of explanatory variables. Thus, it appears that a general model with equilibrium and disequilibrium credit rationing cannot be estimated directly from separate aggregate demand and supply equations, which was the methodology used by the disequilibrium studies of Laffont-Garcia et al. Chapter 5 maintains the reliance on an individual short-side rule to propose an alternative method of estimation.

CHAPTER 5

AN ECONOMETRIC MODEL OF AGGREGATE CREDIT RATIONING

The previous chapter demonstrated the problems involved in attempting to make existing empirical models of credit rationing consistent with the individual short-side rule. An alternative approach is required which incorporates both equilibrium and disequilibrium rationing and is based on a micro explanation of loan determination. A system satisfying these objectives is derived in this chapter by modelling individual loan determination in 5.1 and then using this micro analysis in 5.2 to obtain an aggregate loan equation. This loan relationship, in combination with an interest rate equation, forms a two-equation empirical model which is applied to the Canadian business loans market. The chapter concludes with a comparison of results from aggregate and individual short-side rule models.

5.1 Individual Loan Size Determination (Flows)

The individual short-side rule states that the observed loan size granted to borrower i is the minimum of the demand and supply functions of borrower i at the prevailing interest rate.

$$(1) \quad L_i = \min(L_i^d, L_i^s) \qquad i = 1, \ldots, n$$

$$L_i^d \geq 0, \quad L_i^s \geq 0$$

Due to the absence of micro data for the n loan customers individual demand and supply functions must be specified with aggregate variables as arguments. This approach can be implemented by expressing borrower i's demand and supply characteristics in terms of deviations from the mean position of the total population of borrowers. For example, suppose current period flow demand and supply for borrower i are

(2) $L_i^d = \alpha_o + \tilde{\alpha}_{oi} + \alpha_1 (\bar{X} + \tilde{X}_i) + \alpha_2(\bar{R} + \tilde{R}_i)$

(3) $L_i^s = \beta_o + \tilde{\beta}_{oi} + \beta_1 (\bar{Z} + \tilde{Z}_i) + \beta_2(\bar{R} + \tilde{R}_i)$

with $\bar{X}$ = mean value across all borrowers of exogenous variable X,

$\bar{Z}$ = mean value across all borrowers of exogenous variable Z,

$\bar{R}$ = mean interest rate on loans,

and ($\tilde{\alpha}_{oi}$, $\tilde{\beta}_{oi}$, $\tilde{X}_i$, $\tilde{Z}_i$, $\tilde{R}_i$) measured as borrower-specific (zero mean) deviations from corresponding elements in the vector of population means (α_o, β_o, $\bar{X}$, $\bar{Z}$, $\bar{R}$).[1]

Equations (2) and (3) allow inter-customer heterogeneity to occur through different intercept terms and different values for explanatory variables. From (2), borrower i's demand consists of one term valued at population means of the intercept and explanatory variables and a second term e_{1i} which depends on deviations from means.

(4) $L_i^d = (\alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R}) + e_{1i}$

with $e_{1i} = \tilde{\alpha}_{oi} + \alpha_1\tilde{X}_i + \alpha_2\tilde{R}_i$

It will be assumed that e_1 is normally distributed over the population of borrowers with mean zero and variance σ_1^2.

$$e_1 \sim N(0, \sigma_1^2)$$

1. It is conceivable that a borrower would receive a loan from one bank after being rationed by another lender. In the Jaffee theoretical model this event could occur if lenders have different assessments of the individual's default risk. In such cases the loan supply curve (3) could be interpreted as total loan supply by all lenders to individual i in a given period.

Similarly, equation (3) can be rearranged to show that the lender's supply function to borrower i contains one term composed of population means and a borrower-specific deviation e_{2i}. Assume that the deviation component e_2 is normally distributed across the population of borrowers with mean zero and variance σ_2^2.

$$(5) \quad L_i^s = (\beta_o + \beta_1\bar{Z} + \beta_2\bar{R}) + e_{2i}$$

$$\text{with } e_{2i} = \tilde{\beta}_{oi} + \beta_1\tilde{Z}_i + \beta_2\tilde{R}_i$$

$$e_2 \sim N(0, \sigma_2^2)$$

Explicit recognition of the non-negativity constraint on loan sizes gives (6) and (7) as general representations of individual demand and supply. From (4) and (5),

$$(6) \quad L_i^d = \begin{cases} \alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R} + e_{1i} & \\ \quad \text{if } \alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R} + e_{1i} > 0 & \\ \quad \text{i.e. } e_{1i} > -(\alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R}) & \\ 0 \quad \text{otherwise} & \\ \quad \text{i.e. } e_{1i} \leq -(\alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R}) & \end{cases}$$

$$(7) \quad L_i^s = \begin{cases} \beta_o + \beta_1\bar{Z} + \beta_2\bar{R} + e_{2i} & \\ \quad \text{if } e_{2i} > -(\beta_o + \beta_1\bar{Z} + \beta_2\bar{R}) & \\ 0 \quad \text{otherwise} & \end{cases}$$

It is now possible to state the conditions which establish whether the realized loan size of a particular borrower is supply or demand-determined.

Define r_i as the interest rate at which borrower i's supply and demand functions intersect. If r_i is less than the interest rate charged on borrower i's loan, then according to short-side rule (1) the loan is demand-determined:

$$L_i = L_i^d = \min(L_i^d, L_i^s) \qquad \text{for } r_i < R_i$$

with $r_i = \frac{1}{\beta_2 - \alpha_2} \left(\alpha_o + \tilde{\alpha}_{oi} + \alpha_1 (\bar{X} + \tilde{X}_i) - \beta_o - \tilde{\beta}_{oi} - \beta_1(\bar{Z} + \tilde{Z}_i)\right)$

and $R_i = \bar{R} + \tilde{R}_i$

= interest rate charged on borrower i's loan.

Unfortunately information on the values of r_i and R_i is unavailable for each potential borrower. However the requirement $r_i \leq R_i$ for a demand-determined loan is equivalent to $e_{3i} \leq \eta$

for $e_{3i} = e_{1i} - e_{2i}$

and $\eta = (\beta_2 - \alpha_2)\bar{R} - \alpha_o - \alpha_1\bar{X} + \beta_o + \beta_1\bar{Z}$.

This alternative condition for a demand-determined loan is convenient since all unobservable borrower-specific random components are isolated on the left side of the inequality as e_{3i} whereas the right side depends on more readily obtainable mean values of explanatory variables. Thus, from (1),

$$(8) \quad L_i = \begin{cases} L_i^d \text{ if } e_{3i} \leq \eta & (r_i \leq R_i) \\ L_i^s \text{ if } e_{3i} > \eta & (r_i > R_i) \end{cases}$$

$$e_3 \sim N(0, \sigma_3^2)$$

Equation (9) combines (6) - (8) to summarize loan size determination with a non-negativity constraint.

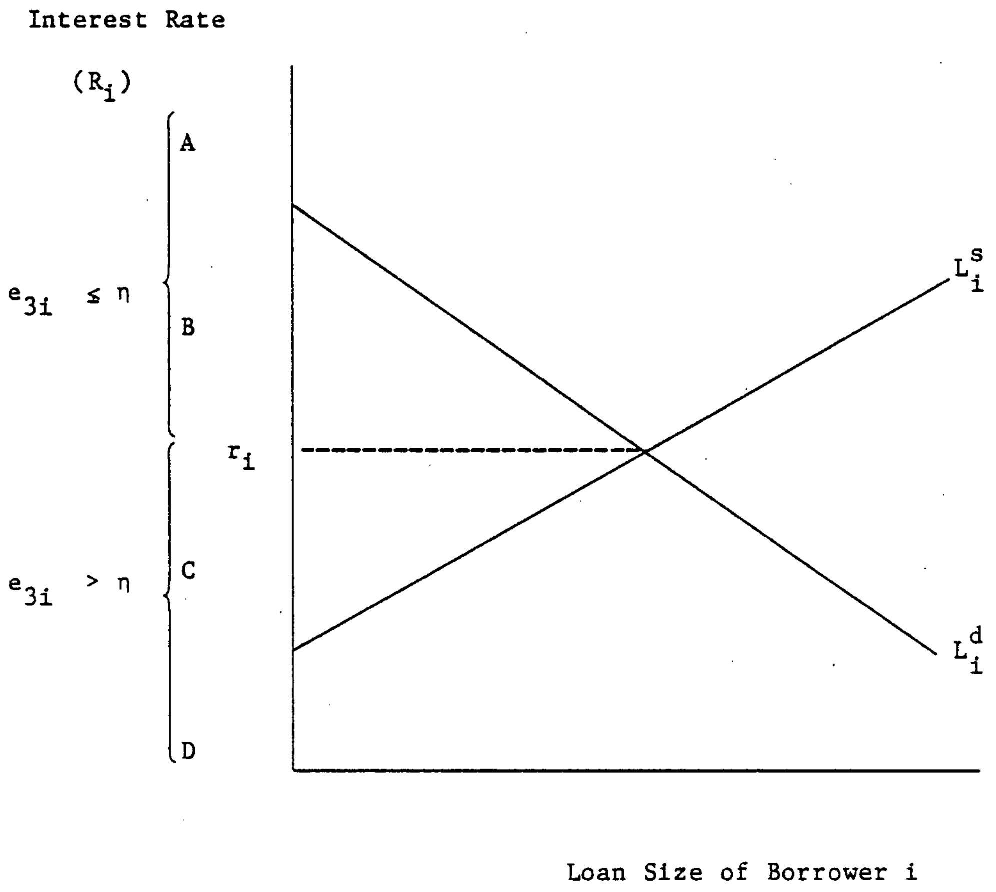
$$
(9)\quad L_i = \begin{cases}
L_i^d = \alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R} + e_{1i} \\
\quad \text{if} \quad \text{(i)}\ e_{3i} \leq \eta \\
\quad \text{and (ii)}\ e_{1i} > -(\alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R}) \equiv \gamma \\
\\
L_i^s = \beta_o + \beta_1\bar{Z} + \beta_2\bar{R} + e_{2i} \\
\quad \text{if} \quad \text{(i)}\ e_{3i} > \eta \\
\quad \text{and (ii)}\ e_{2i} > -(\beta_o + \beta_1\bar{Z} + \beta_2\bar{R}) \equiv \theta. \\
\\
0 \quad \text{if} \quad \text{(i)}\ e_{1i} \leq -(\alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R}) \\
\quad \text{or} \quad \text{(ii)}\ e_{2i} \leq -(\beta_o + \beta_1\bar{Z} + \beta_2\bar{R})
\end{cases}
$$

Figure 5 illustrates (9). The direction of inequality between e_{3i} and η determines whether borrower i's loan is demand-determined (as in regions A and B) or supply-determined (C and D). Non-negativity constraints on loan demand and supply become binding at sufficiently low values of borrower-specific random terms e_{1i} and e_{2i} respectively.

5.2 Derivation of the Aggregate Loan Equation

An aggregate loan flow equation consistent with the individual short-side rule can be derived using the above information on individual loan determination. Order the total set of potential borrowers such that the first n_o $(i = 1,\ldots,n_o)$ receive a new loan in the current period and the last $(n - n_o)$ do not obtain a loan. Expected new loan size of a representative borrower from the set $i = 1,\ldots.,n_o$ is (using (9))

FIGURE 5: Individual Loan Determination



A: $L_i = L_i^d = 0$ $\qquad$ $e_{1i} \leq -(\alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R})$

B: $L_i = L_i^d > 0$

C: $L_i = L_i^s > 0$

D: $L_i = L_i^s = 0$ $\qquad$ $e_{2i} \leq -(\beta_o + \beta_1\bar{Z} + \beta_2\bar{R})$

$$(10)\quad E(L_i|L_i > 0) = \frac{1}{K}\left(\begin{array}{l} E(L_i^d|L_i = L_i^d > 0)\ \text{Prob}\ (L_i = L_i^d > 0) \\ \\ +\ E(L_i^s|L_i = L_i^s > 0)\ \text{Prob}\ (L_i = L_i^s > 0) \end{array}\right)$$

$$= \frac{1}{K}\left(\begin{array}{l} E(L_i^d|e_{1i} > \gamma,\ e_{3i} \leq \eta)\ \text{Prob}\ (e_{1i} > \gamma,\ e_{3i} \leq \eta) \\ \\ +\ E(L_i^s|e_{2i} > \theta,\ e_{3i} > \eta)\ \text{Prob}\ (e_{2i} > \theta,\ e_{3i} > \eta) \end{array}\right)$$

with $K = \text{Prob}(L_i > 0)$

= proportion of potential borrowers that do receive a loan.

After substituting the definitions of L_i^d and L_i^s into (10) and using (i) - (iv) in Appendix 3, the expectation is stated as (11) where $g_1(e_1, e_3)$ and $g_2(e_2, e_3)$ are bivariate normal density functions.

$$(11)\quad E(L_i|L_i > 0) = \frac{1}{K}\left(\begin{array}{l} \displaystyle\int_{-\infty}^{\eta}\int_{\gamma}^{\infty} (\alpha_0 + \alpha_1\bar{X} + \alpha_2\bar{R} + e_1)g_1(e_1,e_3)de_1de_3 \\ \\ +\ \displaystyle\int_{\eta}^{\infty}\int_{\theta}^{\infty} (\beta_0 + \beta_1\bar{Z} + \beta_2\bar{R} + e_2)g_2(e_2,e_3)de_2de_3 \end{array}\right)$$

For a particular borrower i within $i = 1,\ldots,n_o$, realized loan flow L_i varies from the expected value (11) calculated over all n_o borrowers by an amount v_i.

$$(12)\quad L_i = E(L_i | L_i > 0) + v_i \qquad i = 1,\ldots,n_o$$

Summation of (12) over $i = 1,\ldots,n_o$ gives an equation for new loan flows in period t.

$$(13)\quad L^f = n_o\, E(L_i | L_i > 0) + \sum_{i=1}^{n_o} v_i$$

with L^f = aggregate loan flow, and

n_o = number of new loans.

In addition to the new loan flows modelled by (13), the business loans data used in this study includes "predetermined" loans granted in previous periods and still outstanding. Although the precise breakdown between predetermined and new loans is unknown, Appendix 4 details how available information can be used to model predetermined loans by (14):

$$(14)\quad PL_t = \phi(1 + h)TL_{t-1}(>1)$$

with PL_t = value of predetermined loans at quarter t,

ϕ = unknown parameter to be estimated,

$$h = \frac{BL^{\circ}_{t-1}(<1)}{BL_{t-1}(>1)}$$

$BL_{t-1}(<1)$ = total stock of outstanding loans less than \$1 million at the end of t-1,

$BL_{t-1}(>1)$ = total stock of outstanding loans of size \$1 million and greater, and

$TL_{t-1}(>1)$ = total stock of term loans of size \$1 million and greater at the end of t-1.

Briefly, a high proportion of term loans (original maturity at least twelve months) outstanding at the end of quarter t-1 is predetermined for quarter t since many of these long-term loans will not mature during period t. Therefore, the lagged stock of term loans has informational value in assessing the overall amount of current predetermined loans, and the parameter ϕ in (14) may be interpreted as a factor of proportionality between total term loans last period TL_{t-1} and PL_t. The available series on term loans is restricted to loans with a minimum size of \$1 million so $TL_{t-1}(> 1)$ is augmented by the factor $(1 + h)$ to obtain a figure for total term loans of all sizes.[2]

The aggregate loan equation used in estimation combines the flow relationship (13) and the predetermined loan equation (14).

$$(15)\quad BL_t = PL_t + L^f{}_t$$

$$= \phi\ (1 + h)TL_{t-1}(> 1) + n_o E(L_i | L_i > 0) + \varepsilon_L$$

with BL = total outstanding stock of business loans,

PL = predetermined loans,

L^f = new loan flows,

n_o = number of new loans,

ε_L = random error term,

and $E(L_i | L_i > 0)$ is defined by (11).[3]

2. The adjustment is undertaken to remove a potential source of systematic intertemporal parameter variation. Appendix 4 has additional discussion.
3. Equation (15) can be estimated without knowledge of the number of new loans n_o. From (11), $n_o E(L_i | L_i>0)$ involves the term n_o/K which is equal to n (the total number of potential borrowers). The data series for n is described in section 5.5 below.

It should be noted that the proposed model contains a single loan equation for the actual quantity of loans granted. Unlike the disequilibrium models of Chapter 4 separate aggregate demand and aggregate supply equations are not estimated directly. More importantly, the new model is preferred over the earlier approaches since it does not constrain individual loans to be either entirely demand-determined or entirely supply-determined within a given period.

5.3 Interest Rate Equation Specification

A second equation must be added to (15) to make the interest rate endogenous. One possibility is to consider that the current interest rate is a weighted average of the preceding period's rate and the current equilibrium rate.[4]

$$R_t = \mu R_{t-1} + (1 - \mu)\hat{R}_t \qquad 0 \leq \mu \leq 1$$

This approach requires an a priori judgment regarding the precise form of the equilibrium interest rate. For example, following the empirical models derived from the aggregate short-side rule, it could be asserted that $\hat{R}_t$ occurs at the intersection of the aggregate schedules L^d and L^s. However, the theoretical credit rationing literature has presented several reasons why the equilibrium interest rate may not equate notional supply and demand.

An alternative specification was chosen which does not constrain equilibrium to occur at the intersection of aggregate supply and demand, although this result is contained as a special case when $\delta_o = 0$.

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4. This equation was discussed previously in Chapter 4's analysis of the Ito-Ueda model of disequilibrium rationing.

$$(16)\quad \Delta R_t = \delta_o + \delta_1 \frac{(L_t^d - L_t^s)}{n_t} + \varepsilon_R$$

$$\Delta R_t = R_t - R_{t-1}\ ;\ \delta_o \gtrless 0\ ,\ \delta_1 > 0$$

Equation (16) suggests that the current period change in interest rate depends positively on the difference between aggregate demand and supply with the total number of potential borrowers n_t used to scale this difference relative to market size. The sign of the constant term δ_o determines whether equilibrium exists above or below the intersection of L^d and L^s. The equilibrium interest rate for period t can be found from (16) by solving for the interest rate at which $\Delta R_t = 0$.

Expressions for notional aggregate demand and supply are required to complete specification of the interest rate equation. Aggregate flow demand at a given interest rate is the summation of individual flow demands of all borrowers with positive demand at that rate. This quantity can be determined by multiplying the number of potential borrowers with positive demand by the mean value of demand for this group:

$$L^d(R) = n\ \text{Prob}(L_i^d(R) > 0)\quad E(L_i^d | L_i^d(R) > 0)$$

where n = total number of potential borrowers,

$\text{Prob}(L_i^d(R) > 0)$ = proportion of potential borrowers with positive demand at interest rate R,

$n\ \text{Prob}(L_i^d(R) > 0)$ = number of borrowers with positive demand at R, and

$E(L_i^d | L_i^d(R) > 0)$ = average demand for borrowers with positive demand at R.

L^d may be written in terms of known quantities using information from (6).

$$(17)\ L^d = n\,\mathrm{Prob}(e_{1i} > \gamma)\ E(\alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R} + e_{1i} | e_{1i} > \gamma)$$

$$= n\,(1 - F(\gamma/\sigma_1))\ (\alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R}) + n\,\sigma_1 f(\gamma/\sigma_1)$$

with

$$\gamma = -(\alpha_o + \alpha_1\bar{X} + \alpha_2\bar{R}),$$

$$\mathrm{Prob}(L_i^d > 0) = \mathrm{Prob}(e_{1i} > \gamma) = 1 - F(\gamma/\sigma_1)\ ,$$

$$E(e_{1i} | e_{1i} > \gamma) = \frac{\sigma_1\ f(\gamma/\sigma_1)}{1-F\ (\gamma/\sigma_1)}\ ,$$

f( ) = standard normal density function, and

F( ) = standard normal cumulative distribution function.

Similarly, aggregate flow supply at a given interest rate is obtained by summing individual supplies for all borrowers with positive supply at that rate.

$$L^s(R) = n\,\mathrm{Prob}(L_i^s(R) > 0)\quad E(L_i^s | L_i^s(R) > 0)$$

where $n\,\mathrm{Prob}(L_i^s(R) > 0)$ = number of borrowers with positive supply at R, and

$E(L_i^s | L_i^s(R) > 0)$ = average supply to borrowers with positive supply at R.

The aggregate supply function may be expressed as (using (7))

$$(18)\ L^s = n(\mathrm{Prob}(e_{2i} > \theta)\ E(\beta_o + \beta_1\bar{Z} + \beta_2\bar{R} + e_{2i} | e_{2i} > \theta)$$

$$= n\,(1 - F\,(\theta/\sigma_2))\ (\beta_o + \beta_1\bar{Z} + \beta_2\bar{R}) + n\,\sigma_2 f(\theta/\sigma_2)$$

with $\theta = -(\beta_o + \beta_1\bar{Z} + \beta_2\bar{R})$,

$$\text{Prob}(L_i^s > 0) = \text{Prob}(e_{2i} > \theta) = 1 - F(\theta/\sigma_2) \text{ , and}$$

$$E(e_{2i} | e_{2i} > \theta) = \frac{\sigma_2 f(\theta/\sigma_2)}{1-F (\theta/\sigma_2)}$$

The interest rate equation is specified completely with definitions (17) and (18) substituted into (16).

5.4 Equilibrium and Disequilibrium Credit Rationing

Equations (15) and (16) form a system which is suitable for the estimation of both equilibrium and disequilibrium credit rationing. The estimate of the aggregate loan equation (15) gives a "realized loan flow function" $L^f(R)$ which traces the volume of new loans actually granted at alternative rates of interest. Since $L^f(R)$ is derived from the individual short-side rule, it has the property $L^f(R) < \min(L^d(R), L^s(R))$, as shown in Figure 6.[5] The estimated notional aggregate flows L^d and L^s can be constructed using definitions (17) and (18) and parameter estimates.

Credit rationing during period t, defined as total unsatisfied loan demand at the current interest rate, is equal to the difference between aggregate demand and the quantity of new loans granted during period t as determined by $L^f(R)$:

(19) Total rationing = $L^d(R_t) - L^f(R_t)$.

This quantity can be calculated for each sample period by evaluating the estimated aggregate demand function (17) and the loan flow relationship (13) at current period data values.

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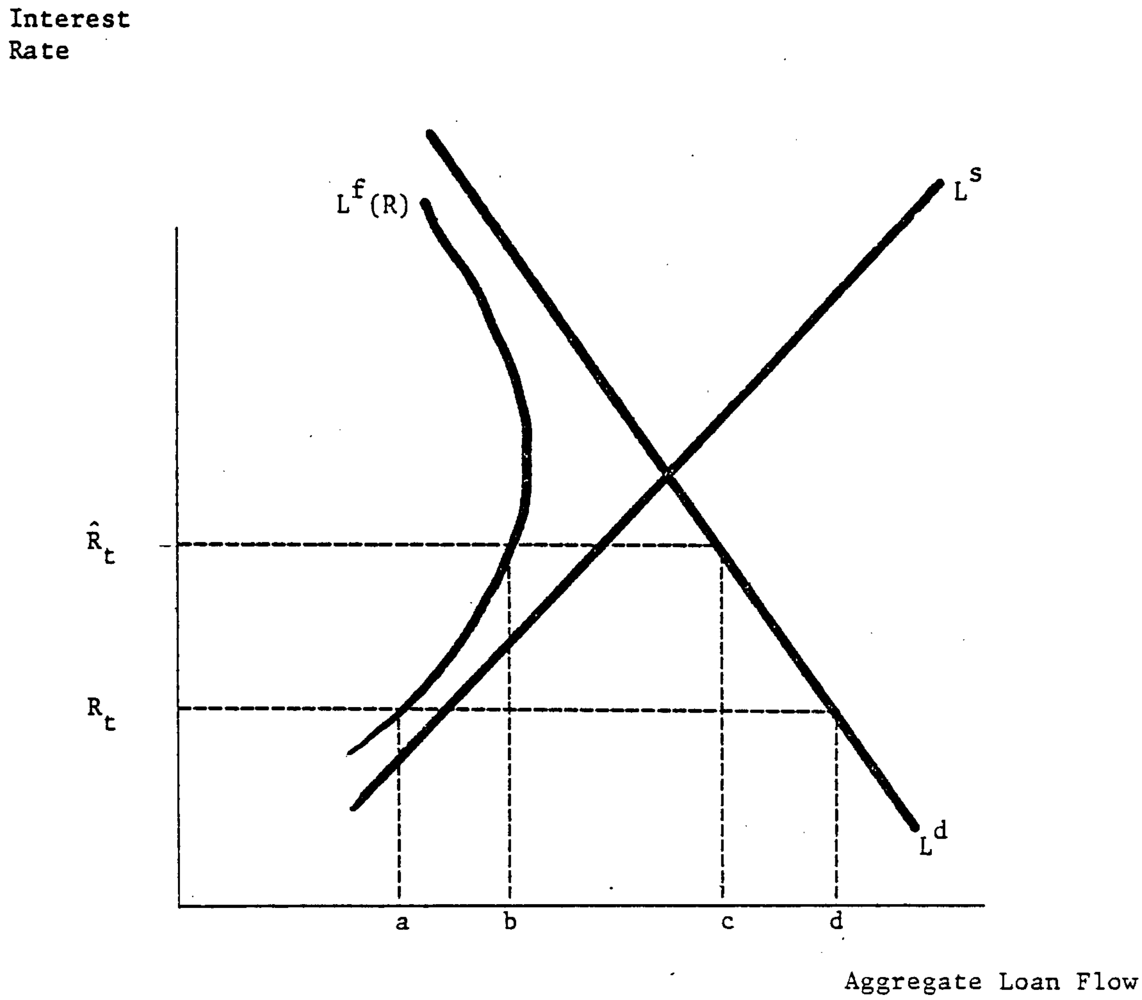
5. In Chapter 4 it is shown that the observed aggregate loan quantity is less than both aggregate demand and aggregate supply when individual loan sizes are determined by the rule $L_i = \min(L_i^d, L_i^s)$.

FIGURE 6: Aggregate Credit Rationing With $L_i = \min(L_i^d, L_i^s)$



The total rationing figure from (19) can be decomposed into separate equilibrium and disequilibrium components. The volume of equilibrium rationing is given by the difference between aggregate demand and the realized loan flow function, where these functions are evaluated at the equilibrium interest rate implied by interest rate equation (16) at $\Delta R_t = 0$.

(20) Equilibrium Rationing = $L^d(\hat{R}_t) - L^f(\hat{R}_t)$

with $\hat{R}_t$ = equilibrium interest rate in period t.

Figure 6 illustrates a situation where the current interest rate is below equilibrium and total rationing (d-a) is greater than equilibrium rationing (c-b). The difference between these quantities is due to positive disequilibrium rationing (b-a) + (d-c) at the disequilibrium interest rate $R_t < \hat{R}_t$.

5.5 Empirical Results

The preceding model was applied to the business loans market of Canadian chartered banks using quarterly data for the period 1968 - 1979. This choice of study period between the 1967 and 1980 Bank Act revisions ensures that lending behavior is examined under a given set of regulatory constraints. Explanatory variables in demand and supply functions were similar to those included in previous empirical studies of disequilibrium rationing by Laffont-Garcia and other authors.

The specification of loan demand reflects the fact that firms have alternative means of financing desired investment expenditures. In addition to securing a bank loan, firms might use retained profits or obtain financing from non-bank sources. The demand for bank loans should be inversely related to the own interest rate and positively related to the rate on non-bank loans. These substitution effects are summarized by the differential between the chartered banks' prime interest rate on business loans and the 90-day rate for prime corporate paper.[6] An increase in this interest differential should reduce loan demand by raising the relative cost of bank loans. The level of retained earnings from the preceding period, which measures the ability of firms to substitute internal sources of financing for loans, is also included as an explanatory variable with an expected negative sign.

A firm's desired investment spending, and therefore desired loan demand, will depend on current economic conditions and anticipated future conditions. The lagged index of real domestic product is used as a proxy for these determinants of loan demand and is anticipated to have a positive effect on demand. The level of predetermined loans is used as another scale factor in flow demand. It is expected that current period demand for new loans would decline if the stock of outstanding loans carried into the period increases.

The banks' desired supply of business loans should increase with the profitability of these loans. This profitability, determined by the loan rate relative to the banks' cost of funds, is measured by the differential between the prime rate on business loans and the interest rate on chartered bank savings deposits. The scale of desired loan supply is also related to the levels of bank deposits and predetermined loans. As deposits increase some proportion of this expansion in bank funds is allocated towards business loans. Conversely, an increase in outstanding predetermined loans should have a negative effect on current period flow supply by chartered banks.

An important feature of the model is that the estimating equations are

6. The corporate paper rate is taken as a representative alternative rate for money market sources of financing. Characteristics of corporate paper are described in Binhammer (1982), p. 147 and Shearer, Chant and Bond (1984), p. 57.

derived from explicit individual borrower demand and supply functions given by (2) and (3). These individual functions express an explanatory variable as a combination of a "population mean" value and a borrower-specific deviation from this mean. The population mean of an explanatory variable such as retained profits is defined as total retained profits of all firms divided by the total number of firms n.[7] Variables in the vectors of population means $\bar{X}$ and $\bar{Z}$ in demand and supply functions are deflated by n wherever necessary to achieve the appropriate scaling.

The empirical model suggests that the average interest rate on all loans $\bar{R}$ is the ideal interest rate variable for demand and supply equations. Data availability requires that the prime rate of interest, representing the minimum rate charged on loans to low risk borrowers, be used instead of $\bar{R}$. Despite this data constraint the prime interest rate variable should be satisfactory since changes in the general levels of rates are related closely to fluctuations in the prime rate.[8]

Other factors minimize potential problems with the prime rate. The supply price variable used to measure net profitability of loans is the differential (R - RD) between the prime rate R and deposit rate RD. Although use of the prime rate in place of $\bar{R}$ tends to understate the lender's revenue, this deficiency causing (R - RD) to understate net lender profitability is at least partially alleviated since administrative loan

7. The total number of firms represents the total number of potential borrowers in a business loans model. Data for the number of firms are constructed from taxation statistics for incorporated and unincorporated businesses.

8. Data are available for the average interest rate on new demand loans for part of the sample period (1968 IV to 1979 IV). The following regression results imply that movements in the prime rate (R) are associated with virtually identical changes in the average rate on new demand loans (AD). Numbers in parentheses are t-statistics.

$$AD = \underset{(1.77)}{.467} + \underset{(33.9)}{.985}\,R \qquad R^2 = .964$$

costs are excluded from the measure (R - RD).[9]

Table 3 presents results from maximum likelihood estimation of equations (15) and (16) under the assumption that the error terms ε_L and ε_R have zero means and are serially uncorrelated.[10] Most coefficients are statistically significant at the 1% level and only the constant term in supply is not significant at the 5% level.[11] The expected signs are obtained except for the retained earnings and index of real domestic product variables in flow demand. The positive coefficient on retained earnings, although unexpected, is also present in previous results of Laffont-Garcia for Canadian business loans. One major result, which will be pertinent to later discussions of rationing, is that flow supply is much more sensitive than flow demand to changes in the relevant interest rate variable.

An indication of the responsiveness of the loan interest rate to demand pressures can be calculated from the interest rate equation. If the final year of the sample period is considered, an increase in quarterly notional excess demand of approximately 625 million dollars (nominal) would induce a one percentage point increase in the loan rate. This figure of 625 million would represent about 8.4% of average estimated realized flows for 1979.

The nature and significance of business loans rationing in Canada can be examined using the estimates of Table 3.

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9. On the demand side, results reported below indicate low sensitivity to the interest rate variable. Therefore, evaluating demand at prime instead of an average rate should not affect conclusions.
10. Non-price variables are measured in hundreds of dollars and expressed in real terms by deflating with the GNE price deflator (1971 = 100). Annual rates of change in the Consumer Price Index are used to construct real interest rates for the interest rate equation.
11. The coefficient on notional excess demand in the interest rate equation is significant at the 5% level using a one-tail test.

Table 3:

Parameter Estimates From The Individual Short-Side Rule Model
(numbers in parentheses are asymptotic t-statistics)

A: <u>Loan Equation</u>

(i) Flow Demand

Constant	RE_{t-1}/n_t	PL_t/n_t	IP_{t-1}/n_t	R_t-CPR_t	σ_1
336.36	.088	-.882	-1.244	-9.612	256.64
(6.51)	(3.19)	(4.23)	(3.31)	(3.01)	(3.70)

(ii) Flow Supply

Constant	D_t/n_t	PL_t/n_t	R_t-RD_t	σ_2
-175.27	.326	-1.474	93.376	142.64
(1.97)	(2.96)	(4.55)	(3.93)	(3.35)

(iii) Predetermined Loans Specification

TL
2.549
(26.69)

B: <u>Interest Rate Equation</u>

Constant	$(L_t^d - L_t^s)/n_t$
-5.193	.255
(2.07)	(1.96)

RE = retained earnings (real)

PL = predetermined loans (real), defined by (14)

IP_{t-1} = lagged index of real domestic product

R = prime rate

CPR = commercial paper rate

D = chartered bank deposits (real)

RD = interest rate on chartered bank non-chequable savings deposits

TL = term loans (real), defined in (14)

L^d = aggregate flow demand, defined by (17)

L^s = aggregate flow supply, defined by (18)

n = number of businessess

Issue 1: Significance of Rationing Relative to Market Size

Column 1 of Table 4 lists the estimated quantity of real per capita rationing PCR for each quarter.[12] These quantities suggest that the general level of rationing is empirically significant relative to various measures of market size. For example, total rationing would have amounted to 9.3% of the outstanding stock of business loans if the average value of PCR (25.1) had prevailed in the final quarter.

Another indicator of the empirical magnitude of rationing is the ratio of rationing to aggregate flow demand. This figure, which measures the proportion of loan demand that is not satisfied by the banks, averages approximately .34 for the 48 quarters in the estimation period. These econometric results on unsatisfied demand are roughly comparable to the Canadian survey results reported by Hatch, Wynant and Grant (1982) when their finding of high informal rejections of business loan requests is considered.[13]

Issue 2: Equilibrium vs Disequilibrium Rationing

Section 5.4 described how total rationing in any quarter can be separated into individual estimates of equilibrium and disequilibrium credit rationing. One objective of this separation is to provide evidence concerning the relative magnitudes of these two categories of rationing. Secondly, this procedure may isolate the dominant cause of the intertemporal variations in total rationing shown in Table 4.

Equilibrium per capita rationing ER, given by column 2 of Table 4, is relatively stable over time with values ranging from 21.0 to 31.2. In contrast, disequilibrium rationing DR (column 3) shows greater variability

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12. Per capita rationing is total rationing (equation (19)) divided by the number of businesses (scaled as hundreds of real dollars).
13. The Hatch, Wynant and Grant (1982) study is discussed in Chapter 1.

Table 4:

Per Capita Rationing Estimates
(hundreds of real dollars)

		(1) PCR	(2) ER	(3) DR
1968	I	31.6	30.9	.7
	II	24.6	31.2	-6.6
	III	33.2	29.8	3.4
	IV	32.9	30.2	2.7
1969	I	17.9	28.8	-10.9
	II	31.6	28.6	3.0
	III	30.8	28.1	2.7
	IV	36.2	28.4	7.8
1970	I	31.0	27.1	3.9
	II	32.0	27.1	4.9
	III	28.4	26.8	1.6
	IV	23.8	27.2	-3.4
1971	I	19.1	25.3	-6.2
	II	21.7	26.1	-4.4
	III	23.5	24.6	-1.1
	IV	22.7	23.6	-.9
1972	I	27.5	23.0	4.5
	II	36.1	22.2	13.9
	III	33.5	21.3	12.2
	IV	33.4	21.2	12.2
1973	I	33.2	21.0	12.2
	II	22.6	21.6	1.0
	III	31.2	21.3	9.9
	IV	17.4	22.0	-4.6
1974	I	25.9	21.6	4.3
	II	33.1	22.2	10.9
	III	25.8	22.0	3.8
	IV	20.3	22.0	-1.7
1975	I	11.5	21.5	-10.0
	II	15.7	22.1	-6.4
	III	19.6	22.3	-2.7
	IV	19.3	22.1	-2.8
1976	I	27.2	21.7	5.5
	II	28.4	21.7	6.7
	III	27.5	21.5	5.8
	IV	41.0	21.5	19.5

1977	I	18.9	21.4	-2.5
	II	22.9	21.2	1.7
	III	21.5	21.4	.1
	IV	20.2	21.5	-1.3
1978	I	10.3	21.9	-11.6
	II	23.0	22.5	.5
	III	11.7	22.5	-10.8
	IV	23.5	22.5	1.0
1979	I	24.1	22.2	1.9
	II	27.5	22.5	5.0
	III	19.0	22.2	-3.2
	IV	13.1	22.5	-9.4

NOTE: All quantities are expressed in "per capita" terms by dividing the relevant total by the number of businesses in the period.

PCR = total per capita rationing (from (19))

ER = per capita equilibrium rationing (from (20))

DR = PCR - ER = per capita disequilibrium rationing with values between 19.5 and -11.6.[14] These two findings indicate that variations in total rationing can be attributed primarily to disequilibrium rationing caused by imperfect adjustment of the current interest rate toward its equilibrium level. Although equilibrium rationing is an important feature of the business loan market (in every period ER exceeds the absolute value of DR), it is not the major source of intertemporal fluctuations in credit rationing.[15]

Further understanding of cyclical variations is possible by examining the determinants of disequilibrium rationing as the loan rate diverges from its equilibrium $\hat{R}_t$. It is recalled that with the individual short-side rule the quantity of new loans is determined by a realized loan flow function $L^f(R_t)$ which is derived from the mix of supply and demand-determined loans. As Figure 6 illustrates, the amount of disequilibrium rationing depends on the change in $(L^d - L^f)$ as R_t varies from $\hat{R}_t$. Interest rate changes and rationing are inversely related through the L^d component of $(L^d - L^f)$ but the separate effect through L^f is uncertain since movements in R_t have opposite repercussions on demand and supply. If the positive supply effect dominates in the neighbourhood of equilibrium

14. As described in Chapter 3 disequilibrium rationing is positive when R_t is less than $\hat{R}_t$. In this situation the lower interest rate creates additional excess demand relative to the level that would have been observed at the equilibrium interest rate. Hence, for these quarters actual rationing PCR exceeds equilibrium rationing. Periods with negative values for disequilibrium rationing have R_t greater than $\hat{R}_t$. Actual rationing in these periods is below the equilibrium level since some borrowers would have been rationed at $\hat{R}_t$ but are unrationed at the higher current interest rate.

15. This conclusion is consistent with Jaffee's (1971) theoretical model.

> "In terms of the comparative static properties of the model, it was shown that parameter changes that would initially lead to more rationing were offset by the adjustment of the loan rate to its new equilibrium level ... equilibrium rationing will generally not be the source of cyclical variations in the total amount of observed rationing." Jaffee (1971), pp. 53-54.

the actual quantity of loans granted is positively related to R_t along L^f.

In the Canadian business loan market the supply effect operating on realized loan flows is sufficiently great that most disequilibrium rationing is related to supply-side responses to current interest rates rather than changes in desired loan demands. This conclusion is demonstrated by referring to the following figures evaluated at R_t and $\hat{R}_t$ (with differences in parentheses) from the final quarter in the study period.

	L^d/n	L^s/n	L^f/n	Rationing
R_t	68.12	66.18	55.07	13.05
$\hat{R}_t$	69.46	49.13	46.94	22.52
	(-1.34)	(17.05)	(8.13)	(-9.47)

During this quarter the interest rate was above equilibrium so actual rationing at R_t was less that the equilibrium quantity. At the higher current rate there was some decrease in loan demand relative to $\hat{R}_t$ but the increase in supply induced by $R_t > \hat{R}_t$ was considerably greater. Thus, through the supply effect on actual loan flows, most of the decrease in rationing at $R_t > \hat{R}_t$ was related to higher realized loan flows (8.13) rather than decreases in loan demand (-1.34).

In addition to the preceding structural factors the magnitude and duration of disequilibrium rationing depends on the characteristics of interest rate adjustment. Table 5 provides some evidence concerning the effectiveness of interest rate flexibility as an equilibrating mechanism. In 39 periods the real interest rate was required to change by at least .25% to achieve equilibrium in the current period (column 1). However, after interest rate adjustments had taken place, there were only 12 quarters for which the actual interest rates were at least .25% different from the equilibrium rates implied by the model (column 2).[16]

Issue 3: Monetary Policy and Intertemporal Fluctuations in Rationing

Results from the model may be used to identify periods when credit rationing was particularly important or unimportant. Table 4 indicates substantial intertemporal variation in PCR and groupings of periods with similar levels of rationing. One question suggested by these trends is whether there exists a stable relationship between fluctuations in rationing and the current direction of monetary policy. For example, ceteris paribus, it might be argued that periods identified with low (high) rationing would be associated with expansionary (tight) monetary conditions. Proposition 1 of Chapter 3 establishes that such a relationship is not expected for equilibrium rationing since changes in the equilibrium interest rate tend to offset the immediate effect on rationing from policy-induced shifts in loan supply schedules.

Nevertheless, by influencing the quantity of disequilibrium rationing, monetary policy is a potential source of cyclical fluctuations in rationing. Consider a contractionary monetary policy which increases the equilibrium loan rate. If the current period's interest rate rises by a smaller amount than $\hat{R}_t$ there will be positive disequilibrium rationing. Therefore, given a tendency for under-adjustment of interest rates, there would be an inverse relationship between total rationing and indicators of monetary expansion. However, if instead the current loan rate reacts to the same disturbance by over-adjusting above $\hat{R}_t$, the contractionary monetary policy would reduce current rationing by creating negative disequilibrium rationing.

This theoretical analysis reveals that monetary policy would not have

16. An additional 3 quarters had (absolute) values of $(\hat{R}_t - R_t)$ within the range .20 - .24.

Table 5:

Interest Rate Adjustment in the Individual Short-Side Rule Model

Interest Rate Differential	(1) Initial Disequilibrium† $(\hat{R}_t - R_{t-1})$	(2) End-of-Period Disequilibrium† $(\hat{R}_t - R_t)$
	number of periods	
0 - .24	9	36
.25 - .49	13	11
.50 - .74	6	1
.75 - .99	8	0
$\geq$ 1.00	12	0

†Interest rate differentials are taken as absolute values.

a uni-directional impact on rationing in a market where both under-adjustment and over-adjustment can occur in the short-run. The Canadian business loan market is such a market. Although 27 quarters were characterized by under-adjustment of the current loan rate toward equilibrium, the remaining 21 quarters experienced an over-shooting phenomenon with the interest rate moving past its equilibrium level. Since there is empirical evidence of both types of adjustment behavior, the cyclical rationing patterns in Table 4 will not exhibit a uniform relationship with monetary trends. It is noteworthy, however, that the model's prediction of above-average rationing throughout much of the 1968-1970 period agrees with J.A. Galbraith's perceptions of tight monetary conditions and excess loan demand during that era.[17]

5.6 Model Comparisons

It is instructive to compare the above results from an individual short-side rule model with those obtained from the standard aggregate short-side rule model represented by equations (17) and (18) of Chapter 4. Two specifications of the latter model were considered since the data series for business loans contains both new and predetermined loans. In model AG-1 the dependent variable is the total stock of business loans and the two-equation system of Chapter 4 is modified to be consistent with

$$\begin{aligned} BL_t &= PL_t + L_t^f \\ &= PL_t + \min(L_t^d, L_t^s) \end{aligned}$$

with BL = total outstanding stock of business loans,

PL = predetermined loans,

17. J.A. Galbraith's description of this period is quoted in Chapter 1.

L^f = new loan flows,

L^d = aggregate flow demand, and

L^s = aggregate flow supply.

The two estimating equations of AG-1 have the general forms

$$BL_t = PL_t + \alpha_1 X + \alpha_2 R_t + \frac{\mu}{(1-\mu)} (\alpha_2 - \beta_2) \quad \Delta R_t^{+} + u_1$$

$$BL_t = PL_t + \beta_1 Z + \beta_2 R_t + \frac{\mu}{(1-\mu)} (\beta_2 - \alpha_2) \quad \Delta R_t^{-} + u_2$$

with PL_t (defined by (14)) a function of term loans TL. One deficiency of the AG-1 specification is that parameters for PL in flow supply and demand could not be identified so PL cannot be used as an explanatory variable in the X and Z vectors. Given this limitation, rationing estimates from AG-1 may not be comparable with estimates in Table 4 for the individual short-side rule model. In response to this concern the levels of PL estimated from the individual short-side rule model were used to construct a series for new loan flows L^f which was then used as the dependent variable in AG-2. Thus AG-2 is also derived from the aggregate short-side rule $L^f = \min(L^d, L^s)$ but with the loan flow series as the dependent variable the coefficients on PL in the X and Z vectors can be identified. Explanatory variables are the same as those used in the previous section. However, following the literature of the conventional models, variables are never deflated by the number of potential borrowers to obtain per capita series.

The three stage least squares estimates reported in Table 6 are similar for AG-1 and AG-2. All parameters possess the expected signs but approximately half of the coefficients are not statistically significant. Although coefficients on interest rate variables are not significant their magnitudes are consistent with the conclusion from the individual short-side rule model that supply is more interest sensitive than demand.

As discussed in Chapter 4 the parameter μ has been proposed for testing the hypothesis that the loan market is in equilibrium. Estimates of .016 and .046 from AG-1 and AG-2 imply that the current interest rate adjusts to eliminate 98.4% and 95.4% respectively of the difference between last period's rate and the current equilibrium rate R*. Since these coefficients are not statistically significant from zero the hypothesis test suggests that interest rate adjustment is sufficiently rapid that the business loan market in Canada reaches equilibrium each period. It is interesting to note that this conclusion in favour of equilibrium exists despite end-of-period disequilibrium $(R_t^* - R_t)$ being at least .25% for 29 quarters in AG-1 and 30 quarters in AG-2 (Table 7).

The evidence summarized in Table 7 suggests that a value of μ not significantly different from zero may not be a satisfactory test of equilibrium. The cause of this deficiency is the prevalence of both under-adjustment and over-adjustment of interest rates throughout the study period. A comparison of actual interest rate changes with model predictions of changes that would give equilibrium reveals 21(18) quarters with over-adjustment in AG-1 (AG-2). This diverse behavior for interest rate movements might indicate that the adjustment speed is variable rather than constant and that the true value of μ in a given period can be positive or negative.[18] If under-adjustment ($\mu > 0$) and over-adjustment ($\mu < 0$) are both common in individual periods, the value of μ estimated for the entire sample period could tend towards zero (and thus the hypothesis test could be biased toward acceptance of equilibrium) even if the market

18. Rearrangement of the interest rate equation $R_t = \mu R_{t-1} + (1 - \mu)R_t^*$ gives $\Delta R_t = R_t - R_{t-1} = (1-\mu)(R_t^* - R_{t-1})$. From this relationship it is seen that for a period of over-adjustment, with the actual change ΔR_t greater than the change $(R_t^* - R_{t-1})$ that would restore equilibrium, the parameter μ is negative.

Table 6:

Parameter Estimates From Aggregate
Short-Side Rule Models
(numbers in parentheses are t-statistics)

Demand

Model	Constant	RE_{t-1}	IP_{t-1}	R_t-CPR_t	PL_t	R^2
AG-1	-5169.46	-.160	137.77	-111.55		.97
	(3.76)	(.41)	(7.28)	(.71)		
AG-2	-8859.76	-.183	190.00	-171.39	-.822	.56
	(2.78)	(.47)	(4.24)	(1.04)	(5.06)	

Supply

Model	Constant	D_t	R_t-RD_t	PL_t	R^2
AG-1	-1547.71	.219	1170.62		.98
	(1.39)	(6.27)	(1.60)		
AG-2	-1272.15	.207	1144.37	-.601	.73
	(1.12)	(5.69)	(1.57)	(9.18)	

Miscellaneous

Model	μ	TL
AG-1	.016	.946
	(.16)	(6.01)
AG-2	.046	
	(.46)	

Table 7:

Interest Rate Adjustment in AG-1 and AG-2

Interest Rate Differential	Initial Disequilibrium† $(R^*_t - R_{t-1})$		End-of-period Disequilibrium† $(R^*_t - R_t)$	
	number of periods			
	AG-1	AG-2	AG-1	AG-2
0 - .24	10	4	19	18
.25 - .49	10	11	16	11
.50 - .74	9	10	8	11
.75 - .99	5	8	2	6
$\geq$ 1.00	14	15	3	2

† Interest rate differentials are taken as absolute values.

was frequently in disequilibrium. Therefore, the proposed test for equilibrium based on μ may be inappropriate in cases such as the business loan market where there is evidence of periodic over-adjustment of the interest rate in the current period. In these situations the estimate of μ might be interpreted as only a measure of the average speed of adjustment.

As described previously aggregate short-side rule models such as AG-1 and AG-2 do not allow for equilibrium rationing but disequilibrium rationing may be calculated as

$$DR = \begin{cases} L^d - L^s & \text{if } L^d - L^s > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Characteristics of disequilibrium rationing in the AG models may be compared with results in Table 4 to show the consequences of using an individual short-side rule to model the loan market. The four possible combinations of outcomes from different models and their implications for relative interest rates are shown below:

$$(21)(a) \quad \left.\begin{array}{l} DR_{Ag} > 0 \\ \\ DR_{Ind} > 0 \end{array}\right\} \quad \begin{array}{l} R_t < R_t^* \\ \\ R_t < \hat{R}_t \end{array}$$

$$(b) \quad \left.\begin{array}{l} DR_{Ag} = 0 \\ \\ DR_{Ind} < 0 \end{array}\right\} \quad \begin{array}{l} R_t > R_t^* \\ \\ R_t > \hat{R}_t \end{array}$$

$$(c) \quad \left.\begin{array}{l} DR_{Ag} = 0 \\ \\ DR_{Ind} > 0 \end{array}\right\} \quad R_t^* < R_t < \hat{R}_t$$

$$(d) \quad \left.\begin{array}{l} DR_{Ag} > 0 \\ \\ DR_{Ind} < 0 \end{array}\right\} \quad \hat{R}_t < R_t < R_t^*$$

with DR_{Ag}, DR_{Ind} = disequilibrium rationing in an aggregate short-side rule model and the individual short-side rule model respectively, and

R_t^*, $\hat{R}_t$ = equilibrium interest rates in the aggregate and individual short-side rule models respectively.

Disequilibrium results from the two types of models are qualitatively in agreement in (21)(a) and (b) where positive (zero) disequilibrium rationing in AG corresponds with positive (negative) values from the individual short-side model IND. In these situations the nature of disequilibrium is identical in each model with the current rate of interest below (above) both equilibrium interest rates R_t^* and $\hat{R}_t$. Periods described in (21)(c) or (d) demonstrate disagreement between models with respect to the direction of interest rate adjustment necessary to reach equilibrium.

Table 8's comparison of disequilibrium rationing estimates shows much greater similarity between AG-1 and AG-2 than between the individual short-side rule model and the aggregate models. In only three periods did AG-1 and AG-2 produce different predictions on whether or not excess demand existed. However, when IND is compared with AG-2 the inconsistencies described by (21)(c)-(d) occur in 21 of 48 quarters, with positive disequilibrium rationing occurring more frequently in the individual short-side rule model (29 periods) than in the aggregate model (22 periods).

Differences persist even if analysis is restricted to periods when both models have positive rationing. During these quarters total rationing averages 29.6 in IND (of which 6.3 is disequilibrium rationing) but only 11.1 and 12.0 in AG-1 and AG-2 respectively. Therefore, relative to the

Table 8:

Per Capita Disequilibrium Rationing in Aggregate and Individual Short-Side Rule Models
(hundreds of real dollars)

		AG-1	AG-2	IND
1968	I	0	0	.7
	II	0	0	-6.6
	III	0	0	3.5
	IV	0	0	2.7
1969	I	0	0	-10.9
	II	0	0	3.0
	III	2.8	5.5	2.7
	IV	1.4	2.9	7.8
1970	I	12.6	15.8	3.9
	II	0	0	4.9
	III	4.9	6.4	1.6
	IV	0	0	-3.4
1971	I	0	0	-6.2
	II	0	0	-4.4
	III	2.4	3.6	-1.1
	IV	0	0	.9
1972	I	3.8	2.3	4.5
	II	0	0	13.9
	III	6.4	0	12.2
	IV	0	0	12.2
1973	I	10.9	3.1	12.2
	II	0	0	1.0
	III	16.5	12.5	9.9
	IV	0	0	-4.6
1974	I	17.5	18.9	4.3
	II	12.1	11.2	10.9
	III	27.9	33.7	3.8
	IV	1.9	4.2	-1.7
1975	I	1.7	4.5	-10.0
	II	0	0	-6.4
	III	11.4	14.5	-2.7
	IV	0	0	-2.8
1976	I	10.0	11.6	5.5
	II	0	0	6.7

	III	19.5	23.9	5.8
	IV	14.0	14.5	19.5
1977	I	5.4	7.5	-2.5
	II	0	0	1.7
	III	9.8	11.7	.1
	IV	0	0	-1.3
1978	I	0	0	-11.6
	II	0	0	.5
	III	0	3.7	-10.8
	IV	0	0	1.0
1979	I	2.6	6.6	1.9
	II	0	0	5.0
	III	0	.8	-3.9
	IV	0	0	-9.4

individual short-side rule model, aggregate models understate total rationing and overstate disequilibrium rationing. The greater disequilibrium rationing in aggregate models is explained by end-of-period interest rates tending to be farther from equilibrium in AG than in IND.[19]

The new model presented and estimated in this Chapter has proven to be useful in studying business loans rationing by Canadian chartered banks. Two of the major results are that equilibrium rationing is significant relative to various measures of market size but that disequilibrium rationing is the primary cause of cyclical fluctuations in total rationing of business loans. Application of the individual short-side rule model to other financial markets and other countries would demonstrate whether these conclusions are replicated in alternative contexts.

19. The average absolute value of $(R_t^* - R_t)$ is .39 in AG-1 and .44 in AG-2. The average absolute value of $(\hat{R}_t - R_t)$ is .18 in IND. Greater efficiency of interest rate adjustment in IND is also evident from a comparison of Tables 5 and 7.

Chapter 6

CONCLUSIONS

It is apparent from survey evidence and casual empiricism that loan interest rates may not adjust to levels at which lenders are willing to supply all quantities demanded by potential borrowers. With this rigidity in interest rate movements some non-price criteria are used to determine which borrowers are rationed with unsatisfied demands at current interest rates. Recent theoretical explanations of credit rationing, focusing on loan size determination from a microeconomic perspective, have demonstrated that rationing is optimal behavior under a variety of conditions regarding the lender's abilities to screen individual default risks and differentiate interest rates charged to different borrowers.

In addition to an emphasis on micro analysis the theoretical literature is noted for its recognition that rationing may develop from two sources. Equilibrium credit rationing, defined as unsatisfied demand at the equilibrium rate of interest, is encountered by some borrowers if there are constraints on the degree of interest rate differentiation among borrowers. In models where lenders can evaluate individual default risks constrained differentiation may result from oligopolistic pricing behavior or attempts by lenders to avoid unsatisfactory bargaining outcomes given imperfect information on individual demand conditions. Higher-risk borrowers are most likely to be rationed in these models.

A second source of rationing exists if there is non-instantaneous movement of interest rates to new equilibrium values after disturbances. Excess demands from this source are referred to as disequilibrium rationing since they are short-run consequences of incomplete interest rate adjustments. Disequilibrium rationing is positive and total rationing exceeds the equilibrium quantity when the prevailing interest rate R_t is below equilibrium $\hat{R}_t$ since lenders are unwilling to satisfy all increases in loan demand arising from R_t being below its equilibrium level. Conversely, disequilibrium rationing is negative with $R_t > \hat{R}_t$ since the high current interest rate raises desired supply and decreases demand, with the result that some borrowers are not rationed at R_t but would have experienced excess demand at $\hat{R}_t$.

Empirical study of credit rationing is made difficult by the complications it imposes on econometric modelling. Typically a market is modelled by assuming it is in market-clearing equilibrium with the observed quantity always corresponding to the intersection of aggregate supply and demand. Such an assumption obviously does not allow rationing to exist. Previous empirical studies of credit rationing respond to this problem by utilizing an aggregate short-side rule which assumes quantity observed is the minimum of aggregate supply and demand. Although this methodology introduces the possibility of rationing the resulting model is subject to serious criticism for ignoring characteristics of loan markets prominent in the theoretical literature. Loan equations constructed from the aggregate short-side rule are not derived from micro foundations whereas theory emphasizes loan determination at the individual borrower level. Another departure from theoretical analysis is a total neglect of equilibrium rationing and a restriction that all excess demand must be disequilibrium rationing. Briefly stated, existing empirical models fail to incorporate factors believed to distinguish the loan market from many other markets, and the inconsistencies with theory suggest that estimates of rationing from these models may be unreliable.

Unlike traditional methods the empirical model developed in this study derives aggregate equations from an explicit micro explanation of loan determination. Loan sizes are determined by an individual short-side rule under which an individual's loan is the minimum of borrower-specific loan supply and demand functions. Since the individual short-side principle allows a mix of supply and demand-determined loans during any given period the aggregate loan quantity is less than both aggregate supply and demand. In contrast to the standard aggregate short-side rule model points along the market's notional demand and supply functions are never observed. Nevertheless, aggregation over all borrowers using the individual short-side rule does yield equations suitable for estimation purposes. An attractive feature of the new model is that it can be used to obtain the first estimates of equilibrium credit rationing. This allowance for both equilibrium and disequilibrium rationing, together with the micro emphasis, means that the proposed empirical model provides greater consistency between theoretical and applied work than has been possible previously.

The new empirical model was applied to the market for business loans from Canadian banks using quarterly data from 1968 to 1979. Selection of this study period corresponds with a time horizon between Bank Act revisions and thus ensures uniform regulatory constraints on lending behavior. Results indicate that business loans rationing occurs in significant quantities in Canada as the average ratio of quarterly rationing to aggregate flow demand is approximately one-third. The estimates of equilibrium rationing reveal that this category, ignored in conventional empirical models, is important since equilibrium rationing exceeds the (absolute) value of disequilibrium rationing in each quarter. However, the evidence indicates that intertemporal fluctuations in total rationing are caused primarily by variations in disequilibrium rationing. With low interest sensitivity of loan demand but high interest sensitivity of supply these variations in rationing are related largely to supply responses of lenders as current interest rates diverge from equilibrium.

A comparison of results from the new approach with those from the traditional model shows that conclusions on the importance of credit rationing do depend on the short-side rule used to represent quantity determination. One difference is that rationing exists every period with the individual short-side rule. For some periods there is no rationing in the aggregate model which is consistent with micro theories of loan determination only if each borrower has loan demand satisfied completely. When periods with positive rationing in both approaches are considered rationing estimates in the aggregate model average about 40% of levels in the individual short-side rule model. Much of the difference corresponds to the size of equilibrium rationing in the new model. Disequilibrium rationing is actually larger in the traditional method where the gap between current and equilibrium interest rates tends to be greater.

Future research may be extended in several directions using the framework presented in this study. One direction is to examine the generality of results obtained for business loans rationing by Canadian banks. Applications of the individual short-side model to the other loan markets and other countries would demonstrate whether conclusions differ in other settings.

The model also could be used to analyze the effects on a given loan market of changes in banking regulations. In Canada the 1980 Bank Act revision relaxed previous limitations on domestic lending activity of foreign-owned banks. Although considerable restrictions remain, notably that total Canadian assets of all foreign banks cannot exceed 8% of total domestic assets of all banks, the 1980 revision gives some increase in competition in certain areas including business loans. Information on levels of rationing under 1980 Bank Act regulations would indicate potential effects of further easing of restrictions on foreign-controlled banks.

Another area for research would be to use rationing estimates from Chapter 5 to examine the impact of rationing on real expenditures by businesses. Firms that do not receive desired loan sizes from chartered banks may face financing constraints that prevent some intended investment spending from being undertaken. Whether rationing by banks imposes these liquidity constraints depends on the ability of rationed firms to substitute non-bank sources of financing for bank loan financing. Previous empirical consideration of real effects has been impeded by problems in obtaining satisfactory measures of rationing. Given the advantages of the individual short-side model described earlier its estimates of rationing should benefit analysis of inter-relationships between financial and real sectors.

The potential extensions listed above show that the individual short-side rule model can make useful contributions to the study of numerous issues dealing with loan markets. In the future the model might also find applications in other markets in which quantity determination is characterized by an individual short-side rule.

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APPENDIX 1: Properties of the Loan Offer Curve

The concept of a loan offer curve was used in Chapter 3's analysis of credit rationing. The properties of this curve, summarized by (12)(a) - (12)(e) in section 3.1, are now presented in greater detail.

The offer curve is defined by

$$(11) \qquad \frac{\delta P_i^s}{\delta L_i} = R_i - \frac{\delta C_i}{\delta L_i}$$

$$= R_i(1-G[\beta]) + q \int_v^\beta yg(y)dy - I = 0.$$

After integration by parts (11) may be denoted as

$$(11)' \qquad \frac{\delta P_i^s}{\delta L_i} = R_i\ (1-G[\beta]) + q\ (\beta G[\beta] - \int_v^\beta G[y]dy) - I$$

$$= R_i - I + (q\beta - R_i)G[\beta] - q \int_v^\beta G[y]dy = 0$$

$$\text{with} \quad (q\,\beta - R_i) = R_i\ (\frac{qL_i}{\rho(L_i+E)} - 1) < 0, \text{ and}$$

$$q = \ = \rho + \rho' A \geq 0.$$

Inspection of (11)' indicates that $\delta P_i^s / \delta L_i < 0$ at all loan sizes for $R_i <$ I. Therefore,

$$(12)(a) \qquad L_i^s = 0 \qquad \text{for} \qquad R_i < I.$$

For $R_i = I$, $(q\beta - R_i)G[\beta] = q\int_v^\beta G[y]dy$ along the offer curve according to (11)'. Since $G[y] \geq 0$ and $(q\beta - R_i) < 0$, (11)' is satisfied only if $G[\beta] = q\int_v^\beta G[y]dy = 0$ which from (5) requires $\beta \leq v$ along the offer curve at $R = I$. Manipulation of $\beta = \frac{R_i L_i}{\rho(L_i + E)} \leq v$ implies

$$\text{(12)(b)} \qquad 0 \leq L_i^s \leq \frac{vpE}{I - v\rho} \text{ for } R_i = I \text{ and } I - v\rho > 0.$$

The requirement that the firm's critical no-default rate of return β can not exceed the minimum rate of return v signifies that default risk is zero along the horizontal section of a loan offer curve.

The curvature of the offer curve at $R_i > I$ is found by differentiating (11).

$$\frac{\delta L_i^s}{\delta R_i} = -\frac{\delta^2 P_i^s/\delta L_i \delta R_i}{\delta^2 P_i^s/\delta L_i^{\,2}}$$

$$\text{(12)(c)} \qquad = \frac{-\left(1 - G[\beta] + (q\beta - R_i)\frac{\delta\beta}{\delta R_i} g(\beta)\right)}{(q\beta - R_i)\frac{\delta\beta}{\delta L_i} g(\beta) + q'\int_v^\beta yg(y)dy}$$

with $\frac{\delta\beta}{\delta R_i} = \frac{L_i}{\rho A} > 0$,

$\frac{\delta\beta}{\delta L_i} = \frac{R_i(\rho A - qL_i)}{(\rho A)^2} > 0$, and

$q' \leq 0$.

The denominator of (12)(c) is negative so the sign of $\delta L_i^s/\delta R_i$ is identical to the sign of $(1-G[\beta] + (q\beta - R_i)\frac{\delta\beta}{\delta R_i} g(\beta))$. Since $0 \leq G[\beta] \leq 1$ the offer curve has a positive slope unless $G[\beta]$ and $g(\beta)$ have significantly large values.

The effect on loan supply of a change in the opportunity cost I is determined by differentiating (11).

$$\frac{\delta L_i^s}{\delta I} = -\frac{\delta^2 P_i^s/\delta L_i \delta I}{\delta^2 P_i^s/\delta L_i^2}$$

$$= \frac{1}{(q\beta - R_i)\frac{\delta\beta}{\delta L_i} g(\beta) + q' \int_v^\beta y g(y) dy} < 0$$

Property (12)(e) of an offer curve is proven in Jaffee (1971), pp. 60-62.

APPENDIX 2: Properties of Iso-profit Curves

Propositions concerning borrower and lender iso-profit curves used in Chapter 3 are proven in this appendix. From section 3.4 an iso-profit curve of the lender Π^s has slope

$$(22) \qquad \left.\frac{\delta R_i}{\delta L_i}\right|_{\Pi^s} = - \frac{\delta P_i^s/\delta L_i}{\delta P_i^s/\delta R_i}$$

$$= \frac{-\left(R_i(1-G[\beta]) + q\int_v^\beta yg(y)dy - I\right)}{L_i\ (1-G[\beta])} .$$

The numerator of (22) is zero at the loan offer curve L_i^s defined by $\delta P_i^s/\delta L_i = 0$. Thus, the slope of an iso-profit curve of the lender is zero as it intersects L_i^s. The sign of $\delta R_i/\delta L_i$ at points not along L_i^s is determined from A(1).

A(1): For a given interest rate factor R_i, the lender's expected profit decreases monotonically as loan size varies from the offer curve in either direction.

Proof: The lender's offer curve is determined by

$$(11) \qquad \frac{\delta P_i^s}{\delta L_i} = R_i - \frac{\delta C_i}{\delta L_i} = 0.$$

Differentiate (11) with respect to L_i to obtain

$$\frac{\delta^2 P_i^s}{\delta L_i^2} = (q\beta - R_i)\frac{\delta\beta}{\delta L_i} g(\beta) + q' \int_v^\beta yg(y)dy.$$

Since $(q\beta - R_i) < 0$, $\delta\beta/\delta L_i > 0$, and $q' \leq 0$, $\delta^2 P_i^s/\delta L_i^2 < 0$ which proves A(1).

A(1) indicates that a change in loan size away from the loan offer curve must be accompanied by an increase in interest rate to maintain a constant level of expected lender profit. Thus,

Proposition 4: An iso-profit curve of the lender is positively-sloped to the right of the offer curve L_i^s and negatively-sloped to the left of L_i^s.

Proposition 5 will now be proven.

Proposition 5: Expected lender profit increases along an offer curve L_i^s as the interest rate increases. Therefore, iso-profit curves intersecting L_i^s at successively higher interest rates represent successively greater levels of expected lender profit.

Proof: Expected lender profit along the loan offer curve to borrower i, denoted by $P_i^s[L_i^s]$, is obtained by substituting (11) into (9) and evaluating L_i as $L_i^s(R_i)$.

$$P_i^s[L_i^s] = (\rho A - qL_i^s) \int_v^\beta yg(y)dy$$

The change in expected profit as the interest rate varies along the offer curve is

$$\frac{\delta P_i^s[L_i^s]}{\delta R_i} = -L_i^s \, z \, \frac{\delta L_i^s}{\delta R_i} + (\rho A - qL_i^s)\beta \, \frac{\delta \beta}{\delta R_i} \; g(\beta)$$

$$\text{with} \quad z = (q\beta - R_i)\,\frac{\delta \beta}{\delta L_i}\; g(\beta) + q' \int_v^{\beta} y g(y)dy$$

Since $z\delta L_i^s/\delta R_i = -(1 - G[\beta] + (q\beta - R_i)\frac{\delta\beta}{\delta R_i} \; g(\beta))$

from (12)(c) and $\beta = RL/\rho A$ from (6), it can be shown that $\delta P_i^s[L_i^s]/\delta R_i = L_i^s(1 - G[\beta]) > 0$ which proves Proposition 5.

An iso-profit curve of the borrower Π^d has the slope

$$(28) \quad \left.\frac{\delta R_i}{\delta L_i}\right|_{\Pi^d} = -\frac{\delta P_i^d/\delta L_i}{\delta P_i^d/\delta R_i}$$

$$= \frac{q\int_{\beta}^{K} yh(y)dy - R_i(1-H[\beta])}{L_i(1-H[\beta])}$$

The numerator of (28) is zero at the loan demand curve L_i^d defined by $\delta P_i^d/\delta L_i = 0$. This indicates that an iso-profit curve of the borrower has a slope equal to zero as it intersects L_i^d. The sign of $\delta R_i/\delta L_i$ at points not along L_i^d is determined from A(2).

A(2): For a given interest rate factor R_i, the borrower's expected profit decreases monotonically as loan size varies from the demand curve in either direction.

Proof: The borrower's demand curve is defined by

$$(26) \qquad \frac{\delta P_i^d}{\delta L_i} = \int_{\beta}^{K} (qy - R_i)h(y)dy = 0$$

Differentiate (26) with respect to L_i to obtain

$$\frac{\delta^2 P_i^d}{\delta L_i^2} = -(q\beta - R_i)\frac{\delta\beta}{\delta L_i} h(\beta) + q' \int_{\beta}^{K} yh(y)dy$$

< 0 from second order conditions which proves A(2).

A(2) implies that a movement in loan size away from the demand curve must be accompanied by a decrease in interest rate to keep expected borrower profit constant. Therefore,

Proposition 6: An iso-profit curve of the borrower is positively-sloped to the left of the loan demand curve L_i^d and negatively-sloped to the right of L_i^d.

Finally, the direction of increasing expected profit on the borrower's iso-profit map is known from Proposition 7.

Proposition 7: Expected profit of borrower i increases as the interest rate falls along L_i^d. Therefore, borrower iso-profit curves intersecting L_i^d at successively lower interest rates represent successively greater levels of expected borrower profit.

Proof: Expected borrower profit along the loan demand curve, denoted by

$P_i^d[L_i^d]$, is calculated by substituting (26) into (25) and evaluating L_i as $L_i^d(R_i)$.

$$P_i^d[L_i^d] = (\rho A - qL_i)\int_\beta^K yh(y)dy$$

The change in expected profit as the interest rate varies along the demand curve is

$$\frac{\delta P_i^d[L_i^d]}{\delta R_i} = - L_i^d \, w \, \frac{\delta L_i^d}{\delta R_i} + L_i^d \, (q\beta - R_i) \, \frac{\delta \beta}{\delta R_i} \, h(\beta)$$

$$\text{with } w = - (q\beta - R_i) \, \frac{\delta\beta}{\delta L_i} \, h(\beta) + q' \int_\beta^K yh(y)dy \quad .$$

Since $w\delta L_i^d/\delta R_i = 1 - H[\beta] + (q\beta - R_i)\delta\beta/\delta R_i \; h(\beta)$ from (27),

$\delta P_i^d[L_i^d]/\delta R_i = - L_i^d(1 - H[\beta]) < 0$ which proves Proposition 7.

APPENDIX 3: Cumulative Densities and Expectations

(i) $\text{Prob}(L_i = L_i^d > 0) = \text{Prob}(e_{1i} > \gamma, e_{3i} \le \eta)$

$$= \int_{-\infty}^{\eta} \int_{\hat{\gamma}}^{\infty} g_1(e_1, e_3)\, de_1\, de_3$$

(ii) $\text{Prob}(L_i = L_i^s > 0) = \text{Prob}(e_{2i} > \theta, e_{3i} \ge \eta)$

$$= \int_{\eta}^{\infty} \int_{\theta}^{\infty} g_2(e_2, e_3)\, de_2\, de_3$$

where $g_1(e_1, e_3)$ and $g_2(e_2, e_3)$ are bivariate normal density functions.

(iii) $E(e_{1i} | e_{1i} > \gamma, e_{3i} \le \eta)$

$$= \frac{\int_{-\infty}^{\eta} \int_{\gamma}^{\infty} e_1 g_1(e_1, e_3)\, de_1\, de_3}{\int_{-\infty}^{\eta} \int_{\gamma}^{\infty} g_1(e_1, e_3)\, de_1\, de_3}$$

(iv) $E(e_{2i} | e_{2i} > \theta, e_{3i} > \eta)$

$$= \frac{\int_{\eta}^{\infty} \int_{\theta}^{\infty} e_2 g_2(e_2, e_3)\, de_2\, de_3}{\int_{\eta}^{\infty} \int_{\theta}^{\infty} g_2(e_2, e_3)\, de_2\, de_3}$$

APPENDIX 4: Modelling Predetermined Loans in an Aggregate Loan Equation

The business loans data series contains both new loans granted during the current period and "predetermined" loans granted in previous periods and still outstanding. Available information can be used to construct a loan equation which accounts for both loan categories.

Predetermined loans at the end of quarter t (PL_t) can be written as the sum of three figures:

(i) total term loans (original term to maturity of twelve months or greater) outstanding at the end of quarter t-1(TL_{t-1}), minus

(ii) loans in TL_{t-1} which matured before the end of quarter t (TL_t^{mat}), plus

(iii) non-term loans (original maturity less than twelve months) which were outstanding at the end of t-1 and did not mature before the end of quarter t(OL).

$$(1) \quad PL_t = TL_{t-1} - TL_t^{mat} + OL$$

Some information is available for the first term loans component of (1). However, prior to 1973 only term loans of \$1 million and over were included in this data series. Therefore, the definition for PL is rewritten by expressing TL_{t-1} as two elements.

$$(2) \quad PL_t = TL_{t-1} (> 1) + TL_{t-1} (< 1) - TL^{mat} + OL$$

with TL_{t-1} (> 1) = total value of term loans of size \$1 million and over,

TL_{t-1} (< 1) = total value of term loans less than $1 million,

and

(3) $TL_{t-1} = TL_{t-1} (> 1) + TL_{t-1} (< 1)$

The only known part of (2) is TL_{t-1} (> 1). It could be specified that the unknown value of predetermined loans is proportional to the known variable TL_{t-1} (> 1).

(4) $PL_t = p_t \, TL_{t-1} (> 1)$

The factor of proportionality p is determined by setting (4) equal to definition (2).

(5) $$p_t = \frac{TL_{t-1} - TL^{mat} + OL}{TL_{t-1} (> 1)} \quad \text{(using (3))}$$

It is expected that the factor p will not be constant over time since only term loans of $1 million and over are included in the denominator. Over time a higher proportion of loans should enter the $1 million and over category (evaluated in nominal dollars) due to the impact of inflation on nominal loan sizes. Consequently, a downward time-trend is expected for the value of p so that (4) should not be used in estimation of the loan equation.

Equation (4) can be modified to avoid this systematic parameter variability. First, (5) is altered to give an equivalent definition of p_t.[1]

1. From (3), $TL_{t-1} = TL_{t-1} (> 1) (1 + k_{t-1})$. Rearrange and substitute into the denominator of (5) to obtain (6).

$$(6) \quad p_t = \frac{(TL_{t-1} - TL^{mat} + OL)\ (1 + k_{t-1})}{TL_{t-1}}$$

with

$$(7) \quad k_{t-1} = \frac{TL_{t-1}\ (< 1)}{TL_{t-1}\ (> 1)}$$

Thus, using (4) and (6), predetermined loans are represented by

$$(8) \quad PL_t = \phi\ (1 + k_{t-1})\ TL_{t-1}\ (> 1)$$

where $\phi = \dfrac{(TL_{t-1} - TL^{mat} + OL)}{TL_{t-1}}$ is an unknown parameter to be estimated.

The parameter ϕ is a factor of proportionality between predetermined loans of period t and total term loans outstanding at the end of t-1. Unlike the original proportionality factor defined by (5), ϕ does not contain any components dependent on the \$1 million critical loan size. Therefore, the adjustment involving $(1 + k_{t-1})$ removes the systematic parameter variation discussed above. However, a proxy is necessary for $k_{t-1} = TL_{t-1}\ (< 1)/TL_{t-1}\ (> 1)$ since complete data on $TL_{t-1}\ (< 1)$ is unavailable. The corresponding ratio for total (term plus non-term) loans is calculated for each period and used as a proxy.

The aggregate loan equation of Section 5.2 uses (8) to represent predetermined loans:

$$BL_t = PL_t + L_t^f$$

with BL_t = total outstanding stock of business loans,

PL_t = predetermined loans

$= \phi\ (1 + h)\ \ TL_{t-1}\ (>1),$

$h = \dfrac{BL_{t-1}\ (<1)}{BL_{t-1}\ (>1)}$, and

L_t^f = new loan flows of period t.